Computer Algebra Independent Integration Tests

Summer 2024

4-Trig-functions/4.1-Sine/178-4.1.0

Nasser M. Abbasi

May 17, 2024 Compiled on May 17, 2024 at 9:18pm

Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**538**]. This is test number [178].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	100.00 (538)	0.00 (0)
Rubi	99.81 (537)	0.19 (1)
Maple	82.90 (446)	17.10 (92)
Fricas	81.60 (439)	18.40 (99)
Mupad	46.10 (248)	53.90 (290)
Maxima	45.17 (243)	54.83 (295)
Giac	39.59 (213)	60.41 (325)
Reduce	30.11 (162)	69.89 (376)
Sympy	18.40 (99)	81.60 (439)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

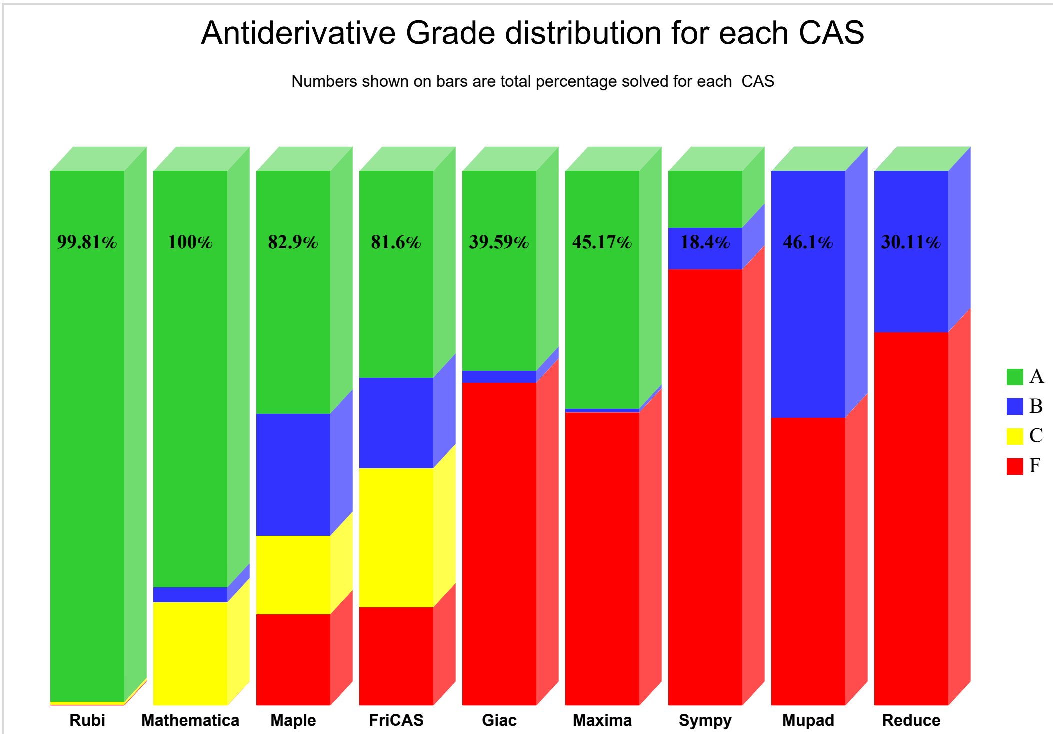
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

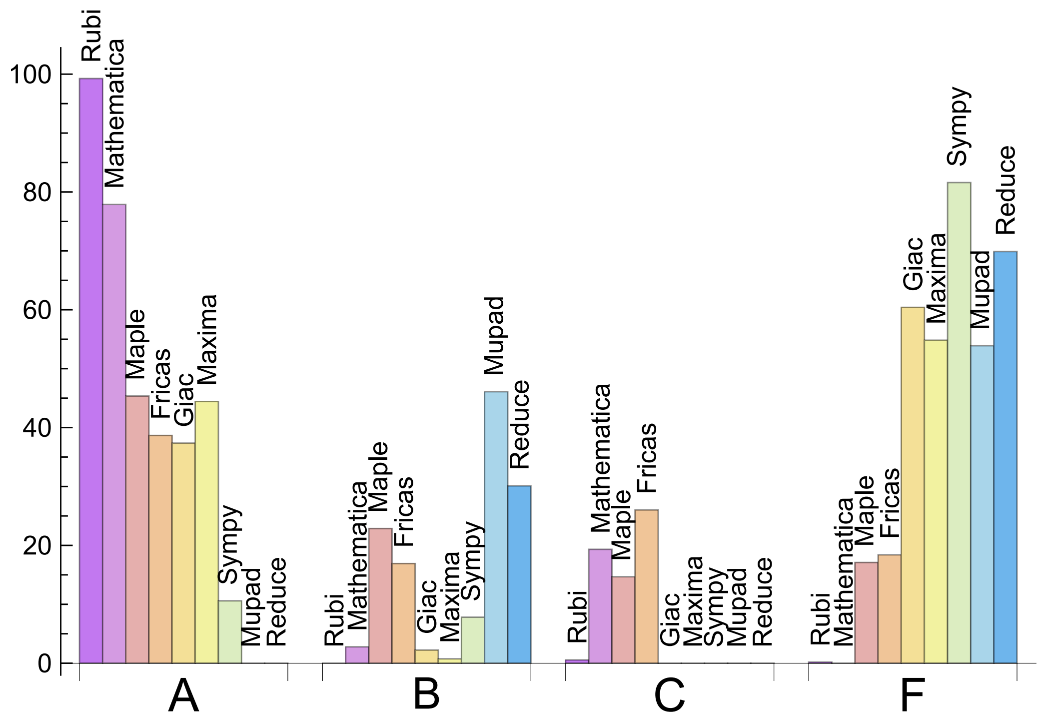
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.257	0.000	0.558	0.186
Mathematica	77.881	2.788	19.331	0.000
Maple	45.353	22.862	14.684	17.100
Maxima	44.424	0.743	0.000	54.833
Fricas	38.662	16.914	26.022	18.401
Giac	37.361	2.230	0.000	60.409
Sympy	10.595	7.807	0.000	81.599
Mupad	0.000	46.097	0.000	53.903
Reduce	0.000	30.112	0.000	69.888

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	0	0.00	0.00	0.00
Rubi	1	100.00	0.00	0.00
Maple	92	100.00	0.00	0.00
Fricas	99	100.00	0.00	0.00
Mupad	290	0.00	100.00	0.00
Maxima	295	99.66	0.34	0.00
Giac	325	97.85	2.15	0.00
Reduce	376	100.00	0.00	0.00
Sympy	439	58.09	41.91	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.05
Fricas	0.11
Giac	0.14
Reduce	0.18
Rubi	0.47
Mathematica	0.67
Sympy	1.60
Maple	9.63
Mupad	22.08

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	50.08	1.01	38.00	0.95
Giac	52.28	1.13	44.00	1.05
Mupad	52.99	1.17	40.00	0.96
Mathematica	61.14	0.95	57.00	0.88
Reduce	65.17	1.58	44.50	1.16
Rubi	78.00	1.01	68.00	1.00
Sympy	114.05	2.62	44.00	1.48
Fricas	121.03	1.52	79.00	1.28
Maple	148.37	1.81	89.50	1.31

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

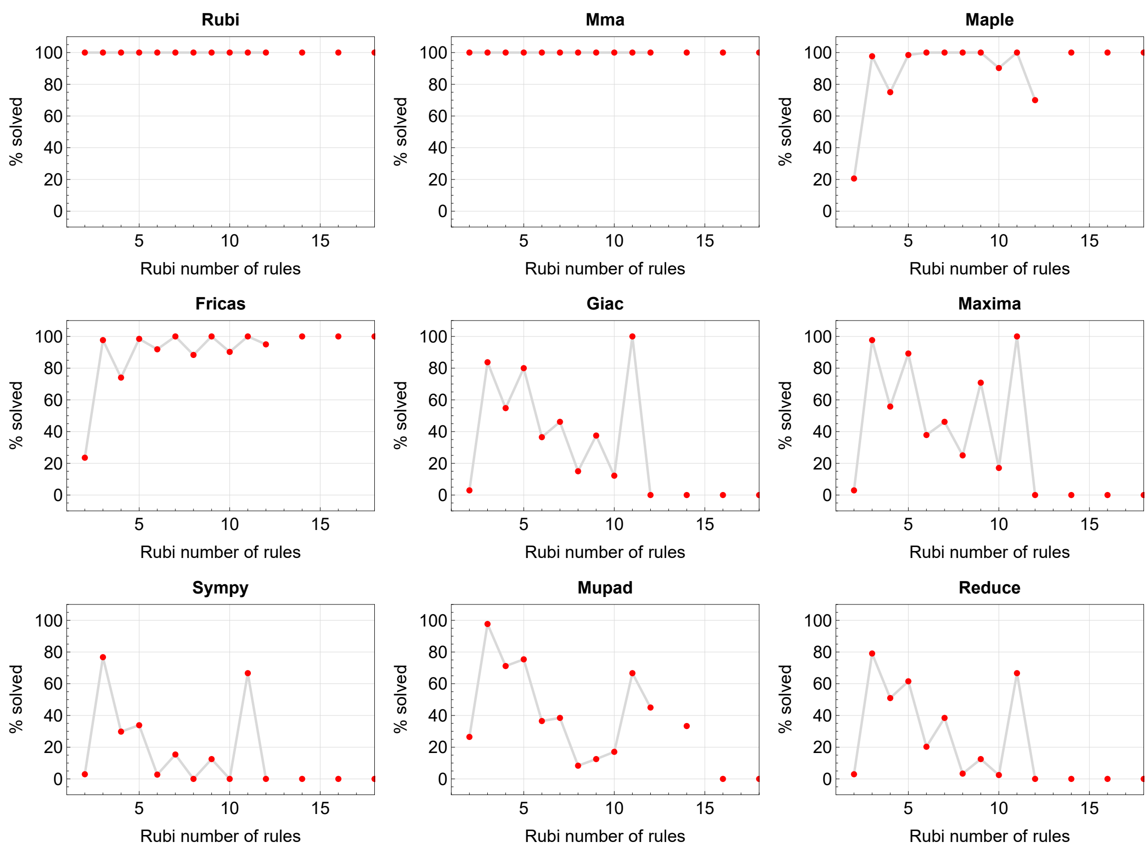


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

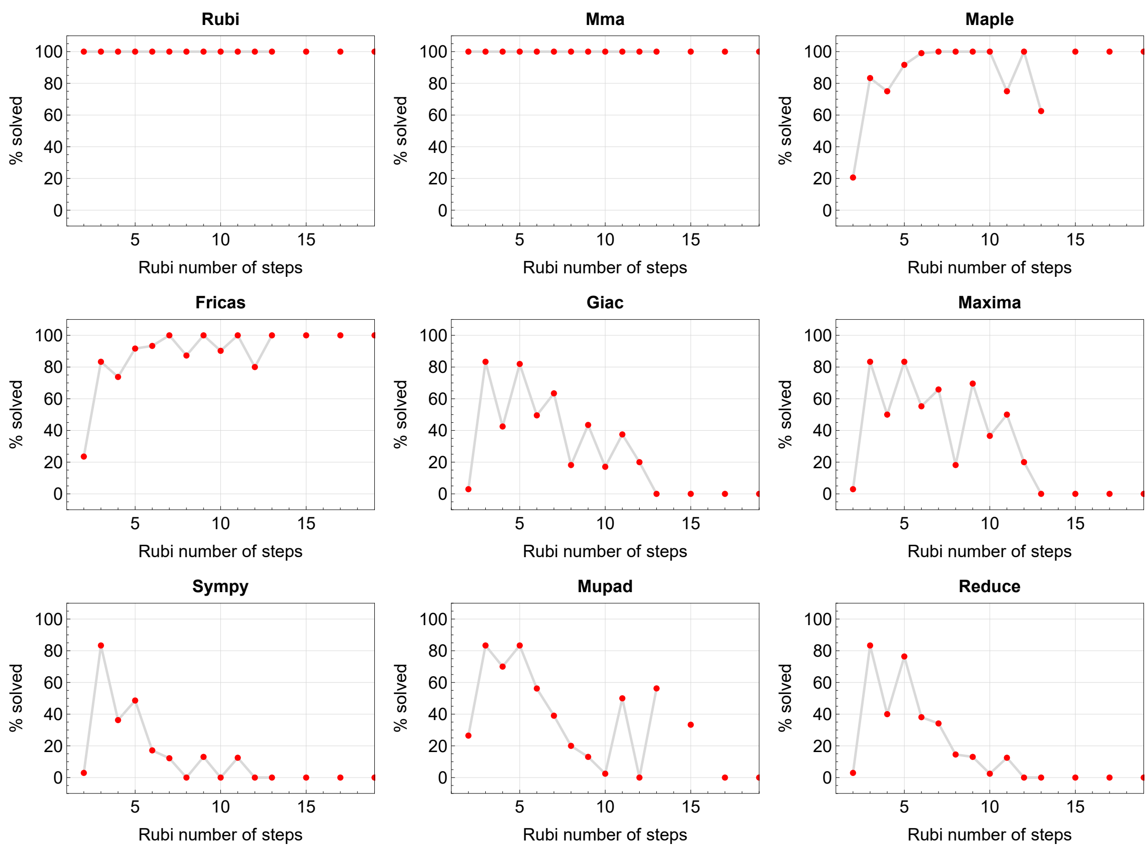


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

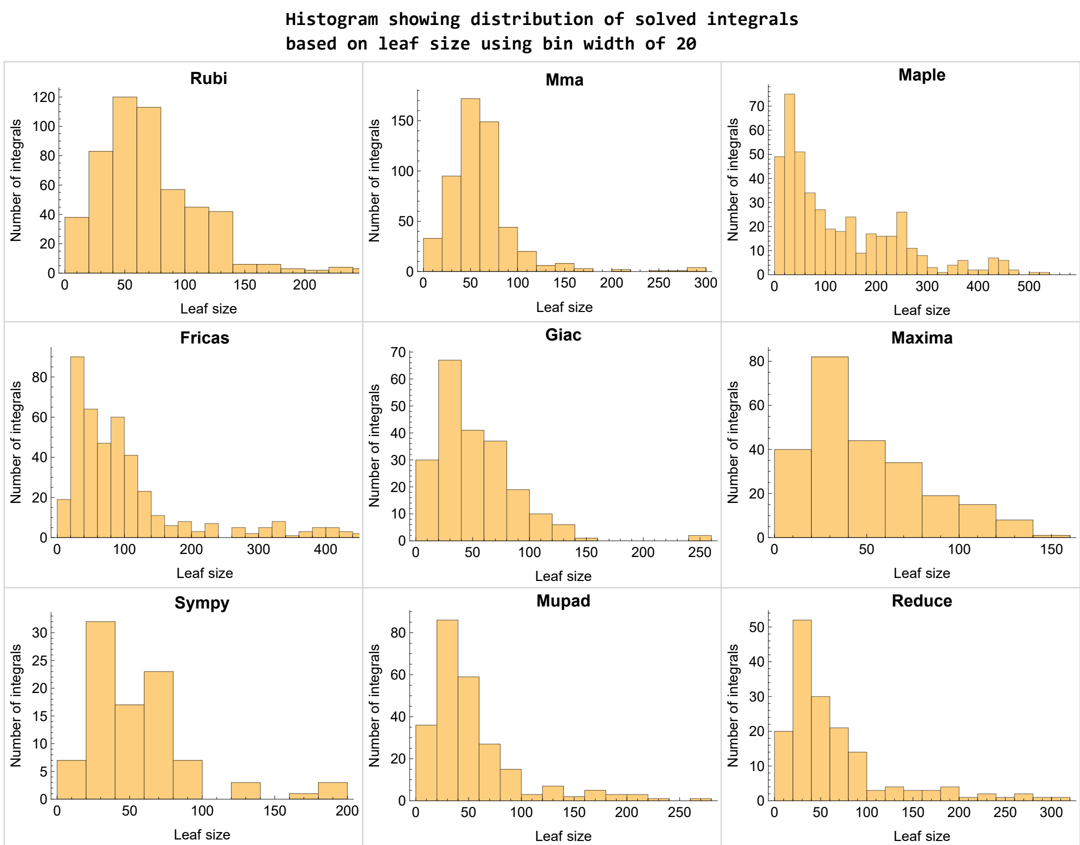


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

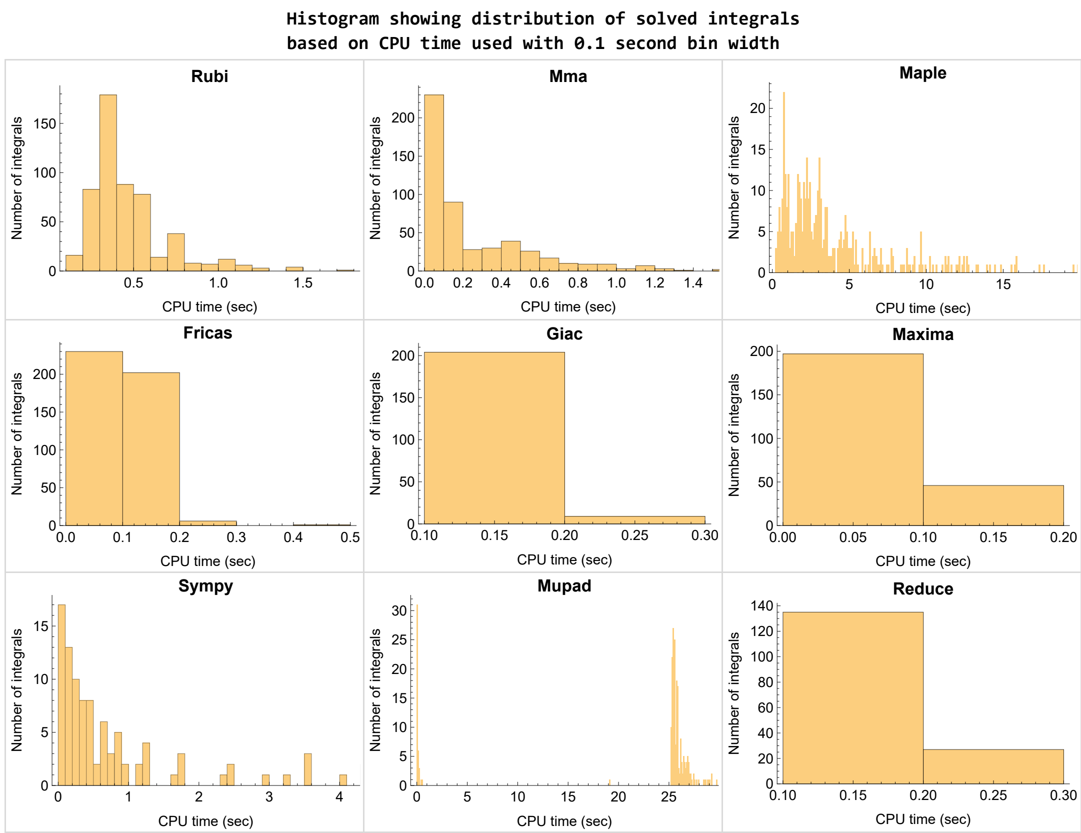


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

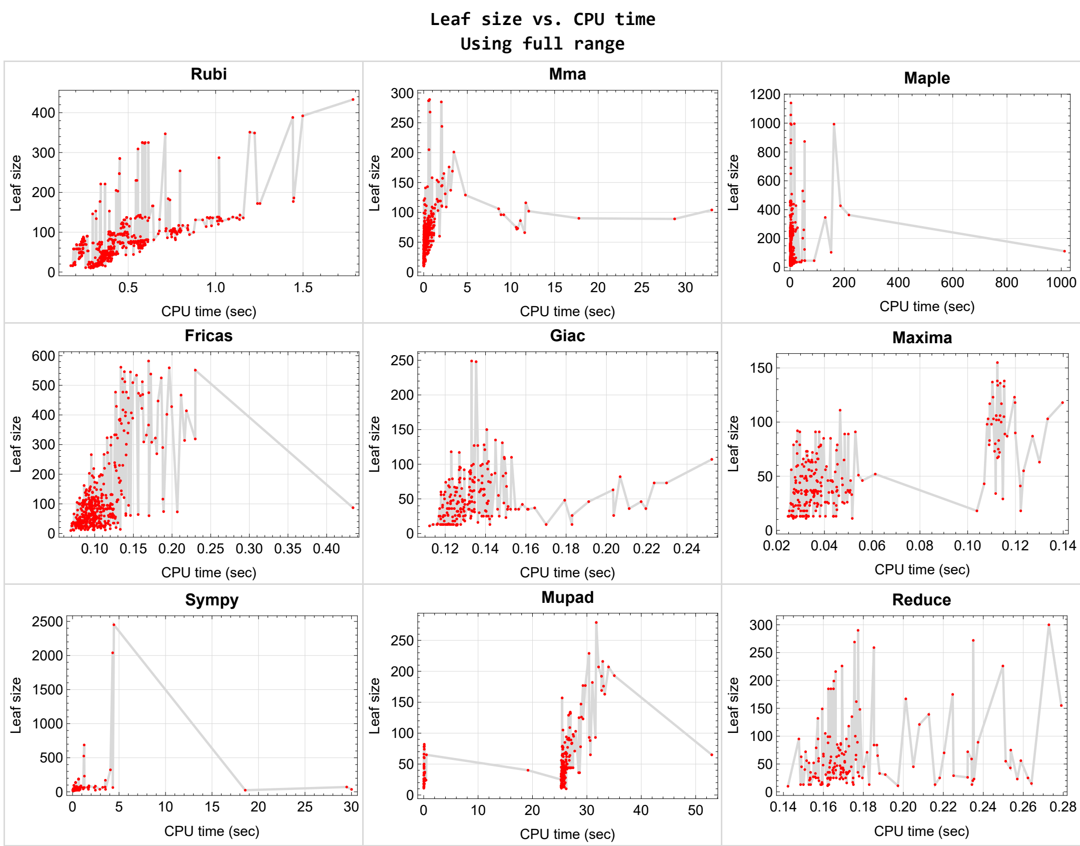


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {107, 121, 145, 147, 154, 156, 171, 173, 180, 182, 184, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 242, 243, 244, 245, 246, 247, 249, 250, 251, 324, 327, 328, 329, 332, 333, 375, 376, 377, 389, 390, 401, 402, 403, 414, 415, 416, 428, 429, 430, 441, 442, 443}

Mathematica {359, 360, 488, 489, 490, 491, 496}

Maple {262, 263, 267, 271, 272, 282, 283, 291, 296, 450, 451, 456, 466, 473, 474, 478}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

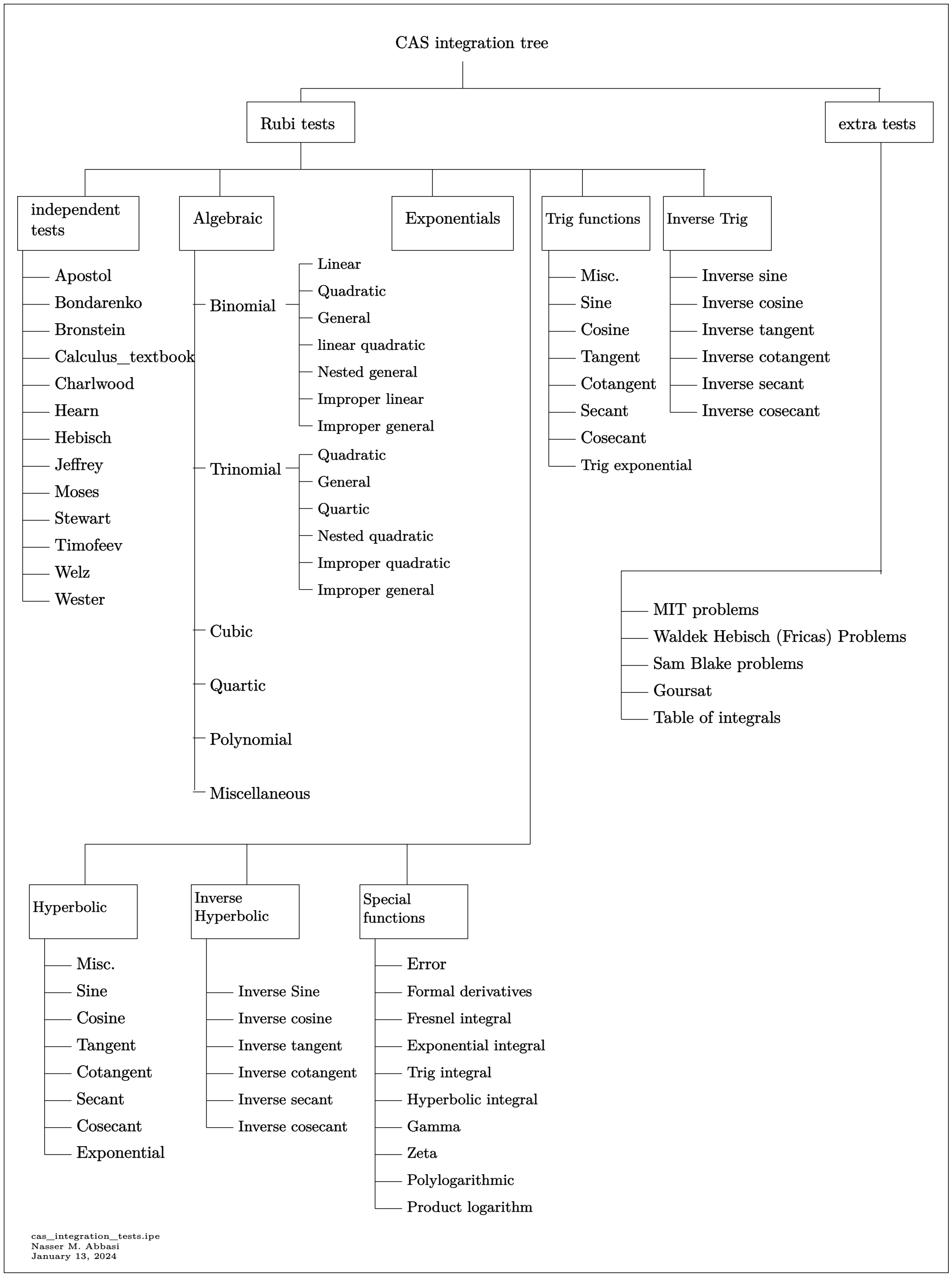
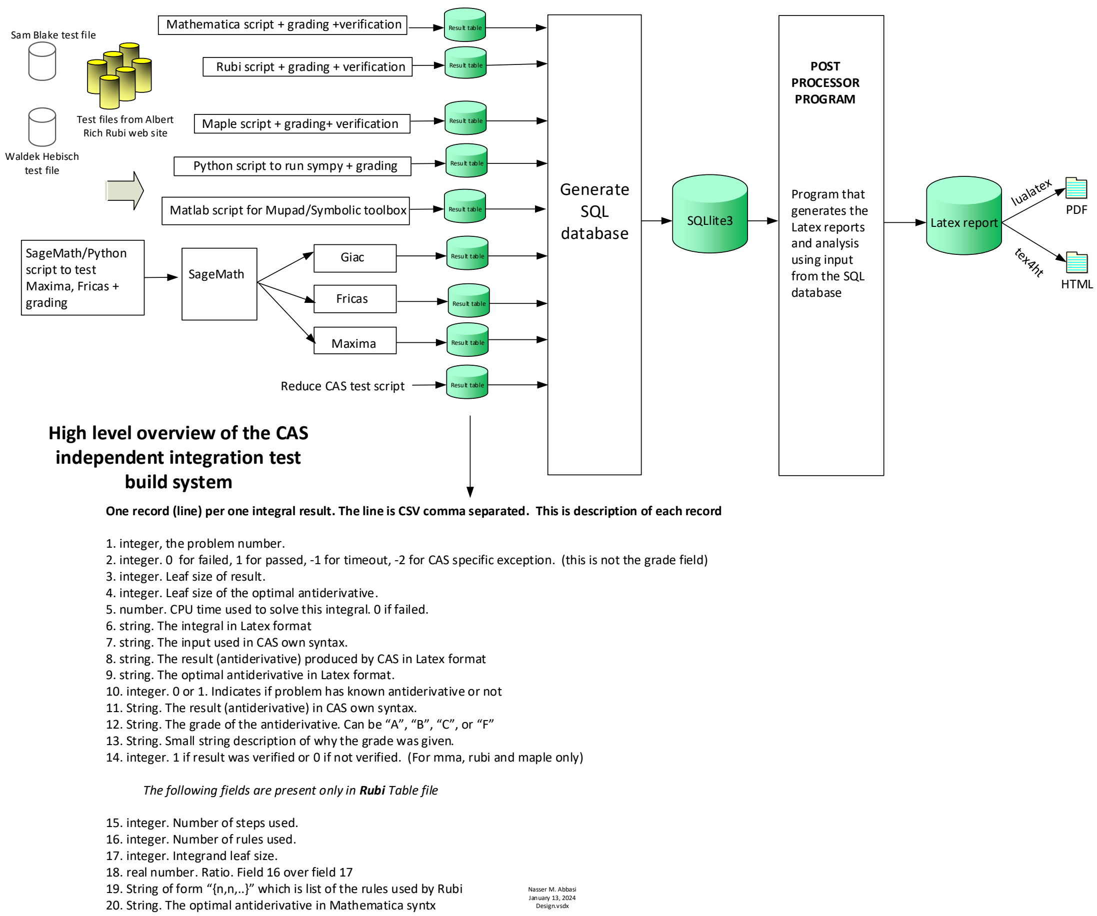


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538 }

B grade { }

C grade { 35, 36, 37 }

F normal fail { 248 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 38, 39, 40, 41, 42, 43, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 139, 141, 143, 145, 146, 147, 148, 149, 151, 152, 154, 156, 157, 158, 159, 160, 161, 163, 164, 165, 167, 169, 171, 172, 173, 174, 175, 177, 179, 180, 181, 182, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 204, 205, 206, 207, 208, 209, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 244, 252, 253, 254, 255, 256, 264, 265, 266, 273, 274, 275, 284, 285, 286, 288, 297, 298, 299, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 361, 362, 363, 364, 365, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 483, 484, 485, 486, 487, 492, 493, 494, 495, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538 }

B grade { 44, 87, 88, 150, 153, 155, 176, 178, 183, 210, 221, 359, 360, 366, 496 }

C grade { 35, 36, 37, 138, 140, 142, 144, 162, 166, 168, 170, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 243, 245, 246, 247, 248, 249, 250, 251, 257, 258, 259, 260, 261, 262, 263, 267, 268, 269, 270, 271, 272, 276, 277, 278, 279, 280, 281, 282, 283, 287, 289, 290, 291, 292, 293, 294, 295, 296, 300, 301, 302, 303, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 428, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 477, 478, 479, 480, 481, 482, 488, 489, 490, 491 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 17, 25, 27, 29, 31, 32, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 137, 138, 139, 140, 141, 142, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 166, 168, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 204, 205, 206, 207, 208, 209, 210, 212, 221, 252, 253, 254, 255, 256, 264, 265, 266, 268, 269, 270, 273, 274, 275, 276, 281, 284, 285, 286, 288, 292, 294, 295, 297, 298, 299, 340, 341, 342, 355, 356, 357, 374, 388, 400, 410, 411, 412, 413, 424, 425, 426, 427, 437, 438, 439, 440, 450, 451, 452, 453, 454, 455, 457, 459, 460, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 479, 480, 481, 482, 492, 493, 494 }

B grade { 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 26, 28, 30, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 211, 213, 214, 215, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 257, 258, 259, 260, 261, 262, 263, 271, 272, 277, 278, 279, 280, 282, 283, 287, 289, 290, 293, 296, 300, 301, 302, 303, 371, 372, 373, 375, 376, 377, 385, 386, 387, 389, 390, 397, 398, 399, 401, 402, 403, 414, 415, 416, 428, 429, 430, 441, 442, 443, 458, 461, 462, 463, 464, 465, 466 }

C grade { 119, 136, 143, 160, 167, 169, 267, 291, 378, 379, 380, 381, 382, 383, 384, 391, 392, 393, 394, 395, 396, 404, 405, 406, 407, 408, 409, 417, 418, 419, 420, 421, 422, 423, 431, 432, 433, 434, 435, 436, 444, 445, 446, 447, 448, 449, 456, 478, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537 }

F normal fail { 33, 34, 35, 36, 37, 38, 39, 40, 41, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 483, 484, 485, 486, 487, 488, 489, 490, 491, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 538 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 129, 130, 131, 132, 133, 134, 135, 136, 137, 139, 141, 143, 144, 145, 146, 147, 149, 151, 157, 158, 159, 160, 161, 163, 165, 167, 169, 170, 171, 172, 173, 187, 188, 189, 190, 191, 192, 193, 204, 205, 206, 207, 208, 209, 210, 221, 222, 223, 230, 231, 242, 251, 252, 253, 254, 255, 256, 264, 265, 266, 273, 274, 275, 284, 285, 286, 288, 297, 298, 299, 324, 327, 328, 329, 332, 333, 334, 335, 340, 341, 342, 355, 356, 357, 371, 372, 373, 374, 385, 386, 387, 388, 397, 398, 399, 400, 410, 411, 412, 413, 424, 425, 426, 427, 437, 438, 439, 440, 450, 453, 454, 455, 466, 468, 469, 470, 471, 472, 473, 492, 493, 494 }

B grade { 54, 63, 83, 86, 104, 111, 126, 127, 128, 138, 140, 142, 148, 150, 152, 153, 154, 155, 156, 162, 164, 166, 168, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 224, 225, 226, 227, 228, 229, 243, 244, 245, 246, 247, 248, 249, 250, 262, 263, 271, 272, 282, 283, 287, 289, 290, 296, 300, 301, 302, 303, 325, 326, 330, 331, 375, 376, 377, 389, 390, 401, 402, 403, 414, 415, 416, 428, 429, 430, 441, 442, 443, 451, 452, 467, 474, 475, 476 }

C grade { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 260, 261, 269, 270, 280, 281, 293, 294, 295, 378, 379, 380, 381, 382, 383, 384, 391, 392, 393, 394, 395, 396, 404, 405, 406, 407, 408, 409, 417, 418, 419, 420, 421, 422, 423, 431, 432, 433, 434, 435, 436, 444, 445, 446, 447, 448, 449, 458, 459, 460, 464, 465, 480, 481, 482, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537 }

F normal fail { 33, 34, 35, 36, 37, 38, 39, 40, 41, 257, 258, 259, 267, 268, 276, 277, 278, 279, 291, 292, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 336, 337, 338, 339, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 456, 457, 461, 462, 463, 477, 478, 479, 483, 484, 485, 486, 487, 488, 489, 490, 491, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 538 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 204, 205, 206, 207, 208, 209, 210, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 340, 341, 342, 355, 356, 357, 371, 372, 373, 374, 375, 376, 377, 385, 386, 387, 388, 389, 390, 397, 398, 399, 400, 401, 402, 403, 410, 411, 412, 413, 414, 415, 416, 424, 425, 426, 427, 428, 429, 430, 437, 438, 439, 440, 441, 442, 443, 492, 493, 494 }

B grade { 79, 113, 117, 126 }

C grade { }

F normal fail { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 378, 379, 380, 381, 382, 383, 384, 391, 392, 393, 394, 395, 396, 404, 405, 406, 407, 408, 409, 417, 418, 419, 420, 421, 422, 423, 431, 432, 433, 434, 435, 436, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538 }

F(-1) timedout fail { 323 }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 188, 189, 204, 205, 226, 227, 247, 248, 252, 253, 254, 255, 256, 342, 357, 371, 372, 373, 374, 375, 376, 377, 385, 386, 387, 388, 389, 390, 397, 398, 399, 401, 402, 403, 410, 411, 412, 414, 415, 416, 424, 425, 426, 428, 429, 430, 437, 438, 439, 441, 442, 443 }

B grade { 111, 126, 138, 162, 340, 341, 355, 356, 400, 413, 427, 440 }

C grade { }

F normal fail { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 187, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 249, 250, 251, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 378, 379, 380, 381, 382, 383, 384, 391, 392, 393, 394, 395, 396, 404, 405, 406, 407, 408, 409, 417, 418, 419, 420, 421, 422, 423, 431, 432, 433, 434, 435, 436, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 466, 467, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538 }

F(-1) timedout fail { 303, 464, 465, 468, 469, 470, 471 }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 28, 29, 39, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 207, 208, 209, 210, 221, 252, 253, 254, 255, 256, 264, 265, 266, 273, 274, 275, 284, 285, 286, 287, 288, 289, 290, 297, 298, 299, 300, 301, 302, 303, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 340, 341, 342, 355, 356, 357, 374, 381, 388, 399, 400, 413, 427, 440, 453, 454, 455, 468, 469, 470, 471, 476, 492, 493, 494, 511 }

C grade { }

F normal fail { }

F(-1) timedout fail { 25, 26, 27, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 257, 258, 259, 260, 261, 262, 263, 267, 268, 269, 270, 271, 272, 276, 277, 278, 279, 280, 281, 282, 283, 291, 292, 293, 294, 295, 296, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 337, 338, 339, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 375, 376, 377, 378, 379, 380, 382, 383, 384, 385, 386, 387, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 472, 473, 474, 475, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538 }

F(-2) exception fail { }

Sympy

A grade { 1, 3, 5, 7, 42, 43, 44, 45, 47, 48, 49, 50, 51, 52, 53, 68, 70, 73, 76, 77, 78, 79, 80, 81, 82, 83, 85, 99, 100, 101, 102, 103, 104, 109, 110, 112, 114, 115, 116, 117, 138, 151, 162, 164, 165, 179, 186, 187, 188, 204, 205, 206, 207, 208, 252, 254, 255 }

B grade { 2, 4, 6, 8, 46, 54, 58, 59, 60, 61, 66, 67, 69, 74, 75, 86, 89, 90, 91, 92, 98, 111, 113, 125, 139, 149, 163, 175, 177, 185, 189, 190, 191, 192, 253, 256, 340, 341, 342, 355, 356, 357 }

C grade { }

F normal fail { 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 55, 56, 57, 62, 63, 64, 65, 71, 72, 84, 87, 88, 93, 94, 95, 96, 97, 105, 106, 107, 108, 119, 121, 122, 123, 124, 126, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 140, 141, 142, 143, 144, 148, 150, 152, 153, 154, 155, 156, 161, 166, 167, 168, 169, 174, 176, 178, 180, 181, 182, 198, 199, 226, 227, 228, 237, 238, 239, 247, 248, 249, 259, 260, 262, 263, 264, 268, 271, 272, 289, 292, 293, 296, 297, 300, 301, 304, 305, 306, 307, 311, 315, 316, 317, 318, 320, 321, 322, 323, 324, 325, 326, 329, 330, 331, 336, 337, 338, 339, 343, 344, 347, 348, 349, 350, 351, 352, 353, 354, 358, 359, 362, 363, 364, 365, 367, 368, 369, 370, 374, 375, 376, 377, 379, 380, 381, 382, 383, 388, 389, 394, 407, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 440, 441, 442, 443, 444, 445, 446, 447, 448, 452, 453, 458, 463, 464, 466, 467, 474, 479, 485, 486, 487, 488, 489, 490, 491, 494, 495, 496, 498, 499, 500, 501, 502, 504, 505, 506, 507, 509, 510, 511, 512, 513, 514, 519, 520, 521, 522, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538 }

F(-1) timedout fail { 9, 17, 25, 118, 120, 133, 145, 146, 147, 157, 158, 159, 160, 170, 171, 172, 173, 183, 184, 193, 194, 195, 196, 197, 200, 201, 202, 203, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 229, 230, 231, 232, 233, 234, 235, 236, 240, 241, 242, 243, 244, 245, 246, 250, 251, 257, 258, 261, 265, 266, 267, 269, 270, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 290, 291, 294, 295, 298, 299, 302, 303, 308, 309, 310, 312, 313, 314, 319, 327, 328, 332, 333, 334, 335, 345, 346, 360, 361, 366, 371, 372, 373, 378, 384, 385, 386, 387, 390, 391, 392, 393, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 408, 409, 410, 411, 412, 424, 425, 426, 437, 438, 439, 449, 450, 451, 454, 455, 456, 457, 459, 460, 461, 462, 465, 468, 469, 470, 471, 472, 473, 475, 476, 477, 478, 480, 481, 482, 483, 484, 492, 493, 497, 503, 508, 515, 516, 517, 518, 523 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 342, 357 }

C grade { }

F normal fail { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	22	12	11	11	14	11	11	11
N.S.	1	1.00	2.00	1.09	1.00	1.00	1.27	1.00	1.00	1.00
time (sec)	N/A	0.257	0.011	2.368	0.032	0.073	0.054	0.112	0.197	0.021

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	19	24	23	46	18	23	18
N.S.	1	1.00	0.92	0.76	0.96	0.92	1.84	0.72	0.92	0.72
time (sec)	N/A	0.277	0.037	3.589	0.026	0.099	0.080	0.120	0.257	25.500

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	24	29	22	22	22	37	25	31	24
N.S.	1	0.89	1.07	0.81	0.81	0.81	1.37	0.93	1.15	0.89
time (sec)	N/A	0.296	0.025	6.322	0.027	0.088	0.103	0.118	0.169	0.043

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	51	33	31	33	36	95	32	40	50
N.S.	1	1.11	0.72	0.67	0.72	0.78	2.07	0.70	0.87	1.09
time (sec)	N/A	0.378	0.062	6.619	0.032	0.075	0.155	0.129	0.151	25.438

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	36	44	32	34	34	60	38	47	32
N.S.	1	0.86	1.05	0.76	0.81	0.81	1.43	0.90	1.12	0.76
time (sec)	N/A	0.312	0.029	8.895	0.030	0.082	0.209	0.119	0.168	0.056

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	77	45	44	48	47	139	46	56	43
N.S.	1	1.15	0.67	0.66	0.72	0.70	2.07	0.69	0.84	0.64
time (sec)	N/A	0.515	0.066	9.667	0.032	0.095	0.327	0.123	0.152	25.486

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	46	59	42	44	44	80	50	63	43
N.S.	1	0.85	1.09	0.78	0.81	0.81	1.48	0.93	1.17	0.80
time (sec)	N/A	0.348	0.025	12.771	0.038	0.084	0.443	0.120	0.149	0.064

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	103	55	55	59	56	184	60	72	90
N.S.	1	1.17	0.62	0.62	0.67	0.64	2.09	0.68	0.82	1.02
time (sec)	N/A	0.678	0.080	14.104	0.028	0.099	0.640	0.118	0.171	26.514

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	65	45	84	0	66	0	0	14	34
N.S.	1	1.08	0.75	1.40	0.00	1.10	0.00	0.00	0.23	0.57
time (sec)	N/A	0.475	0.098	2.428	0.000	0.120	0.000	0.000	0.163	25.386

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	35	118	0	61	0	0	14	34
N.S.	1	1.00	0.85	2.88	0.00	1.49	0.00	0.00	0.34	0.83
time (sec)	N/A	0.367	0.058	2.175	0.000	0.109	0.000	0.000	0.149	25.278

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	33	72	0	55	0	0	12	34
N.S.	1	1.00	0.80	1.76	0.00	1.34	0.00	0.00	0.29	0.83
time (sec)	N/A	0.359	0.052	2.020	0.000	0.103	0.000	0.000	0.165	25.345

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	77	0	50	0	0	7	15
N.S.	1	1.00	1.11	4.05	0.00	2.63	0.00	0.00	0.37	0.79
time (sec)	N/A	0.271	0.037	2.988	0.000	0.088	0.000	0.000	0.232	25.253

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	57	0	42	0	0	14	15
N.S.	1	1.00	1.11	3.00	0.00	2.21	0.00	0.00	0.74	0.79
time (sec)	N/A	0.273	0.039	1.744	0.000	0.085	0.000	0.000	0.253	25.448

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	32	110	0	75	0	0	14	34
N.S.	1	1.00	0.86	2.97	0.00	2.03	0.00	0.00	0.38	0.92
time (sec)	N/A	0.369	0.076	2.341	0.000	0.103	0.000	0.000	0.151	25.423

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	33	72	0	80	0	0	14	34
N.S.	1	1.00	0.80	1.76	0.00	1.95	0.00	0.00	0.34	0.83
time (sec)	N/A	0.365	0.069	2.020	0.000	0.089	0.000	0.000	0.158	25.510

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	61	51	132	0	117	0	0	14	34
N.S.	1	1.02	0.85	2.20	0.00	1.95	0.00	0.00	0.23	0.57
time (sec)	N/A	0.468	0.070	2.125	0.000	0.100	0.000	0.000	0.174	25.527

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	75	55	104	0	80	0	0	18	42
N.S.	1	1.07	0.79	1.49	0.00	1.14	0.00	0.00	0.26	0.60
time (sec)	N/A	0.501	0.134	2.484	0.000	0.095	0.000	0.000	0.150	25.506

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	44	142	0	73	0	0	18	42
N.S.	1	1.00	0.94	3.02	0.00	1.55	0.00	0.00	0.38	0.89
time (sec)	N/A	0.396	0.099	2.377	0.000	0.086	0.000	0.000	0.158	25.462

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	40	88	0	67	0	0	16	42
N.S.	1	1.00	0.85	1.87	0.00	1.43	0.00	0.00	0.34	0.89
time (sec)	N/A	0.389	0.044	2.281	0.000	0.140	0.000	0.000	0.163	25.479

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	24	91	0	58	0	0	9	18
N.S.	1	1.00	1.14	4.33	0.00	2.76	0.00	0.00	0.43	0.86
time (sec)	N/A	0.281	0.016	2.746	0.000	0.101	0.000	0.000	0.152	25.339

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	24	69	0	50	0	0	18	18
N.S.	1	1.00	1.14	3.29	0.00	2.38	0.00	0.00	0.86	0.86
time (sec)	N/A	0.281	0.024	1.735	0.000	0.087	0.000	0.000	0.179	25.351

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	39	132	0	93	0	0	18	42
N.S.	1	1.00	0.91	3.07	0.00	2.16	0.00	0.00	0.42	0.98
time (sec)	N/A	0.383	0.094	2.306	0.000	0.101	0.000	0.000	0.225	25.571

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	43	88	0	98	0	0	18	42
N.S.	1	1.00	0.91	1.87	0.00	2.09	0.00	0.00	0.38	0.89
time (sec)	N/A	0.382	0.127	2.203	0.000	0.104	0.000	0.000	0.159	26.146

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	71	55	160	0	143	0	0	18	42
N.S.	1	1.01	0.79	2.29	0.00	2.04	0.00	0.00	0.26	0.60
time (sec)	N/A	0.516	0.347	2.265	0.000	0.093	0.000	0.000	0.165	26.341

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	109	80	108	0	98	0	0	24	0
N.S.	1	1.06	0.78	1.05	0.00	0.95	0.00	0.00	0.23	0.00
time (sec)	N/A	0.717	0.209	3.659	0.000	0.095	0.000	0.000	0.156	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	66	152	0	94	0	0	24	0
N.S.	1	1.00	0.88	2.03	0.00	1.25	0.00	0.00	0.32	0.00
time (sec)	N/A	0.526	0.138	3.326	0.000	0.100	0.000	0.000	0.163	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	62	97	0	76	0	0	20	0
N.S.	1	1.00	0.83	1.29	0.00	1.01	0.00	0.00	0.27	0.00
time (sec)	N/A	0.530	0.076	2.941	0.000	0.111	0.000	0.000	0.157	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	42	98	0	62	0	0	12	36
N.S.	1	1.00	0.98	2.28	0.00	1.44	0.00	0.00	0.28	0.84
time (sec)	N/A	0.391	0.042	3.749	0.000	0.089	0.000	0.000	0.160	28.498

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	42	74	0	57	0	0	24	36
N.S.	1	1.00	0.98	1.72	0.00	1.33	0.00	0.00	0.56	0.84
time (sec)	N/A	0.390	0.047	2.260	0.000	0.103	0.000	0.000	0.166	28.657

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	54	141	0	102	0	0	24	0
N.S.	1	1.00	0.74	1.93	0.00	1.40	0.00	0.00	0.33	0.00
time (sec)	N/A	0.536	0.076	3.096	0.000	0.085	0.000	0.000	0.158	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	55	105	0	110	0	0	24	0
N.S.	1	1.00	0.71	1.36	0.00	1.43	0.00	0.00	0.31	0.00
time (sec)	N/A	0.516	0.103	2.971	0.000	0.100	0.000	0.000	0.245	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	109	68	168	0	155	0	0	24	0
N.S.	1	1.04	0.65	1.60	0.00	1.48	0.00	0.00	0.23	0.00
time (sec)	N/A	0.717	0.211	3.071	0.000	0.100	0.000	0.000	0.220	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	14	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.325	0.070	0.000	0.000	0.000	0.000	0.000	0.150	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	14	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.332	0.044	0.000	0.000	0.000	0.000	0.000	0.160	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	517	58	55	0	0	0	0	0	14	0
N.S.	1	0.11	0.11	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.320	0.044	0.000	0.000	0.000	0.000	0.000	0.175	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	252	58	55	0	0	0	0	0	14	0
N.S.	1	0.23	0.22	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.323	0.050	0.000	0.000	0.000	0.000	0.000	0.152	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	271	56	53	0	0	0	0	0	14	0
N.S.	1	0.21	0.20	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.331	0.050	0.000	0.000	0.000	0.000	0.000	0.152	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	53	0	0	0	0	0	14	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.329	0.049	0.000	0.000	0.000	0.000	0.000	0.157	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	0	0	0	0	0	10	54
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.16	0.86
time (sec)	N/A	0.337	0.058	0.000	0.000	0.000	0.000	0.000	0.150	26.192

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	63	0	0	0	0	0	14	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.341	0.051	0.000	0.000	0.000	0.000	0.000	0.162	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	76	0	0	0	0	0	19	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.427	0.101	0.000	0.000	0.000	0.000	0.000	0.216	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	22	13	13	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	1.47	0.87	0.87	0.87
time (sec)	N/A	0.309	0.006	4.773	0.031	0.099	0.142	0.119	0.234	0.057

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	22	13	13	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	1.47	0.87	0.87	0.87
time (sec)	N/A	0.311	0.005	4.444	0.031	0.098	0.099	0.121	0.149	0.040

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	37	14	13	13	20	13	13	28
N.S.	1	1.00	2.47	0.93	0.87	0.87	1.33	0.87	0.87	1.87
time (sec)	N/A	0.289	0.020	2.598	0.028	0.082	0.072	0.123	0.154	25.881

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	17	11	18	19	13	16	16
N.S.	1	1.00	1.00	1.42	0.92	1.50	1.58	1.08	1.33	1.33
time (sec)	N/A	0.276	0.010	0.789	0.027	0.108	0.055	0.124	0.172	25.777

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	12	12	15	12	10	20
N.S.	1	1.00	1.00	1.10	1.20	1.20	1.50	1.20	1.00	2.00
time (sec)	N/A	0.286	0.011	1.067	0.028	0.099	0.087	0.125	0.142	26.028

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	20	13	13	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	1.33	0.87	0.87	0.87
time (sec)	N/A	0.305	0.014	1.211	0.033	0.072	0.117	0.121	0.153	25.806

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	20	13	13	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	1.33	0.87	0.87	0.87
time (sec)	N/A	0.306	0.012	1.866	0.027	0.091	0.170	0.126	0.150	25.745

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	53	47	47	46	53	88	46	45	45
N.S.	1	0.87	0.77	0.77	0.75	0.87	1.44	0.75	0.74	0.74
time (sec)	N/A	0.375	0.203	17.695	0.028	0.080	0.903	0.133	0.149	25.853

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	41	37	36	36	43	66	36	35	36
N.S.	1	0.89	0.80	0.78	0.78	0.93	1.43	0.78	0.76	0.78
time (sec)	N/A	0.366	0.102	11.905	0.028	0.094	0.445	0.122	0.154	25.472

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	29	27	37	26	33	44	26	25	26
N.S.	1	0.94	0.87	1.19	0.84	1.06	1.42	0.84	0.81	0.84
time (sec)	N/A	0.352	0.060	9.525	0.033	0.076	0.209	0.130	0.153	0.046

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	21	20	13	13	13
N.S.	1	1.00	1.00	0.93	0.87	1.40	1.33	0.87	0.87	0.87
time (sec)	N/A	0.306	0.005	4.420	0.026	0.080	0.099	0.120	0.216	0.026

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	23	15	18	17	15	21	15	14
N.S.	1	1.00	1.64	1.07	1.29	1.21	1.07	1.50	1.07	1.00
time (sec)	N/A	0.291	0.011	0.733	0.104	0.083	0.066	0.142	0.264	25.447

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	29	22	13	33	13
N.S.	1	1.00	1.00	0.93	0.87	1.93	1.47	0.87	2.20	0.87
time (sec)	N/A	0.321	0.008	1.535	0.025	0.081	0.466	0.170	0.156	25.369

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	29	56	26	26	39	0	26	55	25
N.S.	1	0.94	1.81	0.84	0.84	1.26	0.00	0.84	1.77	0.81
time (sec)	N/A	0.359	0.190	4.460	0.029	0.082	0.000	0.183	0.158	25.392

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	41	77	36	36	51	0	36	75	35
N.S.	1	0.89	1.67	0.78	0.78	1.11	0.00	0.78	1.63	0.76
time (sec)	N/A	0.371	0.156	15.566	0.043	0.090	0.000	0.211	0.168	25.397

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	53	98	46	46	61	0	46	95	45
N.S.	1	0.87	1.61	0.75	0.75	1.00	0.00	0.75	1.56	0.74
time (sec)	N/A	0.368	0.159	54.680	0.033	0.085	0.000	0.217	0.148	25.502

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	103	52	55	48	57	189	60	72	89
N.S.	1	1.17	0.59	0.62	0.55	0.65	2.15	0.68	0.82	1.01
time (sec)	N/A	0.714	0.188	15.714	0.032	0.101	0.654	0.125	0.151	26.735

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	77	40	44	37	47	136	46	56	43
N.S.	1	1.15	0.60	0.66	0.55	0.70	2.03	0.69	0.84	0.64
time (sec)	N/A	0.566	0.091	10.247	0.051	0.081	0.330	0.129	0.164	25.855

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	51	23	19	24	36	92	18	39	50
N.S.	1	1.11	0.50	0.41	0.52	0.78	2.00	0.39	0.85	1.09
time (sec)	N/A	0.434	0.090	4.331	0.027	0.093	0.157	0.125	0.170	25.712

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	19	24	23	46	18	23	18
N.S.	1	1.00	0.92	0.76	0.96	0.92	1.84	0.72	0.92	0.72
time (sec)	N/A	0.290	0.019	3.480	0.037	0.081	0.081	0.130	0.170	0.004

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	20	23	28	34	36	0	42	39	27
N.S.	1	0.87	1.00	1.22	1.48	1.57	0.00	1.83	1.70	1.17
time (sec)	N/A	0.301	0.015	2.200	0.037	0.095	0.000	0.130	0.166	25.738

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	48	46	61	0	48	95	69
N.S.	1	1.00	1.00	1.41	1.35	1.79	0.00	1.41	2.79	2.03
time (sec)	N/A	0.401	0.020	1.504	0.043	0.101	0.000	0.179	0.171	26.134

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	60	55	66	65	71	0	82	155	125
N.S.	1	1.09	1.00	1.20	1.18	1.29	0.00	1.49	2.82	2.27
time (sec)	N/A	0.560	0.021	3.053	0.048	0.089	0.000	0.207	0.279	28.553

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	86	76	84	91	84	0	73	226	177
N.S.	1	1.13	1.00	1.11	1.20	1.11	0.00	0.96	2.97	2.33
time (sec)	N/A	0.724	0.023	8.760	0.038	0.119	0.000	0.224	0.250	29.282

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	30	48	26	36	26	63	26	35	25
N.S.	1	0.97	1.55	0.84	1.16	0.84	2.03	0.84	1.13	0.81
time (sec)	N/A	0.352	0.154	14.457	0.037	0.082	0.623	0.125	0.162	0.059

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	30	27	26	26	26	46	26	63	26
N.S.	1	0.97	0.87	0.84	0.84	0.84	1.48	0.84	2.03	0.84
time (sec)	N/A	0.362	0.117	12.029	0.040	0.078	0.441	0.126	0.167	0.052

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	29	35	26	26	26	42	26	25	37
N.S.	1	0.94	1.13	0.84	0.84	0.84	1.35	0.84	0.81	1.19
time (sec)	N/A	0.353	0.067	7.030	0.034	0.108	0.308	0.121	0.180	25.514

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	30	27	34	26	26	46	26	47	26
N.S.	1	0.97	0.87	1.10	0.84	0.84	1.48	0.84	1.52	0.84
time (sec)	N/A	0.357	0.057	7.708	0.048	0.071	0.209	0.120	0.167	0.031

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	24	20	24	13	13
N.S.	1	1.00	1.00	0.93	0.87	1.60	1.33	1.60	0.87	0.87
time (sec)	N/A	0.309	0.004	4.753	0.044	0.090	0.142	0.119	0.183	25.262

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	25	25	25	25	25	0	27	60	30
N.S.	1	0.89	0.89	0.89	0.89	0.89	0.00	0.96	2.14	1.07
time (sec)	N/A	0.345	0.022	2.471	0.044	0.087	0.000	0.127	0.158	25.274

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	22	21	40	19	22	0	23	31	20
N.S.	1	1.05	1.00	1.90	0.90	1.05	0.00	1.10	1.48	0.95
time (sec)	N/A	0.347	0.108	3.974	0.045	0.107	0.000	0.150	0.161	25.400

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	25	28	31	27	32	30	27	30
N.S.	1	1.00	0.93	1.04	1.15	1.00	1.19	1.11	1.00	1.11
time (sec)	N/A	0.390	0.069	0.828	0.035	0.102	0.078	0.137	0.155	25.252

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	25	27	24	25	25	39	25	21	23
N.S.	1	0.93	1.00	0.89	0.93	0.93	1.44	0.93	0.78	0.85
time (sec)	N/A	0.334	0.116	1.224	0.042	0.094	0.176	0.138	0.153	25.508

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	25	42	25	23	13
N.S.	1	1.00	1.00	0.93	0.87	1.67	2.80	1.67	1.53	0.87
time (sec)	N/A	0.322	0.007	1.945	0.033	0.100	0.251	0.143	0.168	25.322

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	29	31	26	25	25	44	25	25	25
N.S.	1	0.94	1.00	0.84	0.81	0.81	1.42	0.81	0.81	0.81
time (sec)	N/A	0.363	0.178	3.533	0.048	0.077	0.393	0.139	0.218	25.394

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	29	31	34	49	25	42	25	25	25
N.S.	1	0.94	1.00	1.10	1.58	0.81	1.35	0.81	0.81	0.81
time (sec)	N/A	0.360	0.027	6.685	0.039	0.083	0.565	0.141	0.262	25.241

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	29	31	26	25	25	44	25	25	27
N.S.	1	0.94	1.00	0.84	0.81	0.81	1.42	0.81	0.81	0.87
time (sec)	N/A	0.363	0.135	11.602	0.037	0.110	0.816	0.141	0.162	25.597

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	29	31	34	59	25	42	25	25	35
N.S.	1	0.94	1.00	1.10	1.90	0.81	1.35	0.81	0.81	1.13
time (sec)	N/A	0.353	0.026	21.747	0.031	0.106	1.153	0.147	0.163	25.515

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	53	47	47	46	63	88	46	45	45
N.S.	1	0.87	0.77	0.77	0.75	1.03	1.44	0.75	0.74	0.74
time (sec)	N/A	0.379	0.218	54.112	0.041	0.098	1.763	0.129	0.205	0.061

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	41	37	36	36	53	66	36	35	36
N.S.	1	0.89	0.80	0.78	0.78	1.15	1.43	0.78	0.76	0.78
time (sec)	N/A	0.368	0.107	21.329	0.029	0.081	0.896	0.122	0.165	25.445

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	29	27	26	26	41	44	26	25	26
N.S.	1	0.94	0.87	0.84	0.84	1.32	1.42	0.84	0.81	0.84
time (sec)	N/A	0.355	0.067	10.422	0.041	0.106	0.441	0.119	0.171	0.034

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	31	20	13	13	13
N.S.	1	1.00	1.00	0.93	0.87	2.07	1.33	0.87	0.87	0.87
time (sec)	N/A	0.308	0.004	3.891	0.044	0.078	0.202	0.118	0.162	0.038

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	49	31	54	41	42	0	44	51	38
N.S.	1	1.40	0.89	1.54	1.17	1.20	0.00	1.26	1.46	1.09
time (sec)	N/A	0.356	0.258	2.326	0.122	0.083	0.000	0.149	0.166	25.414

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	38	27	29	26	27	39	26	24
N.S.	1	1.00	1.36	0.96	1.04	0.93	0.96	1.39	0.93	0.86
time (sec)	N/A	0.382	0.016	0.435	0.115	0.075	0.089	0.151	0.169	25.341

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	39	22	13	43	13
N.S.	1	1.00	1.00	0.93	0.87	2.60	1.47	0.87	2.87	0.87
time (sec)	N/A	0.314	0.009	1.394	0.033	0.080	0.709	0.183	0.169	25.259

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	29	77	26	26	49	0	26	65	25
N.S.	1	0.94	2.48	0.84	0.84	1.58	0.00	0.84	2.10	0.81
time (sec)	N/A	0.346	0.190	4.694	0.040	0.091	0.000	0.204	0.168	25.306

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	41	98	36	36	61	0	36	85	35
N.S.	1	0.89	2.13	0.78	0.78	1.33	0.00	0.78	1.85	0.76
time (sec)	N/A	0.367	0.164	15.803	0.037	0.122	0.000	0.220	0.169	25.346

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	131	62	66	48	66	231	74	88	109
N.S.	1	1.18	0.56	0.59	0.43	0.59	2.08	0.67	0.79	0.98
time (sec)	N/A	0.926	0.211	21.421	0.047	0.095	1.276	0.127	0.162	26.864

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	105	33	31	33	56	189	32	72	90
N.S.	1	1.17	0.37	0.34	0.37	0.62	2.10	0.36	0.80	1.00
time (sec)	N/A	0.745	0.100	7.570	0.032	0.081	0.646	0.126	0.232	26.450

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	79	40	42	37	46	136	46	56	43
N.S.	1	1.14	0.58	0.61	0.54	0.67	1.97	0.67	0.81	0.62
time (sec)	N/A	0.595	0.062	5.237	0.045	0.090	0.326	0.132	0.259	25.423

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	51	33	31	33	36	95	32	40	50
N.S.	1	1.11	0.72	0.67	0.72	0.78	2.07	0.70	0.87	1.09
time (sec)	N/A	0.399	0.038	4.098	0.051	0.076	0.152	0.126	0.170	0.076

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	32	38	38	46	48	0	60	52	53
N.S.	1	0.84	1.00	1.00	1.21	1.26	0.00	1.58	1.37	1.39
time (sec)	N/A	0.350	0.019	1.762	0.056	0.089	0.000	0.126	0.163	25.435

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	51	53	58	56	74	0	64	71	98
N.S.	1	1.16	1.20	1.32	1.27	1.68	0.00	1.45	1.61	2.23
time (sec)	N/A	0.339	0.021	1.396	0.037	0.097	0.000	0.138	0.182	27.471

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	60	73	76	71	74	0	63	162	126
N.S.	1	1.09	1.33	1.38	1.29	1.35	0.00	1.15	2.95	2.29
time (sec)	N/A	0.571	0.026	1.238	0.037	0.098	0.000	0.203	0.176	29.005

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	88	99	94	91	84	0	73	226	177
N.S.	1	1.13	1.27	1.21	1.17	1.08	0.00	0.94	2.90	2.27
time (sec)	N/A	0.749	0.029	2.986	0.053	0.117	0.000	0.230	0.169	29.717

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	114	120	112	111	94	0	107	290	229
N.S.	1	1.15	1.21	1.13	1.12	0.95	0.00	1.08	2.93	2.31
time (sec)	N/A	0.946	0.030	9.269	0.047	0.096	0.000	0.252	0.177	30.397

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	44	68	47	46	36	83	36	45	35
N.S.	1	0.96	1.48	1.02	1.00	0.78	1.80	0.78	0.98	0.76
time (sec)	N/A	0.380	0.404	43.957	0.051	0.130	2.458	0.123	0.166	0.087

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	42	37	37	36	36	68	36	95	36
N.S.	1	0.91	0.80	0.80	0.78	0.78	1.48	0.78	2.07	0.78
time (sec)	N/A	0.363	0.289	34.793	0.033	0.106	1.757	0.125	0.157	25.795

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	44	50	36	36	36	63	36	35	36
N.S.	1	0.96	1.09	0.78	0.78	0.78	1.37	0.78	0.76	0.78
time (sec)	N/A	0.367	0.084	17.375	0.037	0.107	1.278	0.121	0.171	25.860

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	42	37	37	36	36	68	36	79	36
N.S.	1	0.91	0.80	0.80	0.78	0.78	1.48	0.78	1.72	0.78
time (sec)	N/A	0.366	0.144	12.685	0.050	0.084	0.908	0.122	0.168	25.805

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	29	48	26	26	36	42	36	25	26
N.S.	1	0.94	1.55	0.84	0.84	1.16	1.35	1.16	0.81	0.84
time (sec)	N/A	0.346	0.118	8.481	0.050	0.088	0.628	0.128	0.164	0.062

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	42	37	37	36	36	68	36	63	36
N.S.	1	0.91	0.80	0.80	0.78	0.78	1.48	0.78	1.37	0.78
time (sec)	N/A	0.363	0.090	7.484	0.035	0.080	0.446	0.117	0.164	0.043

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	34	20	13	13	13
N.S.	1	1.00	1.00	0.93	0.87	2.27	1.33	0.87	0.87	0.87
time (sec)	N/A	0.305	0.005	4.908	0.035	0.075	0.290	0.116	0.159	25.856

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	39	35	35	37	35	0	45	70	53
N.S.	1	0.98	0.88	0.88	0.92	0.88	0.00	1.12	1.75	1.32
time (sec)	N/A	0.355	0.040	2.492	0.030	0.096	0.000	0.127	0.220	25.777

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	34	39	50	32	33	0	42	43	31
N.S.	1	0.92	1.05	1.35	0.86	0.89	0.00	1.14	1.16	0.84
time (sec)	N/A	0.385	0.150	3.003	0.039	0.095	0.000	0.158	0.254	25.864

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	35	33	51	41	54	0	47	148	37
N.S.	1	0.81	0.77	1.19	0.95	1.26	0.00	1.09	3.44	0.86
time (sec)	N/A	0.398	0.051	1.688	0.039	0.099	0.000	0.141	0.178	25.707

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	32	38	70	35	35	0	37	185	38
N.S.	1	0.84	1.00	1.84	0.92	0.92	0.00	0.97	4.87	1.00
time (sec)	N/A	0.346	0.131	2.773	0.042	0.097	0.000	0.164	0.163	25.867

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	41	38	54	39	44	44	37	38
N.S.	1	1.00	0.95	0.88	1.26	0.91	1.02	1.02	0.86	0.88
time (sec)	N/A	0.487	0.028	0.549	0.045	0.089	0.106	0.141	0.173	25.545

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	35	41	32	35	35	60	35	33	39
N.S.	1	0.85	1.00	0.78	0.85	0.85	1.46	0.85	0.80	0.95
time (sec)	N/A	0.337	0.159	1.047	0.036	0.085	0.403	0.146	0.172	25.440

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	35	63	35	33	13
N.S.	1	1.00	1.00	0.93	0.87	2.33	4.20	2.33	2.20	0.87
time (sec)	N/A	0.317	0.011	1.952	0.040	0.090	0.549	0.155	0.165	25.277

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	41	46	36	35	35	66	35	35	40
N.S.	1	0.89	1.00	0.78	0.76	0.76	1.43	0.76	0.76	0.87
time (sec)	N/A	0.367	0.145	3.539	0.043	0.124	0.824	0.152	0.177	25.445

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	29	46	26	69	35	63	35	35	25
N.S.	1	0.94	1.48	0.84	2.23	1.13	2.03	1.13	1.13	0.81
time (sec)	N/A	0.350	0.035	6.543	0.033	0.083	1.153	0.161	0.169	25.452

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	41	46	36	35	35	66	35	35	40
N.S.	1	0.89	1.00	0.78	0.76	0.76	1.43	0.76	0.76	0.87
time (sec)	N/A	0.359	0.195	12.164	0.049	0.114	1.618	0.161	0.163	25.876

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	44	46	36	79	35	63	35	35	35
N.S.	1	0.96	1.00	0.78	1.72	0.76	1.37	0.76	0.76	0.76
time (sec)	N/A	0.378	0.032	25.137	0.030	0.117	2.338	0.151	0.172	25.434

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	41	46	36	35	35	66	35	35	40
N.S.	1	0.89	1.00	0.78	0.76	0.76	1.43	0.76	0.76	0.87
time (sec)	N/A	0.360	0.144	40.694	0.044	0.125	3.240	0.151	0.165	19.177

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	44	46	46	89	35	63	35	35	45
N.S.	1	0.96	1.00	1.00	1.93	0.76	1.37	0.76	0.76	0.98
time (sec)	N/A	0.375	0.032	88.975	0.050	0.106	4.335	0.156	0.170	25.458

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	63	78	68	66	84	0	83	118	147
N.S.	1	1.05	1.30	1.13	1.10	1.40	0.00	1.38	1.97	2.45
time (sec)	N/A	0.388	0.028	1.746	0.037	0.103	0.000	0.149	0.173	28.915

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	42	50	92	45	45	0	46	272	50
N.S.	1	0.84	1.00	1.84	0.90	0.90	0.00	0.92	5.44	1.00
time (sec)	N/A	0.353	0.160	3.209	0.037	0.100	0.000	0.191	0.235	25.549

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	45	75	48	56	60	0	74	59	88
N.S.	1	0.85	1.42	0.91	1.06	1.13	0.00	1.40	1.11	1.66
time (sec)	N/A	0.363	0.132	5.013	0.026	0.106	0.000	0.120	0.234	30.600

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	37	40	33	35	35	0	45	52	66
N.S.	1	0.92	1.00	0.82	0.88	0.88	0.00	1.12	1.30	1.65
time (sec)	N/A	0.363	0.020	3.024	0.051	0.098	0.000	0.135	0.153	25.515

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	33	60	38	46	50	0	60	43	62
N.S.	1	0.87	1.58	1.00	1.21	1.32	0.00	1.58	1.13	1.63
time (sec)	N/A	0.370	0.107	2.034	0.043	0.106	0.000	0.120	0.169	26.748

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	26	27	23	25	25	0	31	44	35
N.S.	1	0.96	1.00	0.85	0.93	0.93	0.00	1.15	1.63	1.30
time (sec)	N/A	0.353	0.019	1.741	0.036	0.097	0.000	0.128	0.163	25.422

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	21	42	28	34	38	0	41	22	35
N.S.	1	0.91	1.83	1.22	1.48	1.65	0.00	1.78	0.96	1.52
time (sec)	N/A	0.319	0.045	1.497	0.036	0.089	0.000	0.119	0.163	25.511

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	13	11	17	11	19	29	12	31	24
N.S.	1	1.18	1.00	1.55	1.00	1.73	2.64	1.09	2.82	2.18
time (sec)	N/A	0.287	0.007	0.777	0.052	0.088	0.081	0.126	0.191	0.432

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	15	12	28	30	0	30	43	11
N.S.	1	1.00	1.36	1.09	2.55	2.73	0.00	2.73	3.91	1.00
time (sec)	N/A	0.297	0.008	1.825	0.029	0.099	0.000	0.121	0.165	25.323

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	20	42	30	36	52	0	43	39	23
N.S.	1	0.87	1.83	1.30	1.57	2.26	0.00	1.87	1.70	1.00
time (sec)	N/A	0.326	0.136	1.651	0.038	0.105	0.000	0.123	0.165	25.290

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	24	39	23	40	56	0	47	132	35
N.S.	1	0.89	1.44	0.85	1.48	2.07	0.00	1.74	4.89	1.30
time (sec)	N/A	0.338	0.024	2.266	0.045	0.127	0.000	0.123	0.157	0.087

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	32	57	40	50	67	0	56	105	33
N.S.	1	0.84	1.50	1.05	1.32	1.76	0.00	1.47	2.76	0.87
time (sec)	N/A	0.354	0.090	2.000	0.030	0.123	0.000	0.123	0.161	25.317

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	54	33	65	67	0	56	216	46
N.S.	1	1.00	1.38	0.85	1.67	1.72	0.00	1.44	5.54	1.18
time (sec)	N/A	0.343	0.026	2.427	0.035	0.092	0.000	0.119	0.166	0.062

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	44	72	50	60	77	0	66	167	45
N.S.	1	0.83	1.36	0.94	1.13	1.45	0.00	1.25	3.15	0.85
time (sec)	N/A	0.355	0.109	2.234	0.029	0.096	0.000	0.125	0.201	25.293

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	51	69	43	85	77	0	66	300	56
N.S.	1	0.89	1.21	0.75	1.49	1.35	0.00	1.16	5.26	0.98
time (sec)	N/A	0.359	0.042	3.148	0.043	0.112	0.000	0.126	0.273	25.303

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	42	50	62	42	43	0	42	45	43
N.S.	1	0.84	1.00	1.24	0.84	0.86	0.00	0.84	0.90	0.86
time (sec)	N/A	0.372	0.029	7.746	0.048	0.085	0.000	0.131	0.174	25.842

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	79	41	66	63	52	0	55	64	47
N.S.	1	1.39	0.72	1.16	1.11	0.91	0.00	0.96	1.12	0.82
time (sec)	N/A	0.385	0.318	4.621	0.130	0.090	0.000	0.139	0.160	25.895

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	32	38	52	32	33	0	32	33	35
N.S.	1	0.84	1.00	1.37	0.84	0.87	0.00	0.84	0.87	0.92
time (sec)	N/A	0.366	0.022	3.421	0.036	0.117	0.000	0.134	0.188	25.753

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	50	31	54	43	40	0	43	48	43
N.S.	1	1.39	0.86	1.50	1.19	1.11	0.00	1.19	1.33	1.19
time (sec)	N/A	0.364	0.290	2.674	0.107	0.100	0.000	0.141	0.160	25.994

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	18	23	42	20	22	0	20	24	23
N.S.	1	0.78	1.00	1.83	0.87	0.96	0.00	0.87	1.04	1.00
time (sec)	N/A	0.349	0.016	2.567	0.029	0.113	0.000	0.129	0.162	25.752

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	29	18	18	39	17	35	17	15
N.S.	1	1.00	1.93	1.20	1.20	2.60	1.13	2.33	1.13	1.00
time (sec)	N/A	0.290	0.019	0.497	0.122	0.099	0.062	0.129	0.159	0.062

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	13	13	17	13	11	13
N.S.	1	1.00	1.00	1.09	1.18	1.18	1.55	1.18	1.00	1.18
time (sec)	N/A	0.302	0.012	0.655	0.045	0.087	0.080	0.122	0.162	25.810

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	21	27	30	36	50	0	44	53	22
N.S.	1	0.91	1.17	1.30	1.57	2.17	0.00	1.91	2.30	0.96
time (sec)	N/A	0.315	0.023	1.733	0.036	0.100	0.000	0.121	0.171	0.022

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	19	13	31	22	33	0	16	32	14
N.S.	1	0.86	0.59	1.41	1.00	1.50	0.00	0.73	1.45	0.64
time (sec)	N/A	0.351	0.061	2.950	0.037	0.083	0.000	0.136	0.172	25.788

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	52	27	50	61	85	0	67	121	48
N.S.	1	1.16	0.60	1.11	1.36	1.89	0.00	1.49	2.69	1.07
time (sec)	N/A	0.360	0.018	1.997	0.031	0.099	0.000	0.122	0.208	25.775

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	33	46	46	32	43	0	32	55	33
N.S.	1	0.87	1.21	1.21	0.84	1.13	0.00	0.84	1.45	0.87
time (sec)	N/A	0.359	0.178	3.114	0.046	0.098	0.000	0.140	0.251	25.741

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	83	27	68	79	95	0	80	185	67
N.S.	1	1.26	0.41	1.03	1.20	1.44	0.00	1.21	2.80	1.02
time (sec)	N/A	0.393	0.019	2.061	0.045	0.117	0.000	0.127	0.165	25.842

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	45	58	63	45	71	0	62	89	85
N.S.	1	0.78	1.00	1.09	0.78	1.22	0.00	1.07	1.53	1.47
time (sec)	N/A	0.394	0.028	4.990	0.043	0.099	0.000	0.137	0.177	25.953

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	64	103	70	66	93	0	83	84	129
N.S.	1	1.07	1.72	1.17	1.10	1.55	0.00	1.38	1.40	2.15
time (sec)	N/A	0.420	0.193	2.891	0.031	0.106	0.000	0.138	0.185	26.639

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	33	35	53	35	61	0	48	75	62
N.S.	1	0.77	0.81	1.23	0.81	1.42	0.00	1.12	1.74	1.44
time (sec)	N/A	0.386	0.074	2.048	0.043	0.118	0.000	0.130	0.168	25.661

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	52	86	60	56	83	0	65	68	77
N.S.	1	1.16	1.91	1.33	1.24	1.84	0.00	1.44	1.51	1.71
time (sec)	N/A	0.354	0.106	1.592	0.040	0.098	0.000	0.128	0.160	25.618

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	30	28	29	23	47	53	27	67	78
N.S.	1	1.07	1.00	1.04	0.82	1.68	1.89	0.96	2.39	2.79
time (sec)	N/A	0.404	0.012	0.766	0.045	0.096	0.205	0.121	0.170	28.842

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	75	50	46	72	0	54	42	48
N.S.	1	1.00	2.21	1.47	1.35	2.12	0.00	1.59	1.24	1.41
time (sec)	N/A	0.406	0.091	1.136	0.037	0.107	0.000	0.127	0.169	25.546

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	18	22	13	13	13
N.S.	1	1.00	1.00	0.93	0.87	1.20	1.47	0.87	0.87	0.87
time (sec)	N/A	0.323	0.017	0.597	0.038	0.085	0.109	0.118	0.163	25.356

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	24	39	23	36	65	0	43	87	34
N.S.	1	0.89	1.44	0.85	1.33	2.41	0.00	1.59	3.22	1.26
time (sec)	N/A	0.332	0.025	1.890	0.045	0.084	0.000	0.126	0.169	25.363

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	51	143	52	61	96	0	67	75	49
N.S.	1	1.16	3.25	1.18	1.39	2.18	0.00	1.52	1.70	1.11
time (sec)	N/A	0.359	0.440	1.829	0.045	0.104	0.000	0.125	0.254	0.036

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	35	61	43	64	102	0	69	175	39
N.S.	1	0.81	1.42	1.00	1.49	2.37	0.00	1.60	4.07	0.91
time (sec)	N/A	0.381	0.067	2.144	0.033	0.093	0.000	0.122	0.225	25.368

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	63	205	70	73	112	0	79	139	60
N.S.	1	1.05	3.42	1.17	1.22	1.87	0.00	1.32	2.32	1.00
time (sec)	N/A	0.395	0.605	2.233	0.032	0.085	0.000	0.132	0.213	0.096

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	49	67	61	82	112	0	78	259	74
N.S.	1	0.84	1.16	1.05	1.41	1.93	0.00	1.34	4.47	1.28
time (sec)	N/A	0.381	0.031	2.233	0.028	0.101	0.000	0.122	0.185	0.110

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	58	68	90	56	68	0	56	55	55
N.S.	1	0.85	1.00	1.32	0.82	1.00	0.00	0.82	0.81	0.81
time (sec)	N/A	0.389	0.037	13.319	0.034	0.100	0.000	0.139	0.171	25.594

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	91	53	94	75	89	0	68	82	56
N.S.	1	1.26	0.74	1.31	1.04	1.24	0.00	0.94	1.14	0.78
time (sec)	N/A	0.428	0.462	7.714	0.113	0.090	0.000	0.150	0.173	26.772

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	46	53	80	44	56	0	44	45	45
N.S.	1	0.87	1.00	1.51	0.83	1.06	0.00	0.83	0.85	0.85
time (sec)	N/A	0.374	0.035	4.922	0.028	0.084	0.000	0.132	0.180	25.802

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	62	43	78	55	79	0	55	66	46
N.S.	1	1.22	0.84	1.53	1.08	1.55	0.00	1.08	1.29	0.90
time (sec)	N/A	0.399	0.339	3.382	0.123	0.078	0.000	0.146	0.176	26.156

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	34	37	68	35	48	0	35	35	32
N.S.	1	0.92	1.00	1.84	0.95	1.30	0.00	0.95	0.95	0.86
time (sec)	N/A	0.351	0.026	2.881	0.029	0.077	0.000	0.128	0.163	25.755

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	33	29	34	84	27	62	28	23
N.S.	1	1.00	1.22	1.07	1.26	3.11	1.00	2.30	1.04	0.85
time (sec)	N/A	0.401	0.016	0.631	0.112	0.105	0.076	0.133	0.178	25.580

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	22	25	38	39	25	23	22
N.S.	1	1.00	1.00	0.85	0.96	1.46	1.50	0.96	0.88	0.85
time (sec)	N/A	0.346	0.019	0.857	0.032	0.072	0.151	0.127	0.235	25.592

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	34	24	13	29	13
N.S.	1	1.00	1.00	0.93	0.87	2.27	1.60	0.87	1.93	0.87
time (sec)	N/A	0.321	0.013	0.766	0.025	0.071	0.637	0.142	0.225	25.610

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	26	22	13	13	13
N.S.	1	1.00	1.00	0.93	0.87	1.73	1.47	0.87	0.87	0.87
time (sec)	N/A	0.326	0.013	0.681	0.027	0.070	0.151	0.121	0.159	25.381

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	33	31	40	50	94	0	56	69	32
N.S.	1	0.87	0.82	1.05	1.32	2.47	0.00	1.47	1.82	0.84
time (sec)	N/A	0.350	0.018	1.888	0.032	0.084	0.000	0.118	0.169	0.027

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	31	45	46	35	54	0	35	43	36
N.S.	1	0.84	1.22	1.24	0.95	1.46	0.00	0.95	1.16	0.97
time (sec)	N/A	0.364	0.323	3.010	0.032	0.076	0.000	0.134	0.170	25.545

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	64	31	68	73	130	0	79	135	61
N.S.	1	1.07	0.52	1.13	1.22	2.17	0.00	1.32	2.25	1.02
time (sec)	N/A	0.401	0.020	2.170	0.033	0.110	0.000	0.124	0.174	0.048

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	45	43	46	44	66	0	31	65	45
N.S.	1	0.85	0.81	0.87	0.83	1.25	0.00	0.58	1.23	0.85
time (sec)	N/A	0.369	0.066	3.180	0.030	0.073	0.000	0.135	0.187	25.428

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	95	31	86	91	140	0	92	199	79
N.S.	1	1.17	0.38	1.06	1.12	1.73	0.00	1.14	2.46	0.98
time (sec)	N/A	0.416	0.019	2.493	0.030	0.097	0.000	0.129	0.165	25.505

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	59	69	89	56	100	0	74	99	92
N.S.	1	0.86	1.00	1.29	0.81	1.45	0.00	1.07	1.43	1.33
time (sec)	N/A	0.418	0.045	9.210	0.033	0.101	0.000	0.142	0.162	26.683

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	95	141	98	89	132	0	96	100	157
N.S.	1	1.17	1.74	1.21	1.10	1.63	0.00	1.19	1.23	1.94
time (sec)	N/A	0.443	0.204	5.198	0.048	0.135	0.000	0.127	0.176	25.453

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	45	58	81	49	90	0	60	89	74
N.S.	1	0.78	1.00	1.40	0.84	1.55	0.00	1.03	1.53	1.28
time (sec)	N/A	0.392	0.032	3.019	0.028	0.120	0.000	0.137	0.237	25.560

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	83	123	88	79	122	0	77	84	105
N.S.	1	1.28	1.89	1.35	1.22	1.88	0.00	1.18	1.29	1.62
time (sec)	N/A	0.397	0.128	1.967	0.027	0.098	0.000	0.137	0.187	25.604

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	44	39	39	38	83	66	38	79	182
N.S.	1	1.05	0.93	0.93	0.90	1.98	1.57	0.90	1.88	4.33
time (sec)	N/A	0.530	0.015	0.884	0.027	0.104	0.304	0.129	0.156	31.008

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	60	113	78	71	112	0	67	58	78
N.S.	1	1.09	2.05	1.42	1.29	2.04	0.00	1.22	1.05	1.42
time (sec)	N/A	0.580	0.121	1.230	0.029	0.094	0.000	0.132	0.176	25.456

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	39	42	25	25	25
N.S.	1	1.00	1.00	0.93	0.87	2.60	2.80	1.67	1.67	1.67
time (sec)	N/A	0.369	0.009	0.872	0.025	0.078	0.221	0.129	0.165	25.391

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	60	113	68	65	111	0	88	52	48
N.S.	1	1.09	2.05	1.24	1.18	2.02	0.00	1.60	0.95	0.87
time (sec)	N/A	0.580	0.136	1.348	0.033	0.085	0.000	0.133	0.155	25.628

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	27	22	13	13	23
N.S.	1	1.00	1.00	0.93	0.87	1.80	1.47	0.87	0.87	1.53
time (sec)	N/A	0.324	0.013	0.705	0.030	0.080	0.213	0.128	0.175	25.454

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	37	54	33	51	105	0	55	99	79
N.S.	1	0.92	1.35	0.82	1.28	2.62	0.00	1.38	2.48	1.98
time (sec)	N/A	0.366	0.025	2.018	0.054	0.105	0.000	0.125	0.163	0.114

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	82	129	70	79	132	0	79	85	66
N.S.	1	1.26	1.98	1.08	1.22	2.03	0.00	1.22	1.31	1.02
time (sec)	N/A	0.404	4.781	2.220	0.038	0.121	0.000	0.130	0.167	0.054

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	47	54	61	74	138	0	78	185	82
N.S.	1	0.81	0.93	1.05	1.28	2.38	0.00	1.34	3.19	1.41
time (sec)	N/A	0.389	0.357	2.418	0.045	0.097	0.000	0.127	0.164	0.105

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	94	268	88	91	148	0	92	149	78
N.S.	1	1.16	3.31	1.09	1.12	1.83	0.00	1.14	1.84	0.96
time (sec)	N/A	0.441	0.732	2.345	0.036	0.089	0.000	0.127	0.159	0.074

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	61	91	79	92	148	0	89	269	64
N.S.	1	0.88	1.32	1.14	1.33	2.14	0.00	1.29	3.90	0.93
time (sec)	N/A	0.397	0.113	2.550	0.029	0.102	0.000	0.132	0.176	25.440

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	27	14	14	33	29	14	20	14
N.S.	1	1.00	1.59	0.82	0.82	1.94	1.71	0.82	1.18	0.82
time (sec)	N/A	0.335	0.044	0.927	0.026	0.073	0.026	0.122	0.235	25.611

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	22	14	30	15	14	14	14
N.S.	1	1.00	1.00	1.29	0.82	1.76	0.88	0.82	0.82	0.82
time (sec)	N/A	0.354	0.012	0.933	0.032	0.089	0.045	0.116	0.159	25.483

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	24	37	0	23	18
N.S.	1	1.00	1.00	0.86	0.82	1.09	1.68	0.00	1.05	0.82
time (sec)	N/A	0.325	0.041	1.594	0.026	0.090	2.933	0.000	0.153	0.129

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	21	37	21	20	18
N.S.	1	1.00	1.00	0.86	0.82	0.95	1.68	0.95	0.91	0.82
time (sec)	N/A	0.327	0.020	1.651	0.031	0.099	0.260	0.126	0.158	0.067

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	18	18	36	18	17	18
N.S.	1	1.00	1.00	0.95	0.90	0.90	1.80	0.90	0.85	0.90
time (sec)	N/A	0.318	0.016	1.659	0.027	0.076	0.426	0.124	0.175	25.564

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	18	26	34	0	25	37
N.S.	1	1.00	1.00	0.95	0.90	1.30	1.70	0.00	1.25	1.85
time (sec)	N/A	0.321	0.028	1.674	0.036	0.087	0.810	0.000	0.156	0.203

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	26	36	0	25	53
N.S.	1	1.00	1.00	0.86	0.82	1.18	1.64	0.00	1.14	2.41
time (sec)	N/A	0.333	0.033	1.794	0.025	0.111	3.549	0.000	0.175	25.784

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	26	36	0	25	65
N.S.	1	1.00	1.00	0.86	0.82	1.18	1.64	0.00	1.14	2.95
time (sec)	N/A	0.331	0.150	1.766	0.030	0.083	30.009	0.000	0.166	52.967

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	26	0	0	25	65
N.S.	1	1.00	1.00	0.86	0.82	1.18	0.00	0.00	1.14	2.95
time (sec)	N/A	0.329	0.073	1.730	0.033	0.099	0.000	0.000	0.157	30.652

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	137	60	249	0	119	0	0	32	0
N.S.	1	1.09	0.48	1.98	0.00	0.94	0.00	0.00	0.25	0.00
time (sec)	N/A	0.969	0.168	11.273	0.000	0.097	0.000	0.000	0.208	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	137	60	236	0	107	0	0	32	0
N.S.	1	1.09	0.48	1.87	0.00	0.85	0.00	0.00	0.25	0.00
time (sec)	N/A	0.952	0.148	9.647	0.000	0.094	0.000	0.000	0.200	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	103	57	223	0	106	0	0	32	0
N.S.	1	1.05	0.58	2.28	0.00	1.08	0.00	0.00	0.33	0.00
time (sec)	N/A	0.771	0.058	7.096	0.000	0.096	0.000	0.000	0.201	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	103	57	208	0	90	0	0	28	0
N.S.	1	1.05	0.58	2.12	0.00	0.92	0.00	0.00	0.29	0.00
time (sec)	N/A	0.739	0.074	5.292	0.000	0.085	0.000	0.000	0.244	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	58	194	0	87	0	0	21	0
N.S.	1	1.00	0.84	2.81	0.00	1.26	0.00	0.00	0.30	0.00
time (sec)	N/A	0.570	0.097	4.569	0.000	0.089	0.000	0.000	0.285	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	58	188	0	78	0	0	32	0
N.S.	1	1.00	0.84	2.72	0.00	1.13	0.00	0.00	0.46	0.00
time (sec)	N/A	0.563	0.110	3.521	0.000	0.083	0.000	0.000	0.157	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	60	198	0	105	0	0	32	0
N.S.	1	1.00	0.88	2.91	0.00	1.54	0.00	0.00	0.47	0.00
time (sec)	N/A	0.598	0.100	4.235	0.000	0.077	0.000	0.000	0.169	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	60	242	0	103	0	0	32	0
N.S.	1	1.00	0.83	3.36	0.00	1.43	0.00	0.00	0.44	0.00
time (sec)	N/A	0.595	0.095	4.296	0.000	0.086	0.000	0.000	0.166	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	104	59	365	0	120	0	0	32	0
N.S.	1	1.04	0.59	3.65	0.00	1.20	0.00	0.00	0.32	0.00
time (sec)	N/A	0.760	0.085	5.273	0.000	0.081	0.000	0.000	0.157	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	108	59	396	0	114	0	0	32	0
N.S.	1	1.08	0.59	3.96	0.00	1.14	0.00	0.00	0.32	0.00
time (sec)	N/A	0.764	0.072	6.059	0.000	0.097	0.000	0.000	0.159	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	44	57	37	36	34	73	53	21	0
N.S.	1	0.98	1.27	0.82	0.80	0.76	1.62	1.18	0.47	0.00
time (sec)	N/A	0.392	0.332	2.746	0.033	0.103	0.892	0.122	0.167	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	42	57	37	36	28	71	46	32	0
N.S.	1	0.98	1.33	0.86	0.84	0.65	1.65	1.07	0.74	0.00
time (sec)	N/A	0.394	0.200	2.868	0.032	0.097	0.767	0.126	0.151	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	42	46	36	35	36	70	0	32	0
N.S.	1	0.98	1.07	0.84	0.81	0.84	1.63	0.00	0.74	0.00
time (sec)	N/A	0.403	0.092	2.828	0.027	0.084	0.781	0.000	0.163	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	42	48	35	34	38	71	0	32	66
N.S.	1	0.98	1.12	0.81	0.79	0.88	1.65	0.00	0.74	1.53
time (sec)	N/A	0.405	0.117	2.944	0.027	0.107	3.503	0.000	0.151	26.187

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	42	70	37	37	38	71	0	32	93
N.S.	1	0.98	1.63	0.86	0.86	0.88	1.65	0.00	0.74	2.16
time (sec)	N/A	0.408	0.314	3.070	0.028	0.084	29.485	0.000	0.150	31.574

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	44	70	37	37	38	0	0	32	93
N.S.	1	0.98	1.56	0.82	0.82	0.84	0.00	0.00	0.71	2.07
time (sec)	N/A	0.403	0.305	2.971	0.032	0.099	0.000	0.000	0.161	30.429

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	44	94	37	37	38	0	0	32	279
N.S.	1	0.98	2.09	0.82	0.82	0.84	0.00	0.00	0.71	6.20
time (sec)	N/A	0.412	0.601	3.039	0.050	0.086	0.000	0.000	0.167	31.728

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	172	57	275	0	133	0	0	32	0
N.S.	1	1.10	0.37	1.76	0.00	0.85	0.00	0.00	0.21	0.00
time (sec)	N/A	1.254	0.166	27.663	0.000	0.109	0.000	0.000	0.167	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	172	57	262	0	121	0	0	32	0
N.S.	1	1.10	0.37	1.68	0.00	0.78	0.00	0.00	0.21	0.00
time (sec)	N/A	1.239	0.102	22.325	0.000	0.104	0.000	0.000	0.202	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	138	65	249	0	120	0	0	32	0
N.S.	1	1.08	0.51	1.95	0.00	0.94	0.00	0.00	0.25	0.00
time (sec)	N/A	1.019	0.085	19.829	0.000	0.109	0.000	0.000	0.293	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	138	65	255	0	102	0	0	28	0
N.S.	1	1.08	0.51	1.99	0.00	0.80	0.00	0.00	0.22	0.00
time (sec)	N/A	1.010	0.115	7.815	0.000	0.105	0.000	0.000	0.187	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	104	58	221	0	100	0	0	21	0
N.S.	1	1.05	0.59	2.23	0.00	1.01	0.00	0.00	0.21	0.00
time (sec)	N/A	0.790	0.070	6.861	0.000	0.081	0.000	0.000	0.158	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	104	58	208	0	88	0	0	32	0
N.S.	1	1.05	0.59	2.10	0.00	0.89	0.00	0.00	0.32	0.00
time (sec)	N/A	0.802	0.083	4.384	0.000	0.108	0.000	0.000	0.181	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	103	60	213	0	115	0	0	32	0
N.S.	1	1.03	0.60	2.13	0.00	1.15	0.00	0.00	0.32	0.00
time (sec)	N/A	0.794	0.073	5.188	0.000	0.099	0.000	0.000	0.157	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	105	60	286	0	113	0	0	32	0
N.S.	1	1.03	0.59	2.80	0.00	1.11	0.00	0.00	0.31	0.00
time (sec)	N/A	0.796	0.074	4.748	0.000	0.129	0.000	0.000	0.161	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	106	65	366	0	120	0	0	32	0
N.S.	1	1.04	0.64	3.59	0.00	1.18	0.00	0.00	0.31	0.00
time (sec)	N/A	0.842	0.076	5.138	0.000	0.092	0.000	0.000	0.179	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	110	65	398	0	114	0	0	32	0
N.S.	1	1.08	0.64	3.90	0.00	1.12	0.00	0.00	0.31	0.00
time (sec)	N/A	0.839	0.072	5.420	0.000	0.089	0.000	0.000	0.157	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	48	111	37	36	44	0	0	24	35
N.S.	1	0.92	2.13	0.71	0.69	0.85	0.00	0.00	0.46	0.67
time (sec)	N/A	0.359	0.305	2.193	0.032	0.084	0.000	0.000	0.157	25.637

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	94	83	331	98	322	0	0	30	0
N.S.	1	0.94	0.83	3.31	0.98	3.22	0.00	0.00	0.30	0.00
time (sec)	N/A	0.455	0.531	3.414	0.110	0.178	0.000	0.000	0.174	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	92	80	292	98	308	0	0	30	0
N.S.	1	0.93	0.81	2.95	0.99	3.11	0.00	0.00	0.30	0.00
time (sec)	N/A	0.469	0.440	3.316	0.108	0.174	0.000	0.000	0.176	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	73	68	253	83	290	0	0	30	0
N.S.	1	0.94	0.87	3.24	1.06	3.72	0.00	0.00	0.38	0.00
time (sec)	N/A	0.464	0.483	3.368	0.109	0.188	0.000	0.000	0.183	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	71	65	212	83	269	0	0	26	0
N.S.	1	0.92	0.84	2.75	1.08	3.49	0.00	0.00	0.34	0.00
time (sec)	N/A	0.455	0.251	3.114	0.113	0.180	0.000	0.000	0.182	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	51	51	182	67	230	0	48	19	0
N.S.	1	0.88	0.88	3.14	1.16	3.97	0.00	0.83	0.33	0.00
time (sec)	N/A	0.411	0.233	2.713	0.112	0.136	0.000	0.124	0.160	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	51	50	181	68	236	0	52	30	0
N.S.	1	0.86	0.85	3.07	1.15	4.00	0.00	0.88	0.51	0.00
time (sec)	N/A	0.413	0.209	2.483	0.113	0.119	0.000	0.133	0.166	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	74	66	441	79	301	0	0	30	0
N.S.	1	0.95	0.85	5.65	1.01	3.86	0.00	0.00	0.38	0.00
time (sec)	N/A	0.467	0.272	3.066	0.109	0.141	0.000	0.000	0.172	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	76	70	649	80	309	0	0	30	0
N.S.	1	0.94	0.86	8.01	0.99	3.81	0.00	0.00	0.37	0.00
time (sec)	N/A	0.452	0.340	2.985	0.112	0.163	0.000	0.000	0.247	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	98	81	862	100	333	0	0	30	0
N.S.	1	0.98	0.81	8.62	1.00	3.33	0.00	0.00	0.30	0.00
time (sec)	N/A	0.490	0.425	2.785	0.114	0.128	0.000	0.000	0.161	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	100	87	1057	102	333	0	0	30	0
N.S.	1	0.97	0.84	10.26	0.99	3.23	0.00	0.00	0.29	0.00
time (sec)	N/A	0.495	0.549	2.945	0.113	0.126	0.000	0.000	0.162	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	136	89	242	0	121	0	0	32	0
N.S.	1	1.10	0.72	1.95	0.00	0.98	0.00	0.00	0.26	0.00
time (sec)	N/A	0.983	1.255	9.921	0.000	0.099	0.000	0.000	0.182	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	102	74	229	0	120	0	0	32	0
N.S.	1	1.06	0.77	2.39	0.00	1.25	0.00	0.00	0.33	0.00
time (sec)	N/A	0.791	0.769	8.554	0.000	0.115	0.000	0.000	0.179	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	102	73	216	0	108	0	0	32	0
N.S.	1	1.06	0.76	2.25	0.00	1.12	0.00	0.00	0.33	0.00
time (sec)	N/A	0.756	0.607	7.859	0.000	0.093	0.000	0.000	0.189	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	58	203	0	104	0	0	32	0
N.S.	1	1.00	0.88	3.08	0.00	1.58	0.00	0.00	0.48	0.00
time (sec)	N/A	0.578	0.493	6.529	0.000	0.084	0.000	0.000	0.179	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	56	190	0	90	0	0	28	0
N.S.	1	1.00	0.85	2.88	0.00	1.36	0.00	0.00	0.42	0.00
time (sec)	N/A	0.587	0.369	4.503	0.000	0.096	0.000	0.000	0.181	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	56	203	0	101	0	0	21	0
N.S.	1	1.00	0.86	3.12	0.00	1.55	0.00	0.00	0.32	0.00
time (sec)	N/A	0.562	0.368	4.471	0.000	0.079	0.000	0.000	0.159	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	47	188	0	92	0	0	32	0
N.S.	1	1.00	0.73	2.94	0.00	1.44	0.00	0.00	0.50	0.00
time (sec)	N/A	0.572	0.362	4.078	0.000	0.093	0.000	0.000	0.256	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	99	65	209	0	130	0	0	32	0
N.S.	1	1.05	0.69	2.22	0.00	1.38	0.00	0.00	0.34	0.00
time (sec)	N/A	0.750	0.477	4.869	0.000	0.092	0.000	0.000	0.236	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	103	62	190	0	129	0	0	32	0
N.S.	1	1.05	0.63	1.94	0.00	1.32	0.00	0.00	0.33	0.00
time (sec)	N/A	0.763	0.535	5.295	0.000	0.089	0.000	0.000	0.158	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	135	82	408	0	145	0	0	32	0
N.S.	1	1.07	0.65	3.24	0.00	1.15	0.00	0.00	0.25	0.00
time (sec)	N/A	0.964	0.742	6.372	0.000	0.089	0.000	0.000	0.171	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	134	137	407	133	428	0	0	32	0
N.S.	1	0.99	1.01	3.01	0.99	3.17	0.00	0.00	0.24	0.00
time (sec)	N/A	0.514	3.094	9.641	0.115	0.200	0.000	0.000	0.211	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	B	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	115	78	371	118	414	0	0	32	0
N.S.	1	1.02	0.69	3.28	1.04	3.66	0.00	0.00	0.28	0.00
time (sec)	N/A	0.507	1.209	9.601	0.140	0.219	0.000	0.000	0.208	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	118	310	118	402	0	0	32	0
N.S.	1	1.00	1.04	2.74	1.04	3.56	0.00	0.00	0.28	0.00
time (sec)	N/A	0.477	1.635	9.637	0.120	0.193	0.000	0.000	0.179	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	B	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	93	65	286	103	371	0	0	32	0
N.S.	1	1.02	0.71	3.14	1.13	4.08	0.00	0.00	0.35	0.00
time (sec)	N/A	0.479	0.611	9.773	0.134	0.135	0.000	0.000	0.188	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	B	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	93	76	280	103	339	0	0	28	0
N.S.	1	1.02	0.84	3.08	1.13	3.73	0.00	0.00	0.31	0.00
time (sec)	N/A	0.476	0.449	3.177	0.109	0.143	0.000	0.000	0.260	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	B	F	A	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	99	62	275	102	332	0	95	21	0
N.S.	1	1.06	0.67	2.96	1.10	3.57	0.00	1.02	0.23	0.00
time (sec)	N/A	0.480	0.680	3.481	0.112	0.167	0.000	0.128	0.272	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	A	B	F	A	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	0	69	283	103	325	0	91	32	0
N.S.	1	0.00	0.74	3.04	1.11	3.49	0.00	0.98	0.34	0.00
time (sec)	N/A	0.000	0.499	3.031	0.111	0.121	0.000	0.133	0.149	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	B	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	122	91	689	117	397	0	0	32	0
N.S.	1	1.06	0.79	5.99	1.02	3.45	0.00	0.00	0.28	0.00
time (sec)	N/A	0.472	0.545	3.498	0.109	0.140	0.000	0.000	0.161	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	B	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	124	92	885	117	409	0	0	32	0
N.S.	1	1.08	0.80	7.70	1.02	3.56	0.00	0.00	0.28	0.00
time (sec)	N/A	0.466	0.709	3.510	0.114	0.140	0.000	0.000	0.184	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	A	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	146	102	1140	134	429	0	0	32	0
N.S.	1	1.07	0.74	8.32	0.98	3.13	0.00	0.00	0.23	0.00
time (sec)	N/A	0.297	0.853	3.493	0.113	0.128	0.000	0.000	0.156	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	21	37	21	21	18
N.S.	1	1.00	1.00	0.86	0.82	0.95	1.68	0.95	0.95	0.82
time (sec)	N/A	0.189	0.022	1.003	0.032	0.112	1.710	0.118	0.172	0.092

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	18	14	13	14	170	13	10	25
N.S.	1	1.00	0.86	0.67	0.62	0.67	8.10	0.62	0.48	1.19
time (sec)	N/A	0.193	0.018	1.054	0.027	0.133	3.531	0.119	0.163	25.451

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	18	14	13	20	24	13	12	25
N.S.	1	1.00	0.86	0.67	0.62	0.95	1.14	0.62	0.57	1.19
time (sec)	N/A	0.192	0.017	1.012	0.030	0.128	2.456	0.122	0.150	25.392

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	18	14	13	20	24	13	14	25
N.S.	1	1.00	0.86	0.67	0.62	0.95	1.14	0.62	0.67	1.19
time (sec)	N/A	0.191	0.016	7.110	0.033	0.104	18.574	0.114	0.158	25.483

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	14	13	12	323	13	14	25
N.S.	1	1.00	0.84	0.74	0.68	0.63	17.00	0.68	0.74	1.32
time (sec)	N/A	0.192	0.013	1.037	0.031	0.114	4.076	0.126	0.150	25.343

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	138	70	259	0	0	0	0	33	0
N.S.	1	1.05	0.53	1.96	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.597	0.131	11.052	0.000	0.000	0.000	0.000	0.169	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	70	249	0	0	0	0	33	0
N.S.	1	1.00	0.74	2.62	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.403	0.098	9.157	0.000	0.000	0.000	0.000	0.198	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	67	236	0	0	0	0	22	0
N.S.	1	1.00	1.26	4.45	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.271	0.068	10.648	0.000	0.000	0.000	0.000	0.202	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	70	223	0	172	0	0	33	0
N.S.	1	1.00	0.75	2.40	0.00	1.85	0.00	0.00	0.35	0.00
time (sec)	N/A	0.409	0.119	11.326	0.000	0.132	0.000	0.000	0.250	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	138	70	253	0	193	0	0	33	0
N.S.	1	1.03	0.52	1.89	0.00	1.44	0.00	0.00	0.25	0.00
time (sec)	N/A	0.571	0.153	11.499	0.000	0.106	0.000	0.000	0.158	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	245	325	70	427	0	512	0	0	29	0
N.S.	1	1.33	0.29	1.74	0.00	2.09	0.00	0.00	0.12	0.00
time (sec)	N/A	0.616	0.134	186.003	0.000	0.162	0.000	0.000	0.165	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	203	285	67	363	0	477	0	0	33	0
N.S.	1	1.40	0.33	1.79	0.00	2.35	0.00	0.00	0.16	0.00
time (sec)	N/A	0.452	0.067	217.152	0.000	0.127	0.000	0.000	0.189	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	35	0	42	0	0	33	50
N.S.	1	1.00	1.00	0.95	0.00	1.14	0.00	0.00	0.89	1.35
time (sec)	N/A	0.226	0.088	12.454	0.000	0.118	0.000	0.000	0.153	26.389

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	57	54	0	54	0	0	33	95
N.S.	1	1.00	0.76	0.72	0.00	0.72	0.00	0.00	0.44	1.27
time (sec)	N/A	0.349	0.265	12.691	0.000	0.123	0.000	0.000	0.160	27.564

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	120	67	65	0	64	0	0	33	216
N.S.	1	1.07	0.60	0.58	0.00	0.57	0.00	0.00	0.29	1.93
time (sec)	N/A	0.494	0.266	12.330	0.000	0.156	0.000	0.000	0.158	32.914

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	131	135	71	993	0	0	0	0	36	0
N.S.	1	1.03	0.54	7.58	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.581	0.140	162.135	0.000	0.000	0.000	0.000	0.170	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	67	133	0	0	0	0	40	0
N.S.	1	1.00	0.72	1.43	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.399	0.090	13.929	0.000	0.000	0.000	0.000	0.180	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	67	136	0	108	0	0	82	0
N.S.	1	1.00	0.68	1.39	0.00	1.10	0.00	0.00	0.84	0.00
time (sec)	N/A	0.419	0.177	11.274	0.000	0.105	0.000	0.000	0.168	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	143	70	140	0	121	0	0	82	0
N.S.	1	1.08	0.53	1.05	0.00	0.91	0.00	0.00	0.62	0.00
time (sec)	N/A	0.569	0.161	11.473	0.000	0.138	0.000	0.000	0.165	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	245	325	67	428	0	512	0	0	29	0
N.S.	1	1.33	0.27	1.75	0.00	2.09	0.00	0.00	0.12	0.00
time (sec)	N/A	0.597	0.075	21.862	0.000	0.139	0.000	0.000	0.163	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	236	323	67	873	0	551	0	0	71	0
N.S.	1	1.37	0.28	3.70	0.00	2.33	0.00	0.00	0.30	0.00
time (sec)	N/A	0.594	0.175	53.778	0.000	0.230	0.000	0.000	0.156	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	40	38	0	50	0	0	82	64
N.S.	1	1.00	1.08	1.03	0.00	1.35	0.00	0.00	2.22	1.73
time (sec)	N/A	0.225	0.137	6.399	0.000	0.108	0.000	0.000	0.163	26.886

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	121	57	59	0	61	0	0	82	207
N.S.	1	1.14	0.54	0.56	0.00	0.58	0.00	0.00	0.77	1.95
time (sec)	N/A	0.489	0.433	5.073	0.000	0.146	0.000	0.000	0.225	32.152

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	166	67	68	0	73	0	0	82	193
N.S.	1	1.18	0.48	0.48	0.00	0.52	0.00	0.00	0.58	1.37
time (sec)	N/A	0.639	0.354	5.243	0.000	0.207	0.000	0.000	0.265	35.046

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	181	72	273	0	0	0	0	44	0
N.S.	1	1.09	0.43	1.64	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.739	0.208	8.938	0.000	0.000	0.000	0.000	0.188	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	138	70	262	0	0	0	0	44	0
N.S.	1	1.05	0.53	2.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.566	0.196	5.809	0.000	0.000	0.000	0.000	0.190	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	67	249	0	0	0	0	33	0
N.S.	1	1.00	0.71	2.62	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.388	0.106	5.523	0.000	0.000	0.000	0.000	0.208	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	67	238	0	0	0	0	44	0
N.S.	1	1.00	0.71	2.53	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.413	0.152	5.381	0.000	0.000	0.000	0.000	0.162	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	139	70	259	0	212	0	0	44	0
N.S.	1	1.05	0.53	1.95	0.00	1.59	0.00	0.00	0.33	0.00
time (sec)	N/A	0.548	0.300	4.829	0.000	0.128	0.000	0.000	0.170	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	184	72	275	0	224	0	0	44	0
N.S.	1	1.10	0.43	1.64	0.00	1.33	0.00	0.00	0.26	0.00
time (sec)	N/A	0.729	0.198	4.895	0.000	0.127	0.000	0.000	0.159	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	245	325	67	437	0	534	0	0	44	0
N.S.	1	1.33	0.27	1.78	0.00	2.18	0.00	0.00	0.18	0.00
time (sec)	N/A	0.580	0.128	6.978	0.000	0.154	0.000	0.000	0.160	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	237	325	67	508	0	582	0	0	44	0
N.S.	1	1.37	0.28	2.14	0.00	2.46	0.00	0.00	0.19	0.00
time (sec)	N/A	0.582	0.132	6.217	0.000	0.170	0.000	0.000	0.171	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	40	40	0	60	0	0	44	89
N.S.	1	1.00	1.08	1.08	0.00	1.62	0.00	0.00	1.19	2.41
time (sec)	N/A	0.223	0.158	5.431	0.000	0.170	0.000	0.000	0.164	27.071

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	121	57	61	0	74	0	0	44	176
N.S.	1	1.14	0.54	0.58	0.00	0.70	0.00	0.00	0.42	1.66
time (sec)	N/A	0.461	0.340	5.474	0.000	0.189	0.000	0.000	0.161	33.063

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	166	67	70	0	87	0	0	44	207
N.S.	1	1.18	0.48	0.50	0.00	0.62	0.00	0.00	0.31	1.47
time (sec)	N/A	0.642	0.502	5.844	0.000	0.434	0.000	0.000	0.161	33.986

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	230	57	530	0	447	0	0	33	44
N.S.	1	1.30	0.32	2.99	0.00	2.53	0.00	0.00	0.19	0.25
time (sec)	N/A	0.550	0.066	46.897	0.000	0.182	0.000	0.000	0.154	26.972

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	11	0	16	0	0	38	53
N.S.	1	1.00	1.00	0.69	0.00	1.00	0.00	0.00	2.38	3.31
time (sec)	N/A	0.172	0.024	3.175	0.000	0.120	0.000	0.000	0.162	25.749

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	134	38	202	0	220	0	0	13	25
N.S.	1	1.51	0.43	2.27	0.00	2.47	0.00	0.00	0.15	0.28
time (sec)	N/A	0.318	0.019	46.996	0.000	0.113	0.000	0.000	0.165	25.597

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	153	38	254	0	230	0	0	17	25
N.S.	1	1.37	0.34	2.27	0.00	2.05	0.00	0.00	0.15	0.22
time (sec)	N/A	0.393	0.016	2.335	0.000	0.132	0.000	0.000	0.156	25.734

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	132	135	70	996	0	0	0	0	44	0
N.S.	1	1.02	0.53	7.55	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.540	0.127	15.890	0.000	0.000	0.000	0.000	0.243	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	68	124	0	0	0	0	40	0
N.S.	1	1.00	0.74	1.35	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.398	0.121	3.859	0.000	0.000	0.000	0.000	0.230	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	65	112	0	62	0	0	44	0
N.S.	1	1.00	1.23	2.11	0.00	1.17	0.00	0.00	0.83	0.00
time (sec)	N/A	0.268	0.066	3.220	0.000	0.093	0.000	0.000	0.166	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	65	124	0	108	0	0	44	0
N.S.	1	1.00	0.67	1.28	0.00	1.11	0.00	0.00	0.45	0.00
time (sec)	N/A	0.409	0.113	4.564	0.000	0.103	0.000	0.000	0.178	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	142	70	143	0	122	0	0	44	0
N.S.	1	1.06	0.52	1.07	0.00	0.91	0.00	0.00	0.33	0.00
time (sec)	N/A	0.559	0.143	4.714	0.000	0.096	0.000	0.000	0.172	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	202	285	65	354	0	477	0	0	33	0
N.S.	1	1.41	0.32	1.75	0.00	2.36	0.00	0.00	0.16	0.00
time (sec)	N/A	0.452	0.087	4.319	0.000	0.141	0.000	0.000	0.157	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	36	35	0	39	0	0	44	31
N.S.	1	1.00	1.03	1.00	0.00	1.11	0.00	0.00	1.26	0.89
time (sec)	N/A	0.222	0.065	4.796	0.000	0.088	0.000	0.000	0.181	25.716

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	52	51	0	51	0	0	44	77
N.S.	1	1.00	0.69	0.68	0.00	0.68	0.00	0.00	0.59	1.03
time (sec)	N/A	0.337	0.194	4.730	0.000	0.126	0.000	0.000	0.172	26.459

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	120	67	65	0	61	0	0	44	123
N.S.	1	1.07	0.60	0.58	0.00	0.54	0.00	0.00	0.39	1.10
time (sec)	N/A	0.470	0.272	4.487	0.000	0.140	0.000	0.000	0.170	29.295

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	177	55	353	0	366	0	0	25	44
N.S.	1	1.42	0.44	2.82	0.00	2.93	0.00	0.00	0.20	0.35
time (sec)	N/A	0.339	0.035	2.171	0.000	0.170	0.000	0.000	0.157	26.407

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	203	55	429	0	422	0	0	63	44
N.S.	1	1.35	0.37	2.86	0.00	2.81	0.00	0.00	0.42	0.29
time (sec)	N/A	0.441	0.040	1.570	0.000	0.161	0.000	0.000	0.164	26.475

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	205	57	431	0	475	0	0	33	44
N.S.	1	1.34	0.37	2.82	0.00	3.10	0.00	0.00	0.22	0.29
time (sec)	N/A	0.431	0.041	1.004	0.000	0.170	0.000	0.000	0.156	26.873

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	230	57	460	0	525	0	0	98	44
N.S.	1	1.29	0.32	2.58	0.00	2.95	0.00	0.00	0.55	0.25
time (sec)	N/A	0.544	0.049	1.033	0.000	0.186	0.000	0.000	0.156	26.792

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	23	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.213	0.065	0.000	0.000	0.000	0.000	0.000	0.178	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	23	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.212	0.050	0.000	0.000	0.000	0.000	0.000	0.162	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	14	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.189	0.044	0.000	0.000	0.000	0.000	0.000	0.171	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	23	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.205	0.050	0.000	0.000	0.000	0.000	0.000	0.163	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	23	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.206	0.046	0.000	0.000	0.000	0.000	0.000	0.164	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	23	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.220	0.064	0.000	0.000	0.000	0.000	0.000	0.186	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	23	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.219	0.062	0.000	0.000	0.000	0.000	0.000	0.190	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	14	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.191	0.051	0.000	0.000	0.000	0.000	0.000	0.162	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	23	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.222	0.060	0.000	0.000	0.000	0.000	0.000	0.201	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	23	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.218	0.060	0.000	0.000	0.000	0.000	0.000	0.186	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	23	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.215	0.060	0.000	0.000	0.000	0.000	0.000	0.173	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	23	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.214	0.054	0.000	0.000	0.000	0.000	0.000	0.164	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	14	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.187	0.044	0.000	0.000	0.000	0.000	0.000	0.158	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	23	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.214	0.054	0.000	0.000	0.000	0.000	0.000	0.174	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	23	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.212	0.049	0.000	0.000	0.000	0.000	0.000	0.162	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	23	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.224	0.055	0.000	0.000	0.000	0.000	0.000	0.221	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	23	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.223	0.053	0.000	0.000	0.000	0.000	0.000	0.254	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	14	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.190	0.042	0.000	0.000	0.000	0.000	0.000	0.159	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	23	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.219	0.050	0.000	0.000	0.000	0.000	0.000	0.164	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-1)	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	23	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.218	0.046	0.000	0.000	0.000	0.000	0.000	0.171	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	73	57	0	0	144	0	0	19	44
N.S.	1	0.57	0.45	0.00	0.00	1.12	0.00	0.00	0.15	0.34
time (sec)	N/A	0.297	0.049	0.000	0.000	0.134	0.000	0.000	0.169	26.631

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	163	221	57	0	0	267	0	0	19	44
N.S.	1	1.36	0.35	0.00	0.00	1.64	0.00	0.00	0.12	0.27
time (sec)	N/A	0.368	0.049	0.000	0.000	0.112	0.000	0.000	0.177	26.425

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	188	247	57	0	0	322	0	0	19	44
N.S.	1	1.31	0.30	0.00	0.00	1.71	0.00	0.00	0.10	0.23
time (sec)	N/A	0.450	0.062	0.000	0.000	0.129	0.000	0.000	0.164	26.997

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	155	101	57	0	0	197	0	0	19	44
N.S.	1	0.65	0.37	0.00	0.00	1.27	0.00	0.00	0.12	0.28
time (sec)	N/A	0.393	0.063	0.000	0.000	0.110	0.000	0.000	0.167	26.546

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	155	101	57	0	0	195	0	0	19	44
N.S.	1	0.65	0.37	0.00	0.00	1.26	0.00	0.00	0.12	0.28
time (sec)	N/A	0.389	0.057	0.000	0.000	0.097	0.000	0.000	0.162	27.033

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	73	57	0	0	152	0	0	19	44
N.S.	1	0.57	0.45	0.00	0.00	1.19	0.00	0.00	0.15	0.34
time (sec)	N/A	0.300	0.034	0.000	0.000	0.093	0.000	0.000	0.167	26.885

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	164	221	55	0	0	266	0	0	19	44
N.S.	1	1.35	0.34	0.00	0.00	1.62	0.00	0.00	0.12	0.27
time (sec)	N/A	0.343	0.032	0.000	0.000	0.096	0.000	0.000	0.174	26.535

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	189	247	55	0	0	323	0	0	19	44
N.S.	1	1.31	0.29	0.00	0.00	1.71	0.00	0.00	0.10	0.23
time (sec)	N/A	0.450	0.039	0.000	0.000	0.116	0.000	0.000	0.157	27.195

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	155	101	57	0	0	189	0	0	19	44
N.S.	1	0.65	0.37	0.00	0.00	1.22	0.00	0.00	0.12	0.28
time (sec)	N/A	0.396	0.039	0.000	0.000	0.099	0.000	0.000	0.166	26.597

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	155	101	57	0	0	219	0	0	19	44
N.S.	1	0.65	0.37	0.00	0.00	1.41	0.00	0.00	0.12	0.28
time (sec)	N/A	0.398	0.040	0.000	0.000	0.099	0.000	0.000	0.175	27.459

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	0	0	18	0	0	11	10
N.S.	1	1.00	1.00	0.00	0.00	1.12	0.00	0.00	0.69	0.62
time (sec)	N/A	0.182	0.015	0.000	0.000	0.080	0.000	0.000	0.162	26.198

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	0	0	10	0	0	11	94
N.S.	1	1.00	1.00	0.00	0.00	0.62	0.00	0.00	0.69	5.88
time (sec)	N/A	0.183	0.020	0.000	0.000	0.069	0.000	0.000	0.178	26.227

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	79	0	0	0	0	0	19	71
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.24	0.89
time (sec)	N/A	0.239	0.119	0.000	0.000	0.000	0.000	0.000	0.167	27.764

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	82	0	0	0	0	0	23	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.246	0.131	0.000	0.000	0.000	0.000	0.000	0.171	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	85	0	0	0	0	0	23	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.238	0.084	0.000	0.000	0.000	0.000	0.000	0.211	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	85	0	0	0	0	0	26	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.244	0.105	0.000	0.000	0.000	0.000	0.000	0.171	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	69	55	87	77	70	2040	248	23	132
N.S.	1	0.93	0.74	1.18	1.04	0.95	27.57	3.35	0.31	1.78
time (sec)	N/A	0.260	0.361	5.835	0.038	0.099	4.316	0.135	0.184	26.982

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	48	48	50	53	46	525	118	23	62
N.S.	1	0.96	0.96	1.00	1.06	0.92	10.50	2.36	0.46	1.24
time (sec)	N/A	0.244	0.097	3.721	0.036	0.090	1.212	0.123	0.169	26.265

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	25	25	24	24	56	24	26	25
N.S.	1	1.00	1.04	1.04	1.00	1.00	2.33	1.00	1.08	1.04
time (sec)	N/A	0.193	0.011	8.793	0.032	0.084	0.344	0.124	0.232	25.847

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	51	0	0	0	0	0	21	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.221	0.027	0.000	0.000	0.000	0.000	0.000	0.256	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	51	0	0	0	0	0	23	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.229	0.031	0.000	0.000	0.000	0.000	0.000	0.179	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	63	0	0	0	0	0	23	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.236	0.063	0.000	0.000	0.000	0.000	0.000	0.178	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	63	0	0	0	0	0	23	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.243	0.057	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	63	0	0	0	0	0	14	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.206	0.048	0.000	0.000	0.000	0.000	0.000	0.167	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	63	0	0	0	0	0	23	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.225	0.056	0.000	0.000	0.000	0.000	0.000	0.174	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	63	0	0	0	0	0	23	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.224	0.052	0.000	0.000	0.000	0.000	0.000	0.178	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	78	0	0	0	0	0	31	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	0.241	0.159	0.000	0.000	0.000	0.000	0.000	0.191	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	0	0	0	0	0	24	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.235	0.077	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	0	0	0	0	0	35	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.234	0.081	0.000	0.000	0.000	0.000	0.000	0.169	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	78	0	0	0	0	0	35	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.244	0.103	0.000	0.000	0.000	0.000	0.000	0.175	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	78	0	0	0	0	0	35	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.238	0.100	0.000	0.000	0.000	0.000	0.000	0.173	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	70	83	87	78	84	2451	249	23	132
N.S.	1	0.92	1.09	1.14	1.03	1.11	32.25	3.28	0.30	1.74
time (sec)	N/A	0.253	0.696	6.385	0.038	0.082	4.448	0.133	0.190	26.891

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	49	50	52	52	50	688	117	23	65
N.S.	1	0.98	1.00	1.04	1.04	1.00	13.76	2.34	0.46	1.30
time (sec)	N/A	0.242	0.399	3.116	0.036	0.102	1.251	0.127	0.241	0.510

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	26	26	25	25	60	25	27	26
N.S.	1	1.00	1.04	1.04	1.00	1.00	2.40	1.00	1.08	1.04
time (sec)	N/A	0.195	0.013	6.727	0.026	0.080	0.346	0.117	0.156	0.182

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	52	0	0	0	0	0	21	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.220	0.305	0.000	0.000	0.000	0.000	0.000	0.163	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	154	0	0	0	0	0	23	0
N.S.	1	1.00	3.14	0.00	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.232	1.525	0.000	0.000	0.000	0.000	0.000	0.196	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	244	0	0	0	0	0	23	0
N.S.	1	1.00	4.98	0.00	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.227	2.086	0.000	0.000	0.000	0.000	0.000	0.172	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	68	0	0	0	0	0	23	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.231	0.496	0.000	0.000	0.000	0.000	0.000	0.176	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	68	0	0	0	0	0	23	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.226	0.318	0.000	0.000	0.000	0.000	0.000	0.169	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	64	0	0	0	0	0	14	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.204	0.056	0.000	0.000	0.000	0.000	0.000	0.173	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	80	0	0	0	0	0	23	0
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.225	0.583	0.000	0.000	0.000	0.000	0.000	0.179	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	82	0	0	0	0	0	23	0
N.S.	1	1.00	1.19	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.224	0.523	0.000	0.000	0.000	0.000	0.000	0.191	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	158	0	0	0	0	0	35	0
N.S.	1	1.00	2.08	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.242	0.962	0.000	0.000	0.000	0.000	0.000	0.197	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	0	0	0	0	0	31	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	0.237	0.578	0.000	0.000	0.000	0.000	0.000	0.185	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	82	0	0	0	0	0	24	0
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.232	0.498	0.000	0.000	0.000	0.000	0.000	0.165	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	82	0	0	0	0	0	35	0
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.233	0.658	0.000	0.000	0.000	0.000	0.000	0.174	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	79	0	0	0	0	0	35	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.243	0.647	0.000	0.000	0.000	0.000	0.000	0.167	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	78	58	265	63	56	0	100	21	0
N.S.	1	0.92	0.68	3.12	0.74	0.66	0.00	1.18	0.25	0.00
time (sec)	N/A	0.252	0.921	12.404	0.036	0.100	0.000	0.134	0.175	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	58	48	253	50	46	0	77	21	0
N.S.	1	0.92	0.76	4.02	0.79	0.73	0.00	1.22	0.33	0.00
time (sec)	N/A	0.241	0.548	1.174	0.034	0.078	0.000	0.134	0.175	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	40	36	243	35	34	0	53	21	0
N.S.	1	0.98	0.88	5.93	0.85	0.83	0.00	1.29	0.51	0.00
time (sec)	N/A	0.234	0.482	1.418	0.035	0.075	0.000	0.133	0.167	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	23	23	0	22	19	23
N.S.	1	1.00	1.00	0.94	1.28	1.28	0.00	1.22	1.06	1.28
time (sec)	N/A	0.197	0.196	1.631	0.032	0.094	0.000	0.133	0.180	0.219

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	51	73	151	72	265	0	61	19	0
N.S.	1	0.88	1.26	2.60	1.24	4.57	0.00	1.05	0.33	0.00
time (sec)	N/A	0.234	0.474	2.838	0.113	0.141	0.000	0.134	0.190	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	90	95	462	106	373	0	98	21	0
N.S.	1	0.97	1.02	4.97	1.14	4.01	0.00	1.05	0.23	0.00
time (sec)	N/A	0.267	0.920	3.157	0.112	0.144	0.000	0.133	0.170	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	131	107	309	138	457	0	127	21	0
N.S.	1	1.07	0.87	2.51	1.12	3.72	0.00	1.03	0.17	0.00
time (sec)	N/A	0.481	1.251	2.805	0.112	0.144	0.000	0.134	0.175	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	133	73	117	0	106	0	0	21	0
N.S.	1	1.08	0.59	0.95	0.00	0.86	0.00	0.00	0.17	0.00
time (sec)	N/A	1.076	0.554	12.168	0.000	0.093	0.000	0.000	0.173	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	100	61	105	0	94	0	0	21	0
N.S.	1	1.05	0.64	1.11	0.00	0.99	0.00	0.00	0.22	0.00
time (sec)	N/A	0.768	0.443	4.645	0.000	0.111	0.000	0.000	0.245	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	51	96	0	83	0	0	21	0
N.S.	1	1.00	0.76	1.43	0.00	1.24	0.00	0.00	0.31	0.00
time (sec)	N/A	0.559	0.371	2.477	0.000	0.107	0.000	0.000	0.266	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	77	0	57	0	0	12	35
N.S.	1	1.00	1.00	2.03	0.00	1.50	0.00	0.00	0.32	0.92
time (sec)	N/A	0.392	0.060	2.174	0.000	0.087	0.000	0.000	0.171	0.168

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	47	87	0	98	0	0	21	0
N.S.	1	1.00	0.76	1.40	0.00	1.58	0.00	0.00	0.34	0.00
time (sec)	N/A	0.565	0.463	3.028	0.000	0.089	0.000	0.000	0.174	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	63	105	0	148	0	0	21	0
N.S.	1	1.00	0.66	1.11	0.00	1.56	0.00	0.00	0.22	0.00
time (sec)	N/A	0.780	0.561	3.582	0.000	0.094	0.000	0.000	0.194	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	128	73	117	0	187	0	0	21	0
N.S.	1	1.04	0.59	0.95	0.00	1.52	0.00	0.00	0.17	0.00
time (sec)	N/A	1.037	0.972	4.845	0.000	0.131	0.000	0.000	0.185	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	76	52	255	72	54	0	98	28	0
N.S.	1	0.92	0.63	3.07	0.87	0.65	0.00	1.18	0.34	0.00
time (sec)	N/A	0.416	0.648	6.644	0.035	0.093	0.000	0.138	0.207	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	58	42	247	55	43	0	75	28	0
N.S.	1	0.92	0.67	3.92	0.87	0.68	0.00	1.19	0.44	0.00
time (sec)	N/A	0.405	0.412	1.372	0.035	0.079	0.000	0.141	0.194	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	40	30	236	37	31	0	46	28	0
N.S.	1	0.98	0.73	5.76	0.90	0.76	0.00	1.12	0.68	0.00
time (sec)	N/A	0.396	0.375	1.302	0.036	0.108	0.000	0.132	0.199	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	23	18	0	25	26	18
N.S.	1	1.00	1.00	0.94	1.28	1.00	0.00	1.39	1.44	1.00
time (sec)	N/A	0.319	0.179	0.465	0.034	0.092	0.000	0.133	0.200	25.890

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	71	85	208	87	298	0	75	26	0
N.S.	1	0.92	1.10	2.70	1.13	3.87	0.00	0.97	0.34	0.00
time (sec)	N/A	0.415	0.528	2.175	0.117	0.121	0.000	0.134	0.186	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-1)	A	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	110	97	295	123	407	0	127	28	0
N.S.	1	0.97	0.86	2.61	1.09	3.60	0.00	1.12	0.25	0.00
time (sec)	N/A	0.459	1.111	2.335	0.120	0.147	0.000	0.136	0.200	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	132	70	229	0	108	0	0	28	0
N.S.	1	1.03	0.55	1.79	0.00	0.84	0.00	0.00	0.22	0.00
time (sec)	N/A	1.137	0.687	7.043	0.000	0.100	0.000	0.000	0.183	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	99	60	213	0	98	0	0	28	0
N.S.	1	1.01	0.61	2.17	0.00	1.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.778	0.484	4.794	0.000	0.093	0.000	0.000	0.183	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	48	193	0	85	0	0	28	0
N.S.	1	1.00	0.73	2.92	0.00	1.29	0.00	0.00	0.42	0.00
time (sec)	N/A	0.574	0.349	2.537	0.000	0.093	0.000	0.000	0.204	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	48	184	0	84	0	0	20	0
N.S.	1	1.00	0.73	2.79	0.00	1.27	0.00	0.00	0.30	0.00
time (sec)	N/A	0.533	0.086	2.197	0.000	0.121	0.000	0.000	0.181	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	95	57	159	0	113	0	0	28	0
N.S.	1	1.06	0.63	1.77	0.00	1.26	0.00	0.00	0.31	0.00
time (sec)	N/A	0.746	0.498	3.041	0.000	0.088	0.000	0.000	0.203	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	128	77	169	0	167	0	0	28	0
N.S.	1	1.03	0.62	1.36	0.00	1.35	0.00	0.00	0.23	0.00
time (sec)	N/A	1.010	0.639	4.049	0.000	0.112	0.000	0.000	0.205	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	78	52	264	66	70	0	100	32	0
N.S.	1	0.92	0.61	3.11	0.78	0.82	0.00	1.18	0.38	0.00
time (sec)	N/A	0.412	1.098	5.460	0.037	0.101	0.000	0.138	0.203	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	58	42	254	52	57	0	74	32	0
N.S.	1	0.92	0.67	4.03	0.83	0.90	0.00	1.17	0.51	0.00
time (sec)	N/A	0.396	0.774	1.862	0.061	0.121	0.000	0.134	0.212	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	40	32	260	36	42	0	49	32	50
N.S.	1	0.98	0.78	6.34	0.88	1.02	0.00	1.20	0.78	1.22
time (sec)	N/A	0.393	0.529	50.892	0.035	0.099	0.000	0.128	0.197	26.267

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	23	28	0	33	30	39
N.S.	1	1.00	1.00	0.85	1.15	1.40	0.00	1.65	1.50	1.95
time (sec)	N/A	0.332	0.160	3.941	0.039	0.079	0.000	0.140	0.204	25.995

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-1)	A	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	73	87	193	87	347	0	86	30	0
N.S.	1	0.94	1.12	2.47	1.12	4.45	0.00	1.10	0.38	0.00
time (sec)	N/A	0.413	0.425	4.211	0.127	0.138	0.000	0.133	0.199	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-1)	A	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	112	109	312	123	467	0	121	32	0
N.S.	1	0.99	0.96	2.76	1.09	4.13	0.00	1.07	0.28	0.00
time (sec)	N/A	0.471	2.583	19.529	0.111	0.212	0.000	0.140	0.219	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-1)	A	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	153	119	346	155	561	0	150	32	0
N.S.	1	1.07	0.83	2.42	1.08	3.92	0.00	1.05	0.22	0.00
time (sec)	N/A	0.315	1.739	129.931	0.112	0.134	0.000	0.141	0.199	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	136	74	112	0	129	0	0	32	0
N.S.	1	1.05	0.57	0.86	0.00	0.99	0.00	0.00	0.25	0.00
time (sec)	N/A	0.619	0.692	1012.292	0.000	0.114	0.000	0.000	0.210	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	101	64	104	0	114	0	0	32	0
N.S.	1	1.01	0.64	1.04	0.00	1.14	0.00	0.00	0.32	0.00
time (sec)	N/A	0.483	0.481	151.632	0.000	0.130	0.000	0.000	0.196	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	52	90	0	101	0	0	32	0
N.S.	1	1.00	0.74	1.29	0.00	1.44	0.00	0.00	0.46	0.00
time (sec)	N/A	0.348	0.417	14.858	0.000	0.121	0.000	0.000	0.200	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	51	90	0	101	0	0	24	0
N.S.	1	1.00	0.73	1.29	0.00	1.44	0.00	0.00	0.34	0.00
time (sec)	N/A	0.326	0.114	2.608	0.000	0.085	0.000	0.000	0.189	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	96	67	104	0	131	0	0	32	0
N.S.	1	0.98	0.68	1.06	0.00	1.34	0.00	0.00	0.33	0.00
time (sec)	N/A	0.473	0.580	8.360	0.000	0.123	0.000	0.000	0.201	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	129	79	124	0	193	0	0	32	0
N.S.	1	1.05	0.64	1.01	0.00	1.57	0.00	0.00	0.26	0.00
time (sec)	N/A	0.612	0.790	53.494	0.000	0.106	0.000	0.000	0.200	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	80	52	54	63	61	0	108	32	0
N.S.	1	0.92	0.60	0.62	0.72	0.70	0.00	1.24	0.37	0.00
time (sec)	N/A	0.248	0.636	3.459	0.032	0.092	0.000	0.149	0.171	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	60	42	44	50	51	0	85	32	0
N.S.	1	0.92	0.65	0.68	0.77	0.78	0.00	1.31	0.49	0.00
time (sec)	N/A	0.242	0.438	1.835	0.034	0.089	0.000	0.143	0.179	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	42	32	34	37	41	0	62	32	0
N.S.	1	0.98	0.74	0.79	0.86	0.95	0.00	1.44	0.74	0.00
time (sec)	N/A	0.235	0.392	1.669	0.035	0.113	0.000	0.134	0.178	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	23	28	0	33	30	28
N.S.	1	1.00	1.00	0.85	1.15	1.40	0.00	1.65	1.50	1.40
time (sec)	N/A	0.192	0.173	0.636	0.029	0.114	0.000	0.133	0.166	25.770

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	51	73	142	73	273	0	56	30	0
N.S.	1	0.86	1.24	2.41	1.24	4.63	0.00	0.95	0.51	0.00
time (sec)	N/A	0.230	0.303	1.654	0.111	0.117	0.000	0.132	0.180	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	90	93	281	105	381	0	104	32	0
N.S.	1	0.97	1.00	3.02	1.13	4.10	0.00	1.12	0.34	0.00
time (sec)	N/A	0.264	1.020	1.779	0.115	0.133	0.000	0.142	0.167	0.000

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	131	107	217	138	469	0	135	32	0
N.S.	1	1.07	0.87	1.76	1.12	3.81	0.00	1.10	0.26	0.00
time (sec)	N/A	0.475	1.052	1.865	0.115	0.162	0.000	0.145	0.173	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	133	73	232	0	117	0	0	32	0
N.S.	1	1.08	0.59	1.89	0.00	0.95	0.00	0.00	0.26	0.00
time (sec)	N/A	1.074	0.819	10.229	0.000	0.092	0.000	0.000	0.180	0.000

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	100	63	212	0	107	0	0	32	0
N.S.	1	1.05	0.66	2.23	0.00	1.13	0.00	0.00	0.34	0.00
time (sec)	N/A	0.770	0.658	5.095	0.000	0.087	0.000	0.000	0.182	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	60	192	0	94	0	0	32	0
N.S.	1	1.00	0.90	2.87	0.00	1.40	0.00	0.00	0.48	0.00
time (sec)	N/A	0.566	0.390	3.664	0.000	0.084	0.000	0.000	0.188	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	175	0	66	0	0	24	0
N.S.	1	1.00	1.00	4.61	0.00	1.74	0.00	0.00	0.63	0.00
time (sec)	N/A	0.390	0.067	2.298	0.000	0.089	0.000	0.000	0.172	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	48	152	0	109	0	0	32	0
N.S.	1	1.00	0.76	2.41	0.00	1.73	0.00	0.00	0.51	0.00
time (sec)	N/A	0.560	0.466	2.094	0.000	0.109	0.000	0.000	0.185	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	96	74	170	0	158	0	0	32	0
N.S.	1	1.01	0.78	1.79	0.00	1.66	0.00	0.00	0.34	0.00
time (sec)	N/A	0.779	0.472	2.553	0.000	0.095	0.000	0.000	0.201	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	129	86	186	0	199	0	0	32	0
N.S.	1	1.05	0.70	1.51	0.00	1.62	0.00	0.00	0.26	0.00
time (sec)	N/A	1.024	0.521	3.581	0.000	0.108	0.000	0.000	0.249	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	80	52	59	63	61	0	110	32	0
N.S.	1	0.92	0.60	0.68	0.72	0.70	0.00	1.26	0.37	0.00
time (sec)	N/A	0.411	0.924	2.701	0.034	0.095	0.000	0.149	0.246	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	60	42	49	50	51	0	87	32	0
N.S.	1	0.92	0.65	0.75	0.77	0.78	0.00	1.34	0.49	0.00
time (sec)	N/A	0.408	0.551	1.947	0.031	0.099	0.000	0.150	0.192	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	42	32	39	37	41	0	64	32	0
N.S.	1	0.98	0.74	0.91	0.86	0.95	0.00	1.49	0.74	0.00
time (sec)	N/A	0.395	0.459	1.658	0.034	0.099	0.000	0.139	0.236	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	23	28	0	35	30	28
N.S.	1	1.00	1.00	0.85	1.15	1.40	0.00	1.75	1.50	1.40
time (sec)	N/A	0.327	0.178	0.721	0.030	0.077	0.000	0.132	0.182	25.523

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	B	F	A	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	74	36	197	89	314	0	73	30	0
N.S.	1	0.95	0.46	2.53	1.14	4.03	0.00	0.94	0.38	0.00
time (sec)	N/A	0.417	0.531	1.556	0.116	0.216	0.000	0.138	0.175	0.000

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	96	98	277	106	384	0	93	32	0
N.S.	1	1.03	1.05	2.98	1.14	4.13	0.00	1.00	0.34	0.00
time (sec)	N/A	0.461	0.877	1.668	0.112	0.132	0.000	0.141	0.184	0.000

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	137	109	214	137	473	0	128	32	0
N.S.	1	1.11	0.89	1.74	1.11	3.85	0.00	1.04	0.26	0.00
time (sec)	N/A	0.492	0.961	1.685	0.110	0.157	0.000	0.139	0.199	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	136	81	114	0	109	0	0	32	0
N.S.	1	1.08	0.64	0.90	0.00	0.87	0.00	0.00	0.25	0.00
time (sec)	N/A	1.158	0.503	6.392	0.000	0.104	0.000	0.000	0.194	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	103	71	104	0	99	0	0	32	0
N.S.	1	1.05	0.72	1.06	0.00	1.01	0.00	0.00	0.33	0.00
time (sec)	N/A	0.755	0.351	4.350	0.000	0.096	0.000	0.000	0.266	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	59	92	0	87	0	0	24	0
N.S.	1	1.00	0.82	1.28	0.00	1.21	0.00	0.00	0.33	0.00
time (sec)	N/A	0.551	0.094	2.201	0.000	0.096	0.000	0.000	0.180	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	58	92	0	101	0	0	32	0
N.S.	1	1.00	0.85	1.35	0.00	1.49	0.00	0.00	0.47	0.00
time (sec)	N/A	0.586	0.440	2.354	0.000	0.095	0.000	0.000	0.189	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	100	62	106	0	148	0	0	32	0
N.S.	1	0.98	0.61	1.04	0.00	1.45	0.00	0.00	0.31	0.00
time (sec)	N/A	0.798	0.552	2.205	0.000	0.108	0.000	0.000	0.177	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	133	74	116	0	197	0	0	32	0
N.S.	1	1.01	0.56	0.88	0.00	1.49	0.00	0.00	0.24	0.00
time (sec)	N/A	1.023	0.863	2.894	0.000	0.110	0.000	0.000	0.182	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	80	62	59	63	61	0	110	32	0
N.S.	1	0.92	0.71	0.68	0.72	0.70	0.00	1.26	0.37	0.00
time (sec)	N/A	0.422	0.940	3.157	0.035	0.124	0.000	0.153	0.173	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	60	52	49	50	51	0	87	32	0
N.S.	1	0.92	0.80	0.75	0.77	0.78	0.00	1.34	0.49	0.00
time (sec)	N/A	0.406	0.562	1.893	0.035	0.098	0.000	0.145	0.185	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	42	42	39	37	41	0	64	32	0
N.S.	1	0.98	0.98	0.91	0.86	0.95	0.00	1.49	0.74	0.00
time (sec)	N/A	0.396	0.518	1.655	0.036	0.091	0.000	0.139	0.188	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	23	28	0	35	30	28
N.S.	1	1.00	1.00	0.85	1.15	1.40	0.00	1.75	1.50	1.40
time (sec)	N/A	0.331	0.215	0.444	0.028	0.078	0.000	0.132	0.204	26.061

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	76	90	186	90	319	0	81	30	0
N.S.	1	0.94	1.11	2.30	1.11	3.94	0.00	1.00	0.37	0.00
time (sec)	N/A	0.415	0.470	2.022	0.120	0.230	0.000	0.135	0.286	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	96	98	286	106	389	0	101	32	0
N.S.	1	1.03	1.05	3.08	1.14	4.18	0.00	1.09	0.34	0.00
time (sec)	N/A	0.453	1.101	2.049	0.114	0.149	0.000	0.141	0.177	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	137	110	220	136	477	0	131	32	0
N.S.	1	1.11	0.89	1.79	1.11	3.88	0.00	1.07	0.26	0.00
time (sec)	N/A	0.491	2.090	2.046	0.114	0.136	0.000	0.148	0.185	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	136	83	235	0	117	0	0	32	0
N.S.	1	1.08	0.66	1.87	0.00	0.93	0.00	0.00	0.25	0.00
time (sec)	N/A	1.123	0.927	8.742	0.000	0.104	0.000	0.000	0.197	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	103	66	215	0	107	0	0	32	0
N.S.	1	1.05	0.67	2.19	0.00	1.09	0.00	0.00	0.33	0.00
time (sec)	N/A	0.754	0.715	6.791	0.000	0.129	0.000	0.000	0.176	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	60	191	0	95	0	0	24	0
N.S.	1	1.00	0.83	2.65	0.00	1.32	0.00	0.00	0.33	0.00
time (sec)	N/A	0.552	0.123	3.998	0.000	0.086	0.000	0.000	0.169	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	51	165	0	109	0	0	32	0
N.S.	1	1.00	0.75	2.43	0.00	1.60	0.00	0.00	0.47	0.00
time (sec)	N/A	0.594	0.503	2.937	0.000	0.085	0.000	0.000	0.175	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	101	79	173	0	162	0	0	32	0
N.S.	1	0.99	0.77	1.70	0.00	1.59	0.00	0.00	0.31	0.00
time (sec)	N/A	0.793	0.587	2.612	0.000	0.086	0.000	0.000	0.196	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	134	87	189	0	203	0	0	32	0
N.S.	1	1.02	0.66	1.43	0.00	1.54	0.00	0.00	0.24	0.00
time (sec)	N/A	1.029	0.854	3.017	0.000	0.093	0.000	0.000	0.180	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	350	388	169	458	0	538	0	0	33	0
N.S.	1	1.11	0.48	1.31	0.00	1.54	0.00	0.00	0.09	0.00
time (sec)	N/A	1.441	3.293	51.645	0.000	0.172	0.000	0.000	0.191	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	315	347	157	438	0	522	0	0	33	0
N.S.	1	1.10	0.50	1.39	0.00	1.66	0.00	0.00	0.10	0.00
time (sec)	N/A	0.712	2.163	13.231	0.000	0.136	0.000	0.000	0.184	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	309	122	362	0	465	0	0	22	0
N.S.	1	1.12	0.44	1.32	0.00	1.69	0.00	0.00	0.08	0.00
time (sec)	N/A	0.556	1.524	3.515	0.000	0.159	0.000	0.000	0.181	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	37	35	0	44	0	0	33	36
N.S.	1	1.00	1.12	1.06	0.00	1.33	0.00	0.00	1.00	1.09
time (sec)	N/A	0.218	0.644	0.431	0.000	0.086	0.000	0.000	0.193	26.111

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	52	53	0	73	0	0	33	83
N.S.	1	1.00	0.73	0.75	0.00	1.03	0.00	0.00	0.46	1.17
time (sec)	N/A	0.331	0.883	0.487	0.000	0.087	0.000	0.000	0.169	27.197

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	114	65	65	0	96	0	0	33	169
N.S.	1	1.08	0.61	0.61	0.00	0.91	0.00	0.00	0.31	1.59
time (sec)	N/A	0.471	1.163	0.547	0.000	0.120	0.000	0.000	0.177	32.714

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	128	132	90	990	0	0	0	0	33	0
N.S.	1	1.03	0.70	7.73	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.680	17.784	4.835	0.000	0.000	0.000	0.000	0.193	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	66	133	0	0	0	0	29	0
N.S.	1	1.00	0.73	1.46	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.493	11.575	0.685	0.000	0.000	0.000	0.000	0.184	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	66	112	0	59	0	0	33	0
N.S.	1	1.00	1.25	2.11	0.00	1.11	0.00	0.00	0.62	0.00
time (sec)	N/A	0.359	0.926	0.372	0.000	0.093	0.000	0.000	0.187	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	75	126	0	125	0	0	33	0
N.S.	1	1.00	0.79	1.33	0.00	1.32	0.00	0.00	0.35	0.00
time (sec)	N/A	0.499	1.101	0.503	0.000	0.102	0.000	0.000	0.165	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	138	111	146	0	173	0	0	33	0
N.S.	1	1.06	0.85	1.12	0.00	1.33	0.00	0.00	0.25	0.00
time (sec)	N/A	1.098	2.071	0.609	0.000	0.100	0.000	0.000	0.179	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	120	86	234	0	0	0	0	39	0
N.S.	1	1.04	0.75	2.03	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	1.016	11.080	0.557	0.000	0.000	0.000	0.000	0.175	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	74	224	0	0	0	0	39	0
N.S.	1	1.00	0.87	2.64	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.769	0.645	0.370	0.000	0.000	0.000	0.000	0.178	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	60	214	0	0	0	0	31	0
N.S.	1	1.00	1.18	4.20	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	0.566	1.819	0.347	0.000	0.000	0.000	0.000	0.218	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	63	361	0	171	0	0	39	0
N.S.	1	1.00	0.78	4.46	0.00	2.11	0.00	0.00	0.48	0.00
time (sec)	N/A	0.771	0.579	0.238	0.000	0.103	0.000	0.000	0.171	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	116	82	248	0	239	0	0	39	0
N.S.	1	1.01	0.71	2.16	0.00	2.08	0.00	0.00	0.34	0.00
time (sec)	N/A	0.976	0.847	0.319	0.000	0.126	0.000	0.000	0.177	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	276	287	145	451	0	417	0	0	37	0
N.S.	1	1.04	0.53	1.63	0.00	1.51	0.00	0.00	0.13	0.00
time (sec)	N/A	1.019	2.110	0.436	0.000	0.136	0.000	0.000	0.176	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	254	113	347	0	386	0	0	39	0
N.S.	1	1.07	0.47	1.46	0.00	1.62	0.00	0.00	0.16	0.00
time (sec)	N/A	0.797	1.336	2.235	0.000	0.133	0.000	0.000	0.173	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	30	0	48	0	0	39	57
N.S.	1	1.00	1.00	1.00	0.00	1.60	0.00	0.00	1.30	1.90
time (sec)	N/A	0.328	0.413	0.240	0.000	0.098	0.000	0.000	0.202	26.152

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	42	43	0	72	0	0	39	103
N.S.	1	1.00	0.69	0.70	0.00	1.18	0.00	0.00	0.64	1.69
time (sec)	N/A	0.489	0.491	0.248	0.000	0.108	0.000	0.000	0.175	27.942

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	96	52	53	0	95	0	0	39	163
N.S.	1	1.05	0.57	0.58	0.00	1.04	0.00	0.00	0.43	1.79
time (sec)	N/A	0.672	0.776	0.346	0.000	0.144	0.000	0.000	0.183	33.282

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	131	62	63	0	116	0	0	39	192
N.S.	1	1.08	0.51	0.52	0.00	0.96	0.00	0.00	0.32	1.59
time (sec)	N/A	0.885	1.154	0.448	0.000	0.188	0.000	0.000	0.173	32.687

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	433	176	449	0	559	0	0	44	0
N.S.	1	1.11	0.45	1.15	0.00	1.43	0.00	0.00	0.11	0.00
time (sec)	N/A	1.786	2.913	1.222	0.000	0.196	0.000	0.000	0.179	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	354	392	165	454	0	546	0	0	44	0
N.S.	1	1.11	0.47	1.28	0.00	1.54	0.00	0.00	0.12	0.00
time (sec)	N/A	1.498	2.312	4.665	0.000	0.138	0.000	0.000	0.209	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	319	351	151	430	0	509	0	0	33	0
N.S.	1	1.10	0.47	1.35	0.00	1.60	0.00	0.00	0.10	0.00
time (sec)	N/A	1.196	1.697	3.434	0.000	0.149	0.000	0.000	0.183	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	349	144	457	0	545	0	0	44	0
N.S.	1	1.13	0.47	1.48	0.00	1.76	0.00	0.00	0.14	0.00
time (sec)	N/A	1.223	2.046	3.209	0.000	0.146	0.000	0.000	0.189	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	45	40	0	68	0	0	44	84
N.S.	1	1.00	1.29	1.14	0.00	1.94	0.00	0.00	1.26	2.40
time (sec)	N/A	0.360	0.882	0.653	0.000	0.096	0.000	0.000	0.180	27.255

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	177	103	157	0	0	0	0	44	0
N.S.	1	1.03	0.60	0.91	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	1.444	1.636	2.643	0.000	0.000	0.000	0.000	0.181	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	135	136	87	996	0	0	0	0	40	0
N.S.	1	1.01	0.64	7.38	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	1.120	0.830	2.407	0.000	0.000	0.000	0.000	0.187	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	84	126	0	0	0	0	44	0
N.S.	1	1.00	0.89	1.34	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.851	1.185	1.768	0.000	0.000	0.000	0.000	0.185	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	78	129	0	131	0	0	44	0
N.S.	1	1.00	0.78	1.29	0.00	1.31	0.00	0.00	0.44	0.00
time (sec)	N/A	0.875	1.121	0.827	0.000	0.121	0.000	0.000	0.178	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	143	119	144	0	178	0	0	44	0
N.S.	1	1.04	0.87	1.05	0.00	1.30	0.00	0.00	0.32	0.00
time (sec)	N/A	1.139	1.926	0.962	0.000	0.133	0.000	0.000	0.174	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	186	131	154	0	230	0	0	44	0
N.S.	1	1.07	0.75	0.89	0.00	1.32	0.00	0.00	0.25	0.00
time (sec)	N/A	1.448	2.468	1.154	0.000	0.126	0.000	0.000	0.178	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	96	0	0	0	0	0	35	0
N.S.	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.557	9.174	0.000	0.000	0.000	0.000	0.000	0.209	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	96	0	0	0	0	0	31	0
N.S.	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	0.570	8.860	0.000	0.000	0.000	0.000	0.000	0.247	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	106	0	0	0	0	0	24	0
N.S.	1	1.00	1.38	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.554	8.589	0.000	0.000	0.000	0.000	0.000	0.285	0.000

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	89	0	0	0	0	0	35	0
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.531	28.755	0.000	0.000	0.000	0.000	0.000	0.173	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	116	0	0	0	0	0	35	0
N.S.	1	1.00	1.51	0.00	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.587	11.697	0.000	0.000	0.000	0.000	0.000	0.187	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	285	0	0	0	0	0	19	0
N.S.	1	1.00	3.31	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.541	2.021	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	287	0	0	0	0	0	23	0
N.S.	1	1.00	3.22	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.522	0.561	0.000	0.000	0.000	0.000	0.000	0.180	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	287	0	0	0	0	0	23	0
N.S.	1	1.00	3.22	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.570	0.582	0.000	0.000	0.000	0.000	0.000	0.186	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	289	0	0	0	0	0	26	0
N.S.	1	1.00	3.14	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.533	0.676	0.000	0.000	0.000	0.000	0.000	0.188	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	75	80	87	85	85	0	0	23	134
N.S.	1	0.94	1.00	1.09	1.06	1.06	0.00	0.00	0.29	1.68
time (sec)	N/A	0.441	0.737	2.425	0.040	0.117	0.000	0.000	0.181	26.949

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	51	47	52	59	53	0	0	23	67
N.S.	1	0.98	0.90	1.00	1.13	1.02	0.00	0.00	0.44	1.29
time (sec)	N/A	0.402	0.424	0.983	0.041	0.087	0.000	0.000	0.164	26.300

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	22	26	28	28	0	0	21	27
N.S.	1	1.00	0.88	1.04	1.12	1.12	0.00	0.00	0.84	1.08
time (sec)	N/A	0.331	0.103	0.818	0.034	0.120	0.000	0.000	0.179	0.204

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	50	0	0	0	0	0	21	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.351	0.350	0.000	0.000	0.000	0.000	0.000	0.165	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	201	0	0	0	0	0	23	0
N.S.	1	1.00	4.19	0.00	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.369	3.457	0.000	0.000	0.000	0.000	0.000	0.172	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	63	0	0	0	0	0	23	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.514	0.765	0.000	0.000	0.000	0.000	0.000	0.176	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	63	0	0	0	0	0	23	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.514	0.489	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	63	0	0	0	0	0	23	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.512	0.409	0.000	0.000	0.000	0.000	0.000	0.175	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	61	0	0	0	0	0	14	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.471	0.054	0.000	0.000	0.000	0.000	0.000	0.160	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	57	0	0	0	0	0	23	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.505	0.476	0.000	0.000	0.000	0.000	0.000	0.167	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	63	0	0	0	0	0	23	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.511	0.448	0.000	0.000	0.000	0.000	0.000	0.200	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	104	0	0	0	0	0	31	0
N.S.	1	1.00	1.37	0.00	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	0.613	33.014	0.000	0.000	0.000	0.000	0.000	0.208	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	75	0	0	0	0	0	24	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.531	10.601	0.000	0.000	0.000	0.000	0.000	0.206	0.000

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	72	0	0	0	0	0	35	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.541	10.668	0.000	0.000	0.000	0.000	0.000	0.187	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	73	0	0	0	0	0	35	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.557	10.773	0.000	0.000	0.000	0.000	0.000	0.259	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	117	67	153	0	95	0	0	21	0
N.S.	1	1.17	0.67	1.53	0.00	0.95	0.00	0.00	0.21	0.00
time (sec)	N/A	0.841	0.424	1.003	0.000	0.116	0.000	0.000	0.176	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	81	62	248	0	93	0	0	21	0
N.S.	1	1.08	0.83	3.31	0.00	1.24	0.00	0.00	0.28	0.00
time (sec)	N/A	0.646	0.290	0.950	0.000	0.097	0.000	0.000	0.169	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	81	55	141	0	82	0	0	21	0
N.S.	1	1.12	0.76	1.96	0.00	1.14	0.00	0.00	0.29	0.00
time (sec)	N/A	0.608	0.290	0.776	0.000	0.116	0.000	0.000	0.175	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	43	237	0	59	0	0	19	0
N.S.	1	1.00	0.98	5.39	0.00	1.34	0.00	0.00	0.43	0.00
time (sec)	N/A	0.430	0.079	0.891	0.000	0.112	0.000	0.000	0.169	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	42	120	0	55	0	0	12	63
N.S.	1	1.00	0.98	2.79	0.00	1.28	0.00	0.00	0.28	1.47
time (sec)	N/A	0.379	0.071	0.648	0.000	0.121	0.000	0.000	0.204	25.560

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	75	57	155	0	80	0	0	19	0
N.S.	1	1.10	0.84	2.28	0.00	1.18	0.00	0.00	0.28	0.00
time (sec)	N/A	0.583	0.177	0.719	0.000	0.123	0.000	0.000	0.166	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	79	55	135	0	96	0	0	21	0
N.S.	1	1.07	0.74	1.82	0.00	1.30	0.00	0.00	0.28	0.00
time (sec)	N/A	0.577	0.231	0.751	0.000	0.095	0.000	0.000	0.168	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	109	68	246	0	128	0	0	21	0
N.S.	1	1.09	0.68	2.46	0.00	1.28	0.00	0.00	0.21	0.00
time (sec)	N/A	0.766	0.269	0.841	0.000	0.109	0.000	0.000	0.160	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	117	68	154	0	99	0	0	28	0
N.S.	1	1.14	0.66	1.50	0.00	0.96	0.00	0.00	0.27	0.00
time (sec)	N/A	0.812	0.366	1.006	0.000	0.095	0.000	0.000	0.170	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	81	62	249	0	98	0	0	28	0
N.S.	1	1.05	0.81	3.23	0.00	1.27	0.00	0.00	0.36	0.00
time (sec)	N/A	0.608	0.424	0.956	0.000	0.090	0.000	0.000	0.181	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	81	56	141	0	85	0	0	28	0
N.S.	1	1.08	0.75	1.88	0.00	1.13	0.00	0.00	0.37	0.00
time (sec)	N/A	0.613	0.178	0.800	0.000	0.094	0.000	0.000	0.182	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	45	238	0	61	0	0	28	0
N.S.	1	1.00	0.98	5.17	0.00	1.33	0.00	0.00	0.61	0.00
time (sec)	N/A	0.441	0.018	0.809	0.000	0.096	0.000	0.000	0.170	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	43	121	0	57	0	0	26	0
N.S.	1	1.00	0.98	2.75	0.00	1.30	0.00	0.00	0.59	0.00
time (sec)	N/A	0.418	0.010	0.674	0.000	0.118	0.000	0.000	0.172	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	54	156	0	83	0	0	20	0
N.S.	1	1.00	0.76	2.20	0.00	1.17	0.00	0.00	0.28	0.00
time (sec)	N/A	0.518	0.012	0.726	0.000	0.114	0.000	0.000	0.164	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	79	58	136	0	99	0	0	22	0
N.S.	1	1.10	0.81	1.89	0.00	1.38	0.00	0.00	0.31	0.00
time (sec)	N/A	0.571	0.222	0.770	0.000	0.081	0.000	0.000	0.194	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	109	68	247	0	138	0	0	22	0
N.S.	1	1.06	0.66	2.40	0.00	1.34	0.00	0.00	0.21	0.00
time (sec)	N/A	0.758	0.419	0.817	0.000	0.088	0.000	0.000	0.245	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	117	70	155	0	98	0	0	32	0
N.S.	1	1.15	0.69	1.52	0.00	0.96	0.00	0.00	0.31	0.00
time (sec)	N/A	0.790	0.292	1.032	0.000	0.123	0.000	0.000	0.168	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	81	57	254	0	96	0	0	32	0
N.S.	1	1.12	0.79	3.53	0.00	1.33	0.00	0.00	0.44	0.00
time (sec)	N/A	0.602	0.218	0.918	0.000	0.110	0.000	0.000	0.159	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	79	64	139	0	85	0	0	30	0
N.S.	1	1.07	0.86	1.88	0.00	1.15	0.00	0.00	0.41	0.00
time (sec)	N/A	0.583	0.103	0.757	0.000	0.091	0.000	0.000	0.166	0.000

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	42	243	0	62	0	0	24	0
N.S.	1	1.00	0.98	5.65	0.00	1.44	0.00	0.00	0.56	0.00
time (sec)	N/A	0.371	0.023	0.793	0.000	0.086	0.000	0.000	0.188	0.000

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	45	125	0	58	0	0	15	0
N.S.	1	1.00	0.98	2.72	0.00	1.26	0.00	0.00	0.33	0.00
time (sec)	N/A	0.411	0.010	0.727	0.000	0.101	0.000	0.000	0.159	0.000

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	75	52	161	0	83	0	0	22	0
N.S.	1	1.07	0.74	2.30	0.00	1.19	0.00	0.00	0.31	0.00
time (sec)	N/A	0.581	0.271	0.729	0.000	0.089	0.000	0.000	0.174	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	79	60	145	0	99	0	0	24	0
N.S.	1	1.03	0.78	1.88	0.00	1.29	0.00	0.00	0.31	0.00
time (sec)	N/A	0.572	0.169	0.751	0.000	0.092	0.000	0.000	0.164	0.000

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	117	70	158	0	98	0	0	32	0
N.S.	1	1.14	0.68	1.53	0.00	0.95	0.00	0.00	0.31	0.00
time (sec)	N/A	0.789	0.195	1.023	0.000	0.088	0.000	0.000	0.163	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	79	60	257	0	96	0	0	30	0
N.S.	1	1.07	0.81	3.47	0.00	1.30	0.00	0.00	0.41	0.00
time (sec)	N/A	0.593	0.131	0.926	0.000	0.112	0.000	0.000	0.172	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	63	142	0	85	0	0	24	0
N.S.	1	1.00	0.82	1.84	0.00	1.10	0.00	0.00	0.31	0.00
time (sec)	N/A	0.542	0.012	0.759	0.000	0.111	0.000	0.000	0.168	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	45	246	0	62	0	0	24	0
N.S.	1	1.00	0.98	5.35	0.00	1.35	0.00	0.00	0.52	0.00
time (sec)	N/A	0.420	0.020	0.779	0.000	0.078	0.000	0.000	0.162	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	45	128	0	58	0	0	15	0
N.S.	1	1.00	0.98	2.78	0.00	1.26	0.00	0.00	0.33	0.00
time (sec)	N/A	0.426	0.011	0.747	0.000	0.090	0.000	0.000	0.175	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	75	55	167	0	83	0	0	22	0
N.S.	1	1.03	0.75	2.29	0.00	1.14	0.00	0.00	0.30	0.00
time (sec)	N/A	0.573	0.228	0.749	0.000	0.124	0.000	0.000	0.167	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	79	60	148	0	99	0	0	24	0
N.S.	1	1.03	0.78	1.92	0.00	1.29	0.00	0.00	0.31	0.00
time (sec)	N/A	0.583	0.228	0.753	0.000	0.092	0.000	0.000	0.182	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	109	73	272	0	134	0	0	24	0
N.S.	1	1.04	0.70	2.59	0.00	1.28	0.00	0.00	0.23	0.00
time (sec)	N/A	0.797	0.340	0.844	0.000	0.090	0.000	0.000	0.181	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	102	0	0	0	0	0	26	0
N.S.	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.500	12.037	0.000	0.000	0.000	0.000	0.000	0.167	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [175] had the largest ratio of [1.37500000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	6	0.333
2	A	3	3	1.00	8	0.375
3	A	4	3	0.89	8	0.375
4	A	5	5	1.11	8	0.625
5	A	4	3	0.86	8	0.375
6	A	7	7	1.15	8	0.875
7	A	4	3	0.85	8	0.375
8	A	9	9	1.17	8	1.125
9	A	6	6	1.08	8	0.750
10	A	4	4	1.00	8	0.500
11	A	4	4	1.00	8	0.500
12	A	2	2	1.00	8	0.250
13	A	2	2	1.00	8	0.250
14	A	4	4	1.00	8	0.500
15	A	4	4	1.00	8	0.500
16	A	6	6	1.02	8	0.750
17	A	6	6	1.07	10	0.600
18	A	4	4	1.00	10	0.400
19	A	4	4	1.00	10	0.400
20	A	2	2	1.00	10	0.200
21	A	2	2	1.00	10	0.200

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	4	4	1.00	10	0.400
23	A	4	4	1.00	10	0.400
24	A	6	6	1.01	10	0.600
25	A	8	8	1.06	12	0.667
26	A	6	6	1.00	12	0.500
27	A	6	6	1.00	12	0.500
28	A	4	4	1.00	12	0.333
29	A	4	4	1.00	12	0.333
30	A	6	6	1.00	12	0.500
31	A	6	6	1.00	12	0.500
32	A	8	8	1.04	12	0.667
33	A	2	2	1.00	12	0.167
34	A	2	2	1.00	12	0.167
35	C	2	2	0.11	12	0.167
36	C	2	2	0.23	12	0.167
37	C	2	2	0.21	12	0.167
38	A	2	2	1.00	12	0.167
39	A	2	2	1.00	8	0.250
40	A	2	2	1.00	10	0.200
41	A	3	3	1.00	21	0.143
42	A	4	3	1.00	15	0.200
43	A	4	3	1.00	15	0.200
44	A	4	3	1.00	13	0.231
45	A	2	2	1.00	6	0.333
46	A	4	3	1.00	13	0.231
47	A	4	3	1.00	15	0.200
48	A	4	3	1.00	15	0.200
49	A	5	4	0.87	17	0.235
50	A	5	4	0.89	17	0.235
51	A	5	4	0.94	17	0.235
52	A	4	3	1.00	15	0.200
53	A	3	3	1.00	8	0.375

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	4	3	1.00	17	0.176
55	A	5	4	0.94	17	0.235
56	A	5	4	0.89	17	0.235
57	A	5	4	0.87	17	0.235
58	A	9	9	1.17	17	0.529
59	A	7	7	1.15	17	0.412
60	A	5	5	1.11	17	0.294
61	A	3	3	1.00	8	0.375
62	A	5	4	0.87	13	0.308
63	A	4	4	1.00	15	0.267
64	A	6	6	1.09	17	0.353
65	A	8	8	1.13	17	0.471
66	A	5	4	0.97	17	0.235
67	A	5	4	0.97	17	0.235
68	A	5	4	0.94	17	0.235
69	A	5	4	0.97	17	0.235
70	A	4	3	1.00	15	0.200
71	A	5	4	0.89	15	0.267
72	A	5	4	1.05	15	0.267
73	A	4	4	1.00	8	0.500
74	A	4	3	0.93	15	0.200
75	A	4	3	1.00	17	0.176
76	A	6	5	0.94	17	0.294
77	A	6	5	0.94	17	0.294
78	A	6	5	0.94	17	0.294
79	A	6	5	0.94	17	0.294
80	A	5	4	0.87	17	0.235
81	A	5	4	0.89	17	0.235
82	A	5	4	0.94	17	0.235
83	A	4	3	1.00	15	0.200
84	A	6	5	1.40	17	0.294
85	A	5	5	1.00	8	0.625

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	4	3	1.00	17	0.176
87	A	5	4	0.94	17	0.235
88	A	5	4	0.89	17	0.235
89	A	11	11	1.18	17	0.647
90	A	9	9	1.17	17	0.529
91	A	7	7	1.14	17	0.412
92	A	5	5	1.11	8	0.625
93	A	5	4	0.84	15	0.267
94	A	6	5	1.16	15	0.333
95	A	6	6	1.09	15	0.400
96	A	8	8	1.13	17	0.471
97	A	10	10	1.15	17	0.588
98	A	6	5	0.96	17	0.294
99	A	5	4	0.91	17	0.235
100	A	6	5	0.96	17	0.294
101	A	5	4	0.91	17	0.235
102	A	5	4	0.94	17	0.235
103	A	5	4	0.91	17	0.235
104	A	4	3	1.00	15	0.200
105	A	6	5	0.98	15	0.333
106	A	5	4	0.92	17	0.235
107	A	6	5	0.81	17	0.294
108	A	5	4	0.84	15	0.267
109	A	6	6	1.00	8	0.750
110	A	5	4	0.85	15	0.267
111	A	4	3	1.00	17	0.176
112	A	5	4	0.89	17	0.235
113	A	5	4	0.94	17	0.235
114	A	5	4	0.89	17	0.235
115	A	6	5	0.96	17	0.294
116	A	5	4	0.89	17	0.235
117	A	6	5	0.96	17	0.294

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	6	5	1.05	17	0.294
119	A	5	4	0.84	15	0.267
120	A	6	5	0.85	15	0.333
121	A	7	6	0.92	15	0.400
122	A	6	5	0.87	15	0.333
123	A	6	5	0.96	15	0.333
124	A	6	5	0.91	13	0.385
125	A	3	3	1.18	6	0.500
126	A	4	3	1.00	13	0.231
127	A	6	5	0.87	15	0.333
128	A	5	4	0.89	15	0.267
129	A	6	5	0.84	15	0.333
130	A	6	5	1.00	15	0.333
131	A	6	5	0.83	15	0.333
132	A	6	5	0.89	15	0.333
133	A	5	4	0.84	17	0.235
134	A	7	6	1.39	17	0.353
135	A	5	4	0.84	17	0.235
136	A	6	5	1.39	17	0.294
137	A	5	4	0.78	15	0.267
138	A	3	3	1.00	8	0.375
139	A	5	4	1.00	13	0.308
140	A	6	5	0.91	15	0.333
141	A	5	4	0.86	17	0.235
142	A	6	5	1.16	17	0.294
143	A	5	4	0.87	17	0.235
144	A	8	7	1.26	17	0.412
145	A	7	6	0.78	17	0.353
146	A	7	6	1.07	17	0.353
147	A	7	6	0.77	17	0.353
148	A	7	6	1.16	15	0.400
149	A	7	7	1.07	8	0.875

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	4	4	1.00	15	0.267
151	A	5	4	1.00	15	0.267
152	A	5	4	0.89	15	0.267
153	A	6	5	1.16	17	0.294
154	A	6	5	0.81	17	0.294
155	A	6	5	1.05	17	0.294
156	A	6	5	0.84	17	0.294
157	A	5	4	0.85	17	0.235
158	A	7	6	1.26	17	0.353
159	A	5	4	0.87	17	0.235
160	A	6	5	1.22	17	0.294
161	A	5	4	0.92	15	0.267
162	A	5	5	1.00	8	0.625
163	A	5	4	1.00	15	0.267
164	A	4	3	1.00	17	0.176
165	A	5	4	1.00	15	0.267
166	A	6	5	0.87	15	0.333
167	A	5	4	0.84	17	0.235
168	A	6	5	1.07	17	0.294
169	A	5	4	0.85	17	0.235
170	A	8	7	1.17	17	0.412
171	A	7	6	0.86	17	0.353
172	A	8	7	1.17	17	0.412
173	A	7	6	0.78	17	0.353
174	A	8	7	1.28	15	0.467
175	A	11	11	1.05	8	1.375
176	A	6	6	1.09	15	0.400
177	A	5	4	1.00	17	0.235
178	A	6	6	1.09	17	0.353
179	A	5	4	1.00	15	0.267
180	A	6	5	0.92	15	0.333
181	A	8	7	1.26	17	0.412

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
182	A	6	5	0.81	17	0.294
183	A	8	7	1.16	17	0.412
184	A	6	5	0.88	17	0.294
185	A	5	4	1.00	9	0.444
186	A	7	6	1.00	9	0.667
187	A	4	3	1.00	19	0.158
188	A	4	3	1.00	19	0.158
189	A	4	3	1.00	19	0.158
190	A	4	3	1.00	19	0.158
191	A	4	3	1.00	19	0.158
192	A	4	3	1.00	19	0.158
193	A	4	3	1.00	19	0.158
194	A	10	10	1.09	21	0.476
195	A	10	10	1.09	21	0.476
196	A	8	8	1.05	21	0.381
197	A	8	8	1.05	21	0.381
198	A	6	6	1.00	21	0.286
199	A	6	6	1.00	21	0.286
200	A	6	6	1.00	21	0.286
201	A	6	6	1.00	21	0.286
202	A	8	8	1.04	21	0.381
203	A	8	8	1.08	21	0.381
204	A	6	5	0.98	21	0.238
205	A	6	5	0.98	21	0.238
206	A	6	5	0.98	21	0.238
207	A	6	5	0.98	21	0.238
208	A	6	5	0.98	21	0.238
209	A	6	5	0.98	21	0.238
210	A	6	5	0.98	21	0.238
211	A	12	12	1.10	21	0.571
212	A	12	12	1.10	21	0.571
213	A	10	10	1.08	21	0.476

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	10	10	1.08	21	0.476
215	A	8	8	1.05	21	0.381
216	A	8	8	1.05	21	0.381
217	A	8	8	1.03	21	0.381
218	A	8	8	1.03	21	0.381
219	A	8	8	1.04	21	0.381
220	A	8	8	1.08	21	0.381
221	A	5	4	0.92	19	0.211
222	A	10	9	0.94	19	0.474
223	A	10	9	0.93	19	0.474
224	A	9	8	0.94	19	0.421
225	A	9	8	0.92	19	0.421
226	A	8	7	0.88	19	0.368
227	A	8	7	0.86	19	0.368
228	A	9	8	0.95	19	0.421
229	A	9	8	0.94	19	0.421
230	A	10	9	0.98	19	0.474
231	A	10	9	0.97	19	0.474
232	A	10	10	1.10	21	0.476
233	A	8	8	1.06	21	0.381
234	A	8	8	1.06	21	0.381
235	A	6	6	1.00	21	0.286
236	A	6	6	1.00	21	0.286
237	A	6	6	1.00	21	0.286
238	A	6	6	1.00	21	0.286
239	A	8	8	1.05	21	0.381
240	A	8	8	1.05	21	0.381
241	A	10	10	1.07	21	0.476
242	A	11	10	0.99	21	0.476
243	A	10	9	1.02	21	0.429
244	A	10	9	1.00	21	0.429
245	A	9	8	1.02	21	0.381

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	9	8	1.02	21	0.381
247	A	9	8	1.06	21	0.381
248	F	0	0	N/A	0.000	N/A
249	A	10	9	1.06	21	0.429
250	A	10	9	1.08	21	0.429
251	A	11	10	1.07	21	0.476
252	A	4	3	1.00	19	0.158
253	A	5	4	1.00	11	0.364
254	A	5	4	1.00	11	0.364
255	A	5	4	1.00	11	0.364
256	A	5	4	1.00	11	0.364
257	A	8	8	1.05	25	0.320
258	A	6	6	1.00	25	0.240
259	A	4	4	1.00	25	0.160
260	A	6	6	1.00	25	0.240
261	A	8	8	1.03	25	0.320
262	A	13	12	1.33	25	0.480
263	A	11	10	1.40	25	0.400
264	A	2	2	1.00	25	0.080
265	A	4	4	1.00	25	0.160
266	A	6	6	1.07	25	0.240
267	A	8	8	1.03	25	0.320
268	A	6	6	1.00	25	0.240
269	A	6	6	1.00	25	0.240
270	A	8	8	1.08	25	0.320
271	A	13	12	1.33	25	0.480
272	A	13	12	1.37	25	0.480
273	A	2	2	1.00	25	0.080
274	A	6	6	1.14	25	0.240
275	A	8	8	1.18	25	0.320
276	A	10	10	1.09	25	0.400
277	A	8	8	1.05	25	0.320

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
278	A	6	6	1.00	25	0.240
279	A	6	6	1.00	25	0.240
280	A	8	8	1.05	25	0.320
281	A	10	10	1.10	25	0.400
282	A	13	12	1.33	25	0.480
283	A	13	12	1.37	25	0.480
284	A	2	2	1.00	25	0.080
285	A	6	6	1.14	25	0.240
286	A	8	8	1.18	25	0.320
287	A	15	14	1.30	21	0.667
288	A	2	2	1.00	13	0.154
289	A	11	10	1.51	13	0.769
290	A	13	12	1.37	13	0.923
291	A	8	8	1.02	25	0.320
292	A	6	6	1.00	25	0.240
293	A	4	4	1.00	25	0.160
294	A	6	6	1.00	25	0.240
295	A	8	8	1.06	25	0.320
296	A	11	10	1.41	25	0.400
297	A	2	2	1.00	25	0.080
298	A	4	4	1.00	25	0.160
299	A	6	6	1.07	25	0.240
300	A	11	10	1.42	21	0.476
301	A	13	12	1.35	21	0.571
302	A	13	12	1.34	21	0.571
303	A	15	14	1.29	21	0.667
304	A	2	2	1.00	21	0.095
305	A	2	2	1.00	21	0.095
306	A	2	2	1.00	12	0.167
307	A	2	2	1.00	21	0.095
308	A	2	2	1.00	21	0.095
309	A	2	2	1.00	21	0.095

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
310	A	2	2	1.00	21	0.095
311	A	2	2	1.00	12	0.167
312	A	2	2	1.00	21	0.095
313	A	2	2	1.00	21	0.095
314	A	2	2	1.00	21	0.095
315	A	2	2	1.00	21	0.095
316	A	2	2	1.00	12	0.167
317	A	2	2	1.00	21	0.095
318	A	2	2	1.00	21	0.095
319	A	2	2	1.00	21	0.095
320	A	2	2	1.00	21	0.095
321	A	2	2	1.00	12	0.167
322	A	2	2	1.00	21	0.095
323	A	2	2	1.00	21	0.095
324	A	11	10	0.57	21	0.476
325	A	11	10	1.36	21	0.476
326	A	13	12	1.31	21	0.571
327	A	13	12	0.65	21	0.571
328	A	13	12	0.65	21	0.571
329	A	11	10	0.57	21	0.476
330	A	11	10	1.35	21	0.476
331	A	13	12	1.31	21	0.571
332	A	13	12	0.65	21	0.571
333	A	13	12	0.65	21	0.571
334	A	2	2	1.00	13	0.154
335	A	2	2	1.00	13	0.154
336	A	2	2	1.00	17	0.118
337	A	2	2	1.00	19	0.105
338	A	2	2	1.00	19	0.105
339	A	2	2	1.00	21	0.095
340	A	6	5	0.93	19	0.263
341	A	6	5	0.96	19	0.263

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
342	A	4	3	1.00	17	0.176
343	A	5	4	1.00	17	0.235
344	A	5	4	1.00	19	0.211
345	A	2	2	1.00	19	0.105
346	A	2	2	1.00	19	0.105
347	A	2	2	1.00	10	0.200
348	A	2	2	1.00	19	0.105
349	A	2	2	1.00	19	0.105
350	A	2	2	1.00	23	0.087
351	A	2	2	1.00	23	0.087
352	A	2	2	1.00	23	0.087
353	A	2	2	1.00	23	0.087
354	A	2	2	1.00	23	0.087
355	A	6	5	0.92	19	0.263
356	A	6	5	0.98	19	0.263
357	A	4	3	1.00	17	0.176
358	A	5	4	1.00	17	0.235
359	A	5	4	1.00	19	0.211
360	A	5	4	1.00	19	0.211
361	A	2	2	1.00	19	0.105
362	A	2	2	1.00	19	0.105
363	A	2	2	1.00	10	0.200
364	A	2	2	1.00	19	0.105
365	A	2	2	1.00	19	0.105
366	A	2	2	1.00	23	0.087
367	A	2	2	1.00	23	0.087
368	A	2	2	1.00	23	0.087
369	A	2	2	1.00	23	0.087
370	A	2	2	1.00	23	0.087
371	A	7	6	0.92	21	0.286
372	A	6	5	0.92	21	0.238
373	A	7	6	0.98	21	0.286

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
374	A	4	3	1.00	19	0.158
375	A	9	8	0.88	19	0.421
376	A	9	8	0.97	21	0.381
377	A	11	10	1.07	21	0.476
378	A	10	10	1.08	21	0.476
379	A	8	8	1.05	21	0.381
380	A	6	6	1.00	21	0.286
381	A	4	4	1.00	12	0.333
382	A	6	6	1.00	21	0.286
383	A	8	8	1.00	21	0.381
384	A	10	10	1.04	21	0.476
385	A	7	6	0.92	21	0.286
386	A	6	5	0.92	21	0.238
387	A	7	6	0.98	21	0.286
388	A	4	3	1.00	19	0.158
389	A	10	9	0.92	19	0.474
390	A	10	9	0.97	21	0.429
391	A	10	10	1.03	21	0.476
392	A	8	8	1.01	21	0.381
393	A	6	6	1.00	21	0.286
394	A	6	6	1.00	12	0.500
395	A	8	8	1.06	21	0.381
396	A	10	10	1.03	21	0.476
397	A	7	6	0.92	21	0.286
398	A	6	5	0.92	21	0.238
399	A	7	6	0.98	21	0.286
400	A	4	3	1.00	19	0.158
401	A	10	9	0.94	19	0.474
402	A	10	9	0.99	21	0.429
403	A	12	11	1.07	21	0.524
404	A	10	10	1.05	21	0.476
405	A	8	8	1.01	21	0.381

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
406	A	6	6	1.00	21	0.286
407	A	6	6	1.00	12	0.500
408	A	8	8	0.98	21	0.381
409	A	10	10	1.05	21	0.476
410	A	7	6	0.92	21	0.286
411	A	6	5	0.92	21	0.238
412	A	7	6	0.98	21	0.286
413	A	4	3	1.00	19	0.158
414	A	9	8	0.86	19	0.421
415	A	9	8	0.97	21	0.381
416	A	11	10	1.07	21	0.476
417	A	10	10	1.08	21	0.476
418	A	8	8	1.05	21	0.381
419	A	6	6	1.00	21	0.286
420	A	4	4	1.00	12	0.333
421	A	6	6	1.00	21	0.286
422	A	8	8	1.01	21	0.381
423	A	10	10	1.05	21	0.476
424	A	7	6	0.92	21	0.286
425	A	6	5	0.92	21	0.238
426	A	7	6	0.98	21	0.286
427	A	4	3	1.00	19	0.158
428	A	10	9	0.95	19	0.474
429	A	9	8	1.03	21	0.381
430	A	11	10	1.11	21	0.476
431	A	10	10	1.08	21	0.476
432	A	8	8	1.05	21	0.381
433	A	6	6	1.00	12	0.500
434	A	6	6	1.00	21	0.286
435	A	8	8	0.98	21	0.381
436	A	10	10	1.01	21	0.476
437	A	7	6	0.92	21	0.286

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
438	A	6	5	0.92	21	0.238
439	A	7	6	0.98	21	0.286
440	A	4	3	1.00	19	0.158
441	A	10	9	0.94	19	0.474
442	A	9	8	1.03	21	0.381
443	A	11	10	1.11	21	0.476
444	A	10	10	1.08	21	0.476
445	A	8	8	1.05	21	0.381
446	A	6	6	1.00	12	0.500
447	A	6	6	1.00	21	0.286
448	A	8	8	0.99	21	0.381
449	A	10	10	1.02	21	0.476
450	A	17	16	1.11	25	0.640
451	A	15	14	1.10	25	0.560
452	A	13	12	1.12	25	0.480
453	A	2	2	1.00	25	0.080
454	A	4	4	1.00	25	0.160
455	A	6	6	1.08	25	0.240
456	A	10	10	1.03	25	0.400
457	A	8	8	1.00	25	0.320
458	A	6	6	1.00	25	0.240
459	A	8	8	1.00	25	0.320
460	A	10	10	1.06	25	0.400
461	A	10	10	1.04	23	0.435
462	A	8	8	1.00	23	0.348
463	A	6	6	1.00	23	0.261
464	A	8	8	1.00	23	0.348
465	A	10	10	1.01	23	0.435
466	A	15	14	1.04	23	0.609
467	A	13	12	1.07	23	0.522
468	A	2	2	1.00	23	0.087
469	A	4	4	1.00	23	0.174

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
470	A	6	6	1.05	23	0.261
471	A	8	8	1.08	23	0.348
472	A	19	18	1.11	25	0.720
473	A	17	16	1.11	25	0.640
474	A	15	14	1.10	25	0.560
475	A	15	14	1.13	25	0.560
476	A	2	2	1.00	25	0.080
477	A	12	12	1.03	25	0.480
478	A	10	10	1.01	25	0.400
479	A	8	8	1.00	25	0.320
480	A	8	8	1.00	25	0.320
481	A	10	10	1.04	25	0.400
482	A	12	12	1.07	25	0.480
483	A	4	4	1.00	23	0.174
484	A	4	4	1.00	23	0.174
485	A	4	4	1.00	23	0.174
486	A	4	4	1.00	23	0.174
487	A	4	4	1.00	23	0.174
488	A	4	4	1.00	17	0.235
489	A	4	4	1.00	19	0.211
490	A	4	4	1.00	19	0.211
491	A	4	4	1.00	21	0.190
492	A	6	5	0.94	19	0.263
493	A	7	6	0.98	19	0.316
494	A	4	3	1.00	17	0.176
495	A	6	5	1.00	17	0.294
496	A	5	4	1.00	19	0.211
497	A	4	4	1.00	19	0.211
498	A	4	4	1.00	19	0.211
499	A	4	4	1.00	19	0.211
500	A	4	4	1.00	10	0.400
501	A	4	4	1.00	19	0.211

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
502	A	4	4	1.00	19	0.211
503	A	4	4	1.00	23	0.174
504	A	4	4	1.00	23	0.174
505	A	4	4	1.00	23	0.174
506	A	4	4	1.00	23	0.174
507	A	9	9	1.17	21	0.429
508	A	7	7	1.08	21	0.333
509	A	7	7	1.12	21	0.333
510	A	5	5	1.00	19	0.263
511	A	4	4	1.00	12	0.333
512	A	7	7	1.10	19	0.368
513	A	7	7	1.07	21	0.333
514	A	9	9	1.09	21	0.429
515	A	9	9	1.14	21	0.429
516	A	7	7	1.05	21	0.333
517	A	7	7	1.08	21	0.333
518	A	5	5	1.00	21	0.238
519	A	5	5	1.00	19	0.263
520	A	6	6	1.00	12	0.500
521	A	7	7	1.10	19	0.368
522	A	9	9	1.06	21	0.429
523	A	9	9	1.15	21	0.429
524	A	7	7	1.12	21	0.333
525	A	7	7	1.07	19	0.368
526	A	4	4	1.00	12	0.333
527	A	5	5	1.00	19	0.263
528	A	7	7	1.07	21	0.333
529	A	7	7	1.03	21	0.333
530	A	9	9	1.14	21	0.429
531	A	7	7	1.07	19	0.368
532	A	6	6	1.00	12	0.500
533	A	5	5	1.00	19	0.263

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
534	A	5	5	1.00	21	0.238
535	A	7	7	1.03	21	0.333
536	A	7	7	1.03	21	0.333
537	A	9	9	1.04	21	0.429
538	A	4	4	1.00	21	0.190

CHAPTER 3

Listing of integrals

3.1 $\int \sin(a+bx)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 11

$$\int \sin(a+bx)\,dx = -\frac{\cos(a+bx)}{b}$$

output
```
-cos(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.00

$$\int \sin(a+bx)\,dx = -\frac{\cos(a)\cos(bx)}{b} + \frac{\sin(a)\sin(bx)}{b}$$

input
```
Integrate[Sin[a + b*x],x]
```

output
```
-((Cos[a]*Cos[b*x])/b) + (Sin[a]*Sin[b*x])/b
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)dx$$

$$\Big\downarrow \text{3118}$$

$$-\frac{\cos(a+bx)}{b}$$

input
```
Int[Sin[a + b*x],x]
```

output
```
-(Cos[a + b*x]/b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Maple [A] (verified)

Time = 2.37 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$-\frac{\cos(bx+a)}{b}$	12
default	$-\frac{\cos(bx+a)}{b}$	12
risch	$-\frac{\cos(bx+a)}{b}$	12
orering	$-\frac{\cos(bx+a)}{b}$	12
parallelrisch	$\frac{-1-\cos(bx+a)}{b}$	15
norman	$\frac{2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}{b\left(1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)}$	32
meijerg	$\frac{\sin(a)\sin(bx)}{b}+\frac{\cos(a)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(bx)}{\sqrt{\pi}}\right)}{b}$	34

input
```
int(sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-cos(b*x+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \sin(a+bx)\,dx=-\frac{\cos\left(bx+a\right)}{b}$$

input
```
integrate(sin(b*x+a),x, algorithm="fricas")
```

output
```
-cos(b*x + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \sin(a+bx)\,dx = \begin{cases} -\frac{\cos{(a+bx)}}{b} & \text{for } b \neq 0 \\ x\sin{(a)} & \text{otherwise} \end{cases}$$

input
```
integrate(sin(b*x+a),x)
```

output
```
Piecewise((-cos(a + b*x)/b, Ne(b, 0)), (x*sin(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \sin(a+bx)\,dx = -\frac{\cos{(bx+a)}}{b}$$

input
```
integrate(sin(b*x+a),x, algorithm="maxima")
```

output
```
-cos(b*x + a)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \sin(a+bx)\,dx = -\frac{\cos{(bx+a)}}{b}$$

input
```
integrate(sin(b*x+a),x, algorithm="giac")
```

output
```
-cos(b*x + a)/b
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \sin(a+bx)\,dx = -\frac{\cos(a+b\,x)}{b}$$

input
```
int(sin(a + b*x),x)
```

output
```
-cos(a + b*x)/b
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \sin(a+bx)\,dx = -\frac{\cos(bx+a)}{b}$$

input
```
int(sin(b*x+a),x)
```

output
```
( - cos(a + b*x))/b
```

3.2 $\int \sin^2(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 25

$$\int \sin^2(a+bx)\,dx = \frac{x}{2} - \frac{\cos(a+bx)\sin(a+bx)}{2b}$$

output
```
1/2*x-1/2*cos(b*x+a)*sin(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sin^2(a+bx)\,dx = -\frac{-2(a+bx)+\sin(2(a+bx))}{4b}$$

input
```
Integrate[Sin[a + b*x]^2,x]
```

output
```
-1/4*(-2*(a + b*x) + Sin[2*(a + b*x)])/b
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(a+bx)^2dx$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{\int 1dx}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}$$

input
```
Int[Sin[a + b*x]^2,x]
```

output
```
x/2 - (Cos[a + b*x]*Sin[a + b*x])/(2*b)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 3.59 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{x}{2}-\frac{\sin(2bx+2a)}{4b}$	19
parallelrisch	$\frac{2bx-\sin(2bx+2a)}{4b}$	22
derivativedivides	$\frac{-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}}{b}$	27
default	$\frac{-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}}{b}$	27
orering	$x\sin\left(bx+a\right)^{2}-\frac{\cos(bx+a)\sin(bx+a)}{2b}+\frac{x\left(2\cos(bx+a)^{2}b^{2}-2\sin(bx+a)^{2}b^{2}\right)}{4b^{2}}$	62
norman	$\frac{\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{3}}{b}+x\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}+\frac{x}{2}-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{x\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}}{2}}{\left(1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)^{2}}$	77

input
```
int(sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x-1/4/b*sin(2*b*x+2*a)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sin^2(a+bx)\,dx=\frac{bx-\cos\left(bx+a\right)\sin\left(bx+a\right)}{2\,b}$$

input
```
integrate(sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(b*x - cos(b*x + a)*sin(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(19) = 38$.

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \sin^2(a+bx)\,dx = \begin{cases} \frac{x\sin^2(a+bx)}{2} + \frac{x\cos^2(a+bx)}{2} - \frac{\sin(a+bx)\cos(a+bx)}{2b} & \text{for } b \neq 0 \\ x\sin^2(a) & \text{otherwise} \end{cases}$$

input
```
integrate(sin(b*x+a)**2,x)
```

output
```
Piecewise((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 - sin(a + b*x)*cos(a
+ b*x)/(2*b), Ne(b, 0)), (x*sin(a)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \sin^2(a+bx)\,dx = \frac{2\,bx + 2\,a - \sin(2\,bx + 2\,a)}{4\,b}$$

input
```
integrate(sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/4*(2*b*x + 2*a - sin(2*b*x + 2*a))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \sin^2(a+bx)\,dx = \frac{1}{2}\,x - \frac{\sin\left(2\,bx + 2\,a\right)}{4\,b}$$

input
```
integrate(sin(b*x+a)^2,x, algorithm="giac")
```

output
```
1/2*x - 1/4*sin(2*b*x + 2*a)/b
```

Mupad [B] (verification not implemented)

Time = 25.50 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \sin^2(a+bx)\,dx = \frac{x}{2} - \frac{\sin\left(2\,a + 2\,b\,x\right)}{4\,b}$$

input
```
int(sin(a + b*x)^2,x)
```

output
```
x/2 - sin(2*a + 2*b*x)/(4*b)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sin^2(a+bx)\,dx = \frac{-\cos\left(bx+a\right)\sin\left(bx+a\right)+bx}{2b}$$

input
```
int(sin(b*x+a)^2,x)
```

output
```
( - cos(a + b*x)*sin(a + b*x) + b*x)/(2*b)
```

3.3 $\int \sin^3(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 27

$$\int \sin^3(a+bx)\,dx = -\frac{\cos(a+bx)}{b} + \frac{\cos^3(a+bx)}{3b}$$

output
```
-cos(b*x+a)/b+1/3*cos(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \sin^3(a+bx)\,dx = -\frac{3\cos(a+bx)}{4b} + \frac{\cos(3(a+bx))}{12b}$$

input
```
Integrate[Sin[a + b*x]^3,x]
```

output
```
(-3*Cos[a + b*x])/(4*b) + Cos[3*(a + b*x)]/(12*b)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin(a+bx)^3dx$$
$$\Big\downarrow \text{3113}$$
$$-\frac{\int \left(1-\cos^2(a+bx)\right)d\cos(a+bx)}{b}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{\cos(a+bx)-\frac{1}{3}\cos^3(a+bx)}{b}$$

input
```
Int[Sin[a + b*x]^3,x]
```

output
```
-((Cos[a + b*x] - Cos[a + b*x]^3/3)/b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

Maple [A] (verified)

Time = 6.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$-\frac{\left(2+\sin(bx+a)^{2}\right)\cos(bx+a)}{3b}$	22
default	$-\frac{\left(2+\sin(bx+a)^{2}\right)\cos(bx+a)}{3b}$	22
parallelrisch	$\frac{-8-9\cos(bx+a)+\cos(3bx+3a)}{12b}$	25
risch	$-\frac{3\cos(bx+a)}{4b}+\frac{\cos(3bx+3a)}{12b}$	27
norman	$\frac{-\frac{4\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}{b}-\frac{4}{3b}}{\left(1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)^{3}}$	39
orering	$-\frac{10\sin(bx+a)^{2}\cos(bx+a)}{3b}-\frac{6b^{3}\cos(bx+a)^{3}-21\sin(bx+a)^{2}b^{3}\cos(bx+a)}{9b^{4}}$	59

input
```
int(sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3/b*(2+sin(b*x+a)^2)*cos(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \sin^3(a+bx)\,dx=\frac{\cos\left(bx+a\right)^{3}-3\,\cos\left(bx+a\right)}{3\,b}$$

input
```
integrate(sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/3*(cos(b*x + a)^3 - 3*cos(b*x + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \sin^3(a+bx)\,dx = \begin{cases} -\frac{\sin^2(a+bx)\cos(a+bx)}{b} - \frac{2\cos^3(a+bx)}{3b} & \text{for } b \neq 0 \\ x\sin^3(a) & \text{otherwise} \end{cases}$$

input
```
integrate(sin(b*x+a)**3,x)
```

output
```
Piecewise((-sin(a + b*x)**2*cos(a + b*x)/b - 2*cos(a + b*x)**3/(3*b), Ne(b
, 0)), (x*sin(a)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \sin^3(a+bx)\,dx = \frac{\cos(bx+a)^3 - 3\cos(bx+a)}{3b}$$

input
```
integrate(sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/3*(cos(b*x + a)^3 - 3*cos(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \sin^3(a+bx)\,dx = \frac{\cos(bx+a)^3}{3b} - \frac{\cos(bx+a)}{b}$$

input
```
integrate(sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/3*cos(b*x + a)^3/b - cos(b*x + a)/b
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \sin^3(a+bx)\,dx = -\frac{3\,\cos\left(a+b\,x\right)-{\cos\left(a+b\,x\right)}^3}{3\,b}$$

input
```
int(sin(a + b*x)^3,x)
```

output
```
-(3*cos(a + b*x) - cos(a + b*x)^3)/(3*b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \sin^3(a+bx)\,dx = \frac{-\cos(bx+a)\sin(bx+a)^2-2\cos(bx+a)+2}{3b}$$

input
```
int(sin(b*x+a)^3,x)
```

output
```
( - cos(a + b*x)*sin(a + b*x)**2 - 2*cos(a + b*x) + 2)/(3*b)
```

3.4 $\int \sin^4(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 46

$$\int \sin^4(a+bx)\,dx = \frac{3x}{8} - \frac{3\cos(a+bx)\sin(a+bx)}{8b} - \frac{\cos(a+bx)\sin^3(a+bx)}{4b}$$

output
```
3/8*x-3/8*cos(b*x+a)*sin(b*x+a)/b-1/4*cos(b*x+a)*sin(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int \sin^4(a+bx)\,dx = \frac{12(a+bx) - 8\sin(2(a+bx)) + \sin(4(a+bx))}{32b}$$

input
```
Integrate[Sin[a + b*x]^4,x]
```

output
```
(12*(a + b*x) - 8*Sin[2*(a + b*x)] + Sin[4*(a + b*x)])/(32*b)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^4dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{3}{4}\int \sin^2(a+bx)dx-\frac{\sin^3(a+bx)\cos(a+bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}\int \sin(a+bx)^2dx-\frac{\sin^3(a+bx)\cos(a+bx)}{4b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{3}{4}\left(\frac{\int 1dx}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)-\frac{\sin^3(a+bx)\cos(a+bx)}{4b}$$

$$\Big\downarrow \text{24}$$

$$\frac{3}{4}\left(\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)-\frac{\sin^3(a+bx)\cos(a+bx)}{4b}$$

input
```
Int[Sin[a + b*x]^4,x]
```

output
```
-1/4*(Cos[a + b*x]*Sin[a + b*x]^3)/b + (3*(x/2 - (Cos[a + b*x]*Sin[a + b*x
])/(2*b)))/4
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 6.62 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

method	result
parallelrisch	$\frac{12bx+\sin(4bx+4a)-8\sin(2bx+2a)}{32b}$
risch	$\frac{3x}{8}+\frac{\sin(4bx+4a)}{32b}-\frac{\sin(2bx+2a)}{4b}$
derivativedivides	$\frac{-\frac{\left(\sin(bx+a)^3+\frac{3\sin(bx+a)}{2}\right)\cos(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}}{b}$
default	$\frac{-\frac{\left(\sin(bx+a)^3+\frac{3\sin(bx+a)}{2}\right)\cos(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}}{b}$
norman	$\frac{\frac{3x}{8}-\frac{3\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{4b}-\frac{11\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^3}{4b}+\frac{11\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^5}{4b}+\frac{3\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^7}{4b}+\frac{3x\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}{2}+\frac{9x\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}{4}+\frac{3x\tan\left(\frac{b}{}\right.}{2}}{\left(1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right)^4}$
orering	$x\sin(bx+a)^4-\frac{5\cos(bx+a)\sin(bx+a)^3}{4b}+\frac{5x\left(12\cos(bx+a)^2\sin(bx+a)^2b^2-4\sin(bx+a)^4b^2\right)}{16b^2}-\frac{-40\cos(b}{}$

input
```
int(sin(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/32*(12*b*x+sin(4*b*x+4*a)-8*sin(2*b*x+2*a))/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \sin^4(a+bx)\,dx = \frac{3\,bx + \left(2\,\cos\left(bx+a\right)^3 - 5\,\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{8\,b}$$

input
```
integrate(sin(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/8*(3*b*x + (2*cos(b*x + a)^3 - 5*cos(b*x + a))*sin(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(41) = 82$.

Time = 0.15 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.07

$$\int \sin^4(a+bx)\,dx = \begin{cases} \frac{3x\sin^4(a+bx)}{8} + \frac{3x\sin^2(a+bx)\cos^2(a+bx)}{4} + \frac{3x\cos^4(a+bx)}{8} - \frac{5\sin^3(a+bx)\cos(a+bx)}{8b} - \frac{3\sin(a+bx)\cos^3(a+bx)}{8b} & \text{for } b \neq 0 \\ x\sin^4(a) & \text{otherwise} \end{cases}$$

input
```
integrate(sin(b*x+a)**4,x)
```

output
```
Piecewise((3*x*sin(a + b*x)**4/8 + 3*x*sin(a + b*x)**2*cos(a + b*x)**2/4 +
 3*x*cos(a + b*x)**4/8 - 5*sin(a + b*x)**3*cos(a + b*x)/(8*b) - 3*sin(a +
b*x)*cos(a + b*x)**3/(8*b), Ne(b, 0)), (x*sin(a)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int \sin^4(a+bx)\,dx = \frac{12\,bx + 12\,a + \sin\left(4\,bx + 4\,a\right) - 8\,\sin\left(2\,bx + 2\,a\right)}{32\,b}$$

input
```
integrate(sin(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/32*(12*b*x + 12*a + sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int \sin^4(a+bx)\,dx = \frac{3}{8}\,x + \frac{\sin\left(4\,bx + 4\,a\right)}{32\,b} - \frac{\sin\left(2\,bx + 2\,a\right)}{4\,b}$$

input
```
integrate(sin(b*x+a)^4,x, algorithm="giac")
```

output
```
3/8*x + 1/32*sin(4*b*x + 4*a)/b - 1/4*sin(2*b*x + 2*a)/b
```

Mupad [B] (verification not implemented)

Time = 25.44 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int \sin^4(a+bx)\,dx = \frac{3\,x}{8} - \frac{\frac{5\,\tan(a+b\,x)^3}{8} + \frac{3\,\tan(a+b\,x)}{8}}{b\,\left(\tan\left(a+b\,x\right)^4 + 2\,\tan\left(a+b\,x\right)^2 + 1\right)}$$

input
```
int(sin(a + b*x)^4,x)
```

output
```
(3*x)/8 - ((3*tan(a + b*x))/8 + (5*tan(a + b*x)^3)/8)/(b*(2*tan(a + b*x)^2
 + tan(a + b*x)^4 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \sin^4(a+bx)\,dx = \frac{-2\cos(bx+a)\sin(bx+a)^3 - 3\cos(bx+a)\sin(bx+a) + 3bx}{8b}$$

input
```
int(sin(b*x+a)^4,x)
```

output
```
( - 2*cos(a + b*x)*sin(a + b*x)**3 - 3*cos(a + b*x)*sin(a + b*x) + 3*b*x)/
(8*b)
```

3.5 $\int \sin^5(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 42

$$\int \sin^5(a+bx)\,dx = -\frac{\cos(a+bx)}{b} + \frac{2\cos^3(a+bx)}{3b} - \frac{\cos^5(a+bx)}{5b}$$

output
```
-cos(b*x+a)/b+2/3*cos(b*x+a)^3/b-1/5*cos(b*x+a)^5/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05

$$\int \sin^5(a+bx)\,dx = -\frac{5\cos(a+bx)}{8b} + \frac{5\cos(3(a+bx))}{48b} - \frac{\cos(5(a+bx))}{80b}$$

input
```
Integrate[Sin[a + b*x]^5,x]
```

output
```
(-5*Cos[a + b*x])/(8*b) + (5*Cos[3*(a + b*x)])/(48*b) - Cos[5*(a + b*x)]/(
80*b)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^5dx$$

$$\Big\downarrow \text{3113}$$

$$-\frac{\int \left(\cos^4(a+bx)-2\cos^2(a+bx)+1\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{5}\cos^5(a+bx)-\frac{2}{3}\cos^3(a+bx)+\cos(a+bx)}{b}$$

input
```
Int[Sin[a + b*x]^5,x]
```

output
```
-((Cos[a + b*x] - (2*Cos[a + b*x]^3)/3 + Cos[a + b*x]^5/5)/b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

Maple [A] (verified)

Time = 8.90 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

method	result
derivativedivides	$-\frac{\left(\frac{8}{3}+\sin(bx+a)^4+\frac{4\sin(bx+a)^2}{3}\right)\cos(bx+a)}{5b}$
default	$-\frac{\left(\frac{8}{3}+\sin(bx+a)^4+\frac{4\sin(bx+a)^2}{3}\right)\cos(bx+a)}{5b}$
parallelrisch	$\frac{-150\cos(bx+a)-3\cos(5bx+5a)-128+25\cos(3bx+3a)}{240b}$
risch	$-\frac{5\cos(bx+a)}{8b}-\frac{\cos(5bx+5a)}{80b}+\frac{5\cos(3bx+3a)}{48b}$
norman	$\frac{-\frac{16}{15b}-\frac{16\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}{3b}-\frac{32\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}{3b}}{\left(1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right)^5}$
orering	$-\frac{259\sin(bx+a)^4\cos(bx+a)}{45b}-\frac{7\left(60\sin(bx+a)^2b^3\cos(bx+a)^3-65\sin(bx+a)^4b^3\cos(bx+a)\right)}{45b^4}-\frac{120b^5\cos(bx+a)^5}{}$

input
```
int(sin(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/5/b*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int \sin^5(a+bx)\,dx = -\frac{3\,\cos(bx+a)^5 - 10\,\cos(bx+a)^3 + 15\,\cos(bx+a)}{15\,b}$$

input
```
integrate(sin(b*x+a)^5,x, algorithm="fricas")
```

output
```
-1/15*(3*cos(b*x + a)^5 - 10*cos(b*x + a)^3 + 15*cos(b*x + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.43

$$\int \sin^5(a+bx)\,dx = \begin{cases} -\frac{\sin^4(a+bx)\cos(a+bx)}{b} - \frac{4\sin^2(a+bx)\cos^3(a+bx)}{3b} - \frac{8\cos^5(a+bx)}{15b} & \text{for } b \neq 0 \\ x\sin^5(a) & \text{otherwise} \end{cases}$$

input
```
integrate(sin(b*x+a)**5,x)
```

output
```
Piecewise((-sin(a + b*x)**4*cos(a + b*x)/b - 4*sin(a + b*x)**2*cos(a + b*x
)**3/(3*b) - 8*cos(a + b*x)**5/(15*b), Ne(b, 0)), (x*sin(a)**5, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int \sin^5(a+bx)\,dx = -\frac{3\cos(bx+a)^5 - 10\cos(bx+a)^3 + 15\cos(bx+a)}{15\,b}$$

input
```
integrate(sin(b*x+a)^5,x, algorithm="maxima")
```

output
```
-1/15*(3*cos(b*x + a)^5 - 10*cos(b*x + a)^3 + 15*cos(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \sin^5(a+bx)\,dx = -\frac{\cos(bx+a)^5}{5\,b} + \frac{2\,\cos(bx+a)^3}{3\,b} - \frac{\cos(bx+a)}{b}$$

input
```
integrate(sin(b*x+a)^5,x, algorithm="giac")
```

output
```
-1/5*cos(b*x + a)^5/b + 2/3*cos(b*x + a)^3/b - cos(b*x + a)/b
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int \sin^5(a+bx)\,dx = -\frac{\frac{\cos(a+b\,x)^5}{5} - \frac{2\cos(a+b\,x)^3}{3} + \cos(a+b\,x)}{b}$$

input
```
int(sin(a + b*x)^5,x)
```

output
```
-(cos(a + b*x) - (2*cos(a + b*x)^3)/3 + cos(a + b*x)^5/5)/b
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12

$$\int \sin^5(a+bx)\,dx = \frac{-3\cos(bx+a)\sin(bx+a)^4 - 4\cos(bx+a)\sin(bx+a)^2 - 8\cos(bx+a) + 8}{15b}$$

input
```
int(sin(b*x+a)^5,x)
```

output
```
( - 3*cos(a + b*x)*sin(a + b*x)**4 - 4*cos(a + b*x)*sin(a + b*x)**2 - 8*co
s(a + b*x) + 8)/(15*b)
```

3.6 $\int \sin^6(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 67

$$\int \sin^6(a+bx)\,dx = \frac{5x}{16} - \frac{5\cos(a+bx)\sin(a+bx)}{16b} - \frac{5\cos(a+bx)\sin^3(a+bx)}{24b} - \frac{\cos(a+bx)\sin^5(a+bx)}{6b}$$

output
```
5/16*x-5/16*cos(b*x+a)*sin(b*x+a)/b-5/24*cos(b*x+a)*sin(b*x+a)^3/b-1/6*cos
(b*x+a)*sin(b*x+a)^5/b
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.67

$$\int \sin^6(a+bx)\,dx = \frac{60a+60bx-45\sin(2(a+bx))+9\sin(4(a+bx))-\sin(6(a+bx))}{192b}$$

input
```
Integrate[Sin[a + b*x]^6,x]
```

output
```
(60*a + 60*b*x - 45*Sin[2*(a + b*x)] + 9*Sin[4*(a + b*x)] - Sin[6*(a + b*x
)])/(192*b)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^6(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^6dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{5}{6}\int \sin^4(a+bx)dx-\frac{\sin^5(a+bx)\cos(a+bx)}{6b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\int \sin(a+bx)^4dx-\frac{\sin^5(a+bx)\cos(a+bx)}{6b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{5}{6}\left(\frac{3}{4}\int \sin^2(a+bx)dx-\frac{\sin^3(a+bx)\cos(a+bx)}{4b}\right)-\frac{\sin^5(a+bx)\cos(a+bx)}{6b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\left(\frac{3}{4}\int \sin(a+bx)^2dx-\frac{\sin^3(a+bx)\cos(a+bx)}{4b}\right)-\frac{\sin^5(a+bx)\cos(a+bx)}{6b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)-\frac{\sin^3(a+bx)\cos(a+bx)}{4b}\right)-\frac{\sin^5(a+bx)\cos(a+bx)}{6b}$$

$$\Big\downarrow \text{24}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)-\frac{\sin^3(a+bx)\cos(a+bx)}{4b}\right)-\frac{\sin^5(a+bx)\cos(a+bx)}{6b}$$

input
```
Int[Sin[a + b*x]^6,x]
```

output
```
-1/6*(Cos[a + b*x]*Sin[a + b*x]^5)/b + (5*(-1/4*(Cos[a + b*x]*Sin[a + b*x]
^3)/b + (3*(x/2 - (Cos[a + b*x]*Sin[a + b*x])/(2*b)))/4))/6
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 9.67 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.66

method	result
parallelrisch	$\frac{60bx-\sin(6bx+6a)+9\sin(4bx+4a)-45\sin(2bx+2a)}{192b}$
risch	$\frac{5x}{16}-\frac{\sin(6bx+6a)}{192b}+\frac{3\sin(4bx+4a)}{64b}-\frac{15\sin(2bx+2a)}{64b}$
derivativedivides	$\frac{-\frac{\left(\sin(bx+a)^5+\frac{5\sin(bx+a)^3}{4}+\frac{15\sin(bx+a)}{8}\right)\cos(bx+a)}{6}+\frac{5bx}{16}+\frac{5a}{16}}{b}$
default	$\frac{-\frac{\left(\sin(bx+a)^5+\frac{5\sin(bx+a)^3}{4}+\frac{15\sin(bx+a)}{8}\right)\cos(bx+a)}{6}+\frac{5bx}{16}+\frac{5a}{16}}{b}$
norman	$\frac{\frac{5x}{16}-\frac{5\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{8b}-\frac{85\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^3}{24b}-\frac{33\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^5}{4b}+\frac{33\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^7}{4b}+\frac{85\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^9}{24b}+\frac{5\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{11}}{8b}+\frac{15x\tan\left(\frac{bx}{2}\cdots\right.}{8}\cdots}{\left(1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2\cdots\right.}$
orering	$x\sin\left(bx+a\right)^6-\frac{49\cos(bx+a)\sin(bx+a)^5}{24b}+\frac{49x\left(30\sin(bx+a)^4b^2\cos(bx+a)^2-6\sin(bx+a)^6b^2\right)}{144b^2}-\frac{7\left(120\,\mathrm{s}\cdots\right.}{\cdots}$

input
```
int(sin(b*x+a)^6,x,method=_RETURNVERBOSE)
```

output
```
1/192*(60*b*x-sin(6*b*x+6*a)+9*sin(4*b*x+4*a)-45*sin(2*b*x+2*a))/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

$$\int \sin^6(a+bx)\,dx = \frac{15\,bx - \left(8\,\cos\left(bx+a\right)^5 - 26\,\cos\left(bx+a\right)^3 + 33\,\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{48\,b}$$

input
```
integrate(sin(b*x+a)^6,x, algorithm="fricas")
```

output
```
1/48*(15*b*x - (8*cos(b*x + a)^5 - 26*cos(b*x + a)^3 + 33*cos(b*x + a))*si
n(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(61) = 122.

Time = 0.33 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.07

$$\int \sin^6(a+bx)\,dx = \begin{cases} \frac{5x\sin^6(a+bx)}{16} + \frac{15x\sin^4(a+bx)\cos^2(a+bx)}{16} + \frac{15x\sin^2(a+bx)\cos^4(a+bx)}{16} + \frac{5x\cos^6(a+bx)}{16} - \frac{11\sin^5(a+bx)\cos(a+bx)}{16b} - \frac{5\sin^3}{} \\ x\sin^6(a) \end{cases}$$

input
```
integrate(sin(b*x+a)**6,x)
```

output
```
Piecewise((5*x*sin(a + b*x)**6/16 + 15*x*sin(a + b*x)**4*cos(a + b*x)**2/1
6 + 15*x*sin(a + b*x)**2*cos(a + b*x)**4/16 + 5*x*cos(a + b*x)**6/16 - 11*
sin(a + b*x)**5*cos(a + b*x)/(16*b) - 5*sin(a + b*x)**3*cos(a + b*x)**3/(6
*b) - 5*sin(a + b*x)*cos(a + b*x)**5/(16*b), Ne(b, 0)), (x*sin(a)**6, True
))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.72

$$\int \sin^6(a+bx)\,dx = \frac{4\,\sin\left(2\,bx+2\,a\right)^3+60\,bx+60\,a+9\,\sin\left(4\,bx+4\,a\right)-48\,\sin\left(2\,bx+2\,a\right)}{192\,b}$$

input
```
integrate(sin(b*x+a)^6,x, algorithm="maxima")
```

output
```
1/192*(4*sin(2*b*x + 2*a)^3 + 60*b*x + 60*a + 9*sin(4*b*x + 4*a) - 48*sin(
2*b*x + 2*a))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.69

$$\int \sin^6(a+bx)\,dx = \frac{5}{16}\,x-\frac{\sin\left(6\,bx+6\,a\right)}{192\,b}+\frac{3\,\sin\left(4\,bx+4\,a\right)}{64\,b}-\frac{15\,\sin\left(2\,bx+2\,a\right)}{64\,b}$$

input
```
integrate(sin(b*x+a)^6,x, algorithm="giac")
```

output
```
5/16*x - 1/192*sin(6*b*x + 6*a)/b + 3/64*sin(4*b*x + 4*a)/b - 15/64*sin(2*
b*x + 2*a)/b
```

Mupad [B] (verification not implemented)

Time = 25.49 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.64

$$\int \sin^6(a+bx)\,dx = \frac{5\,x}{16} - \frac{\frac{15\,\sin(2\,a+2\,b\,x)}{64} - \frac{3\,\sin(4\,a+4\,b\,x)}{64} + \frac{\sin(6\,a+6\,b\,x)}{192}}{b}$$

input
```
int(sin(a + b*x)^6,x)
```

output
```
(5*x)/16 - ((15*sin(2*a + 2*b*x))/64 - (3*sin(4*a + 4*b*x))/64 + sin(6*a +
 6*b*x)/192)/b
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.84

$$\int \sin^6(a+bx)\,dx$$
$$= \frac{-8\cos(bx+a)\sin(bx+a)^5 - 10\cos(bx+a)\sin(bx+a)^3 - 15\cos(bx+a)\sin(bx+a) + 15bx}{48b}$$

input
```
int(sin(b*x+a)^6,x)
```

output
```
( - 8*cos(a + b*x)*sin(a + b*x)**5 - 10*cos(a + b*x)*sin(a + b*x)**3 - 15*
cos(a + b*x)*sin(a + b*x) + 15*b*x)/(48*b)
```

3.7 $\int \sin^7(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 54

$$\int \sin^7(a+bx)\,dx = -\frac{\cos(a+bx)}{b}+\frac{\cos^3(a+bx)}{b}-\frac{3\cos^5(a+bx)}{5b}+\frac{\cos^7(a+bx)}{7b}$$

output
```
-cos(b*x+a)/b+cos(b*x+a)^3/b-3/5*cos(b*x+a)^5/b+1/7*cos(b*x+a)^7/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09

$$\int \sin^7(a+bx)\,dx = -\frac{35\cos(a+bx)}{64b}+\frac{7\cos(3(a+bx))}{64b} -\frac{7\cos(5(a+bx))}{320b}+\frac{\cos(7(a+bx))}{448b}$$

input
```
Integrate[Sin[a + b*x]^7,x]
```

output
```
(-35*Cos[a + b*x])/(64*b) + (7*Cos[3*(a + b*x)])/(64*b) - (7*Cos[5*(a + b*
x)])/(320*b) + Cos[7*(a + b*x)]/(448*b)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^7(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^7dx$$

$$\Big\downarrow \text{3113}$$

$$-\frac{\int \left(-\cos^6(a+bx)+3\cos^4(a+bx)-3\cos^2(a+bx)+1\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{1}{7}\cos^7(a+bx)+\frac{3}{5}\cos^5(a+bx)-\cos^3(a+bx)+\cos(a+bx)}{b}$$

input
```
Int[Sin[a + b*x]^7,x]
```

output
```
-((Cos[a + b*x] - Cos[a + b*x]^3 + (3*Cos[a + b*x]^5)/5 - Cos[a + b*x]^7/7
)/b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

Maple [A] (verified)

Time = 12.77 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

method	result
derivativedivides	$-\frac{\left(\frac{16}{5}+\sin(bx+a)^6+\frac{6\sin(bx+a)^4}{5}+\frac{8\sin(bx+a)^2}{5}\right)\cos(bx+a)}{7b}$
default	$-\frac{\left(\frac{16}{5}+\sin(bx+a)^6+\frac{6\sin(bx+a)^4}{5}+\frac{8\sin(bx+a)^2}{5}\right)\cos(bx+a)}{7b}$
parallelrisch	$\frac{-1024-1225\cos(bx+a)-49\cos(5bx+5a)+5\cos(7bx+7a)+245\cos(3bx+3a)}{2240b}$
risch	$-\frac{35\cos(bx+a)}{64b}+\frac{\cos(7bx+7a)}{448b}-\frac{7\cos(5bx+5a)}{320b}+\frac{7\cos(3bx+3a)}{64b}$
norman	$\frac{-\frac{32\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^6}{b}-\frac{32}{35b}-\frac{32\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}{5b}-\frac{96\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}{5b}}{\left(1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right)^7}$
orering	$-\frac{12916\sin(bx+a)^6\cos(bx+a)}{1575b}-\frac{94\left(210\sin(bx+a)^4b^3\cos(bx+a)^3-133\sin(bx+a)^6b^3\cos(bx+a)\right)}{525b^4}-\frac{4\left(2520\sin(b\right.}{}$

input
```
int(sin(b*x+a)^7,x,method=_RETURNVERBOSE)
```

output
```
-1/7/b*(16/5+sin(b*x+a)^6+6/5*sin(b*x+a)^4+8/5*sin(b*x+a)^2)*cos(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \sin^7(a+bx)\,dx = \frac{5\cos(bx+a)^7-21\cos(bx+a)^5+35\cos(bx+a)^3-35\cos(bx+a)}{35\,b}$$

input
```
integrate(sin(b*x+a)^7,x, algorithm="fricas")
```

output
```
1/35*(5*cos(b*x + a)^7 - 21*cos(b*x + a)^5 + 35*cos(b*x + a)^3 - 35*cos(b*
x + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.48

$$\int \sin^7(a+bx)\,dx = \begin{cases} -\frac{\sin^6(a+bx)\cos(a+bx)}{b} - \frac{2\sin^4(a+bx)\cos^3(a+bx)}{b} - \frac{8\sin^2(a+bx)\cos^5(a+bx)}{5b} - \frac{16\cos^7(a+bx)}{35b} & \text{for } b \neq 0 \\ x\sin^7(a) & \text{otherwise} \end{cases}$$

input
```
integrate(sin(b*x+a)**7,x)
```

output
```
Piecewise((-sin(a + b*x)**6*cos(a + b*x)/b - 2*sin(a + b*x)**4*cos(a + b*x
)**3/b - 8*sin(a + b*x)**2*cos(a + b*x)**5/(5*b) - 16*cos(a + b*x)**7/(35*
b), Ne(b, 0)), (x*sin(a)**7, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \sin^7(a+bx)\,dx = \frac{5\cos(bx+a)^7 - 21\cos(bx+a)^5 + 35\cos(bx+a)^3 - 35\cos(bx+a)}{35\,b}$$

input
```
integrate(sin(b*x+a)^7,x, algorithm="maxima")
```

output
```
1/35*(5*cos(b*x + a)^7 - 21*cos(b*x + a)^5 + 35*cos(b*x + a)^3 - 35*cos(b*
x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \sin^7(a+bx)\,dx = \frac{\cos(bx+a)^7}{7\,b} - \frac{3\,\cos(bx+a)^5}{5\,b} + \frac{\cos(bx+a)^3}{b} - \frac{\cos(bx+a)}{b}$$

input
```
integrate(sin(b*x+a)^7,x, algorithm="giac")
```

output
```
1/7*cos(b*x + a)^7/b - 3/5*cos(b*x + a)^5/b + cos(b*x + a)^3/b - cos(b*x +
 a)/b
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int \sin^7(a+bx)\,dx = \frac{\cos(a+b\,x)\,\left(5\cos(a+b\,x)^6 - 21\cos(a+b\,x)^4 + 35\cos(a+b\,x)^2 - 35\right)}{35\,b}$$

input
```
int(sin(a + b*x)^7,x)
```

output
```
(cos(a + b*x)*(35*cos(a + b*x)^2 - 21*cos(a + b*x)^4 + 5*cos(a + b*x)^6 -
35))/(35*b)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.17

$$\int \sin^7(a+bx)\,dx = \frac{-5\cos(bx+a)\sin(bx+a)^6 - 6\cos(bx+a)\sin(bx+a)^4 - 8\cos(bx+a)\sin(bx+a)^2 - 16\cos(bx+a}{35b}$$

input
```
int(sin(b*x+a)^7,x)
```

output
```
( - 5*cos(a + b*x)*sin(a + b*x)**6 - 6*cos(a + b*x)*sin(a + b*x)**4 - 8*co
s(a + b*x)*sin(a + b*x)**2 - 16*cos(a + b*x) + 16)/(35*b)
```

3.8 $\int \sin^8(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 88

$$\int \sin^8(a+bx)\,dx = \frac{35x}{128} - \frac{35\cos(a+bx)\sin(a+bx)}{128b} - \frac{35\cos(a+bx)\sin^3(a+bx)}{192b} - \frac{7\cos(a+bx)\sin^5(a+bx)}{48b} - \frac{\cos(a+bx)\sin^7(a+bx)}{8b}$$

output
```
35/128*x-35/128*cos(b*x+a)*sin(b*x+a)/b-35/192*cos(b*x+a)*sin(b*x+a)^3/b-7
/48*cos(b*x+a)*sin(b*x+a)^5/b-1/8*cos(b*x+a)*sin(b*x+a)^7/b
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.62

$$\int \sin^8(a+bx)\,dx = \frac{840a+840bx-672\sin(2(a+bx))+168\sin(4(a+bx))-32\sin(6(a+bx))+3\sin(8(a+bx))}{3072b}$$

input
```
Integrate[Sin[a + b*x]^8,x]
```

output
```
(840*a + 840*b*x - 672*Sin[2*(a + b*x)] + 168*Sin[4*(a + b*x)] - 32*Sin[6*
(a + b*x)] + 3*Sin[8*(a + b*x)])/(3072*b)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {3042, 3115, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^8(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^8dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{7}{8}\int \sin^6(a+bx)dx-\frac{\sin^7(a+bx)\cos(a+bx)}{8b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{8}\int \sin(a+bx)^6dx-\frac{\sin^7(a+bx)\cos(a+bx)}{8b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{7}{8}\left(\frac{5}{6}\int \sin^4(a+bx)dx-\frac{\sin^5(a+bx)\cos(a+bx)}{6b}\right)-\frac{\sin^7(a+bx)\cos(a+bx)}{8b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{8}\left(\frac{5}{6}\int \sin(a+bx)^4dx-\frac{\sin^5(a+bx)\cos(a+bx)}{6b}\right)-\frac{\sin^7(a+bx)\cos(a+bx)}{8b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \sin^2(a+bx)dx-\frac{\sin^3(a+bx)\cos(a+bx)}{4b}\right)-\frac{\sin^5(a+bx)\cos(a+bx)}{6b}\right)-\frac{\sin^7(a+bx)\cos(a+bx)}{8b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \sin(a+bx)^2dx-\frac{\sin^3(a+bx)\cos(a+bx)}{4b}\right)-\frac{\sin^5(a+bx)\cos(a+bx)}{6b}\right)-\frac{\sin^7(a+bx)\cos(a+bx)}{8b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)-\frac{\sin^3(a+bx)\cos(a+bx)}{4b}\right)-\frac{\sin^5(a+bx)\cos(a+bx)}{6b}\right)-\frac{\sin^7(a+bx)\cos(a+bx)}{8b}$$

$$\Big\downarrow \text{24}$$

$$\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)-\frac{\sin^3(a+bx)\cos(a+bx)}{4b}\right)-\frac{\sin^5(a+bx)\cos(a+bx)}{6b}\right)-\frac{\sin^7(a+bx)\cos(a+bx)}{8b}$$

input
```
Int[Sin[a + b*x]^8,x]
```

output
```
-1/8*(Cos[a + b*x]*Sin[a + b*x]^7)/b + (7*(-1/6*(Cos[a + b*x]*Sin[a + b*x]
^5)/b + (5*(-1/4*(Cos[a + b*x]*Sin[a + b*x]^3)/b + (3*(x/2 - (Cos[a + b*x]
*Sin[a + b*x])/(2*b)))/4))/6))/8
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 14.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.62

method	result
parallelrisch	$\frac{840bx+3\sin(8bx+8a)-32\sin(6bx+6a)+168\sin(4bx+4a)-672\sin(2bx+2a)}{3072b}$
derivativedivides	$\frac{-\frac{\left(\sin(bx+a)^7+\frac{7\sin(bx+a)^5}{6}+\frac{35\sin(bx+a)^3}{24}+\frac{35\sin(bx+a)}{16}\right)\cos(bx+a)}{8}+\frac{35bx}{128}+\frac{35a}{128}}{b}$
default	$\frac{-\frac{\left(\sin(bx+a)^7+\frac{7\sin(bx+a)^5}{6}+\frac{35\sin(bx+a)^3}{24}+\frac{35\sin(bx+a)}{16}\right)\cos(bx+a)}{8}+\frac{35bx}{128}+\frac{35a}{128}}{b}$
risch	$\frac{35x}{128}+\frac{\sin(8bx+8a)}{1024b}-\frac{\sin(6bx+6a)}{96b}+\frac{7\sin(4bx+4a)}{128b}-\frac{7\sin(2bx+2a)}{32b}$
norman	$\frac{\frac{35x}{128}-\frac{35\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{64b}-\frac{805\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^3}{192b}-\frac{2681\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^5}{192b}-\frac{5053\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^7}{192b}+\frac{5053\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^9}{192b}+\frac{2681\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{11}}{192b}$
orering	$x\sin\left(bx+a\right)^8-\frac{205\cos(bx+a)\sin(bx+a)^7}{72b}+\frac{205x\left(56\sin(bx+a)^6b^2\cos(bx+a)^2-8\sin(bx+a)^8b^2\right)}{576b^2}-\frac{91(3$

input
```
int(sin(b*x+a)^8,x,method=_RETURNVERBOSE)
```

output
```
1/3072*(840*b*x+3*sin(8*b*x+8*a)-32*sin(6*b*x+6*a)+168*sin(4*b*x+4*a)-672*
sin(2*b*x+2*a))/b
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

$$\int \sin^8(a+bx)\,dx$$
$$=\frac{105\,bx+\left(48\,\cos\left(bx+a\right)^7-200\,\cos\left(bx+a\right)^5+326\,\cos\left(bx+a\right)^3-279\,\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{384\,b}$$

input
```
integrate(sin(b*x+a)^8,x, algorithm="fricas")
```

output
```
1/384*(105*b*x + (48*cos(b*x + a)^7 - 200*cos(b*x + a)^5 + 326*cos(b*x + a
)^3 - 279*cos(b*x + a))*sin(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(82) = 164.

Time = 0.64 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.09

$$\int \sin^8(a+bx)\,dx$$
$$=\begin{cases}\frac{35x\sin^8(a+bx)}{128}+\frac{35x\sin^6(a+bx)\cos^2(a+bx)}{32}+\frac{105x\sin^4(a+bx)\cos^4(a+bx)}{64}+\frac{35x\sin^2(a+bx)\cos^6(a+bx)}{32}+\frac{35x\cos^8(a+bx)}{128}-\\ x\sin^8(a)\end{cases}$$

input
```
integrate(sin(b*x+a)**8,x)
```

output
```
Piecewise((35*x*sin(a + b*x)**8/128 + 35*x*sin(a + b*x)**6*cos(a + b*x)**2
/32 + 105*x*sin(a + b*x)**4*cos(a + b*x)**4/64 + 35*x*sin(a + b*x)**2*cos(
a + b*x)**6/32 + 35*x*cos(a + b*x)**8/128 - 93*sin(a + b*x)**7*cos(a + b*x
)/(128*b) - 511*sin(a + b*x)**5*cos(a + b*x)**3/(384*b) - 385*sin(a + b*x)
**3*cos(a + b*x)**5/(384*b) - 35*sin(a + b*x)*cos(a + b*x)**7/(128*b), Ne(
b, 0)), (x*sin(a)**8, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.67

$$\int \sin^8(a+bx)\,dx = \frac{128\,\sin(2\,bx+2\,a)^3+840\,bx+840\,a+3\,\sin(8\,bx+8\,a)+168\,\sin(4\,bx+4\,a)-768\,\sin(2\,bx+2\,a)}{3072\,b}$$

input
```
integrate(sin(b*x+a)^8,x, algorithm="maxima")
```

output
```
1/3072*(128*sin(2*b*x + 2*a)^3 + 840*b*x + 840*a + 3*sin(8*b*x + 8*a) + 16
8*sin(4*b*x + 4*a) - 768*sin(2*b*x + 2*a))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.68

$$\int \sin^8(a+bx)\,dx = \frac{35}{128}\,x+\frac{\sin(8\,bx+8\,a)}{1024\,b}-\frac{\sin(6\,bx+6\,a)}{96\,b}+\frac{7\,\sin(4\,bx+4\,a)}{128\,b}-\frac{7\,\sin(2\,bx+2\,a)}{32\,b}$$

input
```
integrate(sin(b*x+a)^8,x, algorithm="giac")
```

output
```
35/128*x + 1/1024*sin(8*b*x + 8*a)/b - 1/96*sin(6*b*x + 6*a)/b + 7/128*sin
(4*b*x + 4*a)/b - 7/32*sin(2*b*x + 2*a)/b
```

Mupad [B] (verification not implemented)

Time = 26.51 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.02

$$\int \sin^8(a+bx)\,dx$$
$$= \frac{35\,x}{128} - \frac{\frac{93\,\tan(a+b\,x)^7}{128} + \frac{511\,\tan(a+b\,x)^5}{384} + \frac{385\,\tan(a+b\,x)^3}{384} + \frac{35\,\tan(a+b\,x)}{128}}{b\,\left(\tan\left(a+b\,x\right)^8 + 4\tan\left(a+b\,x\right)^6 + 6\tan\left(a+b\,x\right)^4 + 4\tan\left(a+b\,x\right)^2 + 1\right)}$$

input
```
int(sin(a + b*x)^8,x)
```

output
```
(35*x)/128 - ((35*tan(a + b*x))/128 + (385*tan(a + b*x)^3)/384 + (511*tan(
a + b*x)^5)/384 + (93*tan(a + b*x)^7)/128)/(b*(4*tan(a + b*x)^2 + 6*tan(a
+ b*x)^4 + 4*tan(a + b*x)^6 + tan(a + b*x)^8 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.82

$$\int \sin^8(a+bx)\,dx$$
$$= \frac{-48\cos\left(bx+a\right)\sin\left(bx+a\right)^7 - 56\cos\left(bx+a\right)\sin\left(bx+a\right)^5 - 70\cos\left(bx+a\right)\sin\left(bx+a\right)^3 - 105\cos\left(bx\right.}{384b}$$

input
```
int(sin(b*x+a)^8,x)
```

output
```
( - 48*cos(a + b*x)*sin(a + b*x)**7 - 56*cos(a + b*x)*sin(a + b*x)**5 - 70
*cos(a + b*x)*sin(a + b*x)**3 - 105*cos(a + b*x)*sin(a + b*x) + 105*b*x)/(
384*b)
```

3.9 $\int \sin^{\frac{7}{2}}(bx)\, dx$

Optimal result

Integrand size = 8, antiderivative size = 60

$$\int \sin^{\frac{7}{2}}(bx)\, dx = -\frac{10\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{bx}{2},2\right)}{21b}-\frac{10\cos(bx)\sqrt{\sin(bx)}}{21b}-\frac{2\cos(bx)\sin^{\frac{5}{2}}(bx)}{7b}$$

output
```
10/21*InverseJacobiAM(-1/4*Pi+1/2*b*x,2^(1/2))/b-10/21*cos(b*x)*sin(b*x)^(
1/2)/b-2/7*cos(b*x)*sin(b*x)^(5/2)/b
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.75

$$\int \sin^{\frac{7}{2}}(bx)\, dx = \frac{-20\,\text{EllipticF}\left(\frac{1}{4}(\pi-2bx),2\right)+(-23\cos(bx)+3\cos(3bx))\sqrt{\sin(bx)}}{42b}$$

input
```
Integrate[Sin[b*x]^(7/2),x]
```

output
```
(-20*EllipticF[(Pi - 2*b*x)/4, 2] + (-23*Cos[b*x] + 3*Cos[3*b*x])*Sqrt[Sin
[b*x]])/(42*b)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 3115, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{7}{2}}(bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sin(bx)^{7/2}dx$$

$$\Big\downarrow \ 3115$$

$$\frac{5}{7}\int \sin^{\frac{3}{2}}(bx)dx-\frac{2\sin^{\frac{5}{2}}(bx)\cos(bx)}{7b}$$

$$\Big\downarrow \ 3042$$

$$\frac{5}{7}\int \sin(bx)^{3/2}dx-\frac{2\sin^{\frac{5}{2}}(bx)\cos(bx)}{7b}$$

$$\Big\downarrow \ 3115$$

$$\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(bx)}}dx-\frac{2\sqrt{\sin(bx)}\cos(bx)}{3b}\right)-\frac{2\sin^{\frac{5}{2}}(bx)\cos(bx)}{7b}$$

$$\Big\downarrow \ 3042$$

$$\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(bx)}}dx-\frac{2\sqrt{\sin(bx)}\cos(bx)}{3b}\right)-\frac{2\sin^{\frac{5}{2}}(bx)\cos(bx)}{7b}$$

$$\Big\downarrow \ 3120$$

$$\frac{5}{7}\left(-\frac{2\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{bx}{2},2\right)}{3b}-\frac{2\sqrt{\sin(bx)}\cos(bx)}{3b}\right)-\frac{2\sin^{\frac{5}{2}}(bx)\cos(bx)}{7b}$$

input
```
Int[Sin[b*x]^(7/2),x]
```

output
```
(5*((-2*EllipticF[Pi/4 - (b*x)/2, 2])/(3*b) - (2*Cos[b*x]*Sqrt[Sin[b*x]])/
(3*b)))/7 - (2*Cos[b*x]*Sin[b*x]^(5/2))/(7*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 2.43 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.40

method	result	size
default	$\frac{\frac{2\cos(bx)^4\sin(bx)}{7}+\frac{5\sqrt{\sin(bx)+1}\,\sqrt{-2\sin(bx)+2}\,\sqrt{-\sin(bx)}\,\operatorname{EllipticF}\left(\sqrt{\sin(bx)+1},\frac{\sqrt{2}}{2}\right)}{21}-\frac{16\cos(bx)^2\sin(bx)}{21}}{\cos(bx)\sqrt{\sin(bx)}\,b}$	84

input
```
int(sin(b*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(2/7*cos(b*x)^4*sin(b*x)+5/21*(sin(b*x)+1)^(1/2)*(-2*sin(b*x)+2)^(1/2)*(-s
in(b*x))^(1/2)*EllipticF((sin(b*x)+1)^(1/2),1/2*2^(1/2))-16/21*cos(b*x)^2*
sin(b*x))/cos(b*x)/sin(b*x)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.10

$$\int \sin^{\frac{7}{2}}(bx)\,dx = \frac{2\left(\left(3\cos(bx)^3 - 8\cos(bx)\right)\sqrt{\sin(bx)} + 5\sqrt{-\frac{1}{2}i}\,\text{weierstrassPInverse}(4,0,\cos(bx) + i\sin(bx)) + 5\sqrt{\frac{1}{2}}\right.}{21\,b}$$

input
```
integrate(sin(b*x)^(7/2),x, algorithm="fricas")
```

output
```
2/21*((3*cos(b*x)^3 - 8*cos(b*x))*sqrt(sin(b*x)) + 5*sqrt(-1/2*I)*weierstr
assPInverse(4, 0, cos(b*x) + I*sin(b*x)) + 5*sqrt(1/2*I)*weierstrassPInver
se(4, 0, cos(b*x) - I*sin(b*x)))/b
```

Sympy [F(-1)]

Timed out.

$$\int \sin^{\frac{7}{2}}(bx)\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sin^{\frac{7}{2}}(bx)\,dx = \int \sin(bx)^{\frac{7}{2}}\,dx$$

input
```
integrate(sin(b*x)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x)^(7/2), x)
```

Giac [F]

$$\int \sin^{\frac{7}{2}}(bx)\,dx = \int \sin(bx)^{\frac{7}{2}}\,dx$$

input
```
integrate(sin(b*x)^(7/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 25.39 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.57

$$\int \sin^{\frac{7}{2}}(bx)\,dx = -\frac{\cos(b\,x)\,\sin(b\,x)^{9/2}\,{}_2F_1\left(-\frac{5}{4},\frac{1}{2};\ \frac{3}{2};\ \cos(b\,x)^2\right)}{b\,\left(\sin(b\,x)^2\right)^{9/4}}$$

input
```
int(sin(b*x)^(7/2),x)
```

output
```
-(cos(b*x)*sin(b*x)^(9/2)*hypergeom([-5/4, 1/2], 3/2, cos(b*x)^2))/(b*(sin
(b*x)^2)^(9/4))
```

Reduce [F]

$$\int \sin^{\frac{7}{2}}(bx)\,dx = \int \sqrt{\sin(bx)}\,\sin(bx)^3\,dx$$

input
```
int(sin(b*x)^(7/2),x)
```

output
```
int(sqrt(sin(b*x))*sin(b*x)**3,x)
```

3.10 $\int \sin^{\frac{5}{2}}(bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 41

$$\int \sin^{\frac{5}{2}}(bx)\,dx = -\frac{6E\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{5b}-\frac{2\cos(bx)\sin^{\frac{3}{2}}(bx)}{5b}$$

output
```
-6/5*EllipticE(cos(1/4*Pi+1/2*b*x),2^(1/2))/b-2/5*cos(b*x)*sin(b*x)^(3/2)/
b
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \sin^{\frac{5}{2}}(bx)\,dx = -\frac{2\left(3E\left(\frac{1}{4}(\pi-2bx)\middle|2\right)+\cos(bx)\sin^{\frac{3}{2}}(bx)\right)}{5b}$$

input
```
Integrate[Sin[b*x]^(5/2),x]
```

output
```
(-2*(3*EllipticE[(Pi - 2*b*x)/4, 2] + Cos[b*x]*Sin[b*x]^(3/2)))/(5*b)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3115, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{5}{2}}(bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(bx)^{5/2}dx$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{3}{5}\int \sqrt{\sin(bx)}dx-\frac{2\sin^{\frac{3}{2}}(bx)\cos(bx)}{5b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3}{5}\int \sqrt{\sin(bx)}dx-\frac{2\sin^{\frac{3}{2}}(bx)\cos(bx)}{5b}$$

$$\Big\downarrow \text{ 3119}$$

$$-\frac{6E\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{5b}-\frac{2\sin^{\frac{3}{2}}(bx)\cos(bx)}{5b}$$

input
```
Int[Sin[b*x]^(5/2),x]
```

output
```
(-6*EllipticE[Pi/4 - (b*x)/2, 2])/(5*b) - (2*Cos[b*x]*Sin[b*x]^(3/2))/(5*b
)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(34) = 68$.

Time = 2.18 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.88

method	result
default	$\frac{\frac{2\sin(bx)^4}{5}-\frac{2\sin(bx)^2}{5}-\frac{6\sqrt{\sin(bx)+1}\,\sqrt{-2\sin(bx)+2}\,\sqrt{-\sin(bx)}\ \mathrm{EllipticE}\left(\sqrt{\sin(bx)+1},\frac{\sqrt{2}}{2}\right)}{5}+\frac{3\sqrt{\sin(bx)+1}\,\sqrt{-2\sin(bx)+2}\,\sqrt{-\sin(bx)}\ \mathrm{Elliptic}}{5}}{\cos(bx)\sqrt{\sin(bx)}\,b}$

input
```
int(sin(b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(2/5*sin(b*x)^4-2/5*sin(b*x)^2-6/5*(sin(b*x)+1)^(1/2)*(-2*sin(b*x)+2)^(1/2
)*(-sin(b*x))^(1/2)*EllipticE((sin(b*x)+1)^(1/2),1/2*2^(1/2))+3/5*(sin(b*x
)+1)^(1/2)*(-2*sin(b*x)+2)^(1/2)*(-sin(b*x))^(1/2)*EllipticF((sin(b*x)+1)^
(1/2),1/2*2^(1/2)))/cos(b*x)/sin(b*x)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.49

$$\int \sin^{\frac{5}{2}}(bx)\,dx =$$
$$-\frac{2\left(\cos(bx)\sin(bx)^{\frac{3}{2}} - 3i\sqrt{-\frac{1}{2}i}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(bx)+i\,\sin(bx))) -\right.}{5b}$$

input
```
integrate(sin(b*x)^(5/2),x, algorithm="fricas")
```

output
```
-2/5*(cos(b*x)*sin(b*x)^(3/2) - 3*I*sqrt(-1/2*I)*weierstrassZeta(4, 0, wei
erstrassPInverse(4, 0, cos(b*x) + I*sin(b*x))) + 3*I*sqrt(1/2*I)*weierstra
ssZeta(4, 0, weierstrassPInverse(4, 0, cos(b*x) - I*sin(b*x))))/b
```

Sympy [F]

$$\int \sin^{\frac{5}{2}}(bx)\,dx = \int \sin^{\frac{5}{2}}(bx)\,dx$$

input
```
integrate(sin(b*x)**(5/2),x)
```

output
```
Integral(sin(b*x)**(5/2), x)
```

Maxima [F]

$$\int \sin^{\frac{5}{2}}(bx)\,dx = \int \sin(bx)^{\frac{5}{2}}\,dx$$

input
```
integrate(sin(b*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x)^(5/2), x)
```

Giac [F]

$$\int \sin^{\frac{5}{2}}(bx)\,dx = \int \sin(bx)^{\frac{5}{2}}\,dx$$

input
```
integrate(sin(b*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 25.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \sin^{\frac{5}{2}}(bx)\,dx = -\frac{\cos(b\,x)\,\sin(b\,x)^{7/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{3}{2};\ \cos(b\,x)^2\right)}{b\,\left(\sin(b\,x)^2\right)^{7/4}}$$

input
```
int(sin(b*x)^(5/2),x)
```

output
```
-(cos(b*x)*sin(b*x)^(7/2)*hypergeom([-3/4, 1/2], 3/2, cos(b*x)^2))/(b*(sin
(b*x)^2)^(7/4))
```

Reduce [F]

$$\int \sin^{\frac{5}{2}}(bx)\,dx = \int \sqrt{\sin(bx)}\,\sin(bx)^2\,dx$$

input
```
int(sin(b*x)^(5/2),x)
```

output
```
int(sqrt(sin(b*x))*sin(b*x)**2,x)
```

3.11 $\int \sin^{\frac{3}{2}}(bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 41

$$\int \sin^{\frac{3}{2}}(bx)\,dx=-\frac{2\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{bx}{2},2\right)}{3b}-\frac{2\cos(bx)\sqrt{\sin(bx)}}{3b}$$

output
```
2/3*InverseJacobiAM(-1/4*Pi+1/2*b*x,2^(1/2))/b-2/3*cos(b*x)*sin(b*x)^(1/2)
/b
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \sin^{\frac{3}{2}}(bx)\,dx=-\frac{2\left(\text{EllipticF}\left(\frac{1}{4}(\pi-2bx),2\right)+\cos(bx)\sqrt{\sin(bx)}\right)}{3b}$$

input
```
Integrate[Sin[b*x]^(3/2),x]
```

output
```
(-2*(EllipticF[(Pi - 2*b*x)/4, 2] + Cos[b*x]*Sqrt[Sin[b*x]]))/(3*b)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{3}{2}}(bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(bx)^{3/2}dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{3}\int \frac{1}{\sqrt{\sin(bx)}}dx - \frac{2\sqrt{\sin(bx)}\cos(bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{1}{\sqrt{\sin(bx)}}dx - \frac{2\sqrt{\sin(bx)}\cos(bx)}{3b}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{2\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{bx}{2},2\right)}{3b} - \frac{2\sqrt{\sin(bx)}\cos(bx)}{3b}$$

input
```
Int[Sin[b*x]^(3/2),x]
```

output
```
(-2*EllipticF[Pi/4 - (b*x)/2, 2])/(3*b) - (2*Cos[b*x]*Sqrt[Sin[b*x]])/(3*b
)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(33) = 66$.

Time = 2.02 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.76

method	result	size
default	$\frac{\frac{\sqrt{\sin(bx)+1}\,\sqrt{-2\sin(bx)+2}\,\sqrt{-\sin(bx)}\,\operatorname{EllipticF}\left(\sqrt{\sin(bx)+1},\frac{\sqrt{2}}{2}\right)}{3}-\frac{2\cos(bx)^{2}\sin(bx)}{3}}{\cos(bx)\sqrt{\sin(bx)}\,b}$	72

input
```
int(sin(b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(1/3*(sin(b*x)+1)^(1/2)*(-2*sin(b*x)+2)^(1/2)*(-sin(b*x))^(1/2)*EllipticF(
(sin(b*x)+1)^(1/2),1/2*2^(1/2))-2/3*cos(b*x)^2*sin(b*x))/cos(b*x)/sin(b*x)
^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.34

$$\int \sin^{\frac{3}{2}}(bx)\,dx =$$
$$-\frac{2\left(\cos(bx)\sqrt{\sin(bx)} - \sqrt{-\frac{1}{2}i}\text{weierstrassPInverse}(4,0,\cos(bx)+i\sin(bx)) - \sqrt{\frac{1}{2}i}\text{weierstrassPInve}\right.}{3\,b}$$

input
```
integrate(sin(b*x)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(cos(b*x)*sqrt(sin(b*x)) - sqrt(-1/2*I)*weierstrassPInverse(4, 0, cos
(b*x) + I*sin(b*x)) - sqrt(1/2*I)*weierstrassPInverse(4, 0, cos(b*x) - I*s
in(b*x)))/b
```

Sympy [F]

$$\int \sin^{\frac{3}{2}}(bx)\,dx = \int \sin^{\frac{3}{2}}(bx)\,dx$$

input
```
integrate(sin(b*x)**(3/2),x)
```

output
```
Integral(sin(b*x)**(3/2), x)
```

Maxima [F]

$$\int \sin^{\frac{3}{2}}(bx)\,dx = \int \sin(bx)^{\frac{3}{2}}\,dx$$

input
```
integrate(sin(b*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x)^(3/2), x)
```

Giac [F]

$$\int \sin^{\frac{3}{2}}(bx)\,dx = \int \sin(bx)^{\frac{3}{2}}\,dx$$

input
```
integrate(sin(b*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 25.34 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \sin^{\frac{3}{2}}(bx)\,dx = -\frac{\cos(b\,x)\,\sin(b\,x)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ \cos(b\,x)^2\right)}{b\left(\sin(b\,x)^2\right)^{5/4}}$$

input
```
int(sin(b*x)^(3/2),x)
```

output
```
-(cos(b*x)*sin(b*x)^(5/2)*hypergeom([-1/4, 1/2], 3/2, cos(b*x)^2))/(b*(sin
(b*x)^2)^(5/4))
```

Reduce [F]

$$\int \sin^{\frac{3}{2}}(bx)\,dx = \int \sqrt{\sin(bx)}\,\sin(bx)\,dx$$

input
```
int(sin(b*x)^(3/2),x)
```

output
```
int(sqrt(sin(b*x))*sin(b*x),x)
```

3.12 $\int \sqrt{\sin(bx)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 19

$$\int \sqrt{\sin(bx)}\,dx = -\frac{2E\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{b}$$

output
```
-2*EllipticE(cos(1/4*Pi+1/2*b*x),2^(1/2))/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \sqrt{\sin(bx)}\,dx = -\frac{2E\left(\frac{1}{2}\left(\frac{\pi}{2}-bx\right)\middle|2\right)}{b}$$

input
```
Integrate[Sqrt[Sin[b*x]],x]
```

output
```
(-2*EllipticE[(Pi/2 - b*x)/2, 2])/b
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sin(bx)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \sqrt{\sin(bx)}dx$$
$$\Big\downarrow \text{ 3119}$$
$$-\frac{2E\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{b}$$

input
```
Int[Sqrt[Sin[b*x]],x]
```

output
```
(-2*EllipticE[Pi/4 - (b*x)/2, 2])/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(18) = 36$.

Time = 2.99 (sec) , antiderivative size = 77, normalized size of antiderivative = 4.05

method	result
default	$-\frac{\sqrt{\sin(bx)+1}\,\sqrt{-2\sin(bx)+2}\,\sqrt{-\sin(bx)}\left(2\,\mathrm{EllipticE}\left(\sqrt{\sin(bx)+1},\frac{\sqrt{2}}{2}\right)-\mathrm{EllipticF}\left(\sqrt{\sin(bx)+1},\frac{\sqrt{2}}{2}\right)\right)}{\cos(bx)\sqrt{\sin(bx)}\,b}$
risch	$-\frac{i\sqrt{2}\,\sqrt{-i\left(e^{2ibx}-1\right)e^{-ibx}}}{b}+\frac{i\left(\frac{2i\left(i-ie^{2ibx}\right)}{\sqrt{e^{ibx}\left(i-ie^{2ibx}\right)}}-\frac{\sqrt{e^{ibx}+1}\,\sqrt{-2\,e^{ibx}+2}\,\sqrt{-e^{ibx}}\left(-2\,\mathrm{EllipticE}\left(\sqrt{e^{ibx}+1},\frac{\sqrt{2}}{2}\right)+\mathrm{EllipticF}\left(\sqrt{e^{ibx}+1},\right.\right.}{\sqrt{-ie^{3ibx}+ie^{ibx}}}\right.}{b\left(e^{2ibx}-1\right)}$

input
```
int(sin(b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(sin(b*x)+1)^(1/2)*(-2*sin(b*x)+2)^(1/2)*(-sin(b*x))^(1/2)*(2*EllipticE((
sin(b*x)+1)^(1/2),1/2*2^(1/2))-EllipticF((sin(b*x)+1)^(1/2),1/2*2^(1/2)))/
cos(b*x)/sin(b*x)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.63

$$\int \sqrt{\sin(bx)}\,dx = -\frac{2\left(-i\sqrt{-\frac{1}{2}i}\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(bx)+i\sin(bx)))+i\sqrt{\frac{1}{2}i}\mathrm{weierstrass}\ldots\right.}{b}$$

input
```
integrate(sin(b*x)^(1/2),x, algorithm="fricas")
```

output
```
-2*(-I*sqrt(-1/2*I)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(b*
x) + I*sin(b*x))) + I*sqrt(1/2*I)*weierstrassZeta(4, 0, weierstrassPInvers
e(4, 0, cos(b*x) - I*sin(b*x))))/b
```

Sympy [F]

$$\int \sqrt{\sin(bx)}\,dx = \int \sqrt{\sin{(bx)}}\,dx$$

input
```
integrate(sin(b*x)**(1/2),x)
```

output
```
Integral(sqrt(sin(b*x)), x)
```

Maxima [F]

$$\int \sqrt{\sin(bx)}\,dx = \int \sqrt{\sin{(bx)}}\,dx$$

input
```
integrate(sin(b*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sin(b*x)), x)
```

Giac [F]

$$\int \sqrt{\sin(bx)}\,dx = \int \sqrt{\sin{(bx)}}\,dx$$

input
```
integrate(sin(b*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sin(b*x)), x)
```

Mupad [B] (verification not implemented)

Time = 25.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \sqrt{\sin(bx)}\,dx = -\frac{2\,\mathrm{E}\left(\frac{\pi}{4}-\frac{b\,x}{2}\middle|2\right)}{b}$$

input
```
int(sin(b*x)^(1/2),x)
```

output
```
-(2*ellipticE(pi/4 - (b*x)/2, 2))/b
```

Reduce [F]

$$\int \sqrt{\sin(bx)}\,dx = \int \sqrt{\sin{(bx)}}dx$$

input
```
int(sin(b*x)^(1/2),x)
```

output
```
int(sqrt(sin(b*x)),x)
```

3.13 $\int \frac{1}{\sqrt{\sin(bx)}}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 19

$$\int \frac{1}{\sqrt{\sin(bx)}}\,dx = -\frac{2\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{bx}{2},2\right)}{b}$$

output
```
2*InverseJacobiAM(-1/4*Pi+1/2*b*x,2^(1/2))/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{\sin(bx)}}\,dx = -\frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(\frac{\pi}{2}-bx\right),2\right)}{b}$$

input
```
Integrate[1/Sqrt[Sin[b*x]],x]
```

output
```
(-2*EllipticF[(Pi/2 - b*x)/2, 2])/b
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sin(bx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sqrt{\sin(bx)}}dx$$
$$\Big\downarrow \text{3120}$$
$$-\frac{2\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{bx}{2},2\right)}{b}$$

input
```
Int[1/Sqrt[Sin[b*x]],x]
```

output
```
(-2*EllipticF[Pi/4 - (b*x)/2, 2])/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(17) = 34$.

Time = 1.74 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.00

method	result	size
default	$\frac{\sqrt{\sin(bx)+1}\,\sqrt{-2\sin(bx)+2}\,\sqrt{-\sin(bx)}\,\operatorname{EllipticF}\left(\sqrt{\sin(bx)+1},\frac{\sqrt{2}}{2}\right)}{\cos(bx)\sqrt{\sin(bx)}\,b}$	57

input
```
int(1/sin(b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(sin(b*x)+1)^(1/2)*(-2*sin(b*x)+2)^(1/2)*(-sin(b*x))^(1/2)*EllipticF((sin(
b*x)+1)^(1/2),1/2*2^(1/2))/cos(b*x)/sin(b*x)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.21

$$\int \frac{1}{\sqrt{\sin(bx)}}\,dx = \frac{2\left(\sqrt{-\frac{1}{2}i}\text{weierstrassPInverse}(4,0,\cos(bx)+i\sin(bx))+\sqrt{\frac{1}{2}i}\text{weierstrassPInverse}(4,0,\cos(bx)-i\sin$$

$$b$$

input
```
integrate(1/sin(b*x)^(1/2),x, algorithm="fricas")
```

output
```
2*(sqrt(-1/2*I)*weierstrassPInverse(4, 0, cos(b*x) + I*sin(b*x)) + sqrt(1/
2*I)*weierstrassPInverse(4, 0, cos(b*x) - I*sin(b*x)))/b
```

Sympy [F]

$$\int \frac{1}{\sqrt{\sin(bx)}}\,dx = \int \frac{1}{\sqrt{\sin{(bx)}}}\,dx$$

input
```
integrate(1/sin(b*x)**(1/2),x)
```

output
```
Integral(1/sqrt(sin(b*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\sin(bx)}}\,dx = \int \frac{1}{\sqrt{\sin{(bx)}}}\,dx$$

input
```
integrate(1/sin(b*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(sin(b*x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\sin(bx)}}\,dx = \int \frac{1}{\sqrt{\sin{(bx)}}}\,dx$$

input
```
integrate(1/sin(b*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(sin(b*x)), x)
```

Mupad [B] (verification not implemented)

Time = 25.45 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{\sin(bx)}}\,dx = -\frac{2\,\mathrm{F}\left(\frac{\pi}{4}-\frac{b\,x}{2}\middle|2\right)}{b}$$

input
```
int(1/sin(b*x)^(1/2),x)
```

output
```
-(2*ellipticF(pi/4 - (b*x)/2, 2))/b
```

Reduce [F]

$$\int \frac{1}{\sqrt{\sin(bx)}}\,dx = \int \frac{\sqrt{\sin(bx)}}{\sin(bx)}dx$$

input
```
int(1/sin(b*x)^(1/2),x)
```

output
```
int(sqrt(sin(b*x))/sin(b*x),x)
```

3.14 $\int \frac{1}{\sin^{\frac{3}{2}}(bx)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 37

$$\int \frac{1}{\sin^{\frac{3}{2}}(bx)}\,dx = \frac{2E\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{b} - \frac{2\cos(bx)}{b\sqrt{\sin(bx)}}$$

output
```
2*EllipticE(cos(1/4*Pi+1/2*b*x),2^(1/2))/b-2*cos(b*x)/b/sin(b*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sin^{\frac{3}{2}}(bx)}\,dx = \frac{2\left(E\left(\frac{1}{4}(\pi-2bx)\middle|2\right)-\frac{\cos(bx)}{\sqrt{\sin(bx)}}\right)}{b}$$

input
```
Integrate[Sin[b*x]^(-3/2),x]
```

output
```
(2*(EllipticE[(Pi - 2*b*x)/4, 2] - Cos[b*x]/Sqrt[Sin[b*x]]))/b
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin^{\frac{3}{2}}(bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(bx)^{3/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$-\int \sqrt{\sin(bx)}dx-\frac{2\cos(bx)}{b\sqrt{\sin(bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\int \sqrt{\sin(bx)}dx-\frac{2\cos(bx)}{b\sqrt{\sin(bx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2E\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{b}-\frac{2\cos(bx)}{b\sqrt{\sin(bx)}}$$

input
```
Int[Sin[b*x]^(-3/2),x]
```

output
```
(2*EllipticE[Pi/4 - (b*x)/2, 2])/b - (2*Cos[b*x])/(b*Sqrt[Sin[b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(34) = 68$.

Time = 2.34 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.97

method	result
default	$\frac{2\sqrt{\sin(bx)+1}\,\sqrt{-2\sin(bx)+2}\,\sqrt{-\sin(bx)}\ \operatorname{EllipticE}\left(\sqrt{\sin(bx)+1},\frac{\sqrt{2}}{2}\right)-\sqrt{\sin(bx)+1}\,\sqrt{-2\sin(bx)+2}\,\sqrt{-\sin(bx)}\ \operatorname{EllipticF}\left(\sqrt{\sin}\right.}{\cos(bx)\sqrt{\sin(bx)}\,b}$

input
```
int(1/sin(b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(2*(sin(b*x)+1)^(1/2)*(-2*sin(b*x)+2)^(1/2)*(-sin(b*x))^(1/2)*EllipticE((s
in(b*x)+1)^(1/2),1/2*2^(1/2))-(sin(b*x)+1)^(1/2)*(-2*sin(b*x)+2)^(1/2)*(-s
in(b*x))^(1/2)*EllipticF((sin(b*x)+1)^(1/2),1/2*2^(1/2))-2*cos(b*x)^2)/cos
(b*x)/sin(b*x)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.03

$$\int \frac{1}{\sin^{\frac{3}{2}}(bx)}\,dx = -\frac{2\left(i\sqrt{-\frac{1}{2}i}\sin(bx)\,\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(bx)+i\sin(bx)))-i\sqrt{\frac{1}{2}i}\sin(b\right.}{b\sin(bx}$$

input
```
integrate(1/sin(b*x)^(3/2),x, algorithm="fricas")
```

output
```
-2*(I*sqrt(-1/2*I)*sin(b*x)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0
, cos(b*x) + I*sin(b*x))) - I*sqrt(1/2*I)*sin(b*x)*weierstrassZeta(4, 0, w
eierstrassPInverse(4, 0, cos(b*x) - I*sin(b*x))) + cos(b*x)*sqrt(sin(b*x))
)/(b*sin(b*x))
```

Sympy [F]

$$\int \frac{1}{\sin^{\frac{3}{2}}(bx)}\,dx = \int \frac{1}{\sin^{\frac{3}{2}}(bx)}\,dx$$

input
```
integrate(1/sin(b*x)**(3/2),x)
```

output
```
Integral(sin(b*x)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\sin^{\frac{3}{2}}(bx)}\,dx = \int \frac{1}{\sin(bx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sin(b*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\sin^{\frac{3}{2}}(bx)}\,dx = \int \frac{1}{\sin(bx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sin(b*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x)^(-3/2), x)
```

Mupad [B] (verification not implemented)

Time = 25.42 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sin^{\frac{3}{2}}(bx)}\,dx = -\frac{\cos(b\,x)\,\left(\sin(b\,x)^2\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{5}{4};\ \frac{3}{2};\ \cos(b\,x)^2\right)}{b\,\sqrt{\sin(b\,x)}}$$

input
```
int(1/sin(b*x)^(3/2),x)
```

output
```
-(cos(b*x)*(sin(b*x)^2)^(1/4)*hypergeom([1/2, 5/4], 3/2, cos(b*x)^2))/(b*s
in(b*x)^(1/2))
```

Reduce [F]

$$\int \frac{1}{\sin^{\frac{3}{2}}(bx)}\,dx = \int \frac{\sqrt{\sin(bx)}}{\sin(bx)^2}\,dx$$

input
```
int(1/sin(b*x)^(3/2),x)
```

output
```
int(sqrt(sin(b*x))/sin(b*x)**2,x)
```

3.15 $\int \frac{1}{\sin^{\frac{5}{2}}(bx)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 41

$$\int \frac{1}{\sin^{\frac{5}{2}}(bx)}\,dx = -\frac{2\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{bx}{2},2\right)}{3b}-\frac{2\cos(bx)}{3b\sin^{\frac{3}{2}}(bx)}$$

output
```
2/3*InverseJacobiAM(-1/4*Pi+1/2*b*x,2^(1/2))/b-2/3*cos(b*x)/b/sin(b*x)^(3/
2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sin^{\frac{5}{2}}(bx)}\,dx = -\frac{2\left(\text{EllipticF}\left(\frac{1}{4}(\pi-2bx),2\right)+\frac{\cos(bx)}{\sin^{\frac{3}{2}}(bx)}\right)}{3b}$$

input
```
Integrate[Sin[b*x]^(-5/2),x]
```

output
```
(-2*(EllipticF[(Pi - 2*b*x)/4, 2] + Cos[b*x]/Sin[b*x]^(3/2)))/(3*b)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3116, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin^{\frac{5}{2}}(bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(bx)^{5/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$\frac{1}{3}\int \frac{1}{\sqrt{\sin(bx)}}dx-\frac{2\cos(bx)}{3b\sin^{\frac{3}{2}}(bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{1}{\sqrt{\sin(bx)}}dx-\frac{2\cos(bx)}{3b\sin^{\frac{3}{2}}(bx)}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{2\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{bx}{2},2\right)}{3b}-\frac{2\cos(bx)}{3b\sin^{\frac{3}{2}}(bx)}$$

input
```
Int[Sin[b*x]^(-5/2),x]
```

output
```
(-2*EllipticF[Pi/4 - (b*x)/2, 2])/(3*b) - (2*Cos[b*x])/(3*b*Sin[b*x]^(3/2)
)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(33) = 66$.

Time = 2.02 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.76

method	result	size
default	$\frac{\sqrt{\sin(bx)+1}\,\sqrt{-2\sin(bx)+2}\,\sqrt{-\sin(bx)}\,\operatorname{EllipticF}\left(\sqrt{\sin(bx)+1},\frac{\sqrt{2}}{2}\right)\sin(bx)-2\cos(bx)^{2}}{3\sin(bx)^{\frac{3}{2}}\cos(bx)b}$	72

input
```
int(1/sin(b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/sin(b*x)^(3/2)*((sin(b*x)+1)^(1/2)*(-2*sin(b*x)+2)^(1/2)*(-sin(b*x))^(
1/2)*EllipticF((sin(b*x)+1)^(1/2),1/2*2^(1/2))*sin(b*x)-2*cos(b*x)^2)/cos(
b*x)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.95

$$\int \frac{1}{\sin^{\frac{5}{2}}(bx)}\,dx = \frac{2\left(\sqrt{-\frac{1}{2}i}\left(\cos(bx)^2-1\right)\text{weierstrassPInverse}(4,0,\cos(bx)+i\,\sin(bx))+\sqrt{\frac{1}{2}i}\left(\cos(bx)^2-1\right)\text{weierstrass}\right.}{3\left(b\cos(bx)^2-b\right)}$$

input
```
integrate(1/sin(b*x)^(5/2),x, algorithm="fricas")
```

output
```
2/3*(sqrt(-1/2*I)*(cos(b*x)^2 - 1)*weierstrassPInverse(4, 0, cos(b*x) + I*
sin(b*x)) + sqrt(1/2*I)*(cos(b*x)^2 - 1)*weierstrassPInverse(4, 0, cos(b*x
) - I*sin(b*x)) + cos(b*x)*sqrt(sin(b*x)))/(b*cos(b*x)^2 - b)
```

Sympy [F]

$$\int \frac{1}{\sin^{\frac{5}{2}}(bx)}\,dx = \int \frac{1}{\sin^{\frac{5}{2}}(bx)}\,dx$$

input
```
integrate(1/sin(b*x)**(5/2),x)
```

output
```
Integral(sin(b*x)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{\sin^{\frac{5}{2}}(bx)}\,dx = \int \frac{1}{\sin(bx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/sin(b*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{\sin^{\frac{5}{2}}(bx)}\,dx = \int \frac{1}{\sin(bx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/sin(b*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x)^(-5/2), x)
```

Mupad [B] (verification not implemented)

Time = 25.51 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sin^{\frac{5}{2}}(bx)}\,dx = -\frac{\cos(b\,x)\,\left(\sin(b\,x)^2\right)^{3/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{3}{2};\ \cos(b\,x)^2\right)}{b\sin(b\,x)^{3/2}}$$

input
```
int(1/sin(b*x)^(5/2),x)
```

output
```
-(cos(b*x)*(sin(b*x)^2)^(3/4)*hypergeom([1/2, 7/4], 3/2, cos(b*x)^2))/(b*s
in(b*x)^(3/2))
```

Reduce [F]

$$\int \frac{1}{\sin^{\frac{5}{2}}(bx)}\,dx = \int \frac{\sqrt{\sin{(bx)}}}{\sin{(bx)}^3}dx$$

input
```
int(1/sin(b*x)^(5/2),x)
```

output
```
int(sqrt(sin(b*x))/sin(b*x)**3,x)
```

3.16 $\int \frac{1}{\sin^{\frac{7}{2}}(bx)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 60

$$\int \frac{1}{\sin^{\frac{7}{2}}(bx)}\,dx = \frac{6E\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{5b}-\frac{2\cos(bx)}{5b\sin^{\frac{5}{2}}(bx)}-\frac{6\cos(bx)}{5b\sqrt{\sin(bx)}}$$

output
```
6/5*EllipticE(cos(1/4*Pi+1/2*b*x),2^(1/2))/b-2/5*cos(b*x)/b/sin(b*x)^(5/2)
-6/5*cos(b*x)/b/sin(b*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sin^{\frac{7}{2}}(bx)}\,dx = \frac{-7\cos(bx)+3\cos(3bx)+12E\left(\frac{1}{4}(\pi-2bx)\middle|2\right)\sin^{\frac{5}{2}}(bx)}{10b\sin^{\frac{5}{2}}(bx)}$$

input
```
Integrate[Sin[b*x]^(-7/2),x]
```

output
```
(-7*Cos[b*x] + 3*Cos[3*b*x] + 12*EllipticE[(Pi - 2*b*x)/4, 2]*Sin[b*x]^(5/
2))/(10*b*Sin[b*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3116, 3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin^{\frac{7}{2}}(bx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin(bx)^{7/2}}dx$$
$$\Big\downarrow \text{3116}$$
$$\frac{3}{5}\int \frac{1}{\sin^{\frac{3}{2}}(bx)}dx-\frac{2\cos(bx)}{5b\sin^{\frac{5}{2}}(bx)}$$
$$\Big\downarrow \text{3042}$$
$$\frac{3}{5}\int \frac{1}{\sin(bx)^{3/2}}dx-\frac{2\cos(bx)}{5b\sin^{\frac{5}{2}}(bx)}$$
$$\Big\downarrow \text{3116}$$
$$\frac{3}{5}\left(-\int \sqrt{\sin(bx)}dx-\frac{2\cos(bx)}{b\sqrt{\sin(bx)}}\right)-\frac{2\cos(bx)}{5b\sin^{\frac{5}{2}}(bx)}$$
$$\Big\downarrow \text{3042}$$
$$\frac{3}{5}\left(-\int \sqrt{\sin(bx)}dx-\frac{2\cos(bx)}{b\sqrt{\sin(bx)}}\right)-\frac{2\cos(bx)}{5b\sin^{\frac{5}{2}}(bx)}$$
$$\Big\downarrow \text{3119}$$
$$\frac{3}{5}\left(\frac{2E\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{b}-\frac{2\cos(bx)}{b\sqrt{\sin(bx)}}\right)-\frac{2\cos(bx)}{5b\sin^{\frac{5}{2}}(bx)}$$

input
```
Int[Sin[b*x]^(-7/2),x]
```

output
```
(3*((2*EllipticE[Pi/4 - (b*x)/2, 2])/b - (2*Cos[b*x])/(b*Sqrt[Sin[b*x]])))
/5 - (2*Cos[b*x])/(5*b*Sin[b*x]^(5/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(49) = 98$.

Time = 2.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.20

method	result
default	$\frac{6\sqrt{\sin(bx)+1}\,\sqrt{-2\sin(bx)+2}\,\sqrt{-\sin(bx)}\,\sin(bx)^{2}\,\mathrm{EllipticE}\left(\sqrt{\sin(bx)+1},\frac{\sqrt{2}}{2}\right)-3\sqrt{\sin(bx)+1}\,\sqrt{-2\sin(bx)+2}\,\sqrt{-\sin(bx)}\,\sin(bx}{5\sin(bx)^{\frac{5}{2}}\cos(bx)b}$

input
```
int(1/sin(b*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/5/sin(b*x)^(5/2)*(6*(sin(b*x)+1)^(1/2)*(-2*sin(b*x)+2)^(1/2)*(-sin(b*x))
^(1/2)*sin(b*x)^2*EllipticE((sin(b*x)+1)^(1/2),1/2*2^(1/2))-3*(sin(b*x)+1)
^(1/2)*(-2*sin(b*x)+2)^(1/2)*(-sin(b*x))^(1/2)*sin(b*x)^2*EllipticF((sin(b
*x)+1)^(1/2),1/2*2^(1/2))+6*sin(b*x)^4-4*sin(b*x)^2-2)/cos(b*x)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.95

$$\int \frac{1}{\sin^{\frac{7}{2}}(bx)}\,dx = -\frac{2\left(3\sqrt{-\frac{1}{2}i}\left(i\cos(bx)^2-i\right)\sin(bx)\,\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(bx)+i\sin(bx)\right.}{}$$

input
```
integrate(1/sin(b*x)^(7/2),x, algorithm="fricas")
```

output
```
-2/5*(3*sqrt(-1/2*I)*(I*cos(b*x)^2 - I)*sin(b*x)*weierstrassZeta(4, 0, wei
erstrassPInverse(4, 0, cos(b*x) + I*sin(b*x))) + 3*sqrt(1/2*I)*(-I*cos(b*x
)^2 + I)*sin(b*x)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(b*x)
 - I*sin(b*x))) + (3*cos(b*x)^3 - 4*cos(b*x))*sqrt(sin(b*x)))/((b*cos(b*x)
^2 - b)*sin(b*x))
```

Sympy [F]

$$\int \frac{1}{\sin^{\frac{7}{2}}(bx)}\,dx = \int \frac{1}{\sin^{\frac{7}{2}}(bx)}\,dx$$

input
```
integrate(1/sin(b*x)**(7/2),x)
```

output
```
Integral(sin(b*x)**(-7/2), x)
```

Maxima [F]

$$\int \frac{1}{\sin^{\frac{7}{2}}(bx)}\,dx = \int \frac{1}{\sin\left(bx\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/sin(b*x)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x)^(-7/2), x)
```

Giac [F]

$$\int \frac{1}{\sin^{\frac{7}{2}}(bx)}\,dx = \int \frac{1}{\sin\left(bx\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/sin(b*x)^(7/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x)^(-7/2), x)
```

Mupad [B] (verification not implemented)

Time = 25.53 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.57

$$\int \frac{1}{\sin^{\frac{7}{2}}(bx)}\,dx = -\frac{\cos\left(b\,x\right)\,{\left({\sin\left(b\,x\right)}^2\right)}^{5/4}\,{{}}_2{\mathrm{F}}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{3}{2};\ {\cos\left(b\,x\right)}^2\right)}{b\,{\sin\left(b\,x\right)}^{5/2}}$$

input
```
int(1/sin(b*x)^(7/2),x)
```

output
```
-(cos(b*x)*(sin(b*x)^2)^(5/4)*hypergeom([1/2, 9/4], 3/2, cos(b*x)^2))/(b*s
in(b*x)^(5/2))
```

Reduce [F]

$$\int \frac{1}{\sin^{\frac{7}{2}}(bx)}\,dx = \int \frac{\sqrt{\sin (bx)}}{\sin (bx)^4}dx$$

input
```
int(1/sin(b*x)^(7/2),x)
```

output
```
int(sqrt(sin(b*x))/sin(b*x)**4,x)
```

3.17 $\int \sin^{\frac{7}{2}}(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 70

$$\int \sin^{\frac{7}{2}}(a+bx)\,dx = \frac{10\,\text{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)}{21b} - \frac{10\cos(a+bx)\sqrt{\sin(a+bx)}}{21b} - \frac{2\cos(a+bx)\sin^{\frac{5}{2}}(a+bx)}{7b}$$

output
```
10/21*InverseJacobiAM(1/2*a-1/4*Pi+1/2*b*x,2^(1/2))/b-10/21*cos(b*x+a)*sin
(b*x+a)^(1/2)/b-2/7*cos(b*x+a)*sin(b*x+a)^(5/2)/b
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.79

$$\int \sin^{\frac{7}{2}}(a+bx)\,dx = \frac{-20\,\text{EllipticF}\left(\frac{1}{4}(-2a+\pi-2bx),2\right)+(-23\cos(a+bx)+3\cos(3(a+bx)))\sqrt{\sin(a+bx)}}{42b}$$

input
```
Integrate[Sin[a + b*x]^(7/2),x]
```

output
```
(-20*EllipticF[(-2*a + Pi - 2*b*x)/4, 2] + (-23*Cos[a + b*x] + 3*Cos[3*(a
+ b*x)])*Sqrt[Sin[a + b*x]])/(42*b)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 3115, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{7}{2}}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^{7/2}dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{5}{7}\int \sin^{\frac{3}{2}}(a+bx)dx-\frac{2\sin^{\frac{5}{2}}(a+bx)\cos(a+bx)}{7b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{7}\int \sin(a+bx)^{3/2}dx-\frac{2\sin^{\frac{5}{2}}(a+bx)\cos(a+bx)}{7b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(a+bx)}}dx-\frac{2\sqrt{\sin(a+bx)}\cos(a+bx)}{3b}\right)-\frac{2\sin^{\frac{5}{2}}(a+bx)\cos(a+bx)}{7b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(a+bx)}}dx-\frac{2\sqrt{\sin(a+bx)}\cos(a+bx)}{3b}\right)-\frac{2\sin^{\frac{5}{2}}(a+bx)\cos(a+bx)}{7b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{5}{7}\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right),2\right)}{3b}-\frac{2\sqrt{\sin(a+bx)}\cos(a+bx)}{3b}\right)-\frac{2\sin^{\frac{5}{2}}(a+bx)\cos(a+bx)}{7b}$$

input
```
Int[Sin[a + b*x]^(7/2),x]
```

output
```
(5*((2*EllipticF[(a - Pi/2 + b*x)/2, 2])/(3*b) - (2*Cos[a + b*x]*Sqrt[Sin[
a + b*x]])/(3*b)))/7 - (2*Cos[a + b*x]*Sin[a + b*x]^(5/2))/(7*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.49

method	result	size
default	$\frac{\frac{2 \cos\left(bx+a\right)^{4} \sin\left(bx+a\right)}{7}+\frac{5 \sqrt{\sin\left(bx+a\right)+1}\, \sqrt{-2 \sin\left(bx+a\right)+2}\, \sqrt{-\sin\left(bx+a\right)}\, \operatorname{EllipticF}\left(\sqrt{\sin\left(bx+a\right)+1}, \frac{\sqrt{2}}{2}\right)}{21}-\frac{16 \cos\left(bx+a\right)^{2} \sin\left(bx+a\right)}{21}}{\cos\left(bx+a\right) \sqrt{\sin\left(bx+a\right)}\, b}$	104

input
```
int(sin(b*x+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(2/7*cos(b*x+a)^4*sin(b*x+a)+5/21*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(
1/2)*(-sin(b*x+a))^(1/2)*EllipticF((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-16/21
*cos(b*x+a)^2*sin(b*x+a))/cos(b*x+a)/sin(b*x+a)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.14

$$\int \sin^{\frac{7}{2}}(a+bx)\,dx = \frac{2\left(\left(3\cos(bx+a)^3-8\cos(bx+a)\right)\sqrt{\sin(bx+a)}+5\sqrt{-\frac{1}{2}i}\text{weierstrassPInverse}(4,0,\cos(bx+a)+i\right.}{21\,b}$$

input
```
integrate(sin(b*x+a)^(7/2),x, algorithm="fricas")
```

output
```
2/21*((3*cos(b*x + a)^3 - 8*cos(b*x + a))*sqrt(sin(b*x + a)) + 5*sqrt(-1/2
*I)*weierstrassPInverse(4, 0, cos(b*x + a) + I*sin(b*x + a)) + 5*sqrt(1/2*
I)*weierstrassPInverse(4, 0, cos(b*x + a) - I*sin(b*x + a)))/b
```

Sympy [F(-1)]

Timed out.

$$\int \sin^{\frac{7}{2}}(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sin^{\frac{7}{2}}(a+bx)\,dx = \int \sin(bx+a)^{\frac{7}{2}}\,dx$$

input
```
integrate(sin(b*x+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^(7/2), x)
```

Giac [F]

$$\int \sin^{\frac{7}{2}}(a+bx)\,dx=\int \sin\left(bx+a\right)^{\frac{7}{2}}\,dx$$

input
```
integrate(sin(b*x+a)^(7/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 25.51 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.60

$$\int \sin^{\frac{7}{2}}(a+bx)\,dx=-\frac{\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^{9/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ \frac{3}{2};\ \cos\left(a+b\,x\right)^2\right)}{b\,\left(\sin\left(a+b\,x\right)^2\right)^{9/4}}$$

input
```
int(sin(a + b*x)^(7/2),x)
```

output
```
-(cos(a + b*x)*sin(a + b*x)^(9/2)*hypergeom([-5/4, 1/2], 3/2, cos(a + b*x)
^2))/(b*(sin(a + b*x)^2)^(9/4))
```

Reduce [F]

$$\int \sin^{\frac{7}{2}}(a+bx)\,dx=\int \sqrt{\sin\left(bx+a\right)}\,\sin\left(bx+a\right)^3dx$$

input
```
int(sin(b*x+a)^(7/2),x)
```

output
```
int(sqrt(sin(a + b*x))*sin(a + b*x)**3,x)
```

3.18 $\int \sin^{\frac{5}{2}}(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 47

$$\int \sin^{\frac{5}{2}}(a+bx)\,dx = \frac{6E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{5b} - \frac{2\cos(a+bx)\sin^{\frac{3}{2}}(a+bx)}{5b}$$

output
```
-6/5*EllipticE(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b-2/5*cos(b*x+a)*sin(b*x
+a)^(3/2)/b
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \sin^{\frac{5}{2}}(a+bx)\,dx = -\frac{6E\left(\frac{1}{4}(-2a+\pi-2bx)\middle|2\right)+\sqrt{\sin(a+bx)}\sin(2(a+bx))}{5b}$$

input
```
Integrate[Sin[a + b*x]^(5/2),x]
```

output
```
-1/5*(6*EllipticE[(-2*a + Pi - 2*b*x)/4, 2] + Sqrt[Sin[a + b*x]]*Sin[2*(a
+ b*x)])/b
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3115, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{5}{2}}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^{5/2}dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{3}{5}\int \sqrt{\sin(a+bx)}dx-\frac{2\sin^{\frac{3}{2}}(a+bx)\cos(a+bx)}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{5}\int \sqrt{\sin(a+bx)}dx-\frac{2\sin^{\frac{3}{2}}(a+bx)\cos(a+bx)}{5b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{6E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{5b}-\frac{2\sin^{\frac{3}{2}}(a+bx)\cos(a+bx)}{5b}$$

input
```
Int[Sin[a + b*x]^(5/2),x]
```

output
```
(6*EllipticE[(a - Pi/2 + b*x)/2, 2])/(5*b) - (2*Cos[a + b*x]*Sin[a + b*x]^
(3/2))/(5*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(41) = 82$.

Time = 2.38 (sec) , antiderivative size = 142, normalized size of antiderivative = 3.02

method	result
default	$\frac{\frac{2 \sin(bx+a)^4}{5}-\frac{2 \sin(bx+a)^2}{5}-\frac{6\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(bx+a)+2}\,\sqrt{-\sin(bx+a)}\,\operatorname{EllipticE}\left(\sqrt{\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)}{5}+\frac{3\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(bx+}}{}}{\cos(bx+a)\sqrt{\sin(bx+a)}\,b}$

input
```
int(sin(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(2/5*sin(b*x+a)^4-2/5*sin(b*x+a)^2-6/5*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)
+2)^(1/2)*(-sin(b*x+a))^(1/2)*EllipticE((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))+
3/5*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*x+a))^(1/2)*Ellip
ticF((sin(b*x+a)+1)^(1/2),1/2*2^(1/2)))/cos(b*x+a)/sin(b*x+a)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.55

$$\int \sin^{\frac{5}{2}}(a+bx)\,dx =$$
$$-\frac{2\left(\cos\left(bx+a\right)\sin\left(bx+a\right)^{\frac{3}{2}} - 3i\sqrt{-\frac{1}{2}i}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos\left(bx+a\right)+$$

input
```
integrate(sin(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
-2/5*(cos(b*x + a)*sin(b*x + a)^(3/2) - 3*I*sqrt(-1/2*I)*weierstrassZeta(4
, 0, weierstrassPInverse(4, 0, cos(b*x + a) + I*sin(b*x + a))) + 3*I*sqrt(
1/2*I)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(b*x + a) - I*si
n(b*x + a))))/b
```

Sympy [F]

$$\int \sin^{\frac{5}{2}}(a+bx)\,dx = \int \sin^{\frac{5}{2}}(a+bx)\,dx$$

input
```
integrate(sin(b*x+a)**(5/2),x)
```

output
```
Integral(sin(a + b*x)**(5/2), x)
```

Maxima [F]

$$\int \sin^{\frac{5}{2}}(a+bx)\,dx = \int \sin{(bx+a)}^{\frac{5}{2}}\,dx$$

input
```
integrate(sin(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^(5/2), x)
```

Giac [F]

$$\int \sin^{\frac{5}{2}}(a+bx)\,dx = \int \sin{(bx+a)}^{\frac{5}{2}}\,dx$$

input
```
integrate(sin(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 25.46 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \sin^{\frac{5}{2}}(a+bx)\,dx = -\frac{\cos{(a+b\,x)}\,\sin{(a+b\,x)}^{7/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{3}{2};\ \cos{(a+b\,x)}^2\right)}{b\,\left(\sin{(a+b\,x)}^2\right)^{7/4}}$$

input
```
int(sin(a + b*x)^(5/2),x)
```

output
```
-(cos(a + b*x)*sin(a + b*x)^(7/2)*hypergeom([-3/4, 1/2], 3/2, cos(a + b*x)
^2))/(b*(sin(a + b*x)^2)^(7/4))
```

Reduce [F]

$$\int \sin^{\frac{5}{2}}(a+bx)\,dx = \int \sqrt{\sin(bx+a)}\,\sin(bx+a)^2\,dx$$

input
```
int(sin(b*x+a)^(5/2),x)
```

output
```
int(sqrt(sin(a + b*x))*sin(a + b*x)**2,x)
```

3.19 $\int \sin^{\frac{3}{2}}(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 47

$$\int \sin^{\frac{3}{2}}(a+bx)\,dx=\frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)}{3b}-\frac{2\cos(a+bx)\sqrt{\sin(a+bx)}}{3b}$$

output
```
2/3*InverseJacobiAM(1/2*a-1/4*Pi+1/2*b*x,2^(1/2))/b-2/3*cos(b*x+a)*sin(b*x
+a)^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \sin^{\frac{3}{2}}(a+bx)\,dx=-\frac{2\left(\text{EllipticF}\left(\frac{1}{4}(-2a+\pi-2bx),2\right)+\cos(a+bx)\sqrt{\sin(a+bx)}\right)}{3b}$$

input
```
Integrate[Sin[a + b*x]^(3/2),x]
```

output
```
(-2*(EllipticF[(-2*a + Pi - 2*b*x)/4, 2] + Cos[a + b*x]*Sqrt[Sin[a + b*x]]
))/(3*b)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{3}{2}}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^{3/2}dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{3}\int \frac{1}{\sqrt{\sin(a+bx)}}dx-\frac{2\sqrt{\sin(a+bx)}\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{1}{\sqrt{\sin(a+bx)}}dx-\frac{2\sqrt{\sin(a+bx)}\cos(a+bx)}{3b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right),2\right)}{3b}-\frac{2\sqrt{\sin(a+bx)}\cos(a+bx)}{3b}$$

input
```
Int[Sin[a + b*x]^(3/2),x]
```

output
```
(2*EllipticF[(a - Pi/2 + b*x)/2, 2])/(3*b) - (2*Cos[a + b*x]*Sqrt[Sin[a +
b*x]])/(3*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(40) = 80$.

Time = 2.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.87

method	result	size
default	$\frac{\frac{\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(bx+a)+2}\,\sqrt{-\sin(bx+a)}\,\operatorname{EllipticF}\left(\sqrt{\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)}{3}-\frac{2\cos(bx+a)^2\sin(bx+a)}{3}}{\cos(bx+a)\sqrt{\sin(bx+a)}\,b}$	88

input
```
int(sin(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(1/3*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*x+a))^(1/2)*Elli
pticF((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-2/3*cos(b*x+a)^2*sin(b*x+a))/cos(b
*x+a)/sin(b*x+a)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.43

$$\int \sin^{\frac{3}{2}}(a+bx)\,dx =$$
$$-\frac{2\left(\cos\left(bx+a\right)\sqrt{\sin\left(bx+a\right)}-\sqrt{-\frac{1}{2}i}\text{weierstrassPInverse}(4,0,\cos\left(bx+a\right)+i\,\sin\left(bx+a\right))-\sqrt{\frac{1}{2}i}\cdots\right.}{3\,b}$$

input
```
integrate(sin(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(cos(b*x + a)*sqrt(sin(b*x + a)) - sqrt(-1/2*I)*weierstrassPInverse(4
, 0, cos(b*x + a) + I*sin(b*x + a)) - sqrt(1/2*I)*weierstrassPInverse(4, 0
, cos(b*x + a) - I*sin(b*x + a)))/b
```

Sympy [F]

$$\int \sin^{\frac{3}{2}}(a+bx)\,dx = \int \sin^{\frac{3}{2}}\left(a+bx\right)dx$$

input
```
integrate(sin(b*x+a)**(3/2),x)
```

output
```
Integral(sin(a + b*x)**(3/2), x)
```

Maxima [F]

$$\int \sin^{\frac{3}{2}}(a+bx)\,dx = \int \sin\left(bx+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(sin(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^(3/2), x)
```

Giac [F]

$$\int \sin^{\frac{3}{2}}(a+bx)\,dx = \int \sin(bx+a)^{\frac{3}{2}}\,dx$$

input
```
integrate(sin(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 25.48 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \sin^{\frac{3}{2}}(a+bx)\,dx = -\frac{\cos(a+b\,x)\,\sin(a+b\,x)^{5/2}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ \cos(a+b\,x)^2\right)}{b\,\left(\sin(a+b\,x)^2\right)^{5/4}}$$

input
```
int(sin(a + b*x)^(3/2),x)
```

output
```
-(cos(a + b*x)*sin(a + b*x)^(5/2)*hypergeom([-1/4, 1/2], 3/2, cos(a + b*x)
^2))/(b*(sin(a + b*x)^2)^(5/4))
```

Reduce [F]

$$\int \sin^{\frac{3}{2}}(a+bx)\,dx = \int \sqrt{\sin(bx+a)}\,\sin(bx+a)\,dx$$

input
```
int(sin(b*x+a)^(3/2),x)
```

output
```
int(sqrt(sin(a + b*x))*sin(a + b*x),x)
```

3.20 $\int \sqrt{\sin(a+bx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 21

$$\int \sqrt{\sin(a+bx)}\,dx = \frac{2E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{b}$$

output
```
-2*EllipticE(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \sqrt{\sin(a+bx)}\,dx = -\frac{2E\left(\frac{1}{2}\left(-a+\frac{\pi}{2}-bx\right)\middle|2\right)}{b}$$

input
```
Integrate[Sqrt[Sin[a + b*x]],x]
```

output
```
(-2*EllipticE[(-a + Pi/2 - b*x)/2, 2])/b
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sin(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin(a+bx)}dx$$

$$\Big\downarrow \text{3119}$$

$$\frac{2E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b}$$

input
```
Int[Sqrt[Sin[a + b*x]],x]
```

output
```
(2*EllipticE[(a - Pi/2 + b*x)/2, 2])/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(21) = 42$.

Time = 2.75 (sec) , antiderivative size = 91, normalized size of antiderivative = 4.33

method	result
default	$-\frac{\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(bx+a)+2}\,\sqrt{-\sin(bx+a)}\,\left(2\,\mathrm{EllipticE}\left(\sqrt{\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)-\mathrm{EllipticF}\left(\sqrt{\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)\right)}{\cos(bx+a)\sqrt{\sin(bx+a)}\,b}$
risch	$-\frac{i\sqrt{2}\,\sqrt{-i\left(e^{2i(bx+a)}-1\right)e^{-i(bx+a)}}}{b}+\frac{i\left(\frac{2i\left(i-ie^{2i(bx+a)}\right)}{\sqrt{e^{i(bx+a)}\left(i-ie^{2i(bx+a)}\right)}}-\frac{\sqrt{e^{i(bx+a)}+1}\,\sqrt{-2\,e^{i(bx+a)}+2}\,\sqrt{-e^{i(bx+a)}}\,\left(-2\,\mathrm{EllipticE}\left(\sqrt{e^{i(}\cdots\right.\right.}{\sqrt{-ie^{3i(bx+a)}+ie^{i(b}\cdots}}\right.}{\cdots}$

input
```
int(sin(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*x+a))^(1/2)*(2*Ellip
ticE((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-EllipticF((sin(b*x+a)+1)^(1/2),1/2*
2^(1/2)))/cos(b*x+a)/sin(b*x+a)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.76

$$\int \sqrt{\sin(a+bx)}\,dx =$$
$$-\frac{2\left(-i\sqrt{-\frac{1}{2}i}\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(bx+a)+i\,\sin(bx+a)))+i\sqrt{\frac{1}{2}i}\mathrm{wei}\cdots\right.}{b}$$

input
```
integrate(sin(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-2*(-I*sqrt(-1/2*I)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(b*
x + a) + I*sin(b*x + a))) + I*sqrt(1/2*I)*weierstrassZeta(4, 0, weierstras
sPInverse(4, 0, cos(b*x + a) - I*sin(b*x + a))))/b
```

Sympy [F]

$$\int \sqrt{\sin(a+bx)}\,dx = \int \sqrt{\sin{\left(a+bx \right)}}\,dx$$

input
```
integrate(sin(b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(sin(a + b*x)), x)
```

Maxima [F]

$$\int \sqrt{\sin(a+bx)}\,dx = \int \sqrt{\sin\left(bx+a\right)}\,dx$$

input
```
integrate(sin(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sin(b*x + a)), x)
```

Giac [F]

$$\int \sqrt{\sin(a+bx)}\,dx = \int \sqrt{\sin\left(bx+a\right)}\,dx$$

input
```
integrate(sin(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sin(b*x + a)), x)
```

Mupad [B] (verification not implemented)

Time = 25.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \sqrt{\sin(a+bx)}\,dx = \frac{2\,\mathrm{E}\left(\frac{a}{2}-\frac{\pi}{4}+\frac{b\,x}{2}\,\middle|\,2\right)}{b}$$

input
```
int(sin(a + b*x)^(1/2),x)
```

output
```
(2*ellipticE(a/2 - pi/4 + (b*x)/2, 2))/b
```

Reduce [F]

$$\int \sqrt{\sin(a+bx)}\,dx = \int \sqrt{\sin(bx+a)}dx$$

input
```
int(sin(b*x+a)^(1/2),x)
```

output
```
int(sqrt(sin(a + b*x)),x)
```

3.21 $\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 21

$$\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx = \frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)}{b}$$

output
```
2*InverseJacobiAM(1/2*a-1/4*Pi+1/2*b*x,2^(1/2))/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx = -\frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(-a+\frac{\pi}{2}-bx\right),2\right)}{b}$$

input
```
Integrate[1/Sqrt[Sin[a + b*x]],x]
```

output
```
(-2*EllipticF[(-a + Pi/2 - b*x)/2, 2])/b
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sin(a+bx)}}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\sqrt{\sin(a+bx)}}dx$$
$$\Big\downarrow \text{ 3120}$$
$$\frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right),2\right)}{b}$$

input
```
Int[1/Sqrt[Sin[a + b*x]],x]
```

output
```
(2*EllipticF[(a - Pi/2 + b*x)/2, 2])/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(20) = 40$.

Time = 1.74 (sec) , antiderivative size = 69, normalized size of antiderivative = 3.29

method	result	size
default	$\frac{\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(bx+a)+2}\,\sqrt{-\sin(bx+a)}\ \mathrm{EllipticF}\left(\sqrt{\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)}{\cos(bx+a)\sqrt{\sin(bx+a)}\,b}$	69

input
```
int(1/sin(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*x+a))^(1/2)*EllipticF
((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))/cos(b*x+a)/sin(b*x+a)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.38

$$\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx = \frac{2\left(\sqrt{-\frac{1}{2}i}\mathrm{weierstrassPInverse}(4,0,\cos(bx+a)+i\,\sin(bx+a))+\sqrt{\frac{1}{2}i}\mathrm{weierstrassPInverse}(4,0,\cos(bx$$
$$b$$

input
```
integrate(1/sin(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2*(sqrt(-1/2*I)*weierstrassPInverse(4, 0, cos(b*x + a) + I*sin(b*x + a)) +
 sqrt(1/2*I)*weierstrassPInverse(4, 0, cos(b*x + a) - I*sin(b*x + a)))/b
```

Sympy [F]

$$\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx = \int \frac{1}{\sqrt{\sin{(a+bx)}}}\,dx$$

input
```
integrate(1/sin(b*x+a)**(1/2),x)
```

output
```
Integral(1/sqrt(sin(a + b*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx = \int \frac{1}{\sqrt{\sin{(bx+a)}}}\,dx$$

input
```
integrate(1/sin(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(sin(b*x + a)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx = \int \frac{1}{\sqrt{\sin{(bx+a)}}}\,dx$$

input
```
integrate(1/sin(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(sin(b*x + a)), x)
```

Mupad [B] (verification not implemented)

Time = 25.35 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx = -\frac{2\,\mathrm{F}\left(\frac{\pi}{4}-\frac{a}{2}-\frac{b\,x}{2}\middle|2\right)}{b}$$

input
```
int(1/sin(a + b*x)^(1/2),x)
```

output
```
-(2*ellipticF(pi/4 - a/2 - (b*x)/2, 2))/b
```

Reduce [F]

$$\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx = \int \frac{\sqrt{\sin(bx+a)}}{\sin(bx+a)}dx$$

input
```
int(1/sin(b*x+a)^(1/2),x)
```

output
```
int(sqrt(sin(a + b*x))/sin(a + b*x),x)
```

3.22 $\int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 43

$$\int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}\,dx = -\frac{2E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{b} - \frac{2\cos(a+bx)}{b\sqrt{\sin(a+bx)}}$$

output
```
2*EllipticE(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b-2*cos(b*x+a)/b/sin(b*x+a)
^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}\,dx = \frac{2\left(E\left(\frac{1}{4}(-2a+\pi-2bx)\middle|2\right)-\frac{\cos(a+bx)}{\sqrt{\sin(a+bx)}}\right)}{b}$$

input
```
Integrate[Sin[a + b*x]^(-3/2),x]
```

output
```
(2*(EllipticE[(-2*a + Pi - 2*b*x)/4, 2] - Cos[a + b*x]/Sqrt[Sin[a + b*x]])
)/b
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^{3/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$-\int \sqrt{\sin(a+bx)}dx-\frac{2\cos(a+bx)}{b\sqrt{\sin(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\int \sqrt{\sin(a+bx)}dx-\frac{2\cos(a+bx)}{b\sqrt{\sin(a+bx)}}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{2E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b}-\frac{2\cos(a+bx)}{b\sqrt{\sin(a+bx)}}$$

input
```
Int[Sin[a + b*x]^(-3/2),x]
```

output
```
(-2*EllipticE[(a - Pi/2 + b*x)/2, 2])/b - (2*Cos[a + b*x])/(b*Sqrt[Sin[a +
b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(41) = 82$.

Time = 2.31 (sec) , antiderivative size = 132, normalized size of antiderivative = 3.07

method	result
default	$\frac{2\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(bx+a)+2}\,\sqrt{-\sin(bx+a)}\ \mathrm{EllipticE}\left(\sqrt{\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)-\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(bx+a)+2}\,\sqrt{-\sin(b}{\cos(bx+a)\sqrt{\sin(bx+a)}\,b}$

input
```
int(1/sin(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(2*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*x+a))^(1/2)*Ellipt
icE((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+
2)^(1/2)*(-sin(b*x+a))^(1/2)*EllipticF((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-2
*cos(b*x+a)^2)/cos(b*x+a)/sin(b*x+a)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.16

$$\int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}\,dx = -\frac{2\left(i\sqrt{-\frac{1}{2}i}\sin\left(bx+a\right)\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos\left(bx+a\right)+i\sin\left(bx+a\right)))-\right.}{}$$

input
```
integrate(1/sin(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
-2*(I*sqrt(-1/2*I)*sin(b*x + a)*weierstrassZeta(4, 0, weierstrassPInverse(
4, 0, cos(b*x + a) + I*sin(b*x + a))) - I*sqrt(1/2*I)*sin(b*x + a)*weierst
rassZeta(4, 0, weierstrassPInverse(4, 0, cos(b*x + a) - I*sin(b*x + a))) +
 cos(b*x + a)*sqrt(sin(b*x + a)))/(b*sin(b*x + a))
```

Sympy [F]

$$\int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}\,dx$$

input
```
integrate(1/sin(b*x+a)**(3/2),x)
```

output
```
Integral(sin(a + b*x)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\sin(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sin(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\sin(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sin(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^(-3/2), x)
```

Mupad [B] (verification not implemented)

Time = 25.57 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}\,dx = -\frac{\cos(a+b\,x)\,\left(\sin(a+b\,x)^2\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{5}{4};\ \frac{3}{2};\ \cos(a+b\,x)^2\right)}{b\,\sqrt{\sin(a+b\,x)}}$$

input
```
int(1/sin(a + b*x)^(3/2),x)
```

output
```
-(cos(a + b*x)*(sin(a + b*x)^2)^(1/4)*hypergeom([1/2, 5/4], 3/2, cos(a + b
*x)^2))/(b*sin(a + b*x)^(1/2))
```

Reduce [F]

$$\int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{\sqrt{\sin{(bx+a)}}}{\sin{(bx+a)}^{2}}dx$$

input
```
int(1/sin(b*x+a)^(3/2),x)
```

output
```
int(sqrt(sin(a + b*x))/sin(a + b*x)**2,x)
```

3.23 $\int \frac{1}{\sin^{\frac{5}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 47

$$\int \frac{1}{\sin^{\frac{5}{2}}(a+bx)}\,dx = \frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)}{3b} - \frac{2\cos(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)}$$

output
```
2/3*InverseJacobiAM(1/2*a-1/4*Pi+1/2*b*x,2^(1/2))/b-2/3*cos(b*x+a)/b/sin(b
*x+a)^(3/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sin^{\frac{5}{2}}(a+bx)}\,dx = \frac{2\left(\text{EllipticF}\left(\frac{1}{4}(2a-\pi+2bx),2\right)-\frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\right)}{3b}$$

input
```
Integrate[Sin[a + b*x]^(-5/2),x]
```

output
```
(2*(EllipticF[(2*a - Pi + 2*b*x)/4, 2] - Cos[a + b*x]/Sin[a + b*x]^(3/2)))
/(3*b)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3116, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin^{\frac{5}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin(a+bx)^{5/2}}dx$$

$$\Big\downarrow \text{ 3116}$$

$$\frac{1}{3}\int \frac{1}{\sqrt{\sin(a+bx)}}dx-\frac{2\cos(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3}\int \frac{1}{\sqrt{\sin(a+bx)}}dx-\frac{2\cos(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right),2\right)}{3b}-\frac{2\cos(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)}$$

input
```
Int[Sin[a + b*x]^(-5/2),x]
```

output
```
(2*EllipticF[(a - Pi/2 + b*x)/2, 2])/(3*b) - (2*Cos[a + b*x])/(3*b*Sin[a +
 b*x]^(3/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(40) = 80$.

Time = 2.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.87

method	result	size
default	$\frac{\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(bx+a)+2}\,\sqrt{-\sin(bx+a)}\,\operatorname{EllipticF}\left(\sqrt{\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)\sin(bx+a)-2\cos(bx+a)^2}{3\sin(bx+a)^{\frac{3}{2}}\cos(bx+a)b}$	88

input
```
int(1/sin(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/sin(b*x+a)^(3/2)*((sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b
*x+a))^(1/2)*EllipticF((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))*sin(b*x+a)-2*cos(
b*x+a)^2)/cos(b*x+a)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.09

$$\int \frac{1}{\sin^{\frac{5}{2}}(a+bx)}\,dx$$
$$= \frac{2\left(\sqrt{-\frac{1}{2}i}\left(\cos\left(bx+a\right)^2-1\right)\text{weierstrassPInverse}(4,0,\cos\left(bx+a\right)+i\,\sin\left(bx+a\right))+\sqrt{\frac{1}{2}i}\left(\cos\left(bx+a\right)\right.}{3\left(b\cos\left(bx+a\right)^2\right.}$$

input
```
integrate(1/sin(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
2/3*(sqrt(-1/2*I)*(cos(b*x + a)^2 - 1)*weierstrassPInverse(4, 0, cos(b*x +
 a) + I*sin(b*x + a)) + sqrt(1/2*I)*(cos(b*x + a)^2 - 1)*weierstrassPInver
se(4, 0, cos(b*x + a) - I*sin(b*x + a)) + cos(b*x + a)*sqrt(sin(b*x + a)))
/(b*cos(b*x + a)^2 - b)
```

Sympy [F]

$$\int \frac{1}{\sin^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{1}{\sin^{\frac{5}{2}}(a+bx)}\,dx$$

input
```
integrate(1/sin(b*x+a)**(5/2),x)
```

output
```
Integral(sin(a + b*x)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{\sin^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{1}{\sin\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/sin(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{\sin^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{1}{\sin\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/sin(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^(-5/2), x)
```

Mupad [B] (verification not implemented)

Time = 26.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sin^{\frac{5}{2}}(a+bx)}\,dx = -\frac{\cos\left(a+b\,x\right)\,\left(\sin\left(a+b\,x\right)^2\right)^{3/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{3}{2};\ \cos\left(a+b\,x\right)^2\right)}{b\sin\left(a+b\,x\right)^{3/2}}$$

input
```
int(1/sin(a + b*x)^(5/2),x)
```

output
```
-(cos(a + b*x)*(sin(a + b*x)^2)^(3/4)*hypergeom([1/2, 7/4], 3/2, cos(a + b
*x)^2))/(b*sin(a + b*x)^(3/2))
```

Reduce [F]

$$\int \frac{1}{\sin^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{\sqrt{\sin(bx+a)}}{\sin(bx+a)^3}dx$$

input
```
int(1/sin(b*x+a)^(5/2),x)
```

output
```
int(sqrt(sin(a + b*x))/sin(a + b*x)**3,x)
```

3.24 $\int \frac{1}{\sin^{\frac{7}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 70

$$\int \frac{1}{\sin^{\frac{7}{2}}(a+bx)}\,dx = -\frac{6E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{5b} - \frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)} - \frac{6\cos(a+bx)}{5b\sqrt{\sin(a+bx)}}$$

output
```
6/5*EllipticE(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b-2/5*cos(b*x+a)/b/sin(b*
x+a)^(5/2)-6/5*cos(b*x+a)/b/sin(b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sin^{\frac{7}{2}}(a+bx)}\,dx = \frac{2\left(3E\left(\frac{1}{4}(-2a+\pi-2bx)\middle|2\right) - \frac{\cos(a+bx)(1+3\sin^2(a+bx))}{\sin^{\frac{5}{2}}(a+bx)}\right)}{5b}$$

input
```
Integrate[Sin[a + b*x]^(-7/2),x]
```

output
```
(2*(3*EllipticE[(-2*a + Pi - 2*b*x)/4, 2] - (Cos[a + b*x]*(1 + 3*Sin[a + b
*x]^2))/Sin[a + b*x]^(5/2)))/(5*b)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 3116, 3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin^{\frac{7}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^{7/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$\frac{3}{5}\int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}dx-\frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{5}\int \frac{1}{\sin(a+bx)^{3/2}}dx-\frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)}$$

$$\Big\downarrow \text{3116}$$

$$\frac{3}{5}\left(-\int \sqrt{\sin(a+bx)}dx-\frac{2\cos(a+bx)}{b\sqrt{\sin(a+bx)}}\right)-\frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{5}\left(-\int \sqrt{\sin(a+bx)}dx-\frac{2\cos(a+bx)}{b\sqrt{\sin(a+bx)}}\right)-\frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{3}{5}\left(-\frac{2E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)|2\right)}{b}-\frac{2\cos(a+bx)}{b\sqrt{\sin(a+bx)}}\right)-\frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)}$$

input
```
Int[Sin[a + b*x]^(-7/2),x]
```

output
```
(3*((-2*EllipticE[(a - Pi/2 + b*x)/2, 2])/b - (2*Cos[a + b*x])/(b*Sqrt[Sin
[a + b*x]])))/5 - (2*Cos[a + b*x])/(5*b*Sin[a + b*x]^(5/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(60) = 120$.

Time = 2.26 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.29

method	result
default	$\frac{6\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(bx+a)+2}\,\sqrt{-\sin(bx+a)}\,\sin(bx+a)^2\,\mathrm{EllipticE}\left(\sqrt{\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)-3\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(bx+a)}}{5\sin(bx+a)^{\frac{5}{2}}\cos(bx+a)b}$

input
```
int(1/sin(b*x+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/5/sin(b*x+a)^(5/2)*(6*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin
(b*x+a))^(1/2)*sin(b*x+a)^2*EllipticE((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-3*
(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*x+a))^(1/2)*sin(b*x+a
)^2*EllipticF((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))+6*sin(b*x+a)^4-4*sin(b*x+a
)^2-2)/cos(b*x+a)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.04

$$\int \frac{1}{\sin^{\frac{7}{2}}(a+bx)}\,dx = -\frac{2\left(3\sqrt{-\frac{1}{2}i}\left(i\cos\left(bx+a\right)^2-i\right)\sin\left(bx+a\right)\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos\left(bx+a\right.\right.}{}$$

input
```
integrate(1/sin(b*x+a)^(7/2),x, algorithm="fricas")
```

output
```
-2/5*(3*sqrt(-1/2*I)*(I*cos(b*x + a)^2 - I)*sin(b*x + a)*weierstrassZeta(4
, 0, weierstrassPInverse(4, 0, cos(b*x + a) + I*sin(b*x + a))) + 3*sqrt(1/
2*I)*(-I*cos(b*x + a)^2 + I)*sin(b*x + a)*weierstrassZeta(4, 0, weierstras
sPInverse(4, 0, cos(b*x + a) - I*sin(b*x + a))) + (3*cos(b*x + a)^3 - 4*co
s(b*x + a))*sqrt(sin(b*x + a)))/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

Sympy [F]

$$\int \frac{1}{\sin^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{1}{\sin^{\frac{7}{2}}(a+bx)}\,dx$$

input
```
integrate(1/sin(b*x+a)**(7/2),x)
```

output
```
Integral(sin(a + b*x)**(-7/2), x)
```

Maxima [F]

$$\int \frac{1}{\sin^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{1}{\sin(bx+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/sin(b*x+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^(-7/2), x)
```

Giac [F]

$$\int \frac{1}{\sin^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{1}{\sin(bx+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/sin(b*x+a)^(7/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^(-7/2), x)
```

Mupad [B] (verification not implemented)

Time = 26.34 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sin^{\frac{7}{2}}(a+bx)}\,dx = -\frac{\cos(a+b\,x)\,\left(\sin(a+b\,x)^2\right)^{5/4}\,{}_2F_1\left(\frac{1}{2},\frac{9}{4};\ \frac{3}{2};\ \cos(a+b\,x)^2\right)}{b\sin(a+b\,x)^{5/2}}$$

input
```
int(1/sin(a + b*x)^(7/2),x)
```

output
```
-(cos(a + b*x)*(sin(a + b*x)^2)^(5/4)*hypergeom([1/2, 9/4], 3/2, cos(a + b
*x)^2))/(b*sin(a + b*x)^(5/2))
```

Reduce [F]

$$\int \frac{1}{\sin^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{\sqrt{\sin(bx+a)}}{\sin(bx+a)^4}dx$$

input
```
int(1/sin(b*x+a)^(7/2),x)
```

output
```
int(sqrt(sin(a + b*x))/sin(a + b*x)**4,x)
```

3.25 $\int (c\sin(a+bx))^{7/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 103

$$\int (c\sin(a+bx))^{7/2}\,dx = \frac{10c^4\,\text{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)\sqrt{\sin(a+bx)}}{21b\sqrt{c\sin(a+bx)}} - \frac{10c^3\cos(a+bx)\sqrt{c\sin(a+bx)}}{21b} - \frac{2c\cos(a+bx)(c\sin(a+bx))^{5/2}}{7b}$$

output
```
10/21*c^4*InverseJacobiAM(1/2*a-1/4*Pi+1/2*b*x,2^(1/2))*sin(b*x+a)^(1/2)/b
/(c*sin(b*x+a))^(1/2)-10/21*c^3*cos(b*x+a)*(c*sin(b*x+a))^(1/2)/b-2/7*c*co
s(b*x+a)*(c*sin(b*x+a))^(5/2)/b
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.78

$$\int (c\sin(a+bx))^{7/2}\,dx = \frac{c^3\left(-20\,\text{EllipticF}\left(\frac{1}{4}(-2a+\pi-2bx),2\right)+(-23\cos(a+bx)+3\cos(3(a+bx)))\sqrt{\sin(a+b}\right.}{42b\sqrt{\sin(a+bx)}}$$

input
```
Integrate[(c*Sin[a + b*x])^(7/2),x]
```

output
```
(c^3*(-20*EllipticF[(-2*a + Pi - 2*b*x)/4, 2] + (-23*Cos[a + b*x] + 3*Cos[
3*(a + b*x)])*Sqrt[Sin[a + b*x]])*Sqrt[c*Sin[a + b*x]])/(42*b*Sqrt[Sin[a +
 b*x]])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3115, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\sin(a+bx))^{7/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (c\sin(a+bx))^{7/2}dx$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{5}{7}c^2\int (c\sin(a+bx))^{3/2}dx-\frac{2c\cos(a+bx)(c\sin(a+bx))^{5/2}}{7b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{5}{7}c^2\int (c\sin(a+bx))^{3/2}dx-\frac{2c\cos(a+bx)(c\sin(a+bx))^{5/2}}{7b}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{5}{7}c^2\left(\frac{1}{3}c^2\int \frac{1}{\sqrt{c\sin(a+bx)}}dx-\frac{2c\cos(a+bx)\sqrt{c\sin(a+bx)}}{3b}\right)-\frac{2c\cos(a+bx)(c\sin(a+bx))^{5/2}}{7b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{5}{7}c^2\left(\frac{1}{3}c^2\int \frac{1}{\sqrt{c\sin(a+bx)}}dx-\frac{2c\cos(a+bx)\sqrt{c\sin(a+bx)}}{3b}\right)-\frac{2c\cos(a+bx)(c\sin(a+bx))^{5/2}}{7b}$$

$$\Big\downarrow \text{3121}$$

$$\frac{5}{7}c^2\left(\frac{c^2\sqrt{\sin(a+bx)}\int\frac{1}{\sqrt{\sin(a+bx)}}dx}{3\sqrt{c\sin(a+bx)}}-\frac{2c\cos(a+bx)\sqrt{c\sin(a+bx)}}{3b}\right)-\frac{2c\cos(a+bx)(c\sin(a+bx))^{5/2}}{7b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{7}c^2\left(\frac{c^2\sqrt{\sin(a+bx)}\int\frac{1}{\sqrt{\sin(a+bx)}}dx}{3\sqrt{c\sin(a+bx)}}-\frac{2c\cos(a+bx)\sqrt{c\sin(a+bx)}}{3b}\right)-\frac{2c\cos(a+bx)(c\sin(a+bx))^{5/2}}{7b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{5}{7}c^2\left(\frac{2c^2\sqrt{\sin(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right),2\right)}{3b\sqrt{c\sin(a+bx)}}-\frac{2c\cos(a+bx)\sqrt{c\sin(a+bx)}}{3b}\right)-\frac{2c\cos(a+bx)(c\sin(a+bx))^{5/2}}{7b}$$

input
```
Int[(c*Sin[a + b*x])^(7/2),x]
```

output
```
(-2*c*Cos[a + b*x]*(c*Sin[a + b*x])^(5/2))/(7*b) + (5*c^2*((2*c^2*Elliptic
F[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(3*b*Sqrt[c*Sin[a + b*x]]) -
(2*c*Cos[a + b*x]*Sqrt[c*Sin[a + b*x]])/(3*b)))/7
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 3.66 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.05

method	result
default	$-\frac{c^4\left(-6\sin(bx+a)^5+5\sqrt{1-\sin(bx+a)}\,\sqrt{2+2\sin(bx+a)}\,\sqrt{\sin(bx+a)}\,\text{EllipticF}\left(\sqrt{1-\sin(bx+a)},\frac{\sqrt{2}}{2}\right)-4\sin(bx+a)^3+10\sin(bx+a)\right)}{21\cos(bx+a)\sqrt{c\sin(bx+a)}\,b}$

input
```
int((c*sin(b*x+a))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/21*c^4*(-6*sin(b*x+a)^5+5*(1-sin(b*x+a))^(1/2)*(2+2*sin(b*x+a))^(1/2)*s
in(b*x+a)^(1/2)*EllipticF((1-sin(b*x+a))^(1/2),1/2*2^(1/2))-4*sin(b*x+a)^3
+10*sin(b*x+a))/cos(b*x+a)/(c*sin(b*x+a))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.95

$$\int (c\sin(a+bx))^{7/2}\,dx = \frac{2\left(5\sqrt{-\frac{1}{2}i\,c}c^3\text{weierstrassPInverse}(4,0,\cos(bx+a)+i\sin(bx+a))+5\sqrt{\frac{1}{2}i\,c}c^3\text{weierstrass}\right.}{}$$

input
```
integrate((c*sin(b*x+a))^(7/2),x, algorithm="fricas")
```

output
```
2/21*(5*sqrt(-1/2*I*c)*c^3*weierstrassPInverse(4, 0, cos(b*x + a) + I*sin(
b*x + a)) + 5*sqrt(1/2*I*c)*c^3*weierstrassPInverse(4, 0, cos(b*x + a) - I
*sin(b*x + a)) + (3*c^3*cos(b*x + a)^3 - 8*c^3*cos(b*x + a))*sqrt(c*sin(b*
x + a)))/b
```

Sympy [F(-1)]

Timed out.

$$\int (c\sin(a+bx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((c*sin(b*x+a))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (c\sin(a+bx))^{7/2}\,dx = \int (c\sin(bx+a))^{\frac{7}{2}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(7/2),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a))^(7/2), x)
```

Giac [F]

$$\int (c\sin(a+bx))^{7/2}\,dx = \int (c\sin(bx+a))^{\frac{7}{2}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(7/2),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c\sin(a+bx))^{7/2}\,dx = \int (c\,\sin(a+b\,x))^{7/2}\,dx$$

input
```
int((c*sin(a + b*x))^(7/2),x)
```

output
```
int((c*sin(a + b*x))^(7/2), x)
```

Reduce [F]

$$\int (c\sin(a+bx))^{7/2}\,dx = \sqrt{c}\left(\int \sqrt{\sin(bx+a)}\,\sin(bx+a)^3\,dx\right)c^3$$

input
```
int((c*sin(b*x+a))^(7/2),x)
```

output
```
sqrt(c)*int(sqrt(sin(a + b*x))*sin(a + b*x)**3,x)*c**3
```

3.26 $\int (c\sin(a+bx))^{5/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 75

$$\int (c\sin(a+bx))^{5/2}\,dx=\frac{6c^2E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{c\sin(a+bx)}}{5b\sqrt{\sin(a+bx)}}-\frac{2c\cos(a+bx)(c\sin(a+bx))^{3/2}}{5b}$$

output
```
-6/5*c^2*EllipticE(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*(c*sin(b*x+a))^(1/2)
/b/sin(b*x+a)^(1/2)-2/5*c*cos(b*x+a)*(c*sin(b*x+a))^(3/2)/b
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.88

$$\int (c\sin(a+bx))^{5/2}\,dx=-\frac{(c\sin(a+bx))^{5/2}\left(6E\left(\frac{1}{4}(-2a+\pi-2bx)\middle|2\right)+\sqrt{\sin(a+bx)}\sin(2(a+bx))\right)}{5b\sin^{\frac{5}{2}}(a+bx)}$$

input
```
Integrate[(c*Sin[a + b*x])^(5/2),x]
```

output
```
-1/5*((c*Sin[a + b*x])^(5/2)*(6*EllipticE[(-2*a + Pi - 2*b*x)/4, 2] + Sqrt
[Sin[a + b*x]]*Sin[2*(a + b*x)]))/(b*Sin[a + b*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3115, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\sin(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c\sin(a+bx))^{5/2}dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{3}{5}c^2\int \sqrt{c\sin(a+bx)}dx-\frac{2c\cos(a+bx)(c\sin(a+bx))^{3/2}}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{5}c^2\int \sqrt{c\sin(a+bx)}dx-\frac{2c\cos(a+bx)(c\sin(a+bx))^{3/2}}{5b}$$

$$\Big\downarrow \text{3121}$$

$$\frac{3c^2\sqrt{c\sin(a+bx)}\int \sqrt{\sin(a+bx)}dx}{5\sqrt{\sin(a+bx)}}-\frac{2c\cos(a+bx)(c\sin(a+bx))^{3/2}}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3c^2\sqrt{c\sin(a+bx)}\int \sqrt{\sin(a+bx)}dx}{5\sqrt{\sin(a+bx)}}-\frac{2c\cos(a+bx)(c\sin(a+bx))^{3/2}}{5b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{6c^2E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)|2\right)\sqrt{c\sin(a+bx)}}{5b\sqrt{\sin(a+bx)}}-\frac{2c\cos(a+bx)(c\sin(a+bx))^{3/2}}{5b}$$

input
```
Int[(c*Sin[a + b*x])^(5/2),x]
```

output
```
(6*c^2*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[c*Sin[a + b*x]])/(5*b*Sqrt[Si
n[a + b*x]]) - (2*c*Cos[a + b*x]*(c*Sin[a + b*x])^(3/2))/(5*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(65) = 130$.

Time = 3.33 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.03

method	result
default	$-\frac{c^3\left(6\sqrt{1-\sin(bx+a)}\,\sqrt{2+2\sin(bx+a)}\,\sqrt{\sin(bx+a)}\ \text{EllipticE}\left(\sqrt{1-\sin(bx+a)},\frac{\sqrt{2}}{2}\right)-3\sqrt{1-\sin(bx+a)}\,\sqrt{2+2\sin(bx+a)}\,\sqrt{\sin(bx}\right.}{5\cos(bx+a)\sqrt{c\sin(bx+a)}\,b}$

input
```
int((c*sin(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*c^3*(6*(1-sin(b*x+a))^(1/2)*(2+2*sin(b*x+a))^(1/2)*sin(b*x+a)^(1/2)*E
llipticE((1-sin(b*x+a))^(1/2),1/2*2^(1/2))-3*(1-sin(b*x+a))^(1/2)*(2+2*sin
(b*x+a))^(1/2)*sin(b*x+a)^(1/2)*EllipticF((1-sin(b*x+a))^(1/2),1/2*2^(1/2)
)-2*sin(b*x+a)^4+2*sin(b*x+a)^2)/cos(b*x+a)/(c*sin(b*x+a))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.25

$$\int (c\sin(a+bx))^{5/2}\,dx =$$
$$-\frac{2\left(\sqrt{c\sin(bx+a)}c^2\cos(bx+a)\sin(bx+a)-3i\sqrt{-\frac{1}{2}i\,c}c^2\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,$$

input
```
integrate((c*sin(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
-2/5*(sqrt(c*sin(b*x + a))*c^2*cos(b*x + a)*sin(b*x + a) - 3*I*sqrt(-1/2*I
*c)*c^2*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(b*x + a) + I*s
in(b*x + a))) + 3*I*sqrt(1/2*I*c)*c^2*weierstrassZeta(4, 0, weierstrassPIn
verse(4, 0, cos(b*x + a) - I*sin(b*x + a))))/b
```

Sympy [F]

$$\int (c\sin(a+bx))^{5/2}\,dx = \int (c\sin(a+bx))^{\frac{5}{2}}\,dx$$

input
```
integrate((c*sin(b*x+a))**(5/2),x)
```

output
```
Integral((c*sin(a + b*x))**(5/2), x)
```

Maxima [F]

$$\int (c\sin(a+bx))^{5/2}\,dx = \int (c\sin{(bx+a)})^{\frac{5}{2}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a))^(5/2), x)
```

Giac [F]

$$\int (c\sin(a+bx))^{5/2}\,dx = \int (c\sin{(bx+a)})^{\frac{5}{2}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c\sin(a+bx))^{5/2}\,dx = \int (c\,\sin{(a+b\,x)})^{5/2}\,dx$$

input
```
int((c*sin(a + b*x))^(5/2),x)
```

output
```
int((c*sin(a + b*x))^(5/2), x)
```

Reduce [F]

$$\int (c\sin(a+bx))^{5/2}\,dx = \sqrt{c}\left(\int \sqrt{\sin(bx+a)}\,\sin(bx+a)^2\,dx\right)c^2$$

input

```
int((c*sin(b*x+a))^(5/2),x)
```

output

```
sqrt(c)*int(sqrt(sin(a + b*x))*sin(a + b*x)**2,x)*c**2
```

3.27 $\int (c\sin(a+bx))^{3/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 75

$$\int (c\sin(a+bx))^{3/2}\,dx=\frac{2c^2\operatorname{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)\sqrt{\sin(a+bx)}}{3b\sqrt{c\sin(a+bx)}}-\frac{2c\cos(a+bx)\sqrt{c\sin(a+bx)}}{3b}$$

output
```
2/3*c^2*InverseJacobiAM(1/2*a-1/4*Pi+1/2*b*x,2^(1/2))*sin(b*x+a)^(1/2)/b/(
c*sin(b*x+a))^(1/2)-2/3*c*cos(b*x+a)*(c*sin(b*x+a))^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.83

$$\int (c\sin(a+bx))^{3/2}\,dx=-\frac{2\left(\operatorname{EllipticF}\left(\frac{1}{4}(-2a+\pi-2bx),2\right)+\cos(a+bx)\sqrt{\sin(a+bx)}\right)(c\sin(a+bx))^{3/2}}{3b\sin^{\frac{3}{2}}(a+bx)}$$

input
```
Integrate[(c*Sin[a + b*x])^(3/2),x]
```

output
```
(-2*(EllipticF[(-2*a + Pi - 2*b*x)/4, 2] + Cos[a + b*x]*Sqrt[Sin[a + b*x]]
)*(c*Sin[a + b*x])^(3/2))/(3*b*Sin[a + b*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\sin(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int (c\sin(a+bx))^{3/2}dx$$

$$\Big\downarrow \ 3115$$

$$\frac{1}{3}c^2\int \frac{1}{\sqrt{c\sin(a+bx)}}dx-\frac{2c\cos(a+bx)\sqrt{c\sin(a+bx)}}{3b}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{3}c^2\int \frac{1}{\sqrt{c\sin(a+bx)}}dx-\frac{2c\cos(a+bx)\sqrt{c\sin(a+bx)}}{3b}$$

$$\Big\downarrow \ 3121$$

$$\frac{c^2\sqrt{\sin(a+bx)}\int \frac{1}{\sqrt{\sin(a+bx)}}dx}{3\sqrt{c\sin(a+bx)}}-\frac{2c\cos(a+bx)\sqrt{c\sin(a+bx)}}{3b}$$

$$\Big\downarrow \ 3042$$

$$\frac{c^2\sqrt{\sin(a+bx)}\int \frac{1}{\sqrt{\sin(a+bx)}}dx}{3\sqrt{c\sin(a+bx)}}-\frac{2c\cos(a+bx)\sqrt{c\sin(a+bx)}}{3b}$$

$$\Big\downarrow \ 3120$$

$$\frac{2c^2\sqrt{\sin(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right),2\right)}{3b\sqrt{c\sin(a+bx)}}-\frac{2c\cos(a+bx)\sqrt{c\sin(a+bx)}}{3b}$$

input
```
Int[(c*Sin[a + b*x])^(3/2),x]
```

output
```
(2*c^2*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(3*b*Sqrt[c*Si
n[a + b*x]]) - (2*c*Cos[a + b*x]*Sqrt[c*Sin[a + b*x]])/(3*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 2.94 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.29

method	result	size
default	$-\frac{c^{2}\left(\sqrt{1-\sin \left(b x +a \right)}\, \sqrt{2+2 \sin \left(b x +a \right)}\, \sqrt{\sin \left(b x +a \right)}\, \operatorname{EllipticF}\left(\sqrt{1-\sin \left(b x +a \right)}, \frac{\sqrt{2}}{2}\right)-2 \sin \left(b x +a \right)^{3}+2 \sin \left(b x +a \right)\right)}{3 \cos \left(b x +a \right) \sqrt{c \sin \left(b x +a \right)}\, b}$	97

input
```
int((c*sin(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*c^2*((1-sin(b*x+a))^(1/2)*(2+2*sin(b*x+a))^(1/2)*sin(b*x+a)^(1/2)*Ell
ipticF((1-sin(b*x+a))^(1/2),1/2*2^(1/2))-2*sin(b*x+a)^3+2*sin(b*x+a))/cos(
b*x+a)/(c*sin(b*x+a))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01

$$\int (c\sin(a+bx))^{3/2}\,dx =$$
$$-\frac{2\left(\sqrt{c\sin\left(bx+a\right)}c\cos\left(bx+a\right)-\sqrt{-\frac{1}{2}i\,c}c\text{weierstrassPInverse}(4,0,\cos\left(bx+a\right)+i\,\sin\left(bx+a\right))-\sqrt{\frac{1}{2}\right.}{3\,b}$$

input
```
integrate((c*sin(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(c*sin(b*x + a))*c*cos(b*x + a) - sqrt(-1/2*I*c)*c*weierstrassPI
nverse(4, 0, cos(b*x + a) + I*sin(b*x + a)) - sqrt(1/2*I*c)*c*weierstrassP
Inverse(4, 0, cos(b*x + a) - I*sin(b*x + a)))/b
```

Sympy [F]

$$\int (c\sin(a+bx))^{3/2}\,dx = \int (c\sin{(a+bx)})^{\frac{3}{2}}\,dx$$

input
```
integrate((c*sin(b*x+a))**(3/2),x)
```

output
```
Integral((c*sin(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int (c\sin(a+bx))^{3/2}\,dx = \int (c\sin{(bx+a)})^{\frac{3}{2}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a))^(3/2), x)
```

Giac [F]

$$\int (c\sin(a+bx))^{3/2}\,dx = \int (c\sin{(bx+a)})^{\frac{3}{2}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c\sin(a+bx))^{3/2}\,dx = \int (c\,\sin{(a+b\,x)})^{3/2}\,dx$$

input
```
int((c*sin(a + b*x))^(3/2),x)
```

output
```
int((c*sin(a + b*x))^(3/2), x)
```

Reduce [F]

$$\int (c\sin(a+bx))^{3/2}\,dx = \sqrt{c}\left(\int \sqrt{\sin(bx+a)}\,\sin(bx+a)\,dx\right)c$$

input
```
int((c*sin(b*x+a))^(3/2),x)
```

output
```
sqrt(c)*int(sqrt(sin(a + b*x))*sin(a + b*x),x)*c
```

3.28 $\int \sqrt{c \sin(a+bx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 43

$$\int \sqrt{c \sin(a+bx)}\,dx = \frac{2E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{c \sin(a+bx)}}{b\sqrt{\sin(a+bx)}}$$

output
```
-2*EllipticE(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*(c*sin(b*x+a))^(1/2)/b/sin
(b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \sqrt{c \sin(a+bx)}\,dx = -\frac{2E\left(\frac{1}{4}(-2a+\pi-2bx)\middle|2\right)\sqrt{c \sin(a+bx)}}{b\sqrt{\sin(a+bx)}}$$

input
```
Integrate[Sqrt[c*Sin[a + b*x]],x]
```

output
```
(-2*EllipticE[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[c*Sin[a + b*x]])/(b*Sqrt[Sin[
a + b*x]])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c\sin(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{c\sin(a+bx)}dx$$

$$\Big\downarrow \text{3121}$$

$$\frac{\sqrt{c\sin(a+bx)}\int \sqrt{\sin(a+bx)}dx}{\sqrt{\sin(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{c\sin(a+bx)}\int \sqrt{\sin(a+bx)}dx}{\sqrt{\sin(a+bx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{c\sin(a+bx)}}{b\sqrt{\sin(a+bx)}}$$

input
```
Int[Sqrt[c*Sin[a + b*x]],x]
```

output
```
(2*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[c*Sin[a + b*x]])/(b*Sqrt[Sin[a +
b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(39) = 78$.

Time = 3.75 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.28

method	result
default	$-\frac{c\sqrt{1-\sin(bx+a)}\,\sqrt{2+2\sin(bx+a)}\,\sqrt{\sin(bx+a)}\left(2\,\text{EllipticE}\left(\sqrt{1-\sin(bx+a)},\frac{\sqrt{2}}{2}\right)-\text{EllipticF}\left(\sqrt{1-\sin(bx+a)},\frac{\sqrt{2}}{2}\right)\right)}{\cos(bx+a)\sqrt{c\sin(bx+a)}\,b}$
risch	$-\frac{i\sqrt{2}\,\sqrt{-ic\left(e^{2i(bx+a)}-1\right)e^{-i(bx+a)}}}{b}+\frac{i\left(\frac{2i\left(-ie^{2i(bx+a)}c+ic\right)}{c\sqrt{e^{i(bx+a)}\left(-ie^{2i(bx+a)}c+ic\right)}}-\frac{\sqrt{e^{i(bx+a)}+1}\,\sqrt{-2\,e^{i(bx+a)}+2}\,\sqrt{-e^{i(bx+a)}}\left(-2\,\text{EllipticE}\ldots\right.}{\sqrt{-ic\,e^{3i(bx+a)}\ldots}}\right.}{\ldots}$

input
```
int((c*sin(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-c*(1-sin(b*x+a))^(1/2)*(2+2*sin(b*x+a))^(1/2)*sin(b*x+a)^(1/2)*(2*Ellipti
cE((1-sin(b*x+a))^(1/2),1/2*2^(1/2))-EllipticF((1-sin(b*x+a))^(1/2),1/2*2^
(1/2)))/cos(b*x+a)/(c*sin(b*x+a))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.44

$$\int \sqrt{c\sin(a+bx)}\,dx =$$
$$-\frac{2\left(-i\sqrt{-\frac{1}{2}i\,c}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos{(bx+a)}+i\,\sin{(bx+a)}))+i\sqrt{\frac{1}{2}i\,c}\text{w}\right.}{b}$$

input
```
integrate((c*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
-2*(-I*sqrt(-1/2*I*c)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(
b*x + a) + I*sin(b*x + a))) + I*sqrt(1/2*I*c)*weierstrassZeta(4, 0, weiers
trassPInverse(4, 0, cos(b*x + a) - I*sin(b*x + a))))/b
```

Sympy [F]

$$\int \sqrt{c\sin(a+bx)}\,dx = \int \sqrt{c\sin{(a+bx)}}\,dx$$

input
```
integrate((c*sin(b*x+a))**(1/2),x)
```

output
```
Integral(sqrt(c*sin(a + b*x)), x)
```

Maxima [F]

$$\int \sqrt{c\sin(a+bx)}\,dx = \int \sqrt{c\sin{(bx+a)}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*sin(b*x + a)), x)
```

Giac [F]

$$\int \sqrt{c\sin(a+bx)}\,dx = \int \sqrt{c\sin(bx+a)}\,dx$$

input
```
integrate((c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*sin(b*x + a)), x)
```

Mupad [B] (verification not implemented)

Time = 28.50 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \sqrt{c\sin(a+bx)}\,dx = \frac{2\sqrt{c\,\sin(a+b\,x)}\,\mathrm{E}\left(\frac{a}{2}-\frac{\pi}{4}+\frac{b\,x}{2}\,\middle|\,2\right)}{b\,\sqrt{\sin(a+b\,x)}}$$

input
```
int((c*sin(a + b*x))^(1/2),x)
```

output
```
(2*(c*sin(a + b*x))^(1/2)*ellipticE(a/2 - pi/4 + (b*x)/2, 2))/(b*sin(a + b
*x)^(1/2))
```

Reduce [F]

$$\int \sqrt{c\sin(a+bx)}\,dx = \sqrt{c}\left(\int \sqrt{\sin(bx+a)}dx\right)$$

input
```
int((c*sin(b*x+a))^(1/2),x)
```

output
```
sqrt(c)*int(sqrt(sin(a + b*x)),x)
```

3.29 $\int \frac{1}{\sqrt{c\sin(a+bx)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 43

$$\int \frac{1}{\sqrt{c\sin(a+bx)}}\,dx=\frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)\sqrt{\sin(a+bx)}}{b\sqrt{c\sin(a+bx)}}$$

output
```
2*InverseJacobiAM(1/2*a-1/4*Pi+1/2*b*x,2^(1/2))*sin(b*x+a)^(1/2)/b/(c*sin(
b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{1}{\sqrt{c\sin(a+bx)}}\,dx=-\frac{2\,\text{EllipticF}\left(\frac{1}{4}(-2a+\pi-2bx),2\right)\sqrt{\sin(a+bx)}}{b\sqrt{c\sin(a+bx)}}$$

input
```
Integrate[1/Sqrt[c*Sin[a + b*x]],x]
```

output
```
(-2*EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a + b*x]])/(b*Sqrt[c*Sin[
a + b*x]])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{c\sin(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{c\sin(a+bx)}}dx$$

$$\Big\downarrow \text{3121}$$

$$\frac{\sqrt{\sin(a+bx)}\int \frac{1}{\sqrt{\sin(a+bx)}}dx}{\sqrt{c\sin(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(a+bx)}\int \frac{1}{\sqrt{\sin(a+bx)}}dx}{\sqrt{c\sin(a+bx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sqrt{\sin(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right),2\right)}{b\sqrt{c\sin(a+bx)}}$$

input
```
Int[1/Sqrt[c*Sin[a + b*x]],x]
```

output
```
(2*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(b*Sqrt[c*Sin[a +
b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.72

method	result	size
default	$-\frac{\sqrt{1-\sin(bx+a)}\,\sqrt{2+2\sin(bx+a)}\,\sqrt{\sin(bx+a)}\,\operatorname{EllipticF}\left(\sqrt{1-\sin(bx+a)},\frac{\sqrt{2}}{2}\right)}{\cos(bx+a)\sqrt{c\sin(bx+a)}\,b}$	74

input
```
int(1/(c*sin(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(1-sin(b*x+a))^(1/2)*(2+2*sin(b*x+a))^(1/2)*sin(b*x+a)^(1/2)*EllipticF((1
-sin(b*x+a))^(1/2),1/2*2^(1/2))/cos(b*x+a)/(c*sin(b*x+a))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.33

$$\int \frac{1}{\sqrt{c\sin(a+bx)}}\,dx$$
$$= \frac{2\left(\sqrt{-\frac{1}{2}i\,c}\text{weierstrassPInverse}(4,0,\cos(bx+a)+i\,\sin(bx+a))+\sqrt{\frac{1}{2}i\,c}\text{weierstrassPInverse}(4,0,\cos(\right.}{bc}$$

input
```
integrate(1/(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
2*(sqrt(-1/2*I*c)*weierstrassPInverse(4, 0, cos(b*x + a) + I*sin(b*x + a))
 + sqrt(1/2*I*c)*weierstrassPInverse(4, 0, cos(b*x + a) - I*sin(b*x + a)))
/(b*c)
```

Sympy [F]

$$\int \frac{1}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{1}{\sqrt{c\sin{(a+bx)}}}\,dx$$

input
```
integrate(1/(c*sin(b*x+a))**(1/2),x)
```

output
```
Integral(1/sqrt(c*sin(a + b*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{1}{\sqrt{c\sin{(bx+a)}}}\,dx$$

input
```
integrate(1/(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(c*sin(b*x + a)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{1}{\sqrt{c\sin{(bx+a)}}}\,dx$$

input
```
integrate(1/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(c*sin(b*x + a)), x)
```

Mupad [B] (verification not implemented)

Time = 28.66 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{c\sin(a+bx)}}\,dx = -\frac{2\sqrt{\sin{(a+b\,x)}}\,\mathrm{F}\left(\frac{\pi}{4}-\frac{a}{2}-\frac{b\,x}{2}\middle|2\right)}{b\,\sqrt{c\,\sin{(a+b\,x)}}}$$

input
```
int(1/(c*sin(a + b*x))^(1/2),x)
```

output
```
-(2*sin(a + b*x)^(1/2)*ellipticF(pi/4 - a/2 - (b*x)/2, 2))/(b*(c*sin(a + b
*x))^(1/2))
```

Reduce [F]

$$\int \frac{1}{\sqrt{c\sin(a+bx)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{\sin(bx+a)}}{\sin(bx+a)}dx\right)}{c}$$

input
```
int(1/(c*sin(b*x+a))^(1/2),x)
```

output
```
(sqrt(c)*int(sqrt(sin(a + b*x))/sin(a + b*x),x))/c
```

3.30 $\int \frac{1}{(c\sin(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 73

$$\int \frac{1}{(c\sin(a+bx))^{3/2}}\,dx = -\frac{2\cos(a+bx)}{bc\sqrt{c\sin(a+bx)}} - \frac{2E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{c\sin(a+bx)}}{bc^2\sqrt{\sin(a+bx)}}$$

output
```
-2*cos(b*x+a)/b/c/(c*sin(b*x+a))^(1/2)+2*EllipticE(cos(1/2*a+1/4*Pi+1/2*b*
x),2^(1/2))*(c*sin(b*x+a))^(1/2)/b/c^2/sin(b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.74

$$\int \frac{1}{(c\sin(a+bx))^{3/2}}\,dx = -\frac{2\left(\cos(a+bx) - E\left(\frac{1}{4}(-2a+\pi-2bx)\middle|2\right)\sqrt{\sin(a+bx)}\right)}{bc\sqrt{c\sin(a+bx)}}$$

input
```
Integrate[(c*Sin[a + b*x])^(-3/2),x]
```

output
```
(-2*(Cos[a + b*x] - EllipticE[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a + b*x]]
))/(b*c*Sqrt[c*Sin[a + b*x]])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c\sin(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(c\sin(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$-\frac{\int \sqrt{c\sin(a+bx)}dx}{c^2}-\frac{2\cos(a+bx)}{bc\sqrt{c\sin(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \sqrt{c\sin(a+bx)}dx}{c^2}-\frac{2\cos(a+bx)}{bc\sqrt{c\sin(a+bx)}}$$

$$\Big\downarrow \text{3121}$$

$$-\frac{\sqrt{c\sin(a+bx)}\int \sqrt{\sin(a+bx)}dx}{c^2\sqrt{\sin(a+bx)}}-\frac{2\cos(a+bx)}{bc\sqrt{c\sin(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{c\sin(a+bx)}\int \sqrt{\sin(a+bx)}dx}{c^2\sqrt{\sin(a+bx)}}-\frac{2\cos(a+bx)}{bc\sqrt{c\sin(a+bx)}}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{2E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)|2\right)\sqrt{c\sin(a+bx)}}{bc^2\sqrt{\sin(a+bx)}}-\frac{2\cos(a+bx)}{bc\sqrt{c\sin(a+bx)}}$$

input
```
Int[(c*Sin[a + b*x])^(-3/2),x]
```

output
```
(-2*Cos[a + b*x])/(b*c*Sqrt[c*Sin[a + b*x]]) - (2*EllipticE[(a - Pi/2 + b*
x)/2, 2]*Sqrt[c*Sin[a + b*x]])/(b*c^2*Sqrt[Sin[a + b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(67) = 134$.

Time = 3.10 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.93

method	result
default	$\frac{2\sqrt{1-\sin(bx+a)}\,\sqrt{2+2\sin(bx+a)}\,\sqrt{\sin(bx+a)}\ \operatorname{EllipticE}\left(\sqrt{1-\sin(bx+a)}, \frac{\sqrt{2}}{2}\right)-\sqrt{1-\sin(bx+a)}\,\sqrt{2+2\sin(bx+a)}\,\sqrt{\sin(bx+a)}\ \mathrm{El}}{c\cos(bx+a)\sqrt{c\sin(bx+a)}\,b}$

input
```
int(1/(c*sin(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/c*(2*(1-sin(b*x+a))^(1/2)*(2+2*sin(b*x+a))^(1/2)*sin(b*x+a)^(1/2)*Ellipt
icE((1-sin(b*x+a))^(1/2),1/2*2^(1/2))-(1-sin(b*x+a))^(1/2)*(2+2*sin(b*x+a)
)^(1/2)*sin(b*x+a)^(1/2)*EllipticF((1-sin(b*x+a))^(1/2),1/2*2^(1/2))-2*cos
(b*x+a)^2)/cos(b*x+a)/(c*sin(b*x+a))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.40

$$\int \frac{1}{(c\sin(a+bx))^{3/2}}\,dx =$$
$$-\frac{2\left(i\sqrt{-\frac{1}{2}i\,c}\sin\left(bx+a\right)\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)))-\right.}{}$$

input
```
integrate(1/(c*sin(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
-2*(I*sqrt(-1/2*I*c)*sin(b*x + a)*weierstrassZeta(4, 0, weierstrassPInvers
e(4, 0, cos(b*x + a) + I*sin(b*x + a))) - I*sqrt(1/2*I*c)*sin(b*x + a)*wei
erstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(b*x + a) - I*sin(b*x + a)
)) + sqrt(c*sin(b*x + a))*cos(b*x + a))/(b*c^2*sin(b*x + a))
```

Sympy [F]

$$\int \frac{1}{(c\sin(a+bx))^{3/2}}\,dx = \int \frac{1}{(c\sin{(a+bx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*sin(b*x+a))**(3/2),x)
```

output
```
Integral((c*sin(a + b*x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(c\sin(a+bx))^{3/2}}\,dx = \int \frac{1}{(c\sin(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*sin(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a))^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(c\sin(a+bx))^{3/2}}\,dx = \int \frac{1}{(c\sin(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*sin(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a))^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c\sin(a+bx))^{3/2}}\,dx = \int \frac{1}{(c\,\sin(a+b\,x))^{3/2}}\,dx$$

input
```
int(1/(c*sin(a + b*x))^(3/2),x)
```

output
```
int(1/(c*sin(a + b*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(c\sin(a+bx))^{3/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{\sin(bx+a)}}{\sin(bx+a)^2}dx\right)}{c^2}$$

input
```
int(1/(c*sin(b*x+a))^(3/2),x)
```

output
```
(sqrt(c)*int(sqrt(sin(a + b*x))/sin(a + b*x)**2,x))/c**2
```

3.31 $\int \frac{1}{(c\sin(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 77

$$\int \frac{1}{(c\sin(a+bx))^{5/2}}\,dx = -\frac{2\cos(a+bx)}{3bc(c\sin(a+bx))^{3/2}} + \frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)\sqrt{\sin(a+bx)}}{3bc^2\sqrt{c\sin(a+bx)}}$$

output
```
-2/3*cos(b*x+a)/b/c/(c*sin(b*x+a))^(3/2)+2/3*InverseJacobiAM(1/2*a-1/4*Pi+
1/2*b*x,2^(1/2))*sin(b*x+a)^(1/2)/b/c^2/(c*sin(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.71

$$\int \frac{1}{(c\sin(a+bx))^{5/2}}\,dx = -\frac{2\left(\cos(a+bx)+\text{EllipticF}\left(\frac{1}{4}(-2a+\pi-2bx),2\right)\sin^{\frac{3}{2}}(a+bx)\right)}{3bc(c\sin(a+bx))^{3/2}}$$

input
```
Integrate[(c*Sin[a + b*x])^(-5/2),x]
```

output
```
(-2*(Cos[a + b*x] + EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sin[a + b*x]^(3/2)
))/(3*b*c*(c*Sin[a + b*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3116, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c\sin(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(c\sin(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$\frac{\int \frac{1}{\sqrt{c\sin(a+bx)}}dx}{3c^2}-\frac{2\cos(a+bx)}{3bc(c\sin(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sqrt{c\sin(a+bx)}}dx}{3c^2}-\frac{2\cos(a+bx)}{3bc(c\sin(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\sqrt{\sin(a+bx)}\int \frac{1}{\sqrt{\sin(a+bx)}}dx}{3c^2\sqrt{c\sin(a+bx)}}-\frac{2\cos(a+bx)}{3bc(c\sin(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(a+bx)}\int \frac{1}{\sqrt{\sin(a+bx)}}dx}{3c^2\sqrt{c\sin(a+bx)}}-\frac{2\cos(a+bx)}{3bc(c\sin(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sqrt{\sin(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right),2\right)}{3bc^2\sqrt{c\sin(a+bx)}}-\frac{2\cos(a+bx)}{3bc(c\sin(a+bx))^{3/2}}$$

input
```
Int[(c*Sin[a + b*x])^(-5/2),x]
```

output
```
(-2*Cos[a + b*x])/(3*b*c*(c*Sin[a + b*x])^(3/2)) + (2*EllipticF[(a - Pi/2
+ b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(3*b*c^2*Sqrt[c*Sin[a + b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 2.97 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.36

method	result	size
default	$-\frac{\sqrt{1-\sin(bx+a)}\,\sqrt{2+2\sin(bx+a)}\,\sin(bx+a)^{\frac{5}{2}}\,\mathrm{EllipticF}\left(\sqrt{1-\sin(bx+a)},\frac{\sqrt{2}}{2}\right)-2\sin(bx+a)^{3}+2\sin(bx+a)}{3c^{2}\sin(bx+a)^{2}\cos(bx+a)\sqrt{c\sin(bx+a)}\,b}$	105

input
```
int(1/(c*sin(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/c^2*((1-sin(b*x+a))^(1/2)*(2+2*sin(b*x+a))^(1/2)*sin(b*x+a)^(5/2)*Ell
ipticF((1-sin(b*x+a))^(1/2),1/2*2^(1/2))-2*sin(b*x+a)^3+2*sin(b*x+a))/sin(
b*x+a)^2/cos(b*x+a)/(c*sin(b*x+a))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.43

$$\int \frac{1}{(c\sin(a+bx))^{5/2}}\,dx = \frac{2\left(\left(\cos\left(bx+a\right)^2-1\right)\sqrt{-\frac{1}{2}i\,c}\,\text{weierstrassPInverse}(4,0,\cos\left(bx+a\right)+i\,\sin\left(bx\right.\right.}{}$$

input
```
integrate(1/(c*sin(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
2/3*((cos(b*x + a)^2 - 1)*sqrt(-1/2*I*c)*weierstrassPInverse(4, 0, cos(b*x
 + a) + I*sin(b*x + a)) + (cos(b*x + a)^2 - 1)*sqrt(1/2*I*c)*weierstrassPI
nverse(4, 0, cos(b*x + a) - I*sin(b*x + a)) + sqrt(c*sin(b*x + a))*cos(b*x
 + a))/(b*c^3*cos(b*x + a)^2 - b*c^3)
```

Sympy [F]

$$\int \frac{1}{(c\sin(a+bx))^{5/2}}\,dx = \int \frac{1}{\left(c\sin\left(a+bx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*sin(b*x+a))**(5/2),x)
```

output
```
Integral((c*sin(a + b*x))**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(c\sin(a+bx))^{5/2}}\,dx = \int \frac{1}{(c\sin(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*sin(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a))^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(c\sin(a+bx))^{5/2}}\,dx = \int \frac{1}{(c\sin(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*sin(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a))^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c\sin(a+bx))^{5/2}}\,dx = \int \frac{1}{(c\,\sin(a+b\,x))^{5/2}}\,dx$$

input
```
int(1/(c*sin(a + b*x))^(5/2),x)
```

output
```
int(1/(c*sin(a + b*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{(c\sin(a+bx))^{5/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{\sin(bx+a)}}{\sin(bx+a)^3}dx\right)}{c^3}$$

input
```
int(1/(c*sin(b*x+a))^(5/2),x)
```

output
```
(sqrt(c)*int(sqrt(sin(a + b*x))/sin(a + b*x)**3,x))/c**3
```

3.32 $\int \frac{1}{(c\sin(a+bx))^{7/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 105

$$\int \frac{1}{(c\sin(a+bx))^{7/2}}\,dx = -\frac{2\cos(a+bx)}{5bc(c\sin(a+bx))^{5/2}} - \frac{6\cos(a+bx)}{5bc^3\sqrt{c\sin(a+bx)}} - \frac{6E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{c\sin(a+bx)}}{5bc^4\sqrt{\sin(a+bx)}}$$

output
```
-2/5*cos(b*x+a)/b/c/(c*sin(b*x+a))^(5/2)-6/5*cos(b*x+a)/b/c^3/(c*sin(b*x+a
))^(1/2)+6/5*EllipticE(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*(c*sin(b*x+a))^(
1/2)/b/c^4/sin(b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.65

$$\int \frac{1}{(c\sin(a+bx))^{7/2}}\,dx = -\frac{2\left(\cot(a+bx)-3E\left(\frac{1}{4}(-2a+\pi-2bx)\middle|2\right)\sin^{\frac{3}{2}}(a+bx)+\frac{3}{2}\sin(2(a+bx))\right)}{5bc^2(c\sin(a+bx))^{3/2}}$$

input
```
Integrate[(c*Sin[a + b*x])^(-7/2),x]
```

output
```
(-2*(Cot[a + b*x] - 3*EllipticE[(-2*a + Pi - 2*b*x)/4, 2]*Sin[a + b*x]^(3/
2) + (3*Sin[2*(a + b*x)])/2))/(5*b*c^2*(c*Sin[a + b*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 3116, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c\sin(a+bx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(c\sin(a+bx))^{7/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$\frac{3\int \frac{1}{(c\sin(a+bx))^{3/2}}dx}{5c^2}-\frac{2\cos(a+bx)}{5bc(c\sin(a+bx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{1}{(c\sin(a+bx))^{3/2}}dx}{5c^2}-\frac{2\cos(a+bx)}{5bc(c\sin(a+bx))^{5/2}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{3\left(-\frac{\int \sqrt{c\sin(a+bx)}dx}{c^2}-\frac{2\cos(a+bx)}{bc\sqrt{c\sin(a+bx)}}\right)}{5c^2}-\frac{2\cos(a+bx)}{5bc(c\sin(a+bx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(-\frac{\int \sqrt{c\sin(a+bx)}dx}{c^2}-\frac{2\cos(a+bx)}{bc\sqrt{c\sin(a+bx)}}\right)}{5c^2}-\frac{2\cos(a+bx)}{5bc(c\sin(a+bx))^{5/2}}$$

$$\Big\downarrow \text{3121}$$

$$\frac{3\left(-\frac{\sqrt{c\sin(a+bx)}\int\sqrt{\sin(a+bx)}dx}{c^2\sqrt{\sin(a+bx)}}-\frac{2\cos(a+bx)}{bc\sqrt{c\sin(a+bx)}}\right)}{5c^2}-\frac{2\cos(a+bx)}{5bc(c\sin(a+bx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(-\frac{\sqrt{c\sin(a+bx)}\int\sqrt{\sin(a+bx)}dx}{c^2\sqrt{\sin(a+bx)}}-\frac{2\cos(a+bx)}{bc\sqrt{c\sin(a+bx)}}\right)}{5c^2}-\frac{2\cos(a+bx)}{5bc(c\sin(a+bx))^{5/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{3\left(-\frac{2E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)|2\right)\sqrt{c\sin(a+bx)}}{bc^2\sqrt{\sin(a+bx)}}-\frac{2\cos(a+bx)}{bc\sqrt{c\sin(a+bx)}}\right)}{5c^2}-\frac{2\cos(a+bx)}{5bc(c\sin(a+bx))^{5/2}}$$

input
```
Int[(c*Sin[a + b*x])^(-7/2),x]
```

output
```
(-2*Cos[a + b*x])/(5*b*c*(c*Sin[a + b*x])^(5/2)) + (3*((-2*Cos[a + b*x])/(
b*c*Sqrt[c*Sin[a + b*x]]) - (2*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[c*Sin
[a + b*x]])/(b*c^2*Sqrt[Sin[a + b*x]])))/(5*c^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 3.07 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.60

method	result
default	$\frac{6\sqrt{1-\sin(bx+a)}\,\sqrt{2+2\sin(bx+a)}\,\sin(bx+a)^{\frac{7}{2}}\,\mathrm{EllipticE}\left(\sqrt{1-\sin(bx+a)},\frac{\sqrt{2}}{2}\right)-3\sqrt{1-\sin(bx+a)}\,\sqrt{2+2\sin(bx+a)}\,\sin(bx+a)^{\frac{7}{2}}\,\mathrm{El}}{5c^{3}\sin(bx+a)^{3}\cos(bx+a)\sqrt{c\sin(bx+a)}\,b}$

input
```
int(1/(c*sin(b*x+a))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/5/c^3*(6*(1-sin(b*x+a))^(1/2)*(2+2*sin(b*x+a))^(1/2)*sin(b*x+a)^(7/2)*El
lipticE((1-sin(b*x+a))^(1/2),1/2*2^(1/2))-3*(1-sin(b*x+a))^(1/2)*(2+2*sin(
b*x+a))^(1/2)*sin(b*x+a)^(7/2)*EllipticF((1-sin(b*x+a))^(1/2),1/2*2^(1/2))
+6*sin(b*x+a)^5-4*sin(b*x+a)^3-2*sin(b*x+a))/sin(b*x+a)^3/cos(b*x+a)/(c*si
n(b*x+a))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.48

$$\int \frac{1}{(c\sin(a+bx))^{7/2}}\,dx =$$
$$-\frac{2\left(3\left(i\cos\left(bx+a\right)^{2}-i\right)\sqrt{-\frac{1}{2}i\,c}\sin\left(bx+a\right)\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos\left(bx+a\right)}{}$$

input
```
integrate(1/(c*sin(b*x+a))^(7/2),x, algorithm="fricas")
```

output
```
-2/5*(3*(I*cos(b*x + a)^2 - I)*sqrt(-1/2*I*c)*sin(b*x + a)*weierstrassZeta
(4, 0, weierstrassPInverse(4, 0, cos(b*x + a) + I*sin(b*x + a))) + 3*(-I*c
os(b*x + a)^2 + I)*sqrt(1/2*I*c)*sin(b*x + a)*weierstrassZeta(4, 0, weiers
trassPInverse(4, 0, cos(b*x + a) - I*sin(b*x + a))) + (3*cos(b*x + a)^3 -
4*cos(b*x + a))*sqrt(c*sin(b*x + a)))/((b*c^4*cos(b*x + a)^2 - b*c^4)*sin(
b*x + a))
```

Sympy [F]

$$\int \frac{1}{(c\sin(a+bx))^{7/2}}\,dx = \int \frac{1}{(c\sin{(a+bx)})^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(c*sin(b*x+a))**(7/2),x)
```

output
```
Integral((c*sin(a + b*x))**(-7/2), x)
```

Maxima [F]

$$\int \frac{1}{(c\sin(a+bx))^{7/2}}\,dx = \int \frac{1}{(c\sin{(bx+a)})^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(c*sin(b*x+a))^(7/2),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a))^(-7/2), x)
```

Giac [F]

$$\int \frac{1}{(c\sin(a+bx))^{7/2}}\,dx = \int \frac{1}{(c\sin(bx+a))^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(c*sin(b*x+a))^(7/2),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a))^(-7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c\sin(a+bx))^{7/2}}\,dx = \int \frac{1}{(c\,\sin(a+b\,x))^{7/2}}\,dx$$

input
```
int(1/(c*sin(a + b*x))^(7/2),x)
```

output
```
int(1/(c*sin(a + b*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{1}{(c\sin(a+bx))^{7/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{\sin(bx+a)}}{\sin(bx+a)^4}dx\right)}{c^4}$$

input
```
int(1/(c*sin(b*x+a))^(7/2),x)
```

output
```
(sqrt(c)*int(sqrt(sin(a + b*x))/sin(a + b*x)**4,x))/c**4
```

3.33 $\int (c\sin(a+bx))^{4/3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 58

$$\int (c\sin(a+bx))^{4/3}\,dx=\frac{3\cos(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\sin^2(a+bx)\right)(c\sin(a+bx))^{7/3}}{7bc\sqrt{\cos^2(a+bx)}}$$

output
```
3/7*cos(b*x+a)*hypergeom([1/2, 7/6],[13/6],sin(b*x+a)^2)*(c*sin(b*x+a))^(7
/3)/b/c/(cos(b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int (c\sin(a+bx))^{4/3}\,dx=\frac{3\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\sin^2(a+bx)\right)(c\sin(a+bx))^{4/3}\tan(a+bx)}{7b}$$

input
```
Integrate[(c*Sin[a + b*x])^(4/3),x]
```

output
```
(3*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, 7/6, 13/6, Sin[a + b*x]^2]*
(c*Sin[a + b*x])^(4/3)*Tan[a + b*x])/(7*b)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\sin(a+bx))^{4/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c\sin(a+bx))^{4/3}dx$$

$$\Big\downarrow \text{3122}$$

$$\frac{3\cos(a+bx)(c\sin(a+bx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\sin^2(a+bx)\right)}{7bc\sqrt{\cos^2(a+bx)}}$$

input
```
Int[(c*Sin[a + b*x])^(4/3),x]
```

output
```
(3*Cos[a + b*x]*Hypergeometric2F1[1/2, 7/6, 13/6, Sin[a + b*x]^2]*(c*Sin[a
 + b*x])^(7/3))/(7*b*c*Sqrt[Cos[a + b*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Maple [F]

$$\int (c\sin(bx+a))^{\frac{4}{3}}\,dx$$

input
```
int((c*sin(b*x+a))^(4/3),x)
```

output
```
int((c*sin(b*x+a))^(4/3),x)
```

Fricas [F]

$$\int (c\sin(a+bx))^{4/3}\,dx = \int (c\sin(bx+a))^{\frac{4}{3}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(4/3),x, algorithm="fricas")
```

output
```
integral((c*sin(b*x + a))^(1/3)*c*sin(b*x + a), x)
```

Sympy [F]

$$\int (c\sin(a+bx))^{4/3}\,dx = \int (c\sin(a+bx))^{\frac{4}{3}}\,dx$$

input
```
integrate((c*sin(b*x+a))**(4/3),x)
```

output
```
Integral((c*sin(a + b*x))**(4/3), x)
```

Maxima [F]

$$\int (c\sin(a+bx))^{4/3}\,dx = \int (c\sin(bx+a))^{\frac{4}{3}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(4/3),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a))^(4/3), x)
```

Giac [F]

$$\int (c\sin(a+bx))^{4/3}\,dx = \int (c\sin(bx+a))^{\frac{4}{3}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(4/3),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c\sin(a+bx))^{4/3}\,dx = \int (c\,\sin(a+b\,x))^{4/3}\,dx$$

input
```
int((c*sin(a + b*x))^(4/3),x)
```

output
```
int((c*sin(a + b*x))^(4/3), x)
```

Reduce [F]

$$\int (c\sin(a+bx))^{4/3}\,dx = c^{\frac{4}{3}}\left(\int \sin(bx+a)^{\frac{4}{3}}\,dx\right)$$

input
```
int((c*sin(b*x+a))^(4/3),x)
```

output
```
c**(1/3)*int(sin(a + b*x)**(1/3)*sin(a + b*x),x)*c
```

3.34 $\int (c\sin(a+bx))^{2/3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 58

$$\int (c\sin(a+bx))^{2/3}\,dx=\frac{3\cos(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\sin^2(a+bx)\right)(c\sin(a+bx))^{5/3}}{5bc\sqrt{\cos^2(a+bx)}}$$

output
```
3/5*cos(b*x+a)*hypergeom([1/2, 5/6],[11/6],sin(b*x+a)^2)*(c*sin(b*x+a))^(5
/3)/b/c/(cos(b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int (c\sin(a+bx))^{2/3}\,dx=\frac{3\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\sin^2(a+bx)\right)(c\sin(a+bx))^{2/3}\tan(a+bx)}{5b}$$

input
```
Integrate[(c*Sin[a + b*x])^(2/3),x]
```

output
```
(3*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, 5/6, 11/6, Sin[a + b*x]^2]*
(c*Sin[a + b*x])^(2/3)*Tan[a + b*x])/(5*b)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\sin(a+bx))^{2/3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (c\sin(a+bx))^{2/3}dx$$

$$\Big\downarrow \; 3122$$

$$\frac{3\cos(a+bx)(c\sin(a+bx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\sin^2(a+bx)\right)}{5bc\sqrt{\cos^2(a+bx)}}$$

input
```
Int[(c*Sin[a + b*x])^(2/3),x]
```

output
```
(3*Cos[a + b*x]*Hypergeometric2F1[1/2, 5/6, 11/6, Sin[a + b*x]^2]*(c*Sin[a
 + b*x])^(5/3))/(5*b*c*Sqrt[Cos[a + b*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Maple [F]

$$\int (c\sin(bx+a))^{\frac{2}{3}}\,dx$$

input
```
int((c*sin(b*x+a))^(2/3),x)
```

output
```
int((c*sin(b*x+a))^(2/3),x)
```

Fricas [F]

$$\int (c\sin(a+bx))^{2/3}\,dx = \int (c\sin(bx+a))^{\frac{2}{3}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(2/3),x, algorithm="fricas")
```

output
```
integral((c*sin(b*x + a))^(2/3), x)
```

Sympy [F]

$$\int (c\sin(a+bx))^{2/3}\,dx = \int (c\sin(a+bx))^{\frac{2}{3}}\,dx$$

input
```
integrate((c*sin(b*x+a))**(2/3),x)
```

output
```
Integral((c*sin(a + b*x))**(2/3), x)
```

Maxima [F]

$$\int (c\sin(a+bx))^{2/3}\,dx = \int (c\sin(bx+a))^{\frac{2}{3}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(2/3),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a))^(2/3), x)
```

Giac [F]

$$\int (c\sin(a+bx))^{2/3}\,dx = \int (c\sin(bx+a))^{\frac{2}{3}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(2/3),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a))^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c\sin(a+bx))^{2/3}\,dx = \int (c\,\sin(a+b\,x))^{2/3}\,dx$$

input
```
int((c*sin(a + b*x))^(2/3),x)
```

output
```
int((c*sin(a + b*x))^(2/3), x)
```

Reduce [F]

$$\int (c\sin(a+bx))^{2/3}\,dx = c^{\frac{2}{3}}\left(\int \sin(bx+a)^{\frac{2}{3}}\,dx\right)$$

input
```
int((c*sin(b*x+a))^(2/3),x)
```

output
```
c**(2/3)*int(sin(a + b*x)**(2/3),x)
```

3.35 $\int \sqrt[3]{c\sin(a+bx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 517

$$\int \sqrt[3]{c\sin(a+bx)}\,dx = -\frac{3\sqrt{\frac{3}{2}\left(3-i\sqrt{3}\right)}\sqrt[3]{c}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{1-\frac{(c\sin(a+bx))^{2/3}}{c^{2/3}}}}{\sqrt{3+i\sqrt{3}}}\right)|\frac{3i-\sqrt{3}}{3i+\sqrt{3}}\right)\sec(a+bx)\sqrt{1-\frac{(c\sin(a+bx))^{2/3}}{c^{2/3}}}\sqrt{\frac{i+\sqrt{3}}{3i+\sqrt{3}}+\cdots}}{b} + \frac{3\left(1-i\sqrt{3}\right)\sqrt{3-i\sqrt{3}}\sqrt[3]{c}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{1-\frac{(c\sin(a+bx))^{2/3}}{c^{2/3}}}}{\sqrt{3-i\sqrt{3}}}\right),\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)\sec(a+bx)\sqrt{1-\frac{(c\sin(a+b\cdots}{c^{2/3}}\cdots}}{2\sqrt{2}b}$$

output
```
-3/2*(18-6*I*3^(1/2))^(1/2)*c^(1/3)*EllipticE(2^(1/2)*(1-(c*sin(b*x+a))^(2
/3)/c^(2/3))^(1/2)/(3+I*3^(1/2))^(1/2),((3*I-3^(1/2))/(3*I+3^(1/2)))^(1/2)
)*sec(b*x+a)*(1-(c*sin(b*x+a))^(2/3)/c^(2/3))^(1/2)*((3^(1/2)+I)/(3*I+3^(1
/2))+2*(c*sin(b*x+a))^(2/3)/(3-I*3^(1/2))/c^(2/3))^(1/2)*((I-3^(1/2))/(3*I
-3^(1/2))+2*(c*sin(b*x+a))^(2/3)/(3+I*3^(1/2))/c^(2/3))^(1/2)/b+3/4*(1-I*3
^(1/2))*(3-I*3^(1/2))^(1/2)*c^(1/3)*EllipticF(2^(1/2)*(1-(c*sin(b*x+a))^(2
/3)/c^(2/3))^(1/2)/(3-I*3^(1/2))^(1/2),((3*I+3^(1/2))/(3*I-3^(1/2)))^(1/2)
)*sec(b*x+a)*(1-(c*sin(b*x+a))^(2/3)/c^(2/3))^(1/2)*((3^(1/2)+I)/(3*I+3^(1
/2))+2*(c*sin(b*x+a))^(2/3)/(3-I*3^(1/2))/c^(2/3))^(1/2)*((I-3^(1/2))/(3*I
-3^(1/2))+2*(c*sin(b*x+a))^(2/3)/(3+I*3^(1/2))/c^(2/3))^(1/2)*2^(1/2)/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.11

$$\int \sqrt[3]{c\sin(a+bx)}\,dx = \frac{3\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\sin^2(a+bx)\right)\sqrt[3]{c\sin(a+bx)}\tan(a+bx)}{4b}$$

input
```
Integrate[(c*Sin[a + b*x])^(1/3),x]
```

output
```
(3*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, 2/3, 5/3, Sin[a + b*x]^2]*(
c*Sin[a + b*x])^(1/3)*Tan[a + b*x])/(4*b)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.32 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{c\sin(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt[3]{c\sin(a+bx)}dx$$

$$\Big\downarrow \text{3122}$$

$$\frac{3\cos(a+bx)(c\sin(a+bx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\sin^2(a+bx)\right)}{4bc\sqrt{\cos^2(a+bx)}}$$

input
```
Int[(c*Sin[a + b*x])^(1/3),x]
```

output
```
(3*Cos[a + b*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Sin[a + b*x]^2]*(c*Sin[a
+ b*x])^(4/3))/(4*b*c*Sqrt[Cos[a + b*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Maple [F]

$$\int (c\sin (bx+a))^{\frac{1}{3}} dx$$

input
```
int((c*sin(b*x+a))^(1/3),x)
```

output
```
int((c*sin(b*x+a))^(1/3),x)
```

Fricas [F]

$$\int \sqrt[3]{c\sin(a+bx)}\,dx = \int (c\sin (bx+a))^{\frac{1}{3}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(1/3),x, algorithm="fricas")
```

output
```
integral((c*sin(b*x + a))^(1/3), x)
```

Sympy [F]

$$\int \sqrt[3]{c\sin(a+bx)}\,dx = \int \sqrt[3]{c\sin(a+bx)}\,dx$$

input
```
integrate((c*sin(b*x+a))**(1/3),x)
```

output
```
Integral((c*sin(a + b*x))**(1/3), x)
```

Maxima [F]

$$\int \sqrt[3]{c\sin(a+bx)}\,dx = \int (c\sin(bx+a))^{\frac{1}{3}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(1/3),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a))^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{c\sin(a+bx)}\,dx = \int (c\sin(bx+a))^{\frac{1}{3}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(1/3),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{c\sin(a+bx)}\,dx = \int (c\,\sin(a+b\,x))^{1/3}\,dx$$

input
```
int((c*sin(a + b*x))^(1/3),x)
```

output
```
int((c*sin(a + b*x))^(1/3), x)
```

Reduce [F]

$$\int \sqrt[3]{c\sin(a+bx)}\,dx = c^{\frac{1}{3}}\left(\int \sin(bx+a)^{\frac{1}{3}}\,dx\right)$$

input
```
int((c*sin(b*x+a))^(1/3),x)
```

output
```
c**(1/3)*int(sin(a + b*x)**(1/3),x)
```

3.36 $\int \frac{1}{\sqrt[3]{c\sin(a+bx)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 252

$$\int \frac{1}{\sqrt[3]{c\sin(a+bx)}}\,dx = -\frac{3\sqrt{3-i\sqrt{3}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{1-\frac{(c\sin(a+bx))^{2/3}}{c^{2/3}}}}{\sqrt{3-i\sqrt{3}}}\right),\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)\sec(a+bx)\sqrt{1-\frac{(c\sin(a+bx))^{2/3}}{c^{2/3}}}\sqrt{\frac{i+\sqrt{3}}{3i+\sqrt{3}}\cdots}}{\sqrt{2}b\sqrt[3]{c}}$$

output
```
-3/2*(3-I*3^(1/2))^(1/2)*EllipticF(2^(1/2)*(1-(c*sin(b*x+a))^(2/3)/c^(2/3)
)^(1/2)/(3-I*3^(1/2))^(1/2),((3*I+3^(1/2))/(3*I-3^(1/2)))^(1/2))*sec(b*x+a
)*(1-(c*sin(b*x+a))^(2/3)/c^(2/3))^(1/2)*((3^(1/2)+I)/(3*I+3^(1/2))+2*(c*s
in(b*x+a))^(2/3)/(3-I*3^(1/2))/c^(2/3))^(1/2)*((I-3^(1/2))/(3*I-3^(1/2))+2
*(c*sin(b*x+a))^(2/3)/(3+I*3^(1/2))/c^(2/3))^(1/2)*2^(1/2)/b/c^(1/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.22

$$\int \frac{1}{\sqrt[3]{c\sin(a+bx)}}\,dx = \frac{3\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\sin^2(a+bx)\right)\tan(a+bx)}{2b\sqrt[3]{c\sin(a+bx)}}$$

input
```
Integrate[(c*Sin[a + b*x])^(-1/3),x]
```

output
```
(3*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/3, 1/2, 4/3, Sin[a + b*x]^2]*T
an[a + b*x])/(2*b*(c*Sin[a + b*x])^(1/3))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.32 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.23, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{c\sin(a+bx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sqrt[3]{c\sin(a+bx)}}\,dx$$
$$\Big\downarrow \text{3122}$$
$$\frac{3\cos(a+bx)(c\sin(a+bx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\sin^2(a+bx)\right)}{2bc\sqrt{\cos^2(a+bx)}}$$

input
```
Int[(c*Sin[a + b*x])^(-1/3),x]
```

output
```
(3*Cos[a + b*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Sin[a + b*x]^2]*(c*Sin[a
+ b*x])^(2/3))/(2*b*c*Sqrt[Cos[a + b*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Maple [F]

$$\int \frac{1}{\left(c \sin \left(b x +a \right)\right)^{\frac{1}{3}}}d x$$

input
```
int(1/(c*sin(b*x+a))^(1/3),x)
```

output
```
int(1/(c*sin(b*x+a))^(1/3),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[3]{c \sin(a+bx)}}\, dx = \int \frac{1}{\left(c \sin \left(b x + a\right)\right)^{\frac{1}{3}}}\, dx$$

input
```
integrate(1/(c*sin(b*x+a))^(1/3),x, algorithm="fricas")
```

output
```
integral((c*sin(b*x + a))^(2/3)/(c*sin(b*x + a)), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{c\sin(a+bx)}}\,dx = \int \frac{1}{\sqrt[3]{c\sin{(a+bx)}}}\,dx$$

input
```
integrate(1/(c*sin(b*x+a))**(1/3),x)
```

output
```
Integral((c*sin(a + b*x))**(-1/3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{c\sin(a+bx)}}\,dx = \int \frac{1}{(c\sin{(bx+a)})^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(c*sin(b*x+a))^(1/3),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a))^(-1/3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{c\sin(a+bx)}}\,dx = \int \frac{1}{(c\sin{(bx+a)})^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(c*sin(b*x+a))^(1/3),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a))^(-1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{c\sin(a+bx)}}\,dx=\int \frac{1}{\left(c\,\sin\left(a+b\,x\right)\right)^{1/3}}\,dx$$

input
```
int(1/(c*sin(a + b*x))^(1/3),x)
```

output
```
int(1/(c*sin(a + b*x))^(1/3), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{c\sin(a+bx)}}\,dx=\frac{\int \frac{1}{\sin(bx+a)^{\frac{1}{3}}}dx}{c^{\frac{1}{3}}}$$

input
```
int(1/(c*sin(b*x+a))^(1/3),x)
```

output
```
int(1/sin(a + b*x)**(1/3),x)/c**(1/3)
```

3.37 $\int \frac{1}{(c\sin(a+bx))^{2/3}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 271

$$\int \frac{1}{(c\sin(a+bx))^{2/3}}\,dx = \frac{3^{3/4}\,\text{EllipticF}\left(\arccos\left(\frac{c^{2/3}-\left(1-\sqrt{3}\right)(c\sin(a+bx))^{2/3}}{c^{2/3}-\left(1+\sqrt{3}\right)(c\sin(a+bx))^{2/3}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)\sec(a+bx)\sqrt[3]{c\,s}}{2bc^{5/3}\sqrt{-\frac{(c\sin(a+bx))^{2/3}(c^{2}}{\left(c^{2/3}-\left(1+\sqrt{3}\right)\right.}}}$$

output
```
1/2*3^(3/4)*InverseJacobiAM(arccos((c^(2/3)-(1-3^(1/2))*(c*sin(b*x+a))^(2/
3))/(c^(2/3)-(1+3^(1/2))*(c*sin(b*x+a))^(2/3))),1/4*6^(1/2)+1/4*2^(1/2))*s
ec(b*x+a)*(c*sin(b*x+a))^(1/3)*(c^(2/3)-(c*sin(b*x+a))^(2/3))*(c^(4/3)*(1+
(c*sin(b*x+a))^(2/3)/c^(2/3)+(c*sin(b*x+a))^(4/3)/c^(4/3))/(c^(2/3)-(1+3^(
1/2))*(c*sin(b*x+a))^(2/3))^2)^(1/2)/b/c^(5/3)/(-(c*sin(b*x+a))^(2/3)*(c^(
2/3)-(c*sin(b*x+a))^(2/3))/(c^(2/3)-(1+3^(1/2))*(c*sin(b*x+a))^(2/3))^2)^(
1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.20

$$\int \frac{1}{(c\sin(a+bx))^{2/3}}\,dx = \frac{3\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\sin^2(a+bx)\right)\tan(a+bx)}{b(c\sin(a+bx))^{2/3}}$$

input
```
Integrate[(c*Sin[a + b*x])^(-2/3),x]
```

output
```
(3*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[a + b*x]^2]*T
an[a + b*x])/(b*(c*Sin[a + b*x])^(2/3))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.33 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.21, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c\sin(a+bx))^{2/3}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(c\sin(a+bx))^{2/3}}dx$$

$$\Big\downarrow \text{3122}$$

$$\frac{3\cos(a+bx)\sqrt[3]{c\sin(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\sin^2(a+bx)\right)}{bc\sqrt{\cos^2(a+bx)}}$$

input
```
Int[(c*Sin[a + b*x])^(-2/3),x]
```

output
```
(3*Cos[a + b*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[a + b*x]^2]*(c*Sin[a
+ b*x])^(1/3))/(b*c*Sqrt[Cos[a + b*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Maple [F]

$$\int \frac{1}{\left(c\sin\left(bx+a\right)\right)^{\frac{2}{3}}}dx$$

input
```
int(1/(c*sin(b*x+a))^(2/3),x)
```

output
```
int(1/(c*sin(b*x+a))^(2/3),x)
```

Fricas [F]

$$\int \frac{1}{(c\sin(a+bx))^{2/3}}\,dx = \int \frac{1}{\left(c\sin\left(bx+a\right)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(c*sin(b*x+a))^(2/3),x, algorithm="fricas")
```

output
```
integral((c*sin(b*x + a))^(1/3)/(c*sin(b*x + a)), x)
```

Sympy [F]

$$\int \frac{1}{(c\sin(a+bx))^{2/3}}\,dx = \int \frac{1}{(c\sin{(a+bx)})^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(c*sin(b*x+a))**(2/3),x)
```

output
```
Integral((c*sin(a + b*x))**(-2/3), x)
```

Maxima [F]

$$\int \frac{1}{(c\sin(a+bx))^{2/3}}\,dx = \int \frac{1}{(c\sin{(bx+a)})^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(c*sin(b*x+a))^(2/3),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a))^(-2/3), x)
```

Giac [F]

$$\int \frac{1}{(c\sin(a+bx))^{2/3}}\,dx = \int \frac{1}{(c\sin{(bx+a)})^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(c*sin(b*x+a))^(2/3),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a))^(-2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c\sin(a+bx))^{2/3}}\,dx = \int \frac{1}{\left(c\,\sin\left(a+b\,x\right)\right)^{2/3}}\,dx$$

input
```
int(1/(c*sin(a + b*x))^(2/3),x)
```

output
```
int(1/(c*sin(a + b*x))^(2/3), x)
```

Reduce [F]

$$\int \frac{1}{(c\sin(a+bx))^{2/3}}\,dx = \frac{\int \frac{1}{\sin(bx+a)^{\frac{2}{3}}}dx}{c^{\frac{2}{3}}}$$

input
```
int(1/(c*sin(b*x+a))^(2/3),x)
```

output
```
int(1/sin(a + b*x)**(2/3),x)/c**(2/3)
```

3.38 $\int \frac{1}{(c\sin(a+bx))^{4/3}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 56

$$\int \frac{1}{(c\sin(a+bx))^{4/3}}\,dx = -\frac{3\cos(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\sin^2(a+bx)\right)}{bc\sqrt{\cos^2(a+bx)}\,\sqrt[3]{c\sin(a+bx)}}$$

output
```
-3*cos(b*x+a)*hypergeom([-1/6, 1/2],[5/6],sin(b*x+a)^2)/b/c/(cos(b*x+a)^2)
^(1/2)/(c*sin(b*x+a))^(1/3)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

$$\int \frac{1}{(c\sin(a+bx))^{4/3}}\,dx = -\frac{3\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\sin^2(a+bx)\right)\tan(a+bx)}{b(c\sin(a+bx))^{4/3}}$$

input
```
Integrate[(c*Sin[a + b*x])^(-4/3),x]
```

output
```
(-3*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[-1/6, 1/2, 5/6, Sin[a + b*x]^2]
*Tan[a + b*x])/(b*(c*Sin[a + b*x])^(4/3))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c\sin(a+bx))^{4/3}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(c\sin(a+bx))^{4/3}}dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{3\cos(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\sin^2(a+bx)\right)}{bc\sqrt{\cos^2(a+bx)}\sqrt[3]{c\sin(a+bx)}}$$

input
```
Int[(c*Sin[a + b*x])^(-4/3),x]
```

output
```
(-3*Cos[a + b*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Sin[a + b*x]^2])/(b*c*S
qrt[Cos[a + b*x]^2]*(c*Sin[a + b*x])^(1/3))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Maple [F]

$$\int \frac{1}{(c\sin(bx+a))^{\frac{4}{3}}}dx$$

input
```
int(1/(c*sin(b*x+a))^(4/3),x)
```

output
```
int(1/(c*sin(b*x+a))^(4/3),x)
```

Fricas [F]

$$\int \frac{1}{(c\sin(a+bx))^{4/3}}\,dx = \int \frac{1}{(c\sin(bx+a))^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(c*sin(b*x+a))^(4/3),x, algorithm="fricas")
```

output
```
integral(-(c*sin(b*x + a))^(2/3)/(c^2*cos(b*x + a)^2 - c^2), x)
```

Sympy [F]

$$\int \frac{1}{(c\sin(a+bx))^{4/3}}\,dx = \int \frac{1}{(c\sin(a+bx))^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(c*sin(b*x+a))**(4/3),x)
```

output
```
Integral((c*sin(a + b*x))**(-4/3), x)
```

Maxima [F]

$$\int \frac{1}{(c\sin(a+bx))^{4/3}}\,dx = \int \frac{1}{(c\sin(bx+a))^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(c*sin(b*x+a))^(4/3),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a))^(-4/3), x)
```

Giac [F]

$$\int \frac{1}{(c\sin(a+bx))^{4/3}}\,dx = \int \frac{1}{(c\sin(bx+a))^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(c*sin(b*x+a))^(4/3),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a))^(-4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c\sin(a+bx))^{4/3}}\,dx = \int \frac{1}{(c\,\sin(a+b\,x))^{4/3}}\,dx$$

input
```
int(1/(c*sin(a + b*x))^(4/3),x)
```

output
```
int(1/(c*sin(a + b*x))^(4/3), x)
```

Reduce [F]

$$\int \frac{1}{(c\sin(a+bx))^{4/3}}\,dx = \frac{\int \frac{1}{\sin(bx+a)^{\frac{4}{3}}}dx}{c^{\frac{4}{3}}}$$

input
```
int(1/(c*sin(b*x+a))^(4/3),x)
```

output
```
int(1/(sin(a + b*x)**(1/3)*sin(a + b*x)),x)/(c**(1/3)*c)
```

3.39 $\int \sin^n(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 63

$$\int \sin^n(a+bx)\,dx = \frac{\cos(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(a+bx)\right)\sin^{1+n}(a+bx)}{b(1+n)\sqrt{\cos^2(a+bx)}}$$

output
```
cos(b*x+a)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],sin(b*x+a)^2)*sin(b*x+a)
^(1+n)/b/(1+n)/(cos(b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \sin^n(a+bx)\,dx = \frac{\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(a+bx)\right)\sec(a+bx)\sin^{1+n}(a+bx)}{b(1+n)}$$

input
```
Integrate[Sin[a + b*x]^n,x]
```

output
```
(Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[a +
 b*x]^2]*Sec[a + b*x]*Sin[a + b*x]^(1 + n))/(b*(1 + n))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^n(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^n dx$$

$$\Big\downarrow \text{3122}$$

$$\frac{\cos(a+bx)\sin^{n+1}(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\sin^2(a+bx)\right)}{b(n+1)\sqrt{\cos^2(a+bx)}}$$

input
```
Int[Sin[a + b*x]^n,x]
```

output
```
(Cos[a + b*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[a + b*x]^2]
*Sin[a + b*x]^(1 + n))/(b*(1 + n)*Sqrt[Cos[a + b*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Maple [F]

$$\int \sin(bx+a)^n\,dx$$

input
```
int(sin(b*x+a)^n,x)
```

output
```
int(sin(b*x+a)^n,x)
```

Fricas [F]

$$\int \sin^n(a+bx)\,dx = \int \sin(bx+a)^n\,dx$$

input
```
integrate(sin(b*x+a)^n,x, algorithm="fricas")
```

output
```
integral(sin(b*x + a)^n, x)
```

Sympy [F]

$$\int \sin^n(a+bx)\,dx = \int \sin^n(a+bx)\,dx$$

input
```
integrate(sin(b*x+a)**n,x)
```

output
```
Integral(sin(a + b*x)**n, x)
```

Maxima [F]

$$\int \sin^n(a+bx)\,dx = \int \sin{(bx+a)}^n \,dx$$

input
```
integrate(sin(b*x+a)^n,x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^n, x)
```

Giac [F]

$$\int \sin^n(a+bx)\,dx = \int \sin{(bx+a)}^n \,dx$$

input
```
integrate(sin(b*x+a)^n,x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^n, x)
```

Mupad [B] (verification not implemented)

Time = 26.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

$$\int \sin^n(a+bx)\,dx = -\frac{\cos{(a+b\,x)}\,\sin{(a+b\,x)}^{n+1}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{1}{2}-\frac{n}{2};\ \frac{3}{2};\ \cos{(a+b\,x)}^2\right)}{b\left(\sin{(a+b\,x)}^2\right)^{\frac{n}{2}+\frac{1}{2}}}$$

input
```
int(sin(a + b*x)^n,x)
```

output
```
-(cos(a + b*x)*sin(a + b*x)^(n + 1)*hypergeom([1/2, 1/2 - n/2], 3/2, cos(a
 + b*x)^2))/(b*(sin(a + b*x)^2)^(n/2 + 1/2))
```

Reduce [F]

$$\int \sin^n(a+bx)\,dx = \int \sin{(bx+a)}^n\,dx$$

input
```
int(sin(b*x+a)^n,x)
```

output
```
int(sin(a + b*x)**n,x)
```

3.40 $\int (c\sin(a+bx))^n\,dx$

Optimal result

Integrand size = 10, antiderivative size = 68

$$\int (c\sin(a+bx))^n\,dx = \frac{\cos(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^{1+n}}{bc(1+n)\sqrt{\cos^2(a+bx)}}$$

output
```
cos(b*x+a)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],sin(b*x+a)^2)*(c*sin(b*x
+a))^(1+n)/b/c/(1+n)/(cos(b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

$$\int (c\sin(a+bx))^n\,dx = \frac{\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^n\tan(a+bx)}{b(1+n)}$$

input
```
Integrate[(c*Sin[a + b*x])^n,x]
```

output
```
(Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[a +
 b*x]^2]*(c*Sin[a + b*x])^n*Tan[a + b*x])/(b*(1 + n))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\sin(a+bx))^n\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (c\sin(a+bx))^n dx$$

$$\Big\downarrow \; 3122$$

$$\frac{\cos(a+bx)(c\sin(a+bx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\sin^2(a+bx)\right)}{bc(n+1)\sqrt{\cos^2(a+bx)}}$$

input
```
Int[(c*Sin[a + b*x])^n,x]
```

output
```
(Cos[a + b*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[a + b*x]^2]
*(c*Sin[a + b*x])^(1 + n))/(b*c*(1 + n)*Sqrt[Cos[a + b*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Maple [F]

$$\int (c\sin(bx+a))^n\,dx$$

input
```
int((c*sin(b*x+a))^n,x)
```

output
```
int((c*sin(b*x+a))^n,x)
```

Fricas [F]

$$\int (c\sin(a+bx))^n\,dx = \int (c\sin(bx+a))^n\,dx$$

input
```
integrate((c*sin(b*x+a))^n,x, algorithm="fricas")
```

output
```
integral((c*sin(b*x + a))^n, x)
```

Sympy [F]

$$\int (c\sin(a+bx))^n\,dx = \int (c\sin(a+bx))^n\,dx$$

input
```
integrate((c*sin(b*x+a))**n,x)
```

output
```
Integral((c*sin(a + b*x))**n, x)
```

Maxima [F]

$$\int (c\sin(a+bx))^n\,dx = \int (c\sin(bx+a))^n\,dx$$

input
```
integrate((c*sin(b*x+a))^n,x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a))^n, x)
```

Giac [F]

$$\int (c\sin(a+bx))^n\,dx = \int (c\sin(bx+a))^n\,dx$$

input
```
integrate((c*sin(b*x+a))^n,x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c\sin(a+bx))^n\,dx = \int (c\,\sin(a+b\,x))^n\,dx$$

input
```
int((c*sin(a + b*x))^n,x)
```

output
```
int((c*sin(a + b*x))^n, x)
```

Reduce [F]

$$\int (c\sin(a+bx))^n\,dx = c^n\left(\int \sin(bx+a)^n\,dx\right)$$

input
```
int((c*sin(b*x+a))^n,x)
```

output
```
c**n*int(sin(a + b*x)**n,x)
```

3.41 $\int (a\sin(e+fx))^m (b\sin(e+fx))^n \,dx$

Optimal result

Integrand size = 21, antiderivative size = 81

$$\int (a\sin(e+fx))^m (b\sin(e+fx))^n \,dx = \frac{\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+m+n),\frac{1}{2}(3+m+n),\sin^2(e+fx)\right)(a\sin(e+fx))^{1+m}(b\sin}{af(1+m+n)\sqrt{\cos^2(e+fx)}}$$

output
```
cos(f*x+e)*hypergeom([1/2, 1/2+1/2*m+1/2*n],[3/2+1/2*m+1/2*n],sin(f*x+e)^2
)*(a*sin(f*x+e))^(1+m)*(b*sin(f*x+e))^n/a/f/(1+m+n)/(cos(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.94

$$\int (a\sin(e+fx))^m (b\sin(e+fx))^n \,dx = \frac{\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+m+n),\frac{1}{2}(3+m+n),\sin^2(e+fx)\right)(a\sin(e+fx))^m(b\,\text{si}}{f(1+m+n)}$$

input
```
Integrate[(a*Sin[e + f*x])^m*(b*Sin[e + f*x])^n,x]
```

output
```
(Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[1/2, (1 + m + n)/2, (3 + m + n)/2,
 Sin[e + f*x]^2]*(a*Sin[e + f*x])^m*(b*Sin[e + f*x])^n*Tan[e + f*x])/(f*(1
 + m + n))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2034, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx))^m(b\sin(e+fx))^n\,dx$$

$$\Big\downarrow \text{2034}$$

$$(a\sin(e+fx))^{-n}(b\sin(e+fx))^n\int (a\sin(e+fx))^{m+n}dx$$

$$\Big\downarrow \text{3042}$$

$$(a\sin(e+fx))^{-n}(b\sin(e+fx))^n\int (a\sin(e+fx))^{m+n}dx$$

$$\Big\downarrow \text{3122}$$

$$\frac{\cos(e+fx)(a\sin(e+fx))^{m+1}(b\sin(e+fx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(m+n+1),\frac{1}{2}(m+n+3),\sin^2(e+fx\right.}{af(m+n+1)\sqrt{\cos^2(e+fx)}}$$

input
```
Int[(a*Sin[e + f*x])^m*(b*Sin[e + f*x])^n,x]
```

output
```
(Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + m + n)/2, (3 + m + n)/2, Sin[e +
 f*x]^2]*(a*Sin[e + f*x])^(1 + m)*(b*Sin[e + f*x])^n)/(a*f*(1 + m + n)*Sqr
t[Cos[e + f*x]^2])
```

Defintions of rubi rules used

rule 2034
```
Int[(Fx_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Simp[b^IntPart
[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n]))   Int[(a*v)^(m + n
)*Fx, x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] &&
 !IntegerQ[m + n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Maple [F]

$$\int (a\sin(fx+e))^m (b\sin(fx+e))^n\,dx$$

input
```
int((a*sin(f*x+e))^m*(b*sin(f*x+e))^n,x)
```

output
```
int((a*sin(f*x+e))^m*(b*sin(f*x+e))^n,x)
```

Fricas [F]

$$\int (a\sin(e+fx))^m(b\sin(e+fx))^n\,dx=\int (a\sin(fx+e))^m (b\sin(fx+e))^n\,dx$$

input
```
integrate((a*sin(f*x+e))^m*(b*sin(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((a*sin(f*x + e))^m*(b*sin(f*x + e))^n, x)
```

Sympy [F]

$$\int (a\sin(e+fx))^m(b\sin(e+fx))^n\,dx = \int (a\sin(e+fx))^m (b\sin(e+fx))^n\,dx$$

input
```
integrate((a*sin(f*x+e))**m*(b*sin(f*x+e))**n,x)
```

output
```
Integral((a*sin(e + f*x))**m*(b*sin(e + f*x))**n, x)
```

Maxima [F]

$$\int (a\sin(e+fx))^m(b\sin(e+fx))^n\,dx = \int (a\sin(fx+e))^m (b\sin(fx+e))^n\,dx$$

input
```
integrate((a*sin(f*x+e))^m*(b*sin(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^m*(b*sin(f*x + e))^n, x)
```

Giac [F]

$$\int (a\sin(e+fx))^m(b\sin(e+fx))^n\,dx = \int (a\sin(fx+e))^m (b\sin(fx+e))^n\,dx$$

input
```
integrate((a*sin(f*x+e))^m*(b*sin(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e))^m*(b*sin(f*x + e))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a\sin(e+fx))^m(b\sin(e+fx))^n\,dx = \int (a\,\sin\left(e+f\,x\right))^m\,(b\,\sin\left(e+f\,x\right))^n\,dx$$

input
```
int((a*sin(e + f*x))^m*(b*sin(e + f*x))^n,x)
```

output
```
int((a*sin(e + f*x))^m*(b*sin(e + f*x))^n, x)
```

Reduce [F]

$$\int (a\sin(e+fx))^m(b\sin(e+fx))^n\,dx = b^n a^m\left(\int \sin\left(fx+e\right)^{m+n}dx\right)$$

input
```
int((a*sin(f*x+e))^m*(b*sin(f*x+e))^n,x)
```

output
```
b**n*a**m*int(sin(e + f*x)**(m + n),x)
```

3.42 $\int \cos^3(a+bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \cos^3(a+bx)\sin(a+bx)\,dx=-\frac{\cos^4(a+bx)}{4b}$$

output
```
-1/4*cos(b*x+a)^4/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cos^3(a+bx)\sin(a+bx)\,dx=-\frac{\cos^4(a+bx)}{4b}$$

input
```
Integrate[Cos[a + b*x]^3*Sin[a + b*x],x]
```

output
```
-1/4*Cos[a + b*x]^4/b
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3045, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)\cos(a+bx)^3dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int \cos^3(a+bx)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\cos^4(a+bx)}{4b}$$

input
```
Int[Cos[a + b*x]^3*Sin[a + b*x],x]
```

output
```
-1/4*Cos[a + b*x]^4/b
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [A] (verified)

Time = 4.77 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result
derivativedivides	$-\frac{\cos(bx+a)^4}{4b}$
default	$-\frac{\cos(bx+a)^4}{4b}$
risch	$-\frac{\cos(4bx+4a)}{32b}-\frac{\cos(2bx+2a)}{8b}$
parallelrisch	$\frac{2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4+1\right)}{b\left(1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right)^4}$
norman	$\frac{\frac{2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}{b}+\frac{2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^6}{b}}{\left(1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right)^4}$
orering	$-\frac{5\left(-3\cos(bx+a)^2\sin(bx+a)^2b+\cos(bx+a)^4b\right)}{16b^2}-\frac{-6\sin(bx+a)^4b^3+48\cos(bx+a)^2\sin(bx+a)^2b^3-10\cos(bx+a)^4b}{64b^4}$

input
```
int(cos(b*x+a)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*cos(b*x+a)^4/b
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos^3(a+bx)\sin(a+bx)\,dx = -\frac{\cos(bx+a)^4}{4\,b}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/4*cos(b*x + a)^4/b
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \cos^3(a+bx)\sin(a+bx)\,dx = \begin{cases} -\frac{\cos^4(a+bx)}{4b} & \text{for } b \neq 0 \\ x\sin(a)\cos^3(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a)**3*sin(b*x+a),x)
```

output
```
Piecewise((-cos(a + b*x)**4/(4*b), Ne(b, 0)), (x*sin(a)*cos(a)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos^3(a+bx)\sin(a+bx)\,dx = -\frac{\cos(bx+a)^4}{4b}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/4*cos(b*x + a)^4/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos^3(a+bx)\sin(a+bx)\,dx = -\frac{\cos(bx+a)^4}{4b}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/4*cos(b*x + a)^4/b
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos^3(a+bx)\sin(a+bx)\,dx = -\frac{\cos{(a+b\,x)}^4}{4\,b}$$

input
```
int(cos(a + b*x)^3*sin(a + b*x),x)
```

output
```
-cos(a + b*x)^4/(4*b)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos^3(a+bx)\sin(a+bx)\,dx = -\frac{\cos{(bx+a)}^4}{4b}$$

input
```
int(cos(b*x+a)^3*sin(b*x+a),x)
```

output
```
( - cos(a + b*x)**4)/(4*b)
```

3.43 $\int \cos^2(a+bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \cos^2(a+bx)\sin(a+bx)\,dx=-\frac{\cos^3(a+bx)}{3b}$$

output
```
-1/3*cos(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cos^2(a+bx)\sin(a+bx)\,dx=-\frac{\cos^3(a+bx)}{3b}$$

input
```
Integrate[Cos[a + b*x]^2*Sin[a + b*x],x]
```

output
```
-1/3*Cos[a + b*x]^3/b
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3045, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)\cos(a+bx)^2dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int \cos^2(a+bx)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\cos^3(a+bx)}{3b}$$

input
```
Int[Cos[a + b*x]^2*Sin[a + b*x],x]
```

output
```
-1/3*Cos[a + b*x]^3/b
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [A] (verified)

Time = 4.44 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{\cos(bx+a)^3}{3b}$	14
default	$-\frac{\cos(bx+a)^3}{3b}$	14
risch	$-\frac{\cos(bx+a)}{4b}-\frac{\cos(3bx+3a)}{12b}$	27
parallelrisch	$\frac{-2-6\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}{3b\left(1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right)^3}$	36
norman	$\frac{-\frac{2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}{b}-\frac{2}{3b}}{\left(1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right)^3}$	39
orering	$-\frac{10\left(-2\sin(bx+a)^2b\cos(bx+a)+\cos(bx+a)^3b\right)}{9b^2}-\frac{-7b^3\cos(bx+a)^3+20\sin(bx+a)^2b^3\cos(bx+a)}{9b^4}$	73

input
```
int(cos(b*x+a)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*cos(b*x+a)^3/b
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos^2(a+bx)\sin(a+bx)\,dx=-\frac{\cos\left(bx+a\right)^3}{3\,b}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/3*cos(b*x + a)^3/b
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \cos^2(a+bx)\sin(a+bx)\,dx = \begin{cases} -\frac{\cos^3(a+bx)}{3b} & \text{for } b \neq 0 \\ x\sin(a)\cos^2(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a)**2*sin(b*x+a),x)
```

output
```
Piecewise((-cos(a + b*x)**3/(3*b), Ne(b, 0)), (x*sin(a)*cos(a)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos^2(a+bx)\sin(a+bx)\,dx = -\frac{\cos(bx+a)^3}{3b}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/3*cos(b*x + a)^3/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos^2(a+bx)\sin(a+bx)\,dx = -\frac{\cos(bx+a)^3}{3b}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/3*cos(b*x + a)^3/b
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos^2(a+bx)\sin(a+bx)\,dx = -\frac{\cos\left(a+b\,x\right)^3}{3\,b}$$

input
```
int(cos(a + b*x)^2*sin(a + b*x),x)
```

output
```
-cos(a + b*x)^3/(3*b)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos^2(a+bx)\sin(a+bx)\,dx = -\frac{\cos\left(bx+a\right)^3}{3b}$$

input
```
int(cos(b*x+a)^2*sin(b*x+a),x)
```

output
```
( - cos(a + b*x)**3)/(3*b)
```

3.44 $\int \cos(a+bx)\sin(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \cos(a+bx)\sin(a+bx)\,dx=\frac{\sin^2(a+bx)}{2b}$$

output
```
1/2*sin(b*x+a)^2/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 37 vs. 2(15) = 30.

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.47

$$\int \cos(a+bx)\sin(a+bx)\,dx=\frac{1}{2}\left(-\frac{\cos(2a)\cos(2bx)}{2b}+\frac{\sin(2a)\sin(2bx)}{2b}\right)$$

input
```
Integrate[Cos[a + b*x]*Sin[a + b*x],x]
```

output
```
(-1/2*(Cos[2*a]*Cos[2*b*x])/b + (Sin[2*a]*Sin[2*b*x])/(2*b))/2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3044, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)\cos(a+bx)dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{\int \sin(a+bx)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{\sin^2(a+bx)}{2b}$$

input
```
Int[Cos[a + b*x]*Sin[a + b*x],x]
```

output
```
Sin[a + b*x]^2/(2*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\sin(bx+a)^2}{2b}$	14
default	$\frac{\sin(bx+a)^2}{2b}$	14
risch	$-\frac{\cos(2bx+2a)}{4b}$	15
parallelrisch	$\frac{1-\cos(2bx+2a)}{4b}$	19
orering	$-\frac{-b\sin(bx+a)^2+\cos(bx+a)^2 b}{4b^2}$	28
norman	$\frac{2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}{b\left(1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right)^2}$	32

input
```
int(cos(b*x+a)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*sin(b*x+a)^2/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\sin(a+bx)\,dx = -\frac{\cos(bx+a)^2}{2\,b}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*cos(b*x + a)^2/b
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \cos(a+bx)\sin(a+bx)\,dx = \begin{cases} -\frac{\cos^2(a+bx)}{2b} & \text{for } b \neq 0 \\ x\sin(a)\cos(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a),x)
```

output
```
Piecewise((-cos(a + b*x)**2/(2*b), Ne(b, 0)), (x*sin(a)*cos(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\sin(a+bx)\,dx = -\frac{\cos(bx+a)^2}{2b}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*cos(b*x + a)^2/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\sin(a+bx)\,dx = -\frac{\cos(bx+a)^2}{2b}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/2*cos(b*x + a)^2/b
```

Mupad [B] (verification not implemented)

Time = 25.88 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.87

$$\int \cos(a+bx)\sin(a+bx)\,dx = \begin{cases} \frac{x\,\sin(2\,a)}{2} & \text{if } b=0 \\ -\frac{\cos(2\,a+2\,b\,x)}{4\,b} & \text{if } b\neq 0 \end{cases}$$

input
```
int(cos(a + b*x)*sin(a + b*x),x)
```

output
```
piecewise(b == 0, (x*sin(2*a))/2, b ~= 0, -cos(2*a + 2*b*x)/(4*b))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\sin(a+bx)\,dx = -\frac{\cos(bx+a)^2}{2b}$$

input
```
int(cos(b*x+a)*sin(b*x+a),x)
```

output
```
( - cos(a + b*x)**2)/(2*b)
```

3.45 $\int \tan(a + bx)\, dx$

Optimal result

Integrand size = 6, antiderivative size = 12

$$\int \tan(a + bx)\, dx = -\frac{\log(\cos(a + bx))}{b}$$

output
```
-ln(cos(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \tan(a + bx)\, dx = -\frac{\log(\cos(a + bx))}{b}$$

input
```
Integrate[Tan[a + b*x],x]
```

output
```
-(Log[Cos[a + b*x]]/b)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(a+bx)dx$$

$$\Big\downarrow \text{3956}$$

$$-\frac{\log(\cos(a+bx))}{b}$$

input
```
Int[Tan[a + b*x],x]
```

output
```
-(Log[Cos[a + b*x]]/b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

method	result	size
derivativedivides	$\frac{\ln\left(1+\tan(bx+a)^2\right)}{2b}$	17
default	$\frac{\ln\left(1+\tan(bx+a)^2\right)}{2b}$	17
norman	$\frac{\ln\left(1+\tan(bx+a)^2\right)}{2b}$	17
parallelrisch	$\frac{\ln\left(1+\tan(bx+a)^2\right)}{2b}$	17
risch	$ix + \frac{2ia}{b} - \frac{\ln(e^{2i(bx+a)}+1)}{b}$	30

input
```
int(tan(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*ln(1+tan(b*x+a)^2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \tan(a+bx)\,dx = -\frac{\log\left(\frac{1}{\tan(bx+a)^2+1}\right)}{2\,b}$$

input
```
integrate(tan(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*log(1/(tan(b*x + a)^2 + 1))/b
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.58

$$\int \tan(a+bx)\,dx = \begin{cases} \frac{\log(\tan^2(a+bx)+1)}{2b} & \text{for } b \neq 0 \\ x\tan(a) & \text{otherwise} \end{cases}$$

input
```
integrate(tan(b*x+a),x)
```

output
```
Piecewise((log(tan(a + b*x)**2 + 1)/(2*b), Ne(b, 0)), (x*tan(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \tan(a+bx)\,dx = \frac{\log(\sec(bx+a))}{b}$$

input
```
integrate(tan(b*x+a),x, algorithm="maxima")
```

output
```
log(sec(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \tan(a+bx)\,dx = -\frac{\log(|\cos(bx+a)|)}{b}$$

input
```
integrate(tan(b*x+a),x, algorithm="giac")
```

output
```
-log(abs(cos(b*x + a)))/b
```

Mupad [B] (verification not implemented)

Time = 25.78 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \tan(a+bx)\,dx = \frac{\ln\left(\tan(a+b\,x)^2+1\right)}{2\,b}$$

input
```
int(tan(a + b*x),x)
```

output
```
log(tan(a + b*x)^2 + 1)/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \tan(a+bx)\,dx = \frac{\log\left(\tan\left(bx+a\right)^2+1\right)}{2b}$$

input
```
int(tan(b*x+a),x)
```

output
```
log(tan(a + b*x)**2 + 1)/(2*b)
```

3.46 $\int \sec(a+bx)\tan(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 10

$$\int \sec(a+bx)\tan(a+bx)\,dx=\frac{\sec(a+bx)}{b}$$

output
```
sec(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \sec(a+bx)\tan(a+bx)\,dx=\frac{\sec(a+bx)}{b}$$

input
```
Integrate[Sec[a + b*x]*Tan[a + b*x],x]
```

output
```
Sec[a + b*x]/b
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3086, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(a+bx)\sec(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(a+bx)\sec(a+bx)dx$$

$$\Big\downarrow \text{ 3086}$$

$$\frac{\int 1d\sec(a+bx)}{b}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\sec(a+bx)}{b}$$

input
```
Int[Sec[a + b*x]*Tan[a + b*x],x]
```

output
```
Sec[a + b*x]/b
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\frac{\sec(bx+a)}{b}$	11
default	$\frac{\sec(bx+a)}{b}$	11
risch	$\frac{2\,\mathrm{e}^{i(bx+a)}}{b\left(\mathrm{e}^{2i(bx+a)}+1\right)}$	28

input
```
int(sec(b*x+a)*tan(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
sec(b*x+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \sec(a+bx)\tan(a+bx)\,dx = \frac{1}{b\cos\left(bx+a\right)}$$

input
```
integrate(sec(b*x+a)*tan(b*x+a),x, algorithm="fricas")
```

output
```
1/(b*cos(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(7) = 14$.

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.50

$$\int \sec(a+bx)\tan(a+bx)\,dx = \begin{cases} \frac{\sec(a+bx)}{b} & \text{for } b \neq 0 \\ x\tan(a)\sec(a) & \text{otherwise} \end{cases}$$

input
```
integrate(sec(b*x+a)*tan(b*x+a),x)
```

output
```
Piecewise((sec(a + b*x)/b, Ne(b, 0)), (x*tan(a)*sec(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \sec(a+bx)\tan(a+bx)\,dx = \frac{1}{b\cos(bx+a)}$$

input
```
integrate(sec(b*x+a)*tan(b*x+a),x, algorithm="maxima")
```

output
```
1/(b*cos(b*x + a))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \sec(a+bx)\tan(a+bx)\,dx = \frac{1}{b\cos(bx+a)}$$

input
```
integrate(sec(b*x+a)*tan(b*x+a),x, algorithm="giac")
```

output
```
1/(b*cos(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 26.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

$$\int \sec(a+bx)\tan(a+bx)\,dx = -\frac{2}{b\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2-1\right)}$$

input
```
int(tan(a + b*x)/cos(a + b*x),x)
```

output
```
-2/(b*(tan(a/2 + (b*x)/2)^2 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \sec(a+bx)\tan(a+bx)\,dx = \frac{\sec(bx+a)}{b}$$

input
```
int(sec(b*x+a)*tan(b*x+a),x)
```

output
```
sec(a + b*x)/b
```

3.47 $\int \sec^2(a+bx)\tan(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \sec^2(a+bx)\tan(a+bx)\,dx=\frac{\sec^2(a+bx)}{2b}$$

output
```
1/2*sec(b*x+a)^2/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \sec^2(a+bx)\tan(a+bx)\,dx=\frac{\sec^2(a+bx)}{2b}$$

input
```
Integrate[Sec[a + b*x]^2*Tan[a + b*x],x]
```

output
```
Sec[a + b*x]^2/(2*b)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3086, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(a+bx)\sec^2(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \tan(a+bx)\sec(a+bx)^2dx$$

$$\Big\downarrow \; 3086$$

$$\frac{\int \sec(a+bx)d\sec(a+bx)}{b}$$

$$\Big\downarrow \; 15$$

$$\frac{\sec^2(a+bx)}{2b}$$

input
```
Int[Sec[a + b*x]^2*Tan[a + b*x],x]
```

output
```
Sec[a + b*x]^2/(2*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\sec(bx+a)^2}{2b}$	14
default	$\frac{\sec(bx+a)^2}{2b}$	14
risch	$\frac{2\,\mathrm{e}^{2i(bx+a)}}{b\left(\mathrm{e}^{2i(bx+a)}+1\right)^2}$	28

input
```
int(sec(b*x+a)^2*tan(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*sec(b*x+a)^2/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sec^2(a+bx)\tan(a+bx)\,dx=\frac{1}{2\,b\cos\left(bx+a\right)^2}$$

input
```
integrate(sec(b*x+a)^2*tan(b*x+a),x, algorithm="fricas")
```

output
```
1/2/(b*cos(b*x + a)^2)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \sec^2(a+bx)\tan(a+bx)\,dx = \begin{cases} \frac{\sec^2(a+bx)}{2b} & \text{for } b \neq 0 \\ x\tan(a)\sec^2(a) & \text{otherwise} \end{cases}$$

input
```
integrate(sec(b*x+a)**2*tan(b*x+a),x)
```

output
```
Piecewise((sec(a + b*x)**2/(2*b), Ne(b, 0)), (x*tan(a)*sec(a)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sec^2(a+bx)\tan(a+bx)\,dx = \frac{\tan(bx+a)^2}{2b}$$

input
```
integrate(sec(b*x+a)^2*tan(b*x+a),x, algorithm="maxima")
```

output
```
1/2*tan(b*x + a)^2/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sec^2(a+bx)\tan(a+bx)\,dx = \frac{1}{2b\cos(bx+a)^2}$$

input
```
integrate(sec(b*x+a)^2*tan(b*x+a),x, algorithm="giac")
```

output
```
1/2/(b*cos(b*x + a)^2)
```

Mupad [B] (verification not implemented)

Time = 25.81 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sec^2(a+bx)\tan(a+bx)\,dx = \frac{\tan(a+b\,x)^2}{2\,b}$$

input
```
int(tan(a + b*x)/cos(a + b*x)^2,x)
```

output
```
tan(a + b*x)^2/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sec^2(a+bx)\tan(a+bx)\,dx = \frac{\sec(bx+a)^2}{2b}$$

input
```
int(sec(b*x+a)^2*tan(b*x+a),x)
```

output
```
sec(a + b*x)**2/(2*b)
```

3.48 $\int \sec^3(a+bx)\tan(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \sec^3(a+bx)\tan(a+bx)\,dx=\frac{\sec^3(a+bx)}{3b}$$

output
```
1/3*sec(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \sec^3(a+bx)\tan(a+bx)\,dx=\frac{\sec^3(a+bx)}{3b}$$

input
```
Integrate[Sec[a + b*x]^3*Tan[a + b*x],x]
```

output
```
Sec[a + b*x]^3/(3*b)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3086, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(a+bx)\sec^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(a+bx)\sec(a+bx)^3dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{\int \sec^2(a+bx)d\sec(a+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{\sec^3(a+bx)}{3b}$$

input
```
Int[Sec[a + b*x]^3*Tan[a + b*x],x]
```

output
```
Sec[a + b*x]^3/(3*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\sec(bx+a)^3}{3b}$	14
default	$\frac{\sec(bx+a)^3}{3b}$	14
risch	$\frac{8\,e^{3i(bx+a)}}{3b\left(e^{2i(bx+a)}+1\right)^3}$	28

input
```
int(sec(b*x+a)^3*tan(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*sec(b*x+a)^3/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sec^3(a+bx)\tan(a+bx)\,dx=\frac{1}{3\,b\cos{(bx+a)}^3}$$

input
```
integrate(sec(b*x+a)^3*tan(b*x+a),x, algorithm="fricas")
```

output
```
1/3/(b*cos(b*x + a)^3)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \sec^3(a+bx)\tan(a+bx)\,dx = \begin{cases} \frac{\sec^3(a+bx)}{3b} & \text{for } b \neq 0 \\ x\tan(a)\sec^3(a) & \text{otherwise} \end{cases}$$

input
```
integrate(sec(b*x+a)**3*tan(b*x+a),x)
```

output
```
Piecewise((sec(a + b*x)**3/(3*b), Ne(b, 0)), (x*tan(a)*sec(a)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sec^3(a+bx)\tan(a+bx)\,dx = \frac{1}{3\,b\cos(bx+a)^3}$$

input
```
integrate(sec(b*x+a)^3*tan(b*x+a),x, algorithm="maxima")
```

output
```
1/3/(b*cos(b*x + a)^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sec^3(a+bx)\tan(a+bx)\,dx = \frac{1}{3\,b\cos(bx+a)^3}$$

input
```
integrate(sec(b*x+a)^3*tan(b*x+a),x, algorithm="giac")
```

output
```
1/3/(b*cos(b*x + a)^3)
```

Mupad [B] (verification not implemented)

Time = 25.75 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sec^3(a+bx)\tan(a+bx)\,dx = \frac{1}{3\,b\cos\left(a+b\,x\right)^3}$$

input
```
int(tan(a + b*x)/cos(a + b*x)^3,x)
```

output
```
1/(3*b*cos(a + b*x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sec^3(a+bx)\tan(a+bx)\,dx = \frac{\sec\left(bx+a\right)^3}{3b}$$

input
```
int(sec(b*x+a)^3*tan(b*x+a),x)
```

output
```
sec(a + b*x)**3/(3*b)
```

3.49 $\int \cos^7(a+bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 61

$$\int \cos^7(a+bx)\sin^2(a+bx)\,dx=\frac{\sin^3(a+bx)}{3b}-\frac{3\sin^5(a+bx)}{5b}+\frac{3\sin^7(a+bx)}{7b}-\frac{\sin^9(a+bx)}{9b}$$

output
```
1/3*sin(b*x+a)^3/b-3/5*sin(b*x+a)^5/b+3/7*sin(b*x+a)^7/b-1/9*sin(b*x+a)^9/
b
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int \cos^7(a+bx)\sin^2(a+bx)\,dx=\frac{(1606+1389\cos(2(a+bx))+330\cos(4(a+bx))+35\cos(6(a+bx)))\sin^3(a+bx)}{10080b}$$

input
```
Integrate[Cos[a + b*x]^7*Sin[a + b*x]^2,x]
```

output
```
((1606 + 1389*Cos[2*(a + b*x)] + 330*Cos[4*(a + b*x)] + 35*Cos[6*(a + b*x)
])*Sin[a + b*x]^3)/(10080*b)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\cos^7(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^2\cos(a+bx)^7dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{\int \sin^2(a+bx)\left(1-\sin^2(a+bx)\right)^3d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(-\sin^8(a+bx)+3\sin^6(a+bx)-3\sin^4(a+bx)+\sin^2(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{9}\sin^9(a+bx)+\frac{3}{7}\sin^7(a+bx)-\frac{3}{5}\sin^5(a+bx)+\frac{1}{3}\sin^3(a+bx)}{b}$$

input
```
Int[Cos[a + b*x]^7*Sin[a + b*x]^2,x]
```

output
```
(Sin[a + b*x]^3/3 - (3*Sin[a + b*x]^5)/5 + (3*Sin[a + b*x]^7)/7 - Sin[a +
b*x]^9/9)/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 17.70 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

method	result
derivativedivides	$-\frac{\frac{\sin(bx+a)^9}{9}-\frac{3\sin(bx+a)^7}{7}+\frac{3\sin(bx+a)^5}{5}-\frac{\sin(bx+a)^3}{3}}{b}$
default	$-\frac{\frac{\sin(bx+a)^9}{9}-\frac{3\sin(bx+a)^7}{7}+\frac{3\sin(bx+a)^5}{5}-\frac{\sin(bx+a)^3}{3}}{b}$
risch	$\frac{7\sin(bx+a)}{128b}-\frac{\sin(9bx+9a)}{2304b}-\frac{5\sin(7bx+7a)}{1792b}-\frac{\sin(5bx+5a)}{160b}$
parallelrisch	$-\frac{\left(\sin\left(\frac{3bx}{2}+\frac{3a}{2}\right)-3\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)(1389\cos(2bx+2a)+35\cos(6bx+6a)+330\cos(4bx+4a)+1606)\left(\cos\left(\frac{3bx}{2}+\frac{3a}{2}\right)+3\cos\right.}{20160b}$
norman	$\frac{\frac{8\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^3}{3b}-\frac{16\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^5}{5b}+\frac{632\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^7}{35b}-\frac{2848\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^9}{315b}+\frac{632\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{11}}{35b}-\frac{16\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{13}}{5b}+\frac{8\tan\left(\frac{b}{2}\right.}{3}}{\left(1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right)^9}$
orering	$-\frac{106444\left(-7\cos(bx+a)^6\sin(bx+a)^3b+2\cos(bx+a)^8\sin(bx+a)b\right)}{99225b^2}-\frac{2458\left(-210\cos(bx+a)^4\sin(bx+a)^5b^3+427\cos\right.}{3}$

input
```
int(cos(b*x+a)^7*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/b*(1/9*sin(b*x+a)^9-3/7*sin(b*x+a)^7+3/5*sin(b*x+a)^5-1/3*sin(b*x+a)^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \cos^7(a+bx)\sin^2(a+bx)\,dx = -\frac{\left(35\,\cos\left(bx+a\right)^8 - 5\,\cos\left(bx+a\right)^6 - 6\,\cos\left(bx+a\right)^4 - 8\,\cos\left(bx+a\right)^2 - 16\right)\sin\left(bx+a\right)}{315\,b}$$

input
```
integrate(cos(b*x+a)^7*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/315*(35*cos(b*x + a)^8 - 5*cos(b*x + a)^6 - 6*cos(b*x + a)^4 - 8*cos(b*
x + a)^2 - 16)*sin(b*x + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.44

$$\int \cos^7(a+bx)\sin^2(a+bx)\,dx = \begin{cases} \frac{16\sin^9\left(a+bx\right)}{315b} + \frac{8\sin^7\left(a+bx\right)\cos^2\left(a+bx\right)}{35b} + \frac{2\sin^5\left(a+bx\right)\cos^4\left(a+bx\right)}{5b} + \frac{\sin^3\left(a+bx\right)\cos^6\left(a+bx\right)}{3b} & \text{for } b \neq 0 \\ x\sin^2\left(a\right)\cos^7\left(a\right) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a)**7*sin(b*x+a)**2,x)
```

output
```
Piecewise((16*sin(a + b*x)**9/(315*b) + 8*sin(a + b*x)**7*cos(a + b*x)**2/
(35*b) + 2*sin(a + b*x)**5*cos(a + b*x)**4/(5*b) + sin(a + b*x)**3*cos(a +
 b*x)**6/(3*b), Ne(b, 0)), (x*sin(a)**2*cos(a)**7, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \cos^7(a+bx)\sin^2(a+bx)\,dx$$
$$= -\frac{35\sin(bx+a)^9 - 135\sin(bx+a)^7 + 189\sin(bx+a)^5 - 105\sin(bx+a)^3}{315\,b}$$

input
```
integrate(cos(b*x+a)^7*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/315*(35*sin(b*x + a)^9 - 135*sin(b*x + a)^7 + 189*sin(b*x + a)^5 - 105*
sin(b*x + a)^3)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \cos^7(a+bx)\sin^2(a+bx)\,dx$$
$$= -\frac{35\sin(bx+a)^9 - 135\sin(bx+a)^7 + 189\sin(bx+a)^5 - 105\sin(bx+a)^3}{315\,b}$$

input
```
integrate(cos(b*x+a)^7*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/315*(35*sin(b*x + a)^9 - 135*sin(b*x + a)^7 + 189*sin(b*x + a)^5 - 105*
sin(b*x + a)^3)/b
```

Mupad [B] (verification not implemented)

Time = 25.85 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \cos^7(a+bx)\sin^2(a+bx)\,dx = \frac{-\frac{\sin(a+b\,x)^9}{9}+\frac{3\sin(a+b\,x)^7}{7}-\frac{3\sin(a+b\,x)^5}{5}+\frac{\sin(a+b\,x)^3}{3}}{b}$$

input
```
int(cos(a + b*x)^7*sin(a + b*x)^2,x)
```

output
```
(sin(a + b*x)^3/3 - (3*sin(a + b*x)^5)/5 + (3*sin(a + b*x)^7)/7 - sin(a +
b*x)^9/9)/b
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \cos^7(a+bx)\sin^2(a+bx)\,dx = \frac{\sin(bx+a)^3\left(-35\sin(bx+a)^6+135\sin(bx+a)^4-189\sin(bx+a)^2+105\right)}{315b}$$

input
```
int(cos(b*x+a)^7*sin(b*x+a)^2,x)
```

output
```
(sin(a + b*x)**3*( - 35*sin(a + b*x)**6 + 135*sin(a + b*x)**4 - 189*sin(a
+ b*x)**2 + 105))/(315*b)
```

3.50 $\int \cos^5(a+bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \cos^5(a+bx)\sin^2(a+bx)\,dx = \frac{\sin^3(a+bx)}{3b} - \frac{2\sin^5(a+bx)}{5b} + \frac{\sin^7(a+bx)}{7b}$$

output
```
1/3*sin(b*x+a)^3/b-2/5*sin(b*x+a)^5/b+1/7*sin(b*x+a)^7/b
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \cos^5(a+bx)\sin^2(a+bx)\,dx = \frac{(157+108\cos(2(a+bx))+15\cos(4(a+bx)))\sin^3(a+bx)}{840b}$$

input
```
Integrate[Cos[a + b*x]^5*Sin[a + b*x]^2,x]
```

output
```
((157 + 108*Cos[2*(a + b*x)] + 15*Cos[4*(a + b*x)])*Sin[a + b*x]^3)/(840*b
)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\cos^5(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^2\cos(a+bx)^5dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{\int \sin^2(a+bx)\left(1-\sin^2(a+bx)\right)^2d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\sin^6(a+bx)-2\sin^4(a+bx)+\sin^2(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{7}\sin^7(a+bx)-\frac{2}{5}\sin^5(a+bx)+\frac{1}{3}\sin^3(a+bx)}{b}$$

input
```
Int[Cos[a + b*x]^5*Sin[a + b*x]^2,x]
```

output
```
(Sin[a + b*x]^3/3 - (2*Sin[a + b*x]^5)/5 + Sin[a + b*x]^7/7)/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 11.90 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{\frac{\sin(bx+a)^7}{7}-\frac{2\sin(bx+a)^5}{5}+\frac{\sin(bx+a)^3}{3}}{b}$
default	$\frac{\frac{\sin(bx+a)^7}{7}-\frac{2\sin(bx+a)^5}{5}+\frac{\sin(bx+a)^3}{3}}{b}$
risch	$\frac{5\sin(bx+a)}{64b}-\frac{\sin(7bx+7a)}{448b}-\frac{3\sin(5bx+5a)}{320b}-\frac{\sin(3bx+3a)}{192b}$
parallelrisch	$-\frac{\left(\sin\left(\frac{3bx}{2}+\frac{3a}{2}\right)-3\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)(157+15\cos(4bx+4a)+108\cos(2bx+2a))\left(\cos\left(\frac{3bx}{2}+\frac{3a}{2}\right)+3\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{1680b}$
norman	$\frac{\frac{8\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^3}{3b}-\frac{32\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^5}{15b}+\frac{304\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^7}{35b}-\frac{32\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^9}{15b}+\frac{8\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{11}}{3b}}{\left(1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right)^7}$
orering	$-\frac{12916\left(-5\cos(bx+a)^4\sin(bx+a)^3b+2\cos(bx+a)^6\sin(bx+a)b\right)}{11025b^2}-\frac{94\left(-60\cos(bx+a)^2\sin(bx+a)^5b^3+215\cos(bx+\right.}{52}$

input
```
int(cos(b*x+a)^5*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/7*sin(b*x+a)^7-2/5*sin(b*x+a)^5+1/3*sin(b*x+a)^3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \cos^5(a+bx)\sin^2(a+bx)\,dx = -\frac{\left(15\cos\left(bx+a\right)^6-3\cos\left(bx+a\right)^4-4\cos\left(bx+a\right)^2-8\right)\sin\left(bx+a\right)}{105\,b}$$

input
```
integrate(cos(b*x+a)^5*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/105*(15*cos(b*x + a)^6 - 3*cos(b*x + a)^4 - 4*cos(b*x + a)^2 - 8)*sin(b
*x + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.43

$$\int \cos^5(a+bx)\sin^2(a+bx)\,dx = \begin{cases} \frac{8\sin^7(a+bx)}{105b}+\frac{4\sin^5(a+bx)\cos^2(a+bx)}{15b}+\frac{\sin^3(a+bx)\cos^4(a+bx)}{3b} & \text{for } b\neq 0 \\ x\sin^2(a)\cos^5(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a)**5*sin(b*x+a)**2,x)
```

output
```
Piecewise((8*sin(a + b*x)**7/(105*b) + 4*sin(a + b*x)**5*cos(a + b*x)**2/(
15*b) + sin(a + b*x)**3*cos(a + b*x)**4/(3*b), Ne(b, 0)), (x*sin(a)**2*cos
(a)**5, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^5(a+bx)\sin^2(a+bx)\,dx = \frac{15\sin(bx+a)^7 - 42\sin(bx+a)^5 + 35\sin(bx+a)^3}{105\,b}$$

input
```
integrate(cos(b*x+a)^5*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/105*(15*sin(b*x + a)^7 - 42*sin(b*x + a)^5 + 35*sin(b*x + a)^3)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^5(a+bx)\sin^2(a+bx)\,dx = \frac{15\sin(bx+a)^7 - 42\sin(bx+a)^5 + 35\sin(bx+a)^3}{105\,b}$$

input
```
integrate(cos(b*x+a)^5*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
1/105*(15*sin(b*x + a)^7 - 42*sin(b*x + a)^5 + 35*sin(b*x + a)^3)/b
```

Mupad [B] (verification not implemented)

Time = 25.47 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^5(a+bx)\sin^2(a+bx)\,dx = \frac{15\sin(a+b\,x)^7 - 42\sin(a+b\,x)^5 + 35\sin(a+b\,x)^3}{105\,b}$$

input
```
int(cos(a + b*x)^5*sin(a + b*x)^2,x)
```

output
```
(35*sin(a + b*x)^3 - 42*sin(a + b*x)^5 + 15*sin(a + b*x)^7)/(105*b)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \cos^5(a+bx)\sin^2(a+bx)\,dx = \frac{\sin(bx+a)^3\left(15\sin(bx+a)^4-42\sin(bx+a)^2+35\right)}{105b}$$

input
```
int(cos(b*x+a)^5*sin(b*x+a)^2,x)
```

output
```
(sin(a + b*x)**3*(15*sin(a + b*x)**4 - 42*sin(a + b*x)**2 + 35))/(105*b)
```

3.51 $\int \cos^3(a+bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \cos^3(a+bx)\sin^2(a+bx)\,dx=\frac{\sin^3(a+bx)}{3b}-\frac{\sin^5(a+bx)}{5b}$$

output
```
1/3*sin(b*x+a)^3/b-1/5*sin(b*x+a)^5/b
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \cos^3(a+bx)\sin^2(a+bx)\,dx=\frac{(7+3\cos(2(a+bx)))\sin^3(a+bx)}{30b}$$

input
```
Integrate[Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output
```
((7 + 3*Cos[2*(a + b*x)])*Sin[a + b*x]^3)/(30*b)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^2\cos(a+bx)^3dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{\int \sin^2(a+bx)\left(1-\sin^2(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\sin^2(a+bx)-\sin^4(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{3}\sin^3(a+bx)-\frac{1}{5}\sin^5(a+bx)}{b}$$

input
```
Int[Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output
```
(Sin[a + b*x]^3/3 - Sin[a + b*x]^5/5)/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 9.52 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

method	result
parallelrisch	$\frac{-3\sin(5bx+5a)+30\sin(bx+a)-5\sin(3bx+3a)}{240b}$
derivativedivides	$\frac{-\frac{\sin(bx+a)\cos(bx+a)^4}{5}+\frac{\left(2+\cos(bx+a)^2\right)\sin(bx+a)}{15}}{b}$
default	$\frac{-\frac{\sin(bx+a)\cos(bx+a)^4}{5}+\frac{\left(2+\cos(bx+a)^2\right)\sin(bx+a)}{15}}{b}$
risch	$\frac{\sin(bx+a)}{8b}-\frac{\sin(5bx+5a)}{80b}-\frac{\sin(3bx+3a)}{48b}$
norman	$\frac{\frac{8\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^3}{3b}-\frac{16\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^5}{15b}+\frac{8\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^7}{3b}}{\left(1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right)^5}$
orering	$-\frac{259\left(-3\cos(bx+a)^2\sin(bx+a)^3b+2\cos(bx+a)^4\sin(bx+a)b\right)}{225b^2}-\frac{7\left(-6b^3\sin(bx+a)^5+75\cos(bx+a)^2\sin(bx+a)^3b^3\right.}{45b^4}$

input
```
int(cos(b*x+a)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/240*(-3*sin(5*b*x+5*a)+30*sin(b*x+a)-5*sin(3*b*x+3*a))/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \cos^3(a+bx)\sin^2(a+bx)\,dx = -\frac{\left(3\,\cos\left(bx+a\right)^4-\cos\left(bx+a\right)^2-2\right)\sin\left(bx+a\right)}{15\,b}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/15*(3*cos(b*x + a)^4 - cos(b*x + a)^2 - 2)*sin(b*x + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int \cos^3(a+bx)\sin^2(a+bx)\,dx = \begin{cases} \frac{2\sin^5\left(a+bx\right)}{15b}+\frac{\sin^3\left(a+bx\right)\cos^2\left(a+bx\right)}{3b} & \text{for}\ b\neq 0 \\ x\sin^2\left(a\right)\cos^3\left(a\right) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a)**3*sin(b*x+a)**2,x)
```

output
```
Piecewise((2*sin(a + b*x)**5/(15*b) + sin(a + b*x)**3*cos(a + b*x)**2/(3*b
), Ne(b, 0)), (x*sin(a)**2*cos(a)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^3(a+bx)\sin^2(a+bx)\,dx = -\frac{3\sin(bx+a)^5 - 5\sin(bx+a)^3}{15\,b}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/15*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^3(a+bx)\sin^2(a+bx)\,dx = -\frac{3\sin(bx+a)^5 - 5\sin(bx+a)^3}{15\,b}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/15*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)/b
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^3(a+bx)\sin^2(a+bx)\,dx = \frac{5\sin(a+b\,x)^3 - 3\sin(a+b\,x)^5}{15\,b}$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)^2,x)
```

output
```
(5*sin(a + b*x)^3 - 3*sin(a + b*x)^5)/(15*b)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \cos^3(a+bx)\sin^2(a+bx)\,dx = \frac{\sin(bx+a)^3\left(-3\sin(bx+a)^2+5\right)}{15b}$$

input
```
int(cos(b*x+a)^3*sin(b*x+a)^2,x)
```

output
```
(sin(a + b*x)**3*( - 3*sin(a + b*x)**2 + 5))/(15*b)
```

3.52 $\int \cos(a+bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \cos(a+bx)\sin^2(a+bx)\,dx = \frac{\sin^3(a+bx)}{3b}$$

output
```
1/3*sin(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cos(a+bx)\sin^2(a+bx)\,dx = \frac{\sin^3(a+bx)}{3b}$$

input
```
Integrate[Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output
```
Sin[a + b*x]^3/(3*b)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3044, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^2\cos(a+bx)dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{\int \sin^2(a+bx)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{\sin^3(a+bx)}{3b}$$

input
```
Int[Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output
```
Sin[a + b*x]^3/(3*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 4.42 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\sin(bx+a)^3}{3b}$	14
default	$\frac{\sin(bx+a)^3}{3b}$	14
parallelrisch	$\frac{-\sin(3bx+3a)+3\sin(bx+a)}{12b}$	26
risch	$\frac{\sin(bx+a)}{4b}-\frac{\sin(3bx+3a)}{12b}$	27
norman	$\frac{8\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^3}{3b\left(1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right)^3}$	32
orering	$-\frac{10\left(-b\sin(bx+a)^3+2\cos(bx+a)^2\sin(bx+a)b\right)}{9b^2}-\frac{-20b^3\sin(bx+a)\cos(bx+a)^2+7b^3\sin(bx+a)^3}{9b^4}$	74

input
```
int(cos(b*x+a)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*sin(b*x+a)^3/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

$$\int \cos(a+bx)\sin^2(a+bx)\,dx=-\frac{\left(\cos\left(bx+a\right)^2-1\right)\sin\left(bx+a\right)}{3\,b}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/3*(cos(b*x + a)^2 - 1)*sin(b*x + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \cos(a+bx)\sin^2(a+bx)\,dx = \begin{cases} \frac{\sin^3(a+bx)}{3b} & \text{for } b \neq 0 \\ x\sin^2(a)\cos(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)**2,x)
```

output
```
Piecewise((sin(a + b*x)**3/(3*b), Ne(b, 0)), (x*sin(a)**2*cos(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\sin^2(a+bx)\,dx = \frac{\sin(bx+a)^3}{3b}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/3*sin(b*x + a)^3/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\sin^2(a+bx)\,dx = \frac{\sin(bx+a)^3}{3b}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
1/3*sin(b*x + a)^3/b
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\sin^2(a+bx)\,dx = \frac{\sin\left(a+b\,x\right)^3}{3\,b}$$

input
```
int(cos(a + b*x)*sin(a + b*x)^2,x)
```

output
```
sin(a + b*x)^3/(3*b)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\sin^2(a+bx)\,dx = \frac{\sin\left(bx+a\right)^3}{3b}$$

input
```
int(cos(b*x+a)*sin(b*x+a)^2,x)
```

output
```
sin(a + b*x)**3/(3*b)
```

3.53 $\int \tan^2(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 14

$$\int \tan^2(a+bx)\,dx=-x+\frac{\tan(a+bx)}{b}$$

output
```
-x+tan(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.64

$$\int \tan^2(a+bx)\,dx=-\frac{\arctan(\tan(a+bx))}{b}+\frac{\tan(a+bx)}{b}$$

input
```
Integrate[Tan[a + b*x]^2,x]
```

output
```
-(ArcTan[Tan[a + b*x]]/b) + Tan[a + b*x]/b
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(a+bx)^2dx$$

$$\Big\downarrow \text{3954}$$

$$\frac{\tan(a+bx)}{b}-\int 1dx$$

$$\Big\downarrow \text{24}$$

$$\frac{\tan(a+bx)}{b}-x$$

input
```
Int[Tan[a + b*x]^2,x]
```

output
```
-x + Tan[a + b*x]/b
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
norman	$-x+\frac{\tan(bx+a)}{b}$	15
parallelrisch	$-\frac{bx-\tan(bx+a)}{b}$	18
derivativedivides	$\frac{\tan(bx+a)-\arctan(\tan(bx+a))}{b}$	21
default	$\frac{\tan(bx+a)-\arctan(\tan(bx+a))}{b}$	21
risch	$-x+\frac{2i}{b\left(\mathrm{e}^{2i(bx+a)}+1\right)}$	24

input
```
int(tan(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-x+tan(b*x+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

$$\int \tan^2(a+bx)\,dx=-\frac{bx-\tan(bx+a)}{b}$$

input
```
integrate(tan(b*x+a)^2,x, algorithm="fricas")
```

output
```
-(b*x - tan(b*x + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \tan^2(a+bx)\,dx = \begin{cases} -x + \frac{\tan(a+bx)}{b} & \text{for } b \neq 0 \\ x\tan^2(a) & \text{otherwise} \end{cases}$$

input
```
integrate(tan(b*x+a)**2,x)
```

output
```
Piecewise((-x + tan(a + b*x)/b, Ne(b, 0)), (x*tan(a)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \tan^2(a+bx)\,dx = -\frac{bx + a - \tan(bx+a)}{b}$$

input
```
integrate(tan(b*x+a)^2,x, algorithm="maxima")
```

output
```
-(b*x + a - tan(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.50

$$\int \tan^2(a+bx)\,dx = -\frac{bx+a}{b} + \frac{\tan(bx+a)}{b}$$

input
```
integrate(tan(b*x+a)^2,x, algorithm="giac")
```

output
```
-(b*x + a)/b + tan(b*x + a)/b
```

Mupad [B] (verification not implemented)

Time = 25.45 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \tan^2(a+bx)\,dx = \frac{\tan(a+b\,x)}{b} - x$$

input
```
int(tan(a + b*x)^2,x)
```

output
```
tan(a + b*x)/b - x
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \tan^2(a+bx)\,dx = \frac{\tan(bx+a) - bx}{b}$$

input
```
int(tan(b*x+a)^2,x)
```

output
```
(tan(a + b*x) - b*x)/b
```

3.54 $\int \sec^2(a+bx)\tan^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 15

$$\int \sec^2(a+bx)\tan^2(a+bx)\,dx = \frac{\tan^3(a+bx)}{3b}$$

output
```
1/3*tan(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \sec^2(a+bx)\tan^2(a+bx)\,dx = \frac{\tan^3(a+bx)}{3b}$$

input
```
Integrate[Sec[a + b*x]^2*Tan[a + b*x]^2,x]
```

output
```
Tan[a + b*x]^3/(3*b)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3042, 3087, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(a+bx)\sec^2(a+bx)\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \tan(a+bx)^2\sec(a+bx)^2dx$$
$$\Big\downarrow \; 3087$$
$$\frac{\int \tan^2(a+bx)d\tan(a+bx)}{b}$$
$$\Big\downarrow \; 15$$
$$\frac{\tan^3(a+bx)}{3b}$$

input
```
Int[Sec[a + b*x]^2*Tan[a + b*x]^2,x]
```

output
```
Tan[a + b*x]^3/(3*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\tan(bx+a)^3}{3b}$	14
default	$\frac{\tan(bx+a)^3}{3b}$	14
risch	$-\frac{2i\left(3\,\mathrm{e}^{4i(bx+a)}+1\right)}{3b\left(\mathrm{e}^{2i(bx+a)}+1\right)^3}$	33

input
```
int(sec(b*x+a)^2*tan(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*tan(b*x+a)^3/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(13) = 26.

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int \sec^2(a+bx)\tan^2(a+bx)\,dx = -\frac{\left(\cos\left(bx+a\right)^2-1\right)\sin\left(bx+a\right)}{3\,b\cos\left(bx+a\right)^3}$$

input
```
integrate(sec(b*x+a)^2*tan(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/3*(cos(b*x + a)^2 - 1)*sin(b*x + a)/(b*cos(b*x + a)^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(10) = 20$.

Time = 0.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \sec^2(a+bx)\tan^2(a+bx)\,dx = \begin{cases} \frac{\tan^3(a+bx)}{3b} & \text{for } b \neq 0 \\ x\tan^2(a)\sec^2(a) & \text{otherwise} \end{cases}$$

input
```
integrate(sec(b*x+a)**2*tan(b*x+a)**2,x)
```

output
```
Piecewise((tan(a + b*x)**3/(3*b), Ne(b, 0)), (x*tan(a)**2*sec(a)**2, True)
)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sec^2(a+bx)\tan^2(a+bx)\,dx = \frac{\tan(bx+a)^3}{3\,b}$$

input
```
integrate(sec(b*x+a)^2*tan(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/3*tan(b*x + a)^3/b
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sec^2(a+bx)\tan^2(a+bx)\,dx = \frac{\tan(bx+a)^3}{3\,b}$$

input
```
integrate(sec(b*x+a)^2*tan(b*x+a)^2,x, algorithm="giac")
```

output
```
1/3*tan(b*x + a)^3/b
```

Mupad [B] (verification not implemented)

Time = 25.37 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sec^2(a+bx)\tan^2(a+bx)\,dx = \frac{\tan(a+b\,x)^3}{3\,b}$$

input
```
int(tan(a + b*x)^2/cos(a + b*x)^2,x)
```

output
```
tan(a + b*x)^3/(3*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.20

$$\int \sec^2(a+bx)\tan^2(a+bx)\,dx = -\frac{\sin(bx+a)^3}{3\cos(bx+a)\,b\left(\sin(bx+a)^2-1\right)}$$

input
```
int(sec(b*x+a)^2*tan(b*x+a)^2,x)
```

output
```
( - sin(a + b*x)**3)/(3*cos(a + b*x)*b*(sin(a + b*x)**2 - 1))
```

3.55 $\int \sec^4(a+bx)\tan^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \sec^4(a+bx)\tan^2(a+bx)\,dx = \frac{\tan^3(a+bx)}{3b} + \frac{\tan^5(a+bx)}{5b}$$

output
```
1/3*tan(b*x+a)^3/b+1/5*tan(b*x+a)^5/b
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.81

$$\int \sec^4(a+bx)\tan^2(a+bx)\,dx = -\frac{2\tan(a+bx)}{15b} - \frac{\sec^2(a+bx)\tan(a+bx)}{15b} + \frac{\sec^4(a+bx)\tan(a+bx)}{5b}$$

input
```
Integrate[Sec[a + b*x]^4*Tan[a + b*x]^2,x]
```

output
```
(-2*Tan[a + b*x])/(15*b) - (Sec[a + b*x]^2*Tan[a + b*x])/(15*b) + (Sec[a +
 b*x]^4*Tan[a + b*x])/(5*b)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(a+bx)\sec^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(a+bx)^2\sec(a+bx)^4dx$$

$$\Big\downarrow \text{3087}$$

$$\frac{\int \tan^2(a+bx)\left(\tan^2(a+bx)+1\right)d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\tan^4(a+bx)+\tan^2(a+bx)\right)d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{5}\tan^5(a+bx)+\frac{1}{3}\tan^3(a+bx)}{b}$$

input
```
Int[Sec[a + b*x]^4*Tan[a + b*x]^2,x]
```

output
```
(Tan[a + b*x]^3/3 + Tan[a + b*x]^5/5)/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

Maple [A] (verified)

Time = 4.46 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\frac{\tan(bx+a)^5}{5}+\frac{\tan(bx+a)^3}{3}}{b}$	26
default	$\frac{\frac{\tan(bx+a)^5}{5}+\frac{\tan(bx+a)^3}{3}}{b}$	26
risch	$-\frac{4i\left(15\,\mathrm{e}^{6i(bx+a)}-5\,\mathrm{e}^{4i(bx+a)}+5\,\mathrm{e}^{2i(bx+a)}+1\right)}{15b\left(\mathrm{e}^{2i(bx+a)}+1\right)^5}$	55

input
```
int(sec(b*x+a)^4*tan(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/5*tan(b*x+a)^5+1/3*tan(b*x+a)^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

$$\int \sec^4(a+bx)\tan^2(a+bx)\,dx = -\frac{\left(2\cos\left(bx+a\right)^4+\cos\left(bx+a\right)^2-3\right)\sin\left(bx+a\right)}{15\,b\cos\left(bx+a\right)^5}$$

input
```
integrate(sec(b*x+a)^4*tan(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/15*(2*cos(b*x + a)^4 + cos(b*x + a)^2 - 3)*sin(b*x + a)/(b*cos(b*x + a)
^5)
```

Sympy [F]

$$\int \sec^4(a+bx)\tan^2(a+bx)\,dx = \int \tan^2\left(a+bx\right)\sec^4\left(a+bx\right)dx$$

input
```
integrate(sec(b*x+a)**4*tan(b*x+a)**2,x)
```

output
```
Integral(tan(a + b*x)**2*sec(a + b*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \sec^4(a+bx)\tan^2(a+bx)\,dx = \frac{3\tan\left(bx+a\right)^5+5\tan\left(bx+a\right)^3}{15\,b}$$

input
```
integrate(sec(b*x+a)^4*tan(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/15*(3*tan(b*x + a)^5 + 5*tan(b*x + a)^3)/b
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \sec^4(a+bx)\tan^2(a+bx)\,dx = \frac{3\tan(bx+a)^5 + 5\tan(bx+a)^3}{15\,b}$$

input
```
integrate(sec(b*x+a)^4*tan(b*x+a)^2,x, algorithm="giac")
```

output
```
1/15*(3*tan(b*x + a)^5 + 5*tan(b*x + a)^3)/b
```

Mupad [B] (verification not implemented)

Time = 25.39 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \sec^4(a+bx)\tan^2(a+bx)\,dx = \frac{\tan(a+b\,x)^3\,\left(3\tan(a+b\,x)^2+5\right)}{15\,b}$$

input
```
int(tan(a + b*x)^2/cos(a + b*x)^4,x)
```

output
```
(tan(a + b*x)^3*(3*tan(a + b*x)^2 + 5))/(15*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.77

$$\int \sec^4(a+bx)\tan^2(a+bx)\,dx = \frac{\sin(bx+a)^3\,\left(-2\sin(bx+a)^2+5\right)}{15\cos(bx+a)\,b\,\left(\sin(bx+a)^4-2\sin(bx+a)^2+1\right)}$$

input
```
int(sec(b*x+a)^4*tan(b*x+a)^2,x)
```

output
```
(sin(a + b*x)**3*( - 2*sin(a + b*x)**2 + 5))/(15*cos(a + b*x)*b*(sin(a + b
*x)**4 - 2*sin(a + b*x)**2 + 1))
```

3.56 $\int \sec^6(a+bx)\tan^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \sec^6(a+bx)\tan^2(a+bx)\,dx=\frac{\tan^3(a+bx)}{3b}+\frac{2\tan^5(a+bx)}{5b}+\frac{\tan^7(a+bx)}{7b}$$

output
```
1/3*tan(b*x+a)^3/b+2/5*tan(b*x+a)^5/b+1/7*tan(b*x+a)^7/b
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.67

$$\int \sec^6(a+bx)\tan^2(a+bx)\,dx=-\frac{8\tan(a+bx)}{105b}-\frac{4\sec^2(a+bx)\tan(a+bx)}{105b}-\frac{\sec^4(a+bx)\tan(a+bx)}{35b}+\frac{\sec^6(a+bx)\tan(a+bx)}{7b}$$

input
```
Integrate[Sec[a + b*x]^6*Tan[a + b*x]^2,x]
```

output
```
(-8*Tan[a + b*x])/(105*b) - (4*Sec[a + b*x]^2*Tan[a + b*x])/(105*b) - (Sec
[a + b*x]^4*Tan[a + b*x])/(35*b) + (Sec[a + b*x]^6*Tan[a + b*x])/(7*b)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(a+bx)\sec^6(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(a+bx)^2\sec(a+bx)^6dx$$

$$\Big\downarrow \text{ 3087}$$

$$\frac{\int \tan^2(a+bx)\left(\tan^2(a+bx)+1\right)^2d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{\int \left(\tan^6(a+bx)+2\tan^4(a+bx)+\tan^2(a+bx)\right)d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{7}\tan^7(a+bx)+\frac{2}{5}\tan^5(a+bx)+\frac{1}{3}\tan^3(a+bx)}{b}$$

input
```
Int[Sec[a + b*x]^6*Tan[a + b*x]^2,x]
```

output
```
(Tan[a + b*x]^3/3 + (2*Tan[a + b*x]^5)/5 + Tan[a + b*x]^7/7)/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

Maple [A] (verified)

Time = 15.57 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\frac{\tan(bx+a)^7}{7}+\frac{2\tan(bx+a)^5}{5}+\frac{\tan(bx+a)^3}{3}}{b}$	36
default	$\frac{\frac{\tan(bx+a)^7}{7}+\frac{2\tan(bx+a)^5}{5}+\frac{\tan(bx+a)^3}{3}}{b}$	36
risch	$-\frac{16i\left(70\,e^{8i(bx+a)}-35\,e^{6i(bx+a)}+21\,e^{4i(bx+a)}+7\,e^{2i(bx+a)}+1\right)}{105b\left(e^{2i(bx+a)}+1\right)^7}$	66

input
```
int(sec(b*x+a)^6*tan(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/7*tan(b*x+a)^7+2/5*tan(b*x+a)^5+1/3*tan(b*x+a)^3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11

$$\int \sec^6(a+bx)\tan^2(a+bx)\,dx = -\frac{\left(8\cos(bx+a)^6+4\cos(bx+a)^4+3\cos(bx+a)^2-15\right)\sin(bx+a)}{105\,b\cos(bx+a)^7}$$

input
```
integrate(sec(b*x+a)^6*tan(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/105*(8*cos(b*x + a)^6 + 4*cos(b*x + a)^4 + 3*cos(b*x + a)^2 - 15)*sin(b
*x + a)/(b*cos(b*x + a)^7)
```

Sympy [F]

$$\int \sec^6(a+bx)\tan^2(a+bx)\,dx = \int \tan^2(a+bx)\sec^6(a+bx)\,dx$$

input
```
integrate(sec(b*x+a)**6*tan(b*x+a)**2,x)
```

output
```
Integral(tan(a + b*x)**2*sec(a + b*x)**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \sec^6(a+bx)\tan^2(a+bx)\,dx = \frac{15\tan(bx+a)^7+42\tan(bx+a)^5+35\tan(bx+a)^3}{105\,b}$$

input
```
integrate(sec(b*x+a)^6*tan(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/105*(15*tan(b*x + a)^7 + 42*tan(b*x + a)^5 + 35*tan(b*x + a)^3)/b
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \sec^6(a+bx)\tan^2(a+bx)\,dx = \frac{15\,\tan\left(bx+a\right)^7+42\,\tan\left(bx+a\right)^5+35\,\tan\left(bx+a\right)^3}{105\,b}$$

input
```
integrate(sec(b*x+a)^6*tan(b*x+a)^2,x, algorithm="giac")
```

output
```
1/105*(15*tan(b*x + a)^7 + 42*tan(b*x + a)^5 + 35*tan(b*x + a)^3)/b
```

Mupad [B] (verification not implemented)

Time = 25.40 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \sec^6(a+bx)\tan^2(a+bx)\,dx = \frac{\tan(a+b\,x)^3\,\left(15\tan(a+b\,x)^4+42\tan(a+b\,x)^2+35\right)}{105\,b}$$

input
```
int(tan(a + b*x)^2/cos(a + b*x)^6,x)
```

output
```
(tan(a + b*x)^3*(42*tan(a + b*x)^2 + 15*tan(a + b*x)^4 + 35))/(105*b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.63

$$\int \sec^6(a+bx)\tan^2(a+bx)\,dx = \frac{\sin\left(bx+a\right)^3\left(-8\sin\left(bx+a\right)^4+28\sin\left(bx+a\right)^2-35\right)}{105\cos\left(bx+a\right)b\left(\sin\left(bx+a\right)^6-3\sin\left(bx+a\right)^4+3\sin\left(bx+a\right)^2-1\right)}$$

input
```
int(sec(b*x+a)^6*tan(b*x+a)^2,x)
```

output
```
(sin(a + b*x)**3*( - 8*sin(a + b*x)**4 + 28*sin(a + b*x)**2 - 35))/(105*co
s(a + b*x)*b*(sin(a + b*x)**6 - 3*sin(a + b*x)**4 + 3*sin(a + b*x)**2 - 1)
)
```

3.57 $\int \sec^8(a+bx)\tan^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 61

$$\int \sec^8(a+bx)\tan^2(a+bx)\,dx = \frac{\tan^3(a+bx)}{3b}+\frac{3\tan^5(a+bx)}{5b}+\frac{3\tan^7(a+bx)}{7b}+\frac{\tan^9(a+bx)}{9b}$$

output
```
1/3*tan(b*x+a)^3/b+3/5*tan(b*x+a)^5/b+3/7*tan(b*x+a)^7/b+1/9*tan(b*x+a)^9/
b
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.61

$$\int \sec^8(a+bx)\tan^2(a+bx)\,dx = -\frac{16\tan(a+bx)}{315b}-\frac{8\sec^2(a+bx)\tan(a+bx)}{315b}-\frac{2\sec^4(a+bx)\tan(a+bx)}{105b}-\frac{\sec^6(a+bx)\tan(a+bx)}{63b}+\frac{\sec^8(a+bx)\tan(a+bx)}{9b}$$

input
```
Integrate[Sec[a + b*x]^8*Tan[a + b*x]^2,x]
```

output
```
(-16*Tan[a + b*x])/(315*b) - (8*Sec[a + b*x]^2*Tan[a + b*x])/(315*b) - (2*
Sec[a + b*x]^4*Tan[a + b*x])/(105*b) - (Sec[a + b*x]^6*Tan[a + b*x])/(63*b
) + (Sec[a + b*x]^8*Tan[a + b*x])/(9*b)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(a+bx)\sec^8(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(a+bx)^2\sec(a+bx)^8dx$$

$$\Big\downarrow \text{3087}$$

$$\frac{\int \tan^2(a+bx)\left(\tan^2(a+bx)+1\right)^3d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\tan^8(a+bx)+3\tan^6(a+bx)+3\tan^4(a+bx)+\tan^2(a+bx)\right)d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{9}\tan^9(a+bx)+\frac{3}{7}\tan^7(a+bx)+\frac{3}{5}\tan^5(a+bx)+\frac{1}{3}\tan^3(a+bx)}{b}$$

input
```
Int[Sec[a + b*x]^8*Tan[a + b*x]^2,x]
```

output
```
(Tan[a + b*x]^3/3 + (3*Tan[a + b*x]^5)/5 + (3*Tan[a + b*x]^7)/7 + Tan[a +
b*x]^9/9)/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

Maple [A] (verified)

Time = 54.68 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{\frac{\tan(bx+a)^9}{9}+\frac{3\tan(bx+a)^7}{7}+\frac{3\tan(bx+a)^5}{5}+\frac{\tan(bx+a)^3}{3}}{b}$	46
default	$\frac{\frac{\tan(bx+a)^9}{9}+\frac{3\tan(bx+a)^7}{7}+\frac{3\tan(bx+a)^5}{5}+\frac{\tan(bx+a)^3}{3}}{b}$	46
risch	$-\frac{32i\left(315\,e^{10i(bx+a)}-189\,e^{8i(bx+a)}+84\,e^{6i(bx+a)}+36\,e^{4i(bx+a)}+9\,e^{2i(bx+a)}+1\right)}{315b\left(e^{2i(bx+a)}+1\right)^9}$	77

input
```
int(sec(b*x+a)^8*tan(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/9*tan(b*x+a)^9+3/7*tan(b*x+a)^7+3/5*tan(b*x+a)^5+1/3*tan(b*x+a)^3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \sec^8(a+bx)\tan^2(a+bx)\,dx = -\frac{(16\cos(bx+a)^8+8\cos(bx+a)^6+6\cos(bx+a)^4+5\cos(bx+a)^2-35)\sin(bx+a)}{315\,b\cos(bx+a)^9}$$

input
```
integrate(sec(b*x+a)^8*tan(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/315*(16*cos(b*x + a)^8 + 8*cos(b*x + a)^6 + 6*cos(b*x + a)^4 + 5*cos(b*
x + a)^2 - 35)*sin(b*x + a)/(b*cos(b*x + a)^9)
```

Sympy [F]

$$\int \sec^8(a+bx)\tan^2(a+bx)\,dx = \int \tan^2(a+bx)\sec^8(a+bx)\,dx$$

input
```
integrate(sec(b*x+a)**8*tan(b*x+a)**2,x)
```

output
```
Integral(tan(a + b*x)**2*sec(a + b*x)**8, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \sec^8(a+bx)\tan^2(a+bx)\,dx = \frac{35\tan(bx+a)^9+135\tan(bx+a)^7+189\tan(bx+a)^5+105\tan(bx+a)^3}{315\,b}$$

input
```
integrate(sec(b*x+a)^8*tan(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/315*(35*tan(b*x + a)^9 + 135*tan(b*x + a)^7 + 189*tan(b*x + a)^5 + 105*t
an(b*x + a)^3)/b
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \sec^8(a+bx)\tan^2(a+bx)\,dx = \frac{35\tan(bx+a)^9 + 135\tan(bx+a)^7 + 189\tan(bx+a)^5 + 105\tan(bx+a)^3}{315\,b}$$

input
```
integrate(sec(b*x+a)^8*tan(b*x+a)^2,x, algorithm="giac")
```

output
```
1/315*(35*tan(b*x + a)^9 + 135*tan(b*x + a)^7 + 189*tan(b*x + a)^5 + 105*t
an(b*x + a)^3)/b
```

Mupad [B] (verification not implemented)

Time = 25.50 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \sec^8(a+bx)\tan^2(a+bx)\,dx = \frac{\frac{\tan(a+b\,x)^9}{9} + \frac{3\tan(a+b\,x)^7}{7} + \frac{3\tan(a+b\,x)^5}{5} + \frac{\tan(a+b\,x)^3}{3}}{b}$$

input
```
int(tan(a + b*x)^2/cos(a + b*x)^8,x)
```

output
```
(tan(a + b*x)^3/3 + (3*tan(a + b*x)^5)/5 + (3*tan(a + b*x)^7)/7 + tan(a +
b*x)^9/9)/b
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.56

$$\int \sec^8(a+bx)\tan^2(a+bx)\,dx$$
$$=\frac{\sin(bx+a)^3\left(-16\sin(bx+a)^6+72\sin(bx+a)^4-126\sin(bx+a)^2+105\right)}{315\cos(bx+a)\,b\left(\sin(bx+a)^8-4\sin(bx+a)^6+6\sin(bx+a)^4-4\sin(bx+a)^2+1\right)}$$

input
```
int(sec(b*x+a)^8*tan(b*x+a)^2,x)
```

output
```
(sin(a + b*x)**3*( - 16*sin(a + b*x)**6 + 72*sin(a + b*x)**4 - 126*sin(a +
 b*x)**2 + 105))/(315*cos(a + b*x)*b*(sin(a + b*x)**8 - 4*sin(a + b*x)**6
+ 6*sin(a + b*x)**4 - 4*sin(a + b*x)**2 + 1))
```

3.58 $\int \cos^6(a+bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 88

$$\int \cos^6(a+bx)\sin^2(a+bx)\,dx = \frac{5x}{128}+\frac{5\cos(a+bx)\sin(a+bx)}{128b}+\frac{5\cos^3(a+bx)\sin(a+bx)}{192b}+\frac{\cos^5(a+bx)\sin(a+bx)}{48b}-\frac{\cos^7(a+bx)\sin(a+bx)}{8b}$$

output
```
5/128*x+5/128*cos(b*x+a)*sin(b*x+a)/b+5/192*cos(b*x+a)^3*sin(b*x+a)/b+1/48
*cos(b*x+a)^5*sin(b*x+a)/b-1/8*cos(b*x+a)^7*sin(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.59

$$\int \cos^6(a+bx)\sin^2(a+bx)\,dx = \frac{120bx+48\sin(2(a+bx))-24\sin(4(a+bx))-16\sin(6(a+bx))-3\sin(8(a+bx))}{3072b}$$

input
```
Integrate[Cos[a + b*x]^6*Sin[a + b*x]^2,x]
```

output
```
(120*b*x + 48*Sin[2*(a + b*x)] - 24*Sin[4*(a + b*x)] - 16*Sin[6*(a + b*x)]
 - 3*Sin[8*(a + b*x)])/(3072*b)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {3042, 3048, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\cos^6(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^2\cos(a+bx)^6dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{1}{8}\int \cos^6(a+bx)dx-\frac{\sin(a+bx)\cos^7(a+bx)}{8b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\int \sin\left(a+bx+\frac{\pi}{2}\right)^6dx-\frac{\sin(a+bx)\cos^7(a+bx)}{8b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{8}\left(\frac{5}{6}\int \cos^4(a+bx)dx+\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)-\frac{\sin(a+bx)\cos^7(a+bx)}{8b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\left(\frac{5}{6}\int \sin\left(a+bx+\frac{\pi}{2}\right)^4dx+\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)-\frac{\sin(a+bx)\cos^7(a+bx)}{8b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \cos^2(a+bx)dx+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)+\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)-\frac{\sin(a+bx)\cos^7(a+bx)}{8b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)+\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)-\frac{\sin(a+bx)\cos^7(a+bx)}{8b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)+\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)-\frac{\sin(a+bx)\cos^7(a+bx)}{8b}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{8}\left(\frac{\sin(a+bx)\cos^5(a+bx)}{6b}+\frac{5}{6}\left(\frac{\sin(a+bx)\cos^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sin(a+bx)\cos(a+bx)}{2b}+\frac{x}{2}\right)\right)\right)-\frac{\sin(a+bx)\cos^7(a+bx)}{8b}$$

input
```
Int[Cos[a + b*x]^6*Sin[a + b*x]^2,x]
```

output
```
-1/8*(Cos[a + b*x]^7*Sin[a + b*x])/b + ((Cos[a + b*x]^5*Sin[a + b*x])/(6*b
) + (5*((Cos[a + b*x]^3*Sin[a + b*x])/(4*b) + (3*(x/2 + (Cos[a + b*x]*Sin[
a + b*x])/(2*b)))/4))/6)/8
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 15.71 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.62

method	result
parallelrisch	$\frac{120bx-3\sin(8bx+8a)-16\sin(6bx+6a)-24\sin(4bx+4a)+48\sin(2bx+2a)}{3072b}$
risch	$\frac{5x}{128}-\frac{\sin(8bx+8a)}{1024b}-\frac{\sin(6bx+6a)}{192b}-\frac{\sin(4bx+4a)}{128b}+\frac{\sin(2bx+2a)}{64b}$
derivativedivides	$\frac{-\frac{\sin(bx+a)\cos(bx+a)^7}{8}+\frac{\left(\cos(bx+a)^5+\frac{5\cos(bx+a)^3}{4}+\frac{15\cos(bx+a)}{8}\right)\sin(bx+a)}{48}+\frac{5bx}{128}+\frac{5a}{128}}{b}$
default	$\frac{-\frac{\sin(bx+a)\cos(bx+a)^7}{8}+\frac{\left(\cos(bx+a)^5+\frac{5\cos(bx+a)^3}{4}+\frac{15\cos(bx+a)}{8}\right)\sin(bx+a)}{48}+\frac{5bx}{128}+\frac{5a}{128}}{b}$
norman	$\frac{5x}{128}-\frac{5\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{64b}+\frac{397\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^3}{192b}-\frac{895\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^5}{192b}+\frac{1765\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^7}{192b}-\frac{1765\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^9}{192b}+\frac{895\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{11}}{192b}-$ 3
orering	$x\cos(bx+a)^6\sin(bx+a)^2-\frac{205\left(-6\cos(bx+a)^5\sin(bx+a)^3b+2\cos(bx+a)^7\sin(bx+a)b\right)}{576b^2}+\frac{205x\left(30\,c\right.}{}$

input
```
int(cos(b*x+a)^6*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3072*(120*b*x-3*sin(8*b*x+8*a)-16*sin(6*b*x+6*a)-24*sin(4*b*x+4*a)+48*si
n(2*b*x+2*a))/b
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.65

$$\int \cos^6(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{15\,bx - \left(48\,\cos\left(bx+a\right)^7 - 8\,\cos\left(bx+a\right)^5 - 10\,\cos\left(bx+a\right)^3 - 15\,\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{384\,b}$$

input
```
integrate(cos(b*x+a)^6*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/384*(15*b*x - (48*cos(b*x + a)^7 - 8*cos(b*x + a)^5 - 10*cos(b*x + a)^3
- 15*cos(b*x + a))*sin(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(80) = 160$.

Time = 0.65 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.15

$$\int \cos^6(a+bx)\sin^2(a+bx)\,dx$$
$$= \begin{cases} \frac{5x\sin^8(a+bx)}{128} + \frac{5x\sin^6(a+bx)\cos^2(a+bx)}{32} + \frac{15x\sin^4(a+bx)\cos^4(a+bx)}{64} + \frac{5x\sin^2(a+bx)\cos^6(a+bx)}{32} + \frac{5x\cos^8(a+bx)}{128} + \frac{5\sin^7 \\ x\sin^2(a)\cos^6(a) \end{cases}$$

input
```
integrate(cos(b*x+a)**6*sin(b*x+a)**2,x)
```

output
```
Piecewise((5*x*sin(a + b*x)**8/128 + 5*x*sin(a + b*x)**6*cos(a + b*x)**2/3
2 + 15*x*sin(a + b*x)**4*cos(a + b*x)**4/64 + 5*x*sin(a + b*x)**2*cos(a +
b*x)**6/32 + 5*x*cos(a + b*x)**8/128 + 5*sin(a + b*x)**7*cos(a + b*x)/(128
*b) + 55*sin(a + b*x)**5*cos(a + b*x)**3/(384*b) + 73*sin(a + b*x)**3*cos(
a + b*x)**5/(384*b) - 5*sin(a + b*x)*cos(a + b*x)**7/(128*b), Ne(b, 0)), (
x*sin(a)**2*cos(a)**6, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.55

$$\int \cos^6(a+bx)\sin^2(a+bx)\,dx = \frac{64\,\sin\left(2\,bx+2\,a\right)^3+120\,bx+120\,a-3\,\sin\left(8\,bx+8\,a\right)-24\,\sin\left(4\,bx+4\,a\right)}{3072\,b}$$

input
```
integrate(cos(b*x+a)^6*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/3072*(64*sin(2*b*x + 2*a)^3 + 120*b*x + 120*a - 3*sin(8*b*x + 8*a) - 24*
sin(4*b*x + 4*a))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.68

$$\int \cos^6(a+bx)\sin^2(a+bx)\,dx = \frac{5}{128}\,x-\frac{\sin\left(8\,bx+8\,a\right)}{1024\,b}-\frac{\sin\left(6\,bx+6\,a\right)}{192\,b}-\frac{\sin\left(4\,bx+4\,a\right)}{128\,b}+\frac{\sin\left(2\,bx+2\,a\right)}{64\,b}$$

input
```
integrate(cos(b*x+a)^6*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
5/128*x - 1/1024*sin(8*b*x + 8*a)/b - 1/192*sin(6*b*x + 6*a)/b - 1/128*sin
(4*b*x + 4*a)/b + 1/64*sin(2*b*x + 2*a)/b
```

Mupad [B] (verification not implemented)

Time = 26.73 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.01

$$\int \cos^6(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{5\,x}{128} + \frac{\frac{5\tan(a+b\,x)^7}{128} + \frac{55\tan(a+b\,x)^5}{384} + \frac{73\tan(a+b\,x)^3}{384} - \frac{5\tan(a+b\,x)}{128}}{b\,\left(\tan\left(a+b\,x\right)^8 + 4\tan\left(a+b\,x\right)^6 + 6\tan\left(a+b\,x\right)^4 + 4\tan\left(a+b\,x\right)^2 + 1\right)}$$

input
```
int(cos(a + b*x)^6*sin(a + b*x)^2,x)
```

output
```
(5*x)/128 + ((73*tan(a + b*x)^3)/384 - (5*tan(a + b*x))/128 + (55*tan(a +
b*x)^5)/384 + (5*tan(a + b*x)^7)/128)/(b*(4*tan(a + b*x)^2 + 6*tan(a + b*x
)^4 + 4*tan(a + b*x)^6 + tan(a + b*x)^8 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.82

$$\int \cos^6(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{48\cos\left(bx+a\right)\sin\left(bx+a\right)^7 - 136\cos\left(bx+a\right)\sin\left(bx+a\right)^5 + 118\cos\left(bx+a\right)\sin\left(bx+a\right)^3 - 15\cos\left(bx\right.}{384b}$$

input
```
int(cos(b*x+a)^6*sin(b*x+a)^2,x)
```

output
```
(48*cos(a + b*x)*sin(a + b*x)**7 - 136*cos(a + b*x)*sin(a + b*x)**5 + 118*
cos(a + b*x)*sin(a + b*x)**3 - 15*cos(a + b*x)*sin(a + b*x) + 15*b*x)/(384
*b)
```

3.59 $\int \cos^4(a+bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 67

$$\int \cos^4(a+bx)\sin^2(a+bx)\,dx = \frac{x}{16}+\frac{\cos(a+bx)\sin(a+bx)}{16b}+\frac{\cos^3(a+bx)\sin(a+bx)}{24b}-\frac{\cos^5(a+bx)\sin(a+bx)}{6b}$$

output
```
1/16*x+1/16*cos(b*x+a)*sin(b*x+a)/b+1/24*cos(b*x+a)^3*sin(b*x+a)/b-1/6*cos
(b*x+a)^5*sin(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.60

$$\int \cos^4(a+bx)\sin^2(a+bx)\,dx = -\frac{-12bx-3\sin(2(a+bx))+3\sin(4(a+bx))+\sin(6(a+bx))}{192b}$$

input
```
Integrate[Cos[a + b*x]^4*Sin[a + b*x]^2,x]
```

output
```
-1/192*(-12*b*x - 3*Sin[2*(a + b*x)] + 3*Sin[4*(a + b*x)] + Sin[6*(a + b*x
)])/b
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {3042, 3048, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\cos^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^2\cos(a+bx)^4dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{1}{6}\int \cos^4(a+bx)dx-\frac{\sin(a+bx)\cos^5(a+bx)}{6b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\int \sin\left(a+bx+\frac{\pi}{2}\right)^4dx-\frac{\sin(a+bx)\cos^5(a+bx)}{6b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{6}\left(\frac{3}{4}\int \cos^2(a+bx)dx+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)-\frac{\sin(a+bx)\cos^5(a+bx)}{6b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(\frac{3}{4}\int \sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)-\frac{\sin(a+bx)\cos^5(a+bx)}{6b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)-\frac{\sin(a+bx)\cos^5(a+bx)}{6b}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{6}\left(\frac{\sin(a+bx)\cos^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sin(a+bx)\cos(a+bx)}{2b}+\frac{x}{2}\right)\right)-\frac{\sin(a+bx)\cos^5(a+bx)}{6b}$$

input
```
Int[Cos[a + b*x]^4*Sin[a + b*x]^2,x]
```

output
```
-1/6*(Cos[a + b*x]^5*Sin[a + b*x])/b + ((Cos[a + b*x]^3*Sin[a + b*x])/(4*b
) + (3*(x/2 + (Cos[a + b*x]*Sin[a + b*x])/(2*b)))/4)/6
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 10.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.66

method	result
parallelrisch	$\frac{12bx-\sin(6bx+6a)-3\sin(4bx+4a)+3\sin(2bx+2a)}{192b}$
risch	$\frac{x}{16}-\frac{\sin(6bx+6a)}{192b}-\frac{\sin(4bx+4a)}{64b}+\frac{\sin(2bx+2a)}{64b}$
derivativedivides	$\frac{-\frac{\sin(bx+a)\cos(bx+a)^5}{6}+\frac{\left(\cos(bx+a)^3+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{24}+\frac{bx}{16}+\frac{a}{16}}{b}$
default	$\frac{-\frac{\sin(bx+a)\cos(bx+a)^5}{6}+\frac{\left(\cos(bx+a)^3+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{24}+\frac{bx}{16}+\frac{a}{16}}{b}$
norman	$\frac{\frac{x}{16}-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{8b}+\frac{47\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^3}{24b}-\frac{13\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^5}{4b}+\frac{13\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^7}{4b}-\frac{47\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^9}{24b}+\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{11}}{8b}+\frac{3x\tan\left(\frac{bx}{2}+\frac{a}{2}\right.}{8}}{\left(1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right)^6}$
orering	$x\cos(bx+a)^4\sin(bx+a)^2-\frac{49\left(-4\cos(bx+a)^3\sin(bx+a)^3b+2\cos(bx+a)^5\sin(bx+a)b\right)}{144b^2}+\frac{49x\left(12\cos\right.}{}$

input
```
int(cos(b*x+a)^4*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/192*(12*b*x-sin(6*b*x+6*a)-3*sin(4*b*x+4*a)+3*sin(2*b*x+2*a))/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

$$\int \cos^4(a+bx)\sin^2(a+bx)\,dx = \frac{3\,bx-\left(8\,\cos(bx+a)^5-2\,\cos(bx+a)^3-3\,\cos(bx+a)\right)\sin(bx+a)}{48\,b}$$

input
```
integrate(cos(b*x+a)^4*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/48*(3*b*x - (8*cos(b*x + a)^5 - 2*cos(b*x + a)^3 - 3*cos(b*x + a))*sin(b
*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(56) = 112$.

Time = 0.33 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.03

$$\int \cos^4(a+bx)\sin^2(a+bx)\,dx$$
$$= \begin{cases} \frac{x\sin^6(a+bx)}{16} + \frac{3x\sin^4(a+bx)\cos^2(a+bx)}{16} + \frac{3x\sin^2(a+bx)\cos^4(a+bx)}{16} + \frac{x\cos^6(a+bx)}{16} + \frac{\sin^5(a+bx)\cos(a+bx)}{16b} + \frac{\sin^3(a+bx)\,c}{6b} \\ x\sin^2(a)\cos^4(a) \end{cases}$$

input
```
integrate(cos(b*x+a)**4*sin(b*x+a)**2,x)
```

output
```
Piecewise((x*sin(a + b*x)**6/16 + 3*x*sin(a + b*x)**4*cos(a + b*x)**2/16 +
 3*x*sin(a + b*x)**2*cos(a + b*x)**4/16 + x*cos(a + b*x)**6/16 + sin(a + b
*x)**5*cos(a + b*x)/(16*b) + sin(a + b*x)**3*cos(a + b*x)**3/(6*b) - sin(a
 + b*x)*cos(a + b*x)**5/(16*b), Ne(b, 0)), (x*sin(a)**2*cos(a)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.55

$$\int \cos^4(a+bx)\sin^2(a+bx)\,dx = \frac{4\,\sin\left(2\,bx+2\,a\right)^3 + 12\,bx + 12\,a - 3\,\sin\left(4\,bx+4\,a\right)}{192\,b}$$

input
```
integrate(cos(b*x+a)^4*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/192*(4*sin(2*b*x + 2*a)^3 + 12*b*x + 12*a - 3*sin(4*b*x + 4*a))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.69

$$\int \cos^4(a+bx)\sin^2(a+bx)\,dx = \frac{1}{16}\,x - \frac{\sin\left(6\,bx+6\,a\right)}{192\,b} - \frac{\sin\left(4\,bx+4\,a\right)}{64\,b} + \frac{\sin\left(2\,bx+2\,a\right)}{64\,b}$$

input
```
integrate(cos(b*x+a)^4*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
1/16*x - 1/192*sin(6*b*x + 6*a)/b - 1/64*sin(4*b*x + 4*a)/b + 1/64*sin(2*b
*x + 2*a)/b
```

Mupad [B] (verification not implemented)

Time = 25.85 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.64

$$\int \cos^4(a+bx)\sin^2(a+bx)\,dx = \frac{x}{16} - \frac{\frac{\sin(4\,a+4\,b\,x)}{64} - \frac{\sin(2\,a+2\,b\,x)}{64} + \frac{\sin(6\,a+6\,b\,x)}{192}}{b}$$

input
```
int(cos(a + b*x)^4*sin(a + b*x)^2,x)
```

output
```
x/16 - (sin(4*a + 4*b*x)/64 - sin(2*a + 2*b*x)/64 + sin(6*a + 6*b*x)/192)/
b
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.84

$$\int \cos^4(a+bx)\sin^2(a+bx)\,dx = \frac{-8\cos\left(bx+a\right)\sin\left(bx+a\right)^5+14\cos\left(bx+a\right)\sin\left(bx+a\right)^3-3\cos\left(bx+a\right)\sin\left(bx+a\right)+3bx}{48b}$$

input
```
int(cos(b*x+a)^4*sin(b*x+a)^2,x)
```

output
```
( - 8*cos(a + b*x)*sin(a + b*x)**5 + 14*cos(a + b*x)*sin(a + b*x)**3 - 3*c
os(a + b*x)*sin(a + b*x) + 3*b*x)/(48*b)
```

3.60 $\int \cos^2(a+bx)\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{x}{8}+\frac{\cos(a+bx)\sin(a+bx)}{8b}-\frac{\cos^3(a+bx)\sin(a+bx)}{4b}$$

output
```
1/8*x+1/8*cos(b*x+a)*sin(b*x+a)/b-1/4*cos(b*x+a)^3*sin(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.50

$$\int \cos^2(a+bx)\sin^2(a+bx)\,dx = -\frac{-4(a+bx)+\sin(4(a+bx))}{32b}$$

input
```
Integrate[Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
-1/32*(-4*(a + b*x) + Sin[4*(a + b*x)])/b
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3042, 3048, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^2\cos(a+bx)^2dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{1}{4}\int \cos^2(a+bx)dx-\frac{\sin(a+bx)\cos^3(a+bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \sin\left(a+bx+\frac{\pi}{2}\right)^2dx-\frac{\sin(a+bx)\cos^3(a+bx)}{4b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)-\frac{\sin(a+bx)\cos^3(a+bx)}{4b}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{4}\left(\frac{\sin(a+bx)\cos(a+bx)}{2b}+\frac{x}{2}\right)-\frac{\sin(a+bx)\cos^3(a+bx)}{4b}$$

input
```
Int[Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
-1/4*(Cos[a + b*x]^3*Sin[a + b*x])/b + (x/2 + (Cos[a + b*x]*Sin[a + b*x])/
(2*b))/4
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 4.33 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.41

method	result
risch	$\frac{x}{8}-\frac{\sin(4bx+4a)}{32b}$
parallelrisch	$\frac{4bx-\sin(4bx+4a)}{32b}$
derivativedivides	$\frac{-\frac{\cos(bx+a)^3\sin(bx+a)}{4}+\frac{\cos(bx+a)\sin(bx+a)}{8}+\frac{bx}{8}+\frac{a}{8}}{b}$
default	$\frac{-\frac{\cos(bx+a)^3\sin(bx+a)}{4}+\frac{\cos(bx+a)\sin(bx+a)}{8}+\frac{bx}{8}+\frac{a}{8}}{b}$
orering	$x\cos(bx+a)^2\sin(bx+a)^2-\frac{-2\sin(bx+a)^3b\cos(bx+a)+2\cos(bx+a)^3\sin(bx+a)b}{16b^2}+\frac{x\left(-12\sin(bx+a)\right.}{}$
norman	$\frac{\frac{x}{8}-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{4b}+\frac{7\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^3}{4b}-\frac{7\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^5}{4b}+\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^7}{4b}+\frac{x\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}{2}+\frac{3x\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}{4}+\frac{x\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^6}{2}+\frac{x}{}}{\left(1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right)^4}$

input
```
int(cos(b*x+a)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*x-1/32/b*sin(4*b*x+4*a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{bx - \left(2\,\cos\left(bx+a\right)^3 - \cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{8\,b}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/8*(b*x - (2*cos(b*x + a)^3 - cos(b*x + a))*sin(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(37) = 74.

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.00

$$\int \cos^2(a+bx)\sin^2(a+bx)\,dx$$
$$= \begin{cases} \frac{x\sin^4(a+bx)}{8} + \frac{x\sin^2(a+bx)\cos^2(a+bx)}{4} + \frac{x\cos^4(a+bx)}{8} + \frac{\sin^3(a+bx)\cos(a+bx)}{8b} - \frac{\sin(a+bx)\cos^3(a+bx)}{8b} & \text{for } b \neq 0 \\ x\sin^2(a)\cos^2(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a)**2*sin(b*x+a)**2,x)
```

output
```
Piecewise((x*sin(a + b*x)**4/8 + x*sin(a + b*x)**2*cos(a + b*x)**2/4 + x*c
os(a + b*x)**4/8 + sin(a + b*x)**3*cos(a + b*x)/(8*b) - sin(a + b*x)*cos(a
 + b*x)**3/(8*b), Ne(b, 0)), (x*sin(a)**2*cos(a)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.52

$$\int \cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{4\,bx+4\,a-\sin\left(4\,bx+4\,a\right)}{32\,b}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/32*(4*b*x + 4*a - sin(4*b*x + 4*a))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.39

$$\int \cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{1}{8}\,x-\frac{\sin\left(4\,bx+4\,a\right)}{32\,b}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
1/8*x - 1/32*sin(4*b*x + 4*a)/b
```

Mupad [B] (verification not implemented)

Time = 25.71 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int \cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{x}{8}-\frac{\frac{\tan(a+b\,x)}{8}-\frac{\tan(a+b\,x)^3}{8}}{b\,\left(\tan\left(a+b\,x\right)^4+2\tan\left(a+b\,x\right)^2+1\right)}$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)^2,x)
```

output
```
x/8 - (tan(a + b*x)/8 - tan(a + b*x)^3/8)/(b*(2*tan(a + b*x)^2 + tan(a + b
*x)^4 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{2\cos(bx+a)\sin(bx+a)^3 - \cos(bx+a)\sin(bx+a) + bx}{8b}$$

input
```
int(cos(b*x+a)^2*sin(b*x+a)^2,x)
```

output
```
(2*cos(a + b*x)*sin(a + b*x)**3 - cos(a + b*x)*sin(a + b*x) + b*x)/(8*b)
```

3.61 $\int \sin^2(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 25

$$\int \sin^2(a+bx)\,dx = \frac{x}{2} - \frac{\cos(a+bx)\sin(a+bx)}{2b}$$

output
```
1/2*x-1/2*cos(b*x+a)*sin(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sin^2(a+bx)\,dx = -\frac{-2(a+bx)+\sin(2(a+bx))}{4b}$$

input
```
Integrate[Sin[a + b*x]^2,x]
```

output
```
-1/4*(-2*(a + b*x) + Sin[2*(a + b*x)])/b
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(a+bx)^2dx$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{\int 1dx}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}$$

input
```
Int[Sin[a + b*x]^2,x]
```

output
```
x/2 - (Cos[a + b*x]*Sin[a + b*x])/(2*b)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 3.48 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{x}{2}-\frac{\sin(2bx+2a)}{4b}$	19
parallelrisch	$\frac{2bx-\sin(2bx+2a)}{4b}$	22
derivativedivides	$\frac{-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}}{b}$	27
default	$\frac{-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}}{b}$	27
orering	$x\sin\left(bx+a\right)^{2}-\frac{\cos(bx+a)\sin(bx+a)}{2b}+\frac{x\left(-2\sin(bx+a)^{2}b^{2}+2\cos(bx+a)^{2}b^{2}\right)}{4b^{2}}$	62
norman	$\frac{\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{3}}{b}+x\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}+\frac{x}{2}-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{x\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}}{2}}{\left(1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)^{2}}$	77

input
```
int(sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x-1/4/b*sin(2*b*x+2*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sin^2(a+bx)\,dx=\frac{bx-\cos\left(bx+a\right)\sin\left(bx+a\right)}{2\,b}$$

input
```
integrate(sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(b*x - cos(b*x + a)*sin(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(19) = 38$.

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \sin^2(a+bx)\,dx = \begin{cases} \frac{x\sin^2(a+bx)}{2} + \frac{x\cos^2(a+bx)}{2} - \frac{\sin(a+bx)\cos(a+bx)}{2b} & \text{for } b \neq 0 \\ x\sin^2(a) & \text{otherwise} \end{cases}$$

input
```
integrate(sin(b*x+a)**2,x)
```

output
```
Piecewise((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 - sin(a + b*x)*cos(a
+ b*x)/(2*b), Ne(b, 0)), (x*sin(a)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \sin^2(a+bx)\,dx = \frac{2\,bx + 2\,a - \sin(2\,bx + 2\,a)}{4\,b}$$

input
```
integrate(sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/4*(2*b*x + 2*a - sin(2*b*x + 2*a))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \sin^2(a+bx)\,dx = \frac{1}{2}\,x - \frac{\sin\left(2\,bx + 2\,a\right)}{4\,b}$$

input
```
integrate(sin(b*x+a)^2,x, algorithm="giac")
```

output
```
1/2*x - 1/4*sin(2*b*x + 2*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \sin^2(a+bx)\,dx = \frac{x}{2} - \frac{\sin\left(2\,a + 2\,b\,x\right)}{4\,b}$$

input
```
int(sin(a + b*x)^2,x)
```

output
```
x/2 - sin(2*a + 2*b*x)/(4*b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sin^2(a+bx)\,dx = \frac{-\cos\left(bx+a\right)\sin\left(bx+a\right)+bx}{2b}$$

input
```
int(sin(b*x+a)^2,x)
```

output
```
( - cos(a + b*x)*sin(a + b*x) + b*x)/(2*b)
```

3.62 $\int \sin(a+bx)\tan(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \sin(a+bx)\tan(a+bx)\,dx=\frac{\operatorname{arctanh}(\sin(a+bx))}{b}-\frac{\sin(a+bx)}{b}$$

output
```
arctanh(sin(b*x+a))/b-sin(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \sin(a+bx)\tan(a+bx)\,dx=\frac{\operatorname{arctanh}(\sin(a+bx))}{b}-\frac{\sin(a+bx)}{b}$$

input
```
Integrate[Sin[a + b*x]*Tan[a + b*x],x]
```

output
```
ArcTanh[Sin[a + b*x]]/b - Sin[a + b*x]/b
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3072, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\tan(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)\tan(a+bx)dx$$

$$\Big\downarrow \text{3072}$$

$$\frac{\int \frac{\sin^2(a+bx)}{1-\sin^2(a+bx)}d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{262}$$

$$\frac{\int \frac{1}{1-\sin^2(a+bx)}d\sin(a+bx)-\sin(a+bx)}{b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}(\sin(a+bx))-\sin(a+bx)}{b}$$

input
```
Int[Sin[a + b*x]*Tan[a + b*x],x]
```

output
```
(ArcTanh[Sin[a + b*x]] - Sin[a + b*x])/b
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Maple [A] (verified)

Time = 2.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22

method	result	size
derivativedivides	$\frac{-\sin(bx+a)+\ln(\sec(bx+a)+\tan(bx+a))}{b}$	28
default	$\frac{-\sin(bx+a)+\ln(\sec(bx+a)+\tan(bx+a))}{b}$	28
risch	$-\frac{i\mathrm{e}^{-i(bx+a)}}{2b}+\frac{i\mathrm{e}^{i(bx+a)}}{2b}-\frac{\ln\left(\mathrm{e}^{i(bx+a)}-i\right)}{b}+\frac{\ln\left(\mathrm{e}^{i(bx+a)}+i\right)}{b}$	67

input
```
int(sin(b*x+a)*tan(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-sin(b*x+a)+ln(sec(b*x+a)+tan(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \sin(a+bx)\tan(a+bx)\,dx = \frac{\log(\sin(bx+a)+1) - \log(-\sin(bx+a)+1) - 2\sin(bx+a)}{2b}$$

input
```
integrate(sin(b*x+a)*tan(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(log(sin(b*x + a) + 1) - log(-sin(b*x + a) + 1) - 2*sin(b*x + a))/b
```

Sympy [F]

$$\int \sin(a+bx)\tan(a+bx)\,dx = \int \sin(a+bx)\tan(a+bx)\,dx$$

input
```
integrate(sin(b*x+a)*tan(b*x+a),x)
```

output
```
Integral(sin(a + b*x)*tan(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.48

$$\int \sin(a+bx)\tan(a+bx)\,dx = \frac{\log(\sin(bx+a)+1) - \log(\sin(bx+a)-1) - 2\sin(bx+a)}{2b}$$

input
```
integrate(sin(b*x+a)*tan(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \sin(a+bx)\tan(a+bx)\,dx = \frac{\log\left(|\sin\left(bx+a\right)+1|\right)}{2\,b} - \frac{\log\left(|\sin\left(bx+a\right)-1|\right)}{2\,b} - \frac{\sin\left(bx+a\right)}{b}$$

input
```
integrate(sin(b*x+a)*tan(b*x+a),x, algorithm="giac")
```

output
```
1/2*log(abs(sin(b*x + a) + 1))/b - 1/2*log(abs(sin(b*x + a) - 1))/b - sin(
b*x + a)/b
```

Mupad [B] (verification not implemented)

Time = 25.74 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \sin(a+bx)\tan(a+bx)\,dx = \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{b} - \frac{\sin\left(a+b\,x\right)}{b}$$

input
```
int(sin(a + b*x)*tan(a + b*x),x)
```

output
```
(2*atanh(tan(a/2 + (b*x)/2)))/b - sin(a + b*x)/b
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

$$\int \sin(a+bx)\tan(a+bx)\,dx = \frac{-\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)-\sin\left(bx+a\right)}{b}$$

input
```
int(sin(b*x+a)*tan(b*x+a),x)
```

output

```
( - log(tan((a + b*x)/2) - 1) + log(tan((a + b*x)/2) + 1) - sin(a + b*x))/
b
```

3.63 $\int \sec(a+bx)\tan^2(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \sec(a+bx)\tan^2(a+bx)\,dx = -\frac{\operatorname{arctanh}(\sin(a+bx))}{2b}+\frac{\sec(a+bx)\tan(a+bx)}{2b}$$

output
```
-1/2*arctanh(sin(b*x+a))/b+1/2*sec(b*x+a)*tan(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \sec(a+bx)\tan^2(a+bx)\,dx = -\frac{\operatorname{arctanh}(\sin(a+bx))}{2b}+\frac{\sec(a+bx)\tan(a+bx)}{2b}$$

input
```
Integrate[Sec[a + b*x]*Tan[a + b*x]^2,x]
```

output
```
-1/2*ArcTanh[Sin[a + b*x]]/b + (Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3091, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(a+bx)\sec(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(a+bx)^2\sec(a+bx)dx$$

$$\Big\downarrow \text{3091}$$

$$\frac{\tan(a+bx)\sec(a+bx)}{2b}-\frac{1}{2}\int \sec(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(a+bx)\sec(a+bx)}{2b}-\frac{1}{2}\int \csc\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{4257}$$

$$\frac{\tan(a+bx)\sec(a+bx)}{2b}-\frac{\text{arctanh}(\sin(a+bx))}{2b}$$

input
```
Int[Sec[a + b*x]*Tan[a + b*x]^2,x]
```

output
```
-1/2*ArcTanh[Sin[a + b*x]]/b + (Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

method	result	size
derivativedivides	$\frac{\frac{\sin(bx+a)^3}{2\cos(bx+a)^2}+\frac{\sin(bx+a)}{2}-\frac{\ln(\sec(bx+a)+\tan(bx+a))}{2}}{b}$	48
default	$\frac{\frac{\sin(bx+a)^3}{2\cos(bx+a)^2}+\frac{\sin(bx+a)}{2}-\frac{\ln(\sec(bx+a)+\tan(bx+a))}{2}}{b}$	48
risch	$-\frac{i\left(e^{3i(bx+a)}-e^{i(bx+a)}\right)}{b\left(e^{2i(bx+a)}+1\right)^2}+\frac{\ln\left(e^{i(bx+a)}-i\right)}{2b}-\frac{\ln\left(e^{i(bx+a)}+i\right)}{2b}$	78

input
```
int(sec(b*x+a)*tan(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2*sin(b*x+a)^3/cos(b*x+a)^2+1/2*sin(b*x+a)-1/2*ln(sec(b*x+a)+tan(b*
x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(30) = 60$.

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.79

$$\int \sec(a+bx)\tan^2(a+bx)\,dx = -\frac{\cos(bx+a)^2\log(\sin(bx+a)+1)-\cos(bx+a)^2\log(-\sin(bx+a)+1)-2\sin(bx+a)}{4\,b\cos(bx+a)^2}$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/4*(cos(b*x + a)^2*log(sin(b*x + a) + 1) - cos(b*x + a)^2*log(-sin(b*x +
 a) + 1) - 2*sin(b*x + a))/(b*cos(b*x + a)^2)
```

Sympy [F]

$$\int \sec(a+bx)\tan^2(a+bx)\,dx = \int \tan^2(a+bx)\sec(a+bx)\,dx$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)**2,x)
```

output
```
Integral(tan(a + b*x)**2*sec(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \sec(a+bx)\tan^2(a+bx)\,dx = -\frac{\frac{2\sin(bx+a)}{\sin(bx+a)^2-1}+\log(\sin(bx+a)+1)-\log(\sin(bx+a)-1)}{4\,b}$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/4*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + log(sin(b*x + a) + 1) - log(si
n(b*x + a) - 1))/b
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int \sec(a+bx)\tan^2(a+bx)\,dx$$
$$= -\frac{\frac{2\sin(bx+a)}{\sin(bx+a)^2-1} + \log(|\sin(bx+a)+1|) - \log(|\sin(bx+a)-1|)}{4b}$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/4*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + log(abs(sin(b*x + a) + 1)) - l
og(abs(sin(b*x + a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 26.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.03

$$\int \sec(a+bx)\tan^2(a+bx)\,dx = \frac{\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^3+\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)}{b\,\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4-2\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2+1\right)} - \frac{\mathrm{atanh}\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{b}$$

input
```
int(tan(a + b*x)^2/cos(a + b*x),x)
```

output
```
(tan(a/2 + (b*x)/2) + tan(a/2 + (b*x)/2)^3)/(b*(tan(a/2 + (b*x)/2)^4 - 2*t
an(a/2 + (b*x)/2)^2 + 1)) - atanh(tan(a/2 + (b*x)/2))/b
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.79

$$\int \sec(a+bx)\tan^2(a+bx)\,dx$$
$$= \frac{\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin\left(bx+a\right)^2-\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)-\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)\sin\left(bx+a\right)^2+\log}{2b\left(\sin\left(bx+a\right)^2-1\right)}$$

input
```
int(sec(b*x+a)*tan(b*x+a)^2,x)
```

output
```
(log(tan((a + b*x)/2) - 1)*sin(a + b*x)**2 - log(tan((a + b*x)/2) - 1) - l
og(tan((a + b*x)/2) + 1)*sin(a + b*x)**2 + log(tan((a + b*x)/2) + 1) - sin
(a + b*x))/(2*b*(sin(a + b*x)**2 - 1))
```

3.64 $\int \sec^3(a+bx)\tan^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 55

$$\int \sec^3(a+bx)\tan^2(a+bx)\,dx=-\frac{\operatorname{arctanh}(\sin(a+bx))}{8b}-\frac{\sec(a+bx)\tan(a+bx)}{8b}+\frac{\sec^3(a+bx)\tan(a+bx)}{4b}$$

output
```
-1/8*arctanh(sin(b*x+a))/b-1/8*sec(b*x+a)*tan(b*x+a)/b+1/4*sec(b*x+a)^3*ta
n(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \sec^3(a+bx)\tan^2(a+bx)\,dx=-\frac{\operatorname{arctanh}(\sin(a+bx))}{8b}-\frac{\sec(a+bx)\tan(a+bx)}{8b}+\frac{\sec^3(a+bx)\tan(a+bx)}{4b}$$

input
```
Integrate[Sec[a + b*x]^3*Tan[a + b*x]^2,x]
```

output
```
-1/8*ArcTanh[Sin[a + b*x]]/b - (Sec[a + b*x]*Tan[a + b*x])/(8*b) + (Sec[a
+ b*x]^3*Tan[a + b*x])/(4*b)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3042, 3091, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(a+bx)\sec^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(a+bx)^2\sec(a+bx)^3dx$$

$$\Big\downarrow \text{3091}$$

$$\frac{\tan(a+bx)\sec^3(a+bx)}{4b}-\frac{1}{4}\int \sec^3(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(a+bx)\sec^3(a+bx)}{4b}-\frac{1}{4}\int \csc\left(a+bx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{4}\left(-\frac{1}{2}\int \sec(a+bx)dx-\frac{\tan(a+bx)\sec(a+bx)}{2b}\right)+\frac{\tan(a+bx)\sec^3(a+bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(-\frac{1}{2}\int \csc\left(a+bx+\frac{\pi}{2}\right)dx-\frac{\tan(a+bx)\sec(a+bx)}{2b}\right)+\frac{\tan(a+bx)\sec^3(a+bx)}{4b}$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{4}\left(-\frac{\text{arctanh}(\sin(a+bx))}{2b}-\frac{\tan(a+bx)\sec(a+bx)}{2b}\right)+\frac{\tan(a+bx)\sec^3(a+bx)}{4b}$$

input
```
Int[Sec[a + b*x]^3*Tan[a + b*x]^2,x]
```

output
```
(Sec[a + b*x]^3*Tan[a + b*x])/(4*b) + (-1/2*ArcTanh[Sin[a + b*x]]/b - (Sec
[a + b*x]*Tan[a + b*x])/(2*b))/4
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 3.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$\frac{\frac{\sin(bx+a)^3}{4\cos(bx+a)^4}+\frac{\sin(bx+a)^3}{8\cos(bx+a)^2}+\frac{\sin(bx+a)}{8}-\frac{\ln(\sec(bx+a)+\tan(bx+a))}{8}}{b}$	66
default	$\frac{\frac{\sin(bx+a)^3}{4\cos(bx+a)^4}+\frac{\sin(bx+a)^3}{8\cos(bx+a)^2}+\frac{\sin(bx+a)}{8}-\frac{\ln(\sec(bx+a)+\tan(bx+a))}{8}}{b}$	66
risch	$\frac{i(e^{7i(bx+a)}-7\,e^{5i(bx+a)}+7\,e^{3i(bx+a)}-e^{i(bx+a)})}{4b(e^{2i(bx+a)}+1)^4}+\frac{\ln(e^{i(bx+a)}-i)}{8b}-\frac{\ln(e^{i(bx+a)}+i)}{8b}$	100

input
```
int(sec(b*x+a)^3*tan(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/4*sin(b*x+a)^3/cos(b*x+a)^4+1/8*sin(b*x+a)^3/cos(b*x+a)^2+1/8*sin(b
*x+a)-1/8*ln(sec(b*x+a)+tan(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.29

$$\int \sec^3(a+bx)\tan^2(a+bx)\,dx = -\frac{\cos(bx+a)^4\log(\sin(bx+a)+1)-\cos(bx+a)^4\log(-\sin(bx+a)+1)+2\left(\cos(bx+a)^2-2\right)\sin}{16\,b\cos(bx+a)^4}$$

input
```
integrate(sec(b*x+a)^3*tan(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/16*(cos(b*x + a)^4*log(sin(b*x + a) + 1) - cos(b*x + a)^4*log(-sin(b*x
+ a) + 1) + 2*(cos(b*x + a)^2 - 2)*sin(b*x + a))/(b*cos(b*x + a)^4)
```

Sympy [F]

$$\int \sec^3(a+bx)\tan^2(a+bx)\,dx = \int \tan^2{(a+bx)}\sec^3{(a+bx)}\,dx$$

input
```
integrate(sec(b*x+a)**3*tan(b*x+a)**2,x)
```

output
```
Integral(tan(a + b*x)**2*sec(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.18

$$\int \sec^3(a+bx)\tan^2(a+bx)\,dx = \frac{\frac{2\left(\sin(bx+a)^3+\sin(bx+a)\right)}{\sin(bx+a)^4-2\sin(bx+a)^2+1} - \log\left(\sin\left(bx+a\right)+1\right) + \log\left(\sin\left(bx+a\right)-1\right)}{16\,b}$$

input
```
integrate(sec(b*x+a)^3*tan(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/16*(2*(sin(b*x + a)^3 + sin(b*x + a))/(sin(b*x + a)^4 - 2*sin(b*x + a)^2
 + 1) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1))/b
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.49

$$\int \sec^3(a+bx)\tan^2(a+bx)\,dx = \frac{\frac{4\left(\frac{1}{\sin(bx+a)}+\sin(bx+a)\right)}{\left(\frac{1}{\sin(bx+a)}+\sin(bx+a)\right)^2-4} - \log\left(\left|\frac{1}{\sin(bx+a)}+\sin\left(bx+a\right)+2\right|\right) + \log\left(\left|\frac{1}{\sin(bx+a)}+\sin\left(bx+a\right)-2\right|\right)}{32\,b}$$

input
```
integrate(sec(b*x+a)^3*tan(b*x+a)^2,x, algorithm="giac")
```

output
```
1/32*(4*(1/sin(b*x + a) + sin(b*x + a))/((1/sin(b*x + a) + sin(b*x + a))^2
 - 4) - log(abs(1/sin(b*x + a) + sin(b*x + a) + 2)) + log(abs(1/sin(b*x +
a) + sin(b*x + a) - 2)))/b
```

Mupad [B] (verification not implemented)

Time = 28.55 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.27

$$\int \sec^3(a+bx)\tan^2(a+bx)\,dx = \frac{\frac{\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^7}{4}+\frac{7\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^5}{4}+\frac{7\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^3}{4}+\frac{\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)}{4}}{b\,\left({\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)}^8-4\,{\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)}^6+6\,{\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)}^4-4\,{\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)}^2+1\right)} - \frac{\mathrm{atanh}\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{4\,b}$$

input
```
int(tan(a + b*x)^2/cos(a + b*x)^3,x)
```

output
```
(tan(a/2 + (b*x)/2)/4 + (7*tan(a/2 + (b*x)/2)^3)/4 + (7*tan(a/2 + (b*x)/2)
^5)/4 + tan(a/2 + (b*x)/2)^7/4)/(b*(6*tan(a/2 + (b*x)/2)^4 - 4*tan(a/2 + (
b*x)/2)^2 - 4*tan(a/2 + (b*x)/2)^6 + tan(a/2 + (b*x)/2)^8 + 1)) - atanh(ta
n(a/2 + (b*x)/2))/(4*b)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.82

$$\int \sec^3(a+bx)\tan^2(a+bx)\,dx = \frac{\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin(bx+a)^4-2\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin(bx+a)^2+\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)-\mathrm{lo}}{8b\,(\sin}$$

input
```
int(sec(b*x+a)^3*tan(b*x+a)^2,x)
```

output
```
(log(tan((a + b*x)/2) - 1)*sin(a + b*x)**4 - 2*log(tan((a + b*x)/2) - 1)*s
in(a + b*x)**2 + log(tan((a + b*x)/2) - 1) - log(tan((a + b*x)/2) + 1)*sin
(a + b*x)**4 + 2*log(tan((a + b*x)/2) + 1)*sin(a + b*x)**2 - log(tan((a +
b*x)/2) + 1) + sin(a + b*x)**3 + sin(a + b*x))/(8*b*(sin(a + b*x)**4 - 2*s
in(a + b*x)**2 + 1))
```

3.65 $\int \sec^5(a+bx)\tan^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 76

$$\int \sec^5(a+bx)\tan^2(a+bx)\,dx = -\frac{\operatorname{arctanh}(\sin(a+bx))}{16b} - \frac{\sec(a+bx)\tan(a+bx)}{16b} - \frac{\sec^3(a+bx)\tan(a+bx)}{24b} + \frac{\sec^5(a+bx)\tan(a+bx)}{6b}$$

output
```
-1/16*arctanh(sin(b*x+a))/b-1/16*sec(b*x+a)*tan(b*x+a)/b-1/24*sec(b*x+a)^3
*tan(b*x+a)/b+1/6*sec(b*x+a)^5*tan(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int \sec^5(a+bx)\tan^2(a+bx)\,dx = -\frac{\operatorname{arctanh}(\sin(a+bx))}{16b} - \frac{\sec(a+bx)\tan(a+bx)}{16b} - \frac{\sec^3(a+bx)\tan(a+bx)}{24b} + \frac{\sec^5(a+bx)\tan(a+bx)}{6b}$$

input
```
Integrate[Sec[a + b*x]^5*Tan[a + b*x]^2,x]
```

output
```
-1/16*ArcTanh[Sin[a + b*x]]/b - (Sec[a + b*x]*Tan[a + b*x])/(16*b) - (Sec[
a + b*x]^3*Tan[a + b*x])/(24*b) + (Sec[a + b*x]^5*Tan[a + b*x])/(6*b)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {3042, 3091, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(a+bx)\sec^5(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(a+bx)^2\sec(a+bx)^5dx$$

$$\Big\downarrow \text{3091}$$

$$\frac{\tan(a+bx)\sec^5(a+bx)}{6b}-\frac{1}{6}\int \sec^5(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(a+bx)\sec^5(a+bx)}{6b}-\frac{1}{6}\int \csc\left(a+bx+\frac{\pi}{2}\right)^5dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{6}\left(-\frac{3}{4}\int \sec^3(a+bx)dx-\frac{\tan(a+bx)\sec^3(a+bx)}{4b}\right)+\frac{\tan(a+bx)\sec^5(a+bx)}{6b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}\left(-\frac{3}{4}\int \csc\left(a+bx+\frac{\pi}{2}\right)^3dx-\frac{\tan(a+bx)\sec^3(a+bx)}{4b}\right)+\frac{\tan(a+bx)\sec^5(a+bx)}{6b}$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{6}\left(-\frac{3}{4}\left(\frac{1}{2}\int \sec(a+bx)dx+\frac{\tan(a+bx)\sec(a+bx)}{2b}\right)-\frac{\tan(a+bx)\sec^3(a+bx)}{4b}\right)+\frac{\tan(a+bx)\sec^5(a+bx)}{6b}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{6}\left(-\frac{3}{4}\left(\frac{1}{2}\int \csc\left(a+bx+\frac{\pi}{2}\right)dx+\frac{\tan(a+bx)\sec(a+bx)}{2b}\right)-\frac{\tan(a+bx)\sec^3(a+bx)}{4b}\right)+\frac{\tan(a+bx)\sec^5(a+bx)}{6b}$$

$$\Big\downarrow \ 4257$$

$$\frac{1}{6}\left(-\frac{3}{4}\left(\frac{\text{arctanh}(\sin(a+bx))}{2b}+\frac{\tan(a+bx)\sec(a+bx)}{2b}\right)-\frac{\tan(a+bx)\sec^3(a+bx)}{4b}\right)+\frac{\tan(a+bx)\sec^5(a+bx)}{6b}$$

input
```
Int[Sec[a + b*x]^5*Tan[a + b*x]^2,x]
```

output
```
(Sec[a + b*x]^5*Tan[a + b*x])/(6*b) + (-1/4*(Sec[a + b*x]^3*Tan[a + b*x])/
b - (3*(ArcTanh[Sin[a + b*x]]/(2*b) + (Sec[a + b*x]*Tan[a + b*x])/(2*b)))/
4)/6
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 8.76 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{\frac{\sin(bx+a)^3}{6\cos(bx+a)^6}+\frac{\sin(bx+a)^3}{8\cos(bx+a)^4}+\frac{\sin(bx+a)^3}{16\cos(bx+a)^2}+\frac{\sin(bx+a)}{16}-\frac{\ln(\sec(bx+a)+\tan(bx+a))}{16}}{b}$
default	$\frac{\frac{\sin(bx+a)^3}{6\cos(bx+a)^6}+\frac{\sin(bx+a)^3}{8\cos(bx+a)^4}+\frac{\sin(bx+a)^3}{16\cos(bx+a)^2}+\frac{\sin(bx+a)}{16}-\frac{\ln(\sec(bx+a)+\tan(bx+a))}{16}}{b}$
risch	$\frac{i\left(3\,\mathrm{e}^{11i(bx+a)}+17\,\mathrm{e}^{9i(bx+a)}-114\,\mathrm{e}^{7i(bx+a)}+114\,\mathrm{e}^{5i(bx+a)}-17\,\mathrm{e}^{3i(bx+a)}-3\,\mathrm{e}^{i(bx+a)}\right)}{24b\left(\mathrm{e}^{2i(bx+a)}+1\right)^6}+\frac{\ln\left(\mathrm{e}^{i(bx+a)}-i\right)}{16b}-\frac{\ln\left(\mathrm{e}^{i(bx-}\right.}{16}$

input
```
int(sec(b*x+a)^5*tan(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/6*sin(b*x+a)^3/cos(b*x+a)^6+1/8*sin(b*x+a)^3/cos(b*x+a)^4+1/16*sin(
b*x+a)^3/cos(b*x+a)^2+1/16*sin(b*x+a)-1/16*ln(sec(b*x+a)+tan(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.11

$$\int \sec^5(a+bx)\tan^2(a+bx)\,dx = -\frac{3\,\cos\left(bx+a\right)^6\log\left(\sin\left(bx+a\right)+1\right)-3\,\cos\left(bx+a\right)^6\log\left(-\sin\left(bx+a\right)+1\right)+2\left(3\,\cos\left(bx+a\right)^4+\right.}{96\,b\cos\left(bx+a\right)^6}$$

input
```
integrate(sec(b*x+a)^5*tan(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/96*(3*cos(b*x + a)^6*log(sin(b*x + a) + 1) - 3*cos(b*x + a)^6*log(-sin(
b*x + a) + 1) + 2*(3*cos(b*x + a)^4 + 2*cos(b*x + a)^2 - 8)*sin(b*x + a))/
(b*cos(b*x + a)^6)
```

Sympy [F]

$$\int \sec^5(a+bx)\tan^2(a+bx)\,dx = \int \tan^2(a+bx)\sec^5(a+bx)\,dx$$

input
```
integrate(sec(b*x+a)**5*tan(b*x+a)**2,x)
```

output
```
Integral(tan(a + b*x)**2*sec(a + b*x)**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.20

$$\int \sec^5(a+bx)\tan^2(a+bx)\,dx = \frac{\frac{2\left(3\sin(bx+a)^5-8\sin(bx+a)^3-3\sin(bx+a)\right)}{\sin(bx+a)^6-3\sin(bx+a)^4+3\sin(bx+a)^2-1} - 3\log(\sin(bx+a)+1) + 3\log(\sin(bx+a)-1)}{96\,b}$$

input
```
integrate(sec(b*x+a)^5*tan(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/96*(2*(3*sin(b*x + a)^5 - 8*sin(b*x + a)^3 - 3*sin(b*x + a))/(sin(b*x +
a)^6 - 3*sin(b*x + a)^4 + 3*sin(b*x + a)^2 - 1) - 3*log(sin(b*x + a) + 1)
+ 3*log(sin(b*x + a) - 1))/b
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96

$$\int \sec^5(a+bx)\tan^2(a+bx)\,dx = \frac{\frac{2\left(3\sin(bx+a)^5-8\sin(bx+a)^3-3\sin(bx+a)\right)}{\left(\sin(bx+a)^2-1\right)^3} - 3\log(|\sin(bx+a)+1|) + 3\log(|\sin(bx+a)-1|)}{96\,b}$$

input
```
integrate(sec(b*x+a)^5*tan(b*x+a)^2,x, algorithm="giac")
```

output
```
1/96*(2*(3*sin(b*x + a)^5 - 8*sin(b*x + a)^3 - 3*sin(b*x + a))/(sin(b*x +
a)^2 - 1)^3 - 3*log(abs(sin(b*x + a) + 1)) + 3*log(abs(sin(b*x + a) - 1)))
/b
```

Mupad [B] (verification not implemented)

Time = 29.28 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.33

$$\int \sec^5(a+bx)\tan^2(a+bx)\,dx$$
$$=\frac{\frac{\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{11}}{8}+\frac{47\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{9}}{24}+\frac{13\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{7}}{4}+\frac{13\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{5}}{4}+\frac{47\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{3}}{24}+\frac{\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)}{8}}{b\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{12}-6\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{10}+15\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{8}-20\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{6}+15\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{4}-6\tan\right.}$$
$$-\frac{\operatorname{atanh}\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{8\,b}$$

input
```
int(tan(a + b*x)^2/cos(a + b*x)^5,x)
```

output
```
(tan(a/2 + (b*x)/2)/8 + (47*tan(a/2 + (b*x)/2)^3)/24 + (13*tan(a/2 + (b*x)
/2)^5)/4 + (13*tan(a/2 + (b*x)/2)^7)/4 + (47*tan(a/2 + (b*x)/2)^9)/24 + ta
n(a/2 + (b*x)/2)^11/8)/(b*(15*tan(a/2 + (b*x)/2)^4 - 6*tan(a/2 + (b*x)/2)^
2 - 20*tan(a/2 + (b*x)/2)^6 + 15*tan(a/2 + (b*x)/2)^8 - 6*tan(a/2 + (b*x)/
2)^10 + tan(a/2 + (b*x)/2)^12 + 1)) - atanh(tan(a/2 + (b*x)/2))/(8*b)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.97

$$\int \sec^5(a+bx)\tan^2(a+bx)\,dx$$
$$=\frac{3\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin\left(bx+a\right)^{6}-9\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin\left(bx+a\right)^{4}+9\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\mathrm{s}}{}$$

input
```
int(sec(b*x+a)^5*tan(b*x+a)^2,x)
```

output
```
(3*log(tan((a + b*x)/2) - 1)*sin(a + b*x)**6 - 9*log(tan((a + b*x)/2) - 1)
*sin(a + b*x)**4 + 9*log(tan((a + b*x)/2) - 1)*sin(a + b*x)**2 - 3*log(tan
((a + b*x)/2) - 1) - 3*log(tan((a + b*x)/2) + 1)*sin(a + b*x)**6 + 9*log(t
an((a + b*x)/2) + 1)*sin(a + b*x)**4 - 9*log(tan((a + b*x)/2) + 1)*sin(a +
 b*x)**2 + 3*log(tan((a + b*x)/2) + 1) + 3*sin(a + b*x)**5 - 8*sin(a + b*x
)**3 - 3*sin(a + b*x))/(48*b*(sin(a + b*x)**6 - 3*sin(a + b*x)**4 + 3*sin(
a + b*x)**2 - 1))
```

3.66 $\int \cos^5(a+bx)\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \cos^5(a+bx)\sin^3(a+bx)\,dx = -\frac{\cos^6(a+bx)}{6b} + \frac{\cos^8(a+bx)}{8b}$$

output
```
-1/6*cos(b*x+a)^6/b+1/8*cos(b*x+a)^8/b
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.55

$$\int \cos^5(a+bx)\sin^3(a+bx)\,dx = \frac{-72\cos(2(a+bx)) - 12\cos(4(a+bx)) + 8\cos(6(a+bx)) + 3\cos(8(a+bx))}{3072b}$$

input
```
Integrate[Cos[a + b*x]^5*Sin[a + b*x]^3,x]
```

output
```
(-72*Cos[2*(a + b*x)] - 12*Cos[4*(a + b*x)] + 8*Cos[6*(a + b*x)] + 3*Cos[8
*(a + b*x)])/(3072*b)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\cos^5(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^3\cos(a+bx)^5dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int \cos^5(a+bx)\left(1-\cos^2(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(\cos^5(a+bx)-\cos^7(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{6}\cos^6(a+bx)-\frac{1}{8}\cos^8(a+bx)}{b}$$

input
```
Int[Cos[a + b*x]^5*Sin[a + b*x]^3,x]
```

output
```
-((Cos[a + b*x]^6/6 - Cos[a + b*x]^8/8)/b)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [A] (verified)

Time = 14.46 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{\frac{\cos(bx+a)^8}{8}-\frac{\cos(bx+a)^6}{6}}{b}$
default	$\frac{\frac{\cos(bx+a)^8}{8}-\frac{\cos(bx+a)^6}{6}}{b}$
parallelrisch	$\frac{8\cos(6bx+6a)-72\cos(2bx+2a)+3\cos(8bx+8a)+73-12\cos(4bx+4a)}{3072b}$
risch	$\frac{\cos(8bx+8a)}{1024b}+\frac{\cos(6bx+6a)}{384b}-\frac{\cos(4bx+4a)}{256b}-\frac{3\cos(2bx+2a)}{128b}$
norman	$\frac{\frac{4\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}{b}+\frac{4\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{12}}{b}-\frac{16\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^6}{3b}-\frac{16\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{10}}{3b}+\frac{40\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^8}{3b}}{\left(1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right)^8}$
orering	$-\frac{205\left(-5\cos(bx+a)^4\sin(bx+a)^4b+3\cos(bx+a)^6\sin(bx+a)^2b\right)}{576b^2}-\frac{91\left(-60\cos(bx+a)^2\sin(bx+a)^6b^3+290\cos(bx+\right.}{}$

input
```
int(cos(b*x+a)^5*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/8*cos(b*x+a)^8-1/6*cos(b*x+a)^6)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^5(a+bx)\sin^3(a+bx)\,dx = \frac{3\cos(bx+a)^8 - 4\cos(bx+a)^6}{24\,b}$$

input
```
integrate(cos(b*x+a)^5*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/24*(3*cos(b*x + a)^8 - 4*cos(b*x + a)^6)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(22) = 44$.

Time = 0.62 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.03

$$\int \cos^5(a+bx)\sin^3(a+bx)\,dx = \begin{cases} \frac{\sin^8(a+bx)}{24b} + \frac{\sin^6(a+bx)\cos^2(a+bx)}{6b} + \frac{\sin^4(a+bx)\cos^4(a+bx)}{4b} & \text{for } b \neq 0 \\ x\sin^3(a)\cos^5(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a)**5*sin(b*x+a)**3,x)
```

output
```
Piecewise((sin(a + b*x)**8/(24*b) + sin(a + b*x)**6*cos(a + b*x)**2/(6*b)
+ sin(a + b*x)**4*cos(a + b*x)**4/(4*b), Ne(b, 0)), (x*sin(a)**3*cos(a)**5
, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \cos^5(a+bx)\sin^3(a+bx)\,dx = \frac{3\,\sin(bx+a)^8 - 8\,\sin(bx+a)^6 + 6\,\sin(bx+a)^4}{24\,b}$$

input
```
integrate(cos(b*x+a)^5*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/24*(3*sin(b*x + a)^8 - 8*sin(b*x + a)^6 + 6*sin(b*x + a)^4)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^5(a+bx)\sin^3(a+bx)\,dx = \frac{3\,\cos(bx+a)^8 - 4\,\cos(bx+a)^6}{24\,b}$$

input
```
integrate(cos(b*x+a)^5*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/24*(3*cos(b*x + a)^8 - 4*cos(b*x + a)^6)/b
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \cos^5(a+bx)\sin^3(a+bx)\,dx = \frac{\cos(a+b\,x)^6\,\left(3\cos(a+b\,x)^2-4\right)}{24\,b}$$

input
```
int(cos(a + b*x)^5*sin(a + b*x)^3,x)
```

output
```
(cos(a + b*x)^6*(3*cos(a + b*x)^2 - 4))/(24*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13

$$\int \cos^5(a+bx)\sin^3(a+bx)\,dx = \frac{\sin(bx+a)^4\left(3\sin(bx+a)^4-8\sin(bx+a)^2+6\right)}{24b}$$

input
```
int(cos(b*x+a)^5*sin(b*x+a)^3,x)
```

output
```
(sin(a + b*x)**4*(3*sin(a + b*x)**4 - 8*sin(a + b*x)**2 + 6))/(24*b)
```

3.67 $\int \cos^4(a+bx)\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \cos^4(a+bx)\sin^3(a+bx)\,dx=-\frac{\cos^5(a+bx)}{5b}+\frac{\cos^7(a+bx)}{7b}$$

output
```
-1/5*cos(b*x+a)^5/b+1/7*cos(b*x+a)^7/b
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \cos^4(a+bx)\sin^3(a+bx)\,dx=\frac{\cos^5(a+bx)(-9+5\cos(2(a+bx)))}{70b}$$

input
```
Integrate[Cos[a + b*x]^4*Sin[a + b*x]^3,x]
```

output
```
(Cos[a + b*x]^5*(-9 + 5*Cos[2*(a + b*x)]))/(70*b)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\cos^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^3\cos(a+bx)^4dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int \cos^4(a+bx)\left(1-\cos^2(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(\cos^4(a+bx)-\cos^6(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{5}\cos^5(a+bx)-\frac{1}{7}\cos^7(a+bx)}{b}$$

input
```
Int[Cos[a + b*x]^4*Sin[a + b*x]^3,x]
```

output
```
-((Cos[a + b*x]^5/5 - Cos[a + b*x]^7/7)/b)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [A] (verified)

Time = 12.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{\frac{\cos(bx+a)^7}{7}-\frac{\cos(bx+a)^5}{5}}{b}$
default	$\frac{\frac{\cos(bx+a)^7}{7}-\frac{\cos(bx+a)^5}{5}}{b}$
parallelrisch	$\frac{-35\cos(3bx+3a)+7\cos(5bx+5a)-128-105\cos(bx+a)+5\cos(7bx+7a)}{2240b}$
risch	$-\frac{3\cos(bx+a)}{64b}+\frac{\cos(7bx+7a)}{448b}+\frac{\cos(5bx+5a)}{320b}-\frac{\cos(3bx+3a)}{64b}$
norman	$\frac{-\frac{4\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{10}}{b}-\frac{8\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{6}}{b}+\frac{4\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{8}}{b}-\frac{4}{35b}-\frac{4\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}{5b}+\frac{8\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}}{5b}}{\left(1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)^{7}}$
orering	$-\frac{12916\left(-4\cos(bx+a)^3\sin(bx+a)^4b+3\cos(bx+a)^5\sin(bx+a)^2b\right)}{11025b^2}-\frac{94\left(-24\cos(bx+a)\sin(bx+a)^6b^3+184\sin(bx+\right.}{}$

input
```
int(cos(b*x+a)^4*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/7*cos(b*x+a)^7-1/5*cos(b*x+a)^5)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^4(a+bx)\sin^3(a+bx)\,dx = \frac{5\,\cos\left(bx+a\right)^7-7\,\cos\left(bx+a\right)^5}{35\,b}$$

input
```
integrate(cos(b*x+a)^4*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/35*(5*cos(b*x + a)^7 - 7*cos(b*x + a)^5)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(22) = 44$.

Time = 0.44 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.48

$$\int \cos^4(a+bx)\sin^3(a+bx)\,dx = \begin{cases} -\frac{\sin^2(a+bx)\cos^5(a+bx)}{5b} - \frac{2\cos^7(a+bx)}{35b} & \text{for } b\neq 0 \\ x\sin^3(a)\cos^4(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a)**4*sin(b*x+a)**3,x)
```

output
```
Piecewise((-sin(a + b*x)**2*cos(a + b*x)**5/(5*b) - 2*cos(a + b*x)**7/(35*
b), Ne(b, 0)), (x*sin(a)**3*cos(a)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^4(a+bx)\sin^3(a+bx)\,dx = \frac{5\,\cos(bx+a)^7 - 7\,\cos(bx+a)^5}{35\,b}$$

input
```
integrate(cos(b*x+a)^4*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/35*(5*cos(b*x + a)^7 - 7*cos(b*x + a)^5)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^4(a+bx)\sin^3(a+bx)\,dx = \frac{5\,\cos(bx+a)^7 - 7\,\cos(bx+a)^5}{35\,b}$$

input
```
integrate(cos(b*x+a)^4*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/35*(5*cos(b*x + a)^7 - 7*cos(b*x + a)^5)/b
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^4(a+bx)\sin^3(a+bx)\,dx = -\frac{7\cos(a+b\,x)^5 - 5\cos(a+b\,x)^7}{35\,b}$$

input
```
int(cos(a + b*x)^4*sin(a + b*x)^3,x)
```

output
```
-(7*cos(a + b*x)^5 - 5*cos(a + b*x)^7)/(35*b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.03

$$\int \cos^4(a+bx)\sin^3(a+bx)\,dx$$
$$= \frac{-5\cos(bx+a)\sin(bx+a)^6 + 8\cos(bx+a)\sin(bx+a)^4 - \cos(bx+a)\sin(bx+a)^2 - 2\cos(bx+a) +}{35b}$$

input
```
int(cos(b*x+a)^4*sin(b*x+a)^3,x)
```

output
```
( - 5*cos(a + b*x)*sin(a + b*x)**6 + 8*cos(a + b*x)*sin(a + b*x)**4 - cos(
a + b*x)*sin(a + b*x)**2 - 2*cos(a + b*x) + 2)/(35*b)
```

3.68 $\int \cos^3(a+bx)\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \cos^3(a+bx)\sin^3(a+bx)\,dx=\frac{\sin^4(a+bx)}{4b}-\frac{\sin^6(a+bx)}{6b}$$

output
```
1/4*sin(b*x+a)^4/b-1/6*sin(b*x+a)^6/b
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13

$$\int \cos^3(a+bx)\sin^3(a+bx)\,dx=\frac{1}{8}\left(-\frac{3\cos(2(a+bx))}{8b}+\frac{\cos(6(a+bx))}{24b}\right)$$

input
```
Integrate[Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
((-3*Cos[2*(a + b*x)])/(8*b) + Cos[6*(a + b*x)]/(24*b))/8
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^3\cos(a+bx)^3dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{\int \sin^3(a+bx)\left(1-\sin^2(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\sin^3(a+bx)-\sin^5(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{4}\sin^4(a+bx)-\frac{1}{6}\sin^6(a+bx)}{b}$$

input
```
Int[Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
(Sin[a + b*x]^4/4 - Sin[a + b*x]^6/6)/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 7.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{-\frac{\sin(bx+a)^6}{6}+\frac{\sin(bx+a)^4}{4}}{b}$
default	$\frac{-\frac{\sin(bx+a)^6}{6}+\frac{\sin(bx+a)^4}{4}}{b}$
parallelrisch	$\frac{\cos(6bx+6a)-9\cos(2bx+2a)+8}{192b}$
risch	$\frac{\cos(6bx+6a)}{192b}-\frac{3\cos(2bx+2a)}{64b}$
norman	$\frac{\frac{4\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}{b}+\frac{4\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^8}{b}-\frac{8\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^6}{3b}}{\left(1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right)^6}$
orering	$-\frac{5\left(-3\cos(bx+a)^2\sin(bx+a)^4b+3\cos(bx+a)^4\sin(bx+a)^2b\right)}{18b^2}-\frac{-6b^3\sin(bx+a)^6+102\cos(bx+a)^2\sin(bx+a)^4b^3-}{144b^4}$

input
```
int(cos(b*x+a)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/6*sin(b*x+a)^6+1/4*sin(b*x+a)^4)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^3(a+bx)\sin^3(a+bx)\,dx=\frac{2\,\cos\left(bx+a\right)^6-3\,\cos\left(bx+a\right)^4}{12\,b}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/12*(2*cos(b*x + a)^6 - 3*cos(b*x + a)^4)/b
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

$$\int \cos^3(a+bx)\sin^3(a+bx)\,dx=\begin{cases}\frac{\sin^6(a+bx)}{12b}+\frac{\sin^4(a+bx)\cos^2(a+bx)}{4b} & \text{for } b\neq 0\\ x\sin^3(a)\cos^3(a) & \text{otherwise}\end{cases}$$

input
```
integrate(cos(b*x+a)**3*sin(b*x+a)**3,x)
```

output
```
Piecewise((sin(a + b*x)**6/(12*b) + sin(a + b*x)**4*cos(a + b*x)**2/(4*b),
 Ne(b, 0)), (x*sin(a)**3*cos(a)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^3(a+bx)\sin^3(a+bx)\,dx = -\frac{2\sin(bx+a)^6 - 3\sin(bx+a)^4}{12\,b}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/12*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^3(a+bx)\sin^3(a+bx)\,dx = \frac{2\cos(bx+a)^6 - 3\cos(bx+a)^4}{12\,b}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/12*(2*cos(b*x + a)^6 - 3*cos(b*x + a)^4)/b
```

Mupad [B] (verification not implemented)

Time = 25.51 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

$$\int \cos^3(a+bx)\sin^3(a+bx)\,dx = \frac{\cos(a+b\,x)^4\,\left(\cos(a+b\,x)^2-1\right)}{4\,b} - \frac{\cos(a+b\,x)^6}{12\,b}$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)^3,x)
```

output
```
(cos(a + b*x)^4*(cos(a + b*x)^2 - 1))/(4*b) - cos(a + b*x)^6/(12*b)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \cos^3(a+bx)\sin^3(a+bx)\,dx = \frac{\sin(bx+a)^4\left(-2\sin(bx+a)^2+3\right)}{12b}$$

input
```
int(cos(b*x+a)^3*sin(b*x+a)^3,x)
```

output
```
(sin(a + b*x)**4*( - 2*sin(a + b*x)**2 + 3))/(12*b)
```

3.69 $\int \cos^2(a+bx)\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \cos^2(a+bx)\sin^3(a+bx)\,dx = -\frac{\cos^3(a+bx)}{3b}+\frac{\cos^5(a+bx)}{5b}$$

output
```
-1/3*cos(b*x+a)^3/b+1/5*cos(b*x+a)^5/b
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \cos^2(a+bx)\sin^3(a+bx)\,dx = \frac{\cos^3(a+bx)(-7+3\cos(2(a+bx)))}{30b}$$

input
```
Integrate[Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output
```
(Cos[a + b*x]^3*(-7 + 3*Cos[2*(a + b*x)]))/(30*b)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^3\cos(a+bx)^2dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int \cos^2(a+bx)\left(1-\cos^2(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(\cos^2(a+bx)-\cos^4(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{3}\cos^3(a+bx)-\frac{1}{5}\cos^5(a+bx)}{b}$$

input
```
Int[Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output
```
-((Cos[a + b*x]^3/3 - Cos[a + b*x]^5/5)/b)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [A] (verified)

Time = 7.71 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{-\frac{\cos(bx+a)^3\sin(bx+a)^2}{5}-\frac{2\cos(bx+a)^3}{15}}{b}$
default	$\frac{-\frac{\cos(bx+a)^3\sin(bx+a)^2}{5}-\frac{2\cos(bx+a)^3}{15}}{b}$
parallelrisch	$\frac{-30\cos(bx+a)+3\cos(5bx+5a)-32-5\cos(3bx+3a)}{240b}$
risch	$-\frac{\cos(bx+a)}{8b}+\frac{\cos(5bx+5a)}{80b}-\frac{\cos(3bx+3a)}{48b}$
norman	$\frac{-\frac{4}{15b}-\frac{4\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^6}{b}-\frac{4\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}{3b}+\frac{4\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}{3b}}{\left(1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right)^5}$
orering	$-\frac{259\left(-2\sin(bx+a)^4b\cos(bx+a)+3\cos(bx+a)^3\sin(bx+a)^2b\right)}{225b^2}-\frac{7\left(-75\sin(bx+a)^2b^3\cos(bx+a)^3+44\sin(bx+a)^4b\right.}{45b^4}$

input
```
int(cos(b*x+a)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/5*cos(b*x+a)^3*sin(b*x+a)^2-2/15*cos(b*x+a)^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^2(a+bx)\sin^3(a+bx)\,dx = \frac{3\,\cos\left(bx+a\right)^5-5\,\cos\left(bx+a\right)^3}{15\,b}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/15*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(22) = 44.

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.48

$$\int \cos^2(a+bx)\sin^3(a+bx)\,dx = \begin{cases} -\frac{\sin^2(a+bx)\cos^3(a+bx)}{3b} - \frac{2\cos^5(a+bx)}{15b} & \text{for } b\neq 0 \\ x\sin^3(a)\cos^2(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a)**2*sin(b*x+a)**3,x)
```

output
```
Piecewise((-sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*cos(a + b*x)**5/(15*
b), Ne(b, 0)), (x*sin(a)**3*cos(a)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^2(a+bx)\sin^3(a+bx)\,dx = \frac{3\,\cos\left(bx+a\right)^5-5\,\cos\left(bx+a\right)^3}{15\,b}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/15*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^2(a+bx)\sin^3(a+bx)\,dx = \frac{3\,\cos\left(bx+a\right)^5-5\,\cos\left(bx+a\right)^3}{15\,b}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/15*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)/b
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^2(a+bx)\sin^3(a+bx)\,dx = -\frac{5\cos\left(a+b\,x\right)^3-3\cos\left(a+b\,x\right)^5}{15\,b}$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)^3,x)
```

output
```
-(5*cos(a + b*x)^3 - 3*cos(a + b*x)^5)/(15*b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.52

$$\int \cos^2(a+bx)\sin^3(a+bx)\,dx = \frac{3\cos(bx+a)\sin(bx+a)^4-\cos(bx+a)\sin(bx+a)^2-2\cos(bx+a)+2}{15b}$$

input
```
int(cos(b*x+a)^2*sin(b*x+a)^3,x)
```

output
```
(3*cos(a + b*x)*sin(a + b*x)**4 - cos(a + b*x)*sin(a + b*x)**2 - 2*cos(a +
 b*x) + 2)/(15*b)
```

3.70 $\int \cos(a+bx)\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \cos(a+bx)\sin^3(a+bx)\,dx=\frac{\sin^4(a+bx)}{4b}$$

output
```
1/4*sin(b*x+a)^4/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cos(a+bx)\sin^3(a+bx)\,dx=\frac{\sin^4(a+bx)}{4b}$$

input
```
Integrate[Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output
```
Sin[a + b*x]^4/(4*b)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3044, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^3\cos(a+bx)dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{\int \sin^3(a+bx)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{\sin^4(a+bx)}{4b}$$

input
```
Int[Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output
```
Sin[a + b*x]^4/(4*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 4.75 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{\sin(bx+a)^4}{4b}$
default	$\frac{\sin(bx+a)^4}{4b}$
parallelrisch	$\frac{\cos(4bx+4a)+3-4\cos(2bx+2a)}{32b}$
risch	$\frac{\cos(4bx+4a)}{32b}-\frac{\cos(2bx+2a)}{8b}$
norman	$\frac{4\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}{b\left(1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right)^4}$
orering	$-\frac{5\left(-b\sin(bx+a)^4+3\cos(bx+a)^2\sin(bx+a)^2b\right)}{16b^2}-\frac{-48\cos(bx+a)^2\sin(bx+a)^2b^3+10\sin(bx+a)^4b^3+6\cos(bx+a)^4b}{64b^4}$

input
```
int(cos(b*x+a)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*sin(b*x+a)^4/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int \cos(a+bx)\sin^3(a+bx)\,dx = \frac{\cos\left(bx+a\right)^4-2\,\cos\left(bx+a\right)^2}{4\,b}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4*(cos(b*x + a)^4 - 2*cos(b*x + a)^2)/b
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \cos(a+bx)\sin^3(a+bx)\,dx = \begin{cases} \frac{\sin^4(a+bx)}{4b} & \text{for } b \neq 0 \\ x\sin^3(a)\cos(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)**3,x)
```

output
```
Piecewise((sin(a + b*x)**4/(4*b), Ne(b, 0)), (x*sin(a)**3*cos(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\sin^3(a+bx)\,dx = \frac{\sin(bx+a)^4}{4b}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/4*sin(b*x + a)^4/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int \cos(a+bx)\sin^3(a+bx)\,dx = \frac{\cos(bx+a)^4 - 2\cos(bx+a)^2}{4b}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/4*(cos(b*x + a)^4 - 2*cos(b*x + a)^2)/b
```

Mupad [B] (verification not implemented)

Time = 25.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\sin^3(a+bx)\,dx=\frac{\sin{(a+b\,x)}^4}{4\,b}$$

input
```
int(cos(a + b*x)*sin(a + b*x)^3,x)
```

output
```
sin(a + b*x)^4/(4*b)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\sin^3(a+bx)\,dx=\frac{\sin{(bx+a)}^4}{4b}$$

input
```
int(cos(b*x+a)*sin(b*x+a)^3,x)
```

output
```
sin(a + b*x)**4/(4*b)
```

3.71 $\int \sin^2(a+bx)\tan(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \sin^2(a+bx)\tan(a+bx)\,dx = \frac{\cos^2(a+bx)}{2b} - \frac{\log(\cos(a+bx))}{b}$$

output
```
1/2*cos(b*x+a)^2/b-ln(cos(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \sin^2(a+bx)\tan(a+bx)\,dx = -\frac{-\frac{1}{2}\cos^2(a+bx)+\log(\cos(a+bx))}{b}$$

input
```
Integrate[Sin[a + b*x]^2*Tan[a + b*x],x]
```

output
```
-((-1/2*Cos[a + b*x]^2 + Log[Cos[a + b*x]])/b)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\tan(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^2\tan(a+bx)dx$$

$$\Big\downarrow \text{3070}$$

$$-\frac{\int (1-\cos^2(a+bx))\sec(a+bx)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int (\sec(a+bx)-\cos(a+bx))d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\log(\cos(a+bx))-\frac{1}{2}\cos^2(a+bx)}{b}$$

input
```
Int[Sin[a + b*x]^2*Tan[a + b*x],x]
```

output
```
-((-1/2*Cos[a + b*x]^2 + Log[Cos[a + b*x]])/b)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Maple [A] (verified)

Time = 2.47 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{-\frac{\sin(bx+a)^2}{2}-\ln(\cos(bx+a))}{b}$	25
default	$\frac{-\frac{\sin(bx+a)^2}{2}-\ln(\cos(bx+a))}{b}$	25
risch	$ix+\frac{e^{2i(bx+a)}}{8b}+\frac{e^{-2i(bx+a)}}{8b}+\frac{2ia}{b}-\frac{\ln\left(e^{2i(bx+a)}+1\right)}{b}$	58

input
```
int(sin(b*x+a)^2*tan(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/2*sin(b*x+a)^2-ln(cos(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \sin^2(a+bx)\tan(a+bx)\,dx = \frac{\cos(bx+a)^2 - 2\log(-\cos(bx+a))}{2b}$$

input
```
integrate(sin(b*x+a)^2*tan(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(cos(b*x + a)^2 - 2*log(-cos(b*x + a)))/b
```

Sympy [F]

$$\int \sin^2(a+bx)\tan(a+bx)\,dx = \int \sin^2(a+bx)\tan(a+bx)\,dx$$

input
```
integrate(sin(b*x+a)**2*tan(b*x+a),x)
```

output
```
Integral(sin(a + b*x)**2*tan(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \sin^2(a+bx)\tan(a+bx)\,dx = -\frac{\sin(bx+a)^2 + \log(\sin(bx+a)^2 - 1)}{2b}$$

input
```
integrate(sin(b*x+a)^2*tan(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \sin^2(a+bx)\tan(a+bx)\,dx = \frac{\cos\left(bx+a\right)^2}{2\,b} - \frac{\log\left(\left|\cos\left(bx+a\right)\right|\right)}{b}$$

input
```
integrate(sin(b*x+a)^2*tan(b*x+a),x, algorithm="giac")
```

output
```
1/2*cos(b*x + a)^2/b - log(abs(cos(b*x + a)))/b
```

Mupad [B] (verification not implemented)

Time = 25.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \sin^2(a+bx)\tan(a+bx)\,dx = \frac{\ln\left(\tan(a+b\,x)^2+1\right)}{2\,b} + \frac{\cos\left(a+b\,x\right)^2}{2\,b}$$

input
```
int(sin(a + b*x)^2*tan(a + b*x),x)
```

output
```
log(tan(a + b*x)^2 + 1)/(2*b) + cos(a + b*x)^2/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.14

$$\int \sin^2(a+bx)\tan(a+bx)\,dx$$
$$= \frac{2\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2+1\right) - 2\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right) - 2\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right) - \sin\left(bx+a\right)^2}{2b}$$

input
```
int(sin(b*x+a)^2*tan(b*x+a),x)
```

output
```
(2*log(tan((a + b*x)/2)**2 + 1) - 2*log(tan((a + b*x)/2) - 1) - 2*log(tan(
(a + b*x)/2) + 1) - sin(a + b*x)**2)/(2*b)
```

3.72 $\int \sin(a+bx)\tan^2(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \sin(a+bx)\tan^2(a+bx)\,dx = \frac{\cos(a+bx)}{b} + \frac{\sec(a+bx)}{b}$$

output
```
cos(b*x+a)/b+sec(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \sin(a+bx)\tan^2(a+bx)\,dx = \frac{\cos(a+bx)}{b} + \frac{\sec(a+bx)}{b}$$

input
```
Integrate[Sin[a + b*x]*Tan[a + b*x]^2,x]
```

output
```
Cos[a + b*x]/b + Sec[a + b*x]/b
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\tan^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)\tan(a+bx)^2dx$$

$$\Big\downarrow \text{3070}$$

$$-\frac{\int \left(1-\cos^2(a+bx)\right)\sec^2(a+bx)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(\sec^2(a+bx)-1\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\cos(a+bx)-\sec(a+bx)}{b}$$

input
```
Int[Sin[a + b*x]*Tan[a + b*x]^2,x]
```

output
```
-((-Cos[a + b*x] - Sec[a + b*x])/b)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Maple [A] (verified)

Time = 3.97 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.90

method	result	size
derivativedivides	$\frac{\frac{\sin(bx+a)^4}{\cos(bx+a)}+\left(2+\sin(bx+a)^2\right)\cos(bx+a)}{b}$	40
default	$\frac{\frac{\sin(bx+a)^4}{\cos(bx+a)}+\left(2+\sin(bx+a)^2\right)\cos(bx+a)}{b}$	40
risch	$\frac{e^{3i(bx+a)}+7\cos(bx+a)+5i\sin(bx+a)}{2b\left(e^{2i(bx+a)}+1\right)}$	46

input
```
int(sin(b*x+a)*tan(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(sin(b*x+a)^4/cos(b*x+a)+(2+sin(b*x+a)^2)*cos(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \sin(a+bx)\tan^2(a+bx)\,dx = \frac{\cos\left(bx+a\right)^2+1}{b\cos\left(bx+a\right)}$$

input
```
integrate(sin(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

output
```
(cos(b*x + a)^2 + 1)/(b*cos(b*x + a))
```

Sympy [F]

$$\int \sin(a+bx)\tan^2(a+bx)\,dx = \int \sin\left(a+bx\right)\tan^2\left(a+bx\right)\,dx$$

input
```
integrate(sin(b*x+a)*tan(b*x+a)**2,x)
```

output
```
Integral(sin(a + b*x)*tan(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \sin(a+bx)\tan^2(a+bx)\,dx = \frac{\frac{1}{\cos(bx+a)}+\cos\left(bx+a\right)}{b}$$

input
```
integrate(sin(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

output
```
(1/cos(b*x + a) + cos(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \sin(a+bx)\tan^2(a+bx)\,dx = \frac{\cos(bx+a)}{b} + \frac{1}{b\cos(bx+a)}$$

input
```
integrate(sin(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

output
```
cos(b*x + a)/b + 1/(b*cos(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 25.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \sin(a+bx)\tan^2(a+bx)\,dx = -\frac{4}{b\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4-1\right)}$$

input
```
int(sin(a + b*x)*tan(a + b*x)^2,x)
```

output
```
-4/(b*(tan(a/2 + (b*x)/2)^4 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.48

$$\int \sin(a+bx)\tan^2(a+bx)\,dx = -\frac{4\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}{b\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4-1\right)}$$

input
```
int(sin(b*x+a)*tan(b*x+a)^2,x)
```

output
```
( - 4*tan((a + b*x)/2)**4)/(b*(tan((a + b*x)/2)**4 - 1))
```

3.73 $\int \tan^3(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 27

$$\int \tan^3(a+bx)\,dx = \frac{\log(\cos(a+bx))}{b} + \frac{\tan^2(a+bx)}{2b}$$

output
```
ln(cos(b*x+a))/b+1/2*tan(b*x+a)^2/b
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \tan^3(a+bx)\,dx = \frac{2\log(\cos(a+bx)) + \sec^2(a+bx)}{2b}$$

input
```
Integrate[Tan[a + b*x]^3,x]
```

output
```
(2*Log[Cos[a + b*x]] + Sec[a + b*x]^2)/(2*b)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3954, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(a+bx)^3dx$$

$$\Big\downarrow \text{ 3954}$$

$$\frac{\tan^2(a+bx)}{2b}-\int \tan(a+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\tan^2(a+bx)}{2b}-\int \tan(a+bx)dx$$

$$\Big\downarrow \text{ 3956}$$

$$\frac{\tan^2(a+bx)}{2b}+\frac{\log(\cos(a+bx))}{b}$$

input
```
Int[Tan[a + b*x]^3,x]
```

output
```
Log[Cos[a + b*x]]/b + Tan[a + b*x]^2/(2*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

method	result	size
parallelrisch	$-\frac{-\tan(bx+a)^2+\ln\left(1+\tan(bx+a)^2\right)}{2b}$	28
derivativedivides	$\frac{\frac{\tan(bx+a)^2}{2}-\frac{\ln\left(1+\tan(bx+a)^2\right)}{2}}{b}$	29
default	$\frac{\frac{\tan(bx+a)^2}{2}-\frac{\ln\left(1+\tan(bx+a)^2\right)}{2}}{b}$	29
norman	$\frac{\tan(bx+a)^2}{2b}-\frac{\ln\left(1+\tan(bx+a)^2\right)}{2b}$	31
risch	$-ix-\frac{2ia}{b}+\frac{2\,e^{2i(bx+a)}}{b\left(e^{2i(bx+a)}+1\right)^2}+\frac{\ln\left(e^{2i(bx+a)}+1\right)}{b}$	56

input
```
int(tan(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-tan(b*x+a)^2+ln(1+tan(b*x+a)^2))/b
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \tan^3(a+bx)\,dx = \frac{\tan(bx+a)^2 + \log\left(\frac{1}{\tan(bx+a)^2+1}\right)}{2\,b}$$

input
```
integrate(tan(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/2*(tan(b*x + a)^2 + log(1/(tan(b*x + a)^2 + 1)))/b
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \tan^3(a+bx)\,dx = \begin{cases} -\frac{\log(\tan^2(a+bx)+1)}{2b} + \frac{\tan^2(a+bx)}{2b} & \text{for } b \neq 0 \\ x\tan^3(a) & \text{otherwise} \end{cases}$$

input
```
integrate(tan(b*x+a)**3,x)
```

output
```
Piecewise((-log(tan(a + b*x)**2 + 1)/(2*b) + tan(a + b*x)**2/(2*b), Ne(b,
0)), (x*tan(a)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \tan^3(a+bx)\,dx = -\frac{\frac{1}{\sin(bx+a)^2-1} - \log\left(\sin(bx+a)^2 - 1\right)}{2\,b}$$

input
```
integrate(tan(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/2*(1/(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2 - 1))/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \tan^3(a+bx)\,dx = \frac{\tan{(bx+a)}^2}{2\,b} - \frac{\log\left(\tan{(bx+a)}^2+1\right)}{2\,b}$$

input
```
integrate(tan(b*x+a)^3,x, algorithm="giac")
```

output
```
1/2*tan(b*x + a)^2/b - 1/2*log(tan(b*x + a)^2 + 1)/b
```

Mupad [B] (verification not implemented)

Time = 25.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \tan^3(a+bx)\,dx = \frac{\tan(a+b\,x)^2}{2\,b} - \frac{\ln\left(\tan(a+b\,x)^2+1\right)}{2\,b}$$

input
```
int(tan(a + b*x)^3,x)
```

output
```
tan(a + b*x)^2/(2*b) - log(tan(a + b*x)^2 + 1)/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \tan^3(a+bx)\,dx = \frac{-\log\left(\tan{(bx+a)}^2+1\right)+\tan{(bx+a)}^2}{2b}$$

input
```
int(tan(b*x+a)^3,x)
```

output
```
( - log(tan(a + b*x)**2 + 1) + tan(a + b*x)**2)/(2*b)
```

3.74 $\int \sec(a+bx)\tan^3(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \sec(a+bx)\tan^3(a+bx)\,dx=-\frac{\sec(a+bx)}{b}+\frac{\sec^3(a+bx)}{3b}$$

output
```
-sec(b*x+a)/b+1/3*sec(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \sec(a+bx)\tan^3(a+bx)\,dx=-\frac{\sec(a+bx)}{b}+\frac{\sec^3(a+bx)}{3b}$$

input
```
Integrate[Sec[a + b*x]*Tan[a + b*x]^3,x]
```

output
```
-(Sec[a + b*x]/b) + Sec[a + b*x]^3/(3*b)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3086, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(a+bx)\sec(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \tan(a+bx)^3\sec(a+bx)dx$$
$$\Big\downarrow \text{3086}$$
$$\frac{\int \left(\sec^2(a+bx)-1\right)d\sec(a+bx)}{b}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\frac{1}{3}\sec^3(a+bx)-\sec(a+bx)}{b}$$

input
```
Int[Sec[a + b*x]*Tan[a + b*x]^3,x]
```

output
```
(-Sec[a + b*x] + Sec[a + b*x]^3/3)/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\frac{\sec(bx+a)^3}{3}-\sec(bx+a)}{b}$	24
default	$\frac{\frac{\sec(bx+a)^3}{3}-\sec(bx+a)}{b}$	24
risch	$-\frac{2\left(3\,\mathrm{e}^{5i(bx+a)}+2\,\mathrm{e}^{3i(bx+a)}+3\,\mathrm{e}^{i(bx+a)}\right)}{3b\left(\mathrm{e}^{2i(bx+a)}+1\right)^3}$	53

input
```
int(sec(b*x+a)*tan(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/3*sec(b*x+a)^3-sec(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \sec(a+bx)\tan^3(a+bx)\,dx = -\frac{3\,\cos\left(bx+a\right)^2-1}{3\,b\cos\left(bx+a\right)^3}$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/3*(3*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(19) = 38$.

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int \sec(a+bx)\tan^3(a+bx)\,dx = \begin{cases} \frac{\tan^2(a+bx)\sec(a+bx)}{3b} - \frac{2\sec(a+bx)}{3b} & \text{for } b \neq 0 \\ x\tan^3(a)\sec(a) & \text{otherwise} \end{cases}$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)**3,x)
```

output
```
Piecewise((tan(a + b*x)**2*sec(a + b*x)/(3*b) - 2*sec(a + b*x)/(3*b), Ne(b
, 0)), (x*tan(a)**3*sec(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \sec(a+bx)\tan^3(a+bx)\,dx = -\frac{3\cos(bx+a)^2-1}{3\,b\cos(bx+a)^3}$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/3*(3*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \sec(a+bx)\tan^3(a+bx)\,dx = -\frac{3\cos(bx+a)^2-1}{3\,b\cos(bx+a)^3}$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/3*(3*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^3)
```

Mupad [B] (verification not implemented)

Time = 25.51 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \sec(a+bx)\tan^3(a+bx)\,dx = -\frac{\cos\left(a+b\,x\right)^2-\frac{1}{3}}{b\cos\left(a+b\,x\right)^3}$$

input
```
int(tan(a + b*x)^3/cos(a + b*x),x)
```

output
```
-(cos(a + b*x)^2 - 1/3)/(b*cos(a + b*x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \sec(a+bx)\tan^3(a+bx)\,dx = \frac{\sec\left(bx+a\right)\left(\tan\left(bx+a\right)^2-2\right)}{3b}$$

input
```
int(sec(b*x+a)*tan(b*x+a)^3,x)
```

output
```
(sec(a + b*x)*(tan(a + b*x)**2 - 2))/(3*b)
```

3.75 $\int \sec^2(a+bx)\tan^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 15

$$\int \sec^2(a+bx)\tan^3(a+bx)\,dx=\frac{\tan^4(a+bx)}{4b}$$

output
```
1/4*tan(b*x+a)^4/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \sec^2(a+bx)\tan^3(a+bx)\,dx=\frac{\tan^4(a+bx)}{4b}$$

input
```
Integrate[Sec[a + b*x]^2*Tan[a + b*x]^3,x]
```

output
```
Tan[a + b*x]^4/(4*b)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3042, 3087, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(a+bx)\sec^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(a+bx)^3\sec(a+bx)^2dx$$

$$\Big\downarrow \text{3087}$$

$$\frac{\int \tan^3(a+bx)d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{\tan^4(a+bx)}{4b}$$

input
```
Int[Sec[a + b*x]^2*Tan[a + b*x]^3,x]
```

output
```
Tan[a + b*x]^4/(4*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

Maple [A] (verified)

Time = 1.94 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\tan(bx+a)^4}{4b}$	14
default	$\frac{\tan(bx+a)^4}{4b}$	14
risch	$-\frac{2\left(e^{6i(bx+a)}+e^{2i(bx+a)}\right)}{b\left(e^{2i(bx+a)}+1\right)^4}$	38

input
```
int(sec(b*x+a)^2*tan(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*tan(b*x+a)^4/b
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.67

$$\int \sec^2(a+bx)\tan^3(a+bx)\,dx = -\frac{2\,\cos\left(bx+a\right)^2-1}{4\,b\cos\left(bx+a\right)^4}$$

input
```
integrate(sec(b*x+a)^2*tan(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/4*(2*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(10) = 20$.

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.80

$$\int \sec^2(a+bx)\tan^3(a+bx)\,dx = \begin{cases} \frac{\tan^2(a+bx)\sec^2(a+bx)}{4b} - \frac{\sec^2(a+bx)}{4b} & \text{for } b \neq 0 \\ x\tan^3(a)\sec^2(a) & \text{otherwise} \end{cases}$$

input
```
integrate(sec(b*x+a)**2*tan(b*x+a)**3,x)
```

output
```
Piecewise((tan(a + b*x)**2*sec(a + b*x)**2/(4*b) - sec(a + b*x)**2/(4*b),
Ne(b, 0)), (x*tan(a)**3*sec(a)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sec^2(a+bx)\tan^3(a+bx)\,dx = \frac{\tan(bx+a)^4}{4\,b}$$

input
```
integrate(sec(b*x+a)^2*tan(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/4*tan(b*x + a)^4/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.67

$$\int \sec^2(a+bx)\tan^3(a+bx)\,dx = -\frac{2\cos(bx+a)^2-1}{4\,b\cos(bx+a)^4}$$

input
```
integrate(sec(b*x+a)^2*tan(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/4*(2*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^4)
```

Mupad [B] (verification not implemented)

Time = 25.32 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sec^2(a+bx)\tan^3(a+bx)\,dx = \frac{\tan(a+b\,x)^4}{4\,b}$$

input
```
int(tan(a + b*x)^3/cos(a + b*x)^2,x)
```

output
```
tan(a + b*x)^4/(4*b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53

$$\int \sec^2(a+bx)\tan^3(a+bx)\,dx = \frac{\sec(bx+a)^2\left(\tan(bx+a)^2-1\right)}{4b}$$

input
```
int(sec(b*x+a)^2*tan(b*x+a)^3,x)
```

output
```
(sec(a + b*x)**2*(tan(a + b*x)**2 - 1))/(4*b)
```

3.76 $\int \sec^3(a+bx)\tan^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \sec^3(a+bx)\tan^3(a+bx)\,dx=-\frac{\sec^3(a+bx)}{3b}+\frac{\sec^5(a+bx)}{5b}$$

output
```
-1/3*sec(b*x+a)^3/b+1/5*sec(b*x+a)^5/b
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \sec^3(a+bx)\tan^3(a+bx)\,dx=-\frac{\sec^3(a+bx)}{3b}+\frac{\sec^5(a+bx)}{5b}$$

input
```
Integrate[Sec[a + b*x]^3*Tan[a + b*x]^3,x]
```

output
```
-1/3*Sec[a + b*x]^3/b + Sec[a + b*x]^5/(5*b)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3042, 3086, 25, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(a+bx)\sec^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(a+bx)^3\sec(a+bx)^3dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{\int -\sec^2(a+bx)\left(1-\sec^2(a+bx)\right)d\sec(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \sec^2(a+bx)\left(1-\sec^2(a+bx)\right)d\sec(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(\sec^2(a+bx)-\sec^4(a+bx)\right)d\sec(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{5}\sec^5(a+bx)-\frac{1}{3}\sec^3(a+bx)}{b}$$

input
```
Int[Sec[a + b*x]^3*Tan[a + b*x]^3,x]
```

output
```
(-1/3*Sec[a + b*x]^3 + Sec[a + b*x]^5/5)/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Maple [A] (verified)

Time = 3.53 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\frac{\sec(bx+a)^5}{5}-\frac{\sec(bx+a)^3}{3}}{b}$	26
default	$\frac{\frac{\sec(bx+a)^5}{5}-\frac{\sec(bx+a)^3}{3}}{b}$	26
risch	$-\frac{8\left(5\,e^{7i(bx+a)}-2\,e^{5i(bx+a)}+5\,e^{3i(bx+a)}\right)}{15b\left(e^{2i(bx+a)}+1\right)^5}$	53

input
```
int(sec(b*x+a)^3*tan(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/5*sec(b*x+a)^5-1/3*sec(b*x+a)^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \sec^3(a+bx)\tan^3(a+bx)\,dx = -\frac{5\,\cos{(bx+a)}^2-3}{15\,b\cos{(bx+a)}^5}$$

input
```
integrate(sec(b*x+a)^3*tan(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/15*(5*cos(b*x + a)^2 - 3)/(b*cos(b*x + a)^5)
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int \sec^3(a+bx)\tan^3(a+bx)\,dx = \begin{cases} \frac{\tan^2{(a+bx)}\sec^3{(a+bx)}}{5b} - \frac{2\sec^3{(a+bx)}}{15b} & \text{for } b \neq 0 \\ x\tan^3{(a)}\sec^3{(a)} & \text{otherwise} \end{cases}$$

input
```
integrate(sec(b*x+a)**3*tan(b*x+a)**3,x)
```

output
```
Piecewise((tan(a + b*x)**2*sec(a + b*x)**3/(5*b) - 2*sec(a + b*x)**3/(15*b
), Ne(b, 0)), (x*tan(a)**3*sec(a)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \sec^3(a+bx)\tan^3(a+bx)\,dx = -\frac{5\,\cos{(bx+a)}^2-3}{15\,b\cos{(bx+a)}^5}$$

input
```
integrate(sec(b*x+a)^3*tan(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/15*(5*cos(b*x + a)^2 - 3)/(b*cos(b*x + a)^5)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \sec^3(a+bx)\tan^3(a+bx)\,dx = -\frac{5\,\cos\left(bx+a\right)^2-3}{15\,b\cos\left(bx+a\right)^5}$$

input
```
integrate(sec(b*x+a)^3*tan(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/15*(5*cos(b*x + a)^2 - 3)/(b*cos(b*x + a)^5)
```

Mupad [B] (verification not implemented)

Time = 25.39 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \sec^3(a+bx)\tan^3(a+bx)\,dx = -\frac{\frac{\cos(a+b\,x)^2}{3}-\frac{1}{5}}{b\cos\left(a+b\,x\right)^5}$$

input
```
int(tan(a + b*x)^3/cos(a + b*x)^3,x)
```

output
```
-(cos(a + b*x)^2/3 - 1/5)/(b*cos(a + b*x)^5)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \sec^3(a+bx)\tan^3(a+bx)\,dx = \frac{\sec\left(bx+a\right)^3\left(3\tan\left(bx+a\right)^2-2\right)}{15b}$$

input
```
int(sec(b*x+a)^3*tan(b*x+a)^3,x)
```

output
```
(sec(a + b*x)**3*(3*tan(a + b*x)**2 - 2))/(15*b)
```

3.77 $\int \sec^4(a+bx)\tan^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \sec^4(a+bx)\tan^3(a+bx)\,dx=-\frac{\sec^4(a+bx)}{4b}+\frac{\sec^6(a+bx)}{6b}$$

output
```
-1/4*sec(b*x+a)^4/b+1/6*sec(b*x+a)^6/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \sec^4(a+bx)\tan^3(a+bx)\,dx=-\frac{\sec^4(a+bx)}{4b}+\frac{\sec^6(a+bx)}{6b}$$

input
```
Integrate[Sec[a + b*x]^4*Tan[a + b*x]^3,x]
```

output
```
-1/4*Sec[a + b*x]^4/b + Sec[a + b*x]^6/(6*b)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3042, 3086, 25, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(a+bx)\sec^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(a+bx)^3\sec(a+bx)^4dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{\int -\sec^3(a+bx)\left(1-\sec^2(a+bx)\right)d\sec(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \sec^3(a+bx)\left(1-\sec^2(a+bx)\right)d\sec(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(\sec^3(a+bx)-\sec^5(a+bx)\right)d\sec(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{6}\sec^6(a+bx)-\frac{1}{4}\sec^4(a+bx)}{b}$$

input
```
Int[Sec[a + b*x]^4*Tan[a + b*x]^3,x]
```

output
```
(-1/4*Sec[a + b*x]^4 + Sec[a + b*x]^6/6)/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Maple [A] (verified)

Time = 6.68 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\frac{\frac{\left(\sec(bx+a)^2-1\right)^3}{6}+\frac{\left(\sec(bx+a)^2-1\right)^2}{4}}{b}$	34
default	$\frac{\frac{\left(\sec(bx+a)^2-1\right)^3}{6}+\frac{\left(\sec(bx+a)^2-1\right)^2}{4}}{b}$	34
risch	$-\frac{4\left(3\,\mathrm{e}^{8i(bx+a)}-2\,\mathrm{e}^{6i(bx+a)}+3\,\mathrm{e}^{4i(bx+a)}\right)}{3b\left(\mathrm{e}^{2i(bx+a)}+1\right)^6}$	53

input
```
int(sec(b*x+a)^4*tan(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/6*(sec(b*x+a)^2-1)^3+1/4*(sec(b*x+a)^2-1)^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \sec^4(a+bx)\tan^3(a+bx)\,dx = -\frac{3\,\cos\left(bx+a\right)^2-2}{12\,b\cos\left(bx+a\right)^6}$$

input
```
integrate(sec(b*x+a)^4*tan(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/12*(3*cos(b*x + a)^2 - 2)/(b*cos(b*x + a)^6)
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

$$\int \sec^4(a+bx)\tan^3(a+bx)\,dx = \begin{cases} \frac{\tan^2\left(a+bx\right)\sec^4\left(a+bx\right)}{6b} - \frac{\sec^4\left(a+bx\right)}{12b} & \text{for } b \neq 0 \\ x\tan^3\left(a\right)\sec^4\left(a\right) & \text{otherwise} \end{cases}$$

input
```
integrate(sec(b*x+a)**4*tan(b*x+a)**3,x)
```

output
```
Piecewise((tan(a + b*x)**2*sec(a + b*x)**4/(6*b) - sec(a + b*x)**4/(12*b),
 Ne(b, 0)), (x*tan(a)**3*sec(a)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.58

$$\int \sec^4(a+bx)\tan^3(a+bx)\,dx = -\frac{3\,\sin\left(bx+a\right)^2-1}{12\,\left(\sin\left(bx+a\right)^6-3\,\sin\left(bx+a\right)^4+3\,\sin\left(bx+a\right)^2-1\right)b}$$

input
```
integrate(sec(b*x+a)^4*tan(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/12*(3*sin(b*x + a)^2 - 1)/((sin(b*x + a)^6 - 3*sin(b*x + a)^4 + 3*sin(b
*x + a)^2 - 1)*b)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \sec^4(a+bx)\tan^3(a+bx)\,dx = -\frac{3\cos(bx+a)^2-2}{12\,b\cos(bx+a)^6}$$

input
```
integrate(sec(b*x+a)^4*tan(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/12*(3*cos(b*x + a)^2 - 2)/(b*cos(b*x + a)^6)
```

Mupad [B] (verification not implemented)

Time = 25.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \sec^4(a+bx)\tan^3(a+bx)\,dx = \frac{\tan(a+b\,x)^4\,\left(2\tan(a+b\,x)^2+3\right)}{12\,b}$$

input
```
int(tan(a + b*x)^3/cos(a + b*x)^4,x)
```

output
```
(tan(a + b*x)^4*(2*tan(a + b*x)^2 + 3))/(12*b)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \sec^4(a+bx)\tan^3(a+bx)\,dx = \frac{\sec(bx+a)^4\left(2\tan(bx+a)^2-1\right)}{12b}$$

input
```
int(sec(b*x+a)^4*tan(b*x+a)^3,x)
```

output
```
(sec(a + b*x)**4*(2*tan(a + b*x)**2 - 1))/(12*b)
```

3.78 $\int \sec^5(a+bx)\tan^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \sec^5(a+bx)\tan^3(a+bx)\,dx = -\frac{\sec^5(a+bx)}{5b}+\frac{\sec^7(a+bx)}{7b}$$

output
```
-1/5*sec(b*x+a)^5/b+1/7*sec(b*x+a)^7/b
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \sec^5(a+bx)\tan^3(a+bx)\,dx = -\frac{\sec^5(a+bx)}{5b}+\frac{\sec^7(a+bx)}{7b}$$

input
```
Integrate[Sec[a + b*x]^5*Tan[a + b*x]^3,x]
```

output
```
-1/5*Sec[a + b*x]^5/b + Sec[a + b*x]^7/(7*b)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3042, 3086, 25, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(a+bx)\sec^5(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(a+bx)^3\sec(a+bx)^5dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{\int -\sec^4(a+bx)\left(1-\sec^2(a+bx)\right)d\sec(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \sec^4(a+bx)\left(1-\sec^2(a+bx)\right)d\sec(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(\sec^4(a+bx)-\sec^6(a+bx)\right)d\sec(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{7}\sec^7(a+bx)-\frac{1}{5}\sec^5(a+bx)}{b}$$

input
```
Int[Sec[a + b*x]^5*Tan[a + b*x]^3,x]
```

output
```
(-1/5*Sec[a + b*x]^5 + Sec[a + b*x]^7/7)/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Maple [A] (verified)

Time = 11.60 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\frac{\sec(bx+a)^7}{7}-\frac{\sec(bx+a)^5}{5}}{b}$	26
default	$\frac{\frac{\sec(bx+a)^7}{7}-\frac{\sec(bx+a)^5}{5}}{b}$	26
risch	$-\frac{32\left(7\,\mathrm{e}^{9i(bx+a)}-6\,\mathrm{e}^{7i(bx+a)}+7\,\mathrm{e}^{5i(bx+a)}\right)}{35b\left(\mathrm{e}^{2i(bx+a)}+1\right)^7}$	53

input
```
int(sec(b*x+a)^5*tan(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/7*sec(b*x+a)^7-1/5*sec(b*x+a)^5)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \sec^5(a+bx)\tan^3(a+bx)\,dx = -\frac{7\cos(bx+a)^2-5}{35\,b\cos(bx+a)^7}$$

input
```
integrate(sec(b*x+a)^5*tan(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/35*(7*cos(b*x + a)^2 - 5)/(b*cos(b*x + a)^7)
```

Sympy [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int \sec^5(a+bx)\tan^3(a+bx)\,dx = \begin{cases} \frac{\tan^2(a+bx)\sec^5(a+bx)}{7b} - \frac{2\sec^5(a+bx)}{35b} & \text{for } b\neq 0 \\ x\tan^3(a)\sec^5(a) & \text{otherwise} \end{cases}$$

input
```
integrate(sec(b*x+a)**5*tan(b*x+a)**3,x)
```

output
```
Piecewise((tan(a + b*x)**2*sec(a + b*x)**5/(7*b) - 2*sec(a + b*x)**5/(35*b
), Ne(b, 0)), (x*tan(a)**3*sec(a)**5, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \sec^5(a+bx)\tan^3(a+bx)\,dx = -\frac{7\cos(bx+a)^2-5}{35\,b\cos(bx+a)^7}$$

input
```
integrate(sec(b*x+a)^5*tan(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/35*(7*cos(b*x + a)^2 - 5)/(b*cos(b*x + a)^7)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \sec^5(a+bx)\tan^3(a+bx)\,dx = -\frac{7\cos(bx+a)^2-5}{35\,b\cos(bx+a)^7}$$

input
```
integrate(sec(b*x+a)^5*tan(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/35*(7*cos(b*x + a)^2 - 5)/(b*cos(b*x + a)^7)
```

Mupad [B] (verification not implemented)

Time = 25.60 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \sec^5(a+bx)\tan^3(a+bx)\,dx = \frac{1}{7\,b\cos(a+b\,x)^7} - \frac{1}{5\,b\cos(a+b\,x)^5}$$

input
```
int(tan(a + b*x)^3/cos(a + b*x)^5,x)
```

output
```
1/(7*b*cos(a + b*x)^7) - 1/(5*b*cos(a + b*x)^5)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \sec^5(a+bx)\tan^3(a+bx)\,dx = \frac{\sec(bx+a)^5\,(5\tan(bx+a)^2-2)}{35b}$$

input
```
int(sec(b*x+a)^5*tan(b*x+a)^3,x)
```

output
```
(sec(a + b*x)**5*(5*tan(a + b*x)**2 - 2))/(35*b)
```

3.79 $\int \sec^6(a+bx)\tan^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \sec^6(a+bx)\tan^3(a+bx)\,dx=-\frac{\sec^6(a+bx)}{6b}+\frac{\sec^8(a+bx)}{8b}$$

output
```
-1/6*sec(b*x+a)^6/b+1/8*sec(b*x+a)^8/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \sec^6(a+bx)\tan^3(a+bx)\,dx=-\frac{\sec^6(a+bx)}{6b}+\frac{\sec^8(a+bx)}{8b}$$

input
```
Integrate[Sec[a + b*x]^6*Tan[a + b*x]^3,x]
```

output
```
-1/6*Sec[a + b*x]^6/b + Sec[a + b*x]^8/(8*b)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3042, 3086, 25, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(a+bx)\sec^6(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \tan(a+bx)^3\sec(a+bx)^6dx$$

$$\Big\downarrow \; 3086$$

$$\frac{\int -\sec^5(a+bx)\left(1-\sec^2(a+bx)\right)d\sec(a+bx)}{b}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \sec^5(a+bx)\left(1-\sec^2(a+bx)\right)d\sec(a+bx)}{b}$$

$$\Big\downarrow \; 244$$

$$-\frac{\int \left(\sec^5(a+bx)-\sec^7(a+bx)\right)d\sec(a+bx)}{b}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{1}{8}\sec^8(a+bx)-\frac{1}{6}\sec^6(a+bx)}{b}$$

input
```
Int[Sec[a + b*x]^6*Tan[a + b*x]^3,x]
```

output
```
(-1/6*Sec[a + b*x]^6 + Sec[a + b*x]^8/8)/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Maple [A] (verified)

Time = 21.75 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\frac{\frac{\left(1+\tan(bx+a)^2\right)^4}{8}-\frac{\left(1+\tan(bx+a)^2\right)^3}{6}}{b}$	34
default	$\frac{\frac{\left(1+\tan(bx+a)^2\right)^4}{8}-\frac{\left(1+\tan(bx+a)^2\right)^3}{6}}{b}$	34
risch	$-\frac{32\left(e^{10i(bx+a)}-e^{8i(bx+a)}+e^{6i(bx+a)}\right)}{3b\left(e^{2i(bx+a)}+1\right)^8}$	49

input
```
int(sec(b*x+a)^6*tan(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/8*(1+tan(b*x+a)^2)^4-1/6*(1+tan(b*x+a)^2)^3)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \sec^6(a+bx)\tan^3(a+bx)\,dx = -\frac{4\cos(bx+a)^2-3}{24\,b\cos(bx+a)^8}$$

input
```
integrate(sec(b*x+a)^6*tan(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/24*(4*cos(b*x + a)^2 - 3)/(b*cos(b*x + a)^8)
```

Sympy [A] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

$$\int \sec^6(a+bx)\tan^3(a+bx)\,dx = \begin{cases} \frac{\tan^2(a+bx)\sec^6(a+bx)}{8b} - \frac{\sec^6(a+bx)}{24b} & \text{for } b \neq 0 \\ x\tan^3(a)\sec^6(a) & \text{otherwise} \end{cases}$$

input
```
integrate(sec(b*x+a)**6*tan(b*x+a)**3,x)
```

output
```
Piecewise((tan(a + b*x)**2*sec(a + b*x)**6/(8*b) - sec(a + b*x)**6/(24*b),
 Ne(b, 0)), (x*tan(a)**3*sec(a)**6, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(27) = 54.

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.90

$$\int \sec^6(a+bx)\tan^3(a+bx)\,dx$$
$$= \frac{4\sin(bx+a)^2-1}{24\left(\sin(bx+a)^8-4\sin(bx+a)^6+6\sin(bx+a)^4-4\sin(bx+a)^2+1\right)b}$$

input
```
integrate(sec(b*x+a)^6*tan(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/24*(4*sin(b*x + a)^2 - 1)/((sin(b*x + a)^8 - 4*sin(b*x + a)^6 + 6*sin(b*
x + a)^4 - 4*sin(b*x + a)^2 + 1)*b)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \sec^6(a+bx)\tan^3(a+bx)\,dx = -\frac{4\cos(bx+a)^2-3}{24\,b\cos(bx+a)^8}$$

input
```
integrate(sec(b*x+a)^6*tan(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/24*(4*cos(b*x + a)^2 - 3)/(b*cos(b*x + a)^8)
```

Mupad [B] (verification not implemented)

Time = 25.51 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13

$$\int \sec^6(a+bx)\tan^3(a+bx)\,dx = \frac{\tan(a+b\,x)^4\,\left(3\tan(a+b\,x)^4+8\tan(a+b\,x)^2+6\right)}{24\,b}$$

input
```
int(tan(a + b*x)^3/cos(a + b*x)^6,x)
```

output
```
(tan(a + b*x)^4*(8*tan(a + b*x)^2 + 3*tan(a + b*x)^4 + 6))/(24*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \sec^6(a+bx)\tan^3(a+bx)\,dx = \frac{\sec(bx+a)^6\left(3\tan(bx+a)^2-1\right)}{24b}$$

input
```
int(sec(b*x+a)^6*tan(b*x+a)^3,x)
```

output
```
(sec(a + b*x)**6*(3*tan(a + b*x)**2 - 1))/(24*b)
```

3.80 $\int \cos^7(a+bx)\sin^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 61

$$\int \cos^7(a+bx)\sin^4(a+bx)\,dx = \frac{\sin^5(a+bx)}{5b}-\frac{3\sin^7(a+bx)}{7b}+\frac{\sin^9(a+bx)}{3b}-\frac{\sin^{11}(a+bx)}{11b}$$

output
```
1/5*sin(b*x+a)^5/b-3/7*sin(b*x+a)^7/b+1/3*sin(b*x+a)^9/b-1/11*sin(b*x+a)^1
1/b
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int \cos^7(a+bx)\sin^4(a+bx)\,dx = \frac{(3042+3335\cos(2(a+bx))+910\cos(4(a+bx))+105\cos(6(a+bx)))\sin^5(a+bx)}{36960b}$$

input
```
Integrate[Cos[a + b*x]^7*Sin[a + b*x]^4,x]
```

output
```
((3042 + 3335*Cos[2*(a + b*x)] + 910*Cos[4*(a + b*x)] + 105*Cos[6*(a + b*x
)])*Sin[a + b*x]^5)/(36960*b)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(a+bx)\cos^7(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin(a+bx)^4\cos(a+bx)^7dx$$
$$\Big\downarrow \text{3044}$$
$$\frac{\int \sin^4(a+bx)\left(1-\sin^2(a+bx)\right)^3d\sin(a+bx)}{b}$$
$$\Big\downarrow \text{244}$$
$$\frac{\int \left(-\sin^{10}(a+bx)+3\sin^8(a+bx)-3\sin^6(a+bx)+\sin^4(a+bx)\right)d\sin(a+bx)}{b}$$
$$\Big\downarrow \text{2009}$$
$$\frac{-\frac{1}{11}\sin^{11}(a+bx)+\frac{1}{3}\sin^9(a+bx)-\frac{3}{7}\sin^7(a+bx)+\frac{1}{5}\sin^5(a+bx)}{b}$$

input
```
Int[Cos[a + b*x]^7*Sin[a + b*x]^4,x]
```

output
```
(Sin[a + b*x]^5/5 - (3*Sin[a + b*x]^7)/7 + Sin[a + b*x]^9/3 - Sin[a + b*x]
^11/11)/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 54.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

method	result
derivativedivides	$-\frac{\frac{\sin(bx+a)^{11}}{11}-\frac{\sin(bx+a)^{9}}{3}+\frac{3\sin(bx+a)^{7}}{7}-\frac{\sin(bx+a)^{5}}{5}}{b}$
default	$-\frac{\frac{\sin(bx+a)^{11}}{11}-\frac{\sin(bx+a)^{9}}{3}+\frac{3\sin(bx+a)^{7}}{7}-\frac{\sin(bx+a)^{5}}{5}}{b}$
risch	$\frac{7\sin(bx+a)}{512b}+\frac{\sin(11bx+11a)}{11264b}+\frac{\sin(9bx+9a)}{3072b}-\frac{\sin(7bx+7a)}{7168b}-\frac{11\sin(5bx+5a)}{5120b}-\frac{\sin(3bx+3a)}{512b}$
parallelrisch	$\frac{\left(\sin\left(\frac{5bx}{2}+\frac{5a}{2}\right)-5\sin\left(\frac{3bx}{2}+\frac{3a}{2}\right)+10\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)(3335\cos(2bx+2a)+105\cos(6bx+6a)+910\cos(4bx+4a)+3042)\left(c\right.}{295680b}$
orering	$-\frac{14312974\left(-7\cos(bx+a)^{6}\sin(bx+a)^{5}b+4\cos(bx+a)^{8}\sin(bx+a)^{3}b\right)}{120062225b^{2}}-\frac{1997021\left(-210\cos(bx+a)^{4}\sin(bx+a)^{7}b^{3}+\right.}{}$

input
```
int(cos(b*x+a)^7*sin(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/b*(1/11*sin(b*x+a)^11-1/3*sin(b*x+a)^9+3/7*sin(b*x+a)^7-1/5*sin(b*x+a)^
5)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.03

$$\int \cos^7(a+bx)\sin^4(a+bx)\,dx$$
$$= \frac{\left(105\,\cos\left(bx+a\right)^{10}-140\,\cos\left(bx+a\right)^{8}+5\,\cos\left(bx+a\right)^{6}+6\,\cos\left(bx+a\right)^{4}+8\,\cos\left(bx+a\right)^{2}+16\right)\sin\left(\right.}{1155\,b}$$

input
```
integrate(cos(b*x+a)^7*sin(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/1155*(105*cos(b*x + a)^10 - 140*cos(b*x + a)^8 + 5*cos(b*x + a)^6 + 6*co
s(b*x + a)^4 + 8*cos(b*x + a)^2 + 16)*sin(b*x + a)/b
```

Sympy [A] (verification not implemented)

Time = 1.76 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.44

$$\int \cos^7(a+bx)\sin^4(a+bx)\,dx$$
$$= \begin{cases} \frac{16\sin^{11}(a+bx)}{1155b}+\frac{8\sin^9(a+bx)\cos^2(a+bx)}{105b}+\frac{6\sin^7(a+bx)\cos^4(a+bx)}{35b}+\frac{\sin^5(a+bx)\cos^6(a+bx)}{5b} & \text{for } b\neq 0 \\ x\sin^4(a)\cos^7(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a)**7*sin(b*x+a)**4,x)
```

output
```
Piecewise((16*sin(a + b*x)**11/(1155*b) + 8*sin(a + b*x)**9*cos(a + b*x)**
2/(105*b) + 6*sin(a + b*x)**7*cos(a + b*x)**4/(35*b) + sin(a + b*x)**5*cos
(a + b*x)**6/(5*b), Ne(b, 0)), (x*sin(a)**4*cos(a)**7, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \cos^7(a+bx)\sin^4(a+bx)\,dx = -\frac{105\,\sin\left(bx+a\right)^{11}-385\,\sin\left(bx+a\right)^{9}+495\,\sin\left(bx+a\right)^{7}-231\,\sin\left(bx+a\right)^{5}}{1155\,b}$$

input
```
integrate(cos(b*x+a)^7*sin(b*x+a)^4,x, algorithm="maxima")
```

output
```
-1/1155*(105*sin(b*x + a)^11 - 385*sin(b*x + a)^9 + 495*sin(b*x + a)^7 - 2
31*sin(b*x + a)^5)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \cos^7(a+bx)\sin^4(a+bx)\,dx = -\frac{105\,\sin\left(bx+a\right)^{11}-385\,\sin\left(bx+a\right)^{9}+495\,\sin\left(bx+a\right)^{7}-231\,\sin\left(bx+a\right)^{5}}{1155\,b}$$

input
```
integrate(cos(b*x+a)^7*sin(b*x+a)^4,x, algorithm="giac")
```

output
```
-1/1155*(105*sin(b*x + a)^11 - 385*sin(b*x + a)^9 + 495*sin(b*x + a)^7 - 2
31*sin(b*x + a)^5)/b
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \cos^7(a+bx)\sin^4(a+bx)\,dx = \frac{-\frac{\sin(a+b\,x)^{11}}{11}+\frac{\sin(a+b\,x)^9}{3}-\frac{3\sin(a+b\,x)^7}{7}+\frac{\sin(a+b\,x)^5}{5}}{b}$$

input
```
int(cos(a + b*x)^7*sin(a + b*x)^4,x)
```

output
```
(sin(a + b*x)^5/5 - (3*sin(a + b*x)^7)/7 + sin(a + b*x)^9/3 - sin(a + b*x)
^11/11)/b
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \cos^7(a+bx)\sin^4(a+bx)\,dx = \frac{\sin(bx+a)^5\left(-105\sin(bx+a)^6+385\sin(bx+a)^4-495\sin(bx+a)^2+231\right)}{1155b}$$

input
```
int(cos(b*x+a)^7*sin(b*x+a)^4,x)
```

output
```
(sin(a + b*x)**5*( - 105*sin(a + b*x)**6 + 385*sin(a + b*x)**4 - 495*sin(a
 + b*x)**2 + 231))/(1155*b)
```

3.81 $\int \cos^5(a+bx)\sin^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \cos^5(a+bx)\sin^4(a+bx)\,dx = \frac{\sin^5(a+bx)}{5b} - \frac{2\sin^7(a+bx)}{7b} + \frac{\sin^9(a+bx)}{9b}$$

output
```
1/5*sin(b*x+a)^5/b-2/7*sin(b*x+a)^7/b+1/9*sin(b*x+a)^9/b
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \cos^5(a+bx)\sin^4(a+bx)\,dx = \frac{(249+220\cos(2(a+bx))+35\cos(4(a+bx)))\sin^5(a+bx)}{2520b}$$

input
```
Integrate[Cos[a + b*x]^5*Sin[a + b*x]^4,x]
```

output
```
((249 + 220*Cos[2*(a + b*x)] + 35*Cos[4*(a + b*x)])*Sin[a + b*x]^5)/(2520*
b)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(a+bx)\cos^5(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^4\cos(a+bx)^5dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{\int \sin^4(a+bx)\left(1-\sin^2(a+bx)\right)^2d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\sin^8(a+bx)-2\sin^6(a+bx)+\sin^4(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{9}\sin^9(a+bx)-\frac{2}{7}\sin^7(a+bx)+\frac{1}{5}\sin^5(a+bx)}{b}$$

input
```
Int[Cos[a + b*x]^5*Sin[a + b*x]^4,x]
```

output
```
(Sin[a + b*x]^5/5 - (2*Sin[a + b*x]^7)/7 + Sin[a + b*x]^9/9)/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 21.33 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{\frac{\sin(bx+a)^9}{9}-\frac{2\sin(bx+a)^7}{7}+\frac{\sin(bx+a)^5}{5}}{b}$
default	$\frac{\frac{\sin(bx+a)^9}{9}-\frac{2\sin(bx+a)^7}{7}+\frac{\sin(bx+a)^5}{5}}{b}$
risch	$\frac{3\sin(bx+a)}{128b}+\frac{\sin(9bx+9a)}{2304b}+\frac{\sin(7bx+7a)}{1792b}-\frac{\sin(5bx+5a)}{320b}-\frac{\sin(3bx+3a)}{192b}$
parallelrisch	$\frac{\left(\sin\left(\frac{5bx}{2}+\frac{5a}{2}\right)-5\sin\left(\frac{3bx}{2}+\frac{3a}{2}\right)+10\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)(249+35\cos(4bx+4a)+220\cos(2bx+2a))\left(\cos\left(\frac{5bx}{2}+\frac{5a}{2}\right)+5\cos\left(\right.\right.}{20160b}$
norman	$\frac{\frac{32\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^5}{5b}-\frac{384\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^7}{35b}+\frac{6976\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^9}{315b}-\frac{384\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{11}}{35b}+\frac{32\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{13}}{5b}}{\left(1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right)^9}$
orering	$-\frac{117469\left(-5\cos(bx+a)^4\sin(bx+a)^5b+4\cos(bx+a)^6\sin(bx+a)^3b\right)}{99225b^2}-\frac{34562\left(-60\cos(bx+a)^2\sin(bx+a)^7b^3+365\cos\right.}{}$

input
```
int(cos(b*x+a)^5*sin(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/9*sin(b*x+a)^9-2/7*sin(b*x+a)^7+1/5*sin(b*x+a)^5)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.15

$$\int \cos^5(a+bx)\sin^4(a+bx)\,dx = \frac{\left(35\,\cos\left(bx+a\right)^{8}-50\,\cos\left(bx+a\right)^{6}+3\,\cos\left(bx+a\right)^{4}+4\,\cos\left(bx+a\right)^{2}+8\right)\sin\left(bx+a\right)}{315\,b}$$

input
```
integrate(cos(b*x+a)^5*sin(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/315*(35*cos(b*x + a)^8 - 50*cos(b*x + a)^6 + 3*cos(b*x + a)^4 + 4*cos(b*
x + a)^2 + 8)*sin(b*x + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.43

$$\int \cos^5(a+bx)\sin^4(a+bx)\,dx = \begin{cases} \frac{8\sin^9{(a+bx)}}{315b}+\frac{4\sin^7{(a+bx)}\cos^2{(a+bx)}}{35b}+\frac{\sin^5{(a+bx)}\cos^4{(a+bx)}}{5b} & \text{for}\ b\neq 0 \\ x\sin^4{(a)}\cos^5{(a)} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a)**5*sin(b*x+a)**4,x)
```

output
```
Piecewise((8*sin(a + b*x)**9/(315*b) + 4*sin(a + b*x)**7*cos(a + b*x)**2/(
35*b) + sin(a + b*x)**5*cos(a + b*x)**4/(5*b), Ne(b, 0)), (x*sin(a)**4*cos
(a)**5, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^5(a+bx)\sin^4(a+bx)\,dx = \frac{35\,\sin(bx+a)^9 - 90\,\sin(bx+a)^7 + 63\,\sin(bx+a)^5}{315\,b}$$

input
```
integrate(cos(b*x+a)^5*sin(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/315*(35*sin(b*x + a)^9 - 90*sin(b*x + a)^7 + 63*sin(b*x + a)^5)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^5(a+bx)\sin^4(a+bx)\,dx = \frac{35\,\sin(bx+a)^9 - 90\,\sin(bx+a)^7 + 63\,\sin(bx+a)^5}{315\,b}$$

input
```
integrate(cos(b*x+a)^5*sin(b*x+a)^4,x, algorithm="giac")
```

output
```
1/315*(35*sin(b*x + a)^9 - 90*sin(b*x + a)^7 + 63*sin(b*x + a)^5)/b
```

Mupad [B] (verification not implemented)

Time = 25.44 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^5(a+bx)\sin^4(a+bx)\,dx = \frac{35\sin(a+b\,x)^9 - 90\sin(a+b\,x)^7 + 63\sin(a+b\,x)^5}{315\,b}$$

input
```
int(cos(a + b*x)^5*sin(a + b*x)^4,x)
```

output
```
(63*sin(a + b*x)^5 - 90*sin(a + b*x)^7 + 35*sin(a + b*x)^9)/(315*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \cos^5(a+bx)\sin^4(a+bx)\,dx = \frac{\sin(bx+a)^5\left(35\sin(bx+a)^4-90\sin(bx+a)^2+63\right)}{315b}$$

input
```
int(cos(b*x+a)^5*sin(b*x+a)^4,x)
```

output
```
(sin(a + b*x)**5*(35*sin(a + b*x)**4 - 90*sin(a + b*x)**2 + 63))/(315*b)
```

3.82 $\int \cos^3(a+bx)\sin^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \cos^3(a+bx)\sin^4(a+bx)\,dx = \frac{\sin^5(a+bx)}{5b} - \frac{\sin^7(a+bx)}{7b}$$

output
```
1/5*sin(b*x+a)^5/b-1/7*sin(b*x+a)^7/b
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \cos^3(a+bx)\sin^4(a+bx)\,dx = \frac{(9+5\cos(2(a+bx)))\sin^5(a+bx)}{70b}$$

input
```
Integrate[Cos[a + b*x]^3*Sin[a + b*x]^4,x]
```

output
```
((9 + 5*Cos[2*(a + b*x)])*Sin[a + b*x]^5)/(70*b)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(a+bx)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin(a+bx)^4\cos(a+bx)^3dx$$

$$\Big\downarrow \; 3044$$

$$\frac{\int \sin^4(a+bx)\left(1-\sin^2(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \; 244$$

$$\frac{\int \left(\sin^4(a+bx)-\sin^6(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{1}{5}\sin^5(a+bx)-\frac{1}{7}\sin^7(a+bx)}{b}$$

input
```
Int[Cos[a + b*x]^3*Sin[a + b*x]^4,x]
```

output
```
(Sin[a + b*x]^5/5 - Sin[a + b*x]^7/7)/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 10.42 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{-\frac{\sin(bx+a)^7}{7}+\frac{\sin(bx+a)^5}{5}}{b}$
default	$\frac{-\frac{\sin(bx+a)^7}{7}+\frac{\sin(bx+a)^5}{5}}{b}$
risch	$\frac{3\sin(bx+a)}{64b}+\frac{\sin(7bx+7a)}{448b}-\frac{\sin(5bx+5a)}{320b}-\frac{\sin(3bx+3a)}{64b}$
norman	$\frac{\frac{32\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^5}{5b}-\frac{192\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^7}{35b}+\frac{32\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^9}{5b}}{\left(1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right)^7}$
parallelrisch	$\frac{\left(\sin\left(\frac{5bx}{2}+\frac{5a}{2}\right)-5\sin\left(\frac{3bx}{2}+\frac{3a}{2}\right)+10\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\right)(9+5\cos(2bx+2a))\left(\cos\left(\frac{5bx}{2}+\frac{5a}{2}\right)+5\cos\left(\frac{3bx}{2}+\frac{3a}{2}\right)+10\cos\left(\frac{bx}{2}+\right.\right.}{560b}$
orering	$-\frac{12916\left(-3\cos(bx+a)^2\sin(bx+a)^5b+4\cos(bx+a)^4\sin(bx+a)^3b\right)}{11025b^2}-\frac{94\left(-6b^3\sin(bx+a)^7+129\cos(bx+a)^2\sin(bx+\right.}{}$

input
```
int(cos(b*x+a)^3*sin(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/7*sin(b*x+a)^7+1/5*sin(b*x+a)^5)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \cos^3(a+bx)\sin^4(a+bx)\,dx = \frac{\left(5\,\cos\left(bx+a\right)^6-8\,\cos\left(bx+a\right)^4+\cos\left(bx+a\right)^2+2\right)\sin\left(bx+a\right)}{35\,b}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/35*(5*cos(b*x + a)^6 - 8*cos(b*x + a)^4 + cos(b*x + a)^2 + 2)*sin(b*x +
a)/b
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int \cos^3(a+bx)\sin^4(a+bx)\,dx = \begin{cases} \frac{2\sin^7(a+bx)}{35b} + \frac{\sin^5(a+bx)\cos^2(a+bx)}{5b} & \text{for}\ b \neq 0 \\ x\sin^4(a)\cos^3(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a)**3*sin(b*x+a)**4,x)
```

output
```
Piecewise((2*sin(a + b*x)**7/(35*b) + sin(a + b*x)**5*cos(a + b*x)**2/(5*b
), Ne(b, 0)), (x*sin(a)**4*cos(a)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^3(a+bx)\sin^4(a+bx)\,dx = -\frac{5\,\sin(bx+a)^7 - 7\,\sin(bx+a)^5}{35\,b}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^4,x, algorithm="maxima")
```

output
```
-1/35*(5*sin(b*x + a)^7 - 7*sin(b*x + a)^5)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^3(a+bx)\sin^4(a+bx)\,dx = -\frac{5\,\sin(bx+a)^7 - 7\,\sin(bx+a)^5}{35\,b}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^4,x, algorithm="giac")
```

output
```
-1/35*(5*sin(b*x + a)^7 - 7*sin(b*x + a)^5)/b
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^3(a+bx)\sin^4(a+bx)\,dx = \frac{7\sin(a+b\,x)^5 - 5\sin(a+b\,x)^7}{35\,b}$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)^4,x)
```

output
```
(7*sin(a + b*x)^5 - 5*sin(a + b*x)^7)/(35*b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \cos^3(a+bx)\sin^4(a+bx)\,dx = \frac{\sin\left(bx+a\right)^5\left(-5\sin\left(bx+a\right)^2+7\right)}{35b}$$

input
```
int(cos(b*x+a)^3*sin(b*x+a)^4,x)
```

output
```
(sin(a + b*x)**5*( - 5*sin(a + b*x)**2 + 7))/(35*b)
```

3.83 $\int \cos(a+bx)\sin^4(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \cos(a+bx)\sin^4(a+bx)\,dx = \frac{\sin^5(a+bx)}{5b}$$

output
```
1/5*sin(b*x+a)^5/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cos(a+bx)\sin^4(a+bx)\,dx = \frac{\sin^5(a+bx)}{5b}$$

input
```
Integrate[Cos[a + b*x]*Sin[a + b*x]^4,x]
```

output
```
Sin[a + b*x]^5/(5*b)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3044, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(a+bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^4\cos(a+bx)dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{\int \sin^4(a+bx)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{\sin^5(a+bx)}{5b}$$

input
```
Int[Cos[a + b*x]*Sin[a + b*x]^4,x]
```

output
```
Sin[a + b*x]^5/(5*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 3.89 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{\sin(bx+a)^5}{5b}$
default	$\frac{\sin(bx+a)^5}{5b}$
norman	$\frac{32\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^5}{5b\left(1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right)^5}$
parallelrisch	$\frac{10\sin(bx+a)-5\sin(3bx+3a)+\sin(5bx+5a)}{80b}$
risch	$\frac{\sin(bx+a)}{8b}+\frac{\sin(5bx+5a)}{80b}-\frac{\sin(3bx+3a)}{16b}$
orering	$-\frac{259\left(-b\sin(bx+a)^5+4\cos(bx+a)^2\sin(bx+a)^3b\right)}{225b^2}-\frac{7\left(-88b^3\sin(bx+a)^3\cos(bx+a)^2+13b^3\sin(bx+a)^5+24\cos(ba\right.}{45b^4}$

input
```
int(cos(b*x+a)*sin(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/5*sin(b*x+a)^5/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(13) = 26.

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \cos(a+bx)\sin^4(a+bx)\,dx = \frac{\left(\cos\left(bx+a\right)^4-2\,\cos\left(bx+a\right)^2+1\right)\sin\left(bx+a\right)}{5\,b}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/5*(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sin(b*x + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \cos(a+bx)\sin^4(a+bx)\,dx = \begin{cases} \frac{\sin^5(a+bx)}{5b} & \text{for } b\neq 0 \\ x\sin^4(a)\cos(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)**4,x)
```

output
```
Piecewise((sin(a + b*x)**5/(5*b), Ne(b, 0)), (x*sin(a)**4*cos(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\sin^4(a+bx)\,dx = \frac{\sin(bx+a)^5}{5\,b}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/5*sin(b*x + a)^5/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\sin^4(a+bx)\,dx = \frac{\sin(bx+a)^5}{5\,b}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^4,x, algorithm="giac")
```

output
```
1/5*sin(b*x + a)^5/b
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\sin^4(a+bx)\,dx = \frac{\sin\left(a+b\,x\right)^5}{5\,b}$$

input
```
int(cos(a + b*x)*sin(a + b*x)^4,x)
```

output
```
sin(a + b*x)^5/(5*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\sin^4(a+bx)\,dx = \frac{\sin\left(bx+a\right)^5}{5b}$$

input
```
int(cos(b*x+a)*sin(b*x+a)^4,x)
```

output
```
sin(a + b*x)**5/(5*b)
```

3.84 $\int \sin^2(a+bx)\tan^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 35

$$\int \sin^2(a+bx)\tan^2(a+bx)\,dx = -\frac{3x}{2}+\frac{\cos(a+bx)\sin(a+bx)}{2b}+\frac{\tan(a+bx)}{b}$$

output
```
-3/2*x+1/2*cos(b*x+a)*sin(b*x+a)/b+tan(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \sin^2(a+bx)\tan^2(a+bx)\,dx = \frac{-6(a+bx)+\sin(2(a+bx))+4\tan(a+bx)}{4b}$$

input
```
Integrate[Sin[a + b*x]^2*Tan[a + b*x]^2,x]
```

output
```
(-6*(a + b*x) + Sin[2*(a + b*x)] + 4*Tan[a + b*x])/(4*b)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.40, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3042, 3071, 252, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\tan^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^2\tan(a+bx)^2dx$$

$$\Big\downarrow \text{3071}$$

$$\frac{\int \frac{\tan^4(a+bx)}{(\tan^2(a+bx)+1)^2}d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{3}{2}\int \frac{\tan^2(a+bx)}{\tan^2(a+bx)+1}d\tan(a+bx)-\frac{\tan^3(a+bx)}{2(\tan^2(a+bx)+1)}}{b}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{3}{2}\left(\tan(a+bx)-\int \frac{1}{\tan^2(a+bx)+1}d\tan(a+bx)\right)-\frac{\tan^3(a+bx)}{2(\tan^2(a+bx)+1)}}{b}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{3}{2}(\tan(a+bx)-\arctan(\tan(a+bx)))-\frac{\tan^3(a+bx)}{2(\tan^2(a+bx)+1)}}{b}$$

input
```
Int[Sin[a + b*x]^2*Tan[a + b*x]^2,x]
```

output
```
((3*(-ArcTan[Tan[a + b*x]] + Tan[a + b*x]))/2 - Tan[a + b*x]^3/(2*(1 + Tan
[a + b*x]^2)))/b
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 2.33 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.54

method	result	size
derivativedivides	$\frac{\frac{\sin(bx+a)^5}{\cos(bx+a)}+\left(\sin(bx+a)^3+\frac{3\sin(bx+a)}{2}\right)\cos(bx+a)-\frac{3bx}{2}-\frac{3a}{2}}{b}$	54
default	$\frac{\frac{\sin(bx+a)^5}{\cos(bx+a)}+\left(\sin(bx+a)^3+\frac{3\sin(bx+a)}{2}\right)\cos(bx+a)-\frac{3bx}{2}-\frac{3a}{2}}{b}$	54
risch	$-\frac{3x}{2}-\frac{i e^{2i(bx+a)}}{8b}+\frac{i e^{-2i(bx+a)}}{8b}+\frac{2i}{b\left(e^{2i(bx+a)}+1\right)}$	54

input
```
int(sin(b*x+a)^2*tan(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(sin(b*x+a)^5/cos(b*x+a)+(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-3/2*
b*x-3/2*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int \sin^2(a+bx)\tan^2(a+bx)\,dx = -\frac{3\,bx\cos(bx+a)-\left(\cos(bx+a)^2+2\right)\sin(bx+a)}{2\,b\cos(bx+a)}$$

input
```
integrate(sin(b*x+a)^2*tan(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(3*b*x*cos(b*x + a) - (cos(b*x + a)^2 + 2)*sin(b*x + a))/(b*cos(b*x +
 a))
```

Sympy [F]

$$\int \sin^2(a+bx)\tan^2(a+bx)\,dx = \int \sin^2(a+bx)\tan^2(a+bx)\,dx$$

input
```
integrate(sin(b*x+a)**2*tan(b*x+a)**2,x)
```

output
```
Integral(sin(a + b*x)**2*tan(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

$$\int \sin^2(a+bx)\tan^2(a+bx)\,dx = -\frac{3\,bx + 3\,a - \frac{\tan(bx+a)}{\tan(bx+a)^2+1} - 2\,\tan(bx+a)}{2\,b}$$

input
```
integrate(sin(b*x+a)^2*tan(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(3*b*x + 3*a - tan(b*x + a)/(tan(b*x + a)^2 + 1) - 2*tan(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int \sin^2(a+bx)\tan^2(a+bx)\,dx = -\frac{3\,(bx+a)}{2\,b} + \frac{\tan(bx+a)}{b} + \frac{\tan(bx+a)}{2\,\left(\tan(bx+a)^2+1\right)b}$$

input
```
integrate(sin(b*x+a)^2*tan(b*x+a)^2,x, algorithm="giac")
```

output
```
-3/2*(b*x + a)/b + tan(b*x + a)/b + 1/2*tan(b*x + a)/((tan(b*x + a)^2 + 1)
*b)
```

Mupad [B] (verification not implemented)

Time = 25.41 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int \sin^2(a+bx)\tan^2(a+bx)\,dx = \frac{\frac{\cos(a+b\,x)\,\sin(a+b\,x)}{2}+\frac{\sin(a+b\,x)}{\cos(a+b\,x)}}{b}-\frac{3\,x}{2}$$

input
```
int(sin(a + b*x)^2*tan(a + b*x)^2,x)
```

output
```
((cos(a + b*x)*sin(a + b*x))/2 + sin(a + b*x)/cos(a + b*x))/b - (3*x)/2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.46

$$\int \sin^2(a+bx)\tan^2(a+bx)\,dx = \frac{-3\cos(bx+a)\,a-3\cos(bx+a)\,bx-\sin(bx+a)^3+3\sin(bx+a)}{2\cos(bx+a)\,b}$$

input
```
int(sin(b*x+a)^2*tan(b*x+a)^2,x)
```

output
```
( - 3*cos(a + b*x)*a - 3*cos(a + b*x)*b*x - sin(a + b*x)**3 + 3*sin(a + b*
x))/(2*cos(a + b*x)*b)
```

3.85 $\int \tan^4(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 28

$$\int \tan^4(a+bx)\,dx = x - \frac{\tan(a+bx)}{b} + \frac{\tan^3(a+bx)}{3b}$$

output
```
x-tan(b*x+a)/b+1/3*tan(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \tan^4(a+bx)\,dx = \frac{\arctan(\tan(a+bx))}{b} - \frac{\tan(a+bx)}{b} + \frac{\tan^3(a+bx)}{3b}$$

input
```
Integrate[Tan[a + b*x]^4,x]
```

output
```
ArcTan[Tan[a + b*x]]/b - Tan[a + b*x]/b + Tan[a + b*x]^3/(3*b)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(a+bx)^4dx$$

$$\Big\downarrow \text{ 3954}$$

$$\frac{\tan^3(a+bx)}{3b}-\int \tan^2(a+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\tan^3(a+bx)}{3b}-\int \tan(a+bx)^2dx$$

$$\Big\downarrow \text{ 3954}$$

$$\int 1dx+\frac{\tan^3(a+bx)}{3b}-\frac{\tan(a+bx)}{b}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\tan^3(a+bx)}{3b}-\frac{\tan(a+bx)}{b}+x$$

input
```
Int[Tan[a + b*x]^4,x]
```

output
```
x - Tan[a + b*x]/b + Tan[a + b*x]^3/(3*b)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

method	result	size
norman	$x-\frac{\tan(bx+a)}{b}+\frac{\tan(bx+a)^{3}}{3b}$	27
parallelrisch	$\frac{\tan(bx+a)^{3}+3bx-3\tan(bx+a)}{3b}$	27
derivativedivides	$\frac{\frac{\tan(bx+a)^{3}}{3}-\tan(bx+a)+\arctan(\tan(bx+a))}{b}$	31
default	$\frac{\frac{\tan(bx+a)^{3}}{3}-\tan(bx+a)+\arctan(\tan(bx+a))}{b}$	31
risch	$x-\frac{4i\left(3\,e^{4i(bx+a)}+3\,e^{2i(bx+a)}+2\right)}{3b\left(e^{2i(bx+a)}+1\right)^{3}}$	46

input
```
int(tan(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
x-tan(b*x+a)/b+1/3*tan(b*x+a)^3/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \tan^4(a+bx)\,dx = \frac{\tan(bx+a)^3 + 3\,bx - 3\,\tan(bx+a)}{3\,b}$$

input
```
integrate(tan(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/3*(tan(b*x + a)^3 + 3*b*x - 3*tan(b*x + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \tan^4(a+bx)\,dx = \begin{cases} x + \frac{\tan^3(a+bx)}{3b} - \frac{\tan(a+bx)}{b} & \text{for } b \neq 0 \\ x\tan^4(a) & \text{otherwise} \end{cases}$$

input
```
integrate(tan(b*x+a)**4,x)
```

output
```
Piecewise((x + tan(a + b*x)**3/(3*b) - tan(a + b*x)/b, Ne(b, 0)), (x*tan(a
)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \tan^4(a+bx)\,dx = \frac{\tan(bx+a)^3 + 3\,bx + 3\,a - 3\,\tan(bx+a)}{3\,b}$$

input
```
integrate(tan(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/3*(tan(b*x + a)^3 + 3*b*x + 3*a - 3*tan(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \tan^4(a+bx)\,dx = \frac{bx+a}{b} + \frac{b^2\tan{(bx+a)}^3 - 3\,b^2\tan{(bx+a)}}{3\,b^3}$$

input
```
integrate(tan(b*x+a)^4,x, algorithm="giac")
```

output
```
(b*x + a)/b + 1/3*(b^2*tan(b*x + a)^3 - 3*b^2*tan(b*x + a))/b^3
```

Mupad [B] (verification not implemented)

Time = 25.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \tan^4(a+bx)\,dx = x - \frac{\tan(a+b\,x) - \frac{\tan(a+b\,x)^3}{3}}{b}$$

input
```
int(tan(a + b*x)^4,x)
```

output
```
x - (tan(a + b*x) - tan(a + b*x)^3/3)/b
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \tan^4(a+bx)\,dx = \frac{\tan{(bx+a)}^3 - 3\tan{(bx+a)} + 3bx}{3b}$$

input
```
int(tan(b*x+a)^4,x)
```

output
```
(tan(a + b*x)**3 - 3*tan(a + b*x) + 3*b*x)/(3*b)
```

3.86 $\int \sec^2(a+bx)\tan^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 15

$$\int \sec^2(a+bx)\tan^4(a+bx)\,dx=\frac{\tan^5(a+bx)}{5b}$$

output
```
1/5*tan(b*x+a)^5/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \sec^2(a+bx)\tan^4(a+bx)\,dx=\frac{\tan^5(a+bx)}{5b}$$

input
```
Integrate[Sec[a + b*x]^2*Tan[a + b*x]^4,x]
```

output
```
Tan[a + b*x]^5/(5*b)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3042, 3087, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(a+bx)\sec^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(a+bx)^4\sec(a+bx)^2dx$$

$$\Big\downarrow \text{3087}$$

$$\frac{\int \tan^4(a+bx)d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{\tan^5(a+bx)}{5b}$$

input
```
Int[Sec[a + b*x]^2*Tan[a + b*x]^4,x]
```

output
```
Tan[a + b*x]^5/(5*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\tan(bx+a)^5}{5b}$	14
default	$\frac{\tan(bx+a)^5}{5b}$	14
risch	$\frac{2i\left(5\,\mathrm{e}^{8i(bx+a)}+10\,\mathrm{e}^{4i(bx+a)}+1\right)}{5b\left(\mathrm{e}^{2i(bx+a)}+1\right)^5}$	44

input
```
int(sec(b*x+a)^2*tan(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/5*tan(b*x+a)^5/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(13) = 26.

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.60

$$\int \sec^2(a+bx)\tan^4(a+bx)\,dx=\frac{\left(\cos\left(bx+a\right)^4-2\,\cos\left(bx+a\right)^2+1\right)\sin\left(bx+a\right)}{5\,b\cos\left(bx+a\right)^5}$$

input
```
integrate(sec(b*x+a)^2*tan(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/5*(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sin(b*x + a)/(b*cos(b*x + a)^5
)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(10) = 20$.

Time = 0.71 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \sec^2(a+bx)\tan^4(a+bx)\,dx = \begin{cases} \frac{\tan^5(a+bx)}{5b} & \text{for } b \neq 0 \\ x\tan^4(a)\sec^2(a) & \text{otherwise} \end{cases}$$

input
```
integrate(sec(b*x+a)**2*tan(b*x+a)**4,x)
```

output
```
Piecewise((tan(a + b*x)**5/(5*b), Ne(b, 0)), (x*tan(a)**4*sec(a)**2, True)
)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sec^2(a+bx)\tan^4(a+bx)\,dx = \frac{\tan(bx+a)^5}{5\,b}$$

input
```
integrate(sec(b*x+a)^2*tan(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/5*tan(b*x + a)^5/b
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sec^2(a+bx)\tan^4(a+bx)\,dx = \frac{\tan(bx+a)^5}{5\,b}$$

input
```
integrate(sec(b*x+a)^2*tan(b*x+a)^4,x, algorithm="giac")
```

output
```
1/5*tan(b*x + a)^5/b
```

Mupad [B] (verification not implemented)

Time = 25.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sec^2(a+bx)\tan^4(a+bx)\,dx = \frac{\tan(a+b\,x)^5}{5\,b}$$

input
```
int(tan(a + b*x)^4/cos(a + b*x)^2,x)
```

output
```
tan(a + b*x)^5/(5*b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.87

$$\int \sec^2(a+bx)\tan^4(a+bx)\,dx = \frac{\sin(bx+a)^5}{5\cos(bx+a)\,b\left(\sin(bx+a)^4-2\sin(bx+a)^2+1\right)}$$

input
```
int(sec(b*x+a)^2*tan(b*x+a)^4,x)
```

output
```
sin(a + b*x)**5/(5*cos(a + b*x)*b*(sin(a + b*x)**4 - 2*sin(a + b*x)**2 + 1
))
```

3.87 $\int \sec^4(a+bx)\tan^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \sec^4(a+bx)\tan^4(a+bx)\,dx = \frac{\tan^5(a+bx)}{5b}+\frac{\tan^7(a+bx)}{7b}$$

output
```
1/5*tan(b*x+a)^5/b+1/7*tan(b*x+a)^7/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 77 vs. 2(31) = 62.

Time = 0.19 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.48

$$\int \sec^4(a+bx)\tan^4(a+bx)\,dx = \frac{2\tan(a+bx)}{35b}+\frac{\sec^2(a+bx)\tan(a+bx)}{35b} - \frac{8\sec^4(a+bx)\tan(a+bx)}{35b} + \frac{\sec^6(a+bx)\tan(a+bx)}{7b}$$

input
```
Integrate[Sec[a + b*x]^4*Tan[a + b*x]^4,x]
```

output
```
(2*Tan[a + b*x])/(35*b) + (Sec[a + b*x]^2*Tan[a + b*x])/(35*b) - (8*Sec[a
+ b*x]^4*Tan[a + b*x])/(35*b) + (Sec[a + b*x]^6*Tan[a + b*x])/(7*b)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(a+bx)\sec^4(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(a+bx)^4\sec(a+bx)^4dx$$

$$\Big\downarrow \text{ 3087}$$

$$\frac{\int \tan^4(a+bx)\left(\tan^2(a+bx)+1\right)d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{\int \left(\tan^6(a+bx)+\tan^4(a+bx)\right)d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{7}\tan^7(a+bx)+\frac{1}{5}\tan^5(a+bx)}{b}$$

input
```
Int[Sec[a + b*x]^4*Tan[a + b*x]^4,x]
```

output
```
(Tan[a + b*x]^5/5 + Tan[a + b*x]^7/7)/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

Maple [A] (verified)

Time = 4.69 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\frac{\tan(bx+a)^7}{7}+\frac{\tan(bx+a)^5}{5}}{b}$	26
default	$\frac{\frac{\tan(bx+a)^7}{7}+\frac{\tan(bx+a)^5}{5}}{b}$	26
risch	$\frac{4i\left(35\,\mathrm{e}^{10i(bx+a)}-35\,\mathrm{e}^{8i(bx+a)}+70\,\mathrm{e}^{6i(bx+a)}-14\,\mathrm{e}^{4i(bx+a)}+7\,\mathrm{e}^{2i(bx+a)}+1\right)}{35b\left(\mathrm{e}^{2i(bx+a)}+1\right)^7}$	77

input
```
int(sec(b*x+a)^4*tan(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/7*tan(b*x+a)^7+1/5*tan(b*x+a)^5)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.58

$$\int \sec^4(a+bx)\tan^4(a+bx)\,dx = \frac{\left(2\cos\left(bx+a\right)^6+\cos\left(bx+a\right)^4-8\cos\left(bx+a\right)^2+5\right)\sin\left(bx+a\right)}{35\,b\cos\left(bx+a\right)^7}$$

input
```
integrate(sec(b*x+a)^4*tan(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/35*(2*cos(b*x + a)^6 + cos(b*x + a)^4 - 8*cos(b*x + a)^2 + 5)*sin(b*x +
a)/(b*cos(b*x + a)^7)
```

Sympy [F]

$$\int \sec^4(a+bx)\tan^4(a+bx)\,dx = \int \tan^4\left(a+bx\right)\sec^4\left(a+bx\right)\,dx$$

input
```
integrate(sec(b*x+a)**4*tan(b*x+a)**4,x)
```

output
```
Integral(tan(a + b*x)**4*sec(a + b*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \sec^4(a+bx)\tan^4(a+bx)\,dx = \frac{5\tan\left(bx+a\right)^7+7\tan\left(bx+a\right)^5}{35\,b}$$

input
```
integrate(sec(b*x+a)^4*tan(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/35*(5*tan(b*x + a)^7 + 7*tan(b*x + a)^5)/b
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \sec^4(a+bx)\tan^4(a+bx)\,dx = \frac{5\,\tan\left(bx+a\right)^7+7\,\tan\left(bx+a\right)^5}{35\,b}$$

input
```
integrate(sec(b*x+a)^4*tan(b*x+a)^4,x, algorithm="giac")
```

output
```
1/35*(5*tan(b*x + a)^7 + 7*tan(b*x + a)^5)/b
```

Mupad [B] (verification not implemented)

Time = 25.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \sec^4(a+bx)\tan^4(a+bx)\,dx = \frac{\tan(a+b\,x)^5\,\left(5\tan(a+b\,x)^2+7\right)}{35\,b}$$

input
```
int(tan(a + b*x)^4/cos(a + b*x)^4,x)
```

output
```
(tan(a + b*x)^5*(5*tan(a + b*x)^2 + 7))/(35*b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.10

$$\int \sec^4(a+bx)\tan^4(a+bx)\,dx$$
$$= \frac{\sin\left(bx+a\right)^5\left(2\sin\left(bx+a\right)^2-7\right)}{35\cos\left(bx+a\right)b\left(\sin\left(bx+a\right)^6-3\sin\left(bx+a\right)^4+3\sin\left(bx+a\right)^2-1\right)}$$

input
```
int(sec(b*x+a)^4*tan(b*x+a)^4,x)
```

output
```
(sin(a + b*x)**5*(2*sin(a + b*x)**2 - 7))/(35*cos(a + b*x)*b*(sin(a + b*x)
**6 - 3*sin(a + b*x)**4 + 3*sin(a + b*x)**2 - 1))
```

3.88 $\int \sec^6(a+bx)\tan^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \sec^6(a+bx)\tan^4(a+bx)\,dx=\frac{\tan^5(a+bx)}{5b}+\frac{2\tan^7(a+bx)}{7b}+\frac{\tan^9(a+bx)}{9b}$$

output
```
1/5*tan(b*x+a)^5/b+2/7*tan(b*x+a)^7/b+1/9*tan(b*x+a)^9/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 98 vs. 2(46) = 92.

Time = 0.16 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.13

$$\int \sec^6(a+bx)\tan^4(a+bx)\,dx=\frac{8\tan(a+bx)}{315b}+\frac{4\sec^2(a+bx)\tan(a+bx)}{315b}+\frac{\sec^4(a+bx)\tan(a+bx)}{105b}-\frac{10\sec^6(a+bx)\tan(a+bx)}{63b}+\frac{\sec^8(a+bx)\tan(a+bx)}{9b}$$

input
```
Integrate[Sec[a + b*x]^6*Tan[a + b*x]^4,x]
```

output
```
(8*Tan[a + b*x])/(315*b) + (4*Sec[a + b*x]^2*Tan[a + b*x])/(315*b) + (Sec[
a + b*x]^4*Tan[a + b*x])/(105*b) - (10*Sec[a + b*x]^6*Tan[a + b*x])/(63*b)
 + (Sec[a + b*x]^8*Tan[a + b*x])/(9*b)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(a+bx)\sec^6(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(a+bx)^4\sec(a+bx)^6dx$$

$$\Big\downarrow \text{3087}$$

$$\frac{\int \tan^4(a+bx)\left(\tan^2(a+bx)+1\right)^2d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\tan^8(a+bx)+2\tan^6(a+bx)+\tan^4(a+bx)\right)d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{9}\tan^9(a+bx)+\frac{2}{7}\tan^7(a+bx)+\frac{1}{5}\tan^5(a+bx)}{b}$$

input
```
Int[Sec[a + b*x]^6*Tan[a + b*x]^4,x]
```

output
```
(Tan[a + b*x]^5/5 + (2*Tan[a + b*x]^7)/7 + Tan[a + b*x]^9/9)/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

Maple [A] (verified)

Time = 15.80 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\frac{\tan(bx+a)^9}{9}+\frac{2\tan(bx+a)^7}{7}+\frac{\tan(bx+a)^5}{5}}{b}$	36
default	$\frac{\frac{\tan(bx+a)^9}{9}+\frac{2\tan(bx+a)^7}{7}+\frac{\tan(bx+a)^5}{5}}{b}$	36
risch	$\frac{16i\left(210\,\mathrm{e}^{12i(bx+a)}-315\,\mathrm{e}^{10i(bx+a)}+441\,\mathrm{e}^{8i(bx+a)}-126\,\mathrm{e}^{6i(bx+a)}+36\,\mathrm{e}^{4i(bx+a)}+9\,\mathrm{e}^{2i(bx+a)}+1\right)}{315b\left(\mathrm{e}^{2i(bx+a)}+1\right)^9}$	88

input
```
int(sec(b*x+a)^6*tan(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/9*tan(b*x+a)^9+2/7*tan(b*x+a)^7+1/5*tan(b*x+a)^5)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.33

$$\int \sec^6(a+bx)\tan^4(a+bx)\,dx = \frac{\left(8\cos(bx+a)^8+4\cos(bx+a)^6+3\cos(bx+a)^4-50\cos(bx+a)^2+35\right)\sin(bx+a)}{315\,b\cos(bx+a)^9}$$

input
```
integrate(sec(b*x+a)^6*tan(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/315*(8*cos(b*x + a)^8 + 4*cos(b*x + a)^6 + 3*cos(b*x + a)^4 - 50*cos(b*x
 + a)^2 + 35)*sin(b*x + a)/(b*cos(b*x + a)^9)
```

Sympy [F]

$$\int \sec^6(a+bx)\tan^4(a+bx)\,dx = \int \tan^4(a+bx)\sec^6(a+bx)\,dx$$

input
```
integrate(sec(b*x+a)**6*tan(b*x+a)**4,x)
```

output
```
Integral(tan(a + b*x)**4*sec(a + b*x)**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \sec^6(a+bx)\tan^4(a+bx)\,dx = \frac{35\tan(bx+a)^9+90\tan(bx+a)^7+63\tan(bx+a)^5}{315\,b}$$

input
```
integrate(sec(b*x+a)^6*tan(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/315*(35*tan(b*x + a)^9 + 90*tan(b*x + a)^7 + 63*tan(b*x + a)^5)/b
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \sec^6(a+bx)\tan^4(a+bx)\,dx = \frac{35\,\tan\left(bx+a\right)^9+90\,\tan\left(bx+a\right)^7+63\,\tan\left(bx+a\right)^5}{315\,b}$$

input
```
integrate(sec(b*x+a)^6*tan(b*x+a)^4,x, algorithm="giac")
```

output
```
1/315*(35*tan(b*x + a)^9 + 90*tan(b*x + a)^7 + 63*tan(b*x + a)^5)/b
```

Mupad [B] (verification not implemented)

Time = 25.35 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \sec^6(a+bx)\tan^4(a+bx)\,dx = \frac{\tan(a+b\,x)^5\,\left(35\tan(a+b\,x)^4+90\tan(a+b\,x)^2+63\right)}{315\,b}$$

input
```
int(tan(a + b*x)^4/cos(a + b*x)^6,x)
```

output
```
(tan(a + b*x)^5*(90*tan(a + b*x)^2 + 35*tan(a + b*x)^4 + 63))/(315*b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.85

$$\int \sec^6(a+bx)\tan^4(a+bx)\,dx$$
$$= \frac{\sin\left(bx+a\right)^5\left(8\sin\left(bx+a\right)^4-36\sin\left(bx+a\right)^2+63\right)}{315\cos\left(bx+a\right)b\left(\sin\left(bx+a\right)^8-4\sin\left(bx+a\right)^6+6\sin\left(bx+a\right)^4-4\sin\left(bx+a\right)^2+1\right)}$$

input
```
int(sec(b*x+a)^6*tan(b*x+a)^4,x)
```

output
```
(sin(a + b*x)**5*(8*sin(a + b*x)**4 - 36*sin(a + b*x)**2 + 63))/(315*cos(a
 + b*x)*b*(sin(a + b*x)**8 - 4*sin(a + b*x)**6 + 6*sin(a + b*x)**4 - 4*sin
(a + b*x)**2 + 1))
```

3.89 $\int \cos^6(a+bx)\sin^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 111

$$\int \cos^6(a+bx)\sin^4(a+bx)\,dx = \frac{3x}{256}+\frac{3\cos(a+bx)\sin(a+bx)}{256b}+\frac{\cos^3(a+bx)\sin(a+bx)}{128b}+\frac{\cos^5(a+bx)\sin(a+bx)}{160b}-\frac{3\cos^7(a+bx)\sin(a+bx)}{80b}-\frac{\cos^7(a+bx)\sin^3(a+bx)}{10b}$$

output
```
3/256*x+3/256*cos(b*x+a)*sin(b*x+a)/b+1/128*cos(b*x+a)^3*sin(b*x+a)/b+1/16
0*cos(b*x+a)^5*sin(b*x+a)/b-3/80*cos(b*x+a)^7*sin(b*x+a)/b-1/10*cos(b*x+a)
^7*sin(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.56

$$\int \cos^6(a+bx)\sin^4(a+bx)\,dx = \frac{120bx+20\sin(2(a+bx))-40\sin(4(a+bx))-10\sin(6(a+bx))+5\sin(8(a+bx))+2\sin(10(a+bx))}{10240b}$$

input
```
Integrate[Cos[a + b*x]^6*Sin[a + b*x]^4,x]
```

output
```
(120*b*x + 20*Sin[2*(a + b*x)] - 40*Sin[4*(a + b*x)] - 10*Sin[6*(a + b*x)]
 + 5*Sin[8*(a + b*x)] + 2*Sin[10*(a + b*x)])/(10240*b)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.18, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {3042, 3048, 3042, 3048, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(a+bx)\cos^6(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^4\cos(a+bx)^6dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{3}{10}\int \cos^6(a+bx)\sin^2(a+bx)dx-\frac{\sin^3(a+bx)\cos^7(a+bx)}{10b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{10}\int \cos(a+bx)^6\sin(a+bx)^2dx-\frac{\sin^3(a+bx)\cos^7(a+bx)}{10b}$$

$$\Big\downarrow \text{3048}$$

$$\frac{3}{10}\left(\frac{1}{8}\int \cos^6(a+bx)dx-\frac{\sin(a+bx)\cos^7(a+bx)}{8b}\right)-\frac{\sin^3(a+bx)\cos^7(a+bx)}{10b}$$

$$\Big\downarrow 3042$$

$$\frac{3}{10}\left(\frac{1}{8}\int \sin\left(a+bx+\frac{\pi}{2}\right)^6dx-\frac{\sin(a+bx)\cos^7(a+bx)}{8b}\right)-\frac{\sin^3(a+bx)\cos^7(a+bx)}{10b}$$

$$\Big\downarrow 3115$$

$$\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\int \cos^4(a+bx)dx+\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)-\frac{\sin(a+bx)\cos^7(a+bx)}{8b}\right)-\frac{\sin^3(a+bx)\cos^7(a+bx)}{10b}$$

$$\Big\downarrow 3042$$

$$\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\int \sin\left(a+bx+\frac{\pi}{2}\right)^4dx+\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)-\frac{\sin(a+bx)\cos^7(a+bx)}{8b}\right)-\frac{\sin^3(a+bx)\cos^7(a+bx)}{10b}$$

$$\Big\downarrow 3115$$

$$\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \cos^2(a+bx)dx+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)+\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)-\frac{\sin(a+bx)\cos^7(a+}{8b}\right.$$
$$\frac{\sin^3(a+bx)\cos^7(a+bx)}{10b}$$

$$\Big\downarrow 3042$$

$$\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)+\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)-\frac{\sin(a+bx)\,c}{8b}\right.$$
$$\frac{\sin^3(a+bx)\cos^7(a+bx)}{10b}$$

$$\Big\downarrow 3115$$

$$\frac{3}{10}\left(\frac{1}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)+\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)-\text{si}\right.$$
$$\frac{\sin^3(a+bx)\cos^7(a+bx)}{10b}$$

$$\Big\downarrow 24$$

$$\frac{3}{10}\left(\frac{1}{8}\left(\frac{\sin(a+bx)\cos^5(a+bx)}{6b}+\frac{5}{6}\left(\frac{\sin(a+bx)\cos^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sin(a+bx)\cos(a+bx)}{2b}+\frac{x}{2}\right)\right)\right)-\frac{\sin(a\right.$$

$$\frac{\sin^3(a+bx)\cos^7(a+bx)}{10b}$$

input
```
Int[Cos[a + b*x]^6*Sin[a + b*x]^4,x]
```

output
```
-1/10*(Cos[a + b*x]^7*Sin[a + b*x]^3)/b + (3*(-1/8*(Cos[a + b*x]^7*Sin[a +
 b*x])/b + ((Cos[a + b*x]^5*Sin[a + b*x])/(6*b) + (5*((Cos[a + b*x]^3*Sin[
a + b*x])/(4*b) + (3*(x/2 + (Cos[a + b*x]*Sin[a + b*x])/(2*b)))/4))/6)/8))
/10
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 21.42 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.59

method	result	siz
parallelrisch	$\frac{120bx+2\sin(10bx+10a)+5\sin(8bx+8a)-10\sin(6bx+6a)-40\sin(4bx+4a)+20\sin(2bx+2a)}{10240b}$	66
risch	$\frac{3x}{256}+\frac{\sin(10bx+10a)}{5120b}+\frac{\sin(8bx+8a)}{2048b}-\frac{\sin(6bx+6a)}{1024b}-\frac{\sin(4bx+4a)}{256b}+\frac{\sin(2bx+2a)}{512b}$	75
derivativedivides	$\frac{-\frac{\sin(bx+a)^3\cos(bx+a)^7}{10}-\frac{3\sin(bx+a)\cos(bx+a)^7}{80}+\frac{\left(\cos(bx+a)^5+\frac{5\cos(bx+a)^3}{4}+\frac{15\cos(bx+a)}{8}\right)\sin(bx+a)}{160}+\frac{3bx}{256}+\frac{3a}{256}}{b}$	82
default	$\frac{-\frac{\sin(bx+a)^3\cos(bx+a)^7}{10}-\frac{3\sin(bx+a)\cos(bx+a)^7}{80}+\frac{\left(\cos(bx+a)^5+\frac{5\cos(bx+a)^3}{4}+\frac{15\cos(bx+a)}{8}\right)\sin(bx+a)}{160}+\frac{3bx}{256}+\frac{3a}{256}}{b}$	82
orering	Expression too large to display	99

input
```
int(cos(b*x+a)^6*sin(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/10240*(120*b*x+2*sin(10*b*x+10*a)+5*sin(8*b*x+8*a)-10*sin(6*b*x+6*a)-40*
sin(4*b*x+4*a)+20*sin(2*b*x+2*a))/b
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.59

$$\int \cos^6(a+bx)\sin^4(a+bx)\,dx$$
$$=\frac{15\,bx+\left(128\,\cos\left(bx+a\right)^9-176\,\cos\left(bx+a\right)^7+8\,\cos\left(bx+a\right)^5+10\,\cos\left(bx+a\right)^3+15\,\cos\left(bx+a\right)\right)\mathrm{s}}{1280\,b}$$

input
```
integrate(cos(b*x+a)^6*sin(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/1280*(15*b*x + (128*cos(b*x + a)^9 - 176*cos(b*x + a)^7 + 8*cos(b*x + a)
^5 + 10*cos(b*x + a)^3 + 15*cos(b*x + a))*sin(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(100) = 200$.

Time = 1.28 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.08

$$\int \cos^6(a+bx)\sin^4(a+bx)\,dx = \begin{cases} \frac{3x\sin^{10}(a+bx)}{256} + \frac{15x\sin^8(a+bx)\cos^2(a+bx)}{256} + \frac{15x\sin^6(a+bx)\cos^4(a+bx)}{128} + \frac{15x\sin^4(a+bx)\cos^6(a+bx)}{128} + \frac{15x\sin^2(a+bx)\cos^8(a+bx)}{256} \\ x\sin^4(a)\cos^6(a) \end{cases}$$

input
```
integrate(cos(b*x+a)**6*sin(b*x+a)**4,x)
```

output
```
Piecewise((3*x*sin(a + b*x)**10/256 + 15*x*sin(a + b*x)**8*cos(a + b*x)**2
/256 + 15*x*sin(a + b*x)**6*cos(a + b*x)**4/128 + 15*x*sin(a + b*x)**4*cos
(a + b*x)**6/128 + 15*x*sin(a + b*x)**2*cos(a + b*x)**8/256 + 3*x*cos(a +
b*x)**10/256 + 3*sin(a + b*x)**9*cos(a + b*x)/(256*b) + 7*sin(a + b*x)**7*
cos(a + b*x)**3/(128*b) + sin(a + b*x)**5*cos(a + b*x)**5/(10*b) - 7*sin(a
 + b*x)**3*cos(a + b*x)**7/(128*b) - 3*sin(a + b*x)*cos(a + b*x)**9/(256*b
), Ne(b, 0)), (x*sin(a)**4*cos(a)**6, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.43

$$\int \cos^6(a+bx)\sin^4(a+bx)\,dx = \frac{32\sin(2bx+2a)^5 + 120bx + 120a + 5\sin(8bx+8a) - 40\sin(4bx+4a)}{10240b}$$

input
```
integrate(cos(b*x+a)^6*sin(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/10240*(32*sin(2*b*x + 2*a)^5 + 120*b*x + 120*a + 5*sin(8*b*x + 8*a) - 40
*sin(4*b*x + 4*a))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.67

$$\int \cos^6(a+bx)\sin^4(a+bx)\,dx = \frac{3}{256}\,x + \frac{\sin\left(10\,bx+10\,a\right)}{5120\,b} + \frac{\sin\left(8\,bx+8\,a\right)}{2048\,b} - \frac{\sin\left(6\,bx+6\,a\right)}{1024\,b} - \frac{\sin\left(4\,bx+4\,a\right)}{256\,b} + \frac{\sin\left(2\,bx+2\,a\right)}{512\,b}$$

input
```
integrate(cos(b*x+a)^6*sin(b*x+a)^4,x, algorithm="giac")
```

output
```
3/256*x + 1/5120*sin(10*b*x + 10*a)/b + 1/2048*sin(8*b*x + 8*a)/b - 1/1024
*sin(6*b*x + 6*a)/b - 1/256*sin(4*b*x + 4*a)/b + 1/512*sin(2*b*x + 2*a)/b
```

Mupad [B] (verification not implemented)

Time = 26.86 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.98

$$\int \cos^6(a+bx)\sin^4(a+bx)\,dx = \frac{3\,x}{256} + \frac{\frac{3\tan(a+b\,x)^9}{256} + \frac{7\tan(a+b\,x)^7}{128} + \frac{\tan(a+b\,x)^5}{10} - \frac{7\tan(a+b\,x)^3}{128} - \frac{3\tan(a+b\,x)}{256}}{b\,\left(\tan\left(a+b\,x\right)^{10} + 5\tan\left(a+b\,x\right)^8 + 10\tan\left(a+b\,x\right)^6 + 10\tan\left(a+b\,x\right)^4 + 5\tan\left(a+b\,x\right)^2 + 1\right)}$$

input
```
int(cos(a + b*x)^6*sin(a + b*x)^4,x)
```

output
```
(3*x)/256 + (tan(a + b*x)^5/10 - (7*tan(a + b*x)^3)/128 - (3*tan(a + b*x))
/256 + (7*tan(a + b*x)^7)/128 + (3*tan(a + b*x)^9)/256)/(b*(5*tan(a + b*x)
^2 + 10*tan(a + b*x)^4 + 10*tan(a + b*x)^6 + 5*tan(a + b*x)^8 + tan(a + b*
x)^10 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.79

$$\int \cos^6(a+bx)\sin^4(a+bx)\,dx$$
$$= \frac{128\cos(bx+a)\sin(bx+a)^9 - 336\cos(bx+a)\sin(bx+a)^7 + 248\cos(bx+a)\sin(bx+a)^5 - 10\cos(b}{1280b}$$

input
```
int(cos(b*x+a)^6*sin(b*x+a)^4,x)
```

output
```
(128*cos(a + b*x)*sin(a + b*x)**9 - 336*cos(a + b*x)*sin(a + b*x)**7 + 248
*cos(a + b*x)*sin(a + b*x)**5 - 10*cos(a + b*x)*sin(a + b*x)**3 - 15*cos(a
 + b*x)*sin(a + b*x) + 15*b*x)/(1280*b)
```

3.90 $\int \cos^4(a+bx)\sin^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 90

$$\int \cos^4(a+bx)\sin^4(a+bx)\,dx = \frac{3x}{128}+\frac{3\cos(a+bx)\sin(a+bx)}{128b}+\frac{\cos^3(a+bx)\sin(a+bx)}{64b}-\frac{\cos^5(a+bx)\sin(a+bx)}{16b}-\frac{\cos^5(a+bx)\sin^3(a+bx)}{8b}$$

output
```
3/128*x+3/128*cos(b*x+a)*sin(b*x+a)/b+1/64*cos(b*x+a)^3*sin(b*x+a)/b-1/16*
cos(b*x+a)^5*sin(b*x+a)/b-1/8*cos(b*x+a)^5*sin(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.37

$$\int \cos^4(a+bx)\sin^4(a+bx)\,dx = \frac{24(a+bx)-8\sin(4(a+bx))+\sin(8(a+bx))}{1024b}$$

input
```
Integrate[Cos[a + b*x]^4*Sin[a + b*x]^4,x]
```

output
```
(24*(a + b*x) - 8*Sin[4*(a + b*x)] + Sin[8*(a + b*x)])/(1024*b)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {3042, 3048, 3042, 3048, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(a+bx)\cos^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^4\cos(a+bx)^4dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{3}{8}\int \cos^4(a+bx)\sin^2(a+bx)dx-\frac{\sin^3(a+bx)\cos^5(a+bx)}{8b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{8}\int \cos(a+bx)^4\sin(a+bx)^2dx-\frac{\sin^3(a+bx)\cos^5(a+bx)}{8b}$$

$$\Big\downarrow \text{3048}$$

$$\frac{3}{8}\left(\frac{1}{6}\int \cos^4(a+bx)dx-\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)-\frac{\sin^3(a+bx)\cos^5(a+bx)}{8b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{8}\left(\frac{1}{6}\int \sin\left(a+bx+\frac{\pi}{2}\right)^4dx-\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)-\frac{\sin^3(a+bx)\cos^5(a+bx)}{8b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{3}{8}\left(\frac{1}{6}\left(\frac{3}{4}\int \cos^2(a+bx)dx+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)-\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)-\frac{\sin^3(a+bx)\cos^5(a+bx)}{8b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{8}\left(\frac{1}{6}\left(\frac{3}{4}\int \sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)-\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)-\frac{\sin^3(a+bx)\cos^5(a+bx)}{8b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{3}{8}\left(\frac{1}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)-\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)-\frac{\sin^3(a+bx)\cos^5(a+bx)}{8b}$$

$$\Big\downarrow \text{24}$$

$$\frac{3}{8}\left(\frac{1}{6}\left(\frac{\sin(a+bx)\cos^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sin(a+bx)\cos(a+bx)}{2b}+\frac{x}{2}\right)\right)-\frac{\sin(a+bx)\cos^5(a+bx)}{6b}\right)-\frac{\sin^3(a+bx)\cos^5(a+bx)}{8b}$$

input
```
Int[Cos[a + b*x]^4*Sin[a + b*x]^4,x]
```

output
```
-1/8*(Cos[a + b*x]^5*Sin[a + b*x]^3)/b + (3*(-1/6*(Cos[a + b*x]^5*Sin[a +
b*x])/b + ((Cos[a + b*x]^3*Sin[a + b*x])/(4*b) + (3*(x/2 + (Cos[a + b*x]*S
in[a + b*x])/(2*b)))/4)/6))/8
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 7.57 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.34

method	result
parallelrisch	$\frac{24bx+\sin(8bx+8a)-8\sin(4bx+4a)}{1024b}$
risch	$\frac{3x}{128}+\frac{\sin(8bx+8a)}{1024b}-\frac{\sin(4bx+4a)}{128b}$
derivativedivides	$\frac{-\frac{\cos(bx+a)^5\sin(bx+a)^3}{8}-\frac{\sin(bx+a)\cos(bx+a)^5}{16}+\frac{\left(\cos(bx+a)^3+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{64}+\frac{3bx}{128}+\frac{3a}{128}}{b}$
default	$\frac{-\frac{\cos(bx+a)^5\sin(bx+a)^3}{8}-\frac{\sin(bx+a)\cos(bx+a)^5}{16}+\frac{\left(\cos(bx+a)^3+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{64}+\frac{3bx}{128}+\frac{3a}{128}}{b}$
norman	$\frac{3x}{128}-\frac{3\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{64b}-\frac{23\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^3}{64b}+\frac{333\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^5}{64b}-\frac{671\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^7}{64b}+\frac{671\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^9}{64b}-\frac{333\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{11}}{64b}+\frac{23\,\mathrm{ta}}{}$
orering	$x\cos(bx+a)^4\sin(bx+a)^4-\frac{5\left(-4\cos(bx+a)^3\sin(bx+a)^5b+4\cos(bx+a)^5\sin(bx+a)^3b\right)}{64b^2}+\frac{5x\left(12\cos(\right.}{}$

input
```
int(cos(b*x+a)^4*sin(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/1024*(24*b*x+sin(8*b*x+8*a)-8*sin(4*b*x+4*a))/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.62

$$\int \cos^4(a+bx)\sin^4(a+bx)\,dx = \frac{3\,bx+\left(16\,\cos(bx+a)^7-24\,\cos(bx+a)^5+2\,\cos(bx+a)^3+3\,\cos(bx+a)\right)\sin(bx+a)}{128\,b}$$

input
```
integrate(cos(b*x+a)^4*sin(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/128*(3*b*x + (16*cos(b*x + a)^7 - 24*cos(b*x + a)^5 + 2*cos(b*x + a)^3 +
 3*cos(b*x + a))*sin(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(80) = 160.

Time = 0.65 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.10

$$\int \cos^4(a+bx)\sin^4(a+bx)\,dx = \begin{cases} \frac{3x\sin^8(a+bx)}{128} + \frac{3x\sin^6(a+bx)\cos^2(a+bx)}{32} + \frac{9x\sin^4(a+bx)\cos^4(a+bx)}{64} + \frac{3x\sin^2(a+bx)\cos^6(a+bx)}{32} + \frac{3x\cos^8(a+bx)}{128} + \frac{3\sin^7 \ldots \\ x\sin^4(a)\cos^4(a) \end{cases}$$

input
```
integrate(cos(b*x+a)**4*sin(b*x+a)**4,x)
```

output
```
Piecewise((3*x*sin(a + b*x)**8/128 + 3*x*sin(a + b*x)**6*cos(a + b*x)**2/3
2 + 9*x*sin(a + b*x)**4*cos(a + b*x)**4/64 + 3*x*sin(a + b*x)**2*cos(a + b
*x)**6/32 + 3*x*cos(a + b*x)**8/128 + 3*sin(a + b*x)**7*cos(a + b*x)/(128*
b) + 11*sin(a + b*x)**5*cos(a + b*x)**3/(128*b) - 11*sin(a + b*x)**3*cos(a
 + b*x)**5/(128*b) - 3*sin(a + b*x)*cos(a + b*x)**7/(128*b), Ne(b, 0)), (x
*sin(a)**4*cos(a)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.37

$$\int \cos^4(a+bx)\sin^4(a+bx)\,dx = \frac{24\,bx + 24\,a + \sin(8\,bx + 8\,a) - 8\,\sin(4\,bx + 4\,a)}{1024\,b}$$

input
```
integrate(cos(b*x+a)^4*sin(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/1024*(24*b*x + 24*a + sin(8*b*x + 8*a) - 8*sin(4*b*x + 4*a))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.36

$$\int \cos^4(a+bx)\sin^4(a+bx)\,dx = \frac{3}{128}\,x + \frac{\sin\left(8\,bx+8\,a\right)}{1024\,b} - \frac{\sin\left(4\,bx+4\,a\right)}{128\,b}$$

input
```
integrate(cos(b*x+a)^4*sin(b*x+a)^4,x, algorithm="giac")
```

output
```
3/128*x + 1/1024*sin(8*b*x + 8*a)/b - 1/128*sin(4*b*x + 4*a)/b
```

Mupad [B] (verification not implemented)

Time = 26.45 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int \cos^4(a+bx)\sin^4(a+bx)\,dx$$
$$= \frac{3\,x}{128} - \frac{-\frac{3\tan(a+b\,x)^7}{128} - \frac{11\tan(a+b\,x)^5}{128} + \frac{11\tan(a+b\,x)^3}{128} + \frac{3\tan(a+b\,x)}{128}}{b\,\left(\tan\left(a+b\,x\right)^8+4\tan\left(a+b\,x\right)^6+6\tan\left(a+b\,x\right)^4+4\tan\left(a+b\,x\right)^2+1\right)}$$

input
```
int(cos(a + b*x)^4*sin(a + b*x)^4,x)
```

output
```
(3*x)/128 - ((3*tan(a + b*x))/128 + (11*tan(a + b*x)^3)/128 - (11*tan(a +
b*x)^5)/128 - (3*tan(a + b*x)^7)/128)/(b*(4*tan(a + b*x)^2 + 6*tan(a + b*x
)^4 + 4*tan(a + b*x)^6 + tan(a + b*x)^8 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.80

$$\int \cos^4(a+bx)\sin^4(a+bx)\,dx$$
$$= \frac{-16\cos(bx+a)\sin(bx+a)^7 + 24\cos(bx+a)\sin(bx+a)^5 - 2\cos(bx+a)\sin(bx+a)^3 - 3\cos(bx+}{128b}$$

input
```
int(cos(b*x+a)^4*sin(b*x+a)^4,x)
```

output
```
( - 16*cos(a + b*x)*sin(a + b*x)**7 + 24*cos(a + b*x)*sin(a + b*x)**5 - 2*
cos(a + b*x)*sin(a + b*x)**3 - 3*cos(a + b*x)*sin(a + b*x) + 3*b*x)/(128*b
)
```

3.91 $\int \cos^2(a+bx)\sin^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 69

$$\int \cos^2(a+bx)\sin^4(a+bx)\,dx = \frac{x}{16}+\frac{\cos(a+bx)\sin(a+bx)}{16b} - \frac{\cos^3(a+bx)\sin(a+bx)}{8b} - \frac{\cos^3(a+bx)\sin^3(a+bx)}{6b}$$

output
```
1/16*x+1/16*cos(b*x+a)*sin(b*x+a)/b-1/8*cos(b*x+a)^3*sin(b*x+a)/b-1/6*cos(
b*x+a)^3*sin(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.58

$$\int \cos^2(a+bx)\sin^4(a+bx)\,dx = \frac{12bx-3\sin(2(a+bx))-3\sin(4(a+bx))+\sin(6(a+bx))}{192b}$$

input
```
Integrate[Cos[a + b*x]^2*Sin[a + b*x]^4,x]
```

output
```
(12*b*x - 3*Sin[2*(a + b*x)] - 3*Sin[4*(a + b*x)] + Sin[6*(a + b*x)])/(192
*b)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3042, 3048, 3042, 3048, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(a+bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^4\cos(a+bx)^2dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{1}{2}\int \cos^2(a+bx)\sin^2(a+bx)dx-\frac{\sin^3(a+bx)\cos^3(a+bx)}{6b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \cos(a+bx)^2\sin(a+bx)^2dx-\frac{\sin^3(a+bx)\cos^3(a+bx)}{6b}$$

$$\Big\downarrow \text{3048}$$

$$\frac{1}{2}\left(\frac{1}{4}\int \cos^2(a+bx)dx-\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)-\frac{\sin^3(a+bx)\cos^3(a+bx)}{6b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{1}{4}\int \sin\left(a+bx+\frac{\pi}{2}\right)^2dx-\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)-\frac{\sin^3(a+bx)\cos^3(a+bx)}{6b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)-\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)-\frac{\sin^3(a+bx)\cos^3(a+bx)}{6b}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{\sin(a+bx)\cos(a+bx)}{2b}+\frac{x}{2}\right)-\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)-\frac{\sin^3(a+bx)\cos^3(a+bx)}{6b}$$

input
```
Int[Cos[a + b*x]^2*Sin[a + b*x]^4,x]
```

output
```
-1/6*(Cos[a + b*x]^3*Sin[a + b*x]^3)/b + (-1/4*(Cos[a + b*x]^3*Sin[a + b*x
])/b + (x/2 + (Cos[a + b*x]*Sin[a + b*x])/(2*b))/4)/2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 5.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.61

method	result
parallelrisch	$\frac{12bx+\sin(6bx+6a)-3\sin(4bx+4a)-3\sin(2bx+2a)}{192b}$
risch	$\frac{x}{16}+\frac{\sin(6bx+6a)}{192b}-\frac{\sin(4bx+4a)}{64b}-\frac{\sin(2bx+2a)}{64b}$
derivativedivides	$\frac{-\frac{\cos(bx+a)^3\sin(bx+a)^3}{6}-\frac{\cos(bx+a)^3\sin(bx+a)}{8}+\frac{\cos(bx+a)\sin(bx+a)}{16}+\frac{bx}{16}+\frac{a}{16}}{b}$
default	$\frac{-\frac{\cos(bx+a)^3\sin(bx+a)^3}{6}-\frac{\cos(bx+a)^3\sin(bx+a)}{8}+\frac{\cos(bx+a)\sin(bx+a)}{16}+\frac{bx}{16}+\frac{a}{16}}{b}$
norman	$\frac{\frac{x}{16}-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{8b}-\frac{17\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^3}{24b}+\frac{19\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^5}{4b}-\frac{19\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^7}{4b}+\frac{17\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^9}{24b}+\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{11}}{8b}+\frac{3x\tan\left(\frac{bx}{2}+\frac{a}{2}\right.}{8}}{\left(1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right)^6}$
orering	$x\cos(bx+a)^2\sin(bx+a)^4-\frac{49\left(-2\cos(bx+a)\sin(bx+a)^5b+4\cos(bx+a)^3\sin(bx+a)^3b\right)}{144b^2}+\frac{49x\left(2b^2\sin\right.}{}$

input
```
int(cos(b*x+a)^2*sin(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/192*(12*b*x+sin(6*b*x+6*a)-3*sin(4*b*x+4*a)-3*sin(2*b*x+2*a))/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.67

$$\int \cos^2(a+bx)\sin^4(a+bx)\,dx = \frac{3\,bx+\left(8\,\cos(bx+a)^5-14\,\cos(bx+a)^3+3\,\cos(bx+a)\right)\sin(bx+a)}{48\,b}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/48*(3*b*x + (8*cos(b*x + a)^5 - 14*cos(b*x + a)^3 + 3*cos(b*x + a))*sin(
b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(58) = 116.

Time = 0.33 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.97

$$\int \cos^2(a+bx)\sin^4(a+bx)\,dx$$
$$= \begin{cases} \frac{x\sin^6(a+bx)}{16} + \frac{3x\sin^4(a+bx)\cos^2(a+bx)}{16} + \frac{3x\sin^2(a+bx)\cos^4(a+bx)}{16} + \frac{x\cos^6(a+bx)}{16} + \frac{\sin^5(a+bx)\cos(a+bx)}{16b} - \frac{\sin^3(a+bx)\cos^3(a+bx)}{6b} \\ x\sin^4(a)\cos^2(a) \end{cases}$$

input
```
integrate(cos(b*x+a)**2*sin(b*x+a)**4,x)
```

output
```
Piecewise((x*sin(a + b*x)**6/16 + 3*x*sin(a + b*x)**4*cos(a + b*x)**2/16 +
 3*x*sin(a + b*x)**2*cos(a + b*x)**4/16 + x*cos(a + b*x)**6/16 + sin(a + b
*x)**5*cos(a + b*x)/(16*b) - sin(a + b*x)**3*cos(a + b*x)**3/(6*b) - sin(a
 + b*x)*cos(a + b*x)**5/(16*b), Ne(b, 0)), (x*sin(a)**4*cos(a)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.54

$$\int \cos^2(a+bx)\sin^4(a+bx)\,dx = -\frac{4\sin(2bx+2a)^3 - 12bx - 12a + 3\sin(4bx+4a)}{192b}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^4,x, algorithm="maxima")
```

output
```
-1/192*(4*sin(2*b*x + 2*a)^3 - 12*b*x - 12*a + 3*sin(4*b*x + 4*a))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.67

$$\int \cos^2(a+bx)\sin^4(a+bx)\,dx = \frac{1}{16}\,x + \frac{\sin\left(6\,bx+6\,a\right)}{192\,b} - \frac{\sin\left(4\,bx+4\,a\right)}{64\,b} - \frac{\sin\left(2\,bx+2\,a\right)}{64\,b}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^4,x, algorithm="giac")
```

output
```
1/16*x + 1/192*sin(6*b*x + 6*a)/b - 1/64*sin(4*b*x + 4*a)/b - 1/64*sin(2*b
*x + 2*a)/b
```

Mupad [B] (verification not implemented)

Time = 25.42 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.62

$$\int \cos^2(a+bx)\sin^4(a+bx)\,dx = \frac{x}{16} - \frac{\frac{\sin(2\,a+2\,b\,x)}{64} + \frac{\sin(4\,a+4\,b\,x)}{64} - \frac{\sin(6\,a+6\,b\,x)}{192}}{b}$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)^4,x)
```

output
```
x/16 - (sin(2*a + 2*b*x)/64 + sin(4*a + 4*b*x)/64 - sin(6*a + 6*b*x)/192)/
b
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.81

$$\int \cos^2(a+bx)\sin^4(a+bx)\,dx = \frac{8\cos\left(bx+a\right)\sin\left(bx+a\right)^5 - 2\cos\left(bx+a\right)\sin\left(bx+a\right)^3 - 3\cos\left(bx+a\right)\sin\left(bx+a\right) + 3bx}{48b}$$

input
```
int(cos(b*x+a)^2*sin(b*x+a)^4,x)
```

output
```
(8*cos(a + b*x)*sin(a + b*x)**5 - 2*cos(a + b*x)*sin(a + b*x)**3 - 3*cos(a
 + b*x)*sin(a + b*x) + 3*b*x)/(48*b)
```

3.92 $\int \sin^4(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 46

$$\int \sin^4(a+bx)\,dx=\frac{3x}{8}-\frac{3\cos(a+bx)\sin(a+bx)}{8b}-\frac{\cos(a+bx)\sin^3(a+bx)}{4b}$$

output
```
3/8*x-3/8*cos(b*x+a)*sin(b*x+a)/b-1/4*cos(b*x+a)*sin(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int \sin^4(a+bx)\,dx=\frac{12(a+bx)-8\sin(2(a+bx))+\sin(4(a+bx))}{32b}$$

input
```
Integrate[Sin[a + b*x]^4,x]
```

output
```
(12*(a + b*x) - 8*Sin[2*(a + b*x)] + Sin[4*(a + b*x)])/(32*b)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(a+bx)^4dx$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{3}{4}\int \sin^2(a+bx)dx-\frac{\sin^3(a+bx)\cos(a+bx)}{4b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3}{4}\int \sin(a+bx)^2dx-\frac{\sin^3(a+bx)\cos(a+bx)}{4b}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{3}{4}\left(\frac{\int 1dx}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)-\frac{\sin^3(a+bx)\cos(a+bx)}{4b}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{3}{4}\left(\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)-\frac{\sin^3(a+bx)\cos(a+bx)}{4b}$$

input
```
Int[Sin[a + b*x]^4,x]
```

output
```
-1/4*(Cos[a + b*x]*Sin[a + b*x]^3)/b + (3*(x/2 - (Cos[a + b*x]*Sin[a + b*x
])/(2*b)))/4
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 4.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

method	result
parallelrisch	$\frac{12bx+\sin(4bx+4a)-8\sin(2bx+2a)}{32b}$
risch	$\frac{3x}{8}+\frac{\sin(4bx+4a)}{32b}-\frac{\sin(2bx+2a)}{4b}$
derivativedivides	$\frac{-\frac{\left(\sin(bx+a)^3+\frac{3\sin(bx+a)}{2}\right)\cos(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}}{b}$
default	$\frac{-\frac{\left(\sin(bx+a)^3+\frac{3\sin(bx+a)}{2}\right)\cos(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}}{b}$
norman	$\frac{\frac{3x}{8}-\frac{3\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{4b}-\frac{11\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^3}{4b}+\frac{11\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^5}{4b}+\frac{3\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^7}{4b}+\frac{3x\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}{2}+\frac{9x\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}{4}+\frac{3x\tan\left(\frac{bx}{2}+\right.}{2}}{\left(1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right)^4}$
orering	$x\sin(bx+a)^4-\frac{5\cos(bx+a)\sin(bx+a)^3}{4b}+\frac{5x\left(12\sin(bx+a)^2\cos(bx+a)^2b^2-4\sin(bx+a)^4b^2\right)}{16b^2}-\frac{24\sin(bx-}{}$

input
```
int(sin(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/32*(12*b*x+sin(4*b*x+4*a)-8*sin(2*b*x+2*a))/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \sin^4(a+bx)\,dx = \frac{3\,bx + \left(2\,\cos\left(bx+a\right)^3 - 5\,\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{8\,b}$$

input
```
integrate(sin(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/8*(3*b*x + (2*cos(b*x + a)^3 - 5*cos(b*x + a))*sin(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(41) = 82.

Time = 0.15 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.07

$$\int \sin^4(a+bx)\,dx = \begin{cases} \frac{3x\sin^4(a+bx)}{8} + \frac{3x\sin^2(a+bx)\cos^2(a+bx)}{4} + \frac{3x\cos^4(a+bx)}{8} - \frac{5\sin^3(a+bx)\cos(a+bx)}{8b} - \frac{3\sin(a+bx)\cos^3(a+bx)}{8b} & \text{for } b \neq 0 \\ x\sin^4(a) & \text{otherwise} \end{cases}$$

input
```
integrate(sin(b*x+a)**4,x)
```

output
```
Piecewise((3*x*sin(a + b*x)**4/8 + 3*x*sin(a + b*x)**2*cos(a + b*x)**2/4 +
 3*x*cos(a + b*x)**4/8 - 5*sin(a + b*x)**3*cos(a + b*x)/(8*b) - 3*sin(a +
b*x)*cos(a + b*x)**3/(8*b), Ne(b, 0)), (x*sin(a)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int \sin^4(a+bx)\,dx = \frac{12\,bx + 12\,a + \sin\left(4\,bx + 4\,a\right) - 8\,\sin\left(2\,bx + 2\,a\right)}{32\,b}$$

input
```
integrate(sin(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/32*(12*b*x + 12*a + sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int \sin^4(a+bx)\,dx = \frac{3}{8}\,x + \frac{\sin\left(4\,bx + 4\,a\right)}{32\,b} - \frac{\sin\left(2\,bx + 2\,a\right)}{4\,b}$$

input
```
integrate(sin(b*x+a)^4,x, algorithm="giac")
```

output
```
3/8*x + 1/32*sin(4*b*x + 4*a)/b - 1/4*sin(2*b*x + 2*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int \sin^4(a+bx)\,dx = \frac{3\,x}{8} - \frac{\frac{5\tan(a+b\,x)^3}{8} + \frac{3\tan(a+b\,x)}{8}}{b\,\left(\tan\left(a+b\,x\right)^4 + 2\tan\left(a+b\,x\right)^2 + 1\right)}$$

input
```
int(sin(a + b*x)^4,x)
```

output
```
(3*x)/8 - ((3*tan(a + b*x))/8 + (5*tan(a + b*x)^3)/8)/(b*(2*tan(a + b*x)^2
 + tan(a + b*x)^4 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \sin^4(a+bx)\,dx = \frac{-2\cos(bx+a)\sin(bx+a)^3 - 3\cos(bx+a)\sin(bx+a) + 3bx}{8b}$$

input
```
int(sin(b*x+a)^4,x)
```

output
```
( - 2*cos(a + b*x)*sin(a + b*x)**3 - 3*cos(a + b*x)*sin(a + b*x) + 3*b*x)/
(8*b)
```

3.93 $\int \sin^3(a+bx)\tan(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \sin^3(a+bx)\tan(a+bx)\,dx=\frac{\operatorname{arctanh}(\sin(a+bx))}{b}-\frac{\sin(a+bx)}{b}-\frac{\sin^3(a+bx)}{3b}$$

output
```
arctanh(sin(b*x+a))/b-sin(b*x+a)/b-1/3*sin(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \sin^3(a+bx)\tan(a+bx)\,dx=\frac{\operatorname{arctanh}(\sin(a+bx))}{b}-\frac{\sin(a+bx)}{b}-\frac{\sin^3(a+bx)}{3b}$$

input
```
Integrate[Sin[a + b*x]^3*Tan[a + b*x],x]
```

output
```
ArcTanh[Sin[a + b*x]]/b - Sin[a + b*x]/b - Sin[a + b*x]^3/(3*b)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 3072, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\tan(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^3\tan(a+bx)dx$$

$$\Big\downarrow \text{3072}$$

$$\frac{\int \frac{\sin^4(a+bx)}{1-\sin^2(a+bx)}d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{254}$$

$$\frac{\int \left(-\sin^2(a+bx)+\frac{1}{1-\sin^2(a+bx)}-1\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\text{arctanh}(\sin(a+bx))-\frac{1}{3}\sin^3(a+bx)-\sin(a+bx)}{b}$$

input
```
Int[Sin[a + b*x]^3*Tan[a + b*x],x]
```

output
```
(ArcTanh[Sin[a + b*x]] - Sin[a + b*x] - Sin[a + b*x]^3/3)/b
```

Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{-\frac{\sin(bx+a)^3}{3}-\sin(bx+a)+\ln(\sec(bx+a)+\tan(bx+a))}{b}$	38
default	$\frac{-\frac{\sin(bx+a)^3}{3}-\sin(bx+a)+\ln(\sec(bx+a)+\tan(bx+a))}{b}$	38
risch	$\frac{5i\mathrm{e}^{i(bx+a)}}{8b}-\frac{5i\mathrm{e}^{-i(bx+a)}}{8b}+\frac{\ln\left(\mathrm{e}^{i(bx+a)}+i\right)}{b}-\frac{\ln\left(\mathrm{e}^{i(bx+a)}-i\right)}{b}+\frac{\sin(3bx+3a)}{12b}$	81

input
```
int(sin(b*x+a)^3*tan(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/3*sin(b*x+a)^3-sin(b*x+a)+ln(sec(b*x+a)+tan(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int \sin^3(a+bx)\tan(a+bx)\,dx = \frac{2\left(\cos\left(bx+a\right)^2-4\right)\sin\left(bx+a\right)+3\,\log\left(\sin\left(bx+a\right)+1\right)-3\,\log\left(-\sin\left(bx+a\right)+1\right)}{6\,b}$$

input
```
integrate(sin(b*x+a)^3*tan(b*x+a),x, algorithm="fricas")
```

output
```
1/6*(2*(cos(b*x + a)^2 - 4)*sin(b*x + a) + 3*log(sin(b*x + a) + 1) - 3*log
(-sin(b*x + a) + 1))/b
```

Sympy [F]

$$\int \sin^3(a+bx)\tan(a+bx)\,dx = \int \sin^3\left(a+bx\right)\tan\left(a+bx\right)dx$$

input
```
integrate(sin(b*x+a)**3*tan(b*x+a),x)
```

output
```
Integral(sin(a + b*x)**3*tan(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

$$\int \sin^3(a+bx)\tan(a+bx)\,dx = -\frac{2\,\sin\left(bx+a\right)^3-3\,\log\left(\sin\left(bx+a\right)+1\right)+3\,\log\left(\sin\left(bx+a\right)-1\right)+6\,\sin\left(bx+a\right)}{6\,b}$$

input
```
integrate(sin(b*x+a)^3*tan(b*x+a),x, algorithm="maxima")
```

output
```
-1/6*(2*sin(b*x + a)^3 - 3*log(sin(b*x + a) + 1) + 3*log(sin(b*x + a) - 1)
 + 6*sin(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.58

$$\int \sin^3(a+bx)\tan(a+bx)\,dx = \frac{\log(|\sin(bx+a)+1|)}{2b} - \frac{\log(|\sin(bx+a)-1|)}{2b} - \frac{b^2\sin(bx+a)^3 + 3b^2\sin(bx+a)}{3b^3}$$

input
```
integrate(sin(b*x+a)^3*tan(b*x+a),x, algorithm="giac")
```

output
```
1/2*log(abs(sin(b*x + a) + 1))/b - 1/2*log(abs(sin(b*x + a) - 1))/b - 1/3*
(b^2*sin(b*x + a)^3 + 3*b^2*sin(b*x + a))/b^3
```

Mupad [B] (verification not implemented)

Time = 25.44 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.39

$$\int \sin^3(a+bx)\tan(a+bx)\,dx = \frac{2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{a}{2}+\frac{b\,x}{2}\right)}{\cos\left(\frac{a}{2}+\frac{b\,x}{2}\right)}\right)}{b} - \frac{5\,\sin(a+b\,x)}{4\,b} + \frac{\sin(3\,a+3\,b\,x)}{12\,b}$$

input
```
int(sin(a + b*x)^3*tan(a + b*x),x)
```

output
```
(2*atanh(sin(a/2 + (b*x)/2)/cos(a/2 + (b*x)/2)))/b - (5*sin(a + b*x))/(4*b
) + sin(3*a + 3*b*x)/(12*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.37

$$\int \sin^3(a+bx)\tan(a+bx)\,dx = \frac{-3\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+3\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)-\sin\left(bx+a\right)^3-3\sin\left(bx+a\right)}{3b}$$

input
```
int(sin(b*x+a)^3*tan(b*x+a),x)
```

output
```
( - 3*log(tan((a + b*x)/2) - 1) + 3*log(tan((a + b*x)/2) + 1) - sin(a + b*
x)**3 - 3*sin(a + b*x))/(3*b)
```

3.94 $\int \sin(a+bx)\tan^3(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \sin(a+bx)\tan^3(a+bx)\,dx = -\frac{3\text{arctanh}(\sin(a+bx))}{2b} + \frac{\sin(a+bx)}{b} + \frac{\sec(a+bx)\tan(a+bx)}{2b}$$

output
```
-3/2*arctanh(sin(b*x+a))/b+sin(b*x+a)/b+1/2*sec(b*x+a)*tan(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.20

$$\int \sin(a+bx)\tan^3(a+bx)\,dx = -\frac{3\text{arctanh}(\sin(a+bx))}{2b} + \frac{3\sec(a+bx)\tan(a+bx)}{2b} - \frac{\sin(a+bx)\tan^2(a+bx)}{b}$$

input
```
Integrate[Sin[a + b*x]*Tan[a + b*x]^3,x]
```

output
```
(-3*ArcTanh[Sin[a + b*x]])/(2*b) + (3*Sec[a + b*x]*Tan[a + b*x])/(2*b) - (
Sin[a + b*x]*Tan[a + b*x]^2)/b
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3072, 252, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\tan^3(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(a+bx)\tan(a+bx)^3dx$$

$$\Big\downarrow \text{ 3072}$$

$$\frac{\int \frac{\sin^4(a+bx)}{(1-\sin^2(a+bx))^2}d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{\frac{\sin^3(a+bx)}{2(1-\sin^2(a+bx))}-\frac{3}{2}\int \frac{\sin^2(a+bx)}{1-\sin^2(a+bx)}d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{\frac{\sin^3(a+bx)}{2(1-\sin^2(a+bx))}-\frac{3}{2}\left(\int \frac{1}{1-\sin^2(a+bx)}d\sin(a+bx)-\sin(a+bx)\right)}{b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{\sin^3(a+bx)}{2(1-\sin^2(a+bx))}-\frac{3}{2}(\text{arctanh}(\sin(a+bx))-\sin(a+bx))}{b}$$

input
```
Int[Sin[a + b*x]*Tan[a + b*x]^3,x]
```

output
```
((-3*(ArcTanh[Sin[a + b*x]] - Sin[a + b*x]))/2 + Sin[a + b*x]^3/(2*(1 - Si
n[a + b*x]^2)))/b
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.32

method	result	size
derivativedivides	$\frac{\frac{\sin(bx+a)^5}{2\cos(bx+a)^2}+\frac{\sin(bx+a)^3}{2}+\frac{3\sin(bx+a)}{2}-\frac{3\ln(\sec(bx+a)+\tan(bx+a))}{2}}{b}$	58
default	$\frac{\frac{\sin(bx+a)^5}{2\cos(bx+a)^2}+\frac{\sin(bx+a)^3}{2}+\frac{3\sin(bx+a)}{2}-\frac{3\ln(\sec(bx+a)+\tan(bx+a))}{2}}{b}$	58
risch	$\frac{i\,e^{-i(bx+a)}}{2b}-\frac{i\,e^{i(bx+a)}}{2b}-\frac{i\left(e^{3i(bx+a)}-e^{i(bx+a)}\right)}{b\left(e^{2i(bx+a)}+1\right)^2}+\frac{3\ln\left(e^{i(bx+a)}-i\right)}{2b}-\frac{3\ln\left(e^{i(bx+a)}+i\right)}{2b}$	108

input
```
int(sin(b*x+a)*tan(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2*sin(b*x+a)^5/cos(b*x+a)^2+1/2*sin(b*x+a)^3+3/2*sin(b*x+a)-3/2*ln(
sec(b*x+a)+tan(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.68

$$\int \sin(a+bx)\tan^3(a+bx)\,dx =$$
$$-\frac{3\cos(bx+a)^2\log(\sin(bx+a)+1)-3\cos(bx+a)^2\log(-\sin(bx+a)+1)-2\left(2\cos(bx+a)^2+\right.}{4\,b\cos(bx+a)^2}$$

input
```
integrate(sin(b*x+a)*tan(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/4*(3*cos(b*x + a)^2*log(sin(b*x + a) + 1) - 3*cos(b*x + a)^2*log(-sin(b
*x + a) + 1) - 2*(2*cos(b*x + a)^2 + 1)*sin(b*x + a))/(b*cos(b*x + a)^2)
```

Sympy [F]

$$\int \sin(a+bx)\tan^3(a+bx)\,dx = \int \sin\left(a+bx\right)\tan^3\left(a+bx\right)dx$$

input
```
integrate(sin(b*x+a)*tan(b*x+a)**3,x)
```

output
```
Integral(sin(a + b*x)*tan(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.27

$$\int \sin(a+bx)\tan^3(a+bx)\,dx = -\frac{\frac{2\sin(bx+a)}{\sin(bx+a)^2-1} + 3\log\left(\sin\left(bx+a\right)+1\right) - 3\log\left(\sin\left(bx+a\right)-1\right) - 4\sin\left(bx+a\right)}{4\,b}$$

input
```
integrate(sin(b*x+a)*tan(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/4*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + 3*log(sin(b*x + a) + 1) - 3*lo
g(sin(b*x + a) - 1) - 4*sin(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.45

$$\int \sin(a+bx)\tan^3(a+bx)\,dx = -\frac{3\log\left(\left|\sin\left(bx+a\right)+1\right|\right)}{4\,b} + \frac{3\log\left(\left|\sin\left(bx+a\right)-1\right|\right)}{4\,b} + \frac{\sin\left(bx+a\right)}{b} - \frac{\sin\left(bx+a\right)}{2\left(\sin\left(bx+a\right)^2-1\right)b}$$

input
```
integrate(sin(b*x+a)*tan(b*x+a)^3,x, algorithm="giac")
```

output
```
-3/4*log(abs(sin(b*x + a) + 1))/b + 3/4*log(abs(sin(b*x + a) - 1))/b + sin
(b*x + a)/b - 1/2*sin(b*x + a)/((sin(b*x + a)^2 - 1)*b)
```

Mupad [B] (verification not implemented)

Time = 27.47 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.23

$$\int \sin(a+bx)\tan^3(a+bx)\,dx = -\frac{3\,\mathrm{atanh}\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{b} - \frac{3\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^5-2\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^3+3\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)}{b\left(-\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^6+\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4+\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2-1\right)}$$

input
```
int(sin(a + b*x)*tan(a + b*x)^3,x)
```

output
```
- (3*atanh(tan(a/2 + (b*x)/2)))/b - (3*tan(a/2 + (b*x)/2) - 2*tan(a/2 + (b
*x)/2)^3 + 3*tan(a/2 + (b*x)/2)^5)/(b*(tan(a/2 + (b*x)/2)^2 + tan(a/2 + (b
*x)/2)^4 - tan(a/2 + (b*x)/2)^6 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.61

$$\int \sin(a+bx)\tan^3(a+bx)\,dx = \frac{-\cos(bx+a)\tan(bx+a)+3\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)-3\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)+\sin(bx+a)\tan(bx+}{2b}$$

input
```
int(sin(b*x+a)*tan(b*x+a)^3,x)
```

output
```
( - cos(a + b*x)*tan(a + b*x) + 3*log(tan((a + b*x)/2) - 1) - 3*log(tan((a
 + b*x)/2) + 1) + sin(a + b*x)*tan(a + b*x)**2 + 4*sin(a + b*x))/(2*b)
```

3.95 $\int \sec(a+bx)\tan^4(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \sec(a+bx)\tan^4(a+bx)\,dx=\frac{3\text{arctanh}(\sin(a+bx))}{8b}-\frac{3\sec(a+bx)\tan(a+bx)}{8b}+\frac{\sec(a+bx)\tan^3(a+bx)}{4b}$$

output
```
3/8*arctanh(sin(b*x+a))/b-3/8*sec(b*x+a)*tan(b*x+a)/b+1/4*sec(b*x+a)*tan(b
*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.33

$$\int \sec(a+bx)\tan^4(a+bx)\,dx=\frac{3\text{arctanh}(\sin(a+bx))}{8b}+\frac{3\sec(a+bx)\tan(a+bx)}{8b}-\frac{3\sec^3(a+bx)\tan(a+bx)}{4b}+\frac{\sec(a+bx)\tan^3(a+bx)}{b}$$

input
```
Integrate[Sec[a + b*x]*Tan[a + b*x]^4,x]
```

output
```
(3*ArcTanh[Sin[a + b*x]])/(8*b) + (3*Sec[a + b*x]*Tan[a + b*x])/(8*b) - (3
*Sec[a + b*x]^3*Tan[a + b*x])/(4*b) + (Sec[a + b*x]*Tan[a + b*x]^3)/b
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3091, 3042, 3091, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(a+bx)\sec(a+bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \tan(a+bx)^4\sec(a+bx)dx$$

$$\Big\downarrow \ 3091$$

$$\frac{\tan^3(a+bx)\sec(a+bx)}{4b}-\frac{3}{4}\int \sec(a+bx)\tan^2(a+bx)dx$$

$$\Big\downarrow \ 3042$$

$$\frac{\tan^3(a+bx)\sec(a+bx)}{4b}-\frac{3}{4}\int \sec(a+bx)\tan(a+bx)^2dx$$

$$\Big\downarrow \ 3091$$

$$\frac{\tan^3(a+bx)\sec(a+bx)}{4b}-\frac{3}{4}\left(\frac{\tan(a+bx)\sec(a+bx)}{2b}-\frac{1}{2}\int \sec(a+bx)dx\right)$$

$$\Big\downarrow \ 3042$$

$$\frac{\tan^3(a+bx)\sec(a+bx)}{4b}-\frac{3}{4}\left(\frac{\tan(a+bx)\sec(a+bx)}{2b}-\frac{1}{2}\int \csc\left(a+bx+\frac{\pi}{2}\right)dx\right)$$

$$\Big\downarrow \ 4257$$

$$\frac{\tan^3(a+bx)\sec(a+bx)}{4b}-\frac{3}{4}\left(\frac{\tan(a+bx)\sec(a+bx)}{2b}-\frac{\text{arctanh}(\sin(a+bx))}{2b}\right)$$

input
```
Int[Sec[a + b*x]*Tan[a + b*x]^4,x]
```

output
```
(Sec[a + b*x]*Tan[a + b*x]^3)/(4*b) - (3*(-1/2*ArcTanh[Sin[a + b*x]]/b + (
Sec[a + b*x]*Tan[a + b*x])/(2*b)))/4
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.38

method	result	size
derivativedivides	$\frac{\frac{\sin(bx+a)^5}{4\cos(bx+a)^4}-\frac{\sin(bx+a)^5}{8\cos(bx+a)^2}-\frac{\sin(bx+a)^3}{8}-\frac{3\sin(bx+a)}{8}+\frac{3\ln(\sec(bx+a)+\tan(bx+a))}{8}}{b}$	76
default	$\frac{\frac{\sin(bx+a)^5}{4\cos(bx+a)^4}-\frac{\sin(bx+a)^5}{8\cos(bx+a)^2}-\frac{\sin(bx+a)^3}{8}-\frac{3\sin(bx+a)}{8}+\frac{3\ln(\sec(bx+a)+\tan(bx+a))}{8}}{b}$	76
risch	$\frac{i\left(5\,e^{7i(bx+a)}-3\,e^{5i(bx+a)}+3\,e^{3i(bx+a)}-5\,e^{i(bx+a)}\right)}{4b\left(e^{2i(bx+a)}+1\right)^4}-\frac{3\ln\left(e^{i(bx+a)}-i\right)}{8b}+\frac{3\ln\left(e^{i(bx+a)}+i\right)}{8b}$	102

input
```
int(sec(b*x+a)*tan(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/4*sin(b*x+a)^5/cos(b*x+a)^4-1/8*sin(b*x+a)^5/cos(b*x+a)^2-1/8*sin(b
*x+a)^3-3/8*sin(b*x+a)+3/8*ln(sec(b*x+a)+tan(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.35

$$\int \sec(a+bx)\tan^4(a+bx)\,dx = \frac{3\cos(bx+a)^4\log(\sin(bx+a)+1) - 3\cos(bx+a)^4\log(-\sin(bx+a)+1) - 2\left(5\cos(bx+a)^2-2\right)}{16\,b\cos(bx+a)^4}$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/16*(3*cos(b*x + a)^4*log(sin(b*x + a) + 1) - 3*cos(b*x + a)^4*log(-sin(b
*x + a) + 1) - 2*(5*cos(b*x + a)^2 - 2)*sin(b*x + a))/(b*cos(b*x + a)^4)
```

Sympy [F]

$$\int \sec(a+bx)\tan^4(a+bx)\,dx = \int \tan^4(a+bx)\sec(a+bx)\,dx$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)**4,x)
```

output
```
Integral(tan(a + b*x)**4*sec(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.29

$$\int \sec(a+bx)\tan^4(a+bx)\,dx = \frac{\frac{2\left(5\sin(bx+a)^3-3\sin(bx+a)\right)}{\sin(bx+a)^4-2\sin(bx+a)^2+1}+3\log\left(\sin\left(bx+a\right)+1\right)-3\log\left(\sin\left(bx+a\right)-1\right)}{16\,b}$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/16*(2*(5*sin(b*x + a)^3 - 3*sin(b*x + a))/(sin(b*x + a)^4 - 2*sin(b*x +
a)^2 + 1) + 3*log(sin(b*x + a) + 1) - 3*log(sin(b*x + a) - 1))/b
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

$$\int \sec(a+bx)\tan^4(a+bx)\,dx = \frac{\frac{2\left(5\sin(bx+a)^3-3\sin(bx+a)\right)}{\left(\sin(bx+a)^2-1\right)^2}+3\log\left(\left|\sin\left(bx+a\right)+1\right|\right)-3\log\left(\left|\sin\left(bx+a\right)-1\right|\right)}{16\,b}$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)^4,x, algorithm="giac")
```

output
```
1/16*(2*(5*sin(b*x + a)^3 - 3*sin(b*x + a))/(sin(b*x + a)^2 - 1)^2 + 3*log
(abs(sin(b*x + a) + 1)) - 3*log(abs(sin(b*x + a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 29.01 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.29

$$\int \sec(a+bx)\tan^4(a+bx)\,dx$$
$$= \frac{3\operatorname{atanh}\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{4b} - \frac{\frac{3\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^7}{4} - \frac{11\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^5}{4} - \frac{11\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^3}{4} + \frac{3\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{4}}{b\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^8 - 4\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^6 + 6\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^4 - 4\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2 + 1\right)}$$

input
```
int(tan(a + b*x)^4/cos(a + b*x),x)
```

output
```
(3*atanh(tan(a/2 + (b*x)/2)))/(4*b) - ((3*tan(a/2 + (b*x)/2))/4 - (11*tan(
a/2 + (b*x)/2)^3)/4 - (11*tan(a/2 + (b*x)/2)^5)/4 + (3*tan(a/2 + (b*x)/2)^
7)/4)/(b*(6*tan(a/2 + (b*x)/2)^4 - 4*tan(a/2 + (b*x)/2)^2 - 4*tan(a/2 + (b
*x)/2)^6 + tan(a/2 + (b*x)/2)^8 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.95

$$\int \sec(a+bx)\tan^4(a+bx)\,dx$$
$$= \frac{-3\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin(bx+a)^4 + 6\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin(bx+a)^2 - 3\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{8b}$$

input
```
int(sec(b*x+a)*tan(b*x+a)^4,x)
```

output
```
( - 3*log(tan((a + b*x)/2) - 1)*sin(a + b*x)**4 + 6*log(tan((a + b*x)/2) -
 1)*sin(a + b*x)**2 - 3*log(tan((a + b*x)/2) - 1) + 3*log(tan((a + b*x)/2)
 + 1)*sin(a + b*x)**4 - 6*log(tan((a + b*x)/2) + 1)*sin(a + b*x)**2 + 3*lo
g(tan((a + b*x)/2) + 1) + 5*sin(a + b*x)**3 - 3*sin(a + b*x))/(8*b*(sin(a
 + b*x)**4 - 2*sin(a + b*x)**2 + 1))
```

3.96 $\int \sec^3(a+bx)\tan^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 78

$$\int \sec^3(a+bx)\tan^4(a+bx)\,dx=\frac{\operatorname{arctanh}(\sin(a+bx))}{16b}+\frac{\sec(a+bx)\tan(a+bx)}{16b}-\frac{\sec^3(a+bx)\tan(a+bx)}{8b}+\frac{\sec^3(a+bx)\tan^3(a+bx)}{6b}$$

output
```
1/16*arctanh(sin(b*x+a))/b+1/16*sec(b*x+a)*tan(b*x+a)/b-1/8*sec(b*x+a)^3*t
an(b*x+a)/b+1/6*sec(b*x+a)^3*tan(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.27

$$\int \sec^3(a+bx)\tan^4(a+bx)\,dx=\frac{\operatorname{arctanh}(\sin(a+bx))}{16b}+\frac{\sec(a+bx)\tan(a+bx)}{16b}+\frac{\sec^3(a+bx)\tan(a+bx)}{24b}-\frac{\sec^5(a+bx)\tan(a+bx)}{6b}+\frac{\sec^3(a+bx)\tan^3(a+bx)}{3b}$$

input
```
Integrate[Sec[a + b*x]^3*Tan[a + b*x]^4,x]
```

output
```
ArcTanh[Sin[a + b*x]]/(16*b) + (Sec[a + b*x]*Tan[a + b*x])/(16*b) + (Sec[a
 + b*x]^3*Tan[a + b*x])/(24*b) - (Sec[a + b*x]^5*Tan[a + b*x])/(6*b) + (Se
c[a + b*x]^3*Tan[a + b*x]^3)/(3*b)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {3042, 3091, 3042, 3091, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(a+bx)\sec^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(a+bx)^4\sec(a+bx)^3dx$$

$$\Big\downarrow \text{3091}$$

$$\frac{\tan^3(a+bx)\sec^3(a+bx)}{6b}-\frac{1}{2}\int \sec^3(a+bx)\tan^2(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^3(a+bx)\sec^3(a+bx)}{6b}-\frac{1}{2}\int \sec(a+bx)^3\tan(a+bx)^2dx$$

$$\Big\downarrow \text{3091}$$

$$\frac{1}{2}\left(\frac{1}{4}\int \sec^3(a+bx)dx-\frac{\tan(a+bx)\sec^3(a+bx)}{4b}\right)+\frac{\tan^3(a+bx)\sec^3(a+bx)}{6b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{1}{4}\int \csc\left(a+bx+\frac{\pi}{2}\right)^3dx-\frac{\tan(a+bx)\sec^3(a+bx)}{4b}\right)+\frac{\tan^3(a+bx)\sec^3(a+bx)}{6b}$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{1}{2}\int \sec(a+bx)dx+\frac{\tan(a+bx)\sec(a+bx)}{2b}\right)-\frac{\tan(a+bx)\sec^3(a+bx)}{4b}\right)+\frac{\tan^3(a+bx)\sec^3(a+bx)}{6b}$$

$\big\downarrow$ 3042

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{1}{2}\int \csc\left(a+bx+\frac{\pi}{2}\right)dx+\frac{\tan(a+bx)\sec(a+bx)}{2b}\right)-\frac{\tan(a+bx)\sec^3(a+bx)}{4b}\right)+\frac{\tan^3(a+bx)\sec^3(a+bx)}{6b}$$

$\big\downarrow$ 4257

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{\text{arctanh}(\sin(a+bx))}{2b}+\frac{\tan(a+bx)\sec(a+bx)}{2b}\right)-\frac{\tan(a+bx)\sec^3(a+bx)}{4b}\right)+\frac{\tan^3(a+bx)\sec^3(a+bx)}{6b}$$

input
```
Int[Sec[a + b*x]^3*Tan[a + b*x]^4,x]
```

output
```
(Sec[a + b*x]^3*Tan[a + b*x]^3)/(6*b) + (-1/4*(Sec[a + b*x]^3*Tan[a + b*x]
)/b + (ArcTanh[Sin[a + b*x]]/(2*b) + (Sec[a + b*x]*Tan[a + b*x])/(2*b))/4)
/2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 2.99 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{\frac{\sin(bx+a)^5}{6\cos(bx+a)^6}+\frac{\sin(bx+a)^5}{24\cos(bx+a)^4}-\frac{\sin(bx+a)^5}{48\cos(bx+a)^2}-\frac{\sin(bx+a)^3}{48}-\frac{\sin(bx+a)}{16}+\frac{\ln(\sec(bx+a)+\tan(bx+a))}{16}}{b}$
default	$\frac{\frac{\sin(bx+a)^5}{6\cos(bx+a)^6}+\frac{\sin(bx+a)^5}{24\cos(bx+a)^4}-\frac{\sin(bx+a)^5}{48\cos(bx+a)^2}-\frac{\sin(bx+a)^3}{48}-\frac{\sin(bx+a)}{16}+\frac{\ln(\sec(bx+a)+\tan(bx+a))}{16}}{b}$
risch	$-\frac{i\left(3\,e^{11i(bx+a)}-47\,e^{9i(bx+a)}+78\,e^{7i(bx+a)}-78\,e^{5i(bx+a)}+47\,e^{3i(bx+a)}-3\,e^{i(bx+a)}\right)}{24b\left(e^{2i(bx+a)}+1\right)^6}+\frac{\ln\left(e^{i(bx+a)}+i\right)}{16b}-\frac{\ln(e^{i(bx}}{16}$

input
```
int(sec(b*x+a)^3*tan(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/6*sin(b*x+a)^5/cos(b*x+a)^6+1/24*sin(b*x+a)^5/cos(b*x+a)^4-1/48*sin
(b*x+a)^5/cos(b*x+a)^2-1/48*sin(b*x+a)^3-1/16*sin(b*x+a)+1/16*ln(sec(b*x+a
)+tan(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.08

$$\int \sec^3(a+bx)\tan^4(a+bx)\,dx$$
$$=\frac{3\,\cos(bx+a)^6\log(\sin(bx+a)+1)-3\,\cos(bx+a)^6\log(-\sin(bx+a)+1)+2\left(3\,\cos(bx+a)^4-14\right.}{96\,b\cos(bx+a)^6}$$

input
```
integrate(sec(b*x+a)^3*tan(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/96*(3*cos(b*x + a)^6*log(sin(b*x + a) + 1) - 3*cos(b*x + a)^6*log(-sin(b
*x + a) + 1) + 2*(3*cos(b*x + a)^4 - 14*cos(b*x + a)^2 + 8)*sin(b*x + a))/
(b*cos(b*x + a)^6)
```

Sympy [F]

$$\int \sec^3(a+bx)\tan^4(a+bx)\,dx = \int \tan^4(a+bx)\sec^3(a+bx)\,dx$$

input
```
integrate(sec(b*x+a)**3*tan(b*x+a)**4,x)
```

output
```
Integral(tan(a + b*x)**4*sec(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.17

$$\int \sec^3(a+bx)\tan^4(a+bx)\,dx = -\frac{\frac{2\left(3\sin(bx+a)^5+8\sin(bx+a)^3-3\sin(bx+a)\right)}{\sin(bx+a)^6-3\sin(bx+a)^4+3\sin(bx+a)^2-1} - 3\log\left(\sin\left(bx+a\right)+1\right) + 3\log\left(\sin\left(bx+a\right)-1\right)}{96\,b}$$

input
```
integrate(sec(b*x+a)^3*tan(b*x+a)^4,x, algorithm="maxima")
```

output
```
-1/96*(2*(3*sin(b*x + a)^5 + 8*sin(b*x + a)^3 - 3*sin(b*x + a))/(sin(b*x +
 a)^6 - 3*sin(b*x + a)^4 + 3*sin(b*x + a)^2 - 1) - 3*log(sin(b*x + a) + 1)
 + 3*log(sin(b*x + a) - 1))/b
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

$$\int \sec^3(a+bx)\tan^4(a+bx)\,dx = -\frac{\frac{2\left(3\sin(bx+a)^5+8\sin(bx+a)^3-3\sin(bx+a)\right)}{\left(\sin(bx+a)^2-1\right)^3} - 3\log\left(\left|\sin\left(bx+a\right)+1\right|\right) + 3\log\left(\left|\sin\left(bx+a\right)-1\right|\right)}{96\,b}$$

input
```
integrate(sec(b*x+a)^3*tan(b*x+a)^4,x, algorithm="giac")
```

output
```
-1/96*(2*(3*sin(b*x + a)^5 + 8*sin(b*x + a)^3 - 3*sin(b*x + a))/(sin(b*x +
 a)^2 - 1)^3 - 3*log(abs(sin(b*x + a) + 1)) + 3*log(abs(sin(b*x + a) - 1))
)/b
```

Mupad [B] (verification not implemented)

Time = 29.72 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.27

$$\int \sec^3(a+bx)\tan^4(a+bx)\,dx = \frac{\mathrm{atanh}\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{8\,b} + \frac{-\frac{\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{11}}{8}+\frac{17\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{9}}{24}+\frac{19\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{7}}{4}+\frac{19\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{5}}{4}+\frac{17\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{3}}{24}-\frac{\tan\left(\frac{a}{2}+\frac{b}{}\right.}{8}}{b\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{12}-6\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{10}+15\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{8}-20\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{6}+15\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{4}-6\,\mathrm{ta}\right.}$$

input
```
int(tan(a + b*x)^4/cos(a + b*x)^3,x)
```

output
```
atanh(tan(a/2 + (b*x)/2))/(8*b) + ((17*tan(a/2 + (b*x)/2)^3)/24 - tan(a/2
+ (b*x)/2)/8 + (19*tan(a/2 + (b*x)/2)^5)/4 + (19*tan(a/2 + (b*x)/2)^7)/4 +
 (17*tan(a/2 + (b*x)/2)^9)/24 - tan(a/2 + (b*x)/2)^11/8)/(b*(15*tan(a/2 +
(b*x)/2)^4 - 6*tan(a/2 + (b*x)/2)^2 - 20*tan(a/2 + (b*x)/2)^6 + 15*tan(a/2
 + (b*x)/2)^8 - 6*tan(a/2 + (b*x)/2)^10 + tan(a/2 + (b*x)/2)^12 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.90

$$\int \sec^3(a+bx)\tan^4(a+bx)\,dx$$
$$= \frac{-3\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin(bx+a)^6+9\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin(bx+a)^4-9\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{}$$

input
```
int(sec(b*x+a)^3*tan(b*x+a)^4,x)
```

output
```
( - 3*log(tan((a + b*x)/2) - 1)*sin(a + b*x)**6 + 9*log(tan((a + b*x)/2) -
 1)*sin(a + b*x)**4 - 9*log(tan((a + b*x)/2) - 1)*sin(a + b*x)**2 + 3*log(
tan((a + b*x)/2) - 1) + 3*log(tan((a + b*x)/2) + 1)*sin(a + b*x)**6 - 9*lo
g(tan((a + b*x)/2) + 1)*sin(a + b*x)**4 + 9*log(tan((a + b*x)/2) + 1)*sin(
a + b*x)**2 - 3*log(tan((a + b*x)/2) + 1) - 3*sin(a + b*x)**5 - 8*sin(a +
b*x)**3 + 3*sin(a + b*x))/(48*b*(sin(a + b*x)**6 - 3*sin(a + b*x)**4 + 3*s
in(a + b*x)**2 - 1))
```

3.97 $\int \sec^5(a+bx)\tan^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 99

$$\int \sec^5(a+bx)\tan^4(a+bx)\,dx = \frac{3\text{arctanh}(\sin(a+bx))}{128b} + \frac{3\sec(a+bx)\tan(a+bx)}{128b} + \frac{\sec^3(a+bx)\tan(a+bx)}{64b} - \frac{\sec^5(a+bx)\tan(a+bx)}{16b} + \frac{\sec^5(a+bx)\tan^3(a+bx)}{8b}$$

output
```
3/128*arctanh(sin(b*x+a))/b+3/128*sec(b*x+a)*tan(b*x+a)/b+1/64*sec(b*x+a)^
3*tan(b*x+a)/b-1/16*sec(b*x+a)^5*tan(b*x+a)/b+1/8*sec(b*x+a)^5*tan(b*x+a)^
3/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.21

$$\int \sec^5(a+bx)\tan^4(a+bx)\,dx = \frac{3\text{arctanh}(\sin(a+bx))}{128b} + \frac{3\sec(a+bx)\tan(a+bx)}{128b} + \frac{\sec^3(a+bx)\tan(a+bx)}{64b} + \frac{\sec^5(a+bx)\tan(a+bx)}{80b} - \frac{3\sec^7(a+bx)\tan(a+bx)}{40b} + \frac{\sec^5(a+bx)\tan^3(a+bx)}{5b}$$

input
```
Integrate[Sec[a + b*x]^5*Tan[a + b*x]^4,x]
```

output
```
(3*ArcTanh[Sin[a + b*x]])/(128*b) + (3*Sec[a + b*x]*Tan[a + b*x])/(128*b)
+ (Sec[a + b*x]^3*Tan[a + b*x])/(64*b) + (Sec[a + b*x]^5*Tan[a + b*x])/(80
*b) - (3*Sec[a + b*x]^7*Tan[a + b*x])/(40*b) + (Sec[a + b*x]^5*Tan[a + b*x
]^3)/(5*b)
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {3042, 3091, 3042, 3091, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(a+bx)\sec^5(a+bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \tan(a+bx)^4\sec(a+bx)^5dx$$

$$\Big\downarrow \ 3091$$

$$\frac{\tan^3(a+bx)\sec^5(a+bx)}{8b} - \frac{3}{8}\int \sec^5(a+bx)\tan^2(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^3(a+bx)\sec^5(a+bx)}{8b}-\frac{3}{8}\int \sec(a+bx)^5\tan(a+bx)^2dx$$

$$\Big\downarrow \text{3091}$$

$$\frac{\tan^3(a+bx)\sec^5(a+bx)}{8b}-\frac{3}{8}\left(\frac{\tan(a+bx)\sec^5(a+bx)}{6b}-\frac{1}{6}\int \sec^5(a+bx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^3(a+bx)\sec^5(a+bx)}{8b}-\frac{3}{8}\left(\frac{\tan(a+bx)\sec^5(a+bx)}{6b}-\frac{1}{6}\int \csc\left(a+bx+\frac{\pi}{2}\right)^5dx\right)$$

$$\Big\downarrow \text{4255}$$

$$\frac{\tan^3(a+bx)\sec^5(a+bx)}{8b}-\frac{3}{8}\left(\frac{1}{6}\left(-\frac{3}{4}\int \sec^3(a+bx)dx-\frac{\tan(a+bx)\sec^3(a+bx)}{4b}\right)+\frac{\tan(a+bx)\sec^5(a+bx)}{6b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^3(a+bx)\sec^5(a+bx)}{8b}-\frac{3}{8}\left(\frac{1}{6}\left(-\frac{3}{4}\int \csc\left(a+bx+\frac{\pi}{2}\right)^3dx-\frac{\tan(a+bx)\sec^3(a+bx)}{4b}\right)+\frac{\tan(a+bx)\sec^5(a+bx)}{6b}\right)$$

$$\Big\downarrow \text{4255}$$

$$\frac{\tan^3(a+bx)\sec^5(a+bx)}{8b}-\frac{3}{8}\left(\frac{1}{6}\left(-\frac{3}{4}\left(\frac{1}{2}\int \sec(a+bx)dx+\frac{\tan(a+bx)\sec(a+bx)}{2b}\right)-\frac{\tan(a+bx)\sec^3(a+bx)}{4b}\right)+\frac{\tan(a+bx)\sec^5(a+}{6b}\right.$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^3(a+bx)\sec^5(a+bx)}{8b}-\frac{3}{8}\left(\frac{1}{6}\left(-\frac{3}{4}\left(\frac{1}{2}\int \csc\left(a+bx+\frac{\pi}{2}\right)dx+\frac{\tan(a+bx)\sec(a+bx)}{2b}\right)-\frac{\tan(a+bx)\sec^3(a+bx)}{4b}\right)+\frac{\tan(a+bx)\,\mathrm{se}}{6b}\right.$$

$$\Big\downarrow \text{4257}$$

$$\frac{\tan^3(a+bx)\sec^5(a+bx)}{8b}-\frac{3}{8}\left(\frac{1}{6}\left(-\frac{3}{4}\left(\frac{\text{arctanh}(\sin(a+bx))}{2b}+\frac{\tan(a+bx)\sec(a+bx)}{2b}\right)-\frac{\tan(a+bx)\sec^3(a+bx)}{4b}\right)+\frac{\tan(a+bx)\sec^5(}{6b}\right.$$

input
```
Int[Sec[a + b*x]^5*Tan[a + b*x]^4,x]
```

output
```
(Sec[a + b*x]^5*Tan[a + b*x]^3)/(8*b) - (3*((Sec[a + b*x]^5*Tan[a + b*x])/
(6*b) + (-1/4*(Sec[a + b*x]^3*Tan[a + b*x])/b - (3*(ArcTanh[Sin[a + b*x]]/
(2*b) + (Sec[a + b*x]*Tan[a + b*x])/(2*b)))/4)/6))/8
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 9.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{\frac{\sin(bx+a)^5}{8\cos(bx+a)^8}+\frac{\sin(bx+a)^5}{16\cos(bx+a)^6}+\frac{\sin(bx+a)^5}{64\cos(bx+a)^4}-\frac{\sin(bx+a)^5}{128\cos(bx+a)^2}-\frac{\sin(bx+a)^3}{128}-\frac{3\sin(bx+a)}{128}+\frac{3\ln(\sec(bx+a)+\tan(bx+a))}{128}}{b}$
default	$\frac{\frac{\sin(bx+a)^5}{8\cos(bx+a)^8}+\frac{\sin(bx+a)^5}{16\cos(bx+a)^6}+\frac{\sin(bx+a)^5}{64\cos(bx+a)^4}-\frac{\sin(bx+a)^5}{128\cos(bx+a)^2}-\frac{\sin(bx+a)^3}{128}-\frac{3\sin(bx+a)}{128}+\frac{3\ln(\sec(bx+a)+\tan(bx+a))}{128}}{b}$
risch	$-\frac{i(3\,e^{15i(bx+a)}+23\,e^{13i(bx+a)}-333\,e^{11i(bx+a)}+671\,e^{9i(bx+a)}-671\,e^{7i(bx+a)}+333\,e^{5i(bx+a)}-23\,e^{3i(bx+a)}-3\,e^{i(bx+a}}{64b\left(e^{2i(bx+a)}+1\right)^8}$

input
```
int(sec(b*x+a)^5*tan(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/8*sin(b*x+a)^5/cos(b*x+a)^8+1/16*sin(b*x+a)^5/cos(b*x+a)^6+1/64*sin
(b*x+a)^5/cos(b*x+a)^4-1/128*sin(b*x+a)^5/cos(b*x+a)^2-1/128*sin(b*x+a)^3-
3/128*sin(b*x+a)+3/128*ln(sec(b*x+a)+tan(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.95

$$\int \sec^5(a+bx)\tan^4(a+bx)\,dx$$
$$= \frac{3\cos(bx+a)^8\log(\sin(bx+a)+1)-3\cos(bx+a)^8\log(-\sin(bx+a)+1)+2\left(3\cos(bx+a)^6+2\,c\right.}{256\,b\cos(bx+a)^8}$$

input
```
integrate(sec(b*x+a)^5*tan(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/256*(3*cos(b*x + a)^8*log(sin(b*x + a) + 1) - 3*cos(b*x + a)^8*log(-sin(
b*x + a) + 1) + 2*(3*cos(b*x + a)^6 + 2*cos(b*x + a)^4 - 24*cos(b*x + a)^2
 + 16)*sin(b*x + a))/(b*cos(b*x + a)^8)
```

Sympy [F]

$$\int \sec^5(a+bx)\tan^4(a+bx)\,dx = \int \tan^4(a+bx)\sec^5(a+bx)\,dx$$

input
```
integrate(sec(b*x+a)**5*tan(b*x+a)**4,x)
```

output
```
Integral(tan(a + b*x)**4*sec(a + b*x)**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.12

$$\int \sec^5(a+bx)\tan^4(a+bx)\,dx = -\frac{\frac{2\left(3\sin(bx+a)^7-11\sin(bx+a)^5-11\sin(bx+a)^3+3\sin(bx+a)\right)}{\sin(bx+a)^8-4\sin(bx+a)^6+6\sin(bx+a)^4-4\sin(bx+a)^2+1}-3\log\left(\sin(bx+a)+1\right)+3\log\left(\sin(bx+a)-1\right)}{256\,b}$$

input
```
integrate(sec(b*x+a)^5*tan(b*x+a)^4,x, algorithm="maxima")
```

output
```
-1/256*(2*(3*sin(b*x + a)^7 - 11*sin(b*x + a)^5 - 11*sin(b*x + a)^3 + 3*si
n(b*x + a))/(sin(b*x + a)^8 - 4*sin(b*x + a)^6 + 6*sin(b*x + a)^4 - 4*sin(
b*x + a)^2 + 1) - 3*log(sin(b*x + a) + 1) + 3*log(sin(b*x + a) - 1))/b
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.08

$$\int \sec^5(a+bx)\tan^4(a+bx)\,dx = -\frac{\frac{4\left(3\left(\frac{1}{\sin(bx+a)}+\sin(bx+a)\right)^3-\frac{20}{\sin(bx+a)}-20\sin(bx+a)\right)}{\left(\left(\frac{1}{\sin(bx+a)}+\sin(bx+a)\right)^2-4\right)^2}-3\log\left(\left|\frac{1}{\sin(bx+a)}+\sin(bx+a)+2\right|\right)+3\log\left(\left|\frac{1}{\sin(bx+a)}\right.\right.}{512\,b}$$

input
```
integrate(sec(b*x+a)^5*tan(b*x+a)^4,x, algorithm="giac")
```

output
```
-1/512*(4*(3*(1/sin(b*x + a) + sin(b*x + a))^3 - 20/sin(b*x + a) - 20*sin(
b*x + a))/((1/sin(b*x + a) + sin(b*x + a))^2 - 4)^2 - 3*log(abs(1/sin(b*x
+ a) + sin(b*x + a) + 2)) + 3*log(abs(1/sin(b*x + a) + sin(b*x + a) - 2)))
/b
```

Mupad [B] (verification not implemented)

Time = 30.40 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.31

$$\int \sec^5(a+bx)\tan^4(a+bx)\,dx = \frac{3\,\mathrm{atanh}\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{64\,b} + \frac{-\frac{3\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{15}}{64}+\frac{23\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{13}}{64}+\frac{333\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{11}}{64}+\frac{671\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{9}}{64}+\frac{671\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{7}}{64}+\frac{3}{}}{b\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{16}-8\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{14}+28\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{12}-56\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{10}+70\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{8}-5\right.}$$

input
```
int(tan(a + b*x)^4/cos(a + b*x)^5,x)
```

output
```
(3*atanh(tan(a/2 + (b*x)/2)))/(64*b) + ((23*tan(a/2 + (b*x)/2)^3)/64 - (3*
tan(a/2 + (b*x)/2))/64 + (333*tan(a/2 + (b*x)/2)^5)/64 + (671*tan(a/2 + (b
*x)/2)^7)/64 + (671*tan(a/2 + (b*x)/2)^9)/64 + (333*tan(a/2 + (b*x)/2)^11)
/64 + (23*tan(a/2 + (b*x)/2)^13)/64 - (3*tan(a/2 + (b*x)/2)^15)/64)/(b*(28
*tan(a/2 + (b*x)/2)^4 - 8*tan(a/2 + (b*x)/2)^2 - 56*tan(a/2 + (b*x)/2)^6 +
 70*tan(a/2 + (b*x)/2)^8 - 56*tan(a/2 + (b*x)/2)^10 + 28*tan(a/2 + (b*x)/2
)^12 - 8*tan(a/2 + (b*x)/2)^14 + tan(a/2 + (b*x)/2)^16 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.93

$$\int \sec^5(a+bx)\tan^4(a+bx)\,dx = \frac{-3\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin(bx+a)^8+12\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin(bx+a)^6-18\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-\right.}{}$$

input
```
int(sec(b*x+a)^5*tan(b*x+a)^4,x)
```

output
```
( - 3*log(tan((a + b*x)/2) - 1)*sin(a + b*x)**8 + 12*log(tan((a + b*x)/2)
- 1)*sin(a + b*x)**6 - 18*log(tan((a + b*x)/2) - 1)*sin(a + b*x)**4 + 12*l
og(tan((a + b*x)/2) - 1)*sin(a + b*x)**2 - 3*log(tan((a + b*x)/2) - 1) + 3
*log(tan((a + b*x)/2) + 1)*sin(a + b*x)**8 - 12*log(tan((a + b*x)/2) + 1)*
sin(a + b*x)**6 + 18*log(tan((a + b*x)/2) + 1)*sin(a + b*x)**4 - 12*log(ta
n((a + b*x)/2) + 1)*sin(a + b*x)**2 + 3*log(tan((a + b*x)/2) + 1) - 3*sin(
a + b*x)**7 + 11*sin(a + b*x)**5 + 11*sin(a + b*x)**3 - 3*sin(a + b*x))/(1
28*b*(sin(a + b*x)**8 - 4*sin(a + b*x)**6 + 6*sin(a + b*x)**4 - 4*sin(a +
b*x)**2 + 1))
```

3.98 $\int \cos^7(a+bx)\sin^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \cos^7(a+bx)\sin^5(a+bx)\,dx=-\frac{\cos^8(a+bx)}{8b}+\frac{\cos^{10}(a+bx)}{5b}-\frac{\cos^{12}(a+bx)}{12b}$$

output
```
-1/8*cos(b*x+a)^8/b+1/5*cos(b*x+a)^10/b-1/12*cos(b*x+a)^12/b
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.48

$$\int \cos^7(a+bx)\sin^5(a+bx)\,dx=$$
$$-\frac{600\cos(2(a+bx))+75\cos(4(a+bx))-100\cos(6(a+bx))-30\cos(8(a+bx))+12\cos(10(a+bx)}{122880b}$$

input
```
Integrate[Cos[a + b*x]^7*Sin[a + b*x]^5,x]
```

output
```
-1/122880*(600*Cos[2*(a + b*x)] + 75*Cos[4*(a + b*x)] - 100*Cos[6*(a + b*x
)] - 30*Cos[8*(a + b*x)] + 12*Cos[10*(a + b*x)] + 5*Cos[12*(a + b*x)])/b
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3042, 3045, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(a+bx)\cos^7(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^5\cos(a+bx)^7dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int \cos^7(a+bx)\left(1-\cos^2(a+bx)\right)^2d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{243}$$

$$-\frac{\int \cos^6(a+bx)\left(1-\cos^2(a+bx)\right)^2d\cos^2(a+bx)}{2b}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\int \left(\cos^{10}(a+bx)-2\cos^8(a+bx)+\cos^6(a+bx)\right)d\cos^2(a+bx)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{6}\cos^{12}(a+bx)-\frac{2}{5}\cos^{10}(a+bx)+\frac{1}{4}\cos^8(a+bx)}{2b}$$

input
```
Int[Cos[a + b*x]^7*Sin[a + b*x]^5,x]
```

output
```
-1/2*(Cos[a + b*x]^8/4 - (2*Cos[a + b*x]^10)/5 + Cos[a + b*x]^12/6)/b
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [A] (verified)

Time = 43.96 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

method	result
derivativedivides	$-\frac{\frac{\sin(bx+a)^{12}}{12}-\frac{3\sin(bx+a)^{10}}{10}+\frac{3\sin(bx+a)^{8}}{8}-\frac{\sin(bx+a)^{6}}{6}}{b}$
default	$-\frac{\frac{\sin(bx+a)^{12}}{12}-\frac{3\sin(bx+a)^{10}}{10}+\frac{3\sin(bx+a)^{8}}{8}-\frac{\sin(bx+a)^{6}}{6}}{b}$
parallelrisch	$\frac{-600\cos(2bx+2a)+100\cos(6bx+6a)+30\cos(8bx+8a)-75\cos(4bx+4a)+562-5\cos(12bx+12a)-12\cos(10bx+10a)}{122880b}$
risch	$-\frac{\cos(12bx+12a)}{24576b}-\frac{\cos(10bx+10a)}{10240b}+\frac{\cos(8bx+8a)}{4096b}+\frac{5\cos(6bx+6a)}{6144b}-\frac{5\cos(4bx+4a)}{8192b}-\frac{5\cos(2bx+2a)}{1024b}$
orering	$-\frac{5369\left(-7\cos(bx+a)^{6}\sin(bx+a)^{6}b+5\cos(bx+a)^{8}\sin(bx+a)^{4}b\right)}{14400b^{2}}-\frac{37037\left(-210\cos(bx+a)^{4}\sin(bx+a)^{8}b^{3}+868\cos\right.}{}$

input
```
int(cos(b*x+a)^7*sin(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/b*(1/12*sin(b*x+a)^12-3/10*sin(b*x+a)^10+3/8*sin(b*x+a)^8-1/6*sin(b*x+a
)^6)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^7(a+bx)\sin^5(a+bx)\,dx$$
$$= -\frac{10\cos(bx+a)^{12} - 24\cos(bx+a)^{10} + 15\cos(bx+a)^{8}}{120\,b}$$

input
```
integrate(cos(b*x+a)^7*sin(b*x+a)^5,x, algorithm="fricas")
```

output
```
-1/120*(10*cos(b*x + a)^12 - 24*cos(b*x + a)^10 + 15*cos(b*x + a)^8)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(34) = 68.

Time = 2.46 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.80

$$\int \cos^7(a+bx)\sin^5(a+bx)\,dx$$
$$= \begin{cases} \frac{\sin^{12}(a+bx)}{120b} + \frac{\sin^{10}(a+bx)\cos^2(a+bx)}{20b} + \frac{\sin^8(a+bx)\cos^4(a+bx)}{8b} + \frac{\sin^6(a+bx)\cos^6(a+bx)}{6b} & \text{for } b \neq 0 \\ x\sin^5(a)\cos^7(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a)**7*sin(b*x+a)**5,x)
```

output
```
Piecewise((sin(a + b*x)**12/(120*b) + sin(a + b*x)**10*cos(a + b*x)**2/(20
*b) + sin(a + b*x)**8*cos(a + b*x)**4/(8*b) + sin(a + b*x)**6*cos(a + b*x)
**6/(6*b), Ne(b, 0)), (x*sin(a)**5*cos(a)**7, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \cos^7(a+bx)\sin^5(a+bx)\,dx = -\frac{10\,\sin\left(bx+a\right)^{12}-36\,\sin\left(bx+a\right)^{10}+45\,\sin\left(bx+a\right)^{8}-20\,\sin\left(bx+a\right)^{6}}{120\,b}$$

input
```
integrate(cos(b*x+a)^7*sin(b*x+a)^5,x, algorithm="maxima")
```

output
```
-1/120*(10*sin(b*x + a)^12 - 36*sin(b*x + a)^10 + 45*sin(b*x + a)^8 - 20*s
in(b*x + a)^6)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^7(a+bx)\sin^5(a+bx)\,dx = -\frac{10\,\cos\left(bx+a\right)^{12}-24\,\cos\left(bx+a\right)^{10}+15\,\cos\left(bx+a\right)^{8}}{120\,b}$$

input
```
integrate(cos(b*x+a)^7*sin(b*x+a)^5,x, algorithm="giac")
```

output
```
-1/120*(10*cos(b*x + a)^12 - 24*cos(b*x + a)^10 + 15*cos(b*x + a)^8)/b
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \cos^7(a+bx)\sin^5(a+bx)\,dx = -\frac{\cos\left(a+b\,x\right)^8\,\left(10\cos\left(a+b\,x\right)^4-24\cos\left(a+b\,x\right)^2+15\right)}{120\,b}$$

input
```
int(cos(a + b*x)^7*sin(a + b*x)^5,x)
```

output
```
-(cos(a + b*x)^8*(10*cos(a + b*x)^4 - 24*cos(a + b*x)^2 + 15))/(120*b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \cos^7(a+bx)\sin^5(a+bx)\,dx = \frac{\sin(bx+a)^6\left(-10\sin(bx+a)^6+36\sin(bx+a)^4-45\sin(bx+a)^2+20\right)}{120b}$$

input
```
int(cos(b*x+a)^7*sin(b*x+a)^5,x)
```

output
```
(sin(a + b*x)**6*( - 10*sin(a + b*x)**6 + 36*sin(a + b*x)**4 - 45*sin(a +
b*x)**2 + 20))/(120*b)
```

3.99 $\int \cos^6(a+bx)\sin^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \cos^6(a+bx)\sin^5(a+bx)\,dx = -\frac{\cos^7(a+bx)}{7b}+\frac{2\cos^9(a+bx)}{9b}-\frac{\cos^{11}(a+bx)}{11b}$$

output
```
-1/7*cos(b*x+a)^7/b+2/9*cos(b*x+a)^9/b-1/11*cos(b*x+a)^11/b
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \cos^6(a+bx)\sin^5(a+bx)\,dx = \frac{\cos^7(a+bx)(-365+364\cos(2(a+bx))-63\cos(4(a+bx)))}{5544b}$$

input
```
Integrate[Cos[a + b*x]^6*Sin[a + b*x]^5,x]
```

output
```
(Cos[a + b*x]^7*(-365 + 364*Cos[2*(a + b*x)] - 63*Cos[4*(a + b*x)]))/(5544
*b)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(a+bx)\cos^6(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^5\cos(a+bx)^6dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int \cos^6(a+bx)\left(1-\cos^2(a+bx)\right)^2d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(\cos^{10}(a+bx)-2\cos^8(a+bx)+\cos^6(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{11}\cos^{11}(a+bx)-\frac{2}{9}\cos^9(a+bx)+\frac{1}{7}\cos^7(a+bx)}{b}$$

input
```
Int[Cos[a + b*x]^6*Sin[a + b*x]^5,x]
```

output
```
-((Cos[a + b*x]^7/7 - (2*Cos[a + b*x]^9)/9 + Cos[a + b*x]^11/11)/b)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [A] (verified)

Time = 34.79 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

method	result
derivativedivides	$-\frac{\frac{\cos(bx+a)^{11}}{11}-\frac{2\cos(bx+a)^{9}}{9}+\frac{\cos(bx+a)^{7}}{7}}{b}$
default	$-\frac{\frac{\cos(bx+a)^{11}}{11}-\frac{2\cos(bx+a)^{9}}{9}+\frac{\cos(bx+a)^{7}}{7}}{b}$
parallelrisch	$\frac{-8192-63\cos(11bx+11a)-2310\cos(3bx+3a)-6930\cos(bx+a)+693\cos(5bx+5a)+495\cos(7bx+7a)-77\cos(9bx+9a)}{709632b}$
risch	$-\frac{5\cos(bx+a)}{512b}-\frac{\cos(11bx+11a)}{11264b}-\frac{\cos(9bx+9a)}{9216b}+\frac{5\cos(7bx+7a)}{7168b}+\frac{\cos(5bx+5a)}{1024b}-\frac{5\cos(3bx+3a)}{1536b}$
orering	$-\frac{14312974\left(-6\cos(bx+a)^{5}\sin(bx+a)^{6}b+5\cos(bx+a)^{7}\sin(bx+a)^{4}b\right)}{120062225b^{2}}-\frac{1997021\left(-120\cos(bx+a)^{3}\sin(bx+a)^{8}b^{3}+\right.}{}$

input
```
int(cos(b*x+a)^6*sin(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/b*(1/11*cos(b*x+a)^11-2/9*cos(b*x+a)^9+1/7*cos(b*x+a)^7)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^6(a+bx)\sin^5(a+bx)\,dx = -\frac{63\,\cos(bx+a)^{11} - 154\,\cos(bx+a)^9 + 99\,\cos(bx+a)^7}{693\,b}$$

input
```
integrate(cos(b*x+a)^6*sin(b*x+a)^5,x, algorithm="fricas")
```

output
```
-1/693*(63*cos(b*x + a)^11 - 154*cos(b*x + a)^9 + 99*cos(b*x + a)^7)/b
```

Sympy [A] (verification not implemented)

Time = 1.76 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.48

$$\int \cos^6(a+bx)\sin^5(a+bx)\,dx = \begin{cases} -\frac{\sin^4(a+bx)\cos^7(a+bx)}{7b} - \frac{4\sin^2(a+bx)\cos^9(a+bx)}{63b} - \frac{8\cos^{11}(a+bx)}{693b} & \text{for } b \neq 0 \\ x\sin^5(a)\cos^6(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a)**6*sin(b*x+a)**5,x)
```

output
```
Piecewise((-sin(a + b*x)**4*cos(a + b*x)**7/(7*b) - 4*sin(a + b*x)**2*cos(
a + b*x)**9/(63*b) - 8*cos(a + b*x)**11/(693*b), Ne(b, 0)), (x*sin(a)**5*c
os(a)**6, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^6(a+bx)\sin^5(a+bx)\,dx = -\frac{63\cos(bx+a)^{11} - 154\cos(bx+a)^9 + 99\cos(bx+a)^7}{693\,b}$$

input
```
integrate(cos(b*x+a)^6*sin(b*x+a)^5,x, algorithm="maxima")
```

output
```
-1/693*(63*cos(b*x + a)^11 - 154*cos(b*x + a)^9 + 99*cos(b*x + a)^7)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^6(a+bx)\sin^5(a+bx)\,dx = -\frac{63\cos(bx+a)^{11} - 154\cos(bx+a)^9 + 99\cos(bx+a)^7}{693\,b}$$

input
```
integrate(cos(b*x+a)^6*sin(b*x+a)^5,x, algorithm="giac")
```

output
```
-1/693*(63*cos(b*x + a)^11 - 154*cos(b*x + a)^9 + 99*cos(b*x + a)^7)/b
```

Mupad [B] (verification not implemented)

Time = 25.79 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^6(a+bx)\sin^5(a+bx)\,dx = -\frac{63\cos(a+b\,x)^{11} - 154\cos(a+b\,x)^9 + 99\cos(a+b\,x)^7}{693\,b}$$

input
```
int(cos(a + b*x)^6*sin(a + b*x)^5,x)
```

output
```
-(99*cos(a + b*x)^7 - 154*cos(a + b*x)^9 + 63*cos(a + b*x)^11)/(693*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.07

$$\int \cos^6(a+bx)\sin^5(a+bx)\,dx = \frac{63\cos(bx+a)\sin(bx+a)^{10} - 161\cos(bx+a)\sin(bx+a)^{8} + 113\cos(bx+a)\sin(bx+a)^{6} - 3\cos(bx}{693b}$$

input
```
int(cos(b*x+a)^6*sin(b*x+a)^5,x)
```

output
```
(63*cos(a + b*x)*sin(a + b*x)**10 - 161*cos(a + b*x)*sin(a + b*x)**8 + 113
*cos(a + b*x)*sin(a + b*x)**6 - 3*cos(a + b*x)*sin(a + b*x)**4 - 4*cos(a +
 b*x)*sin(a + b*x)**2 - 8*cos(a + b*x) + 8)/(693*b)
```

3.100 $\int \cos^5(a+bx)\sin^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \cos^5(a+bx)\sin^5(a+bx)\,dx=\frac{\sin^6(a+bx)}{6b}-\frac{\sin^8(a+bx)}{4b}+\frac{\sin^{10}(a+bx)}{10b}$$

output
```
1/6*sin(b*x+a)^6/b-1/4*sin(b*x+a)^8/b+1/10*sin(b*x+a)^10/b
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int \cos^5(a+bx)\sin^5(a+bx)\,dx=\frac{1}{32}\left(-\frac{5\cos(2(a+bx))}{16b}+\frac{5\cos(6(a+bx))}{96b}-\frac{\cos(10(a+bx))}{160b}\right)$$

input
```
Integrate[Cos[a + b*x]^5*Sin[a + b*x]^5,x]
```

output
```
((-5*Cos[2*(a + b*x)])/(16*b) + (5*Cos[6*(a + b*x)])/(96*b) - Cos[10*(a +
b*x)]/(160*b))/32
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3042, 3044, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(a+bx)\cos^5(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^5\cos(a+bx)^5dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{\int \sin^5(a+bx)\left(1-\sin^2(a+bx)\right)^2d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int \sin^4(a+bx)\left(1-\sin^2(a+bx)\right)^2d\sin^2(a+bx)}{2b}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\sin^8(a+bx)-2\sin^6(a+bx)+\sin^4(a+bx)\right)d\sin^2(a+bx)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{5}\sin^{10}(a+bx)-\frac{1}{2}\sin^8(a+bx)+\frac{1}{3}\sin^6(a+bx)}{2b}$$

input
```
Int[Cos[a + b*x]^5*Sin[a + b*x]^5,x]
```

output
```
(Sin[a + b*x]^6/3 - Sin[a + b*x]^8/2 + Sin[a + b*x]^10/5)/(2*b)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 17.38 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{\frac{\sin(bx+a)^{10}}{10}-\frac{\sin(bx+a)^{8}}{4}+\frac{\sin(bx+a)^{6}}{6}}{b}$
default	$\frac{\frac{\sin(bx+a)^{10}}{10}-\frac{\sin(bx+a)^{8}}{4}+\frac{\sin(bx+a)^{6}}{6}}{b}$
parallelrisch	$\frac{128-150\cos(2bx+2a)+25\cos(6bx+6a)-3\cos(10bx+10a)}{15360b}$
risch	$-\frac{\cos(10bx+10a)}{5120b}+\frac{5\cos(6bx+6a)}{3072b}-\frac{5\cos(2bx+2a)}{512b}$
orering	$-\frac{259\left(-5\cos(bx+a)^{4}\sin(bx+a)^{6}b+5\cos(bx+a)^{6}\sin(bx+a)^{4}b\right)}{900b^{2}}-\frac{7\left(-60\cos(bx+a)^{2}\sin(bx+a)^{8}b^{3}+440\cos(bx+a\right.}{}$

input
```
int(cos(b*x+a)^5*sin(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/10*sin(b*x+a)^10-1/4*sin(b*x+a)^8+1/6*sin(b*x+a)^6)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^5(a+bx)\sin^5(a+bx)\,dx = -\frac{6\,\cos(bx+a)^{10} - 15\,\cos(bx+a)^8 + 10\,\cos(bx+a)^6}{60\,b}$$

input
```
integrate(cos(b*x+a)^5*sin(b*x+a)^5,x, algorithm="fricas")
```

output
```
-1/60*(6*cos(b*x + a)^10 - 15*cos(b*x + a)^8 + 10*cos(b*x + a)^6)/b
```

Sympy [A] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.37

$$\int \cos^5(a+bx)\sin^5(a+bx)\,dx = \begin{cases} \frac{\sin^{10}(a+bx)}{60b} + \frac{\sin^8(a+bx)\cos^2(a+bx)}{12b} + \frac{\sin^6(a+bx)\cos^4(a+bx)}{6b} & \text{for } b \neq 0 \\ x\sin^5(a)\cos^5(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a)**5*sin(b*x+a)**5,x)
```

output
```
Piecewise((sin(a + b*x)**10/(60*b) + sin(a + b*x)**8*cos(a + b*x)**2/(12*b
) + sin(a + b*x)**6*cos(a + b*x)**4/(6*b), Ne(b, 0)), (x*sin(a)**5*cos(a)*
*5, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^5(a+bx)\sin^5(a+bx)\,dx = \frac{6\,\sin(bx+a)^{10} - 15\,\sin(bx+a)^8 + 10\,\sin(bx+a)^6}{60\,b}$$

input
```
integrate(cos(b*x+a)^5*sin(b*x+a)^5,x, algorithm="maxima")
```

output
```
1/60*(6*sin(b*x + a)^10 - 15*sin(b*x + a)^8 + 10*sin(b*x + a)^6)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^5(a+bx)\sin^5(a+bx)\,dx = -\frac{6\,\cos(bx+a)^{10} - 15\,\cos(bx+a)^8 + 10\,\cos(bx+a)^6}{60\,b}$$

input
```
integrate(cos(b*x+a)^5*sin(b*x+a)^5,x, algorithm="giac")
```

output
```
-1/60*(6*cos(b*x + a)^10 - 15*cos(b*x + a)^8 + 10*cos(b*x + a)^6)/b
```

Mupad [B] (verification not implemented)

Time = 25.86 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^5(a+bx)\sin^5(a+bx)\,dx = -\frac{\frac{\cos(a+b\,x)^{10}}{10} - \frac{\cos(a+b\,x)^8}{4} + \frac{\cos(a+b\,x)^6}{6}}{b}$$

input
```
int(cos(a + b*x)^5*sin(a + b*x)^5,x)
```

output
```
-(cos(a + b*x)^6/6 - cos(a + b*x)^8/4 + cos(a + b*x)^10/10)/b
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \cos^5(a+bx)\sin^5(a+bx)\,dx = \frac{\sin(bx+a)^6\left(6\sin(bx+a)^4-15\sin(bx+a)^2+10\right)}{60b}$$

input
```
int(cos(b*x+a)^5*sin(b*x+a)^5,x)
```

output
```
(sin(a + b*x)**6*(6*sin(a + b*x)**4 - 15*sin(a + b*x)**2 + 10))/(60*b)
```

3.101 $\int \cos^4(a+bx)\sin^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \cos^4(a+bx)\sin^5(a+bx)\,dx = -\frac{\cos^5(a+bx)}{5b}+\frac{2\cos^7(a+bx)}{7b}-\frac{\cos^9(a+bx)}{9b}$$

output
```
-1/5*cos(b*x+a)^5/b+2/7*cos(b*x+a)^7/b-1/9*cos(b*x+a)^9/b
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \cos^4(a+bx)\sin^5(a+bx)\,dx = \frac{\cos^5(a+bx)(-249+220\cos(2(a+bx))-35\cos(4(a+bx)))}{2520b}$$

input
```
Integrate[Cos[a + b*x]^4*Sin[a + b*x]^5,x]
```

output
```
(Cos[a + b*x]^5*(-249 + 220*Cos[2*(a + b*x)] - 35*Cos[4*(a + b*x)]))/(2520
*b)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(a+bx)\cos^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^5\cos(a+bx)^4dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int \cos^4(a+bx)\left(1-\cos^2(a+bx)\right)^2d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(\cos^8(a+bx)-2\cos^6(a+bx)+\cos^4(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{9}\cos^9(a+bx)-\frac{2}{7}\cos^7(a+bx)+\frac{1}{5}\cos^5(a+bx)}{b}$$

input
```
Int[Cos[a + b*x]^4*Sin[a + b*x]^5,x]
```

output
```
-((Cos[a + b*x]^5/5 - (2*Cos[a + b*x]^7)/7 + Cos[a + b*x]^9/9)/b)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [A] (verified)

Time = 12.68 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

method	result
derivativedivides	$-\frac{\frac{\cos(bx+a)^9}{9}-\frac{2\cos(bx+a)^7}{7}+\frac{\cos(bx+a)^5}{5}}{b}$
default	$-\frac{\frac{\cos(bx+a)^9}{9}-\frac{2\cos(bx+a)^7}{7}+\frac{\cos(bx+a)^5}{5}}{b}$
parallelrisch	$\frac{-2048-420\cos(3bx+3a)-1890\cos(bx+a)+252\cos(5bx+5a)+45\cos(7bx+7a)-35\cos(9bx+9a)}{80640b}$
risch	$-\frac{3\cos(bx+a)}{128b}-\frac{\cos(9bx+9a)}{2304b}+\frac{\cos(7bx+7a)}{1792b}+\frac{\cos(5bx+5a)}{320b}-\frac{\cos(3bx+3a)}{192b}$
norman	$\frac{-\frac{16}{315b}-\frac{112\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^8}{5b}-\frac{32\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{12}}{3b}+\frac{16\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{10}}{b}+\frac{32\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^6}{5b}-\frac{16\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}{35b}-\frac{64\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}{35b}}{\left(1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right)^9}$
orering	$-\frac{117469\left(-4\cos(bx+a)^3\sin(bx+a)^6b+5\cos(bx+a)^5\sin(bx+a)^4b\right)}{99225b^2}-\frac{34562\left(-24\cos(bx+a)\sin(bx+a)^8b^3+280\cos\right.}{}$

input
```
int(cos(b*x+a)^4*sin(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/b*(1/9*cos(b*x+a)^9-2/7*cos(b*x+a)^7+1/5*cos(b*x+a)^5)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^4(a+bx)\sin^5(a+bx)\,dx = -\frac{35\cos\left(bx+a\right)^9-90\cos\left(bx+a\right)^7+63\cos\left(bx+a\right)^5}{315\,b}$$

input
```
integrate(cos(b*x+a)^4*sin(b*x+a)^5,x, algorithm="fricas")
```

output
```
-1/315*(35*cos(b*x + a)^9 - 90*cos(b*x + a)^7 + 63*cos(b*x + a)^5)/b
```

Sympy [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.48

$$\int \cos^4(a+bx)\sin^5(a+bx)\,dx = \begin{cases} -\frac{\sin^4(a+bx)\cos^5(a+bx)}{5b} - \frac{4\sin^2(a+bx)\cos^7(a+bx)}{35b} - \frac{8\cos^9(a+bx)}{315b} & \text{for } b\neq 0 \\ x\sin^5(a)\cos^4(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a)**4*sin(b*x+a)**5,x)
```

output
```
Piecewise((-sin(a + b*x)**4*cos(a + b*x)**5/(5*b) - 4*sin(a + b*x)**2*cos(
a + b*x)**7/(35*b) - 8*cos(a + b*x)**9/(315*b), Ne(b, 0)), (x*sin(a)**5*co
s(a)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^4(a+bx)\sin^5(a+bx)\,dx = -\frac{35\cos(bx+a)^9 - 90\cos(bx+a)^7 + 63\cos(bx+a)^5}{315\,b}$$

input
```
integrate(cos(b*x+a)^4*sin(b*x+a)^5,x, algorithm="maxima")
```

output
```
-1/315*(35*cos(b*x + a)^9 - 90*cos(b*x + a)^7 + 63*cos(b*x + a)^5)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^4(a+bx)\sin^5(a+bx)\,dx = -\frac{35\cos(bx+a)^9 - 90\cos(bx+a)^7 + 63\cos(bx+a)^5}{315\,b}$$

input
```
integrate(cos(b*x+a)^4*sin(b*x+a)^5,x, algorithm="giac")
```

output
```
-1/315*(35*cos(b*x + a)^9 - 90*cos(b*x + a)^7 + 63*cos(b*x + a)^5)/b
```

Mupad [B] (verification not implemented)

Time = 25.80 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^4(a+bx)\sin^5(a+bx)\,dx = -\frac{35\cos(a+b\,x)^9 - 90\cos(a+b\,x)^7 + 63\cos(a+b\,x)^5}{315\,b}$$

input
```
int(cos(a + b*x)^4*sin(a + b*x)^5,x)
```

output
```
-(63*cos(a + b*x)^5 - 90*cos(a + b*x)^7 + 35*cos(a + b*x)^9)/(315*b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.72

$$\int \cos^4(a+bx)\sin^5(a+bx)\,dx$$
$$= \frac{-35\cos(bx+a)\sin(bx+a)^8+50\cos(bx+a)\sin(bx+a)^6-3\cos(bx+a)\sin(bx+a)^4-4\cos(bx+}{315b}$$

input
```
int(cos(b*x+a)^4*sin(b*x+a)^5,x)
```

output
```
( - 35*cos(a + b*x)*sin(a + b*x)**8 + 50*cos(a + b*x)*sin(a + b*x)**6 - 3*
cos(a + b*x)*sin(a + b*x)**4 - 4*cos(a + b*x)*sin(a + b*x)**2 - 8*cos(a +
b*x) + 8)/(315*b)
```

3.102 $\int \cos^3(a+bx)\sin^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \cos^3(a+bx)\sin^5(a+bx)\,dx=\frac{\sin^6(a+bx)}{6b}-\frac{\sin^8(a+bx)}{8b}$$

output
```
1/6*sin(b*x+a)^6/b-1/8*sin(b*x+a)^8/b
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.55

$$\int \cos^3(a+bx)\sin^5(a+bx)\,dx\\ =\frac{-72\cos(2(a+bx))+12\cos(4(a+bx))+8\cos(6(a+bx))-3\cos(8(a+bx))}{3072b}$$

input
```
Integrate[Cos[a + b*x]^3*Sin[a + b*x]^5,x]
```

output
```
(-72*Cos[2*(a + b*x)] + 12*Cos[4*(a + b*x)] + 8*Cos[6*(a + b*x)] - 3*Cos[8
*(a + b*x)])/(3072*b)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(a+bx)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^5\cos(a+bx)^3dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{\int \sin^5(a+bx)\left(1-\sin^2(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\sin^5(a+bx)-\sin^7(a+bx)\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{6}\sin^6(a+bx)-\frac{1}{8}\sin^8(a+bx)}{b}$$

input
```
Int[Cos[a + b*x]^3*Sin[a + b*x]^5,x]
```

output
```
(Sin[a + b*x]^6/6 - Sin[a + b*x]^8/8)/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 8.48 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{-\frac{\sin(bx+a)^8}{8}+\frac{\sin(bx+a)^6}{6}}{b}$
default	$\frac{-\frac{\sin(bx+a)^8}{8}+\frac{\sin(bx+a)^6}{6}}{b}$
parallelrisch	$\frac{12\cos(4bx+4a)-3\cos(8bx+8a)-72\cos(2bx+2a)+55+8\cos(6bx+6a)}{3072b}$
risch	$-\frac{\cos(8bx+8a)}{1024b}+\frac{\cos(6bx+6a)}{384b}+\frac{\cos(4bx+4a)}{256b}-\frac{3\cos(2bx+2a)}{128b}$
norman	$\frac{\frac{32\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^6}{3b}+\frac{32\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{10}}{3b}-\frac{32\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^8}{3b}}{\left(1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right)^8}$
orering	$-\frac{205\left(-3\cos(bx+a)^2\sin(bx+a)^6b+5\cos(bx+a)^4\sin(bx+a)^4b\right)}{576b^2}-\frac{91\left(-6b^3\sin(bx+a)^8+156\cos(bx+a)^2\sin(bx+a)\right.}{}$

input
```
int(cos(b*x+a)^3*sin(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/8*sin(b*x+a)^8+1/6*sin(b*x+a)^6)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \cos^3(a+bx)\sin^5(a+bx)\,dx = -\frac{3\,\cos{(bx+a)}^8-8\,\cos{(bx+a)}^6+6\,\cos{(bx+a)}^4}{24\,b}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^5,x, algorithm="fricas")
```

output
```
-1/24*(3*cos(b*x + a)^8 - 8*cos(b*x + a)^6 + 6*cos(b*x + a)^4)/b
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

$$\int \cos^3(a+bx)\sin^5(a+bx)\,dx = \begin{cases} \frac{\sin^8{(a+bx)}}{24b}+\frac{\sin^6{(a+bx)}\cos^2{(a+bx)}}{6b} & \text{for } b\neq 0 \\ x\sin^5{(a)}\cos^3{(a)} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a)**3*sin(b*x+a)**5,x)
```

output
```
Piecewise((sin(a + b*x)**8/(24*b) + sin(a + b*x)**6*cos(a + b*x)**2/(6*b),
 Ne(b, 0)), (x*sin(a)**5*cos(a)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^3(a+bx)\sin^5(a+bx)\,dx = -\frac{3\,\sin\left(bx+a\right)^8-4\,\sin\left(bx+a\right)^6}{24\,b}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^5,x, algorithm="maxima")
```

output
```
-1/24*(3*sin(b*x + a)^8 - 4*sin(b*x + a)^6)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \cos^3(a+bx)\sin^5(a+bx)\,dx = -\frac{3\,\cos\left(bx+a\right)^8-8\,\cos\left(bx+a\right)^6+6\,\cos\left(bx+a\right)^4}{24\,b}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^5,x, algorithm="giac")
```

output
```
-1/24*(3*cos(b*x + a)^8 - 8*cos(b*x + a)^6 + 6*cos(b*x + a)^4)/b
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \cos^3(a+bx)\sin^5(a+bx)\,dx = \frac{4\sin\left(a+b\,x\right)^6-3\sin\left(a+b\,x\right)^8}{24\,b}$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)^5,x)
```

output
```
(4*sin(a + b*x)^6 - 3*sin(a + b*x)^8)/(24*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \cos^3(a+bx)\sin^5(a+bx)\,dx = \frac{\sin(bx+a)^6\left(-3\sin(bx+a)^2+4\right)}{24b}$$

input
```
int(cos(b*x+a)^3*sin(b*x+a)^5,x)
```

output
```
(sin(a + b*x)**6*( - 3*sin(a + b*x)**2 + 4))/(24*b)
```

3.103 $\int \cos^2(a+bx)\sin^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \cos^2(a+bx)\sin^5(a+bx)\,dx = -\frac{\cos^3(a+bx)}{3b}+\frac{2\cos^5(a+bx)}{5b}-\frac{\cos^7(a+bx)}{7b}$$

output
```
-1/3*cos(b*x+a)^3/b+2/5*cos(b*x+a)^5/b-1/7*cos(b*x+a)^7/b
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \cos^2(a+bx)\sin^5(a+bx)\,dx = \frac{\cos^3(a+bx)(-157+108\cos(2(a+bx))-15\cos(4(a+bx)))}{840b}$$

input
```
Integrate[Cos[a + b*x]^2*Sin[a + b*x]^5,x]
```

output
```
(Cos[a + b*x]^3*(-157 + 108*Cos[2*(a + b*x)] - 15*Cos[4*(a + b*x)]))/(840*
b)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(a+bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^5\cos(a+bx)^2dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int \cos^2(a+bx)\left(1-\cos^2(a+bx)\right)^2d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(\cos^6(a+bx)-2\cos^4(a+bx)+\cos^2(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{7}\cos^7(a+bx)-\frac{2}{5}\cos^5(a+bx)+\frac{1}{3}\cos^3(a+bx)}{b}$$

input
```
Int[Cos[a + b*x]^2*Sin[a + b*x]^5,x]
```

output
```
-((Cos[a + b*x]^3/3 - (2*Cos[a + b*x]^5)/5 + Cos[a + b*x]^7/7)/b)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [A] (verified)

Time = 7.48 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

method	result
derivativedivides	$-\frac{\frac{\cos(bx+a)^7}{7}-\frac{2\cos(bx+a)^5}{5}+\frac{\cos(bx+a)^3}{3}}{b}$
default	$-\frac{\frac{\cos(bx+a)^7}{7}-\frac{2\cos(bx+a)^5}{5}+\frac{\cos(bx+a)^3}{3}}{b}$
parallelrisch	$\frac{-512-525\cos(bx+a)+63\cos(5bx+5a)-15\cos(7bx+7a)-35\cos(3bx+3a)}{6720b}$
risch	$-\frac{5\cos(bx+a)}{64b}-\frac{\cos(7bx+7a)}{448b}+\frac{3\cos(5bx+5a)}{320b}-\frac{\cos(3bx+3a)}{192b}$
norman	$\frac{-\frac{16}{105b}-\frac{32\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^8}{3b}-\frac{16\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}{15b}-\frac{16\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}{5b}+\frac{16\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^6}{3b}}{\left(1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right)^7}$
orering	$-\frac{12916\left(-2\cos(bx+a)\sin(bx+a)^6b+5\cos(bx+a)^3\sin(bx+a)^4b\right)}{11025b^2}-\frac{94\left(68\cos(bx+a)\sin(bx+a)^6b^3-215\sin(bx+a)\right.}{525}$

input
```
int(cos(b*x+a)^2*sin(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/b*(1/7*cos(b*x+a)^7-2/5*cos(b*x+a)^5+1/3*cos(b*x+a)^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^2(a+bx)\sin^5(a+bx)\,dx = -\frac{15\,\cos\left(bx+a\right)^7-42\,\cos\left(bx+a\right)^5+35\,\cos\left(bx+a\right)^3}{105\,b}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^5,x, algorithm="fricas")
```

output
```
-1/105*(15*cos(b*x + a)^7 - 42*cos(b*x + a)^5 + 35*cos(b*x + a)^3)/b
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.48

$$\int \cos^2(a+bx)\sin^5(a+bx)\,dx = \begin{cases} -\frac{\sin^4{(a+bx)}\cos^3{(a+bx)}}{3b} - \frac{4\sin^2{(a+bx)}\cos^5{(a+bx)}}{15b} - \frac{8\cos^7{(a+bx)}}{105b} & \text{for } b \neq 0 \\ x\sin^5{(a)}\cos^2{(a)} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a)**2*sin(b*x+a)**5,x)
```

output
```
Piecewise((-sin(a + b*x)**4*cos(a + b*x)**3/(3*b) - 4*sin(a + b*x)**2*cos(
a + b*x)**5/(15*b) - 8*cos(a + b*x)**7/(105*b), Ne(b, 0)), (x*sin(a)**5*co
s(a)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^2(a+bx)\sin^5(a+bx)\,dx = -\frac{15\,\cos(bx+a)^7 - 42\,\cos(bx+a)^5 + 35\,\cos(bx+a)^3}{105\,b}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^5,x, algorithm="maxima")
```

output
```
-1/105*(15*cos(b*x + a)^7 - 42*cos(b*x + a)^5 + 35*cos(b*x + a)^3)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^2(a+bx)\sin^5(a+bx)\,dx = -\frac{15\,\cos(bx+a)^7 - 42\,\cos(bx+a)^5 + 35\,\cos(bx+a)^3}{105\,b}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^5,x, algorithm="giac")
```

output
```
-1/105*(15*cos(b*x + a)^7 - 42*cos(b*x + a)^5 + 35*cos(b*x + a)^3)/b
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^2(a+bx)\sin^5(a+bx)\,dx = -\frac{15\cos(a+b\,x)^7 - 42\cos(a+b\,x)^5 + 35\cos(a+b\,x)^3}{105\,b}$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)^5,x)
```

output
```
-(35*cos(a + b*x)^3 - 42*cos(a + b*x)^5 + 15*cos(a + b*x)^7)/(105*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.37

$$\int \cos^2(a+bx)\sin^5(a+bx)\,dx$$
$$= \frac{15\cos(bx+a)\sin(bx+a)^6 - 3\cos(bx+a)\sin(bx+a)^4 - 4\cos(bx+a)\sin(bx+a)^2 - 8\cos(bx+a)}{105b}$$

input
```
int(cos(b*x+a)^2*sin(b*x+a)^5,x)
```

output
```
(15*cos(a + b*x)*sin(a + b*x)**6 - 3*cos(a + b*x)*sin(a + b*x)**4 - 4*cos(
a + b*x)*sin(a + b*x)**2 - 8*cos(a + b*x) + 8)/(105*b)
```

3.104 $\int \cos(a+bx)\sin^5(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \cos(a+bx)\sin^5(a+bx)\,dx=\frac{\sin^6(a+bx)}{6b}$$

output
```
1/6*sin(b*x+a)^6/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cos(a+bx)\sin^5(a+bx)\,dx=\frac{\sin^6(a+bx)}{6b}$$

input
```
Integrate[Cos[a + b*x]*Sin[a + b*x]^5,x]
```

output
```
Sin[a + b*x]^6/(6*b)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3044, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(a+bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^5\cos(a+bx)dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{\int \sin^5(a+bx)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{\sin^6(a+bx)}{6b}$$

input
```
Int[Cos[a + b*x]*Sin[a + b*x]^5,x]
```

output
```
Sin[a + b*x]^6/(6*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 4.91 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{\sin(bx+a)^6}{6b}$
default	$\frac{\sin(bx+a)^6}{6b}$
norman	$\frac{32\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^6}{3b\left(1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right)^6}$
parallelrisch	$\frac{6\cos(4bx+4a)+10-15\cos(2bx+2a)-\cos(6bx+6a)}{192b}$
risch	$-\frac{\cos(6bx+6a)}{192b}+\frac{\cos(4bx+4a)}{32b}-\frac{5\cos(2bx+2a)}{64b}$
orering	$-\frac{49\left(-b\sin(bx+a)^6+5\cos(bx+a)^2\sin(bx+a)^4b\right)}{144b^2}-\frac{7\left(-140b^3\sin(bx+a)^4\cos(bx+a)^2+16b^3\sin(bx+a)^6+60\cos(ba\right.}{288b^4}$

input
```
int(cos(b*x+a)*sin(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/6*sin(b*x+a)^6/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(13) = 26.

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.27

$$\int \cos(a+bx)\sin^5(a+bx)\,dx = -\frac{\cos(bx+a)^6-3\cos(bx+a)^4+3\cos(bx+a)^2}{6b}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^5,x, algorithm="fricas")
```

output
```
-1/6*(cos(b*x + a)^6 - 3*cos(b*x + a)^4 + 3*cos(b*x + a)^2)/b
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \cos(a+bx)\sin^5(a+bx)\,dx = \begin{cases} \frac{\sin^6(a+bx)}{6b} & \text{for } b \neq 0 \\ x\sin^5(a)\cos(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)**5,x)
```

output
```
Piecewise((sin(a + b*x)**6/(6*b), Ne(b, 0)), (x*sin(a)**5*cos(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\sin^5(a+bx)\,dx = \frac{\sin(bx+a)^6}{6\,b}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^5,x, algorithm="maxima")
```

output
```
1/6*sin(b*x + a)^6/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\sin^5(a+bx)\,dx = \frac{\sin(bx+a)^6}{6\,b}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^5,x, algorithm="giac")
```

output
```
1/6*sin(b*x + a)^6/b
```

Mupad [B] (verification not implemented)

Time = 25.86 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\sin^5(a+bx)\,dx = \frac{\sin\left(a+b\,x\right)^6}{6\,b}$$

input
```
int(cos(a + b*x)*sin(a + b*x)^5,x)
```

output
```
sin(a + b*x)^6/(6*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\sin^5(a+bx)\,dx = \frac{\sin\left(bx+a\right)^6}{6b}$$

input
```
int(cos(b*x+a)*sin(b*x+a)^5,x)
```

output
```
sin(a + b*x)**6/(6*b)
```

3.105 $\int \sin^4(a+bx)\tan(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 40

$$\int \sin^4(a+bx)\tan(a+bx)\,dx=\frac{\cos^2(a+bx)}{b}-\frac{\cos^4(a+bx)}{4b}-\frac{\log(\cos(a+bx))}{b}$$

output
```
cos(b*x+a)^2/b-1/4*cos(b*x+a)^4/b-ln(cos(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int \sin^4(a+bx)\tan(a+bx)\,dx=-\frac{-\cos^2(a+bx)+\frac{1}{4}\cos^4(a+bx)+\log(\cos(a+bx))}{b}$$

input
```
Integrate[Sin[a + b*x]^4*Tan[a + b*x],x]
```

output
```
-((-Cos[a + b*x]^2 + Cos[a + b*x]^4/4 + Log[Cos[a + b*x]])/b)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3070, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(a+bx)\tan(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^4\tan(a+bx)dx$$

$$\Big\downarrow \text{3070}$$

$$-\frac{\int \left(1-\cos^2(a+bx)\right)^2\sec(a+bx)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{243}$$

$$-\frac{\int \left(1-\cos^2(a+bx)\right)^2\sec(a+bx)d\cos^2(a+bx)}{2b}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\int \left(\cos^2(a+bx)+\sec(a+bx)-2\right)d\cos^2(a+bx)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{2}\cos^4(a+bx)-2\cos^2(a+bx)+\log\left(\cos^2(a+bx)\right)}{2b}$$

input
```
Int[Sin[a + b*x]^4*Tan[a + b*x],x]
```

output
```
-1/2*(-2*Cos[a + b*x]^2 + Cos[a + b*x]^4/2 + Log[Cos[a + b*x]^2])/b
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Maple [A] (verified)

Time = 2.49 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{-\frac{\sin(bx+a)^4}{4}-\frac{\sin(bx+a)^2}{2}-\ln(\cos(bx+a))}{b}$	35
default	$\frac{-\frac{\sin(bx+a)^4}{4}-\frac{\sin(bx+a)^2}{2}-\ln(\cos(bx+a))}{b}$	35
risch	$ix+\frac{3\,\mathrm{e}^{2i(bx+a)}}{16b}+\frac{3\,\mathrm{e}^{-2i(bx+a)}}{16b}+\frac{2ia}{b}-\frac{\ln\left(\mathrm{e}^{2i(bx+a)}+1\right)}{b}-\frac{\cos(4bx+4a)}{32b}$	72

input
```
int(sin(b*x+a)^4*tan(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/4*sin(b*x+a)^4-1/2*sin(b*x+a)^2-ln(cos(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int \sin^4(a+bx)\tan(a+bx)\,dx = -\frac{\cos(bx+a)^4 - 4\cos(bx+a)^2 + 4\log(-\cos(bx+a))}{4b}$$

input
```
integrate(sin(b*x+a)^4*tan(b*x+a),x, algorithm="fricas")
```

output
```
-1/4*(cos(b*x + a)^4 - 4*cos(b*x + a)^2 + 4*log(-cos(b*x + a)))/b
```

Sympy [F]

$$\int \sin^4(a+bx)\tan(a+bx)\,dx = \int \sin^4(a+bx)\tan(a+bx)\,dx$$

input
```
integrate(sin(b*x+a)**4*tan(b*x+a),x)
```

output
```
Integral(sin(a + b*x)**4*tan(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \sin^4(a+bx)\tan(a+bx)\,dx = -\frac{\sin(bx+a)^4 + 2\sin(bx+a)^2 + 2\log\left(\sin(bx+a)^2 - 1\right)}{4b}$$

input
```
integrate(sin(b*x+a)^4*tan(b*x+a),x, algorithm="maxima")
```

output
```
-1/4*(sin(b*x + a)^4 + 2*sin(b*x + a)^2 + 2*log(sin(b*x + a)^2 - 1))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12

$$\int \sin^4(a+bx)\tan(a+bx)\,dx = -\frac{\log\left(\left|\sin\left(bx+a\right)^2-1\right|\right)}{2\,b} - \frac{b\sin\left(bx+a\right)^4+2\,b\sin\left(bx+a\right)^2}{4\,b^2}$$

input
```
integrate(sin(b*x+a)^4*tan(b*x+a),x, algorithm="giac")
```

output
```
-1/2*log(abs(sin(b*x + a)^2 - 1))/b - 1/4*(b*sin(b*x + a)^4 + 2*b*sin(b*x
+ a)^2)/b^2
```

Mupad [B] (verification not implemented)

Time = 25.78 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.32

$$\int \sin^4(a+bx)\tan(a+bx)\,dx = \frac{\ln\left(\tan(a+b\,x)^2+1\right)}{2\,b} + \frac{\tan(a+b\,x)^2+\frac{3}{4}}{b\,\left(\tan\left(a+b\,x\right)^4+2\tan\left(a+b\,x\right)^2+1\right)}$$

input
```
int(sin(a + b*x)^4*tan(a + b*x),x)
```

output
```
log(tan(a + b*x)^2 + 1)/(2*b) + (tan(a + b*x)^2 + 3/4)/(b*(2*tan(a + b*x)^
2 + tan(a + b*x)^4 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.75

$$\int \sin^4(a+bx)\tan(a+bx)\,dx$$
$$= \frac{4\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2+1\right)-4\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)-4\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)-\sin\left(bx+a\right)^4-2\sin\left(b\right.}{4b}$$

input
```
int(sin(b*x+a)^4*tan(b*x+a),x)
```

output
```
(4*log(tan((a + b*x)/2)**2 + 1) - 4*log(tan((a + b*x)/2) - 1) - 4*log(tan(
(a + b*x)/2) + 1) - sin(a + b*x)**4 - 2*sin(a + b*x)**2)/(4*b)
```

3.106 $\int \sin^3(a+bx)\tan^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 37

$$\int \sin^3(a+bx)\tan^2(a+bx)\,dx = \frac{2\cos(a+bx)}{b} - \frac{\cos^3(a+bx)}{3b} + \frac{\sec(a+bx)}{b}$$

output
```
2*cos(b*x+a)/b-1/3*cos(b*x+a)^3/b+sec(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05

$$\int \sin^3(a+bx)\tan^2(a+bx)\,dx = \frac{7\cos(a+bx)}{4b} - \frac{\cos(3(a+bx))}{12b} + \frac{\sec(a+bx)}{b}$$

input
```
Integrate[Sin[a + b*x]^3*Tan[a + b*x]^2,x]
```

output
```
(7*Cos[a + b*x])/(4*b) - Cos[3*(a + b*x)]/(12*b) + Sec[a + b*x]/b
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\tan^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^3\tan(a+bx)^2dx$$

$$\Big\downarrow \text{3070}$$

$$-\frac{\int \left(1-\cos^2(a+bx)\right)^2\sec^2(a+bx)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(\cos^2(a+bx)+\sec^2(a+bx)-2\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{3}\cos^3(a+bx)-2\cos(a+bx)-\sec(a+bx)}{b}$$

input
```
Int[Sin[a + b*x]^3*Tan[a + b*x]^2,x]
```

output
```
-((-2*Cos[a + b*x] + Cos[a + b*x]^3/3 - Sec[a + b*x])/b)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Maple [A] (verified)

Time = 3.00 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.35

method	result	size
derivativedivides	$\frac{\frac{\sin(bx+a)^6}{\cos(bx+a)}+\left(\frac{8}{3}+\sin(bx+a)^4+\frac{4\sin(bx+a)^2}{3}\right)\cos(bx+a)}{b}$	50
default	$\frac{\frac{\sin(bx+a)^6}{\cos(bx+a)}+\left(\frac{8}{3}+\sin(bx+a)^4+\frac{4\sin(bx+a)^2}{3}\right)\cos(bx+a)}{b}$	50
risch	$\frac{7\,e^{i(bx+a)}}{8b}+\frac{7\,e^{-i(bx+a)}}{8b}+\frac{2\,e^{i(bx+a)}}{b\left(e^{2i(bx+a)}+1\right)}-\frac{\cos(3bx+3a)}{12b}$	71

input
```
int(sin(b*x+a)^3*tan(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(sin(b*x+a)^6/cos(b*x+a)+(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a
))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \sin^3(a+bx)\tan^2(a+bx)\,dx = -\frac{\cos(bx+a)^4 - 6\cos(bx+a)^2 - 3}{3\,b\cos(bx+a)}$$

input
```
integrate(sin(b*x+a)^3*tan(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/3*(cos(b*x + a)^4 - 6*cos(b*x + a)^2 - 3)/(b*cos(b*x + a))
```

Sympy [F]

$$\int \sin^3(a+bx)\tan^2(a+bx)\,dx = \int \sin^3(a+bx)\tan^2(a+bx)\,dx$$

input
```
integrate(sin(b*x+a)**3*tan(b*x+a)**2,x)
```

output
```
Integral(sin(a + b*x)**3*tan(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \sin^3(a+bx)\tan^2(a+bx)\,dx = -\frac{\cos(bx+a)^3 - \frac{3}{\cos(bx+a)} - 6\cos(bx+a)}{3\,b}$$

input
```
integrate(sin(b*x+a)^3*tan(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/3*(cos(b*x + a)^3 - 3/cos(b*x + a) - 6*cos(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \sin^3(a+bx)\tan^2(a+bx)\,dx = \frac{1}{b\cos(bx+a)} - \frac{b^5\cos(bx+a)^3 - 6\,b^5\cos(bx+a)}{3\,b^6}$$

input
```
integrate(sin(b*x+a)^3*tan(b*x+a)^2,x, algorithm="giac")
```

output
```
1/(b*cos(b*x + a)) - 1/3*(b^5*cos(b*x + a)^3 - 6*b^5*cos(b*x + a))/b^6
```

Mupad [B] (verification not implemented)

Time = 25.86 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \sin^3(a+bx)\tan^2(a+bx)\,dx = -\frac{(\cos(a+b\,x)+1)^3\,(\cos(a+b\,x)-3)}{3\,b\,\cos(a+b\,x)}$$

input
```
int(sin(a + b*x)^3*tan(a + b*x)^2,x)
```

output
```
-((cos(a + b*x) + 1)^3*(cos(a + b*x) - 3))/(3*b*cos(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.16

$$\int \sin^3(a+bx)\tan^2(a+bx)\,dx = \frac{-8\cos(bx+a) - \sin(bx+a)^4 - 4\sin(bx+a)^2 + 8}{3\cos(bx+a)\,b}$$

input
```
int(sin(b*x+a)^3*tan(b*x+a)^2,x)
```

output
```
( - 8*cos(a + b*x) - sin(a + b*x)**4 - 4*sin(a + b*x)**2 + 8)/(3*cos(a + b
*x)*b)
```

3.107 $\int \sin^2(a+bx)\tan^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \sin^2(a+bx)\tan^3(a+bx)\,dx=-\frac{\cos^2(a+bx)}{2b}+\frac{2\log(\cos(a+bx))}{b}+\frac{\sec^2(a+bx)}{2b}$$

output
```
-1/2*cos(b*x+a)^2/b+2*ln(cos(b*x+a))/b+1/2*sec(b*x+a)^2/b
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \sin^2(a+bx)\tan^3(a+bx)\,dx=\frac{4\log(\cos(a+bx))+\sec^2(a+bx)+\sin^2(a+bx)}{2b}$$

input
```
Integrate[Sin[a + b*x]^2*Tan[a + b*x]^3,x]
```

output
```
(4*Log[Cos[a + b*x]] + Sec[a + b*x]^2 + Sin[a + b*x]^2)/(2*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.40 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3042, 3070, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\tan^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^2\tan(a+bx)^3dx$$

$$\Big\downarrow \text{3070}$$

$$-\frac{\int \left(1-\cos^2(a+bx)\right)^2\sec^3(a+bx)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{243}$$

$$-\frac{\int \left(1-\cos^2(a+bx)\right)^2\sec^2(a+bx)d\cos^2(a+bx)}{2b}$$

$$\Big\downarrow \text{49}$$

$$-\frac{\int \left(\sec^2(a+bx)-2\sec(a+bx)+1\right)d\cos^2(a+bx)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\cos^2(a+bx)-\sec(a+bx)-2\log\left(\cos^2(a+bx)\right)}{2b}$$

input
```
Int[Sin[a + b*x]^2*Tan[a + b*x]^3,x]
```

output
```
-1/2*(Cos[a + b*x]^2 - 2*Log[Cos[a + b*x]^2] - Sec[a + b*x])/b
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Maple [A] (verified)

Time = 1.69 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

method	result	size
derivativedivides	$\frac{\frac{\sin(bx+a)^6}{2\cos(bx+a)^2}+\frac{\sin(bx+a)^4}{2}+\sin(bx+a)^2+2\ln(\cos(bx+a))}{b}$	51
default	$\frac{\frac{\sin(bx+a)^6}{2\cos(bx+a)^2}+\frac{\sin(bx+a)^4}{2}+\sin(bx+a)^2+2\ln(\cos(bx+a))}{b}$	51
risch	$-2ix-\frac{e^{2i(bx+a)}}{8b}-\frac{e^{-2i(bx+a)}}{8b}-\frac{4ia}{b}+\frac{2\,e^{2i(bx+a)}}{b\left(e^{2i(bx+a)}+1\right)^2}+\frac{2\ln\left(e^{2i(bx+a)}+1\right)}{b}$	85

input
```
int(sin(b*x+a)^2*tan(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2*sin(b*x+a)^6/cos(b*x+a)^2+1/2*sin(b*x+a)^4+sin(b*x+a)^2+2*ln(cos(
b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.26

$$\int \sin^2(a+bx)\tan^3(a+bx)\,dx$$
$$= -\frac{2\,\cos(bx+a)^4 - 8\,\cos(bx+a)^2\log(-\cos(bx+a)) - \cos(bx+a)^2 - 2}{4\,b\cos(bx+a)^2}$$

input
```
integrate(sin(b*x+a)^2*tan(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/4*(2*cos(b*x + a)^4 - 8*cos(b*x + a)^2*log(-cos(b*x + a)) - cos(b*x + a
)^2 - 2)/(b*cos(b*x + a)^2)
```

Sympy [F]

$$\int \sin^2(a+bx)\tan^3(a+bx)\,dx = \int \sin^2(a+bx)\tan^3(a+bx)\,dx$$

input
```
integrate(sin(b*x+a)**2*tan(b*x+a)**3,x)
```

output
```
Integral(sin(a + b*x)**2*tan(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \sin^2(a+bx)\tan^3(a+bx)\,dx = \frac{\sin(bx+a)^2 - \frac{1}{\sin(bx+a)^2-1} + 2\log\left(\sin(bx+a)^2-1\right)}{2b}$$

input
```
integrate(sin(b*x+a)^2*tan(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/2*(sin(b*x + a)^2 - 1/(sin(b*x + a)^2 - 1) + 2*log(sin(b*x + a)^2 - 1))/
b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09

$$\int \sin^2(a+bx)\tan^3(a+bx)\,dx = \frac{\sin(bx+a)^2}{2b} + \frac{\log\left(\left|\sin(bx+a)^2-1\right|\right)}{b} - \frac{1}{2\left(\sin(bx+a)^2-1\right)b}$$

input
```
integrate(sin(b*x+a)^2*tan(b*x+a)^3,x, algorithm="giac")
```

output
```
1/2*sin(b*x + a)^2/b + log(abs(sin(b*x + a)^2 - 1))/b - 1/2/((sin(b*x + a)
^2 - 1)*b)
```

Mupad [B] (verification not implemented)

Time = 25.71 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \sin^2(a+bx)\tan^3(a+bx)\,dx = -\frac{\ln\left(\tan(a+b\,x)^2+1\right) + \frac{\cos(a+b\,x)^2}{2} - \frac{\tan(a+b\,x)^2}{2}}{b}$$

input
```
int(sin(a + b*x)^2*tan(a + b*x)^3,x)
```

output
```
-(log(tan(a + b*x)^2 + 1) + cos(a + b*x)^2/2 - tan(a + b*x)^2/2)/b
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 148, normalized size of antiderivative = 3.44

$$\int \sin^2(a+bx)\tan^3(a+bx)\,dx$$

$$= \frac{-4\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2+1\right)\sin(bx+a)^2+4\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2+1\right)+4\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin(bx+}{}$$

input
```
int(sin(b*x+a)^2*tan(b*x+a)^3,x)
```

output
```
( - 4*log(tan((a + b*x)/2)**2 + 1)*sin(a + b*x)**2 + 4*log(tan((a + b*x)/2
)**2 + 1) + 4*log(tan((a + b*x)/2) - 1)*sin(a + b*x)**2 - 4*log(tan((a + b
*x)/2) - 1) + 4*log(tan((a + b*x)/2) + 1)*sin(a + b*x)**2 - 4*log(tan((a +
 b*x)/2) + 1) + sin(a + b*x)**4 - 2*sin(a + b*x)**2)/(2*b*(sin(a + b*x)**2
 - 1))
```

3.108 $\int \sin(a+bx)\tan^4(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \sin(a+bx)\tan^4(a+bx)\,dx = -\frac{\cos(a+bx)}{b} - \frac{2\sec(a+bx)}{b} + \frac{\sec^3(a+bx)}{3b}$$

output
```
-cos(b*x+a)/b-2*sec(b*x+a)/b+1/3*sec(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \sin(a+bx)\tan^4(a+bx)\,dx = -\frac{\cos(a+bx)}{b} - \frac{2\sec(a+bx)}{b} + \frac{\sec^3(a+bx)}{3b}$$

input
```
Integrate[Sin[a + b*x]*Tan[a + b*x]^4,x]
```

output
```
-(Cos[a + b*x]/b) - (2*Sec[a + b*x])/b + Sec[a + b*x]^3/(3*b)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\tan^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)\tan(a+bx)^4dx$$

$$\Big\downarrow \text{3070}$$

$$-\frac{\int \left(1-\cos^2(a+bx)\right)^2\sec^4(a+bx)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(\sec^4(a+bx)-2\sec^2(a+bx)+1\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\cos(a+bx)-\frac{1}{3}\sec^3(a+bx)+2\sec(a+bx)}{b}$$

input
```
Int[Sin[a + b*x]*Tan[a + b*x]^4,x]
```

output
```
-((Cos[a + b*x] + 2*Sec[a + b*x] - Sec[a + b*x]^3/3)/b)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Maple [A] (verified)

Time = 2.77 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.84

method	result	size
derivativedivides	$\frac{\frac{\sin(bx+a)^6}{3\cos(bx+a)^3}-\frac{\sin(bx+a)^6}{\cos(bx+a)}-\left(\frac{8}{3}+\sin(bx+a)^4+\frac{4\sin(bx+a)^2}{3}\right)\cos(bx+a)}{b}$	70
default	$\frac{\frac{\sin(bx+a)^6}{3\cos(bx+a)^3}-\frac{\sin(bx+a)^6}{\cos(bx+a)}-\left(\frac{8}{3}+\sin(bx+a)^4+\frac{4\sin(bx+a)^2}{3}\right)\cos(bx+a)}{b}$	70
risch	$-\frac{3\,e^{7i(bx+a)}+36\,e^{5i(bx+a)}+50\,e^{3i(bx+a)}+39\cos(bx+a)+33i\sin(bx+a)}{6b\left(e^{2i(bx+a)}+1\right)^3}$	70

input
```
int(sin(b*x+a)*tan(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/3*sin(b*x+a)^6/cos(b*x+a)^3-sin(b*x+a)^6/cos(b*x+a)-(8/3+sin(b*x+a)
^4+4/3*sin(b*x+a)^2)*cos(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \sin(a+bx)\tan^4(a+bx)\,dx = -\frac{3\,\cos\left(bx+a\right)^4+6\,\cos\left(bx+a\right)^2-1}{3\,b\cos\left(bx+a\right)^3}$$

input
```
integrate(sin(b*x+a)*tan(b*x+a)^4,x, algorithm="fricas")
```

output
```
-1/3*(3*cos(b*x + a)^4 + 6*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^3)
```

Sympy [F]

$$\int \sin(a+bx)\tan^4(a+bx)\,dx = \int \sin\left(a+bx\right)\tan^4\left(a+bx\right)\,dx$$

input
```
integrate(sin(b*x+a)*tan(b*x+a)**4,x)
```

output
```
Integral(sin(a + b*x)*tan(a + b*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \sin(a+bx)\tan^4(a+bx)\,dx = -\frac{\frac{6\,\cos(bx+a)^2-1}{\cos(bx+a)^3}+3\,\cos\left(bx+a\right)}{3\,b}$$

input
```
integrate(sin(b*x+a)*tan(b*x+a)^4,x, algorithm="maxima")
```

output
```
-1/3*((6*cos(b*x + a)^2 - 1)/cos(b*x + a)^3 + 3*cos(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \sin(a+bx)\tan^4(a+bx)\,dx = -\frac{\cos(bx+a)}{b} - \frac{6\cos(bx+a)^2-1}{3\,b\cos(bx+a)^3}$$

input
```
integrate(sin(b*x+a)*tan(b*x+a)^4,x, algorithm="giac")
```

output
```
-cos(b*x + a)/b - 1/3*(6*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^3)
```

Mupad [B] (verification not implemented)

Time = 25.87 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \sin(a+bx)\tan^4(a+bx)\,dx = \frac{1}{3\,b\cos(a+b\,x)^3} - \frac{2}{b\,\cos(a+b\,x)} - \frac{\cos(a+b\,x)}{b}$$

input
```
int(sin(a + b*x)*tan(a + b*x)^4,x)
```

output
```
1/(3*b*cos(a + b*x)^3) - 2/(b*cos(a + b*x)) - cos(a + b*x)/b
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 185, normalized size of antiderivative = 4.87

$$\int \sin(a+bx)\tan^4(a+bx)\,dx$$
$$= \frac{-\cos(bx+a)\tan(bx+a)^2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4 + \cos(bx+a)\tan(bx+a)^2 + 2\cos(bx+a)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4 - 2}{}$$

input
```
int(sin(b*x+a)*tan(b*x+a)^4,x)
```

output
```
( - cos(a + b*x)*tan(a + b*x)**2*tan((a + b*x)/2)**4 + cos(a + b*x)*tan(a
+ b*x)**2 + 2*cos(a + b*x)*tan((a + b*x)/2)**4 - 2*cos(a + b*x) + 2*sin(a
+ b*x)*tan(a + b*x)**3*tan((a + b*x)/2)**4 - 2*sin(a + b*x)*tan(a + b*x)**
3 + 2*sin(a + b*x)*tan(a + b*x)*tan((a + b*x)/2)**4 - 2*sin(a + b*x)*tan(a
 + b*x) + 36*tan((a + b*x)/2)**4)/(6*b*(tan((a + b*x)/2)**4 - 1))
```

3.109 $\int \tan^5(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 43

$$\int \tan^5(a+bx)\,dx = -\frac{\log(\cos(a+bx))}{b} - \frac{\tan^2(a+bx)}{2b} + \frac{\tan^4(a+bx)}{4b}$$

output
```
-ln(cos(b*x+a))/b-1/2*tan(b*x+a)^2/b+1/4*tan(b*x+a)^4/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \tan^5(a+bx)\,dx = -\frac{\log(\cos(a+bx))}{b} - \frac{\sec^2(a+bx)}{b} + \frac{\sec^4(a+bx)}{4b}$$

input
```
Integrate[Tan[a + b*x]^5,x]
```

output
```
-(Log[Cos[a + b*x]]/b) - Sec[a + b*x]^2/b + Sec[a + b*x]^4/(4*b)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3954, 3042, 3954, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(a+bx)^5dx$$

$$\Big\downarrow \text{3954}$$

$$\frac{\tan^4(a+bx)}{4b}-\int \tan^3(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^4(a+bx)}{4b}-\int \tan(a+bx)^3dx$$

$$\Big\downarrow \text{3954}$$

$$\int \tan(a+bx)dx+\frac{\tan^4(a+bx)}{4b}-\frac{\tan^2(a+bx)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(a+bx)dx+\frac{\tan^4(a+bx)}{4b}-\frac{\tan^2(a+bx)}{2b}$$

$$\Big\downarrow \text{3956}$$

$$\frac{\tan^4(a+bx)}{4b}-\frac{\tan^2(a+bx)}{2b}-\frac{\log(\cos(a+bx))}{b}$$

input
```
Int[Tan[a + b*x]^5,x]
```

output
```
-(Log[Cos[a + b*x]]/b) - Tan[a + b*x]^2/(2*b) + Tan[a + b*x]^4/(4*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

method	result	size
parallelrisch	$\frac{\tan(bx+a)^4-2\tan(bx+a)^2+2\ln\left(1+\tan(bx+a)^2\right)}{4b}$	38
derivativedivides	$\frac{\frac{\tan(bx+a)^4}{4}-\frac{\tan(bx+a)^2}{2}+\frac{\ln\left(1+\tan(bx+a)^2\right)}{2}}{b}$	39
default	$\frac{\frac{\tan(bx+a)^4}{4}-\frac{\tan(bx+a)^2}{2}+\frac{\ln\left(1+\tan(bx+a)^2\right)}{2}}{b}$	39
norman	$-\frac{\tan(bx+a)^2}{2b}+\frac{\tan(bx+a)^4}{4b}+\frac{\ln\left(1+\tan(bx+a)^2\right)}{2b}$	44
risch	$ix+\frac{2ia}{b}-\frac{4\left(e^{6i(bx+a)}+e^{4i(bx+a)}+e^{2i(bx+a)}\right)}{b\left(e^{2i(bx+a)}+1\right)^4}-\frac{\ln\left(e^{2i(bx+a)}+1\right)}{b}$	76

input
```
int(tan(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/4*(tan(b*x+a)^4-2*tan(b*x+a)^2+2*ln(1+tan(b*x+a)^2))/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \tan^5(a+bx)\,dx = \frac{\tan\left(bx+a\right)^4 - 2\,\tan\left(bx+a\right)^2 - 2\,\log\left(\frac{1}{\tan(bx+a)^2+1}\right)}{4\,b}$$

input
```
integrate(tan(b*x+a)^5,x, algorithm="fricas")
```

output
```
1/4*(tan(b*x + a)^4 - 2*tan(b*x + a)^2 - 2*log(1/(tan(b*x + a)^2 + 1)))/b
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

$$\int \tan^5(a+bx)\,dx = \begin{cases} \frac{\log\left(\tan^2\left(a+bx\right)+1\right)}{2b} + \frac{\tan^4\left(a+bx\right)}{4b} - \frac{\tan^2\left(a+bx\right)}{2b} & \text{for}\; b \neq 0 \\ x\tan^5\left(a\right) & \text{otherwise} \end{cases}$$

input
```
integrate(tan(b*x+a)**5,x)
```

output
```
Piecewise((log(tan(a + b*x)**2 + 1)/(2*b) + tan(a + b*x)**4/(4*b) - tan(a
+ b*x)**2/(2*b), Ne(b, 0)), (x*tan(a)**5, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.26

$$\int \tan^5(a+bx)\,dx = \frac{\frac{4\,\sin(bx+a)^2-3}{\sin(bx+a)^4-2\,\sin(bx+a)^2+1} - 2\,\log\left(\sin\left(bx+a\right)^2-1\right)}{4\,b}$$

input
```
integrate(tan(b*x+a)^5,x, algorithm="maxima")
```

output
```
1/4*((4*sin(b*x + a)^2 - 3)/(sin(b*x + a)^4 - 2*sin(b*x + a)^2 + 1) - 2*lo
g(sin(b*x + a)^2 - 1))/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

$$\int \tan^5(a+bx)\,dx = \frac{\log\left(\tan\left(bx+a\right)^2+1\right)}{2\,b} + \frac{b\tan\left(bx+a\right)^4 - 2\,b\tan\left(bx+a\right)^2}{4\,b^2}$$

input
```
integrate(tan(b*x+a)^5,x, algorithm="giac")
```

output
```
1/2*log(tan(b*x + a)^2 + 1)/b + 1/4*(b*tan(b*x + a)^4 - 2*b*tan(b*x + a)^2
)/b^2
```

Mupad [B] (verification not implemented)

Time = 25.54 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \tan^5(a+bx)\,dx = \frac{\frac{\ln\left(\tan(a+b\,x)^2+1\right)}{2} - \frac{\tan(a+b\,x)^2}{2} + \frac{\tan(a+b\,x)^4}{4}}{b}$$

input
```
int(tan(a + b*x)^5,x)
```

output
```
(log(tan(a + b*x)^2 + 1)/2 - tan(a + b*x)^2/2 + tan(a + b*x)^4/4)/b
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \tan^5(a+bx)\,dx = \frac{2\log\left(\tan\left(bx+a\right)^2+1\right) + \tan\left(bx+a\right)^4 - 2\tan\left(bx+a\right)^2}{4b}$$

input
```
int(tan(b*x+a)^5,x)
```

output
```
(2*log(tan(a + b*x)**2 + 1) + tan(a + b*x)**4 - 2*tan(a + b*x)**2)/(4*b)
```

3.110 $\int \sec(a+bx)\tan^5(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \sec(a+bx)\tan^5(a+bx)\,dx = \frac{\sec(a+bx)}{b} - \frac{2\sec^3(a+bx)}{3b} + \frac{\sec^5(a+bx)}{5b}$$

output
```
sec(b*x+a)/b-2/3*sec(b*x+a)^3/b+1/5*sec(b*x+a)^5/b
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \sec(a+bx)\tan^5(a+bx)\,dx = \frac{\sec(a+bx)}{b} - \frac{2\sec^3(a+bx)}{3b} + \frac{\sec^5(a+bx)}{5b}$$

input
```
Integrate[Sec[a + b*x]*Tan[a + b*x]^5,x]
```

output
```
Sec[a + b*x]/b - (2*Sec[a + b*x]^3)/(3*b) + Sec[a + b*x]^5/(5*b)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 3086, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(a+bx)\sec(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(a+bx)^5\sec(a+bx)dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{\int \left(\sec^2(a+bx)-1\right)^2d\sec(a+bx)}{b}$$

$$\Big\downarrow \text{210}$$

$$\frac{\int \left(\sec^4(a+bx)-2\sec^2(a+bx)+1\right)d\sec(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{5}\sec^5(a+bx)-\frac{2}{3}\sec^3(a+bx)+\sec(a+bx)}{b}$$

input
```
Int[Sec[a + b*x]*Tan[a + b*x]^5,x]
```

output
```
(Sec[a + b*x] - (2*Sec[a + b*x]^3)/3 + Sec[a + b*x]^5/5)/b
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\frac{\sec(bx+a)^5}{5}-\frac{2\sec(bx+a)^3}{3}+\sec(bx+a)}{b}$	32
default	$\frac{\frac{\sec(bx+a)^5}{5}-\frac{2\sec(bx+a)^3}{3}+\sec(bx+a)}{b}$	32
risch	$\frac{2\,e^{9i(bx+a)}+\frac{8\,e^{7i(bx+a)}}{3}+\frac{116\,e^{5i(bx+a)}}{15}+\frac{8\,e^{3i(bx+a)}}{3}+2\,e^{i(bx+a)}}{b\left(e^{2i(bx+a)}+1\right)^5}$	75

input
```
int(sec(b*x+a)*tan(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/5*sec(b*x+a)^5-2/3*sec(b*x+a)^3+sec(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \sec(a+bx)\tan^5(a+bx)\,dx = \frac{15\cos(bx+a)^4 - 10\cos(bx+a)^2 + 3}{15\,b\cos(bx+a)^5}$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)^5,x, algorithm="fricas")
```

output
```
1/15*(15*cos(b*x + a)^4 - 10*cos(b*x + a)^2 + 3)/(b*cos(b*x + a)^5)
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.46

$$\int \sec(a+bx)\tan^5(a+bx)\,dx = \begin{cases} \frac{\tan^4(a+bx)\sec(a+bx)}{5b} - \frac{4\tan^2(a+bx)\sec(a+bx)}{15b} + \frac{8\sec(a+bx)}{15b} & \text{for } b \neq 0 \\ x\tan^5(a)\sec(a) & \text{otherwise} \end{cases}$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)**5,x)
```

output
```
Piecewise((tan(a + b*x)**4*sec(a + b*x)/(5*b) - 4*tan(a + b*x)**2*sec(a +
b*x)/(15*b) + 8*sec(a + b*x)/(15*b), Ne(b, 0)), (x*tan(a)**5*sec(a), True)
)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \sec(a+bx)\tan^5(a+bx)\,dx = \frac{15\cos(bx+a)^4 - 10\cos(bx+a)^2 + 3}{15\,b\cos(bx+a)^5}$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)^5,x, algorithm="maxima")
```

output
```
1/15*(15*cos(b*x + a)^4 - 10*cos(b*x + a)^2 + 3)/(b*cos(b*x + a)^5)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \sec(a+bx)\tan^5(a+bx)\,dx = \frac{15\,\cos\left(bx+a\right)^4-10\,\cos\left(bx+a\right)^2+3}{15\,b\cos\left(bx+a\right)^5}$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)^5,x, algorithm="giac")
```

output
```
1/15*(15*cos(b*x + a)^4 - 10*cos(b*x + a)^2 + 3)/(b*cos(b*x + a)^5)
```

Mupad [B] (verification not implemented)

Time = 25.44 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \sec(a+bx)\tan^5(a+bx)\,dx = \frac{1}{b\,\cos\left(a+b\,x\right)}-\frac{2}{3\,b\cos\left(a+b\,x\right)^3}+\frac{1}{5\,b\cos\left(a+b\,x\right)^5}$$

input
```
int(tan(a + b*x)^5/cos(a + b*x),x)
```

output
```
1/(b*cos(a + b*x)) - 2/(3*b*cos(a + b*x)^3) + 1/(5*b*cos(a + b*x)^5)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \sec(a+bx)\tan^5(a+bx)\,dx = \frac{\sec\left(bx+a\right)\left(3\tan\left(bx+a\right)^4-4\tan\left(bx+a\right)^2+8\right)}{15b}$$

input
```
int(sec(b*x+a)*tan(b*x+a)^5,x)
```

output
```
(sec(a + b*x)*(3*tan(a + b*x)**4 - 4*tan(a + b*x)**2 + 8))/(15*b)
```

3.111 $\int \sec^2(a+bx)\tan^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 15

$$\int \sec^2(a+bx)\tan^5(a+bx)\,dx=\frac{\tan^6(a+bx)}{6b}$$

output
```
1/6*tan(b*x+a)^6/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \sec^2(a+bx)\tan^5(a+bx)\,dx=\frac{\tan^6(a+bx)}{6b}$$

input
```
Integrate[Sec[a + b*x]^2*Tan[a + b*x]^5,x]
```

output
```
Tan[a + b*x]^6/(6*b)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3042, 3087, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(a+bx)\sec^2(a+bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \tan(a+bx)^5\sec(a+bx)^2dx$$

$$\Big\downarrow \ 3087$$

$$\frac{\int \tan^5(a+bx)d\tan(a+bx)}{b}$$

$$\Big\downarrow \ 15$$

$$\frac{\tan^6(a+bx)}{6b}$$

input
```
Int[Sec[a + b*x]^2*Tan[a + b*x]^5,x]
```

output
```
Tan[a + b*x]^6/(6*b)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

Maple [A] (verified)

Time = 1.95 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\tan(bx+a)^6}{6b}$	14
default	$\frac{\tan(bx+a)^6}{6b}$	14
risch	$\frac{2\,\mathrm{e}^{10i(bx+a)}+\frac{20\,\mathrm{e}^{6i(bx+a)}}{3}+2\,\mathrm{e}^{2i(bx+a)}}{b\left(\mathrm{e}^{2i(bx+a)}+1\right)^6}$	53

input
```
int(sec(b*x+a)^2*tan(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/6*tan(b*x+a)^6/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(13) = 26.

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.33

$$\int \sec^2(a+bx)\tan^5(a+bx)\,dx = \frac{3\,\cos\left(bx+a\right)^4-3\,\cos\left(bx+a\right)^2+1}{6\,b\cos\left(bx+a\right)^6}$$

input
```
integrate(sec(b*x+a)^2*tan(b*x+a)^5,x, algorithm="fricas")
```

output
```
1/6*(3*cos(b*x + a)^4 - 3*cos(b*x + a)^2 + 1)/(b*cos(b*x + a)^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(10) = 20$.

Time = 0.55 (sec) , antiderivative size = 63, normalized size of antiderivative = 4.20

$$\int \sec^2(a+bx)\tan^5(a+bx)\,dx = \begin{cases} \frac{\tan^4(a+bx)\sec^2(a+bx)}{6b} - \frac{\tan^2(a+bx)\sec^2(a+bx)}{6b} + \frac{\sec^2(a+bx)}{6b} & \text{for } b \neq 0 \\ x\tan^5(a)\sec^2(a) & \text{otherwise} \end{cases}$$

input
```
integrate(sec(b*x+a)**2*tan(b*x+a)**5,x)
```

output
```
Piecewise((tan(a + b*x)**4*sec(a + b*x)**2/(6*b) - tan(a + b*x)**2*sec(a +
 b*x)**2/(6*b) + sec(a + b*x)**2/(6*b), Ne(b, 0)), (x*tan(a)**5*sec(a)**2,
 True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sec^2(a+bx)\tan^5(a+bx)\,dx = \frac{\tan(bx+a)^6}{6\,b}$$

input
```
integrate(sec(b*x+a)^2*tan(b*x+a)^5,x, algorithm="maxima")
```

output
```
1/6*tan(b*x + a)^6/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(13) = 26$.

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.33

$$\int \sec^2(a+bx)\tan^5(a+bx)\,dx = \frac{3\cos(bx+a)^4 - 3\cos(bx+a)^2 + 1}{6\,b\cos(bx+a)^6}$$

input
```
integrate(sec(b*x+a)^2*tan(b*x+a)^5,x, algorithm="giac")
```

output
```
1/6*(3*cos(b*x + a)^4 - 3*cos(b*x + a)^2 + 1)/(b*cos(b*x + a)^6)
```

Mupad [B] (verification not implemented)

Time = 25.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sec^2(a+bx)\tan^5(a+bx)\,dx = \frac{\tan(a+b\,x)^6}{6\,b}$$

input
```
int(tan(a + b*x)^5/cos(a + b*x)^2,x)
```

output
```
tan(a + b*x)^6/(6*b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.20

$$\int \sec^2(a+bx)\tan^5(a+bx)\,dx = \frac{\sec(bx+a)^2\left(\tan(bx+a)^4 - \tan(bx+a)^2 + 1\right)}{6b}$$

input
```
int(sec(b*x+a)^2*tan(b*x+a)^5,x)
```

output
```
(sec(a + b*x)**2*(tan(a + b*x)**4 - tan(a + b*x)**2 + 1))/(6*b)
```

3.112 $\int \sec^3(a+bx)\tan^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \sec^3(a+bx)\tan^5(a+bx)\,dx=\frac{\sec^3(a+bx)}{3b}-\frac{2\sec^5(a+bx)}{5b}+\frac{\sec^7(a+bx)}{7b}$$

output
```
1/3*sec(b*x+a)^3/b-2/5*sec(b*x+a)^5/b+1/7*sec(b*x+a)^7/b
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \sec^3(a+bx)\tan^5(a+bx)\,dx=\frac{\sec^3(a+bx)}{3b}-\frac{2\sec^5(a+bx)}{5b}+\frac{\sec^7(a+bx)}{7b}$$

input
```
Integrate[Sec[a + b*x]^3*Tan[a + b*x]^5,x]
```

output
```
Sec[a + b*x]^3/(3*b) - (2*Sec[a + b*x]^5)/(5*b) + Sec[a + b*x]^7/(7*b)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 3086, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(a+bx)\sec^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(a+bx)^5\sec(a+bx)^3dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{\int \sec^2(a+bx)\left(1-\sec^2(a+bx)\right)^2d\sec(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\sec^6(a+bx)-2\sec^4(a+bx)+\sec^2(a+bx)\right)d\sec(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{7}\sec^7(a+bx)-\frac{2}{5}\sec^5(a+bx)+\frac{1}{3}\sec^3(a+bx)}{b}$$

input
```
Int[Sec[a + b*x]^3*Tan[a + b*x]^5,x]
```

output
```
(Sec[a + b*x]^3/3 - (2*Sec[a + b*x]^5)/5 + Sec[a + b*x]^7/7)/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Maple [A] (verified)

Time = 3.54 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\frac{\sec(bx+a)^7}{7}-\frac{2\sec(bx+a)^5}{5}+\frac{\sec(bx+a)^3}{3}}{b}$	36
default	$\frac{\frac{\sec(bx+a)^7}{7}-\frac{2\sec(bx+a)^5}{5}+\frac{\sec(bx+a)^3}{3}}{b}$	36
risch	$\frac{\frac{8\,e^{11i(bx+a)}}{3}-\frac{32\,e^{9i(bx+a)}}{15}+\frac{304\,e^{7i(bx+a)}}{35}-\frac{32\,e^{5i(bx+a)}}{15}+\frac{8\,e^{3i(bx+a)}}{3}}{b\left(e^{2i(bx+a)}+1\right)^7}$	75

input
```
int(sec(b*x+a)^3*tan(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/7*sec(b*x+a)^7-2/5*sec(b*x+a)^5+1/3*sec(b*x+a)^3)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \sec^3(a+bx)\tan^5(a+bx)\,dx = \frac{35\,\cos(bx+a)^4 - 42\,\cos(bx+a)^2 + 15}{105\,b\cos(bx+a)^7}$$

input
```
integrate(sec(b*x+a)^3*tan(b*x+a)^5,x, algorithm="fricas")
```

output
```
1/105*(35*cos(b*x + a)^4 - 42*cos(b*x + a)^2 + 15)/(b*cos(b*x + a)^7)
```

Sympy [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.43

$$\int \sec^3(a+bx)\tan^5(a+bx)\,dx = \begin{cases} \frac{\tan^4(a+bx)\sec^3(a+bx)}{7b} - \frac{4\tan^2(a+bx)\sec^3(a+bx)}{35b} + \frac{8\sec^3(a+bx)}{105b} & \text{for } b \neq 0 \\ x\tan^5(a)\sec^3(a) & \text{otherwise} \end{cases}$$

input
```
integrate(sec(b*x+a)**3*tan(b*x+a)**5,x)
```

output
```
Piecewise((tan(a + b*x)**4*sec(a + b*x)**3/(7*b) - 4*tan(a + b*x)**2*sec(a
 + b*x)**3/(35*b) + 8*sec(a + b*x)**3/(105*b), Ne(b, 0)), (x*tan(a)**5*sec
(a)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \sec^3(a+bx)\tan^5(a+bx)\,dx = \frac{35\,\cos(bx+a)^4 - 42\,\cos(bx+a)^2 + 15}{105\,b\cos(bx+a)^7}$$

input
```
integrate(sec(b*x+a)^3*tan(b*x+a)^5,x, algorithm="maxima")
```

output
```
1/105*(35*cos(b*x + a)^4 - 42*cos(b*x + a)^2 + 15)/(b*cos(b*x + a)^7)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \sec^3(a+bx)\tan^5(a+bx)\,dx = \frac{35\cos(bx+a)^4-42\cos(bx+a)^2+15}{105\,b\cos(bx+a)^7}$$

input
```
integrate(sec(b*x+a)^3*tan(b*x+a)^5,x, algorithm="giac")
```

output
```
1/105*(35*cos(b*x + a)^4 - 42*cos(b*x + a)^2 + 15)/(b*cos(b*x + a)^7)
```

Mupad [B] (verification not implemented)

Time = 25.44 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \sec^3(a+bx)\tan^5(a+bx)\,dx = \frac{1}{3\,b\cos(a+b\,x)^3}-\frac{2}{5\,b\cos(a+b\,x)^5}+\frac{1}{7\,b\cos(a+b\,x)^7}$$

input
```
int(tan(a + b*x)^5/cos(a + b*x)^3,x)
```

output
```
1/(3*b*cos(a + b*x)^3) - 2/(5*b*cos(a + b*x)^5) + 1/(7*b*cos(a + b*x)^7)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \sec^3(a+bx)\tan^5(a+bx)\,dx = \frac{\sec(bx+a)^3\left(15\tan(bx+a)^4-12\tan(bx+a)^2+8\right)}{105b}$$

input
```
int(sec(b*x+a)^3*tan(b*x+a)^5,x)
```

output
```
(sec(a + b*x)**3*(15*tan(a + b*x)**4 - 12*tan(a + b*x)**2 + 8))/(105*b)
```

3.113 $\int \sec^4(a+bx)\tan^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \sec^4(a+bx)\tan^5(a+bx)\,dx=\frac{\tan^6(a+bx)}{6b}+\frac{\tan^8(a+bx)}{8b}$$

output
```
1/6*tan(b*x+a)^6/b+1/8*tan(b*x+a)^8/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.48

$$\int \sec^4(a+bx)\tan^5(a+bx)\,dx=\frac{\sec^4(a+bx)}{4b}-\frac{\sec^6(a+bx)}{3b}+\frac{\sec^8(a+bx)}{8b}$$

input
```
Integrate[Sec[a + b*x]^4*Tan[a + b*x]^5,x]
```

output
```
Sec[a + b*x]^4/(4*b) - Sec[a + b*x]^6/(3*b) + Sec[a + b*x]^8/(8*b)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(a+bx)\sec^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(a+bx)^5\sec(a+bx)^4dx$$

$$\Big\downarrow \text{3087}$$

$$\frac{\int \tan^5(a+bx)\left(\tan^2(a+bx)+1\right)d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\tan^7(a+bx)+\tan^5(a+bx)\right)d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{8}\tan^8(a+bx)+\frac{1}{6}\tan^6(a+bx)}{b}$$

input
```
Int[Sec[a + b*x]^4*Tan[a + b*x]^5,x]
```

output
```
(Tan[a + b*x]^6/6 + Tan[a + b*x]^8/8)/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

Maple [A] (verified)

Time = 6.54 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\frac{\tan(bx+a)^8}{8}+\frac{\tan(bx+a)^6}{6}}{b}$	26
default	$\frac{\frac{\tan(bx+a)^8}{8}+\frac{\tan(bx+a)^6}{6}}{b}$	26
risch	$\frac{4\,\mathrm{e}^{12i(bx+a)}-\frac{16\,\mathrm{e}^{10i(bx+a)}}{3}+\frac{40\,\mathrm{e}^{8i(bx+a)}}{3}-\frac{16\,\mathrm{e}^{6i(bx+a)}}{3}+4\,\mathrm{e}^{4i(bx+a)}}{b\left(\mathrm{e}^{2i(bx+a)}+1\right)^{8}}$	75

input
```
int(sec(b*x+a)^4*tan(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/8*tan(b*x+a)^8+1/6*tan(b*x+a)^6)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13

$$\int \sec^4(a+bx)\tan^5(a+bx)\,dx = \frac{6\cos(bx+a)^4 - 8\cos(bx+a)^2 + 3}{24\,b\cos(bx+a)^8}$$

input
```
integrate(sec(b*x+a)^4*tan(b*x+a)^5,x, algorithm="fricas")
```

output
```
1/24*(6*cos(b*x + a)^4 - 8*cos(b*x + a)^2 + 3)/(b*cos(b*x + a)^8)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(22) = 44.

Time = 1.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.03

$$\int \sec^4(a+bx)\tan^5(a+bx)\,dx$$
$$= \begin{cases} \frac{\tan^4(a+bx)\sec^4(a+bx)}{8b} - \frac{\tan^2(a+bx)\sec^4(a+bx)}{12b} + \frac{\sec^4(a+bx)}{24b} & \text{for } b \neq 0 \\ x\tan^5(a)\sec^4(a) & \text{otherwise} \end{cases}$$

input
```
integrate(sec(b*x+a)**4*tan(b*x+a)**5,x)
```

output
```
Piecewise((tan(a + b*x)**4*sec(a + b*x)**4/(8*b) - tan(a + b*x)**2*sec(a +
 b*x)**4/(12*b) + sec(a + b*x)**4/(24*b), Ne(b, 0)), (x*tan(a)**5*sec(a)**
4, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(27) = 54$.

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.23

$$\int \sec^4(a+bx)\tan^5(a+bx)\,dx = \frac{6\sin(bx+a)^4 - 4\sin(bx+a)^2 + 1}{24\left(\sin(bx+a)^8 - 4\sin(bx+a)^6 + 6\sin(bx+a)^4 - 4\sin(bx+a)^2 + 1\right)b}$$

input
```
integrate(sec(b*x+a)^4*tan(b*x+a)^5,x, algorithm="maxima")
```

output
```
1/24*(6*sin(b*x + a)^4 - 4*sin(b*x + a)^2 + 1)/((sin(b*x + a)^8 - 4*sin(b*
x + a)^6 + 6*sin(b*x + a)^4 - 4*sin(b*x + a)^2 + 1)*b)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13

$$\int \sec^4(a+bx)\tan^5(a+bx)\,dx = \frac{6\cos(bx+a)^4 - 8\cos(bx+a)^2 + 3}{24\,b\cos(bx+a)^8}$$

input
```
integrate(sec(b*x+a)^4*tan(b*x+a)^5,x, algorithm="giac")
```

output
```
1/24*(6*cos(b*x + a)^4 - 8*cos(b*x + a)^2 + 3)/(b*cos(b*x + a)^8)
```

Mupad [B] (verification not implemented)

Time = 25.45 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \sec^4(a+bx)\tan^5(a+bx)\,dx = \frac{\tan(a+b\,x)^6\,\left(3\tan(a+b\,x)^2+4\right)}{24\,b}$$

input
```
int(tan(a + b*x)^5/cos(a + b*x)^4,x)
```

output
```
(tan(a + b*x)^6*(3*tan(a + b*x)^2 + 4))/(24*b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13

$$\int \sec^4(a+bx)\tan^5(a+bx)\,dx = \frac{\sec(bx+a)^4\left(3\tan(bx+a)^4-2\tan(bx+a)^2+1\right)}{24b}$$

input
```
int(sec(b*x+a)^4*tan(b*x+a)^5,x)
```

output
```
(sec(a + b*x)**4*(3*tan(a + b*x)**4 - 2*tan(a + b*x)**2 + 1))/(24*b)
```

3.114 $\int \sec^5(a+bx)\tan^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \sec^5(a+bx)\tan^5(a+bx)\,dx=\frac{\sec^5(a+bx)}{5b}-\frac{2\sec^7(a+bx)}{7b}+\frac{\sec^9(a+bx)}{9b}$$

output
```
1/5*sec(b*x+a)^5/b-2/7*sec(b*x+a)^7/b+1/9*sec(b*x+a)^9/b
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \sec^5(a+bx)\tan^5(a+bx)\,dx=\frac{\sec^5(a+bx)}{5b}-\frac{2\sec^7(a+bx)}{7b}+\frac{\sec^9(a+bx)}{9b}$$

input
```
Integrate[Sec[a + b*x]^5*Tan[a + b*x]^5,x]
```

output
```
Sec[a + b*x]^5/(5*b) - (2*Sec[a + b*x]^7)/(7*b) + Sec[a + b*x]^9/(9*b)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 3086, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(a+bx)\sec^5(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(a+bx)^5\sec(a+bx)^5dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{\int \sec^4(a+bx)\left(1-\sec^2(a+bx)\right)^2d\sec(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\sec^8(a+bx)-2\sec^6(a+bx)+\sec^4(a+bx)\right)d\sec(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{9}\sec^9(a+bx)-\frac{2}{7}\sec^7(a+bx)+\frac{1}{5}\sec^5(a+bx)}{b}$$

input
```
Int[Sec[a + b*x]^5*Tan[a + b*x]^5,x]
```

output
```
(Sec[a + b*x]^5/5 - (2*Sec[a + b*x]^7)/7 + Sec[a + b*x]^9/9)/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Maple [A] (verified)

Time = 12.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\frac{\sec(bx+a)^9}{9}-\frac{2\sec(bx+a)^7}{7}+\frac{\sec(bx+a)^5}{5}}{b}$	36
default	$\frac{\frac{\sec(bx+a)^9}{9}-\frac{2\sec(bx+a)^7}{7}+\frac{\sec(bx+a)^5}{5}}{b}$	36
risch	$\frac{\frac{32\,e^{13i(bx+a)}}{5}-\frac{384\,e^{11i(bx+a)}}{35}+\frac{6976\,e^{9i(bx+a)}}{315}-\frac{384\,e^{7i(bx+a)}}{35}+\frac{32\,e^{5i(bx+a)}}{5}}{b\left(e^{2i(bx+a)}+1\right)^9}$	75

input
```
int(sec(b*x+a)^5*tan(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/9*sec(b*x+a)^9-2/7*sec(b*x+a)^7+1/5*sec(b*x+a)^5)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \sec^5(a+bx)\tan^5(a+bx)\,dx = \frac{63\,\cos(bx+a)^4 - 90\,\cos(bx+a)^2 + 35}{315\,b\cos(bx+a)^9}$$

input
```
integrate(sec(b*x+a)^5*tan(b*x+a)^5,x, algorithm="fricas")
```

output
```
1/315*(63*cos(b*x + a)^4 - 90*cos(b*x + a)^2 + 35)/(b*cos(b*x + a)^9)
```

Sympy [A] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.43

$$\int \sec^5(a+bx)\tan^5(a+bx)\,dx$$
$$= \begin{cases} \frac{\tan^4(a+bx)\sec^5(a+bx)}{9b} - \frac{4\tan^2(a+bx)\sec^5(a+bx)}{63b} + \frac{8\sec^5(a+bx)}{315b} & \text{for}\ b \neq 0 \\ x\tan^5(a)\sec^5(a) & \text{otherwise} \end{cases}$$

input
```
integrate(sec(b*x+a)**5*tan(b*x+a)**5,x)
```

output
```
Piecewise((tan(a + b*x)**4*sec(a + b*x)**5/(9*b) - 4*tan(a + b*x)**2*sec(a
 + b*x)**5/(63*b) + 8*sec(a + b*x)**5/(315*b), Ne(b, 0)), (x*tan(a)**5*sec
(a)**5, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \sec^5(a+bx)\tan^5(a+bx)\,dx = \frac{63\,\cos(bx+a)^4 - 90\,\cos(bx+a)^2 + 35}{315\,b\cos(bx+a)^9}$$

input
```
integrate(sec(b*x+a)^5*tan(b*x+a)^5,x, algorithm="maxima")
```

output
```
1/315*(63*cos(b*x + a)^4 - 90*cos(b*x + a)^2 + 35)/(b*cos(b*x + a)^9)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \sec^5(a+bx)\tan^5(a+bx)\,dx = \frac{63\,\cos\left(bx+a\right)^4 - 90\,\cos\left(bx+a\right)^2 + 35}{315\,b\cos\left(bx+a\right)^9}$$

input
```
integrate(sec(b*x+a)^5*tan(b*x+a)^5,x, algorithm="giac")
```

output
```
1/315*(63*cos(b*x + a)^4 - 90*cos(b*x + a)^2 + 35)/(b*cos(b*x + a)^9)
```

Mupad [B] (verification not implemented)

Time = 25.88 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \sec^5(a+bx)\tan^5(a+bx)\,dx = \frac{1}{5\,b\cos\left(a+b\,x\right)^5} - \frac{2}{7\,b\cos\left(a+b\,x\right)^7} + \frac{1}{9\,b\cos\left(a+b\,x\right)^9}$$

input
```
int(tan(a + b*x)^5/cos(a + b*x)^5,x)
```

output
```
1/(5*b*cos(a + b*x)^5) - 2/(7*b*cos(a + b*x)^7) + 1/(9*b*cos(a + b*x)^9)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \sec^5(a+bx)\tan^5(a+bx)\,dx = \frac{\sec\left(bx+a\right)^5\left(35\tan\left(bx+a\right)^4 - 20\tan\left(bx+a\right)^2 + 8\right)}{315b}$$

input
```
int(sec(b*x+a)^5*tan(b*x+a)^5,x)
```

output
```
(sec(a + b*x)**5*(35*tan(a + b*x)**4 - 20*tan(a + b*x)**2 + 8))/(315*b)
```

3.115 $\int \sec^6(a+bx)\tan^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \sec^6(a+bx)\tan^5(a+bx)\,dx=\frac{\sec^6(a+bx)}{6b}-\frac{\sec^8(a+bx)}{4b}+\frac{\sec^{10}(a+bx)}{10b}$$

output
```
1/6*sec(b*x+a)^6/b-1/4*sec(b*x+a)^8/b+1/10*sec(b*x+a)^10/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \sec^6(a+bx)\tan^5(a+bx)\,dx=\frac{\sec^6(a+bx)}{6b}-\frac{\sec^8(a+bx)}{4b}+\frac{\sec^{10}(a+bx)}{10b}$$

input
```
Integrate[Sec[a + b*x]^6*Tan[a + b*x]^5,x]
```

output
```
Sec[a + b*x]^6/(6*b) - Sec[a + b*x]^8/(4*b) + Sec[a + b*x]^10/(10*b)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3042, 3086, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(a+bx)\sec^6(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(a+bx)^5\sec(a+bx)^6dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{\int \sec^5(a+bx)\left(1-\sec^2(a+bx)\right)^2d\sec(a+bx)}{b}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int \sec^4(a+bx)\left(1-\sec^2(a+bx)\right)^2d\sec^2(a+bx)}{2b}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\sec^8(a+bx)-2\sec^6(a+bx)+\sec^4(a+bx)\right)d\sec^2(a+bx)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{5}\sec^{10}(a+bx)-\frac{1}{2}\sec^8(a+bx)+\frac{1}{3}\sec^6(a+bx)}{2b}$$

input
```
Int[Sec[a + b*x]^6*Tan[a + b*x]^5,x]
```

output
```
(Sec[a + b*x]^6/3 - Sec[a + b*x]^8/2 + Sec[a + b*x]^10/5)/(2*b)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Maple [A] (verified)

Time = 25.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\frac{\tan(bx+a)^{10}}{10}+\frac{\tan(bx+a)^{8}}{4}+\frac{\tan(bx+a)^{6}}{6}}{b}$	36
default	$\frac{\frac{\tan(bx+a)^{10}}{10}+\frac{\tan(bx+a)^{8}}{4}+\frac{\tan(bx+a)^{6}}{6}}{b}$	36
risch	$\frac{\frac{32\,{\rm e}^{14i(bx+a)}}{3}-\frac{64\,{\rm e}^{12i(bx+a)}}{3}+\frac{192\,{\rm e}^{10i(bx+a)}}{5}-\frac{64\,{\rm e}^{8i(bx+a)}}{3}+\frac{32\,{\rm e}^{6i(bx+a)}}{3}}{b\left({\rm e}^{2i(bx+a)}+1\right)^{10}}$	75

input
```
int(sec(b*x+a)^6*tan(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/10*tan(b*x+a)^10+1/4*tan(b*x+a)^8+1/6*tan(b*x+a)^6)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \sec^6(a+bx)\tan^5(a+bx)\,dx = \frac{10\,\cos\left(bx+a\right)^4-15\,\cos\left(bx+a\right)^2+6}{60\,b\cos\left(bx+a\right)^{10}}$$

input
```
integrate(sec(b*x+a)^6*tan(b*x+a)^5,x, algorithm="fricas")
```

output
```
1/60*(10*cos(b*x + a)^4 - 15*cos(b*x + a)^2 + 6)/(b*cos(b*x + a)^10)
```

Sympy [A] (verification not implemented)

Time = 2.34 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.37

$$\int \sec^6(a+bx)\tan^5(a+bx)\,dx$$
$$= \begin{cases} \frac{\tan^4(a+bx)\sec^6(a+bx)}{10b} - \frac{\tan^2(a+bx)\sec^6(a+bx)}{20b} + \frac{\sec^6(a+bx)}{60b} & \text{for } b \neq 0 \\ x\tan^5(a)\sec^6(a) & \text{otherwise} \end{cases}$$

input
```
integrate(sec(b*x+a)**6*tan(b*x+a)**5,x)
```

output
```
Piecewise((tan(a + b*x)**4*sec(a + b*x)**6/(10*b) - tan(a + b*x)**2*sec(a
+ b*x)**6/(20*b) + sec(a + b*x)**6/(60*b), Ne(b, 0)), (x*tan(a)**5*sec(a)*
*6, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.72

$$\int \sec^6(a+bx)\tan^5(a+bx)\,dx = -\frac{10\sin(bx+a)^4 - 5\sin(bx+a)^2 + 1}{60\left(\sin(bx+a)^{10} - 5\sin(bx+a)^8 + 10\sin(bx+a)^6 - 10\sin(bx+a)^4 + 5\sin(bx+a)^2 - 1\right)b}$$

input
```
integrate(sec(b*x+a)^6*tan(b*x+a)^5,x, algorithm="maxima")
```

output
```
-1/60*(10*sin(b*x + a)^4 - 5*sin(b*x + a)^2 + 1)/((sin(b*x + a)^10 - 5*sin
(b*x + a)^8 + 10*sin(b*x + a)^6 - 10*sin(b*x + a)^4 + 5*sin(b*x + a)^2 - 1
)*b)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \sec^6(a+bx)\tan^5(a+bx)\,dx = \frac{10\cos(bx+a)^4 - 15\cos(bx+a)^2 + 6}{60\,b\cos(bx+a)^{10}}$$

input
```
integrate(sec(b*x+a)^6*tan(b*x+a)^5,x, algorithm="giac")
```

output
```
1/60*(10*cos(b*x + a)^4 - 15*cos(b*x + a)^2 + 6)/(b*cos(b*x + a)^10)
```

Mupad [B] (verification not implemented)

Time = 25.43 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \sec^6(a+bx)\tan^5(a+bx)\,dx = \frac{\tan(a+b\,x)^6\,\left(6\tan(a+b\,x)^4+15\tan(a+b\,x)^2+10\right)}{60\,b}$$

input
```
int(tan(a + b*x)^5/cos(a + b*x)^6,x)
```

output
```
(tan(a + b*x)^6*(15*tan(a + b*x)^2 + 6*tan(a + b*x)^4 + 10))/(60*b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \sec^6(a+bx)\tan^5(a+bx)\,dx=\frac{\sec\left(bx+a\right)^6\left(6\tan\left(bx+a\right)^4-3\tan\left(bx+a\right)^2+1\right)}{60b}$$

input
```
int(sec(b*x+a)^6*tan(b*x+a)^5,x)
```

output
```
(sec(a + b*x)**6*(6*tan(a + b*x)**4 - 3*tan(a + b*x)**2 + 1))/(60*b)
```

3.116 $\int \sec^7(a+bx)\tan^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \sec^7(a+bx)\tan^5(a+bx)\,dx=\frac{\sec^7(a+bx)}{7b}-\frac{2\sec^9(a+bx)}{9b}+\frac{\sec^{11}(a+bx)}{11b}$$

output
```
1/7*sec(b*x+a)^7/b-2/9*sec(b*x+a)^9/b+1/11*sec(b*x+a)^11/b
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \sec^7(a+bx)\tan^5(a+bx)\,dx=\frac{\sec^7(a+bx)}{7b}-\frac{2\sec^9(a+bx)}{9b}+\frac{\sec^{11}(a+bx)}{11b}$$

input
```
Integrate[Sec[a + b*x]^7*Tan[a + b*x]^5,x]
```

output
```
Sec[a + b*x]^7/(7*b) - (2*Sec[a + b*x]^9)/(9*b) + Sec[a + b*x]^11/(11*b)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 3086, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(a+bx)\sec^7(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(a+bx)^5\sec(a+bx)^7dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{\int \sec^6(a+bx)\left(1-\sec^2(a+bx)\right)^2d\sec(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\sec^{10}(a+bx)-2\sec^8(a+bx)+\sec^6(a+bx)\right)d\sec(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{11}\sec^{11}(a+bx)-\frac{2}{9}\sec^9(a+bx)+\frac{1}{7}\sec^7(a+bx)}{b}$$

input
```
Int[Sec[a + b*x]^7*Tan[a + b*x]^5,x]
```

output
```
(Sec[a + b*x]^7/7 - (2*Sec[a + b*x]^9)/9 + Sec[a + b*x]^11/11)/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Maple [A] (verified)

Time = 40.69 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\frac{\sec(bx+a)^{11}}{11}-\frac{2\sec(bx+a)^{9}}{9}+\frac{\sec(bx+a)^{7}}{7}}{b}$	36
default	$\frac{\frac{\sec(bx+a)^{11}}{11}-\frac{2\sec(bx+a)^{9}}{9}+\frac{\sec(bx+a)^{7}}{7}}{b}$	36
risch	$\frac{\frac{128\,e^{15i(bx+a)}}{7}-\frac{2560\,e^{13i(bx+a)}}{63}+\frac{47360\,e^{11i(bx+a)}}{693}-\frac{2560\,e^{9i(bx+a)}}{63}+\frac{128\,e^{7i(bx+a)}}{7}}{b\left(e^{2i(bx+a)}+1\right)^{11}}$	75

input
```
int(sec(b*x+a)^7*tan(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/11*sec(b*x+a)^11-2/9*sec(b*x+a)^9+1/7*sec(b*x+a)^7)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \sec^7(a+bx)\tan^5(a+bx)\,dx = \frac{99\cos(bx+a)^4 - 154\cos(bx+a)^2 + 63}{693\,b\cos(bx+a)^{11}}$$

input
```
integrate(sec(b*x+a)^7*tan(b*x+a)^5,x, algorithm="fricas")
```

output
```
1/693*(99*cos(b*x + a)^4 - 154*cos(b*x + a)^2 + 63)/(b*cos(b*x + a)^11)
```

Sympy [A] (verification not implemented)

Time = 3.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.43

$$\int \sec^7(a+bx)\tan^5(a+bx)\,dx$$
$$= \begin{cases} \frac{\tan^4(a+bx)\sec^7(a+bx)}{11b} - \frac{4\tan^2(a+bx)\sec^7(a+bx)}{99b} + \frac{8\sec^7(a+bx)}{693b} & \text{for } b \neq 0 \\ x\tan^5(a)\sec^7(a) & \text{otherwise} \end{cases}$$

input
```
integrate(sec(b*x+a)**7*tan(b*x+a)**5,x)
```

output
```
Piecewise((tan(a + b*x)**4*sec(a + b*x)**7/(11*b) - 4*tan(a + b*x)**2*sec(
a + b*x)**7/(99*b) + 8*sec(a + b*x)**7/(693*b), Ne(b, 0)), (x*tan(a)**5*se
c(a)**7, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \sec^7(a+bx)\tan^5(a+bx)\,dx = \frac{99\cos(bx+a)^4 - 154\cos(bx+a)^2 + 63}{693\,b\cos(bx+a)^{11}}$$

input
```
integrate(sec(b*x+a)^7*tan(b*x+a)^5,x, algorithm="maxima")
```

output
```
1/693*(99*cos(b*x + a)^4 - 154*cos(b*x + a)^2 + 63)/(b*cos(b*x + a)^11)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \sec^7(a+bx)\tan^5(a+bx)\,dx = \frac{99\,\cos(bx+a)^4 - 154\,\cos(bx+a)^2 + 63}{693\,b\cos(bx+a)^{11}}$$

input
```
integrate(sec(b*x+a)^7*tan(b*x+a)^5,x, algorithm="giac")
```

output
```
1/693*(99*cos(b*x + a)^4 - 154*cos(b*x + a)^2 + 63)/(b*cos(b*x + a)^11)
```

Mupad [B] (verification not implemented)

Time = 19.18 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \sec^7(a+bx)\tan^5(a+bx)\,dx = \frac{1}{7\,b\cos(a+b\,x)^7} - \frac{2}{9\,b\cos(a+b\,x)^9} + \frac{1}{11\,b\cos(a+b\,x)^{11}}$$

input
```
int(tan(a + b*x)^5/cos(a + b*x)^7,x)
```

output
```
1/(7*b*cos(a + b*x)^7) - 2/(9*b*cos(a + b*x)^9) + 1/(11*b*cos(a + b*x)^11)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \sec^7(a+bx)\tan^5(a+bx)\,dx = \frac{\sec(bx+a)^7\,(63\tan(bx+a)^4 - 28\tan(bx+a)^2 + 8)}{693b}$$

input
```
int(sec(b*x+a)^7*tan(b*x+a)^5,x)
```

output
```
(sec(a + b*x)**7*(63*tan(a + b*x)**4 - 28*tan(a + b*x)**2 + 8))/(693*b)
```

3.117 $\int \sec^8(a+bx)\tan^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \sec^8(a+bx)\tan^5(a+bx)\,dx=\frac{\sec^8(a+bx)}{8b}-\frac{\sec^{10}(a+bx)}{5b}+\frac{\sec^{12}(a+bx)}{12b}$$

output
```
1/8*sec(b*x+a)^8/b-1/5*sec(b*x+a)^10/b+1/12*sec(b*x+a)^12/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \sec^8(a+bx)\tan^5(a+bx)\,dx=\frac{\sec^8(a+bx)}{8b}-\frac{\sec^{10}(a+bx)}{5b}+\frac{\sec^{12}(a+bx)}{12b}$$

input
```
Integrate[Sec[a + b*x]^8*Tan[a + b*x]^5,x]
```

output
```
Sec[a + b*x]^8/(8*b) - Sec[a + b*x]^10/(5*b) + Sec[a + b*x]^12/(12*b)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3042, 3086, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(a+bx)\sec^8(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(a+bx)^5\sec(a+bx)^8dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{\int \sec^7(a+bx)\left(1-\sec^2(a+bx)\right)^2d\sec(a+bx)}{b}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int \sec^6(a+bx)\left(1-\sec^2(a+bx)\right)^2d\sec^2(a+bx)}{2b}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\sec^{10}(a+bx)-2\sec^8(a+bx)+\sec^6(a+bx)\right)d\sec^2(a+bx)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{6}\sec^{12}(a+bx)-\frac{2}{5}\sec^{10}(a+bx)+\frac{1}{4}\sec^8(a+bx)}{2b}$$

input
```
Int[Sec[a + b*x]^8*Tan[a + b*x]^5,x]
```

output
```
(Sec[a + b*x]^8/4 - (2*Sec[a + b*x]^10)/5 + Sec[a + b*x]^12/6)/(2*b)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Maple [A] (verified)

Time = 88.98 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\frac{\tan(bx+a)^{12}}{12}+\frac{3\tan(bx+a)^{10}}{10}+\frac{3\tan(bx+a)^{8}}{8}+\frac{\tan(bx+a)^{6}}{6}}{b}$	46
default	$\frac{\frac{\tan(bx+a)^{12}}{12}+\frac{3\tan(bx+a)^{10}}{10}+\frac{3\tan(bx+a)^{8}}{8}+\frac{\tan(bx+a)^{6}}{6}}{b}$	46
risch	$\frac{32\,\mathrm{e}^{16i(bx+a)}-\frac{384\,\mathrm{e}^{14i(bx+a)}}{5}+\frac{1856\,\mathrm{e}^{12i(bx+a)}}{15}-\frac{384\,\mathrm{e}^{10i(bx+a)}}{5}+32\,\mathrm{e}^{8i(bx+a)}}{b\left(\mathrm{e}^{2i(bx+a)}+1\right)^{12}}$	75

input
```
int(sec(b*x+a)^8*tan(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/12*tan(b*x+a)^12+3/10*tan(b*x+a)^10+3/8*tan(b*x+a)^8+1/6*tan(b*x+a)
^6)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \sec^8(a+bx)\tan^5(a+bx)\,dx = \frac{15\,\cos\left(bx+a\right)^4 - 24\,\cos\left(bx+a\right)^2 + 10}{120\,b\cos\left(bx+a\right)^{12}}$$

input
```
integrate(sec(b*x+a)^8*tan(b*x+a)^5,x, algorithm="fricas")
```

output
```
1/120*(15*cos(b*x + a)^4 - 24*cos(b*x + a)^2 + 10)/(b*cos(b*x + a)^12)
```

Sympy [A] (verification not implemented)

Time = 4.33 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.37

$$\int \sec^8(a+bx)\tan^5(a+bx)\,dx$$
$$= \begin{cases} \frac{\tan^4(a+bx)\sec^8(a+bx)}{12b} - \frac{\tan^2(a+bx)\sec^8(a+bx)}{30b} + \frac{\sec^8(a+bx)}{120b} & \text{for } b \neq 0 \\ x\tan^5(a)\sec^8(a) & \text{otherwise} \end{cases}$$

input
```
integrate(sec(b*x+a)**8*tan(b*x+a)**5,x)
```

output
```
Piecewise((tan(a + b*x)**4*sec(a + b*x)**8/(12*b) - tan(a + b*x)**2*sec(a
+ b*x)**8/(30*b) + sec(a + b*x)**8/(120*b), Ne(b, 0)), (x*tan(a)**5*sec(a)
**8, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(40) = 80$.

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.93

$$\int \sec^8(a+bx)\tan^5(a+bx)\,dx$$
$$= \frac{15\sin(bx+a)^4 - 6\sin(bx+a)^2 + 1}{120\left(\sin(bx+a)^{12} - 6\sin(bx+a)^{10} + 15\sin(bx+a)^8 - 20\sin(bx+a)^6 + 15\sin(bx+a)^4 - 6\sin(bx\right.}$$

input
```
integrate(sec(b*x+a)^8*tan(b*x+a)^5,x, algorithm="maxima")
```

output
```
1/120*(15*sin(b*x + a)^4 - 6*sin(b*x + a)^2 + 1)/((sin(b*x + a)^12 - 6*sin
(b*x + a)^10 + 15*sin(b*x + a)^8 - 20*sin(b*x + a)^6 + 15*sin(b*x + a)^4 -
 6*sin(b*x + a)^2 + 1)*b)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \sec^8(a+bx)\tan^5(a+bx)\,dx = \frac{15\cos(bx+a)^4 - 24\cos(bx+a)^2 + 10}{120\,b\cos(bx+a)^{12}}$$

input
```
integrate(sec(b*x+a)^8*tan(b*x+a)^5,x, algorithm="giac")
```

output
```
1/120*(15*cos(b*x + a)^4 - 24*cos(b*x + a)^2 + 10)/(b*cos(b*x + a)^12)
```

Mupad [B] (verification not implemented)

Time = 25.46 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \sec^8(a+bx)\tan^5(a+bx)\,dx = \frac{\frac{\tan(a+b\,x)^{12}}{12}+\frac{3\tan(a+b\,x)^{10}}{10}+\frac{3\tan(a+b\,x)^8}{8}+\frac{\tan(a+b\,x)^6}{6}}{b}$$

input
```
int(tan(a + b*x)^5/cos(a + b*x)^8,x)
```

output
```
(tan(a + b*x)^6/6 + (3*tan(a + b*x)^8)/8 + (3*tan(a + b*x)^10)/10 + tan(a
+ b*x)^12/12)/b
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \sec^8(a+bx)\tan^5(a+bx)\,dx = \frac{\sec(bx+a)^8\left(10\tan(bx+a)^4-4\tan(bx+a)^2+1\right)}{120b}$$

input
```
int(sec(b*x+a)^8*tan(b*x+a)^5,x)
```

output
```
(sec(a + b*x)**8*(10*tan(a + b*x)**4 - 4*tan(a + b*x)**2 + 1))/(120*b)
```

3.118 $\int \sin^3(a+bx)\tan^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 60

$$\int \sin^3(a+bx)\tan^3(a+bx)\,dx = -\frac{5\text{arctanh}(\sin(a+bx))}{2b}+\frac{2\sin(a+bx)}{b}+\frac{\sin^3(a+bx)}{3b}+\frac{\sec(a+bx)\tan(a+bx)}{2b}$$

output
```
-5/2*arctanh(sin(b*x+a))/b+2*sin(b*x+a)/b+1/3*sin(b*x+a)^3/b+1/2*sec(b*x+a
)*tan(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.30

$$\int \sin^3(a+bx)\tan^3(a+bx)\,dx = -\frac{5\text{arctanh}(\sin(a+bx))}{2b}+\frac{5\sec(a+bx)\tan(a+bx)}{2b}-\frac{5\sin(a+bx)\tan^2(a+bx)}{3b}-\frac{\sin^3(a+bx)\tan^2(a+bx)}{3b}$$

input
```
Integrate[Sin[a + b*x]^3*Tan[a + b*x]^3,x]
```

output
```
(-5*ArcTanh[Sin[a + b*x]])/(2*b) + (5*Sec[a + b*x]*Tan[a + b*x])/(2*b) - (
5*Sin[a + b*x]*Tan[a + b*x]^2)/(3*b) - (Sin[a + b*x]^3*Tan[a + b*x]^2)/(3*
b)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3042, 3072, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\tan^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^3\tan(a+bx)^3dx$$

$$\Big\downarrow \text{3072}$$

$$\frac{\int \frac{\sin^6(a+bx)}{(1-\sin^2(a+bx))^2}d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{\sin^5(a+bx)}{2(1-\sin^2(a+bx))}-\frac{5}{2}\int \frac{\sin^4(a+bx)}{1-\sin^2(a+bx)}d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{254}$$

$$\frac{\frac{\sin^5(a+bx)}{2(1-\sin^2(a+bx))}-\frac{5}{2}\int \left(-\sin^2(a+bx)+\frac{1}{1-\sin^2(a+bx)}-1\right)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\sin^5(a+bx)}{2(1-\sin^2(a+bx))}-\frac{5}{2}\left(\text{arctanh}(\sin(a+bx))-\frac{1}{3}\sin^3(a+bx)-\sin(a+bx)\right)}{b}$$

input
```
Int[Sin[a + b*x]^3*Tan[a + b*x]^3,x]
```

output
```
(Sin[a + b*x]^5/(2*(1 - Sin[a + b*x]^2)) - (5*(ArcTanh[Sin[a + b*x]] - Sin
[a + b*x] - Sin[a + b*x]^3/3))/2)/b
```

Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{\frac{\sin(bx+a)^7}{2\cos(bx+a)^2}+\frac{\sin(bx+a)^5}{2}+\frac{5\sin(bx+a)^3}{6}+\frac{5\sin(bx+a)}{2}-\frac{5\ln(\sec(bx+a)+\tan(bx+a))}{2}}{b}$
default	$\frac{\frac{\sin(bx+a)^7}{2\cos(bx+a)^2}+\frac{\sin(bx+a)^5}{2}+\frac{5\sin(bx+a)^3}{6}+\frac{5\sin(bx+a)}{2}-\frac{5\ln(\sec(bx+a)+\tan(bx+a))}{2}}{b}$
risch	$\frac{i\,\mathrm{e}^{3i(bx+a)}}{24b}-\frac{9i\,\mathrm{e}^{i(bx+a)}}{8b}+\frac{9i\,\mathrm{e}^{-i(bx+a)}}{8b}-\frac{i\,\mathrm{e}^{-3i(bx+a)}}{24b}-\frac{i\left(\mathrm{e}^{3i(bx+a)}-\mathrm{e}^{i(bx+a)}\right)}{b\left(\mathrm{e}^{2i(bx+a)}+1\right)^2}-\frac{5\ln\left(\mathrm{e}^{i(bx+a)}+i\right)}{2b}+\frac{5\ln(\mathrm{e}$

input
```
int(sin(b*x+a)^3*tan(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2*sin(b*x+a)^7/cos(b*x+a)^2+1/2*sin(b*x+a)^5+5/6*sin(b*x+a)^3+5/2*s
in(b*x+a)-5/2*ln(sec(b*x+a)+tan(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.40

$$\int \sin^3(a+bx)\tan^3(a+bx)\,dx = -\frac{15\,\cos(bx+a)^2\log(\sin(bx+a)+1)-15\,\cos(bx+a)^2\log(-\sin(bx+a)+1)+2\left(2\,\cos(bx+a)^4\right.}{12\,b\cos(bx+a)^2}$$

input
```
integrate(sin(b*x+a)^3*tan(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/12*(15*cos(b*x + a)^2*log(sin(b*x + a) + 1) - 15*cos(b*x + a)^2*log(-si
n(b*x + a) + 1) + 2*(2*cos(b*x + a)^4 - 14*cos(b*x + a)^2 - 3)*sin(b*x + a
))/(b*cos(b*x + a)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \sin^3(a+bx)\tan^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**3*tan(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.10

$$\int \sin^3(a+bx)\tan^3(a+bx)\,dx = \frac{4\sin(bx+a)^3 - \frac{6\sin(bx+a)}{\sin(bx+a)^2-1} - 15\log(\sin(bx+a)+1) + 15\log(\sin(bx+a)-1) + 24\sin(bx+a)}{12\,b}$$

input
```
integrate(sin(b*x+a)^3*tan(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/12*(4*sin(b*x + a)^3 - 6*sin(b*x + a)/(sin(b*x + a)^2 - 1) - 15*log(sin(
b*x + a) + 1) + 15*log(sin(b*x + a) - 1) + 24*sin(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.38

$$\int \sin^3(a+bx)\tan^3(a+bx)\,dx = -\frac{5\log(|\sin(bx+a)+1|)}{4\,b} + \frac{5\log(|\sin(bx+a)-1|)}{4\,b} - \frac{\sin(bx+a)}{2\left(\sin(bx+a)^2-1\right)b} + \frac{b^2\sin(bx+a)^3 + 6\,b^2\sin(bx+a)}{3\,b^3}$$

input
```
integrate(sin(b*x+a)^3*tan(b*x+a)^3,x, algorithm="giac")
```

output
```
-5/4*log(abs(sin(b*x + a) + 1))/b + 5/4*log(abs(sin(b*x + a) - 1))/b - 1/2
*sin(b*x + a)/((sin(b*x + a)^2 - 1)*b) + 1/3*(b^2*sin(b*x + a)^3 + 6*b^2*s
in(b*x + a))/b^3
```

Mupad [B] (verification not implemented)

Time = 28.92 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.45

$$\int \sin^3(a+bx)\tan^3(a+bx)\,dx = \frac{5\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^9+\frac{20\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^7}{3}-\frac{22\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^5}{3}+\frac{20\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^3}{3}+5\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)}{b\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{10}+\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^8-2\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^6-2\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4+\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2+1\right)} - \frac{5\,\mathrm{atanh}\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{b}$$

input
```
int(sin(a + b*x)^3*tan(a + b*x)^3,x)
```

output
```
(5*tan(a/2 + (b*x)/2) + (20*tan(a/2 + (b*x)/2)^3)/3 - (22*tan(a/2 + (b*x)/
2)^5)/3 + (20*tan(a/2 + (b*x)/2)^7)/3 + 5*tan(a/2 + (b*x)/2)^9)/(b*(tan(a/
2 + (b*x)/2)^2 - 2*tan(a/2 + (b*x)/2)^4 - 2*tan(a/2 + (b*x)/2)^6 + tan(a/2
 + (b*x)/2)^8 + tan(a/2 + (b*x)/2)^10 + 1)) - (5*atanh(tan(a/2 + (b*x)/2))
)/b
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.97

$$\int \sin^3(a+bx)\tan^3(a+bx)\,dx = \frac{15\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin(bx+a)^2-15\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)-15\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)\sin(bx+a}{6b\left(\sin(bx+a)^2-1\right)}$$

input
```
int(sin(b*x+a)^3*tan(b*x+a)^3,x)
```

output
```
(15*log(tan((a + b*x)/2) - 1)*sin(a + b*x)**2 - 15*log(tan((a + b*x)/2) -
1) - 15*log(tan((a + b*x)/2) + 1)*sin(a + b*x)**2 + 15*log(tan((a + b*x)/2
) + 1) + 2*sin(a + b*x)**5 + 10*sin(a + b*x)**3 - 15*sin(a + b*x))/(6*b*(s
in(a + b*x)**2 - 1))
```

3.119 $\int \sin(a+bx)\tan^6(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 50

$$\int \sin(a+bx)\tan^6(a+bx)\,dx = \frac{\cos(a+bx)}{b} + \frac{3\sec(a+bx)}{b} - \frac{\sec^3(a+bx)}{b} + \frac{\sec^5(a+bx)}{5b}$$

output
```
cos(b*x+a)/b+3*sec(b*x+a)/b-sec(b*x+a)^3/b+1/5*sec(b*x+a)^5/b
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \sin(a+bx)\tan^6(a+bx)\,dx = \frac{\cos(a+bx)}{b} + \frac{3\sec(a+bx)}{b} - \frac{\sec^3(a+bx)}{b} + \frac{\sec^5(a+bx)}{5b}$$

input
```
Integrate[Sin[a + b*x]*Tan[a + b*x]^6,x]
```

output
```
Cos[a + b*x]/b + (3*Sec[a + b*x])/b - Sec[a + b*x]^3/b + Sec[a + b*x]^5/(5
*b)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\tan^6(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)\tan(a+bx)^6dx$$

$$\Big\downarrow \text{3070}$$

$$-\frac{\int \left(1-\cos^2(a+bx)\right)^3\sec^6(a+bx)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(\sec^6(a+bx)-3\sec^4(a+bx)+3\sec^2(a+bx)-1\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\cos(a+bx)-\frac{1}{5}\sec^5(a+bx)+\sec^3(a+bx)-3\sec(a+bx)}{b}$$

input
```
Int[Sin[a + b*x]*Tan[a + b*x]^6,x]
```

output
```
-((-Cos[a + b*x] - 3*Sec[a + b*x] + Sec[a + b*x]^3 - Sec[a + b*x]^5/5)/b)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.84

method	result	size
risch	$\frac{5\,\mathrm{e}^{11i(bx+a)}+90\,\mathrm{e}^{9i(bx+a)}+235\,\mathrm{e}^{7i(bx+a)}+364\,\mathrm{e}^{5i(bx+a)}+235\,\mathrm{e}^{3i(bx+a)}+95\cos(bx+a)+85i\sin(bx+a)}{10b\left(\mathrm{e}^{2i(bx+a)}+1\right)^{5}}$	92
derivativedivides	$\frac{\frac{\sin(bx+a)^{8}}{5\cos(bx+a)^{5}}-\frac{\sin(bx+a)^{8}}{5\cos(bx+a)^{3}}+\frac{\sin(bx+a)^{8}}{\cos(bx+a)}+\left(\frac{16}{5}+\sin(bx+a)^{6}+\frac{6\sin(bx+a)^{4}}{5}+\frac{8\sin(bx+a)^{2}}{5}\right)\cos(bx+a)}{b}$	96
default	$\frac{\frac{\sin(bx+a)^{8}}{5\cos(bx+a)^{5}}-\frac{\sin(bx+a)^{8}}{5\cos(bx+a)^{3}}+\frac{\sin(bx+a)^{8}}{\cos(bx+a)}+\left(\frac{16}{5}+\sin(bx+a)^{6}+\frac{6\sin(bx+a)^{4}}{5}+\frac{8\sin(bx+a)^{2}}{5}\right)\cos(bx+a)}{b}$	96

input
```
int(sin(b*x+a)*tan(b*x+a)^6,x,method=_RETURNVERBOSE)
```

output
```
1/10*(5*exp(11*I*(b*x+a))+90*exp(9*I*(b*x+a))+235*exp(7*I*(b*x+a))+364*exp
(5*I*(b*x+a))+235*exp(3*I*(b*x+a))+95*cos(b*x+a)+85*I*sin(b*x+a))/b/(exp(2
*I*(b*x+a))+1)^5
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int \sin(a+bx)\tan^6(a+bx)\,dx = \frac{5\cos(bx+a)^6+15\cos(bx+a)^4-5\cos(bx+a)^2+1}{5\,b\cos(bx+a)^5}$$

input
```
integrate(sin(b*x+a)*tan(b*x+a)^6,x, algorithm="fricas")
```

output
```
1/5*(5*cos(b*x + a)^6 + 15*cos(b*x + a)^4 - 5*cos(b*x + a)^2 + 1)/(b*cos(b
*x + a)^5)
```

Sympy [F]

$$\int \sin(a+bx)\tan^6(a+bx)\,dx = \int \sin(a+bx)\tan^6(a+bx)\,dx$$

input
```
integrate(sin(b*x+a)*tan(b*x+a)**6,x)
```

output
```
Integral(sin(a + b*x)*tan(a + b*x)**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int \sin(a+bx)\tan^6(a+bx)\,dx = \frac{\frac{15\cos(bx+a)^4-5\cos(bx+a)^2+1}{\cos(bx+a)^5}+5\cos(bx+a)}{5\,b}$$

input
```
integrate(sin(b*x+a)*tan(b*x+a)^6,x, algorithm="maxima")
```

output
```
1/5*((15*cos(b*x + a)^4 - 5*cos(b*x + a)^2 + 1)/cos(b*x + a)^5 + 5*cos(b*x
 + a))/b
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \sin(a+bx)\tan^6(a+bx)\,dx = \frac{\cos(bx+a)}{b} + \frac{15\cos(bx+a)^4 - 5\cos(bx+a)^2 + 1}{5\,b\cos(bx+a)^5}$$

input
```
integrate(sin(b*x+a)*tan(b*x+a)^6,x, algorithm="giac")
```

output
```
cos(b*x + a)/b + 1/5*(15*cos(b*x + a)^4 - 5*cos(b*x + a)^2 + 1)/(b*cos(b*x
 + a)^5)
```

Mupad [B] (verification not implemented)

Time = 25.55 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \sin(a+bx)\tan^6(a+bx)\,dx = \frac{\cos(a+b\,x)}{b} + \frac{3}{b\,\cos(a+b\,x)} - \frac{1}{b\cos(a+b\,x)^3} + \frac{1}{5\,b\cos(a+b\,x)^5}$$

input
```
int(sin(a + b*x)*tan(a + b*x)^6,x)
```

output
```
cos(a + b*x)/b + 3/(b*cos(a + b*x)) - 1/(b*cos(a + b*x)^3) + 1/(5*b*cos(a
+ b*x)^5)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 272, normalized size of antiderivative = 5.44

$$\int \sin(a+bx)\tan^6(a+bx)\,dx$$

$$= \frac{-2\cos(bx+a)\tan(bx+a)^4\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4 + 2\cos(bx+a)\tan(bx+a)^4 + 11\cos(bx+a)\tan(bx+a)^2}{}$$

input
```
int(sin(b*x+a)*tan(b*x+a)^6,x)
```

output
```
( - 2*cos(a + b*x)*tan(a + b*x)**4*tan((a + b*x)/2)**4 + 2*cos(a + b*x)*ta
n(a + b*x)**4 + 11*cos(a + b*x)*tan(a + b*x)**2*tan((a + b*x)/2)**4 - 11*c
os(a + b*x)*tan(a + b*x)**2 - 22*cos(a + b*x)*tan((a + b*x)/2)**4 + 22*cos
(a + b*x) + 8*sin(a + b*x)*tan(a + b*x)**5*tan((a + b*x)/2)**4 - 8*sin(a +
 b*x)*tan(a + b*x)**5 - 14*sin(a + b*x)*tan(a + b*x)**3*tan((a + b*x)/2)**
4 + 14*sin(a + b*x)*tan(a + b*x)**3 - 22*sin(a + b*x)*tan(a + b*x)*tan((a
+ b*x)/2)**4 + 22*sin(a + b*x)*tan(a + b*x) - 300*tan((a + b*x)/2)**4)/(40
*b*(tan((a + b*x)/2)**4 - 1))
```

3.120 $\int \cos^5(a+bx)\cot(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 53

$$\int \cos^5(a+bx)\cot(a+bx)\,dx = -\frac{\operatorname{arctanh}(\cos(a+bx))}{b}+\frac{\cos(a+bx)}{b}+\frac{\cos^3(a+bx)}{3b}+\frac{\cos^5(a+bx)}{5b}$$

output
```
-arctanh(cos(b*x+a))/b+cos(b*x+a)/b+1/3*cos(b*x+a)^3/b+1/5*cos(b*x+a)^5/b
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.42

$$\int \cos^5(a+bx)\cot(a+bx)\,dx = \frac{11\cos(a+bx)}{8b}+\frac{7\cos(3(a+bx))}{48b}+\frac{\cos(5(a+bx))}{80b}-\frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b}+\frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b}$$

input
```
Integrate[Cos[a + b*x]^5*Cot[a + b*x],x]
```

output
```
(11*Cos[a + b*x])/(8*b) + (7*Cos[3*(a + b*x)])/(48*b) + Cos[5*(a + b*x)]/(
80*b) - Log[Cos[(a + b*x)/2]]/b + Log[Sin[(a + b*x)/2]]/b
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 25, 3072, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(a+bx)\cot(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\sin\left(a+bx+\frac{\pi}{2}\right)^5\tan\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \; 25$$

$$-\int \sin\left(\frac{1}{2}(2a+\pi)+bx\right)^5\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx$$

$$\Big\downarrow \; 3072$$

$$-\frac{\int \frac{\cos^6(a+bx)}{1-\cos^2(a+bx)}d\cos(a+bx)}{b}$$

$$\Big\downarrow \; 254$$

$$-\frac{\int \left(-\cos^4(a+bx)-\cos^2(a+bx)+\frac{1}{1-\cos^2(a+bx)}-1\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\text{arctanh}(\cos(a+bx))-\frac{1}{5}\cos^5(a+bx)-\frac{1}{3}\cos^3(a+bx)-\cos(a+bx)}{b}$$

input

```
Int[Cos[a + b*x]^5*Cot[a + b*x],x]
```

output

```
-((ArcTanh[Cos[a + b*x]] - Cos[a + b*x] - Cos[a + b*x]^3/3 - Cos[a + b*x]^
5/5)/b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Maple [A] (verified)

Time = 5.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{\frac{\cos(bx+a)^5}{5}+\frac{\cos(bx+a)^3}{3}+\cos(bx+a)+\ln(\csc(bx+a)-\cot(bx+a))}{b}$	48
default	$\frac{\frac{\cos(bx+a)^5}{5}+\frac{\cos(bx+a)^3}{3}+\cos(bx+a)+\ln(\csc(bx+a)-\cot(bx+a))}{b}$	48
risch	$\frac{11\,e^{i(bx+a)}}{16b}+\frac{11\,e^{-i(bx+a)}}{16b}+\frac{\ln(e^{i(bx+a)}-1)}{b}-\frac{\ln(e^{i(bx+a)}+1)}{b}+\frac{\cos(5bx+5a)}{80b}+\frac{7\cos(3bx+3a)}{48b}$	91

input
```
int(cos(b*x+a)^5*cot(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/5*cos(b*x+a)^5+1/3*cos(b*x+a)^3+cos(b*x+a)+ln(csc(b*x+a)-cot(b*x+a)
))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.13

$$\int \cos^5(a+bx)\cot(a+bx)\,dx$$
$$= \frac{6\cos(bx+a)^5 + 10\cos(bx+a)^3 + 30\cos(bx+a) - 15\log(\frac{1}{2}\cos(bx+a)+\frac{1}{2}) + 15\log(-\frac{1}{2}\cos(bx}{30\,b}$$

input
```
integrate(cos(b*x+a)^5*cot(b*x+a),x, algorithm="fricas")
```

output
```
1/30*(6*cos(b*x + a)^5 + 10*cos(b*x + a)^3 + 30*cos(b*x + a) - 15*log(1/2*
cos(b*x + a) + 1/2) + 15*log(-1/2*cos(b*x + a) + 1/2))/b
```

Sympy [F(-1)]

Timed out.

$$\int \cos^5(a+bx)\cot(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**5*cot(b*x+a),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int \cos^5(a+bx)\cot(a+bx)\,dx$$
$$= \frac{6\cos(bx+a)^5 + 10\cos(bx+a)^3 + 30\cos(bx+a) - 15\log(\cos(bx+a)+1) + 15\log(\cos(bx+a) -}{30\,b}$$

input
```
integrate(cos(b*x+a)^5*cot(b*x+a),x, algorithm="maxima")
```

output
```
1/30*(6*cos(b*x + a)^5 + 10*cos(b*x + a)^3 + 30*cos(b*x + a) - 15*log(cos(
b*x + a) + 1) + 15*log(cos(b*x + a) - 1))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.40

$$\int \cos^5(a+bx)\cot(a+bx)\,dx = -\frac{\log(|\cos(bx+a)+1|)}{2b} + \frac{\log(|\cos(bx+a)-1|)}{2b} + \frac{3b^4\cos(bx+a)^5 + 5b^4\cos(bx+a)^3 + 15b^4\cos(bx+a)}{15b^5}$$

input
```
integrate(cos(b*x+a)^5*cot(b*x+a),x, algorithm="giac")
```

output
```
-1/2*log(abs(cos(b*x + a) + 1))/b + 1/2*log(abs(cos(b*x + a) - 1))/b + 1/1
5*(3*b^4*cos(b*x + a)^5 + 5*b^4*cos(b*x + a)^3 + 15*b^4*cos(b*x + a))/b^5
```

Mupad [B] (verification not implemented)

Time = 30.60 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.66

$$\int \cos^5(a+bx)\cot(a+bx)\,dx = \frac{\ln\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b} + \frac{6\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^8 + 12\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^6 + \frac{56\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^4}{3} + \frac{28\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2}{3} + \frac{46}{15}}{b\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2+1\right)^5}$$

input
```
int(cos(a + b*x)^5*cot(a + b*x),x)
```

output
```
log(tan(a/2 + (b*x)/2))/b + ((28*tan(a/2 + (b*x)/2)^2)/3 + (56*tan(a/2 + (
b*x)/2)^4)/3 + 12*tan(a/2 + (b*x)/2)^6 + 6*tan(a/2 + (b*x)/2)^8 + 46/15)/(
b*(tan(a/2 + (b*x)/2)^2 + 1)^5)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.11

$$\int \cos^5(a+bx)\cot(a+bx)\,dx$$
$$= \frac{3\cos(bx+a)\sin(bx+a)^4 - 11\cos(bx+a)\sin(bx+a)^2 + 23\cos(bx+a) + 15\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right) - 23}{15b}$$

input
```
int(cos(b*x+a)^5*cot(b*x+a),x)
```

output
```
(3*cos(a + b*x)*sin(a + b*x)**4 - 11*cos(a + b*x)*sin(a + b*x)**2 + 23*cos
(a + b*x) + 15*log(tan((a + b*x)/2)) - 23)/(15*b)
```

3.121 $\int \cos^4(a+bx)\cot(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 40

$$\int \cos^4(a+bx)\cot(a+bx)\,dx=\frac{\log(\sin(a+bx))}{b}-\frac{\sin^2(a+bx)}{b}+\frac{\sin^4(a+bx)}{4b}$$

output
```
ln(sin(b*x+a))/b-sin(b*x+a)^2/b+1/4*sin(b*x+a)^4/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \cos^4(a+bx)\cot(a+bx)\,dx=\frac{\log(\sin(a+bx))}{b}-\frac{\sin^2(a+bx)}{b}+\frac{\sin^4(a+bx)}{4b}$$

input
```
Integrate[Cos[a + b*x]^4*Cot[a + b*x],x]
```

output
```
Log[Sin[a + b*x]]/b - Sin[a + b*x]^2/b + Sin[a + b*x]^4/(4*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 25, 3070, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(a+bx)\cot(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\sin\left(a+bx+\frac{\pi}{2}\right)^4\tan\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sin\left(\frac{1}{2}(2a+\pi)+bx\right)^4\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx$$

$$\Big\downarrow \text{3070}$$

$$\frac{\int -\csc(a+bx)\left(1-\sin^2(a+bx)\right)^2d(-\sin(a+bx))}{b}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int -\csc(a+bx)(\sin(a+bx)+1)^2d\sin^2(a+bx)}{2b}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\sin^2(a+bx)-\csc(a+bx)-2\right)d\sin^2(a+bx)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}\sin^2(a+bx)+2\sin(a+bx)+\log\left(\sin^2(a+bx)\right)}{2b}$$

input
```
Int[Cos[a + b*x]^4*Cot[a + b*x],x]
```

output
```
(Log[Sin[a + b*x]^2] + 2*Sin[a + b*x] + Sin[a + b*x]^2/2)/(2*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Maple [A] (verified)

Time = 3.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\frac{\cos(bx+a)^4}{4}+\frac{\cos(bx+a)^2}{2}+\ln(\sin(bx+a))}{b}$	33
default	$\frac{\frac{\cos(bx+a)^4}{4}+\frac{\cos(bx+a)^2}{2}+\ln(\sin(bx+a))}{b}$	33
risch	$-ix+\frac{3\,e^{2i(bx+a)}}{16b}+\frac{3\,e^{-2i(bx+a)}}{16b}-\frac{2ia}{b}+\frac{\ln\left(e^{2i(bx+a)}-1\right)}{b}+\frac{\cos(4bx+4a)}{32b}$	71

input
```
int(cos(b*x+a)^4*cot(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/4*cos(b*x+a)^4+1/2*cos(b*x+a)^2+ln(sin(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int \cos^4(a+bx)\cot(a+bx)\,dx = \frac{\cos{(bx+a)}^4 + 2\,\cos{(bx+a)}^2 + 4\,\log\left(\frac{1}{2}\,\sin{(bx+a)}\right)}{4\,b}$$

input
```
integrate(cos(b*x+a)^4*cot(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(cos(b*x + a)^4 + 2*cos(b*x + a)^2 + 4*log(1/2*sin(b*x + a)))/b
```

Sympy [F]

$$\int \cos^4(a+bx)\cot(a+bx)\,dx = \int \cos^4{(a+bx)}\cot{(a+bx)}\,dx$$

input
```
integrate(cos(b*x+a)**4*cot(b*x+a),x)
```

output
```
Integral(cos(a + b*x)**4*cot(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int \cos^4(a+bx)\cot(a+bx)\,dx = \frac{\sin{(bx+a)}^4 - 4\,\sin{(bx+a)}^2 + 2\,\log\left(\sin{(bx+a)}^2\right)}{4\,b}$$

input
```
integrate(cos(b*x+a)^4*cot(b*x+a),x, algorithm="maxima")
```

output
```
1/4*(sin(b*x + a)^4 - 4*sin(b*x + a)^2 + 2*log(sin(b*x + a)^2))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12

$$\int \cos^4(a+bx)\cot(a+bx)\,dx = \frac{\log\left(\left|\cos\left(bx+a\right)^2-1\right|\right)}{2\,b} + \frac{b\cos\left(bx+a\right)^4+2\,b\cos\left(bx+a\right)^2}{4\,b^2}$$

input
```
integrate(cos(b*x+a)^4*cot(b*x+a),x, algorithm="giac")
```

output
```
1/2*log(abs(cos(b*x + a)^2 - 1))/b + 1/4*(b*cos(b*x + a)^4 + 2*b*cos(b*x +
 a)^2)/b^2
```

Mupad [B] (verification not implemented)

Time = 25.52 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.65

$$\int \cos^4(a+bx)\cot(a+bx)\,dx = \frac{\ln\left(\tan(a+b\,x)\right)}{b} - \frac{\ln\left(\tan(a+b\,x)^2+1\right)}{2\,b} + \frac{\frac{\tan(a+b\,x)^2}{2}+\frac{3}{4}}{b\,\left(\tan\left(a+b\,x\right)^4+2\tan\left(a+b\,x\right)^2+1\right)}$$

input
```
int(cos(a + b*x)^4*cot(a + b*x),x)
```

output
```
log(tan(a + b*x))/b - log(tan(a + b*x)^2 + 1)/(2*b) + (tan(a + b*x)^2/2 +
3/4)/(b*(2*tan(a + b*x)^2 + tan(a + b*x)^4 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.30

$$\int \cos^4(a+bx)\cot(a+bx)\,dx$$
$$= \frac{-4\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2+1\right)+4\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sin(bx+a)^4-4\sin(bx+a)^2}{4b}$$

input
```
int(cos(b*x+a)^4*cot(b*x+a),x)
```

output
```
( - 4*log(tan((a + b*x)/2)**2 + 1) + 4*log(tan((a + b*x)/2)) + sin(a + b*x
)**4 - 4*sin(a + b*x)**2)/(4*b)
```

3.122 $\int \cos^3(a+bx)\cot(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \cos^3(a+bx)\cot(a+bx)\,dx=-\frac{\operatorname{arctanh}(\cos(a+bx))}{b}+\frac{\cos(a+bx)}{b}+\frac{\cos^3(a+bx)}{3b}$$

output
```
-arctanh(cos(b*x+a))/b+cos(b*x+a)/b+1/3*cos(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.58

$$\int \cos^3(a+bx)\cot(a+bx)\,dx=\frac{5\cos(a+bx)}{4b}+\frac{\cos(3(a+bx))}{12b}-\frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b}+\frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b}$$

input
```
Integrate[Cos[a + b*x]^3*Cot[a + b*x],x]
```

output
```
(5*Cos[a + b*x])/(4*b) + Cos[3*(a + b*x)]/(12*b) - Log[Cos[(a + b*x)/2]]/b
 + Log[Sin[(a + b*x)/2]]/b
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 25, 3072, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)\cot(a+bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int -\sin\left(a+bx+\frac{\pi}{2}\right)^3\tan\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \ 25$$

$$-\int \sin\left(\frac{1}{2}(2a+\pi)+bx\right)^3\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx$$

$$\Big\downarrow \ 3072$$

$$-\frac{\int \frac{\cos^4(a+bx)}{1-\cos^2(a+bx)}d\cos(a+bx)}{b}$$

$$\Big\downarrow \ 254$$

$$-\frac{\int \left(-\cos^2(a+bx)+\frac{1}{1-\cos^2(a+bx)}-1\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \ 2009$$

$$-\frac{\text{arctanh}(\cos(a+bx))-\frac{1}{3}\cos^3(a+bx)-\cos(a+bx)}{b}$$

input
```
Int[Cos[a + b*x]^3*Cot[a + b*x],x]
```

output
```
-((ArcTanh[Cos[a + b*x]] - Cos[a + b*x] - Cos[a + b*x]^3/3)/b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\frac{\cos(bx+a)^3}{3}+\cos(bx+a)+\ln(\csc(bx+a)-\cot(bx+a))}{b}$	38
default	$\frac{\frac{\cos(bx+a)^3}{3}+\cos(bx+a)+\ln(\csc(bx+a)-\cot(bx+a))}{b}$	38
risch	$\frac{5\,e^{i(bx+a)}}{8b}+\frac{5\,e^{-i(bx+a)}}{8b}-\frac{\ln\left(e^{i(bx+a)}+1\right)}{b}+\frac{\ln\left(e^{i(bx+a)}-1\right)}{b}+\frac{\cos(3bx+3a)}{12b}$	77

input
```
int(cos(b*x+a)^3*cot(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/3*cos(b*x+a)^3+cos(b*x+a)+ln(csc(b*x+a)-cot(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int \cos^3(a+bx)\cot(a+bx)\,dx$$
$$=\frac{2\cos(bx+a)^3+6\cos(bx+a)-3\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)+3\log\left(-\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)}{6b}$$

input
```
integrate(cos(b*x+a)^3*cot(b*x+a),x, algorithm="fricas")
```

output
```
1/6*(2*cos(b*x + a)^3 + 6*cos(b*x + a) - 3*log(1/2*cos(b*x + a) + 1/2) + 3
*log(-1/2*cos(b*x + a) + 1/2))/b
```

Sympy [F]

$$\int \cos^3(a+bx)\cot(a+bx)\,dx=\int \cos^3(a+bx)\cot(a+bx)\,dx$$

input
```
integrate(cos(b*x+a)**3*cot(b*x+a),x)
```

output
```
Integral(cos(a + b*x)**3*cot(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

$$\int \cos^3(a+bx)\cot(a+bx)\,dx$$
$$=\frac{2\cos(bx+a)^3+6\cos(bx+a)-3\log(\cos(bx+a)+1)+3\log(\cos(bx+a)-1)}{6b}$$

input
```
integrate(cos(b*x+a)^3*cot(b*x+a),x, algorithm="maxima")
```

output
```
1/6*(2*cos(b*x + a)^3 + 6*cos(b*x + a) - 3*log(cos(b*x + a) + 1) + 3*log(c
os(b*x + a) - 1))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.58

$$\int \cos^3(a+bx)\cot(a+bx)\,dx = -\frac{\log\left(|\cos\left(bx+a\right)+1|\right)}{2\,b}+\frac{\log\left(|\cos\left(bx+a\right)-1|\right)}{2\,b}+\frac{b^2\cos\left(bx+a\right)^3+3\,b^2\cos\left(bx+a\right)}{3\,b^3}$$

input
```
integrate(cos(b*x+a)^3*cot(b*x+a),x, algorithm="giac")
```

output
```
-1/2*log(abs(cos(b*x + a) + 1))/b + 1/2*log(abs(cos(b*x + a) - 1))/b + 1/3
*(b^2*cos(b*x + a)^3 + 3*b^2*cos(b*x + a))/b^3
```

Mupad [B] (verification not implemented)

Time = 26.75 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.63

$$\int \cos^3(a+bx)\cot(a+bx)\,dx = \frac{\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{b}+\frac{4\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4+4\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2+\frac{8}{3}}{b\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2+1\right)^3}$$

input
```
int(cos(a + b*x)^3*cot(a + b*x),x)
```

output
```
log(tan(a/2 + (b*x)/2))/b + (4*tan(a/2 + (b*x)/2)^2 + 4*tan(a/2 + (b*x)/2)
^4 + 8/3)/(b*(tan(a/2 + (b*x)/2)^2 + 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13

$$\int \cos^3(a+bx)\cot(a+bx)\,dx$$
$$= \frac{-\cos(bx+a)\sin(bx+a)^2+4\cos(bx+a)+3\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-4}{3b}$$

input
```
int(cos(b*x+a)^3*cot(b*x+a),x)
```

output
```
( - cos(a + b*x)*sin(a + b*x)**2 + 4*cos(a + b*x) + 3*log(tan((a + b*x)/2)
) - 4)/(3*b)
```

3.123 $\int \cos^2(a+bx)\cot(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \cos^2(a+bx)\cot(a+bx)\,dx=\frac{\log(\sin(a+bx))}{b}-\frac{\sin^2(a+bx)}{2b}$$

output
```
ln(sin(b*x+a))/b-1/2*sin(b*x+a)^2/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \cos^2(a+bx)\cot(a+bx)\,dx=\frac{\log(\sin(a+bx))}{b}-\frac{\sin^2(a+bx)}{2b}$$

input
```
Integrate[Cos[a + b*x]^2*Cot[a + b*x],x]
```

output
```
Log[Sin[a + b*x]]/b - Sin[a + b*x]^2/(2*b)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 25, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\cot(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\sin\left(a+bx+\frac{\pi}{2}\right)^2\tan\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sin\left(\frac{1}{2}(2a+\pi)+bx\right)^2\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx$$

$$\Big\downarrow \text{3070}$$

$$\frac{\int -\csc(a+bx)\left(1-\sin^2(a+bx)\right)d(-\sin(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int (\sin(a+bx)-\csc(a+bx))d(-\sin(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log(-\sin(a+bx))-\frac{1}{2}\sin^2(a+bx)}{b}$$

input
```
Int[Cos[a + b*x]^2*Cot[a + b*x],x]
```

output
```
(Log[-Sin[a + b*x]] - Sin[a + b*x]^2/2)/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\frac{\cos(bx+a)^2}{2}+\ln(\sin(bx+a))}{b}$	23
default	$\frac{\frac{\cos(bx+a)^2}{2}+\ln(\sin(bx+a))}{b}$	23
risch	$-ix+\frac{e^{2i(bx+a)}}{8b}+\frac{e^{-2i(bx+a)}}{8b}-\frac{2ia}{b}+\frac{\ln\left(e^{2i(bx+a)}-1\right)}{b}$	57

input
```
int(cos(b*x+a)^2*cot(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2*cos(b*x+a)^2+ln(sin(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \cos^2(a+bx)\cot(a+bx)\,dx = \frac{\cos(bx+a)^2 + 2\log\left(\frac{1}{2}\sin(bx+a)\right)}{2b}$$

input
```
integrate(cos(b*x+a)^2*cot(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(cos(b*x + a)^2 + 2*log(1/2*sin(b*x + a)))/b
```

Sympy [F]

$$\int \cos^2(a+bx)\cot(a+bx)\,dx = \int \cos^2(a+bx)\cot(a+bx)\,dx$$

input
```
integrate(cos(b*x+a)**2*cot(b*x+a),x)
```

output
```
Integral(cos(a + b*x)**2*cot(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \cos^2(a+bx)\cot(a+bx)\,dx = -\frac{\sin(bx+a)^2 - \log\left(\sin(bx+a)^2\right)}{2b}$$

input
```
integrate(cos(b*x+a)^2*cot(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(sin(b*x + a)^2 - log(sin(b*x + a)^2))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \cos^2(a+bx)\cot(a+bx)\,dx = \frac{\cos(bx+a)^2}{2\,b} + \frac{\log\left(\left|\cos(bx+a)^2-1\right|\right)}{2\,b}$$

input
```
integrate(cos(b*x+a)^2*cot(b*x+a),x, algorithm="giac")
```

output
```
1/2*cos(b*x + a)^2/b + 1/2*log(abs(cos(b*x + a)^2 - 1))/b
```

Mupad [B] (verification not implemented)

Time = 25.42 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int \cos^2(a+bx)\cot(a+bx)\,dx = \frac{\frac{\cos(a+b\,x)^2}{2} - \frac{\ln\left(\tan(a+b\,x)^2+1\right)}{2} + \ln\left(\tan(a+b\,x)\right)}{b}$$

input
```
int(cos(a + b*x)^2*cot(a + b*x),x)
```

output
```
(log(tan(a + b*x)) - log(tan(a + b*x)^2 + 1)/2 + cos(a + b*x)^2/2)/b
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.63

$$\int \cos^2(a+bx)\cot(a+bx)\,dx = \frac{-2\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2+1\right) + 2\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right) - \sin(bx+a)^2}{2b}$$

input
```
int(cos(b*x+a)^2*cot(b*x+a),x)
```

output
```
( - 2*log(tan((a + b*x)/2)**2 + 1) + 2*log(tan((a + b*x)/2)) - sin(a + b*x
)**2)/(2*b)
```

3.124 $\int \cos(a+bx)\cot(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \cos(a+bx)\cot(a+bx)\,dx = -\frac{\operatorname{arctanh}(\cos(a+bx))}{b} + \frac{\cos(a+bx)}{b}$$

output
```
-arctanh(cos(b*x+a))/b+cos(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \cos(a+bx)\cot(a+bx)\,dx = \frac{\cos(a+bx)}{b} - \frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b} + \frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b}$$

input
```
Integrate[Cos[a + b*x]*Cot[a + b*x],x]
```

output
```
Cos[a + b*x]/b - Log[Cos[(a + b*x)/2]]/b + Log[Sin[(a + b*x)/2]]/b
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 25, 3072, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\cot(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\sin\left(a+bx+\frac{\pi}{2}\right)\tan\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sin\left(\frac{1}{2}(2a+\pi)+bx\right)\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx$$

$$\Big\downarrow \text{3072}$$

$$-\frac{\int \frac{\cos^2(a+bx)}{1-\cos^2(a+bx)}d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{262}$$

$$-\frac{\int \frac{1}{1-\cos^2(a+bx)}d\cos(a+bx)-\cos(a+bx)}{b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arctanh}(\cos(a+bx))-\cos(a+bx)}{b}$$

input
```
Int[Cos[a + b*x]*Cot[a + b*x],x]
```

output
```
-((ArcTanh[Cos[a + b*x]] - Cos[a + b*x])/b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22

method	result	size
derivativedivides	$\frac{\cos(bx+a)+\ln(\csc(bx+a)-\cot(bx+a))}{b}$	28
default	$\frac{\cos(bx+a)+\ln(\csc(bx+a)-\cot(bx+a))}{b}$	28
risch	$\frac{e^{i(bx+a)}}{2b}+\frac{e^{-i(bx+a)}}{2b}-\frac{\ln(e^{i(bx+a)}+1)}{b}+\frac{\ln(e^{i(bx+a)}-1)}{b}$	63

input
```
int(cos(b*x+a)*cot(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(cos(b*x+a)+ln(csc(b*x+a)-cot(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.65

$$\int \cos(a+bx)\cot(a+bx)\,dx = \frac{2\cos(bx+a) - \log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right) + \log\left(-\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)}{2b}$$

input
```
integrate(cos(b*x+a)*cot(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(2*cos(b*x + a) - log(1/2*cos(b*x + a) + 1/2) + log(-1/2*cos(b*x + a)
+ 1/2))/b
```

Sympy [F]

$$\int \cos(a+bx)\cot(a+bx)\,dx = \int \cos(a+bx)\cot(a+bx)\,dx$$

input
```
integrate(cos(b*x+a)*cot(b*x+a),x)
```

output
```
Integral(cos(a + b*x)*cot(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.48

$$\int \cos(a+bx)\cot(a+bx)\,dx = \frac{2\,\cos\left(bx+a\right)-\log\left(\cos\left(bx+a\right)+1\right)+\log\left(\cos\left(bx+a\right)-1\right)}{2\,b}$$

input
```
integrate(cos(b*x+a)*cot(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.78

$$\int \cos(a+bx)\cot(a+bx)\,dx = \frac{\cos\left(bx+a\right)}{b}-\frac{\log\left(\left|\cos\left(bx+a\right)+1\right|\right)}{2\,b}+\frac{\log\left(\left|\cos\left(bx+a\right)-1\right|\right)}{2\,b}$$

input
```
integrate(cos(b*x+a)*cot(b*x+a),x, algorithm="giac")
```

output
```
cos(b*x + a)/b - 1/2*log(abs(cos(b*x + a) + 1))/b + 1/2*log(abs(cos(b*x +
a) - 1))/b
```

Mupad [B] (verification not implemented)

Time = 25.51 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.52

$$\int \cos(a+bx)\cot(a+bx)\,dx = \frac{2}{b\,\left({\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)}^2+1\right)}+\frac{\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{b}$$

input
```
int(cos(a + b*x)*cot(a + b*x),x)
```

output
```
2/(b*(tan(a/2 + (b*x)/2)^2 + 1)) + log(tan(a/2 + (b*x)/2))/b
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \cos(a+bx)\cot(a+bx)\,dx = \frac{\cos(bx+a)+\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}{b}$$

input
```
int(cos(b*x+a)*cot(b*x+a),x)
```

output
```
(cos(a + b*x) + log(tan((a + b*x)/2)) - 1)/b
```

3.125 $\int \cot(a+bx)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 11

$$\int \cot(a+bx)\,dx = \frac{\log(\sin(a+bx))}{b}$$

output
```
ln(sin(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \cot(a+bx)\,dx = \frac{\log(\sin(a+bx))}{b}$$

input
```
Integrate[Cot[a + b*x],x]
```

output
```
Log[Sin[a + b*x]]/b
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(a+bx)\,dx$$
$$\Big\downarrow \ 3042$$
$$\int -\tan\left(a+bx+\frac{\pi}{2}\right)dx$$
$$\Big\downarrow \ 25$$
$$-\int \tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx$$
$$\Big\downarrow \ 3956$$
$$\frac{\log(-\sin(a+bx))}{b}$$

input
```
Int[Cot[a + b*x],x]
```

output
```
Log[-Sin[a + b*x]]/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

method	result	size
derivativedivides	$-\frac{\ln\left(\cot(bx+a)^2+1\right)}{2b}$	17
default	$-\frac{\ln\left(\cot(bx+a)^2+1\right)}{2b}$	17
parallelrisch	$\frac{\ln(\tan(bx+a))+\ln\left(\frac{1}{\sqrt{\sec(bx+a)^2}}\right)}{b}$	24
norman	$\frac{\ln(\tan(bx+a))}{b}-\frac{\ln\left(1+\tan(bx+a)^2\right)}{2b}$	29
risch	$-ix-\frac{2ia}{b}+\frac{\ln\left(e^{2i(bx+a)}-1\right)}{b}$	29

input
```
int(cot(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b*ln(cot(b*x+a)^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int \cot(a+bx)\,dx=\frac{\log\left(-\frac{1}{2}\,\cos\left(2\,bx+2\,a\right)+\frac{1}{2}\right)}{2\,b}$$

input
```
integrate(cot(b*x+a),x, algorithm="fricas")
```

output
```
1/2*log(-1/2*cos(2*b*x + 2*a) + 1/2)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(8) = 16$.

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.64

$$\int \cot(a+bx)\,dx = \begin{cases} -\frac{\log\left(\tan^2\left(a+bx\right)+1\right)}{2b} + \frac{\log\left(\tan\left(a+bx\right)\right)}{b} & \text{for } b \neq 0 \\ x\cot\left(a\right) & \text{otherwise} \end{cases}$$

input
```
integrate(cot(b*x+a),x)
```

output
```
Piecewise((-log(tan(a + b*x)**2 + 1)/(2*b) + log(tan(a + b*x))/b, Ne(b, 0)
), (x*cot(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \cot(a+bx)\,dx = \frac{\log\left(\sin\left(bx+a\right)\right)}{b}$$

input
```
integrate(cot(b*x+a),x, algorithm="maxima")
```

output
```
log(sin(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \cot(a+bx)\,dx = \frac{\log\left(\left|\sin\left(bx+a\right)\right|\right)}{b}$$

input
```
integrate(cot(b*x+a),x, algorithm="giac")
```

output
```
log(abs(sin(b*x + a)))/b
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.18

$$\int \cot(a+bx)\,dx = -x\,1\mathrm{i} + \frac{\ln\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}-1\right)}{b}$$

input
```
int(cot(a + b*x),x)
```

output
```
log(exp(a*2i)*exp(b*x*2i) - 1)/b - x*1i
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.82

$$\int \cot(a+bx)\,dx = \frac{-\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2+1\right)+\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}$$

input
```
int(cot(b*x+a),x)
```

output
```
( - log(tan((a + b*x)/2)**2 + 1) + log(tan((a + b*x)/2)))/b
```

3.126 $\int \csc(a+bx)\sec(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 11

$$\int \csc(a+bx)\sec(a+bx)\,dx = \frac{\log(\tan(a+bx))}{b}$$

output
```
ln(tan(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \csc(a+bx)\sec(a+bx)\,dx = -\frac{\operatorname{arctanh}(\cos(2a+2bx))}{b}$$

input
```
Integrate[Csc[a + b*x]*Sec[a + b*x],x]
```

output
```
-(ArcTanh[Cos[2*a + 2*b*x]]/b)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3100, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\sec(a+bx)\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \csc(a+bx)\sec(a+bx)dx$$
$$\Big\downarrow \text{ 3100}$$
$$\frac{\int \cot(a+bx)d\tan(a+bx)}{b}$$
$$\Big\downarrow \text{ 14}$$
$$\frac{\log(\tan(a+bx))}{b}$$

input
```
Int[Csc[a + b*x]*Sec[a + b*x],x]
```

output
```
Log[Tan[a + b*x]]/b
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{\ln(\tan(bx+a))}{b}$	12
default	$\frac{\ln(\tan(bx+a))}{b}$	12
risch	$-\frac{\ln(e^{2i(bx+a)}+1)}{b}+\frac{\ln(e^{2i(bx+a)}-1)}{b}$	35
parallelrisch	$\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)-\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{b}$	44
norman	$\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{b}-\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{b}$	50

input
```
int(csc(b*x+a)*sec(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
ln(tan(b*x+a))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(11) = 22.

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.73

$$\int \csc(a+bx)\sec(a+bx)\,dx=-\frac{\log\left(\cos\left(bx+a\right)^{2}\right)-\log\left(-\frac{1}{4}\,\cos\left(bx+a\right)^{2}+\frac{1}{4}\right)}{2\,b}$$

input
```
integrate(csc(b*x+a)*sec(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(log(cos(b*x + a)^2) - log(-1/4*cos(b*x + a)^2 + 1/4))/b
```

Sympy [F]

$$\int \csc(a+bx)\sec(a+bx)\,dx=\int \csc{(a+bx)}\sec{(a+bx)}\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a),x)
```

output
```
Integral(csc(a + b*x)*sec(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(11)=22$.

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.55

$$\int \csc(a+bx)\sec(a+bx)\,dx=-\frac{\log\left(\sin\left(bx+a\right)^{2}-1\right)-\log\left(\sin\left(bx+a\right)^{2}\right)}{2\,b}$$

input
```
integrate(csc(b*x+a)*sec(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(11)=22$.

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.73

$$\int \csc(a+bx)\sec(a+bx)\,dx=-\frac{\log\left(\left|\sin\left(bx+a\right)^{2}-1\right|\right)}{2\,b}+\frac{\log\left(\left|\sin\left(bx+a\right)\right|\right)}{b}$$

input
```
integrate(csc(b*x+a)*sec(b*x+a),x, algorithm="giac")
```

output
```
-1/2*log(abs(sin(b*x + a)^2 - 1))/b + log(abs(sin(b*x + a)))/b
```

Mupad [B] (verification not implemented)

Time = 25.32 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \csc(a+bx)\sec(a+bx)\,dx = \frac{\ln\left(\tan(a+b\,x)\right)}{b}$$

input
```
int(1/(cos(a + b*x)*sin(a + b*x)),x)
```

output
```
log(tan(a + b*x))/b
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 3.91

$$\int \csc(a+bx)\sec(a+bx)\,dx = \frac{-\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)-\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)+\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}$$

input
```
int(csc(b*x+a)*sec(b*x+a),x)
```

output
```
( - log(tan((a + b*x)/2) - 1) - log(tan((a + b*x)/2) + 1) + log(tan((a + b
*x)/2)))/b
```

3.127 $\int \csc(a+bx)\sec^2(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 23

$$\int \csc(a+bx)\sec^2(a+bx)\,dx = -\frac{\operatorname{arctanh}(\cos(a+bx))}{b}+\frac{\sec(a+bx)}{b}$$

output
```
-arctanh(cos(b*x+a))/b+sec(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \csc(a+bx)\sec^2(a+bx)\,dx = -\frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b}+\frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b}+\frac{\sec(a+bx)}{b}$$

input
```
Integrate[Csc[a + b*x]*Sec[a + b*x]^2,x]
```

output
```
-(Log[Cos[(a + b*x)/2]]/b) + Log[Sin[(a + b*x)/2]]/b + Sec[a + b*x]/b
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3102, 25, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\sec^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(a+bx)\sec(a+bx)^2dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{\int -\frac{\sec^2(a+bx)}{1-\sec^2(a+bx)}d\sec(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\sec^2(a+bx)}{1-\sec^2(a+bx)}d\sec(a+bx)}{b}$$

$$\Big\downarrow \text{262}$$

$$\frac{\sec(a+bx)-\int \frac{1}{1-\sec^2(a+bx)}d\sec(a+bx)}{b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sec(a+bx)-\text{arctanh}(\sec(a+bx))}{b}$$

input
```
Int[Csc[a + b*x]*Sec[a + b*x]^2,x]
```

output
```
(-ArcTanh[Sec[a + b*x]] + Sec[a + b*x])/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30

method	result	size
derivativedivides	$\frac{\frac{1}{\cos(bx+a)}+\ln(\csc(bx+a)-\cot(bx+a))}{b}$	30
default	$\frac{\frac{1}{\cos(bx+a)}+\ln(\csc(bx+a)-\cot(bx+a))}{b}$	30
norman	$-\frac{2}{b\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)}+\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}$	36
parallelrisch	$\frac{-2+\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)}{b\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)}$	46
risch	$\frac{2\,\mathrm{e}^{i(bx+a)}}{b\left(\mathrm{e}^{2i(bx+a)}+1\right)}+\frac{\ln\left(\mathrm{e}^{i(bx+a)}-1\right)}{b}-\frac{\ln\left(\mathrm{e}^{i(bx+a)}+1\right)}{b}$	62

input
```
int(csc(b*x+a)*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/cos(b*x+a)+ln(csc(b*x+a)-cot(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(23) = 46$.

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.26

$$\int \csc(a+bx)\sec^2(a+bx)\,dx = -\frac{\cos(bx+a)\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)-\cos(bx+a)\log\left(-\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)-2}{2\,b\cos(bx+a)}$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(cos(b*x + a)*log(1/2*cos(b*x + a) + 1/2) - cos(b*x + a)*log(-1/2*cos
(b*x + a) + 1/2) - 2)/(b*cos(b*x + a))
```

Sympy [F]

$$\int \csc(a+bx)\sec^2(a+bx)\,dx = \int \csc(a+bx)\sec^2(a+bx)\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)**2,x)
```

output
```
Integral(csc(a + b*x)*sec(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \csc(a+bx)\sec^2(a+bx)\,dx = \frac{\frac{2}{\cos(bx+a)} - \log(\cos(bx+a)+1) + \log(\cos(bx+a)-1)}{2\,b}$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.87

$$\int \csc(a+bx)\sec^2(a+bx)\,dx = -\frac{\log(|\cos(bx+a)+1|)}{2\,b} + \frac{\log(|\cos(bx+a)-1|)}{2\,b} + \frac{1}{b\cos(bx+a)}$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/2*log(abs(cos(b*x + a) + 1))/b + 1/2*log(abs(cos(b*x + a) - 1))/b + 1/(
b*cos(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 25.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \csc(a+bx)\sec^2(a+bx)\,dx = -\frac{\operatorname{atanh}(\cos(a+b\,x)) - \frac{1}{\cos(a+b\,x)}}{b}$$

input
```
int(1/(cos(a + b*x)^2*sin(a + b*x)),x)
```

output
```
-(atanh(cos(a + b*x)) - 1/cos(a + b*x))/b
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

$$\int \csc(a+bx)\sec^2(a+bx)\,dx = \frac{\cos(bx+a)\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right) - \cos(bx+a) + 1}{\cos(bx+a)\,b}$$

input
```
int(csc(b*x+a)*sec(b*x+a)^2,x)
```

output
```
(cos(a + b*x)*log(tan((a + b*x)/2)) - cos(a + b*x) + 1)/(cos(a + b*x)*b)
```

3.128 $\int \csc(a+bx)\sec^3(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \csc(a+bx)\sec^3(a+bx)\,dx=\frac{\log(\tan(a+bx))}{b}+\frac{\tan^2(a+bx)}{2b}$$

output
```
ln(tan(b*x+a))/b+1/2*tan(b*x+a)^2/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int \csc(a+bx)\sec^3(a+bx)\,dx=-\frac{\log(\cos(a+bx))}{b}+\frac{\log(\sin(a+bx))}{b}+\frac{\sec^2(a+bx)}{2b}$$

input
```
Integrate[Csc[a + b*x]*Sec[a + b*x]^3,x]
```

output
```
-(Log[Cos[a + b*x]]/b) + Log[Sin[a + b*x]]/b + Sec[a + b*x]^2/(2*b)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\sec^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(a+bx)\sec(a+bx)^3dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{\int \cot(a+bx)\left(\tan^2(a+bx)+1\right)d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int (\cot(a+bx)+\tan(a+bx))d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}\tan^2(a+bx)+\log(\tan(a+bx))}{b}$$

input
```
Int[Csc[a + b*x]*Sec[a + b*x]^3,x]
```

output
```
(Log[Tan[a + b*x]] + Tan[a + b*x]^2/2)/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Maple [A] (verified)

Time = 2.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{\frac{1}{2\cos(bx+a)^2}+\ln(\tan(bx+a))}{b}$
default	$\frac{\frac{1}{2\cos(bx+a)^2}+\ln(\tan(bx+a))}{b}$
risch	$\frac{2\,e^{2i(bx+a)}}{b\left(e^{2i(bx+a)}+1\right)^2}-\frac{\ln\left(e^{2i(bx+a)}+1\right)}{b}+\frac{\ln\left(e^{2i(bx+a)}-1\right)}{b}$
norman	$\frac{2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}{b\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1\right)^2}+\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{b}-\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{b}$
parallelrisch	$\frac{(-2\cos(2bx+2a)-2)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+(-2\cos(2bx+2a)-2)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)+(2\cos(2bx+2a)+2)\ln\left(\tan\left(\frac{bx}{2}\right.\right.}{2b(1+\cos(2bx+2a))}$

input
```
int(csc(b*x+a)*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2/cos(b*x+a)^2+ln(tan(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(25) = 50$.

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.07

$$\int \csc(a+bx)\sec^3(a+bx)\,dx = -\frac{\cos(bx+a)^2\log\left(\cos(bx+a)^2\right) - \cos(bx+a)^2\log\left(-\frac{1}{4}\cos(bx+a)^2+\frac{1}{4}\right) - 1}{2\,b\cos(bx+a)^2}$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/2*(cos(b*x + a)^2*log(cos(b*x + a)^2) - cos(b*x + a)^2*log(-1/4*cos(b*x
 + a)^2 + 1/4) - 1)/(b*cos(b*x + a)^2)
```

Sympy [F]

$$\int \csc(a+bx)\sec^3(a+bx)\,dx = \int \csc(a+bx)\sec^3(a+bx)\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)**3,x)
```

output
```
Integral(csc(a + b*x)*sec(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.48

$$\int \csc(a+bx)\sec^3(a+bx)\,dx = -\frac{\frac{1}{\sin(bx+a)^2-1} + \log\left(\sin(bx+a)^2-1\right) - \log\left(\sin(bx+a)^2\right)}{2\,b}$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/2*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^
2))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.74

$$\int \csc(a+bx)\sec^3(a+bx)\,dx = -\frac{\log\left(\left|\sin\left(bx+a\right)^2-1\right|\right)}{2\,b} + \frac{\log\left(\left|\sin\left(bx+a\right)\right|\right)}{b} - \frac{1}{2\left(\sin\left(bx+a\right)^2-1\right)b}$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/2*log(abs(sin(b*x + a)^2 - 1))/b + log(abs(sin(b*x + a)))/b - 1/2/((sin
(b*x + a)^2 - 1)*b)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int \csc(a+bx)\sec^3(a+bx)\,dx = \frac{\frac{\ln\left(\sin(a+b\,x)^2\right)}{2} - \ln\left(\cos\left(a+b\,x\right)\right) + \frac{1}{2\cos(a+b\,x)^2}}{b}$$

input
```
int(1/(cos(a + b*x)^3*sin(a + b*x)),x)
```

output
```
(log(sin(a + b*x)^2)/2 - log(cos(a + b*x)) + 1/(2*cos(a + b*x)^2))/b
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 132, normalized size of antiderivative = 4.89

$$\int \csc(a+bx)\sec^3(a+bx)\,dx$$
$$= \frac{-2\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin\left(bx+a\right)^2+2\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)-2\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)\sin\left(bx+a\right)^2}{2b\left(\sin\left(bx+\right.\right.}$$

input
```
int(csc(b*x+a)*sec(b*x+a)^3,x)
```

output
```
( - 2*log(tan((a + b*x)/2) - 1)*sin(a + b*x)**2 + 2*log(tan((a + b*x)/2) -
 1) - 2*log(tan((a + b*x)/2) + 1)*sin(a + b*x)**2 + 2*log(tan((a + b*x)/2)
 + 1) + 2*log(tan((a + b*x)/2))*sin(a + b*x)**2 - 2*log(tan((a + b*x)/2))
- sin(a + b*x)**2)/(2*b*(sin(a + b*x)**2 - 1))
```

3.129 $\int \csc(a+bx)\sec^4(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \csc(a+bx)\sec^4(a+bx)\,dx = -\frac{\operatorname{arctanh}(\cos(a+bx))}{b}+\frac{\sec(a+bx)}{b}+\frac{\sec^3(a+bx)}{3b}$$

output
```
-arctanh(cos(b*x+a))/b+sec(b*x+a)/b+1/3*sec(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.50

$$\int \csc(a+bx)\sec^4(a+bx)\,dx = -\frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b}+\frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b}+\frac{\sec(a+bx)}{b}+\frac{\sec^3(a+bx)}{3b}$$

input
```
Integrate[Csc[a + b*x]*Sec[a + b*x]^4,x]
```

output
```
-(Log[Cos[(a + b*x)/2]]/b) + Log[Sin[(a + b*x)/2]]/b + Sec[a + b*x]/b + Se
c[a + b*x]^3/(3*b)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3102, 25, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\sec^4(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc(a+bx)\sec(a+bx)^4dx$$

$$\Big\downarrow \text{ 3102}$$

$$\frac{\int -\frac{\sec^4(a+bx)}{1-\sec^2(a+bx)}d\sec(a+bx)}{b}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{\sec^4(a+bx)}{1-\sec^2(a+bx)}d\sec(a+bx)}{b}$$

$$\Big\downarrow \text{ 254}$$

$$-\frac{\int \left(-\sec^2(a+bx)+\frac{1}{1-\sec^2(a+bx)}-1\right)d\sec(a+bx)}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\text{arctanh}(\sec(a+bx))+\frac{1}{3}\sec^3(a+bx)+\sec(a+bx)}{b}$$

input
```
Int[Csc[a + b*x]*Sec[a + b*x]^4,x]
```

output
```
(-ArcTanh[Sec[a + b*x]] + Sec[a + b*x] + Sec[a + b*x]^3/3)/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$\frac{\frac{1}{3 \cos(bx+a)^3}+\frac{1}{\cos(bx+a)}+\ln(\csc(bx+a)-\cot(bx+a))}{b}$	40
default	$\frac{\frac{1}{3 \cos(bx+a)^3}+\frac{1}{\cos(bx+a)}+\ln(\csc(bx+a)-\cot(bx+a))}{b}$	40
norman	$\frac{\frac{4 \tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}{b}-\frac{8}{3b}-\frac{4 \tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}{b}}{\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1\right)^3}+\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}$	70
risch	$\frac{2 e^{5i(bx+a)}+\frac{20 e^{3i(bx+a)}}{3}+2 e^{i(bx+a)}}{b\left(e^{2i(bx+a)}+1\right)^3}-\frac{\ln\left(e^{i(bx+a)}+1\right)}{b}+\frac{\ln\left(e^{i(bx+a)}-1\right)}{b}$	87
parallelrisch	$\frac{3 \ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^3\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)^3-12 \tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4+12 \tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2-8}{3b\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^3\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)^3}$	98

input
```
int(csc(b*x+a)*sec(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/3/cos(b*x+a)^3+1/cos(b*x+a)+ln(csc(b*x+a)-cot(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.76

$$\int \csc(a+bx)\sec^4(a+bx)\,dx = -\frac{3\,\cos\left(bx+a\right)^3\log\left(\frac{1}{2}\,\cos\left(bx+a\right)+\frac{1}{2}\right)-3\,\cos\left(bx+a\right)^3\log\left(-\frac{1}{2}\,\cos\left(bx+a\right)+\frac{1}{2}\right)-6\,\cos\left(bx+a\right)^2}{6\,b\cos\left(bx+a\right)^3}$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)^4,x, algorithm="fricas")
```

output
```
-1/6*(3*cos(b*x + a)^3*log(1/2*cos(b*x + a) + 1/2) - 3*cos(b*x + a)^3*log(
-1/2*cos(b*x + a) + 1/2) - 6*cos(b*x + a)^2 - 2)/(b*cos(b*x + a)^3)
```

Sympy [F]

$$\int \csc(a+bx)\sec^4(a+bx)\,dx = \int \csc\left(a+bx\right)\sec^4\left(a+bx\right)dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)**4,x)
```

output
```
Integral(csc(a + b*x)*sec(a + b*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int \csc(a+bx)\sec^4(a+bx)\,dx = \frac{\frac{2\left(3\cos(bx+a)^2+1\right)}{\cos(bx+a)^3} - 3\log\left(\cos\left(bx+a\right)+1\right) + 3\log\left(\cos\left(bx+a\right)-1\right)}{6\,b}$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/6*(2*(3*cos(b*x + a)^2 + 1)/cos(b*x + a)^3 - 3*log(cos(b*x + a) + 1) + 3
*log(cos(b*x + a) - 1))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.47

$$\int \csc(a+bx)\sec^4(a+bx)\,dx = -\frac{\log\left(\left|\cos\left(bx+a\right)+1\right|\right)}{2\,b} + \frac{\log\left(\left|\cos\left(bx+a\right)-1\right|\right)}{2\,b} + \frac{3\cos\left(bx+a\right)^2+1}{3\,b\cos\left(bx+a\right)^3}$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)^4,x, algorithm="giac")
```

output
```
-1/2*log(abs(cos(b*x + a) + 1))/b + 1/2*log(abs(cos(b*x + a) - 1))/b + 1/3
*(3*cos(b*x + a)^2 + 1)/(b*cos(b*x + a)^3)
```

Mupad [B] (verification not implemented)

Time = 25.32 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \csc(a+bx)\sec^4(a+bx)\,dx = -\frac{\operatorname{atanh}(\cos(a+b\,x)) - \frac{\cos(a+b\,x)^2+\frac{1}{3}}{\cos(a+b\,x)^3}}{b}$$

input
```
int(1/(cos(a + b*x)^4*sin(a + b*x)),x)
```

output
```
-(atanh(cos(a + b*x)) - (cos(a + b*x)^2 + 1/3)/cos(a + b*x)^3)/b
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.76

$$\int \csc(a+bx)\sec^4(a+bx)\,dx$$
$$= \frac{3\cos(bx+a)\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\sin(bx+a)^2 - 3\cos(bx+a)\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right) - 4\cos(bx+a)\sin(bx}{3\cos(bx+a)\,b\left(\sin(bx+a)^2-1\right)}$$

input
```
int(csc(b*x+a)*sec(b*x+a)^4,x)
```

output
```
(3*cos(a + b*x)*log(tan((a + b*x)/2))*sin(a + b*x)**2 - 3*cos(a + b*x)*log
(tan((a + b*x)/2)) - 4*cos(a + b*x)*sin(a + b*x)**2 + 4*cos(a + b*x) + 3*s
in(a + b*x)**2 - 4)/(3*cos(a + b*x)*b*(sin(a + b*x)**2 - 1))
```

3.130 $\int \csc(a+bx)\sec^5(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 39

$$\int \csc(a+bx)\sec^5(a+bx)\,dx=\frac{\log(\tan(a+bx))}{b}+\frac{\tan^2(a+bx)}{b}+\frac{\tan^4(a+bx)}{4b}$$

output
```
ln(tan(b*x+a))/b+tan(b*x+a)^2/b+1/4*tan(b*x+a)^4/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.38

$$\int \csc(a+bx)\sec^5(a+bx)\,dx=-\frac{\log(\cos(a+bx))}{b}+\frac{\log(\sin(a+bx))}{b}+\frac{\sec^2(a+bx)}{2b}+\frac{\sec^4(a+bx)}{4b}$$

input
```
Integrate[Csc[a + b*x]*Sec[a + b*x]^5,x]
```

output
```
-(Log[Cos[a + b*x]]/b) + Log[Sin[a + b*x]]/b + Sec[a + b*x]^2/(2*b) + Sec[
a + b*x]^4/(4*b)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3100, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\sec^5(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc(a+bx)\sec(a+bx)^5dx$$
$$\Big\downarrow \text{3100}$$
$$\frac{\int \cot(a+bx)\left(\tan^2(a+bx)+1\right)^2d\tan(a+bx)}{b}$$
$$\Big\downarrow \text{243}$$
$$\frac{\int \cot(a+bx)\left(\tan^2(a+bx)+1\right)^2d\tan^2(a+bx)}{2b}$$
$$\Big\downarrow \text{49}$$
$$\frac{\int \left(\tan^2(a+bx)+\cot(a+bx)+2\right)d\tan^2(a+bx)}{2b}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\frac{1}{2}\tan^4(a+bx)+2\tan^2(a+bx)+\log\left(\tan^2(a+bx)\right)}{2b}$$

input
```
Int[Csc[a + b*x]*Sec[a + b*x]^5,x]
```

output
```
(Log[Tan[a + b*x]^2] + 2*Tan[a + b*x]^2 + Tan[a + b*x]^4/2)/(2*b)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Maple [A] (verified)

Time = 2.43 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{\frac{1}{4\cos(bx+a)^4}+\frac{1}{2\cos(bx+a)^2}+\ln(\tan(bx+a))}{b}$
default	$\frac{\frac{1}{4\cos(bx+a)^4}+\frac{1}{2\cos(bx+a)^2}+\ln(\tan(bx+a))}{b}$
risch	$\frac{2\,e^{6i(bx+a)}+8\,e^{4i(bx+a)}+2\,e^{2i(bx+a)}}{b\left(e^{2i(bx+a)}+1\right)^4}+\frac{\ln\left(e^{2i(bx+a)}-1\right)}{b}-\frac{\ln\left(e^{2i(bx+a)}+1\right)}{b}$
norman	$\frac{\frac{2}{3b}+\frac{4\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}{3b}+\frac{4\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^6}{3b}+\frac{2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^8}{3b}}{\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1\right)^4}+\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{b}-\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right.\right.}{b}$
parallelrisch	$\frac{(-4\cos(4bx+4a)-16\cos(2bx+2a)-12)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+(-4\cos(4bx+4a)-16\cos(2bx+2a)-12)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right.\right.}{4b(\cos(4bx+4a)+4\cos(2bx}$

input
```
int(csc(b*x+a)*sec(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/4/cos(b*x+a)^4+1/2/cos(b*x+a)^2+ln(tan(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.72

$$\int \csc(a+bx)\sec^5(a+bx)\,dx = -\frac{2\cos(bx+a)^4\log\left(\cos(bx+a)^2\right) - 2\cos(bx+a)^4\log\left(-\frac{1}{4}\cos(bx+a)^2+\frac{1}{4}\right) - 2\cos(bx+a)^2 - 1}{4\,b\cos(bx+a)^4}$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)^5,x, algorithm="fricas")
```

output
```
-1/4*(2*cos(b*x + a)^4*log(cos(b*x + a)^2) - 2*cos(b*x + a)^4*log(-1/4*cos
(b*x + a)^2 + 1/4) - 2*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^4)
```

Sympy [F]

$$\int \csc(a+bx)\sec^5(a+bx)\,dx = \int \csc(a+bx)\sec^5(a+bx)\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)**5,x)
```

output
```
Integral(csc(a + b*x)*sec(a + b*x)**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.67

$$\int \csc(a+bx)\sec^5(a+bx)\,dx = -\frac{\frac{2\sin(bx+a)^2-3}{\sin(bx+a)^4-2\sin(bx+a)^2+1} + 2\log\left(\sin(bx+a)^2-1\right) - 2\log\left(\sin(bx+a)^2\right)}{4b}$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)^5,x, algorithm="maxima")
```

output
```
-1/4*((2*sin(b*x + a)^2 - 3)/(sin(b*x + a)^4 - 2*sin(b*x + a)^2 + 1) + 2*l
og(sin(b*x + a)^2 - 1) - 2*log(sin(b*x + a)^2))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.44

$$\int \csc(a+bx)\sec^5(a+bx)\,dx = \frac{\log\left(\left|\cos(bx+a)^2-1\right|\right)}{2b} - \frac{\log\left(\left|\cos(bx+a)\right|\right)}{b} + \frac{2\cos(bx+a)^2+1}{4b\cos(bx+a)^4}$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)^5,x, algorithm="giac")
```

output
```
1/2*log(abs(cos(b*x + a)^2 - 1))/b - log(abs(cos(b*x + a)))/b + 1/4*(2*cos
(b*x + a)^2 + 1)/(b*cos(b*x + a)^4)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int \csc(a+bx)\sec^5(a+bx)\,dx = \frac{\frac{\ln\left(\sin(a+b\,x)^2\right)}{2} - \ln\left(\cos\left(a+b\,x\right)\right) + \frac{\frac{\cos(a+b\,x)^2}{2}+\frac{1}{4}}{\cos(a+b\,x)^4}}{b}$$

input
```
int(1/(cos(a + b*x)^5*sin(a + b*x)),x)
```

output
```
(log(sin(a + b*x)^2)/2 - log(cos(a + b*x)) + (cos(a + b*x)^2/2 + 1/4)/cos(
a + b*x)^4)/b
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 216, normalized size of antiderivative = 5.54

$$\int \csc(a+bx)\sec^5(a+bx)\,dx$$
$$= \frac{-4\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin\left(bx+a\right)^4+8\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin\left(bx+a\right)^2-4\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{}$$

input
```
int(csc(b*x+a)*sec(b*x+a)^5,x)
```

output
```
( - 4*log(tan((a + b*x)/2) - 1)*sin(a + b*x)**4 + 8*log(tan((a + b*x)/2) -
 1)*sin(a + b*x)**2 - 4*log(tan((a + b*x)/2) - 1) - 4*log(tan((a + b*x)/2)
 + 1)*sin(a + b*x)**4 + 8*log(tan((a + b*x)/2) + 1)*sin(a + b*x)**2 - 4*lo
g(tan((a + b*x)/2) + 1) + 4*log(tan((a + b*x)/2))*sin(a + b*x)**4 - 8*log(
tan((a + b*x)/2))*sin(a + b*x)**2 + 4*log(tan((a + b*x)/2)) - 3*sin(a + b*
x)**4 + 4*sin(a + b*x)**2)/(4*b*(sin(a + b*x)**4 - 2*sin(a + b*x)**2 + 1))
```

3.131 $\int \csc(a+bx)\sec^6(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 53

$$\int \csc(a+bx)\sec^6(a+bx)\,dx = -\frac{\operatorname{arctanh}(\cos(a+bx))}{b}+\frac{\sec(a+bx)}{b}+\frac{\sec^3(a+bx)}{3b}+\frac{\sec^5(a+bx)}{5b}$$

output
```
-arctanh(cos(b*x+a))/b+sec(b*x+a)/b+1/3*sec(b*x+a)^3/b+1/5*sec(b*x+a)^5/b
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.36

$$\int \csc(a+bx)\sec^6(a+bx)\,dx = -\frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b}+\frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b}+\frac{\sec(a+bx)}{b}+\frac{\sec^3(a+bx)}{3b}+\frac{\sec^5(a+bx)}{5b}$$

input
```
Integrate[Csc[a + b*x]*Sec[a + b*x]^6,x]
```

output
```
-(Log[Cos[(a + b*x)/2]]/b) + Log[Sin[(a + b*x)/2]]/b + Sec[a + b*x]/b + Se
c[a + b*x]^3/(3*b) + Sec[a + b*x]^5/(5*b)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3102, 25, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\sec^6(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(a+bx)\sec(a+bx)^6dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{\int -\frac{\sec^6(a+bx)}{1-\sec^2(a+bx)}d\sec(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\sec^6(a+bx)}{1-\sec^2(a+bx)}d\sec(a+bx)}{b}$$

$$\Big\downarrow \text{254}$$

$$-\frac{\int \left(-\sec^4(a+bx)-\sec^2(a+bx)+\frac{1}{1-\sec^2(a+bx)}-1\right)d\sec(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\text{arctanh}(\sec(a+bx))+\frac{1}{5}\sec^5(a+bx)+\frac{1}{3}\sec^3(a+bx)+\sec(a+bx)}{b}$$

input
```
Int[Csc[a + b*x]*Sec[a + b*x]^6,x]
```

output
```
(-ArcTanh[Sec[a + b*x]] + Sec[a + b*x] + Sec[a + b*x]^3/3 + Sec[a + b*x]^5
/5)/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 2.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{\frac{1}{5\cos(bx+a)^5}+\frac{1}{3\cos(bx+a)^3}+\frac{1}{\cos(bx+a)}+\ln(\csc(bx+a)-\cot(bx+a))}{b}$
default	$\frac{\frac{1}{5\cos(bx+a)^5}+\frac{1}{3\cos(bx+a)^3}+\frac{1}{\cos(bx+a)}+\ln(\csc(bx+a)-\cot(bx+a))}{b}$
norman	$\frac{-\frac{46}{15b}+\frac{12\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^6}{b}-\frac{6\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^8}{b}+\frac{28\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}{3b}-\frac{56\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}{3b}}{\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1\right)^5}+\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}$
risch	$\frac{2\,e^{9i(bx+a)}+\frac{32\,e^{7i(bx+a)}}{3}+\frac{356\,e^{5i(bx+a)}}{15}+\frac{32\,e^{3i(bx+a)}}{3}+2\,e^{i(bx+a)}}{b\left(e^{2i(bx+a)}+1\right)^5}+\frac{\ln\left(e^{i(bx+a)}-1\right)}{b}-\frac{\ln\left(e^{i(bx+a)}+1\right)}{b}$
parallelrisch	$\frac{15\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^5\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)^5\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-90\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^8+180\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^6-280\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}{15b\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^5\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)^5}$

input
```
int(csc(b*x+a)*sec(b*x+a)^6,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/5/cos(b*x+a)^5+1/3/cos(b*x+a)^3+1/cos(b*x+a)+ln(csc(b*x+a)-cot(b*x+
a)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.45

$$\int \csc(a+bx)\sec^6(a+bx)\,dx = -\frac{15\cos\left(bx+a\right)^5\log\left(\frac{1}{2}\cos\left(bx+a\right)+\frac{1}{2}\right)-15\cos\left(bx+a\right)^5\log\left(-\frac{1}{2}\cos\left(bx+a\right)+\frac{1}{2}\right)-30\cos\left(bx+\right.}{30\,b\cos\left(bx+a\right)^5}$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)^6,x, algorithm="fricas")
```

output
```
-1/30*(15*cos(b*x + a)^5*log(1/2*cos(b*x + a) + 1/2) - 15*cos(b*x + a)^5*l
og(-1/2*cos(b*x + a) + 1/2) - 30*cos(b*x + a)^4 - 10*cos(b*x + a)^2 - 6)/(
b*cos(b*x + a)^5)
```

Sympy [F]

$$\int \csc(a+bx)\sec^6(a+bx)\,dx = \int \csc\left(a+bx\right)\sec^6\left(a+bx\right)\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)**6,x)
```

output
```
Integral(csc(a + b*x)*sec(a + b*x)**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.13

$$\int \csc(a+bx)\sec^6(a+bx)\,dx = \frac{\frac{2\left(15\cos(bx+a)^4+5\cos(bx+a)^2+3\right)}{\cos(bx+a)^5} - 15\log\left(\cos\left(bx+a\right)+1\right) + 15\log\left(\cos\left(bx+a\right)-1\right)}{30\,b}$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)^6,x, algorithm="maxima")
```

output
```
1/30*(2*(15*cos(b*x + a)^4 + 5*cos(b*x + a)^2 + 3)/cos(b*x + a)^5 - 15*log
(cos(b*x + a) + 1) + 15*log(cos(b*x + a) - 1))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.25

$$\int \csc(a+bx)\sec^6(a+bx)\,dx = -\frac{\log\left(\left|\cos\left(bx+a\right)+1\right|\right)}{2\,b} + \frac{\log\left(\left|\cos\left(bx+a\right)-1\right|\right)}{2\,b} + \frac{15\cos\left(bx+a\right)^4 + 5\cos\left(bx+a\right)^2 + 3}{15\,b\cos\left(bx+a\right)^5}$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)^6,x, algorithm="giac")
```

output
```
-1/2*log(abs(cos(b*x + a) + 1))/b + 1/2*log(abs(cos(b*x + a) - 1))/b + 1/1
5*(15*cos(b*x + a)^4 + 5*cos(b*x + a)^2 + 3)/(b*cos(b*x + a)^5)
```

Mupad [B] (verification not implemented)

Time = 25.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \csc(a+bx)\sec^6(a+bx)\,dx = \frac{\cos\left(a+b\,x\right)^4+\frac{\cos\left(a+b\,x\right)^2}{3}+\frac{1}{5}}{b\cos\left(a+b\,x\right)^5}-\frac{\mathrm{atanh}(\cos\left(a+b\,x\right))}{b}$$

input
```
int(1/(cos(a + b*x)^6*sin(a + b*x)),x)
```

output
```
(cos(a + b*x)^2/3 + cos(a + b*x)^4 + 1/5)/(b*cos(a + b*x)^5) - atanh(cos(a
 + b*x))/b
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 167, normalized size of antiderivative = 3.15

$$\int \csc(a+bx)\sec^6(a+bx)\,dx$$

$$= \frac{15\cos\left(bx+a\right)\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\sin\left(bx+a\right)^4-30\cos\left(bx+a\right)\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\sin\left(bx+a\right)^2+15\cos}{15\,\mathrm{co}}$$

input
```
int(csc(b*x+a)*sec(b*x+a)^6,x)
```

output
```
(15*cos(a + b*x)*log(tan((a + b*x)/2))*sin(a + b*x)**4 - 30*cos(a + b*x)*l
og(tan((a + b*x)/2))*sin(a + b*x)**2 + 15*cos(a + b*x)*log(tan((a + b*x)/2
)) - 23*cos(a + b*x)*sin(a + b*x)**4 + 46*cos(a + b*x)*sin(a + b*x)**2 - 2
3*cos(a + b*x) + 15*sin(a + b*x)**4 - 35*sin(a + b*x)**2 + 23)/(15*cos(a +
 b*x)*b*(sin(a + b*x)**4 - 2*sin(a + b*x)**2 + 1))
```

3.132 $\int \csc(a+bx)\sec^7(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 57

$$\int \csc(a+bx)\sec^7(a+bx)\,dx = \frac{\log(\tan(a+bx))}{b} + \frac{3\tan^2(a+bx)}{2b} + \frac{3\tan^4(a+bx)}{4b} + \frac{\tan^6(a+bx)}{6b}$$

output
```
ln(tan(b*x+a))/b+3/2*tan(b*x+a)^2/b+3/4*tan(b*x+a)^4/b+1/6*tan(b*x+a)^6/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.21

$$\int \csc(a+bx)\sec^7(a+bx)\,dx = -\frac{\log(\cos(a+bx))}{b} + \frac{\log(\sin(a+bx))}{b} + \frac{\sec^2(a+bx)}{2b} + \frac{\sec^4(a+bx)}{4b} + \frac{\sec^6(a+bx)}{6b}$$

input
```
Integrate[Csc[a + b*x]*Sec[a + b*x]^7,x]
```

output
```
-(Log[Cos[a + b*x]]/b) + Log[Sin[a + b*x]]/b + Sec[a + b*x]^2/(2*b) + Sec[
a + b*x]^4/(4*b) + Sec[a + b*x]^6/(6*b)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3100, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\sec^7(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(a+bx)\sec(a+bx)^7dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{\int \cot(a+bx)\left(\tan^2(a+bx)+1\right)^3d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int \cot(a+bx)\left(\tan^2(a+bx)+1\right)^3d\tan^2(a+bx)}{2b}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\tan^4(a+bx)+3\tan^2(a+bx)+\cot(a+bx)+3\right)d\tan^2(a+bx)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{3}\tan^6(a+bx)+\frac{3}{2}\tan^4(a+bx)+3\tan^2(a+bx)+\log\left(\tan^2(a+bx)\right)}{2b}$$

input
```
Int[Csc[a + b*x]*Sec[a + b*x]^7,x]
```

output
```
(Log[Tan[a + b*x]^2] + 3*Tan[a + b*x]^2 + (3*Tan[a + b*x]^4)/2 + Tan[a + b
*x]^6/3)/(2*b)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Maple [A] (verified)

Time = 3.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{\frac{1}{6\cos(bx+a)^6}+\frac{1}{4\cos(bx+a)^4}+\frac{1}{2\cos(bx+a)^2}+\ln(\tan(bx+a))}{b}$
default	$\frac{\frac{1}{6\cos(bx+a)^6}+\frac{1}{4\cos(bx+a)^4}+\frac{1}{2\cos(bx+a)^2}+\ln(\tan(bx+a))}{b}$
risch	$\frac{2\,e^{10i(bx+a)}+12\,e^{8i(bx+a)}+\frac{92\,e^{6i(bx+a)}}{3}+12\,e^{4i(bx+a)}+2\,e^{2i(bx+a)}}{b\left(e^{2i(bx+a)}+1\right)^6}+\frac{\ln\left(e^{2i(bx+a)}-1\right)}{b}-\frac{\ln\left(e^{2i(bx+a)}+1\right)}{b}$
norman	$\frac{\frac{6\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}{b}+\frac{6\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{10}}{b}-\frac{12\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}{b}-\frac{12\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^8}{b}+\frac{68\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^6}{3b}}{\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1\right)^6}+\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{\ln(\tan}{}$
parallelrisch	$\frac{(-180\cos(2bx+2a)-72\cos(4bx+4a)-12\cos(6bx+6a)-120)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+(-180\cos(2bx+2a)-72\cos(4bx+}{}$

input
```
int(csc(b*x+a)*sec(b*x+a)^7,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/6/cos(b*x+a)^6+1/4/cos(b*x+a)^4+1/2/cos(b*x+a)^2+ln(tan(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.35

$$\int \csc(a+bx)\sec^7(a+bx)\,dx = -\frac{6\cos(bx+a)^6\log\left(\cos(bx+a)^2\right)-6\cos(bx+a)^6\log\left(-\frac{1}{4}\cos(bx+a)^2+\frac{1}{4}\right)-6\cos(bx+a)^4-3}{12\,b\cos(bx+a)^6}$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)^7,x, algorithm="fricas")
```

output
```
-1/12*(6*cos(b*x + a)^6*log(cos(b*x + a)^2) - 6*cos(b*x + a)^6*log(-1/4*co
s(b*x + a)^2 + 1/4) - 6*cos(b*x + a)^4 - 3*cos(b*x + a)^2 - 2)/(b*cos(b*x
+ a)^6)
```

Sympy [F]

$$\int \csc(a+bx)\sec^7(a+bx)\,dx = \int \csc(a+bx)\sec^7(a+bx)\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)**7,x)
```

output
```
Integral(csc(a + b*x)*sec(a + b*x)**7, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.49

$$\int \csc(a+bx)\sec^7(a+bx)\,dx = -\frac{\frac{6\,\sin(bx+a)^4-15\,\sin(bx+a)^2+11}{\sin(bx+a)^6-3\,\sin(bx+a)^4+3\,\sin(bx+a)^2-1}+6\,\log\left(\sin\left(bx+a\right)^2-1\right)-6\,\log\left(\sin\left(bx+a\right)^2\right)}{12\,b}$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)^7,x, algorithm="maxima")
```

output
```
-1/12*((6*sin(b*x + a)^4 - 15*sin(b*x + a)^2 + 11)/(sin(b*x + a)^6 - 3*sin
(b*x + a)^4 + 3*sin(b*x + a)^2 - 1) + 6*log(sin(b*x + a)^2 - 1) - 6*log(si
n(b*x + a)^2))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.16

$$\int \csc(a+bx)\sec^7(a+bx)\,dx = \frac{\log\left(\left|\cos\left(bx+a\right)^2-1\right|\right)}{2\,b} - \frac{\log\left(\left|\cos\left(bx+a\right)\right|\right)}{b} + \frac{6\,\cos\left(bx+a\right)^4+3\,\cos\left(bx+a\right)^2+2}{12\,b\cos\left(bx+a\right)^6}$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)^7,x, algorithm="giac")
```

output
```
1/2*log(abs(cos(b*x + a)^2 - 1))/b - log(abs(cos(b*x + a)))/b + 1/12*(6*co
s(b*x + a)^4 + 3*cos(b*x + a)^2 + 2)/(b*cos(b*x + a)^6)
```

Mupad [B] (verification not implemented)

Time = 25.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98

$$\int \csc(a+bx)\sec^7(a+bx)\,dx = \frac{\frac{\ln\left(\sin(a+b\,x)^2\right)}{2} - \ln\left(\cos\left(a+b\,x\right)\right) + \frac{\frac{\cos(a+b\,x)^4}{2}+\frac{\cos(a+b\,x)^2}{4}+\frac{1}{6}}{\cos(a+b\,x)^6}}{b}$$

input
```
int(1/(cos(a + b*x)^7*sin(a + b*x)),x)
```

output
```
(log(sin(a + b*x)^2)/2 - log(cos(a + b*x)) + (cos(a + b*x)^2/4 + cos(a + b
*x)^4/2 + 1/6)/cos(a + b*x)^6)/b
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 300, normalized size of antiderivative = 5.26

$$\int \csc(a+bx)\sec^7(a+bx)\,dx$$
$$= \frac{-12\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin\left(bx+a\right)^6 + 36\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin\left(bx+a\right)^4 - 36\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right.}{}$$

input
```
int(csc(b*x+a)*sec(b*x+a)^7,x)
```

output
```
( - 12*log(tan((a + b*x)/2) - 1)*sin(a + b*x)**6 + 36*log(tan((a + b*x)/2)
 - 1)*sin(a + b*x)**4 - 36*log(tan((a + b*x)/2) - 1)*sin(a + b*x)**2 + 12*
log(tan((a + b*x)/2) - 1) - 12*log(tan((a + b*x)/2) + 1)*sin(a + b*x)**6 +
 36*log(tan((a + b*x)/2) + 1)*sin(a + b*x)**4 - 36*log(tan((a + b*x)/2) +
1)*sin(a + b*x)**2 + 12*log(tan((a + b*x)/2) + 1) + 12*log(tan((a + b*x)/2
))*sin(a + b*x)**6 - 36*log(tan((a + b*x)/2))*sin(a + b*x)**4 + 36*log(tan
((a + b*x)/2))*sin(a + b*x)**2 - 12*log(tan((a + b*x)/2)) - 11*sin(a + b*x
)**6 + 27*sin(a + b*x)**4 - 18*sin(a + b*x)**2)/(12*b*(sin(a + b*x)**6 - 3
*sin(a + b*x)**4 + 3*sin(a + b*x)**2 - 1))
```

3.133 $\int \cos^5(a+bx)\cot^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 50

$$\int \cos^5(a+bx)\cot^2(a+bx)\,dx=-\frac{\csc(a+bx)}{b}-\frac{3\sin(a+bx)}{b}+\frac{\sin^3(a+bx)}{b}-\frac{\sin^5(a+bx)}{5b}$$

output
```
-csc(b*x+a)/b-3*sin(b*x+a)/b+sin(b*x+a)^3/b-1/5*sin(b*x+a)^5/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \cos^5(a+bx)\cot^2(a+bx)\,dx=-\frac{\csc(a+bx)}{b}-\frac{3\sin(a+bx)}{b}+\frac{\sin^3(a+bx)}{b}-\frac{\sin^5(a+bx)}{5b}$$

input
```
Integrate[Cos[a + b*x]^5*Cot[a + b*x]^2,x]
```

output
```
-(Csc[a + b*x]/b) - (3*Sin[a + b*x])/b + Sin[a + b*x]^3/b - Sin[a + b*x]^5
/(5*b)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(a+bx)\cot^2(a+bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sin\left(a+bx+\frac{\pi}{2}\right)^5\tan\left(a+bx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \ 3070$$

$$-\frac{\int \csc^2(a+bx)\left(1-\sin^2(a+bx)\right)^3d(-\sin(a+bx))}{b}$$

$$\Big\downarrow \ 244$$

$$-\frac{\int \left(-\sin^4(a+bx)+3\sin^2(a+bx)+\csc^2(a+bx)-3\right)d(-\sin(a+bx))}{b}$$

$$\Big\downarrow \ 2009$$

$$-\frac{\frac{1}{5}\sin^5(a+bx)-\sin^3(a+bx)+3\sin(a+bx)+\csc(a+bx)}{b}$$

input
```
Int[Cos[a + b*x]^5*Cot[a + b*x]^2,x]
```

output
```
-((Csc[a + b*x] + 3*Sin[a + b*x] - Sin[a + b*x]^3 + Sin[a + b*x]^5/5)/b)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Maple [A] (verified)

Time = 7.75 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.24

method	result	size
derivativedivides	$\frac{-\frac{\cos(bx+a)^8}{\sin(bx+a)}-\left(\frac{16}{5}+\cos(bx+a)^6+\frac{6\cos(bx+a)^4}{5}+\frac{8\cos(bx+a)^2}{5}\right)\sin(bx+a)}{b}$	62
default	$\frac{-\frac{\cos(bx+a)^8}{\sin(bx+a)}-\left(\frac{16}{5}+\cos(bx+a)^6+\frac{6\cos(bx+a)^4}{5}+\frac{8\cos(bx+a)^2}{5}\right)\sin(bx+a)}{b}$	62
risch	$\frac{19i\,\mathrm{e}^{i(bx+a)}}{16b}-\frac{19i\,\mathrm{e}^{-i(bx+a)}}{16b}-\frac{2i\,\mathrm{e}^{i(bx+a)}}{b\left(\mathrm{e}^{2i(bx+a)}-1\right)}-\frac{\sin(5bx+5a)}{80b}-\frac{3\sin(3bx+3a)}{16b}$	88

input
```
int(cos(b*x+a)^5*cot(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/sin(b*x+a)*cos(b*x+a)^8-(16/5+cos(b*x+a)^6+6/5*cos(b*x+a)^4+8/5*co
s(b*x+a)^2)*sin(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \cos^5(a+bx)\cot^2(a+bx)\,dx = \frac{\cos(bx+a)^6+2\cos(bx+a)^4+8\cos(bx+a)^2-16}{5\,b\sin(bx+a)}$$

input
```
integrate(cos(b*x+a)^5*cot(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/5*(cos(b*x + a)^6 + 2*cos(b*x + a)^4 + 8*cos(b*x + a)^2 - 16)/(b*sin(b*x
 + a))
```

Sympy [F(-1)]

Timed out.

$$\int \cos^5(a+bx)\cot^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**5*cot(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \cos^5(a+bx)\cot^2(a+bx)\,dx = -\frac{\sin(bx+a)^5-5\sin(bx+a)^3+\frac{5}{\sin(bx+a)}+15\sin(bx+a)}{5\,b}$$

input
```
integrate(cos(b*x+a)^5*cot(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/5*(sin(b*x + a)^5 - 5*sin(b*x + a)^3 + 5/sin(b*x + a) + 15*sin(b*x + a)
)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \cos^5(a+bx)\cot^2(a+bx)\,dx = -\frac{\sin(bx+a)^5 - 5\sin(bx+a)^3 + \frac{5}{\sin(bx+a)} + 15\sin(bx+a)}{5b}$$

input
```
integrate(cos(b*x+a)^5*cot(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/5*(sin(b*x + a)^5 - 5*sin(b*x + a)^3 + 5/sin(b*x + a) + 15*sin(b*x + a)
)/b
```

Mupad [B] (verification not implemented)

Time = 25.84 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \cos^5(a+bx)\cot^2(a+bx)\,dx = -\frac{\sin(a+b\,x)^6 - 5\sin(a+b\,x)^4 + 15\sin(a+b\,x)^2 + 5}{5\,b\,\sin(a+b\,x)}$$

input
```
int(cos(a + b*x)^5*cot(a + b*x)^2,x)
```

output
```
-(15*sin(a + b*x)^2 - 5*sin(a + b*x)^4 + sin(a + b*x)^6 + 5)/(5*b*sin(a +
b*x))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int \cos^5(a+bx)\cot^2(a+bx)\,dx = \frac{-\sin(bx+a)^6 + 5\sin(bx+a)^4 - 15\sin(bx+a)^2 - 5}{5\sin(bx+a)\,b}$$

input
```
int(cos(b*x+a)^5*cot(b*x+a)^2,x)
```

output
```
( - sin(a + b*x)**6 + 5*sin(a + b*x)**4 - 15*sin(a + b*x)**2 - 5)/(5*sin(a
 + b*x)*b)
```

3.134 $\int \cos^4(a+bx)\cot^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 57

$$\int \cos^4(a+bx)\cot^2(a+bx)\,dx=-\frac{15x}{8}-\frac{\cot(a+bx)}{b}-\frac{9\cos(a+bx)\sin(a+bx)}{8b}+\frac{\cos(a+bx)\sin^3(a+bx)}{4b}$$

output
```
-15/8*x-cot(b*x+a)/b-9/8*cos(b*x+a)*sin(b*x+a)/b+1/4*cos(b*x+a)*sin(b*x+a)
^3/b
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.72

$$\int \cos^4(a+bx)\cot^2(a+bx)\,dx =-\frac{60a+60bx+32\cot(a+bx)+16\sin(2(a+bx))+\sin(4(a+bx))}{32b}$$

input
```
Integrate[Cos[a + b*x]^4*Cot[a + b*x]^2,x]
```

output
```
-1/32*(60*a + 60*b*x + 32*Cot[a + b*x] + 16*Sin[2*(a + b*x)] + Sin[4*(a +
b*x)])/b
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.39, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3042, 3071, 252, 252, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(a+bx)\cot^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(a+bx+\frac{\pi}{2}\right)^4\tan\left(a+bx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3071}$$

$$-\frac{\int \frac{\cot^6(a+bx)}{(\cot^2(a+bx)+1)^3}d\cot(a+bx)}{b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{\frac{5}{4}\int \frac{\cot^4(a+bx)}{(\cot^2(a+bx)+1)^2}d\cot(a+bx)-\frac{\cot^5(a+bx)}{4(\cot^2(a+bx)+1)^2}}{b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{\frac{5}{4}\left(\frac{3}{2}\int \frac{\cot^2(a+bx)}{\cot^2(a+bx)+1}d\cot(a+bx)-\frac{\cot^3(a+bx)}{2(\cot^2(a+bx)+1)}\right)-\frac{\cot^5(a+bx)}{4(\cot^2(a+bx)+1)^2}}{b}$$

$$\Big\downarrow \text{262}$$

$$-\frac{\frac{5}{4}\left(\frac{3}{2}\left(\cot(a+bx)-\int \frac{1}{\cot^2(a+bx)+1}d\cot(a+bx)\right)-\frac{\cot^3(a+bx)}{2(\cot^2(a+bx)+1)}\right)-\frac{\cot^5(a+bx)}{4(\cot^2(a+bx)+1)^2}}{b}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\frac{5}{4}\left(\frac{3}{2}(\cot(a+bx)-\arctan(\cot(a+bx)))-\frac{\cot^3(a+bx)}{2(\cot^2(a+bx)+1)}\right)-\frac{\cot^5(a+bx)}{4(\cot^2(a+bx)+1)^2}}{b}$$

input
```
Int[Cos[a + b*x]^4*Cot[a + b*x]^2,x]
```

output
```
-((-1/4*Cot[a + b*x]^5/(1 + Cot[a + b*x]^2)^2 + (5*((3*(-ArcTan[Cot[a + b*
x]] + Cot[a + b*x]))/2 - Cot[a + b*x]^3/(2*(1 + Cot[a + b*x]^2))))/4)/b)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 4.62 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.16

method	result	size
derivativedivides	$\frac{-\frac{\cos(bx+a)^7}{\sin(bx+a)}-\left(\cos(bx+a)^5+\frac{5\cos(bx+a)^3}{4}+\frac{15\cos(bx+a)}{8}\right)\sin(bx+a)-\frac{15bx}{8}-\frac{15a}{8}}{b}$	66
default	$\frac{-\frac{\cos(bx+a)^7}{\sin(bx+a)}-\left(\cos(bx+a)^5+\frac{5\cos(bx+a)^3}{4}+\frac{15\cos(bx+a)}{8}\right)\sin(bx+a)-\frac{15bx}{8}-\frac{15a}{8}}{b}$	66
risch	$-\frac{15x}{8}+\frac{i e^{2i(bx+a)}}{4b}-\frac{i e^{-2i(bx+a)}}{4b}-\frac{2i}{b\left(e^{2i(bx+a)}-1\right)}-\frac{\sin(4bx+4a)}{32b}$	68

input
```
int(cos(b*x+a)^4*cot(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/sin(b*x+a)*cos(b*x+a)^7-(cos(b*x+a)^5+5/4*cos(b*x+a)^3+15/8*cos(b*
x+a))*sin(b*x+a)-15/8*b*x-15/8*a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int \cos^4(a+bx)\cot^2(a+bx)\,dx = \frac{2\,\cos(bx+a)^5+5\,\cos(bx+a)^3-15\,bx\sin(bx+a)-15\,\cos(bx+a)}{8\,b\sin(bx+a)}$$

input
```
integrate(cos(b*x+a)^4*cot(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/8*(2*cos(b*x + a)^5 + 5*cos(b*x + a)^3 - 15*b*x*sin(b*x + a) - 15*cos(b*
x + a))/(b*sin(b*x + a))
```

Sympy [F]

$$\int \cos^4(a+bx)\cot^2(a+bx)\,dx = \int \cos^4{(a+bx)}\cot^2{(a+bx)}\,dx$$

input
```
integrate(cos(b*x+a)**4*cot(b*x+a)**2,x)
```

output
```
Integral(cos(a + b*x)**4*cot(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.11

$$\int \cos^4(a+bx)\cot^2(a+bx)\,dx = -\frac{15\,bx+15\,a+\frac{15\,\tan(bx+a)^4+25\,\tan(bx+a)^2+8}{\tan(bx+a)^5+2\,\tan(bx+a)^3+\tan(bx+a)}}{8\,b}$$

input
```
integrate(cos(b*x+a)^4*cot(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/8*(15*b*x + 15*a + (15*tan(b*x + a)^4 + 25*tan(b*x + a)^2 + 8)/(tan(b*x
 + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a)))/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \cos^4(a+bx)\cot^2(a+bx)\,dx = -\frac{15\,bx+15\,a+\frac{7\,\tan(bx+a)^3+9\,\tan(bx+a)}{\left(\tan(bx+a)^2+1\right)^2}+\frac{8}{\tan(bx+a)}}{8\,b}$$

input
```
integrate(cos(b*x+a)^4*cot(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/8*(15*b*x + 15*a + (7*tan(b*x + a)^3 + 9*tan(b*x + a))/(tan(b*x + a)^2
+ 1)^2 + 8/tan(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 25.90 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int \cos^4(a+bx)\cot^2(a+bx)\,dx = -\frac{15\,x}{8} - \frac{\cos\left(a+b\,x\right)^4\,\left(\frac{15\,\tan(a+b\,x)^4}{8}+\frac{25\,\tan(a+b\,x)^2}{8}+1\right)}{b\,\tan\left(a+b\,x\right)}$$

input
```
int(cos(a + b*x)^4*cot(a + b*x)^2,x)
```

output
```
- (15*x)/8 - (cos(a + b*x)^4*((25*tan(a + b*x)^2)/8 + (15*tan(a + b*x)^4)/
8 + 1))/(b*tan(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.12

$$\int \cos^4(a+bx)\cot^2(a+bx)\,dx$$
$$= \frac{2\cos\left(bx+a\right)\sin\left(bx+a\right)^4 - 9\cos\left(bx+a\right)\sin\left(bx+a\right)^2 - 8\cos\left(bx+a\right) - 15\sin\left(bx+a\right)bx}{8\sin\left(bx+a\right)b}$$

input
```
int(cos(b*x+a)^4*cot(b*x+a)^2,x)
```

output
```
(2*cos(a + b*x)*sin(a + b*x)**4 - 9*cos(a + b*x)*sin(a + b*x)**2 - 8*cos(a
 + b*x) - 15*sin(a + b*x)*b*x)/(8*sin(a + b*x)*b)
```

3.135 $\int \cos^3(a+bx)\cot^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 38

$$\int \cos^3(a+bx)\cot^2(a+bx)\,dx = -\frac{\csc(a+bx)}{b}-\frac{2\sin(a+bx)}{b}+\frac{\sin^3(a+bx)}{3b}$$

output
```
-csc(b*x+a)/b-2*sin(b*x+a)/b+1/3*sin(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \cos^3(a+bx)\cot^2(a+bx)\,dx = -\frac{\csc(a+bx)}{b}-\frac{2\sin(a+bx)}{b}+\frac{\sin^3(a+bx)}{3b}$$

input
```
Integrate[Cos[a + b*x]^3*Cot[a + b*x]^2,x]
```

output
```
-(Csc[a + b*x]/b) - (2*Sin[a + b*x])/b + Sin[a + b*x]^3/(3*b)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)\cot^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(a+bx+\frac{\pi}{2}\right)^3\tan\left(a+bx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3070}$$

$$-\frac{\int \csc^2(a+bx)\left(1-\sin^2(a+bx)\right)^2d(-\sin(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(\csc^2(a+bx)+\sin^2(a+bx)-2\right)d(-\sin(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{1}{3}\sin^3(a+bx)+2\sin(a+bx)+\csc(a+bx)}{b}$$

input
```
Int[Cos[a + b*x]^3*Cot[a + b*x]^2,x]
```

output
```
-((Csc[a + b*x] + 2*Sin[a + b*x] - Sin[a + b*x]^3/3)/b)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Maple [A] (verified)

Time = 3.42 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.37

method	result	size
derivativedivides	$\frac{-\frac{\cos(bx+a)^6}{\sin(bx+a)}-\left(\frac{8}{3}+\cos(bx+a)^4+\frac{4\cos(bx+a)^2}{3}\right)\sin(bx+a)}{b}$	52
default	$\frac{-\frac{\cos(bx+a)^6}{\sin(bx+a)}-\left(\frac{8}{3}+\cos(bx+a)^4+\frac{4\cos(bx+a)^2}{3}\right)\sin(bx+a)}{b}$	52
risch	$\frac{7i e^{i(bx+a)}}{8b}-\frac{7i e^{-i(bx+a)}}{8b}-\frac{2i e^{i(bx+a)}}{b\left(e^{2i(bx+a)}-1\right)}-\frac{\sin(3bx+3a)}{12b}$	74

input
```
int(cos(b*x+a)^3*cot(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/sin(b*x+a)*cos(b*x+a)^6-(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*
x+a))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \cos^3(a+bx)\cot^2(a+bx)\,dx = \frac{\cos(bx+a)^4+4\cos(bx+a)^2-8}{3\,b\sin(bx+a)}$$

input
```
integrate(cos(b*x+a)^3*cot(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/3*(cos(b*x + a)^4 + 4*cos(b*x + a)^2 - 8)/(b*sin(b*x + a))
```

Sympy [F]

$$\int \cos^3(a+bx)\cot^2(a+bx)\,dx = \int \cos^3(a+bx)\cot^2(a+bx)\,dx$$

input
```
integrate(cos(b*x+a)**3*cot(b*x+a)**2,x)
```

output
```
Integral(cos(a + b*x)**3*cot(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \cos^3(a+bx)\cot^2(a+bx)\,dx = \frac{\sin(bx+a)^3-\frac{3}{\sin(bx+a)}-6\sin(bx+a)}{3\,b}$$

input
```
integrate(cos(b*x+a)^3*cot(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/3*(sin(b*x + a)^3 - 3/sin(b*x + a) - 6*sin(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \cos^3(a+bx)\cot^2(a+bx)\,dx = \frac{\sin(bx+a)^3 - \frac{3}{\sin(bx+a)} - 6\sin(bx+a)}{3\,b}$$

input
```
integrate(cos(b*x+a)^3*cot(b*x+a)^2,x, algorithm="giac")
```

output
```
1/3*(sin(b*x + a)^3 - 3/sin(b*x + a) - 6*sin(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 25.75 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \cos^3(a+bx)\cot^2(a+bx)\,dx = -\frac{-\sin(a+b\,x)^4 + 6\sin(a+b\,x)^2 + 3}{3\,b\,\sin(a+b\,x)}$$

input
```
int(cos(a + b*x)^3*cot(a + b*x)^2,x)
```

output
```
-(6*sin(a + b*x)^2 - sin(a + b*x)^4 + 3)/(3*b*sin(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \cos^3(a+bx)\cot^2(a+bx)\,dx = \frac{\sin(bx+a)^4 - 6\sin(bx+a)^2 - 3}{3\sin(bx+a)\,b}$$

input
```
int(cos(b*x+a)^3*cot(b*x+a)^2,x)
```

output
```
(sin(a + b*x)**4 - 6*sin(a + b*x)**2 - 3)/(3*sin(a + b*x)*b)
```

3.136 $\int \cos^2(a+bx)\cot^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 36

$$\int \cos^2(a+bx)\cot^2(a+bx)\,dx=-\frac{3x}{2}-\frac{\cot(a+bx)}{b}-\frac{\cos(a+bx)\sin(a+bx)}{2b}$$

output
```
-3/2*x-cot(b*x+a)/b-1/2*cos(b*x+a)*sin(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \cos^2(a+bx)\cot^2(a+bx)\,dx=-\frac{6(a+bx)+4\cot(a+bx)+\sin(2(a+bx))}{4b}$$

input
```
Integrate[Cos[a + b*x]^2*Cot[a + b*x]^2,x]
```

output
```
-1/4*(6*(a + b*x) + 4*Cot[a + b*x] + Sin[2*(a + b*x)])/b
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.39, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3042, 3071, 252, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\cot^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(a+bx+\frac{\pi}{2}\right)^2\tan\left(a+bx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3071}$$

$$-\frac{\int \frac{\cot^4(a+bx)}{(\cot^2(a+bx)+1)^2}d\cot(a+bx)}{b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{\frac{3}{2}\int \frac{\cot^2(a+bx)}{\cot^2(a+bx)+1}d\cot(a+bx)-\frac{\cot^3(a+bx)}{2(\cot^2(a+bx)+1)}}{b}$$

$$\Big\downarrow \text{262}$$

$$-\frac{\frac{3}{2}\left(\cot(a+bx)-\int \frac{1}{\cot^2(a+bx)+1}d\cot(a+bx)\right)-\frac{\cot^3(a+bx)}{2(\cot^2(a+bx)+1)}}{b}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\frac{3}{2}(\cot(a+bx)-\arctan(\cot(a+bx)))-\frac{\cot^3(a+bx)}{2(\cot^2(a+bx)+1)}}{b}$$

input
```
Int[Cos[a + b*x]^2*Cot[a + b*x]^2,x]
```

output
```
-(((3*(-ArcTan[Cot[a + b*x]] + Cot[a + b*x]))/2 - Cot[a + b*x]^3/(2*(1 + C
ot[a + b*x]^2)))/b)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.67 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.50

method	result	size
risch	$-\frac{3x}{2}+\frac{i\,\mathrm{e}^{2i(bx+a)}}{8b}-\frac{i\,\mathrm{e}^{-2i(bx+a)}}{8b}-\frac{2i}{b\left(\mathrm{e}^{2i(bx+a)}-1\right)}$	54
derivativedivides	$\frac{-\frac{\cos(bx+a)^5}{\sin(bx+a)}-\left(\cos(bx+a)^3+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)-\frac{3bx}{2}-\frac{3a}{2}}{b}$	56
default	$\frac{-\frac{\cos(bx+a)^5}{\sin(bx+a)}-\left(\cos(bx+a)^3+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)-\frac{3bx}{2}-\frac{3a}{2}}{b}$	56

input
```
int(cos(b*x+a)^2*cot(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-3/2*x+1/8*I/b*exp(2*I*(b*x+a))-1/8*I/b*exp(-2*I*(b*x+a))-2*I/b/(exp(2*I*(
b*x+a))-1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int \cos^2(a+bx)\cot^2(a+bx)\,dx=\frac{\cos(bx+a)^3-3\,bx\sin(bx+a)-3\,\cos(bx+a)}{2\,b\sin(bx+a)}$$

input
```
integrate(cos(b*x+a)^2*cot(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(cos(b*x + a)^3 - 3*b*x*sin(b*x + a) - 3*cos(b*x + a))/(b*sin(b*x + a)
)
```

Sympy [F]

$$\int \cos^2(a+bx)\cot^2(a+bx)\,dx = \int \cos^2(a+bx)\cot^2(a+bx)\,dx$$

input
```
integrate(cos(b*x+a)**2*cot(b*x+a)**2,x)
```

output
```
Integral(cos(a + b*x)**2*cot(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.19

$$\int \cos^2(a+bx)\cot^2(a+bx)\,dx = -\frac{3\,bx+3\,a+\frac{3\,\tan(bx+a)^2+2}{\tan(bx+a)^3+\tan(bx+a)}}{2\,b}$$

input
```
integrate(cos(b*x+a)^2*cot(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(3*b*x + 3*a + (3*tan(b*x + a)^2 + 2)/(tan(b*x + a)^3 + tan(b*x + a))
)/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.19

$$\int \cos^2(a+bx)\cot^2(a+bx)\,dx = -\frac{3\,bx+3\,a+\frac{3\,\tan(bx+a)^2+2}{\tan(bx+a)^3+\tan(bx+a)}}{2\,b}$$

input
```
integrate(cos(b*x+a)^2*cot(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/2*(3*b*x + 3*a + (3*tan(b*x + a)^2 + 2)/(tan(b*x + a)^3 + tan(b*x + a))
)/b
```

Mupad [B] (verification not implemented)

Time = 25.99 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.19

$$\int \cos^2(a+bx)\cot^2(a+bx)\,dx = -\frac{9\,\cos\left(a+b\,x\right)-\cos\left(3\,a+3\,b\,x\right)+12\,b\,x\,\sin\left(a+b\,x\right)}{8\,b\,\sin\left(a+b\,x\right)}$$

input
```
int(cos(a + b*x)^2*cot(a + b*x)^2,x)
```

output
```
-(9*cos(a + b*x) - cos(3*a + 3*b*x) + 12*b*x*sin(a + b*x))/(8*b*sin(a + b*
x))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.33

$$\int \cos^2(a+bx)\cot^2(a+bx)\,dx = \frac{-\cos\left(bx+a\right)\sin\left(bx+a\right)^2-2\cos\left(bx+a\right)-3\sin\left(bx+a\right)bx}{2\sin\left(bx+a\right)b}$$

input
```
int(cos(b*x+a)^2*cot(b*x+a)^2,x)
```

output
```
( - cos(a + b*x)*sin(a + b*x)**2 - 2*cos(a + b*x) - 3*sin(a + b*x)*b*x)/(2
*sin(a + b*x)*b)
```

3.137 $\int \cos(a+bx)\cot^2(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 23

$$\int \cos(a+bx)\cot^2(a+bx)\,dx = -\frac{\csc(a+bx)}{b} - \frac{\sin(a+bx)}{b}$$

output
```
-csc(b*x+a)/b-sin(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \cos(a+bx)\cot^2(a+bx)\,dx = -\frac{\csc(a+bx)}{b} - \frac{\sin(a+bx)}{b}$$

input
```
Integrate[Cos[a + b*x]*Cot[a + b*x]^2,x]
```

output
```
-(Csc[a + b*x]/b) - Sin[a + b*x]/b
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\cot^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(a+bx+\frac{\pi}{2}\right)\tan\left(a+bx+\frac{\pi}{2}\right)^2 dx$$

$$\Big\downarrow \text{3070}$$

$$-\frac{\int \csc^2(a+bx)\left(1-\sin^2(a+bx)\right)d(-\sin(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(\csc^2(a+bx)-1\right)d(-\sin(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sin(a+bx)+\csc(a+bx)}{b}$$

input
```
Int[Cos[a + b*x]*Cot[a + b*x]^2,x]
```

output
```
-((Csc[a + b*x] + Sin[a + b*x])/b)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Maple [A] (verified)

Time = 2.57 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

method	result	size
derivativedivides	$\frac{-\frac{\cos(bx+a)^4}{\sin(bx+a)}-\left(2+\cos(bx+a)^2\right)\sin(bx+a)}{b}$	42
default	$\frac{-\frac{\cos(bx+a)^4}{\sin(bx+a)}-\left(2+\cos(bx+a)^2\right)\sin(bx+a)}{b}$	42
risch	$\frac{i\left(e^{3i(bx+a)}-5\cos(bx+a)-7i\sin(bx+a)\right)}{2b\left(e^{2i(bx+a)}-1\right)}$	47

input
```
int(cos(b*x+a)*cot(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/sin(b*x+a)*cos(b*x+a)^4-(2+cos(b*x+a)^2)*sin(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \cos(a+bx)\cot^2(a+bx)\,dx = \frac{\cos(bx+a)^2-2}{b\sin(bx+a)}$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)^2,x, algorithm="fricas")
```

output
```
(cos(b*x + a)^2 - 2)/(b*sin(b*x + a))
```

Sympy [F]

$$\int \cos(a+bx)\cot^2(a+bx)\,dx = \int \cos(a+bx)\cot^2(a+bx)\,dx$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)**2,x)
```

output
```
Integral(cos(a + b*x)*cot(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\cot^2(a+bx)\,dx = -\frac{\frac{1}{\sin(bx+a)}+\sin(bx+a)}{b}$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)^2,x, algorithm="maxima")
```

output
```
-(1/sin(b*x + a) + sin(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \cos(a+bx)\cot^2(a+bx)\,dx = -\frac{\frac{1}{\sin(bx+a)} + \sin(bx+a)}{b}$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)^2,x, algorithm="giac")
```

output
```
-(1/sin(b*x + a) + sin(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 25.75 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \cos(a+bx)\cot^2(a+bx)\,dx = -\frac{\sin(a+b\,x)^2+1}{b\,\sin(a+b\,x)}$$

input
```
int(cos(a + b*x)*cot(a + b*x)^2,x)
```

output
```
-(sin(a + b*x)^2 + 1)/(b*sin(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \cos(a+bx)\cot^2(a+bx)\,dx = \frac{-\sin(bx+a)^2-1}{\sin(bx+a)\,b}$$

input
```
int(cos(b*x+a)*cot(b*x+a)^2,x)
```

output
```
( - (sin(a + b*x)**2 + 1))/(sin(a + b*x)*b)
```

3.138 $\int \cot^2(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 15

$$\int \cot^2(a+bx)\,dx=-x-\frac{\cot(a+bx)}{b}$$

output
```
-x-cot(b*x+a)/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int \cot^2(a+bx)\,dx=-\frac{\cot(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(a+bx)\right)}{b}$$

input
```
Integrate[Cot[a + b*x]^2,x]
```

output
```
-((Cot[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[a + b*x]^2])/b)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(a+bx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3954}$$

$$-\int 1dx-\frac{\cot(a+bx)}{b}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\cot(a+bx)}{b}-x$$

input
```
Int[Cot[a + b*x]^2,x]
```

output
```
-x - Cot[a + b*x]/b
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

method	result	size
parallelrisch	$\frac{-bx-\cot(bx+a)}{b}$	18
risch	$-x-\frac{2i}{b\left(\mathrm{e}^{2i(bx+a)}-1\right)}$	24
norman	$\frac{-\frac{1}{b}-x\tan(bx+a)}{\tan(bx+a)}$	25
derivativedivides	$\frac{-\cot(bx+a)+\frac{\pi}{2}-\operatorname{arccot}(\cot(bx+a))}{b}$	26
default	$\frac{-\cot(bx+a)+\frac{\pi}{2}-\operatorname{arccot}(\cot(bx+a))}{b}$	26

input
```
int(cot(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-b*x-cot(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(15) = 30.

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.60

$$\int \cot^2(a+bx)\,dx = -\frac{bx\sin\left(2\,bx+2\,a\right)+\cos\left(2\,bx+2\,a\right)+1}{b\sin\left(2\,bx+2\,a\right)}$$

input
```
integrate(cot(b*x+a)^2,x, algorithm="fricas")
```

output
```
-(b*x*sin(2*b*x + 2*a) + cos(2*b*x + 2*a) + 1)/(b*sin(2*b*x + 2*a))
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \cot^2(a+bx)\,dx = \begin{cases} -x - \frac{\cot(a+bx)}{b} & \text{for } b \neq 0 \\ x\cot^2(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cot(b*x+a)**2,x)
```

output
```
Piecewise((-x - cot(a + b*x)/b, Ne(b, 0)), (x*cot(a)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \cot^2(a+bx)\,dx = -\frac{bx + a + \frac{1}{\tan(bx+a)}}{b}$$

input
```
integrate(cot(b*x+a)^2,x, algorithm="maxima")
```

output
```
-(b*x + a + 1/tan(b*x + a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(15) = 30.

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.33

$$\int \cot^2(a+bx)\,dx = -\frac{2\,bx + 2\,a + \frac{1}{\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)} - \tan\left(\frac{1}{2}\,bx + \frac{1}{2}\,a\right)}{2\,b}$$

input
```
integrate(cot(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/2*(2*b*x + 2*a + 1/tan(1/2*b*x + 1/2*a) - tan(1/2*b*x + 1/2*a))/b
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cot^2(a+bx)\,dx = -x - \frac{\cot(a+b\,x)}{b}$$

input
```
int(cot(a + b*x)^2,x)
```

output
```
- x - cot(a + b*x)/b
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \cot^2(a+bx)\,dx = \frac{-\cot(bx+a) - bx}{b}$$

input
```
int(cot(b*x+a)^2,x)
```

output
```
( - (cot(a + b*x) + b*x))/b
```

3.139 $\int \cot(a+bx)\csc(a+bx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 11

$$\int \cot(a+bx)\csc(a+bx)\,dx=-\frac{\csc(a+bx)}{b}$$

output
```
-csc(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \cot(a+bx)\csc(a+bx)\,dx=-\frac{\csc(a+bx)}{b}$$

input
```
Integrate[Cot[a + b*x]*Csc[a + b*x],x]
```

output
```
-(Csc[a + b*x]/b)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 25, 3086, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(a+bx)\csc(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(a+bx-\frac{\pi}{2}\right)\left(-\sec\left(a+bx-\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sec\left(\frac{1}{2}(2a-\pi)+bx\right)\tan\left(\frac{1}{2}(2a-\pi)+bx\right)dx$$

$$\Big\downarrow \text{3086}$$

$$-\frac{\int 1d\csc(a+bx)}{b}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\csc(a+bx)}{b}$$

input
```
Int[Cot[a + b*x]*Csc[a + b*x],x]
```

output
```
-(Csc[a + b*x]/b)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$-\frac{\csc(bx+a)}{b}$	12
default	$-\frac{\csc(bx+a)}{b}$	12
risch	$-\frac{2i\,{\rm e}^{i(bx+a)}}{b\left({\rm e}^{2i(bx+a)}-1\right)}$	29

input
```
int(cot(b*x+a)*csc(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-csc(b*x+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \cot(a+bx)\csc(a+bx)\,dx = -\frac{1}{b\sin{(bx+a)}}$$

input
```
integrate(cot(b*x+a)*csc(b*x+a),x, algorithm="fricas")
```

output
```
-1/(b*sin(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(8) = 16.

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \cot(a+bx)\csc(a+bx)\,dx = \begin{cases} -\frac{\csc{(a+bx)}}{b} & \text{for}\ b\neq 0 \\ x\cot{(a)}\csc{(a)} & \text{otherwise} \end{cases}$$

input
```
integrate(cot(b*x+a)*csc(b*x+a),x)
```

output
```
Piecewise((-csc(a + b*x)/b, Ne(b, 0)), (x*cot(a)*csc(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \cot(a+bx)\csc(a+bx)\,dx = -\frac{1}{b\sin{(bx+a)}}$$

input
```
integrate(cot(b*x+a)*csc(b*x+a),x, algorithm="maxima")
```

output
```
-1/(b*sin(b*x + a))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \cot(a+bx)\csc(a+bx)\,dx = -\frac{1}{b\sin(bx+a)}$$

input
```
integrate(cot(b*x+a)*csc(b*x+a),x, algorithm="giac")
```

output
```
-1/(b*sin(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 25.81 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \cot(a+bx)\csc(a+bx)\,dx = -\frac{1}{b\,\sin(a+b\,x)}$$

input
```
int(cot(a + b*x)/sin(a + b*x),x)
```

output
```
-1/(b*sin(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \cot(a+bx)\csc(a+bx)\,dx = -\frac{\csc(bx+a)}{b}$$

input
```
int(cot(b*x+a)*csc(b*x+a),x)
```

output
```
( - csc(a + b*x))/b
```

3.140 $\int \csc^2(a+bx)\sec(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 23

$$\int \csc^2(a+bx)\sec(a+bx)\,dx = \frac{\operatorname{arctanh}(\sin(a+bx))}{b} - \frac{\csc(a+bx)}{b}$$

output
```
arctanh(sin(b*x+a))/b-csc(b*x+a)/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \csc^2(a+bx)\sec(a+bx)\,dx = -\frac{\csc(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\sin^2(a+bx)\right)}{b}$$

input
```
Integrate[Csc[a + b*x]^2*Sec[a + b*x],x]
```

output
```
-((Csc[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, Sin[a + b*x]^2])/b)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3101, 25, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)\sec(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(a+bx)^2\sec(a+bx)dx$$

$$\Big\downarrow \text{3101}$$

$$-\frac{\int -\frac{\csc^2(a+bx)}{1-\csc^2(a+bx)}d\csc(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\csc^2(a+bx)}{1-\csc^2(a+bx)}d\csc(a+bx)}{b}$$

$$\Big\downarrow \text{262}$$

$$-\frac{\csc(a+bx)-\int \frac{1}{1-\csc^2(a+bx)}d\csc(a+bx)}{b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\csc(a+bx)-\text{arctanh}(\csc(a+bx))}{b}$$

input
```
Int[Csc[a + b*x]^2*Sec[a + b*x],x]
```

output
```
-((-ArcTanh[Csc[a + b*x]] + Csc[a + b*x])/b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30

method	result	size
derivativedivides	$\frac{-\frac{1}{\sin(bx+a)}+\ln(\sec(bx+a)+\tan(bx+a))}{b}$	30
default	$\frac{-\frac{1}{\sin(bx+a)}+\ln(\sec(bx+a)+\tan(bx+a))}{b}$	30
parallelrisch	$\frac{-\cot\left(\frac{bx}{2}+\frac{a}{2}\right)-2\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+2\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{2b}$	57
risch	$-\frac{2i\,e^{i(bx+a)}}{b\left(e^{2i(bx+a)}-1\right)}-\frac{\ln\left(e^{i(bx+a)}-i\right)}{b}+\frac{\ln\left(e^{i(bx+a)}+i\right)}{b}$	65
norman	$\frac{-\frac{1}{2b}-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}{2b}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}+\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{b}-\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{b}$	69

input
```
int(csc(b*x+a)^2*sec(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/sin(b*x+a)+ln(sec(b*x+a)+tan(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(23) = 46$.

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.17

$$\int \csc^2(a+bx)\sec(a+bx)\,dx = \frac{\log(\sin(bx+a)+1)\sin(bx+a)-\log(-\sin(bx+a)+1)\sin(bx+a)-2}{2\,b\sin(bx+a)}$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(log(sin(b*x + a) + 1)*sin(b*x + a) - log(-sin(b*x + a) + 1)*sin(b*x +
 a) - 2)/(b*sin(b*x + a))
```

Sympy [F]

$$\int \csc^2(a+bx)\sec(a+bx)\,dx = \int \csc^2{(a+bx)}\sec{(a+bx)}\,dx$$

input
```
integrate(csc(b*x+a)**2*sec(b*x+a),x)
```

output
```
Integral(csc(a + b*x)**2*sec(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \csc^2(a+bx)\sec(a+bx)\,dx = -\frac{\frac{2}{\sin(bx+a)} - \log(\sin(bx+a)+1) + \log(\sin(bx+a)-1)}{2\,b}$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.91

$$\int \csc^2(a+bx)\sec(a+bx)\,dx = \frac{\log(|\sin(bx+a)+1|)}{2\,b} - \frac{\log(|\sin(bx+a)-1|)}{2\,b} - \frac{1}{b\sin(bx+a)}$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a),x, algorithm="giac")
```

output
```
1/2*log(abs(sin(b*x + a) + 1))/b - 1/2*log(abs(sin(b*x + a) - 1))/b - 1/(b
*sin(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \csc^2(a+bx)\sec(a+bx)\,dx = \frac{\operatorname{atanh}(\sin(a+b\,x))-\frac{1}{\sin(a+b\,x)}}{b}$$

input
```
int(1/(cos(a + b*x)*sin(a + b*x)^2),x)
```

output
```
(atanh(sin(a + b*x)) - 1/sin(a + b*x))/b
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.30

$$\int \csc^2(a+bx)\sec(a+bx)\,dx$$
$$= \frac{-\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin(bx+a)+\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)\sin(bx+a)-1}{\sin(bx+a)\,b}$$

input
```
int(csc(b*x+a)^2*sec(b*x+a),x)
```

output
```
( - log(tan((a + b*x)/2) - 1)*sin(a + b*x) + log(tan((a + b*x)/2) + 1)*sin
(a + b*x) - 1)/(sin(a + b*x)*b)
```

3.141 $\int \csc^2(a+bx)\sec^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 22

$$\int \csc^2(a+bx)\sec^2(a+bx)\,dx=-\frac{\cot(a+bx)}{b}+\frac{\tan(a+bx)}{b}$$

output
```
-cot(b*x+a)/b+tan(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.59

$$\int \csc^2(a+bx)\sec^2(a+bx)\,dx=-\frac{2\cot(2(a+bx))}{b}$$

input
```
Integrate[Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

output
```
(-2*Cot[2*(a + b*x)])/b
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)\sec^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(a+bx)^2\sec(a+bx)^2dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{\int \cot^2(a+bx)\left(\tan^2(a+bx)+1\right)d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\cot^2(a+bx)+1\right)d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\tan(a+bx)-\cot(a+bx)}{b}$$

input
```
Int[Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

output
```
(-Cot[a + b*x] + Tan[a + b*x])/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Maple [A] (verified)

Time = 2.95 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

method	result	size
derivativedivides	$\frac{\frac{1}{\sin(bx+a)\cos(bx+a)}-2\cot(bx+a)}{b}$	31
default	$\frac{\frac{1}{\sin(bx+a)\cos(bx+a)}-2\cot(bx+a)}{b}$	31
risch	$-\frac{4i}{b\left(e^{2i(bx+a)}+1\right)\left(e^{2i(bx+a)}-1\right)}$	33
parallelrisch	$\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{3}-6\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+\cot\left(\frac{bx}{2}+\frac{a}{2}\right)}{2b\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-2b}$	54
norman	$\frac{\frac{1}{2b}-\frac{3\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}{b}+\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}}{2b}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)}$	66

input
```
int(csc(b*x+a)^2*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/sin(b*x+a)/cos(b*x+a)-2*cot(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.50

$$\int \csc^2(a+bx)\sec^2(a+bx)\,dx = -\frac{2\,\cos(bx+a)^2-1}{b\cos(bx+a)\sin(bx+a)}$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
-(2*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)*sin(b*x + a))
```

Sympy [F]

$$\int \csc^2(a+bx)\sec^2(a+bx)\,dx = \int \csc^2(a+bx)\sec^2(a+bx)\,dx$$

input
```
integrate(csc(b*x+a)**2*sec(b*x+a)**2,x)
```

output
```
Integral(csc(a + b*x)**2*sec(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \csc^2(a+bx)\sec^2(a+bx)\,dx = -\frac{\frac{1}{\tan(bx+a)}-\tan(bx+a)}{b}$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
-(1/tan(b*x + a) - tan(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \csc^2(a+bx)\sec^2(a+bx)\,dx = -\frac{2}{b\tan(2bx+2a)}$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
-2/(b*tan(2*b*x + 2*a))
```

Mupad [B] (verification not implemented)

Time = 25.79 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.64

$$\int \csc^2(a+bx)\sec^2(a+bx)\,dx = -\frac{2\cot(2a+2bx)}{b}$$

input
```
int(1/(cos(a + b*x)^2*sin(a + b*x)^2),x)
```

output
```
-(2*cot(2*a + 2*b*x))/b
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int \csc^2(a+bx)\sec^2(a+bx)\,dx = \frac{2\sin(bx+a)^2-1}{\cos(bx+a)\sin(bx+a)\,b}$$

input
```
int(csc(b*x+a)^2*sec(b*x+a)^2,x)
```

output
```
(2*sin(a + b*x)**2 - 1)/(cos(a + b*x)*sin(a + b*x)*b)
```

3.142 $\int \csc^2(a+bx)\sec^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 45

$$\int \csc^2(a+bx)\sec^3(a+bx)\,dx = \frac{3\operatorname{arctanh}(\sin(a+bx))}{2b} - \frac{\csc(a+bx)}{b} + \frac{\sec(a+bx)\tan(a+bx)}{2b}$$

output
```
3/2*arctanh(sin(b*x+a))/b-csc(b*x+a)/b+1/2*sec(b*x+a)*tan(b*x+a)/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.60

$$\int \csc^2(a+bx)\sec^3(a+bx)\,dx = -\frac{\csc(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},2,\frac{1}{2},\sin^2(a+bx)\right)}{b}$$

input
```
Integrate[Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

output
```
-((Csc[a + b*x]*Hypergeometric2F1[-1/2, 2, 1/2, Sin[a + b*x]^2])/b)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3042, 3101, 252, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)\sec^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(a+bx)^2\sec(a+bx)^3dx$$

$$\Big\downarrow \text{3101}$$

$$-\frac{\int \frac{\csc^4(a+bx)}{(1-\csc^2(a+bx))^2}d\csc(a+bx)}{b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{\frac{\csc^3(a+bx)}{2(1-\csc^2(a+bx))}-\frac{3}{2}\int \frac{\csc^2(a+bx)}{1-\csc^2(a+bx)}d\csc(a+bx)}{b}$$

$$\Big\downarrow \text{262}$$

$$-\frac{\frac{\csc^3(a+bx)}{2(1-\csc^2(a+bx))}-\frac{3}{2}\left(\int \frac{1}{1-\csc^2(a+bx)}d\csc(a+bx)-\csc(a+bx)\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{\csc^3(a+bx)}{2(1-\csc^2(a+bx))}-\frac{3}{2}(\text{arctanh}(\csc(a+bx))-\csc(a+bx))}{b}$$

input
```
Int[Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

output
```
-(((-3*(ArcTanh[Csc[a + b*x]] - Csc[a + b*x]))/2 + Csc[a + b*x]^3/(2*(1 -
Csc[a + b*x]^2)))/b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{\frac{1}{2\sin(bx+a)\cos(bx+a)^2}-\frac{3}{2\sin(bx+a)}+\frac{3\ln(\sec(bx+a)+\tan(bx+a))}{2}}{b}$
default	$\frac{\frac{1}{2\sin(bx+a)\cos(bx+a)^2}-\frac{3}{2\sin(bx+a)}+\frac{3\ln(\sec(bx+a)+\tan(bx+a))}{2}}{b}$
risch	$-\frac{i\left(3\,\mathrm{e}^{5i(bx+a)}+2\,\mathrm{e}^{3i(bx+a)}+3\,\mathrm{e}^{i(bx+a)}\right)}{b\left(\mathrm{e}^{2i(bx+a)}+1\right)^2\left(\mathrm{e}^{2i(bx+a)}-1\right)}-\frac{3\ln\left(\mathrm{e}^{i(bx+a)}-i\right)}{2b}+\frac{3\ln\left(\mathrm{e}^{i(bx+a)}+i\right)}{2b}$
parallelrisch	$\frac{(-3\cos(2bx+2a)-3)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+(3\cos(2bx+2a)+3)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)+(-6\cos(bx+a)+6)\cot\left(\frac{bx}{2}+\frac{a}{2}\right)}{2b(1+\cos(2bx+2a))}$
norman	$\frac{-\frac{1}{2b}+\frac{3\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}{2b}+\frac{3\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}{2b}-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^6}{2b}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1\right)^2}-\frac{3\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{2b}+\frac{3\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{2b}$

input
```
int(csc(b*x+a)^2*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2/sin(b*x+a)/cos(b*x+a)^2-3/2/sin(b*x+a)+3/2*ln(sec(b*x+a)+tan(b*x+
a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(41) = 82.

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.89

$$\int \csc^2(a+bx)\sec^3(a+bx)\,dx$$
$$=\frac{3\cos(bx+a)^2\log(\sin(bx+a)+1)\sin(bx+a)-3\cos(bx+a)^2\log(-\sin(bx+a)+1)\sin(bx+a)-}{4\,b\cos(bx+a)^2\sin(bx+a)}$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4*(3*cos(b*x + a)^2*log(sin(b*x + a) + 1)*sin(b*x + a) - 3*cos(b*x + a)^
2*log(-sin(b*x + a) + 1)*sin(b*x + a) - 6*cos(b*x + a)^2 + 2)/(b*cos(b*x +
 a)^2*sin(b*x + a))
```

Sympy [F]

$$\int \csc^2(a+bx)\sec^3(a+bx)\,dx = \int \csc^2{(a+bx)}\sec^3{(a+bx)}\,dx$$

input
```
integrate(csc(b*x+a)**2*sec(b*x+a)**3,x)
```

output
```
Integral(csc(a + b*x)**2*sec(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.36

$$\int \csc^2(a+bx)\sec^3(a+bx)\,dx = -\frac{\frac{2\left(3\sin(bx+a)^2-2\right)}{\sin(bx+a)^3-\sin(bx+a)} - 3\log\left(\sin\left(bx+a\right)+1\right) + 3\log\left(\sin\left(bx+a\right)-1\right)}{4b}$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/4*(2*(3*sin(b*x + a)^2 - 2)/(sin(b*x + a)^3 - sin(b*x + a)) - 3*log(sin
(b*x + a) + 1) + 3*log(sin(b*x + a) - 1))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.49

$$\int \csc^2(a+bx)\sec^3(a+bx)\,dx = \frac{3\log\left(\left|\sin\left(bx+a\right)+1\right|\right)}{4b} - \frac{3\log\left(\left|\sin\left(bx+a\right)-1\right|\right)}{4b} - \frac{3\sin\left(bx+a\right)^2-2}{2\left(\sin\left(bx+a\right)^3-\sin\left(bx+a\right)\right)b}$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="giac")
```

output
```
3/4*log(abs(sin(b*x + a) + 1))/b - 3/4*log(abs(sin(b*x + a) - 1))/b - 1/2*
(3*sin(b*x + a)^2 - 2)/((sin(b*x + a)^3 - sin(b*x + a))*b)
```

Mupad [B] (verification not implemented)

Time = 25.77 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.07

$$\int \csc^2(a+bx)\sec^3(a+bx)\,dx = \frac{3\,\mathrm{atanh}(\sin(a+b\,x))}{2\,b} + \frac{\frac{3\sin(a+b\,x)^2}{2}-1}{b\,\left(\sin(a+b\,x)-\sin(a+b\,x)^3\right)}$$

input
```
int(1/(cos(a + b*x)^3*sin(a + b*x)^2),x)
```

output
```
(3*atanh(sin(a + b*x)))/(2*b) + ((3*sin(a + b*x)^2)/2 - 1)/(b*(sin(a + b*x
) - sin(a + b*x)^3))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.69

$$\int \csc^2(a+bx)\sec^3(a+bx)\,dx$$
$$= \frac{-3\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin(bx+a)^3+3\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin(bx+a)+3\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{2\sin(bx+a)\,b\left(\sin(bx+a)^2-1\right.}$$

input
```
int(csc(b*x+a)^2*sec(b*x+a)^3,x)
```

output
```
( - 3*log(tan((a + b*x)/2) - 1)*sin(a + b*x)**3 + 3*log(tan((a + b*x)/2) -
 1)*sin(a + b*x) + 3*log(tan((a + b*x)/2) + 1)*sin(a + b*x)**3 - 3*log(tan
((a + b*x)/2) + 1)*sin(a + b*x) - 3*sin(a + b*x)**2 + 2)/(2*sin(a + b*x)*b
*(sin(a + b*x)**2 - 1))
```

3.143 $\int \csc^2(a+bx)\sec^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 38

$$\int \csc^2(a+bx)\sec^4(a+bx)\,dx=-\frac{\cot(a+bx)}{b}+\frac{2\tan(a+bx)}{b}+\frac{\tan^3(a+bx)}{3b}$$

output
```
-cot(b*x+a)/b+2*tan(b*x+a)/b+1/3*tan(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

$$\int \csc^2(a+bx)\sec^4(a+bx)\,dx=-\frac{\cot(a+bx)}{b}+\frac{5\tan(a+bx)}{3b}+\frac{\sec^2(a+bx)\tan(a+bx)}{3b}$$

input
```
Integrate[Csc[a + b*x]^2*Sec[a + b*x]^4,x]
```

output
```
-(Cot[a + b*x]/b) + (5*Tan[a + b*x])/(3*b) + (Sec[a + b*x]^2*Tan[a + b*x])
/(3*b)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)\sec^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(a+bx)^2\sec(a+bx)^4dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{\int \cot^2(a+bx)\left(\tan^2(a+bx)+1\right)^2d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\cot^2(a+bx)+\tan^2(a+bx)+2\right)d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{3}\tan^3(a+bx)+2\tan(a+bx)-\cot(a+bx)}{b}$$

input
```
Int[Csc[a + b*x]^2*Sec[a + b*x]^4,x]
```

output
```
(-Cot[a + b*x] + 2*Tan[a + b*x] + Tan[a + b*x]^3/3)/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

method	result	size
risch	$-\frac{16 i \left(2 \,{\mathrm e}^{2 i \left(b x +a \right)}+1\right)}{3 b \left({\mathrm e}^{2 i \left(b x +a \right)}+1\right)^{3} \left({\mathrm e}^{2 i \left(b x +a \right)}-1\right)}$	46
derivativedivides	$\frac{\frac{1}{3 \sin \left(b x +a \right) \cos \left(b x +a \right)^{3}}+\frac{4}{3 \sin \left(b x +a \right) \cos \left(b x +a \right)}-\frac{8 \cot \left(b x +a \right)}{3}}{b}$	50
default	$\frac{\frac{1}{3 \sin \left(b x +a \right) \cos \left(b x +a \right)^{3}}+\frac{4}{3 \sin \left(b x +a \right) \cos \left(b x +a \right)}-\frac{8 \cot \left(b x +a \right)}{3}}{b}$	50
parallelrisch	$\frac{3 \tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{7}-36 \tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{5}+50 \tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{3}-36 \tan \left(\frac{b x}{2}+\frac{a}{2}\right)+3 \cot \left(\frac{b x}{2}+\frac{a}{2}\right)}{6 b \left(\tan \left(\frac{b x}{2}+\frac{a}{2}\right)-1\right)^{3} \left(\tan \left(\frac{b x}{2}+\frac{a}{2}\right)+1\right)^{3}}$	94
norman	$\frac{\frac{1}{2 b}-\frac{6 \tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}}{b}+\frac{25 \tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{4}}{3 b}-\frac{6 \tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{6}}{b}+\frac{\tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{8}}{2 b}}{\tan \left(\frac{b x}{2}+\frac{a}{2}\right) \left(\tan \left(\frac{b x}{2}+\frac{a}{2}\right)^{2}-1\right)^{3}}$	98

input
```
int(csc(b*x+a)^2*sec(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
-16/3*I*(2*exp(2*I*(b*x+a))+1)/b/(exp(2*I*(b*x+a))+1)^3/(exp(2*I*(b*x+a))-
1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13

$$\int \csc^2(a+bx)\sec^4(a+bx)\,dx = -\frac{8\,\cos\left(bx+a\right)^4-4\,\cos\left(bx+a\right)^2-1}{3\,b\cos\left(bx+a\right)^3\sin\left(bx+a\right)}$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a)^4,x, algorithm="fricas")
```

output
```
-1/3*(8*cos(b*x + a)^4 - 4*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^3*sin(b*x +
 a))
```

Sympy [F]

$$\int \csc^2(a+bx)\sec^4(a+bx)\,dx = \int \csc^2\left(a+bx\right)\sec^4\left(a+bx\right)\,dx$$

input
```
integrate(csc(b*x+a)**2*sec(b*x+a)**4,x)
```

output
```
Integral(csc(a + b*x)**2*sec(a + b*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \csc^2(a+bx)\sec^4(a+bx)\,dx = \frac{\tan\left(bx+a\right)^3-\frac{3}{\tan(bx+a)}+6\,\tan\left(bx+a\right)}{3\,b}$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/3*(tan(b*x + a)^3 - 3/tan(b*x + a) + 6*tan(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \csc^2(a+bx)\sec^4(a+bx)\,dx=\frac{\tan\left(bx+a\right)^3-\frac{3}{\tan(bx+a)}+6\,\tan\left(bx+a\right)}{3\,b}$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a)^4,x, algorithm="giac")
```

output
```
1/3*(tan(b*x + a)^3 - 3/tan(b*x + a) + 6*tan(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 25.74 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \csc^2(a+bx)\sec^4(a+bx)\,dx=\frac{\tan(a+b\,x)^4+6\tan(a+b\,x)^2-3}{3\,b\tan\left(a+b\,x\right)}$$

input
```
int(1/(cos(a + b*x)^4*sin(a + b*x)^2),x)
```

output
```
(6*tan(a + b*x)^2 + tan(a + b*x)^4 - 3)/(3*b*tan(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.45

$$\int \csc^2(a+bx)\sec^4(a+bx)\,dx=\frac{8\sin\left(bx+a\right)^4-12\sin\left(bx+a\right)^2+3}{3\cos\left(bx+a\right)\sin\left(bx+a\right)b\left(\sin\left(bx+a\right)^2-1\right)}$$

input
```
int(csc(b*x+a)^2*sec(b*x+a)^4,x)
```

output
```
(8*sin(a + b*x)**4 - 12*sin(a + b*x)**2 + 3)/(3*cos(a + b*x)*sin(a + b*x)*
b*(sin(a + b*x)**2 - 1))
```

3.144 $\int \csc^2(a+bx)\sec^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 66

$$\int \csc^2(a+bx)\sec^5(a+bx)\,dx = \frac{15\text{arctanh}(\sin(a+bx))}{8b} - \frac{\csc(a+bx)}{b} + \frac{9\sec(a+bx)\tan(a+bx)}{8b} + \frac{\sec(a+bx)\tan^3(a+bx)}{4b}$$

output
```
15/8*arctanh(sin(b*x+a))/b-csc(b*x+a)/b+9/8*sec(b*x+a)*tan(b*x+a)/b+1/4*se
c(b*x+a)*tan(b*x+a)^3/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.41

$$\int \csc^2(a+bx)\sec^5(a+bx)\,dx = -\frac{\csc(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},3,\frac{1}{2},\sin^2(a+bx)\right)}{b}$$

input
```
Integrate[Csc[a + b*x]^2*Sec[a + b*x]^5,x]
```

output
```
-((Csc[a + b*x]*Hypergeometric2F1[-1/2, 3, 1/2, Sin[a + b*x]^2])/b)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.26, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3042, 3101, 25, 252, 252, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)\sec^5(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(a+bx)^2\sec(a+bx)^5dx$$

$$\Big\downarrow \text{3101}$$

$$-\frac{\int -\frac{\csc^6(a+bx)}{(1-\csc^2(a+bx))^3}d\csc(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\csc^6(a+bx)}{(1-\csc^2(a+bx))^3}d\csc(a+bx)}{b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{\frac{5}{4}\int \frac{\csc^4(a+bx)}{(1-\csc^2(a+bx))^2}d\csc(a+bx)-\frac{\csc^5(a+bx)}{4(1-\csc^2(a+bx))^2}}{b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{\frac{5}{4}\left(\frac{\csc^3(a+bx)}{2(1-\csc^2(a+bx))}-\frac{3}{2}\int \frac{\csc^2(a+bx)}{1-\csc^2(a+bx)}d\csc(a+bx)\right)-\frac{\csc^5(a+bx)}{4(1-\csc^2(a+bx))^2}}{b}$$

$$\Big\downarrow \text{262}$$

$$-\frac{\frac{5}{4}\left(\frac{\csc^3(a+bx)}{2(1-\csc^2(a+bx))}-\frac{3}{2}\left(\int \frac{1}{1-\csc^2(a+bx)}d\csc(a+bx)-\csc(a+bx)\right)\right)-\frac{\csc^5(a+bx)}{4(1-\csc^2(a+bx))^2}}{b}$$

$$\Big\downarrow \; 219$$

$$-\frac{\frac{5}{4}\left(\frac{\csc^3(a+bx)}{2(1-\csc^2(a+bx))}-\frac{3}{2}(\operatorname{arctanh}(\csc(a+bx))-\csc(a+bx))\right)-\frac{\csc^5(a+bx)}{4(1-\csc^2(a+bx))^2}}{b}$$

input
```
Int[Csc[a + b*x]^2*Sec[a + b*x]^5,x]
```

output
```
-((-1/4*Csc[a + b*x]^5/(1 - Csc[a + b*x]^2)^2 + (5*((-3*(ArcTanh[Csc[a + b
*x]] - Csc[a + b*x]))/2 + Csc[a + b*x]^3/(2*(1 - Csc[a + b*x]^2))))/4)/b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{\frac{1}{4\sin(bx+a)\cos(bx+a)^4}+\frac{5}{8\sin(bx+a)\cos(bx+a)^2}-\frac{15}{8\sin(bx+a)}+\frac{15\ln(\sec(bx+a)+\tan(bx+a))}{8}}{b}$
default	$\frac{\frac{1}{4\sin(bx+a)\cos(bx+a)^4}+\frac{5}{8\sin(bx+a)\cos(bx+a)^2}-\frac{15}{8\sin(bx+a)}+\frac{15\ln(\sec(bx+a)+\tan(bx+a))}{8}}{b}$
risch	$-\frac{i\left(15\,e^{9i(bx+a)}+40\,e^{7i(bx+a)}+18\,e^{5i(bx+a)}+40\,e^{3i(bx+a)}+15\,e^{i(bx+a)}\right)}{4b\left(e^{2i(bx+a)}+1\right)^4\left(e^{2i(bx+a)}-1\right)}-\frac{15\ln\left(e^{i(bx+a)}-i\right)}{8b}+\frac{15\ln\left(e^{i(bx+a)}+i\right)}{8b}$
norman	$\frac{-\frac{1}{2b}+\frac{15\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}{4b}-\frac{5\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}{4b}-\frac{5\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^6}{4b}+\frac{15\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^8}{4b}-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{10}}{2b}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1\right)^4}-\frac{15\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{8b}$
parallelrisch	$\frac{(-60\cos(2bx+2a)-15\cos(4bx+4a)-45)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+(60\cos(2bx+2a)+15\cos(4bx+4a)+45)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right.\right.}{8b(\cos(4bx+4a)+4\cos}$

input
```
int(csc(b*x+a)^2*sec(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/4/sin(b*x+a)/cos(b*x+a)^4+5/8/sin(b*x+a)/cos(b*x+a)^2-15/8/sin(b*x+
a)+15/8*ln(sec(b*x+a)+tan(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.44

$$\int \csc^2(a+bx)\sec^5(a+bx)\,dx$$
$$= \frac{15\cos(bx+a)^4\log(\sin(bx+a)+1)\sin(bx+a) - 15\cos(bx+a)^4\log(-\sin(bx+a)+1)\sin(bx+a}{16\,b\cos(bx+a)^4\sin(bx+a)}$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a)^5,x, algorithm="fricas")
```

output
```
1/16*(15*cos(b*x + a)^4*log(sin(b*x + a) + 1)*sin(b*x + a) - 15*cos(b*x +
a)^4*log(-sin(b*x + a) + 1)*sin(b*x + a) - 30*cos(b*x + a)^4 + 10*cos(b*x
+ a)^2 + 4)/(b*cos(b*x + a)^4*sin(b*x + a))
```

Sympy [F]

$$\int \csc^2(a+bx)\sec^5(a+bx)\,dx = \int \csc^2(a+bx)\sec^5(a+bx)\,dx$$

input
```
integrate(csc(b*x+a)**2*sec(b*x+a)**5,x)
```

output
```
Integral(csc(a + b*x)**2*sec(a + b*x)**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.20

$$\int \csc^2(a+bx)\sec^5(a+bx)\,dx =$$
$$-\frac{\frac{2\left(15\sin(bx+a)^4-25\sin(bx+a)^2+8\right)}{\sin(bx+a)^5-2\sin(bx+a)^3+\sin(bx+a)} - 15\log(\sin(bx+a)+1) + 15\log(\sin(bx+a)-1)}{16\,b}$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a)^5,x, algorithm="maxima")
```

output
```
-1/16*(2*(15*sin(b*x + a)^4 - 25*sin(b*x + a)^2 + 8)/(sin(b*x + a)^5 - 2*s
in(b*x + a)^3 + sin(b*x + a)) - 15*log(sin(b*x + a) + 1) + 15*log(sin(b*x
+ a) - 1))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.21

$$\int \csc^2(a+bx)\sec^5(a+bx)\,dx = \frac{15\,\log\left(\left|\sin\left(bx+a\right)+1\right|\right)}{16\,b} - \frac{15\,\log\left(\left|\sin\left(bx+a\right)-1\right|\right)}{16\,b} - \frac{7\,\sin\left(bx+a\right)^3 - 9\,\sin\left(bx+a\right)}{8\,\left(\sin\left(bx+a\right)^2-1\right)^2 b} - \frac{1}{b\sin\left(bx+a\right)}$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a)^5,x, algorithm="giac")
```

output
```
15/16*log(abs(sin(b*x + a) + 1))/b - 15/16*log(abs(sin(b*x + a) - 1))/b -
1/8*(7*sin(b*x + a)^3 - 9*sin(b*x + a))/((sin(b*x + a)^2 - 1)^2*b) - 1/(b*
sin(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 25.84 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

$$\int \csc^2(a+bx)\sec^5(a+bx)\,dx = \frac{15\,\mathrm{atanh}(\sin\left(a+b\,x\right))}{8\,b} - \frac{\frac{15\sin(a+b\,x)^4}{8} - \frac{25\sin(a+b\,x)^2}{8} + 1}{b\,\left(\sin\left(a+b\,x\right)^5 - 2\sin\left(a+b\,x\right)^3 + \sin\left(a+b\,x\right)\right)}$$

input
```
int(1/(cos(a + b*x)^5*sin(a + b*x)^2),x)
```

output
```
(15*atanh(sin(a + b*x)))/(8*b) - ((15*sin(a + b*x)^4)/8 - (25*sin(a + b*x)
^2)/8 + 1)/(b*(sin(a + b*x) - 2*sin(a + b*x)^3 + sin(a + b*x)^5))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.80

$$\int \csc^2(a+bx)\sec^5(a+bx)\,dx$$

$$= \frac{-15\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin(bx+a)^5+30\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin(bx+a)^3-15\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right.}{}$$

input
```
int(csc(b*x+a)^2*sec(b*x+a)^5,x)
```

output
```
( - 15*log(tan((a + b*x)/2) - 1)*sin(a + b*x)**5 + 30*log(tan((a + b*x)/2)
 - 1)*sin(a + b*x)**3 - 15*log(tan((a + b*x)/2) - 1)*sin(a + b*x) + 15*log
(tan((a + b*x)/2) + 1)*sin(a + b*x)**5 - 30*log(tan((a + b*x)/2) + 1)*sin(
a + b*x)**3 + 15*log(tan((a + b*x)/2) + 1)*sin(a + b*x) - 15*sin(a + b*x)*
*4 + 25*sin(a + b*x)**2 - 8)/(8*sin(a + b*x)*b*(sin(a + b*x)**4 - 2*sin(a
+ b*x)**2 + 1))
```

3.145 $\int \cos^4(a+bx)\cot^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 58

$$\int \cos^4(a+bx)\cot^3(a+bx)\,dx = -\frac{\csc^2(a+bx)}{2b} - \frac{3\log(\sin(a+bx))}{b} + \frac{3\sin^2(a+bx)}{2b} - \frac{\sin^4(a+bx)}{4b}$$

output
```
-1/2*csc(b*x+a)^2/b-3*ln(sin(b*x+a))/b+3/2*sin(b*x+a)^2/b-1/4*sin(b*x+a)^4
/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \cos^4(a+bx)\cot^3(a+bx)\,dx = -\frac{\csc^2(a+bx)}{2b} - \frac{3\log(\sin(a+bx))}{b} + \frac{3\sin^2(a+bx)}{2b} - \frac{\sin^4(a+bx)}{4b}$$

input
```
Integrate[Cos[a + b*x]^4*Cot[a + b*x]^3,x]
```

output
```
-1/2*Csc[a + b*x]^2/b - (3*Log[Sin[a + b*x]])/b + (3*Sin[a + b*x]^2)/(2*b)
 - Sin[a + b*x]^4/(4*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.78, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3042, 25, 3070, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(a+bx)\cot^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\sin\left(a+bx+\frac{\pi}{2}\right)^4\tan\left(a+bx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sin\left(\frac{1}{2}(2a+\pi)+bx\right)^4\tan\left(\frac{1}{2}(2a+\pi)+bx\right)^3dx$$

$$\Big\downarrow \text{3070}$$

$$\frac{\int -\csc^3(a+bx)\left(1-\sin^2(a+bx)\right)^3d(-\sin(a+bx))}{b}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int \csc^2(a+bx)(\sin(a+bx)+1)^3d\sin^2(a+bx)}{2b}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\csc^2(a+bx)+3\csc(a+bx)+\sin(a+bx)+3\right)d\sin^2(a+bx)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{2}\sin^2(a+bx)-3\sin(a+bx)+\csc(a+bx)-3\log\left(\sin^2(a+bx)\right)}{2b}$$

input
```
Int[Cos[a + b*x]^4*Cot[a + b*x]^3,x]
```

output
```
(Csc[a + b*x] - 3*Log[Sin[a + b*x]^2] - 3*Sin[a + b*x] - Sin[a + b*x]^2/2)
/(2*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Maple [A] (verified)

Time = 4.99 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{-\frac{\cos(bx+a)^8}{2\sin(bx+a)^2}-\frac{\cos(bx+a)^6}{2}-\frac{3\cos(bx+a)^4}{4}-\frac{3\cos(bx+a)^2}{2}-3\ln(\sin(bx+a))}{b}$	63
default	$\frac{-\frac{\cos(bx+a)^8}{2\sin(bx+a)^2}-\frac{\cos(bx+a)^6}{2}-\frac{3\cos(bx+a)^4}{4}-\frac{3\cos(bx+a)^2}{2}-3\ln(\sin(bx+a))}{b}$	63
risch	$3ix-\frac{5\,\mathrm{e}^{2i(bx+a)}}{16b}-\frac{5\,\mathrm{e}^{-2i(bx+a)}}{16b}+\frac{6ia}{b}+\frac{2\,\mathrm{e}^{2i(bx+a)}}{b\left(\mathrm{e}^{2i(bx+a)}-1\right)^2}-\frac{3\ln\left(\mathrm{e}^{2i(bx+a)}-1\right)}{b}-\frac{\cos(4bx+4a)}{32b}$	99

input
```
int(cos(b*x+a)^4*cot(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/2/sin(b*x+a)^2*cos(b*x+a)^8-1/2*cos(b*x+a)^6-3/4*cos(b*x+a)^4-3/2*
cos(b*x+a)^2-3*ln(sin(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.22

$$\int \cos^4(a+bx)\cot^3(a+bx)\,dx = -\frac{8\,\cos\left(bx+a\right)^6+24\,\cos\left(bx+a\right)^4-51\,\cos\left(bx+a\right)^2+96\left(\cos\left(bx+a\right)^2-1\right)\log\left(\frac{1}{2}\,\sin\left(bx+a\right)\right)+3}{32\left(b\cos\left(bx+a\right)^2-b\right)}$$

input
```
integrate(cos(b*x+a)^4*cot(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/32*(8*cos(b*x + a)^6 + 24*cos(b*x + a)^4 - 51*cos(b*x + a)^2 + 96*(cos(
b*x + a)^2 - 1)*log(1/2*sin(b*x + a)) + 3)/(b*cos(b*x + a)^2 - b)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^4(a+bx)\cot^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**4*cot(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.78

$$\int \cos^4(a+bx)\cot^3(a+bx)\,dx = -\frac{\sin(bx+a)^4 - 6\sin(bx+a)^2 + \frac{2}{\sin(bx+a)^2} + 6\log\left(\sin(bx+a)^2\right)}{4b}$$

input
```
integrate(cos(b*x+a)^4*cot(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/4*(sin(b*x + a)^4 - 6*sin(b*x + a)^2 + 2/sin(b*x + a)^2 + 6*log(sin(b*x
 + a)^2))/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.07

$$\int \cos^4(a+bx)\cot^3(a+bx)\,dx = -\frac{3\log\left(\left|\cos(bx+a)^2-1\right|\right)}{2b} - \frac{b\cos(bx+a)^4 + 4b\cos(bx+a)^2}{4b^2} + \frac{1}{2\left(\cos(bx+a)^2-1\right)b}$$

input
```
integrate(cos(b*x+a)^4*cot(b*x+a)^3,x, algorithm="giac")
```

output
```
-3/2*log(abs(cos(b*x + a)^2 - 1))/b - 1/4*(b*cos(b*x + a)^4 + 4*b*cos(b*x
+ a)^2)/b^2 + 1/2/((cos(b*x + a)^2 - 1)*b)
```

Mupad [B] (verification not implemented)

Time = 25.95 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.47

$$\int \cos^4(a+bx)\cot^3(a+bx)\,dx = \frac{3\ln\left(\tan(a+b\,x)^2+1\right)}{2\,b} - \frac{3\ln\left(\tan(a+b\,x)\right)}{b} - \frac{\frac{3\tan(a+b\,x)^4}{2}+\frac{9\tan(a+b\,x)^2}{4}+\frac{1}{2}}{b\,\left(\tan\left(a+b\,x\right)^6+2\tan\left(a+b\,x\right)^4+\tan\left(a+b\,x\right)^2\right)}$$

input
```
int(cos(a + b*x)^4*cot(a + b*x)^3,x)
```

output
```
(3*log(tan(a + b*x)^2 + 1))/(2*b) - (3*log(tan(a + b*x)))/b - ((9*tan(a +
b*x)^2)/4 + (3*tan(a + b*x)^4)/2 + 1/2)/(b*(tan(a + b*x)^2 + 2*tan(a + b*x
)^4 + tan(a + b*x)^6))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.53

$$\int \cos^4(a+bx)\cot^3(a+bx)\,dx$$
$$= \frac{12\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2+1\right)\sin\left(bx+a\right)^2 - 12\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\sin\left(bx+a\right)^2 - \sin\left(bx+a\right)^6 + 6\sin\left(bx-\right.}{4\sin\left(bx+a\right)^2 b}$$

input
```
int(cos(b*x+a)^4*cot(b*x+a)^3,x)
```

output
```
(12*log(tan((a + b*x)/2)**2 + 1)*sin(a + b*x)**2 - 12*log(tan((a + b*x)/2)
)*sin(a + b*x)**2 - sin(a + b*x)**6 + 6*sin(a + b*x)**4 + 3*sin(a + b*x)**
2 - 2)/(4*sin(a + b*x)**2*b)
```

3.146 $\int \cos^3(a+bx)\cot^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 60

$$\int \cos^3(a+bx)\cot^3(a+bx)\,dx = \frac{5\text{arctanh}(\cos(a+bx))}{2b} - \frac{2\cos(a+bx)}{b} - \frac{\cos^3(a+bx)}{3b} - \frac{\cot(a+bx)\csc(a+bx)}{2b}$$

output
```
5/2*arctanh(cos(b*x+a))/b-2*cos(b*x+a)/b-1/3*cos(b*x+a)^3/b-1/2*cot(b*x+a)
*csc(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.72

$$\int \cos^3(a+bx)\cot^3(a+bx)\,dx = -\frac{9\cos(a+bx)}{4b} - \frac{\cos(3(a+bx))}{12b} - \frac{\csc^2\left(\frac{1}{2}(a+bx)\right)}{8b} + \frac{5\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{2b} - \frac{5\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{2b} + \frac{\sec^2\left(\frac{1}{2}(a+bx)\right)}{8b}$$

input
```
Integrate[Cos[a + b*x]^3*Cot[a + b*x]^3,x]
```

output
```
(-9*Cos[a + b*x])/(4*b) - Cos[3*(a + b*x)]/(12*b) - Csc[(a + b*x)/2]^2/(8*
b) + (5*Log[Cos[(a + b*x)/2]])/(2*b) - (5*Log[Sin[(a + b*x)/2]])/(2*b) + S
ec[(a + b*x)/2]^2/(8*b)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3042, 25, 3072, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)\cot^3(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\sin\left(a+bx+\frac{\pi}{2}\right)^3\tan\left(a+bx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \; 25$$

$$-\int \sin\left(\frac{1}{2}(2a+\pi)+bx\right)^3\tan\left(\frac{1}{2}(2a+\pi)+bx\right)^3dx$$

$$\Big\downarrow \; 3072$$

$$-\frac{\int \frac{\cos^6(a+bx)}{(1-\cos^2(a+bx))^2}d\cos(a+bx)}{b}$$

$$\Big\downarrow \; 252$$

$$-\frac{\frac{\cos^5(a+bx)}{2(1-\cos^2(a+bx))}-\frac{5}{2}\int \frac{\cos^4(a+bx)}{1-\cos^2(a+bx)}d\cos(a+bx)}{b}$$

$$\Big\downarrow \; 254$$

$$-\frac{\frac{\cos^5(a+bx)}{2(1-\cos^2(a+bx))}-\frac{5}{2}\int \left(-\cos^2(a+bx)+\frac{1}{1-\cos^2(a+bx)}-1\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\frac{\cos^5(a+bx)}{2(1-\cos^2(a+bx))}-\frac{5}{2}\left(\text{arctanh}(\cos(a+bx))-\frac{1}{3}\cos^3(a+bx)-\cos(a+bx)\right)}{b}$$

input
```
Int[Cos[a + b*x]^3*Cot[a + b*x]^3,x]
```

output
```
-((Cos[a + b*x]^5/(2*(1 - Cos[a + b*x]^2)) - (5*(ArcTanh[Cos[a + b*x]] - C
os[a + b*x] - Cos[a + b*x]^3/3))/2)/b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Maple [A] (verified)

Time = 2.89 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{-\frac{\cos(bx+a)^7}{2\sin(bx+a)^2}-\frac{\cos(bx+a)^5}{2}-\frac{5\cos(bx+a)^3}{6}-\frac{5\cos(bx+a)}{2}-\frac{5\ln(\csc(bx+a)-\cot(bx+a))}{2}}{b}$
default	$\frac{-\frac{\cos(bx+a)^7}{2\sin(bx+a)^2}-\frac{\cos(bx+a)^5}{2}-\frac{5\cos(bx+a)^3}{6}-\frac{5\cos(bx+a)}{2}-\frac{5\ln(\csc(bx+a)-\cot(bx+a))}{2}}{b}$
risch	$-\frac{e^{3i(bx+a)}}{24b}-\frac{9\,e^{i(bx+a)}}{8b}-\frac{9\,e^{-i(bx+a)}}{8b}-\frac{e^{-3i(bx+a)}}{24b}+\frac{e^{3i(bx+a)}+e^{i(bx+a)}}{b\left(e^{2i(bx+a)}-1\right)^2}-\frac{5\ln\left(e^{i(bx+a)}-1\right)}{2b}+\frac{5\ln\left(e^{i(b\cdots}\right.}{2\cdots}$

input
```
int(cos(b*x+a)^3*cot(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/2/sin(b*x+a)^2*cos(b*x+a)^7-1/2*cos(b*x+a)^5-5/6*cos(b*x+a)^3-5/2*
cos(b*x+a)-5/2*ln(csc(b*x+a)-cot(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.55

$$\int \cos^3(a+bx)\cot^3(a+bx)\,dx =$$
$$-\frac{4\cos(bx+a)^5+20\cos(bx+a)^3-15\left(\cos(bx+a)^2-1\right)\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)+15\left(\cos(bx+a\cdots\right.}{12\left(b\cos(bx+a)^2-b\right)}$$

input
```
integrate(cos(b*x+a)^3*cot(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/12*(4*cos(b*x + a)^5 + 20*cos(b*x + a)^3 - 15*(cos(b*x + a)^2 - 1)*log(
1/2*cos(b*x + a) + 1/2) + 15*(cos(b*x + a)^2 - 1)*log(-1/2*cos(b*x + a) +
1/2) - 30*cos(b*x + a))/(b*cos(b*x + a)^2 - b)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\cot^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3*cot(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.10

$$\int \cos^3(a+bx)\cot^3(a+bx)\,dx = -\frac{4\,\cos\left(bx+a\right)^3 - \frac{6\,\cos(bx+a)}{\cos(bx+a)^2-1} + 24\,\cos\left(bx+a\right) - 15\,\log\left(\cos\left(bx+a\right)+1\right) + 15\,\log\left(\cos\left(bx+a\right)-1\right)}{12\,b}$$

input
```
integrate(cos(b*x+a)^3*cot(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/12*(4*cos(b*x + a)^3 - 6*cos(b*x + a)/(cos(b*x + a)^2 - 1) + 24*cos(b*x
 + a) - 15*log(cos(b*x + a) + 1) + 15*log(cos(b*x + a) - 1))/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.38

$$\int \cos^3(a+bx)\cot^3(a+bx)\,dx = \frac{5\,\log\left(\left|\cos\left(bx+a\right)+1\right|\right)}{4\,b} - \frac{5\,\log\left(\left|\cos\left(bx+a\right)-1\right|\right)}{4\,b} + \frac{\cos\left(bx+a\right)}{2\,\left(\cos\left(bx+a\right)^2-1\right)b} - \frac{b^2\cos\left(bx+a\right)^3 + 6\,b^2\cos\left(bx+a\right)}{3\,b^3}$$

input
```
integrate(cos(b*x+a)^3*cot(b*x+a)^3,x, algorithm="giac")
```

output
```
5/4*log(abs(cos(b*x + a) + 1))/b - 5/4*log(abs(cos(b*x + a) - 1))/b + 1/2*
cos(b*x + a)/((cos(b*x + a)^2 - 1)*b) - 1/3*(b^2*cos(b*x + a)^3 + 6*b^2*co
s(b*x + a))/b^3
```

Mupad [B] (verification not implemented)

Time = 26.64 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.15

$$\int \cos^3(a+bx)\cot^3(a+bx)\,dx = \frac{\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2}{8\,b}-\frac{5\,\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{2\,b} - \frac{\frac{49\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^6}{8}+\frac{67\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4}{8}+\frac{121\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2}{24}+\frac{1}{8}}{b\,\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^8+3\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^6+3\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4+\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2\right)}$$

input
```
int(cos(a + b*x)^3*cot(a + b*x)^3,x)
```

output
```
tan(a/2 + (b*x)/2)^2/(8*b) - (5*log(tan(a/2 + (b*x)/2)))/(2*b) - ((121*tan
(a/2 + (b*x)/2)^2)/24 + (67*tan(a/2 + (b*x)/2)^4)/8 + (49*tan(a/2 + (b*x)/
2)^6)/8 + 1/8)/(b*(tan(a/2 + (b*x)/2)^2 + 3*tan(a/2 + (b*x)/2)^4 + 3*tan(a
/2 + (b*x)/2)^6 + tan(a/2 + (b*x)/2)^8))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.40

$$\int \cos^3(a+bx)\cot^3(a+bx)\,dx = \frac{8\cos(bx+a)\sin(bx+a)^4-56\cos(bx+a)\sin(bx+a)^2-12\cos(bx+a)-60\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\sin(b}{24\sin(bx+a)^2 b}$$

input
```
int(cos(b*x+a)^3*cot(b*x+a)^3,x)
```

output
```
(8*cos(a + b*x)*sin(a + b*x)**4 - 56*cos(a + b*x)*sin(a + b*x)**2 - 12*cos
(a + b*x) - 60*log(tan((a + b*x)/2))*sin(a + b*x)**2 + 65*sin(a + b*x)**2)
/(24*sin(a + b*x)**2*b)
```

3.147 $\int \cos^2(a+bx)\cot^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \cos^2(a+bx)\cot^3(a+bx)\,dx = -\frac{\csc^2(a+bx)}{2b} - \frac{2\log(\sin(a+bx))}{b} + \frac{\sin^2(a+bx)}{2b}$$

output
```
-1/2*csc(b*x+a)^2/b-2*ln(sin(b*x+a))/b+1/2*sin(b*x+a)^2/b
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \cos^2(a+bx)\cot^3(a+bx)\,dx = -\frac{\csc^2(a+bx)+4\log(\sin(a+bx))-\sin^2(a+bx)}{2b}$$

input
```
Integrate[Cos[a + b*x]^2*Cot[a + b*x]^3,x]
```

output
```
-1/2*(Csc[a + b*x]^2 + 4*Log[Sin[a + b*x]] - Sin[a + b*x]^2)/b
```

Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3042, 25, 3070, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\cot^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\sin\left(a+bx+\frac{\pi}{2}\right)^2\tan\left(a+bx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sin\left(\frac{1}{2}(2a+\pi)+bx\right)^2\tan\left(\frac{1}{2}(2a+\pi)+bx\right)^3dx$$

$$\Big\downarrow \text{3070}$$

$$\frac{\int -\csc^3(a+bx)\left(1-\sin^2(a+bx)\right)^2d(-\sin(a+bx))}{b}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int \csc^2(a+bx)(\sin(a+bx)+1)^2d\sin^2(a+bx)}{2b}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\csc^2(a+bx)+2\csc(a+bx)+1\right)d\sin^2(a+bx)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin^2(a+bx)+\csc(a+bx)-2\log\left(\sin^2(a+bx)\right)}{2b}$$

input
```
Int[Cos[a + b*x]^2*Cot[a + b*x]^3,x]
```

output
```
(Csc[a + b*x] - 2*Log[Sin[a + b*x]^2] + Sin[a + b*x]^2)/(2*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.23

method	result	size
derivativedivides	$\frac{-\frac{\cos(bx+a)^6}{2\sin(bx+a)^2}-\frac{\cos(bx+a)^4}{2}-\cos(bx+a)^2-2\ln(\sin(bx+a))}{b}$	53
default	$\frac{-\frac{\cos(bx+a)^6}{2\sin(bx+a)^2}-\frac{\cos(bx+a)^4}{2}-\cos(bx+a)^2-2\ln(\sin(bx+a))}{b}$	53
risch	$2ix-\frac{e^{2i(bx+a)}}{8b}-\frac{e^{-2i(bx+a)}}{8b}+\frac{4ia}{b}+\frac{2\,e^{2i(bx+a)}}{b\left(e^{2i(bx+a)}-1\right)^2}-\frac{2\ln\left(e^{2i(bx+a)}-1\right)}{b}$	85

input
```
int(cos(b*x+a)^2*cot(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/2/sin(b*x+a)^2*cos(b*x+a)^6-1/2*cos(b*x+a)^4-cos(b*x+a)^2-2*ln(sin
(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.42

$$\int \cos^2(a+bx)\cot^3(a+bx)\,dx$$
$$= -\frac{2\cos(bx+a)^4 - 3\cos(bx+a)^2 + 8\left(\cos(bx+a)^2 - 1\right)\log\left(\frac{1}{2}\sin(bx+a)\right) - 1}{4\left(b\cos(bx+a)^2 - b\right)}$$

input
```
integrate(cos(b*x+a)^2*cot(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/4*(2*cos(b*x + a)^4 - 3*cos(b*x + a)^2 + 8*(cos(b*x + a)^2 - 1)*log(1/2
*sin(b*x + a)) - 1)/(b*cos(b*x + a)^2 - b)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\cot^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2*cot(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \cos^2(a+bx)\cot^3(a+bx)\,dx = \frac{\sin(bx+a)^2 - \frac{1}{\sin(bx+a)^2} - 2\log\left(\sin(bx+a)^2\right)}{2b}$$

input
```
integrate(cos(b*x+a)^2*cot(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/2*(sin(b*x + a)^2 - 1/sin(b*x + a)^2 - 2*log(sin(b*x + a)^2))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

$$\int \cos^2(a+bx)\cot^3(a+bx)\,dx = -\frac{\cos(bx+a)^2}{2b} - \frac{\log\left(\left|\cos(bx+a)^2-1\right|\right)}{b} + \frac{1}{2\left(\cos(bx+a)^2-1\right)b}$$

input
```
integrate(cos(b*x+a)^2*cot(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/2*cos(b*x + a)^2/b - log(abs(cos(b*x + a)^2 - 1))/b + 1/2/((cos(b*x + a
)^2 - 1)*b)
```

Mupad [B] (verification not implemented)

Time = 25.66 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.44

$$\int \cos^2(a+bx)\cot^3(a+bx)\,dx = \frac{\ln\left(\tan(a+b\,x)^2+1\right)}{b} - \frac{2\ln\left(\tan(a+b\,x)\right)}{b} - \frac{\tan(a+b\,x)^2+\frac{1}{2}}{b\,\left(\tan\left(a+b\,x\right)^4+\tan\left(a+b\,x\right)^2\right)}$$

input
```
int(cos(a + b*x)^2*cot(a + b*x)^3,x)
```

output
```
log(tan(a + b*x)^2 + 1)/b - (2*log(tan(a + b*x)))/b - (tan(a + b*x)^2 + 1/
2)/(b*(tan(a + b*x)^2 + tan(a + b*x)^4))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.74

$$\int \cos^2(a+bx)\cot^3(a+bx)\,dx = \frac{4\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2+1\right)\sin(bx+a)^2-4\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\sin(bx+a)^2+\sin(bx+a)^4+\sin(bx+a)}{2\sin(bx+a)^2 b}$$

input
```
int(cos(b*x+a)^2*cot(b*x+a)^3,x)
```

output
```
(4*log(tan((a + b*x)/2)**2 + 1)*sin(a + b*x)**2 - 4*log(tan((a + b*x)/2))*
sin(a + b*x)**2 + sin(a + b*x)**4 + sin(a + b*x)**2 - 1)/(2*sin(a + b*x)**
2*b)
```

3.148 $\int \cos(a+bx)\cot^3(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 45

$$\int \cos(a+bx)\cot^3(a+bx)\,dx=\frac{3\text{arctanh}(\cos(a+bx))}{2b}-\frac{\cos(a+bx)}{b}-\frac{\cot(a+bx)\csc(a+bx)}{2b}$$

output
```
3/2*arctanh(cos(b*x+a))/b-cos(b*x+a)/b-1/2*cot(b*x+a)*csc(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.91

$$\int \cos(a+bx)\cot^3(a+bx)\,dx=-\frac{\cos(a+bx)}{b}-\frac{\csc^2\left(\frac{1}{2}(a+bx)\right)}{8b}+\frac{3\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{2b}-\frac{3\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{2b}+\frac{\sec^2\left(\frac{1}{2}(a+bx)\right)}{8b}$$

input
```
Integrate[Cos[a + b*x]*Cot[a + b*x]^3,x]
```

output
```
-(Cos[a + b*x]/b) - Csc[(a + b*x)/2]^2/(8*b) + (3*Log[Cos[(a + b*x)/2]])/(
2*b) - (3*Log[Sin[(a + b*x)/2]])/(2*b) + Sec[(a + b*x)/2]^2/(8*b)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 25, 3072, 252, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\cot^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\sin\left(a+bx+\frac{\pi}{2}\right)\tan\left(a+bx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sin\left(\frac{1}{2}(2a+\pi)+bx\right)\tan\left(\frac{1}{2}(2a+\pi)+bx\right)^3dx$$

$$\Big\downarrow \text{3072}$$

$$-\frac{\int \frac{\cos^4(a+bx)}{(1-\cos^2(a+bx))^2}d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{\frac{\cos^3(a+bx)}{2(1-\cos^2(a+bx))}-\frac{3}{2}\int \frac{\cos^2(a+bx)}{1-\cos^2(a+bx)}d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{262}$$

$$-\frac{\frac{\cos^3(a+bx)}{2(1-\cos^2(a+bx))}-\frac{3}{2}\left(\int \frac{1}{1-\cos^2(a+bx)}d\cos(a+bx)-\cos(a+bx)\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{\cos^3(a+bx)}{2(1-\cos^2(a+bx))}-\frac{3}{2}(\text{arctanh}(\cos(a+bx))-\cos(a+bx))}{b}$$

input
```
Int[Cos[a + b*x]*Cot[a + b*x]^3,x]
```

output
```
-(((-3*(ArcTanh[Cos[a + b*x]] - Cos[a + b*x]))/2 + Cos[a + b*x]^3/(2*(1 -
Cos[a + b*x]^2)))/b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.33

method	result	size
derivativedivides	$\frac{-\frac{\cos(bx+a)^5}{2\sin(bx+a)^2}-\frac{\cos(bx+a)^3}{2}-\frac{3\cos(bx+a)}{2}-\frac{3\ln(\csc(bx+a)-\cot(bx+a))}{2}}{b}$	60
default	$\frac{-\frac{\cos(bx+a)^5}{2\sin(bx+a)^2}-\frac{\cos(bx+a)^3}{2}-\frac{3\cos(bx+a)}{2}-\frac{3\ln(\csc(bx+a)-\cot(bx+a))}{2}}{b}$	60
risch	$-\frac{e^{i(bx+a)}}{2b}-\frac{e^{-i(bx+a)}}{2b}+\frac{e^{3i(bx+a)}+e^{i(bx+a)}}{b\left(e^{2i(bx+a)}-1\right)^2}+\frac{3\ln\left(e^{i(bx+a)}+1\right)}{2b}-\frac{3\ln\left(e^{i(bx+a)}-1\right)}{2b}$	100

input
```
int(cos(b*x+a)*cot(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/2/sin(b*x+a)^2*cos(b*x+a)^5-1/2*cos(b*x+a)^3-3/2*cos(b*x+a)-3/2*ln
(csc(b*x+a)-cot(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(41) = 82$.

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.84

$$\int \cos(a+bx)\cot^3(a+bx)\,dx = -\frac{4\cos(bx+a)^3-3\left(\cos(bx+a)^2-1\right)\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)+3\left(\cos(bx+a)^2-1\right)\log\left(-\frac{1}{2}\cos(b\right.}{4\left(b\cos(bx+a)^2-b\right)}$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/4*(4*cos(b*x + a)^3 - 3*(cos(b*x + a)^2 - 1)*log(1/2*cos(b*x + a) + 1/2
) + 3*(cos(b*x + a)^2 - 1)*log(-1/2*cos(b*x + a) + 1/2) - 6*cos(b*x + a))/
(b*cos(b*x + a)^2 - b)
```

Sympy [F]

$$\int \cos(a+bx)\cot^3(a+bx)\,dx = \int \cos\left(a+bx\right)\cot^3\left(a+bx\right)dx$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)**3,x)
```

output
```
Integral(cos(a + b*x)*cot(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.24

$$\int \cos(a+bx)\cot^3(a+bx)\,dx = \frac{\frac{2\cos(bx+a)}{\cos(bx+a)^2-1} - 4\cos\left(bx+a\right) + 3\log\left(\cos\left(bx+a\right)+1\right) - 3\log\left(\cos\left(bx+a\right)-1\right)}{4\,b}$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/4*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) - 4*cos(b*x + a) + 3*log(cos(b*x
+ a) + 1) - 3*log(cos(b*x + a) - 1))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.44

$$\int \cos(a+bx)\cot^3(a+bx)\,dx = -\frac{\cos\left(bx+a\right)}{b} + \frac{3\log\left(\left|\cos\left(bx+a\right)+1\right|\right)}{4\,b} - \frac{3\log\left(\left|\cos\left(bx+a\right)-1\right|\right)}{4\,b} + \frac{\cos\left(bx+a\right)}{2\left(\cos\left(bx+a\right)^2-1\right)b}$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)^3,x, algorithm="giac")
```

output
```
-cos(b*x + a)/b + 3/4*log(abs(cos(b*x + a) + 1))/b - 3/4*log(abs(cos(b*x +
 a) - 1))/b + 1/2*cos(b*x + a)/((cos(b*x + a)^2 - 1)*b)
```

Mupad [B] (verification not implemented)

Time = 25.62 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.71

$$\int \cos(a+bx)\cot^3(a+bx)\,dx = \frac{\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2}{8\,b} - \frac{3\,\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{2\,b} - \frac{\frac{17\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2}{8}+\frac{1}{8}}{b\,\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4+\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2\right)}$$

input
```
int(cos(a + b*x)*cot(a + b*x)^3,x)
```

output
```
tan(a/2 + (b*x)/2)^2/(8*b) - (3*log(tan(a/2 + (b*x)/2)))/(2*b) - ((17*tan(
a/2 + (b*x)/2)^2)/8 + 1/8)/(b*(tan(a/2 + (b*x)/2)^2 + tan(a/2 + (b*x)/2)^4
))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.51

$$\int \cos(a+bx)\cot^3(a+bx)\,dx = \frac{-8\cos(bx+a)\sin(bx+a)^2-4\cos(bx+a)-12\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\sin(bx+a)^2+9\sin(bx+a)^2}{8\sin(bx+a)^2 b}$$

input
```
int(cos(b*x+a)*cot(b*x+a)^3,x)
```

output
```
( - 8*cos(a + b*x)*sin(a + b*x)**2 - 4*cos(a + b*x) - 12*log(tan((a + b*x)
/2))*sin(a + b*x)**2 + 9*sin(a + b*x)**2)/(8*sin(a + b*x)**2*b)
```

3.149 $\int \cot^3(a + bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 28

$$\int \cot^3(a + bx)\,dx = -\frac{\cot^2(a + bx)}{2b} - \frac{\log(\sin(a + bx))}{b}$$

output
```
-1/2*cot(b*x+a)^2/b-ln(sin(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \cot^3(a + bx)\,dx = -\frac{\csc^2(a + bx)}{2b} - \frac{\log(\sin(a + bx))}{b}$$

input
```
Integrate[Cot[a + b*x]^3,x]
```

output
```
-1/2*Csc[a + b*x]^2/b - Log[Sin[a + b*x]]/b
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {3042, 25, 3954, 25, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\tan\left(a+bx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{25}$$

$$-\int \tan\left(\frac{1}{2}(2a+\pi)+bx\right)^3dx$$

$$\Big\downarrow \text{3954}$$

$$\int -\cot(a+bx)dx-\frac{\cot^2(a+bx)}{2b}$$

$$\Big\downarrow \text{25}$$

$$-\int \cot(a+bx)dx-\frac{\cot^2(a+bx)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$-\int -\tan\left(a+bx+\frac{\pi}{2}\right)dx-\frac{\cot^2(a+bx)}{2b}$$

$$\Big\downarrow \text{25}$$

$$\int \tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx-\frac{\cot^2(a+bx)}{2b}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{\cot^2(a+bx)}{2b}-\frac{\log(-\sin(a+bx))}{b}$$

input
```
Int[Cot[a + b*x]^3,x]
```

output
```
-1/2*Cot[a + b*x]^2/b - Log[-Sin[a + b*x]]/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{-\frac{\cot(bx+a)^2}{2}+\frac{\ln\left(\cot(bx+a)^2+1\right)}{2}}{b}$	29
default	$\frac{-\frac{\cot(bx+a)^2}{2}+\frac{\ln\left(\cot(bx+a)^2+1\right)}{2}}{b}$	29
parallelrisch	$\frac{-\cot(bx+a)^2-2\ln(\tan(bx+a))+\ln\left(\sec(bx+a)^2\right)}{2b}$	35
norman	$-\frac{1}{2b\tan(bx+a)^2}-\frac{\ln(\tan(bx+a))}{b}+\frac{\ln\left(1+\tan(bx+a)^2\right)}{2b}$	43
risch	$ix+\frac{2ia}{b}+\frac{2\,e^{2i(bx+a)}}{b\left(e^{2i(bx+a)}-1\right)^2}-\frac{\ln\left(e^{2i(bx+a)}-1\right)}{b}$	57

input
```
int(cot(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/2*cot(b*x+a)^2+1/2*ln(cot(b*x+a)^2+1))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.68

$$\int \cot^3(a+bx)\,dx = -\frac{(\cos(2\,bx+2\,a)-1)\log\left(-\frac{1}{2}\,\cos(2\,bx+2\,a)+\frac{1}{2}\right)-2}{2\,(b\cos(2\,bx+2\,a)-b)}$$

input
```
integrate(cot(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/2*((cos(2*b*x + 2*a) - 1)*log(-1/2*cos(2*b*x + 2*a) + 1/2) - 2)/(b*cos(
2*b*x + 2*a) - b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(22) = 44.

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.89

$$\int \cot^3(a+bx)\,dx = \begin{cases} \tilde{\infty}x & \text{for } a=0\wedge b=0 \\ x\cot^3(a) & \text{for } b=0 \\ \tilde{\infty}x & \text{for } a=-bx \\ \frac{\log(\tan^2(a+bx)+1)}{2b}-\frac{\log(\tan(a+bx))}{b}-\frac{1}{2b\tan^2(a+bx)} & \text{otherwise} \end{cases}$$

input
```
integrate(cot(b*x+a)**3,x)
```

output
```
Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x*cot(a)**3, Eq(b, 0)), (zoo*x, E
q(a, -b*x)), (log(tan(a + b*x)**2 + 1)/(2*b) - log(tan(a + b*x))/b - 1/(2*
b*tan(a + b*x)**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \cot^3(a+bx)\,dx = -\frac{\frac{1}{\sin(bx+a)^2} + \log\left(\sin\left(bx+a\right)^2\right)}{2\,b}$$

input
```
integrate(cot(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/2*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \cot^3(a+bx)\,dx = -\frac{\log\left(\left|\sin\left(bx+a\right)\right|\right)}{b} - \frac{1}{2\,b\sin\left(bx+a\right)^2}$$

input
```
integrate(cot(b*x+a)^3,x, algorithm="giac")
```

output
```
-log(abs(sin(b*x + a)))/b - 1/2/(b*sin(b*x + a)^2)
```

Mupad [B] (verification not implemented)

Time = 28.84 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.79

$$\int \cot^3(a+bx)\,dx = x\,1\mathrm{i} - \frac{\ln\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}-1\right)}{b} + \frac{2}{b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)} + \frac{2}{b\,\left(1+\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}-2\,\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\right)}$$

input
```
int(cot(a + b*x)^3,x)
```

output
```
x*1i - log(exp(a*2i)*exp(b*x*2i) - 1)/b + 2/(b*(exp(a*2i + b*x*2i) - 1)) +
 2/(b*(exp(a*4i + b*x*4i) - 2*exp(a*2i + b*x*2i) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.39

$$\int \cot^3(a+bx)\,dx$$
$$= \frac{4\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2+1\right)\sin(bx+a)^2-4\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\sin(bx+a)^2+\sin(bx+a)^2-2}{4\sin(bx+a)^2 b}$$

input
```
int(cot(b*x+a)^3,x)
```

output
```
(4*log(tan((a + b*x)/2)**2 + 1)*sin(a + b*x)**2 - 4*log(tan((a + b*x)/2))*
sin(a + b*x)**2 + sin(a + b*x)**2 - 2)/(4*sin(a + b*x)**2*b)
```

3.150 $\int \cot^2(a+bx)\csc(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \cot^2(a+bx)\csc(a+bx)\,dx=\frac{\operatorname{arctanh}(\cos(a+bx))}{2b}-\frac{\cot(a+bx)\csc(a+bx)}{2b}$$

output
```
1/2*arctanh(cos(b*x+a))/b-1/2*cot(b*x+a)*csc(b*x+a)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 75 vs. 2(34) = 68.

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.21

$$\int \cot^2(a+bx)\csc(a+bx)\,dx=-\frac{\csc^2\left(\frac{1}{2}(a+bx)\right)}{8b}+\frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{2b}-\frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{2b}+\frac{\sec^2\left(\frac{1}{2}(a+bx)\right)}{8b}$$

input
```
Integrate[Cot[a + b*x]^2*Csc[a + b*x],x]
```

output
```
-1/8*Csc[(a + b*x)/2]^2/b + Log[Cos[(a + b*x)/2]]/(2*b) - Log[Sin[(a + b*x
)/2]]/(2*b) + Sec[(a + b*x)/2]^2/(8*b)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 3091, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(a+bx)\csc(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(a+bx-\frac{\pi}{2}\right)^2\sec\left(a+bx-\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3091}$$

$$-\frac{1}{2}\int \csc(a+bx)dx-\frac{\cot(a+bx)\csc(a+bx)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}\int \csc(a+bx)dx-\frac{\cot(a+bx)\csc(a+bx)}{2b}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\text{arctanh}(\cos(a+bx))}{2b}-\frac{\cot(a+bx)\csc(a+bx)}{2b}$$

input
```
Int[Cot[a + b*x]^2*Csc[a + b*x],x]
```

output
```
ArcTanh[Cos[a + b*x]]/(2*b) - (Cot[a + b*x]*Csc[a + b*x])/(2*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.47

method	result	size
derivativedivides	$\frac{-\frac{\cos(bx+a)^3}{2\sin(bx+a)^2}-\frac{\cos(bx+a)}{2}-\frac{\ln(\csc(bx+a)-\cot(bx+a))}{2}}{b}$	50
default	$\frac{-\frac{\cos(bx+a)^3}{2\sin(bx+a)^2}-\frac{\cos(bx+a)}{2}-\frac{\ln(\csc(bx+a)-\cot(bx+a))}{2}}{b}$	50
risch	$\frac{e^{3i(bx+a)}+e^{i(bx+a)}}{b\left(e^{2i(bx+a)}-1\right)^2}-\frac{\ln\left(e^{i(bx+a)}-1\right)}{2b}+\frac{\ln\left(e^{i(bx+a)}+1\right)}{2b}$	72

input
```
int(cot(b*x+a)^2*csc(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/2/sin(b*x+a)^2*cos(b*x+a)^3-1/2*cos(b*x+a)-1/2*ln(csc(b*x+a)-cot(b
*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(30) = 60$.

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.12

$$\int \cot^2(a+bx)\csc(a+bx)\,dx = \frac{\left(\cos(bx+a)^2-1\right)\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)-\left(\cos(bx+a)^2-1\right)\log\left(-\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)+2\cos(bx}{4\left(b\cos(bx+a)^2-b\right)}$$

input
```
integrate(cot(b*x+a)^2*csc(b*x+a),x, algorithm="fricas")
```

output
```
1/4*((cos(b*x + a)^2 - 1)*log(1/2*cos(b*x + a) + 1/2) - (cos(b*x + a)^2 -
1)*log(-1/2*cos(b*x + a) + 1/2) + 2*cos(b*x + a))/(b*cos(b*x + a)^2 - b)
```

Sympy [F]

$$\int \cot^2(a+bx)\csc(a+bx)\,dx = \int \cot^2(a+bx)\csc(a+bx)\,dx$$

input
```
integrate(cot(b*x+a)**2*csc(b*x+a),x)
```

output
```
Integral(cot(a + b*x)**2*csc(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \cot^2(a+bx)\csc(a+bx)\,dx = \frac{\frac{2\cos(bx+a)}{\cos(bx+a)^2-1}+\log(\cos(bx+a)+1)-\log(\cos(bx+a)-1)}{4b}$$

input
```
integrate(cot(b*x+a)^2*csc(b*x+a),x, algorithm="maxima")
```

output
```
1/4*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1) - log(cos
(b*x + a) - 1))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.59

$$\int \cot^2(a+bx)\csc(a+bx)\,dx = \frac{\log\left(|\cos\left(bx+a\right)+1|\right)}{4\,b} - \frac{\log\left(|\cos\left(bx+a\right)-1|\right)}{4\,b} + \frac{\cos\left(bx+a\right)}{2\,\left(\cos\left(bx+a\right)^2-1\right)b}$$

input
```
integrate(cot(b*x+a)^2*csc(b*x+a),x, algorithm="giac")
```

output
```
1/4*log(abs(cos(b*x + a) + 1))/b - 1/4*log(abs(cos(b*x + a) - 1))/b + 1/2*
cos(b*x + a)/((cos(b*x + a)^2 - 1)*b)
```

Mupad [B] (verification not implemented)

Time = 25.55 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int \cot^2(a+bx)\csc(a+bx)\,dx = \frac{\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2}{8\,b} - \frac{1}{8\,b\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2} - \frac{\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{2\,b}$$

input
```
int(cot(a + b*x)^2/sin(a + b*x),x)
```

output
```
tan(a/2 + (b*x)/2)^2/(8*b) - 1/(8*b*tan(a/2 + (b*x)/2)^2) - log(tan(a/2 +
(b*x)/2))/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \cot^2(a+bx)\csc(a+bx)\,dx = \frac{-\cos(bx+a) - \log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\sin(bx+a)^2}{2\sin(bx+a)^2 b}$$

input
```
int(cot(b*x+a)^2*csc(b*x+a),x)
```

output
```
( - (cos(a + b*x) + log(tan((a + b*x)/2))*sin(a + b*x)**2))/(2*sin(a + b*x
)**2*b)
```

3.151 $\int \cot(a+bx)\csc^2(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \cot(a+bx)\csc^2(a+bx)\,dx = -\frac{\csc^2(a+bx)}{2b}$$

output
```
-1/2*csc(b*x+a)^2/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cot(a+bx)\csc^2(a+bx)\,dx = -\frac{\csc^2(a+bx)}{2b}$$

input
```
Integrate[Cot[a + b*x]*Csc[a + b*x]^2,x]
```

output
```
-1/2*Csc[a + b*x]^2/b
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 25, 3086, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(a+bx)\csc^2(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \tan\left(a+bx-\frac{\pi}{2}\right)\left(-\sec\left(a+bx-\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \; 25$$

$$-\int \sec\left(\frac{1}{2}(2a-\pi)+bx\right)^2\tan\left(\frac{1}{2}(2a-\pi)+bx\right)dx$$

$$\Big\downarrow \; 3086$$

$$-\frac{\int \csc(a+bx)d\csc(a+bx)}{b}$$

$$\Big\downarrow \; 15$$

$$-\frac{\csc^2(a+bx)}{2b}$$

input
```
Int[Cot[a + b*x]*Csc[a + b*x]^2,x]
```

output
```
-1/2*Csc[a + b*x]^2/b
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{\cot(bx+a)^2}{2b}$	14
default	$-\frac{\cot(bx+a)^2}{2b}$	14
risch	$\frac{2\,\mathrm{e}^{2i(bx+a)}}{b\left(\mathrm{e}^{2i(bx+a)}-1\right)^2}$	28

input
```
int(cot(b*x+a)*csc(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*cot(b*x+a)^2/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \cot(a+bx)\csc^2(a+bx)\,dx = \frac{1}{2\left(b\cos\left(bx+a\right)^2-b\right)}$$

input
```
integrate(cot(b*x+a)*csc(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2/(b*cos(b*x + a)^2 - b)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \cot(a+bx)\csc^2(a+bx)\,dx = \begin{cases} -\frac{\csc^2(a+bx)}{2b} & \text{for } b\neq 0 \\ x\cot(a)\csc^2(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cot(b*x+a)*csc(b*x+a)**2,x)
```

output
```
Piecewise((-csc(a + b*x)**2/(2*b), Ne(b, 0)), (x*cot(a)*csc(a)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cot(a+bx)\csc^2(a+bx)\,dx = -\frac{\cot\left(bx+a\right)^2}{2\,b}$$

input
```
integrate(cot(b*x+a)*csc(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*cot(b*x + a)^2/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cot(a+bx)\csc^2(a+bx)\,dx = -\frac{1}{2\,b\sin\left(bx+a\right)^2}$$

input
```
integrate(cot(b*x+a)*csc(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/2/(b*sin(b*x + a)^2)
```

Mupad [B] (verification not implemented)

Time = 25.36 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cot(a+bx)\csc^2(a+bx)\,dx = -\frac{1}{2\,b\sin\left(a+b\,x\right)^2}$$

input
```
int(cot(a + b*x)/sin(a + b*x)^2,x)
```

output
```
-1/(2*b*sin(a + b*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cot(a+bx)\csc^2(a+bx)\,dx = -\frac{\csc\left(bx+a\right)^2}{2b}$$

input
```
int(cot(b*x+a)*csc(b*x+a)^2,x)
```

output
```
( - csc(a + b*x)**2)/(2*b)
```

3.152 $\int \csc^3(a+bx)\sec(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \csc^3(a+bx)\sec(a+bx)\,dx=-\frac{\cot^2(a+bx)}{2b}+\frac{\log(\tan(a+bx))}{b}$$

output
```
-1/2*cot(b*x+a)^2/b+ln(tan(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int \csc^3(a+bx)\sec(a+bx)\,dx=-\frac{\csc^2(a+bx)}{2b}-\frac{\log(\cos(a+bx))}{b}+\frac{\log(\sin(a+bx))}{b}$$

input
```
Integrate[Csc[a + b*x]^3*Sec[a + b*x],x]
```

output
```
-1/2*Csc[a + b*x]^2/b - Log[Cos[a + b*x]]/b + Log[Sin[a + b*x]]/b
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(a+bx)\sec(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(a+bx)^3\sec(a+bx)dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{\int \cot^3(a+bx)\left(\tan^2(a+bx)+1\right)d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\cot^3(a+bx)+\cot(a+bx)\right)d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log(\tan(a+bx))-\frac{1}{2}\cot^2(a+bx)}{b}$$

input
```
Int[Csc[a + b*x]^3*Sec[a + b*x],x]
```

output
```
(-1/2*Cot[a + b*x]^2 + Log[Tan[a + b*x]])/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{-\frac{1}{2 \sin(bx+a)^2}+\ln(\tan(bx+a))}{b}$	23
default	$\frac{-\frac{1}{2 \sin(bx+a)^2}+\ln(\tan(bx+a))}{b}$	23
risch	$\frac{2 \,\mathrm{e}^{2i(bx+a)}}{b\left(\mathrm{e}^{2i(bx+a)}-1\right)^2}+\frac{\ln\left(\mathrm{e}^{2i(bx+a)}-1\right)}{b}-\frac{\ln\left(\mathrm{e}^{2i(bx+a)}+1\right)}{b}$	62
parallelrisch	$\frac{-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2-\cot\left(\frac{bx}{2}+\frac{a}{2}\right)^2+8\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-8\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)-8\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{8b}$	73
norman	$\frac{-\frac{1}{8b}-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}{8b}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}+\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{b}-\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{b}$	84

input
```
int(csc(b*x+a)^3*sec(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/2/sin(b*x+a)^2+ln(tan(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(25) = 50$.

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.41

$$\int \csc^3(a+bx)\sec(a+bx)\,dx = -\frac{\left(\cos\left(bx+a\right)^2-1\right)\log\left(\cos\left(bx+a\right)^2\right)-\left(\cos\left(bx+a\right)^2-1\right)\log\left(-\frac{1}{4}\cos\left(bx+a\right)^2+\frac{1}{4}\right)-1}{2\left(b\cos\left(bx+a\right)^2-b\right)}$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*((cos(b*x + a)^2 - 1)*log(cos(b*x + a)^2) - (cos(b*x + a)^2 - 1)*log(
-1/4*cos(b*x + a)^2 + 1/4) - 1)/(b*cos(b*x + a)^2 - b)
```

Sympy [F]

$$\int \csc^3(a+bx)\sec(a+bx)\,dx = \int \csc^3\left(a+bx\right)\sec\left(a+bx\right)\,dx$$

input
```
integrate(csc(b*x+a)**3*sec(b*x+a),x)
```

output
```
Integral(csc(a + b*x)**3*sec(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

$$\int \csc^3(a+bx)\sec(a+bx)\,dx = -\frac{\frac{1}{\sin(bx+a)^2}+\log\left(\sin\left(bx+a\right)^2-1\right)-\log\left(\sin\left(bx+a\right)^2\right)}{2\,b}$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.59

$$\int \csc^3(a+bx)\sec(a+bx)\,dx = -\frac{\log\left(\left|\sin\left(bx+a\right)^2-1\right|\right)}{2\,b} + \frac{\log\left(\left|\sin\left(bx+a\right)\right|\right)}{b} - \frac{1}{2\,b\sin\left(bx+a\right)^2}$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a),x, algorithm="giac")
```

output
```
-1/2*log(abs(sin(b*x + a)^2 - 1))/b + log(abs(sin(b*x + a)))/b - 1/2/(b*si
n(b*x + a)^2)
```

Mupad [B] (verification not implemented)

Time = 25.36 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \csc^3(a+bx)\sec(a+bx)\,dx = -\frac{\ln\left(\cos\left(a+b\,x\right)\right)-\frac{\ln\left(\sin(a+b\,x)^2\right)}{2}+\frac{1}{2\,\sin(a+b\,x)^2}}{b}$$

input
```
int(1/(cos(a + b*x)*sin(a + b*x)^3),x)
```

output
```
-(log(cos(a + b*x)) - log(sin(a + b*x)^2)/2 + 1/(2*sin(a + b*x)^2))/b
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.22

$$\int \csc^3(a+bx)\sec(a+bx)\,dx$$
$$= \frac{-4\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin(bx+a)^2-4\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)\sin(bx+a)^2+4\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\sin}{4\sin(bx+a)^2 b}$$

input
```
int(csc(b*x+a)^3*sec(b*x+a),x)
```

output
```
( - 4*log(tan((a + b*x)/2) - 1)*sin(a + b*x)**2 - 4*log(tan((a + b*x)/2) +
 1)*sin(a + b*x)**2 + 4*log(tan((a + b*x)/2))*sin(a + b*x)**2 + sin(a + b*
x)**2 - 2)/(4*sin(a + b*x)**2*b)
```

3.153 $\int \csc^3(a+bx)\sec^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 44

$$\int \csc^3(a+bx)\sec^2(a+bx)\,dx = -\frac{3\text{arctanh}(\cos(a+bx))}{2b} - \frac{\cot(a+bx)\csc(a+bx)}{2b} + \frac{\sec(a+bx)}{b}$$

output
```
-3/2*arctanh(cos(b*x+a))/b-1/2*cot(b*x+a)*csc(b*x+a)/b+sec(b*x+a)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 143 vs. 2(44) = 88.

Time = 0.44 (sec) , antiderivative size = 143, normalized size of antiderivative = 3.25

$$\int \csc^3(a+bx)\sec^2(a+bx)\,dx = \frac{\csc^4(a+bx)\left(2-6\cos(2(a+bx))+2\cos(3(a+bx))+3\cos(3(a+bx))\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)-3\cos(3\cdots\right.}{2b\left(\csc^2\left(\frac{1}{2}(a+bx)\right)\cdots\right.}$$

input
```
Integrate[Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

output
```
(Csc[a + b*x]^4*(2 - 6*Cos[2*(a + b*x)] + 2*Cos[3*(a + b*x)] + 3*Cos[3*(a
+ b*x)]*Log[Cos[(a + b*x)/2]] - 3*Cos[3*(a + b*x)]*Log[Sin[(a + b*x)/2]] +
 Cos[a + b*x]*(-2 - 3*Log[Cos[(a + b*x)/2]] + 3*Log[Sin[(a + b*x)/2]])))/(
2*b*(Csc[(a + b*x)/2]^2 - Sec[(a + b*x)/2]^2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3042, 3102, 252, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(a+bx)\sec^2(a+bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \csc(a+bx)^3\sec(a+bx)^2dx$$

$$\Big\downarrow \ 3102$$

$$\frac{\int \frac{\sec^4(a+bx)}{(1-\sec^2(a+bx))^2}d\sec(a+bx)}{b}$$

$$\Big\downarrow \ 252$$

$$\frac{\frac{\sec^3(a+bx)}{2(1-\sec^2(a+bx))}-\frac{3}{2}\int \frac{\sec^2(a+bx)}{1-\sec^2(a+bx)}d\sec(a+bx)}{b}$$

$$\Big\downarrow \ 262$$

$$\frac{\frac{\sec^3(a+bx)}{2(1-\sec^2(a+bx))}-\frac{3}{2}\left(\int \frac{1}{1-\sec^2(a+bx)}d\sec(a+bx)-\sec(a+bx)\right)}{b}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{\sec^3(a+bx)}{2(1-\sec^2(a+bx))}-\frac{3}{2}(\text{arctanh}(\sec(a+bx))-\sec(a+bx))}{b}$$

input
```
Int[Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

output
```
((-3*(ArcTanh[Sec[a + b*x]] - Sec[a + b*x]))/2 + Sec[a + b*x]^3/(2*(1 - Se
c[a + b*x]^2)))/b
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.18

method	result	size
derivativedivides	$\frac{-\frac{1}{2\sin(bx+a)^2\cos(bx+a)}+\frac{3}{2\cos(bx+a)}+\frac{3\ln(\csc(bx+a)-\cot(bx+a))}{2}}{b}$	52
default	$\frac{-\frac{1}{2\sin(bx+a)^2\cos(bx+a)}+\frac{3}{2\cos(bx+a)}+\frac{3\ln(\csc(bx+a)-\cot(bx+a))}{2}}{b}$	52
parallelrisch	$\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4+12\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2+\cot\left(\frac{bx}{2}+\frac{a}{2}\right)^2-12\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-18}{8b\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2-8b}$	81
norman	$\frac{\frac{1}{8b}+\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^6}{8b}-\frac{9\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}{4b}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1\right)}+\frac{3\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2b}$	82
risch	$\frac{3\,e^{5i(bx+a)}-2\,e^{3i(bx+a)}+3\,e^{i(bx+a)}}{b\left(e^{2i(bx+a)}-1\right)^2\left(e^{2i(bx+a)}+1\right)}-\frac{3\ln\left(e^{i(bx+a)}+1\right)}{2b}+\frac{3\ln\left(e^{i(bx+a)}-1\right)}{2b}$	100

input

```
int(csc(b*x+a)^3*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output

```
1/b*(-1/2/sin(b*x+a)^2/cos(b*x+a)+3/2/cos(b*x+a)+3/2*ln(csc(b*x+a)-cot(b*x
+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(40) = 80$.

Time = 0.10 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.18

$$\int \csc^3(a+bx)\sec^2(a+bx)\,dx$$
$$= \frac{6\cos(bx+a)^2-3\left(\cos(bx+a)^3-\cos(bx+a)\right)\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)+3\left(\cos(bx+a)^3-\cos(bx+a)\right)}{4\left(b\cos(bx+a)^3-b\cos(bx+a)\right)}$$

input

```
integrate(csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/4*(6*cos(b*x + a)^2 - 3*(cos(b*x + a)^3 - cos(b*x + a))*log(1/2*cos(b*x
+ a) + 1/2) + 3*(cos(b*x + a)^3 - cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/
2) - 4)/(b*cos(b*x + a)^3 - b*cos(b*x + a))
```

Sympy [F]

$$\int \csc^3(a+bx)\sec^2(a+bx)\,dx = \int \csc^3{(a+bx)}\sec^2{(a+bx)}\,dx$$

input
```
integrate(csc(b*x+a)**3*sec(b*x+a)**2,x)
```

output
```
Integral(csc(a + b*x)**3*sec(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.39

$$\int \csc^3(a+bx)\sec^2(a+bx)\,dx = \frac{\frac{2\left(3\cos(bx+a)^2-2\right)}{\cos(bx+a)^3-\cos(bx+a)} - 3\log(\cos(bx+a)+1) + 3\log(\cos(bx+a)-1)}{4b}$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/4*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log(cos(
b*x + a) + 1) + 3*log(cos(b*x + a) - 1))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.52

$$\int \csc^3(a+bx)\sec^2(a+bx)\,dx = -\frac{3\,\log\left(|\cos\left(bx+a\right)+1|\right)}{4\,b} + \frac{3\,\log\left(|\cos\left(bx+a\right)-1|\right)}{4\,b} + \frac{3\,\cos\left(bx+a\right)^2-2}{2\,\left(\cos\left(bx+a\right)^3-\cos\left(bx+a\right)\right)b}$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
-3/4*log(abs(cos(b*x + a) + 1))/b + 3/4*log(abs(cos(b*x + a) - 1))/b + 1/2
*(3*cos(b*x + a)^2 - 2)/((cos(b*x + a)^3 - cos(b*x + a))*b)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int \csc^3(a+bx)\sec^2(a+bx)\,dx = -\frac{3\,\mathrm{atanh}(\cos\left(a+b\,x\right))}{2\,b} - \frac{\frac{3\cos\left(a+b\,x\right)^2}{2}-1}{b\,\left(\cos\left(a+b\,x\right)-\cos\left(a+b\,x\right)^3\right)}$$

input
```
int(1/(cos(a + b*x)^2*sin(a + b*x)^3),x)
```

output
```
- (3*atanh(cos(a + b*x)))/(2*b) - ((3*cos(a + b*x)^2)/2 - 1)/(b*(cos(a + b
*x) - cos(a + b*x)^3))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.70

$$\int \csc^3(a+bx)\sec^2(a+bx)\,dx = \frac{12\cos(bx+a)\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\sin(bx+a)^2-9\cos(bx+a)\sin(bx+a)^2+12\sin(bx+a)^2-4}{8\cos(bx+a)\sin(bx+a)^2 b}$$

input
```
int(csc(b*x+a)^3*sec(b*x+a)^2,x)
```

output
```
(12*cos(a + b*x)*log(tan((a + b*x)/2))*sin(a + b*x)**2 - 9*cos(a + b*x)*si
n(a + b*x)**2 + 12*sin(a + b*x)**2 - 4)/(8*cos(a + b*x)*sin(a + b*x)**2*b)
```

3.154 $\int \csc^3(a+bx)\sec^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \csc^3(a+bx)\sec^3(a+bx)\,dx = -\frac{\cot^2(a+bx)}{2b} + \frac{2\log(\tan(a+bx))}{b} + \frac{\tan^2(a+bx)}{2b}$$

output
```
-1/2*cot(b*x+a)^2/b+2*ln(tan(b*x+a))/b+1/2*tan(b*x+a)^2/b
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.42

$$\int \csc^3(a+bx)\sec^3(a+bx)\,dx = 8\left(-\frac{\csc^2(a+bx)}{16b} - \frac{\log(\cos(a+bx))}{4b} + \frac{\log(\sin(a+bx))}{4b} + \frac{\sec^2(a+bx)}{16b}\right)$$

input
```
Integrate[Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

output
```
8*(-1/16*Csc[a + b*x]^2/b - Log[Cos[a + b*x]]/(4*b) + Log[Sin[a + b*x]]/(4
*b) + Sec[a + b*x]^2/(16*b))
```

Rubi [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3042, 3100, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(a+bx)\sec^3(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc(a+bx)^3\sec(a+bx)^3dx$$

$$\Big\downarrow \; 3100$$

$$\frac{\int \cot^3(a+bx)\left(\tan^2(a+bx)+1\right)^2d\tan(a+bx)}{b}$$

$$\Big\downarrow \; 243$$

$$\frac{\int \cot^2(a+bx)\left(\tan^2(a+bx)+1\right)^2d\tan^2(a+bx)}{2b}$$

$$\Big\downarrow \; 49$$

$$\frac{\int \left(\cot^2(a+bx)+2\cot(a+bx)+1\right)d\tan^2(a+bx)}{2b}$$

$$\Big\downarrow \; 2009$$

$$\frac{\tan^2(a+bx)-\cot(a+bx)+2\log\left(\tan^2(a+bx)\right)}{2b}$$

input
```
Int[Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

output
```
(-Cot[a + b*x] + 2*Log[Tan[a + b*x]^2] + Tan[a + b*x]^2)/(2*b)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Maple [A] (verified)

Time = 2.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{\frac{1}{2\sin(bx+a)^2\cos(bx+a)^2}-\frac{1}{\sin(bx+a)^2}+2\ln(\tan(bx+a))}{b}$
default	$\frac{\frac{1}{2\sin(bx+a)^2\cos(bx+a)^2}-\frac{1}{\sin(bx+a)^2}+2\ln(\tan(bx+a))}{b}$
risch	$\frac{4\,e^{6i(bx+a)}+4\,e^{2i(bx+a)}}{b\left(e^{2i(bx+a)}+1\right)^2\left(e^{2i(bx+a)}-1\right)^2}+\frac{2\ln\left(e^{2i(bx+a)}-1\right)}{b}-\frac{2\ln\left(e^{2i(bx+a)}+1\right)}{b}$
norman	$\frac{-\frac{1}{8b}-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^8}{8b}+\frac{9\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}{4b}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1\right)^2}+\frac{2\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{2\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{b}-\frac{2\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{b}$
parallelrisch	$\frac{(-8\cos(2bx+2a)-8)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+(-8\cos(2bx+2a)-8)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)+(8\cos(2bx+2a)+8)\ln\left(\tan\left(\frac{bx}{2}\right.\right.}{4b(1+\cos(2bx+2a))}$

input
```
int(csc(b*x+a)^3*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2/sin(b*x+a)^2/cos(b*x+a)^2-1/sin(b*x+a)^2+2*ln(tan(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(39) = 78$.

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.37

$$\int \csc^3(a+bx)\sec^3(a+bx)\,dx = \frac{2\cos(bx+a)^2 - 2\left(\cos(bx+a)^4 - \cos(bx+a)^2\right)\log\left(\cos(bx+a)^2\right) + 2\left(\cos(bx+a)^4 - \cos(bx+a)^2\right)}{2\left(b\cos(bx+a)^4 - b\cos(bx+a)^2\right)}$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/2*(2*cos(b*x + a)^2 - 2*(cos(b*x + a)^4 - cos(b*x + a)^2)*log(cos(b*x +
a)^2) + 2*(cos(b*x + a)^4 - cos(b*x + a)^2)*log(-1/4*cos(b*x + a)^2 + 1/4)
 - 1)/(b*cos(b*x + a)^4 - b*cos(b*x + a)^2)
```

Sympy [F]

$$\int \csc^3(a+bx)\sec^3(a+bx)\,dx = \int \csc^3(a+bx)\sec^3(a+bx)\,dx$$

input
```
integrate(csc(b*x+a)**3*sec(b*x+a)**3,x)
```

output
```
Integral(csc(a + b*x)**3*sec(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.49

$$\int \csc^3(a+bx)\sec^3(a+bx)\,dx = -\frac{\frac{2\,\sin(bx+a)^2-1}{\sin(bx+a)^4-\sin(bx+a)^2} + 2\,\log\left(\sin\left(bx+a\right)^2-1\right) - 2\,\log\left(\sin\left(bx+a\right)^2\right)}{2\,b}$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/2*((2*sin(b*x + a)^2 - 1)/(sin(b*x + a)^4 - sin(b*x + a)^2) + 2*log(sin
(b*x + a)^2 - 1) - 2*log(sin(b*x + a)^2))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.60

$$\int \csc^3(a+bx)\sec^3(a+bx)\,dx = -\frac{\log\left(\left|\sin\left(bx+a\right)^2-1\right|\right)}{b} + \frac{2\,\log\left(\left|\sin\left(bx+a\right)\right|\right)}{b} - \frac{2\,\sin\left(bx+a\right)^2-1}{2\left(\sin\left(bx+a\right)^4-\sin\left(bx+a\right)^2\right)b}$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="giac")
```

output
```
-log(abs(sin(b*x + a)^2 - 1))/b + 2*log(abs(sin(b*x + a)))/b - 1/2*(2*sin(
b*x + a)^2 - 1)/((sin(b*x + a)^4 - sin(b*x + a)^2)*b)
```

Mupad [B] (verification not implemented)

Time = 25.37 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \csc^3(a+bx)\sec^3(a+bx)\,dx = \frac{\tan(a+b\,x)^2}{2\,b} - \frac{1}{2\,b\tan\left(a+b\,x\right)^2} + \frac{2\,\ln\left(\tan(a+b\,x)\right)}{b}$$

input
```
int(1/(cos(a + b*x)^3*sin(a + b*x)^3),x)
```

output
```
tan(a + b*x)^2/(2*b) - 1/(2*b*tan(a + b*x)^2) + (2*log(tan(a + b*x)))/b
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 175, normalized size of antiderivative = 4.07

$$\int \csc^3(a+bx)\sec^3(a+bx)\,dx$$
$$= \frac{-4\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin\left(bx+a\right)^4+4\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin\left(bx+a\right)^2-4\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{}$$

input
```
int(csc(b*x+a)^3*sec(b*x+a)^3,x)
```

output
```
( - 4*log(tan((a + b*x)/2) - 1)*sin(a + b*x)**4 + 4*log(tan((a + b*x)/2) -
 1)*sin(a + b*x)**2 - 4*log(tan((a + b*x)/2) + 1)*sin(a + b*x)**4 + 4*log(
tan((a + b*x)/2) + 1)*sin(a + b*x)**2 + 4*log(tan((a + b*x)/2))*sin(a + b*
x)**4 - 4*log(tan((a + b*x)/2))*sin(a + b*x)**2 - sin(a + b*x)**4 - sin(a
+ b*x)**2 + 1)/(2*sin(a + b*x)**2*b*(sin(a + b*x)**2 - 1))
```

3.155 $\int \csc^3(a+bx)\sec^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 60

$$\int \csc^3(a+bx)\sec^4(a+bx)\,dx = -\frac{5\text{arctanh}(\cos(a+bx))}{2b} - \frac{\cot(a+bx)\csc(a+bx)}{2b} + \frac{2\sec(a+bx)}{b} + \frac{\sec^3(a+bx)}{3b}$$

output
```
-5/2*arctanh(cos(b*x+a))/b-1/2*cot(b*x+a)*csc(b*x+a)/b+2*sec(b*x+a)/b+1/3*
sec(b*x+a)^3/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 205 vs. 2(60) = 120.

Time = 0.61 (sec) , antiderivative size = 205, normalized size of antiderivative = 3.42

$$\int \csc^3(a+bx)\sec^4(a+bx)\,dx = \frac{2\csc^8(a+bx)\,(22-40\cos(2(a+bx))+13\cos(3(a+bx))-30\cos(4(a+bx))+13\cos(5(a+bx))+15}{}$$

input
```
Integrate[Csc[a + b*x]^3*Sec[a + b*x]^4,x]
```

output
```
(2*Csc[a + b*x]^8*(22 - 40*Cos[2*(a + b*x)] + 13*Cos[3*(a + b*x)] - 30*Cos
[4*(a + b*x)] + 13*Cos[5*(a + b*x)] + 15*Cos[3*(a + b*x)]*Log[Cos[(a + b*x
)/2]] + 15*Cos[5*(a + b*x)]*Log[Cos[(a + b*x)/2]] - 15*Cos[3*(a + b*x)]*Lo
g[Sin[(a + b*x)/2]] - 15*Cos[5*(a + b*x)]*Log[Sin[(a + b*x)/2]] + Cos[a +
b*x]*(-26 - 30*Log[Cos[(a + b*x)/2]] + 30*Log[Sin[(a + b*x)/2]])))/(3*b*(C
sc[(a + b*x)/2]^2 - Sec[(a + b*x)/2]^2)^3)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3042, 3102, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(a+bx)\sec^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(a+bx)^3\sec(a+bx)^4dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{\int \frac{\sec^6(a+bx)}{\left(1-\sec^2(a+bx)\right)^2}d\sec(a+bx)}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{\sec^5(a+bx)}{2\left(1-\sec^2(a+bx)\right)}-\frac{5}{2}\int \frac{\sec^4(a+bx)}{1-\sec^2(a+bx)}d\sec(a+bx)}{b}$$

$$\Big\downarrow \text{254}$$

$$\frac{\frac{\sec^5(a+bx)}{2\left(1-\sec^2(a+bx)\right)}-\frac{5}{2}\int \left(-\sec^2(a+bx)+\frac{1}{1-\sec^2(a+bx)}-1\right)d\sec(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\sec^5(a+bx)}{2\left(1-\sec^2(a+bx)\right)}-\frac{5}{2}\left(\text{arctanh}(\sec(a+bx))-\frac{1}{3}\sec^3(a+bx)-\sec(a+bx)\right)}{b}$$

input
```
Int[Csc[a + b*x]^3*Sec[a + b*x]^4,x]
```

output
```
(Sec[a + b*x]^5/(2*(1 - Sec[a + b*x]^2)) - (5*(ArcTanh[Sec[a + b*x]] - Sec
[a + b*x] - Sec[a + b*x]^3/3))/2)/b
```

Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 2.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{\frac{1}{3\sin(bx+a)^2\cos(bx+a)^3}-\frac{5}{6\sin(bx+a)^2\cos(bx+a)}+\frac{5}{2\cos(bx+a)}+\frac{5\ln(\csc(bx+a)-\cot(bx+a))}{2}}{b}$
default	$\frac{\frac{1}{3\sin(bx+a)^2\cos(bx+a)^3}-\frac{5}{6\sin(bx+a)^2\cos(bx+a)}+\frac{5}{2\cos(bx+a)}+\frac{5\ln(\csc(bx+a)-\cot(bx+a))}{2}}{b}$
norman	$\frac{\frac{1}{8b}+\frac{\tan(\frac{bx}{2}+\frac{a}{2})^{10}}{8b}+\frac{75\tan(\frac{bx}{2}+\frac{a}{2})^4}{8b}-\frac{65\tan(\frac{bx}{2}+\frac{a}{2})^2}{12b}-\frac{55\tan(\frac{bx}{2}+\frac{a}{2})^6}{8b}}{\tan(\frac{bx}{2}+\frac{a}{2})^2(\tan(\frac{bx}{2}+\frac{a}{2})^2-1)^3}+\frac{5\ln(\tan(\frac{bx}{2}+\frac{a}{2}))}{2b}$
risch	$\frac{15\,e^{9i(bx+a)}+20\,e^{7i(bx+a)}-22\,e^{5i(bx+a)}+20\,e^{3i(bx+a)}+15\,e^{i(bx+a)}}{3b(e^{2i(bx+a)}+1)^3(e^{2i(bx+a)}-1)^2}+\frac{5\ln(e^{i(bx+a)}-1)}{2b}-\frac{5\ln(e^{i(bx+a)}+1)}{2b}$
parallelrisch	$\frac{60\ln(\tan(\frac{bx}{2}+\frac{a}{2}))(\tan(\frac{bx}{2}+\frac{a}{2})-1)^3(\tan(\frac{bx}{2}+\frac{a}{2})+1)^3+3\tan(\frac{bx}{2}+\frac{a}{2})^8-165\tan(\frac{bx}{2}+\frac{a}{2})^4+3\cot(\frac{bx}{2}+\frac{a}{2})^2+22}{24b(\tan(\frac{bx}{2}+\frac{a}{2})-1)^3(\tan(\frac{bx}{2}+\frac{a}{2})+1)^3}$

input
```
int(csc(b*x+a)^3*sec(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/3/sin(b*x+a)^2/cos(b*x+a)^3-5/6/sin(b*x+a)^2/cos(b*x+a)+5/2/cos(b*x
+a)+5/2*ln(csc(b*x+a)-cot(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(54) = 108.

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.87

$$\int \csc^3(a+bx)\sec^4(a+bx)\,dx$$
$$=\frac{30\cos(bx+a)^4-20\cos(bx+a)^2-15\left(\cos(bx+a)^5-\cos(bx+a)^3\right)\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)+15\,(c}{12\left(b\cos(bx+a)^5-b\cos(bx+a)^3\right)}$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/12*(30*cos(b*x + a)^4 - 20*cos(b*x + a)^2 - 15*(cos(b*x + a)^5 - cos(b*x
 + a)^3)*log(1/2*cos(b*x + a) + 1/2) + 15*(cos(b*x + a)^5 - cos(b*x + a)^3
)*log(-1/2*cos(b*x + a) + 1/2) - 4)/(b*cos(b*x + a)^5 - b*cos(b*x + a)^3)
```

Sympy [F]

$$\int \csc^3(a+bx)\sec^4(a+bx)\,dx = \int \csc^3(a+bx)\sec^4(a+bx)\,dx$$

input
```
integrate(csc(b*x+a)**3*sec(b*x+a)**4,x)
```

output
```
Integral(csc(a + b*x)**3*sec(a + b*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.22

$$\int \csc^3(a+bx)\sec^4(a+bx)\,dx = \frac{\frac{2\left(15\cos(bx+a)^4-10\cos(bx+a)^2-2\right)}{\cos(bx+a)^5-\cos(bx+a)^3} - 15\log(\cos(bx+a)+1) + 15\log(\cos(bx+a)-1)}{12\,b}$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/12*(2*(15*cos(b*x + a)^4 - 10*cos(b*x + a)^2 - 2)/(cos(b*x + a)^5 - cos(
b*x + a)^3) - 15*log(cos(b*x + a) + 1) + 15*log(cos(b*x + a) - 1))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.32

$$\int \csc^3(a+bx)\sec^4(a+bx)\,dx = -\frac{5\,\log\left(|\cos\left(bx+a\right)+1|\right)}{4\,b} + \frac{5\,\log\left(|\cos\left(bx+a\right)-1|\right)}{4\,b} + \frac{\cos\left(bx+a\right)}{2\,\left(\cos\left(bx+a\right)^2-1\right)b} + \frac{6\,\cos\left(bx+a\right)^2+1}{3\,b\cos\left(bx+a\right)^3}$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)^4,x, algorithm="giac")
```

output
```
-5/4*log(abs(cos(b*x + a) + 1))/b + 5/4*log(abs(cos(b*x + a) - 1))/b + 1/2
*cos(b*x + a)/((cos(b*x + a)^2 - 1)*b) + 1/3*(6*cos(b*x + a)^2 + 1)/(b*cos
(b*x + a)^3)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \csc^3(a+bx)\sec^4(a+bx)\,dx = \frac{-\frac{5\cos(a+b\,x)^4}{2}+\frac{5\cos(a+b\,x)^2}{3}+\frac{1}{3}}{b\,\left(\cos\left(a+b\,x\right)^3-\cos\left(a+b\,x\right)^5\right)} - \frac{5\,\mathrm{atanh}(\cos\left(a+b\,x\right))}{2\,b}$$

input
```
int(1/(cos(a + b*x)^4*sin(a + b*x)^3),x)
```

output
```
((5*cos(a + b*x)^2)/3 - (5*cos(a + b*x)^4)/2 + 1/3)/(b*(cos(a + b*x)^3 - c
os(a + b*x)^5)) - (5*atanh(cos(a + b*x)))/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.32

$$\int \csc^3(a+bx)\sec^4(a+bx)\,dx$$
$$= \frac{60\cos(bx+a)\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\sin(bx+a)^4 - 60\cos(bx+a)\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\sin(bx+a)^2 - 65\cos}{24\cos(bx+a)\sin(bx+a)^2\, b}$$

input
```
int(csc(b*x+a)^3*sec(b*x+a)^4,x)
```

output
```
(60*cos(a + b*x)*log(tan((a + b*x)/2))*sin(a + b*x)**4 - 60*cos(a + b*x)*l
og(tan((a + b*x)/2))*sin(a + b*x)**2 - 65*cos(a + b*x)*sin(a + b*x)**4 + 6
5*cos(a + b*x)*sin(a + b*x)**2 + 60*sin(a + b*x)**4 - 80*sin(a + b*x)**2 +
 12)/(24*cos(a + b*x)*sin(a + b*x)**2*b*(sin(a + b*x)**2 - 1))
```

3.156 $\int \csc^3(a+bx)\sec^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 58

$$\int \csc^3(a+bx)\sec^5(a+bx)\,dx=-\frac{\cot^2(a+bx)}{2b}+\frac{3\log(\tan(a+bx))}{b}+\frac{3\tan^2(a+bx)}{2b}+\frac{\tan^4(a+bx)}{4b}$$

output
```
-1/2*cot(b*x+a)^2/b+3*ln(tan(b*x+a))/b+3/2*tan(b*x+a)^2/b+1/4*tan(b*x+a)^4
/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.16

$$\int \csc^3(a+bx)\sec^5(a+bx)\,dx=-\frac{\csc^2(a+bx)}{2b}-\frac{3\log(\cos(a+bx))}{b}+\frac{3\log(\sin(a+bx))}{b}+\frac{\sec^2(a+bx)}{b}+\frac{\sec^4(a+bx)}{4b}$$

input
```
Integrate[Csc[a + b*x]^3*Sec[a + b*x]^5,x]
```

output
```
-1/2*Csc[a + b*x]^2/b - (3*Log[Cos[a + b*x]])/b + (3*Log[Sin[a + b*x]])/b
+ Sec[a + b*x]^2/b + Sec[a + b*x]^4/(4*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.84, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3042, 3100, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(a+bx)\sec^5(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(a+bx)^3\sec(a+bx)^5dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{\int \cot^3(a+bx)\left(\tan^2(a+bx)+1\right)^3d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int \cot^2(a+bx)\left(\tan^2(a+bx)+1\right)^3d\tan^2(a+bx)}{2b}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\cot^2(a+bx)+3\cot(a+bx)+\tan^2(a+bx)+3\right)d\tan^2(a+bx)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}\tan^4(a+bx)+3\tan^2(a+bx)-\cot(a+bx)+3\log\left(\tan^2(a+bx)\right)}{2b}$$

input
```
Int[Csc[a + b*x]^3*Sec[a + b*x]^5,x]
```

output
```
(-Cot[a + b*x] + 3*Log[Tan[a + b*x]^2] + 3*Tan[a + b*x]^2 + Tan[a + b*x]^4
/2)/(2*b)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Maple [A] (verified)

Time = 2.23 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{\frac{1}{4\sin(bx+a)^2\cos(bx+a)^4}+\frac{3}{4\sin(bx+a)^2\cos(bx+a)^2}-\frac{3}{2\sin(bx+a)^2}+3\ln(\tan(bx+a))}{b}$
default	$\frac{\frac{1}{4\sin(bx+a)^2\cos(bx+a)^4}+\frac{3}{4\sin(bx+a)^2\cos(bx+a)^2}-\frac{3}{2\sin(bx+a)^2}+3\ln(\tan(bx+a))}{b}$
risch	$\frac{6\,\mathrm{e}^{10i(bx+a)}+12\,\mathrm{e}^{8i(bx+a)}-4\,\mathrm{e}^{6i(bx+a)}+12\,\mathrm{e}^{4i(bx+a)}+6\,\mathrm{e}^{2i(bx+a)}}{b(\mathrm{e}^{2i(bx+a)}+1)^4(\mathrm{e}^{2i(bx+a)}-1)^2}-\frac{3\ln(\mathrm{e}^{2i(bx+a)}+1)}{b}+\frac{3\ln(\mathrm{e}^{2i(bx+a)}-1)}{b}$
norman	$\frac{-\frac{1}{8b}-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{12}}{8b}-\frac{10\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{6}}{b}+\frac{57\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}}{8b}+\frac{57\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{8}}{8b}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)^{4}}+\frac{3\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{3\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right.\right.}{b}$
parallelrisch	$\frac{(-24\cos(2bx+2a)-6\cos(4bx+4a)-18)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+(-24\cos(2bx+2a)-6\cos(4bx+4a)-18)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right.}{}$

input
```
int(csc(b*x+a)^3*sec(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/4/sin(b*x+a)^2/cos(b*x+a)^4+3/4/sin(b*x+a)^2/cos(b*x+a)^2-3/2/sin(b
*x+a)^2+3*ln(tan(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(52) = 104.

Time = 0.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.93

$$\int \csc^3(a+bx)\sec^5(a+bx)\,dx = \frac{6\cos(bx+a)^4-3\cos(bx+a)^2-6\left(\cos(bx+a)^6-\cos(bx+a)^4\right)\log\left(\cos(bx+a)^2\right)+6\left(\cos(bx+a\right.}{4\left(b\cos(bx+a)^6-b\cos(bx+a)^4\right)}$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)^5,x, algorithm="fricas")
```

output
```
1/4*(6*cos(b*x + a)^4 - 3*cos(b*x + a)^2 - 6*(cos(b*x + a)^6 - cos(b*x + a
)^4)*log(cos(b*x + a)^2) + 6*(cos(b*x + a)^6 - cos(b*x + a)^4)*log(-1/4*co
s(b*x + a)^2 + 1/4) - 1)/(b*cos(b*x + a)^6 - b*cos(b*x + a)^4)
```

Sympy [F]

$$\int \csc^3(a+bx)\sec^5(a+bx)\,dx = \int \csc^3{(a+bx)}\sec^5{(a+bx)}\,dx$$

input
```
integrate(csc(b*x+a)**3*sec(b*x+a)**5,x)
```

output
```
Integral(csc(a + b*x)**3*sec(a + b*x)**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.41

$$\int \csc^3(a+bx)\sec^5(a+bx)\,dx = -\frac{\frac{6\sin(bx+a)^4-9\sin(bx+a)^2+2}{\sin(bx+a)^6-2\sin(bx+a)^4+\sin(bx+a)^2}+6\log\left(\sin(bx+a)^2-1\right)-6\log\left(\sin(bx+a)^2\right)}{4b}$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)^5,x, algorithm="maxima")
```

output
```
-1/4*((6*sin(b*x + a)^4 - 9*sin(b*x + a)^2 + 2)/(sin(b*x + a)^6 - 2*sin(b*
x + a)^4 + sin(b*x + a)^2) + 6*log(sin(b*x + a)^2 - 1) - 6*log(sin(b*x + a
)^2))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.34

$$\int \csc^3(a+bx)\sec^5(a+bx)\,dx = -\frac{3\,\log\left(\left|\sin\left(bx+a\right)^2-1\right|\right)}{2\,b}+\frac{3\,\log\left(\left|\sin\left(bx+a\right)\right|\right)}{b} - \frac{6\,\sin\left(bx+a\right)^4-9\,\sin\left(bx+a\right)^2+2}{4\left(\sin\left(bx+a\right)^2-1\right)^2 b\sin\left(bx+a\right)^2}$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)^5,x, algorithm="giac")
```

output
```
-3/2*log(abs(sin(b*x + a)^2 - 1))/b + 3*log(abs(sin(b*x + a)))/b - 1/4*(6*
sin(b*x + a)^4 - 9*sin(b*x + a)^2 + 2)/((sin(b*x + a)^2 - 1)^2*b*sin(b*x +
 a)^2)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.28

$$\int \csc^3(a+bx)\sec^5(a+bx)\,dx = \frac{3\,\ln\left(\sin\left(a+b\,x\right)^2\right)}{2\,b}-\frac{3\,\ln\left(\cos\left(a+b\,x\right)\right)}{b} + \frac{-\frac{3\cos(a+b\,x)^4}{2}+\frac{3\cos(a+b\,x)^2}{4}+\frac{1}{4}}{b\,\left(\cos\left(a+b\,x\right)^4-\cos\left(a+b\,x\right)^6\right)}$$

input
```
int(1/(cos(a + b*x)^5*sin(a + b*x)^3),x)
```

output
```
(3*log(sin(a + b*x)^2))/(2*b) - (3*log(cos(a + b*x)))/b + ((3*cos(a + b*x)
^2)/4 - (3*cos(a + b*x)^4)/2 + 1/4)/(b*(cos(a + b*x)^4 - cos(a + b*x)^6))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 259, normalized size of antiderivative = 4.47

$$\int \csc^3(a+bx)\sec^5(a+bx)\,dx$$
$$= \frac{-12\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin(bx+a)^6+24\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin(bx+a)^4-12\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right.}{}$$

input
```
int(csc(b*x+a)^3*sec(b*x+a)^5,x)
```

output
```
( - 12*log(tan((a + b*x)/2) - 1)*sin(a + b*x)**6 + 24*log(tan((a + b*x)/2)
 - 1)*sin(a + b*x)**4 - 12*log(tan((a + b*x)/2) - 1)*sin(a + b*x)**2 - 12*
log(tan((a + b*x)/2) + 1)*sin(a + b*x)**6 + 24*log(tan((a + b*x)/2) + 1)*s
in(a + b*x)**4 - 12*log(tan((a + b*x)/2) + 1)*sin(a + b*x)**2 + 12*log(tan
((a + b*x)/2))*sin(a + b*x)**6 - 24*log(tan((a + b*x)/2))*sin(a + b*x)**4
+ 12*log(tan((a + b*x)/2))*sin(a + b*x)**2 - 6*sin(a + b*x)**6 + 6*sin(a +
 b*x)**4 + 3*sin(a + b*x)**2 - 2)/(4*sin(a + b*x)**2*b*(sin(a + b*x)**4 -
2*sin(a + b*x)**2 + 1))
```

3.157 $\int \cos^5(a+bx)\cot^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 68

$$\int \cos^5(a+bx)\cot^4(a+bx)\,dx = \frac{4\csc(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}+\frac{6\sin(a+bx)}{b}-\frac{4\sin^3(a+bx)}{3b}+\frac{\sin^5(a+bx)}{5b}$$

output
```
4*csc(b*x+a)/b-1/3*csc(b*x+a)^3/b+6*sin(b*x+a)/b-4/3*sin(b*x+a)^3/b+1/5*si
n(b*x+a)^5/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int \cos^5(a+bx)\cot^4(a+bx)\,dx = \frac{4\csc(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}+\frac{6\sin(a+bx)}{b}-\frac{4\sin^3(a+bx)}{3b}+\frac{\sin^5(a+bx)}{5b}$$

input
```
Integrate[Cos[a + b*x]^5*Cot[a + b*x]^4,x]
```

output
```
(4*Csc[a + b*x])/b - Csc[a + b*x]^3/(3*b) + (6*Sin[a + b*x])/b - (4*Sin[a
+ b*x]^3)/(3*b) + Sin[a + b*x]^5/(5*b)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(a+bx)\cot^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(a+bx+\frac{\pi}{2}\right)^5\tan\left(a+bx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{3070}$$

$$-\frac{\int \csc^4(a+bx)\left(1-\sin^2(a+bx)\right)^4d(-\sin(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(\csc^4(a+bx)-4\csc^2(a+bx)+\sin^4(a+bx)-4\sin^2(a+bx)+6\right)d(-\sin(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{1}{5}\sin^5(a+bx)+\frac{4}{3}\sin^3(a+bx)-6\sin(a+bx)+\frac{1}{3}\csc^3(a+bx)-4\csc(a+bx)}{b}$$

input
```
Int[Cos[a + b*x]^5*Cot[a + b*x]^4,x]
```

output
```
-((-4*Csc[a + b*x] + Csc[a + b*x]^3/3 - 6*Sin[a + b*x] + (4*Sin[a + b*x]^3
)/3 - Sin[a + b*x]^5/5)/b)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Maple [A] (verified)

Time = 13.32 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.32

method	result
derivativedivides	$\frac{-\frac{\cos(bx+a)^{10}}{3\sin(bx+a)^3}+\frac{7\cos(bx+a)^{10}}{3\sin(bx+a)}+\frac{7\left(\frac{128}{35}+\cos(bx+a)^8+\frac{8\cos(bx+a)^6}{7}+\frac{48\cos(bx+a)^4}{35}+\frac{64\cos(bx+a)^2}{35}\right)\sin(bx+a)}{3}}{b}$
default	$\frac{-\frac{\cos(bx+a)^{10}}{3\sin(bx+a)^3}+\frac{7\cos(bx+a)^{10}}{3\sin(bx+a)}+\frac{7\left(\frac{128}{35}+\cos(bx+a)^8+\frac{8\cos(bx+a)^6}{7}+\frac{48\cos(bx+a)^4}{35}+\frac{64\cos(bx+a)^2}{35}\right)\sin(bx+a)}{3}}{b}$
risch	$-\frac{i\left(3\,e^{11i(bx+a)}+56\,e^{9i(bx+a)}+1044\,e^{7i(bx+a)}-7524\cos(bx+a)-9612i\sin(bx+a)-8565\cos(5bx+5a)-8571i\sin(5bx\right.}{480b\left(e^{2i(bx+a)}-1\right)^3}$

input
```
int(cos(b*x+a)^5*cot(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/3/sin(b*x+a)^3*cos(b*x+a)^10+7/3/sin(b*x+a)*cos(b*x+a)^10+7/3*(128
/35+cos(b*x+a)^8+8/7*cos(b*x+a)^6+48/35*cos(b*x+a)^4+64/35*cos(b*x+a)^2)*s
in(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int \cos^5(a+bx)\cot^4(a+bx)\,dx = -\frac{3\,\cos\left(bx+a\right)^8+8\,\cos\left(bx+a\right)^6+48\,\cos\left(bx+a\right)^4-192\,\cos\left(bx+a\right)^2+128}{15\,\left(b\cos\left(bx+a\right)^2-b\right)\sin\left(bx+a\right)}$$

input
```
integrate(cos(b*x+a)^5*cot(b*x+a)^4,x, algorithm="fricas")
```

output
```
-1/15*(3*cos(b*x + a)^8 + 8*cos(b*x + a)^6 + 48*cos(b*x + a)^4 - 192*cos(b
*x + a)^2 + 128)/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \cos^5(a+bx)\cot^4(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**5*cot(b*x+a)**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

$$\int \cos^5(a+bx)\cot^4(a+bx)\,dx = \frac{3\,\sin\left(bx+a\right)^5-20\,\sin\left(bx+a\right)^3+\frac{5\left(12\,\sin\left(bx+a\right)^2-1\right)}{\sin\left(bx+a\right)^3}+90\,\sin\left(bx+a\right)}{15\,b}$$

input
```
integrate(cos(b*x+a)^5*cot(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/15*(3*sin(b*x + a)^5 - 20*sin(b*x + a)^3 + 5*(12*sin(b*x + a)^2 - 1)/sin
(b*x + a)^3 + 90*sin(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

$$\int \cos^5(a+bx)\cot^4(a+bx)\,dx = \frac{3\sin(bx+a)^5 - 20\sin(bx+a)^3 + \frac{5\left(12\sin(bx+a)^2-1\right)}{\sin(bx+a)^3} + 90\sin(bx+a)}{15\,b}$$

input
```
integrate(cos(b*x+a)^5*cot(b*x+a)^4,x, algorithm="giac")
```

output
```
1/15*(3*sin(b*x + a)^5 - 20*sin(b*x + a)^3 + 5*(12*sin(b*x + a)^2 - 1)/sin
(b*x + a)^3 + 90*sin(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 25.59 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.81

$$\int \cos^5(a+bx)\cot^4(a+bx)\,dx = \frac{3\sin(a+b\,x)^8 - 20\sin(a+b\,x)^6 + 90\sin(a+b\,x)^4 + 60\sin(a+b\,x)^2 - 5}{15\,b\sin(a+b\,x)^3}$$

input
```
int(cos(a + b*x)^5*cot(a + b*x)^4,x)
```

output
```
(60*sin(a + b*x)^2 + 90*sin(a + b*x)^4 - 20*sin(a + b*x)^6 + 3*sin(a + b*x
)^8 - 5)/(15*b*sin(a + b*x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.81

$$\int \cos^5(a+bx)\cot^4(a+bx)\,dx$$
$$=\frac{3\sin(bx+a)^8-20\sin(bx+a)^6+90\sin(bx+a)^4+60\sin(bx+a)^2-5}{15\sin(bx+a)^3b}$$

input
```
int(cos(b*x+a)^5*cot(b*x+a)^4,x)
```

output
```
(3*sin(a + b*x)**8 - 20*sin(a + b*x)**6 + 90*sin(a + b*x)**4 + 60*sin(a +
b*x)**2 - 5)/(15*sin(a + b*x)**3*b)
```

3.158 $\int \cos^4(a+bx)\cot^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 72

$$\int \cos^4(a+bx)\cot^4(a+bx)\,dx = \frac{35x}{8}+\frac{3\cot(a+bx)}{b}-\frac{\cot^3(a+bx)}{3b}+\frac{13\cos(a+bx)\sin(a+bx)}{8b}-\frac{\cos(a+bx)\sin^3(a+bx)}{4b}$$

output
```
35/8*x+3*cot(b*x+a)/b-1/3*cot(b*x+a)^3/b+13/8*cos(b*x+a)*sin(b*x+a)/b-1/4*
cos(b*x+a)*sin(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.74

$$\int \cos^4(a+bx)\cot^4(a+bx)\,dx = \frac{420(a+bx)-32\cot(a+bx)\left(-10+\csc^2(a+bx)\right)+72\sin(2(a+bx))+3\sin(4(a+bx))}{96b}$$

input
```
Integrate[Cos[a + b*x]^4*Cot[a + b*x]^4,x]
```

output
```
(420*(a + b*x) - 32*Cot[a + b*x]*(-10 + Csc[a + b*x]^2) + 72*Sin[2*(a + b*
x)] + 3*Sin[4*(a + b*x)])/(96*b)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.26, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3042, 3071, 252, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(a+bx)\cot^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(a+bx+\frac{\pi}{2}\right)^4\tan\left(a+bx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{3071}$$

$$-\frac{\int \frac{\cot^8(a+bx)}{(\cot^2(a+bx)+1)^3}d\cot(a+bx)}{b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{\frac{7}{4}\int \frac{\cot^6(a+bx)}{(\cot^2(a+bx)+1)^2}d\cot(a+bx)-\frac{\cot^7(a+bx)}{4(\cot^2(a+bx)+1)^2}}{b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{\frac{7}{4}\left(\frac{5}{2}\int \frac{\cot^4(a+bx)}{\cot^2(a+bx)+1}d\cot(a+bx)-\frac{\cot^5(a+bx)}{2(\cot^2(a+bx)+1)}\right)-\frac{\cot^7(a+bx)}{4(\cot^2(a+bx)+1)^2}}{b}$$

$$\Big\downarrow \text{254}$$

$$-\frac{\frac{7}{4}\left(\frac{5}{2}\int \left(\cot^2(a+bx)+\frac{1}{\cot^2(a+bx)+1}-1\right)d\cot(a+bx)-\frac{\cot^5(a+bx)}{2(\cot^2(a+bx)+1)}\right)-\frac{\cot^7(a+bx)}{4(\cot^2(a+bx)+1)^2}}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{7}{4}\left(\frac{5}{2}\left(\arctan(\cot(a+bx))+\frac{1}{3}\cot^3(a+bx)-\cot(a+bx)\right)-\frac{\cot^5(a+bx)}{2(\cot^2(a+bx)+1)}\right)-\frac{\cot^7(a+bx)}{4(\cot^2(a+bx)+1)^2}}{b}$$

input
```
Int[Cos[a + b*x]^4*Cot[a + b*x]^4,x]
```

output
```
-((-1/4*Cot[a + b*x]^7/(1 + Cot[a + b*x]^2)^2 + (7*(-1/2*Cot[a + b*x]^5/(1
 + Cot[a + b*x]^2) + (5*(ArcTan[Cot[a + b*x]] - Cot[a + b*x] + Cot[a + b*x
]^3/3))/2))/4)/b)
```

Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 7.71 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.31

method	result	s
derivativedivides	$\frac{-\frac{\cos(bx+a)^9}{3\sin(bx+a)^3}+\frac{2\cos(bx+a)^9}{\sin(bx+a)}+2\left(\cos(bx+a)^7+\frac{7\cos(bx+a)^5}{6}+\frac{35\cos(bx+a)^3}{24}+\frac{35\cos(bx+a)}{16}\right)\sin(bx+a)+\frac{35bx}{8}+\frac{35a}{8}}{b}$	9
default	$\frac{-\frac{\cos(bx+a)^9}{3\sin(bx+a)^3}+\frac{2\cos(bx+a)^9}{\sin(bx+a)}+2\left(\cos(bx+a)^7+\frac{7\cos(bx+a)^5}{6}+\frac{35\cos(bx+a)^3}{24}+\frac{35\cos(bx+a)}{16}\right)\sin(bx+a)+\frac{35bx}{8}+\frac{35a}{8}}{b}$	9
risch	$\frac{35x}{8}-\frac{ie^{4i(bx+a)}}{64b}-\frac{3ie^{2i(bx+a)}}{8b}+\frac{3ie^{-2i(bx+a)}}{8b}+\frac{ie^{-4i(bx+a)}}{64b}+\frac{4i\left(6\,e^{4i(bx+a)}-9\,e^{2i(bx+a)}+5\right)}{3b\left(e^{2i(bx+a)}-1\right)^3}$	1

input
```
int(cos(b*x+a)^4*cot(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/3/sin(b*x+a)^3*cos(b*x+a)^9+2/sin(b*x+a)*cos(b*x+a)^9+2*(cos(b*x+a
)^7+7/6*cos(b*x+a)^5+35/24*cos(b*x+a)^3+35/16*cos(b*x+a))*sin(b*x+a)+35/8*
b*x+35/8*a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.24

$$\int \cos^4(a+bx)\cot^4(a+bx)\,dx = -\frac{6\,\cos\left(bx+a\right)^7+21\,\cos\left(bx+a\right)^5-140\,\cos\left(bx+a\right)^3-105\,\left(bx\cos\left(bx+a\right)^2-bx\right)\sin\left(bx+a\right)+10}{24\,\left(b\cos\left(bx+a\right)^2-b\right)\sin\left(bx+a\right)}$$

input
```
integrate(cos(b*x+a)^4*cot(b*x+a)^4,x, algorithm="fricas")
```

output
```
-1/24*(6*cos(b*x + a)^7 + 21*cos(b*x + a)^5 - 140*cos(b*x + a)^3 - 105*(b*
x*cos(b*x + a)^2 - b*x)*sin(b*x + a) + 105*cos(b*x + a))/((b*cos(b*x + a)^
2 - b)*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \cos^4(a+bx)\cot^4(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**4*cot(b*x+a)**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04

$$\int \cos^4(a+bx)\cot^4(a+bx)\,dx = \frac{105\,bx + 105\,a + \frac{105\,\tan(bx+a)^6 + 175\,\tan(bx+a)^4 + 56\,\tan(bx+a)^2 - 8}{\tan(bx+a)^7 + 2\,\tan(bx+a)^5 + \tan(bx+a)^3}}{24\,b}$$

input
```
integrate(cos(b*x+a)^4*cot(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/24*(105*b*x + 105*a + (105*tan(b*x + a)^6 + 175*tan(b*x + a)^4 + 56*tan(
b*x + a)^2 - 8)/(tan(b*x + a)^7 + 2*tan(b*x + a)^5 + tan(b*x + a)^3))/b
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94

$$\int \cos^4(a+bx)\cot^4(a+bx)\,dx = \frac{105\,bx + 105\,a + \frac{3\left(11\,\tan(bx+a)^3 + 13\,\tan(bx+a)\right)}{\left(\tan(bx+a)^2 + 1\right)^2} + \frac{8\left(9\,\tan(bx+a)^2 - 1\right)}{\tan(bx+a)^3}}{24\,b}$$

input
```
integrate(cos(b*x+a)^4*cot(b*x+a)^4,x, algorithm="giac")
```

output
```
1/24*(105*b*x + 105*a + 3*(11*tan(b*x + a)^3 + 13*tan(b*x + a))/(tan(b*x +
 a)^2 + 1)^2 + 8*(9*tan(b*x + a)^2 - 1)/tan(b*x + a)^3)/b
```

Mupad [B] (verification not implemented)

Time = 26.77 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \cos^4(a+bx)\cot^4(a+bx)\,dx$$
$$=\frac{35\,x}{8}+\frac{\cos\left(a+b\,x\right)^4\,\left(\frac{35\,\tan\left(a+b\,x\right)^6}{8}+\frac{175\,\tan\left(a+b\,x\right)^4}{24}+\frac{7\,\tan\left(a+b\,x\right)^2}{3}-\frac{1}{3}\right)}{b\,\tan\left(a+b\,x\right)^3}$$

input
```
int(cos(a + b*x)^4*cot(a + b*x)^4,x)
```

output
```
(35*x)/8 + (cos(a + b*x)^4*((7*tan(a + b*x)^2)/3 + (175*tan(a + b*x)^4)/24
 + (35*tan(a + b*x)^6)/8 - 1/3))/(b*tan(a + b*x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.14

$$\int \cos^4(a+bx)\cot^4(a+bx)\,dx$$
$$=\frac{-6\cos\left(bx+a\right)\sin\left(bx+a\right)^6+39\cos\left(bx+a\right)\sin\left(bx+a\right)^4+80\cos\left(bx+a\right)\sin\left(bx+a\right)^2-8\cos\left(bx+\right.}{24\sin\left(bx+a\right)^3b}$$

input
```
int(cos(b*x+a)^4*cot(b*x+a)^4,x)
```

output
```
( - 6*cos(a + b*x)*sin(a + b*x)**6 + 39*cos(a + b*x)*sin(a + b*x)**4 + 80*
cos(a + b*x)*sin(a + b*x)**2 - 8*cos(a + b*x) + 105*sin(a + b*x)**3*b*x)/(
24*sin(a + b*x)**3*b)
```

3.159 $\int \cos^3(a+bx)\cot^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 53

$$\int \cos^3(a+bx)\cot^4(a+bx)\,dx = \frac{3\csc(a+bx)}{b} - \frac{\csc^3(a+bx)}{3b} + \frac{3\sin(a+bx)}{b} - \frac{\sin^3(a+bx)}{3b}$$

output
```
3*csc(b*x+a)/b-1/3*csc(b*x+a)^3/b+3*sin(b*x+a)/b-1/3*sin(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \cos^3(a+bx)\cot^4(a+bx)\,dx = \frac{3\csc(a+bx)}{b} - \frac{\csc^3(a+bx)}{3b} + \frac{3\sin(a+bx)}{b} - \frac{\sin^3(a+bx)}{3b}$$

input
```
Integrate[Cos[a + b*x]^3*Cot[a + b*x]^4,x]
```

output
```
(3*Csc[a + b*x])/b - Csc[a + b*x]^3/(3*b) + (3*Sin[a + b*x])/b - Sin[a + b
*x]^3/(3*b)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)\cot^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(a+bx+\frac{\pi}{2}\right)^3\tan\left(a+bx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{3070}$$

$$-\frac{\int \csc^4(a+bx)\left(1-\sin^2(a+bx)\right)^3d(-\sin(a+bx))}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(\csc^4(a+bx)-3\csc^2(a+bx)-\sin^2(a+bx)+3\right)d(-\sin(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{3}\sin^3(a+bx)-3\sin(a+bx)+\frac{1}{3}\csc^3(a+bx)-3\csc(a+bx)}{b}$$

input
```
Int[Cos[a + b*x]^3*Cot[a + b*x]^4,x]
```

output
```
-((-3*Csc[a + b*x] + Csc[a + b*x]^3/3 - 3*Sin[a + b*x] + Sin[a + b*x]^3/3)
/b)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Maple [A] (verified)

Time = 4.92 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.51

method	result
derivativedivides	$\frac{-\frac{\cos(bx+a)^8}{3\sin(bx+a)^3}+\frac{5\cos(bx+a)^8}{3\sin(bx+a)}+\frac{5\left(\frac{16}{5}+\cos(bx+a)^6+\frac{6\cos(bx+a)^4}{5}+\frac{8\cos(bx+a)^2}{5}\right)\sin(bx+a)}{3}}{b}$
default	$\frac{-\frac{\cos(bx+a)^8}{3\sin(bx+a)^3}+\frac{5\cos(bx+a)^8}{3\sin(bx+a)}+\frac{5\left(\frac{16}{5}+\cos(bx+a)^6+\frac{6\cos(bx+a)^4}{5}+\frac{8\cos(bx+a)^2}{5}\right)\sin(bx+a)}{3}}{b}$
risch	$-\frac{i\left(e^{9i(bx+a)}+30\,e^{7i(bx+a)}-273\,e^{5i(bx+a)}-243\cos(bx+a)-303i\sin(bx+a)+421\cos(3bx+3a)+419i\sin(3bx+3a)\right)}{24b\left(e^{2i(bx+a)}-1\right)^3}$

input
```
int(cos(b*x+a)^3*cot(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/3/sin(b*x+a)^3*cos(b*x+a)^8+5/3/sin(b*x+a)*cos(b*x+a)^8+5/3*(16/5+
cos(b*x+a)^6+6/5*cos(b*x+a)^4+8/5*cos(b*x+a)^2)*sin(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int \cos^3(a+bx)\cot^4(a+bx)\,dx = -\frac{\cos(bx+a)^6 + 6\cos(bx+a)^4 - 24\cos(bx+a)^2 + 16}{3\left(b\cos(bx+a)^2 - b\right)\sin(bx+a)}$$

input
```
integrate(cos(b*x+a)^3*cot(b*x+a)^4,x, algorithm="fricas")
```

output
```
-1/3*(cos(b*x + a)^6 + 6*cos(b*x + a)^4 - 24*cos(b*x + a)^2 + 16)/((b*cos(
b*x + a)^2 - b)*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\cot^4(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3*cot(b*x+a)**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int \cos^3(a+bx)\cot^4(a+bx)\,dx = -\frac{\sin(bx+a)^3 - \frac{9\sin(bx+a)^2 - 1}{\sin(bx+a)^3} - 9\sin(bx+a)}{3b}$$

input
```
integrate(cos(b*x+a)^3*cot(b*x+a)^4,x, algorithm="maxima")
```

output
```
-1/3*(sin(b*x + a)^3 - (9*sin(b*x + a)^2 - 1)/sin(b*x + a)^3 - 9*sin(b*x +
 a))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int \cos^3(a+bx)\cot^4(a+bx)\,dx = -\frac{\sin(bx+a)^3 - \frac{9\sin(bx+a)^2-1}{\sin(bx+a)^3} - 9\sin(bx+a)}{3b}$$

input
```
integrate(cos(b*x+a)^3*cot(b*x+a)^4,x, algorithm="giac")
```

output
```
-1/3*(sin(b*x + a)^3 - (9*sin(b*x + a)^2 - 1)/sin(b*x + a)^3 - 9*sin(b*x +
 a))/b
```

Mupad [B] (verification not implemented)

Time = 25.80 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \cos^3(a+bx)\cot^4(a+bx)\,dx = \frac{-\sin(a+b\,x)^6 + 9\sin(a+b\,x)^4 + 9\sin(a+b\,x)^2 - 1}{3\,b\sin(a+b\,x)^3}$$

input
```
int(cos(a + b*x)^3*cot(a + b*x)^4,x)
```

output
```
(9*sin(a + b*x)^2 + 9*sin(a + b*x)^4 - sin(a + b*x)^6 - 1)/(3*b*sin(a + b*
x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \cos^3(a+bx)\cot^4(a+bx)\,dx = \frac{-\sin(bx+a)^6 + 9\sin(bx+a)^4 + 9\sin(bx+a)^2 - 1}{3\sin(bx+a)^3\,b}$$

input
```
int(cos(b*x+a)^3*cot(b*x+a)^4,x)
```

output
```
( - sin(a + b*x)**6 + 9*sin(a + b*x)**4 + 9*sin(a + b*x)**2 - 1)/(3*sin(a
+ b*x)**3*b)
```

3.160 $\int \cos^2(a+bx)\cot^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 51

$$\int \cos^2(a+bx)\cot^4(a+bx)\,dx = \frac{5x}{2}+\frac{2\cot(a+bx)}{b}-\frac{\cot^3(a+bx)}{3b}+\frac{\cos(a+bx)\sin(a+bx)}{2b}$$

output
```
5/2*x+2*cot(b*x+a)/b-1/3*cot(b*x+a)^3/b+1/2*cos(b*x+a)*sin(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \cos^2(a+bx)\cot^4(a+bx)\,dx = \frac{30(a+bx)-4\cot(a+bx)\left(-7+\csc^2(a+bx)\right)+3\sin(2(a+bx))}{12b}$$

input
```
Integrate[Cos[a + b*x]^2*Cot[a + b*x]^4,x]
```

output
```
(30*(a + b*x) - 4*Cot[a + b*x]*(-7 + Csc[a + b*x]^2) + 3*Sin[2*(a + b*x)])
/(12*b)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3042, 3071, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\cot^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(a+bx+\frac{\pi}{2}\right)^2\tan\left(a+bx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{3071}$$

$$-\frac{\int \frac{\cot^6(a+bx)}{(\cot^2(a+bx)+1)^2}d\cot(a+bx)}{b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{\frac{5}{2}\int \frac{\cot^4(a+bx)}{\cot^2(a+bx)+1}d\cot(a+bx)-\frac{\cot^5(a+bx)}{2(\cot^2(a+bx)+1)}}{b}$$

$$\Big\downarrow \text{254}$$

$$-\frac{\frac{5}{2}\int \left(\cot^2(a+bx)+\frac{1}{\cot^2(a+bx)+1}-1\right)d\cot(a+bx)-\frac{\cot^5(a+bx)}{2(\cot^2(a+bx)+1)}}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{5}{2}\left(\arctan(\cot(a+bx))+\frac{1}{3}\cot^3(a+bx)-\cot(a+bx)\right)-\frac{\cot^5(a+bx)}{2(\cot^2(a+bx)+1)}}{b}$$

input
```
Int[Cos[a + b*x]^2*Cot[a + b*x]^4,x]
```

output
```
-((-1/2*Cot[a + b*x]^5/(1 + Cot[a + b*x]^2) + (5*(ArcTan[Cot[a + b*x]] - C
ot[a + b*x] + Cot[a + b*x]^3/3))/2)/b)
```

Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.38 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.53

method	result	size
risch	$\frac{5x}{2}-\frac{i{\mathrm e}^{2 i (b x +a)}}{8 b}+\frac{i{\mathrm e}^{-2 i (b x +a)}}{8 b}+\frac{2 i \left(9 \,{\mathrm e}^{4 i (b x +a)}-12 \,{\mathrm e}^{2 i (b x +a)}+7\right)}{3 b \left({\mathrm e}^{2 i (b x +a)}-1\right)^{3}}$	78
derivativedivides	$\frac{-\frac{\cos \left(b x +a \right)^{7}}{3 \sin \left(b x +a \right)^{3}}+\frac{4 \cos \left(b x +a \right)^{7}}{3 \sin \left(b x +a \right)}+\frac{4 \left(\cos \left(b x +a \right)^{5}+\frac{5 \cos \left(b x +a \right)^{3}}{4}+\frac{15 \cos \left(b x +a \right)}{8}\right) \sin \left(b x +a \right)}{3}+\frac{5 b x}{2}+\frac{5 a}{2}}{b}$	84
default	$\frac{-\frac{\cos \left(b x +a \right)^{7}}{3 \sin \left(b x +a \right)^{3}}+\frac{4 \cos \left(b x +a \right)^{7}}{3 \sin \left(b x +a \right)}+\frac{4 \left(\cos \left(b x +a \right)^{5}+\frac{5 \cos \left(b x +a \right)^{3}}{4}+\frac{15 \cos \left(b x +a \right)}{8}\right) \sin \left(b x +a \right)}{3}+\frac{5 b x}{2}+\frac{5 a}{2}}{b}$	84

input
```
int(cos(b*x+a)^2*cot(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
5/2*x-1/8*I/b*exp(2*I*(b*x+a))+1/8*I/b*exp(-2*I*(b*x+a))+2/3*I*(9*exp(4*I*
(b*x+a))-12*exp(2*I*(b*x+a))+7)/b/(exp(2*I*(b*x+a))-1)^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.55

$$\int \cos^2(a+bx)\cot^4(a+bx)\,dx = -\frac{3\cos(bx+a)^5 - 20\cos(bx+a)^3 - 15\left(bx\cos(bx+a)^2 - bx\right)\sin(bx+a) + 15\cos(bx+a)}{6\left(b\cos(bx+a)^2 - b\right)\sin(bx+a)}$$

input
```
integrate(cos(b*x+a)^2*cot(b*x+a)^4,x, algorithm="fricas")
```

output
```
-1/6*(3*cos(b*x + a)^5 - 20*cos(b*x + a)^3 - 15*(b*x*cos(b*x + a)^2 - b*x)
*sin(b*x + a) + 15*cos(b*x + a))/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\cot^4(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2*cot(b*x+a)**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

$$\int \cos^2(a+bx)\cot^4(a+bx)\,dx = \frac{15\,bx+15\,a+\frac{15\,\tan(bx+a)^4+10\,\tan(bx+a)^2-2}{\tan(bx+a)^5+\tan(bx+a)^3}}{6\,b}$$

input
```
integrate(cos(b*x+a)^2*cot(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/6*(15*b*x + 15*a + (15*tan(b*x + a)^4 + 10*tan(b*x + a)^2 - 2)/(tan(b*x
+ a)^5 + tan(b*x + a)^3))/b
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

$$\int \cos^2(a+bx)\cot^4(a+bx)\,dx = \frac{15\,bx+15\,a+\frac{3\,\tan(bx+a)}{\tan(bx+a)^2+1}+\frac{2\left(6\,\tan(bx+a)^2-1\right)}{\tan(bx+a)^3}}{6\,b}$$

input
```
integrate(cos(b*x+a)^2*cot(b*x+a)^4,x, algorithm="giac")
```

output
```
1/6*(15*b*x + 15*a + 3*tan(b*x + a)/(tan(b*x + a)^2 + 1) + 2*(6*tan(b*x +
a)^2 - 1)/tan(b*x + a)^3)/b
```

Mupad [B] (verification not implemented)

Time = 26.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int \cos^2(a+bx)\cot^4(a+bx)\,dx = \frac{5\,x}{2}+\frac{\cos\left(a+b\,x\right)^2\,\left(\frac{5\,\tan\left(a+b\,x\right)^4}{2}+\frac{5\,\tan\left(a+b\,x\right)^2}{3}-\frac{1}{3}\right)}{b\,\tan\left(a+b\,x\right)^3}$$

input
```
int(cos(a + b*x)^2*cot(a + b*x)^4,x)
```

output
```
(5*x)/2 + (cos(a + b*x)^2*((5*tan(a + b*x)^2)/3 + (5*tan(a + b*x)^4)/2 - 1
/3))/(b*tan(a + b*x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.29

$$\int \cos^2(a+bx)\cot^4(a+bx)\,dx = \frac{3\cos(bx+a)\sin(bx+a)^4+14\cos(bx+a)\sin(bx+a)^2-2\cos(bx+a)+15\sin(bx+a)^3bx}{6\sin(bx+a)^3b}$$

input
```
int(cos(b*x+a)^2*cot(b*x+a)^4,x)
```

output
```
(3*cos(a + b*x)*sin(a + b*x)**4 + 14*cos(a + b*x)*sin(a + b*x)**2 - 2*cos(
a + b*x) + 15*sin(a + b*x)**3*b*x)/(6*sin(a + b*x)**3*b)
```

3.161 $\int \cos(a+bx)\cot^4(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \cos(a+bx)\cot^4(a+bx)\,dx=\frac{2\csc(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}+\frac{\sin(a+bx)}{b}$$

output
```
2*csc(b*x+a)/b-1/3*csc(b*x+a)^3/b+sin(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \cos(a+bx)\cot^4(a+bx)\,dx=\frac{2\csc(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}+\frac{\sin(a+bx)}{b}$$

input
```
Integrate[Cos[a + b*x]*Cot[a + b*x]^4,x]
```

output
```
(2*Csc[a + b*x])/b - Csc[a + b*x]^3/(3*b) + Sin[a + b*x]/b
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\cot^4(a+bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sin\left(a+bx+\frac{\pi}{2}\right)\tan\left(a+bx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \ 3070$$

$$-\frac{\int \csc^4(a+bx)\left(1-\sin^2(a+bx)\right)^2d(-\sin(a+bx))}{b}$$

$$\Big\downarrow \ 244$$

$$-\frac{\int \left(\csc^4(a+bx)-2\csc^2(a+bx)+1\right)d(-\sin(a+bx))}{b}$$

$$\Big\downarrow \ 2009$$

$$-\frac{-\sin(a+bx)+\frac{1}{3}\csc^3(a+bx)-2\csc(a+bx)}{b}$$

input
```
Int[Cos[a + b*x]*Cot[a + b*x]^4,x]
```

output
```
-((-2*Csc[a + b*x] + Csc[a + b*x]^3/3 - Sin[a + b*x])/b)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Maple [A] (verified)

Time = 2.88 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.84

method	result	size
derivativedivides	$\frac{-\frac{\cos(bx+a)^6}{3\sin(bx+a)^3}+\frac{\cos(bx+a)^6}{\sin(bx+a)}+\left(\frac{8}{3}+\cos(bx+a)^4+\frac{4\cos(bx+a)^2}{3}\right)\sin(bx+a)}{b}$	68
default	$\frac{-\frac{\cos(bx+a)^6}{3\sin(bx+a)^3}+\frac{\cos(bx+a)^6}{\sin(bx+a)}+\left(\frac{8}{3}+\cos(bx+a)^4+\frac{4\cos(bx+a)^2}{3}\right)\sin(bx+a)}{b}$	68
risch	$-\frac{i\left(3\,\mathrm{e}^{7i(bx+a)}-36\,\mathrm{e}^{5i(bx+a)}+50\,\mathrm{e}^{3i(bx+a)}-33\cos(bx+a)-39i\sin(bx+a)\right)}{6b\left(\mathrm{e}^{2i(bx+a)}-1\right)^3}$	71

input
```
int(cos(b*x+a)*cot(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/3/sin(b*x+a)^3*cos(b*x+a)^6+1/sin(b*x+a)*cos(b*x+a)^6+(8/3+cos(b*x
+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.30

$$\int \cos(a+bx)\cot^4(a+bx)\,dx = -\frac{3\cos(bx+a)^4 - 12\cos(bx+a)^2 + 8}{3\left(b\cos(bx+a)^2 - b\right)\sin(bx+a)}$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)^4,x, algorithm="fricas")
```

output
```
-1/3*(3*cos(b*x + a)^4 - 12*cos(b*x + a)^2 + 8)/((b*cos(b*x + a)^2 - b)*si
n(b*x + a))
```

Sympy [F]

$$\int \cos(a+bx)\cot^4(a+bx)\,dx = \int \cos(a+bx)\cot^4(a+bx)\,dx$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)**4,x)
```

output
```
Integral(cos(a + b*x)*cot(a + b*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \cos(a+bx)\cot^4(a+bx)\,dx = \frac{\frac{6\sin(bx+a)^2-1}{\sin(bx+a)^3} + 3\sin(bx+a)}{3b}$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/3*((6*sin(b*x + a)^2 - 1)/sin(b*x + a)^3 + 3*sin(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \cos(a+bx)\cot^4(a+bx)\,dx = \frac{\frac{6\,\sin(bx+a)^2-1}{\sin(bx+a)^3}+3\,\sin(bx+a)}{3\,b}$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)^4,x, algorithm="giac")
```

output
```
1/3*((6*sin(b*x + a)^2 - 1)/sin(b*x + a)^3 + 3*sin(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 25.76 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \cos(a+bx)\cot^4(a+bx)\,dx = \frac{\sin(a+b\,x)^4+2\sin(a+b\,x)^2-\frac{1}{3}}{b\sin(a+b\,x)^3}$$

input
```
int(cos(a + b*x)*cot(a + b*x)^4,x)
```

output
```
(2*sin(a + b*x)^2 + sin(a + b*x)^4 - 1/3)/(b*sin(a + b*x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \cos(a+bx)\cot^4(a+bx)\,dx = \frac{3\sin(bx+a)^4+6\sin(bx+a)^2-1}{3\sin(bx+a)^3\,b}$$

input
```
int(cos(b*x+a)*cot(b*x+a)^4,x)
```

output
```
(3*sin(a + b*x)**4 + 6*sin(a + b*x)**2 - 1)/(3*sin(a + b*x)**3*b)
```

3.162 $\int \cot^4(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 27

$$\int \cot^4(a+bx)\,dx = x+\frac{\cot(a+bx)}{b}-\frac{\cot^3(a+bx)}{3b}$$

output
```
x+cot(b*x+a)/b-1/3*cot(b*x+a)^3/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \cot^4(a+bx)\,dx = -\frac{\cot^3(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(a+bx)\right)}{3b}$$

input
```
Integrate[Cot[a + b*x]^4,x]
```

output
```
-1/3*(Cot[a + b*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[a + b*x]^2])/b
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(a+bx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{3954}$$

$$-\int \cot^2(a+bx)dx-\frac{\cot^3(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$-\int \tan\left(a+bx+\frac{\pi}{2}\right)^2dx-\frac{\cot^3(a+bx)}{3b}$$

$$\Big\downarrow \text{3954}$$

$$\int 1dx-\frac{\cot^3(a+bx)}{3b}+\frac{\cot(a+bx)}{b}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\cot^3(a+bx)}{3b}+\frac{\cot(a+bx)}{b}+x$$

input
```
Int[Cot[a + b*x]^4,x]
```

output
```
x + Cot[a + b*x]/b - Cot[a + b*x]^3/(3*b)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

method	result	size
parallelrisch	$\frac{-\cot(bx+a)^3+3bx+3\cot(bx+a)}{3b}$	29
derivativedivides	$\frac{-\frac{\cot(bx+a)^3}{3}+\cot(bx+a)-\frac{\pi}{2}+\operatorname{arccot}(\cot(bx+a))}{b}$	32
default	$\frac{-\frac{\cot(bx+a)^3}{3}+\cot(bx+a)-\frac{\pi}{2}+\operatorname{arccot}(\cot(bx+a))}{b}$	32
norman	$\frac{x\tan(bx+a)^3+\frac{\tan(bx+a)^2}{b}-\frac{1}{3b}}{\tan(bx+a)^3}$	38
risch	$x+\frac{4i\left(3\,e^{4i(bx+a)}-3\,e^{2i(bx+a)}+2\right)}{3b\left(e^{2i(bx+a)}-1\right)^3}$	46

input
```
int(cot(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*(-cot(b*x+a)^3+3*b*x+3*cot(b*x+a))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(25) = 50$.

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 3.11

$$\int \cot^4(a+bx)\,dx = \frac{4\cos(2\,bx+2\,a)^2 + 3\,(bx\cos(2\,bx+2\,a) - bx)\sin(2\,bx+2\,a) + 2\,\cos(2\,bx+2\,a) - 2}{3\,(b\cos(2\,bx+2\,a) - b)\sin(2\,bx+2\,a)}$$

input
```
integrate(cot(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/3*(4*cos(2*b*x + 2*a)^2 + 3*(b*x*cos(2*b*x + 2*a) - b*x)*sin(2*b*x + 2*a
) + 2*cos(2*b*x + 2*a) - 2)/((b*cos(2*b*x + 2*a) - b)*sin(2*b*x + 2*a))
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \cot^4(a+bx)\,dx = \begin{cases} x - \frac{\cot^3(a+bx)}{3b} + \frac{\cot(a+bx)}{b} & \text{for } b \neq 0 \\ x\cot^4(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cot(b*x+a)**4,x)
```

output
```
Piecewise((x - cot(a + b*x)**3/(3*b) + cot(a + b*x)/b, Ne(b, 0)), (x*cot(a
)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \cot^4(a+bx)\,dx = \frac{3\,bx + 3\,a + \frac{3\,\tan(bx+a)^2-1}{\tan(bx+a)^3}}{3\,b}$$

input
```
integrate(cot(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/3*(3*b*x + 3*a + (3*tan(b*x + a)^2 - 1)/tan(b*x + a)^3)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(25) = 50.

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.30

$$\int \cot^4(a+bx)\,dx$$
$$= \frac{\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^3 + 24\,bx + 24\,a + \frac{15\,\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^2-1}{\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^3} - 15\,\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}{24\,b}$$

input
```
integrate(cot(b*x+a)^4,x, algorithm="giac")
```

output
```
1/24*(tan(1/2*b*x + 1/2*a)^3 + 24*b*x + 24*a + (15*tan(1/2*b*x + 1/2*a)^2
- 1)/tan(1/2*b*x + 1/2*a)^3 - 15*tan(1/2*b*x + 1/2*a))/b
```

Mupad [B] (verification not implemented)

Time = 25.58 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \cot^4(a+bx)\,dx = x + \frac{\cot(a+b\,x) - \frac{\cot(a+b\,x)^3}{3}}{b}$$

input
```
int(cot(a + b*x)^4,x)
```

output
```
x + (cot(a + b*x) - cot(a + b*x)^3/3)/b
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int \cot^4(a+bx)\,dx = \frac{-\cot\left(bx+a\right)^3+3\cot\left(bx+a\right)+3bx}{3b}$$

input
```
int(cot(b*x+a)^4,x)
```

output
```
( - cot(a + b*x)**3 + 3*cot(a + b*x) + 3*b*x)/(3*b)
```

3.163 $\int \cot^3(a+bx)\csc(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 26

$$\int \cot^3(a+bx)\csc(a+bx)\,dx=\frac{\csc(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}$$

output
```
csc(b*x+a)/b-1/3*csc(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \cot^3(a+bx)\csc(a+bx)\,dx=\frac{\csc(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}$$

input
```
Integrate[Cot[a + b*x]^3*Csc[a + b*x],x]
```

output
```
Csc[a + b*x]/b - Csc[a + b*x]^3/(3*b)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 25, 3086, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(a+bx)\csc(a+bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \tan\left(a+bx-\frac{\pi}{2}\right)^3\left(-\sec\left(a+bx-\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \ 25$$

$$-\int \sec\left(\frac{1}{2}(2a-\pi)+bx\right)\tan\left(\frac{1}{2}(2a-\pi)+bx\right)^3dx$$

$$\Big\downarrow \ 3086$$

$$-\frac{\int \left(\csc^2(a+bx)-1\right)d\csc(a+bx)}{b}$$

$$\Big\downarrow \ 2009$$

$$-\frac{\frac{1}{3}\csc^3(a+bx)-\csc(a+bx)}{b}$$

input
```
Int[Cot[a + b*x]^3*Csc[a + b*x],x]
```

output
```
-((-Csc[a + b*x] + Csc[a + b*x]^3/3)/b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{-\frac{\csc(bx+a)^3}{3}+\csc(bx+a)}{b}$	22
default	$\frac{-\frac{\csc(bx+a)^3}{3}+\csc(bx+a)}{b}$	22
risch	$\frac{2i\left(3\,\mathrm{e}^{5i(bx+a)}-2\,\mathrm{e}^{3i(bx+a)}+3\,\mathrm{e}^{i(bx+a)}\right)}{3b\left(\mathrm{e}^{2i(bx+a)}-1\right)^3}$	54

input
```
int(cot(b*x+a)^3*csc(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/3*csc(b*x+a)^3+csc(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.46

$$\int \cot^3(a+bx)\csc(a+bx)\,dx = \frac{3\cos(bx+a)^2-2}{3\left(b\cos(bx+a)^2-b\right)\sin(bx+a)}$$

input
```
integrate(cot(b*x+a)^3*csc(b*x+a),x, algorithm="fricas")
```

output
```
1/3*(3*cos(b*x + a)^2 - 2)/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(19) = 38$.

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.50

$$\int \cot^3(a+bx)\csc(a+bx)\,dx = \begin{cases} -\frac{\cot^2(a+bx)\csc(a+bx)}{3b} + \frac{2\csc(a+bx)}{3b} & \text{for } b \neq 0 \\ x\cot^3(a)\csc(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cot(b*x+a)**3*csc(b*x+a),x)
```

output
```
Piecewise((-cot(a + b*x)**2*csc(a + b*x)/(3*b) + 2*csc(a + b*x)/(3*b), Ne(
b, 0)), (x*cot(a)**3*csc(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \cot^3(a+bx)\csc(a+bx)\,dx = \frac{3\sin(bx+a)^2-1}{3\,b\sin(bx+a)^3}$$

input
```
integrate(cot(b*x+a)^3*csc(b*x+a),x, algorithm="maxima")
```

output
```
1/3*(3*sin(b*x + a)^2 - 1)/(b*sin(b*x + a)^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \cot^3(a+bx)\csc(a+bx)\,dx = \frac{3\,\sin\left(bx+a\right)^2-1}{3\,b\sin\left(bx+a\right)^3}$$

input
```
integrate(cot(b*x+a)^3*csc(b*x+a),x, algorithm="giac")
```

output
```
1/3*(3*sin(b*x + a)^2 - 1)/(b*sin(b*x + a)^3)
```

Mupad [B] (verification not implemented)

Time = 25.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \cot^3(a+bx)\csc(a+bx)\,dx = \frac{\sin\left(a+b\,x\right)^2-\frac{1}{3}}{b\sin\left(a+b\,x\right)^3}$$

input
```
int(cot(a + b*x)^3/sin(a + b*x),x)
```

output
```
(sin(a + b*x)^2 - 1/3)/(b*sin(a + b*x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int \cot^3(a+bx)\csc(a+bx)\,dx = \frac{\csc\left(bx+a\right)\left(-\cot\left(bx+a\right)^2+2\right)}{3b}$$

input
```
int(cot(b*x+a)^3*csc(b*x+a),x)
```

output
```
(csc(a + b*x)*( - cot(a + b*x)**2 + 2))/(3*b)
```

3.164 $\int \cot^2(a+bx)\csc^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 15

$$\int \cot^2(a+bx)\csc^2(a+bx)\,dx=-\frac{\cot^3(a+bx)}{3b}$$

output
```
-1/3*cot(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cot^2(a+bx)\csc^2(a+bx)\,dx=-\frac{\cot^3(a+bx)}{3b}$$

input
```
Integrate[Cot[a + b*x]^2*Csc[a + b*x]^2,x]
```

output
```
-1/3*Cot[a + b*x]^3/b
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3042, 3087, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(a+bx)\csc^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(a+bx-\frac{\pi}{2}\right)^2\sec\left(a+bx-\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3087}$$

$$\frac{\int \cot^2(a+bx)d(-\cot(a+bx))}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\cot^3(a+bx)}{3b}$$

input
```
Int[Cot[a + b*x]^2*Csc[a + b*x]^2,x]
```

output
```
-1/3*Cot[a + b*x]^3/b
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{\cot(bx+a)^3}{3b}$	14
default	$-\frac{\cot(bx+a)^3}{3b}$	14
risch	$\frac{2i\left(3\,\mathrm{e}^{4i(bx+a)}+1\right)}{3b\left(\mathrm{e}^{2i(bx+a)}-1\right)^3}$	33

input
```
int(cot(b*x+a)^2*csc(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*cot(b*x+a)^3/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(13) = 26$.

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.27

$$\int \cot^2(a+bx)\csc^2(a+bx)\,dx = \frac{\cos(bx+a)^3}{3\left(b\cos(bx+a)^2-b\right)\sin(bx+a)}$$

input
```
integrate(cot(b*x+a)^2*csc(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/3*cos(b*x + a)^3/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int \cot^2(a+bx)\csc^2(a+bx)\,dx = \begin{cases} -\frac{\cot^3(a+bx)}{3b} & \text{for } b \neq 0 \\ x\cot^2(a)\csc^2(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cot(b*x+a)**2*csc(b*x+a)**2,x)
```

output
```
Piecewise((-cot(a + b*x)**3/(3*b), Ne(b, 0)), (x*cot(a)**2*csc(a)**2, True
))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cot^2(a+bx)\csc^2(a+bx)\,dx = -\frac{\cot(bx+a)^3}{3\,b}$$

input
```
integrate(cot(b*x+a)^2*csc(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/3*cot(b*x + a)^3/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cot^2(a+bx)\csc^2(a+bx)\,dx = -\frac{1}{3\,b\tan(bx+a)^3}$$

input
```
integrate(cot(b*x+a)^2*csc(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/3/(b*tan(b*x + a)^3)
```

Mupad [B] (verification not implemented)

Time = 25.61 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cot^2(a+bx)\csc^2(a+bx)\,dx = -\frac{\cot(a+b\,x)^3}{3\,b}$$

input
```
int(cot(a + b*x)^2/sin(a + b*x)^2,x)
```

output
```
-cot(a + b*x)^3/(3*b)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int \cot^2(a+bx)\csc^2(a+bx)\,dx = \frac{\cos(bx+a)\left(\sin(bx+a)^2-1\right)}{3\sin(bx+a)^3 b}$$

input
```
int(cot(b*x+a)^2*csc(b*x+a)^2,x)
```

output
```
(cos(a + b*x)*(sin(a + b*x)**2 - 1))/(3*sin(a + b*x)**3*b)
```

3.165 $\int \cot(a+bx)\csc^3(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \cot(a+bx)\csc^3(a+bx)\,dx = -\frac{\csc^3(a+bx)}{3b}$$

output
```
-1/3*csc(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cot(a+bx)\csc^3(a+bx)\,dx = -\frac{\csc^3(a+bx)}{3b}$$

input
```
Integrate[Cot[a + b*x]*Csc[a + b*x]^3,x]
```

output
```
-1/3*Csc[a + b*x]^3/b
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 25, 3086, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(a+bx)\csc^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(a+bx-\frac{\pi}{2}\right)\left(-\sec\left(a+bx-\frac{\pi}{2}\right)^3\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sec\left(\frac{1}{2}(2a-\pi)+bx\right)^3\tan\left(\frac{1}{2}(2a-\pi)+bx\right)dx$$

$$\Big\downarrow \text{3086}$$

$$-\frac{\int \csc^2(a+bx)d\csc(a+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\csc^3(a+bx)}{3b}$$

input
```
Int[Cot[a + b*x]*Csc[a + b*x]^3,x]
```

output
```
-1/3*Csc[a + b*x]^3/b
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{\csc(bx+a)^3}{3b}$	14
default	$-\frac{\csc(bx+a)^3}{3b}$	14
risch	$\frac{8i\,e^{3i(bx+a)}}{3b\left(e^{2i(bx+a)}-1\right)^3}$	29

input
```
int(cot(b*x+a)*csc(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*csc(b*x+a)^3/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \cot(a+bx)\csc^3(a+bx)\,dx = \frac{1}{3\left(b\cos\left(bx+a\right)^2-b\right)\sin\left(bx+a\right)}$$

input
```
integrate(cot(b*x+a)*csc(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/3/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \cot(a+bx)\csc^3(a+bx)\,dx = \begin{cases} -\frac{\csc^3(a+bx)}{3b} & \text{for } b \neq 0 \\ x\cot(a)\csc^3(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cot(b*x+a)*csc(b*x+a)**3,x)
```

output
```
Piecewise((-csc(a + b*x)**3/(3*b), Ne(b, 0)), (x*cot(a)*csc(a)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cot(a+bx)\csc^3(a+bx)\,dx = -\frac{1}{3\,b\sin\left(bx+a\right)^3}$$

input
```
integrate(cot(b*x+a)*csc(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/3/(b*sin(b*x + a)^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cot(a+bx)\csc^3(a+bx)\,dx = -\frac{1}{3\,b\sin\left(bx+a\right)^3}$$

input
```
integrate(cot(b*x+a)*csc(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/3/(b*sin(b*x + a)^3)
```

Mupad [B] (verification not implemented)

Time = 25.38 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cot(a+bx)\csc^3(a+bx)\,dx = -\frac{1}{3\,b\sin\left(a+b\,x\right)^3}$$

input
```
int(cot(a + b*x)/sin(a + b*x)^3,x)
```

output
```
-1/(3*b*sin(a + b*x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cot(a+bx)\csc^3(a+bx)\,dx = -\frac{\csc\left(bx+a\right)^3}{3b}$$

input
```
int(cot(b*x+a)*csc(b*x+a)^3,x)
```

output
```
( - csc(a + b*x)**3)/(3*b)
```

3.166 $\int \csc^4(a+bx)\sec(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \csc^4(a+bx)\sec(a+bx)\,dx=\frac{\operatorname{arctanh}(\sin(a+bx))}{b}-\frac{\csc(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}$$

output
```
arctanh(sin(b*x+a))/b-csc(b*x+a)/b-1/3*csc(b*x+a)^3/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \csc^4(a+bx)\sec(a+bx)\,dx = -\frac{\csc^3(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\sin^2(a+bx)\right)}{3b}$$

input
```
Integrate[Csc[a + b*x]^4*Sec[a + b*x],x]
```

output
```
-1/3*(Csc[a + b*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Sin[a + b*x]^2])/b
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3101, 25, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(a+bx)\sec(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(a+bx)^4\sec(a+bx)dx$$

$$\Big\downarrow \text{3101}$$

$$-\frac{\int -\frac{\csc^4(a+bx)}{1-\csc^2(a+bx)}d\csc(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\csc^4(a+bx)}{1-\csc^2(a+bx)}d\csc(a+bx)}{b}$$

$$\Big\downarrow \text{254}$$

$$\frac{\int \left(-\csc^2(a+bx)+\frac{1}{1-\csc^2(a+bx)}-1\right)d\csc(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\text{arctanh}(\csc(a+bx))+\frac{1}{3}\csc^3(a+bx)+\csc(a+bx)}{b}$$

input
```
Int[Csc[a + b*x]^4*Sec[a + b*x],x]
```

output
```
-((-ArcTanh[Csc[a + b*x]] + Csc[a + b*x] + Csc[a + b*x]^3/3)/b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{-\frac{1}{3 \sin(bx+a)^3}-\frac{1}{\sin(bx+a)}+\ln(\sec(bx+a)+\tan(bx+a))}{b}$
default	$\frac{-\frac{1}{3 \sin(bx+a)^3}-\frac{1}{\sin(bx+a)}+\ln(\sec(bx+a)+\tan(bx+a))}{b}$
parallelrisch	$\frac{-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^3-\cot\left(\frac{bx}{2}+\frac{a}{2}\right)^3+24\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)-24\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)-15\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-15\cot\left(\frac{bx}{2}+\frac{a}{2}\right)}{24b}$
risch	$-\frac{2i\left(3\,\mathrm{e}^{5i(bx+a)}-10\,\mathrm{e}^{3i(bx+a)}+3\,\mathrm{e}^{i(bx+a)}\right)}{3b\left(\mathrm{e}^{2i(bx+a)}-1\right)^3}+\frac{\ln\left(\mathrm{e}^{i(bx+a)}+i\right)}{b}-\frac{\ln\left(\mathrm{e}^{i(bx+a)}-i\right)}{b}$
norman	$\frac{-\frac{1}{24b}-\frac{5\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}{8b}-\frac{5\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}{8b}-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^6}{24b}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^3}+\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{b}-\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{b}$

input
```
int(csc(b*x+a)^4*sec(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/3/sin(b*x+a)^3-1/sin(b*x+a)+ln(sec(b*x+a)+tan(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(36) = 72$.

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.47

$$\int \csc^4(a+bx)\sec(a+bx)\,dx$$
$$= \frac{3\left(\cos\left(bx+a\right)^2-1\right)\log\left(\sin\left(bx+a\right)+1\right)\sin\left(bx+a\right)-3\left(\cos\left(bx+a\right)^2-1\right)\log\left(-\sin\left(bx+a\right)+1\right)\mathrm{si}}{6\left(b\cos\left(bx+a\right)^2-b\right)\sin\left(bx+a\right)}$$

input
```
integrate(csc(b*x+a)^4*sec(b*x+a),x, algorithm="fricas")
```

output
```
1/6*(3*(cos(b*x + a)^2 - 1)*log(sin(b*x + a) + 1)*sin(b*x + a) - 3*(cos(b*
x + a)^2 - 1)*log(-sin(b*x + a) + 1)*sin(b*x + a) - 6*cos(b*x + a)^2 + 8)/
((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

Sympy [F]

$$\int \csc^4(a+bx)\sec(a+bx)\,dx = \int \csc^4\left(a+bx\right)\sec\left(a+bx\right)dx$$

input
```
integrate(csc(b*x+a)**4*sec(b*x+a),x)
```

output
```
Integral(csc(a + b*x)**4*sec(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int \csc^4(a+bx)\sec(a+bx)\,dx = -\frac{\frac{2\left(3\sin(bx+a)^2+1\right)}{\sin(bx+a)^3} - 3\log\left(\sin\left(bx+a\right)+1\right) + 3\log\left(\sin\left(bx+a\right)-1\right)}{6\,b}$$

input
```
integrate(csc(b*x+a)^4*sec(b*x+a),x, algorithm="maxima")
```

output
```
-1/6*(2*(3*sin(b*x + a)^2 + 1)/sin(b*x + a)^3 - 3*log(sin(b*x + a) + 1) +
3*log(sin(b*x + a) - 1))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.47

$$\int \csc^4(a+bx)\sec(a+bx)\,dx = \frac{\log\left(\left|\sin\left(bx+a\right)+1\right|\right)}{2\,b} - \frac{\log\left(\left|\sin\left(bx+a\right)-1\right|\right)}{2\,b} - \frac{3\sin\left(bx+a\right)^2+1}{3\,b\sin\left(bx+a\right)^3}$$

input
```
integrate(csc(b*x+a)^4*sec(b*x+a),x, algorithm="giac")
```

output
```
1/2*log(abs(sin(b*x + a) + 1))/b - 1/2*log(abs(sin(b*x + a) - 1))/b - 1/3*
(3*sin(b*x + a)^2 + 1)/(b*sin(b*x + a)^3)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \csc^4(a+bx)\sec(a+bx)\,dx=\frac{\operatorname{atanh}(\sin(a+b\,x))-\frac{\sin(a+b\,x)^2+\frac{1}{3}}{\sin(a+b\,x)^3}}{b}$$

input
```
int(1/(cos(a + b*x)*sin(a + b*x)^4),x)
```

output
```
(atanh(sin(a + b*x)) - (sin(a + b*x)^2 + 1/3)/sin(a + b*x)^3)/b
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.82

$$\int \csc^4(a+bx)\sec(a+bx)\,dx$$
$$=\frac{-3\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin(bx+a)^3+3\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)\sin(bx+a)^3-3\sin(bx+a)^2-1}{3\sin(bx+a)^3\,b}$$

input
```
int(csc(b*x+a)^4*sec(b*x+a),x)
```

output
```
( - 3*log(tan((a + b*x)/2) - 1)*sin(a + b*x)**3 + 3*log(tan((a + b*x)/2) +
 1)*sin(a + b*x)**3 - 3*sin(a + b*x)**2 - 1)/(3*sin(a + b*x)**3*b)
```

3.167 $\int \csc^4(a+bx)\sec^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 37

$$\int \csc^4(a+bx)\sec^2(a+bx)\,dx = -\frac{2\cot(a+bx)}{b} - \frac{\cot^3(a+bx)}{3b} + \frac{\tan(a+bx)}{b}$$

output
```
-2*cot(b*x+a)/b-1/3*cot(b*x+a)^3/b+tan(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

$$\int \csc^4(a+bx)\sec^2(a+bx)\,dx = -\frac{5\cot(a+bx)}{3b} - \frac{\cot(a+bx)\csc^2(a+bx)}{3b} + \frac{\tan(a+bx)}{b}$$

input
```
Integrate[Csc[a + b*x]^4*Sec[a + b*x]^2,x]
```

output
```
(-5*Cot[a + b*x])/(3*b) - (Cot[a + b*x]*Csc[a + b*x]^2)/(3*b) + Tan[a + b*
x]/b
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(a+bx)\sec^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(a+bx)^4\sec(a+bx)^2dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{\int \cot^4(a+bx)\left(\tan^2(a+bx)+1\right)^2d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\cot^4(a+bx)+2\cot^2(a+bx)+1\right)d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\tan(a+bx)-\frac{1}{3}\cot^3(a+bx)-2\cot(a+bx)}{b}$$

input
```
Int[Csc[a + b*x]^4*Sec[a + b*x]^2,x]
```

output
```
(-2*Cot[a + b*x] - Cot[a + b*x]^3/3 + Tan[a + b*x])/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.24

method	result	size
risch	$\frac{16i\left(2\,\mathrm{e}^{2i(bx+a)}-1\right)}{3b\left(\mathrm{e}^{2i(bx+a)}-1\right)^{3}\left(\mathrm{e}^{2i(bx+a)}+1\right)}$	46
derivativedivides	$\frac{-\frac{1}{3\sin(bx+a)^{3}\cos(bx+a)}+\frac{4}{3\sin(bx+a)\cos(bx+a)}-\frac{8\cot(bx+a)}{3}}{b}$	50
default	$\frac{-\frac{1}{3\sin(bx+a)^{3}\cos(bx+a)}+\frac{4}{3\sin(bx+a)\cos(bx+a)}-\frac{8\cot(bx+a)}{3}}{b}$	50
parallelrisch	$\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{5}+20\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{3}+\cot\left(\frac{bx}{2}+\frac{a}{2}\right)^{3}-90\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+20\cot\left(\frac{bx}{2}+\frac{a}{2}\right)}{24b\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-24b}$	80
norman	$\frac{\frac{1}{24b}+\frac{5\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}{6b}-\frac{15\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}}{4b}+\frac{5\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{6}}{6b}+\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{8}}{24b}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{3}\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)}$	98

input
```
int(csc(b*x+a)^4*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
16/3*I*(2*exp(2*I*(b*x+a))-1)/b/(exp(2*I*(b*x+a))-1)^3/(exp(2*I*(b*x+a))+1
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.46

$$\int \csc^4(a+bx)\sec^2(a+bx)\,dx = -\frac{8\cos(bx+a)^4-12\cos(bx+a)^2+3}{3\left(b\cos(bx+a)^3-b\cos(bx+a)\right)\sin(bx+a)}$$

input
```
integrate(csc(b*x+a)^4*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/3*(8*cos(b*x + a)^4 - 12*cos(b*x + a)^2 + 3)/((b*cos(b*x + a)^3 - b*cos
(b*x + a))*sin(b*x + a))
```

Sympy [F]

$$\int \csc^4(a+bx)\sec^2(a+bx)\,dx = \int \csc^4(a+bx)\sec^2(a+bx)\,dx$$

input
```
integrate(csc(b*x+a)**4*sec(b*x+a)**2,x)
```

output
```
Integral(csc(a + b*x)**4*sec(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \csc^4(a+bx)\sec^2(a+bx)\,dx = -\frac{\frac{6\tan(bx+a)^2+1}{\tan(bx+a)^3}-3\tan(bx+a)}{3b}$$

input
```
integrate(csc(b*x+a)^4*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/3*((6*tan(b*x + a)^2 + 1)/tan(b*x + a)^3 - 3*tan(b*x + a))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \csc^4(a+bx)\sec^2(a+bx)\,dx = -\frac{\frac{6\tan(bx+a)^2+1}{\tan(bx+a)^3} - 3\tan(bx+a)}{3b}$$

input
```
integrate(csc(b*x+a)^4*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/3*((6*tan(b*x + a)^2 + 1)/tan(b*x + a)^3 - 3*tan(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 25.55 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \csc^4(a+bx)\sec^2(a+bx)\,dx = \frac{\tan(a+b\,x)}{b} - \frac{2\tan(a+b\,x)^2+\frac{1}{3}}{b\tan(a+b\,x)^3}$$

input
```
int(1/(cos(a + b*x)^2*sin(a + b*x)^4),x)
```

output
```
tan(a + b*x)/b - (2*tan(a + b*x)^2 + 1/3)/(b*tan(a + b*x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.16

$$\int \csc^4(a+bx)\sec^2(a+bx)\,dx = \frac{8\sin(bx+a)^4 - 4\sin(bx+a)^2 - 1}{3\cos(bx+a)\sin(bx+a)^3 b}$$

input
```
int(csc(b*x+a)^4*sec(b*x+a)^2,x)
```

output
```
(8*sin(a + b*x)**4 - 4*sin(a + b*x)**2 - 1)/(3*cos(a + b*x)*sin(a + b*x)**
3*b)
```

3.168 $\int \csc^4(a+bx)\sec^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 60

$$\int \csc^4(a+bx)\sec^3(a+bx)\,dx=\frac{5\text{arctanh}(\sin(a+bx))}{2b}-\frac{2\csc(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}+\frac{\sec(a+bx)\tan(a+bx)}{2b}$$

output
```
5/2*arctanh(sin(b*x+a))/b-2*csc(b*x+a)/b-1/3*csc(b*x+a)^3/b+1/2*sec(b*x+a)
*tan(b*x+a)/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.52

$$\int \csc^4(a+bx)\sec^3(a+bx)\,dx=-\frac{\csc^3(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},2,-\frac{1}{2},\sin^2(a+bx)\right)}{3b}$$

input
```
Integrate[Csc[a + b*x]^4*Sec[a + b*x]^3,x]
```

output
```
-1/3*(Csc[a + b*x]^3*Hypergeometric2F1[-3/2, 2, -1/2, Sin[a + b*x]^2])/b
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3042, 3101, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(a+bx)\sec^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(a+bx)^4\sec(a+bx)^3dx$$

$$\Big\downarrow \text{3101}$$

$$-\frac{\int \frac{\csc^6(a+bx)}{(1-\csc^2(a+bx))^2}d\csc(a+bx)}{b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{\frac{\csc^5(a+bx)}{2(1-\csc^2(a+bx))}-\frac{5}{2}\int \frac{\csc^4(a+bx)}{1-\csc^2(a+bx)}d\csc(a+bx)}{b}$$

$$\Big\downarrow \text{254}$$

$$-\frac{\frac{\csc^5(a+bx)}{2(1-\csc^2(a+bx))}-\frac{5}{2}\int \left(-\csc^2(a+bx)+\frac{1}{1-\csc^2(a+bx)}-1\right)d\csc(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{\csc^5(a+bx)}{2(1-\csc^2(a+bx))}-\frac{5}{2}\left(\text{arctanh}(\csc(a+bx))-\frac{1}{3}\csc^3(a+bx)-\csc(a+bx)\right)}{b}$$

input
```
Int[Csc[a + b*x]^4*Sec[a + b*x]^3,x]
```

output
```
-((Csc[a + b*x]^5/(2*(1 - Csc[a + b*x]^2)) - (5*(ArcTanh[Csc[a + b*x]] - C
sc[a + b*x] - Csc[a + b*x]^3/3))/2)/b)
```

Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 2.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{-\frac{1}{3\sin(bx+a)^{3}\cos(bx+a)^{2}}+\frac{5}{6\sin(bx+a)\cos(bx+a)^{2}}-\frac{5}{2\sin(bx+a)}+\frac{5\ln(\sec(bx+a)+\tan(bx+a))}{2}}{b}$
default	$\frac{-\frac{1}{3\sin(bx+a)^{3}\cos(bx+a)^{2}}+\frac{5}{6\sin(bx+a)\cos(bx+a)^{2}}-\frac{5}{2\sin(bx+a)}+\frac{5\ln(\sec(bx+a)+\tan(bx+a))}{2}}{b}$
risch	$-\frac{i(15\,e^{9i(bx+a)}-20\,e^{7i(bx+a)}-22\,e^{5i(bx+a)}-20\,e^{3i(bx+a)}+15\,e^{i(bx+a)})}{3b(e^{2i(bx+a)}-1)^{3}(e^{2i(bx+a)}+1)^{2}}-\frac{5\ln(e^{i(bx+a)}-i)}{2b}+\frac{5\ln(e^{i(bx+a)}+i)}{2b}$
norman	$\frac{-\frac{1}{24b}-\frac{25\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}{24b}+\frac{25\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}}{12b}+\frac{25\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{6}}{12b}-\frac{25\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{8}}{24b}-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{10}}{24b}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{3}\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)^{2}}-\frac{5\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{2b}$ …
parallelrisch	$\frac{30(-\cos(2bx+2a)-1)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+30(1+\cos(2bx+2a))\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)+60(-3+\cos(bx+a))\cot\left(\frac{bx}{2}+\cdots\right.}{12b(1+\cos(2bx+2a))}$ …

input
```
int(csc(b*x+a)^4*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/3/sin(b*x+a)^3/cos(b*x+a)^2+5/6/sin(b*x+a)/cos(b*x+a)^2-5/2/sin(b*
x+a)+5/2*ln(sec(b*x+a)+tan(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(54) = 108.

Time = 0.11 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.17

$$\int \csc^{4}(a+bx)\sec^{3}(a+bx)\,dx =$$

$$-\frac{30\cos(bx+a)^{4}-15\left(\cos(bx+a)^{4}-\cos(bx+a)^{2}\right)\log(\sin(bx+a)+1)\sin(bx+a)+15\left(\cos(bx+\cdots\right.}{12\left(b\cos(bx+a)^{4}-b\cos(bx+a)^{2}\cdots\right.}$$

input
```
integrate(csc(b*x+a)^4*sec(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/12*(30*cos(b*x + a)^4 - 15*(cos(b*x + a)^4 - cos(b*x + a)^2)*log(sin(b*
x + a) + 1)*sin(b*x + a) + 15*(cos(b*x + a)^4 - cos(b*x + a)^2)*log(-sin(b
*x + a) + 1)*sin(b*x + a) - 40*cos(b*x + a)^2 + 6)/((b*cos(b*x + a)^4 - b*
cos(b*x + a)^2)*sin(b*x + a))
```

Sympy [F]

$$\int \csc^4(a+bx)\sec^3(a+bx)\,dx = \int \csc^4{(a+bx)}\sec^3{(a+bx)}\,dx$$

input
```
integrate(csc(b*x+a)**4*sec(b*x+a)**3,x)
```

output
```
Integral(csc(a + b*x)**4*sec(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.22

$$\int \csc^4(a+bx)\sec^3(a+bx)\,dx$$
$$= -\frac{\frac{2\left(15\sin(bx+a)^4-10\sin(bx+a)^2-2\right)}{\sin(bx+a)^5-\sin(bx+a)^3} - 15\log(\sin(bx+a)+1) + 15\log(\sin(bx+a)-1)}{12\,b}$$

input
```
integrate(csc(b*x+a)^4*sec(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/12*(2*(15*sin(b*x + a)^4 - 10*sin(b*x + a)^2 - 2)/(sin(b*x + a)^5 - sin
(b*x + a)^3) - 15*log(sin(b*x + a) + 1) + 15*log(sin(b*x + a) - 1))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.32

$$\int \csc^4(a+bx)\sec^3(a+bx)\,dx = \frac{5\log(|\sin(bx+a)+1|)}{4b} - \frac{5\log(|\sin(bx+a)-1|)}{4b} - \frac{\sin(bx+a)}{2\left(\sin(bx+a)^2-1\right)b} - \frac{6\sin(bx+a)^2+1}{3b\sin(bx+a)^3}$$

input
```
integrate(csc(b*x+a)^4*sec(b*x+a)^3,x, algorithm="giac")
```

output
```
5/4*log(abs(sin(b*x + a) + 1))/b - 5/4*log(abs(sin(b*x + a) - 1))/b - 1/2*
sin(b*x + a)/((sin(b*x + a)^2 - 1)*b) - 1/3*(6*sin(b*x + a)^2 + 1)/(b*sin(
b*x + a)^3)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02

$$\int \csc^4(a+bx)\sec^3(a+bx)\,dx = \frac{5\,\mathrm{atanh}(\sin(a+b\,x))}{2\,b} - \frac{-\frac{5\sin(a+b\,x)^4}{2}+\frac{5\sin(a+b\,x)^2}{3}+\frac{1}{3}}{b\,\left(\sin(a+b\,x)^3-\sin(a+b\,x)^5\right)}$$

input
```
int(1/(cos(a + b*x)^3*sin(a + b*x)^4),x)
```

output
```
(5*atanh(sin(a + b*x)))/(2*b) - ((5*sin(a + b*x)^2)/3 - (5*sin(a + b*x)^4)
/2 + 1/3)/(b*(sin(a + b*x)^3 - sin(a + b*x)^5))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.25

$$\int \csc^4(a+bx)\sec^3(a+bx)\,dx$$
$$= \frac{-15\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin(bx+a)^5+15\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin(bx+a)^3+15\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right.}{6\sin(bx+a)^3 b\,(\sin}$$

input
```
int(csc(b*x+a)^4*sec(b*x+a)^3,x)
```

output
```
( - 15*log(tan((a + b*x)/2) - 1)*sin(a + b*x)**5 + 15*log(tan((a + b*x)/2)
 - 1)*sin(a + b*x)**3 + 15*log(tan((a + b*x)/2) + 1)*sin(a + b*x)**5 - 15*
log(tan((a + b*x)/2) + 1)*sin(a + b*x)**3 - 15*sin(a + b*x)**4 + 10*sin(a
+ b*x)**2 + 2)/(6*sin(a + b*x)**3*b*(sin(a + b*x)**2 - 1))
```

3.169 $\int \csc^4(a+bx)\sec^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 53

$$\int \csc^4(a+bx)\sec^4(a+bx)\,dx = -\frac{3\cot(a+bx)}{b}-\frac{\cot^3(a+bx)}{3b}+\frac{3\tan(a+bx)}{b}+\frac{\tan^3(a+bx)}{3b}$$

output
```
-3*cot(b*x+a)/b-1/3*cot(b*x+a)^3/b+3*tan(b*x+a)/b+1/3*tan(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

$$\int \csc^4(a+bx)\sec^4(a+bx)\,dx = 16\left(-\frac{\cot(2(a+bx))}{3b}-\frac{\cot(2(a+bx))\csc^2(2(a+bx))}{6b}\right)$$

input
```
Integrate[Csc[a + b*x]^4*Sec[a + b*x]^4,x]
```

output
```
16*(-1/3*Cot[2*(a + b*x)]/b - (Cot[2*(a + b*x)]*Csc[2*(a + b*x)]^2)/(6*b))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(a+bx)\sec^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(a+bx)^4\sec(a+bx)^4dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{\int \cot^4(a+bx)\left(\tan^2(a+bx)+1\right)^3d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\cot^4(a+bx)+3\cot^2(a+bx)+\tan^2(a+bx)+3\right)d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{3}\tan^3(a+bx)+3\tan(a+bx)-\frac{1}{3}\cot^3(a+bx)-3\cot(a+bx)}{b}$$

input
```
Int[Csc[a + b*x]^4*Sec[a + b*x]^4,x]
```

output
```
(-3*Cot[a + b*x] - Cot[a + b*x]^3/3 + 3*Tan[a + b*x] + Tan[a + b*x]^3/3)/b
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

method	result
risch	$\frac{32i\left(3\,\mathrm{e}^{4i(bx+a)}-1\right)}{3b\left(\mathrm{e}^{2i(bx+a)}-1\right)^{3}\left(\mathrm{e}^{2i(bx+a)}+1\right)^{3}}$
derivativedivides	$\frac{\frac{1}{3\sin(bx+a)^{3}\cos(bx+a)^{3}}-\frac{2}{3\sin(bx+a)^{3}\cos(bx+a)}+\frac{8}{3\sin(bx+a)\cos(bx+a)}-\frac{16\cot(bx+a)}{3}}{b}$
default	$\frac{\frac{1}{3\sin(bx+a)^{3}\cos(bx+a)^{3}}-\frac{2}{3\sin(bx+a)^{3}\cos(bx+a)}+\frac{8}{3\sin(bx+a)\cos(bx+a)}-\frac{16\cot(bx+a)}{3}}{b}$
parallelrisch	$\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{9}+30\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{7}-273\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{5}+420\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{3}+\cot\left(\frac{bx}{2}+\frac{a}{2}\right)^{3}-273\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+30\cot\left(\frac{bx}{2}\right.}{24b\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^{3}\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)^{3}}$
norman	$\frac{\frac{1}{24b}+\frac{5\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}{4b}-\frac{91\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}}{8b}+\frac{35\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{6}}{2b}-\frac{91\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{8}}{8b}+\frac{5\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{10}}{4b}+\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{12}}{24b}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{3}\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)^{3}}$

input
```
int(csc(b*x+a)^4*sec(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
32/3*I*(3*exp(4*I*(b*x+a))-1)/b/(exp(2*I*(b*x+a))-1)^3/(exp(2*I*(b*x+a))+1
)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.25

$$\int \csc^4(a+bx)\sec^4(a+bx)\,dx = -\frac{16\cos(bx+a)^6-24\cos(bx+a)^4+6\cos(bx+a)^2+1}{3\left(b\cos(bx+a)^5-b\cos(bx+a)^3\right)\sin(bx+a)}$$

input
```
integrate(csc(b*x+a)^4*sec(b*x+a)^4,x, algorithm="fricas")
```

output
```
-1/3*(16*cos(b*x + a)^6 - 24*cos(b*x + a)^4 + 6*cos(b*x + a)^2 + 1)/((b*co
s(b*x + a)^5 - b*cos(b*x + a)^3)*sin(b*x + a))
```

Sympy [F]

$$\int \csc^4(a+bx)\sec^4(a+bx)\,dx = \int \csc^4(a+bx)\sec^4(a+bx)\,dx$$

input
```
integrate(csc(b*x+a)**4*sec(b*x+a)**4,x)
```

output
```
Integral(csc(a + b*x)**4*sec(a + b*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int \csc^4(a+bx)\sec^4(a+bx)\,dx = \frac{\tan(bx+a)^3 - \frac{9\,\tan(bx+a)^2+1}{\tan(bx+a)^3} + 9\,\tan(bx+a)}{3\,b}$$

input
```
integrate(csc(b*x+a)^4*sec(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/3*(tan(b*x + a)^3 - (9*tan(b*x + a)^2 + 1)/tan(b*x + a)^3 + 9*tan(b*x +
a))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.58

$$\int \csc^4(a+bx)\sec^4(a+bx)\,dx = -\frac{8\,(3\,\tan(2\,bx+2\,a)^2+1)}{3\,b\tan(2\,bx+2\,a)^3}$$

input
```
integrate(csc(b*x+a)^4*sec(b*x+a)^4,x, algorithm="giac")
```

output
```
-8/3*(3*tan(2*b*x + 2*a)^2 + 1)/(b*tan(2*b*x + 2*a)^3)
```

Mupad [B] (verification not implemented)

Time = 25.43 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \csc^4(a+bx)\sec^4(a+bx)\,dx = -\frac{-\tan(a+b\,x)^6 - 9\tan(a+b\,x)^4 + 9\tan(a+b\,x)^2 + 1}{3\,b\tan(a+b\,x)^3}$$

input
```
int(1/(cos(a + b*x)^4*sin(a + b*x)^4),x)
```

output
```
-(9*tan(a + b*x)^2 - 9*tan(a + b*x)^4 - tan(a + b*x)^6 + 1)/(3*b*tan(a + b
*x)^3)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.23

$$\int \csc^4(a+bx)\sec^4(a+bx)\,dx = \frac{16\sin(bx+a)^6 - 24\sin(bx+a)^4 + 6\sin(bx+a)^2 + 1}{3\cos(bx+a)\sin(bx+a)^3 b\left(\sin(bx+a)^2 - 1\right)}$$

input
```
int(csc(b*x+a)^4*sec(b*x+a)^4,x)
```

output
```
(16*sin(a + b*x)**6 - 24*sin(a + b*x)**4 + 6*sin(a + b*x)**2 + 1)/(3*cos(a
 + b*x)*sin(a + b*x)**3*b*(sin(a + b*x)**2 - 1))
```

3.170 $\int \csc^4(a+bx)\sec^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 81

$$\int \csc^4(a+bx)\sec^5(a+bx)\,dx = \frac{35\text{arctanh}(\sin(a+bx))}{8b} - \frac{3\csc(a+bx)}{b} - \frac{\csc^3(a+bx)}{3b} + \frac{13\sec(a+bx)\tan(a+bx)}{8b} + \frac{\sec(a+bx)\tan^3(a+bx)}{4b}$$

output
```
35/8*arctanh(sin(b*x+a))/b-3*csc(b*x+a)/b-1/3*csc(b*x+a)^3/b+13/8*sec(b*x+
a)*tan(b*x+a)/b+1/4*sec(b*x+a)*tan(b*x+a)^3/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.38

$$\int \csc^4(a+bx)\sec^5(a+bx)\,dx = -\frac{\csc^3(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},3,-\frac{1}{2},\sin^2(a+bx)\right)}{3b}$$

input
```
Integrate[Csc[a + b*x]^4*Sec[a + b*x]^5,x]
```

output
```
-1/3*(Csc[a + b*x]^3*Hypergeometric2F1[-3/2, 3, -1/2, Sin[a + b*x]^2])/b
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3042, 3101, 25, 252, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(a+bx)\sec^5(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(a+bx)^4\sec(a+bx)^5dx$$

$$\Big\downarrow \text{3101}$$

$$-\frac{\int -\frac{\csc^8(a+bx)}{(1-\csc^2(a+bx))^3}d\csc(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\csc^8(a+bx)}{(1-\csc^2(a+bx))^3}d\csc(a+bx)}{b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{\frac{7}{4}\int \frac{\csc^6(a+bx)}{(1-\csc^2(a+bx))^2}d\csc(a+bx)-\frac{\csc^7(a+bx)}{4(1-\csc^2(a+bx))^2}}{b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{\frac{7}{4}\left(\frac{\csc^5(a+bx)}{2(1-\csc^2(a+bx))}-\frac{5}{2}\int \frac{\csc^4(a+bx)}{1-\csc^2(a+bx)}d\csc(a+bx)\right)-\frac{\csc^7(a+bx)}{4(1-\csc^2(a+bx))^2}}{b}$$

$$\Big\downarrow \text{254}$$

$$-\frac{\frac{7}{4}\left(\frac{\csc^5(a+bx)}{2(1-\csc^2(a+bx))}-\frac{5}{2}\int\left(-\csc^2(a+bx)+\frac{1}{1-\csc^2(a+bx)}-1\right)d\csc(a+bx)\right)-\frac{\csc^7(a+bx)}{4(1-\csc^2(a+bx))^2}}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{7}{4}\left(\frac{\csc^5(a+bx)}{2(1-\csc^2(a+bx))}-\frac{5}{2}\left(\operatorname{arctanh}(\csc(a+bx))-\frac{1}{3}\csc^3(a+bx)-\csc(a+bx)\right)\right)-\frac{\csc^7(a+bx)}{4(1-\csc^2(a+bx))^2}}{b}$$

input
```
Int[Csc[a + b*x]^4*Sec[a + b*x]^5,x]
```

output
```
-((-1/4*Csc[a + b*x]^7/(1 - Csc[a + b*x]^2)^2 + (7*(Csc[a + b*x]^5/(2*(1 -
 Csc[a + b*x]^2)) - (5*(ArcTanh[Csc[a + b*x]] - Csc[a + b*x] - Csc[a + b*x
]^3/3))/2))/4)/b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3101
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Simp[-(f*a^n)^(-1)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
 + 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 2.49 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{\frac{1}{4\sin(bx+a)^3\cos(bx+a)^4}-\frac{7}{12\sin(bx+a)^3\cos(bx+a)^2}+\frac{35}{24\sin(bx+a)\cos(bx+a)^2}-\frac{35}{8\sin(bx+a)}+\frac{35\ln(\sec(bx+a)+\tan(bx+a))}{8}}{b}$
default	$\frac{\frac{1}{4\sin(bx+a)^3\cos(bx+a)^4}-\frac{7}{12\sin(bx+a)^3\cos(bx+a)^2}+\frac{35}{24\sin(bx+a)\cos(bx+a)^2}-\frac{35}{8\sin(bx+a)}+\frac{35\ln(\sec(bx+a)+\tan(bx+a))}{8}}{b}$
risch	$-\frac{i\left(105\,e^{13i(bx+a)}+70\,e^{11i(bx+a)}-329\,e^{9i(bx+a)}-204\,e^{7i(bx+a)}-329\,e^{5i(bx+a)}+70\,e^{3i(bx+a)}+105\,e^{i(bx+a)}\right)}{12b\left(e^{2i(bx+a)}+1\right)^4\left(e^{2i(bx+a)}-1\right)^3}-\frac{35\ln\ldots}{}$
norman	$\frac{-\frac{1}{24b}-\frac{35\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}{24b}+\frac{63\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}{8b}-\frac{35\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^6}{8b}-\frac{35\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^8}{8b}+\frac{63\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{10}}{8b}-\frac{35\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{12}}{24b}-\tan\ldots}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^3\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1\right)^4}$
parallelrisch	$\frac{35\left(\left(\sin(7bx+7a)-3\sin(bx+a)-3\sin(3bx+3a)+\sin(5bx+5a)\right)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+\left(3\sin(bx+a)+3\sin(3bx+3a)-\sin\ldots\right.\right.}{512b(\cos(4bx+\ldots}$

input
```
int(csc(b*x+a)^4*sec(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/4/sin(b*x+a)^3/cos(b*x+a)^4-7/12/sin(b*x+a)^3/cos(b*x+a)^2+35/24/si
n(b*x+a)/cos(b*x+a)^2-35/8/sin(b*x+a)+35/8*ln(sec(b*x+a)+tan(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.73

$$\int \csc^4(a+bx)\sec^5(a+bx)\,dx = -\frac{210\,\cos(bx+a)^6-280\,\cos(bx+a)^4-105\left(\cos(bx+a)^6-\cos(bx+a)^4\right)\log(\sin(bx+a)+1)\sin(\ldots}{48\left(b\cos(bx+a)^6-b\ldots\right.}$$

input
```
integrate(csc(b*x+a)^4*sec(b*x+a)^5,x, algorithm="fricas")
```

output
```
-1/48*(210*cos(b*x + a)^6 - 280*cos(b*x + a)^4 - 105*(cos(b*x + a)^6 - cos
(b*x + a)^4)*log(sin(b*x + a) + 1)*sin(b*x + a) + 105*(cos(b*x + a)^6 - co
s(b*x + a)^4)*log(-sin(b*x + a) + 1)*sin(b*x + a) + 42*cos(b*x + a)^2 + 12
)/((b*cos(b*x + a)^6 - b*cos(b*x + a)^4)*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^4(a+bx)\sec^5(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**4*sec(b*x+a)**5,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.12

$$\int \csc^4(a+bx)\sec^5(a+bx)\,dx = -\frac{\frac{2\left(105\sin(bx+a)^6-175\sin(bx+a)^4+56\sin(bx+a)^2+8\right)}{\sin(bx+a)^7-2\sin(bx+a)^5+\sin(bx+a)^3}-105\log\left(\sin\left(bx+a\right)+1\right)+105\log\left(\sin\left(bx+a\right)-1\right)}{48\,b}$$

input
```
integrate(csc(b*x+a)^4*sec(b*x+a)^5,x, algorithm="maxima")
```

output
```
-1/48*(2*(105*sin(b*x + a)^6 - 175*sin(b*x + a)^4 + 56*sin(b*x + a)^2 + 8)
/(sin(b*x + a)^7 - 2*sin(b*x + a)^5 + sin(b*x + a)^3) - 105*log(sin(b*x +
a) + 1) + 105*log(sin(b*x + a) - 1))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.14

$$\int \csc^4(a+bx)\sec^5(a+bx)\,dx = \frac{35\log\left(|\sin\left(bx+a\right)+1|\right)}{16\,b} - \frac{35\log\left(|\sin\left(bx+a\right)-1|\right)}{16\,b} - \frac{11\sin\left(bx+a\right)^3-13\sin\left(bx+a\right)}{8\left(\sin\left(bx+a\right)^2-1\right)^2 b} - \frac{9\sin\left(bx+a\right)^2+1}{3\,b\sin\left(bx+a\right)^3}$$

input
```
integrate(csc(b*x+a)^4*sec(b*x+a)^5,x, algorithm="giac")
```

output
```
35/16*log(abs(sin(b*x + a) + 1))/b - 35/16*log(abs(sin(b*x + a) - 1))/b -
1/8*(11*sin(b*x + a)^3 - 13*sin(b*x + a))/((sin(b*x + a)^2 - 1)^2*b) - 1/3
*(9*sin(b*x + a)^2 + 1)/(b*sin(b*x + a)^3)
```

Mupad [B] (verification not implemented)

Time = 25.51 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98

$$\int \csc^4(a+bx)\sec^5(a+bx)\,dx = \frac{35\,\mathrm{atanh}(\sin\left(a+b\,x\right))}{8\,b} - \frac{\frac{35\sin(a+b\,x)^6}{8}-\frac{175\sin(a+b\,x)^4}{24}+\frac{7\sin(a+b\,x)^2}{3}+\frac{1}{3}}{b\,\left(\sin\left(a+b\,x\right)^7-2\sin\left(a+b\,x\right)^5+\sin\left(a+b\,x\right)^3\right)}$$

input
```
int(1/(cos(a + b*x)^5*sin(a + b*x)^4),x)
```

output
```
(35*atanh(sin(a + b*x)))/(8*b) - ((7*sin(a + b*x)^2)/3 - (175*sin(a + b*x)
^4)/24 + (35*sin(a + b*x)^6)/8 + 1/3)/(b*(sin(a + b*x)^3 - 2*sin(a + b*x)^
5 + sin(a + b*x)^7))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.46

$$\int \csc^4(a+bx)\sec^5(a+bx)\,dx$$

$$= \frac{-105\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin(bx+a)^7+210\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin(bx+a)^5-105\log\left(\tan\left(\frac{bx}{2}+\right.\right.}{}$$

input
```
int(csc(b*x+a)^4*sec(b*x+a)^5,x)
```

output
```
( - 105*log(tan((a + b*x)/2) - 1)*sin(a + b*x)**7 + 210*log(tan((a + b*x)/
2) - 1)*sin(a + b*x)**5 - 105*log(tan((a + b*x)/2) - 1)*sin(a + b*x)**3 +
105*log(tan((a + b*x)/2) + 1)*sin(a + b*x)**7 - 210*log(tan((a + b*x)/2) +
 1)*sin(a + b*x)**5 + 105*log(tan((a + b*x)/2) + 1)*sin(a + b*x)**3 - 105*
sin(a + b*x)**6 + 175*sin(a + b*x)**4 - 56*sin(a + b*x)**2 - 8)/(24*sin(a
+ b*x)**3*b*(sin(a + b*x)**4 - 2*sin(a + b*x)**2 + 1))
```

3.171 $\int \cos^4(a+bx)\cot^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 69

$$\int \cos^4(a+bx)\cot^5(a+bx)\,dx = \frac{2\csc^2(a+bx)}{b} - \frac{\csc^4(a+bx)}{4b} + \frac{6\log(\sin(a+bx))}{b} - \frac{2\sin^2(a+bx)}{b} + \frac{\sin^4(a+bx)}{4b}$$

output
```
2*csc(b*x+a)^2/b-1/4*csc(b*x+a)^4/b+6*ln(sin(b*x+a))/b-2*sin(b*x+a)^2/b+1/
4*sin(b*x+a)^4/b
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \cos^4(a+bx)\cot^5(a+bx)\,dx = \frac{2\csc^2(a+bx)}{b} - \frac{\csc^4(a+bx)}{4b} + \frac{6\log(\sin(a+bx))}{b} - \frac{2\sin^2(a+bx)}{b} + \frac{\sin^4(a+bx)}{4b}$$

input
```
Integrate[Cos[a + b*x]^4*Cot[a + b*x]^5,x]
```

output
```
(2*Csc[a + b*x]^2)/b - Csc[a + b*x]^4/(4*b) + (6*Log[Sin[a + b*x]])/b - (2
*Sin[a + b*x]^2)/b + Sin[a + b*x]^4/(4*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3042, 25, 3070, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(a+bx)\cot^5(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\sin\left(a+bx+\frac{\pi}{2}\right)^4\tan\left(a+bx+\frac{\pi}{2}\right)^5dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sin\left(\frac{1}{2}(2a+\pi)+bx\right)^4\tan\left(\frac{1}{2}(2a+\pi)+bx\right)^5dx$$

$$\Big\downarrow \text{3070}$$

$$\frac{\int -\csc^5(a+bx)\left(1-\sin^2(a+bx)\right)^4d(-\sin(a+bx))}{b}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int -\csc^3(a+bx)(\sin(a+bx)+1)^4d\sin^2(a+bx)}{2b}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(-\csc^3(a+bx)-4\csc^2(a+bx)-6\csc(a+bx)+\sin^2(a+bx)-4\right)d\sin^2(a+bx)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}\sin^2(a+bx)+4\sin(a+bx)-\frac{1}{2}\csc^2(a+bx)-4\csc(a+bx)+6\log\left(\sin^2(a+bx)\right)}{2b}$$

input
```
Int[Cos[a + b*x]^4*Cot[a + b*x]^5,x]
```

output
```
(-4*Csc[a + b*x] - Csc[a + b*x]^2/2 + 6*Log[Sin[a + b*x]^2] + 4*Sin[a + b*
x] + Sin[a + b*x]^2/2)/(2*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Maple [A] (verified)

Time = 9.21 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{-\frac{\cos(bx+a)^{10}}{4\sin(bx+a)^4}+\frac{3\cos(bx+a)^{10}}{4\sin(bx+a)^2}+\frac{3\cos(bx+a)^8}{4}+\cos(bx+a)^6+\frac{3\cos(bx+a)^4}{2}+3\cos(bx+a)^2+6\ln(\sin(bx+a))}{b}$
default	$\frac{-\frac{\cos(bx+a)^{10}}{4\sin(bx+a)^4}+\frac{3\cos(bx+a)^{10}}{4\sin(bx+a)^2}+\frac{3\cos(bx+a)^8}{4}+\cos(bx+a)^6+\frac{3\cos(bx+a)^4}{2}+3\cos(bx+a)^2+6\ln(\sin(bx+a))}{b}$
risch	$-6ix+\frac{e^{4i(bx+a)}}{64b}+\frac{7e^{2i(bx+a)}}{16b}+\frac{7e^{-2i(bx+a)}}{16b}+\frac{e^{-4i(bx+a)}}{64b}-\frac{12ia}{b}-\frac{4(2e^{6i(bx+a)}-3e^{4i(bx+a)}+2e^{2i(bx}}{b(e^{2i(bx+a)}-1)^4}$

input
```
int(cos(b*x+a)^4*cot(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/4/sin(b*x+a)^4*cos(b*x+a)^10+3/4/sin(b*x+a)^2*cos(b*x+a)^10+3/4*co
s(b*x+a)^8+cos(b*x+a)^6+3/2*cos(b*x+a)^4+3*cos(b*x+a)^2+6*ln(sin(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.45

$$\int \cos^4(a+bx)\cot^5(a+bx)\,dx$$
$$=\frac{8\cos(bx+a)^8+32\cos(bx+a)^6-115\cos(bx+a)^4+38\cos(bx+a)^2+192\left(\cos(bx+a)^4-2\cos(b\right.}{32\left(b\cos(bx+a)^4-2b\cos(bx+a)^2+b\right)}$$

input
```
integrate(cos(b*x+a)^4*cot(b*x+a)^5,x, algorithm="fricas")
```

output
```
1/32*(8*cos(b*x + a)^8 + 32*cos(b*x + a)^6 - 115*cos(b*x + a)^4 + 38*cos(b
*x + a)^2 + 192*(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*log(1/2*sin(b*x +
a)) + 29)/(b*cos(b*x + a)^4 - 2*b*cos(b*x + a)^2 + b)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^4(a+bx)\cot^5(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**4*cot(b*x+a)**5,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.81

$$\int \cos^4(a+bx)\cot^5(a+bx)\,dx = \frac{\sin(bx+a)^4 - 8\sin(bx+a)^2 + \frac{8\sin(bx+a)^2-1}{\sin(bx+a)^4} + 12\log\left(\sin(bx+a)^2\right)}{4b}$$

input
```
integrate(cos(b*x+a)^4*cot(b*x+a)^5,x, algorithm="maxima")
```

output
```
1/4*(sin(b*x + a)^4 - 8*sin(b*x + a)^2 + (8*sin(b*x + a)^2 - 1)/sin(b*x +
a)^4 + 12*log(sin(b*x + a)^2))/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07

$$\int \cos^4(a+bx)\cot^5(a+bx)\,dx = \frac{3\log\left(\left|\cos(bx+a)^2-1\right|\right)}{b} + \frac{b\cos(bx+a)^4 + 6b\cos(bx+a)^2}{4b^2} - \frac{8\cos(bx+a)^2-7}{4\left(\cos(bx+a)^2-1\right)^2 b}$$

input
```
integrate(cos(b*x+a)^4*cot(b*x+a)^5,x, algorithm="giac")
```

output
```
3*log(abs(cos(b*x + a)^2 - 1))/b + 1/4*(b*cos(b*x + a)^4 + 6*b*cos(b*x + a
)^2)/b^2 - 1/4*(8*cos(b*x + a)^2 - 7)/((cos(b*x + a)^2 - 1)^2*b)
```

Mupad [B] (verification not implemented)

Time = 26.68 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.33

$$\int \cos^4(a+bx)\cot^5(a+bx)\,dx = \frac{6\,\ln\left(\tan(a+b\,x)\right)}{b} - \frac{3\,\ln\left(\tan(a+b\,x)^2+1\right)}{b} + \frac{3\tan(a+b\,x)^6 + \frac{9\tan(a+b\,x)^4}{2} + \tan(a+b\,x)^2 - \frac{1}{4}}{b\,\left(\tan\left(a+b\,x\right)^8 + 2\tan\left(a+b\,x\right)^6 + \tan\left(a+b\,x\right)^4\right)}$$

input
```
int(cos(a + b*x)^4*cot(a + b*x)^5,x)
```

output
```
(6*log(tan(a + b*x)))/b - (3*log(tan(a + b*x)^2 + 1))/b + (tan(a + b*x)^2
+ (9*tan(a + b*x)^4)/2 + 3*tan(a + b*x)^6 - 1/4)/(b*(tan(a + b*x)^4 + 2*ta
n(a + b*x)^6 + tan(a + b*x)^8))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.43

$$\int \cos^4(a+bx)\cot^5(a+bx)\,dx = \frac{-96\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2+1\right)\sin\left(bx+a\right)^4 + 96\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\sin\left(bx+a\right)^4 + 4\sin\left(bx+a\right)^8 - 32\sin}{16\sin\left(bx+a\right)^4 b}$$

input
```
int(cos(b*x+a)^4*cot(b*x+a)^5,x)
```

output
```
( - 96*log(tan((a + b*x)/2)**2 + 1)*sin(a + b*x)**4 + 96*log(tan((a + b*x)
/2))*sin(a + b*x)**4 + 4*sin(a + b*x)**8 - 32*sin(a + b*x)**6 - 41*sin(a +
 b*x)**4 + 32*sin(a + b*x)**2 - 4)/(16*sin(a + b*x)**4*b)
```

3.172 $\int \cos^3(a+bx)\cot^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 81

$$\int \cos^3(a+bx)\cot^5(a+bx)\,dx = -\frac{35\text{arctanh}(\cos(a+bx))}{8b}+\frac{3\cos(a+bx)}{b}+\frac{\cos^3(a+bx)}{3b}+\frac{13\cot(a+bx)\csc(a+bx)}{8b}-\frac{\cot(a+bx)\csc^3(a+bx)}{4b}$$

output
```
-35/8*arctanh(cos(b*x+a))/b+3*cos(b*x+a)/b+1/3*cos(b*x+a)^3/b+13/8*cot(b*x
+a)*csc(b*x+a)/b-1/4*cot(b*x+a)*csc(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.74

$$\int \cos^3(a+bx)\cot^5(a+bx)\,dx = \frac{13\cos(a+bx)}{4b}+\frac{\cos(3(a+bx))}{12b}+\frac{13\csc^2\left(\frac{1}{2}(a+bx)\right)}{32b}-\frac{\csc^4\left(\frac{1}{2}(a+bx)\right)}{64b}-\frac{35\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{8b}+\frac{35\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{8b}-\frac{13\sec^2\left(\frac{1}{2}(a+bx)\right)}{32b}+\frac{\sec^4\left(\frac{1}{2}(a+bx)\right)}{64b}$$

input
```
Integrate[Cos[a + b*x]^3*Cot[a + b*x]^5,x]
```

output
```
(13*Cos[a + b*x])/(4*b) + Cos[3*(a + b*x)]/(12*b) + (13*Csc[(a + b*x)/2]^2
)/(32*b) - Csc[(a + b*x)/2]^4/(64*b) - (35*Log[Cos[(a + b*x)/2]])/(8*b) +
(35*Log[Sin[(a + b*x)/2]])/(8*b) - (13*Sec[(a + b*x)/2]^2)/(32*b) + Sec[(a
 + b*x)/2]^4/(64*b)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3042, 25, 3072, 252, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)\cot^5(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\sin\left(a+bx+\frac{\pi}{2}\right)^3\tan\left(a+bx+\frac{\pi}{2}\right)^5dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sin\left(\frac{1}{2}(2a+\pi)+bx\right)^3\tan\left(\frac{1}{2}(2a+\pi)+bx\right)^5dx$$

$$\Big\downarrow \text{3072}$$

$$-\frac{\int \frac{\cos^8(a+bx)}{(1-\cos^2(a+bx))^3}d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{\frac{\cos^7(a+bx)}{4(1-\cos^2(a+bx))^2}-\frac{7}{4}\int \frac{\cos^6(a+bx)}{(1-\cos^2(a+bx))^2}d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{\frac{\cos^7(a+bx)}{4(1-\cos^2(a+bx))^2}-\frac{7}{4}\left(\frac{\cos^5(a+bx)}{2(1-\cos^2(a+bx))}-\frac{5}{2}\int \frac{\cos^4(a+bx)}{1-\cos^2(a+bx)}d\cos(a+bx)\right)}{b}$$

$$\Big\downarrow \text{254}$$

$$-\frac{\frac{\cos^7(a+bx)}{4(1-\cos^2(a+bx))^2}-\frac{7}{4}\left(\frac{\cos^5(a+bx)}{2(1-\cos^2(a+bx))}-\frac{5}{2}\int\left(-\cos^2(a+bx)+\frac{1}{1-\cos^2(a+bx)}-1\right)d\cos(a+bx)\right)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{\cos^7(a+bx)}{4(1-\cos^2(a+bx))^2}-\frac{7}{4}\left(\frac{\cos^5(a+bx)}{2(1-\cos^2(a+bx))}-\frac{5}{2}\left(\operatorname{arctanh}(\cos(a+bx))-\frac{1}{3}\cos^3(a+bx)-\cos(a+bx)\right)\right)}{b}$$

input
```
Int[Cos[a + b*x]^3*Cot[a + b*x]^5,x]
```

output
```
-((Cos[a + b*x]^7/(4*(1 - Cos[a + b*x]^2)^2) - (7*(Cos[a + b*x]^5/(2*(1 -
Cos[a + b*x]^2)) - (5*(ArcTanh[Cos[a + b*x]] - Cos[a + b*x] - Cos[a + b*x]
^3/3))/2))/4)/b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Maple [A] (verified)

Time = 5.20 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{-\frac{\cos(bx+a)^9}{4\sin(bx+a)^4}+\frac{5\cos(bx+a)^9}{8\sin(bx+a)^2}+\frac{5\cos(bx+a)^7}{8}+\frac{7\cos(bx+a)^5}{8}+\frac{35\cos(bx+a)^3}{24}+\frac{35\cos(bx+a)}{8}+\frac{35\ln(\csc(bx+a)-\cot(bx+a))}{8}}{b}$
default	$\frac{-\frac{\cos(bx+a)^9}{4\sin(bx+a)^4}+\frac{5\cos(bx+a)^9}{8\sin(bx+a)^2}+\frac{5\cos(bx+a)^7}{8}+\frac{7\cos(bx+a)^5}{8}+\frac{35\cos(bx+a)^3}{24}+\frac{35\cos(bx+a)}{8}+\frac{35\ln(\csc(bx+a)-\cot(bx+a))}{8}}{b}$
risch	$\frac{e^{3i(bx+a)}}{24b}+\frac{13\,e^{i(bx+a)}}{8b}+\frac{13\,e^{-i(bx+a)}}{8b}+\frac{e^{-3i(bx+a)}}{24b}-\frac{13\,e^{7i(bx+a)}-5\,e^{5i(bx+a)}-5\,e^{3i(bx+a)}+13\,e^{i(bx+a)}}{4b\left(e^{2i(bx+a)}-1\right)^4}-$

input
```
int(cos(b*x+a)^3*cot(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/4/sin(b*x+a)^4*cos(b*x+a)^9+5/8/sin(b*x+a)^2*cos(b*x+a)^9+5/8*cos(
b*x+a)^7+7/8*cos(b*x+a)^5+35/24*cos(b*x+a)^3+35/8*cos(b*x+a)+35/8*ln(csc(b
*x+a)-cot(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.63

$$\int \cos^3(a+bx)\cot^5(a+bx)\,dx$$
$$=\frac{16\cos(bx+a)^7+112\cos(bx+a)^5-350\cos(bx+a)^3-105\left(\cos(bx+a)^4-2\cos(bx+a)^2+1\right)\log}{48\left(b\cos(bx+a)^4\right.}$$

input
```
integrate(cos(b*x+a)^3*cot(b*x+a)^5,x, algorithm="fricas")
```

output
```
1/48*(16*cos(b*x + a)^7 + 112*cos(b*x + a)^5 - 350*cos(b*x + a)^3 - 105*(c
os(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*log(1/2*cos(b*x + a) + 1/2) + 105*(c
os(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*log(-1/2*cos(b*x + a) + 1/2) + 210*c
os(b*x + a))/(b*cos(b*x + a)^4 - 2*b*cos(b*x + a)^2 + b)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(a+bx)\cot^5(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3*cot(b*x+a)**5,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.10

$$\int \cos^3(a+bx)\cot^5(a+bx)\,dx$$
$$= \frac{16 \cos(bx+a)^3 - \frac{6\left(13 \cos(bx+a)^3 - 11 \cos(bx+a)\right)}{\cos(bx+a)^4 - 2 \cos(bx+a)^2 + 1} + 144 \cos(bx+a) - 105 \log(\cos(bx+a)+1) + 105 \log(}{48\,b}$$

input
```
integrate(cos(b*x+a)^3*cot(b*x+a)^5,x, algorithm="maxima")
```

output
```
1/48*(16*cos(b*x + a)^3 - 6*(13*cos(b*x + a)^3 - 11*cos(b*x + a))/(cos(b*x
 + a)^4 - 2*cos(b*x + a)^2 + 1) + 144*cos(b*x + a) - 105*log(cos(b*x + a)
+ 1) + 105*log(cos(b*x + a) - 1))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.19

$$\int \cos^3(a+bx)\cot^5(a+bx)\,dx = -\frac{35\,\log\left(|\cos\left(bx+a\right)+1|\right)}{16\,b} + \frac{35\,\log\left(|\cos\left(bx+a\right)-1|\right)}{16\,b} + \frac{b^2\cos\left(bx+a\right)^3+9\,b^2\cos\left(bx+a\right)}{3\,b^3} - \frac{13\,\cos\left(bx+a\right)^3-11\,\cos\left(bx+a\right)}{8\,\left(\cos\left(bx+a\right)^2-1\right)^2 b}$$

input
```
integrate(cos(b*x+a)^3*cot(b*x+a)^5,x, algorithm="giac")
```

output
```
-35/16*log(abs(cos(b*x + a) + 1))/b + 35/16*log(abs(cos(b*x + a) - 1))/b +
 1/3*(b^2*cos(b*x + a)^3 + 9*b^2*cos(b*x + a))/b^3 - 1/8*(13*cos(b*x + a)^
3 - 11*cos(b*x + a))/((cos(b*x + a)^2 - 1)^2*b)
```

Mupad [B] (verification not implemented)

Time = 25.45 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.94

$$\int \cos^3(a+bx)\cot^5(a+bx)\,dx = \frac{\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4}{64\,b} - \frac{3\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2}{8\,b} + \frac{35\,\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{8\,b} + \frac{\frac{67\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^8}{8}+\frac{839\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^6}{64}+\frac{1487\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4}{192}+\frac{21\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2}{64}-\frac{1}{64}}{b\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{10}+3\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^8+3\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^6+\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4\right)}$$

input
```
int(cos(a + b*x)^3*cot(a + b*x)^5,x)
```

output
```
tan(a/2 + (b*x)/2)^4/(64*b) - (3*tan(a/2 + (b*x)/2)^2)/(8*b) + (35*log(tan
(a/2 + (b*x)/2)))/(8*b) + ((21*tan(a/2 + (b*x)/2)^2)/64 + (1487*tan(a/2 +
(b*x)/2)^4)/192 + (839*tan(a/2 + (b*x)/2)^6)/64 + (67*tan(a/2 + (b*x)/2)^8
)/8 - 1/64)/(b*(tan(a/2 + (b*x)/2)^4 + 3*tan(a/2 + (b*x)/2)^6 + 3*tan(a/2
+ (b*x)/2)^8 + tan(a/2 + (b*x)/2)^10))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.23

$$\int \cos^3(a+bx)\cot^5(a+bx)\,dx$$
$$= \frac{-64\cos(bx+a)\sin(bx+a)^6 + 640\cos(bx+a)\sin(bx+a)^4 + 312\cos(bx+a)\sin(bx+a)^2 - 48\cos(}{192\sin(bx+a)^4 b}$$

input
```
int(cos(b*x+a)^3*cot(b*x+a)^5,x)
```

output
```
( - 64*cos(a + b*x)*sin(a + b*x)**6 + 640*cos(a + b*x)*sin(a + b*x)**4 + 3
12*cos(a + b*x)*sin(a + b*x)**2 - 48*cos(a + b*x) + 840*log(tan((a + b*x)/
2))*sin(a + b*x)**4 - 847*sin(a + b*x)**4)/(192*sin(a + b*x)**4*b)
```

3.173 $\int \cos^2(a+bx)\cot^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 58

$$\int \cos^2(a+bx)\cot^5(a+bx)\,dx=\frac{3\csc^2(a+bx)}{2b}-\frac{\csc^4(a+bx)}{4b}+\frac{3\log(\sin(a+bx))}{b}-\frac{\sin^2(a+bx)}{2b}$$

output
```
3/2*csc(b*x+a)^2/b-1/4*csc(b*x+a)^4/b+3*ln(sin(b*x+a))/b-1/2*sin(b*x+a)^2/
b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \cos^2(a+bx)\cot^5(a+bx)\,dx=\frac{3\csc^2(a+bx)}{2b}-\frac{\csc^4(a+bx)}{4b}+\frac{3\log(\sin(a+bx))}{b}-\frac{\sin^2(a+bx)}{2b}$$

input
```
Integrate[Cos[a + b*x]^2*Cot[a + b*x]^5,x]
```

output
```
(3*Csc[a + b*x]^2)/(2*b) - Csc[a + b*x]^4/(4*b) + (3*Log[Sin[a + b*x]])/b
- Sin[a + b*x]^2/(2*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.78, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3042, 25, 3070, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\cot^5(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\sin\left(a+bx+\frac{\pi}{2}\right)^2\tan\left(a+bx+\frac{\pi}{2}\right)^5dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sin\left(\frac{1}{2}(2a+\pi)+bx\right)^2\tan\left(\frac{1}{2}(2a+\pi)+bx\right)^5dx$$

$$\Big\downarrow \text{3070}$$

$$\frac{\int -\csc^5(a+bx)\left(1-\sin^2(a+bx)\right)^3d(-\sin(a+bx))}{b}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int -\csc^3(a+bx)(\sin(a+bx)+1)^3d\sin^2(a+bx)}{2b}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(-\csc^3(a+bx)-3\csc^2(a+bx)-3\csc(a+bx)-1\right)d\sin^2(a+bx)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin(a+bx)-\frac{1}{2}\csc^2(a+bx)-3\csc(a+bx)+3\log\left(\sin^2(a+bx)\right)}{2b}$$

input
```
Int[Cos[a + b*x]^2*Cot[a + b*x]^5,x]
```

output
```
(-3*Csc[a + b*x] - Csc[a + b*x]^2/2 + 3*Log[Sin[a + b*x]^2] + Sin[a + b*x]
)/(2*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Maple [A] (verified)

Time = 3.02 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.40

method	result	si
derivativedivides	$\frac{-\frac{\cos(bx+a)^8}{4\sin(bx+a)^4}+\frac{\cos(bx+a)^8}{2\sin(bx+a)^2}+\frac{\cos(bx+a)^6}{2}+\frac{3\cos(bx+a)^4}{4}+\frac{3\cos(bx+a)^2}{2}+3\ln(\sin(bx+a))}{b}$	81
default	$\frac{-\frac{\cos(bx+a)^8}{4\sin(bx+a)^4}+\frac{\cos(bx+a)^8}{2\sin(bx+a)^2}+\frac{\cos(bx+a)^6}{2}+\frac{3\cos(bx+a)^4}{4}+\frac{3\cos(bx+a)^2}{2}+3\ln(\sin(bx+a))}{b}$	81
risch	$-3ix+\frac{e^{2i(bx+a)}}{8b}+\frac{e^{-2i(bx+a)}}{8b}-\frac{6ia}{b}-\frac{2(3\,e^{6i(bx+a)}-4\,e^{4i(bx+a)}+3\,e^{2i(bx+a)})}{b(e^{2i(bx+a)}-1)^4}+\frac{3\ln(e^{2i(bx+a)}-1)}{b}$	11

input
```
int(cos(b*x+a)^2*cot(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/4/sin(b*x+a)^4*cos(b*x+a)^8+1/2/sin(b*x+a)^2*cos(b*x+a)^8+1/2*cos(
b*x+a)^6+3/4*cos(b*x+a)^4+3/2*cos(b*x+a)^2+3*ln(sin(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.55

$$\int \cos^2(a+bx)\cot^5(a+bx)\,dx$$
$$=\frac{2\cos(bx+a)^6-5\cos(bx+a)^4-2\cos(bx+a)^2+12\left(\cos(bx+a)^4-2\cos(bx+a)^2+1\right)\log\left(\frac{1}{2}\sin\right.}{4\left(b\cos(bx+a)^4-2\,b\cos(bx+a)^2+b\right)}$$

input
```
integrate(cos(b*x+a)^2*cot(b*x+a)^5,x, algorithm="fricas")
```

output
```
1/4*(2*cos(b*x + a)^6 - 5*cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 12*(cos(b*x
+ a)^4 - 2*cos(b*x + a)^2 + 1)*log(1/2*sin(b*x + a)) + 4)/(b*cos(b*x + a)^
4 - 2*b*cos(b*x + a)^2 + b)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)\cot^5(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2*cot(b*x+a)**5,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.84

$$\int \cos^2(a+bx)\cot^5(a+bx)\,dx = -\frac{2\,\sin\left(bx+a\right)^2 - \frac{6\,\sin(bx+a)^2-1}{\sin(bx+a)^4} - 6\,\log\left(\sin\left(bx+a\right)^2\right)}{4\,b}$$

input
```
integrate(cos(b*x+a)^2*cot(b*x+a)^5,x, algorithm="maxima")
```

output
```
-1/4*(2*sin(b*x + a)^2 - (6*sin(b*x + a)^2 - 1)/sin(b*x + a)^4 - 6*log(sin
(b*x + a)^2))/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03

$$\int \cos^2(a+bx)\cot^5(a+bx)\,dx = \frac{\cos\left(bx+a\right)^2}{2\,b} + \frac{3\,\log\left(\left|\cos\left(bx+a\right)^2-1\right|\right)}{2\,b} - \frac{6\,\cos\left(bx+a\right)^2-5}{4\,\left(\cos\left(bx+a\right)^2-1\right)^2 b}$$

input
```
integrate(cos(b*x+a)^2*cot(b*x+a)^5,x, algorithm="giac")
```

output
```
1/2*cos(b*x + a)^2/b + 3/2*log(abs(cos(b*x + a)^2 - 1))/b - 1/4*(6*cos(b*x
 + a)^2 - 5)/((cos(b*x + a)^2 - 1)^2*b)
```

Mupad [B] (verification not implemented)

Time = 25.56 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.28

$$\int \cos^2(a+bx)\cot^5(a+bx)\,dx = \frac{3\,\ln\left(\tan(a+b\,x)\right)}{b}-\frac{3\,\ln\left({\tan(a+b\,x)}^2+1\right)}{2\,b}+\frac{\frac{3\,{\tan(a+b\,x)}^4}{2}+\frac{3\,{\tan(a+b\,x)}^2}{4}-\frac{1}{4}}{b\,\left({\tan\left(a+b\,x\right)}^6+{\tan\left(a+b\,x\right)}^4\right)}$$

input
```
int(cos(a + b*x)^2*cot(a + b*x)^5,x)
```

output
```
(3*log(tan(a + b*x)))/b - (3*log(tan(a + b*x)^2 + 1))/(2*b) + ((3*tan(a +
b*x)^2)/4 + (3*tan(a + b*x)^4)/2 - 1/4)/(b*(tan(a + b*x)^4 + tan(a + b*x)^
6))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.53

$$\int \cos^2(a+bx)\cot^5(a+bx)\,dx = \frac{-192\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2+1\right)\sin\left(bx+a\right)^4+192\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\sin\left(bx+a\right)^4-32\sin\left(bx+a\right)^6-81}{64\sin\left(bx+a\right)^4 b}$$

input
```
int(cos(b*x+a)^2*cot(b*x+a)^5,x)
```

output
```
( - 192*log(tan((a + b*x)/2)**2 + 1)*sin(a + b*x)**4 + 192*log(tan((a + b*
x)/2))*sin(a + b*x)**4 - 32*sin(a + b*x)**6 - 81*sin(a + b*x)**4 + 96*sin(
a + b*x)**2 - 16)/(64*sin(a + b*x)**4*b)
```

3.174 $\int \cos(a+bx)\cot^5(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 65

$$\int \cos(a+bx)\cot^5(a+bx)\,dx = -\frac{15\text{arctanh}(\cos(a+bx))}{8b}+\frac{\cos(a+bx)}{b}+\frac{9\cot(a+bx)\csc(a+bx)}{8b}-\frac{\cot(a+bx)\csc^3(a+bx)}{4b}$$

output
```
-15/8*arctanh(cos(b*x+a))/b+cos(b*x+a)/b+9/8*cot(b*x+a)*csc(b*x+a)/b-1/4*c
ot(b*x+a)*csc(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.89

$$\int \cos(a+bx)\cot^5(a+bx)\,dx = \frac{\cos(a+bx)}{b}+\frac{9\csc^2\left(\frac{1}{2}(a+bx)\right)}{32b}-\frac{\csc^4\left(\frac{1}{2}(a+bx)\right)}{64b}-\frac{15\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{8b}+\frac{15\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{8b}-\frac{9\sec^2\left(\frac{1}{2}(a+bx)\right)}{32b}+\frac{\sec^4\left(\frac{1}{2}(a+bx)\right)}{64b}$$

input
```
Integrate[Cos[a + b*x]*Cot[a + b*x]^5,x]
```

output
```
Cos[a + b*x]/b + (9*Csc[(a + b*x)/2]^2)/(32*b) - Csc[(a + b*x)/2]^4/(64*b)
 - (15*Log[Cos[(a + b*x)/2]])/(8*b) + (15*Log[Sin[(a + b*x)/2]])/(8*b) - (
9*Sec[(a + b*x)/2]^2)/(32*b) + Sec[(a + b*x)/2]^4/(64*b)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.28, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3042, 25, 3072, 252, 252, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\cot^5(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\sin\left(a+bx+\frac{\pi}{2}\right)\tan\left(a+bx+\frac{\pi}{2}\right)^5dx$$

$$\Big\downarrow \; 25$$

$$-\int \sin\left(\frac{1}{2}(2a+\pi)+bx\right)\tan\left(\frac{1}{2}(2a+\pi)+bx\right)^5dx$$

$$\Big\downarrow \; 3072$$

$$-\frac{\int \frac{\cos^6(a+bx)}{(1-\cos^2(a+bx))^3}d\cos(a+bx)}{b}$$

$$\Big\downarrow \; 252$$

$$-\frac{\frac{\cos^5(a+bx)}{4(1-\cos^2(a+bx))^2}-\frac{5}{4}\int \frac{\cos^4(a+bx)}{(1-\cos^2(a+bx))^2}d\cos(a+bx)}{b}$$

$$\Big\downarrow \; 252$$

$$-\frac{\frac{\cos^5(a+bx)}{4(1-\cos^2(a+bx))^2}-\frac{5}{4}\left(\frac{\cos^3(a+bx)}{2(1-\cos^2(a+bx))}-\frac{3}{2}\int \frac{\cos^2(a+bx)}{1-\cos^2(a+bx)}d\cos(a+bx)\right)}{b}$$

$$\Big\downarrow \; 262$$

$$-\frac{\frac{\cos^5(a+bx)}{4(1-\cos^2(a+bx))^2}-\frac{5}{4}\left(\frac{\cos^3(a+bx)}{2(1-\cos^2(a+bx))}-\frac{3}{2}\left(\int \frac{1}{1-\cos^2(a+bx)}d\cos(a+bx)-\cos(a+bx)\right)\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{\cos^5(a+bx)}{4(1-\cos^2(a+bx))^2}-\frac{5}{4}\left(\frac{\cos^3(a+bx)}{2(1-\cos^2(a+bx))}-\frac{3}{2}(\text{arctanh}(\cos(a+bx))-\cos(a+bx))\right)}{b}$$

input
```
Int[Cos[a + b*x]*Cot[a + b*x]^5,x]
```

output
```
-((Cos[a + b*x]^5/(4*(1 - Cos[a + b*x]^2)^2) - (5*((-3*(ArcTanh[Cos[a + b*
x]] - Cos[a + b*x]))/2 + Cos[a + b*x]^3/(2*(1 - Cos[a + b*x]^2))))/4)/b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Maple [A] (verified)

Time = 1.97 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.35

method	result
derivativedivides	$\frac{-\frac{\cos(bx+a)^7}{4\sin(bx+a)^4}+\frac{3\cos(bx+a)^7}{8\sin(bx+a)^2}+\frac{3\cos(bx+a)^5}{8}+\frac{5\cos(bx+a)^3}{8}+\frac{15\cos(bx+a)}{8}+\frac{15\ln(\csc(bx+a)-\cot(bx+a))}{8}}{b}$
default	$\frac{-\frac{\cos(bx+a)^7}{4\sin(bx+a)^4}+\frac{3\cos(bx+a)^7}{8\sin(bx+a)^2}+\frac{3\cos(bx+a)^5}{8}+\frac{5\cos(bx+a)^3}{8}+\frac{15\cos(bx+a)}{8}+\frac{15\ln(\csc(bx+a)-\cot(bx+a))}{8}}{b}$
risch	$\frac{e^{i(bx+a)}}{2b}+\frac{e^{-i(bx+a)}}{2b}-\frac{9\,e^{7i(bx+a)}-e^{5i(bx+a)}-e^{3i(bx+a)}+9\,e^{i(bx+a)}}{4b\left(e^{2i(bx+a)}-1\right)^4}-\frac{15\ln\left(e^{i(bx+a)}+1\right)}{8b}+\frac{15\ln\left(e^{i(bx+a)}-1\right.}{8b}$

input
```
int(cos(b*x+a)*cot(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/4/sin(b*x+a)^4*cos(b*x+a)^7+3/8/sin(b*x+a)^2*cos(b*x+a)^7+3/8*cos(
b*x+a)^5+5/8*cos(b*x+a)^3+15/8*cos(b*x+a)+15/8*ln(csc(b*x+a)-cot(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(59) = 118$.

Time = 0.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.88

$$\int \cos(a+bx)\cot^5(a+bx)\,dx$$
$$=\frac{16\,\cos(bx+a)^5-50\,\cos(bx+a)^3-15\left(\cos(bx+a)^4-2\,\cos(bx+a)^2+1\right)\log\left(\frac{1}{2}\,\cos(bx+a)+\frac{1}{2}\right)+}{16\left(b\cos(bx+a)^4-2\,b\cos(bx\right.}$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)^5,x, algorithm="fricas")
```

output
```
1/16*(16*cos(b*x + a)^5 - 50*cos(b*x + a)^3 - 15*(cos(b*x + a)^4 - 2*cos(b
*x + a)^2 + 1)*log(1/2*cos(b*x + a) + 1/2) + 15*(cos(b*x + a)^4 - 2*cos(b*
x + a)^2 + 1)*log(-1/2*cos(b*x + a) + 1/2) + 30*cos(b*x + a))/(b*cos(b*x +
 a)^4 - 2*b*cos(b*x + a)^2 + b)
```

Sympy [F]

$$\int \cos(a+bx)\cot^5(a+bx)\,dx = \int \cos{(a+bx)}\cot^5{(a+bx)}\,dx$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)**5,x)
```

output
```
Integral(cos(a + b*x)*cot(a + b*x)**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.22

$$\int \cos(a+bx)\cot^5(a+bx)\,dx = -\frac{\frac{2\left(9\cos(bx+a)^3-7\cos(bx+a)\right)}{\cos(bx+a)^4-2\cos(bx+a)^2+1} - 16\cos{(bx+a)} + 15\log{(\cos{(bx+a)}+1)} - 15\log{(\cos{(bx+a)}-1)}}{16\,b}$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)^5,x, algorithm="maxima")
```

output
```
-1/16*(2*(9*cos(b*x + a)^3 - 7*cos(b*x + a))/(cos(b*x + a)^4 - 2*cos(b*x +
 a)^2 + 1) - 16*cos(b*x + a) + 15*log(cos(b*x + a) + 1) - 15*log(cos(b*x +
 a) - 1))/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.18

$$\int \cos(a+bx)\cot^5(a+bx)\,dx = \frac{\cos(bx+a)}{b} - \frac{15\log(|\cos(bx+a)+1|)}{16\,b} + \frac{15\log(|\cos(bx+a)-1|)}{16\,b} - \frac{9\cos(bx+a)^3 - 7\cos(bx+a)}{8\left(\cos(bx+a)^2-1\right)^2 b}$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)^5,x, algorithm="giac")
```

output
```
cos(b*x + a)/b - 15/16*log(abs(cos(b*x + a) + 1))/b + 15/16*log(abs(cos(b*
x + a) - 1))/b - 1/8*(9*cos(b*x + a)^3 - 7*cos(b*x + a))/((cos(b*x + a)^2
- 1)^2*b)
```

Mupad [B] (verification not implemented)

Time = 25.60 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.62

$$\int \cos(a+bx)\cot^5(a+bx)\,dx = \frac{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^4}{64\,b} - \frac{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2}{4\,b} + \frac{15\ln\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{8\,b} + \frac{\frac{9\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^4}{4} + \frac{15\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2}{64} - \frac{1}{64}}{b\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^6 + \tan\left(\frac{a}{2}+\frac{bx}{2}\right)^4\right)}$$

input
```
int(cos(a + b*x)*cot(a + b*x)^5,x)
```

output
```
tan(a/2 + (b*x)/2)^4/(64*b) - tan(a/2 + (b*x)/2)^2/(4*b) + (15*log(tan(a/2
 + (b*x)/2)))/(8*b) + ((15*tan(a/2 + (b*x)/2)^2)/64 + (9*tan(a/2 + (b*x)/2
)^4)/4 - 1/64)/(b*(tan(a/2 + (b*x)/2)^4 + tan(a/2 + (b*x)/2)^6))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.29

$$\int \cos(a+bx)\cot^5(a+bx)\,dx$$
$$= \frac{8\cos(bx+a)\sin(bx+a)^4+9\cos(bx+a)\sin(bx+a)^2-2\cos(bx+a)+15\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\sin(bx+ \cdots}{8\sin(bx+a)^4 b}$$

input
```
int(cos(b*x+a)*cot(b*x+a)^5,x)
```

output
```
(8*cos(a + b*x)*sin(a + b*x)**4 + 9*cos(a + b*x)*sin(a + b*x)**2 - 2*cos(a
 + b*x) + 15*log(tan((a + b*x)/2))*sin(a + b*x)**4 - 10*sin(a + b*x)**4)/(
8*sin(a + b*x)**4*b)
```

3.175 $\int \cot^5(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 42

$$\int \cot^5(a+bx)\,dx = \frac{\cot^2(a+bx)}{2b} - \frac{\cot^4(a+bx)}{4b} + \frac{\log(\sin(a+bx))}{b}$$

output
```
1/2*cot(b*x+a)^2/b-1/4*cot(b*x+a)^4/b+ln(sin(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int \cot^5(a+bx)\,dx = \frac{\csc^2(a+bx)}{b} - \frac{\csc^4(a+bx)}{4b} + \frac{\log(\sin(a+bx))}{b}$$

input
```
Integrate[Cot[a + b*x]^5,x]
```

output
```
Csc[a + b*x]^2/b - Csc[a + b*x]^4/(4*b) + Log[Sin[a + b*x]]/b
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.375, Rules used = {3042, 25, 3954, 25, 3042, 25, 3954, 25, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\tan\left(a+bx+\frac{\pi}{2}\right)^5dx$$

$$\Big\downarrow \text{25}$$

$$-\int \tan\left(\frac{1}{2}(2a+\pi)+bx\right)^5dx$$

$$\Big\downarrow \text{3954}$$

$$\int -\cot^3(a+bx)dx-\frac{\cot^4(a+bx)}{4b}$$

$$\Big\downarrow \text{25}$$

$$-\int \cot^3(a+bx)dx-\frac{\cot^4(a+bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$-\int -\tan\left(a+bx+\frac{\pi}{2}\right)^3dx-\frac{\cot^4(a+bx)}{4b}$$

$$\Big\downarrow \text{25}$$

$$\int \tan\left(\frac{1}{2}(2a+\pi)+bx\right)^3dx-\frac{\cot^4(a+bx)}{4b}$$

$$\Big\downarrow \text{3954}$$

$$-\int -\cot(a+bx)dx-\frac{\cot^4(a+bx)}{4b}+\frac{\cot^2(a+bx)}{2b}$$

$$\Big\downarrow \text{25}$$

$$\int \cot(a+bx)dx - \frac{\cot^4(a+bx)}{4b} + \frac{\cot^2(a+bx)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\int -\tan\left(a+bx+\frac{\pi}{2}\right)dx - \frac{\cot^4(a+bx)}{4b} + \frac{\cot^2(a+bx)}{2b}$$

$$\Big\downarrow \text{25}$$

$$-\int \tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx - \frac{\cot^4(a+bx)}{4b} + \frac{\cot^2(a+bx)}{2b}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{\cot^4(a+bx)}{4b} + \frac{\cot^2(a+bx)}{2b} + \frac{\log(-\sin(a+bx))}{b}$$

input
```
Int[Cot[a + b*x]^5,x]
```

output
```
Cot[a + b*x]^2/(2*b) - Cot[a + b*x]^4/(4*b) + Log[-Sin[a + b*x]]/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{-\frac{\cot(bx+a)^4}{4}+\frac{\cot(bx+a)^2}{2}-\frac{\ln\left(\cot(bx+a)^2+1\right)}{2}}{b}$	39
default	$\frac{-\frac{\cot(bx+a)^4}{4}+\frac{\cot(bx+a)^2}{2}-\frac{\ln\left(\cot(bx+a)^2+1\right)}{2}}{b}$	39
parallelrisch	$\frac{-\cot(bx+a)^4+2\cot(bx+a)^2+4\ln(\tan(bx+a))-2\ln\left(\sec(bx+a)^2\right)}{4b}$	47
norman	$\frac{-\frac{1}{4b}+\frac{\tan(bx+a)^2}{2b}}{\tan(bx+a)^4}+\frac{\ln(\tan(bx+a))}{b}-\frac{\ln\left(1+\tan(bx+a)^2\right)}{2b}$	57
risch	$-ix-\frac{2ia}{b}-\frac{4\left(e^{6i(bx+a)}-e^{4i(bx+a)}+e^{2i(bx+a)}\right)}{b\left(e^{2i(bx+a)}-1\right)^4}+\frac{\ln\left(e^{2i(bx+a)}-1\right)}{b}$	77

input
```
int(cot(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/4*cot(b*x+a)^4+1/2*cot(b*x+a)^2-1/2*ln(cot(b*x+a)^2+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(38) = 76.

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.98

$$\int \cot^5(a+bx)\,dx = \frac{\left(\cos\left(2\,bx+2\,a\right)^2-2\,\cos\left(2\,bx+2\,a\right)+1\right)\log\left(-\frac{1}{2}\,\cos\left(2\,bx+2\,a\right)+\frac{1}{2}\right)-4\,\cos\left(2\,bx+2\,a\right)+2}{2\,\left(b\cos\left(2\,bx+2\,a\right)^2-2\,b\cos\left(2\,bx+2\,a\right)+b\right)}$$

input
```
integrate(cot(b*x+a)^5,x, algorithm="fricas")
```

output
```
1/2*((cos(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(-1/2*cos(2*b*x + 2*
a) + 1/2) - 4*cos(2*b*x + 2*a) + 2)/(b*cos(2*b*x + 2*a)^2 - 2*b*cos(2*b*x
+ 2*a) + b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(32) = 64$.

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.57

$$\int \cot^5(a+bx)\,dx = \begin{cases} \tilde{\infty}x & \text{for } a=0 \wedge b=0 \\ x\cot^5(a) & \text{for } b=0 \\ \tilde{\infty}x & \text{for } a=-bx \\ -\frac{\log(\tan^2(a+bx)+1)}{2b} + \frac{\log(\tan(a+bx))}{b} + \frac{1}{2b\tan^2(a+bx)} - \frac{1}{4b\tan^4(a+bx)} & \text{otherwise} \end{cases}$$

input
```
integrate(cot(b*x+a)**5,x)
```

output
```
Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x*cot(a)**5, Eq(b, 0)), (zoo*x, E
q(a, -b*x)), (-log(tan(a + b*x)**2 + 1)/(2*b) + log(tan(a + b*x))/b + 1/(2
*b*tan(a + b*x)**2) - 1/(4*b*tan(a + b*x)**4), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \cot^5(a+bx)\,dx = \frac{\frac{4\sin(bx+a)^2-1}{\sin(bx+a)^4} + 2\log\left(\sin(bx+a)^2\right)}{4b}$$

input
```
integrate(cot(b*x+a)^5,x, algorithm="maxima")
```

output
```
1/4*((4*sin(b*x + a)^2 - 1)/sin(b*x + a)^4 + 2*log(sin(b*x + a)^2))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \cot^5(a+bx)\,dx = \frac{\log(|\sin(bx+a)|)}{b} + \frac{4\sin(bx+a)^2-1}{4b\sin(bx+a)^4}$$

input
```
integrate(cot(b*x+a)^5,x, algorithm="giac")
```

output
```
log(abs(sin(b*x + a)))/b + 1/4*(4*sin(b*x + a)^2 - 1)/(b*sin(b*x + a)^4)
```

Mupad [B] (verification not implemented)

Time = 31.01 (sec) , antiderivative size = 182, normalized size of antiderivative = 4.33

$$\begin{aligned}\int \cot^5(a+bx)\,dx = {} & -x\,1\mathrm{i} + \frac{\ln\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}-1\right)}{b} - \frac{4}{b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)} \\ & - \frac{8}{b\,\left(1+\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}-2\,\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\right)} \\ & - \frac{8}{b\,\left(3\,\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-3\,\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+\mathrm{e}^{a\,6\mathrm{i}+b\,x\,6\mathrm{i}}-1\right)} \\ & - \frac{4}{b\,\left(1+6\,\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}-4\,\mathrm{e}^{a\,6\mathrm{i}+b\,x\,6\mathrm{i}}+\mathrm{e}^{a\,8\mathrm{i}+b\,x\,8\mathrm{i}}-4\,\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\right)}\end{aligned}$$

input
```
int(cot(a + b*x)^5,x)
```

output
```
log(exp(a*2i)*exp(b*x*2i) - 1)/b - x*1i - 4/(b*(exp(a*2i + b*x*2i) - 1)) -
 8/(b*(exp(a*4i + b*x*4i) - 2*exp(a*2i + b*x*2i) + 1)) - 8/(b*(3*exp(a*2i
+ b*x*2i) - 3*exp(a*4i + b*x*4i) + exp(a*6i + b*x*6i) - 1)) - 4/(b*(6*exp(
a*4i + b*x*4i) - 4*exp(a*2i + b*x*2i) - 4*exp(a*6i + b*x*6i) + exp(a*8i +
b*x*8i) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.88

$$\int \cot^5(a+bx)\,dx$$

$$= \frac{-32\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2+1\right)\sin\left(bx+a\right)^4+32\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\sin\left(bx+a\right)^4-13\sin\left(bx+a\right)^4+32\,\mathrm{si}}{32\sin\left(bx+a\right)^4 b}$$

input
```
int(cot(b*x+a)^5,x)
```

output
```
( - 32*log(tan((a + b*x)/2)**2 + 1)*sin(a + b*x)**4 + 32*log(tan((a + b*x)
/2))*sin(a + b*x)**4 - 13*sin(a + b*x)**4 + 32*sin(a + b*x)**2 - 8)/(32*si
n(a + b*x)**4*b)
```

3.176 $\int \cot^4(a+bx)\csc(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \cot^4(a+bx)\csc(a+bx)\,dx = -\frac{3\text{arctanh}(\cos(a+bx))}{8b}+\frac{3\cot(a+bx)\csc(a+bx)}{8b}-\frac{\cot^3(a+bx)\csc(a+bx)}{4b}$$

output
```
-3/8*arctanh(cos(b*x+a))/b+3/8*cot(b*x+a)*csc(b*x+a)/b-1/4*cot(b*x+a)^3*cs
c(b*x+a)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 113 vs. 2(55) = 110.

Time = 0.12 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.05

$$\int \cot^4(a+bx)\csc(a+bx)\,dx = \frac{5\csc^2\left(\frac{1}{2}(a+bx)\right)}{32b}-\frac{\csc^4\left(\frac{1}{2}(a+bx)\right)}{64b}-\frac{3\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{8b}+\frac{3\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{8b}-\frac{5\sec^2\left(\frac{1}{2}(a+bx)\right)}{32b}+\frac{\sec^4\left(\frac{1}{2}(a+bx)\right)}{64b}$$

input
```
Integrate[Cot[a + b*x]^4*Csc[a + b*x],x]
```

output
```
(5*Csc[(a + b*x)/2]^2)/(32*b) - Csc[(a + b*x)/2]^4/(64*b) - (3*Log[Cos[(a
+ b*x)/2]])/(8*b) + (3*Log[Sin[(a + b*x)/2]])/(8*b) - (5*Sec[(a + b*x)/2]^
2)/(32*b) + Sec[(a + b*x)/2]^4/(64*b)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3091, 3042, 3091, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(a+bx)\csc(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(a+bx-\frac{\pi}{2}\right)^4\sec\left(a+bx-\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3091}$$

$$-\frac{3}{4}\int \cot^2(a+bx)\csc(a+bx)dx-\frac{\cot^3(a+bx)\csc(a+bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3}{4}\int \sec\left(a+bx-\frac{\pi}{2}\right)\tan\left(a+bx-\frac{\pi}{2}\right)^2dx-\frac{\cot^3(a+bx)\csc(a+bx)}{4b}$$

$$\Big\downarrow \text{3091}$$

$$-\frac{3}{4}\left(-\frac{1}{2}\int \csc(a+bx)dx-\frac{\cot(a+bx)\csc(a+bx)}{2b}\right)-\frac{\cot^3(a+bx)\csc(a+bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3}{4}\left(-\frac{1}{2}\int \csc(a+bx)dx-\frac{\cot(a+bx)\csc(a+bx)}{2b}\right)-\frac{\cot^3(a+bx)\csc(a+bx)}{4b}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{3}{4}\left(\frac{\operatorname{arctanh}(\cos(a+bx))}{2b}-\frac{\cot(a+bx)\csc(a+bx)}{2b}\right)-\frac{\cot^3(a+bx)\csc(a+bx)}{4b}$$

input
```
Int[Cot[a + b*x]^4*Csc[a + b*x],x]
```

output
```
-1/4*(Cot[a + b*x]^3*Csc[a + b*x])/b - (3*(ArcTanh[Cos[a + b*x]]/(2*b) - (
Cot[a + b*x]*Csc[a + b*x])/(2*b)))/4
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.42

method	result	size
derivativedivides	$\frac{-\frac{\cos(bx+a)^5}{4\sin(bx+a)^4}+\frac{\cos(bx+a)^5}{8\sin(bx+a)^2}+\frac{\cos(bx+a)^3}{8}+\frac{3\cos(bx+a)}{8}+\frac{3\ln(\csc(bx+a)-\cot(bx+a))}{8}}{b}$	78
default	$\frac{-\frac{\cos(bx+a)^5}{4\sin(bx+a)^4}+\frac{\cos(bx+a)^5}{8\sin(bx+a)^2}+\frac{\cos(bx+a)^3}{8}+\frac{3\cos(bx+a)}{8}+\frac{3\ln(\csc(bx+a)-\cot(bx+a))}{8}}{b}$	78
risch	$-\frac{5\,e^{7i(bx+a)}+3\,e^{5i(bx+a)}+3\,e^{3i(bx+a)}+5\,e^{i(bx+a)}}{4b\left(e^{2i(bx+a)}-1\right)^4}+\frac{3\ln\left(e^{i(bx+a)}-1\right)}{8b}-\frac{3\ln\left(e^{i(bx+a)}+1\right)}{8b}$	99

input
```
int(cot(b*x+a)^4*csc(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/4/sin(b*x+a)^4*cos(b*x+a)^5+1/8/sin(b*x+a)^2*cos(b*x+a)^5+1/8*cos(
b*x+a)^3+3/8*cos(b*x+a)+3/8*ln(csc(b*x+a)-cot(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(49) = 98.

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.04

$$\int \cot^4(a+bx)\csc(a+bx)\,dx = -\frac{10\cos(bx+a)^3+3\left(\cos(bx+a)^4-2\cos(bx+a)^2+1\right)\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)-3\left(\cos(bx+a)^4 \cdots\right.}{16\left(b\cos(bx+a)^4-2b\cos(bx+a)^2+\cdots\right.}$$

input
```
integrate(cot(b*x+a)^4*csc(b*x+a),x, algorithm="fricas")
```

output
```
-1/16*(10*cos(b*x + a)^3 + 3*(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*log(1
/2*cos(b*x + a) + 1/2) - 3*(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*log(-1/
2*cos(b*x + a) + 1/2) - 6*cos(b*x + a))/(b*cos(b*x + a)^4 - 2*b*cos(b*x +
a)^2 + b)
```

Sympy [F]

$$\int \cot^4(a+bx)\csc(a+bx)\,dx = \int \cot^4(a+bx)\csc(a+bx)\,dx$$

input
```
integrate(cot(b*x+a)**4*csc(b*x+a),x)
```

output
```
Integral(cot(a + b*x)**4*csc(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.29

$$\int \cot^4(a+bx)\csc(a+bx)\,dx = -\frac{\frac{2\left(5\cos(bx+a)^3-3\cos(bx+a)\right)}{\cos(bx+a)^4-2\cos(bx+a)^2+1}+3\log\left(\cos\left(bx+a\right)+1\right)-3\log\left(\cos\left(bx+a\right)-1\right)}{16\,b}$$

input
```
integrate(cot(b*x+a)^4*csc(b*x+a),x, algorithm="maxima")
```

output
```
-1/16*(2*(5*cos(b*x + a)^3 - 3*cos(b*x + a))/(cos(b*x + a)^4 - 2*cos(b*x +
 a)^2 + 1) + 3*log(cos(b*x + a) + 1) - 3*log(cos(b*x + a) - 1))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.22

$$\int \cot^4(a+bx)\csc(a+bx)\,dx = -\frac{3\log\left(\left|\cos\left(bx+a\right)+1\right|\right)}{16\,b}+\frac{3\log\left(\left|\cos\left(bx+a\right)-1\right|\right)}{16\,b} - \frac{5\cos\left(bx+a\right)^3-3\cos\left(bx+a\right)}{8\left(\cos\left(bx+a\right)^2-1\right)^2b}$$

input
```
integrate(cot(b*x+a)^4*csc(b*x+a),x, algorithm="giac")
```

output
```
-3/16*log(abs(cos(b*x + a) + 1))/b + 3/16*log(abs(cos(b*x + a) - 1))/b - 1
/8*(5*cos(b*x + a)^3 - 3*cos(b*x + a))/((cos(b*x + a)^2 - 1)^2*b)
```

Mupad [B] (verification not implemented)

Time = 25.46 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.42

$$\int \cot^4(a+bx)\csc(a+bx)\,dx = \frac{\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4}{64\,b} - \frac{\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2}{8\,b} + \frac{3\,\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{8\,b} + \frac{\cot\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4\left(\frac{\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2}{8}-\frac{1}{64}\right)}{b}$$

input
```
int(cot(a + b*x)^4/sin(a + b*x),x)
```

output
```
tan(a/2 + (b*x)/2)^4/(64*b) - tan(a/2 + (b*x)/2)^2/(8*b) + (3*log(tan(a/2
+ (b*x)/2)))/(8*b) + (cot(a/2 + (b*x)/2)^4*(tan(a/2 + (b*x)/2)^2/8 - 1/64)
)/b
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05

$$\int \cot^4(a+bx)\csc(a+bx)\,dx = \frac{5\cos(bx+a)\sin(bx+a)^2 - 2\cos(bx+a) + 3\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\sin(bx+a)^4}{8\sin(bx+a)^4 b}$$

input
```
int(cot(b*x+a)^4*csc(b*x+a),x)
```

output
```
(5*cos(a + b*x)*sin(a + b*x)**2 - 2*cos(a + b*x) + 3*log(tan((a + b*x)/2))
*sin(a + b*x)**4)/(8*sin(a + b*x)**4*b)
```

3.177 $\int \cot^3(a+bx)\csc^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 15

$$\int \cot^3(a+bx)\csc^2(a+bx)\,dx = -\frac{\cot^4(a+bx)}{4b}$$

output
```
-1/4*cot(b*x+a)^4/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cot^3(a+bx)\csc^2(a+bx)\,dx = -\frac{\cot^4(a+bx)}{4b}$$

input
```
Integrate[Cot[a + b*x]^3*Csc[a + b*x]^2,x]
```

output
```
-1/4*Cot[a + b*x]^4/b
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 25, 3087, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(a+bx)\csc^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(a+bx-\frac{\pi}{2}\right)^3\left(-\sec\left(a+bx-\frac{\pi}{2}\right)^2\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sec\left(\frac{1}{2}(2a-\pi)+bx\right)^2\tan\left(\frac{1}{2}(2a-\pi)+bx\right)^3dx$$

$$\Big\downarrow \text{3087}$$

$$-\frac{\int -\cot^3(a+bx)d(-\cot(a+bx))}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\cot^4(a+bx)}{4b}$$

input
```
Int[Cot[a + b*x]^3*Csc[a + b*x]^2,x]
```

output
```
-1/4*Cot[a + b*x]^4/b
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{\cot(bx+a)^4}{4b}$	14
default	$-\frac{\cot(bx+a)^4}{4b}$	14
risch	$-\frac{2\left(e^{6i(bx+a)}+e^{2i(bx+a)}\right)}{b\left(e^{2i(bx+a)}-1\right)^4}$	38

input
```
int(cot(b*x+a)^3*csc(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*cot(b*x+a)^4/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(13) = 26$.

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.60

$$\int \cot^3(a+bx)\csc^2(a+bx)\,dx = -\frac{2\,\cos\left(bx+a\right)^2-1}{4\,\left(b\cos\left(bx+a\right)^4-2\,b\cos\left(bx+a\right)^2+b\right)}$$

input
```
integrate(cot(b*x+a)^3*csc(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/4*(2*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^4 - 2*b*cos(b*x + a)^2 + b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(12) = 24$.

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.80

$$\int \cot^3(a+bx)\csc^2(a+bx)\,dx = \begin{cases} -\frac{\cot^2(a+bx)\csc^2(a+bx)}{4b}+\frac{\csc^2(a+bx)}{4b} & \text{for } b\neq 0 \\ x\cot^3(a)\csc^2(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cot(b*x+a)**3*csc(b*x+a)**2,x)
```

output
```
Piecewise((-cot(a + b*x)**2*csc(a + b*x)**2/(4*b) + csc(a + b*x)**2/(4*b),
 Ne(b, 0)), (x*cot(a)**3*csc(a)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cot^3(a+bx)\csc^2(a+bx)\,dx = -\frac{\cot\left(bx+a\right)^4}{4\,b}$$

input
```
integrate(cot(b*x+a)^3*csc(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/4*cot(b*x + a)^4/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.67

$$\int \cot^3(a+bx)\csc^2(a+bx)\,dx = \frac{2\sin{(bx+a)}^2-1}{4\,b\sin{(bx+a)}^4}$$

input
```
integrate(cot(b*x+a)^3*csc(b*x+a)^2,x, algorithm="giac")
```

output
```
1/4*(2*sin(b*x + a)^2 - 1)/(b*sin(b*x + a)^4)
```

Mupad [B] (verification not implemented)

Time = 25.39 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.67

$$\int \cot^3(a+bx)\csc^2(a+bx)\,dx = -\frac{\left(\sin{(a+b\,x)}^2-1\right)^2}{4\,b\sin{(a+b\,x)}^4}$$

input
```
int(cot(a + b*x)^3/sin(a + b*x)^2,x)
```

output
```
-(sin(a + b*x)^2 - 1)^2/(4*b*sin(a + b*x)^4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.67

$$\int \cot^3(a+bx)\csc^2(a+bx)\,dx = \frac{\csc{(bx+a)}^2\left(-\cot{(bx+a)}^2+1\right)}{4b}$$

input
```
int(cot(b*x+a)^3*csc(b*x+a)^2,x)
```

output
```
(csc(a + b*x)**2*( - cot(a + b*x)**2 + 1))/(4*b)
```

3.178 $\int \cot^2(a+bx)\csc^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 55

$$\int \cot^2(a+bx)\csc^3(a+bx)\,dx = \frac{\operatorname{arctanh}(\cos(a+bx))}{8b} + \frac{\cot(a+bx)\csc(a+bx)}{8b} - \frac{\cot(a+bx)\csc^3(a+bx)}{4b}$$

output
```
1/8*arctanh(cos(b*x+a))/b+1/8*cot(b*x+a)*csc(b*x+a)/b-1/4*cot(b*x+a)*csc(b
*x+a)^3/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 113 vs. 2(55) = 110.

Time = 0.14 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.05

$$\int \cot^2(a+bx)\csc^3(a+bx)\,dx = \frac{\csc^2\left(\frac{1}{2}(a+bx)\right)}{32b} - \frac{\csc^4\left(\frac{1}{2}(a+bx)\right)}{64b} + \frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{8b} - \frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{8b} - \frac{\sec^2\left(\frac{1}{2}(a+bx)\right)}{32b} + \frac{\sec^4\left(\frac{1}{2}(a+bx)\right)}{64b}$$

input
```
Integrate[Cot[a + b*x]^2*Csc[a + b*x]^3,x]
```

output
```
Csc[(a + b*x)/2]^2/(32*b) - Csc[(a + b*x)/2]^4/(64*b) + Log[Cos[(a + b*x)/
2]]/(8*b) - Log[Sin[(a + b*x)/2]]/(8*b) - Sec[(a + b*x)/2]^2/(32*b) + Sec[
(a + b*x)/2]^4/(64*b)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3042, 3091, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(a+bx)\csc^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(a+bx-\frac{\pi}{2}\right)^2\sec\left(a+bx-\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{3091}$$

$$-\frac{1}{4}\int \csc^3(a+bx)dx-\frac{\cot(a+bx)\csc^3(a+bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{4}\int \csc(a+bx)^3dx-\frac{\cot(a+bx)\csc^3(a+bx)}{4b}$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{4}\left(\frac{\cot(a+bx)\csc(a+bx)}{2b}-\frac{1}{2}\int \csc(a+bx)dx\right)-\frac{\cot(a+bx)\csc^3(a+bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{\cot(a+bx)\csc(a+bx)}{2b}-\frac{1}{2}\int \csc(a+bx)dx\right)-\frac{\cot(a+bx)\csc^3(a+bx)}{4b}$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{4}\left(\frac{\operatorname{arctanh}(\cos(a+bx))}{2b}+\frac{\cot(a+bx)\csc(a+bx)}{2b}\right)-\frac{\cot(a+bx)\csc^3(a+bx)}{4b}$$

input
```
Int[Cot[a + b*x]^2*Csc[a + b*x]^3,x]
```

output
```
-1/4*(Cot[a + b*x]*Csc[a + b*x]^3)/b + (ArcTanh[Cos[a + b*x]]/(2*b) + (Cot
[a + b*x]*Csc[a + b*x])/(2*b))/4
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.24

method	result	size
derivativedivides	$\frac{-\frac{\cos(bx+a)^3}{4\sin(bx+a)^4}-\frac{\cos(bx+a)^3}{8\sin(bx+a)^2}-\frac{\cos(bx+a)}{8}-\frac{\ln(\csc(bx+a)-\cot(bx+a))}{8}}{b}$	68
default	$\frac{-\frac{\cos(bx+a)^3}{4\sin(bx+a)^4}-\frac{\cos(bx+a)^3}{8\sin(bx+a)^2}-\frac{\cos(bx+a)}{8}-\frac{\ln(\csc(bx+a)-\cot(bx+a))}{8}}{b}$	68
risch	$-\frac{e^{7i(bx+a)}+7\,e^{5i(bx+a)}+7\,e^{3i(bx+a)}+e^{i(bx+a)}}{4b\left(e^{2i(bx+a)}-1\right)^4}+\frac{\ln\left(e^{i(bx+a)}+1\right)}{8b}-\frac{\ln\left(e^{i(bx+a)}-1\right)}{8b}$	95

input
```
int(cot(b*x+a)^2*csc(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/4/sin(b*x+a)^4*cos(b*x+a)^3-1/8/sin(b*x+a)^2*cos(b*x+a)^3-1/8*cos(
b*x+a)-1/8*ln(csc(b*x+a)-cot(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(49) = 98$.

Time = 0.09 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.02

$$\int \cot^2(a+bx)\csc^3(a+bx)\,dx = -\frac{2\cos(bx+a)^3-\left(\cos(bx+a)^4-2\cos(bx+a)^2+1\right)\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)+\left(\cos(bx+a)^4-2\right.}{16\left(b\cos(bx+a)^4-2\,b\cos(bx+a)^2+b\right.}$$

input
```
integrate(cot(b*x+a)^2*csc(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/16*(2*cos(b*x + a)^3 - (cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*log(1/2*
cos(b*x + a) + 1/2) + (cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*log(-1/2*cos
(b*x + a) + 1/2) + 2*cos(b*x + a))/(b*cos(b*x + a)^4 - 2*b*cos(b*x + a)^2
+ b)
```

Sympy [F]

$$\int \cot^2(a+bx)\csc^3(a+bx)\,dx = \int \cot^2(a+bx)\csc^3(a+bx)\,dx$$

input
```
integrate(cot(b*x+a)**2*csc(b*x+a)**3,x)
```

output
```
Integral(cot(a + b*x)**2*csc(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.18

$$\int \cot^2(a+bx)\csc^3(a+bx)\,dx$$
$$= -\frac{\frac{2\left(\cos(bx+a)^3+\cos(bx+a)\right)}{\cos(bx+a)^4-2\cos(bx+a)^2+1} - \log(\cos(bx+a)+1) + \log(\cos(bx+a)-1)}{16\,b}$$

input
```
integrate(cot(b*x+a)^2*csc(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/16*(2*(cos(b*x + a)^3 + cos(b*x + a))/(cos(b*x + a)^4 - 2*cos(b*x + a)^
2 + 1) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.60

$$\int \cot^2(a+bx)\csc^3(a+bx)\,dx = \frac{\log\left(\left|\frac{1}{\cos(bx+a)}+\cos(bx+a)+2\right|\right)}{32\,b} - \frac{\log\left(\left|\frac{1}{\cos(bx+a)}+\cos(bx+a)-2\right|\right)}{32\,b} - \frac{\frac{1}{\cos(bx+a)}+\cos(bx+a)}{8\left(\left(\frac{1}{\cos(bx+a)}+\cos(bx+a)\right)^2-4\right)b}$$

input
```
integrate(cot(b*x+a)^2*csc(b*x+a)^3,x, algorithm="giac")
```

output
```
1/32*log(abs(1/cos(b*x + a) + cos(b*x + a) + 2))/b - 1/32*log(abs(1/cos(b*
x + a) + cos(b*x + a) - 2))/b - 1/8*(1/cos(b*x + a) + cos(b*x + a))/(((1/c
os(b*x + a) + cos(b*x + a))^2 - 4)*b)
```

Mupad [B] (verification not implemented)

Time = 25.63 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int \cot^2(a+bx)\csc^3(a+bx)\,dx = \frac{\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4}{64\,b} - \frac{1}{64\,b\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4} - \frac{\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{8\,b}$$

input
```
int(cot(a + b*x)^2/sin(a + b*x)^3,x)
```

output
```
tan(a/2 + (b*x)/2)^4/(64*b) - 1/(64*b*tan(a/2 + (b*x)/2)^4) - log(tan(a/2
+ (b*x)/2))/(8*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int \cot^2(a+bx)\csc^3(a+bx)\,dx = \frac{-8\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^8-1}{64\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4 b}$$

input
```
int(cot(b*x+a)^2*csc(b*x+a)^3,x)
```

output
```
( - 8*log(tan((a + b*x)/2))*tan((a + b*x)/2)**4 + tan((a + b*x)/2)**8 - 1)
/(64*tan((a + b*x)/2)**4*b)
```

3.179 $\int \cot(a+bx)\csc^4(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \cot(a+bx)\csc^4(a+bx)\,dx=-\frac{\csc^4(a+bx)}{4b}$$

output
```
-1/4*csc(b*x+a)^4/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cot(a+bx)\csc^4(a+bx)\,dx=-\frac{\csc^4(a+bx)}{4b}$$

input
```
Integrate[Cot[a + b*x]*Csc[a + b*x]^4,x]
```

output
```
-1/4*Csc[a + b*x]^4/b
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 25, 3086, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(a+bx)\csc^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(a+bx-\frac{\pi}{2}\right)\left(-\sec\left(a+bx-\frac{\pi}{2}\right)^4\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sec\left(\frac{1}{2}(2a-\pi)+bx\right)^4\tan\left(\frac{1}{2}(2a-\pi)+bx\right)dx$$

$$\Big\downarrow \text{3086}$$

$$-\frac{\int \csc^3(a+bx)d\csc(a+bx)}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\csc^4(a+bx)}{4b}$$

input
```
Int[Cot[a + b*x]*Csc[a + b*x]^4,x]
```

output
```
-1/4*Csc[a + b*x]^4/b
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{\csc(bx+a)^4}{4b}$	14
default	$-\frac{\csc(bx+a)^4}{4b}$	14
risch	$-\frac{4\,\mathrm{e}^{4i(bx+a)}}{b\left(\mathrm{e}^{2i(bx+a)}-1\right)^4}$	28

input
```
int(cot(b*x+a)*csc(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/4*csc(b*x+a)^4/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(13) = 26$.

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

$$\int \cot(a+bx)\csc^4(a+bx)\,dx = -\frac{1}{4\left(b\cos\left(bx+a\right)^4 - 2\,b\cos\left(bx+a\right)^2 + b\right)}$$

input
```
integrate(cot(b*x+a)*csc(b*x+a)^4,x, algorithm="fricas")
```

output
```
-1/4/(b*cos(b*x + a)^4 - 2*b*cos(b*x + a)^2 + b)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \cot(a+bx)\csc^4(a+bx)\,dx = \begin{cases} -\frac{\csc^4(a+bx)}{4b} & \text{for}\ b \neq 0 \\ x\cot(a)\csc^4(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cot(b*x+a)*csc(b*x+a)**4,x)
```

output
```
Piecewise((-csc(a + b*x)**4/(4*b), Ne(b, 0)), (x*cot(a)*csc(a)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cot(a+bx)\csc^4(a+bx)\,dx = -\frac{1}{4\,b\sin\left(bx+a\right)^4}$$

input
```
integrate(cot(b*x+a)*csc(b*x+a)^4,x, algorithm="maxima")
```

output
```
-1/4/(b*sin(b*x + a)^4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cot(a+bx)\csc^4(a+bx)\,dx = -\frac{1}{4\,b\sin{(bx+a)}^4}$$

input
```
integrate(cot(b*x+a)*csc(b*x+a)^4,x, algorithm="giac")
```

output
```
-1/4/(b*sin(b*x + a)^4)
```

Mupad [B] (verification not implemented)

Time = 25.45 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53

$$\int \cot(a+bx)\csc^4(a+bx)\,dx = -\frac{{\cot(a+b\,x)}^2\,\left({\cot(a+b\,x)}^2+2\right)}{4\,b}$$

input
```
int(cot(a + b*x)/sin(a + b*x)^4,x)
```

output
```
-(cot(a + b*x)^2*(cot(a + b*x)^2 + 2))/(4*b)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \cot(a+bx)\csc^4(a+bx)\,dx = -\frac{\csc{(bx+a)}^4}{4b}$$

input
```
int(cot(b*x+a)*csc(b*x+a)^4,x)
```

output
```
( - csc(a + b*x)**4)/(4*b)
```

3.180 $\int \csc^5(a+bx)\sec(a+bx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 40

$$\int \csc^5(a+bx)\sec(a+bx)\,dx = -\frac{\cot^2(a+bx)}{b}-\frac{\cot^4(a+bx)}{4b}+\frac{\log(\tan(a+bx))}{b}$$

output
```
-cot(b*x+a)^2/b-1/4*cot(b*x+a)^4/b+ln(tan(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.35

$$\int \csc^5(a+bx)\sec(a+bx)\,dx = -\frac{\csc^2(a+bx)}{2b}-\frac{\csc^4(a+bx)}{4b}-\frac{\log(\cos(a+bx))}{b}+\frac{\log(\sin(a+bx))}{b}$$

input
```
Integrate[Csc[a + b*x]^5*Sec[a + b*x],x]
```

output
```
-1/2*Csc[a + b*x]^2/b - Csc[a + b*x]^4/(4*b) - Log[Cos[a + b*x]]/b + Log[S
in[a + b*x]]/b
```

Rubi [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3100, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^5(a+bx)\sec(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(a+bx)^5\sec(a+bx)dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{\int \cot^5(a+bx)\left(\tan^2(a+bx)+1\right)^2d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int \cot^3(a+bx)\left(\tan^2(a+bx)+1\right)^2d\tan^2(a+bx)}{2b}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\cot^3(a+bx)+2\cot^2(a+bx)+\cot(a+bx)\right)d\tan^2(a+bx)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{2}\cot^2(a+bx)-2\cot(a+bx)+\log\left(\tan^2(a+bx)\right)}{2b}$$

input
```
Int[Csc[a + b*x]^5*Sec[a + b*x],x]
```

output
```
(-2*Cot[a + b*x] - Cot[a + b*x]^2/2 + Log[Tan[a + b*x]^2])/(2*b)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{-\frac{1}{4\sin(bx+a)^4}-\frac{1}{2\sin(bx+a)^2}+\ln(\tan(bx+a))}{b}$
default	$\frac{-\frac{1}{4\sin(bx+a)^4}-\frac{1}{2\sin(bx+a)^2}+\ln(\tan(bx+a))}{b}$
risch	$\frac{2\,e^{6i(bx+a)}-8\,e^{4i(bx+a)}+2\,e^{2i(bx+a)}}{b\left(e^{2i(bx+a)}-1\right)^4}+\frac{\ln\left(e^{2i(bx+a)}-1\right)}{b}-\frac{\ln\left(e^{2i(bx+a)}+1\right)}{b}$
parallelrisch	$\frac{-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4-\cot\left(\frac{bx}{2}+\frac{a}{2}\right)^4-12\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2-12\cot\left(\frac{bx}{2}+\frac{a}{2}\right)^2+64\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-64\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)-6}{64b}$
norman	$\frac{-\frac{1}{64b}-\frac{3\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}{16b}-\frac{3\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^6}{16b}-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^8}{64b}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}+\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{b}-\frac{\ln\left(\tan\left(\frac{b}{2}+\right.\right.}{b}$

input
```
int(csc(b*x+a)^5*sec(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/4/sin(b*x+a)^4-1/2/sin(b*x+a)^2+ln(tan(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(38) = 76$.

Time = 0.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.62

$$\int \csc^5(a+bx)\sec(a+bx)\,dx = \frac{2\cos(bx+a)^2 - 2\left(\cos(bx+a)^4 - 2\cos(bx+a)^2 + 1\right)\log\left(\cos(bx+a)^2\right) + 2\left(\cos(bx+a)^4 - 2\cos(b\right.}{4\left(b\cos(bx+a)^4 - 2b\cos(bx+a)^2 + b\right)}$$

input
```
integrate(csc(b*x+a)^5*sec(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(2*cos(b*x + a)^2 - 2*(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*log(cos(
b*x + a)^2) + 2*(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*log(-1/4*cos(b*x +
 a)^2 + 1/4) - 3)/(b*cos(b*x + a)^4 - 2*b*cos(b*x + a)^2 + b)
```

Sympy [F]

$$\int \csc^5(a+bx)\sec(a+bx)\,dx = \int \csc^5(a+bx)\sec(a+bx)\,dx$$

input
```
integrate(csc(b*x+a)**5*sec(b*x+a),x)
```

output
```
Integral(csc(a + b*x)**5*sec(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.28

$$\int \csc^5(a+bx)\sec(a+bx)\,dx = -\frac{\frac{2\sin(bx+a)^2+1}{\sin(bx+a)^4} + 2\log\left(\sin(bx+a)^2-1\right) - 2\log\left(\sin(bx+a)^2\right)}{4b}$$

input
```
integrate(csc(b*x+a)^5*sec(b*x+a),x, algorithm="maxima")
```

output
```
-1/4*((2*sin(b*x + a)^2 + 1)/sin(b*x + a)^4 + 2*log(sin(b*x + a)^2 - 1) -
2*log(sin(b*x + a)^2))/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.38

$$\int \csc^5(a+bx)\sec(a+bx)\,dx = -\frac{\log\left(\left|\sin(bx+a)^2-1\right|\right)}{2b} + \frac{\log\left(\left|\sin(bx+a)\right|\right)}{b} - \frac{2\sin(bx+a)^2+1}{4b\sin(bx+a)^4}$$

input
```
integrate(csc(b*x+a)^5*sec(b*x+a),x, algorithm="giac")
```

output
```
-1/2*log(abs(sin(b*x + a)^2 - 1))/b + log(abs(sin(b*x + a)))/b - 1/4*(2*si
n(b*x + a)^2 + 1)/(b*sin(b*x + a)^4)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.98

$$\int \csc^5(a+bx)\sec(a+bx)\,dx = \frac{\ln\left(\frac{\cos(2\,a+2\,b\,x)}{2}-\frac{1}{2}\right)}{2\,b}-\frac{\ln\left(\cos\left(a+b\,x\right)\right)}{b}-\frac{\frac{\cos(2\,a+2\,b\,x)}{4}-\frac{1}{2}}{b\,\left(\cos\left(2\,a+2\,b\,x\right)-{\left(\frac{\cos(2\,a+2\,b\,x)}{2}+\frac{1}{2}\right)}^2\right)}$$

input
```
int(1/(cos(a + b*x)*sin(a + b*x)^5),x)
```

output
```
log(cos(2*a + 2*b*x)/2 - 1/2)/(2*b) - log(cos(a + b*x))/b - (cos(2*a + 2*b
*x)/4 - 1/2)/(b*(cos(2*a + 2*b*x) - (cos(2*a + 2*b*x)/2 + 1/2)^2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.48

$$\int \csc^5(a+bx)\sec(a+bx)\,dx$$
$$= \frac{-32\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin\left(bx+a\right)^4-32\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)\sin\left(bx+a\right)^4+32\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{32\sin\left(bx+a\right)^4b}$$

input
```
int(csc(b*x+a)^5*sec(b*x+a),x)
```

output
```
( - 32*log(tan((a + b*x)/2) - 1)*sin(a + b*x)**4 - 32*log(tan((a + b*x)/2)
 + 1)*sin(a + b*x)**4 + 32*log(tan((a + b*x)/2))*sin(a + b*x)**4 + 11*sin(
a + b*x)**4 - 16*sin(a + b*x)**2 - 8)/(32*sin(a + b*x)**4*b)
```

3.181 $\int \csc^5(a+bx)\sec^2(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 65

$$\int \csc^5(a+bx)\sec^2(a+bx)\,dx=-\frac{15\text{arctanh}(\cos(a+bx))}{8b}-\frac{9\cot(a+bx)\csc(a+bx)}{8b}-\frac{\cot^3(a+bx)\csc(a+bx)}{4b}+\frac{\sec(a+bx)}{b}$$

output
```
-15/8*arctanh(cos(b*x+a))/b-9/8*cot(b*x+a)*csc(b*x+a)/b-1/4*cot(b*x+a)^3*c
sc(b*x+a)/b+sec(b*x+a)/b
```

Mathematica [A] (verified)

Time = 4.78 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.98

$$\int \csc^5(a+bx)\sec^2(a+bx)\,dx=-\frac{14\csc^2\left(\frac{1}{2}(a+bx)\right)+\csc^4\left(\frac{1}{2}(a+bx)\right)+\frac{\sec^2\left(\frac{1}{2}(a+bx)\right)\left(78+\cos(a+bx)\left(-8\left(8+15\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)-15\log\left(\sin\left(\frac{1}{2}(a+bx\right.\right.\right.\right.\right.}{-1+\tan^2\left(\frac{1}{2}(a+bx)\right)}}{64b}$$

input
```
Integrate[Csc[a + b*x]^5*Sec[a + b*x]^2,x]
```

output
```
-1/64*(14*Csc[(a + b*x)/2]^2 + Csc[(a + b*x)/2]^4 + (Sec[(a + b*x)/2]^2*(7
8 + Cos[a + b*x]*(-8*(8 + 15*Log[Cos[(a + b*x)/2]] - 15*Log[Sin[(a + b*x)/
2]]) + Sec[(a + b*x)/2]^4) - 14*Tan[(a + b*x)/2]^2))/(-1 + Tan[(a + b*x)/2
]^2))/b
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.26, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {3042, 3102, 25, 252, 252, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^5(a+bx)\sec^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(a+bx)^5\sec(a+bx)^2dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{\int -\frac{\sec^6(a+bx)}{(1-\sec^2(a+bx))^3}d\sec(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\sec^6(a+bx)}{(1-\sec^2(a+bx))^3}d\sec(a+bx)}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{5}{4}\int \frac{\sec^4(a+bx)}{(1-\sec^2(a+bx))^2}d\sec(a+bx)-\frac{\sec^5(a+bx)}{4(1-\sec^2(a+bx))^2}}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{5}{4}\left(\frac{\sec^3(a+bx)}{2(1-\sec^2(a+bx))}-\frac{3}{2}\int \frac{\sec^2(a+bx)}{1-\sec^2(a+bx)}d\sec(a+bx)\right)-\frac{\sec^5(a+bx)}{4(1-\sec^2(a+bx))^2}}{b}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{5}{4}\left(\frac{\sec^3(a+bx)}{2(1-\sec^2(a+bx))}-\frac{3}{2}\left(\int\frac{1}{1-\sec^2(a+bx)}d\sec(a+bx)-\sec(a+bx)\right)\right)-\frac{\sec^5(a+bx)}{4(1-\sec^2(a+bx))^2}}{b}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{5}{4}\left(\frac{\sec^3(a+bx)}{2(1-\sec^2(a+bx))}-\frac{3}{2}(\text{arctanh}(\sec(a+bx))-\sec(a+bx))\right)-\frac{\sec^5(a+bx)}{4(1-\sec^2(a+bx))^2}}{b}$$

input
```
Int[Csc[a + b*x]^5*Sec[a + b*x]^2,x]
```

output
```
(-1/4*Sec[a + b*x]^5/(1 - Sec[a + b*x]^2)^2 + (5*((-3*(ArcTanh[Sec[a + b*x
]] - Sec[a + b*x]))/2 + Sec[a + b*x]^3/(2*(1 - Sec[a + b*x]^2))))/4)/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{-\frac{1}{4\sin(bx+a)^4\cos(bx+a)}-\frac{5}{8\sin(bx+a)^2\cos(bx+a)}+\frac{15}{8\cos(bx+a)}+\frac{15\ln(\csc(bx+a)-\cot(bx+a))}{8}}{b}$
default	$\frac{-\frac{1}{4\sin(bx+a)^4\cos(bx+a)}-\frac{5}{8\sin(bx+a)^2\cos(bx+a)}+\frac{15}{8\cos(bx+a)}+\frac{15\ln(\csc(bx+a)-\cot(bx+a))}{8}}{b}$
parallelrisch	$\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^6+15\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4+\cot\left(\frac{bx}{2}+\frac{a}{2}\right)^4+120\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2+15\cot\left(\frac{bx}{2}+\frac{a}{2}\right)^2-120\ln\left(\tan\left(\right.\right.}{64b\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1\right)}$
norman	$\frac{\frac{1}{64b}+\frac{15\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}{64b}+\frac{15\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^8}{64b}+\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{10}}{64b}-\frac{5\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}{2b}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1\right)}+\frac{15\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}$
risch	$\frac{15\,e^{9i(bx+a)}-40\,e^{7i(bx+a)}+18\,e^{5i(bx+a)}-40\,e^{3i(bx+a)}+15\,e^{i(bx+a)}}{4b\left(e^{2i(bx+a)}-1\right)^4\left(e^{2i(bx+a)}+1\right)}+\frac{15\ln\left(e^{i(bx+a)}-1\right)}{8b}-\frac{15\ln\left(e^{i(bx+a)}+1\right)}{8b}$

input
```
int(csc(b*x+a)^5*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/4/sin(b*x+a)^4/cos(b*x+a)-5/8/sin(b*x+a)^2/cos(b*x+a)+15/8/cos(b*x
+a)+15/8*ln(csc(b*x+a)-cot(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(59) = 118$.

Time = 0.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.03

$$\int \csc^5(a+bx)\sec^2(a+bx)\,dx$$
$$= \frac{30\,\cos\left(bx+a\right)^4 - 50\,\cos\left(bx+a\right)^2 - 15\,\left(\cos\left(bx+a\right)^5 - 2\,\cos\left(bx+a\right)^3 + \cos\left(bx+a\right)\right)\log\left(\frac{1}{2}\,\cos\left(bx -\right.\right.}{16\,\left(b\cos\left(bx+a\right)^5 - 2\,b\cos\left(bx -\right.\right.}$$

input
```
integrate(csc(b*x+a)^5*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/16*(30*cos(b*x + a)^4 - 50*cos(b*x + a)^2 - 15*(cos(b*x + a)^5 - 2*cos(b
*x + a)^3 + cos(b*x + a))*log(1/2*cos(b*x + a) + 1/2) + 15*(cos(b*x + a)^5
 - 2*cos(b*x + a)^3 + cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2) + 16)/(b*
cos(b*x + a)^5 - 2*b*cos(b*x + a)^3 + b*cos(b*x + a))
```

Sympy [F]

$$\int \csc^5(a+bx)\sec^2(a+bx)\,dx = \int \csc^5\left(a+bx\right)\sec^2\left(a+bx\right)dx$$

input
```
integrate(csc(b*x+a)**5*sec(b*x+a)**2,x)
```

output
```
Integral(csc(a + b*x)**5*sec(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.22

$$\int \csc^5(a+bx)\sec^2(a+bx)\,dx = \frac{\frac{2\left(15\cos(bx+a)^4-25\cos(bx+a)^2+8\right)}{\cos(bx+a)^5-2\cos(bx+a)^3+\cos(bx+a)} - 15\log\left(\cos(bx+a)+1\right) + 15\log\left(\cos(bx+a)-1\right)}{16\,b}$$

input
```
integrate(csc(b*x+a)^5*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/16*(2*(15*cos(b*x + a)^4 - 25*cos(b*x + a)^2 + 8)/(cos(b*x + a)^5 - 2*co
s(b*x + a)^3 + cos(b*x + a)) - 15*log(cos(b*x + a) + 1) + 15*log(cos(b*x +
 a) - 1))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.22

$$\int \csc^5(a+bx)\sec^2(a+bx)\,dx = -\frac{15\log\left(|\cos(bx+a)+1|\right)}{16\,b} + \frac{15\log\left(|\cos(bx+a)-1|\right)}{16\,b} + \frac{7\cos(bx+a)^3-9\cos(bx+a)}{8\left(\cos(bx+a)^2-1\right)^2 b} + \frac{1}{b\cos(bx+a)}$$

input
```
integrate(csc(b*x+a)^5*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
-15/16*log(abs(cos(b*x + a) + 1))/b + 15/16*log(abs(cos(b*x + a) - 1))/b +
 1/8*(7*cos(b*x + a)^3 - 9*cos(b*x + a))/((cos(b*x + a)^2 - 1)^2*b) + 1/(b
*cos(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02

$$\int \csc^5(a+bx)\sec^2(a+bx)\,dx = \frac{\frac{15\cos(a+b\,x)^4}{8}-\frac{25\cos(a+b\,x)^2}{8}+1}{b\,\left(\cos\left(a+b\,x\right)^5-2\cos\left(a+b\,x\right)^3+\cos\left(a+b\,x\right)\right)} - \frac{15\,\mathrm{atanh}(\cos\left(a+b\,x\right))}{8\,b}$$

input
```
int(1/(cos(a + b*x)^2*sin(a + b*x)^5),x)
```

output
```
((15*cos(a + b*x)^4)/8 - (25*cos(a + b*x)^2)/8 + 1)/(b*(cos(a + b*x) - 2*c
os(a + b*x)^3 + cos(a + b*x)^5)) - (15*atanh(cos(a + b*x)))/(8*b)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.31

$$\int \csc^5(a+bx)\sec^2(a+bx)\,dx$$
$$= \frac{15\cos\left(bx+a\right)\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\sin\left(bx+a\right)^4-10\cos\left(bx+a\right)\sin\left(bx+a\right)^4+15\sin\left(bx+a\right)^4-5\sin\left(\right.}{8\cos\left(bx+a\right)\sin\left(bx+a\right)^4 b}$$

input
```
int(csc(b*x+a)^5*sec(b*x+a)^2,x)
```

output
```
(15*cos(a + b*x)*log(tan((a + b*x)/2))*sin(a + b*x)**4 - 10*cos(a + b*x)*s
in(a + b*x)**4 + 15*sin(a + b*x)**4 - 5*sin(a + b*x)**2 - 2)/(8*cos(a + b*
x)*sin(a + b*x)**4*b)
```

3.182 $\int \csc^5(a+bx)\sec^3(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 58

$$\int \csc^5(a+bx)\sec^3(a+bx)\,dx = -\frac{3\cot^2(a+bx)}{2b} - \frac{\cot^4(a+bx)}{4b} + \frac{3\log(\tan(a+bx))}{b} + \frac{\tan^2(a+bx)}{2b}$$

output
```
-3/2*cot(b*x+a)^2/b-1/4*cot(b*x+a)^4/b+3*ln(tan(b*x+a))/b+1/2*tan(b*x+a)^2
/b
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93

$$\int \csc^5(a+bx)\sec^3(a+bx)\,dx = -\frac{4\csc^2(a+bx)+\csc^4(a+bx)+12\log(\cos(a+bx))-12\log(\sin(a+bx))-2\sec^2(a+bx)}{4b}$$

input
```
Integrate[Csc[a + b*x]^5*Sec[a + b*x]^3,x]
```

output
```
-1/4*(4*Csc[a + b*x]^2 + Csc[a + b*x]^4 + 12*Log[Cos[a + b*x]] - 12*Log[Si
n[a + b*x]] - 2*Sec[a + b*x]^2)/b
```

Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3042, 3100, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^5(a+bx)\sec^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(a+bx)^5\sec(a+bx)^3dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{\int \cot^5(a+bx)\left(\tan^2(a+bx)+1\right)^3d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int \cot^3(a+bx)\left(\tan^2(a+bx)+1\right)^3d\tan^2(a+bx)}{2b}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\cot^3(a+bx)+3\cot^2(a+bx)+3\cot(a+bx)+1\right)d\tan^2(a+bx)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\tan^2(a+bx)-\frac{1}{2}\cot^2(a+bx)-3\cot(a+bx)+3\log\left(\tan^2(a+bx)\right)}{2b}$$

input
```
Int[Csc[a + b*x]^5*Sec[a + b*x]^3,x]
```

output
```
(-3*Cot[a + b*x] - Cot[a + b*x]^2/2 + 3*Log[Tan[a + b*x]^2] + Tan[a + b*x]
^2)/(2*b)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{-\frac{1}{4\sin(bx+a)^4\cos(bx+a)^2}+\frac{3}{4\sin(bx+a)^2\cos(bx+a)^2}-\frac{3}{2\sin(bx+a)^2}+3\ln(\tan(bx+a))}{b}$
default	$\frac{-\frac{1}{4\sin(bx+a)^4\cos(bx+a)^2}+\frac{3}{4\sin(bx+a)^2\cos(bx+a)^2}-\frac{3}{2\sin(bx+a)^2}+3\ln(\tan(bx+a))}{b}$
risch	$\frac{6\,e^{10i(bx+a)}-12\,e^{8i(bx+a)}-4\,e^{6i(bx+a)}-12\,e^{4i(bx+a)}+6\,e^{2i(bx+a)}}{b\left(e^{2i(bx+a)}-1\right)^4\left(e^{2i(bx+a)}+1\right)^2}-\frac{3\ln\left(e^{2i(bx+a)}+1\right)}{b}+\frac{3\ln\left(e^{2i(bx+a)}-1\right)}{b}$
norman	$\frac{-\frac{1}{64b}-\frac{9\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}{32b}-\frac{9\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{10}}{32b}-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{12}}{64b}+\frac{83\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^6}{32b}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1\right)^2}+\frac{3\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}-\frac{3\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right.\right.}{b}$
parallelrisch	$\left(-192\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4+384\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2-192\right)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+\left(-192\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4+384\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2-192\right.$

input
```
int(csc(b*x+a)^5*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/4/sin(b*x+a)^4/cos(b*x+a)^2+3/4/sin(b*x+a)^2/cos(b*x+a)^2-3/2/sin(
b*x+a)^2+3*ln(tan(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(52) = 104$.

Time = 0.10 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.38

$$\int \csc^5(a+bx)\sec^3(a+bx)\,dx$$
$$=\frac{6\cos(bx+a)^4-9\cos(bx+a)^2-6\left(\cos(bx+a)^6-2\cos(bx+a)^4+\cos(bx+a)^2\right)\log\left(\cos(bx+a)^2\right.}{4\left(b\cos(bx+a)^6-2\,b\cos(bx+a)^4\right.}$$

input
```
integrate(csc(b*x+a)^5*sec(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4*(6*cos(b*x + a)^4 - 9*cos(b*x + a)^2 - 6*(cos(b*x + a)^6 - 2*cos(b*x +
 a)^4 + cos(b*x + a)^2)*log(cos(b*x + a)^2) + 6*(cos(b*x + a)^6 - 2*cos(b*
x + a)^4 + cos(b*x + a)^2)*log(-1/4*cos(b*x + a)^2 + 1/4) + 2)/(b*cos(b*x
+ a)^6 - 2*b*cos(b*x + a)^4 + b*cos(b*x + a)^2)
```

Sympy [F]

$$\int \csc^5(a+bx)\sec^3(a+bx)\,dx = \int \csc^5(a+bx)\sec^3(a+bx)\,dx$$

input
```
integrate(csc(b*x+a)**5*sec(b*x+a)**3,x)
```

output
```
Integral(csc(a + b*x)**5*sec(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.28

$$\int \csc^5(a+bx)\sec^3(a+bx)\,dx = -\frac{\frac{6\sin(bx+a)^4-3\sin(bx+a)^2-1}{\sin(bx+a)^6-\sin(bx+a)^4} + 6\log\left(\sin(bx+a)^2-1\right) - 6\log\left(\sin(bx+a)^2\right)}{4b}$$

input
```
integrate(csc(b*x+a)^5*sec(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/4*((6*sin(b*x + a)^4 - 3*sin(b*x + a)^2 - 1)/(sin(b*x + a)^6 - sin(b*x
+ a)^4) + 6*log(sin(b*x + a)^2 - 1) - 6*log(sin(b*x + a)^2))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.34

$$\int \csc^5(a+bx)\sec^3(a+bx)\,dx = -\frac{3\,\log\left(\left|\sin\left(bx+a\right)^2-1\right|\right)}{2\,b}+\frac{3\,\log\left(\left|\sin\left(bx+a\right)\right|\right)}{b} - \frac{6\,\sin\left(bx+a\right)^4-3\,\sin\left(bx+a\right)^2-1}{4\left(\sin\left(bx+a\right)^2-1\right)b\sin\left(bx+a\right)^4}$$

input
```
integrate(csc(b*x+a)^5*sec(b*x+a)^3,x, algorithm="giac")
```

output
```
-3/2*log(abs(sin(b*x + a)^2 - 1))/b + 3*log(abs(sin(b*x + a)))/b - 1/4*(6*
sin(b*x + a)^4 - 3*sin(b*x + a)^2 - 1)/((sin(b*x + a)^2 - 1)*b*sin(b*x + a
)^4)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.41

$$\int \csc^5(a+bx)\sec^3(a+bx)\,dx = \frac{3\,\ln\left(\sin\left(a+b\,x\right)^2\right)}{2\,b}-\frac{3\,\ln\left(\cos\left(a+b\,x\right)\right)}{b} + \frac{\frac{3\cos(a+b\,x)^4}{2}-\frac{9\cos(a+b\,x)^2}{4}+\frac{1}{2}}{b\,\left(\cos\left(a+b\,x\right)^6-2\cos\left(a+b\,x\right)^4+\cos\left(a+b\,x\right)^2\right)}$$

input
```
int(1/(cos(a + b*x)^3*sin(a + b*x)^5),x)
```

output
```
(3*log(sin(a + b*x)^2))/(2*b) - (3*log(cos(a + b*x)))/b + ((3*cos(a + b*x)
^4)/2 - (9*cos(a + b*x)^2)/4 + 1/2)/(b*(cos(a + b*x)^2 - 2*cos(a + b*x)^4
+ cos(a + b*x)^6))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 185, normalized size of antiderivative = 3.19

$$\int \csc^5(a+bx)\sec^3(a+bx)\,dx$$
$$= \frac{-192\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin\left(bx+a\right)^6+192\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin\left(bx+a\right)^4-192\log\left(\tan\left(\frac{bx}{2}+\right.\right.}{}$$

input
```
int(csc(b*x+a)^5*sec(b*x+a)^3,x)
```

output
```
( - 192*log(tan((a + b*x)/2) - 1)*sin(a + b*x)**6 + 192*log(tan((a + b*x)/
2) - 1)*sin(a + b*x)**4 - 192*log(tan((a + b*x)/2) + 1)*sin(a + b*x)**6 +
192*log(tan((a + b*x)/2) + 1)*sin(a + b*x)**4 + 192*log(tan((a + b*x)/2))*
sin(a + b*x)**6 - 192*log(tan((a + b*x)/2))*sin(a + b*x)**4 - 33*sin(a + b
*x)**6 - 63*sin(a + b*x)**4 + 48*sin(a + b*x)**2 + 16)/(64*sin(a + b*x)**4
*b*(sin(a + b*x)**2 - 1))
```

3.183 $\int \csc^5(a+bx)\sec^4(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 81

$$\int \csc^5(a+bx)\sec^4(a+bx)\,dx = -\frac{35\text{arctanh}(\cos(a+bx))}{8b} - \frac{13\cot(a+bx)\csc(a+bx)}{8b} - \frac{\cot^3(a+bx)\csc(a+bx)}{4b} + \frac{3\sec(a+bx)}{b} + \frac{\sec^3(a+bx)}{3b}$$

output
```
-35/8*arctanh(cos(b*x+a))/b-13/8*cot(b*x+a)*csc(b*x+a)/b-1/4*cot(b*x+a)^3*
csc(b*x+a)/b+3*sec(b*x+a)/b+1/3*sec(b*x+a)^3/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 268 vs. 2(81) = 162.

Time = 0.73 (sec) , antiderivative size = 268, normalized size of antiderivative = 3.31

$$\int \csc^5(a+bx)\sec^4(a+bx)\,dx = -\frac{\csc^{10}(a+bx)\left(-204+658\cos(2(a+bx))-228\cos(3(a+bx))+140\cos(4(a+bx))-76\cos(5(a+b\right.}{}$$

input
```
Integrate[Csc[a + b*x]^5*Sec[a + b*x]^4,x]
```

output
```
-1/24*(Csc[a + b*x]^10*(-204 + 658*Cos[2*(a + b*x)] - 228*Cos[3*(a + b*x)]
 + 140*Cos[4*(a + b*x)] - 76*Cos[5*(a + b*x)] - 210*Cos[6*(a + b*x)] + 76*
Cos[7*(a + b*x)] - 315*Cos[3*(a + b*x)]*Log[Cos[(a + b*x)/2]] - 105*Cos[5*
(a + b*x)]*Log[Cos[(a + b*x)/2]] + 105*Cos[7*(a + b*x)]*Log[Cos[(a + b*x)/
2]] + 3*Cos[a + b*x]*(76 + 105*Log[Cos[(a + b*x)/2]] - 105*Log[Sin[(a + b*
x)/2]]) + 315*Cos[3*(a + b*x)]*Log[Sin[(a + b*x)/2]] + 105*Cos[5*(a + b*x)
]*Log[Sin[(a + b*x)/2]] - 105*Cos[7*(a + b*x)]*Log[Sin[(a + b*x)/2]]))/(b*
(Csc[(a + b*x)/2]^2 - Sec[(a + b*x)/2]^2)^3)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3042, 3102, 25, 252, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^5(a+bx)\sec^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(a+bx)^5\sec(a+bx)^4dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{\int -\frac{\sec^8(a+bx)}{(1-\sec^2(a+bx))^3}d\sec(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\sec^8(a+bx)}{(1-\sec^2(a+bx))^3}d\sec(a+bx)}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{7}{4}\int \frac{\sec^6(a+bx)}{(1-\sec^2(a+bx))^2}d\sec(a+bx)-\frac{\sec^7(a+bx)}{4(1-\sec^2(a+bx))^2}}{b}$$

$$\Big\downarrow \ 252$$

$$\frac{\frac{7}{4}\left(\frac{\sec^5(a+bx)}{2(1-\sec^2(a+bx))}-\frac{5}{2}\int \frac{\sec^4(a+bx)}{1-\sec^2(a+bx)}d\sec(a+bx)\right)-\frac{\sec^7(a+bx)}{4(1-\sec^2(a+bx))^2}}{b}$$

$$\Big\downarrow \ 254$$

$$\frac{\frac{7}{4}\left(\frac{\sec^5(a+bx)}{2(1-\sec^2(a+bx))}-\frac{5}{2}\int \left(-\sec^2(a+bx)+\frac{1}{1-\sec^2(a+bx)}-1\right)d\sec(a+bx)\right)-\frac{\sec^7(a+bx)}{4(1-\sec^2(a+bx))^2}}{b}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{7}{4}\left(\frac{\sec^5(a+bx)}{2(1-\sec^2(a+bx))}-\frac{5}{2}\left(\operatorname{arctanh}(\sec(a+bx))-\frac{1}{3}\sec^3(a+bx)-\sec(a+bx)\right)\right)-\frac{\sec^7(a+bx)}{4(1-\sec^2(a+bx))^2}}{b}$$

input
```
Int[Csc[a + b*x]^5*Sec[a + b*x]^4,x]
```

output
```
(-1/4*Sec[a + b*x]^7/(1 - Sec[a + b*x]^2)^2 + (7*(Sec[a + b*x]^5/(2*(1 - S
ec[a + b*x]^2)) - (5*(ArcTanh[Sec[a + b*x]] - Sec[a + b*x] - Sec[a + b*x]^
3/3))/2))/4)/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 2.34 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{-\frac{1}{4 \sin(bx+a)^4 \cos(bx+a)^3}+\frac{7}{12 \sin(bx+a)^2 \cos(bx+a)^3}-\frac{35}{24 \sin(bx+a)^2 \cos(bx+a)}+\frac{35}{8 \cos(bx+a)}+\frac{35 \ln(\csc(bx+a)-\cot(bx+a))}{8}}{b}$
default	$\frac{-\frac{1}{4 \sin(bx+a)^4 \cos(bx+a)^3}+\frac{7}{12 \sin(bx+a)^2 \cos(bx+a)^3}-\frac{35}{24 \sin(bx+a)^2 \cos(bx+a)}+\frac{35}{8 \cos(bx+a)}+\frac{35 \ln(\csc(bx+a)-\cot(bx+a))}{8}}{b}$
risch	$\frac{105 \,\mathrm{e}^{13 i (bx+a)}-70 \,\mathrm{e}^{11 i (bx+a)}-329 \,\mathrm{e}^{9 i (bx+a)}+204 \,\mathrm{e}^{7 i (bx+a)}-329 \,\mathrm{e}^{5 i (bx+a)}-70 \,\mathrm{e}^{3 i (bx+a)}+105 \,\mathrm{e}^{i (bx+a)}}{12 b \left(\mathrm{e}^{2 i (bx+a)}-1\right)^4 \left(\mathrm{e}^{2 i (bx+a)}+1\right)^3}-\frac{35 \ln(\mathrm{e}^{i(bx}}{8b}$
norman	$\frac{\frac{1}{64b}+\frac{21 \tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}{64b}+\frac{21 \tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{12}}{64b}+\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{14}}{64b}-\frac{21 \tan\left(\frac{bx}{2}+\frac{a}{2}\right)^8}{2b}+\frac{511 \tan\left(\frac{bx}{2}+\frac{a}{2}\right)^6}{32b}-\frac{847 \tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4}{96b}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4 \left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1\right)^3}+\frac{35}{}$
parallelrisch	$\frac{840 \ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right) \left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^3 \left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)^3+3 \tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{10}+63 \tan\left(\frac{bx}{2}+\frac{a}{2}\right)^8+3 \cot\left(\frac{bx}{2}+\frac{a}{2}\right)^4-2}{192 b \left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^3 \left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)^3}$

input
```
int(csc(b*x+a)^5*sec(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/4/sin(b*x+a)^4/cos(b*x+a)^3+7/12/sin(b*x+a)^2/cos(b*x+a)^3-35/24/s
in(b*x+a)^2/cos(b*x+a)+35/8/cos(b*x+a)+35/8*ln(csc(b*x+a)-cot(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(73) = 146$.

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.83

$$\int \csc^5(a+bx)\sec^4(a+bx)\,dx$$
$$= \frac{210\cos(bx+a)^6 - 350\cos(bx+a)^4 + 112\cos(bx+a)^2 - 105\left(\cos(bx+a)^7 - 2\cos(bx+a)^5 + \cos(\right.}{48\left(b\cos(bx+a)^7\right.}$$

input
```
integrate(csc(b*x+a)^5*sec(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/48*(210*cos(b*x + a)^6 - 350*cos(b*x + a)^4 + 112*cos(b*x + a)^2 - 105*(
cos(b*x + a)^7 - 2*cos(b*x + a)^5 + cos(b*x + a)^3)*log(1/2*cos(b*x + a) +
 1/2) + 105*(cos(b*x + a)^7 - 2*cos(b*x + a)^5 + cos(b*x + a)^3)*log(-1/2*
cos(b*x + a) + 1/2) + 16)/(b*cos(b*x + a)^7 - 2*b*cos(b*x + a)^5 + b*cos(b
*x + a)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^5(a+bx)\sec^4(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**5*sec(b*x+a)**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.12

$$\int \csc^5(a+bx)\sec^4(a+bx)\,dx = \frac{\frac{2\left(105\cos(bx+a)^6-175\cos(bx+a)^4+56\cos(bx+a)^2+8\right)}{\cos(bx+a)^7-2\cos(bx+a)^5+\cos(bx+a)^3}-105\log\left(\cos(bx+a)+1\right)+105\log\left(\cos(bx+a)-1\right)}{48\,b}$$

input
```
integrate(csc(b*x+a)^5*sec(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/48*(2*(105*cos(b*x + a)^6 - 175*cos(b*x + a)^4 + 56*cos(b*x + a)^2 + 8)/
(cos(b*x + a)^7 - 2*cos(b*x + a)^5 + cos(b*x + a)^3) - 105*log(cos(b*x + a
) + 1) + 105*log(cos(b*x + a) - 1))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.14

$$\int \csc^5(a+bx)\sec^4(a+bx)\,dx = -\frac{35\log\left(|\cos(bx+a)+1|\right)}{16\,b} + \frac{35\log\left(|\cos(bx+a)-1|\right)}{16\,b} + \frac{11\cos(bx+a)^3-13\cos(bx+a)}{8\left(\cos(bx+a)^2-1\right)^2 b} + \frac{9\cos(bx+a)^2+1}{3\,b\cos(bx+a)^3}$$

input
```
integrate(csc(b*x+a)^5*sec(b*x+a)^4,x, algorithm="giac")
```

output
```
-35/16*log(abs(cos(b*x + a) + 1))/b + 35/16*log(abs(cos(b*x + a) - 1))/b +
 1/8*(11*cos(b*x + a)^3 - 13*cos(b*x + a))/((cos(b*x + a)^2 - 1)^2*b) + 1/
3*(9*cos(b*x + a)^2 + 1)/(b*cos(b*x + a)^3)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96

$$\int \csc^5(a+bx)\sec^4(a+bx)\,dx = \frac{\frac{35\cos(a+b\,x)^6}{8}-\frac{175\cos(a+b\,x)^4}{24}+\frac{7\cos(a+b\,x)^2}{3}+\frac{1}{3}}{b\,\left(\cos\left(a+b\,x\right)^7-2\cos\left(a+b\,x\right)^5+\cos\left(a+b\,x\right)^3\right)} - \frac{35\,\mathrm{atanh}(\cos\left(a+b\,x\right))}{8\,b}$$

input
```
int(1/(cos(a + b*x)^4*sin(a + b*x)^5),x)
```

output
```
((7*cos(a + b*x)^2)/3 - (175*cos(a + b*x)^4)/24 + (35*cos(a + b*x)^6)/8 +
1/3)/(b*(cos(a + b*x)^3 - 2*cos(a + b*x)^5 + cos(a + b*x)^7)) - (35*atanh(
cos(a + b*x)))/(8*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.84

$$\int \csc^5(a+bx)\sec^4(a+bx)\,dx = \frac{840\cos\left(bx+a\right)\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\sin\left(bx+a\right)^6-840\cos\left(bx+a\right)\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\sin\left(bx+a\right)^4-847}{192\cos\left(bx+a\right)}$$

input
```
int(csc(b*x+a)^5*sec(b*x+a)^4,x)
```

output
```
(840*cos(a + b*x)*log(tan((a + b*x)/2))*sin(a + b*x)**6 - 840*cos(a + b*x)
*log(tan((a + b*x)/2))*sin(a + b*x)**4 - 847*cos(a + b*x)*sin(a + b*x)**6
+ 847*cos(a + b*x)*sin(a + b*x)**4 + 840*sin(a + b*x)**6 - 1120*sin(a + b*
x)**4 + 168*sin(a + b*x)**2 + 48)/(192*cos(a + b*x)*sin(a + b*x)**4*b*(sin
(a + b*x)**2 - 1))
```

3.184 $\int \csc^5(a+bx)\sec^5(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 69

$$\int \csc^5(a+bx)\sec^5(a+bx)\,dx = -\frac{2\cot^2(a+bx)}{b}-\frac{\cot^4(a+bx)}{4b}+\frac{6\log(\tan(a+bx))}{b}+\frac{2\tan^2(a+bx)}{b}+\frac{\tan^4(a+bx)}{4b}$$

output
```
-2*cot(b*x+a)^2/b-1/4*cot(b*x+a)^4/b+6*ln(tan(b*x+a))/b+2*tan(b*x+a)^2/b+1
/4*tan(b*x+a)^4/b
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.32

$$\int \csc^5(a+bx)\sec^5(a+bx)\,dx = 32\left(-\frac{3\csc^2(a+bx)}{64b}-\frac{\csc^4(a+bx)}{128b}-\frac{3\log(\cos(a+bx))}{16b}+\frac{3\log(\sin(a+bx))}{16b}+\frac{3\sec^2(a+bx)}{64b}+\frac{\sec^4(a+bx)}{128b}\right)$$

input
```
Integrate[Csc[a + b*x]^5*Sec[a + b*x]^5,x]
```

output
```
32*((-3*Csc[a + b*x]^2)/(64*b) - Csc[a + b*x]^4/(128*b) - (3*Log[Cos[a + b
*x]])/(16*b) + (3*Log[Sin[a + b*x]])/(16*b) + (3*Sec[a + b*x]^2)/(64*b) +
Sec[a + b*x]^4/(128*b))
```

Rubi [A] (warning: unable to verify)

Time = 0.40 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3042, 3100, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^5(a+bx)\sec^5(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(a+bx)^5\sec(a+bx)^5dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{\int \cot^5(a+bx)\left(\tan^2(a+bx)+1\right)^4d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int \cot^3(a+bx)\left(\tan^2(a+bx)+1\right)^4d\tan^2(a+bx)}{2b}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\cot^3(a+bx)+4\cot^2(a+bx)+6\cot(a+bx)+\tan^2(a+bx)+4\right)d\tan^2(a+bx)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}\tan^4(a+bx)+4\tan^2(a+bx)-\frac{1}{2}\cot^2(a+bx)-4\cot(a+bx)+6\log\left(\tan^2(a+bx)\right)}{2b}$$

input
```
Int[Csc[a + b*x]^5*Sec[a + b*x]^5,x]
```

output
```
(-4*Cot[a + b*x] - Cot[a + b*x]^2/2 + 6*Log[Tan[a + b*x]^2] + 4*Tan[a + b*
x]^2 + Tan[a + b*x]^4/2)/(2*b)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Maple [A] (verified)

Time = 2.55 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{\frac{1}{4\sin(bx+a)^4\cos(bx+a)^4}-\frac{1}{2\sin(bx+a)^4\cos(bx+a)^2}+\frac{3}{2\sin(bx+a)^2\cos(bx+a)^2}-\frac{3}{\sin(bx+a)^2}+6\ln(\tan(bx+a))}{b}$
default	$\frac{\frac{1}{4\sin(bx+a)^4\cos(bx+a)^4}-\frac{1}{2\sin(bx+a)^4\cos(bx+a)^2}+\frac{3}{2\sin(bx+a)^2\cos(bx+a)^2}-\frac{3}{\sin(bx+a)^2}+6\ln(\tan(bx+a))}{b}$
risch	$\frac{12\,\mathrm{e}^{14i(bx+a)}-44\,\mathrm{e}^{10i(bx+a)}-44\,\mathrm{e}^{6i(bx+a)}+12\,\mathrm{e}^{2i(bx+a)}}{b\left(\mathrm{e}^{2i(bx+a)}-1\right)^4\left(\mathrm{e}^{2i(bx+a)}+1\right)^4}-\frac{6\ln\left(\mathrm{e}^{2i(bx+a)}+1\right)}{b}+\frac{6\ln\left(\mathrm{e}^{2i(bx+a)}-1\right)}{b}$
parallelrisch	$\frac{(-384\cos(2bx+2a)-96\cos(4bx+4a)-288)\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)+(-384\cos(2bx+2a)-96\cos(4bx+4a)-288)\ln\left(\tan\right.}{}$

input
```
int(csc(b*x+a)^5*sec(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/4/sin(b*x+a)^4/cos(b*x+a)^4-1/2/sin(b*x+a)^4/cos(b*x+a)^2+3/2/sin(b
*x+a)^2/cos(b*x+a)^2-3/sin(b*x+a)^2+6*ln(tan(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(65) = 130.

Time = 0.10 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.14

$$\int \csc^5(a+bx)\sec^5(a+bx)\,dx = \frac{12\cos(bx+a)^6-18\cos(bx+a)^4+4\cos(bx+a)^2-12\left(\cos(bx+a)^8-2\cos(bx+a)^6+\cos(bx+a\right.}{4\left(b\cos(bx+a)^8-2\,b\cos\right.}$$

input
```
integrate(csc(b*x+a)^5*sec(b*x+a)^5,x, algorithm="fricas")
```

output
```
1/4*(12*cos(b*x + a)^6 - 18*cos(b*x + a)^4 + 4*cos(b*x + a)^2 - 12*(cos(b*
x + a)^8 - 2*cos(b*x + a)^6 + cos(b*x + a)^4)*log(cos(b*x + a)^2) + 12*(co
s(b*x + a)^8 - 2*cos(b*x + a)^6 + cos(b*x + a)^4)*log(-1/4*cos(b*x + a)^2
+ 1/4) + 1)/(b*cos(b*x + a)^8 - 2*b*cos(b*x + a)^6 + b*cos(b*x + a)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^5(a+bx)\sec^5(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**5*sec(b*x+a)**5,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.33

$$\int \csc^5(a+bx)\sec^5(a+bx)\,dx = -\frac{\frac{12\sin(bx+a)^6-18\sin(bx+a)^4+4\sin(bx+a)^2+1}{\sin(bx+a)^8-2\sin(bx+a)^6+\sin(bx+a)^4}+12\log\left(\sin(bx+a)^2-1\right)-12\log\left(\sin(bx+a)^2\right)}{4b}$$

input
```
integrate(csc(b*x+a)^5*sec(b*x+a)^5,x, algorithm="maxima")
```

output
```
-1/4*((12*sin(b*x + a)^6 - 18*sin(b*x + a)^4 + 4*sin(b*x + a)^2 + 1)/(sin(
b*x + a)^8 - 2*sin(b*x + a)^6 + sin(b*x + a)^4) + 12*log(sin(b*x + a)^2 -
1) - 12*log(sin(b*x + a)^2))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.29

$$\int \csc^5(a+bx)\sec^5(a+bx)\,dx = -\frac{3\log\left(\left|\sin(bx+a)^2-1\right|\right)}{b}+\frac{6\log\left(\left|\sin(bx+a)\right|\right)}{b} - \frac{12\sin(bx+a)^6-18\sin(bx+a)^4+4\sin(bx+a)^2+1}{4\left(\sin(bx+a)^4-\sin(bx+a)^2\right)^2 b}$$

input
```
integrate(csc(b*x+a)^5*sec(b*x+a)^5,x, algorithm="giac")
```

output
```
-3*log(abs(sin(b*x + a)^2 - 1))/b + 6*log(abs(sin(b*x + a)))/b - 1/4*(12*s
in(b*x + a)^6 - 18*sin(b*x + a)^4 + 4*sin(b*x + a)^2 + 1)/((sin(b*x + a)^4
 - sin(b*x + a)^2)^2*b)
```

Mupad [B] (verification not implemented)

Time = 25.44 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.93

$$\int \csc^5(a+bx)\sec^5(a+bx)\,dx = \frac{2\tan(a+b\,x)^2}{b} + \frac{\tan(a+b\,x)^4}{4\,b} + \frac{6\,\ln\left(\tan(a+b\,x)\right)}{b} - \frac{\cot(a+b\,x)^4\left(2\tan(a+b\,x)^2+\frac{1}{4}\right)}{b}$$

input
```
int(1/(cos(a + b*x)^5*sin(a + b*x)^5),x)
```

output
```
(2*tan(a + b*x)^2)/b + tan(a + b*x)^4/(4*b) + (6*log(tan(a + b*x)))/b - (c
ot(a + b*x)^4*(2*tan(a + b*x)^2 + 1/4))/b
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 269, normalized size of antiderivative = 3.90

$$\int \csc^5(a+bx)\sec^5(a+bx)\,dx = \frac{-96\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin\left(bx+a\right)^8 + 192\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin\left(bx+a\right)^6 - 96\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right.}{}$$

input
```
int(csc(b*x+a)^5*sec(b*x+a)^5,x)
```

output
```
( - 96*log(tan((a + b*x)/2) - 1)*sin(a + b*x)**8 + 192*log(tan((a + b*x)/2
) - 1)*sin(a + b*x)**6 - 96*log(tan((a + b*x)/2) - 1)*sin(a + b*x)**4 - 96
*log(tan((a + b*x)/2) + 1)*sin(a + b*x)**8 + 192*log(tan((a + b*x)/2) + 1)
*sin(a + b*x)**6 - 96*log(tan((a + b*x)/2) + 1)*sin(a + b*x)**4 + 96*log(t
an((a + b*x)/2))*sin(a + b*x)**8 - 192*log(tan((a + b*x)/2))*sin(a + b*x)*
*6 + 96*log(tan((a + b*x)/2))*sin(a + b*x)**4 - 41*sin(a + b*x)**8 + 34*si
n(a + b*x)**6 + 31*sin(a + b*x)**4 - 16*sin(a + b*x)**2 - 4)/(16*sin(a + b
*x)**4*b*(sin(a + b*x)**4 - 2*sin(a + b*x)**2 + 1))
```

3.185 $\int \cot^2(x)\csc^4(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \cot^2(x)\csc^4(x)\,dx = -\frac{1}{3}\cot^3(x) - \frac{\cot^5(x)}{5}$$

output
```
-1/3*cot(x)^3-1/5*cot(x)^5
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \cot^2(x)\csc^4(x)\,dx = \frac{2\cot(x)}{15} + \frac{1}{15}\cot(x)\csc^2(x) - \frac{1}{5}\cot(x)\csc^4(x)$$

input
```
Integrate[Cot[x]^2*Csc[x]^4,x]
```

output
```
(2*Cot[x])/15 + (Cot[x]*Csc[x]^2)/15 - (Cot[x]*Csc[x]^4)/5
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3042, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(x)\csc^4(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(x-\frac{\pi}{2}\right)^2\sec\left(x-\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{3087}$$

$$\int \cot^2(x)\left(\cot^2(x)+1\right)d(-\cot(x))$$

$$\Big\downarrow \text{244}$$

$$\int \left(\cot^4(x)+\cot^2(x)\right)d(-\cot(x))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{5}\cot^5(x)-\frac{\cot^3(x)}{3}$$

input
```
Int[Cot[x]^2*Csc[x]^4,x]
```

output
```
-1/3*Cot[x]^3 - Cot[x]^5/5
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$-\frac{\cot(x)^{3}}{3}-\frac{\cot(x)^{5}}{5}$	14
default	$-\frac{\cot(x)^{3}}{3}-\frac{\cot(x)^{5}}{5}$	14
risch	$-\frac{4i\left(15\,\mathrm{e}^{6ix}+5\,\mathrm{e}^{4ix}+5\,\mathrm{e}^{2ix}-1\right)}{15\left(\mathrm{e}^{2ix}-1\right)^{5}}$	36

input
```
int(cot(x)^2*csc(x)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*cot(x)^3-1/5*cot(x)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(13) = 26$.

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.94

$$\int \cot^2(x)\csc^4(x)\,dx = \frac{2\,\cos(x)^5 - 5\,\cos(x)^3}{15\,\left(\cos(x)^4 - 2\,\cos(x)^2 + 1\right)\sin(x)}$$

input
```
integrate(cot(x)^2*csc(x)^4,x, algorithm="fricas")
```

output
```
1/15*(2*cos(x)^5 - 5*cos(x)^3)/((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(14) = 28$.

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.71

$$\int \cot^2(x)\csc^4(x)\,dx = \frac{2\cos(x)}{15\sin(x)} + \frac{\cos(x)}{15\sin^3(x)} - \frac{\cos(x)}{5\sin^5(x)}$$

input
```
integrate(cot(x)**2*csc(x)**4,x)
```

output
```
2*cos(x)/(15*sin(x)) + cos(x)/(15*sin(x)**3) - cos(x)/(5*sin(x)**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \cot^2(x)\csc^4(x)\,dx = -\frac{5\,\tan(x)^2 + 3}{15\,\tan(x)^5}$$

input
```
integrate(cot(x)^2*csc(x)^4,x, algorithm="maxima")
```

output
```
-1/15*(5*tan(x)^2 + 3)/tan(x)^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \cot^2(x)\csc^4(x)\,dx = -\frac{5\,\tan\left(x\right)^2+3}{15\,\tan\left(x\right)^5}$$

input
```
integrate(cot(x)^2*csc(x)^4,x, algorithm="giac")
```

output
```
-1/15*(5*tan(x)^2 + 3)/tan(x)^5
```

Mupad [B] (verification not implemented)

Time = 25.61 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \cot^2(x)\csc^4(x)\,dx = -\frac{\cot(x)^3\,\left(3\cot(x)^2+5\right)}{15}$$

input
```
int(cot(x)^2/sin(x)^4,x)
```

output
```
-(cot(x)^3*(3*cot(x)^2 + 5))/15
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \cot^2(x)\csc^4(x)\,dx = \frac{\cos\left(x\right)\left(2\sin\left(x\right)^4+\sin\left(x\right)^2-3\right)}{15\sin\left(x\right)^5}$$

input
```
int(cot(x)^2*csc(x)^4,x)
```

output
```
(cos(x)*(2*sin(x)**4 + sin(x)**2 - 3))/(15*sin(x)**5)
```

3.186 $\int \cot^3(x)\csc^4(x)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \cot^3(x)\csc^4(x)\,dx = \frac{\csc^4(x)}{4} - \frac{\csc^6(x)}{6}$$

output
```
1/4*csc(x)^4-1/6*csc(x)^6
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cot^3(x)\csc^4(x)\,dx = \frac{\csc^4(x)}{4} - \frac{\csc^6(x)}{6}$$

input
```
Integrate[Cot[x]^3*Csc[x]^4,x]
```

output
```
Csc[x]^4/4 - Csc[x]^6/6
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 25, 3086, 25, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(x)\csc^4(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(x-\frac{\pi}{2}\right)^3\left(-\sec\left(x-\frac{\pi}{2}\right)^4\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sec\left(x-\frac{\pi}{2}\right)^4\tan\left(x-\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{3086}$$

$$-\int -\csc^3(x)\left(1-\csc^2(x)\right)d\csc(x)$$

$$\Big\downarrow \text{25}$$

$$\int \csc^3(x)\left(1-\csc^2(x)\right)d\csc(x)$$

$$\Big\downarrow \text{244}$$

$$\int \left(\csc^3(x)-\csc^5(x)\right)d\csc(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{\csc^4(x)}{4}-\frac{\csc^6(x)}{6}$$

input
```
Int[Cot[x]^3*Csc[x]^4,x]
```

output
```
Csc[x]^4/4 - Csc[x]^6/6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

method	result	size
derivativedivides	$-\frac{\left(1+\cot \left(x \right)^{2}\right)^{3}}{6}+\frac{\left(1+\cot \left(x \right)^{2}\right)^{2}}{4}$	22
default	$-\frac{\left(1+\cot \left(x \right)^{2}\right)^{3}}{6}+\frac{\left(1+\cot \left(x \right)^{2}\right)^{2}}{4}$	22
risch	$\frac{4 \,{\mathrm e}^{8 i x}+\frac{8 \,{\mathrm e}^{6 i x}}{3}+4 \,{\mathrm e}^{4 i x}}{\left({\mathrm e}^{2 i x}-1\right)^{6}}$	34

input
```
int(cot(x)^3*csc(x)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(1+cot(x)^2)^3+1/4*(1+cot(x)^2)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(13) = 26$.

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.76

$$\int \cot^3(x)\csc^4(x)\,dx = \frac{3\cos(x)^2 - 1}{12\left(\cos(x)^6 - 3\cos(x)^4 + 3\cos(x)^2 - 1\right)}$$

input
```
integrate(cot(x)^3*csc(x)^4,x, algorithm="fricas")
```

output
```
1/12*(3*cos(x)^2 - 1)/(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \cot^3(x)\csc^4(x)\,dx = -\frac{2 - 3\sin^2(x)}{12\sin^6(x)}$$

input
```
integrate(cot(x)**3*csc(x)**4,x)
```

output
```
-(2 - 3*sin(x)**2)/(12*sin(x)**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \cot^3(x)\csc^4(x)\,dx = \frac{3\sin(x)^2 - 2}{12\sin(x)^6}$$

input
```
integrate(cot(x)^3*csc(x)^4,x, algorithm="maxima")
```

output
```
1/12*(3*sin(x)^2 - 2)/sin(x)^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \cot^3(x)\csc^4(x)\,dx = \frac{3\,\sin(x)^2 - 2}{12\,\sin(x)^6}$$

input
```
integrate(cot(x)^3*csc(x)^4,x, algorithm="giac")
```

output
```
1/12*(3*sin(x)^2 - 2)/sin(x)^6
```

Mupad [B] (verification not implemented)

Time = 25.48 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \cot^3(x)\csc^4(x)\,dx = -\frac{\cot(x)^4\,(2\cot(x)^2 + 3)}{12}$$

input
```
int(cot(x)^3/sin(x)^4,x)
```

output
```
-(cot(x)^4*(2*cot(x)^2 + 3))/12
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \cot^3(x)\csc^4(x)\,dx = \frac{\csc(x)^4\,(-2\cot(x)^2 + 1)}{12}$$

input
```
int(cot(x)^3*csc(x)^4,x)
```

output
```
(csc(x)**4*( - 2*cot(x)**2 + 1))/12
```

3.187 $\int (d\cos(a+bx))^{3/2}\sin(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 22

$$\int (d\cos(a+bx))^{3/2}\sin(a+bx)\,dx=-\frac{2(d\cos(a+bx))^{5/2}}{5bd}$$

output
```
-2/5*(d*cos(b*x+a))^(5/2)/b/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (d\cos(a+bx))^{3/2}\sin(a+bx)\,dx=-\frac{2(d\cos(a+bx))^{5/2}}{5bd}$$

input
```
Integrate[(d*Cos[a + b*x])^(3/2)*Sin[a + b*x],x]
```

output
```
(-2*(d*Cos[a + b*x])^(5/2))/(5*b*d)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 3045, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)(d\cos(a+bx))^{3/2} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)(d\cos(a+bx))^{3/2}dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int (d\cos(a+bx))^{3/2}d(d\cos(a+bx))}{bd}$$

$$\Big\downarrow \text{15}$$

$$-\frac{2(d\cos(a+bx))^{5/2}}{5bd}$$

input
```
Int[(d*Cos[a + b*x])^(3/2)*Sin[a + b*x],x]
```

output
```
(-2*(d*Cos[a + b*x])^(5/2))/(5*b*d)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$-\frac{2(d\cos(bx+a))^{\frac{5}{2}}}{5bd}$	19
default	$-\frac{2(d\cos(bx+a))^{\frac{5}{2}}}{5bd}$	19

input
```
int((d*cos(b*x+a))^(3/2)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(d*cos(b*x+a))^(5/2)/b/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (d\cos(a+bx))^{3/2}\sin(a+bx)\,dx = -\frac{2\,\sqrt{d\cos\left(bx+a\right)}d\cos\left(bx+a\right)^2}{5\,b}$$

input
```
integrate((d*cos(b*x+a))^(3/2)*sin(b*x+a),x, algorithm="fricas")
```

output
```
-2/5*sqrt(d*cos(b*x + a))*d*cos(b*x + a)^2/b
```

Sympy [A] (verification not implemented)

Time = 2.93 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int (d\cos(a+bx))^{3/2}\sin(a+bx)\,dx = \begin{cases} -\frac{2(d\cos(a+bx))^{\frac{3}{2}}\cos(a+bx)}{5b} & \text{for } b \neq 0 \\ x(d\cos(a))^{\frac{3}{2}}\sin(a) & \text{otherwise} \end{cases}$$

input
```
integrate((d*cos(b*x+a))**(3/2)*sin(b*x+a),x)
```

output
```
Piecewise((-2*(d*cos(a + b*x))**(3/2)*cos(a + b*x)/(5*b), Ne(b, 0)), (x*(d
*cos(a))**(3/2)*sin(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int (d\cos(a+bx))^{3/2}\sin(a+bx)\,dx = -\frac{2\,(d\cos(bx+a))^{\frac{5}{2}}}{5\,bd}$$

input
```
integrate((d*cos(b*x+a))^(3/2)*sin(b*x+a),x, algorithm="maxima")
```

output
```
-2/5*(d*cos(b*x + a))^(5/2)/(b*d)
```

Giac [F]

$$\int (d\cos(a+bx))^{3/2}\sin(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{3}{2}}\sin(bx+a)\,dx$$

input
```
integrate((d*cos(b*x+a))^(3/2)*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*cos(b*x + a))^(3/2)*sin(b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int (d\cos(a+bx))^{3/2}\sin(a+bx)\,dx = -\frac{2\,(d\,\cos\left(a+b\,x\right))^{5/2}}{5\,b\,d}$$

input
```
int(sin(a + b*x)*(d*cos(a + b*x))^(3/2),x)
```

output
```
-(2*(d*cos(a + b*x))^(5/2))/(5*b*d)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int (d\cos(a+bx))^{3/2}\sin(a+bx)\,dx = -\frac{2\sqrt{d}\,\sqrt{\cos\left(bx+a\right)}\,\cos\left(bx+a\right)^{2}d}{5b}$$

input
```
int((d*cos(b*x+a))^(3/2)*sin(b*x+a),x)
```

output
```
( - 2*sqrt(d)*sqrt(cos(a + b*x))*cos(a + b*x)**2*d)/(5*b)
```

3.188 $\int \sqrt{d\cos(a+bx)}\sin(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 22

$$\int \sqrt{d\cos(a+bx)}\sin(a+bx)\,dx = -\frac{2(d\cos(a+bx))^{3/2}}{3bd}$$

output
```
-2/3*(d*cos(b*x+a))^(3/2)/b/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \sqrt{d\cos(a+bx)}\sin(a+bx)\,dx = -\frac{2(d\cos(a+bx))^{3/2}}{3bd}$$

input
```
Integrate[Sqrt[d*Cos[a + b*x]]*Sin[a + b*x],x]
```

output
```
(-2*(d*Cos[a + b*x])^(3/2))/(3*b*d)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 3045, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sqrt{d\cos(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)\sqrt{d\cos(a+bx)}dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int \sqrt{d\cos(a+bx)}d(d\cos(a+bx))}{bd}$$

$$\Big\downarrow \text{15}$$

$$-\frac{2(d\cos(a+bx))^{3/2}}{3bd}$$

input
```
Int[Sqrt[d*Cos[a + b*x]]*Sin[a + b*x],x]
```

output
```
(-2*(d*Cos[a + b*x])^(3/2))/(3*b*d)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$-\frac{2(d\cos(bx+a))^{\frac{3}{2}}}{3bd}$	19
default	$-\frac{2(d\cos(bx+a))^{\frac{3}{2}}}{3bd}$	19

input
```
int((d*cos(b*x+a))^(1/2)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(d*cos(b*x+a))^(3/2)/b/d
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \sqrt{d\cos(a+bx)}\sin(a+bx)\,dx = -\frac{2\,\sqrt{d\cos\left(bx+a\right)}\cos\left(bx+a\right)}{3\,b}$$

input
```
integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a),x, algorithm="fricas")
```

output
```
-2/3*sqrt(d*cos(b*x + a))*cos(b*x + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \sqrt{d\cos(a+bx)}\sin(a+bx)\,dx = \begin{cases} -\frac{2\sqrt{d\cos{(a+bx)}}\cos{(a+bx)}}{3b} & \text{for } b \neq 0 \\ x\sqrt{d\cos{(a)}}\sin{(a)} & \text{otherwise} \end{cases}$$

input
```
integrate((d*cos(b*x+a))**(1/2)*sin(b*x+a),x)
```

output
```
Piecewise((-2*sqrt(d*cos(a + b*x))*cos(a + b*x)/(3*b), Ne(b, 0)), (x*sqrt(
d*cos(a))*sin(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \sqrt{d\cos(a+bx)}\sin(a+bx)\,dx = -\frac{2\,(d\cos{(bx+a)})^{\frac{3}{2}}}{3\,bd}$$

input
```
integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a),x, algorithm="maxima")
```

output
```
-2/3*(d*cos(b*x + a))^(3/2)/(b*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \sqrt{d\cos(a+bx)}\sin(a+bx)\,dx = -\frac{2\,\sqrt{d\cos{(bx+a)}}\cos{(bx+a)}}{3\,b}$$

input
```
integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a),x, algorithm="giac")
```

output
```
-2/3*sqrt(d*cos(b*x + a))*cos(b*x + a)/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \sqrt{d\cos(a+bx)}\sin(a+bx)\,dx = -\frac{2\,(d\,\cos\left(a+b\,x\right))^{3/2}}{3\,b\,d}$$

input
```
int(sin(a + b*x)*(d*cos(a + b*x))^(1/2),x)
```

output
```
-(2*(d*cos(a + b*x))^(3/2))/(3*b*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \sqrt{d\cos(a+bx)}\sin(a+bx)\,dx = -\frac{2\sqrt{d}\,\sqrt{\cos\left(bx+a\right)}\,\cos\left(bx+a\right)}{3b}$$

input
```
int((d*cos(b*x+a))^(1/2)*sin(b*x+a),x)
```

output
```
( - 2*sqrt(d)*sqrt(cos(a + b*x))*cos(a + b*x))/(3*b)
```

3.189 $\int \frac{\sin(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 20

$$\int \frac{\sin(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{2\sqrt{d\cos(a+bx)}}{bd}$$

output
```
-2*(d*cos(b*x+a))^(1/2)/b/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sin(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{2\sqrt{d\cos(a+bx)}}{bd}$$

input
```
Integrate[Sin[a + b*x]/Sqrt[d*Cos[a + b*x]],x]
```

output
```
(-2*Sqrt[d*Cos[a + b*x]])/(b*d)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 3045, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)}{\sqrt{d\cos(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)}{\sqrt{d\cos(a+bx)}}dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int \frac{1}{\sqrt{d\cos(a+bx)}}d(d\cos(a+bx))}{bd}$$

$$\Big\downarrow \text{15}$$

$$-\frac{2\sqrt{d\cos(a+bx)}}{bd}$$

input
```
Int[Sin[a + b*x]/Sqrt[d*Cos[a + b*x]],x]
```

output
```
(-2*Sqrt[d*Cos[a + b*x]])/(b*d)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$-\frac{2\sqrt{d\cos(bx+a)}}{bd}$	19
default	$-\frac{2\sqrt{d\cos(bx+a)}}{bd}$	19
risch	$-\frac{2\cos(bx+a)}{\sqrt{d\cos(bx+a)}\,b}$	22

input
```
int(sin(b*x+a)/(d*cos(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(d*cos(b*x+a))^(1/2)/b/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\sin(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{2\,\sqrt{d\cos\left(bx+a\right)}}{bd}$$

input
```
integrate(sin(b*x+a)/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(d*cos(b*x + a))/(b*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.

Time = 0.43 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{\sin(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \begin{cases} -\frac{2\cos{(a+bx)}}{b\sqrt{d\cos{(a+bx)}}} & \text{for } b \neq 0 \\ \frac{x\sin{(a)}}{\sqrt{d\cos{(a)}}} & \text{otherwise} \end{cases}$$

input
```
integrate(sin(b*x+a)/(d*cos(b*x+a))**(1/2),x)
```

output
```
Piecewise((-2*cos(a + b*x)/(b*sqrt(d*cos(a + b*x))), Ne(b, 0)), (x*sin(a)/
sqrt(d*cos(a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\sin(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{2\sqrt{d\cos{(bx+a)}}}{bd}$$

input
```
integrate(sin(b*x+a)/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
-2*sqrt(d*cos(b*x + a))/(b*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\sin(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{2\sqrt{d\cos{(bx+a)}}}{bd}$$

input
```
integrate(sin(b*x+a)/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(d*cos(b*x + a))/(b*d)
```

Mupad [B] (verification not implemented)

Time = 25.56 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\sin(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{2\,\sqrt{d\,\cos\left(a+b\,x\right)}}{b\,d}$$

input
```
int(sin(a + b*x)/(d*cos(a + b*x))^(1/2),x)
```

output
```
-(2*(d*cos(a + b*x))^(1/2))/(b*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{\sin(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{2\sqrt{d}\,\sqrt{\cos\left(bx+a\right)}}{bd}$$

input
```
int(sin(b*x+a)/(d*cos(b*x+a))^(1/2),x)
```

output
```
( - 2*sqrt(d)*sqrt(cos(a + b*x)))/(b*d)
```

3.190 $\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 20

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{2}{bd\sqrt{d\cos(a+bx)}}$$

output
```
2/b/d/(d*cos(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{2}{bd\sqrt{d\cos(a+bx)}}$$

input
```
Integrate[Sin[a + b*x]/(d*Cos[a + b*x])^(3/2),x]
```

output
```
2/(b*d*Sqrt[d*Cos[a + b*x]])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 3045, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int \frac{1}{(d\cos(a+bx))^{3/2}}d(d\cos(a+bx))}{bd}$$

$$\Big\downarrow \text{15}$$

$$\frac{2}{bd\sqrt{d\cos(a+bx)}}$$

input
```
Int[Sin[a + b*x]/(d*Cos[a + b*x])^(3/2),x]
```

output
```
2/(b*d*Sqrt[d*Cos[a + b*x]])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$\frac{2}{bd\sqrt{d\cos(bx+a)}}$	19
default	$\frac{2}{bd\sqrt{d\cos(bx+a)}}$	19

input
```
int(sin(b*x+a)/(d*cos(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/b/d/(d*cos(b*x+a))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{2\sqrt{d\cos(bx+a)}}{bd^2\cos(bx+a)}$$

input
```
integrate(sin(b*x+a)/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
2*sqrt(d*cos(b*x + a))/(b*d^2*cos(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(15) = 30$.

Time = 0.81 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \begin{cases} \frac{2\cos{(a+bx)}}{b(d\cos{(a+bx)})^{\frac{3}{2}}} & \text{for } b \neq 0 \\ \frac{x\sin{(a)}}{(d\cos{(a)})^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(sin(b*x+a)/(d*cos(b*x+a))**(3/2),x)
```

output
```
Piecewise((2*cos(a + b*x)/(b*(d*cos(a + b*x))**(3/2)), Ne(b, 0)), (x*sin(a
)/(d*cos(a))**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{2}{\sqrt{d\cos{(bx+a)}}bd}$$

input
```
integrate(sin(b*x+a)/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
2/(sqrt(d*cos(b*x + a))*b*d)
```

Giac [F]

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\sin{(bx+a)}}{(d\cos{(bx+a)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)/(d*cos(b*x + a))^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{4\,\cos\left(a+b\,x\right)\,\sqrt{d\,\cos\left(a+b\,x\right)}}{b\,d^2\,\left(\cos\left(2\,a+2\,b\,x\right)+1\right)}$$

input
```
int(sin(a + b*x)/(d*cos(a + b*x))^(3/2),x)
```

output
```
(4*cos(a + b*x)*(d*cos(a + b*x))^(1/2))/(b*d^2*(cos(2*a + 2*b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{2\sqrt{d}\,\sqrt{\cos\left(bx+a\right)}}{\cos\left(bx+a\right)b\,d^2}$$

input
```
int(sin(b*x+a)/(d*cos(b*x+a))^(3/2),x)
```

output
```
(2*sqrt(d)*sqrt(cos(a + b*x)))/(cos(a + b*x)*b*d**2)
```

3.191 $\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 22

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \frac{2}{3bd(d\cos(a+bx))^{3/2}}$$

output
```
2/3/b/d/(d*cos(b*x+a))^(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \frac{2}{3bd(d\cos(a+bx))^{3/2}}$$

input
```
Integrate[Sin[a + b*x]/(d*Cos[a + b*x])^(5/2),x]
```

output
```
2/(3*b*d*(d*Cos[a + b*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 3045, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3045}$$

$$-\frac{\int \frac{1}{(d\cos(a+bx))^{5/2}}d(d\cos(a+bx))}{bd}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{2}{3bd(d\cos(a+bx))^{3/2}}$$

input
```
Int[Sin[a + b*x]/(d*Cos[a + b*x])^(5/2),x]
```

output
```
2/(3*b*d*(d*Cos[a + b*x])^(3/2))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{2}{3bd(d\cos(bx+a))^{\frac{3}{2}}}$	19
default	$\frac{2}{3bd(d\cos(bx+a))^{\frac{3}{2}}}$	19

input
```
int(sin(b*x+a)/(d*cos(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/b/d/(d*cos(b*x+a))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \frac{2\sqrt{d\cos\left(bx+a\right)}}{3\,bd^3\cos\left(bx+a\right)^2}$$

input
```
integrate(sin(b*x+a)/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(d*cos(b*x + a))/(b*d^3*cos(b*x + a)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.

Time = 3.55 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \begin{cases} \frac{2\cos(a+bx)}{3b(d\cos(a+bx))^{\frac{5}{2}}} & \text{for } b \neq 0 \\ \frac{x\sin(a)}{(d\cos(a))^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(sin(b*x+a)/(d*cos(b*x+a))**(5/2),x)
```

output
```
Piecewise((2*cos(a + b*x)/(3*b*(d*cos(a + b*x))**(5/2)), Ne(b, 0)), (x*sin
(a)/(d*cos(a))**(5/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \frac{2}{3\,(d\cos(bx+a))^{\frac{3}{2}}\,bd}$$

input
```
integrate(sin(b*x+a)/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
2/3/((d*cos(b*x + a))^(3/2)*b*d)
```

Giac [F]

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{\sin(bx+a)}{(d\cos(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)/(d*cos(b*x + a))^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 25.78 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.41

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \frac{8\left(\cos\left(2\,a+2\,b\,x\right)+1\right)\,\sqrt{d\,\cos\left(a+b\,x\right)}}{3\,b\,d^3\,\left(4\,\cos\left(2\,a+2\,b\,x\right)+\cos\left(4\,a+4\,b\,x\right)+3\right)}$$

input
```
int(sin(a + b*x)/(d*cos(a + b*x))^(5/2),x)
```

output
```
(8*(cos(2*a + 2*b*x) + 1)*(d*cos(a + b*x))^(1/2))/(3*b*d^3*(4*cos(2*a + 2*
b*x) + cos(4*a + 4*b*x) + 3))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \frac{2\sqrt{d}\,\sqrt{\cos\left(bx+a\right)}}{3\cos\left(bx+a\right)^2 b\,d^3}$$

input
```
int(sin(b*x+a)/(d*cos(b*x+a))^(5/2),x)
```

output
```
(2*sqrt(d)*sqrt(cos(a + b*x)))/(3*cos(a + b*x)**2*b*d**3)
```

3.192 $\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 22

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{2}{5bd(d\cos(a+bx))^{5/2}}$$

output
```
2/5/b/d/(d*cos(b*x+a))^(5/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{2}{5bd(d\cos(a+bx))^{5/2}}$$

input
```
Integrate[Sin[a + b*x]/(d*Cos[a + b*x])^(7/2),x]
```

output
```
2/(5*b*d*(d*Cos[a + b*x])^(5/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 3045, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{7/2}}dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int \frac{1}{(d\cos(a+bx))^{7/2}}d(d\cos(a+bx))}{bd}$$

$$\Big\downarrow \text{15}$$

$$\frac{2}{5bd(d\cos(a+bx))^{5/2}}$$

input
```
Int[Sin[a + b*x]/(d*Cos[a + b*x])^(7/2),x]
```

output
```
2/(5*b*d*(d*Cos[a + b*x])^(5/2))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{2}{5bd(d\cos(bx+a))^{\frac{5}{2}}}$	19
default	$\frac{2}{5bd(d\cos(bx+a))^{\frac{5}{2}}}$	19

input
```
int(sin(b*x+a)/(d*cos(b*x+a))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/b/d/(d*cos(b*x+a))^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{2\,\sqrt{d\cos\left(bx+a\right)}}{5\,bd^4\cos\left(bx+a\right)^3}$$

input
```
integrate(sin(b*x+a)/(d*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

output
```
2/5*sqrt(d*cos(b*x + a))/(b*d^4*cos(b*x + a)^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.

Time = 30.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \begin{cases} \frac{2\cos(a+bx)}{5b(d\cos(a+bx))^{\frac{7}{2}}} & \text{for } b \neq 0 \\ \frac{x\sin(a)}{(d\cos(a))^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(sin(b*x+a)/(d*cos(b*x+a))**(7/2),x)
```

output
```
Piecewise((2*cos(a + b*x)/(5*b*(d*cos(a + b*x))**(7/2)), Ne(b, 0)), (x*sin
(a)/(d*cos(a))**(7/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{2}{5\,(d\cos(bx+a))^{\frac{5}{2}}\,bd}$$

input
```
integrate(sin(b*x+a)/(d*cos(b*x+a))^(7/2),x, algorithm="maxima")
```

output
```
2/5/((d*cos(b*x + a))^(5/2)*b*d)
```

Giac [F]

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{\sin(bx+a)}{(d\cos(bx+a))^{\frac{7}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)/(d*cos(b*x+a))^(7/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)/(d*cos(b*x + a))^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 52.97 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.95

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{16\,e^{a\,3i+b\,x\,3i}\sqrt{d\,\left(\frac{e^{-a\,1i-b\,x\,1i}}{2}+\frac{e^{a\,1i+b\,x\,1i}}{2}\right)}}{5\,b\,d^4\,\left(e^{a\,2i+b\,x\,2i}+1\right)^3}$$

input
```
int(sin(a + b*x)/(d*cos(a + b*x))^(7/2),x)
```

output
```
(16*exp(a*3i + b*x*3i)*(d*(exp(- a*1i - b*x*1i)/2 + exp(a*1i + b*x*1i)/2))
^(1/2))/(5*b*d^4*(exp(a*2i + b*x*2i) + 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{2\sqrt{d}\,\sqrt{\cos(bx+a)}}{5\cos(bx+a)^3\,b\,d^4}$$

input
```
int(sin(b*x+a)/(d*cos(b*x+a))^(7/2),x)
```

output
```
(2*sqrt(d)*sqrt(cos(a + b*x)))/(5*cos(a + b*x)**3*b*d**4)
```

3.193 $\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 22

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \frac{2}{7bd(d\cos(a+bx))^{7/2}}$$

output
```
2/7/b/d/(d*cos(b*x+a))^(7/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \frac{2}{7bd(d\cos(a+bx))^{7/2}}$$

input
```
Integrate[Sin[a + b*x]/(d*Cos[a + b*x])^(9/2),x]
```

output
```
2/(7*b*d*(d*Cos[a + b*x])^(7/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 3045, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{9/2}}dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int \frac{1}{(d\cos(a+bx))^{9/2}}d(d\cos(a+bx))}{bd}$$

$$\Big\downarrow \text{15}$$

$$\frac{2}{7bd(d\cos(a+bx))^{7/2}}$$

input
```
Int[Sin[a + b*x]/(d*Cos[a + b*x])^(9/2),x]
```

output
```
2/(7*b*d*(d*Cos[a + b*x])^(7/2))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{2}{7bd(d\cos(bx+a))^{\frac{7}{2}}}$	19
default	$\frac{2}{7bd(d\cos(bx+a))^{\frac{7}{2}}}$	19

input
```
int(sin(b*x+a)/(d*cos(b*x+a))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/7/b/d/(d*cos(b*x+a))^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \frac{2\sqrt{d\cos(bx+a)}}{7\,bd^5\cos(bx+a)^4}$$

input
```
integrate(sin(b*x+a)/(d*cos(b*x+a))^(9/2),x, algorithm="fricas")
```

output
```
2/7*sqrt(d*cos(b*x + a))/(b*d^5*cos(b*x + a)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)/(d*cos(b*x+a))**(9/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \frac{2}{7\,\left(d\cos\left(bx+a\right)\right)^{\frac{7}{2}}bd}$$

input
```
integrate(sin(b*x+a)/(d*cos(b*x+a))^(9/2),x, algorithm="maxima")
```

output
```
2/7/((d*cos(b*x + a))^(7/2)*b*d)
```

Giac [F]

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \int \frac{\sin\left(bx+a\right)}{\left(d\cos\left(bx+a\right)\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)/(d*cos(b*x+a))^(9/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)/(d*cos(b*x + a))^(9/2), x)
```

Mupad [B] (verification not implemented)

Time = 30.65 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.95

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \frac{32\,e^{a\,4i+b\,x\,4i}\,\sqrt{d\,\left(\frac{e^{-a\,1i-b\,x\,1i}}{2}+\frac{e^{a\,1i+b\,x\,1i}}{2}\right)}}{7\,b\,d^5\,\left(e^{a\,2i+b\,x\,2i}+1\right)^4}$$

input
```
int(sin(a + b*x)/(d*cos(a + b*x))^(9/2),x)
```

output
```
(32*exp(a*4i + b*x*4i)*(d*(exp(- a*1i - b*x*1i)/2 + exp(a*1i + b*x*1i)/2))
^(1/2))/(7*b*d^5*(exp(a*2i + b*x*2i) + 1)^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \frac{2\sqrt{d}\,\sqrt{\cos\left(bx+a\right)}}{7\cos\left(bx+a\right)^4 b\,d^5}$$

input
```
int(sin(b*x+a)/(d*cos(b*x+a))^(9/2),x)
```

output
```
(2*sqrt(d)*sqrt(cos(a + b*x)))/(7*cos(a + b*x)**4*b*d**5)
```

3.194 $\int (d\cos(a+bx))^{9/2}\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 126

$$\int (d\cos(a+bx))^{9/2}\sin^2(a+bx)\,dx=\frac{28d^4\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{195b\sqrt{\cos(a+bx)}}+\frac{28d^3(d\cos(a+bx))^{3/2}\sin(a+bx)}{585b}+\frac{4d(d\cos(a+bx))^{7/2}\sin(a+bx)}{117b}-\frac{2(d\cos(a+bx))^{11/2}\sin(a+bx)}{13bd}$$

output
```
28/195*d^4*(d*cos(b*x+a))^(1/2)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))/b/co
s(b*x+a)^(1/2)+28/585*d^3*(d*cos(b*x+a))^(3/2)*sin(b*x+a)/b+4/117*d*(d*cos
(b*x+a))^(7/2)*sin(b*x+a)/b-2/13*(d*cos(b*x+a))^(11/2)*sin(b*x+a)/b/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.48

$$\int (d\cos(a+bx))^{9/2}\sin^2(a+bx)\,dx=\frac{d^2(d\cos(a+bx))^{5/2}\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{3}{2},\frac{5}{2},\sin^2(a+bx)\right)\tan^3(a+bx)}{3b}$$

input
```
Integrate[(d*Cos[a + b*x])^(9/2)*Sin[a + b*x]^2,x]
```

output
```
(d^2*(d*Cos[a + b*x])^(5/2)*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[-7/4,
 3/2, 5/2, Sin[a + b*x]^2]*Tan[a + b*x]^3)/(3*b)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3048, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)(d\cos(a+bx))^{9/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^2(d\cos(a+bx))^{9/2}dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{2}{13}\int (d\cos(a+bx))^{9/2}dx-\frac{2\sin(a+bx)(d\cos(a+bx))^{11/2}}{13bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{13}\int \left(d\sin\left(a+bx+\frac{\pi}{2}\right)\right)^{9/2}dx-\frac{2\sin(a+bx)(d\cos(a+bx))^{11/2}}{13bd}$$

$$\Big\downarrow \text{3115}$$

$$\frac{2}{13}\left(\frac{7}{9}d^2\int (d\cos(a+bx))^{5/2}dx+\frac{2d\sin(a+bx)(d\cos(a+bx))^{7/2}}{9b}\right)-\frac{2\sin(a+bx)(d\cos(a+bx))^{11/2}}{13bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{13}\left(\frac{7}{9}d^2\int\left(d\sin\left(a+bx+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{2d\sin(a+bx)(d\cos(a+bx))^{7/2}}{9b}\right)-\frac{2\sin(a+bx)(d\cos(a+bx))^{11/2}}{13bd}$$

$\Big\downarrow$ 3115

$$\frac{2}{13}\left(\frac{7}{9}d^2\left(\frac{3}{5}d^2\int\sqrt{d\cos(a+bx)}dx+\frac{2d\sin(a+bx)(d\cos(a+bx))^{3/2}}{5b}\right)+\frac{2d\sin(a+bx)(d\cos(a+bx))^{7/2}}{9b}\right)-\frac{2\sin(a+bx)(d\cos(a+bx))^{11/2}}{13bd}$$

$\Big\downarrow$ 3042

$$\frac{2}{13}\left(\frac{7}{9}d^2\left(\frac{3}{5}d^2\int\sqrt{d\sin\left(a+bx+\frac{\pi}{2}\right)}dx+\frac{2d\sin(a+bx)(d\cos(a+bx))^{3/2}}{5b}\right)+\frac{2d\sin(a+bx)(d\cos(a+bx))^{7/}}{9b}\right.$$
$$\frac{2\sin(a+bx)(d\cos(a+bx))^{11/2}}{13bd}$$

$\Big\downarrow$ 3121

$$\frac{2}{13}\left(\frac{7}{9}d^2\left(\frac{3d^2\sqrt{d\cos(a+bx)}\int\sqrt{\cos(a+bx)}dx}{5\sqrt{\cos(a+bx)}}+\frac{2d\sin(a+bx)(d\cos(a+bx))^{3/2}}{5b}\right)+\frac{2d\sin(a+bx)(d\cos(a+}{9b}\right.$$
$$\frac{2\sin(a+bx)(d\cos(a+bx))^{11/2}}{13bd}$$

$\Big\downarrow$ 3042

$$\frac{2}{13}\left(\frac{7}{9}d^2\left(\frac{3d^2\sqrt{d\cos(a+bx)}\int\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(a+bx)}}+\frac{2d\sin(a+bx)(d\cos(a+bx))^{3/2}}{5b}\right)+\frac{2d\sin(a+bx)(d\,c}{9b}\right.$$
$$\frac{2\sin(a+bx)(d\cos(a+bx))^{11/2}}{13bd}$$

$\Big\downarrow$ 3119

$$\frac{2}{13}\left(\frac{7}{9}d^2\left(\frac{6d^2E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{5b\sqrt{\cos(a+bx)}}+\frac{2d\sin(a+bx)(d\cos(a+bx))^{3/2}}{5b}\right)+\frac{2d\sin(a+bx)(d\cos(a+b}{9b}\right.$$
$$\frac{2\sin(a+bx)(d\cos(a+bx))^{11/2}}{13bd}$$

input
```
Int[(d*Cos[a + b*x])^(9/2)*Sin[a + b*x]^2,x]
```

output
```
(-2*(d*Cos[a + b*x])^(11/2)*Sin[a + b*x])/(13*b*d) + (2*((2*d*(d*Cos[a + b
*x])^(7/2)*Sin[a + b*x])/(9*b) + (7*d^2*((6*d^2*Sqrt[d*Cos[a + b*x]]*Ellip
ticE[(a + b*x)/2, 2])/(5*b*Sqrt[Cos[a + b*x]]) + (2*d*(d*Cos[a + b*x])^(3/
2)*Sin[a + b*x])/(5*b)))/9))/13
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(110) = 220$.

Time = 11.27 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.98

method	result
default	$\frac{4\sqrt{d\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\, d^{5}\left(2880\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{15}-11520\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{13}+19280\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{11}-17520\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\right.}{585\sqrt{-d\left(2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}-\sin\right.}}$

input
```
int((d*cos(b*x+a))^(9/2)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
4/585*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^5*(2880*
cos(1/2*b*x+1/2*a)^15-11520*cos(1/2*b*x+1/2*a)^13+19280*cos(1/2*b*x+1/2*a)
^11-17520*cos(1/2*b*x+1/2*a)^9+9284*cos(1/2*b*x+1/2*a)^7-2808*cos(1/2*b*x+
1/2*a)^5+425*cos(1/2*b*x+1/2*a)^3+21*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(-2*cos(
1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))-21*cos(1/2
*b*x+1/2*a))/(-d*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(
1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.94

$$\int (d\cos(a+bx))^{9/2}\sin^2(a+bx)\,dx =$$
$$-\frac{2\left(-42i\sqrt{\frac{1}{2}}d^{\frac{9}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\,\sin(bx+a)))+42i\sqrt{\right.}}{}$$

input
```
integrate((d*cos(b*x+a))^(9/2)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-2/585*(-42*I*sqrt(1/2)*d^(9/2)*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(b*x + a) + I*sin(b*x + a))) + 42*I*sqrt(1/2)*d^(9/2)*weierstra
ssZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a))) +
 (45*d^4*cos(b*x + a)^5 - 10*d^4*cos(b*x + a)^3 - 14*d^4*cos(b*x + a))*sqr
t(d*cos(b*x + a))*sin(b*x + a))/b
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{9/2}\sin^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*cos(b*x+a))**(9/2)*sin(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\cos(a+bx))^{9/2}\sin^2(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{9}{2}}\sin(bx+a)^2\,dx$$

input
```
integrate((d*cos(b*x+a))^(9/2)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*cos(b*x + a))^(9/2)*sin(b*x + a)^2, x)
```

Giac [F]

$$\int (d\cos(a+bx))^{9/2}\sin^2(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{9}{2}}\sin(bx+a)^2\,dx$$

input
```
integrate((d*cos(b*x+a))^(9/2)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*cos(b*x + a))^(9/2)*sin(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{9/2}\sin^2(a+bx)\,dx = \int \sin(a+b\,x)^2\,(d\,\cos(a+b\,x))^{9/2}\,dx$$

input
```
int(sin(a + b*x)^2*(d*cos(a + b*x))^(9/2),x)
```

output
```
int(sin(a + b*x)^2*(d*cos(a + b*x))^(9/2), x)
```

Reduce [F]

$$\int (d\cos(a+bx))^{9/2}\sin^2(a+bx)\,dx = \sqrt{d}\left(\int \sqrt{\cos(bx+a)}\,\cos(bx+a)^4\sin(bx+a)^2\,dx\right)d^4$$

input
```
int((d*cos(b*x+a))^(9/2)*sin(b*x+a)^2,x)
```

output
```
sqrt(d)*int(sqrt(cos(a + b*x))*cos(a + b*x)**4*sin(a + b*x)**2,x)*d**4
```

3.195 $\int (d\cos(a+bx))^{7/2}\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 126

$$\int (d\cos(a+bx))^{7/2}\sin^2(a+bx)\,dx = \frac{20d^4\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{231b\sqrt{d\cos(a+bx)}} + \frac{20d^3\sqrt{d\cos(a+bx)}\sin(a+bx)}{231b} + \frac{4d(d\cos(a+bx))^{5/2}\sin(a+bx)}{77b} - \frac{2(d\cos(a+bx))^{9/2}\sin(a+bx)}{11bd}$$

output
```
20/231*d^4*cos(b*x+a)^(1/2)*InverseJacobiAM(1/2*a+1/2*b*x,2^(1/2))/b/(d*co
s(b*x+a))^(1/2)+20/231*d^3*(d*cos(b*x+a))^(1/2)*sin(b*x+a)/b+4/77*d*(d*cos
(b*x+a))^(5/2)*sin(b*x+a)/b-2/11*(d*cos(b*x+a))^(9/2)*sin(b*x+a)/b/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.48

$$\int (d\cos(a+bx))^{7/2}\sin^2(a+bx)\,dx = \frac{d^2(d\cos(a+bx))^{3/2}\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{3}{2},\frac{5}{2},\sin^2(a+bx)\right)\tan^3(a+bx)}{3b}$$

input
```
Integrate[(d*Cos[a + b*x])^(7/2)*Sin[a + b*x]^2,x]
```

output
```
(d^2*(d*Cos[a + b*x])^(3/2)*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-5/4,
 3/2, 5/2, Sin[a + b*x]^2]*Tan[a + b*x]^3)/(3*b)
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3048, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)(d\cos(a+bx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^2(d\cos(a+bx))^{7/2}dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{2}{11}\int (d\cos(a+bx))^{7/2}dx-\frac{2\sin(a+bx)(d\cos(a+bx))^{9/2}}{11bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{11}\int \left(d\sin\left(a+bx+\frac{\pi}{2}\right)\right)^{7/2}dx-\frac{2\sin(a+bx)(d\cos(a+bx))^{9/2}}{11bd}$$

$$\Big\downarrow \text{3115}$$

$$\frac{2}{11}\left(\frac{5}{7}d^2\int (d\cos(a+bx))^{3/2}dx+\frac{2d\sin(a+bx)(d\cos(a+bx))^{5/2}}{7b}\right)-\frac{2\sin(a+bx)(d\cos(a+bx))^{9/2}}{11bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{11}\left(\frac{5}{7}d^2\int \left(d\sin\left(a+bx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2d\sin(a+bx)(d\cos(a+bx))^{5/2}}{7b}\right)-\frac{2\sin(a+bx)(d\cos(a+bx))^{9/2}}{11bd}$$

$\downarrow$ 3115

$$\frac{2}{11}\left(\frac{5}{7}d^2\left(\frac{1}{3}d^2\int \frac{1}{\sqrt{d\cos(a+bx)}}dx+\frac{2d\sin(a+bx)\sqrt{d\cos(a+bx)}}{3b}\right)+\frac{2d\sin(a+bx)(d\cos(a+bx))^{5/2}}{7b}\right)-\frac{2\sin(a+bx)(d\cos(a+bx))^{9/2}}{11bd}$$

$\downarrow$ 3042

$$\frac{2}{11}\left(\frac{5}{7}d^2\left(\frac{1}{3}d^2\int \frac{1}{\sqrt{d\sin\left(a+bx+\frac{\pi}{2}\right)}}dx+\frac{2d\sin(a+bx)\sqrt{d\cos(a+bx)}}{3b}\right)+\frac{2d\sin(a+bx)(d\cos(a+bx))^{5/2}}{7b}\right. \qquad \frac{2\sin(a+bx)(d\cos(a+bx))^{9/2}}{11bd}$$

$\downarrow$ 3121

$$\frac{2}{11}\left(\frac{5}{7}d^2\left(\frac{d^2\sqrt{\cos(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}}dx}{3\sqrt{d\cos(a+bx)}}+\frac{2d\sin(a+bx)\sqrt{d\cos(a+bx)}}{3b}\right)+\frac{2d\sin(a+bx)(d\cos(a+bx))^{5/2}}{7b}\right. \qquad \frac{2\sin(a+bx)(d\cos(a+bx))^{9/2}}{11bd}$$

$\downarrow$ 3042

$$\frac{2}{11}\left(\frac{5}{7}d^2\left(\frac{d^2\sqrt{\cos(a+bx)}\int \frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx}{3\sqrt{d\cos(a+bx)}}+\frac{2d\sin(a+bx)\sqrt{d\cos(a+bx)}}{3b}\right)+\frac{2d\sin(a+bx)(d\cos(a+bx)}{7b}\right. \qquad \frac{2\sin(a+bx)(d\cos(a+bx))^{9/2}}{11bd}$$

$\downarrow$ 3120

$$\frac{2}{11}\left(\frac{5}{7}d^2\left(\frac{2d^2\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b\sqrt{d\cos(a+bx)}}+\frac{2d\sin(a+bx)\sqrt{d\cos(a+bx)}}{3b}\right)+\frac{2d\sin(a+bx)(d\cos(a}{7b}\right. \qquad \frac{2\sin(a+bx)(d\cos(a+bx))^{9/2}}{11bd}$$

input
```
Int[(d*Cos[a + b*x])^(7/2)*Sin[a + b*x]^2,x]
```

output
```
(-2*(d*Cos[a + b*x])^(9/2)*Sin[a + b*x])/(11*b*d) + (2*((2*d*(d*Cos[a + b*
x])^(5/2)*Sin[a + b*x])/(7*b) + (5*d^2*((2*d^2*Sqrt[Cos[a + b*x]]*Elliptic
F[(a + b*x)/2, 2])/(3*b*Sqrt[d*Cos[a + b*x]]) + (2*d*Sqrt[d*Cos[a + b*x]]*
Sin[a + b*x])/(3*b)))/7))/11
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(109) = 218$.

Time = 9.65 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.87

method	result
default	$\frac{4\sqrt{d\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\, d^{4}\left(672\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{13}-2352\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{11}+3312\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{9}-2400\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{7}+922\right.}{231\sqrt{-d\left(2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}-\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)}\,\sin}$

input
```
int((d*cos(b*x+a))^(7/2)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
4/231*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^4*(672*c
os(1/2*b*x+1/2*a)^13-2352*cos(1/2*b*x+1/2*a)^11+3312*cos(1/2*b*x+1/2*a)^9-
2400*cos(1/2*b*x+1/2*a)^7+922*cos(1/2*b*x+1/2*a)^5-159*cos(1/2*b*x+1/2*a)^
3-5*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(-2*cos(1/2*b*x+1/2*a)^2+1)^(1/2)*Ellipti
cF(cos(1/2*b*x+1/2*a),2^(1/2))+5*cos(1/2*b*x+1/2*a))/(-d*(2*sin(1/2*b*x+1/
2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1
/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.85

$$\int (d\cos(a+bx))^{7/2}\sin^2(a+bx)\,dx =$$

$$-\frac{2\left(10i\sqrt{\frac{1}{2}}d^{\frac{7}{2}}\,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)+i\,\sin(bx+a))-10i\sqrt{\frac{1}{2}}d^{\frac{7}{2}}\,\mathrm{weierstrassPInverse}(-\right.}{}$$

input
```
integrate((d*cos(b*x+a))^(7/2)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-2/231*(10*I*sqrt(1/2)*d^(7/2)*weierstrassPInverse(-4, 0, cos(b*x + a) + I
*sin(b*x + a)) - 10*I*sqrt(1/2)*d^(7/2)*weierstrassPInverse(-4, 0, cos(b*x
 + a) - I*sin(b*x + a)) + (21*d^3*cos(b*x + a)^4 - 6*d^3*cos(b*x + a)^2 -
10*d^3)*sqrt(d*cos(b*x + a))*sin(b*x + a))/b
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{7/2}\sin^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*cos(b*x+a))**(7/2)*sin(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\cos(a+bx))^{7/2}\sin^2(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{7}{2}}\sin(bx+a)^2\,dx$$

input
```
integrate((d*cos(b*x+a))^(7/2)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*cos(b*x + a))^(7/2)*sin(b*x + a)^2, x)
```

Giac [F]

$$\int (d\cos(a+bx))^{7/2}\sin^2(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{7}{2}}\sin(bx+a)^2\,dx$$

input
```
integrate((d*cos(b*x+a))^(7/2)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*cos(b*x + a))^(7/2)*sin(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{7/2}\sin^2(a+bx)\,dx = \int \sin(a+b\,x)^2\,(d\,\cos(a+b\,x))^{7/2}\,dx$$

input
```
int(sin(a + b*x)^2*(d*cos(a + b*x))^(7/2),x)
```

output
```
int(sin(a + b*x)^2*(d*cos(a + b*x))^(7/2), x)
```

Reduce [F]

$$\int (d\cos(a+bx))^{7/2}\sin^2(a+bx)\,dx = \sqrt{d}\left(\int \sqrt{\cos(bx+a)}\,\cos(bx+a)^3\sin(bx+a)^2\,dx\right)d^3$$

input
```
int((d*cos(b*x+a))^(7/2)*sin(b*x+a)^2,x)
```

output
```
sqrt(d)*int(sqrt(cos(a + b*x))*cos(a + b*x)**3*sin(a + b*x)**2,x)*d**3
```

3.196 $\int (d\cos(a+bx))^{5/2}\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int (d\cos(a+bx))^{5/2}\sin^2(a+bx)\,dx=\frac{4d^2\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{15b\sqrt{\cos(a+bx)}}+\frac{4d(d\cos(a+bx))^{3/2}\sin(a+bx)}{45b}-\frac{2(d\cos(a+bx))^{7/2}\sin(a+bx)}{9bd}$$

output
```
4/15*d^2*(d*cos(b*x+a))^(1/2)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))/b/cos(
b*x+a)^(1/2)+4/45*d*(d*cos(b*x+a))^(3/2)*sin(b*x+a)/b-2/9*(d*cos(b*x+a))^(
7/2)*sin(b*x+a)/b/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.58

$$\int (d\cos(a+bx))^{5/2}\sin^2(a+bx)\,dx=\frac{(d\cos(a+bx))^{5/2}\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{3}{2},\frac{5}{2},\sin^2(a+bx)\right)\tan^3(a+bx)}{3b}$$

input
```
Integrate[(d*Cos[a + b*x])^(5/2)*Sin[a + b*x]^2,x]
```

output
```
((d*Cos[a + b*x])^(5/2)*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[-3/4, 3/2
, 5/2, Sin[a + b*x]^2]*Tan[a + b*x]^3)/(3*b)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3048, 3042, 3115, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)(d\cos(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sin(a+bx)^2(d\cos(a+bx))^{5/2}dx$$

$$\Big\downarrow \ 3048$$

$$\frac{2}{9}\int (d\cos(a+bx))^{5/2}dx-\frac{2\sin(a+bx)(d\cos(a+bx))^{7/2}}{9bd}$$

$$\Big\downarrow \ 3042$$

$$\frac{2}{9}\int \left(d\sin\left(a+bx+\frac{\pi}{2}\right)\right)^{5/2}dx-\frac{2\sin(a+bx)(d\cos(a+bx))^{7/2}}{9bd}$$

$$\Big\downarrow \ 3115$$

$$\frac{2}{9}\left(\frac{3}{5}d^2\int \sqrt{d\cos(a+bx)}dx+\frac{2d\sin(a+bx)(d\cos(a+bx))^{3/2}}{5b}\right)-\frac{2\sin(a+bx)(d\cos(a+bx))^{7/2}}{9bd}$$

$$\Big\downarrow \ 3042$$

$$\frac{2}{9}\left(\frac{3}{5}d^2\int\sqrt{d\sin\left(a+bx+\frac{\pi}{2}\right)}dx+\frac{2d\sin(a+bx)(d\cos(a+bx))^{3/2}}{5b}\right)-\frac{2\sin(a+bx)(d\cos(a+bx))^{7/2}}{9bd}$$

$\Big\downarrow$ 3121

$$\frac{2}{9}\left(\frac{3d^2\sqrt{d\cos(a+bx)}\int\sqrt{\cos(a+bx)}dx}{5\sqrt{\cos(a+bx)}}+\frac{2d\sin(a+bx)(d\cos(a+bx))^{3/2}}{5b}\right)-\frac{2\sin(a+bx)(d\cos(a+bx))^{7/2}}{9bd}$$

$\Big\downarrow$ 3042

$$\frac{2}{9}\left(\frac{3d^2\sqrt{d\cos(a+bx)}\int\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(a+bx)}}+\frac{2d\sin(a+bx)(d\cos(a+bx))^{3/2}}{5b}\right)-\frac{2\sin(a+bx)(d\cos(a+bx))^{7/2}}{9bd}$$

$\Big\downarrow$ 3119

$$\frac{2}{9}\left(\frac{6d^2E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{5b\sqrt{\cos(a+bx)}}+\frac{2d\sin(a+bx)(d\cos(a+bx))^{3/2}}{5b}\right)-\frac{2\sin(a+bx)(d\cos(a+bx))^{7/2}}{9bd}$$

input
```
Int[(d*Cos[a + b*x])^(5/2)*Sin[a + b*x]^2,x]
```

output
```
(-2*(d*Cos[a + b*x])^(7/2)*Sin[a + b*x])/(9*b*d) + (2*((6*d^2*Sqrt[d*Cos[a
 + b*x]]*EllipticE[(a + b*x)/2, 2])/(5*b*Sqrt[Cos[a + b*x]]) + (2*d*(d*Cos
[a + b*x])^(3/2)*Sin[a + b*x])/(5*b)))/9
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(86) = 172$.

Time = 7.10 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.28

method	result
default	$\frac{4\sqrt{d\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\, d^{3}\left(80\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{11}-240\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{9}+272\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{7}-144\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{5}+35\cos\left(\frac{b}{\ }\right.\right.}{45\sqrt{-d\left(2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}-\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)}\,\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{d}}$

input
```
int((d*cos(b*x+a))^(5/2)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
4/45*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^3*(80*cos
(1/2*b*x+1/2*a)^11-240*cos(1/2*b*x+1/2*a)^9+272*cos(1/2*b*x+1/2*a)^7-144*c
os(1/2*b*x+1/2*a)^5+35*cos(1/2*b*x+1/2*a)^3+3*(sin(1/2*b*x+1/2*a)^2)^(1/2)
*(-2*cos(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))-3
*cos(1/2*b*x+1/2*a))/(-d*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1
/2)/sin(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08

$$\int (d\cos(a+bx))^{5/2}\sin^2(a+bx)\,dx =$$
$$-\frac{2\left(-6i\sqrt{\frac{1}{2}}d^{\frac{5}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)))+6i\sqrt{\frac{1}{2}}d\right.}{}$$

input
```
integrate((d*cos(b*x+a))^(5/2)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-2/45*(-6*I*sqrt(1/2)*d^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(b*x + a) + I*sin(b*x + a))) + 6*I*sqrt(1/2)*d^(5/2)*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a))) + (5
*d^2*cos(b*x + a)^3 - 2*d^2*cos(b*x + a))*sqrt(d*cos(b*x + a))*sin(b*x + a
))/b
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{5/2}\sin^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*cos(b*x+a))**(5/2)*sin(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\cos(a+bx))^{5/2}\sin^2(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{5}{2}}\sin(bx+a)^2\,dx$$

input
```
integrate((d*cos(b*x+a))^(5/2)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*cos(b*x + a))^(5/2)*sin(b*x + a)^2, x)
```

Giac [F]

$$\int (d\cos(a+bx))^{5/2}\sin^2(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{5}{2}}\sin(bx+a)^2\,dx$$

input
```
integrate((d*cos(b*x+a))^(5/2)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*cos(b*x + a))^(5/2)*sin(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{5/2}\sin^2(a+bx)\,dx = \int \sin(a+b\,x)^2\,(d\,\cos(a+b\,x))^{5/2}\,dx$$

input
```
int(sin(a + b*x)^2*(d*cos(a + b*x))^(5/2),x)
```

output
```
int(sin(a + b*x)^2*(d*cos(a + b*x))^(5/2), x)
```

Reduce [F]

$$\int (d\cos(a+bx))^{5/2}\sin^2(a+bx)\,dx = \sqrt{d}\left(\int \sqrt{\cos(bx+a)}\,\cos(bx+a)^2\sin(bx+a)^2\,dx\right)d^2$$

input
```
int((d*cos(b*x+a))^(5/2)*sin(b*x+a)^2,x)
```

output
```
sqrt(d)*int(sqrt(cos(a + b*x))*cos(a + b*x)**2*sin(a + b*x)**2,x)*d**2
```

3.197 $\int (d\cos(a+bx))^{3/2}\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int (d\cos(a+bx))^{3/2}\sin^2(a+bx)\,dx = \frac{4d^2\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{21b\sqrt{d\cos(a+bx)}} + \frac{4d\sqrt{d\cos(a+bx)}\sin(a+bx)}{21b} - \frac{2(d\cos(a+bx))^{5/2}\sin(a+bx)}{7bd}$$

output
```
4/21*d^2*cos(b*x+a)^(1/2)*InverseJacobiAM(1/2*a+1/2*b*x,2^(1/2))/b/(d*cos(
b*x+a))^(1/2)+4/21*d*(d*cos(b*x+a))^(1/2)*sin(b*x+a)/b-2/7*(d*cos(b*x+a))^
(5/2)*sin(b*x+a)/b/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.58

$$\int (d\cos(a+bx))^{3/2}\sin^2(a+bx)\,dx = \frac{(d\cos(a+bx))^{3/2}\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{3}{2},\frac{5}{2},\sin^2(a+bx)\right)\tan^3(a+bx)}{3b}$$

input
```
Integrate[(d*Cos[a + b*x])^(3/2)*Sin[a + b*x]^2,x]
```

output
```
((d*Cos[a + b*x])^(3/2)*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-1/4, 3/2
, 5/2, Sin[a + b*x]^2]*Tan[a + b*x]^3)/(3*b)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3048, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)(d\cos(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^2(d\cos(a+bx))^{3/2}dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{2}{7}\int (d\cos(a+bx))^{3/2}dx-\frac{2\sin(a+bx)(d\cos(a+bx))^{5/2}}{7bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{7}\int \left(d\sin\left(a+bx+\frac{\pi}{2}\right)\right)^{3/2}dx-\frac{2\sin(a+bx)(d\cos(a+bx))^{5/2}}{7bd}$$

$$\Big\downarrow \text{3115}$$

$$\frac{2}{7}\left(\frac{1}{3}d^2\int \frac{1}{\sqrt{d\cos(a+bx)}}dx+\frac{2d\sin(a+bx)\sqrt{d\cos(a+bx)}}{3b}\right)-\frac{2\sin(a+bx)(d\cos(a+bx))^{5/2}}{7bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{7}\left(\frac{1}{3}d^2\int\frac{1}{\sqrt{d\sin\left(a+bx+\frac{\pi}{2}\right)}}dx+\frac{2d\sin(a+bx)\sqrt{d\cos(a+bx)}}{3b}\right)-\frac{2\sin(a+bx)(d\cos(a+bx))^{5/2}}{7bd}$$

$$\Big\downarrow \text{3121}$$

$$\frac{2}{7}\left(\frac{d^2\sqrt{\cos(a+bx)}\int\frac{1}{\sqrt{\cos(a+bx)}}dx}{3\sqrt{d\cos(a+bx)}}+\frac{2d\sin(a+bx)\sqrt{d\cos(a+bx)}}{3b}\right)-\frac{2\sin(a+bx)(d\cos(a+bx))^{5/2}}{7bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{7}\left(\frac{d^2\sqrt{\cos(a+bx)}\int\frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx}{3\sqrt{d\cos(a+bx)}}+\frac{2d\sin(a+bx)\sqrt{d\cos(a+bx)}}{3b}\right)-\frac{2\sin(a+bx)(d\cos(a+bx))^{5/2}}{7bd}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2}{7}\left(\frac{2d^2\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b\sqrt{d\cos(a+bx)}}+\frac{2d\sin(a+bx)\sqrt{d\cos(a+bx)}}{3b}\right)-\frac{2\sin(a+bx)(d\cos(a+bx))^{5/2}}{7bd}$$

input
```
Int[(d*Cos[a + b*x])^(3/2)*Sin[a + b*x]^2,x]
```

output
```
(-2*(d*Cos[a + b*x])^(5/2)*Sin[a + b*x])/(7*b*d) + (2*((2*d^2*Sqrt[Cos[a +
 b*x]]*EllipticF[(a + b*x)/2, 2])/(3*b*Sqrt[d*Cos[a + b*x]]) + (2*d*Sqrt[d
*Cos[a + b*x]]*Sin[a + b*x])/(3*b)))/7
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(85) = 170$.

Time = 5.29 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.12

method	result
default	$\frac{4\sqrt{d\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,d^{2}\left(24\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{9}-60\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{7}+50\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{5}-15\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{3}-\sqrt{\frac{1}{2}-\frac{\cos(bx+}{2}}}{21\sqrt{-d\left(2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}-\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)}\,\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{d\left(2\cos\left(\frac{bx}{2}+\right.\right.}}$

input
```
int((d*cos(b*x+a))^(3/2)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
4/21*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^2*(24*cos
(1/2*b*x+1/2*a)^9-60*cos(1/2*b*x+1/2*a)^7+50*cos(1/2*b*x+1/2*a)^5-15*cos(1
/2*b*x+1/2*a)^3-(sin(1/2*b*x+1/2*a)^2)^(1/2)*(-2*cos(1/2*b*x+1/2*a)^2+1)^(
1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))+cos(1/2*b*x+1/2*a))/(-d*(2*sin(
1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(d*(2*cos
(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.92

$$\int (d\cos(a+bx))^{3/2}\sin^2(a+bx)\,dx =$$
$$-\frac{2\left(2i\sqrt{\frac{1}{2}}d^{\frac{3}{2}}\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a))-2i\sqrt{\frac{1}{2}}d^{\frac{3}{2}}\text{weierstrassPInverse}(-4,\right.}{21\,b}$$

input
```
integrate((d*cos(b*x+a))^(3/2)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-2/21*(2*I*sqrt(1/2)*d^(3/2)*weierstrassPInverse(-4, 0, cos(b*x + a) + I*s
in(b*x + a)) - 2*I*sqrt(1/2)*d^(3/2)*weierstrassPInverse(-4, 0, cos(b*x +
a) - I*sin(b*x + a)) + (3*d*cos(b*x + a)^2 - 2*d)*sqrt(d*cos(b*x + a))*sin
(b*x + a))/b
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{3/2}\sin^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*cos(b*x+a))**(3/2)*sin(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\cos(a+bx))^{3/2}\sin^2(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{3}{2}}\sin(bx+a)^2\,dx$$

input
```
integrate((d*cos(b*x+a))^(3/2)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*cos(b*x + a))^(3/2)*sin(b*x + a)^2, x)
```

Giac [F]

$$\int (d\cos(a+bx))^{3/2}\sin^2(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{3}{2}}\sin(bx+a)^2\,dx$$

input
```
integrate((d*cos(b*x+a))^(3/2)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*cos(b*x + a))^(3/2)*sin(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{3/2}\sin^2(a+bx)\,dx = \int \sin(a+b\,x)^2\,(d\,\cos(a+b\,x))^{3/2}\,dx$$

input
```
int(sin(a + b*x)^2*(d*cos(a + b*x))^(3/2),x)
```

output
```
int(sin(a + b*x)^2*(d*cos(a + b*x))^(3/2), x)
```

Reduce [F]

$$\int (d\cos(a+bx))^{3/2}\sin^2(a+bx)\,dx = \sqrt{d}\left(\int \sqrt{\cos(bx+a)}\,\cos(bx+a)\sin(bx+a)^2\,dx\right)d$$

input
```
int((d*cos(b*x+a))^(3/2)*sin(b*x+a)^2,x)
```

output
```
sqrt(d)*int(sqrt(cos(a + b*x))*cos(a + b*x)*sin(a + b*x)**2,x)*d
```

3.198 $\int \sqrt{d\cos(a+bx)}\sin^2(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 69

$$\int \sqrt{d\cos(a+bx)}\sin^2(a+bx)\,dx = \frac{4\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b\sqrt{\cos(a+bx)}} - \frac{2(d\cos(a+bx))^{3/2}\sin(a+bx)}{5bd}$$

output
```
4/5*(d*cos(b*x+a))^(1/2)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))/b/cos(b*x+a
)^(1/2)-2/5*(d*cos(b*x+a))^(3/2)*sin(b*x+a)/b/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

$$\int \sqrt{d\cos(a+bx)}\sin^2(a+bx)\,dx = \frac{d\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{2},\frac{5}{2},\sin^2(a+bx)\right)\sin^3(a+bx)}{3b\sqrt{d\cos(a+bx)}}$$

input
```
Integrate[Sqrt[d*Cos[a + b*x]]*Sin[a + b*x]^2,x]
```

output
```
(d*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, 3/2, 5/2, Sin[a + b*x]^2]
*Sin[a + b*x]^3)/(3*b*Sqrt[d*Cos[a + b*x]])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3048, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\sqrt{d\cos(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^2\sqrt{d\cos(a+bx)}dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{2}{5}\int \sqrt{d\cos(a+bx)}dx-\frac{2\sin(a+bx)(d\cos(a+bx))^{3/2}}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}\int \sqrt{d\sin\left(a+bx+\frac{\pi}{2}\right)}dx-\frac{2\sin(a+bx)(d\cos(a+bx))^{3/2}}{5bd}$$

$$\Big\downarrow \text{3121}$$

$$\frac{2\sqrt{d\cos(a+bx)}\int \sqrt{\cos(a+bx)}dx}{5\sqrt{\cos(a+bx)}}-\frac{2\sin(a+bx)(d\cos(a+bx))^{3/2}}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sqrt{d\cos(a+bx)}\int \sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(a+bx)}}-\frac{2\sin(a+bx)(d\cos(a+bx))^{3/2}}{5bd}$$

$$\Big\downarrow \text{3119}$$

$$\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{5b\sqrt{\cos(a+bx)}}-\frac{2\sin(a+bx)(d\cos(a+bx))^{3/2}}{5bd}$$

input
```
Int[Sqrt[d*Cos[a + b*x]]*Sin[a + b*x]^2,x]
```

output
```
(4*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(5*b*Sqrt[Cos[a + b*x]]
) - (2*(d*Cos[a + b*x])^(3/2)*Sin[a + b*x])/(5*b*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(61) = 122$.

Time = 4.57 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.81

method	result
default	$\frac{4\sqrt{d\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\, d\left(4\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{7}-8\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{5}+5\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{3}+\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\sqrt{-2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\right.}{5\sqrt{-d\left(2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}-\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)}\,\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{d\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)}\, b}$

input
```
int((d*cos(b*x+a))^(1/2)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
4/5*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d*(4*cos(1/2
*b*x+1/2*a)^7-8*cos(1/2*b*x+1/2*a)^5+5*cos(1/2*b*x+1/2*a)^3+(sin(1/2*b*x+1
/2*a)^2)^(1/2)*(-2*cos(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cos(1/2*b*x+1/2
*a),2^(1/2))-cos(1/2*b*x+1/2*a))/(-d*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1
/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.26

$$\int \sqrt{d\cos(a+bx)}\sin^2(a+bx)\,dx =$$

$$-\frac{2\left(\sqrt{d\cos\left(bx+a\right)}\cos\left(bx+a\right)\sin\left(bx+a\right)-2i\sqrt{\frac{1}{2}}\sqrt{d}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-\right.}{}$$

input
```
integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-2/5*(sqrt(d*cos(b*x + a))*cos(b*x + a)*sin(b*x + a) - 2*I*sqrt(1/2)*sqrt(
d)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) + I*sin(
b*x + a))) + 2*I*sqrt(1/2)*sqrt(d)*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, cos(b*x + a) - I*sin(b*x + a))))/b
```

Sympy [F]

$$\int \sqrt{d\cos(a+bx)}\sin^2(a+bx)\,dx = \int \sqrt{d\cos\left(a+bx\right)}\sin^2\left(a+bx\right)\,dx$$

input
```
integrate((d*cos(b*x+a))**(1/2)*sin(b*x+a)**2,x)
```

output
```
Integral(sqrt(d*cos(a + b*x))*sin(a + b*x)**2, x)
```

Maxima [F]

$$\int \sqrt{d\cos(a+bx)}\sin^2(a+bx)\,dx = \int \sqrt{d\cos(bx+a)}\sin(bx+a)^2\,dx$$

input
```
integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*cos(b*x + a))*sin(b*x + a)^2, x)
```

Giac [F]

$$\int \sqrt{d\cos(a+bx)}\sin^2(a+bx)\,dx = \int \sqrt{d\cos(bx+a)}\sin(bx+a)^2\,dx$$

input
```
integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(d*cos(b*x + a))*sin(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\cos(a+bx)}\sin^2(a+bx)\,dx = \int \sin(a+b\,x)^2\,\sqrt{d\,\cos(a+b\,x)}\,dx$$

input
```
int(sin(a + b*x)^2*(d*cos(a + b*x))^(1/2),x)
```

output
```
int(sin(a + b*x)^2*(d*cos(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{d\cos(a+bx)}\sin^2(a+bx)\,dx = \sqrt{d}\left(\int \sqrt{\cos(bx+a)}\,\sin(bx+a)^2\,dx\right)$$

input
```
int((d*cos(b*x+a))^(1/2)*sin(b*x+a)^2,x)
```

output
```
sqrt(d)*int(sqrt(cos(a + b*x))*sin(a + b*x)**2,x)
```

3.199 $\int \frac{\sin^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 69

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \frac{4\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b\sqrt{d\cos(a+bx)}} - \frac{2\sqrt{d\cos(a+bx)}\sin(a+bx)}{3bd}$$

output
```
4/3*cos(b*x+a)^(1/2)*InverseJacobiAM(1/2*a+1/2*b*x,2^(1/2))/b/(d*cos(b*x+a
))^(1/2)-2/3*(d*cos(b*x+a))^(1/2)*sin(b*x+a)/b/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \frac{d\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{5}{2},\sin^2(a+bx)\right)\sin^3(a+bx)}{3b(d\cos(a+bx))^{3/2}}$$

input
```
Integrate[Sin[a + b*x]^2/Sqrt[d*Cos[a + b*x]],x]
```

output
```
(d*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/4, 3/2, 5/2, Sin[a + b*x]^2]
*Sin[a + b*x]^3)/(3*b*(d*Cos[a + b*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3048, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\cos(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^2}{\sqrt{d\cos(a+bx)}}dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{2}{3}\int \frac{1}{\sqrt{d\cos(a+bx)}}dx-\frac{2\sin(a+bx)\sqrt{d\cos(a+bx)}}{3bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\int \frac{1}{\sqrt{d\sin\left(a+bx+\frac{\pi}{2}\right)}}dx-\frac{2\sin(a+bx)\sqrt{d\cos(a+bx)}}{3bd}$$

$$\Big\downarrow \text{3121}$$

$$\frac{2\sqrt{\cos(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}}dx}{3\sqrt{d\cos(a+bx)}}-\frac{2\sin(a+bx)\sqrt{d\cos(a+bx)}}{3bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sqrt{\cos(a+bx)}\int \frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx}{3\sqrt{d\cos(a+bx)}}-\frac{2\sin(a+bx)\sqrt{d\cos(a+bx)}}{3bd}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{4\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b\sqrt{d\cos(a+bx)}}-\frac{2\sin(a+bx)\sqrt{d\cos(a+bx)}}{3bd}$$

input
```
Int[Sin[a + b*x]^2/Sqrt[d*Cos[a + b*x]],x]
```

output
```
(4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(3*b*Sqrt[d*Cos[a + b*x]]
) - (2*Sqrt[d*Cos[a + b*x]]*Sin[a + b*x])/(3*b*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(60) = 120$.

Time = 3.52 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.72

method	result
default	$\frac{4\sqrt{d\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}-\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\sqrt{2\sin\left(\frac{bx}{2}+\frac{a}{2}\right.}}{3\sqrt{-d\left(2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}-\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)}\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{d\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)}b}$

input
```
int(sin(b*x+a)^2/(d*cos(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
4/3*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*cos(1/2*b
*x+1/2*a)*sin(1/2*b*x+1/2*a)^4-cos(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)^2-(si
n(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cos(1
/2*b*x+1/2*a),2^(1/2)))/(-d*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))
^(1/2)/sin(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{2\left(2i\sqrt{\frac{1}{2}}\sqrt{d}\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a))-2i\sqrt{\frac{1}{2}}\sqrt{d}\text{weierstrassPInverse}\right.}{3\,bd}$$

input
```
integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(2*I*sqrt(1/2)*sqrt(d)*weierstrassPInverse(-4, 0, cos(b*x + a) + I*si
n(b*x + a)) - 2*I*sqrt(1/2)*sqrt(d)*weierstrassPInverse(-4, 0, cos(b*x + a
) - I*sin(b*x + a)) + sqrt(d*cos(b*x + a))*sin(b*x + a))/(b*d)
```

Sympy [F]

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{\sin^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$$

input
```
integrate(sin(b*x+a)**2/(d*cos(b*x+a))**(1/2),x)
```

output
```
Integral(sin(a + b*x)**2/sqrt(d*cos(a + b*x)), x)
```

Maxima [F]

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{\sin(bx+a)^2}{\sqrt{d\cos(bx+a)}}\,dx$$

input
```
integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^2/sqrt(d*cos(b*x + a)), x)
```

Giac [F]

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{\sin(bx+a)^2}{\sqrt{d\cos(bx+a)}}\,dx$$

input
```
integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^2/sqrt(d*cos(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{\sin\left(a+b\,x\right)^2}{\sqrt{d\,\cos\left(a+b\,x\right)}}\,dx$$

input
```
int(sin(a + b*x)^2/(d*cos(a + b*x))^(1/2),x)
```

output
```
int(sin(a + b*x)^2/(d*cos(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\cos(bx+a)}\,\sin(bx+a)^2}{\cos(bx+a)}dx\right)}{d}$$

input
```
int(sin(b*x+a)^2/(d*cos(b*x+a))^(1/2),x)
```

output
```
(sqrt(d)*int((sqrt(cos(a + b*x))*sin(a + b*x)**2)/cos(a + b*x),x))/d
```

3.200 $\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 68

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = -\frac{4\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{bd^2\sqrt{\cos(a+bx)}}+\frac{2\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}$$

output
```
-4*(d*cos(b*x+a))^(1/2)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))/b/d^2/cos(b*
x+a)^(1/2)+2*sin(b*x+a)/b/d/(d*cos(b*x+a))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{3}{2},\frac{5}{2},\sin^2(a+bx)\right)\sin^3(a+bx)}{3bd\sqrt{d\cos(a+bx)}}$$

input
```
Integrate[Sin[a + b*x]^2/(d*Cos[a + b*x])^(3/2),x]
```

output
```
((Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[5/4, 3/2, 5/2, Sin[a + b*x]^2]*S
in[a + b*x]^3)/(3*b*d*Sqrt[d*Cos[a + b*x]])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3046, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sin(a+bx)^2}{(d\cos(a+bx))^{3/2}}dx$$

$$\Big\downarrow \ 3046$$

$$\frac{2\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{2\int\sqrt{d\cos(a+bx)}dx}{d^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{2\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{2\int\sqrt{d\sin\left(a+bx+\frac{\pi}{2}\right)}dx}{d^2}$$

$$\Big\downarrow \ 3121$$

$$\frac{2\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{2\sqrt{d\cos(a+bx)}\int\sqrt{\cos(a+bx)}dx}{d^2\sqrt{\cos(a+bx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{2\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{2\sqrt{d\cos(a+bx)}\int\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx}{d^2\sqrt{\cos(a+bx)}}$$

$$\Big\downarrow \ 3119$$

$$\frac{2\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{bd^2\sqrt{\cos(a+bx)}}$$

input
```
Int[Sin[a + b*x]^2/(d*Cos[a + b*x])^(3/2),x]
```

output
```
(-4*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(b*d^2*Sqrt[Cos[a + b*
x]]) + (2*Sin[a + b*x])/(b*d*Sqrt[d*Cos[a + b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3046
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Sin[e + f
*x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(64) = 128$.

Time = 4.24 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.91

method	result
default	$-\frac{4\left(-\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}d+\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}d}\,\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}+\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\sqrt{2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1}\,\sqrt{-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}d-}\right.}{d\sqrt{-d\left(2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}-\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)}\,\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{d\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)}\,b}$

input
```
int(sin(b*x+a)^2/(d*cos(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-4/d*(-cos(1/2*b*x+1/2*a)*(-2*sin(1/2*b*x+1/2*a)^4*d+sin(1/2*b*x+1/2*a)^2*
d)^(1/2)*sin(1/2*b*x+1/2*a)^2+(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+
1/2*a)^2-1)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^4*d+sin(1/2*b*x+1/2*a)^2*d)^(1/2)
*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2)))/(-d*(2*sin(1/2*b*x+1/2*a)^4-sin(1/
2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(
1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.54

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx =$$
$$-\frac{2\left(2i\sqrt{\frac{1}{2}}\sqrt{d}\cos(bx+a)\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)\right)}{}$$

input
```
integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
-2*(2*I*sqrt(1/2)*sqrt(d)*cos(b*x + a)*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(b*x + a) + I*sin(b*x + a))) - 2*I*sqrt(1/2)*sqrt(d)*cos
(b*x + a)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) -
 I*sin(b*x + a))) - sqrt(d*cos(b*x + a))*sin(b*x + a))/(b*d^2*cos(b*x + a)
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**2/(d*cos(b*x+a))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\sin(bx+a)^2}{(d\cos(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^2/(d*cos(b*x + a))^(3/2), x)
```

Giac [F]

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\sin(bx+a)^2}{(d\cos(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^2/(d*cos(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\sin(a+b\,x)^2}{(d\,\cos(a+b\,x))^{3/2}}\,dx$$

input
```
int(sin(a + b*x)^2/(d*cos(a + b*x))^(3/2),x)
```

output
```
int(sin(a + b*x)^2/(d*cos(a + b*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\cos(bx+a)}\,\sin(bx+a)^2}{\cos(bx+a)^2}dx\right)}{d^2}$$

input
```
int(sin(b*x+a)^2/(d*cos(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*int((sqrt(cos(a + b*x))*sin(a + b*x)**2)/cos(a + b*x)**2,x))/d**2
```

3.201 $\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 72

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx=-\frac{4\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3bd^2\sqrt{d\cos(a+bx)}}+\frac{2\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}$$

output
```
-4/3*cos(b*x+a)^(1/2)*InverseJacobiAM(1/2*a+1/2*b*x,2^(1/2))/b/d^2/(d*cos(
b*x+a))^(1/2)+2/3*sin(b*x+a)/b/d/(d*cos(b*x+a))^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx=\frac{\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{7}{4},\frac{5}{2},\sin^2(a+bx)\right)\sin^3(a+bx)}{3bd(d\cos(a+bx))^{3/2}}$$

input
```
Integrate[Sin[a + b*x]^2/(d*Cos[a + b*x])^(5/2),x]
```

output
```
((Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/2, 7/4, 5/2, Sin[a + b*x]^2]*S
in[a + b*x]^3)/(3*b*d*(d*Cos[a + b*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3046, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^2}{(d\cos(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3046}$$

$$\frac{2\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}-\frac{2\int \frac{1}{\sqrt{d\cos(a+bx)}}dx}{3d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}-\frac{2\int \frac{1}{\sqrt{d\sin\left(a+bx+\frac{\pi}{2}\right)}}dx}{3d^2}$$

$$\Big\downarrow \text{3121}$$

$$\frac{2\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}-\frac{2\sqrt{\cos(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}}dx}{3d^2\sqrt{d\cos(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}-\frac{2\sqrt{\cos(a+bx)}\int \frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx}{3d^2\sqrt{d\cos(a+bx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}-\frac{4\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3bd^2\sqrt{d\cos(a+bx)}}$$

input
```
Int[Sin[a + b*x]^2/(d*Cos[a + b*x])^(5/2),x]
```

output
```
(-4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(3*b*d^2*Sqrt[d*Cos[a +
b*x]]) + (2*Sin[a + b*x])/(3*b*d*(d*Cos[a + b*x])^(3/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3046
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Sin[e + f
*x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(63) = 126$.

Time = 4.30 (sec) , antiderivative size = 242, normalized size of antiderivative = 3.36

method	result
default	$-\frac{4\left(2\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\sqrt{2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\right.}{3d^{2}\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)\sqrt{-d\left(2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}-\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)}\,\sin\left(\right.}$

input
```
int(sin(b*x+a)^2/(d*cos(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-4/3*(2*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*Elli
pticF(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^2-cos(1/2*b*x+1/2*a)*
sin(1/2*b*x+1/2*a)^2-(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-
1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2)))/d^2*(d*(2*cos(1/2*b*x+1/2*
a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/(2*cos(1/2*b*x+1/2*a)^2-1)/(-d*(2*sin(
1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(d*(2*cos
(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.43

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx =$$
$$-\frac{2\left(-2i\sqrt{\frac{1}{2}}\sqrt{d}\cos\left(bx+a\right)^2 \text{weierstrassPInverse}(-4,0,\cos\left(bx+a\right)+i\,\sin\left(bx+a\right))+2i\sqrt{\frac{1}{2}}\sqrt{d}\cos\left(bx\right.\right.}{3\,bd^3\cos\left(bx+\right.}$$

input
```
integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(-2*I*sqrt(1/2)*sqrt(d)*cos(b*x + a)^2*weierstrassPInverse(-4, 0, cos
(b*x + a) + I*sin(b*x + a)) + 2*I*sqrt(1/2)*sqrt(d)*cos(b*x + a)^2*weierst
rassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a)) - sqrt(d*cos(b*x + a))*
sin(b*x + a))/(b*d^3*cos(b*x + a)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**2/(d*cos(b*x+a))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{\sin\left(bx+a\right)^2}{\left(d\cos\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^2/(d*cos(b*x + a))^(5/2), x)
```

Giac [F]

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{\sin\left(bx+a\right)^2}{\left(d\cos\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^2/(d*cos(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{\sin\left(a+b\,x\right)^2}{\left(d\,\cos\left(a+b\,x\right)\right)^{5/2}}\,dx$$

input
```
int(sin(a + b*x)^2/(d*cos(a + b*x))^(5/2),x)
```

output
```
int(sin(a + b*x)^2/(d*cos(a + b*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\cos(bx+a)}\,\sin(bx+a)^2}{\cos(bx+a)^3}dx\right)}{d^3}$$

input
```
int(sin(b*x+a)^2/(d*cos(b*x+a))^(5/2),x)
```

output
```
(sqrt(d)*int((sqrt(cos(a + b*x))*sin(a + b*x)**2)/cos(a + b*x)**3,x))/d**3
```

3.202 $\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 100

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{4\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5bd^4\sqrt{\cos(a+bx)}} + \frac{2\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}} - \frac{4\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}}$$

output
```
4/5*(d*cos(b*x+a))^(1/2)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))/b/d^4/cos(b
*x+a)^(1/2)+2/5*sin(b*x+a)/b/d/(d*cos(b*x+a))^(5/2)-4/5*sin(b*x+a)/b/d^3/(
d*cos(b*x+a))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.59

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{9}{4},\frac{5}{2},\sin^2(a+bx)\right)\sin^3(2(a+bx))}{24b(d\cos(a+bx))^{7/2}}$$

input
```
Integrate[Sin[a + b*x]^2/(d*Cos[a + b*x])^(7/2),x]
```

output
```
((Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[3/2, 9/4, 5/2, Sin[a + b*x]^2]*S
in[2*(a + b*x)]^3)/(24*b*(d*Cos[a + b*x])^(7/2))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3042, 3046, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^2}{(d\cos(a+bx))^{7/2}}dx$$

$$\Big\downarrow \text{3046}$$

$$\frac{2\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}}-\frac{2\int \frac{1}{(d\cos(a+bx))^{3/2}}dx}{5d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}}-\frac{2\int \frac{1}{\left(d\sin\left(a+bx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{5d^2}$$

$$\Big\downarrow \text{3116}$$

$$\frac{2\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}}-\frac{2\left(\frac{2\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{\int \sqrt{d\cos(a+bx)}dx}{d^2}\right)}{5d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}}-\frac{2\left(\frac{2\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{\int \sqrt{d\sin\left(a+bx+\frac{\pi}{2}\right)}dx}{d^2}\right)}{5d^2}$$

$$\Big\downarrow \text{3121}$$

$$\frac{2\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}}-\frac{2\left(\frac{2\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{\sqrt{d\cos(a+bx)}\int\sqrt{\cos(a+bx)}dx}{d^2\sqrt{\cos(a+bx)}}\right)}{5d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}}-\frac{2\left(\frac{2\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{\sqrt{d\cos(a+bx)}\int\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx}{d^2\sqrt{\cos(a+bx)}}\right)}{5d^2}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}}-\frac{2\left(\frac{2\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{2E\left(\frac{1}{2}(a+bx)|2\right)\sqrt{d\cos(a+bx)}}{bd^2\sqrt{\cos(a+bx)}}\right)}{5d^2}$$

input
```
Int[Sin[a + b*x]^2/(d*Cos[a + b*x])^(7/2),x]
```

output
```
(2*Sin[a + b*x])/(5*b*d*(d*Cos[a + b*x])^(5/2)) - (2*((-2*Sqrt[d*Cos[a + b
*x]]*EllipticE[(a + b*x)/2, 2])/(b*d^2*Sqrt[Cos[a + b*x]]) + (2*Sin[a + b*
x])/(b*d*Sqrt[d*Cos[a + b*x]])))/(5*d^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3046
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Sin[e + f
*x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 364 vs. 2(88) = 176.

Time = 5.27 (sec) , antiderivative size = 365, normalized size of antiderivative = 3.65

method	result
default	$4\sqrt{d\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^2}\ \bigl(8\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^6\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-4\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\sqrt{2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1}\ \mathrm{EllipticE}\bigl(\cos\bigl(\frac{bx}{2}+$

input
```
int(sin(b*x+a)^2/(d*cos(b*x+a))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
4/5*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/d^4/sin(1/2*
b*x+1/2*a)^3/(8*sin(1/2*b*x+1/2*a)^6-12*sin(1/2*b*x+1/2*a)^4+6*sin(1/2*b*x
+1/2*a)^2-1)*(8*sin(1/2*b*x+1/2*a)^6*cos(1/2*b*x+1/2*a)-4*(sin(1/2*b*x+1/2
*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a)
,2^(1/2))*sin(1/2*b*x+1/2*a)^4-8*cos(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)^4+4
*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticE(c
os(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^2+cos(1/2*b*x+1/2*a)*sin(1/2
*b*x+1/2*a)^2-(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2
)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2)))*(-2*sin(1/2*b*x+1/2*a)^4*d+sin(1/
2*b*x+1/2*a)^2*d)^(1/2)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.20

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = -\frac{2\left(-2i\sqrt{\frac{1}{2}}\sqrt{d}\cos(bx+a)^3\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+\right.}{}$$

input
```
integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

output
```
-2/5*(-2*I*sqrt(1/2)*sqrt(d)*cos(b*x + a)^3*weierstrassZeta(-4, 0, weierst
rassPInverse(-4, 0, cos(b*x + a) + I*sin(b*x + a))) + 2*I*sqrt(1/2)*sqrt(d
)*cos(b*x + a)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x
 + a) - I*sin(b*x + a))) + sqrt(d*cos(b*x + a))*(2*cos(b*x + a)^2 - 1)*sin
(b*x + a))/(b*d^4*cos(b*x + a)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**2/(d*cos(b*x+a))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{\sin(bx+a)^2}{(d\cos(bx+a))^{\frac{7}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(7/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^2/(d*cos(b*x + a))^(7/2), x)
```

Giac [F]

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{\sin(bx+a)^2}{(d\cos(bx+a))^{\frac{7}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(7/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^2/(d*cos(b*x + a))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{\sin(a+b\,x)^2}{(d\,\cos(a+b\,x))^{7/2}}\,dx$$

input
```
int(sin(a + b*x)^2/(d*cos(a + b*x))^(7/2),x)
```

output
```
int(sin(a + b*x)^2/(d*cos(a + b*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\cos(bx+a)}\,\sin(bx+a)^2}{\cos(bx+a)^4}dx\right)}{d^4}$$

input
```
int(sin(b*x+a)^2/(d*cos(b*x+a))^(7/2),x)
```

output
```
(sqrt(d)*int((sqrt(cos(a + b*x))*sin(a + b*x)**2)/cos(a + b*x)**4,x))/d**4
```

3.203 $\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 100

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = -\frac{4\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{21bd^4\sqrt{d\cos(a+bx)}} + \frac{2\sin(a+bx)}{7bd(d\cos(a+bx))^{7/2}} - \frac{4\sin(a+bx)}{21bd^3(d\cos(a+bx))^{3/2}}$$

output
```
-4/21*cos(b*x+a)^(1/2)*InverseJacobiAM(1/2*a+1/2*b*x,2^(1/2))/b/d^4/(d*cos
(b*x+a))^(1/2)+2/7*sin(b*x+a)/b/d/(d*cos(b*x+a))^(7/2)-4/21*sin(b*x+a)/b/d
^3/(d*cos(b*x+a))^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.59

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \frac{\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{11}{4},\frac{5}{2},\sin^2(a+bx)\right)\sin^3(2(a+bx))}{24b(d\cos(a+bx))^{9/2}}$$

input
```
Integrate[Sin[a + b*x]^2/(d*Cos[a + b*x])^(9/2),x]
```

output
```
((Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/2, 11/4, 5/2, Sin[a + b*x]^2]*
Sin[2*(a + b*x)]^3)/(24*b*(d*Cos[a + b*x])^(9/2))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3046, 3042, 3116, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^2}{(d\cos(a+bx))^{9/2}}dx$$

$$\Big\downarrow \text{3046}$$

$$\frac{2\sin(a+bx)}{7bd(d\cos(a+bx))^{7/2}}-\frac{2\int \frac{1}{(d\cos(a+bx))^{5/2}}dx}{7d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(a+bx)}{7bd(d\cos(a+bx))^{7/2}}-\frac{2\int \frac{1}{\left(d\sin\left(a+bx+\frac{\pi}{2}\right)\right)^{5/2}}dx}{7d^2}$$

$$\Big\downarrow \text{3116}$$

$$\frac{2\sin(a+bx)}{7bd(d\cos(a+bx))^{7/2}}-\frac{2\left(\frac{\int \frac{1}{\sqrt{d\cos(a+bx)}}dx}{3d^2}+\frac{2\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}\right)}{7d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(a+bx)}{7bd(d\cos(a+bx))^{7/2}}-\frac{2\left(\frac{\int \frac{1}{\sqrt{d\sin\left(a+bx+\frac{\pi}{2}\right)}}dx}{3d^2}+\frac{2\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}\right)}{7d^2}$$

$$\Big\downarrow \text{3121}$$

$$\frac{2\sin(a+bx)}{7bd(d\cos(a+bx))^{7/2}}-\frac{2\left(\frac{\sqrt{\cos(a+bx)}\int\frac{1}{\sqrt{\cos(a+bx)}}dx}{3d^2\sqrt{d\cos(a+bx)}}+\frac{2\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}\right)}{7d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(a+bx)}{7bd(d\cos(a+bx))^{7/2}}-\frac{2\left(\frac{\sqrt{\cos(a+bx)}\int\frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx}{3d^2\sqrt{d\cos(a+bx)}}+\frac{2\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}\right)}{7d^2}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sin(a+bx)}{7bd(d\cos(a+bx))^{7/2}}-\frac{2\left(\frac{2\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3bd^2\sqrt{d\cos(a+bx)}}+\frac{2\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}\right)}{7d^2}$$

input
```
Int[Sin[a + b*x]^2/(d*Cos[a + b*x])^(9/2),x]
```

output
```
(2*Sin[a + b*x])/(7*b*d*(d*Cos[a + b*x])^(7/2)) - (2*((2*Sqrt[Cos[a + b*x]
]*EllipticF[(a + b*x)/2, 2])/(3*b*d^2*Sqrt[d*Cos[a + b*x]]) + (2*Sin[a + b
*x])/(3*b*d*(d*Cos[a + b*x])^(3/2))))/(7*d^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3046
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Sin[e + f
*x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(87) = 174$.

Time = 6.06 (sec) , antiderivative size = 396, normalized size of antiderivative = 3.96

method	result
default	$4\left(-8\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\sqrt{2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{6}-8\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{6}\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+12\sqrt{\frac{1}{2}-\frac{\cos(bx}{2}}\right.$

input
```
int(sin(b*x+a)^2/(d*cos(b*x+a))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
4/21*(-8*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*Ell
ipticF(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^6-8*sin(1/2*b*x+1/2*
a)^6*cos(1/2*b*x+1/2*a)+12*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2
*a)^2-1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^4+
8*cos(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)^4-6*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(
2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))*sin(
1/2*b*x+1/2*a)^2+cos(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)^2+(sin(1/2*b*x+1/2*
a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),
2^(1/2)))/d^4*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/(-
d*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/(2*cos(1/2*b*x+1/2*
a)^2-1)^3/sin(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.14

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx =$$
$$-\frac{2\left(-2i\sqrt{\frac{1}{2}}\sqrt{d}\cos(bx+a)^4\operatorname{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a))+2i\sqrt{\frac{1}{2}}\sqrt{d}\cos(bx\right.}{21\,b}$$

input
```
integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(9/2),x, algorithm="fricas")
```

output
```
-2/21*(-2*I*sqrt(1/2)*sqrt(d)*cos(b*x + a)^4*weierstrassPInverse(-4, 0, co
s(b*x + a) + I*sin(b*x + a)) + 2*I*sqrt(1/2)*sqrt(d)*cos(b*x + a)^4*weiers
trassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a)) + sqrt(d*cos(b*x + a))
*(2*cos(b*x + a)^2 - 3)*sin(b*x + a))/(b*d^5*cos(b*x + a)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**2/(d*cos(b*x+a))**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \int \frac{\sin\left(bx+a\right)^2}{\left(d\cos\left(bx+a\right)\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(9/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^2/(d*cos(b*x + a))^(9/2), x)
```

Giac [F]

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \int \frac{\sin\left(bx+a\right)^2}{\left(d\cos\left(bx+a\right)\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(9/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^2/(d*cos(b*x + a))^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \int \frac{\sin\left(a+b\,x\right)^2}{\left(d\,\cos\left(a+b\,x\right)\right)^{9/2}}\,dx$$

input
```
int(sin(a + b*x)^2/(d*cos(a + b*x))^(9/2),x)
```

output
```
int(sin(a + b*x)^2/(d*cos(a + b*x))^(9/2), x)
```

Reduce [F]

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\cos(bx+a)}\,\sin(bx+a)^2}{\cos(bx+a)^5}dx\right)}{d^5}$$

input
```
int(sin(b*x+a)^2/(d*cos(b*x+a))^(9/2),x)
```

output
```
(sqrt(d)*int((sqrt(cos(a + b*x))*sin(a + b*x)**2)/cos(a + b*x)**5,x))/d**5
```

3.204 $\int \sqrt{d\cos(a+bx)}\sin^3(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 45

$$\int \sqrt{d\cos(a+bx)}\sin^3(a+bx)\,dx = -\frac{2(d\cos(a+bx))^{3/2}}{3bd}+\frac{2(d\cos(a+bx))^{7/2}}{7bd^3}$$

output
```
-2/3*(d*cos(b*x+a))^(3/2)/b/d+2/7*(d*cos(b*x+a))^(7/2)/b/d^3
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.27

$$\int \sqrt{d\cos(a+bx)}\sin^3(a+bx)\,dx = -\frac{d\left(16\cos^2(a+bx)-16\sqrt[4]{\cos^2(a+bx)}+3\sin^2(2(a+bx))\right)}{42b\sqrt{d\cos(a+bx)}}$$

input
```
Integrate[Sqrt[d*Cos[a + b*x]]*Sin[a + b*x]^3,x]
```

output
```
-1/42*(d*(16*Cos[a + b*x]^2 - 16*(Cos[a + b*x]^2)^(1/4) + 3*Sin[2*(a + b*x
)]^2))/(b*Sqrt[d*Cos[a + b*x]])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3045, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\sqrt{d\cos(a+bx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(a+bx)^3\sqrt{d\cos(a+bx)}dx$$

$$\Big\downarrow \text{ 3045}$$

$$-\frac{\int \frac{\sqrt{d\cos(a+bx)}\left(d^2-d^2\cos^2(a+bx)\right)}{d^2}d(d\cos(a+bx))}{bd}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \sqrt{d\cos(a+bx)}\left(d^2-d^2\cos^2(a+bx)\right)d(d\cos(a+bx))}{bd^3}$$

$$\Big\downarrow \text{ 244}$$

$$-\frac{\int \left(d^2\sqrt{d\cos(a+bx)}-(d\cos(a+bx))^{5/2}\right)d(d\cos(a+bx))}{bd^3}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\frac{2}{3}d^2(d\cos(a+bx))^{3/2}-\frac{2}{7}(d\cos(a+bx))^{7/2}}{bd^3}$$

input
```
Int[Sqrt[d*Cos[a + b*x]]*Sin[a + b*x]^3,x]
```

output
```
-(((2*d^2*(d*Cos[a + b*x])^(3/2))/3 - (2*(d*Cos[a + b*x])^(7/2))/7)/(b*d^3
))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [A] (verified)

Time = 2.75 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\frac{2(d\cos(bx+a))^{\frac{7}{2}}}{7}-\frac{2d^2(d\cos(bx+a))^{\frac{3}{2}}}{3}}{b\,d^3}$	37
default	$\frac{\frac{2(d\cos(bx+a))^{\frac{7}{2}}}{7}-\frac{2d^2(d\cos(bx+a))^{\frac{3}{2}}}{3}}{b\,d^3}$	37

input
```
int((d*cos(b*x+a))^(1/2)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/b/d^3*(1/7*(d*cos(b*x+a))^(7/2)-1/3*d^2*(d*cos(b*x+a))^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int \sqrt{d\cos(a+bx)}\sin^3(a+bx)\,dx = \frac{2\left(3\cos\left(bx+a\right)^3-7\cos\left(bx+a\right)\right)\sqrt{d\cos\left(bx+a\right)}}{21\,b}$$

input
```
integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
2/21*(3*cos(b*x + a)^3 - 7*cos(b*x + a))*sqrt(d*cos(b*x + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.62

$$\int \sqrt{d\cos(a+bx)}\sin^3(a+bx)\,dx = \begin{cases} -\frac{2\sqrt{d\cos\left(a+bx\right)}\sin^2\left(a+bx\right)\cos\left(a+bx\right)}{3b}-\frac{8\sqrt{d\cos\left(a+bx\right)}\cos^3\left(a+bx\right)}{21b} & \text{for } b\neq 0 \\ x\sqrt{d\cos\left(a\right)}\sin^3\left(a\right) & \text{otherwise} \end{cases}$$

input
```
integrate((d*cos(b*x+a))**(1/2)*sin(b*x+a)**3,x)
```

output
```
Piecewise((-2*sqrt(d*cos(a + b*x))*sin(a + b*x)**2*cos(a + b*x)/(3*b) - 8*
sqrt(d*cos(a + b*x))*cos(a + b*x)**3/(21*b), Ne(b, 0)), (x*sqrt(d*cos(a))*
sin(a)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

$$\int \sqrt{d\cos(a+bx)}\sin^3(a+bx)\,dx = \frac{2\left(3\left(d\cos\left(bx+a\right)\right)^{\frac{7}{2}}-7\left(d\cos\left(bx+a\right)\right)^{\frac{3}{2}}d^2\right)}{21\,bd^3}$$

input
```
integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
2/21*(3*(d*cos(b*x + a))^(7/2) - 7*(d*cos(b*x + a))^(3/2)*d^2)/(b*d^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int \sqrt{d\cos(a+bx)}\sin^3(a+bx)\,dx = \frac{2\left(3\sqrt{d\cos(bx+a)}d^3\cos(bx+a)^3 - 7\sqrt{d\cos(bx+a)}d^3\cos(bx+a)\right)}{21\,bd^3}$$

input
```
integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
2/21*(3*sqrt(d*cos(b*x + a))*d^3*cos(b*x + a)^3 - 7*sqrt(d*cos(b*x + a))*d
^3*cos(b*x + a))/(b*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\cos(a+bx)}\sin^3(a+bx)\,dx = \int \sin(a+b\,x)^3\,\sqrt{d\,\cos(a+b\,x)}\,dx$$

input
```
int(sin(a + b*x)^3*(d*cos(a + b*x))^(1/2),x)
```

output
```
int(sin(a + b*x)^3*(d*cos(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{d\cos(a+bx)}\sin^3(a+bx)\,dx = \sqrt{d}\left(\int \sqrt{\cos(bx+a)}\,\sin(bx+a)^3\,dx\right)$$

input
```
int((d*cos(b*x+a))^(1/2)*sin(b*x+a)^3,x)
```

output
```
sqrt(d)*int(sqrt(cos(a + b*x))*sin(a + b*x)**3,x)
```

3.205 $\int \frac{\sin^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 43

$$\int \frac{\sin^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{2\sqrt{d\cos(a+bx)}}{bd}+\frac{2(d\cos(a+bx))^{5/2}}{5bd^3}$$

output
```
-2*(d*cos(b*x+a))^(1/2)/b/d+2/5*(d*cos(b*x+a))^(5/2)/b/d^3
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.33

$$\int \frac{\sin^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \frac{\cos(a+bx)(-9+\cos(2(a+bx)))+8\cos^2(a+bx)^{3/4}\sec(a+bx)}{5b\sqrt{d\cos(a+bx)}}$$

input
```
Integrate[Sin[a + b*x]^3/Sqrt[d*Cos[a + b*x]],x]
```

output
```
(Cos[a + b*x]*(-9 + Cos[2*(a + b*x)]) + 8*(Cos[a + b*x]^2)^(3/4)*Sec[a + b
*x])/(5*b*Sqrt[d*Cos[a + b*x]])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3045, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)}{\sqrt{d\cos(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^3}{\sqrt{d\cos(a+bx)}}dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int \frac{d^2-d^2\cos^2(a+bx)}{d^2\sqrt{d\cos(a+bx)}}d(d\cos(a+bx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{d^2-d^2\cos^2(a+bx)}{\sqrt{d\cos(a+bx)}}d(d\cos(a+bx))}{bd^3}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(\frac{d^2}{\sqrt{d\cos(a+bx)}}-(d\cos(a+bx))^{3/2}\right)d(d\cos(a+bx))}{bd^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2d^2\sqrt{d\cos(a+bx)}-\frac{2}{5}(d\cos(a+bx))^{5/2}}{bd^3}$$

input
```
Int[Sin[a + b*x]^3/Sqrt[d*Cos[a + b*x]],x]
```

output
```
-((2*d^2*Sqrt[d*Cos[a + b*x]] - (2*(d*Cos[a + b*x])^(5/2))/5)/(b*d^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [A] (verified)

Time = 2.87 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{\frac{2(d\cos(bx+a))^{\frac{5}{2}}}{5}-2d^2\sqrt{d\cos(bx+a)}}{b\,d^3}$	37
default	$\frac{\frac{2(d\cos(bx+a))^{\frac{5}{2}}}{5}-2d^2\sqrt{d\cos(bx+a)}}{b\,d^3}$	37

input
```
int(sin(b*x+a)^3/(d*cos(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b/d^3*(1/5*(d*cos(b*x+a))^(5/2)-d^2*(d*cos(b*x+a))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.65

$$\int \frac{\sin^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \frac{2\sqrt{d\cos(bx+a)}\left(\cos(bx+a)^2-5\right)}{5\,bd}$$

input
```
integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
2/5*sqrt(d*cos(b*x + a))*(cos(b*x + a)^2 - 5)/(b*d)
```

Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.65

$$\int \frac{\sin^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \begin{cases} -\frac{2\sin^2(a+bx)\cos(a+bx)}{b\sqrt{d\cos(a+bx)}} - \frac{8\cos^3(a+bx)}{5b\sqrt{d\cos(a+bx)}} & \text{for } b\neq 0 \\ \frac{x\sin^3(a)}{\sqrt{d\cos(a)}} & \text{otherwise} \end{cases}$$

input
```
integrate(sin(b*x+a)**3/(d*cos(b*x+a))**(1/2),x)
```

output
```
Piecewise((-2*sin(a + b*x)**2*cos(a + b*x)/(b*sqrt(d*cos(a + b*x))) - 8*co
s(a + b*x)**3/(5*b*sqrt(d*cos(a + b*x))), Ne(b, 0)), (x*sin(a)**3/sqrt(d*c
os(a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{\sin^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{2\left(5\sqrt{d\cos(bx+a)} - \frac{(d\cos(bx+a))^{\frac{5}{2}}}{d^2}\right)}{5\,bd}$$

input
```
integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
-2/5*(5*sqrt(d*cos(b*x + a)) - (d*cos(b*x + a))^(5/2)/d^2)/(b*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int \frac{\sin^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \frac{2\left(\sqrt{d\cos\left(bx+a\right)}d^2\cos\left(bx+a\right)^2 - 5\sqrt{d\cos\left(bx+a\right)}d^2\right)}{5\,bd^3}$$

input
```
integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
2/5*(sqrt(d*cos(b*x + a))*d^2*cos(b*x + a)^2 - 5*sqrt(d*cos(b*x + a))*d^2)
/(b*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{\sin\left(a+b\,x\right)^3}{\sqrt{d\,\cos\left(a+b\,x\right)}}\,dx$$

input
```
int(sin(a + b*x)^3/(d*cos(a + b*x))^(1/2),x)
```

output
```
int(sin(a + b*x)^3/(d*cos(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sin^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\cos(bx+a)}\,\sin(bx+a)^3}{\cos(bx+a)}dx\right)}{d}$$

input
```
int(sin(b*x+a)^3/(d*cos(b*x+a))^(1/2),x)
```

output
```
(sqrt(d)*int((sqrt(cos(a + b*x))*sin(a + b*x)**3)/cos(a + b*x),x))/d
```

3.206 $\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 43

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{2}{bd\sqrt{d\cos(a+bx)}} + \frac{2(d\cos(a+bx))^{3/2}}{3bd^3}$$

output
```
2/b/d/(d*cos(b*x+a))^(1/2)+2/3*(d*cos(b*x+a))^(3/2)/b/d^3
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = -\frac{2\left(-4+4\sqrt[4]{\cos^2(a+bx)}+\sin^2(a+bx)\right)}{3bd\sqrt{d\cos(a+bx)}}$$

input
```
Integrate[Sin[a + b*x]^3/(d*Cos[a + b*x])^(3/2),x]
```

output
```
(-2*(-4 + 4*(Cos[a + b*x]^2)^(1/4) + Sin[a + b*x]^2))/(3*b*d*Sqrt[d*Cos[a
+ b*x]])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3045, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^3}{(d\cos(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int \frac{d^2-d^2\cos^2(a+bx)}{d^2(d\cos(a+bx))^{3/2}}d(d\cos(a+bx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{d^2-d^2\cos^2(a+bx)}{(d\cos(a+bx))^{3/2}}d(d\cos(a+bx))}{bd^3}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(\frac{d^2}{(d\cos(a+bx))^{3/2}}-\sqrt{d\cos(a+bx)}\right)d(d\cos(a+bx))}{bd^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{2d^2}{\sqrt{d\cos(a+bx)}}-\frac{2}{3}(d\cos(a+bx))^{3/2}}{bd^3}$$

input
```
Int[Sin[a + b*x]^3/(d*Cos[a + b*x])^(3/2),x]
```

output
```
-(((-2*d^2)/Sqrt[d*Cos[a + b*x]] - (2*(d*Cos[a + b*x])^(3/2))/3)/(b*d^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [A] (verified)

Time = 2.83 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\frac{2(d\cos(bx+a))^{\frac{3}{2}}}{3}+\frac{2d^2}{\sqrt{d\cos(bx+a)}}}{b\,d^3}$	36
default	$\frac{\frac{2(d\cos(bx+a))^{\frac{3}{2}}}{3}+\frac{2d^2}{\sqrt{d\cos(bx+a)}}}{b\,d^3}$	36

input
```
int(sin(b*x+a)^3/(d*cos(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/b/d^3*(1/3*(d*cos(b*x+a))^(3/2)+d^2/(d*cos(b*x+a))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{2\sqrt{d\cos(bx+a)}\left(\cos(bx+a)^2+3\right)}{3\,bd^2\cos(bx+a)}$$

input
```
integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(d*cos(b*x + a))*(cos(b*x + a)^2 + 3)/(b*d^2*cos(b*x + a))
```

Sympy [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.63

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \begin{cases} \frac{2\sin^2(a+bx)\cos(a+bx)}{b(d\cos(a+bx))^{\frac{3}{2}}} + \frac{8\cos^3(a+bx)}{3b(d\cos(a+bx))^{\frac{3}{2}}} & \text{for } b \neq 0 \\ \frac{x\sin^3(a)}{(d\cos(a))^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(sin(b*x+a)**3/(d*cos(b*x+a))**(3/2),x)
```

output
```
Piecewise((2*sin(a + b*x)**2*cos(a + b*x)/(b*(d*cos(a + b*x))**(3/2)) + 8*
cos(a + b*x)**3/(3*b*(d*cos(a + b*x))**(3/2)), Ne(b, 0)), (x*sin(a)**3/(d*
cos(a))**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{2\left(\frac{3}{\sqrt{d\cos(bx+a)}} + \frac{(d\cos(bx+a))^{\frac{3}{2}}}{d^2}\right)}{3\,bd}$$

input
```
integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
2/3*(3/sqrt(d*cos(b*x + a)) + (d*cos(b*x + a))^(3/2)/d^2)/(b*d)
```

Giac [F]

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\sin\left(bx+a\right)^3}{\left(d\cos\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^3/(d*cos(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\sin\left(a+b\,x\right)^3}{\left(d\,\cos\left(a+b\,x\right)\right)^{3/2}}\,dx$$

input
```
int(sin(a + b*x)^3/(d*cos(a + b*x))^(3/2),x)
```

output
```
int(sin(a + b*x)^3/(d*cos(a + b*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\cos(bx+a)}\,\sin(bx+a)^3}{\cos(bx+a)^2}dx\right)}{d^2}$$

input
```
int(sin(b*x+a)^3/(d*cos(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*int((sqrt(cos(a + b*x))*sin(a + b*x)**3)/cos(a + b*x)**2,x))/d**2
```

3.207 $\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 43

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \frac{2}{3bd(d\cos(a+bx))^{3/2}} + \frac{2\sqrt{d\cos(a+bx)}}{bd^3}$$

output
```
2/3/b/d/(d*cos(b*x+a))^(3/2)+2*(d*cos(b*x+a))^(1/2)/b/d^3
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = -\frac{2\left(-4+4\cos^2(a+bx)^{3/4}+3\sin^2(a+bx)\right)}{3bd(d\cos(a+bx))^{3/2}}$$

input
```
Integrate[Sin[a + b*x]^3/(d*Cos[a + b*x])^(5/2),x]
```

output
```
(-2*(-4 + 4*(Cos[a + b*x]^2)^(3/4) + 3*Sin[a + b*x]^2))/(3*b*d*(d*Cos[a + 
b*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3045, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^3}{(d\cos(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int \frac{d^2-d^2\cos^2(a+bx)}{d^2(d\cos(a+bx))^{5/2}}d(d\cos(a+bx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{d^2-d^2\cos^2(a+bx)}{(d\cos(a+bx))^{5/2}}d(d\cos(a+bx))}{bd^3}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(\frac{d^2}{(d\cos(a+bx))^{5/2}}-\frac{1}{\sqrt{d\cos(a+bx)}}\right)d(d\cos(a+bx))}{bd^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{2d^2}{3(d\cos(a+bx))^{3/2}}-2\sqrt{d\cos(a+bx)}}{bd^3}$$

input
```
Int[Sin[a + b*x]^3/(d*Cos[a + b*x])^(5/2),x]
```

output
```
-(((-2*d^2)/(3*(d*Cos[a + b*x])^(3/2)) - 2*Sqrt[d*Cos[a + b*x]])/(b*d^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [A] (verified)

Time = 2.94 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{2\sqrt{d\cos(bx+a)}+\frac{2d^2}{3(d\cos(bx+a))^{\frac{3}{2}}}}{b\,d^3}$	35
default	$\frac{2\sqrt{d\cos(bx+a)}+\frac{2d^2}{3(d\cos(bx+a))^{\frac{3}{2}}}}{b\,d^3}$	35

input
```
int(sin(b*x+a)^3/(d*cos(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/b/d^3*((d*cos(b*x+a))^(1/2)+1/3*d^2/(d*cos(b*x+a))^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \frac{2\sqrt{d\cos(bx+a)}\left(3\cos(bx+a)^2+1\right)}{3\,bd^3\cos(bx+a)^2}$$

input
```
integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(d*cos(b*x + a))*(3*cos(b*x + a)^2 + 1)/(b*d^3*cos(b*x + a)^2)
```

Sympy [A] (verification not implemented)

Time = 3.50 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.65

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \begin{cases} \frac{2\sin^2(a+bx)\cos(a+bx)}{3b(d\cos(a+bx))^{\frac{5}{2}}} + \frac{8\cos^3(a+bx)}{3b(d\cos(a+bx))^{\frac{5}{2}}} & \text{for } b \neq 0 \\ \frac{x\sin^3(a)}{(d\cos(a))^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(sin(b*x+a)**3/(d*cos(b*x+a))**(5/2),x)
```

output
```
Piecewise((2*sin(a + b*x)**2*cos(a + b*x)/(3*b*(d*cos(a + b*x))**(5/2)) +
8*cos(a + b*x)**3/(3*b*(d*cos(a + b*x))**(5/2)), Ne(b, 0)), (x*sin(a)**3/(
d*cos(a))**(5/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \frac{2\left(\frac{1}{(d\cos(bx+a))^{\frac{3}{2}}} + \frac{3\sqrt{d\cos(bx+a)}}{d^2}\right)}{3\,bd}$$

input
```
integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
2/3*(1/(d*cos(b*x + a))^(3/2) + 3*sqrt(d*cos(b*x + a))/d^2)/(b*d)
```

Giac [F]

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{\sin(bx+a)^3}{(d\cos(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^3/(d*cos(b*x + a))^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 26.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.53

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \frac{2\sqrt{d\cos(a+bx)}\,(16\cos(2a+2bx)+3\cos(4a+4bx)+13)}{3bd^3\,(4\cos(2a+2bx)+\cos(4a+4bx)+3)}$$

input
```
int(sin(a + b*x)^3/(d*cos(a + b*x))^(5/2),x)
```

output
```
(2*(d*cos(a + b*x))^(1/2)*(16*cos(2*a + 2*b*x) + 3*cos(4*a + 4*b*x) + 13))
/(3*b*d^3*(4*cos(2*a + 2*b*x) + cos(4*a + 4*b*x) + 3))
```

Reduce [F]

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\cos(bx+a)}\,\sin(bx+a)^3}{\cos(bx+a)^3}dx\right)}{d^3}$$

input
```
int(sin(b*x+a)^3/(d*cos(b*x+a))^(5/2),x)
```

output
```
(sqrt(d)*int((sqrt(cos(a + b*x))*sin(a + b*x)**3)/cos(a + b*x)**3,x))/d**3
```

3.208 $\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 43

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{2}{5bd(d\cos(a+bx))^{5/2}} - \frac{2}{bd^3\sqrt{d\cos(a+bx)}}$$

output
```
2/5/b/d/(d*cos(b*x+a))^(5/2)-2/b/d^3/(d*cos(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.63

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{2\left(5-4\sqrt[4]{\cos^2(a+bx)}+4\left(-1+\sqrt[4]{\cos^2(a+bx)}\right)\csc^2(a+bx)\right)\tan^2(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}}$$

input
```
Integrate[Sin[a + b*x]^3/(d*Cos[a + b*x])^(7/2),x]
```

output
```
(2*(5 - 4*(Cos[a + b*x]^2)^(1/4) + 4*(-1 + (Cos[a + b*x]^2)^(1/4))*Csc[a +
 b*x]^2)*Tan[a + b*x]^2)/(5*b*d^3*Sqrt[d*Cos[a + b*x]])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3045, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{7/2}}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sin(a+bx)^3}{(d\cos(a+bx))^{7/2}}dx$$
$$\Big\downarrow \text{ 3045}$$
$$-\frac{\int \frac{d^2-d^2\cos^2(a+bx)}{d^2(d\cos(a+bx))^{7/2}}d(d\cos(a+bx))}{bd}$$
$$\Big\downarrow \text{ 27}$$
$$-\frac{\int \frac{d^2-d^2\cos^2(a+bx)}{(d\cos(a+bx))^{7/2}}d(d\cos(a+bx))}{bd^3}$$
$$\Big\downarrow \text{ 244}$$
$$-\frac{\int \left(\frac{d^2}{(d\cos(a+bx))^{7/2}}-\frac{1}{(d\cos(a+bx))^{3/2}}\right)d(d\cos(a+bx))}{bd^3}$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{\frac{2}{\sqrt{d\cos(a+bx)}}-\frac{2d^2}{5(d\cos(a+bx))^{5/2}}}{bd^3}$$

input
```
Int[Sin[a + b*x]^3/(d*Cos[a + b*x])^(7/2),x]
```

output
```
-(((-2*d^2)/(5*(d*Cos[a + b*x])^(5/2)) + 2/Sqrt[d*Cos[a + b*x]])/(b*d^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [A] (verified)

Time = 3.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{-\frac{2}{\sqrt{d\cos(bx+a)}}+\frac{2d^2}{5(d\cos(bx+a))^{\frac{5}{2}}}}{b\,d^3}$	37
default	$\frac{-\frac{2}{\sqrt{d\cos(bx+a)}}+\frac{2d^2}{5(d\cos(bx+a))^{\frac{5}{2}}}}{b\,d^3}$	37

input
```
int(sin(b*x+a)^3/(d*cos(b*x+a))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/b/d^3*(-1/(d*cos(b*x+a))^(1/2)+1/5*d^2/(d*cos(b*x+a))^(5/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = -\frac{2\,\sqrt{d\cos\left(bx+a\right)}\left(5\,\cos\left(bx+a\right)^2-1\right)}{5\,bd^4\cos\left(bx+a\right)^3}$$

input
```
integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

output
```
-2/5*sqrt(d*cos(b*x + a))*(5*cos(b*x + a)^2 - 1)/(b*d^4*cos(b*x + a)^3)
```

Sympy [A] (verification not implemented)

Time = 29.49 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.65

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \begin{cases} \frac{2\sin^2\left(a+bx\right)\cos\left(a+bx\right)}{5b(d\cos\left(a+bx\right))^{\frac{7}{2}}} - \frac{8\cos^3\left(a+bx\right)}{5b(d\cos\left(a+bx\right))^{\frac{7}{2}}} & \text{for } b\neq 0 \\ \frac{x\sin^3\left(a\right)}{(d\cos\left(a\right))^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(sin(b*x+a)**3/(d*cos(b*x+a))**(7/2),x)
```

output
```
Piecewise((2*sin(a + b*x)**2*cos(a + b*x)/(5*b*(d*cos(a + b*x))**(7/2)) -
8*cos(a + b*x)**3/(5*b*(d*cos(a + b*x))**(7/2)), Ne(b, 0)), (x*sin(a)**3/(
d*cos(a))**(7/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = -\frac{2\left(5\,d^2\cos\left(bx+a\right)^2-d^2\right)}{5\,\left(d\cos\left(bx+a\right)\right)^{\frac{5}{2}}bd^3}$$

input
```
integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(7/2),x, algorithm="maxima")
```

output
```
-2/5*(5*d^2*cos(b*x + a)^2 - d^2)/((d*cos(b*x + a))^(5/2)*b*d^3)
```

Giac [F]

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{\sin(bx+a)^3}{(d\cos(bx+a))^{\frac{7}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(7/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^3/(d*cos(b*x + a))^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 31.57 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.16

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = -\frac{4\,e^{a\,1i+b\,x\,1i}\sqrt{d\left(\frac{e^{-a\,1i-b\,x\,1i}}{2}+\frac{e^{a\,1i+b\,x\,1i}}{2}\right)}\left(6\,e^{a\,2i+b\,x\,2i}+5\,e^{a\,4i+b\,x\,4i}+5\right)}{5\,b\,d^4\left(e^{a\,2i+b\,x\,2i}+1\right)^3}$$

input
```
int(sin(a + b*x)^3/(d*cos(a + b*x))^(7/2),x)
```

output
```
-(4*exp(a*1i + b*x*1i)*(d*(exp(- a*1i - b*x*1i)/2 + exp(a*1i + b*x*1i)/2))
^(1/2)*(6*exp(a*2i + b*x*2i) + 5*exp(a*4i + b*x*4i) + 5))/(5*b*d^4*(exp(a*
2i + b*x*2i) + 1)^3)
```

Reduce [F]

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\cos(bx+a)}\,\sin(bx+a)^3}{\cos(bx+a)^4}dx\right)}{d^4}$$

input
```
int(sin(b*x+a)^3/(d*cos(b*x+a))^(7/2),x)
```

output
```
(sqrt(d)*int((sqrt(cos(a + b*x))*sin(a + b*x)**3)/cos(a + b*x)**4,x))/d**4
```

3.209 $\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 45

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx=\frac{2}{7bd(d\cos(a+bx))^{7/2}}-\frac{2}{3bd^3(d\cos(a+bx))^{3/2}}$$

output
```
2/7/b/d/(d*cos(b*x+a))^(7/2)-2/3/b/d^3/(d*cos(b*x+a))^(3/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.56

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx=\frac{2\left(7-4\cos^2(a+bx)^{3/4}+4\left(-1+\cos^2(a+bx)^{3/4}\right)\csc^2(a+bx)\right)\tan^2(a+bx)}{21bd^3(d\cos(a+bx))^{3/2}}$$

input
```
Integrate[Sin[a + b*x]^3/(d*Cos[a + b*x])^(9/2),x]
```

output
```
(2*(7 - 4*(Cos[a + b*x]^2)^(3/4) + 4*(-1 + (Cos[a + b*x]^2)^(3/4))*Csc[a + 
 b*x]^2)*Tan[a + b*x]^2)/(21*b*d^3*(d*Cos[a + b*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3045, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^3}{(d\cos(a+bx))^{9/2}}dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int \frac{d^2-d^2\cos^2(a+bx)}{d^2(d\cos(a+bx))^{9/2}}d(d\cos(a+bx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{d^2-d^2\cos^2(a+bx)}{(d\cos(a+bx))^{9/2}}d(d\cos(a+bx))}{bd^3}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(\frac{d^2}{(d\cos(a+bx))^{9/2}}-\frac{1}{(d\cos(a+bx))^{5/2}}\right)d(d\cos(a+bx))}{bd^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{2}{3(d\cos(a+bx))^{3/2}}-\frac{2d^2}{7(d\cos(a+bx))^{7/2}}}{bd^3}$$

input
```
Int[Sin[a + b*x]^3/(d*Cos[a + b*x])^(9/2),x]
```

output
```
-(((-2*d^2)/(7*(d*Cos[a + b*x])^(7/2)) + 2/(3*(d*Cos[a + b*x])^(3/2)))/(b*
d^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [A] (verified)

Time = 2.97 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{-\frac{2}{3(d\cos(bx+a))^{\frac{3}{2}}}+\frac{2d^2}{7(d\cos(bx+a))^{\frac{7}{2}}}}{b\,d^3}$	37
default	$\frac{-\frac{2}{3(d\cos(bx+a))^{\frac{3}{2}}}+\frac{2d^2}{7(d\cos(bx+a))^{\frac{7}{2}}}}{b\,d^3}$	37

input
```
int(sin(b*x+a)^3/(d*cos(b*x+a))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/b/d^3*(-1/3/(d*cos(b*x+a))^(3/2)+1/7*d^2/(d*cos(b*x+a))^(7/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = -\frac{2\sqrt{d\cos\left(bx+a\right)}\left(7\cos\left(bx+a\right)^2-3\right)}{21\,bd^5\cos\left(bx+a\right)^4}$$

input
```
integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(9/2),x, algorithm="fricas")
```

output
```
-2/21*sqrt(d*cos(b*x + a))*(7*cos(b*x + a)^2 - 3)/(b*d^5*cos(b*x + a)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**3/(d*cos(b*x+a))**(9/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = -\frac{2\left(7\,d^2\cos\left(bx+a\right)^2-3\,d^2\right)}{21\,\left(d\cos\left(bx+a\right)\right)^{\frac{7}{2}}bd^3}$$

input
```
integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(9/2),x, algorithm="maxima")
```

output
```
-2/21*(7*d^2*cos(b*x + a)^2 - 3*d^2)/((d*cos(b*x + a))^(7/2)*b*d^3)
```

Giac [F]

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \int \frac{\sin\left(bx+a\right)^3}{\left(d\cos\left(bx+a\right)\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(9/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^3/(d*cos(b*x + a))^(9/2), x)
```

Mupad [B] (verification not implemented)

Time = 30.43 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.07

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = -\frac{8\,e^{a\,2i+b\,x\,2i}\sqrt{d\,\left(\frac{e^{-a\,1i-b\,x\,1i}}{2}+\frac{e^{a\,1i+b\,x\,1i}}{2}\right)}\,\left(2\,e^{a\,2i+b\,x\,2i}+7\,e^{a\,4i+b\,x\,4i}+7\right)}{21\,b\,d^5\,\left(e^{a\,2i+b\,x\,2i}+1\right)^4}$$

input
```
int(sin(a + b*x)^3/(d*cos(a + b*x))^(9/2),x)
```

output
```
-(8*exp(a*2i + b*x*2i)*(d*(exp(- a*1i - b*x*1i)/2 + exp(a*1i + b*x*1i)/2))
^(1/2)*(2*exp(a*2i + b*x*2i) + 7*exp(a*4i + b*x*4i) + 7))/(21*b*d^5*(exp(a
*2i + b*x*2i) + 1)^4)
```

Reduce [F]

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\cos(bx+a)}\,\sin(bx+a)^3}{\cos(bx+a)^5}dx\right)}{d^5}$$

input
```
int(sin(b*x+a)^3/(d*cos(b*x+a))^(9/2),x)
```

output
```
(sqrt(d)*int((sqrt(cos(a + b*x))*sin(a + b*x)**3)/cos(a + b*x)**5,x))/d**5
```

3.210 $\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{11/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 45

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{11/2}}\,dx = \frac{2}{9bd(d\cos(a+bx))^{9/2}} - \frac{2}{5bd^3(d\cos(a+bx))^{5/2}}$$

output
```
2/9/b/d/(d*cos(b*x+a))^(9/2)-2/5/b/d^3/(d*cos(b*x+a))^(5/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 94 vs. 2(45) = 90.

Time = 0.60 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.09

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{11/2}}\,dx = \frac{2\left(4\sqrt[4]{\cos^2(a+bx)} + \left(9-8\sqrt[4]{\cos^2(a+bx)}\right)\csc^2(a+bx) + 4\left(-1+\sqrt[4]{\cos^2(a\right.\right.}{45bd^5\sqrt{d\cos(a+bx)}}$$

input
```
Integrate[Sin[a + b*x]^3/(d*Cos[a + b*x])^(11/2),x]
```

output
```
(2*(4*(Cos[a + b*x]^2)^(1/4) + (9 - 8*(Cos[a + b*x]^2)^(1/4))*Csc[a + b*x]
^2 + 4*(-1 + (Cos[a + b*x]^2)^(1/4))*Csc[a + b*x]^4)*Tan[a + b*x]^4)/(45*b
*d^5*Sqrt[d*Cos[a + b*x]])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3045, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{11/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^3}{(d\cos(a+bx))^{11/2}}dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int \frac{d^2-d^2\cos^2(a+bx)}{d^2(d\cos(a+bx))^{11/2}}d(d\cos(a+bx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{d^2-d^2\cos^2(a+bx)}{(d\cos(a+bx))^{11/2}}d(d\cos(a+bx))}{bd^3}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(\frac{d^2}{(d\cos(a+bx))^{11/2}}-\frac{1}{(d\cos(a+bx))^{7/2}}\right)d(d\cos(a+bx))}{bd^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{2}{5(d\cos(a+bx))^{5/2}}-\frac{2d^2}{9(d\cos(a+bx))^{9/2}}}{bd^3}$$

input
```
Int[Sin[a + b*x]^3/(d*Cos[a + b*x])^(11/2),x]
```

output
```
-(((-2*d^2)/(9*(d*Cos[a + b*x])^(9/2)) + 2/(5*(d*Cos[a + b*x])^(5/2)))/(b*
d^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [A] (verified)

Time = 3.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{-\frac{2}{5(d\cos(bx+a))^{\frac{5}{2}}}+\frac{2d^2}{9(d\cos(bx+a))^{\frac{9}{2}}}}{b\,d^3}$	37
default	$\frac{-\frac{2}{5(d\cos(bx+a))^{\frac{5}{2}}}+\frac{2d^2}{9(d\cos(bx+a))^{\frac{9}{2}}}}{b\,d^3}$	37

input
```
int(sin(b*x+a)^3/(d*cos(b*x+a))^(11/2),x,method=_RETURNVERBOSE)
```

output
```
2/b/d^3*(-1/5/(d*cos(b*x+a))^(5/2)+1/9*d^2/(d*cos(b*x+a))^(9/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{11/2}}\,dx = -\frac{2\sqrt{d\cos\left(bx+a\right)}\left(9\cos\left(bx+a\right)^2-5\right)}{45\,bd^6\cos\left(bx+a\right)^5}$$

input
```
integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(11/2),x, algorithm="fricas")
```

output
```
-2/45*sqrt(d*cos(b*x + a))*(9*cos(b*x + a)^2 - 5)/(b*d^6*cos(b*x + a)^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**3/(d*cos(b*x+a))**(11/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{11/2}}\,dx = -\frac{2\left(9\,d^2\cos\left(bx+a\right)^2-5\,d^2\right)}{45\,\left(d\cos\left(bx+a\right)\right)^{\frac{9}{2}}bd^3}$$

input
```
integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(11/2),x, algorithm="maxima")
```

output
```
-2/45*(9*d^2*cos(b*x + a)^2 - 5*d^2)/((d*cos(b*x + a))^(9/2)*b*d^3)
```

Giac [F]

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{11/2}}\,dx = \int \frac{\sin(bx+a)^3}{(d\cos(bx+a))^{\frac{11}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(11/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^3/(d*cos(b*x + a))^(11/2), x)
```

Mupad [B] (verification not implemented)

Time = 31.73 (sec) , antiderivative size = 279, normalized size of antiderivative = 6.20

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{11/2}}\,dx = \frac{16\,e^{a\,1i+b\,x\,1i}\sqrt{d\left(\frac{e^{-a\,1i-b\,x\,1i}}{2}+\frac{e^{a\,1i+b\,x\,1i}}{2}\right)}}{5\,b\,d^6\left(e^{a\,2i+b\,x\,2i}\,1i+1i\right)^2} - \frac{e^{a\,1i+b\,x\,1i}\sqrt{d\left(\frac{e^{-a\,1i-b\,x\,1i}}{2}+\frac{e^{a\,1i+b\,x\,1i}}{2}\right)}\,464i}{45\,b\,d^6\left(e^{a\,2i+b\,x\,2i}\,1i+1i\right)^3} - \frac{128\,e^{a\,1i+b\,x\,1i}\sqrt{d\left(\frac{e^{-a\,1i-b\,x\,1i}}{2}+\frac{e^{a\,1i+b\,x\,1i}}{2}\right)}}{9\,b\,d^6\left(e^{a\,2i+b\,x\,2i}\,1i+1i\right)^4} + \frac{e^{a\,1i+b\,x\,1i}\sqrt{d\left(\frac{e^{-a\,1i-b\,x\,1i}}{2}+\frac{e^{a\,1i+b\,x\,1i}}{2}\right)}\,64i}{9\,b\,d^6\left(e^{a\,2i+b\,x\,2i}\,1i+1i\right)^5}$$

input
```
int(sin(a + b*x)^3/(d*cos(a + b*x))^(11/2),x)
```

output
```
(16*exp(a*1i + b*x*1i)*(d*(exp(- a*1i - b*x*1i)/2 + exp(a*1i + b*x*1i)/2))
^(1/2))/(5*b*d^6*(exp(a*2i + b*x*2i)*1i + 1i)^2) - (exp(a*1i + b*x*1i)*(d*
(exp(- a*1i - b*x*1i)/2 + exp(a*1i + b*x*1i)/2))^(1/2)*464i)/(45*b*d^6*(ex
p(a*2i + b*x*2i)*1i + 1i)^3) - (128*exp(a*1i + b*x*1i)*(d*(exp(- a*1i - b*
x*1i)/2 + exp(a*1i + b*x*1i)/2))^(1/2))/(9*b*d^6*(exp(a*2i + b*x*2i)*1i +
1i)^4) + (exp(a*1i + b*x*1i)*(d*(exp(- a*1i - b*x*1i)/2 + exp(a*1i + b*x*1
i)/2))^(1/2)*64i)/(9*b*d^6*(exp(a*2i + b*x*2i)*1i + 1i)^5)
```

Reduce [F]

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{11/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\cos(bx+a)}\,\sin(bx+a)^3}{\cos(bx+a)^6}dx\right)}{d^6}$$

input
```
int(sin(b*x+a)^3/(d*cos(b*x+a))^(11/2),x)
```

output
```
(sqrt(d)*int((sqrt(cos(a + b*x))*sin(a + b*x)**3)/cos(a + b*x)**6,x))/d**6
```

3.211 $\int (d\cos(a+bx))^{9/2}\sin^4(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 156

$$\int (d\cos(a+bx))^{9/2}\sin^4(a+bx)\,dx = \frac{56d^4\sqrt{d\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|\,2\right)}{1105b\sqrt{\cos(a+bx)}} + \frac{56d^3(d\cos(a+bx))^{3/2}\sin(a+bx)}{3315b} + \frac{8d(d\cos(a+bx))^{7/2}\sin(a+bx)}{663b} - \frac{12(d\cos(a+bx))^{11/2}\sin(a+bx)}{221bd} - \frac{2(d\cos(a+bx))^{11/2}\sin^3(a+bx)}{17bd}$$

output
```
56/1105*d^4*(d*cos(b*x+a))^(1/2)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))/b/c
os(b*x+a)^(1/2)+56/3315*d^3*(d*cos(b*x+a))^(3/2)*sin(b*x+a)/b+8/663*d*(d*c
os(b*x+a))^(7/2)*sin(b*x+a)/b-12/221*(d*cos(b*x+a))^(11/2)*sin(b*x+a)/b/d-
2/17*(d*cos(b*x+a))^(11/2)*sin(b*x+a)^3/b/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.37

$$\int (d\cos(a+bx))^{9/2}\sin^4(a+bx)\,dx = \frac{(d\cos(a+bx))^{9/2}\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{5}{2},\frac{7}{2},\sin^2(a+bx)\right)\tan^5(a+bx)}{5b}$$

input
```
Integrate[(d*Cos[a + b*x])^(9/2)*Sin[a + b*x]^4,x]
```

output
```
((d*Cos[a + b*x])^(9/2)*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[-7/4, 5/2
, 7/2, Sin[a + b*x]^2]*Tan[a + b*x]^5)/(5*b)
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3048, 3042, 3048, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(a+bx)(d\cos(a+bx))^{9/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^4(d\cos(a+bx))^{9/2}dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{6}{17}\int (d\cos(a+bx))^{9/2}\sin^2(a+bx)dx - \frac{2\sin^3(a+bx)(d\cos(a+bx))^{11/2}}{17bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{17}\int (d\cos(a+bx))^{9/2}\sin(a+bx)^2dx - \frac{2\sin^3(a+bx)(d\cos(a+bx))^{11/2}}{17bd}$$

$$\Big\downarrow \text{3048}$$

$$\frac{6}{17}\left(\frac{2}{13}\int (d\cos(a+bx))^{9/2}dx-\frac{2\sin(a+bx)(d\cos(a+bx))^{11/2}}{13bd}\right)-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{11/2}}{17bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{17}\left(\frac{2}{13}\int \left(d\sin\left(a+bx+\frac{\pi}{2}\right)\right)^{9/2}dx-\frac{2\sin(a+bx)(d\cos(a+bx))^{11/2}}{13bd}\right)-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{11/2}}{17bd}$$

$$\Big\downarrow \text{3115}$$

$$\frac{6}{17}\left(\frac{2}{13}\left(\frac{7}{9}d^2\int (d\cos(a+bx))^{5/2}dx+\frac{2d\sin(a+bx)(d\cos(a+bx))^{7/2}}{9b}\right)-\frac{2\sin(a+bx)(d\cos(a+bx))^{11/2}}{13bd}\right)-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{11/2}}{17bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{17}\left(\frac{2}{13}\left(\frac{7}{9}d^2\int \left(d\sin\left(a+bx+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{2d\sin(a+bx)(d\cos(a+bx))^{7/2}}{9b}\right)-\frac{2\sin(a+bx)(d\cos(a+bx))}{13bd}\right.$$
$$\frac{2\sin^3(a+bx)(d\cos(a+bx))^{11/2}}{17bd}$$

$$\Big\downarrow \text{3115}$$

$$\frac{6}{17}\left(\frac{2}{13}\left(\frac{7}{9}d^2\left(\frac{3}{5}d^2\int \sqrt{d\cos(a+bx)}dx+\frac{2d\sin(a+bx)(d\cos(a+bx))^{3/2}}{5b}\right)+\frac{2d\sin(a+bx)(d\cos(a+bx))^{7/2}}{9b}\right.\right.$$
$$\frac{2\sin^3(a+bx)(d\cos(a+bx))^{11/2}}{17bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{17}\left(\frac{2}{13}\left(\frac{7}{9}d^2\left(\frac{3}{5}d^2\int \sqrt{d\sin\left(a+bx+\frac{\pi}{2}\right)}dx+\frac{2d\sin(a+bx)(d\cos(a+bx))^{3/2}}{5b}\right)+\frac{2d\sin(a+bx)(d\cos(a+b}{9b}\right.\right.$$
$$\frac{2\sin^3(a+bx)(d\cos(a+bx))^{11/2}}{17bd}$$

$$\Big\downarrow \text{3121}$$

$$\frac{6}{17}\left(\frac{2}{13}\left(\frac{7}{9}d^2\left(\frac{3d^2\sqrt{d\cos(a+bx)}\int\sqrt{\cos(a+bx)}dx}{5\sqrt{\cos(a+bx)}}+\frac{2d\sin(a+bx)(d\cos(a+bx))^{3/2}}{5b}\right)+\frac{2d\sin(a+bx)(d\cos}{9b}\right.\right.$$
$$\frac{2\sin^3(a+bx)(d\cos(a+bx))^{11/2}}{17bd}$$

$\downarrow$ 3042

$$\frac{6}{17}\left(\frac{2}{13}\left(\frac{7}{9}d^2\left(\frac{3d^2\sqrt{d\cos(a+bx)}\int\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(a+bx)}}+\frac{2d\sin(a+bx)(d\cos(a+bx))^{3/2}}{5b}\right)+\frac{2d\sin(a+bx}{}\right.\right.$$
$$\frac{2\sin^3(a+bx)(d\cos(a+bx))^{11/2}}{17bd}$$

$\downarrow$ 3119

$$\frac{6}{17}\left(\frac{2}{13}\left(\frac{7}{9}d^2\left(\frac{6d^2E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{5b\sqrt{\cos(a+bx)}}+\frac{2d\sin(a+bx)(d\cos(a+bx))^{3/2}}{5b}\right)+\frac{2d\sin(a+bx)(d\cos(}{9b}\right.\right.$$
$$\frac{2\sin^3(a+bx)(d\cos(a+bx))^{11/2}}{17bd}$$

input
```
Int[(d*Cos[a + b*x])^(9/2)*Sin[a + b*x]^4,x]
```

output
```
(-2*(d*Cos[a + b*x])^(11/2)*Sin[a + b*x]^3)/(17*b*d) + (6*((-2*(d*Cos[a +
b*x])^(11/2)*Sin[a + b*x])/(13*b*d) + (2*((2*d*(d*Cos[a + b*x])^(7/2)*Sin[
a + b*x])/(9*b) + (7*d^2*((6*d^2*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/
2, 2])/(5*b*Sqrt[Cos[a + b*x]]) + (2*d*(d*Cos[a + b*x])^(3/2)*Sin[a + b*x]
)/(5*b)))/9))/13))/17
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(136) = 272$.

Time = 27.66 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.76

method	result
default	$-\frac{8\sqrt{d\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,d^{5}\left(24960\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{19}-124800\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{17}+265440\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{15}-312960\cos\left(\cdots\right.\right.}{33\cdots}$

input
```
int((d*cos(b*x+a))^(9/2)*sin(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
-8/3315*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^5*(249
60*cos(1/2*b*x+1/2*a)^19-124800*cos(1/2*b*x+1/2*a)^17+265440*cos(1/2*b*x+1
/2*a)^15-312960*cos(1/2*b*x+1/2*a)^13+222520*cos(1/2*b*x+1/2*a)^11-96360*c
os(1/2*b*x+1/2*a)^9+23866*cos(1/2*b*x+1/2*a)^7-2652*cos(1/2*b*x+1/2*a)^5-3
5*cos(1/2*b*x+1/2*a)^3-21*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(-2*cos(1/2*b*x+1/2
*a)^2+1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))+21*cos(1/2*b*x+1/2*a)
)/(-d*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2
*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.85

$$\int (d\cos(a+bx))^{9/2}\sin^4(a+bx)\,dx =$$

$$-\frac{2\left(-84i\sqrt{\frac{1}{2}}d^{\frac{9}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)))+84i\sqrt{\ }\right.}{}$$

input
```
integrate((d*cos(b*x+a))^(9/2)*sin(b*x+a)^4,x, algorithm="fricas")
```

output
```
-2/3315*(-84*I*sqrt(1/2)*d^(9/2)*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(b*x + a) + I*sin(b*x + a))) + 84*I*sqrt(1/2)*d^(9/2)*weierstr
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a)))
- (195*d^4*cos(b*x + a)^7 - 285*d^4*cos(b*x + a)^5 + 20*d^4*cos(b*x + a)^3
 + 28*d^4*cos(b*x + a))*sqrt(d*cos(b*x + a))*sin(b*x + a))/b
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{9/2}\sin^4(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*cos(b*x+a))**(9/2)*sin(b*x+a)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\cos(a+bx))^{9/2}\sin^4(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{9}{2}}\sin(bx+a)^4\,dx$$

input
```
integrate((d*cos(b*x+a))^(9/2)*sin(b*x+a)^4,x, algorithm="maxima")
```

output
```
integrate((d*cos(b*x + a))^(9/2)*sin(b*x + a)^4, x)
```

Giac [F]

$$\int (d\cos(a+bx))^{9/2}\sin^4(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{9}{2}}\sin(bx+a)^4\,dx$$

input
```
integrate((d*cos(b*x+a))^(9/2)*sin(b*x+a)^4,x, algorithm="giac")
```

output
```
integrate((d*cos(b*x + a))^(9/2)*sin(b*x + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{9/2}\sin^4(a+bx)\,dx = \int \sin\left(a+b\,x\right)^4\left(d\,\cos\left(a+b\,x\right)\right)^{9/2}\,dx$$

input
```
int(sin(a + b*x)^4*(d*cos(a + b*x))^(9/2),x)
```

output
```
int(sin(a + b*x)^4*(d*cos(a + b*x))^(9/2), x)
```

Reduce [F]

$$\int (d\cos(a+bx))^{9/2}\sin^4(a+bx)\,dx = \sqrt{d}\left(\int \sqrt{\cos\left(bx+a\right)}\,\cos\left(bx+a\right)^4\sin\left(bx+a\right)^4dx\right)d^4$$

input
```
int((d*cos(b*x+a))^(9/2)*sin(b*x+a)^4,x)
```

output
```
sqrt(d)*int(sqrt(cos(a + b*x))*cos(a + b*x)**4*sin(a + b*x)**4,x)*d**4
```

3.212 $\int (d\cos(a+bx))^{7/2}\sin^4(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 156

$$\int (d\cos(a+bx))^{7/2}\sin^4(a+bx)\,dx = \frac{8d^4\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{231b\sqrt{d\cos(a+bx)}} + \frac{8d^3\sqrt{d\cos(a+bx)}\sin(a+bx)}{231b} + \frac{8d(d\cos(a+bx))^{5/2}\sin(a+bx)}{385b} - \frac{4(d\cos(a+bx))^{9/2}\sin(a+bx)}{55bd} - \frac{2(d\cos(a+bx))^{9/2}\sin^3(a+bx)}{15bd}$$

output
```
8/231*d^4*cos(b*x+a)^(1/2)*InverseJacobiAM(1/2*a+1/2*b*x,2^(1/2))/b/(d*cos
(b*x+a))^(1/2)+8/231*d^3*(d*cos(b*x+a))^(1/2)*sin(b*x+a)/b+8/385*d*(d*cos(
b*x+a))^(5/2)*sin(b*x+a)/b-4/55*(d*cos(b*x+a))^(9/2)*sin(b*x+a)/b/d-2/15*(
d*cos(b*x+a))^(9/2)*sin(b*x+a)^3/b/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.37

$$\int (d\cos(a+bx))^{7/2}\sin^4(a+bx)\,dx = \frac{(d\cos(a+bx))^{7/2}\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{5}{2},\frac{7}{2},\sin^2(a+bx)\right)\tan^5(a+bx)}{5b}$$

input
```
Integrate[(d*Cos[a + b*x])^(7/2)*Sin[a + b*x]^4,x]
```

output
```
((d*Cos[a + b*x])^(7/2)*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-5/4, 5/2
, 7/2, Sin[a + b*x]^2]*Tan[a + b*x]^5)/(5*b)
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3048, 3042, 3048, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(a+bx)(d\cos(a+bx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^4(d\cos(a+bx))^{7/2}dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{2}{5}\int (d\cos(a+bx))^{7/2}\sin^2(a+bx)dx - \frac{2\sin^3(a+bx)(d\cos(a+bx))^{9/2}}{15bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}\int (d\cos(a+bx))^{7/2}\sin(a+bx)^2dx - \frac{2\sin^3(a+bx)(d\cos(a+bx))^{9/2}}{15bd}$$

$\big\downarrow$ 3048

$$\frac{2}{5}\left(\frac{2}{11}\int (d\cos(a+bx))^{7/2}dx-\frac{2\sin(a+bx)(d\cos(a+bx))^{9/2}}{11bd}\right)-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{9/2}}{15bd}$$

$\big\downarrow$ 3042

$$\frac{2}{5}\left(\frac{2}{11}\int \left(d\sin\left(a+bx+\frac{\pi}{2}\right)\right)^{7/2}dx-\frac{2\sin(a+bx)(d\cos(a+bx))^{9/2}}{11bd}\right)-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{9/2}}{15bd}$$

$\big\downarrow$ 3115

$$\frac{2}{5}\left(\frac{2}{11}\left(\frac{5}{7}d^2\int (d\cos(a+bx))^{3/2}dx+\frac{2d\sin(a+bx)(d\cos(a+bx))^{5/2}}{7b}\right)-\frac{2\sin(a+bx)(d\cos(a+bx))^{9/2}}{11bd}\right)-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{9/2}}{15bd}$$

$\big\downarrow$ 3042

$$\frac{2}{5}\left(\frac{2}{11}\left(\frac{5}{7}d^2\int \left(d\sin\left(a+bx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2d\sin(a+bx)(d\cos(a+bx))^{5/2}}{7b}\right)-\frac{2\sin(a+bx)(d\cos(a+bx))^{9/}}{11bd}\right.$$
$$\frac{2\sin^3(a+bx)(d\cos(a+bx))^{9/2}}{15bd}$$

$\big\downarrow$ 3115

$$\frac{2}{5}\left(\frac{2}{11}\left(\frac{5}{7}d^2\left(\frac{1}{3}d^2\int \frac{1}{\sqrt{d\cos(a+bx)}}dx+\frac{2d\sin(a+bx)\sqrt{d\cos(a+bx)}}{3b}\right)+\frac{2d\sin(a+bx)(d\cos(a+bx))^{5/2}}{7b}\right)\right.$$
$$\frac{2\sin^3(a+bx)(d\cos(a+bx))^{9/2}}{15bd}$$

$\big\downarrow$ 3042

$$\frac{2}{5}\left(\frac{2}{11}\left(\frac{5}{7}d^2\left(\frac{1}{3}d^2\int \frac{1}{\sqrt{d\sin\left(a+bx+\frac{\pi}{2}\right)}}dx+\frac{2d\sin(a+bx)\sqrt{d\cos(a+bx)}}{3b}\right)+\frac{2d\sin(a+bx)(d\cos(a+bx))}{7b}\right.\right.$$
$$\frac{2\sin^3(a+bx)(d\cos(a+bx))^{9/2}}{15bd}$$

$\big\downarrow$ 3121

$$\frac{2}{5}\left(\frac{2}{11}\left(\frac{5}{7}d^2\left(\frac{d^2\sqrt{\cos(a+bx)}\int\frac{1}{\sqrt{\cos(a+bx)}}dx}{3\sqrt{d\cos(a+bx)}}+\frac{2d\sin(a+bx)\sqrt{d\cos(a+bx)}}{3b}\right)+\frac{2d\sin(a+bx)(d\cos(a+bx)}{7b}\right.\right.$$

$$\frac{2\sin^3(a+bx)(d\cos(a+bx))^{9/2}}{15bd}$$

$\big\downarrow$ 3042

$$\frac{2}{5}\left(\frac{2}{11}\left(\frac{5}{7}d^2\left(\frac{d^2\sqrt{\cos(a+bx)}\int\frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx}{3\sqrt{d\cos(a+bx)}}+\frac{2d\sin(a+bx)\sqrt{d\cos(a+bx)}}{3b}\right)+\frac{2d\sin(a+bx)(d\cos(a+}{7b}\right.\right.$$

$$\frac{2\sin^3(a+bx)(d\cos(a+bx))^{9/2}}{15bd}$$

$\big\downarrow$ 3120

$$\frac{2}{5}\left(\frac{2}{11}\left(\frac{5}{7}d^2\left(\frac{2d^2\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b\sqrt{d\cos(a+bx)}}+\frac{2d\sin(a+bx)\sqrt{d\cos(a+bx)}}{3b}\right)+\frac{2d\sin(a+bx)(d\cos}{7b}\right.\right.$$

$$\frac{2\sin^3(a+bx)(d\cos(a+bx))^{9/2}}{15bd}$$

input
```
Int[(d*Cos[a + b*x])^(7/2)*Sin[a + b*x]^4,x]
```

output
```
(-2*(d*Cos[a + b*x])^(9/2)*Sin[a + b*x]^3)/(15*b*d) + (2*((-2*(d*Cos[a + b
*x])^(9/2)*Sin[a + b*x])/(11*b*d) + (2*((2*d*(d*Cos[a + b*x])^(5/2)*Sin[a
+ b*x])/(7*b) + (5*d^2*((2*d^2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2
])/(3*b*Sqrt[d*Cos[a + b*x]]) + (2*d*Sqrt[d*Cos[a + b*x]]*Sin[a + b*x])/(3
*b)))/7))/11))/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 22.32 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.68

method	result
default	$-\frac{8\sqrt{d\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,d^{4}\left(4928\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{17}-22176\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{15}+41216\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{13}-40768\cos\left(\frac{bx}{2}+\right.\right.}{1155\sqrt{-d\left(2\sin\left(\frac{bx}{2}\right.\right.}}$

input
```
int((d*cos(b*x+a))^(7/2)*sin(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
-8/1155*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^4*(492
8*cos(1/2*b*x+1/2*a)^17-22176*cos(1/2*b*x+1/2*a)^15+41216*cos(1/2*b*x+1/2*
a)^13-40768*cos(1/2*b*x+1/2*a)^11+22868*cos(1/2*b*x+1/2*a)^9-6994*cos(1/2*
b*x+1/2*a)^7+926*cos(1/2*b*x+1/2*a)^5+5*cos(1/2*b*x+1/2*a)^3+5*(sin(1/2*b*
x+1/2*a)^2)^(1/2)*(-2*cos(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticF(cos(1/2*b*x+
1/2*a),2^(1/2))-5*cos(1/2*b*x+1/2*a))/(-d*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*
b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/
2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.78

$$\int (d\cos(a+bx))^{7/2}\sin^4(a+bx)\,dx =$$

$$-\frac{2\left(20i\sqrt{\frac{1}{2}}d^{\frac{7}{2}}\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a))-20i\sqrt{\frac{1}{2}}d^{\frac{7}{2}}\text{weierstrassPInverse}(-\right.}{}$$

input
```
integrate((d*cos(b*x+a))^(7/2)*sin(b*x+a)^4,x, algorithm="fricas")
```

output
```
-2/1155*(20*I*sqrt(1/2)*d^(7/2)*weierstrassPInverse(-4, 0, cos(b*x + a) +
I*sin(b*x + a)) - 20*I*sqrt(1/2)*d^(7/2)*weierstrassPInverse(-4, 0, cos(b*
x + a) - I*sin(b*x + a)) - (77*d^3*cos(b*x + a)^6 - 119*d^3*cos(b*x + a)^4
 + 12*d^3*cos(b*x + a)^2 + 20*d^3)*sqrt(d*cos(b*x + a))*sin(b*x + a))/b
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{7/2}\sin^4(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*cos(b*x+a))**(7/2)*sin(b*x+a)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\cos(a+bx))^{7/2}\sin^4(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{7}{2}}\sin(bx+a)^4\,dx$$

input
```
integrate((d*cos(b*x+a))^(7/2)*sin(b*x+a)^4,x, algorithm="maxima")
```

output
```
integrate((d*cos(b*x + a))^(7/2)*sin(b*x + a)^4, x)
```

Giac [F]

$$\int (d\cos(a+bx))^{7/2}\sin^4(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{7}{2}}\sin(bx+a)^4\,dx$$

input
```
integrate((d*cos(b*x+a))^(7/2)*sin(b*x+a)^4,x, algorithm="giac")
```

output
```
integrate((d*cos(b*x + a))^(7/2)*sin(b*x + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{7/2}\sin^4(a+bx)\,dx = \int \sin(a+b\,x)^4\,(d\,\cos(a+b\,x))^{7/2}\,dx$$

input
```
int(sin(a + b*x)^4*(d*cos(a + b*x))^(7/2),x)
```

output
```
int(sin(a + b*x)^4*(d*cos(a + b*x))^(7/2), x)
```

Reduce [F]

$$\int (d\cos(a+bx))^{7/2}\sin^4(a+bx)\,dx = \sqrt{d}\left(\int \sqrt{\cos(bx+a)}\,\cos(bx+a)^3\sin(bx+a)^4\,dx\right)d^3$$

input
```
int((d*cos(b*x+a))^(7/2)*sin(b*x+a)^4,x)
```

output
```
sqrt(d)*int(sqrt(cos(a + b*x))*cos(a + b*x)**3*sin(a + b*x)**4,x)*d**3
```

3.213 $\int (d\cos(a+bx))^{5/2}\sin^4(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 128

$$\int (d\cos(a+bx))^{5/2}\sin^4(a+bx)\,dx = \frac{8d^2\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{65b\sqrt{\cos(a+bx)}} + \frac{8d(d\cos(a+bx))^{3/2}\sin(a+bx)}{195b} - \frac{4(d\cos(a+bx))^{7/2}\sin(a+bx)}{39bd} - \frac{2(d\cos(a+bx))^{7/2}\sin^3(a+bx)}{13bd}$$

output
```
8/65*d^2*(d*cos(b*x+a))^(1/2)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))/b/cos(
b*x+a)^(1/2)+8/195*d*(d*cos(b*x+a))^(3/2)*sin(b*x+a)/b-4/39*(d*cos(b*x+a))
^(7/2)*sin(b*x+a)/b/d-2/13*(d*cos(b*x+a))^(7/2)*sin(b*x+a)^3/b/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.51

$$\int (d\cos(a+bx))^{5/2}\sin^4(a+bx)\,dx = \frac{(d\cos(a+bx))^{5/2}\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{5}{2},\frac{7}{2},\sin^2(a+bx)\right)\sin^2(a+bx)\tan^3(\cdots}{5b}$$

input
```
Integrate[(d*Cos[a + b*x])^(5/2)*Sin[a + b*x]^4,x]
```

output
```
((d*Cos[a + b*x])^(5/2)*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[-3/4, 5/2
, 7/2, Sin[a + b*x]^2]*Sin[a + b*x]^2*Tan[a + b*x]^3)/(5*b)
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3048, 3042, 3048, 3042, 3115, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(a+bx)(d\cos(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^4(d\cos(a+bx))^{5/2}dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{6}{13}\int (d\cos(a+bx))^{5/2}\sin^2(a+bx)dx-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{7/2}}{13bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{13}\int (d\cos(a+bx))^{5/2}\sin(a+bx)^2dx-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{7/2}}{13bd}$$

$$\Big\downarrow \text{3048}$$

$$\frac{6}{13}\left(\frac{2}{9}\int (d\cos(a+bx))^{5/2}dx-\frac{2\sin(a+bx)(d\cos(a+bx))^{7/2}}{9bd}\right)-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{7/2}}{13bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{13}\left(\frac{2}{9}\int \left(d\sin\left(a+bx+\frac{\pi}{2}\right)\right)^{5/2}dx-\frac{2\sin(a+bx)(d\cos(a+bx))^{7/2}}{9bd}\right)-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{7/2}}{13bd}$$

$\Big\downarrow$ 3115

$$\frac{6}{13}\left(\frac{2}{9}\left(\frac{3}{5}d^2\int \sqrt{d\cos(a+bx)}dx+\frac{2d\sin(a+bx)(d\cos(a+bx))^{3/2}}{5b}\right)-\frac{2\sin(a+bx)(d\cos(a+bx))^{7/2}}{9bd}\right)-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{7/2}}{13bd}$$

$\Big\downarrow$ 3042

$$\frac{6}{13}\left(\frac{2}{9}\left(\frac{3}{5}d^2\int \sqrt{d\sin\left(a+bx+\frac{\pi}{2}\right)}dx+\frac{2d\sin(a+bx)(d\cos(a+bx))^{3/2}}{5b}\right)-\frac{2\sin(a+bx)(d\cos(a+bx))^{7/2}}{9bd}\right)\frac{2\sin^3(a+bx)(d\cos(a+bx))^{7/2}}{13bd}$$

$\Big\downarrow$ 3121

$$\frac{6}{13}\left(\frac{2}{9}\left(\frac{3d^2\sqrt{d\cos(a+bx)}\int \sqrt{\cos(a+bx)}dx}{5\sqrt{\cos(a+bx)}}+\frac{2d\sin(a+bx)(d\cos(a+bx))^{3/2}}{5b}\right)-\frac{2\sin(a+bx)(d\cos(a+bx}{9bd}\right.$$
$$\frac{2\sin^3(a+bx)(d\cos(a+bx))^{7/2}}{13bd}$$

$\Big\downarrow$ 3042

$$\frac{6}{13}\left(\frac{2}{9}\left(\frac{3d^2\sqrt{d\cos(a+bx)}\int \sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(a+bx)}}+\frac{2d\sin(a+bx)(d\cos(a+bx))^{3/2}}{5b}\right)-\frac{2\sin(a+bx)(d\cos(}{9bd}\right.$$
$$\frac{2\sin^3(a+bx)(d\cos(a+bx))^{7/2}}{13bd}$$

$\Big\downarrow$ 3119

$$\frac{6}{13}\left(\frac{2}{9}\left(\frac{6d^2E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{5b\sqrt{\cos(a+bx)}}+\frac{2d\sin(a+bx)(d\cos(a+bx))^{3/2}}{5b}\right)-\frac{2\sin(a+bx)(d\cos(a+bx))}{9bd}\right.$$
$$\frac{2\sin^3(a+bx)(d\cos(a+bx))^{7/2}}{13bd}$$

input
```
Int[(d*Cos[a + b*x])^(5/2)*Sin[a + b*x]^4,x]
```

output
```
(-2*(d*Cos[a + b*x])^(7/2)*Sin[a + b*x]^3)/(13*b*d) + (6*((-2*(d*Cos[a + b
*x])^(7/2)*Sin[a + b*x])/(9*b*d) + (2*((6*d^2*Sqrt[d*Cos[a + b*x]]*Ellipti
cE[(a + b*x)/2, 2])/(5*b*Sqrt[Cos[a + b*x]]) + (2*d*(d*Cos[a + b*x])^(3/2)
*Sin[a + b*x])/(5*b)))/9))/13
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(112) = 224$.

Time = 19.83 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.95

method	result
default	$-\frac{8\sqrt{d\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\, d^{3}\left(480\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{15}-1920\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{13}+3040\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{11}-2400\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{9}+\right.}{195\sqrt{-d\left(2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}-\sin\left(\frac{bx}{2}\right.\right.}}$

input
```
int((d*cos(b*x+a))^(5/2)*sin(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
-8/195*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^3*(480*
cos(1/2*b*x+1/2*a)^15-1920*cos(1/2*b*x+1/2*a)^13+3040*cos(1/2*b*x+1/2*a)^1
1-2400*cos(1/2*b*x+1/2*a)^9+958*cos(1/2*b*x+1/2*a)^7-156*cos(1/2*b*x+1/2*a
)^5-5*cos(1/2*b*x+1/2*a)^3-3*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(-2*cos(1/2*b*x+
1/2*a)^2+1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))+3*cos(1/2*b*x+1/2*
a))/(-d*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1
/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.94

$$\int (d\cos(a+bx))^{5/2}\sin^4(a+bx)\,dx =$$

$$-\frac{2\left(-12i\sqrt{\frac{1}{2}}d^{\frac{5}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)))+12i\sqrt{\right.}}{}$$

input
```
integrate((d*cos(b*x+a))^(5/2)*sin(b*x+a)^4,x, algorithm="fricas")
```

output
```
-2/195*(-12*I*sqrt(1/2)*d^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(b*x + a) + I*sin(b*x + a))) + 12*I*sqrt(1/2)*d^(5/2)*weierstra
ssZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a))) -
 (15*d^2*cos(b*x + a)^5 - 25*d^2*cos(b*x + a)^3 + 4*d^2*cos(b*x + a))*sqrt
(d*cos(b*x + a))*sin(b*x + a))/b
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{5/2}\sin^4(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*cos(b*x+a))**(5/2)*sin(b*x+a)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\cos(a+bx))^{5/2}\sin^4(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{5}{2}}\sin(bx+a)^4\,dx$$

input
```
integrate((d*cos(b*x+a))^(5/2)*sin(b*x+a)^4,x, algorithm="maxima")
```

output
```
integrate((d*cos(b*x + a))^(5/2)*sin(b*x + a)^4, x)
```

Giac [F]

$$\int (d\cos(a+bx))^{5/2}\sin^4(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{5}{2}}\sin(bx+a)^4\,dx$$

input
```
integrate((d*cos(b*x+a))^(5/2)*sin(b*x+a)^4,x, algorithm="giac")
```

output
```
integrate((d*cos(b*x + a))^(5/2)*sin(b*x + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{5/2}\sin^4(a+bx)\,dx = \int \sin(a+b\,x)^4\,(d\,\cos(a+b\,x))^{5/2}\,dx$$

input
```
int(sin(a + b*x)^4*(d*cos(a + b*x))^(5/2),x)
```

output
```
int(sin(a + b*x)^4*(d*cos(a + b*x))^(5/2), x)
```

Reduce [F]

$$\int (d\cos(a+bx))^{5/2}\sin^4(a+bx)\,dx = \sqrt{d}\left(\int \sqrt{\cos(bx+a)}\,\cos(bx+a)^2\sin(bx+a)^4\,dx\right)d^2$$

input
```
int((d*cos(b*x+a))^(5/2)*sin(b*x+a)^4,x)
```

output
```
sqrt(d)*int(sqrt(cos(a + b*x))*cos(a + b*x)**2*sin(a + b*x)**4,x)*d**2
```

3.214 $\int (d\cos(a+bx))^{3/2}\sin^4(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 128

$$\int (d\cos(a+bx))^{3/2}\sin^4(a+bx)\,dx = \frac{8d^2\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{77b\sqrt{d\cos(a+bx)}} + \frac{8d\sqrt{d\cos(a+bx)}\sin(a+bx)}{77b} - \frac{12(d\cos(a+bx))^{5/2}\sin(a+bx)}{77bd} - \frac{2(d\cos(a+bx))^{5/2}\sin^3(a+bx)}{11bd}$$

output
```
8/77*d^2*cos(b*x+a)^(1/2)*InverseJacobiAM(1/2*a+1/2*b*x,2^(1/2))/b/(d*cos(
b*x+a))^(1/2)+8/77*d*(d*cos(b*x+a))^(1/2)*sin(b*x+a)/b-12/77*(d*cos(b*x+a)
)^(5/2)*sin(b*x+a)/b/d-2/11*(d*cos(b*x+a))^(5/2)*sin(b*x+a)^3/b/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.51

$$\int (d\cos(a+bx))^{3/2}\sin^4(a+bx)\,dx = \frac{(d\cos(a+bx))^{3/2}\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{5}{2},\frac{7}{2},\sin^2(a+bx)\right)\sin^2(a+bx)\tan^3(}{5b}$$

input
```
Integrate[(d*Cos[a + b*x])^(3/2)*Sin[a + b*x]^4,x]
```

output
```
((d*Cos[a + b*x])^(3/2)*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-1/4, 5/2
, 7/2, Sin[a + b*x]^2]*Sin[a + b*x]^2*Tan[a + b*x]^3)/(5*b)
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3048, 3042, 3048, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(a+bx)(d\cos(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^4(d\cos(a+bx))^{3/2}dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{6}{11}\int (d\cos(a+bx))^{3/2}\sin^2(a+bx)dx-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{5/2}}{11bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{11}\int (d\cos(a+bx))^{3/2}\sin(a+bx)^2dx-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{5/2}}{11bd}$$

$$\Big\downarrow \text{3048}$$

$$\frac{6}{11}\left(\frac{2}{7}\int (d\cos(a+bx))^{3/2}dx-\frac{2\sin(a+bx)(d\cos(a+bx))^{5/2}}{7bd}\right)-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{5/2}}{11bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{11}\left(\frac{2}{7}\int \left(d\sin\left(a+bx+\frac{\pi}{2}\right)\right)^{3/2}dx-\frac{2\sin(a+bx)(d\cos(a+bx))^{5/2}}{7bd}\right)-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{5/2}}{11bd}$$

$\Big\downarrow$ 3115

$$\frac{6}{11}\left(\frac{2}{7}\left(\frac{1}{3}d^2\int \frac{1}{\sqrt{d\cos(a+bx)}}dx+\frac{2d\sin(a+bx)\sqrt{d\cos(a+bx)}}{3b}\right)-\frac{2\sin(a+bx)(d\cos(a+bx))^{5/2}}{7bd}\right)-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{5/2}}{11bd}$$

$\Big\downarrow$ 3042

$$\frac{6}{11}\left(\frac{2}{7}\left(\frac{1}{3}d^2\int \frac{1}{\sqrt{d\sin\left(a+bx+\frac{\pi}{2}\right)}}dx+\frac{2d\sin(a+bx)\sqrt{d\cos(a+bx)}}{3b}\right)-\frac{2\sin(a+bx)(d\cos(a+bx))^{5/2}}{7bd}\right)-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{5/2}}{11bd}$$

$\Big\downarrow$ 3121

$$\frac{6}{11}\left(\frac{2}{7}\left(\frac{d^2\sqrt{\cos(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}}dx}{3\sqrt{d\cos(a+bx)}}+\frac{2d\sin(a+bx)\sqrt{d\cos(a+bx)}}{3b}\right)-\frac{2\sin(a+bx)(d\cos(a+bx))^{5/2}}{7bd}\right)-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{5/2}}{11bd}$$

$\Big\downarrow$ 3042

$$\frac{6}{11}\left(\frac{2}{7}\left(\frac{d^2\sqrt{\cos(a+bx)}\int \frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx}{3\sqrt{d\cos(a+bx)}}+\frac{2d\sin(a+bx)\sqrt{d\cos(a+bx)}}{3b}\right)-\frac{2\sin(a+bx)(d\cos(a+bx))^{5/2}}{7bd}\right.$$
$$\frac{2\sin^3(a+bx)(d\cos(a+bx))^{5/2}}{11bd}$$

$\Big\downarrow$ 3120

$$\frac{6}{11}\left(\frac{2}{7}\left(\frac{2d^2\sqrt{\cos(a+bx)}\operatorname{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b\sqrt{d\cos(a+bx)}}+\frac{2d\sin(a+bx)\sqrt{d\cos(a+bx)}}{3b}\right)-\frac{2\sin(a+bx)(d\cos(a+b}{7bd}\right.$$
$$\frac{2\sin^3(a+bx)(d\cos(a+bx))^{5/2}}{11bd}$$

input
```
Int[(d*Cos[a + b*x])^(3/2)*Sin[a + b*x]^4,x]
```

output
```
(-2*(d*Cos[a + b*x])^(5/2)*Sin[a + b*x]^3)/(11*b*d) + (6*((-2*(d*Cos[a + b
*x])^(5/2)*Sin[a + b*x])/(7*b*d) + (2*((2*d^2*Sqrt[Cos[a + b*x]]*EllipticF
[(a + b*x)/2, 2])/(3*b*Sqrt[d*Cos[a + b*x]]) + (2*d*Sqrt[d*Cos[a + b*x]]*S
in[a + b*x])/(3*b)))/7))/11
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(111) = 222$.

Time = 7.82 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.99

method	result
default	$-\frac{8\sqrt{d\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,d^{2}\left(112\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{12}-280\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{10}\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+228\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{8}\right.}{77\sqrt{-d\left(2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}-\sin\left(\frac{bx}{2}+\right.\right.}}$

input
```
int((d*cos(b*x+a))^(3/2)*sin(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
-8/77*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^2*(112*c
os(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)^12-280*sin(1/2*b*x+1/2*a)^10*cos(1/2*
b*x+1/2*a)+228*sin(1/2*b*x+1/2*a)^8*cos(1/2*b*x+1/2*a)-62*sin(1/2*b*x+1/2*
a)^6*cos(1/2*b*x+1/2*a)+cos(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)^2+(sin(1/2*b
*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cos(1/2*b*x+
1/2*a),2^(1/2)))/(-d*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/
sin(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.80

$$\int (d\cos(a+bx))^{3/2}\sin^4(a+bx)\,dx =$$

$$-\frac{2\left(4i\sqrt{\frac{1}{2}}d^{\frac{3}{2}}\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a))-4i\sqrt{\frac{1}{2}}d^{\frac{3}{2}}\text{weierstrassPInverse}(-4,\right.}{}$$

input
```
integrate((d*cos(b*x+a))^(3/2)*sin(b*x+a)^4,x, algorithm="fricas")
```

output
```
-2/77*(4*I*sqrt(1/2)*d^(3/2)*weierstrassPInverse(-4, 0, cos(b*x + a) + I*s
in(b*x + a)) - 4*I*sqrt(1/2)*d^(3/2)*weierstrassPInverse(-4, 0, cos(b*x +
a) - I*sin(b*x + a)) - (7*d*cos(b*x + a)^4 - 13*d*cos(b*x + a)^2 + 4*d)*sq
rt(d*cos(b*x + a))*sin(b*x + a))/b
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{3/2}\sin^4(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*cos(b*x+a))**(3/2)*sin(b*x+a)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\cos(a+bx))^{3/2}\sin^4(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{3}{2}}\sin(bx+a)^4\,dx$$

input
```
integrate((d*cos(b*x+a))^(3/2)*sin(b*x+a)^4,x, algorithm="maxima")
```

output
```
integrate((d*cos(b*x + a))^(3/2)*sin(b*x + a)^4, x)
```

Giac [F]

$$\int (d\cos(a+bx))^{3/2}\sin^4(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{3}{2}}\sin(bx+a)^4\,dx$$

input
```
integrate((d*cos(b*x+a))^(3/2)*sin(b*x+a)^4,x, algorithm="giac")
```

output
```
integrate((d*cos(b*x + a))^(3/2)*sin(b*x + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{3/2}\sin^4(a+bx)\,dx = \int \sin\left(a+b\,x\right)^4\left(d\,\cos\left(a+b\,x\right)\right)^{3/2}\,dx$$

input
```
int(sin(a + b*x)^4*(d*cos(a + b*x))^(3/2),x)
```

output
```
int(sin(a + b*x)^4*(d*cos(a + b*x))^(3/2), x)
```

Reduce [F]

$$\int (d\cos(a+bx))^{3/2}\sin^4(a+bx)\,dx = \sqrt{d}\left(\int \sqrt{\cos\left(bx+a\right)}\,\cos\left(bx+a\right)\sin\left(bx+a\right)^4dx\right)d$$

input
```
int((d*cos(b*x+a))^(3/2)*sin(b*x+a)^4,x)
```

output
```
sqrt(d)*int(sqrt(cos(a + b*x))*cos(a + b*x)*sin(a + b*x)**4,x)*d
```

3.215 $\int \sqrt{d\cos(a+bx)}\sin^4(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 99

$$\int \sqrt{d\cos(a+bx)}\sin^4(a+bx)\,dx = \frac{8\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{15b\sqrt{\cos(a+bx)}} - \frac{4(d\cos(a+bx))^{3/2}\sin(a+bx)}{15bd} - \frac{2(d\cos(a+bx))^{3/2}\sin^3(a+bx)}{9bd}$$

output
```
8/15*(d*cos(b*x+a))^(1/2)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))/b/cos(b*x+
a)^(1/2)-4/15*(d*cos(b*x+a))^(3/2)*sin(b*x+a)/b/d-2/9*(d*cos(b*x+a))^(3/2)
*sin(b*x+a)^3/b/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.59

$$\int \sqrt{d\cos(a+bx)}\sin^4(a+bx)\,dx = \frac{d\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{5}{2},\frac{7}{2},\sin^2(a+bx)\right)\sin^5(a+bx)}{5b\sqrt{d\cos(a+bx)}}$$

input
```
Integrate[Sqrt[d*Cos[a + b*x]]*Sin[a + b*x]^4,x]
```

output
```
(d*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, 5/2, 7/2, Sin[a + b*x]^2]
*Sin[a + b*x]^5)/(5*b*Sqrt[d*Cos[a + b*x]])
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3048, 3042, 3048, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(a+bx)\sqrt{d\cos(a+bx)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sin(a+bx)^4\sqrt{d\cos(a+bx)}dx$$

$$\Big\downarrow \ 3048$$

$$\frac{2}{3}\int \sqrt{d\cos(a+bx)}\sin^2(a+bx)dx-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{3/2}}{9bd}$$

$$\Big\downarrow \ 3042$$

$$\frac{2}{3}\int \sqrt{d\cos(a+bx)}\sin(a+bx)^2dx-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{3/2}}{9bd}$$

$$\Big\downarrow \ 3048$$

$$\frac{2}{3}\left(\frac{2}{5}\int \sqrt{d\cos(a+bx)}dx-\frac{2\sin(a+bx)(d\cos(a+bx))^{3/2}}{5bd}\right)-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{3/2}}{9bd}$$

$$\Big\downarrow \ 3042$$

$$\frac{2}{3}\left(\frac{2}{5}\int \sqrt{d\sin\left(a+bx+\frac{\pi}{2}\right)}dx-\frac{2\sin(a+bx)(d\cos(a+bx))^{3/2}}{5bd}\right)-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{3/2}}{9bd}$$

$\Big\downarrow$ 3121

$$\frac{2}{3}\left(\frac{2\sqrt{d\cos(a+bx)}\int \sqrt{\cos(a+bx)}dx}{5\sqrt{\cos(a+bx)}}-\frac{2\sin(a+bx)(d\cos(a+bx))^{3/2}}{5bd}\right)-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{3/2}}{9bd}$$

$\Big\downarrow$ 3042

$$\frac{2}{3}\left(\frac{2\sqrt{d\cos(a+bx)}\int \sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(a+bx)}}-\frac{2\sin(a+bx)(d\cos(a+bx))^{3/2}}{5bd}\right)-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{3/2}}{9bd}$$

$\Big\downarrow$ 3119

$$\frac{2}{3}\left(\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{5b\sqrt{\cos(a+bx)}}-\frac{2\sin(a+bx)(d\cos(a+bx))^{3/2}}{5bd}\right)-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{3/2}}{9bd}$$

input
```
Int[Sqrt[d*Cos[a + b*x]]*Sin[a + b*x]^4,x]
```

output
```
(-2*(d*Cos[a + b*x])^(3/2)*Sin[a + b*x]^3)/(9*b*d) + (2*((4*Sqrt[d*Cos[a +
 b*x]]*EllipticE[(a + b*x)/2, 2])/(5*b*Sqrt[Cos[a + b*x]]) - (2*(d*Cos[a +
 b*x])^(3/2)*Sin[a + b*x])/(5*b*d)))/3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(87) = 174$.

Time = 6.86 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.23

method	result
default	$-\frac{8\sqrt{d\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\, d\left(40\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{11}-120\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{9}+118\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{7}-36\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{5}-5\cos\left(\frac{bx}{2}\right.\right.}{45\sqrt{-d\left(2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}-\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)}\,\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{\ldots}}$

input
```
int((d*cos(b*x+a))^(1/2)*sin(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
-8/45*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d*(40*cos(
1/2*b*x+1/2*a)^11-120*cos(1/2*b*x+1/2*a)^9+118*cos(1/2*b*x+1/2*a)^7-36*cos
(1/2*b*x+1/2*a)^5-5*cos(1/2*b*x+1/2*a)^3-3*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(-
2*cos(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))+3*co
s(1/2*b*x+1/2*a))/(-d*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)
/sin(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.01

$$\int \sqrt{d\cos(a+bx)}\sin^4(a+bx)\,dx$$
$$= \frac{2\left(\left(5\cos(bx+a)^3-11\cos(bx+a)\right)\sqrt{d\cos(bx+a)}\sin(bx+a)+12i\sqrt{\frac{1}{2}}\sqrt{d}\text{weierstrassZeta}(-4,0,\text{w}\right.}{}$$

input
```
integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a)^4,x, algorithm="fricas")
```

output
```
2/45*((5*cos(b*x + a)^3 - 11*cos(b*x + a))*sqrt(d*cos(b*x + a))*sin(b*x +
a) + 12*I*sqrt(1/2)*sqrt(d)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
 0, cos(b*x + a) + I*sin(b*x + a))) - 12*I*sqrt(1/2)*sqrt(d)*weierstrassZe
ta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a))))/b
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{d\cos(a+bx)}\sin^4(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*cos(b*x+a))**(1/2)*sin(b*x+a)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{d\cos(a+bx)}\sin^4(a+bx)\,dx = \int \sqrt{d\cos\left(bx+a\right)}\sin\left(bx+a\right)^4\,dx$$

input
```
integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a)^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*cos(b*x + a))*sin(b*x + a)^4, x)
```

Giac [F]

$$\int \sqrt{d\cos(a+bx)}\sin^4(a+bx)\,dx = \int \sqrt{d\cos\left(bx+a\right)}\sin\left(bx+a\right)^4\,dx$$

input
```
integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a)^4,x, algorithm="giac")
```

output
```
integrate(sqrt(d*cos(b*x + a))*sin(b*x + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\cos(a+bx)}\sin^4(a+bx)\,dx = \int \sin\left(a+b\,x\right)^4\,\sqrt{d\,\cos\left(a+b\,x\right)}\,dx$$

input
```
int(sin(a + b*x)^4*(d*cos(a + b*x))^(1/2),x)
```

output
```
int(sin(a + b*x)^4*(d*cos(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{d\cos(a+bx)}\sin^4(a+bx)\,dx = \sqrt{d}\left(\int \sqrt{\cos(bx+a)}\,\sin(bx+a)^4\,dx\right)$$

input
```
int((d*cos(b*x+a))^(1/2)*sin(b*x+a)^4,x)
```

output
```
sqrt(d)*int(sqrt(cos(a + b*x))*sin(a + b*x)**4,x)
```

3.216 $\int \frac{\sin^4(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 99

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \frac{8\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{7b\sqrt{d\cos(a+bx)}} - \frac{4\sqrt{d\cos(a+bx)}\sin(a+bx)}{7bd} - \frac{2\sqrt{d\cos(a+bx)}\sin^3(a+bx)}{7bd}$$

output
```
8/7*cos(b*x+a)^(1/2)*InverseJacobiAM(1/2*a+1/2*b*x,2^(1/2))/b/(d*cos(b*x+a
))^(1/2)-4/7*(d*cos(b*x+a))^(1/2)*sin(b*x+a)/b/d-2/7*(d*cos(b*x+a))^(1/2)*
sin(b*x+a)^3/b/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.59

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \frac{d\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{2},\frac{7}{2},\sin^2(a+bx)\right)\sin^5(a+bx)}{5b(d\cos(a+bx))^{3/2}}$$

input
```
Integrate[Sin[a + b*x]^4/Sqrt[d*Cos[a + b*x]],x]
```

output
```
(d*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/4, 5/2, 7/2, Sin[a + b*x]^2]
*Sin[a + b*x]^5)/(5*b*(d*Cos[a + b*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3048, 3042, 3048, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\cos(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^4}{\sqrt{d\cos(a+bx)}}dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{6}{7}\int \frac{\sin^2(a+bx)}{\sqrt{d\cos(a+bx)}}dx-\frac{2\sin^3(a+bx)\sqrt{d\cos(a+bx)}}{7bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{7}\int \frac{\sin(a+bx)^2}{\sqrt{d\cos(a+bx)}}dx-\frac{2\sin^3(a+bx)\sqrt{d\cos(a+bx)}}{7bd}$$

$$\Big\downarrow \text{3048}$$

$$\frac{6}{7}\left(\frac{2}{3}\int \frac{1}{\sqrt{d\cos(a+bx)}}dx-\frac{2\sin(a+bx)\sqrt{d\cos(a+bx)}}{3bd}\right)-\frac{2\sin^3(a+bx)\sqrt{d\cos(a+bx)}}{7bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{7}\left(\frac{2}{3}\int \frac{1}{\sqrt{d\sin\left(a+bx+\frac{\pi}{2}\right)}}dx-\frac{2\sin(a+bx)\sqrt{d\cos(a+bx)}}{3bd}\right)-\frac{2\sin^3(a+bx)\sqrt{d\cos(a+bx)}}{7bd}$$

$\Big\downarrow$ 3121

$$\frac{6}{7}\left(\frac{2\sqrt{\cos(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}}dx}{3\sqrt{d\cos(a+bx)}}-\frac{2\sin(a+bx)\sqrt{d\cos(a+bx)}}{3bd}\right)-\frac{2\sin^3(a+bx)\sqrt{d\cos(a+bx)}}{7bd}$$

$\Big\downarrow$ 3042

$$\frac{6}{7}\left(\frac{2\sqrt{\cos(a+bx)}\int \frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx}{3\sqrt{d\cos(a+bx)}}-\frac{2\sin(a+bx)\sqrt{d\cos(a+bx)}}{3bd}\right)-\frac{2\sin^3(a+bx)\sqrt{d\cos(a+bx)}}{7bd}$$

$\Big\downarrow$ 3120

$$\frac{6}{7}\left(\frac{4\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b\sqrt{d\cos(a+bx)}}-\frac{2\sin(a+bx)\sqrt{d\cos(a+bx)}}{3bd}\right)-\frac{2\sin^3(a+bx)\sqrt{d\cos(a+bx)}}{7bd}$$

input
```
Int[Sin[a + b*x]^4/Sqrt[d*Cos[a + b*x]],x]
```

output
```
(-2*Sqrt[d*Cos[a + b*x]]*Sin[a + b*x]^3)/(7*b*d) + (6*((4*Sqrt[Cos[a + b*x
]]*EllipticF[(a + b*x)/2, 2])/(3*b*Sqrt[d*Cos[a + b*x]]) - (2*Sqrt[d*Cos[a
 + b*x]]*Sin[a + b*x])/(3*b*d)))/7
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(86) = 172$.

Time = 4.38 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.10

method	result
default	$-\frac{8\sqrt{d\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,\left(4\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{8}\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-6\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{6}\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\cdots}{7\sqrt{-d\left(2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}-\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)}\,\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{d\left(2\cos\left(\frac{bx}{2}+\cdots\right.\right.}}$

input
```
int(sin(b*x+a)^4/(d*cos(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-8/7*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(4*sin(1/2*
b*x+1/2*a)^8*cos(1/2*b*x+1/2*a)-6*sin(1/2*b*x+1/2*a)^6*cos(1/2*b*x+1/2*a)+
cos(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)^2+(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*si
n(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2)))/(-d*(2*
sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(d*(2
*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.89

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \frac{2\left(\sqrt{d\cos\left(bx+a\right)}\left(\cos\left(bx+a\right)^2-3\right)\sin\left(bx+a\right)-4i\,\sqrt{\frac{1}{2}}\sqrt{d}\text{weierstrassPInverse}(-4,0,\cos\left(bx+a\right)+\right.}{7\,bd}$$

input
```
integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
2/7*(sqrt(d*cos(b*x + a))*(cos(b*x + a)^2 - 3)*sin(b*x + a) - 4*I*sqrt(1/2
)*sqrt(d)*weierstrassPInverse(-4, 0, cos(b*x + a) + I*sin(b*x + a)) + 4*I*
sqrt(1/2)*sqrt(d)*weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a)
))/(b*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**4/(d*cos(b*x+a))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{\sin\left(bx+a\right)^4}{\sqrt{d\cos\left(bx+a\right)}}\,dx$$

input
```
integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^4/sqrt(d*cos(b*x + a)), x)
```

Giac [F]

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{\sin\left(bx+a\right)^4}{\sqrt{d\cos\left(bx+a\right)}}\,dx$$

input
```
integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^4/sqrt(d*cos(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{\sin\left(a+b\,x\right)^4}{\sqrt{d\,\cos\left(a+b\,x\right)}}\,dx$$

input
```
int(sin(a + b*x)^4/(d*cos(a + b*x))^(1/2),x)
```

output
```
int(sin(a + b*x)^4/(d*cos(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\cos(bx+a)}\,\sin(bx+a)^4}{\cos(bx+a)}dx\right)}{d}$$

input
```
int(sin(b*x+a)^4/(d*cos(b*x+a))^(1/2),x)
```

output
```
(sqrt(d)*int((sqrt(cos(a + b*x))*sin(a + b*x)**4)/cos(a + b*x),x))/d
```

3.217 $\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 100

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = -\frac{24\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5bd^2\sqrt{\cos(a+bx)}} + \frac{12(d\cos(a+bx))^{3/2}\sin(a+bx)}{5bd^3} + \frac{2\sin^3(a+bx)}{bd\sqrt{d\cos(a+bx)}}$$

output
```
-24/5*(d*cos(b*x+a))^(1/2)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))/b/d^2/cos
(b*x+a)^(1/2)+12/5*(d*cos(b*x+a))^(3/2)*sin(b*x+a)/b/d^3+2*sin(b*x+a)^3/b/
d/(d*cos(b*x+a))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.60

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{5}{2},\frac{7}{2},\sin^2(a+bx)\right)\sin^5(a+bx)}{5bd\sqrt{d\cos(a+bx)}}$$

input
```
Integrate[Sin[a + b*x]^4/(d*Cos[a + b*x])^(3/2),x]
```

output
```
((Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[5/4, 5/2, 7/2, Sin[a + b*x]^2]*S
in[a + b*x]^5)/(5*b*d*Sqrt[d*Cos[a + b*x]])
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3046, 3042, 3048, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^4}{(d\cos(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3046}$$

$$\frac{2\sin^3(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{6\int\sqrt{d\cos(a+bx)}\sin^2(a+bx)dx}{d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin^3(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{6\int\sqrt{d\cos(a+bx)}\sin(a+bx)^2dx}{d^2}$$

$$\Big\downarrow \text{3048}$$

$$\frac{2\sin^3(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{6\left(\frac{2}{5}\int\sqrt{d\cos(a+bx)}dx-\frac{2\sin(a+bx)(d\cos(a+bx))^{3/2}}{5bd}\right)}{d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin^3(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{6\left(\frac{2}{5}\int\sqrt{d\sin\left(a+bx+\frac{\pi}{2}\right)}dx-\frac{2\sin(a+bx)(d\cos(a+bx))^{3/2}}{5bd}\right)}{d^2}$$

$$\Big\downarrow \text{3121}$$

$$\frac{2\sin^3(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{6\left(\frac{2\sqrt{d\cos(a+bx)}\int\sqrt{\cos(a+bx)}dx}{5\sqrt{\cos(a+bx)}}-\frac{2\sin(a+bx)(d\cos(a+bx))^{3/2}}{5bd}\right)}{d^2}$$

$\Big\downarrow$ 3042

$$\frac{2\sin^3(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{6\left(\frac{2\sqrt{d\cos(a+bx)}\int\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(a+bx)}}-\frac{2\sin(a+bx)(d\cos(a+bx))^{3/2}}{5bd}\right)}{d^2}$$

$\Big\downarrow$ 3119

$$\frac{2\sin^3(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{6\left(\frac{4E\left(\frac{1}{2}(a+bx)|2\right)\sqrt{d\cos(a+bx)}}{5b\sqrt{\cos(a+bx)}}-\frac{2\sin(a+bx)(d\cos(a+bx))^{3/2}}{5bd}\right)}{d^2}$$

input
```
Int[Sin[a + b*x]^4/(d*Cos[a + b*x])^(3/2),x]
```

output
```
(2*Sin[a + b*x]^3)/(b*d*Sqrt[d*Cos[a + b*x]]) - (6*((4*Sqrt[d*Cos[a + b*x]
]*EllipticE[(a + b*x)/2, 2])/(5*b*Sqrt[Cos[a + b*x]]) - (2*(d*Cos[a + b*x]
)^(3/2)*Sin[a + b*x])/(5*b*d)))/d^2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3046
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))    Int[(a*Sin[e + f
*x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(90) = 180$.

Time = 5.19 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.13

method	result
default	$\frac{8\sqrt{-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}d+\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}d}\left(2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{6}\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}+3\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-3\cdots\right.}{5d\sqrt{-d\left(2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}-\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)}\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{d\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\cdots\right.}}$

input
```
int(sin(b*x+a)^4/(d*cos(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
8/5/d*(-2*sin(1/2*b*x+1/2*a)^4*d+sin(1/2*b*x+1/2*a)^2*d)^(1/2)*(2*sin(1/2*
b*x+1/2*a)^6*cos(1/2*b*x+1/2*a)-2*cos(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)^4+
3*cos(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)^2-3*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(
2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2)))/(-d
*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(
d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.15

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx =$$
$$-\frac{2\left(12i\sqrt{\frac{1}{2}}\sqrt{d}\cos(bx+a)\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a\right.}{}$$

input
```
integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
-2/5*(12*I*sqrt(1/2)*sqrt(d)*cos(b*x + a)*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(b*x + a) + I*sin(b*x + a))) - 12*I*sqrt(1/2)*sqrt(d)
*cos(b*x + a)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x +
a) - I*sin(b*x + a))) - sqrt(d*cos(b*x + a))*(cos(b*x + a)^2 + 5)*sin(b*x
+ a))/(b*d^2*cos(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**4/(d*cos(b*x+a))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\sin(bx+a)^4}{(d\cos(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^4/(d*cos(b*x + a))^(3/2), x)
```

Giac [F]

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\sin(bx+a)^4}{(d\cos(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^4/(d*cos(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\sin(a+b\,x)^4}{(d\,\cos(a+b\,x))^{3/2}}\,dx$$

input
```
int(sin(a + b*x)^4/(d*cos(a + b*x))^(3/2),x)
```

output
```
int(sin(a + b*x)^4/(d*cos(a + b*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\cos(bx+a)}\,\sin(bx+a)^4}{\cos(bx+a)^2}dx\right)}{d^2}$$

input
```
int(sin(b*x+a)^4/(d*cos(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*int((sqrt(cos(a + b*x))*sin(a + b*x)**4)/cos(a + b*x)**2,x))/d**2
```

3.218 $\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 102

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = -\frac{8\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3bd^2\sqrt{d\cos(a+bx)}} + \frac{4\sqrt{d\cos(a+bx)}\sin(a+bx)}{3bd^3} + \frac{2\sin^3(a+bx)}{3bd(d\cos(a+bx))^{3/2}}$$

output
```
-8/3*cos(b*x+a)^(1/2)*InverseJacobiAM(1/2*a+1/2*b*x,2^(1/2))/b/d^2/(d*cos(
b*x+a))^(1/2)+4/3*(d*cos(b*x+a))^(1/2)*sin(b*x+a)/b/d^3+2/3*sin(b*x+a)^3/b
/d/(d*cos(b*x+a))^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.59

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \frac{\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{7}{4},\frac{5}{2},\frac{7}{2},\sin^2(a+bx)\right)\sin^5(a+bx)}{5bd(d\cos(a+bx))^{3/2}}$$

input
```
Integrate[Sin[a + b*x]^4/(d*Cos[a + b*x])^(5/2),x]
```

output
```
((Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[7/4, 5/2, 7/2, Sin[a + b*x]^2]*S
in[a + b*x]^5)/(5*b*d*(d*Cos[a + b*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3046, 3042, 3048, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^4}{(d\cos(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3046}$$

$$\frac{2\sin^3(a+bx)}{3bd(d\cos(a+bx))^{3/2}}-\frac{2\int \frac{\sin^2(a+bx)}{\sqrt{d\cos(a+bx)}}dx}{d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin^3(a+bx)}{3bd(d\cos(a+bx))^{3/2}}-\frac{2\int \frac{\sin(a+bx)^2}{\sqrt{d\cos(a+bx)}}dx}{d^2}$$

$$\Big\downarrow \text{3048}$$

$$\frac{2\sin^3(a+bx)}{3bd(d\cos(a+bx))^{3/2}}-\frac{2\left(\frac{2}{3}\int \frac{1}{\sqrt{d\cos(a+bx)}}dx-\frac{2\sin(a+bx)\sqrt{d\cos(a+bx)}}{3bd}\right)}{d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin^3(a+bx)}{3bd(d\cos(a+bx))^{3/2}}-\frac{2\left(\frac{2}{3}\int \frac{1}{\sqrt{d\sin\left(a+bx+\frac{\pi}{2}\right)}}dx-\frac{2\sin(a+bx)\sqrt{d\cos(a+bx)}}{3bd}\right)}{d^2}$$

$$\Big\downarrow \text{3121}$$

$$\frac{2\sin^3(a+bx)}{3bd(d\cos(a+bx))^{3/2}}-\frac{2\left(\frac{2\sqrt{\cos(a+bx)}\int\frac{1}{\sqrt{\cos(a+bx)}}dx}{3\sqrt{d\cos(a+bx)}}-\frac{2\sin(a+bx)\sqrt{d\cos(a+bx)}}{3bd}\right)}{d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin^3(a+bx)}{3bd(d\cos(a+bx))^{3/2}}-\frac{2\left(\frac{2\sqrt{\cos(a+bx)}\int\frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx}{3\sqrt{d\cos(a+bx)}}-\frac{2\sin(a+bx)\sqrt{d\cos(a+bx)}}{3bd}\right)}{d^2}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sin^3(a+bx)}{3bd(d\cos(a+bx))^{3/2}}-\frac{2\left(\frac{4\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b\sqrt{d\cos(a+bx)}}-\frac{2\sin(a+bx)\sqrt{d\cos(a+bx)}}{3bd}\right)}{d^2}$$

input
```
Int[Sin[a + b*x]^4/(d*Cos[a + b*x])^(5/2),x]
```

output
```
(2*Sin[a + b*x]^3)/(3*b*d*(d*Cos[a + b*x])^(3/2)) - (2*((4*Sqrt[Cos[a + b*
x]]*EllipticF[(a + b*x)/2, 2])/(3*b*Sqrt[d*Cos[a + b*x]]) - (2*Sqrt[d*Cos[
a + b*x]]*Sin[a + b*x])/(3*b*d)))/d^2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3046
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Sin[e + f
*x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(89) = 178$.

Time = 4.75 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.80

method	result
default	$-\frac{8\left(-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^6\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^4+2\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\sqrt{2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right.}{3d^2\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1\right)\sqrt{-d\left(2\sin\left(\frac{b}{2}\right.}}$

input
```
int(sin(b*x+a)^4/(d*cos(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-8/3*(-2*sin(1/2*b*x+1/2*a)^6*cos(1/2*b*x+1/2*a)+2*cos(1/2*b*x+1/2*a)*sin(
1/2*b*x+1/2*a)^4+2*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)
^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^2-cos(1/2*
b*x+1/2*a)*sin(1/2*b*x+1/2*a)^2-(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*
x+1/2*a)^2-1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2)))/d^2*(d*(2*cos(1
/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/(2*cos(1/2*b*x+1/2*a)^2-1)/
(-d*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a
)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.11

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = -\frac{2\left(-4i\sqrt{\frac{1}{2}}\sqrt{d}\cos(bx+a)^2\,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a))+4i\sqrt{\frac{1}{2}}\sqrt{d}\cos(bx\right.}{3\,bd}$$

input
```
integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(-4*I*sqrt(1/2)*sqrt(d)*cos(b*x + a)^2*weierstrassPInverse(-4, 0, cos
(b*x + a) + I*sin(b*x + a)) + 4*I*sqrt(1/2)*sqrt(d)*cos(b*x + a)^2*weierst
rassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a)) - sqrt(d*cos(b*x + a))*
(cos(b*x + a)^2 + 1)*sin(b*x + a))/(b*d^3*cos(b*x + a)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**4/(d*cos(b*x+a))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{\sin(bx+a)^4}{(d\cos(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^4/(d*cos(b*x + a))^(5/2), x)
```

Giac [F]

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{\sin(bx+a)^4}{(d\cos(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^4/(d*cos(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{\sin(a+b\,x)^4}{(d\,\cos(a+b\,x))^{5/2}}\,dx$$

input
```
int(sin(a + b*x)^4/(d*cos(a + b*x))^(5/2),x)
```

output
```
int(sin(a + b*x)^4/(d*cos(a + b*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\cos(bx+a)}\,\sin(bx+a)^4}{\cos(bx+a)^3}dx\right)}{d^3}$$

input
```
int(sin(b*x+a)^4/(d*cos(b*x+a))^(5/2),x)
```

output
```
(sqrt(d)*int((sqrt(cos(a + b*x))*sin(a + b*x)**4)/cos(a + b*x)**3,x))/d**3
```

3.219 $\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 102

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{24\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5bd^4\sqrt{\cos(a+bx)}} - \frac{12\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}} + \frac{2\sin^3(a+bx)}{5bd(d\cos(a+bx))^{5/2}}$$

output
```
24/5*(d*cos(b*x+a))^(1/2)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))/b/d^4/cos(
b*x+a)^(1/2)-12/5*sin(b*x+a)/b/d^3/(d*cos(b*x+a))^(1/2)+2/5*sin(b*x+a)^3/b
/d/(d*cos(b*x+a))^(5/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.64

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{\cos^3(a+bx)\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{9}{4},\frac{5}{2},\frac{7}{2},\sin^2(a+bx)\right)\sin^5(a+}{5b(d\cos(a+bx))^{7/2}}$$

input
```
Integrate[Sin[a + b*x]^4/(d*Cos[a + b*x])^(7/2),x]
```

output
```
(Cos[a + b*x]^3*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[9/4, 5/2, 7/2, Si
n[a + b*x]^2]*Sin[a + b*x]^5)/(5*b*(d*Cos[a + b*x])^(7/2))
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3042, 3046, 3042, 3046, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^4}{(d\cos(a+bx))^{7/2}}dx$$

$$\Big\downarrow \text{3046}$$

$$\frac{2\sin^3(a+bx)}{5bd(d\cos(a+bx))^{5/2}}-\frac{6\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{3/2}}dx}{5d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin^3(a+bx)}{5bd(d\cos(a+bx))^{5/2}}-\frac{6\int \frac{\sin(a+bx)^2}{(d\cos(a+bx))^{3/2}}dx}{5d^2}$$

$$\Big\downarrow \text{3046}$$

$$\frac{2\sin^3(a+bx)}{5bd(d\cos(a+bx))^{5/2}}-\frac{6\left(\frac{2\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{2\int \sqrt{d\cos(a+bx)}dx}{d^2}\right)}{5d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin^3(a+bx)}{5bd(d\cos(a+bx))^{5/2}}-\frac{6\left(\frac{2\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{2\int \sqrt{d\sin\left(a+bx+\frac{\pi}{2}\right)}dx}{d^2}\right)}{5d^2}$$

$$\Big\downarrow \text{3121}$$

$$\frac{2\sin^3(a+bx)}{5bd(d\cos(a+bx))^{5/2}}-\frac{6\left(\frac{2\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{2\sqrt{d\cos(a+bx)}\int\sqrt{\cos(a+bx)}dx}{d^2\sqrt{\cos(a+bx)}}\right)}{5d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin^3(a+bx)}{5bd(d\cos(a+bx))^{5/2}}-\frac{6\left(\frac{2\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{2\sqrt{d\cos(a+bx)}\int\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx}{d^2\sqrt{\cos(a+bx)}}\right)}{5d^2}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2\sin^3(a+bx)}{5bd(d\cos(a+bx))^{5/2}}-\frac{6\left(\frac{2\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{4E\left(\frac{1}{2}(a+bx)|2\right)\sqrt{d\cos(a+bx)}}{bd^2\sqrt{\cos(a+bx)}}\right)}{5d^2}$$

input
```
Int[Sin[a + b*x]^4/(d*Cos[a + b*x])^(7/2),x]
```

output
```
(2*Sin[a + b*x]^3)/(5*b*d*(d*Cos[a + b*x])^(5/2)) - (6*((-4*Sqrt[d*Cos[a +
 b*x]]*EllipticE[(a + b*x)/2, 2])/(b*d^2*Sqrt[Cos[a + b*x]]) + (2*Sin[a +
b*x])/(b*d*Sqrt[d*Cos[a + b*x]])))/(5*d^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3046
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Sin[e + f
*x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(90) = 180$.

Time = 5.14 (sec) , antiderivative size = 366, normalized size of antiderivative = 3.59

method	result
default	$8\sqrt{d\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\left(14\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{6}\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-12\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\sqrt{2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1}\,\mathrm{EllipticE}\left(\cos\left(\frac{bx}{2}\right.\right.\right.$

input
```
int(sin(b*x+a)^4/(d*cos(b*x+a))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
8/5*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/d^4/sin(1/2*
b*x+1/2*a)^3/(8*sin(1/2*b*x+1/2*a)^6-12*sin(1/2*b*x+1/2*a)^4+6*sin(1/2*b*x
+1/2*a)^2-1)*(14*sin(1/2*b*x+1/2*a)^6*cos(1/2*b*x+1/2*a)-12*(sin(1/2*b*x+1
/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*
a),2^(1/2))*sin(1/2*b*x+1/2*a)^4-14*cos(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)^
4+12*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*Ellipti
cE(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^2+3*cos(1/2*b*x+1/2*a)*s
in(1/2*b*x+1/2*a)^2-3*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2
-1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2)))*(-2*sin(1/2*b*x+1/2*a)^4*
d+sin(1/2*b*x+1/2*a)^2*d)^(1/2)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.18

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx =$$
$$-\frac{2\left(-12i\sqrt{\frac{1}{2}}\sqrt{d}\cos{(bx+a)}^3\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos{(bx+a)}+i\sin{(bx+$$

input
```
integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

output
```
-2/5*(-12*I*sqrt(1/2)*sqrt(d)*cos(b*x + a)^3*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(b*x + a) + I*sin(b*x + a))) + 12*I*sqrt(1/2)*sqrt
(d)*cos(b*x + a)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b
*x + a) - I*sin(b*x + a))) + sqrt(d*cos(b*x + a))*(7*cos(b*x + a)^2 - 1)*s
in(b*x + a))/(b*d^4*cos(b*x + a)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**4/(d*cos(b*x+a))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{\sin\left(bx+a\right)^4}{\left(d\cos\left(bx+a\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(7/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^4/(d*cos(b*x + a))^(7/2), x)
```

Giac [F]

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{\sin\left(bx+a\right)^4}{\left(d\cos\left(bx+a\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(7/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^4/(d*cos(b*x + a))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{\sin\left(a+b\,x\right)^4}{\left(d\,\cos\left(a+b\,x\right)\right)^{7/2}}\,dx$$

input
```
int(sin(a + b*x)^4/(d*cos(a + b*x))^(7/2),x)
```

output
```
int(sin(a + b*x)^4/(d*cos(a + b*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\cos(bx+a)}\,\sin(bx+a)^4}{\cos(bx+a)^4}dx\right)}{d^4}$$

input
```
int(sin(b*x+a)^4/(d*cos(b*x+a))^(7/2),x)
```

output
```
(sqrt(d)*int((sqrt(cos(a + b*x))*sin(a + b*x)**4)/cos(a + b*x)**4,x))/d**4
```

3.220 $\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 102

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \frac{8\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{7bd^4\sqrt{d\cos(a+bx)}} - \frac{4\sin(a+bx)}{7bd^3(d\cos(a+bx))^{3/2}} + \frac{2\sin^3(a+bx)}{7bd(d\cos(a+bx))^{7/2}}$$

output
```
8/7*cos(b*x+a)^(1/2)*InverseJacobiAM(1/2*a+1/2*b*x,2^(1/2))/b/d^4/(d*cos(b
*x+a))^(1/2)-4/7*sin(b*x+a)/b/d^3/(d*cos(b*x+a))^(3/2)+2/7*sin(b*x+a)^3/b/
d/(d*cos(b*x+a))^(7/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.64

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \frac{\cos^3(a+bx)\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{11}{4},\frac{7}{2},\sin^2(a+bx)\right)\sin^5(a+}{5b(d\cos(a+bx))^{9/2}}$$

input
```
Integrate[Sin[a + b*x]^4/(d*Cos[a + b*x])^(9/2),x]
```

output
```
(Cos[a + b*x]^3*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[5/2, 11/4, 7/2, S
in[a + b*x]^2]*Sin[a + b*x]^5)/(5*b*(d*Cos[a + b*x])^(9/2))
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3042, 3046, 3042, 3046, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^4}{(d\cos(a+bx))^{9/2}}dx$$

$$\Big\downarrow \text{3046}$$

$$\frac{2\sin^3(a+bx)}{7bd(d\cos(a+bx))^{7/2}}-\frac{6\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{5/2}}dx}{7d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin^3(a+bx)}{7bd(d\cos(a+bx))^{7/2}}-\frac{6\int \frac{\sin(a+bx)^2}{(d\cos(a+bx))^{5/2}}dx}{7d^2}$$

$$\Big\downarrow \text{3046}$$

$$\frac{2\sin^3(a+bx)}{7bd(d\cos(a+bx))^{7/2}}-\frac{6\left(\frac{2\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}-\frac{2\int \frac{1}{\sqrt{d\cos(a+bx)}}dx}{3d^2}\right)}{7d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin^3(a+bx)}{7bd(d\cos(a+bx))^{7/2}}-\frac{6\left(\frac{2\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}-\frac{2\int \frac{1}{\sqrt{d\sin\left(a+bx+\frac{\pi}{2}\right)}}dx}{3d^2}\right)}{7d^2}$$

$$\Big\downarrow \text{3121}$$

$$\frac{2\sin^3(a+bx)}{7bd(d\cos(a+bx))^{7/2}}-\frac{6\left(\frac{2\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}-\frac{2\sqrt{\cos(a+bx)}\int\frac{1}{\sqrt{\cos(a+bx)}}dx}{3d^2\sqrt{d\cos(a+bx)}}\right)}{7d^2}$$

$\Big\downarrow$ 3042

$$\frac{2\sin^3(a+bx)}{7bd(d\cos(a+bx))^{7/2}}-\frac{6\left(\frac{2\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}-\frac{2\sqrt{\cos(a+bx)}\int\frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx}{3d^2\sqrt{d\cos(a+bx)}}\right)}{7d^2}$$

$\Big\downarrow$ 3120

$$\frac{2\sin^3(a+bx)}{7bd(d\cos(a+bx))^{7/2}}-\frac{6\left(\frac{2\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}-\frac{4\sqrt{\cos(a+bx)}\operatorname{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3bd^2\sqrt{d\cos(a+bx)}}\right)}{7d^2}$$

input
```
Int[Sin[a + b*x]^4/(d*Cos[a + b*x])^(9/2),x]
```

output
```
(2*Sin[a + b*x]^3)/(7*b*d*(d*Cos[a + b*x])^(7/2)) - (6*((-4*Sqrt[Cos[a + b
*x]]*EllipticF[(a + b*x)/2, 2])/(3*b*d^2*Sqrt[d*Cos[a + b*x]]) + (2*Sin[a
+ b*x])/(3*b*d*(d*Cos[a + b*x])^(3/2))))/(7*d^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3046
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))    Int[(a*Sin[e + f
*x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 397 vs. 2(89) = 178.

Time = 5.42 (sec) , antiderivative size = 398, normalized size of antiderivative = 3.90

method	result
default	$\frac{8\left(8\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\sqrt{2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1}\,\text{EllipticF}\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^6-6\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^6\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-12\sqrt{\frac{1}{2}-\frac{\cos(bx+}{2}}\right.}{}$

input
```
int(sin(b*x+a)^4/(d*cos(b*x+a))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
8/7*(8*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*Ellip
ticF(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^6-6*sin(1/2*b*x+1/2*a)
^6*cos(1/2*b*x+1/2*a)-12*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a
)^2-1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^4+6*
cos(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)^4+6*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*
sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/
2*b*x+1/2*a)^2-cos(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)^2-(sin(1/2*b*x+1/2*a)
^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^
(1/2)))/d^4*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/(2*c
os(1/2*b*x+1/2*a)^2-1)^3/(-d*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2)
)^(1/2)/sin(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.12

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx =$$
$$-\frac{2\left(4i\sqrt{\frac{1}{2}}\sqrt{d}\cos(bx+a)^4 \text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)) - 4i\sqrt{\frac{1}{2}}\sqrt{d}\cos(bx+\right.}{7\,bd}$$

input
```
integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(9/2),x, algorithm="fricas")
```

output
```
-2/7*(4*I*sqrt(1/2)*sqrt(d)*cos(b*x + a)^4*weierstrassPInverse(-4, 0, cos(
b*x + a) + I*sin(b*x + a)) - 4*I*sqrt(1/2)*sqrt(d)*cos(b*x + a)^4*weierstr
assPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a)) + sqrt(d*cos(b*x + a))*(
3*cos(b*x + a)^2 - 1)*sin(b*x + a))/(b*d^5*cos(b*x + a)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**4/(d*cos(b*x+a))**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \int \frac{\sin\left(bx+a\right)^4}{\left(d\cos\left(bx+a\right)\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(9/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^4/(d*cos(b*x + a))^(9/2), x)
```

Giac [F]

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \int \frac{\sin\left(bx+a\right)^4}{\left(d\cos\left(bx+a\right)\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(9/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^4/(d*cos(b*x + a))^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \int \frac{\sin\left(a+b\,x\right)^4}{\left(d\,\cos\left(a+b\,x\right)\right)^{9/2}}\,dx$$

input
```
int(sin(a + b*x)^4/(d*cos(a + b*x))^(9/2),x)
```

output
```
int(sin(a + b*x)^4/(d*cos(a + b*x))^(9/2), x)
```

Reduce [F]

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\cos(bx+a)}\,\sin(bx+a)^4}{\cos(bx+a)^5}dx\right)}{d^5}$$

input
```
int(sin(b*x+a)^4/(d*cos(b*x+a))^(9/2),x)
```

output
```
(sqrt(d)*int((sqrt(cos(a + b*x))*sin(a + b*x)**4)/cos(a + b*x)**5,x))/d**5
```

3.221 $\int \cos^{\frac{3}{2}}(a+bx)\sin^5(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 52

$$\int \cos^{\frac{3}{2}}(a+bx)\sin^5(a+bx)\,dx=-\frac{2\cos^{\frac{5}{2}}(a+bx)}{5b}+\frac{4\cos^{\frac{9}{2}}(a+bx)}{9b}-\frac{2\cos^{\frac{13}{2}}(a+bx)}{13b}$$

output
```
-2/5*cos(b*x+a)^(5/2)/b+4/9*cos(b*x+a)^(9/2)/b-2/13*cos(b*x+a)^(13/2)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 111 vs. 2(52) = 104.

Time = 0.31 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.13

$$\int \cos^{\frac{3}{2}}(a+bx)\sin^5(a+bx)\,dx$$
$$=\frac{2\sqrt{\cos(a+bx)}\left(32-32\sqrt[4]{\cos^2(a+bx)}-8\sqrt[4]{\cos^2(a+bx)}\sin^2(a+bx)-5\sqrt[4]{\cos^2(a+bx)}\sin^4(a+bx)\right.}{585b\sqrt[4]{\cos^2(a+bx)}}$$

input
```
Integrate[Cos[a + b*x]^(3/2)*Sin[a + b*x]^5,x]
```

output
```
(2*Sqrt[Cos[a + b*x]]*(32 - 32*(Cos[a + b*x]^2)^(1/4) - 8*(Cos[a + b*x]^2)
^(1/4)*Sin[a + b*x]^2 - 5*(Cos[a + b*x]^2)^(1/4)*Sin[a + b*x]^4 + 45*(Cos[
a + b*x]^2)^(1/4)*Sin[a + b*x]^6))/(585*b*(Cos[a + b*x]^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3045, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(a+bx)\cos^{\frac{3}{2}}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^5\cos(a+bx)^{3/2}dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int \cos^{\frac{3}{2}}(a+bx)\left(1-\cos^2(a+bx)\right)^2d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(\cos^{\frac{11}{2}}(a+bx)-2\cos^{\frac{7}{2}}(a+bx)+\cos^{\frac{3}{2}}(a+bx)\right)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{2}{13}\cos^{\frac{13}{2}}(a+bx)-\frac{4}{9}\cos^{\frac{9}{2}}(a+bx)+\frac{2}{5}\cos^{\frac{5}{2}}(a+bx)}{b}$$

input
```
Int[Cos[a + b*x]^(3/2)*Sin[a + b*x]^5,x]
```

output
```
-(((2*Cos[a + b*x]^(5/2))/5 - (4*Cos[a + b*x]^(9/2))/9 + (2*Cos[a + b*x]^(
13/2))/13)/b)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$-\frac{\frac{2 \cos(bx+a)^{\frac{13}{2}}}{13}-\frac{4 \cos(bx+a)^{\frac{9}{2}}}{9}+\frac{2 \cos(bx+a)^{\frac{5}{2}}}{5}}{b}$	37
default	$-\frac{\frac{2 \cos(bx+a)^{\frac{13}{2}}}{13}-\frac{4 \cos(bx+a)^{\frac{9}{2}}}{9}+\frac{2 \cos(bx+a)^{\frac{5}{2}}}{5}}{b}$	37

input
```
int(cos(b*x+a)^(3/2)*sin(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/b*(2/13*cos(b*x+a)^(13/2)-4/9*cos(b*x+a)^(9/2)+2/5*cos(b*x+a)^(5/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \cos^{\frac{3}{2}}(a+bx)\sin^5(a+bx)\,dx = -\frac{2\left(45\cos(bx+a)^6-130\cos(bx+a)^4+117\cos(bx+a)^2\right)\sqrt{\cos(bx+a)}}{585\,b}$$

input
```
integrate(cos(b*x+a)^(3/2)*sin(b*x+a)^5,x, algorithm="fricas")
```

output
```
-2/585*(45*cos(b*x + a)^6 - 130*cos(b*x + a)^4 + 117*cos(b*x + a)^2)*sqrt(
cos(b*x + a))/b
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(a+bx)\sin^5(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**(3/2)*sin(b*x+a)**5,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.69

$$\int \cos^{\frac{3}{2}}(a+bx)\sin^5(a+bx)\,dx = -\frac{2\left(45\cos(bx+a)^{\frac{13}{2}}-130\cos(bx+a)^{\frac{9}{2}}+117\cos(bx+a)^{\frac{5}{2}}\right)}{585\,b}$$

input
```
integrate(cos(b*x+a)^(3/2)*sin(b*x+a)^5,x, algorithm="maxima")
```

output
```
-2/585*(45*cos(b*x + a)^(13/2) - 130*cos(b*x + a)^(9/2) + 117*cos(b*x + a)
^(5/2))/b
```

Giac [F]

$$\int \cos^{\frac{3}{2}}(a+bx)\sin^5(a+bx)\,dx = \int \cos(bx+a)^{\frac{3}{2}}\sin(bx+a)^5\,dx$$

input
```
integrate(cos(b*x+a)^(3/2)*sin(b*x+a)^5,x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^(3/2)*sin(b*x + a)^5, x)
```

Mupad [B] (verification not implemented)

Time = 25.64 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.67

$$\int \cos^{\frac{3}{2}}(a+bx)\sin^5(a+bx)\,dx = -\frac{2\cos(a+b\,x)^{5/2}\left(\frac{5\cos(a+b\,x)^4}{13}-\frac{10\cos(a+b\,x)^2}{9}+1\right)}{5\,b}$$

input
```
int(cos(a + b*x)^(3/2)*sin(a + b*x)^5,x)
```

output
```
-(2*cos(a + b*x)^(5/2)*((5*cos(a + b*x)^4)/13 - (10*cos(a + b*x)^2)/9 + 1)
)/(5*b)
```

Reduce [F]

$$\int \cos^{\frac{3}{2}}(a+bx)\sin^5(a+bx)\,dx = \int \sqrt{\cos(bx+a)}\,\cos(bx+a)\sin(bx+a)^5\,dx$$

input
```
int(cos(b*x+a)^(3/2)*sin(b*x+a)^5,x)
```

output
```
int(sqrt(cos(a + b*x))*cos(a + b*x)*sin(a + b*x)**5,x)
```

3.222 $\int (d\cos(a+bx))^{9/2}\csc(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 100

$$\int (d\cos(a+bx))^{9/2}\csc(a+bx)\,dx = \frac{d^{9/2}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} - \frac{d^{9/2}\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} + \frac{2d^3(d\cos(a+bx))^{3/2}}{3b} + \frac{2d(d\cos(a+bx))^{7/2}}{7b}$$

output
```
d^(9/2)*arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b-d^(9/2)*arctanh((d*cos(b*x+
a))^(1/2)/d^(1/2))/b+2/3*d^3*(d*cos(b*x+a))^(3/2)/b+2/7*d*(d*cos(b*x+a))^(
7/2)/b
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.83

$$\int (d\cos(a+bx))^{9/2}\csc(a+bx)\,dx = \frac{d^4\sqrt{d\cos(a+bx)}\left(21\arctan\left(\sqrt{\cos(a+bx)}\right) - 21\text{arctanh}\left(\sqrt{\cos(a+bx)}\right) + 2\cos^{\frac{3}{2}}(a+bx)\,(7$$

$$21b\sqrt{\cos(a+bx)}$$

input
```
Integrate[(d*Cos[a + b*x])^(9/2)*Csc[a + b*x],x]
```

output
```
(d^4*Sqrt[d*Cos[a + b*x]]*(21*ArcTan[Sqrt[Cos[a + b*x]]] - 21*ArcTanh[Sqrt
[Cos[a + b*x]]] + 2*Cos[a + b*x]^(3/2)*(7 + 3*Cos[a + b*x]^2)))/(21*b*Sqrt
[Cos[a + b*x]])
```

Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3042, 3045, 27, 262, 262, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)(d\cos(a+bx))^{9/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(d\cos(a+bx))^{9/2}}{\sin(a+bx)}dx$$

$$\Big\downarrow \text{ 3045}$$

$$-\frac{\int \frac{d^2(d\cos(a+bx))^{9/2}}{d^2-d^2\cos^2(a+bx)}d(d\cos(a+bx))}{bd}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{d\int \frac{(d\cos(a+bx))^{9/2}}{d^2-d^2\cos^2(a+bx)}d(d\cos(a+bx))}{b}$$

$$\Big\downarrow \text{ 262}$$

$$-\frac{d\left(d^2\int \frac{(d\cos(a+bx))^{5/2}}{d^2-d^2\cos^2(a+bx)}d(d\cos(a+bx))-\frac{2}{7}(d\cos(a+bx))^{7/2}\right)}{b}$$

$$\Big\downarrow \text{ 262}$$

$$-\frac{d\left(d^2\left(d^2\int \frac{\sqrt{d\cos(a+bx)}}{d^2-d^2\cos^2(a+bx)}d(d\cos(a+bx))-\frac{2}{3}(d\cos(a+bx))^{3/2}\right)-\frac{2}{7}(d\cos(a+bx))^{7/2}\right)}{b}$$

$$\Big\downarrow \text{ 266}$$

$$-\frac{d\left(d^2\left(2d^2\int \frac{d^2\cos^2(a+bx)}{d^2-d^4\cos^4(a+bx)}d\sqrt{d\cos(a+bx)}-\frac{2}{3}(d\cos(a+bx))^{3/2}\right)-\frac{2}{7}(d\cos(a+bx))^{7/2}\right)}{b}$$

$\downarrow$ 827

$$-\frac{d\left(d^2\left(2d^2\left(\frac{1}{2}\int \frac{1}{d-d^2\cos^2(a+bx)}d\sqrt{d\cos(a+bx)}-\frac{1}{2}\int \frac{1}{d^2\cos^2(a+bx)+d}d\sqrt{d\cos(a+bx)}\right)-\frac{2}{3}(d\cos(a+bx))^{3/2}\right)-\right.}{b}$$

$\downarrow$ 216

$$-\frac{d\left(d^2\left(2d^2\left(\frac{1}{2}\int \frac{1}{d-d^2\cos^2(a+bx)}d\sqrt{d\cos(a+bx)}-\frac{\arctan\left(\sqrt{d}\cos(a+bx)\right)}{2\sqrt{d}}\right)-\frac{2}{3}(d\cos(a+bx))^{3/2}\right)-\frac{2}{7}(d\cos(a+b\right.}{b}$$

$\downarrow$ 219

$$-\frac{d\left(d^2\left(2d^2\left(\frac{\text{arctanh}\left(\sqrt{d}\cos(a+bx)\right)}{2\sqrt{d}}-\frac{\arctan\left(\sqrt{d}\cos(a+bx)\right)}{2\sqrt{d}}\right)-\frac{2}{3}(d\cos(a+bx))^{3/2}\right)-\frac{2}{7}(d\cos(a+bx))^{7/2}\right)}{b}$$

input

```
Int[(d*Cos[a + b*x])^(9/2)*Csc[a + b*x],x]
```

output

```
-((d*((-2*(d*Cos[a + b*x])^(7/2))/7 + d^2*(2*d^2*(-1/2*ArcTan[Sqrt[d]*Cos[
a + b*x]]/Sqrt[d] + ArcTanh[Sqrt[d]*Cos[a + b*x]]/(2*Sqrt[d])) - (2*(d*Cos
[a + b*x])^(3/2))/3)))/b)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(80) = 160$.

Time = 3.41 (sec) , antiderivative size = 331, normalized size of antiderivative = 3.31

method	result
default	$-\frac{96\sqrt{-d}\,\sqrt{-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}d+d}\,\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{6}d^{4}+21d^{\frac{9}{2}}\ln\left(-\frac{2\left(2d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-\sqrt{d}\,\sqrt{-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}d+d}+d\right)}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\right)\sqrt{-d}+21d^{\frac{9}{2}}\ln\left(\right.}{}$

input
```
int((d*cos(b*x+a))^(9/2)*csc(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/42/(-d)^(1/2)*(96*(-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*sin(1/
2*b*x+1/2*a)^6*d^4+21*d^(9/2)*ln(-2/(cos(1/2*b*x+1/2*a)+1)*(2*d*cos(1/2*b*
x+1/2*a)-d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+d))*(-d)^(1/2)+21*d^(
9/2)*ln(2/(cos(1/2*b*x+1/2*a)-1)*(2*d*cos(1/2*b*x+1/2*a)+d^(1/2)*(-2*sin(1
/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*(-d)^(1/2)-144*(-d)^(1/2)*(-2*sin(1/2*b*x+1
/2*a)^2*d+d)^(1/2)*sin(1/2*b*x+1/2*a)^4*d^4+128*(-d)^(1/2)*(-2*sin(1/2*b*x
+1/2*a)^2*d+d)^(1/2)*sin(1/2*b*x+1/2*a)^2*d^4-40*d^4*(-2*sin(1/2*b*x+1/2*a
)^2*d+d)^(1/2)*(-d)^(1/2)+42*d^5*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*s
in(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d)))/b
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 322, normalized size of antiderivative = 3.22

$$\int (d\cos(a+bx))^{9/2}\csc(a+bx)\,dx = \left[\frac{42\sqrt{-d}d^{4}\arctan\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)+1)}{2\,d\cos(bx+a)}\right)+21\sqrt{-d}d^{4}\log\left(-\frac{d\cos(bx+a)^{2}+4\sqrt{d\cos(bx+a)}\sqrt{-d}(\cdots}{\cos(bx+a)^{2}+2\cos\cdots}\right.}{84\,b}\right.$$

input
```
integrate((d*cos(b*x+a))^(9/2)*csc(b*x+a),x, algorithm="fricas")
```

output
```
[1/84*(42*sqrt(-d)*d^4*arctan(1/2*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x +
 a) + 1)/(d*cos(b*x + a))) + 21*sqrt(-d)*d^4*log(-(d*cos(b*x + a)^2 + 4*sq
rt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) - 1) - 6*d*cos(b*x + a) + d)/(co
s(b*x + a)^2 + 2*cos(b*x + a) + 1)) + 8*(3*d^4*cos(b*x + a)^3 + 7*d^4*cos(
b*x + a))*sqrt(d*cos(b*x + a)))/b, 1/84*(42*d^(9/2)*arctan(1/2*sqrt(d*cos(
b*x + a))*(cos(b*x + a) - 1)/(sqrt(d)*cos(b*x + a))) + 21*d^(9/2)*log(-(d*
cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*c
os(b*x + a) + d)/(cos(b*x + a)^2 - 2*cos(b*x + a) + 1)) + 8*(3*d^4*cos(b*x
 + a)^3 + 7*d^4*cos(b*x + a))*sqrt(d*cos(b*x + a)))/b]
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{9/2}\csc(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*cos(b*x+a))**(9/2)*csc(b*x+a),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.98

$$\int (d\cos(a+bx))^{9/2}\csc(a+bx)\,dx = \frac{42\,d^{\frac{11}{2}}\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)+21\,d^{\frac{11}{2}}\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)+12\,(d\cos(bx+a))^{\frac{7}{2}}\,d^2+28\,(d\cos(b}{42\,bd}$$

input
```
integrate((d*cos(b*x+a))^(9/2)*csc(b*x+a),x, algorithm="maxima")
```

output
```
1/42*(42*d^(11/2)*arctan(sqrt(d*cos(b*x + a))/sqrt(d)) + 21*d^(11/2)*log((
sqrt(d*cos(b*x + a)) - sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d))) + 12*(d*
cos(b*x + a))^(7/2)*d^2 + 28*(d*cos(b*x + a))^(3/2)*d^4)/(b*d)
```

Giac [F]

$$\int (d\cos(a+bx))^{9/2}\csc(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{9}{2}}\csc(bx+a)\,dx$$

input
```
integrate((d*cos(b*x+a))^(9/2)*csc(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*cos(b*x + a))^(9/2)*csc(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{9/2}\csc(a+bx)\,dx = \int \frac{(d\cos(a+bx))^{9/2}}{\sin(a+bx)}\,dx$$

input
```
int((d*cos(a + b*x))^(9/2)/sin(a + b*x),x)
```

output
```
int((d*cos(a + b*x))^(9/2)/sin(a + b*x), x)
```

Reduce [F]

$$\int (d\cos(a+bx))^{9/2}\csc(a+bx)\,dx = \sqrt{d}\left(\int \sqrt{\cos(bx+a)}\cos(bx+a)^4\csc(bx+a)\,dx\right)d^4$$

input
```
int((d*cos(b*x+a))^(9/2)*csc(b*x+a),x)
```

output
```
sqrt(d)*int(sqrt(cos(a + b*x))*cos(a + b*x)**4*csc(a + b*x),x)*d**4
```

3.223 $\int (d\cos(a+bx))^{7/2}\csc(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 99

$$\int (d\cos(a+bx))^{7/2}\csc(a+bx)\,dx = -\frac{d^{7/2}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} - \frac{d^{7/2}\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} + \frac{2d^3\sqrt{d\cos(a+bx)}}{b} + \frac{2d(d\cos(a+bx))^{5/2}}{5b}$$

output
```
-d^(7/2)*arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b-d^(7/2)*arctanh((d*cos(b*x
+a))^(1/2)/d^(1/2))/b+2*d^3*(d*cos(b*x+a))^(1/2)/b+2/5*d*(d*cos(b*x+a))^(5
/2)/b
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.81

$$\int (d\cos(a+bx))^{7/2}\csc(a+bx)\,dx = \frac{d^3\sqrt{d\cos(a+bx)}\left(-5\arctan\left(\sqrt{\cos(a+bx)}\right) - 5\text{arctanh}\left(\sqrt{\cos(a+bx)}\right) + \sqrt{\cos(a+bx)}(11\right.}{5b\sqrt{\cos(a+bx)}}$$

input
```
Integrate[(d*Cos[a + b*x])^(7/2)*Csc[a + b*x],x]
```

output
```
(d^3*Sqrt[d*Cos[a + b*x]]*(-5*ArcTan[Sqrt[Cos[a + b*x]]] - 5*ArcTanh[Sqrt[
Cos[a + b*x]]] + Sqrt[Cos[a + b*x]]*(11 + Cos[2*(a + b*x)])))/(5*b*Sqrt[Co
s[a + b*x]])
```

Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3042, 3045, 27, 262, 262, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)(d\cos(a+bx))^{7/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\cos(a+bx))^{7/2}}{\sin(a+bx)}dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int \frac{d^2(d\cos(a+bx))^{7/2}}{d^2-d^2\cos^2(a+bx)}d(d\cos(a+bx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$-\frac{d\int \frac{(d\cos(a+bx))^{7/2}}{d^2-d^2\cos^2(a+bx)}d(d\cos(a+bx))}{b}$$

$$\Big\downarrow \text{262}$$

$$-\frac{d\left(d^2\int \frac{(d\cos(a+bx))^{3/2}}{d^2-d^2\cos^2(a+bx)}d(d\cos(a+bx))-\frac{2}{5}(d\cos(a+bx))^{5/2}\right)}{b}$$

$$\Big\downarrow \text{262}$$

$$-\frac{d\left(d^2\left(d^2\int \frac{1}{\sqrt{d\cos(a+bx)}(d^2-d^2\cos^2(a+bx))}d(d\cos(a+bx))-2\sqrt{d\cos(a+bx)}\right)-\frac{2}{5}(d\cos(a+bx))^{5/2}\right)}{b}$$

$$\Big\downarrow \text{266}$$

$$-\frac{d\left(d^2\left(2d^2\int \frac{1}{d^2-d^4\cos^4(a+bx)}d\sqrt{d\cos(a+bx)}-2\sqrt{d\cos(a+bx)}\right)-\frac{2}{5}(d\cos(a+bx))^{5/2}\right)}{b}$$

$\Big\downarrow$ 756

$$-\frac{d\left(d^2\left(2d^2\left(\frac{\int \frac{1}{d-d^2\cos^2(a+bx)}d\sqrt{d\cos(a+bx)}}{2d}+\frac{\int \frac{1}{d^2\cos^2(a+bx)+d}d\sqrt{d\cos(a+bx)}}{2d}\right)-2\sqrt{d\cos(a+bx)}\right)-\frac{2}{5}(d\cos(a+bx))\right.}{b}$$

$\Big\downarrow$ 216

$$-\frac{d\left(d^2\left(2d^2\left(\frac{\int \frac{1}{d-d^2\cos^2(a+bx)}d\sqrt{d\cos(a+bx)}}{2d}+\frac{\arctan\left(\sqrt{d}\cos(a+bx)\right)}{2d^{3/2}}\right)-2\sqrt{d\cos(a+bx)}\right)-\frac{2}{5}(d\cos(a+bx))^{5/2}\right)}{b}$$

$\Big\downarrow$ 219

$$-\frac{d\left(d^2\left(2d^2\left(\frac{\arctan\left(\sqrt{d}\cos(a+bx)\right)}{2d^{3/2}}+\frac{\operatorname{arctanh}\left(\sqrt{d}\cos(a+bx)\right)}{2d^{3/2}}\right)-2\sqrt{d\cos(a+bx)}\right)-\frac{2}{5}(d\cos(a+bx))^{5/2}\right)}{b}$$

input
```
Int[(d*Cos[a + b*x])^(7/2)*Csc[a + b*x],x]
```

output
```
-((d*((-2*(d*Cos[a + b*x])^(5/2))/5 + d^2*(2*d^2*(ArcTan[Sqrt[d]*Cos[a + b
*x]]/(2*d^(3/2)) + ArcTanh[Sqrt[d]*Cos[a + b*x]]/(2*d^(3/2))) - 2*Sqrt[d*C
os[a + b*x]])))/b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. 2(81) = 162.

Time = 3.32 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.95

method	result
default	$\frac{16\sqrt{-d}\,\sqrt{-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}d+d}\,\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}d^{3}-5d^{\frac{7}{2}}\ln\left(-\frac{2\left(2d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-\sqrt{d}\,\sqrt{-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}d+d}+d\right)}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\right)\sqrt{-d}-5d^{\frac{7}{2}}\ln\left(\frac{4d\cos(}{}\right.}{}$

input
```
int((d*cos(b*x+a))^(7/2)*csc(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/10*(16*(-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*sin(1/2*b*x+1/2*a)
^4*d^3-5*d^(7/2)*ln(-2/(cos(1/2*b*x+1/2*a)+1)*(2*d*cos(1/2*b*x+1/2*a)-d^(1
/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+d))*(-d)^(1/2)-5*d^(7/2)*ln(2/(cos
(1/2*b*x+1/2*a)-1)*(2*d*cos(1/2*b*x+1/2*a)+d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^
2*d+d)^(1/2)-d))*(-d)^(1/2)-16*(-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1
/2)*sin(1/2*b*x+1/2*a)^2*d^3+24*d^3*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*(-
d)^(1/2)+10*d^4*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)
^2*d+d)^(1/2)-d)))/(-d)^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 308, normalized size of antiderivative = 3.11

$$\int (d\cos(a+bx))^{7/2}\csc(a+bx)\,dx = \left[\frac{10\sqrt{-d}d^{3}\arctan\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)+1)}{2\,d\cos(bx+a)}\right)+5\sqrt{-d}d^{3}\log\left(-\frac{d\cos(bx+a)^{2}-4\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos}{\cos(bx+a)^{2}+2\cos(}\right.}{20\,b}\right.$$

$$-\frac{10\,d^{\frac{7}{2}}\arctan\left(\frac{\sqrt{d\cos(bx+a)}(\cos(bx+a)-1)}{2\sqrt{d}\cos(bx+a)}\right)-5\,d^{\frac{7}{2}}\log\left(-\frac{d\cos(bx+a)^{2}-4\sqrt{d\cos(bx+a)}\sqrt{d}(\cos(bx+a)+1)+6\,d\cos(bx+a)+d}{\cos(bx+a)^{2}-2\cos(bx+a)+1}\right)}{20\,b}$$

input
```
integrate((d*cos(b*x+a))^(7/2)*csc(b*x+a),x, algorithm="fricas")
```

output
```
[1/20*(10*sqrt(-d)*d^3*arctan(1/2*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x +
 a) + 1)/(d*cos(b*x + a))) + 5*sqrt(-d)*d^3*log(-(d*cos(b*x + a)^2 - 4*sqr
t(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) - 1) - 6*d*cos(b*x + a) + d)/(cos
(b*x + a)^2 + 2*cos(b*x + a) + 1)) + 8*(d^3*cos(b*x + a)^2 + 5*d^3)*sqrt(d
*cos(b*x + a)))/b, -1/20*(10*d^(7/2)*arctan(1/2*sqrt(d*cos(b*x + a))*(cos(
b*x + a) - 1)/(sqrt(d)*cos(b*x + a))) - 5*d^(7/2)*log(-(d*cos(b*x + a)^2 -
 4*sqrt(d*cos(b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)
/(cos(b*x + a)^2 - 2*cos(b*x + a) + 1)) - 8*(d^3*cos(b*x + a)^2 + 5*d^3)*s
qrt(d*cos(b*x + a)))/b]
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{7/2}\csc(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*cos(b*x+a))**(7/2)*csc(b*x+a),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.99

$$\int (d\cos(a+bx))^{7/2}\csc(a+bx)\,dx = -\frac{10\,d^{\frac{9}{2}}\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)-5\,d^{\frac{9}{2}}\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)-4\,(d\cos(bx+a))^{\frac{5}{2}}\,d^2-20\,\sqrt{d\cos(bx+a)}d^4}{10\,bd}$$

input
```
integrate((d*cos(b*x+a))^(7/2)*csc(b*x+a),x, algorithm="maxima")
```

output
```
-1/10*(10*d^(9/2)*arctan(sqrt(d*cos(b*x + a))/sqrt(d)) - 5*d^(9/2)*log((sq
rt(d*cos(b*x + a)) - sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d))) - 4*(d*cos
(b*x + a))^(5/2)*d^2 - 20*sqrt(d*cos(b*x + a))*d^4)/(b*d)
```

Giac [F]

$$\int (d\cos(a+bx))^{7/2}\csc(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{7}{2}}\csc(bx+a)\;dx$$

input
```
integrate((d*cos(b*x+a))^(7/2)*csc(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*cos(b*x + a))^(7/2)*csc(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{7/2}\csc(a+bx)\,dx = \int \frac{(d\cos(a+b\,x))^{7/2}}{\sin(a+b\,x)}\,dx$$

input
```
int((d*cos(a + b*x))^(7/2)/sin(a + b*x),x)
```

output
```
int((d*cos(a + b*x))^(7/2)/sin(a + b*x), x)
```

Reduce [F]

$$\int (d\cos(a+bx))^{7/2}\csc(a+bx)\,dx = \sqrt{d}\left(\int \sqrt{\cos(bx+a)}\,\cos(bx+a)^3\csc(bx+a)\,dx\right)d^3$$

input
```
int((d*cos(b*x+a))^(7/2)*csc(b*x+a),x)
```

output
```
sqrt(d)*int(sqrt(cos(a + b*x))*cos(a + b*x)**3*csc(a + b*x),x)*d**3
```

3.224 $\int (d\cos(a+bx))^{5/2}\csc(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 78

$$\int (d\cos(a+bx))^{5/2}\csc(a+bx)\,dx=\frac{d^{5/2}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}-\frac{d^{5/2}\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}+\frac{2d(d\cos(a+bx))^{3/2}}{3b}$$

output
```
d^(5/2)*arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b-d^(5/2)*arctanh((d*cos(b*x+
a))^(1/2)/d^(1/2))/b+2/3*d*(d*cos(b*x+a))^(3/2)/b
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.87

$$\int (d\cos(a+bx))^{5/2}\csc(a+bx)\,dx=\frac{(d\cos(a+bx))^{5/2}\left(3\arctan\left(\sqrt{\cos(a+bx)}\right)-3\text{arctanh}\left(\sqrt{\cos(a+bx)}\right)+2\cos^{\frac{3}{2}}(a+bx)\right)}{3b\cos^{\frac{5}{2}}(a+bx)}$$

input
```
Integrate[(d*Cos[a + b*x])^(5/2)*Csc[a + b*x],x]
```

output
```
((d*Cos[a + b*x])^(5/2)*(3*ArcTan[Sqrt[Cos[a + b*x]]] - 3*ArcTanh[Sqrt[Cos
[a + b*x]]] + 2*Cos[a + b*x]^(3/2)))/(3*b*Cos[a + b*x]^(5/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 3045, 27, 262, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)(d\cos(a+bx))^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\cos(a+bx))^{5/2}}{\sin(a+bx)}dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int \frac{d^2(d\cos(a+bx))^{5/2}}{d^2-d^2\cos^2(a+bx)}d(d\cos(a+bx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$-\frac{d\int \frac{(d\cos(a+bx))^{5/2}}{d^2-d^2\cos^2(a+bx)}d(d\cos(a+bx))}{b}$$

$$\Big\downarrow \text{262}$$

$$-\frac{d\left(d^2\int \frac{\sqrt{d\cos(a+bx)}}{d^2-d^2\cos^2(a+bx)}d(d\cos(a+bx))-\frac{2}{3}(d\cos(a+bx))^{3/2}\right)}{b}$$

$$\Big\downarrow \text{266}$$

$$-\frac{d\left(2d^2\int \frac{d^2\cos^2(a+bx)}{d^2-d^4\cos^4(a+bx)}d\sqrt{d\cos(a+bx)}-\frac{2}{3}(d\cos(a+bx))^{3/2}\right)}{b}$$

$$\Big\downarrow \text{827}$$

$$-\frac{d\left(2d^2\left(\frac{1}{2}\int \frac{1}{d-d^2\cos^2(a+bx)}d\sqrt{d\cos(a+bx)}-\frac{1}{2}\int \frac{1}{d^2\cos^2(a+bx)+d}d\sqrt{d\cos(a+bx)}\right)-\frac{2}{3}(d\cos(a+bx))^{3/2}\right)}{b}$$

$$\Big\downarrow 216$$

$$-\frac{d\left(2d^2\left(\frac{1}{2}\int\frac{1}{d-d^2\cos^2(a+bx)}d\sqrt{d\cos(a+bx)}-\frac{\arctan\left(\sqrt{d}\cos(a+bx)\right)}{2\sqrt{d}}\right)-\frac{2}{3}(d\cos(a+bx))^{3/2}\right)}{b}$$

$$\Big\downarrow 219$$

$$-\frac{d\left(2d^2\left(\frac{\text{arctanh}\left(\sqrt{d}\cos(a+bx)\right)}{2\sqrt{d}}-\frac{\arctan\left(\sqrt{d}\cos(a+bx)\right)}{2\sqrt{d}}\right)-\frac{2}{3}(d\cos(a+bx))^{3/2}\right)}{b}$$

input
```
Int[(d*Cos[a + b*x])^(5/2)*Csc[a + b*x],x]
```

output
```
-((d*(2*d^2*(-1/2*ArcTan[Sqrt[d]*Cos[a + b*x]]/Sqrt[d] + ArcTanh[Sqrt[d]*C
os[a + b*x]]/(2*Sqrt[d])) - (2*(d*Cos[a + b*x])^(3/2))/3))/b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(62) = 124$.

Time = 3.37 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.24

method	result
default	$-\frac{3d^{\frac{5}{2}}\ln\left(\frac{4d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+2\sqrt{d}\,\sqrt{-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}d+d}-2d}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-1}\right)\sqrt{-d}+3d^{\frac{5}{2}}\ln\left(-\frac{2\left(2d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-\sqrt{d}\,\sqrt{-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}d+d}+d\right)}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\right)\sqrt{-d}+}{6\sqrt{-d}}$

input
```
int((d*cos(b*x+a))^(5/2)*csc(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/6/(-d)^(1/2)*(3*d^(5/2)*ln(2/(cos(1/2*b*x+1/2*a)-1)*(2*d*cos(1/2*b*x+1/
2*a)+d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*(-d)^(1/2)+3*d^(5/2)*
ln(-2/(cos(1/2*b*x+1/2*a)+1)*(2*d*cos(1/2*b*x+1/2*a)-d^(1/2)*(-2*sin(1/2*b
*x+1/2*a)^2*d+d)^(1/2)+d))*(-d)^(1/2)+8*(-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^
2*d+d)^(1/2)*sin(1/2*b*x+1/2*a)^2*d^2-4*d^2*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^
(1/2)*(-d)^(1/2)+6*d^3*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x
+1/2*a)^2*d+d)^(1/2)-d)))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(62) = 124$.

Time = 0.19 (sec) , antiderivative size = 290, normalized size of antiderivative = 3.72

$$\int (d\cos(a+bx))^{5/2}\csc(a+bx)\,dx = \left[\frac{6\sqrt{-d}d^2\arctan\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)+1)}{2\,d\cos(bx+a)}\right)+8\sqrt{d\cos(bx+a)}d^2\cos(bx+a)+3\sqrt{-d}d^2\log}{12\,b}\right.$$

input
```
integrate((d*cos(b*x+a))^(5/2)*csc(b*x+a),x, algorithm="fricas")
```

output
```
[1/12*(6*sqrt(-d)*d^2*arctan(1/2*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x +
a) + 1)/(d*cos(b*x + a))) + 8*sqrt(d*cos(b*x + a))*d^2*cos(b*x + a) + 3*sq
rt(-d)*d^2*log(-(d*cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b
*x + a) - 1) - 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a) + 1)
))/b, 1/12*(6*d^(5/2)*arctan(1/2*sqrt(d*cos(b*x + a))*(cos(b*x + a) - 1)/(
sqrt(d)*cos(b*x + a))) + 8*sqrt(d*cos(b*x + a))*d^2*cos(b*x + a) + 3*d^(5/
2)*log(-(d*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))*sqrt(d)*(cos(b*x + a) +
 1) + 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 - 2*cos(b*x + a) + 1)))/b]
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{5/2}\csc(a+bx)\,dx=\text{Timed out}$$

input
```
integrate((d*cos(b*x+a))**(5/2)*csc(b*x+a),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.06

$$\int (d\cos(a+bx))^{5/2}\csc(a+bx)\,dx=\frac{6\,d^{\frac{7}{2}}\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)+3\,d^{\frac{7}{2}}\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)+4\,(d\cos(bx+a))^{\frac{3}{2}}\,d^2}{6\,bd}$$

input
```
integrate((d*cos(b*x+a))^(5/2)*csc(b*x+a),x, algorithm="maxima")
```

output
```
1/6*(6*d^(7/2)*arctan(sqrt(d*cos(b*x + a))/sqrt(d)) + 3*d^(7/2)*log((sqrt(
d*cos(b*x + a)) - sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d))) + 4*(d*cos(b*
x + a))^(3/2)*d^2)/(b*d)
```

Giac [F]

$$\int (d\cos(a+bx))^{5/2}\csc(a+bx)\,dx=\int (d\cos(bx+a))^{\frac{5}{2}}\csc(bx+a)\,dx$$

input
```
integrate((d*cos(b*x+a))^(5/2)*csc(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*cos(b*x + a))^(5/2)*csc(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{5/2}\csc(a+bx)\,dx = \int \frac{(d\,\cos(a+b\,x))^{5/2}}{\sin(a+b\,x)}\,dx$$

input
```
int((d*cos(a + b*x))^(5/2)/sin(a + b*x),x)
```

output
```
int((d*cos(a + b*x))^(5/2)/sin(a + b*x), x)
```

Reduce [F]

$$\int (d\cos(a+bx))^{5/2}\csc(a+bx)\,dx = \sqrt{d}\left(\int \sqrt{\cos(bx+a)}\,\cos(bx+a)^2\csc(bx+a)\,dx\right)d^2$$

input
```
int((d*cos(b*x+a))^(5/2)*csc(b*x+a),x)
```

output
```
sqrt(d)*int(sqrt(cos(a + b*x))*cos(a + b*x)**2*csc(a + b*x),x)*d**2
```

3.225 $\int (d\cos(a+bx))^{3/2}\csc(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 77

$$\int (d\cos(a+bx))^{3/2}\csc(a+bx)\,dx = -\frac{d^{3/2}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} - \frac{d^{3/2}\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} + \frac{2d\sqrt{d\cos(a+bx)}}{b}$$

output
```
-d^(3/2)*arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b-d^(3/2)*arctanh((d*cos(b*x
+a))^(1/2)/d^(1/2))/b+2*d*(d*cos(b*x+a))^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.84

$$\int (d\cos(a+bx))^{3/2}\csc(a+bx)\,dx = \frac{\left(-\arctan\left(\sqrt{\cos(a+bx)}\right) - \text{arctanh}\left(\sqrt{\cos(a+bx)}\right) + 2\sqrt{\cos(a+bx)}\right)(d\cos(a+bx))^{3/2}}{b\cos^{\frac{3}{2}}(a+bx)}$$

input
```
Integrate[(d*Cos[a + b*x])^(3/2)*Csc[a + b*x],x]
```

output
```
((-ArcTan[Sqrt[Cos[a + b*x]]] - ArcTanh[Sqrt[Cos[a + b*x]]] + 2*Sqrt[Cos[a
 + b*x]])*(d*Cos[a + b*x])^(3/2))/(b*Cos[a + b*x]^(3/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.45 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 3045, 27, 262, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)(d\cos(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\cos(a+bx))^{3/2}}{\sin(a+bx)}dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int \frac{d^2(d\cos(a+bx))^{3/2}}{d^2-d^2\cos^2(a+bx)}d(d\cos(a+bx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$-\frac{d\int \frac{(d\cos(a+bx))^{3/2}}{d^2-d^2\cos^2(a+bx)}d(d\cos(a+bx))}{b}$$

$$\Big\downarrow \text{262}$$

$$-\frac{d\left(d^2\int \frac{1}{\sqrt{d\cos(a+bx)}(d^2-d^2\cos^2(a+bx))}d(d\cos(a+bx))-2\sqrt{d\cos(a+bx)}\right)}{b}$$

$$\Big\downarrow \text{266}$$

$$-\frac{d\left(2d^2\int \frac{1}{d^2-d^4\cos^4(a+bx)}d\sqrt{d\cos(a+bx)}-2\sqrt{d\cos(a+bx)}\right)}{b}$$

$$\Big\downarrow \text{756}$$

$$-\frac{d\left(2d^2\left(\frac{\int \frac{1}{d-d^2\cos^2(a+bx)}d\sqrt{d\cos(a+bx)}}{2d}+\frac{\int \frac{1}{d^2\cos^2(a+bx)+d}d\sqrt{d\cos(a+bx)}}{2d}\right)-2\sqrt{d\cos(a+bx)}\right)}{b}$$

$$\Big\downarrow 216$$

$$-\frac{d\left(2d^2\left(\frac{\int \frac{1}{d-d^2\cos^2(a+bx)}d\sqrt{d\cos(a+bx)}}{2d}+\frac{\arctan\left(\sqrt{d}\cos(a+bx)\right)}{2d^{3/2}}\right)-2\sqrt{d\cos(a+bx)}\right)}{b}$$

$$\Big\downarrow 219$$

$$-\frac{d\left(2d^2\left(\frac{\arctan\left(\sqrt{d}\cos(a+bx)\right)}{2d^{3/2}}+\frac{\text{arctanh}\left(\sqrt{d}\cos(a+bx)\right)}{2d^{3/2}}\right)-2\sqrt{d\cos(a+bx)}\right)}{b}$$

input
```
Int[(d*Cos[a + b*x])^(3/2)*Csc[a + b*x],x]
```

output
```
-((d*(2*d^2*(ArcTan[Sqrt[d]*Cos[a + b*x]]/(2*d^(3/2)) + ArcTanh[Sqrt[d]*Co
s[a + b*x]]/(2*d^(3/2))) - 2*Sqrt[d*Cos[a + b*x]]))/b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(63) = 126$.

Time = 3.11 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.75

method	result
default	$\frac{-d^{\frac{3}{2}} \ln\left(-\frac{2\left(2d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-\sqrt{d}\,\sqrt{-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2} d+d}+d\right)}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\right)\sqrt{-d}-d^{\frac{3}{2}} \ln\left(\frac{4d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+2\sqrt{d}\,\sqrt{-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2} d+d}-2d}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-1}\right)\sqrt{-d}+4d\sqrt{}}{2\sqrt{-d}\,b}$

input
```
int((d*cos(b*x+a))^(3/2)*csc(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-d^(3/2)*ln(-2/(cos(1/2*b*x+1/2*a)+1)*(2*d*cos(1/2*b*x+1/2*a)-d^(1/2)
*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+d))*(-d)^(1/2)-d^(3/2)*ln(2/(cos(1/2*
b*x+1/2*a)-1)*(2*d*cos(1/2*b*x+1/2*a)+d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d
)^(1/2)-d))*(-d)^(1/2)+4*d*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*(-d)^(1/2)+
2*d^2*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1
/2)-d)))/(-d)^(1/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(63) = 126$.

Time = 0.18 (sec) , antiderivative size = 269, normalized size of antiderivative = 3.49

$$\int (d\cos(a+bx))^{3/2}\csc(a+bx)\,dx = \left[\frac{2\sqrt{-d}d\arctan\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)+1)}{2\,d\cos(bx+a)}\right)+\sqrt{-d}d\log\left(-\frac{d\cos(bx+a)^2-4\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a}{\cos(bx+a)^2+2\cos(bx+a)}\right.}{4\,b}\right.$$

$$-\frac{2\,d^{\frac{3}{2}}\arctan\left(\frac{\sqrt{d\cos(bx+a)}(\cos(bx+a)-1)}{2\sqrt{d}\cos(bx+a)}\right)-d^{\frac{3}{2}}\log\left(-\frac{d\cos(bx+a)^2-4\sqrt{d\cos(bx+a)}\sqrt{d}(\cos(bx+a)+1)+6\,d\cos(bx+a)+d}{\cos(bx+a)^2-2\cos(bx+a)+1}\right)-8}{4\,b}$$

input
```
integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(-d)*d*arctan(1/2*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a)
+ 1)/(d*cos(b*x + a))) + sqrt(-d)*d*log(-(d*cos(b*x + a)^2 - 4*sqrt(d*cos(
b*x + a))*sqrt(-d)*(cos(b*x + a) - 1) - 6*d*cos(b*x + a) + d)/(cos(b*x + a
)^2 + 2*cos(b*x + a) + 1)) + 8*sqrt(d*cos(b*x + a))*d)/b, -1/4*(2*d^(3/2)*
arctan(1/2*sqrt(d*cos(b*x + a))*(cos(b*x + a) - 1)/(sqrt(d)*cos(b*x + a)))
 - d^(3/2)*log(-(d*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))*sqrt(d)*(cos(b*
x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 - 2*cos(b*x + a) + 1))
 - 8*sqrt(d*cos(b*x + a))*d)/b]
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{3/2}\csc(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*cos(b*x+a))**(3/2)*csc(b*x+a),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.08

$$\int (d\cos(a+bx))^{3/2}\csc(a+bx)\,dx = -\frac{2\,d^{\frac{5}{2}}\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right) - d^{\frac{5}{2}}\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right) - 4\,\sqrt{d\cos\left(bx+a\right)}d^2}{2\,bd}$$

input
```
integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(2*d^(5/2)*arctan(sqrt(d*cos(b*x + a))/sqrt(d)) - d^(5/2)*log((sqrt(d
*cos(b*x + a)) - sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d))) - 4*sqrt(d*cos
(b*x + a))*d^2)/(b*d)
```

Giac [F]

$$\int (d\cos(a+bx))^{3/2}\csc(a+bx)\,dx = \int (d\cos\left(bx+a\right))^{\frac{3}{2}}\csc\left(bx+a\right)\,dx$$

input
```
integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*cos(b*x + a))^(3/2)*csc(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{3/2}\csc(a+bx)\,dx = \int \frac{(d\,\cos(a+b\,x))^{3/2}}{\sin(a+b\,x)}\,dx$$

input
```
int((d*cos(a + b*x))^(3/2)/sin(a + b*x),x)
```

output
```
int((d*cos(a + b*x))^(3/2)/sin(a + b*x), x)
```

Reduce [F]

$$\int (d\cos(a+bx))^{3/2}\csc(a+bx)\,dx = \sqrt{d}\left(\int \sqrt{\cos(bx+a)}\,\cos(bx+a)\csc(bx+a)\,dx\right)d$$

input
```
int((d*cos(b*x+a))^(3/2)*csc(b*x+a),x)
```

output
```
sqrt(d)*int(sqrt(cos(a + b*x))*cos(a + b*x)*csc(a + b*x),x)*d
```

3.226 $\int \sqrt{d\cos(a+bx)}\csc(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \sqrt{d\cos(a+bx)}\csc(a+bx)\,dx = \frac{\sqrt{d}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} - \frac{\sqrt{d}\,\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}$$

output
```
d^(1/2)*arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b-d^(1/2)*arctanh((d*cos(b*x+
a))^(1/2)/d^(1/2))/b
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

$$\int \sqrt{d\cos(a+bx)}\csc(a+bx)\,dx = \frac{\left(\arctan\left(\sqrt{\cos(a+bx)}\right) - \text{arctanh}\left(\sqrt{\cos(a+bx)}\right)\right)\sqrt{d\cos(a+bx)}}{b\sqrt{\cos(a+bx)}}$$

input
```
Integrate[Sqrt[d*Cos[a + b*x]]*Csc[a + b*x],x]
```

output
```
((ArcTan[Sqrt[Cos[a + b*x]]] - ArcTanh[Sqrt[Cos[a + b*x]]])*Sqrt[d*Cos[a +
 b*x]])/(b*Sqrt[Cos[a + b*x]])
```

Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 3045, 27, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\sqrt{d\cos(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{d\cos(a+bx)}}{\sin(a+bx)}dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int \frac{d^2\sqrt{d\cos(a+bx)}}{d^2-d^2\cos^2(a+bx)}d(d\cos(a+bx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$-\frac{d\int \frac{\sqrt{d\cos(a+bx)}}{d^2-d^2\cos^2(a+bx)}d(d\cos(a+bx))}{b}$$

$$\Big\downarrow \text{266}$$

$$-\frac{2d\int \frac{d^2\cos^2(a+bx)}{d^2-d^4\cos^4(a+bx)}d\sqrt{d\cos(a+bx)}}{b}$$

$$\Big\downarrow \text{827}$$

$$-\frac{2d\left(\frac{1}{2}\int \frac{1}{d-d^2\cos^2(a+bx)}d\sqrt{d\cos(a+bx)}-\frac{1}{2}\int \frac{1}{d^2\cos^2(a+bx)+d}d\sqrt{d\cos(a+bx)}\right)}{b}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2d\left(\frac{1}{2}\int \frac{1}{d-d^2\cos^2(a+bx)}d\sqrt{d\cos(a+bx)}-\frac{\arctan\left(\sqrt{d}\cos(a+bx)\right)}{2\sqrt{d}}\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2d\left(\frac{\operatorname{arctanh}\left(\sqrt{d}\cos(a+bx)\right)}{2\sqrt{d}}-\frac{\arctan\left(\sqrt{d}\cos(a+bx)\right)}{2\sqrt{d}}\right)}{b}$$

input
```
Int[Sqrt[d*Cos[a + b*x]]*Csc[a + b*x],x]
```

output
```
(-2*d*(-1/2*ArcTan[Sqrt[d]*Cos[a + b*x]]/Sqrt[d] + ArcTanh[Sqrt[d]*Cos[a +
 b*x]]/(2*Sqrt[d])))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(46) = 92$.

Time = 2.71 (sec) , antiderivative size = 182, normalized size of antiderivative = 3.14

method	result
default	$-\frac{\sqrt{d}\,\ln\left(-\frac{2\left(2d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-\sqrt{d}\,\sqrt{-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}d+d}+d\right)}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\right)\sqrt{-d}+\sqrt{d}\,\ln\left(\frac{4d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+2\sqrt{d}\,\sqrt{-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}d+d}-2d}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-1}\right)\sqrt{-d}+2}{2\sqrt{-d}\,b}$

input
```
int((d*cos(b*x+a))^(1/2)*csc(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(-d)^(1/2)*(d^(1/2)*ln(-2/(cos(1/2*b*x+1/2*a)+1)*(2*d*cos(1/2*b*x+1/2
*a)-d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+d))*(-d)^(1/2)+d^(1/2)*ln(
2/(cos(1/2*b*x+1/2*a)-1)*(2*d*cos(1/2*b*x+1/2*a)+d^(1/2)*(-2*sin(1/2*b*x+1
/2*a)^2*d+d)^(1/2)-d))*(-d)^(1/2)+2*d*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*
(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d)))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(46) = 92$.

Time = 0.14 (sec) , antiderivative size = 230, normalized size of antiderivative = 3.97

$$\int \sqrt{d\cos(a+bx)}\csc(a+bx)\,dx$$
$$= \left[\frac{2\sqrt{-d}\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{-d}}{d\cos(bx+a)+d}\right)+\sqrt{-d}\log\left(\frac{d\cos(bx+a)^2+4\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)-1)-6\,d\cos(bx+a)+d}{\cos(bx+a)^2+2\cos(bx+a)+1}\right)}{4\,b},\right.$$
$$\left.-\frac{2\sqrt{d}\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{d}}{d\cos(bx+a)-d}\right)-\sqrt{d}\log\left(\frac{d\cos(bx+a)^2-4\sqrt{d\cos(bx+a)}\sqrt{d}(\cos(bx+a)+1)+6\,d\cos(bx+a)+d}{\cos(bx+a)^2-2\cos(bx+a)+1}\right)}{4\,b}\right]$$

input
```
integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(-d)*arctan(2*sqrt(d*cos(b*x + a))*sqrt(-d)/(d*cos(b*x + a) +
d)) + sqrt(-d)*log((d*cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(co
s(b*x + a) - 1) - 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a) +
 1)))/b, -1/4*(2*sqrt(d)*arctan(2*sqrt(d*cos(b*x + a))*sqrt(d)/(d*cos(b*x
+ a) - d)) - sqrt(d)*log((d*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))*sqrt(d
)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 - 2*cos(b*x +
 a) + 1)))/b]
```

Sympy [F]

$$\int \sqrt{d\cos(a+bx)}\csc(a+bx)\,dx = \int \sqrt{d\cos(a+bx)}\csc(a+bx)\,dx$$

input
```
integrate((d*cos(b*x+a))**(1/2)*csc(b*x+a),x)
```

output
```
Integral(sqrt(d*cos(a + b*x))*csc(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.16

$$\int \sqrt{d\cos(a+bx)}\csc(a+bx)\,dx = \frac{2\,d^{\frac{3}{2}}\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)+d^{\frac{3}{2}}\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)}{2\,bd}$$

input
```
integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(2*d^(3/2)*arctan(sqrt(d*cos(b*x + a))/sqrt(d)) + d^(3/2)*log((sqrt(d*
cos(b*x + a)) - sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d))))/(b*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83

$$\int \sqrt{d\cos(a+bx)}\csc(a+bx)\,dx = \frac{d\left(\frac{\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{-d}}\right)}{\sqrt{-d}}+\frac{\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)}{\sqrt{d}}\right)}{b}$$

input
```
integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a),x, algorithm="giac")
```

output
```
d*(arctan(sqrt(d*cos(b*x + a))/sqrt(-d))/sqrt(-d) + arctan(sqrt(d*cos(b*x
+ a))/sqrt(d))/sqrt(d))/b
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\cos(a+bx)}\csc(a+bx)\,dx = \int \frac{\sqrt{d\,\cos\left(a+b\,x\right)}}{\sin\left(a+b\,x\right)}\,dx$$

input
```
int((d*cos(a + b*x))^(1/2)/sin(a + b*x),x)
```

output
```
int((d*cos(a + b*x))^(1/2)/sin(a + b*x), x)
```

Reduce [F]

$$\int \sqrt{d\cos(a+bx)}\csc(a+bx)\,dx=\sqrt{d}\left(\int \sqrt{\cos(bx+a)}\,\csc(bx+a)\,dx\right)$$

input
```
int((d*cos(b*x+a))^(1/2)*csc(b*x+a),x)
```

output
```
sqrt(d)*int(sqrt(cos(a + b*x))*csc(a + b*x),x)
```

3.227 $\int \frac{\csc(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 59

$$\int \frac{\csc(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b\sqrt{d}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b\sqrt{d}}$$

output
```
-arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b/d^(1/2)-arctanh((d*cos(b*x+a))^(1/
2)/d^(1/2))/b/d^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int \frac{\csc(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{\left(\arctan\left(\sqrt{\cos(a+bx)}\right)+\operatorname{arctanh}\left(\sqrt{\cos(a+bx)}\right)\right)\sqrt{\cos(a+bx)}}{b\sqrt{d\cos(a+bx)}}$$

input
```
Integrate[Csc[a + b*x]/Sqrt[d*Cos[a + b*x]],x]
```

output
```
-(((ArcTan[Sqrt[Cos[a + b*x]]] + ArcTanh[Sqrt[Cos[a + b*x]]])*Sqrt[Cos[a +
 b*x]])/(b*Sqrt[d*Cos[a + b*x]]))
```

Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 3045, 27, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(a+bx)}{\sqrt{d\cos(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)\sqrt{d\cos(a+bx)}}dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int \frac{d^2}{\sqrt{d\cos(a+bx)}(d^2-d^2\cos^2(a+bx))}d(d\cos(a+bx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$-\frac{d\int \frac{1}{\sqrt{d\cos(a+bx)}(d^2-d^2\cos^2(a+bx))}d(d\cos(a+bx))}{b}$$

$$\Big\downarrow \text{266}$$

$$-\frac{2d\int \frac{1}{d^2-d^4\cos^4(a+bx)}d\sqrt{d\cos(a+bx)}}{b}$$

$$\Big\downarrow \text{756}$$

$$-\frac{2d\left(\frac{\int \frac{1}{d-d^2\cos^2(a+bx)}d\sqrt{d\cos(a+bx)}}{2d}+\frac{\int \frac{1}{d^2\cos^2(a+bx)+d}d\sqrt{d\cos(a+bx)}}{2d}\right)}{b}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2d\left(\frac{\int \frac{1}{d-d^2\cos^2(a+bx)}d\sqrt{d\cos(a+bx)}}{2d}+\frac{\arctan\left(\sqrt{d}\cos(a+bx)\right)}{2d^{3/2}}\right)}{b}$$

$$\Big\downarrow \; 219$$

$$-\frac{2d\left(\frac{\arctan\left(\sqrt{d}\cos(a+bx)\right)}{2d^{3/2}}+\frac{\operatorname{arctanh}\left(\sqrt{d}\cos(a+bx)\right)}{2d^{3/2}}\right)}{b}$$

input

```
Int[Csc[a + b*x]/Sqrt[d*Cos[a + b*x]],x]
```

output

```
(-2*d*(ArcTan[Sqrt[d]*Cos[a + b*x]]/(2*d^(3/2)) + ArcTanh[Sqrt[d]*Cos[a +
b*x]]/(2*d^(3/2))))/b
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(47) = 94$.

Time = 2.48 (sec) , antiderivative size = 181, normalized size of antiderivative = 3.07

method	result
default	$-\frac{\ln\left(\frac{4d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+2\sqrt{d}\sqrt{-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}d+d}-2d}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-1}\right)\sqrt{-d}-2\ln\left(\frac{2\sqrt{-d}\sqrt{-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}d+d}-2d}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)}\right)\sqrt{d}+\ln\left(-\frac{2\left(2d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-\sqrt{d}\right.}{\cos\left(\right.}\right.}{2\sqrt{-d}\sqrt{d}\,b}$

input
```
int(csc(b*x+a)/(d*cos(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(-d)^(1/2)/d^(1/2)*(ln(2/(cos(1/2*b*x+1/2*a)-1)*(2*d*cos(1/2*b*x+1/2*
a)+d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*(-d)^(1/2)-2*ln(2/cos(1
/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(1/2)+
ln(-2/(cos(1/2*b*x+1/2*a)+1)*(2*d*cos(1/2*b*x+1/2*a)-d^(1/2)*(-2*sin(1/2*b
*x+1/2*a)^2*d+d)^(1/2)+d))*(-d)^(1/2))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(47) = 94$.

Time = 0.12 (sec) , antiderivative size = 236, normalized size of antiderivative = 4.00

$$\int \frac{\csc(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$$
$$= \left[\frac{2\sqrt{-d}\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{-d}}{d\cos(bx+a)+d}\right) - \sqrt{-d}\log\left(\frac{d\cos(bx+a)^2+4\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)-1)-6\,d\cos(bx+a)+d}{\cos(bx+a)^2+2\cos(bx+a)+1}\right)}{4\,bd}, \frac{2}{}\right.$$

input
```
integrate(csc(b*x+a)/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(-d)*arctan(2*sqrt(d*cos(b*x + a))*sqrt(-d)/(d*cos(b*x + a) +
d)) - sqrt(-d)*log((d*cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(co
s(b*x + a) - 1) - 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a) +
 1)))/(b*d), 1/4*(2*sqrt(d)*arctan(2*sqrt(d*cos(b*x + a))*sqrt(d)/(d*cos(b
*x + a) - d)) + sqrt(d)*log((d*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))*sqr
t(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 - 2*cos(b*
x + a) + 1)))/(b*d)]
```

Sympy [F]

$$\int \frac{\csc(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{\csc(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$$

input
```
integrate(csc(b*x+a)/(d*cos(b*x+a))**(1/2),x)
```

output
```
Integral(csc(a + b*x)/sqrt(d*cos(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.15

$$\int \frac{\csc(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{2\sqrt{d}\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right) - \sqrt{d}\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)}{2\,bd}$$

input
```
integrate(csc(b*x+a)/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(2*sqrt(d)*arctan(sqrt(d*cos(b*x + a))/sqrt(d)) - sqrt(d)*log((sqrt(d
*cos(b*x + a)) - sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d))))/(b*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$\int \frac{\csc(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \frac{d\left(\frac{\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{-d}}\right)}{\sqrt{-d}d} - \frac{\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)}{d^{\frac{3}{2}}}\right)}{b}$$

input
```
integrate(csc(b*x+a)/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
d*(arctan(sqrt(d*cos(b*x + a))/sqrt(-d))/(sqrt(-d)*d) - arctan(sqrt(d*cos(
b*x + a))/sqrt(d))/d^(3/2))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{1}{\sin\left(a+b\,x\right)\,\sqrt{d\,\cos\left(a+b\,x\right)}}\,dx$$

input
```
int(1/(sin(a + b*x)*(d*cos(a + b*x))^(1/2)),x)
```

output
```
int(1/(sin(a + b*x)*(d*cos(a + b*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{\csc(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\cos(bx+a)}\,\csc(bx+a)}{\cos(bx+a)}dx\right)}{d}$$

input
```
int(csc(b*x+a)/(d*cos(b*x+a))^(1/2),x)
```

output
```
(sqrt(d)*int((sqrt(cos(a + b*x))*csc(a + b*x))/cos(a + b*x),x))/d
```

3.228 $\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{3/2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{3/2}} + \frac{2}{bd\sqrt{d\cos(a+bx)}}$$

output
```
arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b/d^(3/2)-arctanh((d*cos(b*x+a))^(1/2
)/d^(1/2))/b/d^(3/2)+2/b/d/(d*cos(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{2+\arctan\left(\sqrt{\cos(a+bx)}\right)\sqrt{\cos(a+bx)} - \operatorname{arctanh}\left(\sqrt{\cos(a+bx)}\right)\sqrt{\cos(a-}{bd\sqrt{d\cos(a+bx)}}$$

input
```
Integrate[Csc[a + b*x]/(d*Cos[a + b*x])^(3/2),x]
```

output
```
(2 + ArcTan[Sqrt[Cos[a + b*x]]]*Sqrt[Cos[a + b*x]] - ArcTanh[Sqrt[Cos[a +
b*x]]]*Sqrt[Cos[a + b*x]])/(b*d*Sqrt[d*Cos[a + b*x]])
```

Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 3045, 27, 264, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)(d\cos(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int \frac{d^2}{(d\cos(a+bx))^{3/2}(d^2-d^2\cos^2(a+bx))}d(d\cos(a+bx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$-\frac{d\int \frac{1}{(d\cos(a+bx))^{3/2}(d^2-d^2\cos^2(a+bx))}d(d\cos(a+bx))}{b}$$

$$\Big\downarrow \text{264}$$

$$-\frac{d\left(\frac{\int \frac{\sqrt{d\cos(a+bx)}}{d^2-d^2\cos^2(a+bx)}d(d\cos(a+bx))}{d^2}-\frac{2}{d^2\sqrt{d\cos(a+bx)}}\right)}{b}$$

$$\Big\downarrow \text{266}$$

$$-\frac{d\left(\frac{2\int \frac{d^2\cos^2(a+bx)}{d^2-d^4\cos^4(a+bx)}d\sqrt{d\cos(a+bx)}}{d^2}-\frac{2}{d^2\sqrt{d\cos(a+bx)}}\right)}{b}$$

$$\Big\downarrow \text{827}$$

$$-\frac{d\left(\frac{2\left(\frac{1}{2}\int \frac{1}{d-d^2\cos^2(a+bx)}d\sqrt{d\cos(a+bx)}-\frac{1}{2}\int \frac{1}{d^2\cos^2(a+bx)+d}d\sqrt{d\cos(a+bx)}\right)}{d^2}-\frac{2}{d^2\sqrt{d\cos(a+bx)}}\right)}{b}$$

$$\Big\downarrow \text{216}$$

$$-\frac{d\left(\frac{2\left(\frac{1}{2}\int \frac{1}{d-d^2\cos^2(a+bx)}d\sqrt{d\cos(a+bx)}-\frac{\arctan\left(\sqrt{d}\cos(a+bx)\right)}{2\sqrt{d}}\right)}{d^2}-\frac{2}{d^2\sqrt{d\cos(a+bx)}}\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{d\left(\frac{2\left(\frac{\text{arctanh}\left(\sqrt{d}\cos(a+bx)\right)}{2\sqrt{d}}-\frac{\arctan\left(\sqrt{d}\cos(a+bx)\right)}{2\sqrt{d}}\right)}{d^2}-\frac{2}{d^2\sqrt{d\cos(a+bx)}}\right)}{b}$$

input
```
Int[Csc[a + b*x]/(d*Cos[a + b*x])^(3/2),x]
```

output
```
-((d*((2*(-1/2*ArcTan[Sqrt[d]*Cos[a + b*x]]/Sqrt[d] + ArcTanh[Sqrt[d]*Cos[
a + b*x]]/(2*Sqrt[d])))/d^2 - 2/(d^2*Sqrt[d*Cos[a + b*x]])))/b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 440 vs. $2(64) = 128$.

Time = 3.07 (sec) , antiderivative size = 441, normalized size of antiderivative = 5.65

method	result
default	$-\frac{4 d^{\frac{5}{2}} \ln\left(\frac{2\sqrt{-d}\,\sqrt{-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2} d+d}-2d}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)}\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}+2\sqrt{-d}\,\ln\left(-\frac{2\left(2d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-\sqrt{d}\,\sqrt{-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2} d+d}+d\right)}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\right)\sin\left(\frac{bx}{2}+\right.}{}$

input
```
int(csc(b*x+a)/(d*cos(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(-d)^(1/2)/d^(7/2)/(2*sin(1/2*b*x+1/2*a)^2-1)*(4*d^(5/2)*ln(2/cos(1/2
*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*sin(1/2*b*
x+1/2*a)^2+2*(-d)^(1/2)*ln(-2/(cos(1/2*b*x+1/2*a)+1)*(2*d*cos(1/2*b*x+1/2*
a)-d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+d))*sin(1/2*b*x+1/2*a)^2*d^
2+2*(-d)^(1/2)*ln(2/(cos(1/2*b*x+1/2*a)-1)*(2*d*cos(1/2*b*x+1/2*a)+d^(1/2)
*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*sin(1/2*b*x+1/2*a)^2*d^2-2*d^(5/2
)*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-
d))+4*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*(-d)^(1/2)*d^(3/2)-(-d)^(1/2)*ln
(-2/(cos(1/2*b*x+1/2*a)+1)*(2*d*cos(1/2*b*x+1/2*a)-d^(1/2)*(-2*sin(1/2*b*x
+1/2*a)^2*d+d)^(1/2)+d))*d^2-(-d)^(1/2)*ln(2/(cos(1/2*b*x+1/2*a)-1)*(2*d*c
os(1/2*b*x+1/2*a)+d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(64) = 128$.

Time = 0.14 (sec) , antiderivative size = 301, normalized size of antiderivative = 3.86

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \left[\frac{2\sqrt{-d}\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{-d}}{d\cos(bx+a)+d}\right)\cos(bx+a) - \sqrt{-d}\cos(bx+a)\log\left(\frac{d\cos(bx+\cdots}{\cdots}\right.}{4\,bd^2\cos(bx+\cdots}\right.$$

$$-\frac{2\sqrt{d}\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{d}}{d\cos(bx+a)-d}\right)\cos(bx+a) - \sqrt{d}\cos(bx+a)\log\left(\frac{d\cos(bx+a)^2-4\sqrt{d\cos(bx+a)}\sqrt{d}(\cos(bx+a)+1)+6\cdots}{\cos(bx+a)^2-2\cos(bx+a)+1}\right.}{4\,bd^2\cos(bx+a)}$$

input
```
integrate(csc(b*x+a)/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(-d)*arctan(2*sqrt(d*cos(b*x + a))*sqrt(-d)/(d*cos(b*x + a) +
d))*cos(b*x + a) - sqrt(-d)*cos(b*x + a)*log((d*cos(b*x + a)^2 - 4*sqrt(d*
cos(b*x + a))*sqrt(-d)*(cos(b*x + a) - 1) - 6*d*cos(b*x + a) + d)/(cos(b*x
 + a)^2 + 2*cos(b*x + a) + 1)) + 8*sqrt(d*cos(b*x + a)))/(b*d^2*cos(b*x +
a)), -1/4*(2*sqrt(d)*arctan(2*sqrt(d*cos(b*x + a))*sqrt(d)/(d*cos(b*x + a)
 - d))*cos(b*x + a) - sqrt(d)*cos(b*x + a)*log((d*cos(b*x + a)^2 - 4*sqrt(
d*cos(b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*
x + a)^2 - 2*cos(b*x + a) + 1)) - 8*sqrt(d*cos(b*x + a)))/(b*d^2*cos(b*x +
 a))]
```

Sympy [F]

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\csc{(a+bx)}}{(d\cos{(a+bx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)/(d*cos(b*x+a))**(3/2),x)
```

output
```
Integral(csc(a + b*x)/(d*cos(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{\frac{2\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)}{\sqrt{d}} + \frac{\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)}{\sqrt{d}} + \frac{4}{\sqrt{d\cos(bx+a)}}}{2\,bd}$$

input
```
integrate(csc(b*x+a)/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
1/2*(2*arctan(sqrt(d*cos(b*x + a))/sqrt(d))/sqrt(d) + log((sqrt(d*cos(b*x
+ a)) - sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d)))/sqrt(d) + 4/sqrt(d*cos(
b*x + a)))/(b*d)
```

Giac [F]

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\csc(bx+a)}{(d\cos(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)/(d*cos(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{1}{\sin(a+b\,x)\,\left(d\,\cos(a+b\,x)\right)^{3/2}}\,dx$$

input
```
int(1/(sin(a + b*x)*(d*cos(a + b*x))^(3/2)),x)
```

output
```
int(1/(sin(a + b*x)*(d*cos(a + b*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\cos(bx+a)}\,\csc(bx+a)}{\cos(bx+a)^2}dx\right)}{d^2}$$

input
```
int(csc(b*x+a)/(d*cos(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*int((sqrt(cos(a + b*x))*csc(a + b*x))/cos(a + b*x)**2,x))/d**2
```

3.229 $\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 81

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{5/2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{5/2}} + \frac{2}{3bd(d\cos(a+bx))^{3/2}}$$

output
```
-arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b/d^(5/2)-arctanh((d*cos(b*x+a))^(1/
2)/d^(1/2))/b/d^(5/2)+2/3/b/d/(d*cos(b*x+a))^(3/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = -\frac{-2+3\arctan\left(\sqrt{\cos(a+bx)}\right)\cos^{\frac{3}{2}}(a+bx)+3\operatorname{arctanh}\left(\sqrt{\cos(a+bx)}\right)\cos^{\frac{3}{2}}(a+bx)}{3bd(d\cos(a+bx))^{3/2}}$$

input
```
Integrate[Csc[a + b*x]/(d*Cos[a + b*x])^(5/2),x]
```

output
```
-1/3*(-2 + 3*ArcTan[Sqrt[Cos[a + b*x]]]*Cos[a + b*x]^(3/2) + 3*ArcTanh[Sqr
t[Cos[a + b*x]]]*Cos[a + b*x]^(3/2))/(b*d*(d*Cos[a + b*x])^(3/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.45 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 3045, 27, 264, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)(d\cos(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int \frac{d^2}{(d\cos(a+bx))^{5/2}(d^2-d^2\cos^2(a+bx))}d(d\cos(a+bx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$-\frac{d\int \frac{1}{(d\cos(a+bx))^{5/2}(d^2-d^2\cos^2(a+bx))}d(d\cos(a+bx))}{b}$$

$$\Big\downarrow \text{264}$$

$$-\frac{d\left(\frac{\int \frac{1}{\sqrt{d\cos(a+bx)}\left(d^2-d^2\cos^2(a+bx)\right)}d(d\cos(a+bx))}{d^2}-\frac{2}{3d^2(d\cos(a+bx))^{3/2}}\right)}{b}$$

$$\Big\downarrow \text{266}$$

$$-\frac{d\left(\frac{2\int \frac{1}{d^2-d^4\cos^4(a+bx)}d\sqrt{d\cos(a+bx)}}{d^2}-\frac{2}{3d^2(d\cos(a+bx))^{3/2}}\right)}{b}$$

$$\Big\downarrow \text{756}$$

$$-\frac{d\left(\frac{2\left(\frac{\int \frac{1}{d-d^2\cos^2(a+bx)}d\sqrt{d\cos(a+bx)}}{2d}+\frac{\int \frac{1}{d^2\cos^2(a+bx)+d}d\sqrt{d\cos(a+bx)}}{2d}\right)}{d^2}-\frac{2}{3d^2(d\cos(a+bx))^{3/2}}\right)}{b}$$

$$\Big\downarrow \text{216}$$

$$-\frac{d\left(\frac{2\left(\frac{\int \frac{1}{d-d^2\cos^2(a+bx)}d\sqrt{d\cos(a+bx)}}{2d}+\frac{\arctan\left(\sqrt{d}\cos(a+bx)\right)}{2d^{3/2}}\right)}{d^2}-\frac{2}{3d^2(d\cos(a+bx))^{3/2}}\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{d\left(\frac{2\left(\frac{\arctan\left(\sqrt{d}\cos(a+bx)\right)}{2d^{3/2}}+\frac{\text{arctanh}\left(\sqrt{d}\cos(a+bx)\right)}{2d^{3/2}}\right)}{d^2}-\frac{2}{3d^2(d\cos(a+bx))^{3/2}}\right)}{b}$$

input
```
Int[Csc[a + b*x]/(d*Cos[a + b*x])^(5/2),x]
```

output
```
-((d*((2*(ArcTan[Sqrt[d]*Cos[a + b*x]]/(2*d^(3/2)) + ArcTanh[Sqrt[d]*Cos[a
 + b*x]]/(2*d^(3/2))))/d^2 - 2/(3*d^2*(d*Cos[a + b*x])^(3/2))))/b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 648 vs. $2(65) = 130$.

Time = 2.98 (sec) , antiderivative size = 649, normalized size of antiderivative = 8.01

method	result
default	$\frac{24d^{\frac{3}{2}}\ln\left(\frac{2\sqrt{-d}\,\sqrt{-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}d+d-2d}}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)}\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}-12\sqrt{-d}\,\ln\left(\frac{4d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+2\sqrt{d}\,\sqrt{-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}d+d-2d}}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-1}\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}}{}$

input
```
int(csc(b*x+a)/(d*cos(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/d^(7/2)/(-d)^(1/2)/(4*sin(1/2*b*x+1/2*a)^4-4*sin(1/2*b*x+1/2*a)^2+1)*(
24*d^(3/2)*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+
d)^(1/2)-d))*sin(1/2*b*x+1/2*a)^4-12*(-d)^(1/2)*ln(2/(cos(1/2*b*x+1/2*a)-1
)*(2*d*cos(1/2*b*x+1/2*a)+d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*
sin(1/2*b*x+1/2*a)^4*d-12*(-d)^(1/2)*ln(-2/(cos(1/2*b*x+1/2*a)+1)*(2*d*cos
(1/2*b*x+1/2*a)-d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+d))*sin(1/2*b*
x+1/2*a)^4*d-24*d^(3/2)*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*
x+1/2*a)^2*d+d)^(1/2)-d))*sin(1/2*b*x+1/2*a)^2+12*(-d)^(1/2)*ln(2/(cos(1/2
*b*x+1/2*a)-1)*(2*d*cos(1/2*b*x+1/2*a)+d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+
d)^(1/2)-d))*sin(1/2*b*x+1/2*a)^2*d+12*(-d)^(1/2)*ln(-2/(cos(1/2*b*x+1/2*a
)+1)*(2*d*cos(1/2*b*x+1/2*a)-d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+d
))*sin(1/2*b*x+1/2*a)^2*d+6*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/
2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(3/2)+4*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2
)*(-d)^(1/2)*d^(1/2)-3*ln(2/(cos(1/2*b*x+1/2*a)-1)*(2*d*cos(1/2*b*x+1/2*a)
+d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*(-d)^(1/2)*d-3*ln(-2/(cos
(1/2*b*x+1/2*a)+1)*(2*d*cos(1/2*b*x+1/2*a)-d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^
2*d+d)^(1/2)+d))*(-d)^(1/2)*d)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(65) = 130$.

Time = 0.16 (sec) , antiderivative size = 309, normalized size of antiderivative = 3.81

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \left[\frac{6\sqrt{-d}\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{-d}}{d\cos(bx+a)+d}\right)\cos(bx+a)^2 - 3\sqrt{-d}\cos(bx+a)^2\log\left(\frac{d\cos}{\cdots}\right.}{12\,bd^3\cos(bx}\right.$$

input
```
integrate(csc(b*x+a)/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
[1/12*(6*sqrt(-d)*arctan(2*sqrt(d*cos(b*x + a))*sqrt(-d)/(d*cos(b*x + a) +
 d))*cos(b*x + a)^2 - 3*sqrt(-d)*cos(b*x + a)^2*log((d*cos(b*x + a)^2 + 4*
sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) - 1) - 6*d*cos(b*x + a) + d)/(
cos(b*x + a)^2 + 2*cos(b*x + a) + 1)) + 8*sqrt(d*cos(b*x + a)))/(b*d^3*cos
(b*x + a)^2), 1/12*(6*sqrt(d)*arctan(2*sqrt(d*cos(b*x + a))*sqrt(d)/(d*cos
(b*x + a) - d))*cos(b*x + a)^2 + 3*sqrt(d)*cos(b*x + a)^2*log((d*cos(b*x +
 a)^2 - 4*sqrt(d*cos(b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x +
a) + d)/(cos(b*x + a)^2 - 2*cos(b*x + a) + 1)) + 8*sqrt(d*cos(b*x + a)))/(
b*d^3*cos(b*x + a)^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)/(d*cos(b*x+a))**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = -\frac{\frac{6\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)}{d^{\frac{3}{2}}} - \frac{3\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)}{d^{\frac{3}{2}}} - \frac{4}{(d\cos(bx+a))^{\frac{3}{2}}}}{6\,bd}$$

input
```
integrate(csc(b*x+a)/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
-1/6*(6*arctan(sqrt(d*cos(b*x + a))/sqrt(d))/d^(3/2) - 3*log((sqrt(d*cos(b
*x + a)) - sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d)))/d^(3/2) - 4/(d*cos(b
*x + a))^(3/2))/(b*d)
```

Giac [F]

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{\csc(bx+a)}{(d\cos(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)/(d*cos(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{1}{\sin(a+b\,x)\,(d\,\cos(a+b\,x))^{5/2}}\,dx$$

input
```
int(1/(sin(a + b*x)*(d*cos(a + b*x))^(5/2)),x)
```

output
```
int(1/(sin(a + b*x)*(d*cos(a + b*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\cos(bx+a)}\,\csc(bx+a)}{\cos(bx+a)^3}dx\right)}{d^3}$$

input
```
int(csc(b*x+a)/(d*cos(b*x+a))^(5/2),x)
```

output
```
(sqrt(d)*int((sqrt(cos(a + b*x))*csc(a + b*x))/cos(a + b*x)**3,x))/d**3
```

3.230 $\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 100

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{7/2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{7/2}} + \frac{2}{5bd(d\cos(a+bx))^{5/2}} + \frac{2}{bd^3\sqrt{d\cos(a+bx)}}$$

output
```
arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b/d^(7/2)-arctanh((d*cos(b*x+a))^(1/2
)/d^(1/2))/b/d^(7/2)+2/5/b/d/(d*cos(b*x+a))^(5/2)+2/b/d^3/(d*cos(b*x+a))^(
1/2)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.81

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{5\arctan\left(\sqrt{\cos(a+bx)}\right)\sqrt{\cos(a+bx)} - 5\operatorname{arctanh}\left(\sqrt{\cos(a+bx)}\right)\sqrt{\cos(a+}}{5bd^3\sqrt{d\cos(a+bx)}}$$

input
```
Integrate[Csc[a + b*x]/(d*Cos[a + b*x])^(7/2),x]
```

output
```
(5*ArcTan[Sqrt[Cos[a + b*x]]]*Sqrt[Cos[a + b*x]] - 5*ArcTanh[Sqrt[Cos[a +
b*x]]]*Sqrt[Cos[a + b*x]] + 2*(5 + Sec[a + b*x]^2))/(5*b*d^3*Sqrt[d*Cos[a
+ b*x]])
```

Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3042, 3045, 27, 264, 264, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)(d\cos(a+bx))^{7/2}}dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int \frac{d^2}{(d\cos(a+bx))^{7/2}(d^2-d^2\cos^2(a+bx))}d(d\cos(a+bx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$-\frac{d\int \frac{1}{(d\cos(a+bx))^{7/2}(d^2-d^2\cos^2(a+bx))}d(d\cos(a+bx))}{b}$$

$$\Big\downarrow \text{264}$$

$$-\frac{d\left(\frac{\int \frac{1}{(d\cos(a+bx))^{3/2}\left(d^2-d^2\cos^2(a+bx)\right)}d(d\cos(a+bx))}{d^2}-\frac{2}{5d^2(d\cos(a+bx))^{5/2}}\right)}{b}$$

$$\Big\downarrow \text{264}$$

$$-\frac{d\left(\frac{\frac{\int \frac{\sqrt{d\cos(a+bx)}}{d^2-d^2\cos^2(a+bx)}d(d\cos(a+bx))}{d^2}-\frac{2}{d^2\sqrt{d\cos(a+bx)}}}{d^2}-\frac{2}{5d^2(d\cos(a+bx))^{5/2}}\right)}{b}$$

$$\Big\downarrow \text{266}$$

$$-\frac{d\left(\frac{\frac{2\int \frac{d^2\cos^2(a+bx)}{d^2-d^4\cos^4(a+bx)}d\sqrt{d\cos(a+bx)}}{d^2}-\frac{2}{d^2\sqrt{d\cos(a+bx)}}}{d^2}-\frac{2}{5d^2(d\cos(a+bx))^{5/2}}\right)}{b}$$

$$\Big\downarrow \text{827}$$

$$-\frac{d\left(\frac{\frac{2\left(\frac{1}{2}\int \frac{1}{d-d^2\cos^2(a+bx)}d\sqrt{d\cos(a+bx)}-\frac{1}{2}\int \frac{1}{d^2\cos^2(a+bx)+d}d\sqrt{d\cos(a+bx)}\right)}{d^2}-\frac{2}{d^2\sqrt{d\cos(a+bx)}}}{d^2}-\frac{2}{5d^2(d\cos(a+bx))^{5/2}}\right)}{b}$$

$$\Big\downarrow \text{216}$$

$$-\frac{d\left(\frac{\frac{2\left(\frac{1}{2}\int \frac{1}{d-d^2\cos^2(a+bx)}d\sqrt{d\cos(a+bx)}-\frac{\arctan\left(\sqrt{d}\cos(a+bx)\right)}{2\sqrt{d}}\right)}{d^2}-\frac{2}{d^2\sqrt{d\cos(a+bx)}}}{d^2}-\frac{2}{5d^2(d\cos(a+bx))^{5/2}}\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{d\left(\frac{\frac{2\left(\frac{\text{arctanh}\left(\sqrt{d}\cos(a+bx)\right)}{2\sqrt{d}}-\frac{\arctan\left(\sqrt{d}\cos(a+bx)\right)}{2\sqrt{d}}\right)}{d^2}-\frac{2}{d^2\sqrt{d\cos(a+bx)}}}{d^2}-\frac{2}{5d^2(d\cos(a+bx))^{5/2}}\right)}{b}$$

input
```
Int[Csc[a + b*x]/(d*Cos[a + b*x])^(7/2),x]
```

output
```
-((d*(-2/(5*d^2*(d*Cos[a + b*x])^(5/2)) + ((2*(-1/2*ArcTan[Sqrt[d]*Cos[a +
 b*x]]/Sqrt[d] + ArcTanh[Sqrt[d]*Cos[a + b*x]]/(2*Sqrt[d])))/d^2 - 2/(d^2*
Sqrt[d*Cos[a + b*x]]))/d^2))/b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 861 vs. $2(82) = 164$.

Time = 2.78 (sec) , antiderivative size = 862, normalized size of antiderivative = 8.62

method	result	size
default	Expression too large to display	862

input
```
int(csc(b*x+a)/(d*cos(b*x+a))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/10/d^(9/2)/(-d)^(1/2)/(8*sin(1/2*b*x+1/2*a)^6-12*sin(1/2*b*x+1/2*a)^4+6*
sin(1/2*b*x+1/2*a)^2-1)*(10*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/
2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(3/2)-24*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/
2)*(-d)^(1/2)*d^(1/2)+5*ln(-2/(cos(1/2*b*x+1/2*a)+1)*(2*d*cos(1/2*b*x+1/2*
a)-d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+d))*(-d)^(1/2)*d+5*ln(2/(co
s(1/2*b*x+1/2*a)-1)*(2*d*cos(1/2*b*x+1/2*a)+d^(1/2)*(-2*sin(1/2*b*x+1/2*a)
^2*d+d)^(1/2)-d))*(-d)^(1/2)*d-40*(2*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(
-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(3/2)+ln(2/(cos(1/2*b*x+1/2*a)-1)
*(2*d*cos(1/2*b*x+1/2*a)+d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*(
-d)^(1/2)*d+ln(-2/(cos(1/2*b*x+1/2*a)+1)*(2*d*cos(1/2*b*x+1/2*a)-d^(1/2)*(
-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+d))*(-d)^(1/2)*d)*sin(1/2*b*x+1/2*a)^6-
20*(-6*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(
1/2)-d))*d^(3/2)+4*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*(-d)^(1/2)*d^(1/2)-
3*ln(2/(cos(1/2*b*x+1/2*a)-1)*(2*d*cos(1/2*b*x+1/2*a)+d^(1/2)*(-2*sin(1/2*
b*x+1/2*a)^2*d+d)^(1/2)-d))*(-d)^(1/2)*d-3*ln(-2/(cos(1/2*b*x+1/2*a)+1)*(2
*d*cos(1/2*b*x+1/2*a)-d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+d))*(-d)
^(1/2)*d)*sin(1/2*b*x+1/2*a)^4+10*(-6*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*
(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(3/2)+8*(-2*sin(1/2*b*x+1/2*a)^2
*d+d)^(1/2)*(-d)^(1/2)*d^(1/2)-3*ln(2/(cos(1/2*b*x+1/2*a)-1)*(2*d*cos(1/2*
b*x+1/2*a)+d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*(-d)^(1/2)*d...
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 333, normalized size of antiderivative = 3.33

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \left[\frac{10\sqrt{-d}\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{-d}}{d\cos(bx+a)+d}\right)\cos(bx+a)^3 - 5\sqrt{-d}\cos(bx+a)^3\log\left(\frac{d\cos\ldots}{\ldots}\right.\ldots}{20\ldots}\right.$$

$$-\frac{10\sqrt{d}\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{d}}{d\cos(bx+a)-d}\right)\cos(bx+a)^3 - 5\sqrt{d}\cos(bx+a)^3\log\left(\frac{d\cos(bx+a)^2-4\sqrt{d\cos(bx+a)}\sqrt{d}(\cos(bx+a)-\ldots}{\cos(bx+a)^2-2\cos(bx+a)+\ldots}\right.\ldots}{20\,bd^4\cos(bx+a)^3}$$

input
```
integrate(csc(b*x+a)/(d*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

output
```
[1/20*(10*sqrt(-d)*arctan(2*sqrt(d*cos(b*x + a))*sqrt(-d)/(d*cos(b*x + a)
+ d))*cos(b*x + a)^3 - 5*sqrt(-d)*cos(b*x + a)^3*log((d*cos(b*x + a)^2 - 4
*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) - 1) - 6*d*cos(b*x + a) + d)/
(cos(b*x + a)^2 + 2*cos(b*x + a) + 1)) + 8*sqrt(d*cos(b*x + a))*(5*cos(b*x
 + a)^2 + 1))/(b*d^4*cos(b*x + a)^3), -1/20*(10*sqrt(d)*arctan(2*sqrt(d*co
s(b*x + a))*sqrt(d)/(d*cos(b*x + a) - d))*cos(b*x + a)^3 - 5*sqrt(d)*cos(b
*x + a)^3*log((d*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))*sqrt(d)*(cos(b*x
+ a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 - 2*cos(b*x + a) + 1)) -
 8*sqrt(d*cos(b*x + a))*(5*cos(b*x + a)^2 + 1))/(b*d^4*cos(b*x + a)^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)/(d*cos(b*x+a))**(7/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{\frac{10\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)}{d^{\frac{5}{2}}} + \frac{5\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)}{d^{\frac{5}{2}}} + \frac{4\left(5\,d^2\cos(bx+a)^2+d^2\right)}{(d\cos(bx+a))^{\frac{5}{2}}d^2}}{10\,bd}$$

input
```
integrate(csc(b*x+a)/(d*cos(b*x+a))^(7/2),x, algorithm="maxima")
```

output
```
1/10*(10*arctan(sqrt(d*cos(b*x + a))/sqrt(d))/d^(5/2) + 5*log((sqrt(d*cos(
b*x + a)) - sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d)))/d^(5/2) + 4*(5*d^2*
cos(b*x + a)^2 + d^2)/((d*cos(b*x + a))^(5/2)*d^2))/(b*d)
```

Giac [F]

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{\csc(bx+a)}{(d\cos(bx+a))^{\frac{7}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)/(d*cos(b*x+a))^(7/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)/(d*cos(b*x + a))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{1}{\sin(a+b\,x)\,\left(d\,\cos(a+b\,x)\right)^{7/2}}\,dx$$

input
```
int(1/(sin(a + b*x)*(d*cos(a + b*x))^(7/2)),x)
```

output
```
int(1/(sin(a + b*x)*(d*cos(a + b*x))^(7/2)), x)
```

Reduce [F]

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\cos(bx+a)}\,\csc(bx+a)}{\cos(bx+a)^4}dx\right)}{d^4}$$

input
```
int(csc(b*x+a)/(d*cos(b*x+a))^(7/2),x)
```

output
```
(sqrt(d)*int((sqrt(cos(a + b*x))*csc(a + b*x))/cos(a + b*x)**4,x))/d**4
```

3.231 $\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 103

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{9/2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{9/2}} + \frac{2}{7bd(d\cos(a+bx))^{7/2}} + \frac{2}{3bd^3(d\cos(a+bx))^{3/2}}$$

output
```
-arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b/d^(9/2)-arctanh((d*cos(b*x+a))^(1/
2)/d^(1/2))/b/d^(9/2)+2/7/b/d/(d*cos(b*x+a))^(7/2)+2/3/b/d^3/(d*cos(b*x+a)
)^(3/2)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \frac{-21\arctan\left(\sqrt{\cos(a+bx)}\right)\sqrt{\cos(a+bx)} - 21\operatorname{arctanh}\left(\sqrt{\cos(a+bx)}\right)\sqrt{\cos(}}{21bd^4\sqrt{d\cos(a+bx)}}$$

input
```
Integrate[Csc[a + b*x]/(d*Cos[a + b*x])^(9/2),x]
```

output
```
(-21*ArcTan[Sqrt[Cos[a + b*x]]]*Sqrt[Cos[a + b*x]] - 21*ArcTanh[Sqrt[Cos[a
 + b*x]]]*Sqrt[Cos[a + b*x]] + 14*Sec[a + b*x] + 6*Sec[a + b*x]^3)/(21*b*d
^4*Sqrt[d*Cos[a + b*x]])
```

Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3042, 3045, 27, 264, 264, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)(d\cos(a+bx))^{9/2}}dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int \frac{d^2}{(d\cos(a+bx))^{9/2}(d^2-d^2\cos^2(a+bx))}d(d\cos(a+bx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$-\frac{d\int \frac{1}{(d\cos(a+bx))^{9/2}(d^2-d^2\cos^2(a+bx))}d(d\cos(a+bx))}{b}$$

$$\Big\downarrow \text{264}$$

$$-\frac{d\left(\frac{\int \frac{1}{(d\cos(a+bx))^{5/2}\left(d^2-d^2\cos^2(a+bx)\right)}d(d\cos(a+bx))}{d^2}-\frac{2}{7d^2(d\cos(a+bx))^{7/2}}\right)}{b}$$

$$\Big\downarrow \text{264}$$

$$-\frac{d\left(\frac{\frac{\int \frac{1}{\sqrt{d\cos(a+bx)}\left(d^2-d^2\cos^2(a+bx)\right)}d(d\cos(a+bx))}{d^2}-\frac{2}{3d^2(d\cos(a+bx))^{3/2}}}{d^2}-\frac{2}{7d^2(d\cos(a+bx))^{7/2}}\right)}{b}$$

$$\Big\downarrow \textbf{266}$$

$$-\frac{d\left(\frac{\frac{2\int \frac{1}{d^2-d^4\cos^4(a+bx)}d\sqrt{d\cos(a+bx)}}{d^2}-\frac{2}{3d^2(d\cos(a+bx))^{3/2}}}{d^2}-\frac{2}{7d^2(d\cos(a+bx))^{7/2}}\right)}{b}$$

$$\Big\downarrow \textbf{756}$$

$$-\frac{d\left(\frac{\frac{2\left(\frac{\int \frac{1}{d-d^2\cos^2(a+bx)}d\sqrt{d\cos(a+bx)}}{2d}+\frac{\int \frac{1}{d^2\cos^2(a+bx)+d}d\sqrt{d\cos(a+bx)}}{2d}\right)}{d^2}-\frac{2}{3d^2(d\cos(a+bx))^{3/2}}}{d^2}-\frac{2}{7d^2(d\cos(a+bx))^{7/2}}\right)}{b}$$

$$\Big\downarrow \textbf{216}$$

$$-\frac{d\left(\frac{\frac{2\left(\frac{\int \frac{1}{d-d^2\cos^2(a+bx)}d\sqrt{d\cos(a+bx)}}{2d}+\frac{\arctan\left(\sqrt{d}\cos(a+bx)\right)}{2d^{3/2}}\right)}{d^2}-\frac{2}{3d^2(d\cos(a+bx))^{3/2}}}{d^2}-\frac{2}{7d^2(d\cos(a+bx))^{7/2}}\right)}{b}$$

$$\Big\downarrow \textbf{219}$$

$$-\frac{d\left(\frac{\frac{2\left(\frac{\arctan\left(\sqrt{d}\cos(a+bx)\right)}{2d^{3/2}}+\frac{\text{arctanh}\left(\sqrt{d}\cos(a+bx)\right)}{2d^{3/2}}\right)}{d^2}-\frac{2}{3d^2(d\cos(a+bx))^{3/2}}}{d^2}-\frac{2}{7d^2(d\cos(a+bx))^{7/2}}\right)}{b}$$

input
```
Int[Csc[a + b*x]/(d*Cos[a + b*x])^(9/2),x]
```

output
```
-((d*(-2/(7*d^2*(d*Cos[a + b*x])^(7/2)) + ((2*(ArcTan[Sqrt[d]*Cos[a + b*x]
]/(2*d^(3/2)) + ArcTanh[Sqrt[d]*Cos[a + b*x]]/(2*d^(3/2))))/d^2 - 2/(3*d^2
*(d*Cos[a + b*x])^(3/2)))/d^2))/b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1056 vs. $2(83) = 166$.

Time = 2.94 (sec) , antiderivative size = 1057, normalized size of antiderivative = 10.26

method	result	size
default	Expression too large to display	1057

input
```
int(csc(b*x+a)/(d*cos(b*x+a))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/42/d^(11/2)/(-d)^(1/2)/(16*sin(1/2*b*x+1/2*a)^8-32*sin(1/2*b*x+1/2*a)^6+
24*sin(1/2*b*x+1/2*a)^4-8*sin(1/2*b*x+1/2*a)^2+1)*(42*ln(2/cos(1/2*b*x+1/2
*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(3/2)-21*ln(-2/(
cos(1/2*b*x+1/2*a)+1)*(2*d*cos(1/2*b*x+1/2*a)-d^(1/2)*(-2*sin(1/2*b*x+1/2*
a)^2*d+d)^(1/2)+d))*(-d)^(1/2)*d-21*ln(2/(cos(1/2*b*x+1/2*a)-1)*(2*d*cos(1
/2*b*x+1/2*a)+d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*(-d)^(1/2)*d
+40*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*(-d)^(1/2)*d^(1/2)-336*(-2*ln(2/co
s(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(3/
2)+ln(-2/(cos(1/2*b*x+1/2*a)+1)*(2*d*cos(1/2*b*x+1/2*a)-d^(1/2)*(-2*sin(1/
2*b*x+1/2*a)^2*d+d)^(1/2)+d))*(-d)^(1/2)*d+ln(2/(cos(1/2*b*x+1/2*a)-1)*(2*
d*cos(1/2*b*x+1/2*a)+d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*(-d)^
(1/2)*d)*sin(1/2*b*x+1/2*a)^8+672*(-2*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*
(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(3/2)+ln(-2/(cos(1/2*b*x+1/2*a)+
1)*(2*d*cos(1/2*b*x+1/2*a)-d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+d))
*(-d)^(1/2)*d+ln(2/(cos(1/2*b*x+1/2*a)-1)*(2*d*cos(1/2*b*x+1/2*a)+d^(1/2)*
(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*(-d)^(1/2)*d)*sin(1/2*b*x+1/2*a)^6
-56*(6*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(
1/2)-d))*d^(3/2)+2*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*(-d)^(1/2)*d^(1/2)-
3*ln(-2/(cos(1/2*b*x+1/2*a)+1)*(2*d*cos(1/2*b*x+1/2*a)-d^(1/2)*(-2*sin(1/2
*b*x+1/2*a)^2*d+d)^(1/2)+d))*(-d)^(1/2)*d-3*ln(2/(cos(1/2*b*x+1/2*a)-1)...
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 333, normalized size of antiderivative = 3.23

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \left[\frac{42\sqrt{-d}\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{-d}}{d\cos(bx+a)+d}\right)\cos(bx+a)^4 - 21\sqrt{-d}\cos(bx+a)^4\log\left(\frac{d}{}\right.}{84}\right.$$

input
```
integrate(csc(b*x+a)/(d*cos(b*x+a))^(9/2),x, algorithm="fricas")
```

output
```
[1/84*(42*sqrt(-d)*arctan(2*sqrt(d*cos(b*x + a))*sqrt(-d)/(d*cos(b*x + a)
+ d))*cos(b*x + a)^4 - 21*sqrt(-d)*cos(b*x + a)^4*log((d*cos(b*x + a)^2 +
4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) - 1) - 6*d*cos(b*x + a) + d)
/(cos(b*x + a)^2 + 2*cos(b*x + a) + 1)) + 8*sqrt(d*cos(b*x + a))*(7*cos(b*
x + a)^2 + 3))/(b*d^5*cos(b*x + a)^4), 1/84*(42*sqrt(d)*arctan(2*sqrt(d*co
s(b*x + a))*sqrt(d)/(d*cos(b*x + a) - d))*cos(b*x + a)^4 + 21*sqrt(d)*cos(
b*x + a)^4*log((d*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))*sqrt(d)*(cos(b*x
 + a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 - 2*cos(b*x + a) + 1))
+ 8*sqrt(d*cos(b*x + a))*(7*cos(b*x + a)^2 + 3))/(b*d^5*cos(b*x + a)^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)/(d*cos(b*x+a))**(9/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.99

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = -\frac{\frac{42\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)}{d^{\frac{7}{2}}} - \frac{21\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)}{d^{\frac{7}{2}}} - \frac{4\left(7\,d^2\cos(bx+a)^2+3\,d^2\right)}{(d\cos(bx+a))^{\frac{7}{2}}d^2}}{42\,bd}$$

input
```
integrate(csc(b*x+a)/(d*cos(b*x+a))^(9/2),x, algorithm="maxima")
```

output
```
-1/42*(42*arctan(sqrt(d*cos(b*x + a))/sqrt(d))/d^(7/2) - 21*log((sqrt(d*co
s(b*x + a)) - sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d)))/d^(7/2) - 4*(7*d^
2*cos(b*x + a)^2 + 3*d^2)/((d*cos(b*x + a))^(7/2)*d^2))/(b*d)
```

Giac [F]

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \int \frac{\csc(bx+a)}{(d\cos(bx+a))^{\frac{9}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)/(d*cos(b*x+a))^(9/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)/(d*cos(b*x + a))^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \int \frac{1}{\sin(a+b\,x)\,\left(d\,\cos(a+b\,x)\right)^{9/2}}\,dx$$

input
```
int(1/(sin(a + b*x)*(d*cos(a + b*x))^(9/2)),x)
```

output
```
int(1/(sin(a + b*x)*(d*cos(a + b*x))^(9/2)), x)
```

Reduce [F]

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\cos(bx+a)}\,\csc(bx+a)}{\cos(bx+a)^5}dx\right)}{d^5}$$

input
```
int(csc(b*x+a)/(d*cos(b*x+a))^(9/2),x)
```

output
```
(sqrt(d)*int((sqrt(cos(a + b*x))*csc(a + b*x))/cos(a + b*x)**5,x))/d**5
```

3.232 $\int (d\cos(a+bx))^{11/2}\csc^2(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 124

$$\int (d\cos(a+bx))^{11/2}\csc^2(a+bx)\,dx = -\frac{d(d\cos(a+bx))^{9/2}\csc(a+bx)}{b} - \frac{15d^6\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{7b\sqrt{d\cos(a+bx)}} - \frac{15d^5\sqrt{d\cos(a+bx)}\sin(a+bx)}{7b} - \frac{9d^3(d\cos(a+bx))^{5/2}\sin(a+bx)}{7b}$$

output
```
-d*(d*cos(b*x+a))^(9/2)*csc(b*x+a)/b-15/7*d^6*cos(b*x+a)^(1/2)*InverseJaco
biAM(1/2*a+1/2*b*x,2^(1/2))/b/(d*cos(b*x+a))^(1/2)-15/7*d^5*(d*cos(b*x+a))
^(1/2)*sin(b*x+a)/b-9/7*d^3*(d*cos(b*x+a))^(5/2)*sin(b*x+a)/b
```

Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.72

$$\int (d\cos(a+bx))^{11/2}\csc^2(a+bx)\,dx = \frac{d^5\sqrt{d\cos(a+bx)}\csc(a+bx)\left(\sqrt{\cos(a+bx)}(-45+16\cos(2(a+bx))+\cos(4(a+bx)))-60\,\text{E}\right.}{28b\sqrt{\cos(a+bx)}}$$

input
```
Integrate[(d*Cos[a + b*x])^(11/2)*Csc[a + b*x]^2,x]
```

output
```
(d^5*Sqrt[d*Cos[a + b*x]]*Csc[a + b*x]*(Sqrt[Cos[a + b*x]]*(-45 + 16*Cos[2
*(a + b*x)] + Cos[4*(a + b*x)]) - 60*EllipticF[(a + b*x)/2, 2]*Sin[a + b*x
]))/(28*b*Sqrt[Cos[a + b*x]])
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3047, 3042, 3115, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)(d\cos(a+bx))^{11/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\cos(a+bx))^{11/2}}{\sin(a+bx)^2}dx$$

$$\Big\downarrow \text{3047}$$

$$-\frac{9}{2}d^2\int (d\cos(a+bx))^{7/2}dx-\frac{d\csc(a+bx)(d\cos(a+bx))^{9/2}}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{9}{2}d^2\int \left(d\sin\left(a+bx+\frac{\pi}{2}\right)\right)^{7/2}dx-\frac{d\csc(a+bx)(d\cos(a+bx))^{9/2}}{b}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{9}{2}d^2\left(\frac{5}{7}d^2\int (d\cos(a+bx))^{3/2}dx+\frac{2d\sin(a+bx)(d\cos(a+bx))^{5/2}}{7b}\right)-\frac{d\csc(a+bx)(d\cos(a+bx))^{9/2}}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{9}{2}d^2\left(\frac{5}{7}d^2\int\left(d\sin\left(a+bx+\frac{\pi}{2}\right)\right)^{3/2}dx+\frac{2d\sin(a+bx)(d\cos(a+bx))^{5/2}}{7b}\right)-\frac{d\csc(a+bx)(d\cos(a+bx))^{9/2}}{b}$$

$\big\downarrow$ 3115

$$-\frac{9}{2}d^2\left(\frac{5}{7}d^2\left(\frac{1}{3}d^2\int\frac{1}{\sqrt{d\cos(a+bx)}}dx+\frac{2d\sin(a+bx)\sqrt{d\cos(a+bx)}}{3b}\right)+\frac{2d\sin(a+bx)(d\cos(a+bx))^{5/2}}{7b}\right)-\frac{d\csc(a+bx)(d\cos(a+bx))^{9/2}}{b}$$

$\big\downarrow$ 3042

$$-\frac{9}{2}d^2\left(\frac{5}{7}d^2\left(\frac{1}{3}d^2\int\frac{1}{\sqrt{d\sin\left(a+bx+\frac{\pi}{2}\right)}}dx+\frac{2d\sin(a+bx)\sqrt{d\cos(a+bx)}}{3b}\right)+\frac{2d\sin(a+bx)(d\cos(a+bx))^{5}}{7b}\right.$$
$$\frac{d\csc(a+bx)(d\cos(a+bx))^{9/2}}{b}$$

$\big\downarrow$ 3121

$$-\frac{9}{2}d^2\left(\frac{5}{7}d^2\left(\frac{d^2\sqrt{\cos(a+bx)}\int\frac{1}{\sqrt{\cos(a+bx)}}dx}{3\sqrt{d\cos(a+bx)}}+\frac{2d\sin(a+bx)\sqrt{d\cos(a+bx)}}{3b}\right)+\frac{2d\sin(a+bx)(d\cos(a+bx)}{7b}\right.$$
$$\frac{d\csc(a+bx)(d\cos(a+bx))^{9/2}}{b}$$

$\big\downarrow$ 3042

$$-\frac{9}{2}d^2\left(\frac{5}{7}d^2\left(\frac{d^2\sqrt{\cos(a+bx)}\int\frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx}{3\sqrt{d\cos(a+bx)}}+\frac{2d\sin(a+bx)\sqrt{d\cos(a+bx)}}{3b}\right)+\frac{2d\sin(a+bx)(d\cos(a+b}{7b}\right.$$
$$\frac{d\csc(a+bx)(d\cos(a+bx))^{9/2}}{b}$$

$\big\downarrow$ 3120

$$-\frac{9}{2}d^2\left(\frac{5}{7}d^2\left(\frac{2d^2\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b\sqrt{d\cos(a+bx)}}+\frac{2d\sin(a+bx)\sqrt{d\cos(a+bx)}}{3b}\right)+\frac{2d\sin(a+bx)(d\cos}{7b}\right.$$
$$\frac{d\csc(a+bx)(d\cos(a+bx))^{9/2}}{b}$$

input
```
Int[(d*Cos[a + b*x])^(11/2)*Csc[a + b*x]^2,x]
```

output
```
-((d*(d*Cos[a + b*x])^(9/2)*Csc[a + b*x])/b) - (9*d^2*((2*d*(d*Cos[a + b*x
])^(5/2)*Sin[a + b*x])/(7*b) + (5*d^2*((2*d^2*Sqrt[Cos[a + b*x]]*EllipticF
[(a + b*x)/2, 2])/(3*b*Sqrt[d*Cos[a + b*x]]) + (2*d*Sqrt[d*Cos[a + b*x]]*S
in[a + b*x])/(3*b)))/7))/2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3047
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Cos[e + f*x]
)^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(109) = 218$.

Time = 9.92 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.95

method	result
default	$-\frac{\sqrt{d\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\, d^{7}\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\left(-128\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{12}+384\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{10}-576\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{8}+30\cos\left(\frac{bx}{2}\right.\right.}{14\left(-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}d+\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}d\right.}$

input
```
int((d*cos(b*x+a))^(11/2)*csc(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/14*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^7/(-2*si
n(1/2*b*x+1/2*a)^4*d+sin(1/2*b*x+1/2*a)^2*d)^(3/2)/cos(1/2*b*x+1/2*a)*sin(
1/2*b*x+1/2*a)*(-128*sin(1/2*b*x+1/2*a)^12+384*sin(1/2*b*x+1/2*a)^10-576*s
in(1/2*b*x+1/2*a)^8+30*cos(1/2*b*x+1/2*a)*(2*sin(1/2*b*x+1/2*a)^2-1)^(3/2)
*(sin(1/2*b*x+1/2*a)^2)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))+512*si
n(1/2*b*x+1/2*a)^6-204*sin(1/2*b*x+1/2*a)^4+12*sin(1/2*b*x+1/2*a)^2+7)/(d*
(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98

$$\int (d\cos(a+bx))^{11/2}\csc^2(a+bx)\,dx = \frac{15i\sqrt{\frac{1}{2}}d^{\frac{11}{2}}\sin(bx+a)\,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)) - 15i\sqrt{\frac{1}{2}}d^{\frac{11}{2}}\sin}{}$$

input
```
integrate((d*cos(b*x+a))^(11/2)*csc(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/7*(15*I*sqrt(1/2)*d^(11/2)*sin(b*x + a)*weierstrassPInverse(-4, 0, cos(b
*x + a) + I*sin(b*x + a)) - 15*I*sqrt(1/2)*d^(11/2)*sin(b*x + a)*weierstra
ssPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a)) + (2*d^5*cos(b*x + a)^4 +
 6*d^5*cos(b*x + a)^2 - 15*d^5)*sqrt(d*cos(b*x + a)))/(b*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{11/2}\csc^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*cos(b*x+a))**(11/2)*csc(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\cos(a+bx))^{11/2}\csc^2(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{11}{2}}\csc(bx+a)^2\,dx$$

input
```
integrate((d*cos(b*x+a))^(11/2)*csc(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*cos(b*x + a))^(11/2)*csc(b*x + a)^2, x)
```

Giac [F]

$$\int (d\cos(a+bx))^{11/2}\csc^2(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{11}{2}}\csc(bx+a)^2\,dx$$

input
```
integrate((d*cos(b*x+a))^(11/2)*csc(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*cos(b*x + a))^(11/2)*csc(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{11/2}\csc^2(a+bx)\,dx = \int \frac{(d\,\cos\left(a+b\,x\right))^{11/2}}{\sin\left(a+b\,x\right)^2}\,dx$$

input
```
int((d*cos(a + b*x))^(11/2)/sin(a + b*x)^2,x)
```

output
```
int((d*cos(a + b*x))^(11/2)/sin(a + b*x)^2, x)
```

Reduce [F]

$$\int (d\cos(a+bx))^{11/2}\csc^2(a+bx)\,dx = \sqrt{d}\left(\int \sqrt{\cos\left(bx+a\right)}\,\cos\left(bx+a\right)^5\csc\left(bx+a\right)^2dx\right)d^5$$

input
```
int((d*cos(b*x+a))^(11/2)*csc(b*x+a)^2,x)
```

output
```
sqrt(d)*int(sqrt(cos(a + b*x))*cos(a + b*x)**5*csc(a + b*x)**2,x)*d**5
```

3.233 $\int (d\cos(a+bx))^{9/2}\csc^2(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 96

$$\int (d\cos(a+bx))^{9/2}\csc^2(a+bx)\,dx = -\frac{d(d\cos(a+bx))^{7/2}\csc(a+bx)}{b} - \frac{21d^4\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b\sqrt{\cos(a+bx)}} - \frac{7d^3(d\cos(a+bx))^{3/2}\sin(a+bx)}{5b}$$

output
```
-d*(d*cos(b*x+a))^(7/2)*csc(b*x+a)/b-21/5*d^4*(d*cos(b*x+a))^(1/2)*Ellipti
cE(sin(1/2*a+1/2*b*x),2^(1/2))/b/cos(b*x+a)^(1/2)-7/5*d^3*(d*cos(b*x+a))^(
3/2)*sin(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.77

$$\int (d\cos(a+bx))^{9/2}\csc^2(a+bx)\,dx = -\frac{d^4\sqrt{d\cos(a+bx)}\left(21E\left(\frac{1}{2}(a+bx)\middle|2\right)+\sqrt{\cos(a+bx)}(5\cot(a+bx)+\sin(2(a+bx)))\right)}{5b\sqrt{\cos(a+bx)}}$$

input
```
Integrate[(d*Cos[a + b*x])^(9/2)*Csc[a + b*x]^2,x]
```

output
```
-1/5*(d^4*Sqrt[d*Cos[a + b*x]]*(21*EllipticE[(a + b*x)/2, 2] + Sqrt[Cos[a
+ b*x]]*(5*Cot[a + b*x] + Sin[2*(a + b*x)])))/(b*Sqrt[Cos[a + b*x]])
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3047, 3042, 3115, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)(d\cos(a+bx))^{9/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\cos(a+bx))^{9/2}}{\sin(a+bx)^2}dx$$

$$\Big\downarrow \text{3047}$$

$$-\frac{7}{2}d^2\int (d\cos(a+bx))^{5/2}dx-\frac{d\csc(a+bx)(d\cos(a+bx))^{7/2}}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{7}{2}d^2\int \left(d\sin\left(a+bx+\frac{\pi}{2}\right)\right)^{5/2}dx-\frac{d\csc(a+bx)(d\cos(a+bx))^{7/2}}{b}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{7}{2}d^2\left(\frac{3}{5}d^2\int \sqrt{d\cos(a+bx)}dx+\frac{2d\sin(a+bx)(d\cos(a+bx))^{3/2}}{5b}\right)-\frac{d\csc(a+bx)(d\cos(a+bx))^{7/2}}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{7}{2}d^2\left(\frac{3}{5}d^2\int \sqrt{d\sin\left(a+bx+\frac{\pi}{2}\right)}dx+\frac{2d\sin(a+bx)(d\cos(a+bx))^{3/2}}{5b}\right)-\frac{d\csc(a+bx)(d\cos(a+bx))^{7/2}}{b}$$

$$\Big\downarrow \text{3121}$$

$$-\frac{7}{2}d^2\left(\frac{3d^2\sqrt{d\cos(a+bx)}\int\sqrt{\cos(a+bx)}dx}{5\sqrt{\cos(a+bx)}}+\frac{2d\sin(a+bx)(d\cos(a+bx))^{3/2}}{5b}\right)-\frac{d\csc(a+bx)(d\cos(a+bx))^{7/2}}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{7}{2}d^2\left(\frac{3d^2\sqrt{d\cos(a+bx)}\int\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(a+bx)}}+\frac{2d\sin(a+bx)(d\cos(a+bx))^{3/2}}{5b}\right)-\frac{d\csc(a+bx)(d\cos(a+bx))^{7/2}}{b}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{7}{2}d^2\left(\frac{6d^2E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{5b\sqrt{\cos(a+bx)}}+\frac{2d\sin(a+bx)(d\cos(a+bx))^{3/2}}{5b}\right)-\frac{d\csc(a+bx)(d\cos(a+bx))^{7/2}}{b}$$

input
```
Int[(d*Cos[a + b*x])^(9/2)*Csc[a + b*x]^2,x]
```

output
```
-((d*(d*Cos[a + b*x])^(7/2)*Csc[a + b*x])/b) - (7*d^2*((6*d^2*Sqrt[d*Cos[a
 + b*x]]*EllipticE[(a + b*x)/2, 2])/(5*b*Sqrt[Cos[a + b*x]]) + (2*d*(d*Cos
[a + b*x])^(3/2)*Sin[a + b*x])/(5*b)))/2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3047
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Cos[e + f*x]
)^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(86) = 172$.

Time = 8.55 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.39

method	result
default	$\frac{\sqrt{d\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\, d^{6}\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\left(-64\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{10}+160\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{8}+42\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\left(2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right.\right.}{10\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\left(-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}d+\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}d\right)}$

input
```
int((d*cos(b*x+a))^(9/2)*csc(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/10*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^6/cos(1/2
*b*x+1/2*a)/(-2*sin(1/2*b*x+1/2*a)^4*d+sin(1/2*b*x+1/2*a)^2*d)^(3/2)*sin(1
/2*b*x+1/2*a)*(-64*sin(1/2*b*x+1/2*a)^10+160*sin(1/2*b*x+1/2*a)^8+42*cos(1
/2*b*x+1/2*a)*(2*sin(1/2*b*x+1/2*a)^2-1)^(3/2)*EllipticE(cos(1/2*b*x+1/2*a
),2^(1/2))*(sin(1/2*b*x+1/2*a)^2)^(1/2)-104*sin(1/2*b*x+1/2*a)^6-4*sin(1/2
*b*x+1/2*a)^4+22*sin(1/2*b*x+1/2*a)^2-5)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1
/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.25

$$\int (d\cos(a+bx))^{9/2}\csc^2(a+bx)\,dx = \frac{-21i\sqrt{\frac{1}{2}}d^{\frac{9}{2}}\sin(bx+a)\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(b$$

input
```
integrate((d*cos(b*x+a))^(9/2)*csc(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/5*(-21*I*sqrt(1/2)*d^(9/2)*sin(b*x + a)*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(b*x + a) + I*sin(b*x + a))) + 21*I*sqrt(1/2)*d^(9/2)
*sin(b*x + a)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x +
a) - I*sin(b*x + a))) + (2*d^4*cos(b*x + a)^3 - 7*d^4*cos(b*x + a))*sqrt(d
*cos(b*x + a)))/(b*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{9/2}\csc^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*cos(b*x+a))**(9/2)*csc(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\cos(a+bx))^{9/2}\csc^2(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{9}{2}}\csc(bx+a)^2\,dx$$

input
```
integrate((d*cos(b*x+a))^(9/2)*csc(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*cos(b*x + a))^(9/2)*csc(b*x + a)^2, x)
```

Giac [F]

$$\int (d\cos(a+bx))^{9/2}\csc^2(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{9}{2}}\csc(bx+a)^2\,dx$$

input
```
integrate((d*cos(b*x+a))^(9/2)*csc(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*cos(b*x + a))^(9/2)*csc(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{9/2}\csc^2(a+bx)\,dx = \int \frac{(d\cos(a+bx))^{9/2}}{\sin(a+bx)^2}\,dx$$

input
```
int((d*cos(a + b*x))^(9/2)/sin(a + b*x)^2,x)
```

output
```
int((d*cos(a + b*x))^(9/2)/sin(a + b*x)^2, x)
```

Reduce [F]

$$\int (d\cos(a+bx))^{9/2}\csc^2(a+bx)\,dx = \sqrt{d}\left(\int \sqrt{\cos(bx+a)}\,\cos(bx+a)^4\csc(bx+a)^2\,dx\right)d^4$$

input
```
int((d*cos(b*x+a))^(9/2)*csc(b*x+a)^2,x)
```

output
```
sqrt(d)*int(sqrt(cos(a + b*x))*cos(a + b*x)**4*csc(a + b*x)**2,x)*d**4
```

3.234 $\int (d\cos(a+bx))^{7/2}\csc^2(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 96

$$\int (d\cos(a+bx))^{7/2}\csc^2(a+bx)\,dx = -\frac{d(d\cos(a+bx))^{5/2}\csc(a+bx)}{b} - \frac{5d^4\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b\sqrt{d\cos(a+bx)}} - \frac{5d^3\sqrt{d\cos(a+bx)}\sin(a+bx)}{3b}$$

output
```
-d*(d*cos(b*x+a))^(5/2)*csc(b*x+a)/b-5/3*d^4*cos(b*x+a)^(1/2)*InverseJacob
iAM(1/2*a+1/2*b*x,2^(1/2))/b/(d*cos(b*x+a))^(1/2)-5/3*d^3*(d*cos(b*x+a))^(
1/2)*sin(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.76

$$\int (d\cos(a+bx))^{7/2}\csc^2(a+bx)\,dx = \frac{d^3\sqrt{d\cos(a+bx)}\left(\sqrt{\cos(a+bx)}(-4+\cos(2(a+bx)))\csc(a+bx) - 5\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)\right)}{3b\sqrt{\cos(a+bx)}}$$

input
```
Integrate[(d*Cos[a + b*x])^(7/2)*Csc[a + b*x]^2,x]
```

output
```
(d^3*Sqrt[d*Cos[a + b*x]]*(Sqrt[Cos[a + b*x]]*(-4 + Cos[2*(a + b*x)])*Csc[
a + b*x] - 5*EllipticF[(a + b*x)/2, 2]))/(3*b*Sqrt[Cos[a + b*x]])
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3047, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)(d\cos(a+bx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\cos(a+bx))^{7/2}}{\sin(a+bx)^2}dx$$

$$\Big\downarrow \text{3047}$$

$$-\frac{5}{2}d^2\int (d\cos(a+bx))^{3/2}dx-\frac{d\csc(a+bx)(d\cos(a+bx))^{5/2}}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5}{2}d^2\int \left(d\sin\left(a+bx+\frac{\pi}{2}\right)\right)^{3/2}dx-\frac{d\csc(a+bx)(d\cos(a+bx))^{5/2}}{b}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{5}{2}d^2\left(\frac{1}{3}d^2\int \frac{1}{\sqrt{d\cos(a+bx)}}dx+\frac{2d\sin(a+bx)\sqrt{d\cos(a+bx)}}{3b}\right)-\frac{d\csc(a+bx)(d\cos(a+bx))^{5/2}}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5}{2}d^2\left(\frac{1}{3}d^2\int \frac{1}{\sqrt{d\sin\left(a+bx+\frac{\pi}{2}\right)}}dx+\frac{2d\sin(a+bx)\sqrt{d\cos(a+bx)}}{3b}\right)-\frac{d\csc(a+bx)(d\cos(a+bx))^{5/2}}{b}$$

$$\Big\downarrow \text{3121}$$

$$-\frac{5}{2}d^2\left(\frac{d^2\sqrt{\cos(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}}dx}{3\sqrt{d\cos(a+bx)}}+\frac{2d\sin(a+bx)\sqrt{d\cos(a+bx)}}{3b}\right)-\frac{d\csc(a+bx)(d\cos(a+bx))^{5/2}}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5}{2}d^2\left(\frac{d^2\sqrt{\cos(a+bx)}\int \frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx}{3\sqrt{d\cos(a+bx)}}+\frac{2d\sin(a+bx)\sqrt{d\cos(a+bx)}}{3b}\right)-\frac{d\csc(a+bx)(d\cos(a+bx))^{5/2}}{b}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{5}{2}d^2\left(\frac{2d^2\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b\sqrt{d\cos(a+bx)}}+\frac{2d\sin(a+bx)\sqrt{d\cos(a+bx)}}{3b}\right)-\frac{d\csc(a+bx)(d\cos(a+bx))^{5/2}}{b}$$

input
```
Int[(d*Cos[a + b*x])^(7/2)*Csc[a + b*x]^2,x]
```

output
```
-((d*(d*Cos[a + b*x])^(5/2)*Csc[a + b*x])/b) - (5*d^2*((2*d^2*Sqrt[Cos[a +
 b*x]]*EllipticF[(a + b*x)/2, 2])/(3*b*Sqrt[d*Cos[a + b*x]]) + (2*d*Sqrt[d
*Cos[a + b*x]]*Sin[a + b*x])/(3*b)))/2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3047
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Cos[e + f*x]
)^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(85) = 170$.

Time = 7.86 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.25

method	result
default	$-\frac{\sqrt{d\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\, d^{5}\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\left(-32\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{8}+10\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\left(2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)^{\frac{3}{2}}\sqrt{\frac{1}{2}-\frac{\cos(bx+}{2}}}{6\left(-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}d+\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}d\right)^{\frac{3}{2}}\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{d\left(2\right.}}$

input
```
int((d*cos(b*x+a))^(7/2)*csc(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^5/(-2*sin
(1/2*b*x+1/2*a)^4*d+sin(1/2*b*x+1/2*a)^2*d)^(3/2)/cos(1/2*b*x+1/2*a)*sin(1
/2*b*x+1/2*a)*(-32*sin(1/2*b*x+1/2*a)^8+10*cos(1/2*b*x+1/2*a)*(2*sin(1/2*b
*x+1/2*a)^2-1)^(3/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2)*EllipticF(cos(1/2*b*x+1/
2*a),2^(1/2))+64*sin(1/2*b*x+1/2*a)^6-28*sin(1/2*b*x+1/2*a)^4-4*sin(1/2*b*
x+1/2*a)^2+3)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.12

$$\int (d\cos(a+bx))^{7/2}\csc^2(a+bx)\,dx = \frac{5i\sqrt{\frac{1}{2}}d^{\frac{7}{2}}\sin(bx+a)\,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)) - 5i\sqrt{\frac{1}{2}}d^{\frac{7}{2}}\sin(bx}{3}$$

input
```
integrate((d*cos(b*x+a))^(7/2)*csc(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/3*(5*I*sqrt(1/2)*d^(7/2)*sin(b*x + a)*weierstrassPInverse(-4, 0, cos(b*x
 + a) + I*sin(b*x + a)) - 5*I*sqrt(1/2)*d^(7/2)*sin(b*x + a)*weierstrassPI
nverse(-4, 0, cos(b*x + a) - I*sin(b*x + a)) + (2*d^3*cos(b*x + a)^2 - 5*d
^3)*sqrt(d*cos(b*x + a)))/(b*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{7/2}\csc^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*cos(b*x+a))**(7/2)*csc(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\cos(a+bx))^{7/2}\csc^2(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{7}{2}}\csc(bx+a)^2\,dx$$

input
```
integrate((d*cos(b*x+a))^(7/2)*csc(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*cos(b*x + a))^(7/2)*csc(b*x + a)^2, x)
```

Giac [F]

$$\int (d\cos(a+bx))^{7/2}\csc^2(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{7}{2}}\csc(bx+a)^2\,dx$$

input
```
integrate((d*cos(b*x+a))^(7/2)*csc(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*cos(b*x + a))^(7/2)*csc(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{7/2}\csc^2(a+bx)\,dx = \int \frac{(d\,\cos(a+b\,x))^{7/2}}{\sin(a+b\,x)^2}\,dx$$

input
```
int((d*cos(a + b*x))^(7/2)/sin(a + b*x)^2,x)
```

output
```
int((d*cos(a + b*x))^(7/2)/sin(a + b*x)^2, x)
```

Reduce [F]

$$\int (d\cos(a+bx))^{7/2}\csc^2(a+bx)\,dx = \sqrt{d}\left(\int \sqrt{\cos(bx+a)}\,\cos(bx+a)^3\csc(bx+a)^2\,dx\right)d^3$$

input
```
int((d*cos(b*x+a))^(7/2)*csc(b*x+a)^2,x)
```

output
```
sqrt(d)*int(sqrt(cos(a + b*x))*cos(a + b*x)**3*csc(a + b*x)**2,x)*d**3
```

3.235 $\int (d\cos(a+bx))^{5/2}\csc^2(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 66

$$\int (d\cos(a+bx))^{5/2}\csc^2(a+bx)\,dx = -\frac{d(d\cos(a+bx))^{3/2}\csc(a+bx)}{b} - \frac{3d^2\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{\cos(a+bx)}}$$

output
```
-d*(d*cos(b*x+a))^(3/2)*csc(b*x+a)/b-3*d^2*(d*cos(b*x+a))^(1/2)*EllipticE(
sin(1/2*a+1/2*b*x),2^(1/2))/b/cos(b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.88

$$\int (d\cos(a+bx))^{5/2}\csc^2(a+bx)\,dx = -\frac{(d\cos(a+bx))^{5/2}\left(\cos^{\frac{3}{2}}(a+bx)\csc(a+bx)+3E\left(\frac{1}{2}(a+bx)\middle|2\right)\right)}{b\cos^{\frac{5}{2}}(a+bx)}$$

input
```
Integrate[(d*Cos[a + b*x])^(5/2)*Csc[a + b*x]^2,x]
```

output
```
-(((d*Cos[a + b*x])^(5/2)*(Cos[a + b*x]^(3/2)*Csc[a + b*x] + 3*EllipticE[(
a + b*x)/2, 2]))/(b*Cos[a + b*x]^(5/2)))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3047, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)(d\cos(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\cos(a+bx))^{5/2}}{\sin(a+bx)^2}dx$$

$$\Big\downarrow \text{3047}$$

$$-\frac{3}{2}d^2\int \sqrt{d\cos(a+bx)}dx-\frac{d\csc(a+bx)(d\cos(a+bx))^{3/2}}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3}{2}d^2\int \sqrt{d\sin\left(a+bx+\frac{\pi}{2}\right)}dx-\frac{d\csc(a+bx)(d\cos(a+bx))^{3/2}}{b}$$

$$\Big\downarrow \text{3121}$$

$$-\frac{3d^2\sqrt{d\cos(a+bx)}\int \sqrt{\cos(a+bx)}dx}{2\sqrt{\cos(a+bx)}}-\frac{d\csc(a+bx)(d\cos(a+bx))^{3/2}}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3d^2\sqrt{d\cos(a+bx)}\int \sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx}{2\sqrt{\cos(a+bx)}}-\frac{d\csc(a+bx)(d\cos(a+bx))^{3/2}}{b}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{3d^2E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{b\sqrt{\cos(a+bx)}}-\frac{d\csc(a+bx)(d\cos(a+bx))^{3/2}}{b}$$

input
```
Int[(d*Cos[a + b*x])^(5/2)*Csc[a + b*x]^2,x]
```

output
```
-((d*(d*Cos[a + b*x])^(3/2)*Csc[a + b*x])/b) - (3*d^2*Sqrt[d*Cos[a + b*x]]
*EllipticE[(a + b*x)/2, 2])/(b*Sqrt[Cos[a + b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3047
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Cos[e + f*x]
)^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(62) = 124$.

Time = 6.53 (sec) , antiderivative size = 203, normalized size of antiderivative = 3.08

method	result
default	$\frac{\sqrt{d\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^2}\, d^4 \sin\left(\frac{bx}{2}+\frac{a}{2}\right)\left(6\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\left(2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1\right)^{\frac{3}{2}} \operatorname{EllipticE}\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\cos}\right.}{2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\left(-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^4 d+\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^2 d\right)^{\frac{3}{2}}\sqrt{d\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right.}}$

input
```
int((d*cos(b*x+a))^(5/2)*csc(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^4/cos(1/2*
b*x+1/2*a)/(-2*sin(1/2*b*x+1/2*a)^4*d+sin(1/2*b*x+1/2*a)^2*d)^(3/2)*sin(1/
2*b*x+1/2*a)*(6*cos(1/2*b*x+1/2*a)*(2*sin(1/2*b*x+1/2*a)^2-1)^(3/2)*Ellipt
icE(cos(1/2*b*x+1/2*a),2^(1/2))*(sin(1/2*b*x+1/2*a)^2)^(1/2)+8*sin(1/2*b*x
+1/2*a)^6-12*sin(1/2*b*x+1/2*a)^4+6*sin(1/2*b*x+1/2*a)^2-1)/(d*(2*cos(1/2*
b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.58

$$\int (d\cos(a+bx))^{5/2}\csc^2(a+bx)\,dx = \frac{-3i\,\sqrt{\frac{1}{2}}d^{\frac{5}{2}}\sin{(bx+a)}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos{(bx+a)}+i\,\sin{(bx}}{}$$

input
```
integrate((d*cos(b*x+a))^(5/2)*csc(b*x+a)^2,x, algorithm="fricas")
```

output
```
(-3*I*sqrt(1/2)*d^(5/2)*sin(b*x + a)*weierstrassZeta(-4, 0, weierstrassPIn
verse(-4, 0, cos(b*x + a) + I*sin(b*x + a))) + 3*I*sqrt(1/2)*d^(5/2)*sin(b
*x + a)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) - I
*sin(b*x + a))) - sqrt(d*cos(b*x + a))*d^2*cos(b*x + a))/(b*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{5/2}\csc^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*cos(b*x+a))**(5/2)*csc(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\cos(a+bx))^{5/2}\csc^2(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{5}{2}}\csc(bx+a)^2\,dx$$

input
```
integrate((d*cos(b*x+a))^(5/2)*csc(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*cos(b*x + a))^(5/2)*csc(b*x + a)^2, x)
```

Giac [F]

$$\int (d\cos(a+bx))^{5/2}\csc^2(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{5}{2}}\csc(bx+a)^2\,dx$$

input
```
integrate((d*cos(b*x+a))^(5/2)*csc(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*cos(b*x + a))^(5/2)*csc(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{5/2}\csc^2(a+bx)\,dx = \int \frac{(d\cos(a+b\,x))^{5/2}}{\sin(a+b\,x)^2}\,dx$$

input
```
int((d*cos(a + b*x))^(5/2)/sin(a + b*x)^2,x)
```

output
```
int((d*cos(a + b*x))^(5/2)/sin(a + b*x)^2, x)
```

Reduce [F]

$$\int (d\cos(a+bx))^{5/2}\csc^2(a+bx)\,dx = \sqrt{d}\left(\int \sqrt{\cos(bx+a)}\,\cos(bx+a)^2\csc(bx+a)^2\,dx\right)d^2$$

input
```
int((d*cos(b*x+a))^(5/2)*csc(b*x+a)^2,x)
```

output
```
sqrt(d)*int(sqrt(cos(a + b*x))*cos(a + b*x)**2*csc(a + b*x)**2,x)*d**2
```

3.236 $\int (d\cos(a+bx))^{3/2}\csc^2(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 66

$$\int (d\cos(a+bx))^{3/2}\csc^2(a+bx)\,dx = -\frac{d\sqrt{d\cos(a+bx)}\csc(a+bx)}{b} - \frac{d^2\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{b\sqrt{d\cos(a+bx)}}$$

output
```
-d*(d*cos(b*x+a))^(1/2)*csc(b*x+a)/b-d^2*cos(b*x+a)^(1/2)*InverseJacobiAM(
1/2*a+1/2*b*x,2^(1/2))/b/(d*cos(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int (d\cos(a+bx))^{3/2}\csc^2(a+bx)\,dx = -\frac{(d\cos(a+bx))^{3/2}\left(\sqrt{\cos(a+bx)}\csc(a+bx)+\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)\right)}{b\cos^{\frac{3}{2}}(a+bx)}$$

input
```
Integrate[(d*Cos[a + b*x])^(3/2)*Csc[a + b*x]^2,x]
```

output
```
-(((d*Cos[a + b*x])^(3/2)*(Sqrt[Cos[a + b*x]]*Csc[a + b*x] + EllipticF[(a
+ b*x)/2, 2]))/(b*Cos[a + b*x]^(3/2)))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3047, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)(d\cos(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\cos(a+bx))^{3/2}}{\sin(a+bx)^2}dx$$

$$\Big\downarrow \text{3047}$$

$$-\frac{1}{2}d^2\int \frac{1}{\sqrt{d\cos(a+bx)}}dx-\frac{d\csc(a+bx)\sqrt{d\cos(a+bx)}}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}d^2\int \frac{1}{\sqrt{d\sin\left(a+bx+\frac{\pi}{2}\right)}}dx-\frac{d\csc(a+bx)\sqrt{d\cos(a+bx)}}{b}$$

$$\Big\downarrow \text{3121}$$

$$-\frac{d^2\sqrt{\cos(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}}dx}{2\sqrt{d\cos(a+bx)}}-\frac{d\csc(a+bx)\sqrt{d\cos(a+bx)}}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{d^2\sqrt{\cos(a+bx)}\int \frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx}{2\sqrt{d\cos(a+bx)}}-\frac{d\csc(a+bx)\sqrt{d\cos(a+bx)}}{b}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{d^2\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{b\sqrt{d\cos(a+bx)}}-\frac{d\csc(a+bx)\sqrt{d\cos(a+bx)}}{b}$$

input
```
Int[(d*Cos[a + b*x])^(3/2)*Csc[a + b*x]^2,x]
```

output
```
-((d*Sqrt[d*Cos[a + b*x]]*Csc[a + b*x])/b) - (d^2*Sqrt[Cos[a + b*x]]*Ellip
ticF[(a + b*x)/2, 2])/(b*Sqrt[d*Cos[a + b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3047
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Cos[e + f*x]
)^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(61) = 122.

Time = 4.50 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.88

method	result
default	$-\frac{\sqrt{d\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\, d^{3}\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\left(2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)^{\frac{3}{2}}\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\text{EllipticF}\left(\cos\left(\frac{bx}{2}\right.\right.\right.}{2\left(-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}d+\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}d\right)^{\frac{3}{2}}\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{d\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)}\, b}$

input
```
int((d*cos(b*x+a))^(3/2)*csc(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^3/(-2*sin
(1/2*b*x+1/2*a)^4*d+sin(1/2*b*x+1/2*a)^2*d)^(3/2)/cos(1/2*b*x+1/2*a)*sin(1
/2*b*x+1/2*a)*(2*cos(1/2*b*x+1/2*a)*(2*sin(1/2*b*x+1/2*a)^2-1)^(3/2)*(sin(
1/2*b*x+1/2*a)^2)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))+4*sin(1/2*b*
x+1/2*a)^4-4*sin(1/2*b*x+1/2*a)^2+1)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/
b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.36

$$\int (d\cos(a+bx))^{3/2}\csc^2(a+bx)\,dx = \frac{i\sqrt{\frac{1}{2}}d^{\frac{3}{2}}\sin(bx+a)\,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a))-i\sqrt{\frac{1}{2}}d^{\frac{3}{2}}\sin(bx+}{b\sin(bx+a)}$$

input
```
integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a)^2,x, algorithm="fricas")
```

output
```
(I*sqrt(1/2)*d^(3/2)*sin(b*x + a)*weierstrassPInverse(-4, 0, cos(b*x + a)
+ I*sin(b*x + a)) - I*sqrt(1/2)*d^(3/2)*sin(b*x + a)*weierstrassPInverse(-
4, 0, cos(b*x + a) - I*sin(b*x + a)) - sqrt(d*cos(b*x + a))*d)/(b*sin(b*x
+ a))
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{3/2}\csc^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*cos(b*x+a))**(3/2)*csc(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\cos(a+bx))^{3/2}\csc^2(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{3}{2}}\csc(bx+a)^2\,dx$$

input
```
integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*cos(b*x + a))^(3/2)*csc(b*x + a)^2, x)
```

Giac [F]

$$\int (d\cos(a+bx))^{3/2}\csc^2(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{3}{2}}\csc(bx+a)^2\,dx$$

input
```
integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*cos(b*x + a))^(3/2)*csc(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{3/2}\csc^2(a+bx)\,dx = \int \frac{(d\,\cos(a+b\,x))^{3/2}}{\sin(a+b\,x)^2}\,dx$$

input
```
int((d*cos(a + b*x))^(3/2)/sin(a + b*x)^2,x)
```

output
```
int((d*cos(a + b*x))^(3/2)/sin(a + b*x)^2, x)
```

Reduce [F]

$$\int (d\cos(a+bx))^{3/2}\csc^2(a+bx)\,dx = \sqrt{d}\left(\int \sqrt{\cos(bx+a)}\,\cos(bx+a)\csc(bx+a)^2\,dx\right)d$$

input
```
int((d*cos(b*x+a))^(3/2)*csc(b*x+a)^2,x)
```

output
```
sqrt(d)*int(sqrt(cos(a + b*x))*cos(a + b*x)*csc(a + b*x)**2,x)*d
```

3.237 $\int \sqrt{d\cos(a+bx)}\csc^2(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \sqrt{d\cos(a+bx)}\csc^2(a+bx)\,dx = -\frac{(d\cos(a+bx))^{3/2}\csc(a+bx)}{bd} - \frac{\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{\cos(a+bx)}}$$

output
```
-(d*cos(b*x+a))^(3/2)*csc(b*x+a)/b/d-(d*cos(b*x+a))^(1/2)*EllipticE(sin(1/
2*a+1/2*b*x),2^(1/2))/b/cos(b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int \sqrt{d\cos(a+bx)}\csc^2(a+bx)\,dx = -\frac{\sqrt{d\cos(a+bx)}\left(\cos^{\frac{3}{2}}(a+bx)\csc(a+bx)+E\left(\frac{1}{2}(a+bx)\middle|2\right)\right)}{b\sqrt{\cos(a+bx)}}$$

input
```
Integrate[Sqrt[d*Cos[a + b*x]]*Csc[a + b*x]^2,x]
```

output
```
-((Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^(3/2)*Csc[a + b*x] + EllipticE[(a +
b*x)/2, 2]))/(b*Sqrt[Cos[a + b*x]]))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3050, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)\sqrt{d\cos(a+bx)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sqrt{d\cos(a+bx)}}{\sin(a+bx)^2}dx$$

$$\Big\downarrow \ 3050$$

$$-\frac{1}{2}\int \sqrt{d\cos(a+bx)}dx-\frac{\csc(a+bx)(d\cos(a+bx))^{3/2}}{bd}$$

$$\Big\downarrow \ 3042$$

$$-\frac{1}{2}\int \sqrt{d\sin\left(a+bx+\frac{\pi}{2}\right)}dx-\frac{\csc(a+bx)(d\cos(a+bx))^{3/2}}{bd}$$

$$\Big\downarrow \ 3121$$

$$-\frac{\sqrt{d\cos(a+bx)}\int \sqrt{\cos(a+bx)}dx}{2\sqrt{\cos(a+bx)}}-\frac{\csc(a+bx)(d\cos(a+bx))^{3/2}}{bd}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\sqrt{d\cos(a+bx)}\int \sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx}{2\sqrt{\cos(a+bx)}}-\frac{\csc(a+bx)(d\cos(a+bx))^{3/2}}{bd}$$

$$\Big\downarrow \ 3119$$

$$-\frac{\csc(a+bx)(d\cos(a+bx))^{3/2}}{bd}-\frac{E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{b\sqrt{\cos(a+bx)}}$$

input
```
Int[Sqrt[d*Cos[a + b*x]]*Csc[a + b*x]^2,x]
```

output
```
-(((d*Cos[a + b*x])^(3/2)*Csc[a + b*x])/(b*d)) - (Sqrt[d*Cos[a + b*x]]*Ell
ipticE[(a + b*x)/2, 2])/(b*Sqrt[Cos[a + b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3050
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Simp[(m + n + 2)/(a^2*(m + 1))   Int[(b*Cos[e + f*x])^
n*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(61) = 122$.

Time = 4.47 (sec) , antiderivative size = 203, normalized size of antiderivative = 3.12

method	result
default	$\frac{\sqrt{d\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\, d^{2}\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\left(2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)^{\frac{3}{2}}\operatorname{EllipticE}\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\cdots}\right.}{2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\left(-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}d+\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}d\right)^{\frac{3}{2}}\sqrt{d\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\cdots\right.}}$

input
```
int((d*cos(b*x+a))^(1/2)*csc(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^2/cos(1/2*
b*x+1/2*a)/(-2*sin(1/2*b*x+1/2*a)^4*d+sin(1/2*b*x+1/2*a)^2*d)^(3/2)*sin(1/
2*b*x+1/2*a)*(2*cos(1/2*b*x+1/2*a)*(2*sin(1/2*b*x+1/2*a)^2-1)^(3/2)*Ellipt
icE(cos(1/2*b*x+1/2*a),2^(1/2))*(sin(1/2*b*x+1/2*a)^2)^(1/2)+8*sin(1/2*b*x
+1/2*a)^6-12*sin(1/2*b*x+1/2*a)^4+6*sin(1/2*b*x+1/2*a)^2-1)/(d*(2*cos(1/2*
b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.55

$$\int \sqrt{d\cos(a+bx)}\csc^2(a+bx)\,dx$$
$$= \frac{-i\sqrt{\frac{1}{2}}\sqrt{d}\sin(bx+a)\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)))}{}$$

input
```
integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a)^2,x, algorithm="fricas")
```

output
```
(-I*sqrt(1/2)*sqrt(d)*sin(b*x + a)*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, cos(b*x + a) + I*sin(b*x + a))) + I*sqrt(1/2)*sqrt(d)*sin(b*x +
 a)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin
(b*x + a))) - sqrt(d*cos(b*x + a))*cos(b*x + a))/(b*sin(b*x + a))
```

Sympy [F]

$$\int \sqrt{d\cos(a+bx)}\csc^2(a+bx)\,dx = \int \sqrt{d\cos(a+bx)}\csc^2(a+bx)\,dx$$

input
```
integrate((d*cos(b*x+a))**(1/2)*csc(b*x+a)**2,x)
```

output
```
Integral(sqrt(d*cos(a + b*x))*csc(a + b*x)**2, x)
```

Maxima [F]

$$\int \sqrt{d\cos(a+bx)}\csc^2(a+bx)\,dx = \int \sqrt{d\cos(bx+a)}\csc(bx+a)^2\,dx$$

input
```
integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*cos(b*x + a))*csc(b*x + a)^2, x)
```

Giac [F]

$$\int \sqrt{d\cos(a+bx)}\csc^2(a+bx)\,dx = \int \sqrt{d\cos(bx+a)}\csc(bx+a)^2\,dx$$

input
```
integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(d*cos(b*x + a))*csc(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\cos(a+bx)}\csc^2(a+bx)\,dx = \int \frac{\sqrt{d\cos(a+b\,x)}}{\sin(a+b\,x)^2}\,dx$$

input
```
int((d*cos(a + b*x))^(1/2)/sin(a + b*x)^2,x)
```

output
```
int((d*cos(a + b*x))^(1/2)/sin(a + b*x)^2, x)
```

Reduce [F]

$$\int \sqrt{d\cos(a+bx)}\,\csc^2(a+bx)\,dx = \sqrt{d}\left(\int \sqrt{\cos(bx+a)}\,\csc(bx+a)^2\,dx\right)$$

input
```
int((d*cos(b*x+a))^(1/2)*csc(b*x+a)^2,x)
```

output
```
sqrt(d)*int(sqrt(cos(a + b*x))*csc(a + b*x)**2,x)
```

3.238 $\int \frac{\csc^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 64

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{\sqrt{d\cos(a+bx)}\csc(a+bx)}{bd} + \frac{\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{b\sqrt{d\cos(a+bx)}}$$

output
```
-(d*cos(b*x+a))^(1/2)*csc(b*x+a)/b/d+cos(b*x+a)^(1/2)*InverseJacobiAM(1/2*
a+1/2*b*x,2^(1/2))/b/(d*cos(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.73

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \frac{-\cot(a+bx)+\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{b\sqrt{d\cos(a+bx)}}$$

input
```
Integrate[Csc[a + b*x]^2/Sqrt[d*Cos[a + b*x]],x]
```

output
```
(-Cot[a + b*x] + Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(b*Sqrt[d*C
os[a + b*x]])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3050, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\cos(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^2\sqrt{d\cos(a+bx)}}dx$$

$$\Big\downarrow \text{3050}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{d\cos(a+bx)}}dx-\frac{\csc(a+bx)\sqrt{d\cos(a+bx)}}{bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{d\sin\left(a+bx+\frac{\pi}{2}\right)}}dx-\frac{\csc(a+bx)\sqrt{d\cos(a+bx)}}{bd}$$

$$\Big\downarrow \text{3121}$$

$$\frac{\sqrt{\cos(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}}dx}{2\sqrt{d\cos(a+bx)}}-\frac{\csc(a+bx)\sqrt{d\cos(a+bx)}}{bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(a+bx)}\int \frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx}{2\sqrt{d\cos(a+bx)}}-\frac{\csc(a+bx)\sqrt{d\cos(a+bx)}}{bd}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{b\sqrt{d\cos(a+bx)}}-\frac{\csc(a+bx)\sqrt{d\cos(a+bx)}}{bd}$$

input
```
Int[Csc[a + b*x]^2/Sqrt[d*Cos[a + b*x]],x]
```

output
```
-((Sqrt[d*Cos[a + b*x]]*Csc[a + b*x])/(b*d)) + (Sqrt[Cos[a + b*x]]*Ellipti
cF[(a + b*x)/2, 2])/(b*Sqrt[d*Cos[a + b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3050
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Simp[(m + n + 2)/(a^2*(m + 1))   Int[(b*Cos[e + f*x])^
n*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(59) = 118$.

Time = 4.08 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.94

method	result
default	$\frac{\sqrt{d\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\, d\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\left(2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)^{\frac{3}{2}}\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\mathrm{EllipticF}\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right.\right.\right.}{2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\left(-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}d+\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}d\right)^{\frac{3}{2}}\sqrt{d\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)}\, b}$

input
```
int(csc(b*x+a)^2/(d*cos(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/cos(1/2*b*x+
1/2*a)/(-2*sin(1/2*b*x+1/2*a)^4*d+sin(1/2*b*x+1/2*a)^2*d)^(3/2)*d*sin(1/2*
b*x+1/2*a)*(2*cos(1/2*b*x+1/2*a)*(2*sin(1/2*b*x+1/2*a)^2-1)^(3/2)*(sin(1/2
*b*x+1/2*a)^2)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))-4*sin(1/2*b*x+1
/2*a)^4+4*sin(1/2*b*x+1/2*a)^2-1)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.44

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \frac{-i\sqrt{\frac{1}{2}}\sqrt{d}\sin(bx+a)\,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a))+i\sqrt{\frac{1}{2}}\sqrt{d}\sin(bx+a)\,\mathrm{w}}{bd\sin(bx+a)}$$

input
```
integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(1/2)*sqrt(d)*sin(b*x + a)*weierstrassPInverse(-4, 0, cos(b*x + a)
 + I*sin(b*x + a)) + I*sqrt(1/2)*sqrt(d)*sin(b*x + a)*weierstrassPInverse(
-4, 0, cos(b*x + a) - I*sin(b*x + a)) - sqrt(d*cos(b*x + a)))/(b*d*sin(b*x
 + a))
```

Sympy [F]

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{\csc^2{(a+bx)}}{\sqrt{d\cos{(a+bx)}}}\,dx$$

input
```
integrate(csc(b*x+a)**2/(d*cos(b*x+a))**(1/2),x)
```

output
```
Integral(csc(a + b*x)**2/sqrt(d*cos(a + b*x)), x)
```

Maxima [F]

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{\csc{(bx+a)}^2}{\sqrt{d\cos{(bx+a)}}}\,dx$$

input
```
integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^2/sqrt(d*cos(b*x + a)), x)
```

Giac [F]

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{\csc{(bx+a)}^2}{\sqrt{d\cos{(bx+a)}}}\,dx$$

input
```
integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2/sqrt(d*cos(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{1}{\sin\left(a+b\,x\right)^2\,\sqrt{d\,\cos\left(a+b\,x\right)}}\,dx$$

input
```
int(1/(sin(a + b*x)^2*(d*cos(a + b*x))^(1/2)),x)
```

output
```
int(1/(sin(a + b*x)^2*(d*cos(a + b*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{\csc^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\cos(bx+a)}\,\csc(bx+a)^2}{\cos(bx+a)}dx\right)}{d}$$

input
```
int(csc(b*x+a)^2/(d*cos(b*x+a))^(1/2),x)
```

output
```
(sqrt(d)*int((sqrt(cos(a + b*x))*csc(a + b*x)**2)/cos(a + b*x),x))/d
```

3.239 $\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 94

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = -\frac{\csc(a+bx)}{bd\sqrt{d\cos(a+bx)}} - \frac{3\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{bd^2\sqrt{\cos(a+bx)}} + \frac{3\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}$$

output
```
-csc(b*x+a)/b/d/(d*cos(b*x+a))^(1/2)-3*(d*cos(b*x+a))^(1/2)*EllipticE(sin(
1/2*a+1/2*b*x),2^(1/2))/b/d^2/cos(b*x+a)^(1/2)+3*sin(b*x+a)/b/d/(d*cos(b*x
+a))^(1/2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.69

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{-\cos(a+bx)\cot(a+bx) - 3\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right) + 2\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}$$

input
```
Integrate[Csc[a + b*x]^2/(d*Cos[a + b*x])^(3/2),x]
```

output
```
(-(Cos[a + b*x]*Cot[a + b*x]) - 3*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2
, 2] + 2*Sin[a + b*x])/(b*d*Sqrt[d*Cos[a + b*x]])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3050, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^2(d\cos(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3050}$$

$$\frac{3}{2}\int \frac{1}{(d\cos(a+bx))^{3/2}}dx-\frac{\csc(a+bx)}{bd\sqrt{d\cos(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}\int \frac{1}{\left(d\sin\left(a+bx+\frac{\pi}{2}\right)\right)^{3/2}}dx-\frac{\csc(a+bx)}{bd\sqrt{d\cos(a+bx)}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{3}{2}\left(\frac{2\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{\int \sqrt{d\cos(a+bx)}dx}{d^2}\right)-\frac{\csc(a+bx)}{bd\sqrt{d\cos(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}\left(\frac{2\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{\int \sqrt{d\sin\left(a+bx+\frac{\pi}{2}\right)}dx}{d^2}\right)-\frac{\csc(a+bx)}{bd\sqrt{d\cos(a+bx)}}$$

$$\Big\downarrow \text{3121}$$

$$\frac{3}{2}\left(\frac{2\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{\sqrt{d\cos(a+bx)}\int \sqrt{\cos(a+bx)}dx}{d^2\sqrt{\cos(a+bx)}}\right)-\frac{\csc(a+bx)}{bd\sqrt{d\cos(a+bx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{3}{2}\left(\frac{2\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{\sqrt{d\cos(a+bx)}\int\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx}{d^2\sqrt{\cos(a+bx)}}\right)-\frac{\csc(a+bx)}{bd\sqrt{d\cos(a+bx)}}$$

$$\Big\downarrow \; 3119$$

$$\frac{3}{2}\left(\frac{2\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{bd^2\sqrt{\cos(a+bx)}}\right)-\frac{\csc(a+bx)}{bd\sqrt{d\cos(a+bx)}}$$

input
```
Int[Csc[a + b*x]^2/(d*Cos[a + b*x])^(3/2),x]
```

output
```
-(Csc[a + b*x]/(b*d*Sqrt[d*Cos[a + b*x]])) + (3*((-2*Sqrt[d*Cos[a + b*x]]*
EllipticE[(a + b*x)/2, 2])/(b*d^2*Sqrt[Cos[a + b*x]]) + (2*Sin[a + b*x])/(
b*d*Sqrt[d*Cos[a + b*x]])))/2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3050
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Simp[(m + n + 2)/(a^2*(m + 1))   Int[(b*Cos[e + f*x])^
n*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. 2(88) = 176.

Time = 4.87 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.22

method	result
default	$-\frac{\sqrt{d\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^2}\left(-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^4 d+\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^2 d\right)^{\frac{3}{2}}\left(6\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\text{EllipticE}\left(\cos\left(\frac{b}{2}\right.\right.\right.}{2d^3\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^5\left(2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1\right)^2\sqrt{d\left(2\cos\left(\frac{b}{2}\right.\right.}}$

input
```
int(csc(b*x+a)^2/(d*cos(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/d^3/cos(1/2
*b*x+1/2*a)/sin(1/2*b*x+1/2*a)^5/(2*sin(1/2*b*x+1/2*a)^2-1)^2*(-2*sin(1/2*
b*x+1/2*a)^4*d+sin(1/2*b*x+1/2*a)^2*d)^(3/2)*(6*cos(1/2*b*x+1/2*a)*(sin(1/
2*b*x+1/2*a)^2)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))*(2*sin(1/2*b*x
+1/2*a)^2-1)^(1/2)+12*sin(1/2*b*x+1/2*a)^4-12*sin(1/2*b*x+1/2*a)^2+1)/(d*(
2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.38

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{-3i\sqrt{\frac{1}{2}}\sqrt{d}\cos(bx+a)\sin(bx+a)\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(}{}$$

input
```
integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
(-3*I*sqrt(1/2)*sqrt(d)*cos(b*x + a)*sin(b*x + a)*weierstrassZeta(-4, 0, w
eierstrassPInverse(-4, 0, cos(b*x + a) + I*sin(b*x + a))) + 3*I*sqrt(1/2)*
sqrt(d)*cos(b*x + a)*sin(b*x + a)*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(b*x + a) - I*sin(b*x + a))) - sqrt(d*cos(b*x + a))*(3*cos(b*
x + a)^2 - 2))/(b*d^2*cos(b*x + a)*sin(b*x + a))
```

Sympy [F]

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\csc^2{(a+bx)}}{(d\cos{(a+bx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)**2/(d*cos(b*x+a))**(3/2),x)
```

output
```
Integral(csc(a + b*x)**2/(d*cos(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\csc{(bx+a)}^2}{(d\cos{(bx+a)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^2/(d*cos(b*x + a))^(3/2), x)
```

Giac [F]

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\csc(bx+a)^2}{(d\cos(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2/(d*cos(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{1}{\sin(a+b\,x)^2\,(d\,\cos(a+b\,x))^{3/2}}\,dx$$

input
```
int(1/(sin(a + b*x)^2*(d*cos(a + b*x))^(3/2)),x)
```

output
```
int(1/(sin(a + b*x)^2*(d*cos(a + b*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\cos(bx+a)}\,\csc(bx+a)^2}{\cos(bx+a)^2}dx\right)}{d^2}$$

input
```
int(csc(b*x+a)^2/(d*cos(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*int((sqrt(cos(a + b*x))*csc(a + b*x)**2)/cos(a + b*x)**2,x))/d**2
```

3.240 $\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = -\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{3/2}} + \frac{5\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3bd^2\sqrt{d\cos(a+bx)}} + \frac{5\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}$$

output
```
-csc(b*x+a)/b/d/(d*cos(b*x+a))^(3/2)+5/3*cos(b*x+a)^(1/2)*InverseJacobiAM(
1/2*a+1/2*b*x,2^(1/2))/b/d^2/(d*cos(b*x+a))^(1/2)+5/3*sin(b*x+a)/b/d/(d*co
s(b*x+a))^(3/2)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.63

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \frac{-3\cot(a+bx)+5\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)+2\tan(a+bx)}{3bd^2\sqrt{d\cos(a+bx)}}$$

input
```
Integrate[Csc[a + b*x]^2/(d*Cos[a + b*x])^(5/2),x]
```

output
```
(-3*Cot[a + b*x] + 5*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2] + 2*Tan[
a + b*x])/(3*b*d^2*Sqrt[d*Cos[a + b*x]])
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3042, 3050, 3042, 3116, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^2(d\cos(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3050}$$

$$\frac{5}{2}\int \frac{1}{(d\cos(a+bx))^{5/2}}dx-\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{2}\int \frac{1}{\left(d\sin\left(a+bx+\frac{\pi}{2}\right)\right)^{5/2}}dx-\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{5}{2}\left(\frac{\int \frac{1}{\sqrt{d\cos(a+bx)}}dx}{3d^2}+\frac{2\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}\right)-\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{2}\left(\frac{\int \frac{1}{\sqrt{d\sin\left(a+bx+\frac{\pi}{2}\right)}}dx}{3d^2}+\frac{2\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}\right)-\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3121}$$

$$\frac{5}{2}\left(\frac{\sqrt{\cos(a+bx)}\int\frac{1}{\sqrt{\cos(a+bx)}}dx}{3d^2\sqrt{d\cos(a+bx)}}+\frac{2\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}\right)-\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{2}\left(\frac{\sqrt{\cos(a+bx)}\int\frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx}{3d^2\sqrt{d\cos(a+bx)}}+\frac{2\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}\right)-\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{5}{2}\left(\frac{2\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3bd^2\sqrt{d\cos(a+bx)}}+\frac{2\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}\right)-\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{3/2}}$$

input
```
Int[Csc[a + b*x]^2/(d*Cos[a + b*x])^(5/2),x]
```

output
```
-(Csc[a + b*x]/(b*d*(d*Cos[a + b*x])^(3/2))) + (5*((2*Sqrt[Cos[a + b*x]]*E
llipticF[(a + b*x)/2, 2])/(3*b*d^2*Sqrt[d*Cos[a + b*x]]) + (2*Sin[a + b*x]
)/(3*b*d*(d*Cos[a + b*x])^(3/2))))/2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3050
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Simp[(m + n + 2)/(a^2*(m + 1))   Int[(b*Cos[e + f*x])^
n*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(87) = 174$.

Time = 5.30 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.94

method	result
default	$\frac{\sqrt{d\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^2}\,\Big(10\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\left(2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1\right)^{\frac{3}{2}}\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\text{EllipticF}\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)-20\,\text{si}}{6d\left(-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^4d+\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^2d\right)^{\frac{3}{2}}\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{d\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1\right)}\,b}$

input
```
int(csc(b*x+a)^2/(d*cos(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/d/(-2*sin(1/
2*b*x+1/2*a)^4*d+sin(1/2*b*x+1/2*a)^2*d)^(3/2)/cos(1/2*b*x+1/2*a)*(10*cos(
1/2*b*x+1/2*a)*(2*sin(1/2*b*x+1/2*a)^2-1)^(3/2)*(sin(1/2*b*x+1/2*a)^2)^(1/
2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))-20*sin(1/2*b*x+1/2*a)^4+20*sin(1/
2*b*x+1/2*a)^2-3)*sin(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/
b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.32

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \frac{-5i\sqrt{\frac{1}{2}}\sqrt{d}\cos(bx+a)^2\sin(bx+a)\,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+$$

input
```
integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
1/3*(-5*I*sqrt(1/2)*sqrt(d)*cos(b*x + a)^2*sin(b*x + a)*weierstrassPInvers
e(-4, 0, cos(b*x + a) + I*sin(b*x + a)) + 5*I*sqrt(1/2)*sqrt(d)*cos(b*x +
a)^2*sin(b*x + a)*weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a)
) - sqrt(d*cos(b*x + a))*(5*cos(b*x + a)^2 - 2))/(b*d^3*cos(b*x + a)^2*sin
(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2/(d*cos(b*x+a))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{\csc(bx+a)^2}{(d\cos(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^2/(d*cos(b*x + a))^(5/2), x)
```

Giac [F]

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{\csc(bx+a)^2}{(d\cos(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2/(d*cos(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{1}{\sin(a+b\,x)^2\,(d\,\cos(a+b\,x))^{5/2}}\,dx$$

input
```
int(1/(sin(a + b*x)^2*(d*cos(a + b*x))^(5/2)),x)
```

output
```
int(1/(sin(a + b*x)^2*(d*cos(a + b*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\cos(bx+a)}\,\csc(bx+a)^2}{\cos(bx+a)^3}dx\right)}{d^3}$$

input
```
int(csc(b*x+a)^2/(d*cos(b*x+a))^(5/2),x)
```

output
```
(sqrt(d)*int((sqrt(cos(a + b*x))*csc(a + b*x)**2)/cos(a + b*x)**3,x))/d**3
```

3.241 $\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 126

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = -\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{5/2}} - \frac{21\sqrt{d\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5bd^4\sqrt{\cos(a+bx)}} + \frac{7\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}} + \frac{21\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}}$$

output
```
-csc(b*x+a)/b/d/(d*cos(b*x+a))^(5/2)-21/5*(d*cos(b*x+a))^(1/2)*EllipticE(s
in(1/2*a+1/2*b*x),2^(1/2))/b/d^4/cos(b*x+a)^(1/2)+7/5*sin(b*x+a)/b/d/(d*co
s(b*x+a))^(5/2)+21/5*sin(b*x+a)/b/d^3/(d*cos(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.65

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{-5\cos(a+bx)\cot(a+bx) - 21\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right) + 16\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}}$$

input
```
Integrate[Csc[a + b*x]^2/(d*Cos[a + b*x])^(7/2),x]
```

output
```
(-5*Cos[a + b*x]*Cot[a + b*x] - 21*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/
2, 2] + 16*Sin[a + b*x] + 2*Sec[a + b*x]*Tan[a + b*x])/(5*b*d^3*Sqrt[d*Cos
[a + b*x]])
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {3042, 3050, 3042, 3116, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^2(d\cos(a+bx))^{7/2}}dx$$

$$\Big\downarrow \text{3050}$$

$$\frac{7}{2}\int \frac{1}{(d\cos(a+bx))^{7/2}}dx-\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{2}\int \frac{1}{\left(d\sin\left(a+bx+\frac{\pi}{2}\right)\right)^{7/2}}dx-\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{5/2}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{7}{2}\left(\frac{3\int \frac{1}{(d\cos(a+bx))^{3/2}}dx}{5d^2}+\frac{2\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}}\right)-\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{2}\left(\frac{3\int \frac{1}{\left(d\sin\left(a+bx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{5d^2}+\frac{2\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}}\right)-\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{5/2}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{7}{2}\left(\frac{3\left(\frac{2\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{\int\sqrt{d\cos(a+bx)}dx}{d^2}\right)}{5d^2}+\frac{2\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}}\right)-\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{5/2}}$$

$\big\downarrow$ 3042

$$\frac{7}{2}\left(\frac{3\left(\frac{2\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{\int\sqrt{d\sin\left(a+bx+\frac{\pi}{2}\right)}dx}{d^2}\right)}{5d^2}+\frac{2\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}}\right)-\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{5/2}}$$

$\big\downarrow$ 3121

$$\frac{7}{2}\left(\frac{3\left(\frac{2\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{\sqrt{d\cos(a+bx)}\int\sqrt{\cos(a+bx)}dx}{d^2\sqrt{\cos(a+bx)}}\right)}{5d^2}+\frac{2\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}}\right)-\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{5/2}}$$

$\big\downarrow$ 3042

$$\frac{7}{2}\left(\frac{3\left(\frac{2\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{\sqrt{d\cos(a+bx)}\int\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx}{d^2\sqrt{\cos(a+bx)}}\right)}{5d^2}+\frac{2\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}}\right)-\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{5/2}}$$

$\big\downarrow$ 3119

$$\frac{7}{2}\left(\frac{3\left(\frac{2\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{2E\left(\frac{1}{2}(a+bx)|2\right)\sqrt{d\cos(a+bx)}}{bd^2\sqrt{\cos(a+bx)}}\right)}{5d^2}+\frac{2\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}}\right)-\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{5/2}}$$

input
```
Int[Csc[a + b*x]^2/(d*Cos[a + b*x])^(7/2),x]
```

output
```
-(Csc[a + b*x]/(b*d*(d*Cos[a + b*x])^(5/2))) + (7*((2*Sin[a + b*x])/(5*b*d
*(d*Cos[a + b*x])^(5/2)) + (3*((-2*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x
)/2, 2])/(b*d^2*Sqrt[Cos[a + b*x]]) + (2*Sin[a + b*x])/(b*d*Sqrt[d*Cos[a +
 b*x]])))/(5*d^2)))/2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3050
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Simp[(m + n + 2)/(a^2*(m + 1))   Int[(b*Cos[e + f*x])^
n*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. 2(112) = 224.

Time = 6.37 (sec) , antiderivative size = 408, normalized size of antiderivative = 3.24

method	result
default	$-\frac{\sqrt{d\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\left(168\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\mathrm{EllipticE}\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\sqrt{2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1}\,\sin\left(\right.\right.}{}$

input
```
int(csc(b*x+a)^2/(d*cos(b*x+a))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/10*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/d^5/(2*sin
(1/2*b*x+1/2*a)^2-1)/cos(1/2*b*x+1/2*a)/sin(1/2*b*x+1/2*a)^5/(8*sin(1/2*b*
x+1/2*a)^6-12*sin(1/2*b*x+1/2*a)^4+6*sin(1/2*b*x+1/2*a)^2-1)*(168*cos(1/2*
b*x+1/2*a)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))*(sin(1/2*b*x+1/2*a)^2)^(1
/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*sin(1/2*b*x+1/2*a)^4+336*sin(1/2*b*x+
1/2*a)^8-168*cos(1/2*b*x+1/2*a)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))*(sin
(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*sin(1/2*b*x+1/2*
a)^2-672*sin(1/2*b*x+1/2*a)^6+42*cos(1/2*b*x+1/2*a)*(sin(1/2*b*x+1/2*a)^2)
^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))*(2*sin(1/2*b*x+1/2*a)^2-1)^(1
/2)+448*sin(1/2*b*x+1/2*a)^4-112*sin(1/2*b*x+1/2*a)^2+5)*(-2*sin(1/2*b*x+1
/2*a)^4*d+sin(1/2*b*x+1/2*a)^2*d)^(3/2)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/
2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.15

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{-21i\sqrt{\frac{1}{2}}\sqrt{d}\cos(bx+a)^3\sin(bx+a)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInvers}}{}$$

input
```
integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

output
```
1/5*(-21*I*sqrt(1/2)*sqrt(d)*cos(b*x + a)^3*sin(b*x + a)*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) + I*sin(b*x + a))) + 21*I*sq
rt(1/2)*sqrt(d)*cos(b*x + a)^3*sin(b*x + a)*weierstrassZeta(-4, 0, weierst
rassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a))) - (21*cos(b*x + a)^4 -
 14*cos(b*x + a)^2 - 2)*sqrt(d*cos(b*x + a)))/(b*d^4*cos(b*x + a)^3*sin(b*
x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2/(d*cos(b*x+a))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{\csc(bx+a)^2}{(d\cos(bx+a))^{\frac{7}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(7/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)^2/(d*cos(b*x + a))^(7/2), x)
```

Giac [F]

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{\csc\left(bx+a\right)^2}{\left(d\cos\left(bx+a\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(7/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2/(d*cos(b*x + a))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{1}{\sin\left(a+b\,x\right)^2\,\left(d\,\cos\left(a+b\,x\right)\right)^{7/2}}\,dx$$

input
```
int(1/(sin(a + b*x)^2*(d*cos(a + b*x))^(7/2)),x)
```

output
```
int(1/(sin(a + b*x)^2*(d*cos(a + b*x))^(7/2)), x)
```

Reduce [F]

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\cos(bx+a)}\,\csc(bx+a)^2}{\cos(bx+a)^4}dx\right)}{d^4}$$

input
```
int(csc(b*x+a)^2/(d*cos(b*x+a))^(7/2),x)
```

output
```
(sqrt(d)*int((sqrt(cos(a + b*x))*csc(a + b*x)**2)/cos(a + b*x)**4,x))/d**4
```

3.242 $\int (d\cos(a+bx))^{11/2}\csc^3(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 135

$$\int (d\cos(a+bx))^{11/2}\csc^3(a+bx)\,dx = \frac{9d^{11/2}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} + \frac{9d^{11/2}\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} - \frac{9d^5\sqrt{d\cos(a+bx)}}{2b} - \frac{9d^3(d\cos(a+bx))^{5/2}}{10b} - \frac{d(d\cos(a+bx))^{9/2}\csc^2(a+bx)}{2b}$$

output
```
9/4*d^(11/2)*arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b+9/4*d^(11/2)*arctanh((
d*cos(b*x+a))^(1/2)/d^(1/2))/b-9/2*d^5*(d*cos(b*x+a))^(1/2)/b-9/10*d^3*(d*
cos(b*x+a))^(5/2)/b-1/2*d*(d*cos(b*x+a))^(9/2)*csc(b*x+a)^2/b
```

Mathematica [A] (verified)

Time = 3.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.01

$$\int (d\cos(a+bx))^{11/2}\csc^3(a+bx)\,dx = \frac{d(d\cos(a+bx))^{9/2}\left(45\arctan\left(\sqrt{\cos(a+bx)}\right) + 24\text{arctanh}\left(\sqrt{\cos(a+bx)}\right) - 2\sqrt{\cos(a+bx)}\right.}{}$$

input
```
Integrate[(d*Cos[a + b*x])^(11/2)*Csc[a + b*x]^3,x]
```

output
```
(d*(d*Cos[a + b*x])^(9/2)*(45*ArcTan[Sqrt[Cos[a + b*x]]] + 24*ArcTanh[Sqrt
[Cos[a + b*x]]] - 2*Sqrt[Cos[a + b*x]]*(2*Cos[2*(a + b*x)] + 5*Csc[a + b*x
]^2) - (21*(8*Sqrt[Cos[a + b*x]] + Log[1 - Sqrt[Cos[a + b*x]]] - Log[1 + S
qrt[Cos[a + b*x]]]))/2))/(20*b*Cos[a + b*x]^(9/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.51 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3045, 27, 252, 262, 262, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(a+bx)(d\cos(a+bx))^{11/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\cos(a+bx))^{11/2}}{\sin(a+bx)^3}dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int \frac{d^4(d\cos(a+bx))^{11/2}}{(d^2-d^2\cos^2(a+bx))^2}d(d\cos(a+bx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$-\frac{d^3\int \frac{(d\cos(a+bx))^{11/2}}{(d^2-d^2\cos^2(a+bx))^2}d(d\cos(a+bx))}{b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{d^3\left(\frac{(d\cos(a+bx))^{9/2}}{2(d^2-d^2\cos^2(a+bx))}-\frac{9}{4}\int \frac{(d\cos(a+bx))^{7/2}}{d^2-d^2\cos^2(a+bx)}d(d\cos(a+bx))\right)}{b}$$

$$\Big\downarrow \text{262}$$

$$-\frac{d^3\left(\frac{(d\cos(a+bx))^{9/2}}{2(d^2-d^2\cos^2(a+bx))}-\frac{9}{4}\left(d^2\int\frac{(d\cos(a+bx))^{3/2}}{d^2-d^2\cos^2(a+bx)}d(d\cos(a+bx))-\frac{2}{5}(d\cos(a+bx))^{5/2}\right)\right)}{b}$$

$\Big\downarrow$ 262

$$-\frac{d^3\left(\frac{(d\cos(a+bx))^{9/2}}{2(d^2-d^2\cos^2(a+bx))}-\frac{9}{4}\left(d^2\left(d^2\int\frac{1}{\sqrt{d\cos(a+bx)}(d^2-d^2\cos^2(a+bx))}d(d\cos(a+bx))-2\sqrt{d\cos(a+bx)}\right)-\frac{2}{5}(d\cos(\right.\right.}{b}$$

$\Big\downarrow$ 266

$$-\frac{d^3\left(\frac{(d\cos(a+bx))^{9/2}}{2(d^2-d^2\cos^2(a+bx))}-\frac{9}{4}\left(d^2\left(2d^2\int\frac{1}{d^2-d^4\cos^4(a+bx)}d\sqrt{d\cos(a+bx)}-2\sqrt{d\cos(a+bx)}\right)-\frac{2}{5}(d\cos(a+bx))^{5/2}\right.\right.}{b}$$

$\Big\downarrow$ 756

$$-\frac{d^3\left(\frac{(d\cos(a+bx))^{9/2}}{2(d^2-d^2\cos^2(a+bx))}-\frac{9}{4}\left(d^2\left(2d^2\left(\frac{\int\frac{1}{d-d^2\cos^2(a+bx)}d\sqrt{d\cos(a+bx)}}{2d}+\frac{\int\frac{1}{d^2\cos^2(a+bx)+d}d\sqrt{d\cos(a+bx)}}{2d}\right)-2\sqrt{d\cos(a+\right.\right.\right.}{b}$$

$\Big\downarrow$ 216

$$-\frac{d^3\left(\frac{(d\cos(a+bx))^{9/2}}{2(d^2-d^2\cos^2(a+bx))}-\frac{9}{4}\left(d^2\left(2d^2\left(\frac{\int\frac{1}{d-d^2\cos^2(a+bx)}d\sqrt{d\cos(a+bx)}}{2d}+\frac{\arctan\left(\sqrt{d}\cos(a+bx)\right)}{2d^{3/2}}\right)-2\sqrt{d\cos(a+bx)}\right)-\right.\right.}{b}$$

$\Big\downarrow$ 219

$$-\frac{d^3\left(\frac{(d\cos(a+bx))^{9/2}}{2(d^2-d^2\cos^2(a+bx))}-\frac{9}{4}\left(d^2\left(2d^2\left(\frac{\arctan\left(\sqrt{d}\cos(a+bx)\right)}{2d^{3/2}}+\frac{\text{arctanh}\left(\sqrt{d}\cos(a+bx)\right)}{2d^{3/2}}\right)-2\sqrt{d\cos(a+bx)}\right)-\frac{2}{5}(d\right.\right.}{b}$$

input
```
Int[(d*Cos[a + b*x])^(11/2)*Csc[a + b*x]^3,x]
```

output
```
-((d^3*((d*Cos[a + b*x])^(9/2)/(2*(d^2 - d^2*Cos[a + b*x]^2)) - (9*((-2*(d
*Cos[a + b*x])^(5/2))/5 + d^2*(2*d^2*(ArcTan[Sqrt[d]*Cos[a + b*x]]/(2*d^(3
/2)) + ArcTanh[Sqrt[d]*Cos[a + b*x]]/(2*d^(3/2))) - 2*Sqrt[d*Cos[a + b*x]]
)))/4))/b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(107) = 214$.

Time = 9.64 (sec) , antiderivative size = 407, normalized size of antiderivative = 3.01

method	result
default	$-\frac{d^{5}\sqrt{2d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-d}}{8\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}-\frac{9d^{6}\ln\left(\frac{-2d+2\sqrt{-d}\,\sqrt{2d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-d}}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)}\right)}{4\sqrt{-d}}-6d^{5}\sqrt{d\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)}+\frac{8d^{5}\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\sqrt{2d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)}}{5}$

input
```
int((d*cos(b*x+a))^(11/2)*csc(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/8*d^5/cos(1/2*b*x+1/2*a)^2*(2*d*cos(1/2*b*x+1/2*a)^2-d)^(1/2)-9/4*d^6/
(-d)^(1/2)*ln((-2*d+2*(-d)^(1/2)*(2*d*cos(1/2*b*x+1/2*a)^2-d)^(1/2))/cos(1
/2*b*x+1/2*a))-6*d^5*(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)+8/5*d^5*cos(1/2*
b*x+1/2*a)^2*(2*d*cos(1/2*b*x+1/2*a)^2-d)^(1/2)+8/5*d^5*(2*d*cos(1/2*b*x+1
/2*a)^2-d)^(1/2)-8/5*d^5*cos(1/2*b*x+1/2*a)^4*(2*d*cos(1/2*b*x+1/2*a)^2-d)
^(1/2)-1/16*d^5/(cos(1/2*b*x+1/2*a)+1)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)
+9/8*d^(11/2)*ln((-4*d*cos(1/2*b*x+1/2*a)+2*d^(1/2)*(-2*sin(1/2*b*x+1/2*a)
^2*d+d)^(1/2)-2*d)/(cos(1/2*b*x+1/2*a)+1))+1/16*d^5/(cos(1/2*b*x+1/2*a)-1)
*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+9/8*d^(11/2)*ln((4*d*cos(1/2*b*x+1/2*
a)+2*d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d)/(cos(1/2*b*x+1/2*a)-
1)))/b
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 428, normalized size of antiderivative = 3.17

$$\int (d\cos(a+bx))^{11/2}\csc^3(a+bx)\,dx = \left[-\frac{90\left(d^5\cos(bx+a)^2-d^5\right)\sqrt{-d}\arctan\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)+1)}{2\,d\cos(bx+a)}\right)-45\left(d^5\cos(bx+a)^2-\right.}{}\right.$$

input
```
integrate((d*cos(b*x+a))^(11/2)*csc(b*x+a)^3,x, algorithm="fricas")
```

output
```
[-1/80*(90*(d^5*cos(b*x + a)^2 - d^5)*sqrt(-d)*arctan(1/2*sqrt(d*cos(b*x +
 a))*sqrt(-d)*(cos(b*x + a) + 1)/(d*cos(b*x + a))) - 45*(d^5*cos(b*x + a)^
2 - d^5)*sqrt(-d)*log(-(d*cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(-d)
*(cos(b*x + a) - 1) - 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x +
a) + 1)) + 8*(4*d^5*cos(b*x + a)^4 + 36*d^5*cos(b*x + a)^2 - 45*d^5)*sqrt(
d*cos(b*x + a)))/(b*cos(b*x + a)^2 - b), 1/80*(90*(d^5*cos(b*x + a)^2 - d^
5)*sqrt(d)*arctan(1/2*sqrt(d*cos(b*x + a))*(cos(b*x + a) - 1)/(sqrt(d)*cos
(b*x + a))) + 45*(d^5*cos(b*x + a)^2 - d^5)*sqrt(d)*log(-(d*cos(b*x + a)^2
 + 4*sqrt(d*cos(b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) +
d)/(cos(b*x + a)^2 - 2*cos(b*x + a) + 1)) - 8*(4*d^5*cos(b*x + a)^4 + 36*d
^5*cos(b*x + a)^2 - 45*d^5)*sqrt(d*cos(b*x + a)))/(b*cos(b*x + a)^2 - b)]
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{11/2}\csc^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*cos(b*x+a))**(11/2)*csc(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.99

$$\int (d\cos(a+bx))^{11/2}\csc^3(a+bx)\,dx = \frac{\frac{20\sqrt{d\cos(bx+a)}d^8}{d^2\cos(bx+a)^2-d^2} + 90\,d^{\frac{13}{2}}\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right) - 45\,d^{\frac{13}{2}}\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right) - 16\,(d\cos(bx+a)}{40\,bd}$$

input
```
integrate((d*cos(b*x+a))^(11/2)*csc(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/40*(20*sqrt(d*cos(b*x + a))*d^8/(d^2*cos(b*x + a)^2 - d^2) + 90*d^(13/2)
*arctan(sqrt(d*cos(b*x + a))/sqrt(d)) - 45*d^(13/2)*log((sqrt(d*cos(b*x +
a)) - sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d))) - 16*(d*cos(b*x + a))^(5/
2)*d^4 - 160*sqrt(d*cos(b*x + a))*d^6)/(b*d)
```

Giac [F]

$$\int (d\cos(a+bx))^{11/2}\csc^3(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{11}{2}}\csc(bx+a)^3\,dx$$

input
```
integrate((d*cos(b*x+a))^(11/2)*csc(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*cos(b*x + a))^(11/2)*csc(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{11/2}\csc^3(a+bx)\,dx = \int \frac{(d\cos(a+b\,x))^{11/2}}{\sin(a+b\,x)^3}\,dx$$

input
```
int((d*cos(a + b*x))^(11/2)/sin(a + b*x)^3,x)
```

output
```
int((d*cos(a + b*x))^(11/2)/sin(a + b*x)^3, x)
```

Reduce [F]

$$\int (d\cos(a+bx))^{11/2}\csc^3(a+bx)\,dx = \sqrt{d}\left(\int \sqrt{\cos(bx+a)}\,\cos(bx+a)^5\csc(bx+a)^3\,dx\right)d^5$$

input
```
int((d*cos(b*x+a))^(11/2)*csc(b*x+a)^3,x)
```

output
```
sqrt(d)*int(sqrt(cos(a + b*x))*cos(a + b*x)**5*csc(a + b*x)**3,x)*d**5
```

3.243 $\int (d\cos(a+bx))^{9/2}\csc^3(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 113

$$\int (d\cos(a+bx))^{9/2}\csc^3(a+bx)\,dx = -\frac{7d^{9/2}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}+\frac{7d^{9/2}\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}-\frac{7d^3(d\cos(a+bx))^{3/2}}{6b}-\frac{d(d\cos(a+bx))^{7/2}\csc^2(a+bx)}{2b}$$

output
```
-7/4*d^(9/2)*arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b+7/4*d^(9/2)*arctanh((d
*cos(b*x+a))^(1/2)/d^(1/2))/b-7/6*d^3*(d*cos(b*x+a))^(3/2)/b-1/2*d*(d*cos(
b*x+a))^(7/2)*csc(b*x+a)^2/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.21 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.69

$$\int (d\cos(a+bx))^{9/2}\csc^3(a+bx)\,dx = \frac{d^5\left(\left(-5+2\cos(2(a+bx))\right)\cot^2(a+bx)+21\sqrt[4]{-\cot^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4},\frac{5}{4},\csc\right.\right.}{6b\sqrt{d\cos(a+bx)}}$$

input
```
Integrate[(d*Cos[a + b*x])^(9/2)*Csc[a + b*x]^3,x]
```

output
```
(d^5*((-5 + 2*Cos[2*(a + b*x)])*Cot[a + b*x]^2 + 21*(-Cot[a + b*x]^2)^(1/4
)*Hypergeometric2F1[1/4, 1/4, 5/4, Csc[a + b*x]^2]))/(6*b*Sqrt[d*Cos[a + b
*x]])
```

Rubi [A] (warning: unable to verify)

Time = 0.51 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3045, 27, 252, 262, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(a+bx)(d\cos(a+bx))^{9/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\cos(a+bx))^{9/2}}{\sin(a+bx)^3}dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int \frac{d^4(d\cos(a+bx))^{9/2}}{(d^2-d^2\cos^2(a+bx))^2}d(d\cos(a+bx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$-\frac{d^3\int \frac{(d\cos(a+bx))^{9/2}}{(d^2-d^2\cos^2(a+bx))^2}d(d\cos(a+bx))}{b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{d^3\left(\frac{(d\cos(a+bx))^{7/2}}{2(d^2-d^2\cos^2(a+bx))}-\frac{7}{4}\int \frac{(d\cos(a+bx))^{5/2}}{d^2-d^2\cos^2(a+bx)}d(d\cos(a+bx))\right)}{b}$$

$$\Big\downarrow \text{262}$$

$$-\frac{d^3\left(\frac{(d\cos(a+bx))^{7/2}}{2(d^2-d^2\cos^2(a+bx))}-\frac{7}{4}\left(d^2\int \frac{\sqrt{d\cos(a+bx)}}{d^2-d^2\cos^2(a+bx)}d(d\cos(a+bx))-\frac{2}{3}(d\cos(a+bx))^{3/2}\right)\right)}{b}$$

$$\Big\downarrow \color{blue}{266}$$

$$-\frac{d^3\left(\frac{(d\cos(a+bx))^{7/2}}{2(d^2-d^2\cos^2(a+bx))}-\frac{7}{4}\left(2d^2\int\frac{d^2\cos^2(a+bx)}{d^2-d^4\cos^4(a+bx)}d\sqrt{d\cos(a+bx)}-\frac{2}{3}(d\cos(a+bx))^{3/2}\right)\right)}{b}$$

$$\Big\downarrow \color{blue}{827}$$

$$-\frac{d^3\left(\frac{(d\cos(a+bx))^{7/2}}{2(d^2-d^2\cos^2(a+bx))}-\frac{7}{4}\left(2d^2\left(\frac{1}{2}\int\frac{1}{d-d^2\cos^2(a+bx)}d\sqrt{d\cos(a+bx)}-\frac{1}{2}\int\frac{1}{d^2\cos^2(a+bx)+d}d\sqrt{d\cos(a+bx)}\right)-\frac{2}{3}(\right.\right.}{b}$$

$$\Big\downarrow \color{blue}{216}$$

$$-\frac{d^3\left(\frac{(d\cos(a+bx))^{7/2}}{2(d^2-d^2\cos^2(a+bx))}-\frac{7}{4}\left(2d^2\left(\frac{1}{2}\int\frac{1}{d-d^2\cos^2(a+bx)}d\sqrt{d\cos(a+bx)}-\frac{\arctan\left(\sqrt{d}\cos(a+bx)\right)}{2\sqrt{d}}\right)-\frac{2}{3}(d\cos(a+bx))^{3\ldots}\right.\right.}{b}$$

$$\Big\downarrow \color{blue}{219}$$

$$-\frac{d^3\left(\frac{(d\cos(a+bx))^{7/2}}{2(d^2-d^2\cos^2(a+bx))}-\frac{7}{4}\left(2d^2\left(\frac{\text{arctanh}\left(\sqrt{d}\cos(a+bx)\right)}{2\sqrt{d}}-\frac{\arctan\left(\sqrt{d}\cos(a+bx)\right)}{2\sqrt{d}}\right)-\frac{2}{3}(d\cos(a+bx))^{3/2}\right)\right)}{b}$$

input

```
Int[(d*Cos[a + b*x])^(9/2)*Csc[a + b*x]^3,x]
```

output

```
-((d^3*((d*Cos[a + b*x])^(7/2)/(2*(d^2 - d^2*Cos[a + b*x]^2)) - (7*(2*d^2*
(-1/2*ArcTan[Sqrt[d]*Cos[a + b*x]]/Sqrt[d] + ArcTanh[Sqrt[d]*Cos[a + b*x]]
/(2*Sqrt[d])) - (2*(d*Cos[a + b*x])^(3/2))/3))/4))/b)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(89) = 178.

Time = 9.60 (sec) , antiderivative size = 371, normalized size of antiderivative = 3.28

method	result
default	$\frac{\frac{d^4\sqrt{2d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^2-d}}{8\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^2}+\frac{7d^5\ln\left(\frac{-2d+2\sqrt{-d}\,\sqrt{2d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^2-d}}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)}\right)}{4\sqrt{-d}}+2d^4\sqrt{d\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1\right)}-\frac{4d^4\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^2\sqrt{2d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^2-}}{3}}{}$

input
```
int((d*cos(b*x+a))^(9/2)*csc(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8*d^4/cos(1/2*b*x+1/2*a)^2*(2*d*cos(1/2*b*x+1/2*a)^2-d)^(1/2)+7/4*d^5/(
-d)^(1/2)*ln((-2*d+2*(-d)^(1/2)*(2*d*cos(1/2*b*x+1/2*a)^2-d)^(1/2))/cos(1/
2*b*x+1/2*a))+2*d^4*(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)-4/3*d^4*cos(1/2*b
*x+1/2*a)^2*(2*d*cos(1/2*b*x+1/2*a)^2-d)^(1/2)-4/3*d^4*(2*d*cos(1/2*b*x+1/
2*a)^2-d)^(1/2)-1/16*d^4/(cos(1/2*b*x+1/2*a)+1)*(-2*sin(1/2*b*x+1/2*a)^2*d
+d)^(1/2)+7/8*d^(9/2)*ln((-4*d*cos(1/2*b*x+1/2*a)+2*d^(1/2)*(-2*sin(1/2*b*
x+1/2*a)^2*d+d)^(1/2)-2*d)/(cos(1/2*b*x+1/2*a)+1))+1/16*d^4/(cos(1/2*b*x+1
/2*a)-1)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+7/8*d^(9/2)*ln((4*d*cos(1/2*b
*x+1/2*a)+2*d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d)/(cos(1/2*b*x+
1/2*a)-1)))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(89) = 178.

Time = 0.22 (sec) , antiderivative size = 414, normalized size of antiderivative = 3.66

$$\int (d\cos(a+bx))^{9/2}\csc^3(a+bx)\,dx = \left[-\frac{42\left(d^4\cos\left(bx+a\right)^2-d^4\right)\sqrt{-d}\arctan\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)+1)}{2\,d\cos(bx+a)}\right)-21\left(d^4\cos\left(bx+a\right)^2-\right.}{}\right.$$

$$-\frac{42\left(d^4\cos\left(bx+a\right)^2-d^4\right)\sqrt{d}\arctan\left(\frac{\sqrt{d\cos(bx+a)}(\cos(bx+a)-1)}{2\sqrt{d}\cos(bx+a)}\right)-21\left(d^4\cos\left(bx+a\right)^2-d^4\right)\sqrt{d}\log\left(-\frac{d\cos(}{}\right.}{48\left(b\cos\left(b\right.\right.}$$

input
```
integrate((d*cos(b*x+a))^(9/2)*csc(b*x+a)^3,x, algorithm="fricas")
```

output
```
[-1/48*(42*(d^4*cos(b*x + a)^2 - d^4)*sqrt(-d)*arctan(1/2*sqrt(d*cos(b*x +
 a))*sqrt(-d)*(cos(b*x + a) + 1)/(d*cos(b*x + a))) - 21*(d^4*cos(b*x + a)^
2 - d^4)*sqrt(-d)*log(-(d*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))*sqrt(-d)
*(cos(b*x + a) - 1) - 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x +
a) + 1)) + 8*(4*d^4*cos(b*x + a)^3 - 7*d^4*cos(b*x + a))*sqrt(d*cos(b*x +
a)))/(b*cos(b*x + a)^2 - b), -1/48*(42*(d^4*cos(b*x + a)^2 - d^4)*sqrt(d)*
arctan(1/2*sqrt(d*cos(b*x + a))*(cos(b*x + a) - 1)/(sqrt(d)*cos(b*x + a)))
 - 21*(d^4*cos(b*x + a)^2 - d^4)*sqrt(d)*log(-(d*cos(b*x + a)^2 + 4*sqrt(d
*cos(b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x
 + a)^2 - 2*cos(b*x + a) + 1)) + 8*(4*d^4*cos(b*x + a)^3 - 7*d^4*cos(b*x +
 a))*sqrt(d*cos(b*x + a)))/(b*cos(b*x + a)^2 - b)]
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{9/2}\csc^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*cos(b*x+a))**(9/2)*csc(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.04

$$\int (d\cos(a+bx))^{9/2}\csc^3(a+bx)\,dx = \frac{\frac{12\,(d\cos(bx+a))^{\frac{3}{2}}d^6}{d^2\cos(bx+a)^2-d^2} - 42\,d^{\frac{11}{2}}\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right) - 21\,d^{\frac{11}{2}}\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right) - 16\,(d\cos(bx+a}{24\,bd}$$

input
```
integrate((d*cos(b*x+a))^(9/2)*csc(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/24*(12*(d*cos(b*x + a))^(3/2)*d^6/(d^2*cos(b*x + a)^2 - d^2) - 42*d^(11/
2)*arctan(sqrt(d*cos(b*x + a))/sqrt(d)) - 21*d^(11/2)*log((sqrt(d*cos(b*x
+ a)) - sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d))) - 16*(d*cos(b*x + a))^(
3/2)*d^4)/(b*d)
```

Giac [F]

$$\int (d\cos(a+bx))^{9/2}\csc^3(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{9}{2}}\csc(bx+a)^3\,dx$$

input
```
integrate((d*cos(b*x+a))^(9/2)*csc(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*cos(b*x + a))^(9/2)*csc(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{9/2}\csc^3(a+bx)\,dx = \int \frac{(d\cos(a+b\,x))^{9/2}}{\sin(a+b\,x)^3}\,dx$$

input
```
int((d*cos(a + b*x))^(9/2)/sin(a + b*x)^3,x)
```

output
```
int((d*cos(a + b*x))^(9/2)/sin(a + b*x)^3, x)
```

Reduce [F]

$$\int (d\cos(a+bx))^{9/2}\csc^3(a+bx)\,dx=\sqrt{d}\left(\int \sqrt{\cos(bx+a)}\,\cos(bx+a)^4\csc(bx+a)^3\,dx\right)d^4$$

input
```
int((d*cos(b*x+a))^(9/2)*csc(b*x+a)^3,x)
```

output
```
sqrt(d)*int(sqrt(cos(a + b*x))*cos(a + b*x)**4*csc(a + b*x)**3,x)*d**4
```

3.244 $\int (d\cos(a+bx))^{7/2}\csc^3(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 113

$$\int (d\cos(a+bx))^{7/2}\csc^3(a+bx)\,dx=\frac{5d^{7/2}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}+\frac{5d^{7/2}\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}-\frac{5d^3\sqrt{d\cos(a+bx)}}{2b}-\frac{d(d\cos(a+bx))^{5/2}\csc^2(a+bx)}{2b}$$

output
```
5/4*d^(7/2)*arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b+5/4*d^(7/2)*arctanh((d*
cos(b*x+a))^(1/2)/d^(1/2))/b-5/2*d^3*(d*cos(b*x+a))^(1/2)/b-1/2*d*(d*cos(b
*x+a))^(5/2)*csc(b*x+a)^2/b
```

Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.04

$$\int (d\cos(a+bx))^{7/2}\csc^3(a+bx)\,dx=\frac{(d\cos(a+bx))^{7/2}\left(5\arctan\left(\sqrt{\cos(a+bx)}\right)+3\text{arctanh}\left(\sqrt{\cos(a+bx)}\right)-8\sqrt{\cos(a+bx)}-2\right.}{4b\cos^{\frac{7}{2}}(a+bx}$$

input
```
Integrate[(d*Cos[a + b*x])^(7/2)*Csc[a + b*x]^3,x]
```

output
```
((d*Cos[a + b*x])^(7/2)*(5*ArcTan[Sqrt[Cos[a + b*x]]] + 3*ArcTanh[Sqrt[Cos
[a + b*x]]] - 8*Sqrt[Cos[a + b*x]] - 2*Sqrt[Cos[a + b*x]]*Csc[a + b*x]^2 -
 Log[1 - Sqrt[Cos[a + b*x]]] + Log[1 + Sqrt[Cos[a + b*x]]]))/(4*b*Cos[a +
b*x]^(7/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3045, 27, 252, 262, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(a+bx)(d\cos(a+bx))^{7/2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(d\cos(a+bx))^{7/2}}{\sin(a+bx)^3}dx$$

$$\Big\downarrow \ 3045$$

$$-\frac{\int \frac{d^4(d\cos(a+bx))^{7/2}}{(d^2-d^2\cos^2(a+bx))^2}d(d\cos(a+bx))}{bd}$$

$$\Big\downarrow \ 27$$

$$-\frac{d^3\int \frac{(d\cos(a+bx))^{7/2}}{(d^2-d^2\cos^2(a+bx))^2}d(d\cos(a+bx))}{b}$$

$$\Big\downarrow \ 252$$

$$-\frac{d^3\left(\frac{(d\cos(a+bx))^{5/2}}{2(d^2-d^2\cos^2(a+bx))}-\frac{5}{4}\int \frac{(d\cos(a+bx))^{3/2}}{d^2-d^2\cos^2(a+bx)}d(d\cos(a+bx))\right)}{b}$$

$$\Big\downarrow \ 262$$

$$-\frac{d^3\left(\frac{(d\cos(a+bx))^{5/2}}{2(d^2-d^2\cos^2(a+bx))}-\frac{5}{4}\left(d^2\int \frac{1}{\sqrt{d\cos(a+bx)}(d^2-d^2\cos^2(a+bx))}d(d\cos(a+bx))-2\sqrt{d\cos(a+bx)}\right)\right)}{b}$$

$$\Big\downarrow \ 266$$

$$-\frac{d^3\left(\frac{(d\cos(a+bx))^{5/2}}{2(d^2-d^2\cos^2(a+bx))}-\frac{5}{4}\left(2d^2\int\frac{1}{d^2-d^4\cos^4(a+bx)}d\sqrt{d\cos(a+bx)}-2\sqrt{d\cos(a+bx)}\right)\right)}{b}$$

$\Big\downarrow$ 756

$$-\frac{d^3\left(\frac{(d\cos(a+bx))^{5/2}}{2(d^2-d^2\cos^2(a+bx))}-\frac{5}{4}\left(2d^2\left(\frac{\int\frac{1}{d-d^2\cos^2(a+bx)}d\sqrt{d\cos(a+bx)}}{2d}+\frac{\int\frac{1}{d^2\cos^2(a+bx)+d}d\sqrt{d\cos(a+bx)}}{2d}\right)-2\sqrt{d\cos(a+bx)}\right)\right)}{b}$$

$\Big\downarrow$ 216

$$-\frac{d^3\left(\frac{(d\cos(a+bx))^{5/2}}{2(d^2-d^2\cos^2(a+bx))}-\frac{5}{4}\left(2d^2\left(\frac{\int\frac{1}{d-d^2\cos^2(a+bx)}d\sqrt{d\cos(a+bx)}}{2d}+\frac{\arctan\left(\sqrt{d}\cos(a+bx)\right)}{2d^{3/2}}\right)-2\sqrt{d\cos(a+bx)}\right)\right)}{b}$$

$\Big\downarrow$ 219

$$-\frac{d^3\left(\frac{(d\cos(a+bx))^{5/2}}{2(d^2-d^2\cos^2(a+bx))}-\frac{5}{4}\left(2d^2\left(\frac{\arctan\left(\sqrt{d}\cos(a+bx)\right)}{2d^{3/2}}+\frac{\text{arctanh}\left(\sqrt{d}\cos(a+bx)\right)}{2d^{3/2}}\right)-2\sqrt{d\cos(a+bx)}\right)\right)}{b}$$

input
```
Int[(d*Cos[a + b*x])^(7/2)*Csc[a + b*x]^3,x]
```

output
```
-((d^3*((d*Cos[a + b*x])^(5/2)/(2*(d^2 - d^2*Cos[a + b*x]^2)) - (5*(2*d^2*
(ArcTan[Sqrt[d]*Cos[a + b*x]]/(2*d^(3/2)) + ArcTanh[Sqrt[d]*Cos[a + b*x]]/
(2*d^(3/2))) - 2*Sqrt[d*Cos[a + b*x]]))/4))/b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 309 vs. 2(89) = 178.

Time = 9.64 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.74

method	result
default	$-\frac{d^3\sqrt{2d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^2-d}}{8\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^2}-\frac{5d^4\ln\left(\frac{-2d+2\sqrt{-d}\sqrt{2d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^2-d}}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)}\right)}{4\sqrt{-d}}-2d^3\sqrt{d\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1\right)}-\frac{d^3\sqrt{-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^2d+d}}{16\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}+\frac{5d^{\frac{7}{2}}\ln}{}$

input
```
int((d*cos(b*x+a))^(7/2)*csc(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/8*d^3/cos(1/2*b*x+1/2*a)^2*(2*d*cos(1/2*b*x+1/2*a)^2-d)^(1/2)-5/4*d^4/
(-d)^(1/2)*ln((-2*d+2*(-d)^(1/2)*(2*d*cos(1/2*b*x+1/2*a)^2-d)^(1/2))/cos(1
/2*b*x+1/2*a))-2*d^3*(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)-1/16*d^3/(cos(1/
2*b*x+1/2*a)+1)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+5/8*d^(7/2)*ln((-4*d*c
os(1/2*b*x+1/2*a)+2*d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d)/(cos(
1/2*b*x+1/2*a)+1))+1/16*d^3/(cos(1/2*b*x+1/2*a)-1)*(-2*sin(1/2*b*x+1/2*a)^
2*d+d)^(1/2)+5/8*d^(7/2)*ln((4*d*cos(1/2*b*x+1/2*a)+2*d^(1/2)*(-2*sin(1/2*
b*x+1/2*a)^2*d+d)^(1/2)-2*d)/(cos(1/2*b*x+1/2*a)-1)))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. 2(89) = 178.

Time = 0.19 (sec) , antiderivative size = 402, normalized size of antiderivative = 3.56

$$\int (d\cos(a+bx))^{7/2}\csc^3(a+bx)\,dx = \left[-\frac{10\left(d^3\cos\left(bx+a\right)^2-d^3\right)\sqrt{-d}\arctan\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)+1)}{2\,d\cos(bx+a)}\right)-5\left(d^3\cos\left(bx+a\right)^2-d^3\right.}{1}\right.$$

input
```
integrate((d*cos(b*x+a))^(7/2)*csc(b*x+a)^3,x, algorithm="fricas")
```

output
```
[-1/16*(10*(d^3*cos(b*x + a)^2 - d^3)*sqrt(-d)*arctan(1/2*sqrt(d*cos(b*x +
 a))*sqrt(-d)*(cos(b*x + a) + 1)/(d*cos(b*x + a))) - 5*(d^3*cos(b*x + a)^2
 - d^3)*sqrt(-d)*log(-(d*cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(-d)*
(cos(b*x + a) - 1) - 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a
) + 1)) + 8*(4*d^3*cos(b*x + a)^2 - 5*d^3)*sqrt(d*cos(b*x + a)))/(b*cos(b*
x + a)^2 - b), 1/16*(10*(d^3*cos(b*x + a)^2 - d^3)*sqrt(d)*arctan(1/2*sqrt
(d*cos(b*x + a))*(cos(b*x + a) - 1)/(sqrt(d)*cos(b*x + a))) + 5*(d^3*cos(b
*x + a)^2 - d^3)*sqrt(d)*log(-(d*cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*s
qrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 - 2*cos(
b*x + a) + 1)) - 8*(4*d^3*cos(b*x + a)^2 - 5*d^3)*sqrt(d*cos(b*x + a)))/(b
*cos(b*x + a)^2 - b)]
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{7/2}\csc^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*cos(b*x+a))**(7/2)*csc(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.04

$$\int (d\cos(a+bx))^{7/2}\csc^3(a+bx)\,dx = \frac{\frac{4\sqrt{d\cos(bx+a)}d^6}{d^2\cos(bx+a)^2-d^2} + 10\,d^{\frac{9}{2}}\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right) - 5\,d^{\frac{9}{2}}\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right) - 16\sqrt{d\cos(bx+a)}d^4}{8\,bd}$$

input
```
integrate((d*cos(b*x+a))^(7/2)*csc(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/8*(4*sqrt(d*cos(b*x + a))*d^6/(d^2*cos(b*x + a)^2 - d^2) + 10*d^(9/2)*ar
ctan(sqrt(d*cos(b*x + a))/sqrt(d)) - 5*d^(9/2)*log((sqrt(d*cos(b*x + a)) -
 sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d))) - 16*sqrt(d*cos(b*x + a))*d^4)
/(b*d)
```

Giac [F]

$$\int (d\cos(a+bx))^{7/2}\csc^3(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{7}{2}}\csc(bx+a)^3\,dx$$

input
```
integrate((d*cos(b*x+a))^(7/2)*csc(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*cos(b*x + a))^(7/2)*csc(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{7/2}\csc^3(a+bx)\,dx = \int \frac{(d\cos(a+bx))^{7/2}}{\sin(a+bx)^3}\,dx$$

input
```
int((d*cos(a + b*x))^(7/2)/sin(a + b*x)^3,x)
```

output
```
int((d*cos(a + b*x))^(7/2)/sin(a + b*x)^3, x)
```

Reduce [F]

$$\int (d\cos(a+bx))^{7/2}\csc^3(a+bx)\,dx=\sqrt{d}\left(\int\sqrt{\cos(bx+a)}\,\cos(bx+a)^3\csc(bx+a)^3\,dx\right)d^3$$

input
```
int((d*cos(b*x+a))^(7/2)*csc(b*x+a)^3,x)
```

output
```
sqrt(d)*int(sqrt(cos(a + b*x))*cos(a + b*x)**3*csc(a + b*x)**3,x)*d**3
```

3.245 $\int (d\cos(a+bx))^{5/2}\csc^3(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 91

$$\int (d\cos(a+bx))^{5/2}\csc^3(a+bx)\,dx = -\frac{3d^{5/2}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} + \frac{3d^{5/2}\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} - \frac{d(d\cos(a+bx))^{3/2}\csc^2(a+bx)}{2b}$$

output
```
-3/4*d^(5/2)*arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b+3/4*d^(5/2)*arctanh((d
*cos(b*x+a))^(1/2)/d^(1/2))/b-1/2*d*(d*cos(b*x+a))^(3/2)*csc(b*x+a)^2/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.61 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.71

$$\int (d\cos(a+bx))^{5/2}\csc^3(a+bx)\,dx = -\frac{d^3\left(\cot^2(a+bx)-3\sqrt[4]{-\cot^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4},\frac{5}{4},\csc^2(a+bx)\right)\right)}{2b\sqrt{d\cos(a+bx)}}$$

input
```
Integrate[(d*Cos[a + b*x])^(5/2)*Csc[a + b*x]^3,x]
```

output
```
-1/2*(d^3*(Cot[a + b*x]^2 - 3*(-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/
4, 1/4, 5/4, Csc[a + b*x]^2]))/(b*Sqrt[d*Cos[a + b*x]])
```

Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3045, 27, 252, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(a+bx)(d\cos(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(d\cos(a+bx))^{5/2}}{\sin(a+bx)^3}dx$$

$$\Big\downarrow \text{ 3045}$$

$$-\frac{\int \frac{d^4(d\cos(a+bx))^{5/2}}{(d^2-d^2\cos^2(a+bx))^2}d(d\cos(a+bx))}{bd}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{d^3\int \frac{(d\cos(a+bx))^{5/2}}{(d^2-d^2\cos^2(a+bx))^2}d(d\cos(a+bx))}{b}$$

$$\Big\downarrow \text{ 252}$$

$$-\frac{d^3\left(\frac{(d\cos(a+bx))^{3/2}}{2(d^2-d^2\cos^2(a+bx))}-\frac{3}{4}\int \frac{\sqrt{d\cos(a+bx)}}{d^2-d^2\cos^2(a+bx)}d(d\cos(a+bx))\right)}{b}$$

$$\Big\downarrow \text{ 266}$$

$$-\frac{d^3\left(\frac{(d\cos(a+bx))^{3/2}}{2(d^2-d^2\cos^2(a+bx))}-\frac{3}{2}\int \frac{d^2\cos^2(a+bx)}{d^2-d^4\cos^4(a+bx)}d\sqrt{d\cos(a+bx)}\right)}{b}$$

$$\Big\downarrow \text{ 827}$$

$$-\frac{d^3\left(\frac{(d\cos(a+bx))^{3/2}}{2(d^2-d^2\cos^2(a+bx))}-\frac{3}{2}\left(\frac{1}{2}\int\frac{1}{d-d^2\cos^2(a+bx)}d\sqrt{d\cos(a+bx)}-\frac{1}{2}\int\frac{1}{d^2\cos^2(a+bx)+d}d\sqrt{d\cos(a+bx)}\right)\right)}{b}$$

$$\Big\downarrow \text{216}$$

$$-\frac{d^3\left(\frac{(d\cos(a+bx))^{3/2}}{2(d^2-d^2\cos^2(a+bx))}-\frac{3}{2}\left(\frac{1}{2}\int\frac{1}{d-d^2\cos^2(a+bx)}d\sqrt{d\cos(a+bx)}-\frac{\arctan\left(\sqrt{d}\cos(a+bx)\right)}{2\sqrt{d}}\right)\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{d^3\left(\frac{(d\cos(a+bx))^{3/2}}{2(d^2-d^2\cos^2(a+bx))}-\frac{3}{2}\left(\frac{\text{arctanh}\left(\sqrt{d}\cos(a+bx)\right)}{2\sqrt{d}}-\frac{\arctan\left(\sqrt{d}\cos(a+bx)\right)}{2\sqrt{d}}\right)\right)}{b}$$

input
```
Int[(d*Cos[a + b*x])^(5/2)*Csc[a + b*x]^3,x]
```

output
```
-((d^3*((-3*(-1/2*ArcTan[Sqrt[d]*Cos[a + b*x]]/Sqrt[d] + ArcTanh[Sqrt[d]*C
os[a + b*x]]/(2*Sqrt[d])))/2 + (d*Cos[a + b*x])^(3/2)/(2*(d^2 - d^2*Cos[a
+ b*x]^2))))/b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(71) = 142$.

Time = 9.77 (sec) , antiderivative size = 286, normalized size of antiderivative = 3.14

method	result
default	$\frac{\frac{d^{2}\sqrt{2d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-d}}{8\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}+\frac{3d^{3}\ln\left(\frac{-2d+2\sqrt{-d}\,\sqrt{2d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-d}}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)}\right)}{4\sqrt{-d}}-\frac{d^{2}\sqrt{-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}d+d}}{16\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}+\frac{3d^{\frac{5}{2}}\ln\left(\frac{-4d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+2\sqrt{d}\,\sqrt{-2\sin\left(\cdots\right.}}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\cdots}{8}\cdots}{b}$

input
```
int((d*cos(b*x+a))^(5/2)*csc(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8*d^2/cos(1/2*b*x+1/2*a)^2*(2*d*cos(1/2*b*x+1/2*a)^2-d)^(1/2)+3/4*d^3/(
-d)^(1/2)*ln((-2*d+2*(-d)^(1/2)*(2*d*cos(1/2*b*x+1/2*a)^2-d)^(1/2))/cos(1/
2*b*x+1/2*a))-1/16*d^2/(cos(1/2*b*x+1/2*a)+1)*(-2*sin(1/2*b*x+1/2*a)^2*d+d
)^(1/2)+3/8*d^(5/2)*ln((-4*d*cos(1/2*b*x+1/2*a)+2*d^(1/2)*(-2*sin(1/2*b*x+
1/2*a)^2*d+d)^(1/2)-2*d)/(cos(1/2*b*x+1/2*a)+1))+1/16*d^2/(cos(1/2*b*x+1/2
*a)-1)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+3/8*d^(5/2)*ln((4*d*cos(1/2*b*x
+1/2*a)+2*d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d)/(cos(1/2*b*x+1/
2*a)-1)))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(71) = 142$.

Time = 0.13 (sec) , antiderivative size = 371, normalized size of antiderivative = 4.08

$$\int (d\cos(a+bx))^{5/2}\csc^3(a+bx)\,dx = \left[\frac{8\sqrt{d\cos(bx+a)}d^2\cos(bx+a) - 6\left(d^2\cos(bx+a)^2 - d^2\right)\sqrt{-d}\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{-d}}{d\cos(bx+a)+d}\right) + 3}{16\left(b\cos(bx+\right.}\right.$$

input
```
integrate((d*cos(b*x+a))^(5/2)*csc(b*x+a)^3,x, algorithm="fricas")
```

output
```
[1/16*(8*sqrt(d*cos(b*x + a))*d^2*cos(b*x + a) - 6*(d^2*cos(b*x + a)^2 - d
^2)*sqrt(-d)*arctan(2*sqrt(d*cos(b*x + a))*sqrt(-d)/(d*cos(b*x + a) + d))
+ 3*(d^2*cos(b*x + a)^2 - d^2)*sqrt(-d)*log((d*cos(b*x + a)^2 - 4*sqrt(d*c
os(b*x + a))*sqrt(-d)*(cos(b*x + a) - 1) - 6*d*cos(b*x + a) + d)/(cos(b*x
+ a)^2 + 2*cos(b*x + a) + 1)))/(b*cos(b*x + a)^2 - b), 1/16*(8*sqrt(d*cos(
b*x + a))*d^2*cos(b*x + a) + 6*(d^2*cos(b*x + a)^2 - d^2)*sqrt(d)*arctan(2
*sqrt(d*cos(b*x + a))*sqrt(d)/(d*cos(b*x + a) - d)) + 3*(d^2*cos(b*x + a)^
2 - d^2)*sqrt(d)*log((d*cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(d)*(c
os(b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 - 2*cos(b*x + a)
+ 1)))/(b*cos(b*x + a)^2 - b)]
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{5/2}\csc^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*cos(b*x+a))**(5/2)*csc(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.13

$$\int (d\cos(a+bx))^{5/2}\csc^3(a+bx)\,dx = \frac{\frac{4\,(d\cos(bx+a))^{\frac{3}{2}}d^4}{d^2\cos(bx+a)^2-d^2} - 6\,d^{\frac{7}{2}}\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right) - 3\,d^{\frac{7}{2}}\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)}{8\,bd}$$

input
```
integrate((d*cos(b*x+a))^(5/2)*csc(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/8*(4*(d*cos(b*x + a))^(3/2)*d^4/(d^2*cos(b*x + a)^2 - d^2) - 6*d^(7/2)*a
rctan(sqrt(d*cos(b*x + a))/sqrt(d)) - 3*d^(7/2)*log((sqrt(d*cos(b*x + a))
- sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d))))/(b*d)
```

Giac [F]

$$\int (d\cos(a+bx))^{5/2}\csc^3(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{5}{2}}\csc(bx+a)^3\,dx$$

input
```
integrate((d*cos(b*x+a))^(5/2)*csc(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*cos(b*x + a))^(5/2)*csc(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{5/2}\csc^3(a+bx)\,dx = \int \frac{\left(d\,\cos\left(a+b\,x\right)\right)^{5/2}}{{\sin\left(a+b\,x\right)}^3}\,dx$$

input
```
int((d*cos(a + b*x))^(5/2)/sin(a + b*x)^3,x)
```

output
```
int((d*cos(a + b*x))^(5/2)/sin(a + b*x)^3, x)
```

Reduce [F]

$$\int (d\cos(a+bx))^{5/2}\csc^3(a + bx)\,dx = \sqrt{d}\left(\int \sqrt{\cos\left(bx+a\right)}\,\cos\left(bx+a\right)^2\csc\left(bx+a\right)^3dx\right)d^2$$

input
```
int((d*cos(b*x+a))^(5/2)*csc(b*x+a)^3,x)
```

output
```
sqrt(d)*int(sqrt(cos(a + b*x))*cos(a + b*x)**2*csc(a + b*x)**3,x)*d**2
```

3.246 $\int (d\cos(a+bx))^{3/2}\csc^3(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 91

$$\int (d\cos(a+bx))^{3/2}\csc^3(a+bx)\,dx = \frac{d^{3/2}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} + \frac{d^{3/2}\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} - \frac{d\sqrt{d\cos(a+bx)}\csc^2(a+bx)}{2b}$$

output
```
1/4*d^(3/2)*arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b+1/4*d^(3/2)*arctanh((d*
cos(b*x+a))^(1/2)/d^(1/2))/b-1/2*d*(d*cos(b*x+a))^(1/2)*csc(b*x+a)^2/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.45 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

$$\int (d\cos(a+bx))^{3/2}\csc^3(a+bx)\,dx = \frac{(d\cos(a+bx))^{3/2}\left(-\cot^2(a+bx)\right)^{3/4}\left(3\sqrt[4]{-\cot^2(a+bx)} + \text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{4},\frac{7}{4},\csc^2(a\right.\right.}{6b}$$

input
```
Integrate[(d*Cos[a + b*x])^(3/2)*Csc[a + b*x]^3,x]
```

output
```
((d*Cos[a + b*x])^(3/2)*(-Cot[a + b*x]^2)^(3/4)*(3*(-Cot[a + b*x]^2)^(1/4)
 + Hypergeometric2F1[3/4, 3/4, 7/4, Csc[a + b*x]^2])*Sec[a + b*x]^3)/(6*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3045, 27, 252, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(a+bx)(d\cos(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\cos(a+bx))^{3/2}}{\sin(a+bx)^3}dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int \frac{d^4(d\cos(a+bx))^{3/2}}{(d^2-d^2\cos^2(a+bx))^2}d(d\cos(a+bx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$-\frac{d^3\int \frac{(d\cos(a+bx))^{3/2}}{(d^2-d^2\cos^2(a+bx))^2}d(d\cos(a+bx))}{b}$$

$$\Big\downarrow \text{252}$$

$$-\frac{d^3\left(\frac{\sqrt{d\cos(a+bx)}}{2(d^2-d^2\cos^2(a+bx))}-\frac{1}{4}\int \frac{1}{\sqrt{d\cos(a+bx)}(d^2-d^2\cos^2(a+bx))}d(d\cos(a+bx))\right)}{b}$$

$$\Big\downarrow \text{266}$$

$$-\frac{d^3\left(\frac{\sqrt{d\cos(a+bx)}}{2(d^2-d^2\cos^2(a+bx))}-\frac{1}{2}\int \frac{1}{d^2-d^4\cos^4(a+bx)}d\sqrt{d\cos(a+bx)}\right)}{b}$$

$$\Big\downarrow \text{756}$$

$$-\frac{d^3\left(\frac{1}{2}\left(-\frac{\int \frac{1}{d-d^2\cos^2(a+bx)}d\sqrt{d\cos(a+bx)}}{2d}-\frac{\int \frac{1}{d^2\cos^2(a+bx)+d}d\sqrt{d\cos(a+bx)}}{2d}\right)+\frac{\sqrt{d\cos(a+bx)}}{2(d^2-d^2\cos^2(a+bx))}\right)}{b}$$

$$\Big\downarrow \text{216}$$

$$-\frac{d^3\left(\frac{1}{2}\left(-\frac{\int \frac{1}{d-d^2\cos^2(a+bx)}d\sqrt{d\cos(a+bx)}}{2d}-\frac{\arctan\left(\sqrt{d}\cos(a+bx)\right)}{2d^{3/2}}\right)+\frac{\sqrt{d\cos(a+bx)}}{2(d^2-d^2\cos^2(a+bx))}\right)}{b}$$

$$\Big\downarrow \text{219}$$

$$-\frac{d^3\left(\frac{1}{2}\left(-\frac{\arctan\left(\sqrt{d}\cos(a+bx)\right)}{2d^{3/2}}-\frac{\text{arctanh}\left(\sqrt{d}\cos(a+bx)\right)}{2d^{3/2}}\right)+\frac{\sqrt{d\cos(a+bx)}}{2(d^2-d^2\cos^2(a+bx))}\right)}{b}$$

input
```
Int[(d*Cos[a + b*x])^(3/2)*Csc[a + b*x]^3,x]
```

output
```
-((d^3*((-1/2*ArcTan[Sqrt[d]*Cos[a + b*x]]/d^(3/2) - ArcTanh[Sqrt[d]*Cos[a
 + b*x]]/(2*d^(3/2)))/2 + Sqrt[d*Cos[a + b*x]]/(2*(d^2 - d^2*Cos[a + b*x]^
2))))/b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(71) = 142$.

Time = 3.18 (sec) , antiderivative size = 280, normalized size of antiderivative = 3.08

method	result
default	$\frac{-\frac{d\sqrt{2d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-d}}{8\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}-\frac{d^{2}\ln\left(\frac{-2d+2\sqrt{-d}\,\sqrt{2d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-d}}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)}\right)}{4\sqrt{-d}}+\frac{d^{\frac{3}{2}}\ln\left(\frac{-4d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+2\sqrt{d}\,\sqrt{-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}d+d}-2d}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\right)}{8}+\frac{d^{\frac{3}{2}}\ln\left(\cdots\right.}{\cdots}}{b}$

input
```
int((d*cos(b*x+a))^(3/2)*csc(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/8*d/cos(1/2*b*x+1/2*a)^2*(2*d*cos(1/2*b*x+1/2*a)^2-d)^(1/2)-1/4*d^2/(-
d)^(1/2)*ln((-2*d+2*(-d)^(1/2)*(2*d*cos(1/2*b*x+1/2*a)^2-d)^(1/2))/cos(1/2
*b*x+1/2*a))+1/8*d^(3/2)*ln((-4*d*cos(1/2*b*x+1/2*a)+2*d^(1/2)*(-2*sin(1/2
*b*x+1/2*a)^2*d+d)^(1/2)-2*d)/(cos(1/2*b*x+1/2*a)+1))+1/8*d^(3/2)*ln((4*d*
cos(1/2*b*x+1/2*a)+2*d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d)/(cos
(1/2*b*x+1/2*a)-1))-1/16*d/(cos(1/2*b*x+1/2*a)+1)*(-2*sin(1/2*b*x+1/2*a)^2
*d+d)^(1/2)+1/16*d/(cos(1/2*b*x+1/2*a)-1)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1
/2))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(71) = 142$.

Time = 0.14 (sec) , antiderivative size = 339, normalized size of antiderivative = 3.73

$$\int (d\cos(a+bx))^{3/2}\csc^3(a+bx)\,dx = \left[-\frac{2\left(d\cos\left(bx+a\right)^2-d\right)\sqrt{-d}\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{-d}}{d\cos(bx+a)+d}\right)-\left(d\cos\left(bx+a\right)^2-d\right)\sqrt{-d}\log\left(\frac{d\cos(\ldots}{\ldots}\right.\ldots}{16\left(b\cos\left(bx+a\right)^2-b\right)\ldots}\right.$$

$$-\frac{2\left(d\cos\left(bx+a\right)^2-d\right)\sqrt{d}\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{d}}{d\cos(bx+a)-d}\right)-\left(d\cos\left(bx+a\right)^2-d\right)\sqrt{d}\log\left(\frac{d\cos(bx+a)^2+4\sqrt{d\cos(bx+\ldots}}{\cos(bx+a\ldots}\right.\ldots}{16\left(b\cos\left(bx+a\right)^2-b\right)}$$

input
```
integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a)^3,x, algorithm="fricas")
```

output
```
[-1/16*(2*(d*cos(b*x + a)^2 - d)*sqrt(-d)*arctan(2*sqrt(d*cos(b*x + a))*sq
rt(-d)/(d*cos(b*x + a) + d)) - (d*cos(b*x + a)^2 - d)*sqrt(-d)*log((d*cos(
b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) - 1) - 6*d*cos(
b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a) + 1)) - 8*sqrt(d*cos(b*x +
a))*d)/(b*cos(b*x + a)^2 - b), -1/16*(2*(d*cos(b*x + a)^2 - d)*sqrt(d)*arc
tan(2*sqrt(d*cos(b*x + a))*sqrt(d)/(d*cos(b*x + a) - d)) - (d*cos(b*x + a)
^2 - d)*sqrt(d)*log((d*cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(d)*(co
s(b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 - 2*cos(b*x + a) +
 1)) - 8*sqrt(d*cos(b*x + a))*d)/(b*cos(b*x + a)^2 - b)]
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{3/2}\csc^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*cos(b*x+a))**(3/2)*csc(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.13

$$\int (d\cos(a+bx))^{3/2}\csc^3(a+bx)\,dx = \frac{\frac{4\sqrt{d\cos(bx+a)}d^4}{d^2\cos(bx+a)^2-d^2} + 2\,d^{\frac{5}{2}}\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right) - d^{\frac{5}{2}}\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)}{8\,bd}$$

input
```
integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/8*(4*sqrt(d*cos(b*x + a))*d^4/(d^2*cos(b*x + a)^2 - d^2) + 2*d^(5/2)*arc
tan(sqrt(d*cos(b*x + a))/sqrt(d)) - d^(5/2)*log((sqrt(d*cos(b*x + a)) - sq
rt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d))))/(b*d)
```

Giac [F]

$$\int (d\cos(a+bx))^{3/2}\csc^3(a+bx)\,dx = \int (d\cos(bx+a))^{\frac{3}{2}}\csc(bx+a)^3\,dx$$

input
```
integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*cos(b*x + a))^(3/2)*csc(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{3/2}\csc^3(a+bx)\,dx = \int \frac{(d\,\cos\left(a+b\,x\right))^{3/2}}{\sin\left(a+b\,x\right)^3}\,dx$$

input
```
int((d*cos(a + b*x))^(3/2)/sin(a + b*x)^3,x)
```

output
```
int((d*cos(a + b*x))^(3/2)/sin(a + b*x)^3, x)
```

Reduce [F]

$$\int (d\cos(a+bx))^{3/2}\csc^3(a+bx)\,dx = \sqrt{d}\left(\int \sqrt{\cos\left(bx+a\right)}\,\cos\left(bx+a\right)\csc\left(bx+a\right)^3 dx\right) d$$

input
```
int((d*cos(b*x+a))^(3/2)*csc(b*x+a)^3,x)
```

output
```
sqrt(d)*int(sqrt(cos(a + b*x))*cos(a + b*x)*csc(a + b*x)**3,x)*d
```

3.247 $\int \sqrt{d\cos(a+bx)}\csc^3(a+bx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 93

$$\int \sqrt{d\cos(a+bx)}\csc^3(a+bx)\,dx = \frac{\sqrt{d}\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} - \frac{\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} - \frac{(d\cos(a+bx))^{3/2}\csc^2(a+bx)}{2bd}$$

output
```
1/4*d^(1/2)*arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b-1/4*d^(1/2)*arctanh((d*
cos(b*x+a))^(1/2)/d^(1/2))/b-1/2*(d*cos(b*x+a))^(3/2)*csc(b*x+a)^2/b/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.68 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.67

$$\int \sqrt{d\cos(a+bx)}\csc^3(a+bx)\,dx = -\frac{d\left(\cot^2(a+bx)+\sqrt[4]{-\cot^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4},\frac{5}{4},\csc^2(a+bx)\right)\right)}{2b\sqrt{d\cos(a+bx)}}$$

input
```
Integrate[Sqrt[d*Cos[a + b*x]]*Csc[a + b*x]^3,x]
```

output
```
-1/2*(d*(Cot[a + b*x]^2 + (-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, 1
/4, 5/4, Csc[a + b*x]^2]))/(b*Sqrt[d*Cos[a + b*x]])
```

Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3045, 27, 253, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(a+bx)\sqrt{d\cos(a+bx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{d\cos(a+bx)}}{\sin(a+bx)^3}dx$$

$$\Big\downarrow \; 3045$$

$$-\frac{\int \frac{d^4\sqrt{d\cos(a+bx)}}{(d^2-d^2\cos^2(a+bx))^2}d(d\cos(a+bx))}{bd}$$

$$\Big\downarrow \; 27$$

$$-\frac{d^3\int \frac{\sqrt{d\cos(a+bx)}}{(d^2-d^2\cos^2(a+bx))^2}d(d\cos(a+bx))}{b}$$

$$\Big\downarrow \; 253$$

$$-\frac{d^3\left(\frac{\int \frac{\sqrt{d\cos(a+bx)}}{d^2-d^2\cos^2(a+bx)}d(d\cos(a+bx))}{4d^2}+\frac{(d\cos(a+bx))^{3/2}}{2d^2(d^2-d^2\cos^2(a+bx))}\right)}{b}$$

$$\Big\downarrow \; 266$$

$$-\frac{d^3\left(\frac{\int \frac{d^2\cos^2(a+bx)}{d^2-d^4\cos^4(a+bx)}d\sqrt{d\cos(a+bx)}}{2d^2}+\frac{(d\cos(a+bx))^{3/2}}{2d^2(d^2-d^2\cos^2(a+bx))}\right)}{b}$$

$$\Big\downarrow \; 827$$

$$-\frac{d^3\left(\frac{\frac{1}{2}\int \frac{1}{d-d^2\cos^2(a+bx)}d\sqrt{d\cos(a+bx)}-\frac{1}{2}\int \frac{1}{d^2\cos^2(a+bx)+d}d\sqrt{d\cos(a+bx)}}{2d^2}+\frac{(d\cos(a+bx))^{3/2}}{2d^2(d^2-d^2\cos^2(a+bx))}\right)}{b}$$

$$\Big\downarrow \; 216$$

$$-\frac{d^3\left(\frac{\frac{1}{2}\int \frac{1}{d-d^2\cos^2(a+bx)}d\sqrt{d\cos(a+bx)}-\frac{\arctan\left(\sqrt{d}\cos(a+bx)\right)}{2\sqrt{d}}}{2d^2}+\frac{(d\cos(a+bx))^{3/2}}{2d^2(d^2-d^2\cos^2(a+bx))}\right)}{b}$$

$$\Big\downarrow \; 219$$

$$-\frac{d^3\left(\frac{\frac{\text{arctanh}\left(\sqrt{d}\cos(a+bx)\right)}{2\sqrt{d}}-\frac{\arctan\left(\sqrt{d}\cos(a+bx)\right)}{2\sqrt{d}}}{2d^2}+\frac{(d\cos(a+bx))^{3/2}}{2d^2(d^2-d^2\cos^2(a+bx))}\right)}{b}$$

input
```
Int[Sqrt[d*Cos[a + b*x]]*Csc[a + b*x]^3,x]
```

output
```
-((d^3*((-1/2*ArcTan[Sqrt[d]*Cos[a + b*x]]/Sqrt[d] + ArcTanh[Sqrt[d]*Cos[a
 + b*x]]/(2*Sqrt[d]))/(2*d^2) + (d*Cos[a + b*x])^(3/2)/(2*d^2*(d^2 - d^2*C
os[a + b*x]^2))))/b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(73) = 146$.

Time = 3.48 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.96

method	result
default	$\frac{\frac{\sqrt{2d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^2-d}}{8\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^2}-\frac{d\ln\left(\frac{-2d+2\sqrt{-d}\sqrt{2d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^2-d}}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)}\right)}{4\sqrt{-d}}-\frac{\sqrt{-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^2d+d}}{16\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}-\frac{\sqrt{d}\ln\left(\frac{-4d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+2\sqrt{d}\sqrt{-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^2}}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\right.}{8}}{b}$

input
```
int((d*cos(b*x+a))^(1/2)*csc(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8/cos(1/2*b*x+1/2*a)^2*(2*d*cos(1/2*b*x+1/2*a)^2-d)^(1/2)-1/4*d/(-d)^(1
/2)*ln((-2*d+2*(-d)^(1/2)*(2*d*cos(1/2*b*x+1/2*a)^2-d)^(1/2))/cos(1/2*b*x+
1/2*a))-1/16/(cos(1/2*b*x+1/2*a)+1)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-1/
8*d^(1/2)*ln((-4*d*cos(1/2*b*x+1/2*a)+2*d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d
+d)^(1/2)-2*d)/(cos(1/2*b*x+1/2*a)+1))+1/16/(cos(1/2*b*x+1/2*a)-1)*(-2*sin
(1/2*b*x+1/2*a)^2*d+d)^(1/2)-1/8*d^(1/2)*ln((4*d*cos(1/2*b*x+1/2*a)+2*d^(1
/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d)/(cos(1/2*b*x+1/2*a)-1)))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(73) = 146$.

Time = 0.17 (sec) , antiderivative size = 332, normalized size of antiderivative = 3.57

$$\int \sqrt{d\cos(a+bx)}\csc^3(a+bx)\,dx$$
$$= \left[\frac{2\left(\cos(bx+a)^2-1\right)\sqrt{-d}\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{-d}}{d\cos(bx+a)+d}\right)+\left(\cos(bx+a)^2-1\right)\sqrt{-d}\log\left(\frac{d\cos(bx+a)^2+4\sqrt{d\cos(}}{\cos(}\right.}{16\left(b\cos(bx+a)^2-b\right)}\right.$$
$$-\frac{2\left(\cos(bx+a)^2-1\right)\sqrt{d}\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{d}}{d\cos(bx+a)-d}\right)-\left(\cos(bx+a)^2-1\right)\sqrt{d}\log\left(\frac{d\cos(bx+a)^2-4\sqrt{d\cos(bx+a}}{\cos(bx+a)}\right.}{16\left(b\cos(bx+a)^2-b\right)}$$

input
```
integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a)^3,x, algorithm="fricas")
```

output
```
[1/16*(2*(cos(b*x + a)^2 - 1)*sqrt(-d)*arctan(2*sqrt(d*cos(b*x + a))*sqrt(
-d)/(d*cos(b*x + a) + d)) + (cos(b*x + a)^2 - 1)*sqrt(-d)*log((d*cos(b*x +
 a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) - 1) - 6*d*cos(b*x +
 a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a) + 1)) + 8*sqrt(d*cos(b*x + a))*c
os(b*x + a))/(b*cos(b*x + a)^2 - b), -1/16*(2*(cos(b*x + a)^2 - 1)*sqrt(d)
*arctan(2*sqrt(d*cos(b*x + a))*sqrt(d)/(d*cos(b*x + a) - d)) - (cos(b*x +
a)^2 - 1)*sqrt(d)*log((d*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))*sqrt(d)*(
cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 - 2*cos(b*x + a)
 + 1)) - 8*sqrt(d*cos(b*x + a))*cos(b*x + a))/(b*cos(b*x + a)^2 - b)]
```

Sympy [F]

$$\int \sqrt{d\cos(a+bx)}\csc^3(a+bx)\,dx = \int \sqrt{d\cos(a+bx)}\csc^3(a+bx)\,dx$$

input
```
integrate((d*cos(b*x+a))**(1/2)*csc(b*x+a)**3,x)
```

output
```
Integral(sqrt(d*cos(a + b*x))*csc(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.10

$$\int \sqrt{d\cos(a+bx)}\csc^3(a+bx)\,dx = \frac{\frac{4\,(d\cos(bx+a))^{\frac{3}{2}}d^2}{d^2\cos(bx+a)^2-d^2} + 2\,d^{\frac{3}{2}}\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right) + d^{\frac{3}{2}}\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)}{8\,bd}$$

input
```
integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/8*(4*(d*cos(b*x + a))^(3/2)*d^2/(d^2*cos(b*x + a)^2 - d^2) + 2*d^(3/2)*a
rctan(sqrt(d*cos(b*x + a))/sqrt(d)) + d^(3/2)*log((sqrt(d*cos(b*x + a)) -
sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d))))/(b*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.02

$$\int \sqrt{d\cos(a+bx)}\csc^3(a+bx)\,dx = \frac{d^3\left(\frac{2\sqrt{d\cos(bx+a)}\cos(bx+a)}{\left(d^2\cos(bx+a)^2-d^2\right)d} + \frac{\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{-d}}\right)}{\sqrt{-d}d^2} + \frac{\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)}{d^{\frac{5}{2}}}\right)}{4b}$$

input
```
integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a)^3,x, algorithm="giac")
```

output
```
1/4*d^3*(2*sqrt(d*cos(b*x + a))*cos(b*x + a)/((d^2*cos(b*x + a)^2 - d^2)*d
) + arctan(sqrt(d*cos(b*x + a))/sqrt(-d))/(sqrt(-d)*d^2) + arctan(sqrt(d*c
os(b*x + a))/sqrt(d))/d^(5/2))/b
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\cos(a+bx)}\csc^3(a+bx)\,dx = \int \frac{\sqrt{d\cos(a+bx)}}{\sin(a+bx)^3}\,dx$$

input
```
int((d*cos(a + b*x))^(1/2)/sin(a + b*x)^3,x)
```

output
```
int((d*cos(a + b*x))^(1/2)/sin(a + b*x)^3, x)
```

Reduce [F]

$$\int \sqrt{d\cos(a+bx)}\csc^3(a+bx)\,dx = \sqrt{d}\left(\int \sqrt{\cos(bx+a)}\,\csc(bx+a)^3\,dx\right)$$

input
```
int((d*cos(b*x+a))^(1/2)*csc(b*x+a)^3,x)
```

output
```
sqrt(d)*int(sqrt(cos(a + b*x))*csc(a + b*x)**3,x)
```

3.248 $\int \frac{\csc^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 93

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{3\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b\sqrt{d}} - \frac{3\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b\sqrt{d}} - \frac{\sqrt{d\cos(a+bx)}\csc^2(a+bx)}{2bd}$$

output
```
-3/4*arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b/d^(1/2)-3/4*arctanh((d*cos(b*x
+a))^(1/2)/d^(1/2))/b/d^(1/2)-1/2*(d*cos(b*x+a))^(1/2)*csc(b*x+a)^2/b/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.50 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.74

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \frac{d(-\cot^2(a+bx))^{3/4}\left(\sqrt[4]{-\cot^2(a+bx)} - \text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{4},\frac{7}{4},\csc^2(a+bx)\right)\right)}{2b(d\cos(a+bx))^{3/2}}$$

input
```
Integrate[Csc[a + b*x]^3/Sqrt[d*Cos[a + b*x]],x]
```

output
```
(d*(-Cot[a + b*x]^2)^(3/4)*((-Cot[a + b*x]^2)^(1/4) - Hypergeometric2F1[3/
4, 3/4, 7/4, Csc[a + b*x]^2]))/(2*b*(d*Cos[a + b*x])^(3/2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

Failed to integrate

input
```
Int[Csc[a + b*x]^3/Sqrt[d*Cos[a + b*x]],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 4276
```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Simp[1/b   Int[Csc[e + f*x], x], x] - Simp[a/b   Int[Csc[e + f*x]/(
a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(73) = 146$.

Time = 3.03 (sec) , antiderivative size = 283, normalized size of antiderivative = 3.04

method	result
default	$\frac{-\frac{\sqrt{2d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-d}}{8d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}+\frac{3\ln\left(\frac{-2d+2\sqrt{-d}\,\sqrt{2d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-d}}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)}\right)}{4\sqrt{-d}}-\frac{\sqrt{-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}d+d}}{16d\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}-\frac{3\ln\left(\frac{-4d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+2\sqrt{d}\,\sqrt{-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\right)}{8\sqrt{d}}}{b}$

input
```
int(csc(b*x+a)^3/(d*cos(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-1/8/d/cos(1/2*b*x+1/2*a)^2*(2*d*cos(1/2*b*x+1/2*a)^2-d)^(1/2)+3/4/(-d)^(
1/2)*ln((-2*d+2*(-d)^(1/2)*(2*d*cos(1/2*b*x+1/2*a)^2-d)^(1/2))/cos(1/2*b*x
+1/2*a))-1/16/d/(cos(1/2*b*x+1/2*a)+1)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)
-3/8/d^(1/2)*ln((-4*d*cos(1/2*b*x+1/2*a)+2*d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^
2*d+d)^(1/2)-2*d)/(cos(1/2*b*x+1/2*a)+1))+1/16/d/(cos(1/2*b*x+1/2*a)-1)*(-
2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-3/8/d^(1/2)*ln((4*d*cos(1/2*b*x+1/2*a)+2
*d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d)/(cos(1/2*b*x+1/2*a)-1)))
/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(73) = 146$.

Time = 0.12 (sec) , antiderivative size = 325, normalized size of antiderivative = 3.49

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \left[\frac{6\left(\cos\left(bx+a\right)^2-1\right)\sqrt{-d}\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{-d}}{d\cos(bx+a)+d}\right)-3\left(\cos\left(bx+a\right)^2-1\right)\sqrt{-d}\log\left(\frac{d\cos(bx+a)^2+4\sqrt{d\cos}\cdots}{\cdots}\right.\cdots}{16\left(bd\cos\left(bx+a\right)^2-bd\right)}\right.$$

input
```
integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(6*(cos(b*x + a)^2 - 1)*sqrt(-d)*arctan(2*sqrt(d*cos(b*x + a))*sqrt(
-d)/(d*cos(b*x + a) + d)) - 3*(cos(b*x + a)^2 - 1)*sqrt(-d)*log((d*cos(b*x
 + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) - 1) - 6*d*cos(b*x
 + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a) + 1)) + 8*sqrt(d*cos(b*x + a))
)/(b*d*cos(b*x + a)^2 - b*d), 1/16*(6*(cos(b*x + a)^2 - 1)*sqrt(d)*arctan(
2*sqrt(d*cos(b*x + a))*sqrt(d)/(d*cos(b*x + a) - d)) + 3*(cos(b*x + a)^2 -
 1)*sqrt(d)*log((d*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))*sqrt(d)*(cos(b*
x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 - 2*cos(b*x + a) + 1))
 + 8*sqrt(d*cos(b*x + a)))/(b*d*cos(b*x + a)^2 - b*d)]
```

Sympy [F]

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{\csc^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$$

input
```
integrate(csc(b*x+a)**3/(d*cos(b*x+a))**(1/2),x)
```

output
```
Integral(csc(a + b*x)**3/sqrt(d*cos(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.11

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \frac{\frac{4\sqrt{d\cos(bx+a)}d^2}{d^2\cos(bx+a)^2-d^2} - 6\sqrt{d}\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right) + 3\sqrt{d}\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)}{8\,bd}$$

input
```
integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
1/8*(4*sqrt(d*cos(b*x + a))*d^2/(d^2*cos(b*x + a)^2 - d^2) - 6*sqrt(d)*arc
tan(sqrt(d*cos(b*x + a))/sqrt(d)) + 3*sqrt(d)*log((sqrt(d*cos(b*x + a)) -
sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d))))/(b*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.98

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \frac{d^3\left(\frac{2\sqrt{d\cos(bx+a)}}{\left(d^2\cos(bx+a)^2-d^2\right)d^2} + \frac{3\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{-d}}\right)}{\sqrt{-d}d^3} - \frac{3\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)}{d^{\frac{7}{2}}}\right)}{4\,b}$$

input
```
integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
1/4*d^3*(2*sqrt(d*cos(b*x + a))/((d^2*cos(b*x + a)^2 - d^2)*d^2) + 3*arcta
n(sqrt(d*cos(b*x + a))/sqrt(-d))/(sqrt(-d)*d^3) - 3*arctan(sqrt(d*cos(b*x
+ a))/sqrt(d))/d^(7/2))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{1}{\sin(a+b\,x)^3\,\sqrt{d\,\cos(a+b\,x)}}\,dx$$

input
```
int(1/(sin(a + b*x)^3*(d*cos(a + b*x))^(1/2)),x)
```

output
```
int(1/(sin(a + b*x)^3*(d*cos(a + b*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\cos(bx+a)}\,\csc(bx+a)^3}{\cos(bx+a)}dx\right)}{d}$$

input
```
int(csc(b*x+a)^3/(d*cos(b*x+a))^(1/2),x)
```

output
```
(sqrt(d)*int((sqrt(cos(a + b*x))*csc(a + b*x)**3)/cos(a + b*x),x))/d
```

3.249 $\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 115

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{5\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{3/2}} - \frac{5\operatorname{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{3/2}} + \frac{5}{2bd\sqrt{d\cos(a+bx)}} - \frac{\csc^2(a+bx)}{2bd\sqrt{d\cos(a+bx)}}$$

output
```
5/4*arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b/d^(3/2)-5/4*arctanh((d*cos(b*x+
a))^(1/2)/d^(1/2))/b/d^(3/2)+5/2/b/d/(d*cos(b*x+a))^(1/2)-1/2*csc(b*x+a)^2
/b/d/(d*cos(b*x+a))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.54 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.79

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{-(-\cot^2(a+bx))^{3/4}(-4+\cot^2(a+bx)) + 5\cot^2(a+bx)\,\text{Hypergeometric2F1}}{2bd\sqrt{d\cos(a+bx)}\,(-\cot^2(a+bx))^{3/4}}$$

input
```
Integrate[Csc[a + b*x]^3/(d*Cos[a + b*x])^(3/2),x]
```

output
```
(-((-Cot[a + b*x]^2)^(3/4)*(-4 + Cot[a + b*x]^2)) + 5*Cot[a + b*x]^2*Hyper
geometric2F1[1/4, 1/4, 5/4, Csc[a + b*x]^2])/(2*b*d*Sqrt[d*Cos[a + b*x]]*(
-Cot[a + b*x]^2)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3045, 27, 253, 264, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^3(d\cos(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int \frac{d^4}{(d\cos(a+bx))^{3/2}(d^2-d^2\cos^2(a+bx))^2}d(d\cos(a+bx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$-\frac{d^3\int \frac{1}{(d\cos(a+bx))^{3/2}(d^2-d^2\cos^2(a+bx))^2}d(d\cos(a+bx))}{b}$$

$$\Big\downarrow \text{253}$$

$$-\frac{d^3\left(\frac{5\int \frac{1}{(d\cos(a+bx))^{3/2}\left(d^2-d^2\cos^2(a+bx)\right)}d(d\cos(a+bx))}{4d^2}+\frac{1}{2d^2\sqrt{d\cos(a+bx)}(d^2-d^2\cos^2(a+bx))}\right)}{b}$$

$$\Big\downarrow \text{264}$$

$$-\frac{d^3\left(\frac{5\left(\frac{\int \frac{\sqrt{d\cos(a+bx)}}{d^2-d^2\cos^2(a+bx)}d(d\cos(a+bx))}{d^2}-\frac{2}{d^2\sqrt{d\cos(a+bx)}}\right)}{4d^2}+\frac{1}{2d^2\sqrt{d\cos(a+bx)}(d^2-d^2\cos^2(a+bx))}\right)}{b}$$

$$\Big\downarrow \ 266$$

$$-\frac{d^3\left(\frac{5\left(\frac{2\int \frac{d^2\cos^2(a+bx)}{d^2-d^4\cos^4(a+bx)}d\sqrt{d\cos(a+bx)}}{d^2}-\frac{2}{d^2\sqrt{d\cos(a+bx)}}\right)}{4d^2}+\frac{1}{2d^2\sqrt{d\cos(a+bx)}(d^2-d^2\cos^2(a+bx))}\right)}{b}$$

$$\Big\downarrow \ 827$$

$$-\frac{d^3\left(\frac{5\left(\frac{2\left(\frac{1}{2}\int \frac{1}{d-d^2\cos^2(a+bx)}d\sqrt{d\cos(a+bx)}-\frac{1}{2}\int \frac{1}{d^2\cos^2(a+bx)+d}d\sqrt{d\cos(a+bx)}\right)}{d^2}-\frac{2}{d^2\sqrt{d\cos(a+bx)}}\right)}{4d^2}+\frac{1}{2d^2\sqrt{d\cos(a+bx)}(d^2-d^2\cos^2(a+bx))}\right)}{b}$$

$$\Big\downarrow \ 216$$

$$-\frac{d^3\left(\frac{5\left(\frac{2\left(\frac{1}{2}\int \frac{1}{d-d^2\cos^2(a+bx)}d\sqrt{d\cos(a+bx)}-\frac{\arctan\left(\sqrt{d}\cos(a+bx)\right)}{2\sqrt{d}}\right)}{d^2}-\frac{2}{d^2\sqrt{d\cos(a+bx)}}\right)}{4d^2}+\frac{1}{2d^2\sqrt{d\cos(a+bx)}(d^2-d^2\cos^2(a+bx))}\right)}{b}$$

$$\Big\downarrow \ 219$$

$$-\frac{d^3\left(\frac{5\left(\frac{2\left(\frac{\text{arctanh}\left(\sqrt{d}\cos(a+bx)\right)}{2\sqrt{d}}-\frac{\arctan\left(\sqrt{d}\cos(a+bx)\right)}{2\sqrt{d}}\right)}{d^2}-\frac{2}{d^2\sqrt{d\cos(a+bx)}}\right)}{4d^2}+\frac{1}{2d^2\sqrt{d\cos(a+bx)}(d^2-d^2\cos^2(a+bx))}\right)}{b}$$

input
```
Int[Csc[a + b*x]^3/(d*Cos[a + b*x])^(3/2),x]
```

output
```
-((d^3*(1/(2*d^2*Sqrt[d*Cos[a + b*x]]*(d^2 - d^2*Cos[a + b*x]^2)) + (5*((2
*(-1/2*ArcTan[Sqrt[d]*Cos[a + b*x]]/Sqrt[d] + ArcTanh[Sqrt[d]*Cos[a + b*x]
]/(2*Sqrt[d])))/d^2 - 2/(d^2*Sqrt[d*Cos[a + b*x]])))/(4*d^2)))/b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 688 vs. $2(91) = 182$.

Time = 3.50 (sec) , antiderivative size = 689, normalized size of antiderivative = 5.99

method	result
default	$-\sqrt{-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^2 d+d}\,\sqrt{-d}\,\sqrt{d}-\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^6\left(-20\ln\left(\frac{2\sqrt{-d}\,\sqrt{-2\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^2 d+d}-2d}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)}\right)d^{\frac{3}{2}}-10\ln\left(\frac{4d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+2\sqrt{d}\,\sqrt{-2\,\mathrm{s}}}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right.}\right.\right.$

input
```
int(csc(b*x+a)^3/(d*cos(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/d^(5/2)/(-d)^(1/2)/sin(1/2*b*x+1/2*a)^2/(2*sin(1/2*b*x+1/2*a)^4-3*sin
(1/2*b*x+1/2*a)^2+1)*((-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*(-d)^(1/2)*d^(1/
2)-sin(1/2*b*x+1/2*a)^6*(-20*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1
/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(3/2)-10*ln(2/(cos(1/2*b*x+1/2*a)-1)*(2*d
*cos(1/2*b*x+1/2*a)+d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*(-d)^(
1/2)*d-10*ln(-2/(cos(1/2*b*x+1/2*a)+1)*(2*d*cos(1/2*b*x+1/2*a)-d^(1/2)*(-2
*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+d))*(-d)^(1/2)*d)-5*(6*ln(2/cos(1/2*b*x+1
/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(3/2)+3*ln(2/(
cos(1/2*b*x+1/2*a)-1)*(2*d*cos(1/2*b*x+1/2*a)+d^(1/2)*(-2*sin(1/2*b*x+1/2*
a)^2*d+d)^(1/2)-d))*(-d)^(1/2)*d-4*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*(-d
)^(1/2)*d^(1/2)+3*ln(-2/(cos(1/2*b*x+1/2*a)+1)*(2*d*cos(1/2*b*x+1/2*a)-d^(
1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+d))*(-d)^(1/2)*d)*sin(1/2*b*x+1/2
*a)^4+5*(2*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+
d)^(1/2)-d))*d^(3/2)+ln(2/(cos(1/2*b*x+1/2*a)-1)*(2*d*cos(1/2*b*x+1/2*a)+d
^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*(-d)^(1/2)*d-4*(-2*sin(1/2*
b*x+1/2*a)^2*d+d)^(1/2)*(-d)^(1/2)*d^(1/2)+ln(-2/(cos(1/2*b*x+1/2*a)+1)*(2
*d*cos(1/2*b*x+1/2*a)-d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+d))*(-d)
^(1/2)*d)*sin(1/2*b*x+1/2*a)^2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. 2(91) = 182.

Time = 0.14 (sec) , antiderivative size = 397, normalized size of antiderivative = 3.45

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \left[\frac{10\left(\cos(bx+a)^3-\cos(bx+a)\right)\sqrt{-d}\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{-d}}{d\cos(bx+a)+d}\right)-5\left(\cos(bx+\right.}{}\right.$$

$$-\frac{10\left(\cos(bx+a)^3-\cos(bx+a)\right)\sqrt{d}\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{d}}{d\cos(bx+a)-d}\right)-5\left(\cos(bx+a)^3-\cos(bx+a)\right)\sqrt{d}\log\left(\right.}{16\left(bd^2\cos(bx+a)^3-bd\right.}$$

input
```
integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(10*(cos(b*x + a)^3 - cos(b*x + a))*sqrt(-d)*arctan(2*sqrt(d*cos(b*x
 + a))*sqrt(-d)/(d*cos(b*x + a) + d)) - 5*(cos(b*x + a)^3 - cos(b*x + a))*
sqrt(-d)*log((d*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x
+ a) - 1) - 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a) + 1)) +
 8*sqrt(d*cos(b*x + a))*(5*cos(b*x + a)^2 - 4))/(b*d^2*cos(b*x + a)^3 - b*
d^2*cos(b*x + a)), -1/16*(10*(cos(b*x + a)^3 - cos(b*x + a))*sqrt(d)*arcta
n(2*sqrt(d*cos(b*x + a))*sqrt(d)/(d*cos(b*x + a) - d)) - 5*(cos(b*x + a)^3
 - cos(b*x + a))*sqrt(d)*log((d*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))*sq
rt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 - 2*cos(b
*x + a) + 1)) - 8*sqrt(d*cos(b*x + a))*(5*cos(b*x + a)^2 - 4))/(b*d^2*cos(
b*x + a)^3 - b*d^2*cos(b*x + a))]
```

Sympy [F]

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)**3/(d*cos(b*x+a))**(3/2),x)
```

output
```
Integral(csc(a + b*x)**3/(d*cos(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.02

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{\frac{10\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)}{\sqrt{d}} + \frac{5\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)}{\sqrt{d}} + \frac{4\left(5d^2\cos(bx+a)^2-4d^2\right)}{(d\cos(bx+a))^{\frac{5}{2}}-\sqrt{d\cos(bx+a)}d^2}}{8bd}$$

input
```
integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
1/8*(10*arctan(sqrt(d*cos(b*x + a))/sqrt(d))/sqrt(d) + 5*log((sqrt(d*cos(b
*x + a)) - sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d)))/sqrt(d) + 4*(5*d^2*c
os(b*x + a)^2 - 4*d^2)/((d*cos(b*x + a))^(5/2) - sqrt(d*cos(b*x + a))*d^2)
)/(b*d)
```

Giac [F]

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\csc\left(bx+a\right)^3}{\left(d\cos\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^3/(d*cos(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{1}{\sin\left(a+b\,x\right)^3\,\left(d\,\cos\left(a+b\,x\right)\right)^{3/2}}\,dx$$

input
```
int(1/(sin(a + b*x)^3*(d*cos(a + b*x))^(3/2)),x)
```

output
```
int(1/(sin(a + b*x)^3*(d*cos(a + b*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\cos(bx+a)}\,\csc(bx+a)^3}{\cos(bx+a)^2}dx\right)}{d^2}$$

input
```
int(csc(b*x+a)^3/(d*cos(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*int((sqrt(cos(a + b*x))*csc(a + b*x)**3)/cos(a + b*x)**2,x))/d**2
```

3.250 $\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 115

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = -\frac{7\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{5/2}} - \frac{7\operatorname{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{5/2}} + \frac{7}{6bd(d\cos(a+bx))^{3/2}} - \frac{\csc^2(a+bx)}{2bd(d\cos(a+bx))^{3/2}}$$

output

```
-7/4*arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b/d^(5/2)-7/4*arctanh((d*cos(b*x
+a))^(1/2)/d^(1/2))/b/d^(5/2)+7/6/b/d/(d*cos(b*x+a))^(3/2)-1/2*csc(b*x+a)^
2/b/d/(d*cos(b*x+a))^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.71 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.80

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \frac{\sqrt[4]{-\cot^2(a+bx)}(4-3\cot^2(a+bx)) + 7\cot^2(a+bx)\,\text{Hypergeometric2F1}\left(\frac{3}{4}, \frac{3}{4} \right.}{6bd(d\cos(a+bx))^{3/2}\sqrt[4]{-\cot^2(a+bx)}}$$

input
```
Integrate[Csc[a + b*x]^3/(d*Cos[a + b*x])^(5/2),x]
```

output
```
((-Cot[a + b*x]^2)^(1/4)*(4 - 3*Cot[a + b*x]^2) + 7*Cot[a + b*x]^2*Hyperge
ometric2F1[3/4, 3/4, 7/4, Csc[a + b*x]^2])/(6*b*d*(d*Cos[a + b*x])^(3/2)*(
-Cot[a + b*x]^2)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3045, 27, 253, 264, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^3(d\cos(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int \frac{d^4}{(d\cos(a+bx))^{5/2}(d^2-d^2\cos^2(a+bx))^2}d(d\cos(a+bx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$-\frac{d^3\int \frac{1}{(d\cos(a+bx))^{5/2}(d^2-d^2\cos^2(a+bx))^2}d(d\cos(a+bx))}{b}$$

$$\Big\downarrow \text{253}$$

$$-\frac{d^3\left(\frac{7\int \frac{1}{(d\cos(a+bx))^{5/2}\left(d^2-d^2\cos^2(a+bx)\right)}d(d\cos(a+bx))}{4d^2}+\frac{1}{2d^2(d\cos(a+bx))^{3/2}(d^2-d^2\cos^2(a+bx))}\right)}{b}$$

$$\Big\downarrow \text{264}$$

$$-\frac{d^3\left(\frac{7\left(\frac{\int \frac{1}{\sqrt{d\cos(a+bx)}\left(d^2-d^2\cos^2(a+bx)\right)}d(d\cos(a+bx))}{d^2}-\frac{2}{3d^2(d\cos(a+bx))^{3/2}}\right)}{4d^2}+\frac{1}{2d^2(d\cos(a+bx))^{3/2}(d^2-d^2\cos^2(a+bx))}\right)}{b}$$

$\downarrow$ 266

$$-\frac{d^3\left(\frac{7\left(\frac{2\int \frac{1}{d^2-d^4\cos^4(a+bx)}d\sqrt{d\cos(a+bx)}}{d^2}-\frac{2}{3d^2(d\cos(a+bx))^{3/2}}\right)}{4d^2}+\frac{1}{2d^2(d\cos(a+bx))^{3/2}(d^2-d^2\cos^2(a+bx))}\right)}{b}$$

$\downarrow$ 756

$$-\frac{d^3\left(\frac{7\left(\frac{2\left(\frac{\int \frac{1}{d-d^2\cos^2(a+bx)}d\sqrt{d\cos(a+bx)}}{2d}+\frac{\int \frac{1}{d^2\cos^2(a+bx)+d}d\sqrt{d\cos(a+bx)}}{2d}\right)}{d^2}-\frac{2}{3d^2(d\cos(a+bx))^{3/2}}\right)}{4d^2}+\frac{1}{2d^2(d\cos(a+bx))^{3/2}(d^2-d^2\cos^2(a+bx))}\right)}{b}$$

$\downarrow$ 216

$$-\frac{d^3\left(\frac{7\left(\frac{2\left(\frac{\int \frac{1}{d-d^2\cos^2(a+bx)}d\sqrt{d\cos(a+bx)}}{2d}+\frac{\arctan\left(\sqrt{d}\cos(a+bx)\right)}{2d^{3/2}}\right)}{d^2}-\frac{2}{3d^2(d\cos(a+bx))^{3/2}}\right)}{4d^2}+\frac{1}{2d^2(d\cos(a+bx))^{3/2}(d^2-d^2\cos^2(a+bx))}\right)}{b}$$

$\downarrow$ 219

$$-\frac{d^3\left(\frac{7\left(\frac{2\left(\frac{\arctan\left(\sqrt{d}\cos(a+bx)\right)}{2d^{3/2}}+\frac{\operatorname{arctanh}\left(\sqrt{d}\cos(a+bx)\right)}{2d^{3/2}}\right)}{d^2}-\frac{2}{3d^2(d\cos(a+bx))^{3/2}}\right)}{4d^2}+\frac{1}{2d^2(d\cos(a+bx))^{3/2}(d^2-d^2\cos^2(a+bx))}\right)}{b}$$

input
```
Int[Csc[a + b*x]^3/(d*Cos[a + b*x])^(5/2),x]
```

output
```
-((d^3*(1/(2*d^2*(d*Cos[a + b*x])^(3/2)*(d^2 - d^2*Cos[a + b*x]^2)) + (7*(
(2*(ArcTan[Sqrt[d]*Cos[a + b*x]]/(2*d^(3/2)) + ArcTanh[Sqrt[d]*Cos[a + b*x
]]/(2*d^(3/2))))/d^2 - 2/(3*d^2*(d*Cos[a + b*x])^(3/2))))/(4*d^2)))/b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 884 vs. $2(91) = 182$.

Time = 3.51 (sec) , antiderivative size = 885, normalized size of antiderivative = 7.70

method	result	size
default	Expression too large to display	885

input
```
int(csc(b*x+a)^3/(d*cos(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24/d^(7/2)/(-d)^(1/2)/sin(1/2*b*x+1/2*a)^2/(4*sin(1/2*b*x+1/2*a)^6-8*si
n(1/2*b*x+1/2*a)^4+5*sin(1/2*b*x+1/2*a)^2-1)*(-3*(-2*sin(1/2*b*x+1/2*a)^2*
d+d)^(1/2)*(-d)^(1/2)*d^(1/2)+84*(-2*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(
-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(3/2)+ln(-2/(cos(1/2*b*x+1/2*a)+1
)*(2*d*cos(1/2*b*x+1/2*a)-d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+d))*
(-d)^(1/2)*d+ln(2/(cos(1/2*b*x+1/2*a)-1)*(2*d*cos(1/2*b*x+1/2*a)+d^(1/2)*(
-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*(-d)^(1/2)*d)*sin(1/2*b*x+1/2*a)^8-
168*(-2*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^
(1/2)-d))*d^(3/2)+ln(-2/(cos(1/2*b*x+1/2*a)+1)*(2*d*cos(1/2*b*x+1/2*a)-d^(
1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+d))*(-d)^(1/2)*d+ln(2/(cos(1/2*b*
x+1/2*a)-1)*(2*d*cos(1/2*b*x+1/2*a)+d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^
(1/2)-d))*(-d)^(1/2)*d)*sin(1/2*b*x+1/2*a)^6-7*(-6*ln(2/cos(1/2*b*x+1/2*a)
*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(3/2)+3*ln(-2/(cos(
1/2*b*x+1/2*a)+1)*(2*d*cos(1/2*b*x+1/2*a)-d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2
*d+d)^(1/2)+d))*(-d)^(1/2)*d-4*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*(-d)^(1
/2)*d^(1/2)+3*ln(2/(cos(1/2*b*x+1/2*a)-1)*(2*d*cos(1/2*b*x+1/2*a)+d^(1/2)*
(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*(-d)^(1/2)*d)*sin(1/2*b*x+1/2*a)^2
+7*(-30*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^
(1/2)-d))*d^(3/2)+15*ln(-2/(cos(1/2*b*x+1/2*a)+1)*(2*d*cos(1/2*b*x+1/2*a)-
d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+d))*(-d)^(1/2)*d-4*(-2*sin(...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(91) = 182$.

Time = 0.14 (sec) , antiderivative size = 409, normalized size of antiderivative = 3.56

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \left[\frac{42\left(\cos\left(bx+a\right)^4-\cos\left(bx+a\right)^2\right)\sqrt{-d}\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{-d}}{d\cos(bx+a)+d}\right)-21\left(\cos\left(b\right.\right.}{}\right.$$

input
```
integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
[1/48*(42*(cos(b*x + a)^4 - cos(b*x + a)^2)*sqrt(-d)*arctan(2*sqrt(d*cos(b
*x + a))*sqrt(-d)/(d*cos(b*x + a) + d)) - 21*(cos(b*x + a)^4 - cos(b*x + a
)^2)*sqrt(-d)*log((d*cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos
(b*x + a) - 1) - 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a) +
1)) + 8*sqrt(d*cos(b*x + a))*(7*cos(b*x + a)^2 - 4))/(b*d^3*cos(b*x + a)^4
 - b*d^3*cos(b*x + a)^2), 1/48*(42*(cos(b*x + a)^4 - cos(b*x + a)^2)*sqrt(
d)*arctan(2*sqrt(d*cos(b*x + a))*sqrt(d)/(d*cos(b*x + a) - d)) + 21*(cos(b
*x + a)^4 - cos(b*x + a)^2)*sqrt(d)*log((d*cos(b*x + a)^2 - 4*sqrt(d*cos(b
*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x + a)^
2 - 2*cos(b*x + a) + 1)) + 8*sqrt(d*cos(b*x + a))*(7*cos(b*x + a)^2 - 4))/
(b*d^3*cos(b*x + a)^4 - b*d^3*cos(b*x + a)^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3/(d*cos(b*x+a))**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.02

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \frac{\frac{4\left(7\,d^2\cos(bx+a)^2-4\,d^2\right)}{(d\cos(bx+a))^{\frac{7}{2}}-(d\cos(bx+a))^{\frac{3}{2}}d^2}-\frac{42\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{21\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)}{d^{\frac{3}{2}}}}{24\,bd}$$

input
```
integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
1/24*(4*(7*d^2*cos(b*x + a)^2 - 4*d^2)/((d*cos(b*x + a))^(7/2) - (d*cos(b*
x + a))^(3/2)*d^2) - 42*arctan(sqrt(d*cos(b*x + a))/sqrt(d))/d^(3/2) + 21*
log((sqrt(d*cos(b*x + a)) - sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d)))/d^(
3/2))/(b*d)
```

Giac [F]

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{\csc\left(bx+a\right)^3}{\left(d\cos\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^3/(d*cos(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{1}{\sin\left(a+b\,x\right)^3\,\left(d\,\cos\left(a+b\,x\right)\right)^{5/2}}\,dx$$

input
```
int(1/(sin(a + b*x)^3*(d*cos(a + b*x))^(5/2)),x)
```

output
```
int(1/(sin(a + b*x)^3*(d*cos(a + b*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\cos(bx+a)}\,\csc(bx+a)^3}{\cos(bx+a)^3}dx\right)}{d^3}$$

input
```
int(csc(b*x+a)^3/(d*cos(b*x+a))^(5/2),x)
```

output
```
(sqrt(d)*int((sqrt(cos(a + b*x))*csc(a + b*x)**3)/cos(a + b*x)**3,x))/d**3
```

3.251 $\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 137

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{9\arctan\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{7/2}} - \frac{9\text{arctanh}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{7/2}} + \frac{9}{10bd(d\cos(a+bx))^{5/2}} + \frac{9}{2bd^3\sqrt{d\cos(a+bx)}} - \frac{\csc^2(a+bx)}{2bd(d\cos(a+bx))^{5/2}}$$

output
```
9/4*arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b/d^(7/2)-9/4*arctanh((d*cos(b*x+
a))^(1/2)/d^(1/2))/b/d^(7/2)+9/10/b/d/(d*cos(b*x+a))^(5/2)+9/2/b/d^3/(d*co
s(b*x+a))^(1/2)-1/2*csc(b*x+a)^2/b/d/(d*cos(b*x+a))^(5/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.85 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.74

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{45\cot^2(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4},\frac{5}{4},\csc^2(a+bx)\right) + \left(-\cot^2(a+bx)\right)^{3/4}}{10bd^3\sqrt{d\cos(a+bx)}\left(-\cot^2(a+bx)\right)^{3/4}}$$

input
```
Integrate[Csc[a + b*x]^3/(d*Cos[a + b*x])^(7/2),x]
```

output
```
(45*Cot[a + b*x]^2*Hypergeometric2F1[1/4, 1/4, 5/4, Csc[a + b*x]^2] + (-Co
t[a + b*x]^2)^(3/4)*(40 - 5*Cot[a + b*x]^2 + 4*Sec[a + b*x]^2))/(10*b*d^3*
Sqrt[d*Cos[a + b*x]]*(-Cot[a + b*x]^2)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3045, 27, 253, 264, 264, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(a+bx)^3(d\cos(a+bx))^{7/2}}dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int \frac{d^4}{(d\cos(a+bx))^{7/2}(d^2-d^2\cos^2(a+bx))^2}d(d\cos(a+bx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$-\frac{d^3\int \frac{1}{(d\cos(a+bx))^{7/2}(d^2-d^2\cos^2(a+bx))^2}d(d\cos(a+bx))}{b}$$

$$\Big\downarrow \text{253}$$

$$-\frac{d^3\left(\frac{9\int \frac{1}{(d\cos(a+bx))^{7/2}\left(d^2-d^2\cos^2(a+bx)\right)}d(d\cos(a+bx))}{4d^2}+\frac{1}{2d^2(d\cos(a+bx))^{5/2}(d^2-d^2\cos^2(a+bx))}\right)}{b}$$

$$\Big\downarrow \text{264}$$

$$-\frac{d^3\left(\frac{9\left(\frac{\int \frac{1}{(d\cos(a+bx))^{3/2}\left(d^2-d^2\cos^2(a+bx)\right)}d(d\cos(a+bx))}{d^2}-\frac{2}{5d^2(d\cos(a+bx))^{5/2}}\right)}{4d^2}+\frac{1}{2d^2(d\cos(a+bx))^{5/2}(d^2-d^2\cos^2(a+bx))}\right)}{b}$$

$\downarrow$ 264

$$-\frac{d^3\left(\frac{9\left(\frac{\frac{\int \frac{\sqrt{d\cos(a+bx)}}{d^2-d^2\cos^2(a+bx)}d(d\cos(a+bx))}{d^2}-\frac{2}{d^2\sqrt{d\cos(a+bx)}}}{d^2}-\frac{2}{5d^2(d\cos(a+bx))^{5/2}}\right)}{4d^2}+\frac{1}{2d^2(d\cos(a+bx))^{5/2}(d^2-d^2\cos^2(a+bx))}\right)}{b}$$

$\downarrow$ 266

$$-\frac{d^3\left(\frac{9\left(\frac{\frac{2\int \frac{d^2\cos^2(a+bx)}{d^2-d^4\cos^4(a+bx)}d\sqrt{d\cos(a+bx)}}{d^2}-\frac{2}{d^2\sqrt{d\cos(a+bx)}}}{d^2}-\frac{2}{5d^2(d\cos(a+bx))^{5/2}}\right)}{4d^2}+\frac{1}{2d^2(d\cos(a+bx))^{5/2}(d^2-d^2\cos^2(a+bx))}\right)}{b}$$

$\downarrow$ 827

$$-\frac{d^3\left(\frac{9\left(\frac{\frac{2\left(\frac{1}{2}\int \frac{1}{d-d^2\cos^2(a+bx)}d\sqrt{d\cos(a+bx)}-\frac{1}{2}\int \frac{1}{d^2\cos^2(a+bx)+d}d\sqrt{d\cos(a+bx)}\right)}{d^2}-\frac{2}{d^2\sqrt{d\cos(a+bx)}}}{d^2}-\frac{2}{5d^2(d\cos(a+bx))^{5/2}}\right)}{4d^2}+\frac{1}{2d^2(d\cos(a+bx)}\right.}{b}$$

$\downarrow$ 216

$$-\frac{d^3\left(\frac{9\left(\frac{\frac{2\left(\frac{1}{2}\int \frac{1}{d-d^2\cos^2(a+bx)}d\sqrt{d\cos(a+bx)}-\frac{\arctan\left(\sqrt{d}\cos(a+bx)\right)}{2\sqrt{d}}\right)}{d^2}-\frac{2}{d^2\sqrt{d\cos(a+bx)}}}{d^2}-\frac{2}{5d^2(d\cos(a+bx))^{5/2}}\right)}{4d^2}+\frac{1}{2d^2(d\cos(a+bx))^{5/2}(d^2-d}\right.}{b}$$

$\downarrow$ 219

$$-\frac{d^3\left(\frac{9\left(\frac{\frac{2\left(\frac{\text{arctanh}\left(\sqrt{d}\cos(a+bx)\right)}{2\sqrt{d}}-\frac{\arctan\left(\sqrt{d}\cos(a+bx)\right)}{2\sqrt{d}}\right)}{d^2}-\frac{2}{d^2\sqrt{d\cos(a+bx)}}}{d^2}-\frac{2}{5d^2(d\cos(a+bx))^{5/2}}\right)}{4d^2}+\frac{1}{2d^2(d\cos(a+bx))^{5/2}(d^2-d^2\cos^2(a}\right.}{b}$$

input
```
Int[Csc[a + b*x]^3/(d*Cos[a + b*x])^(7/2),x]
```

output
```
-((d^3*(1/(2*d^2*(d*Cos[a + b*x])^(5/2)*(d^2 - d^2*Cos[a + b*x]^2)) + (9*(
-2/(5*d^2*(d*Cos[a + b*x])^(5/2)) + ((2*(-1/2*ArcTan[Sqrt[d]*Cos[a + b*x]]
/Sqrt[d] + ArcTanh[Sqrt[d]*Cos[a + b*x]]/(2*Sqrt[d])))/d^2 - 2/(d^2*Sqrt[d
*Cos[a + b*x]]))/d^2))/(4*d^2)))/b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1139 vs. $2(109) = 218$.

Time = 3.49 (sec) , antiderivative size = 1140, normalized size of antiderivative = 8.32

method	result	size
default	Expression too large to display	1140

input
```
int(csc(b*x+a)^3/(d*cos(b*x+a))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/40/d^(9/2)/(-d)^(1/2)/sin(1/2*b*x+1/2*a)^2/(8*sin(1/2*b*x+1/2*a)^8-20*s
in(1/2*b*x+1/2*a)^6+18*sin(1/2*b*x+1/2*a)^4-7*sin(1/2*b*x+1/2*a)^2+1)*(5*(
-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*(-d)^(1/2)*d^(1/2)+360*(2*ln(2/cos(1/2*
b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(3/2)+ln(
2/(cos(1/2*b*x+1/2*a)-1)*(2*d*cos(1/2*b*x+1/2*a)+d^(1/2)*(-2*sin(1/2*b*x+1
/2*a)^2*d+d)^(1/2)-d))*(-d)^(1/2)*d+ln(-2/(cos(1/2*b*x+1/2*a)+1)*(2*d*cos(
1/2*b*x+1/2*a)-d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+d))*(-d)^(1/2)*
d)*sin(1/2*b*x+1/2*a)^10-180*(10*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*s
in(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(3/2)+5*ln(2/(cos(1/2*b*x+1/2*a)-1)*(
2*d*cos(1/2*b*x+1/2*a)+d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*(-d
)^(1/2)*d+5*ln(-2/(cos(1/2*b*x+1/2*a)+1)*(2*d*cos(1/2*b*x+1/2*a)-d^(1/2)*(
-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+d))*(-d)^(1/2)*d-4*(-2*sin(1/2*b*x+1/2*
a)^2*d+d)^(1/2)*(-d)^(1/2)*d^(1/2))*sin(1/2*b*x+1/2*a)^8+90*(18*ln(2/cos(1
/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(3/2)+
9*ln(2/(cos(1/2*b*x+1/2*a)-1)*(2*d*cos(1/2*b*x+1/2*a)+d^(1/2)*(-2*sin(1/2*
b*x+1/2*a)^2*d+d)^(1/2)-d))*(-d)^(1/2)*d+9*ln(-2/(cos(1/2*b*x+1/2*a)+1)*(2
*d*cos(1/2*b*x+1/2*a)-d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+d))*(-d)
^(1/2)*d-16*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*(-d)^(1/2)*d^(1/2))*sin(1/
2*b*x+1/2*a)^6-9*(70*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1
/2*a)^2*d+d)^(1/2)-d))*d^(3/2)+35*ln(2/(cos(1/2*b*x+1/2*a)-1)*(2*d*cos(...
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 429, normalized size of antiderivative = 3.13

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \Bigg[\frac{90\left(\cos(bx+a)^5-\cos(bx+a)^3\right)\sqrt{-d}\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{-d}}{d\cos(bx+a)+d}\right)-45\left(\cos(bx\right.}{}$$

$$-\frac{90\left(\cos(bx+a)^5-\cos(bx+a)^3\right)\sqrt{d}\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{d}}{d\cos(bx+a)-d}\right)-45\left(\cos(bx+a)^5-\cos(bx+a)^3\right)\sqrt{d}\log}{80\left(bd^4\cos(bx\right.}$$

input
```
integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

output
```
[1/80*(90*(cos(b*x + a)^5 - cos(b*x + a)^3)*sqrt(-d)*arctan(2*sqrt(d*cos(b
*x + a))*sqrt(-d)/(d*cos(b*x + a) + d)) - 45*(cos(b*x + a)^5 - cos(b*x + a
)^3)*sqrt(-d)*log((d*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos
(b*x + a) - 1) - 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a) +
1)) + 8*(45*cos(b*x + a)^4 - 36*cos(b*x + a)^2 - 4)*sqrt(d*cos(b*x + a)))/
(b*d^4*cos(b*x + a)^5 - b*d^4*cos(b*x + a)^3), -1/80*(90*(cos(b*x + a)^5 -
 cos(b*x + a)^3)*sqrt(d)*arctan(2*sqrt(d*cos(b*x + a))*sqrt(d)/(d*cos(b*x
+ a) - d)) - 45*(cos(b*x + a)^5 - cos(b*x + a)^3)*sqrt(d)*log((d*cos(b*x +
 a)^2 - 4*sqrt(d*cos(b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x +
a) + d)/(cos(b*x + a)^2 - 2*cos(b*x + a) + 1)) - 8*(45*cos(b*x + a)^4 - 36
*cos(b*x + a)^2 - 4)*sqrt(d*cos(b*x + a)))/(b*d^4*cos(b*x + a)^5 - b*d^4*c
os(b*x + a)^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**3/(d*cos(b*x+a))**(7/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.98

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{\frac{4\left(45\,d^4\cos(bx+a)^4-36\,d^4\cos(bx+a)^2-4\,d^4\right)}{(d\cos(bx+a))^{\frac{9}{2}}d^2-(d\cos(bx+a))^{\frac{5}{2}}d^4}+\frac{90\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)}{d^{\frac{5}{2}}}+\frac{45\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)}{d^{\frac{5}{2}}}}{40\,bd}$$

input
```
integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(7/2),x, algorithm="maxima")
```

output
```
1/40*(4*(45*d^4*cos(b*x + a)^4 - 36*d^4*cos(b*x + a)^2 - 4*d^4)/((d*cos(b*
x + a))^(9/2)*d^2 - (d*cos(b*x + a))^(5/2)*d^4) + 90*arctan(sqrt(d*cos(b*x
 + a))/sqrt(d))/d^(5/2) + 45*log((sqrt(d*cos(b*x + a)) - sqrt(d))/(sqrt(d*
cos(b*x + a)) + sqrt(d)))/d^(5/2))/(b*d)
```

Giac [F]

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{\csc(bx+a)^3}{(d\cos(bx+a))^{\frac{7}{2}}}\,dx$$

input
```
integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(7/2),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^3/(d*cos(b*x + a))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{1}{\sin(a+b\,x)^3\,(d\,\cos(a+b\,x))^{7/2}}\,dx$$

input
```
int(1/(sin(a + b*x)^3*(d*cos(a + b*x))^(7/2)),x)
```

output
```
int(1/(sin(a + b*x)^3*(d*cos(a + b*x))^(7/2)), x)
```

Reduce [F]

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\cos(bx+a)}\,\csc(bx+a)^3}{\cos(bx+a)^4}dx\right)}{d^4}$$

input
```
int(csc(b*x+a)^3/(d*cos(b*x+a))^(7/2),x)
```

output
```
(sqrt(d)*int((sqrt(cos(a + b*x))*csc(a + b*x)**3)/cos(a + b*x)**4,x))/d**4
```

3.252 $\int \sqrt[5]{d\cos(a+bx)}\sin(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 22

$$\int \sqrt[5]{d\cos(a+bx)}\sin(a+bx)\,dx = -\frac{5(d\cos(a+bx))^{6/5}}{6bd}$$

output
```
-5/6*(d*cos(b*x+a))^(6/5)/b/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \sqrt[5]{d\cos(a+bx)}\sin(a+bx)\,dx = -\frac{5(d\cos(a+bx))^{6/5}}{6bd}$$

input
```
Integrate[(d*Cos[a + b*x])^(1/5)*Sin[a + b*x],x]
```

output
```
(-5*(d*Cos[a + b*x])^(6/5))/(6*b*d)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 3045, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sqrt[5]{d\cos(a+bx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(a+bx)\sqrt[5]{d\cos(a+bx)}dx$$

$$\Big\downarrow \text{ 3045}$$

$$-\frac{\int \sqrt[5]{d\cos(a+bx)}d(d\cos(a+bx))}{bd}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{5(d\cos(a+bx))^{6/5}}{6bd}$$

input
```
Int[(d*Cos[a + b*x])^(1/5)*Sin[a + b*x],x]
```

output
```
(-5*(d*Cos[a + b*x])^(6/5))/(6*b*d)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$-\frac{5(d\cos(bx+a))^{\frac{6}{5}}}{6bd}$	19
default	$-\frac{5(d\cos(bx+a))^{\frac{6}{5}}}{6bd}$	19

input
```
int((d*cos(b*x+a))^(1/5)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-5/6*(d*cos(b*x+a))^(6/5)/b/d
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \sqrt[5]{d\cos(a+bx)}\sin(a+bx)\,dx = -\frac{5\,(d\cos{(bx+a)})^{\frac{1}{5}}\cos{(bx+a)}}{6\,b}$$

input
```
integrate((d*cos(b*x+a))^(1/5)*sin(b*x+a),x, algorithm="fricas")
```

output
```
-5/6*(d*cos(b*x + a))^(1/5)*cos(b*x + a)/b
```

Sympy [A] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \sqrt[5]{d\cos(a+bx)}\sin(a+bx)\,dx = \begin{cases} -\frac{5\sqrt[5]{d\cos{(a+bx)}}\cos{(a+bx)}}{6b} & \text{for } b\neq 0 \\ x\sqrt[5]{d\cos{(a)}}\sin{(a)} & \text{otherwise} \end{cases}$$

input
```
integrate((d*cos(b*x+a))**(1/5)*sin(b*x+a),x)
```

output
```
Piecewise((-5*(d*cos(a + b*x))**(1/5)*cos(a + b*x)/(6*b), Ne(b, 0)), (x*(d
*cos(a))**(1/5)*sin(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \sqrt[5]{d\cos(a+bx)}\sin(a+bx)\,dx = -\frac{5\,(d\cos{(bx+a)})^{\frac{6}{5}}}{6\,bd}$$

input
```
integrate((d*cos(b*x+a))^(1/5)*sin(b*x+a),x, algorithm="maxima")
```

output
```
-5/6*(d*cos(b*x + a))^(6/5)/(b*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \sqrt[5]{d\cos(a+bx)}\sin(a+bx)\,dx = -\frac{5\,(d\cos{(bx+a)})^{\frac{1}{5}}\cos{(bx+a)}}{6\,b}$$

input
```
integrate((d*cos(b*x+a))^(1/5)*sin(b*x+a),x, algorithm="giac")
```

output
```
-5/6*(d*cos(b*x + a))^(1/5)*cos(b*x + a)/b
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \sqrt[5]{d\cos(a+bx)}\sin(a+bx)\,dx = -\frac{5\,(d\,\cos\left(a+b\,x\right))^{6/5}}{6\,b\,d}$$

input
```
int(sin(a + b*x)*(d*cos(a + b*x))^(1/5),x)
```

output
```
-(5*(d*cos(a + b*x))^(6/5))/(6*b*d)
```

Reduce [F]

$$\int \sqrt[5]{d\cos(a+bx)}\sin(a+bx)\,dx = d^{\frac{1}{5}}\left(\int \cos\left(bx+a\right)^{\frac{1}{5}}\sin\left(bx+a\right)dx\right)$$

input
```
int((d*cos(b*x+a))^(1/5)*sin(b*x+a),x)
```

output
```
d**(1/5)*int(cos(a + b*x)**(1/5)*sin(a + b*x),x)
```

3.253 $\int \cos^3(x)\sqrt{\sin(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 21

$$\int \cos^3(x)\sqrt{\sin(x)}\,dx = \frac{2}{3}\sin^{\frac{3}{2}}(x) - \frac{2}{7}\sin^{\frac{7}{2}}(x)$$

output
```
2/3*sin(x)^(3/2)-2/7*sin(x)^(7/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \cos^3(x)\sqrt{\sin(x)}\,dx = \frac{1}{21}(11 + 3\cos(2x))\sin^{\frac{3}{2}}(x)$$

input
```
Integrate[Cos[x]^3*Sqrt[Sin[x]],x]
```

output
```
((11 + 3*Cos[2*x])*Sin[x]^(3/2))/21
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sin(x)}\cos^3(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin(x)}\cos(x)^3dx$$

$$\Big\downarrow \text{3044}$$

$$\int \sqrt{\sin(x)}\left(1-\sin^2(x)\right)d\sin(x)$$

$$\Big\downarrow \text{244}$$

$$\int \left(\sqrt{\sin(x)}-\sin^{\frac{5}{2}}(x)\right)d\sin(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}\sin^{\frac{3}{2}}(x)-\frac{2}{7}\sin^{\frac{7}{2}}(x)$$

input
```
Int[Cos[x]^3*Sqrt[Sin[x]],x]
```

output
```
(2*Sin[x]^(3/2))/3 - (2*Sin[x]^(7/2))/7
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$\frac{2\sin(x)^{\frac{3}{2}}}{3}-\frac{2\sin(x)^{\frac{7}{2}}}{7}$	14
default	$\frac{2\sin(x)^{\frac{3}{2}}}{3}-\frac{2\sin(x)^{\frac{7}{2}}}{7}$	14

input
```
int(cos(x)^3*sin(x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*sin(x)^(3/2)-2/7*sin(x)^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int \cos^3(x)\sqrt{\sin(x)}\,dx = \frac{2}{21}\left(3\cos\left(x\right)^2+4\right)\sin\left(x\right)^{\frac{3}{2}}$$

input
```
integrate(cos(x)^3*sin(x)^(1/2),x, algorithm="fricas")
```

output
```
2/21*(3*cos(x)^2 + 4)*sin(x)^(3/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(19) = 38.

Time = 3.53 (sec) , antiderivative size = 170, normalized size of antiderivative = 8.10

$$\int \cos^3(x)\sqrt{\sin(x)}\,dx = \frac{28\sqrt{2}\sqrt{\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}}\tan^5\left(\frac{x}{2}\right)}{21\tan^6\left(\frac{x}{2}\right)+63\tan^4\left(\frac{x}{2}\right)+63\tan^2\left(\frac{x}{2}\right)+21} + \frac{8\sqrt{2}\sqrt{\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}}\tan^3\left(\frac{x}{2}\right)}{21\tan^6\left(\frac{x}{2}\right)+63\tan^4\left(\frac{x}{2}\right)+63\tan^2\left(\frac{x}{2}\right)+21} + \frac{28\sqrt{2}\sqrt{\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}}\tan\left(\frac{x}{2}\right)}{21\tan^6\left(\frac{x}{2}\right)+63\tan^4\left(\frac{x}{2}\right)+63\tan^2\left(\frac{x}{2}\right)+21}$$

input
```
integrate(cos(x)**3*sin(x)**(1/2),x)
```

output
```
28*sqrt(2)*sqrt(tan(x/2)/(tan(x/2)**2 + 1))*tan(x/2)**5/(21*tan(x/2)**6 +
63*tan(x/2)**4 + 63*tan(x/2)**2 + 21) + 8*sqrt(2)*sqrt(tan(x/2)/(tan(x/2)*
*2 + 1))*tan(x/2)**3/(21*tan(x/2)**6 + 63*tan(x/2)**4 + 63*tan(x/2)**2 + 2
1) + 28*sqrt(2)*sqrt(tan(x/2)/(tan(x/2)**2 + 1))*tan(x/2)/(21*tan(x/2)**6
+ 63*tan(x/2)**4 + 63*tan(x/2)**2 + 21)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \cos^3(x)\sqrt{\sin(x)}\,dx = -\frac{2}{7}\sin(x)^{\frac{7}{2}} + \frac{2}{3}\sin(x)^{\frac{3}{2}}$$

input
```
integrate(cos(x)^3*sin(x)^(1/2),x, algorithm="maxima")
```

output
```
-2/7*sin(x)^(7/2) + 2/3*sin(x)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \cos^3(x)\sqrt{\sin(x)}\,dx = -\frac{2}{7}\sin(x)^{\frac{7}{2}} + \frac{2}{3}\sin(x)^{\frac{3}{2}}$$

input
```
integrate(cos(x)^3*sin(x)^(1/2),x, algorithm="giac")
```

output
```
-2/7*sin(x)^(7/2) + 2/3*sin(x)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 25.45 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \cos^3(x)\sqrt{\sin(x)}\,dx = -\frac{\cos(x)^4\sin(x)^{3/2}\,{}_2F_1\left(\frac{1}{4},2;\ 3;\ \cos(x)^2\right)}{4\left(\sin(x)^2\right)^{3/4}}$$

input
```
int(cos(x)^3*sin(x)^(1/2),x)
```

output
```
-(cos(x)^4*sin(x)^(3/2)*hypergeom([1/4, 2], 3, cos(x)^2))/(4*(sin(x)^2)^(3
/4))
```

Reduce [F]

$$\int \cos^3(x)\sqrt{\sin(x)}\,dx = \int \sqrt{\sin(x)}\,\cos(x)^3\,dx$$

input
```
int(cos(x)^3*sin(x)^(1/2),x)
```

output
```
int(sqrt(sin(x))*cos(x)**3,x)
```

3.254 $\int \cos^3(x)\sin^{\frac{3}{2}}(x)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 21

$$\int \cos^3(x)\sin^{\frac{3}{2}}(x)\,dx = \frac{2}{5}\sin^{\frac{5}{2}}(x) - \frac{2}{9}\sin^{\frac{9}{2}}(x)$$

output
```
2/5*sin(x)^(5/2)-2/9*sin(x)^(9/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \cos^3(x)\sin^{\frac{3}{2}}(x)\,dx = \frac{1}{45}(13+5\cos(2x))\sin^{\frac{5}{2}}(x)$$

input
```
Integrate[Cos[x]^3*Sin[x]^(3/2),x]
```

output
```
((13 + 5*Cos[2*x])*Sin[x]^(5/2))/45
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{3}{2}}(x)\cos^3(x)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sin(x)^{3/2}\cos(x)^3dx$$

$$\Big\downarrow \ 3044$$

$$\int \sin^{\frac{3}{2}}(x)\left(1-\sin^2(x)\right)d\sin(x)$$

$$\Big\downarrow \ 244$$

$$\int \left(\sin^{\frac{3}{2}}(x)-\sin^{\frac{7}{2}}(x)\right)d\sin(x)$$

$$\Big\downarrow \ 2009$$

$$\frac{2}{5}\sin^{\frac{5}{2}}(x)-\frac{2}{9}\sin^{\frac{9}{2}}(x)$$

input
```
Int[Cos[x]^3*Sin[x]^(3/2),x]
```

output
```
(2*Sin[x]^(5/2))/5 - (2*Sin[x]^(9/2))/9
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$\frac{2 \sin(x)^{\frac{5}{2}}}{5}-\frac{2 \sin(x)^{\frac{9}{2}}}{9}$	14
default	$\frac{2 \sin(x)^{\frac{5}{2}}}{5}-\frac{2 \sin(x)^{\frac{9}{2}}}{9}$	14

input
```
int(cos(x)^3*sin(x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*sin(x)^(5/2)-2/9*sin(x)^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \cos^3(x)\sin^{\frac{3}{2}}(x)\,dx = -\frac{2}{45}\left(5\,\cos(x)^4-\cos(x)^2-4\right)\sqrt{\sin(x)}$$

input
```
integrate(cos(x)^3*sin(x)^(3/2),x, algorithm="fricas")
```

output
```
-2/45*(5*cos(x)^4 - cos(x)^2 - 4)*sqrt(sin(x))
```

Sympy [A] (verification not implemented)

Time = 2.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \cos^3(x)\sin^{\frac{3}{2}}(x)\,dx = \frac{8\sin^{\frac{9}{2}}(x)}{45}+\frac{2\sin^{\frac{5}{2}}(x)\cos^2(x)}{5}$$

input
```
integrate(cos(x)**3*sin(x)**(3/2),x)
```

output
```
8*sin(x)**(9/2)/45 + 2*sin(x)**(5/2)*cos(x)**2/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \cos^3(x)\sin^{\frac{3}{2}}(x)\,dx = -\frac{2}{9}\sin(x)^{\frac{9}{2}}+\frac{2}{5}\sin(x)^{\frac{5}{2}}$$

input
```
integrate(cos(x)^3*sin(x)^(3/2),x, algorithm="maxima")
```

output
```
-2/9*sin(x)^(9/2) + 2/5*sin(x)^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \cos^3(x)\sin^{\frac{3}{2}}(x)\,dx = -\frac{2}{9}\sin(x)^{\frac{9}{2}} + \frac{2}{5}\sin(x)^{\frac{5}{2}}$$

input
```
integrate(cos(x)^3*sin(x)^(3/2),x, algorithm="giac")
```

output
```
-2/9*sin(x)^(9/2) + 2/5*sin(x)^(5/2)
```

Mupad [B] (verification not implemented)

Time = 25.39 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \cos^3(x)\sin^{\frac{3}{2}}(x)\,dx = -\frac{\cos(x)^4\sin(x)^{5/2}\,{}_2F_1\left(-\frac{1}{4},2;\ 3;\ \cos(x)^2\right)}{4\left(\sin(x)^2\right)^{5/4}}$$

input
```
int(cos(x)^3*sin(x)^(3/2),x)
```

output
```
-(cos(x)^4*sin(x)^(5/2)*hypergeom([-1/4, 2], 3, cos(x)^2))/(4*(sin(x)^2)^(
5/4))
```

Reduce [F]

$$\int \cos^3(x)\sin^{\frac{3}{2}}(x)\,dx = \int \sqrt{\sin(x)}\,\cos(x)^3\sin(x)\,dx$$

input
```
int(cos(x)^3*sin(x)^(3/2),x)
```

output
```
int(sqrt(sin(x))*cos(x)**3*sin(x),x)
```

3.255 $\int \cos^3(x) \sin^{\frac{5}{2}}(x) \, dx$

Optimal result

Integrand size = 11, antiderivative size = 21

$$\int \cos^3(x) \sin^{\frac{5}{2}}(x) \, dx = \frac{2}{7} \sin^{\frac{7}{2}}(x) - \frac{2}{11} \sin^{\frac{11}{2}}(x)$$

output
```
2/7*sin(x)^(7/2)-2/11*sin(x)^(11/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \cos^3(x) \sin^{\frac{5}{2}}(x) \, dx = \frac{1}{77}(15 + 7\cos(2x)) \sin^{\frac{7}{2}}(x)$$

input
```
Integrate[Cos[x]^3*Sin[x]^(5/2),x]
```

output
```
((15 + 7*Cos[2*x])*Sin[x]^(7/2))/77
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{5}{2}}(x)\cos^3(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)^{5/2}\cos(x)^3dx$$

$$\Big\downarrow \text{3044}$$

$$\int \sin^{\frac{5}{2}}(x)\left(1-\sin^2(x)\right)d\sin(x)$$

$$\Big\downarrow \text{244}$$

$$\int \left(\sin^{\frac{5}{2}}(x)-\sin^{\frac{9}{2}}(x)\right)d\sin(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{7}\sin^{\frac{7}{2}}(x)-\frac{2}{11}\sin^{\frac{11}{2}}(x)$$

input
```
Int[Cos[x]^3*Sin[x]^(5/2),x]
```

output
```
(2*Sin[x]^(7/2))/7 - (2*Sin[x]^(11/2))/11
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 7.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$\frac{2\sin(x)^{\frac{7}{2}}}{7}-\frac{2\sin(x)^{\frac{11}{2}}}{11}$	14
default	$\frac{2\sin(x)^{\frac{7}{2}}}{7}-\frac{2\sin(x)^{\frac{11}{2}}}{11}$	14

input
```
int(cos(x)^3*sin(x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*sin(x)^(7/2)-2/11*sin(x)^(11/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \cos^3(x)\sin^{\frac{5}{2}}(x)\,dx = -\frac{2}{77}\left(7\,\cos{(x)}^4 - 3\,\cos{(x)}^2 - 4\right)\sin{(x)}^{\frac{3}{2}}$$

input
```
integrate(cos(x)^3*sin(x)^(5/2),x, algorithm="fricas")
```

output
```
-2/77*(7*cos(x)^4 - 3*cos(x)^2 - 4)*sin(x)^(3/2)
```

Sympy [A] (verification not implemented)

Time = 18.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \cos^3(x)\sin^{\frac{5}{2}}(x)\,dx = \frac{8\sin^{\frac{11}{2}}(x)}{77} + \frac{2\sin^{\frac{7}{2}}(x)\cos^2(x)}{7}$$

input
```
integrate(cos(x)**3*sin(x)**(5/2),x)
```

output
```
8*sin(x)**(11/2)/77 + 2*sin(x)**(7/2)*cos(x)**2/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \cos^3(x)\sin^{\frac{5}{2}}(x)\,dx = -\frac{2}{11}\,\sin{(x)}^{\frac{11}{2}} + \frac{2}{7}\,\sin{(x)}^{\frac{7}{2}}$$

input
```
integrate(cos(x)^3*sin(x)^(5/2),x, algorithm="maxima")
```

output
```
-2/11*sin(x)^(11/2) + 2/7*sin(x)^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \cos^3(x)\sin^{\frac{5}{2}}(x)\,dx = -\frac{2}{11}\sin(x)^{\frac{11}{2}} + \frac{2}{7}\sin(x)^{\frac{7}{2}}$$

input
```
integrate(cos(x)^3*sin(x)^(5/2),x, algorithm="giac")
```

output
```
-2/11*sin(x)^(11/2) + 2/7*sin(x)^(7/2)
```

Mupad [B] (verification not implemented)

Time = 25.48 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \cos^3(x)\sin^{\frac{5}{2}}(x)\,dx = -\frac{\cos(x)^4\sin(x)^{7/2}\,{}_2F_1\left(-\frac{3}{4},2;\ 3;\ \cos(x)^2\right)}{4\left(\sin(x)^2\right)^{7/4}}$$

input
```
int(cos(x)^3*sin(x)^(5/2),x)
```

output
```
-(cos(x)^4*sin(x)^(7/2)*hypergeom([-3/4, 2], 3, cos(x)^2))/(4*(sin(x)^2)^(
7/4))
```

Reduce [F]

$$\int \cos^3(x)\sin^{\frac{5}{2}}(x)\,dx = \int \sqrt{\sin(x)}\,\cos(x)^3\sin(x)^2\,dx$$

input
```
int(cos(x)^3*sin(x)^(5/2),x)
```

output
```
int(sqrt(sin(x))*cos(x)**3*sin(x)**2,x)
```

3.256 $\int \frac{\cos^3(x)}{\sqrt{\sin(x)}} dx$

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{\cos^3(x)}{\sqrt{\sin(x)}} dx = 2\sqrt{\sin(x)} - \frac{2}{5}\sin^{\frac{5}{2}}(x)$$

output
```
2*sin(x)^(1/2)-2/5*sin(x)^(5/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{\cos^3(x)}{\sqrt{\sin(x)}} dx = \frac{1}{5}(9 + \cos(2x))\sqrt{\sin(x)}$$

input
```
Integrate[Cos[x]^3/Sqrt[Sin[x]],x]
```

output
```
((9 + Cos[2*x])*Sqrt[Sin[x]])/5
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(x)}{\sqrt{\sin(x)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(x)^3}{\sqrt{\sin(x)}}dx$$

$$\Big\downarrow \text{ 3044}$$

$$\int \frac{1-\sin^2(x)}{\sqrt{\sin(x)}}d\sin(x)$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(\frac{1}{\sqrt{\sin(x)}}-\sin^{\frac{3}{2}}(x)\right)d\sin(x)$$

$$\Big\downarrow \text{ 2009}$$

$$2\sqrt{\sin(x)}-\frac{2}{5}\sin^{\frac{5}{2}}(x)$$

input
```
Int[Cos[x]^3/Sqrt[Sin[x]],x]
```

output
```
2*Sqrt[Sin[x]] - (2*Sin[x]^(5/2))/5
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$2\sqrt{\sin\left(x\right)}-\frac{2\sin\left(x\right)^{\frac{5}{2}}}{5}$	14
default	$2\sqrt{\sin\left(x\right)}-\frac{2\sin\left(x\right)^{\frac{5}{2}}}{5}$	14

input
```
int(cos(x)^3/sin(x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*sin(x)^(1/2)-2/5*sin(x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{\cos^3(x)}{\sqrt{\sin(x)}}\,dx = \frac{2}{5}\left(\cos\left(x\right)^2 + 4\right)\sqrt{\sin\left(x\right)}$$

input
```
integrate(cos(x)^3/sin(x)^(1/2),x, algorithm="fricas")
```

output
```
2/5*(cos(x)^2 + 4)*sqrt(sin(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(17) = 34.

Time = 4.08 (sec) , antiderivative size = 323, normalized size of antiderivative = 17.00

$$\int \frac{\cos^3(x)}{\sqrt{\sin(x)}}\,dx$$
$$= \frac{10\sqrt{2}\tan^5\left(\frac{x}{2}\right)}{5\sqrt{\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}}\tan^6\left(\frac{x}{2}\right) + 15\sqrt{\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}}\tan^4\left(\frac{x}{2}\right) + 15\sqrt{\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}}\tan^2\left(\frac{x}{2}\right) + 5\sqrt{\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}}}$$
$$+ \frac{12\sqrt{2}\tan^3\left(\frac{x}{2}\right)}{5\sqrt{\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}}\tan^6\left(\frac{x}{2}\right) + 15\sqrt{\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}}\tan^4\left(\frac{x}{2}\right) + 15\sqrt{\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}}\tan^2\left(\frac{x}{2}\right) + 5\sqrt{\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}}}$$
$$+ \frac{10\sqrt{2}\tan\left(\frac{x}{2}\right)}{5\sqrt{\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}}\tan^6\left(\frac{x}{2}\right) + 15\sqrt{\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}}\tan^4\left(\frac{x}{2}\right) + 15\sqrt{\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}}\tan^2\left(\frac{x}{2}\right) + 5\sqrt{\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}}}$$

input
```
integrate(cos(x)**3/sin(x)**(1/2),x)
```

output
```
10*sqrt(2)*tan(x/2)**5/(5*sqrt(tan(x/2)/(tan(x/2)**2 + 1))*tan(x/2)**6 + 1
5*sqrt(tan(x/2)/(tan(x/2)**2 + 1))*tan(x/2)**4 + 15*sqrt(tan(x/2)/(tan(x/2
)**2 + 1))*tan(x/2)**2 + 5*sqrt(tan(x/2)/(tan(x/2)**2 + 1))) + 12*sqrt(2)*
tan(x/2)**3/(5*sqrt(tan(x/2)/(tan(x/2)**2 + 1))*tan(x/2)**6 + 15*sqrt(tan(
x/2)/(tan(x/2)**2 + 1))*tan(x/2)**4 + 15*sqrt(tan(x/2)/(tan(x/2)**2 + 1))*
tan(x/2)**2 + 5*sqrt(tan(x/2)/(tan(x/2)**2 + 1))) + 10*sqrt(2)*tan(x/2)/(5
*sqrt(tan(x/2)/(tan(x/2)**2 + 1))*tan(x/2)**6 + 15*sqrt(tan(x/2)/(tan(x/2)
**2 + 1))*tan(x/2)**4 + 15*sqrt(tan(x/2)/(tan(x/2)**2 + 1))*tan(x/2)**2 +
5*sqrt(tan(x/2)/(tan(x/2)**2 + 1)))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{\cos^3(x)}{\sqrt{\sin(x)}}\,dx = -\frac{2}{5}\sin(x)^{\frac{5}{2}} + 2\sqrt{\sin(x)}$$

input
```
integrate(cos(x)^3/sin(x)^(1/2),x, algorithm="maxima")
```

output
```
-2/5*sin(x)^(5/2) + 2*sqrt(sin(x))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{\cos^3(x)}{\sqrt{\sin(x)}}\,dx = -\frac{2}{5}\sin(x)^{\frac{5}{2}} + 2\sqrt{\sin(x)}$$

input
```
integrate(cos(x)^3/sin(x)^(1/2),x, algorithm="giac")
```

output
```
-2/5*sin(x)^(5/2) + 2*sqrt(sin(x))
```

Mupad [B] (verification not implemented)

Time = 25.34 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

$$\int \frac{\cos^3(x)}{\sqrt{\sin(x)}}\,dx = -\frac{\cos(x)^4\sqrt{\sin(x)}\,{}_2F_1\left(\frac{3}{4},2;\ 3;\ \cos(x)^2\right)}{4\left(\sin(x)^2\right)^{1/4}}$$

input
```
int(cos(x)^3/sin(x)^(1/2),x)
```

output
```
-(cos(x)^4*sin(x)^(1/2)*hypergeom([3/4, 2], 3, cos(x)^2))/(4*(sin(x)^2)^(1
/4))
```

Reduce [F]

$$\int \frac{\cos^3(x)}{\sqrt{\sin(x)}}\,dx = \int \frac{\sqrt{\sin(x)}\,\cos(x)^3}{\sin(x)}dx$$

input
```
int(cos(x)^3/sin(x)^(1/2),x)
```

output
```
int((sqrt(sin(x))*cos(x)**3)/sin(x),x)
```

3.257 $\int (d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 132

$$\int (d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}\,dx = \frac{7d^3(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{30bc} + \frac{d(d\cos(a+bx))^{7/2}(c\sin(a+bx))^{3/2}}{5bc} + \frac{7d^4\sqrt{d\cos(a+bx)}E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sin(a+bx)}}{20b\sqrt{\sin(2a+2bx)}}$$

output
```
7/30*d^3*(d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(3/2)/b/c+1/5*d*(d*cos(b*x+a)
)^(7/2)*(c*sin(b*x+a))^(3/2)/b/c-7/20*d^4*(d*cos(b*x+a))^(1/2)*EllipticE(c
os(a+1/4*Pi+b*x),2^(1/2))*(c*sin(b*x+a))^(1/2)/b/sin(2*b*x+2*a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.53

$$\int (d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}\,dx = \frac{2d^4\sqrt{d\cos(a+bx)}\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{3}{4},\frac{7}{4},\sin^2(a+bx)\right)\ldots}{3b}$$

input
```
Integrate[(d*Cos[a + b*x])^(9/2)*Sqrt[c*Sin[a + b*x]],x]
```

output
```
(2*d^4*Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[-7/4,
 3/4, 7/4, Sin[a + b*x]^2]*Sqrt[c*Sin[a + b*x]]*Tan[a + b*x])/(3*b)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3049, 3042, 3049, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c\sin(a+bx)}(d\cos(a+bx))^{9/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{c\sin(a+bx)}(d\cos(a+bx))^{9/2}dx$$

$$\Big\downarrow \text{3049}$$

$$\frac{7}{10}d^2\int (d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}dx+\frac{d(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{7/2}}{5bc}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{10}d^2\int (d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}dx+\frac{d(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{7/2}}{5bc}$$

$$\Big\downarrow \text{3049}$$

$$\frac{7}{10}d^2\left(\frac{1}{2}d^2\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}dx+\frac{d(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{3bc}\right)+\frac{d(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{7/2}}{5bc}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{10}d^2\left(\frac{1}{2}d^2\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}dx+\frac{d(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{3bc}\right)+\frac{d(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{7/2}}{5bc}$$

$$\Big\downarrow \text{3052}$$

$$\frac{7}{10}d^2\left(\frac{d^2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}\int \sqrt{\sin(2a+2bx)}dx}{2\sqrt{\sin(2a+2bx)}}+\frac{d(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{3bc}\right)+\frac{d(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{7/2}}{5bc}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{10}d^2\left(\frac{d^2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}\int \sqrt{\sin(2a+2bx)}dx}{2\sqrt{\sin(2a+2bx)}}+\frac{d(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{3bc}\right)+\frac{d(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{7/2}}{5bc}$$

$$\Big\downarrow \text{3119}$$

$$\frac{7}{10}d^2\left(\frac{d^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{2b\sqrt{\sin(2a+2bx)}}+\frac{d(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{3bc}\right)+\frac{d(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{7/2}}{5bc}$$

input
```
Int[(d*Cos[a + b*x])^(9/2)*Sqrt[c*Sin[a + b*x]],x]
```

output
```
(d*(d*Cos[a + b*x])^(7/2)*(c*Sin[a + b*x])^(3/2))/(5*b*c) + (7*d^2*((d*(d*
Cos[a + b*x])^(3/2)*(c*Sin[a + b*x])^(3/2))/(3*b*c) + (d^2*Sqrt[d*Cos[a +
b*x]]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(2*b*Sqrt[Sin[2*a
 + 2*b*x]])))/10
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3049
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Sin[e + f*x])^n*(a
*Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(113) = 226$.

Time = 11.05 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.96

method	result
default	$-\frac{\left((42\cos(bx+a)+42)\sqrt{2\cot(bx+a)-2\csc(bx+a)+2}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\ \mathrm{EllipticE}\left(\sqrt{-\cot(bx+a)+\csc(bx+a)+1},\frac{\sqrt{2}}{2}\right)\sqrt{-}\right.}{}$

input
```
int((d*cos(b*x+a))^(9/2)*(c*sin(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/120/b*((42*cos(b*x+a)+42)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+
a)-csc(b*x+a))^(1/2)*EllipticE((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2
))*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)+(-21*cos(b*x+a)-21)*(2*cot(b*x+a)-2*cs
c(b*x+a)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-cot(b*x+a)+csc(b*x+a)+1)
^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))+cos(b*x+a)*
(24*cos(b*x+a)^5+4*cos(b*x+a)^3+14*cos(b*x+a)-42))*(c*sin(b*x+a))^(1/2)*(d
*cos(b*x+a))^(1/2)*d^4*sec(b*x+a)*csc(b*x+a)
```

Fricas [F]

$$\int (d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}\,dx = \int (d\cos(bx+a))^{\frac{9}{2}}\sqrt{c\sin(bx+a)}\,dx$$

input
```
integrate((d*cos(b*x+a))^(9/2)*(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*d^4*cos(b*x + a)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate((d*cos(b*x+a))**(9/2)*(c*sin(b*x+a))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}\,dx = \int (d\cos(bx+a))^{\frac{9}{2}}\sqrt{c\sin(bx+a)}\,dx$$

input
```
integrate((d*cos(b*x+a))^(9/2)*(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*cos(b*x + a))^(9/2)*sqrt(c*sin(b*x + a)), x)
```

Giac [F]

$$\int (d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}\,dx = \int (d\cos(bx+a))^{\frac{9}{2}}\sqrt{c\sin(bx+a)}\,dx$$

input
```
integrate((d*cos(b*x+a))^(9/2)*(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*cos(b*x + a))^(9/2)*sqrt(c*sin(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}\,dx = \int (d\,\cos(a+b\,x))^{9/2}\,\sqrt{c\,\sin(a+b\,x)}\,dx$$

input
```
int((d*cos(a + b*x))^(9/2)*(c*sin(a + b*x))^(1/2),x)
```

output
```
int((d*cos(a + b*x))^(9/2)*(c*sin(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int (d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}\,dx=\sqrt{d}\,\sqrt{c}\left(\int\sqrt{\sin(bx+a)}\,\sqrt{\cos(bx+a)}\,\cos(bx+a)^4\,dx\right)d^4$$

input
```
int((d*cos(b*x+a))^(9/2)*(c*sin(b*x+a))^(1/2),x)
```

output
```
sqrt(d)*sqrt(c)*int(sqrt(sin(a + b*x))*sqrt(cos(a + b*x))*cos(a + b*x)**4,
x)*d**4
```

3.258 $\int (d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 95

$$\int (d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}\,dx=\frac{d(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{3bc}+\frac{d^2\sqrt{d\cos(a+bx)}E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sin(a+bx)}}{2b\sqrt{\sin(2a+2bx)}}$$

output
```
1/3*d*(d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(3/2)/b/c-1/2*d^2*(d*cos(b*x+a))
^(1/2)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*(c*sin(b*x+a))^(1/2)/b/sin(2*b
*x+2*a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

$$\int (d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}\,dx=\frac{2d^2\sqrt{d\cos(a+bx)}\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{3}{4},\frac{7}{4},\sin^2(a+bx)\right)\ldots}{3b}$$

input
```
Integrate[(d*Cos[a + b*x])^(5/2)*Sqrt[c*Sin[a + b*x]],x]
```

output
```
(2*d^2*Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[-3/4,
 3/4, 7/4, Sin[a + b*x]^2]*Sqrt[c*Sin[a + b*x]]*Tan[a + b*x])/(3*b)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 3049, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c\sin(a+bx)}(d\cos(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{c\sin(a+bx)}(d\cos(a+bx))^{5/2}dx$$

$$\Big\downarrow \text{3049}$$

$$\frac{1}{2}d^2\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}dx+\frac{d(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{3bc}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}d^2\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}dx+\frac{d(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{3bc}$$

$$\Big\downarrow \text{3052}$$

$$\frac{d^2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}\int \sqrt{\sin(2a+2bx)}dx}{2\sqrt{\sin(2a+2bx)}}+\frac{d(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{3bc}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d^2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}\int \sqrt{\sin(2a+2bx)}dx}{2\sqrt{\sin(2a+2bx)}}+\frac{d(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{3bc}$$

$$\Big\downarrow \text{3119}$$

$$\frac{d^2 E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{2b\sqrt{\sin(2a+2bx)}}+\frac{d(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{3bc}$$

input
```
Int[(d*Cos[a + b*x])^(5/2)*Sqrt[c*Sin[a + b*x]],x]
```

output
```
(d*(d*Cos[a + b*x])^(3/2)*(c*Sin[a + b*x])^(3/2))/(3*b*c) + (d^2*Sqrt[d*Co
s[a + b*x]]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(2*b*Sqrt[S
in[2*a + 2*b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3049
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Sin[e + f*x])^n*(a
*Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(82) = 164$.

Time = 9.16 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.62

method	result
default	$-\frac{\left((6\cos(bx+a)+6)\sqrt{2\cot(bx+a)-2\csc(bx+a)+2}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\ \text{EllipticE}\left(\sqrt{-\cot(bx+a)+\csc(bx+a)+1},\frac{\sqrt{2}}{2}\right)\sqrt{-\text{co}}\right.}{}$

input
```
int((d*cos(b*x+a))^(5/2)*(c*sin(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12/b*((6*cos(b*x+a)+6)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+a)-
csc(b*x+a))^(1/2)*EllipticE((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))*
(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)+(-3*cos(b*x+a)-3)*(2*cot(b*x+a)-2*csc(b*x
+a)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+
a)+1)^(1/2),1/2*2^(1/2))*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)+cos(b*x+a)*(4*co
s(b*x+a)^3+2*cos(b*x+a)-6))*(c*sin(b*x+a))^(1/2)*(d*cos(b*x+a))^(1/2)*d^2*
sec(b*x+a)*csc(b*x+a)
```

Fricas [F]

$$\int (d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}\,dx = \int (d\cos(bx+a))^{\frac{5}{2}}\sqrt{c\sin(bx+a)}\,dx$$

input
```
integrate((d*cos(b*x+a))^(5/2)*(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*d^2*cos(b*x + a)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate((d*cos(b*x+a))**(5/2)*(c*sin(b*x+a))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}\,dx = \int (d\cos(bx+a))^{\frac{5}{2}}\sqrt{c\sin(bx+a)}\,dx$$

input
```
integrate((d*cos(b*x+a))^(5/2)*(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*cos(b*x + a))^(5/2)*sqrt(c*sin(b*x + a)), x)
```

Giac [F]

$$\int (d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}\,dx = \int (d\cos(bx+a))^{\frac{5}{2}}\sqrt{c\sin(bx+a)}\,dx$$

input
```
integrate((d*cos(b*x+a))^(5/2)*(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*cos(b*x + a))^(5/2)*sqrt(c*sin(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}\,dx = \int (d\,\cos(a+b\,x))^{5/2}\,\sqrt{c\,\sin(a+b\,x)}\,dx$$

input
```
int((d*cos(a + b*x))^(5/2)*(c*sin(a + b*x))^(1/2),x)
```

output
```
int((d*cos(a + b*x))^(5/2)*(c*sin(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int (d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}\,dx = \sqrt{d}\,\sqrt{c}\left(\int \sqrt{\sin(bx+a)}\,\sqrt{\cos(bx+a)}\,\cos(bx+a)^2\,dx\right)d^2$$

input
```
int((d*cos(b*x+a))^(5/2)*(c*sin(b*x+a))^(1/2),x)
```

output
```
sqrt(d)*sqrt(c)*int(sqrt(sin(a + b*x))*sqrt(cos(a + b*x))*cos(a + b*x)**2,
x)*d**2
```

3.259 $\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 53

$$\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\,dx = \frac{\sqrt{d\cos(a+bx)}E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sin(a+bx)}}{b\sqrt{\sin(2a+2bx)}}$$

output
```
-(d*cos(b*x+a))^(1/2)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*(c*sin(b*x+a))^
(1/2)/b/sin(2*b*x+2*a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.26

$$\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\,dx = \frac{2\sqrt{d\cos(a+bx)}\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{7}{4},\sin^2(a+bx)\right)\sqrt{c\sin(a+bx)}\tan(a+bx)}{3b}$$

input
```
Integrate[Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]],x]
```

output
```
(2*Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, 3/4,
 7/4, Sin[a + b*x]^2]*Sqrt[c*Sin[a + b*x]]*Tan[a + b*x])/(3*b)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}dx$$
$$\Big\downarrow \text{ 3052}$$
$$\frac{\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}\int \sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}\int \sqrt{\sin(2a+2bx)}dx}{\sqrt{\sin(2a+2bx)}}$$
$$\Big\downarrow \text{ 3119}$$
$$\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{b\sqrt{\sin(2a+2bx)}}$$

input
```
Int[Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]],x]
```

output
```
(Sqrt[d*Cos[a + b*x]]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(
b*Sqrt[Sin[2*a + 2*b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(49) = 98$.

Time = 10.65 (sec) , antiderivative size = 236, normalized size of antiderivative = 4.45

method	result
default	$-\frac{\left((2\cos(bx+a)+2)\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\,\sqrt{2\cot(bx+a)-2\csc(bx+a)+2}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\ \mathrm{EllipticE}\left(\sqrt{-\cot(bx\right.\right.}{}$

input
```
int((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b*((2*cos(b*x+a)+2)*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(2*cot(b*x+a)-2*
csc(b*x+a)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((-cot(b*x+a)+c
sc(b*x+a)+1)^(1/2),1/2*2^(1/2))+(-cos(b*x+a)-1)*(-cot(b*x+a)+csc(b*x+a)+1)
^(1/2)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*E
llipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))+cos(b*x+a)*(-2+2*co
s(b*x+a)))*(d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2)*sec(b*x+a)*csc(b*x+a)
```

Fricas [F]

$$\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\,dx = \int \sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}\,dx$$

input
```
integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)), x)
```

Sympy [F]

$$\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\,dx = \int \sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}\,dx$$

input
```
integrate((d*cos(b*x+a))**(1/2)*(c*sin(b*x+a))**(1/2),x)
```

output
```
Integral(sqrt(c*sin(a + b*x))*sqrt(d*cos(a + b*x)), x)
```

Maxima [F]

$$\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\,dx = \int \sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}\,dx$$

input
```
integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)), x)
```

Giac [F]

$$\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\,dx = \int \sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}\,dx$$

input
```
integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\,dx = \int \sqrt{d\,\cos(a+b\,x)}\,\sqrt{c\,\sin(a+b\,x)}\,dx$$

input
```
int((d*cos(a + b*x))^(1/2)*(c*sin(a + b*x))^(1/2),x)
```

output
```
int((d*cos(a + b*x))^(1/2)*(c*sin(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\,dx = \sqrt{d}\,\sqrt{c}\left(\int \sqrt{\sin(bx+a)}\,\sqrt{\cos(bx+a)}dx\right)$$

input
```
int((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2),x)
```

output
```
sqrt(d)*sqrt(c)*int(sqrt(sin(a + b*x))*sqrt(cos(a + b*x)),x)
```

3.260 $\int \frac{\sqrt{c \sin(a+bx)}}{(d \cos(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 93

$$\int \frac{\sqrt{c \sin(a+bx)}}{(d \cos(a+bx))^{3/2}}\,dx = \frac{2(c \sin(a+bx))^{3/2}}{bcd\sqrt{d \cos(a+bx)}} - \frac{2\sqrt{d \cos(a+bx)}\,E\left(a-\frac{\pi}{4}+bx\middle|\,2\right)\sqrt{c \sin(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}$$

output
```
2*(c*sin(b*x+a))^(3/2)/b/c/d/(d*cos(b*x+a))^(1/2)+2*(d*cos(b*x+a))^(1/2)*E
llipticE(cos(a+1/4*Pi+b*x),2^(1/2))*(c*sin(b*x+a))^(1/2)/b/d^2/sin(2*b*x+2
*a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{c \sin(a+bx)}}{(d \cos(a+bx))^{3/2}}\,dx = \frac{2\sqrt{d \cos(a+bx)}\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},\sin^2(a+bx)\right)\sqrt{c \sin}}{3bd^2}$$

input
```
Integrate[Sqrt[c*Sin[a + b*x]]/(d*Cos[a + b*x])^(3/2),x]
```

output
```
(2*Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[3/4, 5/4,
 7/4, Sin[a + b*x]^2]*Sqrt[c*Sin[a + b*x]]*Tan[a + b*x])/(3*b*d^2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3051, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3051}$$

$$\frac{2(c\sin(a+bx))^{3/2}}{bcd\sqrt{d\cos(a+bx)}}-\frac{2\int\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}dx}{d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2(c\sin(a+bx))^{3/2}}{bcd\sqrt{d\cos(a+bx)}}-\frac{2\int\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}dx}{d^2}$$

$$\Big\downarrow \text{3052}$$

$$\frac{2(c\sin(a+bx))^{3/2}}{bcd\sqrt{d\cos(a+bx)}}-\frac{2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{d^2\sqrt{\sin(2a+2bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2(c\sin(a+bx))^{3/2}}{bcd\sqrt{d\cos(a+bx)}}-\frac{2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{d^2\sqrt{\sin(2a+2bx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2(c\sin(a+bx))^{3/2}}{bcd\sqrt{d\cos(a+bx)}}-\frac{2E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}$$

input
```
Int[Sqrt[c*Sin[a + b*x]]/(d*Cos[a + b*x])^(3/2),x]
```

output
```
(2*(c*Sin[a + b*x])^(3/2))/(b*c*d*Sqrt[d*Cos[a + b*x]]) - (2*Sqrt[d*Cos[a
+ b*x]]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(b*d^2*Sqrt[Sin
[2*a + 2*b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3051
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Simp[(m + n + 2)/(a^2*(m + 1))   Int[(b*Sin[e + f*x
])^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m
, -1] && IntegersQ[2*m, 2*n]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(84) = 168$.

Time = 11.33 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.40

method	result
default	$\frac{\left(2\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\,\sqrt{2\cot(bx+a)-2\csc(bx+a)+2}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,(\cos(bx+a)+1)\operatorname{EllipticE}\left(\sqrt{-\cot(bx+a}\right.\right.}{}$

input
```
int((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*(2*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2
)*(cot(b*x+a)-csc(b*x+a))^(1/2)*(cos(b*x+a)+1)*EllipticE((-cot(b*x+a)+csc(
b*x+a)+1)^(1/2),1/2*2^(1/2))+(-cos(b*x+a)-1)*(-cot(b*x+a)+csc(b*x+a)+1)^(1
/2)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*Elli
pticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))+2-2*cos(b*x+a))*(c*sin
(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2)/d*csc(b*x+a)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.85

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{3/2}}\,dx = \frac{-i\sqrt{i\,cd}\cos\left(bx+a\right)E(\arcsin\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)|-1)+i\sqrt{-i\,cd}\,\mathrm{c}$$

input
```
integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(I*c*d)*cos(b*x + a)*elliptic_e(arcsin(cos(b*x + a) + I*sin(b*x +
a)), -1) + I*sqrt(-I*c*d)*cos(b*x + a)*elliptic_e(arcsin(cos(b*x + a) - I*
sin(b*x + a)), -1) + I*sqrt(I*c*d)*cos(b*x + a)*elliptic_f(arcsin(cos(b*x
+ a) + I*sin(b*x + a)), -1) - I*sqrt(-I*c*d)*cos(b*x + a)*elliptic_f(arcsi
n(cos(b*x + a) - I*sin(b*x + a)), -1) + 2*sqrt(d*cos(b*x + a))*sqrt(c*sin(
b*x + a))*sin(b*x + a))/(b*d^2*cos(b*x + a))
```

Sympy [F]

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\sqrt{c\sin\left(a+bx\right)}}{\left(d\cos\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))**(1/2)/(d*cos(b*x+a))**(3/2),x)
```

output
```
Integral(sqrt(c*sin(a + b*x))/(d*cos(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\sqrt{c\sin\left(bx+a\right)}}{\left(d\cos\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*sin(b*x + a))/(d*cos(b*x + a))^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\sqrt{c\sin\left(bx+a\right)}}{\left(d\cos\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*sin(b*x + a))/(d*cos(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{\sqrt{c\,\sin\left(a+b\,x\right)}}{{\left(d\,\cos\left(a+b\,x\right)\right)}^{3/2}}\,dx$$

input
```
int((c*sin(a + b*x))^(1/2)/(d*cos(a + b*x))^(3/2),x)
```

output
```
int((c*sin(a + b*x))^(1/2)/(d*cos(a + b*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{3/2}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sin(bx+a)}\,\sqrt{\cos(bx+a)}}{\cos(bx+a)^2}dx\right)}{d^2}$$

input
```
int((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sin(a + b*x))*sqrt(cos(a + b*x)))/cos(a + b*x)*
*2,x))/d**2
```

3.261 $\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 134

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{7/2}}\,dx = \frac{2(c\sin(a+bx))^{3/2}}{5bcd(d\cos(a+bx))^{5/2}} + \frac{4(c\sin(a+bx))^{3/2}}{5bcd^3\sqrt{d\cos(a+bx)}} - \frac{4\sqrt{d\cos(a+bx)}E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sin(a+bx)}}{5bd^4\sqrt{\sin(2a+2bx)}}$$

output
```
2/5*(c*sin(b*x+a))^(3/2)/b/c/d/(d*cos(b*x+a))^(5/2)+4/5*(c*sin(b*x+a))^(3/
2)/b/c/d^3/(d*cos(b*x+a))^(1/2)+4/5*(d*cos(b*x+a))^(1/2)*EllipticE(cos(a+1
/4*Pi+b*x),2^(1/2))*(c*sin(b*x+a))^(1/2)/b/d^4/sin(2*b*x+2*a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.15 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{7/2}}\,dx = \frac{2\sqrt{d\cos(a+bx)}\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{9}{4},\frac{7}{4},\sin^2(a+bx)\right)\sqrt{c\,s}}{3bd^4}$$

input
```
Integrate[Sqrt[c*Sin[a + b*x]]/(d*Cos[a + b*x])^(7/2),x]
```

output
```
(2*Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[3/4, 9/4,
 7/4, Sin[a + b*x]^2]*Sqrt[c*Sin[a + b*x]]*Tan[a + b*x])/(3*b*d^4)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3051, 3042, 3051, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{7/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{7/2}}dx$$

$$\Big\downarrow \ 3051$$

$$\frac{2\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{3/2}}dx}{5d^2}+\frac{2(c\sin(a+bx))^{3/2}}{5bcd(d\cos(a+bx))^{5/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{2\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{3/2}}dx}{5d^2}+\frac{2(c\sin(a+bx))^{3/2}}{5bcd(d\cos(a+bx))^{5/2}}$$

$$\Big\downarrow \ 3051$$

$$\frac{2\left(\frac{2(c\sin(a+bx))^{3/2}}{bcd\sqrt{d\cos(a+bx)}}-\frac{2\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}dx}{d^2}\right)}{5d^2}+\frac{2(c\sin(a+bx))^{3/2}}{5bcd(d\cos(a+bx))^{5/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{2\left(\frac{2(c\sin(a+bx))^{3/2}}{bcd\sqrt{d\cos(a+bx)}}-\frac{2\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}dx}{d^2}\right)}{5d^2}+\frac{2(c\sin(a+bx))^{3/2}}{5bcd(d\cos(a+bx))^{5/2}}$$

$$\Big\downarrow \ 3052$$

$$\frac{2\left(\frac{2(c\sin(a+bx))^{3/2}}{bcd\sqrt{d\cos(a+bx)}}-\frac{2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{d^2\sqrt{\sin(2a+2bx)}}\right)}{5d^2}+\frac{2(c\sin(a+bx))^{3/2}}{5bcd(d\cos(a+bx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{2(c\sin(a+bx))^{3/2}}{bcd\sqrt{d\cos(a+bx)}}-\frac{2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{d^2\sqrt{\sin(2a+2bx)}}\right)}{5d^2}+\frac{2(c\sin(a+bx))^{3/2}}{5bcd(d\cos(a+bx))^{5/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2\left(\frac{2(c\sin(a+bx))^{3/2}}{bcd\sqrt{d\cos(a+bx)}}-\frac{2E\left(a+bx-\frac{\pi}{4}|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}\right)}{5d^2}+\frac{2(c\sin(a+bx))^{3/2}}{5bcd(d\cos(a+bx))^{5/2}}$$

input
```
Int[Sqrt[c*Sin[a + b*x]]/(d*Cos[a + b*x])^(7/2),x]
```

output
```
(2*(c*Sin[a + b*x])^(3/2))/(5*b*c*d*(d*Cos[a + b*x])^(5/2)) + (2*((2*(c*Si
n[a + b*x])^(3/2))/(b*c*d*Sqrt[d*Cos[a + b*x]]) - (2*Sqrt[d*Cos[a + b*x]]*
EllipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(b*d^2*Sqrt[Sin[2*a + 2
*b*x]])))/(5*d^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3051
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Simp[(m + n + 2)/(a^2*(m + 1))   Int[(b*Sin[e + f*x
])^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m
, -1] && IntegersQ[2*m, 2*n]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(115) = 230$.

Time = 11.50 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.89

method	result
default	$\frac{2\sqrt{c\sin(bx+a)}\left(\text{EllipticE}\left(\sqrt{-\cot(bx+a)+\csc(bx+a)+1},\frac{\sqrt{2}}{2}\right)\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\,\sqrt{2\cot(bx+a)-2\csc(bx+a)+2}\,\sqrt{\cot\cdots}\right.}{}$

input
```
int((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/b*(c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2)/d^3*(EllipticE((-cot(b*x+a
)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(2*cot
(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*(2*cot(b*x+a)+
2*csc(b*x+a))+EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))*(-co
t(b*x+a)+csc(b*x+a)+1)^(1/2)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+
a)-csc(b*x+a))^(1/2)*(-cot(b*x+a)-csc(b*x+a))-2*cot(b*x+a)+csc(b*x+a)+csc(
b*x+a)*sec(b*x+a)^2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.44

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{7/2}}\,dx = -\frac{2\left(i\sqrt{i\,cd}\cos(bx+a)^3 E(\arcsin(\cos(bx+a)+i\sin(bx+a))\,|-1)-i\sqrt{-i\,cd}\cos(bx+a)^3 E(\arcsin(\cdots\right.}{}$$

input
```
integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

output
```
-2/5*(I*sqrt(I*c*d)*cos(b*x + a)^3*elliptic_e(arcsin(cos(b*x + a) + I*sin(
b*x + a)), -1) - I*sqrt(-I*c*d)*cos(b*x + a)^3*elliptic_e(arcsin(cos(b*x +
 a) - I*sin(b*x + a)), -1) - I*sqrt(I*c*d)*cos(b*x + a)^3*elliptic_f(arcsi
n(cos(b*x + a) + I*sin(b*x + a)), -1) + I*sqrt(-I*c*d)*cos(b*x + a)^3*elli
ptic_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1) - sqrt(d*cos(b*x + a))*(
2*cos(b*x + a)^2 + 1)*sqrt(c*sin(b*x + a))*sin(b*x + a))/(b*d^4*cos(b*x +
a)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*sin(b*x+a))**(1/2)/(d*cos(b*x+a))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{\sqrt{c\sin\left(bx+a\right)}}{\left(d\cos\left(bx+a\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(7/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*sin(b*x + a))/(d*cos(b*x + a))^(7/2), x)
```

Giac [F]

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{\sqrt{c\sin\left(bx+a\right)}}{\left(d\cos\left(bx+a\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(7/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*sin(b*x + a))/(d*cos(b*x + a))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{\sqrt{c\,\sin\left(a+b\,x\right)}}{\left(d\,\cos\left(a+b\,x\right)\right)^{7/2}}\,dx$$

input
```
int((c*sin(a + b*x))^(1/2)/(d*cos(a + b*x))^(7/2),x)
```

output
```
int((c*sin(a + b*x))^(1/2)/(d*cos(a + b*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{7/2}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sin(bx+a)}\,\sqrt{\cos(bx+a)}}{\cos(bx+a)^{4}}dx\right)}{d^{4}}$$

input
```
int((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(7/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sin(a + b*x))*sqrt(cos(a + b*x)))/cos(a + b*x)*
*4,x))/d**4
```

3.262 $\int (d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 245

$$\begin{aligned}\int (d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}\,dx = &-\frac{\sqrt{c}d^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{4\sqrt{2}b}+\frac{\sqrt{c}d^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{4\sqrt{2}b}\\&-\frac{\sqrt{c}d^{3/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}(\sqrt{c}+\sqrt{c}\tan(a+bx))}\right)}{4\sqrt{2}b}\\&+\frac{d\sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}}{2bc}\end{aligned}$$

output
```
-1/8*c^(1/2)*d^(3/2)*arctan(1-2^(1/2)*d^(1/2)*(c*sin(b*x+a))^(1/2)/c^(1/2)
/(d*cos(b*x+a))^(1/2))*2^(1/2)/b+1/8*c^(1/2)*d^(3/2)*arctan(1+2^(1/2)*d^(1
/2)*(c*sin(b*x+a))^(1/2)/c^(1/2)/(d*cos(b*x+a))^(1/2))*2^(1/2)/b-1/8*c^(1/
2)*d^(3/2)*arctanh(2^(1/2)*d^(1/2)*(c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/
2)/(c^(1/2)+c^(1/2)*tan(b*x+a)))*2^(1/2)/b+1/2*d*(d*cos(b*x+a))^(1/2)*(c*s
in(b*x+a))^(3/2)/b/c
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.29

$$\int (d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}\,dx = \frac{2d^2\cos^2(a+bx)^{3/4}\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{3}{4},\frac{7}{4},\sin^2(a+bx)\right)\sqrt{c\sin(a+bx)}\ldots}{3b\sqrt{d\cos(a+bx)}}$$

input
```
Integrate[(d*Cos[a + b*x])^(3/2)*Sqrt[c*Sin[a + b*x]],x]
```

output
```
(2*d^2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-1/4, 3/4, 7/4, Sin[a + b*
x]^2]*Sqrt[c*Sin[a + b*x]]*Tan[a + b*x])/(3*b*Sqrt[d*Cos[a + b*x]])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.33, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 3049, 3042, 3054, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c\sin(a+bx)}(d\cos(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{c\sin(a+bx)}(d\cos(a+bx))^{3/2}dx$$

$$\Big\downarrow \text{3049}$$

$$\frac{1}{4}d^2\int \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}dx + \frac{d(c\sin(a+bx))^{3/2}\sqrt{d\cos(a+bx)}}{2bc}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}d^2\int \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}dx+\frac{d(c\sin(a+bx))^{3/2}\sqrt{d\cos(a+bx)}}{2bc}$$

$$\Big\downarrow \; 3054$$

$$\frac{cd^3\int \frac{c\tan(a+bx)}{d(\tan^2(a+bx)c^2+c^2)}d\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{2b}+\frac{d(c\sin(a+bx))^{3/2}\sqrt{d\cos(a+bx)}}{2bc}$$

$$\Big\downarrow \; 826$$

$$\frac{cd^3\left(\frac{\int\frac{\tan(a+bx)c+c}{\tan^2(a+bx)c^2+c^2}d\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{2d}-\frac{\int\frac{c-c\tan(a+bx)}{\tan^2(a+bx)c^2+c^2}d\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{2d}\right)}{2b}+\frac{d(c\sin(a+bx))^{3/2}\sqrt{d\cos(a+bx)}}{2bc}$$

$$\Big\downarrow \; 1476$$

$$\frac{cd^3\left(\frac{\frac{\int\frac{1}{\frac{\tan(a+bx)c}{d}+\frac{c}{d}-\frac{\sqrt{2}\sqrt{c\sin(a+bx)}\sqrt{c}}{\sqrt{d}\sqrt{d\cos(a+bx)}}}d\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{2d}+\frac{\int\frac{1}{\frac{\tan(a+bx)c}{d}+\frac{c}{d}+\frac{\sqrt{2}\sqrt{c\sin(a+bx)}\sqrt{c}}{\sqrt{d}\sqrt{d\cos(a+bx)}}}d\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{2d}}{2d}-\frac{\int\frac{c-c\tan(a+bx)}{\tan^2(a+bx)c^2+c^2}d\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{2d}\right)}{2b}+\frac{d(c\sin(a+bx))^{3/2}\sqrt{d\cos(a+bx)}}{2bc}$$

$$\Big\downarrow \; 1082$$

$$\frac{cd^3\left(\frac{\frac{\int\frac{1}{-\frac{c\tan(a+bx)}{d}-1}d\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\int\frac{1}{-\frac{c\tan(a+bx)}{d}-1}d\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}}{2d}-\frac{\int\frac{c-c\tan(a+bx)}{\tan^2(a+bx)c^2+c^2}d\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{2d}\right)}{2b}+\frac{d(c\sin(a+bx))^{3/2}\sqrt{d\cos(a+bx)}}{2bc}$$

$$\Big\downarrow \; 217$$

$$\frac{cd^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}}{2d}-\frac{\int\frac{c-c\tan(a+bx)}{\tan^2(a+bx)c^2+c^2}d\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{2d}\right)}{2b}+\frac{d(c\sin(a+bx))^{3/2}\sqrt{d\cos(a+bx)}}{2bc}$$

$$\Big\downarrow \; 1479$$

$$\frac{\frac{cd^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}}{2d}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{c}-\frac{2\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{\sqrt{d}\left(\frac{\tan(a+bx)c}{d}+\frac{c}{d}-\frac{\sqrt{2}\sqrt{c\sin(a+bx)}\sqrt{c}}{\sqrt{d}\sqrt{d\cos(a+bx)}}\right)}d\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{2\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\int -\frac{\cdots}{\sqrt{d}\left(\cdots\right)}\cdots}{\cdots}}{2d}\cdots\right)}{2b}}{\frac{d(c\sin(a+bx))^{3/2}\sqrt{d\cos(a+bx)}}{2bc}}$$

$\downarrow$ 25

$$\frac{\frac{cd^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}}{2d}-\frac{\frac{\int \frac{\sqrt{2}\sqrt{c}-\frac{2\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{\sqrt{d}\left(\frac{\tan(a+bx)c}{d}+\frac{c}{d}-\frac{\sqrt{2}\sqrt{c\sin(a+bx)}\sqrt{c}}{\sqrt{d}\sqrt{d\cos(a+bx)}}\right)}d\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{2\sqrt{2}\sqrt{c}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\cdots\right)}{\sqrt{d}\left(\frac{\tan(a+\cdots}{d}\cdots\right)}\cdots}{\cdots}}{2d}\cdots\right)}{2b}}{\frac{d(c\sin(a+bx))^{3/2}\sqrt{d\cos(a+bx)}}{2bc}}$$

$\downarrow$ 27

$$\frac{\frac{cd^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}}{2d}-\frac{\frac{\int \frac{\sqrt{2}\sqrt{c}-\frac{2\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{\frac{\tan(a+bx)c}{d}+\frac{c}{d}-\frac{\sqrt{2}\sqrt{c\sin(a+bx)}\sqrt{c}}{\sqrt{d}\sqrt{d\cos(a+bx)}}}d\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{2\sqrt{2}\sqrt{c}d}+\frac{\int \frac{\sqrt{c}+\frac{\sqrt{2}\sqrt{d}\cdots}{\sqrt{d}\cdots}}{\frac{\tan(a+bx)c}{d}+\frac{c}{d}+\cdots}\cdots}{\cdots}}{2d}\cdots\right)}{2b}}{\frac{d(c\sin(a+bx))^{3/2}\sqrt{d\cos(a+bx)}}{2bc}}$$

$\downarrow$ 1103

$$\frac{\frac{cd^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}}{2d}-\frac{\frac{\log\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+c\tan(a+bx)+c\right)}{2\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\log\left(-\frac{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\cdots\right)}{2\sqrt{2}\sqrt{c}\sqrt{d}}}{2d}\cdots\right)}{2b}}{\frac{d(c\sin(a+bx))^{3/2}\sqrt{d\cos(a+bx)}}{2bc}}$$

input
```
Int[(d*Cos[a + b*x])^(3/2)*Sqrt[c*Sin[a + b*x]],x]
```

output
```
(c*d^3*((-(ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/(Sqrt[c]*Sqrt
[d*Cos[a + b*x]])]/(Sqrt[2]*Sqrt[c]*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*Sqrt[d
]*Sqrt[c*Sin[a + b*x]])/(Sqrt[c]*Sqrt[d*Cos[a + b*x]])]/(Sqrt[2]*Sqrt[c]*S
qrt[d]))/(2*d) - (-1/2*Log[c - (Sqrt[2]*Sqrt[c]*Sqrt[d]*Sqrt[c*Sin[a + b*x
]])/Sqrt[d*Cos[a + b*x]] + c*Tan[a + b*x]]/(Sqrt[2]*Sqrt[c]*Sqrt[d]) + Log
[c + (Sqrt[2]*Sqrt[c]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/Sqrt[d*Cos[a + b*x]] +
 c*Tan[a + b*x]]/(2*Sqrt[2]*Sqrt[c]*Sqrt[d]))/(2*d)))/(2*b) + (d*Sqrt[d*Co
s[a + b*x]]*(c*Sin[a + b*x])^(3/2))/(2*b*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3049
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Sin[e + f*x])^n*(a
*Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3054
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k*a*(b/f)   Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 426 vs. $2(183) = 366$.

Time = 186.00 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.74

method	result
default	$\sqrt{2}\, d \left(\ln\left(-\frac{\cos(bx+a)\cot(bx+a)-2\cot(bx+a)-2\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}\,\sin(bx+a)-2\cos(bx+a)-\sin(bx+a)+\csc(bx+a)+2}{\cos(bx+a)-1}\right)-\ln\left(-\frac{\cos(b}{}\right.\right.$

input
```
int((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/256/b*2^(1/2)*d*(ln(-(cos(b*x+a)*cot(b*x+a)-2*cot(b*x+a)-2*(-2*sin(b*x+a
)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)-2*cos(b*x+a)-sin(b*x+a)+cs
c(b*x+a)+2)/(cos(b*x+a)-1))-ln(-(cos(b*x+a)*cot(b*x+a)-2*cot(b*x+a)+2*(-2*
sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)-2*cos(b*x+a)-sin(
b*x+a)+csc(b*x+a)+2)/(cos(b*x+a)-1))-2*arctan((-(-2*sin(b*x+a)*cos(b*x+a)/
(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)+cos(b*x+a)-1)/(cos(b*x+a)-1))+2*arctan(
((-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)+cos(b*x+a)-1
)/(cos(b*x+a)-1))+(4*cos(b*x+a)+4)*sin(b*x+a)*(-2*sin(b*x+a)*cos(b*x+a)/(c
os(b*x+a)+1)^2)^(1/2))*(c*sin(b*x+a))^(1/2)*(d*cos(b*x+a))^(1/2)*sin(b*x+a
)^3/(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sec(1/2*b*x+1/2*a)^5*c
sc(1/2*b*x+1/2*a)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 512 vs. $2(183) = 366$.

Time = 0.16 (sec) , antiderivative size = 512, normalized size of antiderivative = 2.09

$$\int (d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}\,dx =$$

$$-\frac{\sqrt{2}\sqrt{cd}\,d\arctan\left(\frac{2\,cd\cos(bx+a)^3-2\,cd\cos(bx+a)^2\sin(bx+a)-2\,cd\cos(bx+a)+\sqrt{2}\sqrt{cd}\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}}{2\left(cd\cos(bx+a)^3+cd\cos(bx+a)^2\sin(bx+a)-cd\cos(bx+a)\right)}\right)+\sqrt{2}\sqrt{cd}\,d\,a}{}$$

input
```
integrate((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
-1/32*(sqrt(2)*sqrt(c*d)*d*arctan(1/2*(2*c*d*cos(b*x + a)^3 - 2*c*d*cos(b*
x + a)^2*sin(b*x + a) - 2*c*d*cos(b*x + a) + sqrt(2)*sqrt(c*d)*sqrt(d*cos(
b*x + a))*sqrt(c*sin(b*x + a)))/(c*d*cos(b*x + a)^3 + c*d*cos(b*x + a)^2*s
in(b*x + a) - c*d*cos(b*x + a))) + sqrt(2)*sqrt(c*d)*d*arctan(-1/2*(2*c*d*
cos(b*x + a)^3 - 2*c*d*cos(b*x + a)^2*sin(b*x + a) - 2*c*d*cos(b*x + a) -
sqrt(2)*sqrt(c*d)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)))/(c*d*cos(b*x
+ a)^3 + c*d*cos(b*x + a)^2*sin(b*x + a) - c*d*cos(b*x + a))) - 2*sqrt(2)*
sqrt(c*d)*d*arctan(-1/2*sqrt(2)*sqrt(c*d)*sqrt(d*cos(b*x + a))*sqrt(c*sin(
b*x + a))*(cos(b*x + a) - sin(b*x + a))/(c*d*cos(b*x + a)*sin(b*x + a))) +
 sqrt(2)*sqrt(c*d)*d*log(4*c*d*cos(b*x + a)*sin(b*x + a) + 2*sqrt(2)*sqrt(
c*d)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*(cos(b*x + a) + sin(b*x + a
)) + c*d) - sqrt(2)*sqrt(c*d)*d*log(4*c*d*cos(b*x + a)*sin(b*x + a) - 2*sq
rt(2)*sqrt(c*d)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*(cos(b*x + a) +
sin(b*x + a)) + c*d) - 16*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*d*sin(
b*x + a))/b
```

Sympy [F]

$$\int (d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}\,dx = \int \sqrt{c\sin{(a+bx)}}(d\cos{(a+bx)})^{\frac{3}{2}}\,dx$$

input
```
integrate((d*cos(b*x+a))**(3/2)*(c*sin(b*x+a))**(1/2),x)
```

output
```
Integral(sqrt(c*sin(a + b*x))*(d*cos(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int (d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}\,dx = \int (d\cos{(bx+a)})^{\frac{3}{2}}\sqrt{c\sin{(bx+a)}}\,dx$$

input
```
integrate((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*cos(b*x + a))^(3/2)*sqrt(c*sin(b*x + a)), x)
```

Giac [F]

$$\int (d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}\,dx = \int (d\cos(bx+a))^{\frac{3}{2}}\sqrt{c\sin(bx+a)}\,dx$$

input
```
integrate((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*cos(b*x + a))^(3/2)*sqrt(c*sin(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}\,dx = \int (d\cos(a+b\,x))^{3/2}\sqrt{c\sin(a+b\,x)}\,dx$$

input
```
int((d*cos(a + b*x))^(3/2)*(c*sin(a + b*x))^(1/2),x)
```

output
```
int((d*cos(a + b*x))^(3/2)*(c*sin(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int (d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}\,dx = \sqrt{d}\sqrt{c}\left(\int \sqrt{\sin(bx+a)}\sqrt{\cos(bx+a)}\cos(bx+a)\,dx\right)d$$

input
```
int((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(1/2),x)
```

output
```
sqrt(d)*sqrt(c)*int(sqrt(sin(a + b*x))*sqrt(cos(a + b*x))*cos(a + b*x),x)*
d
```

3.263 $\int \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 203

$$\int \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{\sqrt{c}\arctan\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}b\sqrt{d}} + \frac{\sqrt{c}\arctan\left(1+\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}b\sqrt{d}} - \frac{\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}\left(\sqrt{c}+\sqrt{c}\tan(a+bx)\right)}\right)}{\sqrt{2}b\sqrt{d}}$$

output
```
-1/2*c^(1/2)*arctan(1-2^(1/2)*d^(1/2)*(c*sin(b*x+a))^(1/2)/c^(1/2)/(d*cos(
b*x+a))^(1/2))*2^(1/2)/b/d^(1/2)+1/2*c^(1/2)*arctan(1+2^(1/2)*d^(1/2)*(c*s
in(b*x+a))^(1/2)/c^(1/2)/(d*cos(b*x+a))^(1/2))*2^(1/2)/b/d^(1/2)-1/2*c^(1/
2)*arctanh(2^(1/2)*d^(1/2)*(c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2)/(c^(1
/2)+c^(1/2)*tan(b*x+a)))*2^(1/2)/b/d^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.33

$$\int \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\,dx = \frac{2\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{4},\frac{7}{4},\sin^2(a+bx)\right)\sqrt{c\sin(a+bx)}\tan(a+bx)}{3b\sqrt{d\cos(a+bx)}}$$

input
```
Integrate[Sqrt[c*Sin[a + b*x]]/Sqrt[d*Cos[a + b*x]],x]
```

output
```
(2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/4, 3/4, 7/4, Sin[a + b*x]^2]
*Sqrt[c*Sin[a + b*x]]*Tan[a + b*x])/(3*b*Sqrt[d*Cos[a + b*x]])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.40, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3054, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}dx$$

$$\Big\downarrow \text{3054}$$

$$\frac{2cd\int \frac{c\tan(a+bx)}{d(\tan^2(a+bx)c^2+c^2)}d\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{b}$$

$$\Big\downarrow \text{826}$$

$$\frac{2cd\left(\frac{\int \frac{\tan(a+bx)c+c}{\tan^2(a+bx)c^2+c^2}d\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{2d}-\frac{\int \frac{c-c\tan(a+bx)}{\tan^2(a+bx)c^2+c^2}d\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{2d}\right)}{b}$$

$\downarrow$ 1476

$$\frac{2cd\left(\frac{\frac{\int \frac{1}{\frac{\tan(a+bx)c}{d}+\frac{c}{d}-\frac{\sqrt{2}\sqrt{c\sin(a+bx)}\sqrt{c}}{\sqrt{d}\sqrt{d\cos(a+bx)}}}d\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{2d}+\frac{\int \frac{1}{\frac{\tan(a+bx)c}{d}+\frac{c}{d}+\frac{\sqrt{2}\sqrt{c\sin(a+bx)}\sqrt{c}}{\sqrt{d}\sqrt{d\cos(a+bx)}}}d\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{2d}}{2d}-\frac{\int \frac{c-c\tan(a+bx)}{\tan^2(a+bx)c^2+c^2}d\frac{\sqrt{c\sin(a+}}{\sqrt{d\cos(a+}}}{2d}\right.}{b}$$

$\downarrow$ 1082

$$\frac{2cd\left(\frac{\frac{\int \frac{1}{-\frac{c\tan(a+bx)}{d}-1}d\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\int \frac{1}{-\frac{c\tan(a+bx)}{d}-1}d\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}}{2d}-\frac{\int \frac{c-c\tan(a+bx)}{\tan^2(a+bx)c^2+c^2}d\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{2d}\right)}{b}$$

$\downarrow$ 217

$$\frac{2cd\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}}{2d}-\frac{\int \frac{c-c\tan(a+bx)}{\tan^2(a+bx)c^2+c^2}d\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{2d}\right)}{b}$$

$\downarrow$ 1479

$$\frac{2cd\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}}{2d}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{c}-\frac{2\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{\sqrt{d}\left(\frac{\tan(a+bx)c}{d}+\frac{c}{d}-\frac{\sqrt{2}\sqrt{c\sin(a+bx)}\sqrt{c}}{\sqrt{d}\sqrt{d\cos(a+bx)}}\right)}d\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{2\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\int -\frac{}{\sqrt{d}\left(\cdots\right.}}{}}{2d}\right.}{b}$$

$\downarrow$ 25

$$\frac{2cd\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}}{2d}-\frac{\frac{\int\frac{\sqrt{2}\sqrt{c}-\frac{2\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{\sqrt{d}\left(\frac{\tan(a+bx)c}{d}+\frac{c}{d}-\frac{\sqrt{2}\sqrt{c\sin(a+bx)}\sqrt{c}}{\sqrt{d}\sqrt{d\cos(a+bx)}}\right)}d\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{2\sqrt{2}\sqrt{c}\sqrt{d}}+\cdots}{2d}\cdots\right)}{b}$$

$\downarrow$ 27

$$\frac{2cd\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}}{2d}-\frac{\frac{\int\frac{\sqrt{2}\sqrt{c}-\frac{2\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{\frac{\tan(a+bx)c}{d}+\frac{c}{d}-\frac{\sqrt{2}\sqrt{c\sin(a+bx)}\sqrt{c}}{\sqrt{d}\sqrt{d\cos(a+bx)}}}d\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{2\sqrt{2}\sqrt{c}d}+\cdots}{2d}\cdots\right)}{b}$$

$\downarrow$ 1103

$$\frac{2cd\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}}{2d}-\frac{\frac{\log\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+c\tan(a+bx)+c\right)}{2\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\log\left(-\frac{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\cdots\right)}{2\sqrt{2}\sqrt{c}\sqrt{d}}}{2d}\cdots\right)}{b}$$

input
```
Int[Sqrt[c*Sin[a + b*x]]/Sqrt[d*Cos[a + b*x]],x]
```

output
```
(2*c*d*((-(ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/(Sqrt[c]*Sqrt
[d*Cos[a + b*x]])]/(Sqrt[2]*Sqrt[c]*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*Sqrt[d
]*Sqrt[c*Sin[a + b*x]])/(Sqrt[c]*Sqrt[d*Cos[a + b*x]])]/(Sqrt[2]*Sqrt[c]*S
qrt[d]))/(2*d) - (-1/2*Log[c - (Sqrt[2]*Sqrt[c]*Sqrt[d]*Sqrt[c*Sin[a + b*x
]])/Sqrt[d*Cos[a + b*x]] + c*Tan[a + b*x]]/(Sqrt[2]*Sqrt[c]*Sqrt[d]) + Log
[c + (Sqrt[2]*Sqrt[c]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/Sqrt[d*Cos[a + b*x]] +
 c*Tan[a + b*x]]/(2*Sqrt[2]*Sqrt[c]*Sqrt[d]))/(2*d)))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3054
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k*a*(b/f)   Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(154) = 308$.

Time = 217.15 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.79

method	result
default	$\frac{\sqrt{2}\,\sqrt{c\sin(bx+a)}\left((1-\cos(bx+a))^2\csc(bx+a)^2-1\right)\left(\ln\left(\frac{(1-\cos(bx+a))^2\csc(bx+a)+2\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}\,\sin(bx+a)+2-2\cos(bx\cdots}{1-\cos(bx+a)}\right.\right.}{}$

input
```
int((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/b*2^(1/2)*(c*sin(b*x+a))^(1/2)*((1-cos(b*x+a))^2*csc(b*x+a)^2-1)*(ln(1
/(1-cos(b*x+a))*((1-cos(b*x+a))^2*csc(b*x+a)+2*(-2*sin(b*x+a)*cos(b*x+a)/(
cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)+2-2*cos(b*x+a)-sin(b*x+a)))-2*arctan(((-
2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)-cos(b*x+a)+1)/(
cos(b*x+a)-1))-ln(-1/(1-cos(b*x+a))*(-(1-cos(b*x+a))^2*csc(b*x+a)+2*(-2*si
n(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)-2+2*cos(b*x+a)+sin(
b*x+a)))-2*arctan(((-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b
*x+a)+cos(b*x+a)-1)/(cos(b*x+a)-1)))/(d*cos(b*x+a))^(1/2)/(-sin(b*x+a)*cos
(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 477 vs. $2(154) = 308$.

Time = 0.13 (sec) , antiderivative size = 477, normalized size of antiderivative = 2.35

$$\int \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\,dx =$$

$$-\frac{\sqrt{2}\sqrt{\frac{c}{d}}\arctan\left(\frac{2\,c\cos(bx+a)^3-2\,c\cos(bx+a)^2\sin(bx+a)+\sqrt{2}\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}\sqrt{\frac{c}{d}}-2\,c\cos(bx+a)}{2\left(c\cos(bx+a)^3+c\cos(bx+a)^2\sin(bx+a)-c\cos(bx+a)\right)}\right)+\sqrt{2}\sqrt{\frac{c}{d}}\,\mathrm{arct}}{}$$

input
```
integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
-1/8*(sqrt(2)*sqrt(c/d)*arctan(1/2*(2*c*cos(b*x + a)^3 - 2*c*cos(b*x + a)^
2*sin(b*x + a) + sqrt(2)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*sqrt(c/
d) - 2*c*cos(b*x + a))/(c*cos(b*x + a)^3 + c*cos(b*x + a)^2*sin(b*x + a) -
 c*cos(b*x + a))) + sqrt(2)*sqrt(c/d)*arctan(-1/2*(2*c*cos(b*x + a)^3 - 2*
c*cos(b*x + a)^2*sin(b*x + a) - sqrt(2)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*
x + a))*sqrt(c/d) - 2*c*cos(b*x + a))/(c*cos(b*x + a)^3 + c*cos(b*x + a)^2
*sin(b*x + a) - c*cos(b*x + a))) - 2*sqrt(2)*sqrt(c/d)*arctan(-1/2*sqrt(2)
*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*sqrt(c/d)*(cos(b*x + a) - sin(b
*x + a))/(c*cos(b*x + a)*sin(b*x + a))) + sqrt(2)*sqrt(c/d)*log(2*sqrt(2)*
sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*sqrt(c/d)*(cos(b*x + a) + sin(b*
x + a)) + 4*c*cos(b*x + a)*sin(b*x + a) + c) - sqrt(2)*sqrt(c/d)*log(-2*sq
rt(2)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*sqrt(c/d)*(cos(b*x + a) +
sin(b*x + a)) + 4*c*cos(b*x + a)*sin(b*x + a) + c))/b
```

Sympy [F]

$$\int \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{\sqrt{c\sin\left(a+bx\right)}}{\sqrt{d\cos\left(a+bx\right)}}\,dx$$

input
```
integrate((c*sin(b*x+a))**(1/2)/(d*cos(b*x+a))**(1/2),x)
```

output
```
Integral(sqrt(c*sin(a + b*x))/sqrt(d*cos(a + b*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{\sqrt{c\sin\left(bx+a\right)}}{\sqrt{d\cos\left(bx+a\right)}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*sin(b*x + a))/sqrt(d*cos(b*x + a)), x)
```

Giac [F]

$$\int \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{\sqrt{c\sin\left(bx+a\right)}}{\sqrt{d\cos\left(bx+a\right)}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*sin(b*x + a))/sqrt(d*cos(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{\sqrt{c\,\sin\left(a+b\,x\right)}}{\sqrt{d\,\cos\left(a+b\,x\right)}}\,dx$$

input
```
int((c*sin(a + b*x))^(1/2)/(d*cos(a + b*x))^(1/2),x)
```

output
```
int((c*sin(a + b*x))^(1/2)/(d*cos(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sin(bx+a)}\,\sqrt{\cos(bx+a)}}{\cos(bx+a)}dx\right)}{d}$$

input
```
int((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sin(a + b*x))*sqrt(cos(a + b*x)))/cos(a + b*x),
x))/d
```

3.264 $\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 37

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{5/2}}\,dx = \frac{2(c\sin(a+bx))^{3/2}}{3bcd(d\cos(a+bx))^{3/2}}$$

output
```
2/3*(c*sin(b*x+a))^(3/2)/b/c/d/(d*cos(b*x+a))^(3/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{5/2}}\,dx = \frac{2(c\sin(a+bx))^{3/2}}{3bcd(d\cos(a+bx))^{3/2}}$$

input
```
Integrate[Sqrt[c*Sin[a + b*x]]/(d*Cos[a + b*x])^(5/2),x]
```

output
```
(2*(c*Sin[a + b*x])^(3/2))/(3*b*c*d*(d*Cos[a + b*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3043}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{5/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{5/2}}dx$$
$$\Big\downarrow \text{3043}$$
$$\frac{2(c\sin(a+bx))^{3/2}}{3bcd(d\cos(a+bx))^{3/2}}$$

input
```
Int[Sqrt[c*Sin[a + b*x]]/(d*Cos[a + b*x])^(5/2),x]
```

output
```
(2*(c*Sin[a + b*x])^(3/2))/(3*b*c*d*(d*Cos[a + b*x])^(3/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3043
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Maple [A] (verified)

Time = 12.45 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{2\sqrt{c\sin(bx+a)}\,\tan(bx+a)}{3b\sqrt{d\cos(bx+a)}\,d^{2}}$	35

input
```
int((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/b*(c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2)/d^2*tan(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{5/2}}\,dx = \frac{2\,\sqrt{d\cos\left(bx+a\right)}\sqrt{c\sin\left(bx+a\right)}\sin\left(bx+a\right)}{3\,bd^{3}\cos\left(bx+a\right)^{2}}$$

input
```
integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*sin(b*x + a)/(b*d^3*cos(b*x
+ a)^2)
```

Sympy [F]

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{\sqrt{c\sin\left(a+bx\right)}}{\left(d\cos\left(a+bx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))**(1/2)/(d*cos(b*x+a))**(5/2),x)
```

output
```
Integral(sqrt(c*sin(a + b*x))/(d*cos(a + b*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{\sqrt{c\sin\left(bx+a\right)}}{\left(d\cos\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*sin(b*x + a))/(d*cos(b*x + a))^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{\sqrt{c\sin\left(bx+a\right)}}{\left(d\cos\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*sin(b*x + a))/(d*cos(b*x + a))^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 26.39 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.35

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{5/2}}\,dx = \frac{2\,\sin\left(2\,a+2\,b\,x\right)\,\sqrt{c\,\sin\left(a+b\,x\right)}}{3\,b\,d^2\,\left(\cos\left(2\,a+2\,b\,x\right)+1\right)\,\sqrt{d\,\cos\left(a+b\,x\right)}}$$

input
```
int((c*sin(a + b*x))^(1/2)/(d*cos(a + b*x))^(5/2),x)
```

output
```
(2*sin(2*a + 2*b*x)*(c*sin(a + b*x))^(1/2))/(3*b*d^2*(cos(2*a + 2*b*x) + 1
)*(d*cos(a + b*x))^(1/2))
```

Reduce [F]

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{5/2}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sin(bx+a)}\,\sqrt{\cos(bx+a)}}{\cos(bx+a)^3}dx\right)}{d^3}$$

input
```
int((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(5/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sin(a + b*x))*sqrt(cos(a + b*x)))/cos(a + b*x)*
*3,x))/d**3
```

3.265 $\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{9/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 75

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{9/2}}\,dx = \frac{2(c\sin(a+bx))^{3/2}}{7bcd(d\cos(a+bx))^{7/2}} + \frac{8(c\sin(a+bx))^{3/2}}{21bcd^3(d\cos(a+bx))^{3/2}}$$

output
```
2/7*(c*sin(b*x+a))^(3/2)/b/c/d/(d*cos(b*x+a))^(7/2)+8/21*(c*sin(b*x+a))^(3
/2)/b/c/d^3/(d*cos(b*x+a))^(3/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{9/2}}\,dx = \frac{2\sqrt{d\cos(a+bx)}(5+2\cos(2(a+bx)))\sec^4(a+bx)(c\sin(a+bx))^{3/2}}{21bcd^5}$$

input
```
Integrate[Sqrt[c*Sin[a + b*x]]/(d*Cos[a + b*x])^(9/2),x]
```

output
```
(2*Sqrt[d*Cos[a + b*x]]*(5 + 2*Cos[2*(a + b*x)])*Sec[a + b*x]^4*(c*Sin[a +
 b*x])^(3/2))/(21*b*c*d^5)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3051, 3042, 3043}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c \sin(a+bx)}}{(d \cos(a+bx))^{9/2}} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{c \sin(a+bx)}}{(d \cos(a+bx))^{9/2}}dx$$

$$\Big\downarrow \text{3051}$$

$$\frac{4 \int \frac{\sqrt{c \sin(a+bx)}}{(d \cos(a+bx))^{5/2}}dx}{7d^2}+\frac{2(c \sin(a+bx))^{3/2}}{7bcd(d \cos(a+bx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4 \int \frac{\sqrt{c \sin(a+bx)}}{(d \cos(a+bx))^{5/2}}dx}{7d^2}+\frac{2(c \sin(a+bx))^{3/2}}{7bcd(d \cos(a+bx))^{7/2}}$$

$$\Big\downarrow \text{3043}$$

$$\frac{8(c \sin(a+bx))^{3/2}}{21bcd^3(d \cos(a+bx))^{3/2}}+\frac{2(c \sin(a+bx))^{3/2}}{7bcd(d \cos(a+bx))^{7/2}}$$

input
```
Int[Sqrt[c*Sin[a + b*x]]/(d*Cos[a + b*x])^(9/2),x]
```

output
```
(2*(c*Sin[a + b*x])^(3/2))/(7*b*c*d*(d*Cos[a + b*x])^(7/2)) + (8*(c*Sin[a
+ b*x])^(3/2))/(21*b*c*d^3*(d*Cos[a + b*x])^(3/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3043
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

rule 3051
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Simp[(m + n + 2)/(a^2*(m + 1))   Int[(b*Sin[e + f*x
])^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m
, -1] && IntegersQ[2*m, 2*n]
```

Maple [A] (verified)

Time = 12.69 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.72

method	result	size
default	$\frac{2\sqrt{c\sin(bx+a)}\,\left(4\tan(bx+a)+3\sec(bx+a)^{2}\tan(bx+a)\right)}{21b\sqrt{d\cos(bx+a)}\,d^{4}}$	54

input
```
int((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/21/b*(c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2)/d^4*(4*tan(b*x+a)+3*sec(b
*x+a)^2*tan(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{9/2}}\,dx = \frac{2\sqrt{d\cos{(bx+a)}}\left(4\cos{(bx+a)}^2+3\right)\sqrt{c\sin{(bx+a)}}\sin{(bx+a)}}{21\,bd^5\cos{(bx+a)}^4}$$

input
```
integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(9/2),x, algorithm="fricas")
```

output
```
2/21*sqrt(d*cos(b*x + a))*(4*cos(b*x + a)^2 + 3)*sqrt(c*sin(b*x + a))*sin(
b*x + a)/(b*d^5*cos(b*x + a)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*sin(b*x+a))**(1/2)/(d*cos(b*x+a))**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{9/2}}\,dx = \int \frac{\sqrt{c\sin{(bx+a)}}}{(d\cos{(bx+a)})^{\frac{9}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(9/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*sin(b*x + a))/(d*cos(b*x + a))^(9/2), x)
```

Giac [F]

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{9/2}}\,dx = \int \frac{\sqrt{c\sin\left(bx+a\right)}}{\left(d\cos\left(bx+a\right)\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(9/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*sin(b*x + a))/(d*cos(b*x + a))^(9/2), x)
```

Mupad [B] (verification not implemented)

Time = 27.56 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{9/2}}\,dx = \frac{8\sqrt{c\,\sin\left(a+b\,x\right)}\left(11\,\sin\left(2\,a+2\,b\,x\right)+7\,\sin\left(4\,a+4\,b\,x\right)+\sin\left(6\,a+6\,b\,x\right)\right)}{21\,b\,d^4\,\sqrt{d\,\cos\left(a+b\,x\right)}\,\left(15\,\cos\left(2\,a+2\,b\,x\right)+6\,\cos\left(4\,a+4\,b\,x\right)+\cos\left(6\,a+6\,b\,x\right)+10\right)}$$

input
```
int((c*sin(a + b*x))^(1/2)/(d*cos(a + b*x))^(9/2),x)
```

output
```
(8*(c*sin(a + b*x))^(1/2)*(11*sin(2*a + 2*b*x) + 7*sin(4*a + 4*b*x) + sin(
6*a + 6*b*x)))/(21*b*d^4*(d*cos(a + b*x))^(1/2)*(15*cos(2*a + 2*b*x) + 6*c
os(4*a + 4*b*x) + cos(6*a + 6*b*x) + 10))
```

Reduce [F]

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{9/2}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sin(bx+a)}\,\sqrt{\cos(bx+a)}}{\cos(bx+a)^5}dx\right)}{d^5}$$

input
```
int((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(9/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sin(a + b*x))*sqrt(cos(a + b*x)))/cos(a + b*x)*
*5,x))/d**5
```

3.266 $\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{13/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 112

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{13/2}}\,dx = \frac{2(c\sin(a+bx))^{3/2}}{11bcd(d\cos(a+bx))^{11/2}} + \frac{16(c\sin(a+bx))^{3/2}}{77bcd^3(d\cos(a+bx))^{7/2}} + \frac{64(c\sin(a+bx))^{3/2}}{231bcd^5(d\cos(a+bx))^{3/2}}$$

output
```
2/11*(c*sin(b*x+a))^(3/2)/b/c/d/(d*cos(b*x+a))^(11/2)+16/77*(c*sin(b*x+a))
^(3/2)/b/c/d^3/(d*cos(b*x+a))^(7/2)+64/231*(c*sin(b*x+a))^(3/2)/b/c/d^5/(d
*cos(b*x+a))^(3/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{13/2}}\,dx = \frac{2\sqrt{d\cos(a+bx)}(45+28\cos(2(a+bx))+4\cos(4(a+bx)))\sec^6(a+bx)(c\sin}{231bcd^7}$$

input
```
Integrate[Sqrt[c*Sin[a + b*x]]/(d*Cos[a + b*x])^(13/2),x]
```

output
```
(2*Sqrt[d*Cos[a + b*x]]*(45 + 28*Cos[2*(a + b*x)] + 4*Cos[4*(a + b*x)])*Se
c[a + b*x]^6*(c*Sin[a + b*x])^(3/2))/(231*b*c*d^7)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 3051, 3042, 3051, 3042, 3043}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{13/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{13/2}}dx$$

$$\Big\downarrow \text{3051}$$

$$\frac{8\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{9/2}}dx}{11d^2}+\frac{2(c\sin(a+bx))^{3/2}}{11bcd(d\cos(a+bx))^{11/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{9/2}}dx}{11d^2}+\frac{2(c\sin(a+bx))^{3/2}}{11bcd(d\cos(a+bx))^{11/2}}$$

$$\Big\downarrow \text{3051}$$

$$\frac{8\left(\frac{4\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{5/2}}dx}{7d^2}+\frac{2(c\sin(a+bx))^{3/2}}{7bcd(d\cos(a+bx))^{7/2}}\right)}{11d^2}+\frac{2(c\sin(a+bx))^{3/2}}{11bcd(d\cos(a+bx))^{11/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8\left(\frac{4\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{5/2}}dx}{7d^2}+\frac{2(c\sin(a+bx))^{3/2}}{7bcd(d\cos(a+bx))^{7/2}}\right)}{11d^2}+\frac{2(c\sin(a+bx))^{3/2}}{11bcd(d\cos(a+bx))^{11/2}}$$

$$\Big\downarrow \text{3043}$$

$$\frac{8\left(\frac{8(c\sin(a+bx))^{3/2}}{21bcd^3(d\cos(a+bx))^{3/2}}+\frac{2(c\sin(a+bx))^{3/2}}{7bcd(d\cos(a+bx))^{7/2}}\right)}{11d^2}+\frac{2(c\sin(a+bx))^{3/2}}{11bcd(d\cos(a+bx))^{11/2}}$$

input
```
Int[Sqrt[c*Sin[a + b*x]]/(d*Cos[a + b*x])^(13/2),x]
```

output
```
(2*(c*Sin[a + b*x])^(3/2))/(11*b*c*d*(d*Cos[a + b*x])^(11/2)) + (8*((2*(c*
Sin[a + b*x])^(3/2))/(7*b*c*d*(d*Cos[a + b*x])^(7/2)) + (8*(c*Sin[a + b*x]
)^(3/2))/(21*b*c*d^3*(d*Cos[a + b*x])^(3/2))))/(11*d^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3043
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

rule 3051
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Simp[(m + n + 2)/(a^2*(m + 1))   Int[(b*Sin[e + f*x
])^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m
, -1] && IntegersQ[2*m, 2*n]
```

Maple [A] (verified)

Time = 12.33 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.58

method	result	size
default	$\frac{2\left(32\cos(bx+a)^4+24\cos(bx+a)^2+21\right)\sqrt{c\sin(bx+a)}\,\tan(bx+a)\sec(bx+a)^4}{231b\sqrt{d\cos(bx+a)}\,d^6}$	65

input
```
int((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(13/2),x,method=_RETURNVERBOSE)
```

output
```
2/231/b*(32*cos(b*x+a)^4+24*cos(b*x+a)^2+21)*(c*sin(b*x+a))^(1/2)/(d*cos(b
*x+a))^(1/2)/d^6*tan(b*x+a)*sec(b*x+a)^4
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{13/2}}\,dx = \frac{2\left(32\,\cos\left(bx+a\right)^{4}+24\,\cos\left(bx+a\right)^{2}+21\right)\sqrt{d\cos\left(bx+a\right)}\sqrt{c\sin\left(bx+a\right)}\sin}{231\,bd^{7}\cos\left(bx+a\right)^{6}}$$

input
```
integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(13/2),x, algorithm="fricas"
)
```

output
```
2/231*(32*cos(b*x + a)^4 + 24*cos(b*x + a)^2 + 21)*sqrt(d*cos(b*x + a))*sq
rt(c*sin(b*x + a))*sin(b*x + a)/(b*d^7*cos(b*x + a)^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{13/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*sin(b*x+a))**(1/2)/(d*cos(b*x+a))**(13/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{13/2}}\,dx = \int \frac{\sqrt{c\sin(bx+a)}}{(d\cos(bx+a))^{\frac{13}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(13/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(c*sin(b*x + a))/(d*cos(b*x + a))^(13/2), x)
```

Giac [F]

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{13/2}}\,dx = \int \frac{\sqrt{c\sin(bx+a)}}{(d\cos(bx+a))^{\frac{13}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(13/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*sin(b*x + a))/(d*cos(b*x + a))^(13/2), x)
```

Mupad [B] (verification not implemented)

Time = 32.91 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.93

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{13/2}}\,dx =$$

$$-\frac{\sqrt{c\,\sin\left(a+b\,x\right)}\,\left(2\sin\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2-1\right)\,\left(2\sin\left(\frac{5\,a}{2}+\frac{5\,b\,x}{2}\right)^2+\sin\left(5\,a+5\,b\,x\right)\,1\mathrm{i}-1\right)\,\left(\frac{1984\,\sin(a+b\,x)\,\left(-2\,\mathrm{si}\right.}{}\right.}{32\,\left(\sin\left(a+b\,x\right.\right.}$$

input
```
int((c*sin(a + b*x))^(1/2)/(d*cos(a + b*x))^(13/2),x)
```

output
```
-((c*sin(a + b*x))^(1/2)*(2*sin(a/2 + (b*x)/2)^2 - 1)*(sin(5*a + 5*b*x)*1i
 + 2*sin((5*a)/2 + (5*b*x)/2)^2 - 1)*((1984*sin(a + b*x)*(sin(5*a + 5*b*x)
*1i - 2*sin((5*a)/2 + (5*b*x)/2)^2 + 1))/(231*b*d^6) + (256*sin(3*a + 3*b*
x)*(sin(5*a + 5*b*x)*1i - 2*sin((5*a)/2 + (5*b*x)/2)^2 + 1))/(77*b*d^6) +
(128*sin(5*a + 5*b*x)*(sin(5*a + 5*b*x)*1i - 2*sin((5*a)/2 + (5*b*x)/2)^2
+ 1))/(231*b*d^6)))/(32*(sin(a + b*x)^2 - 1)^3*(-d*(2*sin(a/2 + (b*x)/2)^2
 - 1))^(1/2))
```

Reduce [F]

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{13/2}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sin(bx+a)}\,\sqrt{\cos(bx+a)}}{\cos(bx+a)^7}dx\right)}{d^7}$$

input
```
int((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(13/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sin(a + b*x))*sqrt(cos(a + b*x)))/cos(a + b*x)*
*7,x))/d**7
```

3.267 $\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 131

$$\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}\,dx = \frac{cd\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}{6b} - \frac{c(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}{3bd} + \frac{c^2d^2\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}}{12b\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}$$

output
```
1/6*c*d*(d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2)/b-1/3*c*(d*cos(b*x+a))^(
5/2)*(c*sin(b*x+a))^(1/2)/b/d+1/12*c^2*d^2*InverseJacobiAM(a-1/4*Pi+b*x,2^
(1/2))*sin(2*b*x+2*a)^(1/2)/b/(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.54

$$\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}\,dx = \frac{2cd\sqrt{d\cos(a+bx)}\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{5}{4},\frac{9}{4},\sin^2(a+b$$
$$5b$$

input
```
Integrate[(d*Cos[a + b*x])^(3/2)*(c*Sin[a + b*x])^(3/2),x]
```

output
```
(2*c*d*Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-1/4,
 5/4, 9/4, Sin[a + b*x]^2]*Sqrt[c*Sin[a + b*x]]*Tan[a + b*x]^2)/(5*b)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3048, 3042, 3049, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{1}{6}c^2\int \frac{(d\cos(a+bx))^{3/2}}{\sqrt{c\sin(a+bx)}}dx-\frac{c\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{5/2}}{3bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}c^2\int \frac{(d\cos(a+bx))^{3/2}}{\sqrt{c\sin(a+bx)}}dx-\frac{c\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{5/2}}{3bd}$$

$$\Big\downarrow \text{3049}$$

$$\frac{1}{6}c^2\left(\frac{1}{2}d^2\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}dx+\frac{d\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{bc}\right)-\frac{c\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{5/2}}{3bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{6}c^2\left(\frac{1}{2}d^2\int\frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}dx+\frac{d\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{bc}\right)-\frac{c\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{5/2}}{3bd}$$

$\Big\downarrow$ 3053

$$\frac{1}{6}c^2\left(\frac{d^2\sqrt{\sin(2a+2bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx}{2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}+\frac{d\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{bc}\right)-\frac{c\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{5/2}}{3bd}$$

$\Big\downarrow$ 3042

$$\frac{1}{6}c^2\left(\frac{d^2\sqrt{\sin(2a+2bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx}{2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}+\frac{d\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{bc}\right)-\frac{c\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{5/2}}{3bd}$$

$\Big\downarrow$ 3120

$$\frac{1}{6}c^2\left(\frac{d^2\sqrt{\sin(2a+2bx)}\operatorname{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{2b\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}+\frac{d\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{bc}\right)-\frac{c\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{5/2}}{3bd}$$

input
```
Int[(d*Cos[a + b*x])^(3/2)*(c*Sin[a + b*x])^(3/2),x]
```

output
```
-1/3*(c*(d*Cos[a + b*x])^(5/2)*Sqrt[c*Sin[a + b*x]])/(b*d) + (c^2*((d*Sqrt
[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]])/(b*c) + (d^2*EllipticF[a - Pi/4 + b
*x, 2]*Sqrt[Sin[2*a + 2*b*x]])/(2*b*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*
x]])))/6
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3049
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Sin[e + f*x])^n*(a
*Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 162.14 (sec) , antiderivative size = 993, normalized size of antiderivative = 7.58

method	result	size
default	Expression too large to display	993

input
```
int((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/384/b*c*d*(sin(b*x+a)*cos(b*x+a)*(16*cos(b*x+a)^2-8)+(3*cos(b*x+a)+3)*(
-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*ln(-(cos(b*x+a)*cot(b*x+a
)-2*cot(b*x+a)+2*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x
+a)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(cos(b*x+a)-1))+(-3*cos(b*x+a)-3
)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*ln(-(cos(b*x+a)*cot(b*
x+a)-2*cot(b*x+a)-2*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(
b*x+a)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(cos(b*x+a)-1))+(6*cos(b*x+a)
+6)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*arctan(((-2*sin(b*x+
a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)+cos(b*x+a)-1)/(cos(b*x+a)
-1))+(-6*cos(b*x+a)-6)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*a
rctan((-(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)+cos(b
*x+a)-1)/(cos(b*x+a)-1))+(8*cos(b*x+a)+8)*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)
*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*Ellipti
cF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))+(-6*cos(b*x+a)-6)*(-cot(b
*x+a)+csc(b*x+a)+1)^(1/2)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+a)-
csc(b*x+a))^(1/2)*EllipticPi((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2-1/2*I,1/
2*2^(1/2))+(-6*cos(b*x+a)-6)*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(2*cot(b*x+a
)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticPi((-cot(b*x
+a)+csc(b*x+a)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))+I*(-6*cos(b*x+a)-6)*(-cot(b
*x+a)+csc(b*x+a)+1)^(1/2)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+...
```

Fricas [F]

$$\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}\,dx = \int (d\cos(bx+a))^{\frac{3}{2}}\,(c\sin(bx+a))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*c*d*cos(b*x + a)*sin(b*
x + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((d*cos(b*x+a))**(3/2)*(c*sin(b*x+a))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}\,dx = \int (d\cos(bx+a))^{\frac{3}{2}}(c\sin(bx+a))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*cos(b*x + a))^(3/2)*(c*sin(b*x + a))^(3/2), x)
```

Giac [F]

$$\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}\,dx = \int (d\cos(bx+a))^{\frac{3}{2}}(c\sin(bx+a))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*cos(b*x + a))^(3/2)*(c*sin(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}\,dx = \int (d\,\cos(a+b\,x))^{3/2}\,(c\,\sin(a+b\,x))^{3/2}\,dx$$

input
```
int((d*cos(a + b*x))^(3/2)*(c*sin(a + b*x))^(3/2),x)
```

output
```
int((d*cos(a + b*x))^(3/2)*(c*sin(a + b*x))^(3/2), x)
```

Reduce [F]

$$\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}\,dx = \sqrt{d}\,\sqrt{c}\left(\int \sqrt{\sin(bx+a)}\,\sqrt{\cos(bx+a)}\,\cos(bx+a)\sin(bx+a)\,dx\right)cd$$

input
```
int((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(3/2),x)
```

output
```
sqrt(d)*sqrt(c)*int(sqrt(sin(a + b*x))*sqrt(cos(a + b*x))*cos(a + b*x)*sin
(a + b*x),x)*c*d
```

3.268 $\int \frac{(c\sin(a+bx))^{3/2}}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 93

$$\int \frac{(c\sin(a+bx))^{3/2}}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{c\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}{bd} + \frac{c^2\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}}{2b\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}$$

output
```
-c*(d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2)/b/d+1/2*c^2*InverseJacobiAM(a
-1/4*Pi+b*x,2^(1/2))*sin(2*b*x+2*a)^(1/2)/b/(d*cos(b*x+a))^(1/2)/(c*sin(b*
x+a))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.72

$$\int \frac{(c\sin(a+bx))^{3/2}}{\sqrt{d\cos(a+bx)}}\,dx = \frac{2\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{9}{4},\sin^2(a+bx)\right)(c\sin(a+bx))^{3/2}\,\text{ta}}{5b\sqrt{d\cos(a+bx)}}$$

input
```
Integrate[(c*Sin[a + b*x])^(3/2)/Sqrt[d*Cos[a + b*x]],x]
```

output
```
(2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/4, 5/4, 9/4, Sin[a + b*x]^2]
*(c*Sin[a + b*x])^(3/2)*Tan[a + b*x])/(5*b*Sqrt[d*Cos[a + b*x]])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3048, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c\sin(a+bx))^{3/2}}{\sqrt{d\cos(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c\sin(a+bx))^{3/2}}{\sqrt{d\cos(a+bx)}}dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{1}{2}c^2\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}dx-\frac{c\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}c^2\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}dx-\frac{c\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{bd}$$

$$\Big\downarrow \text{3053}$$

$$\frac{c^2\sqrt{\sin(2a+2bx)}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx}{2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}-\frac{c\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{c^2\sqrt{\sin(2a+2bx)}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx}{2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}-\frac{c\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{bd}$$

$$\Big\downarrow \text{3120}$$

$$\frac{c^2\sqrt{\sin(2a+2bx)}\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{2b\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}-\frac{c\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{bd}$$

input
```
Int[(c*Sin[a + b*x])^(3/2)/Sqrt[d*Cos[a + b*x]],x]
```

output
```
-((c*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]])/(b*d)) + (c^2*EllipticF[a
- Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]])/(2*b*Sqrt[d*Cos[a + b*x]]*Sqrt[c*
Sin[a + b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 13.93 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.43

method	result
default	$-\frac{\sqrt{c \sin(bx+a)}\, c \left(\text{EllipticF}\left(\sqrt{-\cot(bx+a)+\csc(bx+a)+1}, \frac{\sqrt{2}}{2}\right) \sqrt{-\cot(bx+a)+\csc(bx+a)+1}\, \sqrt{2\cot(bx+a)-2\csc(bx+a)+2}\, \sqrt{c}\right.}{2b\sqrt{d\cos(bx+a)}}$

input
```
int((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b*(c*sin(b*x+a))^(1/2)*c/(d*cos(b*x+a))^(1/2)*(EllipticF((-cot(b*x+a)
+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(2*cot(
b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-cot(b*x+a)-cs
c(b*x+a))+2*cos(b*x+a))
```

Fricas [F]

$$\int \frac{(c\sin(a+bx))^{3/2}}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{3}{2}}}{\sqrt{d\cos(bx+a)}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*c*sin(b*x + a)/(d*cos(b
*x + a)), x)
```

Sympy [F]

$$\int \frac{(c\sin(a+bx))^{3/2}}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{(c\sin(a+bx))^{\frac{3}{2}}}{\sqrt{d\cos(a+bx)}}\,dx$$

input
```
integrate((c*sin(b*x+a))**(3/2)/(d*cos(b*x+a))**(1/2),x)
```

output
```
Integral((c*sin(a + b*x))**(3/2)/sqrt(d*cos(a + b*x)), x)
```

Maxima [F]

$$\int \frac{(c\sin(a+bx))^{3/2}}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{3}{2}}}{\sqrt{d\cos(bx+a)}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a))^(3/2)/sqrt(d*cos(b*x + a)), x)
```

Giac [F]

$$\int \frac{(c\sin(a+bx))^{3/2}}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{3}{2}}}{\sqrt{d\cos(bx+a)}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a))^(3/2)/sqrt(d*cos(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{3/2}}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{(c\,\sin(a+b\,x))^{3/2}}{\sqrt{d\,\cos(a+b\,x)}}\,dx$$

input
```
int((c*sin(a + b*x))^(3/2)/(d*cos(a + b*x))^(1/2),x)
```

output
```
int((c*sin(a + b*x))^(3/2)/(d*cos(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(c\sin(a+bx))^{3/2}}{\sqrt{d\cos(a+bx)}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sin(bx+a)}\,\sqrt{\cos(bx+a)}\,\sin(bx+a)}{\cos(bx+a)}dx\right)c}{d}$$

input
```
int((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(1/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sin(a + b*x))*sqrt(cos(a + b*x))*sin(a + b*x))/
cos(a + b*x),x)*c)/d
```

3.269 $\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 98

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{5/2}}\,dx = \frac{2c\sqrt{c\sin(a+bx)}}{3bd(d\cos(a+bx))^{3/2}} - \frac{c^2\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}}{3bd^2\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}$$

output
```
2/3*c*(c*sin(b*x+a))^(1/2)/b/d/(d*cos(b*x+a))^(3/2)-1/3*c^2*InverseJacobiA
M(a-1/4*Pi+b*x,2^(1/2))*sin(2*b*x+2*a)^(1/2)/b/d^2/(d*cos(b*x+a))^(1/2)/(c
*sin(b*x+a))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.18 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.68

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{5/2}}\,dx = \frac{2\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{7}{4},\frac{9}{4},\sin^2(a+bx)\right)(c\sin(a+bx))^{5/2}}{5bcd(d\cos(a+bx))^{3/2}}$$

input
```
Integrate[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(5/2),x]
```

output
```
(2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[5/4, 7/4, 9/4, Sin[a + b*x]^2]
*(c*Sin[a + b*x])^(5/2))/(5*b*c*d*(d*Cos[a + b*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3046, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3046}$$

$$\frac{2c\sqrt{c\sin(a+bx)}}{3bd(d\cos(a+bx))^{3/2}}-\frac{c^2\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}dx}{3d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c\sqrt{c\sin(a+bx)}}{3bd(d\cos(a+bx))^{3/2}}-\frac{c^2\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}dx}{3d^2}$$

$$\Big\downarrow \text{3053}$$

$$\frac{2c\sqrt{c\sin(a+bx)}}{3bd(d\cos(a+bx))^{3/2}}-\frac{c^2\sqrt{\sin(2a+2bx)}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx}{3d^2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c\sqrt{c\sin(a+bx)}}{3bd(d\cos(a+bx))^{3/2}}-\frac{c^2\sqrt{\sin(2a+2bx)}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx}{3d^2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2c\sqrt{c\sin(a+bx)}}{3bd(d\cos(a+bx))^{3/2}}-\frac{c^2\sqrt{\sin(2a+2bx)}\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{3bd^2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}$$

input
```
Int[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(5/2),x]
```

output
```
(2*c*Sqrt[c*Sin[a + b*x]])/(3*b*d*(d*Cos[a + b*x])^(3/2)) - (c^2*EllipticF
[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]])/(3*b*d^2*Sqrt[d*Cos[a + b*x]]*
Sqrt[c*Sin[a + b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3046
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Sin[e + f
*x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 11.27 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.39

method	result
default	$\frac{\sqrt{c\sin(bx+a)}\, c\left(\operatorname{EllipticF}\left(\sqrt{-\cot(bx+a)+\csc(bx+a)+1}, \frac{\sqrt{2}}{2}\right)\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\,\sqrt{2\cot(bx+a)-2\csc(bx+a)+2}\,\sqrt{\cot \cdots}\right.}{3b\sqrt{d\cos(bx+a)}\, d^2}$

input
```
int((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/b*(c*sin(b*x+a))^(1/2)*c/(d*cos(b*x+a))^(1/2)/d^2*(EllipticF((-cot(b*x
+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(2*c
ot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*(-cot(b*x+a)
-csc(b*x+a))+2*sec(b*x+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.10

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{5/2}}\,dx = \frac{\sqrt{i\,cd}c\cos{(bx+a)}^2 F(\arcsin{(\cos{(bx+a)}+i\,\sin{(bx+a)})}\,|-1)+\sqrt{-i\,cd}c\cos \cdots}{3\,b \cdots}$$

input
```
integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(I*c*d)*c*cos(b*x + a)^2*elliptic_f(arcsin(cos(b*x + a) + I*sin(b
*x + a)), -1) + sqrt(-I*c*d)*c*cos(b*x + a)^2*elliptic_f(arcsin(cos(b*x +
a) - I*sin(b*x + a)), -1) + 2*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*c)
/(b*d^3*cos(b*x + a)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*sin(b*x+a))**(3/2)/(d*cos(b*x+a))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{3}{2}}}{(d\cos(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a))^(3/2)/(d*cos(b*x + a))^(5/2), x)
```

Giac [F]

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{3}{2}}}{(d\cos(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a))^(3/2)/(d*cos(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{\left(c\,\sin\left(a+b\,x\right)\right)^{3/2}}{\left(d\,\cos\left(a+b\,x\right)\right)^{5/2}}\,dx$$

input
```
int((c*sin(a + b*x))^(3/2)/(d*cos(a + b*x))^(5/2),x)
```

output
```
int((c*sin(a + b*x))^(3/2)/(d*cos(a + b*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{5/2}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\,c\left(-\cos\left(bx+a\right)^{2}\left(\int \frac{\sqrt{\sin(bx+a)}\,\sqrt{\cos(bx+a)}}{\cos(bx+a)\sin(bx+a)}dx\right)b+2\sqrt{\sin\left(bx+a\right)}\,\sqrt{\cos\left(b\right.}\right.}{3\cos\left(bx+a\right)^{2}b\,d^{3}}$$

input
```
int((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(5/2),x)
```

output
```
(sqrt(d)*sqrt(c)*c*( - cos(a + b*x)**2*int((sqrt(sin(a + b*x))*sqrt(cos(a
+ b*x)))/(cos(a + b*x)*sin(a + b*x)),x)*b + 2*sqrt(sin(a + b*x))*sqrt(cos(
a + b*x))))/(3*cos(a + b*x)**2*b*d**3)
```

3.270 $\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{9/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 133

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{9/2}}\,dx = \frac{2c\sqrt{c\sin(a+bx)}}{7bd(d\cos(a+bx))^{7/2}} - \frac{2c\sqrt{c\sin(a+bx)}}{21bd^3(d\cos(a+bx))^{3/2}} - \frac{2c^2\,\mathrm{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}}{21bd^4\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}$$

output
```
2/7*c*(c*sin(b*x+a))^(1/2)/b/d/(d*cos(b*x+a))^(7/2)-2/21*c*(c*sin(b*x+a))^
(1/2)/b/d^3/(d*cos(b*x+a))^(3/2)-2/21*c^2*InverseJacobiAM(a-1/4*Pi+b*x,2^(
1/2))*sin(2*b*x+2*a)^(1/2)/b/d^4/(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.53

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{9/2}}\,dx = \frac{2\cos^2(a+bx)^{7/4}\cot(a+bx)\,\mathrm{Hypergeometric2F1}\left(\frac{5}{4},\frac{11}{4},\frac{9}{4},\sin^2(a+bx)\right)(c\sin(a}{5bc^2(d\cos(a+bx))^{9/2}}$$

input
```
Integrate[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(9/2),x]
```

output
```
(2*(Cos[a + b*x]^2)^(7/4)*Cot[a + b*x]*Hypergeometric2F1[5/4, 11/4, 9/4, S
in[a + b*x]^2]*(c*Sin[a + b*x])^(7/2))/(5*b*c^2*(d*Cos[a + b*x])^(9/2))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3046, 3042, 3051, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{9/2}}dx$$

$$\Big\downarrow \text{3046}$$

$$\frac{2c\sqrt{c\sin(a+bx)}}{7bd(d\cos(a+bx))^{7/2}}-\frac{c^2\int \frac{1}{(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}dx}{7d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c\sqrt{c\sin(a+bx)}}{7bd(d\cos(a+bx))^{7/2}}-\frac{c^2\int \frac{1}{(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}dx}{7d^2}$$

$$\Big\downarrow \text{3051}$$

$$\frac{2c\sqrt{c\sin(a+bx)}}{7bd(d\cos(a+bx))^{7/2}}-\frac{c^2\left(\frac{2\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}dx}{3d^2}+\frac{2\sqrt{c\sin(a+bx)}}{3bcd(d\cos(a+bx))^{3/2}}\right)}{7d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c\sqrt{c\sin(a+bx)}}{7bd(d\cos(a+bx))^{7/2}}-\frac{c^2\left(\frac{2\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}dx}{3d^2}+\frac{2\sqrt{c\sin(a+bx)}}{3bcd(d\cos(a+bx))^{3/2}}\right)}{7d^2}$$

$$\Big\downarrow \text{3053}$$

$$\frac{2c\sqrt{c\sin(a+bx)}}{7bd(d\cos(a+bx))^{7/2}}-\frac{c^2\left(\frac{2\sqrt{\sin(2a+2bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx}{3d^2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}+\frac{2\sqrt{c\sin(a+bx)}}{3bcd(d\cos(a+bx))^{3/2}}\right)}{7d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c\sqrt{c\sin(a+bx)}}{7bd(d\cos(a+bx))^{7/2}}-\frac{c^2\left(\frac{2\sqrt{\sin(2a+2bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx}{3d^2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}+\frac{2\sqrt{c\sin(a+bx)}}{3bcd(d\cos(a+bx))^{3/2}}\right)}{7d^2}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2c\sqrt{c\sin(a+bx)}}{7bd(d\cos(a+bx))^{7/2}}-\frac{c^2\left(\frac{2\sqrt{\sin(2a+2bx)}\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{3bd^2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}+\frac{2\sqrt{c\sin(a+bx)}}{3bcd(d\cos(a+bx))^{3/2}}\right)}{7d^2}$$

input
```
Int[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(9/2),x]
```

output
```
(2*c*Sqrt[c*Sin[a + b*x]])/(7*b*d*(d*Cos[a + b*x])^(7/2)) - (c^2*((2*Sqrt[
c*Sin[a + b*x]])/(3*b*c*d*(d*Cos[a + b*x])^(3/2)) + (2*EllipticF[a - Pi/4
+ b*x, 2]*Sqrt[Sin[2*a + 2*b*x]])/(3*b*d^2*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin
[a + b*x]])))/(7*d^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3046
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Sin[e + f
*x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3051
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Simp[(m + n + 2)/(a^2*(m + 1))   Int[(b*Sin[e + f*x
])^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m
, -1] && IntegersQ[2*m, 2*n]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 11.47 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.05

method	result
default	$-\frac{2\sqrt{c\sin(bx+a)}\, c\left(\sec(bx+a)-3\sec(bx+a)^3+\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\ \operatorname{EllipticF}\left(\sqrt{-\cot(bx+a)+\csc(bx+a)+1}, \frac{\sqrt{2}}{2}\right)\sqrt{2\cot(bx+a)-2\csc(bx+a)+2}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,(\cot(bx+a)+\csc(bx+a))\right)}{21b\sqrt{d\cos(bx+a)}\, d^4}$

input
```
int((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21/b*(c*sin(b*x+a))^(1/2)*c/(d*cos(b*x+a))^(1/2)/d^4*(sec(b*x+a)-3*sec(
b*x+a)^3+(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a
)+1)^(1/2),1/2*2^(1/2))*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+a)-cs
c(b*x+a))^(1/2)*(cot(b*x+a)+csc(b*x+a)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.91

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{9/2}}\,dx = \frac{2\left(\sqrt{i\,cd}c\cos{(bx+a)}^4 F(\arcsin{(\cos{(bx+a)}+i\,\sin{(bx+a)})}\,|-1)+\sqrt{-i\,cd}c\right.}{}$$

input
```
integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(9/2),x, algorithm="fricas")
```

output
```
2/21*(sqrt(I*c*d)*c*cos(b*x + a)^4*elliptic_f(arcsin(cos(b*x + a) + I*sin(
b*x + a)), -1) + sqrt(-I*c*d)*c*cos(b*x + a)^4*elliptic_f(arcsin(cos(b*x +
 a) - I*sin(b*x + a)), -1) - (c*cos(b*x + a)^2 - 3*c)*sqrt(d*cos(b*x + a))
*sqrt(c*sin(b*x + a)))/(b*d^5*cos(b*x + a)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*sin(b*x+a))**(3/2)/(d*cos(b*x+a))**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{9/2}}\,dx = \int \frac{(c\sin{(bx+a)})^{\frac{3}{2}}}{(d\cos{(bx+a)})^{\frac{9}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(9/2),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a))^(3/2)/(d*cos(b*x + a))^(9/2), x)
```

Giac [F]

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{9/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{3}{2}}}{(d\cos(bx+a))^{\frac{9}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(9/2),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a))^(3/2)/(d*cos(b*x + a))^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{9/2}}\,dx = \int \frac{(c\,\sin(a+b\,x))^{3/2}}{(d\,\cos(a+b\,x))^{9/2}}\,dx$$

input
```
int((c*sin(a + b*x))^(3/2)/(d*cos(a + b*x))^(9/2),x)
```

output
```
int((c*sin(a + b*x))^(3/2)/(d*cos(a + b*x))^(9/2), x)
```

Reduce [F]

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{9/2}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\,c\left(-\cos(bx+a)^4\left(\int \frac{\sqrt{\sin(bx+a)}\,\sqrt{\cos(bx+a)}}{\cos(bx+a)^3\sin(bx+a)}dx\right)b+2\sqrt{\sin(bx+a)}\,\sqrt{\cos(b}\right.}{7\cos(bx+a)^4\,b\,d^5}$$

input
```
int((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(9/2),x)
```

output

```
(sqrt(d)*sqrt(c)*c*( - cos(a + b*x)**4*int((sqrt(sin(a + b*x))*sqrt(cos(a
+ b*x)))/(cos(a + b*x)**3*sin(a + b*x)),x)*b + 2*sqrt(sin(a + b*x))*sqrt(c
os(a + b*x))))/(7*cos(a + b*x)**4*b*d**5)
```

3.271 $\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 245

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}\,dx = \frac{c^{3/2}\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{4\sqrt{2}b} - \frac{c^{3/2}\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{4\sqrt{2}b} + \frac{c^{3/2}\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\left(\sqrt{d}+\sqrt{d}\cot(a+bx)\right)\sqrt{c\sin(a+bx)}}\right)}{4\sqrt{2}b} - \frac{c(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}{2bd}$$

output
```
-1/8*c^(3/2)*d^(1/2)*arctan(-1+2^(1/2)*c^(1/2)*(d*cos(b*x+a))^(1/2)/d^(1/2
)/(c*sin(b*x+a))^(1/2))*2^(1/2)/b-1/8*c^(3/2)*d^(1/2)*arctan(1+2^(1/2)*c^(
1/2)*(d*cos(b*x+a))^(1/2)/d^(1/2)/(c*sin(b*x+a))^(1/2))*2^(1/2)/b+1/8*c^(3
/2)*d^(1/2)*arctanh(2^(1/2)*c^(1/2)*(d*cos(b*x+a))^(1/2)/(d^(1/2)+d^(1/2)*
cot(b*x+a))/(c*sin(b*x+a))^(1/2))*2^(1/2)/b-1/2*c*(d*cos(b*x+a))^(3/2)*(c*
sin(b*x+a))^(1/2)/b/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.27

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}\,dx = \frac{2\sqrt{d\cos(a+bx)}\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{5}{4},\frac{9}{4},\sin^2(a+bx)\right)(c\sin(a+bx))^{3/2}}{5b}$$

input
```
Integrate[Sqrt[d*Cos[a + b*x]]*(c*Sin[a + b*x])^(3/2),x]
```

output
```
(2*Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, 5/4,
 9/4, Sin[a + b*x]^2]*(c*Sin[a + b*x])^(3/2)*Tan[a + b*x])/(5*b)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.33, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 3048, 3042, 3055, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\sin(a+bx))^{3/2}\sqrt{d\cos(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c\sin(a+bx))^{3/2}\sqrt{d\cos(a+bx)}dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{1}{4}c^2\int \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}dx - \frac{c\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{3/2}}{2bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}c^2\int \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}dx-\frac{c\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{3/2}}{2bd}$$

$\big\downarrow$ 3055

$$-\frac{c^3d\int \frac{d\cot(a+bx)}{c(\cot^2(a+bx)d^2+d^2)}d\frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}}{2b}-\frac{c\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{3/2}}{2bd}$$

$\big\downarrow$ 826

$$-\frac{c^3d\left(\frac{\int\frac{\cot(a+bx)d+d}{\cot^2(a+bx)d^2+d^2}d\frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}}{2c}-\frac{\int\frac{d-d\cot(a+bx)}{\cot^2(a+bx)d^2+d^2}d\frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}}{2c}\right)}{2b}-\frac{c\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{3/2}}{2bd}$$

$\big\downarrow$ 1476

$$-\frac{c^3d\left(\frac{\frac{\int\frac{1}{\frac{\cot(a+bx)d}{c}+\frac{d}{c}-\frac{\sqrt{2}\sqrt{d\cos(a+bx)}\sqrt{d}}{\sqrt{c}\sqrt{c\sin(a+bx)}}}d\frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}}{2c}+\frac{\int\frac{1}{\frac{\cot(a+bx)d}{c}+\frac{d}{c}+\frac{\sqrt{2}\sqrt{d\cos(a+bx)}\sqrt{d}}{\sqrt{c}\sqrt{c\sin(a+bx)}}}d\frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}}{2c}}{2c}-\frac{\int\frac{d-d\cot(a+bx)}{\cot^2(a+bx)d^2+d^2}d\frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}}{2c}\right)}{2b}-\frac{c\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{3/2}}{2bd}$$

$\big\downarrow$ 1082

$$-\frac{c^3d\left(\frac{\frac{\int\frac{1}{-\frac{d\cot(a+bx)}{c}-1}d\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\int\frac{1}{-\frac{d\cot(a+bx)}{c}-1}d\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}+1\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}}{2c}-\frac{\int\frac{d-d\cot(a+bx)}{\cot^2(a+bx)d^2+d^2}d\frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}}{2c}\right)}{2b}-\frac{c\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{3/2}}{2bd}$$

$\big\downarrow$ 217

$$-\frac{c^3d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}+1\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}}{2c}-\frac{\int\frac{d-d\cot(a+bx)}{\cot^2(a+bx)d^2+d^2}d\frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}}{2c}\right)}{2b}-\frac{c\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{3/2}}{2bd}$$

$\big\downarrow$ 1479

$$-\frac{c^3 d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}+1\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}}{2c}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-\frac{2\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}}{\sqrt{c}\left(\frac{\cot(a+bx)d}{c}+\frac{d}{c}-\frac{\sqrt{2}\sqrt{d\cos(a+bx)}\sqrt{d}}{\sqrt{c}\sqrt{c\sin(a+bx)}}\right)}d\frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}}{2\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\int -\cdots}{\cdots}}{2c}\cdots\right)}{2b\cdots}$$

$$\frac{c\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{3/2}}{2bd}$$

$$\Big\downarrow 25$$

$$-\frac{c^3 d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}+1\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}}{2c}-\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-\frac{2\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}}{\sqrt{c}\left(\frac{\cot(a+bx)d}{c}+\frac{d}{c}-\frac{\sqrt{2}\sqrt{d\cos(a+bx)}\sqrt{d}}{\sqrt{c}\sqrt{c\sin(a+bx)}}\right)}d\frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}}{2\sqrt{2}\sqrt{c}\sqrt{d}}+\frac{\int \frac{\sqrt{\cdots}}{\sqrt{c}\left(\cot\cdots\right)}\cdots}{\cdots}}{2c}\cdots\right)}{2b\cdots}$$

$$\frac{c\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{3/2}}{2bd}$$

$$\Big\downarrow 27$$

$$-\frac{c^3 d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}+1\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}}{2c}-\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-\frac{2\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}}{\frac{\cot(a+bx)d}{c}+\frac{d}{c}-\frac{\sqrt{2}\sqrt{d\cos(a+bx)}\sqrt{d}}{\sqrt{c}\sqrt{c\sin(a+bx)}}}d\frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}}{2\sqrt{2}c\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\cdots}{\frac{\cot(a+bx)d}{c}+\cdots}\cdots}{\cdots}}{2c}\cdots\right)}{2b\cdots}$$

$$\frac{c\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{3/2}}{2bd}$$

$$\Big\downarrow 1103$$

$$-\frac{c^3 d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}+1\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}}{2c}-\frac{\frac{\log\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}+d\cot(a+bx)+d\right)}{2\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\log\left(-\frac{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d\cos(a+b\cdots}}{\sqrt{c\sin(a+bx)}}\cdots\right)}{2\sqrt{2}\sqrt{c}\sqrt{\cdots}}}{2c}\cdots\right)}{2b\cdots}$$

$$\frac{c\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{3/2}}{2bd}$$

input
```
Int[Sqrt[d*Cos[a + b*x]]*(c*Sin[a + b*x])^(3/2),x]
```

output
```
-1/2*(c^3*d*((-(ArcTan[1 - (Sqrt[2]*Sqrt[c]*Sqrt[d*Cos[a + b*x]])/(Sqrt[d]
*Sqrt[c*Sin[a + b*x]])]/(Sqrt[2]*Sqrt[c]*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*S
qrt[c]*Sqrt[d*Cos[a + b*x]])/(Sqrt[d]*Sqrt[c*Sin[a + b*x]])]/(Sqrt[2]*Sqrt
[c]*Sqrt[d]))/(2*c) - (-1/2*Log[d + d*Cot[a + b*x] - (Sqrt[2]*Sqrt[c]*Sqrt
[d]*Sqrt[d*Cos[a + b*x]])/Sqrt[c*Sin[a + b*x]]]/(Sqrt[2]*Sqrt[c]*Sqrt[d])
+ Log[d + d*Cot[a + b*x] + (Sqrt[2]*Sqrt[c]*Sqrt[d]*Sqrt[d*Cos[a + b*x]])/
Sqrt[c*Sin[a + b*x]]]/(2*Sqrt[2]*Sqrt[c]*Sqrt[d]))/(2*c)))/b - (c*(d*Cos[a
 + b*x])^(3/2)*Sqrt[c*Sin[a + b*x]])/(2*b*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3055
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[(-k)*a*(b/f)   Subst[Int[x
^(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[
e + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m,
0] && LtQ[m, 1]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(182) = 364$.

Time = 21.86 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.75

method	result
default	$-\frac{\sqrt{2}\,c\left(\ln\left(-\frac{\cos(bx+a)\cot(bx+a)-2\cot(bx+a)+2\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}\,\sin(bx+a)-2\cos(bx+a)-\sin(bx+a)+\csc(bx+a)+2}{\cos(bx+a)-1}\right)-\ln\left(\frac{2\sqrt{-}}{}\right.\right.}{}$

input
```
int((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/256/b*2^(1/2)*c*(ln(-(cos(b*x+a)*cot(b*x+a)-2*cot(b*x+a)+2*(-2*sin(b*x+
a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)-2*cos(b*x+a)-sin(b*x+a)+c
sc(b*x+a)+2)/(cos(b*x+a)-1))-ln((2*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1
)^2)^(1/2)*sin(b*x+a)-cos(b*x+a)*cot(b*x+a)+2*cos(b*x+a)+sin(b*x+a)-csc(b*
x+a)+2*cot(b*x+a)-2)/(cos(b*x+a)-1))+cos(b*x+a)*(4*cos(b*x+a)+4)*(-2*sin(b
*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)+2*arctan(((-2*sin(b*x+a)*cos(b*x+
a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)-cos(b*x+a)+1)/(cos(b*x+a)-1))+2*arct
an(((-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)+cos(b*x+a
)-1)/(cos(b*x+a)-1)))*(c*sin(b*x+a))^(1/2)*(d*cos(b*x+a))^(1/2)*sin(b*x+a)
^3/(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sec(1/2*b*x+1/2*a)^5*cs
c(1/2*b*x+1/2*a)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 512 vs. $2(182) = 364$.

Time = 0.14 (sec) , antiderivative size = 512, normalized size of antiderivative = 2.09

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}\,dx =$$

$$-\frac{\sqrt{2}\sqrt{cd}c\arctan\left(\frac{2\,cd\cos(bx+a)^3-2\,cd\cos(bx+a)^2\sin(bx+a)-2\,cd\cos(bx+a)+\sqrt{2}\sqrt{cd}\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}}{2\left(cd\cos(bx+a)^3+cd\cos(bx+a)^2\sin(bx+a)-cd\cos(bx+a)\right)}\right)+\sqrt{2}\sqrt{cd}c\,\mathrm{a}}{}$$

input
```
integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
-1/32*(sqrt(2)*sqrt(c*d)*c*arctan(1/2*(2*c*d*cos(b*x + a)^3 - 2*c*d*cos(b*
x + a)^2*sin(b*x + a) - 2*c*d*cos(b*x + a) + sqrt(2)*sqrt(c*d)*sqrt(d*cos(
b*x + a))*sqrt(c*sin(b*x + a)))/(c*d*cos(b*x + a)^3 + c*d*cos(b*x + a)^2*s
in(b*x + a) - c*d*cos(b*x + a))) + sqrt(2)*sqrt(c*d)*c*arctan(-1/2*(2*c*d*
cos(b*x + a)^3 - 2*c*d*cos(b*x + a)^2*sin(b*x + a) - 2*c*d*cos(b*x + a) -
sqrt(2)*sqrt(c*d)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)))/(c*d*cos(b*x
+ a)^3 + c*d*cos(b*x + a)^2*sin(b*x + a) - c*d*cos(b*x + a))) - 2*sqrt(2)*
sqrt(c*d)*c*arctan(-1/2*sqrt(2)*sqrt(c*d)*sqrt(d*cos(b*x + a))*sqrt(c*sin(
b*x + a))*(cos(b*x + a) - sin(b*x + a))/(c*d*cos(b*x + a)*sin(b*x + a))) +
 16*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*c*cos(b*x + a) - sqrt(2)*sqr
t(c*d)*c*log(4*c*d*cos(b*x + a)*sin(b*x + a) + 2*sqrt(2)*sqrt(c*d)*sqrt(d*
cos(b*x + a))*sqrt(c*sin(b*x + a))*(cos(b*x + a) + sin(b*x + a)) + c*d) +
sqrt(2)*sqrt(c*d)*c*log(4*c*d*cos(b*x + a)*sin(b*x + a) - 2*sqrt(2)*sqrt(c
*d)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*(cos(b*x + a) + sin(b*x + a)
) + c*d))/b
```

Sympy [F]

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}\,dx = \int (c\sin{(a+bx)})^{\frac{3}{2}}\sqrt{d\cos{(a+bx)}}\,dx$$

input
```
integrate((d*cos(b*x+a))**(1/2)*(c*sin(b*x+a))**(3/2),x)
```

output
```
Integral((c*sin(a + b*x))**(3/2)*sqrt(d*cos(a + b*x)), x)
```

Maxima [F]

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}\,dx = \int \sqrt{d\cos{(bx+a)}}(c\sin{(bx+a)})^{\frac{3}{2}}\,dx$$

input
```
integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*cos(b*x + a))*(c*sin(b*x + a))^(3/2), x)
```

Giac [F]

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}\,dx = \int \sqrt{d\cos(bx+a)}(c\sin(bx+a))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*cos(b*x + a))*(c*sin(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}\,dx = \int \sqrt{d\cos(a+bx)}\,(c\sin(a+bx))^{3/2}\,dx$$

input
```
int((d*cos(a + b*x))^(1/2)*(c*sin(a + b*x))^(3/2),x)
```

output
```
int((d*cos(a + b*x))^(1/2)*(c*sin(a + b*x))^(3/2), x)
```

Reduce [F]

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}\,dx = \sqrt{d}\,\sqrt{c}\left(\int \sqrt{\sin(bx+a)}\,\sqrt{\cos(bx+a)}\,\sin(bx+a)\,dx\right)c$$

input
```
int((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(3/2),x)
```

output
```
sqrt(d)*sqrt(c)*int(sqrt(sin(a + b*x))*sqrt(cos(a + b*x))*sin(a + b*x),x)*
c
```

3.272 $\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 236

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{3/2}}\,dx = -\frac{c^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}bd^{3/2}} + \frac{c^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}bd^{3/2}} - \frac{c^{3/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\left(\sqrt{d}+\sqrt{d}\cot(a+bx)\right)\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}bd^{3/2}} + \frac{2c\sqrt{c\sin(a+bx)}}{bd\sqrt{d\cos(a+bx)}}$$

output

```
1/2*c^(3/2)*arctan(-1+2^(1/2)*c^(1/2)*(d*cos(b*x+a))^(1/2)/d^(1/2)/(c*sin(
b*x+a))^(1/2))*2^(1/2)/b/d^(3/2)+1/2*c^(3/2)*arctan(1+2^(1/2)*c^(1/2)*(d*c
os(b*x+a))^(1/2)/d^(1/2)/(c*sin(b*x+a))^(1/2))*2^(1/2)/b/d^(3/2)-1/2*c^(3/
2)*arctanh(2^(1/2)*c^(1/2)*(d*cos(b*x+a))^(1/2)/(d^(1/2)+d^(1/2)*cot(b*x+a
))/(c*sin(b*x+a))^(1/2))*2^(1/2)/b/d^(3/2)+2*c*(c*sin(b*x+a))^(1/2)/b/d/(d
*cos(b*x+a))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.28

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{3/2}}\,dx=\frac{2\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{5}{4},\frac{9}{4},\sin^2(a+bx)\right)(c\sin(a+bx))^{5/2}}{5bcd\sqrt{d\cos(a+bx)}}$$

input
```
Integrate[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(3/2),x]
```

output
```
(2*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[5/4, 5/4, 9/4, Sin[a + b*x]^2]
*(c*Sin[a + b*x])^(5/2))/(5*b*c*d*Sqrt[d*Cos[a + b*x]])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.37, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3042, 3046, 3042, 3055, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3046}$$

$$\frac{2c\sqrt{c\sin(a+bx)}}{bd\sqrt{d\cos(a+bx)}}-\frac{c^2\int \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}dx}{d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c\sqrt{c\sin(a+bx)}}{bd\sqrt{d\cos(a+bx)}}-\frac{c^2\int \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}dx}{d^2}$$

$$\Big\downarrow \text{3055}$$

$$\frac{2c^3\int \frac{d\cot(a+bx)}{c(\cot^2(a+bx)d^2+d^2)}d\frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}}{bd}+\frac{2c\sqrt{c\sin(a+bx)}}{bd\sqrt{d\cos(a+bx)}}$$

$$\Big\downarrow \text{826}$$

$$\frac{2c^3\left(\frac{\int\frac{\cot(a+bx)d+d}{\cot^2(a+bx)d^2+d^2}d\frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}}{2c}-\frac{\int\frac{d-d\cot(a+bx)}{\cot^2(a+bx)d^2+d^2}d\frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}}{2c}\right)}{bd}+\frac{2c\sqrt{c\sin(a+bx)}}{bd\sqrt{d\cos(a+bx)}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{2c^3\left(\frac{\frac{\int\frac{1}{\frac{\cot(a+bx)d}{c}+\frac{d}{c}-\frac{\sqrt{2}\sqrt{d\cos(a+bx)}\sqrt{d}}{\sqrt{c}\sqrt{c\sin(a+bx)}}}d\frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}}{2c}+\frac{\int\frac{1}{\frac{\cot(a+bx)d}{c}+\frac{d}{c}+\frac{\sqrt{2}\sqrt{d\cos(a+bx)}\sqrt{d}}{\sqrt{c}\sqrt{c\sin(a+bx)}}}d\frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}}{2c}}{2c}-\frac{\int\frac{d-d\cot(a+bx)}{\cot^2(a+bx)d^2+d^2}d\frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}}{2c}\right)}{bd}+\frac{2c\sqrt{c\sin(a+bx)}}{bd\sqrt{d\cos(a+bx)}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2c^3\left(\frac{\frac{\int\frac{1}{-\frac{d\cot(a+bx)}{c}-1}d\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\int\frac{1}{-\frac{d\cot(a+bx)}{c}-1}d\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}+1\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}}{2c}-\frac{\int\frac{d-d\cot(a+bx)}{\cot^2(a+bx)d^2+d^2}d\frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}}{2c}\right)}{bd}+\frac{2c\sqrt{c\sin(a+bx)}}{bd\sqrt{d\cos(a+bx)}}$$

$$\Big\downarrow \text{217}$$

$$\frac{2c^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}+1\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}}{2c}-\frac{\int\frac{d-d\cot(a+bx)}{\cot^2(a+bx)d^2+d^2}d\frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}}{2c}\right)}{bd}+\frac{2c\sqrt{c\sin(a+bx)}}{bd\sqrt{d\cos(a+bx)}}$$

$$\Big\downarrow \text{1479}$$

$$\frac{2c^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}+1\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}}{2c}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt{d}-\frac{2\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}}{\sqrt{c}\left(\frac{\cot(a+bx)d}{c}+\frac{d}{c}-\frac{\sqrt{2}\sqrt{d\cos(a+bx)}\sqrt{d}}{\sqrt{c}\sqrt{c\sin(a+bx)}}\right)}d\frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}}{2\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\int-\frac{\cdots}{\sqrt{c}\left(\cdots\right)}\cdots}{\cdots}}{2c}\cdots\right)}{bd}$$

$$\frac{2c\sqrt{c\sin(a+bx)}}{bd\sqrt{d\cos(a+bx)}}$$

$\downarrow$ 25

$$\frac{2c^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}+1\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}}{2c}-\frac{\frac{\int\frac{\sqrt{2}\sqrt{d}-\frac{2\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}}{\sqrt{c}\left(\frac{\cot(a+bx)d}{c}+\frac{d}{c}-\frac{\sqrt{2}\sqrt{d\cos(a+bx)}\sqrt{d}}{\sqrt{c}\sqrt{c\sin(a+bx)}}\right)}d\frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}}{2\sqrt{2}\sqrt{c}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\cdots\right)}{\sqrt{c}\left(\frac{\cot(a+\cdots}{c}\cdots\right)}\cdots}{\cdots}}{2c}\cdots\right)}{bd}$$

$$\frac{2c\sqrt{c\sin(a+bx)}}{bd\sqrt{d\cos(a+bx)}}$$

$\downarrow$ 27

$$\frac{2c^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}+1\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}}{2c}-\frac{\frac{\int\frac{\sqrt{2}\sqrt{d}-\frac{2\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}}{\frac{\cot(a+bx)d}{c}+\frac{d}{c}-\frac{\sqrt{2}\sqrt{d\cos(a+bx)}\sqrt{d}}{\sqrt{c}\sqrt{c\sin(a+bx)}}}d\frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}}{2\sqrt{2}c\sqrt{d}}+\frac{\int\frac{\sqrt{d}+\frac{\sqrt{2}\sqrt{c}\cdots}{\sqrt{\cdots}}}{\frac{\cot(a+bx)d}{c}+\frac{d}{c}+\cdots}\cdots}{\cdots}}{2c}\cdots\right)}{bd}$$

$$\frac{2c\sqrt{c\sin(a+bx)}}{bd\sqrt{d\cos(a+bx)}}$$

$\downarrow$ 1103

$$\frac{2c^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}+1\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}}{2c}-\frac{\frac{\log\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}+d\cot(a+bx)+d\right)}{2\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\log\left(-\frac{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\cdots\right)}{2\sqrt{2}\sqrt{c}\sqrt{d}}}{2c}\cdots\right)}{bd}$$

$$\frac{2c\sqrt{c\sin(a+bx)}}{bd\sqrt{d\cos(a+bx)}}$$

input
```
Int[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(3/2),x]
```

output
```
(2*c^3*((-(ArcTan[1 - (Sqrt[2]*Sqrt[c]*Sqrt[d*Cos[a + b*x]])/(Sqrt[d]*Sqrt
[c*Sin[a + b*x]])]/(Sqrt[2]*Sqrt[c]*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*Sqrt[c
]*Sqrt[d*Cos[a + b*x]])/(Sqrt[d]*Sqrt[c*Sin[a + b*x]])]/(Sqrt[2]*Sqrt[c]*S
qrt[d]))/(2*c) - (-1/2*Log[d + d*Cot[a + b*x] - (Sqrt[2]*Sqrt[c]*Sqrt[d]*S
qrt[d*Cos[a + b*x]])/Sqrt[c*Sin[a + b*x]]]/(Sqrt[2]*Sqrt[c]*Sqrt[d]) + Log
[d + d*Cot[a + b*x] + (Sqrt[2]*Sqrt[c]*Sqrt[d]*Sqrt[d*Cos[a + b*x]])/Sqrt[
c*Sin[a + b*x]]]/(2*Sqrt[2]*Sqrt[c]*Sqrt[d]))/(2*c)))/(b*d) + (2*c*Sqrt[c*
Sin[a + b*x]])/(b*d*Sqrt[d*Cos[a + b*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3046
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Sin[e + f
*x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3055
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[(-k)*a*(b/f)   Subst[Int[x
^(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[
e + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m,
0] && LtQ[m, 1]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 872 vs. $2(182) = 364$.

Time = 53.78 (sec) , antiderivative size = 873, normalized size of antiderivative = 3.70

method	result	size
default	Expression too large to display	873

input
```
int((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/b*((-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*ln(-(cos(b*x+a)*c
ot(b*x+a)-2*cot(b*x+a)-2*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)
*sin(b*x+a)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(cos(b*x+a)-1))*cos(b*x+
a)-2*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*arctan(((-2*sin(b*x
+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)+cos(b*x+a)-1)/(cos(b*x+a
)-1))*cos(b*x+a)-(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*ln(-(co
s(b*x+a)*cot(b*x+a)-2*cot(b*x+a)+2*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1
)^2)^(1/2)*sin(b*x+a)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(cos(b*x+a)-1)
)*cos(b*x+a)+2*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*arctan((-
(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)+cos(b*x+a)-1)
/(cos(b*x+a)-1))*cos(b*x+a)+(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1
/2)*ln(-(cos(b*x+a)*cot(b*x+a)-2*cot(b*x+a)-2*(-2*sin(b*x+a)*cos(b*x+a)/(c
os(b*x+a)+1)^2)^(1/2)*sin(b*x+a)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(co
s(b*x+a)-1))-2*arctan(((-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*s
in(b*x+a)+cos(b*x+a)-1)/(cos(b*x+a)-1))*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x
+a)+1)^2)^(1/2)-(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*ln(-(cos
(b*x+a)*cot(b*x+a)-2*cot(b*x+a)+2*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)
^2)^(1/2)*sin(b*x+a)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(cos(b*x+a)-1))
+2*arctan((-(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)+c
os(b*x+a)-1)/(cos(b*x+a)-1))*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 551 vs. $2(182) = 364$.

Time = 0.23 (sec) , antiderivative size = 551, normalized size of antiderivative = 2.33

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{3/2}}\,dx = \frac{\sqrt{2}cd\sqrt{\frac{c}{d}}\arctan\left(\frac{2\,c\cos(bx+a)^3-2\,c\cos(bx+a)^2\sin(bx+a)+\sqrt{2}\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}\sqrt{\frac{c}{d}}-2\,c}{2\left(c\cos(bx+a)^3+c\cos(bx+a)^2\sin(bx+a)-c\cos(bx+a)\right)}\right.}{}$$

input
```
integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
1/8*(sqrt(2)*c*d*sqrt(c/d)*arctan(1/2*(2*c*cos(b*x + a)^3 - 2*c*cos(b*x +
a)^2*sin(b*x + a) + sqrt(2)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*sqrt
(c/d) - 2*c*cos(b*x + a))/(c*cos(b*x + a)^3 + c*cos(b*x + a)^2*sin(b*x + a
) - c*cos(b*x + a)))*cos(b*x + a) + sqrt(2)*c*d*sqrt(c/d)*arctan(-1/2*(2*c
*cos(b*x + a)^3 - 2*c*cos(b*x + a)^2*sin(b*x + a) - sqrt(2)*sqrt(d*cos(b*x
 + a))*sqrt(c*sin(b*x + a))*sqrt(c/d) - 2*c*cos(b*x + a))/(c*cos(b*x + a)^
3 + c*cos(b*x + a)^2*sin(b*x + a) - c*cos(b*x + a)))*cos(b*x + a) - 2*sqrt
(2)*c*d*sqrt(c/d)*arctan(-1/2*sqrt(2)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x
+ a))*sqrt(c/d)*(cos(b*x + a) - sin(b*x + a))/(c*cos(b*x + a)*sin(b*x + a)
))*cos(b*x + a) - sqrt(2)*c*d*sqrt(c/d)*cos(b*x + a)*log(2*sqrt(2)*sqrt(d*
cos(b*x + a))*sqrt(c*sin(b*x + a))*sqrt(c/d)*(cos(b*x + a) + sin(b*x + a))
 + 4*c*cos(b*x + a)*sin(b*x + a) + c) + sqrt(2)*c*d*sqrt(c/d)*cos(b*x + a)
*log(-2*sqrt(2)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*sqrt(c/d)*(cos(b
*x + a) + sin(b*x + a)) + 4*c*cos(b*x + a)*sin(b*x + a) + c) + 16*sqrt(d*c
os(b*x + a))*sqrt(c*sin(b*x + a))*c)/(b*d^2*cos(b*x + a))
```

Sympy [F]

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{(c\sin{(a+bx)})^{\frac{3}{2}}}{(d\cos{(a+bx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))**(3/2)/(d*cos(b*x+a))**(3/2),x)
```

output
```
Integral((c*sin(a + b*x))**(3/2)/(d*cos(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{(c\sin{(bx+a)})^{\frac{3}{2}}}{(d\cos{(bx+a)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a))^(3/2)/(d*cos(b*x + a))^(3/2), x)
```

Giac [F]

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{(c\sin{(bx+a)})^{\frac{3}{2}}}{(d\cos{(bx+a)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a))^(3/2)/(d*cos(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{{\left(c\,\sin\left(a+b\,x\right)\right)}^{3/2}}{{\left(d\,\cos\left(a+b\,x\right)\right)}^{3/2}}\,dx$$

input
```
int((c*sin(a + b*x))^(3/2)/(d*cos(a + b*x))^(3/2),x)
```

output
```
int((c*sin(a + b*x))^(3/2)/(d*cos(a + b*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{3/2}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\,c\Big(-\cos{(bx+a)}\left(\int \frac{\sqrt{\sin(bx+a)}\,\sqrt{\cos(bx+a)}}{\sin(bx+a)}dx\right)b+2\sqrt{\sin{(bx+a)}}\,\sqrt{\cos{(bx}}}{\cos{(bx+a)}\,b\,d^2}$$

input
```
int((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*sqrt(c)*c*( - cos(a + b*x)*int((sqrt(sin(a + b*x))*sqrt(cos(a + b
*x)))/sin(a + b*x),x)*b + 2*sqrt(sin(a + b*x))*sqrt(cos(a + b*x))))/(cos(a
 + b*x)*b*d**2)
```

3.273 $\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 37

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{7/2}}\,dx = \frac{2(c\sin(a+bx))^{5/2}}{5bcd(d\cos(a+bx))^{5/2}}$$

output
```
2/5*(c*sin(b*x+a))^(5/2)/b/c/d/(d*cos(b*x+a))^(5/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{7/2}}\,dx = \frac{2\cot(a+bx)(c\sin(a+bx))^{7/2}}{5bc^2(d\cos(a+bx))^{7/2}}$$

input
```
Integrate[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(7/2),x]
```

output
```
(2*Cot[a + b*x]*(c*Sin[a + b*x])^(7/2))/(5*b*c^2*(d*Cos[a + b*x])^(7/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3042, 3043}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{7/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{7/2}}dx$$

$$\Big\downarrow \text{ 3043}$$

$$\frac{2(c\sin(a+bx))^{5/2}}{5bcd(d\cos(a+bx))^{5/2}}$$

input
```
Int[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(7/2),x]
```

output
```
(2*(c*Sin[a + b*x])^(5/2))/(5*b*c*d*(d*Cos[a + b*x])^(5/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3043
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Maple [A] (verified)

Time = 6.40 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{2\sqrt{c\sin(bx+a)}\,c\tan(bx+a)^{2}}{5b\sqrt{d\cos(bx+a)}\,d^{3}}$	38

input
```
int((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/b*(c*sin(b*x+a))^(1/2)*c/(d*cos(b*x+a))^(1/2)/d^3*tan(b*x+a)^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.35

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{7/2}}\,dx = -\frac{2\left(c\cos\left(bx+a\right)^{2}-c\right)\sqrt{d\cos\left(bx+a\right)}\sqrt{c\sin\left(bx+a\right)}}{5\,bd^{4}\cos\left(bx+a\right)^{3}}$$

input
```
integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

output
```
-2/5*(c*cos(b*x + a)^2 - c)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))/(b*d
^4*cos(b*x + a)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*sin(b*x+a))**(3/2)/(d*cos(b*x+a))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{3}{2}}}{(d\cos(bx+a))^{\frac{7}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(7/2),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a))^(3/2)/(d*cos(b*x + a))^(7/2), x)
```

Giac [F]

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{3}{2}}}{(d\cos(bx+a))^{\frac{7}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(7/2),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a))^(3/2)/(d*cos(b*x + a))^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 26.89 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.73

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{7/2}}\,dx = -\frac{2\,c\,(\cos(4\,a+4\,b\,x)-1)\,\sqrt{c\,\sin(a+b\,x)}}{5\,b\,d^3\,\sqrt{d\,\cos(a+b\,x)}\,(4\,\cos(2\,a+2\,b\,x)+\cos(4\,a+4\,b\,x)+3)}$$

input
```
int((c*sin(a + b*x))^(3/2)/(d*cos(a + b*x))^(7/2),x)
```

output
```
-(2*c*(cos(4*a + 4*b*x) - 1)*(c*sin(a + b*x))^(1/2))/(5*b*d^3*(d*cos(a + b
*x))^(1/2)*(4*cos(2*a + 2*b*x) + cos(4*a + 4*b*x) + 3))
```

Reduce [F]

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{7/2}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\,c\left(-\cos(bx+a)^3\left(\int \frac{\sqrt{\sin(bx+a)}\,\sqrt{\cos(bx+a)}}{\cos(bx+a)^2\sin(bx+a)}dx\right)b + 2\sqrt{\sin(bx+a)}\,\sqrt{\cos(b}\right.}{5\cos(bx+a)^3\,b\,d^4}$$

input

```
int((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(7/2),x)
```

output

```
(sqrt(d)*sqrt(c)*c*( - cos(a + b*x)**3*int((sqrt(sin(a + b*x))*sqrt(cos(a
+ b*x)))/(cos(a + b*x)**2*sin(a + b*x)),x)*b + 2*sqrt(sin(a + b*x))*sqrt(c
os(a + b*x))))/(5*cos(a + b*x)**3*b*d**4)
```

3.274 $\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{11/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 106

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{11/2}}\,dx = \frac{2c\sqrt{c\sin(a+bx)}}{9bd(d\cos(a+bx))^{9/2}} - \frac{2c\sqrt{c\sin(a+bx)}}{45bd^3(d\cos(a+bx))^{5/2}} - \frac{8c\sqrt{c\sin(a+bx)}}{45bd^5\sqrt{d\cos(a+bx)}}$$

output
```
2/9*c*(c*sin(b*x+a))^(1/2)/b/d/(d*cos(b*x+a))^(9/2)-2/45*c*(c*sin(b*x+a))^
(1/2)/b/d^3/(d*cos(b*x+a))^(5/2)-8/45*c*(c*sin(b*x+a))^(1/2)/b/d^5/(d*cos(
b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.54

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{11/2}}\,dx = \frac{2\sqrt{d\cos(a+bx)}(7+2\cos(2(a+bx)))\sec^5(a+bx)(c\sin(a+bx))^{5/2}}{45bcd^6}$$

input
```
Integrate[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(11/2),x]
```

output
```
(2*Sqrt[d*Cos[a + b*x]]*(7 + 2*Cos[2*(a + b*x)])*Sec[a + b*x]^5*(c*Sin[a +
 b*x])^(5/2))/(45*b*c*d^6)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3046, 3042, 3051, 3042, 3043}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{11/2}}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{11/2}}dx$$

$$\Big\downarrow \textbf{3046}$$

$$\frac{2c\sqrt{c\sin(a+bx)}}{9bd(d\cos(a+bx))^{9/2}}-\frac{c^2\int \frac{1}{(d\cos(a+bx))^{7/2}\sqrt{c\sin(a+bx)}}dx}{9d^2}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{2c\sqrt{c\sin(a+bx)}}{9bd(d\cos(a+bx))^{9/2}}-\frac{c^2\int \frac{1}{(d\cos(a+bx))^{7/2}\sqrt{c\sin(a+bx)}}dx}{9d^2}$$

$$\Big\downarrow \textbf{3051}$$

$$\frac{2c\sqrt{c\sin(a+bx)}}{9bd(d\cos(a+bx))^{9/2}}-\frac{c^2\left(\frac{4\int \frac{1}{(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}dx}{5d^2}+\frac{2\sqrt{c\sin(a+bx)}}{5bcd(d\cos(a+bx))^{5/2}}\right)}{9d^2}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{2c\sqrt{c\sin(a+bx)}}{9bd(d\cos(a+bx))^{9/2}}-\frac{c^2\left(\frac{4\int \frac{1}{(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}dx}{5d^2}+\frac{2\sqrt{c\sin(a+bx)}}{5bcd(d\cos(a+bx))^{5/2}}\right)}{9d^2}$$

$$\Big\downarrow \textbf{3043}$$

$$\frac{2c\sqrt{c\sin(a+bx)}}{9bd(d\cos(a+bx))^{9/2}}-\frac{c^2\left(\frac{8\sqrt{c\sin(a+bx)}}{5bcd^3\sqrt{d\cos(a+bx)}}+\frac{2\sqrt{c\sin(a+bx)}}{5bcd(d\cos(a+bx))^{5/2}}\right)}{9d^2}$$

input
```
Int[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(11/2),x]
```

output
```
(2*c*Sqrt[c*Sin[a + b*x]])/(9*b*d*(d*Cos[a + b*x])^(9/2)) - (c^2*((2*Sqrt[
c*Sin[a + b*x]])/(5*b*c*d*(d*Cos[a + b*x])^(5/2)) + (8*Sqrt[c*Sin[a + b*x]
])/(5*b*c*d^3*Sqrt[d*Cos[a + b*x]])))/(9*d^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3043
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

rule 3046
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Sin[e + f
*x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3051
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Simp[(m + n + 2)/(a^2*(m + 1))   Int[(b*Sin[e + f*x
])^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m
, -1] && IntegersQ[2*m, 2*n]
```

Maple [A] (verified)

Time = 5.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.56

method	result	size
default	$\frac{2\sqrt{c\sin(bx+a)}\,c\left(4\tan(bx+a)^{2}+5\sec(bx+a)^{2}\tan(bx+a)^{2}\right)}{45b\sqrt{d\cos(bx+a)}\,d^{5}}$	59

input
```
int((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(11/2),x,method=_RETURNVERBOSE)
```

output
```
2/45/b*(c*sin(b*x+a))^(1/2)*c/(d*cos(b*x+a))^(1/2)/d^5*(4*tan(b*x+a)^2+5*s
ec(b*x+a)^2*tan(b*x+a)^2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.58

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{11/2}}\,dx = -\frac{2\left(4\,c\cos\left(bx+a\right)^{4}+c\cos\left(bx+a\right)^{2}-5\,c\right)\sqrt{d\cos\left(bx+a\right)}\sqrt{c\sin\left(bx+a\right)}}{45\,bd^{6}\cos\left(bx+a\right)^{5}}$$

input
```
integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(11/2),x, algorithm="fricas"
)
```

output
```
-2/45*(4*c*cos(b*x + a)^4 + c*cos(b*x + a)^2 - 5*c)*sqrt(d*cos(b*x + a))*s
qrt(c*sin(b*x + a))/(b*d^6*cos(b*x + a)^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*sin(b*x+a))**(3/2)/(d*cos(b*x+a))**(11/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{11/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{3}{2}}}{(d\cos(bx+a))^{\frac{11}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(11/2),x, algorithm="maxima"
)
```

output
```
integrate((c*sin(b*x + a))^(3/2)/(d*cos(b*x + a))^(11/2), x)
```

Giac [F]

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{11/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{3}{2}}}{(d\cos(bx+a))^{\frac{11}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(11/2),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a))^(3/2)/(d*cos(b*x + a))^(11/2), x)
```

Mupad [B] (verification not implemented)

Time = 32.15 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.95

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{11/2}}\,dx = -\frac{\sqrt{c\,\sin\left(a+b\,x\right)}\,\left(2\sin\left(2\,a+2\,b\,x\right)^2+\sin\left(4\,a+4\,b\,x\right)\,1\mathrm{i}-1\right)\,\left(\frac{32\,c\,\left(-2\sin(2\,a+2\,b\,x)^2+\sin(4\,a+4\,b\,x)\,1\mathrm{i}+1\right)}{15\,b\,d^5}+\frac{16\,c}{\ldots}\right.}{16\,\left(\sin\left(a+b\,x\right)^2-1\right)^2\,\sqrt{-\ldots}}$$

input
```
int((c*sin(a + b*x))^(3/2)/(d*cos(a + b*x))^(11/2),x)
```

output
```
-((c*sin(a + b*x))^(1/2)*(sin(4*a + 4*b*x)*1i + 2*sin(2*a + 2*b*x)^2 - 1)*
((32*c*(sin(4*a + 4*b*x)*1i - 2*sin(2*a + 2*b*x)^2 + 1))/(15*b*d^5) + (16*
c*(2*sin(2*a + 2*b*x)^2 - 1)*(sin(4*a + 4*b*x)*1i - 2*sin(2*a + 2*b*x)^2 +
 1))/(45*b*d^5) + (16*c*(2*sin(a + b*x)^2 - 1)*(sin(4*a + 4*b*x)*1i - 2*si
n(2*a + 2*b*x)^2 + 1))/(9*b*d^5)))/(16*(sin(a + b*x)^2 - 1)^2*(-d*(2*sin(a
/2 + (b*x)/2)^2 - 1))^(1/2))
```

Reduce [F]

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{11/2}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\,c\left(-\cos\left(bx+a\right)^5\left(\int\frac{\sqrt{\sin(bx+a)}\,\sqrt{\cos(bx+a)}}{\cos(bx+a)^4\sin(bx+a)}dx\right)b+2\sqrt{\sin\left(bx+a\right)}\,\sqrt{\cos\left(\ldots\right.}\right.}{9\cos\left(bx+a\right)^5 b\,d^6}$$

input
```
int((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(11/2),x)
```

output
```
(sqrt(d)*sqrt(c)*c*( - cos(a + b*x)**5*int((sqrt(sin(a + b*x))*sqrt(cos(a
+ b*x)))/(cos(a + b*x)**4*sin(a + b*x)),x)*b + 2*sqrt(sin(a + b*x))*sqrt(c
os(a + b*x))))/(9*cos(a + b*x)**5*b*d**6)
```

3.275 $\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{15/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 141

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{15/2}}\,dx = \frac{2c\sqrt{c\sin(a+bx)}}{13bd(d\cos(a+bx))^{13/2}} - \frac{2c\sqrt{c\sin(a+bx)}}{117bd^3(d\cos(a+bx))^{9/2}} - \frac{16c\sqrt{c\sin(a+bx)}}{585bd^5(d\cos(a+bx))^{5/2}} - \frac{64c\sqrt{c\sin(a+bx)}}{585bd^7\sqrt{d\cos(a+bx)}}$$

output
```
2/13*c*(c*sin(b*x+a))^(1/2)/b/d/(d*cos(b*x+a))^(13/2)-2/117*c*(c*sin(b*x+a
))^(1/2)/b/d^3/(d*cos(b*x+a))^(9/2)-16/585*c*(c*sin(b*x+a))^(1/2)/b/d^5/(d
*cos(b*x+a))^(5/2)-64/585*c*(c*sin(b*x+a))^(1/2)/b/d^7/(d*cos(b*x+a))^(1/2
)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.48

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{15/2}}\,dx = \frac{2\sqrt{d\cos(a+bx)}(77+36\cos(2(a+bx))+4\cos(4(a+bx)))\sec^7(a+bx)(c\sin(}{585bcd^8}$$

input
```
Integrate[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(15/2),x]
```

output
```
(2*Sqrt[d*Cos[a + b*x]]*(77 + 36*Cos[2*(a + b*x)] + 4*Cos[4*(a + b*x)])*Se
c[a + b*x]^7*(c*Sin[a + b*x])^(5/2))/(585*b*c*d^8)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3046, 3042, 3051, 3042, 3051, 3042, 3043}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{15/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{15/2}}dx$$

$$\Big\downarrow \; 3046$$

$$\frac{2c\sqrt{c\sin(a+bx)}}{13bd(d\cos(a+bx))^{13/2}}-\frac{c^2\int \frac{1}{(d\cos(a+bx))^{11/2}\sqrt{c\sin(a+bx)}}dx}{13d^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{2c\sqrt{c\sin(a+bx)}}{13bd(d\cos(a+bx))^{13/2}}-\frac{c^2\int \frac{1}{(d\cos(a+bx))^{11/2}\sqrt{c\sin(a+bx)}}dx}{13d^2}$$

$$\Big\downarrow \; 3051$$

$$\frac{2c\sqrt{c\sin(a+bx)}}{13bd(d\cos(a+bx))^{13/2}}-\frac{c^2\left(\frac{8\int \frac{1}{(d\cos(a+bx))^{7/2}\sqrt{c\sin(a+bx)}}dx}{9d^2}+\frac{2\sqrt{c\sin(a+bx)}}{9bcd(d\cos(a+bx))^{9/2}}\right)}{13d^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{2c\sqrt{c\sin(a+bx)}}{13bd(d\cos(a+bx))^{13/2}}-\frac{c^2\left(\frac{8\int \frac{1}{(d\cos(a+bx))^{7/2}\sqrt{c\sin(a+bx)}}dx}{9d^2}+\frac{2\sqrt{c\sin(a+bx)}}{9bcd(d\cos(a+bx))^{9/2}}\right)}{13d^2}$$

$$\Big\downarrow \; 3051$$

$$\frac{2c\sqrt{c\sin(a+bx)}}{13bd(d\cos(a+bx))^{13/2}}-\frac{c^2\left(\frac{8\left(\frac{4\int\frac{1}{(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}dx}{5d^2}+\frac{2\sqrt{c\sin(a+bx)}}{5bcd(d\cos(a+bx))^{5/2}}\right)}{9d^2}+\frac{2\sqrt{c\sin(a+bx)}}{9bcd(d\cos(a+bx))^{9/2}}\right)}{13d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c\sqrt{c\sin(a+bx)}}{13bd(d\cos(a+bx))^{13/2}}-\frac{c^2\left(\frac{8\left(\frac{4\int\frac{1}{(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}dx}{5d^2}+\frac{2\sqrt{c\sin(a+bx)}}{5bcd(d\cos(a+bx))^{5/2}}\right)}{9d^2}+\frac{2\sqrt{c\sin(a+bx)}}{9bcd(d\cos(a+bx))^{9/2}}\right)}{13d^2}$$

$$\Big\downarrow \text{3043}$$

$$\frac{2c\sqrt{c\sin(a+bx)}}{13bd(d\cos(a+bx))^{13/2}}-\frac{c^2\left(\frac{8\left(\frac{8\sqrt{c\sin(a+bx)}}{5bcd^3\sqrt{d\cos(a+bx)}}+\frac{2\sqrt{c\sin(a+bx)}}{5bcd(d\cos(a+bx))^{5/2}}\right)}{9d^2}+\frac{2\sqrt{c\sin(a+bx)}}{9bcd(d\cos(a+bx))^{9/2}}\right)}{13d^2}$$

input
```
Int[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(15/2),x]
```

output
```
(2*c*Sqrt[c*Sin[a + b*x]])/(13*b*d*(d*Cos[a + b*x])^(13/2)) - (c^2*((2*Sqr
t[c*Sin[a + b*x]])/(9*b*c*d*(d*Cos[a + b*x])^(9/2)) + (8*((2*Sqrt[c*Sin[a
+ b*x]])/(5*b*c*d*(d*Cos[a + b*x])^(5/2)) + (8*Sqrt[c*Sin[a + b*x]])/(5*b*
c*d^3*Sqrt[d*Cos[a + b*x]]))))/(9*d^2)))/(13*d^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3043
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

rule 3046
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Sin[e + f
*x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3051
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Simp[(m + n + 2)/(a^2*(m + 1))   Int[(b*Sin[e + f*x
])^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m
, -1] && IntegersQ[2*m, 2*n]
```

Maple [A] (verified)

Time = 5.24 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.48

method	result	size
default	$\frac{2\sqrt{c\sin(bx+a)}\left(32\cos(bx+a)^4+40\cos(bx+a)^2+45\right)c\tan(bx+a)^2\sec(bx+a)^4}{585b\sqrt{d\cos(bx+a)}\,d^7}$	68

input
```
int((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(15/2),x,method=_RETURNVERBOSE)
```

output
```
2/585/b*(c*sin(b*x+a))^(1/2)*(32*cos(b*x+a)^4+40*cos(b*x+a)^2+45)*c/(d*cos
(b*x+a))^(1/2)/d^7*tan(b*x+a)^2*sec(b*x+a)^4
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.52

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{15/2}}\,dx = -\frac{2\left(32\,c\cos\left(bx+a\right)^{6}+8\,c\cos\left(bx+a\right)^{4}+5\,c\cos\left(bx+a\right)^{2}-45\,c\right)\sqrt{d\cos\left(bx+a\right)}\sqrt{c\sin\left(bx+a\right)}}{585\,bd^{8}\cos\left(bx+a\right)^{7}}$$

input
```
integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(15/2),x, algorithm="fricas"
)
```

output
```
-2/585*(32*c*cos(b*x + a)^6 + 8*c*cos(b*x + a)^4 + 5*c*cos(b*x + a)^2 - 45
*c)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))/(b*d^8*cos(b*x + a)^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{15/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*sin(b*x+a))**(3/2)/(d*cos(b*x+a))**(15/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{15/2}}\,dx = \int \frac{\left(c\sin\left(bx+a\right)\right)^{\frac{3}{2}}}{\left(d\cos\left(bx+a\right)\right)^{\frac{15}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(15/2),x, algorithm="maxima"
)
```

output
```
integrate((c*sin(b*x + a))^(3/2)/(d*cos(b*x + a))^(15/2), x)
```

Giac [F]

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{15/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{3}{2}}}{(d\cos(bx+a))^{\frac{15}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(15/2),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a))^(3/2)/(d*cos(b*x + a))^(15/2), x)
```

Mupad [B] (verification not implemented)

Time = 35.05 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.37

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{15/2}}\,dx = -\frac{e^{-a\,6i-b\,x\,6i}\sqrt{c\,\left(\frac{e^{-a\,1i-b\,x\,1i}\,1i}{2}-\frac{e^{a\,1i+b\,x\,1i}\,1i}{2}\right)}\,\left(-\frac{3776\,c\,e^{a\,6i+b\,x\,6i}}{585\,b\,d^7}+\frac{2752\,c\,e^{a\,6i+b\,x\,6i}\,\cos(2\,a+2\,b\,x)}{585\,b\,d^7}+\frac{896\,c\,e^{a\,6i+b\,x\,6i}\,\cos(4\,a+}{585\,b\,d^7}\right.}{64\cos\left(a+b\,x\right)^6\sqrt{d\,\left(\frac{e^{-a\,1i-b\,x\,1i}}{2}+\frac{e^{a\,1i+b\,x\,1i}}{2}\right)}}$$

input
```
int((c*sin(a + b*x))^(3/2)/(d*cos(a + b*x))^(15/2),x)
```

output
```
-(exp(- a*6i - b*x*6i)*(c*((exp(- a*1i - b*x*1i)*1i)/2 - (exp(a*1i + b*x*1
i)*1i)/2))^(1/2)*((2752*c*exp(a*6i + b*x*6i)*cos(2*a + 2*b*x))/(585*b*d^7)
 - (3776*c*exp(a*6i + b*x*6i))/(585*b*d^7) + (896*c*exp(a*6i + b*x*6i)*cos
(4*a + 4*b*x))/(585*b*d^7) + (128*c*exp(a*6i + b*x*6i)*cos(6*a + 6*b*x))/(
585*b*d^7)))/(64*cos(a + b*x)^6*(d*(exp(- a*1i - b*x*1i)/2 + exp(a*1i + b*
x*1i)/2))^(1/2))
```

Reduce [F]

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{15/2}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\,c\left(-\cos\left(bx+a\right)^{7}\left(\int \frac{\sqrt{\sin(bx+a)}\,\sqrt{\cos(bx+a)}}{\cos(bx+a)^{6}\sin(bx+a)}dx\right)b+2\sqrt{\sin\left(bx+a\right)}\,\sqrt{\cos(}\right.}{13\cos\left(bx+a\right)^{7}b\,d^{8}}$$

input
```
int((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(15/2),x)
```

output
```
(sqrt(d)*sqrt(c)*c*( - cos(a + b*x)**7*int((sqrt(sin(a + b*x))*sqrt(cos(a
+ b*x)))/(cos(a + b*x)**6*sin(a + b*x)),x)*b + 2*sqrt(sin(a + b*x))*sqrt(c
os(a + b*x))))/(13*cos(a + b*x)**7*b*d**8)
```

3.276 $\int (d\cos(a+bx))^{9/2}(c\sin(a+bx))^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 166

$$\int (d\cos(a+bx))^{9/2}(c\sin(a+bx))^{5/2}\,dx = \frac{cd^3(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{20b} + \frac{3cd(d\cos(a+bx))^{7/2}(c\sin(a+bx))^{3/2}}{70b} - \frac{c(d\cos(a+bx))^{11/2}(c\sin(a+bx))^{3/2}}{7bd} + \frac{3c^2d^4\sqrt{d\cos(a+bx)}E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sin(a+bx)}}{40b\sqrt{\sin(2a+2bx)}}$$

output
```
1/20*c*d^3*(d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(3/2)/b+3/70*c*d*(d*cos(b*x
+a))^(7/2)*(c*sin(b*x+a))^(3/2)/b-1/7*c*(d*cos(b*x+a))^(11/2)*(c*sin(b*x+a
))^(3/2)/b/d-3/40*c^2*d^4*(d*cos(b*x+a))^(1/2)*EllipticE(cos(a+1/4*Pi+b*x)
,2^(1/2))*(c*sin(b*x+a))^(1/2)/b/sin(2*b*x+2*a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.21 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.43

$$\int (d\cos(a+bx))^{9/2}(c\sin(a+bx))^{5/2}\,dx = \frac{2(d\cos(a+bx))^{9/2}\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{7}{4},\frac{11}{4},\sin^2(a+b$$

input
```
Integrate[(d*Cos[a + b*x])^(9/2)*(c*Sin[a + b*x])^(5/2),x]
```

output
```
(2*(d*Cos[a + b*x])^(9/2)*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[-7/4, 7
/4, 11/4, Sin[a + b*x]^2]*Sec[a + b*x]^5*(c*Sin[a + b*x])^(7/2))/(7*b*c)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3048, 3042, 3049, 3042, 3049, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\sin(a+bx))^{5/2}(d\cos(a+bx))^{9/2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int (c\sin(a+bx))^{5/2}(d\cos(a+bx))^{9/2}dx$$

$$\Big\downarrow \ 3048$$

$$\frac{3}{14}c^2\int (d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}dx - \frac{c(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{11/2}}{7bd}$$

$$\Big\downarrow \ 3042$$

$$\frac{3}{14}c^2\int (d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}dx - \frac{c(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{11/2}}{7bd}$$

$$\Big\downarrow \text{3049}$$

$$\frac{3}{14}c^2\left(\frac{7}{10}d^2\int (d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}dx+\frac{d(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{7/2}}{5bc}\right)-\frac{c(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{11/2}}{7bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{14}c^2\left(\frac{7}{10}d^2\int (d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}dx+\frac{d(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{7/2}}{5bc}\right)-\frac{c(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{11/2}}{7bd}$$

$$\Big\downarrow \text{3049}$$

$$\frac{3}{14}c^2\left(\frac{7}{10}d^2\left(\frac{1}{2}d^2\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}dx+\frac{d(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{3bc}\right)+\frac{d(c\sin(a+bx}{}\right.$$
$$\frac{c(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{11/2}}{7bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{14}c^2\left(\frac{7}{10}d^2\left(\frac{1}{2}d^2\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}dx+\frac{d(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{3bc}\right)+\frac{d(c\sin(a+bx}{}\right.$$
$$\frac{c(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{11/2}}{7bd}$$

$$\Big\downarrow \text{3052}$$

$$\frac{3}{14}c^2\left(\frac{7}{10}d^2\left(\frac{d^2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}\int \sqrt{\sin(2a+2bx)}dx}{2\sqrt{\sin(2a+2bx)}}+\frac{d(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{3bc}\right)+\right.$$
$$\frac{c(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{11/2}}{7bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{14}c^2\left(\frac{7}{10}d^2\left(\frac{d^2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}\int \sqrt{\sin(2a+2bx)}dx}{2\sqrt{\sin(2a+2bx)}}+\frac{d(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{3bc}\right)+\right.$$
$$\frac{c(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{11/2}}{7bd}$$

$$\Big\downarrow \text{3119}$$

$$\frac{3}{14}c^2\left(\frac{7}{10}d^2\left(\frac{d^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{2b\sqrt{\sin(2a+2bx)}}+\frac{d(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{3bc}\right)+\frac{d(c}{}\right.$$
$$\frac{c(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{11/2}}{7bd}$$

input
```
Int[(d*Cos[a + b*x])^(9/2)*(c*Sin[a + b*x])^(5/2),x]
```

output
```
-1/7*(c*(d*Cos[a + b*x])^(11/2)*(c*Sin[a + b*x])^(3/2))/(b*d) + (3*c^2*((d
*(d*Cos[a + b*x])^(7/2)*(c*Sin[a + b*x])^(3/2))/(5*b*c) + (7*d^2*((d*(d*Co
s[a + b*x])^(3/2)*(c*Sin[a + b*x])^(3/2))/(3*b*c) + (d^2*Sqrt[d*Cos[a + b*
x]]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(2*b*Sqrt[Sin[2*a +
 2*b*x]])))/10))/14
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3049
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Sin[e + f*x])^n*(a
*Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 8.94 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.64

method	result
default	$\frac{c^2 d^4 \left((21\cos(bx+a)+21)\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\,\sqrt{2\cot(bx+a)-2\csc(bx+a)+2}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\ \operatorname{EllipticF}\left(\sqrt{-\mathrm{co}\right.\right.}{}$

input
```
int((d*cos(b*x+a))^(9/2)*(c*sin(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/560/b*c^2*d^4*((21*cos(b*x+a)+21)*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(2*co
t(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((-c
ot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))+(-42*cos(b*x+a)-42)*(-cot(b*x+a
)+csc(b*x+a)+1)^(1/2)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+a)-csc(
b*x+a))^(1/2)*EllipticE((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))+2*co
s(b*x+a)*(40*cos(b*x+a)^7-52*cos(b*x+a)^5-2*cos(b*x+a)^3-7*cos(b*x+a)+21))
*(c*sin(b*x+a))^(1/2)*(d*cos(b*x+a))^(1/2)*sec(b*x+a)*csc(b*x+a)
```

Fricas [F]

$$\int (d\cos(a+bx))^{9/2}(c\sin(a+bx))^{5/2}\,dx=\int (d\cos(bx+a))^{\frac{9}{2}}\,(c\sin(bx+a))^{\frac{5}{2}}\,dx$$

input
```
integrate((d*cos(b*x+a))^(9/2)*(c*sin(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
integral(-(c^2*d^4*cos(b*x + a)^6 - c^2*d^4*cos(b*x + a)^4)*sqrt(d*cos(b*x
 + a))*sqrt(c*sin(b*x + a)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{9/2}(c\sin(a+bx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((d*cos(b*x+a))**(9/2)*(c*sin(b*x+a))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\cos(a+bx))^{9/2}(c\sin(a+bx))^{5/2}\,dx = \int (d\cos(bx+a))^{\frac{9}{2}}\,(c\sin(bx+a))^{\frac{5}{2}}\,dx$$

input
```
integrate((d*cos(b*x+a))^(9/2)*(c*sin(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*cos(b*x + a))^(9/2)*(c*sin(b*x + a))^(5/2), x)
```

Giac [F]

$$\int (d\cos(a+bx))^{9/2}(c\sin(a+bx))^{5/2}\,dx = \int (d\cos(bx+a))^{\frac{9}{2}}\,(c\sin(bx+a))^{\frac{5}{2}}\,dx$$

input
```
integrate((d*cos(b*x+a))^(9/2)*(c*sin(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate((d*cos(b*x + a))^(9/2)*(c*sin(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{9/2}(c\sin(a+bx))^{5/2}\,dx = \int (d\,\cos(a+b\,x))^{9/2}\,(c\,\sin(a+b\,x))^{5/2}\,dx$$

input
```
int((d*cos(a + b*x))^(9/2)*(c*sin(a + b*x))^(5/2),x)
```

output
```
int((d*cos(a + b*x))^(9/2)*(c*sin(a + b*x))^(5/2), x)
```

Reduce [F]

$$\int (d\cos(a+bx))^{9/2}(c\sin(a+bx))^{5/2}\,dx = \sqrt{d}\,\sqrt{c}\left(\int \sqrt{\sin(bx+a)}\,\sqrt{\cos(bx+a)}\,\cos(bx+a)^4\sin(bx+a)^2\,dx\right)c^2d^4$$

input
```
int((d*cos(b*x+a))^(9/2)*(c*sin(b*x+a))^(5/2),x)
```

output
```
sqrt(d)*sqrt(c)*int(sqrt(sin(a + b*x))*sqrt(cos(a + b*x))*cos(a + b*x)**4*
sin(a + b*x)**2,x)*c**2*d**4
```

3.277 $\int (d\cos(a+bx))^{5/2}(c\sin(a+bx))^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 131

$$\int (d\cos(a+bx))^{5/2}(c\sin(a+bx))^{5/2}\,dx = \frac{cd(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{10b} - \frac{c(d\cos(a+bx))^{7/2}(c\sin(a+bx))^{3/2}}{5bd} + \frac{3c^2d^2\sqrt{d\cos(a+bx)}E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sin(a+bx)}}{20b\sqrt{\sin(2a+2bx)}}$$

output
```
1/10*c*d*(d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(3/2)/b-1/5*c*(d*cos(b*x+a))^
(7/2)*(c*sin(b*x+a))^(3/2)/b/d-3/20*c^2*d^2*(d*cos(b*x+a))^(1/2)*EllipticE
(cos(a+1/4*Pi+b*x),2^(1/2))*(c*sin(b*x+a))^(1/2)/b/sin(2*b*x+2*a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.53

$$\int (d\cos(a+bx))^{5/2}(c\sin(a+bx))^{5/2}\,dx = \frac{2d^2\sqrt{d\cos(a+bx)}\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{7}{4},\frac{11}{4},\sin^2(a+b

input
```
Integrate[(d*Cos[a + b*x])^(5/2)*(c*Sin[a + b*x])^(5/2),x]
```

output
```
(2*d^2*Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[-3/4,
 7/4, 11/4, Sin[a + b*x]^2]*(c*Sin[a + b*x])^(5/2)*Tan[a + b*x])/(7*b)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3048, 3042, 3049, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\sin(a+bx))^{5/2}(d\cos(a+bx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c\sin(a+bx))^{5/2}(d\cos(a+bx))^{5/2}dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{3}{10}c^2\int (d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}dx-\frac{c(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{7/2}}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{10}c^2\int (d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}dx-\frac{c(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{7/2}}{5bd}$$

$$\Big\downarrow \text{3049}$$

$$\frac{3}{10}c^2\left(\frac{1}{2}d^2\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}dx+\frac{d(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{3bc}\right)-\frac{c(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{7/2}}{5bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{10}c^2\left(\frac{1}{2}d^2\int\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}dx+\frac{d(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{3bc}\right)-\frac{c(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{7/2}}{5bd}$$

$\Big\downarrow$ 3052

$$\frac{3}{10}c^2\left(\frac{d^2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{2\sqrt{\sin(2a+2bx)}}+\frac{d(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{3bc}\right)-\frac{c(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{7/2}}{5bd}$$

$\Big\downarrow$ 3042

$$\frac{3}{10}c^2\left(\frac{d^2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{2\sqrt{\sin(2a+2bx)}}+\frac{d(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{3bc}\right)-\frac{c(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{7/2}}{5bd}$$

$\Big\downarrow$ 3119

$$\frac{3}{10}c^2\left(\frac{d^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{2b\sqrt{\sin(2a+2bx)}}+\frac{d(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{3bc}\right)-\frac{c(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{7/2}}{5bd}$$

input
```
Int[(d*Cos[a + b*x])^(5/2)*(c*Sin[a + b*x])^(5/2),x]
```

output
```
-1/5*(c*(d*Cos[a + b*x])^(7/2)*(c*Sin[a + b*x])^(3/2))/(b*d) + (3*c^2*((d*
(d*Cos[a + b*x])^(3/2)*(c*Sin[a + b*x])^(3/2))/(3*b*c) + (d^2*Sqrt[d*Cos[a
 + b*x]]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(2*b*Sqrt[Sin[
2*a + 2*b*x]])))/10
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3049
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Sin[e + f*x])^n*(a
*Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(112) = 224$.

Time = 5.81 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.00

method	result
default	$\frac{\left((3\cos(bx+a)+3)\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\,\sqrt{2\cot(bx+a)-2\csc(bx+a)+2}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\ \operatorname{EllipticF}\left(\sqrt{-\cot(bx+a\right.\right.}{}$

input
```
int((d*cos(b*x+a))^(5/2)*(c*sin(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/40/b*((3*cos(b*x+a)+3)*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(2*cot(b*x+a)-2*
csc(b*x+a)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((-cot(b*x+a)+c
sc(b*x+a)+1)^(1/2),1/2*2^(1/2))+(-6*cos(b*x+a)-6)*(-cot(b*x+a)+csc(b*x+a)+
1)^(1/2)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)
*EllipticE((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))+cos(b*x+a)*(8*cos
(b*x+a)^5-12*cos(b*x+a)^3-2*cos(b*x+a)+6))*(c*sin(b*x+a))^(1/2)*(d*cos(b*x
+a))^(1/2)*c^2*d^2*sec(b*x+a)*csc(b*x+a)
```

Fricas [F]

$$\int (d\cos(a+bx))^{5/2}(c\sin(a+bx))^{5/2}\,dx = \int (d\cos(bx+a))^{\frac{5}{2}}(c\sin(bx+a))^{\frac{5}{2}}\,dx$$

input
```
integrate((d*cos(b*x+a))^(5/2)*(c*sin(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
integral(-(c^2*d^2*cos(b*x + a)^4 - c^2*d^2*cos(b*x + a)^2)*sqrt(d*cos(b*x
 + a))*sqrt(c*sin(b*x + a)), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{5/2}(c\sin(a+bx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((d*cos(b*x+a))**(5/2)*(c*sin(b*x+a))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\cos(a+bx))^{5/2}(c\sin(a+bx))^{5/2}\,dx = \int (d\cos(bx+a))^{\frac{5}{2}}\,(c\sin(bx+a))^{\frac{5}{2}}\,dx$$

input
```
integrate((d*cos(b*x+a))^(5/2)*(c*sin(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*cos(b*x + a))^(5/2)*(c*sin(b*x + a))^(5/2), x)
```

Giac [F]

$$\int (d\cos(a+bx))^{5/2}(c\sin(a+bx))^{5/2}\,dx = \int (d\cos(bx+a))^{\frac{5}{2}}\,(c\sin(bx+a))^{\frac{5}{2}}\,dx$$

input
```
integrate((d*cos(b*x+a))^(5/2)*(c*sin(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate((d*cos(b*x + a))^(5/2)*(c*sin(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{5/2}(c\sin(a+bx))^{5/2}\,dx = \int (d\,\cos(a+b\,x))^{5/2}\,(c\,\sin(a+b\,x))^{5/2}\,dx$$

input
```
int((d*cos(a + b*x))^(5/2)*(c*sin(a + b*x))^(5/2),x)
```

output
```
int((d*cos(a + b*x))^(5/2)*(c*sin(a + b*x))^(5/2), x)
```

Reduce [F]

$$\int (d\cos(a+bx))^{5/2}(c\sin(a+bx))^{5/2}\,dx = \sqrt{d}\,\sqrt{c}\left(\int \sqrt{\sin(bx+a)}\,\sqrt{\cos(bx+a)}\,\cos(bx+a)^2\sin(bx+a)^2\,dx\right)c^2d^2$$

input
```
int((d*cos(b*x+a))^(5/2)*(c*sin(b*x+a))^(5/2),x)
```

output
```
sqrt(d)*sqrt(c)*int(sqrt(sin(a + b*x))*sqrt(cos(a + b*x))*cos(a + b*x)**2*
sin(a + b*x)**2,x)*c**2*d**2
```

3.278 $\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 95

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^{5/2}\,dx = -\frac{c(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{3bd} + \frac{c^2\sqrt{d\cos(a+bx)}E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sin(a+bx)}}{2b\sqrt{\sin(2a+2bx)}}$$

output
```
-1/3*c*(d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(3/2)/b/d-1/2*c^2*(d*cos(b*x+a)
)^(1/2)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*(c*sin(b*x+a))^(1/2)/b/sin(2*
b*x+2*a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.71

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^{5/2}\,dx = \frac{2\sqrt{d\cos(a+bx)}\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{7}{4},\frac{11}{4},\sin^2(a+bx)\right)(c\sin(a+bx))^{5/}}{7b}$$

input
```
Integrate[Sqrt[d*Cos[a + b*x]]*(c*Sin[a + b*x])^(5/2),x]
```

output
```
(2*Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, 7/4,
 11/4, Sin[a + b*x]^2]*(c*Sin[a + b*x])^(5/2)*Tan[a + b*x])/(7*b)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3048, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\sin(a+bx))^{5/2}\sqrt{d\cos(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c\sin(a+bx))^{5/2}\sqrt{d\cos(a+bx)}dx$$

$$\Big\downarrow \text{3048}$$

$$\frac{1}{2}c^2\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}dx-\frac{c(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{3bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}c^2\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}dx-\frac{c(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{3bd}$$

$$\Big\downarrow \text{3052}$$

$$\frac{c^2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}\int \sqrt{\sin(2a+2bx)}dx}{2\sqrt{\sin(2a+2bx)}}-\frac{c(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{3bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{c^2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}\int \sqrt{\sin(2a+2bx)}dx}{2\sqrt{\sin(2a+2bx)}}-\frac{c(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{3bd}$$

$$\Big\downarrow \text{3119}$$

$$\frac{c^2 E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{2b\sqrt{\sin(2a+2bx)}}-\frac{c(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{3bd}$$

input
```
Int[Sqrt[d*Cos[a + b*x]]*(c*Sin[a + b*x])^(5/2),x]
```

output
```
-1/3*(c*(d*Cos[a + b*x])^(3/2)*(c*Sin[a + b*x])^(3/2))/(b*d) + (c^2*Sqrt[d
*Cos[a + b*x]]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(2*b*Sqr
t[Sin[2*a + 2*b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(82) = 164$.

Time = 5.52 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.62

method	result
default	$\frac{\left((3\cos(bx+a)+3)\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\,\sqrt{2\cot(bx+a)-2\csc(bx+a)+2}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,\operatorname{EllipticF}\left(\sqrt{-\cot(bx+a}\right.\right.}{}$

input
```
int((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/12/b*((3*cos(b*x+a)+3)*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(2*cot(b*x+a)-2*
csc(b*x+a)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((-cot(b*x+a)+c
sc(b*x+a)+1)^(1/2),1/2*2^(1/2))+(-6*cos(b*x+a)-6)*(-cot(b*x+a)+csc(b*x+a)+
1)^(1/2)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)
*EllipticE((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))+cos(b*x+a)*(4*cos
(b*x+a)^3-10*cos(b*x+a)+6))*(d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2)*c^2*
sec(b*x+a)*csc(b*x+a)
```

Fricas [F]

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^{5/2}\,dx = \int \sqrt{d\cos(bx+a)}(c\sin(bx+a))^{\frac{5}{2}}\,dx$$

input
```
integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
integral(-(c^2*cos(b*x + a)^2 - c^2)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x +
 a)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((d*cos(b*x+a))**(1/2)*(c*sin(b*x+a))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^{5/2}\,dx = \int \sqrt{d\cos(bx+a)}(c\sin(bx+a))^{\frac{5}{2}}\,dx$$

input
```
integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*cos(b*x + a))*(c*sin(b*x + a))^(5/2), x)
```

Giac [F]

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^{5/2}\,dx = \int \sqrt{d\cos(bx+a)}(c\sin(bx+a))^{\frac{5}{2}}\,dx$$

input
```
integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*cos(b*x + a))*(c*sin(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^{5/2}\,dx = \int \sqrt{d\,\cos\left(a+b\,x\right)}\,\left(c\,\sin\left(a+b\,x\right)\right)^{5/2}\,dx$$

input
```
int((d*cos(a + b*x))^(1/2)*(c*sin(a + b*x))^(5/2),x)
```

output
```
int((d*cos(a + b*x))^(1/2)*(c*sin(a + b*x))^(5/2), x)
```

Reduce [F]

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^{5/2}\,dx = \sqrt{d}\,\sqrt{c}\left(\int \sqrt{\sin(bx+a)}\,\sqrt{\cos(bx+a)}\,\sin(bx+a)^2\,dx\right)c^2$$

input
```
int((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(5/2),x)
```

output
```
sqrt(d)*sqrt(c)*int(sqrt(sin(a + b*x))*sqrt(cos(a + b*x))*sin(a + b*x)**2,
x)*c**2
```

3.279 $\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 94

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{3/2}}\,dx = \frac{2c(c\sin(a+bx))^{3/2}}{bd\sqrt{d\cos(a+bx)}} - \frac{3c^2\sqrt{d\cos(a+bx)}E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sin(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}$$

output
```
2*c*(c*sin(b*x+a))^(3/2)/b/d/(d*cos(b*x+a))^(1/2)+3*c^2*(d*cos(b*x+a))^(1/
2)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*(c*sin(b*x+a))^(1/2)/b/d^2/sin(2*b
*x+2*a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.71

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{3/2}}\,dx = \frac{2\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{7}{4},\frac{11}{4},\sin^2(a+bx)\right)(c\sin(a+bx))^{7/2}}{7bcd\sqrt{d\cos(a+bx)}}$$

input
```
Integrate[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(3/2),x]
```

output
```
(2*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[5/4, 7/4, 11/4, Sin[a + b*x]^2
]*(c*Sin[a + b*x])^(7/2))/(7*b*c*d*Sqrt[d*Cos[a + b*x]])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 3046, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3046}$$

$$\frac{2c(c\sin(a+bx))^{3/2}}{bd\sqrt{d\cos(a+bx)}}-\frac{3c^2\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}dx}{d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c(c\sin(a+bx))^{3/2}}{bd\sqrt{d\cos(a+bx)}}-\frac{3c^2\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}dx}{d^2}$$

$$\Big\downarrow \text{3052}$$

$$\frac{2c(c\sin(a+bx))^{3/2}}{bd\sqrt{d\cos(a+bx)}}-\frac{3c^2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}\int \sqrt{\sin(2a+2bx)}dx}{d^2\sqrt{\sin(2a+2bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c(c\sin(a+bx))^{3/2}}{bd\sqrt{d\cos(a+bx)}}-\frac{3c^2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}\int \sqrt{\sin(2a+2bx)}dx}{d^2\sqrt{\sin(2a+2bx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2c(c\sin(a+bx))^{3/2}}{bd\sqrt{d\cos(a+bx)}}-\frac{3c^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}$$

input
```
Int[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(3/2),x]
```

output
```
(2*c*(c*Sin[a + b*x])^(3/2))/(b*d*Sqrt[d*Cos[a + b*x]]) - (3*c^2*Sqrt[d*Co
s[a + b*x]]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(b*d^2*Sqrt
[Sin[2*a + 2*b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3046
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Sin[e + f
*x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(85) = 170$.

Time = 5.38 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.53

method	result
default	$\frac{\left((-3\cos(bx+a)-3)\sqrt{2\cot(bx+a)-2\csc(bx+a)+2}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\ \mathrm{EllipticF}\left(\sqrt{-\cot(bx+a)+\csc(bx+a)+1},\frac{\sqrt{2}}{2}\right)\sqrt{-\mathrm{co}}\right.}{}$

input
```
int((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*((-3*cos(b*x+a)-3)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+a)-c
sc(b*x+a))^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))*(
-cot(b*x+a)+csc(b*x+a)+1)^(1/2)+(6*cos(b*x+a)+6)*(2*cot(b*x+a)-2*csc(b*x+a
)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((-cot(b*x+a)+csc(b*x+a)
+1)^(1/2),1/2*2^(1/2))*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)+2*cos(b*x+a)^2-6*c
os(b*x+a)+4)*(c*sin(b*x+a))^(1/2)*c^2/(d*cos(b*x+a))^(1/2)/d*csc(b*x+a)
```

Fricas [F]

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{(d\cos(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
integral(-(c^2*cos(b*x + a)^2 - c^2)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x +
 a))/(d^2*cos(b*x + a)^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*sin(b*x+a))**(5/2)/(d*cos(b*x+a))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{(d\cos(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(3/2), x)
```

Giac [F]

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{(d\cos(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{(c\,\sin(a+b\,x))^{5/2}}{(d\,\cos(a+b\,x))^{3/2}}\,dx$$

input
```
int((c*sin(a + b*x))^(5/2)/(d*cos(a + b*x))^(3/2),x)
```

output
```
int((c*sin(a + b*x))^(5/2)/(d*cos(a + b*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{3/2}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sin(bx+a)}\,\sqrt{\cos(bx+a)}\,\sin(bx+a)^2}{\cos(bx+a)^2}dx\right)c^2}{d^2}$$

input
```
int((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sin(a + b*x))*sqrt(cos(a + b*x))*sin(a + b*x)**
2)/cos(a + b*x)**2,x)*c**2)/d**2
```

3.280 $\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 133

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{7/2}}\,dx = \frac{2c(c\sin(a+bx))^{3/2}}{5bd(d\cos(a+bx))^{5/2}} - \frac{6c(c\sin(a+bx))^{3/2}}{5bd^3\sqrt{d\cos(a+bx)}} + \frac{6c^2\sqrt{d\cos(a+bx)}E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sin(a+bx)}}{5bd^4\sqrt{\sin(2a+2bx)}}$$

output
```
2/5*c*(c*sin(b*x+a))^(3/2)/b/d/(d*cos(b*x+a))^(5/2)-6/5*c*(c*sin(b*x+a))^(
3/2)/b/d^3/(d*cos(b*x+a))^(1/2)-6/5*c^2*(d*cos(b*x+a))^(1/2)*EllipticE(cos
(a+1/4*Pi+b*x),2^(1/2))*(c*sin(b*x+a))^(1/2)/b/d^4/sin(2*b*x+2*a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.53

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{7/2}}\,dx = \frac{2\cos^2(a+bx)^{5/4}\cot(a+bx)\,\text{Hypergeometric2F1}\left(\frac{7}{4},\frac{9}{4},\frac{11}{4},\sin^2(a+bx)\right)(c\sin(a}{7bc^2(d\cos(a+bx))^{7/2}}$$

input
```
Integrate[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(7/2),x]
```

output
```
(2*(Cos[a + b*x]^2)^(5/4)*Cot[a + b*x]*Hypergeometric2F1[7/4, 9/4, 11/4, S
in[a + b*x]^2]*(c*Sin[a + b*x])^(9/2))/(7*b*c^2*(d*Cos[a + b*x])^(7/2))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3046, 3042, 3051, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{7/2}}dx$$

$$\Big\downarrow \text{3046}$$

$$\frac{2c(c\sin(a+bx))^{3/2}}{5bd(d\cos(a+bx))^{5/2}}-\frac{3c^2\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{3/2}}dx}{5d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c(c\sin(a+bx))^{3/2}}{5bd(d\cos(a+bx))^{5/2}}-\frac{3c^2\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{3/2}}dx}{5d^2}$$

$$\Big\downarrow \text{3051}$$

$$\frac{2c(c\sin(a+bx))^{3/2}}{5bd(d\cos(a+bx))^{5/2}}-\frac{3c^2\left(\frac{2(c\sin(a+bx))^{3/2}}{bcd\sqrt{d\cos(a+bx)}}-\frac{2\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}dx}{d^2}\right)}{5d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c(c\sin(a+bx))^{3/2}}{5bd(d\cos(a+bx))^{5/2}}-\frac{3c^2\left(\frac{2(c\sin(a+bx))^{3/2}}{bcd\sqrt{d\cos(a+bx)}}-\frac{2\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}dx}{d^2}\right)}{5d^2}$$

$$\Big\downarrow \text{3052}$$

$$\frac{2c(c\sin(a+bx))^{3/2}}{5bd(d\cos(a+bx))^{5/2}}-\frac{3c^2\left(\frac{2(c\sin(a+bx))^{3/2}}{bcd\sqrt{d\cos(a+bx)}}-\frac{2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{d^2\sqrt{\sin(2a+2bx)}}\right)}{5d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c(c\sin(a+bx))^{3/2}}{5bd(d\cos(a+bx))^{5/2}}-\frac{3c^2\left(\frac{2(c\sin(a+bx))^{3/2}}{bcd\sqrt{d\cos(a+bx)}}-\frac{2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{d^2\sqrt{\sin(2a+2bx)}}\right)}{5d^2}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2c(c\sin(a+bx))^{3/2}}{5bd(d\cos(a+bx))^{5/2}}-\frac{3c^2\left(\frac{2(c\sin(a+bx))^{3/2}}{bcd\sqrt{d\cos(a+bx)}}-\frac{2E\left(a+bx-\frac{\pi}{4}|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}\right)}{5d^2}$$

input
```
Int[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(7/2),x]
```

output
```
(2*c*(c*Sin[a + b*x])^(3/2))/(5*b*d*(d*Cos[a + b*x])^(5/2)) - (3*c^2*((2*(
c*Sin[a + b*x])^(3/2))/(b*c*d*Sqrt[d*Cos[a + b*x]]) - (2*Sqrt[d*Cos[a + b*
x]]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(b*d^2*Sqrt[Sin[2*a
 + 2*b*x]])))/(5*d^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3046
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))    Int[(a*Sin[e + f
*x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3051
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Simp[(m + n + 2)/(a^2*(m + 1))   Int[(b*Sin[e + f*x
])^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m
, -1] && IntegersQ[2*m, 2*n]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(114) = 228$.

Time = 4.83 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.95

method	result
default	$\frac{\sqrt{c\sin(bx+a)}\, c^2 \left(\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\,\sqrt{2\cot(bx+a)-2\csc(bx+a)+2}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,\operatorname{EllipticF}\left(\sqrt{-\cot(bx+a}\right.\right.}{}$

input
```
int((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/5/b*(c*sin(b*x+a))^(1/2)*c^2/(d*cos(b*x+a))^(1/2)/d^3*((-cot(b*x+a)+csc(
b*x+a)+1)^(1/2)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a)
)^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))*(3*cot(b*x
+a)+3*csc(b*x+a))+(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(2*cot(b*x+a)-2*csc(b*x
+a)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((-cot(b*x+a)+csc(b*x+
a)+1)^(1/2),1/2*2^(1/2))*(-6*cot(b*x+a)-6*csc(b*x+a))+6*cot(b*x+a)-8*csc(b
*x+a)+2*csc(b*x+a)*sec(b*x+a)^2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.59

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{7/2}}\,dx = \frac{3i\sqrt{i\,cd}c^2\cos(bx+a)^3 E(\arcsin(\cos(bx+a)+i\sin(bx+a))\,|-1)-3i\sqrt{-i\,c}}{}$$

input
```
integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

output
```
1/5*(3*I*sqrt(I*c*d)*c^2*cos(b*x + a)^3*elliptic_e(arcsin(cos(b*x + a) + I
*sin(b*x + a)), -1) - 3*I*sqrt(-I*c*d)*c^2*cos(b*x + a)^3*elliptic_e(arcsi
n(cos(b*x + a) - I*sin(b*x + a)), -1) - 3*I*sqrt(I*c*d)*c^2*cos(b*x + a)^3
*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1) + 3*I*sqrt(-I*c*d)*
c^2*cos(b*x + a)^3*elliptic_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1) -
 2*(3*c^2*cos(b*x + a)^2 - c^2)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*
sin(b*x + a))/(b*d^4*cos(b*x + a)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*sin(b*x+a))**(5/2)/(d*cos(b*x+a))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{(d\cos(bx+a))^{\frac{7}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(7/2),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(7/2), x)
```

Giac [F]

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{(d\cos(bx+a))^{\frac{7}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(7/2),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{7/2}}\,dx = \int \frac{(c\,\sin(a+b\,x))^{5/2}}{(d\,\cos(a+b\,x))^{7/2}}\,dx$$

input
```
int((c*sin(a + b*x))^(5/2)/(d*cos(a + b*x))^(7/2),x)
```

output
```
int((c*sin(a + b*x))^(5/2)/(d*cos(a + b*x))^(7/2), x)
```

Reduce [F]

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{7/2}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sin(bx+a)}\,\sqrt{\cos(bx+a)}\,\sin(bx+a)^2}{\cos(bx+a)^4}dx\right)c^2}{d^4}$$

input
```
int((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(7/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sin(a + b*x))*sqrt(cos(a + b*x))*sin(a + b*x)**
2)/cos(a + b*x)**4,x)*c**2)/d**4
```

3.281 $\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{11/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 168

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{11/2}}\,dx = \frac{2c(c\sin(a+bx))^{3/2}}{9bd(d\cos(a+bx))^{9/2}} - \frac{2c(c\sin(a+bx))^{3/2}}{15bd^3(d\cos(a+bx))^{5/2}} - \frac{4c(c\sin(a+bx))^{3/2}}{15bd^5\sqrt{d\cos(a+bx)}} + \frac{4c^2\sqrt{d\cos(a+bx)}\,E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sin(a+bx)}}{15bd^6\sqrt{\sin(2a+2bx)}}$$

output
```
2/9*c*(c*sin(b*x+a))^(3/2)/b/d/(d*cos(b*x+a))^(9/2)-2/15*c*(c*sin(b*x+a))^
(3/2)/b/d^3/(d*cos(b*x+a))^(5/2)-4/15*c*(c*sin(b*x+a))^(3/2)/b/d^5/(d*cos(
b*x+a))^(1/2)-4/15*c^2*(d*cos(b*x+a))^(1/2)*EllipticE(cos(a+1/4*Pi+b*x),2^
(1/2))*(c*sin(b*x+a))^(1/2)/b/d^6/sin(2*b*x+2*a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.43

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{11/2}}\,dx = \frac{2\cos^5(a+bx)\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{7}{4},\frac{13}{4},\frac{11}{4},\sin^2(a+bx)\right)(c\sin}{7bc(d\cos(a+bx))^{11/2}}$$

input
```
Integrate[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(11/2),x]
```

output
```
(2*Cos[a + b*x]^5*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[7/4, 13/4, 11/4
, Sin[a + b*x]^2]*(c*Sin[a + b*x])^(7/2))/(7*b*c*(d*Cos[a + b*x])^(11/2))
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3046, 3042, 3051, 3042, 3051, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{11/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{11/2}}dx$$

$$\Big\downarrow \text{3046}$$

$$\frac{2c(c\sin(a+bx))^{3/2}}{9bd(d\cos(a+bx))^{9/2}}-\frac{c^2\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{7/2}}dx}{3d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c(c\sin(a+bx))^{3/2}}{9bd(d\cos(a+bx))^{9/2}}-\frac{c^2\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{7/2}}dx}{3d^2}$$

$$\Big\downarrow \text{3051}$$

$$\frac{2c(c\sin(a+bx))^{3/2}}{9bd(d\cos(a+bx))^{9/2}}-\frac{c^2\left(\frac{2\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{3/2}}dx}{5d^2}+\frac{2(c\sin(a+bx))^{3/2}}{5bcd(d\cos(a+bx))^{5/2}}\right)}{3d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c(c\sin(a+bx))^{3/2}}{9bd(d\cos(a+bx))^{9/2}}-\frac{c^2\left(\frac{2\int\frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{3/2}}dx}{5d^2}+\frac{2(c\sin(a+bx))^{3/2}}{5bcd(d\cos(a+bx))^{5/2}}\right)}{3d^2}$$

$\Big\downarrow$ 3051

$$\frac{2c(c\sin(a+bx))^{3/2}}{9bd(d\cos(a+bx))^{9/2}}-\frac{c^2\left(\frac{2\left(\frac{2(c\sin(a+bx))^{3/2}}{bcd\sqrt{d\cos(a+bx)}}-\frac{2\int\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}dx}{d^2}\right)}{5d^2}+\frac{2(c\sin(a+bx))^{3/2}}{5bcd(d\cos(a+bx))^{5/2}}\right)}{3d^2}$$

$\Big\downarrow$ 3042

$$\frac{2c(c\sin(a+bx))^{3/2}}{9bd(d\cos(a+bx))^{9/2}}-\frac{c^2\left(\frac{2\left(\frac{2(c\sin(a+bx))^{3/2}}{bcd\sqrt{d\cos(a+bx)}}-\frac{2\int\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}dx}{d^2}\right)}{5d^2}+\frac{2(c\sin(a+bx))^{3/2}}{5bcd(d\cos(a+bx))^{5/2}}\right)}{3d^2}$$

$\Big\downarrow$ 3052

$$\frac{2c(c\sin(a+bx))^{3/2}}{9bd(d\cos(a+bx))^{9/2}}-\frac{c^2\left(\frac{2\left(\frac{2(c\sin(a+bx))^{3/2}}{bcd\sqrt{d\cos(a+bx)}}-\frac{2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{d^2\sqrt{\sin(2a+2bx)}}\right)}{5d^2}+\frac{2(c\sin(a+bx))^{3/2}}{5bcd(d\cos(a+bx))^{5/2}}\right)}{3d^2}$$

$\Big\downarrow$ 3042

$$\frac{2c(c\sin(a+bx))^{3/2}}{9bd(d\cos(a+bx))^{9/2}}-\frac{c^2\left(\frac{2\left(\frac{2(c\sin(a+bx))^{3/2}}{bcd\sqrt{d\cos(a+bx)}}-\frac{2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}\int\sqrt{\sin(2a+2bx)}dx}{d^2\sqrt{\sin(2a+2bx)}}\right)}{5d^2}+\frac{2(c\sin(a+bx))^{3/2}}{5bcd(d\cos(a+bx))^{5/2}}\right)}{3d^2}$$

$\Big\downarrow$ 3119

$$\frac{2c(c\sin(a+bx))^{3/2}}{9bd(d\cos(a+bx))^{9/2}}-\frac{c^2\left(\frac{2\left(\frac{2(c\sin(a+bx))^{3/2}}{bcd\sqrt{d\cos(a+bx)}}-\frac{2E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}\right)}{5d^2}+\frac{2(c\sin(a+bx))^{3/2}}{5bcd(d\cos(a+bx))^{5/2}}\right)}{3d^2}$$

input
```
Int[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(11/2),x]
```

output
```
(2*c*(c*Sin[a + b*x])^(3/2))/(9*b*d*(d*Cos[a + b*x])^(9/2)) - (c^2*((2*(c*
Sin[a + b*x])^(3/2))/(5*b*c*d*(d*Cos[a + b*x])^(5/2)) + (2*((2*(c*Sin[a +
b*x])^(3/2))/(b*c*d*Sqrt[d*Cos[a + b*x]]) - (2*Sqrt[d*Cos[a + b*x]]*Ellipt
icE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(b*d^2*Sqrt[Sin[2*a + 2*b*x]]
)))/(5*d^2)))/(3*d^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3046
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Sin[e + f
*x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3051
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Simp[(m + n + 2)/(a^2*(m + 1))   Int[(b*Sin[e + f*x
])^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m
, -1] && IntegersQ[2*m, 2*n]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 4.90 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.64

method	result
default	$\frac{2\sqrt{c\sin(bx+a)}\,c^2\left(\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\,\sqrt{2\cot(bx+a)-2\csc(bx+a)+2}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\ \mathrm{EllipticF}\left(\sqrt{-\cot(bx+\right.\right.}{}$

input
```
int((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(11/2),x,method=_RETURNVERBOSE)
```

output
```
2/45/b*(c*sin(b*x+a))^(1/2)*c^2/(d*cos(b*x+a))^(1/2)/d^5*((-cot(b*x+a)+csc
(b*x+a)+1)^(1/2)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a
))^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))*(3*cot(b*
x+a)+3*csc(b*x+a))+(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(2*cot(b*x+a)-2*csc(b*
x+a)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticE((-cot(b*x+a)+csc(b*x
+a)+1)^(1/2),1/2*2^(1/2))*(-6*cot(b*x+a)-6*csc(b*x+a))+6*cot(b*x+a)-3*csc(
b*x+a)-8*csc(b*x+a)*sec(b*x+a)^2+5*csc(b*x+a)*sec(b*x+a)^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.33

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{11/2}}\,dx =$$
$$-\frac{2\left(-3i\sqrt{i\,cd}c^2\cos(bx+a)^5\,E(\arcsin(\cos(bx+a)+i\,\sin(bx+a))\,|-1)+3i\sqrt{-i\,cd}c^2\cos(bx+a)^5\,E(\right.}{}$$

input
```
integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(11/2),x, algorithm="fricas"
)
```

output
```
-2/45*(-3*I*sqrt(I*c*d)*c^2*cos(b*x + a)^5*elliptic_e(arcsin(cos(b*x + a)
+ I*sin(b*x + a)), -1) + 3*I*sqrt(-I*c*d)*c^2*cos(b*x + a)^5*elliptic_e(ar
csin(cos(b*x + a) - I*sin(b*x + a)), -1) + 3*I*sqrt(I*c*d)*c^2*cos(b*x + a
)^5*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1) - 3*I*sqrt(-I*c*
d)*c^2*cos(b*x + a)^5*elliptic_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1
) + (6*c^2*cos(b*x + a)^4 + 3*c^2*cos(b*x + a)^2 - 5*c^2)*sqrt(d*cos(b*x +
 a))*sqrt(c*sin(b*x + a))*sin(b*x + a))/(b*d^6*cos(b*x + a)^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*sin(b*x+a))**(5/2)/(d*cos(b*x+a))**(11/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{11/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{(d\cos(bx+a))^{\frac{11}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(11/2),x, algorithm="maxima"
)
```

output
```
integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(11/2), x)
```

Giac [F]

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{11/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{(d\cos(bx+a))^{\frac{11}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(11/2),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(11/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{11/2}}\,dx = \int \frac{(c\,\sin(a+b\,x))^{5/2}}{(d\,\cos(a+b\,x))^{11/2}}\,dx$$

input
```
int((c*sin(a + b*x))^(5/2)/(d*cos(a + b*x))^(11/2),x)
```

output
```
int((c*sin(a + b*x))^(5/2)/(d*cos(a + b*x))^(11/2), x)
```

Reduce [F]

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{11/2}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sin(bx+a)}\,\sqrt{\cos(bx+a)}\,\sin(bx+a)^2}{\cos(bx+a)^6}dx\right)c^2}{d^6}$$

input
```
int((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(11/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sin(a + b*x))*sqrt(cos(a + b*x))*sin(a + b*x)**
2)/cos(a + b*x)**6,x)*c**2)/d**6
```

3.282 $\int \frac{(c\sin(a+bx))^{5/2}}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 245

$$\int \frac{(c\sin(a+bx))^{5/2}}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{3c^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{4\sqrt{2}b\sqrt{d}} + \frac{3c^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{4\sqrt{2}b\sqrt{d}} - \frac{3c^{5/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}(\sqrt{c}+\sqrt{c}\tan(a+bx))}\right)}{4\sqrt{2}b\sqrt{d}} - \frac{c\sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}}{2bd}$$

output

```
-3/8*c^(5/2)*arctan(1-2^(1/2)*d^(1/2)*(c*sin(b*x+a))^(1/2)/c^(1/2)/(d*cos(
b*x+a))^(1/2))*2^(1/2)/b/d^(1/2)+3/8*c^(5/2)*arctan(1+2^(1/2)*d^(1/2)*(c*s
in(b*x+a))^(1/2)/c^(1/2)/(d*cos(b*x+a))^(1/2))*2^(1/2)/b/d^(1/2)-3/8*c^(5/
2)*arctanh(2^(1/2)*d^(1/2)*(c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2)/(c^(1
/2)+c^(1/2)*tan(b*x+a)))*2^(1/2)/b/d^(1/2)-1/2*c*(d*cos(b*x+a))^(1/2)*(c*s
in(b*x+a))^(3/2)/b/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.27

$$\int \frac{(c\sin(a+bx))^{5/2}}{\sqrt{d\cos(a+bx)}}\,dx = \frac{2\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{7}{4},\frac{11}{4},\sin^2(a+bx)\right)(c\sin(a+bx))^{5/2}\,\text{ta}}{7b\sqrt{d\cos(a+bx)}}$$

input
```
Integrate[(c*Sin[a + b*x])^(5/2)/Sqrt[d*Cos[a + b*x]],x]
```

output
```
(2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/4, 7/4, 11/4, Sin[a + b*x]^2
]*(c*Sin[a + b*x])^(5/2)*Tan[a + b*x])/(7*b*Sqrt[d*Cos[a + b*x]])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.33, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3042, 3048, 3042, 3054, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c\sin(a+bx))^{5/2}}{\sqrt{d\cos(a+bx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(c\sin(a+bx))^{5/2}}{\sqrt{d\cos(a+bx)}}dx$$

$$\Big\downarrow \; 3048$$

$$\frac{3}{4}c^2\int \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}dx-\frac{c(c\sin(a+bx))^{3/2}\sqrt{d\cos(a+bx)}}{2bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{3}{4}c^2\int \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}dx-\frac{c(c\sin(a+bx))^{3/2}\sqrt{d\cos(a+bx)}}{2bd}$$

$$\Big\downarrow \text{3054}$$

$$\frac{3c^3d\int \frac{c\tan(a+bx)}{d(\tan^2(a+bx)c^2+c^2)}d\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{2b}-\frac{c(c\sin(a+bx))^{3/2}\sqrt{d\cos(a+bx)}}{2bd}$$

$$\Big\downarrow \text{826}$$

$$\frac{3c^3d\left(\frac{\int \frac{\tan(a+bx)c+c}{\tan^2(a+bx)c^2+c^2}d\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{2d}-\frac{\int \frac{c-c\tan(a+bx)}{\tan^2(a+bx)c^2+c^2}d\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{2d}\right)}{2b}-\frac{c(c\sin(a+bx))^{3/2}\sqrt{d\cos(a+bx)}}{2bd}$$

$$\Big\downarrow \text{1476}$$

$$\frac{3c^3d\left(\frac{\frac{\int \frac{1}{\frac{\tan(a+bx)c}{d}+\frac{c}{d}-\frac{\sqrt{2}\sqrt{c\sin(a+bx)}\sqrt{c}}{\sqrt{d}\sqrt{d\cos(a+bx)}}}d\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{2d}+\frac{\int \frac{1}{\frac{\tan(a+bx)c}{d}+\frac{c}{d}+\frac{\sqrt{2}\sqrt{c\sin(a+bx)}\sqrt{c}}{\sqrt{d}\sqrt{d\cos(a+bx)}}}d\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{2d}}{2d}-\frac{\int \frac{c-c\tan(a+bx)}{\tan^2(a+bx)c^2+c^2}d\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{2d}\right)}{2b}-\frac{c(c\sin(a+bx))^{3/2}\sqrt{d\cos(a+bx)}}{2bd}$$

$$\Big\downarrow \text{1082}$$

$$\frac{3c^3d\left(\frac{\frac{\int \frac{1}{-\frac{c\tan(a+bx)}{d}-1}d\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\int \frac{1}{-\frac{c\tan(a+bx)}{d}-1}d\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}}{2d}-\frac{\int \frac{c-c\tan(a+bx)}{\tan^2(a+bx)c^2+c^2}d\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{2d}\right)}{2b}-\frac{c(c\sin(a+bx))^{3/2}\sqrt{d\cos(a+bx)}}{2bd}$$

$$\Big\downarrow \text{217}$$

$$\frac{3c^3d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}}{2d}-\frac{\int \frac{c-c\tan(a+bx)}{\tan^2(a+bx)c^2+c^2}d\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{2d}\right)}{2b}-\frac{c(c\sin(a+bx))^{3/2}\sqrt{d\cos(a+bx)}}{2bd}$$

$$\Big\downarrow \text{1479}$$

$$3c^3d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}}{2d}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{c}-\frac{2\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{\sqrt{d}\left(\frac{\tan(a+bx)c}{d}+\frac{c}{d}-\frac{\sqrt{2}\sqrt{c\sin(a+bx)}\sqrt{c}}{\sqrt{d}\sqrt{d\cos(a+bx)}}\right)}d\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{2\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\int -\frac{\cdots}{\sqrt{d}(\cdots}\cdots}{\cdots}}{2d}\cdots}{2b}\cdots\right.$$

$$\frac{c(c\sin(a+bx))^{3/2}\sqrt{d\cos(a+bx)}}{2bd}$$

$$\Big\downarrow \; 25$$

$$3c^3d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}}{2d}-\frac{\frac{\int \frac{\sqrt{2}\sqrt{c}-\frac{2\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{\sqrt{d}\left(\frac{\tan(a+bx)c}{d}+\frac{c}{d}-\frac{\sqrt{2}\sqrt{c\sin(a+bx)}\sqrt{c}}{\sqrt{d}\sqrt{d\cos(a+bx)}}\right)}d\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{2\sqrt{2}\sqrt{c}\sqrt{d}}+\frac{\int \frac{\sqrt{2}(\cdots}{\sqrt{d}\left(\tan(a\cdots\right.}\cdots}{\cdots}}{2d}\cdots}{2b}\cdots\right.$$

$$\frac{c(c\sin(a+bx))^{3/2}\sqrt{d\cos(a+bx)}}{2bd}$$

$$\Big\downarrow \; 27$$

$$3c^3d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}}{2d}-\frac{\frac{\int \frac{\sqrt{2}\sqrt{c}-\frac{2\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{\frac{\tan(a+bx)c}{d}+\frac{c}{d}-\frac{\sqrt{2}\sqrt{c\sin(a+bx)}\sqrt{c}}{\sqrt{d}\sqrt{d\cos(a+bx)}}}d\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{2\sqrt{2}\sqrt{c}d}+\frac{\int \frac{\sqrt{c}+\frac{\sqrt{2}\sqrt{\cdots}}{\sqrt{\cdots}}}{\frac{\tan(a+bx)c}{d}+\frac{c}{d}\cdots}\cdots}{\cdots}}{2d}\cdots}{2b}\cdots\right.$$

$$\frac{c(c\sin(a+bx))^{3/2}\sqrt{d\cos(a+bx)}}{2bd}$$

$$\Big\downarrow \; 1103$$

$$3c^3d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}}{2d}-\frac{\frac{\log\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+c\tan(a+bx)+c\right)}{2\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\log\left(-\frac{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\cdots\right.}{2\sqrt{2}\sqrt{c}\sqrt{d}}}{2d}\cdots}{2b}\cdots\right.$$

$$\frac{c(c\sin(a+bx))^{3/2}\sqrt{d\cos(a+bx)}}{2bd}$$

input
```
Int[(c*Sin[a + b*x])^(5/2)/Sqrt[d*Cos[a + b*x]],x]
```

output
```
(3*c^3*d*((-(ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/(Sqrt[c]*Sq
rt[d*Cos[a + b*x]])]/(Sqrt[2]*Sqrt[c]*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*Sqrt
[d]*Sqrt[c*Sin[a + b*x]])/(Sqrt[c]*Sqrt[d*Cos[a + b*x]])]/(Sqrt[2]*Sqrt[c]
*Sqrt[d]))/(2*d) - (-1/2*Log[c - (Sqrt[2]*Sqrt[c]*Sqrt[d]*Sqrt[c*Sin[a + b
*x]])/Sqrt[d*Cos[a + b*x]] + c*Tan[a + b*x]]/(Sqrt[2]*Sqrt[c]*Sqrt[d]) + L
og[c + (Sqrt[2]*Sqrt[c]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/Sqrt[d*Cos[a + b*x]]
 + c*Tan[a + b*x]]/(2*Sqrt[2]*Sqrt[c]*Sqrt[d]))/(2*d)))/(2*b) - (c*Sqrt[d*
Cos[a + b*x]]*(c*Sin[a + b*x])^(3/2))/(2*b*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3054
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k*a*(b/f)   Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 436 vs. 2(183) = 366.

Time = 6.98 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.78

method	result
default	$-\frac{\sqrt{2}\, c^{2}\left(-3 \ln \left(-\frac{\cos (bx+a) \cot (bx+a)-2 \cot (bx+a)-2 \sqrt{-\frac{2 \sin (bx+a) \cos (bx+a)}{(\cos (bx+a)+1)^{2}}}\, \sin (bx+a)-2 \cos (bx+a)-\sin (bx+a)+\csc (bx+a)+2}{\cos (bx+a)-1}\right)+3 \ln\right.}{}$

input
```
int((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/256/b*2^(1/2)*c^2*(-3*ln(-(cos(b*x+a)*cot(b*x+a)-2*cot(b*x+a)-2*(-2*sin
(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)-2*cos(b*x+a)-sin(b*x
+a)+csc(b*x+a)+2)/(cos(b*x+a)-1))+3*ln(-(cos(b*x+a)*cot(b*x+a)-2*cot(b*x+a
)+2*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)-2*cos(b*x
+a)-sin(b*x+a)+csc(b*x+a)+2)/(cos(b*x+a)-1))+6*arctan((-(-2*sin(b*x+a)*cos
(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)+cos(b*x+a)-1)/(cos(b*x+a)-1))-6
*arctan(((-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)+cos(
b*x+a)-1)/(cos(b*x+a)-1))+(4*cos(b*x+a)+4)*sin(b*x+a)*(-2*sin(b*x+a)*cos(b
*x+a)/(cos(b*x+a)+1)^2)^(1/2))*(c*sin(b*x+a))^(1/2)*cos(b*x+a)*sin(b*x+a)^
3/(d*cos(b*x+a))^(1/2)/(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sec
(1/2*b*x+1/2*a)^5*csc(1/2*b*x+1/2*a)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 534 vs. 2(183) = 366.

Time = 0.15 (sec) , antiderivative size = 534, normalized size of antiderivative = 2.18

$$\int \frac{(c\sin(a+bx))^{5/2}}{\sqrt{d\cos(a+bx)}}\, dx =$$

$$-\frac{3\sqrt{2}c^2 d\sqrt{\frac{c}{d}} \arctan\left(\frac{2\,c\cos(bx+a)^3-2\,c\cos(bx+a)^2\sin(bx+a)+\sqrt{2}\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}\sqrt{\frac{c}{d}}-2\,c\cos(bx+a)}{2\left(c\cos(bx+a)^3+c\cos(bx+a)^2\sin(bx+a)-c\cos(bx+a)\right)}\right)+3\sqrt{2}c^2 d\sqrt{}}{}$$

input
```
integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
-1/32*(3*sqrt(2)*c^2*d*sqrt(c/d)*arctan(1/2*(2*c*cos(b*x + a)^3 - 2*c*cos(
b*x + a)^2*sin(b*x + a) + sqrt(2)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)
)*sqrt(c/d) - 2*c*cos(b*x + a))/(c*cos(b*x + a)^3 + c*cos(b*x + a)^2*sin(b
*x + a) - c*cos(b*x + a))) + 3*sqrt(2)*c^2*d*sqrt(c/d)*arctan(-1/2*(2*c*co
s(b*x + a)^3 - 2*c*cos(b*x + a)^2*sin(b*x + a) - sqrt(2)*sqrt(d*cos(b*x +
a))*sqrt(c*sin(b*x + a))*sqrt(c/d) - 2*c*cos(b*x + a))/(c*cos(b*x + a)^3 +
 c*cos(b*x + a)^2*sin(b*x + a) - c*cos(b*x + a))) - 6*sqrt(2)*c^2*d*sqrt(c
/d)*arctan(-1/2*sqrt(2)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*sqrt(c/d
)*(cos(b*x + a) - sin(b*x + a))/(c*cos(b*x + a)*sin(b*x + a))) + 3*sqrt(2)
*c^2*d*sqrt(c/d)*log(2*sqrt(2)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*s
qrt(c/d)*(cos(b*x + a) + sin(b*x + a)) + 4*c*cos(b*x + a)*sin(b*x + a) + c
) - 3*sqrt(2)*c^2*d*sqrt(c/d)*log(-2*sqrt(2)*sqrt(d*cos(b*x + a))*sqrt(c*s
in(b*x + a))*sqrt(c/d)*(cos(b*x + a) + sin(b*x + a)) + 4*c*cos(b*x + a)*si
n(b*x + a) + c) + 16*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*c^2*sin(b*x
 + a))/(b*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{5/2}}{\sqrt{d\cos(a+bx)}}\,dx = \text{Timed out}$$

input
```
integrate((c*sin(b*x+a))**(5/2)/(d*cos(b*x+a))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c\sin(a+bx))^{5/2}}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{\sqrt{d\cos(bx+a)}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a))^(5/2)/sqrt(d*cos(b*x + a)), x)
```

Giac [F]

$$\int \frac{(c\sin(a+bx))^{5/2}}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{\sqrt{d\cos(bx+a)}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a))^(5/2)/sqrt(d*cos(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{5/2}}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{(c\,\sin(a+b\,x))^{5/2}}{\sqrt{d\,\cos(a+b\,x)}}\,dx$$

input
```
int((c*sin(a + b*x))^(5/2)/(d*cos(a + b*x))^(1/2),x)
```

output
```
int((c*sin(a + b*x))^(5/2)/(d*cos(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(c\sin(a+bx))^{5/2}}{\sqrt{d\cos(a+bx)}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sin(bx+a)}\,\sqrt{\cos(bx+a)}\,\sin(bx+a)^2}{\cos(bx+a)}dx\right)c^2}{d}$$

input
```
int((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(1/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sin(a + b*x))*sqrt(cos(a + b*x))*sin(a + b*x)**
2)/cos(a + b*x),x)*c**2)/d
```

3.283 $\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 237

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{5/2}}\,dx = \frac{c^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}bd^{5/2}} - \frac{c^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}bd^{5/2}} + \frac{c^{5/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}\left(\sqrt{c}+\sqrt{c}\tan(a+bx)\right)}\right)}{\sqrt{2}bd^{5/2}} + \frac{2c(c\sin(a+bx))^{3/2}}{3bd(d\cos(a+bx))^{3/2}}$$

output
```
1/2*c^(5/2)*arctan(1-2^(1/2)*d^(1/2)*(c*sin(b*x+a))^(1/2)/c^(1/2)/(d*cos(b
*x+a))^(1/2))*2^(1/2)/b/d^(5/2)-1/2*c^(5/2)*arctan(1+2^(1/2)*d^(1/2)*(c*si
n(b*x+a))^(1/2)/c^(1/2)/(d*cos(b*x+a))^(1/2))*2^(1/2)/b/d^(5/2)+1/2*c^(5/2
)*arctanh(2^(1/2)*d^(1/2)*(c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2)/(c^(1/
2)+c^(1/2)*tan(b*x+a)))*2^(1/2)/b/d^(5/2)+2/3*c*(c*sin(b*x+a))^(3/2)/b/d/(
d*cos(b*x+a))^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.28

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{5/2}}\,dx=\frac{2\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{7}{4},\frac{7}{4},\frac{11}{4},\sin^2(a+bx)\right)(c\sin(a+bx))^{7/2}}{7bcd(d\cos(a+bx))^{3/2}}$$

input
```
Integrate[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(5/2),x]
```

output
```
(2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[7/4, 7/4, 11/4, Sin[a + b*x]^2
]*(c*Sin[a + b*x])^(7/2))/(7*b*c*d*(d*Cos[a + b*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.37, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3042, 3046, 3042, 3054, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3046}$$

$$\frac{2c(c\sin(a+bx))^{3/2}}{3bd(d\cos(a+bx))^{3/2}}-\frac{c^2\int \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}dx}{d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c(c\sin(a+bx))^{3/2}}{3bd(d\cos(a+bx))^{3/2}}-\frac{c^2\int \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}dx}{d^2}$$

$$\Big\downarrow \text{3054}$$

$$\frac{2c(c\sin(a+bx))^{3/2}}{3bd(d\cos(a+bx))^{3/2}}-\frac{2c^3\int\frac{c\tan(a+bx)}{d(\tan^2(a+bx)c^2+c^2)}d\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{bd}$$

$$\Big\downarrow \text{826}$$

$$\frac{2c(c\sin(a+bx))^{3/2}}{3bd(d\cos(a+bx))^{3/2}}-\frac{2c^3\left(\frac{\int\frac{\tan(a+bx)c+c}{\tan^2(a+bx)c^2+c^2}d\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{2d}-\frac{\int\frac{c-c\tan(a+bx)}{\tan^2(a+bx)c^2+c^2}d\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{2d}\right)}{bd}$$

$$\Big\downarrow \text{1476}$$

$$\frac{2c(c\sin(a+bx))^{3/2}}{3bd(d\cos(a+bx))^{3/2}}-\frac{2c^3\left(\frac{\frac{\int\frac{1}{\frac{\tan(a+bx)c}{d}+\frac{c}{d}-\frac{\sqrt{2}\sqrt{c\sin(a+bx)}\sqrt{c}}{\sqrt{d}\sqrt{d\cos(a+bx)}}}d\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{2d}+\frac{\int\frac{1}{\frac{\tan(a+bx)c}{d}+\frac{c}{d}+\frac{\sqrt{2}\sqrt{c\sin(a+bx)}\sqrt{c}}{\sqrt{d}\sqrt{d\cos(a+bx)}}}d\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{2d}}{2d}-\frac{\int\frac{c-c\tan(a+bx)}{\tan^2(a+bx)c^2+c^2}d\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{2d}\right)}{bd}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2c(c\sin(a+bx))^{3/2}}{3bd(d\cos(a+bx))^{3/2}}-\frac{2c^3\left(\frac{\frac{\int\frac{1}{-\frac{c\tan(a+bx)}{d}-1}d\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\int\frac{1}{-\frac{c\tan(a+bx)}{d}-1}d\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}}{2d}-\frac{\int\frac{c-c\tan(a+bx)}{\tan^2(a+bx)c^2+c^2}d\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{2d}\right)}{bd}$$

$$\Big\downarrow \text{217}$$

$$\frac{2c(c\sin(a+bx))^{3/2}}{3bd(d\cos(a+bx))^{3/2}}-\frac{2c^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}}{2d}-\frac{\int\frac{c-c\tan(a+bx)}{\tan^2(a+bx)c^2+c^2}d\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{2d}\right)}{bd}$$

$$\Big\downarrow \text{1479}$$

$$\frac{2c(c\sin(a+bx))^{3/2}}{3bd(d\cos(a+bx))^{3/2}}-\frac{2c^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}}{2d}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{c}-\frac{2\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{\sqrt{d}\left(\frac{\tan(a+bx)c}{d}+\frac{c}{d}-\frac{\sqrt{2}\sqrt{c\sin(a+bx)}\sqrt{c}}{\sqrt{d}\sqrt{d\cos(a+bx)}}\right)}d\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{2\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\int -\frac{\ldots}{\sqrt{d}\left(\ldots\right)}\ldots}{\ldots}}{2d}\ldots\right)}{bd}$$

$\downarrow$ 25

$$\frac{2c(c\sin(a+bx))^{3/2}}{3bd(d\cos(a+bx))^{3/2}}-\frac{2c^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}}{2d}-\frac{\frac{\int \frac{\sqrt{2}\sqrt{c}-\frac{2\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{\sqrt{d}\left(\frac{\tan(a+bx)c}{d}+\frac{c}{d}-\frac{\sqrt{2}\sqrt{c\sin(a+bx)}\sqrt{c}}{\sqrt{d}\sqrt{d\cos(a+bx)}}\right)}d\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{2\sqrt{2}\sqrt{c}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\ldots\right)}{\sqrt{d}\left(\frac{\tan(a+\ldots}{d}\ldots\right)}\ldots}{\ldots}}{2d}\ldots\right)}{bd}$$

$\downarrow$ 27

$$\frac{2c(c\sin(a+bx))^{3/2}}{3bd(d\cos(a+bx))^{3/2}}-\frac{2c^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}}{2d}-\frac{\frac{\int \frac{\sqrt{2}\sqrt{c}-\frac{2\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{\frac{\tan(a+bx)c}{d}+\frac{c}{d}-\frac{\sqrt{2}\sqrt{c\sin(a+bx)}\sqrt{c}}{\sqrt{d}\sqrt{d\cos(a+bx)}}}d\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}}{2\sqrt{2}\sqrt{c}d}+\frac{\int \frac{\sqrt{c}+\frac{\sqrt{2}\sqrt{d}\ldots}{\sqrt{d\ldots}}}{\frac{\tan(a+bx)c}{d}+\frac{c}{d}+\ldots}\ldots}{\ldots}}{2d}\ldots\right)}{bd}$$

$\downarrow$ 1103

$$\frac{2c(c\sin(a+bx))^{3/2}}{3bd(d\cos(a+bx))^{3/2}}-\frac{2c^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}}{2d}-\frac{\frac{\log\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+c\tan(a+bx)+c\right)}{2\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\log\left(-\frac{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\ldots\right)}{2\sqrt{2}\sqrt{c}\sqrt{d}}}{2d}\ldots\right)}{bd}$$

input
```
Int[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(5/2),x]
```

output
```
(-2*c^3*((-(ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/(Sqrt[c]*Sqr
t[d*Cos[a + b*x]])]/(Sqrt[2]*Sqrt[c]*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*Sqrt[
d]*Sqrt[c*Sin[a + b*x]])/(Sqrt[c]*Sqrt[d*Cos[a + b*x]])]/(Sqrt[2]*Sqrt[c]*
Sqrt[d]))/(2*d) - (-1/2*Log[c - (Sqrt[2]*Sqrt[c]*Sqrt[d]*Sqrt[c*Sin[a + b*
x]])/Sqrt[d*Cos[a + b*x]] + c*Tan[a + b*x]]/(Sqrt[2]*Sqrt[c]*Sqrt[d]) + Lo
g[c + (Sqrt[2]*Sqrt[c]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/Sqrt[d*Cos[a + b*x]]
 + c*Tan[a + b*x]]/(2*Sqrt[2]*Sqrt[c]*Sqrt[d]))/(2*d)))/(b*d) + (2*c*(c*Sin
[a + b*x])^(3/2))/(3*b*d*(d*Cos[a + b*x])^(3/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3046
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Sin[e + f
*x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3054
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k*a*(b/f)   Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 507 vs. 2(183) = 366.

Time = 6.22 (sec) , antiderivative size = 508, normalized size of antiderivative = 2.14

method	result
default	$\frac{\sqrt{c\sin(bx+a)}\,c^2\left(\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}\,\ln\left(-\frac{\cos(bx+a)\cot(bx+a)-2\cot(bx+a)-2\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}\,\sin(bx+a)-2\cos(bx+a)-}{\cos(bx+a)-1}\right.\right.}{}$

input
```
int((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/12/b*(c*sin(b*x+a))^(1/2)*c^2/d^2/(d*cos(b*x+a))^(1/2)*((-2*sin(b*x+a)*c
os(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*ln(-(cos(b*x+a)*cot(b*x+a)-2*cot(b*x+a)-
2*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)-2*cos(b*x+a
)-sin(b*x+a)+csc(b*x+a)+2)/(cos(b*x+a)-1))*(3*cot(b*x+a)+3*csc(b*x+a))+(-2
*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*arctan(((-2*sin(b*x+a)*cos(
b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)+cos(b*x+a)-1)/(cos(b*x+a)-1))*(6
*cot(b*x+a)+6*csc(b*x+a))+(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2
)*ln(-(cos(b*x+a)*cot(b*x+a)-2*cot(b*x+a)+2*(-2*sin(b*x+a)*cos(b*x+a)/(cos
(b*x+a)+1)^2)^(1/2)*sin(b*x+a)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(cos(
b*x+a)-1))*(-3*cot(b*x+a)-3*csc(b*x+a))+(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x
+a)+1)^2)^(1/2)*arctan((-(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)
*sin(b*x+a)+cos(b*x+a)-1)/(cos(b*x+a)-1))*(-6*cot(b*x+a)-6*csc(b*x+a))+8*t
an(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 582 vs. 2(183) = 366.

Time = 0.17 (sec) , antiderivative size = 582, normalized size of antiderivative = 2.46

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
1/24*(3*sqrt(2)*c^2*d*sqrt(c/d)*arctan(1/2*(2*c*cos(b*x + a)^3 - 2*c*cos(b
*x + a)^2*sin(b*x + a) + sqrt(2)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))
*sqrt(c/d) - 2*c*cos(b*x + a))/(c*cos(b*x + a)^3 + c*cos(b*x + a)^2*sin(b*
x + a) - c*cos(b*x + a)))*cos(b*x + a)^2 + 3*sqrt(2)*c^2*d*sqrt(c/d)*arcta
n(-1/2*(2*c*cos(b*x + a)^3 - 2*c*cos(b*x + a)^2*sin(b*x + a) - sqrt(2)*sqr
t(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*sqrt(c/d) - 2*c*cos(b*x + a))/(c*co
s(b*x + a)^3 + c*cos(b*x + a)^2*sin(b*x + a) - c*cos(b*x + a)))*cos(b*x +
a)^2 - 6*sqrt(2)*c^2*d*sqrt(c/d)*arctan(-1/2*sqrt(2)*sqrt(d*cos(b*x + a))*
sqrt(c*sin(b*x + a))*sqrt(c/d)*(cos(b*x + a) - sin(b*x + a))/(c*cos(b*x +
a)*sin(b*x + a)))*cos(b*x + a)^2 + 3*sqrt(2)*c^2*d*sqrt(c/d)*cos(b*x + a)^
2*log(2*sqrt(2)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*sqrt(c/d)*(cos(b
*x + a) + sin(b*x + a)) + 4*c*cos(b*x + a)*sin(b*x + a) + c) - 3*sqrt(2)*c
^2*d*sqrt(c/d)*cos(b*x + a)^2*log(-2*sqrt(2)*sqrt(d*cos(b*x + a))*sqrt(c*s
in(b*x + a))*sqrt(c/d)*(cos(b*x + a) + sin(b*x + a)) + 4*c*cos(b*x + a)*si
n(b*x + a) + c) + 16*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*c^2*sin(b*x
 + a))/(b*d^3*cos(b*x + a)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*sin(b*x+a))**(5/2)/(d*cos(b*x+a))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{(d\cos(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(5/2), x)
```

Giac [F]

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{(d\cos(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{(c\,\sin(a+b\,x))^{5/2}}{(d\,\cos(a+b\,x))^{5/2}}\,dx$$

input
```
int((c*sin(a + b*x))^(5/2)/(d*cos(a + b*x))^(5/2),x)
```

output
```
int((c*sin(a + b*x))^(5/2)/(d*cos(a + b*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{5/2}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sin(bx+a)}\,\sqrt{\cos(bx+a)}\,\sin(bx+a)^{2}}{\cos(bx+a)^{3}}dx\right)c^{2}}{d^{3}}$$

input
```
int((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(5/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sin(a + b*x))*sqrt(cos(a + b*x))*sin(a + b*x)**
2)/cos(a + b*x)**3,x)*c**2)/d**3
```

3.284 $\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{9/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 37

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{9/2}}\,dx=\frac{2(c\sin(a+bx))^{7/2}}{7bcd(d\cos(a+bx))^{7/2}}$$

output
```
2/7*(c*sin(b*x+a))^(7/2)/b/c/d/(d*cos(b*x+a))^(7/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{9/2}}\,dx=\frac{2\cot(a+bx)(c\sin(a+bx))^{9/2}}{7bc^2(d\cos(a+bx))^{9/2}}$$

input
```
Integrate[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(9/2),x]
```

output
```
(2*Cot[a + b*x]*(c*Sin[a + b*x])^(9/2))/(7*b*c^2*(d*Cos[a + b*x])^(9/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3042, 3043}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{9/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{9/2}}dx$$

$$\Big\downarrow \text{3043}$$

$$\frac{2(c\sin(a+bx))^{7/2}}{7bcd(d\cos(a+bx))^{7/2}}$$

input
```
Int[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(9/2),x]
```

output
```
(2*(c*Sin[a + b*x])^(7/2))/(7*b*c*d*(d*Cos[a + b*x])^(7/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3043
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Maple [A] (verified)

Time = 5.43 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{2\sqrt{c\sin(bx+a)}\,c^{2}\tan(bx+a)^{3}}{7b\,d^{4}\sqrt{d\cos(bx+a)}}$	40

input
```
int((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/7/b*(c*sin(b*x+a))^(1/2)*c^2/d^4/(d*cos(b*x+a))^(1/2)*tan(b*x+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{9/2}}\,dx = -\frac{2\left(c^{2}\cos\left(bx+a\right)^{2}-c^{2}\right)\sqrt{d\cos\left(bx+a\right)}\sqrt{c\sin\left(bx+a\right)}\sin\left(bx+a\right)}{7\,bd^{5}\cos\left(bx+a\right)^{4}}$$

input
```
integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(9/2),x, algorithm="fricas")
```

output
```
-2/7*(c^2*cos(b*x + a)^2 - c^2)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*
sin(b*x + a)/(b*d^5*cos(b*x + a)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*sin(b*x+a))**(5/2)/(d*cos(b*x+a))**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{9/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{(d\cos(bx+a))^{\frac{9}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(9/2),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(9/2), x)
```

Giac [F]

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{9/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{(d\cos(bx+a))^{\frac{9}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(9/2),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(9/2), x)
```

Mupad [B] (verification not implemented)

Time = 27.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.41

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{9/2}}\,dx = \frac{2\,c^2\,\sqrt{c\,\sin\left(a+b\,x\right)}\,\left(3\,\sin\left(2\,a+2\,b\,x\right)-\sin\left(6\,a+6\,b\,x\right)\right)}{7\,b\,d^4\,\sqrt{d\,\cos\left(a+b\,x\right)}\,\left(15\,\cos\left(2\,a+2\,b\,x\right)+6\,\cos\left(4\,a+4\,b\,x\right)+\cos\left(6\,a+6\,b\right.}$$

input
```
int((c*sin(a + b*x))^(5/2)/(d*cos(a + b*x))^(9/2),x)
```

output
```
(2*c^2*(c*sin(a + b*x))^(1/2)*(3*sin(2*a + 2*b*x) - sin(6*a + 6*b*x)))/(7*
b*d^4*(d*cos(a + b*x))^(1/2)*(15*cos(2*a + 2*b*x) + 6*cos(4*a + 4*b*x) + c
os(6*a + 6*b*x) + 10))
```

Reduce [F]

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{9/2}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sin(bx+a)}\,\sqrt{\cos(bx+a)}\,\sin(bx+a)^2}{\cos(bx+a)^5}dx\right)c^2}{d^5}$$

input
```
int((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(9/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sin(a + b*x))*sqrt(cos(a + b*x))*sin(a + b*x)**
2)/cos(a + b*x)**5,x)*c**2)/d**5
```

3.285 $\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{13/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 106

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{13/2}}\,dx = \frac{2c(c\sin(a+bx))^{3/2}}{11bd(d\cos(a+bx))^{11/2}} - \frac{6c(c\sin(a+bx))^{3/2}}{77bd^3(d\cos(a+bx))^{7/2}} - \frac{8c(c\sin(a+bx))^{3/2}}{77bd^5(d\cos(a+bx))^{3/2}}$$

output
```
2/11*c*(c*sin(b*x+a))^(3/2)/b/d/(d*cos(b*x+a))^(11/2)-6/77*c*(c*sin(b*x+a)
)^(3/2)/b/d^3/(d*cos(b*x+a))^(7/2)-8/77*c*(c*sin(b*x+a))^(3/2)/b/d^5/(d*co
s(b*x+a))^(3/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.54

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{13/2}}\,dx = \frac{2c^4(9+2\cos(2(a+bx)))\tan^5(a+bx)}{77bd^6\sqrt{d\cos(a+bx)}(c\sin(a+bx))^{3/2}}$$

input
```
Integrate[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(13/2),x]
```

output
```
(2*c^4*(9 + 2*Cos[2*(a + b*x)])*Tan[a + b*x]^5)/(77*b*d^6*Sqrt[d*Cos[a + b
*x]]*(c*Sin[a + b*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 3046, 3042, 3051, 3042, 3043}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{13/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{13/2}}dx$$

$$\Big\downarrow \text{3046}$$

$$\frac{2c(c\sin(a+bx))^{3/2}}{11bd(d\cos(a+bx))^{11/2}}-\frac{3c^2\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{9/2}}dx}{11d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c(c\sin(a+bx))^{3/2}}{11bd(d\cos(a+bx))^{11/2}}-\frac{3c^2\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{9/2}}dx}{11d^2}$$

$$\Big\downarrow \text{3051}$$

$$\frac{2c(c\sin(a+bx))^{3/2}}{11bd(d\cos(a+bx))^{11/2}}-\frac{3c^2\left(\frac{4\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{5/2}}dx}{7d^2}+\frac{2(c\sin(a+bx))^{3/2}}{7bcd(d\cos(a+bx))^{7/2}}\right)}{11d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c(c\sin(a+bx))^{3/2}}{11bd(d\cos(a+bx))^{11/2}}-\frac{3c^2\left(\frac{4\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{5/2}}dx}{7d^2}+\frac{2(c\sin(a+bx))^{3/2}}{7bcd(d\cos(a+bx))^{7/2}}\right)}{11d^2}$$

$$\Big\downarrow \text{3043}$$

$$\frac{2c(c\sin(a+bx))^{3/2}}{11bd(d\cos(a+bx))^{11/2}}-\frac{3c^2\left(\frac{8(c\sin(a+bx))^{3/2}}{21bcd^3(d\cos(a+bx))^{3/2}}+\frac{2(c\sin(a+bx))^{3/2}}{7bcd(d\cos(a+bx))^{7/2}}\right)}{11d^2}$$

input
```
Int[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(13/2),x]
```

output
```
(2*c*(c*Sin[a + b*x])^(3/2))/(11*b*d*(d*Cos[a + b*x])^(11/2)) - (3*c^2*((2
*(c*Sin[a + b*x])^(3/2))/(7*b*c*d*(d*Cos[a + b*x])^(7/2)) + (8*(c*Sin[a +
b*x])^(3/2))/(21*b*c*d^3*(d*Cos[a + b*x])^(3/2))))/(11*d^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3043
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

rule 3046
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Sin[e + f
*x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3051
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Simp[(m + n + 2)/(a^2*(m + 1))   Int[(b*Sin[e + f*x
])^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m
, -1] && IntegersQ[2*m, 2*n]
```

Maple [A] (verified)

Time = 5.47 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.58

method	result	size
default	$\frac{2\sqrt{c\sin(bx+a)}\,c^{2}\left(4\tan(bx+a)^{3}+7\sec(bx+a)^{2}\tan(bx+a)^{3}\right)}{77b\sqrt{d\cos(bx+a)}\,d^{6}}$	61

input
```
int((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(13/2),x,method=_RETURNVERBOSE)
```

output
```
2/77/b*(c*sin(b*x+a))^(1/2)*c^2/(d*cos(b*x+a))^(1/2)/d^6*(4*tan(b*x+a)^3+7
*sec(b*x+a)^2*tan(b*x+a)^3)
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.70

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{13/2}}\,dx = -\frac{2\left(4c^{2}\cos(bx+a)^{4}+3c^{2}\cos(bx+a)^{2}-7c^{2}\right)\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}\sin(bx+a)}{77\,bd^{7}\cos(bx+a)^{6}}$$

input
```
integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(13/2),x, algorithm="fricas"
)
```

output
```
-2/77*(4*c^2*cos(b*x + a)^4 + 3*c^2*cos(b*x + a)^2 - 7*c^2)*sqrt(d*cos(b*x
 + a))*sqrt(c*sin(b*x + a))*sin(b*x + a)/(b*d^7*cos(b*x + a)^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{13/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*sin(b*x+a))**(5/2)/(d*cos(b*x+a))**(13/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{13/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{(d\cos(bx+a))^{\frac{13}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(13/2),x, algorithm="maxima"
)
```

output
```
integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(13/2), x)
```

Giac [F]

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{13/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{(d\cos(bx+a))^{\frac{13}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(13/2),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(13/2), x)
```

Mupad [B] (verification not implemented)

Time = 33.06 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.66

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{13/2}}\,dx =$$

$$-\frac{e^{-a\,5i-b\,x\,5i}\sqrt{c\left(\frac{e^{-a\,1i-b\,x\,1i}\,1i}{2}-\frac{e^{a\,1i+b\,x\,1i}\,1i}{2}\right)}\left(\frac{96\,c^2\,e^{a\,5i+b\,x\,5i}\,\sin(3\,a+3\,b\,x)}{77\,b\,d^6}+\frac{16\,c^2\,e^{a\,5i+b\,x\,5i}\,\sin(5\,a+5\,b\,x)}{77\,b\,d^6}-\frac{368\,c^2\,e^{a\,5i+b\,x}}{77\,b}\right.}{32\cos\left(a+b\,x\right)^5\sqrt{d\left(\frac{e^{-a\,1i-b\,x\,1i}}{2}+\frac{e^{a\,1i+b\,x\,1i}}{2}\right)}}$$

input
```
int((c*sin(a + b*x))^(5/2)/(d*cos(a + b*x))^(13/2),x)
```

output
```
-(exp(- a*5i - b*x*5i)*(c*((exp(- a*1i - b*x*1i)*1i)/2 - (exp(a*1i + b*x*1
i)*1i)/2))^(1/2)*((96*c^2*exp(a*5i + b*x*5i)*sin(3*a + 3*b*x))/(77*b*d^6)
+ (16*c^2*exp(a*5i + b*x*5i)*sin(5*a + 5*b*x))/(77*b*d^6) - (368*c^2*exp(a
*5i + b*x*5i)*sin(a + b*x))/(77*b*d^6)))/(32*cos(a + b*x)^5*(d*(exp(- a*1i
 - b*x*1i)/2 + exp(a*1i + b*x*1i)/2))^(1/2))
```

Reduce [F]

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{13/2}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sin(bx+a)}\,\sqrt{\cos(bx+a)}\,\sin(bx+a)^2}{\cos(bx+a)^7}dx\right)c^2}{d^7}$$

input
```
int((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(13/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sin(a + b*x))*sqrt(cos(a + b*x))*sin(a + b*x)**
2)/cos(a + b*x)**7,x)*c**2)/d**7
```

3.286 $\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{17/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 141

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{17/2}}\,dx = \frac{2c(c\sin(a+bx))^{3/2}}{15bd(d\cos(a+bx))^{15/2}} - \frac{2c(c\sin(a+bx))^{3/2}}{55bd^3(d\cos(a+bx))^{11/2}} - \frac{16c(c\sin(a+bx))^{3/2}}{385bd^5(d\cos(a+bx))^{7/2}} - \frac{64c(c\sin(a+bx))^{3/2}}{1155bd^7(d\cos(a+bx))^{3/2}}$$

output
```
2/15*c*(c*sin(b*x+a))^(3/2)/b/d/(d*cos(b*x+a))^(15/2)-2/55*c*(c*sin(b*x+a)
)^(3/2)/b/d^3/(d*cos(b*x+a))^(11/2)-16/385*c*(c*sin(b*x+a))^(3/2)/b/d^5/(d
*cos(b*x+a))^(7/2)-64/1155*c*(c*sin(b*x+a))^(3/2)/b/d^7/(d*cos(b*x+a))^(3/
2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.48

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{17/2}}\,dx = \frac{2\sqrt{d\cos(a+bx)}(117+44\cos(2(a+bx))+4\cos(4(a+bx)))\sec^8(a+bx)(c\,\mathrm{si}}{1155bcd^9}$$

input
```
Integrate[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(17/2),x]
```

output
```
(2*Sqrt[d*Cos[a + b*x]]*(117 + 44*Cos[2*(a + b*x)] + 4*Cos[4*(a + b*x)])*S
ec[a + b*x]^8*(c*Sin[a + b*x])^(7/2))/(1155*b*c*d^9)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3046, 3042, 3051, 3042, 3051, 3042, 3043}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{17/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{17/2}}dx$$

$$\Big\downarrow \text{3046}$$

$$\frac{2c(c\sin(a+bx))^{3/2}}{15bd(d\cos(a+bx))^{15/2}}-\frac{c^2\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{13/2}}dx}{5d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c(c\sin(a+bx))^{3/2}}{15bd(d\cos(a+bx))^{15/2}}-\frac{c^2\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{13/2}}dx}{5d^2}$$

$$\Big\downarrow \text{3051}$$

$$\frac{2c(c\sin(a+bx))^{3/2}}{15bd(d\cos(a+bx))^{15/2}}-\frac{c^2\left(\frac{8\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{9/2}}dx}{11d^2}+\frac{2(c\sin(a+bx))^{3/2}}{11bcd(d\cos(a+bx))^{11/2}}\right)}{5d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2c(c\sin(a+bx))^{3/2}}{15bd(d\cos(a+bx))^{15/2}}-\frac{c^2\left(\frac{8\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{9/2}}dx}{11d^2}+\frac{2(c\sin(a+bx))^{3/2}}{11bcd(d\cos(a+bx))^{11/2}}\right)}{5d^2}$$

$$\Big\downarrow \text{3051}$$

$$\frac{2c(c\sin(a+bx))^{3/2}}{15bd(d\cos(a+bx))^{15/2}}-\frac{c^2\left(\frac{8\left(\frac{4\int\frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{5/2}}dx}{7d^2}+\frac{2(c\sin(a+bx))^{3/2}}{7bcd(d\cos(a+bx))^{7/2}}\right)}{11d^2}+\frac{2(c\sin(a+bx))^{3/2}}{11bcd(d\cos(a+bx))^{11/2}}\right)}{5d^2}$$

$\Big\downarrow$ 3042

$$\frac{2c(c\sin(a+bx))^{3/2}}{15bd(d\cos(a+bx))^{15/2}}-\frac{c^2\left(\frac{8\left(\frac{4\int\frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{5/2}}dx}{7d^2}+\frac{2(c\sin(a+bx))^{3/2}}{7bcd(d\cos(a+bx))^{7/2}}\right)}{11d^2}+\frac{2(c\sin(a+bx))^{3/2}}{11bcd(d\cos(a+bx))^{11/2}}\right)}{5d^2}$$

$\Big\downarrow$ 3043

$$\frac{2c(c\sin(a+bx))^{3/2}}{15bd(d\cos(a+bx))^{15/2}}-\frac{c^2\left(\frac{8\left(\frac{8(c\sin(a+bx))^{3/2}}{21bcd^3(d\cos(a+bx))^{3/2}}+\frac{2(c\sin(a+bx))^{3/2}}{7bcd(d\cos(a+bx))^{7/2}}\right)}{11d^2}+\frac{2(c\sin(a+bx))^{3/2}}{11bcd(d\cos(a+bx))^{11/2}}\right)}{5d^2}$$

input
```
Int[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(17/2),x]
```

output
```
(2*c*(c*Sin[a + b*x])^(3/2))/(15*b*d*(d*Cos[a + b*x])^(15/2)) - (c^2*((2*(
c*Sin[a + b*x])^(3/2))/(11*b*c*d*(d*Cos[a + b*x])^(11/2)) + (8*((2*(c*Sin[
a + b*x])^(3/2))/(7*b*c*d*(d*Cos[a + b*x])^(7/2)) + (8*(c*Sin[a + b*x])^(3
/2))/(21*b*c*d^3*(d*Cos[a + b*x])^(3/2))))/(11*d^2)))/(5*d^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3043
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

rule 3046
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Sin[e + f
*x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3051
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Simp[(m + n + 2)/(a^2*(m + 1))   Int[(b*Sin[e + f*x
])^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m
, -1] && IntegersQ[2*m, 2*n]
```

Maple [A] (verified)

Time = 5.84 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.50

method	result	size
default	$\frac{2\sqrt{c\sin(bx+a)}\,\left(32\cos(bx+a)^4+56\cos(bx+a)^2+77\right)c^2\tan(bx+a)^3\sec(bx+a)^4}{1155b\sqrt{d\cos(bx+a)}\,d^8}$	70

input
```
int((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(17/2),x,method=_RETURNVERBOSE)
```

output
```
2/1155/b*(c*sin(b*x+a))^(1/2)*(32*cos(b*x+a)^4+56*cos(b*x+a)^2+77)*c^2/(d*
cos(b*x+a))^(1/2)/d^8*tan(b*x+a)^3*sec(b*x+a)^4
```

Fricas [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.62

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{17/2}}\,dx =$$
$$-\frac{2\left(32\,c^2\cos\left(bx+a\right)^6+24\,c^2\cos\left(bx+a\right)^4+21\,c^2\cos\left(bx+a\right)^2-77\,c^2\right)\sqrt{d\cos\left(bx+a\right)}\sqrt{c\sin\left(bx+a\right)}\,\mathrm{s}}{1155\,bd^9\cos\left(bx+a\right)^8}$$

input
```
integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(17/2),x, algorithm="fricas"
)
```

output
```
-2/1155*(32*c^2*cos(b*x + a)^6 + 24*c^2*cos(b*x + a)^4 + 21*c^2*cos(b*x +
a)^2 - 77*c^2)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*sin(b*x + a)/(b*d
^9*cos(b*x + a)^8)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{17/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*sin(b*x+a))**(5/2)/(d*cos(b*x+a))**(17/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{17/2}}\,dx = \int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{(d\cos(bx+a))^{\frac{17}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(17/2),x, algorithm="maxima"
)
```

output
```
integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(17/2), x)
```

Giac [F]

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{17/2}}\,dx=\int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{(d\cos(bx+a))^{\frac{17}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(17/2),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(17/2), x)
```

Mupad [B] (verification not implemented)

Time = 33.99 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.47

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{17/2}}\,dx=-\frac{e^{-a\,7i-b\,x\,7i}\sqrt{c\,\left(\frac{e^{-a\,1i-b\,x\,1i}\,1i}{2}-\frac{e^{a\,1i+b\,x\,1i}\,1i}{2}\right)}\,\left(\frac{1216\,c^2\,e^{a\,7i+b\,x\,7i}\,\sin(3\,a+3\,b\,x)}{385\,b\,d^8}+\frac{1024\,c^2\,e^{a\,7i+b\,x\,7i}\,\sin(5\,a+5\,b\,x)}{1155\,b\,d^8}+\frac{128\,c^2\,e^{a\,7i}}{}\right.}{128\cos(a+b\,x)^7\sqrt{d\,\left(\frac{e^{-a\,1i-b\,x\,1i}}{2}+\frac{e^{a\,1i+b\,x\,1i}}{2}\right)}}$$

input
```
int((c*sin(a + b*x))^(5/2)/(d*cos(a + b*x))^(17/2),x)
```

output
```
-(exp(- a*7i - b*x*7i)*(c*((exp(- a*1i - b*x*1i)*1i)/2 - (exp(a*1i + b*x*1
i)*1i)/2))^(1/2)*((1216*c^2*exp(a*7i + b*x*7i)*sin(3*a + 3*b*x))/(385*b*d^
8) + (1024*c^2*exp(a*7i + b*x*7i)*sin(5*a + 5*b*x))/(1155*b*d^8) + (128*c^
2*exp(a*7i + b*x*7i)*sin(7*a + 7*b*x))/(1155*b*d^8) - (3392*c^2*exp(a*7i +
 b*x*7i)*sin(a + b*x))/(231*b*d^8)))/(128*cos(a + b*x)^7*(d*(exp(- a*1i -
b*x*1i)/2 + exp(a*1i + b*x*1i)/2))^(1/2))
```

Reduce [F]

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{17/2}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sin(bx+a)}\,\sqrt{\cos(bx+a)}\,\sin(bx+a)^2}{\cos(bx+a)^9}dx\right)c^2}{d^9}$$

input
```
int((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(17/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sin(a + b*x))*sqrt(cos(a + b*x))*sin(a + b*x)**
2)/cos(a + b*x)**9,x)*c**2)/d**9
```

3.287 $\int \frac{\sin^{\frac{7}{2}}(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 177

$$\int \frac{\sin^{\frac{7}{2}}(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{(1+\cot(a+bx))\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b} - \frac{2\sqrt{\sin(a+bx)}}{b\sqrt{\cos(a+bx)}} + \frac{2\sin^{\frac{5}{2}}(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}$$

output
```
-1/2*arctan(-1+2^(1/2)*cos(b*x+a)^(1/2)/sin(b*x+a)^(1/2))*2^(1/2)/b-1/2*ar
ctan(1+2^(1/2)*cos(b*x+a)^(1/2)/sin(b*x+a)^(1/2))*2^(1/2)/b+1/2*arctanh(2^
(1/2)*cos(b*x+a)^(1/2)/(1+cot(b*x+a))/sin(b*x+a)^(1/2))*2^(1/2)/b-2*sin(b*
x+a)^(1/2)/b/cos(b*x+a)^(1/2)+2/5*sin(b*x+a)^(5/2)/b/cos(b*x+a)^(5/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.32

$$\int \frac{\sin^{\frac{7}{2}}(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx = \frac{2\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{9}{4},\frac{9}{4},\frac{13}{4},\sin^2(a+bx)\right)\sin^{\frac{9}{2}}(a+bx)}{9b\sqrt{\cos(a+bx)}}$$

input
```
Integrate[Sin[a + b*x]^(7/2)/Cos[a + b*x]^(7/2),x]
```

output
```
(2*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[9/4, 9/4, 13/4, Sin[a + b*x]^2
]*Sin[a + b*x]^(9/2))/(9*b*Sqrt[Cos[a + b*x]])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.30, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3046, 3042, 3046, 3042, 3055, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^{\frac{7}{2}}(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^{7/2}}{\cos(a+bx)^{7/2}}dx$$

$$\Big\downarrow \text{3046}$$

$$\frac{2\sin^{\frac{5}{2}}(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)} - \int \frac{\sin^{\frac{3}{2}}(a+bx)}{\cos^{\frac{3}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin^{\frac{5}{2}}(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}-\int\frac{\sin(a+bx)^{3/2}}{\cos(a+bx)^{3/2}}dx$$

$$\Big\downarrow \text{ 3046}$$

$$\int\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}dx+\frac{2\sin^{\frac{5}{2}}(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\sin(a+bx)}}{b\sqrt{\cos(a+bx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\int\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}dx+\frac{2\sin^{\frac{5}{2}}(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\sin(a+bx)}}{b\sqrt{\cos(a+bx)}}$$

$$\Big\downarrow \text{ 3055}$$

$$-\frac{2\int\frac{\cot(a+bx)}{\cot^2(a+bx)+1}d\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}}{b}+\frac{2\sin^{\frac{5}{2}}(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\sin(a+bx)}}{b\sqrt{\cos(a+bx)}}$$

$$\Big\downarrow \text{ 826}$$

$$-\frac{2\left(\frac{1}{2}\int\frac{\cot(a+bx)+1}{\cot^2(a+bx)+1}d\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}-\frac{1}{2}\int\frac{1-\cot(a+bx)}{\cot^2(a+bx)+1}d\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{b}+\frac{2\sin^{\frac{5}{2}}(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\sin(a+bx)}}{b\sqrt{\cos(a+bx)}}$$

$$\Big\downarrow \text{ 1476}$$

$$-\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1}d\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+\frac{1}{2}\int\frac{1}{\cot(a+bx)+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1}d\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)-\frac{1}{2}\int\frac{1-\cot(a+bx)}{\cot^2(a+bx)+1}d\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{b}+\frac{2\sin^{\frac{5}{2}}(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\sin(a+bx)}}{b\sqrt{\cos(a+bx)}}$$

$$\Big\downarrow \text{ 1082}$$

$$-\frac{2\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\cot(a+bx)-1}d\left(1-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\cot(a+bx)-1}d\left(\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\cot(a+bx)}{\cot^2(a+bx)+1}d\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{b}+\frac{2\sin^{\frac{5}{2}}(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\sin(a+bx)}}{b\sqrt{\cos(a+bx)}}$$

$$\Big\downarrow \text{ 217}$$

$$-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\cot(a+bx)}{\cot^2(a+bx)+1}d\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{b}+\frac{2\sin^{\frac{5}{2}}(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\sin(a+bx)}}{b\sqrt{\cos(a+bx)}}$$

$$\Big\downarrow \text{ 1479}$$

$$-\frac{2\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-\frac{2\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}}{\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1}d\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{\cot(a+bx)+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1}d\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{\sqrt{2}}\right.}{b}$$

$$\frac{2\sin^{\frac{5}{2}}(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\sin(a+bx)}}{b\sqrt{\cos(a+bx)}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-\frac{2\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}}{\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1}d\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{\cot(a+bx)+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1}d\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{\sqrt{2}}\right.}{b}$$

$$\frac{2\sin^{\frac{5}{2}}(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\sin(a+bx)}}{b\sqrt{\cos(a+bx)}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-\frac{2\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}}{\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1}d\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}}{2\sqrt{2}}-\frac{1}{2}\int\frac{\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1}{\cot(a+bx)+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1}d\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right.}{\sqrt{2}}\right.}{b}$$

$$\frac{2\sin^{\frac{5}{2}}(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\sin(a+bx)}}{b\sqrt{\cos(a+bx)}}$$

$$\Big\downarrow \text{ 1103}$$

$$-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\cot(a+bx)+\frac{\sqrt{2}\sqrt{\cdots}}{\sqrt{\cdots}}\cdots\right)}{2\sqrt{2}}\cdots\right)\right)}{b}$$

$$+\frac{2\sin^{\frac{5}{2}}(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\sin(a+bx)}}{b\sqrt{\cos(a+bx)}}$$

input
```
Int[Sin[a + b*x]^(7/2)/Cos[a + b*x]^(7/2),x]
```

output
```
(-2*((-(ArcTan[1 - (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]]]/Sqrt[2
]) + ArcTan[1 + (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]]]/Sqrt[2])/
2 + (Log[1 + Cot[a + b*x] - (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]
]]/(2*Sqrt[2]) - Log[1 + Cot[a + b*x] + (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[
Sin[a + b*x]]]/(2*Sqrt[2]))/2))/b - (2*Sqrt[Sin[a + b*x]])/(b*Sqrt[Cos[a +
 b*x]]) + (2*Sin[a + b*x]^(5/2))/(5*b*Cos[a + b*x]^(5/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3046
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Sin[e + f
*x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3055
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[(-k)*a*(b/f)   Subst[Int[x
^(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[
e + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m,
0] && LtQ[m, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(144) = 288$.

Time = 46.90 (sec) , antiderivative size = 530, normalized size of antiderivative = 2.99

method	result
default	$-\frac{\sqrt{2}\left(\sin(bx+a)\left(24\cos(bx+a)^2-4\right)\sqrt{2}+\cos(bx+a)^2(-10\cos(bx+a)-10)\sqrt{\frac{\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}}{\cos(bx}\right.\right.}{}$

input
```
int(sin(b*x+a)^(7/2)/cos(b*x+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/20/b*2^(1/2)*(sin(b*x+a)*(24*cos(b*x+a)^2-4)*2^(1/2)+cos(b*x+a)^2*(-10*
cos(b*x+a)-10)*(sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*arctan((2^(1
/2)*(sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)+cos(b*x+a)-1
)/(cos(b*x+a)-1))+(sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*ln(-(2*2^
(1/2)*(sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)-cos(b*x+a)
*cot(b*x+a)+2*cot(b*x+a)-csc(b*x+a)-2*cos(b*x+a)+sin(b*x+a)+2)/(cos(b*x+a)
-1))*(-5*cos(b*x+a)^3-5*cos(b*x+a)^2)+(sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1
)^2)^(1/2)*ln((2*2^(1/2)*(sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*si
n(b*x+a)+cos(b*x+a)*cot(b*x+a)-2*cot(b*x+a)+csc(b*x+a)+2*cos(b*x+a)-sin(b*
x+a)-2)/(cos(b*x+a)-1))*(5*cos(b*x+a)^3+5*cos(b*x+a)^2)+cos(b*x+a)^2*(-10*
cos(b*x+a)-10)*(sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*arctan((2^(1
/2)*(sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)-cos(b*x+a)+1
)/(cos(b*x+a)-1)))/sin(b*x+a)^(1/2)/cos(b*x+a)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(144) = 288$.

Time = 0.18 (sec) , antiderivative size = 447, normalized size of antiderivative = 2.53

$$\int \frac{\sin^{\frac{7}{2}}(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx =$$

$$-\frac{5\sqrt{2}\arctan\left(\frac{2\cos(bx+a)^3-2\cos(bx+a)^2\sin(bx+a)+\sqrt{2}\sqrt{\cos(bx+a)}\sqrt{\sin(bx+a)}-2\cos(bx+a)}{2\left(\cos(bx+a)^3+\cos(bx+a)^2\sin(bx+a)-\cos(bx+a)\right)}\right)\cos(bx+a)^3+5\sqrt{2}\,\mathrm{arc}}{}$$

input
```
integrate(sin(b*x+a)^(7/2)/cos(b*x+a)^(7/2),x, algorithm="fricas")
```

output
```
-1/40*(5*sqrt(2)*arctan(1/2*(2*cos(b*x + a)^3 - 2*cos(b*x + a)^2*sin(b*x +
 a) + sqrt(2)*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) - 2*cos(b*x + a))/(cos
(b*x + a)^3 + cos(b*x + a)^2*sin(b*x + a) - cos(b*x + a)))*cos(b*x + a)^3
+ 5*sqrt(2)*arctan(-1/2*(2*cos(b*x + a)^3 - 2*cos(b*x + a)^2*sin(b*x + a)
- sqrt(2)*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) - 2*cos(b*x + a))/(cos(b*x
 + a)^3 + cos(b*x + a)^2*sin(b*x + a) - cos(b*x + a)))*cos(b*x + a)^3 - 10
*sqrt(2)*arctan(-1/2*(sqrt(2)*cos(b*x + a) - sqrt(2)*sin(b*x + a))/(sqrt(c
os(b*x + a))*sqrt(sin(b*x + a))))*cos(b*x + a)^3 - 5*sqrt(2)*cos(b*x + a)^
3*log(2*(sqrt(2)*cos(b*x + a) + sqrt(2)*sin(b*x + a))*sqrt(cos(b*x + a))*s
qrt(sin(b*x + a)) + 4*cos(b*x + a)*sin(b*x + a) + 1) + 5*sqrt(2)*cos(b*x +
 a)^3*log(-2*(sqrt(2)*cos(b*x + a) + sqrt(2)*sin(b*x + a))*sqrt(cos(b*x +
a))*sqrt(sin(b*x + a)) + 4*cos(b*x + a)*sin(b*x + a) + 1) + 16*(6*cos(b*x
+ a)^2 - 1)*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)))/(b*cos(b*x + a)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^{\frac{7}{2}}(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**(7/2)/cos(b*x+a)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sin^{\frac{7}{2}}(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{\sin(bx+a)^{\frac{7}{2}}}{\cos(bx+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^(7/2)/cos(b*x+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^(7/2)/cos(b*x + a)^(7/2), x)
```

Giac [F]

$$\int \frac{\sin^{\frac{7}{2}}(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx=\int \frac{\sin(bx+a)^{\frac{7}{2}}}{\cos(bx+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sin(b*x+a)^(7/2)/cos(b*x+a)^(7/2),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^(7/2)/cos(b*x + a)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 26.97 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.25

$$\int \frac{\sin^{\frac{7}{2}}(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx=\frac{2\sin(a+b\,x)^{9/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{4},-\frac{5}{4};\ -\frac{1}{4};\ \cos(a+b\,x)^2\right)}{5\,b\cos(a+b\,x)^{5/2}\,\left(\sin(a+b\,x)^2\right)^{9/4}}$$

input
```
int(sin(a + b*x)^(7/2)/cos(a + b*x)^(7/2),x)
```

output
```
(2*sin(a + b*x)^(9/2)*hypergeom([-5/4, -5/4], -1/4, cos(a + b*x)^2))/(5*b*
cos(a + b*x)^(5/2)*(sin(a + b*x)^2)^(9/4))
```

Reduce [F]

$$\int \frac{\sin^{\frac{7}{2}}(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx=\int \frac{\sqrt{\sin(bx+a)}\,\sqrt{\cos(bx+a)}\,\sin(bx+a)^3}{\cos(bx+a)^4}dx$$

input
```
int(sin(b*x+a)^(7/2)/cos(b*x+a)^(7/2),x)
```

output
```
int((sqrt(sin(a + b*x))*sqrt(cos(a + b*x))*sin(a + b*x)**3)/cos(a + b*x)**
4,x)
```

3.288 $\int \frac{\sin^{\frac{3}{2}}(x)}{\cos^{\frac{7}{2}}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{\sin^{\frac{3}{2}}(x)}{\cos^{\frac{7}{2}}(x)}\,dx=\frac{2\sin^{\frac{5}{2}}(x)}{5\cos^{\frac{5}{2}}(x)}$$

output
```
2/5*sin(x)^(5/2)/cos(x)^(5/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sin^{\frac{3}{2}}(x)}{\cos^{\frac{7}{2}}(x)}\,dx=\frac{2\sin^{\frac{5}{2}}(x)}{5\cos^{\frac{5}{2}}(x)}$$

input
```
Integrate[Sin[x]^(3/2)/Cos[x]^(7/2),x]
```

output
```
(2*Sin[x]^(5/2))/(5*Cos[x]^(5/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3042, 3043}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^{\frac{3}{2}}(x)}{\cos^{\frac{7}{2}}(x)} \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(x)^{3/2}}{\cos(x)^{7/2}}dx$$

$$\Big\downarrow \text{ 3043}$$

$$\frac{2 \sin^{\frac{5}{2}}(x)}{5 \cos^{\frac{5}{2}}(x)}$$

input
```
Int[Sin[x]^(3/2)/Cos[x]^(7/2),x]
```

output
```
(2*Sin[x]^(5/2))/(5*Cos[x]^(5/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3043
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Maple [A] (verified)

Time = 3.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{2\sin(x)^{\frac{5}{2}}}{5\cos(x)^{\frac{5}{2}}}$	11

input
```
int(sin(x)^(3/2)/cos(x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*sin(x)^(5/2)/cos(x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sin^{\frac{3}{2}}(x)}{\cos^{\frac{7}{2}}(x)}\,dx = -\frac{2\left(\cos(x)^2-1\right)\sqrt{\sin(x)}}{5\cos(x)^{\frac{5}{2}}}$$

input
```
integrate(sin(x)^(3/2)/cos(x)^(7/2),x, algorithm="fricas")
```

output
```
-2/5*(cos(x)^2 - 1)*sqrt(sin(x))/cos(x)^(5/2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^{\frac{3}{2}}(x)}{\cos^{\frac{7}{2}}(x)}\,dx = \text{Timed out}$$

input
```
integrate(sin(x)**(3/2)/cos(x)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sin^{\frac{3}{2}}(x)}{\cos^{\frac{7}{2}}(x)}\,dx = \int \frac{\sin(x)^{\frac{3}{2}}}{\cos(x)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sin(x)^(3/2)/cos(x)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sin(x)^(3/2)/cos(x)^(7/2), x)
```

Giac [F]

$$\int \frac{\sin^{\frac{3}{2}}(x)}{\cos^{\frac{7}{2}}(x)}\,dx = \int \frac{\sin(x)^{\frac{3}{2}}}{\cos(x)^{\frac{7}{2}}}\,dx$$

input
```
integrate(sin(x)^(3/2)/cos(x)^(7/2),x, algorithm="giac")
```

output
```
integrate(sin(x)^(3/2)/cos(x)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 25.75 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.31

$$\int \frac{\sin^{\frac{3}{2}}(x)}{\cos^{\frac{7}{2}}(x)}\,dx = -\frac{8\sqrt{2}\tan\left(\frac{x}{2}\right)^{5/2}\sqrt{1-\tan\left(\frac{x}{2}\right)^2}}{\tan\left(\frac{x}{2}\right)^2\left(\tan\left(\frac{x}{2}\right)^2\left(5\tan\left(\frac{x}{2}\right)^2-15\right)+15\right)-5}$$

input
```
int(sin(x)^(3/2)/cos(x)^(7/2),x)
```

output
```
-(8*2^(1/2)*tan(x/2)^(5/2)*(1 - tan(x/2)^2)^(1/2))/(tan(x/2)^2*(tan(x/2)^2
*(5*tan(x/2)^2 - 15) + 15) - 5)
```

Reduce [F]

$$\int \frac{\sin^{\frac{3}{2}}(x)}{\cos^{\frac{7}{2}}(x)}\,dx = \frac{-\cos(x)^3\left(\int \frac{\sqrt{\sin(x)}\,\sqrt{\cos(x)}}{\cos(x)^2\sin(x)}dx\right)+2\sqrt{\sin(x)}\,\sqrt{\cos(x)}}{5\cos(x)^3}$$

input
```
int(sin(x)^(3/2)/cos(x)^(7/2),x)
```

output
```
( - cos(x)**3*int((sqrt(sin(x))*sqrt(cos(x)))/(cos(x)**2*sin(x)),x) + 2*sq
rt(sin(x))*sqrt(cos(x)))/(5*cos(x)**3)
```

3.289 $\int \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 89

$$\int \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{\sqrt{2}}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{\sqrt{2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}(1+\tan(x))}\right)}{\sqrt{2}}$$

output
```
-1/2*arctan(1-2^(1/2)*sin(x)^(1/2)/cos(x)^(1/2))*2^(1/2)+1/2*arctan(1+2^(1
/2)*sin(x)^(1/2)/cos(x)^(1/2))*2^(1/2)-1/2*arctanh(2^(1/2)*sin(x)^(1/2)/co
s(x)^(1/2)/(1+tan(x)))*2^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.43

$$\int \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\,dx = \frac{2\cos^2(x)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{4},\frac{7}{4},\sin^2(x)\right)\sin^{\frac{3}{2}}(x)}{3\cos^{\frac{3}{2}}(x)}$$

input
```
Integrate[Sqrt[Sin[x]]/Sqrt[Cos[x]],x]
```

output
```
(2*(Cos[x]^2)^(3/4)*Hypergeometric2F1[3/4, 3/4, 7/4, Sin[x]^2]*Sin[x]^(3/2
))/(3*Cos[x]^(3/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.51, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {3042, 3054, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}dx$$

$$\Big\downarrow \text{3054}$$

$$2\int \frac{\tan(x)}{\tan^2(x)+1}d\frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}$$

$$\Big\downarrow \text{826}$$

$$2\left(\frac{1}{2}\int \frac{\tan(x)+1}{\tan^2(x)+1}d\frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}-\frac{1}{2}\int \frac{1-\tan(x)}{\tan^2(x)+1}d\frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)$$

$$\Big\downarrow \text{1476}$$

$$2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(x)-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1}d\frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+\frac{1}{2}\int \frac{1}{\tan(x)+\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1}d\frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)-\frac{1}{2}\int \frac{1-\tan(x)}{\tan^2(x)+1}d\right.$$

$$\Big\downarrow \text{1082}$$

$$2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\tan(x)-1}d\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\tan(x)-1}d\left(\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\tan(x)}{\tan^2(x)+1}d\frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)$$

$\downarrow$ 217

$$2\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\tan(x)}{\tan^2(x)+1}d\frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)$$

$\downarrow$ 1479

$$2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt{\sin(x)}}{\sqrt{\cos(x)}}}{\tan(x)-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1}d\frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{\tan(x)+\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1}d\frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{\sqrt{2}}-\frac{\text{arctan}}{}\right.\right.$$

$\downarrow$ 25

$$2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt{\sin(x)}}{\sqrt{\cos(x)}}}{\tan(x)-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1}d\frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{\tan(x)+\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1}d\frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(\right.}{}\right.\right.$$

$\downarrow$ 27

$$2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt{\sin(x)}}{\sqrt{\cos(x)}}}{\tan(x)-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1}d\frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1}{\tan(x)+\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1}d\frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{\sqrt{2}}\right.\right.$$

$\downarrow$ 1103

$$2\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(x)-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(x)+\right.}{2}\right.\right.$$

input
```
Int[Sqrt[Sin[x]]/Sqrt[Cos[x]],x]
```

output
```
2*((-(ArcTan[1 - (Sqrt[2]*Sqrt[Sin[x]])/Sqrt[Cos[x]]]/Sqrt[2]) + ArcTan[1
+ (Sqrt[2]*Sqrt[Sin[x]])/Sqrt[Cos[x]]]/Sqrt[2])/2 + (Log[1 - (Sqrt[2]*Sqrt
[Sin[x]])/Sqrt[Cos[x]] + Tan[x]]/(2*Sqrt[2]) - Log[1 + (Sqrt[2]*Sqrt[Sin[x
]])/Sqrt[Cos[x]] + Tan[x]]/(2*Sqrt[2]))/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3054
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k*a*(b/f)   Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(66) = 132$.

Time = 47.00 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.27

method	result
default	$-\frac{\sqrt{2}\,\sqrt{\cos(x)}\left(\ln\left(\frac{2\sqrt{2}\,\sqrt{\frac{\sin(x)\cos(x)}{(\cos(x)+1)^2}}\,\sin(x)+\cot(x)\cos(x)-2\cot(x)-\sin(x)+2\cos(x)+\csc(x)-2}{-1+\cos(x)}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{\frac{\sin(x)\cos(x)}{(\cos(x)+1)^2}}\,\sin(x)+}{-1+\cos(x)}\right.\right.}{}$

input
```
int(sin(x)^(1/2)/cos(x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*2^(1/2)*cos(x)^(1/2)*(ln((2*2^(1/2)*(sin(x)*cos(x)/(cos(x)+1)^2)^(1/2
)*sin(x)+cot(x)*cos(x)-2*cot(x)-sin(x)+2*cos(x)+csc(x)-2)/(-1+cos(x)))+2*a
rctan((2^(1/2)*(sin(x)*cos(x)/(cos(x)+1)^2)^(1/2)*sin(x)+cos(x)-1)/(-1+cos
(x)))-ln(-(2*2^(1/2)*(sin(x)*cos(x)/(cos(x)+1)^2)^(1/2)*sin(x)-cot(x)*cos(
x)+2*cot(x)+sin(x)-2*cos(x)-csc(x)+2)/(-1+cos(x)))+2*arctan((2^(1/2)*(sin(
x)*cos(x)/(cos(x)+1)^2)^(1/2)*sin(x)-cos(x)+1)/(-1+cos(x))))*(-1+cos(x))/s
in(x)^(3/2)/(sin(x)*cos(x)/(cos(x)+1)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(66) = 132$.

Time = 0.11 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.47

$$\int \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\,dx$$
$$= -\frac{1}{8}\sqrt{2}\arctan\left(\frac{2\cos(x)^3 - 2\cos(x)^2\sin(x) + \sqrt{2}\sqrt{\cos(x)}\sqrt{\sin(x)} - 2\cos(x)}{2\left(\cos(x)^3 + \cos(x)^2\sin(x) - \cos(x)\right)}\right)$$
$$-\frac{1}{8}\sqrt{2}\arctan\left(-\frac{2\cos(x)^3 - 2\cos(x)^2\sin(x) - \sqrt{2}\sqrt{\cos(x)}\sqrt{\sin(x)} - 2\cos(x)}{2\left(\cos(x)^3 + \cos(x)^2\sin(x) - \cos(x)\right)}\right)$$
$$+\frac{1}{4}\sqrt{2}\arctan\left(-\frac{\sqrt{2}\cos(x) - \sqrt{2}\sin(x)}{2\sqrt{\cos(x)}\sqrt{\sin(x)}}\right)$$
$$-\frac{1}{8}\sqrt{2}\log\left(2\left(\sqrt{2}\cos(x) + \sqrt{2}\sin(x)\right)\sqrt{\cos(x)}\sqrt{\sin(x)} + 4\cos(x)\sin(x) + 1\right)$$
$$+\frac{1}{8}\sqrt{2}\log\left(-2\left(\sqrt{2}\cos(x) + \sqrt{2}\sin(x)\right)\sqrt{\cos(x)}\sqrt{\sin(x)} + 4\cos(x)\sin(x) + 1\right)$$

input
```
integrate(sin(x)^(1/2)/cos(x)^(1/2),x, algorithm="fricas")
```

output
```
-1/8*sqrt(2)*arctan(1/2*(2*cos(x)^3 - 2*cos(x)^2*sin(x) + sqrt(2)*sqrt(cos
(x))*sqrt(sin(x)) - 2*cos(x))/(cos(x)^3 + cos(x)^2*sin(x) - cos(x))) - 1/8
*sqrt(2)*arctan(-1/2*(2*cos(x)^3 - 2*cos(x)^2*sin(x) - sqrt(2)*sqrt(cos(x)
)*sqrt(sin(x)) - 2*cos(x))/(cos(x)^3 + cos(x)^2*sin(x) - cos(x))) + 1/4*sq
rt(2)*arctan(-1/2*(sqrt(2)*cos(x) - sqrt(2)*sin(x))/(sqrt(cos(x))*sqrt(sin
(x)))) - 1/8*sqrt(2)*log(2*(sqrt(2)*cos(x) + sqrt(2)*sin(x))*sqrt(cos(x))*
sqrt(sin(x)) + 4*cos(x)*sin(x) + 1) + 1/8*sqrt(2)*log(-2*(sqrt(2)*cos(x) +
 sqrt(2)*sin(x))*sqrt(cos(x))*sqrt(sin(x)) + 4*cos(x)*sin(x) + 1)
```

Sympy [F]

$$\int \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\,dx = \int \frac{\sqrt{\sin{(x)}}}{\sqrt{\cos{(x)}}}\,dx$$

input
```
integrate(sin(x)**(1/2)/cos(x)**(1/2),x)
```

output
```
Integral(sqrt(sin(x))/sqrt(cos(x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\,dx = \int \frac{\sqrt{\sin{(x)}}}{\sqrt{\cos{(x)}}}\,dx$$

input
```
integrate(sin(x)^(1/2)/cos(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sin(x))/sqrt(cos(x)), x)
```

Giac [F]

$$\int \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\,dx = \int \frac{\sqrt{\sin{(x)}}}{\sqrt{\cos{(x)}}}\,dx$$

input
```
integrate(sin(x)^(1/2)/cos(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sin(x))/sqrt(cos(x)), x)
```

Mupad [B] (verification not implemented)

Time = 25.60 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.28

$$\int \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\,dx = -\frac{2\sqrt{\cos\left(x\right)}\,{\sin\left(x\right)}^{3/2}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ {\cos\left(x\right)}^2\right)}{{\left({\sin\left(x\right)}^2\right)}^{3/4}}$$

input
```
int(sin(x)^(1/2)/cos(x)^(1/2),x)
```

output
```
-(2*cos(x)^(1/2)*sin(x)^(3/2)*hypergeom([1/4, 1/4], 5/4, cos(x)^2))/(sin(x
)^2)^(3/4)
```

Reduce [F]

$$\int \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\,dx = \int \frac{\sqrt{\sin\left(x\right)}\,\sqrt{\cos\left(x\right)}}{\cos\left(x\right)}dx$$

input
```
int(sin(x)^(1/2)/cos(x)^(1/2),x)
```

output
```
int((sqrt(sin(x))*sqrt(cos(x)))/cos(x),x)
```

3.290 $\int \frac{\sin^{\frac{5}{2}}(x)}{\sqrt{\cos(x)}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 112

$$\int \frac{\sin^{\frac{5}{2}}(x)}{\sqrt{\cos(x)}}\,dx = -\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{4\sqrt{2}}+\frac{3\arctan\left(1+\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{4\sqrt{2}} - \frac{3\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}(1+\tan(x))}\right)}{4\sqrt{2}}-\frac{1}{2}\sqrt{\cos(x)}\sin^{\frac{3}{2}}(x)$$

output
```
-3/8*arctan(1-2^(1/2)*sin(x)^(1/2)/cos(x)^(1/2))*2^(1/2)+3/8*arctan(1+2^(1
/2)*sin(x)^(1/2)/cos(x)^(1/2))*2^(1/2)-3/8*arctanh(2^(1/2)*sin(x)^(1/2)/co
s(x)^(1/2)/(1+tan(x)))*2^(1/2)-1/2*cos(x)^(1/2)*sin(x)^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.34

$$\int \frac{\sin^{\frac{5}{2}}(x)}{\sqrt{\cos(x)}}\,dx = \frac{2\cos^2(x)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{7}{4},\frac{11}{4},\sin^2(x)\right)\sin^{\frac{7}{2}}(x)}{7\cos^{\frac{3}{2}}(x)}$$

input
```
Integrate[Sin[x]^(5/2)/Sqrt[Cos[x]],x]
```

output
```
(2*(Cos[x]^2)^(3/4)*Hypergeometric2F1[3/4, 7/4, 11/4, Sin[x]^2]*Sin[x]^(7/
2))/(7*Cos[x]^(3/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.37, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.923$, Rules used = {3042, 3048, 3042, 3054, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^{\frac{5}{2}}(x)}{\sqrt{\cos(x)}}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sin(x)^{5/2}}{\sqrt{\cos(x)}}dx$$
$$\Big\downarrow \text{ 3048}$$
$$\frac{3}{4}\int \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}dx-\frac{1}{2}\sin^{\frac{3}{2}}(x)\sqrt{\cos(x)}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{3}{4}\int \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}dx-\frac{1}{2}\sin^{\frac{3}{2}}(x)\sqrt{\cos(x)}$$
$$\Big\downarrow \text{ 3054}$$
$$\frac{3}{2}\int \frac{\tan(x)}{\tan^2(x)+1}d\frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}-\frac{1}{2}\sin^{\frac{3}{2}}(x)\sqrt{\cos(x)}$$
$$\Big\downarrow \text{ 826}$$
$$\frac{3}{2}\left(\frac{1}{2}\int \frac{\tan(x)+1}{\tan^2(x)+1}d\frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}-\frac{1}{2}\int \frac{1-\tan(x)}{\tan^2(x)+1}d\frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)-\frac{1}{2}\sin^{\frac{3}{2}}(x)\sqrt{\cos(x)}$$

$$\Big\downarrow \; 1476$$

$$\frac{3}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(x)-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1}d\frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+\frac{1}{2}\int \frac{1}{\tan(x)+\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1}d\frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)-\frac{1}{2}\int \frac{1-\tan(x)}{\tan^2(x)+1}d\right.$$

$$\frac{1}{2}\sin^{\frac{3}{2}}(x)\sqrt{\cos(x)}$$

$$\Big\downarrow \; 1082$$

$$\frac{3}{2}\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\tan(x)-1}d\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\tan(x)-1}d\left(\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\tan(x)}{\tan^2(x)+1}d\frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)-$$

$$\frac{1}{2}\sin^{\frac{3}{2}}(x)\sqrt{\cos(x)}$$

$$\Big\downarrow \; 217$$

$$\frac{3}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\tan(x)}{\tan^2(x)+1}d\frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)-$$

$$\frac{1}{2}\sin^{\frac{3}{2}}(x)\sqrt{\cos(x)}$$

$$\Big\downarrow \; 1479$$

$$\frac{3}{2}\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt{\sin(x)}}{\sqrt{\cos(x)}}}{\tan(x)-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1}d\frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{\tan(x)+\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1}d\frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{\sqrt{2}}-\frac{\arctan}{}\right.\right.$$

$$\frac{1}{2}\sin^{\frac{3}{2}}(x)\sqrt{\cos(x)}$$

$$\Big\downarrow \; 25$$

$$\frac{3}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt{\sin(x)}}{\sqrt{\cos(x)}}}{\tan(x)-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1}d\frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{\tan(x)+\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1}d\frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{\sqrt{2}}-\frac{\arctan}{}\right.\right.$$

$$\frac{1}{2}\sin^{\frac{3}{2}}(x)\sqrt{\cos(x)}$$

$$\Big\downarrow \ 27$$

$$\frac{3}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt{\sin(x)}}{\sqrt{\cos(x)}}}{\tan(x)-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1}d\frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1}{\tan(x)+\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1}d\frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{\sqrt{2}}\right.\right.$$

$$\frac{1}{2}\sin^{\frac{3}{2}}(x)\sqrt{\cos(x)}$$

$$\Big\downarrow \ 1103$$

$$\frac{3}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(x)-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(x)\right.}{}\right.\right.$$

$$\frac{1}{2}\sin^{\frac{3}{2}}(x)\sqrt{\cos(x)}$$

input
```
Int[Sin[x]^(5/2)/Sqrt[Cos[x]],x]
```

output
```
(3*((-(ArcTan[1 - (Sqrt[2]*Sqrt[Sin[x]])/Sqrt[Cos[x]]]/Sqrt[2]) + ArcTan[1
 + (Sqrt[2]*Sqrt[Sin[x]])/Sqrt[Cos[x]]]/Sqrt[2])/2 + (Log[1 - (Sqrt[2]*Sqr
t[Sin[x]])/Sqrt[Cos[x]] + Tan[x]]/(2*Sqrt[2]) - Log[1 + (Sqrt[2]*Sqrt[Sin[
x]])/Sqrt[Cos[x]] + Tan[x]]/(2*Sqrt[2]))/2))/2 - (Sqrt[Cos[x]]*Sin[x]^(3/2
))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3054
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k*a*(b/f)   Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(76) = 152$.

Time = 2.34 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.27

method	result
default	$\sqrt{2}\,\sqrt{\cos(x)}\,\sin(x)^{\frac{3}{2}}\left((3\cos(x)-3)\ln\left(-\frac{2\sqrt{2}\,\sqrt{\frac{\sin(x)\cos(x)}{(\cos(x)+1)^{2}}}\,\sin(x)-\cot(x)\cos(x)+2\cot(x)+\sin(x)-2\cos(x)-\csc(x)+2}{-1+\cos(x)}\right)+(-3\cos(x)+3$

input
```
int(sin(x)^(5/2)/cos(x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/128*2^(1/2)*cos(x)^(1/2)*sin(x)^(3/2)*((3*cos(x)-3)*ln(-(2*2^(1/2)*(sin(
x)*cos(x)/(cos(x)+1)^2)^(1/2)*sin(x)-cot(x)*cos(x)+2*cot(x)+sin(x)-2*cos(x
)-csc(x)+2)/(-1+cos(x)))+(-3*cos(x)+3)*ln((2*2^(1/2)*(sin(x)*cos(x)/(cos(x
)+1)^2)^(1/2)*sin(x)+cot(x)*cos(x)-2*cot(x)-sin(x)+2*cos(x)+csc(x)-2)/(-1+
cos(x)))+(-6*cos(x)+6)*arctan((2^(1/2)*(sin(x)*cos(x)/(cos(x)+1)^2)^(1/2)*
sin(x)-cos(x)+1)/(-1+cos(x)))+(-6*cos(x)+6)*arctan((2^(1/2)*(sin(x)*cos(x)
/(cos(x)+1)^2)^(1/2)*sin(x)+cos(x)-1)/(-1+cos(x)))-4*2^(1/2)*(sin(x)*cos(x
)/(cos(x)+1)^2)^(1/2)*sin(x)^3)/(sin(x)*cos(x)/(cos(x)+1)^2)^(1/2)*sec(1/2
*x)^3*csc(1/2*x)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(76) = 152$.

Time = 0.13 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.05

$$\int \frac{\sin^{\frac{5}{2}}(x)}{\sqrt{\cos(x)}}\,dx = -\frac{1}{2}\sqrt{\cos(x)}\sin(x)^{\frac{3}{2}} - \frac{3}{32}\sqrt{2}\arctan\left(\frac{2\cos(x)^3 - 2\cos(x)^2\sin(x) + \sqrt{2}\sqrt{\cos(x)}\sqrt{\sin(x)} - 2\cos(x)}{2\left(\cos(x)^3 + \cos(x)^2\sin(x) - \cos(x)\right)}\right) - \frac{3}{32}\sqrt{2}\arctan\left(-\frac{2\cos(x)^3 - 2\cos(x)^2\sin(x) - \sqrt{2}\sqrt{\cos(x)}\sqrt{\sin(x)} - 2\cos(x)}{2\left(\cos(x)^3 + \cos(x)^2\sin(x) - \cos(x)\right)}\right) + \frac{3}{16}\sqrt{2}\arctan\left(-\frac{\sqrt{2}\cos(x) - \sqrt{2}\sin(x)}{2\sqrt{\cos(x)}\sqrt{\sin(x)}}\right) - \frac{3}{32}\sqrt{2}\log\left(2\left(\sqrt{2}\cos(x) + \sqrt{2}\sin(x)\right)\sqrt{\cos(x)}\sqrt{\sin(x)} + 4\cos(x)\sin(x) + 1\right) + \frac{3}{32}\sqrt{2}\log\left(-2\left(\sqrt{2}\cos(x) + \sqrt{2}\sin(x)\right)\sqrt{\cos(x)}\sqrt{\sin(x)} + 4\cos(x)\sin(x) + 1\right)$$

input
```
integrate(sin(x)^(5/2)/cos(x)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(cos(x))*sin(x)^(3/2) - 3/32*sqrt(2)*arctan(1/2*(2*cos(x)^3 - 2*c
os(x)^2*sin(x) + sqrt(2)*sqrt(cos(x))*sqrt(sin(x)) - 2*cos(x))/(cos(x)^3 +
 cos(x)^2*sin(x) - cos(x))) - 3/32*sqrt(2)*arctan(-1/2*(2*cos(x)^3 - 2*cos
(x)^2*sin(x) - sqrt(2)*sqrt(cos(x))*sqrt(sin(x)) - 2*cos(x))/(cos(x)^3 + c
os(x)^2*sin(x) - cos(x))) + 3/16*sqrt(2)*arctan(-1/2*(sqrt(2)*cos(x) - sqr
t(2)*sin(x))/(sqrt(cos(x))*sqrt(sin(x)))) - 3/32*sqrt(2)*log(2*(sqrt(2)*co
s(x) + sqrt(2)*sin(x))*sqrt(cos(x))*sqrt(sin(x)) + 4*cos(x)*sin(x) + 1) +
3/32*sqrt(2)*log(-2*(sqrt(2)*cos(x) + sqrt(2)*sin(x))*sqrt(cos(x))*sqrt(si
n(x)) + 4*cos(x)*sin(x) + 1)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^{\frac{5}{2}}(x)}{\sqrt{\cos(x)}}\,dx = \text{Timed out}$$

input
```
integrate(sin(x)**(5/2)/cos(x)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sin^{\frac{5}{2}}(x)}{\sqrt{\cos(x)}}\,dx = \int \frac{\sin\left(x\right)^{\frac{5}{2}}}{\sqrt{\cos\left(x\right)}}\,dx$$

input
```
integrate(sin(x)^(5/2)/cos(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(x)^(5/2)/sqrt(cos(x)), x)
```

Giac [F]

$$\int \frac{\sin^{\frac{5}{2}}(x)}{\sqrt{\cos(x)}}\,dx = \int \frac{\sin\left(x\right)^{\frac{5}{2}}}{\sqrt{\cos\left(x\right)}}\,dx$$

input
```
integrate(sin(x)^(5/2)/cos(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sin(x)^(5/2)/sqrt(cos(x)), x)
```

Mupad [B] (verification not implemented)

Time = 25.73 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.22

$$\int \frac{\sin^{\frac{5}{2}}(x)}{\sqrt{\cos(x)}}\,dx = -\frac{2\,\sqrt{\cos\left(x\right)}\,{\sin\left(x\right)}^{7/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{4};\ \frac{5}{4};\ {\cos\left(x\right)}^2\right)}{{\left({\sin\left(x\right)}^2\right)}^{7/4}}$$

input
```
int(sin(x)^(5/2)/cos(x)^(1/2),x)
```

output
```
-(2*cos(x)^(1/2)*sin(x)^(7/2)*hypergeom([-3/4, 1/4], 5/4, cos(x)^2))/(sin(
x)^2)^(7/4)
```

Reduce [F]

$$\int \frac{\sin^{\frac{5}{2}}(x)}{\sqrt{\cos(x)}}\,dx = \int \frac{\sqrt{\sin\left(x\right)}\,\sqrt{\cos\left(x\right)}\,\sin\left(x\right)^{2}}{\cos\left(x\right)}dx$$

input
```
int(sin(x)^(5/2)/cos(x)^(1/2),x)
```

output
```
int((sqrt(sin(x))*sqrt(cos(x))*sin(x)**2)/cos(x),x)
```

3.291 $\int \frac{(d\cos(a+bx))^{7/2}}{\sqrt{c\sin(a+bx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 132

$$\int \frac{(d\cos(a+bx))^{7/2}}{\sqrt{c\sin(a+bx)}}\,dx = \frac{5d^3\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}{6bc} + \frac{d(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}{3bc} + \frac{5d^4\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}}{12b\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}$$

output
```
5/6*d^3*(d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2)/b/c+1/3*d*(d*cos(b*x+a))
^(5/2)*(c*sin(b*x+a))^(1/2)/b/c+5/12*d^4*InverseJacobiAM(a-1/4*Pi+b*x,2^(1
/2))*sin(2*b*x+2*a)^(1/2)/b/(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.53

$$\int \frac{(d\cos(a+bx))^{7/2}}{\sqrt{c\sin(a+bx)}}\,dx = \frac{2(d\cos(a+bx))^{7/2}\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{4},\frac{5}{4},\sin^2(a+bx)\right)}{bc}$$

input
```
Integrate[(d*Cos[a + b*x])^(7/2)/Sqrt[c*Sin[a + b*x]],x]
```

output
```
(2*(d*Cos[a + b*x])^(7/2)*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-5/4, 1
/4, 5/4, Sin[a + b*x]^2]*Sec[a + b*x]^5*Sqrt[c*Sin[a + b*x]])/(b*c)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3049, 3042, 3049, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\cos(a+bx))^{7/2}}{\sqrt{c\sin(a+bx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(d\cos(a+bx))^{7/2}}{\sqrt{c\sin(a+bx)}}dx$$

$$\Big\downarrow \text{ 3049}$$

$$\frac{5}{6}d^2\int \frac{(d\cos(a+bx))^{3/2}}{\sqrt{c\sin(a+bx)}}dx+\frac{d\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{5/2}}{3bc}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{5}{6}d^2\int \frac{(d\cos(a+bx))^{3/2}}{\sqrt{c\sin(a+bx)}}dx+\frac{d\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{5/2}}{3bc}$$

$$\Big\downarrow \text{ 3049}$$

$$\frac{5}{6}d^2\left(\frac{1}{2}d^2\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}dx+\frac{d\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{bc}\right)+\frac{d\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{5/2}}{3bc}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{5}{6}d^2\left(\frac{1}{2}d^2\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}dx+\frac{d\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{bc}\right)+\frac{d\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{5/2}}{3bc}$$

$$\Big\downarrow \; 3053$$

$$\frac{5}{6}d^2\left(\frac{d^2\sqrt{\sin(2a+2bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx}{2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}+\frac{d\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{bc}\right)+\frac{d\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{5/2}}{3bc}$$

$$\Big\downarrow \; 3042$$

$$\frac{5}{6}d^2\left(\frac{d^2\sqrt{\sin(2a+2bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx}{2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}+\frac{d\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{bc}\right)+\frac{d\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{5/2}}{3bc}$$

$$\Big\downarrow \; 3120$$

$$\frac{5}{6}d^2\left(\frac{d^2\sqrt{\sin(2a+2bx)}\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{2b\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}+\frac{d\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{bc}\right)+\frac{d\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{5/2}}{3bc}$$

input
```
Int[(d*Cos[a + b*x])^(7/2)/Sqrt[c*Sin[a + b*x]],x]
```

output
```
(d*(d*Cos[a + b*x])^(5/2)*Sqrt[c*Sin[a + b*x]])/(3*b*c) + (5*d^2*((d*Sqrt[
d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]])/(b*c) + (d^2*EllipticF[a - Pi/4 + b*
x, 2]*Sqrt[Sin[2*a + 2*b*x]])/(2*b*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x
]])))/6
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3049
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Sin[e + f*x])^n*(a
*Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 15.89 (sec) , antiderivative size = 996, normalized size of antiderivative = 7.55

method	result	size
default	Expression too large to display	996

input
```
int((d*cos(b*x+a))^(7/2)/(c*sin(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/384/b*d^3*(sin(b*x+a)*cos(b*x+a)*(-16*cos(b*x+a)^2-40)+(3*cos(b*x+a)+3)
*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*ln(-(cos(b*x+a)*cot(b*x
+a)-2*cot(b*x+a)-2*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b
*x+a)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(cos(b*x+a)-1))+(-6*cos(b*x+a)
-6)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*arctan(((-2*sin(b*x+
a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)+cos(b*x+a)-1)/(cos(b*x+a)
-1))+(-3*cos(b*x+a)-3)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*l
n(-(cos(b*x+a)*cot(b*x+a)-2*cot(b*x+a)+2*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*
x+a)+1)^2)^(1/2)*sin(b*x+a)-2*cos(b*x+a)-sin(b*x+a)+csc(b*x+a)+2)/(cos(b*x
+a)-1))+(6*cos(b*x+a)+6)*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)
*arctan((-(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)+cos
(b*x+a)-1)/(cos(b*x+a)-1))+(-32*cos(b*x+a)-32)*(-cot(b*x+a)+csc(b*x+a)+1)^
(1/2)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*El
lipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))+(6*cos(b*x+a)+6)*(-c
ot(b*x+a)+csc(b*x+a)+1)^(1/2)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x
+a)-csc(b*x+a))^(1/2)*EllipticPi((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2-1/2*
I,1/2*2^(1/2))+(6*cos(b*x+a)+6)*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(2*cot(b*
x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticPi((-cot(
b*x+a)+csc(b*x+a)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))+I*(6*cos(b*x+a)+6)*(-cot
(b*x+a)+csc(b*x+a)+1)^(1/2)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*...
```

Fricas [F]

$$\int \frac{(d\cos(a+bx))^{7/2}}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{(d\cos(bx+a))^{\frac{7}{2}}}{\sqrt{c\sin(bx+a)}}\,dx$$

input
```
integrate((d*cos(b*x+a))^(7/2)/(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*d^3*cos(b*x + a)^3/(c*s
in(b*x + a)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d\cos(a+bx))^{7/2}}{\sqrt{c\sin(a+bx)}}\,dx = \text{Timed out}$$

input
```
integrate((d*cos(b*x+a))**(7/2)/(c*sin(b*x+a))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d\cos(a+bx))^{7/2}}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{(d\cos(bx+a))^{\frac{7}{2}}}{\sqrt{c\sin(bx+a)}}\,dx$$

input
```
integrate((d*cos(b*x+a))^(7/2)/(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*cos(b*x + a))^(7/2)/sqrt(c*sin(b*x + a)), x)
```

Giac [F]

$$\int \frac{(d\cos(a+bx))^{7/2}}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{(d\cos(bx+a))^{\frac{7}{2}}}{\sqrt{c\sin(bx+a)}}\,dx$$

input
```
integrate((d*cos(b*x+a))^(7/2)/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*cos(b*x + a))^(7/2)/sqrt(c*sin(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\cos(a+bx))^{7/2}}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{(d\,\cos(a+b\,x))^{7/2}}{\sqrt{c\,\sin(a+b\,x)}}\,dx$$

input
```
int((d*cos(a + b*x))^(7/2)/(c*sin(a + b*x))^(1/2),x)
```

output
```
int((d*cos(a + b*x))^(7/2)/(c*sin(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(d\cos(a+bx))^{7/2}}{\sqrt{c\sin(a+bx)}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sin(bx+a)}\,\sqrt{\cos(bx+a)}\,\cos(bx+a)^3}{\sin(bx+a)}dx\right)d^3}{c}$$

input
```
int((d*cos(b*x+a))^(7/2)/(c*sin(b*x+a))^(1/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sin(a + b*x))*sqrt(cos(a + b*x))*cos(a + b*x)**
3)/sin(a + b*x),x)*d**3)/c
```

3.292 $\int \frac{(d\cos(a+bx))^{3/2}}{\sqrt{c\sin(a+bx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 92

$$\int \frac{(d\cos(a+bx))^{3/2}}{\sqrt{c\sin(a+bx)}}\,dx = \frac{d\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}{bc} + \frac{d^2\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}}{2b\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}$$

output
```
d*(d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2)/b/c+1/2*d^2*InverseJacobiAM(a-
1/4*Pi+b*x,2^(1/2))*sin(2*b*x+2*a)^(1/2)/b/(d*cos(b*x+a))^(1/2)/(c*sin(b*x
+a))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.74

$$\int \frac{(d\cos(a+bx))^{3/2}}{\sqrt{c\sin(a+bx)}}\,dx = \frac{2d^2\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{5}{4},\sin^2(a+bx)\right)\tan(a+bx)}{b\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}$$

input
```
Integrate[(d*Cos[a + b*x])^(3/2)/Sqrt[c*Sin[a + b*x]],x]
```

output
```
(2*d^2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-1/4, 1/4, 5/4, Sin[a + b*
x]^2]*Tan[a + b*x])/(b*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3049, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\cos(a+bx))^{3/2}}{\sqrt{c\sin(a+bx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(d\cos(a+bx))^{3/2}}{\sqrt{c\sin(a+bx)}}dx$$
$$\Big\downarrow \text{3049}$$
$$\frac{1}{2}d^2\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}dx+\frac{d\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{bc}$$
$$\Big\downarrow \text{3042}$$
$$\frac{1}{2}d^2\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}dx+\frac{d\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{bc}$$
$$\Big\downarrow \text{3053}$$
$$\frac{d^2\sqrt{\sin(2a+2bx)}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx}{2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}+\frac{d\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{bc}$$
$$\Big\downarrow \text{3042}$$
$$\frac{d^2\sqrt{\sin(2a+2bx)}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx}{2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}+\frac{d\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{bc}$$
$$\Big\downarrow \text{3120}$$
$$\frac{d^2\sqrt{\sin(2a+2bx)}\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{2b\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}+\frac{d\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{bc}$$

input
```
Int[(d*Cos[a + b*x])^(3/2)/Sqrt[c*Sin[a + b*x]],x]
```

output
```
(d*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]])/(b*c) + (d^2*EllipticF[a - P
i/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]])/(2*b*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin
[a + b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3049
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Sin[e + f*x])^n*(a
*Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 3.86 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.35

method	result
default	$\frac{\sqrt{d\cos(bx+a)}\, d\left(\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\,\sqrt{2\cot(bx+a)-2\csc(bx+a)+2}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\ \text{EllipticF}\left(\sqrt{-\cot(bx+a)}\right.\right.}{2b\sqrt{c\sin(bx+a)}}$

input
```
int((d*cos(b*x+a))^(3/2)/(c*sin(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*(d*cos(b*x+a))^(1/2)*d/(c*sin(b*x+a))^(1/2)*((-cot(b*x+a)+csc(b*x+a)
+1)^(1/2)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2
)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))*(1+sec(b*x+a))+2
*sin(b*x+a))
```

Fricas [F]

$$\int \frac{(d\cos(a+bx))^{3/2}}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{(d\cos(bx+a))^{\frac{3}{2}}}{\sqrt{c\sin(bx+a)}}\,dx$$

input
```
integrate((d*cos(b*x+a))^(3/2)/(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*d*cos(b*x + a)/(c*sin(b
*x + a)), x)
```

Sympy [F]

$$\int \frac{(d\cos(a+bx))^{3/2}}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{(d\cos(a+bx))^{\frac{3}{2}}}{\sqrt{c\sin(a+bx)}}\,dx$$

input
```
integrate((d*cos(b*x+a))**(3/2)/(c*sin(b*x+a))**(1/2),x)
```

output
```
Integral((d*cos(a + b*x))**(3/2)/sqrt(c*sin(a + b*x)), x)
```

Maxima [F]

$$\int \frac{(d\cos(a+bx))^{3/2}}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{(d\cos{(bx+a)})^{\frac{3}{2}}}{\sqrt{c\sin{(bx+a)}}}\,dx$$

input
```
integrate((d*cos(b*x+a))^(3/2)/(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*cos(b*x + a))^(3/2)/sqrt(c*sin(b*x + a)), x)
```

Giac [F]

$$\int \frac{(d\cos(a+bx))^{3/2}}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{(d\cos{(bx+a)})^{\frac{3}{2}}}{\sqrt{c\sin{(bx+a)}}}\,dx$$

input
```
integrate((d*cos(b*x+a))^(3/2)/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*cos(b*x + a))^(3/2)/sqrt(c*sin(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\cos(a+bx))^{3/2}}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{(d\,\cos{(a+b\,x)})^{3/2}}{\sqrt{c\,\sin{(a+b\,x)}}}\,dx$$

input
```
int((d*cos(a + b*x))^(3/2)/(c*sin(a + b*x))^(1/2),x)
```

output
```
int((d*cos(a + b*x))^(3/2)/(c*sin(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(d\cos(a+bx))^{3/2}}{\sqrt{c\sin(a+bx)}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sin(bx+a)}\,\sqrt{\cos(bx+a)}\,\cos(bx+a)}{\sin(bx+a)}dx\right)d}{c}$$

input
```
int((d*cos(b*x+a))^(3/2)/(c*sin(b*x+a))^(1/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sin(a + b*x))*sqrt(cos(a + b*x))*cos(a + b*x))/
sin(a + b*x),x)*d)/c
```

3.293 $\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 53

$$\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\,dx = \frac{\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}}{b\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}$$

output
```
InverseJacobiAM(a-1/4*Pi+b*x,2^(1/2))*sin(2*b*x+2*a)^(1/2)/b/(d*cos(b*x+a)
)^(1/2)/(c*sin(b*x+a))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.23

$$\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\,dx = \frac{2\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},\sin^2(a+bx)\right)\tan(a+bx)}{b\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}$$

input
```
Integrate[1/(Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]]),x]
```

output
```
(2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, Sin[a + b*x]^2]
*Tan[a + b*x])/(b*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}dx$$

$$\Big\downarrow \text{ 3053}$$

$$\frac{\sqrt{\sin(2a+2bx)}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx}{\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{\sin(2a+2bx)}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx}{\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{\sqrt{\sin(2a+2bx)}\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{b\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}$$

input
```
Int[1/(Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]]),x]
```

output
```
(EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]])/(b*Sqrt[d*Cos[a + b*
x]]*Sqrt[c*Sin[a + b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(47) = 94$.

Time = 3.22 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.11

method	result
default	$\frac{(\cos(bx+a)+1)\,\mathrm{EllipticF}\left(\sqrt{-\cot(bx+a)+\csc(bx+a)+1}, \frac{\sqrt{2}}{2}\right)\sqrt{\cot(bx+a)-\csc(bx+a)}\,\sqrt{2\cot(bx+a)-2\csc(bx+a)+2}\,\sqrt{-\cot(bx}}{b\sqrt{d\cos(bx+a)}\,\sqrt{c\sin(bx+a)}}$

input
```
int(1/(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*(cos(b*x+a)+1)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))
*(cot(b*x+a)-csc(b*x+a))^(1/2)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(-cot(b
*x+a)+csc(b*x+a)+1)^(1/2)/(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\,dx = -\frac{\sqrt{i\,cd}F(\arcsin(\cos(bx+a)+i\,\sin(bx+a))\,|-1)+\sqrt{-i\,cd}F(\arcsin(\cos(bx+a)-i\,\sin(bx+a))\,|}{bcd}$$

input
```
integrate(1/(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2),x, algorithm="fricas
")
```

output
```
-(sqrt(I*c*d)*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*x + a)), -1) + sqrt
(-I*c*d)*elliptic_f(arcsin(cos(b*x + a) - I*sin(b*x + a)), -1))/(b*c*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\,dx = \int \frac{1}{\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}\,dx$$

input
```
integrate(1/(d*cos(b*x+a))**(1/2)/(c*sin(b*x+a))**(1/2),x)
```

output
```
Integral(1/(sqrt(c*sin(a + b*x))*sqrt(d*cos(a + b*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\,dx = \int \frac{1}{\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}}\,dx$$

input
```
integrate(1/(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\,dx = \int \frac{1}{\sqrt{d\cos\left(bx+a\right)}\sqrt{c\sin\left(bx+a\right)}}\,dx$$

input
```
integrate(1/(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\,dx = \int \frac{1}{\sqrt{d\,\cos\left(a+b\,x\right)}\,\sqrt{c\,\sin\left(a+b\,x\right)}}\,dx$$

input
```
int(1/((d*cos(a + b*x))^(1/2)*(c*sin(a + b*x))^(1/2)),x)
```

output
```
int(1/((d*cos(a + b*x))^(1/2)*(c*sin(a + b*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sin(bx+a)}\,\sqrt{\cos(bx+a)}}{\cos(bx+a)\sin(bx+a)}dx\right)}{cd}$$

input
```
int(1/(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sin(a + b*x))*sqrt(cos(a + b*x)))/(cos(a + b*x)
*sin(a + b*x)),x))/(c*d)
```

3.294 $\int \frac{1}{(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 97

$$\int \frac{1}{(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}\,dx = \frac{2\sqrt{c\sin(a+bx)}}{3bcd(d\cos(a+bx))^{3/2}} + \frac{2\operatorname{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}}{3bd^2\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}$$

output
```
2/3*(c*sin(b*x+a))^(1/2)/b/c/d/(d*cos(b*x+a))^(3/2)+2/3*InverseJacobiAM(a-
1/4*Pi+b*x,2^(1/2))*sin(2*b*x+2*a)^(1/2)/b/d^2/(d*cos(b*x+a))^(1/2)/(c*sin
(b*x+a))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.67

$$\int \frac{1}{(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}\,dx = \frac{2\cos^2(a+bx)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{7}{4},\frac{5}{4},\sin^2(a+bx)\right)\sqrt{c s}}{bcd(d\cos(a+bx))^{3/2}}$$

input
```
Integrate[1/((d*Cos[a + b*x])^(5/2)*Sqrt[c*Sin[a + b*x]]),x]
```

output
```
(2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[1/4, 7/4, 5/4, Sin[a + b*x]^2]
*Sqrt[c*Sin[a + b*x]])/(b*c*d*(d*Cos[a + b*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3051, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3051}$$

$$\frac{2\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}dx}{3d^2}+\frac{2\sqrt{c\sin(a+bx)}}{3bcd(d\cos(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}dx}{3d^2}+\frac{2\sqrt{c\sin(a+bx)}}{3bcd(d\cos(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3053}$$

$$\frac{2\sqrt{\sin(2a+2bx)}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx}{3d^2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}+\frac{2\sqrt{c\sin(a+bx)}}{3bcd(d\cos(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sqrt{\sin(2a+2bx)}\int \frac{1}{\sqrt{\sin(2a+2bx)}}dx}{3d^2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}+\frac{2\sqrt{c\sin(a+bx)}}{3bcd(d\cos(a+bx))^{3/2}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sqrt{\sin(2a+2bx)}\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{3bd^2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}+\frac{2\sqrt{c\sin(a+bx)}}{3bcd(d\cos(a+bx))^{3/2}}$$

input
```
Int[1/((d*Cos[a + b*x])^(5/2)*Sqrt[c*Sin[a + b*x]]),x]
```

output
```
(2*Sqrt[c*Sin[a + b*x]])/(3*b*c*d*(d*Cos[a + b*x])^(3/2)) + (2*EllipticF[a
 - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]])/(3*b*d^2*Sqrt[d*Cos[a + b*x]]*Sq
rt[c*Sin[a + b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3051
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Simp[(m + n + 2)/(a^2*(m + 1))   Int[(b*Sin[e + f*x
])^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m
, -1] && IntegersQ[2*m, 2*n]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 4.56 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.28

method	result
default	$\frac{\frac{2(\cos(bx+a)+1)\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\,\sqrt{2\cot(bx+a)-2\csc(bx+a)+2}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,\operatorname{EllipticF}\left(\sqrt{-\cot(bx+a)+\csc(bx+a)+1}, \frac{\sqrt{2}}{2}\right.}{3}}{b\sqrt{c\sin(bx+a)}\,\sqrt{d\cos(bx+a)}\,d^2}$

input
```
int(1/(d*cos(b*x+a))^(5/2)/(c*sin(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/b/(c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2)/d^2*((cos(b*x+a)+1)*(-cot(
b*x+a)+csc(b*x+a)+1)^(1/2)*(2*cot(b*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+a)
-csc(b*x+a))^(1/2)*EllipticF((-cot(b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2))
+tan(b*x+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}\,dx =$$
$$-\frac{2\left(\sqrt{i\,cd}\cos\left(bx+a\right)^{2}F(\arcsin\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)\,|-1)+\sqrt{-i\,cd}\cos\left(bx+a\right)^{2}F(\arcsin\left(\cos\right.\right.}{3\,bcd^{3}\cos\left(bx+a\right)^{2}}$$

input
```
integrate(1/(d*cos(b*x+a))^(5/2)/(c*sin(b*x+a))^(1/2),x, algorithm="fricas
")
```

output
```
-2/3*(sqrt(I*c*d)*cos(b*x + a)^2*elliptic_f(arcsin(cos(b*x + a) + I*sin(b*
x + a)), -1) + sqrt(-I*c*d)*cos(b*x + a)^2*elliptic_f(arcsin(cos(b*x + a)
- I*sin(b*x + a)), -1) - sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)))/(b*c*d
^3*cos(b*x + a)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*cos(b*x+a))**(5/2)/(c*sin(b*x+a))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}\,dx = \int \frac{1}{(d\cos(bx+a))^{\frac{5}{2}}\sqrt{c\sin(bx+a)}}\,dx$$

input
```
integrate(1/(d*cos(b*x+a))^(5/2)/(c*sin(b*x+a))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/((d*cos(b*x + a))^(5/2)*sqrt(c*sin(b*x + a))), x)
```

Giac [F]

$$\int \frac{1}{(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}\,dx = \int \frac{1}{(d\cos(bx+a))^{\frac{5}{2}}\sqrt{c\sin(bx+a)}}\,dx$$

input
```
integrate(1/(d*cos(b*x+a))^(5/2)/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((d*cos(b*x + a))^(5/2)*sqrt(c*sin(b*x + a))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}\,dx = \int \frac{1}{(d\cos(a+b\,x))^{5/2}\,\sqrt{c\,\sin(a+b\,x)}}\,dx$$

input
```
int(1/((d*cos(a + b*x))^(5/2)*(c*sin(a + b*x))^(1/2)),x)
```

output
```
int(1/((d*cos(a + b*x))^(5/2)*(c*sin(a + b*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sin(bx+a)}\,\sqrt{\cos(bx+a)}}{\cos(bx+a)^3\sin(bx+a)}dx\right)}{c\,d^3}$$

input
```
int(1/(d*cos(b*x+a))^(5/2)/(c*sin(b*x+a))^(1/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sin(a + b*x))*sqrt(cos(a + b*x)))/(cos(a + b*x)
**3*sin(a + b*x)),x))/(c*d**3)
```

3.295 $\int \frac{1}{(d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 134

$$\int \frac{1}{(d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}}\,dx = \frac{2\sqrt{c\sin(a+bx)}}{7bcd(d\cos(a+bx))^{7/2}} + \frac{4\sqrt{c\sin(a+bx)}}{7bcd^3(d\cos(a+bx))^{3/2}} + \frac{4\,\text{EllipticF}\left(a-\frac{\pi}{4}+bx,2\right)\sqrt{\sin(2a+2bx)}}{7bd^4\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}$$

output
```
2/7*(c*sin(b*x+a))^(1/2)/b/c/d/(d*cos(b*x+a))^(7/2)+4/7*(c*sin(b*x+a))^(1/
2)/b/c/d^3/(d*cos(b*x+a))^(3/2)+4/7*InverseJacobiAM(a-1/4*Pi+b*x,2^(1/2))*
sin(2*b*x+2*a)^(1/2)/b/d^4/(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.52

$$\int \frac{1}{(d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}}\,dx = \frac{2\cos^3(a+bx)\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{11}{4},\frac{5}{4},\sin^2(\right.}{bc(d\cos(a+bx))^{9/2}}$$

input
```
Integrate[1/((d*Cos[a + b*x])^(9/2)*Sqrt[c*Sin[a + b*x]]),x]
```

output
```
(2*Cos[a + b*x]^3*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[1/4, 11/4, 5/4,
 Sin[a + b*x]^2]*Sqrt[c*Sin[a + b*x]])/(b*c*(d*Cos[a + b*x])^(9/2))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3051, 3042, 3051, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{9/2}}dx$$

$$\Big\downarrow \text{3051}$$

$$\frac{6\int \frac{1}{(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}dx}{7d^2}+\frac{2\sqrt{c\sin(a+bx)}}{7bcd(d\cos(a+bx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6\int \frac{1}{(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}dx}{7d^2}+\frac{2\sqrt{c\sin(a+bx)}}{7bcd(d\cos(a+bx))^{7/2}}$$

$$\Big\downarrow \text{3051}$$

$$\frac{6\left(\frac{2\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}dx}{3d^2}+\frac{2\sqrt{c\sin(a+bx)}}{3bcd(d\cos(a+bx))^{3/2}}\right)}{7d^2}+\frac{2\sqrt{c\sin(a+bx)}}{7bcd(d\cos(a+bx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6\left(\frac{2\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}dx}{3d^2}+\frac{2\sqrt{c\sin(a+bx)}}{3bcd(d\cos(a+bx))^{3/2}}\right)}{7d^2}+\frac{2\sqrt{c\sin(a+bx)}}{7bcd(d\cos(a+bx))^{7/2}}$$

$$\Big\downarrow \text{3053}$$

$$\frac{6\left(\frac{2\sqrt{\sin(2a+2bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx}{3d^2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}+\frac{2\sqrt{c\sin(a+bx)}}{3bcd(d\cos(a+bx))^{3/2}}\right)}{7d^2}+\frac{2\sqrt{c\sin(a+bx)}}{7bcd(d\cos(a+bx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6\left(\frac{2\sqrt{\sin(2a+2bx)}\int\frac{1}{\sqrt{\sin(2a+2bx)}}dx}{3d^2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}+\frac{2\sqrt{c\sin(a+bx)}}{3bcd(d\cos(a+bx))^{3/2}}\right)}{7d^2}+\frac{2\sqrt{c\sin(a+bx)}}{7bcd(d\cos(a+bx))^{7/2}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{6\left(\frac{2\sqrt{\sin(2a+2bx)}\,\text{EllipticF}\left(a+bx-\frac{\pi}{4},2\right)}{3bd^2\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}+\frac{2\sqrt{c\sin(a+bx)}}{3bcd(d\cos(a+bx))^{3/2}}\right)}{7d^2}+\frac{2\sqrt{c\sin(a+bx)}}{7bcd(d\cos(a+bx))^{7/2}}$$

input
```
Int[1/((d*Cos[a + b*x])^(9/2)*Sqrt[c*Sin[a + b*x]]),x]
```

output
```
(2*Sqrt[c*Sin[a + b*x]])/(7*b*c*d*(d*Cos[a + b*x])^(7/2)) + (6*((2*Sqrt[c*
Sin[a + b*x]])/(3*b*c*d*(d*Cos[a + b*x])^(3/2)) + (2*EllipticF[a - Pi/4 +
b*x, 2]*Sqrt[Sin[2*a + 2*b*x]])/(3*b*d^2*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a
 + b*x]])))/(7*d^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3051
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Simp[(m + n + 2)/(a^2*(m + 1))   Int[(b*Sin[e + f*x
])^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m
, -1] && IntegersQ[2*m, 2*n]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 4.71 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.07

method	result
default	$\frac{\frac{4\tan(bx+a)}{7}+\frac{2\sec(bx+a)^2\tan(bx+a)}{7}+\frac{2(2\cos(bx+a)+2)\sqrt{-\cot(bx+a)+\csc(bx+a)+1}\,\sqrt{2\cot(bx+a)-2\csc(bx+a)+2}\,\sqrt{\cot(bx+a)-\csc(bx+a)}\,\mathrm{E}}{7}}{b\sqrt{c\sin(bx+a)}\,\sqrt{d\cos(bx+a)}\,d^4}$

input
```
int(1/(d*cos(b*x+a))^(9/2)/(c*sin(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/7/b/(c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2)/d^4*(2*tan(b*x+a)+sec(b*x+
a)^2*tan(b*x+a)+(2*cos(b*x+a)+2)*(-cot(b*x+a)+csc(b*x+a)+1)^(1/2)*(2*cot(b
*x+a)-2*csc(b*x+a)+2)^(1/2)*(cot(b*x+a)-csc(b*x+a))^(1/2)*EllipticF((-cot(
b*x+a)+csc(b*x+a)+1)^(1/2),1/2*2^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.91

$$\int \frac{1}{(d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}}\,dx =$$
$$-\frac{2\left(2\sqrt{i\,cd}\cos\left(bx+a\right)^4F(\arcsin\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)\mid -1)+2\sqrt{-i\,cd}\cos\left(bx+a\right)^4F(\arcsin\left(\right.\right.}{7\,bcd^5\cos\left(bx+\right.}$$

input
```
integrate(1/(d*cos(b*x+a))^(9/2)/(c*sin(b*x+a))^(1/2),x, algorithm="fricas
")
```

output
```
-2/7*(2*sqrt(I*c*d)*cos(b*x + a)^4*elliptic_f(arcsin(cos(b*x + a) + I*sin(
b*x + a)), -1) + 2*sqrt(-I*c*d)*cos(b*x + a)^4*elliptic_f(arcsin(cos(b*x +
 a) - I*sin(b*x + a)), -1) - sqrt(d*cos(b*x + a))*(2*cos(b*x + a)^2 + 1)*s
qrt(c*sin(b*x + a)))/(b*c*d^5*cos(b*x + a)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*cos(b*x+a))**(9/2)/(c*sin(b*x+a))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}}\,dx = \int \frac{1}{\left(d\cos\left(bx+a\right)\right)^{\frac{9}{2}}\sqrt{c\sin\left(bx+a\right)}}\,dx$$

input
```
integrate(1/(d*cos(b*x+a))^(9/2)/(c*sin(b*x+a))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/((d*cos(b*x + a))^(9/2)*sqrt(c*sin(b*x + a))), x)
```

Giac [F]

$$\int \frac{1}{(d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}}\,dx = \int \frac{1}{(d\cos(bx+a))^{\frac{9}{2}}\sqrt{c\sin(bx+a)}}\,dx$$

input
```
integrate(1/(d*cos(b*x+a))^(9/2)/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((d*cos(b*x + a))^(9/2)*sqrt(c*sin(b*x + a))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}}\,dx = \int \frac{1}{(d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}}\,dx$$

input
```
int(1/((d*cos(a + b*x))^(9/2)*(c*sin(a + b*x))^(1/2)),x)
```

output
```
int(1/((d*cos(a + b*x))^(9/2)*(c*sin(a + b*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sin(bx+a)}\,\sqrt{\cos(bx+a)}}{\cos(bx+a)^5\sin(bx+a)}dx\right)}{c\,d^5}$$

input
```
int(1/(d*cos(b*x+a))^(9/2)/(c*sin(b*x+a))^(1/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sin(a + b*x))*sqrt(cos(a + b*x)))/(cos(a + b*x)
**5*sin(a + b*x)),x))/(c*d**5)
```

3.296 $\int \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 202

$$\int \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\,dx = \frac{\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}b\sqrt{c}} - \frac{\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}b\sqrt{c}} + \frac{\sqrt{d}\,\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\left(\sqrt{d}+\sqrt{d}\cot(a+bx)\right)\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}b\sqrt{c}}$$

output

```
-1/2*d^(1/2)*arctan(-1+2^(1/2)*c^(1/2)*(d*cos(b*x+a))^(1/2)/d^(1/2)/(c*sin
(b*x+a))^(1/2))*2^(1/2)/b/c^(1/2)-1/2*d^(1/2)*arctan(1+2^(1/2)*c^(1/2)*(d*
cos(b*x+a))^(1/2)/d^(1/2)/(c*sin(b*x+a))^(1/2))*2^(1/2)/b/c^(1/2)+1/2*d^(1
/2)*arctanh(2^(1/2)*c^(1/2)*(d*cos(b*x+a))^(1/2)/(d^(1/2)+d^(1/2)*cot(b*x+
a))/(c*sin(b*x+a))^(1/2))*2^(1/2)/b/c^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.32

$$\int \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\,dx = \frac{2\sqrt{d\cos(a+bx)}\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4},\frac{5}{4},\sin^2(a+bx)\right)\tan(a+bx)}{b\sqrt{c\sin(a+bx)}}$$

input
```
Integrate[Sqrt[d*Cos[a + b*x]]/Sqrt[c*Sin[a + b*x]],x]
```

output
```
(2*Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, 1/4,
 5/4, Sin[a + b*x]^2]*Tan[a + b*x])/(b*Sqrt[c*Sin[a + b*x]])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.41, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3055, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}dx$$

$$\Big\downarrow \text{3055}$$

$$-\frac{2cd\int \frac{d\cot(a+bx)}{c(\cot^2(a+bx)d^2+d^2)}d\frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}}{b}$$

$$\Big\downarrow \text{826}$$

$$-\frac{2cd\left(\frac{\int \frac{\cot(a+bx)d+d}{\cot^2(a+bx)d^2+d^2}d\frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}}{2c}-\frac{\int \frac{d-d\cot(a+bx)}{\cot^2(a+bx)d^2+d^2}d\frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}}{2c}\right)}{b}$$

$\Big\downarrow$ 1476

$$-\frac{2cd\left(\frac{\frac{\int \frac{1}{\frac{\cot(a+bx)d}{c}+\frac{d}{c}-\frac{\sqrt{2}\sqrt{d\cos(a+bx)}\sqrt{d}}{\sqrt{c}\sqrt{c\sin(a+bx)}}}d\frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}}{2c}+\frac{\int \frac{1}{\frac{\cot(a+bx)d}{c}+\frac{d}{c}+\frac{\sqrt{2}\sqrt{d\cos(a+bx)}\sqrt{d}}{\sqrt{c}\sqrt{c\sin(a+bx)}}}d\frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}}{2c}}{2c}-\frac{\int \frac{d-d\cot(a+bx)}{\cot^2(a+bx)d^2+d^2}d\frac{\sqrt{d\cos}}{\sqrt{c\sin}}}{2c}\right.}{b}$$

$\Big\downarrow$ 1082

$$-\frac{2cd\left(\frac{\frac{\int \frac{1}{-\frac{d\cot(a+bx)}{c}-1}d\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\int \frac{1}{-\frac{d\cot(a+bx)}{c}-1}d\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}+1\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}}{2c}-\frac{\int \frac{d-d\cot(a+bx)}{\cot^2(a+bx)d^2+d^2}d\frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}}{2c}\right)}{b}$$

$\Big\downarrow$ 217

$$-\frac{2cd\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}+1\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}}{2c}-\frac{\int \frac{d-d\cot(a+bx)}{\cot^2(a+bx)d^2+d^2}d\frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}}{2c}\right)}{b}$$

$\Big\downarrow$ 1479

$$-\frac{2cd\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}+1\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}}{2c}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-\frac{2\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}}{\sqrt{c}\left(\frac{\cot(a+bx)d}{c}+\frac{d}{c}-\frac{\sqrt{2}\sqrt{d\cos(a+bx)}\sqrt{d}}{\sqrt{c}\sqrt{c\sin(a+bx)}}\right)}d\frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}}{2\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\int -}{\sqrt{c}}}{2c}\right.}{b}$$

$\Big\downarrow$ 25

$$-\frac{2cd\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}+1\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}}{2c}-\frac{\frac{\int\frac{\sqrt{2}\sqrt{d}-\frac{2\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}}{\sqrt{c}\left(\frac{\cot(a+bx)d}{c}+\frac{d}{c}-\frac{\sqrt{2}\sqrt{d\cos(a+bx)}\sqrt{d}}{\sqrt{c}\sqrt{c\sin(a+bx)}}\right)}d\frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}}{2\sqrt{2}\sqrt{c}\sqrt{d}}+\cdots}{2c}\right)}{b}$$

$\downarrow$ 27

$$-\frac{2cd\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}+1\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}}{2c}-\frac{\frac{\int\frac{\sqrt{2}\sqrt{d}-\frac{2\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}}{\frac{\cot(a+bx)d}{c}+\frac{d}{c}-\frac{\sqrt{2}\sqrt{d\cos(a+bx)}\sqrt{d}}{\sqrt{c}\sqrt{c\sin(a+bx)}}}d\frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}}{2\sqrt{2}c\sqrt{d}}+\cdots}{2c}\right)}{b}$$

$\downarrow$ 1103

$$-\frac{2cd\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}+1\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{d}}}{2c}-\frac{\frac{\log\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}+d\cot(a+bx)+d\right)}{2\sqrt{2}\sqrt{c}\sqrt{d}}-\frac{\log\left(-\frac{\sqrt{2}\sqrt{c}\sqrt{d}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\cdots\right)}{2\sqrt{2}\sqrt{c}\cdots}}{2c}\right)}{b}$$

input
```
Int[Sqrt[d*Cos[a + b*x]]/Sqrt[c*Sin[a + b*x]],x]
```

output
```
(-2*c*d*((-(ArcTan[1 - (Sqrt[2]*Sqrt[c]*Sqrt[d*Cos[a + b*x]])/(Sqrt[d]*Sqr
t[c*Sin[a + b*x]])]/(Sqrt[2]*Sqrt[c]*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*Sqrt[
c]*Sqrt[d*Cos[a + b*x]])/(Sqrt[d]*Sqrt[c*Sin[a + b*x]])]/(Sqrt[2]*Sqrt[c]*
Sqrt[d]))/(2*c) - (-1/2*Log[d + d*Cot[a + b*x] - (Sqrt[2]*Sqrt[c]*Sqrt[d]*
Sqrt[d*Cos[a + b*x]])/Sqrt[c*Sin[a + b*x]]]/(Sqrt[2]*Sqrt[c]*Sqrt[d]) + Lo
g[d + d*Cot[a + b*x] + (Sqrt[2]*Sqrt[c]*Sqrt[d]*Sqrt[d*Cos[a + b*x]])/Sqrt
[c*Sin[a + b*x]]]/(2*Sqrt[2]*Sqrt[c]*Sqrt[d]))/(2*c)))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3055
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[(-k)*a*(b/f)   Subst[Int[x
^(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[
e + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m,
0] && LtQ[m, 1]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(153) = 306$.

Time = 4.32 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.75

method	result
default	$\frac{\sqrt{2}\,\sqrt{d\cos(bx+a)}\left(\ln\left(-\frac{-(1-\cos(bx+a))^2\csc(bx+a)+2\sqrt{-\frac{2\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}\sin(bx+a)-2+2\cos(bx+a)+\sin(bx+a)}{1-\cos(bx+a)}\right)-2\arctan\left(\sqrt{\cdots}\right.\right.}{\cdots}$

input
```
int((d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/b*2^(1/2)/(c*sin(b*x+a))^(1/2)*(d*cos(b*x+a))^(1/2)*(ln(-1/(1-cos(b*x+
a))*(-(1-cos(b*x+a))^2*csc(b*x+a)+2*(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+
1)^2)^(1/2)*sin(b*x+a)-2+2*cos(b*x+a)+sin(b*x+a)))-2*arctan(((-2*sin(b*x+a
)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)+cos(b*x+a)-1)/(cos(b*x+a)-
1))-ln(1/(1-cos(b*x+a))*((1-cos(b*x+a))^2*csc(b*x+a)+2*(-2*sin(b*x+a)*cos(
b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)+2-2*cos(b*x+a)-sin(b*x+a)))+2*ar
ctan((-(-2*sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)+cos(b*
x+a)-1)/(cos(b*x+a)-1)))/(-sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*(
csc(b*x+a)-cot(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 477 vs. $2(153) = 306$.

Time = 0.14 (sec) , antiderivative size = 477, normalized size of antiderivative = 2.36

$$\int \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\,dx =$$

$$-\frac{\sqrt{2}\sqrt{\frac{d}{c}}\arctan\left(\frac{2\,d\cos(bx+a)^3-2\,d\cos(bx+a)^2\sin(bx+a)+\sqrt{2}\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}\sqrt{\frac{d}{c}}-2\,d\cos(bx+a)}{2\left(d\cos(bx+a)^3+d\cos(bx+a)^2\sin(bx+a)-d\cos(bx+a)\right)}\right)+\sqrt{2}\sqrt{\frac{d}{c}}\,\mathrm{arct}}{}$$

input
```
integrate((d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
-1/8*(sqrt(2)*sqrt(d/c)*arctan(1/2*(2*d*cos(b*x + a)^3 - 2*d*cos(b*x + a)^
2*sin(b*x + a) + sqrt(2)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*sqrt(d/
c) - 2*d*cos(b*x + a))/(d*cos(b*x + a)^3 + d*cos(b*x + a)^2*sin(b*x + a) -
 d*cos(b*x + a))) + sqrt(2)*sqrt(d/c)*arctan(-1/2*(2*d*cos(b*x + a)^3 - 2*
d*cos(b*x + a)^2*sin(b*x + a) - sqrt(2)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*
x + a))*sqrt(d/c) - 2*d*cos(b*x + a))/(d*cos(b*x + a)^3 + d*cos(b*x + a)^2
*sin(b*x + a) - d*cos(b*x + a))) - 2*sqrt(2)*sqrt(d/c)*arctan(-1/2*sqrt(2)
*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*sqrt(d/c)*(cos(b*x + a) - sin(b
*x + a))/(d*cos(b*x + a)*sin(b*x + a))) - sqrt(2)*sqrt(d/c)*log(2*sqrt(2)*
sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*sqrt(d/c)*(cos(b*x + a) + sin(b*
x + a)) + 4*d*cos(b*x + a)*sin(b*x + a) + d) + sqrt(2)*sqrt(d/c)*log(-2*sq
rt(2)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*sqrt(d/c)*(cos(b*x + a) +
sin(b*x + a)) + 4*d*cos(b*x + a)*sin(b*x + a) + d))/b
```

Sympy [F]

$$\int \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{\sqrt{d\cos{(a+bx)}}}{\sqrt{c\sin{(a+bx)}}}\,dx$$

input
```
integrate((d*cos(b*x+a))**(1/2)/(c*sin(b*x+a))**(1/2),x)
```

output
```
Integral(sqrt(d*cos(a + b*x))/sqrt(c*sin(a + b*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{\sqrt{d\cos{(bx+a)}}}{\sqrt{c\sin{(bx+a)}}}\,dx$$

input
```
integrate((d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*cos(b*x + a))/sqrt(c*sin(b*x + a)), x)
```

Giac [F]

$$\int \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{\sqrt{d\cos{(bx+a)}}}{\sqrt{c\sin{(bx+a)}}}\,dx$$

input
```
integrate((d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*cos(b*x + a))/sqrt(c*sin(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{\sqrt{d\,\cos\left(a+b\,x\right)}}{\sqrt{c\,\sin\left(a+b\,x\right)}}\,dx$$

input
```
int((d*cos(a + b*x))^(1/2)/(c*sin(a + b*x))^(1/2),x)
```

output
```
int((d*cos(a + b*x))^(1/2)/(c*sin(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sin(bx+a)}\,\sqrt{\cos(bx+a)}}{\sin(bx+a)}dx\right)}{c}$$

input
```
int((d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sin(a + b*x))*sqrt(cos(a + b*x)))/sin(a + b*x),
x))/c
```

3.297 $\int \frac{1}{(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 35

$$\int \frac{1}{(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}\,dx = \frac{2\sqrt{c\sin(a+bx)}}{bcd\sqrt{d\cos(a+bx)}}$$

output
```
2*(c*sin(b*x+a))^(1/2)/b/c/d/(d*cos(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \frac{1}{(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}\,dx = \frac{\sin(2(a+bx))}{b(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}$$

input
```
Integrate[1/((d*Cos[a + b*x])^(3/2)*Sqrt[c*Sin[a + b*x]]),x]
```

output
```
Sin[2*(a + b*x)]/(b*(d*Cos[a + b*x])^(3/2)*Sqrt[c*Sin[a + b*x]])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3043}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3043}$$

$$\frac{2\sqrt{c\sin(a+bx)}}{bcd\sqrt{d\cos(a+bx)}}$$

input
```
Int[1/((d*Cos[a + b*x])^(3/2)*Sqrt[c*Sin[a + b*x]]),x]
```

output
```
(2*Sqrt[c*Sin[a + b*x]])/(b*c*d*Sqrt[d*Cos[a + b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3043
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Maple [A] (verified)

Time = 4.80 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{2\sin(bx+a)}{bd\sqrt{d\cos(bx+a)}\,\sqrt{c\sin(bx+a)}}$	35

input
```
int(1/(d*cos(b*x+a))^(3/2)/(c*sin(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b*sin(b*x+a)/d/(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}\,dx = \frac{2\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}}{bcd^2\cos(bx+a)}$$

input
```
integrate(1/(d*cos(b*x+a))^(3/2)/(c*sin(b*x+a))^(1/2),x, algorithm="fricas
")
```

output
```
2*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))/(b*c*d^2*cos(b*x + a))
```

Sympy [F]

$$\int \frac{1}{(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}\,dx = \int \frac{1}{\sqrt{c\sin(a+bx)}\,(d\cos(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*cos(b*x+a))**(3/2)/(c*sin(b*x+a))**(1/2),x)
```

output
```
Integral(1/(sqrt(c*sin(a + b*x))*(d*cos(a + b*x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}\,dx = \int \frac{1}{(d\cos(bx+a))^{\frac{3}{2}}\sqrt{c\sin(bx+a)}}\,dx$$

input
```
integrate(1/(d*cos(b*x+a))^(3/2)/(c*sin(b*x+a))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/((d*cos(b*x + a))^(3/2)*sqrt(c*sin(b*x + a))), x)
```

Giac [F]

$$\int \frac{1}{(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}\,dx = \int \frac{1}{(d\cos(bx+a))^{\frac{3}{2}}\sqrt{c\sin(bx+a)}}\,dx$$

input
```
integrate(1/(d*cos(b*x+a))^(3/2)/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((d*cos(b*x + a))^(3/2)*sqrt(c*sin(b*x + a))), x)
```

Mupad [B] (verification not implemented)

Time = 25.72 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}\,dx = \frac{2\sqrt{c\sin(a+bx)}}{b\,c\,d\sqrt{d\cos(a+bx)}}$$

input
```
int(1/((d*cos(a + b*x))^(3/2)*(c*sin(a + b*x))^(1/2)),x)
```

output
```
(2*(c*sin(a + b*x))^(1/2))/(b*c*d*(d*cos(a + b*x))^(1/2))
```

Reduce [F]

$$\int \frac{1}{(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sin(bx+a)}\,\sqrt{\cos(bx+a)}}{\cos(bx+a)^2\sin(bx+a)}dx\right)}{c\,d^2}$$

input
```
int(1/(d*cos(b*x+a))^(3/2)/(c*sin(b*x+a))^(1/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sin(a + b*x))*sqrt(cos(a + b*x)))/(cos(a + b*x)
**2*sin(a + b*x)),x))/(c*d**2)
```

3.298 $\int \frac{1}{(d\cos(a+bx))^{7/2}\sqrt{c\sin(a+bx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 75

$$\int \frac{1}{(d\cos(a+bx))^{7/2}\sqrt{c\sin(a+bx)}}\,dx=\frac{2\sqrt{c\sin(a+bx)}}{5bcd(d\cos(a+bx))^{5/2}}+\frac{8\sqrt{c\sin(a+bx)}}{5bcd^3\sqrt{d\cos(a+bx)}}$$

output
```
2/5*(c*sin(b*x+a))^(1/2)/b/c/d/(d*cos(b*x+a))^(5/2)+8/5*(c*sin(b*x+a))^(1/
2)/b/c/d^3/(d*cos(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.69

$$\int \frac{1}{(d\cos(a+bx))^{7/2}\sqrt{c\sin(a+bx)}}\,dx=\frac{2(3+2\cos(2(a+bx)))\tan(a+bx)}{5bd^2(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}$$

input
```
Integrate[1/((d*Cos[a + b*x])^(7/2)*Sqrt[c*Sin[a + b*x]]),x]
```

output
```
(2*(3 + 2*Cos[2*(a + b*x)])*Tan[a + b*x])/(5*b*d^2*(d*Cos[a + b*x])^(3/2)*
Sqrt[c*Sin[a + b*x]])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3051, 3042, 3043}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{7/2}}dx$$

$$\Big\downarrow \text{3051}$$

$$\frac{4\int \frac{1}{(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}dx}{5d^2}+\frac{2\sqrt{c\sin(a+bx)}}{5bcd(d\cos(a+bx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\int \frac{1}{(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}dx}{5d^2}+\frac{2\sqrt{c\sin(a+bx)}}{5bcd(d\cos(a+bx))^{5/2}}$$

$$\Big\downarrow \text{3043}$$

$$\frac{8\sqrt{c\sin(a+bx)}}{5bcd^3\sqrt{d\cos(a+bx)}}+\frac{2\sqrt{c\sin(a+bx)}}{5bcd(d\cos(a+bx))^{5/2}}$$

input
```
Int[1/((d*Cos[a + b*x])^(7/2)*Sqrt[c*Sin[a + b*x]]),x]
```

output
```
(2*Sqrt[c*Sin[a + b*x]])/(5*b*c*d*(d*Cos[a + b*x])^(5/2)) + (8*Sqrt[c*Sin[
a + b*x]])/(5*b*c*d^3*Sqrt[d*Cos[a + b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3043
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

rule 3051
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Simp[(m + n + 2)/(a^2*(m + 1))   Int[(b*Sin[e + f*x
])^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m
, -1] && IntegersQ[2*m, 2*n]
```

Maple [A] (verified)

Time = 4.73 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.68

method	result	size
default	$\frac{\frac{8\sin(bx+a)}{5}+\frac{2\sec(bx+a)\tan(bx+a)}{5}}{b\sqrt{c\sin(bx+a)}\,\sqrt{d\cos(bx+a)}\,d^{3}}$	51

input
```
int(1/(d*cos(b*x+a))^(7/2)/(c*sin(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/b/(c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2)/d^3*(4*sin(b*x+a)+sec(b*x+
a)*tan(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.68

$$\int \frac{1}{(d\cos(a+bx))^{7/2}\sqrt{c\sin(a+bx)}}\,dx = \frac{2\,\sqrt{d\cos\left(bx+a\right)}\left(4\,\cos\left(bx+a\right)^2+1\right)\sqrt{c\sin\left(bx+a\right)}}{5\,bcd^4\cos\left(bx+a\right)^3}$$

input
```
integrate(1/(d*cos(b*x+a))^(7/2)/(c*sin(b*x+a))^(1/2),x, algorithm="fricas
")
```

output
```
2/5*sqrt(d*cos(b*x + a))*(4*cos(b*x + a)^2 + 1)*sqrt(c*sin(b*x + a))/(b*c*
d^4*cos(b*x + a)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d\cos(a+bx))^{7/2}\sqrt{c\sin(a+bx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*cos(b*x+a))**(7/2)/(c*sin(b*x+a))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(d\cos(a+bx))^{7/2}\sqrt{c\sin(a+bx)}}\,dx = \int \frac{1}{\left(d\cos\left(bx+a\right)\right)^{\frac{7}{2}}\sqrt{c\sin\left(bx+a\right)}}\,dx$$

input
```
integrate(1/(d*cos(b*x+a))^(7/2)/(c*sin(b*x+a))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/((d*cos(b*x + a))^(7/2)*sqrt(c*sin(b*x + a))), x)
```

Giac [F]

$$\int \frac{1}{(d\cos(a+bx))^{7/2}\sqrt{c\sin(a+bx)}}\,dx = \int \frac{1}{(d\cos(bx+a))^{\frac{7}{2}}\sqrt{c\sin(bx+a)}}\,dx$$

input
```
integrate(1/(d*cos(b*x+a))^(7/2)/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((d*cos(b*x + a))^(7/2)*sqrt(c*sin(b*x + a))), x)
```

Mupad [B] (verification not implemented)

Time = 26.46 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03

$$\int \frac{1}{(d\cos(a+bx))^{7/2}\sqrt{c\sin(a+bx)}}\,dx = \frac{8\sqrt{c\sin(a+b\,x)}\,(5\cos(2\,a+2\,b\,x)+\cos(4\,a+4\,b\,x)+4)}{5\,b\,c\,d^3\sqrt{d\cos(a+b\,x)}\,(4\cos(2\,a+2\,b\,x)+\cos(4\,a+4\,b\,x)+}$$

input
```
int(1/((d*cos(a + b*x))^(7/2)*(c*sin(a + b*x))^(1/2)),x)
```

output
```
(8*(c*sin(a + b*x))^(1/2)*(5*cos(2*a + 2*b*x) + cos(4*a + 4*b*x) + 4))/(5*
b*c*d^3*(d*cos(a + b*x))^(1/2)*(4*cos(2*a + 2*b*x) + cos(4*a + 4*b*x) + 3)
)
```

Reduce [F]

$$\int \frac{1}{(d\cos(a+bx))^{7/2}\sqrt{c\sin(a+bx)}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sin(bx+a)}\,\sqrt{\cos(bx+a)}}{\cos(bx+a)^4\sin(bx+a)}dx\right)}{c\,d^4}$$

input
```
int(1/(d*cos(b*x+a))^(7/2)/(c*sin(b*x+a))^(1/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sin(a + b*x))*sqrt(cos(a + b*x)))/(cos(a + b*x)
**4*sin(a + b*x)),x))/(c*d**4)
```

3.299 $\int \frac{1}{(d\cos(a+bx))^{11/2}\sqrt{c\sin(a+bx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 112

$$\int \frac{1}{(d\cos(a+bx))^{11/2}\sqrt{c\sin(a+bx)}}\,dx = \frac{2\sqrt{c\sin(a+bx)}}{9bcd(d\cos(a+bx))^{9/2}} + \frac{16\sqrt{c\sin(a+bx)}}{45bcd^3(d\cos(a+bx))^{5/2}} + \frac{64\sqrt{c\sin(a+bx)}}{45bcd^5\sqrt{d\cos(a+bx)}}$$

output
```
2/9*(c*sin(b*x+a))^(1/2)/b/c/d/(d*cos(b*x+a))^(9/2)+16/45*(c*sin(b*x+a))^(
1/2)/b/c/d^3/(d*cos(b*x+a))^(5/2)+64/45*(c*sin(b*x+a))^(1/2)/b/c/d^5/(d*co
s(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.60

$$\int \frac{1}{(d\cos(a+bx))^{11/2}\sqrt{c\sin(a+bx)}}\,dx = \frac{2\sqrt{d\cos(a+bx)}(21+20\cos(2(a+bx))+4\cos(4(a+bx)))\sec}{45bcd^6}$$

input
```
Integrate[1/((d*Cos[a + b*x])^(11/2)*Sqrt[c*Sin[a + b*x]]),x]
```

output
```
(2*Sqrt[d*Cos[a + b*x]]*(21 + 20*Cos[2*(a + b*x)] + 4*Cos[4*(a + b*x)])*Se
c[a + b*x]^5*Sqrt[c*Sin[a + b*x]])/(45*b*c*d^6)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3042, 3051, 3042, 3051, 3042, 3043}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{11/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{11/2}}dx$$

$$\Big\downarrow \text{3051}$$

$$\frac{8\int \frac{1}{(d\cos(a+bx))^{7/2}\sqrt{c\sin(a+bx)}}dx}{9d^2}+\frac{2\sqrt{c\sin(a+bx)}}{9bcd(d\cos(a+bx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8\int \frac{1}{(d\cos(a+bx))^{7/2}\sqrt{c\sin(a+bx)}}dx}{9d^2}+\frac{2\sqrt{c\sin(a+bx)}}{9bcd(d\cos(a+bx))^{9/2}}$$

$$\Big\downarrow \text{3051}$$

$$\frac{8\left(\frac{4\int \frac{1}{(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}dx}{5d^2}+\frac{2\sqrt{c\sin(a+bx)}}{5bcd(d\cos(a+bx))^{5/2}}\right)}{9d^2}+\frac{2\sqrt{c\sin(a+bx)}}{9bcd(d\cos(a+bx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8\left(\frac{4\int \frac{1}{(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}dx}{5d^2}+\frac{2\sqrt{c\sin(a+bx)}}{5bcd(d\cos(a+bx))^{5/2}}\right)}{9d^2}+\frac{2\sqrt{c\sin(a+bx)}}{9bcd(d\cos(a+bx))^{9/2}}$$

$$\Big\downarrow \text{3043}$$

$$\frac{8\left(\frac{8\sqrt{c\sin(a+bx)}}{5bcd^3\sqrt{d\cos(a+bx)}}+\frac{2\sqrt{c\sin(a+bx)}}{5bcd(d\cos(a+bx))^{5/2}}\right)}{9d^2}+\frac{2\sqrt{c\sin(a+bx)}}{9bcd(d\cos(a+bx))^{9/2}}$$

input
```
Int[1/((d*Cos[a + b*x])^(11/2)*Sqrt[c*Sin[a + b*x]]),x]
```

output
```
(2*Sqrt[c*Sin[a + b*x]])/(9*b*c*d*(d*Cos[a + b*x])^(9/2)) + (8*((2*Sqrt[c*
Sin[a + b*x]])/(5*b*c*d*(d*Cos[a + b*x])^(5/2)) + (8*Sqrt[c*Sin[a + b*x]])
/(5*b*c*d^3*Sqrt[d*Cos[a + b*x]])))/(9*d^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3043
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

rule 3051
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Simp[(m + n + 2)/(a^2*(m + 1))   Int[(b*Sin[e + f*x
])^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m
, -1] && IntegersQ[2*m, 2*n]
```

Maple [A] (verified)

Time = 4.49 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.58

method	result	size
default	$\frac{2\left(32\cos(bx+a)^4+8\cos(bx+a)^2+5\right)\tan(bx+a)\sec(bx+a)^3}{45b\sqrt{c\sin(bx+a)}\,\sqrt{d\cos(bx+a)}\,d^5}$	65

input
```
int(1/(d*cos(b*x+a))^(11/2)/(c*sin(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/45/b*(32*cos(b*x+a)^4+8*cos(b*x+a)^2+5)/(c*sin(b*x+a))^(1/2)/(d*cos(b*x+
a))^(1/2)/d^5*tan(b*x+a)*sec(b*x+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.54

$$\int \frac{1}{(d\cos(a+bx))^{11/2}\sqrt{c\sin(a+bx)}}\,dx = \frac{2\left(32\,\cos\left(bx+a\right)^4+8\,\cos\left(bx+a\right)^2+5\right)\sqrt{d\cos\left(bx+a\right)}\sqrt{c\sin\left(bx+a\right)}}{45\,bcd^6\cos\left(bx+a\right)^5}$$

input
```
integrate(1/(d*cos(b*x+a))^(11/2)/(c*sin(b*x+a))^(1/2),x, algorithm="frica
s")
```

output
```
2/45*(32*cos(b*x + a)^4 + 8*cos(b*x + a)^2 + 5)*sqrt(d*cos(b*x + a))*sqrt(
c*sin(b*x + a))/(b*c*d^6*cos(b*x + a)^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d\cos(a+bx))^{11/2}\sqrt{c\sin(a+bx)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*cos(b*x+a))**(11/2)/(c*sin(b*x+a))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(d\cos(a+bx))^{11/2}\sqrt{c\sin(a+bx)}}\,dx = \int \frac{1}{(d\cos(bx+a))^{\frac{11}{2}}\sqrt{c\sin(bx+a)}}\,dx$$

input
```
integrate(1/(d*cos(b*x+a))^(11/2)/(c*sin(b*x+a))^(1/2),x, algorithm="maxim
a")
```

output
```
integrate(1/((d*cos(b*x + a))^(11/2)*sqrt(c*sin(b*x + a))), x)
```

Giac [F]

$$\int \frac{1}{(d\cos(a+bx))^{11/2}\sqrt{c\sin(a+bx)}}\,dx = \int \frac{1}{(d\cos(bx+a))^{\frac{11}{2}}\sqrt{c\sin(bx+a)}}\,dx$$

input
```
integrate(1/(d*cos(b*x+a))^(11/2)/(c*sin(b*x+a))^(1/2),x, algorithm="giac"
)
```

output
```
integrate(1/((d*cos(b*x + a))^(11/2)*sqrt(c*sin(b*x + a))), x)
```

Mupad [B] (verification not implemented)

Time = 29.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d\cos(a+bx))^{11/2}\sqrt{c\sin(a+bx)}}\,dx = \frac{32\sqrt{c\sin(a+bx)}\,(162\cos(2a+2bx)+73\cos(4a+4bx)-}{45\,b\,c\,d^5\sqrt{d\cos(a+bx)}\,(56\cos(2a+2bx)+28\cos(4a+4}$$

input
```
int(1/((d*cos(a + b*x))^(11/2)*(c*sin(a + b*x))^(1/2)),x)
```

output
```
(32*(c*sin(a + b*x))^(1/2)*(162*cos(2*a + 2*b*x) + 73*cos(4*a + 4*b*x) + 1
8*cos(6*a + 6*b*x) + 2*cos(8*a + 8*b*x) + 105))/(45*b*c*d^5*(d*cos(a + b*x
))^(1/2)*(56*cos(2*a + 2*b*x) + 28*cos(4*a + 4*b*x) + 8*cos(6*a + 6*b*x) +
 cos(8*a + 8*b*x) + 35))
```

Reduce [F]

$$\int \frac{1}{(d\cos(a+bx))^{11/2}\sqrt{c\sin(a+bx)}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\left(\int \frac{\sqrt{\sin(bx+a)}\,\sqrt{\cos(bx+a)}}{\cos(bx+a)^{6}\sin(bx+a)}dx\right)}{c\,d^{6}}$$

input
```
int(1/(d*cos(b*x+a))^(11/2)/(c*sin(b*x+a))^(1/2),x)
```

output
```
(sqrt(d)*sqrt(c)*int((sqrt(sin(a + b*x))*sqrt(cos(a + b*x)))/(cos(a + b*x)
**6*sin(a + b*x)),x))/(c*d**6)
```

3.300 $\int \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 125

$$\int \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{(1+\cot(a+bx))\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b}$$

output

```
-1/2*arctan(-1+2^(1/2)*cos(b*x+a)^(1/2)/sin(b*x+a)^(1/2))*2^(1/2)/b-1/2*ar
ctan(1+2^(1/2)*cos(b*x+a)^(1/2)/sin(b*x+a)^(1/2))*2^(1/2)/b+1/2*arctanh(2^
(1/2)*cos(b*x+a)^(1/2)/(1+cot(b*x+a))/sin(b*x+a)^(1/2))*2^(1/2)/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\,dx = \frac{2\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4},\frac{5}{4},\sin^2(a+bx)\right)\sqrt{\sin(a+bx)}}{b\sqrt{\cos(a+bx)}}$$

input
```
Integrate[Sqrt[Cos[a + b*x]]/Sqrt[Sin[a + b*x]],x]
```

output
```
(2*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, 1/4, 5/4, Sin[a + b*x]^2]
*Sqrt[Sin[a + b*x]])/(b*Sqrt[Cos[a + b*x]])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.42, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3055, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}dx$$

$$\Big\downarrow \text{3055}$$

$$-\frac{2\int \frac{\cot(a+bx)}{\cot^2(a+bx)+1}d\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}}{b}$$

$$\Big\downarrow \text{826}$$

$$-\frac{2\left(\frac{1}{2}\int \frac{\cot(a+bx)+1}{\cot^2(a+bx)+1}d\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}-\frac{1}{2}\int \frac{1-\cot(a+bx)}{\cot^2(a+bx)+1}d\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{b}$$

$$\Big\downarrow \text{1476}$$

$$-\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1}d\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+\frac{1}{2}\int \frac{1}{\cot(a+bx)+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1}d\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)-\frac{1}{2}\int \frac{1-\cot(a+bx)}{\cot^2(a+bx)+1}d\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{b}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\cot(a+bx)-1}d\left(1-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\cot(a+bx)-1}d\left(\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\cot(a+bx)}{\cot^2(a+bx)+1}d\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{b}$$

$\downarrow$ 217

$$-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\cot(a+bx)}{\cot^2(a+bx)+1}d\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{b}$$

$\downarrow$ 1479

$$-\frac{2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}}{\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1}d\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{\cot(a+bx)+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1}d\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{\sqrt{2}}\right.\right.}{b}$$

$\downarrow$ 25

$$-\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}}{\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1}d\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{\cot(a+bx)+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1}d\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{\sqrt{2}}\right.\right.}{b}$$

$\downarrow$ 27

$$-\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}}{\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1}d\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1}{\cot(a+bx)+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1}d\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right.}{\sqrt{2}}\right.\right.}{b}$$

$\downarrow$ 1103

$$-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\cot(a+bx)+\frac{\sqrt{2}\sqrt{}}{\sqrt{\mathrm{si}}}\right.}{2\sqrt{2}}\right.\right.}{b}$$

input
```
Int[Sqrt[Cos[a + b*x]]/Sqrt[Sin[a + b*x]],x]
```

output
```
(-2*((-(ArcTan[1 - (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]]]/Sqrt[2
]) + ArcTan[1 + (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]]]/Sqrt[2])/
2 + (Log[1 + Cot[a + b*x] - (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]
]]/(2*Sqrt[2]) - Log[1 + Cot[a + b*x] + (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[
Sin[a + b*x]]]/(2*Sqrt[2]))/2))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3055
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[(-k)*a*(b/f)   Subst[Int[x
^(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[
e + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m,
0] && LtQ[m, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(102) = 204$.

Time = 2.17 (sec) , antiderivative size = 353, normalized size of antiderivative = 2.82

method	result
default	$\frac{\sqrt{2}\,(1-\cos(bx+a))\sqrt{\cos(bx+a)}\left(\ln\left(\frac{-(1-\cos(bx+a))^2\csc(bx+a)+2\sqrt{2}\sqrt{\frac{\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}\sin(bx+a)+2-2\cos(bx+a)+\sin(bx+a)}{1-\cos(bx+a)}\right)+2}{}$

input
```
int(cos(b*x+a)^(1/2)/sin(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/b*2^(1/2)/sin(b*x+a)^(3/2)*(1-cos(b*x+a))*cos(b*x+a)^(1/2)*(ln(1/(1-co
s(b*x+a))*(-(1-cos(b*x+a))^2*csc(b*x+a)+2*2^(1/2)*(sin(b*x+a)*cos(b*x+a)/(
cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)+2-2*cos(b*x+a)+sin(b*x+a)))+2*arctan((2^
(1/2)*(sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)-cos(b*x+a)
+1)/(cos(b*x+a)-1))-ln(-1/(1-cos(b*x+a))*((1-cos(b*x+a))^2*csc(b*x+a)+2*2^
(1/2)*(sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)-2+2*cos(b*
x+a)-sin(b*x+a)))+2*arctan((2^(1/2)*(sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^
2)^(1/2)*sin(b*x+a)+cos(b*x+a)-1)/(cos(b*x+a)-1)))/(sin(b*x+a)*cos(b*x+a)/
(cos(b*x+a)+1)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(102) = 204$.

Time = 0.17 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.93

$$\int \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{2\cos(bx+a)^3-2\cos(bx+a)^2\sin(bx+a)+\sqrt{2}\sqrt{\cos(bx+a)}\sqrt{\sin(bx+a)}-2\cos(bx+a)}{2\left(\cos(bx+a)^3+\cos(bx+a)^2\sin(bx+a)-\cos(bx+a)\right)}\right)+\sqrt{2}\arctan\left(-\frac{2\cos(bx+a}{}\right.}{}$$

input
```
integrate(cos(b*x+a)^(1/2)/sin(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/8*(sqrt(2)*arctan(1/2*(2*cos(b*x + a)^3 - 2*cos(b*x + a)^2*sin(b*x + a)
 + sqrt(2)*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) - 2*cos(b*x + a))/(cos(b*
x + a)^3 + cos(b*x + a)^2*sin(b*x + a) - cos(b*x + a))) + sqrt(2)*arctan(-
1/2*(2*cos(b*x + a)^3 - 2*cos(b*x + a)^2*sin(b*x + a) - sqrt(2)*sqrt(cos(b
*x + a))*sqrt(sin(b*x + a)) - 2*cos(b*x + a))/(cos(b*x + a)^3 + cos(b*x +
a)^2*sin(b*x + a) - cos(b*x + a))) - 2*sqrt(2)*arctan(-1/2*(sqrt(2)*cos(b*
x + a) - sqrt(2)*sin(b*x + a))/(sqrt(cos(b*x + a))*sqrt(sin(b*x + a)))) -
sqrt(2)*log(2*(sqrt(2)*cos(b*x + a) + sqrt(2)*sin(b*x + a))*sqrt(cos(b*x +
 a))*sqrt(sin(b*x + a)) + 4*cos(b*x + a)*sin(b*x + a) + 1) + sqrt(2)*log(-
2*(sqrt(2)*cos(b*x + a) + sqrt(2)*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(si
n(b*x + a)) + 4*cos(b*x + a)*sin(b*x + a) + 1))/b
```

Sympy [F]

$$\int \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\,dx = \int \frac{\sqrt{\cos\left(a+bx\right)}}{\sqrt{\sin\left(a+bx\right)}}\,dx$$

input
```
integrate(cos(b*x+a)**(1/2)/sin(b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(cos(a + b*x))/sqrt(sin(a + b*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\,dx = \int \frac{\sqrt{\cos\left(bx+a\right)}}{\sqrt{\sin\left(bx+a\right)}}\,dx$$

input
```
integrate(cos(b*x+a)^(1/2)/sin(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(b*x + a))/sqrt(sin(b*x + a)), x)
```

Giac [F]

$$\int \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\,dx = \int \frac{\sqrt{\cos\left(bx+a\right)}}{\sqrt{\sin\left(bx+a\right)}}\,dx$$

input
```
integrate(cos(b*x+a)^(1/2)/sin(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(cos(b*x + a))/sqrt(sin(b*x + a)), x)
```

Mupad [B] (verification not implemented)

Time = 26.41 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.35

$$\int \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\,dx = -\frac{2\cos\left(a+b\,x\right)^{3/2}\,\sqrt{\sin\left(a+b\,x\right)}\,{}_2\mathrm{F}_1\left(\frac{3}{4},\frac{3}{4};\ \frac{7}{4};\ \cos\left(a+b\,x\right)^2\right)}{3\,b\,\left(\sin\left(a+b\,x\right)^2\right)^{1/4}}$$

input
```
int(cos(a + b*x)^(1/2)/sin(a + b*x)^(1/2),x)
```

output
```
-(2*cos(a + b*x)^(3/2)*sin(a + b*x)^(1/2)*hypergeom([3/4, 3/4], 7/4, cos(a
 + b*x)^2))/(3*b*(sin(a + b*x)^2)^(1/4))
```

Reduce [F]

$$\int \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\,dx = \int \frac{\sqrt{\sin\left(bx+a\right)}\,\sqrt{\cos\left(bx+a\right)}}{\sin\left(bx+a\right)}dx$$

input
```
int(cos(b*x+a)^(1/2)/sin(b*x+a)^(1/2),x)
```

output
```
int((sqrt(sin(a + b*x))*sqrt(cos(a + b*x)))/sin(a + b*x),x)
```

3.301 $\int \frac{\cos^{\frac{3}{2}}(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 150

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{\sqrt{2}b} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{\sqrt{2}b} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}(1+\tan(a+bx))}\right)}{\sqrt{2}b} - \frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}}$$

output
```
1/2*arctan(1-2^(1/2)*sin(b*x+a)^(1/2)/cos(b*x+a)^(1/2))*2^(1/2)/b-1/2*arct
an(1+2^(1/2)*sin(b*x+a)^(1/2)/cos(b*x+a)^(1/2))*2^(1/2)/b+1/2*arctanh(2^(1
/2)*sin(b*x+a)^(1/2)/cos(b*x+a)^(1/2)/(1+tan(b*x+a)))*2^(1/2)/b-2*cos(b*x+
a)^(1/2)/b/sin(b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.37

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx = -\frac{2\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},-\frac{1}{4},\frac{3}{4},\sin^2(a+bx)\right)}{b\cos^{\frac{3}{2}}(a+bx)\sqrt{\sin(a+bx)}}$$

input
```
Integrate[Cos[a + b*x]^(3/2)/Sin[a + b*x]^(3/2),x]
```

output
```
(-2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-1/4, -1/4, 3/4, Sin[a + b*x]
^2])/(b*Cos[a + b*x]^(3/2)*Sqrt[Sin[a + b*x]])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.35, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3047, 3042, 3054, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)^{3/2}}{\sin(a+bx)^{3/2}}dx$$

$$\Big\downarrow \text{3047}$$

$$-\int \frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}dx - \frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}dx - \frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}}$$

$$\Big\downarrow \text{3054}$$

$$-\frac{2\int \frac{\tan(a+bx)}{\tan^2(a+bx)+1}d\frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}}{b}-\frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}}$$

$$\Big\downarrow \text{826}$$

$$-\frac{2\left(\frac{1}{2}\int \frac{\tan(a+bx)+1}{\tan^2(a+bx)+1}d\frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}-\frac{1}{2}\int \frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{b}-\frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}}$$

$$\Big\downarrow \text{1476}$$

$$-\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(a+bx)-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1}d\frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+\frac{1}{2}\int \frac{1}{\tan(a+bx)+\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1}d\frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)-\frac{1}{2}\int \frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\frac{\sqrt{}}{\sqrt{}}\right.}{b}$$

$$\frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\tan(a+bx)-1}d\left(1-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\tan(a+bx)-1}d\left(\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{b}-$$

$$\frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}}$$

$$\Big\downarrow \text{217}$$

$$-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{b}-$$

$$\frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}}$$

$$\Big\downarrow \text{1479}$$

$$-\frac{2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}}{\tan(a+bx)-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1}d\frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1\right)}{\tan(a+bx)+\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1}d\frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1\right)}{\sqrt{2}}\cdots\right)\right)}{b}-\frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}}$$

$\Big\downarrow$ 25

$$-\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}}{\tan(a+bx)-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1}d\frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1\right)}{\tan(a+bx)+\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1}d\frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1\right)}{\sqrt{2}}\cdots\right)\right)}{b}-\frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}}$$

$\Big\downarrow$ 27

$$-\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}}{\tan(a+bx)-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1}d\frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1}{\tan(a+bx)+\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1}d\frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\cdots\right)}{\sqrt{2}}\cdots\right)\right)}{b}-\frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}}$$

$\Big\downarrow$ 1103

$$-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(a+bx)-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(a+bx)+\frac{\sqrt{2}\cdots}{\sqrt{\cdots}}\cdots\right)}{2\sqrt{2}}\cdots\right)\right)}{b}-\frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}}$$

input

```
Int[Cos[a + b*x]^(3/2)/Sin[a + b*x]^(3/2),x]
```

output
```
(-2*((-(ArcTan[1 - (Sqrt[2]*Sqrt[Sin[a + b*x]])/Sqrt[Cos[a + b*x]]]/Sqrt[2
]) + ArcTan[1 + (Sqrt[2]*Sqrt[Sin[a + b*x]])/Sqrt[Cos[a + b*x]]]/Sqrt[2])/
2 + (Log[1 - (Sqrt[2]*Sqrt[Sin[a + b*x]])/Sqrt[Cos[a + b*x]] + Tan[a + b*x
]]/(2*Sqrt[2]) - Log[1 + (Sqrt[2]*Sqrt[Sin[a + b*x]])/Sqrt[Cos[a + b*x]] +
 Tan[a + b*x]]/(2*Sqrt[2]))/2))/b - (2*Sqrt[Cos[a + b*x]])/(b*Sqrt[Sin[a +
 b*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3047
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Cos[e + f*x]
)^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3054
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k*a*(b/f)   Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(124) = 248$.

Time = 1.57 (sec) , antiderivative size = 429, normalized size of antiderivative = 2.86

method	result
default	$-\frac{\sqrt{2}\left((\cos(bx+a)-1)\ln\left(-\frac{2\sqrt{2}\sqrt{\frac{\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}\sin(bx+a)-\cos(bx+a)\cot(bx+a)+2\cot(bx+a)-\csc(bx+a)-2\cos(bx+a)+\sin(bx+a)}{\cos(bx+a)-1}\right.\right.}{}$

input
```
int(cos(b*x+a)^(3/2)/sin(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b*2^(1/2)*((cos(b*x+a)-1)*ln(-(2*2^(1/2)*(sin(b*x+a)*cos(b*x+a)/(cos(
b*x+a)+1)^2)^(1/2)*sin(b*x+a)-cos(b*x+a)*cot(b*x+a)+2*cot(b*x+a)-csc(b*x+a
)-2*cos(b*x+a)+sin(b*x+a)+2)/(cos(b*x+a)-1))+(1-cos(b*x+a))*ln((2*2^(1/2)*
(sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)+cos(b*x+a)*cot(b
*x+a)-2*cot(b*x+a)+csc(b*x+a)+2*cos(b*x+a)-sin(b*x+a)-2)/(cos(b*x+a)-1))+(
-2*cos(b*x+a)+2)*arctan((2^(1/2)*(sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^
(1/2)*sin(b*x+a)-cos(b*x+a)+1)/(cos(b*x+a)-1))+(-2*cos(b*x+a)+2)*arctan((2
^(1/2)*(sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)+cos(b*x+a
)-1)/(cos(b*x+a)-1))+4*2^(1/2)*(sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1
/2)*sin(b*x+a))*cos(b*x+a)^(1/2)/sin(b*x+a)^(3/2)/(sin(b*x+a)*cos(b*x+a)/(
cos(b*x+a)+1)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(124) = 248$.

Time = 0.16 (sec) , antiderivative size = 422, normalized size of antiderivative = 2.81

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx$$
$$= \frac{\sqrt{2}\arctan\left(\frac{2\cos(bx+a)^3-2\cos(bx+a)^2\sin(bx+a)+\sqrt{2}\sqrt{\cos(bx+a)}\sqrt{\sin(bx+a)}-2\cos(bx+a)}{2\left(\cos(bx+a)^3+\cos(bx+a)^2\sin(bx+a)-\cos(bx+a)\right)}\right)\sin(bx+a)+\sqrt{2}\arctan\left(-\right.}{}$$

input
```
integrate(cos(b*x+a)^(3/2)/sin(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
1/8*(sqrt(2)*arctan(1/2*(2*cos(b*x + a)^3 - 2*cos(b*x + a)^2*sin(b*x + a)
+ sqrt(2)*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) - 2*cos(b*x + a))/(cos(b*x
 + a)^3 + cos(b*x + a)^2*sin(b*x + a) - cos(b*x + a)))*sin(b*x + a) + sqrt
(2)*arctan(-1/2*(2*cos(b*x + a)^3 - 2*cos(b*x + a)^2*sin(b*x + a) - sqrt(2
)*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) - 2*cos(b*x + a))/(cos(b*x + a)^3
+ cos(b*x + a)^2*sin(b*x + a) - cos(b*x + a)))*sin(b*x + a) - 2*sqrt(2)*ar
ctan(-1/2*(sqrt(2)*cos(b*x + a) - sqrt(2)*sin(b*x + a))/(sqrt(cos(b*x + a)
)*sqrt(sin(b*x + a))))*sin(b*x + a) + sqrt(2)*log(2*(sqrt(2)*cos(b*x + a)
+ sqrt(2)*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) + 4*cos(b*x
+ a)*sin(b*x + a) + 1)*sin(b*x + a) - sqrt(2)*log(-2*(sqrt(2)*cos(b*x + a)
 + sqrt(2)*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) + 4*cos(b*x
 + a)*sin(b*x + a) + 1)*sin(b*x + a) - 16*sqrt(cos(b*x + a))*sqrt(sin(b*x
+ a)))/(b*sin(b*x + a))
```

Sympy [F]

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{\cos^{\frac{3}{2}}(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx$$

input
```
integrate(cos(b*x+a)**(3/2)/sin(b*x+a)**(3/2),x)
```

output
```
Integral(cos(a + b*x)**(3/2)/sin(a + b*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{\cos\left(bx+a\right)^{\frac{3}{2}}}{\sin\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^(3/2)/sin(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^(3/2)/sin(b*x + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx=\int \frac{\cos\left(bx+a\right)^{\frac{3}{2}}}{\sin\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^(3/2)/sin(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^(3/2)/sin(b*x + a)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 26.47 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.29

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx=-\frac{2\cos\left(a+b\,x\right)^{5/2}\,{\left({\sin\left(a+b\,x\right)}^2\right)}^{1/4}\,{{}}_2{\mathrm{F}}_1\left(\frac{5}{4},\frac{5}{4};\ \frac{9}{4};\ {\cos\left(a+b\,x\right)}^2\right)}{5\,b\,\sqrt{\sin\left(a+b\,x\right)}}$$

input
```
int(cos(a + b*x)^(3/2)/sin(a + b*x)^(3/2),x)
```

output
```
-(2*cos(a + b*x)^(5/2)*(sin(a + b*x)^2)^(1/4)*hypergeom([5/4, 5/4], 9/4, c
os(a + b*x)^2))/(5*b*sin(a + b*x)^(1/2))
```

Reduce [F]

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx =\frac{-2\sqrt{\sin\left(bx+a\right)}\,\sqrt{\cos\left(bx+a\right)}-\left(\int \frac{\sqrt{\sin(bx+a)}\,\sqrt{\cos(bx+a)}}{\cos(bx+a)}dx\right)\sin\left(bx+a\right)b}{\sin\left(bx+a\right)b}$$

input
```
int(cos(b*x+a)^(3/2)/sin(b*x+a)^(3/2),x)
```

output
```
( - 2*sqrt(sin(a + b*x))*sqrt(cos(a + b*x)) - int((sqrt(sin(a + b*x))*sqrt
(cos(a + b*x)))/cos(a + b*x),x)*sin(a + b*x)*b)/(sin(a + b*x)*b)
```

3.302 $\int \frac{\cos^{\frac{5}{2}}(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 153

$$\int \frac{\cos^{\frac{5}{2}}(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{(1+\cot(a+bx))\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b}-\frac{2\cos^{\frac{3}{2}}(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)}$$

output
```
1/2*arctan(-1+2^(1/2)*cos(b*x+a)^(1/2)/sin(b*x+a)^(1/2))*2^(1/2)/b+1/2*arc
tan(1+2^(1/2)*cos(b*x+a)^(1/2)/sin(b*x+a)^(1/2))*2^(1/2)/b-1/2*arctanh(2^(
1/2)*cos(b*x+a)^(1/2)/(1+cot(b*x+a))/sin(b*x+a)^(1/2))*2^(1/2)/b-2/3*cos(b
*x+a)^(3/2)/b/sin(b*x+a)^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.37

$$\int \frac{\cos^{\frac{5}{2}}(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx=-\frac{2\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},-\frac{3}{4},\frac{1}{4},\sin^2(a+bx)\right)}{3b\sqrt{\cos(a+bx)}\sin^{\frac{3}{2}}(a+bx)}$$

input
```
Integrate[Cos[a + b*x]^(5/2)/Sin[a + b*x]^(5/2),x]
```

output
```
(-2*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[-3/4, -3/4, 1/4, Sin[a + b*x]
^2])/(3*b*Sqrt[Cos[a + b*x]]*Sin[a + b*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.34, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 3047, 3042, 3055, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)^{5/2}}{\sin(a+bx)^{5/2}}dx$$

$$\Big\downarrow \text{3047}$$

$$-\int \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}dx-\frac{2\cos^{\frac{3}{2}}(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}dx-\frac{2\cos^{\frac{3}{2}}(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \; 3055$$

$$\frac{2\int \frac{\cot(a+bx)}{\cot^2(a+bx)+1}d\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}}{b}-\frac{2\cos^{\frac{3}{2}}(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \; 826$$

$$\frac{2\left(\frac{1}{2}\int \frac{\cot(a+bx)+1}{\cot^2(a+bx)+1}d\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}-\frac{1}{2}\int \frac{1-\cot(a+bx)}{\cot^2(a+bx)+1}d\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{b}-\frac{2\cos^{\frac{3}{2}}(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \; 1476$$

$$\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1}d\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+\frac{1}{2}\int \frac{1}{\cot(a+bx)+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1}d\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)-\frac{1}{2}\int \frac{1-\cot(a+bx)}{\cot^2(a+bx)+1}d\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{b}-\frac{2\cos^{\frac{3}{2}}(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \; 1082$$

$$\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\cot(a+bx)-1}d\left(1-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\cot(a+bx)-1}d\left(\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\cot(a+bx)}{\cot^2(a+bx)+1}d\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{b}-\frac{2\cos^{\frac{3}{2}}(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \; 217$$

$$\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\cot(a+bx)}{\cot^2(a+bx)+1}d\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{b}-\frac{2\cos^{\frac{3}{2}}(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \; 1479$$

$$\frac{2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}}{\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1}d\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{\cot(a+bx)+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1}d\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{\sqrt{2}}-\right.}{b}$$

$$\frac{2\cos^{\frac{3}{2}}(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)}$$

$\big\downarrow$ 25

$$\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}}{\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1}d\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{\cot(a+bx)+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1}d\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{\sqrt{2}}-\right.}{b}$$

$$\frac{2\cos^{\frac{3}{2}}(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)}$$

$\big\downarrow$ 27

$$\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}}{\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1}d\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1}{\cot(a+bx)+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1}d\frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+\right.}{\sqrt{2}}\right.}{b}$$

$$\frac{2\cos^{\frac{3}{2}}(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)}$$

$\big\downarrow$ 1103

$$\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\cot(a+bx)+\frac{\sqrt{2}\sqrt{\cos}}{\sqrt{\sin(}}\right.}{2\sqrt{2}}\right.}{b}$$

$$\frac{2\cos^{\frac{3}{2}}(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)}$$

input
```
Int[Cos[a + b*x]^(5/2)/Sin[a + b*x]^(5/2),x]
```

output
```
(2*((-(ArcTan[1 - (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]]]/Sqrt[2]
) + ArcTan[1 + (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]]]/Sqrt[2])/2
 + (Log[1 + Cot[a + b*x] - (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]]
]/(2*Sqrt[2]) - Log[1 + Cot[a + b*x] + (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[S
in[a + b*x]]]/(2*Sqrt[2]))/2))/b - (2*Cos[a + b*x]^(3/2))/(3*b*Sin[a + b*x
]^(3/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3047
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Cos[e + f*x]
)^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3055
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[(-k)*a*(b/f)   Subst[Int[x
^(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[
e + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m,
0] && LtQ[m, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(123) = 246$.

Time = 1.00 (sec) , antiderivative size = 431, normalized size of antiderivative = 2.82

method	result
default	$-\frac{\sqrt{2}\left((-3\cos(bx+a)+3)\ln\left(-\frac{2\sqrt{2}\sqrt{\frac{\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}\sin(bx+a)-\cos(bx+a)\cot(bx+a)+2\cot(bx+a)-\csc(bx+a)-2\cos(bx+a)+\sin(bx\cdots}{\cos(bx+a)-1}\right.\right.}{}$

input
```
int(cos(b*x+a)^(5/2)/sin(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12/b*2^(1/2)*((-3*cos(b*x+a)+3)*ln(-(2*2^(1/2)*(sin(b*x+a)*cos(b*x+a)/(
cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)-cos(b*x+a)*cot(b*x+a)+2*cot(b*x+a)-csc(b
*x+a)-2*cos(b*x+a)+sin(b*x+a)+2)/(cos(b*x+a)-1))+(3*cos(b*x+a)-3)*ln((2*2^
(1/2)*(sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)+cos(b*x+a)
*cot(b*x+a)-2*cot(b*x+a)+csc(b*x+a)+2*cos(b*x+a)-sin(b*x+a)-2)/(cos(b*x+a)
-1))+(-6*cos(b*x+a)+6)*arctan((2^(1/2)*(sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+
1)^2)^(1/2)*sin(b*x+a)-cos(b*x+a)+1)/(cos(b*x+a)-1))+(-6*cos(b*x+a)+6)*arc
tan((2^(1/2)*(sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)+cos
(b*x+a)-1)/(cos(b*x+a)-1))+4*cos(b*x+a)*(sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)
+1)^2)^(1/2)*2^(1/2))*cos(b*x+a)^(1/2)/sin(b*x+a)^(3/2)/(sin(b*x+a)*cos(b*
x+a)/(cos(b*x+a)+1)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 475 vs. $2(123) = 246$.

Time = 0.17 (sec) , antiderivative size = 475, normalized size of antiderivative = 3.10

$$\int \frac{\cos^{\frac{5}{2}}(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx$$
$$= \frac{3\left(\sqrt{2}\cos(bx+a)^2-\sqrt{2}\right)\arctan\left(\frac{2\cos(bx+a)^3-2\cos(bx+a)^2\sin(bx+a)+\sqrt{2}\sqrt{\cos(bx+a)}\sqrt{\sin(bx+a)}-2\cos(bx+a)}{2\left(\cos(bx+a)^3+\cos(bx+a)^2\sin(bx+a)-\cos(bx+a)\right)}\right)+3\,(\cdots}{}$$

input
```
integrate(cos(b*x+a)^(5/2)/sin(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
1/24*(3*(sqrt(2)*cos(b*x + a)^2 - sqrt(2))*arctan(1/2*(2*cos(b*x + a)^3 -
2*cos(b*x + a)^2*sin(b*x + a) + sqrt(2)*sqrt(cos(b*x + a))*sqrt(sin(b*x +
a)) - 2*cos(b*x + a))/(cos(b*x + a)^3 + cos(b*x + a)^2*sin(b*x + a) - cos(
b*x + a))) + 3*(sqrt(2)*cos(b*x + a)^2 - sqrt(2))*arctan(-1/2*(2*cos(b*x +
 a)^3 - 2*cos(b*x + a)^2*sin(b*x + a) - sqrt(2)*sqrt(cos(b*x + a))*sqrt(si
n(b*x + a)) - 2*cos(b*x + a))/(cos(b*x + a)^3 + cos(b*x + a)^2*sin(b*x + a
) - cos(b*x + a))) - 6*(sqrt(2)*cos(b*x + a)^2 - sqrt(2))*arctan(-1/2*(sqr
t(2)*cos(b*x + a) - sqrt(2)*sin(b*x + a))/(sqrt(cos(b*x + a))*sqrt(sin(b*x
 + a)))) - 3*(sqrt(2)*cos(b*x + a)^2 - sqrt(2))*log(2*(sqrt(2)*cos(b*x + a
) + sqrt(2)*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) + 4*cos(b*
x + a)*sin(b*x + a) + 1) + 3*(sqrt(2)*cos(b*x + a)^2 - sqrt(2))*log(-2*(sq
rt(2)*cos(b*x + a) + sqrt(2)*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x
 + a)) + 4*cos(b*x + a)*sin(b*x + a) + 1) + 16*cos(b*x + a)^(3/2)*sqrt(sin
(b*x + a)))/(b*cos(b*x + a)^2 - b)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**(5/2)/sin(b*x+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{\cos(bx+a)^{\frac{5}{2}}}{\sin(bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^(5/2)/sin(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^(5/2)/sin(b*x + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{\cos(bx+a)^{\frac{5}{2}}}{\sin(bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^(5/2)/sin(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^(5/2)/sin(b*x + a)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 26.87 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.29

$$\int \frac{\cos^{\frac{5}{2}}(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx = -\frac{2\cos(a+b\,x)^{7/2}\left(\sin(a+b\,x)^2\right)^{3/4}\,{}_2F_1\left(\frac{7}{4},\frac{7}{4};\ \frac{11}{4};\ \cos(a+b\,x)^2\right)}{7\,b\sin(a+b\,x)^{3/2}}$$

input
```
int(cos(a + b*x)^(5/2)/sin(a + b*x)^(5/2),x)
```

output
```
-(2*cos(a + b*x)^(7/2)*(sin(a + b*x)^2)^(3/4)*hypergeom([7/4, 7/4], 11/4,
cos(a + b*x)^2))/(7*b*sin(a + b*x)^(3/2))
```

Reduce [F]

$$\int \frac{\cos^{\frac{5}{2}}(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{\sqrt{\sin(bx+a)}\,\sqrt{\cos(bx+a)}\,\cos(bx+a)^2}{\sin(bx+a)^3}dx$$

input
```
int(cos(b*x+a)^(5/2)/sin(b*x+a)^(5/2),x)
```

output
```
int((sqrt(sin(a + b*x))*sqrt(cos(a + b*x))*cos(a + b*x)**2)/sin(a + b*x)**
3,x)
```

3.303 $\int \frac{\cos^{\frac{7}{2}}(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 178

$$\int \frac{\cos^{\frac{7}{2}}(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{\sqrt{2}b}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{\sqrt{2}b} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}(1+\tan(a+bx))}\right)}{\sqrt{2}b} - \frac{2\cos^{\frac{5}{2}}(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)}+\frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}}$$

output

```
-1/2*arctan(1-2^(1/2)*sin(b*x+a)^(1/2)/cos(b*x+a)^(1/2))*2^(1/2)/b+1/2*arc
tan(1+2^(1/2)*sin(b*x+a)^(1/2)/cos(b*x+a)^(1/2))*2^(1/2)/b-1/2*arctanh(2^(
1/2)*sin(b*x+a)^(1/2)/cos(b*x+a)^(1/2)/(1+tan(b*x+a)))*2^(1/2)/b-2/5*cos(b
*x+a)^(5/2)/b/sin(b*x+a)^(5/2)+2*cos(b*x+a)^(1/2)/b/sin(b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.32

$$\int \frac{\cos^{\frac{7}{2}}(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx = -\frac{2\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{5}{4},-\frac{1}{4},\sin^2(a+bx)\right)}{5b\cos^{\frac{3}{2}}(a+bx)\sin^{\frac{5}{2}}(a+bx)}$$

input
```
Integrate[Cos[a + b*x]^(7/2)/Sin[a + b*x]^(7/2),x]
```

output
```
(-2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-5/4, -5/4, -1/4, Sin[a + b*x
]^2])/(5*b*Cos[a + b*x]^(3/2)*Sin[a + b*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.29, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3047, 3042, 3047, 3042, 3054, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{7}{2}}(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)^{7/2}}{\sin(a+bx)^{7/2}}dx$$

$$\Big\downarrow \text{3047}$$

$$-\int \frac{\cos^{\frac{3}{2}}(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}dx-\frac{2\cos^{\frac{5}{2}}(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{\cos(a+bx)^{3/2}}{\sin(a+bx)^{3/2}}dx-\frac{2\cos^{\frac{5}{2}}(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)}$$

$$\Big\downarrow \text{ 3047}$$

$$\int \frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}dx - \frac{2\cos^{\frac{5}{2}}(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)} + \frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}dx - \frac{2\cos^{\frac{5}{2}}(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)} + \frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}}$$

$$\Big\downarrow \text{ 3054}$$

$$\frac{2\int \frac{\tan(a+bx)}{\tan^2(a+bx)+1}d\frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}}{b} - \frac{2\cos^{\frac{5}{2}}(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)} + \frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}}$$

$$\Big\downarrow \text{ 826}$$

$$\frac{2\left(\frac{1}{2}\int \frac{\tan(a+bx)+1}{\tan^2(a+bx)+1}d\frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}} - \frac{1}{2}\int \frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{b} - \frac{2\cos^{\frac{5}{2}}(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)} + \frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(a+bx)-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1}d\frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}} + \frac{1}{2}\int \frac{1}{\tan(a+bx)+\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1}d\frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right) - \frac{1}{2}\int \frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{b} - \frac{2\cos^{\frac{5}{2}}(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)} + \frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\tan(a+bx)-1}d\left(1-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{\sqrt{2}} - \frac{\int \frac{1}{-\tan(a+bx)-1}d\left(\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1\right)}{\sqrt{2}}\right) - \frac{1}{2}\int \frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{b} - \frac{2\cos^{\frac{5}{2}}(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)} + \frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\tan(a+bx)}{\tan^2(a+bx)+1}d\frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{b}-\frac{2\cos^{\frac{5}{2}}(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)}+\frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}}$$

$$\Big\downarrow \text{1479}$$

$$\frac{2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}}{\tan(a+bx)-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1}d\frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1\right)}{\tan(a+bx)+\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1}d\frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1\right)}{\sqrt{2}}-\right.}{b}$$

$$\frac{2\cos^{\frac{5}{2}}(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)}+\frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}}{\tan(a+bx)-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1}d\frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1\right)}{\tan(a+bx)+\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1}d\frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1\right)}{\sqrt{2}}-\right.}{b}$$

$$\frac{2\cos^{\frac{5}{2}}(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)}+\frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-\frac{2\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}}{\tan(a+bx)-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1}d\frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1}{\tan(a+bx)+\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1}d\frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+\right.}{\sqrt{2}}\right.}{b}$$

$$\frac{2\cos^{\frac{5}{2}}(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)}+\frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(a+bx)-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(a+bx)+\frac{\sqrt{2}\sqrt{\sin}}{\sqrt{\cos(}}\right.}{2\sqrt{2}}\right.\right.}{b}$$

$$\frac{2\cos^{\frac{5}{2}}(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)}+\frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}}$$

input
```
Int[Cos[a + b*x]^(7/2)/Sin[a + b*x]^(7/2),x]
```

output
```
(2*((-(ArcTan[1 - (Sqrt[2]*Sqrt[Sin[a + b*x]])/Sqrt[Cos[a + b*x]]]/Sqrt[2]
) + ArcTan[1 + (Sqrt[2]*Sqrt[Sin[a + b*x]])/Sqrt[Cos[a + b*x]]]/Sqrt[2])/2
 + (Log[1 - (Sqrt[2]*Sqrt[Sin[a + b*x]])/Sqrt[Cos[a + b*x]] + Tan[a + b*x]
]/(2*Sqrt[2]) - Log[1 + (Sqrt[2]*Sqrt[Sin[a + b*x]])/Sqrt[Cos[a + b*x]] +
Tan[a + b*x]]/(2*Sqrt[2]))/2))/b - (2*Cos[a + b*x]^(5/2))/(5*b*Sin[a + b*x
]^(5/2)) + (2*Sqrt[Cos[a + b*x]])/(b*Sqrt[Sin[a + b*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3047
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Cos[e + f*x]
)^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3054
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k*a*(b/f)   Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 459 vs. $2(145) = 290$.

Time = 1.03 (sec) , antiderivative size = 460, normalized size of antiderivative = 2.58

method	result
default	$-\frac{\sqrt{2}\left((-5\cos(bx+a)+5)\sin(bx+a)\ln\left(-\frac{2\sqrt{2}\sqrt{\frac{\sin(bx+a)\cos(bx+a)}{(\cos(bx+a)+1)^2}}\sin(bx+a)-\cos(bx+a)\cot(bx+a)+2\cot(bx+a)-\csc(bx+a)-2\cos(bx}{\cos(bx+a)-1}\right.\right.}{}$

input
```
int(cos(b*x+a)^(7/2)/sin(b*x+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/20/b*2^(1/2)*((-5*cos(b*x+a)+5)*sin(b*x+a)*ln(-(2*2^(1/2)*(sin(b*x+a)*c
os(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)-cos(b*x+a)*cot(b*x+a)+2*cot(b
*x+a)-csc(b*x+a)-2*cos(b*x+a)+sin(b*x+a)+2)/(cos(b*x+a)-1))+(5*cos(b*x+a)-
5)*sin(b*x+a)*ln((2*2^(1/2)*(sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)
*sin(b*x+a)+cos(b*x+a)*cot(b*x+a)-2*cot(b*x+a)+csc(b*x+a)+2*cos(b*x+a)-sin
(b*x+a)-2)/(cos(b*x+a)-1))+(10*cos(b*x+a)-10)*sin(b*x+a)*arctan((2^(1/2)*(
sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)-cos(b*x+a)+1)/(co
s(b*x+a)-1))+(10*cos(b*x+a)-10)*sin(b*x+a)*arctan((2^(1/2)*(sin(b*x+a)*cos
(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)*sin(b*x+a)+cos(b*x+a)-1)/(cos(b*x+a)-1))+(
24*cos(b*x+a)^2-20)*2^(1/2)*(sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^2)^(1/2)
)*cos(b*x+a)^(1/2)/sin(b*x+a)^(5/2)/(sin(b*x+a)*cos(b*x+a)/(cos(b*x+a)+1)^
2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 525 vs. $2(145) = 290$.

Time = 0.19 (sec) , antiderivative size = 525, normalized size of antiderivative = 2.95

$$\int \frac{\cos^{\frac{7}{2}}(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^(7/2)/sin(b*x+a)^(7/2),x, algorithm="fricas")
```

output
```
-1/40*(5*(sqrt(2)*cos(b*x + a)^2 - sqrt(2))*arctan(1/2*(2*cos(b*x + a)^3 -
 2*cos(b*x + a)^2*sin(b*x + a) + sqrt(2)*sqrt(cos(b*x + a))*sqrt(sin(b*x +
 a)) - 2*cos(b*x + a))/(cos(b*x + a)^3 + cos(b*x + a)^2*sin(b*x + a) - cos
(b*x + a)))*sin(b*x + a) + 5*(sqrt(2)*cos(b*x + a)^2 - sqrt(2))*arctan(-1/
2*(2*cos(b*x + a)^3 - 2*cos(b*x + a)^2*sin(b*x + a) - sqrt(2)*sqrt(cos(b*x
 + a))*sqrt(sin(b*x + a)) - 2*cos(b*x + a))/(cos(b*x + a)^3 + cos(b*x + a)
^2*sin(b*x + a) - cos(b*x + a)))*sin(b*x + a) - 10*(sqrt(2)*cos(b*x + a)^2
 - sqrt(2))*arctan(-1/2*(sqrt(2)*cos(b*x + a) - sqrt(2)*sin(b*x + a))/(sqr
t(cos(b*x + a))*sqrt(sin(b*x + a))))*sin(b*x + a) + 5*(sqrt(2)*cos(b*x + a
)^2 - sqrt(2))*log(2*(sqrt(2)*cos(b*x + a) + sqrt(2)*sin(b*x + a))*sqrt(co
s(b*x + a))*sqrt(sin(b*x + a)) + 4*cos(b*x + a)*sin(b*x + a) + 1)*sin(b*x
+ a) - 5*(sqrt(2)*cos(b*x + a)^2 - sqrt(2))*log(-2*(sqrt(2)*cos(b*x + a) +
 sqrt(2)*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) + 4*cos(b*x +
 a)*sin(b*x + a) + 1)*sin(b*x + a) - 16*(6*cos(b*x + a)^2 - 5)*sqrt(cos(b*
x + a))*sqrt(sin(b*x + a)))/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**(7/2)/sin(b*x+a)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{7}{2}}(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{\cos\left(bx+a\right)^{\frac{7}{2}}}{\sin\left(bx+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^(7/2)/sin(b*x+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^(7/2)/sin(b*x + a)^(7/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{2}}(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)^(7/2)/sin(b*x+a)^(7/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 26.79 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.25

$$\int \frac{\cos^{\frac{7}{2}}(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx = -\frac{2\cos(a+b\,x)^{9/2}\,\left(\sin(a+b\,x)^2\right)^{5/4}\,{}_2\mathrm{F}_1\left(\frac{9}{4},\frac{9}{4};\ \frac{13}{4};\ \cos(a+b\,x)^2\right)}{9\,b\sin(a+b\,x)^{5/2}}$$

input
```
int(cos(a + b*x)^(7/2)/sin(a + b*x)^(7/2),x)
```

output
```
-(2*cos(a + b*x)^(9/2)*(sin(a + b*x)^2)^(5/4)*hypergeom([9/4, 9/4], 13/4,
cos(a + b*x)^2))/(9*b*sin(a + b*x)^(5/2))
```

Reduce [F]

$$\int \frac{\cos^{\frac{7}{2}}(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx$$
$$= \frac{-2\sqrt{\sin(bx+a)}\,\sqrt{\cos(bx+a)}\,\cos(bx+a)^2+10\sqrt{\sin(bx+a)}\,\sqrt{\cos(bx+a)}\,\sin(bx+a)^2+5\left(\int \frac{\sqrt{\mathrm{si}}}{}\right.}{5\sin(bx+a)^3\,b}$$

input
```
int(cos(b*x+a)^(7/2)/sin(b*x+a)^(7/2),x)
```

output
```
( - 2*sqrt(sin(a + b*x))*sqrt(cos(a + b*x))*cos(a + b*x)**2 + 10*sqrt(sin(
a + b*x))*sqrt(cos(a + b*x))*sin(a + b*x)**2 + 5*int((sqrt(sin(a + b*x))*s
qrt(cos(a + b*x)))/cos(a + b*x),x)*sin(a + b*x)**3*b)/(5*sin(a + b*x)**3*b
)
```

3.304 $\int \cos^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \cos^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \frac{3\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{2}{3},\frac{5}{3},\sin^2(e+fx)\right)(b\sin(e+fx))^{4/3}}{4bf\sqrt{\cos^2(e+fx)}}$$

output
```
3/4*cos(f*x+e)*hypergeom([-3/2, 2/3],[5/3],sin(f*x+e)^2)*(b*sin(f*x+e))^(4
/3)/b/f/(cos(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \cos^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{2}{3},\frac{5}{3},\sin^2(e+fx)\right)\sqrt[3]{b\sin(e+fx)}\tan(e+fx)}{4f}$$

input
```
Integrate[Cos[e + f*x]^4*(b*Sin[e + f*x])^(1/3),x]
```

output
```
(3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-3/2, 2/3, 5/3, Sin[e + f*x]^2]*
(b*Sin[e + f*x])^(1/3)*Tan[e + f*x])/(4*f)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(e+fx)^4\sqrt[3]{b\sin(e+fx)}dx$$

$$\Big\downarrow \text{3057}$$

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{4/3}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{2}{3},\frac{5}{3},\sin^2(e+fx)\right)}{4bf\sqrt{\cos^2(e+fx)}}$$

input
```
Int[Cos[e + f*x]^4*(b*Sin[e + f*x])^(1/3),x]
```

output
```
(3*Cos[e + f*x]*Hypergeometric2F1[-3/2, 2/3, 5/3, Sin[e + f*x]^2]*(b*Sin[e
 + f*x])^(4/3))/(4*b*f*Sqrt[Cos[e + f*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Maple [F]

$$\int \cos (fx+e)^4 (b\sin (fx+e))^{\frac{1}{3}}\, dx$$

input
```
int(cos(f*x+e)^4*(b*sin(f*x+e))^(1/3),x)
```

output
```
int(cos(f*x+e)^4*(b*sin(f*x+e))^(1/3),x)
```

Fricas [F]

$$\int \cos^4(e+fx)\sqrt[3]{b\sin(e+fx)}\, dx = \int (b\sin (fx+e))^{\frac{1}{3}} \cos (fx+e)^4 \, dx$$

input
```
integrate(cos(f*x+e)^4*(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*sin(f*x + e))^(1/3)*cos(f*x + e)^4, x)
```

Sympy [F]

$$\int \cos^4(e+fx)\sqrt[3]{b\sin(e+fx)}\, dx = \int \sqrt[3]{b\sin (e+fx)} \cos^4 (e+fx)\, dx$$

input
```
integrate(cos(f*x+e)**4*(b*sin(f*x+e))**(1/3),x)
```

output
```
Integral((b*sin(e + f*x))**(1/3)*cos(e + f*x)**4, x)
```

Maxima [F]

$$\int \cos^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \int (b\sin(fx+e))^{\frac{1}{3}}\cos(fx+e)^4\,dx$$

input
```
integrate(cos(f*x+e)^4*(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e))^(1/3)*cos(f*x + e)^4, x)
```

Giac [F]

$$\int \cos^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \int (b\sin(fx+e))^{\frac{1}{3}}\cos(fx+e)^4\,dx$$

input
```
integrate(cos(f*x+e)^4*(b*sin(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e))^(1/3)*cos(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \int \cos(e+f\,x)^4\,(b\,\sin(e+f\,x))^{1/3}\,dx$$

input
```
int(cos(e + f*x)^4*(b*sin(e + f*x))^(1/3),x)
```

output
```
int(cos(e + f*x)^4*(b*sin(e + f*x))^(1/3), x)
```

Reduce [F]

$$\int \cos^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = b^{\frac{1}{3}}\left(\int \sin(fx+e)^{\frac{1}{3}}\cos(fx+e)^4\,dx\right)$$

input

```
int(cos(f*x+e)^4*(b*sin(f*x+e))^(1/3),x)
```

output

```
b**(1/3)*int(sin(e + f*x)**(1/3)*cos(e + f*x)**4,x)
```

3.305 $\int \cos^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \cos^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \frac{3\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2}{3},\frac{5}{3},\sin^2(e+fx)\right)(b\sin(e+fx))^{4/3}}{4bf\sqrt{\cos^2(e+fx)}}$$

output
```
3/4*cos(f*x+e)*hypergeom([-1/2, 2/3],[5/3],sin(f*x+e)^2)*(b*sin(f*x+e))^(4
/3)/b/f/(cos(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \cos^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2}{3},\frac{5}{3},\sin^2(e+fx)\right)\sqrt[3]{b\sin(e+fx)}\tan(e+fx)}{4f}$$

input
```
Integrate[Cos[e + f*x]^2*(b*Sin[e + f*x])^(1/3),x]
```

output
```
(3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-1/2, 2/3, 5/3, Sin[e + f*x]^2]*
(b*Sin[e + f*x])^(1/3)*Tan[e + f*x])/(4*f)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(e+fx)^2\sqrt[3]{b\sin(e+fx)}dx$$

$$\Big\downarrow \text{3057}$$

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{4/3}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2}{3},\frac{5}{3},\sin^2(e+fx)\right)}{4bf\sqrt{\cos^2(e+fx)}}$$

input
```
Int[Cos[e + f*x]^2*(b*Sin[e + f*x])^(1/3),x]
```

output
```
(3*Cos[e + f*x]*Hypergeometric2F1[-1/2, 2/3, 5/3, Sin[e + f*x]^2]*(b*Sin[e
 + f*x])^(4/3))/(4*b*f*Sqrt[Cos[e + f*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Maple [F]

$$\int \cos(fx+e)^2 (b\sin(fx+e))^{\frac{1}{3}} dx$$

input
```
int(cos(f*x+e)^2*(b*sin(f*x+e))^(1/3),x)
```

output
```
int(cos(f*x+e)^2*(b*sin(f*x+e))^(1/3),x)
```

Fricas [F]

$$\int \cos^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \int (b\sin(fx+e))^{\frac{1}{3}} \cos(fx+e)^2\,dx$$

input
```
integrate(cos(f*x+e)^2*(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*sin(f*x + e))^(1/3)*cos(f*x + e)^2, x)
```

Sympy [F]

$$\int \cos^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \int \sqrt[3]{b\sin(e+fx)}\cos^2(e+fx)\,dx$$

input
```
integrate(cos(f*x+e)**2*(b*sin(f*x+e))**(1/3),x)
```

output
```
Integral((b*sin(e + f*x))**(1/3)*cos(e + f*x)**2, x)
```

Maxima [F]

$$\int \cos^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx=\int (b\sin(fx+e))^{\frac{1}{3}}\cos(fx+e)^2\,dx$$

input
```
integrate(cos(f*x+e)^2*(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e))^(1/3)*cos(f*x + e)^2, x)
```

Giac [F]

$$\int \cos^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx=\int (b\sin(fx+e))^{\frac{1}{3}}\cos(fx+e)^2\,dx$$

input
```
integrate(cos(f*x+e)^2*(b*sin(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e))^(1/3)*cos(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx=\int \cos(e+f\,x)^2\,(b\,\sin(e+f\,x))^{1/3}\,dx$$

input
```
int(cos(e + f*x)^2*(b*sin(e + f*x))^(1/3),x)
```

output
```
int(cos(e + f*x)^2*(b*sin(e + f*x))^(1/3), x)
```

Reduce [F]

$$\int \cos^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = b^{\frac{1}{3}}\left(\int \sin(fx+e)^{\frac{1}{3}}\cos(fx+e)^2\,dx\right)$$

input
```
int(cos(f*x+e)^2*(b*sin(f*x+e))^(1/3),x)
```

output
```
b**(1/3)*int(sin(e + f*x)**(1/3)*cos(e + f*x)**2,x)
```

3.306 $\int \sqrt[3]{b\sin(e+fx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 58

$$\int \sqrt[3]{b\sin(e+fx)}\,dx = \frac{3\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\sin^2(e+fx)\right)(b\sin(e+fx))^{4/3}}{4bf\sqrt{\cos^2(e+fx)}}$$

output
```
3/4*cos(f*x+e)*hypergeom([1/2, 2/3],[5/3],sin(f*x+e)^2)*(b*sin(f*x+e))^(4/
3)/b/f/(cos(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \sqrt[3]{b\sin(e+fx)}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\sin^2(e+fx)\right)\sqrt[3]{b\sin(e+fx)}\tan(e+fx)}{4f}$$

input
```
Integrate[(b*Sin[e + f*x])^(1/3),x]
```

output
```
(3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[1/2, 2/3, 5/3, Sin[e + f*x]^2]*(
b*Sin[e + f*x])^(1/3)*Tan[e + f*x])/(4*f)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{b\sin(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt[3]{b\sin(e+fx)}dx$$

$$\Big\downarrow \text{3122}$$

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\sin^2(e+fx)\right)}{4bf\sqrt{\cos^2(e+fx)}}$$

input
```
Int[(b*Sin[e + f*x])^(1/3),x]
```

output
```
(3*Cos[e + f*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Sin[e + f*x]^2]*(b*Sin[e
+ f*x])^(4/3))/(4*b*f*Sqrt[Cos[e + f*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Maple [F]

$$\int (b\sin(fx+e))^{\frac{1}{3}}\,dx$$

input
```
int((b*sin(f*x+e))^(1/3),x)
```

output
```
int((b*sin(f*x+e))^(1/3),x)
```

Fricas [F]

$$\int \sqrt[3]{b\sin(e+fx)}\,dx = \int (b\sin(fx+e))^{\frac{1}{3}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*sin(f*x + e))^(1/3), x)
```

Sympy [F]

$$\int \sqrt[3]{b\sin(e+fx)}\,dx = \int \sqrt[3]{b\sin(e+fx)}\,dx$$

input
```
integrate((b*sin(f*x+e))**(1/3),x)
```

output
```
Integral((b*sin(e + f*x))**(1/3), x)
```

Maxima [F]

$$\int \sqrt[3]{b\sin(e+fx)}\,dx = \int (b\sin(fx+e))^{\frac{1}{3}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e))^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{b\sin(e+fx)}\,dx = \int (b\sin(fx+e))^{\frac{1}{3}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{b\sin(e+fx)}\,dx = \int (b\sin(e+fx))^{1/3}\,dx$$

input
```
int((b*sin(e + f*x))^(1/3),x)
```

output
```
int((b*sin(e + f*x))^(1/3), x)
```

Reduce [F]

$$\int \sqrt[3]{b\sin(e+fx)}\,dx = b^{\frac{1}{3}}\left(\int \sin\left(fx+e\right)^{\frac{1}{3}}dx\right)$$

input
```
int((b*sin(f*x+e))^(1/3),x)
```

output
```
b**(1/3)*int(sin(e + f*x)**(1/3),x)
```

3.307 $\int \sec^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \sec^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{3}{2},\frac{5}{3},\sin^2(e+fx)\right)\sec(e+fx)(b\sin(e+fx))^{4/3}}{4bf}$$

output
```
3/4*(cos(f*x+e)^2)^(1/2)*hypergeom([2/3, 3/2],[5/3],sin(f*x+e)^2)*sec(f*x+
e)*(b*sin(f*x+e))^(4/3)/b/f
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \sec^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{3}{2},\frac{5}{3},\sin^2(e+fx)\right)\sqrt[3]{b\sin(e+fx)}\tan(e+fx)}{4f}$$

input
```
Integrate[Sec[e + f*x]^2*(b*Sin[e + f*x])^(1/3),x]
```

output
```
(3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[2/3, 3/2, 5/3, Sin[e + f*x]^2]*(
b*Sin[e + f*x])^(1/3)*Tan[e + f*x])/(4*f)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt[3]{b\sin(e+fx)}}{\cos(e+fx)^2}dx$$

$$\Big\downarrow \text{ 3057}$$

$$\frac{3\sqrt{\cos^2(e+fx)}\sec(e+fx)(b\sin(e+fx))^{4/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{3}{2},\frac{5}{3},\sin^2(e+fx)\right)}{4bf}$$

input
```
Int[Sec[e + f*x]^2*(b*Sin[e + f*x])^(1/3),x]
```

output
```
(3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[2/3, 3/2, 5/3, Sin[e + f*x]^2]*S
ec[e + f*x]*(b*Sin[e + f*x])^(4/3))/(4*b*f)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Maple [F]

$$\int \sec(fx+e)^2 (b\sin(fx+e))^{\frac{1}{3}}\,dx$$

input
```
int(sec(f*x+e)^2*(b*sin(f*x+e))^(1/3),x)
```

output
```
int(sec(f*x+e)^2*(b*sin(f*x+e))^(1/3),x)
```

Fricas [F]

$$\int \sec^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \int (b\sin(fx+e))^{\frac{1}{3}} \sec(fx+e)^2\,dx$$

input
```
integrate(sec(f*x+e)^2*(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*sin(f*x + e))^(1/3)*sec(f*x + e)^2, x)
```

Sympy [F]

$$\int \sec^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx=\int \sqrt[3]{b\sin(e+fx)}\sec^2(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)**2*(b*sin(f*x+e))**(1/3),x)
```

output
```
Integral((b*sin(e + f*x))**(1/3)*sec(e + f*x)**2, x)
```

Maxima [F]

$$\int \sec^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx=\int (b\sin(fx+e))^{\frac{1}{3}}\sec(fx+e)^2\,dx$$

input
```
integrate(sec(f*x+e)^2*(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e))^(1/3)*sec(f*x + e)^2, x)
```

Giac [F]

$$\int \sec^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx=\int (b\sin(fx+e))^{\frac{1}{3}}\sec(fx+e)^2\,dx$$

input
```
integrate(sec(f*x+e)^2*(b*sin(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e))^(1/3)*sec(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx=\int \frac{\left(b\,\sin\left(e+f\,x\right)\right)^{1/3}}{{\cos\left(e+f\,x\right)}^2}\,dx$$

input

```
int((b*sin(e + f*x))^(1/3)/cos(e + f*x)^2,x)
```

output

```
int((b*sin(e + f*x))^(1/3)/cos(e + f*x)^2, x)
```

Reduce [F]

$$\int \sec^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx=b^{\frac{1}{3}}\left(\int \sin\left(fx+e\right)^{\frac{1}{3}}\sec\left(fx+e\right)^{2}dx\right)$$

input

```
int(sec(f*x+e)^2*(b*sin(f*x+e))^(1/3),x)
```

output

```
b**(1/3)*int(sin(e + f*x)**(1/3)*sec(e + f*x)**2,x)
```

3.308 $\int \sec^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \sec^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{2},\frac{5}{3},\sin^2(e+fx)\right)\sec(e+fx)(b\sin(e+fx))^{4/3}}{4bf}$$

output
```
3/4*(cos(f*x+e)^2)^(1/2)*hypergeom([2/3, 5/2],[5/3],sin(f*x+e)^2)*sec(f*x+
e)*(b*sin(f*x+e))^(4/3)/b/f
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \sec^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{2},\frac{5}{3},\sin^2(e+fx)\right)\sqrt[3]{b\sin(e+fx)}\tan(e+fx)}{4f}$$

input
```
Integrate[Sec[e + f*x]^4*(b*Sin[e + f*x])^(1/3),x]
```

output
```
(3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[2/3, 5/2, 5/3, Sin[e + f*x]^2]*(
b*Sin[e + f*x])^(1/3)*Tan[e + f*x])/(4*f)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt[3]{b\sin(e+fx)}}{\cos(e+fx)^4}dx$$

$$\Big\downarrow \text{3057}$$

$$\frac{3\sqrt{\cos^2(e+fx)}\sec(e+fx)(b\sin(e+fx))^{4/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{2},\frac{5}{3},\sin^2(e+fx)\right)}{4bf}$$

input
```
Int[Sec[e + f*x]^4*(b*Sin[e + f*x])^(1/3),x]
```

output
```
(3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[2/3, 5/2, 5/3, Sin[e + f*x]^2]*S
ec[e + f*x]*(b*Sin[e + f*x])^(4/3))/(4*b*f)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Maple [F]

$$\int \sec(fx+e)^4 (b\sin(fx+e))^{\frac{1}{3}} dx$$

input
```
int(sec(f*x+e)^4*(b*sin(f*x+e))^(1/3),x)
```

output
```
int(sec(f*x+e)^4*(b*sin(f*x+e))^(1/3),x)
```

Fricas [F]

$$\int \sec^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \int (b\sin(fx+e))^{\frac{1}{3}} \sec(fx+e)^4 \, dx$$

input
```
integrate(sec(f*x+e)^4*(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*sin(f*x + e))^(1/3)*sec(f*x + e)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)**4*(b*sin(f*x+e))**(1/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sec^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \int (b\sin(fx+e))^{\frac{1}{3}}\sec(fx+e)^4\,dx$$

input
```
integrate(sec(f*x+e)^4*(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e))^(1/3)*sec(f*x + e)^4, x)
```

Giac [F]

$$\int \sec^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \int (b\sin(fx+e))^{\frac{1}{3}}\sec(fx+e)^4\,dx$$

input
```
integrate(sec(f*x+e)^4*(b*sin(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e))^(1/3)*sec(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx=\int \frac{(b\,\sin\left(e+f\,x\right))^{1/3}}{\cos\left(e+f\,x\right)^4}\,dx$$

input
```
int((b*sin(e + f*x))^(1/3)/cos(e + f*x)^4,x)
```

output
```
int((b*sin(e + f*x))^(1/3)/cos(e + f*x)^4, x)
```

Reduce [F]

$$\int \sec^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx=b^{\frac{1}{3}}\left(\int \sin\left(fx+e\right)^{\frac{1}{3}}\sec\left(fx+e\right)^{4}dx\right)$$

input
```
int(sec(f*x+e)^4*(b*sin(f*x+e))^(1/3),x)
```

output
```
b**(1/3)*int(sin(e + f*x)**(1/3)*sec(e + f*x)**4,x)
```

3.309 $\int \cos^4(e+fx)(b\sin(e+fx))^{5/3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \cos^4(e+fx)(b\sin(e+fx))^{5/3}\,dx=\frac{3\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{4}{3},\frac{7}{3},\sin^2(e+fx)\right)(b\sin(e+fx))^{8/3}}{8bf\sqrt{\cos^2(e+fx)}}$$

output
```
3/8*cos(f*x+e)*hypergeom([-3/2, 4/3],[7/3],sin(f*x+e)^2)*(b*sin(f*x+e))^(8
/3)/b/f/(cos(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \cos^4(e+fx)(b\sin(e+fx))^{5/3}\,dx=\frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{4}{3},\frac{7}{3},\sin^2(e+fx)\right)(b\sin(e+fx))^{5/3}\tan(e+fx)}{8f}$$

input
```
Integrate[Cos[e + f*x]^4*(b*Sin[e + f*x])^(5/3),x]
```

output
```
(3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-3/2, 4/3, 7/3, Sin[e + f*x]^2]*
(b*Sin[e + f*x])^(5/3)*Tan[e + f*x])/(8*f)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(e+fx)(b\sin(e+fx))^{5/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(e+fx)^4(b\sin(e+fx))^{5/3}dx$$

$$\Big\downarrow \text{3057}$$

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{8/3}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{4}{3},\frac{7}{3},\sin^2(e+fx)\right)}{8bf\sqrt{\cos^2(e+fx)}}$$

input
```
Int[Cos[e + f*x]^4*(b*Sin[e + f*x])^(5/3),x]
```

output
```
(3*Cos[e + f*x]*Hypergeometric2F1[-3/2, 4/3, 7/3, Sin[e + f*x]^2]*(b*Sin[e
 + f*x])^(8/3))/(8*b*f*Sqrt[Cos[e + f*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Maple [F]

$$\int \cos(fx+e)^4 (b\sin(fx+e))^{\frac{5}{3}} dx$$

input
```
int(cos(f*x+e)^4*(b*sin(f*x+e))^(5/3),x)
```

output
```
int(cos(f*x+e)^4*(b*sin(f*x+e))^(5/3),x)
```

Fricas [F]

$$\int \cos^4(e+fx)(b\sin(e+fx))^{5/3} dx = \int (b\sin(fx+e))^{\frac{5}{3}} \cos(fx+e)^4 dx$$

input
```
integrate(cos(f*x+e)^4*(b*sin(f*x+e))^(5/3),x, algorithm="fricas")
```

output
```
integral((b*sin(f*x + e))^(2/3)*b*cos(f*x + e)^4*sin(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^4(e+fx)(b\sin(e+fx))^{5/3} dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**4*(b*sin(f*x+e))**(5/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^4(e+fx)(b\sin(e+fx))^{5/3}\,dx = \int (b\sin(fx+e))^{\frac{5}{3}}\cos(fx+e)^4\,dx$$

input
```
integrate(cos(f*x+e)^4*(b*sin(f*x+e))^(5/3),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e))^(5/3)*cos(f*x + e)^4, x)
```

Giac [F]

$$\int \cos^4(e+fx)(b\sin(e+fx))^{5/3}\,dx = \int (b\sin(fx+e))^{\frac{5}{3}}\cos(fx+e)^4\,dx$$

input
```
integrate(cos(f*x+e)^4*(b*sin(f*x+e))^(5/3),x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e))^(5/3)*cos(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^4(e+fx)(b\sin(e+fx))^{5/3}\,dx = \int \cos(e+f\,x)^4\,(b\,\sin(e+f\,x))^{5/3}\,dx$$

input
```
int(cos(e + f*x)^4*(b*sin(e + f*x))^(5/3),x)
```

output
```
int(cos(e + f*x)^4*(b*sin(e + f*x))^(5/3), x)
```

Reduce [F]

$$\int \cos^4(e+fx)(b\sin(e+fx))^{5/3}\,dx = b^{\frac{5}{3}}\left(\int \sin\left(fx+e\right)^{\frac{5}{3}}\cos\left(fx+e\right)^4 dx\right)$$

input
```
int(cos(f*x+e)^4*(b*sin(f*x+e))^(5/3),x)
```

output
```
b**(2/3)*int(sin(e + f*x)**(2/3)*cos(e + f*x)**4*sin(e + f*x),x)*b
```

3.310 $\int \cos^2(e+fx)(b\sin(e+fx))^{5/3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \cos^2(e+fx)(b\sin(e+fx))^{5/3}\,dx = \frac{3\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{4}{3},\frac{7}{3},\sin^2(e+fx)\right)(b\sin(e+fx))^{8/3}}{8bf\sqrt{\cos^2(e+fx)}}$$

output
```
3/8*cos(f*x+e)*hypergeom([-1/2, 4/3],[7/3],sin(f*x+e)^2)*(b*sin(f*x+e))^(8
/3)/b/f/(cos(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \cos^2(e+fx)(b\sin(e+fx))^{5/3}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{4}{3},\frac{7}{3},\sin^2(e+fx)\right)(b\sin(e+fx))^{5/3}\tan(e+fx)}{8f}$$

input
```
Integrate[Cos[e + f*x]^2*(b*Sin[e + f*x])^(5/3),x]
```

output
```
(3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-1/2, 4/3, 7/3, Sin[e + f*x]^2]*
(b*Sin[e + f*x])^(5/3)*Tan[e + f*x])/(8*f)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(e+fx)(b\sin(e+fx))^{5/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(e+fx)^2(b\sin(e+fx))^{5/3}dx$$

$$\Big\downarrow \text{3057}$$

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{8/3}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{4}{3},\frac{7}{3},\sin^2(e+fx)\right)}{8bf\sqrt{\cos^2(e+fx)}}$$

input
```
Int[Cos[e + f*x]^2*(b*Sin[e + f*x])^(5/3),x]
```

output
```
(3*Cos[e + f*x]*Hypergeometric2F1[-1/2, 4/3, 7/3, Sin[e + f*x]^2]*(b*Sin[e
 + f*x])^(8/3))/(8*b*f*Sqrt[Cos[e + f*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Maple [F]

$$\int \cos(fx+e)^2 (b\sin(fx+e))^{\frac{5}{3}} dx$$

input
```
int(cos(f*x+e)^2*(b*sin(f*x+e))^(5/3),x)
```

output
```
int(cos(f*x+e)^2*(b*sin(f*x+e))^(5/3),x)
```

Fricas [F]

$$\int \cos^2(e+fx)(b\sin(e+fx))^{5/3}\,dx = \int (b\sin(fx+e))^{\frac{5}{3}} \cos(fx+e)^2\,dx$$

input
```
integrate(cos(f*x+e)^2*(b*sin(f*x+e))^(5/3),x, algorithm="fricas")
```

output
```
integral((b*sin(f*x + e))^(2/3)*b*cos(f*x + e)^2*sin(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(e+fx)(b\sin(e+fx))^{5/3}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**2*(b*sin(f*x+e))**(5/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^2(e+fx)(b\sin(e+fx))^{5/3}\,dx = \int (b\sin(fx+e))^{\frac{5}{3}}\cos(fx+e)^2\,dx$$

input
```
integrate(cos(f*x+e)^2*(b*sin(f*x+e))^(5/3),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e))^(5/3)*cos(f*x + e)^2, x)
```

Giac [F]

$$\int \cos^2(e+fx)(b\sin(e+fx))^{5/3}\,dx = \int (b\sin(fx+e))^{\frac{5}{3}}\cos(fx+e)^2\,dx$$

input
```
integrate(cos(f*x+e)^2*(b*sin(f*x+e))^(5/3),x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e))^(5/3)*cos(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(e+fx)(b\sin(e+fx))^{5/3}\,dx = \int \cos(e+f\,x)^2\,(b\,\sin(e+f\,x))^{5/3}\,dx$$

input
```
int(cos(e + f*x)^2*(b*sin(e + f*x))^(5/3),x)
```

output
```
int(cos(e + f*x)^2*(b*sin(e + f*x))^(5/3), x)
```

Reduce [F]

$$\int \cos^2(e+fx)(b\sin(e+fx))^{5/3}\,dx = b^{\frac{5}{3}}\left(\int \sin(fx+e)^{\frac{5}{3}}\cos(fx+e)^2\,dx\right)$$

input
```
int(cos(f*x+e)^2*(b*sin(f*x+e))^(5/3),x)
```

output
```
b**(2/3)*int(sin(e + f*x)**(2/3)*cos(e + f*x)**2*sin(e + f*x),x)*b
```

3.311 $\int (b\sin(e+fx))^{5/3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 58

$$\int (b\sin(e+fx))^{5/3}\,dx=\frac{3\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\sin^2(e+fx)\right)(b\sin(e+fx))^{8/3}}{8bf\sqrt{\cos^2(e+fx)}}$$

output
```
3/8*cos(f*x+e)*hypergeom([1/2, 4/3],[7/3],sin(f*x+e)^2)*(b*sin(f*x+e))^(8/
3)/b/f/(cos(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int (b\sin(e+fx))^{5/3}\,dx=\frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\sin^2(e+fx)\right)(b\sin(e+fx))^{5/3}\tan(e+fx)}{8f}$$

input
```
Integrate[(b*Sin[e + f*x])^(5/3),x]
```

output
```
(3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[1/2, 4/3, 7/3, Sin[e + f*x]^2]*(
b*Sin[e + f*x])^(5/3)*Tan[e + f*x])/(8*f)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\sin(e+fx))^{5/3}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (b\sin(e+fx))^{5/3}dx$$

$$\Big\downarrow \text{ 3122}$$

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{8/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\sin^2(e+fx)\right)}{8bf\sqrt{\cos^2(e+fx)}}$$

input
```
Int[(b*Sin[e + f*x])^(5/3),x]
```

output
```
(3*Cos[e + f*x]*Hypergeometric2F1[1/2, 4/3, 7/3, Sin[e + f*x]^2]*(b*Sin[e
+ f*x])^(8/3))/(8*b*f*Sqrt[Cos[e + f*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Maple [F]

$$\int (b\sin (fx+e))^{\frac{5}{3}}\,dx$$

input
```
int((b*sin(f*x+e))^(5/3),x)
```

output
```
int((b*sin(f*x+e))^(5/3),x)
```

Fricas [F]

$$\int (b\sin(e+fx))^{5/3}\,dx=\int (b\sin (fx+e))^{\frac{5}{3}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(5/3),x, algorithm="fricas")
```

output
```
integral((b*sin(f*x + e))^(2/3)*b*sin(f*x + e), x)
```

Sympy [F]

$$\int (b\sin(e+fx))^{5/3}\,dx=\int (b\sin (e+fx))^{\frac{5}{3}}\,dx$$

input
```
integrate((b*sin(f*x+e))**(5/3),x)
```

output
```
Integral((b*sin(e + f*x))**(5/3), x)
```

Maxima [F]

$$\int (b\sin(e+fx))^{5/3}\,dx = \int (b\sin(fx+e))^{\frac{5}{3}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(5/3),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e))^(5/3), x)
```

Giac [F]

$$\int (b\sin(e+fx))^{5/3}\,dx = \int (b\sin(fx+e))^{\frac{5}{3}}\,dx$$

input
```
integrate((b*sin(f*x+e))^(5/3),x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e))^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sin(e+fx))^{5/3}\,dx = \int (b\,\sin(e+f\,x))^{5/3}\,dx$$

input
```
int((b*sin(e + f*x))^(5/3),x)
```

output
```
int((b*sin(e + f*x))^(5/3), x)
```

Reduce [F]

$$\int (b\sin(e+fx))^{5/3}\,dx = b^{\frac{5}{3}}\left(\int \sin\left(fx+e\right)^{\frac{5}{3}}dx\right)$$

input
```
int((b*sin(f*x+e))^(5/3),x)
```

output
```
b**(2/3)*int(sin(e + f*x)**(2/3)*sin(e + f*x),x)*b
```

3.312 $\int \sec^2(e+fx)(b\sin(e+fx))^{5/3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \sec^2(e+fx)(b\sin(e+fx))^{5/3}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{3}{2},\frac{7}{3},\sin^2(e+fx)\right)\sec(e+fx)(b\sin(e+fx))^{8/3}}{8bf}$$

output
```
3/8*(cos(f*x+e)^2)^(1/2)*hypergeom([4/3, 3/2],[7/3],sin(f*x+e)^2)*sec(f*x+
e)*(b*sin(f*x+e))^(8/3)/b/f
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \sec^2(e+fx)(b\sin(e+fx))^{5/3}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{3}{2},\frac{7}{3},\sin^2(e+fx)\right)(b\sin(e+fx))^{5/3}\tan(e+fx)}{8f}$$

input
```
Integrate[Sec[e + f*x]^2*(b*Sin[e + f*x])^(5/3),x]
```

output
```
(3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[4/3, 3/2, 7/3, Sin[e + f*x]^2]*(
b*Sin[e + f*x])^(5/3)*Tan[e + f*x])/(8*f)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(e+fx)(b\sin(e+fx))^{5/3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\sin(e+fx))^{5/3}}{\cos(e+fx)^2}dx$$

$$\Big\downarrow \text{3057}$$

$$\frac{3\sqrt{\cos^2(e+fx)}\sec(e+fx)(b\sin(e+fx))^{8/3}\,\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{3}{2},\frac{7}{3},\sin^2(e+fx)\right)}{8bf}$$

input
```
Int[Sec[e + f*x]^2*(b*Sin[e + f*x])^(5/3),x]
```

output
```
(3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[4/3, 3/2, 7/3, Sin[e + f*x]^2]*S
ec[e + f*x]*(b*Sin[e + f*x])^(8/3))/(8*b*f)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Maple [F]

$$\int \sec(fx+e)^2 (b\sin(fx+e))^{\frac{5}{3}}\,dx$$

input
```
int(sec(f*x+e)^2*(b*sin(f*x+e))^(5/3),x)
```

output
```
int(sec(f*x+e)^2*(b*sin(f*x+e))^(5/3),x)
```

Fricas [F]

$$\int \sec^2(e+fx)(b\sin(e+fx))^{5/3}\,dx = \int (b\sin(fx+e))^{\frac{5}{3}}\sec(fx+e)^2\,dx$$

input
```
integrate(sec(f*x+e)^2*(b*sin(f*x+e))^(5/3),x, algorithm="fricas")
```

output
```
integral((b*sin(f*x + e))^(2/3)*b*sec(f*x + e)^2*sin(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^2(e+fx)(b\sin(e+fx))^{5/3}\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)**2*(b*sin(f*x+e))**(5/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sec^2(e+fx)(b\sin(e+fx))^{5/3}\,dx = \int (b\sin(fx+e))^{\frac{5}{3}}\sec(fx+e)^2\,dx$$

input
```
integrate(sec(f*x+e)^2*(b*sin(f*x+e))^(5/3),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e))^(5/3)*sec(f*x + e)^2, x)
```

Giac [F]

$$\int \sec^2(e+fx)(b\sin(e+fx))^{5/3}\,dx = \int (b\sin(fx+e))^{\frac{5}{3}}\sec(fx+e)^2\,dx$$

input
```
integrate(sec(f*x+e)^2*(b*sin(f*x+e))^(5/3),x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e))^(5/3)*sec(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^2(e+fx)(b\sin(e+fx))^{5/3}\,dx = \int \frac{(b\,\sin\left(e+f\,x\right))^{5/3}}{\cos\left(e+f\,x\right)^2}\,dx$$

input
```
int((b*sin(e + f*x))^(5/3)/cos(e + f*x)^2,x)
```

output
```
int((b*sin(e + f*x))^(5/3)/cos(e + f*x)^2, x)
```

Reduce [F]

$$\int \sec^2(e+fx)(b\sin(e+fx))^{5/3}\,dx = b^{\frac{5}{3}}\left(\int \sin\left(fx+e\right)^{\frac{5}{3}}\sec\left(fx+e\right)^2dx\right)$$

input
```
int(sec(f*x+e)^2*(b*sin(f*x+e))^(5/3),x)
```

output
```
b**(2/3)*int(sin(e + f*x)**(2/3)*sec(e + f*x)**2*sin(e + f*x),x)*b
```

3.313 $\int \sec^4(e+fx)(b\sin(e+fx))^{5/3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \sec^4(e+fx)(b\sin(e+fx))^{5/3}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{5}{2},\frac{7}{3},\sin^2(e+fx)\right)\sec(e+fx)(b\sin(e+fx))^{8/3}}{8bf}$$

output
```
3/8*(cos(f*x+e)^2)^(1/2)*hypergeom([4/3, 5/2],[7/3],sin(f*x+e)^2)*sec(f*x+
e)*(b*sin(f*x+e))^(8/3)/b/f
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \sec^4(e+fx)(b\sin(e+fx))^{5/3}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{5}{2},\frac{7}{3},\sin^2(e+fx)\right)(b\sin(e+fx))^{5/3}\tan(e+fx)}{8f}$$

input
```
Integrate[Sec[e + f*x]^4*(b*Sin[e + f*x])^(5/3),x]
```

output
```
(3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[4/3, 5/2, 7/3, Sin[e + f*x]^2]*(
b*Sin[e + f*x])^(5/3)*Tan[e + f*x])/(8*f)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(e+fx)(b\sin(e+fx))^{5/3}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(b\sin(e+fx))^{5/3}}{\cos(e+fx)^4}dx$$

$$\Big\downarrow \text{ 3057}$$

$$\frac{3\sqrt{\cos^2(e+fx)}\sec(e+fx)(b\sin(e+fx))^{8/3}\,\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{5}{2},\frac{7}{3},\sin^2(e+fx)\right)}{8bf}$$

input
```
Int[Sec[e + f*x]^4*(b*Sin[e + f*x])^(5/3),x]
```

output
```
(3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[4/3, 5/2, 7/3, Sin[e + f*x]^2]*S
ec[e + f*x]*(b*Sin[e + f*x])^(8/3))/(8*b*f)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Maple [F]

$$\int \sec(fx+e)^4 (b\sin(fx+e))^{\frac{5}{3}} dx$$

input
```
int(sec(f*x+e)^4*(b*sin(f*x+e))^(5/3),x)
```

output
```
int(sec(f*x+e)^4*(b*sin(f*x+e))^(5/3),x)
```

Fricas [F]

$$\int \sec^4(e+fx)(b\sin(e+fx))^{5/3} dx = \int (b\sin(fx+e))^{\frac{5}{3}} \sec(fx+e)^4 dx$$

input
```
integrate(sec(f*x+e)^4*(b*sin(f*x+e))^(5/3),x, algorithm="fricas")
```

output
```
integral((b*sin(f*x + e))^(2/3)*b*sec(f*x + e)^4*sin(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^4(e+fx)(b\sin(e+fx))^{5/3}\,dx=\text{Timed out}$$

input
```
integrate(sec(f*x+e)**4*(b*sin(f*x+e))**(5/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sec^4(e+fx)(b\sin(e+fx))^{5/3}\,dx=\int (b\sin(fx+e))^{\frac{5}{3}}\sec(fx+e)^4\,dx$$

input
```
integrate(sec(f*x+e)^4*(b*sin(f*x+e))^(5/3),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e))^(5/3)*sec(f*x + e)^4, x)
```

Giac [F]

$$\int \sec^4(e+fx)(b\sin(e+fx))^{5/3}\,dx=\int (b\sin(fx+e))^{\frac{5}{3}}\sec(fx+e)^4\,dx$$

input
```
integrate(sec(f*x+e)^4*(b*sin(f*x+e))^(5/3),x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e))^(5/3)*sec(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^4(e+fx)(b\sin(e+fx))^{5/3}\,dx = \int \frac{(b\sin(e+f\,x))^{5/3}}{\cos(e+f\,x)^4}\,dx$$

input
```
int((b*sin(e + f*x))^(5/3)/cos(e + f*x)^4,x)
```

output
```
int((b*sin(e + f*x))^(5/3)/cos(e + f*x)^4, x)
```

Reduce [F]

$$\int \sec^4(e+fx)(b\sin(e+fx))^{5/3}\,dx = b^{\frac{5}{3}}\left(\int \sin(fx+e)^{\frac{5}{3}}\sec(fx+e)^4\,dx\right)$$

input
```
int(sec(f*x+e)^4*(b*sin(f*x+e))^(5/3),x)
```

output
```
b**(2/3)*int(sin(e + f*x)**(2/3)*sec(e + f*x)**4*sin(e + f*x),x)*b
```

3.314 $\int \frac{\cos^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \frac{\cos^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{3\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{3},\frac{4}{3},\sin^2(e+fx)\right)(b\sin(e+fx))^{2/3}}{2bf\sqrt{\cos^2(e+fx)}}$$

output
```
3/2*cos(f*x+e)*hypergeom([-3/2, 1/3],[4/3],sin(f*x+e)^2)*(b*sin(f*x+e))^(2
/3)/b/f/(cos(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \frac{\cos^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{3},\frac{4}{3},\sin^2(e+fx)\right)\tan(e+fx)}{2f\sqrt[3]{b\sin(e+fx)}}$$

input
```
Integrate[Cos[e + f*x]^4/(b*Sin[e + f*x])^(1/3),x]
```

output
```
(3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-3/2, 1/3, 4/3, Sin[e + f*x]^2]*
Tan[e + f*x])/(2*f*(b*Sin[e + f*x])^(1/3))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos(e+fx)^4}{\sqrt[3]{b\sin(e+fx)}}dx$$

$$\Big\downarrow \; 3057$$

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{2/3}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{3},\frac{4}{3},\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}}$$

input
```
Int[Cos[e + f*x]^4/(b*Sin[e + f*x])^(1/3),x]
```

output
```
(3*Cos[e + f*x]*Hypergeometric2F1[-3/2, 1/3, 4/3, Sin[e + f*x]^2]*(b*Sin[e
 + f*x])^(2/3))/(2*b*f*Sqrt[Cos[e + f*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Maple [F]

$$\int \frac{\cos(fx+e)^4}{(b\sin(fx+e))^{\frac{1}{3}}}dx$$

input
```
int(cos(f*x+e)^4/(b*sin(f*x+e))^(1/3),x)
```

output
```
int(cos(f*x+e)^4/(b*sin(f*x+e))^(1/3),x)
```

Fricas [F]

$$\int \frac{\cos^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{\cos(fx+e)^4}{(b\sin(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(f*x+e)^4/(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*sin(f*x + e))^(2/3)*cos(f*x + e)^4/(b*sin(f*x + e)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**4/(b*sin(f*x+e))**(1/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{\cos(fx+e)^4}{(b\sin(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(f*x+e)^4/(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^4/(b*sin(f*x + e))^(1/3), x)
```

Giac [F]

$$\int \frac{\cos^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{\cos(fx+e)^4}{(b\sin(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(f*x+e)^4/(b*sin(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate(cos(f*x + e)^4/(b*sin(f*x + e))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx=\int \frac{\cos\left(e+f\,x\right)^4}{\left(b\,\sin\left(e+f\,x\right)\right)^{1/3}}\,dx$$

input
```
int(cos(e + f*x)^4/(b*sin(e + f*x))^(1/3),x)
```

output
```
int(cos(e + f*x)^4/(b*sin(e + f*x))^(1/3), x)
```

Reduce [F]

$$\int \frac{\cos^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx=\frac{\int \frac{\cos(fx+e)^4}{\sin(fx+e)^{\frac{1}{3}}}dx}{b^{\frac{1}{3}}}$$

input
```
int(cos(f*x+e)^4/(b*sin(f*x+e))^(1/3),x)
```

output
```
int(cos(e + f*x)**4/sin(e + f*x)**(1/3),x)/b**(1/3)
```

3.315 $\int \frac{\cos^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \frac{\cos^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{3\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{3},\frac{4}{3},\sin^2(e+fx)\right)(b\sin(e+fx))^{2/3}}{2bf\sqrt{\cos^2(e+fx)}}$$

output
```
3/2*cos(f*x+e)*hypergeom([-1/2, 1/3],[4/3],sin(f*x+e)^2)*(b*sin(f*x+e))^(2
/3)/b/f/(cos(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \frac{\cos^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{3},\frac{4}{3},\sin^2(e+fx)\right)\tan(e+fx)}{2f\sqrt[3]{b\sin(e+fx)}}$$

input
```
Integrate[Cos[e + f*x]^2/(b*Sin[e + f*x])^(1/3),x]
```

output
```
(3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-1/2, 1/3, 4/3, Sin[e + f*x]^2]*
Tan[e + f*x])/(2*f*(b*Sin[e + f*x])^(1/3))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(e+fx)^2}{\sqrt[3]{b\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3057}$$

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{2/3}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{3},\frac{4}{3},\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}}$$

input
```
Int[Cos[e + f*x]^2/(b*Sin[e + f*x])^(1/3),x]
```

output
```
(3*Cos[e + f*x]*Hypergeometric2F1[-1/2, 1/3, 4/3, Sin[e + f*x]^2]*(b*Sin[e
 + f*x])^(2/3))/(2*b*f*Sqrt[Cos[e + f*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Maple [F]

$$\int \frac{\cos (fx+e)^2}{(b\sin (fx+e))^{\frac{1}{3}}}dx$$

input
```
int(cos(f*x+e)^2/(b*sin(f*x+e))^(1/3),x)
```

output
```
int(cos(f*x+e)^2/(b*sin(f*x+e))^(1/3),x)
```

Fricas [F]

$$\int \frac{\cos^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{\cos (fx+e)^2}{(b\sin (fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(f*x+e)^2/(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*sin(f*x + e))^(2/3)*cos(f*x + e)^2/(b*sin(f*x + e)), x)
```

Sympy [F]

$$\int \frac{\cos^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx=\int \frac{\cos^2{(e+fx)}}{\sqrt[3]{b\sin{(e+fx)}}}\,dx$$

input
```
integrate(cos(f*x+e)**2/(b*sin(f*x+e))**(1/3),x)
```

output
```
Integral(cos(e + f*x)**2/(b*sin(e + f*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{\cos^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx=\int \frac{\cos{(fx+e)}^2}{(b\sin{(fx+e)})^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(f*x+e)^2/(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^2/(b*sin(f*x + e))^(1/3), x)
```

Giac [F]

$$\int \frac{\cos^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx=\int \frac{\cos{(fx+e)}^2}{(b\sin{(fx+e)})^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(f*x+e)^2/(b*sin(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate(cos(f*x + e)^2/(b*sin(f*x + e))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx=\int \frac{{\cos\left(e+f\,x\right)}^2}{{\left(b\,\sin\left(e+f\,x\right)\right)}^{1/3}} \,d x$$

input
```
int(cos(e + f*x)^2/(b*sin(e + f*x))^(1/3),x)
```

output
```
int(cos(e + f*x)^2/(b*sin(e + f*x))^(1/3), x)
```

Reduce [F]

$$\int \frac{\cos^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx=\frac{\int \frac{\cos(fx+e)^2}{\sin(fx+e)^{\frac{1}{3}}}dx}{b^{\frac{1}{3}}}$$

input
```
int(cos(f*x+e)^2/(b*sin(f*x+e))^(1/3),x)
```

output
```
int(cos(e + f*x)**2/sin(e + f*x)**(1/3),x)/b**(1/3)
```

3.316 $\int \frac{1}{\sqrt[3]{b\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 58

$$\int \frac{1}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{3\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\sin^2(e+fx)\right)(b\sin(e+fx))^{2/3}}{2bf\sqrt{\cos^2(e+fx)}}$$

output
```
3/2*cos(f*x+e)*hypergeom([1/3, 1/2],[4/3],sin(f*x+e)^2)*(b*sin(f*x+e))^(2/
3)/b/f/(cos(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\sin^2(e+fx)\right)\tan(e+fx)}{2f\sqrt[3]{b\sin(e+fx)}}$$

input
```
Integrate[(b*Sin[e + f*x])^(-1/3),x]
```

output
```
(3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[1/3, 1/2, 4/3, Sin[e + f*x]^2]*T
an[e + f*x])/(2*f*(b*Sin[e + f*x])^(1/3))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{b\sin(e+fx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sqrt[3]{b\sin(e+fx)}}dx$$
$$\Big\downarrow \text{3122}$$
$$\frac{3\cos(e+fx)(b\sin(e+fx))^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}}$$

input
```
Int[(b*Sin[e + f*x])^(-1/3),x]
```

output
```
(3*Cos[e + f*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Sin[e + f*x]^2]*(b*Sin[e
+ f*x])^(2/3))/(2*b*f*Sqrt[Cos[e + f*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Maple [F]

$$\int \frac{1}{(b\sin(fx+e))^{\frac{1}{3}}}dx$$

input
```
int(1/(b*sin(f*x+e))^(1/3),x)
```

output
```
int(1/(b*sin(f*x+e))^(1/3),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{1}{(b\sin(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*sin(f*x + e))^(2/3)/(b*sin(f*x + e)), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{b\sin(e+fx)}}\,dx=\int \frac{1}{\sqrt[3]{b\sin{\left(e+fx\right)}}}\,dx$$

input
```
integrate(1/(b*sin(f*x+e))**(1/3),x)
```

output
```
Integral((b*sin(e + f*x))**(-1/3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{b\sin(e+fx)}}\,dx=\int \frac{1}{\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e))^(-1/3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{b\sin(e+fx)}}\,dx=\int \frac{1}{\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(b*sin(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e))^(-1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{1}{(b\,\sin(e+f\,x))^{1/3}}\,dx$$

input
```
int(1/(b*sin(e + f*x))^(1/3),x)
```

output
```
int(1/(b*sin(e + f*x))^(1/3), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{\int \frac{1}{\sin(fx+e)^{\frac{1}{3}}}dx}{b^{\frac{1}{3}}}$$

input
```
int(1/(b*sin(f*x+e))^(1/3),x)
```

output
```
int(1/sin(e + f*x)**(1/3),x)/b**(1/3)
```

3.317 $\int \frac{\sec^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \frac{\sec^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{2},\frac{4}{3},\sin^2(e+fx)\right)\sec(e+fx)(b\sin(e+fx))^{2/3}}{2bf}$$

output
```
3/2*(cos(f*x+e)^2)^(1/2)*hypergeom([1/3, 3/2],[4/3],sin(f*x+e)^2)*sec(f*x+
e)*(b*sin(f*x+e))^(2/3)/b/f
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \frac{\sec^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{2},\frac{4}{3},\sin^2(e+fx)\right)\tan(e+fx)}{2f\sqrt[3]{b\sin(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]^2/(b*Sin[e + f*x])^(1/3),x]
```

output
```
(3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[1/3, 3/2, 4/3, Sin[e + f*x]^2]*T
an[e + f*x])/(2*f*(b*Sin[e + f*x])^(1/3))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\cos(e+fx)^2\sqrt[3]{b\sin(e+fx)}}dx$$
$$\Big\downarrow \text{3057}$$
$$\frac{3\sqrt{\cos^2(e+fx)}\sec(e+fx)(b\sin(e+fx))^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{3}{2},\frac{4}{3},\sin^2(e+fx)\right)}{2bf}$$

input
```
Int[Sec[e + f*x]^2/(b*Sin[e + f*x])^(1/3),x]
```

output
```
(3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[1/3, 3/2, 4/3, Sin[e + f*x]^2]*S
ec[e + f*x]*(b*Sin[e + f*x])^(2/3))/(2*b*f)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Maple [F]

$$\int \frac{\sec(fx+e)^2}{(b\sin(fx+e))^{\frac{1}{3}}}dx$$

input
```
int(sec(f*x+e)^2/(b*sin(f*x+e))^(1/3),x)
```

output
```
int(sec(f*x+e)^2/(b*sin(f*x+e))^(1/3),x)
```

Fricas [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{\sec(fx+e)^2}{(b\sin(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(f*x+e)^2/(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*sin(f*x + e))^(2/3)*sec(f*x + e)^2/(b*sin(f*x + e)), x)
```

Sympy [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx=\int \frac{\sec^2{(e+fx)}}{\sqrt[3]{b\sin{(e+fx)}}}\,dx$$

input
```
integrate(sec(f*x+e)**2/(b*sin(f*x+e))**(1/3),x)
```

output
```
Integral(sec(e + f*x)**2/(b*sin(e + f*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx=\int \frac{\sec{(fx+e)}^2}{(b\sin{(fx+e)})^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(f*x+e)^2/(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^2/(b*sin(f*x + e))^(1/3), x)
```

Giac [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx=\int \frac{\sec{(fx+e)}^2}{(b\sin{(fx+e)})^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(f*x+e)^2/(b*sin(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^2/(b*sin(f*x + e))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{1}{\cos\left(e+f\,x\right)^2\,{\left(b\,\sin\left(e+f\,x\right)\right)}^{1/3}}\,dx$$

input
```
int(1/(cos(e + f*x)^2*(b*sin(e + f*x))^(1/3)),x)
```

output
```
int(1/(cos(e + f*x)^2*(b*sin(e + f*x))^(1/3)), x)
```

Reduce [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{\int \frac{\sec(fx+e)^2}{\sin(fx+e)^{\frac{1}{3}}}dx}{b^{\frac{1}{3}}}$$

input
```
int(sec(f*x+e)^2/(b*sin(f*x+e))^(1/3),x)
```

output
```
int(sec(e + f*x)**2/sin(e + f*x)**(1/3),x)/b**(1/3)
```

3.318 $\int \frac{\sec^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \frac{\sec^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{2},\frac{4}{3},\sin^2(e+fx)\right)\sec(e+fx)(b\sin(e+fx))^{2/3}}{2bf}$$

output
```
3/2*(cos(f*x+e)^2)^(1/2)*hypergeom([1/3, 5/2],[4/3],sin(f*x+e)^2)*sec(f*x+
e)*(b*sin(f*x+e))^(2/3)/b/f
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \frac{\sec^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{2},\frac{4}{3},\sin^2(e+fx)\right)\tan(e+fx)}{2f\sqrt[3]{b\sin(e+fx)}}$$

input
```
Integrate[Sec[e + f*x]^4/(b*Sin[e + f*x])^(1/3),x]
```

output
```
(3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[1/3, 5/2, 4/3, Sin[e + f*x]^2]*T
an[e + f*x])/(2*f*(b*Sin[e + f*x])^(1/3))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\cos(e+fx)^4\sqrt[3]{b\sin(e+fx)}}dx$$

$$\Big\downarrow \text{ 3057}$$

$$\frac{3\sqrt{\cos^2(e+fx)}\sec(e+fx)(b\sin(e+fx))^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{2},\frac{4}{3},\sin^2(e+fx)\right)}{2bf}$$

input
```
Int[Sec[e + f*x]^4/(b*Sin[e + f*x])^(1/3),x]
```

output
```
(3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[1/3, 5/2, 4/3, Sin[e + f*x]^2]*S
ec[e + f*x]*(b*Sin[e + f*x])^(2/3))/(2*b*f)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Maple [F]

$$\int \frac{\sec(fx+e)^4}{(b\sin(fx+e))^{\frac{1}{3}}}dx$$

input
```
int(sec(f*x+e)^4/(b*sin(f*x+e))^(1/3),x)
```

output
```
int(sec(f*x+e)^4/(b*sin(f*x+e))^(1/3),x)
```

Fricas [F]

$$\int \frac{\sec^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{\sec(fx+e)^4}{(b\sin(fx+e))^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(f*x+e)^4/(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral((b*sin(f*x + e))^(2/3)*sec(f*x + e)^4/(b*sin(f*x + e)), x)
```

Sympy [F]

$$\int \frac{\sec^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{\sec^4{(e+fx)}}{\sqrt[3]{b\sin{(e+fx)}}}\,dx$$

input
```
integrate(sec(f*x+e)**4/(b*sin(f*x+e))**(1/3),x)
```

output
```
Integral(sec(e + f*x)**4/(b*sin(e + f*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{\sec^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{\sec{(fx+e)}^4}{(b\sin{(fx+e)})^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(f*x+e)^4/(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^4/(b*sin(f*x + e))^(1/3), x)
```

Giac [F]

$$\int \frac{\sec^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \int \frac{\sec{(fx+e)}^4}{(b\sin{(fx+e)})^{\frac{1}{3}}}\,dx$$

input
```
integrate(sec(f*x+e)^4/(b*sin(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^4/(b*sin(f*x + e))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx=\int \frac{1}{\cos\left(e+f\,x\right)^4\,{\left(b\,\sin\left(e+f\,x\right)\right)}^{1/3}} \,dx$$

input
```
int(1/(cos(e + f*x)^4*(b*sin(e + f*x))^(1/3)),x)
```

output
```
int(1/(cos(e + f*x)^4*(b*sin(e + f*x))^(1/3)), x)
```

Reduce [F]

$$\int \frac{\sec^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx=\frac{\int \frac{\sec\left(fx+e\right)^{4}}{\sin\left(fx+e\right)^{\frac{1}{3}}}dx}{b^{\frac{1}{3}}}$$

input
```
int(sec(f*x+e)^4/(b*sin(f*x+e))^(1/3),x)
```

output
```
int(sec(e + f*x)**4/sin(e + f*x)**(1/3),x)/b**(1/3)
```

3.319 $\int \frac{\cos^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \frac{\cos^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = -\frac{3\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{3},\frac{2}{3},\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}(b\sin(e+fx))^{2/3}}$$

output
```
-3/2*cos(f*x+e)*hypergeom([-3/2, -1/3],[2/3],sin(f*x+e)^2)/b/f/(cos(f*x+e)
^2)^(1/2)/(b*sin(f*x+e))^(2/3)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \frac{\cos^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = -\frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{3},\frac{2}{3},\sin^2(e+fx)\right)\tan(e+fx)}{2f(b\sin(e+fx))^{5/3}}$$

input
```
Integrate[Cos[e + f*x]^4/(b*Sin[e + f*x])^(5/3),x]
```

output
```
(-3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-3/2, -1/3, 2/3, Sin[e + f*x]^2
]*Tan[e + f*x])/(2*f*(b*Sin[e + f*x])^(5/3))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\cos(e+fx)^4}{(b\sin(e+fx))^{5/3}}dx$$
$$\Big\downarrow \text{ 3057}$$
$$-\frac{3\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{3},\frac{2}{3},\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}(b\sin(e+fx))^{2/3}}$$

input
```
Int[Cos[e + f*x]^4/(b*Sin[e + f*x])^(5/3),x]
```

output
```
(-3*Cos[e + f*x]*Hypergeometric2F1[-3/2, -1/3, 2/3, Sin[e + f*x]^2])/(2*b*
f*Sqrt[Cos[e + f*x]^2]*(b*Sin[e + f*x])^(2/3))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Maple [F]

$$\int \frac{\cos \left(f x +e \right)^{4}}{\left(b \sin \left(f x +e \right)\right)^{\frac{5}{3}}}d x$$

input
```
int(cos(f*x+e)^4/(b*sin(f*x+e))^(5/3),x)
```

output
```
int(cos(f*x+e)^4/(b*sin(f*x+e))^(5/3),x)
```

Fricas [F]

$$\int \frac{\cos^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{\cos \left(f x + e\right)^{4}}{\left(b \sin \left(f x + e\right)\right)^{\frac{5}{3}}}\,dx$$

input
```
integrate(cos(f*x+e)^4/(b*sin(f*x+e))^(5/3),x, algorithm="fricas")
```

output
```
integral(-(b*sin(f*x + e))^(1/3)*cos(f*x + e)^4/(b^2*cos(f*x + e)^2 - b^2)
, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \text{Timed out}$$

input
```
integrate(cos(f*x+e)**4/(b*sin(f*x+e))**(5/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{\cos\left(fx+e\right)^4}{\left(b\sin\left(fx+e\right)\right)^{\frac{5}{3}}}\,dx$$

input
```
integrate(cos(f*x+e)^4/(b*sin(f*x+e))^(5/3),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^4/(b*sin(f*x + e))^(5/3), x)
```

Giac [F]

$$\int \frac{\cos^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{\cos\left(fx+e\right)^4}{\left(b\sin\left(fx+e\right)\right)^{\frac{5}{3}}}\,dx$$

input
```
integrate(cos(f*x+e)^4/(b*sin(f*x+e))^(5/3),x, algorithm="giac")
```

output
```
integrate(cos(f*x + e)^4/(b*sin(f*x + e))^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{\cos\left(e+f\,x\right)^4}{\left(b\,\sin\left(e+f\,x\right)\right)^{5/3}}\,dx$$

input
```
int(cos(e + f*x)^4/(b*sin(e + f*x))^(5/3),x)
```

output
```
int(cos(e + f*x)^4/(b*sin(e + f*x))^(5/3), x)
```

Reduce [F]

$$\int \frac{\cos^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \frac{\int \frac{\cos(fx+e)^4}{\sin(fx+e)^{\frac{5}{3}}}dx}{b^{\frac{5}{3}}}$$

input
```
int(cos(f*x+e)^4/(b*sin(f*x+e))^(5/3),x)
```

output
```
int(cos(e + f*x)**4/(sin(e + f*x)**(2/3)*sin(e + f*x)),x)/(b**(2/3)*b)
```

3.320 $\int \frac{\cos^2(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \frac{\cos^2(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx=-\frac{3\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{3},\frac{2}{3},\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}(b\sin(e+fx))^{2/3}}$$

output
```
-3/2*cos(f*x+e)*hypergeom([-1/2, -1/3],[2/3],sin(f*x+e)^2)/b/f/(cos(f*x+e)
^2)^(1/2)/(b*sin(f*x+e))^(2/3)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \frac{\cos^2(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = -\frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{3},\frac{2}{3},\sin^2(e+fx)\right)\tan(e+fx)}{2f(b\sin(e+fx))^{5/3}}$$

input
```
Integrate[Cos[e + f*x]^2/(b*Sin[e + f*x])^(5/3),x]
```

output
```
(-3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-1/2, -1/3, 2/3, Sin[e + f*x]^2
]*Tan[e + f*x])/(2*f*(b*Sin[e + f*x])^(5/3))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = $\{3042, 3057\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(e+fx)}{(b\sin(e+fx))^{5/3}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(e+fx)^2}{(b\sin(e+fx))^{5/3}}dx$$
$$\Big\downarrow \text{3057}$$
$$-\frac{3\cos(e+fx)\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{3},\frac{2}{3},\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}(b\sin(e+fx))^{2/3}}$$

input
```
Int[Cos[e + f*x]^2/(b*Sin[e + f*x])^(5/3),x]
```

output
```
(-3*Cos[e + f*x]*Hypergeometric2F1[-1/2, -1/3, 2/3, Sin[e + f*x]^2])/(2*b*
f*Sqrt[Cos[e + f*x]^2]*(b*Sin[e + f*x])^(2/3))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Maple [F]

$$\int \frac{\cos(fx+e)^2}{(b\sin(fx+e))^{\frac{5}{3}}}dx$$

input
```
int(cos(f*x+e)^2/(b*sin(f*x+e))^(5/3),x)
```

output
```
int(cos(f*x+e)^2/(b*sin(f*x+e))^(5/3),x)
```

Fricas [F]

$$\int \frac{\cos^2(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{\cos(fx+e)^2}{(b\sin(fx+e))^{\frac{5}{3}}}\,dx$$

input
```
integrate(cos(f*x+e)^2/(b*sin(f*x+e))^(5/3),x, algorithm="fricas")
```

output
```
integral(-(b*sin(f*x + e))^(1/3)*cos(f*x + e)^2/(b^2*cos(f*x + e)^2 - b^2)
, x)
```

Sympy [F]

$$\int \frac{\cos^2(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{\cos^2{(e+fx)}}{(b\sin{(e+fx)})^{\frac{5}{3}}}\,dx$$

input
```
integrate(cos(f*x+e)**2/(b*sin(f*x+e))**(5/3),x)
```

output
```
Integral(cos(e + f*x)**2/(b*sin(e + f*x))**(5/3), x)
```

Maxima [F]

$$\int \frac{\cos^2(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{\cos{(fx+e)}^2}{(b\sin{(fx+e)})^{\frac{5}{3}}}\,dx$$

input
```
integrate(cos(f*x+e)^2/(b*sin(f*x+e))^(5/3),x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^2/(b*sin(f*x + e))^(5/3), x)
```

Giac [F]

$$\int \frac{\cos^2(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{\cos{(fx+e)}^2}{(b\sin{(fx+e)})^{\frac{5}{3}}}\,dx$$

input
```
integrate(cos(f*x+e)^2/(b*sin(f*x+e))^(5/3),x, algorithm="giac")
```

output
```
integrate(cos(f*x + e)^2/(b*sin(f*x + e))^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{\cos(e+f\,x)^2}{(b\,\sin(e+f\,x))^{5/3}}\,dx$$

input
```
int(cos(e + f*x)^2/(b*sin(e + f*x))^(5/3),x)
```

output
```
int(cos(e + f*x)^2/(b*sin(e + f*x))^(5/3), x)
```

Reduce [F]

$$\int \frac{\cos^2(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \frac{\int \frac{\cos(fx+e)^2}{\sin(fx+e)^{\frac{5}{3}}}dx}{b^{\frac{5}{3}}}$$

input
```
int(cos(f*x+e)^2/(b*sin(f*x+e))^(5/3),x)
```

output
```
int(cos(e + f*x)**2/(sin(e + f*x)**(2/3)*sin(e + f*x)),x)/(b**(2/3)*b)
```

3.321 $\int \frac{1}{(b\sin(e+fx))^{5/3}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 58

$$\int \frac{1}{(b\sin(e+fx))^{5/3}}\,dx = -\frac{3\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}(b\sin(e+fx))^{2/3}}$$

output
```
-3/2*cos(f*x+e)*hypergeom([-1/3, 1/2],[2/3],sin(f*x+e)^2)/b/f/(cos(f*x+e)^
2)^(1/2)/(b*sin(f*x+e))^(2/3)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \frac{1}{(b\sin(e+fx))^{5/3}}\,dx = -\frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\sin^2(e+fx)\right)\tan(e+fx)}{2f(b\sin(e+fx))^{5/3}}$$

input
```
Integrate[(b*Sin[e + f*x])^(-5/3),x]
```

output
```
(-3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-1/3, 1/2, 2/3, Sin[e + f*x]^2]
*Tan[e + f*x])/(2*f*(b*Sin[e + f*x])^(5/3))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\sin(e+fx))^{5/3}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(b\sin(e+fx))^{5/3}}dx$$
$$\Big\downarrow \text{3122}$$
$$-\frac{3\cos(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}(b\sin(e+fx))^{2/3}}$$

input
```
Int[(b*Sin[e + f*x])^(-5/3),x]
```

output
```
(-3*Cos[e + f*x]*Hypergeometric2F1[-1/3, 1/2, 2/3, Sin[e + f*x]^2])/(2*b*f
*Sqrt[Cos[e + f*x]^2]*(b*Sin[e + f*x])^(2/3))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Maple [F]

$$\int \frac{1}{(b\sin(fx+e))^{\frac{5}{3}}}dx$$

input
```
int(1/(b*sin(f*x+e))^(5/3),x)
```

output
```
int(1/(b*sin(f*x+e))^(5/3),x)
```

Fricas [F]

$$\int \frac{1}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{1}{(b\sin(fx+e))^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/(b*sin(f*x+e))^(5/3),x, algorithm="fricas")
```

output
```
integral(-(b*sin(f*x + e))^(1/3)/(b^2*cos(f*x + e)^2 - b^2), x)
```

Sympy [F]

$$\int \frac{1}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{1}{(b\sin(e+fx))^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/(b*sin(f*x+e))**(5/3),x)
```

output
```
Integral((b*sin(e + f*x))**(-5/3), x)
```

Maxima [F]

$$\int \frac{1}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{1}{(b\sin(fx+e))^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/(b*sin(f*x+e))^(5/3),x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e))^(-5/3), x)
```

Giac [F]

$$\int \frac{1}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{1}{(b\sin(fx+e))^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/(b*sin(f*x+e))^(5/3),x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e))^(-5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{1}{(b\,\sin(e+f\,x))^{5/3}}\,dx$$

input
```
int(1/(b*sin(e + f*x))^(5/3),x)
```

output
```
int(1/(b*sin(e + f*x))^(5/3), x)
```

Reduce [F]

$$\int \frac{1}{(b\sin(e+fx))^{5/3}}\,dx = \frac{\int \frac{1}{\sin(fx+e)^{\frac{5}{3}}}dx}{b^{\frac{5}{3}}}$$

input
```
int(1/(b*sin(f*x+e))^(5/3),x)
```

output
```
int(1/(sin(e + f*x)**(2/3)*sin(e + f*x)),x)/(b**(2/3)*b)
```

3.322 $\int \frac{\sec^2(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \frac{\sec^2(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = -\frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{3}{2},\frac{2}{3},\sin^2(e+fx)\right)\sec(e+fx)}{2bf(b\sin(e+fx))^{2/3}}$$

output
```
-3/2*(cos(f*x+e)^2)^(1/2)*hypergeom([-1/3, 3/2],[2/3],sin(f*x+e)^2)*sec(f*
x+e)/b/f/(b*sin(f*x+e))^(2/3)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \frac{\sec^2(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = -\frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{3}{2},\frac{2}{3},\sin^2(e+fx)\right)\tan(e+fx)}{2f(b\sin(e+fx))^{5/3}}$$

input
```
Integrate[Sec[e + f*x]^2/(b*Sin[e + f*x])^(5/3),x]
```

output
```
(-3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-1/3, 3/2, 2/3, Sin[e + f*x]^2]
*Tan[e + f*x])/(2*f*(b*Sin[e + f*x])^(5/3))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(e+fx)^2(b\sin(e+fx))^{5/3}}dx$$

$$\Big\downarrow \text{3057}$$

$$-\frac{3\sqrt{\cos^2(e+fx)}\sec(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{3}{2},\frac{2}{3},\sin^2(e+fx)\right)}{2bf(b\sin(e+fx))^{2/3}}$$

input
```
Int[Sec[e + f*x]^2/(b*Sin[e + f*x])^(5/3),x]
```

output
```
(-3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-1/3, 3/2, 2/3, Sin[e + f*x]^2]
*Sec[e + f*x])/(2*b*f*(b*Sin[e + f*x])^(2/3))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Maple [F]

$$\int \frac{\sec (fx+e)^2}{(b\sin (fx+e))^{\frac{5}{3}}}dx$$

input
```
int(sec(f*x+e)^2/(b*sin(f*x+e))^(5/3),x)
```

output
```
int(sec(f*x+e)^2/(b*sin(f*x+e))^(5/3),x)
```

Fricas [F]

$$\int \frac{\sec^2(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{\sec (fx+e)^2}{(b\sin (fx+e))^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(f*x+e)^2/(b*sin(f*x+e))^(5/3),x, algorithm="fricas")
```

output
```
integral(-(b*sin(f*x + e))^(1/3)*sec(f*x + e)^2/(b^2*cos(f*x + e)^2 - b^2)
, x)
```

Sympy [F]

$$\int \frac{\sec^2(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{\sec^2{(e+fx)}}{(b\sin{(e+fx)})^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(f*x+e)**2/(b*sin(f*x+e))**(5/3),x)
```

output
```
Integral(sec(e + f*x)**2/(b*sin(e + f*x))**(5/3), x)
```

Maxima [F]

$$\int \frac{\sec^2(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{\sec{(fx+e)}^2}{(b\sin{(fx+e)})^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(f*x+e)^2/(b*sin(f*x+e))^(5/3),x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^2/(b*sin(f*x + e))^(5/3), x)
```

Giac [F]

$$\int \frac{\sec^2(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{\sec{(fx+e)}^2}{(b\sin{(fx+e)})^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(f*x+e)^2/(b*sin(f*x+e))^(5/3),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^2/(b*sin(f*x + e))^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{1}{\cos(e+f\,x)^2\,(b\,\sin(e+f\,x))^{5/3}}\,dx$$

input
```
int(1/(cos(e + f*x)^2*(b*sin(e + f*x))^(5/3)),x)
```

output
```
int(1/(cos(e + f*x)^2*(b*sin(e + f*x))^(5/3)), x)
```

Reduce [F]

$$\int \frac{\sec^2(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \frac{\int \frac{\sec(fx+e)^2}{\sin(fx+e)^{\frac{5}{3}}}dx}{b^{\frac{5}{3}}}$$

input
```
int(sec(f*x+e)^2/(b*sin(f*x+e))^(5/3),x)
```

output
```
int(sec(e + f*x)**2/(sin(e + f*x)**(2/3)*sin(e + f*x)),x)/(b**(2/3)*b)
```

3.323 $\int \frac{\sec^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \frac{\sec^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = -\frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{5}{2},\frac{2}{3},\sin^2(e+fx)\right)\sec(e+fx)}{2bf(b\sin(e+fx))^{2/3}}$$

output
```
-3/2*(cos(f*x+e)^2)^(1/2)*hypergeom([-1/3, 5/2],[2/3],sin(f*x+e)^2)*sec(f*
x+e)/b/f/(b*sin(f*x+e))^(2/3)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \frac{\sec^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = -\frac{3\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{5}{2},\frac{2}{3},\sin^2(e+fx)\right)\tan(e+fx)}{2f(b\sin(e+fx))^{5/3}}$$

input
```
Integrate[Sec[e + f*x]^4/(b*Sin[e + f*x])^(5/3),x]
```

output
```
(-3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-1/3, 5/2, 2/3, Sin[e + f*x]^2]
*Tan[e + f*x])/(2*f*(b*Sin[e + f*x])^(5/3))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\cos(e+fx)^4(b\sin(e+fx))^{5/3}}dx$$
$$\Big\downarrow \text{ 3057}$$
$$-\frac{3\sqrt{\cos^2(e+fx)}\sec(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{5}{2},\frac{2}{3},\sin^2(e+fx)\right)}{2bf(b\sin(e+fx))^{2/3}}$$

input
```
Int[Sec[e + f*x]^4/(b*Sin[e + f*x])^(5/3),x]
```

output
```
(-3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-1/3, 5/2, 2/3, Sin[e + f*x]^2]
*Sec[e + f*x])/(2*b*f*(b*Sin[e + f*x])^(2/3))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Maple [F]

$$\int \frac{\sec(fx+e)^4}{(b\sin(fx+e))^{\frac{5}{3}}}dx$$

input
```
int(sec(f*x+e)^4/(b*sin(f*x+e))^(5/3),x)
```

output
```
int(sec(f*x+e)^4/(b*sin(f*x+e))^(5/3),x)
```

Fricas [F]

$$\int \frac{\sec^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{\sec(fx+e)^4}{(b\sin(fx+e))^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(f*x+e)^4/(b*sin(f*x+e))^(5/3),x, algorithm="fricas")
```

output
```
integral(-(b*sin(f*x + e))^(1/3)*sec(f*x + e)^4/(b^2*cos(f*x + e)^2 - b^2)
, x)
```

Sympy [F]

$$\int \frac{\sec^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{\sec^4{(e+fx)}}{(b\sin{(e+fx)})^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(f*x+e)**4/(b*sin(f*x+e))**(5/3),x)
```

output
```
Integral(sec(e + f*x)**4/(b*sin(e + f*x))**(5/3), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \text{Timed out}$$

input
```
integrate(sec(f*x+e)^4/(b*sin(f*x+e))^(5/3),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\sec^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{\sec{(fx+e)}^4}{(b\sin{(fx+e)})^{\frac{5}{3}}}\,dx$$

input
```
integrate(sec(f*x+e)^4/(b*sin(f*x+e))^(5/3),x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^4/(b*sin(f*x + e))^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \int \frac{1}{\cos\left(e+f\,x\right)^4\,\left(b\,\sin\left(e+f\,x\right)\right)^{5/3}}\,dx$$

input
```
int(1/(cos(e + f*x)^4*(b*sin(e + f*x))^(5/3)),x)
```

output
```
int(1/(cos(e + f*x)^4*(b*sin(e + f*x))^(5/3)), x)
```

Reduce [F]

$$\int \frac{\sec^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = \frac{\int \frac{\sec(fx+e)^4}{\sin(fx+e)^{\frac{5}{3}}}dx}{b^{\frac{5}{3}}}$$

input
```
int(sec(f*x+e)^4/(b*sin(f*x+e))^(5/3),x)
```

output
```
int(sec(e + f*x)**4/(sin(e + f*x)**(2/3)*sin(e + f*x)),x)/(b**(2/3)*b)
```

3.324 $\int \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 128

$$\int \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1-\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+\frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\right)}{4b}$$

output
```
-1/2*3^(1/2)*arctan(1/3*(1-2*sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3))*3^(1/2))/b
-1/2*ln(1+sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3))/b+1/4*ln(1-sin(b*x+a)^(2/3)/c
os(b*x+a)^(2/3)+sin(b*x+a)^(4/3)/cos(b*x+a)^(4/3))/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.45

$$\int \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\,dx = \frac{3\cos^2(a+bx)^{2/3}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{2}{3},\frac{5}{3},\sin^2(a+bx)\right)\sin^{\frac{4}{3}}(a+bx)}{4b\cos^{\frac{4}{3}}(a+bx)}$$

input
```
Integrate[Sin[a + b*x]^(1/3)/Cos[a + b*x]^(1/3),x]
```

output
```
(3*(Cos[a + b*x]^2)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, Sin[a + b*x]^2]
*Sin[a + b*x]^(4/3))/(4*b*Cos[a + b*x]^(4/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.57, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3054, 807, 821, 16, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}dx$$

$$\Big\downarrow \text{3054}$$

$$\frac{3\int \frac{\tan(a+bx)}{\tan^2(a+bx)+1}d\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}}{b}$$

$$\Big\downarrow \text{807}$$

$$\frac{3\int \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)(\tan(a+bx)+1)}d\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}}{2b}$$

$$\Big\downarrow \; 821$$

$$\frac{3\left(\frac{1}{3}\int \left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)d\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-\frac{1}{3}\int \frac{1}{\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1}d\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$\Big\downarrow \; 16$$

$$\frac{3\left(\frac{1}{3}\int \left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)d\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-\frac{1}{3}\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)\right)}{2b}$$

$$\Big\downarrow \; 1142$$

$$\frac{3\left(\frac{1}{3}\left(\frac{3}{2}\int 1d\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+\frac{1}{2}\int \left(\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-1\right)d\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)-\frac{1}{3}\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)\right)}{2b}$$

$$\Big\downarrow \; 25$$

$$\frac{3\left(\frac{1}{3}\left(\frac{3}{2}\int 1d\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-\frac{1}{2}\int \left(1-\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)d\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)-\frac{1}{3}\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)\right)}{2b}$$

$$\Big\downarrow \; 1083$$

$$\frac{3\left(\frac{1}{3}\left(-3\int \frac{1}{-\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-2}d\left(\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-1\right)-\frac{1}{2}\int \left(1-\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)d\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)-\frac{1}{3}\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)\right)}{2b}$$

$$\Big\downarrow \; 217$$

$$\frac{3\left(\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-1}{\sqrt{3}}\right)-\frac{1}{2}\int \left(1-\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)d\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)-\frac{1}{3}\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)\right)}{2b}$$

$$\Big\downarrow \; 1103$$

$$\frac{3\left(\frac{\arctan\left(\frac{\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{3}\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)\right)}{2b}$$

input
```
Int[Sin[a + b*x]^(1/3)/Cos[a + b*x]^(1/3),x]
```

output
```
(3*(ArcTan[(-1 + (2*Sin[a + b*x]^(2/3))/Cos[a + b*x]^(2/3))/Sqrt[3]]/Sqrt[
3] - Log[1 + Sin[a + b*x]^(2/3)/Cos[a + b*x]^(2/3)]/3))/(2*b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3054
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k*a*(b/f)   Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Maple [F]

$$\int \frac{\sin(bx+a)^{\frac{1}{3}}}{\cos(bx+a)^{\frac{1}{3}}}dx$$

input
```
int(sin(b*x+a)^(1/3)/cos(b*x+a)^(1/3),x)
```

output
```
int(sin(b*x+a)^(1/3)/cos(b*x+a)^(1/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\,dx = \frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}\cos(bx+a)-2\sqrt{3}\cos(bx+a)^{\frac{1}{3}}\sin(bx+a)^{\frac{2}{3}}}{3\cos(bx+a)}\right)-2\log\left(\frac{\cos(bx+a)^{\frac{1}{3}}\sin(bx+a)^{\frac{2}{3}}+\cos(bx+a)}{\cos(bx+a)}\right)+\log\left(\frac{\cos(bx-}{}\right.}{4b}$$

input
```
integrate(sin(b*x+a)^(1/3)/cos(b*x+a)^(1/3),x, algorithm="fricas")
```

output
```
1/4*(2*sqrt(3)*arctan(-1/3*(sqrt(3)*cos(b*x + a) - 2*sqrt(3)*cos(b*x + a)^
(1/3)*sin(b*x + a)^(2/3))/cos(b*x + a)) - 2*log((cos(b*x + a)^(1/3)*sin(b*
x + a)^(2/3) + cos(b*x + a))/cos(b*x + a)) + log((cos(b*x + a)^2 - cos(b*x
 + a)^(4/3)*sin(b*x + a)^(2/3) + cos(b*x + a)^(2/3)*sin(b*x + a)^(4/3))/co
s(b*x + a)^2))/b
```

Sympy [F]

$$\int \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\,dx = \int \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\,dx$$

input
```
integrate(sin(b*x+a)**(1/3)/cos(b*x+a)**(1/3),x)
```

output
```
Integral(sin(a + b*x)**(1/3)/cos(a + b*x)**(1/3), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\,dx = \int \frac{\sin\left(bx+a\right)^{\frac{1}{3}}}{\cos\left(bx+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sin(b*x+a)^(1/3)/cos(b*x+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^(1/3)/cos(b*x + a)^(1/3), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\,dx = \int \frac{\sin\left(bx+a\right)^{\frac{1}{3}}}{\cos\left(bx+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(sin(b*x+a)^(1/3)/cos(b*x+a)^(1/3),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^(1/3)/cos(b*x + a)^(1/3), x)
```

Mupad [B] (verification not implemented)

Time = 26.63 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.34

$$\int \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\,dx = -\frac{3\cos\left(a+b\,x\right)^{2/3}\sin\left(a+b\,x\right)^{4/3}{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{3};\ \frac{4}{3};\ \cos\left(a+b\,x\right)^2\right)}{2\,b\,{\left(\sin\left(a+b\,x\right)^2\right)}^{2/3}}$$

input
```
int(sin(a + b*x)^(1/3)/cos(a + b*x)^(1/3),x)
```

output
```
-(3*cos(a + b*x)^(2/3)*sin(a + b*x)^(4/3)*hypergeom([1/3, 1/3], 4/3, cos(a
 + b*x)^2))/(2*b*(sin(a + b*x)^2)^(2/3))
```

Reduce [F]

$$\int \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\,dx = \int \frac{\sin(bx+a)^{\frac{1}{3}}}{\cos(bx+a)^{\frac{1}{3}}}dx$$

input
```
int(sin(b*x+a)^(1/3)/cos(b*x+a)^(1/3),x)
```

output
```
int(sin(a + b*x)**(1/3)/cos(a + b*x)**(1/3),x)
```

3.325 $\int \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 163

$$\int \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\,dx = -\frac{\arctan\left(\sqrt{3}-\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{2b} + \frac{\arctan\left(\sqrt{3}+\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{2b} + \frac{\arctan\left(\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b} - \frac{\sqrt{3}\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}\left(1+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}\right)}{2b}$$

output

```
1/2*arctan(-3^(1/2)+2*sin(b*x+a)^(1/3)/cos(b*x+a)^(1/3))/b+1/2*arctan(3^(1
/2)+2*sin(b*x+a)^(1/3)/cos(b*x+a)^(1/3))/b+arctan(sin(b*x+a)^(1/3)/cos(b*x
+a)^(1/3))/b-1/2*3^(1/2)*arctanh(3^(1/2)*sin(b*x+a)^(1/3)/cos(b*x+a)^(1/3)
/(1+sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3)))/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.35

$$\int \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\,dx = \frac{3\cos^2(a+bx)^{5/6}\,\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{5}{6},\frac{11}{6},\sin^2(a+bx)\right)\sin^{\frac{5}{3}}(a+bx)}{5b\cos^{\frac{5}{3}}(a+bx)}$$

input
```
Integrate[Sin[a + b*x]^(2/3)/Cos[a + b*x]^(2/3),x]
```

output
```
(3*(Cos[a + b*x]^2)^(5/6)*Hypergeometric2F1[5/6, 5/6, 11/6, Sin[a + b*x]^2
]*Sin[a + b*x]^(5/3))/(5*b*Cos[a + b*x]^(5/3))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.36, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3054, 824, 27, 216, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^{2/3}}{\cos(a+bx)^{2/3}}dx$$

$$\Big\downarrow \text{3054}$$

$$\frac{3\int \frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)(\tan^2(a+bx)+1)}d\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}}{b}$$

$$\Big\downarrow \text{824}$$

$$\frac{3\left(\frac{1}{3}\int \frac{1}{\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1}d\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+\frac{1}{3}\int -\frac{1-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}}{2\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1\right)}d\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+\frac{1}{3}\int -\frac{\frac{\sqrt{3}\sqrt[3]{\cdots}}{\sqrt[3]{\cdots}}}{2\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\cdots\right.}\cdots\right.}{b}$$

$\downarrow$ 27

$$\frac{3\left(\frac{1}{3}\int \frac{1}{\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1}d\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}-\frac{1}{6}\int \frac{1-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}}{\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1}d\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}-\frac{1}{6}\int \frac{\frac{\sqrt{3}\sqrt[3]{\sin(a+\cdots}}{\sqrt[3]{\cos(a+\cdots}}}{\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+\frac{\sqrt{3}\sqrt[3]{\sin\cdots}}{\sqrt[3]{\cos(\cdots}}}\cdots\right.}{b}$$

$\downarrow$ 216

$$\frac{3\left(-\frac{1}{6}\int \frac{1-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}}{\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1}d\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}-\frac{1}{6}\int \frac{\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1}{\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1}d\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+\frac{1}{3}\cdots\right.}{b}$$

$\downarrow$ 1142

$$3\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1}d\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+\frac{1}{2}\sqrt{3}\int -\frac{\sqrt{3}-\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}}{\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1}d\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\cdots\right.\right.$$

$\downarrow$ 25

$$3\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1}d\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}}{\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1}d\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\cdots\right.\right.$$

$\downarrow$ 1083

$$3\left(\frac{1}{6}\left(-\int \frac{1}{-\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-1}d\left(\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}-\sqrt{3}\right)-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}}{\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1}d\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)+\frac{1}{6}$$

$\downarrow$ 217

$$3\left(\frac{1}{6}\left(-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}}{\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1}d\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}-\arctan\left(\sqrt{3}-\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)\right)+\frac{1}{6}\left(\arctan\right.\right.$$

$\downarrow$ 1103

$$3\left(\frac{1}{3}\arctan\left(\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)+\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1\right)-\arctan\left(\sqrt{3}-\frac{2\sqrt[3]{\sin(a+b}}{\sqrt[3]{\cos(a+b}}\right.\right.\right.$$
$$b$$

input
```
Int[Sin[a + b*x]^(2/3)/Cos[a + b*x]^(2/3),x]
```

output
```
(3*(ArcTan[Sin[a + b*x]^(1/3)/Cos[a + b*x]^(1/3)]/3 + (-ArcTan[Sqrt[3] - (
2*Sin[a + b*x]^(1/3))/Cos[a + b*x]^(1/3)] + (Sqrt[3]*Log[1 - (Sqrt[3]*Sin[
a + b*x]^(1/3))/Cos[a + b*x]^(1/3) + Sin[a + b*x]^(2/3)/Cos[a + b*x]^(2/3)
])/2)/6 + (ArcTan[Sqrt[3] + (2*Sin[a + b*x]^(1/3))/Cos[a + b*x]^(1/3)] - (
Sqrt[3]*Log[1 + (Sqrt[3]*Sin[a + b*x]^(1/3))/Cos[a + b*x]^(1/3) + Sin[a +
b*x]^(2/3)/Cos[a + b*x]^(2/3)])/2)/6))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3054
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k*a*(b/f)   Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Maple [F]

$$\int \frac{\sin\left(bx+a\right)^{\frac{2}{3}}}{\cos\left(bx+a\right)^{\frac{2}{3}}}dx$$

input
```
int(sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3),x)
```

output
```
int(sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(131) = 262.

Time = 0.11 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.64

$$\int \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\,dx = -\frac{\sqrt{3}b\sqrt{\frac{1}{b^2}}\log\left(\frac{2\left(\sqrt{3}b\sqrt{\frac{1}{b^2}}\cos(bx+a)^{\frac{1}{3}}\sin(bx+a)^{\frac{2}{3}}+\cos(bx+a)^{\frac{2}{3}}\sin(bx+a)^{\frac{1}{3}}+\sin(bx+a)\right)}{\sin(bx+a)}\right)-\sqrt{3}b\sqrt{\frac{1}{b^2}}\log\left(-\frac{2\left(\sqrt{3}b\sqrt{\frac{1}{b^2}}\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
integrate(sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(3)*b*sqrt(b^(-2))*log(2*(sqrt(3)*b*sqrt(b^(-2))*cos(b*x + a)^(1
/3)*sin(b*x + a)^(2/3) + cos(b*x + a)^(2/3)*sin(b*x + a)^(1/3) + sin(b*x +
 a))/sin(b*x + a)) - sqrt(3)*b*sqrt(b^(-2))*log(-2*(sqrt(3)*b*sqrt(b^(-2))
*cos(b*x + a)^(1/3)*sin(b*x + a)^(2/3) - cos(b*x + a)^(2/3)*sin(b*x + a)^(
1/3) - sin(b*x + a))/sin(b*x + a)) + 2*arctan((sqrt(3)*b*sqrt(b^(-2))*sin(
b*x + a) + 2*cos(b*x + a)^(1/3)*sin(b*x + a)^(2/3))/sin(b*x + a)) + 2*arct
an(-(sqrt(3)*b*sqrt(b^(-2))*sin(b*x + a) - 2*cos(b*x + a)^(1/3)*sin(b*x +
a)^(2/3))/sin(b*x + a)) + 4*arctan(cos(b*x + a)^(1/3)/sin(b*x + a)^(1/3)))
/b
```

Sympy [F]

$$\int \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\,dx = \int \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\,dx$$

input
```
integrate(sin(b*x+a)**(2/3)/cos(b*x+a)**(2/3),x)
```

output
```
Integral(sin(a + b*x)**(2/3)/cos(a + b*x)**(2/3), x)
```

Maxima [F]

$$\int \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\,dx = \int \frac{\sin(bx+a)^{\frac{2}{3}}}{\cos(bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^(2/3)/cos(b*x + a)^(2/3), x)
```

Giac [F]

$$\int \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\,dx=\int \frac{\sin(bx+a)^{\frac{2}{3}}}{\cos(bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^(2/3)/cos(b*x + a)^(2/3), x)
```

Mupad [B] (verification not implemented)

Time = 26.43 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.27

$$\int \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\,dx=-\frac{3\cos(a+b\,x)^{1/3}\sin(a+b\,x)^{5/3}\,{}_2\mathrm{F}_1\left(\frac{1}{6},\frac{1}{6};\ \frac{7}{6};\ \cos(a+b\,x)^2\right)}{b\left(\sin(a+b\,x)^2\right)^{5/6}}$$

input
```
int(sin(a + b*x)^(2/3)/cos(a + b*x)^(2/3),x)
```

output
```
-(3*cos(a + b*x)^(1/3)*sin(a + b*x)^(5/3)*hypergeom([1/6, 1/6], 7/6, cos(a
 + b*x)^2))/(b*(sin(a + b*x)^2)^(5/6))
```

Reduce [F]

$$\int \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\,dx=\int \frac{\sin(bx+a)^{\frac{2}{3}}}{\cos(bx+a)^{\frac{2}{3}}}dx$$

input
```
int(sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3),x)
```

output
```
int(sin(a + b*x)**(2/3)/cos(a + b*x)**(2/3),x)
```

3.326 $\int \frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 188

$$\int \frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\,dx = -\frac{\arctan\left(\sqrt{3}-\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{2b} + \frac{\arctan\left(\sqrt{3}+\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{2b} + \frac{\arctan\left(\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b} - \frac{\sqrt{3}\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)\sqrt[3]{\sin(a+bx)}}\right)}{2b} + \frac{3\sqrt[3]{\sin(a+bx)}}{b\sqrt[3]{\cos(a+bx)}}$$

output

```
1/2*arctan(-3^(1/2)+2*cos(b*x+a)^(1/3)/sin(b*x+a)^(1/3))/b+1/2*arctan(3^(1
/2)+2*cos(b*x+a)^(1/3)/sin(b*x+a)^(1/3))/b+arctan(cos(b*x+a)^(1/3)/sin(b*x
+a)^(1/3))/b-1/2*3^(1/2)*arctanh(3^(1/2)*cos(b*x+a)^(1/3)/(1+cos(b*x+a)^(2
/3)/sin(b*x+a)^(2/3))/sin(b*x+a)^(1/3))/b+3*sin(b*x+a)^(1/3)/b/cos(b*x+a)^
(1/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.30

$$\int \frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\,dx = \frac{3\sqrt[6]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{7}{6},\frac{7}{6},\frac{13}{6},\sin^2(a+bx)\right)\sin^{\frac{7}{3}}(a+bx)}{7b\sqrt[3]{\cos(a+bx)}}$$

input
```
Integrate[Sin[a + b*x]^(4/3)/Cos[a + b*x]^(4/3),x]
```

output
```
(3*(Cos[a + b*x]^2)^(1/6)*Hypergeometric2F1[7/6, 7/6, 13/6, Sin[a + b*x]^2
]*Sin[a + b*x]^(7/3))/(7*b*Cos[a + b*x]^(1/3))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.31, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3046, 3042, 3055, 824, 27, 216, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^{4/3}}{\cos(a+bx)^{4/3}}dx$$

$$\Big\downarrow \text{3046}$$

$$\frac{3\sqrt[3]{\sin(a+bx)}}{b\sqrt[3]{\cos(a+bx)}}-\int \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\sqrt[3]{\sin(a+bx)}}{b\sqrt[3]{\cos(a+bx)}}-\int\frac{\cos(a+bx)^{2/3}}{\sin(a+bx)^{2/3}}dx$$

$\downarrow$ 3055

$$\frac{3\int\frac{\cos^{\frac{4}{3}}(a+bx)}{(\cot^2(a+bx)+1)\sin^{\frac{4}{3}}(a+bx)}d\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}}{b}+\frac{3\sqrt[3]{\sin(a+bx)}}{b\sqrt[3]{\cos(a+bx)}}$$

$\downarrow$ 824

$$\frac{3\left(\frac{1}{3}\int\frac{1}{\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1}d\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+\frac{1}{3}\int-\frac{1-\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}}{2\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1\right)}d\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+\frac{1}{3}\int-\frac{\sqrt{3}\ldots}{2\left(\frac{\cos^{\frac{2}{3}}(a+b\ldots}{\sin^{\frac{2}{3}}(a+b\ldots}\right.}\ldots\right.}{b}$$

$$\frac{3\sqrt[3]{\sin(a+bx)}}{b\sqrt[3]{\cos(a+bx)}}$$

$\downarrow$ 27

$$\frac{3\left(\frac{1}{3}\int\frac{1}{\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1}d\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}-\frac{1}{6}\int\frac{1-\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}}{\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1}d\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}-\frac{1}{6}\int\frac{\frac{\sqrt{3}\sqrt[3]{\cos(a+\ldots}}{\sqrt[3]{\sin(a+b\ldots}}}{\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+\frac{\sqrt{3}\sqrt[3]{\cos\ldots}}{\sqrt[3]{\sin\ldots}}}\ldots\right.}{b}$$

$$\frac{3\sqrt[3]{\sin(a+bx)}}{b\sqrt[3]{\cos(a+bx)}}$$

$\downarrow$ 216

$$\frac{3\left(-\frac{1}{6}\int\frac{1-\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}}{\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1}d\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}-\frac{1}{6}\int\frac{\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1}{\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1}d\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+\frac{1}{3}\ldots\right.}{b}$$

$$\frac{3\sqrt[3]{\sin(a+bx)}}{b\sqrt[3]{\cos(a+bx)}}$$

$\downarrow$ 1142

$$\frac{3\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1}d\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+\frac{1}{2}\sqrt{3}\int -\frac{\sqrt{3}-\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}}{\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1}d\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx}}\right.\right.}{\frac{3\sqrt[3]{\sin(a+bx)}}{b\sqrt[3]{\cos(a+bx)}}}$$

$\downarrow$ 25

$$\frac{3\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1}d\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}}{\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1}d\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right.\right.}{\frac{3\sqrt[3]{\sin(a+bx)}}{b\sqrt[3]{\cos(a+bx)}}}$$

$\downarrow$ 1083

$$\frac{3\left(\frac{1}{6}\left(-\int \frac{1}{-\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-1}d\left(\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}-\sqrt{3}\right)-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}}{\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1}d\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)+\frac{1}{6}\right.}{\frac{3\sqrt[3]{\sin(a+bx)}}{b\sqrt[3]{\cos(a+bx)}}}$$

$\downarrow$ 217

$$\frac{3\left(\frac{1}{6}\left(-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}}{\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1}d\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}-\arctan\left(\sqrt{3}-\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)\right)+\frac{1}{6}\left(\arctan\right.\right.}{\frac{3\sqrt[3]{\sin(a+bx)}}{b\sqrt[3]{\cos(a+bx)}}}$$

$\downarrow$ 1103

$$\frac{3\left(\frac{1}{3}\arctan\left(\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)+\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1\right)-\arctan\left(\sqrt{3}-\frac{2\sqrt[3]{\cos(a+b}}{\sqrt[3]{\sin(a+b}}\right.\right.\right.}{b}$$

$$\frac{3\sqrt[3]{\sin(a+bx)}}{b\sqrt[3]{\cos(a+bx)}}$$

input
```
Int[Sin[a + b*x]^(4/3)/Cos[a + b*x]^(4/3),x]
```

output
```
(3*(ArcTan[Cos[a + b*x]^(1/3)/Sin[a + b*x]^(1/3)]/3 + (-ArcTan[Sqrt[3] - (
2*Cos[a + b*x]^(1/3))/Sin[a + b*x]^(1/3)] + (Sqrt[3]*Log[1 + Cos[a + b*x]^
(2/3)/Sin[a + b*x]^(2/3) - (Sqrt[3]*Cos[a + b*x]^(1/3))/Sin[a + b*x]^(1/3)
])/2)/6 + (ArcTan[Sqrt[3] + (2*Cos[a + b*x]^(1/3))/Sin[a + b*x]^(1/3)] - (
Sqrt[3]*Log[1 + Cos[a + b*x]^(2/3)/Sin[a + b*x]^(2/3) + (Sqrt[3]*Cos[a + b
*x]^(1/3))/Sin[a + b*x]^(1/3)])/2)/6))/b + (3*Sin[a + b*x]^(1/3))/(b*Cos[a
 + b*x]^(1/3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3046
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Sin[e + f
*x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3055
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[(-k)*a*(b/f)   Subst[Int[x
^(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[
e + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m,
0] && LtQ[m, 1]
```

Maple [F]

$$\int \frac{\sin \left(bx+a\right)^{\frac{4}{3}}}{\cos \left(bx+a\right)^{\frac{4}{3}}}dx$$

input
```
int(sin(b*x+a)^(4/3)/cos(b*x+a)^(4/3),x)
```

output
```
int(sin(b*x+a)^(4/3)/cos(b*x+a)^(4/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. 2(152) = 304.

Time = 0.13 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.71

$$\int \frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\,dx =$$

$$-\frac{\sqrt{3}b\sqrt{\frac{1}{b^2}}\cos\left(bx+a\right)\log\left(\frac{\sqrt{3}b\sqrt{\frac{1}{b^2}}\cos(bx+a)^{\frac{2}{3}}\sin(bx+a)^{\frac{1}{3}}+\cos(bx+a)^{\frac{1}{3}}\sin(bx+a)^{\frac{2}{3}}+\cos(bx+a)}{\cos(bx+a)}\right)-\sqrt{3}b\sqrt{\frac{1}{b^2}}\cos\left(bx\right.}{}$$

input
```
integrate(sin(b*x+a)^(4/3)/cos(b*x+a)^(4/3),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(3)*b*sqrt(b^(-2))*cos(b*x + a)*log((sqrt(3)*b*sqrt(b^(-2))*cos(
b*x + a)^(2/3)*sin(b*x + a)^(1/3) + cos(b*x + a)^(1/3)*sin(b*x + a)^(2/3)
+ cos(b*x + a))/cos(b*x + a)) - sqrt(3)*b*sqrt(b^(-2))*cos(b*x + a)*log(-(
sqrt(3)*b*sqrt(b^(-2))*cos(b*x + a)^(2/3)*sin(b*x + a)^(1/3) - cos(b*x + a
)^(1/3)*sin(b*x + a)^(2/3) - cos(b*x + a))/cos(b*x + a)) + 2*arctan((sqrt(
3)*b*sqrt(b^(-2))*cos(b*x + a) + 2*cos(b*x + a)^(2/3)*sin(b*x + a)^(1/3))/
cos(b*x + a))*cos(b*x + a) + 2*arctan(-(sqrt(3)*b*sqrt(b^(-2))*cos(b*x + a
) - 2*cos(b*x + a)^(2/3)*sin(b*x + a)^(1/3))/cos(b*x + a))*cos(b*x + a) +
4*arctan(sin(b*x + a)^(1/3)/cos(b*x + a)^(1/3))*cos(b*x + a) - 12*cos(b*x
+ a)^(2/3)*sin(b*x + a)^(1/3))/(b*cos(b*x + a))
```

Sympy [F]

$$\int \frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\,dx$$

input
```
integrate(sin(b*x+a)**(4/3)/cos(b*x+a)**(4/3),x)
```

output
```
Integral(sin(a + b*x)**(4/3)/cos(a + b*x)**(4/3), x)
```

Maxima [F]

$$\int \frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{\sin(bx+a)^{\frac{4}{3}}}{\cos(bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(sin(b*x+a)^(4/3)/cos(b*x+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^(4/3)/cos(b*x + a)^(4/3), x)
```

Giac [F]

$$\int \frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{\sin(bx+a)^{\frac{4}{3}}}{\cos(bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(sin(b*x+a)^(4/3)/cos(b*x+a)^(4/3),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^(4/3)/cos(b*x + a)^(4/3), x)
```

Mupad [B] (verification not implemented)

Time = 27.00 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.23

$$\int \frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\,dx = \frac{3\sin(a+b\,x)^{7/3}\,{}_2F_1\left(-\frac{1}{6},-\frac{1}{6};\ \frac{5}{6};\ \cos(a+b\,x)^2\right)}{b\cos(a+b\,x)^{1/3}\left(\sin(a+b\,x)^2\right)^{7/6}}$$

input
```
int(sin(a + b*x)^(4/3)/cos(a + b*x)^(4/3),x)
```

output
```
(3*sin(a + b*x)^(7/3)*hypergeom([-1/6, -1/6], 5/6, cos(a + b*x)^2))/(b*cos
(a + b*x)^(1/3)*(sin(a + b*x)^2)^(7/6))
```

Reduce [F]

$$\int \frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{\sin(bx+a)^{\frac{4}{3}}}{\cos(bx+a)^{\frac{4}{3}}}dx$$

input
```
int(sin(b*x+a)^(4/3)/cos(b*x+a)^(4/3),x)
```

output
```
int((sin(a + b*x)**(1/3)*sin(a + b*x))/(cos(a + b*x)**(1/3)*cos(a + b*x)),
x)
```

3.327 $\int \frac{\sin^{\frac{5}{3}}(a+bx)}{\cos^{\frac{5}{3}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 155

$$\int \frac{\sin^{\frac{5}{3}}(a+bx)}{\cos^{\frac{5}{3}}(a+bx)}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} + \frac{\log\left(1+\frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}-\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{3\sin^{\frac{2}{3}}(a+bx)}{2b\cos^{\frac{2}{3}}(a+bx)}$$

output
```
-1/2*3^(1/2)*arctan(1/3*(1-2*cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3))*3^(1/2))/b
+1/4*ln(1+cos(b*x+a)^(4/3)/sin(b*x+a)^(4/3)-cos(b*x+a)^(2/3)/sin(b*x+a)^(2
/3))/b-1/2*ln(1+cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3))/b+3/2*sin(b*x+a)^(2/3)/
b/cos(b*x+a)^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.37

$$\int \frac{\sin^{\frac{5}{3}}(a+bx)}{\cos^{\frac{5}{3}}(a+bx)}\,dx = \frac{3\sqrt[3]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{4}{3},\frac{7}{3},\sin^2(a+bx)\right)\sin^{\frac{8}{3}}(a+bx)}{8b\cos^{\frac{2}{3}}(a+bx)}$$

input
```
Integrate[Sin[a + b*x]^(5/3)/Cos[a + b*x]^(5/3),x]
```

output
```
(3*(Cos[a + b*x]^2)^(1/3)*Hypergeometric2F1[4/3, 4/3, 7/3, Sin[a + b*x]^2]
*Sin[a + b*x]^(8/3))/(8*b*Cos[a + b*x]^(2/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.65, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3046, 3042, 3055, 807, 821, 16, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^{\frac{5}{3}}(a+bx)}{\cos^{\frac{5}{3}}(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^{5/3}}{\cos(a+bx)^{5/3}}dx$$

$$\Big\downarrow \text{3046}$$

$$\frac{3\sin^{\frac{2}{3}}(a+bx)}{2b\cos^{\frac{2}{3}}(a+bx)} - \int \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\sin^{\frac{2}{3}}(a+bx)}{2b\cos^{\frac{2}{3}}(a+bx)}-\int\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}dx$$

$\Big\downarrow$ 3055

$$\frac{3\int\frac{\cot(a+bx)}{\cot^2(a+bx)+1}d\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}}{b}+\frac{3\sin^{\frac{2}{3}}(a+bx)}{2b\cos^{\frac{2}{3}}(a+bx)}$$

$\Big\downarrow$ 807

$$\frac{3\int\frac{\cos^{\frac{2}{3}}(a+bx)}{(\cot(a+bx)+1)\sin^{\frac{2}{3}}(a+bx)}d\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}}{2b}+\frac{3\sin^{\frac{2}{3}}(a+bx)}{2b\cos^{\frac{2}{3}}(a+bx)}$$

$\Big\downarrow$ 821

$$\frac{3\left(\frac{1}{3}\int\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)d\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-\frac{1}{3}\int\frac{1}{\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1}d\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b}+\frac{3\sin^{\frac{2}{3}}(a+bx)}{2b\cos^{\frac{2}{3}}(a+bx)}$$

$\Big\downarrow$ 16

$$\frac{3\left(\frac{1}{3}\int\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)d\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-\frac{1}{3}\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)\right)}{2b}+\frac{3\sin^{\frac{2}{3}}(a+bx)}{2b\cos^{\frac{2}{3}}(a+bx)}$$

$\Big\downarrow$ 1142

$$\frac{3\left(\frac{1}{3}\left(\frac{3}{2}\int 1d\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+\frac{1}{2}\int\left(\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-1\right)d\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)-\frac{1}{3}\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)\right)}{2b}+\frac{3\sin^{\frac{2}{3}}(a+bx)}{2b\cos^{\frac{2}{3}}(a+bx)}$$

$\Big\downarrow$ 25

$$\frac{3\left(\frac{1}{3}\left(\frac{3}{2}\int 1d\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-\frac{1}{2}\int\left(1-\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)d\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)-\frac{1}{3}\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)\right)}{2b}+\frac{3\sin^{\frac{2}{3}}(a+bx)}{2b\cos^{\frac{2}{3}}(a+bx)}$$

$\Big\downarrow$ 1083

$$\frac{3\left(\frac{1}{3}\left(-3\int \frac{1}{-\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-2}d\left(\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-1\right)-\frac{1}{2}\int \left(1-\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)d\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)-\frac{1}{3}\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)\right)}{2b}$$

$$\frac{3\sin^{\frac{2}{3}}(a+bx)}{2b\cos^{\frac{2}{3}}(a+bx)}$$

$$\Big\downarrow \ 217$$

$$\frac{3\left(\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-1}{\sqrt{3}}\right)-\frac{1}{2}\int \left(1-\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)d\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)-\frac{1}{3}\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)\right)}{2b}+$$

$$\frac{3\sin^{\frac{2}{3}}(a+bx)}{2b\cos^{\frac{2}{3}}(a+bx)}$$

$$\Big\downarrow \ 1103$$

$$\frac{3\left(\frac{\arctan\left(\frac{\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{3}\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)\right)}{2b}+\frac{3\sin^{\frac{2}{3}}(a+bx)}{2b\cos^{\frac{2}{3}}(a+bx)}$$

input
```
Int[Sin[a + b*x]^(5/3)/Cos[a + b*x]^(5/3),x]
```

output
```
(3*(ArcTan[(-1 + (2*Cos[a + b*x]^(2/3))/Sin[a + b*x]^(2/3))/Sqrt[3]]/Sqrt[
3] - Log[1 + Cos[a + b*x]^(2/3)/Sin[a + b*x]^(2/3)]/3))/(2*b) + (3*Sin[a +
 b*x]^(2/3))/(2*b*Cos[a + b*x]^(2/3))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3046
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Sin[e + f
*x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3055
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[(-k)*a*(b/f)   Subst[Int[x
^(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[
e + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m,
0] && LtQ[m, 1]
```

Maple [F]

$$\int \frac{\sin\left(bx+a\right)^{\frac{5}{3}}}{\cos\left(bx+a\right)^{\frac{5}{3}}}dx$$

input
```
int(sin(b*x+a)^(5/3)/cos(b*x+a)^(5/3),x)
```

output
```
int(sin(b*x+a)^(5/3)/cos(b*x+a)^(5/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.27

$$\int \frac{\sin^{\frac{5}{3}}(a+bx)}{\cos^{\frac{5}{3}}(a+bx)}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{2\sqrt{3}\cos(bx+a)^{\frac{2}{3}}\sin(bx+a)^{\frac{1}{3}}-\sqrt{3}\sin(bx+a)}{3\sin(bx+a)}\right)\cos\left(bx+a\right)+\cos\left(bx+a\right)\log\left(\frac{4\left(\cos(bx+a)^{2}-\cos(bx+a)^{\frac{4}{3}}\right.}{}\right.}{}$$

input
```
integrate(sin(b*x+a)^(5/3)/cos(b*x+a)^(5/3),x, algorithm="fricas")
```

output
```
1/4*(2*sqrt(3)*arctan(1/3*(2*sqrt(3)*cos(b*x + a)^(2/3)*sin(b*x + a)^(1/3)
 - sqrt(3)*sin(b*x + a))/sin(b*x + a))*cos(b*x + a) + cos(b*x + a)*log(4*(
cos(b*x + a)^2 - cos(b*x + a)^(4/3)*sin(b*x + a)^(2/3) + cos(b*x + a)^(2/3
)*sin(b*x + a)^(4/3) - 1)/(cos(b*x + a)^2 - 1)) - 2*cos(b*x + a)*log(-2*(c
os(b*x + a)^(2/3)*sin(b*x + a)^(1/3) + sin(b*x + a))/sin(b*x + a)) + 6*cos
(b*x + a)^(1/3)*sin(b*x + a)^(2/3))/(b*cos(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^{\frac{5}{3}}(a+bx)}{\cos^{\frac{5}{3}}(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**(5/3)/cos(b*x+a)**(5/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sin^{\frac{5}{3}}(a+bx)}{\cos^{\frac{5}{3}}(a+bx)}\,dx = \int \frac{\sin\left(bx+a\right)^{\frac{5}{3}}}{\cos\left(bx+a\right)^{\frac{5}{3}}}\,dx$$

input
```
integrate(sin(b*x+a)^(5/3)/cos(b*x+a)^(5/3),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^(5/3)/cos(b*x + a)^(5/3), x)
```

Giac [F]

$$\int \frac{\sin^{\frac{5}{3}}(a+bx)}{\cos^{\frac{5}{3}}(a+bx)}\,dx = \int \frac{\sin{(bx+a)}^{\frac{5}{3}}}{\cos{(bx+a)}^{\frac{5}{3}}}\,dx$$

input
```
integrate(sin(b*x+a)^(5/3)/cos(b*x+a)^(5/3),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^(5/3)/cos(b*x + a)^(5/3), x)
```

Mupad [B] (verification not implemented)

Time = 26.55 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.28

$$\int \frac{\sin^{\frac{5}{3}}(a+bx)}{\cos^{\frac{5}{3}}(a+bx)}\,dx = \frac{3\sin{(a+b\,x)}^{8/3}\,{}_2\mathrm{F}_1\left(-\frac{1}{3},-\frac{1}{3};\ \frac{2}{3};\ \cos{(a+b\,x)}^2\right)}{2\,b\cos{(a+b\,x)}^{2/3}\,\left(\sin{(a+b\,x)}^2\right)^{4/3}}$$

input
```
int(sin(a + b*x)^(5/3)/cos(a + b*x)^(5/3),x)
```

output
```
(3*sin(a + b*x)^(8/3)*hypergeom([-1/3, -1/3], 2/3, cos(a + b*x)^2))/(2*b*c
os(a + b*x)^(2/3)*(sin(a + b*x)^2)^(4/3))
```

Reduce [F]

$$\int \frac{\sin^{\frac{5}{3}}(a+bx)}{\cos^{\frac{5}{3}}(a+bx)}\,dx = \int \frac{\sin{(bx+a)}^{\frac{5}{3}}}{\cos{(bx+a)}^{\frac{5}{3}}}dx$$

input
```
int(sin(b*x+a)^(5/3)/cos(b*x+a)^(5/3),x)
```

output
```
int((sin(a + b*x)**(2/3)*sin(a + b*x))/(cos(a + b*x)**(2/3)*cos(a + b*x)),
x)
```

3.328 $\int \frac{\sin^{\frac{7}{3}}(a+bx)}{\cos^{\frac{7}{3}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 155

$$\int \frac{\sin^{\frac{7}{3}}(a+bx)}{\cos^{\frac{7}{3}}(a+bx)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} + \frac{\log\left(1+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{\log\left(1-\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+\frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\right)}{4b} + \frac{3\sin^{\frac{4}{3}}(a+bx)}{4b\cos^{\frac{4}{3}}(a+bx)}$$

output
```
1/2*3^(1/2)*arctan(1/3*(1-2*sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3))*3^(1/2))/b+
1/2*ln(1+sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3))/b-1/4*ln(1-sin(b*x+a)^(2/3)/co
s(b*x+a)^(2/3)+sin(b*x+a)^(4/3)/cos(b*x+a)^(4/3))/b+3/4*sin(b*x+a)^(4/3)/b
/cos(b*x+a)^(4/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.37

$$\int \frac{\sin^{\frac{7}{3}}(a+bx)}{\cos^{\frac{7}{3}}(a+bx)}\,dx = \frac{3\cos^2(a+bx)^{2/3}\,\text{Hypergeometric2F1}\left(\frac{5}{3},\frac{5}{3},\frac{8}{3},\sin^2(a+bx)\right)\sin^{\frac{10}{3}}(a+bx)}{10b\cos^{\frac{4}{3}}(a+bx)}$$

input
```
Integrate[Sin[a + b*x]^(7/3)/Cos[a + b*x]^(7/3),x]
```

output
```
(3*(Cos[a + b*x]^2)^(2/3)*Hypergeometric2F1[5/3, 5/3, 8/3, Sin[a + b*x]^2]
*Sin[a + b*x]^(10/3))/(10*b*Cos[a + b*x]^(4/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.65, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3046, 3042, 3054, 807, 821, 16, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^{\frac{7}{3}}(a+bx)}{\cos^{\frac{7}{3}}(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(a+bx)^{7/3}}{\cos(a+bx)^{7/3}}dx$$

$$\Big\downarrow \text{3046}$$

$$\frac{3\sin^{\frac{4}{3}}(a+bx)}{4b\cos^{\frac{4}{3}}(a+bx)} - \int \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\sin^{\frac{4}{3}}(a+bx)}{4b\cos^{\frac{4}{3}}(a+bx)}-\int\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}dx$$

$$\Big\downarrow \text{3054}$$

$$\frac{3\sin^{\frac{4}{3}}(a+bx)}{4b\cos^{\frac{4}{3}}(a+bx)}-\frac{3\int\frac{\tan(a+bx)}{\tan^2(a+bx)+1}d\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}}{b}$$

$$\Big\downarrow \text{807}$$

$$\frac{3\sin^{\frac{4}{3}}(a+bx)}{4b\cos^{\frac{4}{3}}(a+bx)}-\frac{3\int\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)(\tan(a+bx)+1)}d\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}}{2b}$$

$$\Big\downarrow \text{821}$$

$$\frac{3\sin^{\frac{4}{3}}(a+bx)}{4b\cos^{\frac{4}{3}}(a+bx)}-\frac{3\left(\frac{1}{3}\int\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)d\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-\frac{1}{3}\int\frac{1}{\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1}d\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$\Big\downarrow \text{16}$$

$$\frac{3\sin^{\frac{4}{3}}(a+bx)}{4b\cos^{\frac{4}{3}}(a+bx)}-\frac{3\left(\frac{1}{3}\int\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)d\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-\frac{1}{3}\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)\right)}{2b}$$

$$\Big\downarrow \text{1142}$$

$$\frac{3\sin^{\frac{4}{3}}(a+bx)}{4b\cos^{\frac{4}{3}}(a+bx)}-\frac{3\left(\frac{1}{3}\left(\frac{3}{2}\int 1d\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+\frac{1}{2}\int\left(\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-1\right)d\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)-\frac{1}{3}\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)\right)}{2b}$$

$$\Big\downarrow \text{25}$$

$$\frac{3\sin^{\frac{4}{3}}(a+bx)}{4b\cos^{\frac{4}{3}}(a+bx)}-\frac{3\left(\frac{1}{3}\left(\frac{3}{2}\int 1d\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-\frac{1}{2}\int\left(1-\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)d\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)-\frac{1}{3}\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)\right)}{2b}$$

$$\Big\downarrow \text{1083}$$

$$\frac{3\sin^{\frac{4}{3}}(a+bx)}{4b\cos^{\frac{4}{3}}(a+bx)}-\frac{3\left(\frac{1}{3}\left(-3\int\frac{1}{-\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-2}d\left(\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-1\right)-\frac{1}{2}\int\left(1-\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)d\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)-\frac{1}{3}\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)\right)}{2b}$$

$$\Big\downarrow \text{217}$$

$$\frac{3\sin^{\frac{4}{3}}(a+bx)}{4b\cos^{\frac{4}{3}}(a+bx)}-\frac{3\left(\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-1}{\sqrt{3}}\right)-\frac{1}{2}\int\left(1-\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)d\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)-\frac{1}{3}\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)\right)}{2b}$$

$$\Big\downarrow \text{1103}$$

$$\frac{3\sin^{\frac{4}{3}}(a+bx)}{4b\cos^{\frac{4}{3}}(a+bx)}-\frac{3\left(\frac{\arctan\left(\frac{\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{3}\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)\right)}{2b}$$

input
```
Int[Sin[a + b*x]^(7/3)/Cos[a + b*x]^(7/3),x]
```

output
```
(-3*(ArcTan[(-1 + (2*Sin[a + b*x]^(2/3))/Cos[a + b*x]^(2/3))/Sqrt[3]]/Sqrt
[3] - Log[1 + Sin[a + b*x]^(2/3)/Cos[a + b*x]^(2/3)]/3))/(2*b) + (3*Sin[a
+ b*x]^(4/3))/(4*b*Cos[a + b*x]^(4/3))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3046
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Sin[e + f
*x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3054
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k*a*(b/f)   Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Maple [F]

$$\int \frac{\sin{(bx+a)}^{\frac{7}{3}}}{\cos{(bx+a)}^{\frac{7}{3}}}dx$$

input
```
int(sin(b*x+a)^(7/3)/cos(b*x+a)^(7/3),x)
```

output
```
int(sin(b*x+a)^(7/3)/cos(b*x+a)^(7/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.26

$$\int \frac{\sin^{\frac{7}{3}}(a+bx)}{\cos^{\frac{7}{3}}(a+bx)}\,dx = -\frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}\cos(bx+a)-2\sqrt{3}\cos(bx+a)^{\frac{1}{3}}\sin(bx+a)^{\frac{2}{3}}}{3\cos(bx+a)}\right)\cos{(bx+a)}^2-2\cos{(bx+a)}^2\log\left(\frac{\cos(bx+a)^{\frac{1}{3}}\sin(bx}{\cos(b}\right.}{}$$

input
```
integrate(sin(b*x+a)^(7/3)/cos(b*x+a)^(7/3),x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(3)*arctan(-1/3*(sqrt(3)*cos(b*x + a) - 2*sqrt(3)*cos(b*x + a)
^(1/3)*sin(b*x + a)^(2/3))/cos(b*x + a))*cos(b*x + a)^2 - 2*cos(b*x + a)^2
*log((cos(b*x + a)^(1/3)*sin(b*x + a)^(2/3) + cos(b*x + a))/cos(b*x + a))
+ cos(b*x + a)^2*log((cos(b*x + a)^2 - cos(b*x + a)^(4/3)*sin(b*x + a)^(2/
3) + cos(b*x + a)^(2/3)*sin(b*x + a)^(4/3))/cos(b*x + a)^2) - 3*cos(b*x +
a)^(2/3)*sin(b*x + a)^(4/3))/(b*cos(b*x + a)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^{\frac{7}{3}}(a+bx)}{\cos^{\frac{7}{3}}(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(sin(b*x+a)**(7/3)/cos(b*x+a)**(7/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sin^{\frac{7}{3}}(a+bx)}{\cos^{\frac{7}{3}}(a+bx)}\,dx = \int \frac{\sin\left(bx+a\right)^{\frac{7}{3}}}{\cos\left(bx+a\right)^{\frac{7}{3}}}\,dx$$

input
```
integrate(sin(b*x+a)^(7/3)/cos(b*x+a)^(7/3),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)^(7/3)/cos(b*x + a)^(7/3), x)
```

Giac [F]

$$\int \frac{\sin^{\frac{7}{3}}(a+bx)}{\cos^{\frac{7}{3}}(a+bx)}\,dx = \int \frac{\sin\left(bx+a\right)^{\frac{7}{3}}}{\cos\left(bx+a\right)^{\frac{7}{3}}}\,dx$$

input
```
integrate(sin(b*x+a)^(7/3)/cos(b*x+a)^(7/3),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^(7/3)/cos(b*x + a)^(7/3), x)
```

Mupad [B] (verification not implemented)

Time = 27.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.28

$$\int \frac{\sin^{\frac{7}{3}}(a+bx)}{\cos^{\frac{7}{3}}(a+bx)}\,dx = \frac{3\sin\left(a+b\,x\right)^{10/3}\,{}_2\mathrm{F}_1\left(-\frac{2}{3},-\frac{2}{3};\ \frac{1}{3};\ \cos\left(a+b\,x\right)^2\right)}{4\,b\cos\left(a+b\,x\right)^{4/3}\,\left(\sin\left(a+b\,x\right)^2\right)^{5/3}}$$

input
```
int(sin(a + b*x)^(7/3)/cos(a + b*x)^(7/3),x)
```

output
```
(3*sin(a + b*x)^(10/3)*hypergeom([-2/3, -2/3], 1/3, cos(a + b*x)^2))/(4*b*
cos(a + b*x)^(4/3)*(sin(a + b*x)^2)^(5/3))
```

Reduce [F]

$$\int \frac{\sin^{\frac{7}{3}}(a+bx)}{\cos^{\frac{7}{3}}(a+bx)}\,dx = \int \frac{\sin\left(bx+a\right)^{\frac{7}{3}}}{\cos\left(bx+a\right)^{\frac{7}{3}}}dx$$

input
```
int(sin(b*x+a)^(7/3)/cos(b*x+a)^(7/3),x)
```

output
```
int((sin(a + b*x)**(1/3)*sin(a + b*x)**2)/(cos(a + b*x)**(1/3)*cos(a + b*x
)**2),x)
```

3.329 $\int \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 128

$$\int \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1+\frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}-\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{4b} + \frac{\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

output
```
1/2*3^(1/2)*arctan(1/3*(1-2*cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3))*3^(1/2))/b-
1/4*ln(1+cos(b*x+a)^(4/3)/sin(b*x+a)^(4/3)-cos(b*x+a)^(2/3)/sin(b*x+a)^(2/
3))/b+1/2*ln(1+cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3))/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.45

$$\int \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\,dx = \frac{3\sqrt[3]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{3},\frac{4}{3},\sin^2(a+bx)\right)\sin^{\frac{2}{3}}(a+bx)}{2b\cos^{\frac{2}{3}}(a+bx)}$$

input
```
Integrate[Cos[a + b*x]^(1/3)/Sin[a + b*x]^(1/3),x]
```

output
```
(3*(Cos[a + b*x]^2)^(1/3)*Hypergeometric2F1[1/3, 1/3, 4/3, Sin[a + b*x]^2]
*Sin[a + b*x]^(2/3))/(2*b*Cos[a + b*x]^(2/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.57, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3055, 807, 821, 16, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}dx$$

$$\Big\downarrow \text{3055}$$

$$-\frac{3\int \frac{\cot(a+bx)}{\cot^2(a+bx)+1}d\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}}{b}$$

$$\Big\downarrow \text{807}$$

$$-\frac{3\int \frac{\cos^{\frac{2}{3}}(a+bx)}{(\cot(a+bx)+1)\sin^{\frac{2}{3}}(a+bx)}d\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}}{2b}$$

$$\Big\downarrow \; 821$$

$$-\frac{3\left(\frac{1}{3}\int \left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)d\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-\frac{1}{3}\int \frac{1}{\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1}d\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$\Big\downarrow \; 16$$

$$-\frac{3\left(\frac{1}{3}\int \left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)d\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-\frac{1}{3}\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)\right)}{2b}$$

$$\Big\downarrow \; 1142$$

$$-\frac{3\left(\frac{1}{3}\left(\frac{3}{2}\int 1d\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+\frac{1}{2}\int \left(\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-1\right)d\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)-\frac{1}{3}\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)\right)}{2b}$$

$$\Big\downarrow \; 25$$

$$-\frac{3\left(\frac{1}{3}\left(\frac{3}{2}\int 1d\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-\frac{1}{2}\int \left(1-\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)d\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)-\frac{1}{3}\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)\right)}{2b}$$

$$\Big\downarrow \; 1083$$

$$-\frac{3\left(\frac{1}{3}\left(-3\int \frac{1}{-\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-2}d\left(\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-1\right)-\frac{1}{2}\int \left(1-\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)d\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)-\frac{1}{3}\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)\right)}{2b}$$

$$\Big\downarrow \; 217$$

$$-\frac{3\left(\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-1}{\sqrt{3}}\right)-\frac{1}{2}\int \left(1-\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)d\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)-\frac{1}{3}\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)\right)}{2b}$$

$$\Big\downarrow \; 1103$$

$$-\frac{3\left(\frac{\arctan\left(\frac{\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{3}\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)\right)}{2b}$$

input
```
Int[Cos[a + b*x]^(1/3)/Sin[a + b*x]^(1/3),x]
```

output
```
(-3*(ArcTan[(-1 + (2*Cos[a + b*x]^(2/3))/Sin[a + b*x]^(2/3))/Sqrt[3]]/Sqrt
[3] - Log[1 + Cos[a + b*x]^(2/3)/Sin[a + b*x]^(2/3)]/3))/(2*b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3055
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[(-k)*a*(b/f)   Subst[Int[x
^(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[
e + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m,
0] && LtQ[m, 1]
```

Maple [F]

$$\int \frac{\cos (bx+a)^{\frac{1}{3}}}{\sin (bx+a)^{\frac{1}{3}}}dx$$

input
```
int(cos(b*x+a)^(1/3)/sin(b*x+a)^(1/3),x)
```

output
```
int(cos(b*x+a)^(1/3)/sin(b*x+a)^(1/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{2\sqrt{3}\cos(bx+a)^{\frac{2}{3}}\sin(bx+a)^{\frac{1}{3}}-\sqrt{3}\sin(bx+a)}{3\sin(bx+a)}\right)+\log\left(\frac{4\left(\cos(bx+a)^2-\cos(bx+a)^{\frac{4}{3}}\sin(bx+a)^{\frac{2}{3}}+\cos(bx+a)^{\frac{2}{3}}\sin(b\right.}{\cos(bx+a)^2-1}\right.}{4b}$$

input
```
integrate(cos(b*x+a)^(1/3)/sin(b*x+a)^(1/3),x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(3)*arctan(1/3*(2*sqrt(3)*cos(b*x + a)^(2/3)*sin(b*x + a)^(1/3
) - sqrt(3)*sin(b*x + a))/sin(b*x + a)) + log(4*(cos(b*x + a)^2 - cos(b*x
+ a)^(4/3)*sin(b*x + a)^(2/3) + cos(b*x + a)^(2/3)*sin(b*x + a)^(4/3) - 1)
/(cos(b*x + a)^2 - 1)) - 2*log(-2*(cos(b*x + a)^(2/3)*sin(b*x + a)^(1/3) +
 sin(b*x + a))/sin(b*x + a)))/b
```

Sympy [F]

$$\int \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\,dx = \int \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\,dx$$

input
```
integrate(cos(b*x+a)**(1/3)/sin(b*x+a)**(1/3),x)
```

output
```
Integral(cos(a + b*x)**(1/3)/sin(a + b*x)**(1/3), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\,dx = \int \frac{\cos\left(bx+a\right)^{\frac{1}{3}}}{\sin\left(bx+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(b*x+a)^(1/3)/sin(b*x+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^(1/3)/sin(b*x + a)^(1/3), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\,dx = \int \frac{\cos\left(bx+a\right)^{\frac{1}{3}}}{\sin\left(bx+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(b*x+a)^(1/3)/sin(b*x+a)^(1/3),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^(1/3)/sin(b*x + a)^(1/3), x)
```

Mupad [B] (verification not implemented)

Time = 26.88 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.34

$$\int \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\,dx = -\frac{3\cos\left(a+b\,x\right)^{4/3}\,\sin\left(a+b\,x\right)^{2/3}\,{}_2\mathrm{F}_1\left(\frac{2}{3},\frac{2}{3};\ \frac{5}{3};\ \cos\left(a+b\,x\right)^2\right)}{4\,b\,{\left(\sin\left(a+b\,x\right)^2\right)}^{1/3}}$$

input
```
int(cos(a + b*x)^(1/3)/sin(a + b*x)^(1/3),x)
```

output
```
-(3*cos(a + b*x)^(4/3)*sin(a + b*x)^(2/3)*hypergeom([2/3, 2/3], 5/3, cos(a
 + b*x)^2))/(4*b*(sin(a + b*x)^2)^(1/3))
```

Reduce [F]

$$\int \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\,dx = \int \frac{\cos(bx+a)^{\frac{1}{3}}}{\sin(bx+a)^{\frac{1}{3}}}dx$$

input
```
int(cos(b*x+a)^(1/3)/sin(b*x+a)^(1/3),x)
```

output
```
int(cos(a + b*x)**(1/3)/sin(a + b*x)**(1/3),x)
```

3.330 $\int \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 164

$$\int \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\,dx = \frac{\arctan\left(\sqrt{3}-\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{2b} - \frac{\arctan\left(\sqrt{3}+\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{2b} - \frac{\arctan\left(\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b} + \frac{\sqrt{3}\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)\sqrt[3]{\sin(a+bx)}}\right)}{2b}$$

output
```
-1/2*arctan(-3^(1/2)+2*cos(b*x+a)^(1/3)/sin(b*x+a)^(1/3))/b-1/2*arctan(3^(
1/2)+2*cos(b*x+a)^(1/3)/sin(b*x+a)^(1/3))/b-arctan(cos(b*x+a)^(1/3)/sin(b*
x+a)^(1/3))/b+1/2*3^(1/2)*arctanh(3^(1/2)*cos(b*x+a)^(1/3)/(1+cos(b*x+a)^(
2/3)/sin(b*x+a)^(2/3))/sin(b*x+a)^(1/3))/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.34

$$\int \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\,dx = \frac{3\sqrt[6]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{6},\frac{7}{6},\sin^2(a+bx)\right)\sqrt[3]{\sin(a+bx)}}{b\sqrt[3]{\cos(a+bx)}}$$

input
```
Integrate[Cos[a + b*x]^(2/3)/Sin[a + b*x]^(2/3),x]
```

output
```
(3*(Cos[a + b*x]^2)^(1/6)*Hypergeometric2F1[1/6, 1/6, 7/6, Sin[a + b*x]^2]
*Sin[a + b*x]^(1/3))/(b*Cos[a + b*x]^(1/3))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.35, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3055, 824, 27, 216, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)^{2/3}}{\sin(a+bx)^{2/3}}dx$$

$$\Big\downarrow \text{3055}$$

$$-\frac{3\int \frac{\cos^{\frac{4}{3}}(a+bx)}{(\cot^2(a+bx)+1)\sin^{\frac{4}{3}}(a+bx)}d\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}}{b}$$

$$\Big\downarrow \text{824}$$

$$-\frac{3\left(\frac{1}{3}\int \frac{1}{\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1}d\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+\frac{1}{3}\int -\frac{1-\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}}{2\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1\right)}d\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+\frac{1}{3}\int -\frac{}{2\left(\frac{\cos^{\frac{2}{3}}(a\ldots}{\sin^{\frac{2}{3}}(a\ldots}\right.}\right.}{b}$$

$\downarrow$ 27

$$-\frac{3\left(\frac{1}{3}\int \frac{1}{\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1}d\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}-\frac{1}{6}\int \frac{1-\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}}{\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1}d\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}-\frac{1}{6}\int \frac{\frac{\sqrt{3}\sqrt[3]{\cos(a\ldots}}{\sqrt[3]{\sin(a+\ldots}}}{\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+\frac{\sqrt{3}\sqrt[3]{\ldots}}{\sqrt[3]{\ldots}}}\right.}{b}$$

$\downarrow$ 216

$$-\frac{3\left(-\frac{1}{6}\int \frac{1-\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}}{\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1}d\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}-\frac{1}{6}\int \frac{\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1}{\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1}d\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+\right.}{b}$$

$\downarrow$ 1142

$$-\frac{3\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1}d\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+\frac{1}{2}\sqrt{3}\int -\frac{\sqrt{3}-\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}}{\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1}d\frac{\sqrt[3]{\cos(a+\ldots}}{\sqrt[3]{\sin(a+\ldots}}\right.\right.}{}$$

$\downarrow$ 25

$$-\frac{3\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1}d\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}}{\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1}d\frac{\sqrt[3]{\cos(a+b\ldots}}{\sqrt[3]{\sin(a+b\ldots}}\right.\right.}{}$$

$\downarrow$ 1083

$$-\frac{3\left(\frac{1}{6}\left(\left(-\int \frac{1}{-\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-1}d\left(\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}-\sqrt{3}\right)-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}}{\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1}d\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)+\right.\right.}{}$$

$$\Big\downarrow \text{217}$$

$$-\frac{3\left(\frac{1}{6}\left(\left(-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}}{\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1}d\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}-\arctan\left(\sqrt{3}-\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)\right)+\frac{1}{6}\left(\text{arcta}\right.\right.\right.}{}$$

$$\Big\downarrow \text{1103}$$

$$-\frac{3\left(\frac{1}{3}\arctan\left(\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)+\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1\right)-\arctan\left(\sqrt{3}-\frac{2\sqrt[3]{\cos(a+}}{\sqrt[3]{\sin(a+}}\right.\right.\right.}{b}$$

input
```
Int[Cos[a + b*x]^(2/3)/Sin[a + b*x]^(2/3),x]
```

output
```
(-3*(ArcTan[Cos[a + b*x]^(1/3)/Sin[a + b*x]^(1/3)]/3 + (-ArcTan[Sqrt[3] -
(2*Cos[a + b*x]^(1/3))/Sin[a + b*x]^(1/3)] + (Sqrt[3]*Log[1 + Cos[a + b*x]
^(2/3)/Sin[a + b*x]^(2/3) - (Sqrt[3]*Cos[a + b*x]^(1/3))/Sin[a + b*x]^(1/3
)]])/2)/6 + (ArcTan[Sqrt[3] + (2*Cos[a + b*x]^(1/3))/Sin[a + b*x]^(1/3)] -
(Sqrt[3]*Log[1 + Cos[a + b*x]^(2/3)/Sin[a + b*x]^(2/3) + (Sqrt[3]*Cos[a +
b*x]^(1/3))/Sin[a + b*x]^(1/3)])/2)/6))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3055
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[(-k)*a*(b/f)   Subst[Int[x
^(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[
e + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m,
0] && LtQ[m, 1]
```

Maple [F]

$$\int \frac{\cos\left(bx+a\right)^{\frac{2}{3}}}{\sin\left(bx+a\right)^{\frac{2}{3}}}dx$$

input
```
int(cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3),x)
```

output
```
int(cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(132) = 264.

Time = 0.10 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.62

$$\int \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\,dx = \frac{\sqrt{3}b\sqrt{\frac{1}{b^2}}\log\left(\frac{\sqrt{3}b\sqrt{\frac{1}{b^2}}\cos(bx+a)^{\frac{2}{3}}\sin(bx+a)^{\frac{1}{3}}+\cos(bx+a)^{\frac{1}{3}}\sin(bx+a)^{\frac{2}{3}}+\cos(bx+a)}{\cos(bx+a)}\right)-\sqrt{3}b\sqrt{\frac{1}{b^2}}\log\left(-\frac{\sqrt{3}b\sqrt{\frac{1}{b^2}}\cos(bx+a)^{\frac{2}{3}}}{}\right.}{}$$

input
```
integrate(cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3),x, algorithm="fricas")
```

output
```
1/4*(sqrt(3)*b*sqrt(b^(-2))*log((sqrt(3)*b*sqrt(b^(-2))*cos(b*x + a)^(2/3)
*sin(b*x + a)^(1/3) + cos(b*x + a)^(1/3)*sin(b*x + a)^(2/3) + cos(b*x + a)
)/cos(b*x + a)) - sqrt(3)*b*sqrt(b^(-2))*log(-(sqrt(3)*b*sqrt(b^(-2))*cos(
b*x + a)^(2/3)*sin(b*x + a)^(1/3) - cos(b*x + a)^(1/3)*sin(b*x + a)^(2/3)
- cos(b*x + a))/cos(b*x + a)) + 2*arctan((sqrt(3)*b*sqrt(b^(-2))*cos(b*x +
 a) + 2*cos(b*x + a)^(2/3)*sin(b*x + a)^(1/3))/cos(b*x + a)) + 2*arctan(-(
sqrt(3)*b*sqrt(b^(-2))*cos(b*x + a) - 2*cos(b*x + a)^(2/3)*sin(b*x + a)^(1
/3))/cos(b*x + a)) + 4*arctan(sin(b*x + a)^(1/3)/cos(b*x + a)^(1/3)))/b
```

Sympy [F]

$$\int \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\,dx = \int \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\,dx$$

input
```
integrate(cos(b*x+a)**(2/3)/sin(b*x+a)**(2/3),x)
```

output
```
Integral(cos(a + b*x)**(2/3)/sin(a + b*x)**(2/3), x)
```

Maxima [F]

$$\int \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\,dx = \int \frac{\cos(bx+a)^{\frac{2}{3}}}{\sin(bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^(2/3)/sin(b*x + a)^(2/3), x)
```

Giac [F]

$$\int \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\,dx = \int \frac{\cos(bx+a)^{\frac{2}{3}}}{\sin(bx+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^(2/3)/sin(b*x + a)^(2/3), x)
```

Mupad [B] (verification not implemented)

Time = 26.54 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.27

$$\int \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\,dx = -\frac{3\cos(a+b\,x)^{5/3}\sin(a+b\,x)^{1/3}\,{}_2F_1\left(\frac{5}{6},\frac{5}{6};\ \frac{11}{6};\ \cos(a+b\,x)^2\right)}{5\,b\left(\sin(a+b\,x)^2\right)^{1/6}}$$

input
```
int(cos(a + b*x)^(2/3)/sin(a + b*x)^(2/3),x)
```

output
```
-(3*cos(a + b*x)^(5/3)*sin(a + b*x)^(1/3)*hypergeom([5/6, 5/6], 11/6, cos(
a + b*x)^2))/(5*b*(sin(a + b*x)^2)^(1/6))
```

Reduce [F]

$$\int \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\,dx = \int \frac{\cos(bx+a)^{\frac{2}{3}}}{\sin(bx+a)^{\frac{2}{3}}}dx$$

input
```
int(cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3),x)
```

output
```
int(cos(a + b*x)**(2/3)/sin(a + b*x)**(2/3),x)
```

3.331 $\int \frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 189

$$\int \frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}\,dx = \frac{\arctan\left(\sqrt{3}-\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{2b} - \frac{\arctan\left(\sqrt{3}+\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{2b} - \frac{\arctan\left(\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b} + \frac{\sqrt{3}\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}\left(1+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}\right)}{2b} - \frac{3\sqrt[3]{\cos(a+bx)}}{b\sqrt[3]{\sin(a+bx)}}$$

output
```
-1/2*arctan(-3^(1/2)+2*sin(b*x+a)^(1/3)/cos(b*x+a)^(1/3))/b-1/2*arctan(3^(
1/2)+2*sin(b*x+a)^(1/3)/cos(b*x+a)^(1/3))/b-arctan(sin(b*x+a)^(1/3)/cos(b*
x+a)^(1/3))/b+1/2*3^(1/2)*arctanh(3^(1/2)*sin(b*x+a)^(1/3)/cos(b*x+a)^(1/3
)/(1+sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3)))/b-3*cos(b*x+a)^(1/3)/b/sin(b*x+a)
^(1/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.29

$$\int \frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}\,dx = -\frac{3\cos^2(a+bx)^{5/6}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},-\frac{1}{6},\frac{5}{6},\sin^2(a+bx)\right)}{b\cos^{\frac{5}{3}}(a+bx)\sqrt[3]{\sin(a+bx)}}$$

input
```
Integrate[Cos[a + b*x]^(4/3)/Sin[a + b*x]^(4/3),x]
```

output
```
(-3*(Cos[a + b*x]^2)^(5/6)*Hypergeometric2F1[-1/6, -1/6, 5/6, Sin[a + b*x]
^2])/(b*Cos[a + b*x]^(5/3)*Sin[a + b*x]^(1/3))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.31, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3047, 3042, 3054, 824, 27, 216, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)^{4/3}}{\sin(a+bx)^{4/3}}dx$$

$$\Big\downarrow \text{3047}$$

$$-\int \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}dx-\frac{3\sqrt[3]{\cos(a+bx)}}{b\sqrt[3]{\sin(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{\sin(a+bx)^{2/3}}{\cos(a+bx)^{2/3}}dx-\frac{3\sqrt[3]{\cos(a+bx)}}{b\sqrt[3]{\sin(a+bx)}}$$

$\downarrow$ 3054

$$-\frac{3\int \frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)(\tan^2(a+bx)+1)}d\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}}{b}-\frac{3\sqrt[3]{\cos(a+bx)}}{b\sqrt[3]{\sin(a+bx)}}$$

$\downarrow$ 824

$$-\frac{3\left(\frac{1}{3}\int \frac{1}{\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1}d\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+\frac{1}{3}\int -\frac{1-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}}{2\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1\right)}d\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+\frac{1}{3}\int -\frac{\sqrt{}}{2\left(\frac{\sin^{\frac{2}{3}}(a+}{\cos^{\frac{2}{3}}(a+}\right.}\right.}{b}$$

$$-\frac{3\sqrt[3]{\cos(a+bx)}}{b\sqrt[3]{\sin(a+bx)}}$$

$\downarrow$ 27

$$-\frac{3\left(\frac{1}{3}\int \frac{1}{\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1}d\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}-\frac{1}{6}\int \frac{1-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}}{\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1}d\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}-\frac{1}{6}\int \frac{\frac{\sqrt{3}\sqrt[3]{\sin(a}}{\sqrt[3]{\cos(a+}}}{\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+\frac{\sqrt{3}\sqrt[3]{s}}{\sqrt[3]{co}}}\right.}{b}$$

$$-\frac{3\sqrt[3]{\cos(a+bx)}}{b\sqrt[3]{\sin(a+bx)}}$$

$\downarrow$ 216

$$-\frac{3\left(-\frac{1}{6}\int \frac{1-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}}{\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1}d\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}-\frac{1}{6}\int \frac{\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1}{\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1}d\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+\right.}{b}$$

$$-\frac{3\sqrt[3]{\cos(a+bx)}}{b\sqrt[3]{\sin(a+bx)}}$$

$\downarrow$ 1142

$$-\frac{3\left(\frac{1}{6}\left(\left(\frac{1}{2}\int \frac{1}{\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1}d\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+\frac{1}{2}\sqrt{3}\int -\frac{\sqrt{3}-\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}}{\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1}d\frac{\sqrt[3]{\sin(a+\cdots}}{\sqrt[3]{\cos(a+\cdots}}\cdots\right.\right.\right.}{\frac{3\sqrt[3]{\cos(a+bx)}}{b\sqrt[3]{\sin(a+bx)}}}$$

$$\Big\downarrow \; 25$$

$$-\frac{3\left(\frac{1}{6}\left(\left(\frac{1}{2}\int \frac{1}{\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1}d\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}}{\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1}d\frac{\sqrt[3]{\sin(a+bx\cdots}}{\sqrt[3]{\cos(a+bx\cdots}}\cdots\right.\right.\right.}{\frac{3\sqrt[3]{\cos(a+bx)}}{b\sqrt[3]{\sin(a+bx)}}}$$

$$\Big\downarrow \; 1083$$

$$-\frac{3\left(\frac{1}{6}\left(-\int \frac{1}{-\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-1}d\left(\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}-\sqrt{3}\right)-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}}{\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1}d\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)+\cdots\right.}{\frac{3\sqrt[3]{\cos(a+bx)}}{b\sqrt[3]{\sin(a+bx)}}}$$

$$\Big\downarrow \; 217$$

$$-\frac{3\left(\frac{1}{6}\left(-\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}-\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}}{\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1}d\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}-\arctan\left(\sqrt{3}-\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)\right)+\frac{1}{6}\left(\arctan\cdots\right.\right.}{\frac{3\sqrt[3]{\cos(a+bx)}}{b\sqrt[3]{\sin(a+bx)}}}$$

$$\Big\downarrow \; 1103$$

$$-\frac{3\left(\frac{1}{3}\arctan\left(\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)+\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1\right)-\arctan\left(\sqrt{3}-\frac{2\sqrt[3]{\sin(a+}}{\sqrt[3]{\cos(a+}}\right.\right.\right.}{b}$$

$$\frac{3\sqrt[3]{\cos(a+bx)}}{b\sqrt[3]{\sin(a+bx)}}$$

input
```
Int[Cos[a + b*x]^(4/3)/Sin[a + b*x]^(4/3),x]
```

output
```
(-3*(ArcTan[Sin[a + b*x]^(1/3)/Cos[a + b*x]^(1/3)]/3 + (-ArcTan[Sqrt[3] -
(2*Sin[a + b*x]^(1/3))/Cos[a + b*x]^(1/3)] + (Sqrt[3]*Log[1 - (Sqrt[3]*Sin
[a + b*x]^(1/3))/Cos[a + b*x]^(1/3) + Sin[a + b*x]^(2/3)/Cos[a + b*x]^(2/3
)])/2)/6 + (ArcTan[Sqrt[3] + (2*Sin[a + b*x]^(1/3))/Cos[a + b*x]^(1/3)] -
(Sqrt[3]*Log[1 + (Sqrt[3]*Sin[a + b*x]^(1/3))/Cos[a + b*x]^(1/3) + Sin[a +
 b*x]^(2/3)/Cos[a + b*x]^(2/3)])/2)/6))/b - (3*Cos[a + b*x]^(1/3))/(b*Sin[
a + b*x]^(1/3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3047
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Cos[e + f*x]
)^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3054
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k*a*(b/f)   Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Maple [F]

$$\int \frac{\cos(bx+a)^{\frac{4}{3}}}{\sin(bx+a)^{\frac{4}{3}}}dx$$

input
```
int(cos(b*x+a)^(4/3)/sin(b*x+a)^(4/3),x)
```

output
```
int(cos(b*x+a)^(4/3)/sin(b*x+a)^(4/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(153) = 306.

Time = 0.12 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.71

$$\int \frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}\,dx = \frac{\sqrt{3}b\sqrt{\frac{1}{b^2}}\log\left(\frac{2\left(\sqrt{3}b\sqrt{\frac{1}{b^2}}\cos(bx+a)^{\frac{1}{3}}\sin(bx+a)^{\frac{2}{3}}+\cos(bx+a)^{\frac{2}{3}}\sin(bx+a)^{\frac{1}{3}}+\sin(bx+a)\right)}{\sin(bx+a)}\right)\sin(bx+a)-\sqrt{3}b\sqrt{\frac{1}{b^2}}\log\left(-\right.}{}$$

input
```
integrate(cos(b*x+a)^(4/3)/sin(b*x+a)^(4/3),x, algorithm="fricas")
```

output
```
1/4*(sqrt(3)*b*sqrt(b^(-2))*log(2*(sqrt(3)*b*sqrt(b^(-2))*cos(b*x + a)^(1/
3)*sin(b*x + a)^(2/3) + cos(b*x + a)^(2/3)*sin(b*x + a)^(1/3) + sin(b*x +
a))/sin(b*x + a))*sin(b*x + a) - sqrt(3)*b*sqrt(b^(-2))*log(-2*(sqrt(3)*b*
sqrt(b^(-2))*cos(b*x + a)^(1/3)*sin(b*x + a)^(2/3) - cos(b*x + a)^(2/3)*si
n(b*x + a)^(1/3) - sin(b*x + a))/sin(b*x + a))*sin(b*x + a) + 2*arctan((sq
rt(3)*b*sqrt(b^(-2))*sin(b*x + a) + 2*cos(b*x + a)^(1/3)*sin(b*x + a)^(2/3
))/sin(b*x + a))*sin(b*x + a) + 2*arctan(-(sqrt(3)*b*sqrt(b^(-2))*sin(b*x
+ a) - 2*cos(b*x + a)^(1/3)*sin(b*x + a)^(2/3))/sin(b*x + a))*sin(b*x + a)
 + 4*arctan(cos(b*x + a)^(1/3)/sin(b*x + a)^(1/3))*sin(b*x + a) - 12*cos(b
*x + a)^(1/3)*sin(b*x + a)^(2/3))/(b*sin(b*x + a))
```

Sympy [F]

$$\int \frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}\,dx$$

input
```
integrate(cos(b*x+a)**(4/3)/sin(b*x+a)**(4/3),x)
```

output
```
Integral(cos(a + b*x)**(4/3)/sin(a + b*x)**(4/3), x)
```

Maxima [F]

$$\int \frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{\cos(bx+a)^{\frac{4}{3}}}{\sin(bx+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(cos(b*x+a)^(4/3)/sin(b*x+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^(4/3)/sin(b*x + a)^(4/3), x)
```

Giac [F]

$$\int \frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{\cos\left(bx+a\right)^{\frac{4}{3}}}{\sin\left(bx+a\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(cos(b*x+a)^(4/3)/sin(b*x+a)^(4/3),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^(4/3)/sin(b*x + a)^(4/3), x)
```

Mupad [B] (verification not implemented)

Time = 27.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.23

$$\int \frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}\,dx = -\frac{3\cos\left(a+b\,x\right)^{7/3}\,\left(\sin\left(a+b\,x\right)^2\right)^{1/6}\,{}_2\mathrm{F}_1\left(\frac{7}{6},\frac{7}{6};\ \frac{13}{6};\ \cos\left(a+b\,x\right)^2\right)}{7\,b\sin\left(a+b\,x\right)^{1/3}}$$

input
```
int(cos(a + b*x)^(4/3)/sin(a + b*x)^(4/3),x)
```

output
```
-(3*cos(a + b*x)^(7/3)*(sin(a + b*x)^2)^(1/6)*hypergeom([7/6, 7/6], 13/6,
cos(a + b*x)^2))/(7*b*sin(a + b*x)^(1/3))
```

Reduce [F]

$$\int \frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}\,dx = \int \frac{\cos\left(bx+a\right)^{\frac{4}{3}}}{\sin\left(bx+a\right)^{\frac{4}{3}}}dx$$

input
```
int(cos(b*x+a)^(4/3)/sin(b*x+a)^(4/3),x)
```

output
```
int((cos(a + b*x)**(1/3)*cos(a + b*x))/(sin(a + b*x)**(1/3)*sin(a + b*x)),
x)
```

3.332 $\int \frac{\cos^{\frac{5}{3}}(a+bx)}{\sin^{\frac{5}{3}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 155

$$\int \frac{\cos^{\frac{5}{3}}(a+bx)}{\sin^{\frac{5}{3}}(a+bx)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} + \frac{\log\left(1+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{\log\left(1-\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+\frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\right)}{4b} - \frac{3\cos^{\frac{2}{3}}(a+bx)}{2b\sin^{\frac{2}{3}}(a+bx)}$$

output

```
1/2*3^(1/2)*arctan(1/3*(1-2*sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3))*3^(1/2))/b+
1/2*ln(1+sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3))/b-1/4*ln(1-sin(b*x+a)^(2/3)/co
s(b*x+a)^(2/3)+sin(b*x+a)^(4/3)/cos(b*x+a)^(4/3))/b-3/2*cos(b*x+a)^(2/3)/b
/sin(b*x+a)^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.37

$$\int \frac{\cos^{\frac{5}{3}}(a+bx)}{\sin^{\frac{5}{3}}(a+bx)}\,dx = -\frac{3\cos^2(a+bx)^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},-\frac{1}{3},\frac{2}{3},\sin^2(a+bx)\right)}{2b\cos^{\frac{4}{3}}(a+bx)\sin^{\frac{2}{3}}(a+bx)}$$

input
```
Integrate[Cos[a + b*x]^(5/3)/Sin[a + b*x]^(5/3),x]
```

output
```
(-3*(Cos[a + b*x]^2)^(2/3)*Hypergeometric2F1[-1/3, -1/3, 2/3, Sin[a + b*x]
^2])/(2*b*Cos[a + b*x]^(4/3)*Sin[a + b*x]^(2/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.40 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.65, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3047, 3042, 3054, 807, 821, 16, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{3}}(a+bx)}{\sin^{\frac{5}{3}}(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)^{5/3}}{\sin(a+bx)^{5/3}}dx$$

$$\Big\downarrow \text{3047}$$

$$-\int \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}dx-\frac{3\cos^{\frac{2}{3}}(a+bx)}{2b\sin^{\frac{2}{3}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}dx-\frac{3\cos^{\frac{2}{3}}(a+bx)}{2b\sin^{\frac{2}{3}}(a+bx)}$$

$$\Big\downarrow \; 3054$$

$$-\frac{3\int \frac{\tan(a+bx)}{\tan^2(a+bx)+1}d\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}}{b}-\frac{3\cos^{\frac{2}{3}}(a+bx)}{2b\sin^{\frac{2}{3}}(a+bx)}$$

$$\Big\downarrow \; 807$$

$$-\frac{3\int \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)(\tan(a+bx)+1)}d\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}}{2b}-\frac{3\cos^{\frac{2}{3}}(a+bx)}{2b\sin^{\frac{2}{3}}(a+bx)}$$

$$\Big\downarrow \; 821$$

$$-\frac{3\left(\frac{1}{3}\int \left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)d\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-\frac{1}{3}\int \frac{1}{\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1}d\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b}-\frac{3\cos^{\frac{2}{3}}(a+bx)}{2b\sin^{\frac{2}{3}}(a+bx)}$$

$$\Big\downarrow \; 16$$

$$-\frac{3\left(\frac{1}{3}\int \left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)d\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-\frac{1}{3}\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)\right)}{2b}-\frac{3\cos^{\frac{2}{3}}(a+bx)}{2b\sin^{\frac{2}{3}}(a+bx)}$$

$$\Big\downarrow \; 1142$$

$$-\frac{3\left(\frac{1}{3}\left(\frac{3}{2}\int 1d\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+\frac{1}{2}\int \left(\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-1\right)d\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)-\frac{1}{3}\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)\right)}{2b}-\frac{3\cos^{\frac{2}{3}}(a+bx)}{2b\sin^{\frac{2}{3}}(a+bx)}$$

$$\Big\downarrow \; 25$$

$$-\frac{3\left(\frac{1}{3}\left(\frac{3}{2}\int 1d\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-\frac{1}{2}\int \left(1-\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)d\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)-\frac{1}{3}\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)\right)}{2b}-\frac{3\cos^{\frac{2}{3}}(a+bx)}{2b\sin^{\frac{2}{3}}(a+bx)}$$

$$\Big\downarrow \; 1083$$

$$-\frac{3\left(\frac{1}{3}\left(-3\int \frac{1}{-\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-2}d\left(\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-1\right)-\frac{1}{2}\int \left(1-\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)d\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)-\frac{1}{3}\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)}{2b}$$

$$\frac{3\cos^{\frac{2}{3}}(a+bx)}{2b\sin^{\frac{2}{3}}(a+bx)}$$

$\downarrow$ 217

$$-\frac{3\left(\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-1}{\sqrt{3}}\right)-\frac{1}{2}\int \left(1-\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)d\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)-\frac{1}{3}\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)\right)}{2b}-$$

$$\frac{3\cos^{\frac{2}{3}}(a+bx)}{2b\sin^{\frac{2}{3}}(a+bx)}$$

$\downarrow$ 1103

$$-\frac{3\left(\frac{\arctan\left(\frac{\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{3}\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)\right)}{2b}-\frac{3\cos^{\frac{2}{3}}(a+bx)}{2b\sin^{\frac{2}{3}}(a+bx)}$$

input

```
Int[Cos[a + b*x]^(5/3)/Sin[a + b*x]^(5/3),x]
```

output

```
(-3*(ArcTan[(-1 + (2*Sin[a + b*x]^(2/3))/Cos[a + b*x]^(2/3))/Sqrt[3]]/Sqrt
[3] - Log[1 + Sin[a + b*x]^(2/3)/Cos[a + b*x]^(2/3)]/3))/(2*b) - (3*Cos[a
+ b*x]^(2/3))/(2*b*Sin[a + b*x]^(2/3))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3047
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Cos[e + f*x]
)^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3054
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k*a*(b/f)   Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Maple [F]

$$\int \frac{\cos \left(b x+a\right)^{\frac{5}{3}}}{\sin \left(b x+a\right)^{\frac{5}{3}}}dx$$

input
```
int(cos(b*x+a)^(5/3)/sin(b*x+a)^(5/3),x)
```

output
```
int(cos(b*x+a)^(5/3)/sin(b*x+a)^(5/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.22

$$\int \frac{\cos^{\frac{5}{3}}(a+bx)}{\sin^{\frac{5}{3}}(a+bx)}\,dx = -\frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}\cos(bx+a)-2\sqrt{3}\cos(bx+a)^{\frac{1}{3}}\sin(bx+a)^{\frac{2}{3}}}{3\cos(bx+a)}\right)\sin\left(bx+a\right)-2\log\left(\frac{\cos(bx+a)^{\frac{1}{3}}\sin(bx+a)^{\frac{2}{3}}+\cos(bx+a)}{\cos(bx+a)}\right.}{4}$$

input
```
integrate(cos(b*x+a)^(5/3)/sin(b*x+a)^(5/3),x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(3)*arctan(-1/3*(sqrt(3)*cos(b*x + a) - 2*sqrt(3)*cos(b*x + a)
^(1/3)*sin(b*x + a)^(2/3))/cos(b*x + a))*sin(b*x + a) - 2*log((cos(b*x + a
)^(1/3)*sin(b*x + a)^(2/3) + cos(b*x + a))/cos(b*x + a))*sin(b*x + a) + lo
g((cos(b*x + a)^2 - cos(b*x + a)^(4/3)*sin(b*x + a)^(2/3) + cos(b*x + a)^(
2/3)*sin(b*x + a)^(4/3))/cos(b*x + a)^2)*sin(b*x + a) + 6*cos(b*x + a)^(2/
3)*sin(b*x + a)^(1/3))/(b*sin(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{3}}(a+bx)}{\sin^{\frac{5}{3}}(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**(5/3)/sin(b*x+a)**(5/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{5}{3}}(a+bx)}{\sin^{\frac{5}{3}}(a+bx)}\,dx = \int \frac{\cos(bx+a)^{\frac{5}{3}}}{\sin(bx+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(cos(b*x+a)^(5/3)/sin(b*x+a)^(5/3),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^(5/3)/sin(b*x + a)^(5/3), x)
```

Giac [F]

$$\int \frac{\cos^{\frac{5}{3}}(a+bx)}{\sin^{\frac{5}{3}}(a+bx)}\,dx = \int \frac{\cos(bx+a)^{\frac{5}{3}}}{\sin(bx+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(cos(b*x+a)^(5/3)/sin(b*x+a)^(5/3),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^(5/3)/sin(b*x + a)^(5/3), x)
```

Mupad [B] (verification not implemented)

Time = 26.60 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.28

$$\int \frac{\cos^{\frac{5}{3}}(a+bx)}{\sin^{\frac{5}{3}}(a+bx)}\,dx = -\frac{3\cos(a+b\,x)^{8/3}\left(\sin(a+b\,x)^2\right)^{1/3}\,{}_2\mathrm{F}_1\left(\frac{4}{3},\frac{4}{3};\ \frac{7}{3};\ \cos(a+b\,x)^2\right)}{8\,b\sin(a+b\,x)^{2/3}}$$

input
```
int(cos(a + b*x)^(5/3)/sin(a + b*x)^(5/3),x)
```

output
```
-(3*cos(a + b*x)^(8/3)*(sin(a + b*x)^2)^(1/3)*hypergeom([4/3, 4/3], 7/3, c
os(a + b*x)^2))/(8*b*sin(a + b*x)^(2/3))
```

Reduce [F]

$$\int \frac{\cos^{\frac{5}{3}}(a+bx)}{\sin^{\frac{5}{3}}(a+bx)}\,dx = \int \frac{\cos(bx+a)^{\frac{5}{3}}}{\sin(bx+a)^{\frac{5}{3}}}dx$$

input
```
int(cos(b*x+a)^(5/3)/sin(b*x+a)^(5/3),x)
```

output
```
int((cos(a + b*x)**(2/3)*cos(a + b*x))/(sin(a + b*x)**(2/3)*sin(a + b*x)),
x)
```

3.333 $\int \frac{\cos^{\frac{7}{3}}(a+bx)}{\sin^{\frac{7}{3}}(a+bx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 155

$$\int \frac{\cos^{\frac{7}{3}}(a+bx)}{\sin^{\frac{7}{3}}(a+bx)}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b}+\frac{\log\left(1+\frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}-\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{4b}-\frac{\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b}-\frac{3\cos^{\frac{4}{3}}(a+bx)}{4b\sin^{\frac{4}{3}}(a+bx)}$$

output
```
-1/2*3^(1/2)*arctan(1/3*(1-2*cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3))*3^(1/2))/b
+1/4*ln(1+cos(b*x+a)^(4/3)/sin(b*x+a)^(4/3)-cos(b*x+a)^(2/3)/sin(b*x+a)^(2
/3))/b-1/2*ln(1+cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3))/b-3/4*cos(b*x+a)^(4/3)/
b/sin(b*x+a)^(4/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.37

$$\int \frac{\cos^{\frac{7}{3}}(a+bx)}{\sin^{\frac{7}{3}}(a+bx)}\,dx=-\frac{3\sqrt[3]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{2}{3},\frac{1}{3},\sin^2(a+bx)\right)}{4b\cos^{\frac{2}{3}}(a+bx)\sin^{\frac{4}{3}}(a+bx)}$$

input
```
Integrate[Cos[a + b*x]^(7/3)/Sin[a + b*x]^(7/3),x]
```

output
```
(-3*(Cos[a + b*x]^2)^(1/3)*Hypergeometric2F1[-2/3, -2/3, 1/3, Sin[a + b*x]
^2])/(4*b*Cos[a + b*x]^(2/3)*Sin[a + b*x]^(4/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.40 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.65, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 3047, 3042, 3055, 807, 821, 16, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{7}{3}}(a+bx)}{\sin^{\frac{7}{3}}(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)^{7/3}}{\sin(a+bx)^{7/3}}dx$$

$$\Big\downarrow \text{3047}$$

$$-\int \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}dx-\frac{3\cos^{\frac{4}{3}}(a+bx)}{4b\sin^{\frac{4}{3}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}dx-\frac{3\cos^{\frac{4}{3}}(a+bx)}{4b\sin^{\frac{4}{3}}(a+bx)}$$

$$\Big\downarrow \ 3055$$

$$\frac{3\int \frac{\cot(a+bx)}{\cot^2(a+bx)+1}d\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}}{b}-\frac{3\cos^{\frac{4}{3}}(a+bx)}{4b\sin^{\frac{4}{3}}(a+bx)}$$

$$\Big\downarrow \ 807$$

$$\frac{3\int \frac{\cos^{\frac{2}{3}}(a+bx)}{(\cot(a+bx)+1)\sin^{\frac{2}{3}}(a+bx)}d\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}}{2b}-\frac{3\cos^{\frac{4}{3}}(a+bx)}{4b\sin^{\frac{4}{3}}(a+bx)}$$

$$\Big\downarrow \ 821$$

$$\frac{3\left(\frac{1}{3}\int \left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)d\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-\frac{1}{3}\int \frac{1}{\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1}d\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b}-\frac{3\cos^{\frac{4}{3}}(a+bx)}{4b\sin^{\frac{4}{3}}(a+bx)}$$

$$\Big\downarrow \ 16$$

$$\frac{3\left(\frac{1}{3}\int \left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)d\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-\frac{1}{3}\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)\right)}{2b}-\frac{3\cos^{\frac{4}{3}}(a+bx)}{4b\sin^{\frac{4}{3}}(a+bx)}$$

$$\Big\downarrow \ 1142$$

$$\frac{3\left(\frac{1}{3}\left(\frac{3}{2}\int 1d\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+\frac{1}{2}\int \left(\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-1\right)d\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)-\frac{1}{3}\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)\right)}{2b}-\frac{3\cos^{\frac{4}{3}}(a+bx)}{4b\sin^{\frac{4}{3}}(a+bx)}$$

$$\Big\downarrow \ 25$$

$$\frac{3\left(\frac{1}{3}\left(\frac{3}{2}\int 1d\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-\frac{1}{2}\int \left(1-\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)d\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)-\frac{1}{3}\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)\right)}{2b}-\frac{3\cos^{\frac{4}{3}}(a+bx)}{4b\sin^{\frac{4}{3}}(a+bx)}$$

$$\Big\downarrow \ 1083$$

$$\frac{3\left(\frac{1}{3}\left(-3\int \frac{1}{-\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-2}d\left(\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-1\right)-\frac{1}{2}\int \left(1-\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)d\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)-\frac{1}{3}\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)\right)}{2b}$$

$$\frac{3\cos^{\frac{4}{3}}(a+bx)}{4b\sin^{\frac{4}{3}}(a+bx)}$$

$$\Big\downarrow \text{217}$$

$$\frac{3\left(\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-1}{\sqrt{3}}\right)-\frac{1}{2}\int \left(1-\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)d\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)-\frac{1}{3}\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)\right)}{2b}-$$

$$\frac{3\cos^{\frac{4}{3}}(a+bx)}{4b\sin^{\frac{4}{3}}(a+bx)}$$

$$\Big\downarrow \text{1103}$$

$$\frac{3\left(\frac{\arctan\left(\frac{\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{3}\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)\right)}{2b}-\frac{3\cos^{\frac{4}{3}}(a+bx)}{4b\sin^{\frac{4}{3}}(a+bx)}$$

input
```
Int[Cos[a + b*x]^(7/3)/Sin[a + b*x]^(7/3),x]
```

output
```
(3*(ArcTan[(-1 + (2*Cos[a + b*x]^(2/3))/Sin[a + b*x]^(2/3))/Sqrt[3]]/Sqrt[
3] - Log[1 + Cos[a + b*x]^(2/3)/Sin[a + b*x]^(2/3)]/3))/(2*b) - (3*Cos[a +
 b*x]^(4/3))/(4*b*Sin[a + b*x]^(4/3))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3047
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Simp[a^2*((m - 1)/(b^2*(n + 1)))   Int[(a*Cos[e + f*x]
)^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

rule 3055
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[(-k)*a*(b/f)   Subst[Int[x
^(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[
e + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m,
0] && LtQ[m, 1]
```

Maple [F]

$$\int \frac{\cos (bx+a)^{\frac{7}{3}}}{\sin (bx+a)^{\frac{7}{3}}}dx$$

input
```
int(cos(b*x+a)^(7/3)/sin(b*x+a)^(7/3),x)
```

output
```
int(cos(b*x+a)^(7/3)/sin(b*x+a)^(7/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.41

$$\int \frac{\cos^{\frac{7}{3}}(a+bx)}{\sin^{\frac{7}{3}}(a+bx)}\,dx = \frac{2\left(\sqrt{3}\cos (bx+a)^2-\sqrt{3}\right)\arctan\left(\frac{2\sqrt{3}\cos(bx+a)^{\frac{2}{3}}\sin(bx+a)^{\frac{1}{3}}-\sqrt{3}\sin(bx+a)}{3\sin(bx+a)}\right)+\left(\cos (bx+a)^2-1\right)\log\left(\frac{4\left(\cos(\right.}{}\right.}{}$$

input
```
integrate(cos(b*x+a)^(7/3)/sin(b*x+a)^(7/3),x, algorithm="fricas")
```

output
```
1/4*(2*(sqrt(3)*cos(b*x + a)^2 - sqrt(3))*arctan(1/3*(2*sqrt(3)*cos(b*x +
a)^(2/3)*sin(b*x + a)^(1/3) - sqrt(3)*sin(b*x + a))/sin(b*x + a)) + (cos(b
*x + a)^2 - 1)*log(4*(cos(b*x + a)^2 - cos(b*x + a)^(4/3)*sin(b*x + a)^(2/
3) + cos(b*x + a)^(2/3)*sin(b*x + a)^(4/3) - 1)/(cos(b*x + a)^2 - 1)) - 2*
(cos(b*x + a)^2 - 1)*log(-2*(cos(b*x + a)^(2/3)*sin(b*x + a)^(1/3) + sin(b
*x + a))/sin(b*x + a)) + 3*cos(b*x + a)^(4/3)*sin(b*x + a)^(2/3))/(b*cos(b
*x + a)^2 - b)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{7}{3}}(a+bx)}{\sin^{\frac{7}{3}}(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**(7/3)/sin(b*x+a)**(7/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{7}{3}}(a+bx)}{\sin^{\frac{7}{3}}(a+bx)}\,dx = \int \frac{\cos(bx+a)^{\frac{7}{3}}}{\sin(bx+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate(cos(b*x+a)^(7/3)/sin(b*x+a)^(7/3),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^(7/3)/sin(b*x + a)^(7/3), x)
```

Giac [F]

$$\int \frac{\cos^{\frac{7}{3}}(a+bx)}{\sin^{\frac{7}{3}}(a+bx)}\,dx = \int \frac{\cos(bx+a)^{\frac{7}{3}}}{\sin(bx+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate(cos(b*x+a)^(7/3)/sin(b*x+a)^(7/3),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^(7/3)/sin(b*x + a)^(7/3), x)
```

Mupad [B] (verification not implemented)

Time = 27.46 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.28

$$\int \frac{\cos^{\frac{7}{3}}(a+bx)}{\sin^{\frac{7}{3}}(a+bx)}\,dx = -\frac{3\cos(a+b\,x)^{10/3}\left(\sin(a+b\,x)^2\right)^{2/3}{}_2\mathrm{F}_1\left(\frac{5}{3},\frac{5}{3};\ \frac{8}{3};\ \cos(a+b\,x)^2\right)}{10\,b\sin(a+b\,x)^{4/3}}$$

input
```
int(cos(a + b*x)^(7/3)/sin(a + b*x)^(7/3),x)
```

output
```
-(3*cos(a + b*x)^(10/3)*(sin(a + b*x)^2)^(2/3)*hypergeom([5/3, 5/3], 8/3,
cos(a + b*x)^2))/(10*b*sin(a + b*x)^(4/3))
```

Reduce [F]

$$\int \frac{\cos^{\frac{7}{3}}(a+bx)}{\sin^{\frac{7}{3}}(a+bx)}\,dx = \int \frac{\cos(bx+a)^{\frac{7}{3}}}{\sin(bx+a)^{\frac{7}{3}}}dx$$

input
```
int(cos(b*x+a)^(7/3)/sin(b*x+a)^(7/3),x)
```

output
```
int((cos(a + b*x)**(1/3)*cos(a + b*x)**2)/(sin(a + b*x)**(1/3)*sin(a + b*x
)**2),x)
```

3.334 $\int \frac{\cos^{\frac{2}{3}}(x)}{\sin^{\frac{8}{3}}(x)} dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{\cos^{\frac{2}{3}}(x)}{\sin^{\frac{8}{3}}(x)} dx = -\frac{3\cos^{\frac{5}{3}}(x)}{5\sin^{\frac{5}{3}}(x)}$$

output
```
-3/5*cos(x)^(5/3)/sin(x)^(5/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\cos^{\frac{2}{3}}(x)}{\sin^{\frac{8}{3}}(x)} dx = -\frac{3\cos^{\frac{5}{3}}(x)}{5\sin^{\frac{5}{3}}(x)}$$

input
```
Integrate[Cos[x]^(2/3)/Sin[x]^(8/3),x]
```

output
```
(-3*Cos[x]^(5/3))/(5*Sin[x]^(5/3))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3042, 3043}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{2}{3}}(x)}{\sin^{\frac{8}{3}}(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)^{2/3}}{\sin(x)^{8/3}}dx$$

$$\Big\downarrow \text{3043}$$

$$-\frac{3\cos^{\frac{5}{3}}(x)}{5\sin^{\frac{5}{3}}(x)}$$

input
```
Int[Cos[x]^(2/3)/Sin[x]^(8/3),x]
```

output
```
(-3*Cos[x]^(5/3))/(5*Sin[x]^(5/3))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3043
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Maple [F]

$$\int \frac{\cos(x)^{\frac{2}{3}}}{\sin(x)^{\frac{8}{3}}} dx$$

input
```
int(cos(x)^(2/3)/sin(x)^(8/3),x)
```

output
```
int(cos(x)^(2/3)/sin(x)^(8/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cos^{\frac{2}{3}}(x)}{\sin^{\frac{8}{3}}(x)} dx = \frac{3 \cos(x)^{\frac{5}{3}} \sin(x)^{\frac{1}{3}}}{5 (\cos(x)^2 - 1)}$$

input
```
integrate(cos(x)^(2/3)/sin(x)^(8/3),x, algorithm="fricas")
```

output
```
3/5*cos(x)^(5/3)*sin(x)^(1/3)/(cos(x)^2 - 1)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{2}{3}}(x)}{\sin^{\frac{8}{3}}(x)} dx = \text{Timed out}$$

input
```
integrate(cos(x)**(2/3)/sin(x)**(8/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{2}{3}}(x)}{\sin^{\frac{8}{3}}(x)}\,dx = \int \frac{\cos(x)^{\frac{2}{3}}}{\sin(x)^{\frac{8}{3}}}\,dx$$

input
```
integrate(cos(x)^(2/3)/sin(x)^(8/3),x, algorithm="maxima")
```

output
```
integrate(cos(x)^(2/3)/sin(x)^(8/3), x)
```

Giac [F]

$$\int \frac{\cos^{\frac{2}{3}}(x)}{\sin^{\frac{8}{3}}(x)}\,dx = \int \frac{\cos(x)^{\frac{2}{3}}}{\sin(x)^{\frac{8}{3}}}\,dx$$

input
```
integrate(cos(x)^(2/3)/sin(x)^(8/3),x, algorithm="giac")
```

output
```
integrate(cos(x)^(2/3)/sin(x)^(8/3), x)
```

Mupad [B] (verification not implemented)

Time = 26.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{\cos^{\frac{2}{3}}(x)}{\sin^{\frac{8}{3}}(x)}\,dx = -\frac{3\cos(x)^{5/3}}{5\sin(x)^{5/3}}$$

input
```
int(cos(x)^(2/3)/sin(x)^(8/3),x)
```

output
```
-(3*cos(x)^(5/3))/(5*sin(x)^(5/3))
```

Reduce [F]

$$\int \frac{\cos^{\frac{2}{3}}(x)}{\sin^{\frac{8}{3}}(x)}\,dx = \int \frac{\cos(x)^{\frac{2}{3}}}{\sin(x)^{\frac{8}{3}}}dx$$

input
```
int(cos(x)^(2/3)/sin(x)^(8/3),x)
```

output
```
int(cos(x)**(2/3)/(sin(x)**(2/3)*sin(x)**2),x)
```

3.335 $\int \frac{\sin^{\frac{2}{3}}(x)}{\cos^{\frac{8}{3}}(x)} dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{\sin^{\frac{2}{3}}(x)}{\cos^{\frac{8}{3}}(x)} dx = \frac{3\sin^{\frac{5}{3}}(x)}{5\cos^{\frac{5}{3}}(x)}$$

output
```
3/5*sin(x)^(5/3)/cos(x)^(5/3)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sin^{\frac{2}{3}}(x)}{\cos^{\frac{8}{3}}(x)} dx = \frac{3\sin^{\frac{5}{3}}(x)}{5\cos^{\frac{5}{3}}(x)}$$

input
```
Integrate[Sin[x]^(2/3)/Cos[x]^(8/3),x]
```

output
```
(3*Sin[x]^(5/3))/(5*Cos[x]^(5/3))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3043}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^{\frac{2}{3}}(x)}{\cos^{\frac{8}{3}}(x)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sin(x)^{2/3}}{\cos(x)^{8/3}}dx$$

$$\Big\downarrow \ 3043$$

$$\frac{3\sin^{\frac{5}{3}}(x)}{5\cos^{\frac{5}{3}}(x)}$$

input
```
Int[Sin[x]^(2/3)/Cos[x]^(8/3),x]
```

output
```
(3*Sin[x]^(5/3))/(5*Cos[x]^(5/3))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3043
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Maple [F]

$$\int \frac{\sin(x)^{\frac{2}{3}}}{\cos(x)^{\frac{8}{3}}}dx$$

input
```
int(sin(x)^(2/3)/cos(x)^(8/3),x)
```

output
```
int(sin(x)^(2/3)/cos(x)^(8/3),x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{\sin^{\frac{2}{3}}(x)}{\cos^{\frac{8}{3}}(x)}\,dx = \frac{3\,\sin(x)^{\frac{5}{3}}}{5\,\cos(x)^{\frac{5}{3}}}$$

input
```
integrate(sin(x)^(2/3)/cos(x)^(8/3),x, algorithm="fricas")
```

output
```
3/5*sin(x)^(5/3)/cos(x)^(5/3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^{\frac{2}{3}}(x)}{\cos^{\frac{8}{3}}(x)}\,dx = \text{Timed out}$$

input
```
integrate(sin(x)**(2/3)/cos(x)**(8/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sin^{\frac{2}{3}}(x)}{\cos^{\frac{8}{3}}(x)}\,dx = \int \frac{\sin(x)^{\frac{2}{3}}}{\cos(x)^{\frac{8}{3}}}\,dx$$

input
```
integrate(sin(x)^(2/3)/cos(x)^(8/3),x, algorithm="maxima")
```

output
```
integrate(sin(x)^(2/3)/cos(x)^(8/3), x)
```

Giac [F]

$$\int \frac{\sin^{\frac{2}{3}}(x)}{\cos^{\frac{8}{3}}(x)}\,dx = \int \frac{\sin(x)^{\frac{2}{3}}}{\cos(x)^{\frac{8}{3}}}\,dx$$

input
```
integrate(sin(x)^(2/3)/cos(x)^(8/3),x, algorithm="giac")
```

output
```
integrate(sin(x)^(2/3)/cos(x)^(8/3), x)
```

Mupad [B] (verification not implemented)

Time = 26.23 (sec) , antiderivative size = 94, normalized size of antiderivative = 5.88

$$\int \frac{\sin^{\frac{2}{3}}(x)}{\cos^{\frac{8}{3}}(x)}\,dx = \frac{6\,2^{2/3}\tan\left(\frac{x}{2}\right)^{5/3}\left(1-\tan\left(\frac{x}{2}\right)^2\right)^{1/3}+6\,2^{2/3}\tan\left(\frac{x}{2}\right)^{11/3}\left(1-\tan\left(\frac{x}{2}\right)^2\right)^{1/3}}{5\tan\left(\frac{x}{2}\right)^2-\tan\left(\frac{x}{2}\right)^2\left(10\tan\left(\frac{x}{2}\right)^2-5\tan\left(\frac{x}{2}\right)^2\left(\tan\left(\frac{x}{2}\right)^2+1\right)+10\right)+5}$$

input
```
int(sin(x)^(2/3)/cos(x)^(8/3),x)
```

output
```
(6*2^(2/3)*tan(x/2)^(5/3)*(1 - tan(x/2)^2)^(1/3) + 6*2^(2/3)*tan(x/2)^(11/
3)*(1 - tan(x/2)^2)^(1/3))/(5*tan(x/2)^2 - tan(x/2)^2*(10*tan(x/2)^2 - 5*t
an(x/2)^2*(tan(x/2)^2 + 1) + 10) + 5)
```

Reduce [F]

$$\int \frac{\sin^{\frac{2}{3}}(x)}{\cos^{\frac{8}{3}}(x)}\, dx = \int \frac{\sin(x)^{\frac{2}{3}}}{\cos(x)^{\frac{8}{3}}} dx$$

input
```
int(sin(x)^(2/3)/cos(x)^(8/3),x)
```

output
```
int(sin(x)**(2/3)/(cos(x)**(2/3)*cos(x)**2),x)
```

3.336 $\int \cos^n(e+fx)\sin^m(e+fx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 80

$$\int \cos^n(e+fx)\sin^m(e+fx)\,dx = -\frac{\cos^{1+n}(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(e+fx)\right)\sin^{-1+m}(e+fx)\sin^2(e+fx)^{\frac{1-m}{2}}}{f(1+n)}$$

output
```
-cos(f*x+e)^(1+n)*hypergeom([1/2+1/2*n, 1/2-1/2*m],[3/2+1/2*n],cos(f*x+e)^
2)*sin(f*x+e)^(-1+m)*(sin(f*x+e)^2)^(1/2-1/2*m)/f/(1+n)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99

$$\int \cos^n(e+fx)\sin^m(e+fx)\,dx = \frac{\cos^{-1+n}(e+fx)\cos^2(e+fx)^{\frac{1-n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+m}{2},\frac{1-n}{2},\frac{3+m}{2},\sin^2(e+fx)\right)\sin^{1+m}(e+fx)}{f(1+m)}$$

input
```
Integrate[Cos[e + f*x]^n*Sin[e + f*x]^m,x]
```

output
```
(Cos[e + f*x]^(-1 + n)*(Cos[e + f*x]^2)^((1 - n)/2)*Hypergeometric2F1[(1 +
 m)/2, (1 - n)/2, (3 + m)/2, Sin[e + f*x]^2]*Sin[e + f*x]^(1 + m))/(f*(1 +
 m))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^m(e+fx)\cos^n(e+fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^m\cos(e+fx)^n dx$$

$$\Big\downarrow \text{3056}$$

$$-\frac{\sin^{m-1}(e+fx)\sin^2(e+fx)^{\frac{1-m}{2}}\cos^{n+1}(e+fx)\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(e+fx)\right)}{f(n+1)}$$

input
```
Int[Cos[e + f*x]^n*Sin[e + f*x]^m,x]
```

output
```
-((Cos[e + f*x]^(1 + n)*Hypergeometric2F1[(1 - m)/2, (1 + n)/2, (3 + n)/2,
 Cos[e + f*x]^2]*Sin[e + f*x]^(-1 + m)*(Sin[e + f*x]^2)^((1 - m)/2))/(f*(1
 + n)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Maple [F]

$$\int \cos (fx+e)^n \sin (fx+e)^m \, dx$$

input
```
int(cos(f*x+e)^n*sin(f*x+e)^m,x)
```

output
```
int(cos(f*x+e)^n*sin(f*x+e)^m,x)
```

Fricas [F]

$$\int \cos^n(e+fx) \sin^m(e+fx) \, dx = \int \cos (fx+e)^n \sin (fx+e)^m \, dx$$

input
```
integrate(cos(f*x+e)^n*sin(f*x+e)^m,x, algorithm="fricas")
```

output
```
integral(cos(f*x + e)^n*sin(f*x + e)^m, x)
```

Sympy [F]

$$\int \cos^n(e+fx)\sin^m(e+fx)\,dx = \int \sin^m(e+fx)\cos^n(e+fx)\,dx$$

input
```
integrate(cos(f*x+e)**n*sin(f*x+e)**m,x)
```

output
```
Integral(sin(e + f*x)**m*cos(e + f*x)**n, x)
```

Maxima [F]

$$\int \cos^n(e+fx)\sin^m(e+fx)\,dx = \int \cos(fx+e)^n \sin(fx+e)^m\,dx$$

input
```
integrate(cos(f*x+e)^n*sin(f*x+e)^m,x, algorithm="maxima")
```

output
```
integrate(cos(f*x + e)^n*sin(f*x + e)^m, x)
```

Giac [F]

$$\int \cos^n(e+fx)\sin^m(e+fx)\,dx = \int \cos(fx+e)^n \sin(fx+e)^m\,dx$$

input
```
integrate(cos(f*x+e)^n*sin(f*x+e)^m,x, algorithm="giac")
```

output
```
integrate(cos(f*x + e)^n*sin(f*x + e)^m, x)
```

Mupad [B] (verification not implemented)

Time = 27.76 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

$$\int \cos^n(e+fx)\sin^m(e+fx)\,dx = -\frac{\cos(e+f\,x)^{n+1}\sin(e+f\,x)^{m+1}\,{}_2\mathrm{F}_1\left(\frac{1}{2}-\frac{m}{2},\frac{n}{2}+\frac{1}{2};\ \frac{n}{2}+\frac{3}{2};\ \cos(e+f\,x)^2\right)}{f\ (n+1)\ \left(\sin(e+f\,x)^2\right)^{\frac{m}{2}+\frac{1}{2}}}$$

input
```
int(cos(e + f*x)^n*sin(e + f*x)^m,x)
```

output
```
-(cos(e + f*x)^(n + 1)*sin(e + f*x)^(m + 1)*hypergeom([1/2 - m/2, n/2 + 1/
2], n/2 + 3/2, cos(e + f*x)^2))/(f*(n + 1)*(sin(e + f*x)^2)^(m/2 + 1/2))
```

Reduce [F]

$$\int \cos^n(e+fx)\sin^m(e+fx)\,dx = \int \sin(fx+e)^m\cos(fx+e)^n\,dx$$

input
```
int(cos(f*x+e)^n*sin(f*x+e)^m,x)
```

output
```
int(sin(e + f*x)**m*cos(e + f*x)**n,x)
```

3.337 $\int (d\cos(e+fx))^n \sin^m(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 85

$$\int (d\cos(e+fx))^n \sin^m(e+fx)\,dx = -\frac{(d\cos(e+fx))^{1+n}\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(e+fx)\right)\sin^{-1+m}(e+fx)\sin^2(e+fx)^{\frac{1-}{2}}}{df(1+n)}$$

output
```
-(d*cos(f*x+e))^(1+n)*hypergeom([1/2+1/2*n, 1/2-1/2*m],[3/2+1/2*n],cos(f*x
+e)^2)*sin(f*x+e)^(-1+m)*(sin(f*x+e)^2)^(1/2-1/2*m)/d/f/(1+n)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.96

$$\int (d\cos(e+fx))^n \sin^m(e+fx)\,dx = \frac{d(d\cos(e+fx))^{-1+n}\cos^2(e+fx)^{\frac{1-n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+m}{2},\frac{1-n}{2},\frac{3+m}{2},\sin^2(e+fx)\right)\sin^{1+m}(e+fx}{f(1+m)}$$

input
```
Integrate[(d*Cos[e + f*x])^n*Sin[e + f*x]^m,x]
```

output
```
(d*(d*Cos[e + f*x])^(-1 + n)*(Cos[e + f*x]^2)^((1 - n)/2)*Hypergeometric2F
1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[e + f*x]^2]*Sin[e + f*x]^(1 + m))/(
f*(1 + m))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^m(e+fx)(d\cos(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^m(d\cos(e+fx))^n dx$$

$$\Big\downarrow \text{3056}$$

$$-\frac{\sin^{m-1}(e+fx)\sin^2(e+fx)^{\frac{1-m}{2}}(d\cos(e+fx))^{n+1}\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(e+fx)\right)}{df(n+1)}$$

input
```
Int[(d*Cos[e + f*x])^n*Sin[e + f*x]^m,x]
```

output
```
-(((d*Cos[e + f*x])^(1 + n)*Hypergeometric2F1[(1 - m)/2, (1 + n)/2, (3 + n
)/2, Cos[e + f*x]^2]*Sin[e + f*x]^(-1 + m)*(Sin[e + f*x]^2)^((1 - m)/2))/(
d*f*(1 + n)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Maple [F]

$$\int (d\cos (fx+e))^{n} \sin (fx+e)^{m} dx$$

input
```
int((d*cos(f*x+e))^n*sin(f*x+e)^m,x)
```

output
```
int((d*cos(f*x+e))^n*sin(f*x+e)^m,x)
```

Fricas [F]

$$\int (d\cos(e+fx))^n \sin^m(e+fx)\,dx = \int (d\cos (fx+e))^{n} \sin (fx+e)^{m} \,dx$$

input
```
integrate((d*cos(f*x+e))^n*sin(f*x+e)^m,x, algorithm="fricas")
```

output
```
integral((d*cos(f*x + e))^n*sin(f*x + e)^m, x)
```

Sympy [F]

$$\int (d\cos(e+fx))^n\sin^m(e+fx)\,dx=\int (d\cos{(e+fx)})^n\sin^m{(e+fx)}\,dx$$

input
```
integrate((d*cos(f*x+e))**n*sin(f*x+e)**m,x)
```

output
```
Integral((d*cos(e + f*x))**n*sin(e + f*x)**m, x)
```

Maxima [F]

$$\int (d\cos(e+fx))^n\sin^m(e+fx)\,dx=\int (d\cos{(fx+e)})^n\sin{(fx+e)}^m\,dx$$

input
```
integrate((d*cos(f*x+e))^n*sin(f*x+e)^m,x, algorithm="maxima")
```

output
```
integrate((d*cos(f*x + e))^n*sin(f*x + e)^m, x)
```

Giac [F]

$$\int (d\cos(e+fx))^n\sin^m(e+fx)\,dx=\int (d\cos{(fx+e)})^n\sin{(fx+e)}^m\,dx$$

input
```
integrate((d*cos(f*x+e))^n*sin(f*x+e)^m,x, algorithm="giac")
```

output
```
integrate((d*cos(f*x + e))^n*sin(f*x + e)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(e+fx))^n \sin^m(e+fx)\,dx = \int \sin(e+f\,x)^m\,(d\,\cos(e+f\,x))^n\,dx$$

input
```
int(sin(e + f*x)^m*(d*cos(e + f*x))^n,x)
```

output
```
int(sin(e + f*x)^m*(d*cos(e + f*x))^n, x)
```

Reduce [F]

$$\int (d\cos(e+fx))^n \sin^m(e+fx)\,dx = d^n\left(\int \sin(fx+e)^m \cos(fx+e)^n\,dx\right)$$

input
```
int((d*cos(f*x+e))^n*sin(f*x+e)^m,x)
```

output
```
d**n*int(sin(e + f*x)**m*cos(e + f*x)**n,x)
```

3.338 $\int \cos^n(e+fx)(b\sin(e+fx))^m\,dx$

Optimal result

Integrand size = 19, antiderivative size = 83

$$\int \cos^n(e+fx)(b\sin(e+fx))^m\,dx = -\frac{b\cos^{1+n}(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(e+fx)\right)(b\sin(e+fx))^{-1+m}\sin^2(e+fx)^{\frac{1}{\ldots}}}{f(1+n)}$$

output
```
-b*cos(f*x+e)^(1+n)*hypergeom([1/2+1/2*n, 1/2-1/2*m],[3/2+1/2*n],cos(f*x+e
)^2)*(b*sin(f*x+e))^(-1+m)*(sin(f*x+e)^2)^(1/2-1/2*m)/f/(1+n)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02

$$\int \cos^n(e+fx)(b\sin(e+fx))^m\,dx = \frac{\cos^{-1+n}(e+fx)\cos^2(e+fx)^{\frac{1-n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+m}{2},\frac{1-n}{2},\frac{3+m}{2},\sin^2(e+fx)\right)\sin(e+fx)(b\sin(e\ldots}{f(1+m)}$$

input
```
Integrate[Cos[e + f*x]^n*(b*Sin[e + f*x])^m,x]
```

output
```
(Cos[e + f*x]^(-1 + n)*(Cos[e + f*x]^2)^((1 - n)/2)*Hypergeometric2F1[(1 +
 m)/2, (1 - n)/2, (3 + m)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(b*Sin[e + f*x])
^m)/(f*(1 + m))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^n(e+fx)(b\sin(e+fx))^m\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \cos(e+fx)^n(b\sin(e+fx))^m dx$$

$$\Big\downarrow \; 3056$$

$$-\frac{b\sin^2(e+fx)^{\frac{1-m}{2}}\cos^{n+1}(e+fx)(b\sin(e+fx))^{m-1}\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(e+fx)\right)}{f(n+1)}$$

input
```
Int[Cos[e + f*x]^n*(b*Sin[e + f*x])^m,x]
```

output
```
-((b*Cos[e + f*x]^(1 + n)*Hypergeometric2F1[(1 - m)/2, (1 + n)/2, (3 + n)/
2, Cos[e + f*x]^2]*(b*Sin[e + f*x])^(-1 + m)*(Sin[e + f*x]^2)^((1 - m)/2))
/(f*(1 + n)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Maple [F]

$$\int \cos\left(fx+e\right)^{n} \left(b\sin\left(fx+e\right)\right)^{m} dx$$

input
```
int(cos(f*x+e)^n*(b*sin(f*x+e))^m,x)
```

output
```
int(cos(f*x+e)^n*(b*sin(f*x+e))^m,x)
```

Fricas [F]

$$\int \cos^n(e+fx)(b\sin(e+fx))^m\,dx = \int \left(b\sin\left(fx+e\right)\right)^{m}\cos\left(fx+e\right)^{n}\,dx$$

input
```
integrate(cos(f*x+e)^n*(b*sin(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((b*sin(f*x + e))^m*cos(f*x + e)^n, x)
```

Sympy [F]

$$\int \cos^n(e+fx)(b\sin(e+fx))^m\,dx = \int (b\sin(e+fx))^m \cos^n(e+fx)\,dx$$

input
```
integrate(cos(f*x+e)**n*(b*sin(f*x+e))**m,x)
```

output
```
Integral((b*sin(e + f*x))**m*cos(e + f*x)**n, x)
```

Maxima [F]

$$\int \cos^n(e+fx)(b\sin(e+fx))^m\,dx = \int (b\sin(fx+e))^m \cos(fx+e)^n\,dx$$

input
```
integrate(cos(f*x+e)^n*(b*sin(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e))^m*cos(f*x + e)^n, x)
```

Giac [F]

$$\int \cos^n(e+fx)(b\sin(e+fx))^m\,dx = \int (b\sin(fx+e))^m \cos(fx+e)^n\,dx$$

input
```
integrate(cos(f*x+e)^n*(b*sin(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e))^m*cos(f*x + e)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^n(e+fx)(b\sin(e+fx))^m\,dx=\int \cos\left(e+f\,x\right)^n\,\left(b\,\sin\left(e+f\,x\right)\right)^m\,dx$$

input
```
int(cos(e + f*x)^n*(b*sin(e + f*x))^m,x)
```

output
```
int(cos(e + f*x)^n*(b*sin(e + f*x))^m, x)
```

Reduce [F]

$$\int \cos^n(e+fx)(b\sin(e+fx))^m\,dx=b^m\left(\int \sin\left(fx+e\right)^m\cos\left(fx+e\right)^n dx\right)$$

input
```
int(cos(f*x+e)^n*(b*sin(f*x+e))^m,x)
```

output
```
b**m*int(sin(e + f*x)**m*cos(e + f*x)**n,x)
```

3.339 $\int (d\cos(e+fx))^n(b\sin(e+fx))^m\,dx$

Optimal result

Integrand size = 21, antiderivative size = 88

$$\int (d\cos(e+fx))^n(b\sin(e+fx))^m\,dx = -\frac{b(d\cos(e+fx))^{1+n}\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(e+fx)\right)(b\sin(e+fx))^{-1+m}\sin^2(e+f}{df(1+n)}$$

output
```
-b*(d*cos(f*x+e))^(1+n)*hypergeom([1/2+1/2*n, 1/2-1/2*m],[3/2+1/2*n],cos(f
*x+e)^2)*(b*sin(f*x+e))^(-1+m)*(sin(f*x+e)^2)^(1/2-1/2*m)/d/f/(1+n)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97

$$\int (d\cos(e+fx))^n(b\sin(e+fx))^m\,dx = \frac{(d\cos(e+fx))^n\cos^2(e+fx)^{\frac{1-n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+m}{2},\frac{1-n}{2},\frac{3+m}{2},\sin^2(e+fx)\right)(b\sin(e+fx))^m\tan}{f(1+m)}$$

input
```
Integrate[(d*Cos[e + f*x])^n*(b*Sin[e + f*x])^m,x]
```

output
```
((d*Cos[e + f*x])^n*(Cos[e + f*x]^2)^((1 - n)/2)*Hypergeometric2F1[(1 + m)
/2, (1 - n)/2, (3 + m)/2, Sin[e + f*x]^2]*(b*Sin[e + f*x])^m*Tan[e + f*x])
/(f*(1 + m))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\sin(e+fx))^m(d\cos(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (b\sin(e+fx))^m(d\cos(e+fx))^ndx$$

$$\Big\downarrow \text{3056}$$

$$-\frac{b\sin^2(e+fx)^{\frac{1-m}{2}}(b\sin(e+fx))^{m-1}(d\cos(e+fx))^{n+1}\operatorname{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(e+fx)\right)}{df(n+1)}$$

input
```
Int[(d*Cos[e + f*x])^n*(b*Sin[e + f*x])^m,x]
```

output
```
-((b*(d*Cos[e + f*x])^(1 + n)*Hypergeometric2F1[(1 - m)/2, (1 + n)/2, (3 +
 n)/2, Cos[e + f*x]^2]*(b*Sin[e + f*x])^(-1 + m)*(Sin[e + f*x]^2)^((1 - m)
/2))/(d*f*(1 + n)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Maple [F]

$$\int (d\cos(fx+e))^{n} (b\sin(fx+e))^{m} dx$$

input
```
int((d*cos(f*x+e))^n*(b*sin(f*x+e))^m,x)
```

output
```
int((d*cos(f*x+e))^n*(b*sin(f*x+e))^m,x)
```

Fricas [F]

$$\int (d\cos(e+fx))^n(b\sin(e+fx))^m dx = \int (d\cos(fx+e))^{n} (b\sin(fx+e))^{m} dx$$

input
```
integrate((d*cos(f*x+e))^n*(b*sin(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((d*cos(f*x + e))^n*(b*sin(f*x + e))^m, x)
```

Sympy [F]

$$\int (d\cos(e+fx))^n(b\sin(e+fx))^m\,dx=\int (b\sin{(e+fx)})^m\,(d\cos{(e+fx)})^n\,\,dx$$

input
```
integrate((d*cos(f*x+e))**n*(b*sin(f*x+e))**m,x)
```

output
```
Integral((b*sin(e + f*x))**m*(d*cos(e + f*x))**n, x)
```

Maxima [F]

$$\int (d\cos(e+fx))^n(b\sin(e+fx))^m\,dx=\int (d\cos{(fx+e)})^n\,(b\sin{(fx+e)})^m\,\,dx$$

input
```
integrate((d*cos(f*x+e))^n*(b*sin(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((d*cos(f*x + e))^n*(b*sin(f*x + e))^m, x)
```

Giac [F]

$$\int (d\cos(e+fx))^n(b\sin(e+fx))^m\,dx=\int (d\cos{(fx+e)})^n\,(b\sin{(fx+e)})^m\,\,dx$$

input
```
integrate((d*cos(f*x+e))^n*(b*sin(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((d*cos(f*x + e))^n*(b*sin(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(e+fx))^n(b\sin(e+fx))^m\,dx = \int (d\,\cos\left(e+f\,x\right))^n\,(b\,\sin\left(e+f\,x\right))^m\,dx$$

input
```
int((d*cos(e + f*x))^n*(b*sin(e + f*x))^m,x)
```

output
```
int((d*cos(e + f*x))^n*(b*sin(e + f*x))^m, x)
```

Reduce [F]

$$\int (d\cos(e+fx))^n(b\sin(e+fx))^m\,dx = d^n b^m \left(\int \sin\left(fx+e\right)^m \cos\left(fx+e\right)^n dx\right)$$

input
```
int((d*cos(f*x+e))^n*(b*sin(f*x+e))^m,x)
```

output
```
d**n*b**m*int(sin(e + f*x)**m*cos(e + f*x)**n,x)
```

3.340 $\int \cos^5(a+bx)(c\sin(a+bx))^m\,dx$

Optimal result

Integrand size = 19, antiderivative size = 74

$$\int \cos^5(a+bx)(c\sin(a+bx))^m\,dx = \frac{(c\sin(a+bx))^{1+m}}{bc(1+m)} - \frac{2(c\sin(a+bx))^{3+m}}{bc^3(3+m)} + \frac{(c\sin(a+bx))^{5+m}}{bc^5(5+m)}$$

output
```
(c*sin(b*x+a))^(1+m)/b/c/(1+m)-2*(c*sin(b*x+a))^(3+m)/b/c^3/(3+m)+(c*sin(b
*x+a))^(5+m)/b/c^5/(5+m)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.74

$$\int \cos^5(a+bx)(c\sin(a+bx))^m\,dx = \frac{\sin(a+bx)(c\sin(a+bx))^m\left(\frac{1}{1+m} - \frac{2\sin^2(a+bx)}{3+m} + \frac{\sin^4(a+bx)}{5+m}\right)}{b}$$

input
```
Integrate[Cos[a + b*x]^5*(c*Sin[a + b*x])^m,x]
```

output
```
(Sin[a + b*x]*(c*Sin[a + b*x])^m*((1 + m)^(-1) - (2*Sin[a + b*x]^2)/(3 + m
) + Sin[a + b*x]^4/(5 + m)))/b
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3042, 3044, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(a+bx)(c\sin(a+bx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(a+bx)^5(c\sin(a+bx))^m dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{\int \frac{(c\sin(a+bx))^m\left(c^2-c^2\sin^2(a+bx)\right)^2}{c^4}d(c\sin(a+bx))}{bc}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (c\sin(a+bx))^m\left(c^2-c^2\sin^2(a+bx)\right)^2d(c\sin(a+bx))}{bc^5}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(c^4(c\sin(a+bx))^m-2c^2(c\sin(a+bx))^{m+2}+(c\sin(a+bx))^{m+4}\right)d(c\sin(a+bx))}{bc^5}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{c^4(c\sin(a+bx))^{m+1}}{m+1}-\frac{2c^2(c\sin(a+bx))^{m+3}}{m+3}+\frac{(c\sin(a+bx))^{m+5}}{m+5}}{bc^5}$$

input
```
Int[Cos[a + b*x]^5*(c*Sin[a + b*x])^m,x]
```

output
```
((c^4*(c*Sin[a + b*x])^(1 + m))/(1 + m) - (2*c^2*(c*Sin[a + b*x])^(3 + m))
/(3 + m) + (c*Sin[a + b*x])^(5 + m)/(5 + m))/(b*c^5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 5.84 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.18

method	result	size
parallelrisch	$\frac{\left(\left(\frac{3}{2} m^{2}+14 m +\frac{25}{2}\right) \sin \left(3 b x +3 a \right)+\left(\frac{1}{2} m^{2}+2 m +\frac{3}{2}\right) \sin \left(5 b x +5 a \right)+\sin \left(b x +a \right) \left(m^{2}+12 m +75\right)\right) \left(c \sin \left(b x +a \right)\right)^{m}}{8 b \left(m^{3}+9 m^{2}+23 m +15\right)}$	87
derivativedivides	$\frac{\sin \left(b x +a \right) {\mathrm e}^{m \ln \left(c \sin \left(b x +a \right)\right)}}{b \left(1+m \right)}+\frac{\sin \left(b x +a \right)^{5} {\mathrm e}^{m \ln \left(c \sin \left(b x +a \right)\right)}}{b \left(5+m \right)}-\frac{2 \sin \left(b x +a \right)^{3} {\mathrm e}^{m \ln \left(c \sin \left(b x +a \right)\right)}}{b \left(3+m \right)}$	88
default	$\frac{\sin \left(b x +a \right) {\mathrm e}^{m \ln \left(c \sin \left(b x +a \right)\right)}}{b \left(1+m \right)}+\frac{\sin \left(b x +a \right)^{5} {\mathrm e}^{m \ln \left(c \sin \left(b x +a \right)\right)}}{b \left(5+m \right)}-\frac{2 \sin \left(b x +a \right)^{3} {\mathrm e}^{m \ln \left(c \sin \left(b x +a \right)\right)}}{b \left(3+m \right)}$	88

input
```
int(cos(b*x+a)^5*(c*sin(b*x+a))^m,x,method=_RETURNVERBOSE)
```

output
```
1/8*((3/2*m^2+14*m+25/2)*sin(3*b*x+3*a)+(1/2*m^2+2*m+3/2)*sin(5*b*x+5*a)+s
in(b*x+a)*(m^2+12*m+75))*(c*sin(b*x+a))^m/b/(m^3+9*m^2+23*m+15)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

$$\int \cos^5(a+bx)(c\sin(a+bx))^m\,dx = \frac{\left(\left(m^2+4\,m+3\right)\cos\left(bx+a\right)^4+4\,(m+1)\cos\left(bx+a\right)^2+8\right)\left(c\sin\left(bx+a\right)\right)^m\sin\left(bx+a\right)}{bm^3+9\,bm^2+23\,bm+15\,b}$$

input
```
integrate(cos(b*x+a)^5*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

output
```
((m^2 + 4*m + 3)*cos(b*x + a)^4 + 4*(m + 1)*cos(b*x + a)^2 + 8)*(c*sin(b*x
 + a))^m*sin(b*x + a)/(b*m^3 + 9*b*m^2 + 23*b*m + 15*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2040 vs. 2(60) = 120.

Time = 4.32 (sec) , antiderivative size = 2040, normalized size of antiderivative = 27.57

$$\int \cos^5(a+bx)(c\sin(a+bx))^m\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)**5*(c*sin(b*x+a))**m,x)
```

output
```
Piecewise((x*(c*sin(a))**m*cos(a)**5, Eq(b, 0)), ((log(sin(a + b*x))/b + c
os(a + b*x)**2/(2*b*sin(a + b*x)**2) - cos(a + b*x)**4/(4*b*sin(a + b*x)**
4))/c**5, Eq(m, -5)), ((16*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**
6/(8*b*tan(a/2 + b*x/2)**6 + 16*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/
2)**2) + 32*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**4/(8*b*tan(a/2
+ b*x/2)**6 + 16*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) + 16*log
(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**2/(8*b*tan(a/2 + b*x/2)**6 + 1
6*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) - 16*log(tan(a/2 + b*x/
2))*tan(a/2 + b*x/2)**6/(8*b*tan(a/2 + b*x/2)**6 + 16*b*tan(a/2 + b*x/2)**
4 + 8*b*tan(a/2 + b*x/2)**2) - 32*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**
4/(8*b*tan(a/2 + b*x/2)**6 + 16*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/
2)**2) - 16*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**2/(8*b*tan(a/2 + b*x/2
)**6 + 16*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) - tan(a/2 + b*x
/2)**8/(8*b*tan(a/2 + b*x/2)**6 + 16*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 +
 b*x/2)**2) + 18*tan(a/2 + b*x/2)**4/(8*b*tan(a/2 + b*x/2)**6 + 16*b*tan(a
/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) - 1/(8*b*tan(a/2 + b*x/2)**6 + 1
6*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2))/c**3, Eq(m, -3)), ((-l
og(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**8/(b*tan(a/2 + b*x/2)**8 + 4
*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2
 + b) - 4*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**6/(b*tan(a/2 +...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.04

$$\int \cos^5(a+bx)(c\sin(a+bx))^m\,dx = \frac{\frac{c^m\sin(bx+a)^m\sin(bx+a)^5}{m+5} - \frac{2\,c^m\sin(bx+a)^m\sin(bx+a)^3}{m+3} + \frac{(c\sin(bx+a))^{m+1}}{c(m+1)}}{b}$$

input
```
integrate(cos(b*x+a)^5*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

output
```
(c^m*sin(b*x + a)^m*sin(b*x + a)^5/(m + 5) - 2*c^m*sin(b*x + a)^m*sin(b*x
+ a)^3/(m + 3) + (c*sin(b*x + a))^(m + 1)/(c*(m + 1)))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(74) = 148$.

Time = 0.14 (sec) , antiderivative size = 248, normalized size of antiderivative = 3.35

$$\int \cos^5(a+bx)(c\sin(a+bx))^m\,dx$$
$$= \frac{(c\sin{(bx+a)})^m c^5m^2\sin{(bx+a)}^5 + 4\,(c\sin{(bx+a)})^m c^5m\sin{(bx+a)}^5 - 2\,(c\sin{(bx+a)})^m c^5m^2\sin}{}$$

input
```
integrate(cos(b*x+a)^5*(c*sin(b*x+a))^m,x, algorithm="giac")
```

output
```
((c*sin(b*x + a))^m*c^5*m^2*sin(b*x + a)^5 + 4*(c*sin(b*x + a))^m*c^5*m*si
n(b*x + a)^5 - 2*(c*sin(b*x + a))^m*c^5*m^2*sin(b*x + a)^3 + 3*(c*sin(b*x
+ a))^m*c^5*sin(b*x + a)^5 - 12*(c*sin(b*x + a))^m*c^5*m*sin(b*x + a)^3 +
(c*sin(b*x + a))^m*c^5*m^2*sin(b*x + a) - 10*(c*sin(b*x + a))^m*c^5*sin(b*
x + a)^3 + 8*(c*sin(b*x + a))^m*c^5*m*sin(b*x + a) + 15*(c*sin(b*x + a))^m
*c^5*sin(b*x + a))/((c^4*m^3 + 9*c^4*m^2 + 23*c^4*m + 15*c^4)*b*c)
```

Mupad [B] (verification not implemented)

Time = 26.98 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.78

$$\int \cos^5(a+bx)(c\sin(a+bx))^m\,dx$$
$$= \frac{(c\,\sin{(a+b\,x)})^m\,(150\,\sin{(a+b\,x)} + 25\,\sin{(3\,a+3\,b\,x)} + 3\,\sin{(5\,a+5\,b\,x)} + 24\,m\,\sin{(a+b\,x)} + 28\,n}{16\,b\,(m^3+9\,n}$$

input
```
int(cos(a + b*x)^5*(c*sin(a + b*x))^m,x)
```

output
```
((c*sin(a + b*x))^m*(150*sin(a + b*x) + 25*sin(3*a + 3*b*x) + 3*sin(5*a +
5*b*x) + 24*m*sin(a + b*x) + 28*m*sin(3*a + 3*b*x) + 4*m*sin(5*a + 5*b*x)
+ 2*m^2*sin(a + b*x) + 3*m^2*sin(3*a + 3*b*x) + m^2*sin(5*a + 5*b*x)))/(16
*b*(23*m + 9*m^2 + m^3 + 15))
```

Reduce [F]

$$\int \cos^5(a+bx)(c\sin(a+bx))^m\,dx = c^m\left(\int \sin\left(bx+a\right)^m\cos\left(bx+a\right)^5dx\right)$$

input
```
int(cos(b*x+a)^5*(c*sin(b*x+a))^m,x)
```

output
```
c**m*int(sin(a + b*x)**m*cos(a + b*x)**5,x)
```

3.341 $\int \cos^3(a+bx)(c\sin(a+bx))^m\,dx$

Optimal result

Integrand size = 19, antiderivative size = 50

$$\int \cos^3(a+bx)(c\sin(a+bx))^m\,dx=\frac{(c\sin(a+bx))^{1+m}}{bc(1+m)}-\frac{(c\sin(a+bx))^{3+m}}{bc^3(3+m)}$$

output
```
(c*sin(b*x+a))^(1+m)/b/c/(1+m)-(c*sin(b*x+a))^(3+m)/b/c^3/(3+m)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \cos^3(a+bx)(c\sin(a+bx))^m\,dx =\frac{(5+m+(1+m)\cos(2(a+bx)))\sin(a+bx)(c\sin(a+bx))^m}{2b(1+m)(3+m)}$$

input
```
Integrate[Cos[a + b*x]^3*(c*Sin[a + b*x])^m,x]
```

output
```
((5 + m + (1 + m)*Cos[2*(a + b*x)])*Sin[a + b*x]*(c*Sin[a + b*x])^m)/(2*b*
(1 + m)*(3 + m))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3042, 3044, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)(c\sin(a+bx))^m\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \cos(a+bx)^3(c\sin(a+bx))^m dx$$

$$\Big\downarrow \ 3044$$

$$\frac{\int \frac{(c\sin(a+bx))^m(c^2-c^2\sin^2(a+bx))}{c^2}d(c\sin(a+bx))}{bc}$$

$$\Big\downarrow \ 27$$

$$\frac{\int (c\sin(a+bx))^m\left(c^2-c^2\sin^2(a+bx)\right)d(c\sin(a+bx))}{bc^3}$$

$$\Big\downarrow \ 244$$

$$\frac{\int \left(c^2(c\sin(a+bx))^m-(c\sin(a+bx))^{m+2}\right)d(c\sin(a+bx))}{bc^3}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{c^2(c\sin(a+bx))^{m+1}}{m+1}-\frac{(c\sin(a+bx))^{m+3}}{m+3}}{bc^3}$$

input
```
Int[Cos[a + b*x]^3*(c*Sin[a + b*x])^m,x]
```

output
```
((c^2*(c*Sin[a + b*x])^(1 + m))/(1 + m) - (c*Sin[a + b*x])^(3 + m)/(3 + m)
)/(b*c^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 3.72 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{((1+m)\sin(3bx+3a)+\sin(bx+a)(9+m))(c\sin(bx+a))^m}{4b(m^2+4m+3)}$	50
derivativedivides	$\frac{\sin(bx+a)\mathrm{e}^{m\ln(c\sin(bx+a))}}{b(1+m)}-\frac{\sin(bx+a)^3\mathrm{e}^{m\ln(c\sin(bx+a))}}{b(3+m)}$	59
default	$\frac{\sin(bx+a)\mathrm{e}^{m\ln(c\sin(bx+a))}}{b(1+m)}-\frac{\sin(bx+a)^3\mathrm{e}^{m\ln(c\sin(bx+a))}}{b(3+m)}$	59

input
```
int(cos(b*x+a)^3*(c*sin(b*x+a))^m,x,method=_RETURNVERBOSE)
```

output
```
1/4*((1+m)*sin(3*b*x+3*a)+sin(b*x+a)*(9+m))*(c*sin(b*x+a))^m/b/(m^2+4*m+3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \cos^3(a+bx)(c\sin(a+bx))^m\,dx = \frac{\left((m+1)\cos\left(bx+a\right)^2+2\right)\left(c\sin\left(bx+a\right)\right)^m\sin\left(bx+a\right)}{bm^2+4\,bm+3\,b}$$

input
```
integrate(cos(b*x+a)^3*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

output
```
((m + 1)*cos(b*x + a)^2 + 2)*(c*sin(b*x + a))^m*sin(b*x + a)/(b*m^2 + 4*b*
m + 3*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 525 vs. 2(37) = 74.

Time = 1.21 (sec) , antiderivative size = 525, normalized size of antiderivative = 10.50

$$\int \cos^3(a+bx)(c\sin(a+bx))^m\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)**3*(c*sin(b*x+a))**m,x)
```

output
```
Piecewise((x*(c*sin(a))**m*cos(a)**3, Eq(b, 0)), ((-log(sin(a + b*x))/b -
cos(a + b*x)**2/(2*b*sin(a + b*x)**2))/c**3, Eq(m, -3)), ((-log(tan(a/2 +
b*x/2)**2 + 1)*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**4 + 2*b*tan(a/2 +
b*x/2)**2 + b) - 2*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**2/(b*tan
(a/2 + b*x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b) - log(tan(a/2 + b*x/2)**2
+ 1)/(b*tan(a/2 + b*x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b) + log(tan(a/2 +
 b*x/2))*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**4 + 2*b*tan(a/2 + b*x/2)
**2 + b) + 2*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)
**4 + 2*b*tan(a/2 + b*x/2)**2 + b) + log(tan(a/2 + b*x/2))/(b*tan(a/2 + b*
x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b) - 2*tan(a/2 + b*x/2)**2/(b*tan(a/2
+ b*x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b))/c, Eq(m, -1)), (m*(c*sin(a + b
*x))**m*sin(a + b*x)*cos(a + b*x)**2/(b*m**2 + 4*b*m + 3*b) + 2*(c*sin(a +
 b*x))**m*sin(a + b*x)**3/(b*m**2 + 4*b*m + 3*b) + 3*(c*sin(a + b*x))**m*s
in(a + b*x)*cos(a + b*x)**2/(b*m**2 + 4*b*m + 3*b), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \cos^3(a+bx)(c\sin(a+bx))^m\,dx = -\frac{\frac{c^m\sin(bx+a)^m\sin(bx+a)^3}{m+3} - \frac{(c\sin(bx+a))^{m+1}}{c(m+1)}}{b}$$

input
```
integrate(cos(b*x+a)^3*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

output
```
-(c^m*sin(b*x + a)^m*sin(b*x + a)^3/(m + 3) - (c*sin(b*x + a))^(m + 1)/(c*
(m + 1)))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(50) = 100.

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.36

$$\int \cos^3(a+bx)(c\sin(a+bx))^m\,dx = -\frac{(c\sin(bx+a))^m c^3 m\sin(bx+a)^3 + (c\sin(bx+a))^m c^3\sin(bx+a)^3 - (c\sin(bx+a))^m c^3 m\sin(bx+}{(c^2m^2+4c^2m+3c^2)bc}$$

input
```
integrate(cos(b*x+a)^3*(c*sin(b*x+a))^m,x, algorithm="giac")
```

output
```
-((c*sin(b*x + a))^m*c^3*m*sin(b*x + a)^3 + (c*sin(b*x + a))^m*c^3*sin(b*x
 + a)^3 - (c*sin(b*x + a))^m*c^3*m*sin(b*x + a) - 3*(c*sin(b*x + a))^m*c^3
*sin(b*x + a))/((c^2*m^2 + 4*c^2*m + 3*c^2)*b*c)
```

Mupad [B] (verification not implemented)

Time = 26.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.24

$$\int \cos^3(a+bx)(c\sin(a+bx))^m\,dx = \frac{(c\,\sin\left(a+b\,x\right))^m\,\left(9\,\sin\left(a+b\,x\right)+\sin\left(3\,a+3\,b\,x\right)+m\,\sin\left(a+b\,x\right)+m\,\sin\left(3\,a+3\,b\,x\right)\right)}{4\,b\,\left(m^2+4\,m+3\right)}$$

input
```
int(cos(a + b*x)^3*(c*sin(a + b*x))^m,x)
```

output
```
((c*sin(a + b*x))^m*(9*sin(a + b*x) + sin(3*a + 3*b*x) + m*sin(a + b*x) +
m*sin(3*a + 3*b*x)))/(4*b*(4*m + m^2 + 3))
```

Reduce [F]

$$\int \cos^3(a+bx)(c\sin(a+bx))^m\,dx = c^m\left(\int \sin\left(bx+a\right)^m\cos\left(bx+a\right)^3dx\right)$$

input
```
int(cos(b*x+a)^3*(c*sin(b*x+a))^m,x)
```

output
```
c**m*int(sin(a + b*x)**m*cos(a + b*x)**3,x)
```

3.342 $\int \cos(a+bx)(c\sin(a+bx))^m\,dx$

Optimal result

Integrand size = 17, antiderivative size = 24

$$\int \cos(a+bx)(c\sin(a+bx))^m\,dx=\frac{(c\sin(a+bx))^{1+m}}{bc(1+m)}$$

output
```
(c*sin(b*x+a))^(1+m)/b/c/(1+m)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \cos(a+bx)(c\sin(a+bx))^m\,dx=\frac{\sin(a+bx)(c\sin(a+bx))^m}{b(1+m)}$$

input
```
Integrate[Cos[a + b*x]*(c*Sin[a + b*x])^m,x]
```

output
```
(Sin[a + b*x]*(c*Sin[a + b*x])^m)/(b*(1 + m))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3042, 3044, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)(c\sin(a+bx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(a+bx)(c\sin(a+bx))^m dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{\int (c\sin(a+bx))^m d(c\sin(a+bx))}{bc}$$

$$\Big\downarrow \text{15}$$

$$\frac{(c\sin(a+bx))^{m+1}}{bc(m+1)}$$

input
```
Int[Cos[a + b*x]*(c*Sin[a + b*x])^m,x]
```

output
```
(c*Sin[a + b*x])^(1 + m)/(b*c*(1 + m))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 8.79 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{(c\sin(bx+a))^{1+m}}{bc(1+m)}$
default	$\frac{(c\sin(bx+a))^{1+m}}{bc(1+m)}$
parallelrisch	$\frac{\sin(bx+a)(c\sin(bx+a))^{m}}{b(1+m)}$
norman	$\frac{2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\mathrm{e}^{m\ln\left(\frac{2c\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\right)}}{b(1+m)\left(1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)}$
risch	$\left(\mathrm{e}^{i(bx+a)}\right)^{-m}(\sin(bx)\cos(a)+\cos(bx)\sin(a))\left(\mathrm{e}^{2i(bx+a)}-1\right)^{m}\left(\frac{1}{2}\right)^{m}c^{m}\mathrm{e}^{-\frac{i\pi m\left(-\operatorname{csgn}\left(i\left(\mathrm{e}^{2i(bx+a)}-1\right)\right)\operatorname{csgn}(\sin(bx)\cos(a}{}}}$

input
```
int(cos(b*x+a)*(c*sin(b*x+a))^m,x,method=_RETURNVERBOSE)
```

output
```
(c*sin(b*x+a))^(1+m)/b/c/(1+m)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int\cos(a+bx)(c\sin(a+bx))^m\,dx=\frac{(c\sin{(bx+a)})^m\sin{(bx+a)}}{bm+b}$$

input
```
integrate(cos(b*x+a)*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

output
```
(c*sin(b*x + a))^m*sin(b*x + a)/(b*m + b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(17) = 34$.

Time = 0.34 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.33

$$\int \cos(a+bx)(c\sin(a+bx))^m\,dx = \begin{cases} \frac{x\cos(a)}{c\sin(a)} & \text{for } b = 0 \wedge m = -1 \\ x(c\sin(a))^m\cos(a) & \text{for } b = 0 \\ \frac{\log(\sin(a+bx))}{bc} & \text{for } m = -1 \\ \frac{(c\sin(a+bx))^m\sin(a+bx)}{bm+b} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a)*(c*sin(b*x+a))**m,x)
```

output
```
Piecewise((x*cos(a)/(c*sin(a)), Eq(b, 0) & Eq(m, -1)), (x*(c*sin(a))**m*co
s(a), Eq(b, 0)), (log(sin(a + b*x))/(b*c), Eq(m, -1)), ((c*sin(a + b*x))**
m*sin(a + b*x)/(b*m + b), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \cos(a+bx)(c\sin(a+bx))^m\,dx = \frac{(c\sin(bx+a))^{m+1}}{bc(m+1)}$$

input
```
integrate(cos(b*x+a)*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

output
```
(c*sin(b*x + a))^(m + 1)/(b*c*(m + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \cos(a+bx)(c\sin(a+bx))^m\,dx = \frac{(c\sin(bx+a))^{m+1}}{bc(m+1)}$$

input
```
integrate(cos(b*x+a)*(c*sin(b*x+a))^m,x, algorithm="giac")
```

output
```
(c*sin(b*x + a))^(m + 1)/(b*c*(m + 1))
```

Mupad [B] (verification not implemented)

Time = 25.85 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \cos(a+bx)(c\sin(a+bx))^m\,dx = \frac{\sin(a+b\,x)\,\left(c\,\sin(a+b\,x)\right)^m}{b\,(m+1)}$$

input
```
int(cos(a + b*x)*(c*sin(a + b*x))^m,x)
```

output
```
(sin(a + b*x)*(c*sin(a + b*x))^m)/(b*(m + 1))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \cos(a+bx)(c\sin(a+bx))^m\,dx = \frac{c^m\sin(bx+a)^m\sin(bx+a)}{b\,(m+1)}$$

input
```
int(cos(b*x+a)*(c*sin(b*x+a))^m,x)
```

output
```
(c**m*sin(a + b*x)**m*sin(a + b*x))/(b*(m + 1))
```

3.343 $\int \sec(a+bx)(c\sin(a+bx))^m\,dx$

Optimal result

Integrand size = 17, antiderivative size = 48

$$\int \sec(a+bx)(c\sin(a+bx))^m\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bc(1+m)}$$

output
```
hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)*(c*sin(b*x+a))^(1+m)/b/
c/(1+m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \sec(a+bx)(c\sin(a+bx))^m\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},1+\frac{1+m}{2},\sin^2(a+bx)\right)\sin(a+bx)(c\sin(a+bx))^m}{b(1+m)}$$

input
```
Integrate[Sec[a + b*x]*(c*Sin[a + b*x])^m,x]
```

output
```
(Hypergeometric2F1[1, (1 + m)/2, 1 + (1 + m)/2, Sin[a + b*x]^2]*Sin[a + b*
x]*(c*Sin[a + b*x])^m)/(b*(1 + m))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 3044, 27, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(a+bx)(c\sin(a+bx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c\sin(a+bx))^m}{\cos(a+bx)}dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{\int \frac{c^2(c\sin(a+bx))^m}{c^2-c^2\sin^2(a+bx)}d(c\sin(a+bx))}{bc}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\int \frac{(c\sin(a+bx))^m}{c^2-c^2\sin^2(a+bx)}d(c\sin(a+bx))}{b}$$

$$\Big\downarrow \text{278}$$

$$\frac{(c\sin(a+bx))^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},\sin^2(a+bx)\right)}{bc(m+1)}$$

input
```
Int[Sec[a + b*x]*(c*Sin[a + b*x])^m,x]
```

output
```
(Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[a + b*x]^2]*(c*Sin[a + b*x
])^(1 + m))/(b*c*(1 + m))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Maple [F]

$$\int \sec (bx + a) (c \sin (bx + a))^m dx$$

input
```
int(sec(b*x+a)*(c*sin(b*x+a))^m,x)
```

output
```
int(sec(b*x+a)*(c*sin(b*x+a))^m,x)
```

Fricas [F]

$$\int \sec(a+bx)(c\sin(a+bx))^m\,dx = \int (c\sin(bx+a))^m \sec(bx+a)\,dx$$

input
```
integrate(sec(b*x+a)*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

output
```
integral((c*sin(b*x + a))^m*sec(b*x + a), x)
```

Sympy [F]

$$\int \sec(a+bx)(c\sin(a+bx))^m\,dx = \int (c\sin(a+bx))^m \sec(a+bx)\,dx$$

input
```
integrate(sec(b*x+a)*(c*sin(b*x+a))**m,x)
```

output
```
Integral((c*sin(a + b*x))**m*sec(a + b*x), x)
```

Maxima [F]

$$\int \sec(a+bx)(c\sin(a+bx))^m\,dx = \int (c\sin(bx+a))^m \sec(bx+a)\,dx$$

input
```
integrate(sec(b*x+a)*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a))^m*sec(b*x + a), x)
```

Giac [F]

$$\int \sec(a+bx)(c\sin(a+bx))^m\,dx = \int (c\sin(bx+a))^m \sec(bx+a)\,dx$$

input
```
integrate(sec(b*x+a)*(c*sin(b*x+a))^m,x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a))^m*sec(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(a+bx)(c\sin(a+bx))^m\,dx = \int \frac{(c\,\sin(a+b\,x))^m}{\cos(a+b\,x)}\,dx$$

input
```
int((c*sin(a + b*x))^m/cos(a + b*x),x)
```

output
```
int((c*sin(a + b*x))^m/cos(a + b*x), x)
```

Reduce [F]

$$\int \sec(a+bx)(c\sin(a+bx))^m\,dx = c^m\left(\int \sin(bx+a)^m \sec(bx+a)\,dx\right)$$

input
```
int(sec(b*x+a)*(c*sin(b*x+a))^m,x)
```

output
```
c**m*int(sin(a + b*x)**m*sec(a + b*x),x)
```

3.344 $\int \sec^3(a+bx)(c\sin(a+bx))^m\,dx$

Optimal result

Integrand size = 19, antiderivative size = 48

$$\int \sec^3(a+bx)(c\sin(a+bx))^m\,dx = \frac{\text{Hypergeometric2F1}\left(2,\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bc(1+m)}$$

output
```
hypergeom([2, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)*(c*sin(b*x+a))^(1+m)/b/
c/(1+m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \sec^3(a+bx)(c\sin(a+bx))^m\,dx = \frac{\text{Hypergeometric2F1}\left(2,\frac{1+m}{2},1+\frac{1+m}{2},\sin^2(a+bx)\right)\sin(a+bx)(c\sin(a+bx))^m}{b(1+m)}$$

input
```
Integrate[Sec[a + b*x]^3*(c*Sin[a + b*x])^m,x]
```

output
```
(Hypergeometric2F1[2, (1 + m)/2, 1 + (1 + m)/2, Sin[a + b*x]^2]*Sin[a + b*
x]*(c*Sin[a + b*x])^m)/(b*(1 + m))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3044, 27, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(a+bx)(c\sin(a+bx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c\sin(a+bx))^m}{\cos(a+bx)^3}dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{\int \frac{c^4(c\sin(a+bx))^m}{\left(c^2-c^2\sin^2(a+bx)\right)^2}d(c\sin(a+bx))}{bc}$$

$$\Big\downarrow \text{27}$$

$$\frac{c^3\int \frac{(c\sin(a+bx))^m}{\left(c^2-c^2\sin^2(a+bx)\right)^2}d(c\sin(a+bx))}{b}$$

$$\Big\downarrow \text{278}$$

$$\frac{(c\sin(a+bx))^{m+1}\,\text{Hypergeometric2F1}\left(2,\frac{m+1}{2},\frac{m+3}{2},\sin^2(a+bx)\right)}{bc(m+1)}$$

input
```
Int[Sec[a + b*x]^3*(c*Sin[a + b*x])^m,x]
```

output
```
(Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, Sin[a + b*x]^2]*(c*Sin[a + b*x
])^(1 + m))/(b*c*(1 + m))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Maple [F]

$$\int \sec (bx + a)^3 (c \sin (bx + a))^m dx$$

input
```
int(sec(b*x+a)^3*(c*sin(b*x+a))^m,x)
```

output
```
int(sec(b*x+a)^3*(c*sin(b*x+a))^m,x)
```

Fricas [F]

$$\int \sec^3(a+bx)(c\sin(a+bx))^m\,dx = \int (c\sin(bx+a))^m \sec(bx+a)^3\,dx$$

input
```
integrate(sec(b*x+a)^3*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

output
```
integral((c*sin(b*x + a))^m*sec(b*x + a)^3, x)
```

Sympy [F]

$$\int \sec^3(a+bx)(c\sin(a+bx))^m\,dx = \int (c\sin(a+bx))^m \sec^3(a+bx)\,dx$$

input
```
integrate(sec(b*x+a)**3*(c*sin(b*x+a))**m,x)
```

output
```
Integral((c*sin(a + b*x))**m*sec(a + b*x)**3, x)
```

Maxima [F]

$$\int \sec^3(a+bx)(c\sin(a+bx))^m\,dx = \int (c\sin(bx+a))^m \sec(bx+a)^3\,dx$$

input
```
integrate(sec(b*x+a)^3*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a))^m*sec(b*x + a)^3, x)
```

Giac [F]

$$\int \sec^3(a+bx)(c\sin(a+bx))^m\,dx = \int (c\sin(bx+a))^m \sec(bx+a)^3\,dx$$

input
```
integrate(sec(b*x+a)^3*(c*sin(b*x+a))^m,x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a))^m*sec(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^3(a+bx)(c\sin(a+bx))^m\,dx = \int \frac{(c\sin(a+b\,x))^m}{\cos(a+b\,x)^3}\,dx$$

input
```
int((c*sin(a + b*x))^m/cos(a + b*x)^3,x)
```

output
```
int((c*sin(a + b*x))^m/cos(a + b*x)^3, x)
```

Reduce [F]

$$\int \sec^3(a+bx)(c\sin(a+bx))^m\,dx = c^m\left(\int \sin(bx+a)^m \sec(bx+a)^3\,dx\right)$$

input
```
int(sec(b*x+a)^3*(c*sin(b*x+a))^m,x)
```

output
```
c**m*int(sin(a + b*x)**m*sec(a + b*x)**3,x)
```

3.345 $\int \cos^4(a+bx)(c\sin(a+bx))^m\,dx$

Optimal result

Integrand size = 19, antiderivative size = 68

$$\int \cos^4(a+bx)(c\sin(a+bx))^m\,dx = \frac{\cos(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bc(1+m)\sqrt{\cos^2(a+bx)}}$$

output
```
cos(b*x+a)*hypergeom([-3/2, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)*(c*sin(b*
x+a))^(1+m)/b/c/(1+m)/(cos(b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

$$\int \cos^4(a+bx)(c\sin(a+bx))^m\,dx = \frac{\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^m\tan(a+bx)}{b(1+m)}$$

input
```
Integrate[Cos[a + b*x]^4*(c*Sin[a + b*x])^m,x]
```

output
```
(Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[-3/2, (1 + m)/2, (3 + m)/2, Sin[a
+ b*x]^2]*(c*Sin[a + b*x])^m*Tan[a + b*x])/(b*(1 + m))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(a+bx)(c\sin(a+bx))^m\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \cos(a+bx)^4(c\sin(a+bx))^m dx$$

$$\Big\downarrow \ 3057$$

$$\frac{\cos(a+bx)(c\sin(a+bx))^{m+1}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(a+bx)\right)}{bc(m+1)\sqrt{\cos^2(a+bx)}}$$

input
```
Int[Cos[a + b*x]^4*(c*Sin[a + b*x])^m,x]
```

output
```
(Cos[a + b*x]*Hypergeometric2F1[-3/2, (1 + m)/2, (3 + m)/2, Sin[a + b*x]^2
]*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m)*Sqrt[Cos[a + b*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Maple [F]

$$\int \cos\left(bx+a\right)^{4}\left(c\sin\left(bx+a\right)\right)^{m}dx$$

input
```
int(cos(b*x+a)^4*(c*sin(b*x+a))^m,x)
```

output
```
int(cos(b*x+a)^4*(c*sin(b*x+a))^m,x)
```

Fricas [F]

$$\int \cos^4(a+bx)(c\sin(a+bx))^m\,dx = \int \left(c\sin\left(bx+a\right)\right)^{m}\cos\left(bx+a\right)^{4}\,dx$$

input
```
integrate(cos(b*x+a)^4*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

output
```
integral((c*sin(b*x + a))^m*cos(b*x + a)^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^4(a+bx)(c\sin(a+bx))^m\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**4*(c*sin(b*x+a))**m,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^4(a+bx)(c\sin(a+bx))^m\,dx = \int (c\sin(bx+a))^m\cos(bx+a)^4\,dx$$

input
```
integrate(cos(b*x+a)^4*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a))^m*cos(b*x + a)^4, x)
```

Giac [F]

$$\int \cos^4(a+bx)(c\sin(a+bx))^m\,dx = \int (c\sin(bx+a))^m\cos(bx+a)^4\,dx$$

input
```
integrate(cos(b*x+a)^4*(c*sin(b*x+a))^m,x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a))^m*cos(b*x + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^4(a+bx)(c\sin(a+bx))^m\,dx = \int \cos(a+b\,x)^4\,(c\,\sin(a+b\,x))^m\,dx$$

input
```
int(cos(a + b*x)^4*(c*sin(a + b*x))^m,x)
```

output
```
int(cos(a + b*x)^4*(c*sin(a + b*x))^m, x)
```

Reduce [F]

$$\int \cos^4(a+bx)(c\sin(a+bx))^m\,dx = c^m\left(\int \sin\left(bx+a\right)^m\cos\left(bx+a\right)^4dx\right)$$

input
```
int(cos(b*x+a)^4*(c*sin(b*x+a))^m,x)
```

output
```
c**m*int(sin(a + b*x)**m*cos(a + b*x)**4,x)
```

3.346 $\int \cos^2(a+bx)(c\sin(a+bx))^m\,dx$

Optimal result

Integrand size = 19, antiderivative size = 68

$$\int \cos^2(a+bx)(c\sin(a+bx))^m\,dx = \frac{\cos(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bc(1+m)\sqrt{\cos^2(a+bx)}}$$

output
```
cos(b*x+a)*hypergeom([-1/2, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)*(c*sin(b*
x+a))^(1+m)/b/c/(1+m)/(cos(b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

$$\int \cos^2(a+bx)(c\sin(a+bx))^m\,dx = \frac{\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^m\tan(a+bx)}{b(1+m)}$$

input
```
Integrate[Cos[a + b*x]^2*(c*Sin[a + b*x])^m,x]
```

output
```
(Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/2, Sin[a
+ b*x]^2]*(c*Sin[a + b*x])^m*Tan[a + b*x])/(b*(1 + m))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)(c\sin(a+bx))^m\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \cos(a+bx)^2(c\sin(a+bx))^m dx$$

$$\Big\downarrow \text{ 3057}$$

$$\frac{\cos(a+bx)(c\sin(a+bx))^{m+1}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(a+bx)\right)}{bc(m+1)\sqrt{\cos^2(a+bx)}}$$

input
```
Int[Cos[a + b*x]^2*(c*Sin[a + b*x])^m,x]
```

output
```
(Cos[a + b*x]*Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/2, Sin[a + b*x]^2
]*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m)*Sqrt[Cos[a + b*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Maple [F]

$$\int \cos \left(bx+a\right)^{2} \left(c\sin \left(bx+a\right)\right)^{m} dx$$

input
```
int(cos(b*x+a)^2*(c*sin(b*x+a))^m,x)
```

output
```
int(cos(b*x+a)^2*(c*sin(b*x+a))^m,x)
```

Fricas [F]

$$\int \cos^2(a+bx)(c\sin(a+bx))^m\,dx = \int \left(c\sin \left(bx+a\right)\right)^{m} \cos \left(bx+a\right)^{2}\,dx$$

input
```
integrate(cos(b*x+a)^2*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

output
```
integral((c*sin(b*x + a))^m*cos(b*x + a)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(a+bx)(c\sin(a+bx))^m\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2*(c*sin(b*x+a))**m,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^2(a+bx)(c\sin(a+bx))^m\,dx = \int (c\sin(bx+a))^m \cos(bx+a)^2\,dx$$

input
```
integrate(cos(b*x+a)^2*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a))^m*cos(b*x + a)^2, x)
```

Giac [F]

$$\int \cos^2(a+bx)(c\sin(a+bx))^m\,dx = \int (c\sin(bx+a))^m \cos(bx+a)^2\,dx$$

input
```
integrate(cos(b*x+a)^2*(c*sin(b*x+a))^m,x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a))^m*cos(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(a+bx)(c\sin(a+bx))^m\,dx = \int \cos(a+b\,x)^2\,(c\,\sin(a+b\,x))^m\,dx$$

input
```
int(cos(a + b*x)^2*(c*sin(a + b*x))^m,x)
```

output
```
int(cos(a + b*x)^2*(c*sin(a + b*x))^m, x)
```

Reduce [F]

$$\int \cos^2(a+bx)(c\sin(a+bx))^m\,dx = c^m\left(\int \sin(bx+a)^m\cos(bx+a)^2\,dx\right)$$

input
```
int(cos(b*x+a)^2*(c*sin(b*x+a))^m,x)
```

output
```
c**m*int(sin(a + b*x)**m*cos(a + b*x)**2,x)
```

3.347 $\int (c\sin(a+bx))^m\,dx$

Optimal result

Integrand size = 10, antiderivative size = 68

$$\int (c\sin(a+bx))^m\,dx = \frac{\cos(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bc(1+m)\sqrt{\cos^2(a+bx)}}$$

output
```
cos(b*x+a)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)*(c*sin(b*x
+a))^(1+m)/b/c/(1+m)/(cos(b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

$$\int (c\sin(a+bx))^m\,dx = \frac{\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^m\tan(a+bx)}{b(1+m)}$$

input
```
Integrate[(c*Sin[a + b*x])^m,x]
```

output
```
(Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[a +
 b*x]^2]*(c*Sin[a + b*x])^m*Tan[a + b*x])/(b*(1 + m))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\sin(a+bx))^m\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (c\sin(a+bx))^m dx$$

$$\Big\downarrow \text{ 3122}$$

$$\frac{\cos(a+bx)(c\sin(a+bx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(a+bx)\right)}{bc(m+1)\sqrt{\cos^2(a+bx)}}$$

input
```
Int[(c*Sin[a + b*x])^m,x]
```

output
```
(Cos[a + b*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[a + b*x]^2]
*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m)*Sqrt[Cos[a + b*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Maple [F]

$$\int (c\sin (bx+a))^m\,dx$$

input
```
int((c*sin(b*x+a))^m,x)
```

output
```
int((c*sin(b*x+a))^m,x)
```

Fricas [F]

$$\int (c\sin(a+bx))^m\,dx = \int (c\sin (bx+a))^m\,dx$$

input
```
integrate((c*sin(b*x+a))^m,x, algorithm="fricas")
```

output
```
integral((c*sin(b*x + a))^m, x)
```

Sympy [F]

$$\int (c\sin(a+bx))^m\,dx = \int (c\sin (a+bx))^m\,dx$$

input
```
integrate((c*sin(b*x+a))**m,x)
```

output
```
Integral((c*sin(a + b*x))**m, x)
```

Maxima [F]

$$\int (c\sin(a+bx))^m\,dx = \int (c\sin(bx+a))^m\,dx$$

input
```
integrate((c*sin(b*x+a))^m,x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a))^m, x)
```

Giac [F]

$$\int (c\sin(a+bx))^m\,dx = \int (c\sin(bx+a))^m\,dx$$

input
```
integrate((c*sin(b*x+a))^m,x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c\sin(a+bx))^m\,dx = \int (c\,\sin(a+b\,x))^m\,dx$$

input
```
int((c*sin(a + b*x))^m,x)
```

output
```
int((c*sin(a + b*x))^m, x)
```

Reduce [F]

$$\int (c\sin(a+bx))^m\,dx = c^m\left(\int \sin{(bx+a)}^m\,dx\right)$$

input
```
int((c*sin(b*x+a))^m,x)
```

output
```
c**m*int(sin(a + b*x)**m,x)
```

3.348 $\int \sec^2(a+bx)(c\sin(a+bx))^m\,dx$

Optimal result

Integrand size = 19, antiderivative size = 68

$$\int \sec^2(a+bx)(c\sin(a+bx))^m\,dx = \frac{\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)\sec(a+bx)(c\sin(a+bx))^{1+m}}{bc(1+m)}$$

output
```
(cos(b*x+a)^2)^(1/2)*hypergeom([3/2, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)*
sec(b*x+a)*(c*sin(b*x+a))^(1+m)/b/c/(1+m)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

$$\int \sec^2(a+bx)(c\sin(a+bx))^m\,dx = \frac{\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^m\tan(a+bx)}{b(1+m)}$$

input
```
Integrate[Sec[a + b*x]^2*(c*Sin[a + b*x])^m,x]
```

output
```
(Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, Sin[a +
 b*x]^2]*(c*Sin[a + b*x])^m*Tan[a + b*x])/(b*(1 + m))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(a+bx)(c\sin(a+bx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c\sin(a+bx))^m}{\cos(a+bx)^2}dx$$

$$\Big\downarrow \text{3057}$$

$$\frac{\sqrt{\cos^2(a+bx)}\sec(a+bx)(c\sin(a+bx))^{m+1}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(a+bx)\right)}{bc(m+1)}$$

input
```
Int[Sec[a + b*x]^2*(c*Sin[a + b*x])^m,x]
```

output
```
(Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, Sin[a +
 b*x]^2]*Sec[a + b*x]*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Maple [F]

$$\int \sec(bx+a)^2 (c\sin(bx+a))^m\,dx$$

input
```
int(sec(b*x+a)^2*(c*sin(b*x+a))^m,x)
```

output
```
int(sec(b*x+a)^2*(c*sin(b*x+a))^m,x)
```

Fricas [F]

$$\int \sec^2(a+bx)(c\sin(a+bx))^m\,dx = \int (c\sin(bx+a))^m \sec(bx+a)^2\,dx$$

input
```
integrate(sec(b*x+a)^2*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

output
```
integral((c*sin(b*x + a))^m*sec(b*x + a)^2, x)
```

Sympy [F]

$$\int \sec^2(a+bx)(c\sin(a+bx))^m\,dx = \int (c\sin(a+bx))^m \sec^2(a+bx)\,dx$$

input
```
integrate(sec(b*x+a)**2*(c*sin(b*x+a))**m,x)
```

output
```
Integral((c*sin(a + b*x))**m*sec(a + b*x)**2, x)
```

Maxima [F]

$$\int \sec^2(a+bx)(c\sin(a+bx))^m\,dx=\int (c\sin{(bx+a)})^m\sec{(bx+a)}^2\,dx$$

input
```
integrate(sec(b*x+a)^2*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a))^m*sec(b*x + a)^2, x)
```

Giac [F]

$$\int \sec^2(a+bx)(c\sin(a+bx))^m\,dx=\int (c\sin{(bx+a)})^m\sec{(bx+a)}^2\,dx$$

input
```
integrate(sec(b*x+a)^2*(c*sin(b*x+a))^m,x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a))^m*sec(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^2(a+bx)(c\sin(a+bx))^m\,dx=\int \frac{(c\,\sin{(a+b\,x)})^m}{\cos{(a+b\,x)}^2}\,dx$$

input
```
int((c*sin(a + b*x))^m/cos(a + b*x)^2,x)
```

output
```
int((c*sin(a + b*x))^m/cos(a + b*x)^2, x)
```

Reduce [F]

$$\int \sec^2(a+bx)(c\sin(a+bx))^m\,dx = c^m\left(\int \sin(bx+a)^m \sec(bx+a)^2\,dx\right)$$

input
```
int(sec(b*x+a)^2*(c*sin(b*x+a))^m,x)
```

output
```
c**m*int(sin(a + b*x)**m*sec(a + b*x)**2,x)
```

3.349 $\int \sec^4(a+bx)(c\sin(a+bx))^m\,dx$

Optimal result

Integrand size = 19, antiderivative size = 68

$$\int \sec^4(a+bx)(c\sin(a+bx))^m\,dx = \frac{\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)\sec(a+bx)(c\sin(a+bx))^{1+m}}{bc(1+m)}$$

output
```
(cos(b*x+a)^2)^(1/2)*hypergeom([5/2, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)*
sec(b*x+a)*(c*sin(b*x+a))^(1+m)/b/c/(1+m)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

$$\int \sec^4(a+bx)(c\sin(a+bx))^m\,dx = \frac{\sqrt{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^m\tan(a+bx)}{b(1+m)}$$

input
```
Integrate[Sec[a + b*x]^4*(c*Sin[a + b*x])^m,x]
```

output
```
(Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[5/2, (1 + m)/2, (3 + m)/2, Sin[a +
 b*x]^2]*(c*Sin[a + b*x])^m*Tan[a + b*x])/(b*(1 + m))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(a+bx)(c\sin(a+bx))^m\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(c\sin(a+bx))^m}{\cos(a+bx)^4}dx$$

$$\Big\downarrow \ 3057$$

$$\frac{\sqrt{\cos^2(a+bx)}\sec(a+bx)(c\sin(a+bx))^{m+1}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(a+bx)\right)}{bc(m+1)}$$

input
```
Int[Sec[a + b*x]^4*(c*Sin[a + b*x])^m,x]
```

output
```
(Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[5/2, (1 + m)/2, (3 + m)/2, Sin[a +
 b*x]^2]*Sec[a + b*x]*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Maple [F]

$$\int \sec(bx+a)^4 (c\sin(bx+a))^m\,dx$$

input
```
int(sec(b*x+a)^4*(c*sin(b*x+a))^m,x)
```

output
```
int(sec(b*x+a)^4*(c*sin(b*x+a))^m,x)
```

Fricas [F]

$$\int \sec^4(a+bx)(c\sin(a+bx))^m\,dx = \int (c\sin(bx+a))^m \sec(bx+a)^4\,dx$$

input
```
integrate(sec(b*x+a)^4*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

output
```
integral((c*sin(b*x + a))^m*sec(b*x + a)^4, x)
```

Sympy [F]

$$\int \sec^4(a+bx)(c\sin(a+bx))^m\,dx = \int (c\sin(a+bx))^m \sec^4(a+bx)\,dx$$

input
```
integrate(sec(b*x+a)**4*(c*sin(b*x+a))**m,x)
```

output
```
Integral((c*sin(a + b*x))**m*sec(a + b*x)**4, x)
```

Maxima [F]

$$\int \sec^4(a+bx)(c\sin(a+bx))^m\,dx = \int (c\sin(bx+a))^m \sec(bx+a)^4\,dx$$

input
```
integrate(sec(b*x+a)^4*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a))^m*sec(b*x + a)^4, x)
```

Giac [F]

$$\int \sec^4(a+bx)(c\sin(a+bx))^m\,dx = \int (c\sin(bx+a))^m \sec(bx+a)^4\,dx$$

input
```
integrate(sec(b*x+a)^4*(c*sin(b*x+a))^m,x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a))^m*sec(b*x + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^4(a+bx)(c\sin(a+bx))^m\,dx = \int \frac{(c\,\sin(a+b\,x))^m}{\cos(a+b\,x)^4}\,dx$$

input
```
int((c*sin(a + b*x))^m/cos(a + b*x)^4,x)
```

output
```
int((c*sin(a + b*x))^m/cos(a + b*x)^4, x)
```

Reduce [F]

$$\int \sec^4(a+bx)(c\sin(a+bx))^m\,dx = c^m\left(\int \sin(bx+a)^m \sec(bx+a)^4\,dx\right)$$

input
```
int(sec(b*x+a)^4*(c*sin(b*x+a))^m,x)
```

output
```
c**m*int(sin(a + b*x)**m*sec(a + b*x)**4,x)
```

3.350 $\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^m\,dx$

Optimal result

Integrand size = 23, antiderivative size = 75

$$\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^m\,dx = \frac{d\sqrt{d\cos(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+}{bc(1+m)\sqrt[4]{\cos^2(a+bx)}}$$

output
```
d*(d*cos(b*x+a))^(1/2)*hypergeom([-1/4, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^
2)*(c*sin(b*x+a))^(1+m)/b/c/(1+m)/(cos(b*x+a)^2)^(1/4)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.04

$$\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^m\,dx = \frac{d^2\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a}{b(1+m)\sqrt{d\cos(a+bx)}}$$

input
```
Integrate[(d*Cos[a + b*x])^(3/2)*(c*Sin[a + b*x])^m,x]
```

output
```
(d^2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-1/4, (1 + m)/2, (3 + m)/2,
Sin[a + b*x]^2]*(c*Sin[a + b*x])^m*Tan[a + b*x])/(b*(1 + m)*Sqrt[d*Cos[a +
 b*x]])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^m dx$$

$$\Big\downarrow \text{3057}$$

$$\frac{d\sqrt{d\cos(a+bx)}(c\sin(a+bx))^{m+1}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{m+1}{2},\frac{m+3}{2},\sin^2(a+bx)\right)}{bc(m+1)\sqrt[4]{\cos^2(a+bx)}}$$

input
```
Int[(d*Cos[a + b*x])^(3/2)*(c*Sin[a + b*x])^m,x]
```

output
```
(d*Sqrt[d*Cos[a + b*x]]*Hypergeometric2F1[-1/4, (1 + m)/2, (3 + m)/2, Sin[
a + b*x]^2]*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m)*(Cos[a + b*x]^2)^(1/4))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Maple [F]

$$\int (d\cos(bx+a))^{\frac{3}{2}} (c\sin(bx+a))^m\,dx$$

input
```
int((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^m,x)
```

output
```
int((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^m,x)
```

Fricas [F]

$$\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^m\,dx = \int (d\cos(bx+a))^{\frac{3}{2}} (c\sin(bx+a))^m\,dx$$

input
```
integrate((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

output
```
integral(sqrt(d*cos(b*x + a))*(c*sin(b*x + a))^m*d*cos(b*x + a), x)
```

Sympy [F]

$$\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^m\,dx = \int (c\sin(a+bx))^m\,(d\cos(a+bx))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*cos(b*x+a))**(3/2)*(c*sin(b*x+a))**m,x)
```

output
```
Integral((c*sin(a + b*x))**m*(d*cos(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^m\,dx = \int (d\cos(bx+a))^{\frac{3}{2}}\,(c\sin(bx+a))^m\,dx$$

input
```
integrate((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

output
```
integrate((d*cos(b*x + a))^(3/2)*(c*sin(b*x + a))^m, x)
```

Giac [F]

$$\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^m\,dx = \int (d\cos(bx+a))^{\frac{3}{2}}\,(c\sin(bx+a))^m\,dx$$

input
```
integrate((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^m,x, algorithm="giac")
```

output
```
integrate((d*cos(b*x + a))^(3/2)*(c*sin(b*x + a))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^m\,dx = \int (d\,\cos(a+b\,x))^{3/2}\,(c\,\sin(a+b\,x))^m\,dx$$

input
```
int((d*cos(a + b*x))^(3/2)*(c*sin(a + b*x))^m,x)
```

output
```
int((d*cos(a + b*x))^(3/2)*(c*sin(a + b*x))^m, x)
```

Reduce [F]

$$\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^m\,dx = \sqrt{d}\,c^m\left(\int \sin(bx+a)^m\,\sqrt{\cos(bx+a)}\,\cos(bx+a)\,dx\right)d$$

input
```
int((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^m,x)
```

output
```
sqrt(d)*c**m*int(sin(a + b*x)**m*sqrt(cos(a + b*x))*cos(a + b*x),x)*d
```

3.351 $\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^m\,dx$

Optimal result

Integrand size = 23, antiderivative size = 75

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^m\,dx = \frac{d\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bc(1+m)\sqrt{d\cos(a+bx)}}$$

output
```
d*(cos(b*x+a)^2)^(1/4)*hypergeom([1/4, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2
)*(c*sin(b*x+a))^(1+m)/b/c/(1+m)/(d*cos(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^m\,dx = \frac{\sqrt{d\cos(a+bx)}\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^m\tan(a+}{b(1+m)}$$

input
```
Integrate[Sqrt[d*Cos[a + b*x]]*(c*Sin[a + b*x])^m,x]
```

output
```
(Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, (1 + m
)/2, (3 + m)/2, Sin[a + b*x]^2]*(c*Sin[a + b*x])^m*Tan[a + b*x])/(b*(1 + m
))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^m dx$$

$$\Big\downarrow \text{3057}$$

$$\frac{d\sqrt[4]{\cos^2(a+bx)}(c\sin(a+bx))^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{m+1}{2},\frac{m+3}{2},\sin^2(a+bx)\right)}{bc(m+1)\sqrt{d\cos(a+bx)}}$$

input
```
Int[Sqrt[d*Cos[a + b*x]]*(c*Sin[a + b*x])^m,x]
```

output
```
(d*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, (1 + m)/2, (3 + m)/2, Sin
[a + b*x]^2]*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m)*Sqrt[d*Cos[a + b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Maple [F]

$$\int \sqrt{d\cos \left(bx+a\right)}\left(c\sin \left(bx+a\right)\right)^{m}dx$$

input
```
int((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^m,x)
```

output
```
int((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^m,x)
```

Fricas [F]

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^{m}\,dx=\int \sqrt{d\cos \left(bx+a\right)}(c\sin \left(bx+a\right))^{m}\,dx$$

input
```
integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

output
```
integral(sqrt(d*cos(b*x + a))*(c*sin(b*x + a))^m, x)
```

Sympy [F]

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^m\,dx = \int (c\sin{(a+bx)})^m\sqrt{d\cos{(a+bx)}}\,dx$$

input
```
integrate((d*cos(b*x+a))**(1/2)*(c*sin(b*x+a))**m,x)
```

output
```
Integral((c*sin(a + b*x))**m*sqrt(d*cos(a + b*x)), x)
```

Maxima [F]

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^m\,dx = \int \sqrt{d\cos{(bx+a)}}(c\sin{(bx+a)})^m\,dx$$

input
```
integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*cos(b*x + a))*(c*sin(b*x + a))^m, x)
```

Giac [F]

$$\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^m\,dx = \int \sqrt{d\cos{(bx+a)}}(c\sin{(bx+a)})^m\,dx$$

input
```
integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^m,x, algorithm="giac")
```

output
```
integrate(sqrt(d*cos(b*x + a))*(c*sin(b*x + a))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d \cos(a+bx)}(c \sin(a+bx))^m \, dx = \int \sqrt{d\,\cos\left(a+b\,x\right)}\,\left(c\,\sin\left(a+b\,x\right)\right)^m \,dx$$

input
```
int((d*cos(a + b*x))^(1/2)*(c*sin(a + b*x))^m,x)
```

output
```
int((d*cos(a + b*x))^(1/2)*(c*sin(a + b*x))^m, x)
```

Reduce [F]

$$\int \sqrt{d \cos(a+bx)}(c \sin(a+bx))^m \, dx = \sqrt{d}\,c^m \left(\int \sin\left(bx+a\right)^m \sqrt{\cos\left(bx+a\right)}dx\right)$$

input
```
int((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^m,x)
```

output
```
sqrt(d)*c**m*int(sin(a + b*x)**m*sqrt(cos(a + b*x)),x)
```

3.352 $\int \frac{(c\sin(a+bx))^m}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 75

$$\int \frac{(c\sin(a+bx))^m}{\sqrt{d\cos(a+bx)}}\,dx = \frac{d\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bc(1+m)(d\cos(a+bx))^{3/2}}$$

output
```
d*(cos(b*x+a)^2)^(3/4)*hypergeom([3/4, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2
)*(c*sin(b*x+a))^(1+m)/b/c/(1+m)/(d*cos(b*x+a))^(3/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{(c\sin(a+bx))^m}{\sqrt{d\cos(a+bx)}}\,dx = \frac{\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^m\tan(a+bx)}{b(1+m)\sqrt{d\cos(a+bx)}}$$

input
```
Integrate[(c*Sin[a + b*x])^m/Sqrt[d*Cos[a + b*x]],x]
```

output
```
((Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/4, (1 + m)/2, (3 + m)/2, Sin[a
 + b*x]^2]*(c*Sin[a + b*x])^m*Tan[a + b*x])/(b*(1 + m)*Sqrt[d*Cos[a + b*x]
])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c\sin(a+bx))^m}{\sqrt{d\cos(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c\sin(a+bx))^m}{\sqrt{d\cos(a+bx)}}dx$$

$$\Big\downarrow \text{3057}$$

$$\frac{d\cos^2(a+bx)^{3/4}(c\sin(a+bx))^{m+1}\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{m+1}{2},\frac{m+3}{2},\sin^2(a+bx)\right)}{bc(m+1)(d\cos(a+bx))^{3/2}}$$

input
```
Int[(c*Sin[a + b*x])^m/Sqrt[d*Cos[a + b*x]],x]
```

output
```
(d*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/4, (1 + m)/2, (3 + m)/2, Sin
[a + b*x]^2]*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m)*(d*Cos[a + b*x])^(3/2)
)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Maple [F]

$$\int \frac{(c\sin(bx+a))^m}{\sqrt{d\cos(bx+a)}}dx$$

input
```
int((c*sin(b*x+a))^m/(d*cos(b*x+a))^(1/2),x)
```

output
```
int((c*sin(b*x+a))^m/(d*cos(b*x+a))^(1/2),x)
```

Fricas [F]

$$\int \frac{(c\sin(a+bx))^m}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{(c\sin(bx+a))^m}{\sqrt{d\cos(bx+a)}}\,dx$$

input
```
integrate((c*sin(b*x+a))^m/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*cos(b*x + a))*(c*sin(b*x + a))^m/(d*cos(b*x + a)), x)
```

Sympy [F]

$$\int \frac{(c\sin(a+bx))^m}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{(c\sin(a+bx))^m}{\sqrt{d\cos(a+bx)}}\,dx$$

input
```
integrate((c*sin(b*x+a))**m/(d*cos(b*x+a))**(1/2),x)
```

output
```
Integral((c*sin(a + b*x))**m/sqrt(d*cos(a + b*x)), x)
```

Maxima [F]

$$\int \frac{(c\sin(a+bx))^m}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{(c\sin(bx+a))^m}{\sqrt{d\cos(bx+a)}}\,dx$$

input
```
integrate((c*sin(b*x+a))^m/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a))^m/sqrt(d*cos(b*x + a)), x)
```

Giac [F]

$$\int \frac{(c\sin(a+bx))^m}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{(c\sin(bx+a))^m}{\sqrt{d\cos(bx+a)}}\,dx$$

input
```
integrate((c*sin(b*x+a))^m/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a))^m/sqrt(d*cos(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^m}{\sqrt{d\cos(a+bx)}}\,dx = \int \frac{(c\,\sin(a+b\,x))^m}{\sqrt{d\,\cos(a+b\,x)}}\,dx$$

input
```
int((c*sin(a + b*x))^m/(d*cos(a + b*x))^(1/2),x)
```

output
```
int((c*sin(a + b*x))^m/(d*cos(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(c\sin(a+bx))^m}{\sqrt{d\cos(a+bx)}}\,dx = \frac{\sqrt{d}\,c^m \left(\int \frac{\sin(bx+a)^m\sqrt{\cos(bx+a)}}{\cos(bx+a)}dx\right)}{d}$$

input
```
int((c*sin(b*x+a))^m/(d*cos(b*x+a))^(1/2),x)
```

output
```
(sqrt(d)*c**m*int((sin(a + b*x)**m*sqrt(cos(a + b*x)))/cos(a + b*x),x))/d
```

3.353 $\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{3/2}}\,dx = \frac{\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^{1+}}{bcd(1+m)\sqrt{d\cos(a+bx)}}$$

output
```
(cos(b*x+a)^2)^(1/4)*hypergeom([5/4, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)*
(c*sin(b*x+a))^(1+m)/b/c/d/(1+m)/(d*cos(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01

$$\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{3/2}}\,dx = \frac{\sqrt{d\cos(a+bx)}\,\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)}{bd^2(1+m)}$$

input
```
Integrate[(c*Sin[a + b*x])^m/(d*Cos[a + b*x])^(3/2),x]
```

output
```
(Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[5/4, (1 + m
)/2, (3 + m)/2, Sin[a + b*x]^2]*(c*Sin[a + b*x])^m*Tan[a + b*x])/(b*d^2*(1
 + m))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3057}$$

$$\frac{\sqrt[4]{\cos^2(a+bx)}(c\sin(a+bx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{m+1}{2},\frac{m+3}{2},\sin^2(a+bx)\right)}{bcd(m+1)\sqrt{d\cos(a+bx)}}$$

input
```
Int[(c*Sin[a + b*x])^m/(d*Cos[a + b*x])^(3/2),x]
```

output
```
((Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[5/4, (1 + m)/2, (3 + m)/2, Sin[a
 + b*x]^2]*(c*Sin[a + b*x])^(1 + m))/(b*c*d*(1 + m)*Sqrt[d*Cos[a + b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Maple [F]

$$\int \frac{(c\sin(bx+a))^m}{(d\cos(bx+a))^{\frac{3}{2}}}dx$$

input
```
int((c*sin(b*x+a))^m/(d*cos(b*x+a))^(3/2),x)
```

output
```
int((c*sin(b*x+a))^m/(d*cos(b*x+a))^(3/2),x)
```

Fricas [F]

$$\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{(c\sin(bx+a))^m}{(d\cos(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^m/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*cos(b*x + a))*(c*sin(b*x + a))^m/(d^2*cos(b*x + a)^2), x)
```

Sympy [F]

$$\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))**m/(d*cos(b*x+a))**(3/2),x)
```

output
```
Integral((c*sin(a + b*x))**m/(d*cos(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{(c\sin(bx+a))^m}{(d\cos(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^m/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a))^m/(d*cos(b*x + a))^(3/2), x)
```

Giac [F]

$$\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{(c\sin(bx+a))^m}{(d\cos(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^m/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a))^m/(d*cos(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{3/2}}\,dx = \int \frac{(c\,\sin(a+b\,x))^m}{(d\,\cos(a+b\,x))^{3/2}}\,dx$$

input
```
int((c*sin(a + b*x))^m/(d*cos(a + b*x))^(3/2),x)
```

output
```
int((c*sin(a + b*x))^m/(d*cos(a + b*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{3/2}}\,dx = \frac{\sqrt{d}\,c^m\left(\int \frac{\sin(bx+a)^m\sqrt{\cos(bx+a)}}{\cos(bx+a)^2}dx\right)}{d^2}$$

input
```
int((c*sin(b*x+a))^m/(d*cos(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*c**m*int((sin(a + b*x)**m*sqrt(cos(a + b*x)))/cos(a + b*x)**2,x))
/d**2
```

3.354 $\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{5/2}}\,dx = \frac{\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{7}{4},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^{1}}{bcd(1+m)(d\cos(a+bx))^{3/2}}$$

output
```
(cos(b*x+a)^2)^(3/4)*hypergeom([7/4, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)*
(c*sin(b*x+a))^(1+m)/b/c/d/(1+m)/(d*cos(b*x+a))^(3/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01

$$\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{5/2}}\,dx = \frac{\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{7}{4},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^{m}}{bd^2(1+m)\sqrt{d\cos(a+bx)}}$$

input
```
Integrate[(c*Sin[a + b*x])^m/(d*Cos[a + b*x])^(5/2),x]
```

output
```
((Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[7/4, (1 + m)/2, (3 + m)/2, Sin[a
 + b*x]^2]*(c*Sin[a + b*x])^m*Tan[a + b*x])/(b*d^2*(1 + m)*Sqrt[d*Cos[a +
b*x]])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3057}$$

$$\frac{\cos^2(a+bx)^{3/4}(c\sin(a+bx))^{m+1}\text{Hypergeometric2F1}\left(\frac{7}{4},\frac{m+1}{2},\frac{m+3}{2},\sin^2(a+bx)\right)}{bcd(m+1)(d\cos(a+bx))^{3/2}}$$

input
```
Int[(c*Sin[a + b*x])^m/(d*Cos[a + b*x])^(5/2),x]
```

output
```
((Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[7/4, (1 + m)/2, (3 + m)/2, Sin[a
 + b*x]^2]*(c*Sin[a + b*x])^(1 + m))/(b*c*d*(1 + m)*(d*Cos[a + b*x])^(3/2)
)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Maple [F]

$$\int \frac{(c\sin(bx+a))^m}{(d\cos(bx+a))^{\frac{5}{2}}}dx$$

input
```
int((c*sin(b*x+a))^m/(d*cos(b*x+a))^(5/2),x)
```

output
```
int((c*sin(b*x+a))^m/(d*cos(b*x+a))^(5/2),x)
```

Fricas [F]

$$\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{(c\sin(bx+a))^m}{(d\cos(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^m/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*cos(b*x + a))*(c*sin(b*x + a))^m/(d^3*cos(b*x + a)^3), x)
```

Sympy [F]

$$\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))**m/(d*cos(b*x+a))**(5/2),x)
```

output
```
Integral((c*sin(a + b*x))**m/(d*cos(a + b*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{(c\sin(bx+a))^m}{(d\cos(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^m/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a))^m/(d*cos(b*x + a))^(5/2), x)
```

Giac [F]

$$\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{(c\sin(bx+a))^m}{(d\cos(bx+a))^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^m/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a))^m/(d*cos(b*x + a))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{5/2}}\,dx = \int \frac{(c\,\sin(a+b\,x))^m}{(d\,\cos(a+b\,x))^{5/2}}\,dx$$

input
```
int((c*sin(a + b*x))^m/(d*cos(a + b*x))^(5/2),x)
```

output
```
int((c*sin(a + b*x))^m/(d*cos(a + b*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{5/2}}\,dx = \frac{\sqrt{d}\,c^m\left(\int \frac{\sin(bx+a)^m\sqrt{\cos(bx+a)}}{\cos(bx+a)^3}dx\right)}{d^3}$$

input
```
int((c*sin(b*x+a))^m/(d*cos(b*x+a))^(5/2),x)
```

output
```
(sqrt(d)*c**m*int((sin(a + b*x)**m*sqrt(cos(a + b*x)))/cos(a + b*x)**3,x))
/d**3
```

3.355 $\int (d\cos(a+bx))^n \sin^5(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 76

$$\int (d\cos(a+bx))^n \sin^5(a+bx)\,dx = -\frac{(d\cos(a+bx))^{1+n}}{bd(1+n)} + \frac{2(d\cos(a+bx))^{3+n}}{bd^3(3+n)} - \frac{(d\cos(a+bx))^{5+n}}{bd^5(5+n)}$$

output
```
-(d*cos(b*x+a))^(1+n)/b/d/(1+n)+2*(d*cos(b*x+a))^(3+n)/b/d^3/(3+n)-(d*cos(
b*x+a))^(5+n)/b/d^5/(5+n)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.09

$$\int (d\cos(a+bx))^n \sin^5(a+bx)\,dx = -\frac{\cos(a+bx)(d\cos(a+bx))^n\,(89+28n+3n^2-4(7+8n+n^2)\cos(2(a+bx))+(3+4n+n^2)\cos(4(a}{8b(1+n)(3+n)(5+n)}$$

input
```
Integrate[(d*Cos[a + b*x])^n*Sin[a + b*x]^5,x]
```

output
```
-1/8*(Cos[a + b*x]*(d*Cos[a + b*x])^n*(89 + 28*n + 3*n^2 - 4*(7 + 8*n + n^
2)*Cos[2*(a + b*x)] + (3 + 4*n + n^2)*Cos[4*(a + b*x)]))/(b*(1 + n)*(3 + n
)*(5 + n))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 3045, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(a+bx)(d\cos(a+bx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^5(d\cos(a+bx))^n dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int \frac{(d\cos(a+bx))^n\left(d^2-d^2\cos^2(a+bx)\right)^2}{d^4}d(d\cos(a+bx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int (d\cos(a+bx))^n\left(d^2-d^2\cos^2(a+bx)\right)^2d(d\cos(a+bx))}{bd^5}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(d^4(d\cos(a+bx))^n-2d^2(d\cos(a+bx))^{n+2}+(d\cos(a+bx))^{n+4}\right)d(d\cos(a+bx))}{bd^5}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{d^4(d\cos(a+bx))^{n+1}}{n+1}-\frac{2d^2(d\cos(a+bx))^{n+3}}{n+3}+\frac{(d\cos(a+bx))^{n+5}}{n+5}}{bd^5}$$

input
```
Int[(d*Cos[a + b*x])^n*Sin[a + b*x]^5,x]
```

output
```
-(((d^4*(d*Cos[a + b*x])^(1 + n))/(1 + n) - (2*d^2*(d*Cos[a + b*x])^(3 + n
))/(3 + n) + (d*Cos[a + b*x])^(5 + n)/(5 + n))/(b*d^5))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [A] (verified)

Time = 6.38 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.14

method	result	size
parallelrisch	$-\frac{\left(\left(-\frac{3}{2}n^{2}-14n-\frac{25}{2}\right)\cos(3bx+3a)+\left(\frac{1}{2}n^{2}+2n+\frac{3}{2}\right)\cos(5bx+5a)+\cos(bx+a)\left(n^{2}+12n+75\right)\right)\left(d\cos(bx+a)\right)^{n}}{8b\left(n^{3}+9n^{2}+23n+15\right)}$	87
derivativedivides	$-\frac{\cos(bx+a)\,\mathrm{e}^{n\ln(d\cos(bx+a))}}{b(1+n)}+\frac{2\cos(bx+a)^{3}\mathrm{e}^{n\ln(d\cos(bx+a))}}{b(3+n)}-\frac{\cos(bx+a)^{5}\mathrm{e}^{n\ln(d\cos(bx+a))}}{b(5+n)}$	90
default	$-\frac{\cos(bx+a)\,\mathrm{e}^{n\ln(d\cos(bx+a))}}{b(1+n)}+\frac{2\cos(bx+a)^{3}\mathrm{e}^{n\ln(d\cos(bx+a))}}{b(3+n)}-\frac{\cos(bx+a)^{5}\mathrm{e}^{n\ln(d\cos(bx+a))}}{b(5+n)}$	90

input
```
int((d*cos(b*x+a))^n*sin(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/8*((-3/2*n^2-14*n-25/2)*cos(3*b*x+3*a)+(1/2*n^2+2*n+3/2)*cos(5*b*x+5*a)
+cos(b*x+a)*(n^2+12*n+75))*(d*cos(b*x+a))^n/b/(n^3+9*n^2+23*n+15)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.11

$$\int (d\cos(a+bx))^n \sin^5(a+bx)\,dx = -\frac{\left((n^2+4\,n+3)\cos\left(bx+a\right)^5 - 2\,(n^2+6\,n+5)\cos\left(bx+a\right)^3 + (n^2+8\,n+15)\cos\left(bx+a\right)\right)(d\cos\left(bx\right.}{bn^3+9\,bn^2+23\,bn+15\,b}$$

input
```
integrate((d*cos(b*x+a))^n*sin(b*x+a)^5,x, algorithm="fricas")
```

output
```
-((n^2 + 4*n + 3)*cos(b*x + a)^5 - 2*(n^2 + 6*n + 5)*cos(b*x + a)^3 + (n^2
 + 8*n + 15)*cos(b*x + a))*(d*cos(b*x + a))^n/(b*n^3 + 9*b*n^2 + 23*b*n +
15*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2451 vs. $2(60) = 120$.

Time = 4.45 (sec) , antiderivative size = 2451, normalized size of antiderivative = 32.25

$$\int (d\cos(a+bx))^n \sin^5(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*cos(b*x+a))**n*sin(b*x+a)**5,x)
```

output
```
Piecewise((x*(d*cos(a))**n*sin(a)**5, Eq(b, 0)), ((-log(cos(a + b*x))/b +
sin(a + b*x)**4/(4*b*cos(a + b*x)**4) - sin(a + b*x)**2/(2*b*cos(a + b*x)*
*2))/d**5, Eq(n, -5)), ((2*log(tan(a/2 + b*x/2) - 1)*tan(a/2 + b*x/2)**8/(
b*tan(a/2 + b*x/2)**8 - 2*b*tan(a/2 + b*x/2)**4 + b) - 4*log(tan(a/2 + b*x
/2) - 1)*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**8 - 2*b*tan(a/2 + b*x/2)
**4 + b) + 2*log(tan(a/2 + b*x/2) - 1)/(b*tan(a/2 + b*x/2)**8 - 2*b*tan(a/
2 + b*x/2)**4 + b) + 2*log(tan(a/2 + b*x/2) + 1)*tan(a/2 + b*x/2)**8/(b*ta
n(a/2 + b*x/2)**8 - 2*b*tan(a/2 + b*x/2)**4 + b) - 4*log(tan(a/2 + b*x/2)
+ 1)*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**8 - 2*b*tan(a/2 + b*x/2)**4
+ b) + 2*log(tan(a/2 + b*x/2) + 1)/(b*tan(a/2 + b*x/2)**8 - 2*b*tan(a/2 +
b*x/2)**4 + b) - 2*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**8/(b*tan
(a/2 + b*x/2)**8 - 2*b*tan(a/2 + b*x/2)**4 + b) + 4*log(tan(a/2 + b*x/2)**
2 + 1)*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**8 - 2*b*tan(a/2 + b*x/2)**
4 + b) - 2*log(tan(a/2 + b*x/2)**2 + 1)/(b*tan(a/2 + b*x/2)**8 - 2*b*tan(a
/2 + b*x/2)**4 + b) + 4*tan(a/2 + b*x/2)**6/(b*tan(a/2 + b*x/2)**8 - 2*b*t
an(a/2 + b*x/2)**4 + b) + 4*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**8 - 2
*b*tan(a/2 + b*x/2)**4 + b))/d**3, Eq(n, -3)), ((-log(tan(a/2 + b*x/2) - 1
)*tan(a/2 + b*x/2)**8/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6
*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 4*log(tan(a/2 + b*
x/2) - 1)*tan(a/2 + b*x/2)**6/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

$$\int (d\cos(a+bx))^n \sin^5(a+bx)\,dx = -\frac{\frac{d^n \cos(bx+a)^n \cos(bx+a)^5}{n+5} - \frac{2\,d^n \cos(bx+a)^n \cos(bx+a)^3}{n+3} + \frac{(d\cos(bx+a))^{n+1}}{d(n+1)}}{b}$$

input
```
integrate((d*cos(b*x+a))^n*sin(b*x+a)^5,x, algorithm="maxima")
```

output
```
-(d^n*cos(b*x + a)^n*cos(b*x + a)^5/(n + 5) - 2*d^n*cos(b*x + a)^n*cos(b*x
 + a)^3/(n + 3) + (d*cos(b*x + a))^(n + 1)/(d*(n + 1)))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(76) = 152$.

Time = 0.13 (sec) , antiderivative size = 249, normalized size of antiderivative = 3.28

$$\int (d\cos(a+bx))^n \sin^5(a+bx)\,dx =$$
$$-\frac{(d\cos(bx+a))^n d^5n^2\cos(bx+a)^5 + 4(d\cos(bx+a))^n d^5n\cos(bx+a)^5 - 2(d\cos(bx+a))^n d^5n^2 \text{co}}{}$$

input
```
integrate((d*cos(b*x+a))^n*sin(b*x+a)^5,x, algorithm="giac")
```

output
```
-((d*cos(b*x + a))^n*d^5*n^2*cos(b*x + a)^5 + 4*(d*cos(b*x + a))^n*d^5*n*c
os(b*x + a)^5 - 2*(d*cos(b*x + a))^n*d^5*n^2*cos(b*x + a)^3 + 3*(d*cos(b*x
 + a))^n*d^5*cos(b*x + a)^5 - 12*(d*cos(b*x + a))^n*d^5*n*cos(b*x + a)^3 +
 (d*cos(b*x + a))^n*d^5*n^2*cos(b*x + a) - 10*(d*cos(b*x + a))^n*d^5*cos(b
*x + a)^3 + 8*(d*cos(b*x + a))^n*d^5*n*cos(b*x + a) + 15*(d*cos(b*x + a))^
n*d^5*cos(b*x + a))/((d^4*n^3 + 9*d^4*n^2 + 23*d^4*n + 15*d^4)*b*d)
```

Mupad [B] (verification not implemented)

Time = 26.89 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.74

$$\int (d\cos(a+bx))^n \sin^5(a+bx)\,dx =$$
$$-\frac{(d\,\cos(a+b\,x))^n\,(150\,\cos(a+b\,x) - 25\,\cos(3\,a+3\,b\,x) + 3\,\cos(5\,a+5\,b\,x) + 24\,n\,\cos(a+b\,x) - 2}{16\,b\,(n^3 +}$$

input
```
int(sin(a + b*x)^5*(d*cos(a + b*x))^n,x)
```

output
```
-((d*cos(a + b*x))^n*(150*cos(a + b*x) - 25*cos(3*a + 3*b*x) + 3*cos(5*a +
 5*b*x) + 24*n*cos(a + b*x) - 28*n*cos(3*a + 3*b*x) + 4*n*cos(5*a + 5*b*x)
 + 2*n^2*cos(a + b*x) - 3*n^2*cos(3*a + 3*b*x) + n^2*cos(5*a + 5*b*x)))/(1
6*b*(23*n + 9*n^2 + n^3 + 15))
```

Reduce [F]

$$\int (d\cos(a+bx))^n \sin^5(a+bx)\,dx = d^n\left(\int \cos(bx+a)^n \sin(bx+a)^5\,dx\right)$$

input
```
int((d*cos(b*x+a))^n*sin(b*x+a)^5,x)
```

output
```
d**n*int(cos(a + b*x)**n*sin(a + b*x)**5,x)
```

3.356 $\int (d\cos(a+bx))^n \sin^3(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 50

$$\int (d\cos(a+bx))^n \sin^3(a+bx)\,dx = -\frac{(d\cos(a+bx))^{1+n}}{bd(1+n)} + \frac{(d\cos(a+bx))^{3+n}}{bd^3(3+n)}$$

output
```
-(d*cos(b*x+a))^(1+n)/b/d/(1+n)+(d*cos(b*x+a))^(3+n)/b/d^3/(3+n)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int (d\cos(a+bx))^n \sin^3(a+bx)\,dx = \frac{\cos(a+bx)(d\cos(a+bx))^n(-5-n+(1+n)\cos(2(a+bx)))}{2b(1+n)(3+n)}$$

input
```
Integrate[(d*Cos[a + b*x])^n*Sin[a + b*x]^3,x]
```

output
```
(Cos[a + b*x]*(d*Cos[a + b*x])^n*(-5 - n + (1 + n)*Cos[2*(a + b*x)]))/(2*b
*(1 + n)*(3 + n))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 3045, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)(d\cos(a+bx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^3(d\cos(a+bx))^n dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int \frac{(d\cos(a+bx))^n\left(d^2-d^2\cos^2(a+bx)\right)}{d^2}d(d\cos(a+bx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int (d\cos(a+bx))^n\left(d^2-d^2\cos^2(a+bx)\right)d(d\cos(a+bx))}{bd^3}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(d^2(d\cos(a+bx))^n-(d\cos(a+bx))^{n+2}\right)d(d\cos(a+bx))}{bd^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{d^2(d\cos(a+bx))^{n+1}}{n+1}-\frac{(d\cos(a+bx))^{n+3}}{n+3}}{bd^3}$$

input
```
Int[(d*Cos[a + b*x])^n*Sin[a + b*x]^3,x]
```

output
```
-(((d^2*(d*Cos[a + b*x])^(1 + n))/(1 + n) - (d*Cos[a + b*x])^(3 + n)/(3 +
n))/(b*d^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [A] (verified)

Time = 3.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04

method	result	size
parallelrisch	$-\frac{(d\cos(bx+a))^n((-1-n)\cos(3bx+3a)+\cos(bx+a)(n+9))}{4b(n^2+4n+3)}$	52
derivativedivides	$\frac{\cos(bx+a)^3 e^{n\ln(d\cos(bx+a))}}{b(3+n)}-\frac{\cos(bx+a)e^{n\ln(d\cos(bx+a))}}{b(1+n)}$	59
default	$\frac{\cos(bx+a)^3 e^{n\ln(d\cos(bx+a))}}{b(3+n)}-\frac{\cos(bx+a)e^{n\ln(d\cos(bx+a))}}{b(1+n)}$	59

input
```
int((d*cos(b*x+a))^n*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(d*cos(b*x+a))^n*((-1-n)*cos(3*b*x+3*a)+cos(b*x+a)*(n+9))/b/(n^2+4*n+
3)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int (d\cos(a+bx))^n \sin^3(a+bx)\,dx = \frac{\left((n+1)\cos\left(bx+a\right)^3-(n+3)\cos\left(bx+a\right)\right)\left(d\cos\left(bx+a\right)\right)^n}{bn^2+4\,bn+3\,b}$$

input
```
integrate((d*cos(b*x+a))^n*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
((n + 1)*cos(b*x + a)^3 - (n + 3)*cos(b*x + a))*(d*cos(b*x + a))^n/(b*n^2
+ 4*b*n + 3*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 688 vs. 2(37) = 74.

Time = 1.25 (sec) , antiderivative size = 688, normalized size of antiderivative = 13.76

$$\int (d\cos(a+bx))^n \sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*cos(b*x+a))**n*sin(b*x+a)**3,x)
```

output
```
Piecewise((x*(d*cos(a))**n*sin(a)**3, Eq(b, 0)), ((log(cos(a + b*x))/b + s
in(a + b*x)**2/(2*b*cos(a + b*x)**2))/d**3, Eq(n, -3)), ((-log(tan(a/2 + b
*x/2) - 1)*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**4 + 2*b*tan(a/2 + b*x/
2)**2 + b) - 2*log(tan(a/2 + b*x/2) - 1)*tan(a/2 + b*x/2)**2/(b*tan(a/2 +
b*x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b) - log(tan(a/2 + b*x/2) - 1)/(b*ta
n(a/2 + b*x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b) - log(tan(a/2 + b*x/2) +
1)*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 +
b) - 2*log(tan(a/2 + b*x/2) + 1)*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**
4 + 2*b*tan(a/2 + b*x/2)**2 + b) - log(tan(a/2 + b*x/2) + 1)/(b*tan(a/2 +
b*x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b) + log(tan(a/2 + b*x/2)**2 + 1)*ta
n(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b) +
2*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**4
+ 2*b*tan(a/2 + b*x/2)**2 + b) + log(tan(a/2 + b*x/2)**2 + 1)/(b*tan(a/2 +
 b*x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b) - 2*tan(a/2 + b*x/2)**2/(b*tan(a
/2 + b*x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b))/d, Eq(n, -1)), (-n*(d*cos(a
 + b*x))**n*sin(a + b*x)**2*cos(a + b*x)/(b*n**2 + 4*b*n + 3*b) - 3*(d*cos
(a + b*x))**n*sin(a + b*x)**2*cos(a + b*x)/(b*n**2 + 4*b*n + 3*b) - 2*(d*c
os(a + b*x))**n*cos(a + b*x)**3/(b*n**2 + 4*b*n + 3*b), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04

$$\int (d\cos(a+bx))^n \sin^3(a+bx)\,dx = \frac{\frac{d^n\cos(bx+a)^n\cos(bx+a)^3}{n+3} - \frac{(d\cos(bx+a))^{n+1}}{d(n+1)}}{b}$$

input
```
integrate((d*cos(b*x+a))^n*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
(d^n*cos(b*x + a)^n*cos(b*x + a)^3/(n + 3) - (d*cos(b*x + a))^(n + 1)/(d*(
n + 1)))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(50) = 100$.

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.34

$$\int (d\cos(a+bx))^n \sin^3(a+bx)\,dx$$
$$= \frac{(d\cos(bx+a))^n d^3 n\cos(bx+a)^3 + (d\cos(bx+a))^n d^3\cos(bx+a)^3 - (d\cos(bx+a))^n d^3 n\cos(bx+a}{(d^2n^2+4\,d^2n+3\,d^2)bd}$$

input
```
integrate((d*cos(b*x+a))^n*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
((d*cos(b*x + a))^n*d^3*n*cos(b*x + a)^3 + (d*cos(b*x + a))^n*d^3*cos(b*x
+ a)^3 - (d*cos(b*x + a))^n*d^3*n*cos(b*x + a) - 3*(d*cos(b*x + a))^n*d^3*
cos(b*x + a))/((d^2*n^2 + 4*d^2*n + 3*d^2)*b*d)
```

Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.30

$$\int (d\cos(a+bx))^n \sin^3(a+bx)\,dx =$$
$$-\frac{(d\,\cos(a+b\,x))^n\,(9\,\cos(a+b\,x)-\cos(3\,a+3\,b\,x)+n\,\cos(a+b\,x)-n\,\cos(3\,a+3\,b\,x))}{4\,b\,(n^2+4\,n+3)}$$

input
```
int(sin(a + b*x)^3*(d*cos(a + b*x))^n,x)
```

output
```
-((d*cos(a + b*x))^n*(9*cos(a + b*x) - cos(3*a + 3*b*x) + n*cos(a + b*x) -
 n*cos(3*a + 3*b*x)))/(4*b*(4*n + n^2 + 3))
```

Reduce [F]

$$\int (d\cos(a+bx))^n \sin^3(a+bx)\,dx = d^n\left(\int \cos(bx+a)^n \sin(bx+a)^3\,dx\right)$$

input
```
int((d*cos(b*x+a))^n*sin(b*x+a)^3,x)
```

output
```
d**n*int(cos(a + b*x)**n*sin(a + b*x)**3,x)
```

3.357 $\int (d\cos(a+bx))^n \sin(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 25

$$\int (d\cos(a+bx))^n \sin(a+bx)\,dx = -\frac{(d\cos(a+bx))^{1+n}}{bd(1+n)}$$

output
```
-(d*cos(b*x+a))^(1+n)/b/d/(1+n)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int (d\cos(a+bx))^n \sin(a+bx)\,dx = -\frac{\cos(a+bx)(d\cos(a+bx))^n}{b(1+n)}$$

input
```
Integrate[(d*Cos[a + b*x])^n*Sin[a + b*x],x]
```

output
```
-((Cos[a + b*x]*(d*Cos[a + b*x])^n)/(b*(1 + n)))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3042, 3045, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)(d\cos(a+bx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)(d\cos(a+bx))^n dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int (d\cos(a+bx))^n d(d\cos(a+bx))}{bd}$$

$$\Big\downarrow \text{15}$$

$$-\frac{(d\cos(a+bx))^{n+1}}{bd(n+1)}$$

input
```
Int[(d*Cos[a + b*x])^n*Sin[a + b*x],x]
```

output
```
-((d*Cos[a + b*x])^(1 + n)/(b*d*(1 + n)))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [A] (verified)

Time = 6.73 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

method	result
derivativedivides	$-\frac{(d\cos(bx+a))^{1+n}}{bd(1+n)}$
default	$-\frac{(d\cos(bx+a))^{1+n}}{bd(1+n)}$
parallelrisch	$-\frac{\cos(bx+a)(d\cos(bx+a))^{n}}{b(1+n)}$
norman	$\frac{\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\mathrm{e}^{n\ln\left(\frac{d\left(1-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)}{1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\right)}}{b(1+n)}-\frac{\mathrm{e}^{n\ln\left(\frac{d\left(1-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)}{1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\right)}}{b(1+n)}}{1+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}$
risch	$\left(\mathrm{e}^{i(bx+a)}\right)^{-n}(\sin(bx)\sin(a)-\cos(bx)\cos(a))\left(\mathrm{e}^{2i(bx+a)}+1\right)^{n}d^{n}\left(\frac{1}{2}\right)^{n}\mathrm{e}^{-\frac{i\pi n\left(\mathrm{csgn}\left(i\mathrm{e}^{-i(bx+a)}\right)\mathrm{csgn}(i(\cos(bx)\cos(a)-\sin(b\right.}{}}$

input
```
int((d*cos(b*x+a))^n*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-(d*cos(b*x+a))^(1+n)/b/d/(1+n)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (d\cos(a+bx))^n\sin(a+bx)\,dx=-\frac{(d\cos(bx+a))^n\cos(bx+a)}{bn+b}$$

input
```
integrate((d*cos(b*x+a))^n*sin(b*x+a),x, algorithm="fricas")
```

output
```
-(d*cos(b*x + a))^n*cos(b*x + a)/(b*n + b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(19) = 38$.

Time = 0.35 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.40

$$\int (d\cos(a+bx))^n \sin(a+bx)\,dx = \begin{cases} \frac{x\sin(a)}{d\cos(a)} & \text{for } b=0 \wedge n=-1 \\ x(d\cos(a))^n \sin(a) & \text{for } b=0 \\ -\frac{\log(\cos(a+bx))}{bd} & \text{for } n=-1 \\ -\frac{(d\cos(a+bx))^n \cos(a+bx)}{bn+b} & \text{otherwise} \end{cases}$$

input
```
integrate((d*cos(b*x+a))**n*sin(b*x+a),x)
```

output
```
Piecewise((x*sin(a)/(d*cos(a)), Eq(b, 0) & Eq(n, -1)), (x*(d*cos(a))**n*si
n(a), Eq(b, 0)), (-log(cos(a + b*x))/(b*d), Eq(n, -1)), (-(d*cos(a + b*x))
**n*cos(a + b*x)/(b*n + b), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (d\cos(a+bx))^n \sin(a+bx)\,dx = -\frac{(d\cos(bx+a))^{n+1}}{bd(n+1)}$$

input
```
integrate((d*cos(b*x+a))^n*sin(b*x+a),x, algorithm="maxima")
```

output
```
-(d*cos(b*x + a))^(n + 1)/(b*d*(n + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (d\cos(a+bx))^n \sin(a+bx)\,dx = -\frac{(d\cos(bx+a))^{n+1}}{bd(n+1)}$$

input
```
integrate((d*cos(b*x+a))^n*sin(b*x+a),x, algorithm="giac")
```

output
```
-(d*cos(b*x + a))^(n + 1)/(b*d*(n + 1))
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int (d\cos(a+bx))^n \sin(a+bx)\,dx = -\frac{\cos(a+b\,x)\;(d\,\cos(a+b\,x))^n}{b\,(n+1)}$$

input
```
int(sin(a + b*x)*(d*cos(a + b*x))^n,x)
```

output
```
-(cos(a + b*x)*(d*cos(a + b*x))^n)/(b*(n + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (d\cos(a+bx))^n \sin(a+bx)\,dx = -\frac{d^n\cos(bx+a)^n\cos(bx+a)}{b\,(n+1)}$$

input
```
int((d*cos(b*x+a))^n*sin(b*x+a),x)
```

output
```
( - d**n*cos(a + b*x)**n*cos(a + b*x))/(b*(n + 1))
```

3.358 $\int (d\cos(a+bx))^n \csc(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 49

$$\int (d\cos(a+bx))^n \csc(a+bx)\,dx = -\frac{(d\cos(a+bx))^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)}{bd(1+n)}$$

output
```
-(d*cos(b*x+a))^(1+n)*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],cos(b*x+a)^2)/b
/d/(1+n)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

$$\int (d\cos(a+bx))^n \csc(a+bx)\,dx = -\frac{\cos(a+bx)(d\cos(a+bx))^n\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},1+\frac{1+n}{2},\cos^2(a+bx)\right)}{b(1+n)}$$

input
```
Integrate[(d*Cos[a + b*x])^n*Csc[a + b*x],x]
```

output
```
-((Cos[a + b*x]*(d*Cos[a + b*x])^n*Hypergeometric2F1[1, (1 + n)/2, 1 + (1
+ n)/2, Cos[a + b*x]^2])/(b*(1 + n)))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 3045, 27, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)(d\cos(a+bx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\cos(a+bx))^n}{\sin(a+bx)}dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int \frac{d^2(d\cos(a+bx))^n}{d^2-d^2\cos^2(a+bx)}d(d\cos(a+bx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$-\frac{d\int \frac{(d\cos(a+bx))^n}{d^2-d^2\cos^2(a+bx)}d(d\cos(a+bx))}{b}$$

$$\Big\downarrow \text{278}$$

$$-\frac{(d\cos(a+bx))^{n+1}\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},\cos^2(a+bx)\right)}{bd(n+1)}$$

input
```
Int[(d*Cos[a + b*x])^n*Csc[a + b*x],x]
```

output
```
-(((d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Cos
[a + b*x]^2])/(b*d*(1 + n)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [F]

$$\int (d\cos(bx+a))^n \csc(bx+a)\, dx$$

input
```
int((d*cos(b*x+a))^n*csc(b*x+a),x)
```

output
```
int((d*cos(b*x+a))^n*csc(b*x+a),x)
```

Fricas [F]

$$\int (d\cos(a+bx))^n \csc(a+bx)\,dx = \int (d\cos(bx+a))^n \csc(bx+a)\,dx$$

input
```
integrate((d*cos(b*x+a))^n*csc(b*x+a),x, algorithm="fricas")
```

output
```
integral((d*cos(b*x + a))^n*csc(b*x + a), x)
```

Sympy [F]

$$\int (d\cos(a+bx))^n \csc(a+bx)\,dx = \int (d\cos(a+bx))^n \csc(a+bx)\,dx$$

input
```
integrate((d*cos(b*x+a))**n*csc(b*x+a),x)
```

output
```
Integral((d*cos(a + b*x))**n*csc(a + b*x), x)
```

Maxima [F]

$$\int (d\cos(a+bx))^n \csc(a+bx)\,dx = \int (d\cos(bx+a))^n \csc(bx+a)\,dx$$

input
```
integrate((d*cos(b*x+a))^n*csc(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*cos(b*x + a))^n*csc(b*x + a), x)
```

Giac [F]

$$\int (d\cos(a+bx))^n\csc(a+bx)\,dx=\int (d\cos(bx+a))^n\csc(bx+a)\,dx$$

input
```
integrate((d*cos(b*x+a))^n*csc(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*cos(b*x + a))^n*csc(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^n\csc(a+bx)\,dx=\int \frac{(d\cos(a+b\,x))^n}{\sin(a+b\,x)}\,dx$$

input
```
int((d*cos(a + b*x))^n/sin(a + b*x),x)
```

output
```
int((d*cos(a + b*x))^n/sin(a + b*x), x)
```

Reduce [F]

$$\int (d\cos(a+bx))^n\csc(a+bx)\,dx=d^n\left(\int \cos(bx+a)^n\csc(bx+a)\,dx\right)$$

input
```
int((d*cos(b*x+a))^n*csc(b*x+a),x)
```

output
```
d**n*int(cos(a + b*x)**n*csc(a + b*x),x)
```

3.359 $\int (d\cos(a+bx))^n \csc^3(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 49

$$\int (d\cos(a+bx))^n \csc^3(a+bx)\,dx = -\frac{(d\cos(a+bx))^{1+n}\,\text{Hypergeometric2F1}\left(2,\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)}{bd(1+n)}$$

output
```
-(d*cos(b*x+a))^(1+n)*hypergeom([2, 1/2+1/2*n],[3/2+1/2*n],cos(b*x+a)^2)/b
/d/(1+n)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 154 vs. 2(49) = 98.

Time = 1.53 (sec) , antiderivative size = 154, normalized size of antiderivative = 3.14

$$\int (d\cos(a+bx))^n \csc^3(a+bx)\,dx = -\frac{2^{-3-n}\cos(a+bx)(d\cos(a+bx))^n\left(2^{1+n}\,\text{Hypergeometric2F1}(1,1+n,2+n,\cos(a+bx))+2^{1+n}\,\text{Hyp}\right.}{}$$

input
```
Integrate[(d*Cos[a + b*x])^n*Csc[a + b*x]^3,x]
```

output
```
-((2^(-3 - n)*Cos[a + b*x]*(d*Cos[a + b*x])^n*(2^(1 + n)*Hypergeometric2F1
[1, 1 + n, 2 + n, Cos[a + b*x]] + 2^(1 + n)*Hypergeometric2F1[2, 1 + n, 2
+ n, Cos[a + b*x]] + (Hypergeometric2F1[n, 1 + n, 2 + n, (Cos[a + b*x]*Sec
[(a + b*x)/2]^2)/2] + Hypergeometric2F1[1 + n, 1 + n, 2 + n, (Cos[a + b*x]
*Sec[(a + b*x)/2]^2)/2])*(Sec[(a + b*x)/2]^2)^(1 + n)))/(b*(1 + n)))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3045, 27, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(a+bx)(d\cos(a+bx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\cos(a+bx))^n}{\sin(a+bx)^3}dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int \frac{d^4(d\cos(a+bx))^n}{(d^2-d^2\cos^2(a+bx))^2}d(d\cos(a+bx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$-\frac{d^3\int \frac{(d\cos(a+bx))^n}{(d^2-d^2\cos^2(a+bx))^2}d(d\cos(a+bx))}{b}$$

$$\Big\downarrow \text{278}$$

$$-\frac{(d\cos(a+bx))^{n+1}\text{Hypergeometric2F1}\left(2,\frac{n+1}{2},\frac{n+3}{2},\cos^2(a+bx)\right)}{bd(n+1)}$$

input
```
Int[(d*Cos[a + b*x])^n*Csc[a + b*x]^3,x]
```

output
```
-(((d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[2, (1 + n)/2, (3 + n)/2, Cos
[a + b*x]^2])/(b*d*(1 + n)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [F]

$$\int (d\cos(bx+a))^n \csc(bx+a)^3 \, dx$$

input
```
int((d*cos(b*x+a))^n*csc(b*x+a)^3,x)
```

output
```
int((d*cos(b*x+a))^n*csc(b*x+a)^3,x)
```

Fricas [F]

$$\int (d\cos(a+bx))^n \csc^3(a+bx)\,dx = \int (d\cos(bx+a))^n \csc(bx+a)^3\,dx$$

input
```
integrate((d*cos(b*x+a))^n*csc(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral((d*cos(b*x + a))^n*csc(b*x + a)^3, x)
```

Sympy [F]

$$\int (d\cos(a+bx))^n \csc^3(a+bx)\,dx = \int (d\cos(a+bx))^n \csc^3(a+bx)\,dx$$

input
```
integrate((d*cos(b*x+a))**n*csc(b*x+a)**3,x)
```

output
```
Integral((d*cos(a + b*x))**n*csc(a + b*x)**3, x)
```

Maxima [F]

$$\int (d\cos(a+bx))^n \csc^3(a+bx)\,dx = \int (d\cos(bx+a))^n \csc(bx+a)^3\,dx$$

input
```
integrate((d*cos(b*x+a))^n*csc(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate((d*cos(b*x + a))^n*csc(b*x + a)^3, x)
```

Giac [F]

$$\int (d\cos(a+bx))^n \csc^3(a+bx)\,dx = \int (d\cos(bx+a))^n \csc(bx+a)^3\,dx$$

input
```
integrate((d*cos(b*x+a))^n*csc(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*cos(b*x + a))^n*csc(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^n \csc^3(a+bx)\,dx = \int \frac{(d\cos(a+bx))^n}{\sin(a+bx)^3}\,dx$$

input
```
int((d*cos(a + b*x))^n/sin(a + b*x)^3,x)
```

output
```
int((d*cos(a + b*x))^n/sin(a + b*x)^3, x)
```

Reduce [F]

$$\int (d\cos(a+bx))^n \csc^3(a+bx)\,dx = d^n\left(\int \cos(bx+a)^n \csc(bx+a)^3\,dx\right)$$

input
```
int((d*cos(b*x+a))^n*csc(b*x+a)^3,x)
```

output
```
d**n*int(cos(a + b*x)**n*csc(a + b*x)**3,x)
```

3.360 $\int (d\cos(a+bx))^n \csc^5(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 49

$$\int (d\cos(a+bx))^n \csc^5(a+bx)\,dx = -\frac{(d\cos(a+bx))^{1+n}\,\text{Hypergeometric2F1}\left(3,\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)}{bd(1+n)}$$

output
```
-(d*cos(b*x+a))^(1+n)*hypergeom([3, 1/2+1/2*n],[3/2+1/2*n],cos(b*x+a)^2)/b
/d/(1+n)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 244 vs. 2(49) = 98.

Time = 2.09 (sec) , antiderivative size = 244, normalized size of antiderivative = 4.98

$$\int (d\cos(a+bx))^n \csc^5(a+bx)\,dx = -\frac{2^{-5-n}\cos(a+bx)(d\cos(a+bx))^n \left(3\ 2^{1+n}\,\text{Hypergeometric2F1}(1,1+n,2+n,\cos(a+bx)) + 3\ 2^{1+n}\right.}{}$$

input
```
Integrate[(d*Cos[a + b*x])^n*Csc[a + b*x]^5,x]
```

output
```
-((2^(-5 - n)*Cos[a + b*x]*(d*Cos[a + b*x])^n*(3*2^(1 + n)*Hypergeometric2
F1[1, 1 + n, 2 + n, Cos[a + b*x]] + 3*2^(1 + n)*Hypergeometric2F1[2, 1 + n
, 2 + n, Cos[a + b*x]] + 2^(2 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, Cos[
a + b*x]] + 2*Hypergeometric2F1[-1 + n, 1 + n, 2 + n, (Cos[a + b*x]*Sec[(a
 + b*x)/2]^2)/2]*(Sec[(a + b*x)/2]^2)^(1 + n) + 3*Hypergeometric2F1[n, 1 +
 n, 2 + n, (Cos[a + b*x]*Sec[(a + b*x)/2]^2)/2]*(Sec[(a + b*x)/2]^2)^(1 +
n) + 3*Hypergeometric2F1[1 + n, 1 + n, 2 + n, (Cos[a + b*x]*Sec[(a + b*x)/
2]^2)/2]*(Sec[(a + b*x)/2]^2)^(1 + n)))/(b*(1 + n)))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3045, 27, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^5(a+bx)(d\cos(a+bx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\cos(a+bx))^n}{\sin(a+bx)^5}dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{\int \frac{d^6(d\cos(a+bx))^n}{(d^2-d^2\cos^2(a+bx))^3}d(d\cos(a+bx))}{bd}$$

$$\Big\downarrow \text{27}$$

$$-\frac{d^5\int \frac{(d\cos(a+bx))^n}{(d^2-d^2\cos^2(a+bx))^3}d(d\cos(a+bx))}{b}$$

$$\Big\downarrow \text{278}$$

$$-\frac{(d\cos(a+bx))^{n+1}\text{Hypergeometric2F1}\left(3,\frac{n+1}{2},\frac{n+3}{2},\cos^2(a+bx)\right)}{bd(n+1)}$$

input
```
Int[(d*Cos[a + b*x])^n*Csc[a + b*x]^5,x]
```

output
```
-(((d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[3, (1 + n)/2, (3 + n)/2, Cos
[a + b*x]^2])/(b*d*(1 + n)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Maple [F]

$$\int (d\cos (bx + a))^n \csc (bx + a)^5 \, dx$$

input
```
int((d*cos(b*x+a))^n*csc(b*x+a)^5,x)
```

output
```
int((d*cos(b*x+a))^n*csc(b*x+a)^5,x)
```

Fricas [F]

$$\int (d\cos(a+bx))^n \csc^5(a+bx)\,dx = \int (d\cos{(bx+a)})^n \csc{(bx+a)}^5\,dx$$

input
```
integrate((d*cos(b*x+a))^n*csc(b*x+a)^5,x, algorithm="fricas")
```

output
```
integral((d*cos(b*x + a))^n*csc(b*x + a)^5, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^n \csc^5(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*cos(b*x+a))**n*csc(b*x+a)**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\cos(a+bx))^n \csc^5(a+bx)\,dx = \int (d\cos{(bx+a)})^n \csc{(bx+a)}^5\,dx$$

input
```
integrate((d*cos(b*x+a))^n*csc(b*x+a)^5,x, algorithm="maxima")
```

output
```
integrate((d*cos(b*x + a))^n*csc(b*x + a)^5, x)
```

Giac [F]

$$\int (d\cos(a+bx))^n \csc^5(a+bx)\,dx = \int (d\cos(bx+a))^n \csc(bx+a)^5 \,dx$$

input
```
integrate((d*cos(b*x+a))^n*csc(b*x+a)^5,x, algorithm="giac")
```

output
```
integrate((d*cos(b*x + a))^n*csc(b*x + a)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^n \csc^5(a+bx)\,dx = \int \frac{(d\cos(a+bx))^n}{\sin(a+bx)^5}\,dx$$

input
```
int((d*cos(a + b*x))^n/sin(a + b*x)^5,x)
```

output
```
int((d*cos(a + b*x))^n/sin(a + b*x)^5, x)
```

Reduce [F]

$$\int (d\cos(a+bx))^n \csc^5(a+bx)\,dx = d^n \left(\int \cos(bx+a)^n \csc(bx+a)^5 \,dx\right)$$

input
```
int((d*cos(b*x+a))^n*csc(b*x+a)^5,x)
```

output
```
d**n*int(cos(a + b*x)**n*csc(a + b*x)**5,x)
```

3.361 $\int (d\cos(a+bx))^n \sin^4(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 69

$$\int (d\cos(a+bx))^n \sin^4(a+bx)\,dx = -\frac{(d\cos(a+bx))^{1+n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sin(a+bx)}{bd(1+n)\sqrt{\sin^2(a+bx)}}$$

output
```
-(d*cos(b*x+a))^(1+n)*hypergeom([-3/2, 1/2+1/2*n],[3/2+1/2*n],cos(b*x+a)^2
)*sin(b*x+a)/b/d/(1+n)/(sin(b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int (d\cos(a+bx))^n \sin^4(a+bx)\,dx = -\frac{(d\cos(a+bx))^n\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sin(2(a+bx))}{2b(1+n)\sqrt{\sin^2(a+bx)}}$$

input
```
Integrate[(d*Cos[a + b*x])^n*Sin[a + b*x]^4,x]
```

output
```
-1/2*((d*Cos[a + b*x])^n*Hypergeometric2F1[-3/2, (1 + n)/2, (3 + n)/2, Cos
[a + b*x]^2]*Sin[2*(a + b*x)])/(b*(1 + n)*Sqrt[Sin[a + b*x]^2])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(a+bx)(d\cos(a+bx))^n\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(a+bx)^4(d\cos(a+bx))^n dx$$

$$\Big\downarrow \text{ 3056}$$

$$-\frac{\sin(a+bx)(d\cos(a+bx))^{n+1}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(a+bx)\right)}{bd(n+1)\sqrt{\sin^2(a+bx)}}$$

input
```
Int[(d*Cos[a + b*x])^n*Sin[a + b*x]^4,x]
```

output
```
-(((d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[-3/2, (1 + n)/2, (3 + n)/2,
Cos[a + b*x]^2]*Sin[a + b*x])/(b*d*(1 + n)*Sqrt[Sin[a + b*x]^2]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Maple [F]

$$\int (d\cos (bx+a))^n \sin (bx+a)^4 \, dx$$

input
```
int((d*cos(b*x+a))^n*sin(b*x+a)^4,x)
```

output
```
int((d*cos(b*x+a))^n*sin(b*x+a)^4,x)
```

Fricas [F]

$$\int (d\cos(a+bx))^n \sin^4(a+bx) \, dx = \int (d\cos (bx+a))^n \sin (bx+a)^4 \, dx$$

input
```
integrate((d*cos(b*x+a))^n*sin(b*x+a)^4,x, algorithm="fricas")
```

output
```
integral((cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*(d*cos(b*x + a))^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^n \sin^4(a+bx) \, dx = \text{Timed out}$$

input
```
integrate((d*cos(b*x+a))**n*sin(b*x+a)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\cos(a+bx))^n \sin^4(a+bx)\,dx = \int (d\cos(bx+a))^n \sin(bx+a)^4\,dx$$

input
```
integrate((d*cos(b*x+a))^n*sin(b*x+a)^4,x, algorithm="maxima")
```

output
```
integrate((d*cos(b*x + a))^n*sin(b*x + a)^4, x)
```

Giac [F]

$$\int (d\cos(a+bx))^n \sin^4(a+bx)\,dx = \int (d\cos(bx+a))^n \sin(bx+a)^4\,dx$$

input
```
integrate((d*cos(b*x+a))^n*sin(b*x+a)^4,x, algorithm="giac")
```

output
```
integrate((d*cos(b*x + a))^n*sin(b*x + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^n \sin^4(a+bx)\,dx = \int \sin(a+b\,x)^4\,(d\,\cos(a+b\,x))^n\,dx$$

input
```
int(sin(a + b*x)^4*(d*cos(a + b*x))^n,x)
```

output
```
int(sin(a + b*x)^4*(d*cos(a + b*x))^n, x)
```

Reduce [F]

$$\int (d\cos(a+bx))^n \sin^4(a+bx)\,dx = d^n\left(\int \cos(bx+a)^n \sin(bx+a)^4\,dx\right)$$

input
```
int((d*cos(b*x+a))^n*sin(b*x+a)^4,x)
```

output
```
d**n*int(cos(a + b*x)**n*sin(a + b*x)**4,x)
```

3.362 $\int (d\cos(a+bx))^n \sin^2(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 69

$$\int (d\cos(a+bx))^n \sin^2(a+bx)\,dx = -\frac{(d\cos(a+bx))^{1+n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sin(a+bx)}{bd(1+n)\sqrt{\sin^2(a+bx)}}$$

output
```
-(d*cos(b*x+a))^(1+n)*hypergeom([-1/2, 1/2+1/2*n],[3/2+1/2*n],cos(b*x+a)^2
)*sin(b*x+a)/b/d/(1+n)/(sin(b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int (d\cos(a+bx))^n \sin^2(a+bx)\,dx = -\frac{(d\cos(a+bx))^{n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sin(2(a+bx))}{2b(1+n)\sqrt{\sin^2(a+bx)}}$$

input
```
Integrate[(d*Cos[a + b*x])^n*Sin[a + b*x]^2,x]
```

output
```
-1/2*((d*Cos[a + b*x])^n*Hypergeometric2F1[-1/2, (1 + n)/2, (3 + n)/2, Cos
[a + b*x]^2]*Sin[2*(a + b*x)])/(b*(1 + n)*Sqrt[Sin[a + b*x]^2])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)(d\cos(a+bx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^2(d\cos(a+bx))^n dx$$

$$\Big\downarrow \text{3056}$$

$$-\frac{\sin(a+bx)(d\cos(a+bx))^{n+1}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(a+bx)\right)}{bd(n+1)\sqrt{\sin^2(a+bx)}}$$

input
```
Int[(d*Cos[a + b*x])^n*Sin[a + b*x]^2,x]
```

output
```
-(((d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[-1/2, (1 + n)/2, (3 + n)/2,
Cos[a + b*x]^2]*Sin[a + b*x])/(b*d*(1 + n)*Sqrt[Sin[a + b*x]^2]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Maple [F]

$$\int (d\cos(bx+a))^n \sin(bx+a)^2\, dx$$

input
```
int((d*cos(b*x+a))^n*sin(b*x+a)^2,x)
```

output
```
int((d*cos(b*x+a))^n*sin(b*x+a)^2,x)
```

Fricas [F]

$$\int (d\cos(a+bx))^n \sin^2(a+bx)\, dx = \int (d\cos(bx+a))^n \sin(bx+a)^2\, dx$$

input
```
integrate((d*cos(b*x+a))^n*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*(d*cos(b*x + a))^n, x)
```

Sympy [F]

$$\int (d\cos(a+bx))^n \sin^2(a+bx)\, dx = \int (d\cos(a+bx))^n \sin^2(a+bx)\, dx$$

input
```
integrate((d*cos(b*x+a))**n*sin(b*x+a)**2,x)
```

output
```
Integral((d*cos(a + b*x))**n*sin(a + b*x)**2, x)
```

Maxima [F]

$$\int (d\cos(a+bx))^n \sin^2(a+bx)\,dx = \int (d\cos(bx+a))^n \sin(bx+a)^2\,dx$$

input
```
integrate((d*cos(b*x+a))^n*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*cos(b*x + a))^n*sin(b*x + a)^2, x)
```

Giac [F]

$$\int (d\cos(a+bx))^n \sin^2(a+bx)\,dx = \int (d\cos(bx+a))^n \sin(bx+a)^2\,dx$$

input
```
integrate((d*cos(b*x+a))^n*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*cos(b*x + a))^n*sin(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^n \sin^2(a+bx)\,dx = \int \sin(a+b\,x)^2\,(d\,\cos(a+b\,x))^n\,dx$$

input
```
int(sin(a + b*x)^2*(d*cos(a + b*x))^n,x)
```

output
```
int(sin(a + b*x)^2*(d*cos(a + b*x))^n, x)
```

Reduce [F]

$$\int (d\cos(a+bx))^n \sin^2(a+bx)\,dx = d^n\left(\int \cos\left(bx+a\right)^n \sin\left(bx+a\right)^2 dx\right)$$

input
```
int((d*cos(b*x+a))^n*sin(b*x+a)^2,x)
```

output
```
d**n*int(cos(a + b*x)**n*sin(a + b*x)**2,x)
```

3.363 $\int (d\cos(a+bx))^n\,dx$

Optimal result

Integrand size = 10, antiderivative size = 69

$$\int (d\cos(a+bx))^n\,dx = -\frac{(d\cos(a+bx))^{1+n}\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sin(a+bx)}{bd(1+n)\sqrt{\sin^2(a+bx)}}$$

output
```
-(d*cos(b*x+a))^(1+n)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(b*x+a)^2)
*sin(b*x+a)/b/d/(1+n)/(sin(b*x+a)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.93

$$\int (d\cos(a+bx))^n\,dx = -\frac{(d\cos(a+bx))^n\cot(a+bx)\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{b(1+n)}$$

input
```
Integrate[(d*Cos[a + b*x])^n,x]
```

output
```
-(((d*Cos[a + b*x])^n*Cot[a + b*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 +
n)/2, Cos[a + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(b*(1 + n)))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\cos(a+bx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(d\sin\left(a+bx+\frac{\pi}{2}\right)\right)^n\,dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{\sin(a+bx)(d\cos(a+bx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(a+bx)\right)}{bd(n+1)\sqrt{\sin^2(a+bx)}}$$

input
```
Int[(d*Cos[a + b*x])^n,x]
```

output
```
-(((d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, C
os[a + b*x]^2]*Sin[a + b*x])/(b*d*(1 + n)*Sqrt[Sin[a + b*x]^2]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Maple [F]

$$\int (d\cos(bx+a))^n\,dx$$

input
```
int((d*cos(b*x+a))^n,x)
```

output
```
int((d*cos(b*x+a))^n,x)
```

Fricas [F]

$$\int (d\cos(a+bx))^n\,dx = \int (d\cos(bx+a))^n\,dx$$

input
```
integrate((d*cos(b*x+a))^n,x, algorithm="fricas")
```

output
```
integral((d*cos(b*x + a))^n, x)
```

Sympy [F]

$$\int (d\cos(a+bx))^n\,dx = \int (d\cos(a+bx))^n\,dx$$

input
```
integrate((d*cos(b*x+a))**n,x)
```

output
```
Integral((d*cos(a + b*x))**n, x)
```

Maxima [F]

$$\int (d\cos(a+bx))^n\,dx = \int (d\cos(bx+a))^n\,dx$$

input
```
integrate((d*cos(b*x+a))^n,x, algorithm="maxima")
```

output
```
integrate((d*cos(b*x + a))^n, x)
```

Giac [F]

$$\int (d\cos(a+bx))^n\,dx = \int (d\cos(bx+a))^n\,dx$$

input
```
integrate((d*cos(b*x+a))^n,x, algorithm="giac")
```

output
```
integrate((d*cos(b*x + a))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^n\,dx = \int (d\,\cos(a+b\,x))^n\,dx$$

input
```
int((d*cos(a + b*x))^n,x)
```

output
```
int((d*cos(a + b*x))^n, x)
```

Reduce [F]

$$\int (d\cos(a+bx))^n\,dx = d^n\left(\int \cos\left(bx+a\right)^n dx\right)$$

input
```
int((d*cos(b*x+a))^n,x)
```

output
```
d**n*int(cos(a + b*x)**n,x)
```

3.364 $\int (d\cos(a+bx))^n \csc^2(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 69

$$\int (d\cos(a+bx))^n \csc^2(a+bx)\,dx = -\frac{(d\cos(a+bx))^{1+n}\csc(a+bx)\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{bd(1+n)}$$

output
```
-(d*cos(b*x+a))^(1+n)*csc(b*x+a)*hypergeom([3/2, 1/2+1/2*n],[3/2+1/2*n],co
s(b*x+a)^2)*(sin(b*x+a)^2)^(1/2)/b/d/(1+n)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.16

$$\int (d\cos(a+bx))^n \csc^2(a+bx)\,dx = \frac{d(d\cos(a+bx))^{-1+n}\left(-\cot^2(a+bx)\right)^{\frac{1-n}{2}}\csc(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1-n}{2},1-\frac{n}{2},2-\frac{n}{2},\csc^2(a+b\right.}{b(-2+n)}$$

input
```
Integrate[(d*Cos[a + b*x])^n*Csc[a + b*x]^2,x]
```

output
```
(d*(d*Cos[a + b*x])^(-1 + n)*(-Cot[a + b*x]^2)^((1 - n)/2)*Csc[a + b*x]*Hy
pergeometric2F1[(1 - n)/2, 1 - n/2, 2 - n/2, Csc[a + b*x]^2])/(b*(-2 + n))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)(d\cos(a+bx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\cos(a+bx))^n}{\sin(a+bx)^2}dx$$

$$\Big\downarrow \text{3056}$$

$$-\frac{\sqrt{\sin^2(a+bx)}\csc(a+bx)(d\cos(a+bx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(a+bx)\right)}{bd(n+1)}$$

input
```
Int[(d*Cos[a + b*x])^n*Csc[a + b*x]^2,x]
```

output
```
-(((d*Cos[a + b*x])^(1 + n)*Csc[a + b*x]*Hypergeometric2F1[3/2, (1 + n)/2,
 (3 + n)/2, Cos[a + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(b*d*(1 + n)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Maple [F]

$$\int (d\cos (bx + a))^n \csc (bx + a)^2 \, dx$$

input
```
int((d*cos(b*x+a))^n*csc(b*x+a)^2,x)
```

output
```
int((d*cos(b*x+a))^n*csc(b*x+a)^2,x)
```

Fricas [F]

$$\int (d\cos(a + bx))^n \csc^2(a + bx) \, dx = \int (d\cos (bx + a))^n \csc (bx + a)^2 \, dx$$

input
```
integrate((d*cos(b*x+a))^n*csc(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral((d*cos(b*x + a))^n*csc(b*x + a)^2, x)
```

Sympy [F]

$$\int (d\cos(a+bx))^n \csc^2(a+bx)\,dx = \int (d\cos{(a+bx)})^n \csc^2{(a+bx)}\,dx$$

input
```
integrate((d*cos(b*x+a))**n*csc(b*x+a)**2,x)
```

output
```
Integral((d*cos(a + b*x))**n*csc(a + b*x)**2, x)
```

Maxima [F]

$$\int (d\cos(a+bx))^n \csc^2(a+bx)\,dx = \int (d\cos{(bx+a)})^n \csc{(bx+a)}^2\,dx$$

input
```
integrate((d*cos(b*x+a))^n*csc(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*cos(b*x + a))^n*csc(b*x + a)^2, x)
```

Giac [F]

$$\int (d\cos(a+bx))^n \csc^2(a+bx)\,dx = \int (d\cos{(bx+a)})^n \csc{(bx+a)}^2\,dx$$

input
```
integrate((d*cos(b*x+a))^n*csc(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*cos(b*x + a))^n*csc(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^n \csc^2(a+bx)\,dx = \int \frac{(d\,\cos\left(a+b\,x\right))^n}{\sin\left(a+b\,x\right)^2}\,dx$$

input
```
int((d*cos(a + b*x))^n/sin(a + b*x)^2,x)
```

output
```
int((d*cos(a + b*x))^n/sin(a + b*x)^2, x)
```

Reduce [F]

$$\int (d\cos(a+bx))^n \csc^2(a+bx)\,dx = d^n \left(\int \cos\left(bx+a\right)^n \csc\left(bx+a\right)^2 dx\right)$$

input
```
int((d*cos(b*x+a))^n*csc(b*x+a)^2,x)
```

output
```
d**n*int(cos(a + b*x)**n*csc(a + b*x)**2,x)
```

3.365 $\int (d\cos(a+bx))^n \csc^4(a+bx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 69

$$\int (d\cos(a+bx))^n \csc^4(a+bx)\,dx = -\frac{(d\cos(a+bx))^{1+n}\csc(a+bx)\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{bd(1+n)}$$

output
```
-(d*cos(b*x+a))^(1+n)*csc(b*x+a)*hypergeom([5/2, 1/2+1/2*n],[3/2+1/2*n],co
s(b*x+a)^2)*(sin(b*x+a)^2)^(1/2)/b/d/(1+n)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.19

$$\int (d\cos(a+bx))^n \csc^4(a+bx)\,dx = \frac{d(d\cos(a+bx))^{-1+n}\left(-\cot^2(a+bx)\right)^{\frac{1-n}{2}}\csc^3(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1-n}{2},2-\frac{n}{2},3-\frac{n}{2},\csc^2(a+ \right.}{b(-4+n)}$$

input
```
Integrate[(d*Cos[a + b*x])^n*Csc[a + b*x]^4,x]
```

output
```
(d*(d*Cos[a + b*x])^(-1 + n)*(-Cot[a + b*x]^2)^((1 - n)/2)*Csc[a + b*x]^3*
Hypergeometric2F1[(1 - n)/2, 2 - n/2, 3 - n/2, Csc[a + b*x]^2])/(b*(-4 + n
))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(a+bx)(d\cos(a+bx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\cos(a+bx))^n}{\sin(a+bx)^4}dx$$

$$\Big\downarrow \text{3056}$$

$$-\frac{\sqrt{\sin^2(a+bx)}\csc(a+bx)(d\cos(a+bx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2(a+bx)\right)}{bd(n+1)}$$

input
```
Int[(d*Cos[a + b*x])^n*Csc[a + b*x]^4,x]
```

output
```
-(((d*Cos[a + b*x])^(1 + n)*Csc[a + b*x]*Hypergeometric2F1[5/2, (1 + n)/2,
 (3 + n)/2, Cos[a + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(b*d*(1 + n)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Maple [F]

$$\int (d\cos(bx+a))^n \csc(bx+a)^4\, dx$$

input
```
int((d*cos(b*x+a))^n*csc(b*x+a)^4,x)
```

output
```
int((d*cos(b*x+a))^n*csc(b*x+a)^4,x)
```

Fricas [F]

$$\int (d\cos(a+bx))^n \csc^4(a+bx)\, dx = \int (d\cos(bx+a))^n \csc(bx+a)^4\, dx$$

input
```
integrate((d*cos(b*x+a))^n*csc(b*x+a)^4,x, algorithm="fricas")
```

output
```
integral((d*cos(b*x + a))^n*csc(b*x + a)^4, x)
```

Sympy [F]

$$\int (d\cos(a+bx))^n \csc^4(a+bx)\,dx = \int (d\cos{(a+bx)})^n \csc^4{(a+bx)}\,dx$$

input
```
integrate((d*cos(b*x+a))**n*csc(b*x+a)**4,x)
```

output
```
Integral((d*cos(a + b*x))**n*csc(a + b*x)**4, x)
```

Maxima [F]

$$\int (d\cos(a+bx))^n \csc^4(a+bx)\,dx = \int (d\cos{(bx+a)})^n \csc{(bx+a)}^4\,dx$$

input
```
integrate((d*cos(b*x+a))^n*csc(b*x+a)^4,x, algorithm="maxima")
```

output
```
integrate((d*cos(b*x + a))^n*csc(b*x + a)^4, x)
```

Giac [F]

$$\int (d\cos(a+bx))^n \csc^4(a+bx)\,dx = \int (d\cos{(bx+a)})^n \csc{(bx+a)}^4\,dx$$

input
```
integrate((d*cos(b*x+a))^n*csc(b*x+a)^4,x, algorithm="giac")
```

output
```
integrate((d*cos(b*x + a))^n*csc(b*x + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^n \csc^4(a+bx)\,dx = \int \frac{(d\cos(a+b\,x))^n}{\sin(a+b\,x)^4}\,dx$$

input
```
int((d*cos(a + b*x))^n/sin(a + b*x)^4,x)
```

output
```
int((d*cos(a + b*x))^n/sin(a + b*x)^4, x)
```

Reduce [F]

$$\int (d\cos(a+bx))^n \csc^4(a+bx)\,dx = d^n\left(\int \cos(bx+a)^n \csc(bx+a)^4\,dx\right)$$

input
```
int((d*cos(b*x+a))^n*csc(b*x+a)^4,x)
```

output
```
d**n*int(cos(a + b*x)**n*csc(a + b*x)**4,x)
```

3.366 $\int (d\cos(a+bx))^n(c\sin(a+bx))^{5/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int (d\cos(a+bx))^n(c\sin(a+bx))^{5/2}\,dx = -\frac{c(d\cos(a+bx))^{1+n}\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)(c\sin(a+bx))^{3/2}}{bd(1+n)\sin^2(a+bx)^{3/4}}$$

output
```
-c*(d*cos(b*x+a))^(1+n)*hypergeom([-3/4, 1/2+1/2*n],[3/2+1/2*n],cos(b*x+a)
^2)*(c*sin(b*x+a))^(3/2)/b/d/(1+n)/(sin(b*x+a)^2)^(3/4)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 158 vs. 2(76) = 152.

Time = 0.96 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.08

$$\int (d\cos(a+bx))^n(c\sin(a+bx))^{5/2}\,dx = \frac{(d\cos(a+bx))^n\cot(a+bx)\left(-\left((3+n)\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\right) -\right.}{}$$

input
```
Integrate[(d*Cos[a + b*x])^n*(c*Sin[a + b*x])^(5/2),x]
```

output
```
((d*Cos[a + b*x])^n*Cot[a + b*x]*(-((3 + n)*Hypergeometric2F1[-3/4, (1 + n
)/2, (3 + n)/2, Cos[a + b*x]^2]) - (3 + n)*Hypergeometric2F1[1/4, (1 + n)/
2, (3 + n)/2, Cos[a + b*x]^2] + (1 + n)*Cos[a + b*x]^2*Hypergeometric2F1[1
/4, (3 + n)/2, (5 + n)/2, Cos[a + b*x]^2])*(c*Sin[a + b*x])^(5/2))/(2*b*(1
 + n)*(3 + n)*(Sin[a + b*x]^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\sin(a+bx))^{5/2}(d\cos(a+bx))^n\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (c\sin(a+bx))^{5/2}(d\cos(a+bx))^n dx$$

$$\Big\downarrow \; 3056$$

$$-\frac{c(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{n+1}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{n+1}{2},\frac{n+3}{2},\cos^2(a+bx)\right)}{bd(n+1)\sin^2(a+bx)^{3/4}}$$

input
```
Int[(d*Cos[a + b*x])^n*(c*Sin[a + b*x])^(5/2),x]
```

output
```
-((c*(d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[-3/4, (1 + n)/2, (3 + n)/2
, Cos[a + b*x]^2]*(c*Sin[a + b*x])^(3/2))/(b*d*(1 + n)*(Sin[a + b*x]^2)^(3
/4)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Maple [F]

$$\int (d\cos(bx+a))^{n} (c\sin(bx+a))^{\frac{5}{2}} dx$$

input
```
int((d*cos(b*x+a))^n*(c*sin(b*x+a))^(5/2),x)
```

output
```
int((d*cos(b*x+a))^n*(c*sin(b*x+a))^(5/2),x)
```

Fricas [F]

$$\int (d\cos(a+bx))^n (c\sin(a+bx))^{5/2} dx = \int (c\sin(bx+a))^{\frac{5}{2}} (d\cos(bx+a))^{n} dx$$

input
```
integrate((d*cos(b*x+a))^n*(c*sin(b*x+a))^(5/2),x, algorithm="fricas")
```

output
```
integral(-(c^2*cos(b*x + a)^2 - c^2)*sqrt(c*sin(b*x + a))*(d*cos(b*x + a))
^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^n (c\sin(a+bx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((d*cos(b*x+a))**n*(c*sin(b*x+a))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\cos(a+bx))^n (c\sin(a+bx))^{5/2}\,dx = \int (c\sin(bx+a))^{\frac{5}{2}} (d\cos(bx+a))^n\,dx$$

input
```
integrate((d*cos(b*x+a))^n*(c*sin(b*x+a))^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a))^(5/2)*(d*cos(b*x + a))^n, x)
```

Giac [F]

$$\int (d\cos(a+bx))^n (c\sin(a+bx))^{5/2}\,dx = \int (c\sin(bx+a))^{\frac{5}{2}} (d\cos(bx+a))^n\,dx$$

input
```
integrate((d*cos(b*x+a))^n*(c*sin(b*x+a))^(5/2),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a))^(5/2)*(d*cos(b*x + a))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^n(c\sin(a+bx))^{5/2}\,dx = \int (d\,\cos(a+b\,x))^n\,(c\,\sin(a+b\,x))^{5/2}\,dx$$

input
```
int((d*cos(a + b*x))^n*(c*sin(a + b*x))^(5/2),x)
```

output
```
int((d*cos(a + b*x))^n*(c*sin(a + b*x))^(5/2), x)
```

Reduce [F]

$$\int (d\cos(a+bx))^n(c\sin(a+bx))^{5/2}\,dx = d^n\sqrt{c}\left(\int \sqrt{\sin(bx+a)}\,\cos(bx+a)^n\sin(bx+a)^2\,dx\right)c^2$$

input
```
int((d*cos(b*x+a))^n*(c*sin(b*x+a))^(5/2),x)
```

output
```
d**n*sqrt(c)*int(sqrt(sin(a + b*x))*cos(a + b*x)**n*sin(a + b*x)**2,x)*c**
2
```

3.367 $\int (d\cos(a+bx))^n (c\sin(a+bx))^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int (d\cos(a+bx))^n (c\sin(a+bx))^{3/2}\,dx = -\frac{c(d\cos(a+bx))^{1+n}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sqrt{c\sin(a+bx)}}{bd(1+n)\sqrt[4]{\sin^2(a+bx)}}$$

output
```
-c*(d*cos(b*x+a))^(1+n)*hypergeom([-1/4, 1/2+1/2*n],[3/2+1/2*n],cos(b*x+a)
^2)*(c*sin(b*x+a))^(1/2)/b/d/(1+n)/(sin(b*x+a)^2)^(1/4)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int (d\cos(a+bx))^n (c\sin(a+bx))^{3/2}\,dx = -\frac{(d\cos(a+bx))^n\cot(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)(c\sin(a+bx))^{3/2}}{b(1+n)\sqrt[4]{\sin^2(a+bx)}}$$

input
```
Integrate[(d*Cos[a + b*x])^n*(c*Sin[a + b*x])^(3/2),x]
```

output
```
-(((d*Cos[a + b*x])^n*Cot[a + b*x]*Hypergeometric2F1[-1/4, (1 + n)/2, (3 +
 n)/2, Cos[a + b*x]^2]*(c*Sin[a + b*x])^(3/2))/(b*(1 + n)*(Sin[a + b*x]^2)
^(1/4)))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\sin(a+bx))^{3/2}(d\cos(a+bx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c\sin(a+bx))^{3/2}(d\cos(a+bx))^n dx$$

$$\Big\downarrow \text{3056}$$

$$-\frac{c\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{n+1}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{n+1}{2},\frac{n+3}{2},\cos^2(a+bx)\right)}{bd(n+1)\sqrt[4]{\sin^2(a+bx)}}$$

input
```
Int[(d*Cos[a + b*x])^n*(c*Sin[a + b*x])^(3/2),x]
```

output
```
-((c*(d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[-1/4, (1 + n)/2, (3 + n)/2
, Cos[a + b*x]^2]*Sqrt[c*Sin[a + b*x]])/(b*d*(1 + n)*(Sin[a + b*x]^2)^(1/4
)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Maple [F]

$$\int \left(d\cos \left(bx+a\right)\right)^{n} \left(c\sin \left(bx+a\right)\right)^{\frac{3}{2}}dx$$

input
```
int((d*cos(b*x+a))^n*(c*sin(b*x+a))^(3/2),x)
```

output
```
int((d*cos(b*x+a))^n*(c*sin(b*x+a))^(3/2),x)
```

Fricas [F]

$$\int (d\cos(a+bx))^n(c\sin(a+bx))^{3/2}\,dx=\int \left(c\sin \left(bx+a\right)\right)^{\frac{3}{2}}\left(d\cos \left(bx+a\right)\right)^{n}\,dx$$

input
```
integrate((d*cos(b*x+a))^n*(c*sin(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*sin(b*x + a))*(d*cos(b*x + a))^n*c*sin(b*x + a), x)
```

Sympy [F]

$$\int (d\cos(a+bx))^n(c\sin(a+bx))^{3/2}\,dx = \int (c\sin{(a+bx)})^{\frac{3}{2}}\,(d\cos{(a+bx)})^n\,dx$$

input
```
integrate((d*cos(b*x+a))**n*(c*sin(b*x+a))**(3/2),x)
```

output
```
Integral((c*sin(a + b*x))**(3/2)*(d*cos(a + b*x))**n, x)
```

Maxima [F]

$$\int (d\cos(a+bx))^n(c\sin(a+bx))^{3/2}\,dx = \int (c\sin{(bx+a)})^{\frac{3}{2}}\,(d\cos{(bx+a)})^n\,dx$$

input
```
integrate((d*cos(b*x+a))^n*(c*sin(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a))^(3/2)*(d*cos(b*x + a))^n, x)
```

Giac [F]

$$\int (d\cos(a+bx))^n(c\sin(a+bx))^{3/2}\,dx = \int (c\sin{(bx+a)})^{\frac{3}{2}}\,(d\cos{(bx+a)})^n\,dx$$

input
```
integrate((d*cos(b*x+a))^n*(c*sin(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a))^(3/2)*(d*cos(b*x + a))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^n(c\sin(a+bx))^{3/2}\,dx = \int (d\,\cos\left(a+b\,x\right))^n\,\left(c\,\sin\left(a+b\,x\right)\right)^{3/2}\,dx$$

input
```
int((d*cos(a + b*x))^n*(c*sin(a + b*x))^(3/2),x)
```

output
```
int((d*cos(a + b*x))^n*(c*sin(a + b*x))^(3/2), x)
```

Reduce [F]

$$\int (d\cos(a+bx))^n(c\sin(a+bx))^{3/2}\,dx = d^n\sqrt{c}\left(\int \sqrt{\sin\left(bx+a\right)}\,\cos\left(bx+a\right)^n\sin\left(bx+a\right)dx\right)c$$

input
```
int((d*cos(b*x+a))^n*(c*sin(b*x+a))^(3/2),x)
```

output
```
d**n*sqrt(c)*int(sqrt(sin(a + b*x))*cos(a + b*x)**n*sin(a + b*x),x)*c
```

3.368 $\int (d\cos(a+bx))^n\sqrt{c\sin(a+bx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int (d\cos(a+bx))^n\sqrt{c\sin(a+bx)}\,dx = -\frac{c(d\cos(a+bx))^{1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sqrt[4]{\sin^2(a+bx)}}{bd(1+n)\sqrt{c\sin(a+bx)}}$$

output
```
-c*(d*cos(b*x+a))^(1+n)*hypergeom([1/4, 1/2+1/2*n],[3/2+1/2*n],cos(b*x+a)^
2)*(sin(b*x+a)^2)^(1/4)/b/d/(1+n)/(c*sin(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08

$$\int (d\cos(a+bx))^n\sqrt{c\sin(a+bx)}\,dx = -\frac{\cos(a+bx)(d\cos(a+bx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sin(a+bx)\sqrt{c\sin(a+bx)}}{b(1+n)\sin^2(a+bx)^{3/4}}$$

input
```
Integrate[(d*Cos[a + b*x])^n*Sqrt[c*Sin[a + b*x]],x]
```

output
```
-((Cos[a + b*x]*(d*Cos[a + b*x])^n*Hypergeometric2F1[1/4, (1 + n)/2, (3 +
n)/2, Cos[a + b*x]^2]*Sin[a + b*x]*Sqrt[c*Sin[a + b*x]])/(b*(1 + n)*(Sin[a
 + b*x]^2)^(3/4)))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c\sin(a+bx)}(d\cos(a+bx))^n\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{c\sin(a+bx)}(d\cos(a+bx))^n dx$$

$$\Big\downarrow \text{ 3056}$$

$$-\frac{c\sqrt[4]{\sin^2(a+bx)}(d\cos(a+bx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{n+1}{2},\frac{n+3}{2},\cos^2(a+bx)\right)}{bd(n+1)\sqrt{c\sin(a+bx)}}$$

input
```
Int[(d*Cos[a + b*x])^n*Sqrt[c*Sin[a + b*x]],x]
```

output
```
-((c*(d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[1/4, (1 + n)/2, (3 + n)/2,
 Cos[a + b*x]^2]*(Sin[a + b*x]^2)^(1/4))/(b*d*(1 + n)*Sqrt[c*Sin[a + b*x]]
))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Maple [F]

$$\int (d\cos(bx+a))^n \sqrt{c\sin(bx+a)}dx$$

input
```
int((d*cos(b*x+a))^n*(c*sin(b*x+a))^(1/2),x)
```

output
```
int((d*cos(b*x+a))^n*(c*sin(b*x+a))^(1/2),x)
```

Fricas [F]

$$\int (d\cos(a+bx))^n\sqrt{c\sin(a+bx)}\,dx = \int \sqrt{c\sin(bx+a)}(d\cos(bx+a))^n\,dx$$

input
```
integrate((d*cos(b*x+a))^n*(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*sin(b*x + a))*(d*cos(b*x + a))^n, x)
```

Sympy [F]

$$\int (d\cos(a+bx))^n\sqrt{c\sin(a+bx)}\,dx=\int\sqrt{c\sin{(a+bx)}}(d\cos{(a+bx)})^n\,dx$$

input
```
integrate((d*cos(b*x+a))**n*(c*sin(b*x+a))**(1/2),x)
```

output
```
Integral(sqrt(c*sin(a + b*x))*(d*cos(a + b*x))**n, x)
```

Maxima [F]

$$\int (d\cos(a+bx))^n\sqrt{c\sin(a+bx)}\,dx=\int\sqrt{c\sin{(bx+a)}}(d\cos{(bx+a)})^n\,dx$$

input
```
integrate((d*cos(b*x+a))^n*(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*sin(b*x + a))*(d*cos(b*x + a))^n, x)
```

Giac [F]

$$\int (d\cos(a+bx))^n\sqrt{c\sin(a+bx)}\,dx=\int\sqrt{c\sin{(bx+a)}}(d\cos{(bx+a)})^n\,dx$$

input
```
integrate((d*cos(b*x+a))^n*(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*sin(b*x + a))*(d*cos(b*x + a))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(a+bx))^n\sqrt{c\sin(a+bx)}\,dx = \int (d\,\cos\left(a+b\,x\right))^n\,\sqrt{c\,\sin\left(a+b\,x\right)}\,dx$$

input
```
int((d*cos(a + b*x))^n*(c*sin(a + b*x))^(1/2),x)
```

output
```
int((d*cos(a + b*x))^n*(c*sin(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int (d\cos(a+bx))^n\sqrt{c\sin(a+bx)}\,dx = d^n\sqrt{c}\left(\int \sqrt{\sin\left(bx+a\right)}\,\cos\left(bx+a\right)^n dx\right)$$

input
```
int((d*cos(b*x+a))^n*(c*sin(b*x+a))^(1/2),x)
```

output
```
d**n*sqrt(c)*int(sqrt(sin(a + b*x))*cos(a + b*x)**n,x)
```

3.369 $\int \frac{(d\cos(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \frac{(d\cos(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx = -\frac{c(d\cos(a+bx))^{1+n}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sin^2(a+bx)^{3/4}}{bd(1+n)(c\sin(a+bx))^{3/2}}$$

output
```
-c*(d*cos(b*x+a))^(1+n)*hypergeom([3/4, 1/2+1/2*n],[3/2+1/2*n],cos(b*x+a)^
2)*(sin(b*x+a)^2)^(3/4)/b/d/(1+n)/(c*sin(b*x+a))^(3/2)
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08

$$\int \frac{(d\cos(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx = -\frac{\cos(a+bx)(d\cos(a+bx))^n\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sin(a+bx)}{b(1+n)\sqrt{c\sin(a+bx)}\sqrt[4]{\sin^2(a+bx)}}$$

input
```
Integrate[(d*Cos[a + b*x])^n/Sqrt[c*Sin[a + b*x]],x]
```

output
```
-((Cos[a + b*x]*(d*Cos[a + b*x])^n*Hypergeometric2F1[3/4, (1 + n)/2, (3 +
n)/2, Cos[a + b*x]^2]*Sin[a + b*x])/(b*(1 + n)*Sqrt[c*Sin[a + b*x]]*(Sin[a
 + b*x]^2)^(1/4)))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\cos(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(d\cos(a+bx))^n}{\sqrt{c\sin(a+bx)}}dx$$

$$\Big\downarrow \text{ 3056}$$

$$-\frac{c\sin^2(a+bx)^{3/4}(d\cos(a+bx))^{n+1}\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{n+1}{2},\frac{n+3}{2},\cos^2(a+bx)\right)}{bd(n+1)(c\sin(a+bx))^{3/2}}$$

input
```
Int[(d*Cos[a + b*x])^n/Sqrt[c*Sin[a + b*x]],x]
```

output
```
-((c*(d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[3/4, (1 + n)/2, (3 + n)/2,
 Cos[a + b*x]^2]*(Sin[a + b*x]^2)^(3/4))/(b*d*(1 + n)*(c*Sin[a + b*x])^(3/
2)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Maple [F]

$$\int \frac{(d\cos{(bx+a)})^n}{\sqrt{c\sin{(bx+a)}}}dx$$

input
```
int((d*cos(b*x+a))^n/(c*sin(b*x+a))^(1/2),x)
```

output
```
int((d*cos(b*x+a))^n/(c*sin(b*x+a))^(1/2),x)
```

Fricas [F]

$$\int \frac{(d\cos(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx=\int \frac{(d\cos{(bx+a)})^n}{\sqrt{c\sin{(bx+a)}}}\,dx$$

input
```
integrate((d*cos(b*x+a))^n/(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*sin(b*x + a))*(d*cos(b*x + a))^n/(c*sin(b*x + a)), x)
```

Sympy [F]

$$\int \frac{(d\cos(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{(d\cos{(a+bx)})^n}{\sqrt{c\sin{(a+bx)}}}\,dx$$

input
```
integrate((d*cos(b*x+a))**n/(c*sin(b*x+a))**(1/2),x)
```

output
```
Integral((d*cos(a + b*x))**n/sqrt(c*sin(a + b*x)), x)
```

Maxima [F]

$$\int \frac{(d\cos(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{(d\cos{(bx+a)})^n}{\sqrt{c\sin{(bx+a)}}}\,dx$$

input
```
integrate((d*cos(b*x+a))^n/(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*cos(b*x + a))^n/sqrt(c*sin(b*x + a)), x)
```

Giac [F]

$$\int \frac{(d\cos(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{(d\cos{(bx+a)})^n}{\sqrt{c\sin{(bx+a)}}}\,dx$$

input
```
integrate((d*cos(b*x+a))^n/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*cos(b*x + a))^n/sqrt(c*sin(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\cos(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{(d\,\cos\left(a+b\,x\right))^n}{\sqrt{c\,\sin\left(a+b\,x\right)}}\,dx$$

input
```
int((d*cos(a + b*x))^n/(c*sin(a + b*x))^(1/2),x)
```

output
```
int((d*cos(a + b*x))^n/(c*sin(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(d\cos(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx = \frac{d^n\sqrt{c}\left(\int \frac{\sqrt{\sin(bx+a)}\,\cos(bx+a)^n}{\sin(bx+a)}dx\right)}{c}$$

input
```
int((d*cos(b*x+a))^n/(c*sin(b*x+a))^(1/2),x)
```

output
```
(d**n*sqrt(c)*int((sqrt(sin(a + b*x))*cos(a + b*x)**n)/sin(a + b*x),x))/c
```

3.370 $\int \frac{(d\cos(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 78

$$\int \frac{(d\cos(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx = -\frac{(d\cos(a+bx))^{1+n}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sqrt[4]{\sin^2(a+bx)}}{bcd(1+n)\sqrt{c\sin(a+bx)}}$$

output
```
-(d*cos(b*x+a))^(1+n)*hypergeom([5/4, 1/2+1/2*n],[3/2+1/2*n],cos(b*x+a)^2)
*(sin(b*x+a)^2)^(1/4)/b/c/d/(1+n)/(c*sin(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01

$$\int \frac{(d\cos(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx = -\frac{(d\cos(a+bx))^n\cot(a+bx)\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sqrt{c\sin(a+bx)}\sqrt[4]{\sin^2(a+b}}{bc^2(1+n)}$$

input
```
Integrate[(d*Cos[a + b*x])^n/(c*Sin[a + b*x])^(3/2),x]
```

output
```
-(((d*Cos[a + b*x])^n*Cot[a + b*x]*Hypergeometric2F1[5/4, (1 + n)/2, (3 +
n)/2, Cos[a + b*x]^2]*Sqrt[c*Sin[a + b*x]]*(Sin[a + b*x]^2)^(1/4))/(b*c^2*
(1 + n)))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d\cos(a+bx))^n}{(c\sin(a+bx))^{3/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(d\cos(a+bx))^n}{(c\sin(a+bx))^{3/2}}dx$$

$$\Big\downarrow \; 3056$$

$$-\frac{\sqrt[4]{\sin^2(a+bx)}(d\cos(a+bx))^{n+1}\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{n+1}{2},\frac{n+3}{2},\cos^2(a+bx)\right)}{bcd(n+1)\sqrt{c\sin(a+bx)}}$$

input
```
Int[(d*Cos[a + b*x])^n/(c*Sin[a + b*x])^(3/2),x]
```

output
```
-(((d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[5/4, (1 + n)/2, (3 + n)/2, C
os[a + b*x]^2]*(Sin[a + b*x]^2)^(1/4))/(b*c*d*(1 + n)*Sqrt[c*Sin[a + b*x]]
))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Maple [F]

$$\int \frac{(d\cos(bx+a))^{n}}{(c\sin(bx+a))^{\frac{3}{2}}}dx$$

input
```
int((d*cos(b*x+a))^n/(c*sin(b*x+a))^(3/2),x)
```

output
```
int((d*cos(b*x+a))^n/(c*sin(b*x+a))^(3/2),x)
```

Fricas [F]

$$\int \frac{(d\cos(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx = \int \frac{(d\cos(bx+a))^{n}}{(c\sin(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*cos(b*x+a))^n/(c*sin(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(c*sin(b*x + a))*(d*cos(b*x + a))^n/(c^2*cos(b*x + a)^2 - c^
2), x)
```

Sympy [F]

$$\int \frac{(d\cos(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx = \int \frac{(d\cos{(a+bx)})^n}{(c\sin{(a+bx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*cos(b*x+a))**n/(c*sin(b*x+a))**(3/2),x)
```

output
```
Integral((d*cos(a + b*x))**n/(c*sin(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(d\cos(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx = \int \frac{(d\cos{(bx+a)})^n}{(c\sin{(bx+a)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*cos(b*x+a))^n/(c*sin(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*cos(b*x + a))^n/(c*sin(b*x + a))^(3/2), x)
```

Giac [F]

$$\int \frac{(d\cos(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx = \int \frac{(d\cos{(bx+a)})^n}{(c\sin{(bx+a)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*cos(b*x+a))^n/(c*sin(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*cos(b*x + a))^n/(c*sin(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\cos(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx = \int \frac{(d\,\cos(a+b\,x))^n}{(c\,\sin(a+b\,x))^{3/2}}\,dx$$

input
```
int((d*cos(a + b*x))^n/(c*sin(a + b*x))^(3/2),x)
```

output
```
int((d*cos(a + b*x))^n/(c*sin(a + b*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(d\cos(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx = \frac{d^n\sqrt{c}\left(\int \frac{\sqrt{\sin(bx+a)}\,\cos(bx+a)^n}{\sin(bx+a)^2}dx\right)}{c^2}$$

input
```
int((d*cos(b*x+a))^n/(c*sin(b*x+a))^(3/2),x)
```

output
```
(d**n*sqrt(c)*int((sqrt(sin(a + b*x))*cos(a + b*x)**n)/sin(a + b*x)**2,x))
/c**2
```

3.371 $\int \sqrt{b\sec(e+fx)}\sin^7(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 85

$$\int \sqrt{b\sec(e+fx)}\sin^7(e+fx)\,dx=\frac{2b^7}{13f(b\sec(e+fx))^{13/2}}-\frac{2b^5}{3f(b\sec(e+fx))^{9/2}}+\frac{6b^3}{5f(b\sec(e+fx))^{5/2}}-\frac{2b}{f\sqrt{b\sec(e+fx)}}$$

output
```
2/13*b^7/f/(b*sec(f*x+e))^(13/2)-2/3*b^5/f/(b*sec(f*x+e))^(9/2)+6/5*b^3/f/
(b*sec(f*x+e))^(5/2)-2*b/f/(b*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.68

$$\int \sqrt{b\sec(e+fx)}\sin^7(e+fx)\,dx=\frac{(-8939\cos(e+fx)+887\cos(3(e+fx))-155\cos(5(e+fx))+15\cos(7(e+fx)))\sqrt{b\sec(e+fx)}}{6240f}$$

input
```
Integrate[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^7,x]
```

output
```
((-8939*Cos[e + f*x] + 887*Cos[3*(e + f*x)] - 155*Cos[5*(e + f*x)] + 15*Co
s[7*(e + f*x)])*Sqrt[b*Sec[e + f*x]])/(6240*f)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3102, 25, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^7(e+fx)\sqrt{b\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\sec(e+fx)}}{\csc(e+fx)^7}dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{b^7\int -\frac{\left(b^2-b^2\sec^2(e+fx)\right)^3}{b^6(b\sec(e+fx))^{15/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b^7\int \frac{\left(b^2-b^2\sec^2(e+fx)\right)^3}{b^6(b\sec(e+fx))^{15/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\int \frac{\left(b^2-b^2\sec^2(e+fx)\right)^3}{(b\sec(e+fx))^{15/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{244}$$

$$-\frac{b\int \left(\frac{b^6}{(b\sec(e+fx))^{15/2}}-\frac{3b^4}{(b\sec(e+fx))^{11/2}}+\frac{3b^2}{(b\sec(e+fx))^{7/2}}-\frac{1}{(b\sec(e+fx))^{3/2}}\right)d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\left(-\frac{2b^6}{13(b\sec(e+fx))^{13/2}}+\frac{2b^4}{3(b\sec(e+fx))^{9/2}}-\frac{6b^2}{5(b\sec(e+fx))^{5/2}}+\frac{2}{\sqrt{b\sec(e+fx)}}\right)}{f}$$

input
```
Int[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^7,x]
```

output
```
-((b*((-2*b^6)/(13*(b*Sec[e + f*x])^(13/2)) + (2*b^4)/(3*(b*Sec[e + f*x])^
(9/2)) - (6*b^2)/(5*(b*Sec[e + f*x])^(5/2)) + 2/Sqrt[b*Sec[e + f*x]]))/f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(71) = 142$.

Time = 12.40 (sec) , antiderivative size = 265, normalized size of antiderivative = 3.12

method	result
default	$\frac{\left(\frac{(-195\cos(fx+e)-195)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\cos(fx+e)+1}{\cos(fx+e)+1}\right)}{390}+\frac{(195\cos(fx+e)+1}{}\right.}{}$

input
```
int((b*sec(f*x+e))^(1/2)*sin(f*x+e)^7,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/390*(-195*cos(f*x+e)-195)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln((
2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))+1/390*(195*cos(f*x+e)+195)*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos
(f*x+e)+1))+1/390*cos(f*x+e)*(60*cos(f*x+e)^6-260*cos(f*x+e)^4+468*cos(f*x
+e)^2-780))*(b*sec(f*x+e))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.66

$$\int \sqrt{b\sec(e+fx)}\sin^7(e+fx)\,dx = \frac{2\left(15\cos\left(fx+e\right)^7-65\cos\left(fx+e\right)^5+117\cos\left(fx+e\right)^3-195\cos\left(fx+e\right)\right)\sqrt{\frac{b}{\cos(fx+e)}}}{195\,f}$$

input
```
integrate((b*sec(f*x+e))^(1/2)*sin(f*x+e)^7,x, algorithm="fricas")
```

output
```
2/195*(15*cos(f*x + e)^7 - 65*cos(f*x + e)^5 + 117*cos(f*x + e)^3 - 195*co
s(f*x + e))*sqrt(b/cos(f*x + e))/f
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}\sin^7(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((b*sec(f*x+e))**(1/2)*sin(f*x+e)**7,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.74

$$\int \sqrt{b\sec(e+fx)}\sin^7(e+fx)\,dx = \frac{2\left(15\,b^6 - \frac{65\,b^6}{\cos(fx+e)^2} + \frac{117\,b^6}{\cos(fx+e)^4} - \frac{195\,b^6}{\cos(fx+e)^6}\right)b}{195\,f\left(\frac{b}{\cos(fx+e)}\right)^{\frac{13}{2}}}$$

input
```
integrate((b*sec(f*x+e))^(1/2)*sin(f*x+e)^7,x, algorithm="maxima")
```

output
```
2/195*(15*b^6 - 65*b^6/cos(f*x + e)^2 + 117*b^6/cos(f*x + e)^4 - 195*b^6/c
os(f*x + e)^6)*b/(f*(b/cos(f*x + e))^(13/2))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.18

$$\int \sqrt{b\sec(e+fx)}\sin^7(e+fx)\,dx$$
$$= \frac{2\left(15\sqrt{b\cos(fx+e)}b^6\cos(fx+e)^6 - 65\sqrt{b\cos(fx+e)}b^6\cos(fx+e)^4 + 117\sqrt{b\cos(fx+e)}b^6\cos\right.}{195\,b^6 f}$$

input
```
integrate((b*sec(f*x+e))^(1/2)*sin(f*x+e)^7,x, algorithm="giac")
```

output
```
2/195*(15*sqrt(b*cos(f*x + e))*b^6*cos(f*x + e)^6 - 65*sqrt(b*cos(f*x + e)
)*b^6*cos(f*x + e)^4 + 117*sqrt(b*cos(f*x + e))*b^6*cos(f*x + e)^2 - 195*s
qrt(b*cos(f*x + e))*b^6)*sgn(cos(f*x + e))/(b^6*f)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}\sin^7(e+fx)\,dx=\int \sin\left(e+f\,x\right)^7\,\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}\,dx$$

input
```
int(sin(e + f*x)^7*(b/cos(e + f*x))^(1/2),x)
```

output
```
int(sin(e + f*x)^7*(b/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{b\sec(e+fx)}\sin^7(e+fx)\,dx=\sqrt{b}\,\left(\int \sqrt{\sec\left(fx+e\right)}\,\sin\left(fx+e\right)^7dx\right)$$

input
```
int((b*sec(f*x+e))^(1/2)*sin(f*x+e)^7,x)
```

output
```
sqrt(b)*int(sqrt(sec(e + f*x))*sin(e + f*x)**7,x)
```

3.372 $\int \sqrt{b\sec(e+fx)}\sin^5(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 63

$$\int \sqrt{b\sec(e+fx)}\sin^5(e+fx)\,dx = -\frac{2b^5}{9f(b\sec(e+fx))^{9/2}} + \frac{4b^3}{5f(b\sec(e+fx))^{5/2}} - \frac{2b}{f\sqrt{b\sec(e+fx)}}$$

output
```
-2/9*b^5/f/(b*sec(f*x+e))^(9/2)+4/5*b^3/f/(b*sec(f*x+e))^(5/2)-2*b/f/(b*se
c(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.76

$$\int \sqrt{b\sec(e+fx)}\sin^5(e+fx)\,dx = -\frac{(554\cos(e+fx)-47\cos(3(e+fx))+5\cos(5(e+fx)))\sqrt{b\sec(e+fx)}}{360f}$$

input
```
Integrate[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^5,x]
```

output
```
-1/360*((554*Cos[e + f*x] - 47*Cos[3*(e + f*x)] + 5*Cos[5*(e + f*x)])*Sqrt
[b*Sec[e + f*x]])/f
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3102, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(e+fx)\sqrt{b\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\sec(e+fx)}}{\csc(e+fx)^5}dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{b^5\int \frac{\left(b^2-b^2\sec^2(e+fx)\right)^2}{b^4(b\sec(e+fx))^{11/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{\left(b^2-b^2\sec^2(e+fx)\right)^2}{(b\sec(e+fx))^{11/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{244}$$

$$\frac{b\int \left(\frac{b^4}{(b\sec(e+fx))^{11/2}}-\frac{2b^2}{(b\sec(e+fx))^{7/2}}+\frac{1}{(b\sec(e+fx))^{3/2}}\right)d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(-\frac{2b^4}{9(b\sec(e+fx))^{9/2}}+\frac{4b^2}{5(b\sec(e+fx))^{5/2}}-\frac{2}{\sqrt{b\sec(e+fx)}}\right)}{f}$$

input
```
Int[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^5,x]
```

output
```
(b*((-2*b^4)/(9*(b*Sec[e + f*x])^(9/2)) + (4*b^2)/(5*(b*Sec[e + f*x])^(5/2
)) - 2/Sqrt[b*Sec[e + f*x]]))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(53) = 106$.

Time = 1.17 (sec) , antiderivative size = 253, normalized size of antiderivative = 4.02

method	result
default	$-\frac{\left((45\cos(fx+e)+45)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\cos(fx+e)+1}{\cos(fx+e)+1}\right)+(-45\cos(fx+\right.}{}$

input
```
int((b*sec(f*x+e))^(1/2)*sin(f*x+e)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/90/f*((45*cos(f*x+e)+45)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln((2*cos
(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))+(-45*cos(f*x+e)-45)*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))+
cos(f*x+e)*(20*cos(f*x+e)^4-72*cos(f*x+e)^2+180))*(b*sec(f*x+e))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.73

$$\int \sqrt{b\sec(e+fx)}\sin^5(e+fx)\,dx = -\frac{2\left(5\,\cos\left(fx+e\right)^5-18\,\cos\left(fx+e\right)^3+45\,\cos\left(fx+e\right)\right)\sqrt{\frac{b}{\cos(fx+e)}}}{45\,f}$$

input
```
integrate((b*sec(f*x+e))^(1/2)*sin(f*x+e)^5,x, algorithm="fricas")
```

output
```
-2/45*(5*cos(f*x + e)^5 - 18*cos(f*x + e)^3 + 45*cos(f*x + e))*sqrt(b/cos(
f*x + e))/f
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}\sin^5(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((b*sec(f*x+e))**(1/2)*sin(f*x+e)**5,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

$$\int \sqrt{b\sec(e+fx)}\sin^5(e+fx)\,dx = -\frac{2\left(5\,b^4-\frac{18\,b^4}{\cos(fx+e)^2}+\frac{45\,b^4}{\cos(fx+e)^4}\right)b}{45\,f\left(\frac{b}{\cos(fx+e)}\right)^{\frac{9}{2}}}$$

input
```
integrate((b*sec(f*x+e))^(1/2)*sin(f*x+e)^5,x, algorithm="maxima")
```

output
```
-2/45*(5*b^4 - 18*b^4/cos(f*x + e)^2 + 45*b^4/cos(f*x + e)^4)*b/(f*(b/cos(
f*x + e))^(9/2))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.22

$$\int \sqrt{b\sec(e+fx)}\sin^5(e+fx)\,dx =$$

$$-\frac{2\left(5\sqrt{b\cos(fx+e)}b^4\cos(fx+e)^4-18\sqrt{b\cos(fx+e)}b^4\cos(fx+e)^2+45\sqrt{b\cos(fx+e)}b^4\right)\mathrm{sgn}}{45\,b^4 f}$$

input
```
integrate((b*sec(f*x+e))^(1/2)*sin(f*x+e)^5,x, algorithm="giac")
```

output
```
-2/45*(5*sqrt(b*cos(f*x + e))*b^4*cos(f*x + e)^4 - 18*sqrt(b*cos(f*x + e))
*b^4*cos(f*x + e)^2 + 45*sqrt(b*cos(f*x + e))*b^4)*sgn(cos(f*x + e))/(b^4*
f)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}\sin^5(e+fx)\,dx = \int \sin\left(e+f\,x\right)^5\,\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}\,dx$$

input
```
int(sin(e + f*x)^5*(b/cos(e + f*x))^(1/2),x)
```

output
```
int(sin(e + f*x)^5*(b/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{b\sec(e+fx)}\sin^5(e+fx)\,dx = \sqrt{b}\left(\int \sqrt{\sec\left(fx+e\right)}\,\sin\left(fx+e\right)^5 dx\right)$$

input
```
int((b*sec(f*x+e))^(1/2)*sin(f*x+e)^5,x)
```

output
```
sqrt(b)*int(sqrt(sec(e + f*x))*sin(e + f*x)**5,x)
```

3.373 $\int \sqrt{b\sec(e+fx)}\sin^3(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 41

$$\int \sqrt{b\sec(e+fx)}\sin^3(e+fx)\,dx = \frac{2b^3}{5f(b\sec(e+fx))^{5/2}} - \frac{2b}{f\sqrt{b\sec(e+fx)}}$$

output
```
2/5*b^3/f/(b*sec(f*x+e))^(5/2)-2*b/f/(b*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \sqrt{b\sec(e+fx)}\sin^3(e+fx)\,dx = \frac{(-17\cos(e+fx)+\cos(3(e+fx)))\sqrt{b\sec(e+fx)}}{10f}$$

input
```
Integrate[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^3,x]
```

output
```
((-17*Cos[e + f*x] + Cos[3*(e + f*x)])*Sqrt[b*Sec[e + f*x]])/(10*f)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3102, 25, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(e+fx)\sqrt{b\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\sec(e+fx)}}{\csc(e+fx)^3}dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{b^3\int -\frac{b^2-b^2\sec^2(e+fx)}{b^2(b\sec(e+fx))^{7/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b^3\int \frac{b^2-b^2\sec^2(e+fx)}{b^2(b\sec(e+fx))^{7/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\int \frac{b^2-b^2\sec^2(e+fx)}{(b\sec(e+fx))^{7/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{244}$$

$$-\frac{b\int \left(\frac{b^2}{(b\sec(e+fx))^{7/2}}-\frac{1}{(b\sec(e+fx))^{3/2}}\right)d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\left(\frac{2}{\sqrt{b\sec(e+fx)}}-\frac{2b^2}{5(b\sec(e+fx))^{5/2}}\right)}{f}$$

input
```
Int[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^3,x]
```

output
```
-((b*((-2*b^2)/(5*(b*Sec[e + f*x])^(5/2)) + 2/Sqrt[b*Sec[e + f*x]]))/f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(35) = 70$.

Time = 1.42 (sec) , antiderivative size = 243, normalized size of antiderivative = 5.93

method	result
default	$\left((-5\cos(fx+e)-5)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\cos(fx+e)+1}{\cos(fx+e)+1}\right)+(5\cos(fx+e)+5$

input
```
int((b*sec(f*x+e))^(1/2)*sin(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/10/f*((-5*cos(f*x+e)-5)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln((2*cos(f
*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))+(5*cos(f*x+e)+5)*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+
2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))+cos(f
*x+e)*(4*cos(f*x+e)^2-20))*(b*sec(f*x+e))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \sqrt{b\sec(e+fx)}\sin^3(e+fx)\,dx = \frac{2\left(\cos\left(fx+e\right)^3-5\,\cos\left(fx+e\right)\right)\sqrt{\frac{b}{\cos(fx+e)}}}{5\,f}$$

input
```
integrate((b*sec(f*x+e))^(1/2)*sin(f*x+e)^3,x, algorithm="fricas")
```

output
```
2/5*(cos(f*x + e)^3 - 5*cos(f*x + e))*sqrt(b/cos(f*x + e))/f
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}\sin^3(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((b*sec(f*x+e))**(1/2)*sin(f*x+e)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \sqrt{b\sec(e+fx)}\sin^3(e+fx)\,dx = \frac{2\left(b^2-\frac{5\,b^2}{\cos(fx+e)^2}\right)b}{5\,f\left(\frac{b}{\cos(fx+e)}\right)^{\frac{5}{2}}}$$

input
```
integrate((b*sec(f*x+e))^(1/2)*sin(f*x+e)^3,x, algorithm="maxima")
```

output
```
2/5*(b^2 - 5*b^2/cos(f*x + e)^2)*b/(f*(b/cos(f*x + e))^(5/2))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.29

$$\int \sqrt{b\sec(e+fx)}\sin^3(e+fx)\,dx$$
$$= \frac{2\left(\sqrt{b\cos(fx+e)}b^2\cos(fx+e)^2-5\sqrt{b\cos(fx+e)}b^2\right)\mathrm{sgn}(\cos(fx+e))}{5\,b^2f}$$

input
```
integrate((b*sec(f*x+e))^(1/2)*sin(f*x+e)^3,x, algorithm="giac")
```

output
```
2/5*(sqrt(b*cos(f*x + e))*b^2*cos(f*x + e)^2 - 5*sqrt(b*cos(f*x + e))*b^2)
*sgn(cos(f*x + e))/(b^2*f)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}\sin^3(e+fx)\,dx = \int \sin(e+f\,x)^3\sqrt{\frac{b}{\cos(e+f\,x)}}\,dx$$

input
```
int(sin(e + f*x)^3*(b/cos(e + f*x))^(1/2),x)
```

output
```
int(sin(e + f*x)^3*(b/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{b\sec(e+fx)}\sin^3(e+fx)\,dx = \sqrt{b}\left(\int \sqrt{\sec(fx+e)}\,\sin(fx+e)^3\,dx\right)$$

input
```
int((b*sec(f*x+e))^(1/2)*sin(f*x+e)^3,x)
```

output
```
sqrt(b)*int(sqrt(sec(e + f*x))*sin(e + f*x)**3,x)
```

3.374 $\int \sqrt{b\sec(e+fx)}\sin(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 18

$$\int \sqrt{b\sec(e+fx)}\sin(e+fx)\,dx = -\frac{2b}{f\sqrt{b\sec(e+fx)}}$$

output
```
-2*b/f/(b*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \sqrt{b\sec(e+fx)}\sin(e+fx)\,dx = -\frac{2b}{f\sqrt{b\sec(e+fx)}}$$

input
```
Integrate[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x],x]
```

output
```
(-2*b)/(f*Sqrt[b*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 3102, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(e+fx)\sqrt{b\sec(e+fx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{b\sec(e+fx)}}{\csc(e+fx)}dx$$

$$\Big\downarrow \; 3102$$

$$\frac{b\int \frac{1}{(b\sec(e+fx))^{3/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \; 15$$

$$-\frac{2b}{f\sqrt{b\sec(e+fx)}}$$

input
```
Int[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x],x]
```

output
```
(-2*b)/(f*Sqrt[b*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$-\frac{2b}{f\sqrt{b\sec(fx+e)}}$	17
default	$-\frac{2b}{f\sqrt{b\sec(fx+e)}}$	17
risch	$-\frac{2\sqrt{2}\,\sqrt{\frac{b\,e^{i(fx+e)}}{e^{2i(fx+e)}+1}}\,\cos(fx+e)}{f}$	41

input
```
int((b*sec(f*x+e))^(1/2)*sin(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
-2*b/f/(b*sec(f*x+e))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \sqrt{b\sec(e+fx)}\sin(e+fx)\,dx = -\frac{2\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)}{f}$$

input
```
integrate((b*sec(f*x+e))^(1/2)*sin(f*x+e),x, algorithm="fricas")
```

output
```
-2*sqrt(b/cos(f*x + e))*cos(f*x + e)/f
```

Sympy [F]

$$\int \sqrt{b\sec(e+fx)}\sin(e+fx)\,dx = \int \sqrt{b\sec(e+fx)}\sin(e+fx)\,dx$$

input
```
integrate((b*sec(f*x+e))**(1/2)*sin(f*x+e),x)
```

output
```
Integral(sqrt(b*sec(e + f*x))*sin(e + f*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \sqrt{b\sec(e+fx)}\sin(e+fx)\,dx = -\frac{2\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)}{f}$$

input
```
integrate((b*sec(f*x+e))^(1/2)*sin(f*x+e),x, algorithm="maxima")
```

output
```
-2*sqrt(b/cos(f*x + e))*cos(f*x + e)/f
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \sqrt{b\sec(e+fx)}\sin(e+fx)\,dx = -\frac{2\sqrt{b\cos(fx+e)}\operatorname{sgn}(\cos(fx+e))}{f}$$

input
```
integrate((b*sec(f*x+e))^(1/2)*sin(f*x+e),x, algorithm="giac")
```

output
```
-2*sqrt(b*cos(f*x + e))*sgn(cos(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \sqrt{b\sec(e+fx)}\sin(e+fx)\,dx = -\frac{2\cos\left(e+f\,x\right)\,\sqrt{\frac{b}{\cos(e+f\,x)}}}{f}$$

input
```
int(sin(e + f*x)*(b/cos(e + f*x))^(1/2),x)
```

output
```
-(2*cos(e + f*x)*(b/cos(e + f*x))^(1/2))/f
```

Reduce [F]

$$\int \sqrt{b\sec(e+fx)}\sin(e+fx)\,dx = \sqrt{b}\left(\int \sqrt{\sec\left(fx+e\right)}\,\sin\left(fx+e\right)dx\right)$$

input
```
int((b*sec(f*x+e))^(1/2)*sin(f*x+e),x)
```

output
```
sqrt(b)*int(sqrt(sec(e + f*x))*sin(e + f*x),x)
```

3.375 $\int \csc(e+fx)\sqrt{b\sec(e+fx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \csc(e+fx)\sqrt{b\sec(e+fx)}\,dx=\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f}-\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f}$$

output
```
b^(1/2)*arctan((b*sec(f*x+e))^(1/2)/b^(1/2))/f-b^(1/2)*arctanh((b*sec(f*x+
e))^(1/2)/b^(1/2))/f
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.26

$$\int \csc(e+fx)\sqrt{b\sec(e+fx)}\,dx$$
$$=\frac{\left(2\arctan\left(\sqrt{\sec(e+fx)}\right)+\log\left(1-\sqrt{\sec(e+fx)}\right)-\log\left(1+\sqrt{\sec(e+fx)}\right)\right)\sqrt{b\sec(e+fx)}}{2f\sqrt{\sec(e+fx)}}$$

input
```
Integrate[Csc[e + f*x]*Sqrt[b*Sec[e + f*x]],x]
```

output
```
((2*ArcTan[Sqrt[Sec[e + f*x]]] + Log[1 - Sqrt[Sec[e + f*x]]] - Log[1 + Sqr
t[Sec[e + f*x]]])*Sqrt[b*Sec[e + f*x]])/(2*f*Sqrt[Sec[e + f*x]])
```

Rubi [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 3102, 25, 27, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(e+fx)\sqrt{b\sec(e+fx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc(e+fx)\sqrt{b\sec(e+fx)}dx$$

$$\Big\downarrow \; 3102$$

$$\frac{\int -\frac{b^2\sqrt{b\sec(e+fx)}}{b^2-b^2\sec^2(e+fx)}d(b\sec(e+fx))}{bf}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{b^2\sqrt{b\sec(e+fx)}}{b^2-b^2\sec^2(e+fx)}d(b\sec(e+fx))}{bf}$$

$$\Big\downarrow \; 27$$

$$-\frac{b\int \frac{\sqrt{b\sec(e+fx)}}{b^2-b^2\sec^2(e+fx)}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \; 266$$

$$-\frac{2b\int \frac{b^2\sec^2(e+fx)}{b^2-b^4\sec^4(e+fx)}d\sqrt{b\sec(e+fx)}}{f}$$

$$\Big\downarrow \; 827$$

$$-\frac{2b\left(\frac{1}{2}\int \frac{1}{b-b^2\sec^2(e+fx)}d\sqrt{b\sec(e+fx)}-\frac{1}{2}\int \frac{1}{b^2\sec^2(e+fx)+b}d\sqrt{b\sec(e+fx)}\right)}{f}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2b\left(\frac{1}{2}\int \frac{1}{b-b^2\sec^2(e+fx)}d\sqrt{b\sec(e+fx)}-\frac{\arctan\left(\sqrt{b}\sec(e+fx)\right)}{2\sqrt{b}}\right)}{f}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2b\left(\frac{\operatorname{arctanh}\left(\sqrt{b}\sec(e+fx)\right)}{2\sqrt{b}}-\frac{\arctan\left(\sqrt{b}\sec(e+fx)\right)}{2\sqrt{b}}\right)}{f}$$

input
```
Int[Csc[e + f*x]*Sqrt[b*Sec[e + f*x]],x]
```

output
```
(-2*b*(-1/2*ArcTan[Sqrt[b]*Sec[e + f*x]]/Sqrt[b] + ArcTanh[Sqrt[b]*Sec[e +
 f*x]]/(2*Sqrt[b])))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(46) = 92$.

Time = 2.84 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.60

method	result
default	$-\frac{\left(-\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^{2}}}}\right)+\ln\left(\frac{4\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^{2}}}+4\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^{2}}}-2\cos(fx+e)+2}{\cos(fx+e)+1}\right)\right)\sqrt{b\sec(fx+e)}\, c}{2f(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^{2}}}}$

input
```
int(csc(f*x+e)*(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/f*(-arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))+ln(2*(2*cos(f*x+
e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1)))*(b*sec(f*x+e))^(1/2)*cos(f*x+e)/(cos(f
*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(46) = 92$.

Time = 0.14 (sec) , antiderivative size = 265, normalized size of antiderivative = 4.57

$$\int \csc(e+fx)\sqrt{b\sec(e+fx)}\,dx = \left[\frac{2\sqrt{-b}\arctan\left(\frac{2\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)}{b\cos(fx+e)+b}\right)+\sqrt{-b}\log\left(\frac{b\cos(fx+e)^2-4\left(\cos(fx+e)^2-\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}-6\,b\cos}{\cos(fx+e)^2+2\cos(fx+e)+1}\right.}{4f}\right.$$

input
```
integrate(csc(f*x+e)*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(-b)*arctan(2*sqrt(-b)*sqrt(b/cos(f*x + e))*cos(f*x + e)/(b*co
s(f*x + e) + b)) + sqrt(-b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - co
s(f*x + e))*sqrt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x
 + e)^2 + 2*cos(f*x + e) + 1)))/f, 1/4*(2*sqrt(b)*arctan(2*sqrt(b)*sqrt(b/
cos(f*x + e))*cos(f*x + e)/(b*cos(f*x + e) - b)) + sqrt(b)*log((b*cos(f*x
+ e)^2 - 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(b)*sqrt(b/cos(f*x + e)) +
6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)))/f]
```

Sympy [F]

$$\int \csc(e+fx)\sqrt{b\sec(e+fx)}\,dx = \int \sqrt{b\sec{(e+fx)}}\csc{(e+fx)}\,dx$$

input
```
integrate(csc(f*x+e)*(b*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(b*sec(e + f*x))*csc(e + f*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.24

$$\int \csc(e+fx)\sqrt{b\sec(e+fx)}\,dx = \frac{b\left(\frac{2\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}}\right)}{\sqrt{b}}+\frac{\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos(fx+e)}}}\right)}{\sqrt{b}}\right)}{2f}$$

input
```
integrate(csc(f*x+e)*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
1/2*b*(2*arctan(sqrt(b/cos(f*x + e))/sqrt(b))/sqrt(b) + log(-(sqrt(b) - sq
rt(b/cos(f*x + e)))/(sqrt(b) + sqrt(b/cos(f*x + e))))/sqrt(b))/f
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.05

$$\int \csc(e+fx)\sqrt{b\sec(e+fx)}\,dx = \frac{b^2\left(\frac{\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b}b}-\frac{\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{b}}\right)}{b^{\frac{3}{2}}}\right)\operatorname{sgn}(\cos(fx+e))}{f}$$

input
```
integrate(csc(f*x+e)*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
b^2*(arctan(sqrt(b*cos(f*x + e))/sqrt(-b))/(sqrt(-b)*b) - arctan(sqrt(b*co
s(f*x + e))/sqrt(b))/b^(3/2))*sgn(cos(f*x + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \csc(e+fx)\sqrt{b\sec(e+fx)}\,dx = \int \frac{\sqrt{\frac{b}{\cos(e+f\,x)}}}{\sin(e+f\,x)}\,dx$$

input
```
int((b/cos(e + f*x))^(1/2)/sin(e + f*x),x)
```

output
```
int((b/cos(e + f*x))^(1/2)/sin(e + f*x), x)
```

Reduce [F]

$$\int \csc(e+fx)\sqrt{b\sec(e+fx)}\,dx = \sqrt{b}\left(\int \sqrt{\sec(fx+e)}\,\csc(fx+e)\,dx\right)$$

input
```
int(csc(f*x+e)*(b*sec(f*x+e))^(1/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(e + f*x))*csc(e + f*x),x)
```

3.376 $\int \csc^3(e+fx)\sqrt{b\sec(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 93

$$\int \csc^3(e+fx)\sqrt{b\sec(e+fx)}\,dx = \frac{3\sqrt{b}\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f} - \frac{3\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f} - \frac{\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{2bf}$$

output
```
3/4*b^(1/2)*arctan((b*sec(f*x+e))^(1/2)/b^(1/2))/f-3/4*b^(1/2)*arctanh((b*
sec(f*x+e))^(1/2)/b^(1/2))/f-1/2*cot(f*x+e)^2*(b*sec(f*x+e))^(3/2)/b/f
```

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.02

$$\int \csc^3(e+fx)\sqrt{b\sec(e+fx)}\,dx = -\frac{\left(-6\arctan\left(\sqrt{\sec(e+fx)}\right) - 3\log\left(1-\sqrt{\sec(e+fx)}\right) + 3\log\left(1+\sqrt{\sec(e+fx)}\right) + \frac{4\csc^2(e+fx)}{\sqrt{\sec(e+fx)}}\right)}{8f\sqrt{\sec(e+fx)}}$$

input
```
Integrate[Csc[e + f*x]^3*Sqrt[b*Sec[e + f*x]],x]
```

output
```
-1/8*((-6*ArcTan[Sqrt[Sec[e + f*x]]] - 3*Log[1 - Sqrt[Sec[e + f*x]]] + 3*L
og[1 + Sqrt[Sec[e + f*x]]] + (4*Csc[e + f*x]^2)/Sqrt[Sec[e + f*x]])*Sqrt[b
*Sec[e + f*x]])/(f*Sqrt[Sec[e + f*x]])
```

Rubi [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3042, 3102, 27, 252, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(e+fx)\sqrt{b\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(e+fx)^3\sqrt{b\sec(e+fx)}dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{\int \frac{b^4(b\sec(e+fx))^{5/2}}{(b^2-b^2\sec^2(e+fx))^2}d(b\sec(e+fx))}{b^3 f}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{(b\sec(e+fx))^{5/2}}{(b^2-b^2\sec^2(e+fx))^2}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{252}$$

$$\frac{b\left(\frac{(b\sec(e+fx))^{3/2}}{2(b^2-b^2\sec^2(e+fx))}-\frac{3}{4}\int \frac{\sqrt{b\sec(e+fx)}}{b^2-b^2\sec^2(e+fx)}d(b\sec(e+fx))\right)}{f}$$

$$\Big\downarrow \text{266}$$

$$\frac{b\left(\frac{(b\sec(e+fx))^{3/2}}{2(b^2-b^2\sec^2(e+fx))}-\frac{3}{2}\int \frac{b^2\sec^2(e+fx)}{b^2-b^4\sec^4(e+fx)}d\sqrt{b\sec(e+fx)}\right)}{f}$$

$$\Big\downarrow \; 827$$

$$\frac{b\left(\frac{(b\sec(e+fx))^{3/2}}{2(b^2-b^2\sec^2(e+fx))}-\frac{3}{2}\left(\frac{1}{2}\int\frac{1}{b-b^2\sec^2(e+fx)}d\sqrt{b\sec(e+fx)}-\frac{1}{2}\int\frac{1}{b^2\sec^2(e+fx)+b}d\sqrt{b\sec(e+fx)}\right)\right)}{f}$$

$$\Big\downarrow \; 216$$

$$\frac{b\left(\frac{(b\sec(e+fx))^{3/2}}{2(b^2-b^2\sec^2(e+fx))}-\frac{3}{2}\left(\frac{1}{2}\int\frac{1}{b-b^2\sec^2(e+fx)}d\sqrt{b\sec(e+fx)}-\frac{\arctan\left(\sqrt{b}\sec(e+fx)\right)}{2\sqrt{b}}\right)\right)}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{b\left(\frac{(b\sec(e+fx))^{3/2}}{2(b^2-b^2\sec^2(e+fx))}-\frac{3}{2}\left(\frac{\text{arctanh}\left(\sqrt{b}\sec(e+fx)\right)}{2\sqrt{b}}-\frac{\arctan\left(\sqrt{b}\sec(e+fx)\right)}{2\sqrt{b}}\right)\right)}{f}$$

input
```
Int[Csc[e + f*x]^3*Sqrt[b*Sec[e + f*x]],x]
```

output
```
(b*((-3*(-1/2*ArcTan[Sqrt[b]*Sec[e + f*x]]/Sqrt[b] + ArcTanh[Sqrt[b]*Sec[e
 + f*x]]/(2*Sqrt[b])))/2 + (b*Sec[e + f*x])^(3/2)/(2*(b^2 - b^2*Sec[e + f*
x]^2))))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(73) = 146$.

Time = 3.16 (sec) , antiderivative size = 462, normalized size of antiderivative = 4.97

method	result
default	$\frac{\sqrt{-\frac{b\left((-\cos(fx+e)+1)^2\csc(fx+e)^2+1\right)}{(-\cos(fx+e)+1)^2\csc(fx+e)^2-1}}\left((-\cos(fx+e)+1)^2\csc(fx+e)^2-1\right)\left(\sqrt{(-\cos(fx+e)+1)^4\csc(fx+e)^4-1}\,(-\cos(fx+e)+1\right.}{}$

input
```
int(csc(f*x+e)^3*(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/f*(-b*((-cos(f*x+e)+1)^2*csc(f*x+e)^2+1)/((-cos(f*x+e)+1)^2*csc(f*x+e)
^2-1))^(1/2)*((-cos(f*x+e)+1)^2*csc(f*x+e)^2-1)*(((-cos(f*x+e)+1)^4*csc(f*
x+e)^4-1)^(1/2)*(-cos(f*x+e)+1)^4*csc(f*x+e)^4+4*ln(2*(-cos(f*x+e)+1)^2*cs
c(f*x+e)^2+2*((-cos(f*x+e)+1)^4*csc(f*x+e)^4-1)^(1/2))*(-cos(f*x+e)+1)^2*c
sc(f*x+e)^2-ln((-cos(f*x+e)+1)^2*csc(f*x+e)^2+((-cos(f*x+e)+1)^4*csc(f*x+e
)^4-1)^(1/2))*(-cos(f*x+e)+1)^2*csc(f*x+e)^2-((-cos(f*x+e)+1)^4*csc(f*x+e)
^4-1)^(3/2)+(-cos(f*x+e)+1)^2*((-cos(f*x+e)+1)^4*csc(f*x+e)^4-1)^(1/2)*csc
(f*x+e)^2-3*arctan(1/((-cos(f*x+e)+1)^4*csc(f*x+e)^4-1)^(1/2))*(-cos(f*x+e
)+1)^2*csc(f*x+e)^2)/(((-cos(f*x+e)+1)^2*csc(f*x+e)^2+1)*((-cos(f*x+e)+1)^
2*csc(f*x+e)^2-1))^(1/2)/(-cos(f*x+e)+1)^2*sin(f*x+e)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(73) = 146$.

Time = 0.14 (sec) , antiderivative size = 373, normalized size of antiderivative = 4.01

$$\int \csc^3(e+fx)\sqrt{b\sec(e+fx)}\,dx = \left[\frac{6\left(\cos\left(fx+e\right)^2-1\right)\sqrt{-b}\arctan\left(\frac{2\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)}{b\cos(fx+e)+b}\right)+3\left(\cos\left(fx+e\right)^2-1\right)\sqrt{-b}\log\left(\frac{b\cos(fx+e)}{}\right.}{16\left(f\cos\left(fx+e\right)^2-f\right)}\right.$$

input
```
integrate(csc(f*x+e)^3*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(6*(cos(f*x + e)^2 - 1)*sqrt(-b)*arctan(2*sqrt(-b)*sqrt(b/cos(f*x +
e))*cos(f*x + e)/(b*cos(f*x + e) + b)) + 3*(cos(f*x + e)^2 - 1)*sqrt(-b)*l
og((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - cos(f*x + e))*sqrt(-b)*sqrt(b/c
os(f*x + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)
) + 8*sqrt(b/cos(f*x + e))*cos(f*x + e))/(f*cos(f*x + e)^2 - f), 1/16*(6*(
cos(f*x + e)^2 - 1)*sqrt(b)*arctan(2*sqrt(b)*sqrt(b/cos(f*x + e))*cos(f*x
+ e)/(b*cos(f*x + e) - b)) + 3*(cos(f*x + e)^2 - 1)*sqrt(b)*log((b*cos(f*x
 + e)^2 - 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(b)*sqrt(b/cos(f*x + e)) +
 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)) + 8*sqrt(b/c
os(f*x + e))*cos(f*x + e))/(f*cos(f*x + e)^2 - f)]
```

Sympy [F]

$$\int \csc^3(e+fx)\sqrt{b\sec(e+fx)}\,dx=\int \sqrt{b\sec{(e+fx)}}\csc^3{(e+fx)}\,dx$$

input
```
integrate(csc(f*x+e)**3*(b*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(b*sec(e + f*x))*csc(e + f*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.14

$$\int \csc^3(e+fx)\sqrt{b\sec(e+fx)}\,dx$$
$$=\frac{b\left(\frac{4\left(\frac{b}{\cos(fx+e)}\right)^{\frac{3}{2}}}{b^2-\frac{b^2}{\cos(fx+e)^2}}+\frac{6\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}}\right)}{\sqrt{b}}+\frac{3\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos(fx+e)}}}\right)}{\sqrt{b}}\right)}{8f}$$

input
```
integrate(csc(f*x+e)^3*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
1/8*b*(4*(b/cos(f*x + e))^(3/2)/(b^2 - b^2/cos(f*x + e)^2) + 6*arctan(sqrt
(b/cos(f*x + e))/sqrt(b))/sqrt(b) + 3*log(-(sqrt(b) - sqrt(b/cos(f*x + e))
)/(sqrt(b) + sqrt(b/cos(f*x + e))))/sqrt(b))/f
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.05

$$\int \csc^3(e+fx)\sqrt{b\sec(e+fx)}\,dx = \frac{b^4\left(\frac{2\sqrt{b\cos(fx+e)}}{\left(b^2\cos(fx+e)^2-b^2\right)b^2}+\frac{3\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b}b^3}-\frac{3\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{b}}\right)}{b^{\frac{7}{2}}}\right)\operatorname{sgn}(\cos(fx+e))}{4f}$$

input
```
integrate(csc(f*x+e)^3*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
1/4*b^4*(2*sqrt(b*cos(f*x + e))/((b^2*cos(f*x + e)^2 - b^2)*b^2) + 3*arcta
n(sqrt(b*cos(f*x + e))/sqrt(-b))/(sqrt(-b)*b^3) - 3*arctan(sqrt(b*cos(f*x
+ e))/sqrt(b))/b^(7/2))*sgn(cos(f*x + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(e+fx)\sqrt{b\sec(e+fx)}\,dx = \int \frac{\sqrt{\frac{b}{\cos(e+f\,x)}}}{\sin\left(e+f\,x\right)^3}\,dx$$

input
```
int((b/cos(e + f*x))^(1/2)/sin(e + f*x)^3,x)
```

output
```
int((b/cos(e + f*x))^(1/2)/sin(e + f*x)^3, x)
```

Reduce [F]

$$\int \csc^3(e+fx)\sqrt{b\sec(e+fx)}\,dx=\sqrt{b}\left(\int\sqrt{\sec(fx+e)}\,\csc(fx+e)^3\,dx\right)$$

input
```
int(csc(f*x+e)^3*(b*sec(f*x+e))^(1/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(e + f*x))*csc(e + f*x)**3,x)
```

3.377 $\int \csc^5(e+fx)\sqrt{b\sec(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 123

$$\int \csc^5(e+fx)\sqrt{b\sec(e+fx)}\,dx = \frac{21\sqrt{b}\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32f} - \frac{21\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32f} - \frac{7\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{16bf} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{7/2}}{4b^3f}$$

output
```
21/32*b^(1/2)*arctan((b*sec(f*x+e))^(1/2)/b^(1/2))/f-21/32*b^(1/2)*arctanh
((b*sec(f*x+e))^(1/2)/b^(1/2))/f-7/16*cot(f*x+e)^2*(b*sec(f*x+e))^(3/2)/b/
f-1/4*cot(f*x+e)^4*(b*sec(f*x+e))^(7/2)/b^3/f
```

Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.87

$$\int \csc^5(e+fx)\sqrt{b\sec(e+fx)}\,dx$$
$$= \frac{b\left(-28\csc^2(e+fx)-16\csc^4(e+fx)+42\arctan\left(\sqrt{\sec(e+fx)}\right)\sqrt{\sec(e+fx)}+21\left(\log\left(1-\sqrt{\sec(\right.\right.\right.}{64f\sqrt{b\sec(e+fx)}}$$

input
```
Integrate[Csc[e + f*x]^5*Sqrt[b*Sec[e + f*x]],x]
```

output
```
(b*(-28*Csc[e + f*x]^2 - 16*Csc[e + f*x]^4 + 42*ArcTan[Sqrt[Sec[e + f*x]]]
*Sqrt[Sec[e + f*x]] + 21*(Log[1 - Sqrt[Sec[e + f*x]]] - Log[1 + Sqrt[Sec[e
 + f*x]]])*Sqrt[Sec[e + f*x]]))/(64*f*Sqrt[b*Sec[e + f*x]])
```

Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3102, 25, 27, 252, 252, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^5(e+fx)\sqrt{b\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc(e+fx)^5\sqrt{b\sec(e+fx)}dx$$

$$\Big\downarrow \text{ 3102}$$

$$\frac{\int -\frac{b^6(b\sec(e+fx))^{9/2}}{(b^2-b^2\sec^2(e+fx))^3}d(b\sec(e+fx))}{b^5 f}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{b^6 (b\sec(e+fx))^{9/2}}{(b^2-b^2\sec^2(e+fx))^3}d(b\sec(e+fx))}{b^5 f}$$

$$\Big\downarrow \; 27$$

$$-\frac{b\int \frac{(b\sec(e+fx))^{9/2}}{(b^2-b^2\sec^2(e+fx))^3}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \; 252$$

$$-\frac{b\left(\frac{(b\sec(e+fx))^{7/2}}{4(b^2-b^2\sec^2(e+fx))^2}-\frac{7}{8}\int \frac{(b\sec(e+fx))^{5/2}}{(b^2-b^2\sec^2(e+fx))^2}d(b\sec(e+fx))\right)}{f}$$

$$\Big\downarrow \; 252$$

$$-\frac{b\left(\frac{(b\sec(e+fx))^{7/2}}{4(b^2-b^2\sec^2(e+fx))^2}-\frac{7}{8}\left(\frac{(b\sec(e+fx))^{3/2}}{2(b^2-b^2\sec^2(e+fx))}-\frac{3}{4}\int \frac{\sqrt{b\sec(e+fx)}}{b^2-b^2\sec^2(e+fx)}d(b\sec(e+fx))\right)\right)}{f}$$

$$\Big\downarrow \; 266$$

$$-\frac{b\left(\frac{(b\sec(e+fx))^{7/2}}{4(b^2-b^2\sec^2(e+fx))^2}-\frac{7}{8}\left(\frac{(b\sec(e+fx))^{3/2}}{2(b^2-b^2\sec^2(e+fx))}-\frac{3}{2}\int \frac{b^2\sec^2(e+fx)}{b^2-b^4\sec^4(e+fx)}d\sqrt{b\sec(e+fx)}\right)\right)}{f}$$

$$\Big\downarrow \; 827$$

$$-\frac{b\left(\frac{(b\sec(e+fx))^{7/2}}{4(b^2-b^2\sec^2(e+fx))^2}-\frac{7}{8}\left(\frac{(b\sec(e+fx))^{3/2}}{2(b^2-b^2\sec^2(e+fx))}-\frac{3}{2}\left(\frac{1}{2}\int \frac{1}{b-b^2\sec^2(e+fx)}d\sqrt{b\sec(e+fx)}-\frac{1}{2}\int \frac{1}{b^2\sec^2(e+fx)+b}d\sqrt{b\mathrm{se}}\right.\right.\right.}{f}$$

$$\Big\downarrow \; 216$$

$$-\frac{b\left(\frac{(b\sec(e+fx))^{7/2}}{4(b^2-b^2\sec^2(e+fx))^2}-\frac{7}{8}\left(\frac{(b\sec(e+fx))^{3/2}}{2(b^2-b^2\sec^2(e+fx))}-\frac{3}{2}\left(\frac{1}{2}\int \frac{1}{b-b^2\sec^2(e+fx)}d\sqrt{b\sec(e+fx)}-\frac{\arctan\left(\sqrt{b}\sec(e+fx)\right)}{2\sqrt{b}}\right)\right)\right)}{f}$$

$$\Big\downarrow \; 219$$

$$-\frac{b\left(\frac{(b\sec(e+fx))^{7/2}}{4(b^2-b^2\sec^2(e+fx))^2}-\frac{7}{8}\left(\frac{(b\sec(e+fx))^{3/2}}{2(b^2-b^2\sec^2(e+fx))}-\frac{3}{2}\left(\frac{\text{arctanh}\left(\sqrt{b}\sec(e+fx)\right)}{2\sqrt{b}}-\frac{\arctan\left(\sqrt{b}\sec(e+fx)\right)}{2\sqrt{b}}\right)\right)\right)}{f}$$

input
```
Int[Csc[e + f*x]^5*Sqrt[b*Sec[e + f*x]],x]
```

output
```
-((b*((b*Sec[e + f*x])^(7/2)/(4*(b^2 - b^2*Sec[e + f*x]^2)^2) - (7*((-3*(-
1/2*ArcTan[Sqrt[b]*Sec[e + f*x]]/Sqrt[b] + ArcTanh[Sqrt[b]*Sec[e + f*x]]/(
2*Sqrt[b])))/2 + (b*Sec[e + f*x])^(3/2)/(2*(b^2 - b^2*Sec[e + f*x]^2))))/8
))/f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(99) = 198$.

Time = 2.80 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.51

method	result
default	$\frac{\left(\left(28\cos(fx+e)^2-44\right)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+(-11\cos(fx+e)+11)\sin(fx+e)^2\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+}}}{\cos(fx+e)+1}\right.\right.}{}$

input
```
int(csc(f*x+e)^5*(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/64/f*((28*cos(f*x+e)^2-44)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+(-11*cos
(f*x+e)+11)*sin(f*x+e)^2*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))+
(32*cos(f*x+e)-32)*sin(f*x+e)^2*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e
)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*
x+e)+1))+(-21*cos(f*x+e)+21)*sin(f*x+e)^2*arctan(1/2/(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)))*(b*sec(f*x+e))^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)*cot(f*x+e)*csc(f*x+e)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(99) = 198$.

Time = 0.14 (sec) , antiderivative size = 457, normalized size of antiderivative = 3.72

$$\int \csc^5(e+fx)\sqrt{b\sec(e+fx)}\,dx$$
$$= \left[\frac{42\left(\cos\left(fx+e\right)^4-2\cos\left(fx+e\right)^2+1\right)\sqrt{-b}\arctan\left(\frac{2\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)}{b\cos(fx+e)+b}\right)+21\left(\cos\left(fx+e\right)^4-2\right.}{128}\right.$$

input
```
integrate(csc(f*x+e)^5*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
[1/128*(42*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-b)*arctan(2*sqrt(
-b)*sqrt(b/cos(f*x + e))*cos(f*x + e)/(b*cos(f*x + e) + b)) + 21*(cos(f*x
+ e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x
 + e)^2 - cos(f*x + e))*sqrt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos(f*x + e) +
 b)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 8*(7*cos(f*x + e)^3 - 11*cos(
f*x + e))*sqrt(b/cos(f*x + e)))/(f*cos(f*x + e)^4 - 2*f*cos(f*x + e)^2 + f
), 1/128*(42*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(b)*arctan(2*sqrt
(b)*sqrt(b/cos(f*x + e))*cos(f*x + e)/(b*cos(f*x + e) - b)) + 21*(cos(f*x
+ e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x
+ e)^2 + cos(f*x + e))*sqrt(b)*sqrt(b/cos(f*x + e)) + 6*b*cos(f*x + e) + b
)/(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)) + 8*(7*cos(f*x + e)^3 - 11*cos(f*
x + e))*sqrt(b/cos(f*x + e)))/(f*cos(f*x + e)^4 - 2*f*cos(f*x + e)^2 + f)]
```

Sympy [F]

$$\int \csc^5(e+fx)\sqrt{b\sec(e+fx)}\,dx = \int \sqrt{b\sec\left(e+fx\right)}\csc^5\left(e+fx\right)dx$$

input
```
integrate(csc(f*x+e)**5*(b*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(b*sec(e + f*x))*csc(e + f*x)**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.12

$$\int \csc^5(e+fx)\sqrt{b\sec(e+fx)}\,dx = \frac{b\left(\frac{42\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}}\right)}{\sqrt{b}}+\frac{21\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos(fx+e)}}}\right)}{\sqrt{b}}+\frac{4\left(7b^2\left(\frac{b}{\cos(fx+e)}\right)^{\frac{3}{2}}-11\left(\frac{b}{\cos(fx+e)}\right)^{\frac{7}{2}}\right)}{b^4-\frac{2b^4}{\cos(fx+e)^2}+\frac{b^4}{\cos(fx+e)^4}}\right)}{64f}$$

input
```
integrate(csc(f*x+e)^5*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
1/64*b*(42*arctan(sqrt(b/cos(f*x + e))/sqrt(b))/sqrt(b) + 21*log(-(sqrt(b)
 - sqrt(b/cos(f*x + e)))/(sqrt(b) + sqrt(b/cos(f*x + e))))/sqrt(b) + 4*(7*
b^2*(b/cos(f*x + e))^(3/2) - 11*(b/cos(f*x + e))^(7/2))/(b^4 - 2*b^4/cos(f
*x + e)^2 + b^4/cos(f*x + e)^4))/f
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.03

$$\int \csc^5(e+fx)\sqrt{b\sec(e+fx)}\,dx = \frac{b^6\left(\frac{21\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b}b^5}-\frac{21\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{b}}\right)}{b^{\frac{11}{2}}}+\frac{2\left(7\sqrt{b\cos(fx+e)}b^2\cos(fx+e)^2-11\sqrt{b\cos(fx+e)}b^2\right)}{\left(b^2\cos(fx+e)^2-b^2\right)^2b^4}\right)\mathrm{sgn}(\cos(fx}{32f}$$

input
```
integrate(csc(f*x+e)^5*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
1/32*b^6*(21*arctan(sqrt(b*cos(f*x + e))/sqrt(-b))/(sqrt(-b)*b^5) - 21*arc
tan(sqrt(b*cos(f*x + e))/sqrt(b))/b^(11/2) + 2*(7*sqrt(b*cos(f*x + e))*b^2
*cos(f*x + e)^2 - 11*sqrt(b*cos(f*x + e))*b^2)/((b^2*cos(f*x + e)^2 - b^2)
^2*b^4))*sgn(cos(f*x + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \csc^5(e+fx)\sqrt{b\sec(e+fx)}\,dx = \int \frac{\sqrt{\frac{b}{\cos(e+f\,x)}}}{\sin\left(e+f\,x\right)^5}\,dx$$

input
```
int((b/cos(e + f*x))^(1/2)/sin(e + f*x)^5,x)
```

output
```
int((b/cos(e + f*x))^(1/2)/sin(e + f*x)^5, x)
```

Reduce [F]

$$\int \csc^5(e+fx)\sqrt{b\sec(e+fx)}\,dx = \sqrt{b}\left(\int \sqrt{\sec\left(fx+e\right)}\,\csc\left(fx+e\right)^5 dx\right)$$

input
```
int(csc(f*x+e)^5*(b*sec(f*x+e))^(1/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(e + f*x))*csc(e + f*x)**5,x)
```

3.378 $\int \sqrt{b\sec(e+fx)}\sin^6(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 123

$$\int \sqrt{b\sec(e+fx)}\sin^6(e+fx)\,dx = \frac{80\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{77f} - \frac{40b\sin(e+fx)}{77f\sqrt{b\sec(e+fx)}} - \frac{20b\sin^3(e+fx)}{77f\sqrt{b\sec(e+fx)}} - \frac{2b\sin^5(e+fx)}{11f\sqrt{b\sec(e+fx)}}$$

output
```
80/77*cos(f*x+e)^(1/2)*InverseJacobiAM(1/2*f*x+1/2*e,2^(1/2))*(b*sec(f*x+e
))^(1/2)/f-40/77*b*sin(f*x+e)/f/(b*sec(f*x+e))^(1/2)-20/77*b*sin(f*x+e)^3/
f/(b*sec(f*x+e))^(1/2)-2/11*b*sin(f*x+e)^5/f/(b*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.59

$$\int \sqrt{b\sec(e+fx)}\sin^6(e+fx)\,dx = \frac{\sqrt{b\sec(e+fx)}\left(1280\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right) - 435\sin(2(e+fx)) + 68\sin(4(e+fx)) -\right.}{1232f}$$

input
```
Integrate[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^6,x]
```

output
```
(Sqrt[b*Sec[e + f*x]]*(1280*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2] -
 435*Sin[2*(e + f*x)] + 68*Sin[4*(e + f*x)] - 7*Sin[6*(e + f*x)]))/(1232*f
)
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3107, 3042, 3107, 3042, 3107, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^6(e+fx)\sqrt{b\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\sec(e+fx)}}{\csc(e+fx)^6}dx$$

$$\Big\downarrow \text{3107}$$

$$\frac{10}{11}\int \sqrt{b\sec(e+fx)}\sin^4(e+fx)dx-\frac{2b\sin^5(e+fx)}{11f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{10}{11}\int \frac{\sqrt{b\sec(e+fx)}}{\csc(e+fx)^4}dx-\frac{2b\sin^5(e+fx)}{11f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3107}$$

$$\frac{10}{11}\left(\frac{6}{7}\int \sqrt{b\sec(e+fx)}\sin^2(e+fx)dx-\frac{2b\sin^3(e+fx)}{7f\sqrt{b\sec(e+fx)}}\right)-\frac{2b\sin^5(e+fx)}{11f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{10}{11}\left(\frac{6}{7}\int \frac{\sqrt{b\sec(e+fx)}}{\csc(e+fx)^2}dx-\frac{2b\sin^3(e+fx)}{7f\sqrt{b\sec(e+fx)}}\right)-\frac{2b\sin^5(e+fx)}{11f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3107}$$

$$\frac{10}{11}\left(\frac{6}{7}\left(\frac{2}{3}\int\sqrt{b\sec(e+fx)}dx-\frac{2b\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}\right)-\frac{2b\sin^3(e+fx)}{7f\sqrt{b\sec(e+fx)}}\right)-\frac{2b\sin^5(e+fx)}{11f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{10}{11}\left(\frac{6}{7}\left(\frac{2}{3}\int\sqrt{b\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{2b\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}\right)-\frac{2b\sin^3(e+fx)}{7f\sqrt{b\sec(e+fx)}}\right)-\frac{2b\sin^5(e+fx)}{11f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{10}{11}\left(\frac{6}{7}\left(\frac{2}{3}\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\int\frac{1}{\sqrt{\cos(e+fx)}}dx-\frac{2b\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}\right)-\frac{2b\sin^3(e+fx)}{7f\sqrt{b\sec(e+fx)}}\right)-\frac{2b\sin^5(e+fx)}{11f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{10}{11}\left(\frac{6}{7}\left(\frac{2}{3}\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\int\frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{2b\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}\right)-\frac{2b\sin^3(e+fx)}{7f\sqrt{b\sec(e+fx)}}\right)-\frac{2b\sin^5(e+fx)}{11f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{10}{11}\left(\frac{6}{7}\left(\frac{4\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{3f}-\frac{2b\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}\right)-\frac{2b\sin^3(e+fx)}{7f\sqrt{b\sec(e+fx)}}\right)-\frac{2b\sin^5(e+fx)}{11f\sqrt{b\sec(e+fx)}}$$

input
```
Int[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^6,x]
```

output
```
(-2*b*Sin[e + f*x]^5)/(11*f*Sqrt[b*Sec[e + f*x]]) + (10*((-2*b*Sin[e + f*x
]^3)/(7*f*Sqrt[b*Sec[e + f*x]]) + (6*((4*Sqrt[Cos[e + f*x]]*EllipticF[(e +
 f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(3*f) - (2*b*Sin[e + f*x])/(3*f*Sqrt[b*S
ec[e + f*x]])))/7))/11
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3107
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Simp[(m + 1)/(a^2*(m + n))   Int[(a*Csc[e + f*x])^(m
+ 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 12.17 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.95

method	result
default	$\frac{2\left(\sin(fx+e)\cos(fx+e)\left(-7\cos(fx+e)^4+24\cos(fx+e)^2-37\right)+i(40\cos(fx+e)+40)\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\ \text{EllipticF}(i(\cot}{77f}$

input
```
int((b*sec(f*x+e))^(1/2)*sin(f*x+e)^6,x,method=_RETURNVERBOSE)
```

output
```
2/77/f*(sin(f*x+e)*cos(f*x+e)*(-7*cos(f*x+e)^4+24*cos(f*x+e)^2-37)+I*(40*c
os(f*x+e)+40)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*E
llipticF(I*(cot(f*x+e)-csc(f*x+e)),I))*(b*sec(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.86

$$\int \sqrt{b\sec(e+fx)}\sin^6(e+fx)\,dx =$$

$$-\frac{2\left(\left(7\cos\left(fx+e\right)^5-24\cos\left(fx+e\right)^3+37\cos\left(fx+e\right)\right)\sqrt{\frac{b}{\cos(fx+e)}}\sin\left(fx+e\right)+20i\sqrt{2}\sqrt{b}\text{weierstr}}{}$$

input
```
integrate((b*sec(f*x+e))^(1/2)*sin(f*x+e)^6,x, algorithm="fricas")
```

output
```
-2/77*((7*cos(f*x + e)^5 - 24*cos(f*x + e)^3 + 37*cos(f*x + e))*sqrt(b/cos
(f*x + e))*sin(f*x + e) + 20*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0,
cos(f*x + e) + I*sin(f*x + e)) - 20*I*sqrt(2)*sqrt(b)*weierstrassPInverse(
-4, 0, cos(f*x + e) - I*sin(f*x + e)))/f
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}\sin^6(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((b*sec(f*x+e))**(1/2)*sin(f*x+e)**6,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{b\sec(e+fx)}\sin^6(e+fx)\,dx = \int \sqrt{b\sec\left(fx+e\right)}\sin\left(fx+e\right)^6\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)*sin(f*x+e)^6,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e))*sin(f*x + e)^6, x)
```

Giac [F]

$$\int \sqrt{b\sec(e+fx)}\sin^6(e+fx)\,dx = \int \sqrt{b\sec\left(fx+e\right)}\sin\left(fx+e\right)^6\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)*sin(f*x+e)^6,x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e))*sin(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}\sin^6(e+fx)\,dx = \int \sin\left(e+f\,x\right)^6\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}\,dx$$

input
```
int(sin(e + f*x)^6*(b/cos(e + f*x))^(1/2),x)
```

output
```
int(sin(e + f*x)^6*(b/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{b\sec(e+fx)}\sin^6(e+fx)\,dx=\sqrt{b}\left(\int \sqrt{\sec\left(fx+e\right)}\,\sin\left(fx+e\right)^6dx\right)$$

input
```
int((b*sec(f*x+e))^(1/2)*sin(f*x+e)^6,x)
```

output
```
sqrt(b)*int(sqrt(sec(e + f*x))*sin(e + f*x)**6,x)
```

3.379 $\int \sqrt{b\sec(e+fx)}\sin^4(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 95

$$\int \sqrt{b\sec(e+fx)}\sin^4(e+fx)\,dx = \frac{8\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{7f} - \frac{4b\sin(e+fx)}{7f\sqrt{b\sec(e+fx)}} - \frac{2b\sin^3(e+fx)}{7f\sqrt{b\sec(e+fx)}}$$

output
```
8/7*cos(f*x+e)^(1/2)*InverseJacobiAM(1/2*f*x+1/2*e,2^(1/2))*(b*sec(f*x+e))
^(1/2)/f-4/7*b*sin(f*x+e)/f/(b*sec(f*x+e))^(1/2)-2/7*b*sin(f*x+e)^3/f/(b*s
ec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.64

$$\int \sqrt{b\sec(e+fx)}\sin^4(e+fx)\,dx = \frac{\sqrt{b\sec(e+fx)}\left(32\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right) - 10\sin(2(e+fx)) + \sin(4(e+fx))\right)}{28f}$$

input
```
Integrate[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^4,x]
```

output
```
(Sqrt[b*Sec[e + f*x]]*(32*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2] - 1
0*Sin[2*(e + f*x)] + Sin[4*(e + f*x)]))/(28*f)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3107, 3042, 3107, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(e+fx)\sqrt{b\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\sec(e+fx)}}{\csc(e+fx)^4}dx$$

$$\Big\downarrow \text{3107}$$

$$\frac{6}{7}\int \sqrt{b\sec(e+fx)}\sin^2(e+fx)dx-\frac{2b\sin^3(e+fx)}{7f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{7}\int \frac{\sqrt{b\sec(e+fx)}}{\csc(e+fx)^2}dx-\frac{2b\sin^3(e+fx)}{7f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3107}$$

$$\frac{6}{7}\left(\frac{2}{3}\int \sqrt{b\sec(e+fx)}dx-\frac{2b\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}\right)-\frac{2b\sin^3(e+fx)}{7f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{7}\left(\frac{2}{3}\int \sqrt{b\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{2b\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}\right)-\frac{2b\sin^3(e+fx)}{7f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{6}{7}\left(\frac{2}{3}\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\int\frac{1}{\sqrt{\cos(e+fx)}}dx-\frac{2b\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}\right)-\frac{2b\sin^3(e+fx)}{7f\sqrt{b\sec(e+fx)}}$$

$\big\downarrow$ 3042

$$\frac{6}{7}\left(\frac{2}{3}\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\int\frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{2b\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}\right)-\frac{2b\sin^3(e+fx)}{7f\sqrt{b\sec(e+fx)}}$$

$\big\downarrow$ 3120

$$\frac{6}{7}\left(\frac{4\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{3f}-\frac{2b\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}\right)-\frac{2b\sin^3(e+fx)}{7f\sqrt{b\sec(e+fx)}}$$

input
```
Int[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^4,x]
```

output
```
(-2*b*Sin[e + f*x]^3)/(7*f*Sqrt[b*Sec[e + f*x]]) + (6*((4*Sqrt[Cos[e + f*x
]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(3*f) - (2*b*Sin[e + f*
x])/(3*f*Sqrt[b*Sec[e + f*x]])))/7
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3107
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Simp[(m + 1)/(a^2*(m + n))   Int[(a*Csc[e + f*x])^(m
+ 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.64 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.11

method	result
default	$\frac{2\left(\sin(fx+e)\cos(fx+e)\left(\cos(fx+e)^2-3\right)+i(4\cos(fx+e)+4)\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\text{EllipticF}(i(\cot(fx+e)-\csc(fx+e)),i)}{7f}$

input
```
int((b*sec(f*x+e))^(1/2)*sin(f*x+e)^4,x,method=_RETURNVERBOSE)
```

output
```
2/7/f*(sin(f*x+e)*cos(f*x+e)*(cos(f*x+e)^2-3)+I*(4*cos(f*x+e)+4)*(1/(cos(f
*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-
csc(f*x+e)),I))*(b*sec(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.99

$$\int \sqrt{b\sec(e+fx)}\sin^4(e+fx)\,dx$$
$$=\frac{2\left(\left(\cos\left(fx+e\right)^3-3\cos\left(fx+e\right)\right)\sqrt{\frac{b}{\cos(fx+e)}}\sin\left(fx+e\right)-2i\sqrt{2}\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos\left(f\right.\right.}{7f}$$

input
```
integrate((b*sec(f*x+e))^(1/2)*sin(f*x+e)^4,x, algorithm="fricas")
```

output
```
2/7*((cos(f*x + e)^3 - 3*cos(f*x + e))*sqrt(b/cos(f*x + e))*sin(f*x + e) -
 2*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x +
 e)) + 2*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin
(f*x + e)))/f
```

Sympy [F]

$$\int \sqrt{b\sec(e+fx)}\sin^4(e+fx)\,dx = \int \sqrt{b\sec\left(e+fx\right)}\sin^4\left(e+fx\right)dx$$

input
```
integrate((b*sec(f*x+e))**(1/2)*sin(f*x+e)**4,x)
```

output
```
Integral(sqrt(b*sec(e + f*x))*sin(e + f*x)**4, x)
```

Maxima [F]

$$\int \sqrt{b\sec(e+fx)}\sin^4(e+fx)\,dx = \int \sqrt{b\sec\left(fx+e\right)}\sin\left(fx+e\right)^4\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)*sin(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e))*sin(f*x + e)^4, x)
```

Giac [F]

$$\int \sqrt{b\sec(e+fx)}\sin^4(e+fx)\,dx = \int \sqrt{b\sec\left(fx+e\right)}\sin\left(fx+e\right)^4\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)*sin(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e))*sin(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}\sin^4(e+fx)\,dx = \int \sin(e+f\,x)^4\,\sqrt{\frac{b}{\cos(e+f\,x)}}\,dx$$

input
```
int(sin(e + f*x)^4*(b/cos(e + f*x))^(1/2),x)
```

output
```
int(sin(e + f*x)^4*(b/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{b\sec(e+fx)}\sin^4(e+fx)\,dx = \sqrt{b}\left(\int \sqrt{\sec(fx+e)}\,\sin(fx+e)^4\,dx\right)$$

input
```
int((b*sec(f*x+e))^(1/2)*sin(f*x+e)^4,x)
```

output
```
sqrt(b)*int(sqrt(sec(e + f*x))*sin(e + f*x)**4,x)
```

3.380 $\int \sqrt{b \sec(e+fx)} \sin^2(e+fx) \, dx$

Optimal result

Integrand size = 21, antiderivative size = 67

$$\int \sqrt{b \sec(e+fx)} \sin^2(e+fx) \, dx = \frac{4\sqrt{\cos(e+fx)} \operatorname{EllipticF}\left(\frac{1}{2}(e+fx),2\right) \sqrt{b \sec(e+fx)}}{3f} - \frac{2b \sin(e+fx)}{3f\sqrt{b \sec(e+fx)}}$$

output
```
4/3*cos(f*x+e)^(1/2)*InverseJacobiAM(1/2*f*x+1/2*e,2^(1/2))*(b*sec(f*x+e))
^(1/2)/f-2/3*b*sin(f*x+e)/f/(b*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.76

$$\int \sqrt{b \sec(e+fx)} \sin^2(e+fx) \, dx = -\frac{\sqrt{b \sec(e+fx)} \left(-4\sqrt{\cos(e+fx)} \operatorname{EllipticF}\left(\frac{1}{2}(e+fx),2\right) + \sin(2(e+fx))\right)}{3f}$$

input
```
Integrate[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^2,x]
```

output
```
-1/3*(Sqrt[b*Sec[e + f*x]]*(-4*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2
] + Sin[2*(e + f*x)]))/f
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3107, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(e+fx)\sqrt{b\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\sec(e+fx)}}{\csc(e+fx)^2}dx$$

$$\Big\downarrow \text{3107}$$

$$\frac{2}{3}\int \sqrt{b\sec(e+fx)}dx-\frac{2b\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\int \sqrt{b\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{2b\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{2}{3}\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}}dx-\frac{2b\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{2b\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{4\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{3f}-\frac{2b\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}$$

input
```
Int[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^2,x]
```

output
```
(4*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(3*f
) - (2*b*Sin[e + f*x])/(3*f*Sqrt[b*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3107
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Simp[(m + 1)/(a^2*(m + n))   Int[(a*Csc[e + f*x])^(m
+ 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.48 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.43

method	result
default	$\frac{2\left(i(2\cos(fx+e)+2)\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{EllipticF}(i(\cot(fx+e)-\csc(fx+e)),i)-\cos(fx+e)\sin(fx+e)\right)\sqrt{b\sec(fx+e)}}{3f}$

input
```
int((b*sec(f*x+e))^(1/2)*sin(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
2/3/f*(I*(2*cos(f*x+e)+2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)-cos(f*x+e)*sin(f*x+e))*(
b*sec(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.24

$$\int \sqrt{b\sec(e+fx)}\sin^2(e+fx)\,dx = -\frac{2\left(\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+i\sqrt{2}\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx-\right.}{3f}$$

input
```
integrate((b*sec(f*x+e))^(1/2)*sin(f*x+e)^2,x, algorithm="fricas")
```

output
```
-2/3*(sqrt(b/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + I*sqrt(2)*sqrt(b)*w
eierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e)) - I*sqrt(2)*sqrt(
b)*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)))/f
```

Sympy [F]

$$\int \sqrt{b\sec(e+fx)}\sin^2(e+fx)\,dx = \int \sqrt{b\sec(e+fx)}\sin^2(e+fx)\,dx$$

input
```
integrate((b*sec(f*x+e))**(1/2)*sin(f*x+e)**2,x)
```

output
```
Integral(sqrt(b*sec(e + f*x))*sin(e + f*x)**2, x)
```

Maxima [F]

$$\int \sqrt{b\sec(e+fx)}\sin^2(e+fx)\,dx = \int \sqrt{b\sec\left(fx+e\right)}\sin\left(fx+e\right)^2\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)*sin(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e))*sin(f*x + e)^2, x)
```

Giac [F]

$$\int \sqrt{b\sec(e+fx)}\sin^2(e+fx)\,dx = \int \sqrt{b\sec\left(fx+e\right)}\sin\left(fx+e\right)^2\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)*sin(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e))*sin(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}\sin^2(e+fx)\,dx = \int \sin\left(e+f\,x\right)^2\,\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}\,dx$$

input
```
int(sin(e + f*x)^2*(b/cos(e + f*x))^(1/2),x)
```

output
```
int(sin(e + f*x)^2*(b/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{b\sec(e+fx)}\sin^2(e+fx)\,dx = \sqrt{b}\left(\int \sqrt{\sec(fx+e)}\,\sin(fx+e)^2\,dx\right)$$

input
```
int((b*sec(f*x+e))^(1/2)*sin(f*x+e)^2,x)
```

output
```
sqrt(b)*int(sqrt(sec(e + f*x))*sin(e + f*x)**2,x)
```

3.381 $\int \sqrt{b \sec(e+fx)}\, dx$

Optimal result

Integrand size = 12, antiderivative size = 38

$$\int \sqrt{b \sec(e+fx)}\, dx = \frac{2\sqrt{\cos(e+fx)}\, \text{EllipticF}\left(\frac{1}{2}(e+fx), 2\right) \sqrt{b \sec(e+fx)}}{f}$$

output
```
2*cos(f*x+e)^(1/2)*InverseJacobiAM(1/2*f*x+1/2*e,2^(1/2))*(b*sec(f*x+e))^(
1/2)/f
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \sqrt{b \sec(e+fx)}\, dx = \frac{2\sqrt{\cos(e+fx)}\, \text{EllipticF}\left(\frac{1}{2}(e+fx), 2\right) \sqrt{b \sec(e+fx)}}{f}$$

input
```
Integrate[Sqrt[b*Sec[e + f*x]],x]
```

output
```
(2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/f
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{b\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{f}$$

input
```
Int[Sqrt[b*Sec[e + f*x]],x]
```

output
```
(2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/f
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.17 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.03

method	result	size
default	$\frac{2i(\cos(fx+e)+1)\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{EllipticF}(i(\cot(fx+e)-\csc(fx+e)),i)\sqrt{b\sec(fx+e)}}{f}$	77

input
```
int((b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/f*(cos(f*x+e)+1)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*(b*sec(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.50

$$\int \sqrt{b\sec(e+fx)}\,dx = \frac{-i\sqrt{2}\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos{(fx+e)}+i\,\sin{(fx+e)})+i\sqrt{2}\sqrt{b}\text{weierstrassPInverse}(-4,0,}{f}$$

input
```
integrate((b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x +
e)) + I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*
x + e)))/f
```

Sympy [F]

$$\int \sqrt{b\sec(e+fx)}\,dx = \int \sqrt{b\sec{(e+fx)}}\,dx$$

input
```
integrate((b*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(b*sec(e + f*x)), x)
```

Maxima [F]

$$\int \sqrt{b\sec(e+fx)}\,dx = \int \sqrt{b\sec{(fx+e)}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e)), x)
```

Giac [F]

$$\int \sqrt{b\sec(e+fx)}\,dx = \int \sqrt{b\sec\left(fx+e\right)}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e)), x)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \sqrt{b\sec(e+fx)}\,dx = \frac{2\sqrt{\cos\left(e+f\,x\right)}\,\sqrt{\frac{b}{\cos(e+f\,x)}}\,\mathrm{F}\left(\frac{e}{2}+\frac{f\,x}{2}\middle|2\right)}{f}$$

input
```
int((b/cos(e + f*x))^(1/2),x)
```

output
```
(2*cos(e + f*x)^(1/2)*(b/cos(e + f*x))^(1/2)*ellipticF(e/2 + (f*x)/2, 2))/
f
```

Reduce [F]

$$\int \sqrt{b\sec(e+fx)}\,dx = \sqrt{b}\left(\int \sqrt{\sec\left(fx+e\right)}dx\right)$$

input
```
int((b*sec(f*x+e))^(1/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(e + f*x)),x)
```

3.382 $\int \csc^2(e+fx)\sqrt{b\sec(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \csc^2(e+fx)\sqrt{b\sec(e+fx)}\,dx = -\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}+\frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{f}$$

output
```
-b*csc(f*x+e)/f/(b*sec(f*x+e))^(1/2)+cos(f*x+e)^(1/2)*InverseJacobiAM(1/2*
f*x+1/2*e,2^(1/2))*(b*sec(f*x+e))^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.76

$$\int \csc^2(e+fx)\sqrt{b\sec(e+fx)}\,dx = \frac{\left(-\cot(e+fx)+\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\right)\sqrt{b\sec(e+fx)}}{f}$$

input
```
Integrate[Csc[e + f*x]^2*Sqrt[b*Sec[e + f*x]],x]
```

output
```
((-Cot[e + f*x] + Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])*Sqrt[b*Sec
[e + f*x]])/f
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3105, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(e+fx)\sqrt{b\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(e+fx)^2\sqrt{b\sec(e+fx)}dx$$

$$\Big\downarrow \text{3105}$$

$$\frac{1}{2}\int \sqrt{b\sec(e+fx)}dx-\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sqrt{b\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{2}\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}}dx-\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{f}-\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}$$

input
```
Int[Csc[e + f*x]^2*Sqrt[b*Sec[e + f*x]],x]
```

output
```
-((b*Csc[e + f*x])/(f*Sqrt[b*Sec[e + f*x]])) + (Sqrt[Cos[e + f*x]]*Ellipti
cF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/f
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3105
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Simp[a^2*((m + n - 2)/(m - 1))   Int[(a*Csc[e + f*
x])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[
m, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.40

method	result	size
default	$\frac{\sqrt{b\sec(fx+e)}\left(i(\cos(fx+e)+1)\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\ \text{EllipticF}(i(\cot(fx+e)-\csc(fx+e)),i)-\cot(fx+e)\right)}{f}$	87

input
```
int(csc(f*x+e)^2*(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(b*sec(f*x+e))^(1/2)*(I*(cos(f*x+e)+1)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)-cot(f*x
+e))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.58

$$\int \csc^2(e+fx)\sqrt{b\sec(e+fx)}\,dx = \frac{-i\sqrt{2}\sqrt{b}\sin(fx+e)\,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e))+i\sqrt{2}\sqrt{b}\sin(fx+e)\,\mathrm{we}}{2f\sin(fx+e)}$$

input
```
integrate(csc(f*x+e)^2*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(-I*sqrt(2)*sqrt(b)*sin(f*x + e)*weierstrassPInverse(-4, 0, cos(f*x +
e) + I*sin(f*x + e)) + I*sqrt(2)*sqrt(b)*sin(f*x + e)*weierstrassPInverse(
-4, 0, cos(f*x + e) - I*sin(f*x + e)) - 2*sqrt(b/cos(f*x + e))*cos(f*x + e
))/(f*sin(f*x + e))
```

Sympy [F]

$$\int \csc^2(e+fx)\sqrt{b\sec(e+fx)}\,dx = \int \sqrt{b\sec(e+fx)}\csc^2(e+fx)\,dx$$

input
```
integrate(csc(f*x+e)**2*(b*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(b*sec(e + f*x))*csc(e + f*x)**2, x)
```

Maxima [F]

$$\int \csc^2(e+fx)\sqrt{b\sec(e+fx)}\,dx=\int \sqrt{b\sec(fx+e)}\csc(fx+e)^2\,dx$$

input
```
integrate(csc(f*x+e)^2*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e))*csc(f*x + e)^2, x)
```

Giac [F]

$$\int \csc^2(e+fx)\sqrt{b\sec(e+fx)}\,dx=\int \sqrt{b\sec(fx+e)}\csc(fx+e)^2\,dx$$

input
```
integrate(csc(f*x+e)^2*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e))*csc(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(e+fx)\sqrt{b\sec(e+fx)}\,dx=\int \frac{\sqrt{\frac{b}{\cos(e+f\,x)}}}{\sin(e+f\,x)^2}\,dx$$

input
```
int((b/cos(e + f*x))^(1/2)/sin(e + f*x)^2,x)
```

output
```
int((b/cos(e + f*x))^(1/2)/sin(e + f*x)^2, x)
```

Reduce [F]

$$\int \csc^2(e+fx)\sqrt{b\sec(e+fx)}\,dx = \sqrt{b}\left(\int \sqrt{\sec\left(fx+e\right)}\,\csc\left(fx+e\right)^2dx\right)$$

input
```
int(csc(f*x+e)^2*(b*sec(f*x+e))^(1/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(e + f*x))*csc(e + f*x)**2,x)
```

3.383 $\int \csc^4(e+fx)\sqrt{b\sec(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 95

$$\int \csc^4(e+fx)\sqrt{b\sec(e+fx)}\,dx = -\frac{5b\csc(e+fx)}{6f\sqrt{b\sec(e+fx)}} - \frac{b\csc^3(e+fx)}{3f\sqrt{b\sec(e+fx)}} + \frac{5\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{6f}$$

output
```
-5/6*b*csc(f*x+e)/f/(b*sec(f*x+e))^(1/2)-1/3*b*csc(f*x+e)^3/f/(b*sec(f*x+e
))^(1/2)+5/6*cos(f*x+e)^(1/2)*InverseJacobiAM(1/2*f*x+1/2*e,2^(1/2))*(b*se
c(f*x+e))^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.66

$$\int \csc^4(e+fx)\sqrt{b\sec(e+fx)}\,dx = \frac{\left(-\cot(e+fx)\left(5+2\csc^2(e+fx)\right)+5\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\right)\sqrt{b\sec(e+fx)}}{6f}$$

input
```
Integrate[Csc[e + f*x]^4*Sqrt[b*Sec[e + f*x]],x]
```

output
```
((-(Cot[e + f*x]*(5 + 2*Csc[e + f*x]^2)) + 5*Sqrt[Cos[e + f*x]]*EllipticF[
(e + f*x)/2, 2])*Sqrt[b*Sec[e + f*x]])/(6*f)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3042, 3105, 3042, 3105, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(e+fx)\sqrt{b\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(e+fx)^4\sqrt{b\sec(e+fx)}dx$$

$$\Big\downarrow \text{3105}$$

$$\frac{5}{6}\int \csc^2(e+fx)\sqrt{b\sec(e+fx)}dx-\frac{b\csc^3(e+fx)}{3f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\int \csc(e+fx)^2\sqrt{b\sec(e+fx)}dx-\frac{b\csc^3(e+fx)}{3f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3105}$$

$$\frac{5}{6}\left(\frac{1}{2}\int \sqrt{b\sec(e+fx)}dx-\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}\right)-\frac{b\csc^3(e+fx)}{3f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}\left(\frac{1}{2}\int \sqrt{b\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}\right)-\frac{b\csc^3(e+fx)}{3f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{5}{6}\left(\frac{1}{2}\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\int\frac{1}{\sqrt{\cos(e+fx)}}dx-\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}\right)-\frac{b\csc^3(e+fx)}{3f\sqrt{b\sec(e+fx)}}$$

$\big\downarrow$ 3042

$$\frac{5}{6}\left(\frac{1}{2}\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\int\frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}\right)-\frac{b\csc^3(e+fx)}{3f\sqrt{b\sec(e+fx)}}$$

$\big\downarrow$ 3120

$$\frac{5}{6}\left(\frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{f}-\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}\right)-\frac{b\csc^3(e+fx)}{3f\sqrt{b\sec(e+fx)}}$$

input
```
Int[Csc[e + f*x]^4*Sqrt[b*Sec[e + f*x]],x]
```

output
```
-1/3*(b*Csc[e + f*x]^3)/(f*Sqrt[b*Sec[e + f*x]]) + (5*(-((b*Csc[e + f*x])/
(f*Sqrt[b*Sec[e + f*x]])) + (Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*
Sqrt[b*Sec[e + f*x]])/f))/6
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3105
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Simp[a^2*((m + n - 2)/(m - 1))   Int[(a*Csc[e + f*
x])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[
m, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.58 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.11

method	result
default	$\frac{\left(\frac{5\cot(fx+e)^3}{6}-\frac{7\cot(fx+e)\csc(fx+e)^2}{6}+\frac{5i(\cos(fx+e)+1)\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\mathrm{EllipticF}(i(\cot(fx+e)-\csc(fx+e)),i)}{6}\right)\sqrt{b\sec(f}}{f}$

input
```
int(csc(f*x+e)^4*(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(5/6*cot(f*x+e)^3-7/6*cot(f*x+e)*csc(f*x+e)^2+5/6*I*EllipticF(I*(cot(f
*x+e)-csc(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)+1)*(cos(f*x+e)/(
cos(f*x+e)+1))^(1/2))*(b*sec(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.56

$$\int \csc^4(e+fx)\sqrt{b\sec(e+fx)}\,dx = -\frac{5\sqrt{2}\left(i\cos(fx+e)^2-i\right)\sqrt{b}\sin(fx+e)\,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e))+5}{}$$

input
```
integrate(csc(f*x+e)^4*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-1/12*(5*sqrt(2)*(I*cos(f*x + e)^2 - I)*sqrt(b)*sin(f*x + e)*weierstrassPI
nverse(-4, 0, cos(f*x + e) + I*sin(f*x + e)) + 5*sqrt(2)*(-I*cos(f*x + e)^
2 + I)*sqrt(b)*sin(f*x + e)*weierstrassPInverse(-4, 0, cos(f*x + e) - I*si
n(f*x + e)) + 2*(5*cos(f*x + e)^3 - 7*cos(f*x + e))*sqrt(b/cos(f*x + e)))/
((f*cos(f*x + e)^2 - f)*sin(f*x + e))
```

Sympy [F]

$$\int \csc^4(e+fx)\sqrt{b\sec(e+fx)}\,dx = \int \sqrt{b\sec\left(e+fx\right)}\csc^4\left(e+fx\right)dx$$

input
```
integrate(csc(f*x+e)**4*(b*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(b*sec(e + f*x))*csc(e + f*x)**4, x)
```

Maxima [F]

$$\int \csc^4(e+fx)\sqrt{b\sec(e+fx)}\,dx = \int \sqrt{b\sec\left(fx+e\right)}\csc\left(fx+e\right)^4\,dx$$

input
```
integrate(csc(f*x+e)^4*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e))*csc(f*x + e)^4, x)
```

Giac [F]

$$\int \csc^4(e+fx)\sqrt{b\sec(e+fx)}\,dx=\int \sqrt{b\sec\left(fx+e\right)}\csc\left(fx+e\right)^4\,dx$$

input
```
integrate(csc(f*x+e)^4*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e))*csc(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^4(e+fx)\sqrt{b\sec(e+fx)}\,dx=\int \frac{\sqrt{\frac{b}{\cos(e+f\,x)}}}{\sin\left(e+f\,x\right)^4}\,dx$$

input
```
int((b/cos(e + f*x))^(1/2)/sin(e + f*x)^4,x)
```

output
```
int((b/cos(e + f*x))^(1/2)/sin(e + f*x)^4, x)
```

Reduce [F]

$$\int \csc^4(e+fx)\sqrt{b\sec(e+fx)}\,dx=\sqrt{b}\left(\int \sqrt{\sec\left(fx+e\right)}\,\csc\left(fx+e\right)^4dx\right)$$

input
```
int(csc(f*x+e)^4*(b*sec(f*x+e))^(1/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(e + f*x))*csc(e + f*x)**4,x)
```

3.384 $\int \csc^6(e+fx)\sqrt{b\sec(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 123

$$\int \csc^6(e+fx)\sqrt{b\sec(e+fx)}\,dx = -\frac{3b\csc(e+fx)}{4f\sqrt{b\sec(e+fx)}}-\frac{3b\csc^3(e+fx)}{10f\sqrt{b\sec(e+fx)}}-\frac{b\csc^5(e+fx)}{5f\sqrt{b\sec(e+fx)}}+\frac{3\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{4f}$$

output
```
-3/4*b*csc(f*x+e)/f/(b*sec(f*x+e))^(1/2)-3/10*b*csc(f*x+e)^3/f/(b*sec(f*x+
e))^(1/2)-1/5*b*csc(f*x+e)^5/f/(b*sec(f*x+e))^(1/2)+3/4*cos(f*x+e)^(1/2)*I
nverseJacobiAM(1/2*f*x+1/2*e,2^(1/2))*(b*sec(f*x+e))^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.59

$$\int \csc^6(e+fx)\sqrt{b\sec(e+fx)}\,dx = \frac{\left(-\cot(e+fx)\left(15+6\csc^2(e+fx)+4\csc^4(e+fx)\right)+15\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\right)\sqrt{b}}{20f}$$

input
```
Integrate[Csc[e + f*x]^6*Sqrt[b*Sec[e + f*x]],x]
```

output
```
((-(Cot[e + f*x]*(15 + 6*Csc[e + f*x]^2 + 4*Csc[e + f*x]^4)) + 15*Sqrt[Cos
[e + f*x]]*EllipticF[(e + f*x)/2, 2])*Sqrt[b*Sec[e + f*x]])/(20*f)
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3105, 3042, 3105, 3042, 3105, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^6(e+fx)\sqrt{b\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(e+fx)^6\sqrt{b\sec(e+fx)}dx$$

$$\Big\downarrow \text{3105}$$

$$\frac{9}{10}\int \csc^4(e+fx)\sqrt{b\sec(e+fx)}dx-\frac{b\csc^5(e+fx)}{5f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9}{10}\int \csc(e+fx)^4\sqrt{b\sec(e+fx)}dx-\frac{b\csc^5(e+fx)}{5f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3105}$$

$$\frac{9}{10}\left(\frac{5}{6}\int \csc^2(e+fx)\sqrt{b\sec(e+fx)}dx-\frac{b\csc^3(e+fx)}{3f\sqrt{b\sec(e+fx)}}\right)-\frac{b\csc^5(e+fx)}{5f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{9}{10}\left(\frac{5}{6}\int \csc(e+fx)^2\sqrt{b\sec(e+fx)}dx-\frac{b\csc^3(e+fx)}{3f\sqrt{b\sec(e+fx)}}\right)-\frac{b\csc^5(e+fx)}{5f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3105}$$

$$\frac{9}{10}\left(\frac{5}{6}\left(\frac{1}{2}\int \sqrt{b\sec(e+fx)}dx-\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}\right)-\frac{b\csc^3(e+fx)}{3f\sqrt{b\sec(e+fx)}}\right)-\frac{b\csc^5(e+fx)}{5f\sqrt{b\sec(e+fx)}}$$

$\Big\downarrow$ 3042

$$\frac{9}{10}\left(\frac{5}{6}\left(\frac{1}{2}\int \sqrt{b\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}\right)-\frac{b\csc^3(e+fx)}{3f\sqrt{b\sec(e+fx)}}\right)-\frac{b\csc^5(e+fx)}{5f\sqrt{b\sec(e+fx)}}$$

$\Big\downarrow$ 4258

$$\frac{9}{10}\left(\frac{5}{6}\left(\frac{1}{2}\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}}dx-\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}\right)-\frac{b\csc^3(e+fx)}{3f\sqrt{b\sec(e+fx)}}\right)-\frac{b\csc^5(e+fx)}{5f\sqrt{b\sec(e+fx)}}$$

$\Big\downarrow$ 3042

$$\frac{9}{10}\left(\frac{5}{6}\left(\frac{1}{2}\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}\right)-\frac{b\csc^3(e+fx)}{3f\sqrt{b\sec(e+fx)}}\right)-\frac{b\csc^5(e+fx)}{5f\sqrt{b\sec(e+fx)}}$$

$\Big\downarrow$ 3120

$$\frac{9}{10}\left(\frac{5}{6}\left(\frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{f}-\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}\right)-\frac{b\csc^3(e+fx)}{3f\sqrt{b\sec(e+fx)}}\right)-\frac{b\csc^5(e+fx)}{5f\sqrt{b\sec(e+fx)}}$$

input
```
Int[Csc[e + f*x]^6*Sqrt[b*Sec[e + f*x]],x]
```

output

```
-1/5*(b*Csc[e + f*x]^5)/(f*Sqrt[b*Sec[e + f*x]]) + (9*(-1/3*(b*Csc[e + f*x
]^3)/(f*Sqrt[b*Sec[e + f*x]]) + (5*(-((b*Csc[e + f*x])/(f*Sqrt[b*Sec[e + f
*x]])) + (Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]
])/f))/6))/10
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Simp[a^2*((m + n - 2)/(m - 1))   Int[(a*Csc[e + f*
x])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[
m, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

rule 3120

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.84 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.95

method	result
default	$\frac{\left(\frac{\left(-15\cos(fx+e)^4+36\cos(fx+e)^2-25\right)\cot(fx+e)\csc(fx+e)^4}{20}+\frac{3i(\cos(fx+e)+1)\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\text{EllipticF}(i(\cot(fx+e)-\csc(f}{4}\right.}{f}$

input
```
int(csc(f*x+e)^6*(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/20*(-15*cos(f*x+e)^4+36*cos(f*x+e)^2-25)*cot(f*x+e)*csc(f*x+e)^4+3/
4*I*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f
*x+e)+1)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*(b*sec(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.52

$$\int \csc^6(e+fx)\sqrt{b\sec(e+fx)}\,dx = -\frac{15\sqrt{2}\left(i\cos(fx+e)^4-2i\cos(fx+e)^2+i\right)\sqrt{b}\sin(fx+e)\,\text{weierstrassPInverse}(-4,0,\cos(fx+e)}{}$$

input
```
integrate(csc(f*x+e)^6*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-1/40*(15*sqrt(2)*(I*cos(f*x + e)^4 - 2*I*cos(f*x + e)^2 + I)*sqrt(b)*sin(
f*x + e)*weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e)) + 15*sq
rt(2)*(-I*cos(f*x + e)^4 + 2*I*cos(f*x + e)^2 - I)*sqrt(b)*sin(f*x + e)*we
ierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) + 2*(15*cos(f*x +
e)^5 - 36*cos(f*x + e)^3 + 25*cos(f*x + e))*sqrt(b/cos(f*x + e)))/((f*cos(
f*x + e)^4 - 2*f*cos(f*x + e)^2 + f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^6(e+fx)\sqrt{b\sec(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**6*(b*sec(f*x+e))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \csc^6(e+fx)\sqrt{b\sec(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)}\csc(fx+e)^6\,dx$$

input
```
integrate(csc(f*x+e)^6*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e))*csc(f*x + e)^6, x)
```

Giac [F]

$$\int \csc^6(e+fx)\sqrt{b\sec(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)}\csc(fx+e)^6\,dx$$

input
```
integrate(csc(f*x+e)^6*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e))*csc(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^6(e+fx)\sqrt{b\sec(e+fx)}\,dx = \int \frac{\sqrt{\frac{b}{\cos(e+f\,x)}}}{\sin(e+f\,x)^6}\,dx$$

input
```
int((b/cos(e + f*x))^(1/2)/sin(e + f*x)^6,x)
```

output
```
int((b/cos(e + f*x))^(1/2)/sin(e + f*x)^6, x)
```

Reduce [F]

$$\int \csc^6(e+fx)\sqrt{b\sec(e+fx)}\,dx = \sqrt{b}\left(\int \sqrt{\sec(fx+e)}\,\csc(fx+e)^6\,dx\right)$$

input
```
int(csc(f*x+e)^6*(b*sec(f*x+e))^(1/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(e + f*x))*csc(e + f*x)**6,x)
```

3.385 $\int (b\sec(e+fx))^{3/2}\sin^7(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 83

$$\int (b\sec(e+fx))^{3/2}\sin^7(e+fx)\,dx = \frac{2b^7}{11f(b\sec(e+fx))^{11/2}} - \frac{6b^5}{7f(b\sec(e+fx))^{7/2}} + \frac{2b^3}{f(b\sec(e+fx))^{3/2}} + \frac{2b\sqrt{b\sec(e+fx)}}{f}$$

output
```
2/11*b^7/f/(b*sec(f*x+e))^(11/2)-6/7*b^5/f/(b*sec(f*x+e))^(7/2)+2*b^3/f/(b
*sec(f*x+e))^(3/2)+2*b*(b*sec(f*x+e))^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.63

$$\int (b\sec(e+fx))^{3/2}\sin^7(e+fx)\,dx = \frac{b(3370+809\cos(2(e+fx))-90\cos(4(e+fx))+7\cos(6(e+fx)))\sqrt{b\sec(e+fx)}}{1232f}$$

input
```
Integrate[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^7,x]
```

output
```
(b*(3370 + 809*Cos[2*(e + f*x)] - 90*Cos[4*(e + f*x)] + 7*Cos[6*(e + f*x)]
)*Sqrt[b*Sec[e + f*x]])/(1232*f)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3102, 25, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^7(e+fx)(b\sec(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\sec(e+fx))^{3/2}}{\csc(e+fx)^7}dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{b^7\int -\frac{\left(b^2-b^2\sec^2(e+fx)\right)^3}{b^6(b\sec(e+fx))^{13/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b^7\int \frac{\left(b^2-b^2\sec^2(e+fx)\right)^3}{b^6(b\sec(e+fx))^{13/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\int \frac{\left(b^2-b^2\sec^2(e+fx)\right)^3}{(b\sec(e+fx))^{13/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{244}$$

$$-\frac{b\int \left(\frac{b^6}{(b\sec(e+fx))^{13/2}}-\frac{3b^4}{(b\sec(e+fx))^{9/2}}+\frac{3b^2}{(b\sec(e+fx))^{5/2}}-\frac{1}{\sqrt{b\sec(e+fx)}}\right)d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\left(-\frac{2b^6}{11(b\sec(e+fx))^{11/2}}+\frac{6b^4}{7(b\sec(e+fx))^{7/2}}-\frac{2b^2}{(b\sec(e+fx))^{3/2}}-2\sqrt{b\sec(e+fx)}\right)}{f}$$

input
```
Int[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^7,x]
```

output
```
-((b*((-2*b^6)/(11*(b*Sec[e + f*x])^(11/2)) + (6*b^4)/(7*(b*Sec[e + f*x])^
(7/2)) - (2*b^2)/(b*Sec[e + f*x])^(3/2) - 2*Sqrt[b*Sec[e + f*x]]))/f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(71) = 142$.

Time = 6.64 (sec) , antiderivative size = 255, normalized size of antiderivative = 3.07

method	result
default	$b\left(2+\frac{(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\cos(fx+e)+1}{\cos(fx+e)+1}\right)}{2}+\frac{(-\cos(fx+e)-1)\sqrt{-\ldots}}{\ldots}\right.$

input
```
int((b*sec(f*x+e))^(3/2)*sin(f*x+e)^7,x,method=_RETURNVERBOSE)
```

output
```
1/f*b*(2+1/2*(cos(f*x+e)+1)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln((2*cos
(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))+1/2*(-cos(f*x+e)-1)*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))+
2/11*cos(f*x+e)^6-6/7*cos(f*x+e)^4+2*cos(f*x+e)^2)*(b*sec(f*x+e))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.65

$$\int (b\sec(e+fx))^{3/2}\sin^7(e+fx)\,dx = \frac{2\left(7\,b\cos\left(fx+e\right)^6-33\,b\cos\left(fx+e\right)^4+77\,b\cos\left(fx+e\right)^2+77\,b\right)\sqrt{\frac{b}{\cos(fx+e)}}}{77\,f}$$

input
```
integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^7,x, algorithm="fricas")
```

output
```
2/77*(7*b*cos(f*x + e)^6 - 33*b*cos(f*x + e)^4 + 77*b*cos(f*x + e)^2 + 77*
b)*sqrt(b/cos(f*x + e))/f
```

Sympy [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{3/2}\sin^7(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((b*sec(f*x+e))**(3/2)*sin(f*x+e)**7,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.87

$$\int (b\sec(e+fx))^{3/2}\sin^7(e+fx)\,dx = \frac{2\,b\left(\frac{7\,b^6}{\left(\frac{b}{\cos(fx+e)}\right)^{\frac{11}{2}}} - \frac{33\,b^4}{\left(\frac{b}{\cos(fx+e)}\right)^{\frac{7}{2}}} + \frac{77\,b^2}{\left(\frac{b}{\cos(fx+e)}\right)^{\frac{3}{2}}} + 77\sqrt{\frac{b}{\cos(fx+e)}}\right)}{77\,f}$$

input
```
integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^7,x, algorithm="maxima")
```

output
```
2/77*b*(7*b^6/(b/cos(f*x + e))^(11/2) - 33*b^4/(b/cos(f*x + e))^(7/2) + 77
*b^2/(b/cos(f*x + e))^(3/2) + 77*sqrt(b/cos(f*x + e)))/f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.18

$$\int (b\sec(e+fx))^{3/2}\sin^7(e+fx)\,dx = \frac{2\left(7\sqrt{b\cos(fx+e)}b^5\cos(fx+e)^5 - 33\sqrt{b\cos(fx+e)}b^5\cos(fx+e)^3 + 77\sqrt{b\cos(fx+e)}\right.}{77\,b^4 f}$$

input
```
integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^7,x, algorithm="giac")
```

output
```
2/77*(7*sqrt(b*cos(f*x + e))*b^5*cos(f*x + e)^5 - 33*sqrt(b*cos(f*x + e))*
b^5*cos(f*x + e)^3 + 77*sqrt(b*cos(f*x + e))*b^5*cos(f*x + e) + 77*b^6/sqr
t(b*cos(f*x + e)))*sgn(cos(f*x + e))/(b^4*f)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{3/2}\sin^7(e+fx)\,dx = \int \sin(e+f\,x)^7\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}dx$$

input
```
int(sin(e + f*x)^7*(b/cos(e + f*x))^(3/2),x)
```

output
```
int(sin(e + f*x)^7*(b/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int (b\sec(e+fx))^{3/2}\sin^7(e+fx)\,dx = \sqrt{b}\left(\int \sqrt{\sec(fx+e)}\,\sec(fx+e)\sin(fx+e)^7\,dx\right)b$$

input
```
int((b*sec(f*x+e))^(3/2)*sin(f*x+e)^7,x)
```

output
```
sqrt(b)*int(sqrt(sec(e + f*x))*sec(e + f*x)*sin(e + f*x)**7,x)*b
```

3.386 $\int (b\sec(e+fx))^{3/2}\sin^5(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 63

$$\int (b\sec(e+fx))^{3/2}\sin^5(e+fx)\,dx = -\frac{2b^5}{7f(b\sec(e+fx))^{7/2}} + \frac{4b^3}{3f(b\sec(e+fx))^{3/2}} + \frac{2b\sqrt{b\sec(e+fx)}}{f}$$

output
```
-2/7*b^5/f/(b*sec(f*x+e))^(7/2)+4/3*b^3/f/(b*sec(f*x+e))^(3/2)+2*b*(b*sec(
f*x+e))^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.67

$$\int (b\sec(e+fx))^{3/2}\sin^5(e+fx)\,dx = \frac{b(215+44\cos(2(e+fx))-3\cos(4(e+fx)))\sqrt{b\sec(e+fx)}}{84f}$$

input
```
Integrate[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^5,x]
```

output
```
(b*(215 + 44*Cos[2*(e + f*x)] - 3*Cos[4*(e + f*x)])*Sqrt[b*Sec[e + f*x]])/
(84*f)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3102, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(e+fx)(b\sec(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\sec(e+fx))^{3/2}}{\csc(e+fx)^5}dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{b^5\int \frac{\left(b^2-b^2\sec^2(e+fx)\right)^2}{b^4(b\sec(e+fx))^{9/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{\left(b^2-b^2\sec^2(e+fx)\right)^2}{(b\sec(e+fx))^{9/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{244}$$

$$\frac{b\int \left(\frac{b^4}{(b\sec(e+fx))^{9/2}}-\frac{2b^2}{(b\sec(e+fx))^{5/2}}+\frac{1}{\sqrt{b\sec(e+fx)}}\right)d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(-\frac{2b^4}{7(b\sec(e+fx))^{7/2}}+\frac{4b^2}{3(b\sec(e+fx))^{3/2}}+2\sqrt{b\sec(e+fx)}\right)}{f}$$

input
```
Int[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^5,x]
```

output
```
(b*((-2*b^4)/(7*(b*Sec[e + f*x])^(7/2)) + (4*b^2)/(3*(b*Sec[e + f*x])^(3/2
)) + 2*Sqrt[b*Sec[e + f*x]]))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(53) = 106$.

Time = 1.37 (sec) , antiderivative size = 247, normalized size of antiderivative = 3.92

method	result
default	$\left(-\frac{(-21\cos(fx+e)-21)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\cos(fx+e)+1}{\cos(fx+e)+1}\right)}{42}-\frac{(21\cos(fx+e)+21}{}\right.$

input
```
int((b*sec(f*x+e))^(3/2)*sin(f*x+e)^5,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/42*(-21*cos(f*x+e)-21)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln((2*
cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))-1/42*(21*cos(f*x+e)+21)*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+
e)+1))-2/7*cos(f*x+e)^4+4/3*cos(f*x+e)^2+2)*b*(b*sec(f*x+e))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.68

$$\int (b\sec(e+fx))^{3/2}\sin^5(e+fx)\,dx = -\frac{2\left(3\,b\cos\left(fx+e\right)^4-14\,b\cos\left(fx+e\right)^2-21\,b\right)\sqrt{\frac{b}{\cos(fx+e)}}}{21\,f}$$

input
```
integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^5,x, algorithm="fricas")
```

output
```
-2/21*(3*b*cos(f*x + e)^4 - 14*b*cos(f*x + e)^2 - 21*b)*sqrt(b/cos(f*x + e
))/f
```

Sympy [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{3/2}\sin^5(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((b*sec(f*x+e))**(3/2)*sin(f*x+e)**5,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int (b\sec(e+fx))^{3/2}\sin^5(e+fx)\,dx = -\frac{2\,b\left(\frac{3\,b^4}{\left(\frac{b}{\cos(fx+e)}\right)^{\frac{7}{2}}} - \frac{14\,b^2}{\left(\frac{b}{\cos(fx+e)}\right)^{\frac{3}{2}}} - 21\,\sqrt{\frac{b}{\cos(fx+e)}}\right)}{21\,f}$$

input
```
integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^5,x, algorithm="maxima")
```

output
```
-2/21*b*(3*b^4/(b/cos(f*x + e))^(7/2) - 14*b^2/(b/cos(f*x + e))^(3/2) - 21
*sqrt(b/cos(f*x + e)))/f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.19

$$\int (b\sec(e+fx))^{3/2}\sin^5(e+fx)\,dx =$$
$$-\frac{2\left(3\,\sqrt{b\cos(fx+e)}b^3\cos(fx+e)^3 - 14\,\sqrt{b\cos(fx+e)}b^3\cos(fx+e) - \frac{21\,b^4}{\sqrt{b\cos(fx+e)}}\right)\operatorname{sgn}(\cos(fx+e)}{21\,b^2 f}$$

input
```
integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^5,x, algorithm="giac")
```

output
```
-2/21*(3*sqrt(b*cos(f*x + e))*b^3*cos(f*x + e)^3 - 14*sqrt(b*cos(f*x + e))
*b^3*cos(f*x + e) - 21*b^4/sqrt(b*cos(f*x + e)))*sgn(cos(f*x + e))/(b^2*f)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{3/2}\sin^5(e+fx)\,dx=\int \sin\left(e+f\,x\right)^5\,\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^{3/2}\,dx$$

input
```
int(sin(e + f*x)^5*(b/cos(e + f*x))^(3/2),x)
```

output
```
int(sin(e + f*x)^5*(b/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int (b\sec(e+fx))^{3/2}\sin^5(e+fx)\,dx=\sqrt{b}\,\left(\int \sqrt{\sec\left(fx+e\right)}\,\sec\left(fx+e\right)\sin\left(fx+e\right)^{5}dx\right)b$$

input
```
int((b*sec(f*x+e))^(3/2)*sin(f*x+e)^5,x)
```

output
```
sqrt(b)*int(sqrt(sec(e + f*x))*sec(e + f*x)*sin(e + f*x)**5,x)*b
```

3.387 $\int (b\sec(e+fx))^{3/2}\sin^3(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 41

$$\int (b\sec(e+fx))^{3/2}\sin^3(e+fx)\,dx=\frac{2b^3}{3f(b\sec(e+fx))^{3/2}}+\frac{2b\sqrt{b\sec(e+fx)}}{f}$$

output
```
2/3*b^3/f/(b*sec(f*x+e))^(3/2)+2*b*(b*sec(f*x+e))^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int (b\sec(e+fx))^{3/2}\sin^3(e+fx)\,dx=\frac{b(7+\cos(2(e+fx)))\sqrt{b\sec(e+fx)}}{3f}$$

input
```
Integrate[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^3,x]
```

output
```
(b*(7 + Cos[2*(e + f*x)])*Sqrt[b*Sec[e + f*x]])/(3*f)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3102, 25, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(e+fx)(b\sec(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(b\sec(e+fx))^{3/2}}{\csc(e+fx)^3}dx$$

$$\Big\downarrow \ 3102$$

$$\frac{b^3\int -\frac{b^2-b^2\sec^2(e+fx)}{b^2(b\sec(e+fx))^{5/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \ 25$$

$$-\frac{b^3\int \frac{b^2-b^2\sec^2(e+fx)}{b^2(b\sec(e+fx))^{5/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \ 27$$

$$-\frac{b\int \frac{b^2-b^2\sec^2(e+fx)}{(b\sec(e+fx))^{5/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \ 244$$

$$-\frac{b\int \left(\frac{b^2}{(b\sec(e+fx))^{5/2}}-\frac{1}{\sqrt{b\sec(e+fx)}}\right)d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \ 2009$$

$$-\frac{b\left(-\frac{2b^2}{3(b\sec(e+fx))^{3/2}}-2\sqrt{b\sec(e+fx)}\right)}{f}$$

input
```
Int[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^3,x]
```

output
```
-((b*((-2*b^2)/(3*(b*Sec[e + f*x])^(3/2)) - 2*Sqrt[b*Sec[e + f*x]]))/f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(35) = 70$.

Time = 1.30 (sec) , antiderivative size = 236, normalized size of antiderivative = 5.76

method	result
default	$\frac{\left((3\cos(fx+e)+3)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\cos(fx+e)+1}{\cos(fx+e)+1}\right)+(-3\cos(fx+e)-3 \right.}{6}$

input
```
int((b*sec(f*x+e))^(3/2)*sin(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/6/f*((3*cos(f*x+e)+3)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln((2*cos(f*x
+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))+(-3*cos(f*x+e)-3)*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2)*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))+4*cos(
f*x+e)^2+12)*b*(b*sec(f*x+e))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int (b\sec(e+fx))^{3/2}\sin^3(e+fx)\,dx = \frac{2\left(b\cos\left(fx+e\right)^2+3\,b\right)\sqrt{\frac{b}{\cos(fx+e)}}}{3\,f}$$

input
```
integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^3,x, algorithm="fricas")
```

output
```
2/3*(b*cos(f*x + e)^2 + 3*b)*sqrt(b/cos(f*x + e))/f
```

Sympy [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{3/2}\sin^3(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((b*sec(f*x+e))**(3/2)*sin(f*x+e)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int (b\sec(e+fx))^{3/2}\sin^3(e+fx)\,dx = \frac{2\,b\left(\frac{b^2}{\left(\frac{b}{\cos(fx+e)}\right)^{\frac{3}{2}}}+3\,\sqrt{\frac{b}{\cos(fx+e)}}\right)}{3\,f}$$

input
```
integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^3,x, algorithm="maxima")
```

output
```
2/3*b*(b^2/(b/cos(f*x + e))^(3/2) + 3*sqrt(b/cos(f*x + e)))/f
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int (b\sec(e+fx))^{3/2}\sin^3(e+fx)\,dx = \frac{2\left(\sqrt{b\cos(fx+e)}b\cos(fx+e)+\frac{3\,b^2}{\sqrt{b\cos(fx+e)}}\right)\mathrm{sgn}(\cos(fx+e))}{3\,f}$$

input
```
integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^3,x, algorithm="giac")
```

output
```
2/3*(sqrt(b*cos(f*x + e))*b*cos(f*x + e) + 3*b^2/sqrt(b*cos(f*x + e)))*sgn
(cos(f*x + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{3/2}\sin^3(e+fx)\,dx = \int \sin(e+f\,x)^3\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}\,dx$$

input
```
int(sin(e + f*x)^3*(b/cos(e + f*x))^(3/2),x)
```

output
```
int(sin(e + f*x)^3*(b/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int (b\sec(e+fx))^{3/2}\sin^3(e+fx)\,dx = \sqrt{b}\left(\int \sqrt{\sec(fx+e)}\,\sec(fx+e)\sin(fx+e)^3\,dx\right)b$$

input
```
int((b*sec(f*x+e))^(3/2)*sin(f*x+e)^3,x)
```

output
```
sqrt(b)*int(sqrt(sec(e + f*x))*sec(e + f*x)*sin(e + f*x)**3,x)*b
```

3.388 $\int (b\sec(e+fx))^{3/2}\sin(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 18

$$\int (b\sec(e+fx))^{3/2}\sin(e+fx)\,dx=\frac{2b\sqrt{b\sec(e+fx)}}{f}$$

output
```
2*b*(b*sec(f*x+e))^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (b\sec(e+fx))^{3/2}\sin(e+fx)\,dx=\frac{2b\sqrt{b\sec(e+fx)}}{f}$$

input
```
Integrate[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x],x]
```

output
```
(2*b*Sqrt[b*Sec[e + f*x]])/f
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 3102, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(e+fx)(b\sec(e+fx))^{3/2}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{(b\sec(e+fx))^{3/2}}{\csc(e+fx)}dx$$
$$\Big\downarrow \ 3102$$
$$\frac{b\int \frac{1}{\sqrt{b\sec(e+fx)}}d(b\sec(e+fx))}{f}$$
$$\Big\downarrow \ 15$$
$$\frac{2b\sqrt{b\sec(e+fx)}}{f}$$

input
```
Int[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x],x]
```

output
```
(2*b*Sqrt[b*Sec[e + f*x]])/f
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{2b\sqrt{b\sec(fx+e)}}{f}$	17
default	$\frac{2b\sqrt{b\sec(fx+e)}}{f}$	17

input
```
int((b*sec(f*x+e))^(3/2)*sin(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
2*b*(b*sec(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (b\sec(e+fx))^{3/2}\sin(e+fx)\,dx = \frac{2\,b\sqrt{\frac{b}{\cos(fx+e)}}}{f}$$

input
```
integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e),x, algorithm="fricas")
```

output
```
2*b*sqrt(b/cos(f*x + e))/f
```

Sympy [F]

$$\int (b\sec(e+fx))^{3/2}\sin(e+fx)\,dx = \int (b\sec{(e+fx)})^{\frac{3}{2}}\sin{(e+fx)}\,dx$$

input
```
integrate((b*sec(f*x+e))**(3/2)*sin(f*x+e),x)
```

output
```
Integral((b*sec(e + f*x))**(3/2)*sin(e + f*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int (b\sec(e+fx))^{3/2}\sin(e+fx)\,dx = \frac{2\left(\frac{b}{\cos(fx+e)}\right)^{\frac{3}{2}}\cos{(fx+e)}}{f}$$

input
```
integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e),x, algorithm="maxima")
```

output
```
2*(b/cos(f*x + e))^(3/2)*cos(f*x + e)/f
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.39

$$\int (b\sec(e+fx))^{3/2}\sin(e+fx)\,dx = \frac{2\,b^2\mathrm{sgn}(\cos{(fx+e)})}{\sqrt{b\cos{(fx+e)}}f}$$

input
```
integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e),x, algorithm="giac")
```

output
```
2*b^2*sgn(cos(f*x + e))/(sqrt(b*cos(f*x + e))*f)
```

Mupad [B] (verification not implemented)

Time = 25.89 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (b\sec(e+fx))^{3/2}\sin(e+fx)\,dx = \frac{2\,b\,\sqrt{\frac{b}{\cos(e+f\,x)}}}{f}$$

input
```
int(sin(e + f*x)*(b/cos(e + f*x))^(3/2),x)
```

output
```
(2*b*(b/cos(e + f*x))^(1/2))/f
```

Reduce [F]

$$\int (b\sec(e+fx))^{3/2}\sin(e+fx)\,dx = \sqrt{b}\left(\int \sqrt{\sec(fx+e)}\,\sec(fx+e)\sin(fx+e)\,dx\right)b$$

input
```
int((b*sec(f*x+e))^(3/2)*sin(f*x+e),x)
```

output
```
sqrt(b)*int(sqrt(sec(e + f*x))*sec(e + f*x)*sin(e + f*x),x)*b
```

3.389 $\int \csc(e+fx)(b\sec(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 77

$$\int \csc(e+fx)(b\sec(e+fx))^{3/2}\,dx = -\frac{b^{3/2}\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f} - \frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f} + \frac{2b\sqrt{b\sec(e+fx)}}{f}$$

output
```
-b^(3/2)*arctan((b*sec(f*x+e))^(1/2)/b^(1/2))/f-b^(3/2)*arctanh((b*sec(f*x
+e))^(1/2)/b^(1/2))/f+2*b*(b*sec(f*x+e))^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.10

$$\int \csc(e+fx)(b\sec(e+fx))^{3/2}\,dx = \frac{\left(-2\arctan\left(\sqrt{\sec(e+fx)}\right) + \log\left(1-\sqrt{\sec(e+fx)}\right) - \log\left(1+\sqrt{\sec(e+fx)}\right) + 4\sqrt{\text{se}}\right.}{2f\sec^{\frac{3}{2}}(e+fx)}$$

input
```
Integrate[Csc[e + f*x]*(b*Sec[e + f*x])^(3/2),x]
```

output
```
((-2*ArcTan[Sqrt[Sec[e + f*x]]] + Log[1 - Sqrt[Sec[e + f*x]]] - Log[1 + Sq
rt[Sec[e + f*x]]] + 4*Sqrt[Sec[e + f*x]])*(b*Sec[e + f*x])^(3/2))/(2*f*Sec
[e + f*x]^(3/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.92, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3042, 3102, 25, 27, 262, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(e+fx)(b\sec(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc(e+fx)(b\sec(e+fx))^{3/2}dx$$

$$\Big\downarrow \text{ 3102}$$

$$\frac{\int -\frac{b^2(b\sec(e+fx))^{3/2}}{b^2-b^2\sec^2(e+fx)}d(b\sec(e+fx))}{bf}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{b^2(b\sec(e+fx))^{3/2}}{b^2-b^2\sec^2(e+fx)}d(b\sec(e+fx))}{bf}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{b\int \frac{(b\sec(e+fx))^{3/2}}{b^2-b^2\sec^2(e+fx)}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{ 262}$$

$$-\frac{b\left(b^2\int \frac{1}{\sqrt{b\sec(e+fx)}(b^2-b^2\sec^2(e+fx))}d(b\sec(e+fx))-2\sqrt{b\sec(e+fx)}\right)}{f}$$

$$\Big\downarrow \text{ 266}$$

$$-\frac{b\left(2b^2\int \frac{1}{b^2-b^4\sec^4(e+fx)}d\sqrt{b\sec(e+fx)}-2\sqrt{b\sec(e+fx)}\right)}{f}$$

$$\Big\downarrow \; 756$$

$$-\frac{b\left(2b^2\left(\frac{\int \frac{1}{b-b^2\sec^2(e+fx)}d\sqrt{b\sec(e+fx)}}{2b}+\frac{\int \frac{1}{b^2\sec^2(e+fx)+b}d\sqrt{b\sec(e+fx)}}{2b}\right)-2\sqrt{b\sec(e+fx)}\right)}{f}$$

$$\Big\downarrow \; 216$$

$$-\frac{b\left(2b^2\left(\frac{\int \frac{1}{b-b^2\sec^2(e+fx)}d\sqrt{b\sec(e+fx)}}{2b}+\frac{\arctan\left(\sqrt{b}\sec(e+fx)\right)}{2b^{3/2}}\right)-2\sqrt{b\sec(e+fx)}\right)}{f}$$

$$\Big\downarrow \; 219$$

$$-\frac{b\left(2b^2\left(\frac{\arctan\left(\sqrt{b}\sec(e+fx)\right)}{2b^{3/2}}+\frac{\text{arctanh}\left(\sqrt{b}\sec(e+fx)\right)}{2b^{3/2}}\right)-2\sqrt{b\sec(e+fx)}\right)}{f}$$

input
```
Int[Csc[e + f*x]*(b*Sec[e + f*x])^(3/2),x]
```

output
```
-((b*(2*b^2*(ArcTan[Sqrt[b]*Sec[e + f*x]]/(2*b^(3/2)) + ArcTanh[Sqrt[b]*Se
c[e + f*x]]/(2*b^(3/2))) - 2*Sqrt[b*Sec[e + f*x]]))/f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(63) = 126$.

Time = 2.18 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.70

method	result
default	$\frac{\left(4\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+\ln\left(\frac{4\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+4\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-2\cos(fx+e)+2}{\cos(fx+e)+1}\right)\cos(fx+e)+\arctan\left(\cdots\right.}{2f(\cos(fx+e)+1)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}}$

input
```
int(csc(f*x+e)*(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f*(4*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+ln(2*(2*cos(f*x+e
)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*cos(f*x+e)+arctan(1/2/(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2))*cos(f*x+e)+4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*b*(
b*sec(f*x+e))^(1/2)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(63) = 126$.

Time = 0.12 (sec) , antiderivative size = 298, normalized size of antiderivative = 3.87

$$\int \csc(e+fx)(b\sec(e+fx))^{3/2}\,dx = \left[\frac{2\sqrt{-b}b\arctan\left(\frac{2\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)}{b\cos(fx+e)+b}\right)+\sqrt{-b}b\log\left(\frac{b\cos(fx+e)^2+4\left(\cos(fx+e)^2-\cos(fx+e)\right)\sqrt{-b}\cdots}{\cos(fx+e)^2+2\cos(fx+\cdots}\right.}{4f}\right.$$

$$-\frac{2b^{\frac{3}{2}}\arctan\left(\frac{2\sqrt{b}\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)}{b\cos(fx+e)-b}\right)-b^{\frac{3}{2}}\log\left(\frac{b\cos(fx+e)^2-4\left(\cos(fx+e)^2+\cos(fx+e)\right)\sqrt{b}\sqrt{\frac{b}{\cos(fx+e)}}+6b\cos(fx+e)+b}{\cos(fx+e)^2-2\cos(fx+e)+1}\right)\cdots}{4f}$$

input
```
integrate(csc(f*x+e)*(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(-b)*b*arctan(2*sqrt(-b)*sqrt(b/cos(f*x + e))*cos(f*x + e)/(b*
cos(f*x + e) + b)) + sqrt(-b)*b*log((b*cos(f*x + e)^2 + 4*(cos(f*x + e)^2
- cos(f*x + e))*sqrt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos(f*x + e) + b)/(cos
(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 8*b*sqrt(b/cos(f*x + e)))/f, -1/4*(2*
b^(3/2)*arctan(2*sqrt(b)*sqrt(b/cos(f*x + e))*cos(f*x + e)/(b*cos(f*x + e)
 - b)) - b^(3/2)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + cos(f*x + e))
*sqrt(b)*sqrt(b/cos(f*x + e)) + 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*
cos(f*x + e) + 1)) - 8*b*sqrt(b/cos(f*x + e)))/f]
```

Sympy [F]

$$\int \csc(e+fx)(b\sec(e+fx))^{3/2}\,dx = \int (b\sec{(e+fx)})^{\frac{3}{2}}\csc{(e+fx)}\,dx$$

input
```
integrate(csc(f*x+e)*(b*sec(f*x+e))**(3/2),x)
```

output
```
Integral((b*sec(e + f*x))**(3/2)*csc(e + f*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.13

$$\int \csc(e+fx)(b\sec(e+fx))^{3/2}\,dx = -\frac{\left(2\sqrt{b}\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}}\right)-\sqrt{b}\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos(fx+e)}}}\right)-4\sqrt{\frac{b}{\cos(fx+e)}}\right)b}{2f}$$

input
```
integrate(csc(f*x+e)*(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
-1/2*(2*sqrt(b)*arctan(sqrt(b/cos(f*x + e))/sqrt(b)) - sqrt(b)*log(-(sqrt(
b) - sqrt(b/cos(f*x + e)))/(sqrt(b) + sqrt(b/cos(f*x + e)))) - 4*sqrt(b/co
s(f*x + e)))*b/f
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97

$$\int \csc(e+fx)(b\sec(e+fx))^{3/2}\,dx = \frac{b^4\left(\frac{\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b}b^2}+\frac{\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{b}}\right)}{b^{\frac{5}{2}}}+\frac{2}{\sqrt{b\cos(fx+e)}b^2}\right)\mathrm{sgn}(\cos(fx+e))}{f}$$

input
```
integrate(csc(f*x+e)*(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
b^4*(arctan(sqrt(b*cos(f*x + e))/sqrt(-b))/(sqrt(-b)*b^2) + arctan(sqrt(b*
cos(f*x + e))/sqrt(b))/b^(5/2) + 2/(sqrt(b*cos(f*x + e))*b^2))*sgn(cos(f*x
 + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \csc(e+fx)(b\sec(e+fx))^{3/2}\,dx = \int \frac{\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}}{\sin(e+f\,x)}\,dx$$

input
```
int((b/cos(e + f*x))^(3/2)/sin(e + f*x),x)
```

output
```
int((b/cos(e + f*x))^(3/2)/sin(e + f*x), x)
```

Reduce [F]

$$\int \csc(e+fx)(b\sec(e+fx))^{3/2}\,dx = \sqrt{b}\left(\int \sqrt{\sec(fx+e)}\,\csc(fx+e)\sec(fx+e)\,dx\right)b$$

input
```
int(csc(f*x+e)*(b*sec(f*x+e))^(3/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(e + f*x))*csc(e + f*x)*sec(e + f*x),x)*b
```

3.390 $\int \csc^3(e+fx)(b\sec(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 113

$$\int \csc^3(e+fx)(b\sec(e+fx))^{3/2}\,dx = -\frac{5b^{3/2}\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f} - \frac{5b^{3/2}\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f} + \frac{5b\sqrt{b\sec(e+fx)}}{2f} - \frac{\cot^2(e+fx)(b\sec(e+fx))^{5/2}}{2bf}$$

output
```
-5/4*b^(3/2)*arctan((b*sec(f*x+e))^(1/2)/b^(1/2))/f-5/4*b^(3/2)*arctanh((b
*sec(f*x+e))^(1/2)/b^(1/2))/f+5/2*b*(b*sec(f*x+e))^(1/2)/f-1/2*cot(f*x+e)^
2*(b*sec(f*x+e))^(5/2)/b/f
```

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.86

$$\int \csc^3(e+fx)(b\sec(e+fx))^{3/2}\,dx = -\frac{\left(10\arctan\left(\sqrt{\sec(e+fx)}\right) - 5\log\left(1-\sqrt{\sec(e+fx)}\right) + 5\log\left(1+\sqrt{\sec(e+fx)}\right) + 4(-5+\csc^2(e\right.}{8f\sec^{\frac{3}{2}}(e+fx)}$$

input
```
Integrate[Csc[e + f*x]^3*(b*Sec[e + f*x])^(3/2),x]
```

output
```
-1/8*((10*ArcTan[Sqrt[Sec[e + f*x]]] - 5*Log[1 - Sqrt[Sec[e + f*x]]] + 5*L
og[1 + Sqrt[Sec[e + f*x]]] + 4*(-5 + Csc[e + f*x]^2)*Sqrt[Sec[e + f*x]])*(
b*Sec[e + f*x])^(3/2))/(f*Sec[e + f*x]^(3/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3102, 27, 252, 262, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(e+fx)(b\sec(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc(e+fx)^3(b\sec(e+fx))^{3/2}dx$$

$$\Big\downarrow \text{ 3102}$$

$$\frac{\int \frac{b^4(b\sec(e+fx))^{7/2}}{\left(b^2-b^2\sec^2(e+fx)\right)^2}d(b\sec(e+fx))}{b^3f}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{b\int \frac{(b\sec(e+fx))^{7/2}}{\left(b^2-b^2\sec^2(e+fx)\right)^2}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{b\left(\frac{(b\sec(e+fx))^{5/2}}{2(b^2-b^2\sec^2(e+fx))}-\frac{5}{4}\int \frac{(b\sec(e+fx))^{3/2}}{b^2-b^2\sec^2(e+fx)}d(b\sec(e+fx))\right)}{f}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{b\left(\frac{(b\sec(e+fx))^{5/2}}{2(b^2-b^2\sec^2(e+fx))}-\frac{5}{4}\left(b^2\int \frac{1}{\sqrt{b\sec(e+fx)}(b^2-b^2\sec^2(e+fx))}d(b\sec(e+fx))-2\sqrt{b\sec(e+fx)}\right)\right)}{f}$$

$$\Big\downarrow \text{266}$$

$$\frac{b\left(\frac{(b\sec(e+fx))^{5/2}}{2(b^2-b^2\sec^2(e+fx))}-\frac{5}{4}\left(2b^2\int \frac{1}{b^2-b^4\sec^4(e+fx)}d\sqrt{b\sec(e+fx)}-2\sqrt{b\sec(e+fx)}\right)\right)}{f}$$

$$\Big\downarrow \text{756}$$

$$\frac{b\left(\frac{(b\sec(e+fx))^{5/2}}{2(b^2-b^2\sec^2(e+fx))}-\frac{5}{4}\left(2b^2\left(\frac{\int \frac{1}{b-b^2\sec^2(e+fx)}d\sqrt{b\sec(e+fx)}}{2b}+\frac{\int \frac{1}{b^2\sec^2(e+fx)+b}d\sqrt{b\sec(e+fx)}}{2b}\right)-2\sqrt{b\sec(e+fx)}\right)\right)}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{b\left(\frac{(b\sec(e+fx))^{5/2}}{2(b^2-b^2\sec^2(e+fx))}-\frac{5}{4}\left(2b^2\left(\frac{\int \frac{1}{b-b^2\sec^2(e+fx)}d\sqrt{b\sec(e+fx)}}{2b}+\frac{\arctan\left(\sqrt{b}\sec(e+fx)\right)}{2b^{3/2}}\right)-2\sqrt{b\sec(e+fx)}\right)\right)}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{b\left(\frac{(b\sec(e+fx))^{5/2}}{2(b^2-b^2\sec^2(e+fx))}-\frac{5}{4}\left(2b^2\left(\frac{\arctan\left(\sqrt{b}\sec(e+fx)\right)}{2b^{3/2}}+\frac{\text{arctanh}\left(\sqrt{b}\sec(e+fx)\right)}{2b^{3/2}}\right)-2\sqrt{b\sec(e+fx)}\right)\right)}{f}$$

input
```
Int[Csc[e + f*x]^3*(b*Sec[e + f*x])^(3/2),x]
```

output
```
(b*((b*Sec[e + f*x])^(5/2)/(2*(b^2 - b^2*Sec[e + f*x]^2)) - (5*(2*b^2*(Arc
Tan[Sqrt[b]*Sec[e + f*x]]/(2*b^(3/2)) + ArcTanh[Sqrt[b]*Sec[e + f*x]]/(2*b
^(3/2))) - 2*Sqrt[b*Sec[e + f*x]]))/4))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(89) = 178$.

Time = 2.34 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.61

method	result
default	$-\frac{b\left(4\left(-4+5\cos(fx+e)^2\right)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+\cos(fx+e)(\cos(fx+e)-1)\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}}{\cos(fx+e)+1}\right.\right.}{}$

input
```
int(csc(f*x+e)^3*(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/f*b*(4*(-4+5*cos(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+cos(f
*x+e)*(cos(f*x+e)-1)*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))+4*co
s(f*x+e)*(cos(f*x+e)-1)*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))
+5*cos(f*x+e)*(cos(f*x+e)-1)*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)))*(b*sec(f*x+e))^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*csc(f*x+e)^
2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(89) = 178$.

Time = 0.15 (sec) , antiderivative size = 407, normalized size of antiderivative = 3.60

$$\int \csc^3(e+fx)(b\sec(e+fx))^{3/2}\,dx = \left[\frac{10\,\left(b\cos\left(fx+e\right)^2-b\right)\sqrt{-b}\arctan\left(\frac{2\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)}{b\cos(fx+e)+b}\right)+5\,\left(b\cos\left(fx+e\right)^2-b\right)\sqrt{-}}{16\,(}\right.$$

$$-\frac{10\,\left(b\cos\left(fx+e\right)^2-b\right)\sqrt{b}\arctan\left(\frac{2\sqrt{b}\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)}{b\cos(fx+e)-b}\right)-5\,\left(b\cos\left(fx+e\right)^2-b\right)\sqrt{b}\log\left(\frac{b\cos(fx+e)^2-4}{}\right.}{16\,\left(f\cos\left(fx+e\right)^2-\right.}$$

input
```
integrate(csc(f*x+e)^3*(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(10*(b*cos(f*x + e)^2 - b)*sqrt(-b)*arctan(2*sqrt(-b)*sqrt(b/cos(f*x
 + e))*cos(f*x + e)/(b*cos(f*x + e) + b)) + 5*(b*cos(f*x + e)^2 - b)*sqrt(
-b)*log((b*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 - cos(f*x + e))*sqrt(-b)*sqr
t(b/cos(f*x + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 + 2*cos(f*x + e)
 + 1)) + 8*(5*b*cos(f*x + e)^2 - 4*b)*sqrt(b/cos(f*x + e)))/(f*cos(f*x + e
)^2 - f), -1/16*(10*(b*cos(f*x + e)^2 - b)*sqrt(b)*arctan(2*sqrt(b)*sqrt(b
/cos(f*x + e))*cos(f*x + e)/(b*cos(f*x + e) - b)) - 5*(b*cos(f*x + e)^2 -
b)*sqrt(b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(
b)*sqrt(b/cos(f*x + e)) + 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*
x + e) + 1)) - 8*(5*b*cos(f*x + e)^2 - 4*b)*sqrt(b/cos(f*x + e)))/(f*cos(f
*x + e)^2 - f)]
```

Sympy [F(-1)]

Timed out.

$$\int \csc^3(e+fx)(b\sec(e+fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**3*(b*sec(f*x+e))**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.09

$$\int \csc^3(e+fx)(b\sec(e+fx))^{3/2}\,dx = \frac{\left(\frac{4b^2\sqrt{\frac{b}{\cos(fx+e)}}}{b^2-\frac{b^2}{\cos(fx+e)^2}} - 10\sqrt{b}\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}}\right) + 5\sqrt{b}\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos(fx+e)}}}\right) + 16\sqrt{\frac{b}{\cos(fx+e)}}\right)b}{8f}$$

input
```
integrate(csc(f*x+e)^3*(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
1/8*(4*b^2*sqrt(b/cos(f*x + e))/(b^2 - b^2/cos(f*x + e)^2) - 10*sqrt(b)*ar
ctan(sqrt(b/cos(f*x + e))/sqrt(b)) + 5*sqrt(b)*log(-(sqrt(b) - sqrt(b/cos(
f*x + e)))/(sqrt(b) + sqrt(b/cos(f*x + e)))) + 16*sqrt(b/cos(f*x + e)))*b/
f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.12

$$\int \csc^3(e+fx)(b\sec(e+fx))^{3/2}\,dx = \frac{b^6\left(\frac{5\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b}b^4} + \frac{5\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{b}}\right)}{b^{\frac{9}{2}}} + \frac{2\left(5b^2\cos(fx+e)^2-4b^2\right)}{\left(\sqrt{b\cos(fx+e)}b^2\cos(fx+e)^2-\sqrt{b\cos(fx+e)}b^2\right)b^4}\right)\mathrm{sgn}(\cdots}{4f}$$

input
```
integrate(csc(f*x+e)^3*(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
1/4*b^6*(5*arctan(sqrt(b*cos(f*x + e))/sqrt(-b))/(sqrt(-b)*b^4) + 5*arctan
(sqrt(b*cos(f*x + e))/sqrt(b))/b^(9/2) + 2*(5*b^2*cos(f*x + e)^2 - 4*b^2)/
((sqrt(b*cos(f*x + e))*b^2*cos(f*x + e)^2 - sqrt(b*cos(f*x + e))*b^2)*b^4)
)*sgn(cos(f*x + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(e+fx)(b\sec(e+fx))^{3/2}\,dx=\int \frac{\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}}{\sin\left(e+f\,x\right)^3}\,dx$$

input
```
int((b/cos(e + f*x))^(3/2)/sin(e + f*x)^3,x)
```

output
```
int((b/cos(e + f*x))^(3/2)/sin(e + f*x)^3, x)
```

Reduce [F]

$$\int \csc^3(e+fx)(b\sec(e+fx))^{3/2}\,dx=\sqrt{b}\left(\int \sqrt{\sec\left(fx+e\right)}\,\csc\left(fx+e\right)^3\sec\left(fx+e\right)dx\right)b$$

input
```
int(csc(f*x+e)^3*(b*sec(f*x+e))^(3/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(e + f*x))*csc(e + f*x)**3*sec(e + f*x),x)*b
```

3.391 $\int (b\sec(e+fx))^{3/2}\sin^6(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 128

$$\int (b\sec(e+fx))^{3/2}\sin^6(e+fx)\,dx = -\frac{16b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{3f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} + \frac{8b^3\sin(e+fx)}{3f(b\sec(e+fx))^{3/2}} + \frac{20b^3\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}} + \frac{2b\sqrt{b\sec(e+fx)}\sin^5(e+fx)}{f}$$

output
```
-16/3*b^2*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)/(b*sec(
f*x+e))^(1/2)+8/3*b^3*sin(f*x+e)/f/(b*sec(f*x+e))^(3/2)+20/9*b^3*sin(f*x+e
)^3/f/(b*sec(f*x+e))^(3/2)+2*b*(b*sec(f*x+e))^(1/2)*sin(f*x+e)^5/f
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.55

$$\int (b\sec(e+fx))^{3/2}\sin^6(e+fx)\,dx = -\frac{b\sqrt{b\sec(e+fx)}\left(384\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right) - 158\sin(e+fx) - 13\sin(3(e+fx)) + \sin(5(e+\right.}{72f}$$

input
```
Integrate[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^6,x]
```

output
```
-1/72*(b*Sqrt[b*Sec[e + f*x]]*(384*Sqrt[Cos[e + f*x]]*EllipticE[(e + f*x)/
2, 2] - 158*Sin[e + f*x] - 13*Sin[3*(e + f*x)] + Sin[5*(e + f*x)]))/f
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3104, 3042, 3107, 3042, 3107, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^6(e+fx)(b\sec(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\sec(e+fx))^{3/2}}{\csc(e+fx)^6}dx$$

$$\Big\downarrow \text{3104}$$

$$\frac{2b\sin^5(e+fx)\sqrt{b\sec(e+fx)}}{f}-10b^2\int \frac{\sin^4(e+fx)}{\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\sin^5(e+fx)\sqrt{b\sec(e+fx)}}{f}-10b^2\int \frac{1}{\csc(e+fx)^4\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3107}$$

$$\frac{2b\sin^5(e+fx)\sqrt{b\sec(e+fx)}}{f}-10b^2\left(\frac{2}{3}\int \frac{\sin^2(e+fx)}{\sqrt{b\sec(e+fx)}}dx-\frac{2b\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\sin^5(e+fx)\sqrt{b\sec(e+fx)}}{f}-10b^2\left(\frac{2}{3}\int \frac{1}{\csc(e+fx)^2\sqrt{b\sec(e+fx)}}dx-\frac{2b\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \text{3107}$$

$$\frac{2b\sin^5(e+fx)\sqrt{b\sec(e+fx)}}{f}-$$
$$10b^2\left(\frac{2}{3}\left(\frac{2}{5}\int\frac{1}{\sqrt{b\sec(e+fx)}}dx-\frac{2b\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}\right)-\frac{2b\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}}\right)$$

$\Big\downarrow$ 3042

$$\frac{2b\sin^5(e+fx)\sqrt{b\sec(e+fx)}}{f}-$$
$$10b^2\left(\frac{2}{3}\left(\frac{2}{5}\int\frac{1}{\sqrt{b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{2b\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}\right)-\frac{2b\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}}\right)$$

$\Big\downarrow$ 4258

$$\frac{2b\sin^5(e+fx)\sqrt{b\sec(e+fx)}}{f}-$$
$$10b^2\left(\frac{2}{3}\left(\frac{2\int\sqrt{\cos(e+fx)}dx}{5\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{2b\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}\right)-\frac{2b\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}}\right)$$

$\Big\downarrow$ 3042

$$\frac{2b\sin^5(e+fx)\sqrt{b\sec(e+fx)}}{f}-$$
$$10b^2\left(\frac{2}{3}\left(\frac{2\int\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{2b\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}\right)-\frac{2b\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}}\right)$$

$\Big\downarrow$ 3119

$$\frac{2b\sin^5(e+fx)\sqrt{b\sec(e+fx)}}{f}-$$
$$10b^2\left(\frac{2}{3}\left(\frac{4E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{2b\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}\right)-\frac{2b\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}}\right)$$

input
```
Int[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^6,x]
```

output
```
(2*b*Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^5)/f - 10*b^2*((-2*b*Sin[e + f*x]^3
)/(9*f*(b*Sec[e + f*x])^(3/2)) + (2*((4*EllipticE[(e + f*x)/2, 2])/(5*f*Sq
rt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]]) - (2*b*Sin[e + f*x])/(5*f*(b*Sec[e
+ f*x])^(3/2))))/3)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3104
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)/
(f*a*(n - 1))), x] + Simp[b^2*((m + 1)/(a^2*(n - 1)))   Int[(a*Csc[e + f*x]
)^(m + 2)*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[n, 1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3107
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Simp[(m + 1)/(a^2*(m + n))   Int[(a*Csc[e + f*x])^(m
+ 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.04 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.79

method	result
default	$-\frac{2b\left(\left(\cos(fx+e)^5+\cos(fx+e)^4-4\cos(fx+e)^3-4\cos(fx+e)^2+15\cos(fx+e)-9\right)\sin(fx+e)+24i\left(\cos(fx+e)^2+2\cos(fx+e)+1\right)}{}$

input
```
int((b*sec(f*x+e))^(3/2)*sin(f*x+e)^6,x,method=_RETURNVERBOSE)
```

output
```
-2/9/f*b*((cos(f*x+e)^5+cos(f*x+e)^4-4*cos(f*x+e)^3-4*cos(f*x+e)^2+15*cos(
f*x+e)-9)*sin(f*x+e)+24*I*(cos(f*x+e)^2+2*cos(f*x+e)+1)*(1/(cos(f*x+e)+1))
^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e
)),I)+24*I*(-cos(f*x+e)^2-2*cos(f*x+e)-1)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)*(1/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cot(f*x+e)-csc(f*x+e)),I))*(b*sec(
f*x+e))^(1/2)/(cos(f*x+e)+1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.84

$$\int (b\sec(e+fx))^{3/2}\sin^6(e+fx)\,dx = -\frac{2\left(12i\sqrt{2}b^{\frac{3}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)))-12i\sqrt{2}b^{\frac{3}{2}}\right.}{}$$

input
```
integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^6,x, algorithm="fricas")
```

output
```
-2/9*(12*I*sqrt(2)*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(f*x + e) + I*sin(f*x + e))) - 12*I*sqrt(2)*b^(3/2)*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) + (b*cos
(f*x + e)^4 - 4*b*cos(f*x + e)^2 - 9*b)*sqrt(b/cos(f*x + e))*sin(f*x + e))
/f
```

Sympy [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{3/2}\sin^6(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((b*sec(f*x+e))**(3/2)*sin(f*x+e)**6,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\sec(e+fx))^{3/2}\sin^6(e+fx)\,dx=\int (b\sec(fx+e))^{\frac{3}{2}}\sin(fx+e)^6\,dx$$

input
```
integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^6,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^(3/2)*sin(f*x + e)^6, x)
```

Giac [F]

$$\int (b\sec(e+fx))^{3/2}\sin^6(e+fx)\,dx=\int (b\sec(fx+e))^{\frac{3}{2}}\sin(fx+e)^6\,dx$$

input
```
integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^6,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^(3/2)*sin(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{3/2}\sin^6(e+fx)\,dx=\int \sin(e+f\,x)^6\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}dx$$

input
```
int(sin(e + f*x)^6*(b/cos(e + f*x))^(3/2),x)
```

output
```
int(sin(e + f*x)^6*(b/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int (b\sec(e+fx))^{3/2}\sin^6(e+fx)\,dx=\sqrt{b}\left(\int \sqrt{\sec(fx+e)}\,\sec(fx+e)\sin(fx+e)^6\,dx\right)b$$

input
```
int((b*sec(f*x+e))^(3/2)*sin(f*x+e)^6,x)
```

output
```
sqrt(b)*int(sqrt(sec(e + f*x))*sec(e + f*x)*sin(e + f*x)**6,x)*b
```

3.392 $\int (b\sec(e+fx))^{3/2}\sin^4(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int (b\sec(e+fx))^{3/2}\sin^4(e+fx)\,dx = -\frac{24b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} + \frac{12b^3\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}} + \frac{2b\sqrt{b\sec(e+fx)}\sin^3(e+fx)}{f}$$

output
```
-24/5*b^2*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)/(b*sec(
f*x+e))^(1/2)+12/5*b^3*sin(f*x+e)/f/(b*sec(f*x+e))^(3/2)+2*b*(b*sec(f*x+e)
)^(1/2)*sin(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.61

$$\int (b\sec(e+fx))^{3/2}\sin^4(e+fx)\,dx = \frac{b\sqrt{b\sec(e+fx)}\left(-48\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)+21\sin(e+fx)+\sin(3(e+fx))\right)}{10f}$$

input
```
Integrate[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^4,x]
```

output
```
(b*Sqrt[b*Sec[e + f*x]]*(-48*Sqrt[Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2]
+ 21*Sin[e + f*x] + Sin[3*(e + f*x)]))/(10*f)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3104, 3042, 3107, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(e+fx)(b\sec(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\sec(e+fx))^{3/2}}{\csc(e+fx)^4}dx$$

$$\Big\downarrow \text{3104}$$

$$\frac{2b\sin^3(e+fx)\sqrt{b\sec(e+fx)}}{f}-6b^2\int \frac{\sin^2(e+fx)}{\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\sin^3(e+fx)\sqrt{b\sec(e+fx)}}{f}-6b^2\int \frac{1}{\csc(e+fx)^2\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3107}$$

$$\frac{2b\sin^3(e+fx)\sqrt{b\sec(e+fx)}}{f}-6b^2\left(\frac{2}{5}\int \frac{1}{\sqrt{b\sec(e+fx)}}dx-\frac{2b\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\sin^3(e+fx)\sqrt{b\sec(e+fx)}}{f}-6b^2\left(\frac{2}{5}\int \frac{1}{\sqrt{b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{2b\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \text{4258}$$

$$\frac{2b\sin^3(e+fx)\sqrt{b\sec(e+fx)}}{f}-6b^2\left(\frac{2\int \sqrt{\cos(e+fx)}dx}{5\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{2b\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\sin^3(e+fx)\sqrt{b\sec(e+fx)}}{f} - 6b^2\left(\frac{2\int\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{2b\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \text{3119}$$

$$\frac{2b\sin^3(e+fx)\sqrt{b\sec(e+fx)}}{f} - 6b^2\left(\frac{4E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{2b\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}\right)$$

input
```
Int[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^4,x]
```

output
```
(2*b*Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^3)/f - 6*b^2*((4*EllipticE[(e + f*x
)/2, 2])/(5*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]]) - (2*b*Sin[e + f*x]
)/(5*f*(b*Sec[e + f*x])^(3/2)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3104
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)/
(f*a*(n - 1))), x] + Simp[b^2*((m + 1)/(a^2*(n - 1)))   Int[(a*Csc[e + f*x]
)^(m + 2)*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[n, 1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3107
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Simp[(m + 1)/(a^2*(m + n))   Int[(a*Csc[e + f*x])^(m
+ 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.79 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.17

method	result
default	$-\frac{2b\left(\left(-\cos(fx+e)^3-\cos(fx+e)^2+7\cos(fx+e)-5\right)\sin(fx+e)+12i\left(\cos(fx+e)^2+2\cos(fx+e)+1\right)\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+}}}{}$

input
```
int((b*sec(f*x+e))^(3/2)*sin(f*x+e)^4,x,method=_RETURNVERBOSE)
```

output
```
-2/5/f*b*((-cos(f*x+e)^3-cos(f*x+e)^2+7*cos(f*x+e)-5)*sin(f*x+e)+12*I*(cos
(f*x+e)^2+2*cos(f*x+e)+1)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)+12*I*(-cos(f*x+e)^2-2*co
s(f*x+e)-1)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*Ell
ipticE(I*(cot(f*x+e)-csc(f*x+e)),I))*(b*sec(f*x+e))^(1/2)/(cos(f*x+e)+1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int (b\sec(e+fx))^{3/2}\sin^4(e+fx)\,dx =$$
$$-\frac{2\left(6i\sqrt{2}b^{\frac{3}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)))-6i\sqrt{2}b^{\frac{3}{2}}\text{w}\right.}{}$$

input
```
integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^4,x, algorithm="fricas")
```

output
```
-2/5*(6*I*sqrt(2)*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(f*x + e) + I*sin(f*x + e))) - 6*I*sqrt(2)*b^(3/2)*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) - (b*cos(f
*x + e)^2 + 5*b)*sqrt(b/cos(f*x + e))*sin(f*x + e))/f
```

Sympy [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{3/2}\sin^4(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((b*sec(f*x+e))**(3/2)*sin(f*x+e)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\sec(e+fx))^{3/2}\sin^4(e+fx)\,dx=\int (b\sec(fx+e))^{\frac{3}{2}}\sin(fx+e)^4\,dx$$

input
```
integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^(3/2)*sin(f*x + e)^4, x)
```

Giac [F]

$$\int (b\sec(e+fx))^{3/2}\sin^4(e+fx)\,dx=\int (b\sec(fx+e))^{\frac{3}{2}}\sin(fx+e)^4\,dx$$

input
```
integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^(3/2)*sin(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{3/2}\sin^4(e+fx)\,dx=\int \sin(e+f\,x)^4\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}dx$$

input
```
int(sin(e + f*x)^4*(b/cos(e + f*x))^(3/2),x)
```

output
```
int(sin(e + f*x)^4*(b/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int (b\sec(e+fx))^{3/2}\sin^4(e+fx)\,dx = \sqrt{b}\left(\int \sqrt{\sec(fx+e)}\,\sec(fx+e)\sin(fx+e)^4\,dx\right)b$$

input
```
int((b*sec(f*x+e))^(3/2)*sin(f*x+e)^4,x)
```

output
```
sqrt(b)*int(sqrt(sec(e + f*x))*sec(e + f*x)*sin(e + f*x)**4,x)*b
```

3.393 $\int (b\sec(e+fx))^{3/2}\sin^2(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 66

$$\int (b\sec(e+fx))^{3/2}\sin^2(e+fx)\,dx = -\frac{4b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}+\frac{2b\sqrt{b\sec(e+fx)}\sin(e+fx)}{f}$$

output
```
-4*b^2*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)/(b*sec(f*x
+e))^(1/2)+2*b*(b*sec(f*x+e))^(1/2)*sin(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.73

$$\int (b\sec(e+fx))^{3/2}\sin^2(e+fx)\,dx = \frac{2b\sqrt{b\sec(e+fx)}\left(-2\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)+\sin(e+fx)\right)}{f}$$

input
```
Integrate[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^2,x]
```

output
```
(2*b*Sqrt[b*Sec[e + f*x]]*(-2*Sqrt[Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2]
 + Sin[e + f*x]))/f
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3104, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(e+fx)(b\sec(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\sec(e+fx))^{3/2}}{\csc(e+fx)^2}dx$$

$$\Big\downarrow \text{3104}$$

$$\frac{2b\sin(e+fx)\sqrt{b\sec(e+fx)}}{f}-2b^2\int \frac{1}{\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\sin(e+fx)\sqrt{b\sec(e+fx)}}{f}-2b^2\int \frac{1}{\sqrt{b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4258}$$

$$\frac{2b\sin(e+fx)\sqrt{b\sec(e+fx)}}{f}-\frac{2b^2\int \sqrt{\cos(e+fx)}dx}{\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\sin(e+fx)\sqrt{b\sec(e+fx)}}{f}-\frac{2b^2\int \sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2b\sin(e+fx)\sqrt{b\sec(e+fx)}}{f}-\frac{4b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

input
```
Int[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^2,x]
```

output
```
(-4*b^2*EllipticE[(e + f*x)/2, 2])/(f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*
x]]) + (2*b*Sqrt[b*Sec[e + f*x]]*Sin[e + f*x])/f
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3104
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)/
(f*a*(n - 1))), x] + Simp[b^2*((m + 1)/(a^2*(n - 1)))   Int[(a*Csc[e + f*x]
)^(m + 2)*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[n, 1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.54 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.92

method	result
default	$-\frac{2\left((\cos(fx+e)-1)\sin(fx+e)+i\left(2\cos(fx+e)^2+4\cos(fx+e)+2\right)\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\ \text{EllipticF}(i(\cot(fx+e)-\csc(fx+\right.}{f(\cos(f}$

input
```
int((b*sec(f*x+e))^(3/2)*sin(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/f*((cos(f*x+e)-1)*sin(f*x+e)+I*(2*cos(f*x+e)^2+4*cos(f*x+e)+2)*(1/(cos(
f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)
-csc(f*x+e)),I)+I*(-2*cos(f*x+e)^2-4*cos(f*x+e)-2)*(cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cot(f*x+e)-csc(f*x+e)),I)
)*b*(b*sec(f*x+e))^(1/2)/(cos(f*x+e)+1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.29

$$\int (b\sec(e+fx))^{3/2}\sin^2(e+fx)\,dx =$$
$$-\frac{2\left(i\sqrt{2}b^{\frac{3}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)))-i\sqrt{2}b^{\frac{3}{2}}\text{weie}\right.}{}$$

input
```
integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^2,x, algorithm="fricas")
```

output
```
-2*(I*sqrt(2)*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, co
s(f*x + e) + I*sin(f*x + e))) - I*sqrt(2)*b^(3/2)*weierstrassZeta(-4, 0, w
eierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) - b*sqrt(b/cos(f
*x + e))*sin(f*x + e))/f
```

Sympy [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{3/2}\sin^2(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((b*sec(f*x+e))**(3/2)*sin(f*x+e)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\sec(e+fx))^{3/2}\sin^2(e+fx)\,dx = \int (b\sec(fx+e))^{\frac{3}{2}}\sin(fx+e)^2\,dx$$

input
```
integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^(3/2)*sin(f*x + e)^2, x)
```

Giac [F]

$$\int (b\sec(e+fx))^{3/2}\sin^2(e+fx)\,dx = \int (b\sec(fx+e))^{\frac{3}{2}}\sin(fx+e)^2\,dx$$

input
```
integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^(3/2)*sin(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{3/2}\sin^2(e+fx)\,dx = \int \sin(e+f\,x)^2\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}\,dx$$

input
```
int(sin(e + f*x)^2*(b/cos(e + f*x))^(3/2),x)
```

output
```
int(sin(e + f*x)^2*(b/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int (b\sec(e+fx))^{3/2}\sin^2(e+fx)\,dx = \sqrt{b}\left(\int \sqrt{\sec(fx+e)}\,\sec(fx+e)\sin(fx+e)^2\,dx\right)b$$

input
```
int((b*sec(f*x+e))^(3/2)*sin(f*x+e)^2,x)
```

output
```
sqrt(b)*int(sqrt(sec(e + f*x))*sec(e + f*x)*sin(e + f*x)**2,x)*b
```

3.394 $\int (b\sec(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 66

$$\int (b\sec(e+fx))^{3/2}\,dx = -\frac{2b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}+\frac{2b\sqrt{b\sec(e+fx)}\sin(e+fx)}{f}$$

output
```
-2*b^2*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)/(b*sec(f*x
+e))^(1/2)+2*b*(b*sec(f*x+e))^(1/2)*sin(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.73

$$\int (b\sec(e+fx))^{3/2}\,dx = \frac{2b\sqrt{b\sec(e+fx)}\left(-\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)+\sin(e+fx)\right)}{f}$$

input
```
Integrate[(b*Sec[e + f*x])^(3/2),x]
```

output
```
(2*b*Sqrt[b*Sec[e + f*x]]*(-(Sqrt[Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])
 + Sin[e + f*x]))/f
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\sec(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{2b\sin(e+fx)\sqrt{b\sec(e+fx)}}{f}-b^2\int \frac{1}{\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\sin(e+fx)\sqrt{b\sec(e+fx)}}{f}-b^2\int \frac{1}{\sqrt{b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4258}$$

$$\frac{2b\sin(e+fx)\sqrt{b\sec(e+fx)}}{f}-\frac{b^2\int \sqrt{\cos(e+fx)}dx}{\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\sin(e+fx)\sqrt{b\sec(e+fx)}}{f}-\frac{b^2\int \sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2b\sin(e+fx)\sqrt{b\sec(e+fx)}}{f}-\frac{2b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

input
```
Int[(b*Sec[e + f*x])^(3/2),x]
```

output
```
(-2*b^2*EllipticE[(e + f*x)/2, 2])/(f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*
x]]) + (2*b*Sqrt[b*Sec[e + f*x]]*Sin[e + f*x])/f
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.20 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.79

method	result
default	$-\frac{2\left(i\left(\cos(fx+e)^2+2\cos(fx+e)+1\right)\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\ \mathrm{EllipticF}(i(\cot(fx+e)-\csc(fx+e)),i)+i\left(-\cos(fx+e)^2-2\,\mathrm{co}\right.\right.}{f(\cos(fx+e)+1)}$

input
```
int((b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/f*(I*(cos(f*x+e)^2+2*cos(f*x+e)+1)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)
/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)+I*(-cos(f*x+
e)^2-2*cos(f*x+e)-1)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^
(1/2)*EllipticE(I*(cot(f*x+e)-csc(f*x+e)),I)-sin(f*x+e))*b*(b*sec(f*x+e))^
(1/2)/(cos(f*x+e)+1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.27

$$\int (b\sec(e+fx))^{3/2}\,dx = \frac{-i\sqrt{2}b^{\frac{3}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)))+$$

input
```
integrate((b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(
f*x + e) + I*sin(f*x + e))) + I*sqrt(2)*b^(3/2)*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) + 2*b*sqrt(b/cos(f
*x + e))*sin(f*x + e))/f
```

Sympy [F]

$$\int (b\sec(e+fx))^{3/2}\,dx = \int (b\sec(e+fx))^{\frac{3}{2}}\,dx$$

input
```
integrate((b*sec(f*x+e))**(3/2),x)
```

output
```
Integral((b*sec(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int (b\sec(e+fx))^{3/2}\,dx = \int (b\sec(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^(3/2), x)
```

Giac [F]

$$\int (b\sec(e+fx))^{3/2}\,dx = \int (b\sec(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{3/2}\,dx = \int \left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}\,dx$$

input
```
int((b/cos(e + f*x))^(3/2),x)
```

output
```
int((b/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int (b\sec(e+fx))^{3/2}\,dx = \sqrt{b}\left(\int \sqrt{\sec\left(fx+e\right)}\,\sec\left(fx+e\right)dx\right)b$$

input
```
int((b*sec(f*x+e))^(3/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(e + f*x))*sec(e + f*x),x)*b
```

3.395 $\int \csc^2(e+fx)(b\sec(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 90

$$\int \csc^2(e+fx)(b\sec(e+fx))^{3/2}\,dx = -\frac{3b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{b\csc(e+fx)\sqrt{b\sec(e+fx)}}{f} + \frac{3b\sqrt{b\sec(e+fx)}\sin(e+fx)}{f}$$

output
```
-3*b^2*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)/(b*sec(f*x
+e))^(1/2)-b*csc(f*x+e)*(b*sec(f*x+e))^(1/2)/f+3*b*(b*sec(f*x+e))^(1/2)*si
n(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.63

$$\int \csc^2(e+fx)(b\sec(e+fx))^{3/2}\,dx = \frac{b\sqrt{b\sec(e+fx)}\left(-\csc(e+fx)-3\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)+3\sin(e+fx)\right)}{f}$$

input
```
Integrate[Csc[e + f*x]^2*(b*Sec[e + f*x])^(3/2),x]
```

output
```
(b*Sqrt[b*Sec[e + f*x]]*(-Csc[e + f*x] - 3*Sqrt[Cos[e + f*x]]*EllipticE[(e
 + f*x)/2, 2] + 3*Sin[e + f*x]))/f
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3105, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(e+fx)(b\sec(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(e+fx)^2(b\sec(e+fx))^{3/2}dx$$

$$\Big\downarrow \text{3105}$$

$$\frac{3}{2}\int (b\sec(e+fx))^{3/2}dx-\frac{b\csc(e+fx)\sqrt{b\sec(e+fx)}}{f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}\int \left(b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx-\frac{b\csc(e+fx)\sqrt{b\sec(e+fx)}}{f}$$

$$\Big\downarrow \text{4255}$$

$$\frac{3}{2}\left(\frac{2b\sin(e+fx)\sqrt{b\sec(e+fx)}}{f}-b^2\int \frac{1}{\sqrt{b\sec(e+fx)}}dx\right)-\frac{b\csc(e+fx)\sqrt{b\sec(e+fx)}}{f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}\left(\frac{2b\sin(e+fx)\sqrt{b\sec(e+fx)}}{f}-b^2\int \frac{1}{\sqrt{b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx\right)-\frac{b\csc(e+fx)\sqrt{b\sec(e+fx)}}{f}$$

$$\Big\downarrow \text{4258}$$

$$\frac{3}{2}\left(\frac{2b\sin(e+fx)\sqrt{b\sec(e+fx)}}{f}-\frac{b^2\int\sqrt{\cos(e+fx)}dx}{\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}\right)-\frac{b\csc(e+fx)\sqrt{b\sec(e+fx)}}{f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}\left(\frac{2b\sin(e+fx)\sqrt{b\sec(e+fx)}}{f}-\frac{b^2\int\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}\right)-\frac{b\csc(e+fx)\sqrt{b\sec(e+fx)}}{f}$$

$$\Big\downarrow \text{3119}$$

$$\frac{3}{2}\left(\frac{2b\sin(e+fx)\sqrt{b\sec(e+fx)}}{f}-\frac{2b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}\right)-\frac{b\csc(e+fx)\sqrt{b\sec(e+fx)}}{f}$$

input
```
Int[Csc[e + f*x]^2*(b*Sec[e + f*x])^(3/2),x]
```

output
```
-((b*Csc[e + f*x]*Sqrt[b*Sec[e + f*x]])/f) + (3*((-2*b^2*EllipticE[(e + f*
x)/2, 2])/(f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]]) + (2*b*Sqrt[b*Sec[e
+ f*x]]*Sin[e + f*x])/f))/2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3105
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Simp[a^2*((m + n - 2)/(m - 1))   Int[(a*Csc[e + f*
x])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[
m, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.04 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.77

method	result
default	$\frac{\left(-3\cot(fx+e)+2\csc(fx+e)+3i(\cos(fx+e)+1)\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\mathrm{EllipticE}(i(\cot(fx+e)-\csc(fx+e)),i)-3i(\cos(fx+\right.}{f}$

input
```
int(csc(f*x+e)^2*(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-3*cot(f*x+e)+2*csc(f*x+e)+3*I*(cos(f*x+e)+1)*(1/(cos(f*x+e)+1))^(1/2
)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cot(f*x+e)-csc(f*x+e)),I)
-3*I*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(
f*x+e)+1)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*b*(b*sec(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.26

$$\int \csc^2(e+fx)(b\sec(e+fx))^{3/2}\,dx = \frac{-3i\sqrt{2}b^{\frac{3}{2}}\sin\left(fx+e\right)\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\sin\ldots$$

input
```
integrate(csc(f*x+e)^2*(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
1/2*(-3*I*sqrt(2)*b^(3/2)*sin(f*x + e)*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*I*sqrt(2)*b^(3/2)*sin(f
*x + e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I
*sin(f*x + e))) - 2*(3*b*cos(f*x + e)^2 - 2*b)*sqrt(b/cos(f*x + e)))/(f*si
n(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^2(e+fx)(b\sec(e+fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**2*(b*sec(f*x+e))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \csc^2(e+fx)(b\sec(e+fx))^{3/2}\,dx = \int (b\sec(fx+e))^{\frac{3}{2}}\csc(fx+e)^2\,dx$$

input
```
integrate(csc(f*x+e)^2*(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^(3/2)*csc(f*x + e)^2, x)
```

Giac [F]

$$\int \csc^2(e+fx)(b\sec(e+fx))^{3/2}\,dx = \int (b\sec(fx+e))^{\frac{3}{2}}\csc(fx+e)^2\,dx$$

input
```
integrate(csc(f*x+e)^2*(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^(3/2)*csc(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(e+fx)(b\sec(e+fx))^{3/2}\,dx = \int \frac{\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}}{\sin(e+f\,x)^2}\,dx$$

input
```
int((b/cos(e + f*x))^(3/2)/sin(e + f*x)^2,x)
```

output
```
int((b/cos(e + f*x))^(3/2)/sin(e + f*x)^2, x)
```

Reduce [F]

$$\int \csc^2(e + fx)(b\sec(e + fx))^{3/2}\,dx = \sqrt{b}\left(\int \sqrt{\sec(fx + e)}\,\csc(fx + e)^2\sec(fx + e)\,dx\right)b$$

input
```
int(csc(f*x+e)^2*(b*sec(f*x+e))^(3/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(e + f*x))*csc(e + f*x)**2*sec(e + f*x),x)*b
```

3.396 $\int \csc^4(e+fx)(b\sec(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 124

$$\int \csc^4(e+fx)(b\sec(e+fx))^{3/2}\,dx = -\frac{7b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{7b\csc(e+fx)\sqrt{b\sec(e+fx)}}{6f}-\frac{b\csc^3(e+fx)\sqrt{b\sec(e+fx)}}{3f}+\frac{7b\sqrt{b\sec(e+fx)}\sin(e+fx)}{2f}$$

output
```
-7/2*b^2*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)/(b*sec(f
*x+e))^(1/2)-7/6*b*csc(f*x+e)*(b*sec(f*x+e))^(1/2)/f-1/3*b*csc(f*x+e)^3*(b
*sec(f*x+e))^(1/2)/f+7/2*b*(b*sec(f*x+e))^(1/2)*sin(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.62

$$\int \csc^4(e+fx)(b\sec(e+fx))^{3/2}\,dx = -\frac{b\left(-21+7\csc^2(e+fx)+2\csc^4(e+fx)+21\sqrt{\cos(e+fx)}\csc(e+fx)E\left(\frac{1}{2}(e+fx)\middle|2\right)\right)\sqrt{b\sec(e+f}}{6f}$$

input
```
Integrate[Csc[e + f*x]^4*(b*Sec[e + f*x])^(3/2),x]
```

output
```
-1/6*(b*(-21 + 7*Csc[e + f*x]^2 + 2*Csc[e + f*x]^4 + 21*Sqrt[Cos[e + f*x]]
*Csc[e + f*x]*EllipticE[(e + f*x)/2, 2])*Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]
)/f
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3105, 3042, 3105, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(e+fx)(b\sec(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(e+fx)^4(b\sec(e+fx))^{3/2}dx$$

$$\Big\downarrow \text{3105}$$

$$\frac{7}{6}\int \csc^2(e+fx)(b\sec(e+fx))^{3/2}dx-\frac{b\csc^3(e+fx)\sqrt{b\sec(e+fx)}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{6}\int \csc(e+fx)^2(b\sec(e+fx))^{3/2}dx-\frac{b\csc^3(e+fx)\sqrt{b\sec(e+fx)}}{3f}$$

$$\Big\downarrow \text{3105}$$

$$\frac{7}{6}\left(\frac{3}{2}\int (b\sec(e+fx))^{3/2}dx-\frac{b\csc(e+fx)\sqrt{b\sec(e+fx)}}{f}\right)-\frac{b\csc^3(e+fx)\sqrt{b\sec(e+fx)}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{6}\left(\frac{3}{2}\int \left(b\csc \left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}dx-\frac{b\csc(e+fx)\sqrt{b\sec(e+fx)}}{f}\right)-\frac{b\csc^3(e+fx)\sqrt{b\sec(e+fx)}}{3f}$$

$\downarrow$ 4255

$$\frac{7}{6}\left(\frac{3}{2}\left(\frac{2b\sin(e+fx)\sqrt{b\sec(e+fx)}}{f}-b^2\int \frac{1}{\sqrt{b\sec(e+fx)}}dx\right)-\frac{b\csc(e+fx)\sqrt{b\sec(e+fx)}}{f}\right)-\frac{b\csc^3(e+fx)\sqrt{b\sec(e+fx)}}{3f}$$

$\downarrow$ 3042

$$\frac{7}{6}\left(\frac{3}{2}\left(\frac{2b\sin(e+fx)\sqrt{b\sec(e+fx)}}{f}-b^2\int \frac{1}{\sqrt{b\csc \left(e+fx+\frac{\pi}{2}\right)}}dx\right)-\frac{b\csc(e+fx)\sqrt{b\sec(e+fx)}}{f}\right)-\frac{b\csc^3(e+fx)\sqrt{b\sec(e+fx)}}{3f}$$

$\downarrow$ 4258

$$\frac{7}{6}\left(\frac{3}{2}\left(\frac{2b\sin(e+fx)\sqrt{b\sec(e+fx)}}{f}-\frac{b^2\int \sqrt{\cos(e+fx)}dx}{\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}\right)-\frac{b\csc(e+fx)\sqrt{b\sec(e+fx)}}{f}\right)-\frac{b\csc^3(e+fx)\sqrt{b\sec(e+fx)}}{3f}$$

$\downarrow$ 3042

$$\frac{7}{6}\left(\frac{3}{2}\left(\frac{2b\sin(e+fx)\sqrt{b\sec(e+fx)}}{f}-\frac{b^2\int \sqrt{\sin \left(e+fx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}\right)-\frac{b\csc(e+fx)\sqrt{b\sec(e+fx)}}{f}\right)-\frac{b\csc^3(e+fx)\sqrt{b\sec(e+fx)}}{3f}$$

$\downarrow$ 3119

$$\frac{7}{6}\left(\frac{3}{2}\left(\frac{2b\sin(e+fx)\sqrt{b\sec(e+fx)}}{f}-\frac{2b^2E\left(\frac{1}{2}(e+fx)|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}\right)-\frac{b\csc(e+fx)\sqrt{b\sec(e+fx)}}{f}\right)-\frac{b\csc^3(e+fx)\sqrt{b\sec(e+fx)}}{3f}$$

input
```
Int[Csc[e + f*x]^4*(b*Sec[e + f*x])^(3/2),x]
```

output
```
-1/3*(b*Csc[e + f*x]^3*Sqrt[b*Sec[e + f*x]])/f + (7*(-((b*Csc[e + f*x]*Sqr
t[b*Sec[e + f*x]])/f) + (3*((-2*b^2*EllipticE[(e + f*x)/2, 2])/(f*Sqrt[Cos
[e + f*x]]*Sqrt[b*Sec[e + f*x]]) + (2*b*Sqrt[b*Sec[e + f*x]]*Sin[e + f*x])
/f))/2))/6
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3105
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Simp[a^2*((m + n - 2)/(m - 1))   Int[(a*Csc[e + f*
x])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[
m, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.05 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.36

method	result
default	$\frac{\left(-\frac{\csc(fx+e)^3}{3}-\frac{7\cot(fx+e)}{2}+\frac{7\csc(fx+e)}{3}+\frac{7i(\cos(fx+e)+1)\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\text{EllipticE}(i(\cot(fx+e)-\csc(fx+e)),i)}{2}-\frac{7i(\cos(}{}\right.}{f}$

input
```
int(csc(f*x+e)^4*(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/3*csc(f*x+e)^3-7/2*cot(f*x+e)+7/3*csc(f*x+e)+7/2*I*(cos(f*x+e)+1)*
(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(co
t(f*x+e)-csc(f*x+e)),I)-7/2*I*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*(1/(c
os(f*x+e)+1))^(1/2)*(cos(f*x+e)+1)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*b*(b
*sec(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.35

$$\int \csc^4(e+fx)(b\sec(e+fx))^{3/2}\,dx =$$

$$-\frac{21\sqrt{2}\left(i\,b\cos\left(fx+e\right)^2-i\,b\right)\sqrt{b}\sin\left(fx+e\right)\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos\left(fx+\right.}{}$$

input
```
integrate(csc(f*x+e)^4*(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
-1/12*(21*sqrt(2)*(I*b*cos(f*x + e)^2 - I*b)*sqrt(b)*sin(f*x + e)*weierstr
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e)))
+ 21*sqrt(2)*(-I*b*cos(f*x + e)^2 + I*b)*sqrt(b)*sin(f*x + e)*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) + 2*
(21*b*cos(f*x + e)^4 - 35*b*cos(f*x + e)^2 + 12*b)*sqrt(b/cos(f*x + e)))/(
(f*cos(f*x + e)^2 - f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^4(e+fx)(b\sec(e+fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**4*(b*sec(f*x+e))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \csc^4(e+fx)(b\sec(e+fx))^{3/2}\,dx = \int (b\sec(fx+e))^{\frac{3}{2}}\csc(fx+e)^4\,dx$$

input
```
integrate(csc(f*x+e)^4*(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^(3/2)*csc(f*x + e)^4, x)
```

Giac [F]

$$\int \csc^4(e+fx)(b\sec(e+fx))^{3/2}\,dx = \int (b\sec(fx+e))^{\frac{3}{2}}\csc(fx+e)^4\,dx$$

input
```
integrate(csc(f*x+e)^4*(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^(3/2)*csc(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^4(e+fx)(b\sec(e+fx))^{3/2}\,dx = \int \frac{\left(\frac{b}{\cos(e+fx)}\right)^{3/2}}{\sin(e+fx)^4}\,dx$$

input
```
int((b/cos(e + f*x))^(3/2)/sin(e + f*x)^4,x)
```

output
```
int((b/cos(e + f*x))^(3/2)/sin(e + f*x)^4, x)
```

Reduce [F]

$$\int \csc^4(e+fx)(b\sec(e+fx))^{3/2}\,dx = \sqrt{b}\left(\int \sqrt{\sec(fx+e)}\,\csc(fx+e)^4\sec(fx+e)\,dx\right)b$$

input
```
int(csc(f*x+e)^4*(b*sec(f*x+e))^(3/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(e + f*x))*csc(e + f*x)**4*sec(e + f*x),x)*b
```

3.397 $\int (b\sec(e+fx))^{5/2}\sin^7(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 85

$$\int (b\sec(e+fx))^{5/2}\sin^7(e+fx)\,dx=\frac{2b^7}{9f(b\sec(e+fx))^{9/2}}-\frac{6b^5}{5f(b\sec(e+fx))^{5/2}}+\frac{6b^3}{f\sqrt{b\sec(e+fx)}}+\frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

output
```
2/9*b^7/f/(b*sec(f*x+e))^(9/2)-6/5*b^5/f/(b*sec(f*x+e))^(5/2)+6*b^3/f/(b*s
ec(f*x+e))^(1/2)+2/3*b*(b*sec(f*x+e))^(3/2)/f
```

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.61

$$\int (b\sec(e+fx))^{5/2}\sin^7(e+fx)\,dx=\frac{b(2366+1803\cos(2(e+fx))-78\cos(4(e+fx))+5\cos(6(e+fx)))(b\sec(e+fx))^{3/2}}{720f}$$

input
```
Integrate[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^7,x]
```

output
```
(b*(2366 + 1803*Cos[2*(e + f*x)] - 78*Cos[4*(e + f*x)] + 5*Cos[6*(e + f*x)
])*(b*Sec[e + f*x])^(3/2))/(720*f)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3102, 25, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^7(e+fx)(b\sec(e+fx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\sec(e+fx))^{5/2}}{\csc(e+fx)^7}dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{b^7\int -\frac{\left(b^2-b^2\sec^2(e+fx)\right)^3}{b^6(b\sec(e+fx))^{11/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b^7\int \frac{\left(b^2-b^2\sec^2(e+fx)\right)^3}{b^6(b\sec(e+fx))^{11/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\int \frac{\left(b^2-b^2\sec^2(e+fx)\right)^3}{(b\sec(e+fx))^{11/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{244}$$

$$-\frac{b\int \left(\frac{b^6}{(b\sec(e+fx))^{11/2}}-\frac{3b^4}{(b\sec(e+fx))^{7/2}}+\frac{3b^2}{(b\sec(e+fx))^{3/2}}-\sqrt{b\sec(e+fx)}\right)d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\left(-\frac{2b^6}{9(b\sec(e+fx))^{9/2}}+\frac{6b^4}{5(b\sec(e+fx))^{5/2}}-\frac{6b^2}{\sqrt{b\sec(e+fx)}}-\frac{2}{3}(b\sec(e+fx))^{3/2}\right)}{f}$$

input
```
Int[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^7,x]
```

output
```
-((b*((-2*b^6)/(9*(b*Sec[e + f*x])^(9/2)) + (6*b^4)/(5*(b*Sec[e + f*x])^(5
/2)) - (6*b^2)/Sqrt[b*Sec[e + f*x]] - (2*(b*Sec[e + f*x])^(3/2))/3))/f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(71) = 142$.

Time = 5.46 (sec) , antiderivative size = 264, normalized size of antiderivative = 3.11

$$\left(\frac{(135\cos(fx+e)+135)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\cos(fx+e)+1}{\cos(fx+e)+1}\right)}{90} + \frac{(-135\cos(fx+e)-135)\sqrt{}}{} \right.$$

input
```
int((b*sec(f*x+e))^(5/2)*sin(f*x+e)^7,x)
```

output
```
1/f*(1/90*(135*cos(f*x+e)+135)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln((2*
cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))+1/90*(-135*cos(f*x+e)-135)*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f
*x+e)+1))+2/9*cos(f*x+e)^5-6/5*cos(f*x+e)^3+6*cos(f*x+e)+2/3*sec(f*x+e))*(
b*sec(f*x+e))^(1/2)*b^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.82

$$\int (b\sec(e+fx))^{5/2}\sin^7(e+fx)\,dx = \frac{2\left(5b^2\cos(fx+e)^6-27b^2\cos(fx+e)^4+135b^2\cos(fx+e)^2+15b^2\right)\sqrt{\frac{b}{\cos(fx+e)}}}{45f\cos(fx+e)}$$

input
```
integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^7,x, algorithm="fricas")
```

output
```
2/45*(5*b^2*cos(f*x + e)^6 - 27*b^2*cos(f*x + e)^4 + 135*b^2*cos(f*x + e)^
2 + 15*b^2)*sqrt(b/cos(f*x + e))/(f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{5/2}\sin^7(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((b*sec(f*x+e))**(5/2)*sin(f*x+e)**7,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.78

$$\int (b\sec(e+fx))^{5/2}\sin^7(e+fx)\,dx = \frac{2\left(15\left(\frac{b}{\cos(fx+e)}\right)^{\frac{3}{2}} + \frac{5\,b^6 - \frac{27\,b^6}{\cos(fx+e)^2} + \frac{135\,b^6}{\cos(fx+e)^4}}{\left(\frac{b}{\cos(fx+e)}\right)^{\frac{9}{2}}}\right)b}{45\,f}$$

input
```
integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^7,x, algorithm="maxima")
```

output
```
2/45*(15*(b/cos(f*x + e))^(3/2) + (5*b^6 - 27*b^6/cos(f*x + e)^2 + 135*b^6
/cos(f*x + e)^4)/(b/cos(f*x + e))^(9/2))*b/f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.18

$$\int (b\sec(e+fx))^{5/2}\sin^7(e+fx)\,dx = \frac{2\left(5\sqrt{b\cos(fx+e)}b^4\cos(fx+e)^4 - 27\sqrt{b\cos(fx+e)}b^4\cos(fx+e)^2 + 135\sqrt{b\cos(fx+e)}\right.}{45\,b^2 f}$$

input
```
integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^7,x, algorithm="giac")
```

output
```
2/45*(5*sqrt(b*cos(f*x + e))*b^4*cos(f*x + e)^4 - 27*sqrt(b*cos(f*x + e))*
b^4*cos(f*x + e)^2 + 135*sqrt(b*cos(f*x + e))*b^4 + 15*b^5/(sqrt(b*cos(f*x
 + e))*cos(f*x + e)))*sgn(cos(f*x + e))/(b^2*f)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{5/2}\sin^7(e+fx)\,dx=\int \sin\left(e+f\,x\right)^7\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{5/2}\,dx$$

input
```
int(sin(e + f*x)^7*(b/cos(e + f*x))^(5/2),x)
```

output
```
int(sin(e + f*x)^7*(b/cos(e + f*x))^(5/2), x)
```

Reduce [F]

$$\int (b\sec(e+fx))^{5/2}\sin^7(e+fx)\,dx=\sqrt{b}\left(\int \sqrt{\sec\left(fx+e\right)}\,\sec\left(fx+e\right)^2\sin\left(fx+e\right)^7dx\right)b^2$$

input
```
int((b*sec(f*x+e))^(5/2)*sin(f*x+e)^7,x)
```

output
```
sqrt(b)*int(sqrt(sec(e + f*x))*sec(e + f*x)**2*sin(e + f*x)**7,x)*b**2
```

3.398 $\int (b\sec(e+fx))^{5/2}\sin^5(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 63

$$\int (b\sec(e+fx))^{5/2}\sin^5(e+fx)\,dx = -\frac{2b^5}{5f(b\sec(e+fx))^{5/2}} + \frac{4b^3}{f\sqrt{b\sec(e+fx)}} + \frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

output
```
-2/5*b^5/f/(b*sec(f*x+e))^(5/2)+4*b^3/f/(b*sec(f*x+e))^(1/2)+2/3*b*(b*sec(
f*x+e))^(3/2)/f
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.67

$$\int (b\sec(e+fx))^{5/2}\sin^5(e+fx)\,dx = \frac{b(151+108\cos(2(e+fx))-3\cos(4(e+fx)))(b\sec(e+fx))^{3/2}}{60f}$$

input
```
Integrate[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^5,x]
```

output
```
(b*(151 + 108*Cos[2*(e + f*x)] - 3*Cos[4*(e + f*x)])*(b*Sec[e + f*x])^(3/2
))/(60*f)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3102, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(e+fx)(b\sec(e+fx))^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\sec(e+fx))^{5/2}}{\csc(e+fx)^5}dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{b^5\int \frac{\left(b^2-b^2\sec^2(e+fx)\right)^2}{b^4(b\sec(e+fx))^{7/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{\left(b^2-b^2\sec^2(e+fx)\right)^2}{(b\sec(e+fx))^{7/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{244}$$

$$\frac{b\int \left(\frac{b^4}{(b\sec(e+fx))^{7/2}}-\frac{2b^2}{(b\sec(e+fx))^{3/2}}+\sqrt{b\sec(e+fx)}\right)d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(-\frac{2b^4}{5(b\sec(e+fx))^{5/2}}+\frac{4b^2}{\sqrt{b\sec(e+fx)}}+\frac{2}{3}(b\sec(e+fx))^{3/2}\right)}{f}$$

input
```
Int[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^5,x]
```

output
```
(b*((-2*b^4)/(5*(b*Sec[e + f*x])^(5/2)) + (4*b^2)/Sqrt[b*Sec[e + f*x]] + (
2*(b*Sec[e + f*x])^(3/2))/3))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(53) = 106$.

Time = 1.86 (sec) , antiderivative size = 254, normalized size of antiderivative = 4.03

$$\left(-\frac{\left(-15\cos(fx+e)-15\right)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\cos(fx+e)+1}{\cos(fx+e)+1}\right)}{15}-\frac{(15\cos(fx+e)+15)\sqrt{-}}{}\right.$$

input
```
int((b*sec(f*x+e))^(5/2)*sin(f*x+e)^5,x)
```

output
```
1/f*(-1/15*(-15*cos(f*x+e)-15)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln((2*
cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))-1/15*(15*cos(f*x+e)+15)*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+
e)+1))-2/5*cos(f*x+e)^3+4*cos(f*x+e)+2/3*sec(f*x+e))*(b*sec(f*x+e))^(1/2)*
b^2
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int (b\sec(e+fx))^{5/2}\sin^5(e+fx)\,dx = -\frac{2\left(3\,b^2\cos\left(fx+e\right)^4-30\,b^2\cos\left(fx+e\right)^2-5\,b^2\right)\sqrt{\frac{b}{\cos(fx+e)}}}{15\,f\cos\left(fx+e\right)}$$

input
```
integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^5,x, algorithm="fricas")
```

output
```
-2/15*(3*b^2*cos(f*x + e)^4 - 30*b^2*cos(f*x + e)^2 - 5*b^2)*sqrt(b/cos(f*
x + e))/(f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{5/2}\sin^5(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((b*sec(f*x+e))**(5/2)*sin(f*x+e)**5,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int (b\sec(e+fx))^{5/2}\sin^5(e+fx)\,dx = \frac{2\left(5\left(\frac{b}{\cos(fx+e)}\right)^{\frac{3}{2}} - \frac{3\left(b^4-\frac{10\,b^4}{\cos(fx+e)^2}\right)}{\left(\frac{b}{\cos(fx+e)}\right)^{\frac{5}{2}}}\right)b}{15\,f}$$

input
```
integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^5,x, algorithm="maxima")
```

output
```
2/15*(5*(b/cos(f*x + e))^(3/2) - 3*(b^4 - 10*b^4/cos(f*x + e)^2)/(b/cos(f*
x + e))^(5/2))*b/f
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.17

$$\int (b\sec(e+fx))^{5/2}\sin^5(e+fx)\,dx = -\frac{2\left(3\sqrt{b\cos(fx+e)}b^2\cos(fx+e)^2 - 30\sqrt{b\cos(fx+e)}b^2 - \frac{5\,b^3}{\sqrt{b\cos(fx+e)}\cos(fx+e)}\right)\mathrm{sgn}(\cos(fx+e))}{15\,f}$$

input
```
integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^5,x, algorithm="giac")
```

output
```
-2/15*(3*sqrt(b*cos(f*x + e))*b^2*cos(f*x + e)^2 - 30*sqrt(b*cos(f*x + e))
*b^2 - 5*b^3/(sqrt(b*cos(f*x + e))*cos(f*x + e)))*sgn(cos(f*x + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{5/2}\sin^5(e+fx)\,dx=\int \sin\left(e+f\,x\right)^5\,\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^{5/2}\,dx$$

input
```
int(sin(e + f*x)^5*(b/cos(e + f*x))^(5/2),x)
```

output
```
int(sin(e + f*x)^5*(b/cos(e + f*x))^(5/2), x)
```

Reduce [F]

$$\int (b\sec(e+fx))^{5/2}\sin^5(e+fx)\,dx=\sqrt{b}\left(\int \sqrt{\sec\left(fx+e\right)}\,\sec\left(fx+e\right)^2\sin\left(fx+e\right)^5dx\right)b^2$$

input
```
int((b*sec(f*x+e))^(5/2)*sin(f*x+e)^5,x)
```

output
```
sqrt(b)*int(sqrt(sec(e + f*x))*sec(e + f*x)**2*sin(e + f*x)**5,x)*b**2
```

3.399 $\int (b\sec(e+fx))^{5/2}\sin^3(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 41

$$\int (b\sec(e+fx))^{5/2}\sin^3(e+fx)\,dx=\frac{2b^3}{f\sqrt{b\sec(e+fx)}}+\frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

output
```
2*b^3/f/(b*sec(f*x+e))^(1/2)+2/3*b*(b*sec(f*x+e))^(3/2)/f
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int (b\sec(e+fx))^{5/2}\sin^3(e+fx)\,dx=\frac{b(5+3\cos(2(e+fx)))(b\sec(e+fx))^{3/2}}{3f}$$

input
```
Integrate[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^3,x]
```

output
```
(b*(5 + 3*Cos[2*(e + f*x)])*(b*Sec[e + f*x])^(3/2))/(3*f)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3102, 25, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(e+fx)(b\sec(e+fx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\sec(e+fx))^{5/2}}{\csc(e+fx)^3}dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{b^3\int -\frac{b^2-b^2\sec^2(e+fx)}{b^2(b\sec(e+fx))^{3/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b^3\int \frac{b^2-b^2\sec^2(e+fx)}{b^2(b\sec(e+fx))^{3/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\int \frac{b^2-b^2\sec^2(e+fx)}{(b\sec(e+fx))^{3/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{244}$$

$$-\frac{b\int \left(\frac{b^2}{(b\sec(e+fx))^{3/2}}-\sqrt{b\sec(e+fx)}\right)d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\left(-\frac{2b^2}{\sqrt{b\sec(e+fx)}}-\frac{2}{3}(b\sec(e+fx))^{3/2}\right)}{f}$$

input
```
Int[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^3,x]
```

output
```
-((b*((-2*b^2)/Sqrt[b*Sec[e + f*x]] - (2*(b*Sec[e + f*x])^(3/2))/3))/f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(35) = 70$.

Time = 50.89 (sec) , antiderivative size = 260, normalized size of antiderivative = 6.34

method	result
default	$\frac{\sqrt{b\sec(fx+e)}\,b^2\left(\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\left(12\cos(fx+e)^2+12\cos(fx+e)+4+4\sec(fx+e)\right)-3\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{\cdots}}{\cos(fx+e\cdots}\right.\right.}{6f\sqrt{-\frac{\cos(fx+e\cdots}{(\cos(fx+e)+\cdots}}\cdots}$

input
```
int((b*sec(f*x+e))^(5/2)*sin(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/6/f*(b*sec(f*x+e))^(1/2)*b^2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/(cos(f
*x+e)+1)*((-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(12*cos(f*x+e)^2+12*cos(f*x
+e)+4+4*sec(f*x+e))-3*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*cos
(f*x+e)+3*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*cos(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int (b\sec(e+fx))^{5/2}\sin^3(e+fx)\,dx = \frac{2\left(3\,b^2\cos\left(fx+e\right)^2+b^2\right)\sqrt{\frac{b}{\cos(fx+e)}}}{3\,f\cos\left(fx+e\right)}$$

input
```
integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^3,x, algorithm="fricas")
```

output
```
2/3*(3*b^2*cos(f*x + e)^2 + b^2)*sqrt(b/cos(f*x + e))/(f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{5/2}\sin^3(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((b*sec(f*x+e))**(5/2)*sin(f*x+e)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int (b\sec(e+fx))^{5/2}\sin^3(e+fx)\,dx = \frac{2\left(\frac{3\,b^2}{\sqrt{\frac{b}{\cos(fx+e)}}} + \left(\frac{b}{\cos(fx+e)}\right)^{\frac{3}{2}}\right)b}{3\,f}$$

input
```
integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^3,x, algorithm="maxima")
```

output
```
2/3*(3*b^2/sqrt(b/cos(f*x + e)) + (b/cos(f*x + e))^(3/2))*b/f
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

$$\int (b\sec(e+fx))^{5/2}\sin^3(e+fx)\,dx = \frac{2\left(3\,\sqrt{b\cos(fx+e)}b + \frac{b^2}{\sqrt{b\cos(fx+e)}\cos(fx+e)}\right)b\mathrm{sgn}(\cos(fx+e))}{3\,f}$$

input
```
integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^3,x, algorithm="giac")
```

output
```
2/3*(3*sqrt(b*cos(f*x + e))*b + b^2/(sqrt(b*cos(f*x + e))*cos(f*x + e)))*b
*sgn(cos(f*x + e))/f
```

Mupad [B] (verification not implemented)

Time = 26.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.22

$$\int (b\sec(e+fx))^{5/2}\sin^3(e+fx)\,dx = \frac{b^2\sqrt{\frac{b}{\cos(e+f\,x)}}\,\left(\frac{13\,\cos(e+f\,x)}{3}+\cos\left(3\,e+3\,f\,x\right)\right)}{f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

input
```
int(sin(e + f*x)^3*(b/cos(e + f*x))^(5/2),x)
```

output
```
(b^2*(b/cos(e + f*x))^(1/2)*((13*cos(e + f*x))/3 + cos(3*e + 3*f*x)))/(f*(
cos(2*e + 2*f*x) + 1))
```

Reduce [F]

$$\int (b\sec(e+fx))^{5/2}\sin^3(e+fx)\,dx = \sqrt{b}\left(\int \sqrt{\sec\left(fx+e\right)}\,\sec\left(fx+e\right)^2\sin\left(fx+e\right)^3dx\right)b^2$$

input
```
int((b*sec(f*x+e))^(5/2)*sin(f*x+e)^3,x)
```

output
```
sqrt(b)*int(sqrt(sec(e + f*x))*sec(e + f*x)**2*sin(e + f*x)**3,x)*b**2
```

3.400 $\int (b\sec(e+fx))^{5/2}\sin(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 20

$$\int (b\sec(e+fx))^{5/2}\sin(e+fx)\,dx=\frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

output
```
2/3*b*(b*sec(f*x+e))^(3/2)/f
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (b\sec(e+fx))^{5/2}\sin(e+fx)\,dx=\frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

input
```
Integrate[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x],x]
```

output
```
(2*b*(b*Sec[e + f*x])^(3/2))/(3*f)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 3102, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(e+fx)(b\sec(e+fx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\sec(e+fx))^{5/2}}{\csc(e+fx)}dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{b\int \sqrt{b\sec(e+fx)}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{15}$$

$$\frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

input
```
Int[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x],x]
```

output
```
(2*b*(b*Sec[e + f*x])^(3/2))/(3*f)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 3.94 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{2b(b\sec(fx+e))^{\frac{3}{2}}}{3f}$	17
default	$\frac{2b(b\sec(fx+e))^{\frac{3}{2}}}{3f}$	17

input
```
int((b*sec(f*x+e))^(5/2)*sin(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
2/3*b*(b*sec(f*x+e))^(3/2)/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.40

$$\int (b\sec(e+fx))^{5/2}\sin(e+fx)\,dx = \frac{2\,b^2\sqrt{\frac{b}{\cos(fx+e)}}}{3\,f\cos(fx+e)}$$

input
```
integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e),x, algorithm="fricas")
```

output
```
2/3*b^2*sqrt(b/cos(f*x + e))/(f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{5/2}\sin(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((b*sec(f*x+e))**(5/2)*sin(f*x+e),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int (b\sec(e+fx))^{5/2}\sin(e+fx)\,dx = \frac{2\left(\frac{b}{\cos(fx+e)}\right)^{\frac{5}{2}}\cos(fx+e)}{3\,f}$$

input
```
integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e),x, algorithm="maxima")
```

output
```
2/3*(b/cos(f*x + e))^(5/2)*cos(f*x + e)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(16) = 32.

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int (b\sec(e+fx))^{5/2}\sin(e+fx)\,dx = \frac{2\,b^3\mathrm{sgn}(\cos(fx+e))}{3\,\sqrt{b\cos(fx+e)}\,f\cos(fx+e)}$$

input
```
integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e),x, algorithm="giac")
```

output
```
2/3*b^3*sgn(cos(f*x + e))/(sqrt(b*cos(f*x + e))*f*cos(f*x + e))
```

Mupad [B] (verification not implemented)

Time = 26.00 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.95

$$\int (b\sec(e+fx))^{5/2}\sin(e+fx)\,dx = \frac{4\,b^2\,\cos\left(e+f\,x\right)\,\sqrt{\frac{b}{\cos(e+f\,x)}}}{3\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

input
```
int(sin(e + f*x)*(b/cos(e + f*x))^(5/2),x)
```

output
```
(4*b^2*cos(e + f*x)*(b/cos(e + f*x))^(1/2))/(3*f*(cos(2*e + 2*f*x) + 1))
```

Reduce [F]

$$\int (b\sec(e+fx))^{5/2}\sin(e+fx)\,dx = \sqrt{b}\left(\int \sqrt{\sec\left(fx+e\right)}\,\sec\left(fx+e\right)^2\sin\left(fx+e\right)dx\right)b^2$$

input
```
int((b*sec(f*x+e))^(5/2)*sin(f*x+e),x)
```

output
```
sqrt(b)*int(sqrt(sec(e + f*x))*sec(e + f*x)**2*sin(e + f*x),x)*b**2
```

3.401 $\int \csc(e+fx)(b\sec(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \csc(e+fx)(b\sec(e+fx))^{5/2}\,dx = \frac{b^{5/2}\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f} - \frac{b^{5/2}\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f} + \frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

output
```
b^(5/2)*arctan((b*sec(f*x+e))^(1/2)/b^(1/2))/f-b^(5/2)*arctanh((b*sec(f*x+
e))^(1/2)/b^(1/2))/f+2/3*b*(b*sec(f*x+e))^(3/2)/f
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.12

$$\int \csc(e+fx)(b\sec(e+fx))^{5/2}\,dx = \frac{(b\sec(e+fx))^{5/2}\left(6\arctan\left(\sqrt{\sec(e+fx)}\right)+3\log\left(1-\sqrt{\sec(e+fx)}\right)-3\log\left(1+\sqrt{\text{se}\right.\right.}{6f\sec^{\frac{5}{2}}(e+fx)}$$

input
```
Integrate[Csc[e + f*x]*(b*Sec[e + f*x])^(5/2),x]
```

output
```
((b*Sec[e + f*x])^(5/2)*(6*ArcTan[Sqrt[Sec[e + f*x]]] + 3*Log[1 - Sqrt[Sec
[e + f*x]]] - 3*Log[1 + Sqrt[Sec[e + f*x]]] + 4*Sec[e + f*x]^(3/2)))/(6*f*
Sec[e + f*x]^(5/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3042, 3102, 25, 27, 262, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(e+fx)(b\sec(e+fx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(e+fx)(b\sec(e+fx))^{5/2}dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{\int -\frac{b^2(b\sec(e+fx))^{5/2}}{b^2-b^2\sec^2(e+fx)}d(b\sec(e+fx))}{bf}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{b^2(b\sec(e+fx))^{5/2}}{b^2-b^2\sec^2(e+fx)}d(b\sec(e+fx))}{bf}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\int \frac{(b\sec(e+fx))^{5/2}}{b^2-b^2\sec^2(e+fx)}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{262}$$

$$-\frac{b\left(b^2\int \frac{\sqrt{b\sec(e+fx)}}{b^2-b^2\sec^2(e+fx)}d(b\sec(e+fx))-\frac{2}{3}(b\sec(e+fx))^{3/2}\right)}{f}$$

$$\Big\downarrow \text{266}$$

$$-\frac{b\left(2b^2\int \frac{b^2\sec^2(e+fx)}{b^2-b^4\sec^4(e+fx)}d\sqrt{b\sec(e+fx)}-\frac{2}{3}(b\sec(e+fx))^{3/2}\right)}{f}$$

$\Big\downarrow$ 827

$$-\frac{b\left(2b^2\left(\frac{1}{2}\int \frac{1}{b-b^2\sec^2(e+fx)}d\sqrt{b\sec(e+fx)}-\frac{1}{2}\int \frac{1}{b^2\sec^2(e+fx)+b}d\sqrt{b\sec(e+fx)}\right)-\frac{2}{3}(b\sec(e+fx))^{3/2}\right)}{f}$$

$\Big\downarrow$ 216

$$-\frac{b\left(2b^2\left(\frac{1}{2}\int \frac{1}{b-b^2\sec^2(e+fx)}d\sqrt{b\sec(e+fx)}-\frac{\arctan\left(\sqrt{b}\sec(e+fx)\right)}{2\sqrt{b}}\right)-\frac{2}{3}(b\sec(e+fx))^{3/2}\right)}{f}$$

$\Big\downarrow$ 219

$$-\frac{b\left(2b^2\left(\frac{\text{arctanh}\left(\sqrt{b}\sec(e+fx)\right)}{2\sqrt{b}}-\frac{\arctan\left(\sqrt{b}\sec(e+fx)\right)}{2\sqrt{b}}\right)-\frac{2}{3}(b\sec(e+fx))^{3/2}\right)}{f}$$

input
```
Int[Csc[e + f*x]*(b*Sec[e + f*x])^(5/2),x]
```

output
```
-((b*(2*b^2*(-1/2*ArcTan[Sqrt[b]*Sec[e + f*x]]/Sqrt[b] + ArcTanh[Sqrt[b]*S
ec[e + f*x]]/(2*Sqrt[b])) - (2*(b*Sec[e + f*x])^(3/2))/3))/f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(62) = 124$.

Time = 4.21 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.47

method	result
default	$-\frac{\sqrt{b\sec(fx+e)}\, b^2 \left(\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,(-4-4\sec(fx+e))+3\ln\left(\frac{4\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+4\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-2\cos(fx+e}{\cos(fx+e)+1}\right.\right.}{6f\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,(\cos(fx+e)+1)}$

input
```
int(csc(f*x+e)*(b*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6/f*(b*sec(f*x+e))^(1/2)*b^2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/(cos(
f*x+e)+1)*((-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(-4-4*sec(f*x+e))+3*ln(2*(
2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*cos(f*x+e)-3*arctan(1/2/(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*cos(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(62) = 124$.

Time = 0.14 (sec) , antiderivative size = 347, normalized size of antiderivative = 4.45

$$\int \csc(e+fx)(b\sec(e+fx))^{5/2}\,dx = \left[\frac{6\sqrt{-b}b^2\arctan\left(\frac{2\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)}{b\cos(fx+e)+b}\right)\cos(fx+e)+3\sqrt{-b}b^2\cos(fx+e)\log\left(\frac{b\cos(fx\cdots}{\cdots}\right.}{12f\cos(fx+e)}\right.$$

input
```
integrate(csc(f*x+e)*(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
[1/12*(6*sqrt(-b)*b^2*arctan(2*sqrt(-b)*sqrt(b/cos(f*x + e))*cos(f*x + e)/
(b*cos(f*x + e) + b))*cos(f*x + e) + 3*sqrt(-b)*b^2*cos(f*x + e)*log((b*co
s(f*x + e)^2 - 4*(cos(f*x + e)^2 - cos(f*x + e))*sqrt(-b)*sqrt(b/cos(f*x +
 e)) - 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 8*b^
2*sqrt(b/cos(f*x + e)))/(f*cos(f*x + e)), 1/12*(6*b^(5/2)*arctan(2*sqrt(b)
*sqrt(b/cos(f*x + e))*cos(f*x + e)/(b*cos(f*x + e) - b))*cos(f*x + e) + 3*
b^(5/2)*cos(f*x + e)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + cos(f*x +
 e))*sqrt(b)*sqrt(b/cos(f*x + e)) + 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2
- 2*cos(f*x + e) + 1)) + 8*b^2*sqrt(b/cos(f*x + e)))/(f*cos(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \csc(e+fx)(b\sec(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)*(b*sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.12

$$\int \csc(e+fx)(b\sec(e+fx))^{5/2}\,dx = \frac{\left(6\,b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}}\right)+3\,b^{\frac{3}{2}}\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos(fx+e)}}}\right)+4\left(\frac{b}{\cos(fx+e)}\right)^{\frac{3}{2}}\right)b}{6\,f}$$

input
```
integrate(csc(f*x+e)*(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
1/6*(6*b^(3/2)*arctan(sqrt(b/cos(f*x + e))/sqrt(b)) + 3*b^(3/2)*log(-(sqrt
(b) - sqrt(b/cos(f*x + e)))/(sqrt(b) + sqrt(b/cos(f*x + e)))) + 4*(b/cos(f
*x + e))^(3/2))*b/f
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.10

$$\int \csc(e+fx)(b\sec(e+fx))^{5/2}\,dx = \frac{b^6\left(\frac{3\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b}b^3} - \frac{3\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{b}}\right)}{b^{\frac{7}{2}}} + \frac{2}{\sqrt{b\cos(fx+e)}b^3\cos(fx+e)}\right)\mathrm{sgn}(\cos(fx+e))}{3f}$$

input
```
integrate(csc(f*x+e)*(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
1/3*b^6*(3*arctan(sqrt(b*cos(f*x + e))/sqrt(-b))/(sqrt(-b)*b^3) - 3*arctan
(sqrt(b*cos(f*x + e))/sqrt(b))/b^(7/2) + 2/(sqrt(b*cos(f*x + e))*b^3*cos(f
*x + e)))*sgn(cos(f*x + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \csc(e+fx)(b\sec(e+fx))^{5/2}\,dx = \int \frac{\left(\frac{b}{\cos(e+f\,x)}\right)^{5/2}}{\sin(e+f\,x)}\,dx$$

input
```
int((b/cos(e + f*x))^(5/2)/sin(e + f*x),x)
```

output
```
int((b/cos(e + f*x))^(5/2)/sin(e + f*x), x)
```

Reduce [F]

$$\int \csc(e + fx)(b\sec(e + fx))^{5/2}\,dx = \sqrt{b}\left(\int \sqrt{\sec(fx+e)}\,\csc(fx+e)\sec(fx+e)^2\,dx\right)b^2$$

input
```
int(csc(f*x+e)*(b*sec(f*x+e))^(5/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(e + f*x))*csc(e + f*x)*sec(e + f*x)**2,x)*b**2
```

3.402 $\int \csc^3(e+fx)(b\sec(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 113

$$\int \csc^3(e+fx)(b\sec(e+fx))^{5/2}\,dx = \frac{7b^{5/2}\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f} - \frac{7b^{5/2}\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f} + \frac{7b(b\sec(e+fx))^{3/2}}{6f} - \frac{\cot^2(e+fx)(b\sec(e+fx))^{7/2}}{2bf}$$

output
```
7/4*b^(5/2)*arctan((b*sec(f*x+e))^(1/2)/b^(1/2))/f-7/4*b^(5/2)*arctanh((b*
sec(f*x+e))^(1/2)/b^(1/2))/f+7/6*b*(b*sec(f*x+e))^(3/2)/f-1/2*cot(f*x+e)^2
*(b*sec(f*x+e))^(7/2)/b/f
```

Mathematica [A] (verified)

Time = 2.58 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96

$$\int \csc^3(e+fx)(b\sec(e+fx))^{5/2}\,dx = \frac{b^3\left(-12\csc^2(e+fx)+42\arctan\left(\sqrt{\sec(e+fx)}\right)\sqrt{\sec(e+fx)}+21\left(\log\left(1-\sqrt{\sec(e+f\right.\right.\right.}{24f\sqrt{b\sec(e+fx)}}$$

input
```
Integrate[Csc[e + f*x]^3*(b*Sec[e + f*x])^(5/2),x]
```

output
```
(b^3*(-12*Csc[e + f*x]^2 + 42*ArcTan[Sqrt[Sec[e + f*x]]]*Sqrt[Sec[e + f*x]
] + 21*(Log[1 - Sqrt[Sec[e + f*x]]] - Log[1 + Sqrt[Sec[e + f*x]]])*Sqrt[Se
c[e + f*x]] + 16*Sec[e + f*x]^2))/(24*f*Sqrt[b*Sec[e + f*x]])
```

Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3102, 27, 252, 262, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(e+fx)(b\sec(e+fx))^{5/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc(e+fx)^3(b\sec(e+fx))^{5/2}dx$$

$$\Big\downarrow \; 3102$$

$$\frac{\int \frac{b^4(b\sec(e+fx))^{9/2}}{(b^2-b^2\sec^2(e+fx))^2}d(b\sec(e+fx))}{b^3 f}$$

$$\Big\downarrow \; 27$$

$$\frac{b\int \frac{(b\sec(e+fx))^{9/2}}{(b^2-b^2\sec^2(e+fx))^2}d(b\sec(e+fx))}{f}$$

$\Big\downarrow$ 252

$$\frac{b\left(\frac{(b\sec(e+fx))^{7/2}}{2(b^2-b^2\sec^2(e+fx))}-\frac{7}{4}\int \frac{(b\sec(e+fx))^{5/2}}{b^2-b^2\sec^2(e+fx)}d(b\sec(e+fx))\right)}{f}$$

$\Big\downarrow$ 262

$$\frac{b\left(\frac{(b\sec(e+fx))^{7/2}}{2(b^2-b^2\sec^2(e+fx))}-\frac{7}{4}\left(b^2\int \frac{\sqrt{b\sec(e+fx)}}{b^2-b^2\sec^2(e+fx)}d(b\sec(e+fx))-\frac{2}{3}(b\sec(e+fx))^{3/2}\right)\right)}{f}$$

$\Big\downarrow$ 266

$$\frac{b\left(\frac{(b\sec(e+fx))^{7/2}}{2(b^2-b^2\sec^2(e+fx))}-\frac{7}{4}\left(2b^2\int \frac{b^2\sec^2(e+fx)}{b^2-b^4\sec^4(e+fx)}d\sqrt{b\sec(e+fx)}-\frac{2}{3}(b\sec(e+fx))^{3/2}\right)\right)}{f}$$

$\Big\downarrow$ 827

$$\frac{b\left(\frac{(b\sec(e+fx))^{7/2}}{2(b^2-b^2\sec^2(e+fx))}-\frac{7}{4}\left(2b^2\left(\frac{1}{2}\int \frac{1}{b-b^2\sec^2(e+fx)}d\sqrt{b\sec(e+fx)}-\frac{1}{2}\int \frac{1}{b^2\sec^2(e+fx)+b}d\sqrt{b\sec(e+fx)}\right)-\frac{2}{3}(b\sec\right.\right.}{f}$$

$\Big\downarrow$ 216

$$\frac{b\left(\frac{(b\sec(e+fx))^{7/2}}{2(b^2-b^2\sec^2(e+fx))}-\frac{7}{4}\left(2b^2\left(\frac{1}{2}\int \frac{1}{b-b^2\sec^2(e+fx)}d\sqrt{b\sec(e+fx)}-\frac{\arctan\left(\sqrt{b}\sec(e+fx)\right)}{2\sqrt{b}}\right)-\frac{2}{3}(b\sec(e+fx))^{3/2}\right)\right.}{f}$$

$\Big\downarrow$ 219

$$\frac{b\left(\frac{(b\sec(e+fx))^{7/2}}{2(b^2-b^2\sec^2(e+fx))}-\frac{7}{4}\left(2b^2\left(\frac{\text{arctanh}\left(\sqrt{b}\sec(e+fx)\right)}{2\sqrt{b}}-\frac{\arctan\left(\sqrt{b}\sec(e+fx)\right)}{2\sqrt{b}}\right)-\frac{2}{3}(b\sec(e+fx))^{3/2}\right)\right)}{f}$$

input
```
Int[Csc[e + f*x]^3*(b*Sec[e + f*x])^(5/2),x]
```

output
```
(b*((b*Sec[e + f*x])^(7/2)/(2*(b^2 - b^2*Sec[e + f*x]^2)) - (7*(2*b^2*(-1/
2*ArcTan[Sqrt[b]*Sec[e + f*x]]/Sqrt[b] + ArcTanh[Sqrt[b]*Sec[e + f*x]]/(2*
Sqrt[b])) - (2*(b*Sec[e + f*x])^(3/2))/3))/4))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(89) = 178$.

Time = 19.53 (sec) , antiderivative size = 312, normalized size of antiderivative = 2.76

method	result
default	$-\frac{\left(\left(28\cos(fx+e)^2-16\right)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+\cos(fx+e)^2(3\cos(fx+e)-3)\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)}}}{\cos(fx+e)+1}\right.\right.}{}$

input
```
int(csc(f*x+e)^3*(b*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24/f*((28*cos(f*x+e)^2-16)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+cos(f*x
+e)^2*(3*cos(f*x+e)-3)*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))+co
s(f*x+e)^2*(-24*cos(f*x+e)+24)*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x
+e)+1))+cos(f*x+e)^2*(21*cos(f*x+e)-21)*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e
)+1)^2)^(1/2)))*b^2*(b*sec(f*x+e))^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)*sec(f*x+e)*csc(f*x+e)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(89) = 178.

Time = 0.21 (sec) , antiderivative size = 467, normalized size of antiderivative = 4.13

$$\int \csc^3(e+fx)(b\sec(e+fx))^{5/2}\,dx = \left[\frac{42\left(b^2\cos\left(fx+e\right)^3-b^2\cos\left(fx+e\right)\right)\sqrt{-b}\arctan\left(\frac{2\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)}{b\cos(fx+e)+b}\right)+21\left(b^2\cos\left(\right.\right.}{}\right.$$

input
```
integrate(csc(f*x+e)^3*(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
[1/48*(42*(b^2*cos(f*x + e)^3 - b^2*cos(f*x + e))*sqrt(-b)*arctan(2*sqrt(-
b)*sqrt(b/cos(f*x + e))*cos(f*x + e)/(b*cos(f*x + e) + b)) + 21*(b^2*cos(f
*x + e)^3 - b^2*cos(f*x + e))*sqrt(-b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x
+ e)^2 - cos(f*x + e))*sqrt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos(f*x + e) +
b)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 8*(7*b^2*cos(f*x + e)^2 - 4*b^
2)*sqrt(b/cos(f*x + e)))/(f*cos(f*x + e)^3 - f*cos(f*x + e)), 1/48*(42*(b^
2*cos(f*x + e)^3 - b^2*cos(f*x + e))*sqrt(b)*arctan(2*sqrt(b)*sqrt(b/cos(f
*x + e))*cos(f*x + e)/(b*cos(f*x + e) - b)) + 21*(b^2*cos(f*x + e)^3 - b^2
*cos(f*x + e))*sqrt(b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + cos(f*x
 + e))*sqrt(b)*sqrt(b/cos(f*x + e)) + 6*b*cos(f*x + e) + b)/(cos(f*x + e)^
2 - 2*cos(f*x + e) + 1)) + 8*(7*b^2*cos(f*x + e)^2 - 4*b^2)*sqrt(b/cos(f*x
 + e)))/(f*cos(f*x + e)^3 - f*cos(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \csc^3(e+fx)(b\sec(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**3*(b*sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.09

$$\int \csc^3(e+fx)(b\sec(e+fx))^{5/2}\,dx = \frac{\left(\frac{12\,b^2\left(\frac{b}{\cos(fx+e)}\right)^{\frac{3}{2}}}{b^2-\frac{b^2}{\cos(fx+e)^2}} + 42\,b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}}\right) + 21\,b^{\frac{3}{2}}\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos(fx+e)}}}\right) + 16\left(\frac{b}{\cos(fx+e)}\right)}{24\,f}$$

input
```
integrate(csc(f*x+e)^3*(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
1/24*(12*b^2*(b/cos(f*x + e))^(3/2)/(b^2 - b^2/cos(f*x + e)^2) + 42*b^(3/2
)*arctan(sqrt(b/cos(f*x + e))/sqrt(b)) + 21*b^(3/2)*log(-(sqrt(b) - sqrt(b
/cos(f*x + e)))/(sqrt(b) + sqrt(b/cos(f*x + e)))) + 16*(b/cos(f*x + e))^(3
/2))*b/f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07

$$\int \csc^3(e+fx)(b\sec(e+fx))^{5/2}\,dx = \frac{b^8\left(\frac{6\sqrt{b\cos(fx+e)}}{\left(b^2\cos(fx+e)^2-b^2\right)b^4} + \frac{21\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b}b^5} - \frac{21\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{b}}\right)}{b^{\frac{11}{2}}} + \frac{8}{\sqrt{b\cos(fx+e)}b^5\cos(fx+e)}\right)}{12\,f}$$

input
```
integrate(csc(f*x+e)^3*(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
1/12*b^8*(6*sqrt(b*cos(f*x + e))/((b^2*cos(f*x + e)^2 - b^2)*b^4) + 21*arc
tan(sqrt(b*cos(f*x + e))/sqrt(-b))/(sqrt(-b)*b^5) - 21*arctan(sqrt(b*cos(f
*x + e))/sqrt(b))/b^(11/2) + 8/(sqrt(b*cos(f*x + e))*b^5*cos(f*x + e)))*sg
n(cos(f*x + e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(e+fx)(b\sec(e+fx))^{5/2}\,dx = \int \frac{\left(\frac{b}{\cos(e+f\,x)}\right)^{5/2}}{\sin\left(e+f\,x\right)^3}\,dx$$

input
```
int((b/cos(e + f*x))^(5/2)/sin(e + f*x)^3,x)
```

output
```
int((b/cos(e + f*x))^(5/2)/sin(e + f*x)^3, x)
```

Reduce [F]

$$\int \csc^3(e+fx)(b\sec(e+fx))^{5/2}\,dx = \sqrt{b}\left(\int \sqrt{\sec\left(fx+e\right)}\,\csc\left(fx+e\right)^3\sec\left(fx+e\right)^2dx\right)b^2$$

input
```
int(csc(f*x+e)^3*(b*sec(f*x+e))^(5/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(e + f*x))*csc(e + f*x)**3*sec(e + f*x)**2,x)*b**2
```

3.403 $\int \csc^5(e+fx)(b\sec(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 143

$$\int \csc^5(e+fx)(b\sec(e+fx))^{5/2}\,dx = \frac{77b^{5/2}\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32f} - \frac{77b^{5/2}\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32f} + \frac{77b(b\sec(e+fx))^{3/2}}{48f} - \frac{11\cot^2(e+fx)(b\sec(e+fx))^{7/2}}{16bf} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{11/2}}{4b^3f}$$

output
```
77/32*b^(5/2)*arctan((b*sec(f*x+e))^(1/2)/b^(1/2))/f-77/32*b^(5/2)*arctanh
((b*sec(f*x+e))^(1/2)/b^(1/2))/f+77/48*b*(b*sec(f*x+e))^(3/2)/f-11/16*cot(
f*x+e)^2*(b*sec(f*x+e))^(7/2)/b/f-1/4*cot(f*x+e)^4*(b*sec(f*x+e))^(11/2)/b
^3/f
```

Mathematica [A] (verified)

Time = 1.74 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.83

$$\int \csc^5(e+fx)(b\sec(e+fx))^{5/2}\,dx=\frac{b^3\left(-180\csc^2(e+fx)-48\csc^4(e+fx)+462\arctan\left(\sqrt{\sec(e+fx)}\right)\sqrt{\sec(e+fx)}+231\ldots}{192f\sqrt{b\sec\ldots}}$$

input
```
Integrate[Csc[e + f*x]^5*(b*Sec[e + f*x])^(5/2),x]
```

output
```
(b^3*(-180*Csc[e + f*x]^2 - 48*Csc[e + f*x]^4 + 462*ArcTan[Sqrt[Sec[e + f*
x]]]*Sqrt[Sec[e + f*x]] + 231*(Log[1 - Sqrt[Sec[e + f*x]]] - Log[1 + Sqrt[
Sec[e + f*x]]])*Sqrt[Sec[e + f*x]] + 128*Sec[e + f*x]^2))/(192*f*Sqrt[b*Se
c[e + f*x]])
```

Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.524$, Rules used = {3042, 3102, 25, 27, 252, 252, 262, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^5(e+fx)(b\sec(e+fx))^{5/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc(e+fx)^5(b\sec(e+fx))^{5/2}dx$$

$$\Big\downarrow \text{ 3102}$$

$$\frac{\int -\frac{b^6(b\sec(e+fx))^{13/2}}{(b^2-b^2\sec^2(e+fx))^3}d(b\sec(e+fx))}{b^5f}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{b^6 (b\sec(e+fx))^{13/2}}{(b^2-b^2\sec^2(e+fx))^3}d(b\sec(e+fx))}{b^5 f}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\int \frac{(b\sec(e+fx))^{13/2}}{(b^2-b^2\sec^2(e+fx))^3}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{252}$$

$$-\frac{b\left(\frac{(b\sec(e+fx))^{11/2}}{4(b^2-b^2\sec^2(e+fx))^2}-\frac{11}{8}\int \frac{(b\sec(e+fx))^{9/2}}{(b^2-b^2\sec^2(e+fx))^2}d(b\sec(e+fx))\right)}{f}$$

$$\Big\downarrow \text{252}$$

$$-\frac{b\left(\frac{(b\sec(e+fx))^{11/2}}{4(b^2-b^2\sec^2(e+fx))^2}-\frac{11}{8}\left(\frac{(b\sec(e+fx))^{7/2}}{2(b^2-b^2\sec^2(e+fx))}-\frac{7}{4}\int \frac{(b\sec(e+fx))^{5/2}}{b^2-b^2\sec^2(e+fx)}d(b\sec(e+fx))\right)\right)}{f}$$

$$\Big\downarrow \text{262}$$

$$-\frac{b\left(\frac{(b\sec(e+fx))^{11/2}}{4(b^2-b^2\sec^2(e+fx))^2}-\frac{11}{8}\left(\frac{(b\sec(e+fx))^{7/2}}{2(b^2-b^2\sec^2(e+fx))}-\frac{7}{4}\left(b^2\int \frac{\sqrt{b\sec(e+fx)}}{b^2-b^2\sec^2(e+fx)}d(b\sec(e+fx))-\frac{2}{3}(b\sec(e+fx))^{3/2}\right)\right)\right)}{f}$$

$$\Big\downarrow \text{266}$$

$$-\frac{b\left(\frac{(b\sec(e+fx))^{11/2}}{4(b^2-b^2\sec^2(e+fx))^2}-\frac{11}{8}\left(\frac{(b\sec(e+fx))^{7/2}}{2(b^2-b^2\sec^2(e+fx))}-\frac{7}{4}\left(2b^2\int \frac{b^2\sec^2(e+fx)}{b^2-b^4\sec^4(e+fx)}d\sqrt{b\sec(e+fx)}-\frac{2}{3}(b\sec(e+fx))^{3/2}\right)\right.\right.}{f}$$

$$\Big\downarrow \text{827}$$

$$-\frac{b\left(\frac{(b\sec(e+fx))^{11/2}}{4(b^2-b^2\sec^2(e+fx))^2}-\frac{11}{8}\left(\frac{(b\sec(e+fx))^{7/2}}{2(b^2-b^2\sec^2(e+fx))}-\frac{7}{4}\left(2b^2\left(\frac{1}{2}\int \frac{1}{b-b^2\sec^2(e+fx)}d\sqrt{b\sec(e+fx)}-\frac{1}{2}\int \frac{1}{b^2\sec^2(e+fx)+b}d\right.\right.\right.\right.}{f}$$

$$\Big\downarrow \text{216}$$

$$-\frac{b\left(\frac{(b\sec(e+fx))^{11/2}}{4(b^2-b^2\sec^2(e+fx))^2}-\frac{11}{8}\left(\frac{(b\sec(e+fx))^{7/2}}{2(b^2-b^2\sec^2(e+fx))}-\frac{7}{4}\left(2b^2\left(\frac{1}{2}\int \frac{1}{b-b^2\sec^2(e+fx)}d\sqrt{b\sec(e+fx)}-\frac{\arctan\left(\sqrt{b}\sec(e+fx)\right.}{2\sqrt{b}}\right.\right.\right.\right.}{f}$$

$$\Big\downarrow \text{219}$$

$$-\frac{b\left(\frac{(b\sec(e+fx))^{11/2}}{4(b^2-b^2\sec^2(e+fx))^2}-\frac{11}{8}\left(\frac{(b\sec(e+fx))^{7/2}}{2(b^2-b^2\sec^2(e+fx))}-\frac{7}{4}\left(2b^2\left(\frac{\operatorname{arctanh}\left(\sqrt{b}\sec(e+fx)\right)}{2\sqrt{b}}-\frac{\arctan\left(\sqrt{b}\sec(e+fx)\right)}{2\sqrt{b}}\right)-\frac{2}{3}(b\sec\right.\right.\right.}{f}$$

input
```
Int[Csc[e + f*x]^5*(b*Sec[e + f*x])^(5/2),x]
```

output
```
-((b*((b*Sec[e + f*x])^(11/2)/(4*(b^2 - b^2*Sec[e + f*x]^2)^2) - (11*((b*S
ec[e + f*x])^(7/2)/(2*(b^2 - b^2*Sec[e + f*x]^2)) - (7*(2*b^2*(-1/2*ArcTan
[Sqrt[b]*Sec[e + f*x]]/Sqrt[b] + ArcTanh[Sqrt[b]*Sec[e + f*x]]/(2*Sqrt[b])
) - (2*(b*Sec[e + f*x])^(3/2))/3))/4))/8))/f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(115) = 230$.

Time = 129.93 (sec) , antiderivative size = 346, normalized size of antiderivative = 2.42

method	result
default	$\frac{\left(\left(308\cos(fx+e)^4-484\cos(fx+e)^2+128\right)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+\cos(fx+e)^2(-57\cos(fx+e)+57)\sin(fx+e)^2\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{}{(}}}{}\right.\right.}{}$

input
```
int(csc(f*x+e)^5*(b*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/192/f*((308*cos(f*x+e)^4-484*cos(f*x+e)^2+128)*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)+cos(f*x+e)^2*(-57*cos(f*x+e)+57)*sin(f*x+e)^2*ln((2*cos(f*x+e)
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)-cos(f*x+e)+1)/(cos(f*x+e)+1))+cos(f*x+e)^2*(288*cos(f*x+e)-288)*sin(f*x
+e)^2*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))+cos(f*x+e)^2*(-23
1*cos(f*x+e)+231)*sin(f*x+e)^2*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)))*b^2*(b*sec(f*x+e))^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sec(f
*x+e)*csc(f*x+e)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(115) = 230$.

Time = 0.13 (sec) , antiderivative size = 561, normalized size of antiderivative = 3.92

$$\int \csc^5(e+fx)(b\sec(e+fx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(csc(f*x+e)^5*(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
[1/384*(462*(b^2*cos(f*x + e)^5 - 2*b^2*cos(f*x + e)^3 + b^2*cos(f*x + e))
*sqrt(-b)*arctan(2*sqrt(-b)*sqrt(b/cos(f*x + e))*cos(f*x + e)/(b*cos(f*x +
 e) + b)) + 231*(b^2*cos(f*x + e)^5 - 2*b^2*cos(f*x + e)^3 + b^2*cos(f*x +
 e))*sqrt(-b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - cos(f*x + e))*sq
rt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 + 2*co
s(f*x + e) + 1)) + 8*(77*b^2*cos(f*x + e)^4 - 121*b^2*cos(f*x + e)^2 + 32*
b^2)*sqrt(b/cos(f*x + e)))/(f*cos(f*x + e)^5 - 2*f*cos(f*x + e)^3 + f*cos(
f*x + e)), 1/384*(462*(b^2*cos(f*x + e)^5 - 2*b^2*cos(f*x + e)^3 + b^2*cos
(f*x + e))*sqrt(b)*arctan(2*sqrt(b)*sqrt(b/cos(f*x + e))*cos(f*x + e)/(b*c
os(f*x + e) - b)) + 231*(b^2*cos(f*x + e)^5 - 2*b^2*cos(f*x + e)^3 + b^2*c
os(f*x + e))*sqrt(b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + cos(f*x +
 e))*sqrt(b)*sqrt(b/cos(f*x + e)) + 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2
- 2*cos(f*x + e) + 1)) + 8*(77*b^2*cos(f*x + e)^4 - 121*b^2*cos(f*x + e)^2
 + 32*b^2)*sqrt(b/cos(f*x + e)))/(f*cos(f*x + e)^5 - 2*f*cos(f*x + e)^3 +
f*cos(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \csc^5(e+fx)(b\sec(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**5*(b*sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.08

$$\int \csc^5(e+fx)(b\sec(e+fx))^{5/2}\,dx = \frac{\left(462\,b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}}\right)+231\,b^{\frac{3}{2}}\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos(fx+e)}}}\right)+128\left(\frac{b}{\cos(fx+e)}\right)^{\frac{3}{2}}+\frac{12\left(15\,b^4\left(\cdots\right.\right.}{b^4-\cdots}\right.}{192\,f}$$

input
```
integrate(csc(f*x+e)^5*(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
1/192*(462*b^(3/2)*arctan(sqrt(b/cos(f*x + e))/sqrt(b)) + 231*b^(3/2)*log(
-(sqrt(b) - sqrt(b/cos(f*x + e)))/(sqrt(b) + sqrt(b/cos(f*x + e)))) + 128*
(b/cos(f*x + e))^(3/2) + 12*(15*b^4*(b/cos(f*x + e))^(3/2) - 19*b^2*(b/cos
(f*x + e))^(7/2))/(b^4 - 2*b^4/cos(f*x + e)^2 + b^4/cos(f*x + e)^4))*b/f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.05

$$\int \csc^5(e+fx)(b\sec(e+fx))^{5/2}\,dx = \frac{b^{10}\left(\frac{231\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b}b^7} - \frac{231\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{b}}\right)}{b^{\frac{15}{2}}} + \frac{6\left(15\sqrt{b\cos(fx+e)}b^2\cos(fx+e)^2 - 19\sqrt{b\cos(fx+e)}b^2\right.}{\left(b^2\cos(fx+e)^2-b^2\right)^2b^6}\right.}{96f}$$

input
```
integrate(csc(f*x+e)^5*(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
1/96*b^10*(231*arctan(sqrt(b*cos(f*x + e))/sqrt(-b))/(sqrt(-b)*b^7) - 231*
arctan(sqrt(b*cos(f*x + e))/sqrt(b))/b^(15/2) + 6*(15*sqrt(b*cos(f*x + e))
*b^2*cos(f*x + e)^2 - 19*sqrt(b*cos(f*x + e))*b^2)/((b^2*cos(f*x + e)^2 -
b^2)^2*b^6) + 64/(sqrt(b*cos(f*x + e))*b^7*cos(f*x + e)))*sgn(cos(f*x + e)
)/f
```

Mupad [F(-1)]

Timed out.

$$\int \csc^5(e+fx)(b\sec(e+fx))^{5/2}\,dx = \int \frac{\left(\frac{b}{\cos(e+f\,x)}\right)^{5/2}}{\sin\left(e+f\,x\right)^5}\,dx$$

input
```
int((b/cos(e + f*x))^(5/2)/sin(e + f*x)^5,x)
```

output
```
int((b/cos(e + f*x))^(5/2)/sin(e + f*x)^5, x)
```

Reduce [F]

$$\int \csc^5(e+fx)(b\sec(e+fx))^{5/2}\,dx=\sqrt{b}\left(\int \sqrt{\sec(fx+e)}\,\csc(fx+e)^5\sec(fx+e)^2\,dx\right)b^2$$

input
```
int(csc(f*x+e)^5*(b*sec(f*x+e))^(5/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(e + f*x))*csc(e + f*x)**5*sec(e + f*x)**2,x)*b**2
```

3.404 $\int (b\sec(e+fx))^{5/2}\sin^6(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 130

$$\int (b\sec(e+fx))^{5/2}\sin^6(e+fx)\,dx = -\frac{80b^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{21f} + \frac{40b^3\sin(e+fx)}{21f\sqrt{b\sec(e+fx)}} + \frac{20b^3\sin^3(e+fx)}{21f\sqrt{b\sec(e+fx)}} + \frac{2b(b\sec(e+fx))^{3/2}\sin^5(e+fx)}{3f}$$

output
```
-80/21*b^2*cos(f*x+e)^(1/2)*InverseJacobiAM(1/2*f*x+1/2*e,2^(1/2))*(b*sec(
f*x+e))^(1/2)/f+40/21*b^3*sin(f*x+e)/f/(b*sec(f*x+e))^(1/2)+20/21*b^3*sin(
f*x+e)^3/f/(b*sec(f*x+e))^(1/2)+2/3*b*(b*sec(f*x+e))^(3/2)*sin(f*x+e)^5/f
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.57

$$\int (b\sec(e+fx))^{5/2}\sin^6(e+fx)\,dx = -\frac{b^2\sqrt{b\sec(e+fx)}\left(320\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right) - 58\sin(2(e+fx)) + 3\sin(4(e+fx)) - 5\right.}{84f}$$

input
```
Integrate[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^6,x]
```

output
```
-1/84*(b^2*Sqrt[b*Sec[e + f*x]]*(320*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x
)/2, 2] - 58*Sin[2*(e + f*x)] + 3*Sin[4*(e + f*x)] - 56*Tan[e + f*x]))/f
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3104, 3042, 3107, 3042, 3107, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^6(e+fx)(b\sec(e+fx))^{5/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(b\sec(e+fx))^{5/2}}{\csc(e+fx)^6}dx$$

$$\Big\downarrow \; 3104$$

$$\frac{2b\sin^5(e+fx)(b\sec(e+fx))^{3/2}}{3f}-\frac{10}{3}b^2\int \sqrt{b\sec(e+fx)}\sin^4(e+fx)dx$$

$$\Big\downarrow \; 3042$$

$$\frac{2b\sin^5(e+fx)(b\sec(e+fx))^{3/2}}{3f}-\frac{10}{3}b^2\int \frac{\sqrt{b\sec(e+fx)}}{\csc(e+fx)^4}dx$$

$$\Big\downarrow \; 3107$$

$$\frac{2b\sin^5(e+fx)(b\sec(e+fx))^{3/2}}{3f}-\frac{10}{3}b^2\left(\frac{6}{7}\int \sqrt{b\sec(e+fx)}\sin^2(e+fx)dx-\frac{2b\sin^3(e+fx)}{7f\sqrt{b\sec(e+fx)}}\right)$$

$$\Big\downarrow \; 3042$$

$$\frac{2b\sin^5(e+fx)(b\sec(e+fx))^{3/2}}{3f}-\frac{10}{3}b^2\left(\frac{6}{7}\int \frac{\sqrt{b\sec(e+fx)}}{\csc(e+fx)^2}dx-\frac{2b\sin^3(e+fx)}{7f\sqrt{b\sec(e+fx)}}\right)$$

$$\Big\downarrow \textbf{3107}$$

$$\frac{2b\sin^5(e+fx)(b\sec(e+fx))^{3/2}}{3f}-\frac{10}{3}b^2\left(\frac{6}{7}\left(\frac{2}{3}\int\sqrt{b\sec(e+fx)}dx-\frac{2b\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}\right)-\frac{2b\sin^3(e+fx)}{7f\sqrt{b\sec(e+fx)}}\right)$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{2b\sin^5(e+fx)(b\sec(e+fx))^{3/2}}{3f}-\frac{10}{3}b^2\left(\frac{6}{7}\left(\frac{2}{3}\int\sqrt{b\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{2b\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}\right)-\frac{2b\sin^3(e+fx)}{7f\sqrt{b\sec(e+fx)}}\right)$$

$$\Big\downarrow \textbf{4258}$$

$$\frac{2b\sin^5(e+fx)(b\sec(e+fx))^{3/2}}{3f}-\frac{10}{3}b^2\left(\frac{6}{7}\left(\frac{2}{3}\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\int\frac{1}{\sqrt{\cos(e+fx)}}dx-\frac{2b\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}\right)-\frac{2b\sin^3(e+fx)}{7f\sqrt{b\sec(e+fx)}}\right)$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{2b\sin^5(e+fx)(b\sec(e+fx))^{3/2}}{3f}-\frac{10}{3}b^2\left(\frac{6}{7}\left(\frac{2}{3}\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\int\frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{2b\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}\right)-\frac{2b\sin^3(e+fx)}{7f\sqrt{b\sec(e+fx)}}\right)$$

$$\Big\downarrow \textbf{3120}$$

$$\frac{2b\sin^5(e+fx)(b\sec(e+fx))^{3/2}}{3f}-\frac{10}{3}b^2\left(\frac{6}{7}\left(\frac{4\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{3f}-\frac{2b\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}\right)-\frac{2b\sin^3(e+fx)}{7f\sqrt{b\sec(e+fx)}}\right)$$

input
```
Int[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^6,x]
```

output
```
(2*b*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^5)/(3*f) - (10*b^2*((-2*b*Sin[e +
 f*x]^3)/(7*f*Sqrt[b*Sec[e + f*x]]) + (6*((4*Sqrt[Cos[e + f*x]]*EllipticF[
(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(3*f) - (2*b*Sin[e + f*x])/(3*f*Sqrt
[b*Sec[e + f*x]])))/7))/3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3104
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)/
(f*a*(n - 1))), x] + Simp[b^2*((m + 1)/(a^2*(n - 1)))   Int[(a*Csc[e + f*x]
)^(m + 2)*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[n, 1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3107
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Simp[(m + 1)/(a^2*(m + n))   Int[(a*Csc[e + f*x])^(m
+ 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1012.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.86

method	result
default	$\frac{\left(-\frac{2\left(3\cos(fx+e)^4-16\cos(fx+e)^2-7\right)\tan(fx+e)}{21}-\frac{80i(\cos(fx+e)+1)\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\mathrm{EllipticF}(i(\cot(fx+e)-\csc(fx+e)),i)}{21}\right)}{f}$

input
```
int((b*sec(f*x+e))^(5/2)*sin(f*x+e)^6,x,method=_RETURNVERBOSE)
```

output
```
1/f*(-2/21*(3*cos(f*x+e)^4-16*cos(f*x+e)^2-7)*tan(f*x+e)-80/21*I*EllipticF
(I*(cot(f*x+e)-csc(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)+1)*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2))*b^2*(b*sec(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.99

$$\int (b\sec(e+fx))^{5/2}\sin^6(e+fx)\,dx =$$
$$-\frac{2\left(-20i\sqrt{2}b^{\frac{5}{2}}\cos(fx+e)\,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e))+20i\sqrt{2}b^{\frac{5}{2}}\cos(fx\right.}{}$$

input
```
integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^6,x, algorithm="fricas")
```

output
```
-2/21*(-20*I*sqrt(2)*b^(5/2)*cos(f*x + e)*weierstrassPInverse(-4, 0, cos(f
*x + e) + I*sin(f*x + e)) + 20*I*sqrt(2)*b^(5/2)*cos(f*x + e)*weierstrassP
Inverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) + (3*b^2*cos(f*x + e)^4 - 16
*b^2*cos(f*x + e)^2 - 7*b^2)*sqrt(b/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x
 + e))
```

Sympy [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{5/2}\sin^6(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((b*sec(f*x+e))**(5/2)*sin(f*x+e)**6,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\sec(e+fx))^{5/2}\sin^6(e+fx)\,dx = \int (b\sec(fx+e))^{\frac{5}{2}}\sin(fx+e)^6\,dx$$

input
```
integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^6,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^(5/2)*sin(f*x + e)^6, x)
```

Giac [F]

$$\int (b\sec(e+fx))^{5/2}\sin^6(e+fx)\,dx = \int (b\sec(fx+e))^{\frac{5}{2}}\sin(fx+e)^6\,dx$$

input
```
integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^6,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^(5/2)*sin(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{5/2}\sin^6(e+fx)\,dx = \int \sin(e+f\,x)^6\left(\frac{b}{\cos(e+f\,x)}\right)^{5/2}dx$$

input
```
int(sin(e + f*x)^6*(b/cos(e + f*x))^(5/2),x)
```

output
```
int(sin(e + f*x)^6*(b/cos(e + f*x))^(5/2), x)
```

Reduce [F]

$$\int (b\sec(e+fx))^{5/2}\sin^6(e+fx)\,dx = \sqrt{b}\left(\int \sqrt{\sec(fx+e)}\,\sec(fx+e)^2\sin(fx+e)^6\,dx\right)b^2$$

input
```
int((b*sec(f*x+e))^(5/2)*sin(f*x+e)^6,x)
```

output
```
sqrt(b)*int(sqrt(sec(e + f*x))*sec(e + f*x)**2*sin(e + f*x)**6,x)*b**2
```

3.405 $\int (b\sec(e+fx))^{5/2}\sin^4(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 100

$$\int (b\sec(e+fx))^{5/2}\sin^4(e+fx)\,dx = -\frac{8b^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{3f} + \frac{4b^3\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}} + \frac{2b(b\sec(e+fx))^{3/2}\sin^3(e+fx)}{3f}$$

output
```
-8/3*b^2*cos(f*x+e)^(1/2)*InverseJacobiAM(1/2*f*x+1/2*e,2^(1/2))*(b*sec(f*
x+e))^(1/2)/f+4/3*b^3*sin(f*x+e)/f/(b*sec(f*x+e))^(1/2)+2/3*b*(b*sec(f*x+e
))^(3/2)*sin(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.64

$$\int (b\sec(e+fx))^{5/2}\sin^4(e+fx)\,dx = -\frac{b^2\sqrt{b\sec(e+fx)}\left(8\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)-\sin(2(e+fx))-2\tan(e+fx)\right)}{3f}$$

input
```
Integrate[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^4,x]
```

output
```
-1/3*(b^2*Sqrt[b*Sec[e + f*x]]*(8*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2
, 2] - Sin[2*(e + f*x)] - 2*Tan[e + f*x]))/f
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3104, 3042, 3107, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(e+fx)(b\sec(e+fx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\sec(e+fx))^{5/2}}{\csc(e+fx)^4}dx$$

$$\Big\downarrow \text{3104}$$

$$\frac{2b\sin^3(e+fx)(b\sec(e+fx))^{3/2}}{3f}-2b^2\int \sqrt{b\sec(e+fx)}\sin^2(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\sin^3(e+fx)(b\sec(e+fx))^{3/2}}{3f}-2b^2\int \frac{\sqrt{b\sec(e+fx)}}{\csc(e+fx)^2}dx$$

$$\Big\downarrow \text{3107}$$

$$\frac{2b\sin^3(e+fx)(b\sec(e+fx))^{3/2}}{3f}-2b^2\left(\frac{2}{3}\int \sqrt{b\sec(e+fx)}dx-\frac{2b\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\sin^3(e+fx)(b\sec(e+fx))^{3/2}}{3f}-2b^2\left(\frac{2}{3}\int \sqrt{b\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{2b\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}\right)$$

$$\Big\downarrow \text{4258}$$

$$\frac{2b\sin^3(e+fx)(b\sec(e+fx))^{3/2}}{3f}-$$
$$2b^2\left(\frac{2}{3}\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\int\frac{1}{\sqrt{\cos(e+fx)}}dx-\frac{2b\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\sin^3(e+fx)(b\sec(e+fx))^{3/2}}{3f}-$$
$$2b^2\left(\frac{2}{3}\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\int\frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{2b\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}\right)$$

$$\Big\downarrow \text{3120}$$

$$\frac{2b\sin^3(e+fx)(b\sec(e+fx))^{3/2}}{3f}-$$
$$2b^2\left(\frac{4\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{3f}-\frac{2b\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}\right)$$

input
```
Int[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^4,x]
```

output
```
(2*b*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^3)/(3*f) - 2*b^2*((4*Sqrt[Cos[e +
 f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(3*f) - (2*b*Sin[e
+ f*x])/(3*f*Sqrt[b*Sec[e + f*x]]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3104
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)/
(f*a*(n - 1))), x] + Simp[b^2*((m + 1)/(a^2*(n - 1)))   Int[(a*Csc[e + f*x]
)^(m + 2)*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[n, 1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3107
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Simp[(m + 1)/(a^2*(m + n))   Int[(a*Csc[e + f*x])^(m
+ 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 151.63 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04

method	result
default	$-\frac{2b^2\sqrt{b\sec(fx+e)}\left(\left(-\cos(fx+e)^2-1\right)\tan(fx+e)+i(4\cos(fx+e)+4)\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\ \mathrm{EllipticF}(i(\cot(fx+e)-\mathrm{cs}}{3f}$

input
```
int((b*sec(f*x+e))^(5/2)*sin(f*x+e)^4,x,method=_RETURNVERBOSE)
```

output
```
-2/3/f*b^2*(b*sec(f*x+e))^(1/2)*((-cos(f*x+e)^2-1)*tan(f*x+e)+I*(4*cos(f*x
+e)+4)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elliptic
F(I*(cot(f*x+e)-csc(f*x+e)),I))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.14

$$\int (b\sec(e+fx))^{5/2}\sin^4(e+fx)\,dx =$$
$$-\frac{2\left(-2i\sqrt{2}b^{\frac{5}{2}}\cos\left(fx+e\right)\text{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right))+2i\sqrt{2}b^{\frac{5}{2}}\cos\left(fx+\right.\right.}{3\,f\,\text{co}}$$

input
```
integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^4,x, algorithm="fricas")
```

output
```
-2/3*(-2*I*sqrt(2)*b^(5/2)*cos(f*x + e)*weierstrassPInverse(-4, 0, cos(f*x
 + e) + I*sin(f*x + e)) + 2*I*sqrt(2)*b^(5/2)*cos(f*x + e)*weierstrassPInv
erse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) - (b^2*cos(f*x + e)^2 + b^2)*sq
rt(b/cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{5/2}\sin^4(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((b*sec(f*x+e))**(5/2)*sin(f*x+e)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\sec(e+fx))^{5/2}\sin^4(e+fx)\,dx = \int (b\sec(fx+e))^{\frac{5}{2}}\sin(fx+e)^4\,dx$$

input
```
integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^(5/2)*sin(f*x + e)^4, x)
```

Giac [F]

$$\int (b\sec(e+fx))^{5/2}\sin^4(e+fx)\,dx = \int (b\sec(fx+e))^{\frac{5}{2}}\sin(fx+e)^4\,dx$$

input
```
integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^(5/2)*sin(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{5/2}\sin^4(e+fx)\,dx = \int \sin(e+f\,x)^4\left(\frac{b}{\cos(e+f\,x)}\right)^{5/2}dx$$

input
```
int(sin(e + f*x)^4*(b/cos(e + f*x))^(5/2),x)
```

output
```
int(sin(e + f*x)^4*(b/cos(e + f*x))^(5/2), x)
```

Reduce [F]

$$\int (b\sec(e+fx))^{5/2}\sin^4(e+fx)\,dx = \sqrt{b}\left(\int \sqrt{\sec(fx+e)}\,\sec(fx+e)^2\sin(fx+e)^4\,dx\right)b^2$$

input
```
int((b*sec(f*x+e))^(5/2)*sin(f*x+e)^4,x)
```

output
```
sqrt(b)*int(sqrt(sec(e + f*x))*sec(e + f*x)**2*sin(e + f*x)**4,x)*b**2
```

3.406 $\int (b\sec(e+fx))^{5/2}\sin^2(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 70

$$\int (b\sec(e+fx))^{5/2}\sin^2(e+fx)\,dx = -\frac{4b^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{3f} + \frac{2b(b\sec(e+fx))^{3/2}\sin(e+fx)}{3f}$$

output
```
-4/3*b^2*cos(f*x+e)^(1/2)*InverseJacobiAM(1/2*f*x+1/2*e,2^(1/2))*(b*sec(f*
x+e))^(1/2)/f+2/3*b*(b*sec(f*x+e))^(3/2)*sin(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.74

$$\int (b\sec(e+fx))^{5/2}\sin^2(e+fx)\,dx = \frac{2b^2\sqrt{b\sec(e+fx)}\left(-2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)+\tan(e+fx)\right)}{3f}$$

input
```
Integrate[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^2,x]
```

output
```
(2*b^2*Sqrt[b*Sec[e + f*x]]*(-2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2,
2] + Tan[e + f*x]))/(3*f)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3104, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(e+fx)(b\sec(e+fx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\sec(e+fx))^{5/2}}{\csc(e+fx)^2}dx$$

$$\Big\downarrow \text{3104}$$

$$\frac{2b\sin(e+fx)(b\sec(e+fx))^{3/2}}{3f}-\frac{2}{3}b^2\int \sqrt{b\sec(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\sin(e+fx)(b\sec(e+fx))^{3/2}}{3f}-\frac{2}{3}b^2\int \sqrt{b\csc\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4258}$$

$$\frac{2b\sin(e+fx)(b\sec(e+fx))^{3/2}}{3f}-\frac{2}{3}b^2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b\sin(e+fx)(b\sec(e+fx))^{3/2}}{3f}-\frac{2}{3}b^2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3120}$$

$$\frac{2b\sin(e+fx)(b\sec(e+fx))^{3/2}}{3f}-\frac{4b^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{3f}$$

input
```
Int[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^2,x]
```

output
```
(-4*b^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])
/(3*f) + (2*b*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x])/(3*f)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3104
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)/
(f*a*(n - 1))), x] + Simp[b^2*((m + 1)/(a^2*(n - 1)))   Int[(a*Csc[e + f*x]
)^(m + 2)*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[n, 1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 14.86 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.29

method	result	size
default	$\frac{\left(\frac{2\tan(fx+e)}{3}-\frac{4i(\cos(fx+e)+1)\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{EllipticF}(i(\cot(fx+e)-\csc(fx+e)),i)}{3}\right)b^2\sqrt{b\sec(fx+e)}}{f}$	90

input
```
int((b*sec(f*x+e))^(5/2)*sin(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(2/3*tan(f*x+e)-4/3*I*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*(1/(cos(f
*x+e)+1))^(1/2)*(cos(f*x+e)+1)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*b^2*(b*s
ec(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.44

$$\int (b\sec(e+fx))^{5/2}\sin^2(e+fx)\,dx = -\frac{2\left(-i\sqrt{2}b^{\frac{5}{2}}\cos(fx+e)\,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e))+i\sqrt{2}b^{\frac{5}{2}}\cos(fx+e)\right.}{3\,f\cos(fx+e)}$$

input
```
integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^2,x, algorithm="fricas")
```

output
```
-2/3*(-I*sqrt(2)*b^(5/2)*cos(f*x + e)*weierstrassPInverse(-4, 0, cos(f*x +
 e) + I*sin(f*x + e)) + I*sqrt(2)*b^(5/2)*cos(f*x + e)*weierstrassPInverse
(-4, 0, cos(f*x + e) - I*sin(f*x + e)) - b^2*sqrt(b/cos(f*x + e))*sin(f*x
+ e))/(f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{5/2}\sin^2(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((b*sec(f*x+e))**(5/2)*sin(f*x+e)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\sec(e+fx))^{5/2}\sin^2(e+fx)\,dx = \int (b\sec(fx+e))^{\frac{5}{2}}\sin(fx+e)^2\,dx$$

input
```
integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^(5/2)*sin(f*x + e)^2, x)
```

Giac [F]

$$\int (b\sec(e+fx))^{5/2}\sin^2(e+fx)\,dx = \int (b\sec(fx+e))^{\frac{5}{2}}\sin(fx+e)^2\,dx$$

input
```
integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^(5/2)*sin(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{5/2}\sin^2(e+fx)\,dx = \int \sin(e+f\,x)^2\left(\frac{b}{\cos(e+f\,x)}\right)^{5/2}dx$$

input
```
int(sin(e + f*x)^2*(b/cos(e + f*x))^(5/2),x)
```

output
```
int(sin(e + f*x)^2*(b/cos(e + f*x))^(5/2), x)
```

Reduce [F]

$$\int (b\sec(e+fx))^{5/2}\sin^2(e+fx)\,dx = \sqrt{b}\left(\int \sqrt{\sec\left(fx+e\right)}\,\sec\left(fx+e\right)^2\sin\left(fx+e\right)^2dx\right)b^2$$

input
```
int((b*sec(f*x+e))^(5/2)*sin(f*x+e)^2,x)
```

output
```
sqrt(b)*int(sqrt(sec(e + f*x))*sec(e + f*x)**2*sin(e + f*x)**2,x)*b**2
```

3.407 $\int (b\sec(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 70

$$\int (b\sec(e+fx))^{5/2}\,dx = \frac{2b^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{3f} + \frac{2b(b\sec(e+fx))^{3/2}\sin(e+fx)}{3f}$$

output
```
2/3*b^2*cos(f*x+e)^(1/2)*InverseJacobiAM(1/2*f*x+1/2*e,2^(1/2))*(b*sec(f*x
+e))^(1/2)/f+2/3*b*(b*sec(f*x+e))^(3/2)*sin(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.73

$$\int (b\sec(e+fx))^{5/2}\,dx = \frac{2b^2\sqrt{b\sec(e+fx)}\left(\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)+\tan(e+fx)\right)}{3f}$$

input
```
Integrate[(b*Sec[e + f*x])^(5/2),x]
```

output
```
(2*b^2*Sqrt[b*Sec[e + f*x]]*(Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]
+ Tan[e + f*x]))/(3*f)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\sec(e+fx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{4255}$$

$$\frac{1}{3}b^2\int \sqrt{b\sec(e+fx)}dx+\frac{2b\sin(e+fx)(b\sec(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}b^2\int \sqrt{b\csc\left(e+fx+\frac{\pi}{2}\right)}dx+\frac{2b\sin(e+fx)(b\sec(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \text{4258}$$

$$\frac{1}{3}b^2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}}dx+\frac{2b\sin(e+fx)(b\sec(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}b^2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx+\frac{2b\sin(e+fx)(b\sec(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2b^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{3f}+\frac{2b\sin(e+fx)(b\sec(e+fx))^{3/2}}{3f}$$

input
```
Int[(b*Sec[e + f*x])^(5/2),x]
```

output
```
(2*b^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/
(3*f) + (2*b*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x])/(3*f)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.61 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.29

method	result	size
default	$\frac{\left(\frac{2i(\cos(fx+e)+1)\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{EllipticF}(i(\cot(fx+e)-\csc(fx+e)),i)}{3}+\frac{2\tan(fx+e)}{3}\right)b^{2}\sqrt{b\sec(fx+e)}}{f}$	90

input
```
int((b*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(2/3*I*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)+1)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+2/3*tan(f*x+e))*b^2*(b*s
ec(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.44

$$\int (b\sec(e+fx))^{5/2}\,dx = \frac{-i\sqrt{2}b^{\frac{5}{2}}\cos(fx+e)\,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e))+i\sqrt{2}b^{\frac{5}{2}}\cos}{3f\cos}$$

input
```
integrate((b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*b^(5/2)*cos(f*x + e)*weierstrassPInverse(-4, 0, cos(f*x +
e) + I*sin(f*x + e)) + I*sqrt(2)*b^(5/2)*cos(f*x + e)*weierstrassPInverse(
-4, 0, cos(f*x + e) - I*sin(f*x + e)) + 2*b^2*sqrt(b/cos(f*x + e))*sin(f*x
 + e))/(f*cos(f*x + e))
```

Sympy [F]

$$\int (b\sec(e+fx))^{5/2}\,dx = \int (b\sec(e+fx))^{\frac{5}{2}}\,dx$$

input
```
integrate((b*sec(f*x+e))**(5/2),x)
```

output
```
Integral((b*sec(e + f*x))**(5/2), x)
```

Maxima [F]

$$\int (b\sec(e+fx))^{5/2}\,dx = \int (b\sec(fx+e))^{\frac{5}{2}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^(5/2), x)
```

Giac [F]

$$\int (b\sec(e+fx))^{5/2}\,dx = \int (b\sec(fx+e))^{\frac{5}{2}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^{5/2}\,dx = \int \left(\frac{b}{\cos(e+f\,x)}\right)^{5/2}\,dx$$

input
```
int((b/cos(e + f*x))^(5/2),x)
```

output
```
int((b/cos(e + f*x))^(5/2), x)
```

Reduce [F]

$$\int (b\sec(e+fx))^{5/2}\,dx = \sqrt{b}\left(\int \sqrt{\sec(fx+e)}\,\sec(fx+e)^2\,dx\right)b^2$$

input
```
int((b*sec(f*x+e))^(5/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(e + f*x))*sec(e + f*x)**2,x)*b**2
```

3.408 $\int \csc^2(e+fx)(b\sec(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int \csc^2(e+fx)(b\sec(e+fx))^{5/2}\,dx = -\frac{5b^3\csc(e+fx)}{3f\sqrt{b\sec(e+fx)}} + \frac{5b^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{3f} + \frac{2b\csc(e+fx)(b\sec(e+fx))^{3/2}}{3f}$$

output
```
-5/3*b^3*csc(f*x+e)/f/(b*sec(f*x+e))^(1/2)+5/3*b^2*cos(f*x+e)^(1/2)*Invers
eJacobiAM(1/2*f*x+1/2*e,2^(1/2))*(b*sec(f*x+e))^(1/2)/f+2/3*b*csc(f*x+e)*(
b*sec(f*x+e))^(3/2)/f
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.68

$$\int \csc^2(e+fx)(b\sec(e+fx))^{5/2}\,dx = \frac{b\left(2-3\cot^2(e+fx)+5\cos^{\frac{3}{2}}(e+fx)\csc(e+fx)\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\right)(b\sec(e+fx))}{3f}$$

input
```
Integrate[Csc[e + f*x]^2*(b*Sec[e + f*x])^(5/2),x]
```

output
```
(b*(2 - 3*Cot[e + f*x]^2 + 5*Cos[e + f*x]^(3/2)*Csc[e + f*x]*EllipticF[(e
+ f*x)/2, 2])*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x])/(3*f)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3106, 3042, 3105, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(e+fx)(b\sec(e+fx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(e+fx)^2(b\sec(e+fx))^{5/2}dx$$

$$\Big\downarrow \text{3106}$$

$$\frac{5}{3}b^2\int \csc^2(e+fx)\sqrt{b\sec(e+fx)}dx+\frac{2b\csc(e+fx)(b\sec(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{3}b^2\int \csc(e+fx)^2\sqrt{b\sec(e+fx)}dx+\frac{2b\csc(e+fx)(b\sec(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \text{3105}$$

$$\frac{5}{3}b^2\left(\frac{1}{2}\int \sqrt{b\sec(e+fx)}dx-\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}\right)+\frac{2b\csc(e+fx)(b\sec(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{3}b^2\left(\frac{1}{2}\int \sqrt{b\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}\right)+\frac{2b\csc(e+fx)(b\sec(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \text{4258}$$

$$\frac{5}{3}b^2\left(\frac{1}{2}\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\int\frac{1}{\sqrt{\cos(e+fx)}}dx-\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}\right)+\frac{2b\csc(e+fx)(b\sec(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \; 3042$$

$$\frac{5}{3}b^2\left(\frac{1}{2}\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\int\frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}\right)+\frac{2b\csc(e+fx)(b\sec(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \; 3120$$

$$\frac{5}{3}b^2\left(\frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{f}-\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}\right)+\frac{2b\csc(e+fx)(b\sec(e+fx))^{3/2}}{3f}$$

input
```
Int[Csc[e + f*x]^2*(b*Sec[e + f*x])^(5/2),x]
```

output
```
(2*b*Csc[e + f*x]*(b*Sec[e + f*x])^(3/2))/(3*f) + (5*b^2*(-((b*Csc[e + f*x
])/(f*Sqrt[b*Sec[e + f*x]])) + (Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2,
2]*Sqrt[b*Sec[e + f*x]])/f))/3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3105
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Simp[a^2*((m + n - 2)/(m - 1))   Int[(a*Csc[e + f*
x])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[
m, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

rule 3106
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n -
1)/(f*(n - 1))), x] + Simp[b^2*((m + n - 2)/(n - 1))   Int[(a*Csc[e + f*x])
^m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n,
1] && IntegersQ[2*m, 2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.36 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.06

method	result
default	$\frac{\left(\frac{2\sec(fx+e)\csc(fx+e)}{3}-\frac{5\cot(fx+e)}{3}+\frac{5i(\cos(fx+e)+1)\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{EllipticF}(i(\cot(fx+e)-\csc(fx+e)),i)}{3}\right)b^2\sqrt{b\sec(f}}{f}$

input
```
int(csc(f*x+e)^2*(b*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(2/3*sec(f*x+e)*csc(f*x+e)-5/3*cot(f*x+e)+5/3*I*EllipticF(I*(cot(f*x+e
)-csc(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)+1)*(cos(f*x+e)/(cos(
f*x+e)+1))^(1/2))*b^2*(b*sec(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.34

$$\int \csc^2(e+fx)(b\sec(e+fx))^{5/2}\,dx = \frac{-5i\sqrt{2}b^{\frac{5}{2}}\cos(fx+e)\sin(fx+e)\,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e))}{}$$

input
```
integrate(csc(f*x+e)^2*(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
1/6*(-5*I*sqrt(2)*b^(5/2)*cos(f*x + e)*sin(f*x + e)*weierstrassPInverse(-4
, 0, cos(f*x + e) + I*sin(f*x + e)) + 5*I*sqrt(2)*b^(5/2)*cos(f*x + e)*sin
(f*x + e)*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) - 2*(5
*b^2*cos(f*x + e)^2 - 2*b^2)*sqrt(b/cos(f*x + e)))/(f*cos(f*x + e)*sin(f*x
 + e))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^2(e+fx)(b\sec(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**2*(b*sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \csc^2(e+fx)(b\sec(e+fx))^{5/2}\,dx = \int (b\sec(fx+e))^{\frac{5}{2}}\csc(fx+e)^2\,dx$$

input
```
integrate(csc(f*x+e)^2*(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^(5/2)*csc(f*x + e)^2, x)
```

Giac [F]

$$\int \csc^2(e+fx)(b\sec(e+fx))^{5/2}\,dx = \int (b\sec(fx+e))^{\frac{5}{2}}\csc(fx+e)^2\,dx$$

input
```
integrate(csc(f*x+e)^2*(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^(5/2)*csc(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(e+fx)(b\sec(e+fx))^{5/2}\,dx = \int \frac{\left(\frac{b}{\cos(e+f\,x)}\right)^{5/2}}{\sin\left(e+f\,x\right)^2}\,dx$$

input
```
int((b/cos(e + f*x))^(5/2)/sin(e + f*x)^2,x)
```

output
```
int((b/cos(e + f*x))^(5/2)/sin(e + f*x)^2, x)
```

Reduce [F]

$$\int \csc^2(e+fx)(b\sec(e+fx))^{5/2}\,dx = \sqrt{b}\left(\int \sqrt{\sec(fx+e)}\,\csc(fx+e)^2\sec(fx+e)^2\,dx\right)b^2$$

input
```
int(csc(f*x+e)^2*(b*sec(f*x+e))^(5/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(e + f*x))*csc(e + f*x)**2*sec(e + f*x)**2,x)*b**2
```

3.409 $\int \csc^4(e+fx)(b\sec(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 123

$$\int \csc^4(e+fx)(b\sec(e+fx))^{5/2}\,dx = -\frac{5b^3\csc(e+fx)}{2f\sqrt{b\sec(e+fx)}} + \frac{5b^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{2f} + \frac{b\csc(e+fx)(b\sec(e+fx))^{3/2}}{f} - \frac{b\csc^3(e+fx)(b\sec(e+fx))^{3/2}}{3f}$$

output
```
-5/2*b^3*csc(f*x+e)/f/(b*sec(f*x+e))^(1/2)+5/2*b^2*cos(f*x+e)^(1/2)*Invers
eJacobiAM(1/2*f*x+1/2*e,2^(1/2))*(b*sec(f*x+e))^(1/2)/f+b*csc(f*x+e)*(b*se
c(f*x+e))^(3/2)/f-1/3*b*csc(f*x+e)^3*(b*sec(f*x+e))^(3/2)/f
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.64

$$\int \csc^4(e+fx)(b\sec(e+fx))^{5/2}\,dx = \frac{b\Big(4-\cot^2(e+fx)\,(11+2\csc^2(e+fx))+15\cos^{\frac{3}{2}}(e+fx)\csc(e+fx)\,\text{EllipticF}\left(\frac{1}{2}(e+fx\right.}{6f}$$

input
```
Integrate[Csc[e + f*x]^4*(b*Sec[e + f*x])^(5/2),x]
```

output
```
(b*(4 - Cot[e + f*x]^2*(11 + 2*Csc[e + f*x]^2) + 15*Cos[e + f*x]^(3/2)*Csc
[e + f*x]*EllipticF[(e + f*x)/2, 2])*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x])/
(6*f)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3105, 3042, 3106, 3042, 3105, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(e+fx)(b\sec(e+fx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(e+fx)^4(b\sec(e+fx))^{5/2}dx$$

$$\Big\downarrow \text{3105}$$

$$\frac{3}{2}\int \csc^2(e+fx)(b\sec(e+fx))^{5/2}dx-\frac{b\csc^3(e+fx)(b\sec(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}\int \csc(e+fx)^2(b\sec(e+fx))^{5/2}dx-\frac{b\csc^3(e+fx)(b\sec(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \text{3106}$$

$$\frac{3}{2}\left(\frac{5}{3}b^2\int \csc^2(e+fx)\sqrt{b\sec(e+fx)}dx+\frac{2b\csc(e+fx)(b\sec(e+fx))^{3/2}}{3f}\right)-\frac{b\csc^3(e+fx)(b\sec(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}\left(\frac{5}{3}b^2\int \csc(e+fx)^2\sqrt{b\sec(e+fx)}dx+\frac{2b\csc(e+fx)(b\sec(e+fx))^{3/2}}{3f}\right)-\frac{b\csc^3(e+fx)(b\sec(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \text{3105}$$

$$\frac{3}{2}\left(\frac{5}{3}b^2\left(\frac{1}{2}\int \sqrt{b\sec(e+fx)}dx-\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}\right)+\frac{2b\csc(e+fx)(b\sec(e+fx))^{3/2}}{3f}\right)-\frac{b\csc^3(e+fx)(b\sec(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}\left(\frac{5}{3}b^2\left(\frac{1}{2}\int \sqrt{b\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}\right)+\frac{2b\csc(e+fx)(b\sec(e+fx))^{3/2}}{3f}\right)-\frac{b\csc^3(e+fx)(b\sec(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \text{4258}$$

$$\frac{3}{2}\left(\frac{5}{3}b^2\left(\frac{1}{2}\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}}dx-\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}\right)+\frac{2b\csc(e+fx)(b\sec(e+fx\right.}{3f}$$
$$\frac{b\csc^3(e+fx)(b\sec(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{2}\left(\frac{5}{3}b^2\left(\frac{1}{2}\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}\right)+\frac{2b\csc(e+fx)(b\sec(e\right.}{3f}$$
$$\frac{b\csc^3(e+fx)(b\sec(e+fx))^{3/2}}{3f}$$

$$\Big\downarrow \text{3120}$$

$$\frac{3}{2}\left(\frac{5}{3}b^2\left(\frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{f}-\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}\right)+\frac{2b\csc(e+fx)(b\sec(e+\right.}{3f}$$
$$\frac{b\csc^3(e+fx)(b\sec(e+fx))^{3/2}}{3f}$$

input
```
Int[Csc[e + f*x]^4*(b*Sec[e + f*x])^(5/2),x]
```

output
```
-1/3*(b*Csc[e + f*x]^3*(b*Sec[e + f*x])^(3/2))/f + (3*((2*b*Csc[e + f*x]*(
b*Sec[e + f*x])^(3/2))/(3*f) + (5*b^2*(-((b*Csc[e + f*x])/(f*Sqrt[b*Sec[e
+ f*x]])) + (Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f
*x]])/f))/3))/2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3105
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Simp[a^2*((m + n - 2)/(m - 1))   Int[(a*Csc[e + f*
x])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[
m, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

rule 3106
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n -
1)/(f*(n - 1))), x] + Simp[b^2*((m + n - 2)/(n - 1))   Int[(a*Csc[e + f*x])
^m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n,
1] && IntegersQ[2*m, 2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 53.49 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.01

method	result
default	$\frac{\left(\frac{2\sec(fx+e)\csc(fx+e)^3}{3}+\frac{5\cot(fx+e)^3}{2}-\frac{7\cot(fx+e)\csc(fx+e)^2}{2}+\frac{5i(\cos(fx+e)+1)\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\text{EllipticF}(i(\cot(fx+e)-\right.}{f}$

input
```
int(csc(f*x+e)^4*(b*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(2/3*sec(f*x+e)*csc(f*x+e)^3+5/2*cot(f*x+e)^3-7/2*cot(f*x+e)*csc(f*x+e
)^2+5/2*I*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*
(cos(f*x+e)+1)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*b^2*(b*sec(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.57

$$\int \csc^4(e+fx)(b\sec(e+fx))^{5/2}\,dx =$$

$$-\frac{15\sqrt{2}\left(i\,b^2\cos\left(fx+e\right)^3-i\,b^2\cos\left(fx+e\right)\right)\sqrt{b}\sin\left(fx+e\right)\text{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\,\text{s}}{}$$

input
```
integrate(csc(f*x+e)^4*(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(15*sqrt(2)*(I*b^2*cos(f*x + e)^3 - I*b^2*cos(f*x + e))*sqrt(b)*sin(
f*x + e)*weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e)) + 15*sq
rt(2)*(-I*b^2*cos(f*x + e)^3 + I*b^2*cos(f*x + e))*sqrt(b)*sin(f*x + e)*we
ierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) + 2*(15*b^2*cos(f*
x + e)^4 - 21*b^2*cos(f*x + e)^2 + 4*b^2)*sqrt(b/cos(f*x + e)))/((f*cos(f*
x + e)^3 - f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^4(e+fx)(b\sec(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**4*(b*sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \csc^4(e+fx)(b\sec(e+fx))^{5/2}\,dx = \int (b\sec(fx+e))^{\frac{5}{2}}\csc(fx+e)^4\,dx$$

input
```
integrate(csc(f*x+e)^4*(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^(5/2)*csc(f*x + e)^4, x)
```

Giac [F]

$$\int \csc^4(e+fx)(b\sec(e+fx))^{5/2}\,dx = \int (b\sec(fx+e))^{\frac{5}{2}}\csc(fx+e)^4\,dx$$

input
```
integrate(csc(f*x+e)^4*(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^(5/2)*csc(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^4(e+fx)(b\sec(e+fx))^{5/2}\,dx=\int \frac{\left(\frac{b}{\cos(e+f\,x)}\right)^{5/2}}{\sin\left(e+f\,x\right)^4}\,dx$$

input
```
int((b/cos(e + f*x))^(5/2)/sin(e + f*x)^4,x)
```

output
```
int((b/cos(e + f*x))^(5/2)/sin(e + f*x)^4, x)
```

Reduce [F]

$$\int \csc^4(e+fx)(b\sec(e+fx))^{5/2}\,dx=\sqrt{b}\left(\int \sqrt{\sec\left(fx+e\right)}\,\csc\left(fx+e\right)^4\sec\left(fx+e\right)^2dx\right)b^2$$

input
```
int(csc(f*x+e)^4*(b*sec(f*x+e))^(5/2),x)
```

output
```
sqrt(b)*int(sqrt(sec(e + f*x))*csc(e + f*x)**4*sec(e + f*x)**2,x)*b**2
```

3.410 $\int \frac{\sin^7(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \frac{\sin^7(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{2b^7}{15f(b\sec(e+fx))^{15/2}} - \frac{6b^5}{11f(b\sec(e+fx))^{11/2}} + \frac{6b^3}{7f(b\sec(e+fx))^{7/2}} - \frac{2b}{3f(b\sec(e+fx))^{3/2}}$$

output
```
2/15*b^7/f/(b*sec(f*x+e))^(15/2)-6/11*b^5/f/(b*sec(f*x+e))^(11/2)+6/7*b^3/
f/(b*sec(f*x+e))^(7/2)-2/3*b/f/(b*sec(f*x+e))^(3/2)
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.60

$$\int \frac{\sin^7(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{b(-7410+4035\cos(2(e+fx))-798\cos(4(e+fx))+77\cos(6(e+fx)))}{18480f(b\sec(e+fx))^{3/2}}$$

input
```
Integrate[Sin[e + f*x]^7/Sqrt[b*Sec[e + f*x]],x]
```

output
```
(b*(-7410 + 4035*Cos[2*(e + f*x)] - 798*Cos[4*(e + f*x)] + 77*Cos[6*(e + f
*x)]))/(18480*f*(b*Sec[e + f*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3102, 25, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^7(e+fx)}{\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc(e+fx)^7\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{b^7\int -\frac{\left(b^2-b^2\sec^2(e+fx)\right)^3}{b^6(b\sec(e+fx))^{17/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b^7\int \frac{\left(b^2-b^2\sec^2(e+fx)\right)^3}{b^6(b\sec(e+fx))^{17/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\int \frac{\left(b^2-b^2\sec^2(e+fx)\right)^3}{(b\sec(e+fx))^{17/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{244}$$

$$-\frac{b\int \left(\frac{b^6}{(b\sec(e+fx))^{17/2}}-\frac{3b^4}{(b\sec(e+fx))^{13/2}}+\frac{3b^2}{(b\sec(e+fx))^{9/2}}-\frac{1}{(b\sec(e+fx))^{5/2}}\right)d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\left(-\frac{2b^6}{15(b\sec(e+fx))^{15/2}}+\frac{6b^4}{11(b\sec(e+fx))^{11/2}}-\frac{6b^2}{7(b\sec(e+fx))^{7/2}}+\frac{2}{3(b\sec(e+fx))^{3/2}}\right)}{f}$$

input
```
Int[Sin[e + f*x]^7/Sqrt[b*Sec[e + f*x]],x]
```

output
```
-((b*((-2*b^6)/(15*(b*Sec[e + f*x])^(15/2)) + (6*b^4)/(11*(b*Sec[e + f*x])
^(11/2)) - (6*b^2)/(7*(b*Sec[e + f*x])^(7/2)) + 2/(3*(b*Sec[e + f*x])^(3/2
))))/f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.62

method	result	size
default	$\frac{\frac{2\cos(fx+e)^7}{15}-\frac{6\cos(fx+e)^5}{11}+\frac{6\cos(fx+e)^3}{7}-\frac{2\cos(fx+e)}{3}}{f\sqrt{b\sec(fx+e)}}$	54

input
```
int(sin(f*x+e)^7/(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(2/15*cos(f*x+e)^7-6/11*cos(f*x+e)^5+6/7*cos(f*x+e)^3-2/3*cos(f*x+e))/
(b*sec(f*x+e))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.70

$$\int \frac{\sin^7(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{2\left(77\cos\left(fx+e\right)^8-315\cos\left(fx+e\right)^6+495\cos\left(fx+e\right)^4-385\cos\left(fx+e\right)^2\right)\sqrt{\frac{b}{\cos(fx+e)}}}{1155\,bf}$$

input
```
integrate(sin(f*x+e)^7/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
2/1155*(77*cos(f*x + e)^8 - 315*cos(f*x + e)^6 + 495*cos(f*x + e)^4 - 385*
cos(f*x + e)^2)*sqrt(b/cos(f*x + e))/(b*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^7(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**7/(b*sec(f*x+e))**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

$$\int \frac{\sin^7(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{2\left(77\,b^6 - \frac{315\,b^6}{\cos(fx+e)^2} + \frac{495\,b^6}{\cos(fx+e)^4} - \frac{385\,b^6}{\cos(fx+e)^6}\right)b}{1155\,f\left(\frac{b}{\cos(fx+e)}\right)^{\frac{15}{2}}}$$

input
```
integrate(sin(f*x+e)^7/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
2/1155*(77*b^6 - 315*b^6/cos(f*x + e)^2 + 495*b^6/cos(f*x + e)^4 - 385*b^6
/cos(f*x + e)^6)*b/(f*(b/cos(f*x + e))^(15/2))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.24

$$\int \frac{\sin^7(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{2\left(77\sqrt{b\cos(fx+e)}b^7\cos(fx+e)^7 - 315\sqrt{b\cos(fx+e)}b^7\cos(fx+e)^5 + 495\sqrt{b\cos(fx+e)}b^7\cos\right.}{1155\,b^8 f\,\mathrm{sgn}(\cos(fx+e))}$$

input
```
integrate(sin(f*x+e)^7/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
2/1155*(77*sqrt(b*cos(f*x + e))*b^7*cos(f*x + e)^7 - 315*sqrt(b*cos(f*x +
e))*b^7*cos(f*x + e)^5 + 495*sqrt(b*cos(f*x + e))*b^7*cos(f*x + e)^3 - 385
*sqrt(b*cos(f*x + e))*b^7*cos(f*x + e))/(b^8*f*sgn(cos(f*x + e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^7(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx=\int \frac{\sin\left(e+f\,x\right)^7}{\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}}\,dx$$

input
```
int(sin(e + f*x)^7/(b/cos(e + f*x))^(1/2),x)
```

output
```
int(sin(e + f*x)^7/(b/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sin^7(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx=\frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(fx+e)}\,\sin(fx+e)^7}{\sec(fx+e)}dx\right)}{b}$$

input
```
int(sin(f*x+e)^7/(b*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(e + f*x))*sin(e + f*x)**7)/sec(e + f*x),x))/b
```

3.411 $\int \frac{\sin^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \frac{\sin^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{2b^5}{11f(b\sec(e+fx))^{11/2}} + \frac{4b^3}{7f(b\sec(e+fx))^{7/2}} - \frac{2b}{3f(b\sec(e+fx))^{3/2}}$$

output
```
-2/11*b^5/f/(b*sec(f*x+e))^(11/2)+4/7*b^3/f/(b*sec(f*x+e))^(7/2)-2/3*b/f/(
b*sec(f*x+e))^(3/2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.65

$$\int \frac{\sin^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{b(-415+180\cos(2(e+fx))-21\cos(4(e+fx)))}{924f(b\sec(e+fx))^{3/2}}$$

input
```
Integrate[Sin[e + f*x]^5/Sqrt[b*Sec[e + f*x]],x]
```

output
```
(b*(-415 + 180*Cos[2*(e + f*x)] - 21*Cos[4*(e + f*x)]))/(924*f*(b*Sec[e +
f*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 3102, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(e+fx)}{\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc(e+fx)^5\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{b^5\int \frac{\left(b^2-b^2\sec^2(e+fx)\right)^2}{b^4(b\sec(e+fx))^{13/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{\left(b^2-b^2\sec^2(e+fx)\right)^2}{(b\sec(e+fx))^{13/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{244}$$

$$\frac{b\int \left(\frac{b^4}{(b\sec(e+fx))^{13/2}}-\frac{2b^2}{(b\sec(e+fx))^{9/2}}+\frac{1}{(b\sec(e+fx))^{5/2}}\right)d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(-\frac{2b^4}{11(b\sec(e+fx))^{11/2}}+\frac{4b^2}{7(b\sec(e+fx))^{7/2}}-\frac{2}{3(b\sec(e+fx))^{3/2}}\right)}{f}$$

input
```
Int[Sin[e + f*x]^5/Sqrt[b*Sec[e + f*x]],x]
```

output
```
(b*((-2*b^4)/(11*(b*Sec[e + f*x])^(11/2)) + (4*b^2)/(7*(b*Sec[e + f*x])^(7
/2)) - 2/(3*(b*Sec[e + f*x])^(3/2))))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.68

method	result	size
default	$\frac{-\frac{2\cos(fx+e)^5}{11}+\frac{4\cos(fx+e)^3}{7}-\frac{2\cos(fx+e)}{3}}{f\sqrt{b\sec(fx+e)}}$	44

input
```
int(sin(f*x+e)^5/(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-2/11*cos(f*x+e)^5+4/7*cos(f*x+e)^3-2/3*cos(f*x+e))/(b*sec(f*x+e))^(1
/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

$$\int \frac{\sin^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{2\left(21\cos\left(fx+e\right)^6-66\cos\left(fx+e\right)^4+77\cos\left(fx+e\right)^2\right)\sqrt{\frac{b}{\cos(fx+e)}}}{231\,bf}$$

input
```
integrate(sin(f*x+e)^5/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-2/231*(21*cos(f*x + e)^6 - 66*cos(f*x + e)^4 + 77*cos(f*x + e)^2)*sqrt(b/
cos(f*x + e))/(b*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**5/(b*sec(f*x+e))**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$\int \frac{\sin^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{2\left(21\,b^4 - \frac{66\,b^4}{\cos(fx+e)^2} + \frac{77\,b^4}{\cos(fx+e)^4}\right)b}{231\,f\left(\frac{b}{\cos(fx+e)}\right)^{\frac{11}{2}}}$$

input
```
integrate(sin(f*x+e)^5/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
-2/231*(21*b^4 - 66*b^4/cos(f*x + e)^2 + 77*b^4/cos(f*x + e)^4)*b/(f*(b/co
s(f*x + e))^(11/2))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.31

$$\int \frac{\sin^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx =$$
$$-\frac{2\left(21\,\sqrt{b\cos\left(fx+e\right)}b^5\cos\left(fx+e\right)^5 - 66\,\sqrt{b\cos\left(fx+e\right)}b^5\cos\left(fx+e\right)^3 + 77\,\sqrt{b\cos\left(fx+e\right)}b^5\cos\right.}{231\,b^6 f\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}$$

input
```
integrate(sin(f*x+e)^5/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
-2/231*(21*sqrt(b*cos(f*x + e))*b^5*cos(f*x + e)^5 - 66*sqrt(b*cos(f*x + e
))*b^5*cos(f*x + e)^3 + 77*sqrt(b*cos(f*x + e))*b^5*cos(f*x + e))/(b^6*f*s
gn(cos(f*x + e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin\left(e+f\,x\right)^5}{\sqrt{\frac{b}{\cos(e+f\,x)}}}\,dx$$

input
```
int(sin(e + f*x)^5/(b/cos(e + f*x))^(1/2),x)
```

output
```
int(sin(e + f*x)^5/(b/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sin^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(fx+e)}\,\sin(fx+e)^5}{\sec(fx+e)}dx\right)}{b}$$

input
```
int(sin(f*x+e)^5/(b*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(e + f*x))*sin(e + f*x)**5)/sec(e + f*x),x))/b
```

3.412 $\int \frac{\sin^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 43

$$\int \frac{\sin^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{2b^3}{7f(b\sec(e+fx))^{7/2}} - \frac{2b}{3f(b\sec(e+fx))^{3/2}}$$

output
```
2/7*b^3/f/(b*sec(f*x+e))^(7/2)-2/3*b/f/(b*sec(f*x+e))^(3/2)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

$$\int \frac{\sin^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{b(-11+3\cos(2(e+fx)))}{21f(b\sec(e+fx))^{3/2}}$$

input
```
Integrate[Sin[e + f*x]^3/Sqrt[b*Sec[e + f*x]],x]
```

output
```
(b*(-11 + 3*Cos[2*(e + f*x)]))/(21*f*(b*Sec[e + f*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3102, 25, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(e+fx)}{\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc(e+fx)^3\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{b^3\int -\frac{b^2-b^2\sec^2(e+fx)}{b^2(b\sec(e+fx))^{9/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b^3\int \frac{b^2-b^2\sec^2(e+fx)}{b^2(b\sec(e+fx))^{9/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\int \frac{b^2-b^2\sec^2(e+fx)}{(b\sec(e+fx))^{9/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{244}$$

$$-\frac{b\int \left(\frac{b^2}{(b\sec(e+fx))^{9/2}}-\frac{1}{(b\sec(e+fx))^{5/2}}\right)d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\left(\frac{2}{3(b\sec(e+fx))^{3/2}}-\frac{2b^2}{7(b\sec(e+fx))^{7/2}}\right)}{f}$$

input
```
Int[Sin[e + f*x]^3/Sqrt[b*Sec[e + f*x]],x]
```

output
```
-((b*((-2*b^2)/(7*(b*Sec[e + f*x])^(7/2)) + 2/(3*(b*Sec[e + f*x])^(3/2))))
/f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{\frac{2\cos(fx+e)^3}{7}-\frac{2\cos(fx+e)}{3}}{f\sqrt{b\sec(fx+e)}}$	34

input
```
int(sin(f*x+e)^3/(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(2/7*cos(f*x+e)^3-2/3*cos(f*x+e))/(b*sec(f*x+e))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \frac{\sin^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{2\left(3\cos\left(fx+e\right)^4-7\cos\left(fx+e\right)^2\right)\sqrt{\frac{b}{\cos(fx+e)}}}{21\,bf}$$

input
```
integrate(sin(f*x+e)^3/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
2/21*(3*cos(f*x + e)^4 - 7*cos(f*x + e)^2)*sqrt(b/cos(f*x + e))/(b*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**3/(b*sec(f*x+e))**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\sin^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{2\left(3\,b^2 - \frac{7\,b^2}{\cos(fx+e)^2}\right)b}{21\,f\left(\frac{b}{\cos(fx+e)}\right)^{\frac{7}{2}}}$$

input
```
integrate(sin(f*x+e)^3/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
2/21*(3*b^2 - 7*b^2/cos(f*x + e)^2)*b/(f*(b/cos(f*x + e))^(7/2))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.44

$$\int \frac{\sin^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{2\left(3\sqrt{b\cos(fx+e)}b^3\cos(fx+e)^3 - 7\sqrt{b\cos(fx+e)}b^3\cos(fx+e)\right)}{21\,b^4 f\,\mathrm{sgn}(\cos(fx+e))}$$

input
```
integrate(sin(f*x+e)^3/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
2/21*(3*sqrt(b*cos(f*x + e))*b^3*cos(f*x + e)^3 - 7*sqrt(b*cos(f*x + e))*b
^3*cos(f*x + e))/(b^4*f*sgn(cos(f*x + e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin(e+f\,x)^3}{\sqrt{\frac{b}{\cos(e+f\,x)}}}\,dx$$

input
```
int(sin(e + f*x)^3/(b/cos(e + f*x))^(1/2),x)
```

output
```
int(sin(e + f*x)^3/(b/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sin^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(fx+e)}\,\sin(fx+e)^3}{\sec(fx+e)}dx\right)}{b}$$

input
```
int(sin(f*x+e)^3/(b*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(e + f*x))*sin(e + f*x)**3)/sec(e + f*x),x))/b
```

3.413 $\int \frac{\sin(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 20

$$\int \frac{\sin(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{2b}{3f(b\sec(e+fx))^{3/2}}$$

output
```
-2/3*b/f/(b*sec(f*x+e))^(3/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sin(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{2b}{3f(b\sec(e+fx))^{3/2}}$$

input
```
Integrate[Sin[e + f*x]/Sqrt[b*Sec[e + f*x]],x]
```

output
```
(-2*b)/(3*f*(b*Sec[e + f*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 3102, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(e+fx)}{\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc(e+fx)\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{b\int \frac{1}{(b\sec(e+fx))^{5/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{15}$$

$$-\frac{2b}{3f(b\sec(e+fx))^{3/2}}$$

input
```
Int[Sin[e + f*x]/Sqrt[b*Sec[e + f*x]],x]
```

output
```
(-2*b)/(3*f*(b*Sec[e + f*x])^(3/2))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$-\frac{2b}{3f(b\sec(fx+e))^{\frac{3}{2}}}$	17
default	$-\frac{2b}{3f(b\sec(fx+e))^{\frac{3}{2}}}$	17

input
```
int(sin(f*x+e)/(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*b/f/(b*sec(f*x+e))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.40

$$\int \frac{\sin(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{2\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)^2}{3\,bf}$$

input
```
integrate(sin(f*x+e)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(b/cos(f*x + e))*cos(f*x + e)^2/(b*f)
```

Sympy [F]

$$\int \frac{\sin(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin{(e+fx)}}{\sqrt{b\sec{(e+fx)}}}\,dx$$

input
```
integrate(sin(f*x+e)/(b*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sin(e + f*x)/sqrt(b*sec(e + f*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{\sin(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{2\,\cos{(fx+e)}}{3\,f\sqrt{\frac{b}{\cos{(fx+e)}}}}$$

input
```
integrate(sin(f*x+e)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
-2/3*cos(f*x + e)/(f*sqrt(b/cos(f*x + e)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(16) = 32.

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int \frac{\sin(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{2\,\sqrt{b\cos{(fx+e)}}\cos{(fx+e)}}{3\,bf\mathrm{sgn}\left(\cos{(fx+e)}\right)}$$

input
```
integrate(sin(f*x+e)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
-2/3*sqrt(b*cos(f*x + e))*cos(f*x + e)/(b*f*sgn(cos(f*x + e)))
```

Mupad [B] (verification not implemented)

Time = 25.77 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.40

$$\int \frac{\sin(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{2\cos\left(e+f\,x\right)^2\,\sqrt{\frac{b}{\cos(e+f\,x)}}}{3\,b\,f}$$

input
```
int(sin(e + f*x)/(b/cos(e + f*x))^(1/2),x)
```

output
```
-(2*cos(e + f*x)^2*(b/cos(e + f*x))^(1/2))/(3*b*f)
```

Reduce [F]

$$\int \frac{\sin(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(fx+e)}\,\sin(fx+e)}{\sec(fx+e)}dx\right)}{b}$$

input
```
int(sin(f*x+e)/(b*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(e + f*x))*sin(e + f*x))/sec(e + f*x),x))/b
```

3.414 $\int \frac{\csc(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 59

$$\int \frac{\csc(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}f} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}f}$$

output
```
-arctan((b*sec(f*x+e))^(1/2)/b^(1/2))/b^(1/2)/f-arctanh((b*sec(f*x+e))^(1/
2)/b^(1/2))/b^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

$$\int \frac{\csc(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{\left(2\arctan\left(\sqrt{\sec(e+fx)}\right) - \log\left(1-\sqrt{\sec(e+fx)}\right) + \log\left(1+\sqrt{\sec(e+fx)}\right)\right)\sqrt{\sec(e+fx)}}{2f\sqrt{b\sec(e+fx)}}$$

input
```
Integrate[Csc[e + f*x]/Sqrt[b*Sec[e + f*x]],x]
```

output
```
-1/2*((2*ArcTan[Sqrt[Sec[e + f*x]]] - Log[1 - Sqrt[Sec[e + f*x]]] + Log[1
+ Sqrt[Sec[e + f*x]]])*Sqrt[Sec[e + f*x]])/(f*Sqrt[b*Sec[e + f*x]])
```

Rubi [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 3102, 25, 27, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(e+fx)}{\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc(e+fx)}{\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{\int -\frac{b^2}{\sqrt{b\sec(e+fx)}(b^2-b^2\sec^2(e+fx))}d(b\sec(e+fx))}{bf}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{b^2}{\sqrt{b\sec(e+fx)}(b^2-b^2\sec^2(e+fx))}d(b\sec(e+fx))}{bf}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\int \frac{1}{\sqrt{b\sec(e+fx)}(b^2-b^2\sec^2(e+fx))}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{266}$$

$$-\frac{2b\int \frac{1}{b^2-b^4\sec^4(e+fx)}d\sqrt{b\sec(e+fx)}}{f}$$

$$\Big\downarrow \text{756}$$

$$-\frac{2b\left(\frac{\int \frac{1}{b-b^2\sec^2(e+fx)}d\sqrt{b\sec(e+fx)}}{2b}+\frac{\int \frac{1}{b^2\sec^2(e+fx)+b}d\sqrt{b\sec(e+fx)}}{2b}\right)}{f}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2b\left(\frac{\int \frac{1}{b-b^2\sec^2(e+fx)}d\sqrt{b\sec(e+fx)}}{2b}+\frac{\arctan\left(\sqrt{b}\sec(e+fx)\right)}{2b^{3/2}}\right)}{f}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2b\left(\frac{\arctan\left(\sqrt{b}\sec(e+fx)\right)}{2b^{3/2}}+\frac{\text{arctanh}\left(\sqrt{b}\sec(e+fx)\right)}{2b^{3/2}}\right)}{f}$$

input
```
Int[Csc[e + f*x]/Sqrt[b*Sec[e + f*x]],x]
```

output
```
(-2*b*(ArcTan[Sqrt[b]*Sec[e + f*x]]/(2*b^(3/2)) + ArcTanh[Sqrt[b]*Sec[e +
f*x]]/(2*b^(3/2))))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(47) = 94$.

Time = 1.65 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.41

method	result	size
default	$\frac{\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}}\right)+\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\cos(fx+e)+1}{\cos(fx+e)+1}\right)}{2f(\cos(fx+e)+1)\sqrt{b\sec(fx+e)}\,\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}}$	142

input
```
int(csc(f*x+e)/(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f*(arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))+ln((2*cos(f*x+e)*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
-cos(f*x+e)+1)/(cos(f*x+e)+1)))/(cos(f*x+e)+1)/(b*sec(f*x+e))^(1/2)/(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(47) = 94$.

Time = 0.12 (sec) , antiderivative size = 273, normalized size of antiderivative = 4.63

$$\int \frac{\csc(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \left[\frac{2\sqrt{-b}\arctan\left(\frac{2\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)}{b\cos(fx+e)+b}\right)-\sqrt{-b}\log\left(\frac{b\cos(fx+e)^2-4\left(\cos(fx+e)^2-\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}-6\,b\cos\ldots}{\cos(fx+e)^2+2\cos(fx+e)+1}\right.}{4\,bf}\right.$$

$$-\frac{2\sqrt{b}\arctan\left(\frac{2\sqrt{b}\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)}{b\cos(fx+e)-b}\right)-\sqrt{b}\log\left(\frac{b\cos(fx+e)^2-4\left(\cos(fx+e)^2+\cos(fx+e)\right)\sqrt{b}\sqrt{\frac{b}{\cos(fx+e)}}+6\,b\cos(fx+e\ldots}{\cos(fx+e)^2-2\cos(fx+e)+1}\right.}{4\,bf}$$

input
```
integrate(csc(f*x+e)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(-b)*arctan(2*sqrt(-b)*sqrt(b/cos(f*x + e))*cos(f*x + e)/(b*co
s(f*x + e) + b)) - sqrt(-b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - co
s(f*x + e))*sqrt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x
 + e)^2 + 2*cos(f*x + e) + 1)))/(b*f), -1/4*(2*sqrt(b)*arctan(2*sqrt(b)*sq
rt(b/cos(f*x + e))*cos(f*x + e)/(b*cos(f*x + e) - b)) - sqrt(b)*log((b*cos
(f*x + e)^2 - 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(b)*sqrt(b/cos(f*x + e
)) + 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)))/(b*f)]
```

Sympy [F]

$$\int \frac{\csc(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\csc(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$$

input
```
integrate(csc(f*x+e)/(b*sec(f*x+e))**(1/2),x)
```

output
```
Integral(csc(e + f*x)/sqrt(b*sec(e + f*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

$$\int \frac{\csc(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{b\left(\frac{2\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}}\right)}{b^{\frac{3}{2}}} - \frac{\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos(fx+e)}}}\right)}{b^{\frac{3}{2}}}\right)}{2f}$$

input
```
integrate(csc(f*x+e)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
-1/2*b*(2*arctan(sqrt(b/cos(f*x + e))/sqrt(b))/b^(3/2) - log(-(sqrt(b) - s
qrt(b/cos(f*x + e)))/(sqrt(b) + sqrt(b/cos(f*x + e))))/b^(3/2))/f
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int \frac{\csc(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{\frac{\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b}} + \frac{\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{b}}\right)}{\sqrt{b}}}{f\,\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}$$

input
```
integrate(csc(f*x+e)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
(arctan(sqrt(b*cos(f*x + e))/sqrt(-b))/sqrt(-b) + arctan(sqrt(b*cos(f*x +
e))/sqrt(b))/sqrt(b))/(f*sgn(cos(f*x + e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)\,\sqrt{\frac{b}{\cos(e+f\,x)}}}\,dx$$

input
```
int(1/(sin(e + f*x)*(b/cos(e + f*x))^(1/2)),x)
```

output
```
int(1/(sin(e + f*x)*(b/cos(e + f*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{\csc(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(fx+e)}\,\csc(fx+e)}{\sec(fx+e)}dx\right)}{b}$$

input
```
int(csc(f*x+e)/(b*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(e + f*x))*csc(e + f*x))/sec(e + f*x),x))/b
```

3.415 $\int \frac{\csc^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 93

$$\int \frac{\csc^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4\sqrt{b}f} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4\sqrt{b}f} - \frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{2bf}$$

output
```
-1/4*arctan((b*sec(f*x+e))^(1/2)/b^(1/2))/b^(1/2)/f-1/4*arctanh((b*sec(f*x
+e))^(1/2)/b^(1/2))/b^(1/2)/f-1/2*cot(f*x+e)^2*(b*sec(f*x+e))^(1/2)/b/f
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00

$$\int \frac{\csc^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{\left(-2\arctan\left(\sqrt{\sec(e+fx)}\right) + \log\left(1-\sqrt{\sec(e+fx)}\right) - \log\left(1+\sqrt{\sec(e+fx)}\right) - \frac{4\csc^2(e+fx)}{\sec^{\frac{3}{2}}(e+fx)}\right)\sqrt{\mathrm{se}}}{8f\sqrt{b\sec(e+fx)}}$$

input
```
Integrate[Csc[e + f*x]^3/Sqrt[b*Sec[e + f*x]],x]
```

output
```
((-2*ArcTan[Sqrt[Sec[e + f*x]]] + Log[1 - Sqrt[Sec[e + f*x]]] - Log[1 + Sq
rt[Sec[e + f*x]]] - (4*Csc[e + f*x]^2)/Sec[e + f*x]^(3/2))*Sqrt[Sec[e + f*
x]])/(8*f*Sqrt[b*Sec[e + f*x]])
```

Rubi [A] (warning: unable to verify)

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3102, 27, 252, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(e+fx)}{\sqrt{b\sec(e+fx)}}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\csc(e+fx)^3}{\sqrt{b\sec(e+fx)}}dx$$
$$\Big\downarrow \text{ 3102}$$
$$\frac{\int \frac{b^4(b\sec(e+fx))^{3/2}}{\left(b^2-b^2\sec^2(e+fx)\right)^2}d(b\sec(e+fx))}{b^3 f}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{b\int \frac{(b\sec(e+fx))^{3/2}}{\left(b^2-b^2\sec^2(e+fx)\right)^2}d(b\sec(e+fx))}{f}$$
$$\Big\downarrow \text{ 252}$$
$$\frac{b\left(\frac{\sqrt{b\sec(e+fx)}}{2(b^2-b^2\sec^2(e+fx))}-\frac{1}{4}\int \frac{1}{\sqrt{b\sec(e+fx)}(b^2-b^2\sec^2(e+fx))}d(b\sec(e+fx))\right)}{f}$$
$$\Big\downarrow \text{ 266}$$

$$\frac{b\left(\frac{\sqrt{b\sec(e+fx)}}{2(b^2-b^2\sec^2(e+fx))}-\frac{1}{2}\int\frac{1}{b^2-b^4\sec^4(e+fx)}d\sqrt{b\sec(e+fx)}\right)}{f}$$

$$\Big\downarrow \text{756}$$

$$\frac{b\left(\frac{1}{2}\left(-\frac{\int\frac{1}{b-b^2\sec^2(e+fx)}d\sqrt{b\sec(e+fx)}}{2b}-\frac{\int\frac{1}{b^2\sec^2(e+fx)+b}d\sqrt{b\sec(e+fx)}}{2b}\right)+\frac{\sqrt{b\sec(e+fx)}}{2(b^2-b^2\sec^2(e+fx))}\right)}{f}$$

$$\Big\downarrow \text{216}$$

$$\frac{b\left(\frac{1}{2}\left(-\frac{\int\frac{1}{b-b^2\sec^2(e+fx)}d\sqrt{b\sec(e+fx)}}{2b}-\frac{\arctan\left(\sqrt{b}\sec(e+fx)\right)}{2b^{3/2}}\right)+\frac{\sqrt{b\sec(e+fx)}}{2(b^2-b^2\sec^2(e+fx))}\right)}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{b\left(\frac{1}{2}\left(-\frac{\arctan\left(\sqrt{b}\sec(e+fx)\right)}{2b^{3/2}}-\frac{\text{arctanh}\left(\sqrt{b}\sec(e+fx)\right)}{2b^{3/2}}\right)+\frac{\sqrt{b\sec(e+fx)}}{2(b^2-b^2\sec^2(e+fx))}\right)}{f}$$

input
```
Int[Csc[e + f*x]^3/Sqrt[b*Sec[e + f*x]],x]
```

output
```
(b*((-1/2*ArcTan[Sqrt[b]*Sec[e + f*x]]/b^(3/2) - ArcTanh[Sqrt[b]*Sec[e + f
*x]]/(2*b^(3/2)))/2 + Sqrt[b*Sec[e + f*x]]/(2*(b^2 - b^2*Sec[e + f*x]^2)))
)/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(73) = 146$.

Time = 1.78 (sec) , antiderivative size = 281, normalized size of antiderivative = 3.02

method	result
default	$-\frac{\left(4\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\cos(fx+e)+1}{\cos(fx+e)+1}\right)\cos(fx+e)+\arctan\left(\right.}{8f\sqrt{}}$

input
```
int(csc(f*x+e)^3/(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/f*(4*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+ln((2*cos(f*x+e)
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*cos(f*x+e)+arctan(1/2/(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2))*cos(f*x+e)-ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)
+1))-arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)))/(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)/(b*sec(f*x+e))^(1/2)*csc(f*x+e)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(73) = 146$.

Time = 0.13 (sec) , antiderivative size = 381, normalized size of antiderivative = 4.10

$$\int \frac{\csc^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$$

$$= \left[\frac{2\left(\cos\left(fx+e\right)^2-1\right)\sqrt{-b}\arctan\left(\frac{2\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)}{b\cos(fx+e)+b}\right)+8\sqrt{\frac{b}{\cos(fx+e)}}\cos\left(fx+e\right)^2-\left(\cos\left(fx+\right.\right.}{16\left(bf\cos\left(fx+e\right)^2-bf\right.}\right.$$

$$-\frac{2\left(\cos\left(fx+e\right)^2-1\right)\sqrt{b}\arctan\left(\frac{2\sqrt{b}\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)}{b\cos(fx+e)-b}\right)-8\sqrt{\frac{b}{\cos(fx+e)}}\cos\left(fx+e\right)^2-\left(\cos\left(fx+e\right)^{2}\right.}{16\left(bf\cos\left(fx+e\right)^2-bf\right)}$$

input
```
integrate(csc(f*x+e)^3/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(2*(cos(f*x + e)^2 - 1)*sqrt(-b)*arctan(2*sqrt(-b)*sqrt(b/cos(f*x +
e))*cos(f*x + e)/(b*cos(f*x + e) + b)) + 8*sqrt(b/cos(f*x + e))*cos(f*x +
e)^2 - (cos(f*x + e)^2 - 1)*sqrt(-b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x +
e)^2 - cos(f*x + e))*sqrt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos(f*x + e) + b)
/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)))/(b*f*cos(f*x + e)^2 - b*f), -1/16
*(2*(cos(f*x + e)^2 - 1)*sqrt(b)*arctan(2*sqrt(b)*sqrt(b/cos(f*x + e))*cos
(f*x + e)/(b*cos(f*x + e) - b)) - 8*sqrt(b/cos(f*x + e))*cos(f*x + e)^2 -
(cos(f*x + e)^2 - 1)*sqrt(b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + c
os(f*x + e))*sqrt(b)*sqrt(b/cos(f*x + e)) + 6*b*cos(f*x + e) + b)/(cos(f*x
 + e)^2 - 2*cos(f*x + e) + 1)))/(b*f*cos(f*x + e)^2 - b*f)]
```

Sympy [F]

$$\int \frac{\csc^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\csc^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$$

input
```
integrate(csc(f*x+e)**3/(b*sec(f*x+e))**(1/2),x)
```

output
```
Integral(csc(e + f*x)**3/sqrt(b*sec(e + f*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.13

$$\int \frac{\csc^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{b\left(\frac{4\sqrt{\frac{b}{\cos(fx+e)}}}{b^2-\frac{b^2}{\cos(fx+e)^2}} - \frac{2\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}}\right)}{b^{\frac{3}{2}}} + \frac{\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos(fx+e)}}}\right)}{b^{\frac{3}{2}}}\right)}{8f}$$

input
```
integrate(csc(f*x+e)^3/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
1/8*b*(4*sqrt(b/cos(f*x + e))/(b^2 - b^2/cos(f*x + e)^2) - 2*arctan(sqrt(b
/cos(f*x + e))/sqrt(b))/b^(3/2) + log(-(sqrt(b) - sqrt(b/cos(f*x + e)))/(s
qrt(b) + sqrt(b/cos(f*x + e))))/b^(3/2))/f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.12

$$\int \frac{\csc^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{b^2\left(\frac{2\sqrt{b\cos(fx+e)}\cos(fx+e)}{\left(b^2\cos(fx+e)^2-b^2\right)b}+\frac{\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b}b^2}+\frac{\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{b}}\right)}{b^{\frac{5}{2}}}\right)}{4\,f\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}$$

input
```
integrate(csc(f*x+e)^3/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
1/4*b^2*(2*sqrt(b*cos(f*x + e))*cos(f*x + e)/((b^2*cos(f*x + e)^2 - b^2)*b
) + arctan(sqrt(b*cos(f*x + e))/sqrt(-b))/(sqrt(-b)*b^2) + arctan(sqrt(b*c
os(f*x + e))/sqrt(b))/b^(5/2))/(f*sgn(cos(f*x + e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)^3\,\sqrt{\frac{b}{\cos(e+f\,x)}}}\,dx$$

input
```
int(1/(sin(e + f*x)^3*(b/cos(e + f*x))^(1/2)),x)
```

output
```
int(1/(sin(e + f*x)^3*(b/cos(e + f*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{\csc^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(fx+e)}\,\csc(fx+e)^3}{\sec(fx+e)}dx\right)}{b}$$

input
```
int(csc(f*x+e)^3/(b*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(e + f*x))*csc(e + f*x)**3)/sec(e + f*x),x))/b
```

3.416 $\int \frac{\csc^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 123

$$\int \frac{\csc^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{5\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32\sqrt{b}f} - \frac{5\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32\sqrt{b}f} - \frac{5\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{16bf} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{5/2}}{4b^3f}$$

output
```
-5/32*arctan((b*sec(f*x+e))^(1/2)/b^(1/2))/b^(1/2)/f-5/32*arctanh((b*sec(f
*x+e))^(1/2)/b^(1/2))/b^(1/2)/f-5/16*cot(f*x+e)^2*(b*sec(f*x+e))^(1/2)/b/f
-1/4*cot(f*x+e)^4*(b*sec(f*x+e))^(5/2)/b^3/f
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.87

$$\int \frac{\csc^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{\left(10\arctan\left(\sqrt{\sec(e+fx)}\right)-5\log\left(1-\sqrt{\sec(e+fx)}\right)+5\log\left(1+\sqrt{\sec(e+fx)}\right)+4(-5+\csc^2}{64f\sqrt{b\sec(e+fx)}}$$

input
```
Integrate[Csc[e + f*x]^5/Sqrt[b*Sec[e + f*x]],x]
```

output
```
-1/64*((10*ArcTan[Sqrt[Sec[e + f*x]]] - 5*Log[1 - Sqrt[Sec[e + f*x]]] + 5*
Log[1 + Sqrt[Sec[e + f*x]]] + 4*(-5 + Csc[e + f*x]^2 + 4*Csc[e + f*x]^4)*S
qrt[Sec[e + f*x]])*Sqrt[Sec[e + f*x]])/(f*Sqrt[b*Sec[e + f*x]])
```

Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3102, 25, 27, 252, 252, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^5(e+fx)}{\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc(e+fx)^5}{\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{\int -\frac{b^6(b\sec(e+fx))^{7/2}}{(b^2-b^2\sec^2(e+fx))^3}d(b\sec(e+fx))}{b^5f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{b^6(b\sec(e+fx))^{7/2}}{(b^2-b^2\sec^2(e+fx))^3}d(b\sec(e+fx))}{b^5f}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\int \frac{(b\sec(e+fx))^{7/2}}{(b^2-b^2\sec^2(e+fx))^3}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{252}$$

$$-\frac{b\left(\frac{(b\sec(e+fx))^{5/2}}{4(b^2-b^2\sec^2(e+fx))^2}-\frac{5}{8}\int \frac{(b\sec(e+fx))^{3/2}}{(b^2-b^2\sec^2(e+fx))^2}d(b\sec(e+fx))\right)}{f}$$

$$\Big\downarrow \text{252}$$

$$-\frac{b\left(\frac{(b\sec(e+fx))^{5/2}}{4(b^2-b^2\sec^2(e+fx))^2}-\frac{5}{8}\left(\frac{\sqrt{b\sec(e+fx)}}{2(b^2-b^2\sec^2(e+fx))}-\frac{1}{4}\int \frac{1}{\sqrt{b\sec(e+fx)}(b^2-b^2\sec^2(e+fx))}d(b\sec(e+fx))\right)\right)}{f}$$

$$\Big\downarrow \text{266}$$

$$-\frac{b\left(\frac{(b\sec(e+fx))^{5/2}}{4(b^2-b^2\sec^2(e+fx))^2}-\frac{5}{8}\left(\frac{\sqrt{b\sec(e+fx)}}{2(b^2-b^2\sec^2(e+fx))}-\frac{1}{2}\int \frac{1}{b^2-b^4\sec^4(e+fx)}d\sqrt{b\sec(e+fx)}\right)\right)}{f}$$

$$\Big\downarrow \text{756}$$

$$-\frac{b\left(\frac{(b\sec(e+fx))^{5/2}}{4(b^2-b^2\sec^2(e+fx))^2}-\frac{5}{8}\left(\frac{1}{2}\left(-\frac{\int \frac{1}{b-b^2\sec^2(e+fx)}d\sqrt{b\sec(e+fx)}}{2b}-\frac{\int \frac{1}{b^2\sec^2(e+fx)+b}d\sqrt{b\sec(e+fx)}}{2b}\right)+\frac{\sqrt{b\sec(e+fx)}}{2(b^2-b^2\sec^2(e+fx))}\right)\right)}{f}$$

$$\Big\downarrow \text{216}$$

$$-\frac{b\left(\frac{(b\sec(e+fx))^{5/2}}{4(b^2-b^2\sec^2(e+fx))^2}-\frac{5}{8}\left(\frac{1}{2}\left(-\frac{\int \frac{1}{b-b^2\sec^2(e+fx)}d\sqrt{b\sec(e+fx)}}{2b}-\frac{\arctan\left(\sqrt{b}\sec(e+fx)\right)}{2b^{3/2}}\right)+\frac{\sqrt{b\sec(e+fx)}}{2(b^2-b^2\sec^2(e+fx))}\right)\right)}{f}$$

$$\Big\downarrow \text{219}$$

$$-\frac{b\left(\frac{(b\sec(e+fx))^{5/2}}{4(b^2-b^2\sec^2(e+fx))^2}-\frac{5}{8}\left(\frac{1}{2}\left(-\frac{\arctan\left(\sqrt{b}\sec(e+fx)\right)}{2b^{3/2}}-\frac{\text{arctanh}\left(\sqrt{b}\sec(e+fx)\right)}{2b^{3/2}}\right)+\frac{\sqrt{b\sec(e+fx)}}{2(b^2-b^2\sec^2(e+fx))}\right)\right)}{f}$$

input
```
Int[Csc[e + f*x]^5/Sqrt[b*Sec[e + f*x]],x]
```

output
```
-((b*((b*Sec[e + f*x])^(5/2)/(4*(b^2 - b^2*Sec[e + f*x]^2)^2) - (5*((-1/2*
ArcTan[Sqrt[b]*Sec[e + f*x]]/b^(3/2) - ArcTanh[Sqrt[b]*Sec[e + f*x]]/(2*b^
(3/2)))/2 + Sqrt[b*Sec[e + f*x]]/(2*(b^2 - b^2*Sec[e + f*x]^2))))/8))/f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(99) = 198$.

Time = 1.86 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.76

method	result
default	$\frac{\left(20\cos(fx+e)^2-36\right)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,\cot(fx+e)\csc(fx+e)^3+(-5\cos(fx+e)+5)\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\frac{\text{co}}{(\cos(}}}{\cos(fx+e)+1}\right.}{64f\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,\sqrt{b\sec(f}}$

input
```
int(csc(f*x+e)^5/(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/64/f/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/(b*sec(f*x+e))^(1/2)*((20*cos(
f*x+e)^2-36)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cot(f*x+e)*csc(f*x+e)^3+
(-5*cos(f*x+e)+5)*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*csc(f*x
+e)^2+(-5*cos(f*x+e)+5)*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*c
sc(f*x+e)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(99) = 198$.

Time = 0.16 (sec) , antiderivative size = 469, normalized size of antiderivative = 3.81

$$\int \frac{\csc^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$$

$$= \left[\frac{10\left(\cos(fx+e)^4-2\cos(fx+e)^2+1\right)\sqrt{-b}\arctan\left(\frac{2\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)}{b\cos(fx+e)+b}\right)-5\left(\cos(fx+e)^4-2\right.}{128\,(b}\right.$$

$$-\frac{10\left(\cos(fx+e)^4-2\cos(fx+e)^2+1\right)\sqrt{b}\arctan\left(\frac{2\sqrt{b}\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)}{b\cos(fx+e)-b}\right)-5\left(\cos(fx+e)^4-2\right.}{128\,(bf}$$

input
```
integrate(csc(f*x+e)^5/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
[1/128*(10*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-b)*arctan(2*sqrt(
-b)*sqrt(b/cos(f*x + e))*cos(f*x + e)/(b*cos(f*x + e) + b)) - 5*(cos(f*x +
 e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x
+ e)^2 - cos(f*x + e))*sqrt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos(f*x + e) +
b)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 8*(5*cos(f*x + e)^4 - 9*cos(f*
x + e)^2)*sqrt(b/cos(f*x + e)))/(b*f*cos(f*x + e)^4 - 2*b*f*cos(f*x + e)^2
 + b*f), -1/128*(10*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(b)*arctan
(2*sqrt(b)*sqrt(b/cos(f*x + e))*cos(f*x + e)/(b*cos(f*x + e) - b)) - 5*(co
s(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(b)*log((b*cos(f*x + e)^2 - 4*(co
s(f*x + e)^2 + cos(f*x + e))*sqrt(b)*sqrt(b/cos(f*x + e)) + 6*b*cos(f*x +
e) + b)/(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)) - 8*(5*cos(f*x + e)^4 - 9*c
os(f*x + e)^2)*sqrt(b/cos(f*x + e)))/(b*f*cos(f*x + e)^4 - 2*b*f*cos(f*x +
 e)^2 + b*f)]
```

Sympy [F]

$$\int \frac{\csc^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\csc^5\left(e+fx\right)}{\sqrt{b\sec\left(e+fx\right)}}\,dx$$

input
```
integrate(csc(f*x+e)**5/(b*sec(f*x+e))**(1/2),x)
```

output
```
Integral(csc(e + f*x)**5/sqrt(b*sec(e + f*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.12

$$\int \frac{\csc^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{b\left(\frac{4\left(5\,b^2\sqrt{\frac{b}{\cos(fx+e)}}-9\left(\frac{b}{\cos(fx+e)}\right)^{\frac{5}{2}}\right)}{b^4-\frac{2\,b^4}{\cos(fx+e)^2}+\frac{b^4}{\cos(fx+e)^4}}-\frac{10\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}}\right)}{b^{\frac{3}{2}}}+\frac{5\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos(fx+e)}}}\right)}{b^{\frac{3}{2}}}\right)}{64\,f}$$

input
```
integrate(csc(f*x+e)^5/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
1/64*b*(4*(5*b^2*sqrt(b/cos(f*x + e)) - 9*(b/cos(f*x + e))^(5/2))/(b^4 - 2
*b^4/cos(f*x + e)^2 + b^4/cos(f*x + e)^4) - 10*arctan(sqrt(b/cos(f*x + e))
/sqrt(b))/b^(3/2) + 5*log(-(sqrt(b) - sqrt(b/cos(f*x + e)))/(sqrt(b) + sqr
t(b/cos(f*x + e))))/b^(3/2))/f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.10

$$\int \frac{\csc^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{b^4\left(\frac{5\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b}b^4}+\frac{5\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{b}}\right)}{b^{\frac{9}{2}}}+\frac{2\left(5\sqrt{b\cos(fx+e)}b^3\cos(fx+e)^3-9\sqrt{b\cos(fx+e)}b^3\cos(fx+e)\right)}{\left(b^2\cos(fx+e)^2-b^2\right)^2b^4}\right)}{32\,f\operatorname{sgn}\left(\cos\left(fx+e\right)\right)}$$

input
```
integrate(csc(f*x+e)^5/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
1/32*b^4*(5*arctan(sqrt(b*cos(f*x + e))/sqrt(-b))/(sqrt(-b)*b^4) + 5*arcta
n(sqrt(b*cos(f*x + e))/sqrt(b))/b^(9/2) + 2*(5*sqrt(b*cos(f*x + e))*b^3*co
s(f*x + e)^3 - 9*sqrt(b*cos(f*x + e))*b^3*cos(f*x + e))/((b^2*cos(f*x + e)
^2 - b^2)^2*b^4))/(f*sgn(cos(f*x + e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)^5\,\sqrt{\frac{b}{\cos(e+f\,x)}}}\,dx$$

input
```
int(1/(sin(e + f*x)^5*(b/cos(e + f*x))^(1/2)),x)
```

output
```
int(1/(sin(e + f*x)^5*(b/cos(e + f*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{\csc^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(fx+e)}\,\csc(fx+e)^5}{\sec(fx+e)}dx\right)}{b}$$

input
```
int(csc(f*x+e)^5/(b*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(e + f*x))*csc(e + f*x)**5)/sec(e + f*x),x))/b
```

3.417 $\int \frac{\sin^6(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 123

$$\int \frac{\sin^6(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{16E\left(\frac{1}{2}(e+fx)\middle|2\right)}{39f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{8b\sin(e+fx)}{39f(b\sec(e+fx))^{3/2}} - \frac{20b\sin^3(e+fx)}{117f(b\sec(e+fx))^{3/2}} - \frac{2b\sin^5(e+fx)}{13f(b\sec(e+fx))^{3/2}}$$

output
```
16/39*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)/(b*sec(f*x+
e))^(1/2)-8/39*b*sin(f*x+e)/f/(b*sec(f*x+e))^(3/2)-20/117*b*sin(f*x+e)^3/f
/(b*sec(f*x+e))^(3/2)-2/13*b*sin(f*x+e)^5/f/(b*sec(f*x+e))^(3/2)
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.59

$$\int \frac{\sin^6(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{\frac{768E\left(\frac{1}{2}(e+fx)\middle|2\right)}{\sqrt{\cos(e+fx)}} - 317\sin(2(e+fx)) + 76\sin(4(e+fx)) - 9\sin(6(e+fx))}{1872f\sqrt{b\sec(e+fx)}}$$

input
```
Integrate[Sin[e + f*x]^6/Sqrt[b*Sec[e + f*x]],x]
```

output
```
((768*EllipticE[(e + f*x)/2, 2])/Sqrt[Cos[e + f*x]] - 317*Sin[2*(e + f*x)]
 + 76*Sin[4*(e + f*x)] - 9*Sin[6*(e + f*x)])/(1872*f*Sqrt[b*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3107, 3042, 3107, 3042, 3107, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^6(e+fx)}{\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc(e+fx)^6\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3107}$$

$$\frac{10}{13}\int \frac{\sin^4(e+fx)}{\sqrt{b\sec(e+fx)}}dx-\frac{2b\sin^5(e+fx)}{13f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{10}{13}\int \frac{1}{\csc(e+fx)^4\sqrt{b\sec(e+fx)}}dx-\frac{2b\sin^5(e+fx)}{13f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3107}$$

$$\frac{10}{13}\left(\frac{2}{3}\int \frac{\sin^2(e+fx)}{\sqrt{b\sec(e+fx)}}dx-\frac{2b\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}}\right)-\frac{2b\sin^5(e+fx)}{13f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{10}{13}\left(\frac{2}{3}\int \frac{1}{\csc(e+fx)^2\sqrt{b\sec(e+fx)}}dx-\frac{2b\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}}\right)-\frac{2b\sin^5(e+fx)}{13f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3107}$$

$$\frac{10}{13}\left(\frac{2}{3}\left(\frac{2}{5}\int\frac{1}{\sqrt{b\sec(e+fx)}}dx-\frac{2b\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}\right)-\frac{2b\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}}\right)-\frac{2b\sin^5(e+fx)}{13f(b\sec(e+fx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{10}{13}\left(\frac{2}{3}\left(\frac{2}{5}\int\frac{1}{\sqrt{b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{2b\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}\right)-\frac{2b\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}}\right)-\frac{2b\sin^5(e+fx)}{13f(b\sec(e+fx))^{3/2}}$$

$\Big\downarrow$ 4258

$$\frac{10}{13}\left(\frac{2}{3}\left(\frac{2\int\sqrt{\cos(e+fx)}dx}{5\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{2b\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}\right)-\frac{2b\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}}\right)-\frac{2b\sin^5(e+fx)}{13f(b\sec(e+fx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{10}{13}\left(\frac{2}{3}\left(\frac{2\int\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{2b\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}\right)-\frac{2b\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}}\right)-\frac{2b\sin^5(e+fx)}{13f(b\sec(e+fx))^{3/2}}$$

$\Big\downarrow$ 3119

$$\frac{10}{13}\left(\frac{2}{3}\left(\frac{4E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{2b\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}\right)-\frac{2b\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}}\right)-\frac{2b\sin^5(e+fx)}{13f(b\sec(e+fx))^{3/2}}$$

input
```
Int[Sin[e + f*x]^6/Sqrt[b*Sec[e + f*x]],x]
```

output
```
(-2*b*Sin[e + f*x]^5)/(13*f*(b*Sec[e + f*x])^(3/2)) + (10*((-2*b*Sin[e + f
*x]^3)/(9*f*(b*Sec[e + f*x])^(3/2)) + (2*((4*EllipticE[(e + f*x)/2, 2])/(5
*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]]) - (2*b*Sin[e + f*x])/(5*f*(b*S
ec[e + f*x])^(3/2))))/3))/13
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3107
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Simp[(m + 1)/(a^2*(m + n))   Int[(a*Csc[e + f*x])^(m
+ 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 10.23 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.89

method	result
default	$\frac{\frac{2\sin(fx+e)\left(-9\cos(fx+e)^6-9\cos(fx+e)^5+28\cos(fx+e)^4+28\cos(fx+e)^3-31\cos(fx+e)^2-31\cos(fx+e)+24\right)}{117}+\frac{16i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+}{\cos(fx+e}}}{}}{f(c}$

input
```
int(sin(f*x+e)^6/(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/117/f/(cos(f*x+e)+1)/(b*sec(f*x+e))^(1/2)*(sin(f*x+e)*(-9*cos(f*x+e)^6-9
*cos(f*x+e)^5+28*cos(f*x+e)^4+28*cos(f*x+e)^3-31*cos(f*x+e)^2-31*cos(f*x+e
)+24)+24*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(2+c
os(f*x+e)+sec(f*x+e))*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)-24*I*(1/(cos(
f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(2+cos(f*x+e)+sec(f*x+e
))*EllipticE(I*(cot(f*x+e)-csc(f*x+e)),I))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.95

$$\int \frac{\sin^6(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{2\left(\left(9\,\cos\left(fx+e\right)^6-28\,\cos\left(fx+e\right)^4+31\,\cos\left(fx+e\right)^2\right)\sqrt{\frac{b}{\cos(fx+e)}}\sin\left(fx+e\right)-12i\,\sqrt{2}\sqrt{b}\text{weierst}}{}$$

input
```
integrate(sin(f*x+e)^6/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-2/117*((9*cos(f*x + e)^6 - 28*cos(f*x + e)^4 + 31*cos(f*x + e)^2)*sqrt(b/
cos(f*x + e))*sin(f*x + e) - 12*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, w
eierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 12*I*sqrt(2)*s
qrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*
sin(f*x + e))))/(b*f)
```

Sympy [F]

$$\int \frac{\sin^6(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin^6\left(e+fx\right)}{\sqrt{b\sec\left(e+fx\right)}}\,dx$$

input
```
integrate(sin(f*x+e)**6/(b*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sin(e + f*x)**6/sqrt(b*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sin^6(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin\left(fx+e\right)^6}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

input
```
integrate(sin(f*x+e)^6/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^6/sqrt(b*sec(f*x + e)), x)
```

Giac [F]

$$\int \frac{\sin^6(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin\left(fx+e\right)^6}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

input
```
integrate(sin(f*x+e)^6/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sin(f*x + e)^6/sqrt(b*sec(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^6(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin\left(e+f\,x\right)^6}{\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}}\,dx$$

input
```
int(sin(e + f*x)^6/(b/cos(e + f*x))^(1/2),x)
```

output
```
int(sin(e + f*x)^6/(b/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sin^6(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(fx+e)}\,\sin(fx+e)^6}{\sec(fx+e)}dx\right)}{b}$$

input

```
int(sin(f*x+e)^6/(b*sec(f*x+e))^(1/2),x)
```

output

```
(sqrt(b)*int((sqrt(sec(e + f*x))*sin(e + f*x)**6)/sec(e + f*x),x))/b
```

3.418 $\int \frac{\sin^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 95

$$\int \frac{\sin^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{8E\left(\frac{1}{2}(e+fx)\middle|2\right)}{15f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{4b\sin(e+fx)}{15f(b\sec(e+fx))^{3/2}} - \frac{2b\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}}$$

output
```
8/15*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)/(b*sec(f*x+e
))^(1/2)-4/15*b*sin(f*x+e)/f/(b*sec(f*x+e))^(3/2)-2/9*b*sin(f*x+e)^3/f/(b*
sec(f*x+e))^(3/2)
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.66

$$\int \frac{\sin^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{\frac{192E\left(\frac{1}{2}(e+fx)\middle|2\right)}{\sqrt{\cos(e+fx)}} - 68\sin(2(e+fx)) + 10\sin(4(e+fx))}{360f\sqrt{b\sec(e+fx)}}$$

input
```
Integrate[Sin[e + f*x]^4/Sqrt[b*Sec[e + f*x]],x]
```

output
```
((192*EllipticE[(e + f*x)/2, 2])/Sqrt[Cos[e + f*x]] - 68*Sin[2*(e + f*x)]
+ 10*Sin[4*(e + f*x)])/(360*f*Sqrt[b*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3107, 3042, 3107, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(e+fx)}{\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc(e+fx)^4\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3107}$$

$$\frac{2}{3}\int \frac{\sin^2(e+fx)}{\sqrt{b\sec(e+fx)}}dx-\frac{2b\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\int \frac{1}{\csc(e+fx)^2\sqrt{b\sec(e+fx)}}dx-\frac{2b\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3107}$$

$$\frac{2}{3}\left(\frac{2}{5}\int \frac{1}{\sqrt{b\sec(e+fx)}}dx-\frac{2b\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}\right)-\frac{2b\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\left(\frac{2}{5}\int \frac{1}{\sqrt{b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{2b\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}\right)-\frac{2b\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{2}{3}\left(\frac{2\int\sqrt{\cos(e+fx)}dx}{5\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{2b\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}\right)-\frac{2b\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\left(\frac{2\int\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{2b\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}\right)-\frac{2b\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2}{3}\left(\frac{4E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{2b\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}\right)-\frac{2b\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}}$$

input
```
Int[Sin[e + f*x]^4/Sqrt[b*Sec[e + f*x]],x]
```

output
```
(-2*b*Sin[e + f*x]^3)/(9*f*(b*Sec[e + f*x])^(3/2)) + (2*((4*EllipticE[(e +
 f*x)/2, 2])/(5*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]]) - (2*b*Sin[e +
f*x])/(5*f*(b*Sec[e + f*x])^(3/2))))/3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3107
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Simp[(m + 1)/(a^2*(m + n))   Int[(a*Csc[e + f*x])^(m
+ 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.10 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.23

method	result
default	$\frac{\frac{2\sin(fx+e)\left(5\cos(fx+e)^4+5\cos(fx+e)^3-11\cos(fx+e)^2-11\cos(fx+e)+12\right)}{45}+\frac{8i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,(2+\cos(fx+e)+\sec(fx+e))\,\mathrm{Ell}}{15}}{f(\cos(fx+e)+1)\sqrt{}}$

input
```
int(sin(f*x+e)^4/(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/45/f/(cos(f*x+e)+1)/(b*sec(f*x+e))^(1/2)*(sin(f*x+e)*(5*cos(f*x+e)^4+5*c
os(f*x+e)^3-11*cos(f*x+e)^2-11*cos(f*x+e)+12)+12*I*(1/(cos(f*x+e)+1))^(1/2
)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(2+cos(f*x+e)+sec(f*x+e))*EllipticF(I*
(cot(f*x+e)-csc(f*x+e)),I)-12*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)*(2+cos(f*x+e)+sec(f*x+e))*EllipticE(I*(cot(f*x+e)-csc(f*x
+e)),I))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.13

$$\int \frac{\sin^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{2\left(\left(5\cos(fx+e)^4-11\cos(fx+e)^2\right)\sqrt{\frac{b}{\cos(fx+e)}}\sin(fx+e)+6i\sqrt{2}\sqrt{b}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierst}\right.}{}$$

input
```
integrate(sin(f*x+e)^4/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
2/45*((5*cos(f*x + e)^4 - 11*cos(f*x + e)^2)*sqrt(b/cos(f*x + e))*sin(f*x
+ e) + 6*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(f*x + e) + I*sin(f*x + e))) - 6*I*sqrt(2)*sqrt(b)*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))))/(b*f)
```

Sympy [F]

$$\int \frac{\sin^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin^4{(e+fx)}}{\sqrt{b\sec{(e+fx)}}}\,dx$$

input
```
integrate(sin(f*x+e)**4/(b*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sin(e + f*x)**4/sqrt(b*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sin^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin{(fx+e)}^4}{\sqrt{b\sec{(fx+e)}}}\,dx$$

input
```
integrate(sin(f*x+e)^4/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^4/sqrt(b*sec(f*x + e)), x)
```

Giac [F]

$$\int \frac{\sin^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin\left(fx+e\right)^4}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

input
```
integrate(sin(f*x+e)^4/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sin(f*x + e)^4/sqrt(b*sec(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin\left(e+f\,x\right)^4}{\sqrt{\frac{b}{\cos(e+f\,x)}}}\,dx$$

input
```
int(sin(e + f*x)^4/(b/cos(e + f*x))^(1/2),x)
```

output
```
int(sin(e + f*x)^4/(b/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sin^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(fx+e)}\,\sin(fx+e)^4}{\sec(fx+e)}dx\right)}{b}$$

input
```
int(sin(f*x+e)^4/(b*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(e + f*x))*sin(e + f*x)**4)/sec(e + f*x),x))/b
```

3.419 $\int \frac{\sin^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 67

$$\int \frac{\sin^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{4E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{2b\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}$$

output
```
4/5*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)/(b*sec(f*x+e)
)^(1/2)-2/5*b*sin(f*x+e)/f/(b*sec(f*x+e))^(3/2)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.90

$$\int \frac{\sin^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{\sqrt{b\sec(e+fx)}\left(-8\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)+\sin(e+fx)+\sin(3(e+fx))\right)}{10bf}$$

input
```
Integrate[Sin[e + f*x]^2/Sqrt[b*Sec[e + f*x]],x]
```

output
```
-1/10*(Sqrt[b*Sec[e + f*x]]*(-8*Sqrt[Cos[e + f*x]]*EllipticE[(e + f*x)/2,
2] + Sin[e + f*x] + Sin[3*(e + f*x)]))/(b*f)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3107, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(e+fx)}{\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc(e+fx)^2\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3107}$$

$$\frac{2}{5}\int \frac{1}{\sqrt{b\sec(e+fx)}}dx-\frac{2b\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}\int \frac{1}{\sqrt{b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{2b\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{2\int \sqrt{\cos(e+fx)}dx}{5\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{2b\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{5\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{2b\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{4E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{2b\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}$$

input
```
Int[Sin[e + f*x]^2/Sqrt[b*Sec[e + f*x]],x]
```

output
```
(4*EllipticE[(e + f*x)/2, 2])/(5*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]]
) - (2*b*Sin[e + f*x])/(5*f*(b*Sec[e + f*x])^(3/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3107
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Simp[(m + 1)/(a^2*(m + n))   Int[(a*Csc[e + f*x])^(m
+ 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.66 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.87

method	result
default	$\frac{\frac{2\sin(fx+e)\left(-\cos(fx+e)^{2}-\cos(fx+e)+2\right)}{5}+\frac{4i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,(2+\cos(fx+e)+\sec(fx+e))\,\operatorname{EllipticF}(i(\cot(fx+e)-\csc(fx+e)),i)}{5}}{f(\cos(fx+e)+1)\sqrt{b\sec(fx+e)}}$

input
```
int(sin(f*x+e)^2/(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/f/(cos(f*x+e)+1)/(b*sec(f*x+e))^(1/2)*(sin(f*x+e)*(-cos(f*x+e)^2-cos(f
*x+e)+2)+2*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(2
+cos(f*x+e)+sec(f*x+e))*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)-2*I*(1/(cos
(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(2+cos(f*x+e)+sec(f*x+
e))*EllipticE(I*(cot(f*x+e)-csc(f*x+e)),I))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.40

$$\int \frac{\sin^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{2\left(\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)^2\sin(fx+e) - i\sqrt{2}\sqrt{b}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\text{c}\right.}{}$$

input
```
integrate(sin(f*x+e)^2/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-2/5*(sqrt(b/cos(f*x + e))*cos(f*x + e)^2*sin(f*x + e) - I*sqrt(2)*sqrt(b)
*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*
x + e))) + I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(f*x + e) - I*sin(f*x + e))))/(b*f)
```

Sympy [F]

$$\int \frac{\sin^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$$

input
```
integrate(sin(f*x+e)**2/(b*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sin(e + f*x)**2/sqrt(b*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sin^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

input
```
integrate(sin(f*x+e)^2/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^2/sqrt(b*sec(f*x + e)), x)
```

Giac [F]

$$\int \frac{\sin^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

input
```
integrate(sin(f*x+e)^2/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sin(f*x + e)^2/sqrt(b*sec(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin\left(e+f\,x\right)^2}{\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}}\,dx$$

input
```
int(sin(e + f*x)^2/(b/cos(e + f*x))^(1/2),x)
```

output
```
int(sin(e + f*x)^2/(b/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sin^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(fx+e)}\,\sin(fx+e)^2}{\sec(fx+e)}dx\right)}{b}$$

input
```
int(sin(f*x+e)^2/(b*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(e + f*x))*sin(e + f*x)**2)/sec(e + f*x),x))/b
```

3.420 $\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 38

$$\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx = \frac{2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

output
```
2*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)/(b*sec(f*x+e))^
(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx = \frac{2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

input
```
Integrate[1/Sqrt[b*Sec[e + f*x]],x]
```

output
```
(2*EllipticE[(e + f*x)/2, 2])/(f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{4258}$$

$$\frac{\int \sqrt{\cos(e+fx)}dx}{\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

input
```
Int[1/Sqrt[b*Sec[e + f*x]],x]
```

output
```
(2*EllipticE[(e + f*x)/2, 2])/(f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.30 (sec) , antiderivative size = 175, normalized size of antiderivative = 4.61

method	result
default	$\frac{2i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,(2+\cos(fx+e)+\sec(fx+e))\,\mathrm{EllipticF}(i(\cot(fx+e)-\csc(fx+e)),i)+2i\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\sqrt{\frac{1}{\cos(fx+e)\cdots}}}{f(\cos(fx+e)+1)\sqrt{b\sec(fx+e)}}$
risch	$-\frac{i\sqrt{2}}{f\sqrt{\frac{b\,e^{i(fx+e)}}{e^{2i(fx+e)}+1}}}-\frac{i\left(-\frac{2\left(b\,e^{2i(fx+e)}+b\right)}{b\sqrt{e^{i(fx+e)}\left(b\,e^{2i(fx+e)}+b\right)}}+\frac{i\sqrt{-i\left(e^{i(fx+e)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(e^{i(fx+e)}-i\right)}\,\sqrt{ie^{i(fx+e)}}\,\left(-2i\,\mathrm{EllipticE}\left(\sqrt{-i\left(e^{i(fx\cdots}\right.}\right.\right.}{\sqrt{e^{3i(fx+e)}b+b\,e^{i(fx+e)}}}\cdots}{f\sqrt{\frac{b\,e^{i(fx+e)}}{e^{2i(fx+e)}+1}}\,\left(e^{2i(fx+e)}+\cdots\right.}$

input
```
int(1/(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/f/(cos(f*x+e)+1)/(b*sec(f*x+e))^(1/2)*(I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*(2+cos(f*x+e)+sec(f*x+e))*EllipticF(I*(cot(f*x
+e)-csc(f*x+e)),I)+I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^
(1/2)*EllipticE(I*(cot(f*x+e)-csc(f*x+e)),I)*(-2-cos(f*x+e)-sec(f*x+e))+si
n(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.74

$$\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx = \frac{i\sqrt{2}\sqrt{b}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e))) - i\sqrt{2}\sqrt{b}\text{weier}}{bf}$$

input
```
integrate(1/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
(I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f
*x + e) + I*sin(f*x + e))) - I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))))/(b*f)
```

Sympy [F]

$$\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx$$

input
```
integrate(1/(b*sec(f*x+e))**(1/2),x)
```

output
```
Integral(1/sqrt(b*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\sec(fx+e)}}\,dx$$

input
```
integrate(1/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*sec(f*x + e)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

input
```
integrate(1/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*sec(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{1}{\sqrt{\frac{b}{\cos(e+f\,x)}}}\,dx$$

input
```
int(1/(b/cos(e + f*x))^(1/2),x)
```

output
```
int(1/(b/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(fx+e)}}{\sec(fx+e)}dx\right)}{b}$$

input
```
int(1/(b*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(b)*int(sqrt(sec(e + f*x))/sec(e + f*x),x))/b
```

3.421 $\int \frac{\csc^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 63

$$\int \frac{\csc^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{b\csc(e+fx)}{f(b\sec(e+fx))^{3/2}} - \frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

output
```
-b*csc(f*x+e)/f/(b*sec(f*x+e))^(3/2)-EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))
/f/cos(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.76

$$\int \frac{\csc^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{-\cot(e+fx) - \frac{E\left(\frac{1}{2}(e+fx)|2\right)}{\sqrt{\cos(e+fx)}}}{f\sqrt{b\sec(e+fx)}}$$

input
```
Integrate[Csc[e + f*x]^2/Sqrt[b*Sec[e + f*x]],x]
```

output
```
(-Cot[e + f*x] - EllipticE[(e + f*x)/2, 2]/Sqrt[Cos[e + f*x]])/(f*Sqrt[b*S
ec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3105, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(e+fx)}{\sqrt{b\sec(e+fx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\csc(e+fx)^2}{\sqrt{b\sec(e+fx)}}dx$$
$$\Big\downarrow \text{3105}$$
$$-\frac{1}{2}\int \frac{1}{\sqrt{b\sec(e+fx)}}dx-\frac{b\csc(e+fx)}{f(b\sec(e+fx))^{3/2}}$$
$$\Big\downarrow \text{3042}$$
$$-\frac{1}{2}\int \frac{1}{\sqrt{b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{b\csc(e+fx)}{f(b\sec(e+fx))^{3/2}}$$
$$\Big\downarrow \text{4258}$$
$$-\frac{\int \sqrt{\cos(e+fx)}dx}{2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{b\csc(e+fx)}{f(b\sec(e+fx))^{3/2}}$$
$$\Big\downarrow \text{3042}$$
$$-\frac{\int \sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{b\csc(e+fx)}{f(b\sec(e+fx))^{3/2}}$$
$$\Big\downarrow \text{3119}$$
$$-\frac{b\csc(e+fx)}{f(b\sec(e+fx))^{3/2}}-\frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

input
```
Int[Csc[e + f*x]^2/Sqrt[b*Sec[e + f*x]],x]
```

output
```
-((b*Csc[e + f*x])/(f*(b*Sec[e + f*x])^(3/2))) - EllipticE[(e + f*x)/2, 2]
/(f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3105
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Simp[a^2*((m + n - 2)/(m - 1))   Int[(a*Csc[e + f*
x])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[
m, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.09 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.41

method	result
default	$\frac{-i\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\sqrt{\frac{1}{\cos(fx+e)+1}}\,\operatorname{EllipticE}(i(\cot(fx+e)-\csc(fx+e)),i)(-1-\sec(fx+e))-i\,\operatorname{EllipticF}(i(\cot(fx+e)-\csc(fx+e)),i)}{f\sqrt{b\sec(fx+e)}}$

input
```
int(csc(f*x+e)^2/(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*Ellipti
cE(I*(cot(f*x+e)-csc(f*x+e)),I)*(-1-sec(f*x+e))-I*EllipticF(I*(cot(f*x+e)-
csc(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
(1+sec(f*x+e))-csc(f*x+e))/(b*sec(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.73

$$\int \frac{\csc^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{-i\sqrt{2}\sqrt{b}\sin(fx+e)\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)))}{}$$

input
```
integrate(csc(f*x+e)^2/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(-I*sqrt(2)*sqrt(b)*sin(f*x + e)*weierstrassZeta(-4, 0, weierstrassPIn
verse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + I*sqrt(2)*sqrt(b)*sin(f*x +
 e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin
(f*x + e))) - 2*sqrt(b/cos(f*x + e))*cos(f*x + e)^2)/(b*f*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\csc^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\csc^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$$

input
```
integrate(csc(f*x+e)**2/(b*sec(f*x+e))**(1/2),x)
```

output
```
Integral(csc(e + f*x)**2/sqrt(b*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\csc^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\csc(fx+e)^2}{\sqrt{b\sec(fx+e)}}\,dx$$

input
```
integrate(csc(f*x+e)^2/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(f*x + e)^2/sqrt(b*sec(f*x + e)), x)
```

Giac [F]

$$\int \frac{\csc^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\csc(fx+e)^2}{\sqrt{b\sec(fx+e)}}\,dx$$

input
```
integrate(csc(f*x+e)^2/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(csc(f*x + e)^2/sqrt(b*sec(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{1}{\sin(e+f\,x)^2\,\sqrt{\frac{b}{\cos(e+f\,x)}}}\,dx$$

input
```
int(1/(sin(e + f*x)^2*(b/cos(e + f*x))^(1/2)),x)
```

output
```
int(1/(sin(e + f*x)^2*(b/cos(e + f*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{\csc^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(fx+e)}\,\csc(fx+e)^2}{\sec(fx+e)}dx\right)}{b}$$

input
```
int(csc(f*x+e)^2/(b*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(e + f*x))*csc(e + f*x)**2)/sec(e + f*x),x))/b
```

3.422 $\int \frac{\csc^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 95

$$\int \frac{\csc^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{b\csc(e+fx)}{2f(b\sec(e+fx))^{3/2}} - \frac{b\csc^3(e+fx)}{3f(b\sec(e+fx))^{3/2}} - \frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)}{2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

output
```
-1/2*b*csc(f*x+e)/f/(b*sec(f*x+e))^(3/2)-1/3*b*csc(f*x+e)^3/f/(b*sec(f*x+e
))^(3/2)-1/2*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)/(b*s
ec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.78

$$\int \frac{\csc^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{\left(-3+\csc^2(e+fx)+2\csc^4(e+fx)+3\sqrt{\cos(e+fx)}\csc(e+fx)E\left(\frac{1}{2}(e+fx)\middle|2\right)\right)\tan(e+fx)}{6f\sqrt{b\sec(e+fx)}}$$

input
```
Integrate[Csc[e + f*x]^4/Sqrt[b*Sec[e + f*x]],x]
```

output
```
-1/6*((-3 + Csc[e + f*x]^2 + 2*Csc[e + f*x]^4 + 3*Sqrt[Cos[e + f*x]]*Csc[e
 + f*x]*EllipticE[(e + f*x)/2, 2])*Tan[e + f*x])/(f*Sqrt[b*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3105, 3042, 3105, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc(e+fx)^4}{\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3105}$$

$$\frac{1}{2}\int \frac{\csc^2(e+fx)}{\sqrt{b\sec(e+fx)}}dx-\frac{b\csc^3(e+fx)}{3f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{\csc(e+fx)^2}{\sqrt{b\sec(e+fx)}}dx-\frac{b\csc^3(e+fx)}{3f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3105}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{\sqrt{b\sec(e+fx)}}dx-\frac{b\csc(e+fx)}{f(b\sec(e+fx))^{3/2}}\right)-\frac{b\csc^3(e+fx)}{3f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{\sqrt{b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{b\csc(e+fx)}{f(b\sec(e+fx))^{3/2}}\right)-\frac{b\csc^3(e+fx)}{3f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \ 4258$$

$$\frac{1}{2}\left(-\frac{\int \sqrt{\cos(e+fx)}dx}{2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{b\csc(e+fx)}{f(b\sec(e+fx))^{3/2}}\right)-\frac{b\csc^3(e+fx)}{3f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{2}\left(-\frac{\int \sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{b\csc(e+fx)}{f(b\sec(e+fx))^{3/2}}\right)-\frac{b\csc^3(e+fx)}{3f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \ 3119$$

$$\frac{1}{2}\left(-\frac{b\csc(e+fx)}{f(b\sec(e+fx))^{3/2}}-\frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}\right)-\frac{b\csc^3(e+fx)}{3f(b\sec(e+fx))^{3/2}}$$

input
```
Int[Csc[e + f*x]^4/Sqrt[b*Sec[e + f*x]],x]
```

output
```
-1/3*(b*Csc[e + f*x]^3)/(f*(b*Sec[e + f*x])^(3/2)) + (-((b*Csc[e + f*x])/(
f*(b*Sec[e + f*x])^(3/2))) - EllipticE[(e + f*x)/2, 2]/(f*Sqrt[Cos[e + f*x
]]*Sqrt[b*Sec[e + f*x]]))/2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3105
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Simp[a^2*((m + n - 2)/(m - 1))   Int[(a*Csc[e + f*
x])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[
m, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.55 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.79

method	result
default	$\frac{-\frac{i\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\sqrt{\frac{1}{\cos(fx+e)+1}}\,\operatorname{EllipticE}(i(\cot(fx+e)-\csc(fx+e)),i)(-3-3\sec(fx+e))}{6}-\frac{i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{EllipticF}(i(\cot(f}{6}}{f\sqrt{b\sec(fx+e)}}$

input
```
int(csc(f*x+e)^4/(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/6*I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*Ell
ipticE(I*(cot(f*x+e)-csc(f*x+e)),I)*(-3-3*sec(f*x+e))-1/6*I*(1/(cos(f*x+e)
+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f
*x+e)),I)*(3+3*sec(f*x+e))-1/2*csc(f*x+e)-1/3*cot(f*x+e)*csc(f*x+e)^2)/(b*
sec(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.66

$$\int \frac{\csc^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx =$$
$$-\frac{3\sqrt{2}\left(i\cos\left(fx+e\right)^2-i\right)\sqrt{b}\sin\left(fx+e\right)\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos\left(fx+\right.}{}$$

input
```
integrate(csc(f*x+e)^4/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-1/12*(3*sqrt(2)*(I*cos(f*x + e)^2 - I)*sqrt(b)*sin(f*x + e)*weierstrassZe
ta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*s
qrt(2)*(-I*cos(f*x + e)^2 + I)*sqrt(b)*sin(f*x + e)*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) + 2*(3*cos(f*x
 + e)^4 - 5*cos(f*x + e)^2)*sqrt(b/cos(f*x + e)))/((b*f*cos(f*x + e)^2 - b
*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\csc^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\csc^4{(e+fx)}}{\sqrt{b\sec{(e+fx)}}}\,dx$$

input
```
integrate(csc(f*x+e)**4/(b*sec(f*x+e))**(1/2),x)
```

output
```
Integral(csc(e + f*x)**4/sqrt(b*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\csc^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\csc(fx+e)^4}{\sqrt{b\sec(fx+e)}}\,dx$$

input
```
integrate(csc(f*x+e)^4/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(f*x + e)^4/sqrt(b*sec(f*x + e)), x)
```

Giac [F]

$$\int \frac{\csc^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx=\int \frac{\csc\left(fx+e\right)^4}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

input
```
integrate(csc(f*x+e)^4/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(csc(f*x + e)^4/sqrt(b*sec(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx=\int \frac{1}{\sin\left(e+f\,x\right)^4\,\sqrt{\frac{b}{\cos(e+f\,x)}}}\,dx$$

input
```
int(1/(sin(e + f*x)^4*(b/cos(e + f*x))^(1/2)),x)
```

output
```
int(1/(sin(e + f*x)^4*(b/cos(e + f*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{\csc^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx=\frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(fx+e)}\,\csc(fx+e)^4}{\sec(fx+e)}dx\right)}{b}$$

input
```
int(csc(f*x+e)^4/(b*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(e + f*x))*csc(e + f*x)**4)/sec(e + f*x),x))/b
```

3.423 $\int \frac{\csc^6(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 123

$$\int \frac{\csc^6(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{7b\csc(e+fx)}{20f(b\sec(e+fx))^{3/2}} - \frac{7b\csc^3(e+fx)}{30f(b\sec(e+fx))^{3/2}} - \frac{b\csc^5(e+fx)}{5f(b\sec(e+fx))^{3/2}} - \frac{7E\left(\frac{1}{2}(e+fx)\middle|2\right)}{20f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

output
```
-7/20*b*csc(f*x+e)/f/(b*sec(f*x+e))^(3/2)-7/30*b*csc(f*x+e)^3/f/(b*sec(f*x
+e))^(3/2)-1/5*b*csc(f*x+e)^5/f/(b*sec(f*x+e))^(3/2)-7/20*EllipticE(sin(1/
2*f*x+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.70

$$\int \frac{\csc^6(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{\left(-21+7\csc^2(e+fx)+2\csc^4(e+fx)+12\csc^6(e+fx)+21\sqrt{\cos(e+fx)}\csc(e+fx)E\big(\tfrac{1}{2}(e+f}{60f\sqrt{b\sec(e+fx)}}$$

input
```
Integrate[Csc[e + f*x]^6/Sqrt[b*Sec[e + f*x]],x]
```

output
```
-1/60*((-21 + 7*Csc[e + f*x]^2 + 2*Csc[e + f*x]^4 + 12*Csc[e + f*x]^6 + 21
*Sqrt[Cos[e + f*x]]*Csc[e + f*x]*EllipticE[(e + f*x)/2, 2])*Tan[e + f*x])/
(f*Sqrt[b*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3105, 3042, 3105, 3042, 3105, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^6(e+fx)}{\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc(e+fx)^6}{\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3105}$$

$$\frac{7}{10}\int \frac{\csc^4(e+fx)}{\sqrt{b\sec(e+fx)}}dx-\frac{b\csc^5(e+fx)}{5f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{10}\int \frac{\csc(e+fx)^4}{\sqrt{b\sec(e+fx)}}dx-\frac{b\csc^5(e+fx)}{5f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3105}$$

$$\frac{7}{10}\left(\frac{1}{2}\int \frac{\csc^2(e+fx)}{\sqrt{b\sec(e+fx)}}dx-\frac{b\csc^3(e+fx)}{3f(b\sec(e+fx))^{3/2}}\right)-\frac{b\csc^5(e+fx)}{5f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{10}\left(\frac{1}{2}\int \frac{\csc(e+fx)^2}{\sqrt{b\sec(e+fx)}}dx-\frac{b\csc^3(e+fx)}{3f(b\sec(e+fx))^{3/2}}\right)-\frac{b\csc^5(e+fx)}{5f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \; 3105$$

$$\frac{7}{10}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{\sqrt{b\sec(e+fx)}}dx-\frac{b\csc(e+fx)}{f(b\sec(e+fx))^{3/2}}\right)-\frac{b\csc^3(e+fx)}{3f(b\sec(e+fx))^{3/2}}\right)-\frac{b\csc^5(e+fx)}{5f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{7}{10}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{\sqrt{b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{b\csc(e+fx)}{f(b\sec(e+fx))^{3/2}}\right)-\frac{b\csc^3(e+fx)}{3f(b\sec(e+fx))^{3/2}}\right)-\frac{b\csc^5(e+fx)}{5f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \; 4258$$

$$\frac{7}{10}\left(\frac{1}{2}\left(-\frac{\int \sqrt{\cos(e+fx)}dx}{2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{b\csc(e+fx)}{f(b\sec(e+fx))^{3/2}}\right)-\frac{b\csc^3(e+fx)}{3f(b\sec(e+fx))^{3/2}}\right)-\frac{b\csc^5(e+fx)}{5f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{7}{10}\left(\frac{1}{2}\left(-\frac{\int \sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{b\csc(e+fx)}{f(b\sec(e+fx))^{3/2}}\right)-\frac{b\csc^3(e+fx)}{3f(b\sec(e+fx))^{3/2}}\right)-\frac{b\csc^5(e+fx)}{5f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \; 3119$$

$$\frac{7}{10}\left(\frac{1}{2}\left(-\frac{b\csc(e+fx)}{f(b\sec(e+fx))^{3/2}}-\frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}\right)-\frac{b\csc^3(e+fx)}{3f(b\sec(e+fx))^{3/2}}\right)-\frac{b\csc^5(e+fx)}{5f(b\sec(e+fx))^{3/2}}$$

input

```
Int[Csc[e + f*x]^6/Sqrt[b*Sec[e + f*x]],x]
```

output

```
-1/5*(b*Csc[e + f*x]^5)/(f*(b*Sec[e + f*x])^(3/2)) + (7*(-1/3*(b*Csc[e + f
*x]^3)/(f*(b*Sec[e + f*x])^(3/2)) + (-((b*Csc[e + f*x])/(f*(b*Sec[e + f*x]
)^(3/2))) - EllipticE[(e + f*x)/2, 2]/(f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e +
 f*x]]))/2))/10
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3105
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Simp[a^2*((m + n - 2)/(m - 1))   Int[(a*Csc[e + f*
x])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[
m, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.58 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.51

method	result
default	$\frac{-\frac{i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\operatorname{EllipticE}(i(\cot(fx+e)-\csc(fx+e)),i)(-21-21\sec(fx+e))}{60}-\frac{i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\operatorname{EllipticF}(i(\cot}{60}}{f\sqrt{b\sec(fx+e)}}$

input
```
int(csc(f*x+e)^6/(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/60*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*El
lipticE(I*(cot(f*x+e)-csc(f*x+e)),I)*(-21-21*sec(f*x+e))-1/60*I*(1/(cos(f*
x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-c
sc(f*x+e)),I)*(21+21*sec(f*x+e))-7/20*csc(f*x+e)-7/30*cot(f*x+e)*csc(f*x+e
)^2-1/5*cot(f*x+e)*csc(f*x+e)^4)/(b*sec(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.62

$$\int \frac{\csc^6(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx =$$
$$-\frac{21\sqrt{2}\left(i\cos\left(fx+e\right)^4-2i\cos\left(fx+e\right)^2+i\right)\sqrt{b}\sin\left(fx+e\right)\text{weierstrassZeta}(-4,0,\text{weierstrassPInve}$$

input
```
integrate(csc(f*x+e)^6/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-1/120*(21*sqrt(2)*(I*cos(f*x + e)^4 - 2*I*cos(f*x + e)^2 + I)*sqrt(b)*sin
(f*x + e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) +
 I*sin(f*x + e))) + 21*sqrt(2)*(-I*cos(f*x + e)^4 + 2*I*cos(f*x + e)^2 - I
)*sqrt(b)*sin(f*x + e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c
os(f*x + e) - I*sin(f*x + e))) + 2*(21*cos(f*x + e)^6 - 56*cos(f*x + e)^4
+ 47*cos(f*x + e)^2)*sqrt(b/cos(f*x + e)))/((b*f*cos(f*x + e)^4 - 2*b*f*co
s(f*x + e)^2 + b*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\csc^6(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\csc^6\left(e+fx\right)}{\sqrt{b\sec\left(e+fx\right)}}\,dx$$

input
```
integrate(csc(f*x+e)**6/(b*sec(f*x+e))**(1/2),x)
```

output
```
Integral(csc(e + f*x)**6/sqrt(b*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\csc^6(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx=\int \frac{\csc\left(fx+e\right)^6}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

input
```
integrate(csc(f*x+e)^6/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(f*x + e)^6/sqrt(b*sec(f*x + e)), x)
```

Giac [F]

$$\int \frac{\csc^6(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx=\int \frac{\csc\left(fx+e\right)^6}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

input
```
integrate(csc(f*x+e)^6/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(csc(f*x + e)^6/sqrt(b*sec(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^6(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx=\int \frac{1}{\sin\left(e+f\,x\right)^6\,\sqrt{\frac{b}{\cos(e+f\,x)}}}\,dx$$

input
```
int(1/(sin(e + f*x)^6*(b/cos(e + f*x))^(1/2)),x)
```

output
```
int(1/(sin(e + f*x)^6*(b/cos(e + f*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{\csc^6(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(fx+e)}\,\csc(fx+e)^6}{\sec(fx+e)}dx\right)}{b}$$

input
```
int(csc(f*x+e)^6/(b*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(e + f*x))*csc(e + f*x)**6)/sec(e + f*x),x))/b
```

3.424 $\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{2b^7}{17f(b\sec(e+fx))^{17/2}} - \frac{6b^5}{13f(b\sec(e+fx))^{13/2}} + \frac{2b^3}{3f(b\sec(e+fx))^{9/2}} - \frac{2b}{5f(b\sec(e+fx))^{5/2}}$$

output
```
2/17*b^7/f/(b*sec(f*x+e))^(17/2)-6/13*b^5/f/(b*sec(f*x+e))^(13/2)+2/3*b^3/
f/(b*sec(f*x+e))^(9/2)-2/5*b/f/(b*sec(f*x+e))^(5/2)
```

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.60

$$\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{b(-10766+8365\cos(2(e+fx))-1890\cos(4(e+fx))+195\cos(6(e+fx)))}{53040f(b\sec(e+fx))^{5/2}}$$

input
```
Integrate[Sin[e + f*x]^7/(b*Sec[e + f*x])^(3/2),x]
```

output
```
(b*(-10766 + 8365*Cos[2*(e + f*x)] - 1890*Cos[4*(e + f*x)] + 195*Cos[6*(e
+ f*x)]))/(53040*f*(b*Sec[e + f*x])^(5/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3102, 25, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc(e+fx)^7(b\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{b^7\int -\frac{\left(b^2-b^2\sec^2(e+fx)\right)^3}{b^6(b\sec(e+fx))^{19/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b^7\int \frac{\left(b^2-b^2\sec^2(e+fx)\right)^3}{b^6(b\sec(e+fx))^{19/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\int \frac{\left(b^2-b^2\sec^2(e+fx)\right)^3}{(b\sec(e+fx))^{19/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{244}$$

$$-\frac{b\int \left(\frac{b^6}{(b\sec(e+fx))^{19/2}}-\frac{3b^4}{(b\sec(e+fx))^{15/2}}+\frac{3b^2}{(b\sec(e+fx))^{11/2}}-\frac{1}{(b\sec(e+fx))^{7/2}}\right)d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\left(-\frac{2b^6}{17(b\sec(e+fx))^{17/2}}+\frac{6b^4}{13(b\sec(e+fx))^{13/2}}-\frac{2b^2}{3(b\sec(e+fx))^{9/2}}+\frac{2}{5(b\sec(e+fx))^{5/2}}\right)}{f}$$

input
```
Int[Sin[e + f*x]^7/(b*Sec[e + f*x])^(3/2),x]
```

output
```
-((b*((-2*b^6)/(17*(b*Sec[e + f*x])^(17/2)) + (6*b^4)/(13*(b*Sec[e + f*x])
^(13/2)) - (2*b^2)/(3*(b*Sec[e + f*x])^(9/2)) + 2/(5*(b*Sec[e + f*x])^(5/2
))))/f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 2.70 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.68

method	result	size
default	$\frac{\frac{2\cos(fx+e)^8}{17}-\frac{6\cos(fx+e)^6}{13}+\frac{2\cos(fx+e)^4}{3}-\frac{2\cos(fx+e)^2}{5}}{fb\sqrt{b\sec(fx+e)}}$	59

input
```
int(sin(f*x+e)^7/(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(2/17*cos(f*x+e)^8-6/13*cos(f*x+e)^6+2/3*cos(f*x+e)^4-2/5*cos(f*x+e)^2
)/b/(b*sec(f*x+e))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.70

$$\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{2\left(195\cos\left(fx+e\right)^9 - 765\cos\left(fx+e\right)^7 + 1105\cos\left(fx+e\right)^5 - 663\cos\left(fx+\right.\right.}{3315\,b^2 f}$$

input
```
integrate(sin(f*x+e)^7/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
2/3315*(195*cos(f*x + e)^9 - 765*cos(f*x + e)^7 + 1105*cos(f*x + e)^5 - 66
3*cos(f*x + e)^3)*sqrt(b/cos(f*x + e))/(b^2*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**7/(b*sec(f*x+e))**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

$$\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{2\left(195\,b^6 - \frac{765\,b^6}{\cos(fx+e)^2} + \frac{1105\,b^6}{\cos(fx+e)^4} - \frac{663\,b^6}{\cos(fx+e)^6}\right)b}{3315\,f\left(\frac{b}{\cos(fx+e)}\right)^{\frac{17}{2}}}$$

input
```
integrate(sin(f*x+e)^7/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
2/3315*(195*b^6 - 765*b^6/cos(f*x + e)^2 + 1105*b^6/cos(f*x + e)^4 - 663*b
^6/cos(f*x + e)^6)*b/(f*(b/cos(f*x + e))^(17/2))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.26

$$\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{2\left(195\sqrt{b\cos(fx+e)}b^8\cos(fx+e)^8 - 765\sqrt{b\cos(fx+e)}b^8\cos(fx+e)^6 + \right.}{3315\,b^{10}f\,\mathrm{sgn}\,(c}$$

input
```
integrate(sin(f*x+e)^7/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
2/3315*(195*sqrt(b*cos(f*x + e))*b^8*cos(f*x + e)^8 - 765*sqrt(b*cos(f*x +
 e))*b^8*cos(f*x + e)^6 + 1105*sqrt(b*cos(f*x + e))*b^8*cos(f*x + e)^4 - 6
63*sqrt(b*cos(f*x + e))*b^8*cos(f*x + e)^2)/(b^10*f*sgn(cos(f*x + e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\sin\left(e+f\,x\right)^7}{\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int(sin(e + f*x)^7/(b/cos(e + f*x))^(3/2),x)
```

output
```
int(sin(e + f*x)^7/(b/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(fx+e)}\,\sin(fx+e)^7}{\sec(fx+e)^2}dx\right)}{b^2}$$

input
```
int(sin(f*x+e)^7/(b*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(e + f*x))*sin(e + f*x)**7)/sec(e + f*x)**2,x))/b**2
```

3.425 $\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{2b^5}{13f(b\sec(e+fx))^{13/2}} + \frac{4b^3}{9f(b\sec(e+fx))^{9/2}} - \frac{2b}{5f(b\sec(e+fx))^{5/2}}$$

output
```
-2/13*b^5/f/(b*sec(f*x+e))^(13/2)+4/9*b^3/f/(b*sec(f*x+e))^(9/2)-2/5*b/f/(
b*sec(f*x+e))^(5/2)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.65

$$\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{b(-551+340\cos(2(e+fx))-45\cos(4(e+fx)))}{2340f(b\sec(e+fx))^{5/2}}$$

input
```
Integrate[Sin[e + f*x]^5/(b*Sec[e + f*x])^(3/2),x]
```

output
```
(b*(-551 + 340*Cos[2*(e + f*x)] - 45*Cos[4*(e + f*x)]))/(2340*f*(b*Sec[e +
 f*x])^(5/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3102, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\csc(e+fx)^5(b\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3102}$$

$$\frac{b^5\int \frac{(b^2-b^2\sec^2(e+fx))^2}{b^4(b\sec(e+fx))^{15/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{b\int \frac{(b^2-b^2\sec^2(e+fx))^2}{(b\sec(e+fx))^{15/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{b\int \left(\frac{b^4}{(b\sec(e+fx))^{15/2}}-\frac{2b^2}{(b\sec(e+fx))^{11/2}}+\frac{1}{(b\sec(e+fx))^{7/2}}\right)d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b\left(-\frac{2b^4}{13(b\sec(e+fx))^{13/2}}+\frac{4b^2}{9(b\sec(e+fx))^{9/2}}-\frac{2}{5(b\sec(e+fx))^{5/2}}\right)}{f}$$

input
```
Int[Sin[e + f*x]^5/(b*Sec[e + f*x])^(3/2),x]
```

output
```
(b*((-2*b^4)/(13*(b*Sec[e + f*x])^(13/2)) + (4*b^2)/(9*(b*Sec[e + f*x])^(9
/2)) - 2/(5*(b*Sec[e + f*x])^(5/2))))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 1.95 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{-\frac{2\cos(fx+e)^6}{13}+\frac{4\cos(fx+e)^4}{9}-\frac{2\cos(fx+e)^2}{5}}{fb\sqrt{b\sec(fx+e)}}$	49

input
```
int(sin(f*x+e)^5/(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-2/13*cos(f*x+e)^6+4/9*cos(f*x+e)^4-2/5*cos(f*x+e)^2)/b/(b*sec(f*x+e)
)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

$$\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{2\left(45\cos\left(fx+e\right)^7-130\cos\left(fx+e\right)^5+117\cos\left(fx+e\right)^3\right)\sqrt{\frac{b}{\cos(fx+e)}}}{585\,b^2f}$$

input
```
integrate(sin(f*x+e)^5/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
-2/585*(45*cos(f*x + e)^7 - 130*cos(f*x + e)^5 + 117*cos(f*x + e)^3)*sqrt(
b/cos(f*x + e))/(b^2*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**5/(b*sec(f*x+e))**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{2\left(45\,b^4-\frac{130\,b^4}{\cos(fx+e)^2}+\frac{117\,b^4}{\cos(fx+e)^4}\right)b}{585\,f\left(\frac{b}{\cos(fx+e)}\right)^{\frac{13}{2}}}$$

input
```
integrate(sin(f*x+e)^5/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
-2/585*(45*b^4 - 130*b^4/cos(f*x + e)^2 + 117*b^4/cos(f*x + e)^4)*b/(f*(b/
cos(f*x + e))^(13/2))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.34

$$\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{2\left(45\sqrt{b\cos(fx+e)}b^6\cos(fx+e)^6 - 130\sqrt{b\cos(fx+e)}b^6\cos(fx+e)^4 + 117\sqrt{b\cos(fx+e)}b^6\cos\right.}{585\,b^8 f\,\mathrm{sgn}(\cos(fx+e))}$$

input
```
integrate(sin(f*x+e)^5/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
-2/585*(45*sqrt(b*cos(f*x + e))*b^6*cos(f*x + e)^6 - 130*sqrt(b*cos(f*x +
e))*b^6*cos(f*x + e)^4 + 117*sqrt(b*cos(f*x + e))*b^6*cos(f*x + e)^2)/(b^8
*f*sgn(cos(f*x + e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\sin(e+f\,x)^5}{\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int(sin(e + f*x)^5/(b/cos(e + f*x))^(3/2),x)
```

output
```
int(sin(e + f*x)^5/(b/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(fx+e)}\,\sin(fx+e)^5}{\sec(fx+e)^2}dx\right)}{b^2}$$

input
```
int(sin(f*x+e)^5/(b*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(e + f*x))*sin(e + f*x)**5)/sec(e + f*x)**2,x))/b**2
```

3.426 $\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 43

$$\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{2b^3}{9f(b\sec(e+fx))^{9/2}} - \frac{2b}{5f(b\sec(e+fx))^{5/2}}$$

output
```
2/9*b^3/f/(b*sec(f*x+e))^(9/2)-2/5*b/f/(b*sec(f*x+e))^(5/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

$$\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{b(-13+5\cos(2(e+fx)))}{45f(b\sec(e+fx))^{5/2}}$$

input
```
Integrate[Sin[e + f*x]^3/(b*Sec[e + f*x])^(3/2),x]
```

output
```
(b*(-13 + 5*Cos[2*(e + f*x)]))/(45*f*(b*Sec[e + f*x])^(5/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3102, 25, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc(e+fx)^3(b\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{b^3\int -\frac{b^2-b^2\sec^2(e+fx)}{b^2(b\sec(e+fx))^{11/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b^3\int \frac{b^2-b^2\sec^2(e+fx)}{b^2(b\sec(e+fx))^{11/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\int \frac{b^2-b^2\sec^2(e+fx)}{(b\sec(e+fx))^{11/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{244}$$

$$-\frac{b\int \left(\frac{b^2}{(b\sec(e+fx))^{11/2}}-\frac{1}{(b\sec(e+fx))^{7/2}}\right)d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\left(\frac{2}{5(b\sec(e+fx))^{5/2}}-\frac{2b^2}{9(b\sec(e+fx))^{9/2}}\right)}{f}$$

input
```
Int[Sin[e + f*x]^3/(b*Sec[e + f*x])^(3/2),x]
```

output
```
-((b*((-2*b^2)/(9*(b*Sec[e + f*x])^(9/2)) + 2/(5*(b*Sec[e + f*x])^(5/2))))
/f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{\frac{2\cos(fx+e)^4}{9}-\frac{2\cos(fx+e)^2}{5}}{fb\sqrt{b\sec(fx+e)}}$	39

input
```
int(sin(f*x+e)^3/(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(2/9*cos(f*x+e)^4-2/5*cos(f*x+e)^2)/b/(b*sec(f*x+e))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{2\left(5\cos\left(fx+e\right)^5-9\cos\left(fx+e\right)^3\right)\sqrt{\frac{b}{\cos(fx+e)}}}{45\,b^2 f}$$

input
```
integrate(sin(f*x+e)^3/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
2/45*(5*cos(f*x + e)^5 - 9*cos(f*x + e)^3)*sqrt(b/cos(f*x + e))/(b^2*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**3/(b*sec(f*x+e))**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{2\left(5\,b^2-\frac{9\,b^2}{\cos(fx+e)^2}\right)b}{45\,f\left(\frac{b}{\cos(fx+e)}\right)^{\frac{9}{2}}}$$

input
```
integrate(sin(f*x+e)^3/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
2/45*(5*b^2 - 9*b^2/cos(f*x + e)^2)*b/(f*(b/cos(f*x + e))^(9/2))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.49

$$\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{2\left(5\,\sqrt{b\cos(fx+e)}b^4\cos(fx+e)^4-9\,\sqrt{b\cos(fx+e)}b^4\cos(fx+e)^2\right)}{45\,b^6 f\mathrm{sgn}(\cos(fx+e))}$$

input
```
integrate(sin(f*x+e)^3/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
2/45*(5*sqrt(b*cos(f*x + e))*b^4*cos(f*x + e)^4 - 9*sqrt(b*cos(f*x + e))*b
^4*cos(f*x + e)^2)/(b^6*f*sgn(cos(f*x + e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\sin(e+f\,x)^3}{\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int(sin(e + f*x)^3/(b/cos(e + f*x))^(3/2),x)
```

output
```
int(sin(e + f*x)^3/(b/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(fx+e)}\,\sin(fx+e)^3}{\sec(fx+e)^2}dx\right)}{b^2}$$

input
```
int(sin(f*x+e)^3/(b*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(e + f*x))*sin(e + f*x)**3)/sec(e + f*x)**2,x))/b**2
```

3.427 $\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 20

$$\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{2b}{5f(b\sec(e+fx))^{5/2}}$$

output
```
-2/5*b/f/(b*sec(f*x+e))^(5/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{2b}{5f(b\sec(e+fx))^{5/2}}$$

input
```
Integrate[Sin[e + f*x]/(b*Sec[e + f*x])^(3/2),x]
```

output
```
(-2*b)/(5*f*(b*Sec[e + f*x])^(5/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 3102, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{3/2}}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\csc(e+fx)(b\sec(e+fx))^{3/2}}dx$$
$$\Big\downarrow \text{ 3102}$$
$$\frac{b\int \frac{1}{(b\sec(e+fx))^{7/2}}d(b\sec(e+fx))}{f}$$
$$\Big\downarrow \text{ 15}$$
$$-\frac{2b}{5f(b\sec(e+fx))^{5/2}}$$

input
```
Int[Sin[e + f*x]/(b*Sec[e + f*x])^(3/2),x]
```

output
```
(-2*b)/(5*f*(b*Sec[e + f*x])^(5/2))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$-\frac{2b}{5f\left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}}$	17
default	$-\frac{2b}{5f\left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}}$	17

input
```
int(sin(f*x+e)/(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*b/f/(b*sec(f*x+e))^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.40

$$\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{2\sqrt{\frac{b}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)^{3}}{5\,b^{2}f}$$

input
```
integrate(sin(f*x+e)/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
-2/5*sqrt(b/cos(f*x + e))*cos(f*x + e)^3/(b^2*f)
```

Sympy [F]

$$\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\sin(e+fx)}{(b\sec(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)/(b*sec(f*x+e))**(3/2),x)
```

output
```
Integral(sin(e + f*x)/(b*sec(e + f*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{2\cos(fx+e)}{5f\left(\frac{b}{\cos(fx+e)}\right)^{\frac{3}{2}}}$$

input
```
integrate(sin(f*x+e)/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
-2/5*cos(f*x + e)/(f*(b/cos(f*x + e))^(3/2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(16) = 32.

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.75

$$\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{2\sqrt{b\cos(fx+e)}\cos(fx+e)^2}{5b^2f\mathrm{sgn}(\cos(fx+e))}$$

input
```
integrate(sin(f*x+e)/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
-2/5*sqrt(b*cos(f*x + e))*cos(f*x + e)^2/(b^2*f*sgn(cos(f*x + e)))
```

Mupad [B] (verification not implemented)

Time = 25.52 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.40

$$\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{2\cos\left(e+f\,x\right)^3\,\sqrt{\frac{b}{\cos(e+f\,x)}}}{5\,b^2\,f}$$

input
```
int(sin(e + f*x)/(b/cos(e + f*x))^(3/2),x)
```

output
```
-(2*cos(e + f*x)^3*(b/cos(e + f*x))^(1/2))/(5*b^2*f)
```

Reduce [F]

$$\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(fx+e)}\,\sin(fx+e)}{\sec(fx+e)^2}dx\right)}{b^2}$$

input
```
int(sin(f*x+e)/(b*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(e + f*x))*sin(e + f*x))/sec(e + f*x)**2,x))/b**2
```

3.428 $\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 78

$$\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{b^{3/2}f} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{b^{3/2}f} + \frac{2}{bf\sqrt{b\sec(e+fx)}}$$

output
```
arctan((b*sec(f*x+e))^(1/2)/b^(1/2))/b^(3/2)/f-arctanh((b*sec(f*x+e))^(1/2
)/b^(1/2))/b^(3/2)/f+2/b/f/(b*sec(f*x+e))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.53 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.46

$$\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(-\frac{1}{4},1,\frac{3}{4},\sec^2(e+fx)\right)}{bf\sqrt{b\sec(e+fx)}}$$

input
```
Integrate[Csc[e + f*x]/(b*Sec[e + f*x])^(3/2),x]
```

output
```
(2*Hypergeometric2F1[-1/4, 1, 3/4, Sec[e + f*x]^2])/(b*f*Sqrt[b*Sec[e + f*
x]])
```

Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3042, 3102, 25, 27, 264, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{\int -\frac{b^2}{(b\sec(e+fx))^{3/2}(b^2-b^2\sec^2(e+fx))}d(b\sec(e+fx))}{bf}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{b^2}{(b\sec(e+fx))^{3/2}(b^2-b^2\sec^2(e+fx))}d(b\sec(e+fx))}{bf}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\int \frac{1}{(b\sec(e+fx))^{3/2}(b^2-b^2\sec^2(e+fx))}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{264}$$

$$-\frac{b\left(\frac{\int \frac{\sqrt{b\sec(e+fx)}}{b^2-b^2\sec^2(e+fx)}d(b\sec(e+fx))}{b^2}-\frac{2}{b^2\sqrt{b\sec(e+fx)}}\right)}{f}$$

$$\Big\downarrow \text{266}$$

$$-\frac{b\left(\frac{2\int \frac{b^2\sec^2(e+fx)}{b^2-b^4\sec^4(e+fx)}d\sqrt{b\sec(e+fx)}}{b^2}-\frac{2}{b^2\sqrt{b\sec(e+fx)}}\right)}{f}$$

$$\Big\downarrow \textbf{827}$$

$$-\frac{b\left(\frac{2\left(\frac{1}{2}\int \frac{1}{b-b^2\sec^2(e+fx)}d\sqrt{b\sec(e+fx)}-\frac{1}{2}\int \frac{1}{b^2\sec^2(e+fx)+b}d\sqrt{b\sec(e+fx)}\right)}{b^2}-\frac{2}{b^2\sqrt{b\sec(e+fx)}}\right)}{f}$$

$$\Big\downarrow \textbf{216}$$

$$-\frac{b\left(\frac{2\left(\frac{1}{2}\int \frac{1}{b-b^2\sec^2(e+fx)}d\sqrt{b\sec(e+fx)}-\frac{\arctan\left(\sqrt{b}\sec(e+fx)\right)}{2\sqrt{b}}\right)}{b^2}-\frac{2}{b^2\sqrt{b\sec(e+fx)}}\right)}{f}$$

$$\Big\downarrow \textbf{219}$$

$$-\frac{b\left(\frac{2\left(\frac{\text{arctanh}\left(\sqrt{b}\sec(e+fx)\right)}{2\sqrt{b}}-\frac{\arctan\left(\sqrt{b}\sec(e+fx)\right)}{2\sqrt{b}}\right)}{b^2}-\frac{2}{b^2\sqrt{b\sec(e+fx)}}\right)}{f}$$

input
```
Int[Csc[e + f*x]/(b*Sec[e + f*x])^(3/2),x]
```

output
```
-((b*((2*(-1/2*ArcTan[Sqrt[b]*Sec[e + f*x]]/Sqrt[b] + ArcTanh[Sqrt[b]*Sec[
e + f*x]]/(2*Sqrt[b])))/b^2 - 2/(b^2*Sqrt[b*Sec[e + f*x]])))/f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(64) = 128$.

Time = 1.56 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.53

method	result
default	$\frac{4\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+4\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\cos(fx+e)+1}{\cos(fx+e)+1}\right)+\mathrm{ar}}{2f(\cos(fx+e)+1)\sqrt{b\sec(fx+e)}\,\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,b}$

input
```
int(csc(f*x+e)/(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f*(4*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+4*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)-ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))+arc
tan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)))/(cos(f*x+e)+1)/(b*sec(f*x+e
))^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(64) = 128$.

Time = 0.22 (sec) , antiderivative size = 314, normalized size of antiderivative = 4.03

$$\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \left[\frac{2\sqrt{-b}\arctan\left(\frac{2\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)}{b\cos(fx+e)+b}\right)+8\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)-\sqrt{-b}\log}{4b^2f}\right.$$

input
```
integrate(csc(f*x+e)/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(-b)*arctan(2*sqrt(-b)*sqrt(b/cos(f*x + e))*cos(f*x + e)/(b*co
s(f*x + e) + b)) + 8*sqrt(b/cos(f*x + e))*cos(f*x + e) - sqrt(-b)*log(-(b*
cos(f*x + e)^2 + 4*(cos(f*x + e)^2 - cos(f*x + e))*sqrt(-b)*sqrt(b/cos(f*x
 + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)))/(b^
2*f), 1/4*(2*sqrt(b)*arctan(2*sqrt(b)*sqrt(b/cos(f*x + e))*cos(f*x + e)/(b
*cos(f*x + e) - b)) + 8*sqrt(b/cos(f*x + e))*cos(f*x + e) + sqrt(b)*log(-(
b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(b)*sqrt(b/cos(f*
x + e)) + 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)))/(b
^2*f)]
```

Sympy [F]

$$\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\csc{(e+fx)}}{(b\sec{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)/(b*sec(f*x+e))**(3/2),x)
```

output
```
Integral(csc(e + f*x)/(b*sec(e + f*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.14

$$\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{b\left(\frac{2\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}}\right)}{b^{\frac{5}{2}}} + \frac{\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos(fx+e)}}}\right)}{b^{\frac{5}{2}}} + \frac{4}{b^2\sqrt{\frac{b}{\cos(fx+e)}}}\right)}{2f}$$

input
```
integrate(csc(f*x+e)/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
1/2*b*(2*arctan(sqrt(b/cos(f*x + e))/sqrt(b))/b^(5/2) + log(-(sqrt(b) - sq
rt(b/cos(f*x + e)))/(sqrt(b) + sqrt(b/cos(f*x + e))))/b^(5/2) + 4/(b^2*sqr
t(b/cos(f*x + e))))/f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

$$\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{\frac{b\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b}} - \sqrt{b}\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{b}}\right) + 2\sqrt{b\cos(fx+e)}}{b^2 f\,\mathrm{sgn}\left(\cos(fx+e)\right)}$$

input
```
integrate(csc(f*x+e)/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
(b*arctan(sqrt(b*cos(f*x + e))/sqrt(-b))/sqrt(-b) - sqrt(b)*arctan(sqrt(b*
cos(f*x + e))/sqrt(b)) + 2*sqrt(b*cos(f*x + e)))/(b^2*f*sgn(cos(f*x + e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\sin(e+f\,x)\,\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(sin(e + f*x)*(b/cos(e + f*x))^(3/2)),x)
```

output
```
int(1/(sin(e + f*x)*(b/cos(e + f*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(fx+e)}\,\csc(fx+e)}{\sec(fx+e)^2}dx\right)}{b^2}$$

input
```
int(csc(f*x+e)/(b*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(e + f*x))*csc(e + f*x))/sec(e + f*x)**2,x))/b**2
```

3.429 $\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 93

$$\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4b^{3/2}f} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4b^{3/2}f} - \frac{\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{2b^3f}$$

output
```
-1/4*arctan((b*sec(f*x+e))^(1/2)/b^(1/2))/b^(3/2)/f+1/4*arctanh((b*sec(f*x
+e))^(1/2)/b^(1/2))/b^(3/2)/f-1/2*cot(f*x+e)^2*(b*sec(f*x+e))^(3/2)/b^3/f
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.05

$$\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{-4\csc^2(e+fx) - 2\arctan\left(\sqrt{\sec(e+fx)}\right)\sqrt{\sec(e+fx)} + \left(-\log\left(1-\sqrt{\sec\right.\right.}{8bf\sqrt{b\sec(e+fx)}}$$

input
```
Integrate[Csc[e + f*x]^3/(b*Sec[e + f*x])^(3/2),x]
```

output
```
(-4*Csc[e + f*x]^2 - 2*ArcTan[Sqrt[Sec[e + f*x]]]*Sqrt[Sec[e + f*x]] + (-L
og[1 - Sqrt[Sec[e + f*x]]] + Log[1 + Sqrt[Sec[e + f*x]]])*Sqrt[Sec[e + f*x
]])/(8*b*f*Sqrt[b*Sec[e + f*x]])
```

Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3102, 27, 253, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc(e+fx)^3}{(b\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{\int \frac{b^4\sqrt{b\sec(e+fx)}}{(b^2-b^2\sec^2(e+fx))^2}d(b\sec(e+fx))}{b^3f}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{\sqrt{b\sec(e+fx)}}{(b^2-b^2\sec^2(e+fx))^2}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{253}$$

$$\frac{b\left(\frac{\int \frac{\sqrt{b\sec(e+fx)}}{b^2-b^2\sec^2(e+fx)}d(b\sec(e+fx))}{4b^2}+\frac{(b\sec(e+fx))^{3/2}}{2b^2(b^2-b^2\sec^2(e+fx))}\right)}{f}$$

$$\Big\downarrow \text{266}$$

$$\frac{b\left(\frac{\int \frac{b^2\sec^2(e+fx)}{b^2-b^4\sec^4(e+fx)}d\sqrt{b\sec(e+fx)}}{2b^2}+\frac{(b\sec(e+fx))^{3/2}}{2b^2(b^2-b^2\sec^2(e+fx))}\right)}{f}$$

$$\Big\downarrow \ 827$$

$$\frac{b\left(\frac{\frac{1}{2}\int \frac{1}{b-b^2\sec^2(e+fx)}d\sqrt{b\sec(e+fx)}-\frac{1}{2}\int \frac{1}{b^2\sec^2(e+fx)+b}d\sqrt{b\sec(e+fx)}}{2b^2}+\frac{(b\sec(e+fx))^{3/2}}{2b^2(b^2-b^2\sec^2(e+fx))}\right)}{f}$$

$$\Big\downarrow \ 216$$

$$\frac{b\left(\frac{\frac{1}{2}\int \frac{1}{b-b^2\sec^2(e+fx)}d\sqrt{b\sec(e+fx)}-\frac{\arctan\left(\sqrt{b}\sec(e+fx)\right)}{2\sqrt{b}}}{2b^2}+\frac{(b\sec(e+fx))^{3/2}}{2b^2(b^2-b^2\sec^2(e+fx))}\right)}{f}$$

$$\Big\downarrow \ 219$$

$$\frac{b\left(\frac{\frac{\text{arctanh}\left(\sqrt{b}\sec(e+fx)\right)}{2\sqrt{b}}-\frac{\arctan\left(\sqrt{b}\sec(e+fx)\right)}{2\sqrt{b}}}{2b^2}+\frac{(b\sec(e+fx))^{3/2}}{2b^2(b^2-b^2\sec^2(e+fx))}\right)}{f}$$

input
```
Int[Csc[e + f*x]^3/(b*Sec[e + f*x])^(3/2),x]
```

output
```
(b*((-1/2*ArcTan[Sqrt[b]*Sec[e + f*x]]/Sqrt[b] + ArcTanh[Sqrt[b]*Sec[e + f
*x]]/(2*Sqrt[b]))/(2*b^2) + (b*Sec[e + f*x])^(3/2)/(2*b^2*(b^2 - b^2*Sec[e
 + f*x]^2))))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(73) = 146$.

Time = 1.67 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.98

method	result
default	$-\frac{\left(\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\cos(fx+e)+1}{\cos(fx+e)+1}\right)\cos(fx+e)-\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}}\right)\cos(fx+e)}{8f\sqrt{-\frac{\mathrm{co}}{(\cos(}}}$

input
```
int(csc(f*x+e)^3/(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/f*(ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*cos(f*x+e)-arctan
(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*cos(f*x+e)+4*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)-ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))+arctan
(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)))/(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2)/(b*sec(f*x+e))^(1/2)/b*csc(f*x+e)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(73) = 146$.

Time = 0.13 (sec) , antiderivative size = 384, normalized size of antiderivative = 4.13

$$\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \left[-\frac{2\left(\cos\left(fx+e\right)^2-1\right)\sqrt{-b}\arctan\left(\frac{2\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)}{b\cos(fx+e)+b}\right)+\left(\cos\left(fx+e\right)^2\right.}{1}\right.$$

$$-\frac{2\left(\cos\left(fx+e\right)^2-1\right)\sqrt{b}\arctan\left(\frac{2\sqrt{b}\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)}{b\cos(fx+e)-b}\right)-\left(\cos\left(fx+e\right)^2-1\right)\sqrt{b}\log\left(\frac{b\cos(fx+e)^2+4\left(\cos(f\right.}{}\right.}{16\left(b^2f\cos\left(fx+e\right)^2-b^2f\right)}$$

input
```
integrate(csc(f*x+e)^3/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
[-1/16*(2*(cos(f*x + e)^2 - 1)*sqrt(-b)*arctan(2*sqrt(-b)*sqrt(b/cos(f*x +
 e))*cos(f*x + e)/(b*cos(f*x + e) + b)) + (cos(f*x + e)^2 - 1)*sqrt(-b)*lo
g((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - cos(f*x + e))*sqrt(-b)*sqrt(b/co
s(f*x + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))
 - 8*sqrt(b/cos(f*x + e))*cos(f*x + e))/(b^2*f*cos(f*x + e)^2 - b^2*f), -1
/16*(2*(cos(f*x + e)^2 - 1)*sqrt(b)*arctan(2*sqrt(b)*sqrt(b/cos(f*x + e))*
cos(f*x + e)/(b*cos(f*x + e) - b)) - (cos(f*x + e)^2 - 1)*sqrt(b)*log((b*c
os(f*x + e)^2 + 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(b)*sqrt(b/cos(f*x +
 e)) + 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)) - 8*sq
rt(b/cos(f*x + e))*cos(f*x + e))/(b^2*f*cos(f*x + e)^2 - b^2*f)]
```

Sympy [F]

$$\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\csc^3{(e+fx)}}{(b\sec{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**3/(b*sec(f*x+e))**(3/2),x)
```

output
```
Integral(csc(e + f*x)**3/(b*sec(e + f*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.14

$$\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{b\left(\frac{4\left(\frac{b}{\cos(fx+e)}\right)^{\frac{3}{2}}}{b^4-\frac{b^4}{\cos(fx+e)^2}} - \frac{2\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}}\right)}{b^{\frac{5}{2}}} - \frac{\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos(fx+e)}}}\right)}{b^{\frac{5}{2}}}\right)}{8f}$$

input
```
integrate(csc(f*x+e)^3/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
1/8*b*(4*(b/cos(f*x + e))^(3/2)/(b^4 - b^4/cos(f*x + e)^2) - 2*arctan(sqrt
(b/cos(f*x + e))/sqrt(b))/b^(5/2) - log(-(sqrt(b) - sqrt(b/cos(f*x + e)))/
(sqrt(b) + sqrt(b/cos(f*x + e))))/b^(5/2))/f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00

$$\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{\frac{2\sqrt{b\cos(fx+e)}}{b^2\cos(fx+e)^2-b^2} - \frac{\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b}b} + \frac{\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{b}}\right)}{b^{\frac{3}{2}}}}{4\,f\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}$$

input
```
integrate(csc(f*x+e)^3/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
1/4*(2*sqrt(b*cos(f*x + e))/(b^2*cos(f*x + e)^2 - b^2) - arctan(sqrt(b*cos
(f*x + e))/sqrt(-b))/(sqrt(-b)*b) + arctan(sqrt(b*cos(f*x + e))/sqrt(b))/b
^(3/2))/(f*sgn(cos(f*x + e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)^3\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

input
```
int(1/(sin(e + f*x)^3*(b/cos(e + f*x))^(3/2)),x)
```

output
```
int(1/(sin(e + f*x)^3*(b/cos(e + f*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(fx+e)}\,\csc(fx+e)^3}{\sec(fx+e)^2}dx\right)}{b^2}$$

input
```
int(csc(f*x+e)^3/(b*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(e + f*x))*csc(e + f*x)**3)/sec(e + f*x)**2,x))/b**2
```

3.430 $\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 123

$$\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{3\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32b^{3/2}f}+\frac{3\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32b^{3/2}f} - \frac{3\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{16b^3f}-\frac{\cot^4(e+fx)(b\sec(e+fx))^{3/2}}{4b^3f}$$

output
```
-3/32*arctan((b*sec(f*x+e))^(1/2)/b^(1/2))/b^(3/2)/f+3/32*arctanh((b*sec(f
*x+e))^(1/2)/b^(1/2))/b^(3/2)/f-3/16*cot(f*x+e)^2*(b*sec(f*x+e))^(3/2)/b^3
/f-1/4*cot(f*x+e)^4*(b*sec(f*x+e))^(3/2)/b^3/f
```

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.89

$$\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{4\csc^2(e+fx)-16\csc^4(e+fx)-6\arctan\left(\sqrt{\sec(e+fx)}\right)\sqrt{\sec(e+fx)}+3}{64bf\sqrt{b\sec(e}}$$

input
```
Integrate[Csc[e + f*x]^5/(b*Sec[e + f*x])^(3/2),x]
```

output
```
(4*Csc[e + f*x]^2 - 16*Csc[e + f*x]^4 - 6*ArcTan[Sqrt[Sec[e + f*x]]]*Sqrt[
Sec[e + f*x]] + 3*(-Log[1 - Sqrt[Sec[e + f*x]]] + Log[1 + Sqrt[Sec[e + f*x
]]])*Sqrt[Sec[e + f*x]])/(64*b*f*Sqrt[b*Sec[e + f*x]])
```

Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3102, 25, 27, 252, 253, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\csc(e+fx)^5}{(b\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \ 3102$$

$$\frac{\int -\frac{b^6(b\sec(e+fx))^{5/2}}{(b^2-b^2\sec^2(e+fx))^3}d(b\sec(e+fx))}{b^5f}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int \frac{b^6(b\sec(e+fx))^{5/2}}{(b^2-b^2\sec^2(e+fx))^3}d(b\sec(e+fx))}{b^5f}$$

$$\Big\downarrow \ 27$$

$$-\frac{b\int \frac{(b\sec(e+fx))^{5/2}}{(b^2-b^2\sec^2(e+fx))^3}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \ 252$$

$$-\frac{b\left(\frac{(b\sec(e+fx))^{3/2}}{4(b^2-b^2\sec^2(e+fx))^2}-\frac{3}{8}\int \frac{\sqrt{b\sec(e+fx)}}{(b^2-b^2\sec^2(e+fx))^2}d(b\sec(e+fx))\right)}{f}$$

$$\Big\downarrow \ 253$$

$$-\frac{b\left(\frac{(b\sec(e+fx))^{3/2}}{4(b^2-b^2\sec^2(e+fx))^2}-\frac{3}{8}\left(\frac{\int\frac{\sqrt{b\sec(e+fx)}}{b^2-b^2\sec^2(e+fx)}d(b\sec(e+fx))}{4b^2}+\frac{(b\sec(e+fx))^{3/2}}{2b^2(b^2-b^2\sec^2(e+fx))}\right)\right)}{f}$$

$\Big\downarrow$ 266

$$-\frac{b\left(\frac{(b\sec(e+fx))^{3/2}}{4(b^2-b^2\sec^2(e+fx))^2}-\frac{3}{8}\left(\frac{\int\frac{b^2\sec^2(e+fx)}{b^2-b^4\sec^4(e+fx)}d\sqrt{b\sec(e+fx)}}{2b^2}+\frac{(b\sec(e+fx))^{3/2}}{2b^2(b^2-b^2\sec^2(e+fx))}\right)\right)}{f}$$

$\Big\downarrow$ 827

$$-\frac{b\left(\frac{(b\sec(e+fx))^{3/2}}{4(b^2-b^2\sec^2(e+fx))^2}-\frac{3}{8}\left(\frac{\frac{1}{2}\int\frac{1}{b-b^2\sec^2(e+fx)}d\sqrt{b\sec(e+fx)}-\frac{1}{2}\int\frac{1}{b^2\sec^2(e+fx)+b}d\sqrt{b\sec(e+fx)}}{2b^2}+\frac{(b\sec(e+fx))^{3/2}}{2b^2(b^2-b^2\sec^2(e+fx))}\right)\right)}{f}$$

$\Big\downarrow$ 216

$$-\frac{b\left(\frac{(b\sec(e+fx))^{3/2}}{4(b^2-b^2\sec^2(e+fx))^2}-\frac{3}{8}\left(\frac{\frac{1}{2}\int\frac{1}{b-b^2\sec^2(e+fx)}d\sqrt{b\sec(e+fx)}-\frac{\arctan\left(\sqrt{b}\sec(e+fx)\right)}{2\sqrt{b}}}{2b^2}+\frac{(b\sec(e+fx))^{3/2}}{2b^2(b^2-b^2\sec^2(e+fx))}\right)\right)}{f}$$

$\Big\downarrow$ 219

$$-\frac{b\left(\frac{(b\sec(e+fx))^{3/2}}{4(b^2-b^2\sec^2(e+fx))^2}-\frac{3}{8}\left(\frac{\frac{\text{arctanh}\left(\sqrt{b}\sec(e+fx)\right)}{2\sqrt{b}}-\frac{\arctan\left(\sqrt{b}\sec(e+fx)\right)}{2\sqrt{b}}}{2b^2}+\frac{(b\sec(e+fx))^{3/2}}{2b^2(b^2-b^2\sec^2(e+fx))}\right)\right)}{f}$$

input
```
Int[Csc[e + f*x]^5/(b*Sec[e + f*x])^(3/2),x]
```

output
```
-((b*((b*Sec[e + f*x])^(3/2)/(4*(b^2 - b^2*Sec[e + f*x]^2)^2) - (3*((-1/2*
ArcTan[Sqrt[b]*Sec[e + f*x]]/Sqrt[b] + ArcTanh[Sqrt[b]*Sec[e + f*x]]/(2*Sq
rt[b]))/(2*b^2) + (b*Sec[e + f*x])^(3/2)/(2*b^2*(b^2 - b^2*Sec[e + f*x]^2)
))/8))/f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(99) = 198$.

Time = 1.68 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.74

method	result
default	$\frac{\left(-4\cos(fx+e)^2-12\right)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,\csc(fx+e)^4+(-3\cos(fx+e)+3)\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}}{\cos(fx+e)+1}\right.}{64f\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,\sqrt{b\sec(fx+e)}\,b}$

input
```
int(csc(f*x+e)^5/(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/64/f/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/(b*sec(f*x+e))^(1/2)/b*((-4*co
s(f*x+e)^2-12)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*csc(f*x+e)^4+(-3*cos(f
*x+e)+3)*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*csc(f*x+e)^2+(3*
cos(f*x+e)-3)*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*csc(f*x+e)^
2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(99) = 198$.

Time = 0.16 (sec) , antiderivative size = 473, normalized size of antiderivative = 3.85

$$\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \left[-\frac{6\left(\cos\left(fx+e\right)^4 - 2\cos\left(fx+e\right)^2 + 1\right)\sqrt{-b}\arctan\left(\frac{2\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)}{b\cos(fx+e)+b}\right)}{\ldots}\right.$$

$$-\frac{6\left(\cos\left(fx+e\right)^4 - 2\cos\left(fx+e\right)^2 + 1\right)\sqrt{b}\arctan\left(\frac{2\sqrt{b}\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)}{b\cos(fx+e)-b}\right) - 3\left(\cos\left(fx+e\right)^4 - 2\cos\left(f\ldots\right.\right.}{128\left(b^2 f\cos\left(f\ldots\right.\right.}$$

input
```
integrate(csc(f*x+e)^5/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
[-1/128*(6*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-b)*arctan(2*sqrt(
-b)*sqrt(b/cos(f*x + e))*cos(f*x + e)/(b*cos(f*x + e) + b)) + 3*(cos(f*x +
 e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x
+ e)^2 - cos(f*x + e))*sqrt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos(f*x + e) +
b)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 8*(cos(f*x + e)^3 + 3*cos(f*x
+ e))*sqrt(b/cos(f*x + e)))/(b^2*f*cos(f*x + e)^4 - 2*b^2*f*cos(f*x + e)^2
 + b^2*f), -1/128*(6*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(b)*arcta
n(2*sqrt(b)*sqrt(b/cos(f*x + e))*cos(f*x + e)/(b*cos(f*x + e) - b)) - 3*(c
os(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(b)*log((b*cos(f*x + e)^2 + 4*(c
os(f*x + e)^2 + cos(f*x + e))*sqrt(b)*sqrt(b/cos(f*x + e)) + 6*b*cos(f*x +
 e) + b)/(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)) + 8*(cos(f*x + e)^3 + 3*co
s(f*x + e))*sqrt(b/cos(f*x + e)))/(b^2*f*cos(f*x + e)^4 - 2*b^2*f*cos(f*x
+ e)^2 + b^2*f)]
```

Sympy [F]

$$\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\csc^5{(e+fx)}}{(b\sec{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**5/(b*sec(f*x+e))**(3/2),x)
```

output
```
Integral(csc(e + f*x)**5/(b*sec(e + f*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.11

$$\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{b\left(\frac{4\left(b^2\left(\frac{b}{\cos(fx+e)}\right)^{\frac{3}{2}}+3\left(\frac{b}{\cos(fx+e)}\right)^{\frac{7}{2}}\right)}{b^6-\frac{2b^6}{\cos(fx+e)^2}+\frac{b^6}{\cos(fx+e)^4}}+\frac{6\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}}\right)}{b^{\frac{5}{2}}}+\frac{3\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos(fx+e)}}}\right)}{b^{\frac{5}{2}}}\right)}{64f}$$

input
```
integrate(csc(f*x+e)^5/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
-1/64*b*(4*(b^2*(b/cos(f*x + e))^(3/2) + 3*(b/cos(f*x + e))^(7/2))/(b^6 -
2*b^6/cos(f*x + e)^2 + b^6/cos(f*x + e)^4) + 6*arctan(sqrt(b/cos(f*x + e))
/sqrt(b))/b^(5/2) + 3*log(-(sqrt(b) - sqrt(b/cos(f*x + e)))/(sqrt(b) + sqr
t(b/cos(f*x + e))))/b^(5/2))/f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.04

$$\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx =$$

$$-\frac{b^2\left(\frac{3\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b}b^3}-\frac{3\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{b}}\right)}{b^{\frac{7}{2}}}+\frac{2\left(\sqrt{b\cos(fx+e)}b^2\cos(fx+e)^2+3\sqrt{b\cos(fx+e)}b^2\right)}{\left(b^2\cos(fx+e)^2-b^2\right)^2b^2}\right)}{32\,f\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}$$

input
```
integrate(csc(f*x+e)^5/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
-1/32*b^2*(3*arctan(sqrt(b*cos(f*x + e))/sqrt(-b))/(sqrt(-b)*b^3) - 3*arct
an(sqrt(b*cos(f*x + e))/sqrt(b))/b^(7/2) + 2*(sqrt(b*cos(f*x + e))*b^2*cos
(f*x + e)^2 + 3*sqrt(b*cos(f*x + e))*b^2)/((b^2*cos(f*x + e)^2 - b^2)^2*b^
2))/(f*sgn(cos(f*x + e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)^5\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

input
```
int(1/(sin(e + f*x)^5*(b/cos(e + f*x))^(3/2)),x)
```

output
```
int(1/(sin(e + f*x)^5*(b/cos(e + f*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(fx+e)}\,\csc(fx+e)^5}{\sec(fx+e)^2}dx\right)}{b^2}$$

input
```
int(csc(f*x+e)^5/(b*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(e + f*x))*csc(e + f*x)**5)/sec(e + f*x)**2,x))/b**2
```

3.431 $\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 126

$$\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{8\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{77b^2f} - \frac{12b\sin(e+fx)}{77f(b\sec(e+fx))^{5/2}} + \frac{8\sin(e+fx)}{77bf\sqrt{b\sec(e+fx)}} - \frac{2b\sin^3(e+fx)}{11f(b\sec(e+fx))^{5/2}}$$

output
```
8/77*cos(f*x+e)^(1/2)*InverseJacobiAM(1/2*f*x+1/2*e,2^(1/2))*(b*sec(f*x+e)
)^(1/2)/b^2/f-12/77*b*sin(f*x+e)/f/(b*sec(f*x+e))^(5/2)+8/77*sin(f*x+e)/b/
f/(b*sec(f*x+e))^(1/2)-2/11*b*sin(f*x+e)^3/f/(b*sec(f*x+e))^(5/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.64

$$\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{\sec^2(e+fx)\left(128\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right) - 5\sin(2(e+fx)) - 2\right.}{1232f(b\sec(e+fx))^{3/2}}$$

input
```
Integrate[Sin[e + f*x]^4/(b*Sec[e + f*x])^(3/2),x]
```

output
```
(Sec[e + f*x]^2*(128*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2] - 5*Sin[
2*(e + f*x)] - 24*Sin[4*(e + f*x)] + 7*Sin[6*(e + f*x)]))/(1232*f*(b*Sec[e
 + f*x])^(3/2))
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3107, 3042, 3107, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc(e+fx)^4(b\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3107}$$

$$\frac{6}{11}\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{3/2}}dx-\frac{2b\sin^3(e+fx)}{11f(b\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{11}\int \frac{1}{\csc(e+fx)^2(b\sec(e+fx))^{3/2}}dx-\frac{2b\sin^3(e+fx)}{11f(b\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3107}$$

$$\frac{6}{11}\left(\frac{2}{7}\int \frac{1}{(b\sec(e+fx))^{3/2}}dx-\frac{2b\sin(e+fx)}{7f(b\sec(e+fx))^{5/2}}\right)-\frac{2b\sin^3(e+fx)}{11f(b\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{11}\left(\frac{2}{7}\int \frac{1}{\left(b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx-\frac{2b\sin(e+fx)}{7f(b\sec(e+fx))^{5/2}}\right)-\frac{2b\sin^3(e+fx)}{11f(b\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{6}{11}\left(\frac{2}{7}\left(\frac{\int \sqrt{b\sec(e+fx)}dx}{3b^2}+\frac{2\sin(e+fx)}{3bf\sqrt{b\sec(e+fx)}}\right)-\frac{2b\sin(e+fx)}{7f(b\sec(e+fx))^{5/2}}\right)-\frac{2b\sin^3(e+fx)}{11f(b\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{11}\left(\frac{2}{7}\left(\frac{\int \sqrt{b\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{3b^2}+\frac{2\sin(e+fx)}{3bf\sqrt{b\sec(e+fx)}}\right)-\frac{2b\sin(e+fx)}{7f(b\sec(e+fx))^{5/2}}\right)-\frac{2b\sin^3(e+fx)}{11f(b\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{6}{11}\left(\frac{2}{7}\left(\frac{\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}}dx}{3b^2}+\frac{2\sin(e+fx)}{3bf\sqrt{b\sec(e+fx)}}\right)-\frac{2b\sin(e+fx)}{7f(b\sec(e+fx))^{5/2}}\right)-\frac{2b\sin^3(e+fx)}{11f(b\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{11}\left(\frac{2}{7}\left(\frac{\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(e+fx)}{3bf\sqrt{b\sec(e+fx)}}\right)-\frac{2b\sin(e+fx)}{7f(b\sec(e+fx))^{5/2}}\right)-\frac{2b\sin^3(e+fx)}{11f(b\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{6}{11}\left(\frac{2}{7}\left(\frac{2\sqrt{\cos(e+fx)}\operatorname{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{3b^2f}+\frac{2\sin(e+fx)}{3bf\sqrt{b\sec(e+fx)}}\right)-\frac{2b\sin(e+fx)}{7f(b\sec(e+fx))^{5/2}}\right)-\frac{2b\sin^3(e+fx)}{11f(b\sec(e+fx))^{5/2}}$$

input
```
Int[Sin[e + f*x]^4/(b*Sec[e + f*x])^(3/2),x]
```

output
```
(-2*b*Sin[e + f*x]^3)/(11*f*(b*Sec[e + f*x])^(5/2)) + (6*((-2*b*Sin[e + f*
x])/(7*f*(b*Sec[e + f*x])^(5/2)) + (2*((2*Sqrt[Cos[e + f*x]]*EllipticF[(e
+ f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(3*b^2*f) + (2*Sin[e + f*x])/(3*b*f*Sqr
t[b*Sec[e + f*x]])))/7))/11
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3107
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Simp[(m + 1)/(a^2*(m + n))   Int[(a*Csc[e + f*x])^(m
+ 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.39 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.90

method	result
default	$\frac{-\frac{2\sin(fx+e)\left(-7\cos(fx+e)^4+13\cos(fx+e)^2-4\right)}{77}-\frac{2i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\mathrm{EllipticF}(i(\cot(fx+e)-\csc(fx+e)),i)(-4-4\sec(fx+e))}{77}}{f\sqrt{b\sec(fx+e)}\,b}$

input
```
int(sin(f*x+e)^4/(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-2/77*sin(f*x+e)*(-7*cos(f*x+e)^4+13*cos(f*x+e)^2-4)-2/77*I*(1/(cos(f
*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-
csc(f*x+e)),I)*(-4-4*sec(f*x+e)))/(b*sec(f*x+e))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.87

$$\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{2\left(\left(7\cos\left(fx+e\right)^5-13\cos\left(fx+e\right)^3+4\cos\left(fx+e\right)\right)\sqrt{\frac{b}{\cos(fx+e)}}\sin\left(fx+e\right)\right.}{}$$

input
```
integrate(sin(f*x+e)^4/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
2/77*((7*cos(f*x + e)^5 - 13*cos(f*x + e)^3 + 4*cos(f*x + e))*sqrt(b/cos(f
*x + e))*sin(f*x + e) - 2*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos
(f*x + e) + I*sin(f*x + e)) + 2*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4,
0, cos(f*x + e) - I*sin(f*x + e)))/(b^2*f)
```

Sympy [F]

$$\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\sin^4{(e+fx)}}{(b\sec{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)**4/(b*sec(f*x+e))**(3/2),x)
```

output
```
Integral(sin(e + f*x)**4/(b*sec(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\sin{(fx+e)}^4}{(b\sec{(fx+e)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^4/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^4/(b*sec(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\sin{(fx+e)}^4}{(b\sec{(fx+e)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^4/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(sin(f*x + e)^4/(b*sec(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\sin(e+f\,x)^4}{\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int(sin(e + f*x)^4/(b/cos(e + f*x))^(3/2),x)
```

output
```
int(sin(e + f*x)^4/(b/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(fx+e)}\,\sin(fx+e)^4}{\sec(fx+e)^2}dx\right)}{b^2}$$

input
```
int(sin(f*x+e)^4/(b*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(e + f*x))*sin(e + f*x)**4)/sec(e + f*x)**2,x))/b**2
```

3.432 $\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{4\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{21b^2f} - \frac{2b\sin(e+fx)}{7f(b\sec(e+fx))^{5/2}} + \frac{4\sin(e+fx)}{21bf\sqrt{b\sec(e+fx)}}$$

output
```
4/21*cos(f*x+e)^(1/2)*InverseJacobiAM(1/2*f*x+1/2*e,2^(1/2))*(b*sec(f*x+e)
)^(1/2)/b^2/f-2/7*b*sin(f*x+e)/f/(b*sec(f*x+e))^(5/2)+4/21*sin(f*x+e)/b/f/
(b*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.72

$$\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{\sec^2(e+fx)\left(16\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)+2\sin(2(e+fx))-3\,\text{s}\right.}{84f(b\sec(e+fx))^{3/2}}$$

input
```
Integrate[Sin[e + f*x]^2/(b*Sec[e + f*x])^(3/2),x]
```

output
```
(Sec[e + f*x]^2*(16*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2] + 2*Sin[2
*(e + f*x)] - 3*Sin[4*(e + f*x)]))/(84*f*(b*Sec[e + f*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3107, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc(e+fx)^2(b\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3107}$$

$$\frac{2}{7}\int \frac{1}{(b\sec(e+fx))^{3/2}}dx-\frac{2b\sin(e+fx)}{7f(b\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{7}\int \frac{1}{\left(b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx-\frac{2b\sin(e+fx)}{7f(b\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{2}{7}\left(\frac{\int \sqrt{b\sec(e+fx)}dx}{3b^2}+\frac{2\sin(e+fx)}{3bf\sqrt{b\sec(e+fx)}}\right)-\frac{2b\sin(e+fx)}{7f(b\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{7}\left(\frac{\int \sqrt{b\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{3b^2}+\frac{2\sin(e+fx)}{3bf\sqrt{b\sec(e+fx)}}\right)-\frac{2b\sin(e+fx)}{7f(b\sec(e+fx))^{5/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{2}{7}\left(\frac{\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\int\frac{1}{\sqrt{\cos(e+fx)}}dx}{3b^2}+\frac{2\sin(e+fx)}{3bf\sqrt{b\sec(e+fx)}}\right)-\frac{2b\sin(e+fx)}{7f(b\sec(e+fx))^{5/2}}$$

$\big\downarrow$ 3042

$$\frac{2}{7}\left(\frac{\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\int\frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(e+fx)}{3bf\sqrt{b\sec(e+fx)}}\right)-\frac{2b\sin(e+fx)}{7f(b\sec(e+fx))^{5/2}}$$

$\big\downarrow$ 3120

$$\frac{2}{7}\left(\frac{2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{3b^2f}+\frac{2\sin(e+fx)}{3bf\sqrt{b\sec(e+fx)}}\right)-\frac{2b\sin(e+fx)}{7f(b\sec(e+fx))^{5/2}}$$

input
```
Int[Sin[e + f*x]^2/(b*Sec[e + f*x])^(3/2),x]
```

output
```
(-2*b*Sin[e + f*x])/(7*f*(b*Sec[e + f*x])^(5/2)) + (2*((2*Sqrt[Cos[e + f*x
]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(3*b^2*f) + (2*Sin[e +
f*x])/(3*b*f*Sqrt[b*Sec[e + f*x]])))/7
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3107
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Simp[(m + 1)/(a^2*(m + n))   Int[(a*Csc[e + f*x])^(m
+ 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.35 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.06

method	result
default	$-\frac{2\left(\sin(fx+e)\left(3\cos(fx+e)^{2}-2\right)+i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{EllipticF}(i(\cot(fx+e)-\csc(fx+e)),i)(-2-2\sec(fx+e))\right)}{21f\sqrt{b\sec(fx+e)}\,b}$

input
```
int(sin(f*x+e)^2/(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21/f/(b*sec(f*x+e))^(1/2)/b*(sin(f*x+e)*(3*cos(f*x+e)^2-2)+I*(1/(cos(f*
x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-c
sc(f*x+e)),I)*(-2-2*sec(f*x+e)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01

$$\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx =$$
$$-\frac{2\left(\left(3\cos\left(fx+e\right)^3-2\cos\left(fx+e\right)\right)\sqrt{\frac{b}{\cos(fx+e)}}\sin\left(fx+e\right)+i\sqrt{2}\sqrt{b}\text{weierstrassPInverse}(-4,0,\cos\left(f\right.\right.}{21\,b^2 f}$$

input
```
integrate(sin(f*x+e)^2/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
-2/21*((3*cos(f*x + e)^3 - 2*cos(f*x + e))*sqrt(b/cos(f*x + e))*sin(f*x +
e) + I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x
 + e)) - I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin
(f*x + e)))/(b^2*f)
```

Sympy [F]

$$\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\sin^2\left(e+fx\right)}{\left(b\sec\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)**2/(b*sec(f*x+e))**(3/2),x)
```

output
```
Integral(sin(e + f*x)**2/(b*sec(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\sin\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^2/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^2/(b*sec(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\sin\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^2/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(sin(f*x + e)^2/(b*sec(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\sin\left(e+f\,x\right)^2}{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^{3/2}}\,dx$$

input
```
int(sin(e + f*x)^2/(b/cos(e + f*x))^(3/2),x)
```

output
```
int(sin(e + f*x)^2/(b/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(fx+e)}\,\sin(fx+e)^2}{\sec(fx+e)^2}dx\right)}{b^2}$$

input
```
int(sin(f*x+e)^2/(b*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(e + f*x))*sin(e + f*x)**2)/sec(e + f*x)**2,x))/b**2
```

3.433 $\int \frac{1}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 72

$$\int \frac{1}{(b\sec(e+fx))^{3/2}}\,dx = \frac{2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{3b^2f} + \frac{2\sin(e+fx)}{3bf\sqrt{b\sec(e+fx)}}$$

output
```
2/3*cos(f*x+e)^(1/2)*InverseJacobiAM(1/2*f*x+1/2*e,2^(1/2))*(b*sec(f*x+e))
^(1/2)/b^2/f+2/3*sin(f*x+e)/b/f/(b*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82

$$\int \frac{1}{(b\sec(e+fx))^{3/2}}\,dx = \frac{\sec^2(e+fx)\left(2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)+\sin(2(e+fx))\right)}{3f(b\sec(e+fx))^{3/2}}$$

input
```
Integrate[(b*Sec[e + f*x])^(-3/2),x]
```

output
```
(Sec[e + f*x]^2*(2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2] + Sin[2*(e
 + f*x)]))/(3*f*(b*Sec[e + f*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$\frac{\int \sqrt{b\sec(e+fx)}dx}{3b^2}+\frac{2\sin(e+fx)}{3bf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{b\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{3b^2}+\frac{2\sin(e+fx)}{3bf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}}dx}{3b^2}+\frac{2\sin(e+fx)}{3bf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{3b^2}+\frac{2\sin(e+fx)}{3bf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{3b^2f}+\frac{2\sin(e+fx)}{3bf\sqrt{b\sec(e+fx)}}$$

input
```
Int[(b*Sec[e + f*x])^(-3/2),x]
```

output
```
(2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(3*b
^2*f) + (2*Sin[e + f*x])/(3*b*f*Sqrt[b*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.20 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.28

method	result	size
default	$-\frac{\frac{2i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{EllipticF}(i(\cot(fx+e)-\csc(fx+e)),i)(-1-\sec(fx+e))}{3}+\frac{2\sin(fx+e)}{3}}{f\sqrt{b\sec(fx+e)}\,b}$	92

input
```
int(1/(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-2/3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ell
ipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*(-1-sec(f*x+e))+2/3*sin(f*x+e))/(b*sec
(f*x+e))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.21

$$\int \frac{1}{(b\sec(e+fx))^{3/2}}\,dx = \frac{2\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e) - i\sqrt{2}\sqrt{b}\,\text{weierstrassPInverse}(-4,0,\cos(}{}$$

input
```
integrate(1/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
1/3*(2*sqrt(b/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - I*sqrt(2)*sqrt(b)*
weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e)) + I*sqrt(2)*sqrt
(b)*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)))/(b^2*f)
```

Sympy [F]

$$\int \frac{1}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{(b\sec{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*sec(f*x+e))**(3/2),x)
```

output
```
Integral((b*sec(e + f*x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{(b\sec{(fx+e)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{(b\sec{(fx+e)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\sec(e+fx))^{3/2}}\,dx=\int \frac{1}{\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(b/cos(e + f*x))^(3/2),x)
```

output
```
int(1/(b/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(b\sec(e+fx))^{3/2}}\,dx=\frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(fx+e)}}{\sec(fx+e)^2}dx\right)}{b^2}$$

input
```
int(1/(b*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(b)*int(sqrt(sec(e + f*x))/sec(e + f*x)**2,x))/b**2
```

3.434 $\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 68

$$\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{\csc(e+fx)}{bf\sqrt{b\sec(e+fx)}} - \frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{b^2f}$$

output
```
-csc(f*x+e)/b/f/(b*sec(f*x+e))^(1/2)-cos(f*x+e)^(1/2)*InverseJacobiAM(1/2*
f*x+1/2*e,2^(1/2))*(b*sec(f*x+e))^(1/2)/b^2/f
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.85

$$\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{-\sqrt{\cos(e+fx)}\csc(e+fx) - \text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)}{f\cos^{\frac{3}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

input
```
Integrate[Csc[e + f*x]^2/(b*Sec[e + f*x])^(3/2),x]
```

output
```
(-(Sqrt[Cos[e + f*x]]*Csc[e + f*x]) - EllipticF[(e + f*x)/2, 2])/(f*Cos[e
+ f*x]^(3/2)*(b*Sec[e + f*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3103, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc(e+fx)^2}{(b\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3103}$$

$$-\frac{\int \sqrt{b\sec(e+fx)}dx}{2b^2}-\frac{\csc(e+fx)}{bf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \sqrt{b\csc\left(e+fx+\frac{\pi}{2}\right)}dx}{2b^2}-\frac{\csc(e+fx)}{bf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{4258}$$

$$-\frac{\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}}dx}{2b^2}-\frac{\csc(e+fx)}{bf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx}{2b^2}-\frac{\csc(e+fx)}{bf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{b^2f}-\frac{\csc(e+fx)}{bf\sqrt{b\sec(e+fx)}}$$

input
```
Int[Csc[e + f*x]^2/(b*Sec[e + f*x])^(3/2),x]
```

output
```
-(Csc[e + f*x]/(b*f*Sqrt[b*Sec[e + f*x]])) - (Sqrt[Cos[e + f*x]]*EllipticF
[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(b^2*f)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3103
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
1)/(f*b*(m - 1))), x] + Simp[a^2*((n + 1)/(b^2*(m - 1)))   Int[(a*Csc[e + f
*x])^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.35 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.35

method	result	size
default	$\frac{i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{EllipticF}(i(\cot(fx+e)-\csc(fx+e)),i)(-1-\sec(fx+e))-\csc(fx+e)}{f\sqrt{b\sec(fx+e)}\,b}$	92

input
```
int(csc(f*x+e)^2/(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f/(b*sec(f*x+e))^(1/2)/b*(I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*(-1-sec(f*x+e))-csc(
f*x+e))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.49

$$\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{i\sqrt{2}\sqrt{b}\sin(fx+e)\,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e))-}{}$$

input
```
integrate(csc(f*x+e)^2/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
1/2*(I*sqrt(2)*sqrt(b)*sin(f*x + e)*weierstrassPInverse(-4, 0, cos(f*x + e
) + I*sin(f*x + e)) - I*sqrt(2)*sqrt(b)*sin(f*x + e)*weierstrassPInverse(-
4, 0, cos(f*x + e) - I*sin(f*x + e)) - 2*sqrt(b/cos(f*x + e))*cos(f*x + e)
)/(b^2*f*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\csc^2{(e+fx)}}{(b\sec{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**2/(b*sec(f*x+e))**(3/2),x)
```

output
```
Integral(csc(e + f*x)**2/(b*sec(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\csc{(fx+e)}^2}{(b\sec{(fx+e)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^2/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(csc(f*x + e)^2/(b*sec(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\csc{(fx+e)}^2}{(b\sec{(fx+e)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^2/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(csc(f*x + e)^2/(b*sec(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\sin(e+f\,x)^2\,\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(sin(e + f*x)^2*(b/cos(e + f*x))^(3/2)),x)
```

output
```
int(1/(sin(e + f*x)^2*(b/cos(e + f*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(fx+e)}\,\csc(fx+e)^2}{\sec(fx+e)^2}dx\right)}{b^2}$$

input
```
int(csc(f*x+e)^2/(b*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(e + f*x))*csc(e + f*x)**2)/sec(e + f*x)**2,x))/b**2
```

3.435 $\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 102

$$\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{\csc(e+fx)}{6bf\sqrt{b\sec(e+fx)}} - \frac{\csc^3(e+fx)}{3bf\sqrt{b\sec(e+fx)}} - \frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{6b^2f}$$

output
```
1/6*csc(f*x+e)/b/f/(b*sec(f*x+e))^(1/2)-1/3*csc(f*x+e)^3/b/f/(b*sec(f*x+e)
)^(1/2)-1/6*cos(f*x+e)^(1/2)*InverseJacobiAM(1/2*f*x+1/2*e,2^(1/2))*(b*sec
(f*x+e))^(1/2)/b^2/f
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.61

$$\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{\csc(e+fx) - 2\csc^3(e+fx) - \frac{\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)}{\sqrt{\cos(e+fx)}}}{6bf\sqrt{b\sec(e+fx)}}$$

input
```
Integrate[Csc[e + f*x]^4/(b*Sec[e + f*x])^(3/2),x]
```

output
```
(Csc[e + f*x] - 2*Csc[e + f*x]^3 - EllipticF[(e + f*x)/2, 2]/Sqrt[Cos[e +
f*x]])/(6*b*f*Sqrt[b*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3103, 3042, 3105, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc(e+fx)^4}{(b\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3103}$$

$$-\frac{\int \csc^2(e+fx)\sqrt{b\sec(e+fx)}dx}{6b^2}-\frac{\csc^3(e+fx)}{3bf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \csc(e+fx)^2\sqrt{b\sec(e+fx)}dx}{6b^2}-\frac{\csc^3(e+fx)}{3bf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3105}$$

$$-\frac{\frac{1}{2}\int \sqrt{b\sec(e+fx)}dx-\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}}{6b^2}-\frac{\csc^3(e+fx)}{3bf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{1}{2}\int \sqrt{b\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}}{6b^2}-\frac{\csc^3(e+fx)}{3bf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{4258}$$

$$-\frac{\frac{1}{2}\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\int\frac{1}{\sqrt{\cos(e+fx)}}dx-\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}}{6b^2}-\frac{\csc^3(e+fx)}{3bf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{1}{2}\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\int\frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}}{6b^2}-\frac{\csc^3(e+fx)}{3bf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{\frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{f}-\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}}{6b^2}-\frac{\csc^3(e+fx)}{3bf\sqrt{b\sec(e+fx)}}$$

input
```
Int[Csc[e + f*x]^4/(b*Sec[e + f*x])^(3/2),x]
```

output
```
-1/3*Csc[e + f*x]^3/(b*f*Sqrt[b*Sec[e + f*x]]) - (-((b*Csc[e + f*x])/(f*Sq
rt[b*Sec[e + f*x]])) + (Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[
b*Sec[e + f*x]])/f)/(6*b^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3103
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
1)/(f*b*(m - 1))), x] + Simp[a^2*((n + 1)/(b^2*(m - 1)))   Int[(a*Csc[e + f
*x])^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

rule 3105
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Simp[a^2*((m + n - 2)/(m - 1))   Int[(a*Csc[e + f*
x])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[
m, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.20 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{\frac{i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\mathrm{EllipticF}(i(\cot(fx+e)-\csc(fx+e)),i)(-1-\sec(fx+e))}{6}+\frac{\left(-\cos(fx+e)^{2}-1\right)\csc(fx+e)^{3}}{6}}{fb\sqrt{b\sec(fx+e)}}$	106

input
```
int(csc(f*x+e)^4/(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/6*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elli
pticF(I*(cot(f*x+e)-csc(f*x+e)),I)*(-1-sec(f*x+e))+1/6*(-cos(f*x+e)^2-1)*c
sc(f*x+e)^3)/b/(b*sec(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.45

$$\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{2}\left(i\,\cos\left(fx+e\right)^2-i\right)\sqrt{b}\sin\left(fx+e\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)$$

input
```
integrate(csc(f*x+e)^4/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
1/12*(sqrt(2)*(I*cos(f*x + e)^2 - I)*sqrt(b)*sin(f*x + e)*weierstrassPInve
rse(-4, 0, cos(f*x + e) + I*sin(f*x + e)) + sqrt(2)*(-I*cos(f*x + e)^2 + I
)*sqrt(b)*sin(f*x + e)*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x
 + e)) + 2*(cos(f*x + e)^3 + cos(f*x + e))*sqrt(b/cos(f*x + e)))/((b^2*f*c
os(f*x + e)^2 - b^2*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\csc^4\left(e+fx\right)}{\left(b\sec\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**4/(b*sec(f*x+e))**(3/2),x)
```

output
```
Integral(csc(e + f*x)**4/(b*sec(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\csc\left(fx+e\right)^4}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^4/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(csc(f*x + e)^4/(b*sec(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\csc\left(fx+e\right)^4}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^4/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(csc(f*x + e)^4/(b*sec(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)^4\,\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(sin(e + f*x)^4*(b/cos(e + f*x))^(3/2)),x)
```

output
```
int(1/(sin(e + f*x)^4*(b/cos(e + f*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(fx+e)}\,\csc(fx+e)^4}{\sec(fx+e)^2}dx\right)}{b^2}$$

input
```
int(csc(f*x+e)^4/(b*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(e + f*x))*csc(e + f*x)**4)/sec(e + f*x)**2,x))/b**2
```

3.436 $\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 132

$$\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{\csc(e+fx)}{12bf\sqrt{b\sec(e+fx)}}+\frac{\csc^3(e+fx)}{30bf\sqrt{b\sec(e+fx)}} - \frac{\csc^5(e+fx)}{5bf\sqrt{b\sec(e+fx)}}-\frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{12b^2f}$$

output
```
1/12*csc(f*x+e)/b/f/(b*sec(f*x+e))^(1/2)+1/30*csc(f*x+e)^3/b/f/(b*sec(f*x+
e))^(1/2)-1/5*csc(f*x+e)^5/b/f/(b*sec(f*x+e))^(1/2)-1/12*cos(f*x+e)^(1/2)*
InverseJacobiAM(1/2*f*x+1/2*e,2^(1/2))*(b*sec(f*x+e))^(1/2)/b^2/f
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.56

$$\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{5\csc(e+fx)+2\csc^3(e+fx)-12\csc^5(e+fx)-\frac{5\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)}{\sqrt{\cos(e+fx)}}}{60bf\sqrt{b\sec(e+fx)}}$$

input
```
Integrate[Csc[e + f*x]^6/(b*Sec[e + f*x])^(3/2),x]
```

output
```
(5*Csc[e + f*x] + 2*Csc[e + f*x]^3 - 12*Csc[e + f*x]^5 - (5*EllipticF[(e +
 f*x)/2, 2])/Sqrt[Cos[e + f*x]])/(60*b*f*Sqrt[b*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3103, 3042, 3105, 3042, 3105, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc(e+fx)^6}{(b\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3103}$$

$$-\frac{\int \csc^4(e+fx)\sqrt{b\sec(e+fx)}dx}{10b^2}-\frac{\csc^5(e+fx)}{5bf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \csc(e+fx)^4\sqrt{b\sec(e+fx)}dx}{10b^2}-\frac{\csc^5(e+fx)}{5bf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3105}$$

$$-\frac{\frac{5}{6}\int \csc^2(e+fx)\sqrt{b\sec(e+fx)}dx-\frac{b\csc^3(e+fx)}{3f\sqrt{b\sec(e+fx)}}}{10b^2}-\frac{\csc^5(e+fx)}{5bf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{5}{6}\int \csc(e+fx)^2\sqrt{b\sec(e+fx)}dx-\frac{b\csc^3(e+fx)}{3f\sqrt{b\sec(e+fx)}}}{10b^2}-\frac{\csc^5(e+fx)}{5bf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3105}$$

$$-\frac{\frac{5}{6}\left(\frac{1}{2}\int \sqrt{b\sec(e+fx)}dx-\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}\right)-\frac{b\csc^3(e+fx)}{3f\sqrt{b\sec(e+fx)}}}{10b^2}-\frac{\csc^5(e+fx)}{5bf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\frac{5}{6}\left(\frac{1}{2}\int \sqrt{b\csc\left(e+fx+\frac{\pi}{2}\right)}dx-\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}\right)-\frac{b\csc^3(e+fx)}{3f\sqrt{b\sec(e+fx)}}}{10b^2}-\frac{\csc^5(e+fx)}{5bf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{ 4258}$$

$$-\frac{\frac{5}{6}\left(\frac{1}{2}\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{\cos(e+fx)}}dx-\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}\right)-\frac{b\csc^3(e+fx)}{3f\sqrt{b\sec(e+fx)}}}{10b^2}-\frac{\csc^5(e+fx)}{5bf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\frac{5}{6}\left(\frac{1}{2}\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}\right)-\frac{b\csc^3(e+fx)}{3f\sqrt{b\sec(e+fx)}}}{10b^2}-\frac{\csc^5(e+fx)}{5bf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{ 3120}$$

$$-\frac{\frac{5}{6}\left(\frac{\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{b\sec(e+fx)}}{f}-\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}\right)-\frac{b\csc^3(e+fx)}{3f\sqrt{b\sec(e+fx)}}}{10b^2}-\frac{\csc^5(e+fx)}{5bf\sqrt{b\sec(e+fx)}}$$

input
```
Int[Csc[e + f*x]^6/(b*Sec[e + f*x])^(3/2),x]
```

output
```
-1/5*Csc[e + f*x]^5/(b*f*Sqrt[b*Sec[e + f*x]]) - (-1/3*(b*Csc[e + f*x]^3)/
(f*Sqrt[b*Sec[e + f*x]]) + (5*(-((b*Csc[e + f*x])/(f*Sqrt[b*Sec[e + f*x]])
) + (Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/f)
)/6)/(10*b^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3103
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
1)/(f*b*(m - 1))), x] + Simp[a^2*((n + 1)/(b^2*(m - 1)))   Int[(a*Csc[e + f
*x])^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

rule 3105
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Simp[a^2*((m + n - 2)/(m - 1))   Int[(a*Csc[e + f*
x])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[
m, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.89 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.88

method	result	si
default	$\frac{\frac{i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{EllipticF}(i(\cot(fx+e)-\csc(fx+e)),i)(-5-5\sec(fx+e))}{60}+\frac{\left(5\cos(fx+e)^4-12\cos(fx+e)^2-5\right)\csc(fx+e)^5}{60}}{fb\sqrt{b\sec(fx+e)}}$	1

input
```
int(csc(f*x+e)^6/(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/60*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ell
ipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*(-5-5*sec(f*x+e))+1/60*(5*cos(f*x+e)^4
-12*cos(f*x+e)^2-5)*csc(f*x+e)^5)/b/(b*sec(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.49

$$\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{5\sqrt{2}\left(-i\cos(fx+e)^4+2i\cos(fx+e)^2-i\right)\sqrt{b}\sin(fx+e)\,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+$$

input
```
integrate(csc(f*x+e)^6/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
-1/120*(5*sqrt(2)*(-I*cos(f*x + e)^4 + 2*I*cos(f*x + e)^2 - I)*sqrt(b)*sin
(f*x + e)*weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e)) + 5*sq
rt(2)*(I*cos(f*x + e)^4 - 2*I*cos(f*x + e)^2 + I)*sqrt(b)*sin(f*x + e)*wei
erstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) - 2*(5*cos(f*x + e)
^5 - 12*cos(f*x + e)^3 - 5*cos(f*x + e))*sqrt(b/cos(f*x + e)))/((b^2*f*cos
(f*x + e)^4 - 2*b^2*f*cos(f*x + e)^2 + b^2*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**6/(b*sec(f*x+e))**(3/2),x)
```

output
```
Integral(csc(e + f*x)**6/(b*sec(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\csc(fx+e)^6}{(b\sec(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^6/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(csc(f*x + e)^6/(b*sec(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\csc(fx+e)^6}{(b\sec(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^6/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(csc(f*x + e)^6/(b*sec(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\sin(e+f\,x)^6\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(sin(e + f*x)^6*(b/cos(e + f*x))^(3/2)),x)
```

output
```
int(1/(sin(e + f*x)^6*(b/cos(e + f*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(fx+e)}\,\csc(fx+e)^6}{\sec(fx+e)^2}dx\right)}{b^2}$$

input
```
int(csc(f*x+e)^6/(b*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(e + f*x))*csc(e + f*x)**6)/sec(e + f*x)**2,x))/b**2
```

3.437 $\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{2b^7}{19f(b\sec(e+fx))^{19/2}} - \frac{2b^5}{5f(b\sec(e+fx))^{15/2}} + \frac{6b^3}{11f(b\sec(e+fx))^{11/2}} - \frac{2b}{7f(b\sec(e+fx))^{7/2}}$$

output
```
2/19*b^7/f/(b*sec(f*x+e))^(19/2)-2/5*b^5/f/(b*sec(f*x+e))^(15/2)+6/11*b^3/
f/(b*sec(f*x+e))^(11/2)-2/7*b/f/(b*sec(f*x+e))^(7/2)
```

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.71

$$\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{\cos^4(e+fx)(-15226+14287\cos(2(e+fx))-3542\cos(4(e+fx))+385\cos(}{117040b^3f}$$

input
```
Integrate[Sin[e + f*x]^7/(b*Sec[e + f*x])^(5/2),x]
```

output
```
(Cos[e + f*x]^4*(-15226 + 14287*Cos[2*(e + f*x)] - 3542*Cos[4*(e + f*x)] +
 385*Cos[6*(e + f*x)])*Sqrt[b*Sec[e + f*x]])/(117040*b^3*f)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3102, 25, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc(e+fx)^7(b\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{b^7\int -\frac{\left(b^2-b^2\sec^2(e+fx)\right)^3}{b^6(b\sec(e+fx))^{21/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b^7\int \frac{\left(b^2-b^2\sec^2(e+fx)\right)^3}{b^6(b\sec(e+fx))^{21/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\int \frac{\left(b^2-b^2\sec^2(e+fx)\right)^3}{(b\sec(e+fx))^{21/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{244}$$

$$-\frac{b\int \left(\frac{b^6}{(b\sec(e+fx))^{21/2}}-\frac{3b^4}{(b\sec(e+fx))^{17/2}}+\frac{3b^2}{(b\sec(e+fx))^{13/2}}-\frac{1}{(b\sec(e+fx))^{9/2}}\right)d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\left(-\frac{2b^6}{19(b\sec(e+fx))^{19/2}}+\frac{2b^4}{5(b\sec(e+fx))^{15/2}}-\frac{6b^2}{11(b\sec(e+fx))^{11/2}}+\frac{2}{7(b\sec(e+fx))^{7/2}}\right)}{f}$$

input
```
Int[Sin[e + f*x]^7/(b*Sec[e + f*x])^(5/2),x]
```

output
```
-((b*((-2*b^6)/(19*(b*Sec[e + f*x])^(19/2)) + (2*b^4)/(5*(b*Sec[e + f*x])^
(15/2)) - (6*b^2)/(11*(b*Sec[e + f*x])^(11/2)) + 2/(7*(b*Sec[e + f*x])^(7/
2))))/f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 3.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.68

method	result	size
default	$\frac{\frac{2\cos(fx+e)^9}{19}-\frac{2\cos(fx+e)^7}{5}+\frac{6\cos(fx+e)^5}{11}-\frac{2\cos(fx+e)^3}{7}}{f\,b^2\sqrt{b\sec(fx+e)}}$	59

input
```
int(sin(f*x+e)^7/(b*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(2/19*cos(f*x+e)^9-2/5*cos(f*x+e)^7+6/11*cos(f*x+e)^5-2/7*cos(f*x+e)^3
)/b^2/(b*sec(f*x+e))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.70

$$\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{2\left(385\cos(fx+e)^{10}-1463\cos(fx+e)^{8}+1995\cos(fx+e)^{6}-1045\cos(fx\right.}{7315\,b^3 f}$$

input
```
integrate(sin(f*x+e)^7/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
2/7315*(385*cos(f*x + e)^10 - 1463*cos(f*x + e)^8 + 1995*cos(f*x + e)^6 -
1045*cos(f*x + e)^4)*sqrt(b/cos(f*x + e))/(b^3*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**7/(b*sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

$$\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{2\left(385\,b^6 - \frac{1463\,b^6}{\cos(fx+e)^2} + \frac{1995\,b^6}{\cos(fx+e)^4} - \frac{1045\,b^6}{\cos(fx+e)^6}\right)b}{7315\,f\left(\frac{b}{\cos(fx+e)}\right)^{\frac{19}{2}}}$$

input
```
integrate(sin(f*x+e)^7/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
2/7315*(385*b^6 - 1463*b^6/cos(f*x + e)^2 + 1995*b^6/cos(f*x + e)^4 - 1045
*b^6/cos(f*x + e)^6)*b/(f*(b/cos(f*x + e))^(19/2))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.26

$$\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{2\left(385\sqrt{b\cos(fx+e)}b^9\cos(fx+e)^9 - 1463\sqrt{b\cos(fx+e)}b^9\cos(fx+e)^7 - \right.}{7315\,b^{12}f\mathrm{sgn}\,(}$$

input
```
integrate(sin(f*x+e)^7/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
2/7315*(385*sqrt(b*cos(f*x + e))*b^9*cos(f*x + e)^9 - 1463*sqrt(b*cos(f*x
+ e))*b^9*cos(f*x + e)^7 + 1995*sqrt(b*cos(f*x + e))*b^9*cos(f*x + e)^5 -
1045*sqrt(b*cos(f*x + e))*b^9*cos(f*x + e)^3)/(b^12*f*sgn(cos(f*x + e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\sin(e+f\,x)^7}{\left(\frac{b}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int(sin(e + f*x)^7/(b/cos(e + f*x))^(5/2),x)
```

output
```
int(sin(e + f*x)^7/(b/cos(e + f*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(fx+e)}\,\sin(fx+e)^7}{\sec(fx+e)^3}dx\right)}{b^3}$$

input
```
int(sin(f*x+e)^7/(b*sec(f*x+e))^(5/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(e + f*x))*sin(e + f*x)**7)/sec(e + f*x)**3,x))/b**3
```

3.438 $\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = -\frac{2b^5}{15f(b\sec(e+fx))^{15/2}} + \frac{4b^3}{11f(b\sec(e+fx))^{11/2}} - \frac{2b}{7f(b\sec(e+fx))^{7/2}}$$

output
```
-2/15*b^5/f/(b*sec(f*x+e))^(15/2)+4/11*b^3/f/(b*sec(f*x+e))^(11/2)-2/7*b/f
/(b*sec(f*x+e))^(7/2)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.80

$$\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{\cos^4(e+fx)(-711+532\cos(2(e+fx))-77\cos(4(e+fx)))\sqrt{b\sec(e+fx)}}{4620b^3f}$$

input
```
Integrate[Sin[e + f*x]^5/(b*Sec[e + f*x])^(5/2),x]
```

output
```
(Cos[e + f*x]^4*(-711 + 532*Cos[2*(e + f*x)] - 77*Cos[4*(e + f*x)])*Sqrt[b
*Sec[e + f*x]])/(4620*b^3*f)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3102, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc(e+fx)^5(b\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{b^5\int \frac{\left(b^2-b^2\sec^2(e+fx)\right)^2}{b^4(b\sec(e+fx))^{17/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{\left(b^2-b^2\sec^2(e+fx)\right)^2}{(b\sec(e+fx))^{17/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{244}$$

$$\frac{b\int \left(\frac{b^4}{(b\sec(e+fx))^{17/2}}-\frac{2b^2}{(b\sec(e+fx))^{13/2}}+\frac{1}{(b\sec(e+fx))^{9/2}}\right)d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(-\frac{2b^4}{15(b\sec(e+fx))^{15/2}}+\frac{4b^2}{11(b\sec(e+fx))^{11/2}}-\frac{2}{7(b\sec(e+fx))^{7/2}}\right)}{f}$$

input
```
Int[Sin[e + f*x]^5/(b*Sec[e + f*x])^(5/2),x]
```

output
```
(b*((-2*b^4)/(15*(b*Sec[e + f*x])^(15/2)) + (4*b^2)/(11*(b*Sec[e + f*x])^(
11/2)) - 2/(7*(b*Sec[e + f*x])^(7/2))))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{-\frac{2 \cos(fx+e)^{7}}{15}+\frac{4 \cos(fx+e)^{5}}{11}-\frac{2 \cos(fx+e)^{3}}{7}}{f\, b^{2} \sqrt{b \sec(fx+e)}}$	49

input
```
int(sin(f*x+e)^5/(b*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-2/15*cos(f*x+e)^7+4/11*cos(f*x+e)^5-2/7*cos(f*x+e)^3)/b^2/(b*sec(f*x
+e))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

$$\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = -\frac{2\left(77\,\cos\left(fx+e\right)^8-210\,\cos\left(fx+e\right)^6+165\,\cos\left(fx+e\right)^4\right)\sqrt{\frac{b}{\cos(fx+e)}}}{1155\,b^3f}$$

input
```
integrate(sin(f*x+e)^5/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
-2/1155*(77*cos(f*x + e)^8 - 210*cos(f*x + e)^6 + 165*cos(f*x + e)^4)*sqrt
(b/cos(f*x + e))/(b^3*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**5/(b*sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = -\frac{2\left(77\,b^4-\frac{210\,b^4}{\cos(fx+e)^2}+\frac{165\,b^4}{\cos(fx+e)^4}\right)b}{1155\,f\left(\frac{b}{\cos(fx+e)}\right)^{\frac{15}{2}}}$$

input
```
integrate(sin(f*x+e)^5/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
-2/1155*(77*b^4 - 210*b^4/cos(f*x + e)^2 + 165*b^4/cos(f*x + e)^4)*b/(f*(b
/cos(f*x + e))^(15/2))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.34

$$\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = -\frac{2\left(77\sqrt{b\cos\left(fx+e\right)}b^7\cos\left(fx+e\right)^7-210\sqrt{b\cos\left(fx+e\right)}b^7\cos\left(fx+e\right)^5+165\sqrt{b\cos\left(fx+e\right)}b^7\cos\right.}{1155\,b^{10}f\text{sgn}\left(\cos\left(fx+e\right)\right)}$$

input
```
integrate(sin(f*x+e)^5/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
-2/1155*(77*sqrt(b*cos(f*x + e))*b^7*cos(f*x + e)^7 - 210*sqrt(b*cos(f*x +
 e))*b^7*cos(f*x + e)^5 + 165*sqrt(b*cos(f*x + e))*b^7*cos(f*x + e)^3)/(b^
10*f*sgn(cos(f*x + e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\sin\left(e+f\,x\right)^5}{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^{5/2}}\,dx$$

input
```
int(sin(e + f*x)^5/(b/cos(e + f*x))^(5/2),x)
```

output
```
int(sin(e + f*x)^5/(b/cos(e + f*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(fx+e)}\,\sin(fx+e)^5}{\sec(fx+e)^3}dx\right)}{b^3}$$

input
```
int(sin(f*x+e)^5/(b*sec(f*x+e))^(5/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(e + f*x))*sin(e + f*x)**5)/sec(e + f*x)**3,x))/b**3
```

3.439 $\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 43

$$\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{2b^3}{11f(b\sec(e+fx))^{11/2}} - \frac{2b}{7f(b\sec(e+fx))^{7/2}}$$

output
```
2/11*b^3/f/(b*sec(f*x+e))^(11/2)-2/7*b/f/(b*sec(f*x+e))^(7/2)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{\cos^4(e+fx)(-15+7\cos(2(e+fx)))\sqrt{b\sec(e+fx)}}{77b^3f}$$

input
```
Integrate[Sin[e + f*x]^3/(b*Sec[e + f*x])^(5/2),x]
```

output
```
(Cos[e + f*x]^4*(-15 + 7*Cos[2*(e + f*x)])*Sqrt[b*Sec[e + f*x]])/(77*b^3*f
)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3102, 25, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\csc(e+fx)^3(b\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3102}$$

$$\frac{b^3\int -\frac{b^2-b^2\sec^2(e+fx)}{b^2(b\sec(e+fx))^{13/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{b^3\int \frac{b^2-b^2\sec^2(e+fx)}{b^2(b\sec(e+fx))^{13/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{b\int \frac{b^2-b^2\sec^2(e+fx)}{(b\sec(e+fx))^{13/2}}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{ 244}$$

$$-\frac{b\int \left(\frac{b^2}{(b\sec(e+fx))^{13/2}}-\frac{1}{(b\sec(e+fx))^{9/2}}\right)d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{b\left(\frac{2}{7(b\sec(e+fx))^{7/2}}-\frac{2b^2}{11(b\sec(e+fx))^{11/2}}\right)}{f}$$

input
```
Int[Sin[e + f*x]^3/(b*Sec[e + f*x])^(5/2),x]
```

output
```
-((b*((-2*b^2)/(11*(b*Sec[e + f*x])^(11/2)) + 2/(7*(b*Sec[e + f*x])^(7/2))
))/f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{\frac{2 \cos(fx+e)^5}{11}-\frac{2 \cos(fx+e)^3}{7}}{f\, b^2 \sqrt{b \sec(fx+e)}}$	39

input
```
int(sin(f*x+e)^3/(b*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(2/11*cos(f*x+e)^5-2/7*cos(f*x+e)^3)/b^2/(b*sec(f*x+e))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{2\left(7\cos\left(fx+e\right)^6-11\cos\left(fx+e\right)^4\right)\sqrt{\frac{b}{\cos(fx+e)}}}{77\,b^3f}$$

input
```
integrate(sin(f*x+e)^3/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
2/77*(7*cos(f*x + e)^6 - 11*cos(f*x + e)^4)*sqrt(b/cos(f*x + e))/(b^3*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**3/(b*sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{2\left(7\,b^2 - \frac{11\,b^2}{\cos(fx+e)^2}\right)b}{77\,f\left(\frac{b}{\cos(fx+e)}\right)^{\frac{11}{2}}}$$

input
```
integrate(sin(f*x+e)^3/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
2/77*(7*b^2 - 11*b^2/cos(f*x + e)^2)*b/(f*(b/cos(f*x + e))^(11/2))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.49

$$\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{2\left(7\,\sqrt{b\cos(fx+e)}b^5\cos(fx+e)^5 - 11\,\sqrt{b\cos(fx+e)}b^5\cos(fx+e)^3\right)}{77\,b^8 f\mathrm{sgn}\left(\cos(fx+e)\right)}$$

input
```
integrate(sin(f*x+e)^3/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
2/77*(7*sqrt(b*cos(f*x + e))*b^5*cos(f*x + e)^5 - 11*sqrt(b*cos(f*x + e))*
b^5*cos(f*x + e)^3)/(b^8*f*sgn(cos(f*x + e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\sin(e+f\,x)^3}{\left(\frac{b}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int(sin(e + f*x)^3/(b/cos(e + f*x))^(5/2),x)
```

output
```
int(sin(e + f*x)^3/(b/cos(e + f*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(fx+e)}\,\sin(fx+e)^3}{\sec(fx+e)^3}dx\right)}{b^3}$$

input
```
int(sin(f*x+e)^3/(b*sec(f*x+e))^(5/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(e + f*x))*sin(e + f*x)**3)/sec(e + f*x)**3,x))/b**3
```

3.440 $\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 20

$$\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = -\frac{2b}{7f(b\sec(e+fx))^{7/2}}$$

output
```
-2/7*b/f/(b*sec(f*x+e))^(7/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = -\frac{2b}{7f(b\sec(e+fx))^{7/2}}$$

input
```
Integrate[Sin[e + f*x]/(b*Sec[e + f*x])^(5/2),x]
```

output
```
(-2*b)/(7*f*(b*Sec[e + f*x])^(7/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 3102, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{5/2}}dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\csc(e+fx)(b\sec(e+fx))^{5/2}}dx$$
$$\Big\downarrow \; 3102$$
$$\frac{b\int \frac{1}{(b\sec(e+fx))^{9/2}}d(b\sec(e+fx))}{f}$$
$$\Big\downarrow \; 15$$
$$-\frac{2b}{7f(b\sec(e+fx))^{7/2}}$$

input
```
Int[Sin[e + f*x]/(b*Sec[e + f*x])^(5/2),x]
```

output
```
(-2*b)/(7*f*(b*Sec[e + f*x])^(7/2))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$-\frac{2b}{7f\left(b\sec\left(fx+e\right)\right)^{\frac{7}{2}}}$	17
default	$-\frac{2b}{7f\left(b\sec\left(fx+e\right)\right)^{\frac{7}{2}}}$	17

input
```
int(sin(f*x+e)/(b*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/7*b/f/(b*sec(f*x+e))^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.40

$$\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = -\frac{2\sqrt{\frac{b}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)^{4}}{7\,b^{3}f}$$

input
```
integrate(sin(f*x+e)/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
-2/7*sqrt(b/cos(f*x + e))*cos(f*x + e)^4/(b^3*f)
```

Sympy [F]

$$\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\sin(e+fx)}{(b\sec(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)/(b*sec(f*x+e))**(5/2),x)
```

output
```
Integral(sin(e + f*x)/(b*sec(e + f*x))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = -\frac{2\cos(fx+e)}{7f\left(\frac{b}{\cos(fx+e)}\right)^{\frac{5}{2}}}$$

input
```
integrate(sin(f*x+e)/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
-2/7*cos(f*x + e)/(f*(b/cos(f*x + e))^(5/2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(16) = 32.

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.75

$$\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = -\frac{2\sqrt{b\cos(fx+e)}\cos(fx+e)^3}{7b^3f\mathrm{sgn}(\cos(fx+e))}$$

input
```
integrate(sin(f*x+e)/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
-2/7*sqrt(b*cos(f*x + e))*cos(f*x + e)^3/(b^3*f*sgn(cos(f*x + e)))
```

Mupad [B] (verification not implemented)

Time = 26.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.40

$$\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = -\frac{2\cos\left(e+f\,x\right)^4\,\sqrt{\frac{b}{\cos(e+f\,x)}}}{7\,b^3\,f}$$

input
```
int(sin(e + f*x)/(b/cos(e + f*x))^(5/2),x)
```

output
```
-(2*cos(e + f*x)^4*(b/cos(e + f*x))^(1/2))/(7*b^3*f)
```

Reduce [F]

$$\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(fx+e)}\,\sin(fx+e)}{\sec(fx+e)^3}dx\right)}{b^3}$$

input
```
int(sin(f*x+e)/(b*sec(f*x+e))^(5/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(e + f*x))*sin(e + f*x))/sec(e + f*x)**3,x))/b**3
```

3.441 $\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 81

$$\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{b^{5/2}f} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{b^{5/2}f} + \frac{2}{3bf(b\sec(e+fx))^{3/2}}$$

output
```
-arctan((b*sec(f*x+e))^(1/2)/b^(1/2))/b^(5/2)/f-arctanh((b*sec(f*x+e))^(1/
2)/b^(1/2))/b^(5/2)/f+2/3/b/f/(b*sec(f*x+e))^(3/2)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.11

$$\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{\left(-6\arctan\left(\sqrt{\sec(e+fx)}\right) + 3\log\left(1-\sqrt{\sec(e+fx)}\right) - 3\log\left(1+\sqrt{\sec(e\right.\right.}{6b^2f\sqrt{b\sec(e+fx)}}$$

input
```
Integrate[Csc[e + f*x]/(b*Sec[e + f*x])^(5/2),x]
```

output
```
((-6*ArcTan[Sqrt[Sec[e + f*x]]] + 3*Log[1 - Sqrt[Sec[e + f*x]]] - 3*Log[1
+ Sqrt[Sec[e + f*x]]] + 4/Sec[e + f*x]^(3/2))*Sqrt[Sec[e + f*x]])/(6*b^2*f
*Sqrt[b*Sec[e + f*x]])
```

Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3042, 3102, 25, 27, 264, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \textbf{3102}$$

$$\frac{\int -\frac{b^2}{(b\sec(e+fx))^{5/2}(b^2-b^2\sec^2(e+fx))}d(b\sec(e+fx))}{bf}$$

$$\Big\downarrow \textbf{25}$$

$$-\frac{\int \frac{b^2}{(b\sec(e+fx))^{5/2}(b^2-b^2\sec^2(e+fx))}d(b\sec(e+fx))}{bf}$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{b\int \frac{1}{(b\sec(e+fx))^{5/2}(b^2-b^2\sec^2(e+fx))}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \textbf{264}$$

$$-\frac{b\left(\frac{\int \frac{1}{\sqrt{b\sec(e+fx)}\left(b^2-b^2\sec^2(e+fx)\right)}d(b\sec(e+fx))}{b^2}-\frac{2}{3b^2(b\sec(e+fx))^{3/2}}\right)}{f}$$

$$\Big\downarrow \textbf{266}$$

$$-\frac{b\left(\frac{2\int \frac{1}{b^2-b^4\sec^4(e+fx)}d\sqrt{b\sec(e+fx)}}{b^2}-\frac{2}{3b^2(b\sec(e+fx))^{3/2}}\right)}{f}$$

$$\Big\downarrow \text{756}$$

$$-\frac{b\left(\frac{2\left(\frac{\int \frac{1}{b-b^2\sec^2(e+fx)}d\sqrt{b\sec(e+fx)}}{2b}+\frac{\int \frac{1}{b^2\sec^2(e+fx)+b}d\sqrt{b\sec(e+fx)}}{2b}\right)}{b^2}-\frac{2}{3b^2(b\sec(e+fx))^{3/2}}\right)}{f}$$

$$\Big\downarrow \text{216}$$

$$-\frac{b\left(\frac{2\left(\frac{\int \frac{1}{b-b^2\sec^2(e+fx)}d\sqrt{b\sec(e+fx)}}{2b}+\frac{\arctan\left(\sqrt{b}\sec(e+fx)\right)}{2b^{3/2}}\right)}{b^2}-\frac{2}{3b^2(b\sec(e+fx))^{3/2}}\right)}{f}$$

$$\Big\downarrow \text{219}$$

$$-\frac{b\left(\frac{2\left(\frac{\arctan\left(\sqrt{b}\sec(e+fx)\right)}{2b^{3/2}}+\frac{\text{arctanh}\left(\sqrt{b}\sec(e+fx)\right)}{2b^{3/2}}\right)}{b^2}-\frac{2}{3b^2(b\sec(e+fx))^{3/2}}\right)}{f}$$

input
```
Int[Csc[e + f*x]/(b*Sec[e + f*x])^(5/2),x]
```

output
```
-((b*((2*(ArcTan[Sqrt[b]*Sec[e + f*x]]/(2*b^(3/2)) + ArcTanh[Sqrt[b]*Sec[e
 + f*x]]/(2*b^(3/2))))/b^2 - 2/(3*b^2*(b*Sec[e + f*x])^(3/2))))/f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(65) = 130.

Time = 2.02 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.30

method	result
default	$\frac{\frac{\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\cos(fx+e)+1}{\cos(fx+e)+1}\right)(-3-3\sec(fx+e))}{6}+\frac{\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e}}}\right.}{}}{f\sqrt{b\sec(fx+e)}\,b^2}$

input
```
int(csc(f*x+e)/(b*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/6*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln((2*cos(f*x+e)*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e
)+1)/(cos(f*x+e)+1))*(-3-3*sec(f*x+e))+1/6*arctan(1/2/(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(-3-3*sec(f*x+e))+2
/3*cos(f*x+e))/(b*sec(f*x+e))^(1/2)/b^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(65) = 130.

Time = 0.23 (sec) , antiderivative size = 319, normalized size of antiderivative = 3.94

$$\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \left[\frac{8\sqrt{\frac{b}{\cos(fx+e)}}\cos\left(fx+e\right)^2+6\sqrt{-b}\arctan\left(\frac{2\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)}{b\cos(fx+e)+b}\right)-3\sqrt{-b}}{12\,b^3f}\right.$$

input
```
integrate(csc(f*x+e)/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
[1/12*(8*sqrt(b/cos(f*x + e))*cos(f*x + e)^2 + 6*sqrt(-b)*arctan(2*sqrt(-b
)*sqrt(b/cos(f*x + e))*cos(f*x + e)/(b*cos(f*x + e) + b)) - 3*sqrt(-b)*log
(-(b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - cos(f*x + e))*sqrt(-b)*sqrt(b/co
s(f*x + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))
)/(b^3*f), 1/12*(8*sqrt(b/cos(f*x + e))*cos(f*x + e)^2 - 6*sqrt(b)*arctan(
2*sqrt(b)*sqrt(b/cos(f*x + e))*cos(f*x + e)/(b*cos(f*x + e) - b)) + 3*sqrt
(b)*log(-(b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(b)*sqr
t(b/cos(f*x + e)) + 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*x + e)
 + 1)))/(b^3*f)]
```

Sympy [F]

$$\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\csc{(e+fx)}}{(b\sec{(e+fx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)/(b*sec(f*x+e))**(5/2),x)
```

output
```
Integral(csc(e + f*x)/(b*sec(e + f*x))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.11

$$\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = -\frac{b\left(\frac{6\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}}\right)}{b^{\frac{7}{2}}} - \frac{3\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos(fx+e)}}}\right)}{b^{\frac{7}{2}}} - \frac{4}{b^2\left(\frac{b}{\cos(fx+e)}\right)^{\frac{3}{2}}}\right)}{6f}$$

input
```
integrate(csc(f*x+e)/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
-1/6*b*(6*arctan(sqrt(b/cos(f*x + e))/sqrt(b))/b^(7/2) - 3*log(-(sqrt(b) -
 sqrt(b/cos(f*x + e)))/(sqrt(b) + sqrt(b/cos(f*x + e))))/b^(7/2) - 4/(b^2*
(b/cos(f*x + e))^(3/2)))/f
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{\frac{3\,b\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b}} + 3\,\sqrt{b}\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{b}}\right) + 2\,\sqrt{b\cos\left(fx+e\right)}\cos\left(fx+\right.}{3\,b^3 f\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}$$

input
```
integrate(csc(f*x+e)/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
1/3*(3*b*arctan(sqrt(b*cos(f*x + e))/sqrt(-b))/sqrt(-b) + 3*sqrt(b)*arctan
(sqrt(b*cos(f*x + e))/sqrt(b)) + 2*sqrt(b*cos(f*x + e))*cos(f*x + e))/(b^3
*f*sgn(cos(f*x + e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)\,\left(\frac{b}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/(sin(e + f*x)*(b/cos(e + f*x))^(5/2)),x)
```

output
```
int(1/(sin(e + f*x)*(b/cos(e + f*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(fx+e)}\,\csc(fx+e)}{\sec(fx+e)^3}dx\right)}{b^3}$$

input
```
int(csc(f*x+e)/(b*sec(f*x+e))^(5/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(e + f*x))*csc(e + f*x))/sec(e + f*x)**3,x))/b**3
```

3.442 $\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 93

$$\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{3\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4b^{5/2}f} + \frac{3\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4b^{5/2}f} - \frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{2b^3f}$$

output
```
3/4*arctan((b*sec(f*x+e))^(1/2)/b^(1/2))/b^(5/2)/f+3/4*arctanh((b*sec(f*x+
e))^(1/2)/b^(1/2))/b^(5/2)/f-1/2*cot(f*x+e)^2*(b*sec(f*x+e))^(1/2)/b^3/f
```

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.05

$$\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{\left(6\arctan\left(\sqrt{\sec(e+fx)}\right) - 3\log\left(1-\sqrt{\sec(e+fx)}\right) + 3\log\left(1+\sqrt{\sec(e+\right.\right.}{8b^2f\sqrt{b\sec(e+fx)}}$$

input
```
Integrate[Csc[e + f*x]^3/(b*Sec[e + f*x])^(5/2),x]
```

output
```
((6*ArcTan[Sqrt[Sec[e + f*x]]] - 3*Log[1 - Sqrt[Sec[e + f*x]]] + 3*Log[1 +
Sqrt[Sec[e + f*x]]] - (4*Csc[e + f*x]^2)/Sec[e + f*x]^(3/2))*Sqrt[Sec[e +
f*x]])/(8*b^2*f*Sqrt[b*Sec[e + f*x]])
```

Rubi [A] (warning: unable to verify)

Time = 0.45 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3102, 27, 253, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc(e+fx)^3}{(b\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{\int \frac{b^4}{\sqrt{b\sec(e+fx)}(b^2-b^2\sec^2(e+fx))^2}d(b\sec(e+fx))}{b^3f}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{1}{\sqrt{b\sec(e+fx)}(b^2-b^2\sec^2(e+fx))^2}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{253}$$

$$\frac{b\left(\frac{3\int \frac{1}{\sqrt{b\sec(e+fx)}\left(b^2-b^2\sec^2(e+fx)\right)}d(b\sec(e+fx))}{4b^2}+\frac{\sqrt{b\sec(e+fx)}}{2b^2(b^2-b^2\sec^2(e+fx))}\right)}{f}$$

$$\Big\downarrow \text{266}$$

$$\frac{b\left(\frac{3\int \frac{1}{b^2-b^4\sec^4(e+fx)}d\sqrt{b\sec(e+fx)}}{2b^2}+\frac{\sqrt{b\sec(e+fx)}}{2b^2(b^2-b^2\sec^2(e+fx))}\right)}{f}$$

$$\Big\downarrow \text{756}$$

$$\frac{b\left(\frac{3\left(\frac{\int \frac{1}{b-b^2\sec^2(e+fx)}d\sqrt{b\sec(e+fx)}}{2b}+\frac{\int \frac{1}{b^2\sec^2(e+fx)+b}d\sqrt{b\sec(e+fx)}}{2b}\right)}{2b^2}+\frac{\sqrt{b\sec(e+fx)}}{2b^2(b^2-b^2\sec^2(e+fx))}\right)}{f}$$

$$\Big\downarrow \; 216$$

$$\frac{b\left(\frac{3\left(\frac{\int \frac{1}{b-b^2\sec^2(e+fx)}d\sqrt{b\sec(e+fx)}}{2b}+\frac{\arctan\left(\sqrt{b}\sec(e+fx)\right)}{2b^{3/2}}\right)}{2b^2}+\frac{\sqrt{b\sec(e+fx)}}{2b^2(b^2-b^2\sec^2(e+fx))}\right)}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{b\left(\frac{3\left(\frac{\arctan\left(\sqrt{b}\sec(e+fx)\right)}{2b^{3/2}}+\frac{\text{arctanh}\left(\sqrt{b}\sec(e+fx)\right)}{2b^{3/2}}\right)}{2b^2}+\frac{\sqrt{b\sec(e+fx)}}{2b^2(b^2-b^2\sec^2(e+fx))}\right)}{f}$$

input
```
Int[Csc[e + f*x]^3/(b*Sec[e + f*x])^(5/2),x]
```

output
```
(b*((3*(ArcTan[Sqrt[b]*Sec[e + f*x]]/(2*b^(3/2)) + ArcTanh[Sqrt[b]*Sec[e +
 f*x]]/(2*b^(3/2))))/(2*b^2) + Sqrt[b*Sec[e + f*x]]/(2*b^2*(b^2 - b^2*Sec[
e + f*x]^2))))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. 2(73) = 146.

Time = 2.05 (sec) , antiderivative size = 286, normalized size of antiderivative = 3.08

method	result
default	$-\frac{\left(4\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-3\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\cos(fx+e)+1}{\cos(fx+e)+1}\right)\cos(fx+e)-3\,\mathrm{arcta}}{8f}$

input
```
int(csc(f*x+e)^3/(b*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/f*(4*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-3*ln((2*cos(f*x+
e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*cos(f*x+e)-3*arctan(1/2/(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2))*cos(f*x+e)+3*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(
f*x+e)+1))+3*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)))/(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)/b^2/(b*sec(f*x+e))^(1/2)*csc(f*x+e)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(73) = 146.

Time = 0.15 (sec) , antiderivative size = 389, normalized size of antiderivative = 4.18

$$\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \left[-\frac{6\left(\cos(fx+e)^2-1\right)\sqrt{-b}\arctan\left(\frac{2\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)}{b\cos(fx+e)+b}\right)-8\sqrt{\frac{b}{\cos(fx+e)}}\,\mathrm{co}}{}\right.$$

input
```
integrate(csc(f*x+e)^3/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
[-1/16*(6*(cos(f*x + e)^2 - 1)*sqrt(-b)*arctan(2*sqrt(-b)*sqrt(b/cos(f*x +
 e))*cos(f*x + e)/(b*cos(f*x + e) + b)) - 8*sqrt(b/cos(f*x + e))*cos(f*x +
 e)^2 + 3*(cos(f*x + e)^2 - 1)*sqrt(-b)*log((b*cos(f*x + e)^2 + 4*(cos(f*x
 + e)^2 - cos(f*x + e))*sqrt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos(f*x + e) +
 b)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)))/(b^3*f*cos(f*x + e)^2 - b^3*f)
, 1/16*(6*(cos(f*x + e)^2 - 1)*sqrt(b)*arctan(2*sqrt(b)*sqrt(b/cos(f*x + e
))*cos(f*x + e)/(b*cos(f*x + e) - b)) + 8*sqrt(b/cos(f*x + e))*cos(f*x + e
)^2 + 3*(cos(f*x + e)^2 - 1)*sqrt(b)*log((b*cos(f*x + e)^2 + 4*(cos(f*x +
e)^2 + cos(f*x + e))*sqrt(b)*sqrt(b/cos(f*x + e)) + 6*b*cos(f*x + e) + b)/
(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)))/(b^3*f*cos(f*x + e)^2 - b^3*f)]
```

Sympy [F]

$$\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx=\int \frac{\csc^3\left(e+fx\right)}{\left(b\sec\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**3/(b*sec(f*x+e))**(5/2),x)
```

output
```
Integral(csc(e + f*x)**3/(b*sec(e + f*x))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.14

$$\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx=\frac{b\left(\frac{4\sqrt{\frac{b}{\cos(fx+e)}}}{b^4-\frac{b^4}{\cos(fx+e)^2}}+\frac{6\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}}\right)}{b^{\frac{7}{2}}}-\frac{3\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos(fx+e)}}}\right)}{b^{\frac{7}{2}}}\right)}{8\,f}$$

input
```
integrate(csc(f*x+e)^3/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
1/8*b*(4*sqrt(b/cos(f*x + e))/(b^4 - b^4/cos(f*x + e)^2) + 6*arctan(sqrt(b
/cos(f*x + e))/sqrt(b))/b^(7/2) - 3*log(-(sqrt(b) - sqrt(b/cos(f*x + e)))/
(sqrt(b) + sqrt(b/cos(f*x + e))))/b^(7/2))/f
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.09

$$\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{\frac{2\sqrt{b\cos(fx+e)}b\cos(fx+e)}{b^2\cos(fx+e)^2-b^2} - \frac{3\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b}} - \frac{3\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{b}}\right)}{\sqrt{b}}}{4\,b^2 f\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}$$

input
```
integrate(csc(f*x+e)^3/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
1/4*(2*sqrt(b*cos(f*x + e))*b*cos(f*x + e)/(b^2*cos(f*x + e)^2 - b^2) - 3*
arctan(sqrt(b*cos(f*x + e))/sqrt(-b))/sqrt(-b) - 3*arctan(sqrt(b*cos(f*x +
 e))/sqrt(b))/sqrt(b))/(b^2*f*sgn(cos(f*x + e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)^3\,{\left(\frac{b}{\cos(e+f\,x)}\right)}^{5/2}}\,dx$$

input
```
int(1/(sin(e + f*x)^3*(b/cos(e + f*x))^(5/2)),x)
```

output
```
int(1/(sin(e + f*x)^3*(b/cos(e + f*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(fx+e)}\,\csc(fx+e)^3}{\sec(fx+e)^3}dx\right)}{b^3}$$

input
```
int(csc(f*x+e)^3/(b*sec(f*x+e))^(5/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(e + f*x))*csc(e + f*x)**3)/sec(e + f*x)**3,x))/b**3
```

3.443 $\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 123

$$\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{3\arctan\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32b^{5/2}f} + \frac{3\text{arctanh}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32b^{5/2}f} - \frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{16b^3f} - \frac{\cot^4(e+fx)\sqrt{b\sec(e+fx)}}{4b^3f}$$

output
```
3/32*arctan((b*sec(f*x+e))^(1/2)/b^(1/2))/b^(5/2)/f+3/32*arctanh((b*sec(f*
x+e))^(1/2)/b^(1/2))/b^(5/2)/f-1/16*cot(f*x+e)^2*(b*sec(f*x+e))^(1/2)/b^3/
f-1/4*cot(f*x+e)^4*(b*sec(f*x+e))^(1/2)/b^3/f
```

Mathematica [A] (verified)

Time = 2.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.89

$$\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{\left(6\arctan\left(\sqrt{\sec(e+fx)}\right) - 3\log\left(1-\sqrt{\sec(e+fx)}\right) + 3\log\left(1+\sqrt{\sec(e+\right.\right.}{64b^2f\sqrt{b\sec(e+fx)}}$$

input
```
Integrate[Csc[e + f*x]^5/(b*Sec[e + f*x])^(5/2),x]
```

output
```
((6*ArcTan[Sqrt[Sec[e + f*x]]] - 3*Log[1 - Sqrt[Sec[e + f*x]]] + 3*Log[1 +
 Sqrt[Sec[e + f*x]]] - (2*(5 + 3*Cos[2*(e + f*x)])*Csc[e + f*x]^4)/Sec[e +
 f*x]^(3/2))*Sqrt[Sec[e + f*x]])/(64*b^2*f*Sqrt[b*Sec[e + f*x]])
```

Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3102, 25, 27, 252, 253, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc(e+fx)^5}{(b\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{\int -\frac{b^6(b\sec(e+fx))^{3/2}}{(b^2-b^2\sec^2(e+fx))^3}d(b\sec(e+fx))}{b^5f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{b^6(b\sec(e+fx))^{3/2}}{(b^2-b^2\sec^2(e+fx))^3}d(b\sec(e+fx))}{b^5f}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\int \frac{(b\sec(e+fx))^{3/2}}{(b^2-b^2\sec^2(e+fx))^3}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{252}$$

$$-\frac{b\left(\frac{\sqrt{b\sec(e+fx)}}{4(b^2-b^2\sec^2(e+fx))^2}-\frac{1}{8}\int \frac{1}{\sqrt{b\sec(e+fx)}(b^2-b^2\sec^2(e+fx))^2}d(b\sec(e+fx))\right)}{f}$$

$$\Big\downarrow \text{253}$$

$$-\frac{b\left(\frac{1}{8}\left(-\frac{3\int \frac{1}{\sqrt{b\sec(e+fx)}\left(b^2-b^2\sec^2(e+fx)\right)}d(b\sec(e+fx))}{4b^2}-\frac{\sqrt{b\sec(e+fx)}}{2b^2(b^2-b^2\sec^2(e+fx))}\right)+\frac{\sqrt{b\sec(e+fx)}}{4(b^2-b^2\sec^2(e+fx))^2}\right)}{f}$$

$$\Big\downarrow \text{266}$$

$$-\frac{b\left(\frac{1}{8}\left(-\frac{3\int \frac{1}{b^2-b^4\sec^4(e+fx)}d\sqrt{b\sec(e+fx)}}{2b^2}-\frac{\sqrt{b\sec(e+fx)}}{2b^2(b^2-b^2\sec^2(e+fx))}\right)+\frac{\sqrt{b\sec(e+fx)}}{4(b^2-b^2\sec^2(e+fx))^2}\right)}{f}$$

$$\Big\downarrow \text{756}$$

$$-\frac{b\left(\frac{1}{8}\left(-\frac{3\left(\frac{\int \frac{1}{b-b^2\sec^2(e+fx)}d\sqrt{b\sec(e+fx)}}{2b}+\frac{\int \frac{1}{b^2\sec^2(e+fx)+b}d\sqrt{b\sec(e+fx)}}{2b}\right)}{2b^2}-\frac{\sqrt{b\sec(e+fx)}}{2b^2(b^2-b^2\sec^2(e+fx))}\right)+\frac{\sqrt{b\sec(e+fx)}}{4(b^2-b^2\sec^2(e+fx))^2}\right)}{f}$$

$$\Big\downarrow \text{216}$$

$$-\frac{b\left(\frac{1}{8}\left(-\frac{3\left(\frac{\int \frac{1}{b-b^2\sec^2(e+fx)}d\sqrt{b\sec(e+fx)}}{2b}+\frac{\arctan\left(\sqrt{b}\sec(e+fx)\right)}{2b^{3/2}}\right)}{2b^2}-\frac{\sqrt{b\sec(e+fx)}}{2b^2(b^2-b^2\sec^2(e+fx))}\right)+\frac{\sqrt{b\sec(e+fx)}}{4(b^2-b^2\sec^2(e+fx))^2}\right)}{f}$$

$$\Big\downarrow \text{219}$$

$$-\frac{b\left(\frac{1}{8}\left(-\frac{3\left(\frac{\arctan\left(\sqrt{b}\sec(e+fx)\right)}{2b^{3/2}}+\frac{\text{arctanh}\left(\sqrt{b}\sec(e+fx)\right)}{2b^{3/2}}\right)}{2b^2}-\frac{\sqrt{b\sec(e+fx)}}{2b^2(b^2-b^2\sec^2(e+fx))}\right)+\frac{\sqrt{b\sec(e+fx)}}{4(b^2-b^2\sec^2(e+fx))^2}\right)}{f}$$

input
```
Int[Csc[e + f*x]^5/(b*Sec[e + f*x])^(5/2),x]
```

output
```
-((b*(Sqrt[b*Sec[e + f*x]]/(4*(b^2 - b^2*Sec[e + f*x]^2)^2) + ((-3*(ArcTan
[Sqrt[b]*Sec[e + f*x]]/(2*b^(3/2)) + ArcTanh[Sqrt[b]*Sec[e + f*x]]/(2*b^(3
/2))))/(2*b^2) - Sqrt[b*Sec[e + f*x]]/(2*b^2*(b^2 - b^2*Sec[e + f*x]^2)))/
8))/f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(99) = 198$.

Time = 2.05 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.79

method	result
default	$-\frac{\left(12\cos(fx+e)^2+4\right)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,\cot(fx+e)\csc(fx+e)^3+(-3\cos(fx+e)+3)\ln\left(\frac{2\cos(fx+e)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}+2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\cos(fx+e)+1}{\cos(fx+e)+1}\right)\csc(fx+e)^2+\cdots}{64f\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\sqrt{b\sec(fx+e)}\,b^2}$

input
```
int(csc(f*x+e)^5/(b*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/64/f/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/(b*sec(f*x+e))^(1/2)/b^2*((12
*cos(f*x+e)^2+4)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cot(f*x+e)*csc(f*x+e
)^3+(-3*cos(f*x+e)+3)*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*csc
(f*x+e)^2+(-3*cos(f*x+e)+3)*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
))*csc(f*x+e)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(99) = 198$.

Time = 0.14 (sec) , antiderivative size = 477, normalized size of antiderivative = 3.88

$$\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \left[-\frac{6\left(\cos\left(fx+e\right)^4-2\,\cos\left(fx+e\right)^2+1\right)\sqrt{-b}\arctan\left(\frac{2\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)}{b\cos(fx+e)+b}\right)}{}\right.$$

input
```
integrate(csc(f*x+e)^5/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
[-1/128*(6*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-b)*arctan(2*sqrt(
-b)*sqrt(b/cos(f*x + e))*cos(f*x + e)/(b*cos(f*x + e) + b)) + 3*(cos(f*x +
 e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-b)*log((b*cos(f*x + e)^2 + 4*(cos(f*x
+ e)^2 - cos(f*x + e))*sqrt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos(f*x + e) +
b)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 8*(3*cos(f*x + e)^4 + cos(f*x
+ e)^2)*sqrt(b/cos(f*x + e)))/(b^3*f*cos(f*x + e)^4 - 2*b^3*f*cos(f*x + e)
^2 + b^3*f), 1/128*(6*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(b)*arct
an(2*sqrt(b)*sqrt(b/cos(f*x + e))*cos(f*x + e)/(b*cos(f*x + e) - b)) + 3*(
cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(b)*log((b*cos(f*x + e)^2 + 4*(
cos(f*x + e)^2 + cos(f*x + e))*sqrt(b)*sqrt(b/cos(f*x + e)) + 6*b*cos(f*x
+ e) + b)/(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)) - 8*(3*cos(f*x + e)^4 + c
os(f*x + e)^2)*sqrt(b/cos(f*x + e)))/(b^3*f*cos(f*x + e)^4 - 2*b^3*f*cos(f
*x + e)^2 + b^3*f)]
```

Sympy [F]

$$\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\csc^5\left(e+fx\right)}{\left(b\sec\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**5/(b*sec(f*x+e))**(5/2),x)
```

output
```
Integral(csc(e + f*x)**5/(b*sec(e + f*x))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.11

$$\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = -\frac{b\left(\frac{4\left(3\,b^2\sqrt{\frac{b}{\cos(fx+e)}}+\left(\frac{b}{\cos(fx+e)}\right)^{\frac{5}{2}}\right)}{b^6-\frac{2\,b^6}{\cos(fx+e)^2}+\frac{b^6}{\cos(fx+e)^4}}-\frac{6\,\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}}\right)}{b^{\frac{7}{2}}}+\frac{3\,\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos(fx+e)}}}\right)}{b^{\frac{7}{2}}}\right)}{64\,f}$$

input
```
integrate(csc(f*x+e)^5/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
-1/64*b*(4*(3*b^2*sqrt(b/cos(f*x + e)) + (b/cos(f*x + e))^(5/2))/(b^6 - 2*
b^6/cos(f*x + e)^2 + b^6/cos(f*x + e)^4) - 6*arctan(sqrt(b/cos(f*x + e))/s
qrt(b))/b^(7/2) + 3*log(-(sqrt(b) - sqrt(b/cos(f*x + e)))/(sqrt(b) + sqrt(
b/cos(f*x + e))))/b^(7/2))/f
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.07

$$\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = -\frac{\frac{3\,\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{-b}}\right)}{\sqrt{-b}b^2}+\frac{3\,\arctan\left(\frac{\sqrt{b\cos(fx+e)}}{\sqrt{b}}\right)}{b^{\frac{5}{2}}}+\frac{2\left(3\,\sqrt{b\cos(fx+e)}b^3\cos(fx+e)^3+\sqrt{b\cos(fx+e)}b^3\cos(fx+e)\right)}{\left(b^2\cos(fx+e)^2-b^2\right)^2b^2}}{32\,f\mathrm{sgn}\left(\cos\left(fx+e\right)\right)}$$

input
```
integrate(csc(f*x+e)^5/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
-1/32*(3*arctan(sqrt(b*cos(f*x + e))/sqrt(-b))/(sqrt(-b)*b^2) + 3*arctan(s
qrt(b*cos(f*x + e))/sqrt(b))/b^(5/2) + 2*(3*sqrt(b*cos(f*x + e))*b^3*cos(f
*x + e)^3 + sqrt(b*cos(f*x + e))*b^3*cos(f*x + e))/((b^2*cos(f*x + e)^2 -
b^2)^2*b^2))/(f*sgn(cos(f*x + e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx=\int \frac{1}{\sin\left(e+f\,x\right)^5\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{5/2}}\,dx$$

input
```
int(1/(sin(e + f*x)^5*(b/cos(e + f*x))^(5/2)),x)
```

output
```
int(1/(sin(e + f*x)^5*(b/cos(e + f*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx=\frac{\sqrt{b}\,\left(\int \frac{\sqrt{\sec(fx+e)}\,\csc(fx+e)^5}{\sec(fx+e)^3}dx\right)}{b^3}$$

input
```
int(csc(f*x+e)^5/(b*sec(f*x+e))^(5/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(e + f*x))*csc(e + f*x)**5)/sec(e + f*x)**3,x))/b**3
```

3.444 $\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 126

$$\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{8E\left(\frac{1}{2}(e+fx)\middle|2\right)}{65b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{4b\sin(e+fx)}{39f(b\sec(e+fx))^{7/2}} + \frac{8\sin(e+fx)}{195bf(b\sec(e+fx))^{3/2}} - \frac{2b\sin^3(e+fx)}{13f(b\sec(e+fx))^{7/2}}$$

output
```
8/65*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/b^2/f/cos(f*x+e)^(1/2)/(b*sec(f
*x+e))^(1/2)-4/39*b*sin(f*x+e)/f/(b*sec(f*x+e))^(7/2)+8/195*sin(f*x+e)/b/f
/(b*sec(f*x+e))^(3/2)-2/13*b*sin(f*x+e)^3/f/(b*sec(f*x+e))^(7/2)
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.66

$$\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{192E\left(\frac{1}{2}(e+fx)\middle|2\right) + \cos^{\frac{3}{2}}(e+fx)(-6\sin(e+fx) - 55\sin(3(e+fx)) + 15\,\mathrm{si}}{1560f\cos^{\frac{5}{2}}(e+fx)(b\sec(e+fx))^{5/2}}$$

input
```
Integrate[Sin[e + f*x]^4/(b*Sec[e + f*x])^(5/2),x]
```

output
```
(192*EllipticE[(e + f*x)/2, 2] + Cos[e + f*x]^(3/2)*(-6*Sin[e + f*x] - 55*
Sin[3*(e + f*x)] + 15*Sin[5*(e + f*x)]))/(1560*f*Cos[e + f*x]^(5/2)*(b*Sec
[e + f*x])^(5/2))
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3107, 3042, 3107, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc(e+fx)^4(b\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3107}$$

$$\frac{6}{13}\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{5/2}}dx-\frac{2b\sin^3(e+fx)}{13f(b\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{13}\int \frac{1}{\csc(e+fx)^2(b\sec(e+fx))^{5/2}}dx-\frac{2b\sin^3(e+fx)}{13f(b\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3107}$$

$$\frac{6}{13}\left(\frac{2}{9}\int \frac{1}{(b\sec(e+fx))^{5/2}}dx-\frac{2b\sin(e+fx)}{9f(b\sec(e+fx))^{7/2}}\right)-\frac{2b\sin^3(e+fx)}{13f(b\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6}{13}\left(\frac{2}{9}\int \frac{1}{\left(b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx-\frac{2b\sin(e+fx)}{9f(b\sec(e+fx))^{7/2}}\right)-\frac{2b\sin^3(e+fx)}{13f(b\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{6}{13}\left(\frac{2}{9}\left(\frac{3\int \frac{1}{\sqrt{b\sec(e+fx)}}dx}{5b^2}+\frac{2\sin(e+fx)}{5bf(b\sec(e+fx))^{3/2}}\right)-\frac{2b\sin(e+fx)}{9f(b\sec(e+fx))^{7/2}}\right)-\frac{2b\sin^3(e+fx)}{13f(b\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{6}{13}\left(\frac{2}{9}\left(\frac{3\int \frac{1}{\sqrt{b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{5b^2}+\frac{2\sin(e+fx)}{5bf(b\sec(e+fx))^{3/2}}\right)-\frac{2b\sin(e+fx)}{9f(b\sec(e+fx))^{7/2}}\right)-\frac{2b\sin^3(e+fx)}{13f(b\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{6}{13}\left(\frac{2}{9}\left(\frac{3\int \sqrt{\cos(e+fx)}dx}{5b^2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}+\frac{2\sin(e+fx)}{5bf(b\sec(e+fx))^{3/2}}\right)-\frac{2b\sin(e+fx)}{9f(b\sec(e+fx))^{7/2}}\right)-\frac{2b\sin^3(e+fx)}{13f(b\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{6}{13}\left(\frac{2}{9}\left(\frac{3\int \sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{5b^2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}+\frac{2\sin(e+fx)}{5bf(b\sec(e+fx))^{3/2}}\right)-\frac{2b\sin(e+fx)}{9f(b\sec(e+fx))^{7/2}}\right)-\frac{2b\sin^3(e+fx)}{13f(b\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{6}{13}\left(\frac{2}{9}\left(\frac{6E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}+\frac{2\sin(e+fx)}{5bf(b\sec(e+fx))^{3/2}}\right)-\frac{2b\sin(e+fx)}{9f(b\sec(e+fx))^{7/2}}\right)-\frac{2b\sin^3(e+fx)}{13f(b\sec(e+fx))^{7/2}}$$

input
```
Int[Sin[e + f*x]^4/(b*Sec[e + f*x])^(5/2),x]
```

output
```
(-2*b*Sin[e + f*x]^3)/(13*f*(b*Sec[e + f*x])^(7/2)) + (6*((-2*b*Sin[e + f*
x])/(9*f*(b*Sec[e + f*x])^(7/2)) + (2*((6*EllipticE[(e + f*x)/2, 2])/(5*b^
2*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]]) + (2*Sin[e + f*x])/(5*b*f*(b*
Sec[e + f*x])^(3/2))))/9))/13
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3107
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Simp[(m + 1)/(a^2*(m + n))   Int[(a*Csc[e + f*x])^(m
+ 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.74 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.87

method	result
default	$\frac{\frac{2\sin(fx+e)\left(15\cos(fx+e)^6+15\cos(fx+e)^5-25\cos(fx+e)^4-25\cos(fx+e)^3+4\cos(fx+e)^2+4\cos(fx+e)+12\right)}{195}+\frac{8i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+}}}{}}{f(\cos(}$

input
```
int(sin(f*x+e)^4/(b*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/195/f/(cos(f*x+e)+1)/(b*sec(f*x+e))^(1/2)/b^2*(sin(f*x+e)*(15*cos(f*x+e)
^6+15*cos(f*x+e)^5-25*cos(f*x+e)^4-25*cos(f*x+e)^3+4*cos(f*x+e)^2+4*cos(f*
x+e)+12)+12*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(
2+cos(f*x+e)+sec(f*x+e))*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)-12*I*(1/(c
os(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(2+cos(f*x+e)+sec(f*
x+e))*EllipticE(I*(cot(f*x+e)-csc(f*x+e)),I))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.93

$$\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{2\left(\left(15\cos(fx+e)^6-25\cos(fx+e)^4+4\cos(fx+e)^2\right)\sqrt{\frac{b}{\cos(fx+e)}}\sin(fx+\right.}{}$$

input
```
integrate(sin(f*x+e)^4/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
2/195*((15*cos(f*x + e)^6 - 25*cos(f*x + e)^4 + 4*cos(f*x + e)^2)*sqrt(b/c
os(f*x + e))*sin(f*x + e) + 6*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) - 6*I*sqrt(2)*sqrt
(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin
(f*x + e))))/(b^3*f)
```

Sympy [F]

$$\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\sin^4{(e+fx)}}{(b\sec{(e+fx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)**4/(b*sec(f*x+e))**(5/2),x)
```

output
```
Integral(sin(e + f*x)**4/(b*sec(e + f*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\sin{(fx+e)}^4}{(b\sec{(fx+e)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^4/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^4/(b*sec(f*x + e))^(5/2), x)
```

Giac [F]

$$\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\sin{(fx+e)}^4}{(b\sec{(fx+e)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^4/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate(sin(f*x + e)^4/(b*sec(f*x + e))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\sin(e+f\,x)^4}{\left(\frac{b}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int(sin(e + f*x)^4/(b/cos(e + f*x))^(5/2),x)
```

output
```
int(sin(e + f*x)^4/(b/cos(e + f*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(fx+e)}\,\sin(fx+e)^4}{\sec(fx+e)^3}dx\right)}{b^3}$$

input
```
int(sin(f*x+e)^4/(b*sec(f*x+e))^(5/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(e + f*x))*sin(e + f*x)**4)/sec(e + f*x)**3,x))/b**3
```

3.445 $\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{4E\left(\frac{1}{2}(e+fx)\middle|2\right)}{15b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{2b\sin(e+fx)}{9f(b\sec(e+fx))^{7/2}} + \frac{4\sin(e+fx)}{45bf(b\sec(e+fx))^{3/2}}$$

output
```
4/15*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/b^2/f/cos(f*x+e)^(1/2)/(b*sec(f
*x+e))^(1/2)-2/9*b*sin(f*x+e)/f/(b*sec(f*x+e))^(7/2)+4/45*sin(f*x+e)/b/f/(
b*sec(f*x+e))^(3/2)
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.67

$$\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{\frac{96E\left(\frac{1}{2}(e+fx)\middle|2\right)}{\sqrt{\cos(e+fx)}} - 4\sin(2(e+fx)) - 10\sin(4(e+fx))}{360b^2f\sqrt{b\sec(e+fx)}}$$

input
```
Integrate[Sin[e + f*x]^2/(b*Sec[e + f*x])^(5/2),x]
```

output
```
((96*EllipticE[(e + f*x)/2, 2])/Sqrt[Cos[e + f*x]] - 4*Sin[2*(e + f*x)] -
10*Sin[4*(e + f*x)])/(360*b^2*f*Sqrt[b*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3107, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc(e+fx)^2(b\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3107}$$

$$\frac{2}{9}\int \frac{1}{(b\sec(e+fx))^{5/2}}dx-\frac{2b\sin(e+fx)}{9f(b\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{9}\int \frac{1}{\left(b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx-\frac{2b\sin(e+fx)}{9f(b\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{4256}$$

$$\frac{2}{9}\left(\frac{3\int \frac{1}{\sqrt{b\sec(e+fx)}}dx}{5b^2}+\frac{2\sin(e+fx)}{5bf(b\sec(e+fx))^{3/2}}\right)-\frac{2b\sin(e+fx)}{9f(b\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{9}\left(\frac{3\int \frac{1}{\sqrt{b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{5b^2}+\frac{2\sin(e+fx)}{5bf(b\sec(e+fx))^{3/2}}\right)-\frac{2b\sin(e+fx)}{9f(b\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{2}{9}\left(\frac{3\int\sqrt{\cos(e+fx)}dx}{5b^2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}+\frac{2\sin(e+fx)}{5bf(b\sec(e+fx))^{3/2}}\right)-\frac{2b\sin(e+fx)}{9f(b\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{9}\left(\frac{3\int\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{5b^2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}+\frac{2\sin(e+fx)}{5bf(b\sec(e+fx))^{3/2}}\right)-\frac{2b\sin(e+fx)}{9f(b\sec(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2}{9}\left(\frac{6E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}+\frac{2\sin(e+fx)}{5bf(b\sec(e+fx))^{3/2}}\right)-\frac{2b\sin(e+fx)}{9f(b\sec(e+fx))^{7/2}}$$

input
```
Int[Sin[e + f*x]^2/(b*Sec[e + f*x])^(5/2),x]
```

output
```
(-2*b*Sin[e + f*x])/(9*f*(b*Sec[e + f*x])^(7/2)) + (2*((6*EllipticE[(e + f
*x)/2, 2])/(5*b^2*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]]) + (2*Sin[e +
f*x])/(5*b*f*(b*Sec[e + f*x])^(3/2))))/9
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3107
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Csc[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)
/(a*f*(m + n))), x] + Simp[(m + 1)/(a^2*(m + n))   Int[(a*Csc[e + f*x])^(m
+ 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.79 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.19

method	result
default	$\frac{\frac{2\sin(fx+e)\left(-5\cos(fx+e)^4-5\cos(fx+e)^3+2\cos(fx+e)^2+2\cos(fx+e)+6\right)}{45}+\frac{4i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,(2+\cos(fx+e)+\sec(fx+e))\,\text{Ellip}}{15}}{f(\cos(fx+e)+1)\sqrt{b s}}$

input
```
int(sin(f*x+e)^2/(b*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/45/f/(cos(f*x+e)+1)/(b*sec(f*x+e))^(1/2)/b^2*(sin(f*x+e)*(-5*cos(f*x+e)^
4-5*cos(f*x+e)^3+2*cos(f*x+e)^2+2*cos(f*x+e)+6)+6*I*(1/(cos(f*x+e)+1))^(1/
2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(2+cos(f*x+e)+sec(f*x+e))*EllipticF(I
*(cot(f*x+e)-csc(f*x+e)),I)-6*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)*(2+cos(f*x+e)+sec(f*x+e))*EllipticE(I*(cot(f*x+e)-csc(f*x
+e)),I))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.09

$$\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx =$$
$$-\frac{2\left(\left(5\cos\left(fx+e\right)^4-2\cos\left(fx+e\right)^2\right)\sqrt{\frac{b}{\cos(fx+e)}}\sin\left(fx+e\right)-3i\sqrt{2}\sqrt{b}\text{weierstrassZeta}(-4,0,\text{weierstr}$$

input
```
integrate(sin(f*x+e)^2/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
-2/45*((5*cos(f*x + e)^4 - 2*cos(f*x + e)^2)*sqrt(b/cos(f*x + e))*sin(f*x
+ e) - 3*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(f*x + e) + I*sin(f*x + e))) + 3*I*sqrt(2)*sqrt(b)*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))))/(b^3*f)
```

Sympy [F]

$$\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\sin^2\left(e+fx\right)}{\left(b\sec\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)**2/(b*sec(f*x+e))**(5/2),x)
```

output
```
Integral(sin(e + f*x)**2/(b*sec(e + f*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\sin\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^2/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^2/(b*sec(f*x + e))^(5/2), x)
```

Giac [F]

$$\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\sin\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^2/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate(sin(f*x + e)^2/(b*sec(f*x + e))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\sin\left(e+f\,x\right)^2}{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^{5/2}}\,dx$$

input
```
int(sin(e + f*x)^2/(b/cos(e + f*x))^(5/2),x)
```

output
```
int(sin(e + f*x)^2/(b/cos(e + f*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(fx+e)}\,\sin(fx+e)^2}{\sec(fx+e)^3}dx\right)}{b^3}$$

input
```
int(sin(f*x+e)^2/(b*sec(f*x+e))^(5/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(e + f*x))*sin(e + f*x)**2)/sec(e + f*x)**3,x))/b**3
```

3.446 $\int \frac{1}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 72

$$\int \frac{1}{(b\sec(e+fx))^{5/2}}\,dx=\frac{6E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}+\frac{2\sin(e+fx)}{5bf(b\sec(e+fx))^{3/2}}$$

output
```
6/5*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/b^2/f/cos(f*x+e)^(1/2)/(b*sec(f*
x+e))^(1/2)+2/5*sin(f*x+e)/b/f/(b*sec(f*x+e))^(3/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \frac{1}{(b\sec(e+fx))^{5/2}}\,dx=\frac{\sqrt{b\sec(e+fx)}\left(12\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)+\sin(e+fx)+\sin(3(e+fx))\right)}{10b^3f}$$

input
```
Integrate[(b*Sec[e + f*x])^(-5/2),x]
```

output
```
(Sqrt[b*Sec[e + f*x]]*(12*Sqrt[Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2] + S
in[e + f*x] + Sin[3*(e + f*x)]))/(10*b^3*f)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(b\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$\frac{3\int \frac{1}{\sqrt{b\sec(e+fx)}}dx}{5b^2}+\frac{2\sin(e+fx)}{5bf(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{1}{\sqrt{b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{5b^2}+\frac{2\sin(e+fx)}{5bf(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{3\int \sqrt{\cos(e+fx)}dx}{5b^2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}+\frac{2\sin(e+fx)}{5bf(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{5b^2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}+\frac{2\sin(e+fx)}{5bf(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{6E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}+\frac{2\sin(e+fx)}{5bf(b\sec(e+fx))^{3/2}}$$

input
```
Int[(b*Sec[e + f*x])^(-5/2),x]
```

output
```
(6*EllipticE[(e + f*x)/2, 2])/(5*b^2*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f
*x]]) + (2*Sin[e + f*x])/(5*b*f*(b*Sec[e + f*x])^(3/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.00 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.65

method	result
default	$\frac{\frac{2\sin(fx+e)\left(\cos(fx+e)^2+\cos(fx+e)+3\right)}{5}+\frac{6i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,(2+\cos(fx+e)+\sec(fx+e))\,\text{EllipticF}(i(\cot(fx+e)-\csc(fx+e)),i)}{5}-}{f(\cos(fx+e)+1)\sqrt{b\sec(fx+e)}\,b^2}$

input
```
int(1/(b*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/f/(cos(f*x+e)+1)/(b*sec(f*x+e))^(1/2)/b^2*(sin(f*x+e)*(cos(f*x+e)^2+co
s(f*x+e)+3)+3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)
*(2+cos(f*x+e)+sec(f*x+e))*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)-3*I*(1/(
cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(2+cos(f*x+e)+sec(f
*x+e))*EllipticE(I*(cot(f*x+e)-csc(f*x+e)),I))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.32

$$\int \frac{1}{(b\sec(e+fx))^{5/2}}\,dx = \frac{2\sqrt{\frac{b}{\cos(fx+e)}}\cos\left(fx+e\right)^2\sin\left(fx+e\right)+3i\sqrt{2}\sqrt{b}\text{weierstrassZeta}(-4,0,\text{weierst}$$

input
```
integrate(1/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
1/5*(2*sqrt(b/cos(f*x + e))*cos(f*x + e)^2*sin(f*x + e) + 3*I*sqrt(2)*sqrt
(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin
(f*x + e))) - 3*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(f*x + e) - I*sin(f*x + e))))/(b^3*f)
```

Sympy [F]

$$\int \frac{1}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{1}{(b\sec\left(e+fx\right))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*sec(f*x+e))**(5/2),x)
```

output
```
Integral((b*sec(e + f*x))**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{1}{(b\sec(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{1}{(b\sec(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{1}{\left(\frac{b}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

input
```
int(1/(b/cos(e + f*x))^(5/2),x)
```

output
```
int(1/(b/cos(e + f*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{(b\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(fx+e)}}{\sec(fx+e)^3}dx\right)}{b^3}$$

input
```
int(1/(b*sec(f*x+e))^(5/2),x)
```

output
```
(sqrt(b)*int(sqrt(sec(e + f*x))/sec(e + f*x)**3,x))/b**3
```

3.447 $\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 68

$$\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = -\frac{\csc(e+fx)}{bf(b\sec(e+fx))^{3/2}} - \frac{3E\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

output
```
-csc(f*x+e)/b/f/(b*sec(f*x+e))^(3/2)-3*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2
))/b^2/f/cos(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.75

$$\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{-\cot(e+fx) - \frac{3E\left(\frac{1}{2}(e+fx)\middle|2\right)}{\sqrt{\cos(e+fx)}}}{b^2f\sqrt{b\sec(e+fx)}}$$

input
```
Integrate[Csc[e + f*x]^2/(b*Sec[e + f*x])^(5/2),x]
```

output
```
(-Cot[e + f*x] - (3*EllipticE[(e + f*x)/2, 2])/Sqrt[Cos[e + f*x]])/(b^2*f*
Sqrt[b*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3103, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc(e+fx)^2}{(b\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3103}$$

$$-\frac{3\int \frac{1}{\sqrt{b\sec(e+fx)}}dx}{2b^2}-\frac{\csc(e+fx)}{bf(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\int \frac{1}{\sqrt{b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx}{2b^2}-\frac{\csc(e+fx)}{bf(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{4258}$$

$$-\frac{3\int \sqrt{\cos(e+fx)}dx}{2b^2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{\csc(e+fx)}{bf(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\int \sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{2b^2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{\csc(e+fx)}{bf(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{3E\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{\csc(e+fx)}{bf(b\sec(e+fx))^{3/2}}$$

input
```
Int[Csc[e + f*x]^2/(b*Sec[e + f*x])^(5/2),x]
```

output
```
-(Csc[e + f*x]/(b*f*(b*Sec[e + f*x])^(3/2))) - (3*EllipticE[(e + f*x)/2, 2
])/(b^2*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3103
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
1)/(f*b*(m - 1))), x] + Simp[a^2*((n + 1)/(b^2*(m - 1)))   Int[(a*Csc[e + f
*x])^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.94 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.43

method	result
default	$\frac{-i\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\sqrt{\frac{1}{\cos(fx+e)+1}}\,\text{EllipticE}(i(\cot(fx+e)-\csc(fx+e)),i)(-3-3\sec(fx+e))-i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\text{Ellipti}}{f\sqrt{b\sec(fx+e)}\,b^2}$

input
```
int(csc(f*x+e)^2/(b*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*Ellipti
cE(I*(cot(f*x+e)-csc(f*x+e)),I)*(-3-3*sec(f*x+e))-I*(1/(cos(f*x+e)+1))^(1/
2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I
)*(3+3*sec(f*x+e))+2*cot(f*x+e)-3*csc(f*x+e))/(b*sec(f*x+e))^(1/2)/b^2
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.60

$$\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{-3i\sqrt{2}\sqrt{b}\sin(fx+e)\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(f$$

input
```
integrate(csc(f*x+e)^2/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
1/2*(-3*I*sqrt(2)*sqrt(b)*sin(f*x + e)*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*I*sqrt(2)*sqrt(b)*sin(f
*x + e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I
*sin(f*x + e))) - 2*sqrt(b/cos(f*x + e))*cos(f*x + e)^2)/(b^3*f*sin(f*x +
e))
```

Sympy [F]

$$\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**2/(b*sec(f*x+e))**(5/2),x)
```

output
```
Integral(csc(e + f*x)**2/(b*sec(e + f*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\csc\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^2/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate(csc(f*x + e)^2/(b*sec(f*x + e))^(5/2), x)
```

Giac [F]

$$\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\csc\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^2/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate(csc(f*x + e)^2/(b*sec(f*x + e))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)^2\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{5/2}}\,dx$$

input
```
int(1/(sin(e + f*x)^2*(b/cos(e + f*x))^(5/2)),x)
```

output
```
int(1/(sin(e + f*x)^2*(b/cos(e + f*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(fx+e)}\,\csc(fx+e)^2}{\sec(fx+e)^3}dx\right)}{b^3}$$

input
```
int(csc(f*x+e)^2/(b*sec(f*x+e))^(5/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(e + f*x))*csc(e + f*x)**2)/sec(e + f*x)**3,x))/b**3
```

3.448 $\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 102

$$\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx=\frac{\csc(e+fx)}{2bf(b\sec(e+fx))^{3/2}}-\frac{\csc^3(e+fx)}{3bf(b\sec(e+fx))^{3/2}}+\frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)}{2b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

output
```
1/2*csc(f*x+e)/b/f/(b*sec(f*x+e))^(3/2)-1/3*csc(f*x+e)^3/b/f/(b*sec(f*x+e)
)^(3/2)+1/2*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/b^2/f/cos(f*x+e)^(1/2)/(
b*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.77

$$\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx=\frac{\left(-3+5\csc^2(e+fx)-2\csc^4(e+fx)+3\sqrt{\cos(e+fx)}\csc(e+fx)E\left(\frac{1}{2}(e+\right.\right.}{6b^3f}$$

input
```
Integrate[Csc[e + f*x]^4/(b*Sec[e + f*x])^(5/2),x]
```

output
```
((-3 + 5*Csc[e + f*x]^2 - 2*Csc[e + f*x]^4 + 3*Sqrt[Cos[e + f*x]]*Csc[e +
f*x]*EllipticE[(e + f*x)/2, 2])*Sqrt[b*Sec[e + f*x]]*Sin[e + f*x])/(6*b^3*
f)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3103, 3042, 3105, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc(e+fx)^4}{(b\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3103}$$

$$-\frac{\int \frac{\csc^2(e+fx)}{\sqrt{b\sec(e+fx)}}dx}{2b^2}-\frac{\csc^3(e+fx)}{3bf(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int \frac{\csc(e+fx)^2}{\sqrt{b\sec(e+fx)}}dx}{2b^2}-\frac{\csc^3(e+fx)}{3bf(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 3105}$$

$$-\frac{-\frac{1}{2}\int \frac{1}{\sqrt{b\sec(e+fx)}}dx-\frac{b\csc(e+fx)}{f(b\sec(e+fx))^{3/2}}}{2b^2}-\frac{\csc^3(e+fx)}{3bf(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{-\frac{1}{2}\int \frac{1}{\sqrt{b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{b\csc(e+fx)}{f(b\sec(e+fx))^{3/2}}}{2b^2}-\frac{\csc^3(e+fx)}{3bf(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 4258}$$

$$-\frac{-\frac{\int \sqrt{\cos(e+fx)}dx}{2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{b\csc(e+fx)}{f(b\sec(e+fx))^{3/2}}}{2b^2}-\frac{\csc^3(e+fx)}{3bf(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{-\frac{\int \sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{b\csc(e+fx)}{f(b\sec(e+fx))^{3/2}}}{2b^2}-\frac{\csc^3(e+fx)}{3bf(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \ 3119$$

$$-\frac{-\frac{b\csc(e+fx)}{f(b\sec(e+fx))^{3/2}}-\frac{E\left(\frac{1}{2}(e+fx)|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}}{2b^2}-\frac{\csc^3(e+fx)}{3bf(b\sec(e+fx))^{3/2}}$$

input
```
Int[Csc[e + f*x]^4/(b*Sec[e + f*x])^(5/2),x]
```

output
```
-1/3*Csc[e + f*x]^3/(b*f*(b*Sec[e + f*x])^(3/2)) - (-((b*Csc[e + f*x])/(f*
(b*Sec[e + f*x])^(3/2))) - EllipticE[(e + f*x)/2, 2]/(f*Sqrt[Cos[e + f*x]]
*Sqrt[b*Sec[e + f*x]]))/(2*b^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3103
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
1)/(f*b*(m - 1))), x] + Simp[a^2*((n + 1)/(b^2*(m - 1)))   Int[(a*Csc[e + f
*x])^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

rule 3105
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Simp[a^2*((m + n - 2)/(m - 1))   Int[(a*Csc[e + f*
x])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[
m, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.61 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.70

method	result
default	$\frac{\frac{i\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\sqrt{\frac{1}{\cos(fx+e)+1}}\,\text{EllipticE}(i(\cot(fx+e)-\csc(fx+e)),i)(-3-3\sec(fx+e))}{6}+\frac{i\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\text{EllipticF}(i(\cot(fx-}{6}}{f\sqrt{b\sec(fx+e)}\,b^2}$

input
```
int(csc(f*x+e)^4/(b*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/6*I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*Elli
pticE(I*(cot(f*x+e)-csc(f*x+e)),I)*(-3-3*sec(f*x+e))+1/6*I*(1/(cos(f*x+e)+
1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*
x+e)),I)*(3+3*sec(f*x+e))+1/2*csc(f*x+e)-1/3*cot(f*x+e)*csc(f*x+e)^2)/(b*s
ec(f*x+e))^(1/2)/b^2
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.59

$$\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx =$$

$$-\frac{3\sqrt{2}\left(-i\cos\left(fx+e\right)^2+i\right)\sqrt{b}\sin\left(fx+e\right)\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos\left(fx+e\right.}{}$$

input
```
integrate(csc(f*x+e)^4/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(3*sqrt(2)*(-I*cos(f*x + e)^2 + I)*sqrt(b)*sin(f*x + e)*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*
sqrt(2)*(I*cos(f*x + e)^2 - I)*sqrt(b)*sin(f*x + e)*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) - 2*(3*cos(f*x
 + e)^4 - cos(f*x + e)^2)*sqrt(b/cos(f*x + e)))/((b^3*f*cos(f*x + e)^2 - b
^3*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**4/(b*sec(f*x+e))**(5/2),x)
```

output
```
Integral(csc(e + f*x)**4/(b*sec(e + f*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\csc(fx+e)^4}{(b\sec(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^4/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate(csc(f*x + e)^4/(b*sec(f*x + e))^(5/2), x)
```

Giac [F]

$$\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\csc\left(fx+e\right)^4}{\left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^4/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate(csc(f*x + e)^4/(b*sec(f*x + e))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)^4\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{5/2}}\,dx$$

input
```
int(1/(sin(e + f*x)^4*(b/cos(e + f*x))^(5/2)),x)
```

output
```
int(1/(sin(e + f*x)^4*(b/cos(e + f*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(fx+e)}\,\csc(fx+e)^4}{\sec(fx+e)^3}dx\right)}{b^3}$$

input
```
int(csc(f*x+e)^4/(b*sec(f*x+e))^(5/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(e + f*x))*csc(e + f*x)**4)/sec(e + f*x)**3,x))/b**3
```

3.449 $\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{5/2}} dx$

Optimal result

Integrand size = 21, antiderivative size = 132

$$\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{5/2}} dx = \frac{3\csc(e+fx)}{20bf(b\sec(e+fx))^{3/2}} + \frac{\csc^3(e+fx)}{10bf(b\sec(e+fx))^{3/2}} - \frac{\csc^5(e+fx)}{5bf(b\sec(e+fx))^{3/2}} + \frac{3E\left(\frac{1}{2}(e+fx)\middle|2\right)}{20b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

output
```
3/20*csc(f*x+e)/b/f/(b*sec(f*x+e))^(3/2)+1/10*csc(f*x+e)^3/b/f/(b*sec(f*x+
e))^(3/2)-1/5*csc(f*x+e)^5/b/f/(b*sec(f*x+e))^(3/2)+3/20*EllipticE(sin(1/2
*f*x+1/2*e),2^(1/2))/b^2/f/cos(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.66

$$\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{5/2}} dx = \frac{\left(-3+\csc^2(e+fx)+6\csc^4(e+fx)-4\csc^6(e+fx)+3\sqrt{\cos(e+fx)}\csc(e+\right.}{20b^3f}$$

input
```
Integrate[Csc[e + f*x]^6/(b*Sec[e + f*x])^(5/2),x]
```

output
```
((-3 + Csc[e + f*x]^2 + 6*Csc[e + f*x]^4 - 4*Csc[e + f*x]^6 + 3*Sqrt[Cos[e
 + f*x]]*Csc[e + f*x]*EllipticE[(e + f*x)/2, 2])*Sqrt[b*Sec[e + f*x]]*Sin[
e + f*x])/(20*b^3*f)
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {3042, 3103, 3042, 3105, 3042, 3105, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc(e+fx)^6}{(b\sec(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3103}$$

$$-\frac{3\int \frac{\csc^4(e+fx)}{\sqrt{b\sec(e+fx)}}dx}{10b^2}-\frac{\csc^5(e+fx)}{5bf(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\int \frac{\csc(e+fx)^4}{\sqrt{b\sec(e+fx)}}dx}{10b^2}-\frac{\csc^5(e+fx)}{5bf(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3105}$$

$$-\frac{3\left(\frac{1}{2}\int \frac{\csc^2(e+fx)}{\sqrt{b\sec(e+fx)}}dx-\frac{b\csc^3(e+fx)}{3f(b\sec(e+fx))^{3/2}}\right)}{10b^2}-\frac{\csc^5(e+fx)}{5bf(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\left(\frac{1}{2}\int \frac{\csc(e+fx)^2}{\sqrt{b\sec(e+fx)}}dx-\frac{b\csc^3(e+fx)}{3f(b\sec(e+fx))^{3/2}}\right)}{10b^2}-\frac{\csc^5(e+fx)}{5bf(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3105}$$

$$-\frac{3\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{1}{\sqrt{b\sec(e+fx)}}dx-\frac{b\csc(e+fx)}{f(b\sec(e+fx))^{3/2}}\right)-\frac{b\csc^3(e+fx)}{3f(b\sec(e+fx))^{3/2}}\right)}{10b^2}-\frac{\csc^5(e+fx)}{5bf(b\sec(e+fx))^{3/2}}$$

$\Big\downarrow$ 3042

$$-\frac{3\left(\frac{1}{2}\left(-\frac{1}{2}\int\frac{1}{\sqrt{b\csc\left(e+fx+\frac{\pi}{2}\right)}}dx-\frac{b\csc(e+fx)}{f(b\sec(e+fx))^{3/2}}\right)-\frac{b\csc^3(e+fx)}{3f(b\sec(e+fx))^{3/2}}\right)}{10b^2}-\frac{\csc^5(e+fx)}{5bf(b\sec(e+fx))^{3/2}}$$

$\Big\downarrow$ 4258

$$-\frac{3\left(\frac{1}{2}\left(-\frac{\int\sqrt{\cos(e+fx)}dx}{2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{b\csc(e+fx)}{f(b\sec(e+fx))^{3/2}}\right)-\frac{b\csc^3(e+fx)}{3f(b\sec(e+fx))^{3/2}}\right)}{10b^2}-\frac{\csc^5(e+fx)}{5bf(b\sec(e+fx))^{3/2}}$$

$\Big\downarrow$ 3042

$$-\frac{3\left(\frac{1}{2}\left(-\frac{\int\sqrt{\sin\left(e+fx+\frac{\pi}{2}\right)}dx}{2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{b\csc(e+fx)}{f(b\sec(e+fx))^{3/2}}\right)-\frac{b\csc^3(e+fx)}{3f(b\sec(e+fx))^{3/2}}\right)}{10b^2}-\frac{\csc^5(e+fx)}{5bf(b\sec(e+fx))^{3/2}}$$

$\Big\downarrow$ 3119

$$-\frac{3\left(\frac{1}{2}\left(-\frac{b\csc(e+fx)}{f(b\sec(e+fx))^{3/2}}-\frac{E\left(\frac{1}{2}(e+fx)|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}\right)-\frac{b\csc^3(e+fx)}{3f(b\sec(e+fx))^{3/2}}\right)}{10b^2}-\frac{\csc^5(e+fx)}{5bf(b\sec(e+fx))^{3/2}}$$

input
```
Int[Csc[e + f*x]^6/(b*Sec[e + f*x])^(5/2),x]
```

output
```
-1/5*Csc[e + f*x]^5/(b*f*(b*Sec[e + f*x])^(3/2)) - (3*(-1/3*(b*Csc[e + f*x
]^3)/(f*(b*Sec[e + f*x])^(3/2)) + (-((b*Csc[e + f*x])/(f*(b*Sec[e + f*x])^
(3/2))) - EllipticE[(e + f*x)/2, 2]/(f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f
*x]]))/2))/(10*b^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3103
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
1)/(f*b*(m - 1))), x] + Simp[a^2*((n + 1)/(b^2*(m - 1)))   Int[(a*Csc[e + f
*x])^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] &&
GtQ[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

rule 3105
```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(-a)*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
 - 1)/(f*(m - 1))), x] + Simp[a^2*((m + n - 2)/(m - 1))   Int[(a*Csc[e + f*
x])^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[
m, 1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.02 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.43

method	result
default	$\frac{\frac{i \sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\, \sqrt{\frac{1}{\cos(fx+e)+1}}\, \operatorname{EllipticE}(i(\cot(fx+e)-\csc(fx+e)),i)(-3-3\sec(fx+e))}{20}+\frac{i \sqrt{\frac{1}{\cos(fx+e)+1}}\, \sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\, \operatorname{EllipticF}(i(\cot(fx-}{20}}{f \sqrt{b \sec(fx+e)}\, b^2}$

input
```
int(csc(f*x+e)^6/(b*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/20*I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*Ell
ipticE(I*(cot(f*x+e)-csc(f*x+e)),I)*(-3-3*sec(f*x+e))+1/20*I*(1/(cos(f*x+e
)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(
f*x+e)),I)*(3+3*sec(f*x+e))+3/20*csc(f*x+e)+1/10*cot(f*x+e)*csc(f*x+e)^2-1
/5*cot(f*x+e)*csc(f*x+e)^4)/(b*sec(f*x+e))^(1/2)/b^2
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.54

$$\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = -\frac{3\sqrt{2}\left(-i\cos(fx+e)^4+2i\cos(fx+e)^2-i\right)\sqrt{b}\sin(fx+e)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}}{}$$

input
```
integrate(csc(f*x+e)^6/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
-1/40*(3*sqrt(2)*(-I*cos(f*x + e)^4 + 2*I*cos(f*x + e)^2 - I)*sqrt(b)*sin(
f*x + e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) +
I*sin(f*x + e))) + 3*sqrt(2)*(I*cos(f*x + e)^4 - 2*I*cos(f*x + e)^2 + I)*s
qrt(b)*sin(f*x + e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(
f*x + e) - I*sin(f*x + e))) - 2*(3*cos(f*x + e)^6 - 8*cos(f*x + e)^4 + cos
(f*x + e)^2)*sqrt(b/cos(f*x + e)))/((b^3*f*cos(f*x + e)^4 - 2*b^3*f*cos(f*
x + e)^2 + b^3*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(csc(f*x+e)**6/(b*sec(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\csc(fx+e)^6}{(b\sec(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^6/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate(csc(f*x + e)^6/(b*sec(f*x + e))^(5/2), x)
```

Giac [F]

$$\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{\csc(fx+e)^6}{(b\sec(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^6/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate(csc(f*x + e)^6/(b*sec(f*x + e))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)^6\,{\left(\frac{b}{\cos(e+f\,x)}\right)}^{5/2}}\,dx$$

input
```
int(1/(sin(e + f*x)^6*(b/cos(e + f*x))^(5/2)),x)
```

output
```
int(1/(sin(e + f*x)^6*(b/cos(e + f*x))^(5/2)), x)
```

Reduce [F]

$$\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sec(fx+e)}\,\csc(fx+e)^6}{\sec(fx+e)^3}dx\right)}{b^3}$$

input
```
int(csc(f*x+e)^6/(b*sec(f*x+e))^(5/2),x)
```

output
```
(sqrt(b)*int((sqrt(sec(e + f*x))*csc(e + f*x)**6)/sec(e + f*x)**3,x))/b**3
```

3.450 $\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{9/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 350

$$\begin{aligned}\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{9/2}\,dx = &-\frac{21a^{9/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{32\sqrt{2}\sqrt{b}f}\\
&+\frac{21a^{9/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{32\sqrt{2}\sqrt{b}f}\\
&-\frac{21a^{9/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}\left(\sqrt{a}+\sqrt{a}\tan(e+fx)\right)}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{32\sqrt{2}\sqrt{b}f}\\
&-\frac{7a^3b(a\sin(e+fx))^{3/2}}{16f\sqrt{b\sec(e+fx)}}-\frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}}\end{aligned}$$

output
```
-21/64*a^(9/2)*arctan(1-2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/a^(1/2)/(b*co
s(f*x+e))^(1/2))*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)*2^(1/2)/b^(1/2)
/f+21/64*a^(9/2)*arctan(1+2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/a^(1/2)/(b*
cos(f*x+e))^(1/2))*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)*2^(1/2)/b^(1/
2)/f-21/64*a^(9/2)*arctanh(2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/(b*cos(f*x
+e))^(1/2)/(a^(1/2)+a^(1/2)*tan(f*x+e)))*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e
))^(1/2)*2^(1/2)/b^(1/2)/f-7/16*a^3*b*(a*sin(f*x+e))^(3/2)/f/(b*sec(f*x+e)
)^(1/2)-1/4*a*b*(a*sin(f*x+e))^(7/2)/f/(b*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 3.29 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.48

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{9/2}\,dx = \frac{a^4\cot(e+fx)\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}\left(4(-9+2\cos(2(e+fx)))\sin^2(e+fx)+21\right.}{}$$

input
```
Integrate[Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(9/2),x]
```

output
```
(a^4*Cot[e + f*x]*Sqrt[b*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]]*(4*(-9 + 2*Cos
[2*(e + f*x)])*Sin[e + f*x]^2 + 21*Sqrt[2]*ArcTan[(-1 + Sqrt[Tan[e + f*x]^
2])/(Sqrt[2]*(Tan[e + f*x]^2)^(1/4))]*(Tan[e + f*x]^2)^(1/4) - 21*Sqrt[2]*
ArcTanh[(Sqrt[2]*(Tan[e + f*x]^2)^(1/4))/(1 + Sqrt[Tan[e + f*x]^2])]*(Tan[
e + f*x]^2)^(1/4)))/(64*f)
```

Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.11, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3042, 3063, 3042, 3063, 3042, 3065, 3042, 3054, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx))^{9/2}\sqrt{b\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx))^{9/2}\sqrt{b\sec(e+fx)}dx$$

$$\Big\downarrow \text{3063}$$

$$\frac{7}{8}a^2\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}dx-\frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{8}a^2\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}dx-\frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3063}$$

$$\frac{7}{8}a^2\left(\frac{3}{4}a^2\int \sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}dx-\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}}\right)-\frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{8}a^2\left(\frac{3}{4}a^2\int \sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}dx-\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}}\right)-\frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3065}$$

$$\frac{7}{8}a^2\left(\frac{3}{4}a^2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}dx-\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}}\right)-\frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{8}a^2\left(\frac{3}{4}a^2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}dx-\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}}\right)-\frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3054}$$

$$\frac{7}{8}a^2\left(\frac{3a^3b\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{a\tan(e+fx)}{b(\tan^2(e+fx)a^2+a^2)}d\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}}{2f}-\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}}\right)-\frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{826}$$

$$\frac{7}{8}a^2\left(\frac{3a^3b\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\int\frac{\tan(e+fx)a+a}{\tan^2(e+fx)a^2+a^2}d\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}}{2b}-\frac{\int\frac{a-a\tan(e+fx)}{\tan^2(e+fx)a^2+a^2}d\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}}{2b}\right)}{2f}-\frac{ab(a\sin}{2f\sqrt{b}}\right.$$

$$\frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}}$$

$\downarrow$ 1476

$$\frac{7}{8}a^2\left(\frac{3a^3b\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\int\frac{1}{\frac{\tan(e+fx)a}{b}+\frac{a}{b}-\frac{\sqrt{2}\sqrt{a\sin(e+fx)}\sqrt{a}}{\sqrt{b}\sqrt{b\cos(e+fx)}}}d\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}}{2b}+\frac{\int\frac{1}{\frac{\tan(e+fx)a}{b}+\frac{a}{b}+\frac{\sqrt{2}\sqrt{a\sin(e+fx)}\sqrt{}}{\sqrt{b}\sqrt{b\cos(e+fx}}}}{2b}}{2b}\right.}{2f}\right.$$

$$\frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}}$$

$\downarrow$ 1082

$$\frac{7}{8}a^2\left(\frac{3a^3b\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\int\frac{1}{-\frac{a\tan(e+fx)}{b}-1}d\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{\int\frac{1}{-\frac{a\tan(e+fx)}{b}-1}d\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}}{2b}\right.}{2f}\right.$$

$$\frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}}$$

$\downarrow$ 217

$$\frac{7}{8}a^2\left(\frac{3a^3b\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}}{2b}-\frac{\int\frac{a-a\tan(e+fx)}{\tan^2(e+fx)a^2+a^2}d\sqrt{}}{2b}\right.}{2f}\right.$$

$$\frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}}$$

$\Big\downarrow$ 1479

$$\frac{7}{8}a^2\left(\frac{3a^3b\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}}{2b}-\frac{-\int-\frac{\sqrt{2}\sqrt{a}-}{\sqrt{b}\left(\frac{\tan(e+fx)a}{b}+\right.}}{}\right.}{2f}\right.$$

$$\frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}}$$

$\Big\downarrow$ 25

$$\frac{7}{8}a^2\left(\frac{3a^3b\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}}{2b}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{b}}{\sqrt{}}}{\sqrt{b}\left(\frac{\tan(e+fx)a}{b}+\frac{a}{b}-\right.}}{}\right.}{2f}\right.$$

$$\frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \ \textbf{27}$$

$$\frac{7}{8}a^2\left(\frac{3a^3b\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}}{2b}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{b}\sqrt{a\cdots}}{\sqrt{b\cos\cdots}}}{\frac{\tan(e+fx)a}{b}+\frac{a}{b}-\frac{\sqrt{2}\cdots}{\sqrt{\cdots}}}}{\cdots}\cdots\right.}{2f}\cdots\right.$$

$$\frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \ \textbf{1103}$$

$$\frac{7}{8}a^2\left(\frac{3a^3b\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}}{2b}-\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{a\sin(e+\cdots}}{\sqrt{b\cos(e+fx)}}\cdots\right)}{2\sqrt{2}\sqrt{\cdots}}\cdots\right.}{2f}\cdots\right.$$

$$\frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}}$$

input
```
Int[Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(9/2),x]
```

output
```
-1/4*(a*b*(a*Sin[e + f*x])^(7/2))/(f*Sqrt[b*Sec[e + f*x]]) + (7*a^2*((3*a^
3*b*Sqrt[b*Cos[e + f*x]]*((-(ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*
x]])/(Sqrt[a]*Sqrt[b*Cos[e + f*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[b])) + ArcTan[1
 + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a]*Sqrt[b*Cos[e + f*x]])]/
(Sqrt[2]*Sqrt[a]*Sqrt[b]))/(2*b) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[b]*
Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x]] + a*Tan[e + f*x]]/(Sqrt[2]*Sqrt
[a]*Sqrt[b]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt
[b*Cos[e + f*x]] + a*Tan[e + f*x]]/(2*Sqrt[2]*Sqrt[a]*Sqrt[b]))/(2*b))*Sqr
t[b*Sec[e + f*x]])/(2*f) - (a*b*(a*Sin[e + f*x])^(3/2))/(2*f*Sqrt[b*Sec[e
+ f*x]])))/8
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3054
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k*a*(b/f)   Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

rule 3063
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*b*(a*Sin[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
- 1)/(f*(m - n))), x] + Simp[a^2*((m - 1)/(m - n))   Int[(a*Sin[e + f*x])^(
m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

rule 3065
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n   Int[(a*Sin[e
+ f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Int
egerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Maple [A] (warning: unable to verify)

Time = 51.64 (sec) , antiderivative size = 458, normalized size of antiderivative = 1.31

method	result
default	$\frac{\sqrt{2}\, a^{4} \left(-21 \ln \left(-\frac{\cos(fx+e)\cot(fx+e)+2\sqrt{-\frac{2\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^{2}}}\, \sin(fx+e)-2\cot(fx+e)-\sin(fx+e)-2\cos(fx+e)+\csc(fx+e)+2}{\cos(fx+e)-1}\right)+21 \ln$

input
```
int((b*sec(f*x+e))^(1/2)*(a*sin(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/16384/f*2^(1/2)*a^4*(-21*ln(-(cos(f*x+e)*cot(f*x+e)+2*(-2*sin(f*x+e)/(co
s(f*x+e)+1)^2*cos(f*x+e))^(1/2)*sin(f*x+e)-2*cot(f*x+e)-sin(f*x+e)-2*cos(f
*x+e)+csc(f*x+e)+2)/(cos(f*x+e)-1))+21*ln((-cos(f*x+e)*cot(f*x+e)+2*(-2*si
n(f*x+e)/(cos(f*x+e)+1)^2*cos(f*x+e))^(1/2)*sin(f*x+e)+2*cot(f*x+e)+sin(f*
x+e)+2*cos(f*x+e)-csc(f*x+e)-2)/(cos(f*x+e)-1))+42*arctan(((-2*sin(f*x+e)/
(cos(f*x+e)+1)^2*cos(f*x+e))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/(cos(f*x+e)-1)
)+42*arctan(((-2*sin(f*x+e)/(cos(f*x+e)+1)^2*cos(f*x+e))^(1/2)*sin(f*x+e)+
cos(f*x+e)-1)/(cos(f*x+e)-1))+(16*cos(f*x+e)^3+16*cos(f*x+e)^2-44*cos(f*x+
e)-44)*sin(f*x+e)*(-2*sin(f*x+e)/(cos(f*x+e)+1)^2*cos(f*x+e))^(1/2))*(a*si
n(f*x+e))^(1/2)*cos(f*x+e)*sin(f*x+e)^6*(b*sec(f*x+e))^(1/2)/(-sin(f*x+e)/
(cos(f*x+e)+1)^2*cos(f*x+e))^(1/2)*sec(1/2*f*x+1/2*e)^8*csc(1/2*f*x+1/2*e)
^6
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 538, normalized size of antiderivative = 1.54

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{9/2}\,dx =$$

$$-\frac{42\sqrt{2}\sqrt{ab}a^{4}\arctan\left(-\frac{\sqrt{2}\sqrt{ab}\sqrt{a\sin(fx+e)}\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)}{ab\cos(fx+e)-ab\sin(fx+e)}\right)+21\sqrt{2}\sqrt{ab}a^{4}\arctan\left(\frac{2\,ab\cos(fx+e)^{2}-2\,ab\cos(fx}{2\left(ab\cos(f\right.}\right.}{}$$

input
```
integrate((b*sec(f*x+e))^(1/2)*(a*sin(f*x+e))^(9/2),x, algorithm="fricas")
```

output
```
-1/256*(42*sqrt(2)*sqrt(a*b)*a^4*arctan(-sqrt(2)*sqrt(a*b)*sqrt(a*sin(f*x
+ e))*sqrt(b/cos(f*x + e))*cos(f*x + e)/(a*b*cos(f*x + e) - a*b*sin(f*x +
e))) + 21*sqrt(2)*sqrt(a*b)*a^4*arctan(1/2*(2*a*b*cos(f*x + e)^2 - 2*a*b*c
os(f*x + e)*sin(f*x + e) - 2*a*b + sqrt(2)*sqrt(a*b)*sqrt(a*sin(f*x + e))*
sqrt(b/cos(f*x + e)))/(a*b*cos(f*x + e)^2 + a*b*cos(f*x + e)*sin(f*x + e)
- a*b)) + 21*sqrt(2)*sqrt(a*b)*a^4*arctan(-1/2*(2*a*b*cos(f*x + e)^2 - 2*a
*b*cos(f*x + e)*sin(f*x + e) - 2*a*b - sqrt(2)*sqrt(a*b)*sqrt(a*sin(f*x +
e))*sqrt(b/cos(f*x + e)))/(a*b*cos(f*x + e)^2 + a*b*cos(f*x + e)*sin(f*x +
 e) - a*b)) + 21*sqrt(2)*sqrt(a*b)*a^4*log(4*a*b*cos(f*x + e)*sin(f*x + e)
 + 2*sqrt(2)*sqrt(a*b)*(cos(f*x + e)^2 + cos(f*x + e)*sin(f*x + e))*sqrt(a
*sin(f*x + e))*sqrt(b/cos(f*x + e)) + a*b) - 21*sqrt(2)*sqrt(a*b)*a^4*log(
4*a*b*cos(f*x + e)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*b)*(cos(f*x + e)^2 + co
s(f*x + e)*sin(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e)) + a*b)
- 16*(4*a^4*cos(f*x + e)^3 - 11*a^4*cos(f*x + e))*sqrt(a*sin(f*x + e))*sqr
t(b/cos(f*x + e))*sin(f*x + e))/f
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{9/2}\,dx = \text{Timed out}$$

input
```
integrate((b*sec(f*x+e))**(1/2)*(a*sin(f*x+e))**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{9/2}\,dx = \int \sqrt{b\sec(fx+e)}(a\sin(fx+e))^{\frac{9}{2}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)*(a*sin(f*x+e))^(9/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e))*(a*sin(f*x + e))^(9/2), x)
```

Giac [F]

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{9/2}\,dx = \int \sqrt{b\sec\left(fx+e\right)}(a\sin\left(fx+e\right))^{\frac{9}{2}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)*(a*sin(f*x+e))^(9/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e))*(a*sin(f*x + e))^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{9/2}\,dx = \int (a\,\sin\left(e+f\,x\right))^{9/2}\,\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}\,dx$$

input
```
int((a*sin(e + f*x))^(9/2)*(b/cos(e + f*x))^(1/2),x)
```

output
```
int((a*sin(e + f*x))^(9/2)*(b/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{9/2}\,dx = \sqrt{b}\,\sqrt{a}\left(\int \sqrt{\sin\left(fx+e\right)}\,\sqrt{\sec\left(fx+e\right)}\,\sin\left(fx+e\right)^{4}dx\right)a^{4}$$

input
```
int((b*sec(f*x+e))^(1/2)*(a*sin(f*x+e))^(9/2),x)
```

output
```
sqrt(b)*sqrt(a)*int(sqrt(sin(e + f*x))*sqrt(sec(e + f*x))*sin(e + f*x)**4,
x)*a**4
```

3.451 $\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 315

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}\,dx = -\frac{3a^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{4\sqrt{2}\sqrt{b}f} + \frac{3a^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{4\sqrt{2}\sqrt{b}f} - \frac{3a^{5/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}(\sqrt{a}+\sqrt{a}\tan(e+fx))}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{4\sqrt{2}\sqrt{b}f} - \frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}}$$

output
```
-3/8*a^(5/2)*arctan(1-2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/a^(1/2)/(b*cos(
f*x+e))^(1/2))*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)*2^(1/2)/b^(1/2)/f
+3/8*a^(5/2)*arctan(1+2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/a^(1/2)/(b*cos(
f*x+e))^(1/2))*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)*2^(1/2)/b^(1/2)/f
-3/8*a^(5/2)*arctanh(2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/(b*cos(f*x+e))^(
1/2)/(a^(1/2)+a^(1/2)*tan(f*x+e)))*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/
2)*2^(1/2)/b^(1/2)/f-1/2*a*b*(a*sin(f*x+e))^(3/2)/f/(b*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 2.16 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.50

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}\,dx =$$

$$-\frac{a^2\cot(e+fx)\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}\left(4\sin^2(e+fx)-3\sqrt{2}\arctan\left(\frac{-1+\sqrt{\tan^2(e+fx)}}{\sqrt{2}\sqrt[4]{\tan^2(e+fx)}}\right)\sqrt[4]{\text{ta}}\right.}{8f}$$

input
```
Integrate[Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(5/2),x]
```

output
```
-1/8*(a^2*Cot[e + f*x]*Sqrt[b*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]]*(4*Sin[e
+ f*x]^2 - 3*Sqrt[2]*ArcTan[(-1 + Sqrt[Tan[e + f*x]^2])/(Sqrt[2]*(Tan[e +
f*x]^2)^(1/4))]*(Tan[e + f*x]^2)^(1/4) + 3*Sqrt[2]*ArcTanh[(Sqrt[2]*(Tan[e
 + f*x]^2)^(1/4))/(1 + Sqrt[Tan[e + f*x]^2])]*(Tan[e + f*x]^2)^(1/4)))/f
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.10, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 3063, 3042, 3065, 3042, 3054, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx))^{5/2}\sqrt{b\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx))^{5/2}\sqrt{b\sec(e+fx)}dx$$

$$\Big\downarrow \text{3063}$$

$$\frac{3}{4}a^2\int \sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}dx-\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3}{4}a^2\int \sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}dx-\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{ 3065}$$

$$\frac{3}{4}a^2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}dx-\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3}{4}a^2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}dx-\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{ 3054}$$

$$\frac{3a^3b\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{a\tan(e+fx)}{b(\tan^2(e+fx)a^2+a^2)}d\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}}{2f}-\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{ 826}$$

$$\frac{3a^3b\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\int \frac{\tan(e+fx)a+a}{\tan^2(e+fx)a^2+a^2}d\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}}{2b}-\frac{\int \frac{a-a\tan(e+fx)}{\tan^2(e+fx)a^2+a^2}d\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}}{2b}\right)}{2f}-\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{3a^3b\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\int \frac{1}{\frac{\tan(e+fx)a}{b}+\frac{a}{b}-\frac{\sqrt{2}\sqrt{a\sin(e+fx)}\sqrt{a}}{\sqrt{b}\sqrt{b\cos(e+fx)}}}d\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}}{2b}+\frac{\int \frac{1}{\frac{\tan(e+fx)a}{b}+\frac{a}{b}+\frac{\sqrt{2}\sqrt{a\sin(e+fx)}\sqrt{a}}{\sqrt{b}\sqrt{b\cos(e+fx)}}}d\frac{\sqrt{a \ldots}}{\sqrt{b \ldots}}}{2b}}{2b}\right.}{2f}$$

$$\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{3a^3b\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\int \frac{1}{-\frac{a\tan(e+fx)}{b}-1}d\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{\int \frac{1}{-\frac{a\tan(e+fx)}{b}-1}d\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}}{2b}-\frac{\int}{}\right.}{2f}$$

$$\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}}$$

$\Big\downarrow$ 217

$$\frac{3a^3b\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}}{2b}-\frac{\int \frac{a-a\tan(e+fx)}{\tan^2(e+fx)a^2+a^2}d\frac{\sqrt{a\sin(e+}}{\sqrt{b\cos(e+}}}{2b}\right.}{2f}$$

$$\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}}$$

$\Big\downarrow$ 1479

$$\frac{3a^3b\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}}{2b}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{b}\sqrt{a\sin}}{\sqrt{b\cos(}}}{\sqrt{b}\left(\frac{\tan(e+fx)a}{b}+\frac{a}{b}-\frac{\sqrt{2}}{\sqrt{}}\right.}}{2\sqrt{2}}}{}\right.}{2f}$$

$$\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}}$$

$\Big\downarrow$ 25

$$\frac{3a^3b\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}}{2b}-\frac{\int \frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{b}\sqrt{a\sin(e}}{\sqrt{b\cos(e+f}}}{\sqrt{b}\left(\frac{\tan(e+fx)a}{b}+\frac{a}{b}-\frac{\sqrt{2}\sqrt{a\sin}}{\sqrt{b}\sqrt{b}}\right.}}{2\sqrt{2}\sqrt{}}\right.}{2f}$$

$$\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}}$$

$\Big\downarrow$ 27

$$\frac{3a^3b\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}}{2b}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{b}\sqrt{a\sin(e+f}}{\sqrt{b\cos(e+fx)}}}{\frac{\tan(e+fx)a}{b}+\frac{a}{b}-\frac{\sqrt{2}\sqrt{a\sin(e-}}{\sqrt{b}\sqrt{b\cos(}}}}{2\sqrt{2}\sqrt{ab}}\right.}{2f}$$

$$\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{3a^3b\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}}{2b}-\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+a\,\mathrm{ta}\right.}{2\sqrt{2}\sqrt{a}\sqrt{b}}\right.}{2f}$$

$$\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}}$$

input

```
Int[Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(5/2),x]
```

output

```
(3*a^3*b*Sqrt[b*Cos[e + f*x]]*((-(ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e
 + f*x]])/(Sqrt[a]*Sqrt[b*Cos[e + f*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[b])) + Arc
Tan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a]*Sqrt[b*Cos[e + f*x
]])]/(Sqrt[2]*Sqrt[a]*Sqrt[b]))/(2*b) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqr
t[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x]] + a*Tan[e + f*x]]/(Sqrt[2]
*Sqrt[a]*Sqrt[b]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])
/Sqrt[b*Cos[e + f*x]] + a*Tan[e + f*x]]/(2*Sqrt[2]*Sqrt[a]*Sqrt[b]))/(2*b)
)*Sqrt[b*Sec[e + f*x]])/(2*f) - (a*b*(a*Sin[e + f*x])^(3/2))/(2*f*Sqrt[b*S
ec[e + f*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3054
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k*a*(b/f)   Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

rule 3063
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*b*(a*Sin[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
- 1)/(f*(m - n))), x] + Simp[a^2*((m - 1)/(m - n))   Int[(a*Sin[e + f*x])^(
m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

rule 3065
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n   Int[(a*Sin[e
+ f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Int
egerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Maple [A] (warning: unable to verify)

Time = 13.23 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.39

method	result
default	$-\frac{\sqrt{2}\,a^2\left(3\ln\left(-\frac{\cos(fx+e)\cot(fx+e)+2\sqrt{-\frac{2\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,\sin(fx+e)-2\cot(fx+e)-\sin(fx+e)-2\cos(fx+e)+\csc(fx+e)+2}{\cos(fx+e)-1}\right)-3\ln$

input
```
int((b*sec(f*x+e))^(1/2)*(a*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/256/f*2^(1/2)*a^2*(3*ln(-(cos(f*x+e)*cot(f*x+e)+2*(-2*sin(f*x+e)/(cos(f
*x+e)+1)^2*cos(f*x+e))^(1/2)*sin(f*x+e)-2*cot(f*x+e)-sin(f*x+e)-2*cos(f*x+
e)+csc(f*x+e)+2)/(cos(f*x+e)-1))-3*ln((-cos(f*x+e)*cot(f*x+e)+2*(-2*sin(f*
x+e)/(cos(f*x+e)+1)^2*cos(f*x+e))^(1/2)*sin(f*x+e)+2*cot(f*x+e)+sin(f*x+e)
+2*cos(f*x+e)-csc(f*x+e)-2)/(cos(f*x+e)-1))-6*arctan(((-2*sin(f*x+e)/(cos(
f*x+e)+1)^2*cos(f*x+e))^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/(cos(f*x+e)-1))-6*a
rctan(((-2*sin(f*x+e)/(cos(f*x+e)+1)^2*cos(f*x+e))^(1/2)*sin(f*x+e)+cos(f*
x+e)-1)/(cos(f*x+e)-1))+(4*cos(f*x+e)+4)*sin(f*x+e)*(-2*sin(f*x+e)/(cos(f*
x+e)+1)^2*cos(f*x+e))^(1/2))*(a*sin(f*x+e))^(1/2)*cos(f*x+e)*sin(f*x+e)^3*
(b*sec(f*x+e))^(1/2)/(-sin(f*x+e)/(cos(f*x+e)+1)^2*cos(f*x+e))^(1/2)*sec(1
/2*f*x+1/2*e)^5*csc(1/2*f*x+1/2*e)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 522 vs. $2(241) = 482$.

Time = 0.14 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.66

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}\,dx =$$

$$-\frac{16\sqrt{a\sin(fx+e)}a^2\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)+6\sqrt{2}\sqrt{ab}a^2\arctan\left(-\frac{\sqrt{2}\sqrt{ab}\sqrt{a\sin(fx+e)}\sqrt{\frac{b}{\cos(fx}}}{ab\cos(fx+e)-ab\sin(}\right.}{}$$

input
```
integrate((b*sec(f*x+e))^(1/2)*(a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
-1/32*(16*sqrt(a*sin(f*x + e))*a^2*sqrt(b/cos(f*x + e))*cos(f*x + e)*sin(f
*x + e) + 6*sqrt(2)*sqrt(a*b)*a^2*arctan(-sqrt(2)*sqrt(a*b)*sqrt(a*sin(f*x
 + e))*sqrt(b/cos(f*x + e))*cos(f*x + e)/(a*b*cos(f*x + e) - a*b*sin(f*x +
 e))) + 3*sqrt(2)*sqrt(a*b)*a^2*arctan(1/2*(2*a*b*cos(f*x + e)^2 - 2*a*b*c
os(f*x + e)*sin(f*x + e) - 2*a*b + sqrt(2)*sqrt(a*b)*sqrt(a*sin(f*x + e))*
sqrt(b/cos(f*x + e)))/(a*b*cos(f*x + e)^2 + a*b*cos(f*x + e)*sin(f*x + e)
- a*b)) + 3*sqrt(2)*sqrt(a*b)*a^2*arctan(-1/2*(2*a*b*cos(f*x + e)^2 - 2*a*
b*cos(f*x + e)*sin(f*x + e) - 2*a*b - sqrt(2)*sqrt(a*b)*sqrt(a*sin(f*x + e
))*sqrt(b/cos(f*x + e)))/(a*b*cos(f*x + e)^2 + a*b*cos(f*x + e)*sin(f*x +
e) - a*b)) + 3*sqrt(2)*sqrt(a*b)*a^2*log(4*a*b*cos(f*x + e)*sin(f*x + e) +
 2*sqrt(2)*sqrt(a*b)*(cos(f*x + e)^2 + cos(f*x + e)*sin(f*x + e))*sqrt(a*s
in(f*x + e))*sqrt(b/cos(f*x + e)) + a*b) - 3*sqrt(2)*sqrt(a*b)*a^2*log(4*a
*b*cos(f*x + e)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*b)*(cos(f*x + e)^2 + cos(f
*x + e)*sin(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e)) + a*b))/f
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((b*sec(f*x+e))**(1/2)*(a*sin(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}\,dx = \int \sqrt{b\sec(fx+e)}(a\sin(fx+e))^{\frac{5}{2}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)*(a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e))*(a*sin(f*x + e))^(5/2), x)
```

Giac [F]

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}\,dx = \int \sqrt{b\sec(fx+e)}(a\sin(fx+e))^{\frac{5}{2}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)*(a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e))*(a*sin(f*x + e))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}\,dx = \int (a\,\sin(e+f\,x))^{5/2}\,\sqrt{\frac{b}{\cos(e+f\,x)}}\,dx$$

input
```
int((a*sin(e + f*x))^(5/2)*(b/cos(e + f*x))^(1/2),x)
```

output
```
int((a*sin(e + f*x))^(5/2)*(b/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}\,dx = \sqrt{b}\,\sqrt{a}\left(\int \sqrt{\sin(fx+e)}\,\sqrt{\sec(fx+e)}\,\sin(fx+e)^2\,dx\right)a^2$$

input
```
int((b*sec(f*x+e))^(1/2)*(a*sin(f*x+e))^(5/2),x)
```

output
```
sqrt(b)*sqrt(a)*int(sqrt(sin(e + f*x))*sqrt(sec(e + f*x))*sin(e + f*x)**2,
x)*a**2
```

3.452 $\int \sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 275

$$\begin{aligned}\int \sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}\,dx &= -\frac{\sqrt{a}\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{\sqrt{2}\sqrt{b}f}\\ &\quad+\frac{\sqrt{a}\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{\sqrt{2}\sqrt{b}f}\\ &\quad-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}(\sqrt{a}+\sqrt{a}\tan(e+fx))}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{\sqrt{2}\sqrt{b}f}\end{aligned}$$

output
```
-1/2*a^(1/2)*arctan(1-2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/a^(1/2)/(b*cos(
f*x+e))^(1/2))*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)*2^(1/2)/b^(1/2)/f
+1/2*a^(1/2)*arctan(1+2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/a^(1/2)/(b*cos(
f*x+e))^(1/2))*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)*2^(1/2)/b^(1/2)/f
-1/2*a^(1/2)*arctanh(2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/(b*cos(f*x+e))^(
1/2)/(a^(1/2)+a^(1/2)*tan(f*x+e)))*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/
2)*2^(1/2)/b^(1/2)/f
```

Mathematica [A] (verified)

Time = 1.52 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.44

$$\int \sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}\,dx$$
$$= \frac{\left(\arctan\left(\frac{-1+\sqrt{\tan^2(e+fx)}}{\sqrt{2}\sqrt[4]{\tan^2(e+fx)}}\right)-\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{\tan^2(e+fx)}}{1+\sqrt{\tan^2(e+fx)}}\right)\right)\cot(e+fx)\sqrt{b\sec(e+fx)}\sqrt{a\sin(e \ldots}}{\sqrt{2}f}$$

input
```
Integrate[Sqrt[b*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]],x]
```

output
```
((ArcTan[(-1 + Sqrt[Tan[e + f*x]^2])/(Sqrt[2]*(Tan[e + f*x]^2)^(1/4))] - A
rcTanh[(Sqrt[2]*(Tan[e + f*x]^2)^(1/4))/(1 + Sqrt[Tan[e + f*x]^2])])*Cot[e
 + f*x]*Sqrt[b*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]]*(Tan[e + f*x]^2)^(1/4))/
(Sqrt[2]*f)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.12, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 3065, 3042, 3054, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}dx$$
$$\Big\downarrow \text{3065}$$
$$\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}dx$$

$\downarrow$ 3042

$$\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}dx$$

$\downarrow$ 3054

$$\frac{2ab\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{a\tan(e+fx)}{b(\tan^2(e+fx)a^2+a^2)}d\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}}{f}$$

$\downarrow$ 826

$$\frac{2ab\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\int \frac{\tan(e+fx)a+a}{\tan^2(e+fx)a^2+a^2}d\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}}{2b}-\frac{\int \frac{a-a\tan(e+fx)}{\tan^2(e+fx)a^2+a^2}d\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}}{2b}\right)}{f}$$

$\downarrow$ 1476

$$\frac{2ab\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\int \frac{1}{\frac{\tan(e+fx)a}{b}+\frac{a}{b}-\frac{\sqrt{2}\sqrt{a\sin(e+fx)}\sqrt{a}}{\sqrt{b}\sqrt{b\cos(e+fx)}}}d\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}}{2b}+\frac{\int \frac{1}{\frac{\tan(e+fx)a}{b}+\frac{a}{b}+\frac{\sqrt{2}\sqrt{a\sin(e+fx)}\sqrt{a}}{\sqrt{b}\sqrt{b\cos(e+fx)}}}d\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}}{2b}}{2b}\right.}{f}$$

$\downarrow$ 1082

$$\frac{2ab\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\int \frac{1}{-\frac{a\tan(e+fx)}{b}-1}d\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{\int \frac{1}{-\frac{a\tan(e+fx)}{b}-1}d\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}}{2b}-\int \frac{\ }{\mathrm{ta}}\right.}{f}$$

$\downarrow$ 217

$$\frac{2ab\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}}{2b}-\frac{\int \frac{a-a\tan(e+fx)}{\tan^2(e+fx)a^2+a^2}d\frac{\sqrt{a\sin(e+f}}{\sqrt{b\cos(e+f}}}{2b}\right.}{f}$$

$\downarrow$ 1479

$$\frac{2ab\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}}{2b}-\frac{-\int -\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{b}\sqrt{a\sin}\cdots}{\sqrt{b\cos(e}\cdots}}{\sqrt{b}\left(\frac{\tan(e+fx)a}{b}+\frac{a}{b}-\frac{\sqrt{2}\sqrt{}\cdots}{\sqrt{b}\cdots}\right.}\cdots}{2\sqrt{2}\cdots}\cdots\right.}{f}$$

$\Big\downarrow$ 25

$$\frac{2ab\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}}{2b}-\frac{\int \frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{b}\sqrt{a\sin(e+}\cdots}{\sqrt{b\cos(e+fx}\cdots}}{\sqrt{b}\left(\frac{\tan(e+fx)a}{b}+\frac{a}{b}-\frac{\sqrt{2}\sqrt{a\sin}\cdots}{\sqrt{b}\sqrt{b\cos}\cdots}\right.}\cdots}{2\sqrt{2}\sqrt{a}\cdots}\cdots\right.}{f}$$

$\Big\downarrow$ 27

$$\frac{2ab\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}}{2b}-\frac{\int \frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\cdots}{\frac{\tan(e+fx)a}{b}+\frac{a}{b}-\frac{\sqrt{2}\sqrt{a\sin(e+}\cdots}{\sqrt{b}\sqrt{b\cos(e}\cdots}\cdots}\cdots}{2\sqrt{2}\sqrt{ab}\cdots}\cdots\right.}{f}$$

$\Big\downarrow$ 1103

$$\frac{2ab\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}}{2b}-\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+a\tan\cdots\right.}{2\sqrt{2}\sqrt{a}\sqrt{b}}\cdots\right.}{f}$$

input
```
Int[Sqrt[b*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]],x]
```

output
```
(2*a*b*Sqrt[b*Cos[e + f*x]]*((-(ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e +
 f*x]])/(Sqrt[a]*Sqrt[b*Cos[e + f*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[b])) + ArcTa
n[1 + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a]*Sqrt[b*Cos[e + f*x]]
)]/(Sqrt[2]*Sqrt[a]*Sqrt[b]))/(2*b) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[
b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x]] + a*Tan[e + f*x]]/(Sqrt[2]*S
qrt[a]*Sqrt[b]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/S
qrt[b*Cos[e + f*x]] + a*Tan[e + f*x]]/(2*Sqrt[2]*Sqrt[a]*Sqrt[b]))/(2*b))*
Sqrt[b*Sec[e + f*x]])/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3054
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k*a*(b/f)   Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

rule 3065
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n   Int[(a*Sin[e
+ f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Int
egerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Maple [A] (verified)

Time = 3.52 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.32

method	result
default	$\frac{\sqrt{2}\,\sqrt{a\sin(fx+e)}\,\sqrt{b\sec(fx+e)}\left((1-\cos(fx+e))^{2}\csc(fx+e)^{2}-1\right)\left(\ln\left(\frac{(1-\cos(fx+e))^{2}\csc(fx+e)+2\sqrt{-\frac{2\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^{2}}}\,\sin(f}{1-\cos(fx+e)}\right.\right.}{}$

input
```
int((b*sec(f*x+e))^(1/2)*(a*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/f*2^(1/2)*(a*sin(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)*((1-cos(f*x+e))^2*
csc(f*x+e)^2-1)*(ln(1/(1-cos(f*x+e))*((1-cos(f*x+e))^2*csc(f*x+e)+2*(-2*si
n(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+2-2*cos(f*x+e)-sin(
f*x+e)))+2*arctan((-(-2*sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(
f*x+e)+cos(f*x+e)-1)/(-1+cos(f*x+e)))-ln(-1/(1-cos(f*x+e))*(-(1-cos(f*x+e)
)^2*csc(f*x+e)+2*(-2*sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x
+e)-2+2*cos(f*x+e)+sin(f*x+e)))-2*arctan(((-2*sin(f*x+e)*cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/(-1+cos(f*x+e))))/(-sin(f*x+e)*
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(214) = 428$.

Time = 0.16 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.69

$$\int \sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}\,dx =$$

$$-\frac{2\sqrt{2}\sqrt{ab}\arctan\left(-\frac{\sqrt{2}\sqrt{ab}\sqrt{a\sin(fx+e)}\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)}{ab\cos(fx+e)-ab\sin(fx+e)}\right)+\sqrt{2}\sqrt{ab}\arctan\left(\frac{2\,ab\cos(fx+e)^{2}-2\,ab\cos(fx+e)\sin(}{2\left(ab\cos(fx+e)^{2}+\right.}\right.}{}$$

input
```
integrate((b*sec(f*x+e))^(1/2)*(a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-1/8*(2*sqrt(2)*sqrt(a*b)*arctan(-sqrt(2)*sqrt(a*b)*sqrt(a*sin(f*x + e))*s
qrt(b/cos(f*x + e))*cos(f*x + e)/(a*b*cos(f*x + e) - a*b*sin(f*x + e))) +
sqrt(2)*sqrt(a*b)*arctan(1/2*(2*a*b*cos(f*x + e)^2 - 2*a*b*cos(f*x + e)*si
n(f*x + e) - 2*a*b + sqrt(2)*sqrt(a*b)*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x
 + e)))/(a*b*cos(f*x + e)^2 + a*b*cos(f*x + e)*sin(f*x + e) - a*b)) + sqrt
(2)*sqrt(a*b)*arctan(-1/2*(2*a*b*cos(f*x + e)^2 - 2*a*b*cos(f*x + e)*sin(f
*x + e) - 2*a*b - sqrt(2)*sqrt(a*b)*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x +
e)))/(a*b*cos(f*x + e)^2 + a*b*cos(f*x + e)*sin(f*x + e) - a*b)) + sqrt(2)
*sqrt(a*b)*log(4*a*b*cos(f*x + e)*sin(f*x + e) + 2*sqrt(2)*sqrt(a*b)*(cos(
f*x + e)^2 + cos(f*x + e)*sin(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*
x + e)) + a*b) - sqrt(2)*sqrt(a*b)*log(4*a*b*cos(f*x + e)*sin(f*x + e) - 2
*sqrt(2)*sqrt(a*b)*(cos(f*x + e)^2 + cos(f*x + e)*sin(f*x + e))*sqrt(a*sin
(f*x + e))*sqrt(b/cos(f*x + e)) + a*b))/f
```

Sympy [F]

$$\int \sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}\,dx = \int \sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}\,dx$$

input
```
integrate((b*sec(f*x+e))**(1/2)*(a*sin(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(a*sin(e + f*x))*sqrt(b*sec(e + f*x)), x)
```

Maxima [F]

$$\int \sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}\,dx = \int \sqrt{b\sec(fx+e)}\sqrt{a\sin(fx+e)}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)*(a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e))*sqrt(a*sin(f*x + e)), x)
```

Giac [F]

$$\int \sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}\,dx = \int \sqrt{b\sec\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)*(a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e))*sqrt(a*sin(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}\,dx = \int \sqrt{a\,\sin\left(e+f\,x\right)}\,\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}\,dx$$

input
```
int((a*sin(e + f*x))^(1/2)*(b/cos(e + f*x))^(1/2),x)
```

output
```
int((a*sin(e + f*x))^(1/2)*(b/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}\,dx = \sqrt{b}\,\sqrt{a}\left(\int \sqrt{\sin\left(fx+e\right)}\,\sqrt{\sec\left(fx+e\right)}dx\right)$$

input
```
int((b*sec(f*x+e))^(1/2)*(a*sin(f*x+e))^(1/2),x)
```

output
```
sqrt(b)*sqrt(a)*int(sqrt(sin(e + f*x))*sqrt(sec(e + f*x)),x)
```

3.453 $\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 33

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx = -\frac{2b}{af\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}}$$

output
```
-2*b/a/f/(b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx = -\frac{\sqrt{b\sec(e+fx)}\sin(2(e+fx))}{f(a\sin(e+fx))^{3/2}}$$

input
```
Integrate[Sqrt[b*Sec[e + f*x]]/(a*Sin[e + f*x])^(3/2),x]
```

output
```
-((Sqrt[b*Sec[e + f*x]]*Sin[2*(e + f*x)])/(f*(a*Sin[e + f*x])^(3/2)))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3042, 3058}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3058}$$

$$-\frac{2b}{af\sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}}$$

input
```
Int[Sqrt[b*Sec[e + f*x]]/(a*Sin[e + f*x])^(3/2),x]
```

output
```
(-2*b)/(a*f*Sqrt[b*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3058
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1
)/(a*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m - n + 2, 0] &
& NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

method	result	size
default	$-\frac{2\sqrt{b\sec(fx+e)}\,\cos(fx+e)}{fa\sqrt{a\sin(fx+e)}}$	35

input
```
int((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/f*(b*sec(f*x+e))^(1/2)*cos(f*x+e)/a/(a*sin(f*x+e))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx = -\frac{2\sqrt{a\sin(fx+e)}\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)}{a^2f\sin(fx+e)}$$

input
```
integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
-2*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e))*cos(f*x + e)/(a^2*f*sin(f*x +
 e))
```

Sympy [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*sec(f*x+e))**(1/2)/(a*sin(f*x+e))**(3/2),x)
```

output
```
Integral(sqrt(b*sec(e + f*x))/(a*sin(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{b\sec(fx+e)}}{(a\sin(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e))/(a*sin(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{b\sec(fx+e)}}{(a\sin(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e))/(a*sin(f*x + e))^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 26.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx = -\frac{2\,\cos(e+f\,x)\,\sqrt{\frac{b}{\cos(e+f\,x)}}}{a\,f\,\sqrt{a\,\sin(e+f\,x)}}$$

input
```
int((b/cos(e + f*x))^(1/2)/(a*sin(e + f*x))^(3/2),x)
```

output
```
-(2*cos(e + f*x)*(b/cos(e + f*x))^(1/2))/(a*f*(a*sin(e + f*x))^(1/2))
```

Reduce [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\left(\int \frac{\sqrt{\sin(fx+e)}\,\sqrt{\sec(fx+e)}}{\sin(fx+e)^2}dx\right)}{a^2}$$

input
```
int((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(3/2),x)
```

output
```
(sqrt(b)*sqrt(a)*int((sqrt(sin(e + f*x))*sqrt(sec(e + f*x)))/sin(e + f*x)*
*2,x))/a**2
```

3.454 $\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 71

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{7/2}}\,dx = -\frac{2b}{5af\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}} - \frac{8b}{5a^3f\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}}$$

output
```
-2/5*b/a/f/(b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(5/2)-8/5*b/a^3/f/(b*sec(f*
x+e))^(1/2)/(a*sin(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{7/2}}\,dx = \frac{2(-3+2\cos(2(e+fx)))\cot(e+fx)\sqrt{b\sec(e+fx)}}{5a^2f(a\sin(e+fx))^{3/2}}$$

input
```
Integrate[Sqrt[b*Sec[e + f*x]]/(a*Sin[e + f*x])^(7/2),x]
```

output
```
(2*(-3 + 2*Cos[2*(e + f*x)])*Cot[e + f*x]*Sqrt[b*Sec[e + f*x]])/(5*a^2*f*(
a*Sin[e + f*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3064, 3042, 3058}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{3064}$$

$$\frac{4\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{3/2}}dx}{5a^2}-\frac{2b}{5af(a\sin(e+fx))^{5/2}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{3/2}}dx}{5a^2}-\frac{2b}{5af(a\sin(e+fx))^{5/2}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3058}$$

$$-\frac{8b}{5a^3f\sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{2b}{5af(a\sin(e+fx))^{5/2}\sqrt{b\sec(e+fx)}}$$

input
```
Int[Sqrt[b*Sec[e + f*x]]/(a*Sin[e + f*x])^(7/2),x]
```

output
```
(-2*b)/(5*a*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(5/2)) - (8*b)/(5*a^3*
f*Sqrt[b*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3058
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1
)/(a*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m - n + 2, 0] &
& NeQ[m, -1]
```

rule 3064
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)/
(a*f*(m + 1))), x] + Simp[(m - n + 2)/(a^2*(m + 1))   Int[(a*Sin[e + f*x])^
(m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{2\left(4\cos\left(fx+e\right)^{2}-5\right)\sqrt{b\sec\left(fx+e\right)}\,\cot\left(fx+e\right)\csc\left(fx+e\right)}{5f\sqrt{a\sin\left(fx+e\right)}\,a^{3}}$	53

input
```
int((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/f*(4*cos(f*x+e)^2-5)*(b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2)/a^3*cot
(f*x+e)*csc(f*x+e)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{7/2}}\,dx = -\frac{2\left(4\cos\left(fx+e\right)^{3}-5\cos\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)}\sqrt{\frac{b}{\cos(fx+e)}}}{5\left(a^{4}f\cos\left(fx+e\right)^{2}-a^{4}f\right)\sin\left(fx+e\right)}$$

input
```
integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

output
```
-2/5*(4*cos(f*x + e)^3 - 5*cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b/cos(f
*x + e))/((a^4*f*cos(f*x + e)^2 - a^4*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*sec(f*x+e))**(1/2)/(a*sin(f*x+e))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{7/2}}\,dx = \int \frac{\sqrt{b\sec\left(fx+e\right)}}{\left(a\sin\left(fx+e\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(7/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e))/(a*sin(f*x + e))^(7/2), x)
```

Giac [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{7/2}}\,dx = \int \frac{\sqrt{b\sec\left(fx+e\right)}}{\left(a\sin\left(fx+e\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(7/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e))/(a*sin(f*x + e))^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 27.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{7/2}}\,dx = -\frac{4\sqrt{\frac{b}{\cos(e+f\,x)}}\,\left(3\,\cos\left(e+f\,x\right)-4\,\cos\left(3\,e+3\,f\,x\right)+\cos\left(5\,e+5\,f\,x\right)\right)}{5\,a^3\,f\,\sqrt{a\,\sin\left(e+f\,x\right)}\,\left(\cos\left(4\,e+4\,f\,x\right)-4\,\cos\left(2\,e+2\,f\,x\right)+3\right)}$$

input
```
int((b/cos(e + f*x))^(1/2)/(a*sin(e + f*x))^(7/2),x)
```

output
```
-(4*(b/cos(e + f*x))^(1/2)*(3*cos(e + f*x) - 4*cos(3*e + 3*f*x) + cos(5*e
+ 5*f*x)))/(5*a^3*f*(a*sin(e + f*x))^(1/2)*(cos(4*e + 4*f*x) - 4*cos(2*e +
 2*f*x) + 3))
```

Reduce [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{7/2}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\left(\int \frac{\sqrt{\sin(fx+e)}\,\sqrt{\sec(fx+e)}}{\sin(fx+e)^4}dx\right)}{a^4}$$

input
```
int((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(7/2),x)
```

output
```
(sqrt(b)*sqrt(a)*int((sqrt(sin(e + f*x))*sqrt(sec(e + f*x)))/sin(e + f*x)*
*4,x))/a**4
```

3.455 $\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{11/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 106

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{11/2}}\,dx = -\frac{2b}{9af\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{9/2}} - \frac{16b}{45a^3f\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}} - \frac{64b}{45a^5f\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}}$$

output
```
-2/9*b/a/f/(b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(9/2)-16/45*b/a^3/f/(b*sec(
f*x+e))^(1/2)/(a*sin(f*x+e))^(5/2)-64/45*b/a^5/f/(b*sec(f*x+e))^(1/2)/(a*s
in(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{11/2}}\,dx = \frac{2b(-21+20\cos(2(e+fx))-4\cos(4(e+fx)))\csc^5(e+fx)\sqrt{a\sin(e+fx)}}{45a^6f\sqrt{b\sec(e+fx)}}$$

input
```
Integrate[Sqrt[b*Sec[e + f*x]]/(a*Sin[e + f*x])^(11/2),x]
```

output
```
(2*b*(-21 + 20*Cos[2*(e + f*x)] - 4*Cos[4*(e + f*x)])*Csc[e + f*x]^5*Sqrt[
a*Sin[e + f*x]])/(45*a^6*f*Sqrt[b*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3064, 3042, 3064, 3042, 3058}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{11/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{11/2}}dx$$

$$\Big\downarrow \text{3064}$$

$$\frac{8\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{7/2}}dx}{9a^2}-\frac{2b}{9af(a\sin(e+fx))^{9/2}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{7/2}}dx}{9a^2}-\frac{2b}{9af(a\sin(e+fx))^{9/2}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3064}$$

$$\frac{8\left(\frac{4\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{3/2}}dx}{5a^2}-\frac{2b}{5af(a\sin(e+fx))^{5/2}\sqrt{b\sec(e+fx)}}\right)}{9a^2}-\frac{2b}{9af(a\sin(e+fx))^{9/2}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8\left(\frac{4\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{3/2}}dx}{5a^2}-\frac{2b}{5af(a\sin(e+fx))^{5/2}\sqrt{b\sec(e+fx)}}\right)}{9a^2}-\frac{2b}{9af(a\sin(e+fx))^{9/2}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3058}$$

$$\frac{8\left(-\frac{8b}{5a^3f\sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{2b}{5af(a\sin(e+fx))^{5/2}\sqrt{b\sec(e+fx)}}\right)}{9a^2}-\frac{2b}{9af(a\sin(e+fx))^{9/2}\sqrt{b\sec(e+fx)}}$$

input
```
Int[Sqrt[b*Sec[e + f*x]]/(a*Sin[e + f*x])^(11/2),x]
```

output
```
(-2*b)/(9*a*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(9/2)) + (8*((-2*b)/(5
*a*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(5/2)) - (8*b)/(5*a^3*f*Sqrt[b*
Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]])))/(9*a^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3058
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1
)/(a*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m - n + 2, 0] &
& NeQ[m, -1]
```

rule 3064
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)/
(a*f*(m + 1))), x] + Simp[(m - n + 2)/(a^2*(m + 1))   Int[(a*Sin[e + f*x])^
(m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.61

method	result	size
default	$-\frac{2\left(32\cos\left(fx+e\right)^{4}-72\cos\left(fx+e\right)^{2}+45\right)\sqrt{b\sec\left(fx+e\right)}\,\cot\left(fx+e\right)\csc\left(fx+e\right)^{3}}{45f\sqrt{a\sin\left(fx+e\right)}\,a^{5}}$	65

input
```
int((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-2/45/f*(32*cos(f*x+e)^4-72*cos(f*x+e)^2+45)*(b*sec(f*x+e))^(1/2)/(a*sin(f
*x+e))^(1/2)/a^5*cot(f*x+e)*csc(f*x+e)^3
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{11/2}}\,dx = -\frac{2\left(32\cos\left(fx+e\right)^{5}-72\cos\left(fx+e\right)^{3}+45\cos\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)}\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{45\left(a^{6}f\cos\left(fx+e\right)^{4}-2\,a^{6}f\cos\left(fx+e\right)^{2}+a^{6}f\right)\sin\left(fx+e\right)}$$

input
```
integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(11/2),x, algorithm="fricas"
)
```

output
```
-2/45*(32*cos(f*x + e)^5 - 72*cos(f*x + e)^3 + 45*cos(f*x + e))*sqrt(a*sin
(f*x + e))*sqrt(b/cos(f*x + e))/((a^6*f*cos(f*x + e)^4 - 2*a^6*f*cos(f*x +
 e)^2 + a^6*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*sec(f*x+e))**(1/2)/(a*sin(f*x+e))**(11/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{11/2}}\,dx = \int \frac{\sqrt{b\sec(fx+e)}}{(a\sin(fx+e))^{\frac{11}{2}}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(11/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(b*sec(f*x + e))/(a*sin(f*x + e))^(11/2), x)
```

Giac [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{11/2}}\,dx = \int \frac{\sqrt{b\sec(fx+e)}}{(a\sin(fx+e))^{\frac{11}{2}}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(11/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e))/(a*sin(f*x + e))^(11/2), x)
```

Mupad [B] (verification not implemented)

Time = 32.71 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.59

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{11/2}}\,dx =$$
$$-\frac{e^{-e\,5\mathrm{i}-f\,x\,5\mathrm{i}}\,\sqrt{\frac{b}{\frac{e^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{e^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,\left(\frac{352\,\cos(e+f\,x)\,e^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}}{45\,a^5\,f}-\frac{256\,e^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\cos(3\,e+3\,f\,x)}{45\,a^5\,f}+\frac{64\,e^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\cos(5\,e+5\,f\,x)}{45\,a^5\,f}\right)}{16\,\sin\left(e+f\,x\right)^4\,\sqrt{a\,\left(\frac{e^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{e^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}}$$

input
```
int((b/cos(e + f*x))^(1/2)/(a*sin(e + f*x))^(11/2),x)
```

output
```
-(exp(- e*5i - f*x*5i)*(b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))
^(1/2)*((352*cos(e + f*x)*exp(e*5i + f*x*5i))/(45*a^5*f) - (256*exp(e*5i +
 f*x*5i)*cos(3*e + 3*f*x))/(45*a^5*f) + (64*exp(e*5i + f*x*5i)*cos(5*e + 5
*f*x))/(45*a^5*f)))/(16*sin(e + f*x)^4*(a*((exp(- e*1i - f*x*1i)*1i)/2 - (
exp(e*1i + f*x*1i)*1i)/2))^(1/2))
```

Reduce [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{11/2}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\,\left(\int \frac{\sqrt{\sin(fx+e)}\,\sqrt{\sec(fx+e)}}{\sin(fx+e)^6}dx\right)}{a^6}$$

input
```
int((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(11/2),x)
```

output
```
(sqrt(b)*sqrt(a)*int((sqrt(sin(e + f*x))*sqrt(sec(e + f*x)))/sin(e + f*x)*
*6,x))/a**6
```

3.456 $\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 128

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}\,dx = -\frac{5a^3b\sqrt{a\sin(e+fx)}}{6f\sqrt{b\sec(e+fx)}}-\frac{ab(a\sin(e+fx))^{5/2}}{3f\sqrt{b\sec(e+fx)}}+\frac{5a^4\operatorname{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}{12f\sqrt{a\sin(e+fx)}}$$

output
```
-5/6*a^3*b*(a*sin(f*x+e))^(1/2)/f/(b*sec(f*x+e))^(1/2)-1/3*a*b*(a*sin(f*x+
e))^(5/2)/f/(b*sec(f*x+e))^(1/2)+5/12*a^4*InverseJacobiAM(e-1/4*Pi+f*x,2^(
1/2))*(b*sec(f*x+e))^(1/2)*sin(2*f*x+2*e)^(1/2)/f/(a*sin(f*x+e))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 17.78 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.70

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}\,dx = \frac{a^3b\sqrt{a\sin(e+fx)}\left(2(-6+\cos(2(e+fx)))+5\csc^2(e+fx)\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\mathrm{s}\right.\right.}{12f\sqrt{b\sec(e+fx)}}$$

input
```
Integrate[Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(7/2),x]
```

output
```
(a^3*b*Sqrt[a*Sin[e + f*x]]*(2*(-6 + Cos[2*(e + f*x)]) + 5*Csc[e + f*x]^2*
Hypergeometric2F1[1/2, 3/4, 3/2, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(3/4)))
/(12*f*Sqrt[b*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3063, 3042, 3063, 3042, 3065, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx))^{7/2}\sqrt{b\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx))^{7/2}\sqrt{b\sec(e+fx)}dx$$

$$\Big\downarrow \text{3063}$$

$$\frac{5}{6}a^2\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}dx-\frac{ab(a\sin(e+fx))^{5/2}}{3f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}a^2\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}dx-\frac{ab(a\sin(e+fx))^{5/2}}{3f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3063}$$

$$\frac{5}{6}a^2\left(\frac{1}{2}a^2\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}dx-\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}}\right)-\frac{ab(a\sin(e+fx))^{5/2}}{3f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}a^2\left(\frac{1}{2}a^2\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}dx-\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}}\right)-\frac{ab(a\sin(e+fx))^{5/2}}{3f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3065}$$

$$\frac{5}{6}a^2\left(\frac{1}{2}a^2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\int\frac{1}{\sqrt{b\cos(e+fx)}\sqrt{a\sin(e+fx)}}dx-\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}}\right)-\frac{ab(a\sin(e+fx))^{5/2}}{3f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}a^2\left(\frac{1}{2}a^2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\int\frac{1}{\sqrt{b\cos(e+fx)}\sqrt{a\sin(e+fx)}}dx-\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}}\right)-\frac{ab(a\sin(e+fx))^{5/2}}{3f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3053}$$

$$\frac{5}{6}a^2\left(\frac{a^2\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}\int\frac{1}{\sqrt{\sin(2e+2fx)}}dx}{2\sqrt{a\sin(e+fx)}}-\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}}\right)-\frac{ab(a\sin(e+fx))^{5/2}}{3f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{6}a^2\left(\frac{a^2\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}\int\frac{1}{\sqrt{\sin(2e+2fx)}}dx}{2\sqrt{a\sin(e+fx)}}-\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}}\right)-\frac{ab(a\sin(e+fx))^{5/2}}{3f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{5}{6}a^2\left(\frac{a^2\sqrt{\sin(2e+2fx)}\,\text{EllipticF}\left(e+fx-\frac{\pi}{4},2\right)\sqrt{b\sec(e+fx)}}{2f\sqrt{a\sin(e+fx)}}-\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}}\right)-\frac{ab(a\sin(e+fx))^{5/2}}{3f\sqrt{b\sec(e+fx)}}$$

input

```
Int[Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(7/2),x]
```

output
```
-1/3*(a*b*(a*Sin[e + f*x])^(5/2))/(f*Sqrt[b*Sec[e + f*x]]) + (5*a^2*(-((a*
b*Sqrt[a*Sin[e + f*x]])/(f*Sqrt[b*Sec[e + f*x]])) + (a^2*EllipticF[e - Pi/
4 + f*x, 2]*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(2*f*Sqrt[a*Sin[e
 + f*x]])))/6
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3063
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*b*(a*Sin[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
- 1)/(f*(m - n))), x] + Simp[a^2*((m - 1)/(m - n))   Int[(a*Sin[e + f*x])^(
m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

rule 3065
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n   Int[(a*Sin[e
+ f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Int
egerQ[m - 1/2] && IntegerQ[n - 1/2]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.84 (sec) , antiderivative size = 990, normalized size of antiderivative = 7.73

method	result	size
default	Expression too large to display	990

input
```
int((b*sec(f*x+e))^(1/2)*(a*sin(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/384/f*a^3*(sin(f*x+e)*cos(f*x+e)*(16*cos(f*x+e)^2-56)+(-6*cos(f*x+e)-6)*
(csc(f*x+e)-cot(f*x+e)+1)^(1/2)*(-2*csc(f*x+e)+2*cot(f*x+e)+2)^(1/2)*(-csc
(f*x+e)+cot(f*x+e))^(1/2)*EllipticPi((csc(f*x+e)-cot(f*x+e)+1)^(1/2),1/2-1
/2*I,1/2*2^(1/2))+(-6*cos(f*x+e)-6)*(csc(f*x+e)-cot(f*x+e)+1)^(1/2)*(-2*cs
c(f*x+e)+2*cot(f*x+e)+2)^(1/2)*(-csc(f*x+e)+cot(f*x+e))^(1/2)*EllipticPi((
csc(f*x+e)-cot(f*x+e)+1)^(1/2),1/2+1/2*I,1/2*2^(1/2))+(32*cos(f*x+e)+32)*(
csc(f*x+e)-cot(f*x+e)+1)^(1/2)*(-2*csc(f*x+e)+2*cot(f*x+e)+2)^(1/2)*(-csc(
f*x+e)+cot(f*x+e))^(1/2)*EllipticF((csc(f*x+e)-cot(f*x+e)+1)^(1/2),1/2*2^(
1/2))+I*(-6*cos(f*x+e)-6)*(csc(f*x+e)-cot(f*x+e)+1)^(1/2)*(-2*csc(f*x+e)+2
*cot(f*x+e)+2)^(1/2)*(-csc(f*x+e)+cot(f*x+e))^(1/2)*EllipticPi((csc(f*x+e)
-cot(f*x+e)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))+I*(6*cos(f*x+e)+6)*(csc(f*x+e)
-cot(f*x+e)+1)^(1/2)*(-2*csc(f*x+e)+2*cot(f*x+e)+2)^(1/2)*(-csc(f*x+e)+cot
(f*x+e))^(1/2)*EllipticPi((csc(f*x+e)-cot(f*x+e)+1)^(1/2),1/2+1/2*I,1/2*2^
(1/2))+(3*cos(f*x+e)+3)*(-2*sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*
ln(-(cos(f*x+e)*cot(f*x+e)-2*cot(f*x+e)+2*(-2*sin(f*x+e)*cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)*sin(f*x+e)+csc(f*x+e)-2*cos(f*x+e)-sin(f*x+e)+2)/(-1+cos
(f*x+e)))+(-3*cos(f*x+e)-3)*(-2*sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)*ln(-(cos(f*x+e)*cot(f*x+e)-2*cot(f*x+e)-2*(-2*sin(f*x+e)*cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+csc(f*x+e)-2*cos(f*x+e)-sin(f*x+e)+2)/(-1
+cos(f*x+e)))+(-6*cos(f*x+e)-6)*(-2*sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1...
```

Fricas [F]

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}\,dx = \int \sqrt{b\sec\left(fx+e\right)}(a\sin\left(fx+e\right))^{\frac{7}{2}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)*(a*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

output
```
integral(-(a^3*cos(f*x + e)^2 - a^3)*sqrt(b*sec(f*x + e))*sqrt(a*sin(f*x +
 e))*sin(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((b*sec(f*x+e))**(1/2)*(a*sin(f*x+e))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}\,dx = \int \sqrt{b\sec\left(fx+e\right)}(a\sin\left(fx+e\right))^{\frac{7}{2}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)*(a*sin(f*x+e))^(7/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e))*(a*sin(f*x + e))^(7/2), x)
```

Giac [F]

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}\,dx = \int \sqrt{b\sec\left(fx+e\right)}(a\sin\left(fx+e\right))^{\frac{7}{2}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)*(a*sin(f*x+e))^(7/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e))*(a*sin(f*x + e))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}\,dx = \int (a\,\sin\left(e+f\,x\right))^{7/2}\,\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}\,dx$$

input
```
int((a*sin(e + f*x))^(7/2)*(b/cos(e + f*x))^(1/2),x)
```

output
```
int((a*sin(e + f*x))^(7/2)*(b/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}\,dx = \sqrt{b}\,\sqrt{a}\left(\int \sqrt{\sin\left(fx+e\right)}\,\sqrt{\sec\left(fx+e\right)}\,\sin\left(fx+e\right)^{3}dx\right)a^{3}$$

input
```
int((b*sec(f*x+e))^(1/2)*(a*sin(f*x+e))^(7/2),x)
```

output
```
sqrt(b)*sqrt(a)*int(sqrt(sin(e + f*x))*sqrt(sec(e + f*x))*sin(e + f*x)**3,
x)*a**3
```

3.457 $\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 91

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}\,dx = -\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}} + \frac{a^2\,\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}{2f\sqrt{a\sin(e+fx)}}$$

output
```
-a*b*(a*sin(f*x+e))^(1/2)/f/(b*sec(f*x+e))^(1/2)+1/2*a^2*InverseJacobiAM(e
-1/4*Pi+f*x,2^(1/2))*(b*sec(f*x+e))^(1/2)*sin(2*f*x+2*e)^(1/2)/f/(a*sin(f*
x+e))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 11.57 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.73

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}\,dx = \frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{4},\frac{1}{2},\sec^2(e+fx)\right)(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{5/2}}{abf\left(-\tan^2(e+fx)\right)^{5/4}}$$

input
```
Integrate[Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(3/2),x]
```

output
```
(Hypergeometric2F1[-1/2, -1/4, 1/2, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(3/2)
*(a*Sin[e + f*x])^(5/2))/(a*b*f*(-Tan[e + f*x]^2)^(5/4))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3063, 3042, 3065, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}dx$$

$$\Big\downarrow \text{3063}$$

$$\frac{1}{2}a^2\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}dx-\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}a^2\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}dx-\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3065}$$

$$\frac{1}{2}a^2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{b\cos(e+fx)}\sqrt{a\sin(e+fx)}}dx-\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}a^2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{b\cos(e+fx)}\sqrt{a\sin(e+fx)}}dx-\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3053}$$

$$\frac{a^2\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}\int\frac{1}{\sqrt{\sin(2e+2fx)}}dx}{2\sqrt{a\sin(e+fx)}}-\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}\int\frac{1}{\sqrt{\sin(2e+2fx)}}dx}{2\sqrt{a\sin(e+fx)}}-\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{a^2\sqrt{\sin(2e+2fx)}\,\text{EllipticF}\left(e+fx-\frac{\pi}{4},2\right)\sqrt{b\sec(e+fx)}}{2f\sqrt{a\sin(e+fx)}}-\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}}$$

input
```
Int[Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(3/2),x]
```

output
```
-((a*b*Sqrt[a*Sin[e + f*x]])/(f*Sqrt[b*Sec[e + f*x]])) + (a^2*EllipticF[e
- Pi/4 + f*x, 2]*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(2*f*Sqrt[a*
Sin[e + f*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3063
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*b*(a*Sin[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
- 1)/(f*(m - n))), x] + Simp[a^2*((m - 1)/(m - n))   Int[(a*Sin[e + f*x])^(
m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

rule 3065
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n   Int[(a*Sin[e
+ f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Int
egerQ[m - 1/2] && IntegerQ[n - 1/2]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.46

method	result
default	$-\frac{\sqrt{a\sin(fx+e)}\,a\sqrt{b\sec(fx+e)}\left(\sqrt{-2\csc(fx+e)+2\cot(fx+e)+2}\,\sqrt{-\csc(fx+e)+\cot(fx+e)}\,\operatorname{EllipticF}\left(\sqrt{\csc(fx+e)-\cot(fx+\right.\right.}{2f}$

input
```
int((b*sec(f*x+e))^(1/2)*(a*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/f*(a*sin(f*x+e))^(1/2)*a*(b*sec(f*x+e))^(1/2)*((-2*csc(f*x+e)+2*cot(f
*x+e)+2)^(1/2)*(-csc(f*x+e)+cot(f*x+e))^(1/2)*EllipticF((csc(f*x+e)-cot(f*
x+e)+1)^(1/2),1/2*2^(1/2))*(csc(f*x+e)-cot(f*x+e)+1)^(1/2)*(-cot(f*x+e)-cs
c(f*x+e))+2*cos(f*x+e))
```

Fricas [F]

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}\,dx = \int \sqrt{b\sec\left(fx+e\right)}(a\sin\left(fx+e\right))^{\frac{3}{2}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)*(a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e))*sqrt(a*sin(f*x + e))*a*sin(f*x + e), x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((b*sec(f*x+e))**(1/2)*(a*sin(f*x+e))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}\,dx = \int \sqrt{b\sec(fx+e)}(a\sin(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)*(a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e))*(a*sin(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}\,dx = \int \sqrt{b\sec(fx+e)}(a\sin(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)*(a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e))*(a*sin(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}\,dx = \int (a\,\sin{(e+f\,x)})^{3/2}\,\sqrt{\frac{b}{\cos{(e+f\,x)}}}\,dx$$

input
```
int((a*sin(e + f*x))^(3/2)*(b/cos(e + f*x))^(1/2),x)
```

output
```
int((a*sin(e + f*x))^(3/2)*(b/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}\,dx = \sqrt{b}\,\sqrt{a}\,\left(\int \sqrt{\sin{(fx+e)}}\,\sqrt{\sec{(fx+e)}}\,\sin{(fx+e)}\,dx\right)a$$

input
```
int((b*sec(f*x+e))^(1/2)*(a*sin(f*x+e))^(3/2),x)
```

output
```
sqrt(b)*sqrt(a)*int(sqrt(sin(e + f*x))*sqrt(sec(e + f*x))*sin(e + f*x),x)*
a
```

3.458 $\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 53

$$\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx = \frac{\operatorname{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}{f\sqrt{a\sin(e+fx)}}$$

output
```
InverseJacobiAM(e-1/4*Pi+f*x,2^(1/2))*(b*sec(f*x+e))^(1/2)*sin(2*f*x+2*e)^
(1/2)/f/(a*sin(f*x+e))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.93 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx = \frac{\cot(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\sec^2(e+fx)\right)\sqrt{b\sec(e+fx)}(-\tan^2(e+fx))^{3/4}}{f\sqrt{a\sin(e+fx)}}$$

input
```
Integrate[Sqrt[b*Sec[e + f*x]]/Sqrt[a*Sin[e + f*x]],x]
```

output
```
(Cot[e + f*x]*Hypergeometric2F1[1/2, 3/4, 3/2, Sec[e + f*x]^2]*Sqrt[b*Sec[
e + f*x]]*(-Tan[e + f*x]^2)^(3/4))/(f*Sqrt[a*Sin[e + f*x]])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3065, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}dx$$
$$\Big\downarrow \text{ 3065}$$
$$\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{b\cos(e+fx)}\sqrt{a\sin(e+fx)}}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{b\cos(e+fx)}\sqrt{a\sin(e+fx)}}dx$$
$$\Big\downarrow \text{ 3053}$$
$$\frac{\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{\sin(2e+2fx)}}dx}{\sqrt{a\sin(e+fx)}}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{\sin(2e+2fx)}}dx}{\sqrt{a\sin(e+fx)}}$$
$$\Big\downarrow \text{ 3120}$$
$$\frac{\sqrt{\sin(2e+2fx)}\,\text{EllipticF}\left(e+fx-\frac{\pi}{4},2\right)\sqrt{b\sec(e+fx)}}{f\sqrt{a\sin(e+fx)}}$$

input
```
Int[Sqrt[b*Sec[e + f*x]]/Sqrt[a*Sin[e + f*x]],x]
```

output
```
(EllipticF[e - Pi/4 + f*x, 2]*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])
/(f*Sqrt[a*Sin[e + f*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3065
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n   Int[(a*Sin[e
+ f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Int
egerQ[m - 1/2] && IntegerQ[n - 1/2]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(47) = 94$.

Time = 0.37 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.11

method	result
default	$\frac{(\cos(fx+e)+1)\sqrt{b\sec(fx+e)}\,\sqrt{\csc(fx+e)-\cot(fx+e)+1}\,\sqrt{-2\csc(fx+e)+2\cot(fx+e)+2}\,\sqrt{-\csc(fx+e)+\cot(fx+e)}\ \text{EllipticF}}{f\sqrt{a\sin(fx+e)}}$

input
```
int((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*(cos(f*x+e)+1)*(b*sec(f*x+e))^(1/2)*(csc(f*x+e)-cot(f*x+e)+1)^(1/2)*(-
2*csc(f*x+e)+2*cot(f*x+e)+2)^(1/2)*(-csc(f*x+e)+cot(f*x+e))^(1/2)*Elliptic
F((csc(f*x+e)-cot(f*x+e)+1)^(1/2),1/2*2^(1/2))/(a*sin(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx = -\frac{\sqrt{i\,ab}F(\arcsin\left(\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)\right)\,|-1)+\sqrt{-i\,ab}F(\arcsin\left(\cos\left(fx+e\right)-i\,\sin\left(fx+e\right)\right)}{af}$$

input
```
integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(I*a*b)*elliptic_f(arcsin(cos(f*x + e) + I*sin(f*x + e)), -1) + sqrt
(-I*a*b)*elliptic_f(arcsin(cos(f*x + e) - I*sin(f*x + e)), -1))/(a*f)
```

Sympy [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx = \int \frac{\sqrt{b\sec\left(e+fx\right)}}{\sqrt{a\sin\left(e+fx\right)}}\,dx$$

input
```
integrate((b*sec(f*x+e))**(1/2)/(a*sin(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(b*sec(e + f*x))/sqrt(a*sin(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx = \int \frac{\sqrt{b\sec\left(fx+e\right)}}{\sqrt{a\sin\left(fx+e\right)}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e))/sqrt(a*sin(f*x + e)), x)
```

Giac [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx = \int \frac{\sqrt{b\sec\left(fx+e\right)}}{\sqrt{a\sin\left(fx+e\right)}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e))/sqrt(a*sin(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx = \int \frac{\sqrt{\frac{b}{\cos(e+f\,x)}}}{\sqrt{a\,\sin\left(e+f\,x\right)}}\,dx$$

input
```
int((b/cos(e + f*x))^(1/2)/(a*sin(e + f*x))^(1/2),x)
```

output
```
int((b/cos(e + f*x))^(1/2)/(a*sin(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\left(\int \frac{\sqrt{\sin(fx+e)}\,\sqrt{\sec(fx+e)}}{\sin(fx+e)}dx\right)}{a}$$

input
```
int((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2),x)
```

output
```
(sqrt(b)*sqrt(a)*int((sqrt(sin(e + f*x))*sqrt(sec(e + f*x)))/sin(e + f*x),
x))/a
```

3.459 $\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 95

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx = -\frac{2b}{3af\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}} + \frac{2\,\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}{3a^2f\sqrt{a\sin(e+fx)}}$$

output
```
-2/3*b/a/f/(b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(3/2)+2/3*InverseJacobiAM(e
-1/4*Pi+f*x,2^(1/2))*(b*sec(f*x+e))^(1/2)*sin(2*f*x+2*e)^(1/2)/a^2/f/(a*si
n(f*x+e))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx = \frac{2\cot(e+fx)\sqrt{b\sec(e+fx)}\left(-1+\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\sec^2(e+fx)\right)\right.}{3a^2f\sqrt{a\sin(e+fx)}}$$

input
```
Integrate[Sqrt[b*Sec[e + f*x]]/(a*Sin[e + f*x])^(5/2),x]
```

output
```
(2*Cot[e + f*x]*Sqrt[b*Sec[e + f*x]]*(-1 + Hypergeometric2F1[1/2, 3/4, 3/2
, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(3/4)))/(3*a^2*f*Sqrt[a*Sin[e + f*x]])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3064, 3042, 3065, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3064}$$

$$\frac{2\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}dx}{3a^2}-\frac{2b}{3af(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}dx}{3a^2}-\frac{2b}{3af(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3065}$$

$$\frac{2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{b\cos(e+fx)}\sqrt{a\sin(e+fx)}}dx}{3a^2}-\frac{2b}{3af(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{b\cos(e+fx)}\sqrt{a\sin(e+fx)}}dx}{3a^2}-\frac{2b}{3af(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3053}$$

$$\frac{2\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}\int\frac{1}{\sqrt{\sin(2e+2fx)}}dx}{3a^2\sqrt{a\sin(e+fx)}}-\frac{2b}{3af(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}\int\frac{1}{\sqrt{\sin(2e+2fx)}}dx}{3a^2\sqrt{a\sin(e+fx)}}-\frac{2b}{3af(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sqrt{\sin(2e+2fx)}\,\text{EllipticF}\left(e+fx-\frac{\pi}{4},2\right)\sqrt{b\sec(e+fx)}}{3a^2f\sqrt{a\sin(e+fx)}}-\frac{2b}{3af(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}$$

input
```
Int[Sqrt[b*Sec[e + f*x]]/(a*Sin[e + f*x])^(5/2),x]
```

output
```
(-2*b)/(3*a*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(3/2)) + (2*EllipticF[
e - Pi/4 + f*x, 2]*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(3*a^2*f*S
qrt[a*Sin[e + f*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3064
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)/
(a*f*(m + 1))), x] + Simp[(m - n + 2)/(a^2*(m + 1))   Int[(a*Sin[e + f*x])^
(m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

rule 3065
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n   Int[(a*Sin[e
+ f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Int
egerQ[m - 1/2] && IntegerQ[n - 1/2]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.33

method	result
default	$\frac{2\sqrt{b\sec(fx+e)}\left((\cos(fx+e)+1)\sqrt{\csc(fx+e)-\cot(fx+e)+1}\,\sqrt{-2\csc(fx+e)+2\cot(fx+e)+2}\,\sqrt{-\csc(fx+e)+\cot(fx+e)}\ \mathrm{Ellipti}\right.}{3f\sqrt{a\sin(fx+e)}\,a^2}$

input
```
int((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/f*(b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2)/a^2*((cos(f*x+e)+1)*(csc(f
*x+e)-cot(f*x+e)+1)^(1/2)*(-2*csc(f*x+e)+2*cot(f*x+e)+2)^(1/2)*(-csc(f*x+e
)+cot(f*x+e))^(1/2)*EllipticF((csc(f*x+e)-cot(f*x+e)+1)^(1/2),1/2*2^(1/2))
-cot(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx = -\frac{2\left(\sqrt{i\,ab}\left(\cos\left(fx+e\right)^2-1\right)F(\arcsin\left(\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)\right)\,|-1)+\sqrt{-i\,ab}\left(\cos\left(fx+e\right)^2-1\right)F\right.}{3\left(a^3f\cos\left(fx+e\right)^2-\right.}$$

input
```
integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(I*a*b)*(cos(f*x + e)^2 - 1)*elliptic_f(arcsin(cos(f*x + e) + I*
sin(f*x + e)), -1) + sqrt(-I*a*b)*(cos(f*x + e)^2 - 1)*elliptic_f(arcsin(c
os(f*x + e) - I*sin(f*x + e)), -1) - sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x +
 e))*cos(f*x + e))/(a^3*f*cos(f*x + e)^2 - a^3*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*sec(f*x+e))**(1/2)/(a*sin(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{b\sec(fx+e)}}{(a\sin(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e))/(a*sin(f*x + e))^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{b\sec(fx+e)}}{(a\sin(fx+e))^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e))/(a*sin(f*x + e))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{\frac{b}{\cos(e+f\,x)}}}{(a\,\sin(e+f\,x))^{5/2}}\,dx$$

input
```
int((b/cos(e + f*x))^(1/2)/(a*sin(e + f*x))^(5/2),x)
```

output
```
int((b/cos(e + f*x))^(1/2)/(a*sin(e + f*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\left(\int \frac{\sqrt{\sin(fx+e)}\,\sqrt{\sec(fx+e)}}{\sin(fx+e)^3}dx\right)}{a^3}$$

input
```
int((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(5/2),x)
```

output
```
(sqrt(b)*sqrt(a)*int((sqrt(sin(e + f*x))*sqrt(sec(e + f*x)))/sin(e + f*x)*
*3,x))/a**3
```

3.460 $\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{9/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 130

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{9/2}}\,dx = -\frac{2b}{7af\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}} - \frac{4b}{7a^3f\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}} + \frac{4\,\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}{7a^4f\sqrt{a\sin(e+fx)}}$$

output
```
-2/7*b/a/f/(b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(7/2)-4/7*b/a^3/f/(b*sec(f*
x+e))^(1/2)/(a*sin(f*x+e))^(3/2)+4/7*InverseJacobiAM(e-1/4*Pi+f*x,2^(1/2))
*(b*sec(f*x+e))^(1/2)*sin(2*f*x+2*e)^(1/2)/a^4/f/(a*sin(f*x+e))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{9/2}}\,dx = -\frac{2\cos(2(e+fx))(b\sec(e+fx))^{3/2}\left((-2+\cos(2(e+fx)))\csc^2(e+fx)+2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2}\right.\right.}{7a^3bf\left(-2+\sec^2(e+fx)\right)(a\sin(e+fx))^{3/2}}$$

input
```
Integrate[Sqrt[b*Sec[e + f*x]]/(a*Sin[e + f*x])^(9/2),x]
```

output
```
(-2*Cos[2*(e + f*x)]*(b*Sec[e + f*x])^(3/2)*((-2 + Cos[2*(e + f*x)])*Csc[e
 + f*x]^2 + 2*Hypergeometric2F1[1/2, 3/4, 3/2, Sec[e + f*x]^2]*(-Tan[e + f
*x]^2)^(3/4)))/(7*a^3*b*f*(-2 + Sec[e + f*x]^2)*(a*Sin[e + f*x])^(3/2))
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3064, 3042, 3064, 3042, 3065, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{9/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{9/2}}dx$$

$$\Big\downarrow \ 3064$$

$$\frac{6\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{5/2}}dx}{7a^2}-\frac{2b}{7af(a\sin(e+fx))^{7/2}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{5/2}}dx}{7a^2}-\frac{2b}{7af(a\sin(e+fx))^{7/2}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3064}$$

$$\frac{6\left(\frac{2\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}dx}{3a^2}-\frac{2b}{3af(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}\right)}{7a^2}-\frac{2b}{7af(a\sin(e+fx))^{7/2}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6\left(\frac{2\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}dx}{3a^2}-\frac{2b}{3af(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}\right)}{7a^2}-\frac{2b}{7af(a\sin(e+fx))^{7/2}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3065}$$

$$\frac{6\left(\frac{2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{b\cos(e+fx)}\sqrt{a\sin(e+fx)}}dx}{3a^2}-\frac{2b}{3af(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}\right)}{7a^2}-\frac{2b}{7af(a\sin(e+fx))^{7/2}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6\left(\frac{2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{b\cos(e+fx)}\sqrt{a\sin(e+fx)}}dx}{3a^2}-\frac{2b}{3af(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}\right)}{7a^2}-\frac{2b}{7af(a\sin(e+fx))^{7/2}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3053}$$

$$\frac{6\left(\frac{2\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{\sin(2e+2fx)}}dx}{3a^2\sqrt{a\sin(e+fx)}}-\frac{2b}{3af(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}\right)}{7a^2}-\frac{2b}{7af(a\sin(e+fx))^{7/2}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6\left(\frac{2\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}\int\frac{1}{\sqrt{\sin(2e+2fx)}}dx}{3a^2\sqrt{a\sin(e+fx)}}-\frac{2b}{3af(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}\right)}{7a^2}-\frac{2b}{7af(a\sin(e+fx))^{7/2}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{6\left(\frac{2\sqrt{\sin(2e+2fx)}\,\text{EllipticF}\left(e+fx-\frac{\pi}{4},2\right)\sqrt{b\sec(e+fx)}}{3a^2f\sqrt{a\sin(e+fx)}}-\frac{2b}{3af(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}\right)}{7a^2}-\frac{2b}{7af(a\sin(e+fx))^{7/2}\sqrt{b\sec(e+fx)}}$$

input

```
Int[Sqrt[b*Sec[e + f*x]]/(a*Sin[e + f*x])^(9/2),x]
```

output

```
(-2*b)/(7*a*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(7/2)) + (6*((-2*b)/(3
*a*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(3/2)) + (2*EllipticF[e - Pi/4
+ f*x, 2]*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(3*a^2*f*Sqrt[a*Sin
[e + f*x]])))/(7*a^2)
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3064

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)/
(a*f*(m + 1))), x] + Simp[(m - n + 2)/(a^2*(m + 1))   Int[(a*Sin[e + f*x])^
(m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

rule 3065
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n   Int[(a*Sin[e
+ f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Int
egerQ[m - 1/2] && IntegerQ[n - 1/2]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.12

method	result
default	$-\frac{2\sqrt{b\sec(fx+e)}\left((-2\cos(fx+e)-2)\sqrt{\csc(fx+e)-\cot(fx+e)+1}\,\sqrt{-2\csc(fx+e)+2\cot(fx+e)+2}\,\sqrt{-\csc(fx+e)+\cot(fx+e)}\ \mathrm{I}\right.}{7f\sqrt{a\sin(fx+e)}\,a^4}$

input
```
int((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/7/f*(b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2)/a^4*((-2*cos(f*x+e)-2)*(c
sc(f*x+e)-cot(f*x+e)+1)^(1/2)*(-2*csc(f*x+e)+2*cot(f*x+e)+2)^(1/2)*(-csc(f
*x+e)+cot(f*x+e))^(1/2)*EllipticF((csc(f*x+e)-cot(f*x+e)+1)^(1/2),1/2*2^(1
/2))-2*cot(f*x+e)^3+3*cot(f*x+e)*csc(f*x+e)^2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{9/2}}\,dx =$$
$$-\frac{2\left(2\left(\cos\left(fx+e\right)^4-2\cos\left(fx+e\right)^2+1\right)\sqrt{i\,ab}F(\arcsin\left(\cos\left(fx+e\right)+i\sin\left(fx+e\right)\right)|-1)+2\left(\cos\left(f\right.\right.}{7}$$

input
```
integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(9/2),x, algorithm="fricas")
```

output
```
-2/7*(2*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(I*a*b)*elliptic_f(arc
sin(cos(f*x + e) + I*sin(f*x + e)), -1) + 2*(cos(f*x + e)^4 - 2*cos(f*x +
e)^2 + 1)*sqrt(-I*a*b)*elliptic_f(arcsin(cos(f*x + e) - I*sin(f*x + e)), -
1) - (2*cos(f*x + e)^3 - 3*cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b/cos(f
*x + e)))/(a^5*f*cos(f*x + e)^4 - 2*a^5*f*cos(f*x + e)^2 + a^5*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*sec(f*x+e))**(1/2)/(a*sin(f*x+e))**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{9/2}}\,dx = \int \frac{\sqrt{b\sec\left(fx+e\right)}}{\left(a\sin\left(fx+e\right)\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(9/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(f*x + e))/(a*sin(f*x + e))^(9/2), x)
```

Giac [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{9/2}}\,dx = \int \frac{\sqrt{b\sec(fx+e)}}{(a\sin(fx+e))^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(9/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(f*x + e))/(a*sin(f*x + e))^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{9/2}}\,dx = \int \frac{\sqrt{\frac{b}{\cos(e+f\,x)}}}{(a\,\sin(e+f\,x))^{9/2}}\,dx$$

input
```
int((b/cos(e + f*x))^(1/2)/(a*sin(e + f*x))^(9/2),x)
```

output
```
int((b/cos(e + f*x))^(1/2)/(a*sin(e + f*x))^(9/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{9/2}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\left(\int \frac{\sqrt{\sin(fx+e)}\,\sqrt{\sec(fx+e)}}{\sin(fx+e)^5}dx\right)}{a^5}$$

input
```
int((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(9/2),x)
```

output
```
(sqrt(b)*sqrt(a)*int((sqrt(sin(e + f*x))*sqrt(sec(e + f*x)))/sin(e + f*x)*
*5,x))/a**5
```

3.461 $\int \frac{\sin^{\frac{9}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 115

$$\int \frac{\sin^{\frac{9}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{7b\sin^{\frac{3}{2}}(e+fx)}{30f(b\sec(e+fx))^{3/2}} - \frac{b\sin^{\frac{7}{2}}(e+fx)}{5f(b\sec(e+fx))^{3/2}} + \frac{7E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{\sin(e+fx)}}{20f\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}$$

output
```
-7/30*b*sin(f*x+e)^(3/2)/f/(b*sec(f*x+e))^(3/2)-1/5*b*sin(f*x+e)^(7/2)/f/(
b*sec(f*x+e))^(3/2)-7/20*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*sin(f*x+e)^(
1/2)/f/(b*sec(f*x+e))^(1/2)/sin(2*f*x+2*e)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 11.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.75

$$\int \frac{\sin^{\frac{9}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{b\left(23-26\cos(2(e+fx))+3\cos(4(e+fx))+42\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{1}{2},\sec^2(e+fx)\right)\sqrt[4]{-\text{t}\right.}{120f(b\sec(e+fx))^{3/2}\sqrt{\sin(e+fx)}}$$

input
```
Integrate[Sin[e + f*x]^(9/2)/Sqrt[b*Sec[e + f*x]],x]
```

output
```
-1/120*(b*(23 - 26*Cos[2*(e + f*x)] + 3*Cos[4*(e + f*x)] + 42*Hypergeometr
ic2F1[-1/2, 1/4, 1/2, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(1/4)))/(f*(b*Sec[
e + f*x])^(3/2)*Sqrt[Sin[e + f*x]])
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 3063, 3042, 3063, 3042, 3065, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^{\frac{9}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)^{9/2}}{\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3063}$$

$$\frac{7}{10}\int \frac{\sin^{\frac{5}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}dx-\frac{b\sin^{\frac{7}{2}}(e+fx)}{5f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{10}\int \frac{\sin(e+fx)^{5/2}}{\sqrt{b\sec(e+fx)}}dx-\frac{b\sin^{\frac{7}{2}}(e+fx)}{5f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3063}$$

$$\frac{7}{10}\left(\frac{1}{2}\int \frac{\sqrt{\sin(e+fx)}}{\sqrt{b\sec(e+fx)}}dx-\frac{b\sin^{\frac{3}{2}}(e+fx)}{3f(b\sec(e+fx))^{3/2}}\right)-\frac{b\sin^{\frac{7}{2}}(e+fx)}{5f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{10}\left(\frac{1}{2}\int \frac{\sqrt{\sin(e+fx)}}{\sqrt{b\sec(e+fx)}}dx-\frac{b\sin^{\frac{3}{2}}(e+fx)}{3f(b\sec(e+fx))^{3/2}}\right)-\frac{b\sin^{\frac{7}{2}}(e+fx)}{5f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3065}$$

$$\frac{7}{10}\left(\frac{\int \sqrt{b\cos(e+fx)}\sqrt{\sin(e+fx)}dx}{2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{b\sin^{\frac{3}{2}}(e+fx)}{3f(b\sec(e+fx))^{3/2}}\right)-\frac{b\sin^{\frac{7}{2}}(e+fx)}{5f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{10}\left(\frac{\int \sqrt{b\cos(e+fx)}\sqrt{\sin(e+fx)}dx}{2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{b\sin^{\frac{3}{2}}(e+fx)}{3f(b\sec(e+fx))^{3/2}}\right)-\frac{b\sin^{\frac{7}{2}}(e+fx)}{5f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3052}$$

$$\frac{7}{10}\left(\frac{\sqrt{\sin(e+fx)}\int \sqrt{\sin(2e+2fx)}dx}{2\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}}-\frac{b\sin^{\frac{3}{2}}(e+fx)}{3f(b\sec(e+fx))^{3/2}}\right)-\frac{b\sin^{\frac{7}{2}}(e+fx)}{5f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{10}\left(\frac{\sqrt{\sin(e+fx)}\int \sqrt{\sin(2e+2fx)}dx}{2\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}}-\frac{b\sin^{\frac{3}{2}}(e+fx)}{3f(b\sec(e+fx))^{3/2}}\right)-\frac{b\sin^{\frac{7}{2}}(e+fx)}{5f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{7}{10}\left(\frac{\sqrt{\sin(e+fx)}E\left(e+fx-\frac{\pi}{4}\middle|2\right)}{2f\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}}-\frac{b\sin^{\frac{3}{2}}(e+fx)}{3f(b\sec(e+fx))^{3/2}}\right)-\frac{b\sin^{\frac{7}{2}}(e+fx)}{5f(b\sec(e+fx))^{3/2}}$$

input
```
Int[Sin[e + f*x]^(9/2)/Sqrt[b*Sec[e + f*x]],x]
```

output
```
-1/5*(b*Sin[e + f*x]^(7/2))/(f*(b*Sec[e + f*x])^(3/2)) + (7*(-1/3*(b*Sin[e
 + f*x]^(3/2))/(f*(b*Sec[e + f*x])^(3/2)) + (EllipticE[e - Pi/4 + f*x, 2]*
Sqrt[Sin[e + f*x]])/(2*f*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])))/10
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3063
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*b*(a*Sin[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
- 1)/(f*(m - n))), x] + Simp[a^2*((m - 1)/(m - n))   Int[(a*Sin[e + f*x])^(
m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

rule 3065
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n   Int[(a*Sin[e
+ f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Int
egerQ[m - 1/2] && IntegerQ[n - 1/2]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(96) = 192$.

Time = 0.56 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.03

method	result
default	$-\frac{\sqrt{\csc(fx+e)-\cot(fx+e)+1}\,\sqrt{-2\csc(fx+e)+2\cot(fx+e)+2}\,\sqrt{-\csc(fx+e)+\cot(fx+e)}\,\mathrm{EllipticF}\left(\sqrt{\csc(fx+e)-\cot(fx+e)+\right.}{}$

input
```
int(sin(f*x+e)^(9/2)/(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/120/f/(b*sec(f*x+e))^(1/2)/sin(f*x+e)^(1/2)*((csc(f*x+e)-cot(f*x+e)+1)^
(1/2)*(-2*csc(f*x+e)+2*cot(f*x+e)+2)^(1/2)*(-csc(f*x+e)+cot(f*x+e))^(1/2)*
EllipticF((csc(f*x+e)-cot(f*x+e)+1)^(1/2),1/2*2^(1/2))*(-21-21*sec(f*x+e))
+(csc(f*x+e)-cot(f*x+e)+1)^(1/2)*(-2*csc(f*x+e)+2*cot(f*x+e)+2)^(1/2)*(-cs
c(f*x+e)+cot(f*x+e))^(1/2)*EllipticE((csc(f*x+e)-cot(f*x+e)+1)^(1/2),1/2*2
^(1/2))*(42+42*sec(f*x+e))+24*cos(f*x+e)^5-76*cos(f*x+e)^3+94*cos(f*x+e)-4
2)
```

Fricas [F]

$$\int \frac{\sin^{\frac{9}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin\left(fx+e\right)^{\frac{9}{2}}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

input
```
integrate(sin(f*x+e)^(9/2)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(b*sec(f*x + e))*sqrt
(sin(f*x + e))/(b*sec(f*x + e)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^{\frac{9}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**(9/2)/(b*sec(f*x+e))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sin^{\frac{9}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin\left(fx+e\right)^{\frac{9}{2}}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

input
```
integrate(sin(f*x+e)^(9/2)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^(9/2)/sqrt(b*sec(f*x + e)), x)
```

Giac [F]

$$\int \frac{\sin^{\frac{9}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin\left(fx+e\right)^{\frac{9}{2}}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

input
```
integrate(sin(f*x+e)^(9/2)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sin(f*x + e)^(9/2)/sqrt(b*sec(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^{\frac{9}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin\left(e+f\,x\right)^{9/2}}{\sqrt{\frac{b}{\cos(e+f\,x)}}}\,dx$$

input
```
int(sin(e + f*x)^(9/2)/(b/cos(e + f*x))^(1/2),x)
```

output
```
int(sin(e + f*x)^(9/2)/(b/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sin^{\frac{9}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sin(fx+e)}\,\sqrt{\sec(fx+e)}\,\sin(fx+e)^4}{\sec(fx+e)}dx\right)}{b}$$

input
```
int(sin(f*x+e)^(9/2)/(b*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(b)*int((sqrt(sin(e + f*x))*sqrt(sec(e + f*x))*sin(e + f*x)**4)/sec(e
 + f*x),x))/b
```

3.462 $\int \frac{\sin^{\frac{5}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 85

$$\int \frac{\sin^{\frac{5}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx=-\frac{b\sin^{\frac{3}{2}}(e+fx)}{3f(b\sec(e+fx))^{3/2}}+\frac{E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{\sin(e+fx)}}{2f\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}$$

output
```
-1/3*b*sin(f*x+e)^(3/2)/f/(b*sec(f*x+e))^(3/2)-1/2*EllipticE(cos(e+1/4*Pi+
f*x),2^(1/2))*sin(f*x+e)^(1/2)/f/(b*sec(f*x+e))^(1/2)/sin(2*f*x+2*e)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.64 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.87

$$\int \frac{\sin^{\frac{5}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$$
$$=\frac{b\left(-1+\cos(2(e+fx))-3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{1}{2},\sec^2(e+fx)\right)\sqrt[4]{-\tan^2(e+fx)}\right)}{6f(b\sec(e+fx))^{3/2}\sqrt{\sin(e+fx)}}$$

input
```
Integrate[Sin[e + f*x]^(5/2)/Sqrt[b*Sec[e + f*x]],x]
```

output
```
(b*(-1 + Cos[2*(e + f*x)] - 3*Hypergeometric2F1[-1/2, 1/4, 1/2, Sec[e + f*
x]^2]*(-Tan[e + f*x]^2)^(1/4)))/(6*f*(b*Sec[e + f*x])^(3/2)*Sqrt[Sin[e + f
*x]])
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 3063, 3042, 3065, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^{\frac{5}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)^{5/2}}{\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3063}$$

$$\frac{1}{2}\int \frac{\sqrt{\sin(e+fx)}}{\sqrt{b\sec(e+fx)}}dx-\frac{b\sin^{\frac{3}{2}}(e+fx)}{3f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{\sqrt{\sin(e+fx)}}{\sqrt{b\sec(e+fx)}}dx-\frac{b\sin^{\frac{3}{2}}(e+fx)}{3f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3065}$$

$$\frac{\int \sqrt{b\cos(e+fx)}\sqrt{\sin(e+fx)}dx}{2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{b\sin^{\frac{3}{2}}(e+fx)}{3f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{b\cos(e+fx)}\sqrt{\sin(e+fx)}dx}{2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{b\sin^{\frac{3}{2}}(e+fx)}{3f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3052}$$

$$\frac{\sqrt{\sin(e+fx)}\int\sqrt{\sin(2e+2fx)}dx}{2\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}}-\frac{b\sin^{\frac{3}{2}}(e+fx)}{3f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(e+fx)}\int\sqrt{\sin(2e+2fx)}dx}{2\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}}-\frac{b\sin^{\frac{3}{2}}(e+fx)}{3f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\sqrt{\sin(e+fx)}E\left(e+fx-\frac{\pi}{4}\middle|2\right)}{2f\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}}-\frac{b\sin^{\frac{3}{2}}(e+fx)}{3f(b\sec(e+fx))^{3/2}}$$

input

```
Int[Sin[e + f*x]^(5/2)/Sqrt[b*Sec[e + f*x]],x]
```

output

```
-1/3*(b*Sin[e + f*x]^(3/2))/(f*(b*Sec[e + f*x])^(3/2)) + (EllipticE[e - Pi
/4 + f*x, 2]*Sqrt[Sin[e + f*x]])/(2*f*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e +
2*f*x]])
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3063

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*b*(a*Sin[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
- 1)/(f*(m - n))), x] + Simp[a^2*((m - 1)/(m - n))   Int[(a*Sin[e + f*x])^(
m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

rule 3065
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n   Int[(a*Sin[e
+ f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Int
egerQ[m - 1/2] && IntegerQ[n - 1/2]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(72) = 144$.

Time = 0.37 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.64

method	result
default	$\frac{\sqrt{\csc(fx+e)-\cot(fx+e)+1}\,\sqrt{-2\csc(fx+e)+2\cot(fx+e)+2}\,\sqrt{-\csc(fx+e)+\cot(fx+e)}\ \mathrm{EllipticF}\left(\sqrt{\csc(fx+e)-\cot(fx+e)+1},\right.}{}$

input
```
int(sin(f*x+e)^(5/2)/(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12/f/sin(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2)*((csc(f*x+e)-cot(f*x+e)+1)^(1
/2)*(-2*csc(f*x+e)+2*cot(f*x+e)+2)^(1/2)*(-csc(f*x+e)+cot(f*x+e))^(1/2)*El
lipticF((csc(f*x+e)-cot(f*x+e)+1)^(1/2),1/2*2^(1/2))*(3+3*sec(f*x+e))+(csc
(f*x+e)-cot(f*x+e)+1)^(1/2)*(-2*csc(f*x+e)+2*cot(f*x+e)+2)^(1/2)*(-csc(f*x
+e)+cot(f*x+e))^(1/2)*EllipticE((csc(f*x+e)-cot(f*x+e)+1)^(1/2),1/2*2^(1/2
))*(-6-6*sec(f*x+e))+4*cos(f*x+e)^3-10*cos(f*x+e)+6)
```

Fricas [F]

$$\int \frac{\sin^{\frac{5}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin\left(fx+e\right)^{\frac{5}{2}}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

input
```
integrate(sin(f*x+e)^(5/2)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(-(cos(f*x + e)^2 - 1)*sqrt(b*sec(f*x + e))*sqrt(sin(f*x + e))/(b*
sec(f*x + e)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^{\frac{5}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**(5/2)/(b*sec(f*x+e))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sin^{\frac{5}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin(fx+e)^{\frac{5}{2}}}{\sqrt{b\sec(fx+e)}}\,dx$$

input
```
integrate(sin(f*x+e)^(5/2)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^(5/2)/sqrt(b*sec(f*x + e)), x)
```

Giac [F]

$$\int \frac{\sin^{\frac{5}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin(fx+e)^{\frac{5}{2}}}{\sqrt{b\sec(fx+e)}}\,dx$$

input
```
integrate(sin(f*x+e)^(5/2)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sin(f*x + e)^(5/2)/sqrt(b*sec(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^{\frac{5}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin\left(e+f\,x\right)^{5/2}}{\sqrt{\frac{b}{\cos(e+f\,x)}}}\,dx$$

input
```
int(sin(e + f*x)^(5/2)/(b/cos(e + f*x))^(1/2),x)
```

output
```
int(sin(e + f*x)^(5/2)/(b/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sin^{\frac{5}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sin(fx+e)}\,\sqrt{\sec(fx+e)}\,\sin(fx+e)^2}{\sec(fx+e)}dx\right)}{b}$$

input
```
int(sin(f*x+e)^(5/2)/(b*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(b)*int((sqrt(sin(e + f*x))*sqrt(sec(e + f*x))*sin(e + f*x)**2)/sec(e
 + f*x),x))/b
```

3.463 $\int \frac{\sqrt{\sin(e+fx)}}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 51

$$\int \frac{\sqrt{\sin(e+fx)}}{\sqrt{b\sec(e+fx)}}\,dx = \frac{E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}$$

output
```
-EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*sin(f*x+e)^(1/2)/f/(b*sec(f*x+e))^(1
/2)/sin(2*f*x+2*e)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.82 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{\sin(e+fx)}}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{b\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{1}{2},\sec^2(e+fx)\right)\sqrt[4]{-\tan^2(e+fx)}}{f(b\sec(e+fx))^{3/2}\sqrt{\sin(e+fx)}}$$

input
```
Integrate[Sqrt[Sin[e + f*x]]/Sqrt[b*Sec[e + f*x]],x]
```

output
```
-((b*Hypergeometric2F1[-1/2, 1/4, 1/2, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(
1/4))/(f*(b*Sec[e + f*x])^(3/2)*Sqrt[Sin[e + f*x]]))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3065, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sin(e+fx)}}{\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sin(e+fx)}}{\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3065}$$

$$\frac{\int \sqrt{b\cos(e+fx)}\sqrt{\sin(e+fx)}dx}{\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{b\cos(e+fx)}\sqrt{\sin(e+fx)}dx}{\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3052}$$

$$\frac{\sqrt{\sin(e+fx)}\int \sqrt{\sin(2e+2fx)}dx}{\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(e+fx)}\int \sqrt{\sin(2e+2fx)}dx}{\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\sqrt{\sin(e+fx)}E\left(e+fx-\frac{\pi}{4}\middle|2\right)}{f\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}}$$

input
```
Int[Sqrt[Sin[e + f*x]]/Sqrt[b*Sec[e + f*x]],x]
```

output
```
(EllipticE[e - Pi/4 + f*x, 2]*Sqrt[Sin[e + f*x]])/(f*Sqrt[b*Sec[e + f*x]]*
Sqrt[Sin[2*e + 2*f*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3065
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n   Int[(a*Sin[e
+ f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Int
egerQ[m - 1/2] && IntegerQ[n - 1/2]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(47) = 94$.

Time = 0.35 (sec) , antiderivative size = 214, normalized size of antiderivative = 4.20

method	result
default	$-\frac{\sqrt{\csc(fx+e)-\cot(fx+e)+1}\,\sqrt{-2\csc(fx+e)+2\cot(fx+e)+2}\,\sqrt{-\csc(fx+e)+\cot(fx+e)}\ \mathrm{EllipticE}\left(\sqrt{\csc(fx+e)-\cot(fx+e)+\right.}{}$

input
```
int(sin(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/f/sin(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2)*((csc(f*x+e)-cot(f*x+e)+1)^(1
/2)*(-2*csc(f*x+e)+2*cot(f*x+e)+2)^(1/2)*(-csc(f*x+e)+cot(f*x+e))^(1/2)*El
lipticE((csc(f*x+e)-cot(f*x+e)+1)^(1/2),1/2*2^(1/2))*(2+2*sec(f*x+e))+(csc
(f*x+e)-cot(f*x+e)+1)^(1/2)*(-2*csc(f*x+e)+2*cot(f*x+e)+2)^(1/2)*(-csc(f*x
+e)+cot(f*x+e))^(1/2)*EllipticF((csc(f*x+e)-cot(f*x+e)+1)^(1/2),1/2*2^(1/2
))*(-1-sec(f*x+e))-2+2*cos(f*x+e))
```

Fricas [F]

$$\int \frac{\sqrt{\sin(e+fx)}}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sqrt{\sin\left(fx+e\right)}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

input
```
integrate(sin(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e))*sqrt(sin(f*x + e))/(b*sec(f*x + e)), x)
```

Sympy [F]

$$\int \frac{\sqrt{\sin(e+fx)}}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sqrt{\sin\left(e+fx\right)}}{\sqrt{b\sec\left(e+fx\right)}}\,dx$$

input
```
integrate(sin(f*x+e)**(1/2)/(b*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(sin(e + f*x))/sqrt(b*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{\sin(e+fx)}}{\sqrt{b\sec(e+fx)}}\,dx=\int \frac{\sqrt{\sin\left(fx+e\right)}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

input
```
integrate(sin(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sin(f*x + e))/sqrt(b*sec(f*x + e)), x)
```

Giac [F]

$$\int \frac{\sqrt{\sin(e+fx)}}{\sqrt{b\sec(e+fx)}}\,dx=\int \frac{\sqrt{\sin\left(fx+e\right)}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

input
```
integrate(sin(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sin(f*x + e))/sqrt(b*sec(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sin(e+fx)}}{\sqrt{b\sec(e+fx)}}\,dx=\int \frac{\sqrt{\sin\left(e+f\,x\right)}}{\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}}\,dx$$

input
```
int(sin(e + f*x)^(1/2)/(b/cos(e + f*x))^(1/2),x)
```

output
```
int(sin(e + f*x)^(1/2)/(b/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{\sin(e+fx)}}{\sqrt{b\sec(e+fx)}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sin(fx+e)}\,\sqrt{\sec(fx+e)}}{\sec(fx+e)}dx\right)}{b}$$

input
```
int(sin(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(b)*int((sqrt(sin(e + f*x))*sqrt(sec(e + f*x)))/sec(e + f*x),x))/b
```

3.464 $\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{3}{2}}(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 81

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{3}{2}}(e+fx)}\,dx = -\frac{2b}{f(b\sec(e+fx))^{3/2}\sqrt{\sin(e+fx)}} - \frac{2E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}$$

output
```
-2*b/f/(b*sec(f*x+e))^(3/2)/sin(f*x+e)^(1/2)+2*EllipticE(cos(e+1/4*Pi+f*x)
,2^(1/2))*sin(f*x+e)^(1/2)/f/(b*sec(f*x+e))^(1/2)/sin(2*f*x+2*e)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.58 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{3}{2}}(e+fx)}\,dx = \frac{2b\left(-1+\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{1}{2},\sec^2(e+fx)\right)\sqrt[4]{-\tan^2(e+fx)}\right)}{f(b\sec(e+fx))^{3/2}\sqrt{\sin(e+fx)}}$$

input
```
Integrate[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(3/2)),x]
```

output
```
(2*b*(-1 + Hypergeometric2F1[-1/2, 1/4, 1/2, Sec[e + f*x]^2]*(-Tan[e + f*x
]^2)^(1/4)))/(f*(b*Sec[e + f*x])^(3/2)*Sqrt[Sin[e + f*x]])
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 3064, 3042, 3065, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin^{\frac{3}{2}}(e+fx)\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)^{3/2}\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3064}$$

$$-2\int \frac{\sqrt{\sin(e+fx)}}{\sqrt{b\sec(e+fx)}}dx-\frac{2b}{f\sqrt{\sin(e+fx)}(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-2\int \frac{\sqrt{\sin(e+fx)}}{\sqrt{b\sec(e+fx)}}dx-\frac{2b}{f\sqrt{\sin(e+fx)}(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3065}$$

$$-\frac{2\int \sqrt{b\cos(e+fx)}\sqrt{\sin(e+fx)}dx}{\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{2b}{f\sqrt{\sin(e+fx)}(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\int \sqrt{b\cos(e+fx)}\sqrt{\sin(e+fx)}dx}{\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{2b}{f\sqrt{\sin(e+fx)}(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3052}$$

$$-\frac{2\sqrt{\sin(e+fx)}\int\sqrt{\sin(2e+2fx)}dx}{\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}}-\frac{2b}{f\sqrt{\sin(e+fx)}(b\sec(e+fx))^{3/2}}$$

$\Big\downarrow$ 3042

$$-\frac{2\sqrt{\sin(e+fx)}\int\sqrt{\sin(2e+2fx)}dx}{\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}}-\frac{2b}{f\sqrt{\sin(e+fx)}(b\sec(e+fx))^{3/2}}$$

$\Big\downarrow$ 3119

$$-\frac{2b}{f\sqrt{\sin(e+fx)}(b\sec(e+fx))^{3/2}}-\frac{2\sqrt{\sin(e+fx)}E\left(e+fx-\frac{\pi}{4}\middle|2\right)}{f\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}}$$

input
```
Int[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(3/2)),x]
```

output
```
(-2*b)/(f*(b*Sec[e + f*x])^(3/2)*Sqrt[Sin[e + f*x]]) - (2*EllipticE[e - Pi
/4 + f*x, 2]*Sqrt[Sin[e + f*x]])/(f*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*
f*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3064
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)/
(a*f*(m + 1))), x] + Simp[(m - n + 2)/(a^2*(m + 1))   Int[(a*Sin[e + f*x])^
(m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

rule 3065
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n   Int[(a*Sin[e
+ f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Int
egerQ[m - 1/2] && IntegerQ[n - 1/2]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(72) = 144$.

Time = 0.24 (sec) , antiderivative size = 361, normalized size of antiderivative = 4.46

method	result
default	$\frac{2\sqrt{\csc(fx+e)-\cot(fx+e)+1}\,\sqrt{-2\csc(fx+e)+2\cot(fx+e)+2}\,\sqrt{-\csc(fx+e)+\cot(fx+e)}\ \text{EllipticE}\left(\sqrt{\csc(fx+e)-\cot(fx+e)+1}\right.}{}$

input
```
int(1/(b*sec(f*x+e))^(1/2)/sin(f*x+e)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/f/(b*sec(f*x+e))^(1/2)/sin(f*x+e)^(1/2)*(2*(csc(f*x+e)-cot(f*x+e)+1)^(1/
2)*(-2*csc(f*x+e)+2*cot(f*x+e)+2)^(1/2)*(-csc(f*x+e)+cot(f*x+e))^(1/2)*Ell
ipticE((csc(f*x+e)-cot(f*x+e)+1)^(1/2),1/2*2^(1/2))-(csc(f*x+e)-cot(f*x+e)
+1)^(1/2)*(-2*csc(f*x+e)+2*cot(f*x+e)+2)^(1/2)*(-csc(f*x+e)+cot(f*x+e))^(1
/2)*EllipticF((csc(f*x+e)-cot(f*x+e)+1)^(1/2),1/2*2^(1/2))+2*(csc(f*x+e)-c
ot(f*x+e)+1)^(1/2)*(-2*csc(f*x+e)+2*cot(f*x+e)+2)^(1/2)*(-csc(f*x+e)+cot(f
*x+e))^(1/2)*EllipticE((csc(f*x+e)-cot(f*x+e)+1)^(1/2),1/2*2^(1/2))*sec(f*
x+e)-(csc(f*x+e)-cot(f*x+e)+1)^(1/2)*(-2*csc(f*x+e)+2*cot(f*x+e)+2)^(1/2)*
(-csc(f*x+e)+cot(f*x+e))^(1/2)*EllipticF((csc(f*x+e)-cot(f*x+e)+1)^(1/2),1
/2*2^(1/2))*sec(f*x+e)-2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{3}{2}}(e+fx)}\,dx =$$
$$-\frac{2\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)^2\sqrt{\sin(fx+e)}+i\sqrt{i\,b}E(\arcsin(\cos(fx+e)+i\,\sin(fx+e))\,|-1)\sin(fx$$

input
```
integrate(1/(b*sec(f*x+e))^(1/2)/sin(f*x+e)^(3/2),x, algorithm="fricas")
```

output
```
-(2*sqrt(b/cos(f*x + e))*cos(f*x + e)^2*sqrt(sin(f*x + e)) + I*sqrt(I*b)*e
lliptic_e(arcsin(cos(f*x + e) + I*sin(f*x + e)), -1)*sin(f*x + e) - I*sqrt
(-I*b)*elliptic_e(arcsin(cos(f*x + e) - I*sin(f*x + e)), -1)*sin(f*x + e)
- I*sqrt(I*b)*elliptic_f(arcsin(cos(f*x + e) + I*sin(f*x + e)), -1)*sin(f*
x + e) + I*sqrt(-I*b)*elliptic_f(arcsin(cos(f*x + e) - I*sin(f*x + e)), -1
)*sin(f*x + e))/(b*f*sin(f*x + e))
```

Sympy [F]

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{3}{2}}(e+fx)}\,dx = \int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{3}{2}}(e+fx)}\,dx$$

input
```
integrate(1/(b*sec(f*x+e))**(1/2)/sin(f*x+e)**(3/2),x)
```

output
```
Integral(1/(sqrt(b*sec(e + f*x))*sin(e + f*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{3}{2}}(e+fx)}\,dx = \int \frac{1}{\sqrt{b\sec\left(fx+e\right)}\sin\left(fx+e\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*sec(f*x+e))^(1/2)/sin(f*x+e)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*sec(f*x + e))*sin(f*x + e)^(3/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{3}{2}}(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*sec(f*x+e))^(1/2)/sin(f*x+e)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{3}{2}}(e+fx)}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)^{3/2}\,\sqrt{\frac{b}{\cos(e+f\,x)}}}\,dx$$

input
```
int(1/(sin(e + f*x)^(3/2)*(b/cos(e + f*x))^(1/2)),x)
```

output
```
int(1/(sin(e + f*x)^(3/2)*(b/cos(e + f*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{3}{2}}(e+fx)}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sin(fx+e)}\,\sqrt{\sec(fx+e)}}{\sec(fx+e)\sin(fx+e)^2}dx\right)}{b}$$

input
```
int(1/(b*sec(f*x+e))^(1/2)/sin(f*x+e)^(3/2),x)
```

output
```
(sqrt(b)*int((sqrt(sin(e + f*x))*sqrt(sec(e + f*x)))/(sec(e + f*x)*sin(e +
 f*x)**2),x))/b
```

3.465 $\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{7}{2}}(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 115

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{7}{2}}(e+fx)}\,dx = -\frac{2b}{5f(b\sec(e+fx))^{3/2}\sin^{\frac{5}{2}}(e+fx)} - \frac{4b}{5f(b\sec(e+fx))^{3/2}\sqrt{\sin(e+fx)}} - \frac{4E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{\sin(e+fx)}}{5f\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}$$

output
```
-2/5*b/f/(b*sec(f*x+e))^(3/2)/sin(f*x+e)^(5/2)-4/5*b/f/(b*sec(f*x+e))^(3/2
)/sin(f*x+e)^(1/2)+4/5*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*sin(f*x+e)^(1/
2)/f/(b*sec(f*x+e))^(1/2)/sin(2*f*x+2*e)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.85 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{7}{2}}(e+fx)}\,dx = \frac{2b\left(-2+\cos(2(e+fx))+2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{1}{2},\sec^2(e+fx)\right)\sin^2(e+fx)\sqrt[4]{-\tan^2(e+fx)}\right)}{5f(b\sec(e+fx))^{3/2}\sin^{\frac{5}{2}}(e+fx)}$$

input
```
Integrate[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(7/2)),x]
```

output
```
(2*b*(-2 + Cos[2*(e + f*x)] + 2*Hypergeometric2F1[-1/2, 1/4, 1/2, Sec[e +
f*x]^2]*Sin[e + f*x]^2*(-Tan[e + f*x]^2)^(1/4)))/(5*f*(b*Sec[e + f*x])^(3/
2)*Sin[e + f*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 3064, 3042, 3064, 3042, 3065, 3042, 3052, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin^{\frac{7}{2}}(e+fx)\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)^{7/2}\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3064}$$

$$\frac{2}{5}\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{3}{2}}(e+fx)}dx-\frac{2b}{5f\sin^{\frac{5}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}\int \frac{1}{\sqrt{b\sec(e+fx)}\sin(e+fx)^{3/2}}dx-\frac{2b}{5f\sin^{\frac{5}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3064}$$

$$\frac{2}{5}\left(-2\int \frac{\sqrt{\sin(e+fx)}}{\sqrt{b\sec(e+fx)}}dx-\frac{2b}{f\sqrt{\sin(e+fx)}(b\sec(e+fx))^{3/2}}\right)-\frac{2b}{5f\sin^{\frac{5}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}\left(-2\int \frac{\sqrt{\sin(e+fx)}}{\sqrt{b\sec(e+fx)}}dx-\frac{2b}{f\sqrt{\sin(e+fx)}(b\sec(e+fx))^{3/2}}\right)-\frac{2b}{5f\sin^{\frac{5}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3065}$$

$$\frac{2}{5}\left(-\frac{2\int \sqrt{b\cos(e+fx)}\sqrt{\sin(e+fx)}dx}{\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{2b}{f\sqrt{\sin(e+fx)}(b\sec(e+fx))^{3/2}}\right)-\frac{2b}{5f\sin^{\frac{5}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}\left(-\frac{2\int \sqrt{b\cos(e+fx)}\sqrt{\sin(e+fx)}dx}{\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{2b}{f\sqrt{\sin(e+fx)}(b\sec(e+fx))^{3/2}}\right)-\frac{2b}{5f\sin^{\frac{5}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3052}$$

$$\frac{2}{5}\left(-\frac{2\sqrt{\sin(e+fx)}\int \sqrt{\sin(2e+2fx)}dx}{\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}}-\frac{2b}{f\sqrt{\sin(e+fx)}(b\sec(e+fx))^{3/2}}\right)-\frac{2b}{5f\sin^{\frac{5}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}\left(-\frac{2\sqrt{\sin(e+fx)}\int \sqrt{\sin(2e+2fx)}dx}{\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}}-\frac{2b}{f\sqrt{\sin(e+fx)}(b\sec(e+fx))^{3/2}}\right)-\frac{2b}{5f\sin^{\frac{5}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2}{5}\left(-\frac{2b}{f\sqrt{\sin(e+fx)}(b\sec(e+fx))^{3/2}}-\frac{2\sqrt{\sin(e+fx)}E\left(e+fx-\frac{\pi}{4}\middle|2\right)}{f\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}}\right)-\frac{2b}{5f\sin^{\frac{5}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

input
```
Int[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(7/2)),x]
```

output
```
(-2*b)/(5*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^(5/2)) + (2*((-2*b)/(f*(b*
Sec[e + f*x])^(3/2)*Sqrt[Sin[e + f*x]]) - (2*EllipticE[e - Pi/4 + f*x, 2]*
Sqrt[Sin[e + f*x]])/(f*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])))/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3052
```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Simp[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]])   Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

rule 3064
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)/
(a*f*(m + 1))), x] + Simp[(m - n + 2)/(a^2*(m + 1))   Int[(a*Sin[e + f*x])^
(m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

rule 3065
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n   Int[(a*Sin[e
+ f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Int
egerQ[m - 1/2] && IntegerQ[n - 1/2]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. 2(96) = 192.

Time = 0.32 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.16

method	result
default	$-\frac{2\left(\sqrt{\csc(fx+e)-\cot(fx+e)+1}\,\sqrt{-2\csc(fx+e)+2\cot(fx+e)+2}\,\sqrt{-\csc(fx+e)+\cot(fx+e)}\ \operatorname{EllipticE}\left(\sqrt{\csc(fx+e)-\cot(fx+e}\right.\right.}{}$

input
```
int(1/(b*sec(f*x+e))^(1/2)/sin(f*x+e)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/f/(b*sec(f*x+e))^(1/2)/sin(f*x+e)^(5/2)*((csc(f*x+e)-cot(f*x+e)+1)^(1
/2)*(-2*csc(f*x+e)+2*cot(f*x+e)+2)^(1/2)*(-csc(f*x+e)+cot(f*x+e))^(1/2)*El
lipticE((csc(f*x+e)-cot(f*x+e)+1)^(1/2),1/2*2^(1/2))*(-2*sin(f*x+e)^2-2*ta
n(f*x+e)*sin(f*x+e))+(csc(f*x+e)-cot(f*x+e)+1)^(1/2)*(-2*csc(f*x+e)+2*cot(
f*x+e)+2)^(1/2)*(-csc(f*x+e)+cot(f*x+e))^(1/2)*EllipticF((csc(f*x+e)-cot(f
*x+e)+1)^(1/2),1/2*2^(1/2))*(sin(f*x+e)^2+tan(f*x+e)*sin(f*x+e))+2*sin(f*x
+e)^2+cos(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.08

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{7}{2}}(e+fx)}\,dx =$$
$$-\frac{2\left(\left(i\cos\left(fx+e\right)^{2}-i\right)\sqrt{i\,b}E(\arcsin\left(\cos\left(fx+e\right)+i\sin\left(fx+e\right)\right)\,|-1)\sin\left(fx+e\right)+\left(-i\cos\left(fx\right.\right.\right.}{}$$

input
```
integrate(1/(b*sec(f*x+e))^(1/2)/sin(f*x+e)^(7/2),x, algorithm="fricas")
```

output
```
-2/5*((I*cos(f*x + e)^2 - I)*sqrt(I*b)*elliptic_e(arcsin(cos(f*x + e) + I*
sin(f*x + e)), -1)*sin(f*x + e) + (-I*cos(f*x + e)^2 + I)*sqrt(-I*b)*ellip
tic_e(arcsin(cos(f*x + e) - I*sin(f*x + e)), -1)*sin(f*x + e) + (-I*cos(f*
x + e)^2 + I)*sqrt(I*b)*elliptic_f(arcsin(cos(f*x + e) + I*sin(f*x + e)),
-1)*sin(f*x + e) + (I*cos(f*x + e)^2 - I)*sqrt(-I*b)*elliptic_f(arcsin(cos
(f*x + e) - I*sin(f*x + e)), -1)*sin(f*x + e) + (2*cos(f*x + e)^4 - 3*cos(
f*x + e)^2)*sqrt(b/cos(f*x + e))*sqrt(sin(f*x + e)))/((b*f*cos(f*x + e)^2
- b*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{7}{2}}(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*sec(f*x+e))**(1/2)/sin(f*x+e)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{7}{2}}(e+fx)}\,dx = \int \frac{1}{\sqrt{b\sec{(fx+e)}}\sin{(fx+e)}^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(b*sec(f*x+e))^(1/2)/sin(f*x+e)^(7/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*sec(f*x + e))*sin(f*x + e)^(7/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{7}{2}}(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*sec(f*x+e))^(1/2)/sin(f*x+e)^(7/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{7}{2}}(e+fx)}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)^{7/2}\,\sqrt{\frac{b}{\cos(e+f\,x)}}}\,dx$$

input
```
int(1/(sin(e + f*x)^(7/2)*(b/cos(e + f*x))^(1/2)),x)
```

output
```
int(1/(sin(e + f*x)^(7/2)*(b/cos(e + f*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{7}{2}}(e+fx)}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sin(fx+e)}\,\sqrt{\sec(fx+e)}}{\sec(fx+e)\sin(fx+e)^4}dx\right)}{b}$$

input
```
int(1/(b*sec(f*x+e))^(1/2)/sin(f*x+e)^(7/2),x)
```

output
```
(sqrt(b)*int((sqrt(sin(e + f*x))*sqrt(sec(e + f*x)))/(sec(e + f*x)*sin(e + 
 f*x)**4),x))/b
```

3.466 $\int \frac{\sin^{\frac{3}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 276

$$\int \frac{\sin^{\frac{3}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{\sqrt{b}\arctan\left(1-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}\right)}{4\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{\sqrt{b}\arctan\left(1+\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}\right)}{4\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} + \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\left(\sqrt{b}+\sqrt{b}\cot(e+fx)\right)\sqrt{\sin(e+fx)}}\right)}{4\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{b\sqrt{\sin(e+fx)}}{2f(b\sec(e+fx))^{3/2}}$$

output
```
1/8*b^(1/2)*arctan(1-2^(1/2)*(b*cos(f*x+e))^(1/2)/b^(1/2)/sin(f*x+e)^(1/2)
)*2^(1/2)/f/(b*cos(f*x+e))^(1/2)/(b*sec(f*x+e))^(1/2)-1/8*b^(1/2)*arctan(1
+2^(1/2)*(b*cos(f*x+e))^(1/2)/b^(1/2)/sin(f*x+e)^(1/2))*2^(1/2)/f/(b*cos(f
*x+e))^(1/2)/(b*sec(f*x+e))^(1/2)+1/8*b^(1/2)*arctanh(2^(1/2)*(b*cos(f*x+e
))^(1/2)/(b^(1/2)+b^(1/2)*cot(f*x+e))/sin(f*x+e)^(1/2))*2^(1/2)/f/(b*cos(f
*x+e))^(1/2)/(b*sec(f*x+e))^(1/2)-1/2*b*sin(f*x+e)^(1/2)/f/(b*sec(f*x+e))^
(3/2)
```

Mathematica [A] (verified)

Time = 2.11 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.53

$$\int \frac{\sin^{\frac{3}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{b\left(-4\sin^2(e+fx)+\sqrt{2}\arctan\left(\frac{-1+\sqrt{\tan^2(e+fx)}}{\sqrt{2}\sqrt[4]{\tan^2(e+fx)}}\right)\tan^2(e+fx)^{3/4}+\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{\tan^2(e+f}}{1+\sqrt{\tan^2(e+fx)}}\right.\right.}{8f(b\sec(e+fx))^{3/2}\sin^{\frac{3}{2}}(e+fx)}$$

input
```
Integrate[Sin[e + f*x]^(3/2)/Sqrt[b*Sec[e + f*x]],x]
```

output
```
(b*(-4*Sin[e + f*x]^2 + Sqrt[2]*ArcTan[(-1 + Sqrt[Tan[e + f*x]^2])/(Sqrt[2
]*(Tan[e + f*x]^2)^(1/4))]*(Tan[e + f*x]^2)^(3/4) + Sqrt[2]*ArcTanh[(Sqrt[
2]*(Tan[e + f*x]^2)^(1/4))/(1 + Sqrt[Tan[e + f*x]^2])]*(Tan[e + f*x]^2)^(3
/4)))/(8*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^(3/2))
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {3042, 3063, 3042, 3065, 3042, 3055, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^{\frac{3}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(e+fx)^{3/2}}{\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3063}$$

$$\frac{1}{4}\int \frac{1}{\sqrt{b\sec(e+fx)}\sqrt{\sin(e+fx)}}dx-\frac{b\sqrt{\sin(e+fx)}}{2f(b\sec(e+fx))^{3/2}}$$

$$\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{1}{\sqrt{b\sec(e+fx)}\sqrt{\sin(e+fx)}}dx-\frac{b\sqrt{\sin(e+fx)}}{2f(b\sec(e+fx))^{3/2}}$$

$$\downarrow \text{3065}$$

$$\frac{\int \frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}dx}{4\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{b\sqrt{\sin(e+fx)}}{2f(b\sec(e+fx))^{3/2}}$$

$$\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}dx}{4\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{b\sqrt{\sin(e+fx)}}{2f(b\sec(e+fx))^{3/2}}$$

$$\downarrow \text{3055}$$

$$-\frac{b\int \frac{b\cot(e+fx)}{\cot^2(e+fx)b^2+b^2}d\frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}}{2f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{b\sqrt{\sin(e+fx)}}{2f(b\sec(e+fx))^{3/2}}$$

$$\downarrow \text{826}$$

$$-\frac{b\left(\frac{1}{2}\int \frac{\cot(e+fx)b+b}{\cot^2(e+fx)b^2+b^2}d\frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}-\frac{1}{2}\int \frac{b-b\cot(e+fx)}{\cot^2(e+fx)b^2+b^2}d\frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{2f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{b\sqrt{\sin(e+fx)}}{2f(b\sec(e+fx))^{3/2}}$$

$$\downarrow \text{1476}$$

$$-\frac{b\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\cot(e+fx)b+b-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}\sqrt{b}}{\sqrt{\sin(e+fx)}}}d\frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}+\frac{1}{2}\int \frac{1}{\cot(e+fx)b+b+\frac{\sqrt{2}\sqrt{b\cos(e+fx)}\sqrt{b}}{\sqrt{\sin(e+fx)}}}d\frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)-\frac{1}{2}\int \frac{b-b}{\cot^2(}\cdots\right.}{2f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$
$$\frac{b\sqrt{\sin(e+fx)}}{2f(b\sec(e+fx))^{3/2}}$$

$$\downarrow \text{1082}$$

$$-\frac{b\left(\frac{1}{2}\left(\frac{\int \frac{1}{-b\cot(e+fx)-1}d\left(1-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}\right)}{\sqrt{2}\sqrt{b}}-\frac{\int \frac{1}{-b\cot(e+fx)-1}d\left(\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}+1\right)}{\sqrt{2}\sqrt{b}}\right)-\frac{1}{2}\int \frac{b-b\cot(e+fx)}{\cot^2(e+fx)b^2+b^2}d\frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\cdots\right.}{2f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$
$$\frac{b\sqrt{\sin(e+fx)}}{2f(b\sec(e+fx))^{3/2}}$$

$$\downarrow \text{217}$$

$$-\frac{b\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}+1\right)}{\sqrt{2}\sqrt{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}\right)}{\sqrt{2}\sqrt{b}}\right)-\frac{1}{2}\int\frac{b-b\cot(e+fx)}{\cot^2(e+fx)b^2+b^2}d\frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{2f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{b\sqrt{\sin(e+fx)}}{2f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \ 1479$$

$$-\frac{b\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}\sqrt{b}-\frac{2\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}}{\cot(e+fx)b+b-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}\sqrt{b}}{\sqrt{\sin(e+fx)}}}d\frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}}{2\sqrt{2}\sqrt{b}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{b}+\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{\cot(e+fx)b+b+\frac{\sqrt{2}\sqrt{b\cos(e+fx)}\sqrt{b}}{\sqrt{\sin(e+fx)}}}d\frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}}{2\sqrt{2}\sqrt{b}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b\cdots}}{\sqrt{b}\sqrt{\cdots}}\right.}{\sqrt{2}\cdots}\right.\cdots}{2f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}\cdots$$

$$\frac{b\sqrt{\sin(e+fx)}}{2f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \ 25$$

$$-\frac{b\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{b}-\frac{2\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}}{\cot(e+fx)b+b-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}\sqrt{b}}{\sqrt{\sin(e+fx)}}}d\frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}}{2\sqrt{2}\sqrt{b}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{b}+\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{\cot(e+fx)b+b+\frac{\sqrt{2}\sqrt{b\cos(e+fx)}\sqrt{b}}{\sqrt{\sin(e+fx)}}}d\frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}}{2\sqrt{2}\sqrt{b}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b\cdots}}{\sqrt{b}\sqrt{\cdots}}\right.}{\sqrt{2}\cdots}\right.\cdots}{2f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}\cdots$$

$$\frac{b\sqrt{\sin(e+fx)}}{2f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{b\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}\sqrt{b}-\frac{2\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}}{\cot(e+fx)b+b-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}\sqrt{b}}{\sqrt{\sin(e+fx)}}}d\frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}}{2\sqrt{2}\sqrt{b}}-\frac{\int\frac{\sqrt{b}+\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}}{\cot(e+fx)b+b+\frac{\sqrt{2}\sqrt{b\cos(e+fx)}\sqrt{b}}{\sqrt{\sin(e+fx)}}}d\frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}}{2\sqrt{b}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b\cdots}}{\sqrt{b}\sqrt{\cdots}}\right.}{\sqrt{2}\cdots}\right.\cdots}{2f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}\cdots$$

$$\frac{b\sqrt{\sin(e+fx)}}{2f(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \ 1103$$

$$-\frac{b\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}+1\right)}{\sqrt{2}\sqrt{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}\right)}{\sqrt{2}\sqrt{b}}\right)+\frac{1}{2}\left(\frac{\log\left(b\cot(e+fx)-\frac{\sqrt{2}\sqrt{b}\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}+b\right)}{2\sqrt{2}\sqrt{b}}-\frac{\log\left(b\cot(e+\right.}{}\right.\right.}{2f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$
$$\frac{b\sqrt{\sin(e+fx)}}{2f(b\sec(e+fx))^{3/2}}$$

input
```
Int[Sin[e + f*x]^(3/2)/Sqrt[b*Sec[e + f*x]],x]
```

output
```
-1/2*(b*((-(ArcTan[1 - (Sqrt[2]*Sqrt[b*Cos[e + f*x]])/(Sqrt[b]*Sqrt[Sin[e
+ f*x]])]/(Sqrt[2]*Sqrt[b])) + ArcTan[1 + (Sqrt[2]*Sqrt[b*Cos[e + f*x]])/(
Sqrt[b]*Sqrt[Sin[e + f*x]])]/(Sqrt[2]*Sqrt[b]))/2 + (Log[b + b*Cot[e + f*x
] - (Sqrt[2]*Sqrt[b]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[e + f*x]]]/(2*Sqrt[2]*
Sqrt[b]) - Log[b + b*Cot[e + f*x] + (Sqrt[2]*Sqrt[b]*Sqrt[b*Cos[e + f*x]])
/Sqrt[Sin[e + f*x]]]/(2*Sqrt[2]*Sqrt[b]))/2))/(f*Sqrt[b*Cos[e + f*x]]*Sqrt
[b*Sec[e + f*x]]) - (b*Sqrt[Sin[e + f*x]])/(2*f*(b*Sec[e + f*x])^(3/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3055
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[(-k)*a*(b/f)   Subst[Int[x
^(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[
e + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m,
0] && LtQ[m, 1]
```

rule 3063
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*b*(a*Sin[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
- 1)/(f*(m - n))), x] + Simp[a^2*((m - 1)/(m - n))   Int[(a*Sin[e + f*x])^(
m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

rule 3065
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n   Int[(a*Sin[e
+ f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Int
egerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(214) = 428$.

Time = 0.44 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.63

method	result
default	$\frac{\sqrt{2}\left(-4\sqrt{-\frac{2\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\cos(fx+e)\sin(fx+e)^2+(2-2\cos(fx+e))\arctan\left(\frac{-\sqrt{-\frac{2\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\sin(fx+e)+\cos(fx+e}{-1+\cos(fx+e)}\right.\right.}{}$

input
```
int(sin(f*x+e)^(3/2)/(b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/128/f*2^(1/2)*(-4*(-2*sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(
f*x+e)*sin(f*x+e)^2+(2-2*cos(f*x+e))*arctan((-(-2*sin(f*x+e)*cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/(-1+cos(f*x+e)))+(-1+cos(f*
x+e))*ln(-(cos(f*x+e)*cot(f*x+e)-2*cot(f*x+e)+2*(-2*sin(f*x+e)*cos(f*x+e)/
(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+csc(f*x+e)-2*cos(f*x+e)-sin(f*x+e)+2)/(
-1+cos(f*x+e)))+(1-cos(f*x+e))*ln(-(cos(f*x+e)*cot(f*x+e)-2*cot(f*x+e)-2*(
-2*sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+csc(f*x+e)-2*c
os(f*x+e)-sin(f*x+e)+2)/(-1+cos(f*x+e)))+(-2+2*cos(f*x+e))*arctan(((-2*sin
(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/(-1+co
s(f*x+e))))*sin(f*x+e)^(3/2)/(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)/(b*sec(f*x+e))^(1/2)*sec(1/2*f*x+1/2*e)^3*csc(1/2*f*x+1/2*e)^3
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.51

$$\int \frac{\sin^{\frac{3}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx =$$

$$-\frac{16\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)^2\sqrt{\sin(fx+e)} - 2\sqrt{2}\sqrt{b}\arctan\left(-\frac{\sqrt{2}\sqrt{\frac{b}{\cos(fx+e)}}(\cos(fx+e)-\sin(fx+e))}{2\sqrt{b}\sqrt{\sin(fx+e)}}\right) + \sqrt{2}}{}$$

input
```
integrate(sin(f*x+e)^(3/2)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-1/32*(16*sqrt(b/cos(f*x + e))*cos(f*x + e)^2*sqrt(sin(f*x + e)) - 2*sqrt(
2)*sqrt(b)*arctan(-1/2*sqrt(2)*sqrt(b/cos(f*x + e))*(cos(f*x + e) - sin(f*
x + e))/(sqrt(b)*sqrt(sin(f*x + e)))) + sqrt(2)*sqrt(b)*arctan(1/2*(2*cos(
f*x + e)^2 - 2*cos(f*x + e)*sin(f*x + e) + sqrt(2)*sqrt(b/cos(f*x + e))*sq
rt(sin(f*x + e))/sqrt(b) - 2)/(cos(f*x + e)^2 + cos(f*x + e)*sin(f*x + e)
- 1)) + sqrt(2)*sqrt(b)*arctan(-1/2*(2*cos(f*x + e)^2 - 2*cos(f*x + e)*sin
(f*x + e) - sqrt(2)*sqrt(b/cos(f*x + e))*sqrt(sin(f*x + e))/sqrt(b) - 2)/(
cos(f*x + e)^2 + cos(f*x + e)*sin(f*x + e) - 1)) - sqrt(2)*sqrt(b)*log(2*s
qrt(2)*(cos(f*x + e)^2 + cos(f*x + e)*sin(f*x + e))*sqrt(b/cos(f*x + e))*s
qrt(sin(f*x + e))/sqrt(b) + 4*cos(f*x + e)*sin(f*x + e) + 1) + sqrt(2)*sqr
t(b)*log(-2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e)*sin(f*x + e))*sqrt(b/co
s(f*x + e))*sqrt(sin(f*x + e))/sqrt(b) + 4*cos(f*x + e)*sin(f*x + e) + 1))
/(b*f)
```

Sympy [F]

$$\int \frac{\sin^{\frac{3}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin^{\frac{3}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$$

input
```
integrate(sin(f*x+e)**(3/2)/(b*sec(f*x+e))**(1/2),x)
```

output
```
Integral(sin(e + f*x)**(3/2)/sqrt(b*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sin^{\frac{3}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin\left(fx+e\right)^{\frac{3}{2}}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

input
```
integrate(sin(f*x+e)^(3/2)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^(3/2)/sqrt(b*sec(f*x + e)), x)
```

Giac [F]

$$\int \frac{\sin^{\frac{3}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin\left(fx+e\right)^{\frac{3}{2}}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

input
```
integrate(sin(f*x+e)^(3/2)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sin(f*x + e)^(3/2)/sqrt(b*sec(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^{\frac{3}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \int \frac{\sin\left(e+f\,x\right)^{3/2}}{\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}}\,dx$$

input
```
int(sin(e + f*x)^(3/2)/(b/cos(e + f*x))^(1/2),x)
```

output
```
int(sin(e + f*x)^(3/2)/(b/cos(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sin^{\frac{3}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sin(fx+e)}\,\sqrt{\sec(fx+e)}\,\sin(fx+e)}{\sec(fx+e)}dx\right)}{b}$$

input
```
int(sin(f*x+e)^(3/2)/(b*sec(f*x+e))^(1/2),x)
```

output
```
(sqrt(b)*int((sqrt(sin(e + f*x))*sqrt(sec(e + f*x))*sin(e + f*x))/sec(e +
f*x),x))/b
```

3.467 $\int \frac{1}{\sqrt{b\sec(e+fx)}\sqrt{\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 238

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sqrt{\sin(e+fx)}}\,dx = \frac{\sqrt{b}\arctan\left(1-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}\right)}{\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{\sqrt{b}\arctan\left(1+\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}\right)}{\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} + \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\left(\sqrt{b}+\sqrt{b}\cot(e+fx)\right)\sqrt{\sin(e+fx)}}\right)}{\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

output
```
1/2*b^(1/2)*arctan(1-2^(1/2)*(b*cos(f*x+e))^(1/2)/b^(1/2)/sin(f*x+e)^(1/2)
)*2^(1/2)/f/(b*cos(f*x+e))^(1/2)/(b*sec(f*x+e))^(1/2)-1/2*b^(1/2)*arctan(1
+2^(1/2)*(b*cos(f*x+e))^(1/2)/b^(1/2)/sin(f*x+e)^(1/2))*2^(1/2)/f/(b*cos(f
*x+e))^(1/2)/(b*sec(f*x+e))^(1/2)+1/2*b^(1/2)*arctanh(2^(1/2)*(b*cos(f*x+e
))^(1/2)/(b^(1/2)+b^(1/2)*cot(f*x+e))/sin(f*x+e)^(1/2))*2^(1/2)/f/(b*cos(f
*x+e))^(1/2)/(b*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.47

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sqrt{\sin(e+fx)}}\,dx = \frac{b\left(\arctan\left(\frac{-1+\sqrt{\tan^2(e+fx)}}{\sqrt{2}\sqrt[4]{\tan^2(e+fx)}}\right)+\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{\tan^2(e+fx)}}{1+\sqrt{\tan^2(e+fx)}}\right)\right)\tan^2(e+fx)^{3/4}}{\sqrt{2}f(b\sec(e+fx))^{3/2}\sin^{\frac{3}{2}}(e+fx)}$$

input

```
Integrate[1/(Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[e + f*x]]),x]
```

output

```
(b*(ArcTan[(-1 + Sqrt[Tan[e + f*x]^2])/(Sqrt[2]*(Tan[e + f*x]^2)^(1/4))] +
 ArcTanh[(Sqrt[2]*(Tan[e + f*x]^2)^(1/4))/(1 + Sqrt[Tan[e + f*x]^2])])*(Ta
n[e + f*x]^2)^(3/4))/(Sqrt[2]*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 3065, 3042, 3055, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sin(e+fx)}\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{\sin(e+fx)}\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{ 3065}$$

$$\frac{\int \frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}dx}{\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}dx}{\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3055}$$

$$-\frac{2b\int \frac{b\cot(e+fx)}{\cot^2(e+fx)b^2+b^2}d\frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}}{f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{826}$$

$$-\frac{2b\left(\frac{1}{2}\int \frac{\cot(e+fx)b+b}{\cot^2(e+fx)b^2+b^2}d\frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}-\frac{1}{2}\int \frac{b-b\cot(e+fx)}{\cot^2(e+fx)b^2+b^2}d\frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{1476}$$

$$-\frac{2b\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\cot(e+fx)b+b-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}\sqrt{b}}{\sqrt{\sin(e+fx)}}}d\frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}+\frac{1}{2}\int \frac{1}{\cot(e+fx)b+b+\frac{\sqrt{2}\sqrt{b\cos(e+fx)}\sqrt{b}}{\sqrt{\sin(e+fx)}}}d\frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)-\frac{1}{2}\int \frac{b-}{\cot^2}\right.}{f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{2b\left(\frac{1}{2}\left(\frac{\int \frac{1}{-b\cot(e+fx)-1}d\left(1-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}\right)}{\sqrt{2}\sqrt{b}}-\frac{\int \frac{1}{-b\cot(e+fx)-1}d\left(\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}+1\right)}{\sqrt{2}\sqrt{b}}\right)-\frac{1}{2}\int \frac{b-b\cot(e+fx)}{\cot^2(e+fx)b^2+b^2}d\frac{\sqrt{b\cos(e+fx}}{\sqrt{\sin(e+fx)}}\right.}{f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{217}$$

$$-\frac{2b\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}+1\right)}{\sqrt{2}\sqrt{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}\right)}{\sqrt{2}\sqrt{b}}\right)-\frac{1}{2}\int \frac{b-b\cot(e+fx)}{\cot^2(e+fx)b^2+b^2}d\frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{1479}$$

$$-\frac{2b\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt{b}-\frac{2\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}}{\cot(e+fx)b+b-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}\sqrt{b}}{\sqrt{\sin(e+fx)}}}d\frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}}{2\sqrt{2}\sqrt{b}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{b}+\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{\cot(e+fx)b+b+\frac{\sqrt{2}\sqrt{b\cos(e+fx)}\sqrt{b}}{\sqrt{\sin(e+fx)}}}d\frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}}{2\sqrt{2}\sqrt{b}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{}}{\sqrt{b}\sqrt{}}\right.}{\sqrt{}}\right.\right.}{f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \; 25$$

$$-\frac{2b\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{b}-\frac{2\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}}{\cot(e+fx)b+b-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}\sqrt{b}}{\sqrt{\sin(e+fx)}}}d\frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}}{2\sqrt{2}\sqrt{b}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{b}+\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{\cot(e+fx)b+b+\frac{\sqrt{2}\sqrt{b\cos(e+fx)}\sqrt{b}}{\sqrt{\sin(e+fx)}}}d\frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}}{2\sqrt{2}\sqrt{b}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b\cdots}}{\sqrt{b}\sqrt{\mathrm{s}\cdots}}\cdots\right.}{\sqrt{2}\cdots}\cdots\right.\right.}{f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \; 27$$

$$-\frac{2b\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt{b}-\frac{2\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}}{\cot(e+fx)b+b-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}\sqrt{b}}{\sqrt{\sin(e+fx)}}}d\frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}}{2\sqrt{2}\sqrt{b}}-\frac{\int \frac{\sqrt{b}+\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}}{\cot(e+fx)b+b+\frac{\sqrt{2}\sqrt{b\cos(e+fx)}\sqrt{b}}{\sqrt{\sin(e+fx)}}}d\frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}}{2\sqrt{b}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b\cdots}}{\sqrt{b}\sqrt{\mathrm{s}\cdots}}\cdots\right.}{\sqrt{2}\cdots}\cdots\right.\right.}{f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \; 1103$$

$$-\frac{2b\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}+1\right)}{\sqrt{2}\sqrt{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}\right)}{\sqrt{2}\sqrt{b}}\right)+\frac{1}{2}\left(\frac{\log\left(b\cot(e+fx)-\frac{\sqrt{2}\sqrt{b}\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}+b\right)}{2\sqrt{2}\sqrt{b}}-\frac{\log\left(b\cot(e+\cdots\right.}{\cdots}\cdots\right.\right.}{f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

input
```
Int[1/(Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[e + f*x]]),x]
```

output
```
(-2*b*((-(ArcTan[1 - (Sqrt[2]*Sqrt[b*Cos[e + f*x]])/(Sqrt[b]*Sqrt[Sin[e +
f*x]])]/(Sqrt[2]*Sqrt[b])) + ArcTan[1 + (Sqrt[2]*Sqrt[b*Cos[e + f*x]])/(Sq
rt[b]*Sqrt[Sin[e + f*x]])]/(Sqrt[2]*Sqrt[b]))/2 + (Log[b + b*Cot[e + f*x]
- (Sqrt[2]*Sqrt[b]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[e + f*x]]]/(2*Sqrt[2]*Sq
rt[b]) - Log[b + b*Cot[e + f*x] + (Sqrt[2]*Sqrt[b]*Sqrt[b*Cos[e + f*x]])/S
qrt[Sin[e + f*x]]]/(2*Sqrt[2]*Sqrt[b]))/2))/(f*Sqrt[b*Cos[e + f*x]]*Sqrt[b
*Sec[e + f*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3055
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[(-k)*a*(b/f)   Subst[Int[x
^(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[
e + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m,
0] && LtQ[m, 1]
```

rule 3065
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n   Int[(a*Sin[e
+ f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Int
egerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Maple [A] (verified)

Time = 2.24 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.46

method	result
default	$-\frac{\sqrt{2}\,(1-\cos(fx+e))\left(\ln\left(\frac{(1-\cos(fx+e))^2\csc(fx+e)+2\sqrt{-\frac{2\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,\sin(fx+e)+2-2\cos(fx+e)-\sin(fx+e)}{1-\cos(fx+e)}\right)-2\arctan\left(\cdots\right.\right.}{\cdots}$

input
```
int(1/(b*sec(f*x+e))^(1/2)/sin(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/f*2^(1/2)/sin(f*x+e)^(3/2)*(1-cos(f*x+e))*(ln(1/(1-cos(f*x+e))*((1-co
s(f*x+e))^2*csc(f*x+e)+2*(-2*sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
*sin(f*x+e)+2-2*cos(f*x+e)-sin(f*x+e)))-2*arctan((-(-2*sin(f*x+e)*cos(f*x+
e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/(-1+cos(f*x+e)))-ln(-1
/(1-cos(f*x+e))*(-(1-cos(f*x+e))^2*csc(f*x+e)+2*(-2*sin(f*x+e)*cos(f*x+e)/
(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)-2+2*cos(f*x+e)+sin(f*x+e)))+2*arctan(((
-2*sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/
(-1+cos(f*x+e))))/(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/(b*sec(f
*x+e))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(190) = 380$.

Time = 0.13 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.62

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sqrt{\sin(e+fx)}}\,dx$$

$$= \frac{\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\sqrt{\frac{b}{\cos(fx+e)}}(\cos(fx+e)-\sin(fx+e))}{2\sqrt{b}\sqrt{\sin(fx+e)}}\right)}{\sqrt{b}} - \frac{\sqrt{2}\arctan\left(\frac{2\cos(fx+e)^2-2\cos(fx+e)\sin(fx+e)+\frac{\sqrt{2}\sqrt{\frac{b}{\cos(fx+e)}}\sqrt{\sin(fx+e)}}{\sqrt{b}}-2}{2\left(\cos(fx+e)^2+\cos(fx+e)\sin(fx+e)-1\right)}\right)}{\sqrt{b}} -}{}$$

input
```
integrate(1/(b*sec(f*x+e))^(1/2)/sin(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
1/8*(2*sqrt(2)*arctan(-1/2*sqrt(2)*sqrt(b/cos(f*x + e))*(cos(f*x + e) - si
n(f*x + e))/(sqrt(b)*sqrt(sin(f*x + e))))/sqrt(b) - sqrt(2)*arctan(1/2*(2*
cos(f*x + e)^2 - 2*cos(f*x + e)*sin(f*x + e) + sqrt(2)*sqrt(b/cos(f*x + e)
)*sqrt(sin(f*x + e))/sqrt(b) - 2)/(cos(f*x + e)^2 + cos(f*x + e)*sin(f*x +
 e) - 1))/sqrt(b) - sqrt(2)*arctan(-1/2*(2*cos(f*x + e)^2 - 2*cos(f*x + e)
*sin(f*x + e) - sqrt(2)*sqrt(b/cos(f*x + e))*sqrt(sin(f*x + e))/sqrt(b) -
2)/(cos(f*x + e)^2 + cos(f*x + e)*sin(f*x + e) - 1))/sqrt(b) + sqrt(2)*log
(2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e)*sin(f*x + e))*sqrt(b/cos(f*x + e
))*sqrt(sin(f*x + e))/sqrt(b) + 4*cos(f*x + e)*sin(f*x + e) + 1)/sqrt(b) -
 sqrt(2)*log(-2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e)*sin(f*x + e))*sqrt(
b/cos(f*x + e))*sqrt(sin(f*x + e))/sqrt(b) + 4*cos(f*x + e)*sin(f*x + e) +
 1)/sqrt(b))/f
```

Sympy [F]

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sqrt{\sin(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\sec{(e+fx)}}\sqrt{\sin{(e+fx)}}}\,dx$$

input
```
integrate(1/(b*sec(f*x+e))**(1/2)/sin(f*x+e)**(1/2),x)
```

output
```
Integral(1/(sqrt(b*sec(e + f*x))*sqrt(sin(e + f*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sqrt{\sin(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\sec{(fx+e)}}\sqrt{\sin{(fx+e)}}}\,dx$$

input
```
integrate(1/(b*sec(f*x+e))^(1/2)/sin(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*sec(f*x + e))*sqrt(sin(f*x + e))), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sqrt{\sin(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\sec{(fx+e)}}\sqrt{\sin{(fx+e)}}}\,dx$$

input
```
integrate(1/(b*sec(f*x+e))^(1/2)/sin(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*sec(f*x + e))*sqrt(sin(f*x + e))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sqrt{\sin(e+fx)}}\,dx = \int \frac{1}{\sqrt{\sin(e+f\,x)}\,\sqrt{\frac{b}{\cos(e+f\,x)}}}\,dx$$

input
```
int(1/(sin(e + f*x)^(1/2)*(b/cos(e + f*x))^(1/2)),x)
```

output
```
int(1/(sin(e + f*x)^(1/2)*(b/cos(e + f*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sqrt{\sin(e+fx)}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sin(fx+e)}\,\sqrt{\sec(fx+e)}}{\sec(fx+e)\sin(fx+e)}dx\right)}{b}$$

input
```
int(1/(b*sec(f*x+e))^(1/2)/sin(f*x+e)^(1/2),x)
```

output
```
(sqrt(b)*int((sqrt(sin(e + f*x))*sqrt(sec(e + f*x)))/(sec(e + f*x)*sin(e +
 f*x)),x))/b
```

3.468 $\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{5}{2}}(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 30

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{5}{2}}(e+fx)}\,dx = -\frac{2b}{3f(b\sec(e+fx))^{3/2}\sin^{\frac{3}{2}}(e+fx)}$$

output
```
-2/3*b/f/(b*sec(f*x+e))^(3/2)/sin(f*x+e)^(3/2)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{5}{2}}(e+fx)}\,dx = -\frac{2b}{3f(b\sec(e+fx))^{3/2}\sin^{\frac{3}{2}}(e+fx)}$$

input
```
Integrate[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(5/2)),x]
```

output
```
(-2*b)/(3*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3042, 3058}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin^{\frac{5}{2}}(e+fx)\sqrt{b\sec(e+fx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin(e+fx)^{5/2}\sqrt{b\sec(e+fx)}}dx$$
$$\Big\downarrow \text{3058}$$
$$-\frac{2b}{3f\sin^{\frac{3}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

input
```
Int[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(5/2)),x]
```

output
```
(-2*b)/(3*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^(3/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3058
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1
)/(a*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m - n + 2, 0] &
& NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{2\cos(fx+e)}{3f\sin(fx+e)^{\frac{3}{2}}\sqrt{b\sec(fx+e)}}$	30

input
```
int(1/(b*sec(f*x+e))^(1/2)/sin(f*x+e)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/f*cos(f*x+e)/sin(f*x+e)^(3/2)/(b*sec(f*x+e))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.60

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{5}{2}}(e+fx)}\,dx = \frac{2\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)^2\sqrt{\sin(fx+e)}}{3\left(bf\cos(fx+e)^2-bf\right)}$$

input
```
integrate(1/(b*sec(f*x+e))^(1/2)/sin(f*x+e)^(5/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(b/cos(f*x + e))*cos(f*x + e)^2*sqrt(sin(f*x + e))/(b*f*cos(f*x +
e)^2 - b*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{5}{2}}(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*sec(f*x+e))**(1/2)/sin(f*x+e)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{5}{2}}(e+fx)}\,dx = \int \frac{1}{\sqrt{b\sec(fx+e)}\sin(fx+e)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*sec(f*x+e))^(1/2)/sin(f*x+e)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*sec(f*x + e))*sin(f*x + e)^(5/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{5}{2}}(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*sec(f*x+e))^(1/2)/sin(f*x+e)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 26.15 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.90

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{5}{2}}(e+fx)}\,dx = \frac{\sqrt{\frac{b}{\cos(e+f\,x)}}\,(\sin(e+f\,x)+\sin(3\,e+3\,f\,x))}{3\,b\,f\,\sqrt{\sin(e+f\,x)}\,(\cos(2\,e+2\,f\,x)-1)}$$

input
```
int(1/(sin(e + f*x)^(5/2)*(b/cos(e + f*x))^(1/2)),x)
```

output
```
((b/cos(e + f*x))^(1/2)*(sin(e + f*x) + sin(3*e + 3*f*x)))/(3*b*f*sin(e +
f*x)^(1/2)*(cos(2*e + 2*f*x) - 1))
```

Reduce [F]

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{5}{2}}(e+fx)}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sin(fx+e)}\,\sqrt{\sec(fx+e)}}{\sec(fx+e)\sin(fx+e)^{3}}dx\right)}{b}$$

input
```
int(1/(b*sec(f*x+e))^(1/2)/sin(f*x+e)^(5/2),x)
```

output
```
(sqrt(b)*int((sqrt(sin(e + f*x))*sqrt(sec(e + f*x)))/(sec(e + f*x)*sin(e +
 f*x)**3),x))/b
```

3.469 $\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{9}{2}}(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 61

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{9}{2}}(e+fx)}\,dx = -\frac{2b}{7f(b\sec(e+fx))^{3/2}\sin^{\frac{7}{2}}(e+fx)} - \frac{8b}{21f(b\sec(e+fx))^{3/2}\sin^{\frac{3}{2}}(e+fx)}$$

output
```
-2/7*b/f/(b*sec(f*x+e))^(3/2)/sin(f*x+e)^(7/2)-8/21*b/f/(b*sec(f*x+e))^(3/
2)/sin(f*x+e)^(3/2)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{9}{2}}(e+fx)}\,dx = \frac{2b(-5+2\cos(2(e+fx)))}{21f(b\sec(e+fx))^{3/2}\sin^{\frac{7}{2}}(e+fx)}$$

input
```
Integrate[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(9/2)),x]
```

output
```
(2*b*(-5 + 2*Cos[2*(e + f*x)]))/(21*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^
(7/2))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3064, 3042, 3058}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin^{\frac{9}{2}}(e+fx)\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)^{9/2}\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3064}$$

$$\frac{4}{7}\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{5}{2}}(e+fx)}dx-\frac{2b}{7f\sin^{\frac{7}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{7}\int \frac{1}{\sqrt{b\sec(e+fx)}\sin(e+fx)^{5/2}}dx-\frac{2b}{7f\sin^{\frac{7}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3058}$$

$$-\frac{8b}{21f\sin^{\frac{3}{2}}(e+fx)(b\sec(e+fx))^{3/2}}-\frac{2b}{7f\sin^{\frac{7}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

input
```
Int[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(9/2)),x]
```

output
```
(-2*b)/(7*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^(7/2)) - (8*b)/(21*f*(b*Se
c[e + f*x])^(3/2)*Sin[e + f*x]^(3/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3058
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1
)/(a*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m - n + 2, 0] &
& NeQ[m, -1]
```

rule 3064
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)/
(a*f*(m + 1))), x] + Simp[(m - n + 2)/(a^2*(m + 1))   Int[(a*Sin[e + f*x])^
(m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{\frac{8\cos(fx+e)^3}{21}-\frac{2\cos(fx+e)}{3}}{f\sin(fx+e)^{\frac{7}{2}}\sqrt{b\sec(fx+e)}}$	43

input
```
int(1/(b*sec(f*x+e))^(1/2)/sin(f*x+e)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/21/f/sin(f*x+e)^(7/2)/(b*sec(f*x+e))^(1/2)*(4*cos(f*x+e)^3-7*cos(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.18

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{9}{2}}(e+fx)}\,dx = \frac{2\left(4\cos\left(fx+e\right)^{4}-7\cos\left(fx+e\right)^{2}\right)\sqrt{\frac{b}{\cos(fx+e)}}\sqrt{\sin\left(fx+e\right)}}{21\left(bf\cos\left(fx+e\right)^{4}-2\,bf\cos\left(fx+e\right)^{2}+bf\right)}$$

input
```
integrate(1/(b*sec(f*x+e))^(1/2)/sin(f*x+e)^(9/2),x, algorithm="fricas")
```

output
```
2/21*(4*cos(f*x + e)^4 - 7*cos(f*x + e)^2)*sqrt(b/cos(f*x + e))*sqrt(sin(f
*x + e))/(b*f*cos(f*x + e)^4 - 2*b*f*cos(f*x + e)^2 + b*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{9}{2}}(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*sec(f*x+e))**(1/2)/sin(f*x+e)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{9}{2}}(e+fx)}\,dx = \int \frac{1}{\sqrt{b\sec\left(fx+e\right)}\sin\left(fx+e\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/(b*sec(f*x+e))^(1/2)/sin(f*x+e)^(9/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*sec(f*x + e))*sin(f*x + e)^(9/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{9}{2}}(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*sec(f*x+e))^(1/2)/sin(f*x+e)^(9/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 27.94 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.69

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{9}{2}}(e+fx)}\,dx = \frac{4\sqrt{\frac{b}{\cos(e+f\,x)}}\,(11\,\sin(e+f\,x)+4\,\sin(3\,e+3\,f\,x)-6\,\sin(5\,e+5\,f\,x)+\sin(7\,e+7\,f\,x))}{21\,b\,f\,\sqrt{\sin(e+f\,x)}\,(15\,\cos(2\,e+2\,f\,x)-6\,\cos(4\,e+4\,f\,x)+\cos(6\,e+6\,f\,x)-10)}$$

input
```
int(1/(sin(e + f*x)^(9/2)*(b/cos(e + f*x))^(1/2)),x)
```

output
```
(4*(b/cos(e + f*x))^(1/2)*(11*sin(e + f*x) + 4*sin(3*e + 3*f*x) - 6*sin(5*
e + 5*f*x) + sin(7*e + 7*f*x)))/(21*b*f*sin(e + f*x)^(1/2)*(15*cos(2*e + 2
*f*x) - 6*cos(4*e + 4*f*x) + cos(6*e + 6*f*x) - 10))
```

Reduce [F]

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{9}{2}}(e+fx)}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sin(fx+e)}\,\sqrt{\sec(fx+e)}}{\sec(fx+e)\sin(fx+e)^5}dx\right)}{b}$$

input
```
int(1/(b*sec(f*x+e))^(1/2)/sin(f*x+e)^(9/2),x)
```

output
```
(sqrt(b)*int((sqrt(sin(e + f*x))*sqrt(sec(e + f*x)))/(sec(e + f*x)*sin(e +
 f*x)**5),x))/b
```

3.470 $\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{13}{2}}(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{13}{2}}(e+fx)}\,dx = -\frac{2b}{11f(b\sec(e+fx))^{3/2}\sin^{\frac{11}{2}}(e+fx)} - \frac{16b}{77f(b\sec(e+fx))^{3/2}\sin^{\frac{7}{2}}(e+fx)} - \frac{64b}{231f(b\sec(e+fx))^{3/2}\sin^{\frac{3}{2}}(e+fx)}$$

output
```
-2/11*b/f/(b*sec(f*x+e))^(3/2)/sin(f*x+e)^(11/2)-16/77*b/f/(b*sec(f*x+e))^
(3/2)/sin(f*x+e)^(7/2)-64/231*b/f/(b*sec(f*x+e))^(3/2)/sin(f*x+e)^(3/2)
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.57

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{13}{2}}(e+fx)}\,dx = \frac{2b(-45+28\cos(2(e+fx))-4\cos(4(e+fx)))}{231f(b\sec(e+fx))^{3/2}\sin^{\frac{11}{2}}(e+fx)}$$

input
```
Integrate[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(13/2)),x]
```

output
```
(2*b*(-45 + 28*Cos[2*(e + f*x)] - 4*Cos[4*(e + f*x)]))/(231*f*(b*Sec[e + f
*x])^(3/2)*Sin[e + f*x]^(11/2))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 3064, 3042, 3064, 3042, 3058}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin^{\frac{13}{2}}(e+fx)\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)^{13/2}\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3064}$$

$$\frac{8}{11}\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{9}{2}}(e+fx)}dx-\frac{2b}{11f\sin^{\frac{11}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{11}\int \frac{1}{\sqrt{b\sec(e+fx)}\sin(e+fx)^{9/2}}dx-\frac{2b}{11f\sin^{\frac{11}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3064}$$

$$\frac{8}{11}\left(\frac{4}{7}\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{5}{2}}(e+fx)}dx-\frac{2b}{7f\sin^{\frac{7}{2}}(e+fx)(b\sec(e+fx))^{3/2}}\right)-\frac{2b}{11f\sin^{\frac{11}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{11}\left(\frac{4}{7}\int \frac{1}{\sqrt{b\sec(e+fx)}\sin(e+fx)^{5/2}}dx-\frac{2b}{7f\sin^{\frac{7}{2}}(e+fx)(b\sec(e+fx))^{3/2}}\right)-\frac{2b}{11f\sin^{\frac{11}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3058}$$

$$\frac{8}{11}\left(-\frac{8b}{21f\sin^{\frac{3}{2}}(e+fx)(b\sec(e+fx))^{3/2}}-\frac{2b}{7f\sin^{\frac{7}{2}}(e+fx)(b\sec(e+fx))^{3/2}}\right)-\frac{2b}{11f\sin^{\frac{11}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

input
```
Int[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(13/2)),x]
```

output
```
(8*((-2*b)/(7*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^(7/2)) - (8*b)/(21*f*(
b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^(3/2))))/11 - (2*b)/(11*f*(b*Sec[e + f*
x])^(3/2)*Sin[e + f*x]^(11/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3058
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1
)/(a*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m - n + 2, 0] &
& NeQ[m, -1]
```

rule 3064
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)/
(a*f*(m + 1))), x] + Simp[(m - n + 2)/(a^2*(m + 1))   Int[(a*Sin[e + f*x])^
(m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.58

method	result	size
default	$-\frac{2\left(32\cos(fx+e)^{5}-88\cos(fx+e)^{3}+77\cos(fx+e)\right)}{231f\sin(fx+e)^{\frac{11}{2}}\sqrt{b\sec(fx+e)}}$	53

input
```
int(1/(b*sec(f*x+e))^(1/2)/sin(f*x+e)^(13/2),x,method=_RETURNVERBOSE)
```

output
```
-2/231/f/sin(f*x+e)^(11/2)/(b*sec(f*x+e))^(1/2)*(32*cos(f*x+e)^5-88*cos(f*
x+e)^3+77*cos(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{13}{2}}(e+fx)}\,dx = \frac{2\left(32\cos\left(fx+e\right)^{6}-88\cos\left(fx+e\right)^{4}+77\cos\left(fx+e\right)^{2}\right)\sqrt{\frac{b}{\cos(fx+e)}}\sqrt{\sin\left(fx+e\right)}}{231\left(bf\cos\left(fx+e\right)^{6}-3\,bf\cos\left(fx+e\right)^{4}+3\,bf\cos\left(fx+e\right)^{2}-bf\right)}$$

input
```
integrate(1/(b*sec(f*x+e))^(1/2)/sin(f*x+e)^(13/2),x, algorithm="fricas")
```

output
```
2/231*(32*cos(f*x + e)^6 - 88*cos(f*x + e)^4 + 77*cos(f*x + e)^2)*sqrt(b/c
os(f*x + e))*sqrt(sin(f*x + e))/(b*f*cos(f*x + e)^6 - 3*b*f*cos(f*x + e)^4
 + 3*b*f*cos(f*x + e)^2 - b*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{13}{2}}(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*sec(f*x+e))**(1/2)/sin(f*x+e)**(13/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{13}{2}}(e+fx)}\,dx = \int \frac{1}{\sqrt{b\sec(fx+e)}\sin(fx+e)^{\frac{13}{2}}}\,dx$$

input
```
integrate(1/(b*sec(f*x+e))^(1/2)/sin(f*x+e)^(13/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*sec(f*x + e))*sin(f*x + e)^(13/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{13}{2}}(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*sec(f*x+e))^(1/2)/sin(f*x+e)^(13/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 33.28 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.79

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{13}{2}}(e+fx)}\,dx = \frac{e^{-e\,6i-f\,x\,6i}\,\sqrt{\frac{b}{\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}}}\,\left(\frac{e^{e\,6i+f\,x\,6i}\,992i}{231\,b\,f}+\frac{e^{e\,6i+f\,x\,6i}\,\cos(2\,e+2\,f\,x)\,608i}{231\,b\,f}-\frac{e^{e\,6i+f\,x\,6i}\,\cos(4\,e+4\,f\,x)\,320i}{231\,b\,f}+\frac{e^{e\,6i+f\,x}\cdots}{\cdots}\right.}{32\sin\left(e+f\,x\right)^{11/2}}$$

input
```
int(1/(sin(e + f*x)^(13/2)*(b/cos(e + f*x))^(1/2)),x)
```

output
```
(exp(- e*6i - f*x*6i)*(b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^
(1/2)*((exp(e*6i + f*x*6i)*992i)/(231*b*f) + (exp(e*6i + f*x*6i)*cos(2*e +
 2*f*x)*608i)/(231*b*f) - (exp(e*6i + f*x*6i)*cos(4*e + 4*f*x)*320i)/(231*
b*f) + (exp(e*6i + f*x*6i)*cos(6*e + 6*f*x)*64i)/(231*b*f))*1i)/(32*sin(e
+ f*x)^(11/2))
```

Reduce [F]

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{13}{2}}(e+fx)}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sin(fx+e)}\,\sqrt{\sec(fx+e)}}{\sec(fx+e)\sin(fx+e)^{7}}dx\right)}{b}$$

input
```
int(1/(b*sec(f*x+e))^(1/2)/sin(f*x+e)^(13/2),x)
```

output
```
(sqrt(b)*int((sqrt(sin(e + f*x))*sqrt(sec(e + f*x)))/(sec(e + f*x)*sin(e +
 f*x)**7),x))/b
```

3.471 $\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{17}{2}}(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 121

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{17}{2}}(e+fx)}\,dx = -\frac{2b}{15f(b\sec(e+fx))^{3/2}\sin^{\frac{15}{2}}(e+fx)} - \frac{8b}{55f(b\sec(e+fx))^{3/2}\sin^{\frac{11}{2}}(e+fx)} - \frac{64b}{385f(b\sec(e+fx))^{3/2}\sin^{\frac{7}{2}}(e+fx)} - \frac{256b}{1155f(b\sec(e+fx))^{3/2}\sin^{\frac{3}{2}}(e+fx)}$$

output
```
-2/15*b/f/(b*sec(f*x+e))^(3/2)/sin(f*x+e)^(15/2)-8/55*b/f/(b*sec(f*x+e))^(
3/2)/sin(f*x+e)^(11/2)-64/385*b/f/(b*sec(f*x+e))^(3/2)/sin(f*x+e)^(7/2)-25
6/1155*b/f/(b*sec(f*x+e))^(3/2)/sin(f*x+e)^(3/2)
```

Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.51

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{17}{2}}(e+fx)}\,dx = \frac{2b(-195+150\cos(2(e+fx))-36\cos(4(e+fx))+4\cos(6(e+fx)))}{1155f(b\sec(e+fx))^{3/2}\sin^{\frac{15}{2}}(e+fx)}$$

input
```
Integrate[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(17/2)),x]
```

output
```
(2*b*(-195 + 150*Cos[2*(e + f*x)] - 36*Cos[4*(e + f*x)] + 4*Cos[6*(e + f*x
)]))/(1155*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^(15/2))
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 3064, 3042, 3064, 3042, 3064, 3042, 3058}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin^{\frac{17}{2}}(e+fx)\sqrt{b\sec(e+fx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(e+fx)^{17/2}\sqrt{b\sec(e+fx)}}dx$$

$$\Big\downarrow \text{3064}$$

$$\frac{4}{5}\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{13}{2}}(e+fx)}dx-\frac{2b}{15f\sin^{\frac{15}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{5}\int \frac{1}{\sqrt{b\sec(e+fx)}\sin(e+fx)^{13/2}}dx-\frac{2b}{15f\sin^{\frac{15}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

$\big\downarrow$ 3064

$$\frac{4}{5}\left(\frac{8}{11}\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{9}{2}}(e+fx)}dx-\frac{2b}{11f\sin^{\frac{11}{2}}(e+fx)(b\sec(e+fx))^{3/2}}\right)-\frac{2b}{15f\sin^{\frac{15}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

$\big\downarrow$ 3042

$$\frac{4}{5}\left(\frac{8}{11}\int \frac{1}{\sqrt{b\sec(e+fx)}\sin(e+fx)^{9/2}}dx-\frac{2b}{11f\sin^{\frac{11}{2}}(e+fx)(b\sec(e+fx))^{3/2}}\right)-\frac{2b}{15f\sin^{\frac{15}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

$\big\downarrow$ 3064

$$\frac{4}{5}\left(\frac{8}{11}\left(\frac{4}{7}\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{5}{2}}(e+fx)}dx-\frac{2b}{7f\sin^{\frac{7}{2}}(e+fx)(b\sec(e+fx))^{3/2}}\right)-\frac{2b}{11f\sin^{\frac{11}{2}}(e+fx)(b\sec(e\right.$$
$$\frac{2b}{15f\sin^{\frac{15}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

$\big\downarrow$ 3042

$$\frac{4}{5}\left(\frac{8}{11}\left(\frac{4}{7}\int \frac{1}{\sqrt{b\sec(e+fx)}\sin(e+fx)^{5/2}}dx-\frac{2b}{7f\sin^{\frac{7}{2}}(e+fx)(b\sec(e+fx))^{3/2}}\right)-\frac{2b}{11f\sin^{\frac{11}{2}}(e+fx)(b\sec(\right.$$
$$\frac{2b}{15f\sin^{\frac{15}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

$\big\downarrow$ 3058

$$\frac{4}{5}\left(\frac{8}{11}\left(-\frac{8b}{21f\sin^{\frac{3}{2}}(e+fx)(b\sec(e+fx))^{3/2}}-\frac{2b}{7f\sin^{\frac{7}{2}}(e+fx)(b\sec(e+fx))^{3/2}}\right)-\frac{2b}{11f\sin^{\frac{11}{2}}(e+fx)(b\sec(e\right.$$
$$\frac{2b}{15f\sin^{\frac{15}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

input
```
Int[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(17/2)),x]
```

output
```
(4*((8*((-2*b)/(7*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^(7/2)) - (8*b)/(21
*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^(3/2))))/11 - (2*b)/(11*f*(b*Sec[e
+ f*x])^(3/2)*Sin[e + f*x]^(11/2))))/5 - (2*b)/(15*f*(b*Sec[e + f*x])^(3/2
)*Sin[e + f*x]^(15/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3058
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1
)/(a*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m - n + 2, 0] &
& NeQ[m, -1]
```

rule 3064
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)/
(a*f*(m + 1))), x] + Simp[(m - n + 2)/(a^2*(m + 1))   Int[(a*Sin[e + f*x])^
(m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.52

method	result	size
default	$\frac{\frac{256 \cos(fx+e)^{7}}{1155}-\frac{64 \cos(fx+e)^{5}}{77}+\frac{8 \cos(fx+e)^{3}}{7}-\frac{2 \cos(fx+e)}{3}}{f \sin(fx+e)^{\frac{15}{2}} \sqrt{b \sec(fx+e)}}$	63

input
```
int(1/(b*sec(f*x+e))^(1/2)/sin(f*x+e)^(17/2),x,method=_RETURNVERBOSE)
```

output
```
2/1155/f/sin(f*x+e)^(15/2)/(b*sec(f*x+e))^(1/2)*(128*cos(f*x+e)^7-480*cos(
f*x+e)^5+660*cos(f*x+e)^3-385*cos(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{17}{2}}(e+fx)}\,dx = \frac{2\left(128\cos(fx+e)^8 - 480\cos(fx+e)^6 + 660\cos(fx+e)^4 - 385\cos(fx+e)^2\right)\sqrt{\frac{b}{\cos(fx+e)}}\sqrt{\sin(fx+e)}}{1155\left(bf\cos(fx+e)^8 - 4\,bf\cos(fx+e)^6 + 6\,bf\cos(fx+e)^4 - 4\,bf\cos(fx+e)^2 + bf\right)}$$

input
```
integrate(1/(b*sec(f*x+e))^(1/2)/sin(f*x+e)^(17/2),x, algorithm="fricas")
```

output
```
2/1155*(128*cos(f*x + e)^8 - 480*cos(f*x + e)^6 + 660*cos(f*x + e)^4 - 385
*cos(f*x + e)^2)*sqrt(b/cos(f*x + e))*sqrt(sin(f*x + e))/(b*f*cos(f*x + e)
^8 - 4*b*f*cos(f*x + e)^6 + 6*b*f*cos(f*x + e)^4 - 4*b*f*cos(f*x + e)^2 +
b*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{17}{2}}(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*sec(f*x+e))**(1/2)/sin(f*x+e)**(17/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{17}{2}}(e+fx)}\,dx = \int \frac{1}{\sqrt{b\sec\left(fx+e\right)}\sin\left(fx+e\right)^{\frac{17}{2}}}\,dx$$

input
```
integrate(1/(b*sec(f*x+e))^(1/2)/sin(f*x+e)^(17/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*sec(f*x + e))*sin(f*x + e)^(17/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{17}{2}}(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*sec(f*x+e))^(1/2)/sin(f*x+e)^(17/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 32.69 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.59

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{17}{2}}(e+fx)}\,dx = \frac{e^{-e\,8i-f\,x\,8i}\,\sqrt{\frac{b}{\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}}}\,\left(\frac{e^{e\,8i+f\,x\,8i}\,1024i}{77\,b\,f}+\frac{e^{e\,8i+f\,x\,8i}\,\cos(2\,e+2\,f\,x)\,384i}{55\,b\,f}-\frac{e^{e\,8i+f\,x\,8i}\,\cos(4\,e+4\,f\,x)\,5248i}{1155\,b\,f}+\frac{e^{e\,8i+}}{}\right.}{128\sin\left(e+f\,x\right)^{15/2}}$$

input
```
int(1/(sin(e + f*x)^(17/2)*(b/cos(e + f*x))^(1/2)),x)
```

output
```
(exp(- e*8i - f*x*8i)*(b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^
(1/2)*((exp(e*8i + f*x*8i)*1024i)/(77*b*f) + (exp(e*8i + f*x*8i)*cos(2*e +
 2*f*x)*384i)/(55*b*f) - (exp(e*8i + f*x*8i)*cos(4*e + 4*f*x)*5248i)/(1155
*b*f) + (exp(e*8i + f*x*8i)*cos(6*e + 6*f*x)*256i)/(165*b*f) - (exp(e*8i +
 f*x*8i)*cos(8*e + 8*f*x)*256i)/(1155*b*f))*1i)/(128*sin(e + f*x)^(15/2))
```

Reduce [F]

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{17}{2}}(e+fx)}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\sin(fx+e)}\sqrt{\sec(fx+e)}}{\sec(fx+e)\sin(fx+e)^9}dx\right)}{b}$$

input
```
int(1/(b*sec(f*x+e))^(1/2)/sin(f*x+e)^(17/2),x)
```

output
```
(sqrt(b)*int((sqrt(sin(e + f*x))*sqrt(sec(e + f*x)))/(sec(e + f*x)*sin(e +
 f*x)**9),x))/b
```

3.472 $\int \frac{(a\sin(e+fx))^{9/2}}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 391

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{7a^{9/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{128\sqrt{2}b^{5/2}f} + \frac{7a^{9/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{128\sqrt{2}b^{5/2}f} - \frac{7a^{9/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}\left(\sqrt{a}+\sqrt{a}\tan(e+fx)\right)}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{128\sqrt{2}b^{5/2}f} - \frac{7a^3(a\sin(e+fx))^{3/2}}{192bf\sqrt{b\sec(e+fx)}} - \frac{a(a\sin(e+fx))^{7/2}}{48bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{11/2}}{6abf\sqrt{b\sec(e+fx)}}$$

output
```
-7/256*a^(9/2)*arctan(1-2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/a^(1/2)/(b*co
s(f*x+e))^(1/2))*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)*2^(1/2)/b^(5/2)
/f+7/256*a^(9/2)*arctan(1+2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/a^(1/2)/(b*
cos(f*x+e))^(1/2))*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)*2^(1/2)/b^(5/
2)/f-7/256*a^(9/2)*arctanh(2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/(b*cos(f*x
+e))^(1/2)/(a^(1/2)+a^(1/2)*tan(f*x+e)))*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e
))^(1/2)*2^(1/2)/b^(5/2)/f-7/192*a^3*(a*sin(f*x+e))^(3/2)/b/f/(b*sec(f*x+e
))^(1/2)-1/48*a*(a*sin(f*x+e))^(7/2)/b/f/(b*sec(f*x+e))^(1/2)+1/6*(a*sin(f
*x+e))^(11/2)/a/b/f/(b*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 2.91 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.45

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\sec(e+fx))^{3/2}}\,dx =$$

$$-\frac{a^5\left(4(-3+14\cos(2(e+fx))-4\cos(4(e+fx)))\sin^2(e+fx)-21\sqrt{2}\arctan\left(\frac{-1+\sqrt{\tan^2(e+fx)}}{\sqrt{2}\sqrt[4]{\tan^2(e+fx)}}\right)\sqrt[4]{\cdots}\right.}{768bf\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+f\cdots}}$$

input
```
Integrate[(a*Sin[e + f*x])^(9/2)/(b*Sec[e + f*x])^(3/2),x]
```

output
```
-1/768*(a^5*(4*(-3 + 14*Cos[2*(e + f*x)] - 4*Cos[4*(e + f*x)])*Sin[e + f*x
]^2 - 21*Sqrt[2]*ArcTan[(-1 + Sqrt[Tan[e + f*x]^2])/(Sqrt[2]*(Tan[e + f*x]
^2)^(1/4))]*(Tan[e + f*x]^2)^(1/4) + 21*Sqrt[2]*ArcTanh[(Sqrt[2]*(Tan[e +
f*x]^2)^(1/4))/(1 + Sqrt[Tan[e + f*x]^2])]*(Tan[e + f*x]^2)^(1/4)))/(b*f*S
qrt[b*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]])
```

Rubi [A] (verified)

Time = 1.79 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.11, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.720, Rules used = {3042, 3062, 3042, 3063, 3042, 3063, 3042, 3065, 3042, 3054, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3062}$$

$$\frac{\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{9/2}dx}{12b^2}+\frac{(a\sin(e+fx))^{11/2}}{6abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{9/2}dx}{12b^2}+\frac{(a\sin(e+fx))^{11/2}}{6abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3063}$$

$$\frac{\frac{7}{8}a^2\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}dx-\frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}}}{12b^2}+\frac{(a\sin(e+fx))^{11/2}}{6abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{7}{8}a^2\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}dx-\frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}}}{12b^2}+\frac{(a\sin(e+fx))^{11/2}}{6abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3063}$$

$$\frac{\frac{7}{8}a^2\left(\frac{3}{4}a^2\int \sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}dx-\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}}\right)-\frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}}}{12b^2}+\frac{(a\sin(e+fx))^{11/2}}{6abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{7}{8}a^2\left(\frac{3}{4}a^2\int \sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}dx-\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}}\right)-\frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}}}{12b^2}+\frac{(a\sin(e+fx))^{11/2}}{6abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3065}$$

$$\frac{\frac{7}{8}a^2\left(\frac{3}{4}a^2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}dx-\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}}\right)-\frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}}}{12b^2}+\frac{(a\sin(e+fx))^{11/2}}{6abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{7}{8}a^2\left(\frac{3}{4}a^2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}dx-\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}}\right)-\frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}}}{12b^2}+\frac{(a\sin(e+fx))^{11/2}}{6abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3054}$$

$$\frac{\frac{7}{8}a^2\left(\frac{3a^3b\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{a\tan(e+fx)}{b\left(\tan^2(e+fx)a^2+a^2\right)}d\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}}{2f}-\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}}\right)-\frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}}}{12b^2}+\frac{(a\sin(e+fx))^{11/2}}{6abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{826}$$

$$\frac{\frac{7}{8}a^2\left(\frac{3a^3b\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\int \frac{\tan(e+fx)a+a}{\tan^2(e+fx)a^2+a^2}d\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}}{2b}-\frac{\int \frac{a-a\tan(e+fx)}{\tan^2(e+fx)a^2+a^2}d\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}}{2b}\right)}{2f}-\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}}\right)-}{12b^2}$$

$$\frac{(a\sin(e+fx))^{11/2}}{6abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\frac{7}{8}a^2\left(\frac{3a^3b\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\int \frac{1}{\frac{\tan(e+fx)a}{b}+\frac{a}{b}-\frac{\sqrt{2}\sqrt{a\sin(e+fx)}\sqrt{a}}{\sqrt{b}\sqrt{b\cos(e+fx)}}}d\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}}{2b}+\frac{\int \frac{1}{\frac{\tan(e+fx)a}{b}+\frac{a}{b}+\frac{\sqrt{2}\sqrt{a\sin(e+fx)}\sqrt{a}}{\sqrt{b}\sqrt{b\cos(e+fx)}}}d\frac{\sqrt{a\sin(e+\cdots}}{\sqrt{b\cos(e+\cdots}}}{2b}}{2b}\cdots\right.}{2f}\cdots\right.}{12b^2\cdots}$$

$$\frac{(a\sin(e+fx))^{11/2}}{6abf\sqrt{b\sec(e+fx)}}$$

$\Big\downarrow$ 1082

$$\frac{\frac{7}{8}a^2\left(\frac{3a^3b\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\int \frac{1}{-\frac{a\tan(e+fx)}{b}-1}d\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{\int \frac{1}{-\frac{a\tan(e+fx)}{b}-1}d\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}}{2b}-\frac{\int\frac{a-a\tan\cdots}{\tan^2(e+\cdots}\cdots}{\cdots}\right.}{2f}\cdots\right.}{12b^2\cdots}$$

$$\frac{(a\sin(e+fx))^{11/2}}{6abf\sqrt{b\sec(e+fx)}}$$

$\Big\downarrow$ 217

$$\frac{\frac{7}{8}a^2\left(\frac{3a^3b\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}}{2b}-\frac{\int\frac{a-a\tan(e+fx)}{\tan^2(e+fx)a^2+a^2}d\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}}{2b}\right)}{2f}\cdots\right.}{12b^2\cdots}$$

$$\frac{(a\sin(e+fx))^{11/2}}{6abf\sqrt{b\sec(e+fx)}}$$

$\Big\downarrow$ 1479

$$\frac{7}{8}a^2\left(\frac{3a^3b\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}}{2b}-\frac{-\int -\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}}{\sqrt{b}\left(\frac{\tan(e+fx)a}{b}+\frac{a}{b}-\frac{\sqrt{2}\sqrt{a\sin(e\cdots}}{\sqrt{b}\sqrt{b\cos(\cdots}}\right.}}{2\sqrt{2}\sqrt{a}\sqrt{b}}\cdots\right.}{2f}\cdots}{12b^2}\right.$$

$$\frac{(a\sin(e+fx))^{11/2}}{6abf\sqrt{b\sec(e+fx)}}$$

$\Big\downarrow$ 25

$$\frac{7}{8}a^2\left(\frac{3a^3b\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}}{2b}-\frac{\int \frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}}{\sqrt{b}\left(\frac{\tan(e+fx)a}{b}+\frac{a}{b}-\frac{\sqrt{2}\sqrt{a\sin(e+fx\cdots}}{\sqrt{b}\sqrt{b\cos(e+f\cdots}}\right.}}{2\sqrt{2}\sqrt{a}\sqrt{b}}\cdots\right.}{2f}\cdots}{12b^2}\right.$$

$$\frac{(a\sin(e+fx))^{11/2}}{6abf\sqrt{b\sec(e+fx)}}$$

$\Big\downarrow$ 27

$$\frac{7}{8}a^2\left(\frac{3a^3b\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}}{2b}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}}{\frac{\tan(e+fx)a}{b}+\frac{a}{b}-\frac{\sqrt{2}\sqrt{a\sin(e+fx)}\sqrt{a}}{\sqrt{b}\sqrt{b\cos(e+fx)}}}\cdots}{2\sqrt{2}\sqrt{ab}}\cdots\right.}{2f}\cdots}{12b^2}\cdots\right.$$

$$\frac{(a\sin(e+fx))^{11/2}}{6abf\sqrt{b\sec(e+fx)}}$$

$\big\downarrow$ 1103

$$\frac{7}{8}a^2\left(\frac{3a^3b\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}}{2b}-\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+a\tan(e+fx\cdots\right.}{2\sqrt{2}\sqrt{a}\sqrt{b}}\cdots\right.}{2f}\cdots}{12b^2}\cdots\right.$$

$$\frac{(a\sin(e+fx))^{11/2}}{6abf\sqrt{b\sec(e+fx)}}$$

input
```
Int[(a*Sin[e + f*x])^(9/2)/(b*Sec[e + f*x])^(3/2),x]
```

output
```
(a*Sin[e + f*x])^(11/2)/(6*a*b*f*Sqrt[b*Sec[e + f*x]]) + (-1/4*(a*b*(a*Sin
[e + f*x])^(7/2))/(f*Sqrt[b*Sec[e + f*x]]) + (7*a^2*((3*a^3*b*Sqrt[b*Cos[e
 + f*x]]*((-(ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a]*Sq
rt[b*Cos[e + f*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[b])) + ArcTan[1 + (Sqrt[2]*Sqrt
[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a]*Sqrt[b*Cos[e + f*x]])]/(Sqrt[2]*Sqrt[a]
*Sqrt[b]))/(2*b) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[b]*Sqrt[a*Sin[e + f
*x]])/Sqrt[b*Cos[e + f*x]] + a*Tan[e + f*x]]/(Sqrt[2]*Sqrt[a]*Sqrt[b]) + L
og[a + (Sqrt[2]*Sqrt[a]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x]]
 + a*Tan[e + f*x]]/(2*Sqrt[2]*Sqrt[a]*Sqrt[b]))/(2*b))*Sqrt[b*Sec[e + f*x]
])/(2*f) - (a*b*(a*Sin[e + f*x])^(3/2))/(2*f*Sqrt[b*Sec[e + f*x]])))/8)/(1
2*b^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3054
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k*a*(b/f)   Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

rule 3062
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n + 1)/(a
*b*f*(m - n))), x] - Simp[(n + 1)/(b^2*(m - n))   Int[(a*Sin[e + f*x])^m*(b
*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

rule 3063
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*b*(a*Sin[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
- 1)/(f*(m - n))), x] + Simp[a^2*((m - 1)/(m - n))   Int[(a*Sin[e + f*x])^(
m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

rule 3065
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n   Int[(a*Sin[e
+ f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Int
egerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.15

method	result
default	$\frac{\sqrt{2}\,a^4\left(21\ln\left(-2\sqrt{-\frac{2\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,\cot(fx+e)+2-2\sqrt{-\frac{2\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,\csc(fx+e)-2\cot(fx+e)\right)-21\ln\left(2\sqrt{-\frac{2\sin(\cdots}{(\cdots}}\right.\right.}{}$

input
```
int((a*sin(f*x+e))^(9/2)/(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/1572864/f*2^(1/2)*a^4/b*(21*ln(-2*(-2*sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)*cot(f*x+e)+2-2*(-2*sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)*csc(f*x+e)-2*cot(f*x+e))-21*ln(2*(-2*sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)*cot(f*x+e)+2*(-2*sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
*csc(f*x+e)-2*cot(f*x+e)+2)-42*arctan((-(-2*sin(f*x+e)*cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/(-1+cos(f*x+e)))+42*arctan(((-2*s
in(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/(-1+
cos(f*x+e)))+4*(21+32*cos(f*x+e)^5+32*cos(f*x+e)^4-60*cos(f*x+e)^3-60*cos(
f*x+e)^2+21*cos(f*x+e))*sin(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2))*(a*sin(f*x+e))^(1/2)*sin(f*x+e)^9/(-sin(f*x+e)*cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)/(b*sec(f*x+e))^(1/2)*sec(1/2*f*x+1/2*e)^11*csc(1/2*f*x
+1/2*e)^9
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 559, normalized size of antiderivative = 1.43

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\sec(e+fx))^{3/2}}\,dx =$$

$$-\frac{42\sqrt{2}a^4b\sqrt{\frac{a}{b}}\arctan\left(-\frac{\sqrt{2}\sqrt{a\sin(fx+e)}\sqrt{\frac{a}{b}}\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)}{a\cos(fx+e)-a\sin(fx+e)}\right)+21\sqrt{2}a^4b\sqrt{\frac{a}{b}}\arctan\left(\frac{2a\cos(fx+e)^2-2a\cos(fx\cdots}{2\left(a\cos(\cdots\right.}\right.}{\cdots}$$

input
```
integrate((a*sin(f*x+e))^(9/2)/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
-1/3072*(42*sqrt(2)*a^4*b*sqrt(a/b)*arctan(-sqrt(2)*sqrt(a*sin(f*x + e))*s
qrt(a/b)*sqrt(b/cos(f*x + e))*cos(f*x + e)/(a*cos(f*x + e) - a*sin(f*x + e
))) + 21*sqrt(2)*a^4*b*sqrt(a/b)*arctan(1/2*(2*a*cos(f*x + e)^2 - 2*a*cos(
f*x + e)*sin(f*x + e) + sqrt(2)*sqrt(a*sin(f*x + e))*sqrt(a/b)*sqrt(b/cos(
f*x + e)) - 2*a)/(a*cos(f*x + e)^2 + a*cos(f*x + e)*sin(f*x + e) - a)) + 2
1*sqrt(2)*a^4*b*sqrt(a/b)*arctan(-1/2*(2*a*cos(f*x + e)^2 - 2*a*cos(f*x +
e)*sin(f*x + e) - sqrt(2)*sqrt(a*sin(f*x + e))*sqrt(a/b)*sqrt(b/cos(f*x +
e)) - 2*a)/(a*cos(f*x + e)^2 + a*cos(f*x + e)*sin(f*x + e) - a)) + 21*sqrt
(2)*a^4*b*sqrt(a/b)*log(2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e)*sin(f*x +
 e))*sqrt(a*sin(f*x + e))*sqrt(a/b)*sqrt(b/cos(f*x + e)) + 4*a*cos(f*x + e
)*sin(f*x + e) + a) - 21*sqrt(2)*a^4*b*sqrt(a/b)*log(-2*sqrt(2)*(cos(f*x +
 e)^2 + cos(f*x + e)*sin(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(a/b)*sqrt(b/c
os(f*x + e)) + 4*a*cos(f*x + e)*sin(f*x + e) + a) - 16*(32*a^4*cos(f*x + e
)^5 - 60*a^4*cos(f*x + e)^3 + 21*a^4*cos(f*x + e))*sqrt(a*sin(f*x + e))*sq
rt(b/cos(f*x + e))*sin(f*x + e))/(b^2*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\sec(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))**(9/2)/(b*sec(f*x+e))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{9}{2}}}{(b\sec(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(9/2)/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^(9/2)/(b*sec(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{9}{2}}}{(b\sec(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(9/2)/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e))^(9/2)/(b*sec(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{(a\,\sin(e+f\,x))^{9/2}}{\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((a*sin(e + f*x))^(9/2)/(b/cos(e + f*x))^(3/2),x)
```

output
```
int((a*sin(e + f*x))^(9/2)/(b/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\left(\int \frac{\sqrt{\sin(fx+e)}\,\sqrt{\sec(fx+e)}\,\sin(fx+e)^4}{\sec(fx+e)^2}dx\right)a^4}{b^2}$$

input
```
int((a*sin(f*x+e))^(9/2)/(b*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(b)*sqrt(a)*int((sqrt(sin(e + f*x))*sqrt(sec(e + f*x))*sin(e + f*x)**
4)/sec(e + f*x)**2,x)*a**4)/b**2
```

3.473 $\int \frac{(a\sin(e+fx))^{5/2}}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 354

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{3a^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{32\sqrt{2}b^{5/2}f} + \frac{3a^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{32\sqrt{2}b^{5/2}f} - \frac{3a^{5/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}\left(\sqrt{a}+\sqrt{a}\tan(e+fx)\right)}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{32\sqrt{2}b^{5/2}f} - \frac{a(a\sin(e+fx))^{3/2}}{16bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{7/2}}{4abf\sqrt{b\sec(e+fx)}}$$

output
```
-3/64*a^(5/2)*arctan(1-2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/a^(1/2)/(b*cos
(f*x+e))^(1/2))*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)*2^(1/2)/b^(5/2)/
f+3/64*a^(5/2)*arctan(1+2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/a^(1/2)/(b*co
s(f*x+e))^(1/2))*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)*2^(1/2)/b^(5/2)
/f-3/64*a^(5/2)*arctanh(2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/(b*cos(f*x+e)
)^(1/2)/(a^(1/2)+a^(1/2)*tan(f*x+e)))*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^
(1/2)*2^(1/2)/b^(5/2)/f-1/16*a*(a*sin(f*x+e))^(3/2)/b/f/(b*sec(f*x+e))^(1/
2)+1/4*(a*sin(f*x+e))^(7/2)/a/b/f/(b*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 2.31 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.47

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\sec(e+fx))^{3/2}}\,dx = \frac{a^3\left(4-6\cos(2(e+fx))+2\cos(4(e+fx))+3\sqrt{2}\arctan\left(\frac{-1+\sqrt{\tan^2(e+fx)}}{\sqrt{2}\sqrt[4]{\tan^2(e+fx)}}\right.\right.}{64bf\sqrt{b\sec(e+fx)}}$$

input
```
Integrate[(a*Sin[e + f*x])^(5/2)/(b*Sec[e + f*x])^(3/2),x]
```

output
```
(a^3*(4 - 6*Cos[2*(e + f*x)] + 2*Cos[4*(e + f*x)] + 3*Sqrt[2]*ArcTan[(-1 +
 Sqrt[Tan[e + f*x]^2])/(Sqrt[2]*(Tan[e + f*x]^2)^(1/4))]*(Tan[e + f*x]^2)^
(1/4) - 3*Sqrt[2]*ArcTanh[(Sqrt[2]*(Tan[e + f*x]^2)^(1/4))/(1 + Sqrt[Tan[e
 + f*x]^2])]*(Tan[e + f*x]^2)^(1/4)))/(64*b*f*Sqrt[b*Sec[e + f*x]]*Sqrt[a*
Sin[e + f*x]])
```

Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.11, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3042, 3062, 3042, 3063, 3042, 3065, 3042, 3054, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3062}$$

$$\frac{\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}dx}{8b^2}+\frac{(a\sin(e+fx))^{7/2}}{4abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{5/2}dx}{8b^2}+\frac{(a\sin(e+fx))^{7/2}}{4abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3063}$$

$$\frac{\frac{3}{4}a^2\int \sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}dx-\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}}}{8b^2}+\frac{(a\sin(e+fx))^{7/2}}{4abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{4}a^2\int \sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}dx-\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}}}{8b^2}+\frac{(a\sin(e+fx))^{7/2}}{4abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3065}$$

$$\frac{\frac{3}{4}a^2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}dx-\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}}}{8b^2}+\frac{(a\sin(e+fx))^{7/2}}{4abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{4}a^2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}dx-\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}}}{8b^2}+\frac{(a\sin(e+fx))^{7/2}}{4abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3054}$$

$$\frac{\frac{3a^3b\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{a\tan(e+fx)}{b\left(\tan^2(e+fx)a^2+a^2\right)}d\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}}{2f}-\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}}}{8b^2}+\frac{(a\sin(e+fx))^{7/2}}{4abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{826}$$

$$\frac{\frac{3a^3b\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\int \frac{\tan(e+fx)a+a}{\tan^2(e+fx)a^2+a^2}d\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}}{2b}-\frac{\int\frac{a-a\tan(e+fx)}{\tan^2(e+fx)a^2+a^2}d\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}}{2b}\right)}{2f}-\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}}}{8b^2}+\frac{(a\sin(e+fx))^{7/2}}{4abf\sqrt{b\sec(e+fx)}}$$

$\downarrow$ 1476

$$\frac{\frac{3a^3b\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\int\frac{1}{\frac{\tan(e+fx)a}{b}+\frac{a}{b}-\frac{\sqrt{2}\sqrt{a\sin(e+fx)}\sqrt{a}}{\sqrt{b}\sqrt{b\cos(e+fx)}}}d\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}}{2b}+\frac{\int\frac{1}{\frac{\tan(e+fx)a}{b}+\frac{a}{b}+\frac{\sqrt{2}\sqrt{a\sin(e+fx)}\sqrt{a}}{\sqrt{b}\sqrt{b\cos(e+fx)}}}d\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}}{2b}}{2b}-\right.}{2f}}{8b^2}$$

$$\frac{(a\sin(e+fx))^{7/2}}{4abf\sqrt{b\sec(e+fx)}}$$

$\downarrow$ 1082

$$\frac{\frac{3a^3b\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\int\frac{1}{-\frac{a\tan(e+fx)}{b}-1}d\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{\int\frac{1}{-\frac{a\tan(e+fx)}{b}-1}d\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}}{2b}-\frac{\int\frac{a-a\tan(e+fx)}{\tan^2(e+fx)a^2+a}}{2b}\right.}{2f}}{8b^2}$$

$$\frac{(a\sin(e+fx))^{7/2}}{4abf\sqrt{b\sec(e+fx)}}$$

$\downarrow$ 217

$$\frac{\frac{3a^3b\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}}{2b}-\frac{\int\frac{a-a\tan(e+fx)}{\tan^2(e+fx)a^2+a^2}d\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}}{2b}\right)}{2f}-\frac{ab(a}{2f}}{8b^2}$$

$$\frac{(a\sin(e+fx))^{7/2}}{4abf\sqrt{b\sec(e+fx)}}$$

$\downarrow$ 1479

$$\frac{\frac{3a^3b\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}}{2b}-\frac{\int -\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}}{\sqrt{b}\left(\frac{\tan(e+fx)a}{b}+\frac{a}{b}-\frac{\sqrt{2}\sqrt{a\sin(e+fx)}\sqrt{a}}{\sqrt{b}\sqrt{b\cos(e+fx)}}\right.}}{2\sqrt{2}\sqrt{a}\sqrt{b}}}{2f}}{8b^2}$$

$$\frac{(a\sin(e+fx))^{7/2}}{4abf\sqrt{b\sec(e+fx)}}$$

$\downarrow$ 25

$$\frac{\frac{3a^3b\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}}{2b}-\frac{\int \frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}}{\sqrt{b}\left(\frac{\tan(e+fx)a}{b}+\frac{a}{b}-\frac{\sqrt{2}\sqrt{a\sin(e+fx)}\sqrt{a}}{\sqrt{b}\sqrt{b\cos(e+fx)}}\right)}d}{2\sqrt{2}\sqrt{a}\sqrt{b}}}{2f}}{8b^2}$$

$$\frac{(a\sin(e+fx))^{7/2}}{4abf\sqrt{b\sec(e+fx)}}$$

$\downarrow$ 27

$$\frac{\frac{3a^3b\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}}{2b}-\frac{\int \frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}}{\frac{\tan(e+fx)a}{b}+\frac{a}{b}-\frac{\sqrt{2}\sqrt{a\sin(e+fx)}\sqrt{a}}{\sqrt{b}\sqrt{b\cos(e+fx)}}}d\frac{\sqrt{a\sin}}{\sqrt{b\cos}}}{2\sqrt{2}\sqrt{a}b}}{2f}}{8b^2}$$

$$\frac{(a\sin(e+fx))^{7/2}}{4abf\sqrt{b\sec(e+fx)}}$$

$\downarrow$ 1103

$$\frac{\frac{3a^3b\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}}{2b}-\frac{\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+a\tan(e+fx)+a\right)}{2\sqrt{2}\sqrt{a}\sqrt{b}}-}{2}\right.}{2f}}{8b^2}$$

$$\frac{(a\sin(e+fx))^{7/2}}{4abf\sqrt{b\sec(e+fx)}}$$

input
```
Int[(a*Sin[e + f*x])^(5/2)/(b*Sec[e + f*x])^(3/2),x]
```

output
```
(a*Sin[e + f*x])^(7/2)/(4*a*b*f*Sqrt[b*Sec[e + f*x]]) + ((3*a^3*b*Sqrt[b*C
os[e + f*x]]*((-(ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a
]*Sqrt[b*Cos[e + f*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[b])) + ArcTan[1 + (Sqrt[2]*
Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a]*Sqrt[b*Cos[e + f*x]])]/(Sqrt[2]*Sqr
t[a]*Sqrt[b]))/(2*b) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[b]*Sqrt[a*Sin[e
 + f*x]])/Sqrt[b*Cos[e + f*x]] + a*Tan[e + f*x]]/(Sqrt[2]*Sqrt[a]*Sqrt[b])
 + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f
*x]] + a*Tan[e + f*x]]/(2*Sqrt[2]*Sqrt[a]*Sqrt[b]))/(2*b))*Sqrt[b*Sec[e +
f*x]])/(2*f) - (a*b*(a*Sin[e + f*x])^(3/2))/(2*f*Sqrt[b*Sec[e + f*x]]))/(8
*b^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3054
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k*a*(b/f)   Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

rule 3062
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n + 1)/(a
*b*f*(m - n))), x] - Simp[(n + 1)/(b^2*(m - n))   Int[(a*Sin[e + f*x])^m*(b
*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

rule 3063
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*b*(a*Sin[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
- 1)/(f*(m - n))), x] + Simp[a^2*((m - 1)/(m - n))   Int[(a*Sin[e + f*x])^(
m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

rule 3065
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n   Int[(a*Sin[e
+ f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Int
egerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Maple [A] (warning: unable to verify)

Time = 4.66 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.28

method	result
default	$-\frac{\sqrt{2}\, a^2 \left(-3 \ln \left(-\frac{\cos(fx+e)\cot(fx+e)-2\cot(fx+e)-2\sqrt{-\frac{2\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,\sin(fx+e)+\csc(fx+e)-2\cos(fx+e)-\sin(fx+e)+2}{-1+\cos(fx+e)}\right)+3\,\mathrm{l}$

input
```
int((a*sin(f*x+e))^(5/2)/(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16384/f*2^(1/2)*a^2/b*(-3*ln(-(cos(f*x+e)*cot(f*x+e)-2*cot(f*x+e)-2*(-2
*sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+csc(f*x+e)-2*cos
(f*x+e)-sin(f*x+e)+2)/(-1+cos(f*x+e)))+3*ln(-(cos(f*x+e)*cot(f*x+e)-2*cot(
f*x+e)+2*(-2*sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+csc(
f*x+e)-2*cos(f*x+e)-sin(f*x+e)+2)/(-1+cos(f*x+e)))+6*arctan((-(-2*sin(f*x+
e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/(-1+cos(f*x
+e)))-6*arctan(((-2*sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+
e)+cos(f*x+e)-1)/(-1+cos(f*x+e)))+(16*cos(f*x+e)^3+16*cos(f*x+e)^2-12*cos(
f*x+e)-12)*sin(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*(
a*sin(f*x+e))^(1/2)*sin(f*x+e)^6/(-sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2)/(b*sec(f*x+e))^(1/2)*sec(1/2*f*x+1/2*e)^8*csc(1/2*f*x+1/2*e)^6
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 546, normalized size of antiderivative = 1.54

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\sec(e+fx))^{3/2}}\,dx =$$

$$-\frac{6\sqrt{2}a^2b\sqrt{\frac{a}{b}}\arctan\left(-\frac{\sqrt{2}\sqrt{a\sin(fx+e)}\sqrt{\frac{a}{b}}\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)}{a\cos(fx+e)-a\sin(fx+e)}\right)+3\sqrt{2}a^2b\sqrt{\frac{a}{b}}\arctan\left(\frac{2a\cos(fx+e)^2-2a\cos(fx+e)}{2\left(a\cos(fx+\right.}\right.}{}$$

input
```
integrate((a*sin(f*x+e))^(5/2)/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
-1/256*(6*sqrt(2)*a^2*b*sqrt(a/b)*arctan(-sqrt(2)*sqrt(a*sin(f*x + e))*sqr
t(a/b)*sqrt(b/cos(f*x + e))*cos(f*x + e)/(a*cos(f*x + e) - a*sin(f*x + e))
) + 3*sqrt(2)*a^2*b*sqrt(a/b)*arctan(1/2*(2*a*cos(f*x + e)^2 - 2*a*cos(f*x
 + e)*sin(f*x + e) + sqrt(2)*sqrt(a*sin(f*x + e))*sqrt(a/b)*sqrt(b/cos(f*x
 + e)) - 2*a)/(a*cos(f*x + e)^2 + a*cos(f*x + e)*sin(f*x + e) - a)) + 3*sq
rt(2)*a^2*b*sqrt(a/b)*arctan(-1/2*(2*a*cos(f*x + e)^2 - 2*a*cos(f*x + e)*s
in(f*x + e) - sqrt(2)*sqrt(a*sin(f*x + e))*sqrt(a/b)*sqrt(b/cos(f*x + e))
- 2*a)/(a*cos(f*x + e)^2 + a*cos(f*x + e)*sin(f*x + e) - a)) + 3*sqrt(2)*a
^2*b*sqrt(a/b)*log(2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e)*sin(f*x + e))*
sqrt(a*sin(f*x + e))*sqrt(a/b)*sqrt(b/cos(f*x + e)) + 4*a*cos(f*x + e)*sin
(f*x + e) + a) - 3*sqrt(2)*a^2*b*sqrt(a/b)*log(-2*sqrt(2)*(cos(f*x + e)^2
+ cos(f*x + e)*sin(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(a/b)*sqrt(b/cos(f*x
 + e)) + 4*a*cos(f*x + e)*sin(f*x + e) + a) + 16*(4*a^2*cos(f*x + e)^3 - 3
*a^2*cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e))*sin(f*x + e))
/(b^2*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\sec(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))**(5/2)/(b*sec(f*x+e))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{5}{2}}}{(b\sec(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(5/2)/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^(5/2)/(b*sec(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{5}{2}}}{(b\sec(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(5/2)/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e))^(5/2)/(b*sec(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{(a\,\sin(e+f\,x))^{5/2}}{\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int((a*sin(e + f*x))^(5/2)/(b/cos(e + f*x))^(3/2),x)
```

output
```
int((a*sin(e + f*x))^(5/2)/(b/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\left(\int \frac{\sqrt{\sin(fx+e)}\,\sqrt{\sec(fx+e)}\,\sin(fx+e)^2}{\sec(fx+e)^2}dx\right)a^2}{b^2}$$

input
```
int((a*sin(f*x+e))^(5/2)/(b*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(b)*sqrt(a)*int((sqrt(sin(e + f*x))*sqrt(sec(e + f*x))*sin(e + f*x)**
2)/sec(e + f*x)**2,x)*a**2)/b**2
```

3.474 $\int \frac{\sqrt{a\sin(e+fx)}}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 319

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{\sqrt{a}\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{4\sqrt{2}b^{5/2}f} + \frac{\sqrt{a}\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{4\sqrt{2}b^{5/2}f} - \frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}(\sqrt{a}+\sqrt{a}\tan(e+fx))}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{4\sqrt{2}b^{5/2}f} + \frac{(a\sin(e+fx))^{3/2}}{2abf\sqrt{b\sec(e+fx)}}$$

output
```
-1/8*a^(1/2)*arctan(1-2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/a^(1/2)/(b*cos(
f*x+e))^(1/2))*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)*2^(1/2)/b^(5/2)/f
+1/8*a^(1/2)*arctan(1+2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/a^(1/2)/(b*cos(
f*x+e))^(1/2))*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)*2^(1/2)/b^(5/2)/f
-1/8*a^(1/2)*arctanh(2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/(b*cos(f*x+e))^(
1/2)/(a^(1/2)+a^(1/2)*tan(f*x+e)))*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/
2)*2^(1/2)/b^(5/2)/f+1/2*(a*sin(f*x+e))^(3/2)/a/b/f/(b*sec(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 1.70 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\sec(e+fx))^{3/2}}\,dx = \frac{a\left(4\sin^2(e+fx)+\sqrt{2}\arctan\left(\frac{-1+\sqrt{\tan^2(e+fx)}}{\sqrt{2}\sqrt[4]{\tan^2(e+fx)}}\right)\sqrt[4]{\tan^2(e+fx)}-\sqrt{2}\text{arct}\right.}{8bf\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}}$$

input
```
Integrate[Sqrt[a*Sin[e + f*x]]/(b*Sec[e + f*x])^(3/2),x]
```

output
```
(a*(4*Sin[e + f*x]^2 + Sqrt[2]*ArcTan[(-1 + Sqrt[Tan[e + f*x]^2])/(Sqrt[2]
*(Tan[e + f*x]^2)^(1/4))]*(Tan[e + f*x]^2)^(1/4) - Sqrt[2]*ArcTanh[(Sqrt[2
]*(Tan[e + f*x]^2)^(1/4))/(1 + Sqrt[Tan[e + f*x]^2])]*(Tan[e + f*x]^2)^(1/
4)))/(8*b*f*Sqrt[b*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]])
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.10, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 3062, 3042, 3065, 3042, 3054, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3062}$$

$$\frac{\int \sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}dx}{4b^2}+\frac{(a\sin(e+fx))^{3/2}}{2abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}dx}{4b^2}+\frac{(a\sin(e+fx))^{3/2}}{2abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3065}$$

$$\frac{\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}dx}{4b^2}+\frac{(a\sin(e+fx))^{3/2}}{2abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}dx}{4b^2}+\frac{(a\sin(e+fx))^{3/2}}{2abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3054}$$

$$\frac{a\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{a\tan(e+fx)}{b(\tan^2(e+fx)a^2+a^2)}d\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}}{2bf}+\frac{(a\sin(e+fx))^{3/2}}{2abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{826}$$

$$\frac{a\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\int \frac{\tan(e+fx)a+a}{\tan^2(e+fx)a^2+a^2}d\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}}{2b}-\frac{\int \frac{a-a\tan(e+fx)}{\tan^2(e+fx)a^2+a^2}d\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}}{2b}\right)}{2bf}+$$

$$\frac{(a\sin(e+fx))^{3/2}}{2abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{a\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\int \frac{1}{\frac{\tan(e+fx)a}{b}+\frac{a}{b}-\frac{\sqrt{2}\sqrt{a\sin(e+fx)}\sqrt{a}}{\sqrt{b}\sqrt{b\cos(e+fx)}}}d\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}}{2b}+\frac{\int \frac{1}{\frac{\tan(e+fx)a}{b}+\frac{a}{b}+\frac{\sqrt{2}\sqrt{a\sin(e+fx)}\sqrt{a}}{\sqrt{b}\sqrt{b\cos(e+fx)}}}d\frac{\sqrt{a\sin(e}{\sqrt{b\cos(e}}{2b}}{2b}\right.}{2bf}$$

$$\frac{(a\sin(e+fx))^{3/2}}{2abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{a\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\int \frac{1}{-\frac{a\tan(e+fx)}{b}-1}d\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{\int \frac{1}{-\frac{a\tan(e+fx)}{b}-1}d\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}}{2b}-\frac{\int \frac{a-\cdots}{\tan^2(\cdots}\cdots}{\cdots}\cdots\right.}{2bf}$$

$$\frac{(a\sin(e+fx))^{3/2}}{2abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \; 217$$

$$\frac{a\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}}{2b}-\frac{\int \frac{a-a\tan(e+fx)}{\tan^2(e+fx)a^2+a^2}d\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\cdots}{2b}\cdots\right.}{2bf}$$

$$\frac{(a\sin(e+fx))^{3/2}}{2abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \; 1479$$

$$\frac{a\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}}{2b}-\frac{-\int -\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{b}\sqrt{a\sin(e+\cdots}}{\sqrt{b\cos(e+f\cdots}}}{\sqrt{b}\left(\frac{\tan(e+fx)a}{b}+\frac{a}{b}-\frac{\sqrt{2}\sqrt{a\sin\cdots}}{\sqrt{b}\sqrt{b\cdots}}\cdots\right)}\cdots}{2\sqrt{2}\sqrt{a}\cdots}\cdots\right.}{2bf}$$

$$\frac{(a\sin(e+fx))^{3/2}}{2abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \; 25$$

$$\frac{a\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}}{2b}-\frac{\int \frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}}{\sqrt{b}\left(\frac{\tan(e+fx)a}{b}+\frac{a}{b}-\frac{\sqrt{2}\sqrt{a\sin(e-\cdots}}{\sqrt{b}\sqrt{b\cos(\cdots}}\cdots\right)}\cdots}{2\sqrt{2}\sqrt{a}\sqrt{b}\cdots}\cdots\right.}{2bf}$$

$$\frac{(a\sin(e+fx))^{3/2}}{2abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \; 27$$

$$\frac{a\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}}{2b}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}}{\frac{\tan(e+fx)a}{b}+\frac{a}{b}-\frac{\sqrt{2}\sqrt{a\sin(e+fx)}}{\sqrt{b}\sqrt{b\cos(e+fx)}}}}{2\sqrt{2}\sqrt{a}b}\right.}{2bf}$$

$$\frac{(a\sin(e+fx))^{3/2}}{2abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{a\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}}{2b}-\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+a\tan(e\ldots\right.}{2\sqrt{2}\sqrt{a}\sqrt{b}}\right.}{2bf}$$

$$\frac{(a\sin(e+fx))^{3/2}}{2abf\sqrt{b\sec(e+fx)}}$$

input
```
Int[Sqrt[a*Sin[e + f*x]]/(b*Sec[e + f*x])^(3/2),x]
```

output
```
(a*Sqrt[b*Cos[e + f*x]]*((-(ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x
]])/(Sqrt[a]*Sqrt[b*Cos[e + f*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[b])) + ArcTan[1
+ (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a]*Sqrt[b*Cos[e + f*x]])]/(
Sqrt[2]*Sqrt[a]*Sqrt[b]))/(2*b) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[b]*S
qrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x]] + a*Tan[e + f*x]]/(Sqrt[2]*Sqrt[
a]*Sqrt[b]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[
b*Cos[e + f*x]] + a*Tan[e + f*x]]/(2*Sqrt[2]*Sqrt[a]*Sqrt[b]))/(2*b))*Sqrt
[b*Sec[e + f*x]])/(2*b*f) + (a*Sin[e + f*x])^(3/2)/(2*a*b*f*Sqrt[b*Sec[e +
 f*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3054
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k*a*(b/f)   Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

rule 3062
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n + 1)/(a
*b*f*(m - n))), x] - Simp[(n + 1)/(b^2*(m - n))   Int[(a*Sin[e + f*x])^m*(b
*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

rule 3065
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n   Int[(a*Sin[e
+ f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Int
egerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Maple [A] (warning: unable to verify)

Time = 3.43 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.35

method	result
default	$\frac{\sqrt{2}\left(-\ln\left(-\frac{\cos(fx+e)\cot(fx+e)-2\cot(fx+e)+2\sqrt{-\frac{2\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,\sin(fx+e)+\csc(fx+e)-2\cos(fx+e)-\sin(fx+e)+2}{-1+\cos(fx+e)}\right)+\ln\left(\frac{2\sqrt{-}\cdots}{\cdots}\right.\right.}{\cdots}$

input
```
int((a*sin(f*x+e))^(1/2)/(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/256/f*2^(1/2)/b*(-ln(-(cos(f*x+e)*cot(f*x+e)-2*cot(f*x+e)+2*(-2*sin(f*x+
e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+csc(f*x+e)-2*cos(f*x+e)-s
in(f*x+e)+2)/(-1+cos(f*x+e)))+ln((2*(-2*sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)*sin(f*x+e)-cos(f*x+e)*cot(f*x+e)+sin(f*x+e)+2*cos(f*x+e)-csc(f
*x+e)+2*cot(f*x+e)-2)/(-1+cos(f*x+e)))+2*arctan(((-2*sin(f*x+e)*cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/(-1+cos(f*x+e)))+2*arcta
n(((-2*sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+cos(f*x+e)
-1)/(-1+cos(f*x+e)))+(4*cos(f*x+e)+4)*sin(f*x+e)*(-2*sin(f*x+e)*cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2))*(a*sin(f*x+e))^(1/2)*sin(f*x+e)^3/(-sin(f*x+e)*c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/(b*sec(f*x+e))^(1/2)*sec(1/2*f*x+1/2*e)^
5*csc(1/2*f*x+1/2*e)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 509 vs. $2(245) = 490$.

Time = 0.15 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.60

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\sec(e+fx))^{3/2}}\,dx =$$

$$-\frac{2\sqrt{2}b\sqrt{\frac{a}{b}}\arctan\left(-\frac{\sqrt{2}\sqrt{a\sin(fx+e)}\sqrt{\frac{a}{b}}\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)}{a\cos(fx+e)-a\sin(fx+e)}\right)+\sqrt{2}b\sqrt{\frac{a}{b}}\arctan\left(\frac{2\,a\cos(fx+e)^2-2\,a\cos(fx+e)\sin(fx+}{2\left(a\cos(fx+e)^2+a\,c\right.}\right.}{}$$

input
```
integrate((a*sin(f*x+e))^(1/2)/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
-1/32*(2*sqrt(2)*b*sqrt(a/b)*arctan(-sqrt(2)*sqrt(a*sin(f*x + e))*sqrt(a/b
)*sqrt(b/cos(f*x + e))*cos(f*x + e)/(a*cos(f*x + e) - a*sin(f*x + e))) + s
qrt(2)*b*sqrt(a/b)*arctan(1/2*(2*a*cos(f*x + e)^2 - 2*a*cos(f*x + e)*sin(f
*x + e) + sqrt(2)*sqrt(a*sin(f*x + e))*sqrt(a/b)*sqrt(b/cos(f*x + e)) - 2*
a)/(a*cos(f*x + e)^2 + a*cos(f*x + e)*sin(f*x + e) - a)) + sqrt(2)*b*sqrt(
a/b)*arctan(-1/2*(2*a*cos(f*x + e)^2 - 2*a*cos(f*x + e)*sin(f*x + e) - sqr
t(2)*sqrt(a*sin(f*x + e))*sqrt(a/b)*sqrt(b/cos(f*x + e)) - 2*a)/(a*cos(f*x
 + e)^2 + a*cos(f*x + e)*sin(f*x + e) - a)) + sqrt(2)*b*sqrt(a/b)*log(2*sq
rt(2)*(cos(f*x + e)^2 + cos(f*x + e)*sin(f*x + e))*sqrt(a*sin(f*x + e))*sq
rt(a/b)*sqrt(b/cos(f*x + e)) + 4*a*cos(f*x + e)*sin(f*x + e) + a) - sqrt(2
)*b*sqrt(a/b)*log(-2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e)*sin(f*x + e))*
sqrt(a*sin(f*x + e))*sqrt(a/b)*sqrt(b/cos(f*x + e)) + 4*a*cos(f*x + e)*sin
(f*x + e) + a) - 16*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e))*cos(f*x + e)
*sin(f*x + e))/(b^2*f)
```

Sympy [F]

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\sec(e+fx))^{3/2}}\,dx=\int \frac{\sqrt{a\sin{\left(e+fx \right)}}}{\left(b\sec{\left(e+fx \right)}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*sin(f*x+e))**(1/2)/(b*sec(f*x+e))**(3/2),x)
```

output
```
Integral(sqrt(a*sin(e + f*x))/(b*sec(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\sec(e+fx))^{3/2}}\,dx=\int \frac{\sqrt{a\sin\left(fx+e\right)}}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(1/2)/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sin(f*x + e))/(b*sec(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{a\sin\left(fx+e\right)}}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(1/2)/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*sin(f*x + e))/(b*sec(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{a\,\sin\left(e+f\,x\right)}}{{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

input
```
int((a*sin(e + f*x))^(1/2)/(b/cos(e + f*x))^(3/2),x)
```

output
```
int((a*sin(e + f*x))^(1/2)/(b/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\left(\int \frac{\sqrt{\sin(fx+e)}\,\sqrt{\sec(fx+e)}}{\sec(fx+e)^2}dx\right)}{b^2}$$

input
```
int((a*sin(f*x+e))^(1/2)/(b*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(b)*sqrt(a)*int((sqrt(sin(e + f*x))*sqrt(sec(e + f*x)))/sec(e + f*x)*
*2,x))/b**2
```

3.475 $\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 309

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{3/2}}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{\sqrt{2}a^{3/2}b^{5/2}f} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{\sqrt{2}a^{3/2}b^{5/2}f} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}(\sqrt{a}+\sqrt{a}\tan(e+fx))}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{\sqrt{2}a^{3/2}b^{5/2}f} - \frac{2}{abf\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}}$$

output
```
1/2*arctan(1-2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/a^(1/2)/(b*cos(f*x+e))^(
1/2))*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)*2^(1/2)/a^(3/2)/b^(5/2)/f-
1/2*arctan(1+2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/a^(1/2)/(b*cos(f*x+e))^(
1/2))*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)*2^(1/2)/a^(3/2)/b^(5/2)/f+
1/2*arctanh(2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/(b*cos(f*x+e))^(1/2)/(a^(
1/2)+a^(1/2)*tan(f*x+e)))*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)*2^(1/2
)/a^(3/2)/b^(5/2)/f-2/a/b/f/(b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 2.05 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.47

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{3/2}}\,dx = -\frac{4+\sqrt{2}\arctan\left(\frac{-1+\sqrt{\tan^2(e+fx)}}{\sqrt{2}\sqrt[4]{\tan^2(e+fx)}}\right)\sqrt[4]{\tan^2(e+fx)}-\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{\tan^2(e+fx)}}{1+\sqrt{\tan^2(e+fx)}}\right)\sqrt[4]{\tan^2(e+fx)}}{2abf\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}}$$

input
```
Integrate[1/((b*Sec[e + f*x])^(3/2)*(a*Sin[e + f*x])^(3/2)),x]
```

output
```
-1/2*(4 + Sqrt[2]*ArcTan[(-1 + Sqrt[Tan[e + f*x]^2])/(Sqrt[2]*(Tan[e + f*x
]^2)^(1/4))]*(Tan[e + f*x]^2)^(1/4) - Sqrt[2]*ArcTanh[(Sqrt[2]*(Tan[e + f*
x]^2)^(1/4))/(1 + Sqrt[Tan[e + f*x]^2])]*(Tan[e + f*x]^2)^(1/4))/(a*b*f*Sq
rt[b*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]])
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.13, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {3042, 3061, 3042, 3065, 3042, 3054, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sin(e+fx))^{3/2}(b\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\sin(e+fx))^{3/2}(b\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3061}$$

$$-\frac{\int \sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}dx}{a^2b^2}-\frac{2}{abf\sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\int \sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}dx}{a^2b^2}-\frac{2}{abf\sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \ 3065$$

$$-\frac{\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}dx}{a^2b^2}-\frac{2}{abf\sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}dx}{a^2b^2}-\frac{2}{abf\sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \ 3054$$

$$-\frac{2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{a\tan(e+fx)}{b(\tan^2(e+fx)a^2+a^2)}d\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}}{abf}-\frac{2}{abf\sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \ 826$$

$$-\frac{2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\int \frac{\tan(e+fx)a+a}{\tan^2(e+fx)a^2+a^2}d\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}}{2b}-\frac{\int \frac{a-a\tan(e+fx)}{\tan^2(e+fx)a^2+a^2}d\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}}{2b}\right)}{abf}-\frac{2}{abf\sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \ 1476$$

$$-\frac{2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\int \frac{1}{\frac{\tan(e+fx)a}{b}+\frac{a}{b}-\frac{\sqrt{2}\sqrt{a\sin(e+fx)}\sqrt{a}}{\sqrt{b}\sqrt{b\cos(e+fx)}}}d\frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}}{2b}+\frac{\int \frac{1}{\frac{\tan(e+fx)a}{b}+\frac{a}{b}+\frac{\sqrt{2}\sqrt{a\sin(e+fx)}\sqrt{a}}{\sqrt{b}\sqrt{b\cos(e+fx)}}}d\frac{\sqrt{a\sin}\cdots}{\sqrt{b\cos}\cdots}}{2b}}{2b}\cdots\right)}{abf}-\frac{2}{abf\sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \ 1082$$

$$-\frac{2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\int\frac{1}{-\frac{a\tan(e+fx)}{b}-1}d\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{\int\frac{1}{-\frac{a\tan(e+fx)}{b}-1}d\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}}{2b}-\frac{\int\frac{a}{\tan\cdots}}{}\right.}{abf}$$

$$\frac{2}{abf\sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}}$$

$\big\downarrow$ 217

$$-\frac{2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}}{2b}-\frac{\int\frac{a-a\tan(e+fx)}{\tan^2(e+fx)a^2+a^2}d\frac{\sqrt{a\sin(e+f\cdots}}{\sqrt{b\cos(e+f\cdots}}}{2b}\right.}{abf}$$

$$\frac{2}{abf\sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}}$$

$\big\downarrow$ 1479

$$-\frac{2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}}{2b}-\frac{-\int-\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{b}\sqrt{a\sin(\cdots}}{\sqrt{b\cos(e-\cdots}}}{\sqrt{b}\left(\frac{\tan(e+fx)a}{b}+\frac{a}{b}-\frac{\sqrt{2}\sqrt{a\cdots}}{\sqrt{b}\cdots}\right.}}{2\sqrt{2}\cdots}\right.}{abf}$$

$$\frac{2}{abf\sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}}$$

$\big\downarrow$ 25

$$-\frac{2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}}{2b}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{b}\sqrt{a\sin(e+f\cdots}}{\sqrt{b\cos(e+fx\cdots}}}{\sqrt{b}\left(\frac{\tan(e+fx)a}{b}+\frac{a}{b}-\frac{\sqrt{2}\sqrt{a\sin\cdots}}{\sqrt{b}\sqrt{b\cos\cdots}}\right.}}{2\sqrt{2}\sqrt{a}\cdots}\right.}{abf}$$

$$\frac{2}{abf\sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}}$$

$\big\downarrow$ 27

$$-\frac{2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}}{2b}-\frac{\int\frac{\sqrt{2}\sqrt{a}-\frac{2\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}}{\frac{\tan(e+fx)a}{b}+\frac{a}{b}-\frac{\sqrt{2}\sqrt{a\sin(e+f}}{\sqrt{b}\sqrt{b\cos(e+}}}}{2\sqrt{2}\sqrt{a}b}\right.}{abf}$$

$$\frac{2}{abf\sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{1103}$$

$$-\frac{2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}+1\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{a}\sqrt{b}}}{2b}-\frac{\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+a\tan\right.}{2\sqrt{2}\sqrt{a}\sqrt{b}}\right.}{abf}$$

$$\frac{2}{abf\sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}}$$

input
```
Int[1/((b*Sec[e + f*x])^(3/2)*(a*Sin[e + f*x])^(3/2)),x]
```

output
```
(-2*Sqrt[b*Cos[e + f*x]]*((-(ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*
x]])/(Sqrt[a]*Sqrt[b*Cos[e + f*x]])]/(Sqrt[2]*Sqrt[a]*Sqrt[b])) + ArcTan[1
 + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a]*Sqrt[b*Cos[e + f*x]])]/
(Sqrt[2]*Sqrt[a]*Sqrt[b]))/(2*b) - (-1/2*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[b]*
Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x]] + a*Tan[e + f*x]]/(Sqrt[2]*Sqrt
[a]*Sqrt[b]) + Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt
[b*Cos[e + f*x]] + a*Tan[e + f*x]]/(2*Sqrt[2]*Sqrt[a]*Sqrt[b]))/(2*b))*Sqr
t[b*Sec[e + f*x]])/(a*b*f) - 2/(a*b*f*Sqrt[b*Sec[e + f*x]]*Sqrt[a*Sin[e +
f*x]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3054
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k*a*(b/f)   Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

rule 3061
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n + 1)/(a
*b*f*(m + 1))), x] - Simp[(n + 1)/(a^2*b^2*(m + 1))   Int[(a*Sin[e + f*x])^
(m + 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n
, -1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3065
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n   Int[(a*Sin[e
+ f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Int
egerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Maple [A] (verified)

Time = 3.21 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.48

method	result
default	$-\frac{\sqrt{2}\left(\ln\left(\frac{2\sqrt{-\frac{2\sin(fx+e)\cos(fx+e)}{(\cos(fx+e)+1)^2}}\,\sin(fx+e)-\cos(fx+e)\cot(fx+e)+\sin(fx+e)+2\cos(fx+e)-\csc(fx+e)+2\cot(fx+e)-2}{-1+\cos(fx+e)}\right)\sin(fx+e)-\right.}{}$

input
```
int(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/f*2^(1/2)*(ln((2*(-2*sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*si
n(f*x+e)-cos(f*x+e)*cot(f*x+e)+sin(f*x+e)+2*cos(f*x+e)-csc(f*x+e)+2*cot(f*
x+e)-2)/(-1+cos(f*x+e)))*sin(f*x+e)+2*arctan(((-2*sin(f*x+e)*cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+cos(f*x+e)-1)/(-1+cos(f*x+e)))*sin(f*x+e)
-ln(-(cos(f*x+e)*cot(f*x+e)-2*cot(f*x+e)+2*(-2*sin(f*x+e)*cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)*sin(f*x+e)+csc(f*x+e)-2*cos(f*x+e)-sin(f*x+e)+2)/(-1+co
s(f*x+e)))*sin(f*x+e)+2*arctan(((-2*sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/(-1+cos(f*x+e)))*sin(f*x+e)+4*(-2*sin(f*x
+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)+4*(-2*sin(f*x+e)*cos(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2))/(cos(f*x+e)+1)/a/(a*sin(f*x+e))^(1/2)/(-sin(f
*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/b/(b*sec(f*x+e))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 545 vs. $2(245) = 490$.

Time = 0.15 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.76

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{3/2}}\,dx = \frac{2\sqrt{2}ab\sqrt{\frac{1}{ab}}\arctan\left(-\frac{\sqrt{2}\sqrt{a\sin(fx+e)}\sqrt{\frac{b}{\cos(fx+e)}}\sqrt{\frac{1}{ab}}\cos(fx+e)}{\cos(fx+e)-\sin(fx+e)}\right)\sin}{}$$

input
```
integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(3/2),x, algorithm="fricas
")
```

output
```
1/8*(2*sqrt(2)*a*b*sqrt(1/(a*b))*arctan(-sqrt(2)*sqrt(a*sin(f*x + e))*sqrt
(b/cos(f*x + e))*sqrt(1/(a*b))*cos(f*x + e)/(cos(f*x + e) - sin(f*x + e)))
*sin(f*x + e) + sqrt(2)*a*b*sqrt(1/(a*b))*arctan(1/2*(sqrt(2)*sqrt(a*sin(f
*x + e))*sqrt(b/cos(f*x + e))*sqrt(1/(a*b)) + 2*cos(f*x + e)^2 - 2*cos(f*x
 + e)*sin(f*x + e) - 2)/(cos(f*x + e)^2 + cos(f*x + e)*sin(f*x + e) - 1))*
sin(f*x + e) + sqrt(2)*a*b*sqrt(1/(a*b))*arctan(1/2*(sqrt(2)*sqrt(a*sin(f*
x + e))*sqrt(b/cos(f*x + e))*sqrt(1/(a*b)) - 2*cos(f*x + e)^2 + 2*cos(f*x
+ e)*sin(f*x + e) + 2)/(cos(f*x + e)^2 + cos(f*x + e)*sin(f*x + e) - 1))*s
in(f*x + e) + sqrt(2)*a*b*sqrt(1/(a*b))*log(2*sqrt(2)*(cos(f*x + e)^2 + co
s(f*x + e)*sin(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e))*sqrt(1/
(a*b)) + 4*cos(f*x + e)*sin(f*x + e) + 1)*sin(f*x + e) - sqrt(2)*a*b*sqrt(
1/(a*b))*log(-2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e)*sin(f*x + e))*sqrt(
a*sin(f*x + e))*sqrt(b/cos(f*x + e))*sqrt(1/(a*b)) + 4*cos(f*x + e)*sin(f*
x + e) + 1)*sin(f*x + e) - 16*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e))*co
s(f*x + e))/(a^2*b^2*f*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*sec(f*x+e))**(3/2)/(a*sin(f*x+e))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{3/2}}\,dx = \int \frac{1}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(3/2),x, algorithm="maxima
")
```

output
```
integrate(1/((b*sec(f*x + e))^(3/2)*(a*sin(f*x + e))^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{3/2}}\,dx=\int \frac{1}{(b\sec{(fx+e)})^{\frac{3}{2}}\,(a\sin{(fx+e)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(f*x + e))^(3/2)*(a*sin(f*x + e))^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{3/2}}\,dx=\int \frac{1}{(a\,\sin{(e+f\,x)})^{3/2}\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/((a*sin(e + f*x))^(3/2)*(b/cos(e + f*x))^(3/2)),x)
```

output
```
int(1/((a*sin(e + f*x))^(3/2)*(b/cos(e + f*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{3/2}}\,dx=\frac{\sqrt{b}\,\sqrt{a}\left(\int \frac{\sqrt{\sin(fx+e)}\,\sqrt{\sec(fx+e)}}{\sec(fx+e)^2\sin(fx+e)^2}dx\right)}{a^2b^2}$$

input
```
int(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(3/2),x)
```

output
```
(sqrt(b)*sqrt(a)*int((sqrt(sin(e + f*x))*sqrt(sec(e + f*x)))/(sec(e + f*x)
**2*sin(e + f*x)**2),x))/(a**2*b**2)
```

3.476 $\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 35

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{7/2}}\,dx = -\frac{2b}{5af(b\sec(e+fx))^{5/2}(a\sin(e+fx))^{5/2}}$$

output
```
-2/5*b/a/f/(b*sec(f*x+e))^(5/2)/(a*sin(f*x+e))^(5/2)
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.29

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{7/2}}\,dx = -\frac{2\cot^3(e+fx)\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}}{5a^4b^2f}$$

input
```
Integrate[1/((b*Sec[e + f*x])^(3/2)*(a*Sin[e + f*x])^(7/2)),x]
```

output
```
(-2*Cot[e + f*x]^3*Sqrt[b*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]])/(5*a^4*b^2*f
)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3042, 3058}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sin(e+fx))^{7/2}(b\sec(e+fx))^{3/2}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(a\sin(e+fx))^{7/2}(b\sec(e+fx))^{3/2}}dx$$
$$\Big\downarrow \text{3058}$$
$$-\frac{2b}{5af(a\sin(e+fx))^{5/2}(b\sec(e+fx))^{5/2}}$$

input
```
Int[1/((b*Sec[e + f*x])^(3/2)*(a*Sin[e + f*x])^(7/2)),x]
```

output
```
(-2*b)/(5*a*f*(b*Sec[e + f*x])^(5/2)*(a*Sin[e + f*x])^(5/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3058
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1
)/(a*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m - n + 2, 0] &
& NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14

method	result	size
default	$-\frac{2\cot(fx+e)^2}{5f\sqrt{a\sin(fx+e)}\,b\sqrt{b\sec(fx+e)}\,a^3}$	40

input
```
int(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/f/(a*sin(f*x+e))^(1/2)/b/(b*sec(f*x+e))^(1/2)/a^3*cot(f*x+e)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(29) = 58$.

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.94

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{7/2}}\,dx = \frac{2\sqrt{a\sin(fx+e)}\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)^3}{5\left(a^4b^2f\cos(fx+e)^2 - a^4b^2f\right)\sin(fx+e)}$$

input
```
integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(7/2),x, algorithm="fricas
")
```

output
```
2/5*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e))*cos(f*x + e)^3/((a^4*b^2*f*c
os(f*x + e)^2 - a^4*b^2*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*sec(f*x+e))**(3/2)/(a*sin(f*x+e))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{7/2}}\,dx = \int \frac{1}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(7/2),x, algorithm="maxima
")
```

output
```
integrate(1/((b*sec(f*x + e))^(3/2)*(a*sin(f*x + e))^(7/2)), x)
```

Giac [F]

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{7/2}}\,dx = \int \frac{1}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(7/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(f*x + e))^(3/2)*(a*sin(f*x + e))^(7/2)), x)
```

Mupad [B] (verification not implemented)

Time = 27.26 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.40

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{7/2}}\,dx = \frac{\sqrt{\frac{b}{\cos(e+f\,x)}}\,\left(\cos\left(3\,e+3\,f\,x\right)-2\,\cos\left(e+f\,x\right)+\cos\left(5\,e+5\right.\right.}{5\,a^3\,b^2\,f\,\sqrt{a\,\sin\left(e+f\,x\right)}\,\left(\cos\left(4\,e+4\,f\,x\right)-4\,\cos\left(2\,e+2\,f\,x\right.\right.}$$

input
```
int(1/((a*sin(e + f*x))^(7/2)*(b/cos(e + f*x))^(3/2)),x)
```

output
```
((b/cos(e + f*x))^(1/2)*(cos(3*e + 3*f*x) - 2*cos(e + f*x) + cos(5*e + 5*f
*x)))/(5*a^3*b^2*f*(a*sin(e + f*x))^(1/2)*(cos(4*e + 4*f*x) - 4*cos(2*e +
2*f*x) + 3))
```

Reduce [F]

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{7/2}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\left(\int \frac{\sqrt{\sin(fx+e)}\,\sqrt{\sec(fx+e)}}{\sec(fx+e)^2\sin(fx+e)^4}dx\right)}{a^4b^2}$$

input
```
int(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(7/2),x)
```

output
```
(sqrt(b)*sqrt(a)*int((sqrt(sin(e + f*x))*sqrt(sec(e + f*x)))/(sec(e + f*x)
**2*sin(e + f*x)**4),x))/(a**4*b**2)
```

3.477 $\int \frac{(a\sin(e+fx))^{7/2}}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 172

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{a^3\sqrt{a\sin(e+fx)}}{12bf\sqrt{b\sec(e+fx)}} - \frac{a(a\sin(e+fx))^{5/2}}{30bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{9/2}}{5abf\sqrt{b\sec(e+fx)}} + \frac{a^4\,\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}{24b^2f\sqrt{a\sin(e+fx)}}$$

output
```
-1/12*a^3*(a*sin(f*x+e))^(1/2)/b/f/(b*sec(f*x+e))^(1/2)-1/30*a*(a*sin(f*x+
e))^(5/2)/b/f/(b*sec(f*x+e))^(1/2)+1/5*(a*sin(f*x+e))^(9/2)/a/b/f/(b*sec(f
*x+e))^(1/2)+1/24*a^4*InverseJacobiAM(e-1/4*Pi+f*x,2^(1/2))*(b*sec(f*x+e))
^(1/2)*sin(2*f*x+2*e)^(1/2)/b^2/f/(a*sin(f*x+e))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.64 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.60

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{a^5\left(-4+17\cos(2(e+fx))-16\cos(4(e+fx))+3\cos(6(e+fx))-20\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\text{sec}\right.\right.}{480bf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}}$$

input
```
Integrate[(a*Sin[e + f*x])^(7/2)/(b*Sec[e + f*x])^(3/2),x]
```

output
```
-1/480*(a^5*(-4 + 17*Cos[2*(e + f*x)] - 16*Cos[4*(e + f*x)] + 3*Cos[6*(e +
 f*x)] - 20*Hypergeometric2F1[1/2, 3/4, 3/2, Sec[e + f*x]^2]*(-Tan[e + f*x
]^2)^(3/4)))/(b*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(3/2))
```

Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 3062, 3042, 3063, 3042, 3063, 3042, 3065, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3062}$$

$$\frac{\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}dx}{10b^2}+\frac{(a\sin(e+fx))^{9/2}}{5abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}dx}{10b^2}+\frac{(a\sin(e+fx))^{9/2}}{5abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3063}$$

$$\frac{\frac{5}{6}a^2\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}dx-\frac{ab(a\sin(e+fx))^{5/2}}{3f\sqrt{b\sec(e+fx)}}}{10b^2}+\frac{(a\sin(e+fx))^{9/2}}{5abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{5}{6}a^2\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}dx-\frac{ab(a\sin(e+fx))^{5/2}}{3f\sqrt{b\sec(e+fx)}}}{10b^2}+\frac{(a\sin(e+fx))^{9/2}}{5abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \ 3063$$

$$\frac{\frac{5}{6}a^2\left(\frac{1}{2}a^2\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}dx-\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}}\right)-\frac{ab(a\sin(e+fx))^{5/2}}{3f\sqrt{b\sec(e+fx)}}}{10b^2}+\frac{(a\sin(e+fx))^{9/2}}{5abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{5}{6}a^2\left(\frac{1}{2}a^2\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}dx-\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}}\right)-\frac{ab(a\sin(e+fx))^{5/2}}{3f\sqrt{b\sec(e+fx)}}}{10b^2}+\frac{(a\sin(e+fx))^{9/2}}{5abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \ 3065$$

$$\frac{\frac{5}{6}a^2\left(\frac{1}{2}a^2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{b\cos(e+fx)}\sqrt{a\sin(e+fx)}}dx-\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}}\right)-\frac{ab(a\sin(e+fx))^{5/2}}{3f\sqrt{b\sec(e+fx)}}}{10b^2}+\frac{(a\sin(e+fx))^{9/2}}{5abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{5}{6}a^2\left(\frac{1}{2}a^2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{b\cos(e+fx)}\sqrt{a\sin(e+fx)}}dx-\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}}\right)-\frac{ab(a\sin(e+fx))^{5/2}}{3f\sqrt{b\sec(e+fx)}}}{10b^2}+\frac{(a\sin(e+fx))^{9/2}}{5abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \ 3053$$

$$\frac{\frac{5}{6}a^2\left(\frac{a^2\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{\sin(2e+2fx)}}dx}{2\sqrt{a\sin(e+fx)}}-\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}}\right)-\frac{ab(a\sin(e+fx))^{5/2}}{3f\sqrt{b\sec(e+fx)}}}{10b^2}+\frac{(a\sin(e+fx))^{9/2}}{5abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{5}{6}a^2\left(\frac{a^2\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{\sin(2e+2fx)}}dx}{2\sqrt{a\sin(e+fx)}}-\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}}\right)-\frac{ab(a\sin(e+fx))^{5/2}}{3f\sqrt{b\sec(e+fx)}}}{10b^2}+\frac{(a\sin(e+fx))^{9/2}}{5abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \ 3120$$

$$\frac{\frac{5}{6}a^2\left(\frac{a^2\sqrt{\sin(2e+2fx)}\,\text{EllipticF}\left(e+fx-\frac{\pi}{4},2\right)\sqrt{b\sec(e+fx)}}{2f\sqrt{a\sin(e+fx)}}-\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}}\right)-\frac{ab(a\sin(e+fx))^{5/2}}{3f\sqrt{b\sec(e+fx)}}}{10b^2}+\frac{(a\sin(e+fx))^{9/2}}{5abf\sqrt{b\sec(e+fx)}}$$

input
```
Int[(a*Sin[e + f*x])^(7/2)/(b*Sec[e + f*x])^(3/2),x]
```

output
```
(a*Sin[e + f*x])^(9/2)/(5*a*b*f*Sqrt[b*Sec[e + f*x]]) + (-1/3*(a*b*(a*Sin[
e + f*x])^(5/2))/(f*Sqrt[b*Sec[e + f*x]]) + (5*a^2*(-((a*b*Sqrt[a*Sin[e +
f*x]])/(f*Sqrt[b*Sec[e + f*x]])) + (a^2*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[
b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(2*f*Sqrt[a*Sin[e + f*x]])))/6)/(1
0*b^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3062
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n + 1)/(a
*b*f*(m - n))), x] - Simp[(n + 1)/(b^2*(m - n))   Int[(a*Sin[e + f*x])^m*(b
*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

rule 3063
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*b*(a*Sin[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
- 1)/(f*(m - n))), x] + Simp[a^2*((m - 1)/(m - n))   Int[(a*Sin[e + f*x])^(
m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

rule 3065
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n   Int[(a*Sin[e
+ f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Int
egerQ[m - 1/2] && IntegerQ[n - 1/2]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 2.64 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.91

method	result
default	$\frac{\sqrt{a\sin(fx+e)}\,a^{3}\left(24\cos(fx+e)^{4}-44\cos(fx+e)^{2}+10+\sqrt{-2\csc(fx+e)+2\cot(fx+e)+2}\,\sqrt{-\csc(fx+e)+\cot(fx+e)}\ \operatorname{EllipticF}\left(\sqrt{\right.\right.}}{120f\sqrt{b\sec(fx+e)}\,b}$

input
```
int((a*sin(f*x+e))^(7/2)/(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/120/f*(a*sin(f*x+e))^(1/2)*a^3/(b*sec(f*x+e))^(1/2)/b*(24*cos(f*x+e)^4-4
4*cos(f*x+e)^2+10+(-2*csc(f*x+e)+2*cot(f*x+e)+2)^(1/2)*(-csc(f*x+e)+cot(f*
x+e))^(1/2)*EllipticF((csc(f*x+e)-cot(f*x+e)+1)^(1/2),1/2*2^(1/2))*(csc(f*
x+e)-cot(f*x+e)+1)^(1/2)*(5*csc(f*x+e)+5*sec(f*x+e)*csc(f*x+e)))
```

Fricas [F]

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{(a\sin\left(fx+e\right))^{\frac{7}{2}}}{(b\sec\left(fx+e\right))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(7/2)/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(-(a^3*cos(f*x + e)^2 - a^3)*sqrt(b*sec(f*x + e))*sqrt(a*sin(f*x +
 e))*sin(f*x + e)/(b^2*sec(f*x + e)^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\sec(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))**(7/2)/(b*sec(f*x+e))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{(a\sin\left(fx+e\right))^{\frac{7}{2}}}{(b\sec\left(fx+e\right))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(7/2)/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^(7/2)/(b*sec(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{7}{2}}}{(b\sec(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(7/2)/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e))^(7/2)/(b*sec(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{(a\sin(e+fx))^{7/2}}{\left(\frac{b}{\cos(e+fx)}\right)^{3/2}}\,dx$$

input
```
int((a*sin(e + f*x))^(7/2)/(b/cos(e + f*x))^(3/2),x)
```

output
```
int((a*sin(e + f*x))^(7/2)/(b/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\left(\int \frac{\sqrt{\sin(fx+e)}\,\sqrt{\sec(fx+e)}\,\sin(fx+e)^3}{\sec(fx+e)^2}dx\right)a^3}{b^2}$$

input
```
int((a*sin(f*x+e))^(7/2)/(b*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(b)*sqrt(a)*int((sqrt(sin(e + f*x))*sqrt(sec(e + f*x))*sin(e + f*x)**
3)/sec(e + f*x)**2,x)*a**3)/b**2
```

3.478 $\int \frac{(a \sin(e+fx))^{3/2}}{(b \sec(e+fx))^{3/2}}\, dx$

Optimal result

Integrand size = 25, antiderivative size = 135

$$\int \frac{(a \sin(e+fx))^{3/2}}{(b \sec(e+fx))^{3/2}}\, dx = -\frac{a\sqrt{a \sin(e+fx)}}{6bf\sqrt{b \sec(e+fx)}} + \frac{(a \sin(e+fx))^{5/2}}{3abf\sqrt{b \sec(e+fx)}} + \frac{a^2 \operatorname{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sqrt{b \sec(e+fx)}\sqrt{\sin(2e+2fx)}}{12b^2f\sqrt{a \sin(e+fx)}}$$

output
```
-1/6*a*(a*sin(f*x+e))^(1/2)/b/f/(b*sec(f*x+e))^(1/2)+1/3*(a*sin(f*x+e))^(5
/2)/a/b/f/(b*sec(f*x+e))^(1/2)+1/12*a^2*InverseJacobiAM(e-1/4*Pi+f*x,2^(1/
2))*(b*sec(f*x+e))^(1/2)*sin(2*f*x+2*e)^(1/2)/b^2/f/(a*sin(f*x+e))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.83 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.64

$$\int \frac{(a \sin(e+fx))^{3/2}}{(b \sec(e+fx))^{3/2}}\, dx = \frac{a\sqrt{a \sin(e+fx)}\left(-2\cos(2(e+fx)) + \csc^2(e+fx) \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\right.\right.}{12bf\sqrt{b \sec(e+fx)}}$$

input
```
Integrate[(a*Sin[e + f*x])^(3/2)/(b*Sec[e + f*x])^(3/2),x]
```

output
```
(a*Sqrt[a*Sin[e + f*x]]*(-2*Cos[2*(e + f*x)] + Csc[e + f*x]^2*Hypergeometr
ic2F1[1/2, 3/4, 3/2, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(3/4)))/(12*b*f*Sqr
t[b*Sec[e + f*x]])
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3062, 3042, 3063, 3042, 3065, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx))^{3/2}}{(b\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx))^{3/2}}{(b\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3062}$$

$$\frac{\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}dx}{6b^2}+\frac{(a\sin(e+fx))^{5/2}}{3abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}dx}{6b^2}+\frac{(a\sin(e+fx))^{5/2}}{3abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3063}$$

$$\frac{\frac{1}{2}a^2\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}dx-\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}}}{6b^2}+\frac{(a\sin(e+fx))^{5/2}}{3abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{2}a^2\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}dx-\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}}}{6b^2}+\frac{(a\sin(e+fx))^{5/2}}{3abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3065}$$

$$\frac{\frac{1}{2}a^2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\int\frac{1}{\sqrt{b\cos(e+fx)}\sqrt{a\sin(e+fx)}}dx-\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}}}{6b^2}+\frac{(a\sin(e+fx))^{5/2}}{3abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{2}a^2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\int\frac{1}{\sqrt{b\cos(e+fx)}\sqrt{a\sin(e+fx)}}dx-\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}}}{6b^2}+\frac{(a\sin(e+fx))^{5/2}}{3abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3053}$$

$$\frac{\frac{a^2\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}\int\frac{1}{\sqrt{\sin(2e+2fx)}}dx}{2\sqrt{a\sin(e+fx)}}-\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}}}{6b^2}+\frac{(a\sin(e+fx))^{5/2}}{3abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{a^2\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}\int\frac{1}{\sqrt{\sin(2e+2fx)}}dx}{2\sqrt{a\sin(e+fx)}}-\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}}}{6b^2}+\frac{(a\sin(e+fx))^{5/2}}{3abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{a^2\sqrt{\sin(2e+2fx)}\,\text{EllipticF}\left(e+fx-\frac{\pi}{4},2\right)\sqrt{b\sec(e+fx)}}{2f\sqrt{a\sin(e+fx)}}-\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}}}{6b^2}+\frac{(a\sin(e+fx))^{5/2}}{3abf\sqrt{b\sec(e+fx)}}$$

input
```
Int[(a*Sin[e + f*x])^(3/2)/(b*Sec[e + f*x])^(3/2),x]
```

output
```
(a*Sin[e + f*x])^(5/2)/(3*a*b*f*Sqrt[b*Sec[e + f*x]]) + (-((a*b*Sqrt[a*Sin
[e + f*x]])/(f*Sqrt[b*Sec[e + f*x]])) + (a^2*EllipticF[e - Pi/4 + f*x, 2]*
Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(2*f*Sqrt[a*Sin[e + f*x]]))/(
6*b^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3062
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n + 1)/(a
*b*f*(m - n))), x] - Simp[(n + 1)/(b^2*(m - n))   Int[(a*Sin[e + f*x])^m*(b
*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

rule 3063
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*b*(a*Sin[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n
- 1)/(f*(m - n))), x] + Simp[a^2*((m - 1)/(m - n))   Int[(a*Sin[e + f*x])^(
m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

rule 3065
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n   Int[(a*Sin[e
+ f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Int
egerQ[m - 1/2] && IntegerQ[n - 1/2]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.41 (sec) , antiderivative size = 996, normalized size of antiderivative = 7.38

method	result	size
default	Expression too large to display	996

input
```
int((a*sin(f*x+e))^(3/2)/(b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/384/f*a/b*(sin(f*x+e)*cos(f*x+e)*(-16*cos(f*x+e)^2+8)+(-3*cos(f*x+e)-3)*
ln(-(cos(f*x+e)*cot(f*x+e)-2*cot(f*x+e)+2*(-2*sin(f*x+e)*cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)*sin(f*x+e)+csc(f*x+e)-2*cos(f*x+e)-sin(f*x+e)+2)/(-1+cos
(f*x+e)))*(-2*sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+(3*cos(f*x+e)+
3)*ln(-(cos(f*x+e)*cot(f*x+e)-2*cot(f*x+e)-2*(-2*sin(f*x+e)*cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)*sin(f*x+e)+csc(f*x+e)-2*cos(f*x+e)-sin(f*x+e)+2)/(-1+
cos(f*x+e)))*(-2*sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+(-6*cos(f*x
+e)-6)*arctan(((-2*sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*sin(f*x+e
)+cos(f*x+e)-1)/(-1+cos(f*x+e)))*(-2*sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)+(-6*cos(f*x+e)-6)*arctan(((-2*sin(f*x+e)*cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)*sin(f*x+e)-cos(f*x+e)+1)/(-1+cos(f*x+e)))*(-2*sin(f*x+e)*cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)+(6*cos(f*x+e)+6)*(csc(f*x+e)-cot(f*x+e)+1)^(1
/2)*(-2*csc(f*x+e)+2*cot(f*x+e)+2)^(1/2)*(-csc(f*x+e)+cot(f*x+e))^(1/2)*El
lipticPi((csc(f*x+e)-cot(f*x+e)+1)^(1/2),1/2-1/2*I,1/2*2^(1/2))+(6*cos(f*x
+e)+6)*(csc(f*x+e)-cot(f*x+e)+1)^(1/2)*(-2*csc(f*x+e)+2*cot(f*x+e)+2)^(1/2
)*(-csc(f*x+e)+cot(f*x+e))^(1/2)*EllipticPi((csc(f*x+e)-cot(f*x+e)+1)^(1/2
),1/2+1/2*I,1/2*2^(1/2))+(-8*cos(f*x+e)-8)*(csc(f*x+e)-cot(f*x+e)+1)^(1/2)
*(-2*csc(f*x+e)+2*cot(f*x+e)+2)^(1/2)*(-csc(f*x+e)+cot(f*x+e))^(1/2)*Ellip
ticF((csc(f*x+e)-cot(f*x+e)+1)^(1/2),1/2*2^(1/2))+I*(6*cos(f*x+e)+6)*(csc(
f*x+e)-cot(f*x+e)+1)^(1/2)*(-2*csc(f*x+e)+2*cot(f*x+e)+2)^(1/2)*(-csc(f...
```

Fricas [F]

$$\int \frac{(a\sin(e+fx))^{3/2}}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{3}{2}}}{(b\sec(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(3/2)/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(f*x + e))*sqrt(a*sin(f*x + e))*a*sin(f*x + e)/(b^2*sec
(f*x + e)^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{3/2}}{(b\sec(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a*sin(f*x+e))**(3/2)/(b*sec(f*x+e))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a\sin(e+fx))^{3/2}}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{3}{2}}}{(b\sec(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(3/2)/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^(3/2)/(b*sec(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{(a\sin(e+fx))^{3/2}}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{(a\sin(fx+e))^{\frac{3}{2}}}{(b\sec(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*sin(f*x+e))^(3/2)/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e))^(3/2)/(b*sec(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a\sin(e+fx))^{3/2}}{(b\sec(e+fx))^{3/2}}\,dx = \int \frac{(a\sin(e+fx))^{3/2}}{\left(\frac{b}{\cos(e+fx)}\right)^{3/2}}\,dx$$

input
```
int((a*sin(e + f*x))^(3/2)/(b/cos(e + f*x))^(3/2),x)
```

output
```
int((a*sin(e + f*x))^(3/2)/(b/cos(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(a\sin(e+fx))^{3/2}}{(b\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\left(\int \frac{\sqrt{\sin(fx+e)}\,\sqrt{\sec(fx+e)}\,\sin(fx+e)}{\sec(fx+e)^2}dx\right)a}{b^2}$$

input
```
int((a*sin(f*x+e))^(3/2)/(b*sec(f*x+e))^(3/2),x)
```

output
```
(sqrt(b)*sqrt(a)*int((sqrt(sin(e + f*x))*sqrt(sec(e + f*x))*sin(e + f*x))/
sec(e + f*x)**2,x)*a)/b**2
```

3.479 $\int \frac{1}{(b\sec(e+fx))^{3/2}\sqrt{a\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 94

$$\int \frac{1}{(b\sec(e+fx))^{3/2}\sqrt{a\sin(e+fx)}}\,dx = \frac{\sqrt{a\sin(e+fx)}}{abf\sqrt{b\sec(e+fx)}} + \frac{\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}{2b^2f\sqrt{a\sin(e+fx)}}$$

output
```
(a*sin(f*x+e))^(1/2)/a/b/f/(b*sec(f*x+e))^(1/2)+1/2*InverseJacobiAM(e-1/4*
Pi+f*x,2^(1/2))*(b*sec(f*x+e))^(1/2)*sin(2*f*x+2*e)^(1/2)/b^2/f/(a*sin(f*x
+e))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.89

$$\int \frac{1}{(b\sec(e+fx))^{3/2}\sqrt{a\sin(e+fx)}}\,dx = -\frac{\cot(e+fx)\sqrt{b\sec(e+fx)}\left(-1+\cos(2(e+fx))-\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\sec^2(e+fx)\right)\left(-\tan^2(\right.\right.}{2b^2f\sqrt{a\sin(e+fx)}}$$

input
```
Integrate[1/((b*Sec[e + f*x])^(3/2)*Sqrt[a*Sin[e + f*x]]),x]
```

output
```
-1/2*(Cot[e + f*x]*Sqrt[b*Sec[e + f*x]]*(-1 + Cos[2*(e + f*x)] - Hypergeom
etric2F1[1/2, 3/4, 3/2, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(3/4)))/(b^2*f*S
qrt[a*Sin[e + f*x]])
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3062, 3042, 3065, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\sin(e+fx)}(b\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\sin(e+fx)}(b\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3062}$$

$$\frac{\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}dx}{2b^2}+\frac{\sqrt{a\sin(e+fx)}}{abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}dx}{2b^2}+\frac{\sqrt{a\sin(e+fx)}}{abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3065}$$

$$\frac{\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{b\cos(e+fx)}\sqrt{a\sin(e+fx)}}dx}{2b^2}+\frac{\sqrt{a\sin(e+fx)}}{abf\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{b\cos(e+fx)}\sqrt{a\sin(e+fx)}}dx}{2b^2}+\frac{\sqrt{a\sin(e+fx)}}{abf\sqrt{b\sec(e+fx)}}$$

$\big\downarrow$ 3053

$$\frac{\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{\sin(2e+2fx)}}dx}{2b^2\sqrt{a\sin(e+fx)}}+\frac{\sqrt{a\sin(e+fx)}}{abf\sqrt{b\sec(e+fx)}}$$

$\big\downarrow$ 3042

$$\frac{\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{\sin(2e+2fx)}}dx}{2b^2\sqrt{a\sin(e+fx)}}+\frac{\sqrt{a\sin(e+fx)}}{abf\sqrt{b\sec(e+fx)}}$$

$\big\downarrow$ 3120

$$\frac{\sqrt{\sin(2e+2fx)}\,\text{EllipticF}\left(e+fx-\frac{\pi}{4},2\right)\sqrt{b\sec(e+fx)}}{2b^2f\sqrt{a\sin(e+fx)}}+\frac{\sqrt{a\sin(e+fx)}}{abf\sqrt{b\sec(e+fx)}}$$

input
```
Int[1/((b*Sec[e + f*x])^(3/2)*Sqrt[a*Sin[e + f*x]]),x]
```

output
```
Sqrt[a*Sin[e + f*x]]/(a*b*f*Sqrt[b*Sec[e + f*x]]) + (EllipticF[e - Pi/4 +
f*x, 2]*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(2*b^2*f*Sqrt[a*Sin[e
 + f*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3062
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n + 1)/(a
*b*f*(m - n))), x] - Simp[(n + 1)/(b^2*(m - n))   Int[(a*Sin[e + f*x])^m*(b
*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &
& NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

rule 3065
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n   Int[(a*Sin[e
+ f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Int
egerQ[m - 1/2] && IntegerQ[n - 1/2]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.34

method	result
default	$\frac{\sqrt{-2\csc(fx+e)+2\cot(fx+e)+2}\,\sqrt{-\csc(fx+e)+\cot(fx+e)}\,\text{EllipticF}\left(\sqrt{\csc(fx+e)-\cot(fx+e)+1},\frac{\sqrt{2}}{2}\right)\sqrt{\csc(fx+e)-\cot(fx+e}}{2f\sqrt{a\sin(fx+e)}\,\sqrt{b\sec(fx+e)}\,b}$

input
```
int(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/(a*sin(f*x+e))^(1/2)/(b*sec(f*x+e))^(1/2)/b*((-2*csc(f*x+e)+2*cot(f*
x+e)+2)^(1/2)*(-csc(f*x+e)+cot(f*x+e))^(1/2)*EllipticF((csc(f*x+e)-cot(f*x
+e)+1)^(1/2),1/2*2^(1/2))*(csc(f*x+e)-cot(f*x+e)+1)^(1/2)*(sec(f*x+e)+1)+2
*sin(f*x+e))
```

Fricas [F]

$$\int \frac{1}{(b\sec(e+fx))^{3/2}\sqrt{a\sin(e+fx)}}\,dx = \int \frac{1}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{a\sin\left(fx+e\right)}}\,dx$$

input
```
integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(1/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(b*sec(f*x + e))*sqrt(a*sin(f*x + e))/(a*b^2*sec(f*x + e)^2*s
in(f*x + e)), x)
```

Sympy [F]

$$\int \frac{1}{(b\sec(e+fx))^{3/2}\sqrt{a\sin(e+fx)}}\,dx = \int \frac{1}{\sqrt{a\sin\left(e+fx\right)}\left(b\sec\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*sec(f*x+e))**(3/2)/(a*sin(f*x+e))**(1/2),x)
```

output
```
Integral(1/(sqrt(a*sin(e + f*x))*(b*sec(e + f*x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(b\sec(e+fx))^{3/2}\sqrt{a\sin(e+fx)}}\,dx = \int \frac{1}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{a\sin\left(fx+e\right)}}\,dx$$

input
```
integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/((b*sec(f*x + e))^(3/2)*sqrt(a*sin(f*x + e))), x)
```

Giac [F]

$$\int \frac{1}{(b\sec(e+fx))^{3/2}\sqrt{a\sin(e+fx)}}\,dx = \int \frac{1}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{a\sin\left(fx+e\right)}}\,dx$$

input
```
integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(f*x + e))^(3/2)*sqrt(a*sin(f*x + e))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\sec(e+fx))^{3/2}\sqrt{a\sin(e+fx)}}\,dx = \int \frac{1}{\sqrt{a\,\sin\left(e+f\,x\right)}\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

input
```
int(1/((a*sin(e + f*x))^(1/2)*(b/cos(e + f*x))^(3/2)),x)
```

output
```
int(1/((a*sin(e + f*x))^(1/2)*(b/cos(e + f*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(b\sec(e+fx))^{3/2}\sqrt{a\sin(e+fx)}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\left(\int \frac{\sqrt{\sin(fx+e)}\,\sqrt{\sec(fx+e)}}{\sec(fx+e)^{2}\sin(fx+e)}dx\right)}{a\,b^{2}}$$

input
```
int(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(1/2),x)
```

output
```
(sqrt(b)*sqrt(a)*int((sqrt(sin(e + f*x))*sqrt(sec(e + f*x)))/(sec(e + f*x)
**2*sin(e + f*x)),x))/(a*b**2)
```

3.480 $\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 100

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{5/2}}\,dx = -\frac{2}{3abf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}} - \frac{\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}{3a^2b^2f\sqrt{a\sin(e+fx)}}$$

output
```
-2/3/a/b/f/(b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(3/2)-1/3*InverseJacobiAM(e
-1/4*Pi+f*x,2^(1/2))*(b*sec(f*x+e))^(1/2)*sin(2*f*x+2*e)^(1/2)/a^2/b^2/f/(
a*sin(f*x+e))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.78

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{5/2}}\,dx = -\frac{\cot(e+fx)\sqrt{b\sec(e+fx)}\left(2+\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\sec^2(e+fx)\right)\left(-\tan^2(e+fx)\right)^{3/4}\right)}{3a^2b^2f\sqrt{a\sin(e+fx)}}$$

input
```
Integrate[1/((b*Sec[e + f*x])^(3/2)*(a*Sin[e + f*x])^(5/2)),x]
```

output
```
-1/3*(Cot[e + f*x]*Sqrt[b*Sec[e + f*x]]*(2 + Hypergeometric2F1[1/2, 3/4, 3
/2, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(3/4)))/(a^2*b^2*f*Sqrt[a*Sin[e + f*
x]])
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3061, 3042, 3065, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sin(e+fx))^{5/2}(b\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\sin(e+fx))^{5/2}(b\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3061}$$

$$-\frac{\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}dx}{3a^2b^2}-\frac{2}{3abf(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}dx}{3a^2b^2}-\frac{2}{3abf(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3065}$$

$$-\frac{\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{b\cos(e+fx)}\sqrt{a\sin(e+fx)}}dx}{3a^2b^2}-\frac{2}{3abf(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\int\frac{1}{\sqrt{b\cos(e+fx)}\sqrt{a\sin(e+fx)}}dx}{3a^2b^2}-\frac{2}{3abf(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}$$

$\big\downarrow$ 3053

$$-\frac{\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}\int\frac{1}{\sqrt{\sin(2e+2fx)}}dx}{3a^2b^2\sqrt{a\sin(e+fx)}}-\frac{2}{3abf(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}$$

$\big\downarrow$ 3042

$$-\frac{\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}\int\frac{1}{\sqrt{\sin(2e+2fx)}}dx}{3a^2b^2\sqrt{a\sin(e+fx)}}-\frac{2}{3abf(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}$$

$\big\downarrow$ 3120

$$-\frac{\sqrt{\sin(2e+2fx)}\,\text{EllipticF}\left(e+fx-\frac{\pi}{4},2\right)\sqrt{b\sec(e+fx)}}{3a^2b^2f\sqrt{a\sin(e+fx)}}-\frac{2}{3abf(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}$$

input
```
Int[1/((b*Sec[e + f*x])^(3/2)*(a*Sin[e + f*x])^(5/2)),x]
```

output
```
-2/(3*a*b*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(3/2)) - (EllipticF[e -
Pi/4 + f*x, 2]*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(3*a^2*b^2*f*S
qrt[a*Sin[e + f*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3061
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n + 1)/(a
*b*f*(m + 1))), x] - Simp[(n + 1)/(a^2*b^2*(m + 1))   Int[(a*Sin[e + f*x])^
(m + 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n
, -1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3065
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n   Int[(a*Sin[e
+ f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Int
egerQ[m - 1/2] && IntegerQ[n - 1/2]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.29

method	result
default	$-\frac{\sqrt{-2\csc(fx+e)+2\cot(fx+e)+2}\,\sqrt{-\csc(fx+e)+\cot(fx+e)}\,\text{EllipticF}\left(\sqrt{\csc(fx+e)-\cot(fx+e)+1},\frac{\sqrt{2}}{2}\right)\sqrt{\csc(fx+e)-\cot(fx}{3f\sqrt{a\sin(fx+e)}\,\sqrt{b\sec(fx+e)}\,a^2b}$

input
```
int(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/f/(a*sin(f*x+e))^(1/2)/(b*sec(f*x+e))^(1/2)/a^2/b*((-2*csc(f*x+e)+2*c
ot(f*x+e)+2)^(1/2)*(-csc(f*x+e)+cot(f*x+e))^(1/2)*EllipticF((csc(f*x+e)-co
t(f*x+e)+1)^(1/2),1/2*2^(1/2))*(csc(f*x+e)-cot(f*x+e)+1)^(1/2)*(sec(f*x+e)
+1)+2*csc(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.31

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{5/2}}\,dx = \frac{\sqrt{i\,ab}\big(\cos{(fx+e)}^2-1\big)F(\arcsin{(\cos{(fx+e)}+i\,\sin{(fx+}}{}$$

input
```
integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(5/2),x, algorithm="fricas
")
```

output
```
1/3*(sqrt(I*a*b)*(cos(f*x + e)^2 - 1)*elliptic_f(arcsin(cos(f*x + e) + I*s
in(f*x + e)), -1) + sqrt(-I*a*b)*(cos(f*x + e)^2 - 1)*elliptic_f(arcsin(co
s(f*x + e) - I*sin(f*x + e)), -1) + 2*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x
+ e))*cos(f*x + e))/(a^3*b^2*f*cos(f*x + e)^2 - a^3*b^2*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*sec(f*x+e))**(3/2)/(a*sin(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{5/2}}\,dx = \int \frac{1}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(5/2),x, algorithm="maxima
")
```

output
```
integrate(1/((b*sec(f*x + e))^(3/2)*(a*sin(f*x + e))^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{5/2}}\,dx = \int \frac{1}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(f*x + e))^(3/2)*(a*sin(f*x + e))^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{5/2}}\,dx = \int \frac{1}{\left(a\,\sin\left(e+f\,x\right)\right)^{5/2}\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

input
```
int(1/((a*sin(e + f*x))^(5/2)*(b/cos(e + f*x))^(3/2)),x)
```

output
```
int(1/((a*sin(e + f*x))^(5/2)*(b/cos(e + f*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{5/2}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\left(\int \frac{\sqrt{\sin(fx+e)}\,\sqrt{\sec(fx+e)}}{\sec(fx+e)^2\sin(fx+e)^3}dx\right)}{a^3b^2}$$

input
```
int(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(5/2),x)
```

output
```
(sqrt(b)*sqrt(a)*int((sqrt(sin(e + f*x))*sqrt(sec(e + f*x)))/(sec(e + f*x)
**2*sin(e + f*x)**3),x))/(a**3*b**2)
```

3.481 $\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{9/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 137

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{9/2}}\,dx = -\frac{2}{7abf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}} + \frac{2}{21a^3bf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}} - \frac{2\,\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}{21a^4b^2f\sqrt{a\sin(e+fx)}}$$

output
```
-2/7/a/b/f/(b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(7/2)+2/21/a^3/b/f/(b*sec(f
*x+e))^(1/2)/(a*sin(f*x+e))^(3/2)-2/21*InverseJacobiAM(e-1/4*Pi+f*x,2^(1/2
))*(b*sec(f*x+e))^(1/2)*sin(2*f*x+2*e)^(1/2)/a^4/b^2/f/(a*sin(f*x+e))^(1/2
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.93 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.87

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{9/2}}\,dx = \frac{\cos(2(e+fx))\csc^4(e+fx)\sqrt{a\sin(e+fx)}\Big((5+\cos(2(e+f}{21a^5bf\sqrt{}}$$

input
```
Integrate[1/((b*Sec[e + f*x])^(3/2)*(a*Sin[e + f*x])^(9/2)),x]
```

output
```
(Cos[2*(e + f*x)]*Csc[e + f*x]^4*Sqrt[a*Sin[e + f*x]]*((5 + Cos[2*(e + f*x
)])*Sec[e + f*x]^2 - 2*Hypergeometric2F1[1/2, 3/4, 3/2, Sec[e + f*x]^2]*(-
Tan[e + f*x]^2)^(7/4)))/(21*a^5*b*f*Sqrt[b*Sec[e + f*x]]*(-2 + Sec[e + f*x
]^2))
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3061, 3042, 3064, 3042, 3065, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sin(e+fx))^{9/2}(b\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\sin(e+fx))^{9/2}(b\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3061}$$

$$-\frac{\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{5/2}}dx}{7a^2b^2} - \frac{2}{7abf(a\sin(e+fx))^{7/2}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{5/2}}dx}{7a^2b^2}-\frac{2}{7abf(a\sin(e+fx))^{7/2}\sqrt{b\sec(e+fx)}}$$

$\downarrow$ 3064

$$-\frac{\frac{2\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}dx}{3a^2}-\frac{2b}{3af(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}}{7a^2b^2}-\frac{2}{7abf(a\sin(e+fx))^{7/2}\sqrt{b\sec(e+fx)}}$$

$\downarrow$ 3042

$$-\frac{\frac{2\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}dx}{3a^2}-\frac{2b}{3af(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}}{7a^2b^2}-\frac{2}{7abf(a\sin(e+fx))^{7/2}\sqrt{b\sec(e+fx)}}$$

$\downarrow$ 3065

$$-\frac{\frac{2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{b\cos(e+fx)}\sqrt{a\sin(e+fx)}}dx}{3a^2}-\frac{2b}{3af(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}}{7a^2b^2}-\frac{2}{7abf(a\sin(e+fx))^{7/2}\sqrt{b\sec(e+fx)}}$$

$\downarrow$ 3042

$$-\frac{\frac{2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{b\cos(e+fx)}\sqrt{a\sin(e+fx)}}dx}{3a^2}-\frac{2b}{3af(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}}{7a^2b^2}-\frac{2}{7abf(a\sin(e+fx))^{7/2}\sqrt{b\sec(e+fx)}}$$

$\downarrow$ 3053

$$-\frac{\frac{2\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{\sin(2e+2fx)}}dx}{3a^2\sqrt{a\sin(e+fx)}}-\frac{2b}{3af(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}}{7a^2b^2}-\frac{2}{7abf(a\sin(e+fx))^{7/2}\sqrt{b\sec(e+fx)}}$$

$\downarrow$ 3042

$$-\frac{\frac{2\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{\sin(2e+2fx)}}dx}{3a^2\sqrt{a\sin(e+fx)}}-\frac{2b}{3af(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}}{7a^2b^2}-\frac{2}{7abf(a\sin(e+fx))^{7/2}\sqrt{b\sec(e+fx)}}$$

$\downarrow$ 3120

$$-\frac{\frac{2\sqrt{\sin(2e+2fx)}\,\text{EllipticF}\left(e+fx-\frac{\pi}{4},2\right)\sqrt{b\sec(e+fx)}}{3a^2f\sqrt{a\sin(e+fx)}}-\frac{2b}{3af(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}}{7a^2b^2}-\frac{2}{7abf(a\sin(e+fx))^{7/2}\sqrt{b\sec(e+fx)}}$$

input
```
Int[1/((b*Sec[e + f*x])^(3/2)*(a*Sin[e + f*x])^(9/2)),x]
```

output
```
-2/(7*a*b*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(7/2)) - ((-2*b)/(3*a*f*
Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(3/2)) + (2*EllipticF[e - Pi/4 + f*x
, 2]*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(3*a^2*f*Sqrt[a*Sin[e +
f*x]]))/(7*a^2*b^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3061
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n + 1)/(a
*b*f*(m + 1))), x] - Simp[(n + 1)/(a^2*b^2*(m + 1))   Int[(a*Sin[e + f*x])^
(m + 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n
, -1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3064
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)/
(a*f*(m + 1))), x] + Simp[(m - n + 2)/(a^2*(m + 1))   Int[(a*Sin[e + f*x])^
(m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

rule 3065
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n   Int[(a*Sin[e
+ f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Int
egerQ[m - 1/2] && IntegerQ[n - 1/2]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.05

method	result
default	$\frac{2\sqrt{\csc(fx+e)-\cot(fx+e)+1}\,\sqrt{-2\csc(fx+e)+2\cot(fx+e)+2}\,\sqrt{-\csc(fx+e)+\cot(fx+e)}\ \text{EllipticF}\left(\sqrt{\csc(fx+e)-\cot(fx+e)+1},\frac{\sqrt{2}}{2}\right)(-1-\sec(fx-}{21}$ / $f\sqrt{a\sin(fx+e)}\,\sqrt{b\sec(fx+e)}\,a^4 b$

input
```
int(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/21/f/(a*sin(f*x+e))^(1/2)/(b*sec(f*x+e))^(1/2)/a^4/b*((csc(f*x+e)-cot(f*
x+e)+1)^(1/2)*(-2*csc(f*x+e)+2*cot(f*x+e)+2)^(1/2)*(-csc(f*x+e)+cot(f*x+e)
)^(1/2)*EllipticF((csc(f*x+e)-cot(f*x+e)+1)^(1/2),1/2*2^(1/2))*(-1-sec(f*x
+e))+(-cos(f*x+e)^2-2)*csc(f*x+e)^3)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.30

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{9/2}}\,dx = \frac{2\left(\left(\cos\left(fx+e\right)^4-2\,\cos\left(fx+e\right)^2+1\right)\sqrt{i\,ab}F(\arcsin\left(\cos\left(f\right.\right.}{}$$

input
```
integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(9/2),x, algorithm="fricas
")
```

output
```
2/21*((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(I*a*b)*elliptic_f(arcsi
n(cos(f*x + e) + I*sin(f*x + e)), -1) + (cos(f*x + e)^4 - 2*cos(f*x + e)^2
 + 1)*sqrt(-I*a*b)*elliptic_f(arcsin(cos(f*x + e) - I*sin(f*x + e)), -1) -
 (cos(f*x + e)^3 + 2*cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e
)))/(a^5*b^2*f*cos(f*x + e)^4 - 2*a^5*b^2*f*cos(f*x + e)^2 + a^5*b^2*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*sec(f*x+e))**(3/2)/(a*sin(f*x+e))**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{9/2}}\,dx = \int \frac{1}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(9/2),x, algorithm="maxima
")
```

output
```
integrate(1/((b*sec(f*x + e))^(3/2)*(a*sin(f*x + e))^(9/2)), x)
```

Giac [F]

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{9/2}}\,dx = \int \frac{1}{(b\sec(fx+e))^{\frac{3}{2}}(a\sin(fx+e))^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(9/2),x, algorithm="giac")
```

output
```
integrate(1/((b*sec(f*x + e))^(3/2)*(a*sin(f*x + e))^(9/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{9/2}}\,dx = \int \frac{1}{(a\sin(e+fx))^{9/2}\left(\frac{b}{\cos(e+fx)}\right)^{3/2}}\,dx$$

input
```
int(1/((a*sin(e + f*x))^(9/2)*(b/cos(e + f*x))^(3/2)),x)
```

output
```
int(1/((a*sin(e + f*x))^(9/2)*(b/cos(e + f*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{9/2}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\left(\int \frac{\sqrt{\sin(fx+e)}\,\sqrt{\sec(fx+e)}}{\sec(fx+e)^2\sin(fx+e)^5}dx\right)}{a^5b^2}$$

input
```
int(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(9/2),x)
```

output
```
(sqrt(b)*sqrt(a)*int((sqrt(sin(e + f*x))*sqrt(sec(e + f*x)))/(sec(e + f*x)
**2*sin(e + f*x)**5),x))/(a**5*b**2)
```

3.482 $\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{13/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 174

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{13/2}}\,dx = -\frac{2}{11abf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{11/2}}+\frac{2}{77a^3bf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{7/2}}+\frac{4}{77a^5bf\sqrt{b\sec(e+fx)}(a\sin(e+fx))^{3/2}}-\frac{4\,\text{EllipticF}\left(e-\frac{\pi}{4}+fx,2\right)\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}{77a^6b^2f\sqrt{a\sin(e+fx)}}$$

output
```
-2/11/a/b/f/(b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(11/2)+2/77/a^3/b/f/(b*sec
(f*x+e))^(1/2)/(a*sin(f*x+e))^(7/2)+4/77/a^5/b/f/(b*sec(f*x+e))^(1/2)/(a*s
in(f*x+e))^(3/2)-4/77*InverseJacobiAM(e-1/4*Pi+f*x,2^(1/2))*(b*sec(f*x+e))
^(1/2)*sin(2*f*x+2*e)^(1/2)/a^6/b^2/f/(a*sin(f*x+e))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.47 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.75

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{13/2}}\,dx = \frac{2\cot(2(e+fx))\csc(2(e+fx))\sqrt{a\sin(e+fx)}\Big((23+6\cos(2}{}$$

input
```
Integrate[1/((b*Sec[e + f*x])^(3/2)*(a*Sin[e + f*x])^(13/2)),x]
```

output
```
(2*Cot[2*(e + f*x)]*Csc[2*(e + f*x)]*Sqrt[a*Sin[e + f*x]]*((23 + 6*Cos[2*(
e + f*x)] - Cos[4*(e + f*x)])*Csc[e + f*x]^4 + 8*Hypergeometric2F1[1/2, 3/
4, 3/2, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(3/4)))/(77*a^7*b*f*Sqrt[b*Sec[e
 + f*x]]*(-2 + Sec[e + f*x]^2))
```

Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3042, 3061, 3042, 3064, 3042, 3064, 3042, 3065, 3042, 3053, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sin(e+fx))^{13/2}(b\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\sin(e+fx))^{13/2}(b\sec(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3061}$$

$$-\frac{\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{9/2}}dx}{11a^2b^2}-\frac{2}{11abf(a\sin(e+fx))^{11/2}\sqrt{b\sec(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{9/2}}dx}{11a^2b^2}-\frac{2}{11abf(a\sin(e+fx))^{11/2}\sqrt{b\sec(e+fx)}}$$

$\Big\downarrow$ 3064

$$-\frac{\frac{6\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{5/2}}dx}{7a^2}-\frac{2b}{7af(a\sin(e+fx))^{7/2}\sqrt{b\sec(e+fx)}}}{11a^2b^2}-\frac{2}{11abf(a\sin(e+fx))^{11/2}\sqrt{b\sec(e+fx)}}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{6\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{5/2}}dx}{7a^2}-\frac{2b}{7af(a\sin(e+fx))^{7/2}\sqrt{b\sec(e+fx)}}}{11a^2b^2}-\frac{2}{11abf(a\sin(e+fx))^{11/2}\sqrt{b\sec(e+fx)}}$$

$\Big\downarrow$ 3064

$$-\frac{\frac{6\left(\frac{2\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}dx}{3a^2}-\frac{2b}{3af(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}\right)}{7a^2}-\frac{2b}{7af(a\sin(e+fx))^{7/2}\sqrt{b\sec(e+fx)}}}{11a^2b^2}-\frac{2}{11abf(a\sin(e+fx))^{11/2}\sqrt{b\sec(e+fx)}}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{6\left(\frac{2\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}dx}{3a^2}-\frac{2b}{3af(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}\right)}{7a^2}-\frac{2b}{7af(a\sin(e+fx))^{7/2}\sqrt{b\sec(e+fx)}}}{11a^2b^2}-\frac{2}{11abf(a\sin(e+fx))^{11/2}\sqrt{b\sec(e+fx)}}$$

$\Big\downarrow$ 3065

$$-\frac{\frac{6\left(\frac{2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\int \frac{1}{\sqrt{b\cos(e+fx)}\sqrt{a\sin(e+fx)}}dx}{3a^2}-\frac{2b}{3af(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}\right)}{7a^2}-\frac{2b}{7af(a\sin(e+fx))^{7/2}\sqrt{b\sec(e+fx)}}}{11a^2b^2}-\frac{2}{11abf(a\sin(e+fx))^{11/2}\sqrt{b\sec(e+fx)}}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{6\left(\frac{2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\int\frac{1}{\sqrt{b\cos(e+fx)}\sqrt{a\sin(e+fx)}}dx}{3a^2}-\frac{2b}{3af(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}\right)}{7a^2}-\frac{2b}{7af(a\sin(e+fx))^{7/2}\sqrt{b\sec(e+fx)}}}{11a^2b^2}-\frac{2}{11abf(a\sin(e+fx))^{11/2}\sqrt{b\sec(e+fx)}}$$

$\Big\downarrow$ 3053

$$-\frac{\frac{6\left(\frac{2\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}\int\frac{1}{\sqrt{\sin(2e+2fx)}}dx}{3a^2\sqrt{a\sin(e+fx)}}-\frac{2b}{3af(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}\right)}{7a^2}-\frac{2b}{7af(a\sin(e+fx))^{7/2}\sqrt{b\sec(e+fx)}}}{11a^2b^2}-\frac{2}{11abf(a\sin(e+fx))^{11/2}\sqrt{b\sec(e+fx)}}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{6\left(\frac{2\sqrt{\sin(2e+2fx)}\sqrt{b\sec(e+fx)}\int\frac{1}{\sqrt{\sin(2e+2fx)}}dx}{3a^2\sqrt{a\sin(e+fx)}}-\frac{2b}{3af(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}\right)}{7a^2}-\frac{2b}{7af(a\sin(e+fx))^{7/2}\sqrt{b\sec(e+fx)}}}{11a^2b^2}-\frac{2}{11abf(a\sin(e+fx))^{11/2}\sqrt{b\sec(e+fx)}}$$

$\Big\downarrow$ 3120

$$-\frac{\frac{6\left(\frac{2\sqrt{\sin(2e+2fx)}\,\text{EllipticF}\left(e+fx-\frac{\pi}{4},2\right)\sqrt{b\sec(e+fx)}}{3a^2f\sqrt{a\sin(e+fx)}}-\frac{2b}{3af(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}\right)}{7a^2}-\frac{2b}{7af(a\sin(e+fx))^{7/2}\sqrt{b\sec(e+fx)}}}{11a^2b^2}-\frac{2}{11abf(a\sin(e+fx))^{11/2}\sqrt{b\sec(e+fx)}}$$

input
```
Int[1/((b*Sec[e + f*x])^(3/2)*(a*Sin[e + f*x])^(13/2)),x]
```

output
```
-2/(11*a*b*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(11/2)) - ((-2*b)/(7*a*
f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(7/2)) + (6*((-2*b)/(3*a*f*Sqrt[b*
Sec[e + f*x]]*(a*Sin[e + f*x])^(3/2)) + (2*EllipticF[e - Pi/4 + f*x, 2]*Sq
rt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(3*a^2*f*Sqrt[a*Sin[e + f*x]]))
)/(7*a^2))/(11*a^2*b^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3053
```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Simp[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]])   Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f
}, x]
```

rule 3061
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n + 1)/(a
*b*f*(m + 1))), x] - Simp[(n + 1)/(a^2*b^2*(m + 1))   Int[(a*Sin[e + f*x])^
(m + 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n
, -1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

rule 3064
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 1)*((b*Sec[e + f*x])^(n - 1)/
(a*f*(m + 1))), x] + Simp[(m - n + 2)/(a^2*(m + 1))   Int[(a*Sin[e + f*x])^
(m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

rule 3065
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n   Int[(a*Sin[e
+ f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Int
egerQ[m - 1/2] && IntegerQ[n - 1/2]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.89

method	result
default	$-\frac{2\left(\sqrt{\csc(fx+e)-\cot(fx+e)+1}\,\sqrt{-2\csc(fx+e)+2\cot(fx+e)+2}\,\sqrt{-\csc(fx+e)+\cot(fx+e)}\ \mathrm{EllipticF}\left(\sqrt{\csc(fx+e)-\cot(fx+e}\right.\right.}{77f\sqrt{a\sin(fx+e)}\,\sqrt{b\sec(fx+e)}\,a^6b}$

input
```
int(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(13/2),x,method=_RETURNVERBOSE)
```

output
```
-2/77/f/(a*sin(f*x+e))^(1/2)/(b*sec(f*x+e))^(1/2)/a^6/b*((csc(f*x+e)-cot(f
*x+e)+1)^(1/2)*(-2*csc(f*x+e)+2*cot(f*x+e)+2)^(1/2)*(-csc(f*x+e)+cot(f*x+e
))^(1/2)*EllipticF((csc(f*x+e)-cot(f*x+e)+1)^(1/2),1/2*2^(1/2))*(2+2*sec(f
*x+e))+(-2*cos(f*x+e)^4+5*cos(f*x+e)^2+4)*csc(f*x+e)^5)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.32

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{13/2}}\,dx = \frac{2\left(2\left(\cos(fx+e)^6-3\cos(fx+e)^4+3\cos(fx+e)^2-1\right)\right.}{}$$

input
```
integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(13/2),x, algorithm="frica
s")
```

output
```
2/77*(2*(cos(f*x + e)^6 - 3*cos(f*x + e)^4 + 3*cos(f*x + e)^2 - 1)*sqrt(I*
a*b)*elliptic_f(arcsin(cos(f*x + e) + I*sin(f*x + e)), -1) + 2*(cos(f*x +
e)^6 - 3*cos(f*x + e)^4 + 3*cos(f*x + e)^2 - 1)*sqrt(-I*a*b)*elliptic_f(ar
csin(cos(f*x + e) - I*sin(f*x + e)), -1) - (2*cos(f*x + e)^5 - 5*cos(f*x +
 e)^3 - 4*cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e)))/(a^7*b^
2*f*cos(f*x + e)^6 - 3*a^7*b^2*f*cos(f*x + e)^4 + 3*a^7*b^2*f*cos(f*x + e)
^2 - a^7*b^2*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{13/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*sec(f*x+e))**(3/2)/(a*sin(f*x+e))**(13/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{13/2}}\,dx = \int \frac{1}{(b\sec(fx+e))^{\frac{3}{2}}(a\sin(fx+e))^{\frac{13}{2}}}\,dx$$

input
```
integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(13/2),x, algorithm="maxim
a")
```

output
```
integrate(1/((b*sec(f*x + e))^(3/2)*(a*sin(f*x + e))^(13/2)), x)
```

Giac [F]

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{13/2}}\,dx = \int \frac{1}{(b\sec(fx+e))^{\frac{3}{2}}(a\sin(fx+e))^{\frac{13}{2}}}\,dx$$

input
```
integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(13/2),x, algorithm="giac"
)
```

output
```
integrate(1/((b*sec(f*x + e))^(3/2)*(a*sin(f*x + e))^(13/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{13/2}}\,dx = \int \frac{1}{(a\,\sin(e+f\,x))^{13/2}\,\left(\frac{b}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/((a*sin(e + f*x))^(13/2)*(b/cos(e + f*x))^(3/2)),x)
```

output
```
int(1/((a*sin(e + f*x))^(13/2)*(b/cos(e + f*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{13/2}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\,\left(\int \frac{\sqrt{\sin(fx+e)}\,\sqrt{\sec(fx+e)}}{\sec(fx+e)^2\sin(fx+e)^7}dx\right)}{a^7b^2}$$

input
```
int(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(13/2),x)
```

output
```
(sqrt(b)*sqrt(a)*int((sqrt(sin(e + f*x))*sqrt(sec(e + f*x)))/(sec(e + f*x)
**2*sin(e + f*x)**7),x))/(a**7*b**2)
```

3.483 $\int (d\sec(a+bx))^{5/2}(c\sin(a+bx))^m\,dx$

Optimal result

Integrand size = 23, antiderivative size = 75

$$\int (d\sec(a+bx))^{5/2}(c\sin(a+bx))^m\,dx = \frac{d\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{7}{4},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(d\sec(a+b}{bc(1+m)}$$

output
```
d*(cos(b*x+a)^2)^(3/4)*hypergeom([7/4, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2
)*(d*sec(b*x+a))^(3/2)*(c*sin(b*x+a))^(1+m)/b/c/(1+m)
```

Mathematica [A] (verified)

Time = 9.17 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.28

$$\int (d\sec(a+bx))^{5/2}(c\sin(a+bx))^m\,dx = -\frac{2\cot(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{4}(5-2m),\frac{1-m}{2},\frac{1}{4}(9-2m),\sec^2(a+bx)\right)(d\sec(a+bx))^{5/2}(c\sin(a+}{b(-5+2m)}$$

input
```
Integrate[(d*Sec[a + b*x])^(5/2)*(c*Sin[a + b*x])^m,x]
```

output
```
(-2*Cot[a + b*x]*Hypergeometric2F1[(5 - 2*m)/4, (1 - m)/2, (9 - 2*m)/4, Se
c[a + b*x]^2]*(d*Sec[a + b*x])^(5/2)*(c*Sin[a + b*x])^m*(-Tan[a + b*x]^2)^
((1 - m)/2))/(b*(-5 + 2*m))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3067, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\sec(a+bx))^{5/2}(c\sin(a+bx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\sec(a+bx))^{5/2}(c\sin(a+bx))^m dx$$

$$\Big\downarrow \text{3067}$$

$$d^2(d\cos(a+bx))^{3/2}(d\sec(a+bx))^{3/2}\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$d^2(d\cos(a+bx))^{3/2}(d\sec(a+bx))^{3/2}\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{5/2}}dx$$

$$\Big\downarrow \text{3057}$$

$$\frac{d\cos^2(a+bx)^{3/4}(d\sec(a+bx))^{3/2}(c\sin(a+bx))^{m+1}\text{Hypergeometric2F1}\left(\frac{7}{4},\frac{m+1}{2},\frac{m+3}{2},\sin^2(a+bx)\right)}{bc(m+1)}$$

input
```
Int[(d*Sec[a + b*x])^(5/2)*(c*Sin[a + b*x])^m,x]
```

output
```
(d*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[7/4, (1 + m)/2, (3 + m)/2, Sin
[a + b*x]^2]*(d*Sec[a + b*x])^(3/2)*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m)
)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3067
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
   Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f,
m, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Maple [F]

$$\int (d\sec(bx+a))^{\frac{5}{2}} (c\sin(bx+a))^m \, dx$$

input
```
int((d*sec(b*x+a))^(5/2)*(c*sin(b*x+a))^m,x)
```

output
```
int((d*sec(b*x+a))^(5/2)*(c*sin(b*x+a))^m,x)
```

Fricas [F]

$$\int (d\sec(a+bx))^{5/2}(c\sin(a+bx))^m \, dx = \int (d\sec(bx+a))^{\frac{5}{2}} (c\sin(bx+a))^m \, dx$$

input
```
integrate((d*sec(b*x+a))^(5/2)*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

output
```
integral(sqrt(d*sec(b*x + a))*(c*sin(b*x + a))^m*d^2*sec(b*x + a)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d\sec(a+bx))^{5/2}(c\sin(a+bx))^m\,dx = \text{Timed out}$$

input
```
integrate((d*sec(b*x+a))**(5/2)*(c*sin(b*x+a))**m,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\sec(a+bx))^{5/2}(c\sin(a+bx))^m\,dx = \int (d\sec(bx+a))^{\frac{5}{2}}(c\sin(bx+a))^m\,dx$$

input
```
integrate((d*sec(b*x+a))^(5/2)*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

output
```
integrate((d*sec(b*x + a))^(5/2)*(c*sin(b*x + a))^m, x)
```

Giac [F]

$$\int (d\sec(a+bx))^{5/2}(c\sin(a+bx))^m\,dx = \int (d\sec(bx+a))^{\frac{5}{2}}(c\sin(bx+a))^m\,dx$$

input
```
integrate((d*sec(b*x+a))^(5/2)*(c*sin(b*x+a))^m,x, algorithm="giac")
```

output
```
integrate((d*sec(b*x + a))^(5/2)*(c*sin(b*x + a))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(a+bx))^{5/2}(c\sin(a+bx))^m\,dx = \int (c\,\sin(a+b\,x))^m\,\left(\frac{d}{\cos(a+b\,x)}\right)^{5/2}\,dx$$

input
```
int((c*sin(a + b*x))^m*(d/cos(a + b*x))^(5/2),x)
```

output
```
int((c*sin(a + b*x))^m*(d/cos(a + b*x))^(5/2), x)
```

Reduce [F]

$$\int (d\sec(a+bx))^{5/2}(c\sin(a+bx))^m\,dx = \sqrt{d}\,c^m\left(\int \sin(bx+a)^m\,\sqrt{\sec(bx+a)}\,\sec(bx+a)^2\,dx\right)d^2$$

input
```
int((d*sec(b*x+a))^(5/2)*(c*sin(b*x+a))^m,x)
```

output
```
sqrt(d)*c**m*int(sin(a + b*x)**m*sqrt(sec(a + b*x))*sec(a + b*x)**2,x)*d**
2
```

3.484 $\int (d\sec(a+bx))^{3/2}(c\sin(a+bx))^m\,dx$

Optimal result

Integrand size = 23, antiderivative size = 75

$$\int (d\sec(a+bx))^{3/2}(c\sin(a+bx))^m\,dx = \frac{d\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)\sqrt{d\sec(a+}{bc(1+m)}$$

output
```
d*(cos(b*x+a)^2)^(1/4)*hypergeom([5/4, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2
)*(d*sec(b*x+a))^(1/2)*(c*sin(b*x+a))^(1+m)/b/c/(1+m)
```

Mathematica [A] (verified)

Time = 8.86 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.28

$$\int (d\sec(a+bx))^{3/2}(c\sin(a+bx))^m\,dx =$$
$$-\frac{2\cot(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{4}(3-2m),\frac{1-m}{2},\frac{1}{4}(7-2m),\sec^2(a+bx)\right)(d\sec(a+bx))^{3/2}(c\sin(a+}{b(-3+2m)}$$

input
```
Integrate[(d*Sec[a + b*x])^(3/2)*(c*Sin[a + b*x])^m,x]
```

output
```
(-2*Cot[a + b*x]*Hypergeometric2F1[(3 - 2*m)/4, (1 - m)/2, (7 - 2*m)/4, Se
c[a + b*x]^2]*(d*Sec[a + b*x])^(3/2)*(c*Sin[a + b*x])^m*(-Tan[a + b*x]^2)^
((1 - m)/2))/(b*(-3 + 2*m))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3067, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\sec(a+bx))^{3/2}(c\sin(a+bx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\sec(a+bx))^{3/2}(c\sin(a+bx))^m dx$$

$$\Big\downarrow \text{3067}$$

$$d^2\sqrt{d\cos(a+bx)}\sqrt{d\sec(a+bx)}\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$d^2\sqrt{d\cos(a+bx)}\sqrt{d\sec(a+bx)}\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3057}$$

$$\frac{d\sqrt[4]{\cos^2(a+bx)}\sqrt{d\sec(a+bx)}(c\sin(a+bx))^{m+1}\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{m+1}{2},\frac{m+3}{2},\sin^2(a+bx)\right)}{bc(m+1)}$$

input
```
Int[(d*Sec[a + b*x])^(3/2)*(c*Sin[a + b*x])^m,x]
```

output
```
(d*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[5/4, (1 + m)/2, (3 + m)/2, Sin
[a + b*x]^2]*Sqrt[d*Sec[a + b*x]]*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3067
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
   Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

Maple [F]

$$\int (d\sec (bx+a))^{\frac{3}{2}} (c\sin (bx+a))^{m} dx$$

input
```
int((d*sec(b*x+a))^(3/2)*(c*sin(b*x+a))^m,x)
```

output
```
int((d*sec(b*x+a))^(3/2)*(c*sin(b*x+a))^m,x)
```

Fricas [F]

$$\int (d\sec(a+bx))^{3/2}(c\sin(a+bx))^m\,dx = \int (d\sec (bx+a))^{\frac{3}{2}} (c\sin (bx+a))^{m}\,dx$$

input
```
integrate((d*sec(b*x+a))^(3/2)*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

output
```
integral(sqrt(d*sec(b*x + a))*(c*sin(b*x + a))^m*d*sec(b*x + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d\sec(a+bx))^{3/2}(c\sin(a+bx))^m\,dx = \text{Timed out}$$

input
```
integrate((d*sec(b*x+a))**(3/2)*(c*sin(b*x+a))**m,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\sec(a+bx))^{3/2}(c\sin(a+bx))^m\,dx = \int (d\sec(bx+a))^{\frac{3}{2}}(c\sin(bx+a))^m\,dx$$

input
```
integrate((d*sec(b*x+a))^(3/2)*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

output
```
integrate((d*sec(b*x + a))^(3/2)*(c*sin(b*x + a))^m, x)
```

Giac [F]

$$\int (d\sec(a+bx))^{3/2}(c\sin(a+bx))^m\,dx = \int (d\sec(bx+a))^{\frac{3}{2}}(c\sin(bx+a))^m\,dx$$

input
```
integrate((d*sec(b*x+a))^(3/2)*(c*sin(b*x+a))^m,x, algorithm="giac")
```

output
```
integrate((d*sec(b*x + a))^(3/2)*(c*sin(b*x + a))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(a+bx))^{3/2}(c\sin(a+bx))^m\,dx = \int (c\,\sin(a+b\,x))^m\,\left(\frac{d}{\cos(a+b\,x)}\right)^{3/2}\,dx$$

input
```
int((c*sin(a + b*x))^m*(d/cos(a + b*x))^(3/2),x)
```

output
```
int((c*sin(a + b*x))^m*(d/cos(a + b*x))^(3/2), x)
```

Reduce [F]

$$\int (d\sec(a+bx))^{3/2}(c\sin(a+bx))^m\,dx = \sqrt{d}\,c^m\left(\int \sin(bx+a)^m\,\sqrt{\sec(bx+a)}\,\sec(bx+a)\,dx\right)d$$

input
```
int((d*sec(b*x+a))^(3/2)*(c*sin(b*x+a))^m,x)
```

output
```
sqrt(d)*c**m*int(sin(a + b*x)**m*sqrt(sec(a + b*x))*sec(a + b*x),x)*d
```

3.485 $\int \sqrt{d\sec(a+bx)}(c\sin(a+bx))^m\,dx$

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \sqrt{d\sec(a+bx)}(c\sin(a+bx))^m\,dx = \frac{\cos^2(a+bx)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(d\sec(a+bx))^{3/2}(c\sin(a+bx))^{1+m}}{bcd(1+m)}$$

output
```
(cos(b*x+a)^2)^(3/4)*hypergeom([3/4, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)*
(d*sec(b*x+a))^(3/2)*(c*sin(b*x+a))^(1+m)/b/c/d/(1+m)
```

Mathematica [A] (verified)

Time = 8.59 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.38

$$\int \sqrt{d\sec(a+bx)}(c\sin(a+bx))^m\,dx = -\frac{\csc^2(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{4}(1-2m),\frac{1-m}{2},\frac{1}{4}(5-2m),\sec^2(a+bx)\right)\sqrt{d\sec(a+bx)}(c\sin(a+}{b(-1+2m)}$$

input
```
Integrate[Sqrt[d*Sec[a + b*x]]*(c*Sin[a + b*x])^m,x]
```

output
```
-((Csc[a + b*x]^2*Hypergeometric2F1[(1 - 2*m)/4, (1 - m)/2, (5 - 2*m)/4, S
ec[a + b*x]^2]*Sqrt[d*Sec[a + b*x]]*(c*Sin[a + b*x])^m*Sin[2*(a + b*x)]*(-
Tan[a + b*x]^2)^((1 - m)/2))/(b*(-1 + 2*m)))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3066, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d\sec(a+bx)}(c\sin(a+bx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{d\sec(a+bx)}(c\sin(a+bx))^m dx$$

$$\Big\downarrow \text{3066}$$

$$\frac{(d\cos(a+bx))^{3/2}(d\sec(a+bx))^{3/2}\int \frac{(c\sin(a+bx))^m}{\sqrt{d\cos(a+bx)}}dx}{d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(d\cos(a+bx))^{3/2}(d\sec(a+bx))^{3/2}\int \frac{(c\sin(a+bx))^m}{\sqrt{d\cos(a+bx)}}dx}{d^2}$$

$$\Big\downarrow \text{3057}$$

$$\frac{\cos^2(a+bx)^{3/4}(d\sec(a+bx))^{3/2}(c\sin(a+bx))^{m+1}\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{m+1}{2},\frac{m+3}{2},\sin^2(a+bx)\right)}{bcd(m+1)}$$

input
```
Int[Sqrt[d*Sec[a + b*x]]*(c*Sin[a + b*x])^m,x]
```

output
```
((Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/4, (1 + m)/2, (3 + m)/2, Sin[a
 + b*x]^2]*(d*Sec[a + b*x])^(3/2)*(c*Sin[a + b*x])^(1 + m))/(b*c*d*(1 + m)
)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3066
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(1/b^2)*(b*Cos[e + f*x])^(n + 1)*(b*Sec[e + f*x])^(n
+ 1)   Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && LtQ[n, 1]
```

Maple [F]

$$\int \sqrt{d \sec (bx+a)}\, (c \sin (bx+a))^m \, dx$$

input
```
int((d*sec(b*x+a))^(1/2)*(c*sin(b*x+a))^m,x)
```

output
```
int((d*sec(b*x+a))^(1/2)*(c*sin(b*x+a))^m,x)
```

Fricas [F]

$$\int \sqrt{d\sec(a+bx)}(c\sin(a+bx))^m\,dx = \int \sqrt{d\sec\left(bx+a\right)}(c\sin\left(bx+a\right))^m\,dx$$

input
```
integrate((d*sec(b*x+a))^(1/2)*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

output
```
integral(sqrt(d*sec(b*x + a))*(c*sin(b*x + a))^m, x)
```

Sympy [F]

$$\int \sqrt{d\sec(a+bx)}(c\sin(a+bx))^m\,dx = \int (c\sin\left(a+bx\right))^m \sqrt{d\sec\left(a+bx\right)}\,dx$$

input
```
integrate((d*sec(b*x+a))**(1/2)*(c*sin(b*x+a))**m,x)
```

output
```
Integral((c*sin(a + b*x))**m*sqrt(d*sec(a + b*x)), x)
```

Maxima [F]

$$\int \sqrt{d\sec(a+bx)}(c\sin(a+bx))^m\,dx = \int \sqrt{d\sec\left(bx+a\right)}(c\sin\left(bx+a\right))^m\,dx$$

input
```
integrate((d*sec(b*x+a))^(1/2)*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*sec(b*x + a))*(c*sin(b*x + a))^m, x)
```

Giac [F]

$$\int \sqrt{d\sec(a+bx)}(c\sin(a+bx))^m\,dx = \int \sqrt{d\sec\left(bx+a\right)}(c\sin\left(bx+a\right))^m\,dx$$

input
```
integrate((d*sec(b*x+a))^(1/2)*(c*sin(b*x+a))^m,x, algorithm="giac")
```

output
```
integrate(sqrt(d*sec(b*x + a))*(c*sin(b*x + a))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\sec(a+bx)}(c\sin(a+bx))^m\,dx = \int (c\,\sin\left(a+b\,x\right))^m\,\sqrt{\frac{d}{\cos\left(a+b\,x\right)}}\,dx$$

input
```
int((c*sin(a + b*x))^m*(d/cos(a + b*x))^(1/2),x)
```

output
```
int((c*sin(a + b*x))^m*(d/cos(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{d\sec(a+bx)}(c\sin(a+bx))^m\,dx = \sqrt{d}\,c^m\left(\int \sin\left(bx+a\right)^m\sqrt{\sec\left(bx+a\right)}dx\right)$$

input
```
int((d*sec(b*x+a))^(1/2)*(c*sin(b*x+a))^m,x)
```

output
```
sqrt(d)*c**m*int(sin(a + b*x)**m*sqrt(sec(a + b*x)),x)
```

3.486 $\int \frac{(c\sin(a+bx))^m}{\sqrt{d\sec(a+bx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{(c\sin(a+bx))^m}{\sqrt{d\sec(a+bx)}}\,dx = \frac{\sqrt[4]{\cos^2(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)\sqrt{d\sec(a+bx)}(c\sin(a+bx))^{1+m}}{bcd(1+m)}$$

output
```
(cos(b*x+a)^2)^(1/4)*hypergeom([1/4, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)*
(d*sec(b*x+a))^(1/2)*(c*sin(b*x+a))^(1+m)/b/c/d/(1+m)
```

Mathematica [A] (verified)

Time = 28.76 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.16

$$\int \frac{(c\sin(a+bx))^m}{\sqrt{d\sec(a+bx)}}\,dx = \frac{\text{Hypergeometric2F1}\left(\frac{1+m}{2},\frac{1}{4}(5+2m),\frac{3+m}{2},-\tan^2(a+bx)\right)\sec^2(a+bx)^{\frac{1}{4}+\frac{m}{2}}(c\sin(a+bx))^m\tan(a+b}{b(1+m)\sqrt{d\sec(a+bx)}}$$

input
```
Integrate[(c*Sin[a + b*x])^m/Sqrt[d*Sec[a + b*x]],x]
```

output
```
(Hypergeometric2F1[(1 + m)/2, (5 + 2*m)/4, (3 + m)/2, -Tan[a + b*x]^2]*(Se
c[a + b*x]^2)^(1/4 + m/2)*(c*Sin[a + b*x])^m*Tan[a + b*x])/(b*(1 + m)*Sqrt
[d*Sec[a + b*x]])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3066, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c\sin(a+bx))^m}{\sqrt{d\sec(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c\sin(a+bx))^m}{\sqrt{d\sec(a+bx)}}dx$$

$$\Big\downarrow \text{3066}$$

$$\frac{\sqrt{d\cos(a+bx)}\sqrt{d\sec(a+bx)}\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^m dx}{d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{d\cos(a+bx)}\sqrt{d\sec(a+bx)}\int \sqrt{d\cos(a+bx)}(c\sin(a+bx))^m dx}{d^2}$$

$$\Big\downarrow \text{3057}$$

$$\frac{\sqrt[4]{\cos^2(a+bx)}\sqrt{d\sec(a+bx)}(c\sin(a+bx))^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{m+1}{2},\frac{m+3}{2},\sin^2(a+bx)\right)}{bcd(m+1)}$$

input
```
Int[(c*Sin[a + b*x])^m/Sqrt[d*Sec[a + b*x]],x]
```

output
```
((Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, (1 + m)/2, (3 + m)/2, Sin[a
 + b*x]^2]*Sqrt[d*Sec[a + b*x]]*(c*Sin[a + b*x])^(1 + m))/(b*c*d*(1 + m))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3066
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(1/b^2)*(b*Cos[e + f*x])^(n + 1)*(b*Sec[e + f*x])^(n
+ 1)   Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && LtQ[n, 1]
```

Maple [F]

$$\int \frac{(c\sin(bx+a))^m}{\sqrt{d\sec(bx+a)}}dx$$

input
```
int((c*sin(b*x+a))^m/(d*sec(b*x+a))^(1/2),x)
```

output
```
int((c*sin(b*x+a))^m/(d*sec(b*x+a))^(1/2),x)
```

Fricas [F]

$$\int \frac{(c\sin(a+bx))^m}{\sqrt{d\sec(a+bx)}}\,dx = \int \frac{(c\sin(bx+a))^m}{\sqrt{d\sec(bx+a)}}\,dx$$

input
```
integrate((c*sin(b*x+a))^m/(d*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*sec(b*x + a))*(c*sin(b*x + a))^m/(d*sec(b*x + a)), x)
```

Sympy [F]

$$\int \frac{(c\sin(a+bx))^m}{\sqrt{d\sec(a+bx)}}\,dx = \int \frac{(c\sin{(a+bx)})^m}{\sqrt{d\sec{(a+bx)}}}\,dx$$

input
```
integrate((c*sin(b*x+a))**m/(d*sec(b*x+a))**(1/2),x)
```

output
```
Integral((c*sin(a + b*x))**m/sqrt(d*sec(a + b*x)), x)
```

Maxima [F]

$$\int \frac{(c\sin(a+bx))^m}{\sqrt{d\sec(a+bx)}}\,dx = \int \frac{(c\sin{(bx+a)})^m}{\sqrt{d\sec{(bx+a)}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^m/(d*sec(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a))^m/sqrt(d*sec(b*x + a)), x)
```

Giac [F]

$$\int \frac{(c\sin(a+bx))^m}{\sqrt{d\sec(a+bx)}}\,dx = \int \frac{(c\sin{(bx+a)})^m}{\sqrt{d\sec{(bx+a)}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^m/(d*sec(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a))^m/sqrt(d*sec(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^m}{\sqrt{d\sec(a+bx)}}\,dx = \int \frac{(c\,\sin\left(a+b\,x\right))^m}{\sqrt{\frac{d}{\cos(a+b\,x)}}}\,dx$$

input
```
int((c*sin(a + b*x))^m/(d/cos(a + b*x))^(1/2),x)
```

output
```
int((c*sin(a + b*x))^m/(d/cos(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(c\sin(a+bx))^m}{\sqrt{d\sec(a+bx)}}\,dx = \frac{\sqrt{d}\,c^m\left(\int \frac{\sin(bx+a)^m\sqrt{\sec(bx+a)}}{\sec(bx+a)}dx\right)}{d}$$

input
```
int((c*sin(b*x+a))^m/(d*sec(b*x+a))^(1/2),x)
```

output
```
(sqrt(d)*c**m*int((sin(a + b*x)**m*sqrt(sec(a + b*x)))/sec(a + b*x),x))/d
```

3.487 $\int \frac{(c\sin(a+bx))^m}{(d\sec(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{(c\sin(a+bx))^m}{(d\sec(a+bx))^{3/2}}\,dx = \frac{\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1+m}{2},\frac{3+m}{2},\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bcd(1+m)\sqrt[4]{\cos^2(a+bx)}\sqrt{d\sec(a+bx)}}$$

output
```
hypergeom([-1/4, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)*(c*sin(b*x+a))^(1+m)
/b/c/d/(1+m)/(cos(b*x+a)^2)^(1/4)/(d*sec(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 11.70 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.51

$$\int \frac{(c\sin(a+bx))^m}{(d\sec(a+bx))^{3/2}}\,dx = \frac{2c\cos(2(a+bx))\,\text{Hypergeometric2F1}\left(\frac{1}{4}(-3-2m),\frac{1-m}{2},\frac{1}{4}(1-2m),\sec^2(a+b\right.}{bd(3+2m)\sqrt{d\sec(a+bx)}\left(-2+\sec^2(a\right.}$$

input
```
Integrate[(c*Sin[a + b*x])^m/(d*Sec[a + b*x])^(3/2),x]
```

output
```
(2*c*Cos[2*(a + b*x)]*Hypergeometric2F1[(-3 - 2*m)/4, (1 - m)/2, (1 - 2*m)
/4, Sec[a + b*x]^2]*(c*Sin[a + b*x])^(-1 + m)*(-Tan[a + b*x]^2)^((1 - m)/2
))/(b*d*(3 + 2*m)*Sqrt[d*Sec[a + b*x]]*(-2 + Sec[a + b*x]^2))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3066, 3042, 3057}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c\sin(a+bx))^m}{(d\sec(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c\sin(a+bx))^m}{(d\sec(a+bx))^{3/2}}dx$$

$$\Big\downarrow \text{3066}$$

$$\frac{\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^m dx}{d^2\sqrt{d\cos(a+bx)}\sqrt{d\sec(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^m dx}{d^2\sqrt{d\cos(a+bx)}\sqrt{d\sec(a+bx)}}$$

$$\Big\downarrow \text{3057}$$

$$\frac{(c\sin(a+bx))^{m+1}\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{m+1}{2},\frac{m+3}{2},\sin^2(a+bx)\right)}{bcd(m+1)\sqrt[4]{\cos^2(a+bx)}\sqrt{d\sec(a+bx)}}$$

input
```
Int[(c*Sin[a + b*x])^m/(d*Sec[a + b*x])^(3/2),x]
```

output
```
(Hypergeometric2F1[-1/4, (1 + m)/2, (3 + m)/2, Sin[a + b*x]^2]*(c*Sin[a +
b*x])^(1 + m))/(b*c*d*(1 + m)*(Cos[a + b*x]^2)^(1/4)*Sqrt[d*Sec[a + b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3057
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

rule 3066
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(1/b^2)*(b*Cos[e + f*x])^(n + 1)*(b*Sec[e + f*x])^(n
+ 1)   Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && LtQ[n, 1]
```

Maple [F]

$$\int \frac{(c\sin(bx+a))^m}{(d\sec(bx+a))^{\frac{3}{2}}}dx$$

input
```
int((c*sin(b*x+a))^m/(d*sec(b*x+a))^(3/2),x)
```

output
```
int((c*sin(b*x+a))^m/(d*sec(b*x+a))^(3/2),x)
```

Fricas [F]

$$\int \frac{(c\sin(a+bx))^m}{(d\sec(a+bx))^{3/2}}\,dx = \int \frac{(c\sin(bx+a))^m}{(d\sec(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^m/(d*sec(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*sec(b*x + a))*(c*sin(b*x + a))^m/(d^2*sec(b*x + a)^2), x)
```

Sympy [F]

$$\int \frac{(c\sin(a+bx))^m}{(d\sec(a+bx))^{3/2}}\,dx = \int \frac{(c\sin{(a+bx)})^m}{(d\sec{(a+bx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))**m/(d*sec(b*x+a))**(3/2),x)
```

output
```
Integral((c*sin(a + b*x))**m/(d*sec(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(c\sin(a+bx))^m}{(d\sec(a+bx))^{3/2}}\,dx = \int \frac{(c\sin{(bx+a)})^m}{(d\sec{(bx+a)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^m/(d*sec(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a))^m/(d*sec(b*x + a))^(3/2), x)
```

Giac [F]

$$\int \frac{(c\sin(a+bx))^m}{(d\sec(a+bx))^{3/2}}\,dx = \int \frac{(c\sin{(bx+a)})^m}{(d\sec{(bx+a)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*sin(b*x+a))^m/(d*sec(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a))^m/(d*sec(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c\sin(a+bx))^m}{(d\sec(a+bx))^{3/2}}\,dx = \int \frac{(c\,\sin\left(a+b\,x\right))^m}{\left(\frac{d}{\cos(a+b\,x)}\right)^{3/2}}\,dx$$

input
```
int((c*sin(a + b*x))^m/(d/cos(a + b*x))^(3/2),x)
```

output
```
int((c*sin(a + b*x))^m/(d/cos(a + b*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(c\sin(a+bx))^m}{(d\sec(a+bx))^{3/2}}\,dx = \frac{\sqrt{d}\,c^m\left(\int \frac{\sin(bx+a)^m\sqrt{\sec(bx+a)}}{\sec(bx+a)^2}dx\right)}{d^2}$$

input
```
int((c*sin(b*x+a))^m/(d*sec(b*x+a))^(3/2),x)
```

output
```
(sqrt(d)*c**m*int((sin(a + b*x)**m*sqrt(sec(a + b*x)))/sec(a + b*x)**2,x))
/d**2
```

3.488 $\int \sec^n(e+fx)\sin^m(e+fx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 86

$$\int \sec^n(e+fx)\sin^m(e+fx)\,dx = -\frac{\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)\sec^{-1+n}(e+fx)\sin^{-1+m}(e+fx)\sin^2(e+fx)^{\frac{1-m}{2}}}{f(1-n)}$$

output
```
-hypergeom([1/2-1/2*m, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*sec(f*x+e)^(-1
+n)*sin(f*x+e)^(-1+m)*(sin(f*x+e)^2)^(1/2-1/2*m)/f/(1-n)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 2.02 (sec) , antiderivative size = 285, normalized size of antiderivative = 3.31

$$\int \sec^n(e+fx)\sin^m(e+fx)\,dx = \frac{4(3+m)\,\text{AppellF1}\left(\frac{1}{}\right.}{f(1+m)\left((3+m)\,\text{AppellF1}\left(\frac{1+m}{2},n,1+m-n,\frac{3+m}{2},\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)(1+\cos(e\right.}$$

input
```
Integrate[Sec[e + f*x]^n*Sin[e + f*x]^m,x]
```

output
```
(4*(3 + m)*AppellF1[(1 + m)/2, n, 1 + m - n, (3 + m)/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^3*Sec[e + f*x]^n*Sin[(e + f*x)/2]*
Sin[e + f*x]^m)/(f*(1 + m)*((3 + m)*AppellF1[(1 + m)/2, n, 1 + m - n, (3 +
 m)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + Cos[e + f*x]) - 4*((1
 + m - n)*AppellF1[(3 + m)/2, n, 2 + m - n, (5 + m)/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2] - n*AppellF1[(3 + m)/2, 1 + n, 1 + m - n, (5 + m)/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sin[(e + f*x)/2]^2))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 3067, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^m(e+fx)\sec^n(e+fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^m\sec(e+fx)^n dx$$

$$\Big\downarrow \text{3067}$$

$$\cos^n(e+fx)\sec^n(e+fx)\int \cos^{-n}(e+fx)\sin^m(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^n(e+fx)\sec^n(e+fx)\int \cos(e+fx)^{-n}\sin(e+fx)^m dx$$

$$\Big\downarrow \text{3056}$$

$$-\frac{\sin^{m-1}(e+fx)\sin^2(e+fx)^{\frac{1-m}{2}}\sec^{n-1}(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f(1-n)}$$

input
```
Int[Sec[e + f*x]^n*Sin[e + f*x]^m,x]
```

output
```
-((Hypergeometric2F1[(1 - m)/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*Sec[
e + f*x]^(-1 + n)*Sin[e + f*x]^(-1 + m)*(Sin[e + f*x]^2)^((1 - m)/2))/(f*(
1 - n)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3067
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
   Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f,
m, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Maple [F]

$$\int \sec\left(fx+e\right)^{n} \sin\left(fx+e\right)^{m} dx$$

input
```
int(sec(f*x+e)^n*sin(f*x+e)^m,x)
```

output
```
int(sec(f*x+e)^n*sin(f*x+e)^m,x)
```

Fricas [F]

$$\int \sec^n(e + fx)\sin^m(e + fx)\,dx = \int \sec(fx + e)^n \sin(fx + e)^m\,dx$$

input
```
integrate(sec(f*x+e)^n*sin(f*x+e)^m,x, algorithm="fricas")
```

output
```
integral(sec(f*x + e)^n*sin(f*x + e)^m, x)
```

Sympy [F]

$$\int \sec^n(e + fx)\sin^m(e + fx)\,dx = \int \sin^m(e + fx)\sec^n(e + fx)\,dx$$

input
```
integrate(sec(f*x+e)**n*sin(f*x+e)**m,x)
```

output
```
Integral(sin(e + f*x)**m*sec(e + f*x)**n, x)
```

Maxima [F]

$$\int \sec^n(e + fx)\sin^m(e + fx)\,dx = \int \sec(fx + e)^n \sin(fx + e)^m\,dx$$

input
```
integrate(sec(f*x+e)^n*sin(f*x+e)^m,x, algorithm="maxima")
```

output
```
integrate(sec(f*x + e)^n*sin(f*x + e)^m, x)
```

Giac [F]

$$\int \sec^n(e+fx)\sin^m(e+fx)\,dx = \int \sec(fx+e)^n \sin(fx+e)^m\,dx$$

input
```
integrate(sec(f*x+e)^n*sin(f*x+e)^m,x, algorithm="giac")
```

output
```
integrate(sec(f*x + e)^n*sin(f*x + e)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^n(e+fx)\sin^m(e+fx)\,dx = \int \sin(e+f\,x)^m \left(\frac{1}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int(sin(e + f*x)^m*(1/cos(e + f*x))^n,x)
```

output
```
int(sin(e + f*x)^m*(1/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int \sec^n(e+fx)\sin^m(e+fx)\,dx = \int \sin(fx+e)^m \sec(fx+e)^n\,dx$$

input
```
int(sec(f*x+e)^n*sin(f*x+e)^m,x)
```

output
```
int(sin(e + f*x)**m*sec(e + f*x)**n,x)
```

3.489 $\int \sec^n(e+fx)(a\sin(e+fx))^m\,dx$

Optimal result

Integrand size = 19, antiderivative size = 89

$$\int \sec^n(e+fx)(a\sin(e+fx))^m\,dx = -\frac{a\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)\sec^{-1+n}(e+fx)(a\sin(e+fx))^{-1+m}\sin^2(e+fx}{f(1-n)}$$

output
```
-a*hypergeom([1/2-1/2*m, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*sec(f*x+e)^(
-1+n)*(a*sin(f*x+e))^(-1+m)*(sin(f*x+e)^2)^(1/2-1/2*m)/f/(1-n)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.56 (sec) , antiderivative size = 287, normalized size of antiderivative = 3.22

$$\int \sec^n(e+fx)(a\sin(e+fx))^m\,dx = \frac{4(3+m)\,\text{AppellF1}\left(\frac{1+m}{2}\right.}{f(1+m)\left((3+m)\,\text{AppellF1}\left(\frac{1+m}{2},n,1+m-n,\frac{3+m}{2},\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)(1+\cos(e\right.}$$

input
```
Integrate[Sec[e + f*x]^n*(a*Sin[e + f*x])^m,x]
```

output
```
(4*(3 + m)*AppellF1[(1 + m)/2, n, 1 + m - n, (3 + m)/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^3*Sec[e + f*x]^n*Sin[(e + f*x)/2]*
(a*Sin[e + f*x])^m)/(f*(1 + m)*((3 + m)*AppellF1[(1 + m)/2, n, 1 + m - n,
(3 + m)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + Cos[e + f*x]) - 4
*((1 + m - n)*AppellF1[(3 + m)/2, n, 2 + m - n, (5 + m)/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] - n*AppellF1[(3 + m)/2, 1 + n, 1 + m - n, (5 + m
)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sin[(e + f*x)/2]^2))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3067, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^n(e+fx)(a\sin(e+fx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(e+fx)^n(a\sin(e+fx))^m dx$$

$$\Big\downarrow \text{3067}$$

$$\cos^n(e+fx)\sec^n(e+fx)\int \cos^{-n}(e+fx)(a\sin(e+fx))^m dx$$

$$\Big\downarrow \text{3042}$$

$$\cos^n(e+fx)\sec^n(e+fx)\int \cos(e+fx)^{-n}(a\sin(e+fx))^m dx$$

$$\Big\downarrow \text{3056}$$

$$-\frac{a\sin^2(e+fx)^{\frac{1-m}{2}}\sec^{n-1}(e+fx)(a\sin(e+fx))^{m-1}\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f(1-n)}$$

input
```
Int[Sec[e + f*x]^n*(a*Sin[e + f*x])^m,x]
```

output
```
-((a*Hypergeometric2F1[(1 - m)/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*Se
c[e + f*x]^(-1 + n)*(a*Sin[e + f*x])^(-1 + m)*(Sin[e + f*x]^2)^((1 - m)/2)
)/(f*(1 - n)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3067
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
   Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f,
m, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Maple [F]

$$\int \sec (fx+e)^n (a\sin (fx+e))^m \, dx$$

input
```
int(sec(f*x+e)^n*(a*sin(f*x+e))^m,x)
```

output
```
int(sec(f*x+e)^n*(a*sin(f*x+e))^m,x)
```

Fricas [F]

$$\int \sec^n(e+fx)(a\sin(e+fx))^m\,dx = \int (a\sin(fx+e))^m\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a*sin(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((a*sin(f*x + e))^m*sec(f*x + e)^n, x)
```

Sympy [F]

$$\int \sec^n(e+fx)(a\sin(e+fx))^m\,dx = \int (a\sin(e+fx))^m\sec^n(e+fx)\,dx$$

input
```
integrate(sec(f*x+e)**n*(a*sin(f*x+e))**m,x)
```

output
```
Integral((a*sin(e + f*x))**m*sec(e + f*x)**n, x)
```

Maxima [F]

$$\int \sec^n(e+fx)(a\sin(e+fx))^m\,dx = \int (a\sin(fx+e))^m\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a*sin(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e))^m*sec(f*x + e)^n, x)
```

Giac [F]

$$\int \sec^n(e+fx)(a\sin(e+fx))^m\,dx=\int (a\sin(fx+e))^m\sec(fx+e)^n\,dx$$

input
```
integrate(sec(f*x+e)^n*(a*sin(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e))^m*sec(f*x + e)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^n(e+fx)(a\sin(e+fx))^m\,dx=\int (a\,\sin(e+f\,x))^m\left(\frac{1}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((a*sin(e + f*x))^m*(1/cos(e + f*x))^n,x)
```

output
```
int((a*sin(e + f*x))^m*(1/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int \sec^n(e+fx)(a\sin(e+fx))^m\,dx=a^m\left(\int \sin(fx+e)^m\sec(fx+e)^n\,dx\right)$$

input
```
int(sec(f*x+e)^n*(a*sin(f*x+e))^m,x)
```

output
```
a**m*int(sin(e + f*x)**m*sec(e + f*x)**n,x)
```

3.490 $\int (b\sec(e+fx))^n\sin^m(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 89

$$\int (b\sec(e+fx))^n\sin^m(e+fx)\,dx = -\frac{b\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)(b\sec(e+fx))^{-1+n}\sin^{-1+m}(e+fx)\sin^2(e+fx)}{f(1-n)}$$

output
```
-b*hypergeom([1/2-1/2*m, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*(b*sec(f*x+e
))^(-1+n)*sin(f*x+e)^(-1+m)*(sin(f*x+e)^2)^(1/2-1/2*m)/f/(1-n)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.58 (sec) , antiderivative size = 287, normalized size of antiderivative = 3.22

$$\int (b\sec(e+fx))^n\sin^m(e+fx)\,dx = \frac{4(3+m)\,\text{AppellF1}\left(\frac{1+m}{2}\right.}{f(1+m)\left((3+m)\,\text{AppellF1}\left(\frac{1+m}{2},n,1+m-n,\frac{3+m}{2},\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)(1+\cos(e\right.}$$

input
```
Integrate[(b*Sec[e + f*x])^n*Sin[e + f*x]^m,x]
```

output
```
(4*(3 + m)*AppellF1[(1 + m)/2, n, 1 + m - n, (3 + m)/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^3*(b*Sec[e + f*x])^n*Sin[(e + f*x)
/2]*Sin[e + f*x]^m)/(f*(1 + m)*((3 + m)*AppellF1[(1 + m)/2, n, 1 + m - n,
(3 + m)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + Cos[e + f*x]) - 4
*((1 + m - n)*AppellF1[(3 + m)/2, n, 2 + m - n, (5 + m)/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] - n*AppellF1[(3 + m)/2, 1 + n, 1 + m - n, (5 + m
)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sin[(e + f*x)/2]^2))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3067, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^m(e+fx)(b\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(e+fx)^m(b\sec(e+fx))^n dx$$

$$\Big\downarrow \text{3067}$$

$$b^2(b\cos(e+fx))^{n-1}(b\sec(e+fx))^{n-1}\int (b\cos(e+fx))^{-n}\sin^m(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$b^2(b\cos(e+fx))^{n-1}(b\sec(e+fx))^{n-1}\int (b\cos(e+fx))^{-n}\sin(e+fx)^m dx$$

$$\Big\downarrow \text{3056}$$

$$-\frac{b\sin^{m-1}(e+fx)\sin^2(e+fx)^{\frac{1-m}{2}}(b\sec(e+fx))^{n-1}\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f(1-n)}$$

input
```
Int[(b*Sec[e + f*x])^n*Sin[e + f*x]^m,x]
```

output
```
-((b*Hypergeometric2F1[(1 - m)/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*(b
*Sec[e + f*x])^(-1 + n)*Sin[e + f*x]^(-1 + m)*(Sin[e + f*x]^2)^((1 - m)/2)
)/(f*(1 - n)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3067
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
   Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f,
m, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Maple [F]

$$\int (b\sec(fx+e))^n \sin(fx+e)^m\,dx$$

input
```
int((b*sec(f*x+e))^n*sin(f*x+e)^m,x)
```

output
```
int((b*sec(f*x+e))^n*sin(f*x+e)^m,x)
```

Fricas [F]

$$\int (b\sec(e+fx))^n \sin^m(e+fx)\,dx = \int (b\sec(fx+e))^n \sin(fx+e)^m\,dx$$

input
```
integrate((b*sec(f*x+e))^n*sin(f*x+e)^m,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e))^n*sin(f*x + e)^m, x)
```

Sympy [F]

$$\int (b\sec(e+fx))^n \sin^m(e+fx)\,dx = \int (b\sec(e+fx))^n \sin^m(e+fx)\,dx$$

input
```
integrate((b*sec(f*x+e))**n*sin(f*x+e)**m,x)
```

output
```
Integral((b*sec(e + f*x))**n*sin(e + f*x)**m, x)
```

Maxima [F]

$$\int (b\sec(e+fx))^n \sin^m(e+fx)\,dx = \int (b\sec(fx+e))^n \sin(fx+e)^m\,dx$$

input
```
integrate((b*sec(f*x+e))^n*sin(f*x+e)^m,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^n*sin(f*x + e)^m, x)
```

Giac [F]

$$\int (b\sec(e+fx))^n \sin^m(e+fx)\, dx = \int (b\sec(fx+e))^n \sin(fx+e)^m\, dx$$

input
```
integrate((b*sec(f*x+e))^n*sin(f*x+e)^m,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^n*sin(f*x + e)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^n \sin^m(e+fx)\, dx = \int \sin(e+f\,x)^m \left(\frac{b}{\cos(e+f\,x)}\right)^n dx$$

input
```
int(sin(e + f*x)^m*(b/cos(e + f*x))^n,x)
```

output
```
int(sin(e + f*x)^m*(b/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int (b\sec(e+fx))^n \sin^m(e+fx)\, dx = b^n \left(\int \sin(fx+e)^m \sec(fx+e)^n\, dx\right)$$

input
```
int((b*sec(f*x+e))^n*sin(f*x+e)^m,x)
```

output
```
b**n*int(sin(e + f*x)**m*sec(e + f*x)**n,x)
```

3.491 $\int (b\sec(e+fx))^n(a\sin(e+fx))^m\,dx$

Optimal result

Integrand size = 21, antiderivative size = 92

$$\int (b\sec(e+fx))^n(a\sin(e+fx))^m\,dx = -\frac{ab\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)(b\sec(e+fx))^{-1+n}(a\sin(e+fx))^{-1+m}\sin^2(e+ \ldots}{f(1-n)}$$

output
```
-a*b*hypergeom([1/2-1/2*m, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*(b*sec(f*x
+e))^(-1+n)*(a*sin(f*x+e))^(-1+m)*(sin(f*x+e)^2)^(1/2-1/2*m)/f/(1-n)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.68 (sec) , antiderivative size = 289, normalized size of antiderivative = 3.14

$$\int (b\sec(e+fx))^n(a\sin(e+fx))^m\,dx = \frac{4(3+m)\,\text{AppellF1}\left(\frac{1+m}{2}, \ldots\right.}{f(1+m)\left((3+m)\,\text{AppellF1}\left(\frac{1+m}{2},n,1+m-n,\frac{3+m}{2},\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)(1+\cos(\ldots\right.}$$

input
```
Integrate[(b*Sec[e + f*x])^n*(a*Sin[e + f*x])^m,x]
```

output
```
(4*(3 + m)*AppellF1[(1 + m)/2, n, 1 + m - n, (3 + m)/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^3*(b*Sec[e + f*x])^n*Sin[(e + f*x)
/2]*(a*Sin[e + f*x])^m)/(f*(1 + m)*((3 + m)*AppellF1[(1 + m)/2, n, 1 + m -
 n, (3 + m)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + Cos[e + f*x])
 - 4*((1 + m - n)*AppellF1[(3 + m)/2, n, 2 + m - n, (5 + m)/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2] - n*AppellF1[(3 + m)/2, 1 + n, 1 + m - n, (5
 + m)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sin[(e + f*x)/2]^2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3067, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx))^m(b\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx))^m(b\sec(e+fx))^n dx$$

$$\Big\downarrow \text{3067}$$

$$b^2(b\cos(e+fx))^{n-1}(b\sec(e+fx))^{n-1}\int (b\cos(e+fx))^{-n}(a\sin(e+fx))^m dx$$

$$\Big\downarrow \text{3042}$$

$$b^2(b\cos(e+fx))^{n-1}(b\sec(e+fx))^{n-1}\int (b\cos(e+fx))^{-n}(a\sin(e+fx))^m dx$$

$$\Big\downarrow \text{3056}$$

$$-\frac{ab\sin^2(e+fx)^{\frac{1-m}{2}}(a\sin(e+fx))^{m-1}(b\sec(e+fx))^{n-1}\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f(1-n)}$$

input
```
Int[(b*Sec[e + f*x])^n*(a*Sin[e + f*x])^m,x]
```

output
```
-((a*b*Hypergeometric2F1[(1 - m)/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*
(b*Sec[e + f*x])^(-1 + n)*(a*Sin[e + f*x])^(-1 + m)*(Sin[e + f*x]^2)^((1 -
 m)/2))/(f*(1 - n)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3067
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
   Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

Maple [F]

$$\int (b\sec(fx+e))^n (a\sin(fx+e))^m \, dx$$

input
```
int((b*sec(f*x+e))^n*(a*sin(f*x+e))^m,x)
```

output
```
int((b*sec(f*x+e))^n*(a*sin(f*x+e))^m,x)
```

Fricas [F]

$$\int (b\sec(e+fx))^n(a\sin(e+fx))^m\,dx = \int (b\sec(fx+e))^n\,(a\sin(fx+e))^m\,dx$$

input
```
integrate((b*sec(f*x+e))^n*(a*sin(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e))^n*(a*sin(f*x + e))^m, x)
```

Sympy [F]

$$\int (b\sec(e+fx))^n(a\sin(e+fx))^m\,dx = \int (a\sin(e+fx))^m\,(b\sec(e+fx))^n\,dx$$

input
```
integrate((b*sec(f*x+e))**n*(a*sin(f*x+e))**m,x)
```

output
```
Integral((a*sin(e + f*x))**m*(b*sec(e + f*x))**n, x)
```

Maxima [F]

$$\int (b\sec(e+fx))^n(a\sin(e+fx))^m\,dx = \int (b\sec(fx+e))^n\,(a\sin(fx+e))^m\,dx$$

input
```
integrate((b*sec(f*x+e))^n*(a*sin(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^n*(a*sin(f*x + e))^m, x)
```

Giac [F]

$$\int (b\sec(e+fx))^n(a\sin(e+fx))^m\,dx=\int (b\sec(fx+e))^n\,(a\sin(fx+e))^m\,dx$$

input
```
integrate((b*sec(f*x+e))^n*(a*sin(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^n*(a*sin(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^n(a\sin(e+fx))^m\,dx=\int (a\,\sin(e+f\,x))^m\,\left(\frac{b}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int((a*sin(e + f*x))^m*(b/cos(e + f*x))^n,x)
```

output
```
int((a*sin(e + f*x))^m*(b/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int (b\sec(e+fx))^n(a\sin(e+fx))^m\,dx=b^n a^m\left(\int \sin(fx+e)^m\sec(fx+e)^n\,dx\right)$$

input
```
int((b*sec(f*x+e))^n*(a*sin(f*x+e))^m,x)
```

output
```
b**n*a**m*int(sin(e + f*x)**m*sec(e + f*x)**n,x)
```

3.492 $\int (b\sec(e+fx))^n \sin^5(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 80

$$\int (b\sec(e+fx))^n \sin^5(e+fx)\,dx = -\frac{b^5(b\sec(e+fx))^{-5+n}}{f(5-n)} + \frac{2b^3(b\sec(e+fx))^{-3+n}}{f(3-n)} - \frac{b(b\sec(e+fx))^{-1+n}}{f(1-n)}$$

output
```
-b^5*(b*sec(f*x+e))^(-5+n)/f/(5-n)+2*b^3*(b*sec(f*x+e))^(-3+n)/f/(3-n)-b*(
b*sec(f*x+e))^(-1+n)/f/(1-n)
```

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int (b\sec(e+fx))^n \sin^5(e+fx)\,dx = \frac{b(89-28n+3n^2-4(7-8n+n^2)\cos(2(e+fx))+(3-4n+n^2)\cos(4(e+fx)))\,(b\sec(e+fx))^{-1+n}}{8f(-5+n)(-3+n)(-1+n)}$$

input
```
Integrate[(b*Sec[e + f*x])^n*Sin[e + f*x]^5,x]
```

output
```
(b*(89 - 28*n + 3*n^2 - 4*(7 - 8*n + n^2)*Cos[2*(e + f*x)] + (3 - 4*n + n^
2)*Cos[4*(e + f*x)])*(b*Sec[e + f*x])^(-1 + n))/(8*f*(-5 + n)*(-3 + n)*(-1
 + n))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 3102, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(e+fx)(b\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\sec(e+fx))^n}{\csc(e+fx)^5}dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{b^5\int \frac{(b\sec(e+fx))^{n-6}\left(b^2-b^2\sec^2(e+fx)\right)^2}{b^4}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int (b\sec(e+fx))^{n-6}\left(b^2-b^2\sec^2(e+fx)\right)^2d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{244}$$

$$\frac{b\int \left(b^4(b\sec(e+fx))^{n-6}-2b^2(b\sec(e+fx))^{n-4}+(b\sec(e+fx))^{n-2}\right)d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(-\frac{b^4(b\sec(e+fx))^{n-5}}{5-n}+\frac{2b^2(b\sec(e+fx))^{n-3}}{3-n}-\frac{(b\sec(e+fx))^{n-1}}{1-n}\right)}{f}$$

input
```
Int[(b*Sec[e + f*x])^n*Sin[e + f*x]^5,x]
```

output
```
(b*(-((b^4*(b*Sec[e + f*x])^(-5 + n))/(5 - n)) + (2*b^2*(b*Sec[e + f*x])^(
-3 + n))/(3 - n) - (b*Sec[e + f*x])^(-1 + n)/(1 - n)))/f
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.09

method	result	size
parallelrisch	$\frac{(b\sec(fx+e))^n\left(\left(-\frac{3}{2}n^2+14n-\frac{25}{2}\right)\cos(3fx+3e)+\left(\frac{1}{2}n^2-2n+\frac{3}{2}\right)\cos(5fx+5e)+\cos(fx+e)\left(n^2-12n+75\right)\right)}{8(n^3-9n^2+23n-15)f}$	87
default	$\frac{\cos(fx+e)\mathrm{e}^{n\ln\left(\frac{b}{\cos(fx+e)}\right)}}{f(-1+n)}-\frac{2\cos(fx+e)^3\mathrm{e}^{n\ln\left(\frac{b}{\cos(fx+e)}\right)}}{f(-3+n)}+\frac{\cos(fx+e)^5\mathrm{e}^{n\ln\left(\frac{b}{\cos(fx+e)}\right)}}{f(-5+n)}$	94

input
```
int((b*sec(f*x+e))^n*sin(f*x+e)^5,x,method=_RETURNVERBOSE)
```

output
```
1/8*(b*sec(f*x+e))^n*((-3/2*n^2+14*n-25/2)*cos(3*f*x+3*e)+(1/2*n^2-2*n+3/2
)*cos(5*f*x+5*e)+cos(f*x+e)*(n^2-12*n+75))/(n^3-9*n^2+23*n-15)/f
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.06

$$\int (b\sec(e+fx))^n\sin^5(e+fx)\,dx = \frac{\left((n^2-4\,n+3)\cos\left(fx+e\right)^5-2\,(n^2-6\,n+5)\cos\left(fx+e\right)^3+(n^2-8\,n+15)\cos\left(fx+e\right)\right)\left(\frac{b}{\cos(fx+e)}\right)}{fn^3-9\,fn^2+23\,fn-15\,f}$$

input
```
integrate((b*sec(f*x+e))^n*sin(f*x+e)^5,x, algorithm="fricas")
```

output
```
((n^2 - 4*n + 3)*cos(f*x + e)^5 - 2*(n^2 - 6*n + 5)*cos(f*x + e)^3 + (n^2
- 8*n + 15)*cos(f*x + e))*(b/cos(f*x + e))^n/(f*n^3 - 9*f*n^2 + 23*f*n - 1
5*f)
```

Sympy [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^n\sin^5(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((b*sec(f*x+e))**n*sin(f*x+e)**5,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.06

$$\int (b\sec(e+fx))^n\sin^5(e+fx)\,dx = \frac{\frac{b^n\cos(fx+e)^{-n}\cos(fx+e)^5}{n-5} - \frac{2\,b^n\cos(fx+e)^{-n}\cos(fx+e)^3}{n-3} + \frac{b^n\cos(fx+e)^{-n}\cos(fx+e)}{n-1}}{f}$$

input
```
integrate((b*sec(f*x+e))^n*sin(f*x+e)^5,x, algorithm="maxima")
```

output
```
(b^n*cos(f*x + e)^(-n)*cos(f*x + e)^5/(n - 5) - 2*b^n*cos(f*x + e)^(-n)*co
s(f*x + e)^3/(n - 3) + b^n*cos(f*x + e)^(-n)*cos(f*x + e)/(n - 1))/f
```

Giac [F]

$$\int (b\sec(e+fx))^n\sin^5(e+fx)\,dx = \int (b\sec(fx+e))^n\sin(fx+e)^5\,dx$$

input
```
integrate((b*sec(f*x+e))^n*sin(f*x+e)^5,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^n*sin(f*x + e)^5, x)
```

Mupad [B] (verification not implemented)

Time = 26.95 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.68

$$\int (b\sec(e+fx))^n\sin^5(e+fx)\,dx = \frac{\left(\frac{b}{\cos(e+f\,x)}\right)^n (150\cos(e+f\,x) - 25\cos(3\,e+3\,f\,x) + 3\cos(5\,e+5\,f\,x) - 24\,n\cos(e+f\,x) + 28\,n\,\mathrm{co}}{16\,f\,(n^3-9\,n^2}$$

input
```
int(sin(e + f*x)^5*(b/cos(e + f*x))^n,x)
```

output
```
((b/cos(e + f*x))^n*(150*cos(e + f*x) - 25*cos(3*e + 3*f*x) + 3*cos(5*e +
5*f*x) - 24*n*cos(e + f*x) + 28*n*cos(3*e + 3*f*x) - 4*n*cos(5*e + 5*f*x)
+ 2*n^2*cos(e + f*x) - 3*n^2*cos(3*e + 3*f*x) + n^2*cos(5*e + 5*f*x)))/(16
*f*(23*n - 9*n^2 + n^3 - 15))
```

Reduce [F]

$$\int (b\sec(e+fx))^n \sin^5(e+fx)\,dx = b^n\left(\int \sec(fx+e)^n \sin(fx+e)^5\,dx\right)$$

input
```
int((b*sec(f*x+e))^n*sin(f*x+e)^5,x)
```

output
```
b**n*int(sec(e + f*x)**n*sin(e + f*x)**5,x)
```

3.493 $\int (b\sec(e+fx))^n \sin^3(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 52

$$\int (b\sec(e+fx))^n \sin^3(e+fx)\,dx = \frac{b^3(b\sec(e+fx))^{-3+n}}{f(3-n)} - \frac{b(b\sec(e+fx))^{-1+n}}{f(1-n)}$$

output
```
b^3*(b*sec(f*x+e))^(-3+n)/f/(3-n)-b*(b*sec(f*x+e))^(-1+n)/f/(1-n)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.90

$$\int (b\sec(e+fx))^n \sin^3(e+fx)\,dx = -\frac{b(5-n+(-1+n)\cos(2(e+fx)))(b\sec(e+fx))^{-1+n}}{2f(-3+n)(-1+n)}$$

input
```
Integrate[(b*Sec[e + f*x])^n*Sin[e + f*x]^3,x]
```

output
```
-1/2*(b*(5 - n + (-1 + n)*Cos[2*(e + f*x)])*(b*Sec[e + f*x])^(-1 + n))/(f*
(-3 + n)*(-1 + n))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 3102, 25, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(e+fx)(b\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\sec(e+fx))^n}{\csc(e+fx)^3}dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{b^3\int -\frac{(b\sec(e+fx))^{n-4}(b^2-b^2\sec^2(e+fx))}{b^2}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b^3\int \frac{(b\sec(e+fx))^{n-4}(b^2-b^2\sec^2(e+fx))}{b^2}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\int (b\sec(e+fx))^{n-4}\left(b^2-b^2\sec^2(e+fx)\right)d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{244}$$

$$-\frac{b\int \left(b^2(b\sec(e+fx))^{n-4}-(b\sec(e+fx))^{n-2}\right)d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\left(\frac{(b\sec(e+fx))^{n-1}}{1-n}-\frac{b^2(b\sec(e+fx))^{n-3}}{3-n}\right)}{f}$$

input
```
Int[(b*Sec[e + f*x])^n*Sin[e + f*x]^3,x]
```

output
```
-((b*(-((b^2*(b*Sec[e + f*x])^(-3 + n))/(3 - n)) + (b*Sec[e + f*x])^(-1 +
n)/(1 - n)))/f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{\left(\left(1-n\right)\cos\left(3fx+3e\right)+\cos\left(fx+e\right)\left(n-9\right)\right)\left(b\sec\left(fx+e\right)\right)^{n}}{4\left(n^{2}-4n+3\right)f}$	52
default	$\frac{\cos\left(fx+e\right){\mathrm e}^{n\ln\left(\frac{b}{\cos\left(fx+e\right)}\right)}}{f\left(-1+n\right)}-\frac{\cos\left(fx+e\right)^{3}{\mathrm e}^{n\ln\left(\frac{b}{\cos\left(fx+e\right)}\right)}}{f\left(-3+n\right)}$	63

input
```
int((b*sec(f*x+e))^n*sin(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*((1-n)*cos(3*f*x+3*e)+cos(f*x+e)*(n-9))*(b*sec(f*x+e))^n/(n^2-4*n+3)/f
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int (b\sec(e+fx))^n \sin^3(e+fx)\,dx = -\frac{\left((n-1)\cos{(fx+e)}^3-(n-3)\cos{(fx+e)}\right)\left(\frac{b}{\cos(fx+e)}\right)^n}{fn^2-4\,fn+3\,f}$$

input
```
integrate((b*sec(f*x+e))^n*sin(f*x+e)^3,x, algorithm="fricas")
```

output
```
-((n - 1)*cos(f*x + e)^3 - (n - 3)*cos(f*x + e))*(b/cos(f*x + e))^n/(f*n^2
 - 4*f*n + 3*f)
```

Sympy [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^n \sin^3(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((b*sec(f*x+e))**n*sin(f*x+e)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13

$$\int (b\sec(e+fx))^n \sin^3(e+fx)\,dx = -\frac{\frac{b^n \cos(fx+e)^{-n}\cos(fx+e)^3}{n-3} - \frac{b^n \cos(fx+e)^{-n}\cos(fx+e)}{n-1}}{f}$$

input
```
integrate((b*sec(f*x+e))^n*sin(f*x+e)^3,x, algorithm="maxima")
```

output
```
-(b^n*cos(f*x + e)^(-n)*cos(f*x + e)^3/(n - 3) - b^n*cos(f*x + e)^(-n)*cos
(f*x + e)/(n - 1))/f
```

Giac [F]

$$\int (b\sec(e+fx))^n \sin^3(e+fx)\,dx = \int (b\sec(fx+e))^n \sin(fx+e)^3\,dx$$

input
```
integrate((b*sec(f*x+e))^n*sin(f*x+e)^3,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^n*sin(f*x + e)^3, x)
```

Mupad [B] (verification not implemented)

Time = 26.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.29

$$\int (b\sec(e+fx))^n \sin^3(e+fx)\,dx = -\frac{\left(\frac{b}{\cos(e+f\,x)}\right)^n (9\cos(e+f\,x) - \cos(3\,e+3\,f\,x) - n\cos(e+f\,x) + n\cos(3\,e+3\,f\,x))}{4\,f\,\left(n^2-4\,n+3\right)}$$

input
```
int(sin(e + f*x)^3*(b/cos(e + f*x))^n,x)
```

output
```
-((b/cos(e + f*x))^n*(9*cos(e + f*x) - cos(3*e + 3*f*x) - n*cos(e + f*x) +
 n*cos(3*e + 3*f*x)))/(4*f*(n^2 - 4*n + 3))
```

Reduce [F]

$$\int (b\sec(e+fx))^n\sin^3(e+fx)\,dx = b^n\left(\int \sec\left(fx+e\right)^n\sin\left(fx+e\right)^3dx\right)$$

input
```
int((b*sec(f*x+e))^n*sin(f*x+e)^3,x)
```

output
```
b**n*int(sec(e + f*x)**n*sin(e + f*x)**3,x)
```

3.494 $\int (b\sec(e+fx))^n \sin(e+fx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 25

$$\int (b\sec(e+fx))^n \sin(e+fx)\,dx = -\frac{b(b\sec(e+fx))^{-1+n}}{f(1-n)}$$

output
```
-b*(b*sec(f*x+e))^(-1+n)/f/(1-n)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int (b\sec(e+fx))^n \sin(e+fx)\,dx = \frac{b(b\sec(e+fx))^{-1+n}}{f(-1+n)}$$

input
```
Integrate[(b*Sec[e + f*x])^n*Sin[e + f*x],x]
```

output
```
(b*(b*Sec[e + f*x])^(-1 + n))/(f*(-1 + n))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3042, 3102, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(e+fx)(b\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\sec(e+fx))^n}{\csc(e+fx)}dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{b\int (b\sec(e+fx))^{n-2}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{15}$$

$$-\frac{b(b\sec(e+fx))^{n-1}}{f(1-n)}$$

input
```
Int[(b*Sec[e + f*x])^n*Sin[e + f*x],x]
```

output
```
-((b*(b*Sec[e + f*x])^(-1 + n))/(f*(1 - n)))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

method	result
parallelrisch	$\frac{\cos(fx+e)(b\sec(fx+e))^n}{f(-1+n)}$
derivativedivides	$\frac{e^{n\ln(b\sec(fx+e))}}{f(-1+n)\sec(fx+e)}$
default	$\frac{e^{n\ln(b\sec(fx+e))}}{f(-1+n)\sec(fx+e)}$
norman	$\frac{\frac{e^{n\ln\left(\frac{b\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)}{1-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}\right)}}{f(-1+n)}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2 e^{n\ln\left(\frac{b\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)}{1-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}\right)}}{f(-1+n)}}{1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}$
risch	$\left(e^{2i(fx+e)}+1\right)^{-n}\cos(fx+e)2^n\left(e^{i(fx+e)}\right)^n b^n e^{\frac{i\pi n\left(-\operatorname{csgn}\left(\frac{ie^{i(fx+e)}}{e^{2i(fx+e)}+1}\right)^3+\operatorname{csgn}\left(\frac{ie^{i(fx+e)}}{e^{2i(fx+e)}+1}\right)^2\operatorname{csgn}\left(ie^{i(fx+e)}\right)+\operatorname{csgn}\right.}{}}$

input
```
int((b*sec(f*x+e))^n*sin(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
1/f/(-1+n)*cos(f*x+e)*(b*sec(f*x+e))^n
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int (b\sec(e+fx))^n\sin(e+fx)\,dx=\frac{\left(\frac{b}{\cos(fx+e)}\right)^n\cos(fx+e)}{fn-f}$$

input
```
integrate((b*sec(f*x+e))^n*sin(f*x+e),x, algorithm="fricas")
```

output
```
(b/cos(f*x + e))^n*cos(f*x + e)/(f*n - f)
```

Sympy [F]

$$\int (b\sec(e+fx))^n \sin(e+fx)\,dx = \int (b\sec(e+fx))^n \sin(e+fx)\,dx$$

input
```
integrate((b*sec(f*x+e))**n*sin(f*x+e),x)
```

output
```
Integral((b*sec(e + f*x))**n*sin(e + f*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int (b\sec(e+fx))^n \sin(e+fx)\,dx = \frac{b^n \cos(fx+e)^{-n} \cos(fx+e)}{f(n-1)}$$

input
```
integrate((b*sec(f*x+e))^n*sin(f*x+e),x, algorithm="maxima")
```

output
```
b^n*cos(f*x + e)^(-n)*cos(f*x + e)/(f*(n - 1))
```

Giac [F]

$$\int (b\sec(e+fx))^n \sin(e+fx)\,dx = \int (b\sec(fx+e))^n \sin(fx+e)\,dx$$

input
```
integrate((b*sec(f*x+e))^n*sin(f*x+e),x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^n*sin(f*x + e), x)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (b\sec(e+fx))^n\sin(e+fx)\,dx=\frac{\cos\left(e+f\,x\right)\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^n}{f\,\left(n-1\right)}$$

input
```
int(sin(e + f*x)*(b/cos(e + f*x))^n,x)
```

output
```
(cos(e + f*x)*(b/cos(e + f*x))^n)/(f*(n - 1))
```

Reduce [F]

$$\int (b\sec(e+fx))^n\sin(e+fx)\,dx=b^n\left(\int \sec\left(fx+e\right)^n\sin\left(fx+e\right)dx\right)$$

input
```
int((b*sec(f*x+e))^n*sin(f*x+e),x)
```

output
```
b**n*int(sec(e + f*x)**n*sin(e + f*x),x)
```

3.495 $\int \csc(e+fx)(b\sec(e+fx))^n\,dx$

Optimal result

Integrand size = 17, antiderivative size = 49

$$\int \csc(e+fx)(b\sec(e+fx))^n\,dx = -\frac{\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},\sec^2(e+fx)\right)(b\sec(e+fx))^{1+n}}{bf(1+n)}$$

output
```
-hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],sec(f*x+e)^2)*(b*sec(f*x+e))^(1+n)/b
/f/(1+n)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

$$\int \csc(e+fx)(b\sec(e+fx))^n\,dx = -\frac{\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},\sec^2(e+fx)\right)\sec(e+fx)(b\sec(e+fx))^n}{f(1+n)}$$

input
```
Integrate[Csc[e + f*x]*(b*Sec[e + f*x])^n,x]
```

output
```
-((Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Sec[e + f*x]^2]*Sec[e + f*x]
*(b*Sec[e + f*x])^n)/(f*(1 + n)))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3042, 3102, 25, 27, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(e+fx)(b\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(e+fx)(b\sec(e+fx))^n dx$$

$$\Big\downarrow \text{3102}$$

$$\frac{\int -\frac{b^2(b\sec(e+fx))^n}{b^2-b^2\sec^2(e+fx)}d(b\sec(e+fx))}{bf}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{b^2(b\sec(e+fx))^n}{b^2-b^2\sec^2(e+fx)}d(b\sec(e+fx))}{bf}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\int \frac{(b\sec(e+fx))^n}{b^2-b^2\sec^2(e+fx)}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{278}$$

$$-\frac{(b\sec(e+fx))^{n+1}\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},\sec^2(e+fx)\right)}{bf(n+1)}$$

input
```
Int[Csc[e + f*x]*(b*Sec[e + f*x])^n,x]
```

output
```
-((Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Sec[e + f*x]^2]*(b*Sec[e + f
*x])^(1 + n))/(b*f*(1 + n)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [F]

$$\int \csc(fx+e)\,(b\sec(fx+e))^{n}\,dx$$

input
```
int(csc(f*x+e)*(b*sec(f*x+e))^n,x)
```

output
```
int(csc(f*x+e)*(b*sec(f*x+e))^n,x)
```

Fricas [F]

$$\int \csc(e+fx)(b\sec(e+fx))^n\,dx=\int (b\sec(fx+e))^n\csc(fx+e)\,dx$$

input
```
integrate(csc(f*x+e)*(b*sec(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e))^n*csc(f*x + e), x)
```

Sympy [F]

$$\int \csc(e+fx)(b\sec(e+fx))^n\,dx=\int (b\sec(e+fx))^n\csc(e+fx)\,dx$$

input
```
integrate(csc(f*x+e)*(b*sec(f*x+e))**n,x)
```

output
```
Integral((b*sec(e + f*x))**n*csc(e + f*x), x)
```

Maxima [F]

$$\int \csc(e+fx)(b\sec(e+fx))^n\,dx=\int (b\sec(fx+e))^n\csc(fx+e)\,dx$$

input
```
integrate(csc(f*x+e)*(b*sec(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^n*csc(f*x + e), x)
```

Giac [F]

$$\int \csc(e+fx)(b\sec(e+fx))^n\,dx=\int (b\sec(fx+e))^n\csc(fx+e)\,dx$$

input
```
integrate(csc(f*x+e)*(b*sec(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^n*csc(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc(e+fx)(b\sec(e+fx))^n\,dx=\int \frac{\left(\frac{b}{\cos(e+f\,x)}\right)^n}{\sin(e+f\,x)}\,dx$$

input
```
int((b/cos(e + f*x))^n/sin(e + f*x),x)
```

output
```
int((b/cos(e + f*x))^n/sin(e + f*x), x)
```

Reduce [F]

$$\int \csc(e+fx)(b\sec(e+fx))^n\,dx=b^n\left(\int \sec(fx+e)^n\csc(fx+e)\,dx\right)$$

input
```
int(csc(f*x+e)*(b*sec(f*x+e))^n,x)
```

output
```
b**n*int(sec(e + f*x)**n*csc(e + f*x),x)
```

3.496 $\int \csc^3(e+fx)(b\sec(e+fx))^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 48

$$\int \csc^3(e+fx)(b\sec(e+fx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(2,\frac{3+n}{2},\frac{5+n}{2},\sec^2(e+fx)\right)(b\sec(e+fx))^{3+n}}{b^3f(3+n)}$$

output
```
hypergeom([2, 3/2+1/2*n],[5/2+1/2*n],sec(f*x+e)^2)*(b*sec(f*x+e))^(3+n)/b^
3/f/(3+n)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 201 vs. 2(48) = 96.

Time = 3.46 (sec) , antiderivative size = 201, normalized size of antiderivative = 4.19

$$\int \csc^3(e+fx)(b\sec(e+fx))^n\,dx = \frac{b(b\sec(e+fx))^{-1+n}\left(2\,\text{Hypergeometric2F1}(1,1-n,2-n,\cos(e+fx))+2\,\text{Hypergeometric2F1}(2,1-\right.}{}$$

input
```
Integrate[Csc[e + f*x]^3*(b*Sec[e + f*x])^n,x]
```

output
```
(b*(b*Sec[e + f*x])^(-1 + n)*(2*Hypergeometric2F1[1, 1 - n, 2 - n, Cos[e +
 f*x]] + 2*Hypergeometric2F1[2, 1 - n, 2 - n, Cos[e + f*x]] + 2^n*Hypergeo
metric2F1[1 - n, -n, 2 - n, (Cos[e + f*x]*Sec[(e + f*x)/2]^2)/2]*(Sec[(e +
 f*x)/2]^2)^(1 - n) + 2^n*Hypergeometric2F1[1 - n, 1 - n, 2 - n, (Cos[e +
f*x]*Sec[(e + f*x)/2]^2)/2]*Sec[e + f*x]^(1 - n)*(Cos[(e + f*x)/2]^2*Sec[e
 + f*x])^(-1 + n)))/(8*f*(-1 + n))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3102, 27, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(e+fx)(b\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc(e+fx)^3(b\sec(e+fx))^n dx$$

$$\Big\downarrow \text{ 3102}$$

$$\frac{\int \frac{b^4(b\sec(e+fx))^{n+2}}{(b^2-b^2\sec^2(e+fx))^2}d(b\sec(e+fx))}{b^3 f}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{b\int \frac{(b\sec(e+fx))^{n+2}}{(b^2-b^2\sec^2(e+fx))^2}d(b\sec(e+fx))}{f}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{(b\sec(e+fx))^{n+3}\,\text{Hypergeometric2F1}\left(2,\frac{n+3}{2},\frac{n+5}{2},\sec^2(e+fx)\right)}{b^3 f(n+3)}$$

input
```
Int[Csc[e + f*x]^3*(b*Sec[e + f*x])^n,x]
```

output
```
(Hypergeometric2F1[2, (3 + n)/2, (5 + n)/2, Sec[e + f*x]^2]*(b*Sec[e + f*x
])^(3 + n))/(b^3*f*(3 + n))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Maple [F]

$$\int \csc (fx+e)^3 (b\sec (fx+e))^n \, dx$$

input
```
int(csc(f*x+e)^3*(b*sec(f*x+e))^n,x)
```

output
```
int(csc(f*x+e)^3*(b*sec(f*x+e))^n,x)
```

Fricas [F]

$$\int \csc^3(e+fx)(b\sec(e+fx))^n\,dx = \int (b\sec(fx+e))^n\csc(fx+e)^3\,dx$$

input
```
integrate(csc(f*x+e)^3*(b*sec(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e))^n*csc(f*x + e)^3, x)
```

Sympy [F]

$$\int \csc^3(e+fx)(b\sec(e+fx))^n\,dx = \int (b\sec(e+fx))^n\csc^3(e+fx)\,dx$$

input
```
integrate(csc(f*x+e)**3*(b*sec(f*x+e))**n,x)
```

output
```
Integral((b*sec(e + f*x))**n*csc(e + f*x)**3, x)
```

Maxima [F]

$$\int \csc^3(e+fx)(b\sec(e+fx))^n\,dx = \int (b\sec(fx+e))^n\csc(fx+e)^3\,dx$$

input
```
integrate(csc(f*x+e)^3*(b*sec(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^n*csc(f*x + e)^3, x)
```

Giac [F]

$$\int \csc^3(e+fx)(b\sec(e+fx))^n\,dx=\int (b\sec(fx+e))^n\csc(fx+e)^3\,dx$$

input
```
integrate(csc(f*x+e)^3*(b*sec(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^n*csc(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(e+fx)(b\sec(e+fx))^n\,dx=\int \frac{\left(\frac{b}{\cos(e+f\,x)}\right)^n}{\sin(e+f\,x)^3}\,dx$$

input
```
int((b/cos(e + f*x))^n/sin(e + f*x)^3,x)
```

output
```
int((b/cos(e + f*x))^n/sin(e + f*x)^3, x)
```

Reduce [F]

$$\int \csc^3(e+fx)(b\sec(e+fx))^n\,dx=b^n\left(\int \sec(fx+e)^n\csc(fx+e)^3\,dx\right)$$

input
```
int(csc(f*x+e)^3*(b*sec(f*x+e))^n,x)
```

output
```
b**n*int(sec(e + f*x)**n*csc(e + f*x)**3,x)
```

3.497 $\int (b\sec(e+fx))^n \sin^6(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 73

$$\int (b\sec(e+fx))^n \sin^6(e+fx)\,dx = -\frac{b\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)(b\sec(e+fx))^{-1+n}\sin(e+fx)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

output
```
-b*hypergeom([-5/2, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*(b*sec(f*x+e))^(-
1+n)*sin(f*x+e)/f/(1-n)/(sin(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int (b\sec(e+fx))^n \sin^6(e+fx)\,dx = \frac{\text{Hypergeometric2F1}\left(\frac{7}{2},4-\frac{n}{2},\frac{9}{2},-\tan^2(e+fx)\right)(b\sec(e+fx))^n\sec^2(e+fx)^{-n/2}\tan^7(e+fx)}{7f}$$

input
```
Integrate[(b*Sec[e + f*x])^n*Sin[e + f*x]^6,x]
```

output
```
(Hypergeometric2F1[7/2, 4 - n/2, 9/2, -Tan[e + f*x]^2]*(b*Sec[e + f*x])^n*
Tan[e + f*x]^7)/(7*f*(Sec[e + f*x]^2)^(n/2))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3112, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^6(e+fx)(b\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\sec(e+fx))^n}{\csc(e+fx)^6}dx$$

$$\Big\downarrow \text{3112}$$

$$b^2(b\cos(e+fx))^{n-1}(b\sec(e+fx))^{n-1}\int (b\cos(e+fx))^{-n}\sin^6(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$b^2(b\cos(e+fx))^{n-1}(b\sec(e+fx))^{n-1}\int (b\cos(e+fx))^{-n}\sin(e+fx)^6dx$$

$$\Big\downarrow \text{3056}$$

$$-\frac{b\sin(e+fx)(b\sec(e+fx))^{n-1}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

input
```
Int[(b*Sec[e + f*x])^n*Sin[e + f*x]^6,x]
```

output
```
-((b*Hypergeometric2F1[-5/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*(b*Sec[
e + f*x])^(-1 + n)*Sin[e + f*x])/(f*(1 - n)*Sqrt[Sin[e + f*x]^2]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3112
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1)   Int[1/((a*Cos[e
+ f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Maple [F]

$$\int (b\sec(fx+e))^n \sin(fx+e)^6\,dx$$

input
```
int((b*sec(f*x+e))^n*sin(f*x+e)^6,x)
```

output
```
int((b*sec(f*x+e))^n*sin(f*x+e)^6,x)
```

Fricas [F]

$$\int (b\sec(e+fx))^n \sin^6(e+fx)\,dx = \int (b\sec(fx+e))^n \sin(fx+e)^6\,dx$$

input
```
integrate((b*sec(f*x+e))^n*sin(f*x+e)^6,x, algorithm="fricas")
```

output
```
integral(-(cos(f*x + e)^6 - 3*cos(f*x + e)^4 + 3*cos(f*x + e)^2 - 1)*(b*se
c(f*x + e))^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^n \sin^6(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((b*sec(f*x+e))**n*sin(f*x+e)**6,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\sec(e+fx))^n \sin^6(e+fx)\,dx = \int (b\sec(fx+e))^n \sin(fx+e)^6\,dx$$

input
```
integrate((b*sec(f*x+e))^n*sin(f*x+e)^6,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^n*sin(f*x + e)^6, x)
```

Giac [F]

$$\int (b\sec(e+fx))^n \sin^6(e+fx)\,dx = \int (b\sec(fx+e))^n \sin(fx+e)^6\,dx$$

input
```
integrate((b*sec(f*x+e))^n*sin(f*x+e)^6,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^n*sin(f*x + e)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^n \sin^6(e+fx)\,dx = \int \sin(e+f\,x)^6 \left(\frac{b}{\cos(e+f\,x)}\right)^n\,dx$$

input
```
int(sin(e + f*x)^6*(b/cos(e + f*x))^n,x)
```

output
```
int(sin(e + f*x)^6*(b/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int (b\sec(e+fx))^n \sin^6(e+fx)\,dx = b^n \left(\int \sec(fx+e)^n \sin(fx+e)^6\,dx\right)$$

input
```
int((b*sec(f*x+e))^n*sin(f*x+e)^6,x)
```

output
```
b**n*int(sec(e + f*x)**n*sin(e + f*x)**6,x)
```

3.498 $\int (b\sec(e+fx))^n \sin^4(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 73

$$\int (b\sec(e+fx))^n \sin^4(e+fx)\,dx = -\frac{b\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)(b\sec(e+fx))^{-1+n}\sin(e+fx)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

output
```
-b*hypergeom([-3/2, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*(b*sec(f*x+e))^(-
1+n)*sin(f*x+e)/f/(1-n)/(sin(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int (b\sec(e+fx))^n \sin^4(e+fx)\,dx = \frac{\text{Hypergeometric2F1}\left(\frac{5}{2},3-\frac{n}{2},\frac{7}{2},-\tan^2(e+fx)\right)(b\sec(e+fx))^n\sec^2(e+fx)^{-n/2}\tan^5(e+fx)}{5f}$$

input
```
Integrate[(b*Sec[e + f*x])^n*Sin[e + f*x]^4,x]
```

output
```
(Hypergeometric2F1[5/2, 3 - n/2, 7/2, -Tan[e + f*x]^2]*(b*Sec[e + f*x])^n*
Tan[e + f*x]^5)/(5*f*(Sec[e + f*x]^2)^(n/2))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3112, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(e+fx)(b\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\sec(e+fx))^n}{\csc(e+fx)^4}dx$$

$$\Big\downarrow \text{3112}$$

$$b^2(b\cos(e+fx))^{n-1}(b\sec(e+fx))^{n-1}\int (b\cos(e+fx))^{-n}\sin^4(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$b^2(b\cos(e+fx))^{n-1}(b\sec(e+fx))^{n-1}\int (b\cos(e+fx))^{-n}\sin(e+fx)^4dx$$

$$\Big\downarrow \text{3056}$$

$$-\frac{b\sin(e+fx)(b\sec(e+fx))^{n-1}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

input
```
Int[(b*Sec[e + f*x])^n*Sin[e + f*x]^4,x]
```

output
```
-((b*Hypergeometric2F1[-3/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*(b*Sec[
e + f*x])^(-1 + n)*Sin[e + f*x])/(f*(1 - n)*Sqrt[Sin[e + f*x]^2]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3112
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1)   Int[1/((a*Cos[e
+ f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Maple [F]

$$\int (b\sec (fx+e))^n \sin (fx+e)^4 dx$$

input
```
int((b*sec(f*x+e))^n*sin(f*x+e)^4,x)
```

output
```
int((b*sec(f*x+e))^n*sin(f*x+e)^4,x)
```

Fricas [F]

$$\int (b\sec(e+fx))^n \sin^4(e+fx)\,dx=\int (b\sec (fx+e))^n \sin (fx+e)^4\,dx$$

input
```
integrate((b*sec(f*x+e))^n*sin(f*x+e)^4,x, algorithm="fricas")
```

output
```
integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*(b*sec(f*x + e))^n, x)
```

Sympy [F]

$$\int (b\sec(e+fx))^n\sin^4(e+fx)\,dx=\int (b\sec{(e+fx)})^n\sin^4{(e+fx)}\,dx$$

input
```
integrate((b*sec(f*x+e))**n*sin(f*x+e)**4,x)
```

output
```
Integral((b*sec(e + f*x))**n*sin(e + f*x)**4, x)
```

Maxima [F]

$$\int (b\sec(e+fx))^n\sin^4(e+fx)\,dx=\int (b\sec{(fx+e)})^n\sin{(fx+e)}^4\,dx$$

input
```
integrate((b*sec(f*x+e))^n*sin(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^n*sin(f*x + e)^4, x)
```

Giac [F]

$$\int (b\sec(e+fx))^n\sin^4(e+fx)\,dx=\int (b\sec{(fx+e)})^n\sin{(fx+e)}^4\,dx$$

input
```
integrate((b*sec(f*x+e))^n*sin(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^n*sin(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^n \sin^4(e+fx)\,dx = \int \sin\left(e+f\,x\right)^4\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^n \,d x$$

input
```
int(sin(e + f*x)^4*(b/cos(e + f*x))^n,x)
```

output
```
int(sin(e + f*x)^4*(b/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int (b\sec(e+fx))^n \sin^4(e+fx)\,dx = b^n \left(\int \sec\left(fx+e\right)^n \sin\left(fx+e\right)^4dx\right)$$

input
```
int((b*sec(f*x+e))^n*sin(f*x+e)^4,x)
```

output
```
b**n*int(sec(e + f*x)**n*sin(e + f*x)**4,x)
```

3.499 $\int (b\sec(e+fx))^n \sin^2(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 73

$$\int (b\sec(e+fx))^n \sin^2(e+fx)\,dx = -\frac{b\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)(b\sec(e+fx))^{-1+n}\sin(e+fx)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

output
```
-b*hypergeom([-1/2, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*(b*sec(f*x+e))^(-
1+n)*sin(f*x+e)/f/(1-n)/(sin(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int (b\sec(e+fx))^n \sin^2(e+fx)\,dx = \frac{\text{Hypergeometric2F1}\left(\frac{3}{2},2-\frac{n}{2},\frac{5}{2},-\tan^2(e+fx)\right)(b\sec(e+fx))^n\sec^2(e+fx)^{-n/2}\tan^3(e+fx)}{3f}$$

input
```
Integrate[(b*Sec[e + f*x])^n*Sin[e + f*x]^2,x]
```

output
```
(Hypergeometric2F1[3/2, 2 - n/2, 5/2, -Tan[e + f*x]^2]*(b*Sec[e + f*x])^n*
Tan[e + f*x]^3)/(3*f*(Sec[e + f*x]^2)^(n/2))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3112, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(e+fx)(b\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\sec(e+fx))^n}{\csc(e+fx)^2}dx$$

$$\Big\downarrow \text{3112}$$

$$b^2(b\cos(e+fx))^{n-1}(b\sec(e+fx))^{n-1}\int (b\cos(e+fx))^{-n}\sin^2(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$b^2(b\cos(e+fx))^{n-1}(b\sec(e+fx))^{n-1}\int (b\cos(e+fx))^{-n}\sin(e+fx)^2dx$$

$$\Big\downarrow \text{3056}$$

$$-\frac{b\sin(e+fx)(b\sec(e+fx))^{n-1}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

input
```
Int[(b*Sec[e + f*x])^n*Sin[e + f*x]^2,x]
```

output
```
-((b*Hypergeometric2F1[-1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*(b*Sec[
e + f*x])^(-1 + n)*Sin[e + f*x])/(f*(1 - n)*Sqrt[Sin[e + f*x]^2]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3112
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1)   Int[1/((a*Cos[e
+ f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Maple [F]

$$\int (b\sec (fx+e))^n \sin (fx+e)^2 \, dx$$

input
```
int((b*sec(f*x+e))^n*sin(f*x+e)^2,x)
```

output
```
int((b*sec(f*x+e))^n*sin(f*x+e)^2,x)
```

Fricas [F]

$$\int (b\sec(e+fx))^n \sin^2(e+fx)\, dx = \int (b\sec (fx+e))^n \sin (fx+e)^2 \, dx$$

input
```
integrate((b*sec(f*x+e))^n*sin(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral(-(cos(f*x + e)^2 - 1)*(b*sec(f*x + e))^n, x)
```

Sympy [F]

$$\int (b\sec(e+fx))^n \sin^2(e+fx)\,dx = \int (b\sec(e+fx))^n \sin^2(e+fx)\,dx$$

input
```
integrate((b*sec(f*x+e))**n*sin(f*x+e)**2,x)
```

output
```
Integral((b*sec(e + f*x))**n*sin(e + f*x)**2, x)
```

Maxima [F]

$$\int (b\sec(e+fx))^n \sin^2(e+fx)\,dx = \int (b\sec(fx+e))^n \sin(fx+e)^2\,dx$$

input
```
integrate((b*sec(f*x+e))^n*sin(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^n*sin(f*x + e)^2, x)
```

Giac [F]

$$\int (b\sec(e+fx))^n \sin^2(e+fx)\,dx = \int (b\sec(fx+e))^n \sin(fx+e)^2\,dx$$

input
```
integrate((b*sec(f*x+e))^n*sin(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^n*sin(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^n\sin^2(e+fx)\,dx=\int \sin\left(e+f\,x\right)^2\,\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^n\,dx$$

input
```
int(sin(e + f*x)^2*(b/cos(e + f*x))^n,x)
```

output
```
int(sin(e + f*x)^2*(b/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int (b\sec(e+fx))^n\sin^2(e+fx)\,dx=b^n\left(\int \sec\left(fx+e\right)^n\sin\left(fx+e\right)^2dx\right)$$

input
```
int((b*sec(f*x+e))^n*sin(f*x+e)^2,x)
```

output
```
b**n*int(sec(e + f*x)**n*sin(e + f*x)**2,x)
```

3.500 $\int (b\sec(e+fx))^n\,dx$

Optimal result

Integrand size = 10, antiderivative size = 73

$$\int (b\sec(e+fx))^n\,dx = -\frac{b\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)(b\sec(e+fx))^{-1+n}\sin(e+fx)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

output
```
-b*hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*(b*sec(f*x+e))^(-1
+n)*sin(f*x+e)/f/(1-n)/(sin(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

$$\int (b\sec(e+fx))^n\,dx = \frac{\cot(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\sec^2(e+fx)\right)(b\sec(e+fx))^n\sqrt{-\tan^2(e+fx)}}{fn}$$

input
```
Integrate[(b*Sec[e + f*x])^n,x]
```

output
```
(Cot[e + f*x]*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Sec[e + f*x]^2]*(b*Se
c[e + f*x])^n*Sqrt[-Tan[e + f*x]^2])/(f*n)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 4259, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\csc\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{4259}$$

$$\left(\frac{\cos(e+fx)}{b}\right)^n (b\sec(e+fx))^n \int \left(\frac{\cos(e+fx)}{b}\right)^{-n} dx$$

$$\Big\downarrow \text{3042}$$

$$\left(\frac{\cos(e+fx)}{b}\right)^n (b\sec(e+fx))^n \int \left(\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{b}\right)^{-n} dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{b\sin(e+fx)(b\sec(e+fx))^{n-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

input
```
Int[(b*Sec[e + f*x])^n,x]
```

output
```
-((b*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*(b*Sec[e
 + f*x])^(-1 + n)*Sin[e + f*x])/(f*(1 - n)*Sqrt[Sin[e + f*x]^2]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 4259
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)   Int[1/(Sin[c + d*x]/b)^n, x]), x] /;
FreeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Maple [F]

$$\int (b\sec(fx+e))^n\,dx$$

input
```
int((b*sec(f*x+e))^n,x)
```

output
```
int((b*sec(f*x+e))^n,x)
```

Fricas [F]

$$\int (b\sec(e+fx))^n\,dx = \int (b\sec(fx+e))^n\,dx$$

input
```
integrate((b*sec(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e))^n, x)
```

Sympy [F]

$$\int (b\sec(e+fx))^n\,dx = \int (b\sec(e+fx))^n\,dx$$

input
```
integrate((b*sec(f*x+e))**n,x)
```

output
```
Integral((b*sec(e + f*x))**n, x)
```

Maxima [F]

$$\int (b\sec(e+fx))^n\,dx = \int (b\sec(fx+e))^n\,dx$$

input
```
integrate((b*sec(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^n, x)
```

Giac [F]

$$\int (b\sec(e+fx))^n\,dx = \int (b\sec(fx+e))^n\,dx$$

input
```
integrate((b*sec(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(e+fx))^n\,dx = \int \left(\frac{b}{\cos\left(e+f\,x\right)}\right)^n\,dx$$

input
```
int((b/cos(e + f*x))^n,x)
```

output
```
int((b/cos(e + f*x))^n, x)
```

Reduce [F]

$$\int (b\sec(e+fx))^n\,dx = b^n\left(\int \sec\left(fx+e\right)^n dx\right)$$

input
```
int((b*sec(f*x+e))^n,x)
```

output
```
b**n*int(sec(e + f*x)**n,x)
```

3.501 $\int \csc^2(e+fx)(b\sec(e+fx))^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 73

$$\int \csc^2(e+fx)(b\sec(e+fx))^n\,dx = -\frac{b\csc(e+fx)\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)(b\sec(e+fx))^{-1+n}\sqrt{\sin^2(e+fx)}}{f(1-n)}$$

output
```
-b*csc(f*x+e)*hypergeom([3/2, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*(b*sec(
f*x+e))^(-1+n)*(sin(f*x+e)^2)^(1/2)/f/(1-n)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int \csc^2(e+fx)(b\sec(e+fx))^n\,dx = -\frac{\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{n}{2},\frac{1}{2},-\tan^2(e+fx)\right)(b\sec(e+fx))^n\sec^2(e+fx)^{-n/2}}{f}$$

input
```
Integrate[Csc[e + f*x]^2*(b*Sec[e + f*x])^n,x]
```

output
```
-((Cot[e + f*x]*Hypergeometric2F1[-1/2, -1/2*n, 1/2, -Tan[e + f*x]^2]*(b*S
ec[e + f*x])^n)/(f*(Sec[e + f*x]^2)^(n/2)))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3112, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(e+fx)(b\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(e+fx)^2(b\sec(e+fx))^ndx$$

$$\Big\downarrow \text{3112}$$

$$b^2(b\cos(e+fx))^{n-1}(b\sec(e+fx))^{n-1}\int (b\cos(e+fx))^{-n}\csc^2(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$b^2(b\cos(e+fx))^{n-1}(b\sec(e+fx))^{n-1}\int \frac{(b\cos(e+fx))^{-n}}{\sin(e+fx)^2}dx$$

$$\Big\downarrow \text{3056}$$

$$-\frac{b\sqrt{\sin^2(e+fx)}\csc(e+fx)(b\sec(e+fx))^{n-1}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f(1-n)}$$

input
```
Int[Csc[e + f*x]^2*(b*Sec[e + f*x])^n,x]
```

output
```
-((b*Csc[e + f*x]*Hypergeometric2F1[3/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x
]^2]*(b*Sec[e + f*x])^(-1 + n)*Sqrt[Sin[e + f*x]^2])/(f*(1 - n)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3112
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1)   Int[1/((a*Cos[e
+ f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Maple [F]

$$\int \csc \left(fx+e\right)^{2} \left(b \sec \left(fx+e\right)\right)^{n} dx$$

input
```
int(csc(f*x+e)^2*(b*sec(f*x+e))^n,x)
```

output
```
int(csc(f*x+e)^2*(b*sec(f*x+e))^n,x)
```

Fricas [F]

$$\int \csc^2(e+fx)(b\sec(e+fx))^n\,dx = \int \left(b \sec \left(fx+e\right)\right)^{n} \csc \left(fx+e\right)^{2}\,dx$$

input
```
integrate(csc(f*x+e)^2*(b*sec(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e))^n*csc(f*x + e)^2, x)
```

Sympy [F]

$$\int \csc^2(e+fx)(b\sec(e+fx))^n\,dx = \int (b\sec{(e+fx)})^n \csc^2{(e+fx)}\,dx$$

input
```
integrate(csc(f*x+e)**2*(b*sec(f*x+e))**n,x)
```

output
```
Integral((b*sec(e + f*x))**n*csc(e + f*x)**2, x)
```

Maxima [F]

$$\int \csc^2(e+fx)(b\sec(e+fx))^n\,dx = \int (b\sec{(fx+e)})^n \csc{(fx+e)}^2\,dx$$

input
```
integrate(csc(f*x+e)^2*(b*sec(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^n*csc(f*x + e)^2, x)
```

Giac [F]

$$\int \csc^2(e+fx)(b\sec(e+fx))^n\,dx = \int (b\sec{(fx+e)})^n \csc{(fx+e)}^2\,dx$$

input
```
integrate(csc(f*x+e)^2*(b*sec(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^n*csc(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(e+fx)(b\sec(e+fx))^n\,dx=\int \frac{\left(\frac{b}{\cos(e+f\,x)}\right)^n}{\sin\left(e+f\,x\right)^2}\,dx$$

input
```
int((b/cos(e + f*x))^n/sin(e + f*x)^2,x)
```

output
```
int((b/cos(e + f*x))^n/sin(e + f*x)^2, x)
```

Reduce [F]

$$\int \csc^2(e+fx)(b\sec(e+fx))^n\,dx=b^n\left(\int \sec\left(fx+e\right)^n\csc\left(fx+e\right)^2dx\right)$$

input
```
int(csc(f*x+e)^2*(b*sec(f*x+e))^n,x)
```

output
```
b**n*int(sec(e + f*x)**n*csc(e + f*x)**2,x)
```

3.502 $\int \csc^4(e+fx)(b\sec(e+fx))^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 73

$$\int \csc^4(e+fx)(b\sec(e+fx))^n\,dx = -\frac{b\csc(e+fx)\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)(b\sec(e+fx))^{-1+n}\sqrt{\sin^2(e+fx)}}{f(1-n)}$$

output
```
-b*csc(f*x+e)*hypergeom([5/2, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*(b*sec(
f*x+e))^(-1+n)*(sin(f*x+e)^2)^(1/2)/f/(1-n)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int \csc^4(e+fx)(b\sec(e+fx))^n\,dx = -\frac{\cot^3(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-1-\frac{n}{2},-\frac{1}{2},-\tan^2(e+fx)\right)(b\sec(e+fx))^n\sec^2(e+fx)^{-n/}}{3f}$$

input
```
Integrate[Csc[e + f*x]^4*(b*Sec[e + f*x])^n,x]
```

output
```
-1/3*(Cot[e + f*x]^3*Hypergeometric2F1[-3/2, -1 - n/2, -1/2, -Tan[e + f*x]
^2]*(b*Sec[e + f*x])^n)/(f*(Sec[e + f*x]^2)^(n/2))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3112, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(e+fx)(b\sec(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(e+fx)^4(b\sec(e+fx))^n dx$$

$$\Big\downarrow \text{3112}$$

$$b^2(b\cos(e+fx))^{n-1}(b\sec(e+fx))^{n-1}\int (b\cos(e+fx))^{-n}\csc^4(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$b^2(b\cos(e+fx))^{n-1}(b\sec(e+fx))^{n-1}\int \frac{(b\cos(e+fx))^{-n}}{\sin(e+fx)^4}dx$$

$$\Big\downarrow \text{3056}$$

$$-\frac{b\sqrt{\sin^2(e+fx)}\csc(e+fx)(b\sec(e+fx))^{n-1}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1-n}{2},\frac{3-n}{2},\cos^2(e+fx)\right)}{f(1-n)}$$

input
```
Int[Csc[e + f*x]^4*(b*Sec[e + f*x])^n,x]
```

output
```
-((b*Csc[e + f*x]*Hypergeometric2F1[5/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x
]^2]*(b*Sec[e + f*x])^(-1 + n)*Sqrt[Sin[e + f*x]^2])/(f*(1 - n)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3112
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(a^2/b^2)*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(
n + 1)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1)   Int[1/((a*Cos[e
+ f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Maple [F]

$$\int \csc (fx+e)^4 (b\sec (fx+e))^n \, dx$$

input
```
int(csc(f*x+e)^4*(b*sec(f*x+e))^n,x)
```

output
```
int(csc(f*x+e)^4*(b*sec(f*x+e))^n,x)
```

Fricas [F]

$$\int \csc^4(e+fx)(b\sec(e+fx))^n \, dx = \int (b\sec (fx+e))^n \csc (fx+e)^4 \, dx$$

input
```
integrate(csc(f*x+e)^4*(b*sec(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((b*sec(f*x + e))^n*csc(f*x + e)^4, x)
```

Sympy [F]

$$\int \csc^4(e+fx)(b\sec(e+fx))^n\,dx = \int (b\sec{(e+fx)})^n\csc^4{(e+fx)}\,dx$$

input
```
integrate(csc(f*x+e)**4*(b*sec(f*x+e))**n,x)
```

output
```
Integral((b*sec(e + f*x))**n*csc(e + f*x)**4, x)
```

Maxima [F]

$$\int \csc^4(e+fx)(b\sec(e+fx))^n\,dx = \int (b\sec{(fx+e)})^n\csc{(fx+e)}^4\,dx$$

input
```
integrate(csc(f*x+e)^4*(b*sec(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((b*sec(f*x + e))^n*csc(f*x + e)^4, x)
```

Giac [F]

$$\int \csc^4(e+fx)(b\sec(e+fx))^n\,dx = \int (b\sec{(fx+e)})^n\csc{(fx+e)}^4\,dx$$

input
```
integrate(csc(f*x+e)^4*(b*sec(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((b*sec(f*x + e))^n*csc(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^4(e+fx)(b\sec(e+fx))^n\,dx=\int \frac{\left(\frac{b}{\cos(e+f\,x)}\right)^n}{\sin\left(e+f\,x\right)^4}\,dx$$

input
```
int((b/cos(e + f*x))^n/sin(e + f*x)^4,x)
```

output
```
int((b/cos(e + f*x))^n/sin(e + f*x)^4, x)
```

Reduce [F]

$$\int \csc^4(e+fx)(b\sec(e+fx))^n\,dx=b^n\left(\int \sec\left(fx+e\right)^n\csc\left(fx+e\right)^4dx\right)$$

input
```
int(csc(f*x+e)^4*(b*sec(f*x+e))^n,x)
```

output
```
b**n*int(sec(e + f*x)**n*csc(e + f*x)**4,x)
```

3.503 $\int (b\sec(a+bx))^n (c\sin(a+bx))^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int (b\sec(a+bx))^n (c\sin(a+bx))^{3/2}\,dx = -\frac{c\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1-n}{2},\frac{3-n}{2},\cos^2(a+bx)\right)(b\sec(a+bx))^{-1+n}\sqrt{c\sin(a+bx)}}{(1-n)\sqrt[4]{\sin^2(a+bx)}}$$

output
```
-c*hypergeom([-1/4, 1/2-1/2*n],[3/2-1/2*n],cos(b*x+a)^2)*(b*sec(b*x+a))^(-
1+n)*(c*sin(b*x+a))^(1/2)/(1-n)/(sin(b*x+a)^2)^(1/4)
```

Mathematica [A] (verified)

Time = 33.01 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.37

$$\int (b\sec(a+bx))^n (c\sin(a+bx))^{3/2}\,dx = \frac{2\cos^2(a+bx)^{\frac{1}{2}(-1+n)}(b\sec(a+bx))^{-1+n}(c\sin(a+bx))^{5/2}\left(9\,\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{1}{2}(-1+\right.\right.}{45c}$$

input
```
Integrate[(b*Sec[a + b*x])^n*(c*Sin[a + b*x])^(3/2),x]
```

output
```
(2*(Cos[a + b*x]^2)^((-1 + n)/2)*(b*Sec[a + b*x])^(-1 + n)*(c*Sin[a + b*x]
)^(5/2)*(9*Hypergeometric2F1[5/4, (-1 + n)/2, 9/4, Sin[a + b*x]^2] + 5*Hyp
ergeometric2F1[9/4, (1 + n)/2, 13/4, Sin[a + b*x]^2]*Sin[a + b*x]^2))/(45*
c)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3067, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\sin(a+bx))^{3/2}(b\sec(a+bx))^n\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (c\sin(a+bx))^{3/2}(b\sec(a+bx))^n dx$$

$$\Big\downarrow \text{ 3067}$$

$$b^2(b\cos(a+bx))^{n-1}(b\sec(a+bx))^{n-1}\int (b\cos(a+bx))^{-n}(c\sin(a+bx))^{3/2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$b^2(b\cos(a+bx))^{n-1}(b\sec(a+bx))^{n-1}\int (b\cos(a+bx))^{-n}(c\sin(a+bx))^{3/2}dx$$

$$\Big\downarrow \text{ 3056}$$

$$-\frac{c\sqrt{c\sin(a+bx)}(b\sec(a+bx))^{n-1}\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1-n}{2},\frac{3-n}{2},\cos^2(a+bx)\right)}{(1-n)\sqrt[4]{\sin^2(a+bx)}}$$

input
```
Int[(b*Sec[a + b*x])^n*(c*Sin[a + b*x])^(3/2),x]
```

output
```
-((c*Hypergeometric2F1[-1/4, (1 - n)/2, (3 - n)/2, Cos[a + b*x]^2]*(b*Sec[
a + b*x])^(-1 + n)*Sqrt[c*Sin[a + b*x]])/((1 - n)*(Sin[a + b*x]^2)^(1/4)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3067
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
   Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

Maple [F]

$$\int (b\sec (bx+a))^{n} (c\sin (bx+a))^{\frac{3}{2}}\, dx$$

input
```
int((b*sec(b*x+a))^n*(c*sin(b*x+a))^(3/2),x)
```

output
```
int((b*sec(b*x+a))^n*(c*sin(b*x+a))^(3/2),x)
```

Fricas [F]

$$\int (b\sec(a+bx))^n(c\sin(a+bx))^{3/2}\, dx = \int (c\sin (bx+a))^{\frac{3}{2}} (b\sec (bx+a))^{n}\, dx$$

input
```
integrate((b*sec(b*x+a))^n*(c*sin(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*sin(b*x + a))*(b*sec(b*x + a))^n*c*sin(b*x + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int (b\sec(a+bx))^n (c\sin(a+bx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((b*sec(b*x+a))**n*(c*sin(b*x+a))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (b\sec(a+bx))^n (c\sin(a+bx))^{3/2}\,dx = \int (c\sin(bx+a))^{\frac{3}{2}} (b\sec(bx+a))^n\,dx$$

input
```
integrate((b*sec(b*x+a))^n*(c*sin(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*sin(b*x + a))^(3/2)*(b*sec(b*x + a))^n, x)
```

Giac [F]

$$\int (b\sec(a+bx))^n (c\sin(a+bx))^{3/2}\,dx = \int (c\sin(bx+a))^{\frac{3}{2}} (b\sec(bx+a))^n\,dx$$

input
```
integrate((b*sec(b*x+a))^n*(c*sin(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate((c*sin(b*x + a))^(3/2)*(b*sec(b*x + a))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(a+bx))^n(c\sin(a+bx))^{3/2}\,dx = \int (c\,\sin(a+b\,x))^{3/2}\left(\frac{b}{\cos(a+b\,x)}\right)^n\,dx$$

input
```
int((c*sin(a + b*x))^(3/2)*(b/cos(a + b*x))^n,x)
```

output
```
int((c*sin(a + b*x))^(3/2)*(b/cos(a + b*x))^n, x)
```

Reduce [F]

$$\int (b\sec(a+bx))^n(c\sin(a+bx))^{3/2}\,dx = \sqrt{c}\,b^n\left(\int \sqrt{\sin(bx+a)}\,\sec(bx+a)^n\sin(bx+a)\,dx\right)c$$

input
```
int((b*sec(b*x+a))^n*(c*sin(b*x+a))^(3/2),x)
```

output
```
sqrt(c)*b**n*int(sqrt(sin(a + b*x))*sec(a + b*x)**n*sin(a + b*x),x)*c
```

3.504 $\int (b\sec(a+bx))^n\sqrt{c\sin(a+bx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int (b\sec(a+bx))^n\sqrt{c\sin(a+bx)}\,dx = -\frac{c\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1-n}{2},\frac{3-n}{2},\cos^2(a+bx)\right)(b\sec(a+bx))^{-1+n}\sqrt[4]{\sin^2(a+bx)}}{(1-n)\sqrt{c\sin(a+bx)}}$$

output
```
-c*hypergeom([1/4, 1/2-1/2*n],[3/2-1/2*n],cos(b*x+a)^2)*(b*sec(b*x+a))^(-1
+n)*(sin(b*x+a)^2)^(1/4)/(1-n)/(c*sin(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 10.60 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99

$$\int (b\sec(a+bx))^n\sqrt{c\sin(a+bx)}\,dx = \frac{\cos^2(a+bx)^{\frac{1}{2}(-1+n)}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1+n}{2},\frac{7}{4},\sin^2(a+bx)\right)(b\sec(a+bx))^n\sqrt{c\sin(a+bx)}\sin(2(a}{3b}$$

input
```
Integrate[(b*Sec[a + b*x])^n*Sqrt[c*Sin[a + b*x]],x]
```

output
```
((Cos[a + b*x]^2)^((-1 + n)/2)*Hypergeometric2F1[3/4, (1 + n)/2, 7/4, Sin[
a + b*x]^2]*(b*Sec[a + b*x])^n*Sqrt[c*Sin[a + b*x]]*Sin[2*(a + b*x)])/(3*b
)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3067, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c\sin(a+bx)}(b\sec(a+bx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{c\sin(a+bx)}(b\sec(a+bx))^n dx$$

$$\Big\downarrow \text{3067}$$

$$b^2(b\cos(a+bx))^{n-1}(b\sec(a+bx))^{n-1}\int (b\cos(a+bx))^{-n}\sqrt{c\sin(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$b^2(b\cos(a+bx))^{n-1}(b\sec(a+bx))^{n-1}\int (b\cos(a+bx))^{-n}\sqrt{c\sin(a+bx)}dx$$

$$\Big\downarrow \text{3056}$$

$$-\frac{c\sqrt[4]{\sin^2(a+bx)}(b\sec(a+bx))^{n-1}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1-n}{2},\frac{3-n}{2},\cos^2(a+bx)\right)}{(1-n)\sqrt{c\sin(a+bx)}}$$

input
```
Int[(b*Sec[a + b*x])^n*Sqrt[c*Sin[a + b*x]],x]
```

output
```
-((c*Hypergeometric2F1[1/4, (1 - n)/2, (3 - n)/2, Cos[a + b*x]^2]*(b*Sec[a
 + b*x])^(-1 + n)*(Sin[a + b*x]^2)^(1/4))/((1 - n)*Sqrt[c*Sin[a + b*x]]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3067
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
   Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f,
m, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Maple [F]

$$\int (b\sec (bx+a))^n \sqrt{c\sin (bx+a)}dx$$

input
```
int((b*sec(b*x+a))^n*(c*sin(b*x+a))^(1/2),x)
```

output
```
int((b*sec(b*x+a))^n*(c*sin(b*x+a))^(1/2),x)
```

Fricas [F]

$$\int (b\sec(a+bx))^n\sqrt{c\sin(a+bx)}\,dx = \int \sqrt{c\sin (bx+a)}(b\sec (bx+a))^n\,dx$$

input
```
integrate((b*sec(b*x+a))^n*(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*sin(b*x + a))*(b*sec(b*x + a))^n, x)
```

Sympy [F]

$$\int (b\sec(a+bx))^n\sqrt{c\sin(a+bx)}\,dx = \int (b\sec{(a+bx)})^n\sqrt{c\sin{(a+bx)}}\,dx$$

input
```
integrate((b*sec(b*x+a))**n*(c*sin(b*x+a))**(1/2),x)
```

output
```
Integral((b*sec(a + b*x))**n*sqrt(c*sin(a + b*x)), x)
```

Maxima [F]

$$\int (b\sec(a+bx))^n\sqrt{c\sin(a+bx)}\,dx = \int \sqrt{c\sin{(bx+a)}}(b\sec{(bx+a)})^n\,dx$$

input
```
integrate((b*sec(b*x+a))^n*(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*sin(b*x + a))*(b*sec(b*x + a))^n, x)
```

Giac [F]

$$\int (b\sec(a+bx))^n\sqrt{c\sin(a+bx)}\,dx = \int \sqrt{c\sin{(bx+a)}}(b\sec{(bx+a)})^n\,dx$$

input
```
integrate((b*sec(b*x+a))^n*(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*sin(b*x + a))*(b*sec(b*x + a))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sec(a+bx))^n\sqrt{c\sin(a+bx)}\,dx=\int \sqrt{c\,\sin\left(a+b\,x\right)}\,{\left(\frac{b}{\cos\left(a+b\,x\right)}\right)}^n\,dx$$

input
```
int((c*sin(a + b*x))^(1/2)*(b/cos(a + b*x))^n,x)
```

output
```
int((c*sin(a + b*x))^(1/2)*(b/cos(a + b*x))^n, x)
```

Reduce [F]

$$\int (b\sec(a+bx))^n\sqrt{c\sin(a+bx)}\,dx=\sqrt{c}\,b^n\left(\int \sqrt{\sin\left(bx+a\right)}\,\sec\left(bx+a\right)^{n}dx\right)$$

input
```
int((b*sec(b*x+a))^n*(c*sin(b*x+a))^(1/2),x)
```

output
```
sqrt(c)*b**n*int(sqrt(sin(a + b*x))*sec(a + b*x)**n,x)
```

3.505 $\int \frac{(b\sec(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \frac{(b\sec(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx = -\frac{c\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1-n}{2},\frac{3-n}{2},\cos^2(a+bx)\right)(b\sec(a+bx))^{-1+n}\sin^2(a+bx)^{3/4}}{(1-n)(c\sin(a+bx))^{3/2}}$$

output
```
-c*hypergeom([3/4, 1/2-1/2*n],[3/2-1/2*n],cos(b*x+a)^2)*(b*sec(b*x+a))^(-1
+n)*(sin(b*x+a)^2)^(3/4)/(1-n)/(c*sin(b*x+a))^(3/2)
```

Mathematica [A] (verified)

Time = 10.67 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.95

$$\int \frac{(b\sec(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx = \frac{\cos^2(a+bx)^{\frac{1}{2}(-1+n)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1+n}{2},\frac{5}{4},\sin^2(a+bx)\right)(b\sec(a+bx))^n\sin(2(a+bx))}{b\sqrt{c\sin(a+bx)}}$$

input
```
Integrate[(b*Sec[a + b*x])^n/Sqrt[c*Sin[a + b*x]],x]
```

output
```
((Cos[a + b*x]^2)^((-1 + n)/2)*Hypergeometric2F1[1/4, (1 + n)/2, 5/4, Sin[
a + b*x]^2]*(b*Sec[a + b*x])^n*Sin[2*(a + b*x)])/(b*Sqrt[c*Sin[a + b*x]])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3067, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\sec(a+bx))^n}{\sqrt{c\sin(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(b\sec(a+bx))^n}{\sqrt{c\sin(a+bx)}}dx$$

$$\Big\downarrow \text{3067}$$

$$b^2(b\cos(a+bx))^{n-1}(b\sec(a+bx))^{n-1}\int \frac{(b\cos(a+bx))^{-n}}{\sqrt{c\sin(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$b^2(b\cos(a+bx))^{n-1}(b\sec(a+bx))^{n-1}\int \frac{(b\cos(a+bx))^{-n}}{\sqrt{c\sin(a+bx)}}dx$$

$$\Big\downarrow \text{3056}$$

$$-\frac{c\sin^2(a+bx)^{3/4}(b\sec(a+bx))^{n-1}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1-n}{2},\frac{3-n}{2},\cos^2(a+bx)\right)}{(1-n)(c\sin(a+bx))^{3/2}}$$

input
```
Int[(b*Sec[a + b*x])^n/Sqrt[c*Sin[a + b*x]],x]
```

output
```
-((c*Hypergeometric2F1[3/4, (1 - n)/2, (3 - n)/2, Cos[a + b*x]^2]*(b*Sec[a
 + b*x])^(-1 + n)*(Sin[a + b*x]^2)^(3/4))/((1 - n)*(c*Sin[a + b*x])^(3/2))
)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3067
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
   Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f,
m, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Maple [F]

$$\int \frac{(b\sec(bx+a))^n}{\sqrt{c\sin(bx+a)}}dx$$

input
```
int((b*sec(b*x+a))^n/(c*sin(b*x+a))^(1/2),x)
```

output
```
int((b*sec(b*x+a))^n/(c*sin(b*x+a))^(1/2),x)
```

Fricas [F]

$$\int \frac{(b\sec(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{(b\sec(bx+a))^n}{\sqrt{c\sin(bx+a)}}\,dx$$

input
```
integrate((b*sec(b*x+a))^n/(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*sin(b*x + a))*(b*sec(b*x + a))^n/(c*sin(b*x + a)), x)
```

Sympy [F]

$$\int \frac{(b\sec(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{(b\sec{(a+bx)})^n}{\sqrt{c\sin{(a+bx)}}}\,dx$$

input
```
integrate((b*sec(b*x+a))**n/(c*sin(b*x+a))**(1/2),x)
```

output
```
Integral((b*sec(a + b*x))**n/sqrt(c*sin(a + b*x)), x)
```

Maxima [F]

$$\int \frac{(b\sec(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{(b\sec{(bx+a)})^n}{\sqrt{c\sin{(bx+a)}}}\,dx$$

input
```
integrate((b*sec(b*x+a))^n/(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(b*x + a))^n/sqrt(c*sin(b*x + a)), x)
```

Giac [F]

$$\int \frac{(b\sec(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{(b\sec{(bx+a)})^n}{\sqrt{c\sin{(bx+a)}}}\,dx$$

input
```
integrate((b*sec(b*x+a))^n/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate((b*sec(b*x + a))^n/sqrt(c*sin(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\sec(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx = \int \frac{\left(\frac{b}{\cos(a+b\,x)}\right)^n}{\sqrt{c\,\sin\left(a+b\,x\right)}}\,dx$$

input
```
int((b/cos(a + b*x))^n/(c*sin(a + b*x))^(1/2),x)
```

output
```
int((b/cos(a + b*x))^n/(c*sin(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{(b\sec(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx = \frac{\sqrt{c}\,b^n\left(\int \frac{\sqrt{\sin(bx+a)}\,\sec(bx+a)^n}{\sin(bx+a)}dx\right)}{c}$$

input
```
int((b*sec(b*x+a))^n/(c*sin(b*x+a))^(1/2),x)
```

output
```
(sqrt(c)*b**n*int((sqrt(sin(a + b*x))*sec(a + b*x)**n)/sin(a + b*x),x))/c
```

3.506 $\int \frac{(b\sec(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 78

$$\int \frac{(b\sec(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{1-n}{2},\frac{3-n}{2},\cos^2(a+bx)\right)(b\sec(a+bx))^{-1+n}\sqrt[4]{\sin^2(a+bx)}}{c(1-n)\sqrt{c\sin(a+bx)}}$$

output
```
-hypergeom([5/4, 1/2-1/2*n],[3/2-1/2*n],cos(b*x+a)^2)*(b*sec(b*x+a))^(-1+n
)*(sin(b*x+a)^2)^(1/4)/c/(1-n)/(c*sin(b*x+a))^(1/2)
```

Mathematica [A] (verified)

Time = 10.77 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

$$\int \frac{(b\sec(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx = -\frac{\cos^2(a+bx)^{\frac{1}{2}(-1+n)}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1+n}{2},\frac{3}{4},\sin^2(a+bx)\right)(b\sec(a+bx))^n\sin(2(a+bx))}{b(c\sin(a+bx))^{3/2}}$$

input
```
Integrate[(b*Sec[a + b*x])^n/(c*Sin[a + b*x])^(3/2),x]
```

output
```
-(((Cos[a + b*x]^2)^((-1 + n)/2)*Hypergeometric2F1[-1/4, (1 + n)/2, 3/4, S
in[a + b*x]^2]*(b*Sec[a + b*x])^n*Sin[2*(a + b*x)])/(b*(c*Sin[a + b*x])^(3
/2)))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3067, 3042, 3056}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\sec(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(b\sec(a+bx))^n}{(c\sin(a+bx))^{3/2}}dx$$
$$\Big\downarrow \text{3067}$$
$$b^2(b\cos(a+bx))^{n-1}(b\sec(a+bx))^{n-1}\int \frac{(b\cos(a+bx))^{-n}}{(c\sin(a+bx))^{3/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$b^2(b\cos(a+bx))^{n-1}(b\sec(a+bx))^{n-1}\int \frac{(b\cos(a+bx))^{-n}}{(c\sin(a+bx))^{3/2}}dx$$
$$\Big\downarrow \text{3056}$$
$$-\frac{\sqrt[4]{\sin^2(a+bx)}(b\sec(a+bx))^{n-1}\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{1-n}{2},\frac{3-n}{2},\cos^2(a+bx)\right)}{c(1-n)\sqrt{c\sin(a+bx)}}$$

input
```
Int[(b*Sec[a + b*x])^n/(c*Sin[a + b*x])^(3/2),x]
```

output
```
-((Hypergeometric2F1[5/4, (1 - n)/2, (3 - n)/2, Cos[a + b*x]^2]*(b*Sec[a +
 b*x])^(-1 + n)*(Sin[a + b*x]^2)^(1/4))/(c*(1 - n)*Sqrt[c*Sin[a + b*x]]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3056
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

rule 3067
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
   Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f,
m, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Maple [F]

$$\int \frac{(b\sec(bx+a))^n}{(c\sin(bx+a))^{\frac{3}{2}}}dx$$

input
```
int((b*sec(b*x+a))^n/(c*sin(b*x+a))^(3/2),x)
```

output
```
int((b*sec(b*x+a))^n/(c*sin(b*x+a))^(3/2),x)
```

Fricas [F]

$$\int \frac{(b\sec(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx = \int \frac{(b\sec(bx+a))^n}{(c\sin(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*sec(b*x+a))^n/(c*sin(b*x+a))^(3/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(c*sin(b*x + a))*(b*sec(b*x + a))^n/(c^2*cos(b*x + a)^2 - c^
2), x)
```

Sympy [F]

$$\int \frac{(b\sec(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx = \int \frac{(b\sec(a+bx))^n}{(c\sin(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*sec(b*x+a))**n/(c*sin(b*x+a))**(3/2),x)
```

output
```
Integral((b*sec(a + b*x))**n/(c*sin(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(b\sec(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx = \int \frac{(b\sec(bx+a))^n}{(c\sin(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*sec(b*x+a))^n/(c*sin(b*x+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*sec(b*x + a))^n/(c*sin(b*x + a))^(3/2), x)
```

Giac [F]

$$\int \frac{(b\sec(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx = \int \frac{(b\sec(bx+a))^n}{(c\sin(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*sec(b*x+a))^n/(c*sin(b*x+a))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*sec(b*x + a))^n/(c*sin(b*x + a))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(b\sec(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx = \int \frac{\left(\frac{b}{\cos(a+b\,x)}\right)^n}{(c\,\sin{(a+b\,x)})^{3/2}}\,dx$$

input
```
int((b/cos(a + b*x))^n/(c*sin(a + b*x))^(3/2),x)
```

output
```
int((b/cos(a + b*x))^n/(c*sin(a + b*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{(b\sec(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx = \frac{\sqrt{c}\,b^n\left(\int \frac{\sqrt{\sin(bx+a)}\,\sec(bx+a)^n}{\sin(bx+a)^2}dx\right)}{c^2}$$

input
```
int((b*sec(b*x+a))^n/(c*sin(b*x+a))^(3/2),x)
```

output
```
(sqrt(c)*b**n*int((sqrt(sin(a + b*x))*sec(a + b*x)**n)/sin(a + b*x)**2,x))
/c**2
```

3.507 $\int \sqrt{d\csc(e+fx)}\sin^4(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 100

$$\int \sqrt{d\csc(e+fx)}\sin^4(e+fx)\,dx = -\frac{2d^3\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} - \frac{10d\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}} + \frac{10\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{21f}$$

output
```
-2/7*d^3*cos(f*x+e)/f/(d*csc(f*x+e))^(5/2)-10/21*d*cos(f*x+e)/f/(d*csc(f*x
+e))^(1/2)+10/21*(d*csc(f*x+e))^(1/2)*InverseJacobiAM(1/2*e-1/4*Pi+1/2*f*x
,2^(1/2))*sin(f*x+e)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.67

$$\int \sqrt{d\csc(e+fx)}\sin^4(e+fx)\,dx = -\frac{\sqrt{d\csc(e+fx)}\left(40\,\text{EllipticF}\left(\frac{1}{4}(-2e+\pi-2fx),2\right)\sqrt{\sin(e+fx)}+26\sin(2(e+fx))-3\sin(4(e+\right.}{84f}$$

input
```
Integrate[Sqrt[d*Csc[e + f*x]]*Sin[e + f*x]^4,x]
```

output
```
-1/84*(Sqrt[d*Csc[e + f*x]]*(40*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[S
in[e + f*x]] + 26*Sin[2*(e + f*x)] - 3*Sin[4*(e + f*x)]))/f
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 2030, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(e+fx)\sqrt{d\csc(e+fx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{d\csc(e+fx)}}{\csc(e+fx)^4}dx$$

$$\Big\downarrow \; 2030$$

$$d^4\int \frac{1}{(d\csc(e+fx))^{7/2}}dx$$

$$\Big\downarrow \; 4256$$

$$d^4\left(\frac{5\int \frac{1}{(d\csc(e+fx))^{3/2}}dx}{7d^2}-\frac{2\cos(e+fx)}{7df(d\csc(e+fx))^{5/2}}\right)$$

$$\Big\downarrow \; 3042$$

$$d^4\left(\frac{5\int \frac{1}{(d\csc(e+fx))^{3/2}}dx}{7d^2}-\frac{2\cos(e+fx)}{7df(d\csc(e+fx))^{5/2}}\right)$$

$$\Big\downarrow \; 4256$$

$$d^4\left(\frac{5\left(\frac{\int \sqrt{d\csc(e+fx)}dx}{3d^2}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}\right)}{7d^2}-\frac{2\cos(e+fx)}{7df(d\csc(e+fx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$d^4\left(\frac{5\left(\frac{\int \sqrt{d\csc(e+fx)}dx}{3d^2}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}\right)}{7d^2}-\frac{2\cos(e+fx)}{7df(d\csc(e+fx))^{5/2}}\right)$$

$$\Big\downarrow \text{4258}$$

$$d^4\left(\frac{5\left(\frac{\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}\int \frac{1}{\sqrt{\sin(e+fx)}}dx}{3d^2}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}\right)}{7d^2}-\frac{2\cos(e+fx)}{7df(d\csc(e+fx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$d^4\left(\frac{5\left(\frac{\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}\int \frac{1}{\sqrt{\sin(e+fx)}}dx}{3d^2}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}\right)}{7d^2}-\frac{2\cos(e+fx)}{7df(d\csc(e+fx))^{5/2}}\right)$$

$$\Big\downarrow \text{3120}$$

$$d^4\left(\frac{5\left(\frac{2\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\csc(e+fx)}}{3d^2f}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}\right)}{7d^2}-\frac{2\cos(e+fx)}{7df(d\csc(e+fx))^{5/2}}\right)$$

input

```
Int[Sqrt[d*Csc[e + f*x]]*Sin[e + f*x]^4,x]
```

output

```
d^4*((-2*Cos[e + f*x])/(7*d*f*(d*Csc[e + f*x])^(5/2)) + (5*((-2*Cos[e + f*
x])/(3*d*f*Sqrt[d*Csc[e + f*x]]) + (2*Sqrt[d*Csc[e + f*x]]*EllipticF[(e -
Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*d^2*f)))/(7*d^2))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.53

method	result
default	$\frac{\sqrt{2}\left(\sin(fx+e)\cos(fx+e)\sqrt{2}\left(3\cos(fx+e)^{2}-8\right)+i(5\cos(fx+e)+5)\sqrt{1+i\cot(fx+e)-i\csc(fx+e)}\,\sqrt{1-i\cot(fx+e)+i\csc(fx+e)}}{21f}$

input
```
int((d*csc(f*x+e))^(1/2)*sin(f*x+e)^4,x,method=_RETURNVERBOSE)
```

output
```
1/21/f*2^(1/2)*(sin(f*x+e)*cos(f*x+e)*2^(1/2)*(3*cos(f*x+e)^2-8)+I*(5*cos(
f*x+e)+5)*(1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2)*(1-I*cot(f*x+e)+I*csc(f*x+e)
)^(1/2)*(-I*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*EllipticF((1+I*cot(f*x+e)-I*cs
c(f*x+e))^(1/2),1/2*2^(1/2)))*(d*csc(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.95

$$\int \sqrt{d\csc(e+fx)}\sin^4(e+fx)\,dx = \frac{2\left(3\cos\left(fx+e\right)^3-8\cos\left(fx+e\right)\right)\sqrt{\frac{d}{\sin(fx+e)}}\sin\left(fx+e\right)-5i\sqrt{2i\,d}\text{weierstrassPInverse}(4,0,\cos\left(fx-\right.}{21\,f}$$

input
```
integrate((d*csc(f*x+e))^(1/2)*sin(f*x+e)^4,x, algorithm="fricas")
```

output
```
1/21*(2*(3*cos(f*x + e)^3 - 8*cos(f*x + e))*sqrt(d/sin(f*x + e))*sin(f*x +
 e) - 5*I*sqrt(2*I*d)*weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x +
 e)) + 5*I*sqrt(-2*I*d)*weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x
 + e)))/f
```

Sympy [F]

$$\int \sqrt{d\csc(e+fx)}\sin^4(e+fx)\,dx = \int \sqrt{d\csc\left(e+fx\right)}\sin^4\left(e+fx\right)dx$$

input
```
integrate((d*csc(f*x+e))**(1/2)*sin(f*x+e)**4,x)
```

output
```
Integral(sqrt(d*csc(e + f*x))*sin(e + f*x)**4, x)
```

Maxima [F]

$$\int \sqrt{d\csc(e+fx)}\sin^4(e+fx)\,dx = \int \sqrt{d\csc\left(fx+e\right)}\sin\left(fx+e\right)^4\,dx$$

input
```
integrate((d*csc(f*x+e))^(1/2)*sin(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*csc(f*x + e))*sin(f*x + e)^4, x)
```

Giac [F]

$$\int \sqrt{d\csc(e+fx)}\sin^4(e+fx)\,dx = \int \sqrt{d\csc\left(fx+e\right)}\sin\left(fx+e\right)^4\,dx$$

input
```
integrate((d*csc(f*x+e))^(1/2)*sin(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate(sqrt(d*csc(f*x + e))*sin(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\csc(e+fx)}\sin^4(e+fx)\,dx = \int \sin\left(e+f\,x\right)^4\sqrt{\frac{d}{\sin\left(e+f\,x\right)}}\,dx$$

input
```
int(sin(e + f*x)^4*(d/sin(e + f*x))^(1/2),x)
```

output
```
int(sin(e + f*x)^4*(d/sin(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{d\csc(e+fx)}\sin^4(e+fx)\,dx = \sqrt{d}\left(\int \sqrt{\csc\left(fx+e\right)}\,\sin\left(fx+e\right)^4 dx\right)$$

input
```
int((d*csc(f*x+e))^(1/2)*sin(f*x+e)^4,x)
```

output
```
sqrt(d)*int(sqrt(csc(e + f*x))*sin(e + f*x)**4,x)
```

3.508 $\int \sqrt{d \csc(e+fx)} \sin^3(e+fx)\, dx$

Optimal result

Integrand size = 21, antiderivative size = 75

$$\int \sqrt{d \csc(e+fx)} \sin^3(e+fx)\, dx = -\frac{2d^2 \cos(e+fx)}{5f(d\csc(e+fx))^{3/2}} + \frac{6dE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{5f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$

output
```
-2/5*d^2*cos(f*x+e)/f/(d*csc(f*x+e))^(3/2)-6/5*d*EllipticE(cos(1/2*e+1/4*P
i+1/2*f*x),2^(1/2))/f/(d*csc(f*x+e))^(1/2)/sin(f*x+e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.83

$$\int \sqrt{d \csc(e+fx)} \sin^3(e+fx)\, dx = -\frac{2\sqrt{d\csc(e+fx)}\left(3E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|2\right)\sqrt{\sin(e+fx)}+\cos(e+fx)\sin^2(e+fx)\right)}{5f}$$

input
```
Integrate[Sqrt[d*Csc[e + f*x]]*Sin[e + f*x]^3,x]
```

output
```
(-2*Sqrt[d*Csc[e + f*x]]*(3*EllipticE[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Sin[e
 + f*x]] + Cos[e + f*x]*Sin[e + f*x]^2))/(5*f)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 2030, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(e+fx)\sqrt{d\csc(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{d\csc(e+fx)}}{\csc(e+fx)^3}dx$$

$$\Big\downarrow \text{2030}$$

$$d^3\int \frac{1}{(d\csc(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$d^3\left(\frac{3\int \frac{1}{\sqrt{d\csc(e+fx)}}dx}{5d^2}-\frac{2\cos(e+fx)}{5df(d\csc(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$d^3\left(\frac{3\int \frac{1}{\sqrt{d\csc(e+fx)}}dx}{5d^2}-\frac{2\cos(e+fx)}{5df(d\csc(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \text{4258}$$

$$d^3\left(\frac{3\int \sqrt{\sin(e+fx)}dx}{5d^2\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2\cos(e+fx)}{5df(d\csc(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$d^3\left(\frac{3\int\sqrt{\sin(e+fx)}dx}{5d^2\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2\cos(e+fx)}{5df(d\csc(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \text{3119}$$

$$d^3\left(\frac{6E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|2\right)}{5d^2f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2\cos(e+fx)}{5df(d\csc(e+fx))^{3/2}}\right)$$

input
```
Int[Sqrt[d*Csc[e + f*x]]*Sin[e + f*x]^3,x]
```

output
```
d^3*((-2*Cos[e + f*x])/(5*d*f*(d*Csc[e + f*x])^(3/2)) + (6*EllipticE[(e -
Pi/2 + f*x)/2, 2])/(5*d^2*f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]]))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.95 (sec) , antiderivative size = 248, normalized size of antiderivative = 3.31

method	result
default	$\frac{\sqrt{2}\left((-6\cos(fx+e)-6)\sqrt{1+i\cot(fx+e)-i\csc(fx+e)}\,\sqrt{i(\csc(fx+e)-\cot(fx+e))}\,\text{EllipticE}\left(\sqrt{1+i\cot(fx+e)-i\csc(fx+e)},\frac{\sqrt{2}}{2}\right)\right.}{}$

input
```
int((d*csc(f*x+e))^(1/2)*sin(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/5/f*2^(1/2)*((-6*cos(f*x+e)-6)*(1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2)*(I*(c
sc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticE((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2)
,1/2*2^(1/2))*(1-I*cot(f*x+e)+I*csc(f*x+e))^(1/2)+(3*cos(f*x+e)+3)*(1+I*co
t(f*x+e)-I*csc(f*x+e))^(1/2)*(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*(1-I*cot(f*
x+e)+I*csc(f*x+e))^(1/2)*EllipticF((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2
*2^(1/2))+(cos(f*x+e)^3-4*cos(f*x+e)+3)*2^(1/2))*(d*csc(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.24

$$\int \sqrt{d\csc(e+fx)}\sin^3(e+fx)\,dx$$
$$= \frac{2\left(\cos(fx+e)^3-\cos(fx+e)\right)\sqrt{\frac{d}{\sin(fx+e)}}+3\sqrt{2i\,d}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(}{}$$

input
```
integrate((d*csc(f*x+e))^(1/2)*sin(f*x+e)^3,x, algorithm="fricas")
```

output
```
1/5*(2*(cos(f*x + e)^3 - cos(f*x + e))*sqrt(d/sin(f*x + e)) + 3*sqrt(2*I*d
)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x
 + e))) + 3*sqrt(-2*I*d)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, c
os(f*x + e) - I*sin(f*x + e))))/f
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{d\csc(e+fx)}\sin^3(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((d*csc(f*x+e))**(1/2)*sin(f*x+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{d\csc(e+fx)}\sin^3(e+fx)\,dx = \int \sqrt{d\csc(fx+e)}\sin(fx+e)^3\,dx$$

input
```
integrate((d*csc(f*x+e))^(1/2)*sin(f*x+e)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*csc(f*x + e))*sin(f*x + e)^3, x)
```

Giac [F]

$$\int \sqrt{d\csc(e+fx)}\sin^3(e+fx)\,dx = \int \sqrt{d\csc(fx+e)}\sin(fx+e)^3\,dx$$

input
```
integrate((d*csc(f*x+e))^(1/2)*sin(f*x+e)^3,x, algorithm="giac")
```

output
```
integrate(sqrt(d*csc(f*x + e))*sin(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d \csc(e+fx)} \sin^3(e+fx) \, dx = \int \sin(e+fx)^3 \sqrt{\frac{d}{\sin(e+fx)}} \, dx$$

input
```
int(sin(e + f*x)^3*(d/sin(e + f*x))^(1/2),x)
```

output
```
int(sin(e + f*x)^3*(d/sin(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{d \csc(e+fx)} \sin^3(e+fx) \, dx = \sqrt{d} \left(\int \sqrt{\csc(fx+e)} \sin(fx+e)^3 \, dx \right)$$

input
```
int((d*csc(f*x+e))^(1/2)*sin(f*x+e)^3,x)
```

output
```
sqrt(d)*int(sqrt(csc(e + f*x))*sin(e + f*x)**3,x)
```

3.509 $\int \sqrt{d\csc(e+fx)}\sin^2(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 72

$$\int \sqrt{d\csc(e+fx)}\sin^2(e+fx)\,dx = -\frac{2d\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}}+\frac{2\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{3f}$$

output
```
-2/3*d*cos(f*x+e)/f/(d*csc(f*x+e))^(1/2)+2/3*(d*csc(f*x+e))^(1/2)*InverseJ
acobiAM(1/2*e-1/4*Pi+1/2*f*x,2^(1/2))*sin(f*x+e)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.76

$$\int \sqrt{d\csc(e+fx)}\sin^2(e+fx)\,dx = -\frac{\sqrt{d\csc(e+fx)}\left(2\,\text{EllipticF}\left(\frac{1}{4}(-2e+\pi-2fx),2\right)\sqrt{\sin(e+fx)}+\sin(2(e+fx))\right)}{3f}$$

input
```
Integrate[Sqrt[d*Csc[e + f*x]]*Sin[e + f*x]^2,x]
```

output
```
-1/3*(Sqrt[d*Csc[e + f*x]]*(2*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Sin
[e + f*x]] + Sin[2*(e + f*x)]))/f
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 2030, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(e+fx)\sqrt{d\csc(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{d\csc(e+fx)}}{\csc(e+fx)^2}dx$$

$$\Big\downarrow \text{2030}$$

$$d^2\int \frac{1}{(d\csc(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$d^2\left(\frac{\int \sqrt{d\csc(e+fx)}dx}{3d^2}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$d^2\left(\frac{\int \sqrt{d\csc(e+fx)}dx}{3d^2}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}\right)$$

$$\Big\downarrow \text{4258}$$

$$d^2\left(\frac{\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}\int \frac{1}{\sqrt{\sin(e+fx)}}dx}{3d^2}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$d^2\left(\frac{\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}\int\frac{1}{\sqrt{\sin(e+fx)}}dx}{3d^2}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}\right)$$

$$\Big\downarrow \text{ 3120}$$

$$d^2\left(\frac{2\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\csc(e+fx)}}{3d^2f}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}\right)$$

input
```
Int[Sqrt[d*Csc[e + f*x]]*Sin[e + f*x]^2,x]
```

output
```
d^2*((-2*Cos[e + f*x])/(3*d*f*Sqrt[d*Csc[e + f*x]]) + (2*Sqrt[d*Csc[e + f*
x]]*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*d^2*f))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.78 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.96

method	result
default	$-\frac{\sqrt{2}\left(i(-\cos(fx+e)-1)\sqrt{1+i\cot(fx+e)-i\csc(fx+e)}\,\sqrt{1-i\cot(fx+e)+i\csc(fx+e)}\,\sqrt{i(\csc(fx+e)-\cot(fx+e))}\ \mathrm{EllipticF}\left(\sqrt{\right.\right.}{3f}$

input
```
int((d*csc(f*x+e))^(1/2)*sin(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3/f*2^(1/2)*(I*(-cos(f*x+e)-1)*(1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2)*(1-I
*cot(f*x+e)+I*csc(f*x+e))^(1/2)*(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*Elliptic
F((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2))+2^(1/2)*cos(f*x+e)*sin(
f*x+e))*(d*csc(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.14

$$\int \sqrt{d\csc(e+fx)}\sin^2(e+fx)\,dx = -\frac{2\sqrt{\frac{d}{\sin(fx+e)}}\cos(fx+e)\sin(fx+e)+i\sqrt{2i\,d}\mathrm{weierstrassPInverse}(4,0,\cos(fx+e)+i\,\sin(fx+e))}{3f}$$

input
```
integrate((d*csc(f*x+e))^(1/2)*sin(f*x+e)^2,x, algorithm="fricas")
```

output
```
-1/3*(2*sqrt(d/sin(f*x + e))*cos(f*x + e)*sin(f*x + e) + I*sqrt(2*I*d)*wei
erstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e)) - I*sqrt(-2*I*d)*wei
erstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e)))/f
```

Sympy [F]

$$\int \sqrt{d\csc(e+fx)}\sin^2(e+fx)\,dx = \int \sqrt{d\csc\left(e+fx\right)}\sin^2\left(e+fx\right)dx$$

input
```
integrate((d*csc(f*x+e))**(1/2)*sin(f*x+e)**2,x)
```

output
```
Integral(sqrt(d*csc(e + f*x))*sin(e + f*x)**2, x)
```

Maxima [F]

$$\int \sqrt{d\csc(e+fx)}\sin^2(e+fx)\,dx = \int \sqrt{d\csc\left(fx+e\right)}\sin\left(fx+e\right)^2\,dx$$

input
```
integrate((d*csc(f*x+e))^(1/2)*sin(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*csc(f*x + e))*sin(f*x + e)^2, x)
```

Giac [F]

$$\int \sqrt{d\csc(e+fx)}\sin^2(e+fx)\,dx = \int \sqrt{d\csc\left(fx+e\right)}\sin\left(fx+e\right)^2\,dx$$

input
```
integrate((d*csc(f*x+e))^(1/2)*sin(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(d*csc(f*x + e))*sin(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\csc(e+fx)}\sin^2(e+fx)\,dx = \int \sin\left(e+f\,x\right)^2\,\sqrt{\frac{d}{\sin\left(e+f\,x\right)}}\,dx$$

input
```
int(sin(e + f*x)^2*(d/sin(e + f*x))^(1/2),x)
```

output
```
int(sin(e + f*x)^2*(d/sin(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{d\csc(e+fx)}\sin^2(e+fx)\,dx = \sqrt{d}\left(\int \sqrt{\csc\left(fx+e\right)}\,\sin\left(fx+e\right)^2dx\right)$$

input
```
int((d*csc(f*x+e))^(1/2)*sin(f*x+e)^2,x)
```

output
```
sqrt(d)*int(sqrt(csc(e + f*x))*sin(e + f*x)**2,x)
```

3.510 $\int \sqrt{d \csc(e+fx)} \sin(e+fx) \, dx$

Optimal result

Integrand size = 19, antiderivative size = 44

$$\int \sqrt{d \csc(e+fx)} \sin(e+fx) \, dx = \frac{2dE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{f\sqrt{d \csc(e+fx)}\sqrt{\sin(e+fx)}}$$

output
```
-2*d*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))/f/(d*csc(f*x+e))^(1/2)/s
in(f*x+e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \sqrt{d \csc(e+fx)} \sin(e+fx) \, dx = -\frac{2dE\left(\frac{1}{4}(-2e+\pi-2fx)\middle|2\right)}{f\sqrt{d \csc(e+fx)}\sqrt{\sin(e+fx)}}$$

input
```
Integrate[Sqrt[d*Csc[e + f*x]]*Sin[e + f*x],x]
```

output
```
(-2*d*EllipticE[(-2*e + Pi - 2*f*x)/4, 2])/(f*Sqrt[d*Csc[e + f*x]]*Sqrt[Si
n[e + f*x]])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 2030, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(e+fx)\sqrt{d\csc(e+fx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{d\csc(e+fx)}}{\csc(e+fx)}dx$$

$$\Big\downarrow \text{ 2030}$$

$$d\int \frac{1}{\sqrt{d\csc(e+fx)}}dx$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{d\int \sqrt{\sin(e+fx)}dx}{\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{d\int \sqrt{\sin(e+fx)}dx}{\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{2dE\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

input
```
Int[Sqrt[d*Csc[e + f*x]]*Sin[e + f*x],x]
```

output
```
(2*d*EllipticE[(e - Pi/2 + f*x)/2, 2])/(f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e
+ f*x]])
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 237, normalized size of antiderivative = 5.39

method	result
default	$-\frac{\sqrt{2}\left(2\sqrt{1+i(-\csc(fx+e)+\cot(fx+e))}\sqrt{1-i(-\csc(fx+e)+\cot(fx+e))}\sqrt{-i(-\csc(fx+e)+\cot(fx+e))}(\cos(fx+e)+1)\,\mathrm{Ellipt}\right.}{}$
risch	$-\frac{\left(e^{2i(fx+e)}-1\right)\sqrt{2}\sqrt{\frac{i d\,e^{i(fx+e)}}{e^{2i(fx+e)}-1}}\,e^{-i(fx+e)}}{f}+\left(-\frac{2i\left(i d\,e^{2i(fx+e)}-i d\right)}{d\sqrt{e^{i(fx+e)}\left(i d\,e^{2i(fx+e)}-i d\right)}}-\frac{\sqrt{e^{i(fx+e)}+1}\sqrt{-2\,e^{i(fx+e)}+2}\sqrt{-e^{i(fx+e)}}\left(-\right.}{\sqrt{i d\,e}}\right.$

input
```
int((d*csc(f*x+e))^(1/2)*sin(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
-1/f*2^(1/2)*(2*(1+I*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*(1-I*(-csc(f*x+e)+cot
(f*x+e)))^(1/2)*(-I*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*(cos(f*x+e)+1)*Ellipti
cE((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2))+(1+I*(-csc(f*x+e)+cot(
f*x+e)))^(1/2)*(1-I*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*(-I*(-csc(f*x+e)+cot(f
*x+e)))^(1/2)*(-cos(f*x+e)-1)*EllipticF((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2
),1/2*2^(1/2))+2^(1/2)*(-1+cos(f*x+e)))*(d*csc(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.34

$$\int \sqrt{d\csc(e+fx)}\sin(e+fx)\,dx$$
$$= \frac{\sqrt{2i\,d}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos{(fx+e)}+i\,\sin{(fx+e)}))+\sqrt{-2i\,d}\text{weierstrass}}{f}$$

input
```
integrate((d*csc(f*x+e))^(1/2)*sin(f*x+e),x, algorithm="fricas")
```

output
```
(sqrt(2*I*d)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e)
+ I*sin(f*x + e))) + sqrt(-2*I*d)*weierstrassZeta(4, 0, weierstrassPInvers
e(4, 0, cos(f*x + e) - I*sin(f*x + e))))/f
```

Sympy [F]

$$\int \sqrt{d\csc(e+fx)}\sin(e+fx)\,dx = \int \sqrt{d\csc{(e+fx)}}\sin{(e+fx)}\,dx$$

input
```
integrate((d*csc(f*x+e))**(1/2)*sin(f*x+e),x)
```

output
```
Integral(sqrt(d*csc(e + f*x))*sin(e + f*x), x)
```

Maxima [F]

$$\int \sqrt{d\csc(e+fx)}\sin(e+fx)\,dx = \int \sqrt{d\csc\left(fx+e\right)}\sin\left(fx+e\right)\,dx$$

input
```
integrate((d*csc(f*x+e))^(1/2)*sin(f*x+e),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*csc(f*x + e))*sin(f*x + e), x)
```

Giac [F]

$$\int \sqrt{d\csc(e+fx)}\sin(e+fx)\,dx = \int \sqrt{d\csc\left(fx+e\right)}\sin\left(fx+e\right)\,dx$$

input
```
integrate((d*csc(f*x+e))^(1/2)*sin(f*x+e),x, algorithm="giac")
```

output
```
integrate(sqrt(d*csc(f*x + e))*sin(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\csc(e+fx)}\sin(e+fx)\,dx = \int \sin\left(e+f\,x\right)\,\sqrt{\frac{d}{\sin\left(e+f\,x\right)}}\,dx$$

input
```
int(sin(e + f*x)*(d/sin(e + f*x))^(1/2),x)
```

output
```
int(sin(e + f*x)*(d/sin(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{d\csc(e+fx)}\sin(e+fx)\,dx = \sqrt{d}\left(\int \sqrt{\csc(fx+e)}\,\sin(fx+e)\,dx\right)$$

input
```
int((d*csc(f*x+e))^(1/2)*sin(f*x+e),x)
```

output
```
sqrt(d)*int(sqrt(csc(e + f*x))*sin(e + f*x),x)
```

3.511 $\int \sqrt{d \csc(e+fx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 43

$$\int \sqrt{d \csc(e+fx)}\,dx = \frac{2\sqrt{d \csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{f}$$

output
```
2*(d*csc(f*x+e))^(1/2)*InverseJacobiAM(1/2*e-1/4*Pi+1/2*f*x,2^(1/2))*sin(f
*x+e)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \sqrt{d \csc(e+fx)}\,dx = -\frac{2\sqrt{d \csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{4}(-2e+\pi-2fx),2\right)\sqrt{\sin(e+fx)}}{f}$$

input
```
Integrate[Sqrt[d*Csc[e + f*x]],x]
```

output
```
(-2*Sqrt[d*Csc[e + f*x]]*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Sin[e +
f*x]])/f
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d\csc(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{d\csc(e+fx)}dx$$

$$\Big\downarrow \text{4258}$$

$$\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}\int \frac{1}{\sqrt{\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}\int \frac{1}{\sqrt{\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\csc(e+fx)}}{f}$$

input
```
Int[Sqrt[d*Csc[e + f*x]],x]
```

output
```
(2*Sqrt[d*Csc[e + f*x]]*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]
])/f
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.79

method	result
default	$\frac{i(\cos(fx+e)+1)\sqrt{1-i\cot(fx+e)+i\csc(fx+e)}\,\sqrt{-i(-\csc(fx+e)+\cot(fx+e))}\ \text{EllipticF}\left(\sqrt{1+i\cot(fx+e)-i\csc(fx+e)},\frac{\sqrt{2}}{2}\right)\sqrt{2}}{f}$

input
```
int((d*csc(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
I/f*(cos(f*x+e)+1)*(1-I*cot(f*x+e)+I*csc(f*x+e))^(1/2)*(-I*(-csc(f*x+e)+co
t(f*x+e)))^(1/2)*EllipticF((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2)
)*2^(1/2)*(d*csc(f*x+e))^(1/2)*(1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.28

$$\int \sqrt{d \csc(e+fx)}\,dx$$
$$= \frac{-i\sqrt{2i\,d}\,\text{weierstrassPInverse}(4,0,\cos(fx+e)+i\,\sin(fx+e)) + i\sqrt{-2i\,d}\,\text{weierstrassPInverse}(4,0,\cos}{f}$$

input
```
integrate((d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2*I*d)*weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e)) +
 I*sqrt(-2*I*d)*weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e)))/
f
```

Sympy [F]

$$\int \sqrt{d \csc(e+fx)}\,dx = \int \sqrt{d \csc(e+fx)}\,dx$$

input
```
integrate((d*csc(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(d*csc(e + f*x)), x)
```

Maxima [F]

$$\int \sqrt{d \csc(e+fx)}\,dx = \int \sqrt{d \csc(fx+e)}\,dx$$

input
```
integrate((d*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*csc(f*x + e)), x)
```

Giac [F]

$$\int \sqrt{d\csc(e+fx)}\,dx = \int \sqrt{d\csc(fx+e)}\,dx$$

input
```
integrate((d*csc(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*csc(f*x + e)), x)
```

Mupad [B] (verification not implemented)

Time = 25.56 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.47

$$\int \sqrt{d\csc(e+fx)}\,dx = -\frac{2\sqrt{\sin(e+fx)}\,\mathrm{F}\left(\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{1-\sin(e+fx)}}{2}\right)\middle|2\right)\sqrt{\frac{d}{\sin(e+fx)}}\sqrt{\cos(e+fx)^2}}{f\cos(e+fx)}$$

input
```
int((d/sin(e + f*x))^(1/2),x)
```

output
```
-(2*sin(e + f*x)^(1/2)*ellipticF(asin((2^(1/2)*(1 - sin(e + f*x))^(1/2))/2
), 2)*(d/sin(e + f*x))^(1/2)*(cos(e + f*x)^2)^(1/2))/(f*cos(e + f*x))
```

Reduce [F]

$$\int \sqrt{d\csc(e+fx)}\,dx = \sqrt{d}\left(\int \sqrt{\csc(fx+e)}dx\right)$$

input
```
int((d*csc(f*x+e))^(1/2),x)
```

output
```
sqrt(d)*int(sqrt(csc(e + f*x)),x)
```

3.512 $\int \csc(e+fx)\sqrt{d\csc(e+fx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 68

$$\int \csc(e+fx)\sqrt{d\csc(e+fx)}\,dx = -\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{f} - \frac{2dE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$

output
```
-2*cos(f*x+e)*(d*csc(f*x+e))^(1/2)/f+2*d*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*
x),2^(1/2))/f/(d*csc(f*x+e))^(1/2)/sin(f*x+e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.84

$$\int \csc(e+fx)\sqrt{d\csc(e+fx)}\,dx = \frac{(d\csc(e+fx))^{3/2}\left(2E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|2\right)\sin^{\frac{3}{2}}(e+fx)-\sin(2(e+fx))\right)}{df}$$

input
```
Integrate[Csc[e + f*x]*Sqrt[d*Csc[e + f*x]],x]
```

output
```
((d*Csc[e + f*x])^(3/2)*(2*EllipticE[(-2*e + Pi - 2*f*x)/4, 2]*Sin[e + f*x
]^(3/2) - Sin[2*(e + f*x)]))/(d*f)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {2030, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(e+fx)\sqrt{d\csc(e+fx)}\,dx$$

$$\Big\downarrow \; 2030$$

$$\frac{\int (d\csc(e+fx))^{3/2}dx}{d}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int (d\csc(e+fx))^{3/2}dx}{d}$$

$$\Big\downarrow \; 4255$$

$$\frac{d^2\left(-\int \frac{1}{\sqrt{d\csc(e+fx)}}dx\right)-\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}}{d}$$

$$\Big\downarrow \; 3042$$

$$\frac{d^2\left(-\int \frac{1}{\sqrt{d\csc(e+fx)}}dx\right)-\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}}{d}$$

$$\Big\downarrow \; 4258$$

$$\frac{-\frac{d^2\int \sqrt{\sin(e+fx)}dx}{\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}}{d}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{d^2\int \sqrt{\sin(e+fx)}dx}{\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}}{d}$$

$$\Big\downarrow \text{3119}$$

$$\frac{-\frac{2d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}}{d}$$

input
```
Int[Csc[e + f*x]*Sqrt[d*Csc[e + f*x]],x]
```

output
```
((-2*d*Cos[e + f*x]*Sqrt[d*Csc[e + f*x]])/f - (2*d^2*EllipticE[(e - Pi/2 +
 f*x)/2, 2])/(f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]]))/d
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.28

method	result
default	$-\frac{\sqrt{2}\left(\sqrt{2}+\left(\operatorname{EllipticF}\left(\sqrt{1+i\cot(fx+e)-i\csc(fx+e)},\frac{\sqrt{2}}{2}\right)-2\operatorname{EllipticE}\left(\sqrt{1+i\cot(fx+e)-i\csc(fx+e)},\frac{\sqrt{2}}{2}\right)\right)\sqrt{1+i(-\csc(fx+}}{f}$

input
```
int(csc(f*x+e)*(d*csc(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f*2^(1/2)*(2^(1/2)+(EllipticF((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2*2
^(1/2))-2*EllipticE((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2)))*(1+I
*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*(1-I*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*(-I*
(-csc(f*x+e)+cot(f*x+e)))^(1/2)*(cos(f*x+e)+1))*(d*csc(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.18

$$\int \csc(e+fx)\sqrt{d\csc(e+fx)}\,dx = -\frac{2\sqrt{\frac{d}{\sin(fx+e)}}\cos\left(fx+e\right)+\sqrt{2i\,d}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos\left(fx+e\right)+i\,\sin\left(\right.}{f}$$

input
```
integrate(csc(f*x+e)*(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-(2*sqrt(d/sin(f*x + e))*cos(f*x + e) + sqrt(2*I*d)*weierstrassZeta(4, 0,
weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e))) + sqrt(-2*I*d)*w
eierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x +
e))))/f
```

Sympy [F]

$$\int \csc(e+fx)\sqrt{d\csc(e+fx)}\,dx = \int \sqrt{d\csc{(e+fx)}}\csc{(e+fx)}\,dx$$

input
```
integrate(csc(f*x+e)*(d*csc(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(d*csc(e + f*x))*csc(e + f*x), x)
```

Maxima [F]

$$\int \csc(e+fx)\sqrt{d\csc(e+fx)}\,dx = \int \sqrt{d\csc{(fx+e)}}\csc{(fx+e)}\,dx$$

input
```
integrate(csc(f*x+e)*(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*csc(f*x + e))*csc(f*x + e), x)
```

Giac [F]

$$\int \csc(e+fx)\sqrt{d\csc(e+fx)}\,dx = \int \sqrt{d\csc{(fx+e)}}\csc{(fx+e)}\,dx$$

input
```
integrate(csc(f*x+e)*(d*csc(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*csc(f*x + e))*csc(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc(e+fx)\sqrt{d\csc(e+fx)}\,dx = \int \frac{\sqrt{\frac{d}{\sin(e+f\,x)}}}{\sin\left(e+f\,x\right)}\,dx$$

input
```
int((d/sin(e + f*x))^(1/2)/sin(e + f*x),x)
```

output
```
int((d/sin(e + f*x))^(1/2)/sin(e + f*x), x)
```

Reduce [F]

$$\int \csc(e+fx)\sqrt{d\csc(e+fx)}\,dx = \sqrt{d}\left(\int \sqrt{\csc\left(fx+e\right)}\,\csc\left(fx+e\right)dx\right)$$

input
```
int(csc(f*x+e)*(d*csc(f*x+e))^(1/2),x)
```

output
```
sqrt(d)*int(sqrt(csc(e + f*x))*csc(e + f*x),x)
```

3.513 $\int \csc^2(e+fx)\sqrt{d\csc(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 74

$$\int \csc^2(e+fx)\sqrt{d\csc(e+fx)}\,dx = -\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3df} + \frac{2\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{3f}$$

output
```
-2/3*cos(f*x+e)*(d*csc(f*x+e))^(3/2)/d/f+2/3*(d*csc(f*x+e))^(1/2)*InverseJ
acobiAM(1/2*e-1/4*Pi+1/2*f*x,2^(1/2))*sin(f*x+e)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.74

$$\int \csc^2(e+fx)\sqrt{d\csc(e+fx)}\,dx = -\frac{2(d\csc(e+fx))^{3/2}\left(\cos(e+fx)+\text{EllipticF}\left(\frac{1}{4}(-2e+\pi-2fx),2\right)\sin^{\frac{3}{2}}(e+fx)\right)}{3df}$$

input
```
Integrate[Csc[e + f*x]^2*Sqrt[d*Csc[e + f*x]],x]
```

output
```
(-2*(d*Csc[e + f*x])^(3/2)*(Cos[e + f*x] + EllipticF[(-2*e + Pi - 2*f*x)/4
, 2]*Sin[e + f*x]^(3/2)))/(3*d*f)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2030, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(e+fx)\sqrt{d\csc(e+fx)}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (d\csc(e+fx))^{5/2}dx}{d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (d\csc(e+fx))^{5/2}dx}{d^2}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{1}{3}d^2\int \sqrt{d\csc(e+fx)}dx-\frac{2d\cos(e+fx)(d\csc(e+fx))^{3/2}}{3f}}{d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{3}d^2\int \sqrt{d\csc(e+fx)}dx-\frac{2d\cos(e+fx)(d\csc(e+fx))^{3/2}}{3f}}{d^2}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{1}{3}d^2\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}\int \frac{1}{\sqrt{\sin(e+fx)}}dx-\frac{2d\cos(e+fx)(d\csc(e+fx))^{3/2}}{3f}}{d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{3}d^2\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}\int\frac{1}{\sqrt{\sin(e+fx)}}dx-\frac{2d\cos(e+fx)(d\csc(e+fx))^{3/2}}{3f}}{d^2}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{\frac{2d^2\sqrt{\sin(e+fx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\csc(e+fx)}}{3f}-\frac{2d\cos(e+fx)(d\csc(e+fx))^{3/2}}{3f}}{d^2}$$

input
```
Int[Csc[e + f*x]^2*Sqrt[d*Csc[e + f*x]],x]
```

output
```
((-2*d*Cos[e + f*x]*(d*Csc[e + f*x])^(3/2))/(3*f) + (2*d^2*Sqrt[d*Csc[e +
f*x]]*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*f))/d^2
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.82

method	result
default	$-\frac{\sqrt{2}\,\sqrt{d\csc(fx+e)}\,\left(i(-\cos(fx+e)-1)\sqrt{1+i\cot(fx+e)-i\csc(fx+e)}\,\sqrt{1-i\cot(fx+e)+i\csc(fx+e)}\,\sqrt{-i(-\csc(fx+e)+\cot(fx}\right.}{3f}$

input
```
int(csc(f*x+e)^2*(d*csc(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/f*2^(1/2)*(d*csc(f*x+e))^(1/2)*(I*(-cos(f*x+e)-1)*(1+I*cot(f*x+e)-I*c
sc(f*x+e))^(1/2)*(1-I*cot(f*x+e)+I*csc(f*x+e))^(1/2)*(-I*(-csc(f*x+e)+cot(
f*x+e)))^(1/2)*EllipticF((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2))+
2^(1/2)*cot(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.30

$$\int \csc^2(e+fx)\sqrt{d\csc(e+fx)}\,dx = \frac{-i\sqrt{2i\,d}\sin(fx+e)\,\mathrm{weierstrassPInverse}(4,0,\cos(fx+e)+i\,\sin(fx+e))+i\sqrt{-2i\,d}\sin(fx+e)\,\mathrm{wei}}{3\,f\sin(fx+e)}$$

input
```
integrate(csc(f*x+e)^2*(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2*I*d)*sin(f*x + e)*weierstrassPInverse(4, 0, cos(f*x + e) +
I*sin(f*x + e)) + I*sqrt(-2*I*d)*sin(f*x + e)*weierstrassPInverse(4, 0, co
s(f*x + e) - I*sin(f*x + e)) - 2*sqrt(d/sin(f*x + e))*cos(f*x + e))/(f*sin
(f*x + e))
```

Sympy [F]

$$\int \csc^2(e+fx)\sqrt{d\csc(e+fx)}\,dx=\int \sqrt{d\csc\left(e+fx\right)}\csc^2\left(e+fx\right)dx$$

input
```
integrate(csc(f*x+e)**2*(d*csc(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(d*csc(e + f*x))*csc(e + f*x)**2, x)
```

Maxima [F]

$$\int \csc^2(e+fx)\sqrt{d\csc(e+fx)}\,dx=\int \sqrt{d\csc\left(fx+e\right)}\csc\left(fx+e\right)^2\,dx$$

input
```
integrate(csc(f*x+e)^2*(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*csc(f*x + e))*csc(f*x + e)^2, x)
```

Giac [F]

$$\int \csc^2(e+fx)\sqrt{d\csc(e+fx)}\,dx=\int \sqrt{d\csc\left(fx+e\right)}\csc\left(fx+e\right)^2\,dx$$

input
```
integrate(csc(f*x+e)^2*(d*csc(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*csc(f*x + e))*csc(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(e+fx)\sqrt{d\csc(e+fx)}\,dx = \int \frac{\sqrt{\frac{d}{\sin(e+f\,x)}}}{\sin\left(e+f\,x\right)^2}\,dx$$

input
```
int((d/sin(e + f*x))^(1/2)/sin(e + f*x)^2,x)
```

output
```
int((d/sin(e + f*x))^(1/2)/sin(e + f*x)^2, x)
```

Reduce [F]

$$\int \csc^2(e+fx)\sqrt{d\csc(e+fx)}\,dx = \sqrt{d}\left(\int \sqrt{\csc\left(fx+e\right)}\,\csc\left(fx+e\right)^2dx\right)$$

input
```
int(csc(f*x+e)^2*(d*csc(f*x+e))^(1/2),x)
```

output
```
sqrt(d)*int(sqrt(csc(e + f*x))*csc(e + f*x)**2,x)
```

3.514 $\int \csc^3(e+fx)\sqrt{d\csc(e+fx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 100

$$\int \csc^3(e+fx)\sqrt{d\csc(e+fx)}\,dx=-\frac{6\cos(e+fx)\sqrt{d\csc(e+fx)}}{5f}-\frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5d^2f}-\frac{6dE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{5f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$

output
```
-6/5*cos(f*x+e)*(d*csc(f*x+e))^(1/2)/f-2/5*cos(f*x+e)*(d*csc(f*x+e))^(5/2)
/d^2/f+6/5*d*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))/f/(d*csc(f*x+e))
^(1/2)/sin(f*x+e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.68

$$\int \csc^3(e+fx)\sqrt{d\csc(e+fx)}\,dx=$$

$$-\frac{2\sqrt{d\csc(e+fx)}\left(3\cos(e+fx)+\cot(e+fx)\csc(e+fx)-3E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|2\right)\sqrt{\sin(e+fx}\right.}{5f}$$

input
```
Integrate[Csc[e + f*x]^3*Sqrt[d*Csc[e + f*x]],x]
```

output
```
(-2*Sqrt[d*Csc[e + f*x]]*(3*Cos[e + f*x] + Cot[e + f*x]*Csc[e + f*x] - 3*E
llipticE[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Sin[e + f*x]]))/(5*f)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2030, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(e+fx)\sqrt{d\csc(e+fx)}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (d\csc(e+fx))^{7/2}dx}{d^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (d\csc(e+fx))^{7/2}dx}{d^3}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{3}{5}d^2\int (d\csc(e+fx))^{3/2}dx-\frac{2d\cos(e+fx)(d\csc(e+fx))^{5/2}}{5f}}{d^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{5}d^2\int (d\csc(e+fx))^{3/2}dx-\frac{2d\cos(e+fx)(d\csc(e+fx))^{5/2}}{5f}}{d^3}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{3}{5}d^2\left(d^2\left(-\int \frac{1}{\sqrt{d\csc(e+fx)}}dx\right)-\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}\right)-\frac{2d\cos(e+fx)(d\csc(e+fx))^{5/2}}{5f}}{d^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{5}d^2\left(d^2\left(-\int \frac{1}{\sqrt{d\csc(e+fx)}}dx\right)-\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}\right)-\frac{2d\cos(e+fx)(d\csc(e+fx))^{5/2}}{5f}}{d^3}$$

$$\Big\downarrow \; 4258$$

$$\frac{\frac{3}{5}d^2\left(-\frac{d^2\int \sqrt{\sin(e+fx)}dx}{\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}\right)-\frac{2d\cos(e+fx)(d\csc(e+fx))^{5/2}}{5f}}{d^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{3}{5}d^2\left(-\frac{d^2\int \sqrt{\sin(e+fx)}dx}{\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}\right)-\frac{2d\cos(e+fx)(d\csc(e+fx))^{5/2}}{5f}}{d^3}$$

$$\Big\downarrow \; 3119$$

$$\frac{\frac{3}{5}d^2\left(-\frac{2d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}\right)-\frac{2d\cos(e+fx)(d\csc(e+fx))^{5/2}}{5f}}{d^3}$$

input
```
Int[Csc[e + f*x]^3*Sqrt[d*Csc[e + f*x]],x]
```

output
```
((-2*d*Cos[e + f*x]*(d*Csc[e + f*x])^(5/2))/(5*f) + (3*d^2*((-2*d*Cos[e +
f*x]*Sqrt[d*Csc[e + f*x]])/f - (2*d^2*EllipticE[(e - Pi/2 + f*x)/2, 2])/(f
*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])))/5)/d^3
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.46

method	result
default	$\frac{\sqrt{2}\,\sqrt{d\csc(fx+e)}\left((6\cos(fx+e)+6)\sqrt{i(\csc(fx+e)-\cot(fx+e))}\ \mathrm{EllipticE}\left(\sqrt{1+i\cot(fx+e)-i\csc(fx+e)}, \frac{\sqrt{2}}{2}\right)\sqrt{1-i\cot(fx+e)-}\right.}{}$

input
```
int(csc(f*x+e)^3*(d*csc(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5/f*2^(1/2)*(d*csc(f*x+e))^(1/2)*((6*cos(f*x+e)+6)*(I*(csc(f*x+e)-cot(f*
x+e)))^(1/2)*EllipticE((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2))*(1
-I*cot(f*x+e)+I*csc(f*x+e))^(1/2)*(1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2)+(-3*
cos(f*x+e)-3)*(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*(1-I*cot(f*x+e)+I*csc(f*x+
e))^(1/2)*EllipticF((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2))*(1+I*
cot(f*x+e)-I*csc(f*x+e))^(1/2)+2^(1/2)*(-3-csc(f*x+e)*cot(f*x+e)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.28

$$\int \csc^3(e+fx)\sqrt{d\csc(e+fx)}\,dx =$$
$$-\frac{3\left(\cos\left(fx+e\right)^2-1\right)\sqrt{2i\,d}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)$$

input
```
integrate(csc(f*x+e)^3*(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-1/5*(3*(cos(f*x + e)^2 - 1)*sqrt(2*I*d)*weierstrassZeta(4, 0, weierstrass
PInverse(4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*(cos(f*x + e)^2 - 1)*sq
rt(-2*I*d)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) -
I*sin(f*x + e))) + 2*(3*cos(f*x + e)^3 - 4*cos(f*x + e))*sqrt(d/sin(f*x +
e)))/(f*cos(f*x + e)^2 - f)
```

Sympy [F]

$$\int \csc^3(e+fx)\sqrt{d\csc(e+fx)}\,dx = \int \sqrt{d\csc\left(e+fx\right)}\csc^3\left(e+fx\right)dx$$

input
```
integrate(csc(f*x+e)**3*(d*csc(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(d*csc(e + f*x))*csc(e + f*x)**3, x)
```

Maxima [F]

$$\int \csc^3(e+fx)\sqrt{d\csc(e+fx)}\,dx=\int \sqrt{d\csc\left(fx+e\right)}\csc\left(fx+e\right)^3\,dx$$

input
```
integrate(csc(f*x+e)^3*(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*csc(f*x + e))*csc(f*x + e)^3, x)
```

Giac [F]

$$\int \csc^3(e+fx)\sqrt{d\csc(e+fx)}\,dx=\int \sqrt{d\csc\left(fx+e\right)}\csc\left(fx+e\right)^3\,dx$$

input
```
integrate(csc(f*x+e)^3*(d*csc(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d*csc(f*x + e))*csc(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(e+fx)\sqrt{d\csc(e+fx)}\,dx=\int \frac{\sqrt{\frac{d}{\sin(e+f\,x)}}}{\sin\left(e+f\,x\right)^3}\,dx$$

input
```
int((d/sin(e + f*x))^(1/2)/sin(e + f*x)^3,x)
```

output
```
int((d/sin(e + f*x))^(1/2)/sin(e + f*x)^3, x)
```

Reduce [F]

$$\int \csc^3(e+fx)\sqrt{d\csc(e+fx)}\,dx = \sqrt{d}\left(\int \sqrt{\csc\left(fx+e\right)}\,\csc\left(fx+e\right)^3 dx\right)$$

input
```
int(csc(f*x+e)^3*(d*csc(f*x+e))^(1/2),x)
```

output
```
sqrt(d)*int(sqrt(csc(e + f*x))*csc(e + f*x)**3,x)
```

3.515 $\int (d\csc(e+fx))^{3/2}\sin^5(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 103

$$\int (d\csc(e+fx))^{3/2}\sin^5(e+fx)\,dx = -\frac{2d^4\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}}-\frac{10d^2\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}}+\frac{10d\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{21f}$$

output
```
-2/7*d^4*cos(f*x+e)/f/(d*csc(f*x+e))^(5/2)-10/21*d^2*cos(f*x+e)/f/(d*csc(f
*x+e))^(1/2)+10/21*d*(d*csc(f*x+e))^(1/2)*InverseJacobiAM(1/2*e-1/4*Pi+1/2
*f*x,2^(1/2))*sin(f*x+e)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.66

$$\int (d\csc(e+fx))^{3/2}\sin^5(e+fx)\,dx = -\frac{d\sqrt{d\csc(e+fx)}\Big(40\,\text{EllipticF}\left(\frac{1}{4}(-2e+\pi-2fx),2\right)\sqrt{\sin(e+fx)}+26\sin(2(e+fx))-3\sin(4(e+f}{84f}$$

input
```
Integrate[(d*Csc[e + f*x])^(3/2)*Sin[e + f*x]^5,x]
```

output
```
-1/84*(d*Sqrt[d*Csc[e + f*x]]*(40*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt
[Sin[e + f*x]] + 26*Sin[2*(e + f*x)] - 3*Sin[4*(e + f*x)]))/f
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 2030, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(e+fx)(d\csc(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\csc(e+fx))^{3/2}}{\csc(e+fx)^5}dx$$

$$\Big\downarrow \text{2030}$$

$$d^5\int \frac{1}{(d\csc(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$d^5\left(\frac{5\int \frac{1}{(d\csc(e+fx))^{3/2}}dx}{7d^2}-\frac{2\cos(e+fx)}{7df(d\csc(e+fx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$d^5\left(\frac{5\int \frac{1}{(d\csc(e+fx))^{3/2}}dx}{7d^2}-\frac{2\cos(e+fx)}{7df(d\csc(e+fx))^{5/2}}\right)$$

$$\Big\downarrow \text{4256}$$

$$d^5\left(\frac{5\left(\frac{\int \sqrt{d\csc(e+fx)}dx}{3d^2}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}\right)}{7d^2}-\frac{2\cos(e+fx)}{7df(d\csc(e+fx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$d^5\left(\frac{5\left(\frac{\int \sqrt{d\csc(e+fx)}dx}{3d^2}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}\right)}{7d^2}-\frac{2\cos(e+fx)}{7df(d\csc(e+fx))^{5/2}}\right)$$

$\big\downarrow$ 4258

$$d^5\left(\frac{5\left(\frac{\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}\int \frac{1}{\sqrt{\sin(e+fx)}}dx}{3d^2}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}\right)}{7d^2}-\frac{2\cos(e+fx)}{7df(d\csc(e+fx))^{5/2}}\right)$$

$\big\downarrow$ 3042

$$d^5\left(\frac{5\left(\frac{\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}\int \frac{1}{\sqrt{\sin(e+fx)}}dx}{3d^2}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}\right)}{7d^2}-\frac{2\cos(e+fx)}{7df(d\csc(e+fx))^{5/2}}\right)$$

$\big\downarrow$ 3120

$$d^5\left(\frac{5\left(\frac{2\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\csc(e+fx)}}{3d^2f}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}\right)}{7d^2}-\frac{2\cos(e+fx)}{7df(d\csc(e+fx))^{5/2}}\right)$$

input
```
Int[(d*Csc[e + f*x])^(3/2)*Sin[e + f*x]^5,x]
```

output
```
d^5*((-2*Cos[e + f*x])/(7*d*f*(d*Csc[e + f*x])^(5/2)) + (5*((-2*Cos[e + f*
x])/(3*d*f*Sqrt[d*Csc[e + f*x]]) + (2*Sqrt[d*Csc[e + f*x]]*EllipticF[(e -
Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*d^2*f)))/(7*d^2))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.01 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.50

method	result
default	$\frac{\sqrt{2}\left(\sin(fx+e)\cos(fx+e)\sqrt{2}\left(3\cos(fx+e)^2-8\right)+i(5\cos(fx+e)+5)\sqrt{1-i\cot(fx+e)+i\csc(fx+e)}\,\sqrt{i(\csc(fx+e)-\cot(fx+e))}\ \mathrm{E}}{21f}$

input
```
int((d*csc(f*x+e))^(3/2)*sin(f*x+e)^5,x,method=_RETURNVERBOSE)
```

output
```
1/21/f*2^(1/2)*(sin(f*x+e)*cos(f*x+e)*2^(1/2)*(3*cos(f*x+e)^2-8)+I*(5*cos(
f*x+e)+5)*(1-I*cot(f*x+e)+I*csc(f*x+e))^(1/2)*(I*(csc(f*x+e)-cot(f*x+e)))^
(1/2)*EllipticF((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2))*(1+I*cot(
f*x+e)-I*csc(f*x+e))^(1/2))*d*(d*csc(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.96

$$\int (d\csc(e+fx))^{3/2}\sin^5(e+fx)\,dx = \frac{2\left(3\,d\cos\left(fx+e\right)^3-8\,d\cos\left(fx+e\right)\right)\sqrt{\frac{d}{\sin(fx+e)}}\sin\left(fx+e\right)-5i\,\sqrt{2i\,d}d\text{weierstrassPInverse}(4$$

input
```
integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^5,x, algorithm="fricas")
```

output
```
1/21*(2*(3*d*cos(f*x + e)^3 - 8*d*cos(f*x + e))*sqrt(d/sin(f*x + e))*sin(f
*x + e) - 5*I*sqrt(2*I*d)*d*weierstrassPInverse(4, 0, cos(f*x + e) + I*sin
(f*x + e)) + 5*I*sqrt(-2*I*d)*d*weierstrassPInverse(4, 0, cos(f*x + e) - I
*sin(f*x + e)))/f
```

Sympy [F(-1)]

Timed out.

$$\int (d\csc(e+fx))^{3/2}\sin^5(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((d*csc(f*x+e))**(3/2)*sin(f*x+e)**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\csc(e+fx))^{3/2}\sin^5(e+fx)\,dx = \int (d\csc(fx+e))^{\frac{3}{2}}\sin(fx+e)^5\,dx$$

input
```
integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^5,x, algorithm="maxima")
```

output
```
integrate((d*csc(f*x + e))^(3/2)*sin(f*x + e)^5, x)
```

Giac [F]

$$\int (d\csc(e+fx))^{3/2}\sin^5(e+fx)\,dx = \int (d\csc(fx+e))^{\frac{3}{2}}\sin(fx+e)^5\,dx$$

input
```
integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^5,x, algorithm="giac")
```

output
```
integrate((d*csc(f*x + e))^(3/2)*sin(f*x + e)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\csc(e+fx))^{3/2}\sin^5(e+fx)\,dx = \int \sin(e+f\,x)^5\left(\frac{d}{\sin(e+f\,x)}\right)^{3/2}dx$$

input
```
int(sin(e + f*x)^5*(d/sin(e + f*x))^(3/2),x)
```

output
```
int(sin(e + f*x)^5*(d/sin(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int (d\csc(e+fx))^{3/2}\sin^5(e+fx)\,dx=\sqrt{d}\left(\int \sqrt{\csc(fx+e)}\,\csc(fx+e)\sin(fx+e)^5\,dx\right)d$$

input
```
int((d*csc(f*x+e))^(3/2)*sin(f*x+e)^5,x)
```

output
```
sqrt(d)*int(sqrt(csc(e + f*x))*csc(e + f*x)*sin(e + f*x)**5,x)*d
```

3.516 $\int (d\csc(e+fx))^{3/2}\sin^4(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 77

$$\int (d\csc(e+fx))^{3/2}\sin^4(e+fx)\,dx = -\frac{2d^3\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}}+\frac{6d^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{5f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$

output
```
-2/5*d^3*cos(f*x+e)/f/(d*csc(f*x+e))^(3/2)-6/5*d^2*EllipticE(cos(1/2*e+1/4
*Pi+1/2*f*x),2^(1/2))/f/(d*csc(f*x+e))^(1/2)/sin(f*x+e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.81

$$\int (d\csc(e+fx))^{3/2}\sin^4(e+fx)\,dx = -\frac{2(d\csc(e+fx))^{3/2}\left(3E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|2\right)\sin^{\frac{3}{2}}(e+fx)+\cos(e+fx)\sin^3(e+fx)\right)}{5f}$$

input
```
Integrate[(d*Csc[e + f*x])^(3/2)*Sin[e + f*x]^4,x]
```

output
```
(-2*(d*Csc[e + f*x])^(3/2)*(3*EllipticE[(-2*e + Pi - 2*f*x)/4, 2]*Sin[e +
f*x]^(3/2) + Cos[e + f*x]*Sin[e + f*x]^3))/(5*f)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 2030, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4(e+fx)(d\csc(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\csc(e+fx))^{3/2}}{\csc(e+fx)^4}dx$$

$$\Big\downarrow \text{2030}$$

$$d^4\int \frac{1}{(d\csc(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$d^4\left(\frac{3\int \frac{1}{\sqrt{d\csc(e+fx)}}dx}{5d^2}-\frac{2\cos(e+fx)}{5df(d\csc(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$d^4\left(\frac{3\int \frac{1}{\sqrt{d\csc(e+fx)}}dx}{5d^2}-\frac{2\cos(e+fx)}{5df(d\csc(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \text{4258}$$

$$d^4\left(\frac{3\int \sqrt{\sin(e+fx)}dx}{5d^2\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2\cos(e+fx)}{5df(d\csc(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$d^4\left(\frac{3\int\sqrt{\sin(e+fx)}dx}{5d^2\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2\cos(e+fx)}{5df(d\csc(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \text{3119}$$

$$d^4\left(\frac{6E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{5d^2f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2\cos(e+fx)}{5df(d\csc(e+fx))^{3/2}}\right)$$

input
```
Int[(d*Csc[e + f*x])^(3/2)*Sin[e + f*x]^4,x]
```

output
```
d^4*((-2*Cos[e + f*x])/(5*d*f*(d*Csc[e + f*x])^(3/2)) + (6*EllipticE[(e -
Pi/2 + f*x)/2, 2])/(5*d^2*f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]]))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
 + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 249, normalized size of antiderivative = 3.23

method	result
default	$\frac{\sqrt{2}\left((-6\cos(fx+e)-6)\sqrt{1+i\cot(fx+e)-i\csc(fx+e)}\,\sqrt{-i(-\csc(fx+e)+\cot(fx+e))}\ \mathrm{EllipticE}\left(\sqrt{1+i\cot(fx+e)-i\csc(fx+e)},\right.\right.}{}$

input
```
int((d*csc(f*x+e))^(3/2)*sin(f*x+e)^4,x,method=_RETURNVERBOSE)
```

output
```
1/5/f*2^(1/2)*((-6*cos(f*x+e)-6)*(1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2)*(-I*(
-csc(f*x+e)+cot(f*x+e)))^(1/2)*EllipticE((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/
2),1/2*2^(1/2))*(1-I*cot(f*x+e)+I*csc(f*x+e))^(1/2)+(3*cos(f*x+e)+3)*(1+I*
cot(f*x+e)-I*csc(f*x+e))^(1/2)*(-I*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*(1-I*co
t(f*x+e)+I*csc(f*x+e))^(1/2)*EllipticF((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2)
,1/2*2^(1/2))+(cos(f*x+e)^3-4*cos(f*x+e)+3)*2^(1/2))*d*(d*csc(f*x+e))^(1/2
)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.27

$$\int (d\csc(e+fx))^{3/2}\sin^4(e+fx)\,dx = \frac{3\sqrt{2i\,d}\,d\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(fx+e)+i\sin(fx+e)))+3\sqrt{-2}}{}$$

input
```
integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^4,x, algorithm="fricas")
```

output
```
1/5*(3*sqrt(2*I*d)*d*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f
*x + e) + I*sin(f*x + e))) + 3*sqrt(-2*I*d)*d*weierstrassZeta(4, 0, weiers
trassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e))) + 2*(d*cos(f*x + e)^3
- d*cos(f*x + e))*sqrt(d/sin(f*x + e)))/f
```

Sympy [F(-1)]

Timed out.

$$\int (d \csc(e + fx))^{3/2} \sin^4(e + fx) \, dx = \text{Timed out}$$

input
```
integrate((d*csc(f*x+e))**(3/2)*sin(f*x+e)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d \csc(e + fx))^{3/2} \sin^4(e + fx) \, dx = \int (d \csc(fx + e))^{\frac{3}{2}} \sin(fx + e)^4 \, dx$$

input
```
integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate((d*csc(f*x + e))^(3/2)*sin(f*x + e)^4, x)
```

Giac [F]

$$\int (d \csc(e + fx))^{3/2} \sin^4(e + fx) \, dx = \int (d \csc(fx + e))^{\frac{3}{2}} \sin(fx + e)^4 \, dx$$

input
```
integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate((d*csc(f*x + e))^(3/2)*sin(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\csc(e+fx))^{3/2}\sin^4(e+fx)\,dx = \int \sin(e+f\,x)^4\left(\frac{d}{\sin(e+f\,x)}\right)^{3/2}dx$$

input
```
int(sin(e + f*x)^4*(d/sin(e + f*x))^(3/2),x)
```

output
```
int(sin(e + f*x)^4*(d/sin(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int (d\csc(e+fx))^{3/2}\sin^4(e+fx)\,dx = \sqrt{d}\left(\int \sqrt{\csc(fx+e)}\,\csc(fx+e)\sin(fx+e)^4\,dx\right)d$$

input
```
int((d*csc(f*x+e))^(3/2)*sin(f*x+e)^4,x)
```

output
```
sqrt(d)*int(sqrt(csc(e + f*x))*csc(e + f*x)*sin(e + f*x)**4,x)*d
```

3.517 $\int (d\csc(e+fx))^{3/2}\sin^3(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 75

$$\int (d\csc(e+fx))^{3/2}\sin^3(e+fx)\,dx = -\frac{2d^2\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}} + \frac{2d\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{3f}$$

output
```
-2/3*d^2*cos(f*x+e)/f/(d*csc(f*x+e))^(1/2)+2/3*d*(d*csc(f*x+e))^(1/2)*Inve
rseJacobiAM(1/2*e-1/4*Pi+1/2*f*x,2^(1/2))*sin(f*x+e)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.75

$$\int (d\csc(e+fx))^{3/2}\sin^3(e+fx)\,dx = -\frac{d\sqrt{d\csc(e+fx)}\left(2\,\text{EllipticF}\left(\frac{1}{4}(-2e+\pi-2fx),2\right)\sqrt{\sin(e+fx)}+\sin(2(e+fx))\right)}{3f}$$

input
```
Integrate[(d*Csc[e + f*x])^(3/2)*Sin[e + f*x]^3,x]
```

output
```
-1/3*(d*Sqrt[d*Csc[e + f*x]]*(2*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[S
in[e + f*x]] + Sin[2*(e + f*x)]))/f
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 2030, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(e+fx)(d\csc(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(d\csc(e+fx))^{3/2}}{\csc(e+fx)^3}dx$$

$$\Big\downarrow \text{2030}$$

$$d^3\int \frac{1}{(d\csc(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$d^3\left(\frac{\int \sqrt{d\csc(e+fx)}dx}{3d^2}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$d^3\left(\frac{\int \sqrt{d\csc(e+fx)}dx}{3d^2}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}\right)$$

$$\Big\downarrow \text{4258}$$

$$d^3\left(\frac{\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}\int \frac{1}{\sqrt{\sin(e+fx)}}dx}{3d^2}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$d^3\left(\frac{\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}\int\frac{1}{\sqrt{\sin(e+fx)}}dx}{3d^2}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}\right)$$

$$\Big\downarrow \text{3120}$$

$$d^3\left(\frac{2\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\csc(e+fx)}}{3d^2f}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}\right)$$

input
```
Int[(d*Csc[e + f*x])^(3/2)*Sin[e + f*x]^3,x]
```

output
```
d^3*((-2*Cos[e + f*x])/(3*d*f*Sqrt[d*Csc[e + f*x]]) + (2*Sqrt[d*Csc[e + f*
x]]*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*d^2*f))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.88

method	result
default	$\frac{\sqrt{2}\left(i(\cos(fx+e)+1)\sqrt{i(\csc(fx+e)-\cot(fx+e))}\ \text{EllipticF}\left(\sqrt{1+i\cot(fx+e)-i\csc(fx+e)},\frac{\sqrt{2}}{2}\right)\sqrt{1-i\cot(fx+e)+i\csc(fx+e)}\ \sqrt{}\right.}{3f}$

input
```
int((d*csc(f*x+e))^(3/2)*sin(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3/f*2^(1/2)*(I*(cos(f*x+e)+1)*(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*Elliptic
F((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2))*(1-I*cot(f*x+e)+I*csc(f
*x+e))^(1/2)*(1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2)-2^(1/2)*cos(f*x+e)*sin(f*
x+e))*d*(d*csc(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.13

$$\int (d\csc(e+fx))^{3/2}\sin^3(e+fx)\,dx =$$
$$-\frac{2\,d\sqrt{\frac{d}{\sin(fx+e)}}\cos(fx+e)\sin(fx+e)+i\,\sqrt{2i\,d}d\text{weierstrassPInverse}(4,0,\cos(fx+e)+i\,\sin(fx+e))}{3\,f}$$

input
```
integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^3,x, algorithm="fricas")
```

output
```
-1/3*(2*d*sqrt(d/sin(f*x + e))*cos(f*x + e)*sin(f*x + e) + I*sqrt(2*I*d)*d
*weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e)) - I*sqrt(-2*I*d)
*d*weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e)))/f
```

Sympy [F(-1)]

Timed out.

$$\int (d\csc(e+fx))^{3/2}\sin^3(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((d*csc(f*x+e))**(3/2)*sin(f*x+e)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\csc(e+fx))^{3/2}\sin^3(e+fx)\,dx = \int (d\csc(fx+e))^{\frac{3}{2}}\sin(fx+e)^3\,dx$$

input
```
integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^3,x, algorithm="maxima")
```

output
```
integrate((d*csc(f*x + e))^(3/2)*sin(f*x + e)^3, x)
```

Giac [F]

$$\int (d\csc(e+fx))^{3/2}\sin^3(e+fx)\,dx = \int (d\csc(fx+e))^{\frac{3}{2}}\sin(fx+e)^3\,dx$$

input
```
integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^3,x, algorithm="giac")
```

output
```
integrate((d*csc(f*x + e))^(3/2)*sin(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\csc(e+fx))^{3/2}\sin^3(e+fx)\,dx = \int \sin\left(e+f\,x\right)^3\left(\frac{d}{\sin\left(e+f\,x\right)}\right)^{3/2}\,dx$$

input
```
int(sin(e + f*x)^3*(d/sin(e + f*x))^(3/2),x)
```

output
```
int(sin(e + f*x)^3*(d/sin(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int (d\csc(e+fx))^{3/2}\sin^3(e+fx)\,dx = \sqrt{d}\left(\int \sqrt{\csc\left(fx+e\right)}\,\csc\left(fx+e\right)\sin\left(fx+e\right)^3dx\right)d$$

input
```
int((d*csc(f*x+e))^(3/2)*sin(f*x+e)^3,x)
```

output
```
sqrt(d)*int(sqrt(csc(e + f*x))*csc(e + f*x)*sin(e + f*x)**3,x)*d
```

3.518 $\int (d\csc(e+fx))^{3/2}\sin^2(e+fx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 46

$$\int (d\csc(e+fx))^{3/2}\sin^2(e+fx)\,dx = \frac{2d^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$

output
```
-2*d^2*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))/f/(d*csc(f*x+e))^(1/2)
/sin(f*x+e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int (d\csc(e+fx))^{3/2}\sin^2(e+fx)\,dx = -\frac{2d^2E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|2\right)}{f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$

input
```
Integrate[(d*Csc[e + f*x])^(3/2)*Sin[e + f*x]^2,x]
```

output
```
(-2*d^2*EllipticE[(-2*e + Pi - 2*f*x)/4, 2])/(f*Sqrt[d*Csc[e + f*x]]*Sqrt[
Sin[e + f*x]])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 2030, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(e+fx)(d\csc(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(d\csc(e+fx))^{3/2}}{\csc(e+fx)^2}dx$$

$$\Big\downarrow \text{ 2030}$$

$$d^2\int \frac{1}{\sqrt{d\csc(e+fx)}}dx$$

$$\Big\downarrow \text{ 4258}$$

$$\frac{d^2\int \sqrt{\sin(e+fx)}dx}{\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{d^2\int \sqrt{\sin(e+fx)}dx}{\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{2d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

input
```
Int[(d*Csc[e + f*x])^(3/2)*Sin[e + f*x]^2,x]
```

output
```
(2*d^2*EllipticE[(e - Pi/2 + f*x)/2, 2])/(f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[
e + f*x]])
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 238, normalized size of antiderivative = 5.17

method	result
default	$-\frac{\sqrt{2}\left(2\sqrt{1+i(-\csc(fx+e)+\cot(fx+e))}\,\sqrt{1-i(-\csc(fx+e)+\cot(fx+e))}\,\sqrt{-i(-\csc(fx+e)+\cot(fx+e))}\,(\cos(fx+e)+1)\,\mathrm{Ellipt}\cdots\right.}{\cdots}$
risch	$-\frac{\left(\mathrm{e}^{2i(fx+e)}-1\right)\sqrt{2}\,d\sqrt{\frac{id\,\mathrm{e}^{i(fx+e)}}{\mathrm{e}^{2i(fx+e)}-1}}\,\mathrm{e}^{-i(fx+e)}}{f}+\frac{\left(-\frac{2i\left(id\,\mathrm{e}^{2i(fx+e)}-id\right)}{d\sqrt{\mathrm{e}^{i(fx+e)}\left(id\,\mathrm{e}^{2i(fx+e)}-id\right)}}-\frac{\sqrt{\mathrm{e}^{i(fx+e)}+1}\,\sqrt{-2\,\mathrm{e}^{i(fx+e)}+2}\,\sqrt{-\mathrm{e}^{i(fx+e)}}\,\left(\cdots\right.}{\sqrt{id\cdots}}\cdots\right.}{\cdots}$

input
```
int((d*csc(f*x+e))^(3/2)*sin(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/f*2^(1/2)*(2*(1+I*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*(1-I*(-csc(f*x+e)+cot
(f*x+e)))^(1/2)*(-I*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*(cos(f*x+e)+1)*Ellipti
cE((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2))+(1+I*(-csc(f*x+e)+cot(
f*x+e)))^(1/2)*(1-I*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*(-I*(-csc(f*x+e)+cot(f
*x+e)))^(1/2)*(-cos(f*x+e)-1)*EllipticF((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2
),1/2*2^(1/2))+2^(1/2)*(-1+cos(f*x+e)))*d*(d*csc(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.33

$$\int (d\csc(e+fx))^{3/2}\sin^2(e+fx)\,dx = \frac{\sqrt{2i\,d}d\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos{(fx+e)}+i\,\sin{(fx+e)}))+\sqrt{-2i\,d}d}{f}$$

input
```
integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^2,x, algorithm="fricas")
```

output
```
(sqrt(2*I*d)*d*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e
) + I*sin(f*x + e))) + sqrt(-2*I*d)*d*weierstrassZeta(4, 0, weierstrassPIn
verse(4, 0, cos(f*x + e) - I*sin(f*x + e))))/f
```

Sympy [F(-1)]

Timed out.

$$\int (d\csc(e+fx))^{3/2}\sin^2(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((d*csc(f*x+e))**(3/2)*sin(f*x+e)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\csc(e+fx))^{3/2}\sin^2(e+fx)\,dx = \int (d\csc(fx+e))^{\frac{3}{2}}\sin(fx+e)^2\,dx$$

input
```
integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate((d*csc(f*x + e))^(3/2)*sin(f*x + e)^2, x)
```

Giac [F]

$$\int (d\csc(e+fx))^{3/2}\sin^2(e+fx)\,dx = \int (d\csc(fx+e))^{\frac{3}{2}}\sin(fx+e)^2\,dx$$

input
```
integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate((d*csc(f*x + e))^(3/2)*sin(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\csc(e+fx))^{3/2}\sin^2(e+fx)\,dx = \int \sin(e+f\,x)^2\left(\frac{d}{\sin(e+f\,x)}\right)^{3/2}dx$$

input
```
int(sin(e + f*x)^2*(d/sin(e + f*x))^(3/2),x)
```

output
```
int(sin(e + f*x)^2*(d/sin(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int (d\csc(e+fx))^{3/2}\sin^2(e+fx)\,dx=\sqrt{d}\left(\int\sqrt{\csc(fx+e)}\,\csc(fx+e)\sin(fx+e)^2\,dx\right)d$$

input
```
int((d*csc(f*x+e))^(3/2)*sin(f*x+e)^2,x)
```

output
```
sqrt(d)*int(sqrt(csc(e + f*x))*csc(e + f*x)*sin(e + f*x)**2,x)*d
```

3.519 $\int (d \csc(e+fx))^{3/2} \sin(e+fx) \, dx$

Optimal result

Integrand size = 19, antiderivative size = 44

$$\int (d \csc(e+fx))^{3/2} \sin(e+fx) \, dx = \frac{2d\sqrt{d \csc(e+fx)} \operatorname{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{f}$$

output
```
2*d*(d*csc(f*x+e))^(1/2)*InverseJacobiAM(1/2*e-1/4*Pi+1/2*f*x,2^(1/2))*sin
(f*x+e)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int (d \csc(e+fx))^{3/2} \sin(e+fx) \, dx = -\frac{2d\sqrt{d \csc(e+fx)} \operatorname{EllipticF}\left(\frac{1}{4}(-2e+\pi-2fx),2\right)\sqrt{\sin(e+fx)}}{f}$$

input
```
Integrate[(d*Csc[e + f*x])^(3/2)*Sin[e + f*x],x]
```

output
```
(-2*d*Sqrt[d*Csc[e + f*x]]*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Sin[e
+ f*x]])/f
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 2030, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(e+fx)(d\csc(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(d\csc(e+fx))^{3/2}}{\csc(e+fx)}dx$$

$$\Big\downarrow \; 2030$$

$$d\int \sqrt{d\csc(e+fx)}dx$$

$$\Big\downarrow \; 4258$$

$$d\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}\int \frac{1}{\sqrt{\sin(e+fx)}}dx$$

$$\Big\downarrow \; 3042$$

$$d\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}\int \frac{1}{\sqrt{\sin(e+fx)}}dx$$

$$\Big\downarrow \; 3120$$

$$\frac{2d\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\csc(e+fx)}}{f}$$

input
```
Int[(d*Csc[e + f*x])^(3/2)*Sin[e + f*x],x]
```

output
```
(2*d*Sqrt[d*Csc[e + f*x]]*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*
x]])/f
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.75

method	result
default	$\frac{id\sqrt{2}\,\sqrt{d\csc(fx+e)}\,\sqrt{1+i\cot(fx+e)-i\csc(fx+e)}\,\sqrt{1-i\cot(fx+e)+i\csc(fx+e)}\,\sqrt{i(\csc(fx+e)-\cot(fx+e))}\ \operatorname{EllipticF}\left(\sqrt{1+i\cot\right.}{f}$

input
```
int((d*csc(f*x+e))^(3/2)*sin(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
I*d/f*2^(1/2)*(d*csc(f*x+e))^(1/2)*(1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2)*(1-
I*cot(f*x+e)+I*csc(f*x+e))^(1/2)*(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*Ellipti
cF((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2))*(cos(f*x+e)+1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.30

$$\int (d\csc(e+fx))^{3/2}\sin(e+fx)\,dx = \frac{-i\sqrt{2i\,d}d\text{weierstrassPInverse}(4,0,\cos(fx+e)+i\sin(fx+e))+i\sqrt{-2i\,d}d\text{weierstrassPInver}}{f}$$

input
```
integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e),x, algorithm="fricas")
```

output
```
(-I*sqrt(2*I*d)*d*weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e))
 + I*sqrt(-2*I*d)*d*weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e
)))/f
```

Sympy [F]

$$\int (d\csc(e+fx))^{3/2}\sin(e+fx)\,dx = \int (d\csc(e+fx))^{\frac{3}{2}}\sin(e+fx)\,dx$$

input
```
integrate((d*csc(f*x+e))**(3/2)*sin(f*x+e),x)
```

output
```
Integral((d*csc(e + f*x))**(3/2)*sin(e + f*x), x)
```

Maxima [F]

$$\int (d\csc(e+fx))^{3/2}\sin(e+fx)\,dx = \int (d\csc(fx+e))^{\frac{3}{2}}\sin(fx+e)\,dx$$

input
```
integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e),x, algorithm="maxima")
```

output
```
integrate((d*csc(f*x + e))^(3/2)*sin(f*x + e), x)
```

Giac [F]

$$\int (d\csc(e+fx))^{3/2}\sin(e+fx)\,dx = \int (d\csc(fx+e))^{\frac{3}{2}}\sin(fx+e)\,dx$$

input
```
integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e),x, algorithm="giac")
```

output
```
integrate((d*csc(f*x + e))^(3/2)*sin(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\csc(e+fx))^{3/2}\sin(e+fx)\,dx = \int \sin(e+f\,x)\left(\frac{d}{\sin(e+f\,x)}\right)^{3/2}dx$$

input
```
int(sin(e + f*x)*(d/sin(e + f*x))^(3/2),x)
```

output
```
int(sin(e + f*x)*(d/sin(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int (d\csc(e+fx))^{3/2}\sin(e+fx)\,dx = \sqrt{d}\left(\int \sqrt{\csc(fx+e)}\,\csc(fx+e)\sin(fx+e)\,dx\right)d$$

input
```
int((d*csc(f*x+e))^(3/2)*sin(f*x+e),x)
```

output
```
sqrt(d)*int(sqrt(csc(e + f*x))*csc(e + f*x)*sin(e + f*x),x)*d
```

3.520 $\int (d \csc(e+fx))^{3/2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 71

$$\int (d\csc(e+fx))^{3/2}\,dx=-\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}-\frac{2d^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$

output
```
-2*d*cos(f*x+e)*(d*csc(f*x+e))^(1/2)/f+2*d^2*EllipticE(cos(1/2*e+1/4*Pi+1/
2*f*x),2^(1/2))/f/(d*csc(f*x+e))^(1/2)/sin(f*x+e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.76

$$\int (d\csc(e+fx))^{3/2}\,dx=\frac{(d\csc(e+fx))^{3/2}\left(2E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|2\right)\sin^{\frac{3}{2}}(e+fx)-\sin(2(e+fx))\right)}{f}$$

input
```
Integrate[(d*Csc[e + f*x])^(3/2),x]
```

output
```
((d*Csc[e + f*x])^(3/2)*(2*EllipticE[(-2*e + Pi - 2*f*x)/4, 2]*Sin[e + f*x
]^(3/2) - Sin[2*(e + f*x)]))/f
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d\csc(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (d\csc(e+fx))^{3/2}dx$$

$$\Big\downarrow \text{4255}$$

$$d^2\left(-\int \frac{1}{\sqrt{d\csc(e+fx)}}dx\right)-\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}$$

$$\Big\downarrow \text{3042}$$

$$d^2\left(-\int \frac{1}{\sqrt{d\csc(e+fx)}}dx\right)-\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}$$

$$\Big\downarrow \text{4258}$$

$$-\frac{d^2\int \sqrt{\sin(e+fx)}dx}{\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{d^2\int \sqrt{\sin(e+fx)}dx}{\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{2d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}$$

input
```
Int[(d*Csc[e + f*x])^(3/2),x]
```

output
```
(-2*d*Cos[e + f*x]*Sqrt[d*Csc[e + f*x]])/f - (2*d^2*EllipticE[(e - Pi/2 +
f*x)/2, 2])/(f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.20

method	result
default	$-\frac{\sqrt{2}\left(\sqrt{2}+\left(\operatorname{EllipticF}\left(\sqrt{1+i\cot(fx+e)-i\csc(fx+e)},\frac{\sqrt{2}}{2}\right)-2\operatorname{EllipticE}\left(\sqrt{1+i\cot(fx+e)-i\csc(fx+e)},\frac{\sqrt{2}}{2}\right)\right)\sqrt{1+i(-\csc(fx+}\right.}{f}$

input
```
int((d*csc(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f*2^(1/2)*(2^(1/2)+(EllipticF((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2*2
^(1/2))-2*EllipticE((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2)))*(1+I
*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*(1-I*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*(-I*
(-csc(f*x+e)+cot(f*x+e)))^(1/2)*(cos(f*x+e)+1))*d*(d*csc(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.17

$$\int (d\csc(e+fx))^{3/2}\,dx = -\frac{2d\sqrt{\frac{d}{\sin(fx+e)}}\cos(fx+e) + \sqrt{2i\,d}\,d\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(fx+e)+i\sin(f}{f}$$

input
```
integrate((d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
-(2*d*sqrt(d/sin(f*x + e))*cos(f*x + e) + sqrt(2*I*d)*d*weierstrassZeta(4,
 0, weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e))) + sqrt(-2*I*
d)*d*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(
f*x + e))))/f
```

Sympy [F]

$$\int (d\csc(e+fx))^{3/2}\,dx = \int (d\csc(e+fx))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*csc(f*x+e))**(3/2),x)
```

output
```
Integral((d*csc(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int (d\csc(e+fx))^{3/2}\,dx = \int (d\csc(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*csc(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*csc(f*x + e))^(3/2), x)
```

Giac [F]

$$\int (d\csc(e+fx))^{3/2}\,dx = \int (d\csc(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*csc(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*csc(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\csc(e+fx))^{3/2}\,dx = \int \left(\frac{d}{\sin(e+f\,x)}\right)^{3/2}\,dx$$

input
```
int((d/sin(e + f*x))^(3/2),x)
```

output
```
int((d/sin(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int (d\csc(e+fx))^{3/2}\,dx = \sqrt{d}\left(\int \sqrt{\csc(fx+e)}\,\csc(fx+e)\,dx\right)d$$

input
```
int((d*csc(f*x+e))^(3/2),x)
```

output
```
sqrt(d)*int(sqrt(csc(e + f*x))*csc(e + f*x),x)*d
```

3.521 $\int \csc(e+fx)(d\csc(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 72

$$\int \csc(e+fx)(d\csc(e+fx))^{3/2}\,dx=-\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3f}+\frac{2d\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{3f}$$

output
```
-2/3*cos(f*x+e)*(d*csc(f*x+e))^(3/2)/f+2/3*d*(d*csc(f*x+e))^(1/2)*InverseJ
acobiAM(1/2*e-1/4*Pi+1/2*f*x,2^(1/2))*sin(f*x+e)^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

$$\int \csc(e+fx)(d\csc(e+fx))^{3/2}\,dx=-\frac{(d\csc(e+fx))^{5/2}\left(2\,\text{EllipticF}\left(\frac{1}{4}(-2e+\pi-2fx),2\right)\sin^{\frac{5}{2}}(e+fx)+\sin(2(e+fx))\right)}{3df}$$

input
```
Integrate[Csc[e + f*x]*(d*Csc[e + f*x])^(3/2),x]
```

output
```
-1/3*((d*Csc[e + f*x])^(5/2)*(2*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sin[e
+ f*x]^(5/2) + Sin[2*(e + f*x)]))/(d*f)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {2030, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(e+fx)(d\csc(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (d\csc(e+fx))^{5/2}dx}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (d\csc(e+fx))^{5/2}dx}{d}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{1}{3}d^2\int \sqrt{d\csc(e+fx)}dx-\frac{2d\cos(e+fx)(d\csc(e+fx))^{3/2}}{3f}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{3}d^2\int \sqrt{d\csc(e+fx)}dx-\frac{2d\cos(e+fx)(d\csc(e+fx))^{3/2}}{3f}}{d}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{1}{3}d^2\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}\int \frac{1}{\sqrt{\sin(e+fx)}}dx-\frac{2d\cos(e+fx)(d\csc(e+fx))^{3/2}}{3f}}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{3}d^2\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}\int \frac{1}{\sqrt{\sin(e+fx)}}dx-\frac{2d\cos(e+fx)(d\csc(e+fx))^{3/2}}{3f}}{d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{2d^2\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\csc(e+fx)}}{3f}-\frac{2d\cos(e+fx)(d\csc(e+fx))^{3/2}}{3f}}{d}$$

input
```
Int[Csc[e + f*x]*(d*Csc[e + f*x])^(3/2),x]
```

output
```
((-2*d*Cos[e + f*x]*(d*Csc[e + f*x])^(3/2))/(3*f) + (2*d^2*Sqrt[d*Csc[e +
f*x]]*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*f))/d
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.89

method	result
default	$-\frac{\sqrt{2}\, d \sqrt{d \csc(fx+e)}\, \left(i(-\cos(fx+e)-1)\sqrt{1+i\cot(fx+e)-i\csc(fx+e)}\, \sqrt{1-i\cot(fx+e)+i\csc(fx+e)}\, \sqrt{i(\csc(fx+e)-\cot(fx+e}\right.}{3f}$

input
```
int(csc(f*x+e)*(d*csc(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/f*2^(1/2)*d*(d*csc(f*x+e))^(1/2)*(I*(-cos(f*x+e)-1)*(1+I*cot(f*x+e)-I
*csc(f*x+e))^(1/2)*(1-I*cot(f*x+e)+I*csc(f*x+e))^(1/2)*(I*(csc(f*x+e)-cot(
f*x+e)))^(1/2)*EllipticF((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2))+
2^(1/2)*cot(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.38

$$\int \csc(e+fx)(d\csc(e+fx))^{3/2}\,dx = \frac{-i\sqrt{2i\,d}d\sin(fx+e)\,\text{weierstrassPInverse}(4,0,\cos(fx+e)+i\,\sin(fx+e))+i\sqrt{-2i\,d}d\,\text{s}}{3\,f\,\text{sin}}$$

input
```
integrate(csc(f*x+e)*(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2*I*d)*d*sin(f*x + e)*weierstrassPInverse(4, 0, cos(f*x + e)
+ I*sin(f*x + e)) + I*sqrt(-2*I*d)*d*sin(f*x + e)*weierstrassPInverse(4, 0
, cos(f*x + e) - I*sin(f*x + e)) - 2*d*sqrt(d/sin(f*x + e))*cos(f*x + e))/
(f*sin(f*x + e))
```

Sympy [F]

$$\int \csc(e+fx)(d\csc(e+fx))^{3/2}\,dx = \int (d\csc(e+fx))^{\frac{3}{2}}\csc(e+fx)\,dx$$

input
```
integrate(csc(f*x+e)*(d*csc(f*x+e))**(3/2),x)
```

output
```
Integral((d*csc(e + f*x))**(3/2)*csc(e + f*x), x)
```

Maxima [F]

$$\int \csc(e+fx)(d\csc(e+fx))^{3/2}\,dx = \int (d\csc(fx+e))^{\frac{3}{2}}\csc(fx+e)\,dx$$

input
```
integrate(csc(f*x+e)*(d*csc(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*csc(f*x + e))^(3/2)*csc(f*x + e), x)
```

Giac [F]

$$\int \csc(e+fx)(d\csc(e+fx))^{3/2}\,dx = \int (d\csc(fx+e))^{\frac{3}{2}}\csc(fx+e)\,dx$$

input
```
integrate(csc(f*x+e)*(d*csc(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*csc(f*x + e))^(3/2)*csc(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc(e+fx)(d\csc(e+fx))^{3/2}\,dx = \int \frac{\left(\frac{d}{\sin(e+f\,x)}\right)^{3/2}}{\sin\left(e+f\,x\right)}\,dx$$

input
```
int((d/sin(e + f*x))^(3/2)/sin(e + f*x),x)
```

output
```
int((d/sin(e + f*x))^(3/2)/sin(e + f*x), x)
```

Reduce [F]

$$\int \csc(e+fx)(d\csc(e+fx))^{3/2}\,dx = \sqrt{d}\left(\int \sqrt{\csc\left(fx+e\right)}\,\csc\left(fx+e\right)^2 dx\right) d$$

input
```
int(csc(f*x+e)*(d*csc(f*x+e))^(3/2),x)
```

output
```
sqrt(d)*int(sqrt(csc(e + f*x))*csc(e + f*x)**2,x)*d
```

3.522 $\int \csc^2(e+fx)(d\csc(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 103

$$\int \csc^2(e+fx)(d\csc(e+fx))^{3/2}\,dx = -\frac{6d\cos(e+fx)\sqrt{d\csc(e+fx)}}{5f} - \frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5df} - \frac{6d^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{5f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$

output
```
-6/5*d*cos(f*x+e)*(d*csc(f*x+e))^(1/2)/f-2/5*cos(f*x+e)*(d*csc(f*x+e))^(5/
2)/d/f+6/5*d^2*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))/f/(d*csc(f*x+e
))^(1/2)/sin(f*x+e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.66

$$\int \csc^2(e+fx)(d\csc(e+fx))^{3/2}\,dx = \frac{(d\csc(e+fx))^{5/2}\left(-7\cos(e+fx)+3\cos(3(e+fx))+12E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|2\right)\sin^{\frac{5}{2}}(e\right.}{10df}$$

input
```
Integrate[Csc[e + f*x]^2*(d*Csc[e + f*x])^(3/2),x]
```

output
```
((d*Csc[e + f*x])^(5/2)*(-7*Cos[e + f*x] + 3*Cos[3*(e + f*x)] + 12*Ellipti
cE[(-2*e + Pi - 2*f*x)/4, 2]*Sin[e + f*x]^(5/2)))/(10*d*f)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2030, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(e+fx)(d\csc(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (d\csc(e+fx))^{7/2}dx}{d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (d\csc(e+fx))^{7/2}dx}{d^2}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{3}{5}d^2\int (d\csc(e+fx))^{3/2}dx-\frac{2d\cos(e+fx)(d\csc(e+fx))^{5/2}}{5f}}{d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{5}d^2\int (d\csc(e+fx))^{3/2}dx-\frac{2d\cos(e+fx)(d\csc(e+fx))^{5/2}}{5f}}{d^2}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{3}{5}d^2\left(d^2\left(-\int \frac{1}{\sqrt{d\csc(e+fx)}}dx\right)-\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}\right)-\frac{2d\cos(e+fx)(d\csc(e+fx))^{5/2}}{5f}}{d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{5}d^2\left(d^2\left(-\int \frac{1}{\sqrt{d\csc(e+fx)}}dx\right)-\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}\right)-\frac{2d\cos(e+fx)(d\csc(e+fx))^{5/2}}{5f}}{d^2}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{3}{5}d^2\left(-\frac{d^2\int\sqrt{\sin(e+fx)}dx}{\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}\right)-\frac{2d\cos(e+fx)(d\csc(e+fx))^{5/2}}{5f}}{d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{5}d^2\left(-\frac{d^2\int\sqrt{\sin(e+fx)}dx}{\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}\right)-\frac{2d\cos(e+fx)(d\csc(e+fx))^{5/2}}{5f}}{d^2}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{3}{5}d^2\left(-\frac{2d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}\right)-\frac{2d\cos(e+fx)(d\csc(e+fx))^{5/2}}{5f}}{d^2}$$

input
```
Int[Csc[e + f*x]^2*(d*Csc[e + f*x])^(3/2),x]
```

output
```
((-2*d*Cos[e + f*x]*(d*Csc[e + f*x])^(5/2))/(5*f) + (3*d^2*((-2*d*Cos[e +
f*x]*Sqrt[d*Csc[e + f*x]])/f - (2*d^2*EllipticE[(e - Pi/2 + f*x)/2, 2])/(f
*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])))/5)/d^2
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.40

method	result
default	$\frac{\sqrt{2}\,\sqrt{d\csc(fx+e)}\,d\Big((6\cos(fx+e)+6)\sqrt{1+i\cot(fx+e)-i\csc(fx+e)}\ \text{EllipticE}\Big(\sqrt{1+i\cot(fx+e)-i\csc(fx+e)},\frac{\sqrt{2}}{2}\Big)\sqrt{1-i\cot(fx-}$

input
```
int(csc(f*x+e)^2*(d*csc(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/5/f*2^(1/2)*(d*csc(f*x+e))^(1/2)*d*((6*cos(f*x+e)+6)*(1+I*cot(f*x+e)-I*c
sc(f*x+e))^(1/2)*EllipticE((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2)
)*(1-I*cot(f*x+e)+I*csc(f*x+e))^(1/2)*(-I*(-csc(f*x+e)+cot(f*x+e)))^(1/2)+
(-3*cos(f*x+e)-3)*(1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2)*(1-I*cot(f*x+e)+I*cs
c(f*x+e))^(1/2)*(-I*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*EllipticF((1+I*cot(f*x
+e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2))+2^(1/2)*(-3-csc(f*x+e)*cot(f*x+e)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.34

$$\int \csc^2(e+fx)(d\csc(e+fx))^{3/2}\,dx =$$

$$-\frac{3\left(d\cos\left(fx+e\right)^2-d\right)\sqrt{2i\,d}\,\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)\right)}{}$$

input
```
integrate(csc(f*x+e)^2*(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
-1/5*(3*(d*cos(f*x + e)^2 - d)*sqrt(2*I*d)*weierstrassZeta(4, 0, weierstra
ssPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*(d*cos(f*x + e)^2 - d
)*sqrt(-2*I*d)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e
) - I*sin(f*x + e))) + 2*(3*d*cos(f*x + e)^3 - 4*d*cos(f*x + e))*sqrt(d/si
n(f*x + e)))/(f*cos(f*x + e)^2 - f)
```

Sympy [F]

$$\int \csc^2(e+fx)(d\csc(e+fx))^{3/2}\,dx = \int \left(d\csc\left(e+fx\right)\right)^{\frac{3}{2}}\csc^2\left(e+fx\right)dx$$

input
```
integrate(csc(f*x+e)**2*(d*csc(f*x+e))**(3/2),x)
```

output
```
Integral((d*csc(e + f*x))**(3/2)*csc(e + f*x)**2, x)
```

Maxima [F]

$$\int \csc^2(e+fx)(d\csc(e+fx))^{3/2}\,dx = \int (d\csc(fx+e))^{\frac{3}{2}}\csc(fx+e)^2\,dx$$

input
```
integrate(csc(f*x+e)^2*(d*csc(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*csc(f*x + e))^(3/2)*csc(f*x + e)^2, x)
```

Giac [F]

$$\int \csc^2(e+fx)(d\csc(e+fx))^{3/2}\,dx = \int (d\csc(fx+e))^{\frac{3}{2}}\csc(fx+e)^2\,dx$$

input
```
integrate(csc(f*x+e)^2*(d*csc(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*csc(f*x + e))^(3/2)*csc(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(e+fx)(d\csc(e+fx))^{3/2}\,dx = \int \frac{\left(\frac{d}{\sin(e+f\,x)}\right)^{3/2}}{\sin\left(e+f\,x\right)^2}\,dx$$

input
```
int((d/sin(e + f*x))^(3/2)/sin(e + f*x)^2,x)
```

output
```
int((d/sin(e + f*x))^(3/2)/sin(e + f*x)^2, x)
```

Reduce [F]

$$\int \csc^2(e+fx)(d\csc(e+fx))^{3/2}\,dx = \sqrt{d}\left(\int \sqrt{\csc(fx+e)}\,\csc(fx+e)^3\,dx\right)d$$

input
```
int(csc(f*x+e)^2*(d*csc(f*x+e))^(3/2),x)
```

output
```
sqrt(d)*int(sqrt(csc(e + f*x))*csc(e + f*x)**3,x)*d
```

3.523 $\int \frac{\sin^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 102

$$\int \frac{\sin^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = -\frac{2d^2\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} - \frac{10\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}} + \frac{10\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{21df}$$

output
```
-2/7*d^2*cos(f*x+e)/f/(d*csc(f*x+e))^(5/2)-10/21*cos(f*x+e)/f/(d*csc(f*x+e
))^(1/2)+10/21*(d*csc(f*x+e))^(1/2)*InverseJacobiAM(1/2*e-1/4*Pi+1/2*f*x,2
^(1/2))*sin(f*x+e)^(1/2)/d/f
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.69

$$\int \frac{\sin^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = -\frac{\sqrt{d\csc(e+fx)}\left(40\,\text{EllipticF}\left(\frac{1}{4}(-2e+\pi-2fx),2\right)\sqrt{\sin(e+fx)}+26\sin(2(e+fx))-3\sin(4(e+\right.}{84df}$$

input
```
Integrate[Sin[e + f*x]^3/Sqrt[d*Csc[e + f*x]],x]
```

output
```
-1/84*(Sqrt[d*Csc[e + f*x]]*(40*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[S
in[e + f*x]] + 26*Sin[2*(e + f*x)] - 3*Sin[4*(e + f*x)]))/(d*f)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 2030, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(e+fx)}{\sqrt{d\csc(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc(e+fx)^3\sqrt{d\csc(e+fx)}}dx$$

$$\Big\downarrow \text{2030}$$

$$d^3\int \frac{1}{(d\csc(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$d^3\left(\frac{5\int \frac{1}{(d\csc(e+fx))^{3/2}}dx}{7d^2}-\frac{2\cos(e+fx)}{7df(d\csc(e+fx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$d^3\left(\frac{5\int \frac{1}{(d\csc(e+fx))^{3/2}}dx}{7d^2}-\frac{2\cos(e+fx)}{7df(d\csc(e+fx))^{5/2}}\right)$$

$$\Big\downarrow \text{4256}$$

$$d^3\left(\frac{5\left(\frac{\int \sqrt{d\csc(e+fx)}dx}{3d^2}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}\right)}{7d^2}-\frac{2\cos(e+fx)}{7df(d\csc(e+fx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$d^3\left(\frac{5\left(\frac{\int \sqrt{d\csc(e+fx)}dx}{3d^2}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}\right)}{7d^2}-\frac{2\cos(e+fx)}{7df(d\csc(e+fx))^{5/2}}\right)$$

$\Big\downarrow$ 4258

$$d^3\left(\frac{5\left(\frac{\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}\int \frac{1}{\sqrt{\sin(e+fx)}}dx}{3d^2}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}\right)}{7d^2}-\frac{2\cos(e+fx)}{7df(d\csc(e+fx))^{5/2}}\right)$$

$\Big\downarrow$ 3042

$$d^3\left(\frac{5\left(\frac{\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}\int \frac{1}{\sqrt{\sin(e+fx)}}dx}{3d^2}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}\right)}{7d^2}-\frac{2\cos(e+fx)}{7df(d\csc(e+fx))^{5/2}}\right)$$

$\Big\downarrow$ 3120

$$d^3\left(\frac{5\left(\frac{2\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\csc(e+fx)}}{3d^2f}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}\right)}{7d^2}-\frac{2\cos(e+fx)}{7df(d\csc(e+fx))^{5/2}}\right)$$

input
```
Int[Sin[e + f*x]^3/Sqrt[d*Csc[e + f*x]],x]
```

output
```
d^3*((-2*Cos[e + f*x])/(7*d*f*(d*Csc[e + f*x])^(5/2)) + (5*((-2*Cos[e + f*
x])/(3*d*f*Sqrt[d*Csc[e + f*x]]) + (2*Sqrt[d*Csc[e + f*x]]*EllipticF[(e -
Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*d^2*f)))/(7*d^2))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.03 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.52

method	result
default	$\frac{\sqrt{2}\left(\sqrt{2}\left(3\cos(fx+e)^{3}-8\cos(fx+e)\right)+i\sqrt{1-i\cot(fx+e)+i\csc(fx+e)}\,\sqrt{-i(-\csc(fx+e)+\cot(fx+e))}\ \text{EllipticF}\left(\sqrt{1+i\cot(fx+}\right.\right.}{21f\sqrt{d\csc(fx+e)}}$

input
```
int(sin(f*x+e)^3/(d*csc(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/21/f*2^(1/2)/(d*csc(f*x+e))^(1/2)*(2^(1/2)*(3*cos(f*x+e)^3-8*cos(f*x+e))
+I*(1-I*cot(f*x+e)+I*csc(f*x+e))^(1/2)*(-I*(-csc(f*x+e)+cot(f*x+e)))^(1/2)
*EllipticF((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2))*(1+I*cot(f*x+e
)-I*csc(f*x+e))^(1/2)*(5*cot(f*x+e)+5*csc(f*x+e)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.96

$$\int \frac{\sin^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \frac{2\left(3\cos(fx+e)^3-8\cos(fx+e)\right)\sqrt{\frac{d}{\sin(fx+e)}}\sin(fx+e)-5i\sqrt{2i\,d}\,\text{weierstrassPInverse}(4,0,\cos(fx-}{21\,df}$$

input
```
integrate(sin(f*x+e)^3/(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
1/21*(2*(3*cos(f*x + e)^3 - 8*cos(f*x + e))*sqrt(d/sin(f*x + e))*sin(f*x +
 e) - 5*I*sqrt(2*I*d)*weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x +
 e)) + 5*I*sqrt(-2*I*d)*weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x
 + e)))/(d*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**3/(d*csc(f*x+e))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sin^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{\sin\left(fx+e\right)^3}{\sqrt{d\csc\left(fx+e\right)}}\,dx$$

input
```
integrate(sin(f*x+e)^3/(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^3/sqrt(d*csc(f*x + e)), x)
```

Giac [F]

$$\int \frac{\sin^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{\sin\left(fx+e\right)^3}{\sqrt{d\csc\left(fx+e\right)}}\,dx$$

input
```
integrate(sin(f*x+e)^3/(d*csc(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sin(f*x + e)^3/sqrt(d*csc(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{\sin\left(e+f\,x\right)^3}{\sqrt{\frac{d}{\sin(e+f\,x)}}}\,dx$$

input
```
int(sin(e + f*x)^3/(d/sin(e + f*x))^(1/2),x)
```

output
```
int(sin(e + f*x)^3/(d/sin(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sin^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\csc(fx+e)}\,\sin(fx+e)^3}{\csc(fx+e)}dx\right)}{d}$$

input
```
int(sin(f*x+e)^3/(d*csc(f*x+e))^(1/2),x)
```

output
```
(sqrt(d)*int((sqrt(csc(e + f*x))*sin(e + f*x)**3)/csc(e + f*x),x))/d
```

3.524 $\int \frac{\sin^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 72

$$\int \frac{\sin^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = -\frac{2d\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}} + \frac{6E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{5f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$

output
```
-2/5*d*cos(f*x+e)/f/(d*csc(f*x+e))^(3/2)-6/5*EllipticE(cos(1/2*e+1/4*Pi+1/
2*f*x),2^(1/2))/f/(d*csc(f*x+e))^(1/2)/sin(f*x+e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.79

$$\int \frac{\sin^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \frac{-\frac{12E\left(\frac{1}{4}(-2e+\pi-2fx)|2\right)}{\sqrt{\sin(e+fx)}} - 2\sin(2(e+fx))}{10f\sqrt{d\csc(e+fx)}}$$

input
```
Integrate[Sin[e + f*x]^2/Sqrt[d*Csc[e + f*x]],x]
```

output
```
((-12*EllipticE[(-2*e + Pi - 2*f*x)/4, 2])/Sqrt[Sin[e + f*x]] - 2*Sin[2*(e
 + f*x)])/(10*f*Sqrt[d*Csc[e + f*x]])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 2030, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(e+fx)}{\sqrt{d\csc(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc(e+fx)^2\sqrt{d\csc(e+fx)}}dx$$

$$\Big\downarrow \text{2030}$$

$$d^2\int \frac{1}{(d\csc(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$d^2\left(\frac{3\int \frac{1}{\sqrt{d\csc(e+fx)}}dx}{5d^2}-\frac{2\cos(e+fx)}{5df(d\csc(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$d^2\left(\frac{3\int \frac{1}{\sqrt{d\csc(e+fx)}}dx}{5d^2}-\frac{2\cos(e+fx)}{5df(d\csc(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \text{4258}$$

$$d^2\left(\frac{3\int \sqrt{\sin(e+fx)}dx}{5d^2\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2\cos(e+fx)}{5df(d\csc(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$d^2\left(\frac{3\int \sqrt{\sin(e+fx)}dx}{5d^2\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2\cos(e+fx)}{5df(d\csc(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \text{3119}$$

$$d^2\left(\frac{6E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{5d^2f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2\cos(e+fx)}{5df(d\csc(e+fx))^{3/2}}\right)$$

input
```
Int[Sin[e + f*x]^2/Sqrt[d*Csc[e + f*x]],x]
```

output
```
d^2*((-2*Cos[e + f*x])/(5*d*f*(d*Csc[e + f*x])^(3/2)) + (6*EllipticE[(e -
Pi/2 + f*x)/2, 2])/(5*d^2*f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]]))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 254, normalized size of antiderivative = 3.53

method	result
default	$\frac{\sqrt{2}\left((-6\cos(fx+e)-6)\sqrt{1+i\cot(fx+e)-i\csc(fx+e)}\,\sqrt{i(\csc(fx+e)-\cot(fx+e))}\,\mathrm{EllipticE}\left(\sqrt{1+i\cot(fx+e)-i\csc(fx+e)},\frac{\sqrt{2}}{2}\right)\right.}{}$

input
```
int(sin(f*x+e)^2/(d*csc(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5/f*2^(1/2)*((-6*cos(f*x+e)-6)*(1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2)*(I*(c
sc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticE((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2)
,1/2*2^(1/2))*(1-I*cot(f*x+e)+I*csc(f*x+e))^(1/2)+(3*cos(f*x+e)+3)*(1+I*co
t(f*x+e)-I*csc(f*x+e))^(1/2)*(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*(1-I*cot(f*
x+e)+I*csc(f*x+e))^(1/2)*EllipticF((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2
*2^(1/2))+(cos(f*x+e)^3-4*cos(f*x+e)+3)*2^(1/2))/(d*csc(f*x+e))^(1/2)*csc(
f*x+e)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.33

$$\int \frac{\sin^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \frac{2\left(\cos(fx+e)^3-\cos(fx+e)\right)\sqrt{\frac{d}{\sin(fx+e)}}+3\sqrt{2i\,d}\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(}{}$$

input
```
integrate(sin(f*x+e)^2/(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
1/5*(2*(cos(f*x + e)^3 - cos(f*x + e))*sqrt(d/sin(f*x + e)) + 3*sqrt(2*I*d
)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x
 + e))) + 3*sqrt(-2*I*d)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, c
os(f*x + e) - I*sin(f*x + e))))/(d*f)
```

Sympy [F]

$$\int \frac{\sin^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{\sin^2{(e+fx)}}{\sqrt{d\csc{(e+fx)}}}\,dx$$

input
```
integrate(sin(f*x+e)**2/(d*csc(f*x+e))**(1/2),x)
```

output
```
Integral(sin(e + f*x)**2/sqrt(d*csc(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sin^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{\sin{(fx+e)}^2}{\sqrt{d\csc{(fx+e)}}}\,dx$$

input
```
integrate(sin(f*x+e)^2/(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^2/sqrt(d*csc(f*x + e)), x)
```

Giac [F]

$$\int \frac{\sin^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{\sin{(fx+e)}^2}{\sqrt{d\csc{(fx+e)}}}\,dx$$

input
```
integrate(sin(f*x+e)^2/(d*csc(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sin(f*x + e)^2/sqrt(d*csc(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx=\int \frac{\sin\left(e+f\,x\right)^2}{\sqrt{\frac{d}{\sin(e+f\,x)}}}\,dx$$

input
```
int(sin(e + f*x)^2/(d/sin(e + f*x))^(1/2),x)
```

output
```
int(sin(e + f*x)^2/(d/sin(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sin^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx=\frac{\sqrt{d}\left(\int \frac{\sqrt{\csc(fx+e)}\,\sin(fx+e)^2}{\csc(fx+e)}dx\right)}{d}$$

input
```
int(sin(f*x+e)^2/(d*csc(f*x+e))^(1/2),x)
```

output
```
(sqrt(d)*int((sqrt(csc(e + f*x))*sin(e + f*x)**2)/csc(e + f*x),x))/d
```

3.525 $\int \frac{\sin(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 74

$$\int \frac{\sin(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = -\frac{2\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}} + \frac{2\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{3df}$$

output
```
-2/3*cos(f*x+e)/f/(d*csc(f*x+e))^(1/2)+2/3*(d*csc(f*x+e))^(1/2)*InverseJac
obiAM(1/2*e-1/4*Pi+1/2*f*x,2^(1/2))*sin(f*x+e)^(1/2)/d/f
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \frac{\sin(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = -\frac{d\csc^2(e+fx)\left(2\,\text{EllipticF}\left(\frac{1}{4}(-2e+\pi-2fx),2\right)\sqrt{\sin(e+fx)}+\sin(2(e+fx))\right)}{3f(d\csc(e+fx))^{3/2}}$$

input
```
Integrate[Sin[e + f*x]/Sqrt[d*Csc[e + f*x]],x]
```

output
```
-1/3*(d*Csc[e + f*x]^2*(2*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Sin[e +
 f*x]] + Sin[2*(e + f*x)]))/(f*(d*Csc[e + f*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3042, 2030, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(e+fx)}{\sqrt{d\csc(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc(e+fx)\sqrt{d\csc(e+fx)}}dx$$

$$\Big\downarrow \text{2030}$$

$$d\int \frac{1}{(d\csc(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$d\left(\frac{\int \sqrt{d\csc(e+fx)}dx}{3d^2}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$d\left(\frac{\int \sqrt{d\csc(e+fx)}dx}{3d^2}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}\right)$$

$$\Big\downarrow \text{4258}$$

$$d\left(\frac{\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}\int \frac{1}{\sqrt{\sin(e+fx)}}dx}{3d^2}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$d\left(\frac{\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}\int\frac{1}{\sqrt{\sin(e+fx)}}dx}{3d^2}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}\right)$$

$\big\downarrow$ 3120

$$d\left(\frac{2\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\csc(e+fx)}}{3d^2f}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}\right)$$

input
```
Int[Sin[e + f*x]/Sqrt[d*Csc[e + f*x]],x]
```

output
```
d*((-2*Cos[e + f*x])/(3*d*f*Sqrt[d*Csc[e + f*x]]) + (2*Sqrt[d*Csc[e + f*x]
]*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*d^2*f))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.76 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.88

method	result
default	$\frac{\sqrt{2}\left(i\sqrt{i(\csc(fx+e)-\cot(fx+e))}\ \operatorname{EllipticF}\left(\sqrt{1+i\cot(fx+e)-i\csc(fx+e)},\frac{\sqrt{2}}{2}\right)\sqrt{1-i\cot(fx+e)+i\csc(fx+e)}\ \sqrt{1+i\cot(fx+e)-}\right.}{3f\sqrt{d\csc(fx+e)}}$

input
```
int(sin(f*x+e)/(d*csc(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/f*2^(1/2)/(d*csc(f*x+e))^(1/2)*(I*(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*El
lipticF((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2))*(1-I*cot(f*x+e)+I
*csc(f*x+e))^(1/2)*(1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2)*(cot(f*x+e)+csc(f*x
+e))-2^(1/2)*cos(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15

$$\int \frac{\sin(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = -\frac{2\sqrt{\frac{d}{\sin(fx+e)}}\cos(fx+e)\sin(fx+e)+i\sqrt{2i\,d}\text{weierstrassPInverse}(4,0,\cos(fx+e)+i\,\sin(fx+e))}{3\,df}$$

input
```
integrate(sin(f*x+e)/(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(2*sqrt(d/sin(f*x + e))*cos(f*x + e)*sin(f*x + e) + I*sqrt(2*I*d)*wei
erstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e)) - I*sqrt(-2*I*d)*wei
erstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e)))/(d*f)
```

Sympy [F]

$$\int \frac{\sin(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{\sin(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx$$

input
```
integrate(sin(f*x+e)/(d*csc(f*x+e))**(1/2),x)
```

output
```
Integral(sin(e + f*x)/sqrt(d*csc(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sin(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{\sin(fx+e)}{\sqrt{d\csc(fx+e)}}\,dx$$

input
```
integrate(sin(f*x+e)/(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)/sqrt(d*csc(f*x + e)), x)
```

Giac [F]

$$\int \frac{\sin(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{\sin(fx+e)}{\sqrt{d\csc(fx+e)}}\,dx$$

input
```
integrate(sin(f*x+e)/(d*csc(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(sin(f*x + e)/sqrt(d*csc(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{\sin\left(e+f\,x\right)}{\sqrt{\frac{d}{\sin(e+f\,x)}}}\,dx$$

input
```
int(sin(e + f*x)/(d/sin(e + f*x))^(1/2),x)
```

output
```
int(sin(e + f*x)/(d/sin(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\sin(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\csc(fx+e)}\,\sin(fx+e)}{\csc(fx+e)}dx\right)}{d}$$

input
```
int(sin(f*x+e)/(d*csc(f*x+e))^(1/2),x)
```

output
```
(sqrt(d)*int((sqrt(csc(e + f*x))*sin(e + f*x))/csc(e + f*x),x))/d
```

3.526 $\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 43

$$\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx = \frac{2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$

output
```
-2*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))/f/(d*csc(f*x+e))^(1/2)/sin
(f*x+e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx = -\frac{2E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|2\right)}{f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$

input
```
Integrate[1/Sqrt[d*Csc[e + f*x]],x]
```

output
```
(-2*EllipticE[(-2*e + Pi - 2*f*x)/4, 2])/(f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[
e + f*x]])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{d \csc(e+fx)}} dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\sqrt{d \csc(e+fx)}}dx$$
$$\Big\downarrow \text{ 4258}$$
$$\frac{\int \sqrt{\sin(e+fx)}dx}{\sqrt{\sin(e+fx)}\sqrt{d \csc(e+fx)}}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{\int \sqrt{\sin(e+fx)}dx}{\sqrt{\sin(e+fx)}\sqrt{d \csc(e+fx)}}$$
$$\Big\downarrow \text{ 3119}$$
$$\frac{2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d \csc(e+fx)}}$$

input
```
Int[1/Sqrt[d*Csc[e + f*x]],x]
```

output
```
(2*EllipticE[(e - Pi/2 + f*x)/2, 2])/(f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e +
f*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 243, normalized size of antiderivative = 5.65

method	result
default	$-\frac{\sqrt{2}\left(2\sqrt{1+i(-\csc(fx+e)+\cot(fx+e))}\,\sqrt{1-i(-\csc(fx+e)+\cot(fx+e))}\,\sqrt{-i(-\csc(fx+e)+\cot(fx+e))}\,(\cos(fx+e)+1)\,\mathrm{Ellipt}\cdots\right.}{\cdots}$
risch	$-\frac{i\sqrt{2}}{f\sqrt{\frac{id\,\mathrm{e}^{i(fx+e)}}{\mathrm{e}^{2i(fx+e)}-1}}}+\frac{i\left(-\frac{2i\left(id\,\mathrm{e}^{2i(fx+e)}-id\right)}{d\sqrt{\mathrm{e}^{i(fx+e)}\left(id\,\mathrm{e}^{2i(fx+e)}-id\right)}}-\frac{\sqrt{\mathrm{e}^{i(fx+e)}+1}\,\sqrt{-2\,\mathrm{e}^{i(fx+e)}+2}\,\sqrt{-\mathrm{e}^{i(fx+e)}}\left(-2\,\mathrm{EllipticE}\left(\sqrt{\mathrm{e}^{i(fx+e)}+1},\frac{\sqrt{\cdots}}{2}\cdots\right.\right.}{\sqrt{id\,\mathrm{e}^{3i(fx+e)}-id\,\mathrm{e}^{i(fx+e)}}}\cdots\right)}{f\sqrt{\frac{id\,\mathrm{e}^{i(fx+e)}}{\mathrm{e}^{2i(fx+e)}-1}}\left(\mathrm{e}^{2i(fx+e)}-1\right)}$

input
```
int(1/(d*csc(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f*2^(1/2)*(2*(1+I*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*(1-I*(-csc(f*x+e)+cot
(f*x+e)))^(1/2)*(-I*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*(cos(f*x+e)+1)*Ellipti
cE((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2))+(1+I*(-csc(f*x+e)+cot(
f*x+e)))^(1/2)*(1-I*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*(-I*(-csc(f*x+e)+cot(f
*x+e)))^(1/2)*(-cos(f*x+e)-1)*EllipticF((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2
),1/2*2^(1/2))+2^(1/2)*(-1+cos(f*x+e)))/(d*csc(f*x+e))^(1/2)*csc(f*x+e)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.44

$$\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx$$
$$= \frac{\sqrt{2i\,d}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos{(fx+e)}+i\,\sin{(fx+e)}))+\sqrt{-2i\,d}\text{weierstrass}}{df}$$

input
```
integrate(1/(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(2*I*d)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e)
+ I*sin(f*x + e))) + sqrt(-2*I*d)*weierstrassZeta(4, 0, weierstrassPInvers
e(4, 0, cos(f*x + e) - I*sin(f*x + e))))/(d*f)
```

Sympy [F]

$$\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{1}{\sqrt{d\csc{(e+fx)}}}\,dx$$

input
```
integrate(1/(d*csc(f*x+e))**(1/2),x)
```

output
```
Integral(1/sqrt(d*csc(e + f*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{1}{\sqrt{d\csc{(fx+e)}}}\,dx$$

input
```
integrate(1/(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(d*csc(f*x + e)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{1}{\sqrt{d\csc\left(fx+e\right)}}\,dx$$

input
```
integrate(1/(d*csc(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(d*csc(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{1}{\sqrt{\frac{d}{\sin(e+f\,x)}}}\,dx$$

input
```
int(1/(d/sin(e + f*x))^(1/2),x)
```

output
```
int(1/(d/sin(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\csc(fx+e)}}{\csc(fx+e)}dx\right)}{d}$$

input
```
int(1/(d*csc(f*x+e))^(1/2),x)
```

output
```
(sqrt(d)*int(sqrt(csc(e + f*x))/csc(e + f*x),x))/d
```

3.527 $\int \frac{\csc(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 46

$$\int \frac{\csc(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \frac{2\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{df}$$

output
```
2*(d*csc(f*x+e))^(1/2)*InverseJacobiAM(1/2*e-1/4*Pi+1/2*f*x,2^(1/2))*sin(f
*x+e)^(1/2)/d/f
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{\csc(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = -\frac{2\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{4}(-2e+\pi-2fx),2\right)\sqrt{\sin(e+fx)}}{df}$$

input
```
Integrate[Csc[e + f*x]/Sqrt[d*Csc[e + f*x]],x]
```

output
```
(-2*Sqrt[d*Csc[e + f*x]]*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Sin[e +
f*x]])/(d*f)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2030, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(e+fx)}{\sqrt{d\csc(e+fx)}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int \sqrt{d\csc(e+fx)}dx}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{d\csc(e+fx)}dx}{d}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}\int \frac{1}{\sqrt{\sin(e+fx)}}dx}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}\int \frac{1}{\sqrt{\sin(e+fx)}}dx}{d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\csc(e+fx)}}{df}$$

input
```
Int[Csc[e + f*x]/Sqrt[d*Csc[e + f*x]],x]
```

output
```
(2*Sqrt[d*Csc[e + f*x]]*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]
])/(d*f)
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.72

method	result
default	$\frac{i\sqrt{2}\,\sqrt{1+i\cot(fx+e)-i\csc(fx+e)}\,\sqrt{1-i\cot(fx+e)+i\csc(fx+e)}\,\sqrt{i(\csc(fx+e)-\cot(fx+e))}\,\operatorname{EllipticF}\left(\sqrt{1+i\cot(fx+e)-i\csc(\right.}{f\sqrt{d\csc(fx+e)}}$

input
```
int(csc(f*x+e)/(d*csc(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
I/f*2^(1/2)*(1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2)*(1-I*cot(f*x+e)+I*csc(f*x+
e))^(1/2)*(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticF((1+I*cot(f*x+e)-I*cs
c(f*x+e))^(1/2),1/2*2^(1/2))/(d*csc(f*x+e))^(1/2)*(cot(f*x+e)+csc(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int \frac{\csc(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx$$
$$= \frac{-i\sqrt{2i\,d}\,\text{weierstrassPInverse}(4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right))+i\sqrt{-2i\,d}\,\text{weierstrassPInverse}(4,0,\cos}{df}$$

input
```
integrate(csc(f*x+e)/(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2*I*d)*weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e)) +
 I*sqrt(-2*I*d)*weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e)))/
(d*f)
```

Sympy [F]

$$\int \frac{\csc(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{\csc\left(e+fx\right)}{\sqrt{d\csc\left(e+fx\right)}}\,dx$$

input
```
integrate(csc(f*x+e)/(d*csc(f*x+e))**(1/2),x)
```

output
```
Integral(csc(e + f*x)/sqrt(d*csc(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\csc(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{\csc\left(fx+e\right)}{\sqrt{d\csc\left(fx+e\right)}}\,dx$$

input
```
integrate(csc(f*x+e)/(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(f*x + e)/sqrt(d*csc(f*x + e)), x)
```

Giac [F]

$$\int \frac{\csc(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{\csc\left(fx+e\right)}{\sqrt{d\csc\left(fx+e\right)}}\,dx$$

input
```
integrate(csc(f*x+e)/(d*csc(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(csc(f*x + e)/sqrt(d*csc(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)\,\sqrt{\frac{d}{\sin(e+f\,x)}}}\,dx$$

input
```
int(1/(sin(e + f*x)*(d/sin(e + f*x))^(1/2)),x)
```

output
```
int(1/(sin(e + f*x)*(d/sin(e + f*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{\csc(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \frac{\sqrt{d}\left(\int \sqrt{\csc\left(fx+e\right)}dx\right)}{d}$$

input
```
int(csc(f*x+e)/(d*csc(f*x+e))^(1/2),x)
```

output
```
(sqrt(d)*int(sqrt(csc(e + f*x)),x))/d
```

3.528 $\int \frac{\csc^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 70

$$\int \frac{\csc^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx=-\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{df}-\frac{2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$

output
```
-2*cos(f*x+e)*(d*csc(f*x+e))^(1/2)/d/f+2*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*
x),2^(1/2))/f/(d*csc(f*x+e))^(1/2)/sin(f*x+e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.74

$$\int \frac{\csc^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx=\frac{-2\cot(e+fx)+\frac{2E\left(\frac{1}{4}(-2e+\pi-2fx)|2\right)}{\sqrt{\sin(e+fx)}}}{f\sqrt{d\csc(e+fx)}}$$

input
```
Integrate[Csc[e + f*x]^2/Sqrt[d*Csc[e + f*x]],x]
```

output
```
(-2*Cot[e + f*x] + (2*EllipticE[(-2*e + Pi - 2*f*x)/4, 2])/Sqrt[Sin[e + f*
x]])/(f*Sqrt[d*Csc[e + f*x]])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2030, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(e+fx)}{\sqrt{d\csc(e+fx)}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (d\csc(e+fx))^{3/2}dx}{d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (d\csc(e+fx))^{3/2}dx}{d^2}$$

$$\Big\downarrow \text{4255}$$

$$\frac{d^2\left(-\int \frac{1}{\sqrt{d\csc(e+fx)}}dx\right)-\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}}{d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d^2\left(-\int \frac{1}{\sqrt{d\csc(e+fx)}}dx\right)-\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}}{d^2}$$

$$\Big\downarrow \text{4258}$$

$$\frac{-\frac{d^2\int \sqrt{\sin(e+fx)}dx}{\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}}{d^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{d^2\int \sqrt{\sin(e+fx)}dx}{\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}}{d^2}$$

$$\Big\downarrow \text{3119}$$

$$\frac{-\frac{2d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}}{d^2}$$

input
```
Int[Csc[e + f*x]^2/Sqrt[d*Csc[e + f*x]],x]
```

output
```
((-2*d*Cos[e + f*x]*Sqrt[d*Csc[e + f*x]])/f - (2*d^2*EllipticE[(e - Pi/2 +
 f*x)/2, 2])/(f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]]))/d^2
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.30

method	result
default	$-\frac{\sqrt{2}\left(\sqrt{2}+\left(\mathrm{EllipticF}\left(\sqrt{1+i\cot(fx+e)-i\csc(fx+e)},\frac{\sqrt{2}}{2}\right)-2\,\mathrm{EllipticE}\left(\sqrt{1+i\cot(fx+e)-i\csc(fx+e)},\frac{\sqrt{2}}{2}\right)\right)\sqrt{1+i(-\csc(fx+e}{f\sqrt{d\csc(fx+e)}}$

input
```
int(csc(f*x+e)^2/(d*csc(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f*2^(1/2)*(2^(1/2)+(EllipticF((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2*2
^(1/2))-2*EllipticE((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2)))*(1+I
*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*(1-I*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*(-I*
(-csc(f*x+e)+cot(f*x+e)))^(1/2)*(cos(f*x+e)+1))/(d*csc(f*x+e))^(1/2)*csc(f
*x+e)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.19

$$\int \frac{\csc^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = -\frac{2\sqrt{\frac{d}{\sin(fx+e)}}\cos(fx+e)+\sqrt{2i\,d}\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(fx+e)+i\,\sin(}{df}$$

input
```
integrate(csc(f*x+e)^2/(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-(2*sqrt(d/sin(f*x + e))*cos(f*x + e) + sqrt(2*I*d)*weierstrassZeta(4, 0,
weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e))) + sqrt(-2*I*d)*w
eierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x +
e))))/(d*f)
```

Sympy [F]

$$\int \frac{\csc^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{\csc^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx$$

input
```
integrate(csc(f*x+e)**2/(d*csc(f*x+e))**(1/2),x)
```

output
```
Integral(csc(e + f*x)**2/sqrt(d*csc(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\csc^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{\csc(fx+e)^2}{\sqrt{d\csc(fx+e)}}\,dx$$

input
```
integrate(csc(f*x+e)^2/(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(f*x + e)^2/sqrt(d*csc(f*x + e)), x)
```

Giac [F]

$$\int \frac{\csc^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{\csc(fx+e)^2}{\sqrt{d\csc(fx+e)}}\,dx$$

input
```
integrate(csc(f*x+e)^2/(d*csc(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(csc(f*x + e)^2/sqrt(d*csc(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)^2\,\sqrt{\frac{d}{\sin(e+f\,x)}}}\,dx$$

input
```
int(1/(sin(e + f*x)^2*(d/sin(e + f*x))^(1/2)),x)
```

output
```
int(1/(sin(e + f*x)^2*(d/sin(e + f*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{\csc^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \frac{\sqrt{d}\left(\int \sqrt{\csc\left(fx+e\right)}\,\csc\left(fx+e\right)dx\right)}{d}$$

input
```
int(csc(f*x+e)^2/(d*csc(f*x+e))^(1/2),x)
```

output
```
(sqrt(d)*int(sqrt(csc(e + f*x))*csc(e + f*x),x))/d
```

3.529 $\int \frac{\csc^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 77

$$\int \frac{\csc^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = -\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3d^2f} + \frac{2\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{3df}$$

output
```
-2/3*cos(f*x+e)*(d*csc(f*x+e))^(3/2)/d^2/f+2/3*(d*csc(f*x+e))^(1/2)*Invers
eJacobiAM(1/2*e-1/4*Pi+1/2*f*x,2^(1/2))*sin(f*x+e)^(1/2)/d/f
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.78

$$\int \frac{\csc^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = -\frac{2\csc^2(e+fx)\left(\cos(e+fx)+\text{EllipticF}\left(\frac{1}{4}(-2e+\pi-2fx),2\right)\sin^{\frac{3}{2}}(e+fx)\right)}{3f\sqrt{d\csc(e+fx)}}$$

input
```
Integrate[Csc[e + f*x]^3/Sqrt[d*Csc[e + f*x]],x]
```

output
```
(-2*Csc[e + f*x]^2*(Cos[e + f*x] + EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sin
[e + f*x]^(3/2)))/(3*f*Sqrt[d*Csc[e + f*x]])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2030, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(e+fx)}{\sqrt{d\csc(e+fx)}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (d\csc(e+fx))^{5/2}dx}{d^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (d\csc(e+fx))^{5/2}dx}{d^3}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{1}{3}d^2\int \sqrt{d\csc(e+fx)}dx-\frac{2d\cos(e+fx)(d\csc(e+fx))^{3/2}}{3f}}{d^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{3}d^2\int \sqrt{d\csc(e+fx)}dx-\frac{2d\cos(e+fx)(d\csc(e+fx))^{3/2}}{3f}}{d^3}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{1}{3}d^2\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}\int \frac{1}{\sqrt{\sin(e+fx)}}dx-\frac{2d\cos(e+fx)(d\csc(e+fx))^{3/2}}{3f}}{d^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{3}d^2\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}\int \frac{1}{\sqrt{\sin(e+fx)}}dx-\frac{2d\cos(e+fx)(d\csc(e+fx))^{3/2}}{3f}}{d^3}$$

$$\Big\downarrow 3120$$

$$\frac{\frac{2d^2\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\csc(e+fx)}}{3f}-\frac{2d\cos(e+fx)(d\csc(e+fx))^{3/2}}{3f}}{d^3}$$

input
```
Int[Csc[e + f*x]^3/Sqrt[d*Csc[e + f*x]],x]
```

output
```
((-2*d*Cos[e + f*x]*(d*Csc[e + f*x])^(3/2))/(3*f) + (2*d^2*Sqrt[d*Csc[e +
f*x]]*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*f))/d^3
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.88

method	result
default	$\frac{\sqrt{2}\left(i\sqrt{1-i\cot(fx+e)+i\csc(fx+e)}\,\sqrt{-i(-\csc(fx+e)+\cot(fx+e))}\,\text{EllipticF}\left(\sqrt{1+i\cot(fx+e)-i\csc(fx+e)},\frac{\sqrt{2}}{2}\right)\sqrt{1+i\cot(fx-}\right.}{3f\sqrt{d\csc(fx+e)}}$

input
```
int(csc(f*x+e)^3/(d*csc(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/f*2^(1/2)/(d*csc(f*x+e))^(1/2)*(I*(1-I*cot(f*x+e)+I*csc(f*x+e))^(1/2)*
(-I*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*EllipticF((1+I*cot(f*x+e)-I*csc(f*x+e)
)^(1/2),1/2*2^(1/2))*(1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2)*(cot(f*x+e)+csc(f
*x+e))-2^(1/2)*cot(f*x+e)*csc(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.29

$$\int \frac{\csc^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx$$
$$= \frac{-i\sqrt{2i\,d}\sin(fx+e)\,\text{weierstrassPInverse}(4,0,\cos(fx+e)+i\,\sin(fx+e))+i\sqrt{-2i\,d}\sin(fx+e)\,\text{wei}}{3\,df\sin(fx+e)}$$

input
```
integrate(csc(f*x+e)^3/(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2*I*d)*sin(f*x + e)*weierstrassPInverse(4, 0, cos(f*x + e) +
I*sin(f*x + e)) + I*sqrt(-2*I*d)*sin(f*x + e)*weierstrassPInverse(4, 0, co
s(f*x + e) - I*sin(f*x + e)) - 2*sqrt(d/sin(f*x + e))*cos(f*x + e))/(d*f*s
in(f*x + e))
```

Sympy [F]

$$\int \frac{\csc^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{\csc^3{(e+fx)}}{\sqrt{d\csc{(e+fx)}}}\,dx$$

input
```
integrate(csc(f*x+e)**3/(d*csc(f*x+e))**(1/2),x)
```

output
```
Integral(csc(e + f*x)**3/sqrt(d*csc(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\csc^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{\csc{(fx+e)}^3}{\sqrt{d\csc{(fx+e)}}}\,dx$$

input
```
integrate(csc(f*x+e)^3/(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(csc(f*x + e)^3/sqrt(d*csc(f*x + e)), x)
```

Giac [F]

$$\int \frac{\csc^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{\csc{(fx+e)}^3}{\sqrt{d\csc{(fx+e)}}}\,dx$$

input
```
integrate(csc(f*x+e)^3/(d*csc(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(csc(f*x + e)^3/sqrt(d*csc(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)^3\,\sqrt{\frac{d}{\sin(e+f\,x)}}}\,dx$$

input
```
int(1/(sin(e + f*x)^3*(d/sin(e + f*x))^(1/2)),x)
```

output
```
int(1/(sin(e + f*x)^3*(d/sin(e + f*x))^(1/2)), x)
```

Reduce [F]

$$\int \frac{\csc^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \frac{\sqrt{d}\left(\int \sqrt{\csc\left(fx+e\right)}\,\csc\left(fx+e\right)^2 dx\right)}{d}$$

input
```
int(csc(f*x+e)^3/(d*csc(f*x+e))^(1/2),x)
```

output
```
(sqrt(d)*int(sqrt(csc(e + f*x))*csc(e + f*x)**2,x))/d
```

3.530 $\int \frac{\sin^2(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 103

$$\int \frac{\sin^2(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = -\frac{2d\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} - \frac{10\cos(e+fx)}{21df\sqrt{d\csc(e+fx)}} + \frac{10\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{21d^2f}$$

output
```
-2/7*d*cos(f*x+e)/f/(d*csc(f*x+e))^(5/2)-10/21*cos(f*x+e)/d/f/(d*csc(f*x+e
))^(1/2)+10/21*(d*csc(f*x+e))^(1/2)*InverseJacobiAM(1/2*e-1/4*Pi+1/2*f*x,2
^(1/2))*sin(f*x+e)^(1/2)/d^2/f
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.68

$$\int \frac{\sin^2(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = -\frac{\sqrt{d\csc(e+fx)}\left(40\,\text{EllipticF}\left(\frac{1}{4}(-2e+\pi-2fx),2\right)\sqrt{\sin(e+fx)}+26\sin(2(e+fx))-3\sin(4(e+fx}{84d^2f}$$

input
```
Integrate[Sin[e + f*x]^2/(d*Csc[e + f*x])^(3/2),x]
```

output
```
-1/84*(Sqrt[d*Csc[e + f*x]]*(40*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[S
in[e + f*x]] + 26*Sin[2*(e + f*x)] - 3*Sin[4*(e + f*x)]))/(d^2*f)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 2030, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc(e+fx)^2(d\csc(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$d^2\int \frac{1}{(d\csc(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$d^2\left(\frac{5\int \frac{1}{(d\csc(e+fx))^{3/2}}dx}{7d^2}-\frac{2\cos(e+fx)}{7df(d\csc(e+fx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$d^2\left(\frac{5\int \frac{1}{(d\csc(e+fx))^{3/2}}dx}{7d^2}-\frac{2\cos(e+fx)}{7df(d\csc(e+fx))^{5/2}}\right)$$

$$\Big\downarrow \text{4256}$$

$$d^2\left(\frac{5\left(\frac{\int \sqrt{d\csc(e+fx)}dx}{3d^2}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}\right)}{7d^2}-\frac{2\cos(e+fx)}{7df(d\csc(e+fx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$d^2\left(\frac{5\left(\frac{\int\sqrt{d\csc(e+fx)}dx}{3d^2}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}\right)}{7d^2}-\frac{2\cos(e+fx)}{7df(d\csc(e+fx))^{5/2}}\right)$$

$\Big\downarrow$ 4258

$$d^2\left(\frac{5\left(\frac{\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}\int\frac{1}{\sqrt{\sin(e+fx)}}dx}{3d^2}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}\right)}{7d^2}-\frac{2\cos(e+fx)}{7df(d\csc(e+fx))^{5/2}}\right)$$

$\Big\downarrow$ 3042

$$d^2\left(\frac{5\left(\frac{\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}\int\frac{1}{\sqrt{\sin(e+fx)}}dx}{3d^2}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}\right)}{7d^2}-\frac{2\cos(e+fx)}{7df(d\csc(e+fx))^{5/2}}\right)$$

$\Big\downarrow$ 3120

$$d^2\left(\frac{5\left(\frac{2\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\csc(e+fx)}}{3d^2f}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}\right)}{7d^2}-\frac{2\cos(e+fx)}{7df(d\csc(e+fx))^{5/2}}\right)$$

input
```
Int[Sin[e + f*x]^2/(d*Csc[e + f*x])^(3/2),x]
```

output
```
d^2*((-2*Cos[e + f*x])/(7*d*f*(d*Csc[e + f*x])^(5/2)) + (5*((-2*Cos[e + f*
x])/(3*d*f*Sqrt[d*Csc[e + f*x]]) + (2*Sqrt[d*Csc[e + f*x]]*EllipticF[(e -
Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*d^2*f)))/(7*d^2))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.02 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.53

method	result
default	$\frac{\sqrt{2}\left(\sqrt{2}\left(3\cos(fx+e)^3-8\cos(fx+e)\right)+i\sqrt{1+i\cot(fx+e)-i\csc(fx+e)}\,\sqrt{1-i\cot(fx+e)+i\csc(fx+e)}\,\sqrt{i(\csc(fx+e)-\cot(fx+e)}}{21fd\sqrt{d\csc(fx+e)}}$

input
```
int(sin(f*x+e)^2/(d*csc(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/21/f*2^(1/2)/d/(d*csc(f*x+e))^(1/2)*(2^(1/2)*(3*cos(f*x+e)^3-8*cos(f*x+e
))+I*(1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2)*(1-I*cot(f*x+e)+I*csc(f*x+e))^(1/
2)*(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticF((1+I*cot(f*x+e)-I*csc(f*x+e
))^(1/2),1/2*2^(1/2))*(5*cot(f*x+e)+5*csc(f*x+e)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.95

$$\int \frac{\sin^2(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \frac{2\left(3\cos\left(fx+e\right)^3-8\cos\left(fx+e\right)\right)\sqrt{\frac{d}{\sin(fx+e)}}\sin\left(fx+e\right)-5i\sqrt{2i\,d}\text{weierstra}}{}$$

input
```
integrate(sin(f*x+e)^2/(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
1/21*(2*(3*cos(f*x + e)^3 - 8*cos(f*x + e))*sqrt(d/sin(f*x + e))*sin(f*x +
 e) - 5*I*sqrt(2*I*d)*weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x +
 e)) + 5*I*sqrt(-2*I*d)*weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x
 + e)))/(d^2*f)
```

Sympy [F]

$$\int \frac{\sin^2(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{\sin^2\left(e+fx\right)}{\left(d\csc\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)**2/(d*csc(f*x+e))**(3/2),x)
```

output
```
Integral(sin(e + f*x)**2/(d*csc(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sin^2(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{\sin\left(fx+e\right)^2}{\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^2/(d*csc(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)^2/(d*csc(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{\sin^2(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{\sin\left(fx+e\right)^2}{\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)^2/(d*csc(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(sin(f*x + e)^2/(d*csc(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{\sin\left(e+f\,x\right)^2}{\left(\frac{d}{\sin(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int(sin(e + f*x)^2/(d/sin(e + f*x))^(3/2),x)
```

output
```
int(sin(e + f*x)^2/(d/sin(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\sin^2(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\csc(fx+e)}\,\sin(fx+e)^2}{\csc(fx+e)^2}dx\right)}{d^2}$$

input
```
int(sin(f*x+e)^2/(d*csc(f*x+e))^(3/2),x)
```

output
```
(sqrt(d)*int((sqrt(csc(e + f*x))*sin(e + f*x)**2)/csc(e + f*x)**2,x))/d**2
```

3.531 $\int \frac{\sin(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 74

$$\int \frac{\sin(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = -\frac{2\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}} + \frac{6E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{5df\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$

output
```
-2/5*cos(f*x+e)/f/(d*csc(f*x+e))^(3/2)-6/5*EllipticE(cos(1/2*e+1/4*Pi+1/2*
f*x),2^(1/2))/d/f/(d*csc(f*x+e))^(1/2)/sin(f*x+e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.81

$$\int \frac{\sin(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \frac{-\frac{12E\left(\frac{1}{4}(-2e+\pi-2fx)|2\right)}{\sqrt{\sin(e+fx)}} - 2\sin(2(e+fx))}{10df\sqrt{d\csc(e+fx)}}$$

input
```
Integrate[Sin[e + f*x]/(d*Csc[e + f*x])^(3/2),x]
```

output
```
((-12*EllipticE[(-2*e + Pi - 2*f*x)/4, 2])/Sqrt[Sin[e + f*x]] - 2*Sin[2*(e
 + f*x)])/(10*d*f*Sqrt[d*Csc[e + f*x]])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3042, 2030, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(e+fx)}{(d\csc(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc(e+fx)(d\csc(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$d\int \frac{1}{(d\csc(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$d\left(\frac{3\int \frac{1}{\sqrt{d\csc(e+fx)}}dx}{5d^2}-\frac{2\cos(e+fx)}{5df(d\csc(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$d\left(\frac{3\int \frac{1}{\sqrt{d\csc(e+fx)}}dx}{5d^2}-\frac{2\cos(e+fx)}{5df(d\csc(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \text{4258}$$

$$d\left(\frac{3\int \sqrt{\sin(e+fx)}dx}{5d^2\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2\cos(e+fx)}{5df(d\csc(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$d\left(\frac{3\int \sqrt{\sin(e+fx)}dx}{5d^2\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2\cos(e+fx)}{5df(d\csc(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \text{3119}$$

$$d\left(\frac{6E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{5d^2f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2\cos(e+fx)}{5df(d\csc(e+fx))^{3/2}}\right)$$

input
```
Int[Sin[e + f*x]/(d*Csc[e + f*x])^(3/2),x]
```

output
```
d*((-2*Cos[e + f*x])/(5*d*f*(d*Csc[e + f*x])^(3/2)) + (6*EllipticE[(e - Pi
/2 + f*x)/2, 2])/(5*d^2*f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]]))
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 257, normalized size of antiderivative = 3.47

method	result
default	$\frac{\sqrt{2}\left((-6\cos(fx+e)-6)\sqrt{1+i\cot(fx+e)-i\csc(fx+e)}\,\sqrt{-i(-\csc(fx+e)+\cot(fx+e))}\ \mathrm{EllipticE}\left(\sqrt{1+i\cot(fx+e)-i\csc(fx+e)},\right.\right.}{}$

input
```
int(sin(f*x+e)/(d*csc(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/5/f*2^(1/2)*((-6*cos(f*x+e)-6)*(1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2)*(-I*(
-csc(f*x+e)+cot(f*x+e)))^(1/2)*EllipticE((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/
2),1/2*2^(1/2))*(1-I*cot(f*x+e)+I*csc(f*x+e))^(1/2)+(3*cos(f*x+e)+3)*(1+I*
cot(f*x+e)-I*csc(f*x+e))^(1/2)*(-I*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*(1-I*co
t(f*x+e)+I*csc(f*x+e))^(1/2)*EllipticF((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2)
,1/2*2^(1/2))+(cos(f*x+e)^3-4*cos(f*x+e)+3)*2^(1/2))/(d*csc(f*x+e))^(1/2)/
d*csc(f*x+e)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.30

$$\int \frac{\sin(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \frac{2\left(\cos\left(fx+e\right)^3-\cos\left(fx+e\right)\right)\sqrt{\frac{d}{\sin(fx+e)}}+3\sqrt{2i\,d}\mathrm{weierstrassZeta}(4,0,\mathrm{weier}}{}$$

input
```
integrate(sin(f*x+e)/(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
1/5*(2*(cos(f*x + e)^3 - cos(f*x + e))*sqrt(d/sin(f*x + e)) + 3*sqrt(2*I*d
)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x
 + e))) + 3*sqrt(-2*I*d)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, c
os(f*x + e) - I*sin(f*x + e))))/(d^2*f)
```

Sympy [F]

$$\int \frac{\sin(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{\sin(e+fx)}{(d\csc(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)/(d*csc(f*x+e))**(3/2),x)
```

output
```
Integral(sin(e + f*x)/(d*csc(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sin(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{\sin(fx+e)}{(d\csc(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)/(d*csc(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sin(f*x + e)/(d*csc(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{\sin(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{\sin(fx+e)}{(d\csc(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sin(f*x+e)/(d*csc(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(sin(f*x + e)/(d*csc(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{\sin\left(e+f\,x\right)}{{\left(\frac{d}{\sin\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

input
```
int(sin(e + f*x)/(d/sin(e + f*x))^(3/2),x)
```

output
```
int(sin(e + f*x)/(d/sin(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{\sin(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \frac{\sqrt{d}\,\left(\int \frac{\sqrt{\csc(fx+e)}\,\sin(fx+e)}{\csc(fx+e)^2}dx\right)}{d^2}$$

input
```
int(sin(f*x+e)/(d*csc(f*x+e))^(3/2),x)
```

output
```
(sqrt(d)*int((sqrt(csc(e + f*x))*sin(e + f*x))/csc(e + f*x)**2,x))/d**2
```

3.532 $\int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 77

$$\int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx = -\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}} + \frac{2\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{3d^2f}$$

output
```
-2/3*cos(f*x+e)/d/f/(d*csc(f*x+e))^(1/2)+2/3*(d*csc(f*x+e))^(1/2)*InverseJ
acobiAM(1/2*e-1/4*Pi+1/2*f*x,2^(1/2))*sin(f*x+e)^(1/2)/d^2/f
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.82

$$\int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx = -\frac{\csc^2(e+fx)\left(2\,\text{EllipticF}\left(\frac{1}{4}(-2e+\pi-2fx),2\right)\sqrt{\sin(e+fx)}+\sin(2(e+fx))\right)}{3f(d\csc(e+fx))^{3/2}}$$

input
```
Integrate[(d*Csc[e + f*x])^(-3/2),x]
```

output
```
-1/3*(Csc[e + f*x]^2*(2*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Sin[e + f
*x]] + Sin[2*(e + f*x)]))/(f*(d*Csc[e + f*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(d\csc(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{4256}$$

$$\frac{\int \sqrt{d\csc(e+fx)}dx}{3d^2}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{d\csc(e+fx)}dx}{3d^2}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}\int \frac{1}{\sqrt{\sin(e+fx)}}dx}{3d^2}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}\int \frac{1}{\sqrt{\sin(e+fx)}}dx}{3d^2}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\csc(e+fx)}}{3d^2f}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}$$

input
```
Int[(d*Csc[e + f*x])^(-3/2),x]
```

output
```
(-2*Cos[e + f*x])/(3*d*f*Sqrt[d*Csc[e + f*x]]) + (2*Sqrt[d*Csc[e + f*x]]*E
llipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*d^2*f)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.76 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.84

method	result
default	$\frac{\sqrt{2}\left(i\sqrt{i(\csc(fx+e)-\cot(fx+e))}\ \text{EllipticF}\left(\sqrt{1+i\cot(fx+e)-i\csc(fx+e)},\frac{\sqrt{2}}{2}\right)\sqrt{1-i\cot(fx+e)+i\csc(fx+e)}\ \sqrt{1+i\cot(fx+e)-}\right.}{3f\sqrt{d\csc(fx+e)}\,d}$

input
```
int(1/(d*csc(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/f*2^(1/2)/(d*csc(f*x+e))^(1/2)/d*(I*(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*
EllipticF((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2))*(1-I*cot(f*x+e)
+I*csc(f*x+e))^(1/2)*(1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2)*(cot(f*x+e)+csc(f
*x+e))-2^(1/2)*cos(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx = -\frac{2\sqrt{\frac{d}{\sin(fx+e)}}\cos(fx+e)\sin(fx+e)+i\sqrt{2i\,d}\,\text{weierstrassPInverse}(4,0,\cos(fx+e)+i\,\sin(fx+e))-}{3\,d^2f}$$

input
```
integrate(1/(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(2*sqrt(d/sin(f*x + e))*cos(f*x + e)*sin(f*x + e) + I*sqrt(2*I*d)*wei
erstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e)) - I*sqrt(-2*I*d)*wei
erstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e)))/(d^2*f)
```

Sympy [F]

$$\int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{1}{(d\csc(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*csc(f*x+e))**(3/2),x)
```

output
```
Integral((d*csc(e + f*x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{1}{(d\csc(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*csc(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*csc(f*x + e))^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{1}{(d\csc(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*csc(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*csc(f*x + e))^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{1}{\left(\frac{d}{\sin(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(d/sin(e + f*x))^(3/2),x)
```

output
```
int(1/(d/sin(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\csc(fx+e)}}{\csc(fx+e)^2}dx\right)}{d^2}$$

input
```
int(1/(d*csc(f*x+e))^(3/2),x)
```

output
```
(sqrt(d)*int(sqrt(csc(e + f*x))/csc(e + f*x)**2,x))/d**2
```

3.533 $\int \frac{\csc(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 46

$$\int \frac{\csc(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \frac{2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{df\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$

output
```
-2*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))/d/f/(d*csc(f*x+e))^(1/2)/s
in(f*x+e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{\csc(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = -\frac{2E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|2\right)}{df\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$

input
```
Integrate[Csc[e + f*x]/(d*Csc[e + f*x])^(3/2),x]
```

output
```
(-2*EllipticE[(-2*e + Pi - 2*f*x)/4, 2])/(d*f*Sqrt[d*Csc[e + f*x]]*Sqrt[Si
n[e + f*x]])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2030, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int \frac{1}{\sqrt{d\csc(e+fx)}}dx}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sqrt{d\csc(e+fx)}}dx}{d}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\int \sqrt{\sin(e+fx)}dx}{d\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{\sin(e+fx)}dx}{d\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|2\right)}{df\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

input
```
Int[Csc[e + f*x]/(d*Csc[e + f*x])^(3/2),x]
```

output
```
(2*EllipticE[(e - Pi/2 + f*x)/2, 2])/(d*f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e
+ f*x]])
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.78 (sec) , antiderivative size = 246, normalized size of antiderivative = 5.35

method	result
default	$-\frac{\sqrt{2}\left(2\sqrt{1+i(-\csc(fx+e)+\cot(fx+e))}\sqrt{1-i(-\csc(fx+e)+\cot(fx+e))}\sqrt{-i(-\csc(fx+e)+\cot(fx+e))}\,(\cos(fx+e)+1)\,\mathrm{Ellipt}\ldots\right.}{\ldots}$
risch	$-\frac{i\sqrt{2}}{fd\sqrt{\frac{id\,e^{i(fx+e)}}{e^{2i(fx+e)}-1}}}+\frac{i\left(-\frac{2i\left(id\,e^{2i(fx+e)}-id\right)}{d\sqrt{e^{i(fx+e)}\left(id\,e^{2i(fx+e)}-id\right)}}-\frac{\sqrt{e^{i(fx+e)}+1}\,\sqrt{-2\,e^{i(fx+e)}+2}\,\sqrt{-e^{i(fx+e)}}\,\left(-2\,\mathrm{EllipticE}\left(\sqrt{e^{i(fx+e)}+1},\ldots\right.\right.}{\sqrt{id\,e^{3i(fx+e)}-id\,e^{i(fx+e)}}}\ldots\right)}{fd\sqrt{\frac{id\,e^{i(fx+e)}}{e^{2i(fx+e)}-1}}\,\left(e^{2i(fx+e)}-1\right)}$

input
```
int(csc(f*x+e)/(d*csc(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/d/f*2^(1/2)*(2*(1+I*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*(1-I*(-csc(f*x+e)+c
ot(f*x+e)))^(1/2)*(-I*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*(cos(f*x+e)+1)*Ellip
ticE((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2))+(1+I*(-csc(f*x+e)+co
t(f*x+e)))^(1/2)*(1-I*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*(-I*(-csc(f*x+e)+cot
(f*x+e)))^(1/2)*(-cos(f*x+e)-1)*EllipticF((1+I*cot(f*x+e)-I*csc(f*x+e))^(1
/2),1/2*2^(1/2))+2^(1/2)*(-1+cos(f*x+e)))/(d*csc(f*x+e))^(1/2)*csc(f*x+e)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.35

$$\int \frac{\csc(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \frac{\sqrt{2i\,d}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(fx+e)+i\,\sin(fx+e$$

input
```
integrate(csc(f*x+e)/(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
(sqrt(2*I*d)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e)
+ I*sin(f*x + e))) + sqrt(-2*I*d)*weierstrassZeta(4, 0, weierstrassPInvers
e(4, 0, cos(f*x + e) - I*sin(f*x + e))))/(d^2*f)
```

Sympy [F]

$$\int \frac{\csc(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{\csc(e+fx)}{(d\csc(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)/(d*csc(f*x+e))**(3/2),x)
```

output
```
Integral(csc(e + f*x)/(d*csc(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\csc(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{\csc\left(fx+e\right)}{\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)/(d*csc(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(csc(f*x + e)/(d*csc(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{\csc(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{\csc\left(fx+e\right)}{\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)/(d*csc(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(csc(f*x + e)/(d*csc(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)\,{\left(\frac{d}{\sin(e+f\,x)}\right)}^{3/2}}\,dx$$

input
```
int(1/(sin(e + f*x)*(d/sin(e + f*x))^(3/2)),x)
```

output
```
int(1/(sin(e + f*x)*(d/sin(e + f*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{\csc(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \frac{\sqrt{d}\left(\int \frac{\sqrt{\csc(fx+e)}}{\csc(fx+e)}dx\right)}{d^2}$$

input
```
int(csc(f*x+e)/(d*csc(f*x+e))^(3/2),x)
```

output
```
(sqrt(d)*int(sqrt(csc(e + f*x))/csc(e + f*x),x))/d**2
```

3.534 $\int \frac{\csc^2(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 46

$$\int \frac{\csc^2(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \frac{2\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{d^2f}$$

output
```
2*(d*csc(f*x+e))^(1/2)*InverseJacobiAM(1/2*e-1/4*Pi+1/2*f*x,2^(1/2))*sin(f
*x+e)^(1/2)/d^2/f
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{\csc^2(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = -\frac{2\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{4}(-2e+\pi-2fx),2\right)\sqrt{\sin(e+fx)}}{d^2f}$$

input
```
Integrate[Csc[e + f*x]^2/(d*Csc[e + f*x])^(3/2),x]
```

output
```
(-2*Sqrt[d*Csc[e + f*x]]*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Sin[e +
f*x]])/(d^2*f)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2030, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(e+fx)}{(d\csc(e+fx))^{3/2}}dx$$
$$\Big\downarrow \text{2030}$$
$$\frac{\int \sqrt{d\csc(e+fx)}dx}{d^2}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \sqrt{d\csc(e+fx)}dx}{d^2}$$
$$\Big\downarrow \text{4258}$$
$$\frac{\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}\int \frac{1}{\sqrt{\sin(e+fx)}}dx}{d^2}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}\int \frac{1}{\sqrt{\sin(e+fx)}}dx}{d^2}$$
$$\Big\downarrow \text{3120}$$
$$\frac{2\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\csc(e+fx)}}{d^2f}$$

input
```
Int[Csc[e + f*x]^2/(d*Csc[e + f*x])^(3/2),x]
```

output
```
(2*Sqrt[d*Csc[e + f*x]]*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]
])/(d^2*f)
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.78

method	result
default	$\frac{i\sqrt{2}\,\sqrt{1+i\cot(fx+e)-i\csc(fx+e)}\,\sqrt{1-i\cot(fx+e)+i\csc(fx+e)}\,\sqrt{i(\csc(fx+e)-\cot(fx+e))}\,\operatorname{EllipticF}\left(\sqrt{1+i\cot(fx+e)-i\csc(\right.}{fd\sqrt{d\csc(fx+e)}}$

input
```
int(csc(f*x+e)^2/(d*csc(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
I/f*2^(1/2)*(1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2)*(1-I*cot(f*x+e)+I*csc(f*x+
e))^(1/2)*(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticF((1+I*cot(f*x+e)-I*cs
c(f*x+e))^(1/2),1/2*2^(1/2))/d/(d*csc(f*x+e))^(1/2)*(cot(f*x+e)+csc(f*x+e)
)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int \frac{\csc^2(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \frac{-i\sqrt{2i\,d}\text{weierstrassPInverse}(4,0,\cos(fx+e)+i\,\sin(fx+e))+i\sqrt{-2i\,d}\text{weie}}{d^2f}$$

input
```
integrate(csc(f*x+e)^2/(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2*I*d)*weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e)) +
 I*sqrt(-2*I*d)*weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x + e)))/
(d^2*f)
```

Sympy [F]

$$\int \frac{\csc^2(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{\csc^2(e+fx)}{(d\csc(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**2/(d*csc(f*x+e))**(3/2),x)
```

output
```
Integral(csc(e + f*x)**2/(d*csc(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\csc^2(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{\csc(fx+e)^2}{(d\csc(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^2/(d*csc(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(csc(f*x + e)^2/(d*csc(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{\csc^2(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{\csc(fx+e)^2}{(d\csc(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^2/(d*csc(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(csc(f*x + e)^2/(d*csc(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^2(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{1}{\sin(e+f\,x)^2\left(\frac{d}{\sin(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(sin(e + f*x)^2*(d/sin(e + f*x))^(3/2)),x)
```

output
```
int(1/(sin(e + f*x)^2*(d/sin(e + f*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{\csc^2(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \frac{\sqrt{d}\left(\int \sqrt{\csc(fx+e)}dx\right)}{d^2}$$

input
```
int(csc(f*x+e)^2/(d*csc(f*x+e))^(3/2),x)
```

output
```
(sqrt(d)*int(sqrt(csc(e + f*x)),x))/d**2
```

3.535 $\int \frac{\csc^3(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 73

$$\int \frac{\csc^3(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = -\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{d^2f} - \frac{2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{df\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$

output
```
-2*cos(f*x+e)*(d*csc(f*x+e))^(1/2)/d^2/f+2*EllipticE(cos(1/2*e+1/4*Pi+1/2*
f*x),2^(1/2))/d/f/(d*csc(f*x+e))^(1/2)/sin(f*x+e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.75

$$\int \frac{\csc^3(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \frac{-2\cot(e+fx)+\frac{2E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|2\right)}{\sqrt{\sin(e+fx)}}}{df\sqrt{d\csc(e+fx)}}$$

input
```
Integrate[Csc[e + f*x]^3/(d*Csc[e + f*x])^(3/2),x]
```

output
```
(-2*Cot[e + f*x] + (2*EllipticE[(-2*e + Pi - 2*f*x)/4, 2])/Sqrt[Sin[e + f*
x]])/(d*f*Sqrt[d*Csc[e + f*x]])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2030, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(e+fx)}{(d\csc(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (d\csc(e+fx))^{3/2}dx}{d^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (d\csc(e+fx))^{3/2}dx}{d^3}$$

$$\Big\downarrow \text{4255}$$

$$\frac{d^2\left(-\int \frac{1}{\sqrt{d\csc(e+fx)}}dx\right)-\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}}{d^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d^2\left(-\int \frac{1}{\sqrt{d\csc(e+fx)}}dx\right)-\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}}{d^3}$$

$$\Big\downarrow \text{4258}$$

$$\frac{-\frac{d^2\int \sqrt{\sin(e+fx)}dx}{\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}}{d^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{d^2\int \sqrt{\sin(e+fx)}dx}{\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}}{d^3}$$

$$\Big\downarrow \text{3119}$$

$$\frac{-\frac{2d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}}{d^3}$$

input
```
Int[Csc[e + f*x]^3/(d*Csc[e + f*x])^(3/2),x]
```

output
```
((-2*d*Cos[e + f*x]*Sqrt[d*Csc[e + f*x]])/f - (2*d^2*EllipticE[(e - Pi/2 +
 f*x)/2, 2])/(f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]]))/d^3
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.29

method	result
default	$-\frac{\sqrt{2}\left(\sqrt{2}-\left(2\,\mathrm{EllipticE}\left(\sqrt{1+i\cot(fx+e)-i\csc(fx+e)},\frac{\sqrt{2}}{2}\right)-\mathrm{EllipticF}\left(\sqrt{1+i\cot(fx+e)-i\csc(fx+e)},\frac{\sqrt{2}}{2}\right)\right)\sqrt{1+i(-\csc(fx+e}{f\sqrt{d\csc(fx+e)}\,d}$

input
```
int(csc(f*x+e)^3/(d*csc(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/f*2^(1/2)*(2^(1/2)-(2*EllipticE((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2
*2^(1/2))-EllipticF((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2)))*(1+I
*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*(1-I*(-csc(f*x+e)+cot(f*x+e)))^(1/2)*(-I*
(-csc(f*x+e)+cot(f*x+e)))^(1/2)*(cos(f*x+e)+1))/(d*csc(f*x+e))^(1/2)/d*csc
(f*x+e)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.14

$$\int \frac{\csc^3(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx =$$
$$-\frac{2\sqrt{\frac{d}{\sin(fx+e)}}\cos(fx+e)+\sqrt{2i\,d}\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(fx+e)+i\,\sin(fx}{d^2f}$$

input
```
integrate(csc(f*x+e)^3/(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
-(2*sqrt(d/sin(f*x + e))*cos(f*x + e) + sqrt(2*I*d)*weierstrassZeta(4, 0,
weierstrassPInverse(4, 0, cos(f*x + e) + I*sin(f*x + e))) + sqrt(-2*I*d)*w
eierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) - I*sin(f*x +
e))))/(d^2*f)
```

Sympy [F]

$$\int \frac{\csc^3(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{\csc^3(e+fx)}{(d\csc(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**3/(d*csc(f*x+e))**(3/2),x)
```

output
```
Integral(csc(e + f*x)**3/(d*csc(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\csc^3(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{\csc(fx+e)^3}{(d\csc(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^3/(d*csc(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(csc(f*x + e)^3/(d*csc(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{\csc^3(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{\csc(fx+e)^3}{(d\csc(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^3/(d*csc(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(csc(f*x + e)^3/(d*csc(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^3(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)^3\,{\left(\frac{d}{\sin(e+f\,x)}\right)}^{3/2}}\,dx$$

input
```
int(1/(sin(e + f*x)^3*(d/sin(e + f*x))^(3/2)),x)
```

output
```
int(1/(sin(e + f*x)^3*(d/sin(e + f*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{\csc^3(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \frac{\sqrt{d}\left(\int \sqrt{\csc\left(fx+e\right)}\,\csc\left(fx+e\right)dx\right)}{d^2}$$

input
```
int(csc(f*x+e)^3/(d*csc(f*x+e))^(3/2),x)
```

output
```
(sqrt(d)*int(sqrt(csc(e + f*x))*csc(e + f*x),x))/d**2
```

3.536 $\int \frac{\csc^4(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 77

$$\int \frac{\csc^4(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx=-\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3d^3f}+\frac{2\sqrt{d\csc(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right),2\right)\sqrt{\sin(e+fx)}}{3d^2f}$$

output
```
-2/3*cos(f*x+e)*(d*csc(f*x+e))^(3/2)/d^3/f+2/3*(d*csc(f*x+e))^(1/2)*Invers
eJacobiAM(1/2*e-1/4*Pi+1/2*f*x,2^(1/2))*sin(f*x+e)^(1/2)/d^2/f
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.78

$$\int \frac{\csc^4(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx=-\frac{2\csc^3(e+fx)\left(\cos(e+fx)+\text{EllipticF}\left(\frac{1}{4}(-2e+\pi-2fx),2\right)\sin^{\frac{3}{2}}(e+fx)\right)}{3f(d\csc(e+fx))^{3/2}}$$

input
```
Integrate[Csc[e + f*x]^4/(d*Csc[e + f*x])^(3/2),x]
```

output
```
(-2*Csc[e + f*x]^3*(Cos[e + f*x] + EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sin
[e + f*x]^(3/2)))/(3*f*(d*Csc[e + f*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2030, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(e+fx)}{(d\csc(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (d\csc(e+fx))^{5/2}dx}{d^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (d\csc(e+fx))^{5/2}dx}{d^4}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{1}{3}d^2\int \sqrt{d\csc(e+fx)}dx-\frac{2d\cos(e+fx)(d\csc(e+fx))^{3/2}}{3f}}{d^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{3}d^2\int \sqrt{d\csc(e+fx)}dx-\frac{2d\cos(e+fx)(d\csc(e+fx))^{3/2}}{3f}}{d^4}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{1}{3}d^2\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}\int \frac{1}{\sqrt{\sin(e+fx)}}dx-\frac{2d\cos(e+fx)(d\csc(e+fx))^{3/2}}{3f}}{d^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{3}d^2\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}\int \frac{1}{\sqrt{\sin(e+fx)}}dx-\frac{2d\cos(e+fx)(d\csc(e+fx))^{3/2}}{3f}}{d^4}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{2d^2\sqrt{\sin(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right),2\right)\sqrt{d\csc(e+fx)}}{3f}-\frac{2d\cos(e+fx)(d\csc(e+fx))^{3/2}}{3f}}{d^4}$$

input
```
Int[Csc[e + f*x]^4/(d*Csc[e + f*x])^(3/2),x]
```

output
```
((-2*d*Cos[e + f*x]*(d*Csc[e + f*x])^(3/2))/(3*f) + (2*d^2*Sqrt[d*Csc[e +
f*x]]*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*f))/d^4
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.92

method	result
default	$\frac{\sqrt{2}\left(i\sqrt{i(\csc(fx+e)-\cot(fx+e))}\ \mathrm{EllipticF}\left(\sqrt{1+i\cot(fx+e)-i\csc(fx+e)},\frac{\sqrt{2}}{2}\right)\sqrt{1-i\cot(fx+e)+i\csc(fx+e)}\ \sqrt{1+i\cot(fx+e)-}\right.}{3fd\sqrt{d\csc(fx+e)}}$

input
```
int(csc(f*x+e)^4/(d*csc(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/f*2^(1/2)/d/(d*csc(f*x+e))^(1/2)*(I*(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*
EllipticF((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2),1/2*2^(1/2))*(1-I*cot(f*x+e)
+I*csc(f*x+e))^(1/2)*(1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2)*(cot(f*x+e)+csc(f
*x+e))-2^(1/2)*cot(f*x+e)*csc(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.29

$$\int \frac{\csc^4(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \frac{-i\sqrt{2i\,d}\sin(fx+e)\ \mathrm{weierstrassPInverse}(4,0,\cos(fx+e)+i\ \sin(fx+e))+}{}$$

input
```
integrate(csc(f*x+e)^4/(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2*I*d)*sin(f*x + e)*weierstrassPInverse(4, 0, cos(f*x + e) +
I*sin(f*x + e)) + I*sqrt(-2*I*d)*sin(f*x + e)*weierstrassPInverse(4, 0, co
s(f*x + e) - I*sin(f*x + e)) - 2*sqrt(d/sin(f*x + e))*cos(f*x + e))/(d^2*f
*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\csc^4(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{\csc^4(e+fx)}{(d\csc(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**4/(d*csc(f*x+e))**(3/2),x)
```

output
```
Integral(csc(e + f*x)**4/(d*csc(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\csc^4(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{\csc(fx+e)^4}{(d\csc(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^4/(d*csc(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(csc(f*x + e)^4/(d*csc(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{\csc^4(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{\csc(fx+e)^4}{(d\csc(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^4/(d*csc(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(csc(f*x + e)^4/(d*csc(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^4(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{1}{\sin(e+f\,x)^4\,\left(\frac{d}{\sin(e+f\,x)}\right)^{3/2}}\,dx$$

input
```
int(1/(sin(e + f*x)^4*(d/sin(e + f*x))^(3/2)),x)
```

output
```
int(1/(sin(e + f*x)^4*(d/sin(e + f*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{\csc^4(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \frac{\sqrt{d}\left(\int \sqrt{\csc(fx+e)}\,\csc(fx+e)^2\,dx\right)}{d^2}$$

input
```
int(csc(f*x+e)^4/(d*csc(f*x+e))^(3/2),x)
```

output
```
(sqrt(d)*int(sqrt(csc(e + f*x))*csc(e + f*x)**2,x))/d**2
```

3.537 $\int \frac{\csc^5(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 105

$$\int \frac{\csc^5(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = -\frac{6\cos(e+fx)\sqrt{d\csc(e+fx)}}{5d^2f} - \frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5d^4f} - \frac{6E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|\,2\right)}{5df\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$

output
```
-6/5*cos(f*x+e)*(d*csc(f*x+e))^(1/2)/d^2/f-2/5*cos(f*x+e)*(d*csc(f*x+e))^(
5/2)/d^4/f+6/5*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))/d/f/(d*csc(f*x
+e))^(1/2)/sin(f*x+e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.70

$$\int \frac{\csc^5(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \frac{\csc^4(e+fx)\left(-7\cos(e+fx)+3\cos(3(e+fx))+12E\left(\frac{1}{4}(-2e+\pi-2fx)\middle|\,2\right)\right.}{10f(d\csc(e+fx))^{3/2}}$$

input
```
Integrate[Csc[e + f*x]^5/(d*Csc[e + f*x])^(3/2),x]
```

output
```
(Csc[e + f*x]^4*(-7*Cos[e + f*x] + 3*Cos[3*(e + f*x)] + 12*EllipticE[(-2*e
 + Pi - 2*f*x)/4, 2]*Sin[e + f*x]^(5/2)))/(10*f*(d*Csc[e + f*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2030, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^5(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{2030}$$

$$\frac{\int (d\csc(e+fx))^{7/2}dx}{d^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (d\csc(e+fx))^{7/2}dx}{d^5}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{3}{5}d^2\int (d\csc(e+fx))^{3/2}dx-\frac{2d\cos(e+fx)(d\csc(e+fx))^{5/2}}{5f}}{d^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{5}d^2\int (d\csc(e+fx))^{3/2}dx-\frac{2d\cos(e+fx)(d\csc(e+fx))^{5/2}}{5f}}{d^5}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{3}{5}d^2\left(d^2\left(-\int \frac{1}{\sqrt{d\csc(e+fx)}}dx\right)-\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}\right)-\frac{2d\cos(e+fx)(d\csc(e+fx))^{5/2}}{5f}}{d^5}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{5}d^2\left(d^2\left(-\int \frac{1}{\sqrt{d\csc(e+fx)}}dx\right)-\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}\right)-\frac{2d\cos(e+fx)(d\csc(e+fx))^{5/2}}{5f}}{d^5}$$

$$\Big\downarrow \textcolor{blue}{4258}$$

$$\frac{\frac{3}{5}d^2\left(-\frac{d^2\int\sqrt{\sin(e+fx)}dx}{\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}\right)-\frac{2d\cos(e+fx)(d\csc(e+fx))^{5/2}}{5f}}{d^5}$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$\frac{\frac{3}{5}d^2\left(-\frac{d^2\int\sqrt{\sin(e+fx)}dx}{\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}\right)-\frac{2d\cos(e+fx)(d\csc(e+fx))^{5/2}}{5f}}{d^5}$$

$$\Big\downarrow \textcolor{blue}{3119}$$

$$\frac{\frac{3}{5}d^2\left(-\frac{2d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}\right)-\frac{2d\cos(e+fx)(d\csc(e+fx))^{5/2}}{5f}}{d^5}$$

input
```
Int[Csc[e + f*x]^5/(d*Csc[e + f*x])^(3/2),x]
```

output
```
((-2*d*Cos[e + f*x]*(d*Csc[e + f*x])^(5/2))/(5*f) + (3*d^2*((-2*d*Cos[e +
f*x]*Sqrt[d*Csc[e + f*x]])/f - (2*d^2*EllipticE[(e - Pi/2 + f*x)/2, 2])/(f
*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])))/5)/d^5
```

Defintions of rubi rules used

rule 2030
```
Int[(Fx_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Simp[1/b^m   Int[(b*v)
^(m + n)*Fx, x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.59

method	result
default	$\frac{\sqrt{2}\left(\sqrt{1+i\cot(fx+e)-i\csc(fx+e)}\,\sqrt{i(\csc(fx+e)-\cot(fx+e))}\ \text{EllipticE}\left(\sqrt{1+i\cot(fx+e)-i\csc(fx+e)}, \frac{\sqrt{2}}{2}\right)\sqrt{1-i\cot(fx+e)+}\right.}{}$

input
```
int(csc(f*x+e)^5/(d*csc(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/5/f*2^(1/2)/d/(d*csc(f*x+e))^(1/2)*((1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2)*
(I*(csc(f*x+e)-cot(f*x+e)))^(1/2)*EllipticE((1+I*cot(f*x+e)-I*csc(f*x+e))^
(1/2),1/2*2^(1/2))*(1-I*cot(f*x+e)+I*csc(f*x+e))^(1/2)*(6*cot(f*x+e)+6*csc
(f*x+e))+(1+I*cot(f*x+e)-I*csc(f*x+e))^(1/2)*(I*(csc(f*x+e)-cot(f*x+e)))^(
1/2)*(1-I*cot(f*x+e)+I*csc(f*x+e))^(1/2)*EllipticF((1+I*cot(f*x+e)-I*csc(f
*x+e))^(1/2),1/2*2^(1/2))*(-3*cot(f*x+e)-3*csc(f*x+e))+2^(1/2)*(-3*csc(f*x
+e)-cot(f*x+e)*csc(f*x+e)^2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.28

$$\int \frac{\csc^5(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = -\frac{3\left(\cos\left(fx+e\right)^2-1\right)\sqrt{2i\,d}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)))}{}$$

input
```
integrate(csc(f*x+e)^5/(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
-1/5*(3*(cos(f*x + e)^2 - 1)*sqrt(2*I*d)*weierstrassZeta(4, 0, weierstrass
PInverse(4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*(cos(f*x + e)^2 - 1)*sq
rt(-2*I*d)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(f*x + e) -
I*sin(f*x + e))) + 2*(3*cos(f*x + e)^3 - 4*cos(f*x + e))*sqrt(d/sin(f*x +
e)))/(d^2*f*cos(f*x + e)^2 - d^2*f)
```

Sympy [F]

$$\int \frac{\csc^5(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{\csc^5\left(e+fx\right)}{\left(d\csc\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)**5/(d*csc(f*x+e))**(3/2),x)
```

output
```
Integral(csc(e + f*x)**5/(d*csc(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\csc^5(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{\csc\left(fx+e\right)^5}{\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^5/(d*csc(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(csc(f*x + e)^5/(d*csc(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{\csc^5(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{\csc\left(fx+e\right)^5}{\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(f*x+e)^5/(d*csc(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(csc(f*x + e)^5/(d*csc(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^5(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \int \frac{1}{\sin\left(e+f\,x\right)^5\,{\left(\frac{d}{\sin(e+f\,x)}\right)}^{3/2}}\,dx$$

input
```
int(1/(sin(e + f*x)^5*(d/sin(e + f*x))^(3/2)),x)
```

output
```
int(1/(sin(e + f*x)^5*(d/sin(e + f*x))^(3/2)), x)
```

Reduce [F]

$$\int \frac{\csc^5(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \frac{\sqrt{d}\left(\int \sqrt{\csc\left(fx+e\right)}\,\csc\left(fx+e\right)^3 dx\right)}{d^2}$$

input
```
int(csc(f*x+e)^5/(d*csc(f*x+e))^(3/2),x)
```

output
```
(sqrt(d)*int(sqrt(csc(e + f*x))*csc(e + f*x)**3,x))/d**2
```

3.538 $\int (b\csc(e+fx))^n(a\sin(e+fx))^m\,dx$

Optimal result

Integrand size = 21, antiderivative size = 87

$$\int (b\csc(e+fx))^n(a\sin(e+fx))^m\,dx = \frac{\cos(e+fx)(b\csc(e+fx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+m-n),\frac{1}{2}(3+m-n),\sin^2(e+fx)\right)(a\sin(e}{af(1+m-n)\sqrt{\cos^2(e+fx)}}$$

output
```
cos(f*x+e)*(b*csc(f*x+e))^n*hypergeom([1/2, 1/2+1/2*m-1/2*n],[3/2+1/2*m-1/
2*n],sin(f*x+e)^2)*(a*sin(f*x+e))^(1+m)/a/f/(1+m-n)/(cos(f*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 12.04 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.17

$$\int (b\csc(e+fx))^n(a\sin(e+fx))^m\,dx = \frac{2(b\csc(e+fx))^n \text{Hypergeometric2F1}\left(\frac{1}{2}(1+m-n),1+m-n,\frac{1}{2}(3+m-n),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\text{se}}{f(1+m-n)}$$

input
```
Integrate[(b*Csc[e + f*x])^n*(a*Sin[e + f*x])^m,x]
```

output
```
(2*(b*Csc[e + f*x])^n*Hypergeometric2F1[(1 + m - n)/2, 1 + m - n, (3 + m -
 n)/2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(m - n)*(a*Sin[e + f*x])^
m*Tan[(e + f*x)/2])/(f*(1 + m - n))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3068, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx))^m(b\csc(e+fx))^n\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a\sin(e+fx))^m(b\csc(e+fx))^n dx$$

$$\Big\downarrow \text{ 3068}$$

$$(a\sin(e+fx))^n(b\csc(e+fx))^n\int (a\sin(e+fx))^{m-n}dx$$

$$\Big\downarrow \text{ 3042}$$

$$(a\sin(e+fx))^n(b\csc(e+fx))^n\int (a\sin(e+fx))^{m-n}dx$$

$$\Big\downarrow \text{ 3122}$$

$$\frac{\cos(e+fx)(a\sin(e+fx))^{m+1}(b\csc(e+fx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(m-n+1),\frac{1}{2}(m-n+3),\sin^2(e+fx\right)}{af(m-n+1)\sqrt{\cos^2(e+fx)}}$$

input
```
Int[(b*Csc[e + f*x])^n*(a*Sin[e + f*x])^m,x]
```

output
```
(Cos[e + f*x]*(b*Csc[e + f*x])^n*Hypergeometric2F1[1/2, (1 + m - n)/2, (3
+ m - n)/2, Sin[e + f*x]^2]*(a*Sin[e + f*x])^(1 + m))/(a*f*(1 + m - n)*Sqr
t[Cos[e + f*x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3068
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[(a*b)^IntPart[n]*(a*Sin[e + f*x])^FracPart[n]*(b*Csc
[e + f*x])^FracPart[n]   Int[(a*Sin[e + f*x])^(m - n), x], x] /; FreeQ[{a,
b, e, f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Maple [F]

$$\int (b\csc(fx+e))^n (a\sin(fx+e))^m dx$$

input
```
int((b*csc(f*x+e))^n*(a*sin(f*x+e))^m,x)
```

output
```
int((b*csc(f*x+e))^n*(a*sin(f*x+e))^m,x)
```

Fricas [F]

$$\int (b\csc(e+fx))^n(a\sin(e+fx))^m\,dx = \int (b\csc(fx+e))^n\,(a\sin(fx+e))^m\,dx$$

input
```
integrate((b*csc(f*x+e))^n*(a*sin(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((b*csc(f*x + e))^n*(a*sin(f*x + e))^m, x)
```

Sympy [F]

$$\int (b\csc(e+fx))^n(a\sin(e+fx))^m\,dx = \int (a\sin(e+fx))^m\,(b\csc(e+fx))^n\,dx$$

input
```
integrate((b*csc(f*x+e))**n*(a*sin(f*x+e))**m,x)
```

output
```
Integral((a*sin(e + f*x))**m*(b*csc(e + f*x))**n, x)
```

Maxima [F]

$$\int (b\csc(e+fx))^n(a\sin(e+fx))^m\,dx = \int (b\csc(fx+e))^n\,(a\sin(fx+e))^m\,dx$$

input
```
integrate((b*csc(f*x+e))^n*(a*sin(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((b*csc(f*x + e))^n*(a*sin(f*x + e))^m, x)
```

Giac [F]

$$\int (b\csc(e+fx))^n(a\sin(e+fx))^m\,dx = \int (b\csc(fx+e))^n\,(a\sin(fx+e))^m\,dx$$

input
```
integrate((b*csc(f*x+e))^n*(a*sin(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((b*csc(f*x + e))^n*(a*sin(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\csc(e+fx))^n(a\sin(e+fx))^m\,dx=\int (a\,\sin\left(e+f\,x\right))^m\,\left(\frac{b}{\sin\left(e+f\,x\right)}\right)^n\,dx$$

input
```
int((a*sin(e + f*x))^m*(b/sin(e + f*x))^n,x)
```

output
```
int((a*sin(e + f*x))^m*(b/sin(e + f*x))^n, x)
```

Reduce [F]

$$\int (b\csc(e+fx))^n(a\sin(e+fx))^m\,dx=b^na^m\left(\int \sin\left(fx+e\right)^m\csc\left(fx+e\right)^n dx\right)$$

input
```
int((b*csc(f*x+e))^n*(a*sin(f*x+e))^m,x)
```

output
```
b**n*a**m*int(sin(e + f*x)**m*csc(e + f*x)**n,x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1]))  #max(3,ExpnType(op(1,expn)),Expn
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file