Computer Algebra Independent Integration Tests

Summer 2024

4-Trig-functions/4.1-Sine/185-4.1.3.1

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**358**]. This is test number [185].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.44 (356)	0.56 (2)
Mathematica	93.30 (334)	6.70 (24)
Maple	82.68 (296)	17.32 (62)
Fricas	76.82 (275)	23.18 (83)
Giac	56.15 (201)	43.85 (157)
Mupad	49.72 (178)	50.28 (180)
Maxima	37.15 (133)	62.85 (225)
Reduce	36.03 (129)	63.97 (229)
Sympy	28.49 (102)	71.51 (256)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

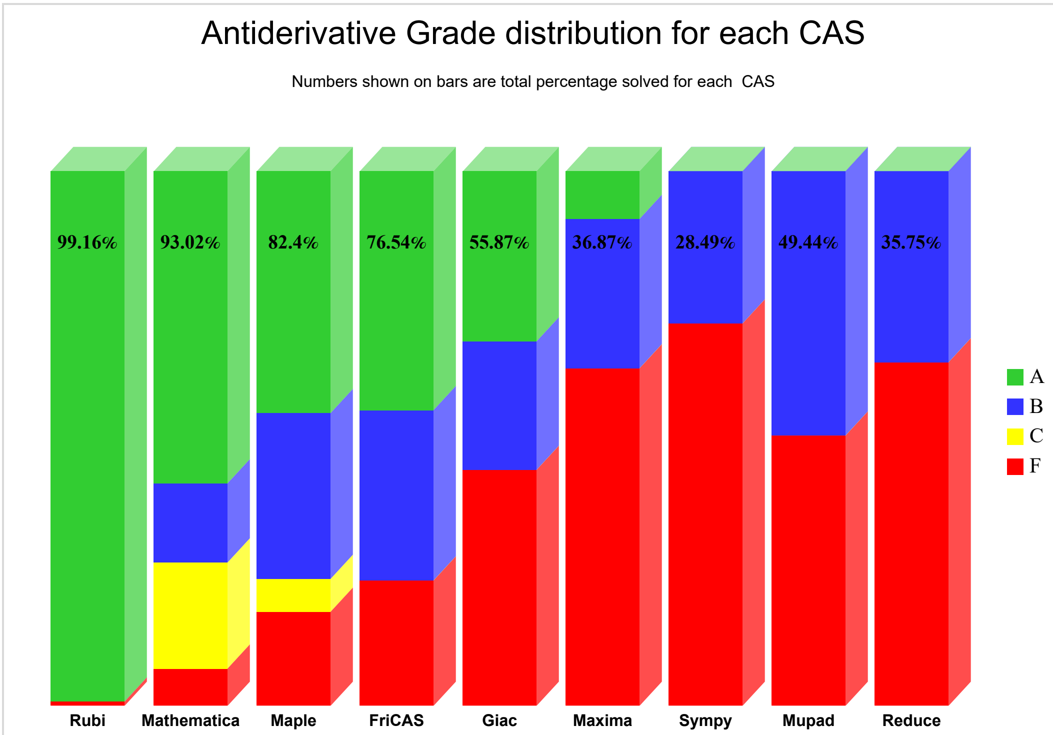
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

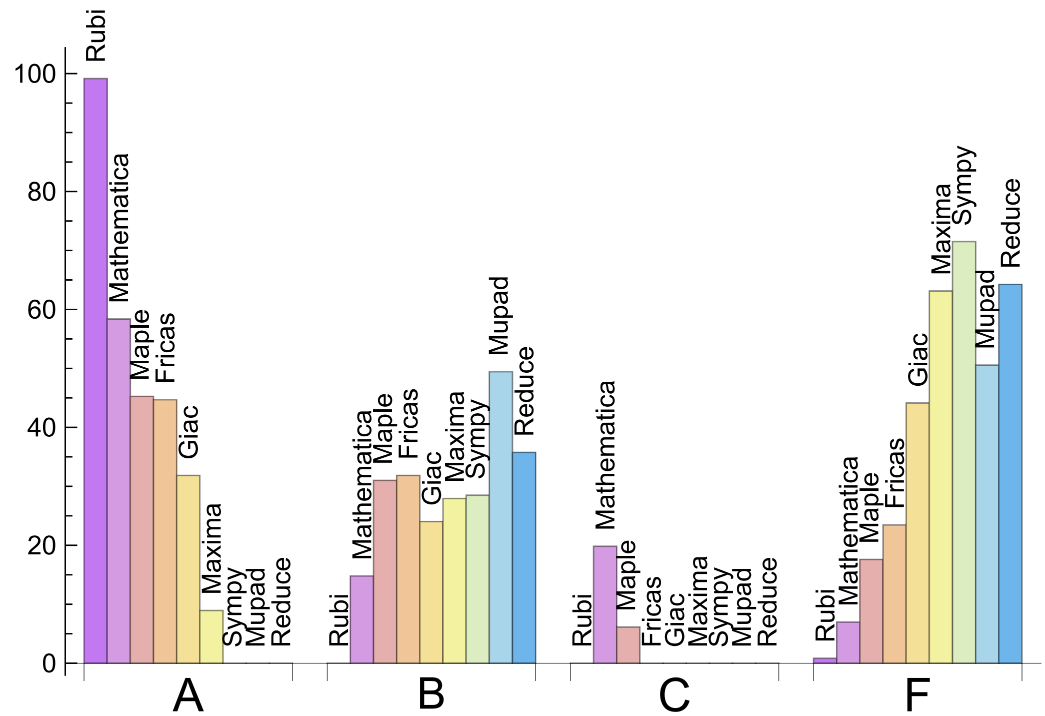
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.162	0.000	0.000	0.838
Mathematica	58.380	14.804	19.832	6.983
Maple	45.251	31.006	6.145	17.598
Fricas	44.693	31.844	0.000	23.464
Giac	31.844	24.022	0.000	44.134
Maxima	8.939	27.933	0.000	63.128
Mupad	0.000	49.441	0.000	50.559
Reduce	0.000	35.754	0.000	64.246
Sympy	0.000	28.492	0.000	71.508

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	2	100.00	0.00	0.00
Mathematica	24	100.00	0.00	0.00
Maple	62	100.00	0.00	0.00
Fricas	83	98.80	1.20	0.00
Giac	157	27.39	5.73	66.88
Mupad	180	0.00	100.00	0.00
Maxima	225	85.78	5.33	8.89
Reduce	229	100.00	0.00	0.00
Sympy	256	41.80	57.81	0.39

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Reduce	0.21
Maxima	0.30
Fricas	0.34
Rubi	0.89
Giac	1.55
Mathematica	8.23
Maple	12.96
Sympy	16.31
Mupad	37.74

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	177.22	0.99	150.00	1.00
Mathematica	330.40	1.75	204.00	1.36
Fricas	527.74	2.59	234.00	1.77
Giac	588.38	10.32	247.00	1.67
Reduce	659.55	3.09	262.00	1.94
Maxima	782.45	5.48	506.00	4.15
Mupad	833.52	4.61	298.50	2.09
Sympy	2842.08	19.47	1616.50	14.42
Maple	8451.71	13.03	223.50	1.47

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

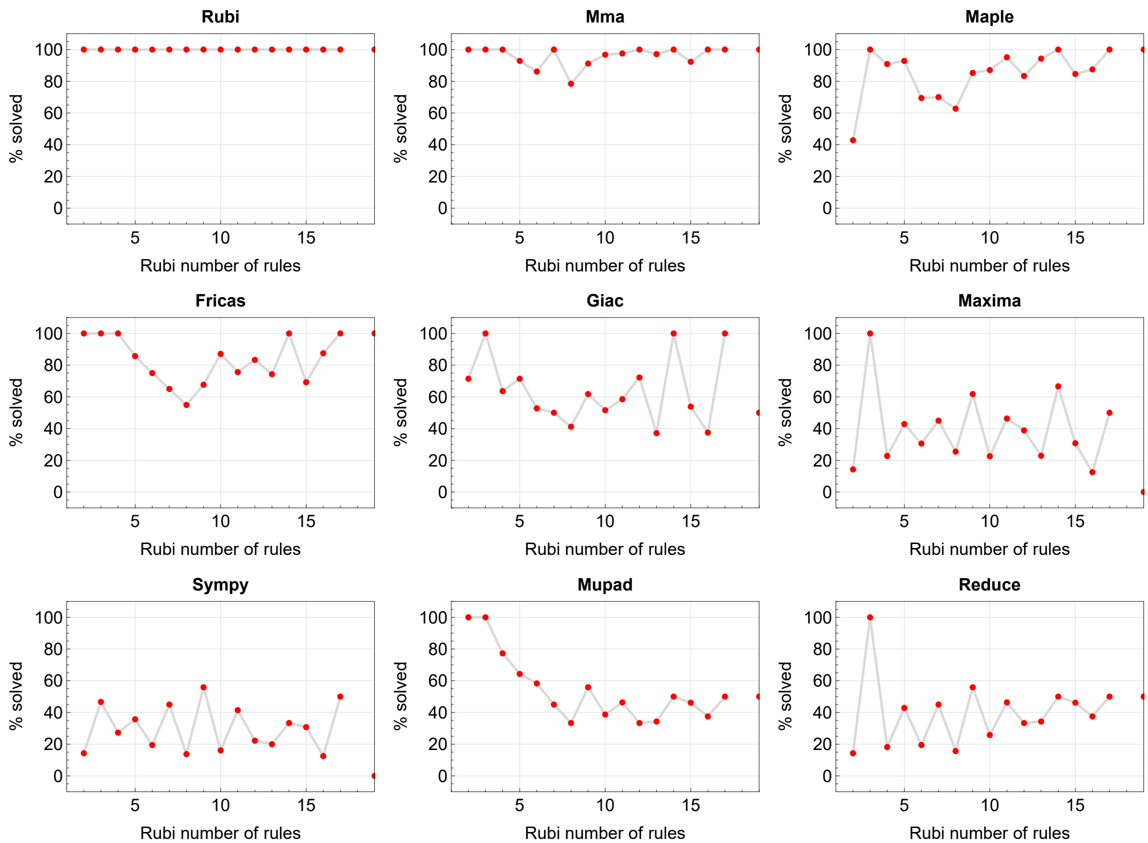


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

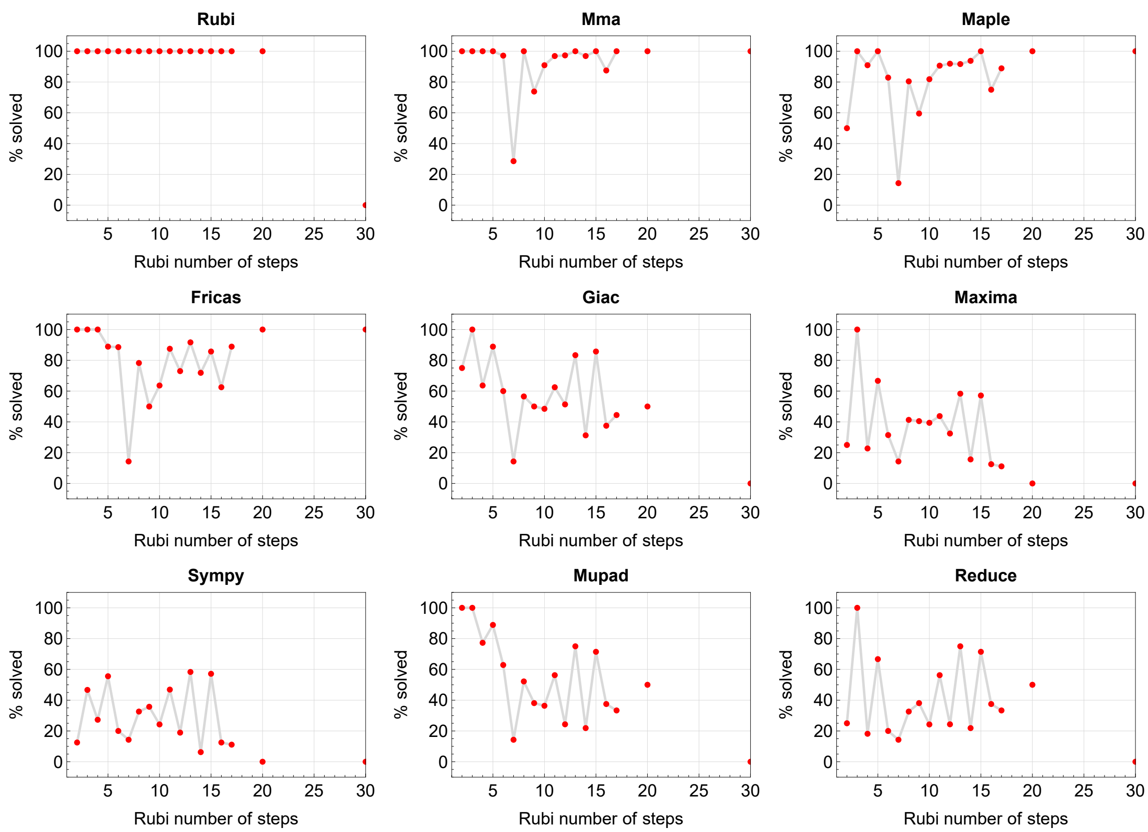


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

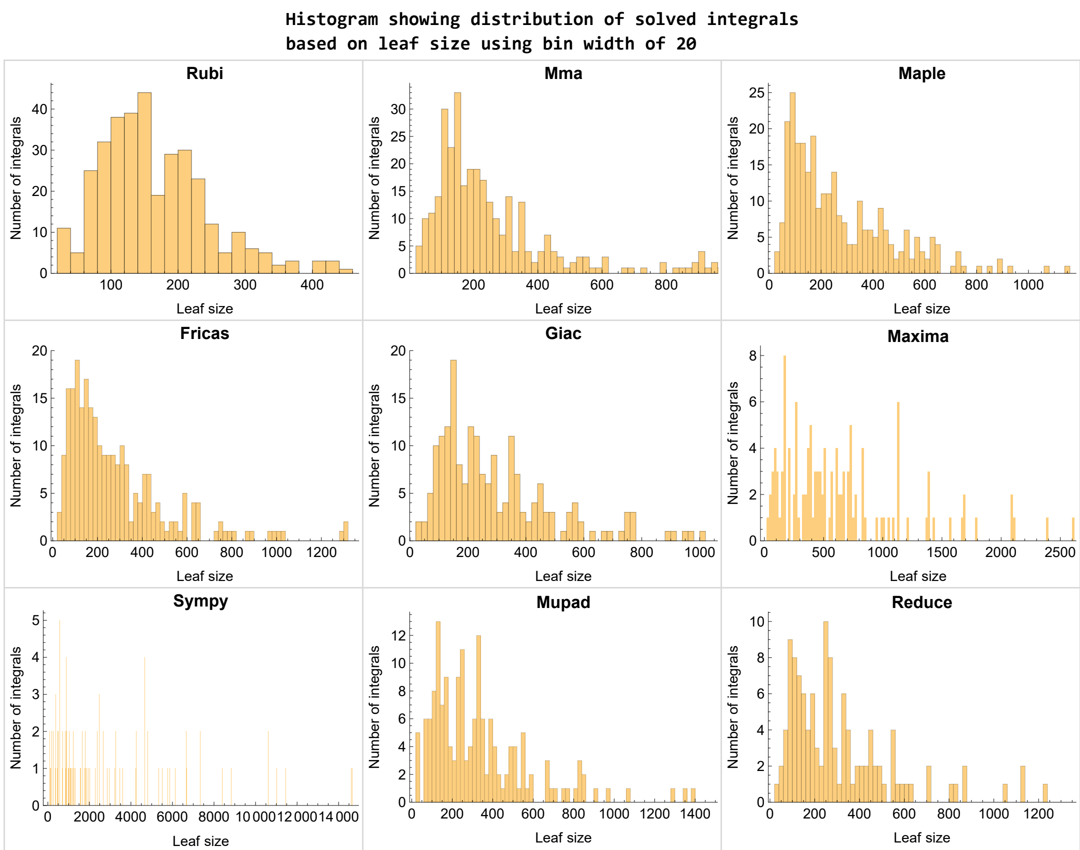


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

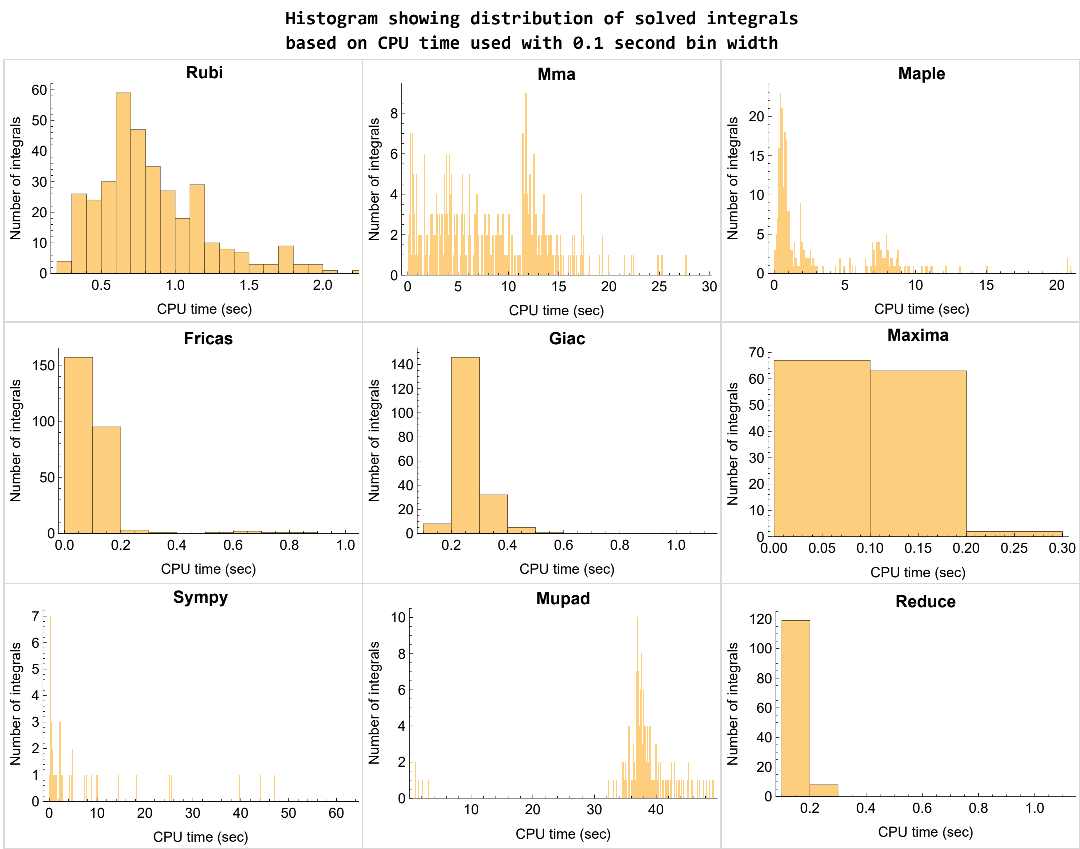


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

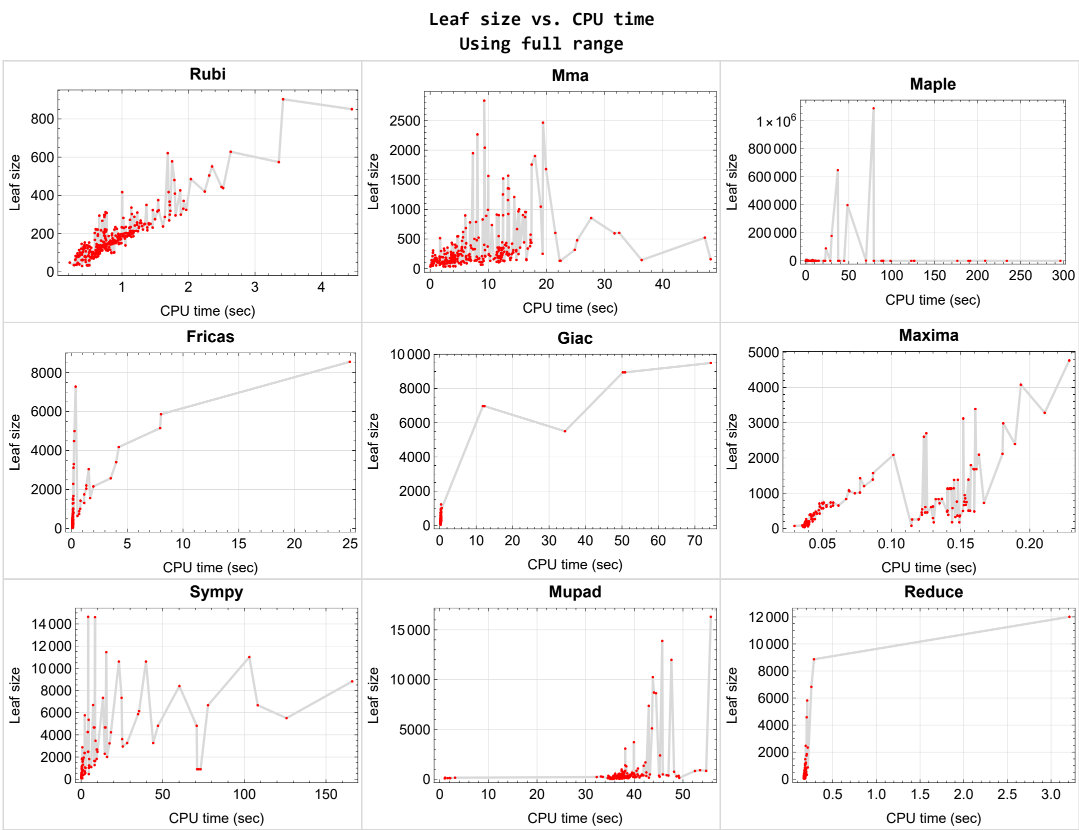


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{358}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {11, 216, 334, 335, 339, 340, 341, 342, 343, 344, 345, 346, 347, 349}

Mathematica {7, 8, 9, 10, 11, 88, 95, 96, 97, 102, 103, 104, 105, 106, 107, 130, 162, 196, 197, 198, 214, 215, 248, 249, 250, 251, 258, 259, 260, 292, 297, 298, 299, 304, 305, 306, 311, 312, 313, 318, 319, 320, 321, 325, 326, 327, 332, 334, 335, 336, 337, 353, 354, 355, 356, 357}

Maple $\{168, 177, 179, 353, 354, 355, 356, 357\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

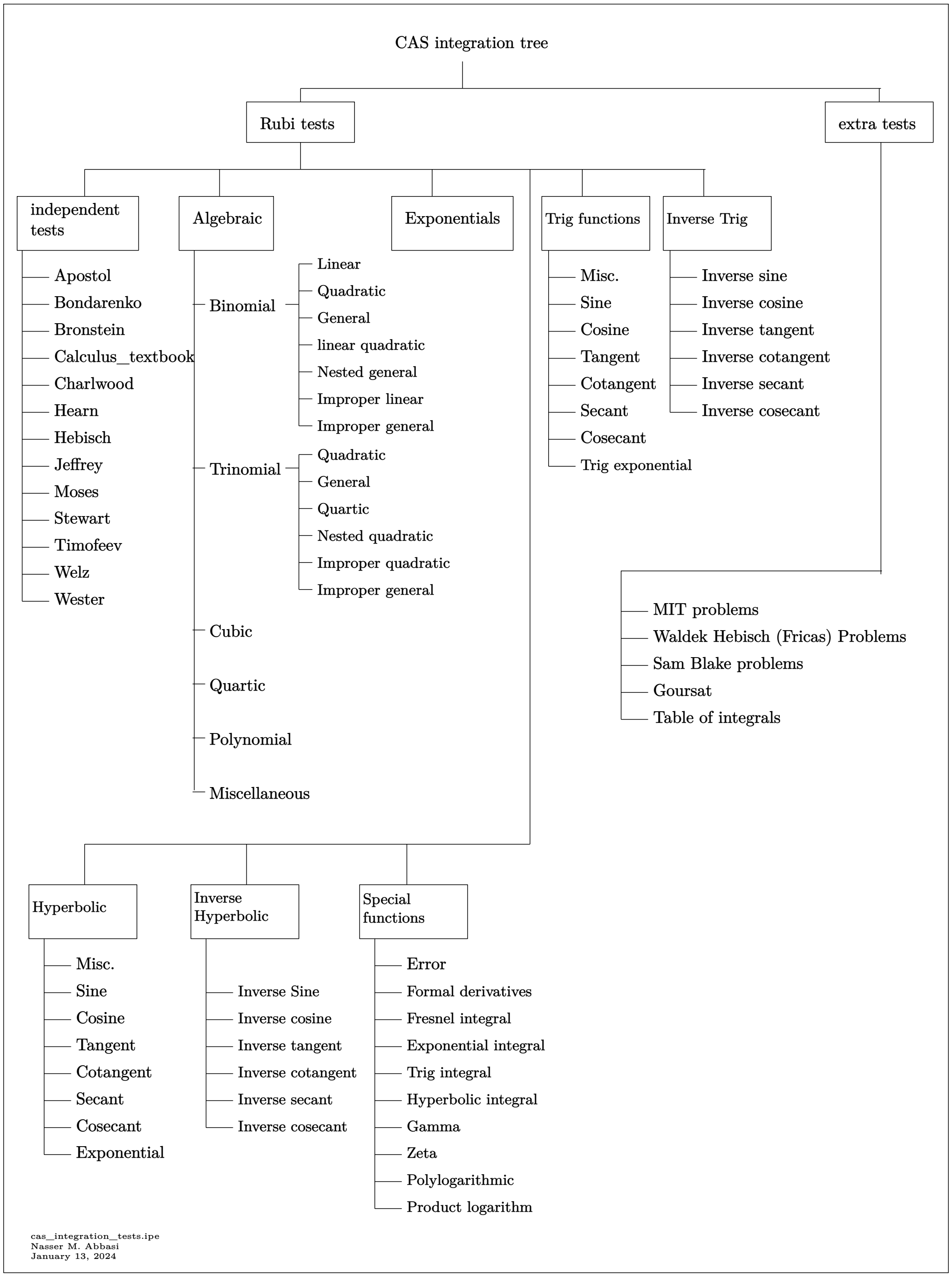
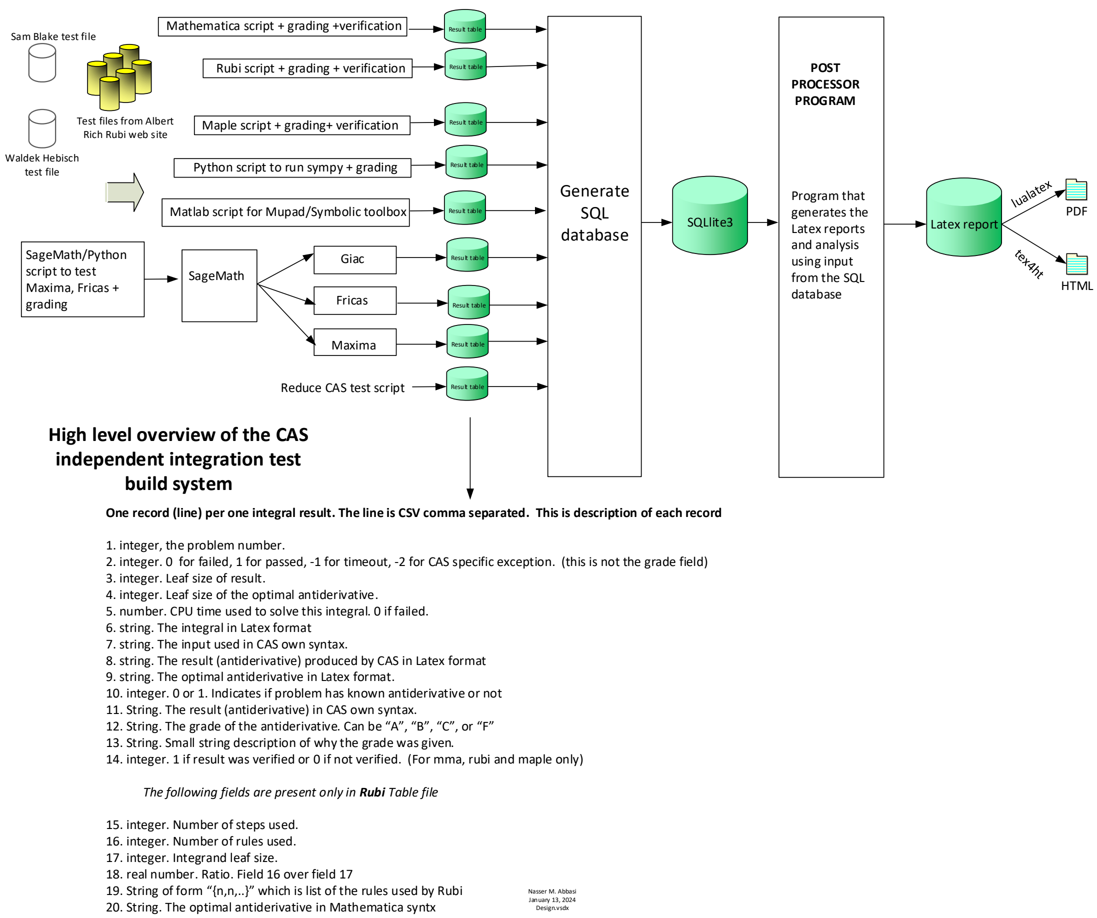


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 308, 309, 310, 311, 312, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357 }

B grade { }

C grade { }

F normal fail { 307, 313 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 10, 15, 16, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 51, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 65, 66, 69, 70, 71, 74, 75, 77, 80, 81, 82, 83, 85, 86, 87, 88, 90, 91, 92, 101, 108, 109, 110, 111, 116, 117, 118, 119, 123, 124, 125, 126, 127, 131, 132, 133, 134, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 166, 167, 168, 169, 173, 174, 175, 177, 178, 179, 180, 181, 182, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 203, 204, 206, 207, 208, 209, 210, 211, 212, 213, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 236, 238, 239, 240, 241, 242, 244, 245, 246, 247, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 266, 267, 269, 270, 271, 275, 276, 277, 279, 281, 282, 285, 286, 287, 288, 289, 293, 294, 295, 296, 302, 303, 332, 334, 350, 351, 352 }

B grade { 11, 14, 22, 32, 33, 34, 35, 46, 47, 48, 49, 50, 55, 63, 64, 67, 68, 72, 73, 76, 78, 79, 84, 89, 98, 99, 100, 115, 160, 170, 171, 172, 232, 233, 234, 243, 264, 265, 268, 272, 273, 274, 278, 280, 283, 284, 301, 335, 353, 354, 355, 356, 357 }

C grade { 9, 93, 94, 95, 96, 97, 102, 103, 104, 105, 106, 107, 112, 113, 114, 120, 121, 122, 128, 129, 130, 135, 136, 176, 183, 196, 197, 198, 199, 205, 214, 215, 235, 237, 248, 249, 250, 290, 291, 292, 297, 298, 299, 300, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 336, 337, 338 }

F normal fail { 12, 13, 195, 200, 201, 202, 216, 217, 328, 329, 330, 331, 333, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 16, 18, 19, 20, 21, 23, 24, 31, 32, 33, 34, 35, 36, 43, 44, 45, 46, 47, 49, 50, 52, 53, 54, 55, 56, 60, 61, 62, 63, 70, 71, 72, 73, 74, 81, 82, 83, 84, 85, 89, 90, 91, 92, 93, 94, 98, 99, 100, 101, 102, 103, 108, 109, 110, 111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 130, 133, 134, 140, 141, 142, 149, 150, 151, 160, 161, 162, 180, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 238, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 251, 252, 253, 254, 255, 256, 258, 259, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 283, 284, 285, 286, 287, 288, 289, 290, 293, 294, 295, 296, 300, 301, 302, 303, 310, 311, 352 }

B grade { 17, 26, 27, 28, 29, 30, 38, 39, 40, 41, 42, 48, 86, 87, 88, 95, 96, 97, 104, 105, 106, 107, 114, 122, 131, 132, 135, 136, 137, 138, 139, 143, 144, 145, 146, 147, 148, 152, 153, 154, 155, 156, 157, 158, 159, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 250, 257, 260, 291, 292, 297, 298, 299, 304, 305, 306, 307, 308, 309, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 353, 354, 355, 356, 357 }

C grade { 22, 25, 37, 51, 57, 58, 59, 64, 65, 66, 67, 68, 69, 75, 76, 77, 78, 79, 80, 237, 239, 282 }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 14, 15, 17, 18, 19, 20, 23, 24, 25, 26, 27, 28, 29, 30, 31, 38, 39, 40, 41, 42, 43, 44, 45, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 65, 66, 67, 68, 69, 70, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 89, 102, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 137, 138, 139, 140, 141, 142, 146, 147, 148, 149, 150, 151, 152, 157, 158, 159, 160, 161, 162, 163, 164, 171, 172, 173, 178, 179, 184, 185, 186, 191, 192, 193, 194, 206, 207, 211, 212, 213, 222, 223, 224, 225, 226, 227, 228, 229, 244, 245, 246, 247, 248, 251, 252, 253, 254, 255, 256, 258, 259, 260, 261, 262, 263, 268, 275, 282, 286, 287, 288, 289, 290, 293, 294, 295, 296, 300, 301, 302, 303, 350, 351 }

B grade { 16, 21, 22, 32, 33, 34, 35, 36, 37, 46, 47, 48, 49, 50, 51, 55, 63, 64, 71, 72, 73, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 170, 205, 218, 219, 220, 221, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 249, 250, 257, 264, 265, 266, 267, 269, 270, 271, 272, 273, 274, 276, 277, 278, 279, 280, 281, 283, 284, 285, 291, 292, 297, 298, 299, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 352 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 135, 136, 143, 144, 145, 153, 154, 155, 156, 165, 166, 167, 168, 169, 174, 175, 176, 177, 180, 181, 182, 183, 187, 188, 189, 190, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 208, 209, 210, 214, 215, 216, 217, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 354, 355, 356, 357 }

F(-1) timedout fail { 353 }

F(-2) exception fail { }

Maxima

A grade { 17, 18, 20, 30, 43, 56, 66, 77, 135, 176, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 244, 245, 246, 247, 251, 252, 253, 254, 258, 259, 261 }

B grade { 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 108, 109, 110, 111, 115, 116, 117, 118, 119, 123, 124, 125, 126, 127, 144, 155, 168, 182, 189, 205, 206, 207, 235, 236, 237, 238, 239, 240, 241, 242, 243, 260, 265, 266, 267, 268, 272, 273, 274, 275, 279, 280, 281, 282 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 112, 113, 114, 120, 128, 131, 132, 133, 134, 136, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 169, 170, 171, 174, 175, 177, 178, 179, 180, 181, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 307, 308, 309, 310, 311, 312, 314, 315, 316, 317, 318, 321, 322, 323, 324, 325, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 353, 354, 355, 356, 357 }

F(-1) timedout fail { 121, 122, 129, 130, 172, 173, 306, 313, 319, 320, 326, 327 }

F(-2) exception fail { 16, 248, 249, 250, 255, 256, 257, 262, 263, 264, 269, 270, 271, 276, 277, 278, 283, 284, 285, 352 }

Giac

A grade { 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 52, 53, 54, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 69, 70, 71, 72, 73, 74, 75, 77, 81, 82, 83, 84, 89, 123, 131, 132, 133, 134, 139, 140, 142, 152, 163, 164, 224, 225, 226, 227, 228, 230, 231, 233, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 251, 252, 253, 254, 255, 258, 259, 260, 261, 266, 268, 269, 274, 275, 276, 277, 281, 282, 287, 288, 289, 290, 291, 294, 295, 296, 301, 303 }

B grade { 14, 15, 16, 21, 34, 35, 36, 37, 48, 49, 50, 51, 55, 61, 67, 68, 76, 78, 79, 80, 90, 91, 92, 98, 99, 100, 101, 108, 109, 110, 111, 115, 116, 117, 118, 119, 124, 125, 126, 127, 141, 149, 150, 151, 160, 161, 162, 218, 219, 220, 221, 222, 223, 229, 232, 234, 250, 256, 257, 262, 263, 264, 265, 267, 270, 271, 272, 273, 278, 279, 280, 283, 284, 285, 286, 292, 293, 297, 298, 299, 300, 302, 304, 305, 306, 352 }

C grade { }

F normal fail { 1, 2, 3, 7, 8, 9, 12, 13, 195, 196, 197, 198, 199, 205, 206, 207, 211, 212, 213, 214, 215, 216, 217, 328, 329, 332, 333, 336, 337, 338, 342, 343, 344, 345, 346, 347, 348, 349, 353, 354, 355, 356, 357 }

F(-1) timedout fail { 10, 11, 203, 204, 208, 334, 335, 350, 351 }

F(-2) exception fail { 4, 5, 6, 85, 86, 87, 88, 93, 94, 95, 96, 97, 102, 103, 104, 105, 106, 107, 112, 113, 114, 120, 121, 122, 128, 129, 130, 135, 136, 137, 138, 143, 144, 145, 146, 147, 148, 153, 154, 155, 156, 157, 158, 159, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 200, 201, 202, 209, 210, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 330, 331, 339, 340, 341 }

Mupad

A grade { }

B grade { 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 111, 118, 119, 125, 126, 127, 131, 132, 133, 134, 138, 139, 140, 141, 142, 147, 148, 149, 150, 151, 152, 158, 159, 160, 161, 162, 163, 164, 171, 172, 173, 191, 205, 206, 207, 211, 212, 213, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 310, 350, 351, 352 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 117, 120, 121, 122, 123, 124, 128, 129, 130, 135, 136, 137, 143, 144, 145, 146, 153, 154, 155, 156, 157, 165, 166, 167, 168, 169, 170, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 208, 209, 210, 214, 215, 216, 217, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 353, 354, 355, 356, 357 }

F(-2) exception fail { }

Sympy

A grade { }

B grade { 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 218, 219, 220, 221, 224, 225, 226, 227, 235, 236, 237, 238, 239, 244, 245, 246, 247, 248, 251, 252, 253, 254, 258, 259, 260, 261, 265, 266, 267, 268, 272, 273, 274, 275, 279, 280, 281, 282 }

C grade { }

F normal fail { 8, 9, 10, 11, 12, 13, 14, 15, 82, 83, 84, 85, 86, 90, 91, 92, 93, 94, 100, 101, 102, 110, 111, 112, 113, 119, 120, 133, 134, 135, 136, 137, 142, 143, 144, 145, 175, 176, 177, 178, 179, 182, 183, 184, 185, 190, 191, 192, 195, 196, 197, 198, 199, 200, 201, 203, 204, 207, 208, 209, 211, 212, 213, 214, 216, 217, 222, 223, 228, 229, 230, 231, 240, 241, 242, 243, 286, 287, 288, 289, 293, 294, 295, 296, 302, 303, 307, 308, 309, 310, 315, 316, 317, 324, 333, 334, 335, 336, 337, 338, 343, 344, 345, 353, 354, 355, 356 }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 16, 81, 87, 88, 89, 95, 96, 97, 98, 99, 103, 104, 105, 106, 107, 108, 109, 114, 115, 116, 117, 118, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 138, 139, 140, 141, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 180, 181, 186, 187, 188, 189, 193, 194, 202, 205, 206, 210, 215, 232, 233, 234, 249, 250, 255, 256, 257, 262, 263, 264, 269, 270, 271, 276, 277, 278, 283, 284, 285, 290, 291, 292, 297, 298, 299, 300, 301, 304, 305, 306, 311, 312, 313, 314, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 331, 332, 339, 340, 341, 342, 346, 347, 348, 349, 350, 351, 352, 357 }

F(-2) exception fail { 358 }

Reduce

A grade { }

B grade { 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 352 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 353, 354, 355, 356, 357 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	373	367	248	0	0	0	0	0	167	0
N.S.	1	0.98	0.66	0.00	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	1.716	2.724	0.000	0.000	0.000	0.000	0.000	0.195	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	277	272	204	0	0	0	0	0	123	0
N.S.	1	0.98	0.74	0.00	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	1.121	1.617	0.000	0.000	0.000	0.000	0.000	0.191	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	191	188	145	0	0	0	0	0	77	0
N.S.	1	0.98	0.76	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.637	1.128	0.000	0.000	0.000	0.000	0.000	0.195	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	202	203	159	0	0	0	0	0	60	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.590	0.838	0.000	0.000	0.000	0.000	0.000	0.185	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	279	282	220	0	0	0	0	0	80	0
N.S.	1	1.01	0.79	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	1.021	4.362	0.000	0.000	0.000	0.000	0.000	0.187	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	362	375	426	0	0	0	0	0	100	0
N.S.	1	1.04	1.18	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	1.545	6.198	0.000	0.000	0.000	0.000	0.000	0.191	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	358	323	596	0	0	0	0	0	180	0
N.S.	1	0.90	1.66	0.00	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	1.463	31.692	0.000	0.000	0.000	0.000	0.000	0.227	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	251	225	478	0	0	0	0	0	116	0
N.S.	1	0.90	1.90	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.923	25.267	0.000	0.000	0.000	0.000	0.000	0.230	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	161	131	409	0	0	0	0	0	57	0
N.S.	1	0.81	2.54	0.00	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.515	3.836	0.000	0.000	0.000	0.000	0.000	0.188	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	161	152	250	0	0	0	0	0	80	0
N.S.	1	0.94	1.55	0.00	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	0.954	19.336	0.000	0.000	0.000	0.000	0.000	0.192	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	246	231	523	0	0	0	0	0	100	0
N.S.	1	0.94	2.13	0.00	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	1.461	47.220	0.000	0.000	0.000	0.000	0.000	0.179	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	0	0	0	0	0	0	61	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.886	0.000	0.000	0.000	0.000	0.000	0.000	0.190	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	0	0	0	0	0	0	60	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	0.356	0.000	0.000	0.000	0.000	0.000	0.000	0.193	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	A	F	B	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	107	0	0	41	0	9496	294	61
N.S.	1	1.00	2.89	0.00	0.00	1.11	0.00	256.65	7.95	1.65
time (sec)	N/A	0.315	8.737	0.000	0.000	0.098	0.000	74.366	0.193	35.224

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	B	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	0	0	41	0	5502	105	38
N.S.	1	1.00	1.00	0.00	0.00	1.17	0.00	157.20	3.00	1.09
time (sec)	N/A	0.285	2.627	0.000	0.000	0.101	0.000	34.360	0.196	35.038

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	189	147	212	0	806	0	356	97	3718
N.S.	1	1.24	0.96	1.39	0.00	5.27	0.00	2.33	0.63	24.30
time (sec)	N/A	0.905	3.820	1.018	0.000	0.186	0.000	0.207	0.172	39.921

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	159	141	342	336	123	853	178	193	454
N.S.	1	0.87	0.77	1.88	1.85	0.68	4.69	0.98	1.06	2.49
time (sec)	N/A	0.838	2.592	0.098	0.042	0.095	0.496	0.201	0.187	37.343

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	124	123	98	200	102	486	140	159	389
N.S.	1	0.87	0.87	0.69	1.41	0.72	3.42	0.99	1.12	2.74
time (sec)	N/A	0.670	2.467	176.259	0.043	0.086	0.306	0.261	0.183	36.133

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	86	105	78	179	82	396	110	125	345
N.S.	1	0.89	1.08	0.80	1.85	0.85	4.08	1.13	1.29	3.56
time (sec)	N/A	0.497	1.618	22.471	0.039	0.088	0.214	0.219	0.179	35.761

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	47	48	50	73	43	138	55	57	122
N.S.	1	0.96	0.98	1.02	1.49	0.88	2.82	1.12	1.16	2.49
time (sec)	N/A	0.334	0.459	44.558	0.030	0.081	0.123	0.195	0.184	36.990

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	61	107	67	265	116	828	117	109	111
N.S.	1	1.09	1.91	1.20	4.73	2.07	14.79	2.09	1.95	1.98
time (sec)	N/A	0.410	5.327	0.588	0.120	0.079	1.109	0.207	0.171	34.691

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	77	160	80	456	162	700	87	155	132
N.S.	1	1.07	2.22	1.11	6.33	2.25	9.72	1.21	2.15	1.83
time (sec)	N/A	0.585	6.612	0.714	0.126	0.084	2.233	0.213	0.182	34.879

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	107	147	94	737	183	1035	131	173	172
N.S.	1	1.03	1.41	0.90	7.09	1.76	9.95	1.26	1.66	1.65
time (sec)	N/A	0.620	6.660	0.816	0.056	0.074	4.735	0.248	0.184	35.434

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	143	174	130	1080	251	1831	176	243	228
N.S.	1	1.01	1.23	0.92	7.61	1.77	12.89	1.24	1.71	1.61
time (sec)	N/A	0.755	6.839	1.116	0.069	0.078	9.542	0.227	0.173	35.770

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	175	200	166	1425	305	3232	251	339	310
N.S.	1	0.99	1.14	0.94	8.10	1.73	18.36	1.43	1.93	1.76
time (sec)	N/A	0.858	6.828	1.438	0.077	0.082	17.415	0.254	0.193	36.549

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	189	219	569	571	158	1586	270	263	661
N.S.	1	0.83	0.96	2.48	2.49	0.69	6.93	1.18	1.15	2.89
time (sec)	N/A	1.014	5.530	0.112	0.049	0.104	0.898	0.218	0.173	37.821

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	154	163	463	460	135	1210	237	229	553
N.S.	1	0.81	0.86	2.45	2.43	0.71	6.40	1.25	1.21	2.93
time (sec)	N/A	0.809	8.252	0.109	0.045	0.098	0.655	0.278	0.183	37.929

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	116	137	365	360	114	910	202	195	542
N.S.	1	0.79	0.93	2.48	2.45	0.78	6.19	1.37	1.33	3.69
time (sec)	N/A	0.633	7.976	0.083	0.042	0.089	0.467	0.248	0.178	36.956

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	77	54	166	164	75	372	113	95	238
N.S.	1	0.87	0.61	1.87	1.84	0.84	4.18	1.27	1.07	2.67
time (sec)	N/A	0.459	0.314	0.062	0.038	0.083	0.291	0.215	0.173	36.935

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	83	115	186	179	77	396	107	125	339
N.S.	1	0.85	1.17	1.90	1.83	0.79	4.04	1.09	1.28	3.46
time (sec)	N/A	0.493	1.319	0.231	0.040	0.089	0.212	0.234	0.176	35.699

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	108	191	92	624	177	2365	156	203	244
N.S.	1	0.92	1.63	0.79	5.33	1.51	20.21	1.33	1.74	2.09
time (sec)	N/A	0.663	12.281	0.767	0.130	0.083	2.159	0.217	0.178	37.235

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	106	238	107	839	237	2474	129	277	246
N.S.	1	0.97	2.18	0.98	7.70	2.17	22.70	1.18	2.54	2.26
time (sec)	N/A	0.669	11.420	0.763	0.136	0.085	4.371	0.245	0.180	37.402

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	117	278	126	1139	277	1647	151	257	233
N.S.	1	1.04	2.48	1.12	10.17	2.47	14.71	1.35	2.29	2.08
time (sec)	N/A	0.673	11.482	0.712	0.145	0.087	8.591	0.271	0.176	33.517

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	191	133	1571	263	2008	217	257	269
N.S.	1	1.00	2.55	1.77	20.95	3.51	26.77	2.89	3.43	3.59
time (sec)	N/A	0.529	11.759	0.864	0.087	0.080	15.864	0.253	0.183	33.152

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	113	261	171	2087	335	3262	285	362	331
N.S.	1	0.98	2.27	1.49	18.15	2.91	28.37	2.48	3.15	2.88
time (sec)	N/A	0.681	12.169	1.036	0.101	0.083	28.185	0.290	0.183	36.281

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	149	285	219	2604	407	4816	353	463	423
N.S.	1	0.96	1.83	1.40	16.69	2.61	30.87	2.26	2.97	2.71
time (sec)	N/A	0.837	12.733	1.401	0.123	0.085	47.085	0.300	0.179	36.999

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	185	313	248	3120	475	6669	421	543	500
N.S.	1	0.94	1.59	1.26	15.84	2.41	33.85	2.14	2.76	2.54
time (sec)	N/A	1.015	14.815	1.488	0.152	0.098	77.724	0.287	0.186	37.501

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	215	255	651	661	181	1948	337	331	812
N.S.	1	0.81	0.96	2.46	2.49	0.68	7.35	1.27	1.25	3.06
time (sec)	N/A	1.124	12.468	0.269	0.053	0.118	1.589	0.351	0.181	38.072

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	180	232	611	617	158	1753	292	297	705
N.S.	1	0.81	1.05	2.75	2.78	0.71	7.90	1.32	1.34	3.18
time (sec)	N/A	0.931	11.453	0.260	0.053	0.108	1.180	0.276	0.189	37.967

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	142	209	568	571	137	1579	265	263	661
N.S.	1	0.78	1.15	3.14	3.15	0.76	8.72	1.46	1.45	3.65
time (sec)	N/A	0.720	8.904	0.217	0.051	0.096	0.928	0.269	0.183	37.770

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	103	64	263	264	92	682	155	129	325
N.S.	1	0.88	0.55	2.25	2.26	0.79	5.83	1.32	1.10	2.78
time (sec)	N/A	0.541	0.405	0.171	0.042	0.094	0.554	0.243	0.173	37.428

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	113	133	364	360	106	910	198	195	536
N.S.	1	0.82	0.96	2.64	2.61	0.77	6.59	1.43	1.41	3.88
time (sec)	N/A	0.603	7.650	0.153	0.043	0.124	0.483	0.254	0.182	36.922

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	122	141	208	200	100	486	140	159	390
N.S.	1	0.87	1.01	1.49	1.43	0.71	3.47	1.00	1.14	2.79
time (sec)	N/A	0.642	1.659	0.149	0.041	0.090	0.321	0.217	0.172	36.702

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	131	223	110	1139	218	4255	223	260	323
N.S.	1	0.84	1.43	0.71	7.30	1.40	27.28	1.43	1.67	2.07
time (sec)	N/A	0.752	12.144	1.724	0.143	0.086	4.151	0.242	0.183	37.025

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	148	280	166	1386	286	4665	222	345	341
N.S.	1	0.91	1.72	1.02	8.50	1.75	28.62	1.36	2.12	2.09
time (sec)	N/A	0.851	11.551	1.891	0.156	0.129	8.211	0.246	0.180	36.734

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	145	316	157	1685	337	4665	215	433	336
N.S.	1	0.95	2.07	1.03	11.01	2.20	30.49	1.41	2.83	2.20
time (sec)	N/A	0.821	11.783	1.976	0.159	0.090	14.970	0.197	0.172	36.769

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	154	356	177	2118	363	2951	202	357	316
N.S.	1	1.02	2.36	1.17	14.03	2.40	19.54	1.34	2.36	2.09
time (sec)	N/A	0.806	11.875	1.857	0.180	0.093	25.534	0.286	0.184	38.371

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	75	283	173	2701	331	3262	285	339	346
N.S.	1	0.97	3.68	2.25	35.08	4.30	42.36	3.70	4.40	4.49
time (sec)	N/A	0.540	13.221	1.993	0.125	0.092	44.157	0.297	0.174	36.430

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	113	313	211	3390	405	4816	353	452	408
N.S.	1	0.96	2.65	1.79	28.73	3.43	40.81	2.99	3.83	3.46
time (sec)	N/A	0.659	14.305	2.161	0.161	0.086	70.801	0.243	0.180	37.177

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	149	339	250	4078	475	6669	421	543	500
N.S.	1	0.96	2.17	1.60	26.14	3.04	42.75	2.70	3.48	3.21
time (sec)	N/A	0.838	16.781	2.434	0.194	0.097	108.172	0.316	0.190	37.401

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	185	378	298	4765	541	8821	489	627	577
N.S.	1	0.94	1.92	1.51	24.19	2.75	44.78	2.48	3.18	2.93
time (sec)	N/A	1.041	17.184	2.846	0.229	0.095	166.056	0.302	0.175	37.563

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	171	274	209	1796	261	6690	326	318	397
N.S.	1	0.90	1.44	1.10	9.45	1.37	35.21	1.72	1.67	2.09
time (sec)	N/A	0.951	14.235	10.824	0.157	0.092	7.346	0.236	0.187	39.094

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	132	220	110	1120	218	4255	224	262	319
N.S.	1	0.84	1.40	0.70	7.13	1.39	27.10	1.43	1.67	2.03
time (sec)	N/A	0.766	12.189	1.681	0.143	0.087	3.975	0.236	0.174	37.353

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	108	188	94	608	179	2365	157	204	241
N.S.	1	0.92	1.59	0.80	5.15	1.52	20.04	1.33	1.73	2.04
time (sec)	N/A	0.664	11.622	0.564	0.124	0.090	2.086	0.236	0.177	39.097

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	63	127	67	256	117	828	115	109	110
N.S.	1	1.11	2.23	1.18	4.49	2.05	14.53	2.02	1.91	1.93
time (sec)	N/A	0.408	5.662	0.381	0.120	0.079	1.092	0.192	0.181	37.911

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	30	35	37	35	28	83	39	37	39
N.S.	1	0.86	1.00	1.06	1.00	0.80	2.37	1.11	1.06	1.11
time (sec)	N/A	0.400	0.046	0.244	0.037	0.071	0.735	0.212	0.171	36.904

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	60	108	86	266	73	578	97	122	118
N.S.	1	0.95	1.71	1.37	4.22	1.16	9.17	1.54	1.94	1.87
time (sec)	N/A	0.511	3.132	0.445	0.043	0.073	2.332	0.218	0.177	37.112

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	94	157	111	423	107	1236	167	190	178
N.S.	1	0.92	1.54	1.09	4.15	1.05	12.12	1.64	1.86	1.75
time (sec)	N/A	0.654	3.893	0.628	0.048	0.076	4.824	0.229	0.171	37.300

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	128	240	136	619	141	2468	223	256	239
N.S.	1	0.90	1.69	0.96	4.36	0.99	17.38	1.57	1.80	1.68
time (sec)	N/A	0.807	4.994	0.865	0.056	0.078	10.028	0.236	0.172	37.694

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	209	354	252	2982	370	10608	391	457	500
N.S.	1	0.87	1.48	1.05	12.42	1.54	44.20	1.63	1.90	2.08
time (sec)	N/A	1.155	12.568	88.195	0.181	0.100	23.133	0.244	0.193	39.911

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	166	311	193	2094	322	7337	349	401	414
N.S.	1	0.92	1.73	1.07	11.63	1.79	40.76	1.94	2.23	2.30
time (sec)	N/A	0.939	11.874	11.174	0.163	0.090	13.367	0.252	0.183	39.066

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	146	274	162	1378	291	4665	222	345	336
N.S.	1	0.90	1.69	1.00	8.51	1.80	28.80	1.37	2.13	2.07
time (sec)	N/A	0.806	11.534	1.951	0.145	0.090	7.780	0.237	0.182	38.712

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	105	234	107	833	242	2474	130	277	242
N.S.	1	0.97	2.17	0.99	7.71	2.24	22.91	1.20	2.56	2.24
time (sec)	N/A	0.651	11.410	0.728	0.134	0.116	4.218	0.212	0.172	38.568

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	78	156	81	452	166	702	87	155	133
N.S.	1	1.08	2.17	1.12	6.28	2.31	9.75	1.21	2.15	1.85
time (sec)	N/A	0.572	6.517	0.473	0.124	0.116	2.159	0.227	0.172	37.114

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	59	110	86	265	69	578	97	118	117
N.S.	1	0.95	1.77	1.39	4.27	1.11	9.32	1.56	1.90	1.89
time (sec)	N/A	0.504	3.191	0.450	0.044	0.072	2.306	0.239	0.183	37.153

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	51	53	70	47	41	469	82	80	82
N.S.	1	0.82	0.85	1.13	0.76	0.66	7.56	1.32	1.29	1.32
time (sec)	N/A	0.454	0.155	0.493	0.037	0.077	2.139	0.235	0.177	37.500

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	77	237	136	651	116	2674	221	256	183
N.S.	1	0.83	2.55	1.46	7.00	1.25	28.75	2.38	2.75	1.97
time (sec)	N/A	0.557	4.979	1.006	0.062	0.077	9.516	0.228	0.189	36.991

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	113	285	161	835	151	4228	277	278	197
N.S.	1	0.84	2.11	1.19	6.19	1.12	31.32	2.05	2.06	1.46
time (sec)	N/A	0.691	5.740	1.543	0.067	0.094	18.282	0.226	0.176	36.984

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	149	329	184	998	187	5868	333	384	337
N.S.	1	0.85	1.88	1.05	5.70	1.07	33.53	1.90	2.19	1.93
time (sec)	N/A	0.862	7.678	2.242	0.073	0.081	34.896	0.282	0.174	36.795

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	206	388	240	3282	429	10608	357	547	501
N.S.	1	0.85	1.60	0.99	13.51	1.77	43.65	1.47	2.25	2.06
time (sec)	N/A	1.117	13.088	87.959	0.211	0.105	39.737	0.294	0.194	39.655

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	184	348	201	2394	392	7337	290	491	419
N.S.	1	0.92	1.73	1.00	11.91	1.95	36.50	1.44	2.44	2.08
time (sec)	N/A	0.978	12.168	11.059	0.189	0.098	24.793	0.253	0.187	40.060

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	143	308	157	1679	338	4665	215	435	333
N.S.	1	0.93	2.01	1.03	10.97	2.21	30.49	1.41	2.84	2.18
time (sec)	N/A	0.823	11.800	2.063	0.160	0.111	14.584	0.240	0.186	38.876

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	115	272	127	1134	279	1647	151	257	230
N.S.	1	1.05	2.47	1.15	10.31	2.54	14.97	1.37	2.34	2.09
time (sec)	N/A	0.649	11.497	0.711	0.142	0.086	8.081	0.265	0.182	39.752

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	106	139	92	733	191	1035	130	175	172
N.S.	1	1.03	1.35	0.89	7.12	1.85	10.05	1.26	1.70	1.67
time (sec)	N/A	0.607	6.923	0.527	0.051	0.076	4.851	0.247	0.179	37.294

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	94	156	111	423	106	1236	165	190	178
N.S.	1	0.92	1.53	1.09	4.15	1.04	12.12	1.62	1.86	1.75
time (sec)	N/A	0.623	3.926	0.619	0.046	0.113	4.948	0.209	0.177	36.650

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	76	237	136	650	108	2674	221	254	183
N.S.	1	0.84	2.63	1.51	7.22	1.20	29.71	2.46	2.82	2.03
time (sec)	N/A	0.520	4.793	0.998	0.058	0.074	9.822	0.270	0.182	36.714

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	63	65	83	60	56	1098	126	119	126
N.S.	1	0.75	0.77	0.99	0.71	0.67	13.07	1.50	1.42	1.50
time (sec)	N/A	0.450	0.255	0.958	0.040	0.076	6.206	0.241	0.186	38.829

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	89	325	186	1019	150	6135	333	386	217
N.S.	1	0.74	2.69	1.54	8.42	1.24	50.70	2.75	3.19	1.79
time (sec)	N/A	0.548	7.381	2.211	0.077	0.104	35.497	0.299	0.186	37.452

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	125	373	211	1201	185	8396	389	454	231
N.S.	1	0.77	2.30	1.30	7.41	1.14	51.83	2.40	2.80	1.43
time (sec)	N/A	0.690	9.892	3.059	0.080	0.096	60.199	0.317	0.186	38.304

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	161	401	236	1387	221	11011	445	518	474
N.S.	1	0.79	1.96	1.15	6.77	1.08	53.71	2.17	2.53	2.31
time (sec)	N/A	0.860	12.849	4.312	0.086	0.094	103.024	0.284	0.183	39.293

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	185	149	119	0	287	0	296	193	0
N.S.	1	0.93	0.75	0.60	0.00	1.45	0.00	1.49	0.97	0.00
time (sec)	N/A	1.076	6.757	7.075	0.000	0.089	0.000	0.436	0.203	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	148	123	103	0	243	0	262	191	0
N.S.	1	0.94	0.78	0.66	0.00	1.55	0.00	1.67	1.22	0.00
time (sec)	N/A	0.875	4.709	6.957	0.000	0.088	0.000	0.354	0.215	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	111	104	81	0	184	0	194	93	0
N.S.	1	0.96	0.90	0.70	0.00	1.59	0.00	1.67	0.80	0.00
time (sec)	N/A	0.694	3.771	0.628	0.000	0.087	0.000	0.288	0.184	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	71	191	63	0	130	0	118	88	0
N.S.	1	0.97	2.62	0.86	0.00	1.78	0.00	1.62	1.21	0.00
time (sec)	N/A	0.543	2.205	0.622	0.000	0.080	0.000	0.257	0.198	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	129	166	158	0	254	0	0	133	0
N.S.	1	1.06	1.36	1.30	0.00	2.08	0.00	0.00	1.09	0.00
time (sec)	N/A	0.734	4.678	0.549	0.000	0.092	0.000	0.000	0.187	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	123	157	227	0	318	0	0	172	0
N.S.	1	1.07	1.37	1.97	0.00	2.77	0.00	0.00	1.50	0.00
time (sec)	N/A	0.739	6.203	0.569	0.000	0.094	0.000	0.000	0.199	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	135	199	267	0	394	0	0	213	0
N.S.	1	1.07	1.58	2.12	0.00	3.13	0.00	0.00	1.69	0.00
time (sec)	N/A	0.733	8.012	0.590	0.000	0.137	0.000	0.000	0.192	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	168	217	352	0	490	0	0	252	0
N.S.	1	1.03	1.33	2.16	0.00	3.01	0.00	0.00	1.55	0.00
time (sec)	N/A	0.869	9.444	0.596	0.000	0.110	0.000	0.000	0.191	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	189	1355	121	0	358	0	373	293	0
N.S.	1	0.90	6.45	0.58	0.00	1.70	0.00	1.78	1.40	0.00
time (sec)	N/A	1.126	13.363	37.612	0.000	0.091	0.000	0.410	0.224	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	152	132	105	0	313	0	340	145	0
N.S.	1	0.91	0.79	0.63	0.00	1.87	0.00	2.04	0.87	0.00
time (sec)	N/A	0.910	12.766	37.686	0.000	0.090	0.000	0.385	0.194	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	113	106	83	0	228	0	238	191	0
N.S.	1	0.94	0.88	0.69	0.00	1.90	0.00	1.98	1.59	0.00
time (sec)	N/A	0.761	11.800	0.844	0.000	0.111	0.000	0.334	0.215	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	77	89	65	0	193	0	200	140	0
N.S.	1	0.95	1.10	0.80	0.00	2.38	0.00	2.47	1.73	0.00
time (sec)	N/A	0.618	1.625	0.815	0.000	0.087	0.000	0.296	0.191	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	160	175	197	0	310	0	0	208	0
N.S.	1	0.99	1.09	1.22	0.00	1.93	0.00	0.00	1.29	0.00
time (sec)	N/A	0.925	12.052	0.734	0.000	0.098	0.000	0.000	0.200	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	172	355	282	0	385	0	0	264	0
N.S.	1	0.98	2.02	1.60	0.00	2.19	0.00	0.00	1.50	0.00
time (sec)	N/A	0.985	11.590	0.746	0.000	0.103	0.000	0.000	0.194	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	169	344	386	0	449	0	0	328	0
N.S.	1	0.97	1.97	2.21	0.00	2.57	0.00	0.00	1.87	0.00
time (sec)	N/A	0.972	11.919	0.770	0.000	0.162	0.000	0.000	0.201	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	171	342	354	0	521	0	0	384	0
N.S.	1	0.98	1.95	2.02	0.00	2.98	0.00	0.00	2.19	0.00
time (sec)	N/A	0.984	12.587	0.763	0.000	0.101	0.000	0.000	0.200	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	214	357	440	0	654	0	0	448	0
N.S.	1	0.96	1.61	1.98	0.00	2.95	0.00	0.00	2.02	0.00
time (sec)	N/A	1.156	13.503	0.787	0.000	0.104	0.000	0.000	0.206	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	189	1569	121	0	405	0	458	197	0
N.S.	1	0.90	7.47	0.58	0.00	1.93	0.00	2.18	0.94	0.00
time (sec)	N/A	1.113	13.415	189.082	0.000	0.098	0.000	0.373	0.213	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	150	1351	105	0	334	0	372	289	0
N.S.	1	0.93	8.39	0.65	0.00	2.07	0.00	2.31	1.80	0.00
time (sec)	N/A	0.992	13.542	191.261	0.000	0.092	0.000	0.402	0.214	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	115	1157	83	0	287	0	294	193	0
N.S.	1	0.93	9.33	0.67	0.00	2.31	0.00	2.37	1.56	0.00
time (sec)	N/A	0.822	13.368	2.010	0.000	0.095	0.000	0.373	0.208	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	77	89	65	0	232	0	253	190	0
N.S.	1	0.95	1.10	0.80	0.00	2.86	0.00	3.12	2.35	0.00
time (sec)	N/A	0.606	2.350	1.977	0.000	0.084	0.000	0.342	0.196	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	199	193	233	0	353	0	0	278	0
N.S.	1	1.00	0.96	1.16	0.00	1.76	0.00	0.00	1.39	0.00
time (sec)	N/A	1.101	12.031	1.829	0.000	0.102	0.000	0.000	0.202	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	211	444	354	0	430	0	0	354	0
N.S.	1	0.97	2.04	1.62	0.00	1.97	0.00	0.00	1.62	0.00
time (sec)	N/A	1.162	12.386	1.882	0.000	0.097	0.000	0.000	0.199	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	210	434	434	0	505	0	0	438	0
N.S.	1	0.93	1.93	1.93	0.00	2.24	0.00	0.00	1.95	0.00
time (sec)	N/A	1.132	13.089	1.863	0.000	0.102	0.000	0.000	0.208	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	210	422	524	0	554	0	0	514	0
N.S.	1	0.97	1.94	2.41	0.00	2.55	0.00	0.00	2.37	0.00
time (sec)	N/A	1.140	14.084	1.852	0.000	0.103	0.000	0.000	0.209	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	212	355	432	0	633	0	0	598	0
N.S.	1	0.98	1.64	1.99	0.00	2.92	0.00	0.00	2.76	0.00
time (sec)	N/A	1.171	15.462	1.888	0.000	0.136	0.000	0.000	0.218	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	255	485	526	0	760	0	0	674	0
N.S.	1	0.96	1.82	1.98	0.00	2.86	0.00	0.00	2.53	0.00
time (sec)	N/A	1.290	17.248	1.892	0.000	0.148	0.000	0.000	0.215	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	182	157	111	478	115	0	777	276	0
N.S.	1	0.91	0.78	0.56	2.39	0.58	0.00	3.88	1.38	0.00
time (sec)	N/A	0.960	16.536	2.799	0.140	0.085	0.000	0.435	0.197	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	147	134	95	386	95	0	573	204	0
N.S.	1	0.92	0.84	0.60	2.43	0.60	0.00	3.60	1.28	0.00
time (sec)	N/A	0.812	12.563	2.799	0.141	0.082	0.000	0.356	0.200	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	112	113	73	294	67	0	353	134	0
N.S.	1	0.95	0.96	0.62	2.49	0.57	0.00	2.99	1.14	0.00
time (sec)	N/A	0.705	11.432	0.349	0.130	0.096	0.000	0.287	0.183	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	44	53	174	44	0	175	65	128
N.S.	1	1.00	0.60	0.73	2.38	0.60	0.00	2.40	0.89	1.75
time (sec)	N/A	0.576	3.187	0.352	0.131	0.077	0.000	0.264	0.191	38.590

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	89	140	130	0	162	0	0	73	0
N.S.	1	0.98	1.54	1.43	0.00	1.78	0.00	0.00	0.80	0.00
time (sec)	N/A	0.597	2.833	0.319	0.000	0.099	0.000	0.000	0.173	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	130	284	225	0	231	0	0	108	0
N.S.	1	0.96	2.09	1.65	0.00	1.70	0.00	0.00	0.79	0.00
time (sec)	N/A	0.710	3.561	0.320	0.000	0.092	0.000	0.000	0.185	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	164	404	350	0	282	0	0	109	0
N.S.	1	0.91	2.24	1.94	0.00	1.57	0.00	0.00	0.61	0.00
time (sec)	N/A	0.888	4.293	0.347	0.000	0.094	0.000	0.000	0.180	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	217	953	143	762	153	0	978	444	0
N.S.	1	0.90	3.94	0.59	3.15	0.63	0.00	4.04	1.83	0.00
time (sec)	N/A	1.381	16.456	20.970	0.147	0.091	0.000	0.445	0.216	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	182	159	121	670	133	0	774	356	0
N.S.	1	0.91	0.79	0.60	3.33	0.66	0.00	3.85	1.77	0.00
time (sec)	N/A	1.141	10.919	20.734	0.154	0.103	0.000	0.399	0.217	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	145	130	105	577	110	0	293	264	0
N.S.	1	0.94	0.84	0.68	3.75	0.71	0.00	1.90	1.71	0.00
time (sec)	N/A	0.952	9.194	20.708	0.144	0.084	0.000	0.340	0.190	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	110	113	81	482	80	0	283	174	492
N.S.	1	0.96	0.98	0.70	4.19	0.70	0.00	2.46	1.51	4.28
time (sec)	N/A	0.786	8.471	0.392	0.160	0.081	0.000	0.312	0.194	40.640

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	79	87	63	343	60	0	223	85	137
N.S.	1	1.01	1.12	0.81	4.40	0.77	0.00	2.86	1.09	1.76
time (sec)	N/A	0.632	2.856	0.424	0.149	0.079	0.000	0.235	0.188	49.241

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	125	176	168	0	217	0	0	105	0
N.S.	1	0.93	1.30	1.24	0.00	1.61	0.00	0.00	0.78	0.00
time (sec)	N/A	0.758	3.664	0.359	0.000	0.093	0.000	0.000	0.192	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-1)	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	164	300	280	0	206	0	0	92	0
N.S.	1	0.94	1.71	1.60	0.00	1.18	0.00	0.00	0.53	0.00
time (sec)	N/A	0.890	4.886	0.368	0.000	0.096	0.000	0.000	0.175	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	202	430	426	0	279	0	0	145	0
N.S.	1	0.90	1.91	1.89	0.00	1.24	0.00	0.00	0.64	0.00
time (sec)	N/A	1.057	6.196	0.366	0.000	0.100	0.000	0.000	0.178	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	221	176	143	945	166	0	395	544	0
N.S.	1	0.91	0.73	0.59	3.90	0.69	0.00	1.63	2.25	0.00
time (sec)	N/A	1.315	13.757	126.200	0.153	0.093	0.000	0.380	0.228	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	186	158	121	854	148	0	774	436	0
N.S.	1	0.89	0.76	0.58	4.09	0.71	0.00	3.70	2.09	0.00
time (sec)	N/A	1.172	12.176	125.861	0.153	0.099	0.000	0.358	0.215	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	149	132	105	761	125	0	449	324	904
N.S.	1	0.93	0.82	0.66	4.76	0.78	0.00	2.81	2.02	5.65
time (sec)	N/A	0.970	10.326	125.747	0.153	0.087	0.000	0.310	0.211	53.516

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	116	113	83	663	95	0	373	214	683
N.S.	1	0.96	0.93	0.69	5.48	0.79	0.00	3.08	1.77	5.64
time (sec)	N/A	0.785	8.970	0.378	0.152	0.083	0.000	0.291	0.201	42.482

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	83	89	65	505	77	0	368	105	479
N.S.	1	0.98	1.05	0.76	5.94	0.91	0.00	4.33	1.24	5.64
time (sec)	N/A	0.642	6.283	0.398	0.140	0.078	0.000	0.307	0.196	41.562

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	161	204	200	0	262	0	0	109	0
N.S.	1	0.93	1.17	1.15	0.00	1.51	0.00	0.00	0.63	0.00
time (sec)	N/A	0.923	4.296	0.375	0.000	0.101	0.000	0.000	0.199	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-1)	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	202	357	308	0	277	0	0	140	0
N.S.	1	0.90	1.59	1.38	0.00	1.24	0.00	0.00	0.62	0.00
time (sec)	N/A	1.070	6.019	0.378	0.000	0.103	0.000	0.000	0.200	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-1)	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	236	479	410	0	238	0	0	113	0
N.S.	1	0.91	1.86	1.59	0.00	0.92	0.00	0.00	0.44	0.00
time (sec)	N/A	1.257	8.023	0.385	0.000	0.102	0.000	0.000	0.190	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	118	238	0	140	0	150	267	173
N.S.	1	1.00	1.26	2.53	0.00	1.49	0.00	1.60	2.84	1.84
time (sec)	N/A	0.612	3.727	8.315	0.000	0.096	0.000	0.250	0.251	40.070

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	102	220	0	121	0	150	197	149
N.S.	1	1.00	1.09	2.34	0.00	1.29	0.00	1.60	2.10	1.59
time (sec)	N/A	0.619	3.219	7.237	0.000	0.092	0.000	0.213	0.223	38.335

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	84	153	0	92	0	144	129	122
N.S.	1	1.00	0.89	1.63	0.00	0.98	0.00	1.53	1.37	1.30
time (sec)	N/A	0.626	2.519	7.050	0.000	0.086	0.000	0.235	0.208	2.037

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	57	111	0	61	0	140	62	75
N.S.	1	1.00	0.62	1.21	0.00	0.66	0.00	1.52	0.67	0.82
time (sec)	N/A	0.586	0.179	5.314	0.000	0.084	0.000	0.233	0.200	1.038

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	101	120	315	175	0	0	0	86	0
N.S.	1	1.01	1.20	3.15	1.75	0.00	0.00	0.00	0.86	0.00
time (sec)	N/A	0.840	3.474	4.668	0.149	0.000	0.000	0.000	0.179	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	100	147	260	0	0	0	0	105	0
N.S.	1	1.01	1.48	2.63	0.00	0.00	0.00	0.00	1.06	0.00
time (sec)	N/A	0.881	4.308	7.796	0.000	0.000	0.000	0.000	0.191	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	101	176	0	87	0	0	126	0
N.S.	1	1.00	1.10	1.91	0.00	0.95	0.00	0.00	1.37	0.00
time (sec)	N/A	0.626	4.157	8.050	0.000	0.091	0.000	0.000	0.185	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	103	279	0	106	0	0	145	153
N.S.	1	1.00	1.10	2.97	0.00	1.13	0.00	0.00	1.54	1.63
time (sec)	N/A	0.616	5.882	8.712	0.000	0.091	0.000	0.000	0.193	42.999

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	144	205	282	0	148	0	247	268	323
N.S.	1	0.99	1.40	1.93	0.00	1.01	0.00	1.69	1.84	2.21
time (sec)	N/A	0.791	9.490	7.953	0.000	0.115	0.000	0.264	0.249	42.466

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	144	172	253	0	126	0	247	266	174
N.S.	1	0.99	1.18	1.73	0.00	0.86	0.00	1.69	1.82	1.19
time (sec)	N/A	0.771	8.438	7.416	0.000	0.095	0.000	0.246	0.252	41.057

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	135	99	170	0	83	0	237	132	103
N.S.	1	1.01	0.74	1.27	0.00	0.62	0.00	1.77	0.99	0.77
time (sec)	N/A	0.734	2.860	6.463	0.000	0.101	0.000	0.257	0.205	2.170

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	81	151	0	87	0	144	127	122
N.S.	1	1.00	0.84	1.57	0.00	0.91	0.00	1.50	1.32	1.27
time (sec)	N/A	0.643	2.303	7.498	0.000	0.096	0.000	0.228	0.216	37.170

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	136	361	0	0	0	0	171	0
N.S.	1	1.00	0.94	2.49	0.00	0.00	0.00	0.00	1.18	0.00
time (sec)	N/A	0.851	6.938	6.537	0.000	0.000	0.000	0.000	0.199	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	155	210	407	366	0	0	0	210	0
N.S.	1	0.98	1.33	2.58	2.32	0.00	0.00	0.00	1.33	0.00
time (sec)	N/A	0.885	11.622	6.445	0.150	0.000	0.000	0.000	0.200	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	150	198	381	0	0	0	0	251	0
N.S.	1	1.01	1.33	2.56	0.00	0.00	0.00	0.00	1.68	0.00
time (sec)	N/A	0.908	11.681	6.853	0.000	0.000	0.000	0.000	0.199	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	125	254	0	125	0	0	290	0
N.S.	1	1.00	1.30	2.65	0.00	1.30	0.00	0.00	3.02	0.00
time (sec)	N/A	0.580	11.709	7.124	0.000	0.093	0.000	0.000	0.203	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	123	307	0	134	0	0	331	245
N.S.	1	1.00	0.84	2.10	0.00	0.92	0.00	0.00	2.27	1.68
time (sec)	N/A	0.766	12.063	7.398	0.000	0.101	0.000	0.000	0.212	45.208

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	152	126	441	0	155	0	0	370	279
N.S.	1	0.99	0.82	2.86	0.00	1.01	0.00	0.00	2.40	1.81
time (sec)	N/A	0.798	12.741	8.645	0.000	0.104	0.000	0.000	0.216	48.686

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	193	223	341	0	160	0	355	404	383
N.S.	1	0.97	1.13	1.72	0.00	0.81	0.00	1.79	2.04	1.93
time (sec)	N/A	0.954	13.421	79.272	0.000	0.109	0.000	0.295	0.290	41.889

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	184	116	216	0	117	0	355	202	131
N.S.	1	1.02	0.64	1.20	0.00	0.65	0.00	1.97	1.12	0.73
time (sec)	N/A	0.944	5.402	79.076	0.000	0.100	0.000	0.250	0.233	38.453

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	165	237	0	119	0	341	266	174
N.S.	1	1.00	1.16	1.67	0.00	0.84	0.00	2.40	1.87	1.23
time (sec)	N/A	0.744	9.119	7.538	0.000	0.103	0.000	0.320	0.249	38.579

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	102	176	0	117	0	150	197	149
N.S.	1	1.00	1.06	1.83	0.00	1.22	0.00	1.56	2.05	1.55
time (sec)	N/A	0.659	2.835	8.506	0.000	0.090	0.000	0.265	0.228	3.154

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	192	177	476	0	0	0	0	264	0
N.S.	1	0.99	0.92	2.47	0.00	0.00	0.00	0.00	1.37	0.00
time (sec)	N/A	1.090	11.153	6.985	0.000	0.000	0.000	0.000	0.201	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	200	231	470	0	0	0	0	320	0
N.S.	1	0.95	1.10	2.24	0.00	0.00	0.00	0.00	1.52	0.00
time (sec)	N/A	1.156	12.414	6.958	0.000	0.000	0.000	0.000	0.208	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	202	207	582	506	0	0	0	384	0
N.S.	1	0.95	0.98	2.75	2.39	0.00	0.00	0.00	1.81	0.00
time (sec)	N/A	1.167	11.787	7.283	0.156	0.000	0.000	0.000	0.217	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	200	204	537	0	0	0	0	440	0
N.S.	1	1.02	1.04	2.74	0.00	0.00	0.00	0.00	2.24	0.00
time (sec)	N/A	1.135	11.741	7.322	0.000	0.000	0.000	0.000	0.215	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	145	344	0	165	0	0	504	0
N.S.	1	1.00	1.51	3.58	0.00	1.72	0.00	0.00	5.25	0.00
time (sec)	N/A	0.554	13.781	8.063	0.000	0.096	0.000	0.000	0.228	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	146	429	0	182	0	0	560	341
N.S.	1	1.00	1.00	2.94	0.00	1.25	0.00	0.00	3.84	2.34
time (sec)	N/A	0.759	14.834	8.996	0.000	0.110	0.000	0.000	0.222	49.113

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	144	495	0	196	0	0	624	357
N.S.	1	1.00	0.73	2.53	0.00	1.00	0.00	0.00	3.18	1.82
time (sec)	N/A	0.984	16.557	10.492	0.000	0.112	0.000	0.000	0.233	45.082

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	242	870	427	0	183	0	453	540	482
N.S.	1	0.97	3.48	1.71	0.00	0.73	0.00	1.81	2.16	1.93
time (sec)	N/A	1.193	16.324	90.079	0.000	0.147	0.000	0.303	0.328	45.851

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	233	138	258	0	134	0	453	272	384
N.S.	1	1.03	0.61	1.14	0.00	0.59	0.00	2.00	1.20	1.70
time (sec)	N/A	1.124	7.029	90.028	0.000	0.113	0.000	0.285	0.255	40.763

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	191	232	325	0	150	0	453	400	383
N.S.	1	0.99	1.21	1.69	0.00	0.78	0.00	2.36	2.08	1.99
time (sec)	N/A	0.948	10.073	89.152	0.000	0.112	0.000	0.314	0.286	41.700

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	212	250	0	142	0	247	268	321
N.S.	1	1.00	1.49	1.76	0.00	1.00	0.00	1.74	1.89	2.26
time (sec)	N/A	0.747	10.339	8.217	0.000	0.099	0.000	0.265	0.239	41.277

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	121	238	0	139	0	150	265	173
N.S.	1	1.00	1.26	2.48	0.00	1.45	0.00	1.56	2.76	1.80
time (sec)	N/A	0.666	3.312	9.566	0.000	0.096	0.000	0.275	0.258	38.805

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	183	528	0	0	0	0	352	0
N.S.	1	1.00	0.77	2.21	0.00	0.00	0.00	0.00	1.47	0.00
time (sec)	N/A	1.303	13.903	7.500	0.000	0.000	0.000	0.000	0.210	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	251	292	527	0	0	0	0	428	0
N.S.	1	0.93	1.08	1.94	0.00	0.00	0.00	0.00	1.58	0.00
time (sec)	N/A	1.342	14.780	7.507	0.000	0.000	0.000	0.000	0.222	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	249	251	626	0	0	0	0	512	0
N.S.	1	0.95	0.95	2.38	0.00	0.00	0.00	0.00	1.95	0.00
time (sec)	N/A	1.335	13.506	7.795	0.000	0.000	0.000	0.000	0.218	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	264	254	244	706	749	0	0	0	588	0
N.S.	1	0.96	0.92	2.67	2.84	0.00	0.00	0.00	2.23	0.00
time (sec)	N/A	1.401	14.026	7.940	0.154	0.000	0.000	0.000	0.231	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	250	238	659	0	0	0	0	672	0
N.S.	1	1.01	0.96	2.67	0.00	0.00	0.00	0.00	2.72	0.00
time (sec)	N/A	1.347	13.583	7.899	0.000	0.000	0.000	0.000	0.227	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	434	414	0	199	0	0	748	0
N.S.	1	1.00	4.52	4.31	0.00	2.07	0.00	0.00	7.79	0.00
time (sec)	N/A	0.556	17.291	9.454	0.000	0.102	0.000	0.000	0.241	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	442	429	0	214	0	0	832	406
N.S.	1	1.00	3.03	2.94	0.00	1.47	0.00	0.00	5.70	2.78
time (sec)	N/A	0.770	17.276	9.856	0.000	0.111	0.000	0.000	0.242	46.669

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-1)	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	198	442	517	0	234	0	0	908	827
N.S.	1	0.98	2.19	2.56	0.00	1.16	0.00	0.00	4.50	4.09
time (sec)	N/A	1.004	17.370	12.119	0.000	0.118	0.000	0.000	0.260	52.463

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-1)	A	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	436	574	0	243	0	0	992	841
N.S.	1	1.00	1.77	2.33	0.00	0.99	0.00	0.00	4.03	3.42
time (sec)	N/A	1.226	17.411	15.052	0.000	0.130	0.000	0.000	0.251	54.774

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	192	185	442	0	0	0	0	260	0
N.S.	1	0.97	0.94	2.24	0.00	0.00	0.00	0.00	1.32	0.00
time (sec)	N/A	1.099	11.967	7.303	0.000	0.000	0.000	0.000	0.207	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	145	146	327	0	0	0	0	172	0
N.S.	1	0.99	1.00	2.24	0.00	0.00	0.00	0.00	1.18	0.00
time (sec)	N/A	0.843	7.873	7.087	0.000	0.000	0.000	0.000	0.190	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	98	119	283	176	0	0	0	85	0
N.S.	1	1.02	1.24	2.95	1.83	0.00	0.00	0.00	0.89	0.00
time (sec)	N/A	0.842	3.323	5.337	0.144	0.000	0.000	0.000	0.192	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	113	116	83	325	0	0	0	0	93	0
N.S.	1	1.03	0.73	2.88	0.00	0.00	0.00	0.00	0.82	0.00
time (sec)	N/A	0.775	1.866	4.674	0.000	0.000	0.000	0.000	0.185	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	191	518	0	335	0	0	128	0
N.S.	1	1.00	1.85	5.03	0.00	3.25	0.00	0.00	1.24	0.00
time (sec)	N/A	0.649	3.833	7.970	0.000	0.121	0.000	0.000	0.192	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	222	647	0	422	0	0	129	0
N.S.	1	1.00	1.45	4.23	0.00	2.76	0.00	0.00	0.84	0.00
time (sec)	N/A	0.851	4.139	8.243	0.000	0.125	0.000	0.000	0.188	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	251	271	479	0	0	0	0	430	0
N.S.	1	0.93	1.00	1.77	0.00	0.00	0.00	0.00	1.59	0.00
time (sec)	N/A	1.429	14.283	7.970	0.000	0.000	0.000	0.000	0.212	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	200	212	440	0	0	0	0	320	0
N.S.	1	0.95	1.01	2.10	0.00	0.00	0.00	0.00	1.52	0.00
time (sec)	N/A	1.138	12.269	7.211	0.000	0.000	0.000	0.000	0.216	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	155	190	389	367	0	0	0	212	0
N.S.	1	0.97	1.19	2.45	2.31	0.00	0.00	0.00	1.33	0.00
time (sec)	N/A	0.901	11.610	6.928	0.148	0.000	0.000	0.000	0.199	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	102	143	284	0	0	0	0	105	0
N.S.	1	1.02	1.43	2.84	0.00	0.00	0.00	0.00	1.05	0.00
time (sec)	N/A	0.883	3.650	8.805	0.000	0.000	0.000	0.000	0.192	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	186	517	0	327	0	0	125	0
N.S.	1	1.00	1.81	5.02	0.00	3.17	0.00	0.00	1.21	0.00
time (sec)	N/A	0.632	3.850	8.018	0.000	0.121	0.000	0.000	0.188	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	148	142	389	0	270	0	0	112	0
N.S.	1	0.99	0.95	2.59	0.00	1.80	0.00	0.00	0.75	0.00
time (sec)	N/A	0.855	4.008	7.698	0.000	0.160	0.000	0.000	0.183	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	207	306	724	0	435	0	0	165	0
N.S.	1	0.95	1.41	3.34	0.00	2.00	0.00	0.00	0.76	0.00
time (sec)	N/A	1.175	5.347	8.424	0.000	0.138	0.000	0.000	0.190	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	299	573	638	0	0	0	0	636	0
N.S.	1	0.93	1.77	1.98	0.00	0.00	0.00	0.00	1.97	0.00
time (sec)	N/A	1.719	17.271	8.760	0.000	0.000	0.000	0.000	0.244	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	250	243	578	0	0	0	0	510	0
N.S.	1	0.95	0.92	2.20	0.00	0.00	0.00	0.00	1.94	0.00
time (sec)	N/A	1.406	13.128	8.132	0.000	0.000	0.000	0.000	0.217	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	203	199	534	504	0	0	0	380	0
N.S.	1	0.96	0.94	2.53	2.39	0.00	0.00	0.00	1.80	0.00
time (sec)	N/A	1.124	11.772	7.604	0.157	0.000	0.000	0.000	0.202	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	152	179	425	0	0	0	0	252	0
N.S.	1	1.02	1.20	2.85	0.00	0.00	0.00	0.00	1.69	0.00
time (sec)	N/A	0.861	11.750	7.422	0.000	0.000	0.000	0.000	0.200	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	99	206	0	85	0	0	125	156
N.S.	1	1.00	1.05	2.19	0.00	0.90	0.00	0.00	1.33	1.66
time (sec)	N/A	0.626	3.284	9.231	0.000	0.114	0.000	0.000	0.183	37.093

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	153	214	647	0	406	0	0	129	0
N.S.	1	1.01	1.42	4.28	0.00	2.69	0.00	0.00	0.85	0.00
time (sec)	N/A	0.859	4.144	8.330	0.000	0.136	0.000	0.000	0.199	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	202	305	724	0	417	0	0	160	0
N.S.	1	0.97	1.47	3.48	0.00	2.00	0.00	0.00	0.77	0.00
time (sec)	N/A	1.102	5.411	8.664	0.000	0.134	0.000	0.000	0.183	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	248	91	455	0	304	0	0	133	0
N.S.	1	1.01	0.37	1.86	0.00	1.24	0.00	0.00	0.54	0.00
time (sec)	N/A	1.406	4.958	7.931	0.000	0.130	0.000	0.000	0.190	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	173	211	0	0	0	0	0	0	67	0
N.S.	1	1.22	0.00	0.00	0.00	0.00	0.00	0.00	0.39	0.00
time (sec)	N/A	0.759	0.000	0.000	0.000	0.000	0.000	0.000	0.391	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	137	604	0	0	0	0	0	198	0
N.S.	1	0.94	4.17	0.00	0.00	0.00	0.00	0.00	1.37	0.00
time (sec)	N/A	0.669	32.504	0.000	0.000	0.000	0.000	0.000	0.220	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	137	443	0	0	0	0	0	144	0
N.S.	1	0.94	3.06	0.00	0.00	0.00	0.00	0.00	0.99	0.00
time (sec)	N/A	0.640	16.146	0.000	0.000	0.000	0.000	0.000	0.195	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	137	133	468	0	0	0	0	0	92	0
N.S.	1	0.97	3.42	0.00	0.00	0.00	0.00	0.00	0.67	0.00
time (sec)	N/A	0.617	5.479	0.000	0.000	0.000	0.000	0.000	0.194	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	101	0	0	0	0	0	40	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.422	0.596	0.000	0.000	0.000	0.000	0.000	0.213	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	122	0	0	0	0	0	0	67	0
N.S.	1	0.95	0.00	0.00	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.644	0.000	0.000	0.000	0.000	0.000	0.000	0.185	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	138	0	0	0	0	0	0	85	0
N.S.	1	0.93	0.00	0.00	0.00	0.00	0.00	0.00	0.57	0.00
time (sec)	N/A	0.637	0.000	0.000	0.000	0.000	0.000	0.000	0.187	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	138	0	0	0	0	0	0	107	0
N.S.	1	0.93	0.00	0.00	0.00	0.00	0.00	0.00	0.72	0.00
time (sec)	N/A	0.642	0.000	0.000	0.000	0.000	0.000	0.000	0.190	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	118	123	132	0	0	0	0	0	90	0
N.S.	1	1.04	1.12	0.00	0.00	0.00	0.00	0.00	0.76	0.00
time (sec)	N/A	0.654	22.413	0.000	0.000	0.000	0.000	0.000	0.202	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	118	123	132	0	0	0	0	0	90	0
N.S.	1	1.04	1.12	0.00	0.00	0.00	0.00	0.00	0.76	0.00
time (sec)	N/A	0.673	22.273	0.000	0.000	0.000	0.000	0.000	0.187	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	B	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	275	231	667	0	725	562	0	0	213	749
N.S.	1	0.84	2.43	0.00	2.64	2.04	0.00	0.00	0.77	2.72
time (sec)	N/A	1.015	14.937	0.000	0.167	0.133	0.000	0.000	0.257	48.171

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	166	174	174	0	498	313	0	0	139	480
N.S.	1	1.05	1.05	0.00	3.00	1.89	0.00	0.00	0.84	2.89
time (sec)	N/A	0.752	6.777	0.000	0.152	0.109	0.000	0.000	0.213	43.211

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	A	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	116	0	323	165	0	0	66	105
N.S.	1	1.00	1.12	0.00	3.11	1.59	0.00	0.00	0.63	1.01
time (sec)	N/A	0.561	2.118	0.000	0.146	0.100	0.000	0.000	0.203	1.598

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	118	123	132	0	0	0	0	0	90	0
N.S.	1	1.04	1.12	0.00	0.00	0.00	0.00	0.00	0.76	0.00
time (sec)	N/A	0.648	0.514	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	139	144	0	0	0	0	0	109	0
N.S.	1	1.04	1.07	0.00	0.00	0.00	0.00	0.00	0.81	0.00
time (sec)	N/A	0.674	36.363	0.000	0.000	0.000	0.000	0.000	0.190	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	139	159	0	0	0	0	0	130	0
N.S.	1	1.04	1.19	0.00	0.00	0.00	0.00	0.00	0.97	0.00
time (sec)	N/A	0.733	48.215	0.000	0.000	0.000	0.000	0.000	0.190	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	267	233	183	0	0	205	0	0	255	368
N.S.	1	0.87	0.69	0.00	0.00	0.77	0.00	0.00	0.96	1.38
time (sec)	N/A	0.996	1.074	0.000	0.000	0.111	0.000	0.000	0.197	47.021

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	191	176	141	0	0	137	0	0	215	239
N.S.	1	0.92	0.74	0.00	0.00	0.72	0.00	0.00	1.13	1.25
time (sec)	N/A	0.748	0.723	0.000	0.000	0.096	0.000	0.000	0.198	39.013

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	92	0	0	89	0	0	163	134
N.S.	1	1.00	0.81	0.00	0.00	0.78	0.00	0.00	1.43	1.18
time (sec)	N/A	0.517	0.568	0.000	0.000	0.096	0.000	0.000	0.185	37.593

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	168	201	310	0	0	0	0	0	123	0
N.S.	1	1.20	1.85	0.00	0.00	0.00	0.00	0.00	0.73	0.00
time (sec)	N/A	0.733	12.304	0.000	0.000	0.000	0.000	0.000	0.197	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	159	196	402	0	0	0	0	0	71	0
N.S.	1	1.23	2.53	0.00	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.639	9.295	0.000	0.000	0.000	0.000	0.000	0.186	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	171	209	0	0	0	0	0	0	153	0
N.S.	1	1.22	0.00	0.00	0.00	0.00	0.00	0.00	0.89	0.00
time (sec)	N/A	0.723	0.000	0.000	0.000	0.000	0.000	0.000	0.195	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	174	211	0	0	0	0	0	0	235	0
N.S.	1	1.21	0.00	0.00	0.00	0.00	0.00	0.00	1.35	0.00
time (sec)	N/A	0.750	0.000	0.000	0.000	0.000	0.000	0.000	0.192	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	63	62	0	78	898	6973	249	64
N.S.	1	1.00	1.85	1.82	0.00	2.29	26.41	205.09	7.32	1.88
time (sec)	N/A	0.510	2.286	8.720	0.000	0.088	73.186	12.269	0.206	37.510

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	67	60	0	77	898	6974	239	61
N.S.	1	1.00	1.97	1.76	0.00	2.26	26.41	205.12	7.03	1.79
time (sec)	N/A	0.473	8.992	7.866	0.000	0.091	71.837	11.843	0.205	37.143

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	66	61	0	78	898	6973	237	61
N.S.	1	1.00	2.00	1.85	0.00	2.36	27.21	211.30	7.18	1.85
time (sec)	N/A	0.483	8.731	7.449	0.000	0.093	71.041	11.876	0.210	36.929

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	61	61	0	77	898	6974	247	64
N.S.	1	1.00	1.74	1.74	0.00	2.20	25.66	199.26	7.06	1.83
time (sec)	N/A	0.493	1.656	7.266	0.000	0.091	72.108	12.208	0.217	36.354

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	37	0	36	0	8947	174	36
N.S.	1	1.00	1.00	1.03	0.00	1.00	0.00	248.53	4.83	1.00
time (sec)	N/A	0.316	3.427	5.747	0.000	0.094	0.000	50.837	12.413	35.475

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	38	0	37	0	8948	170	37
N.S.	1	1.00	1.00	1.03	0.00	1.00	0.00	241.84	4.59	1.00
time (sec)	N/A	0.316	2.295	5.552	0.000	0.092	0.000	50.236	12.736	35.478

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	87	158	157	105	440	131	116	300
N.S.	1	1.00	0.62	1.13	1.12	0.75	3.14	0.94	0.83	2.14
time (sec)	N/A	0.435	0.614	0.800	0.039	0.087	0.544	0.269	0.184	37.805

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	77	79	138	91	359	113	100	256
N.S.	1	1.00	0.64	0.65	1.14	0.75	2.97	0.93	0.83	2.12
time (sec)	N/A	0.387	0.403	122.639	0.039	0.089	0.385	0.212	0.185	37.689

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	106	57	112	77	267	77	84	292
N.S.	1	1.00	1.10	0.59	1.17	0.80	2.78	0.80	0.88	3.04
time (sec)	N/A	0.330	1.229	28.499	0.040	0.080	0.267	0.231	0.173	37.467

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	54	89	86	63	196	77	68	250
N.S.	1	1.00	0.66	1.09	1.05	0.77	2.39	0.94	0.83	3.05
time (sec)	N/A	0.447	1.035	0.421	0.035	0.099	0.185	0.262	0.182	38.168

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	54	58	85	92	0	107	66	212
N.S.	1	1.00	0.71	0.76	1.12	1.21	0.00	1.41	0.87	2.79
time (sec)	N/A	0.319	0.266	0.553	0.038	0.091	0.000	0.241	0.183	37.595

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	62	80	83	111	0	153	90	226
N.S.	1	1.00	0.78	1.01	1.05	1.41	0.00	1.94	1.14	2.86
time (sec)	N/A	0.438	0.450	0.595	0.037	0.096	0.000	0.204	0.183	40.426

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	142	78	90	152	0	137	97	220
N.S.	1	1.00	1.82	1.00	1.15	1.95	0.00	1.76	1.24	2.82
time (sec)	N/A	0.342	0.231	0.773	0.039	0.088	0.000	0.254	0.171	32.268

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	141	99	117	175	0	150	87	245
N.S.	1	1.00	1.81	1.27	1.50	2.24	0.00	1.92	1.12	3.14
time (sec)	N/A	0.345	0.919	0.743	0.037	0.095	0.000	0.247	0.170	37.838

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	210	114	145	166	0	174	91	244
N.S.	1	1.00	2.44	1.33	1.69	1.93	0.00	2.02	1.06	2.84
time (sec)	N/A	0.364	0.404	1.024	0.038	0.088	0.000	0.253	0.180	37.676

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	107	268	121	175	201	0	174	107	244
N.S.	1	1.02	2.55	1.15	1.67	1.91	0.00	1.66	1.02	2.32
time (sec)	N/A	0.514	0.472	1.174	0.037	0.085	0.000	0.272	0.178	34.912

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	306	163	207	240	0	242	123	340
N.S.	1	1.00	2.35	1.25	1.59	1.85	0.00	1.86	0.95	2.62
time (sec)	N/A	0.421	0.507	1.263	0.040	0.091	0.000	0.291	0.175	34.652

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	211	154	715	248	3614	156	277	326
N.S.	1	1.00	1.64	1.19	5.54	1.92	28.02	1.21	2.15	2.53
time (sec)	N/A	0.442	6.177	0.976	0.135	0.085	25.144	0.286	0.172	38.837

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	155	115	543	225	2290	113	251	261
N.S.	1	1.00	1.50	1.12	5.27	2.18	22.23	1.10	2.44	2.53
time (sec)	N/A	0.412	4.387	0.680	0.122	0.090	14.482	0.273	0.174	38.476

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	112	85	392	204	1268	93	202	178
N.S.	1	1.00	1.26	0.96	4.40	2.29	14.25	1.04	2.27	2.00
time (sec)	N/A	0.398	4.162	0.543	0.122	0.081	8.275	0.256	0.180	36.358

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	107	62	348	156	461	63	112	110
N.S.	1	1.00	1.30	0.76	4.24	1.90	5.62	0.77	1.37	1.34
time (sec)	N/A	0.359	4.272	0.440	0.043	0.122	4.713	0.255	0.172	34.998

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	61	92	62	387	154	573	79	125	134
N.S.	1	1.05	1.59	1.07	6.67	2.66	9.88	1.36	2.16	2.31
time (sec)	N/A	0.392	3.454	0.414	0.043	0.070	2.558	0.183	0.173	35.151

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	71	95	433	310	0	99	250	199
N.S.	1	1.00	0.72	0.97	4.42	3.16	0.00	1.01	2.55	2.03
time (sec)	N/A	0.395	5.312	0.672	0.044	0.086	0.000	0.264	0.175	37.128

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	167	120	519	406	0	146	298	210
N.S.	1	1.00	1.48	1.06	4.59	3.59	0.00	1.29	2.64	1.86
time (sec)	N/A	0.642	4.272	0.723	0.047	0.092	0.000	0.235	0.172	37.958

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	245	148	622	498	0	180	326	288
N.S.	1	1.00	1.78	1.07	4.51	3.61	0.00	1.30	2.36	2.09
time (sec)	N/A	0.470	5.294	0.878	0.048	0.089	0.000	0.294	0.187	37.248

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	348	174	706	594	0	213	352	314
N.S.	1	1.00	2.27	1.14	4.61	3.88	0.00	1.39	2.30	2.05
time (sec)	N/A	0.506	7.087	0.980	0.048	0.103	0.000	0.282	0.173	37.055

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	336	267	244	406	239	996	309	541	830
N.S.	1	1.03	0.82	0.75	1.24	0.73	3.05	0.94	1.65	2.54
time (sec)	N/A	1.141	3.965	98.550	0.041	0.096	0.378	0.266	0.194	37.390

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	220	185	172	264	160	571	194	324	547
N.S.	1	1.03	0.87	0.81	1.24	0.75	2.68	0.91	1.52	2.57
time (sec)	N/A	0.766	2.691	13.109	0.045	0.092	0.257	0.213	0.178	36.902

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	123	104	147	143	84	277	98	153	134
N.S.	1	1.11	0.94	1.32	1.29	0.76	2.50	0.88	1.38	1.21
time (sec)	N/A	0.483	1.286	0.297	0.040	0.081	0.146	0.225	0.170	34.586

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	45	44	57	43	94	46	55	100
N.S.	1	1.00	0.94	0.92	1.19	0.90	1.96	0.96	1.15	2.08
time (sec)	N/A	0.215	0.277	0.582	0.036	0.079	0.097	0.224	0.180	34.566

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	109	196	120	0	290	5508	137	147	3074
N.S.	1	1.11	2.00	1.22	0.00	2.96	56.20	1.40	1.50	31.37
time (sec)	N/A	0.652	1.942	0.456	0.000	0.103	125.832	0.236	0.170	38.145

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	155	217	174	0	655	0	197	599	5102
N.S.	1	1.25	1.75	1.40	0.00	5.28	0.00	1.59	4.83	41.15
time (sec)	N/A	0.731	3.627	0.464	0.000	0.116	0.000	0.250	0.181	43.618

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	191	345	424	0	967	0	570	1477	554
N.S.	1	1.09	1.96	2.41	0.00	5.49	0.00	3.24	8.39	3.15
time (sec)	N/A	0.848	5.410	0.729	0.000	0.125	0.000	0.294	0.186	37.528

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	464	480	437	745	724	364	1865	468	705	1291
N.S.	1	1.03	0.94	1.61	1.56	0.78	4.02	1.01	1.52	2.78
time (sec)	N/A	1.788	3.710	0.828	0.050	0.119	0.551	0.307	0.180	38.366

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	350	296	222	478	245	1129	306	442	765
N.S.	1	1.04	0.88	0.66	1.42	0.73	3.36	0.91	1.32	2.28
time (sec)	N/A	1.368	1.874	296.270	0.045	0.099	0.370	0.255	0.183	37.561

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	145	144	278	268	144	571	168	225	492
N.S.	1	0.87	0.87	1.67	1.61	0.87	3.44	1.01	1.36	2.96
time (sec)	N/A	0.618	0.820	0.487	0.038	0.097	0.240	0.238	0.173	36.731

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	83	106	70	114	70	199	85	90	91
N.S.	1	0.88	1.13	0.74	1.21	0.74	2.12	0.90	0.96	0.97
time (sec)	N/A	0.302	0.417	233.872	0.037	0.078	0.143	0.220	0.174	35.415

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	185	177	235	0	452	0	305	400	7371
N.S.	1	1.08	1.04	1.37	0.00	2.64	0.00	1.78	2.34	43.11
time (sec)	N/A	1.063	0.853	0.752	0.000	0.109	0.000	0.240	0.191	42.971

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	213	192	251	0	731	0	480	872	8706
N.S.	1	1.08	0.97	1.27	0.00	3.69	0.00	2.42	4.40	43.97
time (sec)	N/A	1.179	6.157	1.021	0.000	0.124	0.000	0.234	0.183	44.029

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	255	226	431	0	1483	0	678	1748	8632
N.S.	1	1.19	1.05	2.00	0.00	6.90	0.00	3.15	8.13	40.15
time (sec)	N/A	1.271	6.611	0.837	0.000	0.148	0.000	0.248	0.199	44.459

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	604	628	528	1077	1056	432	2878	559	869	1395
N.S.	1	1.04	0.87	1.78	1.75	0.72	4.76	0.93	1.44	2.31
time (sec)	N/A	2.634	5.649	1.069	0.070	0.126	0.760	0.304	0.215	38.132

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	485	355	725	704	299	1804	374	560	976
N.S.	1	1.05	0.77	1.57	1.52	0.65	3.90	0.81	1.21	2.11
time (sec)	N/A	2.035	2.980	0.869	0.050	0.110	0.534	0.248	0.190	37.391

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	B	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	163	156	414	398	178	960	212	297	550
N.S.	1	0.81	0.78	2.06	1.98	0.89	4.78	1.05	1.48	2.74
time (sec)	N/A	0.686	1.176	0.673	0.044	0.092	0.349	0.241	0.177	36.065

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	101	120	178	171	93	371	112	124	330
N.S.	1	0.80	0.94	1.40	1.35	0.73	2.92	0.88	0.98	2.60
time (sec)	N/A	0.367	0.684	0.451	0.038	0.081	0.207	0.216	0.175	35.797

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	269	233	378	0	626	0	597	714	10256
N.S.	1	1.09	0.95	1.54	0.00	2.54	0.00	2.43	2.90	41.69
time (sec)	N/A	1.710	3.867	2.089	0.000	0.118	0.000	0.237	0.192	43.808

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	295	244	406	0	1027	0	571	1223	11993
N.S.	1	1.04	0.86	1.43	0.00	3.63	0.00	2.02	4.32	42.38
time (sec)	N/A	1.805	6.762	2.522	0.000	0.149	0.000	0.247	0.193	47.635

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	330	830	586	0	1670	0	953	2339	13891
N.S.	1	1.08	2.72	1.92	0.00	5.48	0.00	3.12	7.67	45.54
time (sec)	N/A	1.922	9.026	2.954	0.000	0.169	0.000	0.279	0.214	45.748

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	788	337	1124	470	14644	460	810	839
N.S.	1	1.00	3.58	1.53	5.11	2.14	66.56	2.09	3.68	3.81
time (sec)	N/A	0.745	7.887	2.161	0.141	0.145	4.307	0.217	0.182	38.143

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	137	200	193	606	303	5763	214	466	297
N.S.	1	0.96	1.40	1.35	4.24	2.12	40.30	1.50	3.26	2.08
time (sec)	N/A	0.487	6.856	0.825	0.129	0.092	2.277	0.228	0.183	40.113

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	69	126	81	256	154	1307	151	161	122
N.S.	1	1.03	1.88	1.21	3.82	2.30	19.51	2.25	2.40	1.82
time (sec)	N/A	0.572	0.855	0.497	0.115	0.082	1.148	0.199	0.176	36.445

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	79	42	78	66	109	38	62	35
N.S.	1	1.00	2.26	1.20	2.23	1.89	3.11	1.09	1.77	1.00
time (sec)	N/A	0.277	0.266	0.220	0.114	0.075	0.604	0.223	0.179	35.786

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	107	148	94	0	595	0	110	324	154
N.S.	1	1.06	1.47	0.93	0.00	5.89	0.00	1.09	3.21	1.52
time (sec)	N/A	0.456	6.966	0.535	0.000	0.106	0.000	0.231	0.204	36.235

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	189	209	197	0	1538	0	425	2468	437
N.S.	1	1.04	1.15	1.09	0.00	8.50	0.00	2.35	13.64	2.41
time (sec)	N/A	0.733	4.811	0.856	0.000	0.126	0.000	0.245	0.190	37.782

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	302	313	482	0	3303	0	727	5820	1076
N.S.	1	1.07	1.11	1.70	0.00	11.67	0.00	2.57	20.57	3.80
time (sec)	N/A	1.148	5.511	1.815	0.000	0.206	0.000	0.261	0.208	40.746

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	234	547	340	1382	584	14612	472	1041	663
N.S.	1	1.03	2.40	1.49	6.06	2.56	64.09	2.07	4.57	2.91
time (sec)	N/A	0.901	4.813	2.581	0.148	0.098	8.521	0.244	0.184	39.537

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	138	338	193	831	375	5358	264	618	365
N.S.	1	1.05	2.56	1.46	6.30	2.84	40.59	2.00	4.68	2.77
time (sec)	N/A	1.052	2.380	1.036	0.132	0.090	4.605	0.227	0.184	39.330

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	89	180	110	454	208	1062	133	188	94
N.S.	1	1.05	2.12	1.29	5.34	2.45	12.49	1.56	2.21	1.11
time (sec)	N/A	0.588	2.734	0.517	0.122	0.079	2.319	0.213	0.182	36.591

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	64	43	60	214	117	372	64	79	97
N.S.	1	0.98	0.66	0.92	3.29	1.80	5.72	0.98	1.22	1.49
time (sec)	N/A	0.297	0.071	0.349	0.038	0.069	1.256	0.206	0.181	36.217

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	165	229	159	0	1285	0	249	837	302
N.S.	1	1.09	1.51	1.05	0.00	8.45	0.00	1.64	5.51	1.99
time (sec)	N/A	0.761	4.174	0.865	0.000	0.112	0.000	0.267	0.197	37.969

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	288	313	263	0	3123	0	411	4579	844
N.S.	1	1.05	1.14	0.96	0.00	11.36	0.00	1.49	16.65	3.07
time (sec)	N/A	1.187	9.427	1.510	0.000	0.180	0.000	0.212	0.201	40.452

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	417	1522	547	0	4997	0	911	8867	1686
N.S.	1	1.08	3.94	1.42	0.00	12.95	0.00	2.36	22.97	4.37
time (sec)	N/A	1.701	12.548	3.471	0.000	0.265	0.000	0.287	0.285	42.287

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	237	366	349	1682	649	11456	567	1123	593
N.S.	1	1.05	1.63	1.55	7.48	2.88	50.92	2.52	4.99	2.64
time (sec)	N/A	1.617	5.462	2.446	0.161	0.109	15.430	0.213	0.182	38.548

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	174	514	240	1132	432	3468	362	481	286
N.S.	1	1.06	3.13	1.46	6.90	2.63	21.15	2.21	2.93	1.74
time (sec)	N/A	1.042	1.700	0.759	0.140	0.095	8.819	0.266	0.184	40.047

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	129	176	139	733	271	1819	210	247	245
N.S.	1	1.02	1.39	1.09	5.77	2.13	14.32	1.65	1.94	1.93
time (sec)	N/A	0.616	3.388	0.602	0.058	0.073	4.937	0.198	0.180	37.422

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	99	63	95	387	190	1015	122	160	150
N.S.	1	0.97	0.62	0.93	3.79	1.86	9.95	1.20	1.57	1.47
time (sec)	N/A	0.384	0.105	0.420	0.041	0.072	2.724	0.189	0.180	36.796

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	258	502	252	0	2292	0	553	1863	591
N.S.	1	1.13	2.19	1.10	0.00	10.01	0.00	2.41	8.14	2.58
time (sec)	N/A	1.227	7.615	1.205	0.000	0.155	0.000	0.281	0.203	39.947

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	409	1253	355	0	4486	0	743	6828	1349
N.S.	1	1.07	3.29	0.93	0.00	11.77	0.00	1.95	17.92	3.54
time (sec)	N/A	1.795	12.516	2.507	0.000	0.229	0.000	0.316	0.255	41.920

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	508	551	548	639	0	7283	0	1224	12007	2387
N.S.	1	1.08	1.08	1.26	0.00	14.34	0.00	2.41	23.64	4.70
time (sec)	N/A	2.354	11.779	4.997	0.000	0.371	0.000	0.365	3.211	45.300

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	235	305	242	0	467	0	550	199	0
N.S.	1	0.92	1.19	0.95	0.00	1.82	0.00	2.15	0.78	0.00
time (sec)	N/A	1.057	2.466	1.275	0.000	0.103	0.000	0.386	0.202	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	187	176	161	0	306	0	348	141	0
N.S.	1	0.97	0.92	0.84	0.00	1.59	0.00	1.81	0.73	0.00
time (sec)	N/A	0.800	1.626	1.002	0.000	0.087	0.000	0.258	0.209	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	129	117	102	0	175	0	186	83	0
N.S.	1	1.09	0.99	0.86	0.00	1.48	0.00	1.58	0.70	0.00
time (sec)	N/A	0.654	1.372	0.868	0.000	0.084	0.000	0.308	0.182	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	82	58	0	85	0	85	37	0
N.S.	1	1.00	1.32	0.94	0.00	1.37	0.00	1.37	0.60	0.00
time (sec)	N/A	0.300	0.297	0.520	0.000	0.087	0.000	0.225	0.189	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	903	139	0	651	0	125	62	0
N.S.	1	1.00	9.03	1.39	0.00	6.51	0.00	1.25	0.62	0.00
time (sec)	N/A	0.505	11.795	0.446	0.000	0.537	0.000	0.229	0.200	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	901	274	0	1012	0	212	94	0
N.S.	1	1.00	7.15	2.17	0.00	8.03	0.00	1.68	0.75	0.00
time (sec)	N/A	0.538	12.542	0.485	0.000	0.779	0.000	0.211	0.190	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	186	967	628	0	1750	0	422	126	0
N.S.	1	0.97	5.04	3.27	0.00	9.11	0.00	2.20	0.66	0.00
time (sec)	N/A	0.747	15.168	0.511	0.000	1.142	0.000	0.271	0.201	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	374	316	390	312	0	637	0	755	406	0
N.S.	1	0.84	1.04	0.83	0.00	1.70	0.00	2.02	1.09	0.00
time (sec)	N/A	1.532	6.974	1.433	0.000	0.114	0.000	0.374	0.229	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	268	267	207	0	430	0	497	290	0
N.S.	1	0.91	0.91	0.70	0.00	1.46	0.00	1.69	0.99	0.00
time (sec)	N/A	1.280	4.668	1.129	0.000	0.099	0.000	0.337	0.226	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	163	144	150	0	257	0	285	174	0
N.S.	1	0.99	0.87	0.91	0.00	1.56	0.00	1.73	1.05	0.00
time (sec)	N/A	0.826	3.655	0.972	0.000	0.091	0.000	0.215	0.199	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	99	101	77	0	137	0	139	80	0
N.S.	1	0.98	1.00	0.76	0.00	1.36	0.00	1.38	0.79	0.00
time (sec)	N/A	0.397	1.991	0.594	0.000	0.084	0.000	0.214	0.193	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	159	898	292	0	880	0	274	129	0
N.S.	1	1.04	5.87	1.91	0.00	5.75	0.00	1.79	0.84	0.00
time (sec)	N/A	0.869	6.028	0.477	0.000	0.687	0.000	0.207	0.198	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	199	922	592	0	1428	0	364	193	0
N.S.	1	1.04	4.83	3.10	0.00	7.48	0.00	1.91	1.01	0.00
time (sec)	N/A	0.946	13.431	0.541	0.000	0.809	0.000	0.241	0.216	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	229	957	895	0	2208	0	624	257	0
N.S.	1	1.04	4.33	4.05	0.00	9.99	0.00	2.82	1.16	0.00
time (sec)	N/A	1.030	16.344	0.552	0.000	1.330	0.000	0.287	0.212	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	534	420	1565	374	0	863	0	1014	620	0
N.S.	1	0.79	2.93	0.70	0.00	1.62	0.00	1.90	1.16	0.00
time (sec)	N/A	2.244	10.000	208.688	0.000	0.132	0.000	0.571	0.259	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	429	371	891	257	0	593	0	684	446	0
N.S.	1	0.86	2.08	0.60	0.00	1.38	0.00	1.59	1.04	0.00
time (sec)	N/A	1.927	9.638	39.146	0.000	0.111	0.000	0.348	0.228	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	197	202	152	0	361	0	404	272	0
N.S.	1	0.93	0.95	0.72	0.00	1.70	0.00	1.91	1.28	0.00
time (sec)	N/A	0.944	8.009	7.334	0.000	0.106	0.000	0.298	0.223	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	133	119	99	0	191	0	202	128	0
N.S.	1	0.96	0.86	0.72	0.00	1.38	0.00	1.46	0.93	0.00
time (sec)	N/A	0.509	4.030	1.312	0.000	0.083	0.000	0.253	0.197	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	232	992	543	0	1314	0	523	201	0
N.S.	1	1.06	4.55	2.49	0.00	6.03	0.00	2.40	0.92	0.00
time (sec)	N/A	1.379	9.930	2.602	0.000	1.141	0.000	0.270	0.223	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	275	1002	932	0	2046	0	597	297	0
N.S.	1	1.04	3.78	3.52	0.00	7.72	0.00	2.25	1.12	0.00
time (sec)	N/A	1.497	15.291	10.770	0.000	1.325	0.000	0.314	0.223	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	322	1046	1587	0	3046	0	895	393	0
N.S.	1	1.05	3.40	5.15	0.00	9.89	0.00	2.91	1.28	0.00
time (sec)	N/A	1.552	19.015	70.021	0.000	1.545	0.000	0.324	0.235	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	0	375	562	0	629	0	0	283	0
N.S.	1	0.00	1.32	1.98	0.00	2.21	0.00	0.00	1.00	0.00
time (sec)	N/A	0.000	4.574	2.174	0.000	0.111	0.000	0.000	0.195	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	214	246	396	0	448	0	0	205	0
N.S.	1	1.07	1.23	1.98	0.00	2.24	0.00	0.00	1.02	0.00
time (sec)	N/A	1.214	3.626	0.846	0.000	0.137	0.000	0.000	0.201	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	137	135	232	0	303	0	0	127	0
N.S.	1	1.05	1.04	1.78	0.00	2.33	0.00	0.00	0.98	0.00
time (sec)	N/A	0.678	2.936	0.727	0.000	0.096	0.000	0.000	0.193	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	106	128	0	210	0	0	61	151
N.S.	1	1.00	1.34	1.62	0.00	2.66	0.00	0.00	0.77	1.91
time (sec)	N/A	0.324	0.593	0.676	0.000	0.086	0.000	0.000	0.183	1.095

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	619	199	0	744	0	0	101	0
N.S.	1	1.00	4.55	1.46	0.00	5.47	0.00	0.00	0.74	0.00
time (sec)	N/A	0.623	5.669	0.428	0.000	0.683	0.000	0.000	0.196	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	230	736	899	0	2159	0	0	165	0
N.S.	1	1.11	3.56	4.34	0.00	10.43	0.00	0.00	0.80	0.00
time (sec)	N/A	1.108	10.608	0.450	0.000	1.961	0.000	0.000	0.193	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F(-1)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	0	1209	2277	0	4180	0	0	229	0
N.S.	1	0.00	3.91	7.37	0.00	13.53	0.00	0.00	0.74	0.00
time (sec)	N/A	0.000	14.456	0.506	0.000	4.240	0.000	0.000	0.209	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	299	684	817	0	784	0	0	363	0
N.S.	1	1.06	2.42	2.89	0.00	2.77	0.00	0.00	1.28	0.00
time (sec)	N/A	1.884	6.139	2.316	0.000	0.120	0.000	0.000	0.208	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	212	357	612	0	584	0	0	265	0
N.S.	1	1.04	1.76	3.01	0.00	2.88	0.00	0.00	1.31	0.00
time (sec)	N/A	1.241	4.360	0.978	0.000	0.109	0.000	0.000	0.200	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	137	246	389	0	407	0	0	167	0
N.S.	1	1.03	1.85	2.92	0.00	3.06	0.00	0.00	1.26	0.00
time (sec)	N/A	0.701	2.483	0.829	0.000	0.105	0.000	0.000	0.195	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	150	176	0	293	0	0	81	0
N.S.	1	1.00	1.72	2.02	0.00	3.37	0.00	0.00	0.93	0.00
time (sec)	N/A	0.335	0.646	0.467	0.000	0.095	0.000	0.000	0.179	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	200	781	624	0	1561	0	0	145	0
N.S.	1	1.07	4.18	3.34	0.00	8.35	0.00	0.00	0.78	0.00
time (sec)	N/A	1.033	6.851	0.453	0.000	1.671	0.000	0.000	0.207	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	319	904	2049	0	3403	0	0	241	0
N.S.	1	1.09	3.10	7.02	0.00	11.65	0.00	0.00	0.83	0.00
time (sec)	N/A	1.726	15.948	0.517	0.000	4.007	0.000	0.000	0.196	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	444	1757	4707	0	5864	0	0	337	0
N.S.	1	1.10	4.37	11.71	0.00	14.59	0.00	0.00	0.84	0.00
time (sec)	N/A	2.496	17.448	0.601	0.000	8.026	0.000	0.000	0.212	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	324	523	1157	0	980	0	0	443	0
N.S.	1	1.05	1.70	3.76	0.00	3.18	0.00	0.00	1.44	0.00
time (sec)	N/A	1.966	8.323	2.311	0.000	0.138	0.000	0.000	0.211	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	229	544	852	0	744	0	0	325	0
N.S.	1	1.05	2.48	3.89	0.00	3.40	0.00	0.00	1.48	0.00
time (sec)	N/A	1.253	4.670	0.999	0.000	0.104	0.000	0.000	0.210	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	157	267	448	0	536	0	0	207	0
N.S.	1	1.04	1.77	2.97	0.00	3.55	0.00	0.00	1.37	0.00
time (sec)	N/A	0.716	2.979	0.845	0.000	0.104	0.000	0.000	0.188	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	124	227	279	0	392	0	0	101	0
N.S.	1	0.98	1.80	2.21	0.00	3.11	0.00	0.00	0.80	0.00
time (sec)	N/A	0.444	0.852	0.493	0.000	0.094	0.000	0.000	0.183	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	287	912	1418	0	2577	0	0	189	0
N.S.	1	1.10	3.49	5.43	0.00	9.87	0.00	0.00	0.72	0.00
time (sec)	N/A	1.644	11.447	0.457	0.000	3.519	0.000	0.000	0.203	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	395	438	1680	4092	0	5151	0	0	317	0
N.S.	1	1.11	4.25	10.36	0.00	13.04	0.00	0.00	0.80	0.00
time (sec)	N/A	2.520	19.918	0.556	0.000	7.948	0.000	0.000	0.210	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-1)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	519	574	2465	7322	0	8555	0	0	445	0
N.S.	1	1.11	4.75	14.11	0.00	16.48	0.00	0.00	0.86	0.00
time (sec)	N/A	3.357	19.395	0.639	0.000	24.953	0.000	0.000	0.225	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	0	0	0	0	0	0	144	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.65	0.00
time (sec)	N/A	0.614	0.000	0.000	0.000	0.000	0.000	0.000	0.233	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	0	0	0	0	0	0	90	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	0.684	0.000	0.000	0.000	0.000	0.000	0.000	0.206	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	0	0	0	0	0	0	65	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.635	0.000	0.000	0.000	0.000	0.000	0.000	0.194	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	0	0	0	0	0	0	85	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.604	0.000	0.000	0.000	0.000	0.000	0.000	0.219	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	427	426	245	0	0	0	0	0	129	0
N.S.	1	1.00	0.57	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	1.876	15.189	0.000	0.000	0.000	0.000	0.000	0.239	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	0	0	0	0	0	0	62	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.592	0.000	0.000	0.000	0.000	0.000	0.000	0.212	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	190	222	244	0	0	0	0	0	85	0
N.S.	1	1.17	1.28	0.00	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.744	10.069	0.000	0.000	0.000	0.000	0.000	0.184	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	213	267	603	0	0	0	0	0	105	0
N.S.	1	1.25	2.83	0.00	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	0.704	21.514	0.000	0.000	0.000	0.000	0.000	0.203	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	351	349	854	0	0	0	0	0	156	0
N.S.	1	0.99	2.43	0.00	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	1.718	27.679	0.000	0.000	0.000	0.000	0.000	0.197	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	199	200	317	0	0	0	0	0	92	0
N.S.	1	1.01	1.59	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.851	24.865	0.000	0.000	0.000	0.000	0.000	0.195	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	101	0	0	0	0	0	40	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.414	0.275	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	189	205	0	0	0	0	0	0	65	0
N.S.	1	1.08	0.00	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.725	0.000	0.000	0.000	0.000	0.000	0.000	0.198	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	295	310	0	0	0	0	0	0	97	0
N.S.	1	1.05	0.00	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	1.231	0.000	0.000	0.000	0.000	0.000	0.000	0.216	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	469	504	0	0	0	0	0	0	129	0
N.S.	1	1.07	0.00	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	2.311	0.000	0.000	0.000	0.000	0.000	0.000	0.223	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	272	308	0	0	0	0	0	0	137	0
N.S.	1	1.13	0.00	0.00	0.00	0.00	0.00	0.00	0.50	0.00
time (sec)	N/A	0.770	0.000	0.000	0.000	0.000	0.000	0.000	0.247	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	266	298	0	0	0	0	0	0	63	0
N.S.	1	1.12	0.00	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.733	0.000	0.000	0.000	0.000	0.000	0.000	0.201	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	266	298	0	0	0	0	0	0	87	0
N.S.	1	1.12	0.00	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.742	0.000	0.000	0.000	0.000	0.000	0.000	0.203	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	276	312	0	0	0	0	0	0	119	0
N.S.	1	1.13	0.00	0.00	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.753	0.000	0.000	0.000	0.000	0.000	0.000	0.192	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	262	294	0	0	0	0	0	0	65	0
N.S.	1	1.12	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.659	0.000	0.000	0.000	0.000	0.000	0.000	0.221	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	267	287	0	0	0	0	0	0	115	0
N.S.	1	1.07	0.00	0.00	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.736	0.000	0.000	0.000	0.000	0.000	0.000	0.203	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	144	146	0	0	0	0	0	0	65	0
N.S.	1	1.01	0.00	0.00	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.394	0.000	0.000	0.000	0.000	0.000	0.000	0.206	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	149	153	0	0	0	0	0	0	115	0
N.S.	1	1.03	0.00	0.00	0.00	0.00	0.00	0.00	0.77	0.00
time (sec)	N/A	0.441	0.000	0.000	0.000	0.000	0.000	0.000	0.202	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	0	0	56	0	0	517	98
N.S.	1	1.00	1.00	0.00	0.00	1.44	0.00	0.00	13.26	2.51
time (sec)	N/A	0.359	4.339	0.000	0.000	0.120	0.000	0.000	0.265	37.917

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	0	0	57	0	0	521	99
N.S.	1	1.00	1.00	0.00	0.00	1.42	0.00	0.00	13.02	2.48
time (sec)	N/A	0.358	3.978	0.000	0.000	0.124	0.000	0.000	0.261	38.758

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	234	188	374	0	1308	0	749	1138	16312
N.S.	1	1.18	0.94	1.88	0.00	6.57	0.00	3.76	5.72	81.97
time (sec)	N/A	1.130	4.156	0.979	0.000	0.170	0.000	0.195	0.190	55.730

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	840	851	2042	1089305	0	0	0	0	85	0
N.S.	1	1.01	2.43	1296.79	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	4.455	9.391	78.862	0.000	0.000	0.000	0.000	0.198	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	630	621	1901	647398	0	0	0	0	37	0
N.S.	1	0.99	3.02	1027.62	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	1.688	18.019	37.216	0.000	0.000	0.000	0.000	0.184	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	417	417	1949	88770	0	0	0	0	60	0
N.S.	1	1.00	4.67	212.88	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	1.003	7.359	23.299	0.000	0.000	0.000	0.000	0.202	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	544	578	2266	177547	0	0	0	0	94	0
N.S.	1	1.06	4.17	326.37	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	1.753	8.128	29.971	0.000	0.000	0.000	0.000	0.180	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	858	903	2837	397569	0	0	0	0	128	0
N.S.	1	1.05	3.31	463.37	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	3.422	9.297	48.460	0.000	0.000	0.000	0.000	0.190	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	37	35	37	37	0	37	65	37
N.S.	1	1.00	1.06	1.00	1.06	1.06	0.00	1.06	1.86	1.06
time (sec)	N/A	0.269	20.919	0.352	26.557	0.350	0.000	2.509	0.240	44.077

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [285] had the largest ratio of [.5428570000000000034]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	12	12	0.98	33	0.364
2	A	10	10	0.98	33	0.303
3	A	8	8	0.98	31	0.258
4	A	6	6	1.00	33	0.182
5	A	9	9	1.01	33	0.273
6	A	11	11	1.04	33	0.333
7	A	13	12	0.90	35	0.343
8	A	10	9	0.90	35	0.257
9	A	7	6	0.81	35	0.171
10	A	14	13	0.94	35	0.371
11	A	17	16	0.94	35	0.457
12	A	10	9	1.00	33	0.273
13	A	6	5	1.00	34	0.147
14	A	2	2	1.00	43	0.047
15	A	2	2	1.00	37	0.054
16	A	11	10	1.24	31	0.323
17	A	13	13	0.87	34	0.382
18	A	11	11	0.87	34	0.324
19	A	9	9	0.89	34	0.265
20	A	7	7	0.96	32	0.219
21	A	5	5	1.09	34	0.147

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	9	9	1.07	34	0.265
23	A	9	9	1.03	34	0.265
24	A	11	11	1.01	34	0.324
25	A	13	13	0.99	34	0.382
26	A	15	15	0.83	36	0.417
27	A	13	13	0.81	36	0.361
28	A	11	11	0.79	36	0.306
29	A	9	9	0.87	36	0.250
30	A	9	9	0.85	34	0.265
31	A	9	9	0.92	36	0.250
32	A	9	9	0.97	36	0.250
33	A	9	9	1.04	36	0.250
34	A	6	6	1.00	36	0.167
35	A	8	8	0.98	36	0.222
36	A	10	10	0.96	36	0.278
37	A	12	12	0.94	36	0.333
38	A	17	17	0.81	36	0.472
39	A	15	15	0.81	36	0.417
40	A	13	13	0.78	36	0.361
41	A	11	11	0.88	36	0.306
42	A	11	11	0.82	36	0.306
43	A	11	11	0.87	34	0.324
44	A	11	11	0.84	36	0.306
45	A	11	11	0.91	36	0.306
46	A	11	11	0.95	36	0.306
47	A	11	11	1.02	36	0.306
48	A	6	6	0.97	36	0.167
49	A	8	8	0.96	36	0.222
50	A	10	10	0.96	36	0.278
51	A	12	12	0.94	36	0.333
52	A	13	13	0.90	36	0.361
53	A	11	11	0.84	36	0.306

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	9	9	0.92	36	0.250
55	A	5	5	1.11	34	0.147
56	A	8	7	0.86	36	0.194
57	A	8	7	0.95	36	0.194
58	A	10	9	0.92	36	0.250
59	A	12	11	0.90	36	0.306
60	A	15	15	0.87	36	0.417
61	A	13	13	0.92	36	0.361
62	A	11	11	0.90	36	0.306
63	A	9	9	0.97	36	0.250
64	A	8	8	1.08	34	0.235
65	A	8	7	0.95	36	0.194
66	A	8	7	0.82	36	0.194
67	A	8	7	0.83	36	0.194
68	A	10	9	0.84	36	0.250
69	A	12	11	0.85	36	0.306
70	A	15	15	0.85	36	0.417
71	A	13	13	0.92	36	0.361
72	A	11	11	0.93	36	0.306
73	A	9	9	1.05	36	0.250
74	A	8	8	1.03	34	0.235
75	A	10	9	0.92	36	0.250
76	A	8	7	0.84	36	0.194
77	A	8	7	0.75	36	0.194
78	A	8	7	0.74	36	0.194
79	A	10	9	0.77	36	0.250
80	A	12	11	0.79	36	0.306
81	A	12	12	0.93	36	0.333
82	A	10	10	0.94	36	0.278
83	A	8	8	0.96	36	0.222
84	A	6	6	0.97	36	0.167
85	A	11	10	1.06	36	0.278

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	11	10	1.07	36	0.278
87	A	11	10	1.07	36	0.278
88	A	13	12	1.03	36	0.333
89	A	12	12	0.90	38	0.316
90	A	10	10	0.91	38	0.263
91	A	8	8	0.94	38	0.211
92	A	6	6	0.95	38	0.158
93	A	12	11	0.99	38	0.289
94	A	12	11	0.98	38	0.289
95	A	12	11	0.97	38	0.289
96	A	12	11	0.98	38	0.289
97	A	14	13	0.96	38	0.342
98	A	12	12	0.90	38	0.316
99	A	10	10	0.93	38	0.263
100	A	8	8	0.93	38	0.211
101	A	6	6	0.95	38	0.158
102	A	14	13	1.00	38	0.342
103	A	14	13	0.97	38	0.342
104	A	14	13	0.93	38	0.342
105	A	14	13	0.97	38	0.342
106	A	14	13	0.98	38	0.342
107	A	16	15	0.96	38	0.395
108	A	12	12	0.91	38	0.316
109	A	10	10	0.92	38	0.263
110	A	8	8	0.95	38	0.211
111	A	6	6	1.00	38	0.158
112	A	8	7	0.98	38	0.184
113	A	10	9	0.96	38	0.237
114	A	12	11	0.91	38	0.289
115	A	14	14	0.90	38	0.368
116	A	12	12	0.91	38	0.316
117	A	10	10	0.94	38	0.263

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	8	8	0.96	38	0.211
119	A	6	6	1.01	38	0.158
120	A	10	9	0.93	38	0.237
121	A	12	11	0.94	38	0.289
122	A	14	13	0.90	38	0.342
123	A	14	14	0.91	38	0.368
124	A	12	12	0.89	38	0.316
125	A	10	10	0.93	38	0.263
126	A	8	8	0.96	38	0.211
127	A	6	6	0.98	38	0.158
128	A	12	11	0.93	38	0.289
129	A	14	13	0.90	38	0.342
130	A	16	15	0.91	38	0.395
131	A	4	4	1.00	40	0.100
132	A	4	4	1.00	40	0.100
133	A	4	4	1.00	40	0.100
134	A	4	4	1.00	40	0.100
135	A	9	8	1.01	40	0.200
136	A	9	8	1.01	40	0.200
137	A	4	4	1.00	40	0.100
138	A	4	4	1.00	40	0.100
139	A	6	6	0.99	40	0.150
140	A	6	6	0.99	40	0.150
141	A	6	6	1.01	40	0.150
142	A	4	4	1.00	40	0.100
143	A	10	9	1.00	40	0.225
144	A	10	9	0.98	40	0.225
145	A	10	9	1.01	40	0.225
146	A	4	4	1.00	40	0.100
147	A	6	6	1.00	40	0.150
148	A	6	6	0.99	40	0.150
149	A	8	8	0.97	40	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	8	8	1.02	40	0.200
151	A	6	6	1.00	40	0.150
152	A	4	4	1.00	40	0.100
153	A	12	11	0.99	40	0.275
154	A	12	11	0.95	40	0.275
155	A	12	11	0.95	40	0.275
156	A	12	11	1.02	40	0.275
157	A	4	4	1.00	40	0.100
158	A	6	6	1.00	40	0.150
159	A	8	8	1.00	40	0.200
160	A	10	10	0.97	40	0.250
161	A	10	10	1.03	40	0.250
162	A	8	8	0.99	40	0.200
163	A	6	6	1.00	40	0.150
164	A	4	4	1.00	40	0.100
165	A	14	13	1.00	40	0.325
166	A	14	13	0.93	40	0.325
167	A	14	13	0.95	40	0.325
168	A	14	13	0.96	40	0.325
169	A	14	13	1.01	40	0.325
170	A	4	4	1.00	40	0.100
171	A	6	6	1.00	40	0.150
172	A	8	8	0.98	40	0.200
173	A	10	10	1.00	40	0.250
174	A	12	11	0.97	40	0.275
175	A	10	9	0.99	40	0.225
176	A	9	8	1.02	40	0.200
177	A	8	7	1.03	40	0.175
178	A	6	6	1.00	40	0.150
179	A	8	8	1.00	40	0.200
180	A	14	13	0.93	40	0.325
181	A	12	11	0.95	40	0.275

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
182	A	10	9	0.97	40	0.225
183	A	9	8	1.02	40	0.200
184	A	6	6	1.00	40	0.150
185	A	8	8	0.99	40	0.200
186	A	10	10	0.95	40	0.250
187	A	16	15	0.93	40	0.375
188	A	14	13	0.95	40	0.325
189	A	12	11	0.96	40	0.275
190	A	10	9	1.02	40	0.225
191	A	4	4	1.00	40	0.100
192	A	8	8	1.01	40	0.200
193	A	10	10	0.97	40	0.250
194	A	12	12	1.01	40	0.300
195	A	9	8	1.22	36	0.222
196	A	9	8	0.94	36	0.222
197	A	9	8	0.94	36	0.222
198	A	9	8	0.97	34	0.235
199	A	6	6	1.00	23	0.261
200	A	9	8	0.95	36	0.222
201	A	9	8	0.93	36	0.222
202	A	9	8	0.93	36	0.222
203	A	8	7	1.04	38	0.184
204	A	8	7	1.04	38	0.184
205	A	8	8	0.84	38	0.211
206	A	6	6	1.05	38	0.158
207	A	4	4	1.00	38	0.105
208	A	8	7	1.04	38	0.184
209	A	8	7	1.04	38	0.184
210	A	8	7	1.04	38	0.184
211	A	8	8	0.87	40	0.200
212	A	6	6	0.92	40	0.150
213	A	4	4	1.00	40	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	9	8	1.20	40	0.200
215	A	9	8	1.23	38	0.211
216	A	9	8	1.22	40	0.200
217	A	9	8	1.21	40	0.200
218	A	4	4	1.00	46	0.087
219	A	4	4	1.00	45	0.089
220	A	5	5	1.00	44	0.114
221	A	5	5	1.00	43	0.116
222	A	2	2	1.00	47	0.043
223	A	2	2	1.00	46	0.043
224	A	3	3	1.00	32	0.094
225	A	3	3	1.00	32	0.094
226	A	3	3	1.00	30	0.100
227	A	9	9	1.00	24	0.375
228	A	3	3	1.00	30	0.100
229	A	5	5	1.00	32	0.156
230	A	3	3	1.00	32	0.094
231	A	3	3	1.00	32	0.094
232	A	3	3	1.00	32	0.094
233	A	5	5	1.02	32	0.156
234	A	3	3	1.00	32	0.094
235	A	3	3	1.00	32	0.094
236	A	3	3	1.00	32	0.094
237	A	3	3	1.00	32	0.094
238	A	3	3	1.00	30	0.100
239	A	6	6	1.05	24	0.250
240	A	3	3	1.00	30	0.100
241	A	5	5	1.00	32	0.156
242	A	3	3	1.00	32	0.094
243	A	3	3	1.00	32	0.094
244	A	10	10	1.03	33	0.303
245	A	8	8	1.03	33	0.242

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	6	6	1.11	31	0.194
247	A	2	2	1.00	21	0.095
248	A	11	10	1.11	33	0.303
249	A	12	11	1.25	33	0.333
250	A	14	13	1.09	33	0.394
251	A	14	14	1.03	35	0.400
252	A	12	12	1.04	35	0.343
253	A	8	8	0.87	33	0.242
254	A	4	4	0.88	23	0.174
255	A	13	12	1.08	35	0.343
256	A	14	13	1.08	35	0.371
257	A	14	13	1.19	35	0.371
258	A	16	16	1.04	35	0.457
259	A	14	14	1.05	35	0.400
260	A	9	9	0.81	33	0.273
261	A	5	5	0.80	23	0.217
262	A	16	15	1.09	35	0.429
263	A	16	15	1.04	35	0.429
264	A	17	16	1.08	35	0.457
265	A	6	6	1.00	35	0.171
266	A	4	4	0.96	35	0.114
267	A	8	8	1.03	33	0.242
268	A	4	4	1.00	23	0.174
269	A	9	8	1.06	35	0.229
270	A	11	10	1.04	35	0.286
271	A	14	13	1.07	35	0.371
272	A	6	6	1.03	35	0.171
273	A	10	10	1.05	35	0.286
274	A	9	9	1.05	33	0.273
275	A	4	4	0.98	23	0.174
276	A	11	10	1.09	35	0.286
277	A	13	12	1.05	35	0.343

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
278	A	15	14	1.08	35	0.400
279	A	12	12	1.05	35	0.343
280	A	11	11	1.06	35	0.314
281	A	9	9	1.02	33	0.273
282	A	6	6	0.97	23	0.261
283	A	14	13	1.13	35	0.371
284	A	17	16	1.07	35	0.457
285	A	20	19	1.08	35	0.543
286	A	11	11	0.92	37	0.297
287	A	9	9	0.97	37	0.243
288	A	9	9	1.09	35	0.257
289	A	4	4	1.00	25	0.160
290	A	6	5	1.00	37	0.135
291	A	6	5	1.00	37	0.135
292	A	8	7	0.97	37	0.189
293	A	14	14	0.84	37	0.378
294	A	12	12	0.91	37	0.324
295	A	11	11	0.99	35	0.314
296	A	6	6	0.98	25	0.240
297	A	9	8	1.04	37	0.216
298	A	9	8	1.04	37	0.216
299	A	9	8	1.04	37	0.216
300	A	17	17	0.79	37	0.459
301	A	15	15	0.86	37	0.405
302	A	13	13	0.93	35	0.371
303	A	8	8	0.96	25	0.320
304	A	12	11	1.06	37	0.297
305	A	12	11	1.04	37	0.297
306	A	12	11	1.05	37	0.297
307	F	0	0	N/A	0.000	N/A
308	A	14	13	1.07	37	0.351
309	A	11	10	1.05	35	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
310	A	6	5	1.00	25	0.200
311	A	8	7	1.00	37	0.189
312	A	11	10	1.11	37	0.270
313	F	0	0	N/A	0.000	N/A
314	A	17	16	1.06	37	0.432
315	A	14	13	1.04	37	0.351
316	A	11	10	1.03	35	0.286
317	A	6	5	1.00	25	0.200
318	A	11	10	1.07	37	0.270
319	A	14	13	1.09	37	0.351
320	A	17	16	1.10	37	0.432
321	A	17	16	1.05	37	0.432
322	A	14	13	1.05	37	0.351
323	A	11	10	1.04	35	0.286
324	A	8	7	0.98	25	0.280
325	A	14	13	1.10	37	0.351
326	A	17	16	1.11	37	0.432
327	A	20	19	1.11	37	0.514
328	A	7	6	1.00	35	0.171
329	A	12	11	1.00	33	0.333
330	A	10	9	1.00	35	0.257
331	A	7	6	1.00	35	0.171
332	A	16	15	1.00	37	0.405
333	A	7	6	1.00	37	0.162
334	A	11	10	1.17	37	0.270
335	A	8	7	1.25	37	0.189
336	A	12	12	0.99	35	0.343
337	A	10	10	1.01	33	0.303
338	A	6	6	1.00	23	0.261
339	A	11	10	1.08	35	0.286
340	A	14	13	1.05	35	0.371
341	A	16	15	1.07	35	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
342	A	9	8	1.13	37	0.216
343	A	9	8	1.12	37	0.216
344	A	9	8	1.12	37	0.216
345	A	9	8	1.13	37	0.216
346	A	9	8	1.12	35	0.229
347	A	10	9	1.07	39	0.231
348	A	7	6	1.01	36	0.167
349	A	7	6	1.03	40	0.150
350	A	2	2	1.00	55	0.036
351	A	2	2	1.00	51	0.039
352	A	12	11	1.18	35	0.314
353	A	15	15	1.01	39	0.385
354	A	8	8	0.99	39	0.205
355	A	5	5	1.00	39	0.128
356	A	8	8	1.06	39	0.205
357	A	11	11	1.05	39	0.282
358	N/A	2	0	1.00	35	0.000

CHAPTER 3

Listing of integrals

3.1 $\int (d\sin(e+fx))^n (a+a\sin(e+fx))^3 (A+B\sin(e+fx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 373

$$\int (d\sin(e+fx))^n (a+a\sin(e+fx))^3 (A+B\sin(e+fx))\,dx$$
$$= -\frac{a^3(B(27+14n+2n^2)+A(28+15n+2n^2))\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(2+n)(3+n)(4+n)}$$
$$+\frac{a^3(B(15+19n+4n^2)+A(20+21n+4n^2))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(e+f\right.}{df(1+n)(2+n)(4+n)\sqrt{\cos^2(e+fx)}}$$
$$+\frac{a^3(B(9+4n)+A(11+4n))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\sin^2(e+fx)\right)(d\sin(e+f}{d^2f(2+n)(3+n)\sqrt{\cos^2(e+fx)}}$$
$$-\frac{aB\cos(e+fx)(d\sin(e+fx))^{1+n}(a+a\sin(e+fx))^2}{df(4+n)}$$
$$-\frac{(A(4+n)+B(6+n))\cos(e+fx)(d\sin(e+fx))^{1+n}\left(a^3+a^3\sin(e+fx)\right)}{df(3+n)(4+n)}$$

output
```
-a^3*(B*(2*n^2+14*n+27)+A*(2*n^2+15*n+28))*cos(f*x+e)*(d*sin(f*x+e))^(1+n)
/d/f/(2+n)/(3+n)/(4+n)+a^3*(B*(4*n^2+19*n+15)+A*(4*n^2+21*n+20))*cos(f*x+e
)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],sin(f*x+e)^2)*(d*sin(f*x+e))^(1+n
)/d/f/(1+n)/(2+n)/(4+n)/(cos(f*x+e)^2)^(1/2)+a^3*(B*(9+4*n)+A*(11+4*n))*co
s(f*x+e)*hypergeom([1/2, 1+1/2*n],[2+1/2*n],sin(f*x+e)^2)*(d*sin(f*x+e))^(
2+n)/d^2/f/(2+n)/(3+n)/(cos(f*x+e)^2)^(1/2)-a*B*cos(f*x+e)*(d*sin(f*x+e))^
(1+n)*(a+a*sin(f*x+e))^2/d/f/(4+n)-(A*(4+n)+B*(6+n))*cos(f*x+e)*(d*sin(f*x
+e))^(1+n)*(a^3+a^3*sin(f*x+e))/d/f/(3+n)/(4+n)
```

Mathematica [A] (verified)

Time = 2.72 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.66

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx$$
$$= \frac{a^3\cos(e+fx)\sin(e+fx)(d\sin(e+fx))^n\left(\frac{A\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)}{1+n}+\sin(e+fx)\left(\frac{(3A+B)\,\text{H}}{}\right.\right.}{}$$

input
```
Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]),x
]
```

output
```
(a^3*Cos[e + f*x]*Sin[e + f*x]*(d*Sin[e + f*x])^n*((A*Hypergeometric2F1[1/
2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2])/(1 + n) + Sin[e + f*x]*(((3*A +
B)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x]^2])/(2 + n) +
 Sin[e + f*x]*((3*(A + B)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Sin
[e + f*x]^2])/(3 + n) + Sin[e + f*x]*(((A + 3*B)*Hypergeometric2F1[1/2, (4
 + n)/2, (6 + n)/2, Sin[e + f*x]^2])/(4 + n) + (B*Hypergeometric2F1[1/2, (
5 + n)/2, (7 + n)/2, Sin[e + f*x]^2]*Sin[e + f*x])/(5 + n))))))/(f*Sqrt[Co
s[e + f*x]^2])
```

Rubi [A] (verified)

Time = 1.72 (sec) , antiderivative size = 367, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 3455, 3042, 3455, 3042, 3447, 3042, 3502, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^3(A+B\sin(e+fx))(d\sin(e+fx))^n\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (a\sin(e+fx)+a)^3(A+B\sin(e+fx))(d\sin(e+fx))^n dx$$

$$\Big\downarrow \; 3455$$

$$\frac{\int (d\sin(e+fx))^n(\sin(e+fx)a+a)^2(ad(B(n+1)+A(n+4))+ad(A(n+4)+B(n+6))\sin(e+fx))dx}{d(n+4)}-\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(d\sin(e+fx))^{n+1}}{df(n+4)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int (d\sin(e+fx))^n(\sin(e+fx)a+a)^2(ad(B(n+1)+A(n+4))+ad(A(n+4)+B(n+6))\sin(e+fx))dx}{d(n+4)}-\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(d\sin(e+fx))^{n+1}}{df(n+4)}$$

$$\Big\downarrow \; 3455$$

$$\frac{\frac{\int (d\sin(e+fx))^n(\sin(e+fx)a+a)\left(a^2\left(2A\left(n^2+6n+8\right)+B\left(2n^2+11n+9\right)\right)d^2+a^2\left(B\left(2n^2+14n+27\right)+A\left(2n^2+15n+28\right)\right)\sin(e+fx)d^2\right)dx}{d(n+3)}-\frac{(A(n}{}}{d(n+4)}-\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(d\sin(e+fx))^{n+1}}{df(n+4)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int (d\sin(e+fx))^n(\sin(e+fx)a+a)\left(a^2\left(2A\left(n^2+6n+8\right)+B\left(2n^2+11n+9\right)\right)d^2+a^2\left(B\left(2n^2+14n+27\right)+A\left(2n^2+15n+28\right)\right)\sin(e+fx)d^2\right)dx}{d(n+3)} - \frac{(A(n \ldots}{\ldots}$$

$$d(n+4)$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(d\sin(e+fx))^{n+1}}{df(n+4)}$$

$\big\downarrow$ 3447

$$\frac{\int (d\sin(e+fx))^n\left(d^2\left(B\left(2n^2+14n+27\right)+A\left(2n^2+15n+28\right)\right)\sin^2(e+fx)a^3+d^2\left(2A\left(n^2+6n+8\right)+B\left(2n^2+11n+9\right)\right)a^3+\left(d^2\left(2A\left(n^2+6n+8\right)+B(2n \ldots\right.\right.}{d(n+3)}$$

$$d(\ldots$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(d\sin(e+fx))^{n+1}}{df(n+4)}$$

$\big\downarrow$ 3042

$$\frac{\int (d\sin(e+fx))^n\left(d^2\left(B\left(2n^2+14n+27\right)+A\left(2n^2+15n+28\right)\right)\sin(e+fx)^2a^3+d^2\left(2A\left(n^2+6n+8\right)+B\left(2n^2+11n+9\right)\right)a^3+\left(d^2\left(2A\left(n^2+6n+8\right)+B(2n \ldots\right.\right.}{d(n+3)}$$

$$d(\ldots$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(d\sin(e+fx))^{n+1}}{df(n+4)}$$

$\big\downarrow$ 3502

$$\frac{\frac{\int (d\sin(e+fx))^n\left(a^3(n+3)\left(B\left(4n^2+19n+15\right)+A\left(4n^2+21n+20\right)\right)d^3+a^3(n+2)(n+4)(B(4n+9)+A(4n+11))\sin(e+fx)d^3\right)dx}{d(n+2)} - \frac{a^3d\left(A\left(2n^2+15n+28\right)+B\left(2n^2+ \ldots\right.\right.}{f \ldots}}{d(n+3)}$$

$$d(n+4)$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(d\sin(e+fx))^{n+1}}{df(n+4)}$$

$\big\downarrow$ 3042

$$\frac{\frac{\int (d\sin(e+fx))^n\left(a^3(n+3)\left(B\left(4n^2+19n+15\right)+A\left(4n^2+21n+20\right)\right)d^3+a^3(n+2)(n+4)(B(4n+9)+A(4n+11))\sin(e+fx)d^3\right)dx}{d(n+2)} - \frac{a^3d\left(A\left(2n^2+15n+28\right)+B\left(2n^2+ \ldots\right.\right.}{f \ldots}}{d(n+3)}$$

$$d(n+4)$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(d\sin(e+fx))^{n+1}}{df(n+4)}$$

$\big\downarrow$ 3227

$$\frac{\frac{a^3d^3(n+3)\left(A\left(4n^2+21n+20\right)+B\left(4n^2+19n+15\right)\right)\int (d\sin(e+fx))^ndx+a^3d^2(n+2)(n+4)(A(4n+11)+B(4n+9))\int (d\sin(e+fx))^{n+1}dx}{d(n+2)} - \frac{a^3d\left(A\left(2n^2+15n+28\right)+ \ldots\right.}{\ldots}}{d(n+3)}$$

$$d(n+4)$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(d\sin(e+fx))^{n+1}}{df(n+4)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{a^3d^3(n+3)\left(A\left(4n^2+21n+20\right)+B\left(4n^2+19n+15\right)\right)\int (d\sin(e+fx))^n dx+a^3d^2(n+2)(n+4)(A(4n+11)+B(4n+9))\int (d\sin(e+fx))^{n+1}dx}{d(n+2)}-\frac{a^3d\left(A\left(2n^2+15n+28\right)+\ldots\right.}{\ldots}}{d(n+3)}\ldots}{d(n+4)}$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(d\sin(e+fx))^{n+1}}{df(n+4)}$$

$$\Big\downarrow \text{3122}$$

$$\frac{\frac{\frac{a^3d^2(n+3)\left(A\left(4n^2+21n+20\right)+B\left(4n^2+19n+15\right)\right)\cos(e+fx)(d\sin(e+fx))^{n+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\sin^2(e+fx)\right)}{f(n+1)\sqrt{\cos^2(e+fx)}}+\frac{a^3d(n+4)(A(4n+11)+B(4n+9\ldots}{\ldots}}{d(n+2)}\ldots}{d(n+3\ldots}$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(d\sin(e+fx))^{n+1}}{df(n+4)}$$

input

```
Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]),x]
```

output

```
-((a*B*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n)*(a + a*Sin[e + f*x])^2)/(d*f*
(4 + n))) + (-(((A*(4 + n) + B*(6 + n))*Cos[e + f*x]*(d*Sin[e + f*x])^(1 +
 n)*(a^3 + a^3*Sin[e + f*x]))/(f*(3 + n))) + (-((a^3*d*(B*(27 + 14*n + 2*n
^2) + A*(28 + 15*n + 2*n^2))*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(f*(2
+ n))) + ((a^3*d^2*(3 + n)*(B*(15 + 19*n + 4*n^2) + A*(20 + 21*n + 4*n^2))
*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2]
*(d*Sin[e + f*x])^(1 + n))/(f*(1 + n)*Sqrt[Cos[e + f*x]^2]) + (a^3*d*(4 +
n)*(B*(9 + 4*n) + A*(11 + 4*n))*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n
)/2, (4 + n)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(2 + n))/(f*Sqrt[Cos[e +
f*x]^2]))/(d*(2 + n)))/(d*(3 + n)))/(d*(4 + n))
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 && !LtQ[m, -1]
```

Maple [F]

$$\int (d \sin (fx+e))^n (a+a \sin (fx+e))^3 (A+B \sin (fx+e)) \, dx$$

input
```
int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x)
```

output
```
int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x)
```

Fricas [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx = \int (B \sin (fx+e) + A)(a \sin (fx+e) + a)^3 (d \sin (fx+e))^n \, dx$$

input
```
integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
integral((B*a^3*cos(f*x + e)^4 - (3*A + 5*B)*a^3*cos(f*x + e)^2 + 4*(A + B
)*a^3 - ((A + 3*B)*a^3*cos(f*x + e)^2 - 4*(A + B)*a^3)*sin(f*x + e))*(d*si
n(f*x + e))^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx = \text{Timed out}$$

input
```
integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**3*(A+B*sin(f*x+e)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx$$
$$=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^3(d\sin(fx+e))^n\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x, algorith
m="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*(d*sin(f*x + e))^n,
x)
```

Giac [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx$$
$$=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^3(d\sin(fx+e))^n\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x, algorith
m="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*(d*sin(f*x + e))^n,
x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx = \int (d\,\sin(e+f\,x))^n\,(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^3\,dx$$

input
```
int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3,x)
```

output
```
int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3, x)
```

Reduce [F]

$$\begin{aligned}\int (d\sin(e+fx))^n(a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx &= d^na^3\Bigg(\left(\left(\int\sin(fx+e)^n\,dx\right)a+\left(\int\sin(fx+e)^n\sin(fx+e)^4\,dx\right)b\right.\\&+\left(\int\sin(fx+e)^n\sin(fx+e)^3\,dx\right)a+3\left(\int\sin(fx+e)^n\sin(fx+e)^3\,dx\right)b\\&+3\left(\int\sin(fx+e)^n\sin(fx+e)^2\,dx\right)a+3\left(\int\sin(fx+e)^n\sin(fx+e)^2\,dx\right)b\\&+3\left(\int\sin(fx+e)^n\sin(fx+e)\,dx\right)a+\left(\int\sin(fx+e)^n\sin(fx+e)\,dx\right)b\Bigg)\end{aligned}$$

input
```
int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x)
```

output
```
d**n*a**3*(int(sin(e + f*x)**n,x)*a + int(sin(e + f*x)**n*sin(e + f*x)**4,
x)*b + int(sin(e + f*x)**n*sin(e + f*x)**3,x)*a + 3*int(sin(e + f*x)**n*si
n(e + f*x)**3,x)*b + 3*int(sin(e + f*x)**n*sin(e + f*x)**2,x)*a + 3*int(si
n(e + f*x)**n*sin(e + f*x)**2,x)*b + 3*int(sin(e + f*x)**n*sin(e + f*x),x)
*a + int(sin(e + f*x)**n*sin(e + f*x),x)*b)
```

3.2 $\int (d\sin(e+fx))^n(a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 277

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx$$
$$= -\frac{a^2(A(3+n)+B(4+n))\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(2+n)(3+n)}$$
$$+\frac{a^2(2B(1+n)+A(3+2n))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)(d\sin(e+fx}{df(1+n)(2+n)\sqrt{\cos^2(e+fx)}}$$
$$+\frac{a^2(2A(3+n)+B(5+2n))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\sin^2(e+fx)\right)(d\sin(e+fx}{d^2f(2+n)(3+n)\sqrt{\cos^2(e+fx)}}$$
$$-\frac{B\cos(e+fx)(d\sin(e+fx))^{1+n}\left(a^2+a^2\sin(e+fx)\right)}{df(3+n)}$$

output
```
-a^2*(A*(3+n)+B*(4+n))*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(2+n)/(3+n)+a^2
*(2*B*(1+n)+A*(3+2*n))*cos(f*x+e)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],s
in(f*x+e)^2)*(d*sin(f*x+e))^(1+n)/d/f/(1+n)/(2+n)/(cos(f*x+e)^2)^(1/2)+a^2
*(2*A*(3+n)+B*(5+2*n))*cos(f*x+e)*hypergeom([1/2, 1+1/2*n],[2+1/2*n],sin(f
*x+e)^2)*(d*sin(f*x+e))^(2+n)/d^2/f/(2+n)/(3+n)/(cos(f*x+e)^2)^(1/2)-B*cos
(f*x+e)*(d*sin(f*x+e))^(1+n)*(a^2+a^2*sin(f*x+e))/d/f/(3+n)
```

Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.74

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx$$
$$= \frac{a^2\cos(e+fx)\sin(e+fx)(d\sin(e+fx))^n\left(\frac{A\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)}{1+n}+\sin(e+fx)\left(\frac{(2A+B)\,\text{H}}{}\right.\right.}{}$$

input
```
Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]),x
]
```

output
```
(a^2*Cos[e + f*x]*Sin[e + f*x]*(d*Sin[e + f*x])^n*((A*Hypergeometric2F1[1/
2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2])/(1 + n) + Sin[e + f*x]*(((2*A +
B)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x]^2])/(2 + n) +
 Sin[e + f*x]*(((A + 2*B)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Sin
[e + f*x]^2])/(3 + n) + (B*Hypergeometric2F1[1/2, (4 + n)/2, (6 + n)/2, Si
n[e + f*x]^2]*Sin[e + f*x])/(4 + n)))))/(f*Sqrt[Cos[e + f*x]^2])
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {3042, 3455, 3042, 3447, 3042, 3502, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^2(A+B\sin(e+fx))(d\sin(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^2(A+B\sin(e+fx))(d\sin(e+fx))^n dx$$

$$\Big\downarrow \text{3455}$$

$$\frac{\int (d\sin(e+fx))^n(\sin(e+fx)a+a)(ad(B(n+1)+A(n+3))+ad(A(n+3)+B(n+4))\sin(e+fx))dx}{d(n+3)}-\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(d\sin(e+fx))^{n+1}}{df(n+3)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int (d\sin(e+fx))^n(\sin(e+fx)a+a)(ad(B(n+1)+A(n+3))+ad(A(n+3)+B(n+4))\sin(e+fx))dx}{d(n+3)}-\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(d\sin(e+fx))^{n+1}}{df(n+3)}$$

$$\Big\downarrow \text{ 3447}$$

$$\frac{\int (d\sin(e+fx))^n\left(d(A(n+3)+B(n+4))\sin^2(e+fx)a^2+d(B(n+1)+A(n+3))a^2+\left(d(B(n+1)+A(n+\right.\right.}{d(n+3)}$$
$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(d\sin(e+fx))^{n+1}}{df(n+3)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int (d\sin(e+fx))^n\left(d(A(n+3)+B(n+4))\sin(e+fx)^2a^2+d(B(n+1)+A(n+3))a^2+\left(d(B(n+1)+A(n+\right.\right.}{d(n+3)}$$
$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(d\sin(e+fx))^{n+1}}{df(n+3)}$$

$$\Big\downarrow \text{ 3502}$$

$$\frac{\frac{\int (d\sin(e+fx))^n\left(a^2(n+3)(2B(n+1)+A(2n+3))d^2+a^2(n+2)(2A(n+3)+B(2n+5))\sin(e+fx)d^2\right)dx}{d(n+2)}-\frac{a^2(A(n+3)+B(n+4))\cos(e+fx)(d\sin}{f(n+2)}}{d(n+3)}$$
$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(d\sin(e+fx))^{n+1}}{df(n+3)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\int (d\sin(e+fx))^n\left(a^2(n+3)(2B(n+1)+A(2n+3))d^2+a^2(n+2)(2A(n+3)+B(2n+5))\sin(e+fx)d^2\right)dx}{d(n+2)}-\frac{a^2(A(n+3)+B(n+4))\cos(e+fx)(d\sin}{f(n+2)}}{d(n+3)}$$
$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(d\sin(e+fx))^{n+1}}{df(n+3)}$$

$$\Big\downarrow \text{ 3227}$$

$$\frac{\frac{a^2d^2(n+3)(A(2n+3)+2B(n+1))\int (d\sin(e+fx))^n dx+a^2d(n+2)(2A(n+3)+B(2n+5))\int (d\sin(e+fx))^{n+1}dx}{d(n+2)}-\frac{a^2(A(n+3)+B(n+4))\cos(e+}{f(n+2)}}{d(n+3)}$$

$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(d\sin(e+fx))^{n+1}}{df(n+3)}$$

$\downarrow$ 3042

$$\frac{\frac{a^2d^2(n+3)(A(2n+3)+2B(n+1))\int (d\sin(e+fx))^n dx+a^2d(n+2)(2A(n+3)+B(2n+5))\int (d\sin(e+fx))^{n+1}dx}{d(n+2)}-\frac{a^2(A(n+3)+B(n+4))\cos(e+}{f(n+2)}}{d(n+3)}$$

$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(d\sin(e+fx))^{n+1}}{df(n+3)}$$

$\downarrow$ 3122

$$\frac{\frac{\frac{a^2d(n+3)(A(2n+3)+2B(n+1))\cos(e+fx)(d\sin(e+fx))^{n+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\sin^2(e+fx)\right)}{f(n+1)\sqrt{\cos^2(e+fx)}}+\frac{a^2(2A(n+3)+B(2n+5))\cos(e+fx)(d\sin(e+fx))^{n}}{f\sqrt{\cos^2}}}{d(n+2)}}{d(n+3)}$$

$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(d\sin(e+fx))^{n+1}}{df(n+3)}$$

input
```
Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]),x]
```

output
```
-((B*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n)*(a^2 + a^2*Sin[e + f*x]))/(d*f*
(3 + n))) + (-((a^2*(A*(3 + n) + B*(4 + n))*Cos[e + f*x]*(d*Sin[e + f*x])^
(1 + n))/(f*(2 + n))) + ((a^2*d*(3 + n)*(2*B*(1 + n) + A*(3 + 2*n))*Cos[e
+ f*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2]*(d*Sin
[e + f*x])^(1 + n))/(f*(1 + n)*Sqrt[Cos[e + f*x]^2]) + (a^2*(2*A*(3 + n) +
 B*(5 + 2*n))*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Si
n[e + f*x]^2]*(d*Sin[e + f*x])^(2 + n))/(f*Sqrt[Cos[e + f*x]^2]))/(d*(2 +
n)))/(d*(3 + n))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int (d\sin (fx+e))^n (a+a\sin (fx+e))^2 (A+B\sin (fx+e))\,dx$$

input
```
int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x)
```

output
```
int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x)
```

Fricas [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx$$
$$= \int (B\sin (fx+e)+A)(a\sin (fx+e)+a)^2(d\sin (fx+e))^n\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
integral(-((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x +
 e)^2 - 2*(A + B)*a^2)*sin(f*x + e))*(d*sin(f*x + e))^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx = \text{Timed out}$$

input
```
integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**2*(A+B*sin(f*x+e)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx = \int (B\sin{(fx+e)}+A)(a\sin{(fx+e)}+a)^2(d\sin{(fx+e)})^n\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x, algorith
m="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(d*sin(f*x + e))^n,
x)
```

Giac [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx = \int (B\sin{(fx+e)}+A)(a\sin{(fx+e)}+a)^2(d\sin{(fx+e)})^n\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x, algorith
m="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(d*sin(f*x + e))^n,
x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx$$
$$=\int (d\,\sin(e+f\,x))^n\,(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^2\,dx$$

input
```
int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2,x)
```

output
```
int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2, x)
```

Reduce [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx$$
$$=d^na^2\left(\left(\int \sin(fx+e)^n\,dx\right)a+\left(\int \sin(fx+e)^n\sin(fx+e)^3\,dx\right)b+\left(\int \sin(fx+e)^n\sin(fx+e)^2\,dx\right)a+2\left(\int \sin(fx+e)^n\sin(fx+e)^2\,dx\right)b+2\left(\int \sin(fx+e)^n\sin(fx+e)\,dx\right)a+\left(\int \sin(fx+e)^n\sin(fx+e)\,dx\right)b\right)$$

input
```
int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x)
```

output
```
d**n*a**2*(int(sin(e + f*x)**n,x)*a + int(sin(e + f*x)**n*sin(e + f*x)**3,
x)*b + int(sin(e + f*x)**n*sin(e + f*x)**2,x)*a + 2*int(sin(e + f*x)**n*si
n(e + f*x)**2,x)*b + 2*int(sin(e + f*x)**n*sin(e + f*x),x)*a + int(sin(e +
 f*x)**n*sin(e + f*x),x)*b)
```

3.3 $\int (d\sin(e+fx))^n(a+a\sin(e+fx))(A+B\sin(e+fx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 191

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))(A+B\sin(e+fx))\,dx$$
$$= -\frac{aB\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(2+n)}$$
$$+\frac{a(B(1+n)+A(2+n))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)(d\sin(e+fx))^{1+}}{df(1+n)(2+n)\sqrt{\cos^2(e+fx)}}$$
$$+\frac{a(A+B)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\sin^2(e+fx)\right)(d\sin(e+fx))^{2+n}}{d^2f(2+n)\sqrt{\cos^2(e+fx)}}$$

output
```
-a*B*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(2+n)+a*(B*(1+n)+A*(2+n))*cos(f*x
+e)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],sin(f*x+e)^2)*(d*sin(f*x+e))^(1
+n)/d/f/(1+n)/(2+n)/(cos(f*x+e)^2)^(1/2)+a*(A+B)*cos(f*x+e)*hypergeom([1/2
, 1+1/2*n],[2+1/2*n],sin(f*x+e)^2)*(d*sin(f*x+e))^(2+n)/d^2/f/(2+n)/(cos(f
*x+e)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.76

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))(A+B\sin(e+fx))\,dx$$
$$= \frac{a\cos(e+fx)\sin(e+fx)(d\sin(e+fx))^n\left((B(1+n)+A(2+n))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2\right.\right.}{f(1+n)(2}$$

input
```
Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]),x]
```

output
```
(a*Cos[e + f*x]*Sin[e + f*x]*(d*Sin[e + f*x])^n*((B*(1 + n) + A*(2 + n))*H
ypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2] - (1 + n)*(B*S
qrt[Cos[e + f*x]^2] - (A + B)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2,
 Sin[e + f*x]^2]*Sin[e + f*x])))/(f*(1 + n)*(2 + n)*Sqrt[Cos[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {3042, 3447, 3042, 3502, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)(A+B\sin(e+fx))(d\sin(e+fx))^n\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (a\sin(e+fx)+a)(A+B\sin(e+fx))(d\sin(e+fx))^n dx$$
$$\Big\downarrow \text{3447}$$
$$\int (d\sin(e+fx))^n\left((aA+aB)\sin(e+fx)+aA+aB\sin^2(e+fx)\right)dx$$
$$\Big\downarrow \text{3042}$$

$$\int (d\sin(e+fx))^n \left((aA+aB)\sin(e+fx)+aA+aB\sin(e+fx)^2\right)dx$$

$$\Big\downarrow \; 3502$$

$$\frac{\int (d\sin(e+fx))^n (ad(B(n+1)+A(n+2))+a(A+B)d(n+2)\sin(e+fx))dx}{d(n+2)} - \frac{aB\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(n+2)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int (d\sin(e+fx))^n (ad(B(n+1)+A(n+2))+a(A+B)d(n+2)\sin(e+fx))dx}{d(n+2)} - \frac{aB\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(n+2)}$$

$$\Big\downarrow \; 3227$$

$$\frac{ad(A(n+2)+B(n+1))\int (d\sin(e+fx))^n dx+a(n+2)(A+B)\int (d\sin(e+fx))^{n+1}dx}{d(n+2)} - \frac{aB\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(n+2)}$$

$$\Big\downarrow \; 3042$$

$$\frac{ad(A(n+2)+B(n+1))\int (d\sin(e+fx))^n dx+a(n+2)(A+B)\int (d\sin(e+fx))^{n+1}dx}{d(n+2)} - \frac{aB\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(n+2)}$$

$$\Big\downarrow \; 3122$$

$$\frac{\frac{a(A(n+2)+B(n+1))\cos(e+fx)(d\sin(e+fx))^{n+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\sin^2(e+fx)\right)}{f(n+1)\sqrt{\cos^2(e+fx)}} + \frac{a(A+B)\cos(e+fx)(d\sin(e+fx))^{n+2}\text{Hype}}{df\sqrt{\cos^2}}}{d(n+2)}$$

$$\frac{aB\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(n+2)}$$

input
```
Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]),x]
```

output
```
-((a*B*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*(2 + n))) + ((a*(B*(1 +
 n) + A*(2 + n))*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2,
 Sin[e + f*x]^2]*(d*Sin[e + f*x])^(1 + n))/(f*(1 + n)*Sqrt[Cos[e + f*x]^2]
) + (a*(A + B)*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, S
in[e + f*x]^2]*(d*Sin[e + f*x])^(2 + n))/(d*f*Sqrt[Cos[e + f*x]^2]))/(d*(2
 + n))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int (d\sin(fx+e))^n (a+a\sin(fx+e)) (A+B\sin(fx+e))\, dx$$

input
```
int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x)
```

output
```
int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x)
```

Fricas [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))(A+B\sin(e+fx))\,dx$$
$$=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)(d\sin(fx+e))^n\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x, algorithm=
"fricas")
```

output
```
integral(-(B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*(d*sin
(f*x + e))^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))(A+B\sin(e+fx))\,dx = \text{Timed out}$$

input
```
integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))(A+B\sin(e+fx))\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)(d\sin(fx+e))^n\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x, algorithm=
"maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n, x)
```

Giac [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))(A+B\sin(e+fx))\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)(d\sin(fx+e))^n\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x, algorithm=
"giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))(A+B\sin(e+fx))\,dx = \int (d\sin(e+fx))^n\,(A+B\sin(e+fx))\,(a+a\sin(e+fx))\,dx$$

input
```
int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x)),x)
```

output
```
int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x)), x)
```

Reduce [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))(A+B\sin(e+fx))\,dx$$
$$= d^n a\left(\left(\int \sin(fx+e)^n\,dx\right)a+\left(\int \sin(fx+e)^n\sin(fx+e)^2\,dx\right)b + \left(\int \sin(fx+e)^n\sin(fx+e)\,dx\right)a+\left(\int \sin(fx+e)^n\sin(fx+e)\,dx\right)b\right)$$

input
```
int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x)
```

output
```
d**n*a*(int(sin(e + f*x)**n,x)*a + int(sin(e + f*x)**n*sin(e + f*x)**2,x)*
b + int(sin(e + f*x)**n*sin(e + f*x),x)*a + int(sin(e + f*x)**n*sin(e + f*
x),x)*b)
```

3.4 $\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{a+a\sin(e+fx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 202

$$\begin{aligned}\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{a+a\sin(e+fx)}\,dx &= \frac{(B-An+Bn)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)(d\sin(e+fx))^{1+n}}{adf(1+n)\sqrt{\cos^2(e+fx)}}\\ &\quad+\frac{(A-B)(1+n)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\sin^2(e+fx)\right)(d\sin(e+fx))^{2+n}}{ad^2f(2+n)\sqrt{\cos^2(e+fx)}}\\ &\quad+\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(a+a\sin(e+fx))}\end{aligned}$$

output
```
(-A*n+B*n+B)*cos(f*x+e)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],sin(f*x+e)^
2)*(d*sin(f*x+e))^(1+n)/a/d/f/(1+n)/(cos(f*x+e)^2)^(1/2)+(A-B)*(1+n)*cos(f
*x+e)*hypergeom([1/2, 1+1/2*n],[2+1/2*n],sin(f*x+e)^2)*(d*sin(f*x+e))^(2+n
)/a/d^2/f/(2+n)/(cos(f*x+e)^2)^(1/2)+(A-B)*cos(f*x+e)*(d*sin(f*x+e))^(1+n)
/d/f/(a+a*sin(f*x+e))
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.79

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{a+a\sin(e+fx)}\,dx$$
$$= \frac{(d\sin(e+fx))^n\left(\frac{(B-An+Bn)\sqrt{\cos^2(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)}{1+n}+\frac{(A-B)(1+n)\sqrt{\cos^2(e+fx)}\,\text{Hypergeo}}{}\right.}{af}$$

input
```
Integrate[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x]),x
]
```

output
```
((d*Sin[e + f*x])^n*(((B - A*n + B*n)*Sqrt[Cos[e + f*x]^2]*Hypergeometric2
F1[1/2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2])/(1 + n) + ((A - B)*(1 + n)*
Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e +
f*x]^2]*Sin[e + f*x])/(2 + n) + ((A - B)*Cos[e + f*x]^2)/(1 + Sin[e + f*x]
))*Tan[e + f*x])/(a*f)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 3457, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(d\sin(e+fx))^n}{a\sin(e+fx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(A+B\sin(e+fx))(d\sin(e+fx))^n}{a\sin(e+fx)+a}dx$$

$$\Big\downarrow \text{3457}$$

$$\frac{\int (d\sin(e+fx))^n(ad(nB+B-An)+a(A-B)d(n+1)\sin(e+fx))dx}{a^2d}+\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(a\sin(e+fx)+a)}$$

$\Big\downarrow$ 3042

$$\frac{\int (d\sin(e+fx))^n(ad(nB+B-An)+a(A-B)d(n+1)\sin(e+fx))dx}{a^2d}+\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(a\sin(e+fx)+a)}$$

$\Big\downarrow$ 3227

$$\frac{ad(-An+Bn+B)\int (d\sin(e+fx))^ndx+a(n+1)(A-B)\int (d\sin(e+fx))^{n+1}dx}{a^2d}+\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(a\sin(e+fx)+a)}$$

$\Big\downarrow$ 3042

$$\frac{ad(-An+Bn+B)\int (d\sin(e+fx))^ndx+a(n+1)(A-B)\int (d\sin(e+fx))^{n+1}dx}{a^2d}+\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(a\sin(e+fx)+a)}$$

$\Big\downarrow$ 3122

$$\frac{\frac{a(-An+Bn+B)\cos(e+fx)(d\sin(e+fx))^{n+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\sin^2(e+fx)\right)}{f(n+1)\sqrt{\cos^2(e+fx)}}+\frac{a(n+1)(A-B)\cos(e+fx)(d\sin(e+fx))^{n+2}\text{Hy}}{df(n+2)\sqrt{\text{co}}}}{a^2d}+\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(a\sin(e+fx)+a)}$$

input
```
Int[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x]),x]
```

output
```
((A - B)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*(a + a*Sin[e + f*x]))
 + ((a*(B - A*n + B*n)*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 +
 n)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(1 + n))/(f*(1 + n)*Sqrt[Cos[e + f
*x]^2]) + (a*(A - B)*(1 + n)*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n)/2
, (4 + n)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(2 + n))/(d*f*(2 + n)*Sqrt[C
os[e + f*x]^2]))/(a^2*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Maple [F]

$$\int \frac{(d \sin (fx+e))^n (A+B \sin (fx+e))}{a+a \sin (fx+e)} dx$$

input
```
int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

output
```
int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

Fricas [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{a+a\sin(e+fx)}\,dx=\int \frac{(B\sin(fx+e)+A)(d\sin(fx+e))^n}{a\sin(fx+e)+a}\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm=
"fricas")
```

output
```
integral((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{a+a\sin(e+fx)}\,dx=\text{Timed out}$$

input
```
integrate((d*sin(f*x+e))**n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{a+a\sin(e+fx)}\,dx=\int \frac{(B\sin(fx+e)+A)(d\sin(fx+e))^n}{a\sin(fx+e)+a}\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm=
"maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{a+a\sin(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,0]%%%} / %%%{1,[0,0,1]%%%} Error: Bad Argument Valu
e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{a+a\sin(e+fx)}\,dx = \int \frac{(d\,\sin(e+f\,x))^n\,(A+B\,\sin(e+f\,x))}{a+a\,\sin(e+f\,x)}\,dx$$

input
```
int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x)),x)
```

output
```
int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x)), x)
```

Reduce [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{a+a\sin(e+fx)}\,dx = \frac{d^n\left(\left(\int \frac{\sin(fx+e)^n}{\sin(fx+e)+1}dx\right)a+\left(\int \frac{\sin(fx+e)^n\sin(fx+e)}{\sin(fx+e)+1}dx\right)b\right)}{a}$$

input
```
int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

output
```
(d**n*(int(sin(e + f*x)**n/(sin(e + f*x) + 1),x)*a + int((sin(e + f*x)**n*
sin(e + f*x))/(sin(e + f*x) + 1),x)*b))/a
```

3.5 $\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 279

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = -\frac{n(A-2An+2B(1+n))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)(d\sin(e+fx))}{3a^2df(1+n)\sqrt{\cos^2(e+fx)}}$$
$$+\frac{(1+n)(B+2A(1-n)+2Bn)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\sin^2(e+fx)\right)(d\sin(e+}{3a^2d^2f(2+n)\sqrt{\cos^2(e+fx)}}$$
$$+\frac{(B+2A(1-n)+2Bn)\cos(e+fx)(d\sin(e+fx))^{1+n}}{3a^2df(1+\sin(e+fx))}$$
$$+\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{3df(a+a\sin(e+fx))^2}$$

output
```
-1/3*n*(A-2*A*n+2*B*(1+n))*cos(f*x+e)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*
n],sin(f*x+e)^2)*(d*sin(f*x+e))^(1+n)/a^2/d/f/(1+n)/(cos(f*x+e)^2)^(1/2)+1
/3*(1+n)*(B+2*A*(1-n)+2*B*n)*cos(f*x+e)*hypergeom([1/2, 1+1/2*n],[2+1/2*n]
,sin(f*x+e)^2)*(d*sin(f*x+e))^(2+n)/a^2/d^2/f/(2+n)/(cos(f*x+e)^2)^(1/2)+1
/3*(B+2*A*(1-n)+2*B*n)*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/a^2/d/f/(1+sin(f*x+
e))+1/3*(A-B)*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(a+a*sin(f*x+e))^2
```

Mathematica [A] (verified)

Time = 4.36 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.79

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx$$

$$= \frac{(d\sin(e+fx))^n\left((A-B)\sin(2(e+fx)) - \frac{2(1+\sin(e+fx))\left((1+n)(2+n)(2A(-1+n)-B(1+2n))\cos^2(e+fx)+\sqrt{\cos^2(e+fx)}\right.}{}\right.}{}$$

input
```
Integrate[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x])^2
,x]
```

output
```
((d*Sin[e + f*x])^n*((A - B)*Sin[2*(e + f*x)] - (2*(1 + Sin[e + f*x])*((1
+ n)*(2 + n)*(2*A*(-1 + n) - B*(1 + 2*n))*Cos[e + f*x]^2 + Sqrt[Cos[e + f*
x]^2]*(1 + Sin[e + f*x])*(n*(2 + n)*(A - 2*A*n + 2*B*(1 + n))*Hypergeometr
ic2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2] + (1 + n)^2*(2*A*(-1 + n)
 - B*(1 + 2*n))*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x]^
2]*Sin[e + f*x]))*Tan[e + f*x])/((1 + n)*(2 + n))))/(6*a^2*f*(1 + Sin[e +
f*x])^2)
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 3457, 3042, 3457, 25, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(d\sin(e+fx))^n}{(a\sin(e+fx)+a)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(A+B\sin(e+fx))(d\sin(e+fx))^n}{(a\sin(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3457}$$

$$\frac{\int \frac{(d\sin(e+fx))^n(ad(-nA+2A+B+Bn)+a(A-B)dn\sin(e+fx))}{\sin(e+fx)a+a}dx}{3a^2d}+\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{3df(a\sin(e+fx)+a)^2}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{(d\sin(e+fx))^n(ad(-nA+2A+B+Bn)+a(A-B)dn\sin(e+fx))}{\sin(e+fx)a+a}dx}{3a^2d}+\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{3df(a\sin(e+fx)+a)^2}$$

$\Big\downarrow$ 3457

$$\frac{\frac{\int -(d\sin(e+fx))^n\left(a^2d^2n(-2nA+A+2B(n+1))-a^2d^2(n+1)(2nB+B+2A(1-n))\sin(e+fx)\right)dx}{a^2d}+\frac{(2A(1-n)+B(2n+1))\cos(e+fx)(d\sin(e+$$

$$3a^2d$$

$$\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{3df(a\sin(e+fx)+a)^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{(2A(1-n)+B(2n+1))\cos(e+fx)(d\sin(e+fx))^{n+1}}{f(\sin(e+fx)+1)}-\frac{\int(d\sin(e+fx))^n\left(a^2d^2n(-2nA+A+2B(n+1))-a^2d^2(n+1)(2nB+B+2A(1-n))\sin(e+\right.}{a^2d}$$

$$3a^2d$$

$$\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{3df(a\sin(e+fx)+a)^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{(2A(1-n)+B(2n+1))\cos(e+fx)(d\sin(e+fx))^{n+1}}{f(\sin(e+fx)+1)}-\frac{\int(d\sin(e+fx))^n\left(a^2d^2n(-2nA+A+2B(n+1))-a^2d^2(n+1)(2nB+B+2A(1-n))\sin(e+\right.}{a^2d}$$

$$3a^2d$$

$$\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{3df(a\sin(e+fx)+a)^2}$$

$\Big\downarrow$ 3227

$$\frac{\frac{(2A(1-n)+B(2n+1))\cos(e+fx)(d\sin(e+fx))^{n+1}}{f(\sin(e+fx)+1)}-\frac{a^2d^2n(-2An+A+2B(n+1))\int(d\sin(e+fx))^ndx-a^2d(n+1)(2A(1-n)+2Bn+B)\int(d\sin}{a^2d}$$

$$3a^2d$$

$$\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{3df(a\sin(e+fx)+a)^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{(2A(1-n)+B(2n+1))\cos(e+fx)(d\sin(e+fx))^{n+1}}{f(\sin(e+fx)+1)}-\frac{a^2d^2n(-2An+A+2B(n+1))\int(d\sin(e+fx))^ndx-a^2d(n+1)(2A(1-n)+2Bn+B)\int(d\sin}{a^2d}}{3a^2d}$$
$$\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{3df(a\sin(e+fx)+a)^2}$$

$\Big\downarrow$ 3122

$$\frac{\frac{(2A(1-n)+B(2n+1))\cos(e+fx)(d\sin(e+fx))^{n+1}}{f(\sin(e+fx)+1)}-\frac{\frac{a^2dn(-2An+A+2B(n+1))\cos(e+fx)(d\sin(e+fx))^{n+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\sin^2(e+\right.}{f(n+1)\sqrt{\cos^2(e+fx)}}}{}}{3a^2d}$$
$$\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{3df(a\sin(e+fx)+a)^2}$$

input
```
Int[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x])^2,x]
```

output
```
((A - B)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(3*d*f*(a + a*Sin[e + f*x]
)^2) + (((2*A*(1 - n) + B*(1 + 2*n))*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n)
)/(f*(1 + Sin[e + f*x])) - ((a^2*d*n*(A - 2*A*n + 2*B*(1 + n))*Cos[e + f*x
]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2]*(d*Sin[e +
f*x])^(1 + n))/(f*(1 + n)*Sqrt[Cos[e + f*x]^2]) - (a^2*(1 + n)*(B + 2*A*(1
 - n) + 2*B*n)*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, S
in[e + f*x]^2]*(d*Sin[e + f*x])^(2 + n))/(f*(2 + n)*Sqrt[Cos[e + f*x]^2]))
/(a^2*d))/(3*a^2*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Maple [F]

$$\int \frac{(d\sin(fx+e))^{n} (A+B\sin(fx+e))}{(a+a\sin(fx+e))^{2}}dx$$

input
```
int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x)
```

output
```
int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x)
```

Fricas [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e))^{n}}{(a\sin(fx+e)+a)^{2}}\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorith
m="fricas")
```

output
```
integral(-(B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a^2*cos(f*x + e)^2 - 2*
a^2*sin(f*x + e) - 2*a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((d*sin(f*x+e))**n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e))^n}{(a\sin(fx+e)+a)^2}\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorith
m="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a)^2,
x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,0]%%%} / %%%{1,[0,0,2]%%%} Error: Bad Argument Valu
e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = \int \frac{\left(d\,\sin\left(e+f\,x\right)\right)^n\,\left(A+B\,\sin\left(e+f\,x\right)\right)}{\left(a+a\,\sin\left(e+f\,x\right)\right)^2}\,dx$$

input
```
int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^2,x)
```

output
```
int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^2, x)
```

Reduce [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = \frac{d^n\left(\left(\int \frac{\sin(fx+e)^n}{\sin(fx+e)^2+2\sin(fx+e)+1}dx\right)a+\left(\int \frac{\sin(fx+e)^n\sin(fx+e)}{\sin(fx+e)^2+2\sin(fx+e)+1}dx\right)b\right)}{a^2}$$

input
```
int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x)
```

output
```
(d**n*(int(sin(e + f*x)**n/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),x)*a + i
nt((sin(e + f*x)**n*sin(e + f*x))/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),x
)*b))/a**2
```

3.6 $\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 362

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx =$$
$$-\frac{n(B(3-n-4n^2)+A(2-9n+4n^2))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\sin^2(e+fx)\right)(d}{15a^3df(1+n)\sqrt{\cos^2(e+fx)}}$$
$$+\frac{(1-n)(1+n)(7A+3B-4An+4Bn)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\sin^2(e+fx)\right)(}{15a^3d^2f(2+n)\sqrt{\cos^2(e+fx)}}$$
$$+\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{5df(a+a\sin(e+fx))^3}$$
$$+\frac{(A(5-2n)+2Bn)\cos(e+fx)(d\sin(e+fx))^{1+n}}{15adf(a+a\sin(e+fx))^2}$$
$$+\frac{(1-n)(7A+3B-4An+4Bn)\cos(e+fx)(d\sin(e+fx))^{1+n}}{15df\,(a^3+a^3\sin(e+fx))}$$

output
```
-1/15*n*(B*(-4*n^2-n+3)+A*(4*n^2-9*n+2))*cos(f*x+e)*hypergeom([1/2, 1/2+1/
2*n],[3/2+1/2*n],sin(f*x+e)^2)*(d*sin(f*x+e))^(1+n)/a^3/d/f/(1+n)/(cos(f*x
+e)^2)^(1/2)+1/15*(1-n)*(1+n)*(-4*A*n+4*B*n+7*A+3*B)*cos(f*x+e)*hypergeom(
[1/2, 1+1/2*n],[2+1/2*n],sin(f*x+e)^2)*(d*sin(f*x+e))^(2+n)/a^3/d^2/f/(2+n
)/(cos(f*x+e)^2)^(1/2)+1/5*(A-B)*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(a+a*
sin(f*x+e))^3+1/15*(A*(5-2*n)+2*B*n)*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/a/d/f
/(a+a*sin(f*x+e))^2+1/15*(1-n)*(-4*A*n+4*B*n+7*A+3*B)*cos(f*x+e)*(d*sin(f*
x+e))^(1+n)/d/f/(a^3+a^3*sin(f*x+e))
```

Mathematica [A] (verified)

Time = 6.20 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.18

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = \frac{(aA-aB)\cos(e+fx)\sin(e+fx)(d\sin(e+fx))^n}{5af(a+a\sin(e+fx))^3} + \frac{\sin^{-n}(e+fx)(d\sin(e+fx))^n\left(\frac{(a^2(A-B)(1-n)+a^2(4A+B-An+Bn))\cos(e+fx)\sin^{1+n}(e+fx)}{3af(a+a\sin(e+fx))^2}+\frac{\left(-a^3n(A(5-2n)+2Bn)+\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x])^3
,x]
```

output
```
((a*A - a*B)*Cos[e + f*x]*Sin[e + f*x]*(d*Sin[e + f*x])^n)/(5*a*f*(a + a*S
in[e + f*x])^3) + ((d*Sin[e + f*x])^n*(((a^2*(A - B)*(1 - n) + a^2*(4*A +
B - A*n + B*n))*Cos[e + f*x]*Sin[e + f*x]^(1 + n))/(3*a*f*(a + a*Sin[e + f
*x])^2) + (((-(a^3*n*(A*(5 - 2*n) + 2*B*n)) + a^3*(B*(3 + n - 2*n^2) + A*(
7 - 6*n + 2*n^2)))*Cos[e + f*x]*Sin[e + f*x]^(1 + n))/(a*f*(a + a*Sin[e +
f*x])) + (-((a^3*n*(B*(3 - n - 4*n^2) + A*(2 - 9*n + 4*n^2))*Sqrt[Cos[e +
f*x]^2]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2]*Sec[e
 + f*x]*Sin[e + f*x]^(1 + n))/(f*(1 + n))) + (a^3*(1 - n)*(1 + n)*(7*A + 3
*B - 4*A*n + 4*B*n)*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[1/2, (2 + n)/2,
 (4 + n)/2, Sin[e + f*x]^2]*Sec[e + f*x]*Sin[e + f*x]^(2 + n))/(f*(2 + n))
)/a^2)/(3*a^2)))/(5*a^2*Sin[e + f*x]^n)
```

Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3457, 3042, 3457, 3042, 3457, 25, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(d\sin(e+fx))^n}{(a\sin(e+fx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(A+B\sin(e+fx))(d\sin(e+fx))^n}{(a\sin(e+fx)+a)^3}dx$$

$$\Big\downarrow \text{3457}$$

$$\frac{\int \frac{(d\sin(e+fx))^n(ad(-nA+4A+B+Bn)-a(A-B)d(1-n)\sin(e+fx))}{(\sin(e+fx)a+a)^2}dx}{5a^2d}+\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{5df(a\sin(e+fx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(d\sin(e+fx))^n(ad(-nA+4A+B+Bn)-a(A-B)d(1-n)\sin(e+fx))}{(\sin(e+fx)a+a)^2}dx}{5a^2d}+\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{5df(a\sin(e+fx)+a)^3}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\int \frac{(d\sin(e+fx))^n\left(a^2\left(B\left(-2n^2+n+3\right)+A\left(2n^2-6n+7\right)\right)d^2+a^2n(A(5-2n)+2Bn)\sin(e+fx)d^2\right)}{\sin(e+fx)a+a}dx}{3a^2d}+\frac{a(A(5-2n)+2Bn)\cos(e+fx)(d\sin(e+fx))^{n+1}}{3f(a\sin(e+fx)+a)^2}}{5a^2d}+\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{5df(a\sin(e+fx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{(d\sin(e+fx))^n\left(a^2\left(B\left(-2n^2+n+3\right)+A\left(2n^2-6n+7\right)\right)d^2+a^2n(A(5-2n)+2Bn)\sin(e+fx)d^2\right)}{\sin(e+fx)a+a}dx}{3a^2d}+\frac{a(A(5-2n)+2Bn)\cos(e+fx)(d\sin(e+fx))^{n+1}}{3f(a\sin(e+fx)+a)^2}}{5a^2d}+\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{5df(a\sin(e+fx)+a)^3}$$

$\downarrow$ 3457

$$\frac{\frac{\frac{\int -(d\sin(e+fx))^n\left(a^3d^3n\left(B\left(-4n^2-n+3\right)+A\left(4n^2-9n+2\right)\right)-a^3d^3(1-n)(n+1)(-4nA+7A+3B+4Bn)\sin(e+fx)\right)dx}{a^2d}+\frac{a^2d(1-n)(-4An+7A+4Bn+3B)\cos(e\cdots}{f(a\sin(e+fx)+a\cdots}}{3a^2d}\cdots}{5a^2d}$$

$$\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{5df(a\sin(e+fx)+a)^3}$$

$\downarrow$ 25

$$\frac{\frac{\frac{a^2d(1-n)(-4An+7A+4Bn+3B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{f(a\sin(e+fx)+a)}-\frac{\int(d\sin(e+fx))^n\left(a^3d^3n\left(B\left(-4n^2-n+3\right)+A\left(4n^2-9n+2\right)\right)-a^3d^3(1-n)(n+1)(-4nA+7A+3B\cdots\right.}{a^2d}}{3a^2d}\cdots}{5a^2d}$$

$$\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{5df(a\sin(e+fx)+a)^3}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{a^2d(1-n)(-4An+7A+4Bn+3B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{f(a\sin(e+fx)+a)}-\frac{\int(d\sin(e+fx))^n\left(a^3d^3n\left(B\left(-4n^2-n+3\right)+A\left(4n^2-9n+2\right)\right)-a^3d^3(1-n)(n+1)(-4nA+7A+3B\cdots\right.}{a^2d}}{3a^2d}\cdots}{5a^2d}$$

$$\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{5df(a\sin(e+fx)+a)^3}$$

$\downarrow$ 3227

$$\frac{\frac{\frac{a^2d(1-n)(-4An+7A+4Bn+3B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{f(a\sin(e+fx)+a)}-\frac{a^3d^3n\left(A\left(4n^2-9n+2\right)+B\left(-4n^2-n+3\right)\right)\int(d\sin(e+fx))^ndx-a^3d^2(1-n)(n+1)(-4An+7A+4\cdots}{a^2d}}{3a^2d}\cdots}{5a^2d}$$

$$\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{5df(a\sin(e+fx)+a)^3}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{a^2d(1-n)(-4An+7A+4Bn+3B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{f(a\sin(e+fx)+a)}-\frac{a^3d^3n\left(A\left(4n^2-9n+2\right)+B\left(-4n^2-n+3\right)\right)\int(d\sin(e+fx))^ndx-a^3d^2(1-n)(n+1)(-4An+7A+4\cdots}{a^2d}}{3a^2d}\cdots}{5a^2d}$$

$$\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{5df(a\sin(e+fx)+a)^3}$$

$\downarrow$ 3122

$$\frac{\frac{\frac{a^2 d(1-n)(-4An+7A+4Bn+3B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{f(a\sin(e+fx)+a)} - \frac{a^3 d^2 n\left(A\left(4n^2-9n+2\right)+B\left(-4n^2-n+3\right)\right)\cos(e+fx)(d\sin(e+fx))^{n+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\right.}{f(n+1)\sqrt{\cos^2(e+fx)}}}{3a^2d}}{}$$

$$\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{5df(a\sin(e+fx)+a)^3}$$

input
```
Int[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x])^3,x]
```

output
```
((A - B)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(5*d*f*(a + a*Sin[e + f*x]
)^3) + ((a*(A*(5 - 2*n) + 2*B*n)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(3
*f*(a + a*Sin[e + f*x])^2) + ((a^2*d*(1 - n)*(7*A + 3*B - 4*A*n + 4*B*n)*C
os[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(f*(a + a*Sin[e + f*x])) - ((a^3*d^2
*n*(B*(3 - n - 4*n^2) + A*(2 - 9*n + 4*n^2))*Cos[e + f*x]*Hypergeometric2F
1[1/2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(1 + n))/(f*
(1 + n)*Sqrt[Cos[e + f*x]^2]) - (a^3*d*(1 - n)*(1 + n)*(7*A + 3*B - 4*A*n
+ 4*B*n)*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e +
 f*x]^2]*(d*Sin[e + f*x])^(2 + n))/(f*(2 + n)*Sqrt[Cos[e + f*x]^2]))/(a^2*
d))/(3*a^2*d))/(5*a^2*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Maple [F]

$$\int \frac{(d\sin(fx+e))^n (A+B\sin(fx+e))}{(a+a\sin(fx+e))^3} dx$$

input
```
int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x)
```

output
```
int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x)
```

Fricas [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^3} dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e))^n}{(a\sin(fx+e)+a)^3} dx$$

input
```
integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorith
m="fricas")
```

output
```
integral(-(B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(3*a^3*cos(f*x + e)^2 -
4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((d*sin(f*x+e))**n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e))^n}{(a\sin(fx+e)+a)^3}\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorith
m="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a)^3,
x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,0]%%%} / %%%{1,[0,0,3]%%%} Error: Bad Argument Valu
e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = \int \frac{(d\,\sin\left(e+f\,x\right))^n\,\left(A+B\,\sin\left(e+f\,x\right)\right)}{\left(a+a\,\sin\left(e+f\,x\right)\right)^3}\,dx$$

input
```
int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^3,x)
```

output
```
int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^3, x)
```

Reduce [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = \frac{d^n\left(\left(\int \frac{\sin(fx+e)^n}{\sin(fx+e)^3+3\sin(fx+e)^2+3\sin(fx+e)+1}dx\right)a+\left(\int \frac{\sin(fx+e)^n\sin(fx+e)}{\sin(fx+e)^3+3\sin(fx+e)^2+3\sin(fx+e)+1}dx\right)b\right)}{a^3}$$

input
```
int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x)
```

output
```
(d**n*(int(sin(e + f*x)**n/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e
+ f*x) + 1),x)*a + int((sin(e + f*x)**n*sin(e + f*x))/(sin(e + f*x)**3 + 3
*sin(e + f*x)**2 + 3*sin(e + f*x) + 1),x)*b))/a**3
```

3.7 $\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 358

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx =$$

$$-\frac{2a^3(2B(35+23n+4n^2)+A(77+50n+8n^2))\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)(5+2n)(7+2n)\sqrt{a+a\sin(e+fx)}}$$

$$+\frac{a^4(2B(115+203n+104n^2+16n^3)+A(301+478n+224n^2+32n^3))\cos(e+fx)\,\text{Hypergeometric2F1}}{df(5+2n)(7+2n)\left(3+5n+2n^2\right)(a-a\sin(e+fx))\sqrt{}}$$

$$-\frac{2a^2(2B(5+n)+A(7+2n))\cos(e+fx)(d\sin(e+fx))^{1+n}\sqrt{a+a\sin(e+fx)}}{df(5+2n)(7+2n)}$$

$$-\frac{2aB\cos(e+fx)(d\sin(e+fx))^{1+n}(a+a\sin(e+fx))^{3/2}}{df(7+2n)}$$

output
```
-2*a^3*(2*B*(4*n^2+23*n+35)+A*(8*n^2+50*n+77))*cos(f*x+e)*(d*sin(f*x+e))^(
1+n)/d/f/(3+2*n)/(5+2*n)/(7+2*n)/(a+a*sin(f*x+e))^(1/2)+a^4*(2*B*(16*n^3+1
04*n^2+203*n+115)+A*(32*n^3+224*n^2+478*n+301))*cos(f*x+e)*hypergeom([1/2,
 1+n],[2+n],sin(f*x+e))*(1-sin(f*x+e))^(1/2)*(d*sin(f*x+e))^(1+n)/d/f/(5+2
*n)/(7+2*n)/(2*n^2+5*n+3)/(a-a*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)-2*a^2*(2
*B*(5+n)+A*(7+2*n))*cos(f*x+e)*(d*sin(f*x+e))^(1+n)*(a+a*sin(f*x+e))^(1/2)
/d/f/(5+2*n)/(7+2*n)-2*a*B*cos(f*x+e)*(d*sin(f*x+e))^(1+n)*(a+a*sin(f*x+e)
)^(3/2)/d/f/(7+2*n)
```

Mathematica [A] (warning: unable to verify)

Time = 31.69 (sec) , antiderivative size = 596, normalized size of antiderivative = 1.66

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx = \frac{2^{1+n}\sec\left(\frac{1}{2}(e+fx)\right)\sin^{-n}(e+fx)(d\sin(e+fx))^n(a(1+\sin(e+fx)))^{5/2}\tan\left(\frac{1}{2}(e+\right.}{}$$

input
```
Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x
]),x]
```

output
```
(2^(1 + n)*Sec[(e + f*x)/2]*(d*Sin[e + f*x])^n*(a*(1 + Sin[e + f*x]))^(5/2
)*Tan[(e + f*x)/2]*(Tan[(e + f*x)/2]/(1 + Tan[(e + f*x)/2]^2))^n*(1 + Tan[
(e + f*x)/2]^2)^n*((A*Hypergeometric2F1[(1 + n)/2, 9/2 + n, (3 + n)/2, -Ta
n[(e + f*x)/2]^2])/(1 + n) + (A*Hypergeometric2F1[4 + n/2, 9/2 + n, 5 + n/
2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^7)/(8 + n) + Tan[(e + f*x)/2]*(((
5*A + 2*B)*Hypergeometric2F1[(2 + n)/2, 9/2 + n, (4 + n)/2, -Tan[(e + f*x)
/2]^2])/(2 + n) + Tan[(e + f*x)/2]*(((11*A + 10*B)*Hypergeometric2F1[(3 +
n)/2, 9/2 + n, (5 + n)/2, -Tan[(e + f*x)/2]^2])/(3 + n) + Tan[(e + f*x)/2]
*((5*(3*A + 4*B)*Hypergeometric2F1[(4 + n)/2, 9/2 + n, (6 + n)/2, -Tan[(e
+ f*x)/2]^2])/(4 + n) + Tan[(e + f*x)/2]*((5*(3*A + 4*B)*Hypergeometric2F1
[9/2 + n, (5 + n)/2, (7 + n)/2, -Tan[(e + f*x)/2]^2])/(5 + n) + Tan[(e + f
*x)/2]*(((11*A + 10*B)*Hypergeometric2F1[9/2 + n, (6 + n)/2, (8 + n)/2, -T
an[(e + f*x)/2]^2])/(6 + n) + ((5*A + 2*B)*Hypergeometric2F1[9/2 + n, (7 +
 n)/2, (9 + n)/2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2])/(7 + n))))))))/(f
*Sqrt[Sec[(e + f*x)/2]^2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*Sin[e +
f*x]^n)
```

Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.90, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {3042, 3455, 27, 3042, 3455, 27, 3042, 3460, 3042, 3255, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))(d\sin(e+fx))^n\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))(d\sin(e+fx))^n dx$$

$$\Big\downarrow \text{ 3455}$$

$$\frac{2\int \frac{1}{2}(d\sin(e+fx))^n(\sin(e+fx)a+a)^{3/2}\left(ad\left(2B(n+1)+2A\left(n+\frac{7}{2}\right)\right)+ad(2B(n+5)+A(2n+7))\sin(e+\right.}{d(2n+7)}$$
$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(d\sin(e+fx))^{n+1}}{df(2n+7)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int (d\sin(e+fx))^n(\sin(e+fx)a+a)^{3/2}(ad(2B(n+1)+A(2n+7))+ad(2B(n+5)+A(2n+7))\sin(e+fx))}{d(2n+7)}$$
$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(d\sin(e+fx))^{n+1}}{df(2n+7)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int (d\sin(e+fx))^n(\sin(e+fx)a+a)^{3/2}(ad(2B(n+1)+A(2n+7))+ad(2B(n+5)+A(2n+7))\sin(e+fx))}{d(2n+7)}$$
$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(d\sin(e+fx))^{n+1}}{df(2n+7)}$$

$$\Big\downarrow \text{ 3455}$$

$$\frac{\frac{2\int \frac{1}{2}(d\sin(e+fx))^n\sqrt{\sin(e+fx)a+a}\left(a^2\left(2B\left(4n^2+19n+15\right)+A\left(8n^2+42n+49\right)\right)d^2+a^2\left(2B\left(4n^2+23n+35\right)+A\left(8n^2+50n+77\right)\right)\sin(e+fx)d^2\right)d}{d(2n+5)}}{d(2n+7)}$$
$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(d\sin(e+fx))^{n+1}}{df(2n+7)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int (d\sin(e+fx))^n\sqrt{\sin(e+fx)a+a}\left(a^2\left(2B\left(4n^2+19n+15\right)+A\left(8n^2+42n+49\right)\right)d^2+a^2\left(2B\left(4n^2+23n+35\right)+A\left(8n^2+50n+77\right)\right)\sin(e+fx)d^2\right)dx}{d(2n+5)} -}{d(2n+7)}$$
$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(d\sin(e+fx))^{n+1}}{df(2n+7)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\int (d\sin(e+fx))^n\sqrt{\sin(e+fx)a+a}\left(a^2\left(2B\left(4n^2+19n+15\right)+A\left(8n^2+42n+49\right)\right)d^2+a^2\left(2B\left(4n^2+23n+35\right)+A\left(8n^2+50n+77\right)\right)\sin(e+fx)d^2\right)dx}{d(2n+5)}-}{d(2n+7)}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(d\sin(e+fx))^{n+1}}{df(2n+7)}$$

$\Big\downarrow$ 3460

$$\frac{\frac{\frac{a^2d^2\left(A\left(32n^3+224n^2+478n+301\right)+2B\left(16n^3+104n^2+203n+115\right)\right)\int (d\sin(e+fx))^n\sqrt{\sin(e+fx)a+a}dx}{2n+3}-\frac{2a^3d\left(A\left(8n^2+50n+77\right)+2B\left(4n^2+23n+35\right)\right)\cos(e+f}{f(2n+3)\sqrt{a\sin(e+fx)+a}}}{d(2n+5)}}{d(2n+7)}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(d\sin(e+fx))^{n+1}}{df(2n+7)}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{a^2d^2\left(A\left(32n^3+224n^2+478n+301\right)+2B\left(16n^3+104n^2+203n+115\right)\right)\int (d\sin(e+fx))^n\sqrt{\sin(e+fx)a+a}dx}{2n+3}-\frac{2a^3d\left(A\left(8n^2+50n+77\right)+2B\left(4n^2+23n+35\right)\right)\cos(e+f}{f(2n+3)\sqrt{a\sin(e+fx)+a}}}{d(2n+5)}}{d(2n+7)}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(d\sin(e+fx))^{n+1}}{df(2n+7)}$$

$\Big\downarrow$ 3255

$$\frac{\frac{\frac{a^4d^2\left(A\left(32n^3+224n^2+478n+301\right)+2B\left(16n^3+104n^2+203n+115\right)\right)\cos(e+fx)\int\frac{(d\sin(e+fx))^n}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f(2n+3)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}-\frac{2a^3d\left(A\left(8n^2+50n+77\right)+2B\left(4n^2+23n+35\right)\right)c}{f(2n+3)\sqrt{a\sin(e+fx)}}}{d(2n+5)}}{d(2n+7)}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(d\sin(e+fx))^{n+1}}{df(2n+7)}$$

$\Big\downarrow$ 77

$$\frac{\frac{\frac{a^4d^2\left(A\left(32n^3+224n^2+478n+301\right)+2B\left(16n^3+104n^2+203n+115\right)\right)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\int\frac{\sin^n(e+fx)}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f(2n+3)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}-2a^3d\left(A\left(8n^2+50n\right.\right.}{d(2n+5)}}{d(2n+7)}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(d\sin(e+fx))^{n+1}}{df(2n+7)}$$

$\Big\downarrow$ 75

$$\frac{\frac{-\frac{2a^3d^2\left(A\left(32n^3+224n^2+478n+301\right)+2B\left(16n^3+104n^2+203n+115\right)\right)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},1-\sin(e+fx)\right)}{f(2n+3)\sqrt{a\sin(e+fx)+a}}-\frac{2a}{}}{d(2n+5)}}{d(2n}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(d\sin(e+fx))^{n+1}}{df(2n+7)}$$

input
```
Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]),x]
```

output
```
(-2*a*B*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n)*(a + a*Sin[e + f*x])^(3/2))/
(d*f*(7 + 2*n)) + ((-2*a^2*(2*B*(5 + n) + A*(7 + 2*n))*Cos[e + f*x]*(d*Sin
[e + f*x])^(1 + n)*Sqrt[a + a*Sin[e + f*x]])/(f*(5 + 2*n)) + ((-2*a^3*d^2*
(2*B*(115 + 203*n + 104*n^2 + 16*n^3) + A*(301 + 478*n + 224*n^2 + 32*n^3)
)*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]]*(d*Sin[e
+ f*x])^n)/(f*(3 + 2*n)*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x]]) - (2*a^3*
d*(2*B*(35 + 23*n + 4*n^2) + A*(77 + 50*n + 8*n^2))*Cos[e + f*x]*(d*Sin[e
+ f*x])^(1 + n))/(f*(3 + 2*n)*Sqrt[a + a*Sin[e + f*x]]))/(d*(5 + 2*n)))/(d
*(7 + 2*n))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3255
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x]
, x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [F]

$$\int (d\sin(fx+e))^n (a+a\sin(fx+e))^{\frac{5}{2}} (A+B\sin(fx+e))\, dx$$

input
```
int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x)
```

output
```
int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x)
```

Fricas [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}(d\sin(fx+e))^n\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x, algo
rithm="fricas")
```

output
```
integral(-((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x +
 e)^2 - 2*(A + B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x +
 e))^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx=\text{Timed out}$$

input
```
integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}(d\sin(fx+e))^n\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e))
^n, x)
```

Giac [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}(d\sin(fx+e))^n\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x, algo
rithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e))
^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx=\int (d\sin(e+f\,x))^n\,(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{5/2}\,dx$$

input
```
int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2),x)
```

output
```
int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2), x)
```

Reduce [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx = d^n\sqrt{a}\,a^2\left(\left(\left(\int \sin(fx+e)^n\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^3\,dx\right)b + \left(\int \sin(fx+e)^n\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)a + 2\left(\int \sin(fx+e)^n\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)b + 2\left(\int \sin(fx+e)^n\sqrt{\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)a + \left(\int \sin(fx+e)^n\sqrt{\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)b + \left(\int \sin(fx+e)^n\sqrt{\sin(fx+e)+1}dx\right)a\right)$$

input
```
int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x)
```

output
```
d**n*sqrt(a)*a**2*(int(sin(e + f*x)**n*sqrt(sin(e + f*x) + 1)*sin(e + f*x)
**3,x)*b + int(sin(e + f*x)**n*sqrt(sin(e + f*x) + 1)*sin(e + f*x)**2,x)*a
 + 2*int(sin(e + f*x)**n*sqrt(sin(e + f*x) + 1)*sin(e + f*x)**2,x)*b + 2*i
nt(sin(e + f*x)**n*sqrt(sin(e + f*x) + 1)*sin(e + f*x),x)*a + int(sin(e +
f*x)**n*sqrt(sin(e + f*x) + 1)*sin(e + f*x),x)*b + int(sin(e + f*x)**n*sqr
t(sin(e + f*x) + 1),x)*a)
```

3.8 $\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 251

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx = -\frac{2a^2(2B(3+n)+A(5+2n))\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)(5+2n)\sqrt{a+a\sin(e+fx)}} + \frac{a^3(2B(9+13n+4n^2)+A(25+30n+8n^2))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1+n,2+n,\sin(e+f\right.}{df(5+2n)\left(3+5n+2n^2\right)(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx}} - \frac{2aB\cos(e+fx)(d\sin(e+fx))^{1+n}\sqrt{a+a\sin(e+fx)}}{df(5+2n)}$$

output
```
-2*a^2*(2*B*(3+n)+A*(5+2*n))*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(3+2*n)/(
5+2*n)/(a+a*sin(f*x+e))^(1/2)+a^3*(2*B*(4*n^2+13*n+9)+A*(8*n^2+30*n+25))*c
os(f*x+e)*hypergeom([1/2, 1+n],[2+n],sin(f*x+e))*(1-sin(f*x+e))^(1/2)*(d*s
in(f*x+e))^(1+n)/d/f/(5+2*n)/(2*n^2+5*n+3)/(a-a*sin(f*x+e))/(a+a*sin(f*x+e
))^(1/2)-2*a*B*cos(f*x+e)*(d*sin(f*x+e))^(1+n)*(a+a*sin(f*x+e))^(1/2)/d/f/
(5+2*n)
```

Mathematica [A] (warning: unable to verify)

Time = 25.27 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.90

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx = \frac{2^{1+n}\sec\left(\frac{1}{2}(e+fx)\right)\sin^{-n}(e+fx)(d\sin(e+fx))^n(a(1+\sin(e+fx)))^{3/2}\tan\left(\frac{1}{2}(e+\right.}{}$$

input
```
Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x
]),x]
```

output
```
(2^(1 + n)*Sec[(e + f*x)/2]*(d*Sin[e + f*x])^n*(a*(1 + Sin[e + f*x]))^(3/2
)*Tan[(e + f*x)/2]*(Tan[(e + f*x)/2]/(1 + Tan[(e + f*x)/2]^2))^n*(1 + Tan[
(e + f*x)/2]^2)^n*((A*Hypergeometric2F1[(1 + n)/2, 7/2 + n, (3 + n)/2, -Ta
n[(e + f*x)/2]^2])/(1 + n) + Tan[(e + f*x)/2]*(((3*A + 2*B)*Hypergeometric
2F1[(2 + n)/2, 7/2 + n, (4 + n)/2, -Tan[(e + f*x)/2]^2])/(2 + n) + Tan[(e
+ f*x)/2]*((2*(2*A + 3*B)*Hypergeometric2F1[(3 + n)/2, 7/2 + n, (5 + n)/2,
 -Tan[(e + f*x)/2]^2])/(3 + n) + Tan[(e + f*x)/2]*((2*(2*A + 3*B)*Hypergeo
metric2F1[7/2 + n, (4 + n)/2, (6 + n)/2, -Tan[(e + f*x)/2]^2])/(4 + n) + T
an[(e + f*x)/2]*(((3*A + 2*B)*Hypergeometric2F1[7/2 + n, (5 + n)/2, (7 + n
)/2, -Tan[(e + f*x)/2]^2])/(5 + n) + (A*Hypergeometric2F1[7/2 + n, (6 + n)
/2, (8 + n)/2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2])/(6 + n)))))))/(f*Sqr
t[Sec[(e + f*x)/2]^2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sin[e + f*x]
^n)
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.90, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {3042, 3455, 27, 3042, 3460, 3042, 3255, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))(d\sin(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))(d\sin(e+fx))^n dx$$

$$\Big\downarrow \text{3455}$$

$$\frac{2\int \frac{1}{2}(d\sin(e+fx))^n\sqrt{\sin(e+fx)a+a}\left(ad\left(2B(n+1)+2A\left(n+\frac{5}{2}\right)\right)+ad(2B(n+3)+A(2n+5))\sin(e+fx\right.}{d(2n+5)}$$
$$\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(d\sin(e+fx))^{n+1}}{df(2n+5)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (d\sin(e+fx))^n\sqrt{\sin(e+fx)a+a}(ad(2B(n+1)+A(2n+5))+ad(2B(n+3)+A(2n+5))\sin(e+fx))dx}{d(2n+5)}$$
$$\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(d\sin(e+fx))^{n+1}}{df(2n+5)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (d\sin(e+fx))^n\sqrt{\sin(e+fx)a+a}(ad(2B(n+1)+A(2n+5))+ad(2B(n+3)+A(2n+5))\sin(e+fx))dx}{d(2n+5)}$$
$$\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(d\sin(e+fx))^{n+1}}{df(2n+5)}$$

$$\Big\downarrow \text{3460}$$

$$\frac{\frac{ad(A(8n^2+30n+25)+2B(4n^2+13n+9))\int (d\sin(e+fx))^n\sqrt{\sin(e+fx)a+a}dx}{2n+3}-\frac{2a^2(A(2n+5)+2B(n+3))\cos(e+fx)(d\sin(e+fx))^{n+1}}{f(2n+3)\sqrt{a\sin(e+fx)+a}}}{d(2n+5)}-$$
$$\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(d\sin(e+fx))^{n+1}}{df(2n+5)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{ad(A(8n^2+30n+25)+2B(4n^2+13n+9))\int (d\sin(e+fx))^n\sqrt{\sin(e+fx)a+a}dx}{2n+3}-\frac{2a^2(A(2n+5)+2B(n+3))\cos(e+fx)(d\sin(e+fx))^{n+1}}{f(2n+3)\sqrt{a\sin(e+fx)+a}}}{d(2n+5)}-$$
$$\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(d\sin(e+fx))^{n+1}}{df(2n+5)}$$

$\Big\downarrow$ 3255

$$\frac{\frac{a^3 d\left(A\left(8n^2+30n+25\right)+2B\left(4n^2+13n+9\right)\right)\cos(e+fx)\int\frac{(d\sin(e+fx))^n}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f(2n+3)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}-\frac{2a^2(A(2n+5)+2B(n+3))\cos(e+fx)(d\sin(e+fx))^{n+1}}{f(2n+3)\sqrt{a\sin(e+fx)+a}}}{d(2n+5)}$$
$$\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(d\sin(e+fx))^{n+1}}{df(2n+5)}$$

$\Big\downarrow$ 77

$$\frac{\frac{a^3 d\left(A\left(8n^2+30n+25\right)+2B\left(4n^2+13n+9\right)\right)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\int\frac{\sin^n(e+fx)}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f(2n+3)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}-\frac{2a^2(A(2n+5)+2B(n+3))\,\mathrm{c}}{f(2n+3)\sqrt{a}}}{d(2n+5)}$$
$$\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(d\sin(e+fx))^{n+1}}{df(2n+5)}$$

$\Big\downarrow$ 75

$$\frac{-\frac{2a^2 d\left(A\left(8n^2+30n+25\right)+2B\left(4n^2+13n+9\right)\right)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},1-\sin(e+fx)\right)}{f(2n+3)\sqrt{a\sin(e+fx)+a}}-2a^2(A(2}{d(2n+5)}$$
$$\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(d\sin(e+fx))^{n+1}}{df(2n+5)}$$

input
```
Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]),x]
```

output
```
(-2*a*B*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n)*Sqrt[a + a*Sin[e + f*x]])/(d
*f*(5 + 2*n)) + ((-2*a^2*d*(2*B*(9 + 13*n + 4*n^2) + A*(25 + 30*n + 8*n^2)
)*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]]*(d*Sin[e
+ f*x])^n)/(f*(3 + 2*n)*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x]]) - (2*a^2*
(2*B*(3 + n) + A*(5 + 2*n))*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(f*(3 +
 2*n)*Sqrt[a + a*Sin[e + f*x]]))/(d*(5 + 2*n))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3255
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x]
, x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [F]

$$\int (d\sin(fx+e))^n (a+a\sin(fx+e))^{\frac{3}{2}} (A+B\sin(fx+e))\, dx$$

input
```
int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x)
```

output
```
int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x)
```

Fricas [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}(d\sin(fx+e))^n\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x, algo
rithm="fricas")
```

output
```
integral(-(B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*sqrt(a
*sin(f*x + e) + a)*(d*sin(f*x + e))^n, x)
```

Sympy [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx=\int (a(\sin(e+fx)+1))^{\frac{3}{2}}(d\sin(e+fx))^n(A+B\sin(e+fx))\,dx$$

input
```
integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e)),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**(3/2)*(d*sin(e + f*x))**n*(A + B*sin(e +
f*x)), x)
```

Maxima [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}(d\sin(fx+e))^n\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e))
^n, x)
```

Giac [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}(d\sin(fx+e))^n\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x, algo
rithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e))
^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx=\int (d\,\sin(e+f\,x))^n\,(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{3/2}\,dx$$

input
```
int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2),x)
```

output
```
int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx = d^n\sqrt{a}\,a\left(\left(\int \sin(fx+e)^n\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)b + \left(\int \sin(fx+e)^n\sqrt{\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)a + \left(\int \sin(fx+e)^n\sqrt{\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)b + \left(\int \sin(fx+e)^n\sqrt{\sin(fx+e)+1}dx\right)a\right)$$

input
```
int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x)
```

output
```
d**n*sqrt(a)*a*(int(sin(e + f*x)**n*sqrt(sin(e + f*x) + 1)*sin(e + f*x)**2
,x)*b + int(sin(e + f*x)**n*sqrt(sin(e + f*x) + 1)*sin(e + f*x),x)*a + int
(sin(e + f*x)**n*sqrt(sin(e + f*x) + 1)*sin(e + f*x),x)*b + int(sin(e + f*
x)**n*sqrt(sin(e + f*x) + 1),x)*a)
```

3.9 $\int (d\sin(e+fx))^n\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 161

$$\int (d\sin(e+fx))^n\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx = -\frac{2aB\cos(e+fx)(d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}} + \frac{a^2(2B(1+n)+A(3+2n))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},1+n,2+n,\sin(e+fx)\right)\sqrt{1-\sin(e}{df(1+n)(3+2n)(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

output
```
-2*a*B*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(3+2*n)/(a+a*sin(f*x+e))^(1/2)+
a^2*(2*B*(1+n)+A*(3+2*n))*cos(f*x+e)*hypergeom([1/2, 1+n],[2+n],sin(f*x+e)
)*(1-sin(f*x+e))^(1/2)*(d*sin(f*x+e))^(1+n)/d/f/(1+n)/(3+2*n)/(a-a*sin(f*x
+e))/(a+a*sin(f*x+e))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.84 (sec) , antiderivative size = 409, normalized size of antiderivative = 2.54

$$\int (d\sin(e+fx))^n\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx =$$

$$-\frac{(1+i)2^{-2-n}e^{-\frac{3ie}{2}+ifnx}\left(1-e^{2i(e+fx)}\right)^{-n}\left(-ie^{-i(e+fx)}\left(-1+e^{2i(e+fx)}\right)\right)^n\left(\frac{2Be^{-\frac{1}{2}if(3+2n)x}\text{Hypergeometric2F1}(}{f(3+2}\right.}{}$$

input
```
Integrate[(d*Sin[e + f*x])^n*Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])
,x]
```

output
```
((-1 - I)*2^(-2 - n)*E^(((-3*I)/2)*e + I*f*n*x)*(((-I)*(-1 + E^((2*I)*(e +
 f*x))))/E^(I*(e + f*x)))^n*((2*B*Hypergeometric2F1[(-3 - 2*n)/4, -n, (1 -
 2*n)/4, E^((2*I)*(e + f*x))])/(E^((I/2)*f*(3 + 2*n)*x)*f*(3 + 2*n)) + 2*E
^(I*e)*(((-I)*(2*A + B)*Hypergeometric2F1[(-1 - 2*n)/4, -n, (3 - 2*n)/4, E
^((2*I)*(e + f*x))])/(E^((I/2)*f*(1 + 2*n)*x)*(f + 2*f*n)) + (E^((I/2)*(2*
e + f*(1 - 2*n)*x))*(-((2*A + B)*(-3 + 2*n)*Hypergeometric2F1[(1 - 2*n)/4,
 -n, (5 - 2*n)/4, E^((2*I)*(e + f*x))]) + I*B*E^(I*(e + f*x))*(-1 + 2*n)*H
ypergeometric2F1[(3 - 2*n)/4, -n, (7 - 2*n)/4, E^((2*I)*(e + f*x))]))/(f*(
-3 + 2*n)*(-1 + 2*n))))*(d*Sin[e + f*x])^n*Sqrt[a*(1 + Sin[e + f*x]])/((1
 - E^((2*I)*(e + f*x)))^n*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sin[e + f*
x]^n)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.81, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3042, 3460, 3042, 3255, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))(d\sin(e+fx))^n\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))(d\sin(e+fx))^n dx$$

$$\Big\downarrow \; 3460$$

$$\left(A+\frac{2B(n+1)}{2n+3}\right)\int (d\sin(e+fx))^n\sqrt{\sin(e+fx)a+a}dx-\frac{2aB\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \; 3042$$

$$\left(A+\frac{2B(n+1)}{2n+3}\right)\int (d\sin(e+fx))^n\sqrt{\sin(e+fx)a+a}dx-\frac{2aB\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \; 3255$$

$$\frac{a^2\left(A+\frac{2B(n+1)}{2n+3}\right)\cos(e+fx)\int \frac{(d\sin(e+fx))^n}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}-\frac{2aB\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \; 77$$

$$\frac{a^2\left(A+\frac{2B(n+1)}{2n+3}\right)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\int \frac{\sin^n(e+fx)}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}-\frac{2aB\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \; 75$$

$$-\frac{2a\left(A+\frac{2B(n+1)}{2n+3}\right)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},1-\sin(e+fx)\right)}{f\sqrt{a\sin(e+fx)+a}}-\frac{2aB\cos(e+fx)(d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}$$

input
```
Int[(d*Sin[e + f*x])^n*Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]),x]
```

output
```
(-2*a*(A + (2*B*(1 + n))/(3 + 2*n))*Cos[e + f*x]*Hypergeometric2F1[1/2, -n
, 3/2, 1 - Sin[e + f*x]]*(d*Sin[e + f*x])^n)/(f*Sin[e + f*x]^n*Sqrt[a + a*
Sin[e + f*x]]) - (2*a*B*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*(3 + 2
*n)*Sqrt[a + a*Sin[e + f*x]])
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3255
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x]
, x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [F]

$$\int (d\sin(fx+e))^n \sqrt{a+a\sin(fx+e)}\,(A+B\sin(fx+e))\,dx$$

input
```
int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e)),x)
```

output
```
int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e)),x)
```

Fricas [F]

$$\int (d\sin(e+fx))^n\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx = \int (B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}(d\sin(fx+e))^n\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e)),x, algo
rithm="fricas")
```

output
```
integral((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n,
 x)
```

Sympy [F]

$$\int (d\sin(e+fx))^n\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx = \int \sqrt{a(\sin(e+fx)+1)}(d\sin(e+fx))^n\,(A+B\sin(e+fx))\,dx$$

input
```
integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**(1/2)*(A+B*sin(f*x+e)),x)
```

output
```
Integral(sqrt(a*(sin(e + f*x) + 1))*(d*sin(e + f*x))**n*(A + B*sin(e + f*x
)), x)
```

Maxima [F]

$$\int (d\sin(e+fx))^n\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx = \int (B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}(d\sin(fx+e))^n\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e)),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n
, x)
```

Giac [F]

$$\int (d\sin(e+fx))^n\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx = \int (B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}(d\sin(fx+e))^n\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e)),x, algo
rithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n
, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sin(e+fx))^n\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx$$
$$=\int (d\,\sin(e+f\,x))^n\,(A+B\,\sin(e+f\,x))\,\sqrt{a+a\,\sin(e+f\,x)}\,dx$$

input
```
int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2),x)
```

output
```
int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int (d\sin(e+fx))^n\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx$$
$$=d^n\sqrt{a}\left(\left(\int \sin(fx+e)^n\,\sqrt{\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)b+\left(\int \sin(fx+e)^n\,\sqrt{\sin(fx+e)+1}dx\right)a\right)$$

input
```
int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e)),x)
```

output
```
d**n*sqrt(a)*(int(sin(e + f*x)**n*sqrt(sin(e + f*x) + 1)*sin(e + f*x),x)*b
 + int(sin(e + f*x)**n*sqrt(sin(e + f*x) + 1),x)*a)
```

3.10 $\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 161

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{(A-B)\operatorname{AppellF1}\left(\frac{1}{2},-n,1,\frac{3}{2},1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e}{f\sqrt{a+a\sin(e+fx)}} + \frac{B\cos(e+fx)\operatorname{Hypergeometric2F1}\left(\frac{1}{2},1+n,2+n,\sin(e+fx)\right)(d\sin(e+fx))^{1+n}}{df(1+n)\sqrt{1-\sin(e+fx)}\sqrt{a+a\sin(e+fx)}}$$

output
```
-(A-B)*AppellF1(1/2,-n,1,3/2,1-sin(f*x+e),1/2-1/2*sin(f*x+e))*cos(f*x+e)*(
d*sin(f*x+e))^n/f/(sin(f*x+e)^n)/(a+a*sin(f*x+e))^(1/2)+B*cos(f*x+e)*hyper
geom([1/2, 1+n],[2+n],sin(f*x+e))*(d*sin(f*x+e))^(1+n)/d/f/(1+n)/(1-sin(f*
x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 19.34 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.55

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx$$
$$= \frac{\cos(e+fx)\sin^n(e+fx)(d\sin(e+fx))^n\left(-\sin^2(e+fx)\right)^{-n}\sqrt{a(1+\sin(e+fx))}\left(1-\frac{1}{1+\sin(e+fx)}\right)^{-n}\left(\cdots\right.}{}$$

input
```
Integrate[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/Sqrt[a + a*Sin[e + f*x
]],x]
```

output
```
(Cos[e + f*x]*Sin[e + f*x]^n*(d*Sin[e + f*x])^n*Sqrt[a*(1 + Sin[e + f*x])]
*(4*(A - B)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (1
 + Sin[e + f*x])^(-1)]*(-Sin[e + f*x])^n*Sqrt[(-1 + Sin[e + f*x])/(1 + Sin
[e + f*x])] - (A + B)*(1 + 2*n)*AppellF1[1, 1/2, -n, 2, (1 + Sin[e + f*x])
/2, 1 + Sin[e + f*x]]*Sqrt[2 - 2*Sin[e + f*x]]*(1 - (1 + Sin[e + f*x])^(-1
))^n))/(4*a*f*(1 + 2*n)*(-1 + Sin[e + f*x])*(-Sin[e + f*x]^2)^n*(1 - (1 +
Sin[e + f*x])^(-1))^n)
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.94, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 3466, 3042, 3255, 77, 75, 3266, 3042, 3265, 3042, 3264, 148, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(d\sin(e+fx))^n}{\sqrt{a\sin(e+fx)+a}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(A+B\sin(e+fx))(d\sin(e+fx))^n}{\sqrt{a\sin(e+fx)+a}}dx$$
$$\Big\downarrow \text{3466}$$

$$(A-B)\int \frac{(d\sin(e+fx))^n}{\sqrt{\sin(e+fx)a+a}}dx+\frac{B\int (d\sin(e+fx))^n\sqrt{\sin(e+fx)a+a}dx}{a}$$

$\Big\downarrow$ 3042

$$(A-B)\int \frac{(d\sin(e+fx))^n}{\sqrt{\sin(e+fx)a+a}}dx+\frac{B\int (d\sin(e+fx))^n\sqrt{\sin(e+fx)a+a}dx}{a}$$

$\Big\downarrow$ 3255

$$(A-B)\int \frac{(d\sin(e+fx))^n}{\sqrt{\sin(e+fx)a+a}}dx+\frac{aB\cos(e+fx)\int \frac{(d\sin(e+fx))^n}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}$$

$\Big\downarrow$ 77

$$(A-B)\int \frac{(d\sin(e+fx))^n}{\sqrt{\sin(e+fx)a+a}}dx+\frac{aB\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\int \frac{\sin^n(e+fx)}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}$$

$\Big\downarrow$ 75

$$(A-B)\int \frac{(d\sin(e+fx))^n}{\sqrt{\sin(e+fx)a+a}}dx-\frac{2B\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},1-\sin(e+fx)\right)}{f\sqrt{a\sin(e+fx)+a}}$$

$\Big\downarrow$ 3266

$$\frac{(A-B)\sqrt{\sin(e+fx)+1}\int \frac{(d\sin(e+fx))^n}{\sqrt{\sin(e+fx)+1}}dx}{\sqrt{a\sin(e+fx)+a}}-\frac{2B\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},1-\sin(e+fx)\right)}{f\sqrt{a\sin(e+fx)+a}}$$

$\Big\downarrow$ 3042

$$\frac{(A-B)\sqrt{\sin(e+fx)+1}\int \frac{(d\sin(e+fx))^n}{\sqrt{\sin(e+fx)+1}}dx}{\sqrt{a\sin(e+fx)+a}}-\frac{2B\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},1-\sin(e+fx)\right)}{f\sqrt{a\sin(e+fx)+a}}$$

$\Big\downarrow$ 3265

$$\frac{(A-B)\sqrt{\sin(e+fx)+1}\sin^{-n}(e+fx)(d\sin(e+fx))^n\int\frac{\sin^n(e+fx)}{\sqrt{\sin(e+fx)+1}}dx}{\sqrt{a\sin(e+fx)+a}}-\frac{2B\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},1-\sin(e+fx)\right)}{f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-B)\sqrt{\sin(e+fx)+1}\sin^{-n}(e+fx)(d\sin(e+fx))^n\int\frac{\sin(e+fx)^n}{\sqrt{\sin(e+fx)+1}}dx}{\sqrt{a\sin(e+fx)+a}}-\frac{2B\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},1-\sin(e+fx)\right)}{f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3264}$$

$$-\frac{(A-B)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\int\frac{\sin^n(e+fx)}{\sqrt{1-\sin(e+fx)}(\sin(e+fx)+1)}d(1-\sin(e+fx))}{f\sqrt{1-\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}-\frac{2B\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},1-\sin(e+fx)\right)}{f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{148}$$

$$-\frac{2(A-B)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\int\frac{\sin^n(e+fx)}{\sin(e+fx)+1}d\sqrt{1-\sin(e+fx)}}{f\sqrt{1-\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}-\frac{2B\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},1-\sin(e+fx)\right)}{f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{333}$$

$$-\frac{(A-B)\cos(e+fx)\sin^{-n}(e+fx)\text{AppellF1}\left(\frac{1}{2},-n,1,\frac{3}{2},1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)(d\sin(e+fx))^n}{f\sqrt{a\sin(e+fx)+a}}-\frac{2B\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},1-\sin(e+fx)\right)}{f\sqrt{a\sin(e+fx)+a}}$$

input
```
Int[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/Sqrt[a + a*Sin[e + f*x]],x]
```

output
```
-(((A - B)*AppellF1[1/2, -n, 1, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/
2]*Cos[e + f*x]*(d*Sin[e + f*x])^n)/(f*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f
*x]])) - (2*B*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x
]]*(d*Sin[e + f*x])^n)/(f*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x]])
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3255
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x]
, x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

rule 3264
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(a - x)^n*((2*a - x)^(m - 1
/2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}
, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 3265
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Simp[(d/b)^IntPart[n]*((d*Sin[e + f*x])^FracPart[n
]/(b*Sin[e + f*x])^FracPart[n])   Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x
])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !I
ntegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

rule 3266
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Sin[e + f*x])^FracPart[m
]/(1 + (b/a)*Sin[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Sin[e + f*x])^m*(d
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

rule 3466
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)/b   Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x
] + Simp[B/b   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [F]

$$\int \frac{(d\sin(fx+e))^n (A+B\sin(fx+e))}{\sqrt{a+a\sin(fx+e)}}dx$$

input
```
int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x)
```

output
```
int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e))^n}{\sqrt{a\sin(fx+e)+a}}\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algo
rithm="fricas")
```

output
```
integral((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/sqrt(a*sin(f*x + e) + a),
 x)
```

Sympy [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(d\sin(e+fx))^n\,(A+B\sin(e+fx))}{\sqrt{a(\sin(e+fx)+1)}}\,dx$$

input
```
integrate((d*sin(f*x+e))**n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(1/2),x)
```

output
```
Integral((d*sin(e + f*x))**n*(A + B*sin(e + f*x))/sqrt(a*(sin(e + f*x) + 1
)), x)
```

Maxima [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e))^n}{\sqrt{a\sin(fx+e)+a}}\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/sqrt(a*sin(f*x + e) + a)
, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(d\,\sin\left(e+f\,x\right))^n\,\left(A+B\,\sin\left(e+f\,x\right)\right)}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

input
```
int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^(1/2),x
)
```

output
```
int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^(1/2),
x)
```

Reduce [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{d^n\sqrt{a}\left(\left(\int \frac{\sin(fx+e)^n\sqrt{\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)+1}dx\right)b+\left(\int \frac{\sin(fx+e)^n\sqrt{\sin(fx+e)+1}}{\sin(fx+e)+1}dx\right)a\right)}{a}$$

input
```
int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x)
```

output
```
(d**n*sqrt(a)*(int((sin(e + f*x)**n*sqrt(sin(e + f*x) + 1)*sin(e + f*x))/(
sin(e + f*x) + 1),x)*b + int((sin(e + f*x)**n*sqrt(sin(e + f*x) + 1))/(sin
(e + f*x) + 1),x)*a))/a
```

3.11 $\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 246

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{1+n}}{2df(a+a\sin(e+fx))^{3/2}}$$
$$-\frac{(A-4An+B(3+4n))\operatorname{AppellF1}\left(\frac{1}{2},-n,1,\frac{3}{2},1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)\sin^{-n}(e}{4af\sqrt{a+a\sin(e+fx)}}$$
$$+\frac{(A-B)(1+2n)\cos(e+fx)\operatorname{Hypergeometric2F1}\left(\frac{1}{2},1+n,2+n,\sin(e+fx)\right)\sqrt{1-\sin(e+fx)}(d\sin(}{4df(1+n)(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}}$$

output
```
1/2*(A-B)*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(a+a*sin(f*x+e))^(3/2)-1/4*(
A-4*A*n+B*(3+4*n))*AppellF1(1/2,-n,1,3/2,1-sin(f*x+e),1/2-1/2*sin(f*x+e))*
cos(f*x+e)*(d*sin(f*x+e))^n/a/f/(sin(f*x+e)^n)/(a+a*sin(f*x+e))^(1/2)+1/4*
(A-B)*(1+2*n)*cos(f*x+e)*hypergeom([1/2, 1+n],[2+n],sin(f*x+e))*(1-sin(f*x
+e))^(1/2)*(d*sin(f*x+e))^(1+n)/d/f/(1+n)/(a-a*sin(f*x+e))/(a+a*sin(f*x+e)
)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 523 vs. $2(246) = 492$.

Time = 47.22 (sec) , antiderivative size = 523, normalized size of antiderivative = 2.13

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\sec(e+fx)(d\sin(e+fx))^n\left(aB(1+\sin(e+fx))\left(a\,\mathrm{Appel}\right.\right.}{}$$

input
```
Integrate[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x])^(
3/2),x]
```

output
```
(Sec[e + f*x]*(d*Sin[e + f*x])^n*(a*B*(1 + Sin[e + f*x])*((a*AppellF1[1, 1
/2, -n, 2, (1 + Sin[e + f*x])/2, 1 + Sin[e + f*x]]*Sqrt[2 - 2*Sin[e + f*x]
]*(1 + Sin[e + f*x]))/(-Sin[e + f*x])^n - (4*Sqrt[(-1 + Sin[e + f*x])/(1 +
 Sin[e + f*x])]*(-2*a*(1 + 2*n)*AppellF1[1/2 - n, -1/2, -n, 3/2 - n, 2/(1
+ Sin[e + f*x]), (1 + Sin[e + f*x])^(-1)] + a*(-1 + 2*n)*AppellF1[-1/2 - n
, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[e + f*x])^(-1)]*(1 + S
in[e + f*x])))/((-1 + 4*n^2)*(1 - (1 + Sin[e + f*x])^(-1))^n)) + A*((a^2*A
ppellF1[1, 1/2, -n, 2, (1 + Sin[e + f*x])/2, 1 + Sin[e + f*x]]*Sqrt[2 - 2*
Sin[e + f*x]]*(1 + Sin[e + f*x])^2)/(-Sin[e + f*x])^n - (4*a*Sqrt[(-1 + Si
n[e + f*x])/(1 + Sin[e + f*x])]*(1 + Sin[e + f*x])*(2*a*(1 + 2*n)*AppellF1
[1/2 - n, -1/2, -n, 3/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[e + f*x])^(-1)
] + a*(-1 + 2*n)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]
), (1 + Sin[e + f*x])^(-1)]*(1 + Sin[e + f*x])))/((-1 + 4*n^2)*(1 - (1 + S
in[e + f*x])^(-1))^n))))/(8*a^3*f*Sqrt[a*(1 + Sin[e + f*x])])
```

Rubi [A] (warning: unable to verify)

Time = 1.46 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.94, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.457$, Rules used = {3042, 3457, 27, 3042, 3466, 3042, 3255, 77, 75, 3266, 3042, 3265, 3042, 3264, 148, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(d\sin(e+fx))^n}{(a\sin(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(A+B\sin(e+fx))(d\sin(e+fx))^n}{(a\sin(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{ 3457}$$

$$\frac{\int \frac{(d\sin(e+fx))^n(2ad(-nA+A+B+Bn)+a(A-B)d(2n+1)\sin(e+fx))}{2\sqrt{\sin(e+fx)a+a}}dx}{2a^2d}+\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{2df(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{(d\sin(e+fx))^n(2ad(-nA+A+B+Bn)+a(A-B)d(2n+1)\sin(e+fx))}{\sqrt{\sin(e+fx)a+a}}dx}{4a^2d}+\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{2df(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{(d\sin(e+fx))^n(2ad(-nA+A+B+Bn)+a(A-B)d(2n+1)\sin(e+fx))}{\sqrt{\sin(e+fx)a+a}}dx}{4a^2d}+\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{2df(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 3466}$$

$$\frac{ad(-4An+A+4Bn+3B)\int \frac{(d\sin(e+fx))^n}{\sqrt{\sin(e+fx)a+a}}dx+d(2n+1)(A-B)\int (d\sin(e+fx))^n\sqrt{\sin(e+fx)a+a}dx}{4a^2d}+\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{2df(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{ad(-4An+A+4Bn+3B)\int \frac{(d\sin(e+fx))^n}{\sqrt{\sin(e+fx)a+a}}dx+d(2n+1)(A-B)\int (d\sin(e+fx))^n\sqrt{\sin(e+fx)a+a}dx}{4a^2d}+\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{2df(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 3255}$$

$$\frac{\frac{a^2d(2n+1)(A-B)\cos(e+fx)\int\frac{(d\sin(e+fx))^n}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}+ad(-4An+A+4Bn+3B)\int\frac{(d\sin(e+fx))^n}{\sqrt{\sin(e+fx)a+a}}dx}{4a^2d}+$$
$$\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{2df(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow 77$$

$$\frac{\frac{a^2d(2n+1)(A-B)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\int\frac{\sin^n(e+fx)}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}+ad(-4An+A+4Bn+3B)\int\frac{(d\sin(e+f}{\sqrt{\sin(e+fx}}}{4a^2d}$$
$$\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{2df(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow 75$$

$$\frac{ad(-4An+A+4Bn+3B)\int\frac{(d\sin(e+fx))^n}{\sqrt{\sin(e+fx)a+a}}dx-\frac{2ad(2n+1)(A-B)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\text{Hypergeometric2F1}(}{f\sqrt{a\sin(e+fx)+a}}}{4a^2d}$$
$$\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{2df(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow 3266$$

$$\frac{\frac{ad(-4An+A+4Bn+3B)\sqrt{\sin(e+fx)+1}\int\frac{(d\sin(e+fx))^n}{\sqrt{\sin(e+fx)+1}}dx}{\sqrt{a\sin(e+fx)+a}}-\frac{2ad(2n+1)(A-B)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\text{Hypergeometric2F1}(}{f\sqrt{a\sin(e+fx)+a}}}{4a^2d}$$
$$\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{2df(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow 3042$$

$$\frac{\frac{ad(-4An+A+4Bn+3B)\sqrt{\sin(e+fx)+1}\int\frac{(d\sin(e+fx))^n}{\sqrt{\sin(e+fx)+1}}dx}{\sqrt{a\sin(e+fx)+a}}-\frac{2ad(2n+1)(A-B)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\text{Hypergeometric2F1}(}{f\sqrt{a\sin(e+fx)+a}}}{4a^2d}$$
$$\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{2df(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow 3265$$

$$\frac{\frac{ad(-4An+A+4Bn+3B)\sqrt{\sin(e+fx)+1}\sin^{-n}(e+fx)(d\sin(e+fx))^n\int\frac{\sin^n(e+fx)}{\sqrt{\sin(e+fx)+1}}dx}{\sqrt{a\sin(e+fx)+a}}-\frac{2ad(2n+1)(A-B)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e}{f\sqrt{a\sin}}}{4a^2d}$$
$$\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{2df(a\sin(e+fx)+a)^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{ad(-4An+A+4Bn+3B)\sqrt{\sin(e+fx)+1}\sin^{-n}(e+fx)(d\sin(e+fx))^n\int\frac{\sin(e+fx)^n}{\sqrt{\sin(e+fx)+1}}dx}{\sqrt{a\sin(e+fx)+a}}-\frac{2ad(2n+1)(A-B)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e}{f\sqrt{a\sin}}}{4a^2d}$$

$$\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{2df(a\sin(e+fx)+a)^{3/2}}$$

$\downarrow$ 3264

$$\frac{-\frac{ad(-4An+A+4Bn+3B)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\int\frac{\sin^n(e+fx)}{\sqrt{1-\sin(e+fx)}(\sin(e+fx)+1)}d(1-\sin(e+fx))}{f\sqrt{1-\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}-\frac{2ad(2n+1)(A-B)\cos(e+}{}}{4a^2d}$$

$$\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{2df(a\sin(e+fx)+a)^{3/2}}$$

$\downarrow$ 148

$$\frac{-\frac{2ad(-4An+A+4Bn+3B)\cos(e+fx)\sin^{-n}(e+fx)(d\sin(e+fx))^n\int\frac{\sin^n(e+fx)}{\sin(e+fx)+1}d\sqrt{1-\sin(e+fx)}}{f\sqrt{1-\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}-\frac{2ad(2n+1)(A-B)\cos(e+fx)\sin^{-n}(e+}{}}{4a^2d}$$

$$\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{2df(a\sin(e+fx)+a)^{3/2}}$$

$\downarrow$ 333

$$\frac{-\frac{ad(-4An+A+4Bn+3B)\cos(e+fx)\sin^{-n}(e+fx)\operatorname{AppellF1}\left(\frac{1}{2},-n,1,\frac{3}{2},1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)(d\sin(e+fx))^n}{f\sqrt{a\sin(e+fx)+a}}-\frac{2ad(2n+1)(A-B)}{}}{4a^2d}$$

$$\frac{(A-B)\cos(e+fx)(d\sin(e+fx))^{n+1}}{2df(a\sin(e+fx)+a)^{3/2}}$$

input
```
Int[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x])^(3/2),x
]
```

output
```
((A - B)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(2*d*f*(a + a*Sin[e + f*x]
)^(3/2)) + (-((a*d*(A + 3*B - 4*A*n + 4*B*n)*AppellF1[1/2, -n, 1, 3/2, 1 -
 Sin[e + f*x], (1 - Sin[e + f*x])/2]*Cos[e + f*x]*(d*Sin[e + f*x])^n)/(f*S
in[e + f*x]^n*Sqrt[a + a*Sin[e + f*x]])) - (2*a*(A - B)*d*(1 + 2*n)*Cos[e
+ f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]]*(d*Sin[e + f*x])^
n)/(f*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x]]))/(4*a^2*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

rule 148
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))^(p_.),
x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1) - 1)*(c
 + d*(x^k/b))^n*(e + f*(x^k/b))^p, x], x, (b*x)^(1/k)], x]] /; FreeQ[{b, c,
 d, e, f, n, p}, x] && FractionQ[m] && IntegerQ[p]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3255
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x]
, x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

rule 3264
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(a - x)^n*((2*a - x)^(m - 1
/2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}
, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 3265
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Simp[(d/b)^IntPart[n]*((d*Sin[e + f*x])^FracPart[n
]/(b*Sin[e + f*x])^FracPart[n])   Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x
])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !I
ntegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

rule 3266
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Sin[e + f*x])^FracPart[m
]/(1 + (b/a)*Sin[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Sin[e + f*x])^m*(d
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3466
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)/b   Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x
] + Simp[B/b   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [F]

$$\int \frac{(d\sin(fx+e))^{n}(A+B\sin(fx+e))}{(a+a\sin(fx+e))^{\frac{3}{2}}}dx$$

input
```
int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x)
```

output
```
int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int \frac{(d\sin(e+fx))^{n}(A+B\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e))^{n}}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algo
rithm="fricas")
```

output
```
integral(-(B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n
/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

Sympy [F]

$$\int \frac{(d\sin(e+fx))^{n}(A+B\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(d\sin(e+fx))^{n}(A+B\sin(e+fx))}{(a(\sin(e+fx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sin(f*x+e))**n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2),x)
```

output
```
Integral((d*sin(e + f*x))**n*(A + B*sin(e + f*x))/(a*(sin(e + f*x) + 1))**
(3/2), x)
```

Maxima [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e))^n}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a)^(3/
2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algo
rithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(d\,\sin(e+f\,x))^n\,(A+B\,\sin(e+f\,x))}{(a+a\,\sin(e+f\,x))^{3/2}}\,dx$$

input
```
int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^(3/2),x
)
```

output
```
int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^(3/2),
x)
```

Reduce [F]

$$\int \frac{(d\sin(e+fx))^n(A+B\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{d^n\sqrt{a}\left(\left(\int \frac{\sin(fx+e)^n\sqrt{\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)^2+2\sin(fx+e)+1}dx\right)b+\left(\int \frac{\sin(fx+e)^n}{\sin(fx+e)^2+}\right.\right.}{a^2}$$

input
```
int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x)
```

output
```
(d**n*sqrt(a)*(int((sin(e + f*x)**n*sqrt(sin(e + f*x) + 1)*sin(e + f*x))/(
sin(e + f*x)**2 + 2*sin(e + f*x) + 1),x)*b + int((sin(e + f*x)**n*sqrt(sin
(e + f*x) + 1))/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),x)*a))/a**2
```

3.12 $\int (d\sin(e+fx))^n(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 221

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = -\frac{2^{\frac{3}{2}+m}B\,\text{AppellF1}\left(\frac{1}{2},-n,-\frac{1}{2}-m,\frac{3}{2},1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)\sin^{-n}(e+fx)(}{f} - \frac{2^{\frac{1}{2}+m}(A-B)\,\text{AppellF1}\left(\frac{1}{2},-n,\frac{1}{2}-m,\frac{3}{2},1-\sin(e+fx),\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)\sin^{-n}(e+}{f}$$

output
```
-2^(3/2+m)*B*AppellF1(1/2,-n,-1/2-m,3/2,1-sin(f*x+e),1/2-1/2*sin(f*x+e))*c
os(f*x+e)*(d*sin(f*x+e))^n*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m/f/(s
in(f*x+e)^n)-2^(1/2+m)*(A-B)*AppellF1(1/2,-n,1/2-m,3/2,1-sin(f*x+e),1/2-1/
2*sin(f*x+e))*cos(f*x+e)*(d*sin(f*x+e))^n*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin
(f*x+e))^m/f/(sin(f*x+e)^n)
```

Mathematica [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$
$$=\int (d\sin(e+fx))^n(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$

input
```
Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x
]
```

output
```
Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),
x]
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 3466, 3042, 3266, 3042, 3265, 3042, 3264, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(d\sin(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(d\sin(e+fx))^ndx$$

$$\Big\downarrow \text{3466}$$

$$(A-B)\int (d\sin(e+fx))^n(\sin(e+fx)a+a)^mdx+\frac{B\int (d\sin(e+fx))^n(\sin(e+fx)a+a)^{m+1}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$(A-B)\int (d\sin(e+fx))^n(\sin(e+fx)a+a)^mdx+\frac{B\int (d\sin(e+fx))^n(\sin(e+fx)a+a)^{m+1}dx}{a}$$

$$\Big\downarrow 3266$$

$$(A-B)(\sin(e+fx)+1)^{-m}(a\sin(e+fx)+a)^m\int(d\sin(e+fx))^n(\sin(e+fx)+1)^m dx+B(\sin(e+fx)+1)^{-m}(a\sin(e+fx)+a)^m\int(d\sin(e+fx))^n(\sin(e+fx)+1)^{m+1}dx$$

$$\Big\downarrow 3042$$

$$(A-B)(\sin(e+fx)+1)^{-m}(a\sin(e+fx)+a)^m\int(d\sin(e+fx))^n(\sin(e+fx)+1)^m dx+B(\sin(e+fx)+1)^{-m}(a\sin(e+fx)+a)^m\int(d\sin(e+fx))^n(\sin(e+fx)+1)^{m+1}dx$$

$$\Big\downarrow 3265$$

$$(A-B)(\sin(e+fx)+1)^{-m}\sin^{-n}(e+fx)(a\sin(e+fx)+a)^m(d\sin(e+fx))^n\int\sin^n(e+fx)(\sin(e+fx)+1)^m dx+B(\sin(e+fx)+1)^{-m}\sin^{-n}(e+fx)(a\sin(e+fx)+a)^m(d\sin(e+fx))^n\int\sin^n(e+fx)(\sin(e+fx)+1)^{m+1}dx$$

$$\Big\downarrow 3042$$

$$(A-B)(\sin(e+fx)+1)^{-m}\sin^{-n}(e+fx)(a\sin(e+fx)+a)^m(d\sin(e+fx))^n\int\sin(e+fx)^n(\sin(e+fx)+1)^m dx+B(\sin(e+fx)+1)^{-m}\sin^{-n}(e+fx)(a\sin(e+fx)+a)^m(d\sin(e+fx))^n\int\sin(e+fx)^n(\sin(e+fx)+1)^{m+1}dx$$

$$\Big\downarrow 3264$$

$$-\frac{(A-B)\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}\sin^{-n}(e+fx)(a\sin(e+fx)+a)^m(d\sin(e+fx))^n\int\frac{\sin^n(e+fx)(\sin(e+fx)+1)^{m-\frac{1}{2}}}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{1-\sin(e+fx)}}-\frac{B\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}\sin^{-n}(e+fx)(a\sin(e+fx)+a)^m(d\sin(e+fx))^n\int\frac{\sin^n(e+fx)(\sin(e+fx)+1)^{m+\frac{1}{2}}}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{1-\sin(e+fx)}}$$

$$\Big\downarrow 150$$

$$-\frac{2^{m+\frac{1}{2}}(A-B)\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}\sin^{-n}(e+fx)(a\sin(e+fx)+a)^m(d\sin(e+fx))^n\operatorname{AppellF1}\ldots}{f}-\frac{B2^{m+\frac{3}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}\sin^{-n}(e+fx)(a\sin(e+fx)+a)^m(d\sin(e+fx))^n\operatorname{AppellF1}\left(\frac{1}{2},-n,\ldots\right)}{f}$$

input
```
Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

output
```
-((2^(3/2 + m)*B*AppellF1[1/2, -n, -1/2 - m, 3/2, 1 - Sin[e + f*x], (1 - S
in[e + f*x])/2]*Cos[e + f*x]*(d*Sin[e + f*x])^n*(1 + Sin[e + f*x])^(-1/2 -
 m)*(a + a*Sin[e + f*x])^m)/(f*Sin[e + f*x]^n)) - (2^(1/2 + m)*(A - B)*App
ellF1[1/2, -n, 1/2 - m, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2]*Cos[e
 + f*x]*(d*Sin[e + f*x])^n*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*
x])^m)/(f*Sin[e + f*x]^n)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3264
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(a - x)^n*((2*a - x)^(m - 1
/2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}
, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 3265
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Simp[(d/b)^IntPart[n]*((d*Sin[e + f*x])^FracPart[n
]/(b*Sin[e + f*x])^FracPart[n])   Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x
])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !I
ntegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

rule 3266
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Simp[a^IntPart[m]*((a + b*Sin[e + f*x])^FracPart[m
]/(1 + (b/a)*Sin[e + f*x])^FracPart[m])   Int[(1 + (b/a)*Sin[e + f*x])^m*(d
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

rule 3466
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)/b   Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x
] + Simp[B/b   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [F]

$$\int (d\sin(fx+e))^n (a+a\sin(fx+e))^m (A+B\sin(fx+e))\,dx$$

input
```
int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

output
```
int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Fricas [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$
$$=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m(d\sin(fx+e))^n\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x
)
```

Sympy [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$
$$=\int (a(\sin(e+fx)+1))^m\,(d\sin(e+fx))^n\,(A+B\sin(e+fx))\;dx$$

input
```
integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*(d*sin(e + f*x))**n*(A + B*sin(e + f*x)
), x)
```

Maxima [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$
$$=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m(d\sin(fx+e))^n\;dx$$

input
```
integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorith
m="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n,
x)
```

Giac [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$
$$=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m(d\sin(fx+e))^n\;dx$$

input
```
integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorith
m="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n,
x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$
$$= \int (d\,\sin(e+f\,x))^n\,(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m\,dx$$

input
```
int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m,x)
```

output
```
int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m, x)
```

Reduce [F]

$$\int (d\sin(e+fx))^n(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$
$$= d^n\left(\left(\int \sin(fx+e)^n\,(a+a\sin(fx+e))^m\sin(fx+e)\,dx\right)b + \left(\int \sin(fx+e)^n\,(a+a\sin(fx+e))^m\,dx\right)a\right)$$

input
```
int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

output
```
d**n*(int(sin(e + f*x)**n*(sin(e + f*x)*a + a)**m*sin(e + f*x),x)*b + int(
sin(e + f*x)**n*(sin(e + f*x)*a + a)**m,x)*a)
```

3.13 $\int (d\sin(e+fx))^n(a-a\sin(e+fx))(a+a\sin(e+fx))^m\,dx$

Optimal result

Integrand size = 34, antiderivative size = 114

$$\int (d\sin(e+fx))^n(a-a\sin(e+fx))(a+a\sin(e+fx))^m\,dx$$
$$= \frac{\text{AppellF1}\left(1+n,-\frac{1}{2},\frac{1}{2}-m,2+n,\sin(e+fx),-\sin(e+fx)\right)\sec(e+fx)(d\sin(e+fx))^{1+n}(1+\sin(e}{df(1+n)\sqrt{1-\sin(e+fx)}}$$

output
```
AppellF1(1+n,1/2-m,-1/2,2+n,-sin(f*x+e),sin(f*x+e))*sec(f*x+e)*(d*sin(f*x+
e))^(1+n)*(1+sin(f*x+e))^(1/2-m)*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m/d/f/(
1+n)/(1-sin(f*x+e))^(1/2)
```

Mathematica [F]

$$\int (d\sin(e+fx))^n(a-a\sin(e+fx))(a+a\sin(e+fx))^m\,dx$$
$$= \int (d\sin(e+fx))^n(a-a\sin(e+fx))(a+a\sin(e+fx))^m\,dx$$

input
```
Integrate[(d*Sin[e + f*x])^n*(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m,x
]
```

output
```
Integrate[(d*Sin[e + f*x])^n*(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m,
x]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {3042, 3487, 152, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-a\sin(e+fx))(a\sin(e+fx)+a)^m(d\sin(e+fx))^n\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (a-a\sin(e+fx))(a\sin(e+fx)+a)^m(d\sin(e+fx))^ndx$$

$$\Big\downarrow \; 3487$$

$$\frac{\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\int(d\sin(e+fx))^n\sqrt{a-a\sin(e+fx)}(\sin(e+fx)a+a)^{m-\frac{1}{2}}}{f}$$

$$\Big\downarrow \; 152$$

$$\frac{\sec(e+fx)(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}\int\sqrt{1-\sin(e+fx)}(d\sin(e+fx))^n(\sin(e+fx)a+a)^{m-\frac{1}{2}}d\sin}{f\sqrt{1-\sin(e+fx)}}$$

$$\Big\downarrow \; 152$$

$$\frac{\sec(e+fx)(a-a\sin(e+fx))(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m\int\sqrt{1-\sin(e+fx)}(d\sin(e+fx))^n(\sin}{f\sqrt{1-\sin(e+fx)}}$$

$$\Big\downarrow \; 150$$

$$\frac{\sec(e+fx)(a-a\sin(e+fx))(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m(d\sin(e+fx))^{n+1}\operatorname{AppellF1}\left(n+1,-\right.}{df(n+1)\sqrt{1-\sin(e+fx)}}$$

input
```
Int[(d*Sin[e + f*x])^n*(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m,x]
```

output
```
(AppellF1[1 + n, -1/2, 1/2 - m, 2 + n, Sin[e + f*x], -Sin[e + f*x]]*Sec[e
+ f*x]*(d*Sin[e + f*x])^(1 + n)*(1 + Sin[e + f*x])^(1/2 - m)*(a - a*Sin[e
+ f*x])*(a + a*Sin[e + f*x])^m)/(d*f*(1 + n)*Sqrt[1 - Sin[e + f*x]])
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3487
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])^(p_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol]
:> Simp[Sqrt[a + b*Sin[e + f*x]]*(Sqrt[c + d*Sin[e + f*x]]/(f*Cos[e + f*x])
)   Subst[Int[(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2)*(A + B*x)^p, x], x, S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n, p}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int (d\sin (fx+e))^{n} (a-a\sin (fx+e)) (a+a\sin (fx+e))^{m} dx$$

input
```
int((d*sin(f*x+e))^n*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m,x)
```

output
```
int((d*sin(f*x+e))^n*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m,x)
```

Fricas [F]

$$\int (d\sin(e+fx))^n(a-a\sin(e+fx))(a+a\sin(e+fx))^m\,dx$$
$$=\int -(a\sin (fx+e)-a)(a\sin (fx+e)+a)^{m}(d\sin (fx+e))^{n}\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m,x, algorith
m="fricas")
```

output
```
integral(-(a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n,
x)
```

Sympy [F]

$$\int (d\sin(e+fx))^n(a-a\sin(e+fx))(a+a\sin(e+fx))^m\,dx$$
$$=-a\left(\int \left(-(d\sin (e+fx))^{n} (a\sin (e+fx)+a)^{m}\right)\,dx + \int (d\sin (e+fx))^{n} (a\sin (e+fx)+a)^{m} \sin (e+fx)\,dx\right)$$

input
```
integrate((d*sin(f*x+e))**n*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))**m,x)
```

output
```
-a*(Integral(-(d*sin(e + f*x))**n*(a*sin(e + f*x) + a)**m, x) + Integral((
d*sin(e + f*x))**n*(a*sin(e + f*x) + a)**m*sin(e + f*x), x))
```

Maxima [F]

$$\int (d\sin(e+fx))^n(a-a\sin(e+fx))(a+a\sin(e+fx))^m\,dx$$
$$=\int -(a\sin(fx+e)-a)(a\sin(fx+e)+a)^m(d\sin(fx+e))^n\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m,x, algorith
m="maxima")
```

output
```
-integrate((a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n,
 x)
```

Giac [F]

$$\int (d\sin(e+fx))^n(a-a\sin(e+fx))(a+a\sin(e+fx))^m\,dx$$
$$=\int -(a\sin(fx+e)-a)(a\sin(fx+e)+a)^m(d\sin(fx+e))^n\,dx$$

input
```
integrate((d*sin(f*x+e))^n*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m,x, algorith
m="giac")
```

output
```
integrate(-(a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n,
 x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sin(e+fx))^n(a-a\sin(e+fx))(a+a\sin(e+fx))^m\,dx$$
$$=\int (d\,\sin\left(e+f\,x\right))^n\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,\left(a-a\,\sin\left(e+f\,x\right)\right)\,dx$$

input
```
int((d*sin(e + f*x))^n*(a + a*sin(e + f*x))^m*(a - a*sin(e + f*x)),x)
```

output
```
int((d*sin(e + f*x))^n*(a + a*sin(e + f*x))^m*(a - a*sin(e + f*x)), x)
```

Reduce [F]

$$\int (d\sin(e+fx))^n(a-a\sin(e+fx))(a+a\sin(e+fx))^m\,dx$$
$$=d^na\left(-\left(\int \sin\left(fx+e\right)^n\left(a+a\sin\left(fx+e\right)\right)^m\sin\left(fx+e\right)dx\right)+\int \sin\left(fx+e\right)^n\left(a+a\sin\left(fx+e\right)\right)^m dx\right)$$

input
```
int((d*sin(f*x+e))^n*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m,x)
```

output
```
d**n*a*( - int(sin(e + f*x)**n*(sin(e + f*x)*a + a)**m*sin(e + f*x),x) + i
nt(sin(e + f*x)**n*(sin(e + f*x)*a + a)**m,x))
```

3.14 $\int \sin^n(c+dx)(a+a\sin(c+dx))^{-2-n}(-1-n-(-2-n)\sin(c+dx))\,dx$

Optimal result

Integrand size = 43, antiderivative size = 37

$$\int \sin^n(c+dx)(a+a\sin(c+dx))^{-2-n}(-1-n-(-2-n)\sin(c+dx))\,dx = -\frac{\cos(c+dx)\sin^{1+n}(c+dx)(a+a\sin(c+dx))^{-2-n}}{d}$$

output
```
-cos(d*x+c)*sin(d*x+c)^(1+n)*(a+a*sin(d*x+c))^(-2-n)/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 107 vs. 2(37) = 74.

Time = 8.74 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.89

$$\int \sin^n(c+dx)(a+a\sin(c+dx))^{-2-n}(-1-n-(-2-n)\sin(c+dx))\,dx = -\frac{2^n\sin\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{4}(c+dx)\right)\left(-\sin\left(\frac{1}{4}(c+dx)\right)+\sin\left(\frac{3}{4}(c\right.\right.\right.}{d}$$

input
```
Integrate[Sin[c + d*x]^n*(a + a*Sin[c + d*x])^(-2 - n)*(-1 - n - (-2 - n)*
Sin[c + d*x]),x]
```

output
```
-((2^n*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(Cos[(c + d*
x)/4]*(-Sin[(c + d*x)/4] + Sin[(3*(c + d*x))/4]))^n*(1 + Cos[c + d*x] - Si
n[c + d*x])*(a*(1 + Sin[c + d*x]))^(-2 - n))/d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.047$, Rules used = {3042, 3453}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^n(c+dx)(-(-n-2)\sin(c+dx)-n-1)(a\sin(c+dx)+a)^{-n-2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(c+dx)^n(-(-n-2)\sin(c+dx)-n-1)(a\sin(c+dx)+a)^{-n-2}dx$$

$$\Big\downarrow \text{3453}$$

$$-\frac{\cos(c+dx)\sin^{n+1}(c+dx)(a\sin(c+dx)+a)^{-n-2}}{d}$$

input
```
Int[Sin[c + d*x]^n*(a + a*Sin[c + d*x])^(-2 - n)*(-1 - n - (-2 - n)*Sin[c
+ d*x]),x]
```

output
```
-((Cos[c + d*x]*Sin[c + d*x]^(1 + n)*(a + a*Sin[c + d*x])^(-2 - n))/d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3453
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n},
x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ[m
+ n + 2, 0] && EqQ[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)), 0]
```

Maple [F]

$$\int \sin (dx+c)^{n} (a+a\sin (dx+c))^{-2-n} (-1-n-(-2-n)\sin (dx+c))\, dx$$

input
```
int(sin(d*x+c)^n*(a+a*sin(d*x+c))^(-2-n)*(-1-n-(-2-n)*sin(d*x+c)),x)
```

output
```
int(sin(d*x+c)^n*(a+a*sin(d*x+c))^(-2-n)*(-1-n-(-2-n)*sin(d*x+c)),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

$$\int \sin^n(c+dx)(a+a\sin(c+dx))^{-2-n}(-1-n-(-2-n)\sin(c+dx))\, dx = -\frac{(a\sin (dx+c)+a)^{-n-2} \sin (dx+c)^{n} \cos (dx+c) \sin (dx+c)}{d}$$

input
```
integrate(sin(d*x+c)^n*(a+a*sin(d*x+c))^(-2-n)*(-1-n-(-2-n)*sin(d*x+c)),x,
 algorithm="fricas")
```

output
```
-(a*sin(d*x + c) + a)^(-n - 2)*sin(d*x + c)^n*cos(d*x + c)*sin(d*x + c)/d
```

Sympy [F]

$$\int \sin^n(c+dx)(a+a\sin(c+dx))^{-2-n}(-1-n-(-2-n)\sin(c+dx))\,dx$$
$$= \int (a(\sin(c+dx)+1))^{-n-2}\,(n\sin(c+dx)-n+2\sin(c+dx)-1)\sin^n(c+dx)\,dx$$

input
```
integrate(sin(d*x+c)**n*(a+a*sin(d*x+c))**(-2-n)*(-1-n-(-2-n)*sin(d*x+c)),
x)
```

output
```
Integral((a*(sin(c + d*x) + 1))**(-n - 2)*(n*sin(c + d*x) - n + 2*sin(c +
d*x) - 1)*sin(c + d*x)**n, x)
```

Maxima [F]

$$\int \sin^n(c+dx)(a+a\sin(c+dx))^{-2-n}(-1-n-(-2-n)\sin(c+dx))\,dx$$
$$= \int ((n+2)\sin(dx+c)-n-1)(a\sin(dx+c)+a)^{-n-2}\sin(dx+c)^n\,dx$$

input
```
integrate(sin(d*x+c)^n*(a+a*sin(d*x+c))^(-2-n)*(-1-n-(-2-n)*sin(d*x+c)),x,
 algorithm="maxima")
```

output
```
integrate(((n + 2)*sin(d*x + c) - n - 1)*(a*sin(d*x + c) + a)^(-n - 2)*sin
(d*x + c)^n, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9496 vs. 2(37) = 74.

Time = 74.37 (sec) , antiderivative size = 9496, normalized size of antiderivative = 256.65

$$\int \sin^n(c+dx)(a+a\sin(c+dx))^{-2-n}(-1-n-(-2-n)\sin(c+dx))\,dx$$
$$= \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^n*(a+a*sin(d*x+c))^(-2-n)*(-1-n-(-2-n)*sin(d*x+c)),x,
 algorithm="giac")
```

output
```
-8*(cos(-1/2*pi + 2*pi*n*floor(-1/8*sgn(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2
*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1
/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) + 5/8) + 1/4*pi*n*sgn(4*ta
n(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c
) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) +
 2) - 1/4*pi*n + 4*pi*floor(-1/8*sgn(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)
^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*
d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) + 5/8) + 1/2*pi*sgn(4*tan(d*x
 + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4
*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2))*
e^(-n*log(sqrt(2)*sqrt(abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan
(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*
c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 +
 abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*
x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x +
 1/2*c) + 2)*tan(d*x + c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2
+ 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x
 + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(1/2*d*x + 1/2*c)^2 + abs(4*t
an(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*
c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*...
```

Mupad [B] (verification not implemented)

Time = 35.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.65

$$\int \sin^n(c+dx)(a+a\sin(c+dx))^{-2-n}(-1-n-(-2-n)\sin(c+dx))\,dx$$
$$= -\frac{\sin(c+d\,x)^n\,\sin(2\,c+2\,d\,x)}{a^2\,d\,(a\,(\sin(c+d\,x)+1))^n\,\left(2\sin(c+d\,x)^2+4\,\sin(c+d\,x)+2\right)}$$

input
```
int(-(sin(c + d*x)^n*(n - sin(c + d*x)*(n + 2) + 1))/(a + a*sin(c + d*x))^
(n + 2),x)
```

output
```
-(sin(c + d*x)^n*sin(2*c + 2*d*x))/(a^2*d*(a*(sin(c + d*x) + 1))^n*(4*sin(
c + d*x) + 2*sin(c + d*x)^2 + 2))
```

Reduce [F]

$$\int \sin^n(c+dx)(a+a\sin(c+dx))^{-2-n}(-1-n-(-2-n)\sin(c+dx))\,dx$$
$$= \frac{-\left(\int \frac{\sin(dx+c)^n}{(\sin(dx+c)a+a)^n\sin(dx+c)^2+2(\sin(dx+c)a+a)^n\sin(dx+c)+(\sin(dx+c)a+a)^n}dx\right)n-\left(\int \frac{\sin(}{(\sin(dx+c)a+a)^n\sin(dx+c)^2+2(\sin(}\right.}{}$$

input
```
int(sin(d*x+c)^n*(a+a*sin(d*x+c))^(-2-n)*(-1-n-(-2-n)*sin(d*x+c)),x)
```

output
```
( - int(sin(c + d*x)**n/((sin(c + d*x)*a + a)**n*sin(c + d*x)**2 + 2*(sin(
c + d*x)*a + a)**n*sin(c + d*x) + (sin(c + d*x)*a + a)**n),x)*n - int(sin(
c + d*x)**n/((sin(c + d*x)*a + a)**n*sin(c + d*x)**2 + 2*(sin(c + d*x)*a +
 a)**n*sin(c + d*x) + (sin(c + d*x)*a + a)**n),x) + int((sin(c + d*x)**n*s
in(c + d*x))/((sin(c + d*x)*a + a)**n*sin(c + d*x)**2 + 2*(sin(c + d*x)*a
+ a)**n*sin(c + d*x) + (sin(c + d*x)*a + a)**n),x)*n + 2*int((sin(c + d*x)
**n*sin(c + d*x))/((sin(c + d*x)*a + a)**n*sin(c + d*x)**2 + 2*(sin(c + d*
x)*a + a)**n*sin(c + d*x) + (sin(c + d*x)*a + a)**n),x))/a**2
```

3.15 $\int \sin^{-2-m}(c+dx)(a+a\sin(c+dx))^m(1+m-m\sin(c+dx))\,dx$

Optimal result

Integrand size = 37, antiderivative size = 35

$$\int \sin^{-2-m}(c+dx)(a+a\sin(c+dx))^m(1+m-m\sin(c+dx))\,dx = -\frac{\cos(c+dx)\sin^{-1-m}(c+dx)(a+a\sin(c+dx))^m}{d}$$

output
```
-cos(d*x+c)*sin(d*x+c)^(-1-m)*(a+a*sin(d*x+c))^m/d
```

Mathematica [A] (verified)

Time = 2.63 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \sin^{-2-m}(c+dx)(a+a\sin(c+dx))^m(1+m-m\sin(c+dx))\,dx = -\frac{\cos(c+dx)\sin^{-1-m}(c+dx)(a(1+\sin(c+dx)))^m}{d}$$

input
```
Integrate[Sin[c + d*x]^(-2 - m)*(a + a*Sin[c + d*x])^m*(1 + m - m*Sin[c + 
d*x]),x]
```

output
```
-((Cos[c + d*x]*Sin[c + d*x]^(-1 - m)*(a*(1 + Sin[c + d*x]))^m)/d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {3042, 3453}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{-m-2}(c+dx)(m(-\sin(c+dx))+m+1)(a\sin(c+dx)+a)^m\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin(c+dx)^{-m-2}(m(-\sin(c+dx))+m+1)(a\sin(c+dx)+a)^m dx$$
$$\Big\downarrow \text{3453}$$
$$-\frac{\cos(c+dx)\sin^{-m-1}(c+dx)(a\sin(c+dx)+a)^m}{d}$$

input
```
Int[Sin[c + d*x]^(-2 - m)*(a + a*Sin[c + d*x])^m*(1 + m - m*Sin[c + d*x]),
x]
```

output
```
-((Cos[c + d*x]*Sin[c + d*x]^(-1 - m)*(a + a*Sin[c + d*x])^m)/d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3453
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n},
x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ[m
+ n + 2, 0] && EqQ[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)), 0]
```

Maple [F]

$$\int \sin(dx+c)^{-2-m}(a+a\sin(dx+c))^m(1+m-m\sin(dx+c))\,dx$$

input
```
int(sin(d*x+c)^(-2-m)*(a+a*sin(d*x+c))^m*(1+m-m*sin(d*x+c)),x)
```

output
```
int(sin(d*x+c)^(-2-m)*(a+a*sin(d*x+c))^m*(1+m-m*sin(d*x+c)),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

$$\int \sin^{-2-m}(c+dx)(a+a\sin(c+dx))^m(1+m-m\sin(c+dx))\,dx = -\frac{(a\sin(dx+c)+a)^m\sin(dx+c)^{-m-2}\cos(dx+c)\sin(dx+c)}{d}$$

input
```
integrate(sin(d*x+c)^(-2-m)*(a+a*sin(d*x+c))^m*(1+m-m*sin(d*x+c)),x, algor
ithm="fricas")
```

output
```
-(a*sin(d*x + c) + a)^m*sin(d*x + c)^(-m - 2)*cos(d*x + c)*sin(d*x + c)/d
```

Sympy [F]

$$\int \sin^{-2-m}(c+dx)(a+a\sin(c+dx))^m(1+m-m\sin(c+dx))\,dx$$
$$= -\int \left(-(a\sin(c+dx)+a)^m\sin^{-m-2}(c+dx)\right)\,dx - \int \left(-m(a\sin(c+dx)+a)^m\sin^{-m-2}(c+dx)\right)\,dx - \int m(a\sin(c+dx)+a)^m\sin(c+dx)\sin^{-m-2}(c+dx)\,dx$$

input
```
integrate(sin(d*x+c)**(-2-m)*(a+a*sin(d*x+c))**m*(1+m-m*sin(d*x+c)),x)
```

output
```
-Integral(-(a*sin(c + d*x) + a)**m*sin(c + d*x)**(-m - 2), x) - Integral(-
m*(a*sin(c + d*x) + a)**m*sin(c + d*x)**(-m - 2), x) - Integral(m*(a*sin(c
 + d*x) + a)**m*sin(c + d*x)*sin(c + d*x)**(-m - 2), x)
```

Maxima [F]

$$\int \sin^{-2-m}(c+dx)(a+a\sin(c+dx))^m(1+m-m\sin(c+dx))\,dx$$
$$= \int -(m\sin(dx+c)-m-1)(a\sin(dx+c)+a)^m\sin(dx+c)^{-m-2}\,dx$$

input
```
integrate(sin(d*x+c)^(-2-m)*(a+a*sin(d*x+c))^m*(1+m-m*sin(d*x+c)),x, algor
ithm="maxima")
```

output
```
-integrate((m*sin(d*x + c) - m - 1)*(a*sin(d*x + c) + a)^m*sin(d*x + c)^(-
m - 2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5502 vs. 2(35) = 70.

Time = 34.36 (sec) , antiderivative size = 5502, normalized size of antiderivative = 157.20

$$\int \sin^{-2-m}(c+dx)(a+a\sin(c+dx))^m(1+m-m\sin(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^(-2-m)*(a+a*sin(d*x+c))^m*(1+m-m*sin(d*x+c)),x, algor
ithm="giac")
```

output
```
-8*(cos(2*pi*m*floor(-1/8*sgn(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*
tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1
/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) + 5/8) + 1/4*pi*m*sgn(4*tan(d*x + c)
^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(
d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) - 1/4*
pi*m)*e^(m*log(sqrt(2)*sqrt(abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 +
8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x +
 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c
)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1
/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*
d*x + 1/2*c) + 2)*tan(d*x + c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*
c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/
2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2)*tan(1/2*d*x + 1/2*c)^2 + ab
s(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x +
 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/
2*c) + 2))*abs(a)/(tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + tan(d*x + c)^2
+ tan(1/2*d*x + 1/2*c)^2 + 1)) - m*log(4*abs(tan(1/2*d*x + 1/2*c))/(tan(1/
2*d*x + 1/2*c)^2 + 1)) - 2*log(4*abs(tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x +
1/2*c)^2 + 1)))*tan(-1/2*pi + 1/4*pi*m*sgn(2*a*tan(1/2*d*x + 1/2*c)^4 + 4*
a*tan(1/2*d*x + 1/2*c)^3 - 4*a*tan(1/2*d*x + 1/2*c) - 2*a)*sgn(4*a*tan(...
```

Mupad [B] (verification not implemented)

Time = 35.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int \sin^{-2-m}(c+dx)(a+a\sin(c+dx))^m(1+m-m\sin(c+dx))\,dx$$
$$=-\frac{\sin(2\,c+2\,d\,x)\ \left(a\,(\sin(c+d\,x)+1)\right)^m}{2\,d\sin(c+d\,x)^{m+2}}$$

input
```
int(((a + a*sin(c + d*x))^m*(m - m*sin(c + d*x) + 1))/sin(c + d*x)^(m + 2)
,x)
```

output
```
-(sin(2*c + 2*d*x)*(a*(sin(c + d*x) + 1))^m)/(2*d*sin(c + d*x)^(m + 2))
```

Reduce [F]

$$\int \sin^{-2-m}(c+dx)(a+a\sin(c+dx))^m(1+m-m\sin(c+dx))\,dx$$
$$=\left(\int \frac{(\sin(dx+c)\,a+a)^m}{\sin(dx+c)^m\sin(dx+c)^2}dx\right)m+\int \frac{(\sin(dx+c)\,a+a)^m}{\sin(dx+c)^m\sin(dx+c)^2}dx$$
$$-\left(\int \frac{(\sin(dx+c)\,a+a)^m}{\sin(dx+c)^m\sin(dx+c)}dx\right)m$$

input
```
int(sin(d*x+c)^(-2-m)*(a+a*sin(d*x+c))^m*(1+m-m*sin(d*x+c)),x)
```

output
```
int((sin(c + d*x)*a + a)**m/(sin(c + d*x)**m*sin(c + d*x)**2),x)*m + int((
sin(c + d*x)*a + a)**m/(sin(c + d*x)**m*sin(c + d*x)**2),x) - int((sin(c +
 d*x)*a + a)**m/(sin(c + d*x)**m*sin(c + d*x)),x)*m
```

3.16 $\int \frac{\sin^2(e+fx)(A+B\sin(e+fx))}{(a+b\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 153

$$\begin{aligned}\int \frac{\sin^2(e+fx)(A+B\sin(e+fx))}{(a+b\sin(e+fx))^2}\,dx &= \frac{(Ab-2aB)x}{b^3} - \frac{2a(a^2Ab-2Ab^3-2a^3B+3ab^2B)\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{b^3\left(a^2-b^2\right)^{3/2}f} \\ &\quad - \frac{B\cos(e+fx)}{b^2f} + \frac{a^2(Ab-aB)\cos(e+fx)}{b^2\left(a^2-b^2\right)f(a+b\sin(e+fx))}\end{aligned}$$

output
```
(A*b-2*B*a)*x/b^3-2*a*(A*a^2*b-2*A*b^3-2*B*a^3+3*B*a*b^2)*arctan((b+a*tan(
1/2*f*x+1/2*e))/(a^2-b^2)^(1/2))/b^3/(a^2-b^2)^(3/2)/f-B*cos(f*x+e)/b^2/f+
a^2*(A*b-B*a)*cos(f*x+e)/b^2/(a^2-b^2)/f/(a+b*sin(f*x+e))
```

Mathematica [A] (verified)

Time = 3.82 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.96

$$\int \frac{\sin^2(e+fx)(A+B\sin(e+fx))}{(a+b\sin(e+fx))^2}\,dx$$
$$= \frac{(Ab-2aB)(e+fx)+\frac{2a(-a^2Ab+2Ab^3+2a^3B-3ab^2B)\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-bB\cos(e+fx)+\frac{a^2b(Ab-aB)\,\mathrm{co}}{(a-b)(a+b)(a+b\,\mathrm{s}}}{b^3f}$$

input
```
Integrate[(Sin[e + f*x]^2*(A + B*Sin[e + f*x]))/(a + b*Sin[e + f*x])^2,x]
```

output
```
((A*b - 2*a*B)*(e + f*x) + (2*a*(-(a^2*A*b) + 2*A*b^3 + 2*a^3*B - 3*a*b^2*
B)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) - b
*B*Cos[e + f*x] + (a^2*b*(A*b - a*B)*Cos[e + f*x])/((a - b)*(a + b)*(a + b
*Sin[e + f*x])))/(b^3*f)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.24, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {3042, 3467, 3042, 3502, 3042, 3214, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(e+fx)(A+B\sin(e+fx))}{(a+b\sin(e+fx))^2}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{\sin(e+fx)^2(A+B\sin(e+fx))}{(a+b\sin(e+fx))^2}dx$$
$$\Big\downarrow \ 3467$$
$$\frac{\int \frac{b(a^2-b^2)B\sin^2(e+fx)+(a^2-b^2)(Ab-aB)\sin(e+fx)+ab(Ab-aB)}{a+b\sin(e+fx)}dx}{b^2(a^2-b^2)}+\frac{a^2(Ab-aB)\cos(e+fx)}{b^2f(a^2-b^2)(a+b\sin(e+fx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{b(a^2-b^2)B\sin(e+fx)^2+(a^2-b^2)(Ab-aB)\sin(e+fx)+ab(Ab-aB)}{a+b\sin(e+fx)}dx}{b^2(a^2-b^2)}+\frac{a^2(Ab-aB)\cos(e+fx)}{b^2f(a^2-b^2)(a+b\sin(e+fx))}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\frac{\int \frac{a(Ab-aB)b^2+\left(a^2-b^2\right)(Ab-2aB)\sin(e+fx)b}{a+b\sin(e+fx)}dx}{b}-\frac{B(a^2-b^2)\cos(e+fx)}{f}}{b^2(a^2-b^2)}+\frac{a^2(Ab-aB)\cos(e+fx)}{b^2f(a^2-b^2)(a+b\sin(e+fx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{a(Ab-aB)b^2+\left(a^2-b^2\right)(Ab-2aB)\sin(e+fx)b}{a+b\sin(e+fx)}dx}{b}-\frac{B(a^2-b^2)\cos(e+fx)}{f}}{b^2(a^2-b^2)}+\frac{a^2(Ab-aB)\cos(e+fx)}{b^2f(a^2-b^2)(a+b\sin(e+fx))}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\frac{x(a^2-b^2)(Ab-2aB)-a(-2a^3B+a^2Ab+3ab^2B-2Ab^3)\int \frac{1}{a+b\sin(e+fx)}dx}{b}-\frac{B(a^2-b^2)\cos(e+fx)}{f}}{b^2(a^2-b^2)}+\frac{a^2(Ab-aB)\cos(e+fx)}{b^2f(a^2-b^2)(a+b\sin(e+fx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{x(a^2-b^2)(Ab-2aB)-a(-2a^3B+a^2Ab+3ab^2B-2Ab^3)\int \frac{1}{a+b\sin(e+fx)}dx}{b}-\frac{B(a^2-b^2)\cos(e+fx)}{f}}{b^2(a^2-b^2)}+\frac{a^2(Ab-aB)\cos(e+fx)}{b^2f(a^2-b^2)(a+b\sin(e+fx))}$$

$$\Big\downarrow \text{3139}$$

$$\frac{\frac{x(a^2-b^2)(Ab-2aB)-\frac{2a\left(-2a^3B+a^2Ab+3ab^2B-2Ab^3\right)\int \frac{1}{a\tan^2\left(\frac{1}{2}(e+fx)\right)+2b\tan\left(\frac{1}{2}(e+fx)\right)+a}d\tan\left(\frac{1}{2}(e+fx)\right)}{f}}{b}-\frac{B(a^2-b^2)\cos(e+fx)}{f}}{b^2(a^2-b^2)}+\frac{a^2(Ab-aB)\cos(e+fx)}{b^2f(a^2-b^2)(a+b\sin(e+fx))}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\frac{\frac{4a\left(-2a^3B+a^2Ab+3ab^2B-2Ab^3\right)\int \frac{1}{-\left(2b+2a\tan\left(\frac{1}{2}(e+fx)\right)\right)^2-4\left(a^2-b^2\right)}d\left(2b+2a\tan\left(\frac{1}{2}(e+fx)\right)\right)}{f}+x\left(a^2-b^2\right)(Ab-2aB)}{b}-\frac{B\left(a^2-b^2\right)\cos(e+fx)}{f}}{b^2\left(a^2-b^2\right)}+\frac{a^2(Ab-aB)\cos(e+fx)}{b^2f\left(a^2-b^2\right)(a+b\sin(e+fx))}$$

$$\Big\downarrow \; 217$$

$$\frac{a^2(Ab-aB)\cos(e+fx)}{b^2f\left(a^2-b^2\right)(a+b\sin(e+fx))}+\frac{\frac{x\left(a^2-b^2\right)(Ab-2aB)-\frac{2a\left(-2a^3B+a^2Ab+3ab^2B-2Ab^3\right)\arctan\left(\frac{2a\tan\left(\frac{1}{2}(e+fx)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{f\sqrt{a^2-b^2}}}{b}-\frac{B\left(a^2-b^2\right)\cos(e+fx)}{f}}{b^2\left(a^2-b^2\right)}$$

input
```
Int[(Sin[e + f*x]^2*(A + B*Sin[e + f*x]))/(a + b*Sin[e + f*x])^2,x]
```

output
```
(((a^2 - b^2)*(A*b - 2*a*B)*x - (2*a*(a^2*A*b - 2*A*b^3 - 2*a^3*B + 3*a*b^
2*B)*ArcTan[(2*b + 2*a*Tan[(e + f*x)/2])/(2*Sqrt[a^2 - b^2])])/(Sqrt[a^2 -
 b^2]*f))/b - ((a^2 - b^2)*B*Cos[e + f*x])/f)/(b^2*(a^2 - b^2)) + (a^2*(A*
b - a*B)*Cos[e + f*x])/(b^2*(a^2 - b^2)*f*(a + b*Sin[e + f*x]))
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3467
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*d^2
*(n + 1)*(c^2 - d^2))), x] - Simp[1/(d^2*(n + 1)*(c^2 - d^2))   Int[(c + d*
Sin[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c
- 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*
a*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n +
1)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[
n, -1]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.39

method	result
derivativedivides	$\frac{\frac{-\frac{2Bb}{1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}+2(Ab-2Ba)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{b^3}-\frac{2a\left(\frac{-\frac{b^2(Ab-Ba)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a^2-b^2}-\frac{ba(Ab-Ba)}{a^2-b^2}}{a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+2b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+a}+\frac{(A\,a^2b-2A\,b^3-2B\,a^3+}{}\right.}{b^3}}{f}$
default	$\frac{\frac{-\frac{2Bb}{1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}+2(Ab-2Ba)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{b^3}-\frac{2a\left(\frac{-\frac{b^2(Ab-Ba)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a^2-b^2}-\frac{ba(Ab-Ba)}{a^2-b^2}}{a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+2b\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+a}+\frac{(A\,a^2b-2A\,b^3-2B\,a^3+}{}\right.}{b^3}}{f}$
risch	$\frac{xA}{b^2}-\frac{2xBa}{b^3}-\frac{B\,e^{i(fx+e)}}{2b^2f}-\frac{B\,e^{-i(fx+e)}}{2b^2f}+\frac{2ia^2(-Ab+Ba)(ib+a\,e^{i(fx+e)})}{b^3(a^2-b^2)f(-ib\,e^{2i(fx+e)}+ib+2a\,e^{i(fx+e)})}+\frac{a^3\ln\left(e^{i(fx+e)}+\right.}{\sqrt{-a^2+b^2}}$

input
```
int(sin(f*x+e)^2*(A+B*sin(f*x+e))/(a+b*sin(f*x+e))^2,x,method=_RETURNVERBO
SE)
```

output
```
1/f*(2/b^3*(-B*b/(1+tan(1/2*f*x+1/2*e)^2)+(A*b-2*B*a)*arctan(tan(1/2*f*x+1
/2*e)))-2*a/b^3*((-b^2*(A*b-B*a)/(a^2-b^2)*tan(1/2*f*x+1/2*e)-b*a*(A*b-B*a
)/(a^2-b^2))/(a*tan(1/2*f*x+1/2*e)^2+2*b*tan(1/2*f*x+1/2*e)+a)+(A*a^2*b-2*
A*b^3-2*B*a^3+3*B*a*b^2)/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*f*x+1/2*e
)+2*b)/(a^2-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. 2(152) = 304.

Time = 0.19 (sec) , antiderivative size = 806, normalized size of antiderivative = 5.27

$$\int \frac{\sin^2(e+fx)(A+B\sin(e+fx))}{(a+b\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sin(f*x+e)^2*(A+B*sin(f*x+e))/(a+b*sin(f*x+e))^2,x, algorithm="f
ricas")
```

output
```
[-1/2*(2*(2*B*a^6 - A*a^5*b - 4*B*a^4*b^2 + 2*A*a^3*b^3 + 2*B*a^2*b^4 - A*
a*b^5)*f*x - (2*B*a^5 - A*a^4*b - 3*B*a^3*b^2 + 2*A*a^2*b^3 + (2*B*a^4*b -
 A*a^3*b^2 - 3*B*a^2*b^3 + 2*A*a*b^4)*sin(f*x + e))*sqrt(-a^2 + b^2)*log(-
((2*a^2 - b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 - 2*(a*cos(
f*x + e)*sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 + b^2))/(b^2*cos(f*x + e
)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)) + 2*(2*B*a^5*b - A*a^4*b^2 - 3*B*a^
3*b^3 + A*a^2*b^4 + B*a*b^5)*cos(f*x + e) + 2*((2*B*a^5*b - A*a^4*b^2 - 4*
B*a^3*b^3 + 2*A*a^2*b^4 + 2*B*a*b^5 - A*b^6)*f*x + (B*a^4*b^2 - 2*B*a^2*b^
4 + B*b^6)*cos(f*x + e))*sin(f*x + e))/((a^4*b^4 - 2*a^2*b^6 + b^8)*f*sin(
f*x + e) + (a^5*b^3 - 2*a^3*b^5 + a*b^7)*f), -((2*B*a^6 - A*a^5*b - 4*B*a^
4*b^2 + 2*A*a^3*b^3 + 2*B*a^2*b^4 - A*a*b^5)*f*x + (2*B*a^5 - A*a^4*b - 3*
B*a^3*b^2 + 2*A*a^2*b^3 + (2*B*a^4*b - A*a^3*b^2 - 3*B*a^2*b^3 + 2*A*a*b^4
)*sin(f*x + e))*sqrt(a^2 - b^2)*arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b
^2)*cos(f*x + e))) + (2*B*a^5*b - A*a^4*b^2 - 3*B*a^3*b^3 + A*a^2*b^4 + B*
a*b^5)*cos(f*x + e) + ((2*B*a^5*b - A*a^4*b^2 - 4*B*a^3*b^3 + 2*A*a^2*b^4
+ 2*B*a*b^5 - A*b^6)*f*x + (B*a^4*b^2 - 2*B*a^2*b^4 + B*b^6)*cos(f*x + e))
*sin(f*x + e))/((a^4*b^4 - 2*a^2*b^6 + b^8)*f*sin(f*x + e) + (a^5*b^3 - 2*
a^3*b^5 + a*b^7)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^2(e+fx)(A+B\sin(e+fx))}{(a+b\sin(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(f*x+e)**2*(A+B*sin(f*x+e))/(a+b*sin(f*x+e))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^2(e+fx)(A+B\sin(e+fx))}{(a+b\sin(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sin(f*x+e)^2*(A+B*sin(f*x+e))/(a+b*sin(f*x+e))^2,x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(152) = 304$.

Time = 0.21 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.33

$$\int \frac{\sin^2(e+fx)(A+B\sin(e+fx))}{(a+b\sin(e+fx))^2}\,dx$$
$$= \frac{\frac{2\,(2\,Ba^4-Aa^3b-3\,Ba^2b^2+2\,Aab^3)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^2b^3-b^5)\sqrt{a^2-b^2}} - \frac{2\left(Ba^2b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-Aab^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3\right.}{}}{}$$

input
```
integrate(sin(f*x+e)^2*(A+B*sin(f*x+e))/(a+b*sin(f*x+e))^2,x, algorithm="g
iac")
```

output
```
(2*(2*B*a^4 - A*a^3*b - 3*B*a^2*b^2 + 2*A*a*b^3)*(pi*floor(1/2*(f*x + e)/p
i + 1/2)*sgn(a) + arctan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/((
a^2*b^3 - b^5)*sqrt(a^2 - b^2)) - 2*(B*a^2*b*tan(1/2*f*x + 1/2*e)^3 - A*a*
b^2*tan(1/2*f*x + 1/2*e)^3 + 2*B*a^3*tan(1/2*f*x + 1/2*e)^2 - A*a^2*b*tan(
1/2*f*x + 1/2*e)^2 - B*a*b^2*tan(1/2*f*x + 1/2*e)^2 + 3*B*a^2*b*tan(1/2*f*
x + 1/2*e) - A*a*b^2*tan(1/2*f*x + 1/2*e) - 2*B*b^3*tan(1/2*f*x + 1/2*e) +
 2*B*a^3 - A*a^2*b - B*a*b^2)/((a*tan(1/2*f*x + 1/2*e)^4 + 2*b*tan(1/2*f*x
 + 1/2*e)^3 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e) + a)*(
a^2*b^2 - b^4)) - (2*B*a - A*b)*(f*x + e)/b^3)/f
```

Mupad [B] (verification not implemented)

Time = 39.92 (sec) , antiderivative size = 3718, normalized size of antiderivative = 24.30

$$\int \frac{\sin^2(e+fx)(A+B\sin(e+fx))}{(a+b\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int((sin(e + f*x)^2*(A + B*sin(e + f*x)))/(a + b*sin(e + f*x))^2,x)
```

output
```
((2*(A*a^2*b - 2*B*a^3 + B*a*b^2))/(b^2*(a^2 - b^2)) - (2*tan(e/2 + (f*x)/
2)^3*(B*a^2 - A*a*b))/(b*(a^2 - b^2)) + (2*tan(e/2 + (f*x)/2)*(2*B*b^2 - 3
*B*a^2 + A*a*b))/(b*(a^2 - b^2)) + (2*tan(e/2 + (f*x)/2)^2*(A*a^2*b - 2*B*
a^3 + B*a*b^2))/(b^2*(a^2 - b^2)))/(f*(a + 2*b*tan(e/2 + (f*x)/2) + 2*a*ta
n(e/2 + (f*x)/2)^2 + a*tan(e/2 + (f*x)/2)^4 + 2*b*tan(e/2 + (f*x)/2)^3)) +
 (log(tan(e/2 + (f*x)/2) + 1i)*(A*b - 2*B*a)*1i)/(b^3*f) - (log(tan(e/2 +
(f*x)/2) - 1i)*(A*b*1i - B*a*2i))/(b^3*f) - (a*atan(((a*(-(a + b)^3*(a - b
)^3)^(1/2)*((32*(A^2*a^2*b^8 - 2*A^2*a^4*b^6 + A^2*a^6*b^4 + 4*B^2*a^4*b^6
 - 8*B^2*a^6*b^4 + 4*B^2*a^8*b^2 - 4*A*B*a^3*b^7 + 8*A*B*a^5*b^5 - 4*A*B*a
^7*b^3))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(e/2 + (f*x)/2)*(2*A^2*a*b^1
0 - 9*A^2*a^3*b^8 + 8*A^2*a^5*b^6 - 2*A^2*a^7*b^4 + 8*B^2*a^3*b^8 - 29*B^2
*a^5*b^6 + 28*B^2*a^7*b^4 - 8*B^2*a^9*b^2 - 8*A*B*a^2*b^9 + 32*A*B*a^4*b^7
 - 30*A*B*a^6*b^5 + 8*A*B*a^8*b^3))/(b^10 - 2*a^2*b^8 + a^4*b^6) + (a*(-(a
 + b)^3*(a - b)^3)^(1/2)*((32*tan(e/2 + (f*x)/2)*(4*A*a^2*b^11 - 6*A*a^4*b
^9 + 2*A*a^6*b^7 - 6*B*a^3*b^10 + 10*B*a^5*b^8 - 4*B*a^7*b^6))/(b^10 - 2*a
^2*b^8 + a^4*b^6) - (32*(A*a^3*b^9 + 2*B*a^2*b^10 - 3*B*a^4*b^8 + B*a^6*b^
6 - A*a*b^11))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (a*((32*(a^2*b^12 - 2*a^4*b^1
0 + a^6*b^8))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (32*tan(e/2 + (f*x)/2)*(3*a*b^
14 - 8*a^3*b^12 + 7*a^5*b^10 - 2*a^7*b^8))/(b^10 - 2*a^2*b^8 + a^4*b^6))*(
-(a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2))/...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.63

$$\int \frac{\sin^2(e+fx)(A+B\sin(e+fx))}{(a+b\sin(e+fx))^2}\,dx = \frac{2\sqrt{a^2-b^2}\,atan\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)a+b}{\sqrt{a^2-b^2}}\right)a^2-\cos(fx+e)\,a^2b+\cos(fx+e)\,b^3-a^3fx+a\,b^2fx}{b^2f\,(a^2-b^2)}$$

input
```
int(sin(f*x+e)^2*(A+B*sin(f*x+e))/(a+b*sin(f*x+e))^2,x)
```

output
```
(2*sqrt(a**2 - b**2)*atan((tan((e + f*x)/2)*a + b)/sqrt(a**2 - b**2))*a**2
 - cos(e + f*x)*a**2*b + cos(e + f*x)*b**3 - a**3*f*x + a*b**2*f*x)/(b**2*
f*(a**2 - b**2))
```

3.17 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$

Optimal result

Integrand size = 34, antiderivative size = 182

$$\begin{aligned}\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx &= \frac{7}{16}a(2A-B)c^4x+\frac{7a(2A-B)c^4\cos^3(e+fx)}{24f}\\ &+\frac{7a(2A-B)c^4\cos(e+fx)\sin(e+fx)}{16f}-\frac{aBc\cos^3(e+fx)(c-c\sin(e+fx))^3}{6f}\\ &+\frac{a(2A-B)\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)^2}{10f}\\ &+\frac{7a(2A-B)\cos^3(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{40f}\end{aligned}$$

output
```
7/16*a*(2*A-B)*c^4*x+7/24*a*(2*A-B)*c^4*cos(f*x+e)^3/f+7/16*a*(2*A-B)*c^4*
cos(f*x+e)*sin(f*x+e)/f-1/6*a*B*c*cos(f*x+e)^3*(c-c*sin(f*x+e))^3/f+1/10*a
*(2*A-B)*cos(f*x+e)^3*(c^2-c^2*sin(f*x+e))^2/f+7/40*a*(2*A-B)*cos(f*x+e)^3
*(c^4-c^4*sin(f*x+e))/f
```

Mathematica [A] (verified)

Time = 2.59 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.77

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$$
$$=\frac{ac^4\cos(e+fx)\left(272A-176B-\frac{210(2A-B)\arcsin\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)}{\sqrt{\cos^2(e+fx)}}+15(2A+7B)\sin(e+fx)-32(7A-B)\right.}{240f}$$

input
```
Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4
,x]
```

output
```
(a*c^4*Cos[e + f*x]*(272*A - 176*B - (210*(2*A - B)*ArcSin[Sqrt[1 - Sin[e
+ f*x]]/Sqrt[2]])/Sqrt[Cos[e + f*x]^2] + 15*(2*A + 7*B)*Sin[e + f*x] - 32*
(7*A - B)*Sin[e + f*x]^2 + 10*(18*A - 17*B)*Sin[e + f*x]^3 - 48*(A - 3*B)*
Sin[e + f*x]^4 - 40*B*Sin[e + f*x]^5))/(240*f)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.87, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.382$, Rules used = {3042, 3446, 3042, 3339, 3042, 3157, 3042, 3157, 3042, 3148, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)(c-c\sin(e+fx))^4(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)(c-c\sin(e+fx))^4(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3446}$$

$$ac\int \cos^2(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^3dx$$

$$\Big\downarrow \; 3042$$

$$ac\int \cos(e+fx)^2(A+B\sin(e+fx))(c-c\sin(e+fx))^3dx$$

$$\Big\downarrow \; 3339$$

$$ac\left(\frac{1}{2}(2A-B)\int \cos^2(e+fx)(c-c\sin(e+fx))^3dx-\frac{B\cos^3(e+fx)(c-c\sin(e+fx))^3}{6f}\right)$$

$$\Big\downarrow \; 3042$$

$$ac\left(\frac{1}{2}(2A-B)\int \cos(e+fx)^2(c-c\sin(e+fx))^3dx-\frac{B\cos^3(e+fx)(c-c\sin(e+fx))^3}{6f}\right)$$

$$\Big\downarrow \; 3157$$

$$ac\left(\frac{1}{2}(2A-B)\left(\frac{7}{5}c\int \cos^2(e+fx)(c-c\sin(e+fx))^2dx+\frac{c\cos^3(e+fx)(c-c\sin(e+fx))^2}{5f}\right)-\frac{B\cos^3(e+ \right.$$

$$\Big\downarrow \; 3042$$

$$ac\left(\frac{1}{2}(2A-B)\left(\frac{7}{5}c\int \cos(e+fx)^2(c-c\sin(e+fx))^2dx+\frac{c\cos^3(e+fx)(c-c\sin(e+fx))^2}{5f}\right)-\frac{B\cos^3(e+ \right.$$

$$\Big\downarrow \; 3157$$

$$ac\left(\frac{1}{2}(2A-B)\left(\frac{7}{5}c\left(\frac{5}{4}c\int \cos^2(e+fx)(c-c\sin(e+fx))dx+\frac{\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{4f}\right)+\frac{c\cos^3(e \right.\right.$$

$$\Big\downarrow \; 3042$$

$$ac\left(\frac{1}{2}(2A-B)\left(\frac{7}{5}c\left(\frac{5}{4}c\int \cos(e+fx)^2(c-c\sin(e+fx))dx+\frac{\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{4f}\right)+\frac{c\cos^3(e \right.\right.$$

$$\Big\downarrow \; 3148$$

$$ac\left(\frac{1}{2}(2A-B)\left(\frac{7}{5}c\left(\frac{5}{4}c\left(c\int \cos^2(e+fx)dx+\frac{c\cos^3(e+fx)}{3f}\right)+\frac{\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{4f}\right)+\frac{c\cos \right.\right.$$

$$\Big\downarrow \; 3042$$

$$ac\left(\frac{1}{2}(2A-B)\left(\frac{7}{5}c\left(\frac{5}{4}c\left(c\int \sin\left(e+fx+\frac{\pi}{2}\right)^2dx+\frac{c\cos^3(e+fx)}{3f}\right)+\frac{\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{4f}\right) \right.\right.$$

$$\Big\downarrow \; 3115$$

$$ac\left(\frac{1}{2}(2A-B)\left(\frac{7}{5}c\left(\frac{5}{4}c\left(c\left(\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}\right)+\frac{c\cos^3(e+fx)}{3f}\right)+\frac{\cos^3(e+fx)\left(c^2-c^2\sin(\right.}{4f}\right.\right.\right.$$

$$\Big\downarrow \; 24$$

$$ac\left(\frac{1}{2}(2A-B)\left(\frac{7}{5}c\left(\frac{\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{4f}+\frac{5}{4}c\left(\frac{c\cos^3(e+fx)}{3f}+c\left(\frac{\sin(e+fx)\cos(e+fx)}{2f}+\frac{x}{2}\right.\right.\right.\right.\right.$$

input
```
Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4,x]
```

output
```
a*c*(-1/6*(B*Cos[e + f*x]^3*(c - c*Sin[e + f*x])^3)/f + ((2*A - B)*((c*Cos
[e + f*x]^3*(c - c*Sin[e + f*x])^2)/(5*f) + (7*c*((Cos[e + f*x]^3*(c^2 - c
^2*Sin[e + f*x]))/(4*f) + (5*c*((c*Cos[e + f*x]^3)/(3*f) + c*(x/2 + (Cos[e
 + f*x]*Sin[e + f*x])/(2*f))))/4))/5))/2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3157
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && Integers
Q[2*m, 2*p]
```

rule 3339
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(170) = 340$.

Time = 0.10 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.88

$$-3Aa\,c^4\left(-\frac{\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{2Aa\,c^4\left(2+\sin(fx+e)^2\right)\cos(fx+e)}{3}+2Aa\,c^4\left(-\frac{\sin(fx+e)\cos(}{2}\right.$$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x)
```

output
```
1/f*(-3*A*a*c^4*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8
*e)-2/3*A*a*c^4*(2+sin(f*x+e)^2)*cos(f*x+e)+2*A*a*c^4*(-1/2*sin(f*x+e)*cos
(f*x+e)+1/2*f*x+1/2*e)+3*A*a*c^4*cos(f*x+e)+A*a*c^4*(f*x+e)+3/5*B*a*c^4*(8
/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+2*B*a*c^4*(-1/4*(sin(f*x+e)^3
+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2/3*B*a*c^4*(2+sin(f*x+e)^2)*co
s(f*x+e)-3*B*a*c^4*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-B*a*c^4*cos(
f*x+e)-1/5*A*a*c^4*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+B*a*c^4*
(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+
5/16*e))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.68

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx = -\frac{48\,(A-3\,B)ac^4\cos{(fx+e)}^5-320\,(A-B)ac^4\cos{(fx+e)}^3-105\,(2\,A-B)ac^4fx+5\,\big(8\,Bac^4\cos}{240\,f}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorith
m="fricas")
```

output
```
-1/240*(48*(A - 3*B)*a*c^4*cos(f*x + e)^5 - 320*(A - B)*a*c^4*cos(f*x + e)
^3 - 105*(2*A - B)*a*c^4*f*x + 5*(8*B*a*c^4*cos(f*x + e)^5 + 2*(18*A - 25*
B)*a*c^4*cos(f*x + e)^3 - 21*(2*A - B)*a*c^4*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 853 vs. 2(163) = 326.

Time = 0.50 (sec) , antiderivative size = 853, normalized size of antiderivative = 4.69

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**4,x)
```

output
```
Piecewise((-9*A*a*c**4*x*sin(e + f*x)**4/8 - 9*A*a*c**4*x*sin(e + f*x)**2*
cos(e + f*x)**2/4 + A*a*c**4*x*sin(e + f*x)**2 - 9*A*a*c**4*x*cos(e + f*x)
**4/8 + A*a*c**4*x*cos(e + f*x)**2 + A*a*c**4*x - A*a*c**4*sin(e + f*x)**4
*cos(e + f*x)/f + 15*A*a*c**4*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 4*A*a*c
**4*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 2*A*a*c**4*sin(e + f*x)**2*cos
(e + f*x)/f + 9*A*a*c**4*sin(e + f*x)*cos(e + f*x)**3/(8*f) - A*a*c**4*sin
(e + f*x)*cos(e + f*x)/f - 8*A*a*c**4*cos(e + f*x)**5/(15*f) - 4*A*a*c**4*
cos(e + f*x)**3/(3*f) + 3*A*a*c**4*cos(e + f*x)/f + 5*B*a*c**4*x*sin(e + f
*x)**6/16 + 15*B*a*c**4*x*sin(e + f*x)**4*cos(e + f*x)**2/16 + 3*B*a*c**4*
x*sin(e + f*x)**4/4 + 15*B*a*c**4*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 3
*B*a*c**4*x*sin(e + f*x)**2*cos(e + f*x)**2/2 - 3*B*a*c**4*x*sin(e + f*x)*
*2/2 + 5*B*a*c**4*x*cos(e + f*x)**6/16 + 3*B*a*c**4*x*cos(e + f*x)**4/4 -
3*B*a*c**4*x*cos(e + f*x)**2/2 - 11*B*a*c**4*sin(e + f*x)**5*cos(e + f*x)/
(16*f) + 3*B*a*c**4*sin(e + f*x)**4*cos(e + f*x)/f - 5*B*a*c**4*sin(e + f*
x)**3*cos(e + f*x)**3/(6*f) - 5*B*a*c**4*sin(e + f*x)**3*cos(e + f*x)/(4*f
) + 4*B*a*c**4*sin(e + f*x)**2*cos(e + f*x)**3/f - 2*B*a*c**4*sin(e + f*x)
**2*cos(e + f*x)/f - 5*B*a*c**4*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 3*B*
a*c**4*sin(e + f*x)*cos(e + f*x)**3/(4*f) + 3*B*a*c**4*sin(e + f*x)*cos(e
+ f*x)/(2*f) + 8*B*a*c**4*cos(e + f*x)**5/(5*f) - 4*B*a*c**4*cos(e + f*x)*
*3/(3*f) - B*a*c**4*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.85

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^4\,dx =$$
$$-\frac{64\left(3\,\cos\left(fx+e\right)^5 - 10\,\cos\left(fx+e\right)^3 + 15\,\cos\left(fx+e\right)\right)Aac^4 - 640\left(\cos\left(fx+e\right)^3 - 3\,\cos\left(fx+e\right)\right)}{}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorith
m="maxima")
```

output
```
-1/960*(64*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*A*a*c^
4 - 640*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a*c^4 + 90*(12*f*x + 12*e + si
n(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a*c^4 - 480*(2*f*x + 2*e - sin(2*f*
x + 2*e))*A*a*c^4 - 960*(f*x + e)*A*a*c^4 - 192*(3*cos(f*x + e)^5 - 10*cos
(f*x + e)^3 + 15*cos(f*x + e))*B*a*c^4 - 640*(cos(f*x + e)^3 - 3*cos(f*x +
 e))*B*a*c^4 - 5*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e
) - 48*sin(2*f*x + 2*e))*B*a*c^4 - 60*(12*f*x + 12*e + sin(4*f*x + 4*e) -
8*sin(2*f*x + 2*e))*B*a*c^4 + 720*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c^4
 - 2880*A*a*c^4*cos(f*x + e) + 960*B*a*c^4*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.98

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx = -\frac{Bac^4\sin(6\,fx+6\,e)}{192\,f}+\frac{7}{16}\,(2\,Aac^4-Bac^4)x-\frac{(Aac^4-3\,Bac^4)\cos(5\,fx+5\,e)}{80\,f} + \frac{(13\,Aac^4-7\,Bac^4)\cos(3\,fx+3\,e)}{48\,f}+\frac{(7\,Aac^4-5\,Bac^4)\cos(fx+e)}{8\,f} - \frac{(6\,Aac^4-7\,Bac^4)\sin(4\,fx+4\,e)}{64\,f}+\frac{(16\,Aac^4+Bac^4)\sin(2\,fx+2\,e)}{64\,f}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorith
m="giac")
```

output
```
-1/192*B*a*c^4*sin(6*f*x + 6*e)/f + 7/16*(2*A*a*c^4 - B*a*c^4)*x - 1/80*(A
*a*c^4 - 3*B*a*c^4)*cos(5*f*x + 5*e)/f + 1/48*(13*A*a*c^4 - 7*B*a*c^4)*cos
(3*f*x + 3*e)/f + 1/8*(7*A*a*c^4 - 5*B*a*c^4)*cos(f*x + e)/f - 1/64*(6*A*a
*c^4 - 7*B*a*c^4)*sin(4*f*x + 4*e)/f + 1/64*(16*A*a*c^4 + B*a*c^4)*sin(2*f
*x + 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 37.34 (sec) , antiderivative size = 454, normalized size of antiderivative = 2.49

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$$
$$= \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{A\,a\,c^4}{4}+\frac{7\,B\,a\,c^4}{8}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}\left(6\,A\,a\,c^4-2\,B\,a\,c^4\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{4}\left(12\,A\,a\,c^4-4\,B\,a\,c^4\right)}{}$$
$$+\frac{7\,a\,c^4\,\mathrm{atan}\left(\frac{7\,a\,c^4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(2\,A-B)}{8\left(\frac{7\,A\,a\,c^4}{4}-\frac{7\,B\,a\,c^4}{8}\right)}\right)(2\,A-B)}{8\,f}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^4,x)
```

output
```
(tan(e/2 + (f*x)/2)*((A*a*c^4)/4 + (7*B*a*c^4)/8) + tan(e/2 + (f*x)/2)^10*
(6*A*a*c^4 - 2*B*a*c^4) + tan(e/2 + (f*x)/2)^4*(12*A*a*c^4 - 4*B*a*c^4) -
tan(e/2 + (f*x)/2)^11*((A*a*c^4)/4 + (7*B*a*c^4)/8) + tan(e/2 + (f*x)/2)^8
*(22*A*a*c^4 - 18*B*a*c^4) + tan(e/2 + (f*x)/2)^5*((13*A*a*c^4)/2 - (37*B*
a*c^4)/4) - tan(e/2 + (f*x)/2)^7*((13*A*a*c^4)/2 - (37*B*a*c^4)/4) + tan(e
/2 + (f*x)/2)^2*((38*A*a*c^4)/5 - (34*B*a*c^4)/5) + tan(e/2 + (f*x)/2)^6*(
(68*A*a*c^4)/3 - (44*B*a*c^4)/3) + tan(e/2 + (f*x)/2)^3*((27*A*a*c^4)/4 -
(73*B*a*c^4)/24) - tan(e/2 + (f*x)/2)^9*((27*A*a*c^4)/4 - (73*B*a*c^4)/24)
 + (34*A*a*c^4)/15 - (22*B*a*c^4)/15)/(f*(6*tan(e/2 + (f*x)/2)^2 + 15*tan(
e/2 + (f*x)/2)^4 + 20*tan(e/2 + (f*x)/2)^6 + 15*tan(e/2 + (f*x)/2)^8 + 6*t
an(e/2 + (f*x)/2)^10 + tan(e/2 + (f*x)/2)^12 + 1)) + (7*a*c^4*atan((7*a*c^
4*tan(e/2 + (f*x)/2)*(2*A - B))/(8*((7*A*a*c^4)/4 - (7*B*a*c^4)/8)))*(2*A
- B))/(8*f)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.06

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$$
$$= \frac{a\,c^4\left(-40\cos(fx+e)\sin(fx+e)^5 b-48\cos(fx+e)\sin(fx+e)^4 a+144\cos(fx+e)\sin(fx+e)^4 b\right.}{}$$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x)
```

output
```
(a*c**4*( - 40*cos(e + f*x)*sin(e + f*x)**5*b - 48*cos(e + f*x)*sin(e + f*
x)**4*a + 144*cos(e + f*x)*sin(e + f*x)**4*b + 180*cos(e + f*x)*sin(e + f*
x)**3*a - 170*cos(e + f*x)*sin(e + f*x)**3*b - 224*cos(e + f*x)*sin(e + f*
x)**2*a + 32*cos(e + f*x)*sin(e + f*x)**2*b + 30*cos(e + f*x)*sin(e + f*x)
*a + 105*cos(e + f*x)*sin(e + f*x)*b + 272*cos(e + f*x)*a - 176*cos(e + f*
x)*b + 210*a*f*x - 272*a - 105*b*f*x + 176*b))/(240*f)
```

3.18 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$

Optimal result

Integrand size = 34, antiderivative size = 142

$$\begin{aligned}
&\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx \\
&= \frac{1}{8}a(5A-2B)c^3x+\frac{a(5A-2B)c^3\cos^3(e+fx)}{12f} \\
&\quad +\frac{a(5A-2B)c^3\cos(e+fx)\sin(e+fx)}{8f}-\frac{aBc\cos^3(e+fx)(c-c\sin(e+fx))^2}{5f} \\
&\quad +\frac{a(5A-2B)\cos^3(e+fx)\left(c^3-c^3\sin(e+fx)\right)}{20f}
\end{aligned}$$

output
```
1/8*a*(5*A-2*B)*c^3*x+1/12*a*(5*A-2*B)*c^3*cos(f*x+e)^3/f+1/8*a*(5*A-2*B)*
c^3*cos(f*x+e)*sin(f*x+e)/f-1/5*a*B*c*cos(f*x+e)^3*(c-c*sin(f*x+e))^2/f+1/
20*a*(5*A-2*B)*cos(f*x+e)^3*(c^3-c^3*sin(f*x+e))/f
```

Mathematica [A] (verified)

Time = 2.47 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.87

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^3 \, dx$$
$$= \frac{ac^3\cos(e+fx)\left(80A - 56B - \frac{30(5A-2B)\arcsin\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)}{\sqrt{\cos^2(e+fx)}} + 15(3A+2B)\sin(e+fx) + (-80A+32B\right.}{120f}$$

input
```
Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3
,x]
```

output
```
(a*c^3*Cos[e + f*x]*(80*A - 56*B - (30*(5*A - 2*B)*ArcSin[Sqrt[1 - Sin[e +
 f*x]]/Sqrt[2]])/Sqrt[Cos[e + f*x]^2] + 15*(3*A + 2*B)*Sin[e + f*x] + (-80
*A + 32*B)*Sin[e + f*x]^2 + 30*(A - 2*B)*Sin[e + f*x]^3 + 24*B*Sin[e + f*x
]^4))/(120*f)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.87, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {3042, 3446, 3042, 3339, 3042, 3157, 3042, 3148, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)(c-c\sin(e+fx))^3(A+B\sin(e+fx)) \, dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int (a\sin(e+fx)+a)(c-c\sin(e+fx))^3(A+B\sin(e+fx))dx$$
$$\Big\downarrow \text{ 3446}$$
$$ac\int \cos^2(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^2dx$$

$$\Big\downarrow \ 3042$$

$$ac\int \cos(e+fx)^2(A+B\sin(e+fx))(c-c\sin(e+fx))^2dx$$

$$\Big\downarrow \ 3339$$

$$ac\left(\frac{1}{5}(5A-2B)\int \cos^2(e+fx)(c-c\sin(e+fx))^2dx-\frac{B\cos^3(e+fx)(c-c\sin(e+fx))^2}{5f}\right)$$

$$\Big\downarrow \ 3042$$

$$ac\left(\frac{1}{5}(5A-2B)\int \cos(e+fx)^2(c-c\sin(e+fx))^2dx-\frac{B\cos^3(e+fx)(c-c\sin(e+fx))^2}{5f}\right)$$

$$\Big\downarrow \ 3157$$

$$ac\left(\frac{1}{5}(5A-2B)\left(\frac{5}{4}c\int \cos^2(e+fx)(c-c\sin(e+fx))dx+\frac{\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{4f}\right)-\frac{B\cos^3(e+\right.$$

$$\Big\downarrow \ 3042$$

$$ac\left(\frac{1}{5}(5A-2B)\left(\frac{5}{4}c\int \cos(e+fx)^2(c-c\sin(e+fx))dx+\frac{\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{4f}\right)-\frac{B\cos^3(e+\right.$$

$$\Big\downarrow \ 3148$$

$$ac\left(\frac{1}{5}(5A-2B)\left(\frac{5}{4}c\left(c\int \cos^2(e+fx)dx+\frac{c\cos^3(e+fx)}{3f}\right)+\frac{\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{4f}\right)-\frac{B\cos^3(\right.$$

$$\Big\downarrow \ 3042$$

$$ac\left(\frac{1}{5}(5A-2B)\left(\frac{5}{4}c\left(c\int \sin\left(e+fx+\frac{\pi}{2}\right)^2dx+\frac{c\cos^3(e+fx)}{3f}\right)+\frac{\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{4f}\right)-\frac{B}{}\right.$$

$$\Big\downarrow \ 3115$$

$$ac\left(\frac{1}{5}(5A-2B)\left(\frac{5}{4}c\left(c\left(\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}\right)+\frac{c\cos^3(e+fx)}{3f}\right)+\frac{\cos^3(e+fx)\left(c^2-c^2\sin(e+\right.}{4f}\right.\right.$$

$$\Big\downarrow \ 24$$

$$ac\left(\frac{1}{5}(5A-2B)\left(\frac{\cos^3(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{4f}+\frac{5}{4}c\left(\frac{c\cos^3(e+fx)}{3f}+c\left(\frac{\sin(e+fx)\cos(e+fx)}{2f}+\frac{x}{2}\right)\right.\right.\right.$$

input
```
Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3,x]
```

output
```
a*c*(-1/5*(B*Cos[e + f*x]^3*(c - c*Sin[e + f*x])^2)/f + ((5*A - 2*B)*((Cos
[e + f*x]^3*(c^2 - c^2*Sin[e + f*x]))/(4*f) + (5*c*((c*Cos[e + f*x]^3)/(3*
f) + c*(x/2 + (Cos[e + f*x]*Sin[e + f*x])/(2*f))))/4))/5)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3157
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && Integers
Q[2*m, 2*p]
```

rule 3339
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 176.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.69

method	result
parallelrisch	$\frac{c^3 a\left(\left(\frac{2A}{3}-\frac{5B}{12}\right)\cos(3fx+3e)+\left(-\frac{A}{8}+\frac{B}{4}\right)\sin(4fx+4e)+\frac{\cos(5fx+5e)B}{20}+\sin(2fx+2e)A+\left(2A-\frac{3B}{2}\right)\cos(fx+e)+\frac{5fxA}{2}\right.}{4f}$
risch	$\frac{5a\,c^3xA}{8}-\frac{a\,c^3xB}{4}+\frac{a\,c^3\cos(fx+e)A}{2f}-\frac{3a\,c^3\cos(fx+e)B}{8f}+\frac{B a\,c^3\cos(5fx+5e)}{80f}-\frac{\sin(4fx+4e)Aa\,c^3}{32f}+$
parts	$-\frac{(-2Aa\,c^3+Ba\,c^3)\cos(fx+e)}{f}+\frac{(-Aa\,c^3+2Ba\,c^3)\left(-\frac{\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)}{f}+a\,c^3$
derivativedivides	$\frac{-\frac{2Aa\,c^3\left(2+\sin(fx+e)^2\right)\cos(fx+e)}{3}+2\cos(fx+e)Aa\,c^3+Aa\,c^3(fx+e)+2Ba\,c^3\left(-\frac{\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}\right.}{}$
default	$\frac{-\frac{2Aa\,c^3\left(2+\sin(fx+e)^2\right)\cos(fx+e)}{3}+2\cos(fx+e)Aa\,c^3+Aa\,c^3(fx+e)+2Ba\,c^3\left(-\frac{\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}\right.}{}$
norman	$\frac{(\frac{5}{8}Aa\,c^3-\frac{1}{4}Ba\,c^3)x+(\frac{5}{8}Aa\,c^3-\frac{1}{4}Ba\,c^3)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}+(\frac{25}{4}Aa\,c^3-\frac{5}{2}Ba\,c^3)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+(\frac{25}{4}Aa\,c^3-\frac{5}{2}Ba\,c^3)x}{}$
orering	Expression too large to display

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x,method=_RETURNV
ERBOSE)
```

output
```
1/4*c^3*a*((2/3*A-5/12*B)*cos(3*f*x+3*e)+(-1/8*A+1/4*B)*sin(4*f*x+4*e)+1/2
0*cos(5*f*x+5*e)*B+sin(2*f*x+2*e)*A+(2*A-3/2*B)*cos(f*x+e)+5/2*f*x*A-f*x*B
+8/3*A-28/15*B)/f
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.72

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$$
$$=\frac{24\,Bac^3\cos{(fx+e)}^5+80\,(A-B)ac^3\cos{(fx+e)}^3+15\,(5\,A-2\,B)ac^3fx-15\,\big(2\,(A-2\,B)ac^3\cos{(f}}{120\,f}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorith
m="fricas")
```

output
```
1/120*(24*B*a*c^3*cos(f*x + e)^5 + 80*(A - B)*a*c^3*cos(f*x + e)^3 + 15*(5
*A - 2*B)*a*c^3*f*x - 15*(2*(A - 2*B)*a*c^3*cos(f*x + e)^3 - (5*A - 2*B)*a
*c^3*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 486 vs. 2(129) = 258.

Time = 0.31 (sec) , antiderivative size = 486, normalized size of antiderivative = 3.42

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$$
$$=\begin{cases}-\frac{3Aac^3x\sin^4{(e+fx)}}{8}-\frac{3Aac^3x\sin^2{(e+fx)}\cos^2{(e+fx)}}{4}-\frac{3Aac^3x\cos^4{(e+fx)}}{8}+Aac^3x+\frac{5Aac^3\sin^3{(e+fx)}\cos{(e+fx)}}{8f}-\frac{2A}{}\\ x(A+B\sin{(e)})\,(a\sin{(e)}+a)\,(-c\sin{(e)}+c)^3\end{cases}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3,x)
```

output
```
Piecewise((-3*A*a*c**3*x*sin(e + f*x)**4/8 - 3*A*a*c**3*x*sin(e + f*x)**2*
cos(e + f*x)**2/4 - 3*A*a*c**3*x*cos(e + f*x)**4/8 + A*a*c**3*x + 5*A*a*c*
*3*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 2*A*a*c**3*sin(e + f*x)**2*cos(e +
 f*x)/f + 3*A*a*c**3*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 4*A*a*c**3*cos(e
 + f*x)**3/(3*f) + 2*A*a*c**3*cos(e + f*x)/f + 3*B*a*c**3*x*sin(e + f*x)**
4/4 + 3*B*a*c**3*x*sin(e + f*x)**2*cos(e + f*x)**2/2 - B*a*c**3*x*sin(e +
f*x)**2 + 3*B*a*c**3*x*cos(e + f*x)**4/4 - B*a*c**3*x*cos(e + f*x)**2 + B*
a*c**3*sin(e + f*x)**4*cos(e + f*x)/f - 5*B*a*c**3*sin(e + f*x)**3*cos(e +
 f*x)/(4*f) + 4*B*a*c**3*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 3*B*a*c**
3*sin(e + f*x)*cos(e + f*x)**3/(4*f) + B*a*c**3*sin(e + f*x)*cos(e + f*x)/
f + 8*B*a*c**3*cos(e + f*x)**5/(15*f) - B*a*c**3*cos(e + f*x)/f, Ne(f, 0))
, (x*(A + B*sin(e))*(a*sin(e) + a)*(-c*sin(e) + c)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.41

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$$
$$= \frac{320\left(\cos\left(fx+e\right)^3-3\cos\left(fx+e\right)\right)Aac^3-15\left(12\,fx+12\,e+\sin\left(4\,fx+4\,e\right)-8\,\sin\left(2\,fx+2\,e\right)\right)Aac^3}{}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorith
m="maxima")
```

output
```
1/480*(320*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a*c^3 - 15*(12*f*x + 12*e +
 sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a*c^3 + 480*(f*x + e)*A*a*c^3 +
32*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a*c^3 + 30*(
12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a*c^3 - 240*(2*f*
x + 2*e - sin(2*f*x + 2*e))*B*a*c^3 + 960*A*a*c^3*cos(f*x + e) - 480*B*a*c
^3*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.99

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx = \frac{Bac^3\cos(5fx+5e)}{80f} + \frac{Aac^3\sin(2fx+2e)}{4f} + \frac{1}{8}(5Aac^3-2Bac^3)x + \frac{(8Aac^3-5Bac^3)\cos(3fx+3e)}{48f} + \frac{(4Aac^3-3Bac^3)\cos(fx+e)}{8f} - \frac{(Aac^3-2Bac^3)\sin(4fx+4e)}{32f}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorith
m="giac")
```

output
```
1/80*B*a*c^3*cos(5*f*x + 5*e)/f + 1/4*A*a*c^3*sin(2*f*x + 2*e)/f + 1/8*(5*
A*a*c^3 - 2*B*a*c^3)*x + 1/48*(8*A*a*c^3 - 5*B*a*c^3)*cos(3*f*x + 3*e)/f +
 1/8*(4*A*a*c^3 - 3*B*a*c^3)*cos(f*x + e)/f - 1/32*(A*a*c^3 - 2*B*a*c^3)*s
in(4*f*x + 4*e)/f
```

Mupad [B] (verification not implemented)

Time = 36.13 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.74

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx = \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{3Aac^3}{4}+\frac{Bac^3}{2}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8(4Aac^3-2Bac^3)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(\frac{7Aac^3}{2}-3Bac^3\right)}{f\,(\mathrm{t}} + \frac{ac^3\,\mathrm{atan}\left(\frac{ac^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(5A-2B)}{4\left(\frac{5Aac^3}{4}-\frac{Bac^3}{2}\right)}\right)(5A-2B)}{4f} - \frac{ac^3(5A-2B)\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)-\frac{fx}{2}\right)}{4f}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^3,x)
```

output
```
(tan(e/2 + (f*x)/2)*((3*A*a*c^3)/4 + (B*a*c^3)/2) + tan(e/2 + (f*x)/2)^8*(
4*A*a*c^3 - 2*B*a*c^3) + tan(e/2 + (f*x)/2)^3*((7*A*a*c^3)/2 - 3*B*a*c^3)
- tan(e/2 + (f*x)/2)^7*((7*A*a*c^3)/2 - 3*B*a*c^3) - tan(e/2 + (f*x)/2)^9*
((3*A*a*c^3)/4 + (B*a*c^3)/2) + tan(e/2 + (f*x)/2)^6*(8*A*a*c^3 - 8*B*a*c^
3) + tan(e/2 + (f*x)/2)^2*((8*A*a*c^3)/3 - (8*B*a*c^3)/3) + tan(e/2 + (f*x
)/2)^4*((16*A*a*c^3)/3 - (4*B*a*c^3)/3) + (4*A*a*c^3)/3 - (14*B*a*c^3)/15)
/(f*(5*tan(e/2 + (f*x)/2)^2 + 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)
/2)^6 + 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^10 + 1)) + (a*c^3*atan
((a*c^3*tan(e/2 + (f*x)/2)*(5*A - 2*B))/(4*((5*A*a*c^3)/4 - (B*a*c^3)/2)))
*(5*A - 2*B))/(4*f) - (a*c^3*(5*A - 2*B)*(atan(tan(e/2 + (f*x)/2)) - (f*x)
/2))/(4*f)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.12

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^3\,dx$$
$$= \frac{a\,c^3\big(24\cos(fx+e)\sin(fx+e)^4 b + 30\cos(fx+e)\sin(fx+e)^3 a - 60\cos(fx+e)\sin(fx+e)^3 b - 8}{}$$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x)
```

output
```
(a*c**3*(24*cos(e + f*x)*sin(e + f*x)**4*b + 30*cos(e + f*x)*sin(e + f*x)*
*3*a - 60*cos(e + f*x)*sin(e + f*x)**3*b - 80*cos(e + f*x)*sin(e + f*x)**2
*a + 32*cos(e + f*x)*sin(e + f*x)**2*b + 45*cos(e + f*x)*sin(e + f*x)*a +
30*cos(e + f*x)*sin(e + f*x)*b + 80*cos(e + f*x)*a - 56*cos(e + f*x)*b + 7
5*a*f*x - 80*a - 30*b*f*x + 56*b))/(120*f)
```

3.19 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$

Optimal result

Integrand size = 34, antiderivative size = 97

$$\begin{aligned}\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx\\ =\frac{1}{8}a(4A-B)c^2x+\frac{a(A-B)c^2\cos^3(e+fx)}{3f}\\ +\frac{a(4A-B)c^2\cos(e+fx)\sin(e+fx)}{8f}+\frac{aBc^2\cos^3(e+fx)\sin(e+fx)}{4f}\end{aligned}$$

output
```
1/8*a*(4*A-B)*c^2*x+1/3*a*(A-B)*c^2*cos(f*x+e)^3/f+1/8*a*(4*A-B)*c^2*cos(f
*x+e)*sin(f*x+e)/f+1/4*a*B*c^2*cos(f*x+e)^3*sin(f*x+e)/f
```

Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.08

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$$

$$=\frac{ac^2\cos(e+fx)\left(8A-8B-\frac{6(4A-B)\arcsin\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)}{\sqrt{\cos^2(e+fx)}}+3(4A+B)\sin(e+fx)-8(A-B)\sin^2(e+f\right.}{24f}$$

input
```
Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2
,x]
```

output
```
(a*c^2*Cos[e + f*x]*(8*A - 8*B - (6*(4*A - B)*ArcSin[Sqrt[1 - Sin[e + f*x]
]/Sqrt[2]])/Sqrt[Cos[e + f*x]^2] + 3*(4*A + B)*Sin[e + f*x] - 8*(A - B)*Si
n[e + f*x]^2 - 6*B*Sin[e + f*x]^3))/(24*f)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.89, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {3042, 3446, 3042, 3339, 3042, 3148, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)(c-c\sin(e+fx))^2(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)(c-c\sin(e+fx))^2(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3446}$$

$$ac\int \cos^2(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))dx$$

$$\Big\downarrow \text{3042}$$

$$ac\int \cos(e+fx)^2(A+B\sin(e+fx))(c-c\sin(e+fx))dx$$

$$\Big\downarrow \text{3339}$$

$$ac\left(\frac{1}{4}(4A-B)\int \cos^2(e+fx)(c-c\sin(e+fx))dx-\frac{B\cos^3(e+fx)(c-c\sin(e+fx))}{4f}\right)$$

$$\Big\downarrow \text{3042}$$

$$ac\left(\frac{1}{4}(4A-B)\int \cos(e+fx)^2(c-c\sin(e+fx))dx-\frac{B\cos^3(e+fx)(c-c\sin(e+fx))}{4f}\right)$$

$$\Big\downarrow \text{3148}$$

$$ac\left(\frac{1}{4}(4A-B)\left(c\int \cos^2(e+fx)dx+\frac{c\cos^3(e+fx)}{3f}\right)-\frac{B\cos^3(e+fx)(c-c\sin(e+fx))}{4f}\right)$$

$$\Big\downarrow \text{3042}$$

$$ac\left(\frac{1}{4}(4A-B)\left(c\int \sin\left(e+fx+\frac{\pi}{2}\right)^2dx+\frac{c\cos^3(e+fx)}{3f}\right)-\frac{B\cos^3(e+fx)(c-c\sin(e+fx))}{4f}\right)$$

$$\Big\downarrow \text{3115}$$

$$ac\left(\frac{1}{4}(4A-B)\left(c\left(\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}\right)+\frac{c\cos^3(e+fx)}{3f}\right)-\frac{B\cos^3(e+fx)(c-c\sin(e+fx))}{4f}\right)$$

$$\Big\downarrow \text{24}$$

$$ac\left(\frac{1}{4}(4A-B)\left(\frac{c\cos^3(e+fx)}{3f}+c\left(\frac{\sin(e+fx)\cos(e+fx)}{2f}+\frac{x}{2}\right)\right)-\frac{B\cos^3(e+fx)(c-c\sin(e+fx))}{4f}\right)$$

input
```
Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2,x]
```

output
```
a*c*(-1/4*(B*Cos[e + f*x]^3*(c - c*Sin[e + f*x]))/f + ((4*A - B)*((c*Cos[e
 + f*x]^3)/(3*f) + c*(x/2 + (Cos[e + f*x]*Sin[e + f*x])/(2*f))))/4)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3339
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 22.47 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

method	result
parallelrisch	$\frac{c^2 a\left(\frac{(A-B)\cos(3fx+3e)}{3}+\sin(2fx+2e)A+\frac{\sin(4fx+4e)B}{8}+(A-B)\cos(fx+e)+2fxA-\frac{fxB}{2}+\frac{4A}{3}-\frac{4B}{3}\right)}{4f}$
risch	$\frac{a\,c^2xA}{2}-\frac{a\,c^2xB}{8}+\frac{Aa\,c^2\cos(fx+e)}{4f}-\frac{a\,c^2\cos(fx+e)B}{4f}+\frac{Ba\,c^2\sin(4fx+4e)}{32f}+\frac{a\,c^2\cos(3fx+3e)A}{12f}-\frac{a\,c^2}{}$
parts	$\frac{(-Aa\,c^2-Ba\,c^2)\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{(-Aa\,c^2+Ba\,c^2)\cos(fx+e)}{f}-\frac{(Aa\,c^2-Ba\,c^2)\left(2+\sin(fx+e)\right.}{3f}$
derivativedivides	$\frac{-Aa\,c^2\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+\cos(fx+e)Aa\,c^2+Aa\,c^2(fx+e)+\frac{Ba\,c^2\left(2+\sin(fx+e)^2\right)\cos(fx+e)}{3}-Ba\,c^2\left(-\frac{s}{}\right.}{}$
default	$\frac{-Aa\,c^2\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+\cos(fx+e)Aa\,c^2+Aa\,c^2(fx+e)+\frac{Ba\,c^2\left(2+\sin(fx+e)^2\right)\cos(fx+e)}{3}-Ba\,c^2\left(-\frac{s}{}\right.}{}$
norman	$\frac{\left(\frac{1}{2}Aa\,c^2-\frac{1}{8}Ba\,c^2\right)x+\left(2Aa\,c^2-\frac{1}{2}Ba\,c^2\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\left(2Aa\,c^2-\frac{1}{2}Ba\,c^2\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+\left(3Aa\,c^2-\frac{3}{4}Ba\,c^2\right)x\tan\left(\right.}{}$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x,method=_RETURNV
ERBOSE)
```

output
```
1/4*c^2*a*(1/3*(A-B)*cos(3*f*x+3*e)+sin(2*f*x+2*e)*A+1/8*sin(4*f*x+4*e)*B+
(A-B)*cos(f*x+e)+2*f*x*A-1/2*f*x*B+4/3*A-4/3*B)/f
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$$
$$=\frac{8\,(A-B)ac^2\cos\left(fx+e\right)^3+3\,(4\,A-B)ac^2fx+3\,\left(2\,Bac^2\cos\left(fx+e\right)^3+(4\,A-B)ac^2\cos\left(fx+e\right)\right)\,s}{24\,f}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorith
m="fricas")
```

output
```
1/24*(8*(A - B)*a*c^2*cos(f*x + e)^3 + 3*(4*A - B)*a*c^2*f*x + 3*(2*B*a*c^
2*cos(f*x + e)^3 + (4*A - B)*a*c^2*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(85) = 170$.

Time = 0.21 (sec) , antiderivative size = 396, normalized size of antiderivative = 4.08

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^2\,dx$$
$$= \begin{cases} -\frac{Aac^2x\sin^2(e+fx)}{2} - \frac{Aac^2x\cos^2(e+fx)}{2} + Aac^2x - \frac{Aac^2\sin^2(e+fx)\cos(e+fx)}{f} + \frac{Aac^2\sin(e+fx)\cos(e+fx)}{2f} - \frac{2Aac^2\cos^3}{3f} \\ x(A + B\sin(e))\,(a\sin(e) + a)\,(-c\sin(e) + c)^2 \end{cases}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2,x)
```

output
```
Piecewise((-A*a*c**2*x*sin(e + f*x)**2/2 - A*a*c**2*x*cos(e + f*x)**2/2 +
A*a*c**2*x - A*a*c**2*sin(e + f*x)**2*cos(e + f*x)/f + A*a*c**2*sin(e + f*
x)*cos(e + f*x)/(2*f) - 2*A*a*c**2*cos(e + f*x)**3/(3*f) + A*a*c**2*cos(e
+ f*x)/f + 3*B*a*c**2*x*sin(e + f*x)**4/8 + 3*B*a*c**2*x*sin(e + f*x)**2*c
os(e + f*x)**2/4 - B*a*c**2*x*sin(e + f*x)**2/2 + 3*B*a*c**2*x*cos(e + f*x
)**4/8 - B*a*c**2*x*cos(e + f*x)**2/2 - 5*B*a*c**2*sin(e + f*x)**3*cos(e +
 f*x)/(8*f) + B*a*c**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a*c**2*sin(e +
 f*x)*cos(e + f*x)**3/(8*f) + B*a*c**2*sin(e + f*x)*cos(e + f*x)/(2*f) + 2
*B*a*c**2*cos(e + f*x)**3/(3*f) - B*a*c**2*cos(e + f*x)/f, Ne(f, 0)), (x*(
A + B*sin(e))*(a*sin(e) + a)*(-c*sin(e) + c)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(89) = 178$.

Time = 0.04 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.85

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^2\,dx$$
$$= \frac{32\left(\cos(fx + e)^3 - 3\cos(fx + e)\right)Aac^2 - 24\,(2\,fx + 2\,e - \sin(2\,fx + 2\,e))Aac^2 + 96\,(fx + e)Aac^2 - 3}{}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorith
m="maxima")
```

output
```
1/96*(32*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a*c^2 - 24*(2*f*x + 2*e - sin
(2*f*x + 2*e))*A*a*c^2 + 96*(f*x + e)*A*a*c^2 - 32*(cos(f*x + e)^3 - 3*cos
(f*x + e))*B*a*c^2 + 3*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2
*e))*B*a*c^2 - 24*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c^2 + 96*A*a*c^2*co
s(f*x + e) - 96*B*a*c^2*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.13

$$\begin{aligned}\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx \\ =\frac{Bac^2\sin(4\,fx+4\,e)}{32\,f}+\frac{Aac^2\sin(2\,fx+2\,e)}{4\,f}+\frac{1}{8}\left(4\,Aac^2-Bac^2\right)x \\ +\frac{(Aac^2-Bac^2)\cos(3\,fx+3\,e)}{12\,f}+\frac{(Aac^2-Bac^2)\cos(fx+e)}{4\,f}\end{aligned}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorith
m="giac")
```

output
```
1/32*B*a*c^2*sin(4*f*x + 4*e)/f + 1/4*A*a*c^2*sin(2*f*x + 2*e)/f + 1/8*(4*
A*a*c^2 - B*a*c^2)*x + 1/12*(A*a*c^2 - B*a*c^2)*cos(3*f*x + 3*e)/f + 1/4*(
A*a*c^2 - B*a*c^2)*cos(f*x + e)/f
```

Mupad [B] (verification not implemented)

Time = 35.76 (sec) , antiderivative size = 345, normalized size of antiderivative = 3.56

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^2\, dx$$
$$= \frac{\tan\left(\frac{e}{2} + \frac{fx}{2}\right)\left(A a c^2 + \frac{B a c^2}{4}\right) + \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^4 (2 A a c^2 - 2 B a c^2) + \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^6 (2 A a c^2 - 2 B a c^2)}{f\left(\tan\left(\frac{e}{2} + \frac{f}{}\right.\right.}$$
$$+ \frac{a c^2 \operatorname{atan}\left(\frac{a c^2 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)(4A - B)}{4\left(A a c^2 - \frac{B a c^2}{4}\right)}\right)(4A - B)}{4f}$$
$$- \frac{a c^2 (4A - B)\left(\operatorname{atan}\left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right)\right) - \frac{fx}{2}\right)}{4f}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^2,x)
```

output
```
(tan(e/2 + (f*x)/2)*(A*a*c^2 + (B*a*c^2)/4) + tan(e/2 + (f*x)/2)^4*(2*A*a*
c^2 - 2*B*a*c^2) + tan(e/2 + (f*x)/2)^6*(2*A*a*c^2 - 2*B*a*c^2) + tan(e/2
+ (f*x)/2)^2*((2*A*a*c^2)/3 - (2*B*a*c^2)/3) - tan(e/2 + (f*x)/2)^7*(A*a*c
^2 + (B*a*c^2)/4) + tan(e/2 + (f*x)/2)^3*(A*a*c^2 - (7*B*a*c^2)/4) - tan(e
/2 + (f*x)/2)^5*(A*a*c^2 - (7*B*a*c^2)/4) + (2*A*a*c^2)/3 - (2*B*a*c^2)/3)
/(f*(4*tan(e/2 + (f*x)/2)^2 + 6*tan(e/2 + (f*x)/2)^4 + 4*tan(e/2 + (f*x)/2
)^6 + tan(e/2 + (f*x)/2)^8 + 1)) + (a*c^2*atan((a*c^2*tan(e/2 + (f*x)/2)*(
4*A - B))/(4*(A*a*c^2 - (B*a*c^2)/4)))*(4*A - B))/(4*f) - (a*c^2*(4*A - B)
*(atan(tan(e/2 + (f*x)/2)) - (f*x)/2))/(4*f)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.29

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^2\, dx$$
$$= \frac{a c^2 \left(-6\cos(fx + e)\sin(fx + e)^3 b - 8\cos(fx + e)\sin(fx + e)^2 a + 8\cos(fx + e)\sin(fx + e)^2 b + 12\right.}{}$$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x)
```

output
```
(a*c**2*( - 6*cos(e + f*x)*sin(e + f*x)**3*b - 8*cos(e + f*x)*sin(e + f*x)
**2*a + 8*cos(e + f*x)*sin(e + f*x)**2*b + 12*cos(e + f*x)*sin(e + f*x)*a
+ 3*cos(e + f*x)*sin(e + f*x)*b + 8*cos(e + f*x)*a - 8*cos(e + f*x)*b + 12
*a*f*x - 8*a - 3*b*f*x + 8*b))/(24*f)
```

3.20 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$

Optimal result

Integrand size = 32, antiderivative size = 49

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx = \frac{1}{2}aAcx - \frac{aBc\cos^3(e+fx)}{3f} + \frac{aAc\cos(e+fx)\sin(e+fx)}{2f}$$

output
```
1/2*a*A*c*x-1/3*a*B*c*cos(f*x+e)^3/f+1/2*a*A*c*cos(f*x+e)*sin(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx = -\frac{ac(3B\cos(e+fx)+B\cos(3(e+fx))-3A(-2e+2fx+\sin(2(e+fx))))}{12f}$$

input
```
Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),x
]
```

output
```
-1/12*(a*c*(3*B*Cos[e + f*x] + B*Cos[3*(e + f*x)] - 3*A*(-2*e + 2*f*x + Si
n[2*(e + f*x)])))/f
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {3042, 3446, 3042, 3148, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)(c-c\sin(e+fx))(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)(c-c\sin(e+fx))(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3446}$$

$$ac\int \cos^2(e+fx)(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3042}$$

$$ac\int \cos(e+fx)^2(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3148}$$

$$ac\left(A\int \cos^2(e+fx)dx-\frac{B\cos^3(e+fx)}{3f}\right)$$

$$\Big\downarrow \text{3042}$$

$$ac\left(A\int \sin\left(e+fx+\frac{\pi}{2}\right)^2dx-\frac{B\cos^3(e+fx)}{3f}\right)$$

$$\Big\downarrow \text{3115}$$

$$ac\left(A\left(\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}\right)-\frac{B\cos^3(e+fx)}{3f}\right)$$

$$\Big\downarrow \text{24}$$

$$ac\left(A\left(\frac{\sin(e+fx)\cos(e+fx)}{2f}+\frac{x}{2}\right)-\frac{B\cos^3(e+fx)}{3f}\right)$$

input
```
Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),x]
```

output
```
a*c*(-1/3*(B*Cos[e + f*x]^3)/f + A*(x/2 + (Cos[e + f*x]*Sin[e + f*x])/(2*f
)))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 44.56 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

method	result
parallelrisch	$\frac{ac(6fxA+3\sin(2fx+2e)A-3\cos(fx+e)B-\cos(3fx+3e)B-4B)}{12f}$
risch	$\frac{aAcx}{2}-\frac{Bac\cos(fx+e)}{4f}-\frac{Bac\cos(3fx+3e)}{12f}+\frac{Aac\sin(2fx+2e)}{4f}$
derivativedivides	$\frac{\frac{Bac\left(2+\sin(fx+e)^2\right)\cos(fx+e)}{3}-Aac\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-Bac\cos(fx+e)+Aac(fx+e)}{f}$
default	$\frac{\frac{Bac\left(2+\sin(fx+e)^2\right)\cos(fx+e)}{3}-Aac\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-Bac\cos(fx+e)+Aac(fx+e)}{f}$
parts	$aAcx-\frac{Bac\cos(fx+e)}{f}-\frac{Aac\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{Bac\left(2+\sin(fx+e)^2\right)\cos(fx+e)}{3f}$
norman	$\frac{\frac{Aac\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{2Bac}{3f}-\frac{2Bac\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{f}+\frac{aAcx}{2}-\frac{Aac\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{f}+\frac{3aAcx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{2}+\frac{3aAcx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{2}+\cdots}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^3}$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x,method=_RETURNVER
BOSE)
```

output
```
1/12*a*c*(6*f*x*A+3*sin(2*f*x+2*e)*A-3*cos(f*x+e)*B-cos(3*f*x+3*e)*B-4*B)/
f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$$
$$=-\frac{2\,Bac\cos\left(fx+e\right)^3-3\,Aacfx-3\,Aac\cos\left(fx+e\right)\sin\left(fx+e\right)}{6\,f}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm=
"fricas")
```

output
```
-1/6*(2*B*a*c*cos(f*x + e)^3 - 3*A*a*c*f*x - 3*A*a*c*cos(f*x + e)*sin(f*x
+ e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(46) = 92$.

Time = 0.12 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.82

$$\int (a + a\sin(e+fx))(A + B\sin(e+fx))(c - c\sin(e+fx))\,dx$$
$$= \begin{cases} -\frac{Aacx\sin^2(e+fx)}{2} - \frac{Aacx\cos^2(e+fx)}{2} + Aacx + \frac{Aac\sin(e+fx)\cos(e+fx)}{2f} + \frac{Bac\sin^2(e+fx)\cos(e+fx)}{f} + \frac{2Bac\cos^3(e+fx)}{3f} \\ x(A + B\sin(e))(a\sin(e) + a)(-c\sin(e) + c) \end{cases}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

output
```
Piecewise((-A*a*c*x*sin(e + f*x)**2/2 - A*a*c*x*cos(e + f*x)**2/2 + A*a*c*
x + A*a*c*sin(e + f*x)*cos(e + f*x)/(2*f) + B*a*c*sin(e + f*x)**2*cos(e +
f*x)/f + 2*B*a*c*cos(e + f*x)**3/(3*f) - B*a*c*cos(e + f*x)/f, Ne(f, 0)),
(x*(A + B*sin(e))*(a*sin(e) + a)*(-c*sin(e) + c), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.49

$$\int (a + a\sin(e+fx))(A + B\sin(e+fx))(c - c\sin(e+fx))\,dx =$$
$$-\frac{3(2fx + 2e - \sin(2fx + 2e))Aac - 12(fx+e)Aac + 4\left(\cos(fx+e)^3 - 3\cos(fx+e)\right)Bac + 12\,I}{12f}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm=
"maxima")
```

output
```
-1/12*(3*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a*c - 12*(f*x + e)*A*a*c + 4*(
cos(f*x + e)^3 - 3*cos(f*x + e))*B*a*c + 12*B*a*c*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.12

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))\,dx = \frac{1}{2}Aacx - \frac{Bac\cos(3\,fx + 3\,e)}{12\,f} - \frac{Bac\cos(fx + e)}{4\,f} + \frac{Aac\sin(2\,fx + 2\,e)}{4\,f}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm=
"giac")
```

output
```
1/2*A*a*c*x - 1/12*B*a*c*cos(3*f*x + 3*e)/f - 1/4*B*a*c*cos(f*x + e)/f + 1
/4*A*a*c*sin(2*f*x + 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 36.99 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.49

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))\,dx = \frac{A\,a\,c\,x}{2} - \frac{A\,a\,c\tan\left(\frac{e}{2} + \frac{f\,x}{2}\right)^5 + \left(\frac{a\,c\,(12\,B - 9\,A\,(e + f\,x))}{6} + \frac{3\,A\,a\,c\,(e + f\,x)}{2}\right)\tan\left(\frac{e}{2} + \frac{f\,x}{2}\right)^4 - A\,a\,c\tan\left(\frac{e}{2} + \frac{f\,x}{2}\right) + \frac{a\,c\,(4\,B - 3}{}}{f\left(\tan\left(\frac{e}{2} + \frac{f\,x}{2}\right)^2 + 1\right)^3}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x)),x)
```

output
```
(A*a*c*x)/2 - (tan(e/2 + (f*x)/2)^4*((a*c*(12*B - 9*A*(e + f*x)))/6 + (3*A
*a*c*(e + f*x))/2) + (a*c*(4*B - 3*A*(e + f*x)))/6 - A*a*c*tan(e/2 + (f*x)
/2) + (A*a*c*(e + f*x))/2 + A*a*c*tan(e/2 + (f*x)/2)^5)/(f*(tan(e/2 + (f*x
)/2)^2 + 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.16

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))\,dx$$
$$= \frac{ac\left(2\cos(fx + e)\sin(fx + e)^2 b + 3\cos(fx + e)\sin(fx + e)\,a - 2\cos(fx + e)\,b + 3afx + 2b\right)}{6f}$$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

output
```
(a*c*(2*cos(e + f*x)*sin(e + f*x)**2*b + 3*cos(e + f*x)*sin(e + f*x)*a - 2
*cos(e + f*x)*b + 3*a*f*x + 2*b))/(6*f)
```

3.21 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 56

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = -\frac{a(A+2B)x}{c}+\frac{aB\cos(e+fx)}{cf}+\frac{2a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))}$$

output
```
-a*(A+2*B)*x/c+a*B*cos(f*x+e)/c/f+2*a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))
```

Mathematica [A] (verified)

Time = 5.33 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.91

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = \frac{a\cos(e+fx)\left(-2(A+2B)\arcsin\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)\sqrt{1-\sin(e+fx)}+\sqrt{1+\sin(e+fx)}(-2A-3B+\right.}{cf(-1+\sin(e+fx))\sqrt{1+\sin(e+fx)}}$$

input
```
Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
,x]
```

output
```
(a*Cos[e + f*x]*(-2*(A + 2*B)*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]]*Sqrt[
1 - Sin[e + f*x]] + Sqrt[1 + Sin[e + f*x]]*(-2*A - 3*B + B*Sin[e + f*x])))
/(c*f*(-1 + Sin[e + f*x])*Sqrt[1 + Sin[e + f*x]])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {3042, 3446, 3042, 3336, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{c-c\sin(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{c-c\sin(e+fx)}dx$$

$$\Big\downarrow \text{3446}$$

$$ac\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$ac\int \frac{\cos(e+fx)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}dx$$

$$\Big\downarrow \text{3336}$$

$$ac\left(\frac{\int(-((A+2B)c)-B\sin(e+fx)c)dx}{c^3}+\frac{2(A+B)\cos(e+fx)}{f\left(c^2-c^2\sin(e+fx)\right)}\right)$$

$$\Big\downarrow \text{2009}$$

$$ac\left(\frac{\frac{Bc\cos(e+fx)}{f}-cx(A+2B)}{c^3}+\frac{2(A+B)\cos(e+fx)}{f\left(c^2-c^2\sin(e+fx)\right)}\right)$$

input
```
Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]),x]
```

output
```
a*c*((-((A + 2*B)*c*x) + (B*c*Cos[e + f*x])/f)/c^3 + (2*(A + B)*Cos[e + f*
x])/(f*(c^2 - c^2*Sin[e + f*x])))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3336
```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Simp[1/(b^
3*(2*m + 3))   Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*
(2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2
- b^2, 0] && LtQ[m, -3/2]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.20

method	result
derivativedivides	$\frac{2a\left(-\frac{2A+2B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}+\frac{B}{1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}-(A+2B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{fc}$
default	$\frac{2a\left(-\frac{2A+2B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}+\frac{B}{1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}-(A+2B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{fc}$
parallelrisch	$\frac{2\left(\frac{B\cos(2fx+2e)}{4}+(-\frac{1}{2}fxA-fxB+A+\frac{3}{2}B)\cos(fx+e)+(A+B)\sin(fx+e)+A+\frac{5B}{4}\right)a}{cf\cos(fx+e)}$
risch	$-\frac{axA}{c}-\frac{2axB}{c}+\frac{Ba\,e^{i(fx+e)}}{2cf}+\frac{Ba\,e^{-i(fx+e)}}{2cf}+\frac{4aA}{fc(e^{i(fx+e)}-i)}+\frac{4aB}{fc(e^{i(fx+e)}-i)}$
norman	$\frac{\frac{a(A+2B)x}{c}+\frac{a(A+2B)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{c}-\frac{4Aa+4Ba}{fc}-\frac{2Ba\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{fc}-\frac{2Ba\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{fc}-\frac{(4Aa+2Ba)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{fc}-\frac{2(4A}{}}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x,method=_RETURNVER
BOSE)
```

output
```
2/f*a/c*(-(2*A+2*B)/(tan(1/2*f*x+1/2*e)-1)+B/(1+tan(1/2*f*x+1/2*e)^2)-(A+2
*B)*arctan(tan(1/2*f*x+1/2*e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(57) = 114.

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.07

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = -\frac{(A+2B)afx-Ba\cos(fx+e)^2-2(A+B)a+((A+2B)afx-(2A+3B)a)\cos(fx+e)-((A}{cf\cos(fx+e)-cf\sin(fx+e)+cf}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm=
"fricas")
```

output
```
-((A + 2*B)*a*f*x - B*a*cos(f*x + e)^2 - 2*(A + B)*a + ((A + 2*B)*a*f*x -
(2*A + 3*B)*a)*cos(f*x + e) - ((A + 2*B)*a*f*x - B*a*cos(f*x + e) + 2*(A +
 B)*a)*sin(f*x + e))/(c*f*cos(f*x + e) - c*f*sin(f*x + e) + c*f)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 828 vs. $2(48) = 96$.

Time = 1.11 (sec) , antiderivative size = 828, normalized size of antiderivative = 14.79

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{c - c\sin(e + fx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

output
```
Piecewise((-A*a*f*x*tan(e/2 + f*x/2)**3/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan
(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) + A*a*f*x*tan(e/2 + f*x/2)*
*2/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/
2) - c*f) - A*a*f*x*tan(e/2 + f*x/2)/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/
2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) + A*a*f*x/(c*f*tan(e/2 + f*x/2
)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) - 4*A*a*tan(e
/2 + f*x/2)**2/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*ta
n(e/2 + f*x/2) - c*f) - 4*A*a/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x
/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) - 2*B*a*f*x*tan(e/2 + f*x/2)**3/(c*f*
tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f
) + 2*B*a*f*x*tan(e/2 + f*x/2)**2/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 +
 f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) - 2*B*a*f*x*tan(e/2 + f*x/2)/(c*f
*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*
f) + 2*B*a*f*x/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*ta
n(e/2 + f*x/2) - c*f) - 4*B*a*tan(e/2 + f*x/2)**2/(c*f*tan(e/2 + f*x/2)**3
 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) + 2*B*a*tan(e/2 +
 f*x/2)/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 +
 f*x/2) - c*f) - 6*B*a/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2
+ c*f*tan(e/2 + f*x/2) - c*f), Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)
/(-c*sin(e) + c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(57) = 114$.

Time = 0.12 (sec) , antiderivative size = 265, normalized size of antiderivative = 4.73

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{c - c\sin(e + fx)}\,dx =$$
$$-\frac{2\left(Ba\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1} - \frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - 2}{c - \frac{c\sin(fx+e)}{\cos(fx+e)+1} + \frac{c\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}} + \frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c}\right) + Aa\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c} - \frac{1}{c - \frac{c\sin(fx}{\cos(fx+}}\right.\right.}{f}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm=
"maxima")
```

output
```
-2*(B*a*((sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 - 2)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1) + c*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x
+ e)/(cos(f*x + e) + 1))/c) + A*a*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))
/c - 1/(c - c*sin(f*x + e)/(cos(f*x + e) + 1))) + B*a*(arctan(sin(f*x + e)
/(cos(f*x + e) + 1))/c - 1/(c - c*sin(f*x + e)/(cos(f*x + e) + 1))) - A*a/
(c - c*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(57) = 114$.

Time = 0.21 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.09

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{c - c\sin(e + fx)}\,dx$$
$$= -\frac{\frac{(Aa+2Ba)(fx+e)}{c} + \frac{2\left(2Aa\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 + 2Ba\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 - Ba\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right) + 2Aa + 3Ba\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3 - \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 + \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right) - 1\right)c}}{f}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm=
"giac")
```

output
```
-((A*a + 2*B*a)*(f*x + e)/c + 2*(2*A*a*tan(1/2*f*x + 1/2*e)^2 + 2*B*a*tan(
1/2*f*x + 1/2*e)^2 - B*a*tan(1/2*f*x + 1/2*e) + 2*A*a + 3*B*a)/((tan(1/2*f
*x + 1/2*e)^3 - tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e) - 1)*c))/f
```

Mupad [B] (verification not implemented)

Time = 34.69 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.98

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = \frac{(4\,A\,a+4\,B\,a)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-2\,B\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+4\,A\,a+6\,B\,a}{f\,\left(-c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+c\right)} - \frac{A\,a\,f\,x+2\,B\,a\,f\,x}{c\,f}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x)),x)
```

output
```
(4*A*a + 6*B*a + tan(e/2 + (f*x)/2)^2*(4*A*a + 4*B*a) - 2*B*a*tan(e/2 + (f
*x)/2))/(f*(c - c*tan(e/2 + (f*x)/2) + c*tan(e/2 + (f*x)/2)^2 - c*tan(e/2
+ (f*x)/2)^3)) - (A*a*f*x + 2*B*a*f*x)/(c*f)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.95

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = \frac{a\big(\cos\left(fx+e\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)b-\cos\left(fx+e\right)b-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)afx-4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)a-2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)b}{cf\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}$$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

output
```
(a*(cos(e + f*x)*tan((e + f*x)/2)*b - cos(e + f*x)*b - tan((e + f*x)/2)*a*
f*x - 4*tan((e + f*x)/2)*a - 2*tan((e + f*x)/2)*b*f*x - 4*tan((e + f*x)/2)
*b + a*f*x + 2*b*f*x))/(c*f*(tan((e + f*x)/2) - 1))
```

3.22 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 72

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \frac{aBx}{c^2}-\frac{a(A+7B)\cos(e+fx)}{3c^2f(1-\sin(e+fx))}+\frac{2a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^2}$$

output
```
a*B*x/c^2-1/3*a*(A+7*B)*cos(f*x+e)/c^2/f/(1-sin(f*x+e))+2/3*a*(A+B)*cos(f*
x+e)/f/(c-c*sin(f*x+e))^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 160 vs. 2(72) = 144.

Time = 6.61 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.22

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = -\frac{a\left(-9Bfx\cos\left(\frac{fx}{2}\right)-6(A+3B)\cos\left(e+\frac{fx}{2}\right)+2A\cos\left(e+\frac{3fx}{2}\right)+14B\cos\left(e+\frac{3fx}{2}\right)+3Bfx\cos\left(2e\right.\right.}{6c^2f\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\right.}$$

input
```
Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^2,x]
```

output
```
-1/6*(a*(-9*B*f*x*Cos[(f*x)/2] - 6*(A + 3*B)*Cos[e + (f*x)/2] + 2*A*Cos[e
+ (3*f*x)/2] + 14*B*Cos[e + (3*f*x)/2] + 3*B*f*x*Cos[2*e + (3*f*x)/2] + 24
*B*Sin[(f*x)/2] + 9*B*f*x*Sin[e + (f*x)/2] + 3*B*f*x*Sin[e + (3*f*x)/2]))/
(c^2*f*(Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.265, Rules used = {3042, 3446, 3042, 3336, 25, 3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}dx$$

$$\Big\downarrow \text{3446}$$

$$ac\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$ac\int \frac{\cos(e+fx)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}dx$$

$$\Big\downarrow \text{3336}$$

$$ac\left(\frac{\int -\frac{(A+4B)c+3B\sin(e+fx)c}{c-c\sin(e+fx)}dx}{3c^3}+\frac{2(A+B)\cos(e+fx)}{3cf(c-c\sin(e+fx))^2}\right)$$

$$\Big\downarrow \text{25}$$

$$ac\left(\frac{2(A+B)\cos(e+fx)}{3cf(c-c\sin(e+fx))^2}-\frac{\int\frac{(A+4B)c+3B\sin(e+fx)c}{c-c\sin(e+fx)}dx}{3c^3}\right)$$

$$\Big\downarrow \text{3042}$$

$$ac\left(\frac{2(A+B)\cos(e+fx)}{3cf(c-c\sin(e+fx))^2}-\frac{\int\frac{(A+4B)c+3B\sin(e+fx)c}{c-c\sin(e+fx)}dx}{3c^3}\right)$$

$$\Big\downarrow \text{3214}$$

$$ac\left(\frac{2(A+B)\cos(e+fx)}{3cf(c-c\sin(e+fx))^2}-\frac{c(A+7B)\int\frac{1}{c-c\sin(e+fx)}dx-3Bx}{3c^3}\right)$$

$$\Big\downarrow \text{3042}$$

$$ac\left(\frac{2(A+B)\cos(e+fx)}{3cf(c-c\sin(e+fx))^2}-\frac{c(A+7B)\int\frac{1}{c-c\sin(e+fx)}dx-3Bx}{3c^3}\right)$$

$$\Big\downarrow \text{3127}$$

$$ac\left(\frac{2(A+B)\cos(e+fx)}{3cf(c-c\sin(e+fx))^2}-\frac{\frac{c(A+7B)\cos(e+fx)}{f(c-c\sin(e+fx))}-3Bx}{3c^3}\right)$$

input
```
Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^2,x]
```

output
```
a*c*((2*(A + B)*Cos[e + f*x])/(3*c*f*(c - c*Sin[e + f*x])^2) - (-3*B*x + (
(A + 7*B)*c*Cos[e + f*x])/(f*(c - c*Sin[e + f*x])))/(3*c^3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3336
```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Simp[1/(b^
3*(2*m + 3))   Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*
(2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2
- b^2, 0] && LtQ[m, -3/2]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.11

method	result
risch	$\frac{aBx}{c^2}-\frac{2(3Aa\,\mathrm{e}^{2i(fx+e)}-12iBa\,\mathrm{e}^{i(fx+e)}+9Ba\,\mathrm{e}^{2i(fx+e)}-Aa-7Ba)}{3(\mathrm{e}^{i(fx+e)}-i)^3 f c^2}$
derivativedivides	$\frac{2a\left(-\frac{A-B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{4A+4B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{4A+4B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}+B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f c^2}$
default	$\frac{2a\left(-\frac{A-B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{4A+4B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{4A+4B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}+B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f c^2}$
parallelrisch	$-\frac{2a\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3 xfB}{2}+(\frac{3}{2}fxB+A-B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+B\left(-\frac{3fx}{2}+4\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{fxB}{2}+\frac{A}{3}-\frac{5B}{3}\right)}{f c^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}$
norman	$\frac{\frac{axB\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{c}-\frac{2Aa-10Ba}{3cf}-\frac{axB}{c}-\frac{16Ba\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{fc}-\frac{8Ba\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{fc}-\frac{8Ba\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{cf}-\frac{(2Aa-2Ba)\tan\left(\frac{fx}{2}+\frac{e}{2}\right.}{cf}}{}$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x,method=_RETURNV
ERBOSE)
```

output
```
a*B*x/c^2-2/3*(3*A*a*exp(2*I*(f*x+e))-12*I*B*a*exp(I*(f*x+e))+9*B*a*exp(2*
I*(f*x+e))-A*a-7*B*a)/(exp(I*(f*x+e))-I)^3/f/c^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(67) = 134$.

Time = 0.08 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.25

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = -\frac{6\,Bafx-(3\,Bafx+(A+7\,B)a)\cos(fx+e)^2+2\,(A+B)a+(3\,Bafx+(A-5\,B)a)\cos(fx+e)}{3\left(c^2f\cos(fx+e)^2-c^2f\cos(fx+e)-2\,c^2f+(c^2f\cos\right.}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorith
m="fricas")
```

output
```
-1/3*(6*B*a*f*x - (3*B*a*f*x + (A + 7*B)*a)*cos(f*x + e)^2 + 2*(A + B)*a +
 (3*B*a*f*x + (A - 5*B)*a)*cos(f*x + e) - (6*B*a*f*x - 2*(A + B)*a + (3*B*
a*f*x - (A + 7*B)*a)*cos(f*x + e))*sin(f*x + e))/(c^2*f*cos(f*x + e)^2 - c
^2*f*cos(f*x + e) - 2*c^2*f + (c^2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 700 vs. $2(65) = 130$.

Time = 2.23 (sec) , antiderivative size = 700, normalized size of antiderivative = 9.72

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{(c - c\sin(e + fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**2,x)
```

output
```
Piecewise((-6*A*a*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c*
*2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 2*A*a/(
3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan
(e/2 + f*x/2) - 3*c**2*f) + 3*B*a*f*x*tan(e/2 + f*x/2)**3/(3*c**2*f*tan(e/
2 + f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) -
 3*c**2*f) - 9*B*a*f*x*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**3 -
 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 9*
B*a*f*x*tan(e/2 + f*x/2)/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2
+ f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 3*B*a*f*x/(3*c**2*f*
tan(e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*
x/2) - 3*c**2*f) + 6*B*a*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**3
 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) -
24*B*a*tan(e/2 + f*x/2)/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2 +
 f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 10*B*a/(3*c**2*f*tan(
e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2)
 - 3*c**2*f), Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)/(-c*sin(e) + c)*
*2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(67) = 134$.

Time = 0.13 (sec) , antiderivative size = 456, normalized size of antiderivative = 6.33

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \frac{2\left(Ba\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}-\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-4}{c^2-\frac{3c^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3c^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{c^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^2}\right)-\frac{Aa\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-2\cdots\right.}{c^2-\frac{3c^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3c^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{c^2\sin\cdots}{(\cos(fx\cdots}}\cdots\right.}{3f}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorith
m="maxima")
```

output
```
2/3*(B*a*((9*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 - 4)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)
+ 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^2) - A*a*(3*sin(f*x + e)/(co
s(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2)/(c^2 - 3*c^2*
sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + A*a*(3*sin(f*x + e)/(cos(f*
x + e) + 1) - 1)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)
 + B*a*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 1)/(c^2 - 3*c^2*sin(f*x + e)/(
cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*
x + e)^3/(cos(f*x + e) + 1)^3))/f
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.21

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \frac{\frac{3(fx+e)Ba}{c^2}-\frac{2\left(3Aa\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-3Ba\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+12Ba\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+Aa-5Ba\right)}{c^2\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3}}{3f}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorith
m="giac")
```

output
```
1/3*(3*(f*x + e)*B*a/c^2 - 2*(3*A*a*tan(1/2*f*x + 1/2*e)^2 - 3*B*a*tan(1/2
*f*x + 1/2*e)^2 + 12*B*a*tan(1/2*f*x + 1/2*e) + A*a - 5*B*a)/(c^2*(tan(1/2
*f*x + 1/2*e) - 1)^3))/f
```

Mupad [B] (verification not implemented)

Time = 34.88 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.83

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \frac{B\,a\,x}{c^2} - \frac{\left(\frac{a\,(6\,A-6\,B+9\,B\,(e+f\,x))}{3} - 3\,B\,a\,(e+f\,x)\right)\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2 + \left(\frac{a\,(24\,B-9\,B\,(e+f\,x))}{3} + 3\,B\,a\,(e+f\,x)\right)\tan\left(\frac{e}{2}-\right.}{c^2\,f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-1\right)^3}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^2,x)
```

output
```
(B*a*x)/c^2 - ((a*(2*A - 10*B + 3*B*(e + f*x)))/3 + tan(e/2 + (f*x)/2)^2*(
(a*(6*A - 6*B + 9*B*(e + f*x)))/3 - 3*B*a*(e + f*x)) + tan(e/2 + (f*x)/2)*
((a*(24*B - 9*B*(e + f*x)))/3 + 3*B*a*(e + f*x)) - B*a*(e + f*x))/(c^2*f*(
tan(e/2 + (f*x)/2) - 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.15

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \frac{a\left(-2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3 a + 3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3 bfx + 2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3 b - 9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2 bfx - 6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}{3c^2 f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3 - 3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2 + 3\tan\left(\frac{fx}{2}+\right.\right.}$$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x)
```

output
```
(a*( - 2*tan((e + f*x)/2)**3*a + 3*tan((e + f*x)/2)**3*b*f*x + 2*tan((e +
f*x)/2)**3*b - 9*tan((e + f*x)/2)**2*b*f*x - 6*tan((e + f*x)/2)*a + 9*tan(
(e + f*x)/2)*b*f*x - 18*tan((e + f*x)/2)*b - 3*b*f*x + 8*b))/(3*c**2*f*(ta
n((e + f*x)/2)**3 - 3*tan((e + f*x)/2)**2 + 3*tan((e + f*x)/2) - 1))
```

3.23 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 104

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = \frac{2a(A+B)\cos(e+fx)}{5f(c-c\sin(e+fx))^3} - \frac{a(A+11B)c\cos(e+fx)}{15f\left(c^2-c^2\sin(e+fx)\right)^2} - \frac{a(A-4B)\cos(e+fx)}{15f\left(c^3-c^3\sin(e+fx)\right)}$$

output
```
2/5*a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^3-1/15*a*(A+11*B)*c*cos(f*x+e)/f
/(c^2-c^2*sin(f*x+e))^2-1/15*a*(A-4*B)*cos(f*x+e)/f/(c^3-c^3*sin(f*x+e))
```

Mathematica [A] (verified)

Time = 6.66 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.41

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = \frac{a\left(15(A-B)\cos\left(e+\frac{fx}{2}\right)-5(A-B)\cos\left(e+\frac{3fx}{2}\right)+5A\sin\left(\frac{fx}{2}\right)+25B\sin\left(\frac{fx}{2}\right)+15B\sin\left(2e+\frac{3fx}{2}\right)\right.}{30c^3f\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

input
```
Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^3,x]
```

output
```
(a*(15*(A - B)*Cos[e + (f*x)/2] - 5*(A - B)*Cos[e + (3*f*x)/2] + 5*A*Sin[(
f*x)/2] + 25*B*Sin[(f*x)/2] + 15*B*Sin[2*e + (3*f*x)/2] + A*Sin[2*e + (5*f
*x)/2] - 4*B*Sin[2*e + (5*f*x)/2]))/(30*c^3*f*(Cos[e/2] - Sin[e/2])*(Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])^5)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {3042, 3446, 3042, 3336, 25, 3042, 3229, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}dx$$

$$\Big\downarrow \text{3446}$$

$$ac\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$ac\int \frac{\cos(e+fx)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}dx$$

$$\Big\downarrow \text{3336}$$

$$ac\left(\frac{\int -\frac{(A+6B)c+5B\sin(e+fx)c}{(c-c\sin(e+fx))^2}dx}{5c^3}+\frac{2(A+B)\cos(e+fx)}{5cf(c-c\sin(e+fx))^3}\right)$$

$$\Big\downarrow \text{25}$$

$$ac\left(\frac{2(A+B)\cos(e+fx)}{5cf(c-c\sin(e+fx))^3}-\frac{\int \frac{(A+6B)c+5B\sin(e+fx)c}{(c-c\sin(e+fx))^2}dx}{5c^3}\right)$$

$$\Big\downarrow \text{3042}$$

$$ac\left(\frac{2(A+B)\cos(e+fx)}{5cf(c-c\sin(e+fx))^3}-\frac{\int\frac{(A+6B)c+5B\sin(e+fx)c}{(c-c\sin(e+fx))^2}dx}{5c^3}\right)$$

$$\Big\downarrow \text{ 3229}$$

$$ac\left(\frac{2(A+B)\cos(e+fx)}{5cf(c-c\sin(e+fx))^3}-\frac{\frac{1}{3}(A-4B)\int\frac{1}{c-c\sin(e+fx)}dx+\frac{c(A+11B)\cos(e+fx)}{3f(c-c\sin(e+fx))^2}}{5c^3}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$ac\left(\frac{2(A+B)\cos(e+fx)}{5cf(c-c\sin(e+fx))^3}-\frac{\frac{1}{3}(A-4B)\int\frac{1}{c-c\sin(e+fx)}dx+\frac{c(A+11B)\cos(e+fx)}{3f(c-c\sin(e+fx))^2}}{5c^3}\right)$$

$$\Big\downarrow \text{ 3127}$$

$$ac\left(\frac{2(A+B)\cos(e+fx)}{5cf(c-c\sin(e+fx))^3}-\frac{\frac{(A-4B)\cos(e+fx)}{3f(c-c\sin(e+fx))}+\frac{c(A+11B)\cos(e+fx)}{3f(c-c\sin(e+fx))^2}}{5c^3}\right)$$

input
```
Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^3,x]
```

output
```
a*c*((2*(A + B)*Cos[e + f*x])/(5*c*f*(c - c*Sin[e + f*x])^3) - (((A + 11*B
)*c*Cos[e + f*x])/(3*f*(c - c*Sin[e + f*x])^2) + ((A - 4*B)*Cos[e + f*x])/
(3*f*(c - c*Sin[e + f*x])))/(5*c^3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 3336
```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Simp[1/(b^
3*(2*m + 3))   Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*
(2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2
- b^2, 0] && LtQ[m, -3/2]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

method	result
parallelrisch	$-\frac{2\left(A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}+(-A+B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+\frac{(5A+B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{3}+\frac{(-A+B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3}+\frac{4A}{15}-\frac{B}{15}\right)a}{f\,c^{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}}$
derivativedivides	$\frac{2a\left(-\frac{14A+10B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{6A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{8A+8B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{16A+16B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}\right)}{f\,c^{3}}$
default	$\frac{2a\left(-\frac{14A+10B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{6A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{8A+8B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{16A+16B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}\right)}{f\,c^{3}}$
risch	$\frac{-\frac{2Aa\,e^{2i(fx+e)}}{3}-\frac{2iAa\,e^{i(fx+e)}}{3}-\frac{10Ba\,e^{2i(fx+e)}}{3}-2iBa\,e^{3i(fx+e)}+\frac{2iBa\,e^{i(fx+e)}}{3}+2Ba\,e^{4i(fx+e)}-\frac{2Aa}{15}+\frac{8Ba}{15}+2iAa\,e}{\left(e^{i(fx+e)}-i\right)^{5}f\,c^{3}}$
norman	$\frac{-\frac{8Aa-2Ba}{15fc}-\frac{2Aa\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}}{fc}+\frac{10(Aa-Ba)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3fc}+\frac{2(Aa-Ba)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}{fc}+\frac{(2Aa-2Ba)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3fc}-\frac{2(11Aa+}{}}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\right)^{2}c^{2}\left(\tan\left(\right.\right.}$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x,method=_RETURNV
ERBOSE)
```

output
```
-2*(A*tan(1/2*f*x+1/2*e)^4+(-A+B)*tan(1/2*f*x+1/2*e)^3+1/3*(5*A+B)*tan(1/2
*f*x+1/2*e)^2+1/3*(-A+B)*tan(1/2*f*x+1/2*e)+4/15*A-1/15*B)*a/f/c^3/(tan(1/
2*f*x+1/2*e)-1)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.76

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = -\frac{(A-4B)a\cos(fx+e)^3-(2A+7B)a\cos(fx+e)^2+3(A+B)a\cos(fx+e)+6(A+B)a+((A}{15\left(c^3f\cos(fx+e)^3+3c^3f\cos(fx+e)^2-2c^3f\cos(fx+e)-4c^3f-(c^3f\,c\right.}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorith
m="fricas")
```

output
```
-1/15*((A - 4*B)*a*cos(f*x + e)^3 - (2*A + 7*B)*a*cos(f*x + e)^2 + 3*(A +
B)*a*cos(f*x + e) + 6*(A + B)*a + ((A - 4*B)*a*cos(f*x + e)^2 + 3*(A + B)*
a*cos(f*x + e) + 6*(A + B)*a)*sin(f*x + e))/(c^3*f*cos(f*x + e)^3 + 3*c^3*
f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f - (c^3*f*cos(f*x + e)^2
- 2*c^3*f*cos(f*x + e) - 4*c^3*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1035 vs. $2(92) = 184$.

Time = 4.74 (sec) , antiderivative size = 1035, normalized size of antiderivative = 9.95

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{(c - c\sin(e + fx))^3} \, dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**3,x)
```

output
```
Piecewise((-30*A*a*tan(e/2 + f*x/2)**4/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75
*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*
tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 30*A*a*tan
(e/2 + f*x/2)**3/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/
2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 +
75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 50*A*a*tan(e/2 + f*x/2)**2/(15*c
**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan
(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*
x/2) - 15*c**3*f) + 10*A*a*tan(e/2 + f*x/2)/(15*c**3*f*tan(e/2 + f*x/2)**5
 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c*
*3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 8*A*a
/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3
*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/
2 + f*x/2) - 15*c**3*f) - 30*B*a*tan(e/2 + f*x/2)**3/(15*c**3*f*tan(e/2 +
f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3
 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f
) - 10*B*a*tan(e/2 + f*x/2)**2/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*
tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2
+ f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 10*B*a*tan(e/2 + f
*x/2)/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + ...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 737 vs. $2(101) = 202$.

Time = 0.06 (sec) , antiderivative size = 737, normalized size of antiderivative = 7.09

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{(c - c\sin(e + fx))^3} dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorith
m="maxima")
```

output
```
-2/15*(A*a*(20*sin(f*x + e)/(cos(f*x + e) + 1) - 40*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 15*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 - 7)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) +
10*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x +
e)^5/(cos(f*x + e) + 1)^5) - 3*A*a*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 5*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^
3 - 1)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^
3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) +
 1)^5) - 3*B*a*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/(c^3 - 5*c^3*
sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 2*B*a*(5*si
n(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1
)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
))/f
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.26

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = -\frac{2\left(15\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-15\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+15\,Ba\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+25\,Aa\tan\left(\frac{1}{2}\,fx+\right.}{15\,c^3f\left(\tan\left(\frac{1}{2}\,fx+\right.\right.}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorith
m="giac")
```

output
```
-2/15*(15*A*a*tan(1/2*f*x + 1/2*e)^4 - 15*A*a*tan(1/2*f*x + 1/2*e)^3 + 15*
B*a*tan(1/2*f*x + 1/2*e)^3 + 25*A*a*tan(1/2*f*x + 1/2*e)^2 + 5*B*a*tan(1/2
*f*x + 1/2*e)^2 - 5*A*a*tan(1/2*f*x + 1/2*e) + 5*B*a*tan(1/2*f*x + 1/2*e)
+ 4*A*a - B*a)/(c^3*f*(tan(1/2*f*x + 1/2*e) - 1)^5)
```

Mupad [B] (verification not implemented)

Time = 35.43 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.65

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = \frac{2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{11\,A\,a\,\cos(e+f\,x)}{2}-\frac{B\,a}{4}-\frac{41\,A\,a}{4}+\frac{B\,a\,\cos(e+f\,x)}{2}+5\,A\,a\,\sin\left(e+f\,x\right)-5\,B\,a\,\sin\left(e+f\,x\right)+\right.}{15\,c^3\,f\,\left(\frac{5\,\sqrt{2}\,\cos\left(\frac{3\,e}{2}-\frac{\pi}{4}+\frac{3\,f\,x}{2}\right)}{4}-\frac{5\,\sqrt{2}\,\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{f\,x}{2}\right)}{2}+\right.}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^3,x)
```

output
```
(2*cos(e/2 + (f*x)/2)*((11*A*a*cos(e + f*x))/2 - (B*a)/4 - (41*A*a)/4 + (B
*a*cos(e + f*x))/2 + 5*A*a*sin(e + f*x) - 5*B*a*sin(e + f*x) + (3*A*a*cos(
2*e + 2*f*x))/4 + (3*B*a*cos(2*e + 2*f*x))/4 - (5*A*a*sin(2*e + 2*f*x))/4
+ (5*B*a*sin(2*e + 2*f*x))/4))/(15*c^3*f*((5*2^(1/2)*cos((3*e)/2 - pi/4 +
(3*f*x)/2))/4 - (5*2^(1/2)*cos(e/2 + pi/4 + (f*x)/2))/2 + (2^(1/2)*cos((5*
e)/2 + pi/4 + (5*f*x)/2))/4))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.66

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$$
$$= \frac{2a\left(-3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5 a-15\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3 a-15\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3 b+5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2 a-5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2 b\right.}{15c^3f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5-5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+10\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3-10\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right.}$$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x)
```

output
```
(2*a*( - 3*tan((e + f*x)/2)**5*a - 15*tan((e + f*x)/2)**3*a - 15*tan((e +
f*x)/2)**3*b + 5*tan((e + f*x)/2)**2*a - 5*tan((e + f*x)/2)**2*b - 10*tan(
(e + f*x)/2)*a - 5*tan((e + f*x)/2)*b - a + b))/(15*c**3*f*(tan((e + f*x)/
2)**5 - 5*tan((e + f*x)/2)**4 + 10*tan((e + f*x)/2)**3 - 10*tan((e + f*x)/
2)**2 + 5*tan((e + f*x)/2) - 1))
```

3.24 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 142

$$\begin{aligned}
&\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx \\
&= \frac{2a(A+B)\cos(e+fx)}{7f(c-c\sin(e+fx))^4} - \frac{a(A+15B)\cos(e+fx)}{35cf(c-c\sin(e+fx))^3} \\
&\quad - \frac{a(2A-5B)\cos(e+fx)}{105f\left(c^2-c^2\sin(e+fx)\right)^2} - \frac{a(2A-5B)\cos(e+fx)}{105f\left(c^4-c^4\sin(e+fx)\right)}
\end{aligned}$$

output
```
2/7*a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^4-1/35*a*(A+15*B)*cos(f*x+e)/c/f
/(c-c*sin(f*x+e))^3-1/105*a*(2*A-5*B)*cos(f*x+e)/f/(c^2-c^2*sin(f*x+e))^2-
1/105*a*(2*A-5*B)*cos(f*x+e)/f/(c^4-c^4*sin(f*x+e))
```

Mathematica [A] (verified)

Time = 6.84 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.23

$$\begin{aligned}
&\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx \\
&= \frac{a\left(35(4A-B)\cos\left(e+\frac{fx}{2}\right)-42A\cos\left(e+\frac{3fx}{2}\right)+2A\cos\left(3e+\frac{7fx}{2}\right)-5B\cos\left(3e+\frac{7fx}{2}\right)+70A\sin\left(\frac{fx}{2}\right.\right.}{420c^4f\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx\right.\right.}
\end{aligned}$$

input
```
Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^4,x]
```

output
```
(a*(35*(4*A - B)*Cos[e + (f*x)/2] - 42*A*Cos[e + (3*f*x)/2] + 2*A*Cos[3*e
+ (7*f*x)/2] - 5*B*Cos[3*e + (7*f*x)/2] + 70*A*Sin[(f*x)/2] + 140*B*Sin[(f
*x)/2] + 105*B*Sin[2*e + (3*f*x)/2] + 14*A*Sin[2*e + (5*f*x)/2] - 35*B*Sin
[2*e + (5*f*x)/2]))/(420*c^4*f*(Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])^7)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {3042, 3446, 3042, 3336, 25, 3042, 3229, 3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}dx$$

$$\Big\downarrow \text{3446}$$

$$ac\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}dx$$

$$\Big\downarrow \text{3042}$$

$$ac\int \frac{\cos(e+fx)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}dx$$

$$\Big\downarrow \text{3336}$$

$$ac\left(\frac{\int -\frac{(A+8B)c+7B\sin(e+fx)c}{(c-c\sin(e+fx))^3}dx}{7c^3}+\frac{2(A+B)\cos(e+fx)}{7cf(c-c\sin(e+fx))^4}\right)$$

$$\Big\downarrow \text{25}$$

$$ac\left(\frac{2(A+B)\cos(e+fx)}{7cf(c-c\sin(e+fx))^4}-\frac{\int\frac{(A+8B)c+7B\sin(e+fx)c}{(c-c\sin(e+fx))^3}dx}{7c^3}\right)$$

$\Big\downarrow$ 3042

$$ac\left(\frac{2(A+B)\cos(e+fx)}{7cf(c-c\sin(e+fx))^4}-\frac{\int\frac{(A+8B)c+7B\sin(e+fx)c}{(c-c\sin(e+fx))^3}dx}{7c^3}\right)$$

$\Big\downarrow$ 3229

$$ac\left(\frac{2(A+B)\cos(e+fx)}{7cf(c-c\sin(e+fx))^4}-\frac{\frac{1}{5}(2A-5B)\int\frac{1}{(c-c\sin(e+fx))^2}dx+\frac{c(A+15B)\cos(e+fx)}{5f(c-c\sin(e+fx))^3}}{7c^3}\right)$$

$\Big\downarrow$ 3042

$$ac\left(\frac{2(A+B)\cos(e+fx)}{7cf(c-c\sin(e+fx))^4}-\frac{\frac{1}{5}(2A-5B)\int\frac{1}{(c-c\sin(e+fx))^2}dx+\frac{c(A+15B)\cos(e+fx)}{5f(c-c\sin(e+fx))^3}}{7c^3}\right)$$

$\Big\downarrow$ 3129

$$ac\left(\frac{2(A+B)\cos(e+fx)}{7cf(c-c\sin(e+fx))^4}-\frac{\frac{1}{5}(2A-5B)\left(\frac{\int\frac{1}{c-c\sin(e+fx)}dx}{3c}+\frac{\cos(e+fx)}{3f(c-c\sin(e+fx))^2}\right)+\frac{c(A+15B)\cos(e+fx)}{5f(c-c\sin(e+fx))^3}}{7c^3}\right)$$

$\Big\downarrow$ 3042

$$ac\left(\frac{2(A+B)\cos(e+fx)}{7cf(c-c\sin(e+fx))^4}-\frac{\frac{1}{5}(2A-5B)\left(\frac{\int\frac{1}{c-c\sin(e+fx)}dx}{3c}+\frac{\cos(e+fx)}{3f(c-c\sin(e+fx))^2}\right)+\frac{c(A+15B)\cos(e+fx)}{5f(c-c\sin(e+fx))^3}}{7c^3}\right)$$

$\Big\downarrow$ 3127

$$ac\left(\frac{2(A+B)\cos(e+fx)}{7cf(c-c\sin(e+fx))^4}-\frac{\frac{c(A+15B)\cos(e+fx)}{5f(c-c\sin(e+fx))^3}+\frac{1}{5}(2A-5B)\left(\frac{\cos(e+fx)}{3cf(c-c\sin(e+fx))}+\frac{\cos(e+fx)}{3f(c-c\sin(e+fx))^2}\right)}{7c^3}\right)$$

input
```
Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^4,x]
```

output
```
a*c*((2*(A + B)*Cos[e + f*x])/(7*c*f*(c - c*Sin[e + f*x])^4) - (((A + 15*B
)*c*Cos[e + f*x])/(5*f*(c - c*Sin[e + f*x])^3) + ((2*A - 5*B)*(Cos[e + f*x
]/(3*f*(c - c*Sin[e + f*x])^2) + Cos[e + f*x]/(3*c*f*(c - c*Sin[e + f*x]))
))/5)/(7*c^3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 3336
```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Simp[1/(b^
3*(2*m + 3))   Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*
(2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2
- b^2, 0] && LtQ[m, -3/2]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.92

method	result
parallelrisch	$-\frac{2\left(A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}+(-2A+B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}+\frac{(13A-B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{3}+\frac{2(-5A+2B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+\frac{13A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{5}\cdots}{f\,c^{4}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}$
risch	$-\frac{2ia\left(140iA\,e^{4i(fx+e)}-35iB\,e^{4i(fx+e)}+105B\,e^{5i(fx+e)}-42iA\,e^{2i(fx+e)}-70A\,e^{3i(fx+e)}-140B\,e^{3i(fx+e)}+2iA-14A\cdots\right.}{105f\,c^{4}\left(e^{i(fx+e)}-i\right)^{7}}$
derivativedivides	$\frac{2a\left(-\frac{28A+14B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{68A+60B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{56A+40B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{8A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{48A+\cdots}{6\left(\tan\left(\frac{fx}{2}\cdots\right.\right.}\cdots\right.}{f\,c^{4}}$
default	$\frac{2a\left(-\frac{28A+14B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{68A+60B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{56A+40B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{8A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{48A+\cdots}{6\left(\tan\left(\frac{fx}{2}\cdots\right.\right.}\cdots\right.}{f\,c^{4}}$
norman	$\frac{(4Aa-2Ba)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}{fc}-\frac{46Aa-10Ba}{105fc}-\frac{2Aa\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{fc}+\frac{(16Aa-10Ba)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{15fc}+\frac{2(22Aa-10Ba)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}{3fc}-\cdots$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x,method=_RETURNV
ERBOSE)
```

output
```
-2*(A*tan(1/2*f*x+1/2*e)^6+(-2*A+B)*tan(1/2*f*x+1/2*e)^5+1/3*(13*A-B)*tan(
1/2*f*x+1/2*e)^4+2/3*(-5*A+2*B)*tan(1/2*f*x+1/2*e)^3+13/5*A*tan(1/2*f*x+1/
2*e)^2+1/3*(-8/5*A+B)*tan(1/2*f*x+1/2*e)+23/105*A-1/21*B)*a/f/c^4/(tan(1/2
*f*x+1/2*e)-1)^7
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.77

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$$
$$= \frac{(2A-5B)a\cos(fx+e)^4+4(2A-5B)a\cos(fx+e)^3-3(3A+10B)a\cos(fx+e)^2+15(A+B)}{105\left(c^4f\cos(fx+e)^4-3c^4f\cos(fx+e)^3-8c^4f\cos(fx+e)^2+\right.}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorith
m="fricas")
```

output
```
1/105*((2*A - 5*B)*a*cos(f*x + e)^4 + 4*(2*A - 5*B)*a*cos(f*x + e)^3 - 3*(
3*A + 10*B)*a*cos(f*x + e)^2 + 15*(A + B)*a*cos(f*x + e) + 30*(A + B)*a -
((2*A - 5*B)*a*cos(f*x + e)^3 - 3*(2*A - 5*B)*a*cos(f*x + e)^2 - 15*(A + B
)*a*cos(f*x + e) - 30*(A + B)*a)*sin(f*x + e))/(c^4*f*cos(f*x + e)^4 - 3*c
^4*f*cos(f*x + e)^3 - 8*c^4*f*cos(f*x + e)^2 + 4*c^4*f*cos(f*x + e) + 8*c^
4*f + (c^4*f*cos(f*x + e)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*c^4*f*cos(f*x + e
) - 8*c^4*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1831 vs. 2(124) = 248.

Time = 9.54 (sec) , antiderivative size = 1831, normalized size of antiderivative = 12.89

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**4,x)
```

output
```
Piecewise((-210*A*a*tan(e/2 + f*x/2)**6/(105*c**4*f*tan(e/2 + f*x/2)**7 -
735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c*
*4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*t
an(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) + 420*A*a*t
an(e/2 + f*x/2)**5/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 +
f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)
**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 +
735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 910*A*a*tan(e/2 + f*x/2)**4/(1
05*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4
*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan
(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 +
f*x/2) - 105*c**4*f) + 700*A*a*tan(e/2 + f*x/2)**3/(105*c**4*f*tan(e/2 + f
*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**
5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 22
05*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f)
- 546*A*a*tan(e/2 + f*x/2)**2/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f
*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e
/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f
*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) + 112*A*a*tan(e/2 + f
*x/2)/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 ...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1080 vs. $2(138) = 276$.

Time = 0.07 (sec) , antiderivative size = 1080, normalized size of antiderivative = 7.61

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{(c - c\sin(e + fx))^4}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorith
m="maxima")
```

output
```
2/105*(A*a*(91*sin(f*x + e)/(cos(f*x + e) + 1) - 168*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 280*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 175*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 13)/(
c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f
*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^
5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*
x + e) + 1)^7) + B*a*(91*sin(f*x + e)/(cos(f*x + e) + 1) - 168*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 280*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 175*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)
^5 - 13)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35
*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x
+ e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)
^7/(cos(f*x + e) + 1)^7) - 3*A*a*(49*sin(f*x + e)/(cos(f*x + e) + 1) - 147
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 210*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 - 210*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f
*x + e) + 1)^5 - 35*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 12)/(c^4 - 7*c^4
*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^...
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.24

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx =$$

$$-\frac{2\left(105\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-210\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+105\,Ba\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{5}+455\,Aa\tan\left(\frac{1}{2}\,f\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorith
m="giac")
```

output
```
-2/105*(105*A*a*tan(1/2*f*x + 1/2*e)^6 - 210*A*a*tan(1/2*f*x + 1/2*e)^5 +
105*B*a*tan(1/2*f*x + 1/2*e)^5 + 455*A*a*tan(1/2*f*x + 1/2*e)^4 - 35*B*a*t
an(1/2*f*x + 1/2*e)^4 - 350*A*a*tan(1/2*f*x + 1/2*e)^3 + 140*B*a*tan(1/2*f
*x + 1/2*e)^3 + 273*A*a*tan(1/2*f*x + 1/2*e)^2 - 56*A*a*tan(1/2*f*x + 1/2*
e) + 35*B*a*tan(1/2*f*x + 1/2*e) + 23*A*a - 5*B*a)/(c^4*f*(tan(1/2*f*x + 1
/2*e) - 1)^7)
```

Mupad [B] (verification not implemented)

Time = 35.77 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.61

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx = -\frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{15\,B\,a}{4}-\frac{171\,A\,a}{2}+\frac{353\,A\,a\,\cos(e+f\,x)}{8}+\frac{5\,B\,a\,\cos(e+f\,x)}{4}+\frac{595\,A\,a\,\sin(e+f\,x)}{8}-35\,B\,a\,\sin\left(e+f\,\dots\right.\right.}{105\,c^4\,f\left(\frac{35\,\sqrt{2}\,\cos\left(\frac{e}{2}+\frac{\pi}{4}+\frac{f\,x}{2}\right)}{8}-\dots\right.}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^4,x)
```

output
```
-(2*cos(e/2 + (f*x)/2)*((15*B*a)/4 - (171*A*a)/2 + (353*A*a*cos(e + f*x))/
8 + (5*B*a*cos(e + f*x))/4 + (595*A*a*sin(e + f*x))/8 - 35*B*a*sin(e + f*x
) + (43*A*a*cos(2*e + 2*f*x))/2 - (25*A*a*cos(3*e + 3*f*x))/8 - (5*B*a*cos
(2*e + 2*f*x))/4 + (5*B*a*cos(3*e + 3*f*x))/4 - (77*A*a*sin(2*e + 2*f*x))/
4 - (21*A*a*sin(3*e + 3*f*x))/8 + (35*B*a*sin(2*e + 2*f*x))/4))/(105*c^4*f
*((35*2^(1/2)*cos(e/2 + pi/4 + (f*x)/2))/8 - (21*2^(1/2)*cos((3*e)/2 - pi/
4 + (3*f*x)/2))/8 - (7*2^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2))/8 + (2^(1/
2)*cos((7*e)/2 - pi/4 + (7*f*x)/2))/8))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.71

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$$
$$= \frac{2a\left(-15\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7 a-105\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5 a-105\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5 b+70\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4 a+35\tan\left(\frac{fx}{2}+\right.\right.}{105c^4 f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7-7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+21\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5\right.}$$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x)
```

output
```
(2*a*( - 15*tan((e + f*x)/2)**7*a - 105*tan((e + f*x)/2)**5*a - 105*tan((e
 + f*x)/2)**5*b + 70*tan((e + f*x)/2)**4*a + 35*tan((e + f*x)/2)**4*b - 17
5*tan((e + f*x)/2)**3*a - 140*tan((e + f*x)/2)**3*b + 42*tan((e + f*x)/2)*
*2*a - 49*tan((e + f*x)/2)*a - 35*tan((e + f*x)/2)*b - 8*a + 5*b))/(105*c*
*4*f*(tan((e + f*x)/2)**7 - 7*tan((e + f*x)/2)**6 + 21*tan((e + f*x)/2)**5
 - 35*tan((e + f*x)/2)**4 + 35*tan((e + f*x)/2)**3 - 21*tan((e + f*x)/2)**
2 + 7*tan((e + f*x)/2) - 1))
```

3.25 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 176

$$\begin{aligned}
&\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx \\
&= \frac{2a(A+B)\cos(e+fx)}{9f(c-c\sin(e+fx))^5} - \frac{a(A+19B)\cos(e+fx)}{63cf(c-c\sin(e+fx))^4} - \frac{a(A-2B)c\cos(e+fx)}{105f\,(c^2-c^2\sin(e+fx))^3} \\
&\quad - \frac{2a(A-2B)c\cos(e+fx)}{315f\,(c^3-c^3\sin(e+fx))^2} - \frac{2a(A-2B)\cos(e+fx)}{315f\,(c^5-c^5\sin(e+fx))}
\end{aligned}$$

output

```
2/9*a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^5-1/63*a*(A+19*B)*cos(f*x+e)/c/f
/(c-c*sin(f*x+e))^4-1/105*a*(A-2*B)*c*cos(f*x+e)/f/(c^2-c^2*sin(f*x+e))^3-
2/315*a*(A-2*B)*c*cos(f*x+e)/f/(c^3-c^3*sin(f*x+e))^2-2/315*a*(A-2*B)*cos(
f*x+e)/f/(c^5-c^5*sin(f*x+e))
```

Mathematica [A] (verified)

Time = 6.83 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.14

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx$$
$$= \frac{a\left(315A\cos\left(e+\frac{fx}{2}\right)-42(2A+B)\cos\left(e+\frac{3fx}{2}\right)+9A\cos\left(3e+\frac{7fx}{2}\right)-18B\cos\left(3e+\frac{7fx}{2}\right)+189A\sin\left(\right.\right.}{1260c^5f\left(\cos\left(\frac{e}{2}\right)-\mathrm{s}\right.}$$

input
```
Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^5,x]
```

output
```
(a*(315*A*Cos[e + (f*x)/2] - 42*(2*A + B)*Cos[e + (3*f*x)/2] + 9*A*Cos[3*e
 + (7*f*x)/2] - 18*B*Cos[3*e + (7*f*x)/2] + 189*A*Sin[(f*x)/2] + 252*B*Sin
[(f*x)/2] + 210*B*Sin[2*e + (3*f*x)/2] + 36*A*Sin[2*e + (5*f*x)/2] - 72*B*
Sin[2*e + (5*f*x)/2] - A*Sin[4*e + (9*f*x)/2] + 2*B*Sin[4*e + (9*f*x)/2]))
/(1260*c^5*f*(Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9
)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.99, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.382, Rules used = {3042, 3446, 3042, 3336, 25, 3042, 3229, 3042, 3129, 3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}dx$$
$$\Big\downarrow \text{3446}$$
$$ac\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}dx$$

$$\Big\downarrow \text{3042}$$

$$ac\int \frac{\cos(e+fx)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}dx$$

$$\Big\downarrow \text{3336}$$

$$ac\left(\frac{\int -\frac{(A+10B)c+9B\sin(e+fx)c}{(c-c\sin(e+fx))^4}dx}{9c^3}+\frac{2(A+B)\cos(e+fx)}{9cf(c-c\sin(e+fx))^5}\right)$$

$$\Big\downarrow \text{25}$$

$$ac\left(\frac{2(A+B)\cos(e+fx)}{9cf(c-c\sin(e+fx))^5}-\frac{\int \frac{(A+10B)c+9B\sin(e+fx)c}{(c-c\sin(e+fx))^4}dx}{9c^3}\right)$$

$$\Big\downarrow \text{3042}$$

$$ac\left(\frac{2(A+B)\cos(e+fx)}{9cf(c-c\sin(e+fx))^5}-\frac{\int \frac{(A+10B)c+9B\sin(e+fx)c}{(c-c\sin(e+fx))^4}dx}{9c^3}\right)$$

$$\Big\downarrow \text{3229}$$

$$ac\left(\frac{2(A+B)\cos(e+fx)}{9cf(c-c\sin(e+fx))^5}-\frac{\frac{3}{7}(A-2B)\int \frac{1}{(c-c\sin(e+fx))^3}dx+\frac{c(A+19B)\cos(e+fx)}{7f(c-c\sin(e+fx))^4}}{9c^3}\right)$$

$$\Big\downarrow \text{3042}$$

$$ac\left(\frac{2(A+B)\cos(e+fx)}{9cf(c-c\sin(e+fx))^5}-\frac{\frac{3}{7}(A-2B)\int \frac{1}{(c-c\sin(e+fx))^3}dx+\frac{c(A+19B)\cos(e+fx)}{7f(c-c\sin(e+fx))^4}}{9c^3}\right)$$

$$\Big\downarrow \text{3129}$$

$$ac\left(\frac{2(A+B)\cos(e+fx)}{9cf(c-c\sin(e+fx))^5}-\frac{\frac{3}{7}(A-2B)\left(\frac{2\int \frac{1}{(c-c\sin(e+fx))^2}dx}{5c}+\frac{\cos(e+fx)}{5f(c-c\sin(e+fx))^3}\right)+\frac{c(A+19B)\cos(e+fx)}{7f(c-c\sin(e+fx))^4}}{9c^3}\right)$$

$$\Big\downarrow \text{3042}$$

$$ac\left(\frac{2(A+B)\cos(e+fx)}{9cf(c-c\sin(e+fx))^5}-\frac{\frac{3}{7}(A-2B)\left(\frac{2\int \frac{1}{(c-c\sin(e+fx))^2}dx}{5c}+\frac{\cos(e+fx)}{5f(c-c\sin(e+fx))^3}\right)+\frac{c(A+19B)\cos(e+fx)}{7f(c-c\sin(e+fx))^4}}{9c^3}\right)$$

$\Big\downarrow$ 3129

$$ac\left(\frac{2(A+B)\cos(e+fx)}{9cf(c-c\sin(e+fx))^5}-\frac{\frac{3}{7}(A-2B)\left(\frac{2\left(\frac{\int\frac{1}{c-c\sin(e+fx)}dx}{3c}+\frac{\cos(e+fx)}{3f(c-c\sin(e+fx))^2}\right)}{5c}+\frac{\cos(e+fx)}{5f(c-c\sin(e+fx))^3}\right)+\frac{c(A+19B)\,c}{7f(c-c\sin(}}{9c^3}\right.$$

$\Big\downarrow$ 3042

$$ac\left(\frac{2(A+B)\cos(e+fx)}{9cf(c-c\sin(e+fx))^5}-\frac{\frac{3}{7}(A-2B)\left(\frac{2\left(\frac{\int\frac{1}{c-c\sin(e+fx)}dx}{3c}+\frac{\cos(e+fx)}{3f(c-c\sin(e+fx))^2}\right)}{5c}+\frac{\cos(e+fx)}{5f(c-c\sin(e+fx))^3}\right)+\frac{c(A+19B)\,c}{7f(c-c\sin(}}{9c^3}\right.$$

$\Big\downarrow$ 3127

$$ac\left(\frac{2(A+B)\cos(e+fx)}{9cf(c-c\sin(e+fx))^5}-\frac{\frac{c(A+19B)\cos(e+fx)}{7f(c-c\sin(e+fx))^4}+\frac{3}{7}(A-2B)\left(\frac{\cos(e+fx)}{5f(c-c\sin(e+fx))^3}+\frac{2\left(\frac{\cos(e+fx)}{3cf(c-c\sin(e+fx))}+\frac{\cos(e+fx)}{3f(c-c\sin(e+}\right.}{5c}\right.}{9c^3}\right.$$

input
```
Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^5,x]
```

output
```
a*c*((2*(A + B)*Cos[e + f*x])/(9*c*f*(c - c*Sin[e + f*x])^5) - (((A + 19*B
)*c*Cos[e + f*x])/(7*f*(c - c*Sin[e + f*x])^4) + (3*(A - 2*B)*(Cos[e + f*x
]/(5*f*(c - c*Sin[e + f*x])^3) + (2*(Cos[e + f*x]/(3*f*(c - c*Sin[e + f*x]
)^2) + Cos[e + f*x]/(3*c*f*(c - c*Sin[e + f*x]))))/(5*c)))/7)/(9*c^3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 3336
```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Simp[1/(b^
3*(2*m + 3))   Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*
(2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2
- b^2, 0] && LtQ[m, -3/2]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.44 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.94

method	result
risch	$-\frac{4(-42iBa\,e^{3i(fx+e)}+Aa-2Ba-189Aa\,e^{4i(fx+e)}+210Ba\,e^{6i(fx+e)}-252Ba\,e^{4i(fx+e)}-36Aa\,e^{2i(fx+e)}+72Ba\,e^{2i(}}{315(e^{i(fx+e)}-i)^9 f c^5}$
parallelrisch	$-\frac{2\left(A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8+(-3A+B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7+\left(\frac{25A}{3}-B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+(-11A+3B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5+\frac{(61A-7B)\,\mathrm{ta}}{}\right.}{f c^5\left(\tan\left(\frac{fx}{2}+\right.\right.}$
derivativedivides	$\frac{2a\left(-\frac{46A+18B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{128A+72B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{236A+168B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{10A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{296A+}{6\left(\tan\left(\frac{fx}{2}\right.\right.}\right.}{f c^5}$
default	$\frac{2a\left(-\frac{46A+18B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{128A+72B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{236A+168B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{10A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{296A+}{6\left(\tan\left(\frac{fx}{2}\right.\right.}\right.}{f c^5}$
norman	$\frac{\frac{(34Aa-10Ba)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{fc}-\frac{116Aa-22Ba}{315fc}-\frac{2Aa\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{12}}{fc}+\frac{2(3Aa-Ba)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}{fc}-\frac{2(31Aa-3Ba)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{3fc}}{}$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x,method=_RETURNV
ERBOSE)
```

output
```
-4/315*(-42*I*B*a*exp(3*I*(f*x+e))+A*a-2*B*a-189*A*a*exp(4*I*(f*x+e))+210*
B*a*exp(6*I*(f*x+e))-252*B*a*exp(4*I*(f*x+e))-36*A*a*exp(2*I*(f*x+e))+72*B
*a*exp(2*I*(f*x+e))+315*I*A*a*exp(5*I*(f*x+e))-84*I*A*a*exp(3*I*(f*x+e))+9
*I*A*a*exp(I*(f*x+e))-18*I*B*a*exp(I*(f*x+e)))/(exp(I*(f*x+e))-I)^9/f/c^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.73

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{(c - c\sin(e + fx))^5}\,dx = -\frac{2\,(A - 2\,B)a\cos\left(fx + e\right)^5 - 8\,(A - 2\,B)a\cos\left(fx + e\right)^4 - 25\,(A - 2\,B)a\cos\left(fx + e\right)^3 + 5\,(4\,A + 13\,}{315\,\left(c^5f\cos\left(fx + e\right)^5 + 5\,c^5f\cos\left(fx + e\right)^4 - 8\,c^5f\cos\left(fx\right.\right.}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorith
m="fricas")
```

output
```
-1/315*(2*(A - 2*B)*a*cos(f*x + e)^5 - 8*(A - 2*B)*a*cos(f*x + e)^4 - 25*(
A - 2*B)*a*cos(f*x + e)^3 + 5*(4*A + 13*B)*a*cos(f*x + e)^2 - 35*(A + B)*a
*cos(f*x + e) - 70*(A + B)*a + (2*(A - 2*B)*a*cos(f*x + e)^4 + 10*(A - 2*B
)*a*cos(f*x + e)^3 - 15*(A - 2*B)*a*cos(f*x + e)^2 - 35*(A + B)*a*cos(f*x
+ e) - 70*(A + B)*a)*sin(f*x + e))/(c^5*f*cos(f*x + e)^5 + 5*c^5*f*cos(f*x
 + e)^4 - 8*c^5*f*cos(f*x + e)^3 - 20*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f
*x + e) + 16*c^5*f - (c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*x + e)^3 - 12*c
^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3232 vs. 2(160) = 320.

Time = 17.42 (sec) , antiderivative size = 3232, normalized size of antiderivative = 18.36

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{(c - c\sin(e + fx))^5}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**5,x)
```

output
```
Piecewise((-630*A*a*tan(e/2 + f*x/2)**8/(315*c**5*f*tan(e/2 + f*x/2)**9 -
2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460
*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**
5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*
tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) + 1890*A*
a*tan(e/2 + f*x/2)**7/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/
2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 +
f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/
2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**
2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 5250*A*a*tan(e/2 + f*x/2)
**6/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11
340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*
c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5
*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*ta
n(e/2 + f*x/2) - 315*c**5*f) + 6930*A*a*tan(e/2 + f*x/2)**5/(315*c**5*f*ta
n(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2
 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f
*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2
)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 3
15*c**5*f) - 7686*A*a*tan(e/2 + f*x/2)**4/(315*c**5*f*tan(e/2 + f*x/2)*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1425 vs. $2(171) = 342$.

Time = 0.08 (sec) , antiderivative size = 1425, normalized size of antiderivative = 8.10

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{(c - c\sin(e + fx))^5}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorith
m="maxima")
```

output
```
-2/315*(A*a*(432*sin(f*x + e)/(cos(f*x + e) + 1) - 1728*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 3612*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5418*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 5040*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 -
 3360*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1260*sin(f*x + e)^7/(cos(f*x +
 e) + 1)^7 - 315*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 83)/(c^5 - 9*c^5*si
n(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*s
in(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) +
 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(co
s(f*x + e) + 1)^9) - 5*A*a*(45*sin(f*x + e)/(cos(f*x + e) + 1) - 117*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 273*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 -
 315*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 315*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 - 147*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63*sin(f*x + e)^7/(co
s(f*x + e) + 1)^7 - 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c
^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x
+ e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 -
 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*
x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) - 5*B*a*(45*si...
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.43

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx =$$
$$-\frac{2\left(315\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8-945\,Aa\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7+315\,Ba\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7+2625\,Aa\tan\left(\frac{1}{2}\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorith
m="giac")
```

output
```
-2/315*(315*A*a*tan(1/2*f*x + 1/2*e)^8 - 945*A*a*tan(1/2*f*x + 1/2*e)^7 +
315*B*a*tan(1/2*f*x + 1/2*e)^7 + 2625*A*a*tan(1/2*f*x + 1/2*e)^6 - 315*B*a
*tan(1/2*f*x + 1/2*e)^6 - 3465*A*a*tan(1/2*f*x + 1/2*e)^5 + 945*B*a*tan(1/
2*f*x + 1/2*e)^5 + 3843*A*a*tan(1/2*f*x + 1/2*e)^4 - 441*B*a*tan(1/2*f*x +
 1/2*e)^4 - 2247*A*a*tan(1/2*f*x + 1/2*e)^3 + 609*B*a*tan(1/2*f*x + 1/2*e)
^3 + 1143*A*a*tan(1/2*f*x + 1/2*e)^2 - 81*B*a*tan(1/2*f*x + 1/2*e)^2 - 207
*A*a*tan(1/2*f*x + 1/2*e) + 99*B*a*tan(1/2*f*x + 1/2*e) + 58*A*a - 11*B*a)
/(c^5*f*(tan(1/2*f*x + 1/2*e) - 1)^9)
```

Mupad [B] (verification not implemented)

Time = 36.55 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.76

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx$$
$$= \frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{1357\,A\,a}{4}-\frac{461\,B\,a}{16}-\frac{635\,A\,a\,\cos(e+f\,x)}{4}+\frac{5\,B\,a\,\cos(e+f\,x)}{2}-\frac{1575\,A\,a\,\sin(e+f\,x)}{4}+\frac{945\,B\,a\,\sin(e+f\,x)}{8}-\right.}{}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^5,x)
```

output
```
(2*cos(e/2 + (f*x)/2)*((1357*A*a)/4 - (461*B*a)/16 - (635*A*a*cos(e + f*x)
)/4 + (5*B*a*cos(e + f*x))/2 - (1575*A*a*sin(e + f*x))/4 + (945*B*a*sin(e
+ f*x))/8 - (625*A*a*cos(2*e + 2*f*x))/4 + (121*A*a*cos(3*e + 3*f*x))/4 +
(7*A*a*cos(4*e + 4*f*x))/2 + (95*B*a*cos(2*e + 2*f*x))/4 - 8*B*a*cos(3*e +
 3*f*x) - (7*B*a*cos(4*e + 4*f*x))/16 + (399*A*a*sin(2*e + 2*f*x))/4 + (14
1*A*a*sin(3*e + 3*f*x))/4 - (15*A*a*sin(4*e + 4*f*x))/4 - (231*B*a*sin(2*e
 + 2*f*x))/8 - (39*B*a*sin(3*e + 3*f*x))/8 + (15*B*a*sin(4*e + 4*f*x))/16)
)/(315*c^5*f*((63*2^(1/2)*cos(e/2 + pi/4 + (f*x)/2))/8 - (21*2^(1/2)*cos((
3*e)/2 - pi/4 + (3*f*x)/2))/4 - (9*2^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2)
)/4 + (9*2^(1/2)*cos((7*e)/2 - pi/4 + (7*f*x)/2))/16 + (2^(1/2)*cos((9*e)/
2 + pi/4 + (9*f*x)/2))/16))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.93

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx = \frac{2a\left(-35\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9 a-315\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7 a-315\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7 b+315\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6 a+315\tan\left(\frac{fx}{2}\right.\right.}{315c^5 f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9\right.}$$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x)
```

output
```
(2*a*( - 35*tan((e + f*x)/2)**9*a - 315*tan((e + f*x)/2)**7*a - 315*tan((e
 + f*x)/2)**7*b + 315*tan((e + f*x)/2)**6*a + 315*tan((e + f*x)/2)**6*b -
945*tan((e + f*x)/2)**5*a - 945*tan((e + f*x)/2)**5*b + 567*tan((e + f*x)/
2)**4*a + 441*tan((e + f*x)/2)**4*b - 693*tan((e + f*x)/2)**3*a - 609*tan(
(e + f*x)/2)**3*b + 117*tan((e + f*x)/2)**2*a + 81*tan((e + f*x)/2)**2*b -
 108*tan((e + f*x)/2)*a - 99*tan((e + f*x)/2)*b - 23*a + 11*b))/(315*c**5*
f*(tan((e + f*x)/2)**9 - 9*tan((e + f*x)/2)**8 + 36*tan((e + f*x)/2)**7 -
84*tan((e + f*x)/2)**6 + 126*tan((e + f*x)/2)**5 - 126*tan((e + f*x)/2)**4
 + 84*tan((e + f*x)/2)**3 - 36*tan((e + f*x)/2)**2 + 9*tan((e + f*x)/2) -
1))
```

3.26 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx$

Optimal result

Integrand size = 36, antiderivative size = 229

$$\begin{aligned}\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx &= \frac{9}{128}a^2(8A-3B)c^5x+\frac{3a^2(8A-3B)c^5\cos^5(e+fx)}{80f}\\ &\quad+\frac{9a^2(8A-3B)c^5\cos(e+fx)\sin(e+fx)}{128f}\\ &\quad+\frac{3a^2(8A-3B)c^5\cos^3(e+fx)\sin(e+fx)}{64f}\\ &\quad+\frac{a^2(8A-3B)c^3\cos^5(e+fx)(c-c\sin(e+fx))^2}{56f}\\ &\quad-\frac{a^2Bc^2\cos^5(e+fx)(c-c\sin(e+fx))^3}{8f}\\ &\quad+\frac{3a^2(8A-3B)\cos^5(e+fx)\left(c^5-c^5\sin(e+fx)\right)}{112f}\end{aligned}$$

output
```
9/128*a^2*(8*A-3*B)*c^5*x+3/80*a^2*(8*A-3*B)*c^5*cos(f*x+e)^5/f+9/128*a^2*
(8*A-3*B)*c^5*cos(f*x+e)*sin(f*x+e)/f+3/64*a^2*(8*A-3*B)*c^5*cos(f*x+e)^3*
sin(f*x+e)/f+1/56*a^2*(8*A-3*B)*c^3*cos(f*x+e)^5*(c-c*sin(f*x+e))^2/f-1/8*
a^2*B*c^2*cos(f*x+e)^5*(c-c*sin(f*x+e))^3/f+3/112*a^2*(8*A-3*B)*cos(f*x+e)
^5*(c^5-c^5*sin(f*x+e))/f
```

Mathematica [A] (verified)

Time = 5.53 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.96

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx$$
$$=\frac{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5(2520(8A-3B)(e+fx)+560(27A-17B)\cos(e+fx)+560(\ldots}{}$$

input
```
Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^5,x]
```

output
```
((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^5*(2520*(8*A - 3*B)*(e + f*x)
 + 560*(27*A - 17*B)*Cos[e + f*x] + 560*(13*A - 7*B)*Cos[3*(e + f*x)] + 11
2*(11*A - B)*Cos[5*(e + f*x)] - 80*(A - 3*B)*Cos[7*(e + f*x)] + 560*(19*A
- 3*B)*Sin[2*(e + f*x)] - 280*(2*A - 7*B)*Sin[4*(e + f*x)] - 560*(A - B)*S
in[6*(e + f*x)] - 35*B*Sin[8*(e + f*x)]))/(35840*f*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^10*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.83, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 3446, 3042, 3339, 3042, 3157, 3042, 3157, 3042, 3148, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^2(c-c\sin(e+fx))^5(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^2(c-c\sin(e+fx))^5(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3446}$$

$$a^2c^2\int \cos^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^3dx$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\int \cos(e+fx)^4(A+B\sin(e+fx))(c-c\sin(e+fx))^3dx$$

$$\Big\downarrow \text{3339}$$

$$a^2c^2\left(\frac{1}{8}(8A-3B)\int \cos^4(e+fx)(c-c\sin(e+fx))^3dx-\frac{B\cos^5(e+fx)(c-c\sin(e+fx))^3}{8f}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left(\frac{1}{8}(8A-3B)\int \cos(e+fx)^4(c-c\sin(e+fx))^3dx-\frac{B\cos^5(e+fx)(c-c\sin(e+fx))^3}{8f}\right)$$

$$\Big\downarrow \text{3157}$$

$$a^2c^2\left(\frac{1}{8}(8A-3B)\left(\frac{9}{7}c\int \cos^4(e+fx)(c-c\sin(e+fx))^2dx+\frac{c\cos^5(e+fx)(c-c\sin(e+fx))^2}{7f}\right)-\frac{B\cos^5(e\right.$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left(\frac{1}{8}(8A-3B)\left(\frac{9}{7}c\int \cos(e+fx)^4(c-c\sin(e+fx))^2dx+\frac{c\cos^5(e+fx)(c-c\sin(e+fx))^2}{7f}\right)-\frac{B\cos^5(e\right.$$

$$\Big\downarrow \text{3157}$$

$$a^2c^2\left(\frac{1}{8}(8A-3B)\left(\frac{9}{7}c\left(\frac{7}{6}c\int \cos^4(e+fx)(c-c\sin(e+fx))dx+\frac{\cos^5(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{6f}\right)+\frac{c\cos\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left(\frac{1}{8}(8A-3B)\left(\frac{9}{7}c\left(\frac{7}{6}c\int\cos(e+fx)^4(c-c\sin(e+fx))dx+\frac{\cos^5(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{6f}\right)+\frac{c\cos\right.\right.$$

$\downarrow$ 3148

$$a^2c^2\left(\frac{1}{8}(8A-3B)\left(\frac{9}{7}c\left(\frac{7}{6}c\left(c\int\cos^4(e+fx)dx+\frac{c\cos^5(e+fx)}{5f}\right)+\frac{\cos^5(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{6f}\right)+\frac{c}{}\right.\right.$$

$\downarrow$ 3042

$$a^2c^2\left(\frac{1}{8}(8A-3B)\left(\frac{9}{7}c\left(\frac{7}{6}c\left(c\int\sin\left(e+fx+\frac{\pi}{2}\right)^4dx+\frac{c\cos^5(e+fx)}{5f}\right)+\frac{\cos^5(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{6f}\right.\right.\right.$$

$\downarrow$ 3115

$$a^2c^2\left(\frac{1}{8}(8A-3B)\left(\frac{9}{7}c\left(\frac{7}{6}c\left(c\left(\frac{3}{4}\int\cos^2(e+fx)dx+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{c\cos^5(e+fx)}{5f}\right)+\frac{\cos^5(e}{}\right.\right.\right.$$

$\downarrow$ 3042

$$a^2c^2\left(\frac{1}{8}(8A-3B)\left(\frac{9}{7}c\left(\frac{7}{6}c\left(c\left(\frac{3}{4}\int\sin\left(e+fx+\frac{\pi}{2}\right)^2dx+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{c\cos^5(e+fx)}{5f}\right)+\frac{c}{}\right.\right.\right.$$

$\downarrow$ 3115

$$a^2c^2\left(\frac{1}{8}(8A-3B)\left(\frac{9}{7}c\left(\frac{7}{6}c\left(c\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}\right)+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{c\cos^5(e}{5f}\right.\right.\right.\right.$$

$\downarrow$ 24

$$a^2c^2\left(\frac{1}{8}(8A-3B)\left(\frac{9}{7}c\left(\frac{\cos^5(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{6f}+\frac{7}{6}c\left(\frac{c\cos^5(e+fx)}{5f}+c\left(\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right.\right.\right.\right.\right.$$

input
```
Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5,x]
```

output
```
a^2*c^2*(-1/8*(B*Cos[e + f*x]^5*(c - c*Sin[e + f*x])^3)/f + ((8*A - 3*B)*(
(c*Cos[e + f*x]^5*(c - c*Sin[e + f*x])^2)/(7*f) + (9*c*((Cos[e + f*x]^5*(c
^2 - c^2*Sin[e + f*x]))/(6*f) + (7*c*((c*Cos[e + f*x]^5)/(5*f) + c*((Cos[e
 + f*x]^3*Sin[e + f*x])/(4*f) + (3*(x/2 + (Cos[e + f*x]*Sin[e + f*x])/(2*f
)))/4)))/6))/7))/8)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3157
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && Integers
Q[2*m, 2*p]
```

rule 3339
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 568 vs. $2(215) = 430$.

Time = 0.11 (sec) , antiderivative size = 569, normalized size of antiderivative = 2.48

$$a^2 A\,c^5 (fx+e) - a^2 B\,c^5 \cos(fx+e) + a^2 A\,c^5 \left(-\frac{\sin(fx+e)\cos(fx+e)}{2} + \frac{fx}{2} + \frac{e}{2}\right) - \frac{a^2 B\,c^5 \left(2+\sin(fx+e)^2\right)\cos(fx+e)}{3}$$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x)
```

output
```
1/f*(a^2*A*c^5*(f*x+e)-a^2*B*c^5*cos(f*x+e)+a^2*A*c^5*(-1/2*sin(f*x+e)*cos
(f*x+e)+1/2*f*x+1/2*e)-1/3*a^2*B*c^5*(2+sin(f*x+e)^2)*cos(f*x+e)+3*a^2*A*c
^5*cos(f*x+e)-3*a^2*B*c^5*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+1/7*a
^2*A*c^5*(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e)+
3*a^2*A*c^5*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+
e)+5/16*f*x+5/16*e)+1/5*a^2*A*c^5*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(
f*x+e)-5*a^2*A*c^5*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+
3/8*e)-5/3*a^2*A*c^5*(2+sin(f*x+e)^2)*cos(f*x+e)-a^2*B*c^5*(-1/8*(sin(f*x+
e)^7+7/6*sin(f*x+e)^5+35/24*sin(f*x+e)^3+35/16*sin(f*x+e))*cos(f*x+e)+35/1
28*f*x+35/128*e)-3/7*a^2*B*c^5*(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin
(f*x+e)^2)*cos(f*x+e)-a^2*B*c^5*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*
sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)+a^2*B*c^5*(8/3+sin(f*x+e)^4+4/3*si
n(f*x+e)^2)*cos(f*x+e)+5*a^2*B*c^5*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos
(f*x+e)+3/8*f*x+3/8*e))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.69

$$\int (a + a\sin(e + fx))^2 (A + B\sin(e + fx))(c - c\sin(e + fx))^5 \, dx =$$
$$-\frac{640\,(A - 3\,B)a^2c^5\cos{(fx + e)}^7 - 3584\,(A - B)a^2c^5\cos{(fx + e)}^5 - 315\,(8\,A - 3\,B)a^2c^5fx + 35\,(16}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x, algori
thm="fricas")
```

output
```
-1/4480*(640*(A - 3*B)*a^2*c^5*cos(f*x + e)^7 - 3584*(A - B)*a^2*c^5*cos(f
*x + e)^5 - 315*(8*A - 3*B)*a^2*c^5*f*x + 35*(16*B*a^2*c^5*cos(f*x + e)^7
+ 8*(8*A - 11*B)*a^2*c^5*cos(f*x + e)^5 - 6*(8*A - 3*B)*a^2*c^5*cos(f*x +
e)^3 - 9*(8*A - 3*B)*a^2*c^5*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1586 vs. $2(218) = 436$.

Time = 0.90 (sec) , antiderivative size = 1586, normalized size of antiderivative = 6.93

$$\int (a + a\sin(e + fx))^2(A + B\sin(e + fx))(c - c\sin(e + fx))^5\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**5,x)
```

output
```
Piecewise((15*A*a**2*c**5*x*sin(e + f*x)**6/16 + 45*A*a**2*c**5*x*sin(e +
f*x)**4*cos(e + f*x)**2/16 - 15*A*a**2*c**5*x*sin(e + f*x)**4/8 + 45*A*a**
2*c**5*x*sin(e + f*x)**2*cos(e + f*x)**4/16 - 15*A*a**2*c**5*x*sin(e + f*x
)**2*cos(e + f*x)**2/4 + A*a**2*c**5*x*sin(e + f*x)**2/2 + 15*A*a**2*c**5*
x*cos(e + f*x)**6/16 - 15*A*a**2*c**5*x*cos(e + f*x)**4/8 + A*a**2*c**5*x*
cos(e + f*x)**2/2 + A*a**2*c**5*x + A*a**2*c**5*sin(e + f*x)**6*cos(e + f*
x)/f - 33*A*a**2*c**5*sin(e + f*x)**5*cos(e + f*x)/(16*f) + 2*A*a**2*c**5*
sin(e + f*x)**4*cos(e + f*x)**3/f + A*a**2*c**5*sin(e + f*x)**4*cos(e + f*
x)/f - 5*A*a**2*c**5*sin(e + f*x)**3*cos(e + f*x)**3/(2*f) + 25*A*a**2*c**
5*sin(e + f*x)**3*cos(e + f*x)/(8*f) + 8*A*a**2*c**5*sin(e + f*x)**2*cos(e
 + f*x)**5/(5*f) + 4*A*a**2*c**5*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 5
*A*a**2*c**5*sin(e + f*x)**2*cos(e + f*x)/f - 15*A*a**2*c**5*sin(e + f*x)*
cos(e + f*x)**5/(16*f) + 15*A*a**2*c**5*sin(e + f*x)*cos(e + f*x)**3/(8*f)
 - A*a**2*c**5*sin(e + f*x)*cos(e + f*x)/(2*f) + 16*A*a**2*c**5*cos(e + f*
x)**7/(35*f) + 8*A*a**2*c**5*cos(e + f*x)**5/(15*f) - 10*A*a**2*c**5*cos(e
 + f*x)**3/(3*f) + 3*A*a**2*c**5*cos(e + f*x)/f - 35*B*a**2*c**5*x*sin(e +
 f*x)**8/128 - 35*B*a**2*c**5*x*sin(e + f*x)**6*cos(e + f*x)**2/32 - 5*B*a
**2*c**5*x*sin(e + f*x)**6/16 - 105*B*a**2*c**5*x*sin(e + f*x)**4*cos(e +
f*x)**4/64 - 15*B*a**2*c**5*x*sin(e + f*x)**4*cos(e + f*x)**2/16 + 15*B*a*
*2*c**5*x*sin(e + f*x)**4/8 - 35*B*a**2*c**5*x*sin(e + f*x)**2*cos(e + ...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. 2(218) = 436.

Time = 0.05 (sec) , antiderivative size = 571, normalized size of antiderivative = 2.49

$$\int (a + a\sin(e + fx))^2(A + B\sin(e + fx))(c - c\sin(e + fx))^5 \, dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x, algori
thm="maxima")
```

output
```
-1/107520*(3072*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3
- 35*cos(f*x + e))*A*a^2*c^5 - 7168*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3
+ 15*cos(f*x + e))*A*a^2*c^5 - 179200*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*
a^2*c^5 - 1680*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e)
- 48*sin(2*f*x + 2*e))*A*a^2*c^5 + 16800*(12*f*x + 12*e + sin(4*f*x + 4*e)
 - 8*sin(2*f*x + 2*e))*A*a^2*c^5 - 26880*(2*f*x + 2*e - sin(2*f*x + 2*e))*
A*a^2*c^5 - 107520*(f*x + e)*A*a^2*c^5 - 9216*(5*cos(f*x + e)^7 - 21*cos(f
*x + e)^5 + 35*cos(f*x + e)^3 - 35*cos(f*x + e))*B*a^2*c^5 - 35840*(3*cos(
f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^2*c^5 - 35840*(cos(f
*x + e)^3 - 3*cos(f*x + e))*B*a^2*c^5 + 35*(128*sin(2*f*x + 2*e)^3 + 840*f
*x + 840*e + 3*sin(8*f*x + 8*e) + 168*sin(4*f*x + 4*e) - 768*sin(2*f*x + 2
*e))*B*a^2*c^5 + 560*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x +
 4*e) - 48*sin(2*f*x + 2*e))*B*a^2*c^5 - 16800*(12*f*x + 12*e + sin(4*f*x
+ 4*e) - 8*sin(2*f*x + 2*e))*B*a^2*c^5 + 80640*(2*f*x + 2*e - sin(2*f*x +
2*e))*B*a^2*c^5 - 322560*A*a^2*c^5*cos(f*x + e) + 107520*B*a^2*c^5*cos(f*x
 + e))/f
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.18

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx$$
$$= -\frac{Ba^2c^5\sin(8\,fx+8\,e)}{1024\,f} + \frac{9}{128}\left(8\,Aa^2c^5-3\,Ba^2c^5\right)x - \frac{(Aa^2c^5-3\,Ba^2c^5)\cos(7\,fx+7\,e)}{448\,f} + \frac{(11\,Aa^2c^5-Ba^2c^5)\cos(5\,fx+5\,e)}{320\,f} + \frac{(13\,Aa^2c^5-7\,Ba^2c^5)\cos(3\,fx+3\,e)}{64\,f} + \frac{(27\,Aa^2c^5-17\,Ba^2c^5)\cos(fx+e)}{64\,f} - \frac{(Aa^2c^5-Ba^2c^5)\sin(6\,fx+6\,e)}{64\,f} - \frac{(2\,Aa^2c^5-7\,Ba^2c^5)\sin(4\,fx+4\,e)}{128\,f} + \frac{(19\,Aa^2c^5-3\,Ba^2c^5)\sin(2\,fx+2\,e)}{64\,f}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x, algori
thm="giac")
```

output
```
-1/1024*B*a^2*c^5*sin(8*f*x + 8*e)/f + 9/128*(8*A*a^2*c^5 - 3*B*a^2*c^5)*x
 - 1/448*(A*a^2*c^5 - 3*B*a^2*c^5)*cos(7*f*x + 7*e)/f + 1/320*(11*A*a^2*c^
5 - B*a^2*c^5)*cos(5*f*x + 5*e)/f + 1/64*(13*A*a^2*c^5 - 7*B*a^2*c^5)*cos(
3*f*x + 3*e)/f + 1/64*(27*A*a^2*c^5 - 17*B*a^2*c^5)*cos(f*x + e)/f - 1/64*
(A*a^2*c^5 - B*a^2*c^5)*sin(6*f*x + 6*e)/f - 1/128*(2*A*a^2*c^5 - 7*B*a^2*
c^5)*sin(4*f*x + 4*e)/f + 1/64*(19*A*a^2*c^5 - 3*B*a^2*c^5)*sin(2*f*x + 2*
e)/f
```

Mupad [B] (verification not implemented)

Time = 37.82 (sec) , antiderivative size = 661, normalized size of antiderivative = 2.89

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx = \text{Too large to display}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^5,x)
```

output
```
(tan(e/2 + (f*x)/2)^14*(6*A*a^2*c^5 - 2*B*a^2*c^5) + tan(e/2 + (f*x)/2)^10
*(30*A*a^2*c^5 - 10*B*a^2*c^5) + tan(e/2 + (f*x)/2)^12*(22*A*a^2*c^5 - 18*
B*a^2*c^5) + tan(e/2 + (f*x)/2)^8*(46*A*a^2*c^5 - 26*B*a^2*c^5) + tan(e/2
+ (f*x)/2)^4*((74*A*a^2*c^5)/5 - (14*B*a^2*c^5)/5) - tan(e/2 + (f*x)/2)^15
*((7*A*a^2*c^5)/8 + (27*B*a^2*c^5)/64) + tan(e/2 + (f*x)/2)^2*((158*A*a^2*
c^5)/35 - (138*B*a^2*c^5)/35) + tan(e/2 + (f*x)/2)^6*((218*A*a^2*c^5)/5 -
(158*B*a^2*c^5)/5) + tan(e/2 + (f*x)/2)^3*((75*A*a^2*c^5)/8 - (305*B*a^2*c
^5)/64) - tan(e/2 + (f*x)/2)^13*((75*A*a^2*c^5)/8 - (305*B*a^2*c^5)/64) +
tan(e/2 + (f*x)/2)^5*((55*A*a^2*c^5)/8 - (437*B*a^2*c^5)/64) - tan(e/2 + (
f*x)/2)^11*((55*A*a^2*c^5)/8 - (437*B*a^2*c^5)/64) - tan(e/2 + (f*x)/2)^7*
((13*A*a^2*c^5)/8 - (919*B*a^2*c^5)/64) + tan(e/2 + (f*x)/2)^9*((13*A*a^2*
c^5)/8 - (919*B*a^2*c^5)/64) + tan(e/2 + (f*x)/2)*((7*A*a^2*c^5)/8 + (27*B
*a^2*c^5)/64) + (46*A*a^2*c^5)/35 - (26*B*a^2*c^5)/35)/(f*(8*tan(e/2 + (f*
x)/2)^2 + 28*tan(e/2 + (f*x)/2)^4 + 56*tan(e/2 + (f*x)/2)^6 + 70*tan(e/2 +
 (f*x)/2)^8 + 56*tan(e/2 + (f*x)/2)^10 + 28*tan(e/2 + (f*x)/2)^12 + 8*tan(
e/2 + (f*x)/2)^14 + tan(e/2 + (f*x)/2)^16 + 1)) + (9*a^2*c^5*atan((9*a^2*c
^5*tan(e/2 + (f*x)/2)*(8*A - 3*B))/(64*((9*A*a^2*c^5)/8 - (27*B*a^2*c^5)/6
4)))*(8*A - 3*B))/(64*f)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.15

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx$$
$$=\frac{a^2c^5\big(560\cos(fx+e)\sin(fx+e)^7b+640\cos(fx+e)\sin(fx+e)^6a-1920\cos(fx+e)\sin(fx+e)^6}{}$$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x)
```

output
```
(a**2*c**5*(560*cos(e + f*x)*sin(e + f*x)**7*b + 640*cos(e + f*x)*sin(e +
f*x)**6*a - 1920*cos(e + f*x)*sin(e + f*x)**6*b - 2240*cos(e + f*x)*sin(e
+ f*x)**5*a + 1400*cos(e + f*x)*sin(e + f*x)**5*b + 1664*cos(e + f*x)*sin(
e + f*x)**4*a + 2176*cos(e + f*x)*sin(e + f*x)**4*b + 2800*cos(e + f*x)*si
n(e + f*x)**3*a - 3850*cos(e + f*x)*sin(e + f*x)**3*b - 5248*cos(e + f*x)*
sin(e + f*x)**2*a + 1408*cos(e + f*x)*sin(e + f*x)**2*b + 1960*cos(e + f*x
)*sin(e + f*x)*a + 945*cos(e + f*x)*sin(e + f*x)*b + 2944*cos(e + f*x)*a -
 1664*cos(e + f*x)*b + 2520*a*f*x - 2944*a - 945*b*f*x + 1664*b))/(4480*f)
```

3.27 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$

Optimal result

Integrand size = 36, antiderivative size = 189

$$\begin{aligned}
&\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx \\
&= \frac{1}{16}a^2(7A-2B)c^4x+\frac{a^2(7A-2B)c^4\cos^5(e+fx)}{30f} \\
&\quad+\frac{a^2(7A-2B)c^4\cos(e+fx)\sin(e+fx)}{16f}+\frac{a^2(7A-2B)c^4\cos^3(e+fx)\sin(e+fx)}{24f} \\
&\quad-\frac{a^2B\cos^5(e+fx)\left(c^2-c^2\sin(e+fx)\right)^2}{7f} \\
&\quad+\frac{a^2(7A-2B)\cos^5(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{42f}
\end{aligned}$$

output
```
1/16*a^2*(7*A-2*B)*c^4*x+1/30*a^2*(7*A-2*B)*c^4*cos(f*x+e)^5/f+1/16*a^2*(7
*A-2*B)*c^4*cos(f*x+e)*sin(f*x+e)/f+1/24*a^2*(7*A-2*B)*c^4*cos(f*x+e)^3*si
n(f*x+e)/f-1/7*a^2*B*cos(f*x+e)^5*(c^2-c^2*sin(f*x+e))^2/f+1/42*a^2*(7*A-2
*B)*cos(f*x+e)^5*(c^4-c^4*sin(f*x+e))/f
```

Mathematica [A] (verified)

Time = 8.25 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.86

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$$
$$=\frac{a^2c^4(2940Ae-840Be+2940Afx-840Bfx+105(16A-11B)\cos(e+fx)+105(8A-5B)\cos(3(e-$$

input
```
Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^4,x]
```

output
```
(a^2*c^4*(2940*A*e - 840*B*e + 2940*A*f*x - 840*B*f*x + 105*(16*A - 11*B)*
Cos[e + f*x] + 105*(8*A - 5*B)*Cos[3*(e + f*x)] + 168*A*Cos[5*(e + f*x)] -
 63*B*Cos[5*(e + f*x)] + 15*B*Cos[7*(e + f*x)] + 1785*A*Sin[2*(e + f*x)] -
 210*B*Sin[2*(e + f*x)] + 105*A*Sin[4*(e + f*x)] + 210*B*Sin[4*(e + f*x)]
- 35*A*Sin[6*(e + f*x)] + 70*B*Sin[6*(e + f*x)]))/(6720*f)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.81, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.361, Rules used = {3042, 3446, 3042, 3339, 3042, 3157, 3042, 3148, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^2(c-c\sin(e+fx))^4(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^2(c-c\sin(e+fx))^4(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3446}$$

$$a^2c^2\int \cos^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^2dx$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\int \cos(e+fx)^4(A+B\sin(e+fx))(c-c\sin(e+fx))^2dx$$

$$\Big\downarrow \text{3339}$$

$$a^2c^2\left(\frac{1}{7}(7A-2B)\int \cos^4(e+fx)(c-c\sin(e+fx))^2dx-\frac{B\cos^5(e+fx)(c-c\sin(e+fx))^2}{7f}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left(\frac{1}{7}(7A-2B)\int \cos(e+fx)^4(c-c\sin(e+fx))^2dx-\frac{B\cos^5(e+fx)(c-c\sin(e+fx))^2}{7f}\right)$$

$$\Big\downarrow \text{3157}$$

$$a^2c^2\left(\frac{1}{7}(7A-2B)\left(\frac{7}{6}c\int \cos^4(e+fx)(c-c\sin(e+fx))dx+\frac{\cos^5(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{6f}\right)-\frac{B\cos^5(e$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left(\frac{1}{7}(7A-2B)\left(\frac{7}{6}c\int \cos(e+fx)^4(c-c\sin(e+fx))dx+\frac{\cos^5(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{6f}\right)-\frac{B\cos^5(e$$

$$\Big\downarrow \text{3148}$$

$$a^2c^2\left(\frac{1}{7}(7A-2B)\left(\frac{7}{6}c\left(c\int \cos^4(e+fx)dx+\frac{c\cos^5(e+fx)}{5f}\right)+\frac{\cos^5(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{6f}\right)-\frac{B\cos$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left(\frac{1}{7}(7A-2B)\left(\frac{7}{6}c\left(c\int \sin\left(e+fx+\frac{\pi}{2}\right)^4dx+\frac{c\cos^5(e+fx)}{5f}\right)+\frac{\cos^5(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{6f}\right)-$$

$$\Big\downarrow \text{3115}$$

$$a^2c^2\left(\frac{1}{7}(7A-2B)\left(\frac{7}{6}c\left(c\left(\frac{3}{4}\int \cos^2(e+fx)dx+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{c\cos^5(e+fx)}{5f}\right)+\frac{\cos^5(e+fx}{}$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left(\frac{1}{7}(7A-2B)\left(\frac{7}{6}c\left(c\left(\frac{3}{4}\int \sin\left(e+fx+\frac{\pi}{2}\right)^2dx+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{c\cos^5(e+fx)}{5f}\right)+\frac{\cos^5(e}{}$$

$$\Big\downarrow \text{3115}$$

$$a^2c^2\left(\frac{1}{7}(7A-2B)\left(\frac{7}{6}c\left(c\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}\right)+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{c\cos^5(e+fx)}{5f}\right.\right.\right.$$

$$\Big\downarrow \text{24}$$

$$a^2c^2\left(\frac{1}{7}(7A-2B)\left(\frac{\cos^5(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{6f}+\frac{7}{6}c\left(\frac{c\cos^5(e+fx)}{5f}+c\left(\frac{\sin(e+fx)\cos^3(e+fx)}{4f}+\frac{3}{4}\right.\right.\right.\right.$$

input
```
Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4,x]
```

output
```
a^2*c^2*(-1/7*(B*Cos[e + f*x]^5*(c - c*Sin[e + f*x])^2)/f + ((7*A - 2*B)*(
(Cos[e + f*x]^5*(c^2 - c^2*Sin[e + f*x]))/(6*f) + (7*c*((c*Cos[e + f*x]^5)
/(5*f) + c*((Cos[e + f*x]^3*Sin[e + f*x])/(4*f) + (3*(x/2 + (Cos[e + f*x]*
Sin[e + f*x])/(2*f)))/4)))/6))/7)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3157
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && Integers
Q[2*m, 2*p]
```

rule 3339
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(177) = 354$.

Time = 0.11 (sec) , antiderivative size = 463, normalized size of antiderivative = 2.45

$$a^2 A\, c^4 (fx+e) - \frac{4a^2 A\, c^4 \left(2+\sin(fx+e)^2\right) \cos(fx+e)}{3} - \frac{a^2 B\, c^4 \left(\frac{16}{5}+\sin(fx+e)^6+\frac{6\sin(fx+e)^4}{5}+\frac{8\sin(fx+e)^2}{5}\right) \cos(fx+e)}{7} - 2a^2 B\, c^4$$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x)
```

output
```
1/f*(a^2*A*c^4*(f*x+e)-4/3*a^2*A*c^4*(2+sin(f*x+e)^2)*cos(f*x+e)-1/7*a^2*B
*c^4*(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e)-2*a^
2*B*c^4*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5
/16*f*x+5/16*e)+1/5*a^2*B*c^4*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+
e)+4*a^2*B*c^4*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*
e)-a^2*B*c^4*cos(f*x+e)-a^2*A*c^4*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*
e)+1/3*a^2*B*c^4*(2+sin(f*x+e)^2)*cos(f*x+e)+2*a^2*A*c^4*cos(f*x+e)-2*a^2*
B*c^4*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+a^2*A*c^4*(-1/6*(sin(f*x+
e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)+2/5*a^2
*A*c^4*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-a^2*A*c^4*(-1/4*(sin
(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.71

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$$
$$= \frac{240\,Ba^2c^4\cos{(fx+e)}^7+672\,(A-B)a^2c^4\cos{(fx+e)}^5+105\,(7\,A-2\,B)a^2c^4fx-35\,\big(8\,(A-2\,B)a^2c}{1680\,f}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algori
thm="fricas")
```

output
```
1/1680*(240*B*a^2*c^4*cos(f*x + e)^7 + 672*(A - B)*a^2*c^4*cos(f*x + e)^5
+ 105*(7*A - 2*B)*a^2*c^4*f*x - 35*(8*(A - 2*B)*a^2*c^4*cos(f*x + e)^5 - 2
*(7*A - 2*B)*a^2*c^4*cos(f*x + e)^3 - 3*(7*A - 2*B)*a^2*c^4*cos(f*x + e))*
sin(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1210 vs. $2(172) = 344$.

Time = 0.66 (sec) , antiderivative size = 1210, normalized size of antiderivative = 6.40

$$\int (a + a\sin(e + fx))^2(A + B\sin(e + fx))(c - c\sin(e + fx))^4 \, dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**4,x)
```

output
```
Piecewise((5*A*a**2*c**4*x*sin(e + f*x)**6/16 + 15*A*a**2*c**4*x*sin(e + f
*x)**4*cos(e + f*x)**2/16 - 3*A*a**2*c**4*x*sin(e + f*x)**4/8 + 15*A*a**2*
c**4*x*sin(e + f*x)**2*cos(e + f*x)**4/16 - 3*A*a**2*c**4*x*sin(e + f*x)**
2*cos(e + f*x)**2/4 - A*a**2*c**4*x*sin(e + f*x)**2/2 + 5*A*a**2*c**4*x*co
s(e + f*x)**6/16 - 3*A*a**2*c**4*x*cos(e + f*x)**4/8 - A*a**2*c**4*x*cos(e
 + f*x)**2/2 + A*a**2*c**4*x - 11*A*a**2*c**4*sin(e + f*x)**5*cos(e + f*x)
/(16*f) + 2*A*a**2*c**4*sin(e + f*x)**4*cos(e + f*x)/f - 5*A*a**2*c**4*sin
(e + f*x)**3*cos(e + f*x)**3/(6*f) + 5*A*a**2*c**4*sin(e + f*x)**3*cos(e +
 f*x)/(8*f) + 8*A*a**2*c**4*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 4*A*a*
*2*c**4*sin(e + f*x)**2*cos(e + f*x)/f - 5*A*a**2*c**4*sin(e + f*x)*cos(e
+ f*x)**5/(16*f) + 3*A*a**2*c**4*sin(e + f*x)*cos(e + f*x)**3/(8*f) + A*a*
*2*c**4*sin(e + f*x)*cos(e + f*x)/(2*f) + 16*A*a**2*c**4*cos(e + f*x)**5/(
15*f) - 8*A*a**2*c**4*cos(e + f*x)**3/(3*f) + 2*A*a**2*c**4*cos(e + f*x)/f
 - 5*B*a**2*c**4*x*sin(e + f*x)**6/8 - 15*B*a**2*c**4*x*sin(e + f*x)**4*co
s(e + f*x)**2/8 + 3*B*a**2*c**4*x*sin(e + f*x)**4/2 - 15*B*a**2*c**4*x*sin
(e + f*x)**2*cos(e + f*x)**4/8 + 3*B*a**2*c**4*x*sin(e + f*x)**2*cos(e + f
*x)**2 - B*a**2*c**4*x*sin(e + f*x)**2 - 5*B*a**2*c**4*x*cos(e + f*x)**6/8
 + 3*B*a**2*c**4*x*cos(e + f*x)**4/2 - B*a**2*c**4*x*cos(e + f*x)**2 - B*a
**2*c**4*sin(e + f*x)**6*cos(e + f*x)/f + 11*B*a**2*c**4*sin(e + f*x)**5*c
os(e + f*x)/(8*f) - 2*B*a**2*c**4*sin(e + f*x)**4*cos(e + f*x)**3/f + B...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 460 vs. $2(179) = 358$.

Time = 0.05 (sec) , antiderivative size = 460, normalized size of antiderivative = 2.43

$$\int (a + a\sin(e + fx))^2 (A + B\sin(e + fx))(c - c\sin(e + fx))^4 \, dx$$
$$= \frac{896 \left(3 \cos\left(fx + e\right)^5 - 10 \cos\left(fx + e\right)^3 + 15 \cos\left(fx + e\right)\right) A a^2 c^4 + 8960 \left(\cos\left(fx + e\right)^3 - 3 \cos\left(fx + e\right)\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algori
thm="maxima")
```

output
```
1/6720*(896*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*A*a^2
*c^4 + 8960*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*c^4 + 35*(4*sin(2*f*x
+ 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*A*a^2
*c^4 - 210*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^2*c
^4 - 1680*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c^4 + 6720*(f*x + e)*A*a^
2*c^4 + 192*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3 - 35
*cos(f*x + e))*B*a^2*c^4 + 448*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*
cos(f*x + e))*B*a^2*c^4 - 2240*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*c^4
 - 70*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(
2*f*x + 2*e))*B*a^2*c^4 + 840*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*
f*x + 2*e))*B*a^2*c^4 - 3360*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^2*c^4 +
13440*A*a^2*c^4*cos(f*x + e) - 6720*B*a^2*c^4*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.25

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$$
$$= \frac{Ba^2c^4\cos(7\,fx+7\,e)}{448\,f} + \frac{1}{16}\left(7\,Aa^2c^4-2\,Ba^2c^4\right)x + \frac{(8\,Aa^2c^4-3\,Ba^2c^4)\cos(5\,fx+5\,e)}{320\,f} + \frac{(8\,Aa^2c^4-5\,Ba^2c^4)\cos(3\,fx+3\,e)}{64\,f} + \frac{(16\,Aa^2c^4-11\,Ba^2c^4)\cos(fx+e)}{64\,f} - \frac{(Aa^2c^4-2\,Ba^2c^4)\sin(6\,fx+6\,e)}{192\,f} + \frac{(Aa^2c^4+2\,Ba^2c^4)\sin(4\,fx+4\,e)}{64\,f} + \frac{(17\,Aa^2c^4-2\,Ba^2c^4)\sin(2\,fx+2\,e)}{64\,f}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algori
thm="giac")
```

output
```
1/448*B*a^2*c^4*cos(7*f*x + 7*e)/f + 1/16*(7*A*a^2*c^4 - 2*B*a^2*c^4)*x +
1/320*(8*A*a^2*c^4 - 3*B*a^2*c^4)*cos(5*f*x + 5*e)/f + 1/64*(8*A*a^2*c^4 -
 5*B*a^2*c^4)*cos(3*f*x + 3*e)/f + 1/64*(16*A*a^2*c^4 - 11*B*a^2*c^4)*cos(
f*x + e)/f - 1/192*(A*a^2*c^4 - 2*B*a^2*c^4)*sin(6*f*x + 6*e)/f + 1/64*(A*
a^2*c^4 + 2*B*a^2*c^4)*sin(4*f*x + 4*e)/f + 1/64*(17*A*a^2*c^4 - 2*B*a^2*c
^4)*sin(2*f*x + 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 37.93 (sec) , antiderivative size = 553, normalized size of antiderivative = 2.93

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx = \text{Too large to display}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^4,x)
```

output
```
(tan(e/2 + (f*x)/2)^12*(4*A*a^2*c^4 - 2*B*a^2*c^4) + tan(e/2 + (f*x)/2)^8*
(12*A*a^2*c^4 - 2*B*a^2*c^4) + tan(e/2 + (f*x)/2)^10*(8*A*a^2*c^4 - 8*B*a^
2*c^4) + tan(e/2 + (f*x)/2)^2*((8*A*a^2*c^4)/5 - (8*B*a^2*c^4)/5) - tan(e/
2 + (f*x)/2)^13*((9*A*a^2*c^4)/8 + (B*a^2*c^4)/4) + tan(e/2 + (f*x)/2)^6*(
16*A*a^2*c^4 - 16*B*a^2*c^4) + tan(e/2 + (f*x)/2)^3*((29*A*a^2*c^4)/6 - (1
1*B*a^2*c^4)/3) - tan(e/2 + (f*x)/2)^11*((29*A*a^2*c^4)/6 - (11*B*a^2*c^4)
/3) + tan(e/2 + (f*x)/2)^4*((44*A*a^2*c^4)/5 - (14*B*a^2*c^4)/5) + tan(e/2
 + (f*x)/2)^5*((23*A*a^2*c^4)/24 + (31*B*a^2*c^4)/12) - tan(e/2 + (f*x)/2)
^9*((23*A*a^2*c^4)/24 + (31*B*a^2*c^4)/12) + tan(e/2 + (f*x)/2)*((9*A*a^2*
c^4)/8 + (B*a^2*c^4)/4) + (4*A*a^2*c^4)/5 - (18*B*a^2*c^4)/35)/(f*(7*tan(e
/2 + (f*x)/2)^2 + 21*tan(e/2 + (f*x)/2)^4 + 35*tan(e/2 + (f*x)/2)^6 + 35*t
an(e/2 + (f*x)/2)^8 + 21*tan(e/2 + (f*x)/2)^10 + 7*tan(e/2 + (f*x)/2)^12 +
 tan(e/2 + (f*x)/2)^14 + 1)) + (a^2*c^4*atan((a^2*c^4*tan(e/2 + (f*x)/2)*(
7*A - 2*B))/(8*((7*A*a^2*c^4)/8 - (B*a^2*c^4)/4)))*(7*A - 2*B))/(8*f)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.21

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$$
$$= \frac{a^2c^4\left(-240\cos(fx+e)\sin(fx+e)^6 b-280\cos(fx+e)\sin(fx+e)^5 a+560\cos(fx+e)\sin(fx+e)^5\right.}{}$$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x)
```

output
```
(a**2*c**4*( - 240*cos(e + f*x)*sin(e + f*x)**6*b - 280*cos(e + f*x)*sin(e
 + f*x)**5*a + 560*cos(e + f*x)*sin(e + f*x)**5*b + 672*cos(e + f*x)*sin(e
 + f*x)**4*a + 48*cos(e + f*x)*sin(e + f*x)**4*b + 70*cos(e + f*x)*sin(e +
 f*x)**3*a - 980*cos(e + f*x)*sin(e + f*x)**3*b - 1344*cos(e + f*x)*sin(e
+ f*x)**2*a + 624*cos(e + f*x)*sin(e + f*x)**2*b + 945*cos(e + f*x)*sin(e
+ f*x)*a + 210*cos(e + f*x)*sin(e + f*x)*b + 672*cos(e + f*x)*a - 432*cos(
e + f*x)*b + 735*a*f*x - 672*a - 210*b*f*x + 432*b))/(1680*f)
```

3.28 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$

Optimal result

Integrand size = 36, antiderivative size = 147

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx = \frac{1}{16}a^2(6A-B)c^3x+\frac{a^2(6A-B)c^3\cos^5(e+fx)}{30f} + \frac{a^2(6A-B)c^3\cos(e+fx)\sin(e+fx)}{16f}+\frac{a^2(6A-B)c^3\cos^3(e+fx)\sin(e+fx)}{24f} - \frac{a^2B\cos^5(e+fx)\left(c^3-c^3\sin(e+fx)\right)}{6f}$$

output
```
1/16*a^2*(6*A-B)*c^3*x+1/30*a^2*(6*A-B)*c^3*cos(f*x+e)^5/f+1/16*a^2*(6*A-B
)*c^3*cos(f*x+e)*sin(f*x+e)/f+1/24*a^2*(6*A-B)*c^3*cos(f*x+e)^3*sin(f*x+e)
/f-1/6*a^2*B*cos(f*x+e)^5*(c^3-c^3*sin(f*x+e))/f
```

Mathematica [A] (verified)

Time = 7.98 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.93

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$$
$$= \frac{a^2c^3(360Ae-60Be+360Afx-60Bfx+120(A-B)\cos(e+fx)+60(A-B)\cos(3(e+fx))+12A}{}$$

input
```
Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^3,x]
```

output
```
(a^2*c^3*(360*A*e - 60*B*e + 360*A*f*x - 60*B*f*x + 120*(A - B)*Cos[e + f*
x] + 60*(A - B)*Cos[3*(e + f*x)] + 12*A*Cos[5*(e + f*x)] - 12*B*Cos[5*(e +
 f*x)] + 240*A*Sin[2*(e + f*x)] - 15*B*Sin[2*(e + f*x)] + 30*A*Sin[4*(e +
f*x)] + 15*B*Sin[4*(e + f*x)] + 5*B*Sin[6*(e + f*x)]))/(960*f)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.79, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.306$, Rules used = {3042, 3446, 3042, 3339, 3042, 3148, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^2(c-c\sin(e+fx))^3(A+B\sin(e+fx))\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (a\sin(e+fx)+a)^2(c-c\sin(e+fx))^3(A+B\sin(e+fx))dx$$
$$\Big\downarrow \text{3446}$$
$$a^2c^2\int \cos^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))dx$$
$$\Big\downarrow \text{3042}$$

$$a^2c^2\int \cos(e+fx)^4(A+B\sin(e+fx))(c-c\sin(e+fx))dx$$

$\Big\downarrow$ 3339

$$a^2c^2\left(\frac{1}{6}(6A-B)\int \cos^4(e+fx)(c-c\sin(e+fx))dx-\frac{B\cos^5(e+fx)(c-c\sin(e+fx))}{6f}\right)$$

$\Big\downarrow$ 3042

$$a^2c^2\left(\frac{1}{6}(6A-B)\int \cos(e+fx)^4(c-c\sin(e+fx))dx-\frac{B\cos^5(e+fx)(c-c\sin(e+fx))}{6f}\right)$$

$\Big\downarrow$ 3148

$$a^2c^2\left(\frac{1}{6}(6A-B)\left(c\int \cos^4(e+fx)dx+\frac{c\cos^5(e+fx)}{5f}\right)-\frac{B\cos^5(e+fx)(c-c\sin(e+fx))}{6f}\right)$$

$\Big\downarrow$ 3042

$$a^2c^2\left(\frac{1}{6}(6A-B)\left(c\int \sin\left(e+fx+\frac{\pi}{2}\right)^4dx+\frac{c\cos^5(e+fx)}{5f}\right)-\frac{B\cos^5(e+fx)(c-c\sin(e+fx))}{6f}\right)$$

$\Big\downarrow$ 3115

$$a^2c^2\left(\frac{1}{6}(6A-B)\left(c\left(\frac{3}{4}\int \cos^2(e+fx)dx+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{c\cos^5(e+fx)}{5f}\right)-\frac{B\cos^5(e+fx)(c}{6f}\right.$$

$\Big\downarrow$ 3042

$$a^2c^2\left(\frac{1}{6}(6A-B)\left(c\left(\frac{3}{4}\int \sin\left(e+fx+\frac{\pi}{2}\right)^2dx+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{c\cos^5(e+fx)}{5f}\right)-\frac{B\cos^5(e+}{}\right.$$

$\Big\downarrow$ 3115

$$a^2c^2\left(\frac{1}{6}(6A-B)\left(c\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}\right)+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{c\cos^5(e+fx)}{5f}\right)-\right.$$

$\Big\downarrow$ 24

$$a^2c^2\left(\frac{1}{6}(6A-B)\left(\frac{c\cos^5(e+fx)}{5f}+c\left(\frac{\sin(e+fx)\cos^3(e+fx)}{4f}+\frac{3}{4}\left(\frac{\sin(e+fx)\cos(e+fx)}{2f}+\frac{x}{2}\right)\right)\right)-\frac{B\text{ co}}{}\right.$$

input
```
Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3,x]
```

output
```
a^2*c^2*(-1/6*(B*Cos[e + f*x]^5*(c - c*Sin[e + f*x]))/f + ((6*A - B)*((c*C
os[e + f*x]^5)/(5*f) + c*((Cos[e + f*x]^3*Sin[e + f*x])/(4*f) + (3*(x/2 +
(Cos[e + f*x]*Sin[e + f*x])/(2*f)))/4)))/6)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3339
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(137) = 274$.

Time = 0.08 (sec) , antiderivative size = 365, normalized size of antiderivative = 2.48

$$\frac{\frac{a^2 A c^3 \left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(fx+e)}{5}+a^2 A c^3\left(-\frac{\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{2a^2 A c^3\left(2+\sin(\right.}{}}{}$$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x)
```

output
```
1/f*(1/5*a^2*A*c^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+a^2*A*c^
3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2/3*a^2*A*
c^3*(2+sin(f*x+e)^2)*cos(f*x+e)-2*a^2*A*c^3*(-1/2*sin(f*x+e)*cos(f*x+e)+1/
2*f*x+1/2*e)-a^2*B*c^3*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e
))*cos(f*x+e)+5/16*f*x+5/16*e)-1/5*a^2*B*c^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x
+e)^2)*cos(f*x+e)+2*a^2*B*c^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+
e)+3/8*f*x+3/8*e)+2/3*a^2*B*c^3*(2+sin(f*x+e)^2)*cos(f*x+e)+a^2*A*c^3*cos(
f*x+e)-a^2*B*c^3*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+a^2*A*c^3*(f*x
+e)-a^2*B*c^3*cos(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.78

$$\int (a + a\sin(e + fx))^2 (A + B\sin(e + fx))(c - c\sin(e + fx))^3 \, dx$$
$$= \frac{48 (A - B)a^2c^3 \cos(fx + e)^5 + 15 (6 A - B)a^2c^3 fx + 5 \left(8 Ba^2c^3 \cos(fx + e)^5 + 2 (6 A - B)a^2c^3 \cos(f\right.}{240 f}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algori
thm="fricas")
```

output
```
1/240*(48*(A - B)*a^2*c^3*cos(f*x + e)^5 + 15*(6*A - B)*a^2*c^3*f*x + 5*(8
*B*a^2*c^3*cos(f*x + e)^5 + 2*(6*A - B)*a^2*c^3*cos(f*x + e)^3 + 3*(6*A -
B)*a^2*c^3*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 910 vs. 2(128) = 256.

Time = 0.47 (sec) , antiderivative size = 910, normalized size of antiderivative = 6.19

$$\int (a + a\sin(e + fx))^2 (A + B\sin(e + fx))(c - c\sin(e + fx))^3 \, dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3,x)
```

output
```
Piecewise((3*A*a**2*c**3*x*sin(e + f*x)**4/8 + 3*A*a**2*c**3*x*sin(e + f*x
)**2*cos(e + f*x)**2/4 - A*a**2*c**3*x*sin(e + f*x)**2 + 3*A*a**2*c**3*x*c
os(e + f*x)**4/8 - A*a**2*c**3*x*cos(e + f*x)**2 + A*a**2*c**3*x + A*a**2*
c**3*sin(e + f*x)**4*cos(e + f*x)/f - 5*A*a**2*c**3*sin(e + f*x)**3*cos(e
+ f*x)/(8*f) + 4*A*a**2*c**3*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 2*A*a
**2*c**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**2*c**3*sin(e + f*x)*cos(e
 + f*x)**3/(8*f) + A*a**2*c**3*sin(e + f*x)*cos(e + f*x)/f + 8*A*a**2*c**3
*cos(e + f*x)**5/(15*f) - 4*A*a**2*c**3*cos(e + f*x)**3/(3*f) + A*a**2*c**
3*cos(e + f*x)/f - 5*B*a**2*c**3*x*sin(e + f*x)**6/16 - 15*B*a**2*c**3*x*s
in(e + f*x)**4*cos(e + f*x)**2/16 + 3*B*a**2*c**3*x*sin(e + f*x)**4/4 - 15
*B*a**2*c**3*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 3*B*a**2*c**3*x*sin(e
+ f*x)**2*cos(e + f*x)**2/2 - B*a**2*c**3*x*sin(e + f*x)**2/2 - 5*B*a**2*c
**3*x*cos(e + f*x)**6/16 + 3*B*a**2*c**3*x*cos(e + f*x)**4/4 - B*a**2*c**3
*x*cos(e + f*x)**2/2 + 11*B*a**2*c**3*sin(e + f*x)**5*cos(e + f*x)/(16*f)
- B*a**2*c**3*sin(e + f*x)**4*cos(e + f*x)/f + 5*B*a**2*c**3*sin(e + f*x)*
*3*cos(e + f*x)**3/(6*f) - 5*B*a**2*c**3*sin(e + f*x)**3*cos(e + f*x)/(4*f
) - 4*B*a**2*c**3*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) + 2*B*a**2*c**3*si
n(e + f*x)**2*cos(e + f*x)/f + 5*B*a**2*c**3*sin(e + f*x)*cos(e + f*x)**5/
(16*f) - 3*B*a**2*c**3*sin(e + f*x)*cos(e + f*x)**3/(4*f) + B*a**2*c**3*si
n(e + f*x)*cos(e + f*x)/(2*f) - 8*B*a**2*c**3*cos(e + f*x)**5/(15*f) + ...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(138) = 276$.

Time = 0.04 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.45

$$\int (a + a\sin(e+fx))^2 (A + B\sin(e+fx))(c - c\sin(e+fx))^3\,dx$$
$$= \frac{64\left(3\,\cos\left(fx+e\right)^5 - 10\,\cos\left(fx+e\right)^3 + 15\,\cos\left(fx+e\right)\right)Aa^2c^3 + 640\left(\cos\left(fx+e\right)^3 - 3\,\cos\left(fx+e\right)\right)A}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algori
thm="maxima")
```

output
```
1/960*(64*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*A*a^2*c
^3 + 640*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*c^3 + 30*(12*f*x + 12*e +
 sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^2*c^3 - 480*(2*f*x + 2*e - sin
(2*f*x + 2*e))*A*a^2*c^3 + 960*(f*x + e)*A*a^2*c^3 - 64*(3*cos(f*x + e)^5
- 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^2*c^3 - 640*(cos(f*x + e)^3 - 3
*cos(f*x + e))*B*a^2*c^3 - 5*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin
(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*B*a^2*c^3 + 60*(12*f*x + 12*e + sin(4
*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^2*c^3 - 240*(2*f*x + 2*e - sin(2*f*x
 + 2*e))*B*a^2*c^3 + 960*A*a^2*c^3*cos(f*x + e) - 960*B*a^2*c^3*cos(f*x +
e))/f
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.37

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$$
$$=\frac{Ba^2c^3\sin(6fx+6e)}{192f}+\frac{1}{16}\left(6Aa^2c^3-Ba^2c^3\right)x+\frac{(Aa^2c^3-Ba^2c^3)\cos(5fx+5e)}{80f}+\frac{(Aa^2c^3-Ba^2c^3)\cos(3fx+3e)}{16f}+\frac{(Aa^2c^3-Ba^2c^3)\cos(fx+e)}{8f}+\frac{(2Aa^2c^3+Ba^2c^3)\sin(4fx+4e)}{64f}+\frac{(16Aa^2c^3-Ba^2c^3)\sin(2fx+2e)}{64f}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algori
thm="giac")
```

output
```
1/192*B*a^2*c^3*sin(6*f*x + 6*e)/f + 1/16*(6*A*a^2*c^3 - B*a^2*c^3)*x + 1/
80*(A*a^2*c^3 - B*a^2*c^3)*cos(5*f*x + 5*e)/f + 1/16*(A*a^2*c^3 - B*a^2*c^
3)*cos(3*f*x + 3*e)/f + 1/8*(A*a^2*c^3 - B*a^2*c^3)*cos(f*x + e)/f + 1/64*
(2*A*a^2*c^3 + B*a^2*c^3)*sin(4*f*x + 4*e)/f + 1/64*(16*A*a^2*c^3 - B*a^2*
c^3)*sin(2*f*x + 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 36.96 (sec) , antiderivative size = 542, normalized size of antiderivative = 3.69

$$\int (a + a\sin(e + fx))^2(A + B\sin(e + fx))(c - c\sin(e + fx))^3 dx = \text{Too large to display}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^3,x)
```

output
```
(tan(e/2 + (f*x)/2)^4*(4*A*a^2*c^3 - 4*B*a^2*c^3) + tan(e/2 + (f*x)/2)^8*(
2*A*a^2*c^3 - 2*B*a^2*c^3) + tan(e/2 + (f*x)/2)^6*(4*A*a^2*c^3 - 4*B*a^2*c
^3) + tan(e/2 + (f*x)/2)^10*(2*A*a^2*c^3 - 2*B*a^2*c^3) + tan(e/2 + (f*x)/
2)^2*((2*A*a^2*c^3)/5 - (2*B*a^2*c^3)/5) + tan(e/2 + (f*x)/2)^5*((A*a^2*c^
3)/2 + (13*B*a^2*c^3)/4) - tan(e/2 + (f*x)/2)^7*((A*a^2*c^3)/2 + (13*B*a^2
*c^3)/4) - tan(e/2 + (f*x)/2)^11*((5*A*a^2*c^3)/4 + (B*a^2*c^3)/8) + tan(e
/2 + (f*x)/2)^3*((7*A*a^2*c^3)/4 - (47*B*a^2*c^3)/24) - tan(e/2 + (f*x)/2)
^9*((7*A*a^2*c^3)/4 - (47*B*a^2*c^3)/24) + tan(e/2 + (f*x)/2)*((5*A*a^2*c^
3)/4 + (B*a^2*c^3)/8) + (2*A*a^2*c^3)/5 - (2*B*a^2*c^3)/5)/(f*(6*tan(e/2 +
 (f*x)/2)^2 + 15*tan(e/2 + (f*x)/2)^4 + 20*tan(e/2 + (f*x)/2)^6 + 15*tan(e
/2 + (f*x)/2)^8 + 6*tan(e/2 + (f*x)/2)^10 + tan(e/2 + (f*x)/2)^12 + 1)) +
(a^2*c^3*atan((a^2*c^3*tan(e/2 + (f*x)/2)*(6*A - B))/(8*((3*A*a^2*c^3)/4 -
 (B*a^2*c^3)/8)))*(6*A - B))/(8*f) - (a^2*c^3*(6*A - B)*(atan(tan(e/2 + (f
*x)/2)) - (f*x)/2))/(8*f)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.33

$$\int (a + a\sin(e + fx))^2(A + B\sin(e + fx))(c - c\sin(e + fx))^3 dx$$
$$= \frac{a^2c^3(40\cos(fx + e)\sin(fx + e)^5 b + 48\cos(fx + e)\sin(fx + e)^4 a - 48\cos(fx + e)\sin(fx + e)^4 b - 6}{}$$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x)
```

output
```
(a**2*c**3*(40*cos(e + f*x)*sin(e + f*x)**5*b + 48*cos(e + f*x)*sin(e + f*
x)**4*a - 48*cos(e + f*x)*sin(e + f*x)**4*b - 60*cos(e + f*x)*sin(e + f*x)
**3*a - 70*cos(e + f*x)*sin(e + f*x)**3*b - 96*cos(e + f*x)*sin(e + f*x)**
2*a + 96*cos(e + f*x)*sin(e + f*x)**2*b + 150*cos(e + f*x)*sin(e + f*x)*a
+ 15*cos(e + f*x)*sin(e + f*x)*b + 48*cos(e + f*x)*a - 48*cos(e + f*x)*b +
 90*a*f*x - 48*a - 15*b*f*x + 48*b))/(240*f)
```

3.29 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$

Optimal result

Integrand size = 36, antiderivative size = 89

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx = \frac{3}{8}a^2Ac^2x-\frac{a^2Bc^2\cos^5(e+fx)}{5f}+\frac{3a^2Ac^2\cos(e+fx)\sin(e+fx)}{8f}+\frac{a^2Ac^2\cos^3(e+fx)\sin(e+fx)}{4f}$$

output
```
3/8*a^2*A*c^2*x-1/5*a^2*B*c^2*cos(f*x+e)^5/f+3/8*a^2*A*c^2*cos(f*x+e)*sin(
f*x+e)/f+1/4*a^2*A*c^2*cos(f*x+e)^3*sin(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.61

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx = \frac{a^2c^2(-32B\cos^5(e+fx)+5A(12(e+fx)+8\sin(2(e+fx))+\sin(4(e+fx))))}{160f}$$

input
```
Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^2,x]
```

output
```
(a^2*c^2*(-32*B*Cos[e + f*x]^5 + 5*A*(12*(e + f*x) + 8*Sin[2*(e + f*x)] +
Sin[4*(e + f*x)])))/(160*f)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.87, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3446, 3042, 3148, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^2(c-c\sin(e+fx))^2(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^2(c-c\sin(e+fx))^2(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3446}$$

$$a^2c^2\int \cos^4(e+fx)(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\int \cos(e+fx)^4(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3148}$$

$$a^2c^2\left(A\int \cos^4(e+fx)dx-\frac{B\cos^5(e+fx)}{5f}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left(A\int \sin\left(e+fx+\frac{\pi}{2}\right)^4dx-\frac{B\cos^5(e+fx)}{5f}\right)$$

$$\Big\downarrow \text{3115}$$

$$a^2c^2\left(A\left(\frac{3}{4}\int \cos^2(e+fx)dx+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)-\frac{B\cos^5(e+fx)}{5f}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left(A\left(\frac{3}{4}\int \sin\left(e+fx+\frac{\pi}{2}\right)^2dx+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)-\frac{B\cos^5(e+fx)}{5f}\right)$$

$$\Big\downarrow \text{3115}$$

$$a^2c^2\left(A\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}\right)+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)-\frac{B\cos^5(e+fx)}{5f}\right)$$

$$\Big\downarrow \text{24}$$

$$a^2c^2\left(A\left(\frac{\sin(e+fx)\cos^3(e+fx)}{4f}+\frac{3}{4}\left(\frac{\sin(e+fx)\cos(e+fx)}{2f}+\frac{x}{2}\right)\right)-\frac{B\cos^5(e+fx)}{5f}\right)$$

input
```
Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2,x]
```

output
```
a^2*c^2*(-1/5*(B*Cos[e + f*x]^5)/f + A*((Cos[e + f*x]^3*Sin[e + f*x])/(4*f
) + (3*(x/2 + (Cos[e + f*x]*Sin[e + f*x])/(2*f)))/4))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(81) = 162$.

Time = 0.06 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.87

$$\frac{a^2 A\, c^2 \left(-\frac{\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-2a^2 A\, c^2\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{a^2 B\, c^2\left(\frac{8}{3}+\sin\cdots\right.}{\cdots}}{f}$$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x)
```

output
```
1/f*(a^2*A*c^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*
e)-2*a^2*A*c^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/5*a^2*B*c^2*(8
/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+2/3*a^2*B*c^2*(2+sin(f*x+e)^2
)*cos(f*x+e)+a^2*A*c^2*(f*x+e)-a^2*B*c^2*cos(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.84

$$\int (a + a\sin(e + fx))^2(A + B\sin(e + fx))(c - c\sin(e + fx))^2 \, dx =$$
$$-\frac{8\,Ba^2c^2\cos\left(fx+e\right)^5 - 15\,Aa^2c^2fx - 5\,\left(2\,Aa^2c^2\cos\left(fx+e\right)^3 + 3\,Aa^2c^2\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{40\,f}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
-1/40*(8*B*a^2*c^2*cos(f*x + e)^5 - 15*A*a^2*c^2*f*x - 5*(2*A*a^2*c^2*cos(
f*x + e)^3 + 3*A*a^2*c^2*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. 2(87) = 174.

Time = 0.29 (sec) , antiderivative size = 372, normalized size of antiderivative = 4.18

$$\int (a + a\sin(e + fx))^2(A + B\sin(e + fx))(c - c\sin(e + fx))^2 \, dx$$
$$= \begin{cases} \frac{3Aa^2c^2x\sin^4(e+fx)}{8} + \frac{3Aa^2c^2x\sin^2(e+fx)\cos^2(e+fx)}{4} - Aa^2c^2x\sin^2(e+fx) + \frac{3Aa^2c^2x\cos^4(e+fx)}{8} - Aa^2c^2x\cos^2(\\ x(A + B\sin(e))\left(a\sin(e) + a\right)^2\left(-c\sin(e) + c\right)^2 \end{cases}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2,x)
```

output
```
Piecewise((3*A*a**2*c**2*x*sin(e + f*x)**4/8 + 3*A*a**2*c**2*x*sin(e + f*x
)**2*cos(e + f*x)**2/4 - A*a**2*c**2*x*sin(e + f*x)**2 + 3*A*a**2*c**2*x*c
os(e + f*x)**4/8 - A*a**2*c**2*x*cos(e + f*x)**2 + A*a**2*c**2*x - 5*A*a**
2*c**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*A*a**2*c**2*sin(e + f*x)*cos
(e + f*x)**3/(8*f) + A*a**2*c**2*sin(e + f*x)*cos(e + f*x)/f - B*a**2*c**2
*sin(e + f*x)**4*cos(e + f*x)/f - 4*B*a**2*c**2*sin(e + f*x)**2*cos(e + f*
x)**3/(3*f) + 2*B*a**2*c**2*sin(e + f*x)**2*cos(e + f*x)/f - 8*B*a**2*c**2
*cos(e + f*x)**5/(15*f) + 4*B*a**2*c**2*cos(e + f*x)**3/(3*f) - B*a**2*c**
2*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)**2*(-c*sin(e
) + c)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(81) = 162$.

Time = 0.04 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.84

$$\int (a + a\sin(e + fx))^2 (A + B\sin(e + fx))(c - c\sin(e + fx))^2 \, dx$$
$$= \frac{15\,(12\,fx + 12\,e + \sin(4\,fx + 4\,e) - 8\,\sin(2\,fx + 2\,e))Aa^2c^2 - 240\,(2\,fx + 2\,e - \sin(2\,fx + 2\,e))Aa^2c^2}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algori
thm="maxima")
```

output
```
1/480*(15*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^2*c^
2 - 240*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c^2 + 480*(f*x + e)*A*a^2*c
^2 - 32*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^2*c^2
 - 320*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*c^2 - 480*B*a^2*c^2*cos(f*x
 + e))/f
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.27

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx = \frac{3}{8}\,Aa^2c^2x - \frac{Ba^2c^2\cos\left(5\,fx+5\,e\right)}{80\,f} - \frac{Ba^2c^2\cos\left(3\,fx+3\,e\right)}{16\,f} - \frac{Ba^2c^2\cos\left(fx+e\right)}{8\,f} + \frac{Aa^2c^2\sin\left(4\,fx+4\,e\right)}{32\,f} + \frac{Aa^2c^2\sin\left(2\,fx+2\,e\right)}{4\,f}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
3/8*A*a^2*c^2*x - 1/80*B*a^2*c^2*cos(5*f*x + 5*e)/f - 1/16*B*a^2*c^2*cos(3
*f*x + 3*e)/f - 1/8*B*a^2*c^2*cos(f*x + e)/f + 1/32*A*a^2*c^2*sin(4*f*x +
4*e)/f + 1/4*A*a^2*c^2*sin(2*f*x + 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 36.94 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.67

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx = \frac{3\,A\,a^2\,c^2\,x}{8} - \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8\left(\frac{a^2\,c^2\,(80\,B-75\,A\,(e+f\,x))}{40}+\frac{15\,A\,a^2\,c^2\,(e+f\,x)}{8}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\left(\frac{a^2\,c^2\,(160\,B-150\,A\,(e+f\,x))}{40}+\frac{15\,A\,a^2}{}\right.}{}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^2,x)
```

output
```
(3*A*a^2*c^2*x)/8 - (tan(e/2 + (f*x)/2)^8*((a^2*c^2*(80*B - 75*A*(e + f*x)
))/40 + (15*A*a^2*c^2*(e + f*x))/8) + tan(e/2 + (f*x)/2)^4*((a^2*c^2*(160*
B - 150*A*(e + f*x)))/40 + (15*A*a^2*c^2*(e + f*x))/4) + (a^2*c^2*(16*B -
15*A*(e + f*x)))/40 + (3*A*a^2*c^2*(e + f*x))/8 - (A*a^2*c^2*tan(e/2 + (f*
x)/2)^3)/2 + (A*a^2*c^2*tan(e/2 + (f*x)/2)^7)/2 + (5*A*a^2*c^2*tan(e/2 + (
f*x)/2)^9)/4 - (5*A*a^2*c^2*tan(e/2 + (f*x)/2))/4)/(f*(tan(e/2 + (f*x)/2)^
2 + 1)^5)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.07

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$$
$$=\frac{a^2c^2\left(-8\cos(fx+e)\sin(fx+e)^4b-10\cos(fx+e)\sin(fx+e)^3a+16\cos(fx+e)\sin(fx+e)^2b+\right.}{40f}$$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x)
```

output
```
(a**2*c**2*( - 8*cos(e + f*x)*sin(e + f*x)**4*b - 10*cos(e + f*x)*sin(e +
f*x)**3*a + 16*cos(e + f*x)*sin(e + f*x)**2*b + 25*cos(e + f*x)*sin(e + f*
x)*a - 8*cos(e + f*x)*b + 15*a*f*x + 8*b))/(40*f)
```

3.30 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 98

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx = \frac{1}{8}a^2(4A+B)cx - \frac{a^2(4A+B)c\cos^3(e+fx)}{12f} + \frac{a^2(4A+B)c\cos(e+fx)\sin(e+fx)}{8f} - \frac{Bc\cos^3(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{4f}$$

output
```
1/8*a^2*(4*A+B)*c*x-1/12*a^2*(4*A+B)*c*cos(f*x+e)^3/f+1/8*a^2*(4*A+B)*c*co
s(f*x+e)*sin(f*x+e)/f-1/4*B*c*cos(f*x+e)^3*(a^2+a^2*sin(f*x+e))/f
```

Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.17

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx = -\frac{a^2c\cos(e+fx)\left(12(4A+B)\arcsin\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}(8A+8B+8(A+B)\cos(2(e+\right.}{48f\sqrt{\cos^2(e+fx)}}$$

input
```
Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
,x]
```

output
```
-1/48*(a^2*c*Cos[e + f*x]*(12*(4*A + B)*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt
[2]] + Sqrt[Cos[e + f*x]^2]*(8*A + 8*B + 8*(A + B)*Cos[2*(e + f*x)] - 3*(8
*A + B)*Sin[e + f*x] + 3*B*Sin[3*(e + f*x)])))/(f*Sqrt[Cos[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.85, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {3042, 3446, 3042, 3339, 3042, 3148, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^2(c-c\sin(e+fx))(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^2(c-c\sin(e+fx))(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3446}$$

$$ac\int \cos^2(e+fx)(\sin(e+fx)a+a)(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3042}$$

$$ac\int \cos(e+fx)^2(\sin(e+fx)a+a)(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3339}$$

$$ac\left(\frac{1}{4}(4A+B)\int \cos^2(e+fx)(\sin(e+fx)a+a)dx-\frac{B\cos^3(e+fx)(a\sin(e+fx)+a)}{4f}\right)$$

$$\Big\downarrow \text{3042}$$

$$ac\left(\frac{1}{4}(4A+B)\int \cos(e+fx)^2(\sin(e+fx)a+a)dx-\frac{B\cos^3(e+fx)(a\sin(e+fx)+a)}{4f}\right)$$

$$\Big\downarrow \text{3148}$$

$$ac\left(\frac{1}{4}(4A+B)\left(a\int\cos^2(e+fx)dx-\frac{a\cos^3(e+fx)}{3f}\right)-\frac{B\cos^3(e+fx)(a\sin(e+fx)+a)}{4f}\right)$$

$$\Big\downarrow \text{3042}$$

$$ac\left(\frac{1}{4}(4A+B)\left(a\int\sin\left(e+fx+\frac{\pi}{2}\right)^2dx-\frac{a\cos^3(e+fx)}{3f}\right)-\frac{B\cos^3(e+fx)(a\sin(e+fx)+a)}{4f}\right)$$

$$\Big\downarrow \text{3115}$$

$$ac\left(\frac{1}{4}(4A+B)\left(a\left(\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}\right)-\frac{a\cos^3(e+fx)}{3f}\right)-\frac{B\cos^3(e+fx)(a\sin(e+fx)+a)}{4f}\right)$$

$$\Big\downarrow \text{24}$$

$$ac\left(\frac{1}{4}(4A+B)\left(a\left(\frac{\sin(e+fx)\cos(e+fx)}{2f}+\frac{x}{2}\right)-\frac{a\cos^3(e+fx)}{3f}\right)-\frac{B\cos^3(e+fx)(a\sin(e+fx)+a)}{4f}\right)$$

input
```
Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),x]
```

output
```
a*c*(-1/4*(B*Cos[e + f*x]^3*(a + a*Sin[e + f*x]))/f + ((4*A + B)*(-1/3*(a*
Cos[e + f*x]^3)/f + a*(x/2 + (Cos[e + f*x]*Sin[e + f*x])/(2*f))))/4)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3339
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(90) = 180$.

Time = 0.23 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.90

$$\frac{\frac{a^2 Ac\left(2+\sin(fx+e)^2\right)\cos(fx+e)}{3} - a^2 Ac\left(-\frac{\sin(fx+e)\cos(fx+e)}{2} + \frac{fx}{2} + \frac{e}{2}\right) - a^2 Bc\left(-\frac{\left(\sin(fx+e)^3 + \frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4} + \right.}{}$$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

output
```
1/f*(1/3*a^2*A*c*(2+sin(f*x+e)^2)*cos(f*x+e)-a^2*A*c*(-1/2*sin(f*x+e)*cos(
f*x+e)+1/2*f*x+1/2*e)-a^2*B*c*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+
e)+3/8*f*x+3/8*e)+1/3*a^2*B*c*(2+sin(f*x+e)^2)*cos(f*x+e)-A*cos(f*x+e)*a^2
*c+a^2*B*c*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+a^2*A*c*(f*x+e)-B*co
s(f*x+e)*a^2*c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.79

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx =$$
$$-\frac{8\,(A+B)a^2c\cos\left(fx+e\right)^3-3\,(4\,A+B)a^2cfx+3\,\left(2\,Ba^2c\cos\left(fx+e\right)^3-(4\,A+B)a^2c\cos\left(fx+e\right.\right.}{24\,f}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
-1/24*(8*(A + B)*a^2*c*cos(f*x + e)^3 - 3*(4*A + B)*a^2*c*f*x + 3*(2*B*a^2
*c*cos(f*x + e)^3 - (4*A + B)*a^2*c*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. 2(90) = 180.

Time = 0.21 (sec) , antiderivative size = 396, normalized size of antiderivative = 4.04

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$$
$$=\begin{cases}-\frac{Aa^2cx\sin^2(e+fx)}{2}-\frac{Aa^2cx\cos^2(e+fx)}{2}+Aa^2cx+\frac{Aa^2c\sin^2(e+fx)\cos(e+fx)}{f}+\frac{Aa^2c\sin(e+fx)\cos(e+fx)}{2f}+\frac{2Aa^2c\cos^3}{3f} \\ x(A+B\sin(e))\left(a\sin(e)+a\right)^2\left(-c\sin(e)+c\right)\end{cases}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

output
```
Piecewise((-A*a**2*c*x*sin(e + f*x)**2/2 - A*a**2*c*x*cos(e + f*x)**2/2 +
A*a**2*c*x + A*a**2*c*sin(e + f*x)**2*cos(e + f*x)/f + A*a**2*c*sin(e + f*
x)*cos(e + f*x)/(2*f) + 2*A*a**2*c*cos(e + f*x)**3/(3*f) - A*a**2*c*cos(e
+ f*x)/f - 3*B*a**2*c*x*sin(e + f*x)**4/8 - 3*B*a**2*c*x*sin(e + f*x)**2*c
os(e + f*x)**2/4 + B*a**2*c*x*sin(e + f*x)**2/2 - 3*B*a**2*c*x*cos(e + f*x
)**4/8 + B*a**2*c*x*cos(e + f*x)**2/2 + 5*B*a**2*c*sin(e + f*x)**3*cos(e +
 f*x)/(8*f) + B*a**2*c*sin(e + f*x)**2*cos(e + f*x)/f + 3*B*a**2*c*sin(e +
 f*x)*cos(e + f*x)**3/(8*f) - B*a**2*c*sin(e + f*x)*cos(e + f*x)/(2*f) + 2
*B*a**2*c*cos(e + f*x)**3/(3*f) - B*a**2*c*cos(e + f*x)/f, Ne(f, 0)), (x*(
A + B*sin(e))*(a*sin(e) + a)**2*(-c*sin(e) + c), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.83

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx =$$

$$-\frac{32\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)Aa^2c+24\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)Aa^2c-96\left(fx+e\right)Aa^2c\cdots}{\cdots}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorith
m="maxima")
```

output
```
-1/96*(32*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*c + 24*(2*f*x + 2*e - si
n(2*f*x + 2*e))*A*a^2*c - 96*(f*x + e)*A*a^2*c + 32*(cos(f*x + e)^3 - 3*co
s(f*x + e))*B*a^2*c + 3*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x +
2*e))*B*a^2*c - 24*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^2*c + 96*A*a^2*c*c
os(f*x + e) + 96*B*a^2*c*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.09

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$$
$$=-\frac{Ba^2c\sin(4fx+4e)}{32f}+\frac{Aa^2c\sin(2fx+2e)}{4f}+\frac{1}{8}(4Aa^2c+Ba^2c)x-\frac{(Aa^2c+Ba^2c)\cos(3fx+3e)}{12f}-\frac{(Aa^2c+Ba^2c)\cos(fx+e)}{4f}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorith
m="giac")
```

output
```
-1/32*B*a^2*c*sin(4*f*x + 4*e)/f + 1/4*A*a^2*c*sin(2*f*x + 2*e)/f + 1/8*(4
*A*a^2*c + B*a^2*c)*x - 1/12*(A*a^2*c + B*a^2*c)*cos(3*f*x + 3*e)/f - 1/4*
(A*a^2*c + B*a^2*c)*cos(f*x + e)/f
```

Mupad [B] (verification not implemented)

Time = 35.70 (sec) , antiderivative size = 339, normalized size of antiderivative = 3.46

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$$
$$=\frac{a^2\,c\,\mathrm{atan}\left(\frac{a^2\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)(4\,A+B)}{4\left(A\,a^2\,c+\frac{B\,a^2\,c}{4}\right)}\right)(4\,A+B)}{4\,f}-\frac{a^2\,c\,(4\,A+B)\,\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-\frac{f\,x}{2}\right)}{4\,f}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4(2\,A\,a^2\,c+2\,B\,a^2\,c)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(A\,a^2\,c-\frac{B\,a^2\,c}{4}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6(2\,A\,a^2\,c+2\,B\,a^2}{f\,\left(\tan\left(\frac{e}{2}+\right.\right.}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x)),x)
```

output
```
(a^2*c*atan((a^2*c*tan(e/2 + (f*x)/2)*(4*A + B))/(4*(A*a^2*c + (B*a^2*c)/4
)))*(4*A + B))/(4*f) - (a^2*c*(4*A + B)*(atan(tan(e/2 + (f*x)/2)) - (f*x)/
2))/(4*f) - (tan(e/2 + (f*x)/2)^4*(2*A*a^2*c + 2*B*a^2*c) - tan(e/2 + (f*x
)/2)*(A*a^2*c - (B*a^2*c)/4) + tan(e/2 + (f*x)/2)^6*(2*A*a^2*c + 2*B*a^2*c
) + tan(e/2 + (f*x)/2)^2*((2*A*a^2*c)/3 + (2*B*a^2*c)/3) + tan(e/2 + (f*x)
/2)^7*(A*a^2*c - (B*a^2*c)/4) - tan(e/2 + (f*x)/2)^3*(A*a^2*c + (7*B*a^2*c
)/4) + tan(e/2 + (f*x)/2)^5*(A*a^2*c + (7*B*a^2*c)/4) + (2*A*a^2*c)/3 + (2
*B*a^2*c)/3)/(f*(4*tan(e/2 + (f*x)/2)^2 + 6*tan(e/2 + (f*x)/2)^4 + 4*tan(e
/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.28

$$\int (a + a\sin(e + fx))^2 (A + B\sin(e + fx))(c - c\sin(e + fx))\,dx$$
$$= \frac{a^2 c\big(6\cos(fx+e)\sin(fx+e)^3 b + 8\cos(fx+e)\sin(fx+e)^2 a + 8\cos(fx+e)\sin(fx+e)^2 b + 12\,\mathrm{co}}{}$$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

output
```
(a**2*c*(6*cos(e + f*x)*sin(e + f*x)**3*b + 8*cos(e + f*x)*sin(e + f*x)**2
*a + 8*cos(e + f*x)*sin(e + f*x)**2*b + 12*cos(e + f*x)*sin(e + f*x)*a - 3
*cos(e + f*x)*sin(e + f*x)*b - 8*cos(e + f*x)*a - 8*cos(e + f*x)*b + 12*a*
f*x + 8*a + 3*b*f*x + 8*b))/(24*f)
```

3.31 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 117

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = -\frac{3a^2(2A+3B)x}{2c}+\frac{3a^2(2A+3B)\cos(e+fx)}{2cf}+\frac{a^2(A+B)c^2\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}+\frac{a^2(2A+3B)\cos^3(e+fx)}{2f(c-c\sin(e+fx))}$$

output
```
-3/2*a^2*(2*A+3*B)*x/c+3/2*a^2*(2*A+3*B)*cos(f*x+e)/c/f+a^2*(A+B)*c^2*cos(
f*x+e)^5/f/(c-c*sin(f*x+e))^3+1/2*a^2*(2*A+3*B)*cos(f*x+e)^3/f/(c-c*sin(f*
x+e))
```

Mathematica [A] (verified)

Time = 12.28 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.63

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = \frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^2\left(\cos\left(\frac{1}{2}(e+fx)\right)(6(2A+3B)(e+fx)-4(A\right.}{4cf\left(\cos\right.}$$

input
```
Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
]),x]
```

output
```
(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*(Cos[(e +
f*x)/2]*(6*(2*A + 3*B)*(e + f*x) - 4*(A + 3*B)*Cos[e + f*x] - B*Sin[2*(e +
 f*x)]) - Sin[(e + f*x)/2]*(4*A*(8 + 3*e + 3*f*x) + 2*B*(16 + 9*e + 9*f*x)
 - 4*(A + 3*B)*Cos[e + f*x] - B*Sin[2*(e + f*x)])))/(4*c*f*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^4*(-1 + Sin[e + f*x]))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3446, 3042, 3338, 3042, 3158, 3042, 3161, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{c-c\sin(e+fx)}dx$$

$$\Big\downarrow \; 3446$$

$$a^2c^2\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}dx$$

$$\Big\downarrow \; 3042$$

$$a^2c^2\int \frac{\cos(e+fx)^4(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}dx$$

$$\Big\downarrow \; 3338$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}-\frac{(2A+3B)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^2}dx}{c}\right)$$

$$\Big\downarrow \; 3042$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}-\frac{(2A+3B)\int\frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^2}dx}{c}\right)$$

$\Big\downarrow$ 3158

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}-\frac{(2A+3B)\left(\frac{3\int\frac{\cos^2(e+fx)}{c-c\sin(e+fx)}dx}{2c}-\frac{\cos^3(e+fx)}{2f(c^2-c^2\sin(e+fx))}\right)}{c}\right)$$

$\Big\downarrow$ 3042

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}-\frac{(2A+3B)\left(\frac{3\int\frac{\cos(e+fx)^2}{c-c\sin(e+fx)}dx}{2c}-\frac{\cos^3(e+fx)}{2f(c^2-c^2\sin(e+fx))}\right)}{c}\right)$$

$\Big\downarrow$ 3161

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}-\frac{(2A+3B)\left(\frac{3\left(\frac{\int 1dx}{c}-\frac{\cos(e+fx)}{cf}\right)}{2c}-\frac{\cos^3(e+fx)}{2f(c^2-c^2\sin(e+fx))}\right)}{c}\right)$$

$\Big\downarrow$ 24

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{f(c-c\sin(e+fx))^3}-\frac{(2A+3B)\left(\frac{3\left(\frac{x}{c}-\frac{\cos(e+fx)}{cf}\right)}{2c}-\frac{\cos^3(e+fx)}{2f(c^2-c^2\sin(e+fx))}\right)}{c}\right)$$

input
```
Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]),x]
```

output
```
a^2*c^2*(((A + B)*Cos[e + f*x]^5)/(f*(c - c*Sin[e + f*x])^3) - ((2*A + 3*B
)*((3*(x/c - Cos[e + f*x]/(c*f)))/(2*c) - Cos[e + f*x]^3/(2*f*(c^2 - c^2*S
in[e + f*x]))))/c)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3158
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(a*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
 EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && In
tegersQ[2*m, 2*p]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.79

method	result
parallelrisch	$\frac{4a^2\left(\frac{(A+3B)\cos(2fx+2e)}{8}+\frac{B\sin(3fx+3e)}{32}+\frac{\left(-3fxA-\frac{9}{2}fxB+5A+7B\right)\cos(fx+e)}{4}+\left(A+\frac{33B}{32}\right)\sin(fx+e)+\frac{9A}{8}+\frac{11B}{8}\right)}{cf\cos(fx+e)}$
derivativedivides	$\frac{2a^2\left(-\frac{4A+4B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2}+(-A-3B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}-A-3B}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^2}-\frac{3(2A+3B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}{2}\right.}{fc}$
default	$\frac{2a^2\left(-\frac{4A+4B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2}+(-A-3B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}-A-3B}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^2}-\frac{3(2A+3B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}{2}\right.}{fc}$
risch	$-\frac{3a^2xA}{c}-\frac{9a^2xB}{2c}+\frac{a^2e^{i(fx+e)}A}{2cf}+\frac{3a^2e^{i(fx+e)}B}{2cf}+\frac{a^2e^{-i(fx+e)}A}{2cf}+\frac{3a^2e^{-i(fx+e)}B}{2cf}+\frac{8a^2A}{fc\left(e^{i(fx+e)}-i\right)}$
norman	$\frac{-\frac{2a^2A+5a^2B}{cf}+\frac{3a^2(2A+3B)x}{2c}-\frac{\left(2a^2A+3a^2B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{cf}-\frac{\left(4a^2A+8a^2B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{cf}-\frac{\left(6a^2A+4a^2B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{cf}-}{}$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x,method=_RETURNV
ERBOSE)
```

output
```
4*a^2*(1/8*(A+3*B)*cos(2*f*x+2*e)+1/32*B*sin(3*f*x+3*e)+1/4*(-3*f*x*A-9/2*
f*x*B+5*A+7*B)*cos(f*x+e)+(A+33/32*B)*sin(f*x+e)+9/8*A+11/8*B)/c/f/cos(f*x
+e)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.51

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$$
$$=\frac{Ba^2\cos(fx+e)^3-3(2A+3B)a^2fx+2(A+3B)a^2\cos(fx+e)^2+8(A+B)a^2-(3(2A+3B)a^2}{2(cf}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
1/2*(B*a^2*cos(f*x + e)^3 - 3*(2*A + 3*B)*a^2*f*x + 2*(A + 3*B)*a^2*cos(f*
x + e)^2 + 8*(A + B)*a^2 - (3*(2*A + 3*B)*a^2*f*x - (10*A + 13*B)*a^2)*cos
(f*x + e) + (3*(2*A + 3*B)*a^2*f*x + B*a^2*cos(f*x + e)^2 - (2*A + 5*B)*a^
2*cos(f*x + e) + 8*(A + B)*a^2)*sin(f*x + e))/(c*f*cos(f*x + e) - c*f*sin(
f*x + e) + c*f)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2365 vs. $2(104) = 208$.

Time = 2.16 (sec) , antiderivative size = 2365, normalized size of antiderivative = 20.21

$$\int \frac{(a + a\sin(e + fx))^2 (A + B\sin(e + fx))}{c - c\sin(e + fx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

output
```
Piecewise((-6*A*a**2*f*x*tan(e/2 + f*x/2)**5/(2*c*f*tan(e/2 + f*x/2)**5 -
2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*
x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) + 6*A*a**2*f*x*tan(e/2 + f*x/2)*
*4/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2
+ f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f)
- 12*A*a**2*f*x*tan(e/2 + f*x/2)**3/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan
(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 +
 2*c*f*tan(e/2 + f*x/2) - 2*c*f) + 12*A*a**2*f*x*tan(e/2 + f*x/2)**2/(2*c*
f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)
**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 6*A*a*
*2*f*x*tan(e/2 + f*x/2)/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2
)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e
/2 + f*x/2) - 2*c*f) + 6*A*a**2*f*x/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan
(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 +
 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 16*A*a**2*tan(e/2 + f*x/2)**4/(2*c*f*ta
n(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3
- 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) + 4*A*a**2*t
an(e/2 + f*x/2)**3/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4
+ 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 +
f*x/2) - 2*c*f) - 36*A*a**2*tan(e/2 + f*x/2)**2/(2*c*f*tan(e/2 + f*x/2)...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(114) = 228$.

Time = 0.13 (sec) , antiderivative size = 624, normalized size of antiderivative = 5.33

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{c - c\sin(e + fx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorith
m="maxima")
```

output
```
-(2*A*a^2*((sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 - 2)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1) + c*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*
x + e)/(cos(f*x + e) + 1))/c) + 4*B*a^2*((sin(f*x + e)/(cos(f*x + e) + 1)
- sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2)/(c - c*sin(f*x + e)/(cos(f*x +
e) + 1) + c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c) + B*a^2*((sin(
f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3 - 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
- 4)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1) + 2*c*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 - 2*c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + c*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 - c*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(si
n(f*x + e)/(cos(f*x + e) + 1))/c) + 4*A*a^2*(arctan(sin(f*x + e)/(cos(f*x
+ e) + 1))/c - 1/(c - c*sin(f*x + e)/(cos(f*x + e) + 1))) + 2*B*a^2*(arcta
n(sin(f*x + e)/(cos(f*x + e) + 1))/c - 1/(c - c*sin(f*x + e)/(cos(f*x + e)
 + 1))) - 2*A*a^2/(c - c*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.33

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx =$$
$$-\frac{\frac{3\,(2\,Aa^2+3\,Ba^2)(fx+e)}{c}+\frac{16\,(Aa^2+Ba^2)}{c\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)}+\frac{2\left(Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-2\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-6\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-Ba^2\tan\left(\right.\right.}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2c}}{2\,f}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorith
m="giac")
```

output
```
-1/2*(3*(2*A*a^2 + 3*B*a^2)*(f*x + e)/c + 16*(A*a^2 + B*a^2)/(c*(tan(1/2*f
*x + 1/2*e) - 1)) + 2*(B*a^2*tan(1/2*f*x + 1/2*e)^3 - 2*A*a^2*tan(1/2*f*x
+ 1/2*e)^2 - 6*B*a^2*tan(1/2*f*x + 1/2*e)^2 - B*a^2*tan(1/2*f*x + 1/2*e) -
 2*A*a^2 - 6*B*a^2)/((tan(1/2*f*x + 1/2*e)^2 + 1)^2*c))/f
```

Mupad [B] (verification not implemented)

Time = 37.23 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.09

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$$
$$= \frac{10\,A\,a^2-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,A\,a^2+5\,B\,a^2\right)+14\,B\,a^2-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\left(2\,A\,a^2+7\,B\,a^2\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\left(8\right.}{f\,\left(-c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-2\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+2\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right.}$$
$$-\frac{3\,a^2\,\mathrm{atan}\left(\frac{3\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(2\,A+3\,B)}{6\,A\,a^2+9\,B\,a^2}\right)\,(2\,A+3\,B)}{c\,f}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x)),x)
```

output
```
(10*A*a^2 - tan(e/2 + (f*x)/2)*(2*A*a^2 + 5*B*a^2) + 14*B*a^2 - tan(e/2 +
(f*x)/2)^3*(2*A*a^2 + 7*B*a^2) + tan(e/2 + (f*x)/2)^4*(8*A*a^2 + 9*B*a^2)
+ tan(e/2 + (f*x)/2)^2*(18*A*a^2 + 21*B*a^2))/(f*(c - c*tan(e/2 + (f*x)/2)
 + 2*c*tan(e/2 + (f*x)/2)^2 - 2*c*tan(e/2 + (f*x)/2)^3 + c*tan(e/2 + (f*x)
/2)^4 - c*tan(e/2 + (f*x)/2)^5)) - (3*a^2*atan((3*a^2*tan(e/2 + (f*x)/2)*(
2*A + 3*B))/(6*A*a^2 + 9*B*a^2))*(2*A + 3*B))/(c*f)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.74

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$$
$$= \frac{a^2\left(\cos\left(fx+e\right)\sin\left(fx+e\right)^2b+2\cos\left(fx+e\right)\sin\left(fx+e\right)a+5\cos\left(fx+e\right)\sin\left(fx+e\right)b-6\cos\left(fx\right.\right.}{}$$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

output
```
(a**2*(cos(e + f*x)*sin(e + f*x)**2*b + 2*cos(e + f*x)*sin(e + f*x)*a + 5*
cos(e + f*x)*sin(e + f*x)*b - 6*cos(e + f*x)*a*f*x - 16*cos(e + f*x)*a - 9
*cos(e + f*x)*b*f*x - 18*cos(e + f*x)*b - sin(e + f*x)**3*b - 2*sin(e + f*
x)**2*a - 6*sin(e + f*x)**2*b - 6*sin(e + f*x)*a*f*x + 2*sin(e + f*x)*a -
9*sin(e + f*x)*b*f*x + 5*sin(e + f*x)*b + 6*a*f*x + 16*a + 9*b*f*x + 18*b)
)/(2*c*f*(cos(e + f*x) + sin(e + f*x) - 1))
```

3.32 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 109

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \frac{a^2(A+4B)x}{c^2} - \frac{a^2(A+4B)\cos(e+fx)}{c^2f} + \frac{a^2(A+B)c^2\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4} - \frac{2a^2(A+4B)\cos^3(e+fx)}{3f(c-c\sin(e+fx))^2}$$

output
```
a^2*(A+4*B)*x/c^2-a^2*(A+4*B)*cos(f*x+e)/c^2/f+1/3*a^2*(A+B)*c^2*cos(f*x+e
)^5/f/(c-c*sin(f*x+e))^4-2/3*a^2*(A+4*B)*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 238 vs. 2(109) = 218.

Time = 11.42 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.18

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(4(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+3(A+4B)(e-\right.}{}$$

input
```
Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^2,x]
```

output
```
(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(4*(A + B)*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]) + 3*(A + 4*B)*(e + f*x)*(Cos[(e + f*x)/2] - Sin[(e + f*x
)/2])^3 - 3*B*Cos[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 + 8*(A
+ B)*Sin[(e + f*x)/2] - 8*(2*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
)^2*Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2)/(3*f*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^4*(c - c*Sin[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3161, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}dx$$

$$\Big\downarrow \text{3446}$$

$$a^2c^2\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\int \frac{\cos(e+fx)^4(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}dx$$

$$\Big\downarrow \text{3338}$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4}-\frac{(A+4B)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^3}dx}{3c}\right)$$

$$\Big\downarrow \textbf{3042}$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4}-\frac{(A+4B)\int\frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^3}dx}{3c}\right)$$

$$\Big\downarrow \textbf{3159}$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4}-\frac{(A+4B)\left(\frac{2\cos^3(e+fx)}{cf(c-c\sin(e+fx))^2}-\frac{3\int\frac{\cos^2(e+fx)}{c-c\sin(e+fx)}dx}{c^2}\right)}{3c}\right)$$

$$\Big\downarrow \textbf{3042}$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4}-\frac{(A+4B)\left(\frac{2\cos^3(e+fx)}{cf(c-c\sin(e+fx))^2}-\frac{3\int\frac{\cos(e+fx)^2}{c-c\sin(e+fx)}dx}{c^2}\right)}{3c}\right)$$

$$\Big\downarrow \textbf{3161}$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4}-\frac{(A+4B)\left(\frac{2\cos^3(e+fx)}{cf(c-c\sin(e+fx))^2}-\frac{3\left(\frac{\int 1dx}{c}-\frac{\cos(e+fx)}{cf}\right)}{c^2}\right)}{3c}\right)$$

$$\Big\downarrow \textbf{24}$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{3f(c-c\sin(e+fx))^4}-\frac{(A+4B)\left(\frac{2\cos^3(e+fx)}{cf(c-c\sin(e+fx))^2}-\frac{3\left(\frac{x}{c}-\frac{\cos(e+fx)}{cf}\right)}{c^2}\right)}{3c}\right)$$

input
```
Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^2,x
]
```

output
```
a^2*c^2*(((A + B)*Cos[e + f*x]^5)/(3*f*(c - c*Sin[e + f*x])^4) - ((A + 4*B
)*((-3*(x/c - Cos[e + f*x]/(c*f)))/c^2 + (2*Cos[e + f*x]^3)/(c*f*(c - c*Si
n[e + f*x])^2)))/(3*c))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{2a^2\left(-\frac{B}{1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}+(A+4B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{8A+8B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{8A+8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}+\frac{4B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}\right)}{f\,c^2}$
default	$\frac{2a^2\left(-\frac{B}{1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}+(A+4B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{8A+8B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{8A+8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}+\frac{4B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}\right)}{f\,c^2}$
risch	$\frac{a^2xA}{c^2}+\frac{4a^2xB}{c^2}-\frac{B\,a^2\mathrm{e}^{i(fx+e)}}{2c^2f}-\frac{B\,a^2\mathrm{e}^{-i(fx+e)}}{2c^2f}-\frac{8\left(-3iA\,a^2\mathrm{e}^{i(fx+e)}+3A\,a^2\mathrm{e}^{2i(fx+e)}-9iB\,a^2\mathrm{e}^{i(fx+e)}+6B\right.}{3\left(\mathrm{e}^{i(fx+e)}-i\right)^3f\,c^2}$
parallelrisch	$\frac{3a^2\left(\left(4(-fx+2)B-fxA+\frac{4A}{3}\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{\left(\left(\frac{11}{6}+4fx\right)B+fxA+\frac{4A}{3}\right)\cos\left(\frac{3fx}{2}+\frac{3e}{2}\right)}{3}+\left(fxA+4fxB-\frac{4}{3}A-\frac{14}{3}B\right)\sin\left(\right.\right.}{f\,c^2\left(-3\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\cos\left(\frac{3fx}{2}+\frac{3e}{2}\right)+\sin\left(\frac{3fx}{2}+\frac{3e}{2}\right.\right.}$
norman	$\frac{\frac{8a^2B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{cf}+\frac{a^2(A+4B)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{c}+\frac{8a^2A+38a^2B}{3cf}-\frac{a^2(A+4B)x}{c}-\frac{2\left(4a^2A+13a^2B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{cf}+\frac{2\left(4a^2A+25\right.}{}}{}$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x,method=_RETUR
NVERBOSE)
```

output
```
2/f*a^2/c^2*(-B/(1+tan(1/2*f*x+1/2*e)^2)+(A+4*B)*arctan(tan(1/2*f*x+1/2*e)
)-1/3*(8*A+8*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/2*(8*A+8*B)/(tan(1/2*f*x+1/2*e)
-1)^2+4*B/(tan(1/2*f*x+1/2*e)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(107) = 214.

Time = 0.09 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.17

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx =$$

$$-\frac{3\,Ba^2\cos\left(fx+e\right)^3+6\,(A+4\,B)a^2fx+4\,(A+B)a^2-(3\,(A+4\,B)a^2fx+(8\,A+23\,B)a^2)\cos\left(fx\right.}{3\,(\ldots}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
-1/3*(3*B*a^2*cos(f*x + e)^3 + 6*(A + 4*B)*a^2*f*x + 4*(A + B)*a^2 - (3*(A
 + 4*B)*a^2*f*x + (8*A + 23*B)*a^2)*cos(f*x + e)^2 + (3*(A + 4*B)*a^2*f*x
- 2*(2*A + 11*B)*a^2)*cos(f*x + e) - (6*(A + 4*B)*a^2*f*x - 3*B*a^2*cos(f*
x + e)^2 - 4*(A + B)*a^2 + (3*(A + 4*B)*a^2*f*x - 2*(4*A + 13*B)*a^2)*cos(
f*x + e))*sin(f*x + e))/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e) - 2*c^2
*f + (c^2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2474 vs. $2(100) = 200$.

Time = 4.37 (sec) , antiderivative size = 2474, normalized size of antiderivative = 22.70

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**2,x)
```

output
```
Piecewise((3*A*a**2*f*x*tan(e/2 + f*x/2)**5/(3*c**2*f*tan(e/2 + f*x/2)**5
- 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f
*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 9*A*a**2*f*
x*tan(e/2 + f*x/2)**4/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f
*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 +
 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 12*A*a**2*f*x*tan(e/2 + f*x/2)**3
/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*
tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f
*x/2) - 3*c**2*f) - 12*A*a**2*f*x*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 +
f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 -
 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 9
*A*a**2*f*x*tan(e/2 + f*x/2)/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(
e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/
2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 3*A*a**2*f*x/(3*c**2*f*tan
(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/
2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2
*f) - 24*A*a**2*tan(e/2 + f*x/2)**3/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2
*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2
 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 8*A*a**2*tan(e/2 +
f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 ...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 839 vs. $2(107) = 214$.

Time = 0.14 (sec) , antiderivative size = 839, normalized size of antiderivative = 7.70

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algori
thm="maxima")
```

output
```
2/3*(2*B*a^2*((12*sin(f*x + e)/(cos(f*x + e) + 1) - 11*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 3*sin(f*x + e)^
4/(cos(f*x + e) + 1)^4 - 5)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) +
 4*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4*c^2*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 3*c^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^2*sin(f*x + e
)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^2)
 + A*a^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 - 4)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)
+ 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^2) + 2*B*a^2*((9*sin(f*x + e
)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4)/(c^2 - 3
*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)/
(cos(f*x + e) + 1))/c^2) - A*a^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 3*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x
+ e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3
/(cos(f*x + e) + 1)^3) + 2*A*a^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 1)/(
c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + B*a^2*(3*sin(f*
x + e)/(cos(f*x + e) + 1) - 1)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) ...
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.18

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$$
$$= \frac{\frac{3\,(Aa^2+4\,Ba^2)(fx+e)}{c^2} - \frac{6\,Ba^2}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)c^2} + \frac{8\left(3\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-3\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-9\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+Aa^2+4\,Ba^2\right)}{c^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^3}}{3\,f}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
1/3*(3*(A*a^2 + 4*B*a^2)*(f*x + e)/c^2 - 6*B*a^2/((tan(1/2*f*x + 1/2*e)^2
+ 1)*c^2) + 8*(3*B*a^2*tan(1/2*f*x + 1/2*e)^2 - 3*A*a^2*tan(1/2*f*x + 1/2*
e) - 9*B*a^2*tan(1/2*f*x + 1/2*e) + A*a^2 + 4*B*a^2)/(c^2*(tan(1/2*f*x + 1
/2*e) - 1)^3))/f
```

Mupad [B] (verification not implemented)

Time = 37.40 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.26

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$$
$$= \frac{2\,a^2\,\mathrm{atan}\left(\frac{2\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(A+4\,B)}{2\,A\,a^2+8\,B\,a^2}\right)\,(A+4\,B)}{c^2\,f} - \frac{\frac{8\,A\,a^2}{3}-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(8\,A\,a^2+30\,B\,a^2\right)+\frac{38\,B\,a^2}{3}-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\left(8\,A\,a^2+26\,B\,a^2\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(\right.}{f\,\left(-c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+3\,c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-4\,c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+4\,c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-\right.}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^2,x
)
```

output
```
(2*a^2*atan((2*a^2*tan(e/2 + (f*x)/2)*(A + 4*B))/(2*A*a^2 + 8*B*a^2))*(A +
 4*B))/(c^2*f) - ((8*A*a^2)/3 - tan(e/2 + (f*x)/2)*(8*A*a^2 + 30*B*a^2) +
(38*B*a^2)/3 - tan(e/2 + (f*x)/2)^3*(8*A*a^2 + 26*B*a^2) + tan(e/2 + (f*x)
/2)^2*((8*A*a^2)/3 + (74*B*a^2)/3) + 8*B*a^2*tan(e/2 + (f*x)/2)^4)/(f*(4*c
^2*tan(e/2 + (f*x)/2)^2 - 4*c^2*tan(e/2 + (f*x)/2)^3 + 3*c^2*tan(e/2 + (f*
x)/2)^4 - c^2*tan(e/2 + (f*x)/2)^5 + c^2 - 3*c^2*tan(e/2 + (f*x)/2)))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.54

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$$
$$= \frac{a^2\left(-3\cos(fx+e)\sin(fx+e)^2\,b+3\cos(fx+e)\sin(fx+e)\,afx+4\cos(fx+e)\sin(fx+e)\,a+12\right.}{}$$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x)
```

output
```
(a**2*( - 3*cos(e + f*x)*sin(e + f*x)**2*b + 3*cos(e + f*x)*sin(e + f*x)*a
*f*x + 4*cos(e + f*x)*sin(e + f*x)*a + 12*cos(e + f*x)*sin(e + f*x)*b*f*x
+ 15*cos(e + f*x)*sin(e + f*x)*b - 3*cos(e + f*x)*a*f*x - 12*cos(e + f*x)*
b*f*x - 8*cos(e + f*x)*b + 3*sin(e + f*x)**3*b + 3*sin(e + f*x)**2*a*f*x -
 12*sin(e + f*x)**2*a + 12*sin(e + f*x)**2*b*f*x - 34*sin(e + f*x)**2*b -
6*sin(e + f*x)*a*f*x + 4*sin(e + f*x)*a - 24*sin(e + f*x)*b*f*x + 15*sin(e
 + f*x)*b + 3*a*f*x + 12*b*f*x + 8*b))/(3*c**2*f*(cos(e + f*x)*sin(e + f*x
) - cos(e + f*x) + sin(e + f*x)**2 - 2*sin(e + f*x) + 1))
```

3.33 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 112

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx &= -\frac{a^2Bx}{c^3}+\frac{a^2(A+B)c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5} \\ &\quad -\frac{2a^2B\cos^3(e+fx)}{3f(c-c\sin(e+fx))^3}+\frac{2a^2B\cos(e+fx)}{f\left(c^3-c^3\sin(e+fx)\right)}\end{aligned}$$

output
```
-a^2*B*x/c^3+1/5*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^5-2/3*a^2*B
*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^3+2*a^2*B*cos(f*x+e)/f/(c^3-c^3*sin(f*x+e
))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 278 vs. 2(112) = 224.

Time = 11.48 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.48

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$$

$$= \frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\Big(12(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-4(3A+8B)}{}$$

input
```
Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^3,x]
```

output
```
(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(12*(A + B)*(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2]) - 4*(3*A + 8*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3
 - 15*B*(e + f*x)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 + 24*(A + B)*Sin
[(e + f*x)/2] - 8*(3*A + 8*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[
(e + f*x)/2] + 2*(3*A + 43*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[
(e + f*x)/2])*(1 + Sin[e + f*x])^2)/(15*f*(Cos[(e + f*x)/2] + Sin[(e + f*x
)/2])^4*(c - c*Sin[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3159, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}dx$$

$$\Big\downarrow \text{3446}$$

$$a^2c^2\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}dx$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\int \frac{\cos(e+fx)^4(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}dx$$

$$\Big\downarrow \text{3338}$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}-\frac{B\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^4}dx}{c}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}-\frac{B\int\frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^4}dx}{c}\right)$$

$$\Big\downarrow \text{3159}$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}-\frac{B\left(\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^3}-\frac{\int\frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^2}dx}{c^2}\right)}{c}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}-\frac{B\left(\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^3}-\frac{\int\frac{\cos(e+fx)^2}{(c-c\sin(e+fx))^2}dx}{c^2}\right)}{c}\right)$$

$$\Big\downarrow \text{3159}$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}-\frac{B\left(\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^3}-\frac{\frac{2\cos(e+fx)}{f\left(c^2-c^2\sin(e+fx)\right)}-\frac{\int 1dx}{c^2}}{c^2}\right)}{c}\right)$$

$$\Big\downarrow \text{24}$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}-\frac{B\left(\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^3}-\frac{\frac{2\cos(e+fx)}{f\left(c^2-c^2\sin(e+fx)\right)}-\frac{x}{c^2}}{c^2}\right)}{c}\right)$$

input
```
Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^3,x
]
```

output
```
a^2*c^2*(((A + B)*Cos[e + f*x]^5)/(5*f*(c - c*Sin[e + f*x])^5) - (B*((2*Co
s[e + f*x]^3)/(3*c*f*(c - c*Sin[e + f*x])^3) - (-(x/c^2) + (2*Cos[e + f*x]
)/(f*(c^2 - c^2*Sin[e + f*x])))/c^2))/c)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{2a^2\left(-\frac{A+B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{16A+16B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{24A+16B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{32A+32B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{4A}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-B\arctan(\right.}{f\,c^3}$
default	$\frac{2a^2\left(-\frac{A+B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{16A+16B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{24A+16B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{32A+32B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{4A}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-B\arctan(\right.}{f\,c^3}$
parallelrisch	$-\frac{2\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5 xfB}{2}+\left(-\frac{5}{2}fxB+A+B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+B(5fx-4)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3+\left(-5fxB+2A+\frac{34}{3}B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}{f\,c^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}$
risch	$-\frac{a^2Bx}{c^3}+\frac{-4A\,a^2\mathrm{e}^{2i(fx+e)}+2A\,a^2\mathrm{e}^{4i(fx+e)}-\frac{100B\,a^2\mathrm{e}^{2i(fx+e)}}{3}-24iB\,a^2\mathrm{e}^{3i(fx+e)}+\frac{56iB\,a^2\mathrm{e}^{i(fx+e)}}{3}+10B\,a^2\mathrm{e}^{4i(f}}{\left(\mathrm{e}^{i(fx+e)}-i\right)^5 f\,c^3}$
norman	$\frac{\frac{a^2xB}{c}+\frac{8a^2B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{fc}+\frac{48a^2B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{fc}-\frac{6a^2A+46a^2B}{15fc}+\frac{40a^2B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3fc}+\frac{64a^2B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{fc}+\frac{112a^2B\tan(}{3f}}{}$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x,method=_RETUR
NVERBOSE)
```

output
```
2/f*a^2/c^3*(-(A+B)/(tan(1/2*f*x+1/2*e)-1)-1/5*(16*A+16*B)/(tan(1/2*f*x+1/
2*e)-1)^5-1/3*(24*A+16*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/4*(32*A+32*B)/(tan(1/
2*f*x+1/2*e)-1)^4-4*A/(tan(1/2*f*x+1/2*e)-1)^2-B*arctan(tan(1/2*f*x+1/2*e)
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. 2(111) = 222.

Time = 0.09 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.47

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$$
$$=\frac{60\,Ba^2fx-(15\,Ba^2fx-(3\,A+43\,B)a^2)\cos{(fx+e)}^3-12\,(A+B)a^2-(45\,Ba^2fx-(9\,A-11\,B)a^2}{15\,(c^3f\cos{(fx+e)}^3+3}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algori
thm="fricas")
```

output
```
1/15*(60*B*a^2*f*x - (15*B*a^2*f*x - (3*A + 43*B)*a^2)*cos(f*x + e)^3 - 12
*(A + B)*a^2 - (45*B*a^2*f*x - (9*A - 11*B)*a^2)*cos(f*x + e)^2 + 6*(5*B*a
^2*f*x - (A + 11*B)*a^2)*cos(f*x + e) - (60*B*a^2*f*x + 12*(A + B)*a^2 - (
15*B*a^2*f*x + (3*A + 43*B)*a^2)*cos(f*x + e)^2 + 6*(5*B*a^2*f*x + (A - 9*
B)*a^2)*cos(f*x + e))*sin(f*x + e))/(c^3*f*cos(f*x + e)^3 + 3*c^3*f*cos(f*
x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f - (c^3*f*cos(f*x + e)^2 - 2*c^3*
f*cos(f*x + e) - 4*c^3*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1647 vs. $2(102) = 204$.

Time = 8.59 (sec) , antiderivative size = 1647, normalized size of antiderivative = 14.71

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**3,x)
```

output
```
Piecewise((-30*A*a**2*tan(e/2 + f*x/2)**4/(15*c**3*f*tan(e/2 + f*x/2)**5 -
 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3
*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 60*A*a*
*2*tan(e/2 + f*x/2)**2/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2
+ f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)
**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 6*A*a**2/(15*c**3*f*tan(e/
2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2
)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c*
*3*f) - 15*B*a**2*f*x*tan(e/2 + f*x/2)**5/(15*c**3*f*tan(e/2 + f*x/2)**5 -
 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3
*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 75*B*a*
*2*f*x*tan(e/2 + f*x/2)**4/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(
e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*
x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 150*B*a**2*f*x*tan(e/2
 + f*x/2)**3/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**
4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c
**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 150*B*a**2*f*x*tan(e/2 + f*x/2)**2/(
15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f
*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2
+ f*x/2) - 15*c**3*f) - 75*B*a**2*f*x*tan(e/2 + f*x/2)/(15*c**3*f*tan(e...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1139 vs. $2(111) = 222$.

Time = 0.15 (sec) , antiderivative size = 1139, normalized size of antiderivative = 10.17

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algori
thm="maxima")
```

output
```
-2/15*(B*a^2*((95*sin(f*x + e)/(cos(f*x + e) + 1) - 145*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 15*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 - 22)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) +
1) + 10*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1
))/c^3) + A*a^2*(20*sin(f*x + e)/(cos(f*x + e) + 1) - 40*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 15*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 - 7)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) +
1) + 10*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5) - 6*A*a^2*(5*sin(f*x + e)/(cos(f*x + e) +
1) - 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 - 1)/(c^3 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
 + 5*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x
 + e) + 1)^5) - 3*B*a^2*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/(c^3
 - 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*...
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.35

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = -\frac{\frac{15\,(fx+e)Ba^2}{c^3} + \frac{2\left(15\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+15\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-60\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+30\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+170\,Ba^2\tan\left(\frac{1}{2}\right.\right.}{c^3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^5}}{15\,f}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algori
thm="giac")
```

output
```
-1/15*(15*(f*x + e)*B*a^2/c^3 + 2*(15*A*a^2*tan(1/2*f*x + 1/2*e)^4 + 15*B*
a^2*tan(1/2*f*x + 1/2*e)^4 - 60*B*a^2*tan(1/2*f*x + 1/2*e)^3 + 30*A*a^2*ta
n(1/2*f*x + 1/2*e)^2 + 170*B*a^2*tan(1/2*f*x + 1/2*e)^2 - 100*B*a^2*tan(1/
2*f*x + 1/2*e) + 3*A*a^2 + 23*B*a^2)/(c^3*(tan(1/2*f*x + 1/2*e) - 1)^5))/f
```

Mupad [B] (verification not implemented)

Time = 33.52 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.08

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^3} \, dx = -\frac{B\,a^2\,x}{c^3} - \frac{\frac{a^2\,(6\,A+46\,B-15\,B\,(e+f\,x))}{15} - \tan\left(\frac{e}{2} + \frac{f\,x}{2}\right)^3 \left(\frac{a^2\,(120\,B-150\,B\,(e+f\,x))}{15} + 10\,B\,a^2\,(e+f\,x)\right) + \tan\left(\frac{e}{2} + \frac{f\,x}{2}\right)^4 \left(\frac{a^2}{}\right.}{}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^3,x
)
```

output
```
- (B*a^2*x)/c^3 - ((a^2*(6*A + 46*B - 15*B*(e + f*x)))/15 - tan(e/2 + (f*x
)/2)^3*((a^2*(120*B - 150*B*(e + f*x)))/15 + 10*B*a^2*(e + f*x)) + tan(e/2
 + (f*x)/2)^4*((a^2*(30*A + 30*B - 75*B*(e + f*x)))/15 + 5*B*a^2*(e + f*x)
) + tan(e/2 + (f*x)/2)^2*((a^2*(60*A + 340*B - 150*B*(e + f*x)))/15 + 10*B
*a^2*(e + f*x)) - tan(e/2 + (f*x)/2)*((a^2*(200*B - 75*B*(e + f*x)))/15 +
5*B*a^2*(e + f*x)) + B*a^2*(e + f*x))/(c^3*f*(tan(e/2 + (f*x)/2) - 1)^5)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.29

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^3} \, dx = \frac{a^2\left(-6\tan\left(\frac{fx}{2} + \frac{e}{2}\right)^5 a - 15\tan\left(\frac{fx}{2} + \frac{e}{2}\right)^5 bfx - 6\tan\left(\frac{fx}{2} + \frac{e}{2}\right)^5 b + 75\tan\left(\frac{fx}{2} + \frac{e}{2}\right)^4 bfx - 60\tan\left(\frac{fx}{2} \right.\right.}{15c^3 f\left(\tan\left(\right.\right.}$$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x)
```

output
```
(a**2*( - 6*tan((e + f*x)/2)**5*a - 15*tan((e + f*x)/2)**5*b*f*x - 6*tan((
e + f*x)/2)**5*b + 75*tan((e + f*x)/2)**4*b*f*x - 60*tan((e + f*x)/2)**3*a
 - 150*tan((e + f*x)/2)**3*b*f*x + 60*tan((e + f*x)/2)**3*b + 150*tan((e +
 f*x)/2)**2*b*f*x - 280*tan((e + f*x)/2)**2*b - 30*tan((e + f*x)/2)*a - 75
*tan((e + f*x)/2)*b*f*x + 170*tan((e + f*x)/2)*b + 15*b*f*x - 40*b))/(15*c
**3*f*(tan((e + f*x)/2)**5 - 5*tan((e + f*x)/2)**4 + 10*tan((e + f*x)/2)**
3 - 10*tan((e + f*x)/2)**2 + 5*tan((e + f*x)/2) - 1))
```

3.34 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 75

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx = \frac{a^2(A+B)c^2\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}+\frac{a^2(A-6B)c\cos^5(e+fx)}{35f(c-c\sin(e+fx))^5}$$

output
```
1/7*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^6+1/35*a^2*(A-6*B)*c*cos
(f*x+e)^5/f/(c-c*sin(f*x+e))^5
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 191 vs. 2(75) = 150.

Time = 11.76 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.55

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx = -\frac{a^2\left(-35(A+4B)\cos\left(\frac{1}{2}(e+fx)\right)+7(2A+13B)\cos\left(\frac{3}{2}(e+fx)\right)+35B\cos\left(\frac{5}{2}(e+fx)\right)+A\cos\left(\frac{7}{2}(e\right.\right.}{}$$

input
```
Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^4,x]
```

output
```
-1/140*(a^2*(-35*(A + 4*B)*Cos[(e + f*x)/2] + 7*(2*A + 13*B)*Cos[(3*(e + f
*x))/2] + 35*B*Cos[(5*(e + f*x))/2] + A*Cos[(7*(e + f*x))/2] - 6*B*Cos[(7*
(e + f*x))/2] - 70*A*Sin[(e + f*x)/2] + 70*B*Sin[(e + f*x)/2] - 35*A*Sin[(
3*(e + f*x))/2] + 35*B*Sin[(3*(e + f*x))/2] + 7*A*Sin[(5*(e + f*x))/2] - 7
*B*Sin[(5*(e + f*x))/2]))/(c^4*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3446, 3042, 3338, 3042, 3150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a \sin(e+fx)+a)^2 (A+B \sin(e+fx))}{(c-c \sin(e+fx))^4} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a \sin(e+fx)+a)^2 (A+B \sin(e+fx))}{(c-c \sin(e+fx))^4}dx$$

$$\Big\downarrow \text{3446}$$

$$a^2c^2 \int \frac{\cos^4(e+fx)(A+B \sin(e+fx))}{(c-c \sin(e+fx))^6}dx$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2 \int \frac{\cos(e+fx)^4(A+B \sin(e+fx))}{(c-c \sin(e+fx))^6}dx$$

$$\Big\downarrow \text{3338}$$

$$a^2c^2 \left(\frac{(A-6B)\int \frac{\cos^4(e+fx)}{(c-c \sin(e+fx))^5}dx}{7c}+\frac{(A+B)\cos^5(e+fx)}{7f(c-c \sin(e+fx))^6}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left(\frac{(A-6B)\int \frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^5}dx}{7c}+\frac{(A+B)\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}\right)$$

$\Big\downarrow$ 3150

$$a^2c^2\left(\frac{(A-6B)\cos^5(e+fx)}{35cf(c-c\sin(e+fx))^5}+\frac{(A+B)\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}\right)$$

input
```
Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^4,x
]
```

output
```
a^2*c^2*(((A + B)*Cos[e + f*x]^5)/(7*f*(c - c*Sin[e + f*x])^6) + ((A - 6*B
)*Cos[e + f*x]^5)/(35*c*f*(c - c*Sin[e + f*x])^5))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3150
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.77

method	result
parallelrisch	$-\frac{2a^2\left(A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+(-A+B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5+(4A+B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+2(-A+B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3+\frac{(13A+2B)\tan(}{5}\right.}{f\,c^4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}$
derivativedivides	$\frac{2a^2\left(-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{128A+112B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{32A+32B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{96A+64B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{42A+18B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{10A}{2(\tan(}\right.}{f\,c^4}$
default	$\frac{2a^2\left(-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{128A+112B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{32A+32B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{96A+64B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{42A+18B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{10A}{2(\tan(}\right.}{f\,c^4}$
risch	$-\frac{2(-6a^2B+a^2A-35A\,a^2e^{4i(fx+e)}-140B\,a^2e^{4i(fx+e)}+91B\,a^2e^{2i(fx+e)}+14A\,a^2e^{2i(fx+e)}-35iB\,a^2e^{5i(fx+e)}+70i}{35(e^{i(fx+e)}-i)}$
norman	$\frac{\frac{(10a^2A-10a^2B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{fc}-\frac{12a^2A-2a^2B}{35fc}-\frac{2a^2A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{12}}{fc}+\frac{2(a^2A-a^2B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}{fc}+\frac{(2a^2A-2a^2B)\tan(}{5fc}}{}$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x,method=_RETUR
NVERBOSE)
```

output
```
-2*a^2*(A*tan(1/2*f*x+1/2*e)^6+(-A+B)*tan(1/2*f*x+1/2*e)^5+(4*A+B)*tan(1/2
*f*x+1/2*e)^4+2*(-A+B)*tan(1/2*f*x+1/2*e)^3+1/5*(13*A+2*B)*tan(1/2*f*x+1/2
*e)^2+1/5*(-A+B)*tan(1/2*f*x+1/2*e)+6/35*A-1/35*B)/f/c^4/(tan(1/2*f*x+1/2*
e)-1)^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(73) = 146$.

Time = 0.08 (sec) , antiderivative size = 263, normalized size of antiderivative = 3.51

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx = -\frac{(A-6B)a^2\cos(fx+e)^4+(4A+11B)a^2\cos(fx+e)^3+(13A+27B)a^2\cos(fx+e)^2-10(A+}{35\left(c^4f\cos(fx+e)^4-3c^4f\cos(fx+e)^3-8c^4f\cos(fx+e)\right.}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algori
thm="fricas")
```

output
```
-1/35*((A - 6*B)*a^2*cos(f*x + e)^4 + (4*A + 11*B)*a^2*cos(f*x + e)^3 + (1
3*A + 27*B)*a^2*cos(f*x + e)^2 - 10*(A + B)*a^2*cos(f*x + e) - 20*(A + B)*
a^2 - ((A - 6*B)*a^2*cos(f*x + e)^3 - (3*A + 17*B)*a^2*cos(f*x + e)^2 + 10
*(A + B)*a^2*cos(f*x + e) + 20*(A + B)*a^2)*sin(f*x + e))/(c^4*f*cos(f*x +
 e)^4 - 3*c^4*f*cos(f*x + e)^3 - 8*c^4*f*cos(f*x + e)^2 + 4*c^4*f*cos(f*x
+ e) + 8*c^4*f + (c^4*f*cos(f*x + e)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*c^4*f*
cos(f*x + e) - 8*c^4*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2008 vs. $2(66) = 132$.

Time = 15.86 (sec) , antiderivative size = 2008, normalized size of antiderivative = 26.77

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**4,x)
```

output
```
Piecewise((-70*A*a**2*tan(e/2 + f*x/2)**6/(35*c**4*f*tan(e/2 + f*x/2)**7 -
 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c*
*4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*ta
n(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) + 70*A*a**2*t
an(e/2 + f*x/2)**5/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f
*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**
4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)**2 + 245
*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) - 280*A*a**2*tan(e/2 + f*x/2)**4/(35
*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*
tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/
2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/
2) - 35*c**4*f) + 140*A*a**2*tan(e/2 + f*x/2)**3/(35*c**4*f*tan(e/2 + f*x/
2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 -
1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c*
*4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) - 182*
A*a**2*tan(e/2 + f*x/2)**2/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan
(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 +
f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)*
*2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) + 14*A*a**2*tan(e/2 + f*x/2)
/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1571 vs. $2(73) = 146$.

Time = 0.09 (sec) , antiderivative size = 1571, normalized size of antiderivative = 20.95

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^4}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algori
thm="maxima")
```

output
```
2/105*(2*A*a^2*(91*sin(f*x + e)/(cos(f*x + e) + 1) - 168*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 280*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 175*sin(f*x
 + e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 1
3)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) +
 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(co
s(f*x + e) + 1)^7) + B*a^2*(91*sin(f*x + e)/(cos(f*x + e) + 1) - 168*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 280*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 -
 175*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 - 13)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^
3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(co
s(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*
x + e)^7/(cos(f*x + e) + 1)^7) - 3*A*a^2*(49*sin(f*x + e)/(cos(f*x + e) +
1) - 147*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 210*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 - 210*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^
5/(cos(f*x + e) + 1)^5 - 35*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 12)/(c^4
 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(73) = 146$.

Time = 0.25 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.89

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^4} dx =$$
$$-\frac{2\left(35\,Aa^2\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^6 - 35\,Aa^2\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^5 + 35\,Ba^2\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^5 + 140\,Aa^2\tan\left(\frac{1}{2}\,f\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algori
thm="giac")
```

output
```
-2/35*(35*A*a^2*tan(1/2*f*x + 1/2*e)^6 - 35*A*a^2*tan(1/2*f*x + 1/2*e)^5 +
 35*B*a^2*tan(1/2*f*x + 1/2*e)^5 + 140*A*a^2*tan(1/2*f*x + 1/2*e)^4 + 35*B
*a^2*tan(1/2*f*x + 1/2*e)^4 - 70*A*a^2*tan(1/2*f*x + 1/2*e)^3 + 70*B*a^2*t
an(1/2*f*x + 1/2*e)^3 + 91*A*a^2*tan(1/2*f*x + 1/2*e)^2 + 14*B*a^2*tan(1/2
*f*x + 1/2*e)^2 - 7*A*a^2*tan(1/2*f*x + 1/2*e) + 7*B*a^2*tan(1/2*f*x + 1/2
*e) + 6*A*a^2 - B*a^2)/(c^4*f*(tan(1/2*f*x + 1/2*e) - 1)^7)
```

Mupad [B] (verification not implemented)

Time = 33.15 (sec) , antiderivative size = 269, normalized size of antiderivative = 3.59

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx = \frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{109\,A\,a^2}{4}+\frac{11\,B\,a^2}{4}-\frac{27\,A\,a^2\,\cos(2\,e+2\,f\,x)}{4}+\frac{5\,A\,a^2\,\cos(3\,e+3\,f\,x)}{8}-\frac{13\,B\,a^2\,\cos(2\,e+2\,f\,x)}{4}+\frac{5\,B\,a^2\,\cos(3\,e\cdots}{8}\right.}{35\,c^4\,f\,\left(\frac{35\,\sqrt{2}\,\cos\left(\frac{e}{2}+\frac{\pi}{4}\cdots\right.}{8}\right.}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^4,x
)
```

output
```
(2*cos(e/2 + (f*x)/2)*((109*A*a^2)/4 + (11*B*a^2)/4 - (27*A*a^2*cos(2*e +
2*f*x))/4 + (5*A*a^2*cos(3*e + 3*f*x))/8 - (13*B*a^2*cos(2*e + 2*f*x))/4 +
 (5*B*a^2*cos(3*e + 3*f*x))/8 + (7*A*a^2*sin(2*e + 2*f*x))/2 + (7*A*a^2*si
n(3*e + 3*f*x))/8 - (7*B*a^2*sin(2*e + 2*f*x))/2 - (7*B*a^2*sin(3*e + 3*f*
x))/8 - (121*A*a^2*cos(e + f*x))/8 - (9*B*a^2*cos(e + f*x))/8 - (105*A*a^2
*sin(e + f*x))/8 + (105*B*a^2*sin(e + f*x))/8))/(35*c^4*f*((35*2^(1/2)*cos
(e/2 + pi/4 + (f*x)/2))/8 - (21*2^(1/2)*cos((3*e)/2 - pi/4 + (3*f*x)/2))/8
 - (7*2^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2))/8 + (2^(1/2)*cos((7*e)/2 -
pi/4 + (7*f*x)/2))/8))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 257, normalized size of antiderivative = 3.43

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$$
$$= \frac{2a^2\left(-5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7 a-70\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5 a-35\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5 b+35\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4 a-35\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}{35c^4f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7-7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+21\tan\left(\frac{fx}{2}\right.\right.}$$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x)
```

output
```
(2*a**2*( - 5*tan((e + f*x)/2)**7*a - 70*tan((e + f*x)/2)**5*a - 35*tan((e
 + f*x)/2)**5*b + 35*tan((e + f*x)/2)**4*a - 35*tan((e + f*x)/2)**4*b - 10
5*tan((e + f*x)/2)**3*a - 70*tan((e + f*x)/2)**3*b + 14*tan((e + f*x)/2)**
2*a - 14*tan((e + f*x)/2)**2*b - 28*tan((e + f*x)/2)*a - 7*tan((e + f*x)/2
)*b - a + b))/(35*c**4*f*(tan((e + f*x)/2)**7 - 7*tan((e + f*x)/2)**6 + 21
*tan((e + f*x)/2)**5 - 35*tan((e + f*x)/2)**4 + 35*tan((e + f*x)/2)**3 - 2
1*tan((e + f*x)/2)**2 + 7*tan((e + f*x)/2) - 1))
```

3.35 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 115

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx = \frac{a^2(A+B)c^2\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}+\frac{a^2(2A-7B)c\cos^5(e+fx)}{63f(c-c\sin(e+fx))^6}+\frac{a^2(2A-7B)\cos^5(e+fx)}{315f(c-c\sin(e+fx))^5}$$

output
```
1/9*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^7+1/63*a^2*(2*A-7*B)*c*c
os(f*x+e)^5/f/(c-c*sin(f*x+e))^6+1/315*a^2*(2*A-7*B)*cos(f*x+e)^5/f/(c-c*s
in(f*x+e))^5
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 261 vs. 2(115) = 230.

Time = 12.17 (sec) , antiderivative size = 261, normalized size of antiderivative = 2.27

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx = -\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^2\left(315(2A+3B)\cos\left(\frac{1}{2}(e+fx)\right)-63(4A+\right.}{}$$

input
```
Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^5,x]
```

output
```
-1/2520*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*(3
15*(2*A + 3*B)*Cos[(e + f*x)/2] - 63*(4*A + 11*B)*Cos[(3*(e + f*x))/2] - 3
15*B*Cos[(5*(e + f*x))/2] - 18*A*Cos[(7*(e + f*x))/2] + 63*B*Cos[(7*(e + f
*x))/2] + 882*A*Sin[(e + f*x)/2] + 63*B*Sin[(e + f*x)/2] + 420*A*Sin[(3*(e
 + f*x))/2] + 105*B*Sin[(3*(e + f*x))/2] - 72*A*Sin[(5*(e + f*x))/2] - 63*
B*Sin[(5*(e + f*x))/2] + 2*A*Sin[(9*(e + f*x))/2] - 7*B*Sin[(9*(e + f*x))/
2]))/(c^5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(-1 + Sin[e + f*x])^5)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3042, 3446, 3042, 3338, 3042, 3151, 3042, 3150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}dx$$

$$\Big\downarrow \text{3446}$$

$$a^2c^2\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}dx$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\int \frac{\cos(e+fx)^4(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}dx$$

$$\Big\downarrow \text{3338}$$

$$a^2c^2\left(\frac{(2A-7B)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^6}dx}{9c}+\frac{(A+B)\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left(\frac{(2A-7B)\int \frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^6}dx}{9c}+\frac{(A+B)\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}\right)$$

$$\Big\downarrow \text{3151}$$

$$a^2c^2\left(\frac{(2A-7B)\left(\frac{\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^5}dx}{7c}+\frac{\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}\right)}{9c}+\frac{(A+B)\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left(\frac{(2A-7B)\left(\frac{\int \frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^5}dx}{7c}+\frac{\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}\right)}{9c}+\frac{(A+B)\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}\right)$$

$$\Big\downarrow \text{3150}$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}+\frac{(2A-7B)\left(\frac{\cos^5(e+fx)}{35cf(c-c\sin(e+fx))^5}+\frac{\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}\right)}{9c}\right)$$

input
```
Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^5,x
]
```

output
```
a^2*c^2*(((A + B)*Cos[e + f*x]^5)/(9*f*(c - c*Sin[e + f*x])^7) + ((2*A - 7
*B)*(Cos[e + f*x]^5/(7*f*(c - c*Sin[e + f*x])^6) + Cos[e + f*x]^5/(35*c*f*
(c - c*Sin[e + f*x])^5)))/(9*c))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3150
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

rule 3151
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Simp[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1])   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simpli
fy[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.49

method	result
parallelrisch	$-\frac{2a^2\left(A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8+(-2A+B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7+\frac{(22A+B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{3}+(-8A+3B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5+\frac{(54A+B)\tan(}{5}\right.}{f\,c^5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}$
risch	$\frac{2ia^2\left(420iA\,e^{6i(fx+e)}+105iB\,e^{6i(fx+e)}+315B\,e^{7i(fx+e)}-882iA\,e^{4i(fx+e)}-630A\,e^{5i(fx+e)}-63iB\,e^{4i(fx+e)}-945B\,e^{5}\right.}{315f\,c^5\left(e^{i(}\right.}$
derivativedivides	$\frac{2a^2\left(-\frac{64A+64B}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^9}-\frac{544A+448B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{404A+276B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{256A+256B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{64A+22B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{}{7(\mathrm{ta}}\right.}{f\,c^5}$
default	$\frac{2a^2\left(-\frac{64A+64B}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^9}-\frac{544A+448B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{404A+276B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{256A+256B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{64A+22B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{}{7(\mathrm{ta}}\right.}{f\,c^5}$
norman	$\frac{\frac{\left(4a^2A-2a^2B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{13}}{fc}+\frac{\left(28a^2A-12a^2B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}{fc}-\frac{94a^2A-14a^2B}{315fc}-\frac{2a^2A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{14}}{fc}+\frac{\left(24a^2A-14a^2B\right)\mathrm{t}}{35fc}}{}$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x,method=_RETUR
NVERBOSE)
```

output
```
-2*a^2*(A*tan(1/2*f*x+1/2*e)^8+(-2*A+B)*tan(1/2*f*x+1/2*e)^7+1/3*(22*A+B)*
tan(1/2*f*x+1/2*e)^6+(-8*A+3*B)*tan(1/2*f*x+1/2*e)^5+1/5*(54*A+B)*tan(1/2*
f*x+1/2*e)^4+1/5*(-26*A+11*B)*tan(1/2*f*x+1/2*e)^3+1/5*(118/7*A+B)*tan(1/2
*f*x+1/2*e)^2+1/5*(-12/7*A+B)*tan(1/2*f*x+1/2*e)+47/315*A-1/45*B)/f/c^5/(t
an(1/2*f*x+1/2*e)-1)^9
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. 2(112) = 224.

Time = 0.08 (sec) , antiderivative size = 335, normalized size of antiderivative = 2.91

$$\int\frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx$$
$$=\frac{(2A-7B)a^2\cos(fx+e)^5-4(2A-7B)a^2\cos(fx+e)^4-5(5A+14B)a^2\cos(fx+e)^3-5(17A-}{315\left(c^5f\cos(fx+e)^5+5c^5f\cos(fx+e)^4-8c^5f\,\mathrm{co}\right.}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algori
thm="fricas")
```

output
```
1/315*((2*A - 7*B)*a^2*cos(f*x + e)^5 - 4*(2*A - 7*B)*a^2*cos(f*x + e)^4 -
 5*(5*A + 14*B)*a^2*cos(f*x + e)^3 - 5*(17*A + 35*B)*a^2*cos(f*x + e)^2 +
70*(A + B)*a^2*cos(f*x + e) + 140*(A + B)*a^2 + ((2*A - 7*B)*a^2*cos(f*x +
 e)^4 + 5*(2*A - 7*B)*a^2*cos(f*x + e)^3 - 15*(A + 7*B)*a^2*cos(f*x + e)^2
 + 70*(A + B)*a^2*cos(f*x + e) + 140*(A + B)*a^2)*sin(f*x + e))/(c^5*f*cos
(f*x + e)^5 + 5*c^5*f*cos(f*x + e)^4 - 8*c^5*f*cos(f*x + e)^3 - 20*c^5*f*c
os(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f - (c^5*f*cos(f*x + e)^4 -
4*c^5*f*cos(f*x + e)^3 - 12*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) +
16*c^5*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3262 vs. $2(102) = 204$.

Time = 28.19 (sec) , antiderivative size = 3262, normalized size of antiderivative = 28.37

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^5}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**5,x)
```

output
```
Piecewise((-630*A*a**2*tan(e/2 + f*x/2)**8/(315*c**5*f*tan(e/2 + f*x/2)**9
 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26
460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*
c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5
*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) + 1260
*A*a**2*tan(e/2 + f*x/2)**7/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*
tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(
e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2
+ f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*
x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 4620*A*a**2*tan(e/2
 + f*x/2)**6/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2
)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6
 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 2
6460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*
c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) + 5040*A*a**2*tan(e/2 + f*x/2)**5/(3
15*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c*
*5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f
*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan
(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2
+ f*x/2) - 315*c**5*f) - 6804*A*a**2*tan(e/2 + f*x/2)**4/(315*c**5*f*ta...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2087 vs. $2(112) = 224$.

Time = 0.10 (sec) , antiderivative size = 2087, normalized size of antiderivative = 18.15

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^5}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algori
thm="maxima")
```

output
```
-2/315*(A*a^2*(432*sin(f*x + e)/(cos(f*x + e) + 1) - 1728*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 3612*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5418*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 + 5040*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
 - 3360*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1260*sin(f*x + e)^7/(cos(f*x
 + e) + 1)^7 - 315*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 83)/(c^5 - 9*c^5*
sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5
*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e)
 + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(
cos(f*x + e) + 1)^9) - 10*A*a^2*(45*sin(f*x + e)/(cos(f*x + e) + 1) - 117*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 273*sin(f*x + e)^3/(cos(f*x + e) + 1
)^3 - 315*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 315*sin(f*x + e)^5/(cos(f*
x + e) + 1)^5 - 147*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63*sin(f*x + e)^
7/(cos(f*x + e) + 1)^7 - 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) +
 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin
(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1
)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(c
os(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) - 5*B*a^2...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(112) = 224$.

Time = 0.29 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.48

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^5}\,dx =$$

$$-\frac{2\left(315\,Aa^2\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^8 - 630\,Aa^2\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^7 + 315\,Ba^2\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^7 + 2310\,Aa^2\tan\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algori
thm="giac")
```

output
```
-2/315*(315*A*a^2*tan(1/2*f*x + 1/2*e)^8 - 630*A*a^2*tan(1/2*f*x + 1/2*e)^
7 + 315*B*a^2*tan(1/2*f*x + 1/2*e)^7 + 2310*A*a^2*tan(1/2*f*x + 1/2*e)^6 +
 105*B*a^2*tan(1/2*f*x + 1/2*e)^6 - 2520*A*a^2*tan(1/2*f*x + 1/2*e)^5 + 94
5*B*a^2*tan(1/2*f*x + 1/2*e)^5 + 3402*A*a^2*tan(1/2*f*x + 1/2*e)^4 + 63*B*
a^2*tan(1/2*f*x + 1/2*e)^4 - 1638*A*a^2*tan(1/2*f*x + 1/2*e)^3 + 693*B*a^2
*tan(1/2*f*x + 1/2*e)^3 + 1062*A*a^2*tan(1/2*f*x + 1/2*e)^2 + 63*B*a^2*tan
(1/2*f*x + 1/2*e)^2 - 108*A*a^2*tan(1/2*f*x + 1/2*e) + 63*B*a^2*tan(1/2*f*
x + 1/2*e) + 47*A*a^2 - 7*B*a^2)/(c^5*f*(tan(1/2*f*x + 1/2*e) - 1)^9)
```

Mupad [B] (verification not implemented)

Time = 36.28 (sec) , antiderivative size = 331, normalized size of antiderivative = 2.88

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx = -\frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{265\,A\,a^2\,\cos(2\,e+2\,f\,x)}{2}-\frac{49\,B\,a^2}{8}-\frac{4967\,A\,a^2}{16}-\frac{89\,A\,a^2\,\cos(3\,e+3\,f\,x)}{4}-\frac{49\,A\,a^2\,\cos(4\,e+4\,f\,x)}{16}+\frac{35\,B\,a^2}{\cdots}\right.}{}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^5,x
)
```

output
```
-(2*cos(e/2 + (f*x)/2)*((265*A*a^2*cos(2*e + 2*f*x))/2 - (49*B*a^2)/8 - (4
967*A*a^2)/16 - (89*A*a^2*cos(3*e + 3*f*x))/4 - (49*A*a^2*cos(4*e + 4*f*x)
)/16 + (35*B*a^2*cos(2*e + 2*f*x))/4 - (7*B*a^2*cos(3*e + 3*f*x))/8 + (7*B
*a^2*cos(4*e + 4*f*x))/8 - (567*A*a^2*sin(2*e + 2*f*x))/8 - (243*A*a^2*sin
(3*e + 3*f*x))/8 + (45*A*a^2*sin(4*e + 4*f*x))/16 + (63*B*a^2*sin(2*e + 2*
f*x))/2 + (63*B*a^2*sin(3*e + 3*f*x))/8 + (625*A*a^2*cos(e + f*x))/4 + (35
*B*a^2*cos(e + f*x))/8 + (2205*A*a^2*sin(e + f*x))/8 - (945*B*a^2*sin(e +
f*x))/8))/(315*c^5*f*((63*2^(1/2)*cos(e/2 + pi/4 + (f*x)/2))/8 - (21*2^(1/
2)*cos((3*e)/2 - pi/4 + (3*f*x)/2))/4 - (9*2^(1/2)*cos((5*e)/2 + pi/4 + (5
*f*x)/2))/4 + (9*2^(1/2)*cos((7*e)/2 - pi/4 + (7*f*x)/2))/16 + (2^(1/2)*co
s((9*e)/2 + pi/4 + (9*f*x)/2))/16))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 362, normalized size of antiderivative = 3.15

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx$$
$$= \frac{a^2\left(-25\cos\left(fx+e\right)\sin\left(fx+e\right)^4a+102\cos\left(fx+e\right)\sin\left(fx+e\right)^3a-7\cos\left(fx+e\right)\sin\left(fx+e\right)^3b-\right.}{}$$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x)
```

output
```
(a**2*( - 25*cos(e + f*x)*sin(e + f*x)**4*a + 102*cos(e + f*x)*sin(e + f*x
)**3*a - 7*cos(e + f*x)*sin(e + f*x)**3*b - 159*cos(e + f*x)*sin(e + f*x)*
*2*a - 126*cos(e + f*x)*sin(e + f*x)**2*b + 12*cos(e + f*x)*sin(e + f*x)*a
 - 7*cos(e + f*x)*sin(e + f*x)*b - 70*cos(e + f*x)*a - 21*sin(e + f*x)**5*
a - 14*sin(e + f*x)**5*b + 107*sin(e + f*x)**4*a + 63*sin(e + f*x)**4*b -
219*sin(e + f*x)**3*a + 49*sin(e + f*x)**3*b + 331*sin(e + f*x)**2*a + 189
*sin(e + f*x)**2*b + 12*sin(e + f*x)*a - 7*sin(e + f*x)*b + 70*a))/(315*c*
*5*f*(cos(e + f*x)*sin(e + f*x)**4 - 4*cos(e + f*x)*sin(e + f*x)**3 + 6*co
s(e + f*x)*sin(e + f*x)**2 - 4*cos(e + f*x)*sin(e + f*x) + cos(e + f*x) +
sin(e + f*x)**5 - 5*sin(e + f*x)**4 + 10*sin(e + f*x)**3 - 10*sin(e + f*x)
**2 + 5*sin(e + f*x) - 1))
```

3.36 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 156

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx = \frac{a^2(A+B)c^2\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}+\frac{a^2(3A-8B)c\cos^5(e+fx)}{99f(c-c\sin(e+fx))^7} + \frac{2a^2(3A-8B)\cos^5(e+fx)}{693f(c-c\sin(e+fx))^6}+\frac{2a^2(3A-8B)\cos^5(e+fx)}{3465cf(c-c\sin(e+fx))^5}$$

output
```
1/11*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^8+1/99*a^2*(3*A-8*B)*c*
cos(f*x+e)^5/f/(c-c*sin(f*x+e))^7+2/693*a^2*(3*A-8*B)*cos(f*x+e)^5/f/(c-c*
sin(f*x+e))^6+2/3465*a^2*(3*A-8*B)*cos(f*x+e)^5/c/f/(c-c*sin(f*x+e))^5
```

Mathematica [A] (verified)

Time = 12.73 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.83

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx = \frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^2\left(231(27A+28B)\cos\left(\frac{1}{2}(e+fx)\right)-2475(A+\right.}{}$$

input
```
Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^6,x]
```

output
```
(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*(231*(27*A
 + 28*B)*Cos[(e + f*x)/2] - 2475*(A + 2*B)*Cos[(3*(e + f*x))/2] - 2310*B*C
os[(5*(e + f*x))/2] - 165*A*Cos[(7*(e + f*x))/2] + 440*B*Cos[(7*(e + f*x))
/2] + 3*A*Cos[(11*(e + f*x))/2] - 8*B*Cos[(11*(e + f*x))/2] + 7623*A*Sin[(
e + f*x)/2] + 2772*B*Sin[(e + f*x)/2] + 3465*A*Sin[(3*(e + f*x))/2] + 2310
*B*Sin[(3*(e + f*x))/2] - 495*A*Sin[(5*(e + f*x))/2] - 990*B*Sin[(5*(e + f
*x))/2] + 33*A*Sin[(9*(e + f*x))/2] - 88*B*Sin[(9*(e + f*x))/2]))/(27720*c
^6*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(-1 + Sin[e + f*x])^6)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3042, 3446, 3042, 3338, 3042, 3151, 3042, 3151, 3042, 3150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}dx$$

$$\Big\downarrow \text{3446}$$

$$a^2c^2\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}dx$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\int \frac{\cos(e+fx)^4(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}dx$$

$$\Big\downarrow \text{3338}$$

$$a^2c^2\left(\frac{(3A-8B)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^7}dx}{11c}+\frac{(A+B)\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}\right)$$

$\downarrow$ 3042

$$a^2c^2\left(\frac{(3A-8B)\int \frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^7}dx}{11c}+\frac{(A+B)\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}\right)$$

$\downarrow$ 3151

$$a^2c^2\left(\frac{(3A-8B)\left(\frac{2\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^6}dx}{9c}+\frac{\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}\right)}{11c}+\frac{(A+B)\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}\right)$$

$\downarrow$ 3042

$$a^2c^2\left(\frac{(3A-8B)\left(\frac{2\int \frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^6}dx}{9c}+\frac{\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}\right)}{11c}+\frac{(A+B)\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}\right)$$

$\downarrow$ 3151

$$a^2c^2\left(\frac{(3A-8B)\left(\frac{2\left(\frac{\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^5}dx}{7c}+\frac{\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}\right)}{9c}+\frac{\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}\right)}{11c}+\frac{(A+B)\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}\right)$$

$\downarrow$ 3042

$$a^2c^2\left(\frac{(3A-8B)\left(\frac{2\left(\frac{\int \frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^5}dx}{7c}+\frac{\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}\right)}{9c}+\frac{\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}\right)}{11c}+\frac{(A+B)\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}\right)$$

$$\Big\downarrow \text{3150}$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}+\frac{(3A-8B)\left(\frac{\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}+\frac{2\left(\frac{\cos^5(e+fx)}{35cf(c-c\sin(e+fx))^5}+\frac{\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}\right)}{9c}\right)}{11c}\right)$$

input
```
Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^6,x
]
```

output
```
a^2*c^2*(((A + B)*Cos[e + f*x]^5)/(11*f*(c - c*Sin[e + f*x])^8) + ((3*A -
8*B)*(Cos[e + f*x]^5/(9*f*(c - c*Sin[e + f*x])^7) + (2*(Cos[e + f*x]^5/(7*
f*(c - c*Sin[e + f*x])^6) + Cos[e + f*x]^5/(35*c*f*(c - c*Sin[e + f*x])^5)
))/(9*c)))/(11*c))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3150
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]
```

rule 3151
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Simp[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1])   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simpli
fy[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.40

method	result
parallelrisch	$-\frac{2\left(A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}+(-3A+B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}+\left(12A-\frac{B}{3}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}+2\left(-10A+\frac{7B}{3}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}+\frac{2\left(81A-\right.}{}\right.}{}$
derivativedivides	$\frac{2a^2\left(-\frac{932A+528B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{1752A+1208B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{640A+640B}{10\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{10}}-\frac{2304A+2048B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{2376A+1896B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\right.}{}$
default	$\frac{2a^2\left(-\frac{932A+528B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{1752A+1208B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{640A+640B}{10\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{10}}-\frac{2304A+2048B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{2376A+1896B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\right.}{}$
risch	$\frac{32a^2B}{3465}-\frac{4a^2A}{1155}+\frac{20A\,a^2e^{4i(fx+e)}}{7}+\frac{40B\,a^2e^{4i(fx+e)}}{7}+\frac{4A\,a^2e^{2i(fx+e)}}{21}-\frac{32B\,a^2e^{2i(fx+e)}}{63}+\frac{8B\,a^2e^{8i(fx+e)}}{3}-\frac{36A\,a^2e^{6i(fx-}}{5}$
norman	$-\frac{912a^2A-122a^2B}{3465fc}-\frac{2a^2A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{16}}{fc}+\frac{2\left(3a^2A-a^2B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{15}}{fc}-\frac{2\left(45a^2A-a^2B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{14}}{3fc}+\frac{2\left(87a^2A-23a^2\right.}{3}$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x,method=_RETUR
NVERBOSE)
```

output
```
-2*(A*tan(1/2*f*x+1/2*e)^10+(-3*A+B)*tan(1/2*f*x+1/2*e)^9+(12*A-1/3*B)*tan
(1/2*f*x+1/2*e)^8+2*(-10*A+7/3*B)*tan(1/2*f*x+1/2*e)^7+2/5*(81*A-13/3*B)*t
an(1/2*f*x+1/2*e)^6+2/5*(-71*A+16*B)*tan(1/2*f*x+1/2*e)^5+4/7*(41*A-2*B)*t
an(1/2*f*x+1/2*e)^4+2/7*(-34*A+9*B)*tan(1/2*f*x+1/2*e)^3+1/21*(89*A+2/3*B)
*tan(1/2*f*x+1/2*e)^2+1/105*(-47*A+61/3*B)*tan(1/2*f*x+1/2*e)+152/1155*A-6
1/3465*B)*a^2/f/c^6/(tan(1/2*f*x+1/2*e)-1)^11
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(152) = 304$.

Time = 0.09 (sec) , antiderivative size = 407, normalized size of antiderivative = 2.61

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx =$$
$$-\frac{2\,(3\,A-8\,B)a^2\cos{(fx+e)}^6+12\,(3\,A-8\,B)a^2\cos{(fx+e)}^5-25\,(3\,A-8\,B)a^2\cos{(fx+e)}^4-35\,(}{3465\,(c^6f\cos{(fx+e)}^6-5\,c^6f\cos}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x, algori
thm="fricas")
```

output
```
-1/3465*(2*(3*A - 8*B)*a^2*cos(f*x + e)^6 + 12*(3*A - 8*B)*a^2*cos(f*x + e
)^5 - 25*(3*A - 8*B)*a^2*cos(f*x + e)^4 - 35*(6*A + 17*B)*a^2*cos(f*x + e)
^3 - 35*(21*A + 43*B)*a^2*cos(f*x + e)^2 + 630*(A + B)*a^2*cos(f*x + e) +
1260*(A + B)*a^2 - (2*(3*A - 8*B)*a^2*cos(f*x + e)^5 - 10*(3*A - 8*B)*a^2*
cos(f*x + e)^4 - 35*(3*A - 8*B)*a^2*cos(f*x + e)^3 + 35*(3*A + 25*B)*a^2*c
os(f*x + e)^2 - 630*(A + B)*a^2*cos(f*x + e) - 1260*(A + B)*a^2)*sin(f*x +
 e))/(c^6*f*cos(f*x + e)^6 - 5*c^6*f*cos(f*x + e)^5 - 18*c^6*f*cos(f*x + e
)^4 + 20*c^6*f*cos(f*x + e)^3 + 48*c^6*f*cos(f*x + e)^2 - 16*c^6*f*cos(f*x
 + e) - 32*c^6*f + (c^6*f*cos(f*x + e)^5 + 6*c^6*f*cos(f*x + e)^4 - 12*c^6
*f*cos(f*x + e)^3 - 32*c^6*f*cos(f*x + e)^2 + 16*c^6*f*cos(f*x + e) + 32*c
^6*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4816 vs. $2(141) = 282$.

Time = 47.09 (sec) , antiderivative size = 4816, normalized size of antiderivative = 30.87

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^6}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**6,x)
```

output
```
Piecewise((-6930*A*a**2*tan(e/2 + f*x/2)**10/(3465*c**6*f*tan(e/2 + f*x/2)
**11 - 38115*c**6*f*tan(e/2 + f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)*
*9 - 571725*c**6*f*tan(e/2 + f*x/2)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**
7 - 1600830*c**6*f*tan(e/2 + f*x/2)**6 + 1600830*c**6*f*tan(e/2 + f*x/2)**
5 - 1143450*c**6*f*tan(e/2 + f*x/2)**4 + 571725*c**6*f*tan(e/2 + f*x/2)**3
 - 190575*c**6*f*tan(e/2 + f*x/2)**2 + 38115*c**6*f*tan(e/2 + f*x/2) - 346
5*c**6*f) + 20790*A*a**2*tan(e/2 + f*x/2)**9/(3465*c**6*f*tan(e/2 + f*x/2)
**11 - 38115*c**6*f*tan(e/2 + f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)*
*9 - 571725*c**6*f*tan(e/2 + f*x/2)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**
7 - 1600830*c**6*f*tan(e/2 + f*x/2)**6 + 1600830*c**6*f*tan(e/2 + f*x/2)**
5 - 1143450*c**6*f*tan(e/2 + f*x/2)**4 + 571725*c**6*f*tan(e/2 + f*x/2)**3
 - 190575*c**6*f*tan(e/2 + f*x/2)**2 + 38115*c**6*f*tan(e/2 + f*x/2) - 346
5*c**6*f) - 83160*A*a**2*tan(e/2 + f*x/2)**8/(3465*c**6*f*tan(e/2 + f*x/2)
**11 - 38115*c**6*f*tan(e/2 + f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)*
*9 - 571725*c**6*f*tan(e/2 + f*x/2)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**
7 - 1600830*c**6*f*tan(e/2 + f*x/2)**6 + 1600830*c**6*f*tan(e/2 + f*x/2)**
5 - 1143450*c**6*f*tan(e/2 + f*x/2)**4 + 571725*c**6*f*tan(e/2 + f*x/2)**3
 - 190575*c**6*f*tan(e/2 + f*x/2)**2 + 38115*c**6*f*tan(e/2 + f*x/2) - 346
5*c**6*f) + 138600*A*a**2*tan(e/2 + f*x/2)**7/(3465*c**6*f*tan(e/2 + f*x/2
)**11 - 38115*c**6*f*tan(e/2 + f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2604 vs. $2(152) = 304$.

Time = 0.12 (sec) , antiderivative size = 2604, normalized size of antiderivative = 16.69

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^6}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x, algori
thm="maxima")
```

output
```
-2/3465*(5*A*a^2*(913*sin(f*x + e)/(cos(f*x + e) + 1) - 4565*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 + 12540*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 25080
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 33726*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5 - 33726*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 23100*sin(f*x + e)^7/
(cos(f*x + e) + 1)^7 - 11550*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 3465*si
n(f*x + e)^9/(cos(f*x + e) + 1)^9 - 693*sin(f*x + e)^10/(cos(f*x + e) + 1)
^10 - 146)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
+ 330*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(co
s(f*x + e) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*
sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e)
 + 1)^8 - 55*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)
^10/(cos(f*x + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 6
*A*a^2*(671*sin(f*x + e)/(cos(f*x + e) + 1) - 2200*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 6600*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 10890*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 + 15246*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 12
936*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 9240*sin(f*x + e)^7/(cos(f*x + e
) + 1)^7 - 3465*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 1155*sin(f*x + e)^9/
(cos(f*x + e) + 1)^9 - 61)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) +
 55*c^6*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(c...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(152) = 304$.

Time = 0.30 (sec) , antiderivative size = 353, normalized size of antiderivative = 2.26

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^6}\,dx =$$

$$-\frac{2\left(3465\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10} - 10395\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9 + 3465\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9 + 41580\,A\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x, algori
thm="giac")
```

output
```
-2/3465*(3465*A*a^2*tan(1/2*f*x + 1/2*e)^10 - 10395*A*a^2*tan(1/2*f*x + 1/
2*e)^9 + 3465*B*a^2*tan(1/2*f*x + 1/2*e)^9 + 41580*A*a^2*tan(1/2*f*x + 1/2
*e)^8 - 1155*B*a^2*tan(1/2*f*x + 1/2*e)^8 - 69300*A*a^2*tan(1/2*f*x + 1/2*
e)^7 + 16170*B*a^2*tan(1/2*f*x + 1/2*e)^7 + 112266*A*a^2*tan(1/2*f*x + 1/2
*e)^6 - 6006*B*a^2*tan(1/2*f*x + 1/2*e)^6 - 98406*A*a^2*tan(1/2*f*x + 1/2*
e)^5 + 22176*B*a^2*tan(1/2*f*x + 1/2*e)^5 + 81180*A*a^2*tan(1/2*f*x + 1/2*
e)^4 - 3960*B*a^2*tan(1/2*f*x + 1/2*e)^4 - 33660*A*a^2*tan(1/2*f*x + 1/2*e
)^3 + 8910*B*a^2*tan(1/2*f*x + 1/2*e)^3 + 14685*A*a^2*tan(1/2*f*x + 1/2*e)
^2 + 110*B*a^2*tan(1/2*f*x + 1/2*e)^2 - 1551*A*a^2*tan(1/2*f*x + 1/2*e) +
671*B*a^2*tan(1/2*f*x + 1/2*e) + 456*A*a^2 - 61*B*a^2)/(c^6*f*(tan(1/2*f*x
 + 1/2*e) - 1)^11)
```

Mupad [B] (verification not implemented)

Time = 37.00 (sec) , antiderivative size = 423, normalized size of antiderivative = 2.71

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx$$
$$= \frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{38163\,A\,a^2}{8}-\frac{1283\,B\,a^2}{8}-\frac{11931\,A\,a^2\,\cos(2\,e+2\,f\,x)}{4}+\frac{9609\,A\,a^2\,\cos(3\,e+3\,f\,x)}{16}+\frac{1383\,A\,a^2\,\cos(4\,e+4\,f\,x)}{8}-\right.}{}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^6,x
)
```

output
```
(2*cos(e/2 + (f*x)/2)*((38163*A*a^2)/8 - (1283*B*a^2)/8 - (11931*A*a^2*cos
(2*e + 2*f*x))/4 + (9609*A*a^2*cos(3*e + 3*f*x))/16 + (1383*A*a^2*cos(4*e
+ 4*f*x))/8 - (225*A*a^2*cos(5*e + 5*f*x))/16 + (631*B*a^2*cos(2*e + 2*f*x
))/4 - (1583*B*a^2*cos(3*e + 3*f*x))/32 - (223*B*a^2*cos(4*e + 4*f*x))/8 +
 (45*B*a^2*cos(5*e + 5*f*x))/32 + 1386*A*a^2*sin(2*e + 2*f*x) + (14949*A*a
^2*sin(3*e + 3*f*x))/16 - (561*A*a^2*sin(4*e + 4*f*x))/4 - (231*A*a^2*sin(
5*e + 5*f*x))/16 - (3003*B*a^2*sin(2*e + 2*f*x))/8 - (4653*B*a^2*sin(3*e +
 3*f*x))/32 + (209*B*a^2*sin(4*e + 4*f*x))/16 + (77*B*a^2*sin(5*e + 5*f*x)
)/32 - 2091*A*a^2*cos(e + f*x) + (281*B*a^2*cos(e + f*x))/16 - (22869*A*a^
2*sin(e + f*x))/4 + (23331*B*a^2*sin(e + f*x))/16))/(3465*c^6*f*((231*2^(1
/2)*cos(e/2 + pi/4 + (f*x)/2))/16 - (165*2^(1/2)*cos((3*e)/2 - pi/4 + (3*f
*x)/2))/16 - (165*2^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2))/32 + (55*2^(1/2
)*cos((7*e)/2 - pi/4 + (7*f*x)/2))/32 + (11*2^(1/2)*cos((9*e)/2 + pi/4 + (
9*f*x)/2))/32 - (2^(1/2)*cos((11*e)/2 - pi/4 + (11*f*x)/2))/32))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 463, normalized size of antiderivative = 2.97

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx = \frac{a^2\left(-180\cos(fx+e)\sin(fx+e)^5a-45\cos(fx+e)\sin(fx+e)^5b+906\cos(fx+e)\sin(fx+e)^4a+\cdots\right.}{}$$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x)
```

output

```
(a**2*( - 180*cos(e + f*x)*sin(e + f*x)**5*a - 45*cos(e + f*x)*sin(e + f*x
)**5*b + 906*cos(e + f*x)*sin(e + f*x)**4*a + 209*cos(e + f*x)*sin(e + f*x
)**4*b - 1833*cos(e + f*x)*sin(e + f*x)**3*a - 362*cos(e + f*x)*sin(e + f*
x)**3*b + 1878*cos(e + f*x)*sin(e + f*x)**2*a + 1397*cos(e + f*x)*sin(e +
f*x)**2*b - 141*cos(e + f*x)*sin(e + f*x)*a + 61*cos(e + f*x)*sin(e + f*x)
*b + 630*cos(e + f*x)*a - 168*sin(e + f*x)**6*a - 77*sin(e + f*x)**6*b + 1
014*sin(e + f*x)**5*a + 446*sin(e + f*x)**5*b - 2553*sin(e + f*x)**4*a - 1
067*sin(e + f*x)**4*b + 3435*sin(e + f*x)**3*a + 185*sin(e + f*x)**3*b - 3
477*sin(e + f*x)**2*a - 2068*sin(e + f*x)**2*b - 141*sin(e + f*x)*a + 61*s
in(e + f*x)*b - 630*a))/(3465*c**6*f*(cos(e + f*x)*sin(e + f*x)**5 - 5*cos
(e + f*x)*sin(e + f*x)**4 + 10*cos(e + f*x)*sin(e + f*x)**3 - 10*cos(e + f
*x)*sin(e + f*x)**2 + 5*cos(e + f*x)*sin(e + f*x) - cos(e + f*x) + sin(e +
 f*x)**6 - 6*sin(e + f*x)**5 + 15*sin(e + f*x)**4 - 20*sin(e + f*x)**3 + 1
5*sin(e + f*x)**2 - 6*sin(e + f*x) + 1))
```

3.37 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 197

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx = \frac{a^2(A+B)c^2\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9}+\frac{a^2(4A-9B)c\cos^5(e+fx)}{143f(c-c\sin(e+fx))^8}+\frac{a^2(4A-9B)\cos^5(e+fx)}{429f(c-c\sin(e+fx))^7}+\frac{2a^2(4A-9B)\cos^5(e+fx)}{3003cf(c-c\sin(e+fx))^6}+\frac{2a^2(4A-9B)\cos^5(e+fx)}{15015c^2f(c-c\sin(e+fx))^5}$$

output
```
1/13*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^9+1/143*a^2*(4*A-9*B)*c
*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^8+1/429*a^2*(4*A-9*B)*cos(f*x+e)^5/f/(c-c
*sin(f*x+e))^7+2/3003*a^2*(4*A-9*B)*cos(f*x+e)^5/c/f/(c-c*sin(f*x+e))^6+2/
15015*a^2*(4*A-9*B)*cos(f*x+e)^5/c^2/f/(c-c*sin(f*x+e))^5
```

Mathematica [A] (verified)

Time = 14.81 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.59

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^7} dx =$$
$$-\frac{a^2\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right)(1 + \sin(e + fx))^2\left(6006(8A + 7B)\cos\left(\frac{1}{2}(e + fx)\right) - 1716(11\right.}{}$$

input
```
Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^7,x]
```

output
```
-1/240240*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*
(6006*(8*A + 7*B)*Cos[(e + f*x)/2] - 1716*(11*A + 19*B)*Cos[(3*(e + f*x))/
2] - 15015*B*Cos[(5*(e + f*x))/2] - 1144*A*Cos[(7*(e + f*x))/2] + 2574*B*C
os[(7*(e + f*x))/2] + 52*A*Cos[(11*(e + f*x))/2] - 117*B*Cos[(11*(e + f*x)
)/2] + 54912*A*Sin[(e + f*x)/2] + 26598*B*Sin[(e + f*x)/2] + 24024*A*Sin[(
3*(e + f*x))/2] + 21021*B*Sin[(3*(e + f*x))/2] - 2860*A*Sin[(5*(e + f*x))/
2] - 8580*B*Sin[(5*(e + f*x))/2] + 312*A*Sin[(9*(e + f*x))/2] - 702*B*Sin[
(9*(e + f*x))/2] - 4*A*Sin[(13*(e + f*x))/2] + 9*B*Sin[(13*(e + f*x))/2]))
/(c^7*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(-1 + Sin[e + f*x])^7)
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3446, 3042, 3338, 3042, 3151, 3042, 3151, 3042, 3151, 3042, 3150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e + fx) + a)^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^7} dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a\sin(e + fx) + a)^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^7} dx$$

$$\Big\downarrow \text{3446}$$

$$a^2c^2\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^9}dx$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\int \frac{\cos(e+fx)^4(A+B\sin(e+fx))}{(c-c\sin(e+fx))^9}dx$$

$$\Big\downarrow \text{3338}$$

$$a^2c^2\left(\frac{(4A-9B)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^8}dx}{13c}+\frac{(A+B)\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left(\frac{(4A-9B)\int \frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^8}dx}{13c}+\frac{(A+B)\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9}\right)$$

$$\Big\downarrow \text{3151}$$

$$a^2c^2\left(\frac{(4A-9B)\left(\frac{3\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^7}dx}{11c}+\frac{\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}\right)}{13c}+\frac{(A+B)\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left(\frac{(4A-9B)\left(\frac{3\int \frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^7}dx}{11c}+\frac{\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}\right)}{13c}+\frac{(A+B)\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9}\right)$$

$$\Big\downarrow \text{3151}$$

$$a^2c^2\left(\frac{(4A-9B)\left(\frac{3\left(\frac{2\int\frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^6}dx}{9c}+\frac{\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}\right)}{11c}+\frac{\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}\right)}{13c}+\frac{(A+B)\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9}\right)$$

$$\Big\downarrow \ 3042$$

$$a^2c^2\left(\frac{(4A-9B)\left(\frac{3\left(\frac{2\int\frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^6}dx}{9c}+\frac{\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}\right)}{11c}+\frac{\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}\right)}{13c}+\frac{(A+B)\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9}\right)$$

$$\Big\downarrow \ 3151$$

$$a^2c^2\left(\frac{(4A-9B)\left(\frac{3\left(\frac{2\left(\frac{\int\frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^5}dx}{7c}+\frac{\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}\right)}{9c}+\frac{\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}\right)}{11c}+\frac{\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}\right)}{13c}+\frac{(A+B)\,\mathrm{c}}{13f(c-c\,\mathrm{s}}\right.$$

$\downarrow$ 3042

$$a^2c^2\left(\frac{(4A-9B)\left(\frac{3\left(\frac{2\left(\frac{\int\frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^5}dx}{7c}+\frac{\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}\right)}{9c}+\frac{\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}\right)}{11c}+\frac{\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}\right)}{13c}+\frac{(A+B)\,\mathrm{c}}{13f(c-c\,\mathrm{s}}\right.$$

$\downarrow$ 3150

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{13f(c-c\sin(e+fx))^9}+\frac{(4A-9B)\left(\frac{\cos^5(e+fx)}{11f(c-c\sin(e+fx))^8}+\frac{3\left(\frac{\cos^5(e+fx)}{9f(c-c\sin(e+fx))^7}+\frac{2\left(\frac{\cos^5(e+fx)}{35cf(c-c\sin(e+fx))^5}+\frac{\cos^5(e+fx)}{7f(c-c\sin(e+fx))^6}\right)}{9c}\right)}{11c}\right)}{13c}\right)$$

input
```
Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^7,x
]
```

output
```
a^2*c^2*(((A + B)*Cos[e + f*x]^5)/(13*f*(c - c*Sin[e + f*x])^9) + ((4*A -
9*B)*(Cos[e + f*x]^5/(11*f*(c - c*Sin[e + f*x])^8) + (3*(Cos[e + f*x]^5/(9
*f*(c - c*Sin[e + f*x])^7) + (2*(Cos[e + f*x]^5/(7*f*(c - c*Sin[e + f*x])^
6) + Cos[e + f*x]^5/(35*c*f*(c - c*Sin[e + f*x])^5)))/(9*c)))/(11*c)))/(13
*c))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3150
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]
```

rule 3151
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Simp[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1])   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simpli
fy[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.49 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.26

method	result
risch	$-\frac{4ia^2(-26598iB\,e^{6i(fx+e)}+24024iA\,e^{8i(fx+e)}+15015B\,e^{9i(fx+e)}+8580iB\,e^{4i(fx+e)}-48048A\,e^{7i(fx+e)}+2860iA\,e^{}}{}$
parallelrisch	$-\frac{2a^2\left(A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{12}+(-4A+B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}+(18A-B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}+(-40A+7B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}+\frac{(391A-}{}\right.}{}$
derivativedivides	$\frac{2a^2\left(-\frac{4480A+4352B}{11\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{11}}-\frac{7744A+5368B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}-\frac{1536A+1536B}{12\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{12}}-\frac{4320A+2568B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{6}}-\frac{10560A+8256B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{8}}-\right.}{}$
default	$\frac{2a^2\left(-\frac{4480A+4352B}{11\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{11}}-\frac{7744A+5368B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}-\frac{1536A+1536B}{12\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{12}}-\frac{4320A+2568B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{6}}-\frac{10560A+8256B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{8}}-\right.}{}$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x,method=_RETUR
NVERBOSE)
```

output
```
-4/15015*I*a^2*(-26598*I*B*exp(6*I*(f*x+e))+24024*I*A*exp(8*I*(f*x+e))+150
15*B*exp(9*I*(f*x+e))+8580*I*B*exp(4*I*(f*x+e))-48048*A*exp(7*I*(f*x+e))+2
860*I*A*exp(4*I*(f*x+e))-42042*B*exp(7*I*(f*x+e))-312*I*A*exp(2*I*(f*x+e))
+18876*A*exp(5*I*(f*x+e))+21021*I*B*exp(8*I*(f*x+e))+32604*B*exp(5*I*(f*x+
e))+702*I*B*exp(2*I*(f*x+e))+1144*A*exp(3*I*(f*x+e))-9*I*B-2574*B*exp(3*I*
(f*x+e))-54912*I*A*exp(6*I*(f*x+e))-52*A*exp(I*(f*x+e))+4*I*A+117*B*exp(I*
(f*x+e)))/f/c^7/(exp(I*(f*x+e))-I)^13
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 475 vs. $2(192) = 384$.

Time = 0.10 (sec) , antiderivative size = 475, normalized size of antiderivative = 2.41

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx$$
$$=\frac{2\,(4\,A-9\,B)a^2\cos{(fx+e)}^7-12\,(4\,A-9\,B)a^2\cos{(fx+e)}^6-49\,(4\,A-9\,B)a^2\cos{(fx+e)}^5+70\,(4\,A}{15015\,\left(c^7f\cos{(fx+e)}^7\right.}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x, algori
thm="fricas")
```

output
```
1/15015*(2*(4*A - 9*B)*a^2*cos(f*x + e)^7 - 12*(4*A - 9*B)*a^2*cos(f*x + e
)^6 - 49*(4*A - 9*B)*a^2*cos(f*x + e)^5 + 70*(4*A - 9*B)*a^2*cos(f*x + e)^
4 + 105*(7*A + 20*B)*a^2*cos(f*x + e)^3 + 105*(25*A + 51*B)*a^2*cos(f*x +
e)^2 - 2310*(A + B)*a^2*cos(f*x + e) - 4620*(A + B)*a^2 + (2*(4*A - 9*B)*a
^2*cos(f*x + e)^6 + 14*(4*A - 9*B)*a^2*cos(f*x + e)^5 - 35*(4*A - 9*B)*a^2
*cos(f*x + e)^4 - 105*(4*A - 9*B)*a^2*cos(f*x + e)^3 + 105*(3*A + 29*B)*a^
2*cos(f*x + e)^2 - 2310*(A + B)*a^2*cos(f*x + e) - 4620*(A + B)*a^2)*sin(f
*x + e))/(c^7*f*cos(f*x + e)^7 + 7*c^7*f*cos(f*x + e)^6 - 18*c^7*f*cos(f*x
 + e)^5 - 56*c^7*f*cos(f*x + e)^4 + 48*c^7*f*cos(f*x + e)^3 + 112*c^7*f*co
s(f*x + e)^2 - 32*c^7*f*cos(f*x + e) - 64*c^7*f - (c^7*f*cos(f*x + e)^6 -
6*c^7*f*cos(f*x + e)^5 - 24*c^7*f*cos(f*x + e)^4 + 32*c^7*f*cos(f*x + e)^3
 + 80*c^7*f*cos(f*x + e)^2 - 32*c^7*f*cos(f*x + e) - 64*c^7*f)*sin(f*x + e
))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6669 vs. $2(178) = 356$.

Time = 77.72 (sec) , antiderivative size = 6669, normalized size of antiderivative = 33.85

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^7}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**7,x)
```

output
```
Piecewise((-30030*A*a**2*tan(e/2 + f*x/2)**12/(15015*c**7*f*tan(e/2 + f*x/
2)**13 - 195195*c**7*f*tan(e/2 + f*x/2)**12 + 1171170*c**7*f*tan(e/2 + f*x
/2)**11 - 4294290*c**7*f*tan(e/2 + f*x/2)**10 + 10735725*c**7*f*tan(e/2 +
f*x/2)**9 - 19324305*c**7*f*tan(e/2 + f*x/2)**8 + 25765740*c**7*f*tan(e/2
+ f*x/2)**7 - 25765740*c**7*f*tan(e/2 + f*x/2)**6 + 19324305*c**7*f*tan(e/
2 + f*x/2)**5 - 10735725*c**7*f*tan(e/2 + f*x/2)**4 + 4294290*c**7*f*tan(e
/2 + f*x/2)**3 - 1171170*c**7*f*tan(e/2 + f*x/2)**2 + 195195*c**7*f*tan(e/
2 + f*x/2) - 15015*c**7*f) + 120120*A*a**2*tan(e/2 + f*x/2)**11/(15015*c**
7*f*tan(e/2 + f*x/2)**13 - 195195*c**7*f*tan(e/2 + f*x/2)**12 + 1171170*c*
*7*f*tan(e/2 + f*x/2)**11 - 4294290*c**7*f*tan(e/2 + f*x/2)**10 + 10735725
*c**7*f*tan(e/2 + f*x/2)**9 - 19324305*c**7*f*tan(e/2 + f*x/2)**8 + 257657
40*c**7*f*tan(e/2 + f*x/2)**7 - 25765740*c**7*f*tan(e/2 + f*x/2)**6 + 1932
4305*c**7*f*tan(e/2 + f*x/2)**5 - 10735725*c**7*f*tan(e/2 + f*x/2)**4 + 42
94290*c**7*f*tan(e/2 + f*x/2)**3 - 1171170*c**7*f*tan(e/2 + f*x/2)**2 + 19
5195*c**7*f*tan(e/2 + f*x/2) - 15015*c**7*f) - 540540*A*a**2*tan(e/2 + f*x
/2)**10/(15015*c**7*f*tan(e/2 + f*x/2)**13 - 195195*c**7*f*tan(e/2 + f*x/2
)**12 + 1171170*c**7*f*tan(e/2 + f*x/2)**11 - 4294290*c**7*f*tan(e/2 + f*x
/2)**10 + 10735725*c**7*f*tan(e/2 + f*x/2)**9 - 19324305*c**7*f*tan(e/2 +
f*x/2)**8 + 25765740*c**7*f*tan(e/2 + f*x/2)**7 - 25765740*c**7*f*tan(e/2
+ f*x/2)**6 + 19324305*c**7*f*tan(e/2 + f*x/2)**5 - 10735725*c**7*f*tan...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3120 vs. $2(192) = 384$.

Time = 0.15 (sec) , antiderivative size = 3120, normalized size of antiderivative = 15.84

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^7}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x, algori
thm="maxima")
```

output
```
-2/45045*(2*A*a^2*(4771*sin(f*x + e)/(cos(f*x + e) + 1) - 28626*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 74932*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 18
7330*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 265122*sin(f*x + e)^5/(cos(f*x
+ e) + 1)^5 - 353496*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 276276*sin(f*x
+ e)^7/(cos(f*x + e) + 1)^7 - 207207*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 +
 75075*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 30030*sin(f*x + e)^10/(cos(f*
x + e) + 1)^10 - 367)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c
^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1)
^6 - 1716*c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^
8/(cos(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286
*c^7*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f
*x + e) + 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f
*x + e)^13/(cos(f*x + e) + 1)^13) + 4*B*a^2*(4771*sin(f*x + e)/(cos(f*x +
e) + 1) - 28626*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 74932*sin(f*x + e)^3
/(cos(f*x + e) + 1)^3 - 187330*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 26512
2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 353496*sin(f*x + e)^6/(cos(f*x + e
) + 1)^6 + 276276*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 207207*sin(f*x + e
)^8/(cos(f*x + e) + 1)^8 + 75075*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 421 vs. $2(192) = 384$.

Time = 0.29 (sec) , antiderivative size = 421, normalized size of antiderivative = 2.14

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx =$$

$$-\frac{2\left(15015\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{12}-60060\,Aa^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}+15015\,Ba^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}+270\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x, algori
thm="giac")
```

output
```
-2/15015*(15015*A*a^2*tan(1/2*f*x + 1/2*e)^12 - 60060*A*a^2*tan(1/2*f*x +
1/2*e)^11 + 15015*B*a^2*tan(1/2*f*x + 1/2*e)^11 + 270270*A*a^2*tan(1/2*f*x
 + 1/2*e)^10 - 15015*B*a^2*tan(1/2*f*x + 1/2*e)^10 - 600600*A*a^2*tan(1/2*
f*x + 1/2*e)^9 + 105105*B*a^2*tan(1/2*f*x + 1/2*e)^9 + 1174173*A*a^2*tan(1
/2*f*x + 1/2*e)^8 - 93093*B*a^2*tan(1/2*f*x + 1/2*e)^8 - 1465464*A*a^2*tan
(1/2*f*x + 1/2*e)^7 + 234234*B*a^2*tan(1/2*f*x + 1/2*e)^7 + 1559844*A*a^2*
tan(1/2*f*x + 1/2*e)^6 - 131274*B*a^2*tan(1/2*f*x + 1/2*e)^6 - 1094808*A*a
^2*tan(1/2*f*x + 1/2*e)^5 + 181038*B*a^2*tan(1/2*f*x + 1/2*e)^5 + 659945*A
*a^2*tan(1/2*f*x + 1/2*e)^4 - 47190*B*a^2*tan(1/2*f*x + 1/2*e)^4 - 233948*
A*a^2*tan(1/2*f*x + 1/2*e)^3 + 45903*B*a^2*tan(1/2*f*x + 1/2*e)^3 + 77454*
A*a^2*tan(1/2*f*x + 1/2*e)^2 - 1599*B*a^2*tan(1/2*f*x + 1/2*e)^2 - 7904*A*
a^2*tan(1/2*f*x + 1/2*e) + 2769*B*a^2*tan(1/2*f*x + 1/2*e) + 1763*A*a^2 -
213*B*a^2)/(c^7*f*(tan(1/2*f*x + 1/2*e) - 1)^13)
```

Mupad [B] (verification not implemented)

Time = 37.50 (sec) , antiderivative size = 500, normalized size of antiderivative = 2.54

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx = \text{Too large to display}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^7,x
)
```

output
```
-(2*cos(e/2 + (f*x)/2)*((994249*A*a^2)/32 - (63639*B*a^2)/32 - (1609013*A*
a^2*cos(2*e + 2*f*x))/64 + (85687*A*a^2*cos(3*e + 3*f*x))/16 + (79591*A*a^
2*cos(4*e + 4*f*x))/32 - (5261*A*a^2*cos(5*e + 5*f*x))/16 - (1771*A*a^2*co
s(6*e + 6*f*x))/64 + (140553*B*a^2*cos(2*e + 2*f*x))/64 - (4431*B*a^2*cos(
3*e + 3*f*x))/8 - (10161*B*a^2*cos(4*e + 4*f*x))/32 + 36*B*a^2*cos(5*e + 5
*f*x) + (231*B*a^2*cos(6*e + 6*f*x))/64 + (636207*A*a^2*sin(2*e + 2*f*x))/
64 + (309309*A*a^2*sin(3*e + 3*f*x))/32 - (7007*A*a^2*sin(4*e + 4*f*x))/4
- (12389*A*a^2*sin(5*e + 5*f*x))/32 + (1755*A*a^2*sin(6*e + 6*f*x))/64 - (
121407*B*a^2*sin(2*e + 2*f*x))/64 - (39039*B*a^2*sin(3*e + 3*f*x))/32 + (3
003*B*a^2*sin(4*e + 4*f*x))/16 + (1599*B*a^2*sin(5*e + 5*f*x))/32 - (195*B
*a^2*sin(6*e + 6*f*x))/64 - (93221*A*a^2*cos(e + f*x))/8 + (3291*B*a^2*cos
(e + f*x))/8 - (704847*A*a^2*sin(e + f*x))/16 + (125697*B*a^2*sin(e + f*x)
)/16))/(15015*c^7*f*((1287*2^(1/2)*cos((3*e)/2 - pi/4 + (3*f*x)/2))/64 - (
429*2^(1/2)*cos(e/2 + pi/4 + (f*x)/2))/16 + (715*2^(1/2)*cos((5*e)/2 + pi/
4 + (5*f*x)/2))/64 - (143*2^(1/2)*cos((7*e)/2 - pi/4 + (7*f*x)/2))/32 - (3
9*2^(1/2)*cos((9*e)/2 + pi/4 + (9*f*x)/2))/32 + (13*2^(1/2)*cos((11*e)/2 -
 pi/4 + (11*f*x)/2))/64 + (2^(1/2)*cos((13*e)/2 + pi/4 + (13*f*x)/2))/64))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 543, normalized size of antiderivative = 2.76

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx$$
$$= \frac{a^2\left(2310a-231\sin\left(fx+e\right)^7b+3781\sin\left(fx+e\right)^6a+1599\sin\left(fx+e\right)^6b-11371\sin\left(fx+e\right)^5a-55\right.}{}$$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x)
```

output

```
(a**2*( - 555*cos(e + f*x)*sin(e + f*x)**6*a - 195*cos(e + f*x)*sin(e + f*
x)**6*b + 3338*cos(e + f*x)*sin(e + f*x)**5*a + 1152*cos(e + f*x)*sin(e +
f*x)**5*b - 8377*cos(e + f*x)*sin(e + f*x)**4*a - 2808*cos(e + f*x)*sin(e
+ f*x)**4*b + 11248*cos(e + f*x)*sin(e + f*x)**3*a + 3567*cos(e + f*x)*sin
(e + f*x)**3*b - 8572*cos(e + f*x)*sin(e + f*x)**2*a - 6123*cos(e + f*x)*s
in(e + f*x)**2*b + 608*cos(e + f*x)*sin(e + f*x)*a - 213*cos(e + f*x)*sin(
e + f*x)*b - 2310*cos(e + f*x)*a - 539*sin(e + f*x)**7*a - 231*sin(e + f*x
)**7*b + 3781*sin(e + f*x)**6*a + 1599*sin(e + f*x)**6*b - 11371*sin(e + f
*x)**5*a - 4734*sin(e + f*x)**5*b + 19009*sin(e + f*x)**4*a + 7761*sin(e +
 f*x)**4*b - 19086*sin(e + f*x)**3*a - 3834*sin(e + f*x)**3*b + 14528*sin(
e + f*x)**2*a + 8892*sin(e + f*x)**2*b + 608*sin(e + f*x)*a - 213*sin(e +
f*x)*b + 2310*a))/(15015*c**7*f*(cos(e + f*x)*sin(e + f*x)**6 - 6*cos(e +
f*x)*sin(e + f*x)**5 + 15*cos(e + f*x)*sin(e + f*x)**4 - 20*cos(e + f*x)*s
in(e + f*x)**3 + 15*cos(e + f*x)*sin(e + f*x)**2 - 6*cos(e + f*x)*sin(e +
f*x) + cos(e + f*x) + sin(e + f*x)**7 - 7*sin(e + f*x)**6 + 21*sin(e + f*x
)**5 - 35*sin(e + f*x)**4 + 35*sin(e + f*x)**3 - 21*sin(e + f*x)**2 + 7*si
n(e + f*x) - 1))
```

3.38 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^6\,dx$

Optimal result

Integrand size = 36, antiderivative size = 265

$$\begin{aligned}
&\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^6\,dx \\
&= \frac{11}{256}a^3(10A-3B)c^6x+\frac{11a^3(10A-3B)c^6\cos^7(e+fx)}{560f} \\
&\quad+\frac{11a^3(10A-3B)c^6\cos(e+fx)\sin(e+fx)}{256f} \\
&\quad+\frac{11a^3(10A-3B)c^6\cos^3(e+fx)\sin(e+fx)}{384f} \\
&\quad+\frac{11a^3(10A-3B)c^6\cos^5(e+fx)\sin(e+fx)}{480f} \\
&\quad-\frac{a^3B\cos^7(e+fx)\left(c^2-c^2\sin(e+fx)\right)^3}{10f} \\
&\quad+\frac{a^3(10A-3B)\cos^7(e+fx)\left(c^3-c^3\sin(e+fx)\right)^2}{90f} \\
&\quad+\frac{11a^3(10A-3B)\cos^7(e+fx)\left(c^6-c^6\sin(e+fx)\right)}{720f}
\end{aligned}$$

output
```
11/256*a^3*(10*A-3*B)*c^6*x+11/560*a^3*(10*A-3*B)*c^6*cos(f*x+e)^7/f+11/25
6*a^3*(10*A-3*B)*c^6*cos(f*x+e)*sin(f*x+e)/f+11/384*a^3*(10*A-3*B)*c^6*cos
(f*x+e)^3*sin(f*x+e)/f+11/480*a^3*(10*A-3*B)*c^6*cos(f*x+e)^5*sin(f*x+e)/f
-1/10*a^3*B*cos(f*x+e)^7*(c^2-c^2*sin(f*x+e))^3/f+1/90*a^3*(10*A-3*B)*cos(
f*x+e)^7*(c^3-c^3*sin(f*x+e))^2/f+11/720*a^3*(10*A-3*B)*cos(f*x+e)^7*(c^6-
c^6*sin(f*x+e))/f
```

Mathematica [A] (verified)

Time = 12.47 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.96

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^6\,dx$$
$$= \frac{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^6(27720(10A-3B)(e+fx)+5040(33A-19B)\cos(e+fx)+3}{}$$

input
```
Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^6,x]
```

output
```
((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^6*(27720*(10*A - 3*B)*(e + f*
x) + 5040*(33*A - 19*B)*Cos[e + f*x] + 3360*(29*A - 15*B)*Cos[3*(e + f*x)]
 + 10080*(3*A - B)*Cos[5*(e + f*x)] + 360*(9*A + 5*B)*Cos[7*(e + f*x)] - 2
80*(A - 3*B)*Cos[9*(e + f*x)] + 1260*(144*A - 25*B)*Sin[2*(e + f*x)] + 252
0*(6*A + 7*B)*Sin[4*(e + f*x)] - 210*(32*A - 51*B)*Sin[6*(e + f*x)] - 315*
(6*A - 5*B)*Sin[8*(e + f*x)] - 126*B*Sin[10*(e + f*x)]))/(645120*f*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])^12*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.81, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.472$, Rules used = {3042, 3446, 3042, 3339, 3042, 3157, 3042, 3157, 3042, 3148, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^3(c-c\sin(e+fx))^6(A+B\sin(e+fx))\,dx$$

$\Big\downarrow$ 3042

$$\int (a\sin(e+fx)+a)^3(c-c\sin(e+fx))^6(A+B\sin(e+fx))dx$$

$\Big\downarrow$ 3446

$$a^3c^3\int \cos^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^3dx$$

$\Big\downarrow$ 3042

$$a^3c^3\int \cos(e+fx)^6(A+B\sin(e+fx))(c-c\sin(e+fx))^3dx$$

$\Big\downarrow$ 3339

$$a^3c^3\left(\frac{1}{10}(10A-3B)\int \cos^6(e+fx)(c-c\sin(e+fx))^3dx-\frac{B\cos^7(e+fx)(c-c\sin(e+fx))^3}{10f}\right)$$

$\Big\downarrow$ 3042

$$a^3c^3\left(\frac{1}{10}(10A-3B)\int \cos(e+fx)^6(c-c\sin(e+fx))^3dx-\frac{B\cos^7(e+fx)(c-c\sin(e+fx))^3}{10f}\right)$$

$\Big\downarrow$ 3157

$$a^3c^3\left(\frac{1}{10}(10A-3B)\left(\frac{11}{9}c\int \cos^6(e+fx)(c-c\sin(e+fx))^2dx+\frac{c\cos^7(e+fx)(c-c\sin(e+fx))^2}{9f}\right)-\frac{B\cos}{}\right.$$

$\Big\downarrow$ 3042

$$a^3c^3\left(\frac{1}{10}(10A-3B)\left(\frac{11}{9}c\int \cos(e+fx)^6(c-c\sin(e+fx))^2dx+\frac{c\cos^7(e+fx)(c-c\sin(e+fx))^2}{9f}\right)-\frac{B\cos}{}\right.$$

$\Big\downarrow$ 3157

$$a^3c^3\left(\frac{1}{10}(10A-3B)\left(\frac{11}{9}c\left(\frac{9}{8}c\int \cos^6(e+fx)(c-c\sin(e+fx))dx+\frac{\cos^7(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{8f}\right)+\frac{c}{}\right.\right.$$

$\Big\downarrow$ 3042

$$a^3c^3\left(\frac{1}{10}(10A-3B)\left(\frac{11}{9}c\left(\frac{9}{8}c\int\cos(e+fx)^6(c-c\sin(e+fx))dx+\frac{\cos^7(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{8f}\right)+\frac{c}{}\right.\right.$$

$\Big\downarrow$ 3148

$$a^3c^3\left(\frac{1}{10}(10A-3B)\left(\frac{11}{9}c\left(\frac{9}{8}c\left(c\int\cos^6(e+fx)dx+\frac{c\cos^7(e+fx)}{7f}\right)+\frac{\cos^7(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{8f}\right)\right.\right.$$

$\Big\downarrow$ 3042

$$a^3c^3\left(\frac{1}{10}(10A-3B)\left(\frac{11}{9}c\left(\frac{9}{8}c\left(c\int\sin\left(e+fx+\frac{\pi}{2}\right)^6dx+\frac{c\cos^7(e+fx)}{7f}\right)+\frac{\cos^7(e+fx)\left(c^2-c^2\sin(e+f\right.}{8f}\right.\right.\right.$$

$\Big\downarrow$ 3115

$$a^3c^3\left(\frac{1}{10}(10A-3B)\left(\frac{11}{9}c\left(\frac{9}{8}c\left(c\left(\frac{5}{6}\int\cos^4(e+fx)dx+\frac{\sin(e+fx)\cos^5(e+fx)}{6f}\right)+\frac{c\cos^7(e+fx)}{7f}\right)+\frac{\cos^7}{}\right.\right.\right.$$

$\Big\downarrow$ 3042

$$a^3c^3\left(\frac{1}{10}(10A-3B)\left(\frac{11}{9}c\left(\frac{9}{8}c\left(c\left(\frac{5}{6}\int\sin\left(e+fx+\frac{\pi}{2}\right)^4dx+\frac{\sin(e+fx)\cos^5(e+fx)}{6f}\right)+\frac{c\cos^7(e+fx)}{7f}\right)\right.\right.\right.$$

$\Big\downarrow$ 3115

$$a^3c^3\left(\frac{1}{10}(10A-3B)\left(\frac{11}{9}c\left(\frac{9}{8}c\left(c\left(\frac{5}{6}\left(\frac{3}{4}\int\cos^2(e+fx)dx+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{\sin(e+fx)\cos^5(e+}{6f}\right.\right.\right.\right.\right.$$

$\Big\downarrow$ 3042

$$a^3c^3\left(\frac{1}{10}(10A-3B)\left(\frac{11}{9}c\left(\frac{9}{8}c\left(c\left(\frac{5}{6}\left(\frac{3}{4}\int\sin\left(e+fx+\frac{\pi}{2}\right)^2dx+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{\sin(e+fx)\,\mathrm{c}}{6f}\right.\right.\right.\right.\right.$$

$\Big\downarrow$ 3115

$$a^3c^3\left(\frac{1}{10}(10A-3B)\left(\frac{11}{9}c\left(\frac{9}{8}c\left(c\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}\right)+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{\mathrm{si}}{}\right.\right.\right.\right.\right.$$

$\Big\downarrow$ 24

$$a^3c^3\left(\frac{1}{10}(10A-3B)\left(\frac{11}{9}c\left(\frac{\cos^7(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{8f}+\frac{9}{8}c\left(\frac{c\cos^7(e+fx)}{7f}+c\left(\frac{\sin(e+fx)\cos^5(e+\right.\right.\right.\right.\right.$$

input
```
Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^6,x]
```

output
```
a^3*c^3*(-1/10*(B*Cos[e + f*x]^7*(c - c*Sin[e + f*x])^3)/f + ((10*A - 3*B)
*((c*Cos[e + f*x]^7*(c - c*Sin[e + f*x])^2)/(9*f) + (11*c*((Cos[e + f*x]^7
*(c^2 - c^2*Sin[e + f*x]))/(8*f) + (9*c*((c*Cos[e + f*x]^7)/(7*f) + c*((Co
s[e + f*x]^5*Sin[e + f*x])/(6*f) + (5*((Cos[e + f*x]^3*Sin[e + f*x])/(4*f)
 + (3*(x/2 + (Cos[e + f*x]*Sin[e + f*x])/(2*f)))/4))/6)))/8))/9))/10)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3157
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && Integers
Q[2*m, 2*p]
```

rule 3339
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 650 vs. $2(249) = 498$.

Time = 0.27 (sec) , antiderivative size = 651, normalized size of antiderivative = 2.46

$$a^3 A\, c^6 (fx+e) - a^3 B\, c^6 \cos\left(fx+e\right) - \frac{8a^3 A\, c^6 \left(2+\sin(fx+e)^2\right)\cos(fx+e)}{3} + 8a^3 B\, c^6 \left(-\frac{\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx}{4}\right.$$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^6,x)
```

output
```
1/f*(a^3*A*c^6*(f*x+e)-a^3*B*c^6*cos(f*x+e)-8/3*a^3*A*c^6*(2+sin(f*x+e)^2)
*cos(f*x+e)+8*a^3*B*c^6*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8
*f*x+3/8*e)+3*a^3*A*c^6*cos(f*x+e)-3*a^3*B*c^6*(-1/2*sin(f*x+e)*cos(f*x+e)
+1/2*f*x+1/2*e)-1/9*a^3*A*c^6*(128/35+sin(f*x+e)^8+8/7*sin(f*x+e)^6+48/35*
sin(f*x+e)^4+64/35*sin(f*x+e)^2)*cos(f*x+e)-3*a^3*A*c^6*(-1/8*(sin(f*x+e)^
7+7/6*sin(f*x+e)^5+35/24*sin(f*x+e)^3+35/16*sin(f*x+e))*cos(f*x+e)+35/128*
f*x+35/128*e)+8*a^3*A*c^6*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*
x+e))*cos(f*x+e)+5/16*f*x+5/16*e)+6/5*a^3*A*c^6*(8/3+sin(f*x+e)^4+4/3*sin(
f*x+e)^2)*cos(f*x+e)-6*a^3*A*c^6*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f
*x+e)+3/8*f*x+3/8*e)+a^3*B*c^6*(-1/10*(sin(f*x+e)^9+9/8*sin(f*x+e)^7+21/16
*sin(f*x+e)^5+105/64*sin(f*x+e)^3+315/128*sin(f*x+e))*cos(f*x+e)+63/256*f*
x+63/256*e)+1/3*a^3*B*c^6*(128/35+sin(f*x+e)^8+8/7*sin(f*x+e)^6+48/35*sin(
f*x+e)^4+64/35*sin(f*x+e)^2)*cos(f*x+e)-8/7*a^3*B*c^6*(16/5+sin(f*x+e)^6+6
/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e)-6*a^3*B*c^6*(-1/6*(sin(f*x+e)
^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)+6/5*a^3*B
*c^6*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.68

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^6\,dx =$$
$$-\frac{8960\,(A-3\,B)a^3c^6\cos{(fx+e)}^9-46080\,(A-B)a^3c^6\cos{(fx+e)}^7-3465\,(10\,A-3\,B)a^3c^6fx+21$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^6,x, algori
thm="fricas")
```

output
```
-1/80640*(8960*(A - 3*B)*a^3*c^6*cos(f*x + e)^9 - 46080*(A - B)*a^3*c^6*co
s(f*x + e)^7 - 3465*(10*A - 3*B)*a^3*c^6*f*x + 21*(384*B*a^3*c^6*cos(f*x +
 e)^9 + 48*(30*A - 41*B)*a^3*c^6*cos(f*x + e)^7 - 88*(10*A - 3*B)*a^3*c^6*
cos(f*x + e)^5 - 110*(10*A - 3*B)*a^3*c^6*cos(f*x + e)^3 - 165*(10*A - 3*B
)*a^3*c^6*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1948 vs. $2(252) = 504$.

Time = 1.59 (sec) , antiderivative size = 1948, normalized size of antiderivative = 7.35

$$\int (a + a\sin(e + fx))^3 (A + B\sin(e + fx))(c - c\sin(e + fx))^6 \, dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**6,x)
```

output
```
Piecewise((-105*A*a**3*c**6*x*sin(e + f*x)**8/128 - 105*A*a**3*c**6*x*sin(
e + f*x)**6*cos(e + f*x)**2/32 + 5*A*a**3*c**6*x*sin(e + f*x)**6/2 - 315*A
*a**3*c**6*x*sin(e + f*x)**4*cos(e + f*x)**4/64 + 15*A*a**3*c**6*x*sin(e +
 f*x)**4*cos(e + f*x)**2/2 - 9*A*a**3*c**6*x*sin(e + f*x)**4/4 - 105*A*a**
3*c**6*x*sin(e + f*x)**2*cos(e + f*x)**6/32 + 15*A*a**3*c**6*x*sin(e + f*x
)**2*cos(e + f*x)**4/2 - 9*A*a**3*c**6*x*sin(e + f*x)**2*cos(e + f*x)**2/2
 - 105*A*a**3*c**6*x*cos(e + f*x)**8/128 + 5*A*a**3*c**6*x*cos(e + f*x)**6
/2 - 9*A*a**3*c**6*x*cos(e + f*x)**4/4 + A*a**3*c**6*x - A*a**3*c**6*sin(e
 + f*x)**8*cos(e + f*x)/f + 279*A*a**3*c**6*sin(e + f*x)**7*cos(e + f*x)/(
128*f) - 8*A*a**3*c**6*sin(e + f*x)**6*cos(e + f*x)**3/(3*f) + 511*A*a**3*
c**6*sin(e + f*x)**5*cos(e + f*x)**3/(128*f) - 11*A*a**3*c**6*sin(e + f*x)
**5*cos(e + f*x)/(2*f) - 16*A*a**3*c**6*sin(e + f*x)**4*cos(e + f*x)**5/(5
*f) + 6*A*a**3*c**6*sin(e + f*x)**4*cos(e + f*x)/f + 385*A*a**3*c**6*sin(e
 + f*x)**3*cos(e + f*x)**5/(128*f) - 20*A*a**3*c**6*sin(e + f*x)**3*cos(e
+ f*x)**3/(3*f) + 15*A*a**3*c**6*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 64*A
*a**3*c**6*sin(e + f*x)**2*cos(e + f*x)**7/(35*f) + 8*A*a**3*c**6*sin(e +
f*x)**2*cos(e + f*x)**3/f - 8*A*a**3*c**6*sin(e + f*x)**2*cos(e + f*x)/f +
 105*A*a**3*c**6*sin(e + f*x)*cos(e + f*x)**7/(128*f) - 5*A*a**3*c**6*sin(
e + f*x)*cos(e + f*x)**5/(2*f) + 9*A*a**3*c**6*sin(e + f*x)*cos(e + f*x)**
3/(4*f) - 128*A*a**3*c**6*cos(e + f*x)**9/(315*f) + 16*A*a**3*c**6*cos(...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 661 vs. $2(252) = 504$.

Time = 0.05 (sec) , antiderivative size = 661, normalized size of antiderivative = 2.49

$$\int (a + a\sin(e + fx))^3 (A + B\sin(e + fx))(c - c\sin(e + fx))^6 \, dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^6,x, algori
thm="maxima")
```

output
```
-1/645120*(2048*(35*cos(f*x + e)^9 - 180*cos(f*x + e)^7 + 378*cos(f*x + e)
^5 - 420*cos(f*x + e)^3 + 315*cos(f*x + e))*A*a^3*c^6 - 258048*(3*cos(f*x
+ e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*A*a^3*c^6 - 1720320*(cos(f*x
 + e)^3 - 3*cos(f*x + e))*A*a^3*c^6 + 630*(128*sin(2*f*x + 2*e)^3 + 840*f*
x + 840*e + 3*sin(8*f*x + 8*e) + 168*sin(4*f*x + 4*e) - 768*sin(2*f*x + 2*
e))*A*a^3*c^6 - 26880*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x
+ 4*e) - 48*sin(2*f*x + 2*e))*A*a^3*c^6 + 120960*(12*f*x + 12*e + sin(4*f*
x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^3*c^6 - 645120*(f*x + e)*A*a^3*c^6 - 61
44*(35*cos(f*x + e)^9 - 180*cos(f*x + e)^7 + 378*cos(f*x + e)^5 - 420*cos(
f*x + e)^3 + 315*cos(f*x + e))*B*a^3*c^6 - 147456*(5*cos(f*x + e)^7 - 21*c
os(f*x + e)^5 + 35*cos(f*x + e)^3 - 35*cos(f*x + e))*B*a^3*c^6 - 258048*(3
*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*c^6 + 63*(32*
sin(2*f*x + 2*e)^5 - 640*sin(2*f*x + 2*e)^3 - 2520*f*x - 2520*e - 25*sin(8
*f*x + 8*e) - 600*sin(4*f*x + 4*e) + 2560*sin(2*f*x + 2*e))*B*a^3*c^6 + 20
160*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*
f*x + 2*e))*B*a^3*c^6 - 161280*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2
*f*x + 2*e))*B*a^3*c^6 + 483840*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3*c^6
 - 1935360*A*a^3*c^6*cos(f*x + e) + 645120*B*a^3*c^6*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.27

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^6\,dx$$
$$= -\frac{Ba^3c^6\sin(10\,fx+10\,e)}{5120\,f} + \frac{11}{256}\,(10\,Aa^3c^6-3\,Ba^3c^6)x - \frac{(Aa^3c^6-3\,Ba^3c^6)\cos(9\,fx+9\,e)}{2304\,f} + \frac{(9\,Aa^3c^6+5\,Ba^3c^6)\cos(7\,fx+7\,e)}{1792\,f} + \frac{(3\,Aa^3c^6-Ba^3c^6)\cos(5\,fx+5\,e)}{64\,f} + \frac{(29\,Aa^3c^6-15\,Ba^3c^6)\cos(3\,fx+3\,e)}{192\,f} + \frac{(33\,Aa^3c^6-19\,Ba^3c^6)\cos(fx+e)}{128\,f} - \frac{(6\,Aa^3c^6-5\,Ba^3c^6)\sin(8\,fx+8\,e)}{2048\,f} - \frac{(32\,Aa^3c^6-51\,Ba^3c^6)\sin(6\,fx+6\,e)}{3072\,f} + \frac{(6\,Aa^3c^6+7\,Ba^3c^6)\sin(4\,fx+4\,e)}{256\,f} + \frac{(144\,Aa^3c^6-25\,Ba^3c^6)\sin(2\,fx+2\,e)}{512\,f}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^6,x, algori
thm="giac")
```

output
```
-1/5120*B*a^3*c^6*sin(10*f*x + 10*e)/f + 11/256*(10*A*a^3*c^6 - 3*B*a^3*c^
6)*x - 1/2304*(A*a^3*c^6 - 3*B*a^3*c^6)*cos(9*f*x + 9*e)/f + 1/1792*(9*A*a
^3*c^6 + 5*B*a^3*c^6)*cos(7*f*x + 7*e)/f + 1/64*(3*A*a^3*c^6 - B*a^3*c^6)*
cos(5*f*x + 5*e)/f + 1/192*(29*A*a^3*c^6 - 15*B*a^3*c^6)*cos(3*f*x + 3*e)/
f + 1/128*(33*A*a^3*c^6 - 19*B*a^3*c^6)*cos(f*x + e)/f - 1/2048*(6*A*a^3*c
^6 - 5*B*a^3*c^6)*sin(8*f*x + 8*e)/f - 1/3072*(32*A*a^3*c^6 - 51*B*a^3*c^6
)*sin(6*f*x + 6*e)/f + 1/256*(6*A*a^3*c^6 + 7*B*a^3*c^6)*sin(4*f*x + 4*e)/
f + 1/512*(144*A*a^3*c^6 - 25*B*a^3*c^6)*sin(2*f*x + 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 38.07 (sec) , antiderivative size = 812, normalized size of antiderivative = 3.06

$$\int (a + a\sin(e + fx))^3(A + B\sin(e + fx))(c - c\sin(e + fx))^6\,dx = \text{Too large to display}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^6,x)
```

output
```
(tan(e/2 + (f*x)/2)^18*(6*A*a^3*c^6 - 2*B*a^3*c^6) + tan(e/2 + (f*x)/2)^16
*(22*A*a^3*c^6 - 18*B*a^3*c^6) + tan(e/2 + (f*x)/2)^8*(84*A*a^3*c^6 - 28*B
*a^3*c^6) + tan(e/2 + (f*x)/2)^14*((136*A*a^3*c^6)/3 - 8*B*a^3*c^6) + tan(
e/2 + (f*x)/2)^4*((136*A*a^3*c^6)/7 - (24*B*a^3*c^6)/7) + tan(e/2 + (f*x)/
2)^10*(116*A*a^3*c^6 - 60*B*a^3*c^6) - tan(e/2 + (f*x)/2)^19*((73*A*a^3*c^
6)/64 + (33*B*a^3*c^6)/128) + tan(e/2 + (f*x)/2)^2*((202*A*a^3*c^6)/63 - (
58*B*a^3*c^6)/21) + tan(e/2 + (f*x)/2)^12*((328*A*a^3*c^6)/3 - 72*B*a^3*c^
6) + tan(e/2 + (f*x)/2)^7*((341*A*a^3*c^6)/16 + (333*B*a^3*c^6)/32) - tan(
e/2 + (f*x)/2)^13*((341*A*a^3*c^6)/16 + (333*B*a^3*c^6)/32) + tan(e/2 + (f
*x)/2)^6*((456*A*a^3*c^6)/7 - (344*B*a^3*c^6)/7) + tan(e/2 + (f*x)/2)^5*((
449*A*a^3*c^6)/48 - (577*B*a^3*c^6)/160) - tan(e/2 + (f*x)/2)^15*((449*A*a
^3*c^6)/48 - (577*B*a^3*c^6)/160) + tan(e/2 + (f*x)/2)^3*((2117*A*a^3*c^6)
/192 - (705*B*a^3*c^6)/128) - tan(e/2 + (f*x)/2)^17*((2117*A*a^3*c^6)/192
- (705*B*a^3*c^6)/128) + tan(e/2 + (f*x)/2)^9*((699*A*a^3*c^6)/32 - (2749*
B*a^3*c^6)/64) - tan(e/2 + (f*x)/2)^11*((699*A*a^3*c^6)/32 - (2749*B*a^3*c
^6)/64) + tan(e/2 + (f*x)/2)*((73*A*a^3*c^6)/64 + (33*B*a^3*c^6)/128) + (5
8*A*a^3*c^6)/63 - (10*B*a^3*c^6)/21)/(f*(10*tan(e/2 + (f*x)/2)^2 + 45*tan(
e/2 + (f*x)/2)^4 + 120*tan(e/2 + (f*x)/2)^6 + 210*tan(e/2 + (f*x)/2)^8 + 2
52*tan(e/2 + (f*x)/2)^10 + 210*tan(e/2 + (f*x)/2)^12 + 120*tan(e/2 + (f*x)
/2)^14 + 45*tan(e/2 + (f*x)/2)^16 + 10*tan(e/2 + (f*x)/2)^18 + tan(e/2 ...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.25

$$\int (a + a\sin(e + fx))^3(A + B\sin(e + fx))(c - c\sin(e + fx))^6\,dx$$

$$= \frac{a^3c^6(-8064\cos(fx+e)\sin(fx+e)^9 b - 8960\cos(fx+e)\sin(fx+e)^8 a + 26880\cos(fx+e)\sin(fx}{}$$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^6,x)
```

output
```
(a**3*c**6*( - 8064*cos(e + f*x)*sin(e + f*x)**9*b - 8960*cos(e + f*x)*sin
(e + f*x)**8*a + 26880*cos(e + f*x)*sin(e + f*x)**8*b + 30240*cos(e + f*x)
*sin(e + f*x)**7*a - 9072*cos(e + f*x)*sin(e + f*x)**7*b - 10240*cos(e + f
*x)*sin(e + f*x)**6*a - 61440*cos(e + f*x)*sin(e + f*x)**6*b - 72240*cos(e
 + f*x)*sin(e + f*x)**5*a + 70056*cos(e + f*x)*sin(e + f*x)**5*b + 84480*c
os(e + f*x)*sin(e + f*x)**4*a + 23040*cos(e + f*x)*sin(e + f*x)**4*b + 306
60*cos(e + f*x)*sin(e + f*x)**3*a - 73710*cos(e + f*x)*sin(e + f*x)**3*b -
 102400*cos(e + f*x)*sin(e + f*x)**2*a + 30720*cos(e + f*x)*sin(e + f*x)**
2*b + 45990*cos(e + f*x)*sin(e + f*x)*a + 10395*cos(e + f*x)*sin(e + f*x)*
b + 37120*cos(e + f*x)*a - 19200*cos(e + f*x)*b + 34650*a*f*x - 37120*a -
10395*b*f*x + 19200*b))/(80640*f)
```

3.39 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx$

Optimal result

Integrand size = 36, antiderivative size = 222

$$\begin{aligned}\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx\\
&=\frac{5}{128}a^3(9A-2B)c^5x+\frac{a^3(9A-2B)c^5\cos^7(e+fx)}{56f}\\
&\quad+\frac{5a^3(9A-2B)c^5\cos(e+fx)\sin(e+fx)}{128f}\\
&\quad+\frac{5a^3(9A-2B)c^5\cos^3(e+fx)\sin(e+fx)}{192f}\\
&\quad+\frac{a^3(9A-2B)c^5\cos^5(e+fx)\sin(e+fx)}{48f}-\frac{a^3Bc^3\cos^7(e+fx)(c-c\sin(e+fx))^2}{9f}\\
&\quad+\frac{a^3(9A-2B)\cos^7(e+fx)\left(c^5-c^5\sin(e+fx)\right)}{72f}\end{aligned}$$

output
```
5/128*a^3*(9*A-2*B)*c^5*x+1/56*a^3*(9*A-2*B)*c^5*cos(f*x+e)^7/f+5/128*a^3*
(9*A-2*B)*c^5*cos(f*x+e)*sin(f*x+e)/f+5/192*a^3*(9*A-2*B)*c^5*cos(f*x+e)^3
*sin(f*x+e)/f+1/48*a^3*(9*A-2*B)*c^5*cos(f*x+e)^5*sin(f*x+e)/f-1/9*a^3*B*c
^3*cos(f*x+e)^7*(c-c*sin(f*x+e))^2/f+1/72*a^3*(9*A-2*B)*cos(f*x+e)^7*(c^5-
c^5*sin(f*x+e))/f
```

Mathematica [A] (verified)

Time = 11.45 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.05

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx$$
$$= \frac{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^5(2520(9A-2B)(e+fx)+504(20A-13B)\cos(e+fx)+336(}{}$$

input
```
Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^5,x]
```

output
```
((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^5*(2520*(9*A - 2*B)*(e + f*x)
 + 504*(20*A - 13*B)*Cos[e + f*x] + 336*(18*A - 11*B)*Cos[3*(e + f*x)] + 1
008*(2*A - B)*Cos[5*(e + f*x)] + 36*(8*A - B)*Cos[7*(e + f*x)] + 28*B*Cos[
9*(e + f*x)] + 2016*(8*A - B)*Sin[2*(e + f*x)] + 504*(5*A + 2*B)*Sin[4*(e
+ f*x)] + 672*B*Sin[6*(e + f*x)] - 63*(A - 2*B)*Sin[8*(e + f*x)]))/(64512*
f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^10*(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2])^6)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.81, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 3446, 3042, 3339, 3042, 3157, 3042, 3148, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^3(c-c\sin(e+fx))^5(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^3(c-c\sin(e+fx))^5(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3446}$$

$$a^3c^3\int \cos^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^2dx$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\int \cos(e+fx)^6(A+B\sin(e+fx))(c-c\sin(e+fx))^2dx$$

$$\Big\downarrow \text{3339}$$

$$a^3c^3\left(\frac{1}{9}(9A-2B)\int \cos^6(e+fx)(c-c\sin(e+fx))^2dx-\frac{B\cos^7(e+fx)(c-c\sin(e+fx))^2}{9f}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left(\frac{1}{9}(9A-2B)\int \cos(e+fx)^6(c-c\sin(e+fx))^2dx-\frac{B\cos^7(e+fx)(c-c\sin(e+fx))^2}{9f}\right)$$

$$\Big\downarrow \text{3157}$$

$$a^3c^3\left(\frac{1}{9}(9A-2B)\left(\frac{9}{8}c\int \cos^6(e+fx)(c-c\sin(e+fx))dx+\frac{\cos^7(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{8f}\right)-\frac{B\cos^7(e\right.$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left(\frac{1}{9}(9A-2B)\left(\frac{9}{8}c\int \cos(e+fx)^6(c-c\sin(e+fx))dx+\frac{\cos^7(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{8f}\right)-\frac{B\cos^7(e\right.$$

$$\Big\downarrow \text{3148}$$

$$a^3c^3\left(\frac{1}{9}(9A-2B)\left(\frac{9}{8}c\left(c\int \cos^6(e+fx)dx+\frac{c\cos^7(e+fx)}{7f}\right)+\frac{\cos^7(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{8f}\right)-\frac{B\cos}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left(\frac{1}{9}(9A-2B)\left(\frac{9}{8}c\left(c\int \sin\left(e+fx+\frac{\pi}{2}\right)^6dx+\frac{c\cos^7(e+fx)}{7f}\right)+\frac{\cos^7(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{8f}\right)-\right.$$

$$\Big\downarrow \text{3115}$$

$$a^3c^3\left(\frac{1}{9}(9A-2B)\left(\frac{9}{8}c\left(c\left(\frac{5}{6}\int \cos^4(e+fx)dx+\frac{\sin(e+fx)\cos^5(e+fx)}{6f}\right)+\frac{c\cos^7(e+fx)}{7f}\right)+\frac{\cos^7(e+fx}{}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left(\frac{1}{9}(9A-2B)\left(\frac{9}{8}c\left(c\left(\frac{5}{6}\int \sin\left(e+fx+\frac{\pi}{2}\right)^4dx+\frac{\sin(e+fx)\cos^5(e+fx)}{6f}\right)+\frac{c\cos^7(e+fx)}{7f}\right)+\frac{\cos^7(e\right.\right.$$

$$\Big\downarrow \text{3115}$$

$$a^3c^3\left(\frac{1}{9}(9A-2B)\left(\frac{9}{8}c\left(c\left(\frac{5}{6}\left(\frac{3}{4}\int \cos^2(e+fx)dx+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{\sin(e+fx)\cos^5(e+fx)}{6f}\right)\right.\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left(\frac{1}{9}(9A-2B)\left(\frac{9}{8}c\left(c\left(\frac{5}{6}\left(\frac{3}{4}\int \sin\left(e+fx+\frac{\pi}{2}\right)^2dx+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{\sin(e+fx)\cos^5(e+\right.\right.\right.\right.$$

$$\Big\downarrow \text{3115}$$

$$a^3c^3\left(\frac{1}{9}(9A-2B)\left(\frac{9}{8}c\left(c\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}\right)+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{\sin(e+f\right.\right.\right.\right.$$

$$\Big\downarrow \text{24}$$

$$a^3c^3\left(\frac{1}{9}(9A-2B)\left(\frac{\cos^7(e+fx)\left(c^2-c^2\sin(e+fx)\right)}{8f}+\frac{9}{8}c\left(\frac{c\cos^7(e+fx)}{7f}+c\left(\frac{\sin(e+fx)\cos^5(e+fx)}{6f}+\frac{5}{6}\right.\right.\right.\right.$$

input
```
Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5,x]
```

output
```
a^3*c^3*(-1/9*(B*Cos[e + f*x]^7*(c - c*Sin[e + f*x])^2)/f + ((9*A - 2*B)*(
(Cos[e + f*x]^7*(c^2 - c^2*Sin[e + f*x]))/(8*f) + (9*c*((c*Cos[e + f*x]^7)
/(7*f) + c*((Cos[e + f*x]^5*Sin[e + f*x])/(6*f) + (5*((Cos[e + f*x]^3*Sin[
e + f*x])/(4*f) + (3*(x/2 + (Cos[e + f*x]*Sin[e + f*x])/(2*f)))/4))/6)))/8
))/9)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3157
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && Integers
Q[2*m, 2*p]
```

rule 3339
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 610 vs. $2(208) = 416$.

Time $= 0.26$ (sec) , antiderivative size $= 611$, normalized size of antiderivative $= 2.75$

$$a^3 A\, c^5 (fx+e) - 2a^3 A\, c^5 \left(2+\sin\left(fx+e\right)^2\right)\cos\left(fx+e\right) + 6a^3 B\, c^5 \left(-\frac{\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}\right.$$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x)
```

output
```
1/f*(a^3*A*c^5*(f*x+e)-2*a^3*A*c^5*(2+sin(f*x+e)^2)*cos(f*x+e)+6*a^3*B*c^5
*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2*a^3*A*c^5
*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+2/3*a^3*B*c^5*(2+sin(f*x+e)^2)
*cos(f*x+e)+2*a^3*A*c^5*cos(f*x+e)-2*a^3*B*c^5*(-1/2*sin(f*x+e)*cos(f*x+e)
+1/2*f*x+1/2*e)-a^3*A*c^5*(-1/8*(sin(f*x+e)^7+7/6*sin(f*x+e)^5+35/24*sin(f
*x+e)^3+35/16*sin(f*x+e))*cos(f*x+e)+35/128*f*x+35/128*e)-2/7*a^3*A*c^5*(1
6/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e)+2*a^3*A*c^5
*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x
+5/16*e)+6/5*a^3*A*c^5*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+1/9*
a^3*B*c^5*(128/35+sin(f*x+e)^8+8/7*sin(f*x+e)^6+48/35*sin(f*x+e)^4+64/35*s
in(f*x+e)^2)*cos(f*x+e)+2*a^3*B*c^5*(-1/8*(sin(f*x+e)^7+7/6*sin(f*x+e)^5+3
5/24*sin(f*x+e)^3+35/16*sin(f*x+e))*cos(f*x+e)+35/128*f*x+35/128*e)-2/7*a^
3*B*c^5*(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e)-6
*a^3*B*c^5*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e
)+5/16*f*x+5/16*e)-a^3*B*c^5*cos(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.71

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx$$
$$=\frac{896\,Ba^3c^5\cos(fx+e)^9+2304\,(A-B)a^3c^5\cos(fx+e)^7+315\,(9\,A-2\,B)a^3c^5fx-21\,(48\,(A-2\,B)a^3c^5}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x, algori
thm="fricas")
```

output
```
1/8064*(896*B*a^3*c^5*cos(f*x + e)^9 + 2304*(A - B)*a^3*c^5*cos(f*x + e)^7
 + 315*(9*A - 2*B)*a^3*c^5*f*x - 21*(48*(A - 2*B)*a^3*c^5*cos(f*x + e)^7 -
 8*(9*A - 2*B)*a^3*c^5*cos(f*x + e)^5 - 10*(9*A - 2*B)*a^3*c^5*cos(f*x + e
)^3 - 15*(9*A - 2*B)*a^3*c^5*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1753 vs. 2(209) = 418.

Time = 1.18 (sec) , antiderivative size = 1753, normalized size of antiderivative = 7.90

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**5,x)
```

output
```
Piecewise((-35*A*a**3*c**5*x*sin(e + f*x)**8/128 - 35*A*a**3*c**5*x*sin(e
+ f*x)**6*cos(e + f*x)**2/32 + 5*A*a**3*c**5*x*sin(e + f*x)**6/8 - 105*A*a
**3*c**5*x*sin(e + f*x)**4*cos(e + f*x)**4/64 + 15*A*a**3*c**5*x*sin(e + f
*x)**4*cos(e + f*x)**2/8 - 35*A*a**3*c**5*x*sin(e + f*x)**2*cos(e + f*x)**
6/32 + 15*A*a**3*c**5*x*sin(e + f*x)**2*cos(e + f*x)**4/8 - A*a**3*c**5*x*
sin(e + f*x)**2 - 35*A*a**3*c**5*x*cos(e + f*x)**8/128 + 5*A*a**3*c**5*x*c
os(e + f*x)**6/8 - A*a**3*c**5*x*cos(e + f*x)**2 + A*a**3*c**5*x + 93*A*a*
*3*c**5*sin(e + f*x)**7*cos(e + f*x)/(128*f) - 2*A*a**3*c**5*sin(e + f*x)*
*6*cos(e + f*x)/f + 511*A*a**3*c**5*sin(e + f*x)**5*cos(e + f*x)**3/(384*f
) - 11*A*a**3*c**5*sin(e + f*x)**5*cos(e + f*x)/(8*f) - 4*A*a**3*c**5*sin(
e + f*x)**4*cos(e + f*x)**3/f + 6*A*a**3*c**5*sin(e + f*x)**4*cos(e + f*x)
/f + 385*A*a**3*c**5*sin(e + f*x)**3*cos(e + f*x)**5/(384*f) - 5*A*a**3*c*
*5*sin(e + f*x)**3*cos(e + f*x)**3/(3*f) - 16*A*a**3*c**5*sin(e + f*x)**2*
cos(e + f*x)**5/(5*f) + 8*A*a**3*c**5*sin(e + f*x)**2*cos(e + f*x)**3/f -
6*A*a**3*c**5*sin(e + f*x)**2*cos(e + f*x)/f + 35*A*a**3*c**5*sin(e + f*x)
*cos(e + f*x)**7/(128*f) - 5*A*a**3*c**5*sin(e + f*x)*cos(e + f*x)**5/(8*f
) + A*a**3*c**5*sin(e + f*x)*cos(e + f*x)/f - 32*A*a**3*c**5*cos(e + f*x)*
*7/(35*f) + 16*A*a**3*c**5*cos(e + f*x)**5/(5*f) - 4*A*a**3*c**5*cos(e + f
*x)**3/f + 2*A*a**3*c**5*cos(e + f*x)/f + 35*B*a**3*c**5*x*sin(e + f*x)**8
/64 + 35*B*a**3*c**5*x*sin(e + f*x)**6*cos(e + f*x)**2/16 - 15*B*a**3*c...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 617 vs. $2(210) = 420$.

Time = 0.05 (sec) , antiderivative size = 617, normalized size of antiderivative = 2.78

$$\int (a + a\sin(e + fx))^3(A + B\sin(e + fx))(c - c\sin(e + fx))^5\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x, algori
thm="maxima")
```

output
```
1/322560*(18432*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3
- 35*cos(f*x + e))*A*a^3*c^5 + 129024*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^
3 + 15*cos(f*x + e))*A*a^3*c^5 + 645120*(cos(f*x + e)^3 - 3*cos(f*x + e))*
A*a^3*c^5 - 105*(128*sin(2*f*x + 2*e)^3 + 840*f*x + 840*e + 3*sin(8*f*x +
8*e) + 168*sin(4*f*x + 4*e) - 768*sin(2*f*x + 2*e))*A*a^3*c^5 + 3360*(4*si
n(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e
))*A*a^3*c^5 - 161280*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^3*c^5 + 322560*
(f*x + e)*A*a^3*c^5 + 1024*(35*cos(f*x + e)^9 - 180*cos(f*x + e)^7 + 378*c
os(f*x + e)^5 - 420*cos(f*x + e)^3 + 315*cos(f*x + e))*B*a^3*c^5 + 18432*(
5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3 - 35*cos(f*x + e)
)*B*a^3*c^5 - 215040*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^3*c^5 + 210*(12
8*sin(2*f*x + 2*e)^3 + 840*f*x + 840*e + 3*sin(8*f*x + 8*e) + 168*sin(4*f*
x + 4*e) - 768*sin(2*f*x + 2*e))*B*a^3*c^5 - 10080*(4*sin(2*f*x + 2*e)^3 +
 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*B*a^3*c^5 + 604
80*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*c^5 - 161
280*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3*c^5 + 645120*A*a^3*c^5*cos(f*x
+ e) - 322560*B*a^3*c^5*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.32

$$
\begin{aligned}
&\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx \\
&= \frac{Ba^3c^5\cos(9\,fx+9\,e)}{2304\,f} + \frac{Ba^3c^5\sin(6\,fx+6\,e)}{96\,f} \\
&\quad + \frac{5}{128}\left(9\,Aa^3c^5 - 2\,Ba^3c^5\right)x + \frac{\left(8\,Aa^3c^5 - Ba^3c^5\right)\cos(7\,fx+7\,e)}{1792\,f} \\
&\quad + \frac{\left(2\,Aa^3c^5 - Ba^3c^5\right)\cos(5\,fx+5\,e)}{64\,f} + \frac{\left(18\,Aa^3c^5 - 11\,Ba^3c^5\right)\cos(3\,fx+3\,e)}{192\,f} \\
&\quad + \frac{\left(20\,Aa^3c^5 - 13\,Ba^3c^5\right)\cos(fx+e)}{128\,f} - \frac{\left(Aa^3c^5 - 2\,Ba^3c^5\right)\sin(8\,fx+8\,e)}{1024\,f} \\
&\quad + \frac{\left(5\,Aa^3c^5 + 2\,Ba^3c^5\right)\sin(4\,fx+4\,e)}{128\,f} + \frac{\left(8\,Aa^3c^5 - Ba^3c^5\right)\sin(2\,fx+2\,e)}{32\,f}
\end{aligned}
$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x, algori
thm="giac")
```

output
```
1/2304*B*a^3*c^5*cos(9*f*x + 9*e)/f + 1/96*B*a^3*c^5*sin(6*f*x + 6*e)/f +
5/128*(9*A*a^3*c^5 - 2*B*a^3*c^5)*x + 1/1792*(8*A*a^3*c^5 - B*a^3*c^5)*cos
(7*f*x + 7*e)/f + 1/64*(2*A*a^3*c^5 - B*a^3*c^5)*cos(5*f*x + 5*e)/f + 1/19
2*(18*A*a^3*c^5 - 11*B*a^3*c^5)*cos(3*f*x + 3*e)/f + 1/128*(20*A*a^3*c^5 -
 13*B*a^3*c^5)*cos(f*x + e)/f - 1/1024*(A*a^3*c^5 - 2*B*a^3*c^5)*sin(8*f*x
 + 8*e)/f + 1/128*(5*A*a^3*c^5 + 2*B*a^3*c^5)*sin(4*f*x + 4*e)/f + 1/32*(8
*A*a^3*c^5 - B*a^3*c^5)*sin(2*f*x + 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 37.97 (sec) , antiderivative size = 705, normalized size of antiderivative = 3.18

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^5\,dx = \text{Too large to display}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^5,x)
```

output
```
(tan(e/2 + (f*x)/2)^16*(4*A*a^3*c^5 - 2*B*a^3*c^5) + tan(e/2 + (f*x)/2)^14
*(8*A*a^3*c^5 - 8*B*a^3*c^5) + tan(e/2 + (f*x)/2)^2*((8*A*a^3*c^5)/7 - (8*
B*a^3*c^5)/7) + tan(e/2 + (f*x)/2)^8*(32*A*a^3*c^5 - 4*B*a^3*c^5) + tan(e/
2 + (f*x)/2)^6*(24*A*a^3*c^5 - 24*B*a^3*c^5) + tan(e/2 + (f*x)/2)^12*(24*A
*a^3*c^5 - (16*B*a^3*c^5)/3) + tan(e/2 + (f*x)/2)^10*(40*A*a^3*c^5 - 40*B*
a^3*c^5) + tan(e/2 + (f*x)/2)^4*((88*A*a^3*c^5)/7 - (32*B*a^3*c^5)/7) - ta
n(e/2 + (f*x)/2)^17*((83*A*a^3*c^5)/64 + (5*B*a^3*c^5)/32) + tan(e/2 + (f*
x)/2)^5*((149*A*a^3*c^5)/32 + (83*B*a^3*c^5)/16) - tan(e/2 + (f*x)/2)^13*(
(149*A*a^3*c^5)/32 + (83*B*a^3*c^5)/16) + tan(e/2 + (f*x)/2)^3*((189*A*a^3
*c^5)/32 - (191*B*a^3*c^5)/48) - tan(e/2 + (f*x)/2)^15*((189*A*a^3*c^5)/32
 - (191*B*a^3*c^5)/48) + tan(e/2 + (f*x)/2)^7*((409*A*a^3*c^5)/32 - (145*B
*a^3*c^5)/16) - tan(e/2 + (f*x)/2)^11*((409*A*a^3*c^5)/32 - (145*B*a^3*c^5
)/16) + tan(e/2 + (f*x)/2)*((83*A*a^3*c^5)/64 + (5*B*a^3*c^5)/32) + (4*A*a
^3*c^5)/7 - (22*B*a^3*c^5)/63)/(f*(9*tan(e/2 + (f*x)/2)^2 + 36*tan(e/2 + (
f*x)/2)^4 + 84*tan(e/2 + (f*x)/2)^6 + 126*tan(e/2 + (f*x)/2)^8 + 126*tan(e
/2 + (f*x)/2)^10 + 84*tan(e/2 + (f*x)/2)^12 + 36*tan(e/2 + (f*x)/2)^14 + 9
*tan(e/2 + (f*x)/2)^16 + tan(e/2 + (f*x)/2)^18 + 1)) + (5*a^3*c^5*atan((5*
a^3*c^5*tan(e/2 + (f*x)/2)*(9*A - 2*B))/(64*((45*A*a^3*c^5)/64 - (5*B*a^3*
c^5)/32)))*(9*A - 2*B))/(64*f)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.34

$$\int (a + a\sin(e + fx))^3(A + B\sin(e + fx))(c - c\sin(e + fx))^5\,dx$$
$$= \frac{a^3c^5\left(896\cos\left(fx+e\right)\sin\left(fx+e\right)^8 b + 1008\cos\left(fx+e\right)\sin\left(fx+e\right)^7 a - 2016\cos\left(fx+e\right)\sin\left(fx+e\right)\right.}{}$$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x)
```

output
```
(a**3*c**5*(896*cos(e + f*x)*sin(e + f*x)**8*b + 1008*cos(e + f*x)*sin(e +
 f*x)**7*a - 2016*cos(e + f*x)*sin(e + f*x)**7*b - 2304*cos(e + f*x)*sin(e
 + f*x)**6*a - 1280*cos(e + f*x)*sin(e + f*x)**6*b - 1512*cos(e + f*x)*sin
(e + f*x)**5*a + 5712*cos(e + f*x)*sin(e + f*x)**5*b + 6912*cos(e + f*x)*s
in(e + f*x)**4*a - 1536*cos(e + f*x)*sin(e + f*x)**4*b - 1890*cos(e + f*x)
*sin(e + f*x)**3*a - 4956*cos(e + f*x)*sin(e + f*x)**3*b - 6912*cos(e + f*
x)*sin(e + f*x)**2*a + 3328*cos(e + f*x)*sin(e + f*x)**2*b + 5229*cos(e +
f*x)*sin(e + f*x)*a + 630*cos(e + f*x)*sin(e + f*x)*b + 2304*cos(e + f*x)*
a - 1408*cos(e + f*x)*b + 2835*a*f*x - 2304*a - 630*b*f*x + 1408*b))/(8064
*f)
```

3.40 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$

Optimal result

Integrand size = 36, antiderivative size = 181

$$\begin{aligned}\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx &= \frac{5}{128}a^3(8A-B)c^4x+\frac{a^3(8A-B)c^4\cos^7(e+fx)}{56f} \\ &+\frac{5a^3(8A-B)c^4\cos(e+fx)\sin(e+fx)}{128f}+\frac{5a^3(8A-B)c^4\cos^3(e+fx)\sin(e+fx)}{192f} \\ &+\frac{a^3(8A-B)c^4\cos^5(e+fx)\sin(e+fx)}{48f}-\frac{a^3B\cos^7(e+fx)\left(c^4-c^4\sin(e+fx)\right)}{8f}\end{aligned}$$

output
```
5/128*a^3*(8*A-B)*c^4*x+1/56*a^3*(8*A-B)*c^4*cos(f*x+e)^7/f+5/128*a^3*(8*A
-B)*c^4*cos(f*x+e)*sin(f*x+e)/f+5/192*a^3*(8*A-B)*c^4*cos(f*x+e)^3*sin(f*x
+e)/f+1/48*a^3*(8*A-B)*c^4*cos(f*x+e)^5*sin(f*x+e)/f-1/8*a^3*B*cos(f*x+e)^
7*(c^4-c^4*sin(f*x+e))/f
```

Mathematica [A] (verified)

Time = 8.90 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.15

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$$
$$= \frac{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^4(840(8A-B)(e+fx)+1680(A-B)\cos(e+fx)+1008(A-}{}$$

input
```
Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^4,x]
```

output
```
((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^4*(840*(8*A - B)*(e + f*x) +
1680*(A - B)*Cos[e + f*x] + 1008*(A - B)*Cos[3*(e + f*x)] + 336*(A - B)*Co
s[5*(e + f*x)] + 48*(A - B)*Cos[7*(e + f*x)] + 336*(15*A - B)*Sin[2*(e + f
*x)] + 168*(6*A + B)*Sin[4*(e + f*x)] + 112*(A + B)*Sin[6*(e + f*x)] + 21*
B*Sin[8*(e + f*x)]))/(21504*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^8*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.78, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.361$, Rules used = {3042, 3446, 3042, 3339, 3042, 3148, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^3(c-c\sin(e+fx))^4(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^3(c-c\sin(e+fx))^4(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3446}$$

$$a^3c^3\int \cos^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))dx$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\int \cos(e+fx)^6(A+B\sin(e+fx))(c-c\sin(e+fx))dx$$

$$\Big\downarrow \text{3339}$$

$$a^3c^3\left(\frac{1}{8}(8A-B)\int \cos^6(e+fx)(c-c\sin(e+fx))dx-\frac{B\cos^7(e+fx)(c-c\sin(e+fx))}{8f}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left(\frac{1}{8}(8A-B)\int \cos(e+fx)^6(c-c\sin(e+fx))dx-\frac{B\cos^7(e+fx)(c-c\sin(e+fx))}{8f}\right)$$

$$\Big\downarrow \text{3148}$$

$$a^3c^3\left(\frac{1}{8}(8A-B)\left(c\int \cos^6(e+fx)dx+\frac{c\cos^7(e+fx)}{7f}\right)-\frac{B\cos^7(e+fx)(c-c\sin(e+fx))}{8f}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left(\frac{1}{8}(8A-B)\left(c\int \sin\left(e+fx+\frac{\pi}{2}\right)^6dx+\frac{c\cos^7(e+fx)}{7f}\right)-\frac{B\cos^7(e+fx)(c-c\sin(e+fx))}{8f}\right)$$

$$\Big\downarrow \text{3115}$$

$$a^3c^3\left(\frac{1}{8}(8A-B)\left(c\left(\frac{5}{6}\int \cos^4(e+fx)dx+\frac{\sin(e+fx)\cos^5(e+fx)}{6f}\right)+\frac{c\cos^7(e+fx)}{7f}\right)-\frac{B\cos^7(e+fx)(c}{8f}$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left(\frac{1}{8}(8A-B)\left(c\left(\frac{5}{6}\int \sin\left(e+fx+\frac{\pi}{2}\right)^4dx+\frac{\sin(e+fx)\cos^5(e+fx)}{6f}\right)+\frac{c\cos^7(e+fx)}{7f}\right)-\frac{B\cos^7(e+}{}$$

$$\Big\downarrow \text{3115}$$

$$a^3c^3\left(\frac{1}{8}(8A-B)\left(c\left(\frac{5}{6}\left(\frac{3}{4}\int \cos^2(e+fx)dx+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{\sin(e+fx)\cos^5(e+fx)}{6f}\right)+\frac{c\cos}{}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left(\frac{1}{8}(8A-B)\left(c\left(\frac{5}{6}\left(\frac{3}{4}\int \sin\left(e+fx+\frac{\pi}{2}\right)^2dx+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{\sin(e+fx)\cos^5(e+fx)}{6f}\right)\right.\right.$$

$$\Big\downarrow \text{3115}$$

$$a^3c^3\left(\frac{1}{8}(8A-B)\left(c\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}\right)+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{\sin(e+fx)\cos^5}{6f}\right.\right.\right.$$

$$\Big\downarrow 24$$

$$a^3c^3\left(\frac{1}{8}(8A-B)\left(\frac{c\cos^7(e+fx)}{7f}+c\left(\frac{\sin(e+fx)\cos^5(e+fx)}{6f}+\frac{5}{6}\left(\frac{\sin(e+fx)\cos^3(e+fx)}{4f}+\frac{3}{4}\left(\frac{\sin(e+f}{}\right.\right.\right.\right.\right.$$

input
```
Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4,x]
```

output
```
a^3*c^3*(-1/8*(B*Cos[e + f*x]^7*(c - c*Sin[e + f*x]))/f + ((8*A - B)*((c*C
os[e + f*x]^7)/(7*f) + c*((Cos[e + f*x]^5*Sin[e + f*x])/(6*f) + (5*((Cos[e
 + f*x]^3*Sin[e + f*x])/(4*f) + (3*(x/2 + (Cos[e + f*x]*Sin[e + f*x])/(2*f
)))/4))/6)))/8)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3339
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 567 vs. $2(169) = 338$.

Time = 0.22 (sec) , antiderivative size = 568, normalized size of antiderivative = 3.14

$$\frac{a^3 A c^4 (fx+e) - a^3 B c^4 \cos(fx+e) - a^3 A c^4 \left(2+\sin(fx+e)^2\right)\cos(fx+e) + 3a^3 B c^4 \left(-\frac{\left(\sin(fx+e)^3 + \frac{3 \sin}{}\right.}{}\right.}{}$$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x)
```

output
```
1/f*(a^3*A*c^4*(f*x+e)-a^3*B*c^4*cos(f*x+e)-a^3*A*c^4*(2+sin(f*x+e)^2)*cos
(f*x+e)+3*a^3*B*c^4*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x
+3/8*e)-3*a^3*A*c^4*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+a^3*B*c^4*(
2+sin(f*x+e)^2)*cos(f*x+e)+a^3*A*c^4*cos(f*x+e)-a^3*B*c^4*(-1/2*sin(f*x+e)
*cos(f*x+e)+1/2*f*x+1/2*e)-1/7*a^3*A*c^4*(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)
^4+8/5*sin(f*x+e)^2)*cos(f*x+e)-a^3*A*c^4*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+
e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)+3/5*a^3*A*c^4*(8/3+sin(f
*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+3*a^3*A*c^4*(-1/4*(sin(f*x+e)^3+3/2*s
in(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+a^3*B*c^4*(-1/8*(sin(f*x+e)^7+7/6*sin
(f*x+e)^5+35/24*sin(f*x+e)^3+35/16*sin(f*x+e))*cos(f*x+e)+35/128*f*x+35/12
8*e)+1/7*a^3*B*c^4*(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*c
os(f*x+e)-3*a^3*B*c^4*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e)
)*cos(f*x+e)+5/16*f*x+5/16*e)-3/5*a^3*B*c^4*(8/3+sin(f*x+e)^4+4/3*sin(f*x+
e)^2)*cos(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.76

$$\int (a + a\sin(e+fx))^3 (A + B\sin(e+fx))(c - c\sin(e+fx))^4 \, dx$$
$$= \frac{384\,(A-B)a^3c^4\cos{(fx+e)}^7 + 105\,(8\,A-B)a^3c^4fx + 7\,\big(48\,Ba^3c^4\cos{(fx+e)}^7 + 8\,(8\,A-B)a^3c^4\cos}{2688\,f}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algori
thm="fricas")
```

output
```
1/2688*(384*(A - B)*a^3*c^4*cos(f*x + e)^7 + 105*(8*A - B)*a^3*c^4*f*x + 7
*(48*B*a^3*c^4*cos(f*x + e)^7 + 8*(8*A - B)*a^3*c^4*cos(f*x + e)^5 + 10*(8
*A - B)*a^3*c^4*cos(f*x + e)^3 + 15*(8*A - B)*a^3*c^4*cos(f*x + e))*sin(f*
x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1579 vs. $2(163) = 326$.

Time = 0.93 (sec) , antiderivative size = 1579, normalized size of antiderivative = 8.72

$$\int (a + a\sin(e + fx))^3 (A + B\sin(e + fx))(c - c\sin(e + fx))^4 \, dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**4,x)
```

output
```
Piecewise((-5*A*a**3*c**4*x*sin(e + f*x)**6/16 - 15*A*a**3*c**4*x*sin(e +
f*x)**4*cos(e + f*x)**2/16 + 9*A*a**3*c**4*x*sin(e + f*x)**4/8 - 15*A*a**3
*c**4*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 9*A*a**3*c**4*x*sin(e + f*x)*
*2*cos(e + f*x)**2/4 - 3*A*a**3*c**4*x*sin(e + f*x)**2/2 - 5*A*a**3*c**4*x
*cos(e + f*x)**6/16 + 9*A*a**3*c**4*x*cos(e + f*x)**4/8 - 3*A*a**3*c**4*x*
cos(e + f*x)**2/2 + A*a**3*c**4*x - A*a**3*c**4*sin(e + f*x)**6*cos(e + f*
x)/f + 11*A*a**3*c**4*sin(e + f*x)**5*cos(e + f*x)/(16*f) - 2*A*a**3*c**4*
sin(e + f*x)**4*cos(e + f*x)**3/f + 3*A*a**3*c**4*sin(e + f*x)**4*cos(e +
f*x)/f + 5*A*a**3*c**4*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) - 15*A*a**3*c
**4*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 8*A*a**3*c**4*sin(e + f*x)**2*cos
(e + f*x)**5/(5*f) + 4*A*a**3*c**4*sin(e + f*x)**2*cos(e + f*x)**3/f - 3*A
*a**3*c**4*sin(e + f*x)**2*cos(e + f*x)/f + 5*A*a**3*c**4*sin(e + f*x)*cos
(e + f*x)**5/(16*f) - 9*A*a**3*c**4*sin(e + f*x)*cos(e + f*x)**3/(8*f) + 3
*A*a**3*c**4*sin(e + f*x)*cos(e + f*x)/(2*f) - 16*A*a**3*c**4*cos(e + f*x)
**7/(35*f) + 8*A*a**3*c**4*cos(e + f*x)**5/(5*f) - 2*A*a**3*c**4*cos(e + f
*x)**3/f + A*a**3*c**4*cos(e + f*x)/f + 35*B*a**3*c**4*x*sin(e + f*x)**8/1
28 + 35*B*a**3*c**4*x*sin(e + f*x)**6*cos(e + f*x)**2/32 - 15*B*a**3*c**4*
x*sin(e + f*x)**6/16 + 105*B*a**3*c**4*x*sin(e + f*x)**4*cos(e + f*x)**4/6
4 - 45*B*a**3*c**4*x*sin(e + f*x)**4*cos(e + f*x)**2/16 + 9*B*a**3*c**4*x*
sin(e + f*x)**4/8 + 35*B*a**3*c**4*x*sin(e + f*x)**2*cos(e + f*x)**6/32...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(170) = 340$.

Time = 0.05 (sec) , antiderivative size = 571, normalized size of antiderivative = 3.15

$$\int (a + a\sin(e + fx))^3 (A + B\sin(e + fx))(c - c\sin(e + fx))^4 \, dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algori
thm="maxima")
```

output
```
1/107520*(3072*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3 -
 35*cos(f*x + e))*A*a^3*c^4 + 21504*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3
+ 15*cos(f*x + e))*A*a^3*c^4 + 107520*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*
a^3*c^4 - 560*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) -
 48*sin(2*f*x + 2*e))*A*a^3*c^4 + 10080*(12*f*x + 12*e + sin(4*f*x + 4*e)
- 8*sin(2*f*x + 2*e))*A*a^3*c^4 - 80640*(2*f*x + 2*e - sin(2*f*x + 2*e))*A
*a^3*c^4 + 107520*(f*x + e)*A*a^3*c^4 - 3072*(5*cos(f*x + e)^7 - 21*cos(f*
x + e)^5 + 35*cos(f*x + e)^3 - 35*cos(f*x + e))*B*a^3*c^4 - 21504*(3*cos(f
*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*c^4 - 107520*(cos(f
*x + e)^3 - 3*cos(f*x + e))*B*a^3*c^4 + 35*(128*sin(2*f*x + 2*e)^3 + 840*f
*x + 840*e + 3*sin(8*f*x + 8*e) + 168*sin(4*f*x + 4*e) - 768*sin(2*f*x + 2
*e))*B*a^3*c^4 - 1680*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x
+ 4*e) - 48*sin(2*f*x + 2*e))*B*a^3*c^4 + 10080*(12*f*x + 12*e + sin(4*f*x
 + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*c^4 - 26880*(2*f*x + 2*e - sin(2*f*x +
 2*e))*B*a^3*c^4 + 107520*A*a^3*c^4*cos(f*x + e) - 107520*B*a^3*c^4*cos(f*
x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.46

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx$$
$$= \frac{Ba^3c^4\sin(8\,fx+8\,e)}{1024\,f} + \frac{5}{128}\,(8\,Aa^3c^4-Ba^3c^4)x + \frac{(Aa^3c^4-Ba^3c^4)\cos(7\,fx+7\,e)}{448\,f} + \frac{(Aa^3c^4-Ba^3c^4)\cos(5\,fx+5\,e)}{64\,f} + \frac{3\,(Aa^3c^4-Ba^3c^4)\cos(3\,fx+3\,e)}{64\,f} + \frac{5\,(Aa^3c^4-Ba^3c^4)\cos(fx+e)}{64\,f} + \frac{(Aa^3c^4+Ba^3c^4)\sin(6\,fx+6\,e)}{192\,f} + \frac{(6\,Aa^3c^4+Ba^3c^4)\sin(4\,fx+4\,e)}{128\,f} + \frac{(15\,Aa^3c^4-Ba^3c^4)\sin(2\,fx+2\,e)}{64\,f}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algori
thm="giac")
```

output
```
1/1024*B*a^3*c^4*sin(8*f*x + 8*e)/f + 5/128*(8*A*a^3*c^4 - B*a^3*c^4)*x +
1/448*(A*a^3*c^4 - B*a^3*c^4)*cos(7*f*x + 7*e)/f + 1/64*(A*a^3*c^4 - B*a^3
*c^4)*cos(5*f*x + 5*e)/f + 3/64*(A*a^3*c^4 - B*a^3*c^4)*cos(3*f*x + 3*e)/f
 + 5/64*(A*a^3*c^4 - B*a^3*c^4)*cos(f*x + e)/f + 1/192*(A*a^3*c^4 + B*a^3*
c^4)*sin(6*f*x + 6*e)/f + 1/128*(6*A*a^3*c^4 + B*a^3*c^4)*sin(4*f*x + 4*e)
/f + 1/64*(15*A*a^3*c^4 - B*a^3*c^4)*sin(2*f*x + 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 37.77 (sec) , antiderivative size = 661, normalized size of antiderivative = 3.65

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^4\,dx = \text{Too large to display}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^4,x)
```

output
```
(tan(e/2 + (f*x)/2)^4*(6*A*a^3*c^4 - 6*B*a^3*c^4) + tan(e/2 + (f*x)/2)^12*
(2*A*a^3*c^4 - 2*B*a^3*c^4) + tan(e/2 + (f*x)/2)^6*(6*A*a^3*c^4 - 6*B*a^3*
c^4) + tan(e/2 + (f*x)/2)^14*(2*A*a^3*c^4 - 2*B*a^3*c^4) + tan(e/2 + (f*x)
/2)^2*((2*A*a^3*c^4)/7 - (2*B*a^3*c^4)/7) + tan(e/2 + (f*x)/2)^8*(10*A*a^3
*c^4 - 10*B*a^3*c^4) + tan(e/2 + (f*x)/2)^10*(10*A*a^3*c^4 - 10*B*a^3*c^4)
 - tan(e/2 + (f*x)/2)^15*((11*A*a^3*c^4)/8 + (5*B*a^3*c^4)/64) + tan(e/2 +
 (f*x)/2)^3*((61*A*a^3*c^4)/24 - (397*B*a^3*c^4)/192) - tan(e/2 + (f*x)/2)
^13*((61*A*a^3*c^4)/24 - (397*B*a^3*c^4)/192) + tan(e/2 + (f*x)/2)^5*((113
*A*a^3*c^4)/24 + (895*B*a^3*c^4)/192) - tan(e/2 + (f*x)/2)^11*((113*A*a^3*
c^4)/24 + (895*B*a^3*c^4)/192) + tan(e/2 + (f*x)/2)^7*((85*A*a^3*c^4)/24 -
 (1765*B*a^3*c^4)/192) - tan(e/2 + (f*x)/2)^9*((85*A*a^3*c^4)/24 - (1765*B
*a^3*c^4)/192) + tan(e/2 + (f*x)/2)*((11*A*a^3*c^4)/8 + (5*B*a^3*c^4)/64)
+ (2*A*a^3*c^4)/7 - (2*B*a^3*c^4)/7)/(f*(8*tan(e/2 + (f*x)/2)^2 + 28*tan(e
/2 + (f*x)/2)^4 + 56*tan(e/2 + (f*x)/2)^6 + 70*tan(e/2 + (f*x)/2)^8 + 56*t
an(e/2 + (f*x)/2)^10 + 28*tan(e/2 + (f*x)/2)^12 + 8*tan(e/2 + (f*x)/2)^14
+ tan(e/2 + (f*x)/2)^16 + 1)) + (5*a^3*c^4*atan((5*a^3*c^4*tan(e/2 + (f*x)
/2)*(8*A - B))/(64*((5*A*a^3*c^4)/8 - (5*B*a^3*c^4)/64)))*(8*A - B))/(64*f
)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.45

$$\int (a + a\sin(e + fx))^3(A + B\sin(e + fx))(c - c\sin(e + fx))^4\,dx$$
$$= \frac{a^3c^4\left(-336\cos(fx+e)\sin(fx+e)^7 b - 384\cos(fx+e)\sin(fx+e)^6 a + 384\cos(fx+e)\sin(fx+e)^6\right.}{}$$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x)
```

output
```
(a**3*c**4*( - 336*cos(e + f*x)*sin(e + f*x)**7*b - 384*cos(e + f*x)*sin(e
 + f*x)**6*a + 384*cos(e + f*x)*sin(e + f*x)**6*b + 448*cos(e + f*x)*sin(e
 + f*x)**5*a + 952*cos(e + f*x)*sin(e + f*x)**5*b + 1152*cos(e + f*x)*sin(
e + f*x)**4*a - 1152*cos(e + f*x)*sin(e + f*x)**4*b - 1456*cos(e + f*x)*si
n(e + f*x)**3*a - 826*cos(e + f*x)*sin(e + f*x)**3*b - 1152*cos(e + f*x)*s
in(e + f*x)**2*a + 1152*cos(e + f*x)*sin(e + f*x)**2*b + 1848*cos(e + f*x)
*sin(e + f*x)*a + 105*cos(e + f*x)*sin(e + f*x)*b + 384*cos(e + f*x)*a - 3
84*cos(e + f*x)*b + 840*a*f*x - 384*a - 105*b*f*x + 384*b))/(2688*f)
```

3.41 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$

Optimal result

Integrand size = 36, antiderivative size = 117

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx = \frac{5}{16}a^3Ac^3x-\frac{a^3Bc^3\cos^7(e+fx)}{7f}+\frac{5a^3Ac^3\cos(e+fx)\sin(e+fx)}{16f}+\frac{5a^3Ac^3\cos^3(e+fx)\sin(e+fx)}{24f}+\frac{a^3Ac^3\cos^5(e+fx)\sin(e+fx)}{6f}$$

output
```
5/16*a^3*A*c^3*x-1/7*a^3*B*c^3*cos(f*x+e)^7/f+5/16*a^3*A*c^3*cos(f*x+e)*si
n(f*x+e)/f+5/24*a^3*A*c^3*cos(f*x+e)^3*sin(f*x+e)/f+1/6*a^3*A*c^3*cos(f*x+
e)^5*sin(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.55

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx = \frac{a^3c^3(-192B\cos^7(e+fx)+7A(60e+60fx+45\sin(2(e+fx))+9\sin(4(e+fx))+\sin(6(e+fx))))}{1344f}$$

input
```
Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^3,x]
```

output
```
(a^3*c^3*(-192*B*Cos[e + f*x]^7 + 7*A*(60*e + 60*f*x + 45*Sin[2*(e + f*x)]
 + 9*Sin[4*(e + f*x)] + Sin[6*(e + f*x)])))/(1344*f)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.88, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.306$, Rules used = {3042, 3446, 3042, 3148, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^3(c-c\sin(e+fx))^3(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^3(c-c\sin(e+fx))^3(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3446}$$

$$a^3c^3\int \cos^6(e+fx)(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\int \cos(e+fx)^6(A+B\sin(e+fx))dx$$

$$\Big\downarrow \; 3148$$

$$a^3c^3\left(A\int \cos^6(e+fx)dx-\frac{B\cos^7(e+fx)}{7f}\right)$$

$$\Big\downarrow \; 3042$$

$$a^3c^3\left(A\int \sin\left(e+fx+\frac{\pi}{2}\right)^6dx-\frac{B\cos^7(e+fx)}{7f}\right)$$

$$\Big\downarrow \; 3115$$

$$a^3c^3\left(A\left(\frac{5}{6}\int \cos^4(e+fx)dx+\frac{\sin(e+fx)\cos^5(e+fx)}{6f}\right)-\frac{B\cos^7(e+fx)}{7f}\right)$$

$$\Big\downarrow \; 3042$$

$$a^3c^3\left(A\left(\frac{5}{6}\int \sin\left(e+fx+\frac{\pi}{2}\right)^4dx+\frac{\sin(e+fx)\cos^5(e+fx)}{6f}\right)-\frac{B\cos^7(e+fx)}{7f}\right)$$

$$\Big\downarrow \; 3115$$

$$a^3c^3\left(A\left(\frac{5}{6}\left(\frac{3}{4}\int \cos^2(e+fx)dx+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{\sin(e+fx)\cos^5(e+fx)}{6f}\right)-\frac{B\cos^7(e+fx)}{7f}\right)$$

$$\Big\downarrow \; 3042$$

$$a^3c^3\left(A\left(\frac{5}{6}\left(\frac{3}{4}\int \sin\left(e+fx+\frac{\pi}{2}\right)^2dx+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{\sin(e+fx)\cos^5(e+fx)}{6f}\right)-\frac{B\cos^7(e+}{7f}$$

$$\Big\downarrow \; 3115$$

$$a^3c^3\left(A\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}\right)+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)+\frac{\sin(e+fx)\cos^5(e+fx)}{6f}\right)-$$

$$\Big\downarrow \; 24$$

$$a^3c^3\left(A\left(\frac{\sin(e+fx)\cos^5(e+fx)}{6f}+\frac{5}{6}\left(\frac{\sin(e+fx)\cos^3(e+fx)}{4f}+\frac{3}{4}\left(\frac{\sin(e+fx)\cos(e+fx)}{2f}+\frac{x}{2}\right)\right)\right)-\frac{B}{}$$

input
```
Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3,x]
```

output
```
a^3*c^3*(-1/7*(B*Cos[e + f*x]^7)/f + A*((Cos[e + f*x]^5*Sin[e + f*x])/(6*f
) + (5*((Cos[e + f*x]^3*Sin[e + f*x])/(4*f) + (3*(x/2 + (Cos[e + f*x]*Sin[
e + f*x])/(2*f)))/4))/6))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(107) = 214.

Time = 0.17 (sec) , antiderivative size = 263, normalized size of antiderivative = 2.25

$$a^3 A\, c^3 (fx+e) - a^3 B\, c^3 \cos(fx+e) - 3a^3 A\, c^3 \left(-\frac{\sin(fx+e)\cos(fx+e)}{2} + \frac{fx}{2} + \frac{e}{2}\right) + a^3 B\, c^3 \left(2 + \sin(fx+e)^2\right)$$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x)
```

output
```
1/f*(a^3*A*c^3*(f*x+e)-a^3*B*c^3*cos(f*x+e)-3*a^3*A*c^3*(-1/2*sin(f*x+e)*c
os(f*x+e)+1/2*f*x+1/2*e)+a^3*B*c^3*(2+sin(f*x+e)^2)*cos(f*x+e)-a^3*A*c^3*(
-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5
/16*e)+3*a^3*A*c^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+
3/8*e)+1/7*a^3*B*c^3*(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)
*cos(f*x+e)-3/5*a^3*B*c^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.79

$$\int (a + a\sin(e+fx))^3 (A + B\sin(e+fx))(c - c\sin(e+fx))^3\, dx =$$
$$-\frac{48\, Ba^3c^3 \cos(fx+e)^7 - 105\, Aa^3c^3 fx - 7\left(8\, Aa^3c^3 \cos(fx+e)^5 + 10\, Aa^3c^3 \cos(fx+e)^3 + 15\, Aa^3c^3\right.}{336\, f}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algori
thm="fricas")
```

output
```
-1/336*(48*B*a^3*c^3*cos(f*x + e)^7 - 105*A*a^3*c^3*f*x - 7*(8*A*a^3*c^3*c
os(f*x + e)^5 + 10*A*a^3*c^3*cos(f*x + e)^3 + 15*A*a^3*c^3*cos(f*x + e))*s
in(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 682 vs. $2(116) = 232$.

Time = 0.55 (sec) , antiderivative size = 682, normalized size of antiderivative = 5.83

$$\int (a + a\sin(e + fx))^3(A + B\sin(e + fx))(c - c\sin(e + fx))^3\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3,x)
```

output
```
Piecewise((-5*A*a**3*c**3*x*sin(e + f*x)**6/16 - 15*A*a**3*c**3*x*sin(e +
f*x)**4*cos(e + f*x)**2/16 + 9*A*a**3*c**3*x*sin(e + f*x)**4/8 - 15*A*a**3
*c**3*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 9*A*a**3*c**3*x*sin(e + f*x)*
*2*cos(e + f*x)**2/4 - 3*A*a**3*c**3*x*sin(e + f*x)**2/2 - 5*A*a**3*c**3*x
*cos(e + f*x)**6/16 + 9*A*a**3*c**3*x*cos(e + f*x)**4/8 - 3*A*a**3*c**3*x*
cos(e + f*x)**2/2 + A*a**3*c**3*x + 11*A*a**3*c**3*sin(e + f*x)**5*cos(e +
 f*x)/(16*f) + 5*A*a**3*c**3*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) - 15*A*
a**3*c**3*sin(e + f*x)**3*cos(e + f*x)/(8*f) + 5*A*a**3*c**3*sin(e + f*x)*
cos(e + f*x)**5/(16*f) - 9*A*a**3*c**3*sin(e + f*x)*cos(e + f*x)**3/(8*f)
+ 3*A*a**3*c**3*sin(e + f*x)*cos(e + f*x)/(2*f) + B*a**3*c**3*sin(e + f*x)
**6*cos(e + f*x)/f + 2*B*a**3*c**3*sin(e + f*x)**4*cos(e + f*x)**3/f - 3*B
*a**3*c**3*sin(e + f*x)**4*cos(e + f*x)/f + 8*B*a**3*c**3*sin(e + f*x)**2*
cos(e + f*x)**5/(5*f) - 4*B*a**3*c**3*sin(e + f*x)**2*cos(e + f*x)**3/f +
3*B*a**3*c**3*sin(e + f*x)**2*cos(e + f*x)/f + 16*B*a**3*c**3*cos(e + f*x)
**7/(35*f) - 8*B*a**3*c**3*cos(e + f*x)**5/(5*f) + 2*B*a**3*c**3*cos(e + f
*x)**3/f - B*a**3*c**3*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin
(e) + a)**3*(-c*sin(e) + c)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(107) = 214$.

Time = 0.04 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.26

$$\int (a + a\sin(e + fx))^3(A + B\sin(e + fx))(c - c\sin(e + fx))^3\,dx =$$

$$-\frac{35\left(4\sin\left(2fx + 2e\right)^3 + 60fx + 60e + 9\sin\left(4fx + 4e\right) - 48\sin\left(2fx + 2e\right)\right)Aa^3c^3 - 630\left(12fx + \right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algori
thm="maxima")
```

output
```
-1/6720*(35*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 4
8*sin(2*f*x + 2*e))*A*a^3*c^3 - 630*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*
sin(2*f*x + 2*e))*A*a^3*c^3 + 5040*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^3*
c^3 - 6720*(f*x + e)*A*a^3*c^3 + 192*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5
 + 35*cos(f*x + e)^3 - 35*cos(f*x + e))*B*a^3*c^3 + 1344*(3*cos(f*x + e)^5
 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*c^3 + 6720*(cos(f*x + e)^3 -
 3*cos(f*x + e))*B*a^3*c^3 + 6720*B*a^3*c^3*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.32

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$$
$$= \frac{5}{16}Aa^3c^3x - \frac{Ba^3c^3\cos(7fx+7e)}{448f} - \frac{Ba^3c^3\cos(5fx+5e)}{64f} - \frac{3Ba^3c^3\cos(3fx+3e)}{64f} - \frac{5Ba^3c^3\cos(fx+e)}{64f} + \frac{Aa^3c^3\sin(6fx+6e)}{192f} + \frac{3Aa^3c^3\sin(4fx+4e)}{64f} + \frac{15Aa^3c^3\sin(2fx+2e)}{64f}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algori
thm="giac")
```

output
```
5/16*A*a^3*c^3*x - 1/448*B*a^3*c^3*cos(7*f*x + 7*e)/f - 1/64*B*a^3*c^3*cos
(5*f*x + 5*e)/f - 3/64*B*a^3*c^3*cos(3*f*x + 3*e)/f - 5/64*B*a^3*c^3*cos(f
*x + e)/f + 1/192*A*a^3*c^3*sin(6*f*x + 6*e)/f + 3/64*A*a^3*c^3*sin(4*f*x
+ 4*e)/f + 15/64*A*a^3*c^3*sin(2*f*x + 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 37.43 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.78

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx = \frac{5\,A\,a^3\,c^3\,x}{16}$$

$$-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}\left(\frac{a^3\,c^3\,(672\,B-735\,A\,(e+f\,x))}{336}+\frac{35\,A\,a^3\,c^3\,(e+f\,x)}{16}\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\left(\frac{a^3\,c^3\,(2016\,B-2205\,A\,(e+f\,x))}{336}+\ldots\right.}{}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^3,x)
```

output
```
(5*A*a^3*c^3*x)/16 - (tan(e/2 + (f*x)/2)^12*((a^3*c^3*(672*B - 735*A*(e +
f*x)))/336 + (35*A*a^3*c^3*(e + f*x))/16) + tan(e/2 + (f*x)/2)^4*((a^3*c^3
*(2016*B - 2205*A*(e + f*x)))/336 + (105*A*a^3*c^3*(e + f*x))/16) + tan(e/
2 + (f*x)/2)^8*((a^3*c^3*(3360*B - 3675*A*(e + f*x)))/336 + (175*A*a^3*c^3
*(e + f*x))/16) + (a^3*c^3*(96*B - 105*A*(e + f*x)))/336 + (5*A*a^3*c^3*(e
 + f*x))/16 - (7*A*a^3*c^3*tan(e/2 + (f*x)/2)^3)/6 - (85*A*a^3*c^3*tan(e/2
 + (f*x)/2)^5)/24 + (85*A*a^3*c^3*tan(e/2 + (f*x)/2)^9)/24 + (7*A*a^3*c^3*
tan(e/2 + (f*x)/2)^11)/6 + (11*A*a^3*c^3*tan(e/2 + (f*x)/2)^13)/8 - (11*A*
a^3*c^3*tan(e/2 + (f*x)/2))/8)/(f*(tan(e/2 + (f*x)/2)^2 + 1)^7)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.10

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$$

$$=\frac{a^3c^3\left(48\cos(fx+e)\sin(fx+e)^6b+56\cos(fx+e)\sin(fx+e)^5a-144\cos(fx+e)\sin(fx+e)^4b-\ldots\right.}{}$$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x)
```

output
```
(a**3*c**3*(48*cos(e + f*x)*sin(e + f*x)**6*b + 56*cos(e + f*x)*sin(e + f*
x)**5*a - 144*cos(e + f*x)*sin(e + f*x)**4*b - 182*cos(e + f*x)*sin(e + f*
x)**3*a + 144*cos(e + f*x)*sin(e + f*x)**2*b + 231*cos(e + f*x)*sin(e + f*
x)*a - 48*cos(e + f*x)*b + 105*a*f*x + 48*b))/(336*f)
```

3.42 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$

Optimal result

Integrand size = 36, antiderivative size = 138

$$\begin{aligned}\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx\\ =\frac{1}{16}a^3(6A+B)c^2x-\frac{a^3(6A+B)c^2\cos^5(e+fx)}{30f}\\ +\frac{a^3(6A+B)c^2\cos(e+fx)\sin(e+fx)}{16f}+\frac{a^3(6A+B)c^2\cos^3(e+fx)\sin(e+fx)}{24f}\\ -\frac{Bc^2\cos^5(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{6f}\end{aligned}$$

output
```
1/16*a^3*(6*A+B)*c^2*x-1/30*a^3*(6*A+B)*c^2*cos(f*x+e)^5/f+1/16*a^3*(6*A+B
)*c^2*cos(f*x+e)*sin(f*x+e)/f+1/24*a^3*(6*A+B)*c^2*cos(f*x+e)^3*sin(f*x+e)
/f-1/6*B*c^2*cos(f*x+e)^5*(a^3+a^3*sin(f*x+e))/f
```

Mathematica [A] (verified)

Time = 7.65 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.96

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$$
$$= \frac{a^3c^2(360Ae+60Be+360Afx+60Bfx-120(A+B)\cos(e+fx)-60(A+B)\cos(3(e+fx))-12A}{}$$

input
```
Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^2,x]
```

output
```
(a^3*c^2*(360*A*e + 60*B*e + 360*A*f*x + 60*B*f*x - 120*(A + B)*Cos[e + f*
x] - 60*(A + B)*Cos[3*(e + f*x)] - 12*A*Cos[5*(e + f*x)] - 12*B*Cos[5*(e +
 f*x)] + 240*A*Sin[2*(e + f*x)] + 15*B*Sin[2*(e + f*x)] + 30*A*Sin[4*(e +
f*x)] - 15*B*Sin[4*(e + f*x)] - 5*B*Sin[6*(e + f*x)]))/(960*f)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.82, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.306$, Rules used = {3042, 3446, 3042, 3339, 3042, 3148, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^3(c-c\sin(e+fx))^2(A+B\sin(e+fx))\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (a\sin(e+fx)+a)^3(c-c\sin(e+fx))^2(A+B\sin(e+fx))dx$$
$$\Big\downarrow \text{3446}$$
$$a^2c^2\int \cos^4(e+fx)(\sin(e+fx)a+a)(A+B\sin(e+fx))dx$$
$$\Big\downarrow \text{3042}$$

$$a^2c^2\int \cos(e+fx)^4(\sin(e+fx)a+a)(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3339}$$

$$a^2c^2\left(\frac{1}{6}(6A+B)\int \cos^4(e+fx)(\sin(e+fx)a+a)dx-\frac{B\cos^5(e+fx)(a\sin(e+fx)+a)}{6f}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left(\frac{1}{6}(6A+B)\int \cos(e+fx)^4(\sin(e+fx)a+a)dx-\frac{B\cos^5(e+fx)(a\sin(e+fx)+a)}{6f}\right)$$

$$\Big\downarrow \text{3148}$$

$$a^2c^2\left(\frac{1}{6}(6A+B)\left(a\int \cos^4(e+fx)dx-\frac{a\cos^5(e+fx)}{5f}\right)-\frac{B\cos^5(e+fx)(a\sin(e+fx)+a)}{6f}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left(\frac{1}{6}(6A+B)\left(a\int \sin\left(e+fx+\frac{\pi}{2}\right)^4dx-\frac{a\cos^5(e+fx)}{5f}\right)-\frac{B\cos^5(e+fx)(a\sin(e+fx)+a)}{6f}\right)$$

$$\Big\downarrow \text{3115}$$

$$a^2c^2\left(\frac{1}{6}(6A+B)\left(a\left(\frac{3}{4}\int \cos^2(e+fx)dx+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)-\frac{a\cos^5(e+fx)}{5f}\right)-\frac{B\cos^5(e+fx)(a\sin(e+fx)+a)}{6f}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left(\frac{1}{6}(6A+B)\left(a\left(\frac{3}{4}\int \sin\left(e+fx+\frac{\pi}{2}\right)^2dx+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)-\frac{a\cos^5(e+fx)}{5f}\right)-\frac{B\cos^5(e+fx)(a\sin(e+fx)+a)}{6f}\right)$$

$$\Big\downarrow \text{3115}$$

$$a^2c^2\left(\frac{1}{6}(6A+B)\left(a\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}\right)+\frac{\sin(e+fx)\cos^3(e+fx)}{4f}\right)-\frac{a\cos^5(e+fx)}{5f}\right)-\frac{B\cos^5(e+fx)(a\sin(e+fx)+a)}{6f}\right)$$

$$\Big\downarrow \text{24}$$

$$a^2c^2\left(\frac{1}{6}(6A+B)\left(a\left(\frac{\sin(e+fx)\cos^3(e+fx)}{4f}+\frac{3}{4}\left(\frac{\sin(e+fx)\cos(e+fx)}{2f}+\frac{x}{2}\right)\right)-\frac{a\cos^5(e+fx)}{5f}\right)-\frac{B\cos^5(e+fx)(a\sin(e+fx)+a)}{6f}\right)$$

input
```
Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2,x]
```

output
```
a^2*c^2*(-1/6*(B*Cos[e + f*x]^5*(a + a*Sin[e + f*x]))/f + ((6*A + B)*(-1/5
*(a*Cos[e + f*x]^5)/f + a*((Cos[e + f*x]^3*Sin[e + f*x])/(4*f) + (3*(x/2 +
 (Cos[e + f*x]*Sin[e + f*x])/(2*f)))/4)))/6)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3339
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(128) = 256$.

Time = 0.15 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.64

$$-\frac{a^3 A c^2 \left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx+e)^2}{3}\right)\cos(fx+e)}{5}+a^3 A c^2 \left(-\frac{\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+\frac{2a^3 A c^2 \left(2+\mathrm{s}\right.}{}$$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x)
```

output
```
1/f*(-1/5*a^3*A*c^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+a^3*A*c
^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+2/3*a^3*A
*c^2*(2+sin(f*x+e)^2)*cos(f*x+e)+a^3*B*c^2*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x
+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)-1/5*a^3*B*c^2*(8/3+sin(
f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-2*a^3*B*c^2*(-1/4*(sin(f*x+e)^3+3/2*
sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2*a^3*A*c^2*(-1/2*sin(f*x+e)*cos(f*x
+e)+1/2*f*x+1/2*e)+2/3*a^3*B*c^2*(2+sin(f*x+e)^2)*cos(f*x+e)-a^3*A*c^2*cos
(f*x+e)+a^3*B*c^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+a^3*A*c^2*(f*
x+e)-a^3*B*c^2*cos(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.77

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx =$$
$$-\frac{48\,(A+B)a^3c^2\cos\left(fx+e\right)^5-15\,(6\,A+B)a^3c^2fx+5\,\left(8\,Ba^3c^2\cos\left(fx+e\right)^5-2\,(6\,A+B)a^3c^2\cos\right.}{240\,f}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
-1/240*(48*(A + B)*a^3*c^2*cos(f*x + e)^5 - 15*(6*A + B)*a^3*c^2*f*x + 5*(
8*B*a^3*c^2*cos(f*x + e)^5 - 2*(6*A + B)*a^3*c^2*cos(f*x + e)^3 - 3*(6*A +
 B)*a^3*c^2*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 910 vs. 2(128) = 256.

Time = 0.48 (sec) , antiderivative size = 910, normalized size of antiderivative = 6.59

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2,x)
```

output
```
Piecewise((3*A*a**3*c**2*x*sin(e + f*x)**4/8 + 3*A*a**3*c**2*x*sin(e + f*x
)**2*cos(e + f*x)**2/4 - A*a**3*c**2*x*sin(e + f*x)**2 + 3*A*a**3*c**2*x*c
os(e + f*x)**4/8 - A*a**3*c**2*x*cos(e + f*x)**2 + A*a**3*c**2*x - A*a**3*
c**2*sin(e + f*x)**4*cos(e + f*x)/f - 5*A*a**3*c**2*sin(e + f*x)**3*cos(e
+ f*x)/(8*f) - 4*A*a**3*c**2*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) + 2*A*a
**3*c**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**3*c**2*sin(e + f*x)*cos(e
 + f*x)**3/(8*f) + A*a**3*c**2*sin(e + f*x)*cos(e + f*x)/f - 8*A*a**3*c**2
*cos(e + f*x)**5/(15*f) + 4*A*a**3*c**2*cos(e + f*x)**3/(3*f) - A*a**3*c**
2*cos(e + f*x)/f + 5*B*a**3*c**2*x*sin(e + f*x)**6/16 + 15*B*a**3*c**2*x*s
in(e + f*x)**4*cos(e + f*x)**2/16 - 3*B*a**3*c**2*x*sin(e + f*x)**4/4 + 15
*B*a**3*c**2*x*sin(e + f*x)**2*cos(e + f*x)**4/16 - 3*B*a**3*c**2*x*sin(e
+ f*x)**2*cos(e + f*x)**2/2 + B*a**3*c**2*x*sin(e + f*x)**2/2 + 5*B*a**3*c
**2*x*cos(e + f*x)**6/16 - 3*B*a**3*c**2*x*cos(e + f*x)**4/4 + B*a**3*c**2
*x*cos(e + f*x)**2/2 - 11*B*a**3*c**2*sin(e + f*x)**5*cos(e + f*x)/(16*f)
- B*a**3*c**2*sin(e + f*x)**4*cos(e + f*x)/f - 5*B*a**3*c**2*sin(e + f*x)*
*3*cos(e + f*x)**3/(6*f) + 5*B*a**3*c**2*sin(e + f*x)**3*cos(e + f*x)/(4*f
) - 4*B*a**3*c**2*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) + 2*B*a**3*c**2*si
n(e + f*x)**2*cos(e + f*x)/f - 5*B*a**3*c**2*sin(e + f*x)*cos(e + f*x)**5/
(16*f) + 3*B*a**3*c**2*sin(e + f*x)*cos(e + f*x)**3/(4*f) - B*a**3*c**2*si
n(e + f*x)*cos(e + f*x)/(2*f) - 8*B*a**3*c**2*cos(e + f*x)**5/(15*f) + ...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(128) = 256$.

Time = 0.04 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.61

$$\int (a + a\sin(e + fx))^3 (A + B\sin(e + fx))(c - c\sin(e + fx))^2 \, dx =$$
$$-\frac{64 \left(3 \cos\left(fx+e\right)^5 - 10 \cos\left(fx+e\right)^3 + 15 \cos\left(fx+e\right)\right) A a^3 c^2 + 640 \left(\cos\left(fx+e\right)^3 - 3 \cos\left(fx+e\right)\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algori
thm="maxima")
```

output
```
-1/960*(64*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*A*a^3*
c^2 + 640*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c^2 - 30*(12*f*x + 12*e
+ sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^3*c^2 + 480*(2*f*x + 2*e - si
n(2*f*x + 2*e))*A*a^3*c^2 - 960*(f*x + e)*A*a^3*c^2 + 64*(3*cos(f*x + e)^5
 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*c^2 + 640*(cos(f*x + e)^3 -
3*cos(f*x + e))*B*a^3*c^2 - 5*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*si
n(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*B*a^3*c^2 + 60*(12*f*x + 12*e + sin(
4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*c^2 - 240*(2*f*x + 2*e - sin(2*f*
x + 2*e))*B*a^3*c^2 + 960*A*a^3*c^2*cos(f*x + e) + 960*B*a^3*c^2*cos(f*x +
 e))/f
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.43

$$\begin{aligned}
&\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx \\
&= -\frac{Ba^3c^2\sin(6\,fx+6\,e)}{192\,f} + \frac{1}{16}\left(6\,Aa^3c^2+Ba^3c^2\right)x - \frac{\left(Aa^3c^2+Ba^3c^2\right)\cos(5\,fx+5\,e)}{80\,f} \\
&\quad - \frac{\left(Aa^3c^2+Ba^3c^2\right)\cos(3\,fx+3\,e)}{16\,f} - \frac{\left(Aa^3c^2+Ba^3c^2\right)\cos(fx+e)}{8\,f} \\
&\quad + \frac{\left(2\,Aa^3c^2-Ba^3c^2\right)\sin(4\,fx+4\,e)}{64\,f} + \frac{\left(16\,Aa^3c^2+Ba^3c^2\right)\sin(2\,fx+2\,e)}{64\,f}
\end{aligned}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
-1/192*B*a^3*c^2*sin(6*f*x + 6*e)/f + 1/16*(6*A*a^3*c^2 + B*a^3*c^2)*x - 1
/80*(A*a^3*c^2 + B*a^3*c^2)*cos(5*f*x + 5*e)/f - 1/16*(A*a^3*c^2 + B*a^3*c
^2)*cos(3*f*x + 3*e)/f - 1/8*(A*a^3*c^2 + B*a^3*c^2)*cos(f*x + e)/f + 1/64
*(2*A*a^3*c^2 - B*a^3*c^2)*sin(4*f*x + 4*e)/f + 1/64*(16*A*a^3*c^2 + B*a^3
*c^2)*sin(2*f*x + 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 36.92 (sec) , antiderivative size = 536, normalized size of antiderivative = 3.88

$$\int (a + a\sin(e + fx))^3 (A + B\sin(e + fx))(c - c\sin(e + fx))^2 \, dx = \text{Too large to display}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^2,x)
```

output
```
(a^3*c^2*atan((a^3*c^2*tan(e/2 + (f*x)/2)*(6*A + B))/(8*((3*A*a^3*c^2)/4 +
 (B*a^3*c^2)/8)))*(6*A + B))/(8*f) - (tan(e/2 + (f*x)/2)^4*(4*A*a^3*c^2 +
4*B*a^3*c^2) + tan(e/2 + (f*x)/2)^8*(2*A*a^3*c^2 + 2*B*a^3*c^2) + tan(e/2
+ (f*x)/2)^6*(4*A*a^3*c^2 + 4*B*a^3*c^2) + tan(e/2 + (f*x)/2)^10*(2*A*a^3*
c^2 + 2*B*a^3*c^2) + tan(e/2 + (f*x)/2)^2*((2*A*a^3*c^2)/5 + (2*B*a^3*c^2)
/5) - tan(e/2 + (f*x)/2)^5*((A*a^3*c^2)/2 - (13*B*a^3*c^2)/4) + tan(e/2 +
(f*x)/2)^7*((A*a^3*c^2)/2 - (13*B*a^3*c^2)/4) + tan(e/2 + (f*x)/2)^11*((5*
A*a^3*c^2)/4 - (B*a^3*c^2)/8) - tan(e/2 + (f*x)/2)^3*((7*A*a^3*c^2)/4 + (4
7*B*a^3*c^2)/24) + tan(e/2 + (f*x)/2)^9*((7*A*a^3*c^2)/4 + (47*B*a^3*c^2)/
24) - tan(e/2 + (f*x)/2)*((5*A*a^3*c^2)/4 - (B*a^3*c^2)/8) + (2*A*a^3*c^2)
/5 + (2*B*a^3*c^2)/5)/(f*(6*tan(e/2 + (f*x)/2)^2 + 15*tan(e/2 + (f*x)/2)^4
 + 20*tan(e/2 + (f*x)/2)^6 + 15*tan(e/2 + (f*x)/2)^8 + 6*tan(e/2 + (f*x)/2
)^10 + tan(e/2 + (f*x)/2)^12 + 1)) - (a^3*c^2*(6*A + B)*(atan(tan(e/2 + (f
*x)/2)) - (f*x)/2))/(8*f)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.41

$$\int (a + a\sin(e + fx))^3 (A + B\sin(e + fx))(c - c\sin(e + fx))^2 \, dx$$
$$= \frac{a^3c^2(-40\cos(fx+e)\sin(fx+e)^5 b - 48\cos(fx+e)\sin(fx+e)^4 a - 48\cos(fx+e)\sin(fx+e)^4 b -}{}$$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x)
```

output
```
(a**3*c**2*( - 40*cos(e + f*x)*sin(e + f*x)**5*b - 48*cos(e + f*x)*sin(e +
 f*x)**4*a - 48*cos(e + f*x)*sin(e + f*x)**4*b - 60*cos(e + f*x)*sin(e + f
*x)**3*a + 70*cos(e + f*x)*sin(e + f*x)**3*b + 96*cos(e + f*x)*sin(e + f*x
)**2*a + 96*cos(e + f*x)*sin(e + f*x)**2*b + 150*cos(e + f*x)*sin(e + f*x)
*a - 15*cos(e + f*x)*sin(e + f*x)*b - 48*cos(e + f*x)*a - 48*cos(e + f*x)*
b + 90*a*f*x + 48*a + 15*b*f*x + 48*b))/(240*f)
```

3.43 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 140

$$\begin{aligned}\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx\\ =\frac{1}{8}a^3(5A+2B)cx-\frac{a^3(5A+2B)c\cos^3(e+fx)}{12f}\\ +\frac{a^3(5A+2B)c\cos(e+fx)\sin(e+fx)}{8f}-\frac{aBc\cos^3(e+fx)(a+a\sin(e+fx))^2}{5f}\\ -\frac{(5A+2B)c\cos^3(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{20f}\end{aligned}$$

output
```
1/8*a^3*(5*A+2*B)*c*x-1/12*a^3*(5*A+2*B)*c*cos(f*x+e)^3/f+1/8*a^3*(5*A+2*B
)*c*cos(f*x+e)*sin(f*x+e)/f-1/5*a*B*c*cos(f*x+e)^3*(a+a*sin(f*x+e))^2/f-1/
20*(5*A+2*B)*c*cos(f*x+e)^3*(a^3+a^3*sin(f*x+e))/f
```

Mathematica [A] (verified)

Time = 1.66 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.01

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$$
$$= \frac{a^3c\cos(e+fx)\left(-30(5A+2B)\arcsin\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}(-8(10A+7B)+15(3A-2B)\right.}{120f\sqrt{\cos^2(e+}}$$

input
```
Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
,x]
```

output
```
(a^3*c*Cos[e + f*x]*(-30*(5*A + 2*B)*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]
] + Sqrt[Cos[e + f*x]^2]*(-8*(10*A + 7*B) + 15*(3*A - 2*B)*Sin[e + f*x] +
16*(5*A + 2*B)*Sin[e + f*x]^2 + 30*(A + 2*B)*Sin[e + f*x]^3 + 24*B*Sin[e +
 f*x]^4)))/(120*f*Sqrt[Cos[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.87, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {3042, 3446, 3042, 3339, 3042, 3157, 3042, 3148, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^3(c-c\sin(e+fx))(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a\sin(e+fx)+a)^3(c-c\sin(e+fx))(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{ 3446}$$

$$ac\int \cos^2(e+fx)(\sin(e+fx)a+a)^2(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{ 3042}$$

$$ac\int \cos(e+fx)^2(\sin(e+fx)a+a)^2(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3339}$$

$$ac\left(\frac{1}{5}(5A+2B)\int \cos^2(e+fx)(\sin(e+fx)a+a)^2dx-\frac{B\cos^3(e+fx)(a\sin(e+fx)+a)^2}{5f}\right)$$

$$\Big\downarrow \text{3042}$$

$$ac\left(\frac{1}{5}(5A+2B)\int \cos(e+fx)^2(\sin(e+fx)a+a)^2dx-\frac{B\cos^3(e+fx)(a\sin(e+fx)+a)^2}{5f}\right)$$

$$\Big\downarrow \text{3157}$$

$$ac\left(\frac{1}{5}(5A+2B)\left(\frac{5}{4}a\int \cos^2(e+fx)(\sin(e+fx)a+a)dx-\frac{\cos^3(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{4f}\right)-\frac{B\cos^3(e+\right.$$

$$\Big\downarrow \text{3042}$$

$$ac\left(\frac{1}{5}(5A+2B)\left(\frac{5}{4}a\int \cos(e+fx)^2(\sin(e+fx)a+a)dx-\frac{\cos^3(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{4f}\right)-\frac{B\cos^3(e+\right.$$

$$\Big\downarrow \text{3148}$$

$$ac\left(\frac{1}{5}(5A+2B)\left(\frac{5}{4}a\left(a\int \cos^2(e+fx)dx-\frac{a\cos^3(e+fx)}{3f}\right)-\frac{\cos^3(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{4f}\right)-\frac{B\cos^3}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$ac\left(\frac{1}{5}(5A+2B)\left(\frac{5}{4}a\left(a\int \sin\left(e+fx+\frac{\pi}{2}\right)^2dx-\frac{a\cos^3(e+fx)}{3f}\right)-\frac{\cos^3(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{4f}\right)-\right.$$

$$\Big\downarrow \text{3115}$$

$$ac\left(\frac{1}{5}(5A+2B)\left(\frac{5}{4}a\left(a\left(\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}\right)-\frac{a\cos^3(e+fx)}{3f}\right)-\frac{\cos^3(e+fx)\left(a^2\sin(e+fx)\right.}{4f}\right.\right.$$

$$\Big\downarrow \text{24}$$

$$ac\left(\frac{1}{5}(5A+2B)\left(\frac{5}{4}a\left(a\left(\frac{\sin(e+fx)\cos(e+fx)}{2f}+\frac{x}{2}\right)-\frac{a\cos^3(e+fx)}{3f}\right)-\frac{\cos^3(e+fx)\left(a^2\sin(e+fx)+a\right.}{4f}\right.\right.$$

input
```
Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),x]
```

output
```
a*c*(-1/5*(B*Cos[e + f*x]^3*(a + a*Sin[e + f*x])^2)/f + ((5*A + 2*B)*(-1/4
*(Cos[e + f*x]^3*(a^2 + a^2*Sin[e + f*x]))/f + (5*a*(-1/3*(a*Cos[e + f*x]^
3)/f + a*(x/2 + (Cos[e + f*x]*Sin[e + f*x])/(2*f))))/4))/5)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3157
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && Integers
Q[2*m, 2*p]
```

rule 3339
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.49

$$-a^3Ac\left(-\frac{\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+\frac{2a^3Ac\left(2+\sin(fx+e)^2\right)\cos(fx+e)}{3}+\frac{a^3Bc\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4\sin(fx}{3}\cdots\right.}{5}$$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

output
```
1/f*(-a^3*A*c*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e
)+2/3*a^3*A*c*(2+sin(f*x+e)^2)*cos(f*x+e)+1/5*a^3*B*c*(8/3+sin(f*x+e)^4+4/
3*sin(f*x+e)^2)*cos(f*x+e)-2*a^3*B*c*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*c
os(f*x+e)+3/8*f*x+3/8*e)-2*cos(f*x+e)*A*a^3*c+2*a^3*B*c*(-1/2*sin(f*x+e)*c
os(f*x+e)+1/2*f*x+1/2*e)+a^3*A*c*(f*x+e)-cos(f*x+e)*B*a^3*c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.71

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$$
$$=\frac{24\,Ba^3c\cos\left(fx+e\right)^5-80\,(A+B)a^3c\cos\left(fx+e\right)^3+15\,(5\,A+2\,B)a^3cfx-15\,\left(2\,(A+2\,B)a^3c\cos\left(f\right.\right.}{120\,f}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
1/120*(24*B*a^3*c*cos(f*x + e)^5 - 80*(A + B)*a^3*c*cos(f*x + e)^3 + 15*(5
*A + 2*B)*a^3*c*f*x - 15*(2*(A + 2*B)*a^3*c*cos(f*x + e)^3 - (5*A + 2*B)*a
^3*c*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 486 vs. 2(128) = 256.

Time = 0.32 (sec) , antiderivative size = 486, normalized size of antiderivative = 3.47

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$$
$$=\begin{cases}-\frac{3Aa^3cx\sin^4(e+fx)}{8}-\frac{3Aa^3cx\sin^2(e+fx)\cos^2(e+fx)}{4}-\frac{3Aa^3cx\cos^4(e+fx)}{8}+Aa^3cx+\frac{5Aa^3c\sin^3(e+fx)\cos(e+fx)}{8f}+\frac{2A}{} \\ x(A+B\sin(e))\left(a\sin(e)+a\right)^3\left(-c\sin(e)+c\right)\end{cases}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

output
```
Piecewise((-3*A*a**3*c*x*sin(e + f*x)**4/8 - 3*A*a**3*c*x*sin(e + f*x)**2*
cos(e + f*x)**2/4 - 3*A*a**3*c*x*cos(e + f*x)**4/8 + A*a**3*c*x + 5*A*a**3
*c*sin(e + f*x)**3*cos(e + f*x)/(8*f) + 2*A*a**3*c*sin(e + f*x)**2*cos(e +
 f*x)/f + 3*A*a**3*c*sin(e + f*x)*cos(e + f*x)**3/(8*f) + 4*A*a**3*c*cos(e
 + f*x)**3/(3*f) - 2*A*a**3*c*cos(e + f*x)/f - 3*B*a**3*c*x*sin(e + f*x)**
4/4 - 3*B*a**3*c*x*sin(e + f*x)**2*cos(e + f*x)**2/2 + B*a**3*c*x*sin(e +
f*x)**2 - 3*B*a**3*c*x*cos(e + f*x)**4/4 + B*a**3*c*x*cos(e + f*x)**2 + B*
a**3*c*sin(e + f*x)**4*cos(e + f*x)/f + 5*B*a**3*c*sin(e + f*x)**3*cos(e +
 f*x)/(4*f) + 4*B*a**3*c*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) + 3*B*a**3*
c*sin(e + f*x)*cos(e + f*x)**3/(4*f) - B*a**3*c*sin(e + f*x)*cos(e + f*x)/
f + 8*B*a**3*c*cos(e + f*x)**5/(15*f) - B*a**3*c*cos(e + f*x)/f, Ne(f, 0))
, (x*(A + B*sin(e))*(a*sin(e) + a)**3*(-c*sin(e) + c), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.43

$$\int (a + a\sin(e + fx))^3 (A + B\sin(e + fx))(c - c\sin(e + fx))\, dx =$$

$$-\frac{320\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)Aa^3c+15\left(12\,fx+12\,e+\sin\left(4\,fx+4\,e\right)-8\,\sin\left(2\,fx+2\,e\right)\right)A\ldots}{\ldots}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorith
m="maxima")
```

output
```
-1/480*(320*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c + 15*(12*f*x + 12*e
+ sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^3*c - 480*(f*x + e)*A*a^3*c -
 32*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*c + 30*
(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*c - 240*(2*f
*x + 2*e - sin(2*f*x + 2*e))*B*a^3*c + 960*A*a^3*c*cos(f*x + e) + 480*B*a^
3*c*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$$
$$= \frac{Ba^3c\cos(5fx+5e)}{80f} + \frac{Aa^3c\sin(2fx+2e)}{4f} + \frac{1}{8}(5Aa^3c+2Ba^3c)x - \frac{(8Aa^3c+5Ba^3c)\cos(3fx+3e)}{48f} - \frac{(4Aa^3c+3Ba^3c)\cos(fx+e)}{8f} - \frac{(Aa^3c+2Ba^3c)\sin(4fx+4e)}{32f}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorith
m="giac")
```

output
```
1/80*B*a^3*c*cos(5*f*x + 5*e)/f + 1/4*A*a^3*c*sin(2*f*x + 2*e)/f + 1/8*(5*
A*a^3*c + 2*B*a^3*c)*x - 1/48*(8*A*a^3*c + 5*B*a^3*c)*cos(3*f*x + 3*e)/f -
 1/8*(4*A*a^3*c + 3*B*a^3*c)*cos(f*x + e)/f - 1/32*(A*a^3*c + 2*B*a^3*c)*s
in(4*f*x + 4*e)/f
```

Mupad [B] (verification not implemented)

Time = 36.70 (sec) , antiderivative size = 390, normalized size of antiderivative = 2.79

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$$
$$= \frac{a^3\,c\,\mathrm{atan}\left(\frac{a^3\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)(5\,A+2\,B)}{4\left(\frac{5\,A\,a^3\,c}{4}+\frac{B\,a^3\,c}{2}\right)}\right)(5\,A+2\,B)}{4\,f} - \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8(4\,A\,a^3\,c+2\,B\,a^3\,c) - \tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(\frac{3\,A\,a^3\,c}{4}-\frac{B\,a^3\,c}{2}\right) - \tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\left(\frac{7\,A\,a^3\,c}{2}+3\,B\,a^3\,c\right.}{f} - \frac{a^3\,c\,(5\,A+2\,B)\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-\frac{f\,x}{2}\right)}{4\,f}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x)),x)
```

output
```
(a^3*c*atan((a^3*c*tan(e/2 + (f*x)/2)*(5*A + 2*B))/(4*((5*A*a^3*c)/4 + (B*
a^3*c)/2)))*(5*A + 2*B))/(4*f) - (tan(e/2 + (f*x)/2)^8*(4*A*a^3*c + 2*B*a^
3*c) - tan(e/2 + (f*x)/2)*((3*A*a^3*c)/4 - (B*a^3*c)/2) - tan(e/2 + (f*x)/
2)^3*((7*A*a^3*c)/2 + 3*B*a^3*c) + tan(e/2 + (f*x)/2)^7*((7*A*a^3*c)/2 + 3
*B*a^3*c) + tan(e/2 + (f*x)/2)^9*((3*A*a^3*c)/4 - (B*a^3*c)/2) + tan(e/2 +
 (f*x)/2)^6*(8*A*a^3*c + 8*B*a^3*c) + tan(e/2 + (f*x)/2)^2*((8*A*a^3*c)/3
+ (8*B*a^3*c)/3) + tan(e/2 + (f*x)/2)^4*((16*A*a^3*c)/3 + (4*B*a^3*c)/3) +
 (4*A*a^3*c)/3 + (14*B*a^3*c)/15)/(f*(5*tan(e/2 + (f*x)/2)^2 + 10*tan(e/2
+ (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 + 5*tan(e/2 + (f*x)/2)^8 + tan(e/2
+ (f*x)/2)^10 + 1)) - (a^3*c*(5*A + 2*B)*(atan(tan(e/2 + (f*x)/2)) - (f*x)
/2))/(4*f)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.14

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$$
$$= \frac{a^3c\big(24\cos(fx+e)\sin(fx+e)^4b+30\cos(fx+e)\sin(fx+e)^3a+60\cos(fx+e)\sin(fx+e)^3b+80}{}$$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

output
```
(a**3*c*(24*cos(e + f*x)*sin(e + f*x)**4*b + 30*cos(e + f*x)*sin(e + f*x)*
*3*a + 60*cos(e + f*x)*sin(e + f*x)**3*b + 80*cos(e + f*x)*sin(e + f*x)**2
*a + 32*cos(e + f*x)*sin(e + f*x)**2*b + 45*cos(e + f*x)*sin(e + f*x)*a -
30*cos(e + f*x)*sin(e + f*x)*b - 80*cos(e + f*x)*a - 56*cos(e + f*x)*b + 7
5*a*f*x + 80*a + 30*b*f*x + 56*b))/(120*f)
```

3.44 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 156

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx &= -\frac{5a^3(3A+4B)x}{2c}+\frac{5a^3(3A+4B)\cos^3(e+fx)}{3cf}\\&\quad-\frac{5a^3(3A+4B)\cos(e+fx)\sin(e+fx)}{2cf}\\&\quad+\frac{a^3(A+B)c^3\cos^7(e+fx)}{f(c-c\sin(e+fx))^4}+\frac{2a^3(3A+4B)c^3\cos^5(e+fx)}{f\left(c^2-c^2\sin(e+fx)\right)^2}\end{aligned}$$

output
```
-5/2*a^3*(3*A+4*B)*x/c+5/3*a^3*(3*A+4*B)*cos(f*x+e)^3/c/f-5/2*a^3*(3*A+4*B
)*cos(f*x+e)*sin(f*x+e)/c/f+a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^
4+2*a^3*(3*A+4*B)*c^3*cos(f*x+e)^5/f/(c^2-c^2*sin(f*x+e))^2
```

Mathematica [A] (verified)

Time = 12.14 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.43

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$$
$$= \frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3\left(\cos\left(\frac{1}{2}(e+fx)\right)(30(3A+4B)(e+fx)-3(1\right.}{}$$

input
```
Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
]),x]
```

output
```
(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(Cos[(e +
f*x)/2]*(30*(3*A + 4*B)*(e + f*x) - 3*(16*A + 31*B)*Cos[e + f*x] + B*Cos[3
*(e + f*x)] - 3*(A + 4*B)*Sin[2*(e + f*x)]) - Sin[(e + f*x)/2]*(24*B*(8 +
5*e + 5*f*x) + 6*A*(32 + 15*e + 15*f*x) - 3*(16*A + 31*B)*Cos[e + f*x] + B
*Cos[3*(e + f*x)] - 3*(A + 4*B)*Sin[2*(e + f*x)])))/(12*c*f*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^6*(-1 + Sin[e + f*x]))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.84, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.306$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3161, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{c-c\sin(e+fx)}dx$$
$$\Big\downarrow \text{3446}$$
$$a^3c^3\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\int \frac{\cos(e+fx)^6(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}dx$$

$$\Big\downarrow \text{3338}$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{f(c-c\sin(e+fx))^4}-\frac{(3A+4B)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^3}dx}{c}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{f(c-c\sin(e+fx))^4}-\frac{(3A+4B)\int \frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^3}dx}{c}\right)$$

$$\Big\downarrow \text{3159}$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{f(c-c\sin(e+fx))^4}-\frac{(3A+4B)\left(\frac{5\int \frac{\cos^4(e+fx)}{c-c\sin(e+fx)}dx}{c^2}-\frac{2\cos^5(e+fx)}{cf(c-c\sin(e+fx))^2}\right)}{c}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{f(c-c\sin(e+fx))^4}-\frac{(3A+4B)\left(\frac{5\int \frac{\cos(e+fx)^4}{c-c\sin(e+fx)}dx}{c^2}-\frac{2\cos^5(e+fx)}{cf(c-c\sin(e+fx))^2}\right)}{c}\right)$$

$$\Big\downarrow \text{3161}$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{f(c-c\sin(e+fx))^4}-\frac{(3A+4B)\left(\frac{5\left(\frac{\int \cos^2(e+fx)dx}{c}-\frac{\cos^3(e+fx)}{3cf}\right)}{c^2}-\frac{2\cos^5(e+fx)}{cf(c-c\sin(e+fx))^2}\right)}{c}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{f(c-c\sin(e+fx))^4}-\frac{(3A+4B)\left(\frac{5\left(\frac{\int \sin\left(e+fx+\frac{\pi}{2}\right)^2dx}{c}-\frac{\cos^3(e+fx)}{3cf}\right)}{c^2}-\frac{2\cos^5(e+fx)}{cf(c-c\sin(e+fx))^2}\right)}{c}\right)$$

$$\Big\downarrow \ 3115$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{f(c-c\sin(e+fx))^4}-\frac{(3A+4B)\left(\frac{5\left(\frac{\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}}{c}-\frac{\cos^3(e+fx)}{3cf}\right)}{c^2}-\frac{2\cos^5(e+fx)}{cf(c-c\sin(e+fx))^2}\right)}{c}\right)$$

$$\Big\downarrow \ 24$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{f(c-c\sin(e+fx))^4}-\frac{(3A+4B)\left(\frac{5\left(\frac{\frac{\sin(e+fx)\cos(e+fx)}{2f}+\frac{x}{2}}{c}-\frac{\cos^3(e+fx)}{3cf}\right)}{c^2}-\frac{2\cos^5(e+fx)}{cf(c-c\sin(e+fx))^2}\right)}{c}\right)$$

input
```
Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]),x]
```

output
```
a^3*c^3*(((A + B)*Cos[e + f*x]^7)/(f*(c - c*Sin[e + f*x])^4) - ((3*A + 4*B
)*((-2*Cos[e + f*x]^5)/(c*f*(c - c*Sin[e + f*x])^2) + (5*(-1/3*Cos[e + f*x
]^3/(c*f) + (x/2 + (Cos[e + f*x]*Sin[e + f*x])/(2*f))/c))/c^2))/c)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.71

method	result
parallelrisch	$\frac{65\left(\frac{4\left(4A+\frac{23B}{3}\right)\cos(2fx+2e)}{65}+\frac{(A+4B)\sin(3fx+3e)}{65}-\frac{B\cos(4fx+4e)}{195}+\frac{4\left(-3fxA-4fxB+\frac{24}{5}A+\frac{94}{15}B\right)\cos(fx+e)}{13}+\left(A+\frac{68B}{65}\right.\right.}{8cf\cos(fx+e)}$
derivativedivides	$\frac{2a^3\left(-\frac{\left(\frac{A}{2}+2B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5+(-4A-7B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+(-8A-16B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\left(-\frac{A}{2}-2B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-4A-\frac{23B}{3}}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^3}-\frac{5}{}\right.}{fc}$
default	$\frac{2a^3\left(-\frac{\left(\frac{A}{2}+2B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5+(-4A-7B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+(-8A-16B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\left(-\frac{A}{2}-2B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-4A-\frac{23B}{3}}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^3}-\frac{5}{}\right.}{fc}$
risch	$-\frac{15a^3xA}{2c}-\frac{10a^3xB}{c}+\frac{2a^3e^{i(fx+e)}A}{cf}+\frac{31a^3e^{i(fx+e)}B}{8cf}+\frac{2a^3e^{-i(fx+e)}A}{cf}+\frac{31a^3e^{-i(fx+e)}B}{8cf}+\frac{16a^3}{fc(e^{i(fx+}}$
norman	$\frac{-\frac{17a^3A+20a^3B}{cf}+\frac{5a^3(3A+4B)x}{2c}-\frac{\left(5a^3A+2a^3B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{cf}-\frac{\left(17a^3A+18a^3B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{cf}-\frac{\left(21a^3A+34a^3B\right)\tan\left(\frac{fx}{2}\right.}{3cf}}{}$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x,method=_RETURNV
ERBOSE)
```

output
```
65/8*(4/65*(4*A+23/3*B)*cos(2*f*x+2*e)+1/65*(A+4*B)*sin(3*f*x+3*e)-1/195*B
*cos(4*f*x+4*e)+4/13*(-3*f*x*A-4*f*x*B+24/5*A+94/15*B)*cos(f*x+e)+(A+68/65
*B)*sin(f*x+e)+16/13*A+19/13*B)*a^3/c/f/cos(f*x+e)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.40

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = -\frac{2\,Ba^3\cos\left(fx+e\right)^4-(3\,A+10\,B)a^3\cos\left(fx+e\right)^3+15\,(3\,A+4\,B)a^3fx-24\,(A+2\,B)a^3\cos\left(fx+\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
-1/6*(2*B*a^3*cos(f*x + e)^4 - (3*A + 10*B)*a^3*cos(f*x + e)^3 + 15*(3*A +
 4*B)*a^3*f*x - 24*(A + 2*B)*a^3*cos(f*x + e)^2 - 48*(A + B)*a^3 + 3*(5*(3
*A + 4*B)*a^3*f*x - (23*A + 28*B)*a^3)*cos(f*x + e) - (2*B*a^3*cos(f*x + e
)^3 + 15*(3*A + 4*B)*a^3*f*x + 3*(A + 4*B)*a^3*cos(f*x + e)^2 - 3*(7*A + 1
2*B)*a^3*cos(f*x + e) + 48*(A + B)*a^3)*sin(f*x + e))/(c*f*cos(f*x + e) -
c*f*sin(f*x + e) + c*f)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4255 vs. 2(144) = 288.

Time = 4.15 (sec) , antiderivative size = 4255, normalized size of antiderivative = 27.28

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

output
```
Piecewise((-45*A*a**3*f*x*tan(e/2 + f*x/2)**7/(6*c*f*tan(e/2 + f*x/2)**7 -
 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 +
 f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c
*f*tan(e/2 + f*x/2) - 6*c*f) + 45*A*a**3*f*x*tan(e/2 + f*x/2)**6/(6*c*f*ta
n(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5
 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/
2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 135*A*a**3*f*x*tan(e/2 +
 f*x/2)**5/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f
*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2
)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) + 135*
A*a**3*f*x*tan(e/2 + f*x/2)**4/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2
+ f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18
*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*
x/2) - 6*c*f) - 135*A*a**3*f*x*tan(e/2 + f*x/2)**3/(6*c*f*tan(e/2 + f*x/2)
**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(
e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2
+ 6*c*f*tan(e/2 + f*x/2) - 6*c*f) + 135*A*a**3*f*x*tan(e/2 + f*x/2)**2/(6*
c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x
/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*
tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 45*A*a**3*f*x*t...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1139 vs. $2(152) = 304$.

Time = 0.14 (sec) , antiderivative size = 1139, normalized size of antiderivative = 7.30

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{c - c\sin(e + fx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorith
m="maxima")
```

output
```
-1/3*(B*a^3*((7*sin(f*x + e)/(cos(f*x + e) + 1) - 39*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 24*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 24*sin(f*x + e)^
4/(cos(f*x + e) + 1)^4 + 9*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 9*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 - 16)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1)
+ 3*c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 3*c*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 3*c*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3*c*sin(f*x + e)^5/(
cos(f*x + e) + 1)^5 + c*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c*sin(f*x +
e)^7/(cos(f*x + e) + 1)^7) + 9*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c)
+ 18*A*a^3*((sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 - 2)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1) + c*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f
*x + e)/(cos(f*x + e) + 1))/c) + 18*B*a^3*((sin(f*x + e)/(cos(f*x + e) + 1
) - sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2)/(c - c*sin(f*x + e)/(cos(f*x
+ e) + 1) + c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c) + 3*A*a^3*((
sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 3*sin(f*x + e)^4/(cos(f*x + e) + 1
)^4 - 4)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1) + 2*c*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 - 2*c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + c*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 - c*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*ar...
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.43

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx =$$

$$-\frac{\frac{15\,(3\,Aa^3+4\,Ba^3)(fx+e)}{c}+\frac{96\,(Aa^3+Ba^3)}{c\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)}+\frac{2\left(3\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+12\,Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-24\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-42\,B}{}}{6}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorith
m="giac")
```

output
```
-1/6*(15*(3*A*a^3 + 4*B*a^3)*(f*x + e)/c + 96*(A*a^3 + B*a^3)/(c*(tan(1/2*
f*x + 1/2*e) - 1)) + 2*(3*A*a^3*tan(1/2*f*x + 1/2*e)^5 + 12*B*a^3*tan(1/2*
f*x + 1/2*e)^5 - 24*A*a^3*tan(1/2*f*x + 1/2*e)^4 - 42*B*a^3*tan(1/2*f*x +
1/2*e)^4 - 48*A*a^3*tan(1/2*f*x + 1/2*e)^2 - 96*B*a^3*tan(1/2*f*x + 1/2*e)
^2 - 3*A*a^3*tan(1/2*f*x + 1/2*e) - 12*B*a^3*tan(1/2*f*x + 1/2*e) - 24*A*a
^3 - 46*B*a^3)/((tan(1/2*f*x + 1/2*e)^2 + 1)^3*c))/f
```

Mupad [B] (verification not implemented)

Time = 37.02 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.07

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$$
$$= \frac{24\,A\,a^3 - \tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\left(7\,A\,a^3+\frac{34\,B\,a^3}{3}\right)+\frac{94\,B\,a^3}{3}-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\left(9\,A\,a^3+18\,B\,a^3\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6(1}{f\,\left(-c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7+c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-3\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\right.}$$
$$- \frac{5\,a^3\,\mathrm{atan}\left(\frac{5\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)(3\,A+4\,B)}{15\,A\,a^3+20\,B\,a^3}\right)(3\,A+4\,B)}{c\,f}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x)),x)
```

output
```
(24*A*a^3 - tan(e/2 + (f*x)/2)*(7*A*a^3 + (34*B*a^3)/3) + (94*B*a^3)/3 - t
an(e/2 + (f*x)/2)^5*(9*A*a^3 + 18*B*a^3) + tan(e/2 + (f*x)/2)^6*(17*A*a^3
+ 20*B*a^3) - tan(e/2 + (f*x)/2)^3*(16*A*a^3 + 32*B*a^3) + tan(e/2 + (f*x)
/2)^4*(56*A*a^3 + 62*B*a^3) + tan(e/2 + (f*x)/2)^2*(63*A*a^3 + 76*B*a^3))/
(f*(c - c*tan(e/2 + (f*x)/2) + 3*c*tan(e/2 + (f*x)/2)^2 - 3*c*tan(e/2 + (f
*x)/2)^3 + 3*c*tan(e/2 + (f*x)/2)^4 - 3*c*tan(e/2 + (f*x)/2)^5 + c*tan(e/2
 + (f*x)/2)^6 - c*tan(e/2 + (f*x)/2)^7)) - (5*a^3*atan((5*a^3*tan(e/2 + (f
*x)/2)*(3*A + 4*B))/(15*A*a^3 + 20*B*a^3))*(3*A + 4*B))/(c*f)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.67

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = \frac{a^3\left(2\cos\left(fx+e\right)\sin\left(fx+e\right)^3b+3\cos\left(fx+e\right)\sin\left(fx+e\right)^2a+10\cos\left(fx+e\right)\sin\left(fx+e\right)^2b+21\,\mathrm{co}\right.}{}$$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

output
```
(a**3*(2*cos(e + f*x)*sin(e + f*x)**3*b + 3*cos(e + f*x)*sin(e + f*x)**2*a
 + 10*cos(e + f*x)*sin(e + f*x)**2*b + 21*cos(e + f*x)*sin(e + f*x)*a + 34
*cos(e + f*x)*sin(e + f*x)*b - 45*cos(e + f*x)*a*f*x - 102*cos(e + f*x)*a
- 60*cos(e + f*x)*b*f*x - 120*cos(e + f*x)*b - 2*sin(e + f*x)**4*b - 3*sin
(e + f*x)**3*a - 12*sin(e + f*x)**3*b - 24*sin(e + f*x)**2*a - 44*sin(e +
f*x)**2*b - 45*sin(e + f*x)*a*f*x + 21*sin(e + f*x)*a - 60*sin(e + f*x)*b*
f*x + 34*sin(e + f*x)*b + 45*a*f*x + 102*a + 60*b*f*x + 120*b))/(6*c*f*(co
s(e + f*x) + sin(e + f*x) - 1))
```

3.45 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 163

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \frac{5a^3(2A+5B)x}{2c^2} - \frac{5a^3(2A+5B)\cos(e+fx)}{2c^2f} + \frac{a^3(A+B)c^3\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5} - \frac{2a^3(2A+5B)c\cos^5(e+fx)}{3f(c-c\sin(e+fx))^3} - \frac{5a^3(2A+5B)\cos^3(e+fx)}{6f\left(c^2-c^2\sin(e+fx)\right)}$$

output

```
5/2*a^3*(2*A+5*B)*x/c^2-5/2*a^3*(2*A+5*B)*cos(f*x+e)/c^2/f+1/3*a^3*(A+B)*c
^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^5-2/3*a^3*(2*A+5*B)*c*cos(f*x+e)^5/f/(c
-c*sin(f*x+e))^3-5/6*a^3*(2*A+5*B)*cos(f*x+e)^3/f/(c^2-c^2*sin(f*x+e))
```

Mathematica [A] (verified)

Time = 11.55 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.72

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3\left(32(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx\right.\right.\right.}{}$$

input
```
Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^2,x]
```

output
```
(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(32*(A + B
)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) + 30*(2*A + 5*B)*(e + f*x)*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])^3 - 12*(A + 5*B)*Cos[e + f*x]*(Cos[(e + f*x
)/2] - Sin[(e + f*x)/2])^3 + 64*(A + B)*Sin[(e + f*x)/2] - 32*(7*A + 13*B)
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] - 3*B*(Cos[(e +
f*x)/2] - Sin[(e + f*x)/2])^3*Sin[2*(e + f*x)]))/(12*f*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^6*(c - c*Sin[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.91, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.306$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3158, 3042, 3161, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}dx$$

$$\Big\downarrow \ 3446$$

$$a^3c^3\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}dx$$

$$\Big\downarrow \ 3042$$

$$a^3c^3\int \frac{\cos(e+fx)^6(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}dx$$

$$\Big\downarrow \ 3338$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5}-\frac{(2A+5B)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^4}dx}{3c}\right)$$

$$\Big\downarrow \textbf{3042}$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5}-\frac{(2A+5B)\int\frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^4}dx}{3c}\right)$$

$$\Big\downarrow \textbf{3159}$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5}-\frac{(2A+5B)\left(\frac{2\cos^5(e+fx)}{cf(c-c\sin(e+fx))^3}-\frac{5\int\frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^2}dx}{c^2}\right)}{3c}\right)$$

$$\Big\downarrow \textbf{3042}$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5}-\frac{(2A+5B)\left(\frac{2\cos^5(e+fx)}{cf(c-c\sin(e+fx))^3}-\frac{5\int\frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^2}dx}{c^2}\right)}{3c}\right)$$

$$\Big\downarrow \textbf{3158}$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5}-\frac{(2A+5B)\left(\frac{2\cos^5(e+fx)}{cf(c-c\sin(e+fx))^3}-\frac{5\left(\frac{3\int\frac{\cos^2(e+fx)}{c-c\sin(e+fx)}dx}{2c}-\frac{\cos^3(e+fx)}{2f\left(c^2-c^2\sin(e+fx)\right)}\right)}{c^2}\right)}{3c}\right)$$

$$\Big\downarrow \textbf{3042}$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5}-\frac{(2A+5B)\left(\frac{2\cos^5(e+fx)}{cf(c-c\sin(e+fx))^3}-\frac{5\left(\frac{3\int\frac{\cos(e+fx)^2}{c-c\sin(e+fx)}dx}{2c}-\frac{\cos^3(e+fx)}{2f\left(c^2-c^2\sin(e+fx)\right)}\right)}{c^2}\right)}{3c}\right)$$

$$\Big\downarrow \textbf{3161}$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5}-\frac{(2A+5B)\left(\frac{2\cos^5(e+fx)}{cf(c-c\sin(e+fx))^3}-\frac{5\left(\frac{3\left(\frac{\int 1dx}{c}-\frac{\cos(e+fx)}{cf}\right)}{2c}-\frac{\cos^3(e+fx)}{2f\left(c^2-c^2\sin(e+fx)\right)}\right)}{c^2}\right)}{3c}\right)$$

$$\Big\downarrow \text{ 24}$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{3f(c-c\sin(e+fx))^5}-\frac{(2A+5B)\left(\frac{2\cos^5(e+fx)}{cf(c-c\sin(e+fx))^3}-\frac{5\left(\frac{3\left(\frac{x}{c}-\frac{\cos(e+fx)}{cf}\right)}{2c}-\frac{\cos^3(e+fx)}{2f\left(c^2-c^2\sin(e+fx)\right)}\right)}{c^2}\right)}{3c}\right)$$

input
```
Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^2,x
]
```

output
```
a^3*c^3*(((A + B)*Cos[e + f*x]^7)/(3*f*(c - c*Sin[e + f*x])^5) - ((2*A + 5
*B)*((2*Cos[e + f*x]^5)/(c*f*(c - c*Sin[e + f*x])^3) - (5*((3*(x/c - Cos[e
 + f*x]/(c*f)))/(2*c) - Cos[e + f*x]^3/(2*f*(c^2 - c^2*Sin[e + f*x]))))/c^
2))/(3*c))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3158
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(a*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
 EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && In
tegersQ[2*m, 2*p]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{2a^3\left(-\frac{-4A-12B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{16A+16B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{16A+16B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}+\frac{\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2}+(-A-5B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\frac{B\tan\left(\frac{fx}{2}\right.}{2}}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^2}\right.}{f\,c^2}$
default	$\frac{2a^3\left(-\frac{-4A-12B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{16A+16B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{16A+16B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}+\frac{\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2}+(-A-5B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\frac{B\tan\left(\frac{fx}{2}\right.}{2}}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^2}\right.}{f\,c^2}$
parallelrisch	$-\frac{\left(\left(30fxA+75fxB-56A-\frac{607}{4}B\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\left(-10fxA-25fxB-\frac{19}{3}A-\frac{115}{12}B\right)\cos\left(\frac{3fx}{2}+\frac{3e}{2}\right)+(-30fxA-75fx\right.}{2f\,c^2\left(-\right.}$
risch	$\frac{5a^3xA}{c^2}+\frac{25a^3xB}{2c^2}+\frac{ia^3B\,e^{2i(fx+e)}}{8c^2f}-\frac{a^3e^{i(fx+e)}A}{2c^2f}-\frac{5a^3e^{i(fx+e)}B}{2c^2f}-\frac{a^3e^{-i(fx+e)}A}{2c^2f}-\frac{5a^3e^{-i(fx+e)}B}{2c^2f}-$
norman	$\frac{\frac{\left(8a^3A+25a^3B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{cf}+\frac{46a^3A+118a^3B}{3cf}-\frac{5a^3(2A+5B)x}{2c}-\frac{\left(34a^3A+77a^3B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{cf}-\frac{\left(38a^3A+93a^3B\right)\tan\left(\frac{fx}{2}\right.}{cf}}{}$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x,method=_RETUR
NVERBOSE)
```

output
```
2/f*a^3/c^2*(-(-4*A-12*B)/(tan(1/2*f*x+1/2*e)-1)-1/3*(16*A+16*B)/(tan(1/2*
f*x+1/2*e)-1)^3-1/2*(16*A+16*B)/(tan(1/2*f*x+1/2*e)-1)^2+(1/2*B*tan(1/2*f*
x+1/2*e)^3+(-A-5*B)*tan(1/2*f*x+1/2*e)^2-1/2*B*tan(1/2*f*x+1/2*e)-A-5*B)/(
1+tan(1/2*f*x+1/2*e)^2)^2+5/2*(2*A+5*B)*arctan(tan(1/2*f*x+1/2*e)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.75

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$$
$$=\frac{3\,Ba^3\cos\left(fx+e\right)^4-6\,(A+4\,B)a^3\cos\left(fx+e\right)^3-30\,(2\,A+5\,B)a^3fx-16\,(A+B)a^3+(15\,(2\,A+5}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
1/6*(3*B*a^3*cos(f*x + e)^4 - 6*(A + 4*B)*a^3*cos(f*x + e)^3 - 30*(2*A + 5
*B)*a^3*f*x - 16*(A + B)*a^3 + (15*(2*A + 5*B)*a^3*f*x + (62*A + 131*B)*a^
3)*cos(f*x + e)^2 - (15*(2*A + 5*B)*a^3*f*x - 2*(26*A + 71*B)*a^3)*cos(f*x
 + e) - (3*B*a^3*cos(f*x + e)^3 - 30*(2*A + 5*B)*a^3*f*x + 3*(2*A + 9*B)*a
^3*cos(f*x + e)^2 + 16*(A + B)*a^3 - (15*(2*A + 5*B)*a^3*f*x - 2*(34*A + 7
9*B)*a^3)*cos(f*x + e))*sin(f*x + e))/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*
x + e) - 2*c^2*f + (c^2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4665 vs. $2(151) = 302$.

Time = 8.21 (sec) , antiderivative size = 4665, normalized size of antiderivative = 28.62

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**2,x)
```

output
```
Piecewise((30*A*a**3*f*x*tan(e/2 + f*x/2)**7/(6*c**2*f*tan(e/2 + f*x/2)**7
 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2
*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2
 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) - 90*A*a**3*f*x*tan(
e/2 + f*x/2)**6/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)
**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c
**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(
e/2 + f*x/2) - 6*c**2*f) + 150*A*a**3*f*x*tan(e/2 + f*x/2)**5/(6*c**2*f*ta
n(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*
x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 -
30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) - 2
10*A*a**3*f*x*tan(e/2 + f*x/2)**4/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*
f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2
+ f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**
2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) + 210*A*a**3*f*x*tan(e/2 + f*x/
2)**3/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c
**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(
e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/
2) - 6*c**2*f) - 150*A*a**3*f*x*tan(e/2 + f*x/2)**2/(6*c**2*f*tan(e/2 + f*
x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1386 vs. $2(156) = 312$.

Time $= 0.16$ (sec) , antiderivative size $= 1386$, normalized size of antiderivative $= 8.50$

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c - c\sin(e + fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algori
thm="maxima")
```

output
```
1/3*(B*a^3*((75*sin(f*x + e)/(cos(f*x + e) + 1) - 97*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 98*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 21*sin(
f*x + e)^6/(cos(f*x + e) + 1)^6 - 32)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x +
 e) + 1) + 5*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 7*c^2*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 7*c^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 5*c^2
*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 3*c^2*sin(f*x + e)^6/(cos(f*x + e)
+ 1)^6 - c^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f*x + e)
/(cos(f*x + e) + 1))/c^2) + 4*A*a^3*((12*sin(f*x + e)/(cos(f*x + e) + 1) -
 11*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 - 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 5)/(c^2 - 3*c^2*sin(f*x +
e)/(cos(f*x + e) + 1) + 4*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4*c^2*
sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*c^2*sin(f*x + e)^4/(cos(f*x + e) +
 1)^4 - c^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(
cos(f*x + e) + 1))/c^2) + 12*B*a^3*((12*sin(f*x + e)/(cos(f*x + e) + 1) -
11*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 - 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 5)/(c^2 - 3*c^2*sin(f*x + e
)/(cos(f*x + e) + 1) + 4*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4*c^2*s
in(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*c^2*sin(f*x + e)^4/(cos(f*x + e) +
1)^4 - c^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)...
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.36

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$$
$$= \frac{\frac{15\,(2\,Aa^3+5\,Ba^3)(fx+e)}{c^2}+\frac{6\left(Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-2\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-10\,Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-2\,Aa^3-10\,Ba^3\cdots\right.}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2c^2}}{6\,f}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
1/6*(15*(2*A*a^3 + 5*B*a^3)*(f*x + e)/c^2 + 6*(B*a^3*tan(1/2*f*x + 1/2*e)^
3 - 2*A*a^3*tan(1/2*f*x + 1/2*e)^2 - 10*B*a^3*tan(1/2*f*x + 1/2*e)^2 - B*a
^3*tan(1/2*f*x + 1/2*e) - 2*A*a^3 - 10*B*a^3)/((tan(1/2*f*x + 1/2*e)^2 + 1
)^2*c^2) + 16*(3*A*a^3*tan(1/2*f*x + 1/2*e)^2 + 9*B*a^3*tan(1/2*f*x + 1/2*
e)^2 - 12*A*a^3*tan(1/2*f*x + 1/2*e) - 24*B*a^3*tan(1/2*f*x + 1/2*e) + 5*A
*a^3 + 11*B*a^3)/(c^2*(tan(1/2*f*x + 1/2*e) - 1)^3))/f
```

Mupad [B] (verification not implemented)

Time = 36.73 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.09

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$$
$$= \frac{5\,a^3\,\mathrm{atan}\left(\frac{5\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(2\,A+5\,B)}{10\,A\,a^3+25\,B\,a^3}\right)\,(2\,A+5\,B)}{c^2\,f}$$
$$-\frac{\frac{46\,A\,a^3}{3}-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(38\,A\,a^3+93\,B\,a^3)+\frac{118\,B\,a^3}{3}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\,(8\,A\,a^3+25\,B\,a^3)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(-c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7+3\,c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-5\,c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right.}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^2,x
)
```

output
```
(5*a^3*atan((5*a^3*tan(e/2 + (f*x)/2)*(2*A + 5*B))/(10*A*a^3 + 25*B*a^3))*
(2*A + 5*B))/(c^2*f) - ((46*A*a^3)/3 - tan(e/2 + (f*x)/2)*(38*A*a^3 + 93*B
*a^3) + (118*B*a^3)/3 + tan(e/2 + (f*x)/2)^6*(8*A*a^3 + 25*B*a^3) - tan(e/
2 + (f*x)/2)^5*(34*A*a^3 + 77*B*a^3) - tan(e/2 + (f*x)/2)^3*(72*A*a^3 + 16
6*B*a^3) + tan(e/2 + (f*x)/2)^4*((106*A*a^3)/3 + (328*B*a^3)/3) + tan(e/2
+ (f*x)/2)^2*((128*A*a^3)/3 + (359*B*a^3)/3))/(f*(5*c^2*tan(e/2 + (f*x)/2)
^2 - 7*c^2*tan(e/2 + (f*x)/2)^3 + 7*c^2*tan(e/2 + (f*x)/2)^4 - 5*c^2*tan(e
/2 + (f*x)/2)^5 + 3*c^2*tan(e/2 + (f*x)/2)^6 - c^2*tan(e/2 + (f*x)/2)^7 +
c^2 - 3*c^2*tan(e/2 + (f*x)/2)))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 345, normalized size of antiderivative = 2.12

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$$
$$= \frac{a^3\big(16a+50b+30\cos(fx+e)\sin(fx+e)\,afx+75\cos(fx+e)\sin(fx+e)\,bfx-50\cos(fx+e)\,b+$$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x)
```

output
```
(a**3*( - 3*cos(e + f*x)*sin(e + f*x)**3*b - 6*cos(e + f*x)*sin(e + f*x)**
2*a - 24*cos(e + f*x)*sin(e + f*x)**2*b + 30*cos(e + f*x)*sin(e + f*x)*a*f
*x + 38*cos(e + f*x)*sin(e + f*x)*a + 75*cos(e + f*x)*sin(e + f*x)*b*f*x +
 93*cos(e + f*x)*sin(e + f*x)*b - 30*cos(e + f*x)*a*f*x - 16*cos(e + f*x)*
a - 75*cos(e + f*x)*b*f*x - 50*cos(e + f*x)*b + 3*sin(e + f*x)**4*b + 6*si
n(e + f*x)**3*a + 27*sin(e + f*x)**3*b + 30*sin(e + f*x)**2*a*f*x - 92*sin
(e + f*x)**2*a + 75*sin(e + f*x)**2*b*f*x - 205*sin(e + f*x)**2*b - 60*sin
(e + f*x)*a*f*x + 38*sin(e + f*x)*a - 150*sin(e + f*x)*b*f*x + 93*sin(e +
f*x)*b + 30*a*f*x + 16*a + 75*b*f*x + 50*b))/(6*c**2*f*(cos(e + f*x)*sin(e
 + f*x) - cos(e + f*x) + sin(e + f*x)**2 - 2*sin(e + f*x) + 1))
```

3.46 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 153

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = -\frac{a^3(A+6B)x}{c^3}+\frac{a^3(A+6B)\cos(e+fx)}{c^3f}+\frac{a^3(A+B)c^3\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6} - \frac{2a^3(A+6B)c\cos^5(e+fx)}{15f(c-c\sin(e+fx))^4}+\frac{2a^3(A+6B)c^3\cos^3(e+fx)}{3f\left(c^3-c^3\sin(e+fx)\right)^2}$$

output
```
-a^3*(A+6*B)*x/c^3+a^3*(A+6*B)*cos(f*x+e)/c^3/f+1/5*a^3*(A+B)*c^3*cos(f*x+
e)^7/f/(c-c*sin(f*x+e))^6-2/15*a^3*(A+6*B)*c*cos(f*x+e)^5/f/(c-c*sin(f*x+e
))^4+2/3*a^3*(A+6*B)*c^3*cos(f*x+e)^3/f/(c^3-c^3*sin(f*x+e))^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 316 vs. 2(153) = 306.

Time = 11.78 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.07

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = \frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(24(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-4(11A+21B\right.}{}$$

input
```
Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^3,x]
```

output
```
(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(24*(A + B)*(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2]) - 4*(11*A + 21*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^3 - 15*(A + 6*B)*(e + f*x)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 + 15*B
*Cos[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 + 48*(A + B)*Sin[(e
+ f*x)/2] - 8*(11*A + 21*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e
 + f*x)/2] + 4*(23*A + 93*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(
e + f*x)/2])*(1 + Sin[e + f*x])^3)/(15*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)
/2])^6*(c - c*Sin[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.95, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.306$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3159, 3042, 3161, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}dx$$

$$\Big\downarrow \text{3446}$$

$$a^3c^3\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}dx$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\int \frac{\cos(e+fx)^6(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}dx$$

$$\Big\downarrow \text{3338}$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6}-\frac{(A+6B)\int\frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^5}dx}{5c}\right)$$

$\big\downarrow$ 3042

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6}-\frac{(A+6B)\int\frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^5}dx}{5c}\right)$$

$\big\downarrow$ 3159

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6}-\frac{(A+6B)\left(\frac{2\cos^5(e+fx)}{3cf(c-c\sin(e+fx))^4}-\frac{5\int\frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^3}dx}{3c^2}\right)}{5c}\right)$$

$\big\downarrow$ 3042

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6}-\frac{(A+6B)\left(\frac{2\cos^5(e+fx)}{3cf(c-c\sin(e+fx))^4}-\frac{5\int\frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^3}dx}{3c^2}\right)}{5c}\right)$$

$\big\downarrow$ 3159

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6}-\frac{(A+6B)\left(\frac{2\cos^5(e+fx)}{3cf(c-c\sin(e+fx))^4}-\frac{5\left(\frac{2\cos^3(e+fx)}{cf(c-c\sin(e+fx))^2}-\frac{3\int\frac{\cos^2(e+fx)}{c-c\sin(e+fx)}dx}{c^2}\right)}{3c^2}\right)}{5c}\right)$$

$\big\downarrow$ 3042

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6}-\frac{(A+6B)\left(\frac{2\cos^5(e+fx)}{3cf(c-c\sin(e+fx))^4}-\frac{5\left(\frac{2\cos^3(e+fx)}{cf(c-c\sin(e+fx))^2}-\frac{3\int\frac{\cos(e+fx)^2}{c-c\sin(e+fx)}dx}{c^2}\right)}{3c^2}\right)}{5c}\right)$$

$\downarrow$ 3161

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6}-\frac{(A+6B)\left(\frac{2\cos^5(e+fx)}{3cf(c-c\sin(e+fx))^4}-\frac{5\left(\frac{2\cos^3(e+fx)}{cf(c-c\sin(e+fx))^2}-\frac{3\left(\frac{\int 1dx}{c}-\frac{\cos(e+fx)}{cf}\right)}{c^2}\right)}{3c^2}\right)}{5c}\right)$$

$\downarrow$ 24

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{5f(c-c\sin(e+fx))^6}-\frac{(A+6B)\left(\frac{2\cos^5(e+fx)}{3cf(c-c\sin(e+fx))^4}-\frac{5\left(\frac{2\cos^3(e+fx)}{cf(c-c\sin(e+fx))^2}-\frac{3\left(\frac{x}{c}-\frac{\cos(e+fx)}{cf}\right)}{c^2}\right)}{3c^2}\right)}{5c}\right)$$

input
```
Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^3,x
]
```

output
```
a^3*c^3*(((A + B)*Cos[e + f*x]^7)/(5*f*(c - c*Sin[e + f*x])^6) - ((A + 6*B
)*((2*Cos[e + f*x]^5)/(3*c*f*(c - c*Sin[e + f*x])^4) - (5*((-3*(x/c - Cos[
e + f*x]/(c*f)))/c^2 + (2*Cos[e + f*x]^3)/(c*f*(c - c*Sin[e + f*x])^2)))/(
3*c^2)))/(5*c))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 1.98 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{2a^3\left(-\frac{2A+6B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{8A-8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{32A+32B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{40A+24B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{64A+64B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}+\frac{B}{1+\tan\left(\frac{f}{2}\right.}\right.}{f\,c^3}$
default	$\frac{2a^3\left(-\frac{2A+6B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{8A-8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{32A+32B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{40A+24B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{64A+64B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}+\frac{B}{1+\tan\left(\frac{f}{2}\right.}\right.}{f\,c^3}$
risch	$-\frac{a^3xA}{c^3}-\frac{6a^3xB}{c^3}+\frac{B\,a^3\mathrm{e}^{i(fx+e)}}{2c^3f}+\frac{B\,a^3\mathrm{e}^{-i(fx+e)}}{2c^3f}+\frac{-\frac{112A\,a^3\mathrm{e}^{2i(fx+e)}}{3}-24iA\,a^3\mathrm{e}^{3i(fx+e)}+\frac{56iA\,a^3\mathrm{e}^{i(fx+e)}}{3}}{}$
parallelrisch	$-\frac{a^3\left(\left(\left(\frac{233}{2}-60fx\right)B-10fxA+24A\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\left(\left(-\frac{33}{2}+30fx\right)B-\frac{16A}{3}+5fxA\right)\cos\left(\frac{3fx}{2}+\frac{3e}{2}\right)+\left(-\frac{243}{10}B-\frac{24}{5}A\right.\right.}{f\,c^3\left(5\right.}$
norman	$\frac{\left(40a^3A+246a^3B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3fc}-\frac{2\left(304a^3A+1539a^3B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{15fc}+\frac{2\left(92a^3A+591a^3B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3fc}-\frac{\left(1972a^3A+9552a^3\right.}{15f}$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x,method=_RETUR
NVERBOSE)
```

output
```
2/f*a^3/c^3*(-(2*A+6*B)/(tan(1/2*f*x+1/2*e)-1)-1/2*(8*A-8*B)/(tan(1/2*f*x+
1/2*e)-1)^2-1/5*(32*A+32*B)/(tan(1/2*f*x+1/2*e)-1)^5-1/3*(40*A+24*B)/(tan(
1/2*f*x+1/2*e)-1)^3-1/4*(64*A+64*B)/(tan(1/2*f*x+1/2*e)-1)^4+B/(1+tan(1/2*
f*x+1/2*e)^2)-(A+6*B)*arctan(tan(1/2*f*x+1/2*e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(150) = 300.

Time = 0.09 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.20

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$$
$$=\frac{15\,Ba^3\cos\left(fx+e\right)^4+60\,(A+6\,B)a^3fx-24\,(A+B)a^3-(15\,(A+6\,B)a^3fx-(46\,A+231\,B)a^3)\cos}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algori
thm="fricas")
```

output
```
1/15*(15*B*a^3*cos(f*x + e)^4 + 60*(A + 6*B)*a^3*f*x - 24*(A + B)*a^3 - (1
5*(A + 6*B)*a^3*f*x - (46*A + 231*B)*a^3)*cos(f*x + e)^3 - (45*(A + 6*B)*a
^3*f*x + 2*(A + 66*B)*a^3)*cos(f*x + e)^2 + 6*(5*(A + 6*B)*a^3*f*x - 2*(6*
A + 31*B)*a^3)*cos(f*x + e) - (15*B*a^3*cos(f*x + e)^3 + 60*(A + 6*B)*a^3*
f*x + 24*(A + B)*a^3 - (15*(A + 6*B)*a^3*f*x + 2*(23*A + 108*B)*a^3)*cos(f
*x + e)^2 + 6*(5*(A + 6*B)*a^3*f*x - 2*(4*A + 29*B)*a^3)*cos(f*x + e))*sin
(f*x + e))/(c^3*f*cos(f*x + e)^3 + 3*c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*
x + e) - 4*c^3*f - (c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f)
*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4665 vs. $2(143) = 286$.

Time = 14.97 (sec) , antiderivative size = 4665, normalized size of antiderivative = 30.49

$$\int \frac{(a + a\sin(e + fx))^3 (A + B\sin(e + fx))}{(c - c\sin(e + fx))^3}\, dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**3,x)
```

output
```
Piecewise((-15*A*a**3*f*x*tan(e/2 + f*x/2)**7/(15*c**3*f*tan(e/2 + f*x/2)*
*7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*
c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*t
an(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 75*A*a**3*f
*x*tan(e/2 + f*x/2)**6/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2
+ f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)
**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75
*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 165*A*a**3*f*x*tan(e/2 + f*x/2)**5
/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3
*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e
/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/
2) - 15*c**3*f) + 225*A*a**3*f*x*tan(e/2 + f*x/2)**4/(15*c**3*f*tan(e/2 +
f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5
 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c
**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 225*
A*a**3*f*x*tan(e/2 + f*x/2)**3/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*
tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2
+ f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)
**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 165*A*a**3*f*x*tan(e/2 + f
*x/2)**2/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1685 vs. $2(150) = 300$.

Time = 0.16 (sec) , antiderivative size = 1685, normalized size of antiderivative = 11.01

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c - c\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algori
thm="maxima")
```

output
```
-2/15*(3*B*a^3*((105*sin(f*x + e)/(cos(f*x + e) + 1) - 189*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 160*sin(f
*x + e)^4/(cos(f*x + e) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 -
15*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 24)/(c^3 - 5*c^3*sin(f*x + e)/(co
s(f*x + e) + 1) + 11*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 15*c^3*sin(
f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4 - 11*c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5*c^3*sin(f*x + e)^6/(co
s(f*x + e) + 1)^6 - c^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(s
in(f*x + e)/(cos(f*x + e) + 1))/c^3) + A*a^3*((95*sin(f*x + e)/(cos(f*x +
e) + 1) - 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos
(f*x + e) + 1)^3 - 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 22)/(c^3 - 5*c
^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arcta
n(sin(f*x + e)/(cos(f*x + e) + 1))/c^3) + 3*B*a^3*((95*sin(f*x + e)/(cos(f
*x + e) + 1) - 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3
/(cos(f*x + e) + 1)^3 - 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 22)/(c^3
- 5*c^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*c^3*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 - 10*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*c^3*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 - c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + ...
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.41

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$$

$$= \frac{\frac{30\,Ba^3}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)c^3} - \frac{15\,(Aa^3+6\,Ba^3)(fx+e)}{c^3} - \frac{4\left(15\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+45\,Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-30\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-210\ldots}{\ldots}}{15\ldots}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algori
thm="giac")
```

output
```
1/15*(30*B*a^3/((tan(1/2*f*x + 1/2*e)^2 + 1)*c^3) - 15*(A*a^3 + 6*B*a^3)*(
f*x + e)/c^3 - 4*(15*A*a^3*tan(1/2*f*x + 1/2*e)^4 + 45*B*a^3*tan(1/2*f*x +
 1/2*e)^4 - 30*A*a^3*tan(1/2*f*x + 1/2*e)^3 - 210*B*a^3*tan(1/2*f*x + 1/2*
e)^3 + 100*A*a^3*tan(1/2*f*x + 1/2*e)^2 + 420*B*a^3*tan(1/2*f*x + 1/2*e)^2
 - 50*A*a^3*tan(1/2*f*x + 1/2*e) - 270*B*a^3*tan(1/2*f*x + 1/2*e) + 13*A*a
^3 + 63*B*a^3)/(c^3*(tan(1/2*f*x + 1/2*e) - 1)^5))/f
```

Mupad [B] (verification not implemented)

Time = 36.77 (sec) , antiderivative size = 336, normalized size of antiderivative = 2.20

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$$
$$=\frac{\frac{52\,A\,a^3}{15}-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{40\,A\,a^3}{3}+82\,B\,a^3\right)+\frac{94\,B\,a^3}{5}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\,\left(4\,A\,a^3+12\,B\,a^3\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,(8}{f\,\left(-c^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7+5\,c^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-11\,c^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+\right.}$$
$$-\frac{2\,a^3\,\mathrm{atan}\left(\frac{2\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(A+6\,B)}{2\,A\,a^3+12\,B\,a^3}\right)\,(A+6\,B)}{c^3\,f}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^3,x
)
```

output
```
((52*A*a^3)/15 - tan(e/2 + (f*x)/2)*((40*A*a^3)/3 + 82*B*a^3) + (94*B*a^3)
/5 + tan(e/2 + (f*x)/2)^6*(4*A*a^3 + 12*B*a^3) - tan(e/2 + (f*x)/2)^5*(8*A
*a^3 + 58*B*a^3) - tan(e/2 + (f*x)/2)^3*((64*A*a^3)/3 + 148*B*a^3) + tan(e
/2 + (f*x)/2)^4*((92*A*a^3)/3 + 134*B*a^3) + tan(e/2 + (f*x)/2)^2*((452*A*
a^3)/15 + (744*B*a^3)/5))/(f*(11*c^3*tan(e/2 + (f*x)/2)^2 - 15*c^3*tan(e/2
 + (f*x)/2)^3 + 15*c^3*tan(e/2 + (f*x)/2)^4 - 11*c^3*tan(e/2 + (f*x)/2)^5
+ 5*c^3*tan(e/2 + (f*x)/2)^6 - c^3*tan(e/2 + (f*x)/2)^7 + c^3 - 5*c^3*tan(
e/2 + (f*x)/2))) - (2*a^3*atan((2*a^3*tan(e/2 + (f*x)/2)*(A + 6*B))/(2*A*a
^3 + 12*B*a^3))*(A + 6*B))/(c^3*f)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 433, normalized size of antiderivative = 2.83

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$$
$$= \frac{a^3\big(12a+36b+30\cos(fx+e)\sin(fx+e)\,afx+180\cos(fx+e)\sin(fx+e)\,bfx-36\cos(fx+e)\,b\ \cdots}{}$$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x)
```

output
```
(a**3*(15*cos(e + f*x)*sin(e + f*x)**3*b - 15*cos(e + f*x)*sin(e + f*x)**2
*a*f*x - 32*cos(e + f*x)*sin(e + f*x)**2*a - 90*cos(e + f*x)*sin(e + f*x)*
*2*b*f*x - 126*cos(e + f*x)*sin(e + f*x)**2*b + 30*cos(e + f*x)*sin(e + f*
x)*a*f*x + 20*cos(e + f*x)*sin(e + f*x)*a + 180*cos(e + f*x)*sin(e + f*x)*
b*f*x + 123*cos(e + f*x)*sin(e + f*x)*b - 15*cos(e + f*x)*a*f*x - 12*cos(e
 + f*x)*a - 90*cos(e + f*x)*b*f*x - 36*cos(e + f*x)*b - 15*sin(e + f*x)**4
*b - 15*sin(e + f*x)**3*a*f*x + 60*sin(e + f*x)**3*a - 90*sin(e + f*x)**3*
b*f*x + 321*sin(e + f*x)**3*b + 45*sin(e + f*x)**2*a*f*x - 44*sin(e + f*x)
**2*a + 270*sin(e + f*x)**2*b*f*x - 417*sin(e + f*x)**2*b - 45*sin(e + f*x
)*a*f*x + 20*sin(e + f*x)*a - 270*sin(e + f*x)*b*f*x + 123*sin(e + f*x)*b
+ 15*a*f*x + 12*a + 90*b*f*x + 36*b))/(15*c**3*f*(cos(e + f*x)*sin(e + f*x
)**2 - 2*cos(e + f*x)*sin(e + f*x) + cos(e + f*x) + sin(e + f*x)**3 - 3*si
n(e + f*x)**2 + 3*sin(e + f*x) - 1))
```

3.47 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 151

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx &= \frac{a^3Bx}{c^4}+\frac{a^3(A+B)c^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7}-\frac{2a^3Bc\cos^5(e+fx)}{5f(c-c\sin(e+fx))^5}\\ &\quad+\frac{2a^3Bc^2\cos^3(e+fx)}{3f\left(c^2-c^2\sin(e+fx)\right)^3}-\frac{2a^3B\cos(e+fx)}{f\left(c^4-c^4\sin(e+fx)\right)}\end{aligned}$$

output
```
a^3*B*x/c^4+1/7*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^7-2/5*a^3*B*
c*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^5+2/3*a^3*B*c^2*cos(f*x+e)^3/f/(c^2-c^2*
sin(f*x+e))^3-2*a^3*B*cos(f*x+e)/f/(c^4-c^4*sin(f*x+e))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 356 vs. 2(151) = 302.

Time = 11.88 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.36

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$$
$$= \frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(120(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-12(15A+2\right.}{}$$

input
```
Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^4,x]
```

output
```
(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(120*(A + B)*(Cos[(e + f*x)/2]
- Sin[(e + f*x)/2]) - 12*(15*A + 29*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2
])^3 + 2*(45*A + 199*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 + 105*B*(e
 + f*x)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7 + 240*(A + B)*Sin[(e + f*x
)/2] - 24*(15*A + 29*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f
*x)/2] + 4*(45*A + 199*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(e +
 f*x)/2] - 2*(15*A + 337*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6*Sin[(e
 + f*x)/2])*(1 + Sin[e + f*x])^3)/(105*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)
/2])^6*(c - c*Sin[e + f*x])^4)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.306$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3159, 3042, 3159, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}dx$$

$$\Big\downarrow \text{3446}$$

$$a^3c^3\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}dx$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\int \frac{\cos(e+fx)^6(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}dx$$

$$\Big\downarrow \text{3338}$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7}-\frac{B\int\frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^6}dx}{c}\right)$$

$\downarrow$ 3042

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7}-\frac{B\int\frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^6}dx}{c}\right)$$

$\downarrow$ 3159

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7}-\frac{B\left(\frac{2\cos^5(e+fx)}{5cf(c-c\sin(e+fx))^5}-\frac{\int\frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^4}dx}{c^2}\right)}{c}\right)$$

$\downarrow$ 3042

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7}-\frac{B\left(\frac{2\cos^5(e+fx)}{5cf(c-c\sin(e+fx))^5}-\frac{\int\frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^4}dx}{c^2}\right)}{c}\right)$$

$\downarrow$ 3159

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7}-\frac{B\left(\frac{2\cos^5(e+fx)}{5cf(c-c\sin(e+fx))^5}-\frac{\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^3}-\frac{\int\frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^2}dx}{c^2}}{c^2}\right)}{c}\right)$$

$\downarrow$ 3042

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7}-\frac{B\left(\frac{2\cos^5(e+fx)}{5cf(c-c\sin(e+fx))^5}-\frac{\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^3}-\frac{\int\frac{\cos(e+fx)^2}{(c-c\sin(e+fx))^2}dx}{c^2}}{c^2}\right)}{c}\right)$$

$\downarrow$ 3159

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7}-\frac{B\left(\frac{2\cos^5(e+fx)}{5cf(c-c\sin(e+fx))^5}-\frac{\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^3}-\frac{\frac{2\cos(e+fx)}{f\left(c^2-c^2\sin(e+fx)\right)}-\frac{\int 1dx}{c^2}}{c^2}}{c^2}\right)}{c}\right)$$

$$\Big\downarrow \; 24$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{7f(c-c\sin(e+fx))^7}-\frac{B\left(\frac{2\cos^5(e+fx)}{5cf(c-c\sin(e+fx))^5}-\frac{\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^3}-\frac{\frac{2\cos(e+fx)}{f\left(c^2-c^2\sin(e+fx)\right)}-\frac{x}{c^2}}{c^2}}{c^2}\right)}{c}\right)$$

input
```
Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^4,x
]
```

output
```
a^3*c^3*(((A + B)*Cos[e + f*x]^7)/(7*f*(c - c*Sin[e + f*x])^7) - (B*((2*Co
s[e + f*x]^5)/(5*c*f*(c - c*Sin[e + f*x])^5) - ((2*Cos[e + f*x]^3)/(3*c*f*
(c - c*Sin[e + f*x])^3) - (-(x/c^2) + (2*Cos[e + f*x])/(f*(c^2 - c^2*Sin[e
 + f*x])))/c^2)/c^2))/c)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{2a^3\left(-\frac{A-B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{12A+4B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{60A+20B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{64A+64B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{160A+96B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{192A}{6\left(\tan\left(\frac{fx}{2}\right.\right.}\right.}{f\,c^4}$
default	$\frac{2a^3\left(-\frac{A-B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{12A+4B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{60A+20B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{64A+64B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{160A+96B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{192A}{6\left(\tan\left(\frac{fx}{2}\right.\right.}\right.}{f\,c^4}$
parallelrisch	$-\frac{2\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7 xfB}{2}+\left(\frac{7}{2}fxB+A-B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+B\left(-\frac{21fx}{2}+8\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5+\left(\frac{35}{2}fxB+5A-\frac{55}{3}B\right)\tan\left(\frac{f}{2}\right.\right.}{f\,c^4}$
risch	$\frac{a^3Bx}{c^4}-\frac{2\left(-15a^3A-337a^3B-1624iB\,a^3e^{i(fx+e)}-2520iB\,a^3e^{5i(fx+e)}+6160iB\,a^3e^{3i(fx+e)}+105A\,a^3e^{6i(fx+e)}+73\right.}{105\left(e^{i(fx+e)}-i\right.}$
norman	$-\frac{a^3xB}{c}-\frac{\left(150a^3A-1334a^3B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{21fc}-\frac{\left(750a^3A-5438a^3B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{21fc}-\frac{\left(1662a^3A-9790a^3B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{21fc}-\frac{(19}{}$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x,method=_RETUR
NVERBOSE)
```

output
```
2/f*a^3/c^4*(-(A-B)/(tan(1/2*f*x+1/2*e)-1)-1/2*(12*A+4*B)/(tan(1/2*f*x+1/2
*e)-1)^2-1/3*(60*A+20*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/7*(64*A+64*B)/(tan(1/2
*f*x+1/2*e)-1)^7-1/4*(160*A+96*B)/(tan(1/2*f*x+1/2*e)-1)^4-1/6*(192*A+192*
B)/(tan(1/2*f*x+1/2*e)-1)^6-1/5*(240*A+208*B)/(tan(1/2*f*x+1/2*e)-1)^5+B*a
rctan(tan(1/2*f*x+1/2*e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(149) = 298$.

Time = 0.09 (sec) , antiderivative size = 363, normalized size of antiderivative = 2.40

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$$
$$= \frac{840\,Ba^3fx+(105\,Ba^3fx+(15\,A+337\,B)a^3)\cos(fx+e)^4+120\,(A+B)a^3-(315\,Ba^3fx+(45\,A-}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algori
thm="fricas")
```

output
```
1/105*(840*B*a^3*f*x + (105*B*a^3*f*x + (15*A + 337*B)*a^3)*cos(f*x + e)^4
 + 120*(A + B)*a^3 - (315*B*a^3*f*x + (45*A - 613*B)*a^3)*cos(f*x + e)^3 -
 24*(35*B*a^3*f*x + (5*A + 26*B)*a^3)*cos(f*x + e)^2 + 60*(7*B*a^3*f*x + (
A - 13*B)*a^3)*cos(f*x + e) - (840*B*a^3*f*x - 120*(A + B)*a^3 - (105*B*a^
3*f*x - (15*A + 337*B)*a^3)*cos(f*x + e)^3 - 12*(35*B*a^3*f*x - (5*A - 23*
B)*a^3)*cos(f*x + e)^2 + 60*(7*B*a^3*f*x - (A + 15*B)*a^3)*cos(f*x + e))*s
in(f*x + e))/(c^4*f*cos(f*x + e)^4 - 3*c^4*f*cos(f*x + e)^3 - 8*c^4*f*cos(
f*x + e)^2 + 4*c^4*f*cos(f*x + e) + 8*c^4*f + (c^4*f*cos(f*x + e)^3 + 4*c^
4*f*cos(f*x + e)^2 - 4*c^4*f*cos(f*x + e) - 8*c^4*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2951 vs. $2(141) = 282$.

Time = 25.53 (sec) , antiderivative size = 2951, normalized size of antiderivative = 19.54

$$\int \frac{(a + a\sin(e + fx))^3 (A + B\sin(e + fx))}{(c - c\sin(e + fx))^4} \, dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**4,x)
```

output
```
Piecewise((-210*A*a**3*tan(e/2 + f*x/2)**6/(105*c**4*f*tan(e/2 + f*x/2)**7
 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675
*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*
f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 1050*A
*a**3*tan(e/2 + f*x/2)**4/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan
(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 +
 f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2
)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 630*A*a**3*tan(e/2 + f*
x/2)**2/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 +
 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*
c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*
tan(e/2 + f*x/2) - 105*c**4*f) - 30*A*a**3/(105*c**4*f*tan(e/2 + f*x/2)**7
 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675
*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*
f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) + 105*B*
a**3*f*x*tan(e/2 + f*x/2)**7/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*
tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/
2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*
x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 735*B*a**3*f*x*tan(e
/2 + f*x/2)**6/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2118 vs. $2(149) = 298$.

Time = 0.18 (sec) , antiderivative size = 2118, normalized size of antiderivative = 14.03

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c - c\sin(e + fx))^4} \, dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algori
thm="maxima")
```

output
```
2/105*(5*B*a^3*((203*sin(f*x + e)/(cos(f*x + e) + 1) - 525*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 686*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 434*sin(f
*x + e)^4/(cos(f*x + e) + 1)^4 + 147*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 -
 21*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 32)/(c^4 - 7*c^4*sin(f*x + e)/(c
os(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin
(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1
)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(c
os(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(
sin(f*x + e)/(cos(f*x + e) + 1))/c^4) + 3*A*a^3*(91*sin(f*x + e)/(cos(f*x
+ e) + 1) - 168*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 280*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3 - 175*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5 - 13)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e
) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c
^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e
) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + B*a^3*(91*sin(f*x +
e)/(cos(f*x + e) + 1) - 168*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 280*sin(
f*x + e)^3/(cos(f*x + e) + 1)^3 - 175*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
+ 105*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 13)/(c^4 - 7*c^4*sin(f*x + e)/
(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^...
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.34

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$$

$$= \frac{\frac{105\,(fx+e)Ba^3}{c^4} - \frac{2\left(105\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-105\,Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6+840\,Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+525\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-1925\,Ba^3\,\mathrm{ta}}{c}}{10}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algori
thm="giac")
```

output
```
1/105*(105*(f*x + e)*B*a^3/c^4 - 2*(105*A*a^3*tan(1/2*f*x + 1/2*e)^6 - 105
*B*a^3*tan(1/2*f*x + 1/2*e)^6 + 840*B*a^3*tan(1/2*f*x + 1/2*e)^5 + 525*A*a
^3*tan(1/2*f*x + 1/2*e)^4 - 1925*B*a^3*tan(1/2*f*x + 1/2*e)^4 + 3920*B*a^3
*tan(1/2*f*x + 1/2*e)^3 + 315*A*a^3*tan(1/2*f*x + 1/2*e)^2 - 2667*B*a^3*ta
n(1/2*f*x + 1/2*e)^2 + 1064*B*a^3*tan(1/2*f*x + 1/2*e) + 15*A*a^3 - 167*B*
a^3)/(c^4*(tan(1/2*f*x + 1/2*e) - 1)^7))/f
```

Mupad [B] (verification not implemented)

Time = 38.37 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.09

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c - c\sin(e + fx))^4}\,dx = \frac{B\,a^3\,x}{c^4}$$
$$-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\left(\frac{a^3\,(1680\,B-2205\,B\,(e+f\,x))}{105}+21\,B\,a^3\,(e+f\,x)\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\left(\frac{a^3\,(7840\,B-3675\,B\,(e+f\,x))}{105}+3\right.}{}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^4,x
)
```

output
```
(B*a^3*x)/c^4 - (tan(e/2 + (f*x)/2)^5*((a^3*(1680*B - 2205*B*(e + f*x)))/1
05 + 21*B*a^3*(e + f*x)) + tan(e/2 + (f*x)/2)^3*((a^3*(7840*B - 3675*B*(e
+ f*x)))/105 + 35*B*a^3*(e + f*x)) + (a^3*(30*A - 334*B + 105*B*(e + f*x))
)/105 + tan(e/2 + (f*x)/2)^6*((a^3*(210*A - 210*B + 735*B*(e + f*x)))/105
- 7*B*a^3*(e + f*x)) + tan(e/2 + (f*x)/2)^2*((a^3*(630*A - 5334*B + 2205*B
*(e + f*x)))/105 - 21*B*a^3*(e + f*x)) + tan(e/2 + (f*x)/2)^4*((a^3*(1050*
A - 3850*B + 3675*B*(e + f*x)))/105 - 35*B*a^3*(e + f*x)) + tan(e/2 + (f*x
)/2)*((a^3*(2128*B - 735*B*(e + f*x)))/105 + 7*B*a^3*(e + f*x)) - B*a^3*(e
 + f*x))/(c^4*f*(tan(e/2 + (f*x)/2) - 1)^7)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.36

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^4}\,dx$$
$$= \frac{a^3\left(-30\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7 a+105\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7 bfx+30\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7 b-735\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6 bfx-630\tan\right.}{}$$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x)
```

output
```
(a**3*( - 30*tan((e + f*x)/2)**7*a + 105*tan((e + f*x)/2)**7*b*f*x + 30*ta
n((e + f*x)/2)**7*b - 735*tan((e + f*x)/2)**6*b*f*x - 630*tan((e + f*x)/2)
**5*a + 2205*tan((e + f*x)/2)**5*b*f*x - 1050*tan((e + f*x)/2)**5*b - 3675
*tan((e + f*x)/2)**4*b*f*x + 2800*tan((e + f*x)/2)**4*b - 1050*tan((e + f*
x)/2)**3*a + 3675*tan((e + f*x)/2)**3*b*f*x - 6790*tan((e + f*x)/2)**3*b -
 2205*tan((e + f*x)/2)**2*b*f*x + 4704*tan((e + f*x)/2)**2*b - 210*tan((e
+ f*x)/2)*a + 735*tan((e + f*x)/2)*b*f*x - 1918*tan((e + f*x)/2)*b - 105*b
*f*x + 304*b))/(105*c**4*f*(tan((e + f*x)/2)**7 - 7*tan((e + f*x)/2)**6 +
21*tan((e + f*x)/2)**5 - 35*tan((e + f*x)/2)**4 + 35*tan((e + f*x)/2)**3 -
 21*tan((e + f*x)/2)**2 + 7*tan((e + f*x)/2) - 1))
```

3.48 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 77

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx = \frac{a^3(A+B)c^3\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}+\frac{a^3(A-8B)c^2\cos^7(e+fx)}{63f(c-c\sin(e+fx))^7}$$

output
```
1/9*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^8+1/63*a^3*(A-8*B)*c^2*c
os(f*x+e)^7/f/(c-c*sin(f*x+e))^7
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 283 vs. 2(77) = 154.

Time = 13.22 (sec) , antiderivative size = 283, normalized size of antiderivative = 3.68

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx = -\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3\left(315(A-B)\cos\left(\frac{1}{2}(e+fx)\right)-189(A-B\right.}{}$$

input
```
Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^5,x]
```

output
```
-1/504*(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(31
5*(A - B)*Cos[(e + f*x)/2] - 189*(A - B)*Cos[(3*(e + f*x))/2] - 63*A*Cos[(
5*(e + f*x))/2] + 63*B*Cos[(5*(e + f*x))/2] + 9*A*Cos[(7*(e + f*x))/2] - 9
*B*Cos[(7*(e + f*x))/2] + 189*A*Sin[(e + f*x)/2] + 693*B*Sin[(e + f*x)/2]
+ 105*A*Sin[(3*(e + f*x))/2] + 483*B*Sin[(3*(e + f*x))/2] - 27*A*Sin[(5*(e
 + f*x))/2] - 225*B*Sin[(5*(e + f*x))/2] - 63*B*Sin[(7*(e + f*x))/2] - A*S
in[(9*(e + f*x))/2] + 8*B*Sin[(9*(e + f*x))/2]))/(c^5*f*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])^6*(-1 + Sin[e + f*x])^5)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3446, 3042, 3338, 3042, 3150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}dx$$

$$\Big\downarrow \text{3446}$$

$$a^3c^3\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}dx$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\int \frac{\cos(e+fx)^6(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}dx$$

$$\Big\downarrow \text{3338}$$

$$a^3c^3\left(\frac{(A-8B)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^7}dx}{9c}+\frac{(A+B)\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$a^3c^3\left(\frac{(A-8B)\int \frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^7}dx}{9c}+\frac{(A+B)\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}\right)$$

$$\Big\downarrow \text{ 3150}$$

$$a^3c^3\left(\frac{(A-8B)\cos^7(e+fx)}{63cf(c-c\sin(e+fx))^7}+\frac{(A+B)\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}\right)$$

input
```
Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^5,x
]
```

output
```
a^3*c^3*(((A + B)*Cos[e + f*x]^7)/(9*f*(c - c*Sin[e + f*x])^8) + ((A - 8*B
)*Cos[e + f*x]^7)/(63*c*f*(c - c*Sin[e + f*x])^7))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3150
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(73) = 146$.

Time = 1.99 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.25

method	result
parallelrisch	$-\frac{2a^3\left(A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8+(-A+B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7+\frac{(23A+5B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{3}+5(-A+B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5+(11A+3B)\tan}{f\,c^5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right.}$
derivativedivides	$\frac{2a^3\left(-\frac{128A+128B}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^9}-\frac{928A+864B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{992A+800B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{512A+512B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{680A}{5\left(\tan\left(\frac{f}{}\right.\right.}\right.}{f\,c^5}$
default	$\frac{2a^3\left(-\frac{128A+128B}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^9}-\frac{928A+864B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{992A+800B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6}-\frac{512A+512B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{680A}{5\left(\tan\left(\frac{f}{}\right.\right.}\right.}{f\,c^5}$
risch	$-\frac{2a^3A}{63}+\frac{16a^3B}{63}-10iA\,a^3\mathrm{e}^{5i(fx+e)}+2iA\,a^3\mathrm{e}^{7i(fx+e)}-\frac{2iA\,a^3\mathrm{e}^{i(fx+e)}}{7}-\frac{6A\,a^3\mathrm{e}^{2i(fx+e)}}{7}-\frac{50B\,a^3\mathrm{e}^{2i(fx+e)}}{7}-\frac{10A\,a^3\mathrm{e}^{6i(f}}{3}$
norman	$-\frac{16a^3A-2a^3B}{63fc}-\frac{2a^3A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{16}}{fc}+\frac{2\left(a^3A-a^3B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{15}}{fc}+\frac{\left(2a^3A-2a^3B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{7fc}+\frac{6\left(3a^3A-3a^3B\right)\tan\left(\frac{f}{}\right.}{fc}$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x,method=_RETUR
NVERBOSE)
```

output
```
-2*a^3*(A*tan(1/2*f*x+1/2*e)^8+(-A+B)*tan(1/2*f*x+1/2*e)^7+1/3*(23*A+5*B)*
tan(1/2*f*x+1/2*e)^6+5*(-A+B)*tan(1/2*f*x+1/2*e)^5+(11*A+3*B)*tan(1/2*f*x+
1/2*e)^4+3*(-A+B)*tan(1/2*f*x+1/2*e)^3+1/7*(25*A+3*B)*tan(1/2*f*x+1/2*e)^2
+1/7*(-A+B)*tan(1/2*f*x+1/2*e)+8/63*A-1/63*B)/f/c^5/(tan(1/2*f*x+1/2*e)-1)
^9
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 331 vs. $2(75) = 150$.

Time = 0.09 (sec) , antiderivative size = 331, normalized size of antiderivative = 4.30

$$\int \frac{(a + a\sin(e + fx))^3 (A + B\sin(e + fx))}{(c - c\sin(e + fx))^5} \, dx = -\frac{(A - 8\,B)a^3 \cos{(fx + e)}^5 - (4\,A + 31\,B)a^3 \cos{(fx + e)}^4 + (19\,A + 37\,B)a^3 \cos{(fx + e)}^3 + 4\,(13\,A - \cdots}{63\,\left(c^5 f \cos{(fx + e)}^5 + 5\,c^5 f \cos{(fx + e)}^4 - 8\,c^5 f \cos \cdots\right.}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algori
thm="fricas")
```

output
```
-1/63*((A - 8*B)*a^3*cos(f*x + e)^5 - (4*A + 31*B)*a^3*cos(f*x + e)^4 + (1
9*A + 37*B)*a^3*cos(f*x + e)^3 + 4*(13*A + 22*B)*a^3*cos(f*x + e)^2 - 28*(
A + B)*a^3*cos(f*x + e) - 56*(A + B)*a^3 + ((A - 8*B)*a^3*cos(f*x + e)^4 +
 (5*A + 23*B)*a^3*cos(f*x + e)^3 + 12*(2*A + 5*B)*a^3*cos(f*x + e)^2 - 28*
(A + B)*a^3*cos(f*x + e) - 56*(A + B)*a^3)*sin(f*x + e))/(c^5*f*cos(f*x +
e)^5 + 5*c^5*f*cos(f*x + e)^4 - 8*c^5*f*cos(f*x + e)^3 - 20*c^5*f*cos(f*x
+ e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f - (c^5*f*cos(f*x + e)^4 - 4*c^5*f
*cos(f*x + e)^3 - 12*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*
f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3262 vs. $2(68) = 136$.

Time = 44.16 (sec) , antiderivative size = 3262, normalized size of antiderivative = 42.36

$$\int \frac{(a + a\sin(e + fx))^3 (A + B\sin(e + fx))}{(c - c\sin(e + fx))^5} \, dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**5,x)
```

output
```
Piecewise((-126*A*a**3*tan(e/2 + f*x/2)**8/(63*c**5*f*tan(e/2 + f*x/2)**9
- 567*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*
c**5*f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f
*tan(e/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e
/2 + f*x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c**5*f) + 126*A*a**3*tan
(e/2 + f*x/2)**7/(63*c**5*f*tan(e/2 + f*x/2)**9 - 567*c**5*f*tan(e/2 + f*x
/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f*x/2)**6
 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**4 + 529
2*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 + f*x/2)**2 + 567*c**5*
f*tan(e/2 + f*x/2) - 63*c**5*f) - 966*A*a**3*tan(e/2 + f*x/2)**6/(63*c**5*
f*tan(e/2 + f*x/2)**9 - 567*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan(e
/2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 + f
*x/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)*
*3 - 2268*c**5*f*tan(e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c*
*5*f) + 630*A*a**3*tan(e/2 + f*x/2)**5/(63*c**5*f*tan(e/2 + f*x/2)**9 - 56
7*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c**5
*f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*tan
(e/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 +
 f*x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c**5*f) - 1386*A*a**3*tan(e/
2 + f*x/2)**4/(63*c**5*f*tan(e/2 + f*x/2)**9 - 567*c**5*f*tan(e/2 + f*x...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2701 vs. $2(75) = 150$.

Time = 0.13 (sec) , antiderivative size = 2701, normalized size of antiderivative = 35.08

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c - c\sin(e + fx))^5} \, dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algori
thm="maxima")
```

output
```
-2/315*(A*a^3*(432*sin(f*x + e)/(cos(f*x + e) + 1) - 1728*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 3612*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5418*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 + 5040*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
 - 3360*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1260*sin(f*x + e)^7/(cos(f*x
 + e) + 1)^7 - 315*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 83)/(c^5 - 9*c^5*
sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5
*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e)
 + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(
cos(f*x + e) + 1)^9) - 15*A*a^3*(45*sin(f*x + e)/(cos(f*x + e) + 1) - 117*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 273*sin(f*x + e)^3/(cos(f*x + e) + 1
)^3 - 315*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 315*sin(f*x + e)^5/(cos(f*
x + e) + 1)^5 - 147*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63*sin(f*x + e)^
7/(cos(f*x + e) + 1)^7 - 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) +
 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin
(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1
)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(c
os(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) - 5*B*a^3...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(75) = 150$.

Time = 0.30 (sec) , antiderivative size = 285, normalized size of antiderivative = 3.70

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c - c\sin(e + fx))^5}\,dx =$$

$$-\frac{2\left(63\,Aa^3\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^8 - 63\,Aa^3\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^7 + 63\,Ba^3\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^7 + 483\,Aa^3\tan\left(\frac{1}{2}\,f\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algori
thm="giac")
```

output
```
-2/63*(63*A*a^3*tan(1/2*f*x + 1/2*e)^8 - 63*A*a^3*tan(1/2*f*x + 1/2*e)^7 +
 63*B*a^3*tan(1/2*f*x + 1/2*e)^7 + 483*A*a^3*tan(1/2*f*x + 1/2*e)^6 + 105*
B*a^3*tan(1/2*f*x + 1/2*e)^6 - 315*A*a^3*tan(1/2*f*x + 1/2*e)^5 + 315*B*a^
3*tan(1/2*f*x + 1/2*e)^5 + 693*A*a^3*tan(1/2*f*x + 1/2*e)^4 + 189*B*a^3*ta
n(1/2*f*x + 1/2*e)^4 - 189*A*a^3*tan(1/2*f*x + 1/2*e)^3 + 189*B*a^3*tan(1/
2*f*x + 1/2*e)^3 + 225*A*a^3*tan(1/2*f*x + 1/2*e)^2 + 27*B*a^3*tan(1/2*f*x
 + 1/2*e)^2 - 9*A*a^3*tan(1/2*f*x + 1/2*e) + 9*B*a^3*tan(1/2*f*x + 1/2*e)
+ 8*A*a^3 - B*a^3)/(c^5*f*(tan(1/2*f*x + 1/2*e) - 1)^9)
```

Mupad [B] (verification not implemented)

Time = 36.43 (sec) , antiderivative size = 346, normalized size of antiderivative = 4.49

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx = \frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{1013\,A\,a^3}{16}+\frac{149\,B\,a^3}{16}-\frac{113\,A\,a^3\cos(2\,e+2\,f\,x)}{4}+\frac{37\,A\,a^3\cos(3\,e+3\,f\,x)}{8}+\frac{7\,A\,a^3\cos(4\,e+4\,f\,x)}{16}-\frac{41\,B\,a^3\cos\cdots}{\cdots}\right.}{\cdots}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^5,x
)
```

output
```
(2*cos(e/2 + (f*x)/2)*((1013*A*a^3)/16 + (149*B*a^3)/16 - (113*A*a^3*cos(2
*e + 2*f*x))/4 + (37*A*a^3*cos(3*e + 3*f*x))/8 + (7*A*a^3*cos(4*e + 4*f*x)
)/16 - (41*B*a^3*cos(2*e + 2*f*x))/4 + (19*B*a^3*cos(3*e + 3*f*x))/8 + (7*
B*a^3*cos(4*e + 4*f*x))/16 + (63*A*a^3*sin(2*e + 2*f*x))/8 + (9*A*a^3*sin(
3*e + 3*f*x))/2 - (9*A*a^3*sin(4*e + 4*f*x))/16 - (63*B*a^3*sin(2*e + 2*f*
x))/8 - (9*B*a^3*sin(3*e + 3*f*x))/2 + (9*B*a^3*sin(4*e + 4*f*x))/16 - (25
7*A*a^3*cos(e + f*x))/8 - (23*B*a^3*cos(e + f*x))/8 - (63*A*a^3*sin(e + f*
x))/2 + (63*B*a^3*sin(e + f*x))/2))/(63*c^5*f*((63*2^(1/2)*cos(e/2 + pi/4
+ (f*x)/2))/8 - (21*2^(1/2)*cos((3*e)/2 - pi/4 + (3*f*x)/2))/4 - (9*2^(1/2
)*cos((5*e)/2 + pi/4 + (5*f*x)/2))/4 + (9*2^(1/2)*cos((7*e)/2 - pi/4 + (7*
f*x)/2))/16 + (2^(1/2)*cos((9*e)/2 + pi/4 + (9*f*x)/2))/16))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 339, normalized size of antiderivative = 4.40

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^5}\,dx$$
$$= \frac{2a^3\left(-7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9 a-189\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7 a-63\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7 b+105\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6 a-105\tan\left(\frac{fx}{2}\right.\right.}{63c^5 f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9-9\,\mathrm{ta}\right.}$$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x)
```

output
```
(2*a**3*( - 7*tan((e + f*x)/2)**9*a - 189*tan((e + f*x)/2)**7*a - 63*tan((
e + f*x)/2)**7*b + 105*tan((e + f*x)/2)**6*a - 105*tan((e + f*x)/2)**6*b -
 567*tan((e + f*x)/2)**5*a - 315*tan((e + f*x)/2)**5*b + 189*tan((e + f*x)
/2)**4*a - 189*tan((e + f*x)/2)**4*b - 399*tan((e + f*x)/2)**3*a - 189*tan
((e + f*x)/2)**3*b + 27*tan((e + f*x)/2)**2*a - 27*tan((e + f*x)/2)**2*b -
 54*tan((e + f*x)/2)*a - 9*tan((e + f*x)/2)*b - a + b))/(63*c**5*f*(tan((e
 + f*x)/2)**9 - 9*tan((e + f*x)/2)**8 + 36*tan((e + f*x)/2)**7 - 84*tan((e
 + f*x)/2)**6 + 126*tan((e + f*x)/2)**5 - 126*tan((e + f*x)/2)**4 + 84*tan
((e + f*x)/2)**3 - 36*tan((e + f*x)/2)**2 + 9*tan((e + f*x)/2) - 1))
```

3.49 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 118

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx &= \frac{a^3(A+B)c^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}+\frac{a^3(2A-9B)c^2\cos^7(e+fx)}{99f(c-c\sin(e+fx))^8}\\&\quad+\frac{a^3(2A-9B)c\cos^7(e+fx)}{693f(c-c\sin(e+fx))^7}\end{aligned}$$

output
```
1/11*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^9+1/99*a^3*(2*A-9*B)*c^
2*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^8+1/693*a^3*(2*A-9*B)*c*cos(f*x+e)^7/f/(
c-c*sin(f*x+e))^7
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 313 vs. 2(118) = 236.

Time = 14.31 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.65

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx$$
$$= \frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3\left(462(11A+3B)\cos\left(\frac{1}{2}(e+fx)\right)-594(5A+\right.}{}$$

input
```
Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^6,x]
```

output
```
(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(462*(11*A
 + 3*B)*Cos[(e + f*x)/2] - 594*(5*A + 2*B)*Cos[(3*(e + f*x))/2] - 924*A*Co
s[(5*(e + f*x))/2] - 693*B*Cos[(5*(e + f*x))/2] + 110*A*Cos[(7*(e + f*x))/
2] + 198*B*Cos[(7*(e + f*x))/2] - 2*A*Cos[(11*(e + f*x))/2] + 9*B*Cos[(11*
(e + f*x))/2] + 4158*A*Sin[(e + f*x)/2] + 5544*B*Sin[(e + f*x)/2] + 2310*A
*Sin[(3*(e + f*x))/2] + 4158*B*Sin[(3*(e + f*x))/2] - 594*A*Sin[(5*(e + f*
x))/2] - 2178*B*Sin[(5*(e + f*x))/2] - 693*B*Sin[(7*(e + f*x))/2] - 22*A*S
in[(9*(e + f*x))/2] + 99*B*Sin[(9*(e + f*x))/2]))/(11088*c^6*f*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2])^6*(-1 + Sin[e + f*x])^6)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 3446, 3042, 3338, 3042, 3151, 3042, 3150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}dx$$

$$\Big\downarrow \text{ 3446}$$

$$a^3c^3\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^9}dx$$

$$\Big\downarrow \text{ 3042}$$

$$a^3c^3\int \frac{\cos(e+fx)^6(A+B\sin(e+fx))}{(c-c\sin(e+fx))^9}dx$$

$$\Big\downarrow \text{ 3338}$$

$$a^3c^3\left(\frac{(2A-9B)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^8}dx}{11c}+\frac{(A+B)\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left(\frac{(2A-9B)\int \frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^8}dx}{11c}+\frac{(A+B)\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}\right)$$

$$\Big\downarrow \text{3151}$$

$$a^3c^3\left(\frac{(2A-9B)\left(\frac{\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^7}dx}{9c}+\frac{\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}\right)}{11c}+\frac{(A+B)\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left(\frac{(2A-9B)\left(\frac{\int \frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^7}dx}{9c}+\frac{\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}\right)}{11c}+\frac{(A+B)\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}\right)$$

$$\Big\downarrow \text{3150}$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}+\frac{(2A-9B)\left(\frac{\cos^7(e+fx)}{63cf(c-c\sin(e+fx))^7}+\frac{\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}\right)}{11c}\right)$$

input
```
Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^6,x
]
```

output
```
a^3*c^3*(((A + B)*Cos[e + f*x]^7)/(11*f*(c - c*Sin[e + f*x])^9) + ((2*A -
9*B)*(Cos[e + f*x]^7/(9*f*(c - c*Sin[e + f*x])^8) + Cos[e + f*x]^7/(63*c*f
*(c - c*Sin[e + f*x])^7)))/(11*c))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3150
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

rule 3151
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Simp[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1])   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simpli
fy[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.79

method	result
parallelrisch	$-\frac{2a^3\left(A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}+(-2A+B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}+\left(\frac{35A}{3}+B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}+2\left(-\frac{23A}{3}+3B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}+2\left(\frac{46A}{3}\right.\right.}{}$
risch	$-\frac{2ia^3\left(693iB\,e^{8i(fx+e)}-110iA\,e^{2i(fx+e)}+693B\,e^{9i(fx+e)}-1386iB\,e^{6i(fx+e)}-2310A\,e^{7i(fx+e)}-198iB\,e^{2i(fx+e)}-4\right.}{}$
derivativedivides	$\frac{2a^3\left(-\frac{256A+256B}{11\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{11}}-\frac{1460A+780B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{116A+30B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}-\frac{16A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{1280A+1280B}{10\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{10}}-\right.}{}$
default	$\frac{2a^3\left(-\frac{256A+256B}{11\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{11}}-\frac{1460A+780B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{116A+30B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}-\frac{16A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{1280A+1280B}{10\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{10}}-\right.}{}$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x,method=_RETUR
NVERBOSE)
```

output
```
-2*a^3*(A*tan(1/2*f*x+1/2*e)^10+(-2*A+B)*tan(1/2*f*x+1/2*e)^9+(35/3*A+B)*t
an(1/2*f*x+1/2*e)^8+2*(-23/3*A+3*B)*tan(1/2*f*x+1/2*e)^7+2*(46/3*A+B)*tan(
1/2*f*x+1/2*e)^6+2*(-11*A+4*B)*tan(1/2*f*x+1/2*e)^5+12/7*(13*A+B)*tan(1/2*
f*x+1/2*e)^4+2/7*(-25*A+11*B)*tan(1/2*f*x+1/2*e)^3+1/7*(269/9*A+2*B)*tan(1
/2*f*x+1/2*e)^2+1/7*(-16/9*A+B)*tan(1/2*f*x+1/2*e)+79/693*A-1/77*B)/f/c^6/
(tan(1/2*f*x+1/2*e)-1)^11
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. 2(115) = 230.

Time = 0.09 (sec) , antiderivative size = 405, normalized size of antiderivative = 3.43

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx$$
$$=\frac{(2\,A-9\,B)a^3\cos\left(fx+e\right)^6+6\,(2\,A-9\,B)a^3\cos\left(fx+e\right)^5-(25\,A+234\,B)a^3\cos\left(fx+e\right)^4+7\,(23\,A}{693\,\left(c^6f\cos\left(fx+e\right)^6-5\,c^6f\cos\left(fx+\right.\right.}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x, algori
thm="fricas")
```

output
```
1/693*((2*A - 9*B)*a^3*cos(f*x + e)^6 + 6*(2*A - 9*B)*a^3*cos(f*x + e)^5 -
 (25*A + 234*B)*a^3*cos(f*x + e)^4 + 7*(23*A + 45*B)*a^3*cos(f*x + e)^3 +
28*(16*A + 27*B)*a^3*cos(f*x + e)^2 - 252*(A + B)*a^3*cos(f*x + e) - 504*(
A + B)*a^3 - ((2*A - 9*B)*a^3*cos(f*x + e)^5 - 5*(2*A - 9*B)*a^3*cos(f*x +
 e)^4 - 7*(5*A + 27*B)*a^3*cos(f*x + e)^3 - 28*(7*A + 18*B)*a^3*cos(f*x +
e)^2 + 252*(A + B)*a^3*cos(f*x + e) + 504*(A + B)*a^3)*sin(f*x + e))/(c^6*
f*cos(f*x + e)^6 - 5*c^6*f*cos(f*x + e)^5 - 18*c^6*f*cos(f*x + e)^4 + 20*c
^6*f*cos(f*x + e)^3 + 48*c^6*f*cos(f*x + e)^2 - 16*c^6*f*cos(f*x + e) - 32
*c^6*f + (c^6*f*cos(f*x + e)^5 + 6*c^6*f*cos(f*x + e)^4 - 12*c^6*f*cos(f*x
 + e)^3 - 32*c^6*f*cos(f*x + e)^2 + 16*c^6*f*cos(f*x + e) + 32*c^6*f)*sin(
f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4816 vs. $2(105) = 210$.

Time = 70.80 (sec) , antiderivative size = 4816, normalized size of antiderivative = 40.81

$$\int \frac{(a + a\sin(e + fx))^3 (A + B\sin(e + fx))}{(c - c\sin(e + fx))^6}\, dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**6,x)
```

output
```
Piecewise((-1386*A*a**3*tan(e/2 + f*x/2)**10/(693*c**6*f*tan(e/2 + f*x/2)*
*11 - 7623*c**6*f*tan(e/2 + f*x/2)**10 + 38115*c**6*f*tan(e/2 + f*x/2)**9
- 114345*c**6*f*tan(e/2 + f*x/2)**8 + 228690*c**6*f*tan(e/2 + f*x/2)**7 -
320166*c**6*f*tan(e/2 + f*x/2)**6 + 320166*c**6*f*tan(e/2 + f*x/2)**5 - 22
8690*c**6*f*tan(e/2 + f*x/2)**4 + 114345*c**6*f*tan(e/2 + f*x/2)**3 - 3811
5*c**6*f*tan(e/2 + f*x/2)**2 + 7623*c**6*f*tan(e/2 + f*x/2) - 693*c**6*f)
+ 2772*A*a**3*tan(e/2 + f*x/2)**9/(693*c**6*f*tan(e/2 + f*x/2)**11 - 7623*
c**6*f*tan(e/2 + f*x/2)**10 + 38115*c**6*f*tan(e/2 + f*x/2)**9 - 114345*c*
*6*f*tan(e/2 + f*x/2)**8 + 228690*c**6*f*tan(e/2 + f*x/2)**7 - 320166*c**6
*f*tan(e/2 + f*x/2)**6 + 320166*c**6*f*tan(e/2 + f*x/2)**5 - 228690*c**6*f
*tan(e/2 + f*x/2)**4 + 114345*c**6*f*tan(e/2 + f*x/2)**3 - 38115*c**6*f*ta
n(e/2 + f*x/2)**2 + 7623*c**6*f*tan(e/2 + f*x/2) - 693*c**6*f) - 16170*A*a
**3*tan(e/2 + f*x/2)**8/(693*c**6*f*tan(e/2 + f*x/2)**11 - 7623*c**6*f*tan
(e/2 + f*x/2)**10 + 38115*c**6*f*tan(e/2 + f*x/2)**9 - 114345*c**6*f*tan(e
/2 + f*x/2)**8 + 228690*c**6*f*tan(e/2 + f*x/2)**7 - 320166*c**6*f*tan(e/2
 + f*x/2)**6 + 320166*c**6*f*tan(e/2 + f*x/2)**5 - 228690*c**6*f*tan(e/2 +
 f*x/2)**4 + 114345*c**6*f*tan(e/2 + f*x/2)**3 - 38115*c**6*f*tan(e/2 + f*
x/2)**2 + 7623*c**6*f*tan(e/2 + f*x/2) - 693*c**6*f) + 21252*A*a**3*tan(e/
2 + f*x/2)**7/(693*c**6*f*tan(e/2 + f*x/2)**11 - 7623*c**6*f*tan(e/2 + f*x
/2)**10 + 38115*c**6*f*tan(e/2 + f*x/2)**9 - 114345*c**6*f*tan(e/2 + f*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3390 vs. $2(115) = 230$.

Time = 0.16 (sec) , antiderivative size = 3390, normalized size of antiderivative = 28.73

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c - c\sin(e + fx))^6} dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x, algori
thm="maxima")
```

output
```
-2/3465*(5*A*a^3*(913*sin(f*x + e)/(cos(f*x + e) + 1) - 4565*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 + 12540*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 25080
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 33726*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5 - 33726*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 23100*sin(f*x + e)^7/
(cos(f*x + e) + 1)^7 - 11550*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 3465*si
n(f*x + e)^9/(cos(f*x + e) + 1)^9 - 693*sin(f*x + e)^10/(cos(f*x + e) + 1)
^10 - 146)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
+ 330*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*sin(f*x + e)^5/(co
s(f*x + e) + 1)^5 + 462*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*
sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*sin(f*x + e)^8/(cos(f*x + e)
 + 1)^8 - 55*c^6*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*sin(f*x + e)
^10/(cos(f*x + e) + 1)^10 - c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 9
*A*a^3*(671*sin(f*x + e)/(cos(f*x + e) + 1) - 2200*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 6600*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 10890*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 + 15246*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 12
936*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 9240*sin(f*x + e)^7/(cos(f*x + e
) + 1)^7 - 3465*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 1155*sin(f*x + e)^9/
(cos(f*x + e) + 1)^9 - 61)/(c^6 - 11*c^6*sin(f*x + e)/(cos(f*x + e) + 1) +
 55*c^6*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 165*c^6*sin(f*x + e)^3/(c...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(115) = 230$.

Time = 0.24 (sec) , antiderivative size = 353, normalized size of antiderivative = 2.99

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx =$$
$$-\frac{2\left(693\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}-1386\,Aa^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{9}+693\,Ba^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{9}+8085\,Aa^3\,\mathrm{t}\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x, algori
thm="giac")
```

output
```
-2/693*(693*A*a^3*tan(1/2*f*x + 1/2*e)^10 - 1386*A*a^3*tan(1/2*f*x + 1/2*e
)^9 + 693*B*a^3*tan(1/2*f*x + 1/2*e)^9 + 8085*A*a^3*tan(1/2*f*x + 1/2*e)^8
 + 693*B*a^3*tan(1/2*f*x + 1/2*e)^8 - 10626*A*a^3*tan(1/2*f*x + 1/2*e)^7 +
 4158*B*a^3*tan(1/2*f*x + 1/2*e)^7 + 21252*A*a^3*tan(1/2*f*x + 1/2*e)^6 +
1386*B*a^3*tan(1/2*f*x + 1/2*e)^6 - 15246*A*a^3*tan(1/2*f*x + 1/2*e)^5 + 5
544*B*a^3*tan(1/2*f*x + 1/2*e)^5 + 15444*A*a^3*tan(1/2*f*x + 1/2*e)^4 + 11
88*B*a^3*tan(1/2*f*x + 1/2*e)^4 - 4950*A*a^3*tan(1/2*f*x + 1/2*e)^3 + 2178
*B*a^3*tan(1/2*f*x + 1/2*e)^3 + 2959*A*a^3*tan(1/2*f*x + 1/2*e)^2 + 198*B*
a^3*tan(1/2*f*x + 1/2*e)^2 - 176*A*a^3*tan(1/2*f*x + 1/2*e) + 99*B*a^3*tan
(1/2*f*x + 1/2*e) + 79*A*a^3 - 9*B*a^3)/(c^6*f*(tan(1/2*f*x + 1/2*e) - 1)^
11)
```

Mupad [B] (verification not implemented)

Time = 37.18 (sec) , antiderivative size = 408, normalized size of antiderivative = 3.46

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c - c\sin(e + fx))^6}\,dx =$$

$$-\frac{2\cos\left(\frac{e}{2} + \frac{fx}{2}\right)\left(565\,A\,a^3\cos\left(2\,e + 2\,f\,x\right) - \frac{837\,B\,a^3}{16} - 922\,A\,a^3 - \frac{3527\,A\,a^3\cos(3\,e+3\,f\,x)}{32} - 29\,A\,a^3\cos\left(4\right.\right.}{}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^6,x
)
```

output
```
-(2*cos(e/2 + (f*x)/2)*(565*A*a^3*cos(2*e + 2*f*x) - (837*B*a^3)/16 - 922*
A*a^3 - (3527*A*a^3*cos(3*e + 3*f*x))/32 - 29*A*a^3*cos(4*e + 4*f*x) + (81
*A*a^3*cos(5*e + 5*f*x))/32 + (225*B*a^3*cos(2*e + 2*f*x))/4 - (207*B*a^3*
cos(3*e + 3*f*x))/16 + (9*B*a^3*cos(4*e + 4*f*x))/16 - (9*B*a^3*cos(5*e +
5*f*x))/16 - (1617*A*a^3*sin(2*e + 2*f*x))/8 - (5049*A*a^3*sin(3*e + 3*f*x
))/32 + (407*A*a^3*sin(4*e + 4*f*x))/16 + (77*A*a^3*sin(5*e + 5*f*x))/32 +
 (693*B*a^3*sin(2*e + 2*f*x))/8 + (99*B*a^3*sin(3*e + 3*f*x))/2 - (99*B*a^
3*sin(4*e + 4*f*x))/16 + (6635*A*a^3*cos(e + f*x))/16 + 18*B*a^3*cos(e + f
*x) + (13629*A*a^3*sin(e + f*x))/16 - (693*B*a^3*sin(e + f*x))/2))/(693*c^
6*f*((231*2^(1/2)*cos(e/2 + pi/4 + (f*x)/2))/16 - (165*2^(1/2)*cos((3*e)/2
 - pi/4 + (3*f*x)/2))/16 - (165*2^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2))/3
2 + (55*2^(1/2)*cos((7*e)/2 - pi/4 + (7*f*x)/2))/32 + (11*2^(1/2)*cos((9*e
)/2 + pi/4 + (9*f*x)/2))/32 - (2^(1/2)*cos((11*e)/2 - pi/4 + (11*f*x)/2))/
32))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 452, normalized size of antiderivative = 3.83

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^6}\,dx$$
$$= \frac{a^3\left(-45\cos(fx+e)\sin(fx+e)^5 a-18\cos(fx+e)\sin(fx+e)^5 b+223\cos(fx+e)\sin(fx+e)^4 a+\right.}{}$$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x)
```

output
```
(a**3*( - 45*cos(e + f*x)*sin(e + f*x)**5*a - 18*cos(e + f*x)*sin(e + f*x)
**5*b + 223*cos(e + f*x)*sin(e + f*x)**4*a + 99*cos(e + f*x)*sin(e + f*x)*
*4*b - 439*cos(e + f*x)*sin(e + f*x)**3*a + 117*cos(e + f*x)*sin(e + f*x)*
*3*b + 655*cos(e + f*x)*sin(e + f*x)**2*a + 297*cos(e + f*x)*sin(e + f*x)*
*2*b - 16*cos(e + f*x)*sin(e + f*x)*a + 9*cos(e + f*x)*sin(e + f*x)*b + 12
6*cos(e + f*x)*a - 49*sin(e + f*x)**6*a + 292*sin(e + f*x)**5*a + 9*sin(e
+ f*x)**5*b - 724*sin(e + f*x)**4*a - 396*sin(e + f*x)**4*b + 724*sin(e +
f*x)**3*a - 234*sin(e + f*x)**3*b - 1109*sin(e + f*x)**2*a - 396*sin(e + f
*x)**2*b - 16*sin(e + f*x)*a + 9*sin(e + f*x)*b - 126*a))/(693*c**6*f*(cos
(e + f*x)*sin(e + f*x)**5 - 5*cos(e + f*x)*sin(e + f*x)**4 + 10*cos(e + f*
x)*sin(e + f*x)**3 - 10*cos(e + f*x)*sin(e + f*x)**2 + 5*cos(e + f*x)*sin(
e + f*x) - cos(e + f*x) + sin(e + f*x)**6 - 6*sin(e + f*x)**5 + 15*sin(e +
 f*x)**4 - 20*sin(e + f*x)**3 + 15*sin(e + f*x)**2 - 6*sin(e + f*x) + 1))
```

3.50 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 156

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx &= \frac{a^3(A+B)c^3\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}+\frac{a^3(3A-10B)c^2\cos^7(e+fx)}{143f(c-c\sin(e+fx))^9}\\&\quad+\frac{2a^3(3A-10B)c\cos^7(e+fx)}{1287f(c-c\sin(e+fx))^8}+\frac{2a^3(3A-10B)\cos^7(e+fx)}{9009f(c-c\sin(e+fx))^7}\end{aligned}$$

output
```
1/13*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^10+1/143*a^3*(3*A-10*B)
*c^2*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^9+2/1287*a^3*(3*A-10*B)*c*cos(f*x+e)^
7/f/(c-c*sin(f*x+e))^8+2/9009*a^3*(3*A-10*B)*cos(f*x+e)^7/f/(c-c*sin(f*x+e
))^7
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 339 vs. $2(156) = 312$.

Time = 16.78 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.17

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx = -\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3\left(6006(9A+5B)\cos\left(\frac{1}{2}(e+fx)\right)-7722(4A\right.}{}$$

input
```
Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^7,x]
```

output
```
-1/144144*(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*
(6006*(9*A + 5*B)*Cos[(e + f*x)/2] - 7722*(4*A + 3*B)*Cos[(3*(e + f*x))/2]
 - 9009*A*Cos[(5*(e + f*x))/2] - 12012*B*Cos[(5*(e + f*x))/2] + 858*A*Cos[
(7*(e + f*x))/2] + 3146*B*Cos[(7*(e + f*x))/2] - 39*A*Cos[(11*(e + f*x))/2
] + 130*B*Cos[(11*(e + f*x))/2] + 48906*A*Sin[(e + f*x)/2] + 47190*B*Sin[(
e + f*x)/2] + 27027*A*Sin[(3*(e + f*x))/2] + 36036*B*Sin[(3*(e + f*x))/2]
- 6864*A*Sin[(5*(e + f*x))/2] - 19162*B*Sin[(5*(e + f*x))/2] - 6006*B*Sin[
(7*(e + f*x))/2] - 234*A*Sin[(9*(e + f*x))/2] + 780*B*Sin[(9*(e + f*x))/2]
 + 3*A*Sin[(13*(e + f*x))/2] - 10*B*Sin[(13*(e + f*x))/2]))/(c^7*f*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^6*(-1 + Sin[e + f*x])^7)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3042, 3446, 3042, 3338, 3042, 3151, 3042, 3151, 3042, 3150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}dx$$

$$\Big\downarrow \text{3446}$$

$$a^3c^3\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{10}}dx$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\int \frac{\cos(e+fx)^6(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{10}}dx$$

$$\Big\downarrow \text{3338}$$

$$a^3c^3\left(\frac{(3A-10B)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^9}dx}{13c}+\frac{(A+B)\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left(\frac{(3A-10B)\int \frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^9}dx}{13c}+\frac{(A+B)\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}\right)$$

$$\Big\downarrow \text{3151}$$

$$a^3c^3\left(\frac{(3A-10B)\left(\frac{2\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^8}dx}{11c}+\frac{\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}\right)}{13c}+\frac{(A+B)\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left(\frac{(3A-10B)\left(\frac{2\int \frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^8}dx}{11c}+\frac{\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}\right)}{13c}+\frac{(A+B)\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}\right)$$

$$\Big\downarrow \text{3151}$$

$$a^3c^3\left(\frac{(3A-10B)\left(\frac{2\left(\frac{\int\frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^7}dx}{9c}+\frac{\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}\right)}{11c}+\frac{\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}\right)}{13c}+\frac{(A+B)\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}\right)$$

$\Big\downarrow$ 3042

$$a^3c^3\left(\frac{(3A-10B)\left(\frac{2\left(\frac{\int\frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^7}dx}{9c}+\frac{\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}\right)}{11c}+\frac{\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}\right)}{13c}+\frac{(A+B)\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}\right)$$

$\Big\downarrow$ 3150

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}+\frac{(3A-10B)\left(\frac{\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}+\frac{2\left(\frac{\cos^7(e+fx)}{63cf(c-c\sin(e+fx))^7}+\frac{\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}\right)}{11c}\right)}{13c}\right)$$

input
```
Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^7,x
]
```

output
```
a^3*c^3*(((A + B)*Cos[e + f*x]^7)/(13*f*(c - c*Sin[e + f*x])^10) + ((3*A -
 10*B)*(Cos[e + f*x]^7/(11*f*(c - c*Sin[e + f*x])^9) + (2*(Cos[e + f*x]^7/
(9*f*(c - c*Sin[e + f*x])^8) + Cos[e + f*x]^7/(63*c*f*(c - c*Sin[e + f*x])
^7)))/(11*c)))/(13*c))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3150
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

rule 3151
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Simp[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1])   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simpli
fy[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 2.43 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.60

method	result
parallelrisch	$-\frac{2\left(A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{12}+(-3A+B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}+\left(17A+\frac{B}{3}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}+\left(-33A+\frac{23B}{3}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}+(72A-}{}$
derivativedivides	$\frac{2a^3\left(-\frac{18816A+14464B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{3072A+3072B}{12\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{12}}-\frac{13112A+8840B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{18A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{15}{3(\tan(}\right.}{}$
default	$\frac{2a^3\left(-\frac{18816A+14464B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{3072A+3072B}{12\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{12}}-\frac{13112A+8840B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7}-\frac{18A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{15}{3(\tan(}\right.}{}$
risch	$-\frac{4\left(3a^3A-10a^3B+780Ba^3e^{2i(fx+e)}-234Aa^3e^{2i(fx+e)}+48906Aa^3e^{6i(fx+e)}+47190Ba^3e^{6i(fx+e)}-6864Aa^3e^{4i(f}\right.}{}$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x,method=_RETUR
NVERBOSE)
```

output
```
-2*(A*tan(1/2*f*x+1/2*e)^12+(-3*A+B)*tan(1/2*f*x+1/2*e)^11+(17*A+1/3*B)*ta
n(1/2*f*x+1/2*e)^10+(-33*A+23/3*B)*tan(1/2*f*x+1/2*e)^9+(72*A-B)*tan(1/2*f
*x+1/2*e)^8+(-82*A+50/3*B)*tan(1/2*f*x+1/2*e)^7+(666/7*A-38/21*B)*tan(1/2*
f*x+1/2*e)^6+(-426/7*A+90/7*B)*tan(1/2*f*x+1/2*e)^5+(-2/63*B+857/21*A)*tan
(1/2*f*x+1/2*e)^4+(215/63*B-263/21*A)*tan(1/2*f*x+1/2*e)^3+(389/77*A+37/23
1*B)*tan(1/2*f*x+1/2*e)^2+(-79/231*A+97/693*B)*tan(1/2*f*x+1/2*e)+310/3003
*A-97/9009*B)*a^3/f/c^7/(tan(1/2*f*x+1/2*e)-1)^13
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 475 vs. $2(152) = 304$.

Time = 0.10 (sec) , antiderivative size = 475, normalized size of antiderivative = 3.04

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx = -\frac{2\,(3\,A-10\,B)a^3\cos\left(fx+e\right)^7-12\,(3\,A-10\,B)a^3\cos\left(fx+e\right)^6-49\,(3\,A-10\,B)a^3\cos\left(fx+e\right)^5+}{9009\,\left(c^7f\cos\left(fx\right.\right.}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x, algori
thm="fricas")
```

output
```
-1/9009*(2*(3*A - 10*B)*a^3*cos(f*x + e)^7 - 12*(3*A - 10*B)*a^3*cos(f*x +
 e)^6 - 49*(3*A - 10*B)*a^3*cos(f*x + e)^5 + 7*(30*A + 329*B)*a^3*cos(f*x
+ e)^4 - 63*(27*A + 53*B)*a^3*cos(f*x + e)^3 - 252*(19*A + 32*B)*a^3*cos(f
*x + e)^2 + 2772*(A + B)*a^3*cos(f*x + e) + 5544*(A + B)*a^3 + (2*(3*A - 1
0*B)*a^3*cos(f*x + e)^6 + 14*(3*A - 10*B)*a^3*cos(f*x + e)^5 - 35*(3*A - 1
0*B)*a^3*cos(f*x + e)^4 - 63*(5*A + 31*B)*a^3*cos(f*x + e)^3 - 252*(8*A +
21*B)*a^3*cos(f*x + e)^2 + 2772*(A + B)*a^3*cos(f*x + e) + 5544*(A + B)*a^
3)*sin(f*x + e))/(c^7*f*cos(f*x + e)^7 + 7*c^7*f*cos(f*x + e)^6 - 18*c^7*f
*cos(f*x + e)^5 - 56*c^7*f*cos(f*x + e)^4 + 48*c^7*f*cos(f*x + e)^3 + 112*
c^7*f*cos(f*x + e)^2 - 32*c^7*f*cos(f*x + e) - 64*c^7*f - (c^7*f*cos(f*x +
 e)^6 - 6*c^7*f*cos(f*x + e)^5 - 24*c^7*f*cos(f*x + e)^4 + 32*c^7*f*cos(f*
x + e)^3 + 80*c^7*f*cos(f*x + e)^2 - 32*c^7*f*cos(f*x + e) - 64*c^7*f)*sin
(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6669 vs. $2(143) = 286$.

Time = 108.17 (sec) , antiderivative size = 6669, normalized size of antiderivative = 42.75

$$\int \frac{(a + a\sin(e + fx))^3 (A + B\sin(e + fx))}{(c - c\sin(e + fx))^7}\, dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**7,x)
```

output
```
Piecewise((-18018*A*a**3*tan(e/2 + f*x/2)**12/(9009*c**7*f*tan(e/2 + f*x/2
)**13 - 117117*c**7*f*tan(e/2 + f*x/2)**12 + 702702*c**7*f*tan(e/2 + f*x/2
)**11 - 2576574*c**7*f*tan(e/2 + f*x/2)**10 + 6441435*c**7*f*tan(e/2 + f*x
/2)**9 - 11594583*c**7*f*tan(e/2 + f*x/2)**8 + 15459444*c**7*f*tan(e/2 + f
*x/2)**7 - 15459444*c**7*f*tan(e/2 + f*x/2)**6 + 11594583*c**7*f*tan(e/2 +
 f*x/2)**5 - 6441435*c**7*f*tan(e/2 + f*x/2)**4 + 2576574*c**7*f*tan(e/2 +
 f*x/2)**3 - 702702*c**7*f*tan(e/2 + f*x/2)**2 + 117117*c**7*f*tan(e/2 + f
*x/2) - 9009*c**7*f) + 54054*A*a**3*tan(e/2 + f*x/2)**11/(9009*c**7*f*tan(
e/2 + f*x/2)**13 - 117117*c**7*f*tan(e/2 + f*x/2)**12 + 702702*c**7*f*tan(
e/2 + f*x/2)**11 - 2576574*c**7*f*tan(e/2 + f*x/2)**10 + 6441435*c**7*f*ta
n(e/2 + f*x/2)**9 - 11594583*c**7*f*tan(e/2 + f*x/2)**8 + 15459444*c**7*f*
tan(e/2 + f*x/2)**7 - 15459444*c**7*f*tan(e/2 + f*x/2)**6 + 11594583*c**7*
f*tan(e/2 + f*x/2)**5 - 6441435*c**7*f*tan(e/2 + f*x/2)**4 + 2576574*c**7*
f*tan(e/2 + f*x/2)**3 - 702702*c**7*f*tan(e/2 + f*x/2)**2 + 117117*c**7*f*
tan(e/2 + f*x/2) - 9009*c**7*f) - 306306*A*a**3*tan(e/2 + f*x/2)**10/(9009
*c**7*f*tan(e/2 + f*x/2)**13 - 117117*c**7*f*tan(e/2 + f*x/2)**12 + 702702
*c**7*f*tan(e/2 + f*x/2)**11 - 2576574*c**7*f*tan(e/2 + f*x/2)**10 + 64414
35*c**7*f*tan(e/2 + f*x/2)**9 - 11594583*c**7*f*tan(e/2 + f*x/2)**8 + 1545
9444*c**7*f*tan(e/2 + f*x/2)**7 - 15459444*c**7*f*tan(e/2 + f*x/2)**6 + 11
594583*c**7*f*tan(e/2 + f*x/2)**5 - 6441435*c**7*f*tan(e/2 + f*x/2)**4 ...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4078 vs. $2(152) = 304$.

Time = 0.19 (sec) , antiderivative size = 4078, normalized size of antiderivative = 26.14

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x, algori
thm="maxima")
```

output
```
-2/45045*(6*A*a^3*(4771*sin(f*x + e)/(cos(f*x + e) + 1) - 28626*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 74932*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 18
7330*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 265122*sin(f*x + e)^5/(cos(f*x
+ e) + 1)^5 - 353496*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 276276*sin(f*x
+ e)^7/(cos(f*x + e) + 1)^7 - 207207*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 +
 75075*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 30030*sin(f*x + e)^10/(cos(f*
x + e) + 1)^10 - 367)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c
^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1)
^6 - 1716*c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^
8/(cos(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286
*c^7*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f
*x + e) + 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f
*x + e)^13/(cos(f*x + e) + 1)^13) + 6*B*a^3*(4771*sin(f*x + e)/(cos(f*x +
e) + 1) - 28626*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 74932*sin(f*x + e)^3
/(cos(f*x + e) + 1)^3 - 187330*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 26512
2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 353496*sin(f*x + e)^6/(cos(f*x + e
) + 1)^6 + 276276*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 207207*sin(f*x + e
)^8/(cos(f*x + e) + 1)^8 + 75075*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 421 vs. $2(152) = 304$.

Time = 0.32 (sec) , antiderivative size = 421, normalized size of antiderivative = 2.70

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c - c\sin(e + fx))^7}\,dx =$$
$$-\frac{2\left(9009\,Aa^3\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^{12} - 27027\,Aa^3\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^{11} + 9009\,Ba^3\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^{11} + 15315\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x, algori
thm="giac")
```

output
```
-2/9009*(9009*A*a^3*tan(1/2*f*x + 1/2*e)^12 - 27027*A*a^3*tan(1/2*f*x + 1/
2*e)^11 + 9009*B*a^3*tan(1/2*f*x + 1/2*e)^11 + 153153*A*a^3*tan(1/2*f*x +
1/2*e)^10 + 3003*B*a^3*tan(1/2*f*x + 1/2*e)^10 - 297297*A*a^3*tan(1/2*f*x
+ 1/2*e)^9 + 69069*B*a^3*tan(1/2*f*x + 1/2*e)^9 + 648648*A*a^3*tan(1/2*f*x
 + 1/2*e)^8 - 9009*B*a^3*tan(1/2*f*x + 1/2*e)^8 - 738738*A*a^3*tan(1/2*f*x
 + 1/2*e)^7 + 150150*B*a^3*tan(1/2*f*x + 1/2*e)^7 + 857142*A*a^3*tan(1/2*f
*x + 1/2*e)^6 - 16302*B*a^3*tan(1/2*f*x + 1/2*e)^6 - 548262*A*a^3*tan(1/2*
f*x + 1/2*e)^5 + 115830*B*a^3*tan(1/2*f*x + 1/2*e)^5 + 367653*A*a^3*tan(1/
2*f*x + 1/2*e)^4 - 286*B*a^3*tan(1/2*f*x + 1/2*e)^4 - 112827*A*a^3*tan(1/2
*f*x + 1/2*e)^3 + 30745*B*a^3*tan(1/2*f*x + 1/2*e)^3 + 45513*A*a^3*tan(1/2
*f*x + 1/2*e)^2 + 1443*B*a^3*tan(1/2*f*x + 1/2*e)^2 - 3081*A*a^3*tan(1/2*f
*x + 1/2*e) + 1261*B*a^3*tan(1/2*f*x + 1/2*e) + 930*A*a^3 - 97*B*a^3)/(c^7
*f*(tan(1/2*f*x + 1/2*e) - 1)^13)
```

Mupad [B] (verification not implemented)

Time = 37.40 (sec) , antiderivative size = 500, normalized size of antiderivative = 3.21

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx = \text{Too large to display}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^7,x
)
```

output
```
(2*cos(e/2 + (f*x)/2)*((2363*B*a^3)/32 - (279183*A*a^3)/16 + (220269*A*a^3
*cos(2*e + 2*f*x))/16 - (46095*A*a^3*cos(3*e + 3*f*x))/16 - (20829*A*a^3*c
os(4*e + 4*f*x))/16 + (2811*A*a^3*cos(5*e + 5*f*x))/16 + (231*A*a^3*cos(6*
e + 6*f*x))/16 - (8995*B*a^3*cos(2*e + 2*f*x))/64 + (497*B*a^3*cos(3*e + 3
*f*x))/16 + (3725*B*a^3*cos(4*e + 4*f*x))/32 - (361*B*a^3*cos(5*e + 5*f*x)
)/16 - (77*B*a^3*cos(6*e + 6*f*x))/64 - (19305*A*a^3*sin(2*e + 2*f*x))/4 -
 (81081*A*a^3*sin(3*e + 3*f*x))/16 + (15015*A*a^3*sin(4*e + 4*f*x))/16 + (
3237*A*a^3*sin(5*e + 5*f*x))/16 - (117*A*a^3*sin(6*e + 6*f*x))/8 + (77649*
B*a^3*sin(2*e + 2*f*x))/64 + (27027*B*a^3*sin(3*e + 3*f*x))/32 - (1001*B*a
^3*sin(4*e + 4*f*x))/8 - (559*B*a^3*sin(5*e + 5*f*x))/32 + (117*B*a^3*sin(
6*e + 6*f*x))/64 + (26979*A*a^3*cos(e + f*x))/4 + 40*B*a^3*cos(e + f*x) +
(173745*A*a^3*sin(e + f*x))/8 - (80223*B*a^3*sin(e + f*x))/16))/(9009*c^7*
f*((1287*2^(1/2)*cos((3*e)/2 - pi/4 + (3*f*x)/2))/64 - (429*2^(1/2)*cos(e/
2 + pi/4 + (f*x)/2))/16 + (715*2^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2))/64
 - (143*2^(1/2)*cos((7*e)/2 - pi/4 + (7*f*x)/2))/32 - (39*2^(1/2)*cos((9*e
)/2 + pi/4 + (9*f*x)/2))/32 + (13*2^(1/2)*cos((11*e)/2 - pi/4 + (11*f*x)/2
))/64 + (2^(1/2)*cos((13*e)/2 + pi/4 + (13*f*x)/2))/64))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 543, normalized size of antiderivative = 3.48

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^7}\,dx = \frac{a^3\left(1386a-77\sin\left(fx+e\right)^7b+3228\sin\left(fx+e\right)^6a+559\sin\left(fx+e\right)^6b-9663\sin\left(fx+e\right)^5a-450\,\mathrm{c}\right.}{}$$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x)
```

output

```
(a**3*( - 450*cos(e + f*x)*sin(e + f*x)**6*a - 117*cos(e + f*x)*sin(e + f*
x)**6*b + 2694*cos(e + f*x)*sin(e + f*x)**5*a + 722*cos(e + f*x)*sin(e + f
*x)**5*b - 6711*cos(e + f*x)*sin(e + f*x)**4*a - 1885*cos(e + f*x)*sin(e +
 f*x)**4*b + 8889*cos(e + f*x)*sin(e + f*x)**3*a - 293*cos(e + f*x)*sin(e
+ f*x)**3*b - 8817*cos(e + f*x)*sin(e + f*x)**2*a - 3874*cos(e + f*x)*sin(
e + f*x)**2*b + 237*cos(e + f*x)*sin(e + f*x)*a - 97*cos(e + f*x)*sin(e +
f*x)*b - 1386*cos(e + f*x)*a - 462*sin(e + f*x)**7*a - 77*sin(e + f*x)**7*
b + 3228*sin(e + f*x)**6*a + 559*sin(e + f*x)**6*b - 9663*sin(e + f*x)**5*
a - 1747*sin(e + f*x)**5*b + 16062*sin(e + f*x)**4*a + 6058*sin(e + f*x)**
4*b - 13752*sin(e + f*x)**3*a + 1257*sin(e + f*x)**3*b + 14052*sin(e + f*x
)**2*a + 5135*sin(e + f*x)**2*b + 237*sin(e + f*x)*a - 97*sin(e + f*x)*b +
 1386*a))/(9009*c**7*f*(cos(e + f*x)*sin(e + f*x)**6 - 6*cos(e + f*x)*sin(
e + f*x)**5 + 15*cos(e + f*x)*sin(e + f*x)**4 - 20*cos(e + f*x)*sin(e + f*
x)**3 + 15*cos(e + f*x)*sin(e + f*x)**2 - 6*cos(e + f*x)*sin(e + f*x) + co
s(e + f*x) + sin(e + f*x)**7 - 7*sin(e + f*x)**6 + 21*sin(e + f*x)**5 - 35
*sin(e + f*x)**4 + 35*sin(e + f*x)**3 - 21*sin(e + f*x)**2 + 7*sin(e + f*x
) - 1))
```

3.51 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 197

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}\,dx = \frac{a^3(A+B)c^3\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}} + \frac{a^3(4A-11B)c^2\cos^7(e+fx)}{195f(c-c\sin(e+fx))^{10}} + \frac{a^3(4A-11B)c\cos^7(e+fx)}{715f(c-c\sin(e+fx))^9} + \frac{2a^3(4A-11B)\cos^7(e+fx)}{6435f(c-c\sin(e+fx))^8} + \frac{2a^3(4A-11B)\cos^7(e+fx)}{45045cf(c-c\sin(e+fx))^7}$$

output
```
1/15*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^11+1/195*a^3*(4*A-11*B)
*c^2*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^10+1/715*a^3*(4*A-11*B)*c*cos(f*x+e)^
7/f/(c-c*sin(f*x+e))^9+2/6435*a^3*(4*A-11*B)*cos(f*x+e)^7/f/(c-c*sin(f*x+e
))^8+2/45045*a^3*(4*A-11*B)*cos(f*x+e)^7/c/f/(c-c*sin(f*x+e))^7
```

Mathematica [A] (verified)

Time = 17.18 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.92

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a+a\sin(e+fx))^3\left(463320A\cos\left(\frac{1}{2}(e+fx)\right)+302445B\cos\left(\frac{1}{2}(e\right.\right.}{}$$

input
```
Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^8,x]
```

output
```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a + a*Sin[e + f*x])^3*(463320*A*Co
s[(e + f*x)/2] + 302445*B*Cos[(e + f*x)/2] - 260260*A*Cos[(3*(e + f*x))/2]
 - 230230*B*Cos[(3*(e + f*x))/2] - 72072*A*Cos[(5*(e + f*x))/2] - 117117*B
*Cos[(5*(e + f*x))/2] + 5460*A*Cos[(7*(e + f*x))/2] + 30030*B*Cos[(7*(e +
f*x))/2] - 420*A*Cos[(11*(e + f*x))/2] + 1155*B*Cos[(11*(e + f*x))/2] + 4*
A*Cos[(15*(e + f*x))/2] - 11*B*Cos[(15*(e + f*x))/2] + 437580*A*Sin[(e + f
*x)/2] + 373230*B*Sin[(e + f*x)/2] + 240240*A*Sin[(3*(e + f*x))/2] + 28528
5*B*Sin[(3*(e + f*x))/2] - 60060*A*Sin[(5*(e + f*x))/2] - 150150*B*Sin[(5*
(e + f*x))/2] - 45045*B*Sin[(7*(e + f*x))/2] - 1820*A*Sin[(9*(e + f*x))/2]
 + 5005*B*Sin[(9*(e + f*x))/2] + 60*A*Sin[(13*(e + f*x))/2] - 165*B*Sin[(1
3*(e + f*x))/2]))/(1441440*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(c -
c*Sin[e + f*x])^8)
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3446, 3042, 3338, 3042, 3151, 3042, 3151, 3042, 3151, 3042, 3150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}dx$$

$$\Big\downarrow \text{3446}$$

$$a^3c^3\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11}}dx$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\int \frac{\cos(e+fx)^6(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11}}dx$$

$$\Big\downarrow \text{3338}$$

$$a^3c^3\left(\frac{(4A-11B)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{10}}dx}{15c}+\frac{(A+B)\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left(\frac{(4A-11B)\int \frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^{10}}dx}{15c}+\frac{(A+B)\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}}\right)$$

$$\Big\downarrow \text{3151}$$

$$a^3c^3\left(\frac{(4A-11B)\left(\frac{3\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^9}dx}{13c}+\frac{\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}\right)}{15c}+\frac{(A+B)\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left(\frac{(4A-11B)\left(\frac{3\int \frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^9}dx}{13c}+\frac{\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}\right)}{15c}+\frac{(A+B)\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}}\right)$$

$$\Big\downarrow \text{3151}$$

$$a^3c^3\left(\frac{(4A-11B)\left(\frac{3\left(\frac{2\int\frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^8}dx}{11c}+\frac{\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}\right)}{13c}+\frac{\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}\right)}{15c}+\frac{(A+B)\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}}\right)$$

$\downarrow$ 3042

$$a^3c^3\left(\frac{(4A-11B)\left(\frac{3\left(\frac{2\int\frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^8}dx}{11c}+\frac{\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}\right)}{13c}+\frac{\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}\right)}{15c}+\frac{(A+B)\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}}\right)$$

$\downarrow$ 3151

$$a^3c^3\left(\frac{(4A-11B)\left(\frac{3\left(\frac{2\left(\frac{\int\frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^7}dx}{9c}+\frac{\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}\right)}{11c}+\frac{\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}\right)}{13c}+\frac{\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}\right)}{15c}+\frac{(A+B}{15f(c-}\right.$$

$\downarrow$ 3042

$$a^3c^3\left(\frac{(4A-11B)\left(\frac{3\left(\frac{2\left(\frac{\int\frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^7}dx}{9c}+\frac{\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}\right)}{11c}+\frac{\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}\right)}{13c}+\frac{\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}\right)}{15c}+\frac{(A+B}{15f(c-}\right.$$

$\downarrow$ 3150

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{15f(c-c\sin(e+fx))^{11}}+\frac{(4A-11B)\left(\frac{\cos^7(e+fx)}{13f(c-c\sin(e+fx))^{10}}+\frac{3\left(\frac{\cos^7(e+fx)}{11f(c-c\sin(e+fx))^9}+\frac{2\left(\frac{\cos^7(e+fx)}{63cf(c-c\sin(e+fx))^7}+\frac{\cos^7(e+fx)}{9f(c-c\sin(e+fx))^8}\right)}{11c}\right)}{13c}\right)}{15c}\right)$$

input
```
Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^8,x
]
```

output
```
a^3*c^3*(((A + B)*Cos[e + f*x]^7)/(15*f*(c - c*Sin[e + f*x])^11) + ((4*A -
 11*B)*(Cos[e + f*x]^7/(13*f*(c - c*Sin[e + f*x])^10) + (3*(Cos[e + f*x]^7
/(11*f*(c - c*Sin[e + f*x])^9) + (2*(Cos[e + f*x]^7/(9*f*(c - c*Sin[e + f*
x])^8) + Cos[e + f*x]^7/(63*c*f*(c - c*Sin[e + f*x])^7)))/(11*c)))/(13*c))
)/(15*c))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3150
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

rule 3151
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Simp[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1])   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simpli
fy[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.85 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.51

method	result
risch	$\frac{4ia^3(-30030iB\,e^{4i(fx+e)}-302445iB\,e^{8i(fx+e)}+45045B\,e^{11i(fx+e)}+117117iB\,e^{10i(fx+e)}-240240A\,e^{9i(fx+e)}+2302}{}$
parallelrisch	$-\frac{2a^3\left(A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{14}+(-4A+B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{13}+\frac{(71A-B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{12}}{3}+10(-6A+B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}+\frac{(741A-3}{}\right.}{}$
derivativedivides	$\frac{2a^3\left(-\frac{188A+38B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{20A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{58816A+40000B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{81344A+72512B}{11\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{11}}-\frac{52736A+49664B}{12\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{12}}-\right.}{}$
default	$\frac{2a^3\left(-\frac{188A+38B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{20A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{58816A+40000B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8}-\frac{81344A+72512B}{11\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{11}}-\frac{52736A+49664B}{12\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{12}}-\right.}{}$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^8,x,method=_RETUR
NVERBOSE)
```

output
```
4/45045*I*a^3*(-30030*I*B*exp(4*I*(f*x+e))-302445*I*B*exp(8*I*(f*x+e))+450
45*B*exp(11*I*(f*x+e))+117117*I*B*exp(10*I*(f*x+e))-240240*A*exp(9*I*(f*x+
e))+230230*I*B*exp(6*I*(f*x+e))-285285*B*exp(9*I*(f*x+e))+420*I*A*exp(2*I*
(f*x+e))+437580*A*exp(7*I*(f*x+e))+260260*I*A*exp(6*I*(f*x+e))+373230*B*ex
p(7*I*(f*x+e))-1155*I*B*exp(2*I*(f*x+e))-60060*A*exp(5*I*(f*x+e))-5460*I*A
*exp(4*I*(f*x+e))-150150*B*exp(5*I*(f*x+e))+72072*I*A*exp(10*I*(f*x+e))-18
20*A*exp(3*I*(f*x+e))+11*I*B+5005*B*exp(3*I*(f*x+e))-463320*I*A*exp(8*I*(f
*x+e))+60*A*exp(I*(f*x+e))-4*I*A-165*B*exp(I*(f*x+e)))/f/c^8/(exp(I*(f*x+e
))-I)^15
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 541 vs. $2(192) = 384$.

Time = 0.09 (sec) , antiderivative size = 541, normalized size of antiderivative = 2.75

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}\,dx$$
$$= \frac{2\,(4\,A-11\,B)a^3\cos{(fx+e)}^8+16\,(4\,A-11\,B)a^3\cos{(fx+e)}^7-49\,(4\,A-11\,B)a^3\cos{(fx+e)}^6-16}{45045\,(}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^8,x, algori
thm="fricas")
```

output
```
1/45045*(2*(4*A - 11*B)*a^3*cos(f*x + e)^8 + 16*(4*A - 11*B)*a^3*cos(f*x +
 e)^7 - 49*(4*A - 11*B)*a^3*cos(f*x + e)^6 - 168*(4*A - 11*B)*a^3*cos(f*x
+ e)^5 + 105*(7*A + 88*B)*a^3*cos(f*x + e)^4 - 231*(31*A + 61*B)*a^3*cos(f
*x + e)^3 - 924*(22*A + 37*B)*a^3*cos(f*x + e)^2 + 12012*(A + B)*a^3*cos(f
*x + e) + 24024*(A + B)*a^3 - (2*(4*A - 11*B)*a^3*cos(f*x + e)^7 - 14*(4*A
 - 11*B)*a^3*cos(f*x + e)^6 - 63*(4*A - 11*B)*a^3*cos(f*x + e)^5 + 105*(4*
A - 11*B)*a^3*cos(f*x + e)^4 + 1155*(A + 7*B)*a^3*cos(f*x + e)^3 + 2772*(3
*A + 8*B)*a^3*cos(f*x + e)^2 - 12012*(A + B)*a^3*cos(f*x + e) - 24024*(A +
 B)*a^3)*sin(f*x + e))/(c^8*f*cos(f*x + e)^8 - 7*c^8*f*cos(f*x + e)^7 - 32
*c^8*f*cos(f*x + e)^6 + 56*c^8*f*cos(f*x + e)^5 + 160*c^8*f*cos(f*x + e)^4
 - 112*c^8*f*cos(f*x + e)^3 - 256*c^8*f*cos(f*x + e)^2 + 64*c^8*f*cos(f*x
+ e) + 128*c^8*f + (c^8*f*cos(f*x + e)^7 + 8*c^8*f*cos(f*x + e)^6 - 24*c^8
*f*cos(f*x + e)^5 - 80*c^8*f*cos(f*x + e)^4 + 80*c^8*f*cos(f*x + e)^3 + 19
2*c^8*f*cos(f*x + e)^2 - 64*c^8*f*cos(f*x + e) - 128*c^8*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8821 vs. $2(178) = 356$.

Time = 166.06 (sec) , antiderivative size = 8821, normalized size of antiderivative = 44.78

$$\int \frac{(a + a\sin(e + fx))^3 (A + B\sin(e + fx))}{(c - c\sin(e + fx))^8}\, dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**8,x)
```

output
```
Piecewise((-90090*A*a**3*tan(e/2 + f*x/2)**14/(45045*c**8*f*tan(e/2 + f*x/
2)**15 - 675675*c**8*f*tan(e/2 + f*x/2)**14 + 4729725*c**8*f*tan(e/2 + f*x
/2)**13 - 20495475*c**8*f*tan(e/2 + f*x/2)**12 + 61486425*c**8*f*tan(e/2 +
 f*x/2)**11 - 135270135*c**8*f*tan(e/2 + f*x/2)**10 + 225450225*c**8*f*tan
(e/2 + f*x/2)**9 - 289864575*c**8*f*tan(e/2 + f*x/2)**8 + 289864575*c**8*f
*tan(e/2 + f*x/2)**7 - 225450225*c**8*f*tan(e/2 + f*x/2)**6 + 135270135*c*
*8*f*tan(e/2 + f*x/2)**5 - 61486425*c**8*f*tan(e/2 + f*x/2)**4 + 20495475*
c**8*f*tan(e/2 + f*x/2)**3 - 4729725*c**8*f*tan(e/2 + f*x/2)**2 + 675675*c
**8*f*tan(e/2 + f*x/2) - 45045*c**8*f) + 360360*A*a**3*tan(e/2 + f*x/2)**1
3/(45045*c**8*f*tan(e/2 + f*x/2)**15 - 675675*c**8*f*tan(e/2 + f*x/2)**14
+ 4729725*c**8*f*tan(e/2 + f*x/2)**13 - 20495475*c**8*f*tan(e/2 + f*x/2)**
12 + 61486425*c**8*f*tan(e/2 + f*x/2)**11 - 135270135*c**8*f*tan(e/2 + f*x
/2)**10 + 225450225*c**8*f*tan(e/2 + f*x/2)**9 - 289864575*c**8*f*tan(e/2
+ f*x/2)**8 + 289864575*c**8*f*tan(e/2 + f*x/2)**7 - 225450225*c**8*f*tan(
e/2 + f*x/2)**6 + 135270135*c**8*f*tan(e/2 + f*x/2)**5 - 61486425*c**8*f*t
an(e/2 + f*x/2)**4 + 20495475*c**8*f*tan(e/2 + f*x/2)**3 - 4729725*c**8*f*
tan(e/2 + f*x/2)**2 + 675675*c**8*f*tan(e/2 + f*x/2) - 45045*c**8*f) - 213
2130*A*a**3*tan(e/2 + f*x/2)**12/(45045*c**8*f*tan(e/2 + f*x/2)**15 - 6756
75*c**8*f*tan(e/2 + f*x/2)**14 + 4729725*c**8*f*tan(e/2 + f*x/2)**13 - 204
95475*c**8*f*tan(e/2 + f*x/2)**12 + 61486425*c**8*f*tan(e/2 + f*x/2)**1...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4765 vs. $2(192) = 384$.

Time = 0.23 (sec) , antiderivative size = 4765, normalized size of antiderivative = 24.19

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c - c\sin(e + fx))^8}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^8,x, algori
thm="maxima")
```

output
```
2/45045*(3*A*a^3*(17715*sin(f*x + e)/(cos(f*x + e) + 1) - 78960*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 342160*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 8
91345*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1960959*sin(f*x + e)^5/(cos(f*
x + e) + 1)^5 - 3043040*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 3912480*sin(
f*x + e)^7/(cos(f*x + e) + 1)^7 - 3687255*sin(f*x + e)^8/(cos(f*x + e) + 1
)^8 + 2867865*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 1585584*sin(f*x + e)^1
0/(cos(f*x + e) + 1)^10 + 720720*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - 1
95195*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 + 45045*sin(f*x + e)^13/(cos(f
*x + e) + 1)^13 - 1181)/(c^8 - 15*c^8*sin(f*x + e)/(cos(f*x + e) + 1) + 10
5*c^8*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 455*c^8*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 1365*c^8*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3003*c^8*si
n(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5005*c^8*sin(f*x + e)^6/(cos(f*x + e)
+ 1)^6 - 6435*c^8*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 6435*c^8*sin(f*x +
 e)^8/(cos(f*x + e) + 1)^8 - 5005*c^8*sin(f*x + e)^9/(cos(f*x + e) + 1)^9
+ 3003*c^8*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 1365*c^8*sin(f*x + e)^1
1/(cos(f*x + e) + 1)^11 + 455*c^8*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 -
105*c^8*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 + 15*c^8*sin(f*x + e)^14/(co
s(f*x + e) + 1)^14 - c^8*sin(f*x + e)^15/(cos(f*x + e) + 1)^15) + B*a^3*(1
7715*sin(f*x + e)/(cos(f*x + e) + 1) - 78960*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 342160*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 891345*sin(f*x + ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 489 vs. $2(192) = 384$.

Time = 0.30 (sec) , antiderivative size = 489, normalized size of antiderivative = 2.48

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c - c\sin(e + fx))^8} dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^8,x, algori
thm="giac")
```

output
```
-2/45045*(45045*A*a^3*tan(1/2*f*x + 1/2*e)^14 - 180180*A*a^3*tan(1/2*f*x +
 1/2*e)^13 + 45045*B*a^3*tan(1/2*f*x + 1/2*e)^13 + 1066065*A*a^3*tan(1/2*f
*x + 1/2*e)^12 - 15015*B*a^3*tan(1/2*f*x + 1/2*e)^12 - 2702700*A*a^3*tan(1
/2*f*x + 1/2*e)^11 + 450450*B*a^3*tan(1/2*f*x + 1/2*e)^11 + 6675669*A*a^3*
tan(1/2*f*x + 1/2*e)^10 - 306306*B*a^3*tan(1/2*f*x + 1/2*e)^10 - 10210200*
A*a^3*tan(1/2*f*x + 1/2*e)^9 + 1456455*B*a^3*tan(1/2*f*x + 1/2*e)^9 + 1412
4825*A*a^3*tan(1/2*f*x + 1/2*e)^8 - 791505*B*a^3*tan(1/2*f*x + 1/2*e)^8 -
13178880*A*a^3*tan(1/2*f*x + 1/2*e)^7 + 1827540*B*a^3*tan(1/2*f*x + 1/2*e)
^7 + 11026015*A*a^3*tan(1/2*f*x + 1/2*e)^6 - 580580*B*a^3*tan(1/2*f*x + 1/
2*e)^6 - 6066060*A*a^3*tan(1/2*f*x + 1/2*e)^5 + 915915*B*a^3*tan(1/2*f*x +
 1/2*e)^5 + 3088995*A*a^3*tan(1/2*f*x + 1/2*e)^4 - 105105*B*a^3*tan(1/2*f*
x + 1/2*e)^4 - 864500*A*a^3*tan(1/2*f*x + 1/2*e)^3 + 170170*B*a^3*tan(1/2*
f*x + 1/2*e)^3 + 265335*A*a^3*tan(1/2*f*x + 1/2*e)^2 + 2310*B*a^3*tan(1/2*
f*x + 1/2*e)^2 - 18600*A*a^3*tan(1/2*f*x + 1/2*e) + 6105*B*a^3*tan(1/2*f*x
 + 1/2*e) + 4243*A*a^3 - 407*B*a^3)/(c^8*f*(tan(1/2*f*x + 1/2*e) - 1)^15)
```

Mupad [B] (verification not implemented)

Time = 37.56 (sec) , antiderivative size = 577, normalized size of antiderivative = 2.93

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^8}\,dx = \text{Too large to display}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^8,x
)
```

output
```
(2*cos(e/2 + (f*x)/2)*((544369*A*a^3)/4 - (21791*B*a^3)/4 - (257861*A*a^3*
cos(2*e + 2*f*x))/2 + (3497111*A*a^3*cos(3*e + 3*f*x))/128 + (72047*A*a^3*
cos(4*e + 4*f*x))/4 - (378579*A*a^3*cos(5*e + 5*f*x))/128 - (1059*A*a^3*co
s(6*e + 6*f*x))/2 + (4251*A*a^3*cos(7*e + 7*f*x))/128 + (219769*B*a^3*cos(
2*e + 2*f*x))/32 - (191389*B*a^3*cos(3*e + 3*f*x))/128 - 1672*B*a^3*cos(4*
e + 4*f*x) + (38841*B*a^3*cos(5*e + 5*f*x))/128 + (1551*B*a^3*cos(6*e + 6*
f*x))/32 - (429*B*a^3*cos(7*e + 7*f*x))/128 + (2633345*A*a^3*sin(2*e + 2*f
*x))/64 + (7210775*A*a^3*sin(3*e + 3*f*x))/128 - (89375*A*a^3*sin(4*e + 4*
f*x))/8 - (504205*A*a^3*sin(5*e + 5*f*x))/128 + (29765*A*a^3*sin(6*e + 6*f
*x))/64 + (4235*A*a^3*sin(7*e + 7*f*x))/128 - (451165*B*a^3*sin(2*e + 2*f*
x))/64 - (854425*B*a^3*sin(3*e + 3*f*x))/128 + (9295*B*a^3*sin(4*e + 4*f*x
))/8 + (46475*B*a^3*sin(5*e + 5*f*x))/128 - (3025*B*a^3*sin(6*e + 6*f*x))/
64 - (385*B*a^3*sin(7*e + 7*f*x))/128 - (5734111*A*a^3*cos(e + f*x))/128 +
 (126929*B*a^3*cos(e + f*x))/128 - (25501905*A*a^3*sin(e + f*x))/128 + (39
70395*B*a^3*sin(e + f*x))/128))/(45045*c^8*f*((6435*2^(1/2)*cos(e/2 + pi/4
 + (f*x)/2))/128 - (5005*2^(1/2)*cos((3*e)/2 - pi/4 + (3*f*x)/2))/128 - (3
003*2^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2))/128 + (1365*2^(1/2)*cos((7*e)
/2 - pi/4 + (7*f*x)/2))/128 + (455*2^(1/2)*cos((9*e)/2 + pi/4 + (9*f*x)/2)
)/128 - (105*2^(1/2)*cos((11*e)/2 - pi/4 + (11*f*x)/2))/128 - (15*2^(1/2)*
cos((13*e)/2 + pi/4 + (13*f*x)/2))/128 + (2^(1/2)*cos((15*e)/2 - pi/4 +...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 627, normalized size of antiderivative = 3.18

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c - c\sin(e + fx))^8}\,dx = \text{Too large to display}$$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^8,x)
```

output
```
(a**3*( - 1755*cos(e + f*x)*sin(e + f*x)**7*a - 429*cos(e + f*x)*sin(e + f
*x)**7*b + 12277*cos(e + f*x)*sin(e + f*x)**6*a + 3025*cos(e + f*x)*sin(e
+ f*x)**6*b - 36795*cos(e + f*x)*sin(e + f*x)**5*a - 9174*cos(e + f*x)*sin
(e + f*x)**5*b + 61225*cos(e + f*x)*sin(e + f*x)**4*a + 15565*cos(e + f*x)
*sin(e + f*x)**4*b - 61030*cos(e + f*x)*sin(e + f*x)**3*a - 4840*cos(e + f
*x)*sin(e + f*x)**3*b + 45336*cos(e + f*x)*sin(e + f*x)**2*a + 19470*cos(e
 + f*x)*sin(e + f*x)**2*b - 1240*cos(e + f*x)*sin(e + f*x)*a + 407*cos(e +
 f*x)*sin(e + f*x)*b + 6006*cos(e + f*x)*a - 1771*sin(e + f*x)**8*a - 385*
sin(e + f*x)**8*b + 14160*sin(e + f*x)**7*a + 3102*sin(e + f*x)**7*b - 495
28*sin(e + f*x)**6*a - 10945*sin(e + f*x)**6*b + 98980*sin(e + f*x)**5*a +
 22099*sin(e + f*x)**5*b - 123605*sin(e + f*x)**4*a - 39215*sin(e + f*x)**
4*b + 89740*sin(e + f*x)**3*a + 2464*sin(e + f*x)**3*b - 68778*sin(e + f*x
)**2*a - 25575*sin(e + f*x)**2*b - 1240*sin(e + f*x)*a + 407*sin(e + f*x)*
b - 6006*a))/(45045*c**8*f*(cos(e + f*x)*sin(e + f*x)**7 - 7*cos(e + f*x)*
sin(e + f*x)**6 + 21*cos(e + f*x)*sin(e + f*x)**5 - 35*cos(e + f*x)*sin(e
+ f*x)**4 + 35*cos(e + f*x)*sin(e + f*x)**3 - 21*cos(e + f*x)*sin(e + f*x)
**2 + 7*cos(e + f*x)*sin(e + f*x) - cos(e + f*x) + sin(e + f*x)**8 - 8*sin
(e + f*x)**7 + 28*sin(e + f*x)**6 - 56*sin(e + f*x)**5 + 70*sin(e + f*x)**
4 - 56*sin(e + f*x)**3 + 28*sin(e + f*x)**2 - 8*sin(e + f*x) + 1))
```

3.52 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{a+a\sin(e+fx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 190

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{a+a\sin(e+fx)}\,dx = -\frac{35(4A-5B)c^4x}{8a}-\frac{35(4A-5B)c^4\cos^3(e+fx)}{12af} - \frac{35(4A-5B)c^4\cos(e+fx)\sin(e+fx)}{8af}-\frac{a^4(A-B)c^4\cos^9(e+fx)}{f(a+a\sin(e+fx))^5} - \frac{2a^2(4A-5B)c^4\cos^7(e+fx)}{f(a+a\sin(e+fx))^3}-\frac{7(4A-5B)c^4\cos^5(e+fx)}{4f(a+a\sin(e+fx))}$$

output
```
-35/8*(4*A-5*B)*c^4*x/a-35/12*(4*A-5*B)*c^4*cos(f*x+e)^3/a/f-35/8*(4*A-5*B
)*c^4*cos(f*x+e)*sin(f*x+e)/a/f-a^4*(A-B)*c^4*cos(f*x+e)^9/f/(a+a*sin(f*x+
e))^5-2*a^2*(4*A-5*B)*c^4*cos(f*x+e)^7/f/(a+a*sin(f*x+e))^3-7/4*(4*A-5*B)*
c^4*cos(f*x+e)^5/f/(a+a*sin(f*x+e))
```

Mathematica [A] (verified)

Time = 14.23 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.44

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^4}{a + a\sin(e + fx)}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e + fx)\right) + \sin\left(\frac{1}{2}(e + fx)\right)\right)(c - c\sin(e + fx))^4\left(3072(A - B)\sin\left(\frac{1}{2}(e + fx)\right) - 420(4A - 5B\right.}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4)/(a + a*Sin[e + f*x
]),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^4*(3072*(A - B
)*Sin[(e + f*x)/2] - 420*(4*A - 5*B)*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2]) - 24*(47*A - 75*B)*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2]) + 8*(A - 5*B)*Cos[3*(e + f*x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
) + 24*(5*A - 12*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sin[2*(e + f*x)]
 + 3*B*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sin[4*(e + f*x)]))/(96*a*f*(C
os[(e + f*x)/2] - Sin[(e + f*x)/2])^8*(1 + Sin[e + f*x]))
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.90, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.361$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3158, 3042, 3161, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c - c\sin(e + fx))^4(A + B\sin(e + fx))}{a\sin(e + fx) + a}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(c - c\sin(e + fx))^4(A + B\sin(e + fx))}{a\sin(e + fx) + a}dx$$
$$\Big\downarrow \text{3446}$$

$$a^4c^4\int \frac{\cos^8(e+fx)(A+B\sin(e+fx))}{(\sin(e+fx)a+a)^5}dx$$

$\big\downarrow$ 3042

$$a^4c^4\int \frac{\cos(e+fx)^8(A+B\sin(e+fx))}{(\sin(e+fx)a+a)^5}dx$$

$\big\downarrow$ 3338

$$a^4c^4\left(-\frac{(4A-5B)\int \frac{\cos^8(e+fx)}{(\sin(e+fx)a+a)^4}dx}{a}-\frac{(A-B)\cos^9(e+fx)}{f(a\sin(e+fx)+a)^5}\right)$$

$\big\downarrow$ 3042

$$a^4c^4\left(-\frac{(4A-5B)\int \frac{\cos(e+fx)^8}{(\sin(e+fx)a+a)^4}dx}{a}-\frac{(A-B)\cos^9(e+fx)}{f(a\sin(e+fx)+a)^5}\right)$$

$\big\downarrow$ 3159

$$a^4c^4\left(-\frac{(4A-5B)\left(\frac{7\int \frac{\cos^6(e+fx)}{(\sin(e+fx)a+a)^2}dx}{a^2}+\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{a}-\frac{(A-B)\cos^9(e+fx)}{f(a\sin(e+fx)+a)^5}\right)$$

$\big\downarrow$ 3042

$$a^4c^4\left(-\frac{(4A-5B)\left(\frac{7\int \frac{\cos(e+fx)^6}{(\sin(e+fx)a+a)^2}dx}{a^2}+\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{a}-\frac{(A-B)\cos^9(e+fx)}{f(a\sin(e+fx)+a)^5}\right)$$

$\big\downarrow$ 3158

$$a^4c^4\left(-\frac{(4A-5B)\left(\frac{7\left(\frac{5\int \frac{\cos^4(e+fx)}{\sin(e+fx)a+a}dx}{4a}+\frac{\cos^5(e+fx)}{4f\left(a^2\sin(e+fx)+a^2\right)}\right)}{a^2}+\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{a}-\frac{(A-B)\cos^9(e+fx)}{f(a\sin(e+fx)+a)^5}\right)$$

$\big\downarrow$ 3042

$$a^4c^4\left(-\frac{(4A-5B)\left(\frac{7\left(\frac{5\int\frac{\cos(e+fx)^4}{\sin(e+fx)a+a}dx}{4a}+\frac{\cos^5(e+fx)}{4f\left(a^2\sin(e+fx)+a^2\right)}\right)}{a^2}+\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{a}-\frac{(A-B)\cos^9(e+fx)}{f(a\sin(e+fx)+a)^5}\right)$$

$\downarrow$ 3161

$$a^4c^4\left(-\frac{(4A-5B)\left(\frac{7\left(\frac{5\left(\frac{\int\cos^2(e+fx)dx}{a}+\frac{\cos^3(e+fx)}{3af}\right)}{4a}+\frac{\cos^5(e+fx)}{4f\left(a^2\sin(e+fx)+a^2\right)}\right)}{a^2}+\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{a}-\frac{(A-B)\cos^9(e+fx)}{f(a\sin(e+fx)+a)^5}\right)$$

$\downarrow$ 3042

$$a^4c^4\left(-\frac{(4A-5B)\left(\frac{7\left(\frac{5\left(\frac{\int\sin\left(e+fx+\frac{\pi}{2}\right)^2dx}{a}+\frac{\cos^3(e+fx)}{3af}\right)}{4a}+\frac{\cos^5(e+fx)}{4f\left(a^2\sin(e+fx)+a^2\right)}\right)}{a^2}+\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{a}-\frac{(A-B)\cos^9(e+fx)}{f(a\sin(e+fx)+a)^5}\right)$$

$\downarrow$ 3115

$$a^4c^4\left(-\frac{(4A-5B)\left(\frac{7\left(\frac{5\left(\frac{\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}}{a}+\frac{\cos^3(e+fx)}{3af}\right)}{4a}+\frac{\cos^5(e+fx)}{4f\left(a^2\sin(e+fx)+a^2\right)}\right)}{a^2}+\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{a}-\frac{(A-B)\,c}{f(a\sin(e}\right.$$

$\downarrow$ 24

$$a^4c^4\left(-\frac{(4A-5B)\left(\frac{7\left(\frac{\cos^5(e+fx)}{4f\left(a^2\sin(e+fx)+a^2\right)}+\frac{5\left(\frac{\cos^3(e+fx)}{3af}+\frac{\frac{\sin(e+fx)\cos(e+fx)}{2f}+\frac{x}{2}}{a}\right)}{4a}\right)}{a^2}+\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{a}-\frac{(A-B)\cos}{f(a\sin(e+}\right.$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4)/(a + a*Sin[e + f*x]),x]
```

output
```
a^4*c^4*(-(((A - B)*Cos[e + f*x]^9)/(f*(a + a*Sin[e + f*x])^5)) - ((4*A -
5*B)*((2*Cos[e + f*x]^7)/(a*f*(a + a*Sin[e + f*x])^3) + (7*(Cos[e + f*x]^5
/(4*f*(a^2 + a^2*Sin[e + f*x])) + (5*(Cos[e + f*x]^3/(3*a*f) + (x/2 + (Cos
[e + f*x]*Sin[e + f*x])/(2*f))/a))/(4*a)))/a^2))/a)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3158
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(a*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
 EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && In
tegersQ[2*m, 2*p]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 10.82 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{2c^4\left(-\frac{\left(\frac{5A}{2}-\frac{47B}{8}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7+(11A-15B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+\left(\frac{5A}{2}-\frac{55B}{8}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5+(35A-55B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+\left(-\frac{5A}{2}+\right.}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^4}\right.}{}$
default	$\frac{2c^4\left(-\frac{\left(\frac{5A}{2}-\frac{47B}{8}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7+(11A-15B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+\left(\frac{5A}{2}-\frac{55B}{8}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5+(35A-55B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+\left(-\frac{5A}{2}+\right.}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^4}\right.}{}$
parallelrisch	$-\frac{7\left(\left(30fxA-\frac{75}{2}fxB+\frac{1189}{14}A-\frac{1433}{14}B\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\left(30fxA-\frac{75}{2}fxB+\frac{139}{14}A-\frac{215}{14}B\right)\sin\left(\frac{fx}{2}+\frac{e}{2}\right)+9\left(A-\frac{3B}{2}\right)\cos\right.}{}$
risch	$-\frac{35c^4xA}{2a}+\frac{175c^4xB}{8a}-\frac{47c^4e^{i(fx+e)}A}{8af}+\frac{75c^4e^{i(fx+e)}B}{8af}-\frac{47c^4e^{-i(fx+e)}A}{8af}+\frac{75c^4e^{-i(fx+e)}B}{8af}-\frac{32c}{fa(e^{i(f}}$
norman	$\frac{-\frac{166Ac^4-206Bc^4}{3af}-\frac{35(4A-5B)c^4x}{8a}-\frac{\left(108Ac^4-167Bc^4\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{4af}-\frac{\left(148Ac^4-175Bc^4\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{4af}-\frac{\left(204Ac^4-331\right.}{}}{}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e)),x,method=_RETURNV
ERBOSE)
```

output
```
2/f*c^4/a*(-((5/2*A-47/8*B)*tan(1/2*f*x+1/2*e)^7+(11*A-15*B)*tan(1/2*f*x+1
/2*e)^6+(5/2*A-55/8*B)*tan(1/2*f*x+1/2*e)^5+(35*A-55*B)*tan(1/2*f*x+1/2*e)
^4+(-5/2*A+55/8*B)*tan(1/2*f*x+1/2*e)^3+(107/3*A-175/3*B)*tan(1/2*f*x+1/2*
e)^2+(-5/2*A+47/8*B)*tan(1/2*f*x+1/2*e)+35/3*A-55/3*B)/(1+tan(1/2*f*x+1/2*
e)^2)^4-35/8*(4*A-5*B)*arctan(tan(1/2*f*x+1/2*e))-(16*A-16*B)/(tan(1/2*f*x
+1/2*e)+1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.37

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{a+a\sin(e+fx)}\,dx = -\frac{6\,Bc^4\cos{(fx+e)}^5 - 8\,(A-5\,B)c^4\cos{(fx+e)}^4 + (52\,A-113\,B)c^4\cos{(fx+e)}^3 + 105\,(4\,A-5\,B)}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
-1/24*(6*B*c^4*cos(f*x + e)^5 - 8*(A - 5*B)*c^4*cos(f*x + e)^4 + (52*A - 1
13*B)*c^4*cos(f*x + e)^3 + 105*(4*A - 5*B)*c^4*f*x + 96*(3*A - 5*B)*c^4*co
s(f*x + e)^2 + 384*(A - B)*c^4 + 3*(35*(4*A - 5*B)*c^4*f*x + (204*A - 239*
B)*c^4)*cos(f*x + e) - (6*B*c^4*cos(f*x + e)^4 + 2*(4*A - 17*B)*c^4*cos(f*
x + e)^3 - 105*(4*A - 5*B)*c^4*f*x + 3*(20*A - 49*B)*c^4*cos(f*x + e)^2 -
3*(76*A - 111*B)*c^4*cos(f*x + e) + 384*(A - B)*c^4)*sin(f*x + e))/(a*f*co
s(f*x + e) + a*f*sin(f*x + e) + a*f)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6690 vs. $2(173) = 346$.

Time = 7.35 (sec) , antiderivative size = 6690, normalized size of antiderivative = 35.21

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{a+a\sin(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**4/(a+a*sin(f*x+e)),x)
```

output
```
Piecewise((-420*A*c**4*f*x*tan(e/2 + f*x/2)**9/(24*a*f*tan(e/2 + f*x/2)**9
 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/
2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4
+ 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2
 + f*x/2) + 24*a*f) - 420*A*c**4*f*x*tan(e/2 + f*x/2)**8/(24*a*f*tan(e/2 +
 f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*
a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 +
f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a
*f*tan(e/2 + f*x/2) + 24*a*f) - 1680*A*c**4*f*x*tan(e/2 + f*x/2)**7/(24*a*
f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/
2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f
*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2
)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 1680*A*c**4*f*x*tan(e/2 + f*x/2
)**6/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan
(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**
5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(
e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 2520*A*c**4*f*x*tan(
e/2 + f*x/2)**5/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 +
 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2
 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1796 vs. $2(182) = 364$.

Time = 0.16 (sec) , antiderivative size = 1796, normalized size of antiderivative = 9.45

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^4}{a + a\sin(e + fx)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e)),x, algorith
m="maxima")
```

output
```
1/12*(B*c^4*((19*sin(f*x + e)/(cos(f*x + e) + 1) + 211*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 91*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 219*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 + 165*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 165*
sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 45*sin(f*x + e)^7/(cos(f*x + e) + 1)
^7 + 45*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 64)/(a + a*sin(f*x + e)/(cos
(f*x + e) + 1) + 4*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3 + 6*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 6*a*s
in(f*x + e)^5/(cos(f*x + e) + 1)^5 + 4*a*sin(f*x + e)^6/(cos(f*x + e) + 1)
^6 + 4*a*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + a*sin(f*x + e)^8/(cos(f*x +
 e) + 1)^8 + a*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) + 45*arctan(sin(f*x +
e)/(cos(f*x + e) + 1))/a) - 4*A*c^4*((7*sin(f*x + e)/(cos(f*x + e) + 1) +
39*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 24*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 24*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 9*sin(f*x + e)^5/(cos(f*x
 + e) + 1)^5 + 9*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 16)/(a + a*sin(f*x
+ e)/(cos(f*x + e) + 1) + 3*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*a*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^
4 + 3*a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + a*sin(f*x + e)^6/(cos(f*x +
e) + 1)^6 + a*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 9*arctan(sin(f*x + e)
/(cos(f*x + e) + 1))/a) + 16*B*c^4*((7*sin(f*x + e)/(cos(f*x + e) + 1) + 3
9*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 24*sin(f*x + e)^3/(cos(f*x + e)...
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.72

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{a+a\sin(e+fx)}\,dx =$$

$$-\frac{\frac{105\,(4\,Ac^4-5\,Bc^4)(fx+e)}{a}+\frac{768\,(Ac^4-Bc^4)}{a(\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)+1)}+\frac{2\left(60\,Ac^4\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^7-141\,Bc^4\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^7+264\,Ac^4\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)^6-\right.}{}}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e)),x, algorith
m="giac")
```

output
```
-1/24*(105*(4*A*c^4 - 5*B*c^4)*(f*x + e)/a + 768*(A*c^4 - B*c^4)/(a*(tan(1
/2*f*x + 1/2*e) + 1)) + 2*(60*A*c^4*tan(1/2*f*x + 1/2*e)^7 - 141*B*c^4*tan
(1/2*f*x + 1/2*e)^7 + 264*A*c^4*tan(1/2*f*x + 1/2*e)^6 - 360*B*c^4*tan(1/2
*f*x + 1/2*e)^6 + 60*A*c^4*tan(1/2*f*x + 1/2*e)^5 - 165*B*c^4*tan(1/2*f*x
+ 1/2*e)^5 + 840*A*c^4*tan(1/2*f*x + 1/2*e)^4 - 1320*B*c^4*tan(1/2*f*x + 1
/2*e)^4 - 60*A*c^4*tan(1/2*f*x + 1/2*e)^3 + 165*B*c^4*tan(1/2*f*x + 1/2*e)
^3 + 856*A*c^4*tan(1/2*f*x + 1/2*e)^2 - 1400*B*c^4*tan(1/2*f*x + 1/2*e)^2
- 60*A*c^4*tan(1/2*f*x + 1/2*e) + 141*B*c^4*tan(1/2*f*x + 1/2*e) + 280*A*c
^4 - 440*B*c^4)/((tan(1/2*f*x + 1/2*e)^2 + 1)^4*a))/f
```

Mupad [B] (verification not implemented)

Time = 39.09 (sec) , antiderivative size = 397, normalized size of antiderivative = 2.09

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{a+a\sin(e+fx)}\,dx = -\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{55\,A\,c^4}{3}-\frac{299\,B\,c^4}{12}\right)+\frac{166\,A\,c^4}{3}-\frac{206\,B\,c^4}{3}+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7\left(27\,A\,c^4-\frac{167\,B\,c^4}{4}\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\left(a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^9+a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8\right.} - \frac{35\,c^4\,\mathrm{atan}\left(\frac{35\,c^4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(4\,A-5\,B)}{140\,A\,c^4-175\,B\,c^4}\right)(4\,A-5\,B)}{4\,a\,f}$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^4)/(a + a*sin(e + f*x)),x)
```

output
```
- (tan(e/2 + (f*x)/2)*((55*A*c^4)/3 - (299*B*c^4)/12) + (166*A*c^4)/3 - (2
06*B*c^4)/3 + tan(e/2 + (f*x)/2)^7*(27*A*c^4 - (167*B*c^4)/4) + tan(e/2 +
(f*x)/2)^8*(37*A*c^4 - (175*B*c^4)/4) + tan(e/2 + (f*x)/2)^5*(75*A*c^4 - (
495*B*c^4)/4) + tan(e/2 + (f*x)/2)^6*(155*A*c^4 - (687*B*c^4)/4) + tan(e/2
 + (f*x)/2)^4*(257*A*c^4 - (1153*B*c^4)/4) + tan(e/2 + (f*x)/2)^3*((199*A*
c^4)/3 - (1235*B*c^4)/12) + tan(e/2 + (f*x)/2)^2*((583*A*c^4)/3 - (2795*B*
c^4)/12))/(f*(a + a*tan(e/2 + (f*x)/2) + 4*a*tan(e/2 + (f*x)/2)^2 + 4*a*ta
n(e/2 + (f*x)/2)^3 + 6*a*tan(e/2 + (f*x)/2)^4 + 6*a*tan(e/2 + (f*x)/2)^5 +
 4*a*tan(e/2 + (f*x)/2)^6 + 4*a*tan(e/2 + (f*x)/2)^7 + a*tan(e/2 + (f*x)/2
)^8 + a*tan(e/2 + (f*x)/2)^9)) - (35*c^4*atan((35*c^4*tan(e/2 + (f*x)/2)*(
4*A - 5*B))/(140*A*c^4 - 175*B*c^4))*(4*A - 5*B))/(4*a*f)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.67

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{a+a\sin(e+fx)}\,dx$$
$$= \frac{c^4(-888a+1050b-1050\cos(fx+e)\,b-299\cos(fx+e)\sin(fx+e)\,b+420afx-525bfx-420\cos}{}$$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e)),x)
```

output
```
(c**4*(6*cos(e + f*x)*sin(e + f*x)**4*b + 8*cos(e + f*x)*sin(e + f*x)**3*a
 - 34*cos(e + f*x)*sin(e + f*x)**3*b - 52*cos(e + f*x)*sin(e + f*x)**2*a +
 101*cos(e + f*x)*sin(e + f*x)**2*b + 220*cos(e + f*x)*sin(e + f*x)*a - 29
9*cos(e + f*x)*sin(e + f*x)*b - 420*cos(e + f*x)*a*f*x + 888*cos(e + f*x)*
a + 525*cos(e + f*x)*b*f*x - 1050*cos(e + f*x)*b + 6*sin(e + f*x)**5*b + 8
*sin(e + f*x)**4*a - 40*sin(e + f*x)**4*b - 60*sin(e + f*x)**3*a + 135*sin
(e + f*x)**3*b + 272*sin(e + f*x)**2*a - 400*sin(e + f*x)**2*b + 420*sin(e
 + f*x)*a*f*x + 220*sin(e + f*x)*a - 525*sin(e + f*x)*b*f*x - 299*sin(e +
f*x)*b + 420*a*f*x - 888*a - 525*b*f*x + 1050*b))/(24*a*f*(cos(e + f*x) -
sin(e + f*x) - 1))
```

3.53 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 157

$$\begin{aligned}\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx &= -\frac{5(3A-4B)c^3x}{2a}-\frac{5(3A-4B)c^3\cos^3(e+fx)}{3af}\\ &\quad-\frac{5(3A-4B)c^3\cos(e+fx)\sin(e+fx)}{2af}\\ &\quad-\frac{a^3(A-B)c^3\cos^7(e+fx)}{f(a+a\sin(e+fx))^4}-\frac{2a^3(3A-4B)c^3\cos^5(e+fx)}{f\left(a^2+a^2\sin(e+fx)\right)^2}\end{aligned}$$

output
```
-5/2*(3*A-4*B)*c^3*x/a-5/3*(3*A-4*B)*c^3*cos(f*x+e)^3/a/f-5/2*(3*A-4*B)*c^
3*cos(f*x+e)*sin(f*x+e)/a/f-a^3*(A-B)*c^3*cos(f*x+e)^7/f/(a+a*sin(f*x+e))^
4-2*a^3*(3*A-4*B)*c^3*cos(f*x+e)^5/f/(a^2+a^2*sin(f*x+e))^2
```

Mathematica [A] (verified)

Time = 12.19 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.40

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx$$
$$= \frac{c^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^3\left(\cos\left(\frac{1}{2}(e+fx)\right)(30(3A-4B)(e+fx)+(4\right.}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3)/(a + a*Sin[e + f*x
]),x]
```

output
```
(c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^3*(Cos[(e +
 f*x)/2]*(30*(3*A - 4*B)*(e + f*x) + (48*A - 93*B)*Cos[e + f*x] + B*Cos[3*
(e + f*x)] - 3*(A - 4*B)*Sin[2*(e + f*x)]) + Sin[(e + f*x)/2]*(-24*B*(-8 +
 5*e + 5*f*x) + 6*A*(-32 + 15*e + 15*f*x) + (48*A - 93*B)*Cos[e + f*x] + B
*Cos[3*(e + f*x)] - 3*(A - 4*B)*Sin[2*(e + f*x)])))/(12*a*f*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])^6*(1 + Sin[e + f*x]))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.84, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.306$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3161, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^3(A+B\sin(e+fx))}{a\sin(e+fx)+a}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(c-c\sin(e+fx))^3(A+B\sin(e+fx))}{a\sin(e+fx)+a}\,dx$$
$$\Big\downarrow \text{3446}$$
$$a^3c^3\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(\sin(e+fx)a+a)^4}\,dx$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\int \frac{\cos(e+fx)^6(A+B\sin(e+fx))}{(\sin(e+fx)a+a)^4}dx$$

$$\Big\downarrow \text{3338}$$

$$a^3c^3\left(-\frac{(3A-4B)\int \frac{\cos^6(e+fx)}{(\sin(e+fx)a+a)^3}dx}{a}-\frac{(A-B)\cos^7(e+fx)}{f(a\sin(e+fx)+a)^4}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left(-\frac{(3A-4B)\int \frac{\cos(e+fx)^6}{(\sin(e+fx)a+a)^3}dx}{a}-\frac{(A-B)\cos^7(e+fx)}{f(a\sin(e+fx)+a)^4}\right)$$

$$\Big\downarrow \text{3159}$$

$$a^3c^3\left(-\frac{(3A-4B)\left(\frac{5\int \frac{\cos^4(e+fx)}{\sin(e+fx)a+a}dx}{a^2}+\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{a}-\frac{(A-B)\cos^7(e+fx)}{f(a\sin(e+fx)+a)^4}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left(-\frac{(3A-4B)\left(\frac{5\int \frac{\cos(e+fx)^4}{\sin(e+fx)a+a}dx}{a^2}+\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{a}-\frac{(A-B)\cos^7(e+fx)}{f(a\sin(e+fx)+a)^4}\right)$$

$$\Big\downarrow \text{3161}$$

$$a^3c^3\left(-\frac{(3A-4B)\left(\frac{5\left(\frac{\int \cos^2(e+fx)dx}{a}+\frac{\cos^3(e+fx)}{3af}\right)}{a^2}+\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{a}-\frac{(A-B)\cos^7(e+fx)}{f(a\sin(e+fx)+a)^4}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left(-\frac{(3A-4B)\left(\frac{5\left(\frac{\int \sin\left(e+fx+\frac{\pi}{2}\right)^2dx}{a}+\frac{\cos^3(e+fx)}{3af}\right)}{a^2}+\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{a}-\frac{(A-B)\cos^7(e+fx)}{f(a\sin(e+fx)+a)^4}\right)$$

$$\Big\downarrow \text{3115}$$

$$a^3c^3\left(-\frac{(3A-4B)\left(\frac{5\left(\frac{\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}}{a}+\frac{\cos^3(e+fx)}{3af}\right)}{a^2}+\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{a}-\frac{(A-B)\cos^7(e+fx)}{f(a\sin(e+fx)+a)^4}\right)$$

$$\Big\downarrow \text{24}$$

$$a^3c^3\left(-\frac{(3A-4B)\left(\frac{5\left(\frac{\cos^3(e+fx)}{3af}+\frac{\frac{\sin(e+fx)\cos(e+fx)}{2f}+\frac{x}{2}}{a}\right)}{a^2}+\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{a}-\frac{(A-B)\cos^7(e+fx)}{f(a\sin(e+fx)+a)^4}\right)$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]),x]
```

output
```
a^3*c^3*(-(((A - B)*Cos[e + f*x]^7)/(f*(a + a*Sin[e + f*x])^4)) - ((3*A -
4*B)*((2*Cos[e + f*x]^5)/(a*f*(a + a*Sin[e + f*x])^2) + (5*(Cos[e + f*x]^3
/(3*a*f) + (x/2 + (Cos[e + f*x]*Sin[e + f*x])/(2*f))/a))/a^2))/a)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.70

method	result
parallelrisch	$\frac{65c^3\left(\frac{4\left(-4A+\frac{23B}{3}\right)\cos(2fx+2e)}{65}+\frac{(A-4B)\sin(3fx+3e)}{65}-\frac{B\cos(4fx+4e)}{195}+\frac{4\left(-3fxA+4fxB-\frac{24}{5}A+\frac{94}{15}B\right)\cos(fx+e)}{13}+\left(A-6\right.\right.}{8af\cos(fx+e)}$
derivativedivides	$\frac{2c^3\left(-\frac{\left(\frac{A}{2}-2B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5+(4A-7B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+(8A-16B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\left(-\frac{A}{2}+2B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+4A-\frac{23B}{3}}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^3}-\frac{5(3A\right.}{fa}$
default	$\frac{2c^3\left(-\frac{\left(\frac{A}{2}-2B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5+(4A-7B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+(8A-16B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\left(-\frac{A}{2}+2B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+4A-\frac{23B}{3}}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^3}-\frac{5(3A\right.}{fa}$
risch	$-\frac{15c^3xA}{2a}+\frac{10c^3xB}{a}-\frac{2c^3e^{i(fx+e)}A}{af}+\frac{31c^3e^{i(fx+e)}B}{8af}-\frac{2c^3e^{-i(fx+e)}A}{af}+\frac{31c^3e^{-i(fx+e)}B}{8af}-\frac{16c^3A}{fa\left(e^{i(fx+e}\right.}$
norman	$\frac{-\frac{72Ac^3-94Bc^3}{3af}-\frac{5(3A-4B)c^3x}{2a}-\frac{\left(9Ac^3-18Bc^3\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{af}-\frac{\left(17Ac^3-20Bc^3\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{af}-\frac{\left(21Ac^3-34Bc^3\right)\tan\left(\frac{f}{}\right.}{3af}}{}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e)),x,method=_RETURNV
ERBOSE)
```

output
```
65/8*c^3*(4/65*(-4*A+23/3*B)*cos(2*f*x+2*e)+1/65*(A-4*B)*sin(3*f*x+3*e)-1/
195*B*cos(4*f*x+4*e)+4/13*(-3*f*x*A+4*f*x*B-24/5*A+94/15*B)*cos(f*x+e)+(A-
68/65*B)*sin(f*x+e)-16/13*A+19/13*B)/a/f/cos(f*x+e)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.39

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^3}{a + a\sin(e + fx)}\,dx = -\frac{2\,Bc^3\cos{(fx+e)}^4 + (3\,A - 10\,B)c^3\cos{(fx+e)}^3 + 15\,(3\,A - 4\,B)c^3fx + 24\,(A - 2\,B)c^3\cos{(fx+e}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
-1/6*(2*B*c^3*cos(f*x + e)^4 + (3*A - 10*B)*c^3*cos(f*x + e)^3 + 15*(3*A -
 4*B)*c^3*f*x + 24*(A - 2*B)*c^3*cos(f*x + e)^2 + 48*(A - B)*c^3 + 3*(5*(3
*A - 4*B)*c^3*f*x + (23*A - 28*B)*c^3)*cos(f*x + e) + (2*B*c^3*cos(f*x + e
)^3 + 15*(3*A - 4*B)*c^3*f*x - 3*(A - 4*B)*c^3*cos(f*x + e)^2 + 3*(7*A - 1
2*B)*c^3*cos(f*x + e) - 48*(A - B)*c^3)*sin(f*x + e))/(a*f*cos(f*x + e) +
a*f*sin(f*x + e) + a*f)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4255 vs. $2(139) = 278$.

Time = 3.98 (sec) , antiderivative size = 4255, normalized size of antiderivative = 27.10

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^3}{a + a\sin(e + fx)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3/(a+a*sin(f*x+e)),x)
```

output
```
Piecewise((-45*A*c**3*f*x*tan(e/2 + f*x/2)**7/(6*a*f*tan(e/2 + f*x/2)**7 +
 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 +
 f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a
*f*tan(e/2 + f*x/2) + 6*a*f) - 45*A*c**3*f*x*tan(e/2 + f*x/2)**6/(6*a*f*ta
n(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5
 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/
2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 135*A*c**3*f*x*tan(e/2 +
 f*x/2)**5/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f
*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2
)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 135*
A*c**3*f*x*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2
+ f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18
*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*
x/2) + 6*a*f) - 135*A*c**3*f*x*tan(e/2 + f*x/2)**3/(6*a*f*tan(e/2 + f*x/2)
**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(
e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2
+ 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 135*A*c**3*f*x*tan(e/2 + f*x/2)**2/(6*
a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x
/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*
tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 45*A*c**3*f*x*t...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1120 vs. $2(151) = 302$.

Time = 0.14 (sec) , antiderivative size = 1120, normalized size of antiderivative = 7.13

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^3}{a + a\sin(e + fx)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorith
m="maxima")
```

output
```
1/3*(B*c^3*((7*sin(f*x + e)/(cos(f*x + e) + 1) + 39*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 24*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 24*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 9*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 9*sin(f*x
+ e)^6/(cos(f*x + e) + 1)^6 + 16)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) +
 3*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*a*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 3*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3*a*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5 + a*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a*sin(f*x + e
)^7/(cos(f*x + e) + 1)^7) + 9*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) -
 3*A*c^3*((sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + 4)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) + 9*B*c^3*((sin(f*x + e)/
(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 4)/(a +
 a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e
)/(cos(f*x + e) + 1))/a) - 18*A*c^3*((sin(f*x + e)/(cos(f*x + e) + 1) +...
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.43

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx =$$

$$-\frac{\frac{15\,(3\,Ac^3-4\,Bc^3)(fx+e)}{a}+\frac{96\,(Ac^3-Bc^3)}{a\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}+\frac{2\left(3\,Ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-12\,Bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+24\,Ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-42\,B\cdots\right.}{\cdots}}{6\,f\cdots}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorith
m="giac")
```

output
```
-1/6*(15*(3*A*c^3 - 4*B*c^3)*(f*x + e)/a + 96*(A*c^3 - B*c^3)/(a*(tan(1/2*
f*x + 1/2*e) + 1)) + 2*(3*A*c^3*tan(1/2*f*x + 1/2*e)^5 - 12*B*c^3*tan(1/2*
f*x + 1/2*e)^5 + 24*A*c^3*tan(1/2*f*x + 1/2*e)^4 - 42*B*c^3*tan(1/2*f*x +
1/2*e)^4 + 48*A*c^3*tan(1/2*f*x + 1/2*e)^2 - 96*B*c^3*tan(1/2*f*x + 1/2*e)
^2 - 3*A*c^3*tan(1/2*f*x + 1/2*e) + 12*B*c^3*tan(1/2*f*x + 1/2*e) + 24*A*c
^3 - 46*B*c^3)/((tan(1/2*f*x + 1/2*e)^2 + 1)^3*a))/f
```

Mupad [B] (verification not implemented)

Time = 37.35 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.03

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx = -\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(7\,A\,c^3-\frac{34\,B\,c^3}{3}\right)+24\,A\,c^3-\frac{94\,B\,c^3}{3}+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5\left(9\,A\,c^3-18\,B\,c^3\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6\,(}{f\,\left(a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7+a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6+3\,a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5\right.} - \frac{5\,c^3\,\mathrm{atan}\left(\frac{5\,c^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(3\,A-4\,B)}{15\,A\,c^3-20\,B\,c^3}\right)(3\,A-4\,B)}{a\,f}$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^3)/(a + a*sin(e + f*x)),x)
```

output
```
- (tan(e/2 + (f*x)/2)*(7*A*c^3 - (34*B*c^3)/3) + 24*A*c^3 - (94*B*c^3)/3 +
 tan(e/2 + (f*x)/2)^5*(9*A*c^3 - 18*B*c^3) + tan(e/2 + (f*x)/2)^6*(17*A*c^
3 - 20*B*c^3) + tan(e/2 + (f*x)/2)^3*(16*A*c^3 - 32*B*c^3) + tan(e/2 + (f*
x)/2)^4*(56*A*c^3 - 62*B*c^3) + tan(e/2 + (f*x)/2)^2*(63*A*c^3 - 76*B*c^3)
)/(f*(a + a*tan(e/2 + (f*x)/2) + 3*a*tan(e/2 + (f*x)/2)^2 + 3*a*tan(e/2 +
(f*x)/2)^3 + 3*a*tan(e/2 + (f*x)/2)^4 + 3*a*tan(e/2 + (f*x)/2)^5 + a*tan(e
/2 + (f*x)/2)^6 + a*tan(e/2 + (f*x)/2)^7)) - (5*c^3*atan((5*c^3*tan(e/2 +
(f*x)/2)*(3*A - 4*B))/(15*A*c^3 - 20*B*c^3))*(3*A - 4*B))/(a*f)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.67

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^3}{a + a\sin(e + fx)}\,dx$$
$$= \frac{c^3(-2\cos(fx + e)\sin(fx + e)^3 b - 3\cos(fx + e)\sin(fx + e)^2 a + 10\cos(fx + e)\sin(fx + e)^2 b + 21 c}{}$$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e)),x)
```

output
```
(c**3*( - 2*cos(e + f*x)*sin(e + f*x)**3*b - 3*cos(e + f*x)*sin(e + f*x)**
2*a + 10*cos(e + f*x)*sin(e + f*x)**2*b + 21*cos(e + f*x)*sin(e + f*x)*a -
 34*cos(e + f*x)*sin(e + f*x)*b - 45*cos(e + f*x)*a*f*x + 102*cos(e + f*x)
*a + 60*cos(e + f*x)*b*f*x - 120*cos(e + f*x)*b - 2*sin(e + f*x)**4*b - 3*
sin(e + f*x)**3*a + 12*sin(e + f*x)**3*b + 24*sin(e + f*x)**2*a - 44*sin(e
 + f*x)**2*b + 45*sin(e + f*x)*a*f*x + 21*sin(e + f*x)*a - 60*sin(e + f*x)
*b*f*x - 34*sin(e + f*x)*b + 45*a*f*x - 102*a - 60*b*f*x + 120*b))/(6*a*f*
(cos(e + f*x) - sin(e + f*x) - 1))
```

3.54 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 118

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx = -\frac{3(2A-3B)c^2x}{2a}-\frac{3(2A-3B)c^2\cos(e+fx)}{2af} - \frac{a^2(A-B)c^2\cos^5(e+fx)}{f(a+a\sin(e+fx))^3}-\frac{(2A-3B)c^2\cos^3(e+fx)}{2f(a+a\sin(e+fx))}$$

output
```
-3/2*(2*A-3*B)*c^2*x/a-3/2*(2*A-3*B)*c^2*cos(f*x+e)/a/f-a^2*(A-B)*c^2*cos(
f*x+e)^5/f/(a+a*sin(f*x+e))^3-1/2*(2*A-3*B)*c^2*cos(f*x+e)^3/f/(a+a*sin(f*
x+e))
```

Mathematica [A] (verified)

Time = 11.62 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.59

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx = -\frac{c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^2\left(\cos\left(\frac{1}{2}(e+fx)\right)(6(2A-3B)(e+fx)+\right.}{4a}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2)/(a + a*Sin[e + f*x
]),x]
```

output
```
-1/4*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^2*(Cos
[(e + f*x)/2]*(6*(2*A - 3*B)*(e + f*x) + 4*(A - 3*B)*Cos[e + f*x] + B*Sin[
2*(e + f*x)]) + Sin[(e + f*x)/2]*(4*A*(-8 + 3*e + 3*f*x) - 2*B*(-16 + 9*e
+ 9*f*x) + 4*(A - 3*B)*Cos[e + f*x] + B*Sin[2*(e + f*x)])))/(a*f*(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2])^4*(1 + Sin[e + f*x]))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3446, 3042, 3338, 3042, 3158, 3042, 3161, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^2(A+B\sin(e+fx))}{a\sin(e+fx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-c\sin(e+fx))^2(A+B\sin(e+fx))}{a\sin(e+fx)+a}dx$$

$$\Big\downarrow \text{3446}$$

$$a^2c^2\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(\sin(e+fx)a+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\int \frac{\cos(e+fx)^4(A+B\sin(e+fx))}{(\sin(e+fx)a+a)^3}dx$$

$$\Big\downarrow \text{3338}$$

$$a^2c^2\left(-\frac{(2A-3B)\int \frac{\cos^4(e+fx)}{(\sin(e+fx)a+a)^2}dx}{a}-\frac{(A-B)\cos^5(e+fx)}{f(a\sin(e+fx)+a)^3}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left(-\frac{(2A-3B)\int\frac{\cos(e+fx)^4}{(\sin(e+fx)a+a)^2}dx}{a}-\frac{(A-B)\cos^5(e+fx)}{f(a\sin(e+fx)+a)^3}\right)$$

$$\Big\downarrow \text{3158}$$

$$a^2c^2\left(-\frac{(2A-3B)\left(\frac{3\int\frac{\cos^2(e+fx)}{\sin(e+fx)a+a}dx}{2a}+\frac{\cos^3(e+fx)}{2f(a^2\sin(e+fx)+a^2)}\right)}{a}-\frac{(A-B)\cos^5(e+fx)}{f(a\sin(e+fx)+a)^3}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left(-\frac{(2A-3B)\left(\frac{3\int\frac{\cos(e+fx)^2}{\sin(e+fx)a+a}dx}{2a}+\frac{\cos^3(e+fx)}{2f(a^2\sin(e+fx)+a^2)}\right)}{a}-\frac{(A-B)\cos^5(e+fx)}{f(a\sin(e+fx)+a)^3}\right)$$

$$\Big\downarrow \text{3161}$$

$$a^2c^2\left(-\frac{(2A-3B)\left(\frac{3\left(\frac{\int 1dx}{a}+\frac{\cos(e+fx)}{af}\right)}{2a}+\frac{\cos^3(e+fx)}{2f(a^2\sin(e+fx)+a^2)}\right)}{a}-\frac{(A-B)\cos^5(e+fx)}{f(a\sin(e+fx)+a)^3}\right)$$

$$\Big\downarrow \text{24}$$

$$a^2c^2\left(-\frac{(2A-3B)\left(\frac{\cos^3(e+fx)}{2f(a^2\sin(e+fx)+a^2)}+\frac{3\left(\frac{\cos(e+fx)}{af}+\frac{x}{a}\right)}{2a}\right)}{a}-\frac{(A-B)\cos^5(e+fx)}{f(a\sin(e+fx)+a)^3}\right)$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]),x]
```

output
```
a^2*c^2*(-(((A - B)*Cos[e + f*x]^5)/(f*(a + a*Sin[e + f*x])^3)) - ((2*A -
3*B)*((3*(x/a + Cos[e + f*x]/(a*f)))/(2*a) + Cos[e + f*x]^3/(2*f*(a^2 + a^
2*Sin[e + f*x]))))/a)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3158
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(a*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
 EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && In
tegersQ[2*m, 2*p]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.80

method	result
parallelrisch	$\frac{4c^2\left(\frac{(-A+3B)\cos(2fx+2e)}{8}-\frac{B\sin(3fx+3e)}{32}+\frac{\left(-3fxA+\frac{9}{2}fxB-5A+7B\right)\cos(fx+e)}{4}+\left(A-\frac{33B}{32}\right)\sin(fx+e)-\frac{9A}{8}+\frac{11B}{8}\right)}{af\cos(fx+e)}$
derivativedivides	$\frac{2c^2\left(-\frac{-\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2}+(A-3B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}+A-3B}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^2}-\frac{3(2A-3B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{4A-4B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+}\right.}{fa}$
default	$\frac{2c^2\left(-\frac{-\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2}+(A-3B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}+A-3B}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^2}-\frac{3(2A-3B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}-\frac{4A-4B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+}\right.}{fa}$
risch	$-\frac{3c^2xA}{a}+\frac{9c^2xB}{2a}-\frac{c^2e^{i(fx+e)}A}{2af}+\frac{3c^2e^{i(fx+e)}B}{2af}-\frac{c^2e^{-i(fx+e)}A}{2af}+\frac{3c^2e^{-i(fx+e)}B}{2af}-\frac{8c^2A}{fa\left(e^{i(fx+e)}+i\right)}-$
norman	$\frac{\frac{\left(6Ac^2-4Bc^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{\left(8Ac^2-9Bc^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{af}+\frac{\left(20Ac^2-15Bc^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{af}+\frac{\left(22Ac^2-20Bc^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}}{}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e)),x,method=_RETURNV
ERBOSE)
```

output
```
4*c^2*(1/8*(-A+3*B)*cos(2*f*x+2*e)-1/32*B*sin(3*f*x+3*e)+1/4*(-3*f*x*A+9/2
*f*x*B-5*A+7*B)*cos(f*x+e)+(A-33/32*B)*sin(f*x+e)-9/8*A+11/8*B)/a/f/cos(f*
x+e)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.52

$$\int\frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx$$
$$=\frac{Bc^2\cos(fx+e)^3-3(2A-3B)c^2fx-2(A-3B)c^2\cos(fx+e)^2-8(A-B)c^2-(3(2A-3B)c^2f}{2(af}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
1/2*(B*c^2*cos(f*x + e)^3 - 3*(2*A - 3*B)*c^2*f*x - 2*(A - 3*B)*c^2*cos(f*
x + e)^2 - 8*(A - B)*c^2 - (3*(2*A - 3*B)*c^2*f*x + (10*A - 13*B)*c^2)*cos
(f*x + e) - (3*(2*A - 3*B)*c^2*f*x + B*c^2*cos(f*x + e)^2 + (2*A - 5*B)*c^
2*cos(f*x + e) - 8*(A - B)*c^2)*sin(f*x + e))/(a*f*cos(f*x + e) + a*f*sin(
f*x + e) + a*f)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2365 vs. $2(99) = 198$.

Time = 2.09 (sec) , antiderivative size = 2365, normalized size of antiderivative = 20.04

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^2}{a + a\sin(e + fx)}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2/(a+a*sin(f*x+e)),x)
```

output
```
Piecewise((-6*A*c**2*f*x*tan(e/2 + f*x/2)**5/(2*a*f*tan(e/2 + f*x/2)**5 +
2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*
x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 6*A*c**2*f*x*tan(e/2 + f*x/2)*
*4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2
+ f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f)
- 12*A*c**2*f*x*tan(e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan
(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 +
 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 12*A*c**2*f*x*tan(e/2 + f*x/2)**2/(2*a*
f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)
**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 6*A*c*
*2*f*x*tan(e/2 + f*x/2)/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2
)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e
/2 + f*x/2) + 2*a*f) - 6*A*c**2*f*x/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan
(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 +
 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 16*A*c**2*tan(e/2 + f*x/2)**4/(2*a*f*ta
n(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3
+ 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 4*A*c**2*t
an(e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4
+ 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 +
f*x/2) + 2*a*f) - 36*A*c**2*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 608 vs. $2(112) = 224$.

Time = 0.12 (sec) , antiderivative size = 608, normalized size of antiderivative = 5.15

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^2}{a + a\sin(e + fx)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorith
m="maxima")
```

output
```
(B*c^2*((sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x
 + e) + 1)^4 + 4)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)
+ 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 2*A*c^2*((sin(f*x + e)/(c
os(f*x + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x
 + e)/(cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/
a) + 4*B*c^2*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin
(f*x + e)/(cos(f*x + e) + 1))/a) - 4*A*c^2*(arctan(sin(f*x + e)/(cos(f*x +
 e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) + 2*B*c^2*(arctan
(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f*x + e)
+ 1))) - 2*A*c^2/(a + a*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.33

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx =$$

$$-\frac{\frac{3\left(2\,Ac^2-3\,Bc^2\right)(fx+e)}{a}+\frac{16\left(Ac^2-Bc^2\right)}{a\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}-\frac{2\left(Bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-2\,Ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+6\,Bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-Bc^2\tan\left(\cdots\right.\right.}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2a}}{2\,f}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorith
m="giac")
```

output
```
-1/2*(3*(2*A*c^2 - 3*B*c^2)*(f*x + e)/a + 16*(A*c^2 - B*c^2)/(a*(tan(1/2*f
*x + 1/2*e) + 1)) - 2*(B*c^2*tan(1/2*f*x + 1/2*e)^3 - 2*A*c^2*tan(1/2*f*x
+ 1/2*e)^2 + 6*B*c^2*tan(1/2*f*x + 1/2*e)^2 - B*c^2*tan(1/2*f*x + 1/2*e) -
 2*A*c^2 + 6*B*c^2)/((tan(1/2*f*x + 1/2*e)^2 + 1)^2*a))/f
```

Mupad [B] (verification not implemented)

Time = 39.10 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.04

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx =$$
$$-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\,(2\,A\,c^2-5\,B\,c^2)+10\,A\,c^2-14\,B\,c^2+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3(2\,A\,c^2-7\,B\,c^2)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4(8}{f\,\left(a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5+a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4+2\,a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3+2\,a\tan\left(\frac{e}{2}+\frac{fx}{2}\right.}$$
$$-\frac{3\,c^2\,\mathrm{atan}\left(\frac{3\,c^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(2\,A-3\,B)}{6\,A\,c^2-9\,B\,c^2}\right)\,(2\,A-3\,B)}{a\,f}$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^2)/(a + a*sin(e + f*x)),x)
```

output
```
- (tan(e/2 + (f*x)/2)*(2*A*c^2 - 5*B*c^2) + 10*A*c^2 - 14*B*c^2 + tan(e/2
+ (f*x)/2)^3*(2*A*c^2 - 7*B*c^2) + tan(e/2 + (f*x)/2)^4*(8*A*c^2 - 9*B*c^2
) + tan(e/2 + (f*x)/2)^2*(18*A*c^2 - 21*B*c^2))/(f*(a + a*tan(e/2 + (f*x)/
2) + 2*a*tan(e/2 + (f*x)/2)^2 + 2*a*tan(e/2 + (f*x)/2)^3 + a*tan(e/2 + (f*
x)/2)^4 + a*tan(e/2 + (f*x)/2)^5)) - (3*c^2*atan((3*c^2*tan(e/2 + (f*x)/2)
*(2*A - 3*B))/(6*A*c^2 - 9*B*c^2))*(2*A - 3*B))/(a*f)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.73

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx$$
$$=\frac{c^2\left(\cos\left(fx+e\right)\sin\left(fx+e\right)^2b+2\cos\left(fx+e\right)\sin\left(fx+e\right)a-5\cos\left(fx+e\right)\sin\left(fx+e\right)b-6\cos\left(fx+\right.\right.}{}$$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e)),x)
```

output
```
(c**2*(cos(e + f*x)*sin(e + f*x)**2*b + 2*cos(e + f*x)*sin(e + f*x)*a - 5*
cos(e + f*x)*sin(e + f*x)*b - 6*cos(e + f*x)*a*f*x + 16*cos(e + f*x)*a + 9
*cos(e + f*x)*b*f*x - 18*cos(e + f*x)*b + sin(e + f*x)**3*b + 2*sin(e + f*
x)**2*a - 6*sin(e + f*x)**2*b + 6*sin(e + f*x)*a*f*x + 2*sin(e + f*x)*a -
9*sin(e + f*x)*b*f*x - 5*sin(e + f*x)*b + 6*a*f*x - 16*a - 9*b*f*x + 18*b)
)/(2*a*f*(cos(e + f*x) - sin(e + f*x) - 1))
```

3.55 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{a+a\sin(e+fx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 57

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{a+a\sin(e+fx)}\,dx = -\frac{(A-2B)cx}{a}+\frac{Bc\cos(e+fx)}{af}-\frac{2(A-B)c\cos(e+fx)}{f(a+a\sin(e+fx))}$$

output
```
-(A-2*B)*c*x/a+B*c*cos(f*x+e)/a/f-2*(A-B)*c*cos(f*x+e)/f/(a+a*sin(f*x+e))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 127 vs. 2(57) = 114.

Time = 5.66 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.23

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{a+a\sin(e+fx)}\,dx = \frac{\left(-((A-2B)x)+\frac{B\cos(e)\cos(fx)}{f}-\frac{B\sin(e)\sin(fx)}{f}+\frac{4(A-B)\sin\left(\frac{fx}{2}\right)}{f\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}\right)(c-c\sin(e+}{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))/(a + a*Sin[e + f*x])
,x]
```

output
```
((-((A - 2*B)*x) + (B*Cos[e]*Cos[f*x])/f - (B*Sin[e]*Sin[f*x])/f + (4*(A -
 B)*Sin[(f*x)/2])/(f*(Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2])))*(c - c*Sin[e + f*x]))/(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {3042, 3446, 3042, 3336, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))(A+B\sin(e+fx))}{a\sin(e+fx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-c\sin(e+fx))(A+B\sin(e+fx))}{a\sin(e+fx)+a}dx$$

$$\Big\downarrow \text{3446}$$

$$ac\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(\sin(e+fx)a+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$ac\int \frac{\cos(e+fx)^2(A+B\sin(e+fx))}{(\sin(e+fx)a+a)^2}dx$$

$$\Big\downarrow \text{3336}$$

$$ac\left(-\frac{\int (a(A-2B)+aB\sin(e+fx))dx}{a^3}-\frac{2(A-B)\cos(e+fx)}{f\left(a^2\sin(e+fx)+a^2\right)}\right)$$

$$\Big\downarrow \text{2009}$$

$$ac\left(-\frac{ax(A-2B)-\frac{aB\cos(e+fx)}{f}}{a^3}-\frac{2(A-B)\cos(e+fx)}{f\left(a^2\sin(e+fx)+a^2\right)}\right)$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))/(a + a*Sin[e + f*x]),x]
```

output
```
a*c*(-((a*(A - 2*B)*x - (a*B*Cos[e + f*x])/f)/a^3) - (2*(A - B)*Cos[e + f*
x])/(f*(a^2 + a^2*Sin[e + f*x])))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3336
```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Simp[1/(b^
3*(2*m + 3))   Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*
(2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2
- b^2, 0] && LtQ[m, -3/2]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.18

method	result
derivativedivides	$\frac{2c\left(\frac{B}{1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}-(A-2B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{2A-2B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)}{fa}$
default	$\frac{2c\left(\frac{B}{1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}-(A-2B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{2A-2B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)}{fa}$
parallelrisch	$\frac{2c\left(\frac{B\cos(2fx+2e)}{4}+\left(-\frac{1}{2}fxA+fxB-A+\frac{3}{2}B\right)\cos(fx+e)+(A-B)\sin(fx+e)-A+\frac{5B}{4}\right)}{af\cos(fx+e)}$
risch	$-\frac{cxA}{a}+\frac{2cxB}{a}+\frac{Bc\,e^{i(fx+e)}}{2af}+\frac{Bc\,e^{-i(fx+e)}}{2af}-\frac{4cA}{fa\left(e^{i(fx+e)}+i\right)}+\frac{4cB}{fa\left(e^{i(fx+e)}+i\right)}$
norman	$\frac{\frac{2Bc\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{4Ac-6Bc}{af}-\frac{(A-2B)cx}{a}+\frac{2Bc\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{af}-\frac{2(4Ac-5Bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{af}-\frac{(4Ac-4Bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{af}-\frac{(A-}{}}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\right.}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x,method=_RETURNVER
BOSE)
```

output
```
2/f*c/a*(B/(1+tan(1/2*f*x+1/2*e)^2)-(A-2*B)*arctan(tan(1/2*f*x+1/2*e))-(2*
A-2*B)/(tan(1/2*f*x+1/2*e)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(57) = 114.

Time = 0.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.05

$$\int\frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{a+a\sin(e+fx)}\,dx = -\frac{(A-2B)cfx-Bc\cos(fx+e)^{2}+2(A-B)c+((A-2B)cfx+(2A-3B)c)\cos(fx+e)+((A}{af\cos(fx+e)+af\sin(fx+e)+af}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm=
"fricas")
```

output
```
-((A - 2*B)*c*f*x - B*c*cos(f*x + e)^2 + 2*(A - B)*c + ((A - 2*B)*c*f*x +
(2*A - 3*B)*c)*cos(f*x + e) + ((A - 2*B)*c*f*x - B*c*cos(f*x + e) - 2*(A -
 B)*c)*sin(f*x + e))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 828 vs. $2(49) = 98$.

Time = 1.09 (sec) , antiderivative size = 828, normalized size of antiderivative = 14.53

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))}{a + a\sin(e + fx)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

output
```
Piecewise((-A*c*f*x*tan(e/2 + f*x/2)**3/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan
(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - A*c*f*x*tan(e/2 + f*x/2)*
*2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/
2) + a*f) - A*c*f*x*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/
2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - A*c*f*x/(a*f*tan(e/2 + f*x/2
)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - 4*A*c*tan(e
/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*ta
n(e/2 + f*x/2) + a*f) - 4*A*c/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x
/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*B*c*f*x*tan(e/2 + f*x/2)**3/(a*f*
tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f
) + 2*B*c*f*x*tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 +
 f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*B*c*f*x*tan(e/2 + f*x/2)/(a*f
*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*
f) + 2*B*c*f*x/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*ta
n(e/2 + f*x/2) + a*f) + 4*B*c*tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3
 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*B*c*tan(e/2 +
 f*x/2)/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 +
 f*x/2) + a*f) + 6*B*c/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2
+ a*f*tan(e/2 + f*x/2) + a*f), Ne(f, 0)), (x*(A + B*sin(e))*(-c*sin(e) + c
)/(a*sin(e) + a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(57) = 114$.

Time = 0.12 (sec) , antiderivative size = 256, normalized size of antiderivative = 4.49

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{a+a\sin(e+fx)}\,dx = \frac{2\left(Bc\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+2}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}+\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}\right)-Ac\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right.}{f}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm=
"maxima")
```

output
```
2*(B*c*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x +
 e)/(cos(f*x + e) + 1))/a) - A*c*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/
a + 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) + B*c*(arctan(sin(f*x + e)/
(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) - A*c/(
a + a*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(57) = 114$.

Time = 0.19 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.02

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{a+a\sin(e+fx)}\,dx = -\frac{\frac{(Ac-2\,Bc)(fx+e)}{a}+\frac{2\left(2\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-2\,Bc\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-Bc\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+2\,Ac-3\,Bc\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)a}}{f}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm=
"giac")
```

output
```
-((A*c - 2*B*c)*(f*x + e)/a + 2*(2*A*c*tan(1/2*f*x + 1/2*e)^2 - 2*B*c*tan(
1/2*f*x + 1/2*e)^2 - B*c*tan(1/2*f*x + 1/2*e) + 2*A*c - 3*B*c)/((tan(1/2*f
*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e) + 1)*a))/f
```

Mupad [B] (verification not implemented)

Time = 37.91 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.93

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{a+a\sin(e+fx)}\,dx = -\frac{(4\,A\,c-4\,B\,c)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-2\,B\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+4\,A\,c-6\,B\,c}{f\,\left(a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a\right)} - \frac{A\,c\,f\,x-2\,B\,c\,f\,x}{a\,f}$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x)))/(a + a*sin(e + f*x)),x)
```

output
```
- (4*A*c - 6*B*c + tan(e/2 + (f*x)/2)^2*(4*A*c - 4*B*c) - 2*B*c*tan(e/2 +
(f*x)/2))/(f*(a + a*tan(e/2 + (f*x)/2) + a*tan(e/2 + (f*x)/2)^2 + a*tan(e/
2 + (f*x)/2)^3)) - (A*c*f*x - 2*B*c*f*x)/(a*f)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.91

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{a+a\sin(e+fx)}\,dx = \frac{c\left(\cos\left(fx+e\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)b+\cos\left(fx+e\right)b-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)afx+4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)a+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)bf}{af\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}$$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

output

```
(c*(cos(e + f*x)*tan((e + f*x)/2)*b + cos(e + f*x)*b - tan((e + f*x)/2)*a*
f*x + 4*tan((e + f*x)/2)*a + 2*tan((e + f*x)/2)*b*f*x - 4*tan((e + f*x)/2)
*b - a*f*x + 2*b*f*x))/(a*f*(tan((e + f*x)/2) + 1))
```

3.56 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 35

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))}\,dx=\frac{B\sec(e+fx)}{acf}+\frac{A\tan(e+fx)}{acf}$$

output
```
B*sec(f*x+e)/a/c/f+A*tan(f*x+e)/a/c/f
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))}\,dx=\frac{B\sec(e+fx)}{acf}+\frac{A\tan(e+fx)}{acf}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x]))
,x]
```

output
```
(B*Sec[e + f*x])/(a*c*f) + (A*Tan[e + f*x])/(a*c*f)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3042, 3446, 3042, 3148, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)(c-c\sin(e+fx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)(c-c\sin(e+fx))}dx$$

$$\Big\downarrow \text{3446}$$

$$\frac{\int \sec^2(e+fx)(A+B\sin(e+fx))dx}{ac}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{A+B\sin(e+fx)}{\cos(e+fx)^2}dx}{ac}$$

$$\Big\downarrow \text{3148}$$

$$\frac{A\int \sec^2(e+fx)dx+\frac{B\sec(e+fx)}{f}}{ac}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\int \csc\left(e+fx+\frac{\pi}{2}\right)^2dx+\frac{B\sec(e+fx)}{f}}{ac}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\frac{B\sec(e+fx)}{f}-\frac{A\int 1d(-\tan(e+fx))}{f}}{ac}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{A\tan(e+fx)}{f}+\frac{B\sec(e+fx)}{f}}{ac}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])),x]
```

output
```
((B*Sec[e + f*x])/f + (A*Tan[e + f*x])/f)/(a*c)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

method	result	size
parallelrisch	$\frac{A\sin(fx+e)+\cos(fx+e)B+B}{\cos(fx+e)fac}$	37
risch	$\frac{2iA+2B\,e^{i(fx+e)}}{\left(e^{i(fx+e)}-i\right)\left(e^{i(fx+e)}+i\right)fac}$	56
derivativedivides	$\frac{-\frac{2\left(\frac{A}{2}+\frac{B}{2}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{2\left(\frac{A}{2}-\frac{B}{2}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{fac}$	57
default	$\frac{-\frac{2\left(\frac{A}{2}+\frac{B}{2}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{2\left(\frac{A}{2}-\frac{B}{2}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{fac}$	57
norman	$\frac{-\frac{2B}{fac}-\frac{2A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{fac}-\frac{2A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{fac}-\frac{2B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{fac}}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}$	123

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x,method=_RETURNVER
BOSE)
```

output
```
(A*sin(f*x+e)+cos(f*x+e)*B+B)/cos(f*x+e)/f/a/c
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))}\,dx=\frac{A\sin\left(fx+e\right)+B}{acf\cos\left(fx+e\right)}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm=
"fricas")
```

output
```
(A*sin(f*x + e) + B)/(a*c*f*cos(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(26) = 52$.

Time = 0.73 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.37

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))(c - c\sin(e + fx))}\,dx$$
$$= \begin{cases} -\frac{2A\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{acf\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)-acf} - \frac{2B}{acf\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)-acf} & \text{for } f \neq 0 \\ \frac{x(A+B\sin(e))}{(a\sin(e)+a)(-c\sin(e)+c)} & \text{otherwise} \end{cases}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

output
```
Piecewise((-2*A*tan(e/2 + f*x/2)/(a*c*f*tan(e/2 + f*x/2)**2 - a*c*f) - 2*B
/(a*c*f*tan(e/2 + f*x/2)**2 - a*c*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin
(e) + a)*(-c*sin(e) + c)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))(c - c\sin(e + fx))}\,dx = \frac{\frac{A\tan(fx+e)}{ac} + \frac{B}{ac\cos(fx+e)}}{f}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm=
"maxima")
```

output
```
(A*tan(f*x + e)/(a*c) + B/(a*c*cos(f*x + e)))/f
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))}\,dx=-\frac{2\left(A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+B\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)acf}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm=
"giac")
```

output
```
-2*(A*tan(1/2*f*x + 1/2*e) + B)/((tan(1/2*f*x + 1/2*e)^2 - 1)*a*c*f)
```

Mupad [B] (verification not implemented)

Time = 36.90 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))}\,dx=-\frac{2\left(B+A\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{a\,c\,f\,\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2-1\right)}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))),x)
```

output
```
-(2*(B + A*tan(e/2 + (f*x)/2)))/(a*c*f*(tan(e/2 + (f*x)/2)^2 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))}\,dx=\frac{-\cos(fx+e)\,b+a\sin(fx+e)+b}{\cos(fx+e)\,acf}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

output
```
( - cos(e + f*x)*b + sin(e + f*x)*a + b)/(cos(e + f*x)*a*c*f)
```

3.57 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 63

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^2}\,dx = \frac{(A+B)\sec(e+fx)}{3af\,(c^2-c^2\sin(e+fx))}+\frac{(2A-B)\tan(e+fx)}{3ac^2f}$$

output
```
1/3*(A+B)*sec(f*x+e)/a/f/(c^2-c^2*sin(f*x+e))+1/3*(2*A-B)*tan(f*x+e)/a/c^2
/f
```

Mathematica [A] (verified)

Time = 3.13 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.71

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^2}\,dx = \frac{\cos(e+fx)(6B-2(A+B)\cos(e+fx)+(4A-2B)\cos(2(e+fx))+8A\sin(e+fx)-4B\sin(e+f}{12ac^2f(-1+\sin(e+fx))^2(1+\sin(e+fx))}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^
2),x]
```

output
```
(Cos[e + f*x]*(6*B - 2*(A + B)*Cos[e + f*x] + (4*A - 2*B)*Cos[2*(e + f*x)]
 + 8*A*Sin[e + f*x] - 4*B*Sin[e + f*x] + A*Sin[2*(e + f*x)] + B*Sin[2*(e +
 f*x)]))/(12*a*c^2*f*(-1 + Sin[e + f*x])^2*(1 + Sin[e + f*x]))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3042, 3446, 3042, 3338, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)(c-c\sin(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)(c-c\sin(e+fx))^2}dx$$

$$\Big\downarrow \text{3446}$$

$$\frac{\int \frac{\sec^2(e+fx)(A+B\sin(e+fx))}{c-c\sin(e+fx)}dx}{ac}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{A+B\sin(e+fx)}{\cos(e+fx)^2(c-c\sin(e+fx))}dx}{ac}$$

$$\Big\downarrow \text{3338}$$

$$\frac{\frac{(2A-B)\int \sec^2(e+fx)dx}{3c}+\frac{(A+B)\sec(e+fx)}{3f(c-c\sin(e+fx))}}{ac}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(2A-B)\int \csc\left(e+fx+\frac{\pi}{2}\right)^2dx}{3c}+\frac{(A+B)\sec(e+fx)}{3f(c-c\sin(e+fx))}}{ac}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\frac{(A+B)\sec(e+fx)}{3f(c-c\sin(e+fx))}-\frac{(2A-B)\int 1d(-\tan(e+fx))}{3cf}}{ac}$$

$$\Big\downarrow \color{blue}{24}$$

$$\frac{\frac{(2A-B)\tan(e+fx)}{3cf}+\frac{(A+B)\sec(e+fx)}{3f(c-c\sin(e+fx))}}{ac}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^2),x]
```

output
```
(((A + B)*Sec[e + f*x])/(3*f*(c - c*Sin[e + f*x])) + ((2*A - B)*Tan[e + f*
x])/(3*c*f))/(a*c)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.37

method	result
risch	$-\frac{2i\left(4iA\,e^{i(fx+e)}-2iB\,e^{i(fx+e)}+3B\,e^{2i(fx+e)}+2A-B\right)}{3\left(e^{i(fx+e)}-i\right)^{3}\left(e^{i(fx+e)}+i\right)f a c^{2}}$
derivativedivides	$\frac{-\frac{2\left(\frac{A}{4}-\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(A+B)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}-\frac{A+B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{2\left(\frac{3A}{4}+\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}}{f a c^{2}}$
default	$\frac{-\frac{2\left(\frac{A}{4}-\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(A+B)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}-\frac{A+B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{2\left(\frac{3A}{4}+\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}}{f a c^{2}}$
parallelrisch	$\frac{-6A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+(6A-6B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}+(-2A+4B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-2A-2B}{3f a c^{2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}$
norman	$\frac{\frac{2A-4B}{6acf}-\frac{4(2A-B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3acf}-\frac{A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{acf}+\frac{(2A-4B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{2acf}-\frac{(8A-4B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3acf}+\frac{(14A-16B)\tan\left(\frac{fx}{2}+\cdots\right.}{6acf}}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)c\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x,method=_RETURNV
ERBOSE)
```

output
```
-2/3*I*(4*I*A*exp(I*(f*x+e))-2*I*B*exp(I*(f*x+e))+3*B*exp(2*I*(f*x+e))+2*A
-B)/(exp(I*(f*x+e))-I)^3/(exp(I*(f*x+e))+I)/f/a/c^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.16

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))(c - c\sin(e + fx))^2}\,dx$$
$$= -\frac{(2A - B)\cos(fx + e)^2 + (2A - B)\sin(fx + e) - A + 2B}{3\,(ac^2 f\cos(fx + e)\sin(fx + e) - ac^2 f\cos(fx + e))}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorith
m="fricas")
```

output
```
-1/3*((2*A - B)*cos(f*x + e)^2 + (2*A - B)*sin(f*x + e) - A + 2*B)/(a*c^2*
f*cos(f*x + e)*sin(f*x + e) - a*c^2*f*cos(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 578 vs. 2(51) = 102.

Time = 2.33 (sec) , antiderivative size = 578, normalized size of antiderivative = 9.17

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))(c - c\sin(e + fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**2,x)
```

output
```
Piecewise((-6*A*tan(e/2 + f*x/2)**3/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*
c**2*f*tan(e/2 + f*x/2)**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f) + 6
*A*tan(e/2 + f*x/2)**2/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c**2*f*tan(e/
2 + f*x/2)**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f) - 2*A*tan(e/2 +
f*x/2)/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c**2*f*tan(e/2 + f*x/2)**3 +
6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f) - 2*A/(3*a*c**2*f*tan(e/2 + f*x/
2)**4 - 6*a*c**2*f*tan(e/2 + f*x/2)**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a
*c**2*f) - 6*B*tan(e/2 + f*x/2)**2/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c
**2*f*tan(e/2 + f*x/2)**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f) + 4*
B*tan(e/2 + f*x/2)/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c**2*f*tan(e/2 +
f*x/2)**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f) - 2*B/(3*a*c**2*f*ta
n(e/2 + f*x/2)**4 - 6*a*c**2*f*tan(e/2 + f*x/2)**3 + 6*a*c**2*f*tan(e/2 +
f*x/2) - 3*a*c**2*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(e) + a)*(-c*sin
(e) + c)**2), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(60) = 120$.

Time = 0.04 (sec) , antiderivative size = 266, normalized size of antiderivative = 4.22

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))(c - c\sin(e + fx))^2}\,dx = -\frac{2\left(\frac{B\left(\frac{2\sin(fx+e)}{\cos(fx+e)+1} - \frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - 1\right)}{ac^2 - \frac{2ac^2\sin(fx+e)}{\cos(fx+e)+1} + \frac{2ac^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3} - \frac{ac^2\sin(fx+e)^4}{(\cos(fx+e)+1)^4}} - \frac{A\left(\frac{\sin(fx+e)}{\cos(fx+e)+1} - \frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + 1\right)}{ac^2 - \frac{2ac^2\sin(fx+e)}{\cos(fx+e)+1} + \frac{2ac^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3} - \frac{ac^2\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}\right)}{3f}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorith
m="maxima")
```

output
```
-2/3*(B*(2*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 - 1)/(a*c^2 - 2*a*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*c^2*s
in(f*x + e)^3/(cos(f*x + e) + 1)^3 - a*c^2*sin(f*x + e)^4/(cos(f*x + e) +
1)^4) - A*(sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a*c^2 - 2*a*c^2*sin
(f*x + e)/(cos(f*x + e) + 1) + 2*a*c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
 - a*c^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4))/f
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.54

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^2}\,dx = -\frac{\frac{3\,(A-B)}{ac^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}+\frac{9\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-12\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+7\,A+B}{ac^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^3}}{6\,f}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorith
m="giac")
```

output
```
-1/6*(3*(A - B)/(a*c^2*(tan(1/2*f*x + 1/2*e) + 1)) + (9*A*tan(1/2*f*x + 1/
2*e)^2 + 3*B*tan(1/2*f*x + 1/2*e)^2 - 12*A*tan(1/2*f*x + 1/2*e) + 7*A + B)
/(a*c^2*(tan(1/2*f*x + 1/2*e) - 1)^3))/f
```

Mupad [B] (verification not implemented)

Time = 37.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.87

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^2}\,dx = \frac{2\left(\frac{3\,B}{2}+A\,\cos\left(e+f\,x\right)+B\,\cos\left(e+f\,x\right)+2\,A\,\sin\left(e+f\,x\right)-B\,\sin\left(e+f\,x\right)+A\,\cos\left(2\,e+2\,f\,x\right)-\right.}{3\,a\,c^2\,f\,\left(2\,\cos\left(e+f\,x\right)-\sin\left(2\,e+2\,f\,x\right)\right)}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^2),x)
```

output
```
(2*((3*B)/2 + A*cos(e + f*x) + B*cos(e + f*x) + 2*A*sin(e + f*x) - B*sin(e
 + f*x) + A*cos(2*e + 2*f*x) - (B*cos(2*e + 2*f*x))/2 - (A*sin(2*e + 2*f*x
))/2 - (B*sin(2*e + 2*f*x))/2))/(3*a*c^2*f*(2*cos(e + f*x) - sin(2*e + 2*f
*x)))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.94

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^2}\,dx$$
$$= \frac{-2\cos(fx+e)\sin(fx+e)\,a+\cos(fx+e)\sin(fx+e)\,b+2\cos(fx+e)\,a-\cos(fx+e)\,b+2\sin(f}{3\cos(fx+e)\,a\,c^2 f\,(\sin(fx+e)-1}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x)
```

output
```
( - 2*cos(e + f*x)*sin(e + f*x)*a + cos(e + f*x)*sin(e + f*x)*b + 2*cos(e
+ f*x)*a - cos(e + f*x)*b + 2*sin(e + f*x)**2*a - sin(e + f*x)**2*b - 2*si
n(e + f*x)*a + sin(e + f*x)*b - a - b)/(3*cos(e + f*x)*a*c**2*f*(sin(e + f
*x) - 1))
```

3.58 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 102

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^3}\,dx = \frac{(A+B)\sec(e+fx)}{5acf(c-c\sin(e+fx))^2}+\frac{(3A-2B)\sec(e+fx)}{15af\left(c^3-c^3\sin(e+fx)\right)}+\frac{2(3A-2B)\tan(e+fx)}{15ac^3f}$$

output
```
1/5*(A+B)*sec(f*x+e)/a/c/f/(c-c*sin(f*x+e))^2+1/15*(3*A-2*B)*sec(f*x+e)/a/
f/(c^3-c^3*sin(f*x+e))+2/15*(3*A-2*B)*tan(f*x+e)/a/c^3/f
```

Mathematica [A] (verified)

Time = 3.89 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.54

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^3}\,dx = -\frac{\cos(e+fx)(80B+5(-9A+B)\cos(e+fx)+32(3A-2B)\cos(2(e+fx))+9A\cos(3(e+fx))-B}{24}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^
3),x]
```

output
```
-1/240*(Cos[e + f*x]*(80*B + 5*(-9*A + B)*Cos[e + f*x] + 32*(3*A - 2*B)*Co
s[2*(e + f*x)] + 9*A*Cos[3*(e + f*x)] - B*Cos[3*(e + f*x)] + 120*A*Sin[e +
 f*x] - 80*B*Sin[e + f*x] + 36*A*Sin[2*(e + f*x)] - 4*B*Sin[2*(e + f*x)] -
 24*A*Sin[3*(e + f*x)] + 16*B*Sin[3*(e + f*x)]))/(a*c^3*f*(-1 + Sin[e + f*
x])^3*(1 + Sin[e + f*x]))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.92, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3446, 3042, 3338, 3042, 3151, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)(c-c\sin(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)(c-c\sin(e+fx))^3}dx$$

$$\Big\downarrow \text{3446}$$

$$\frac{\int \frac{\sec^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}dx}{ac}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{A+B\sin(e+fx)}{\cos(e+fx)^2(c-c\sin(e+fx))^2}dx}{ac}$$

$$\Big\downarrow \text{3338}$$

$$\frac{\frac{(3A-2B)\int \frac{\sec^2(e+fx)}{c-c\sin(e+fx)}dx}{5c}+\frac{(A+B)\sec(e+fx)}{5f(c-c\sin(e+fx))^2}}{ac}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(3A-2B)\int \frac{1}{\cos(e+fx)^2(c-c\sin(e+fx))}dx}{5c}+\frac{(A+B)\sec(e+fx)}{5f(c-c\sin(e+fx))^2}}{ac}$$

$$\Big\downarrow \text{3151}$$

$$\frac{\frac{(3A-2B)\left(\frac{2\int \sec^2(e+fx)dx}{3c}+\frac{\sec(e+fx)}{3f(c-c\sin(e+fx))}\right)}{5c}+\frac{(A+B)\sec(e+fx)}{5f(c-c\sin(e+fx))^2}}{ac}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(3A-2B)\left(\frac{2\int \csc\left(e+fx+\frac{\pi}{2}\right)^2dx}{3c}+\frac{\sec(e+fx)}{3f(c-c\sin(e+fx))}\right)}{5c}+\frac{(A+B)\sec(e+fx)}{5f(c-c\sin(e+fx))^2}}{ac}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\frac{(3A-2B)\left(\frac{\sec(e+fx)}{3f(c-c\sin(e+fx))}-\frac{2\int 1d(-\tan(e+fx))}{3cf}\right)}{5c}+\frac{(A+B)\sec(e+fx)}{5f(c-c\sin(e+fx))^2}}{ac}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{(A+B)\sec(e+fx)}{5f(c-c\sin(e+fx))^2}+\frac{(3A-2B)\left(\frac{2\tan(e+fx)}{3cf}+\frac{\sec(e+fx)}{3f(c-c\sin(e+fx))}\right)}{5c}}{ac}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^3),x]
```

output
```
(((A + B)*Sec[e + f*x])/(5*f*(c - c*Sin[e + f*x])^2) + ((3*A - 2*B)*(Sec[e
 + f*x]/(3*f*(c - c*Sin[e + f*x])) + (2*Tan[e + f*x])/(3*c*f)))/(5*c))/(a*
c)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3151
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Simp[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1])   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simpli
fy[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.09

method	result
risch	$-\frac{4(15iA\,e^{2i(fx+e)}+12A\,e^{i(fx+e)}-10iB\,e^{2i(fx+e)}+10B\,e^{3i(fx+e)}+2iB-8B\,e^{i(fx+e)}-3iA)}{15(e^{i(fx+e)}-i)^5(e^{i(fx+e)}+i)fa\,c^3}$
parallelrisch	$\frac{-30A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5+(60A-30B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+(-60A+40B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3-40B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+(18A+8B)\tan(}{15fa\,c^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}$
derivativedivides	$\frac{-\frac{2\left(\frac{A}{8}-\frac{B}{8}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(2A+2B)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{4A+4B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{\frac{5A}{2}+\frac{3B}{2}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{2\left(\frac{7A}{8}+\frac{B}{8}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{2\left(\frac{9A}{2}+\frac{7B}{2}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}}{fa\,c^3}$
default	$\frac{-\frac{2\left(\frac{A}{8}-\frac{B}{8}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(2A+2B)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5}-\frac{4A+4B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4}-\frac{\frac{5A}{2}+\frac{3B}{2}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{2\left(\frac{7A}{8}+\frac{B}{8}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{2\left(\frac{9A}{2}+\frac{7B}{2}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}}{fa\,c^3}$
norman	$\frac{-\frac{12A+2B}{15afc}+\frac{2(6A-7B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{3afc}-\frac{2A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{afc}-\frac{2(-8B+7A)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{5afc}+\frac{2(2A-B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{afc}-\frac{2(9A-4B)\,\mathrm{ta}}{3a}}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)c^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x,method=_RETURNV
ERBOSE)
```

output
```
-4/15*(15*I*A*exp(2*I*(f*x+e))+12*A*exp(I*(f*x+e))-10*I*B*exp(2*I*(f*x+e))
+10*B*exp(3*I*(f*x+e))+2*I*B-8*B*exp(I*(f*x+e))-3*I*A)/(exp(I*(f*x+e))-I)^
5/(exp(I*(f*x+e))+I)/f/a/c^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.05

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^3}\,dx = -\frac{4\,(3\,A-2\,B)\cos{(fx+e)}^2-\left(2\,(3\,A-2\,B)\cos{(fx+e)}^2-9\,A+6\,B\right)\sin{(fx+e)}-6\,A+9\,B}{15\,\left(ac^3f\cos{(fx+e)}^3+2\,ac^3f\cos{(fx+e)}\sin{(fx+e)}-2\,ac^3f\cos{(fx+e)}\right)}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorith
m="fricas")
```

output
```
-1/15*(4*(3*A - 2*B)*cos(f*x + e)^2 - (2*(3*A - 2*B)*cos(f*x + e)^2 - 9*A
+ 6*B)*sin(f*x + e) - 6*A + 9*B)/(a*c^3*f*cos(f*x + e)^3 + 2*a*c^3*f*cos(f
*x + e)*sin(f*x + e) - 2*a*c^3*f*cos(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1236 vs. $2(85) = 170$.

Time = 4.82 (sec) , antiderivative size = 1236, normalized size of antiderivative = 12.12

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**3,x)
```

output
```
Piecewise((-30*A*tan(e/2 + f*x/2)**5/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60
*a*c**3*f*tan(e/2 + f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**
3*f*tan(e/2 + f*x/2)**2 + 60*a*c**3*f*tan(e/2 + f*x/2) - 15*a*c**3*f) + 60
*A*tan(e/2 + f*x/2)**4/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(
e/2 + f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 +
f*x/2)**2 + 60*a*c**3*f*tan(e/2 + f*x/2) - 15*a*c**3*f) - 60*A*tan(e/2 + f
*x/2)**3/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2 + f*x/2)**
5 + 75*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/2)**2 + 60
*a*c**3*f*tan(e/2 + f*x/2) - 15*a*c**3*f) + 18*A*tan(e/2 + f*x/2)/(15*a*c*
*3*f*tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2 + f*x/2)**5 + 75*a*c**3*f*t
an(e/2 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/2)**2 + 60*a*c**3*f*tan(e/2
 + f*x/2) - 15*a*c**3*f) - 12*A/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*a*c*
*3*f*tan(e/2 + f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*f*t
an(e/2 + f*x/2)**2 + 60*a*c**3*f*tan(e/2 + f*x/2) - 15*a*c**3*f) - 30*B*ta
n(e/2 + f*x/2)**4/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2 +
 f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/2
)**2 + 60*a*c**3*f*tan(e/2 + f*x/2) - 15*a*c**3*f) + 40*B*tan(e/2 + f*x/2)
**3/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2 + f*x/2)**5 + 7
5*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/2)**2 + 60*a*c*
*3*f*tan(e/2 + f*x/2) - 15*a*c**3*f) - 40*B*tan(e/2 + f*x/2)**2/(15*a*c...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(98) = 196$.

Time = 0.05 (sec) , antiderivative size = 423, normalized size of antiderivative = 4.15

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))(c - c\sin(e + fx))^3}\,dx = -\frac{2\left(\frac{B\left(\frac{4\sin(fx+e)}{\cos(fx+e)+1} - \frac{20\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{20\sin(fx+e)^3}{(\cos(fx+e)+1)^3} - \frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4} - 1\right)}{ac^3 - \frac{4ac^3\sin(fx+e)}{\cos(fx+e)+1} + \frac{5ac^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{5ac^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{4ac^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5} - \frac{ac^3\sin(fx+e)^6}{(\cos(fx+e)+1)^6}} + \frac{3A\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1} - \frac{10\sin(fx+e}{(\cos(fx+e)+1}\right.}{ac^3 - \frac{4ac^3\sin(fx+e)}{\cos(fx+e)+1} + \frac{5ac^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - }\right.}{15f}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorith
m="maxima")
```

output
```
-2/15*(B*(4*sin(f*x + e)/(cos(f*x + e) + 1) - 20*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 15*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 - 1)/(a*c^3 - 4*a*c^3*sin(f*x + e)/(cos(f*x + e) + 1) +
 5*a*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 5*a*c^3*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 + 4*a*c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - a*c^3*sin
(f*x + e)^6/(cos(f*x + e) + 1)^6) + 3*A*(3*sin(f*x + e)/(cos(f*x + e) + 1)
 - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 10*sin(f*x + e)^4/(cos(f*x + e
) + 1)^4 - 5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 2)/(a*c^3 - 4*a*c^3*sin
(f*x + e)/(cos(f*x + e) + 1) + 5*a*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 - 5*a*c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 4*a*c^3*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5 - a*c^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6))/f
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.64

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^3}\,dx =$$

$$-\frac{\frac{15\,(A-B)}{ac^3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}+\frac{105\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+15\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-270\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+30\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+360\,A\tan\left(\frac{1}{2}\,fx\right.}{ac^3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)}}{60\,f}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorith
m="giac")
```

output
```
-1/60*(15*(A - B)/(a*c^3*(tan(1/2*f*x + 1/2*e) + 1)) + (105*A*tan(1/2*f*x
+ 1/2*e)^4 + 15*B*tan(1/2*f*x + 1/2*e)^4 - 270*A*tan(1/2*f*x + 1/2*e)^3 +
30*B*tan(1/2*f*x + 1/2*e)^3 + 360*A*tan(1/2*f*x + 1/2*e)^2 - 40*B*tan(1/2*
f*x + 1/2*e)^2 - 210*A*tan(1/2*f*x + 1/2*e) + 50*B*tan(1/2*f*x + 1/2*e) +
63*A - 7*B)/(a*c^3*(tan(1/2*f*x + 1/2*e) - 1)^5))/f
```

Mupad [B] (verification not implemented)

Time = 37.30 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.75

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^3}\,dx$$
$$= \frac{2\left(\frac{5\,B\,\sin(e+f\,x)}{2}-\frac{15\,A\,\cos(e+f\,x)}{4}-\frac{5\,B\,\cos(e+f\,x)}{8}-\frac{15\,A\,\sin(e+f\,x)}{4}-\frac{5\,B}{2}-3\,A\,\cos\left(2\,e+2\,f\,x\right)+\frac{3\,A\,\cos(3\,e+3\,f\,x}{4}\right.}{15\,a\,c^3\,f\,\left(\frac{\cos(3\,e+3\,f\,x)}{4}-\right.}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^3),x)
```

output
```
(2*((5*B*sin(e + f*x))/2 - (15*A*cos(e + f*x))/4 - (5*B*cos(e + f*x))/8 -
(15*A*sin(e + f*x))/4 - (5*B)/2 - 3*A*cos(2*e + 2*f*x) + (3*A*cos(3*e + 3*
f*x))/4 + 2*B*cos(2*e + 2*f*x) + (B*cos(3*e + 3*f*x))/8 + 3*A*sin(2*e + 2*
f*x) + (3*A*sin(3*e + 3*f*x))/4 + (B*sin(2*e + 2*f*x))/2 - (B*sin(3*e + 3*
f*x))/2))/(15*a*c^3*f*(cos(3*e + 3*f*x)/4 - (5*cos(e + f*x))/4 + sin(2*e +
 2*f*x)))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.86

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^3}\,dx$$
$$= \frac{-3\cos\left(fx+e\right)\sin\left(fx+e\right)^2a+2\cos\left(fx+e\right)\sin\left(fx+e\right)^2b+6\cos\left(fx+e\right)\sin\left(fx+e\right)a-4\cos\left(fx\right.}{}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x)
```

output
```
( - 3*cos(e + f*x)*sin(e + f*x)**2*a + 2*cos(e + f*x)*sin(e + f*x)**2*b +
6*cos(e + f*x)*sin(e + f*x)*a - 4*cos(e + f*x)*sin(e + f*x)*b - 3*cos(e +
f*x)*a + 2*cos(e + f*x)*b + 12*sin(e + f*x)**3*a - 8*sin(e + f*x)**3*b - 2
4*sin(e + f*x)**2*a + 16*sin(e + f*x)**2*b + 6*sin(e + f*x)*a - 4*sin(e +
f*x)*b + 12*a + 2*b)/(30*cos(e + f*x)*a*c**3*f*(sin(e + f*x)**2 - 2*sin(e
+ f*x) + 1))
```

3.59 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^4}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 142

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^4}\,dx = \frac{(A+B)\sec(e+fx)}{7acf(c-c\sin(e+fx))^3}+\frac{(4A-3B)\sec(e+fx)}{35af\left(c^2-c^2\sin(e+fx)\right)^2}+\frac{(4A-3B)\sec(e+fx)}{35af\left(c^4-c^4\sin(e+fx)\right)}+\frac{2(4A-3B)\tan(e+fx)}{35ac^4f}$$

output
```
1/7*(A+B)*sec(f*x+e)/a/c/f/(c-c*sin(f*x+e))^3+1/35*(4*A-3*B)*sec(f*x+e)/a/
f/(c^2-c^2*sin(f*x+e))^2+1/35*(4*A-3*B)*sec(f*x+e)/a/f/(c^4-c^4*sin(f*x+e)
)+2/35*(4*A-3*B)*tan(f*x+e)/a/c^4/f
```

Mathematica [A] (verified)

Time = 4.99 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.69

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^4}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(560B+(-406A+182B)\cos}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^
4),x]
```

output
```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])*(560*B + (-406*A + 182*B)*Cos[e + f*x] + 224*(4*A - 3*B)*Cos[2*(e + f*x
)] + 174*A*Cos[3*(e + f*x)] - 78*B*Cos[3*(e + f*x)] - 64*A*Cos[4*(e + f*x)
] + 48*B*Cos[4*(e + f*x)] + 896*A*Sin[e + f*x] - 672*B*Sin[e + f*x] + 406*
A*Sin[2*(e + f*x)] - 182*B*Sin[2*(e + f*x)] - 384*A*Sin[3*(e + f*x)] + 288
*B*Sin[3*(e + f*x)] - 29*A*Sin[4*(e + f*x)] + 13*B*Sin[4*(e + f*x)]))/(224
0*a*c^4*f*(-1 + Sin[e + f*x])^4*(1 + Sin[e + f*x]))
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.90, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.306$, Rules used = {3042, 3446, 3042, 3338, 3042, 3151, 3042, 3151, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)(c-c\sin(e+fx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)(c-c\sin(e+fx))^4}dx$$

$$\Big\downarrow \text{3446}$$

$$\frac{\int \frac{\sec^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}dx}{ac}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{A+B\sin(e+fx)}{\cos(e+fx)^2(c-c\sin(e+fx))^3}dx}{ac}$$

$$\Big\downarrow \text{3338}$$

$$\frac{\frac{(4A-3B)\int \frac{\sec^2(e+fx)}{(c-c\sin(e+fx))^2}dx}{7c}+\frac{(A+B)\sec(e+fx)}{7f(c-c\sin(e+fx))^3}}{ac}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(4A-3B)\int \frac{1}{\cos(e+fx)^2(c-c\sin(e+fx))^2}dx}{7c}+\frac{(A+B)\sec(e+fx)}{7f(c-c\sin(e+fx))^3}}{ac}$$

$$\Big\downarrow \text{3151}$$

$$\frac{\frac{(4A-3B)\left(\frac{3\int \frac{\sec^2(e+fx)}{c-c\sin(e+fx)}dx}{5c}+\frac{\sec(e+fx)}{5f(c-c\sin(e+fx))^2}\right)}{7c}+\frac{(A+B)\sec(e+fx)}{7f(c-c\sin(e+fx))^3}}{ac}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(4A-3B)\left(\frac{3\int \frac{1}{\cos(e+fx)^2(c-c\sin(e+fx))}dx}{5c}+\frac{\sec(e+fx)}{5f(c-c\sin(e+fx))^2}\right)}{7c}+\frac{(A+B)\sec(e+fx)}{7f(c-c\sin(e+fx))^3}}{ac}$$

$$\Big\downarrow \text{3151}$$

$$\frac{\frac{(4A-3B)\left(\frac{3\left(\frac{2\int \sec^2(e+fx)dx}{3c}+\frac{\sec(e+fx)}{3f(c-c\sin(e+fx))}\right)}{5c}+\frac{\sec(e+fx)}{5f(c-c\sin(e+fx))^2}\right)}{7c}+\frac{(A+B)\sec(e+fx)}{7f(c-c\sin(e+fx))^3}}{ac}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(4A-3B)\left(\frac{3\left(\frac{2\int \csc\left(e+fx+\frac{\pi}{2}\right)^2dx}{3c}+\frac{\sec(e+fx)}{3f(c-c\sin(e+fx))}\right)}{5c}+\frac{\sec(e+fx)}{5f(c-c\sin(e+fx))^2}\right)}{7c}+\frac{(A+B)\sec(e+fx)}{7f(c-c\sin(e+fx))^3}}{ac}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\frac{(4A-3B)\left(\frac{3\left(\frac{\sec(e+fx)}{3f(c-c\sin(e+fx))}-\frac{2\int 1d(-\tan(e+fx))}{3cf}\right)}{5c}+\frac{\sec(e+fx)}{5f(c-c\sin(e+fx))^2}\right)}{7c}+\frac{(A+B)\sec(e+fx)}{7f(c-c\sin(e+fx))^3}}{ac}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{(A+B)\sec(e+fx)}{7f(c-c\sin(e+fx))^3}+\frac{(4A-3B)\left(\frac{\sec(e+fx)}{5f(c-c\sin(e+fx))^2}+\frac{3\left(\frac{2\tan(e+fx)}{3cf}+\frac{\sec(e+fx)}{3f(c-c\sin(e+fx))}\right)}{5c}\right)}{7c}}{ac}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^4),x]
```

output
```
(((A + B)*Sec[e + f*x])/(7*f*(c - c*Sin[e + f*x])^3) + ((4*A - 3*B)*(Sec[e
 + f*x]/(5*f*(c - c*Sin[e + f*x])^2) + (3*(Sec[e + f*x]/(3*f*(c - c*Sin[e
+ f*x])) + (2*Tan[e + f*x])/(3*c*f)))/(5*c)))/(7*c))/(a*c)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3151
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Simp[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1])   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simpli
fy[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.86 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.96

method	result
risch	$\frac{4i\left(56iA\,\mathrm{e}^{3i(fx+e)}-42iB\,\mathrm{e}^{3i(fx+e)}+35B\,\mathrm{e}^{4i(fx+e)}-24iA\,\mathrm{e}^{i(fx+e)}+56A\,\mathrm{e}^{2i(fx+e)}+18iB\,\mathrm{e}^{i(fx+e)}-42B\,\mathrm{e}^{2i(fx+e)}-4A}{35\left(\mathrm{e}^{i(fx+e)}-i\right)^{7}\left(\mathrm{e}^{i(fx+e)}+i\right)fac^{4}}$
parallelrisch	$\frac{-70A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}+(210A-70B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}+(-350A+140B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}+(210A-210B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}+(14}{35fac^{4}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\left(\tan\left(\frac{f}{2}\right.\right.}$
derivativedivides	$\frac{-\frac{2\left(\frac{A}{16}-\frac{B}{16}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(4A+4B)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}-\frac{12A+12B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{6}}-\frac{18A+14B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{2(19A+17B)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{2\left(\frac{15A}{16}+\frac{B}{16}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}}{fac^{4}}$
default	$\frac{-\frac{2\left(\frac{A}{16}-\frac{B}{16}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(4A+4B)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}-\frac{12A+12B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{6}}-\frac{18A+14B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{2(19A+17B)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}}-\frac{2\left(\frac{15A}{16}+\frac{B}{16}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}}{fac^{4}}$
norman	$\frac{\frac{(6A-2B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}}{afc}-\frac{26A-2B}{35afc}-\frac{12(4A-3B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5afc}-\frac{2A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}{afc}+\frac{2(4A-18B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{5afc}-\frac{4(3A-B)\tan\left(\right.}{afc}}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\right.}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x,method=_RETURNV
ERBOSE)
```

output
```
4/35*I*(56*I*A*exp(3*I*(f*x+e))-42*I*B*exp(3*I*(f*x+e))+35*B*exp(4*I*(f*x+
e))-24*I*A*exp(I*(f*x+e))+56*A*exp(2*I*(f*x+e))+18*I*B*exp(I*(f*x+e))-42*B
*exp(2*I*(f*x+e))-4*A+3*B)/(exp(I*(f*x+e))-I)^7/(exp(I*(f*x+e))+I)/f/a/c^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.99

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^4}\,dx$$
$$= \frac{2\,(4\,A-3\,B)\cos\left(fx+e\right)^4-9\,(4\,A-3\,B)\cos\left(fx+e\right)^2+\left(6\,(4\,A-3\,B)\cos\left(fx+e\right)^2-20\,A+15\,B\right)}{35\,\left(3\,ac^4f\cos\left(fx+e\right)^3-4\,ac^4f\cos\left(fx+e\right)-\left(ac^4f\cos\left(fx+e\right)^3-4\,ac^4f\cos\left(fx+e\right)\right)\right.}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorith
m="fricas")
```

output
```
1/35*(2*(4*A - 3*B)*cos(f*x + e)^4 - 9*(4*A - 3*B)*cos(f*x + e)^2 + (6*(4*
A - 3*B)*cos(f*x + e)^2 - 20*A + 15*B)*sin(f*x + e) + 15*A - 20*B)/(3*a*c^
4*f*cos(f*x + e)^3 - 4*a*c^4*f*cos(f*x + e) - (a*c^4*f*cos(f*x + e)^3 - 4*
a*c^4*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2468 vs. $2(122) = 244$.

Time = 10.03 (sec) , antiderivative size = 2468, normalized size of antiderivative = 17.38

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**4,x)
```

output
```
Piecewise((-70*A*tan(e/2 + f*x/2)**7/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 21
0*a*c**4*f*tan(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*
c**4*f*tan(e/2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4
*f*tan(e/2 + f*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) + 21
0*A*tan(e/2 + f*x/2)**6/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*ta
n(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/
2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2 +
f*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) - 350*A*tan(e/2 +
 f*x/2)**5/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*tan(e/2 + f*x/2
)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/2 + f*x/2)**5
 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2 + f*x/2)**2 + 2
10*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) + 210*A*tan(e/2 + f*x/2)**4/(3
5*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*tan(e/2 + f*x/2)**7 + 490*a*
c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/2 + f*x/2)**5 + 490*a*c**4
*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2 + f*x/2)**2 + 210*a*c**4*f*t
an(e/2 + f*x/2) - 35*a*c**4*f) + 14*A*tan(e/2 + f*x/2)**3/(35*a*c**4*f*tan
(e/2 + f*x/2)**8 - 210*a*c**4*f*tan(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/2
 + f*x/2)**6 - 490*a*c**4*f*tan(e/2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f
*x/2)**3 - 490*a*c**4*f*tan(e/2 + f*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2
) - 35*a*c**4*f) - 154*A*tan(e/2 + f*x/2)**2/(35*a*c**4*f*tan(e/2 + f*x...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 619 vs. $2(137) = 274$.

Time = 0.06 (sec) , antiderivative size = 619, normalized size of antiderivative = 4.36

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))(c - c\sin(e + fx))^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorith
m="maxima")
```

output
```
-2/35*(A*(43*sin(f*x + e)/(cos(f*x + e) + 1) - 77*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 105*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 - 175*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 105*sin(f*
x + e)^6/(cos(f*x + e) + 1)^6 - 35*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 1
3)/(a*c^4 - 6*a*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 14*a*c^4*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 - 14*a*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
14*a*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 14*a*c^4*sin(f*x + e)^6/(co
s(f*x + e) + 1)^6 + 6*a*c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - a*c^4*si
n(f*x + e)^8/(cos(f*x + e) + 1)^8) - B*(6*sin(f*x + e)/(cos(f*x + e) + 1)
+ 21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 56*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 105*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 70*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5 + 35*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1)/(a*c^4 - 6*
a*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 14*a*c^4*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 - 14*a*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 14*a*c^4*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5 - 14*a*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1
)^6 + 6*a*c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - a*c^4*sin(f*x + e)^8/(
cos(f*x + e) + 1)^8))/f
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.57

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^4}\,dx = -\frac{\frac{35\,(A-B)}{ac^4\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)} + \frac{525\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6+35\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-1960\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+280\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+4025\,A\tan(\ldots}{\ldots}}{\ldots}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorith
m="giac")
```

output
```
-1/280*(35*(A - B)/(a*c^4*(tan(1/2*f*x + 1/2*e) + 1)) + (525*A*tan(1/2*f*x
 + 1/2*e)^6 + 35*B*tan(1/2*f*x + 1/2*e)^6 - 1960*A*tan(1/2*f*x + 1/2*e)^5
+ 280*B*tan(1/2*f*x + 1/2*e)^5 + 4025*A*tan(1/2*f*x + 1/2*e)^4 - 665*B*tan
(1/2*f*x + 1/2*e)^4 - 4480*A*tan(1/2*f*x + 1/2*e)^3 + 1120*B*tan(1/2*f*x +
 1/2*e)^3 + 3143*A*tan(1/2*f*x + 1/2*e)^2 - 791*B*tan(1/2*f*x + 1/2*e)^2 -
 1176*A*tan(1/2*f*x + 1/2*e) + 392*B*tan(1/2*f*x + 1/2*e) + 243*A - 51*B)/
(a*c^4*(tan(1/2*f*x + 1/2*e) - 1)^7))/f
```

Mupad [B] (verification not implemented)

Time = 37.69 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.68

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))(c - c\sin(e + fx))^4}\,dx$$
$$= \frac{2\left(\frac{35\,B}{4} + \frac{91\,A\,\cos(e+f\,x)}{4} - \frac{7\,B\,\cos(e+f\,x)}{4} + 14\,A\,\sin\left(e + f\,x\right) - \frac{21\,B\,\sin(e+f\,x)}{2} + 14\,A\,\cos\left(2\,e + 2\,f\,x\right) - \frac{39\,A\,\cdots}{}\right.}{}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^4),x)
```

output
```
(2*((35*B)/4 + (91*A*cos(e + f*x))/4 - (7*B*cos(e + f*x))/4 + 14*A*sin(e +
 f*x) - (21*B*sin(e + f*x))/2 + 14*A*cos(2*e + 2*f*x) - (39*A*cos(3*e + 3*
f*x))/4 - A*cos(4*e + 4*f*x) - (21*B*cos(2*e + 2*f*x))/2 + (3*B*cos(3*e +
3*f*x))/4 + (3*B*cos(4*e + 4*f*x))/4 - (91*A*sin(2*e + 2*f*x))/4 - 6*A*sin
(3*e + 3*f*x) + (13*A*sin(4*e + 4*f*x))/8 + (7*B*sin(2*e + 2*f*x))/4 + (9*
B*sin(3*e + 3*f*x))/2 - (B*sin(4*e + 4*f*x))/8))/(35*a*c^4*f*((7*cos(e + f
*x))/2 - (3*cos(3*e + 3*f*x))/2 - (7*sin(2*e + 2*f*x))/2 + sin(4*e + 4*f*x
)/4))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.80

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))(c - c\sin(e + fx))^4}\,dx$$
$$= \frac{4\cos\left(fx + e\right)\sin\left(fx + e\right)^3 a - 3\cos\left(fx + e\right)\sin\left(fx + e\right)^3 b - 12\cos\left(fx + e\right)\sin\left(fx + e\right)^2 a + 9\cos\left(f\cdots\right.}{}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x)
```

output

```
(4*cos(e + f*x)*sin(e + f*x)**3*a - 3*cos(e + f*x)*sin(e + f*x)**3*b - 12*
cos(e + f*x)*sin(e + f*x)**2*a + 9*cos(e + f*x)*sin(e + f*x)**2*b + 12*cos
(e + f*x)*sin(e + f*x)*a - 9*cos(e + f*x)*sin(e + f*x)*b - 4*cos(e + f*x)*
a + 3*cos(e + f*x)*b + 24*sin(e + f*x)**4*a - 18*sin(e + f*x)**4*b - 72*si
n(e + f*x)**3*a + 54*sin(e + f*x)**3*b + 60*sin(e + f*x)**2*a - 45*sin(e +
 f*x)**2*b + 12*sin(e + f*x)*a - 9*sin(e + f*x)*b - 39*a + 3*b)/(105*cos(e
 + f*x)*a*c**4*f*(sin(e + f*x)**3 - 3*sin(e + f*x)**2 + 3*sin(e + f*x) - 1
))
```

3.60 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^2} \, dx$

Optimal result

Integrand size = 36, antiderivative size = 240

$$\begin{aligned} &\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^2} \, dx \\ &= \frac{105(4A-7B)c^5x}{8a^2} + \frac{35(4A-7B)c^5\cos^3(e+fx)}{4a^2f} \\ &\quad + \frac{105(4A-7B)c^5\cos(e+fx)\sin(e+fx)}{8a^2f} \\ &\quad - \frac{a^5(A-B)c^5\cos^{11}(e+fx)}{3f(a+a\sin(e+fx))^7} + \frac{2a^3(4A-7B)c^5\cos^9(e+fx)}{3f(a+a\sin(e+fx))^5} \\ &\quad + \frac{6a^4(4A-7B)c^5\cos^7(e+fx)}{f\left(a^2+a^2\sin(e+fx)\right)^3} + \frac{21(4A-7B)c^5\cos^5(e+fx)}{4f\left(a^2+a^2\sin(e+fx)\right)} \end{aligned}$$

output
```
105/8*(4*A-7*B)*c^5*x/a^2+35/4*(4*A-7*B)*c^5*cos(f*x+e)^3/a^2/f+105/8*(4*A
-7*B)*c^5*cos(f*x+e)*sin(f*x+e)/a^2/f-1/3*a^5*(A-B)*c^5*cos(f*x+e)^11/f/(a
+a*sin(f*x+e))^7+2/3*a^3*(4*A-7*B)*c^5*cos(f*x+e)^9/f/(a+a*sin(f*x+e))^5+6
*a^4*(4*A-7*B)*c^5*cos(f*x+e)^7/f/(a^2+a^2*sin(f*x+e))^3+21/4*(4*A-7*B)*c^
5*cos(f*x+e)^5/f/(a^2+a^2*sin(f*x+e))
```

Mathematica [A] (verified)

Time = 12.57 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.48

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^2}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c-c\sin(e+fx))^5\left(2048(A-B)\sin\left(\frac{1}{2}(e+fx)\right)-1024(A-B)\right.}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5)/(a + a*Sin[e + f*x
])^2,x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^5*(2048*(A - B
)*Sin[(e + f*x)/2] - 1024*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) -
1024*(13*A - 19*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
2 + 1260*(4*A - 7*B)*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 2
4*(95*A - 217*B)*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 8*
(A - 7*B)*Cos[3*(e + f*x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 24*(7
*A - 24*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sin[2*(e + f*x)] - 3*B*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sin[4*(e + f*x)]))/(96*a^2*f*(Cos[
(e + f*x)/2] - Sin[(e + f*x)/2])^10*(1 + Sin[e + f*x])^2)
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.87, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3159, 3042, 3158, 3042, 3161, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^5(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-c\sin(e+fx))^5(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3446}$$

$$a^5c^5\int \frac{\cos^{10}(e+fx)(A+B\sin(e+fx))}{(\sin(e+fx)a+a)^7}dx$$

$$\Big\downarrow \text{3042}$$

$$a^5c^5\int \frac{\cos(e+fx)^{10}(A+B\sin(e+fx))}{(\sin(e+fx)a+a)^7}dx$$

$$\Big\downarrow \text{3338}$$

$$a^5c^5\left(-\frac{(4A-7B)\int \frac{\cos^{10}(e+fx)}{(\sin(e+fx)a+a)^6}dx}{3a}-\frac{(A-B)\cos^{11}(e+fx)}{3f(a\sin(e+fx)+a)^7}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^5c^5\left(-\frac{(4A-7B)\int \frac{\cos(e+fx)^{10}}{(\sin(e+fx)a+a)^6}dx}{3a}-\frac{(A-B)\cos^{11}(e+fx)}{3f(a\sin(e+fx)+a)^7}\right)$$

$$\Big\downarrow \text{3159}$$

$$a^5c^5\left(-\frac{(4A-7B)\left(-\frac{9\int \frac{\cos^8(e+fx)}{(\sin(e+fx)a+a)^4}dx}{a^2}-\frac{2\cos^9(e+fx)}{af(a\sin(e+fx)+a)^5}\right)}{3a}-\frac{(A-B)\cos^{11}(e+fx)}{3f(a\sin(e+fx)+a)^7}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^5c^5\left(-\frac{(4A-7B)\left(-\frac{9\int \frac{\cos(e+fx)^8}{(\sin(e+fx)a+a)^4}dx}{a^2}-\frac{2\cos^9(e+fx)}{af(a\sin(e+fx)+a)^5}\right)}{3a}-\frac{(A-B)\cos^{11}(e+fx)}{3f(a\sin(e+fx)+a)^7}\right)$$

$$\Big\downarrow \text{3159}$$

$$a^5c^5\left(-\frac{(4A-7B)\left(-\frac{9\left(\frac{7\int\frac{\cos^6(e+fx)}{(\sin(e+fx)a+a)^2}dx}{a^2}+\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{a^2}-\frac{2\cos^9(e+fx)}{af(a\sin(e+fx)+a)^5}\right)}{3a}-\frac{(A-B)\cos^{11}(e+fx)}{3f(a\sin(e+fx)+a)^7}\right)$$

$\downarrow$ 3042

$$a^5c^5\left(-\frac{(4A-7B)\left(-\frac{9\left(\frac{7\int\frac{\cos(e+fx)^6}{(\sin(e+fx)a+a)^2}dx}{a^2}+\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{a^2}-\frac{2\cos^9(e+fx)}{af(a\sin(e+fx)+a)^5}\right)}{3a}-\frac{(A-B)\cos^{11}(e+fx)}{3f(a\sin(e+fx)+a)^7}\right)$$

$\downarrow$ 3158

$$a^5c^5\left(-\frac{(4A-7B)\left(-\frac{9\left(\frac{7\left(\frac{5\int\frac{\cos^4(e+fx)}{\sin(e+fx)a+a}dx}{4a}+\frac{\cos^5(e+fx)}{4f\left(a^2\sin(e+fx)+a^2\right)}\right)}{a^2}+\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{a^2}-\frac{2\cos^9(e+fx)}{af(a\sin(e+fx)+a)^5}\right)}{3a}-\frac{(A-B}{3f(a\sin}\right.$$

$\Big\downarrow$ 3042

$$a^5c^5\left(-\frac{(4A-7B)\left(-\frac{9\left(\frac{7\left(\frac{5\int\frac{\cos(e+fx)^4}{\sin(e+fx)a+a}dx}{4a}+\frac{\cos^5(e+fx)}{4f\left(a^2\sin(e+fx)+a^2\right)}\right)}{a^2}+\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{a^2}-\frac{2\cos^9(e+fx)}{af(a\sin(e+fx)+a)^5}\right)}{3a}-\frac{(A-B}{3f(a\sin}\right.$$

$\Big\downarrow$ 3161

$$a^5c^5\left(-\frac{(4A-7B)\left(-\frac{9\left(\frac{7\left(\frac{5\left(\frac{\int\cos^2(e+fx)dx}{a}+\frac{\cos^3(e+fx)}{3af}\right)}{4a}+\frac{\cos^5(e+fx)}{4f\left(a^2\sin(e+fx)+a^2\right)}\right)}{a^2}+\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{a^2}-\frac{2\cos^9(e+fx)}{af(a\sin(e+fx)+a)^5}\right)}{3a}\right.$$

$$\Big\downarrow \; 3042$$

$$a^5c^5\left(-\frac{(4A-7B)\left(-\frac{9\left(\frac{7\left(\frac{5\left(\frac{\int \sin\left(e+fx+\frac{\pi}{2}\right)^2dx}{a}+\frac{\cos^3(e+fx)}{3af}\right)}{4a}+\frac{\cos^5(e+fx)}{4f\left(a^2\sin(e+fx)+a^2\right)}\right)}{a^2}+\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{a^2}-\frac{2\cos^9(e+fx)}{af(a\sin(e+fx)+a)^5}\right)}{3a}\right.$$

$$\Big\downarrow \; 3115$$

$$a^5c^5\left(-\frac{(4A-7B)\left(-\frac{9\left(\frac{7\left(\frac{5\left(\frac{\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}}{a}+\frac{\cos^3(e+fx)}{3af}\right)}{4a}+\frac{\cos^5(e+fx)}{4f\left(a^2\sin(e+fx)+a^2\right)}\right)}{a^2}+\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{a^2}-\frac{2\cos^9(e\cdots}{af(a\sin(e+\cdots}\right.}{3a}\right.$$

$\downarrow$ 24

$$a^5c^5\left(-\frac{(4A-7B)\left(-\frac{9\left(\frac{7\left(\frac{\cos^5(e+fx)}{4f\left(a^2\sin(e+fx)+a^2\right)}+\frac{5\left(\frac{\cos^3(e+fx)}{3af}+\frac{\frac{\sin(e+fx)\cos(e+fx)}{2f}+\frac{x}{2}}{a}\right)}{4a}\right)}{a^2}+\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{a^2}-\frac{2\cos^9(e+fx)}{af(a\sin(e+fx))}\right)}{3a}\right)$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5)/(a + a*Sin[e + f*x])^2,x
]
```

output
```
a^5*c^5*(-1/3*((A - B)*Cos[e + f*x]^11)/(f*(a + a*Sin[e + f*x])^7) - ((4*A
 - 7*B)*((-2*Cos[e + f*x]^9)/(a*f*(a + a*Sin[e + f*x])^5) - (9*((2*Cos[e +
 f*x]^7)/(a*f*(a + a*Sin[e + f*x])^3) + (7*(Cos[e + f*x]^5/(4*f*(a^2 + a^2
*Sin[e + f*x])) + (5*(Cos[e + f*x]^3/(3*a*f) + (x/2 + (Cos[e + f*x]*Sin[e
+ f*x])/(2*f))/a))/(4*a)))/a^2))/a^2))/(3*a))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3158
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(a*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
 EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && In
tegersQ[2*m, 2*p]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 88.20 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{2c^5\left(\frac{\left(\frac{7A}{2}-\frac{95B}{8}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7+(23A-49B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+\left(\frac{7A}{2}-\frac{103B}{8}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5+(71A-161B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+\left(-\frac{7A}{2}+\right.}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^4}\right.}{}$
default	$\frac{2c^5\left(\frac{\left(\frac{7A}{2}-\frac{95B}{8}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7+(23A-49B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6+\left(\frac{7A}{2}-\frac{103B}{8}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5+(71A-161B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+\left(-\frac{7A}{2}+\right.}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^4}\right.}{}$
parallelrisch	$-\frac{75\left(\frac{\left(-\frac{2543}{15}A+\frac{4514}{15}B-84fxA+147fxB\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)}{5}+\frac{\left(-\frac{481}{45}A+\frac{712}{45}B+28fxA-49fxB\right)\cos\left(\frac{3fx}{2}+\frac{3e}{2}\right)}{5}+\frac{(147fxB-84fx\right.}{}\right.}{}$
risch	$\frac{105c^5xA}{2a^2}-\frac{735c^5xB}{8a^2}+\frac{7ic^5e^{2i(fx+e)}A}{8a^2f}-\frac{3ic^5e^{2i(fx+e)}B}{a^2f}+\frac{95c^5e^{i(fx+e)}A}{8a^2f}-\frac{217c^5e^{i(fx+e)}B}{8a^2f}+\frac{95c^5e^{-i(f}}{8a^2}$
norman	Expression too large to display

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x,method=_RETUR
NVERBOSE)
```

output
```
2/f*c^5/a^2*(((7/2*A-95/8*B)*tan(1/2*f*x+1/2*e)^7+(23*A-49*B)*tan(1/2*f*x+
1/2*e)^6+(7/2*A-103/8*B)*tan(1/2*f*x+1/2*e)^5+(71*A-161*B)*tan(1/2*f*x+1/2
*e)^4+(-7/2*A+103/8*B)*tan(1/2*f*x+1/2*e)^3+(215/3*A-497/3*B)*tan(1/2*f*x+
1/2*e)^2+(-7/2*A+95/8*B)*tan(1/2*f*x+1/2*e)+71/3*A-161/3*B)/(1+tan(1/2*f*x
+1/2*e)^2)^4+105/8*(4*A-7*B)*arctan(tan(1/2*f*x+1/2*e))-1/2*(-64*A+64*B)/(
tan(1/2*f*x+1/2*e)+1)^2-(-48*A+80*B)/(tan(1/2*f*x+1/2*e)+1)-1/3*(64*A-64*B
)/(tan(1/2*f*x+1/2*e)+1)^3)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.54

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^2}\,dx = -\frac{6\,Bc^5\cos{(fx+e)}^6+4\,(2\,A-11\,B)c^5\cos{(fx+e)}^5+(76\,A-241\,B)c^5\cos{(fx+e)}^4-2\,(212\,A-43}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
-1/24*(6*B*c^5*cos(f*x + e)^6 + 4*(2*A - 11*B)*c^5*cos(f*x + e)^5 + (76*A
- 241*B)*c^5*cos(f*x + e)^4 - 2*(212*A - 431*B)*c^5*cos(f*x + e)^3 + 630*(
4*A - 7*B)*c^5*f*x - 256*(A - B)*c^5 - (315*(4*A - 7*B)*c^5*f*x - (2156*A
- 3485*B)*c^5)*cos(f*x + e)^2 + (315*(4*A - 7*B)*c^5*f*x + 2*(1196*A - 214
1*B)*c^5)*cos(f*x + e) + (6*B*c^5*cos(f*x + e)^5 - 2*(4*A - 25*B)*c^5*cos(
f*x + e)^4 + (68*A - 191*B)*c^5*cos(f*x + e)^3 + 630*(4*A - 7*B)*c^5*f*x +
 3*(164*A - 351*B)*c^5*cos(f*x + e)^2 + 256*(A - B)*c^5 + (315*(4*A - 7*B)
*c^5*f*x + 2*(1324*A - 2269*B)*c^5)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos
(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f
)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10608 vs. $2(224) = 448$.

Time = 23.13 (sec) , antiderivative size = 10608, normalized size of antiderivative = 44.20

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^5}{(a + a\sin(e + fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**5/(a+a*sin(f*x+e))**2,x)
```

output
```
Piecewise((1260*A*c**5*f*x*tan(e/2 + f*x/2)**11/(24*a**2*f*tan(e/2 + f*x/2
)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 +
312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2
*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e
/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x
/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) + 3780*A*c**5*f*x*tan(e/2
 + f*x/2)**10/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)
**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 4
32*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*
f*tan(e/2 + f*x/2)**5 + 432*a**2*f*tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/
2 + f*x/2)**3 + 168*a**2*f*tan(e/2 + f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2
) + 24*a**2*f) + 8820*A*c**5*f*x*tan(e/2 + f*x/2)**9/(24*a**2*f*tan(e/2 +
f*x/2)**11 + 72*a**2*f*tan(e/2 + f*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)*
*9 + 312*a**2*f*tan(e/2 + f*x/2)**8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528
*a**2*f*tan(e/2 + f*x/2)**6 + 528*a**2*f*tan(e/2 + f*x/2)**5 + 432*a**2*f*
tan(e/2 + f*x/2)**4 + 312*a**2*f*tan(e/2 + f*x/2)**3 + 168*a**2*f*tan(e/2
+ f*x/2)**2 + 72*a**2*f*tan(e/2 + f*x/2) + 24*a**2*f) + 16380*A*c**5*f*x*t
an(e/2 + f*x/2)**8/(24*a**2*f*tan(e/2 + f*x/2)**11 + 72*a**2*f*tan(e/2 + f
*x/2)**10 + 168*a**2*f*tan(e/2 + f*x/2)**9 + 312*a**2*f*tan(e/2 + f*x/2)**
8 + 432*a**2*f*tan(e/2 + f*x/2)**7 + 528*a**2*f*tan(e/2 + f*x/2)**6 + 5...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2982 vs. $2(228) = 456$.

Time = 0.18 (sec) , antiderivative size = 2982, normalized size of antiderivative = 12.42

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^5}{(a + a\sin(e + fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x, algori
thm="maxima")
```

output
```
-1/12*(B*c^5*((603*sin(f*x + e)/(cos(f*x + e) + 1) + 1297*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 2228*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 2628*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 + 3014*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
 + 2618*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1980*sin(f*x + e)^7/(cos(f*x
 + e) + 1)^7 + 1100*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 495*sin(f*x + e)
^9/(cos(f*x + e) + 1)^9 + 165*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 256)
/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 7*a^2*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 13*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 18*a^2*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 + 22*a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)
^5 + 22*a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 18*a^2*sin(f*x + e)^7/(c
os(f*x + e) + 1)^7 + 13*a^2*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 7*a^2*si
n(f*x + e)^9/(cos(f*x + e) + 1)^9 + 3*a^2*sin(f*x + e)^10/(cos(f*x + e) +
1)^10 + a^2*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) + 165*arctan(sin(f*x +
e)/(cos(f*x + e) + 1))/a^2) - 20*A*c^5*((75*sin(f*x + e)/(cos(f*x + e) + 1
) + 97*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 98*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5 + 21*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 + 3
*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 7*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 + 5*a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 3*a^...
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.63

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^2}\,dx$$

$$= \frac{\frac{315\,(4\,Ac^5-7\,Bc^5)(fx+e)}{a^2} + \frac{256\left(9\,Ac^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-15\,Bc^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+24\,Ac^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-36\,Bc^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+11\,Ac^5-}{a^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
1/24*(315*(4*A*c^5 - 7*B*c^5)*(f*x + e)/a^2 + 256*(9*A*c^5*tan(1/2*f*x + 1
/2*e)^2 - 15*B*c^5*tan(1/2*f*x + 1/2*e)^2 + 24*A*c^5*tan(1/2*f*x + 1/2*e)
- 36*B*c^5*tan(1/2*f*x + 1/2*e) + 11*A*c^5 - 17*B*c^5)/(a^2*(tan(1/2*f*x +
 1/2*e) + 1)^3) + 2*(84*A*c^5*tan(1/2*f*x + 1/2*e)^7 - 285*B*c^5*tan(1/2*f
*x + 1/2*e)^7 + 552*A*c^5*tan(1/2*f*x + 1/2*e)^6 - 1176*B*c^5*tan(1/2*f*x
+ 1/2*e)^6 + 84*A*c^5*tan(1/2*f*x + 1/2*e)^5 - 309*B*c^5*tan(1/2*f*x + 1/2
*e)^5 + 1704*A*c^5*tan(1/2*f*x + 1/2*e)^4 - 3864*B*c^5*tan(1/2*f*x + 1/2*e
)^4 - 84*A*c^5*tan(1/2*f*x + 1/2*e)^3 + 309*B*c^5*tan(1/2*f*x + 1/2*e)^3 +
 1720*A*c^5*tan(1/2*f*x + 1/2*e)^2 - 3976*B*c^5*tan(1/2*f*x + 1/2*e)^2 - 8
4*A*c^5*tan(1/2*f*x + 1/2*e) + 285*B*c^5*tan(1/2*f*x + 1/2*e) + 568*A*c^5
- 1288*B*c^5)/((tan(1/2*f*x + 1/2*e)^2 + 1)^4*a^2))/f
```

Mupad [B] (verification not implemented)

Time = 39.91 (sec) , antiderivative size = 500, normalized size of antiderivative = 2.08

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^5)/(a + a*sin(e + f*x))^2,x
)
```

output
```
(tan(e/2 + (f*x)/2)*(391*A*c^5 - (2729*B*c^5)/4) + (494*A*c^5)/3 - (866*B*
c^5)/3 + tan(e/2 + (f*x)/2)^10*(103*A*c^5 - (735*B*c^5)/4) + tan(e/2 + (f*
x)/2)^9*(323*A*c^5 - (2213*B*c^5)/4) + tan(e/2 + (f*x)/2)^7*(1332*A*c^5 -
2253*B*c^5) + tan(e/2 + (f*x)/2)^4*(1632*A*c^5 - 2943*B*c^5) + tan(e/2 + (
f*x)/2)^8*((2002*A*c^5)/3 - (3637*B*c^5)/3) + tan(e/2 + (f*x)/2)^3*((4420*
A*c^5)/3 - (7621*B*c^5)/3) + tan(e/2 + (f*x)/2)^2*((2489*A*c^5)/3 - (17609
*B*c^5)/12) + tan(e/2 + (f*x)/2)^6*((4594*A*c^5)/3 - (16805*B*c^5)/6) + ta
n(e/2 + (f*x)/2)^5*((6274*A*c^5)/3 - (21299*B*c^5)/6))/(f*(7*a^2*tan(e/2 +
 (f*x)/2)^2 + 13*a^2*tan(e/2 + (f*x)/2)^3 + 18*a^2*tan(e/2 + (f*x)/2)^4 +
22*a^2*tan(e/2 + (f*x)/2)^5 + 22*a^2*tan(e/2 + (f*x)/2)^6 + 18*a^2*tan(e/2
 + (f*x)/2)^7 + 13*a^2*tan(e/2 + (f*x)/2)^8 + 7*a^2*tan(e/2 + (f*x)/2)^9 +
 3*a^2*tan(e/2 + (f*x)/2)^10 + a^2*tan(e/2 + (f*x)/2)^11 + a^2 + 3*a^2*tan
(e/2 + (f*x)/2))) + (105*c^5*atan((105*c^5*tan(e/2 + (f*x)/2)*(4*A - 7*B))
/(420*A*c^5 - 735*B*c^5))*(4*A - 7*B))/(4*a^2*f)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.90

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^2}\,dx$$
$$= \frac{c^5(824a - 1470b + 1260\cos(fx+e)\sin(fx+e)\,afx - 2205\cos(fx+e)\sin(fx+e)\,bfx + 1470\cos(f}{}$$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x)
```

output
```
(c**5*( - 6*cos(e + f*x)*sin(e + f*x)**5*b - 8*cos(e + f*x)*sin(e + f*x)**
4*a + 44*cos(e + f*x)*sin(e + f*x)**4*b + 68*cos(e + f*x)*sin(e + f*x)**3*
a - 179*cos(e + f*x)*sin(e + f*x)**3*b - 408*cos(e + f*x)*sin(e + f*x)**2*
a + 774*cos(e + f*x)*sin(e + f*x)**2*b + 1260*cos(e + f*x)*sin(e + f*x)*a*
f*x - 1564*cos(e + f*x)*sin(e + f*x)*a - 2205*cos(e + f*x)*sin(e + f*x)*b*
f*x + 2729*cos(e + f*x)*sin(e + f*x)*b + 1260*cos(e + f*x)*a*f*x - 824*cos
(e + f*x)*a - 2205*cos(e + f*x)*b*f*x + 1470*cos(e + f*x)*b - 6*sin(e + f*
x)**6*b - 8*sin(e + f*x)**5*a + 50*sin(e + f*x)**5*b + 76*sin(e + f*x)**4*
a - 223*sin(e + f*x)**4*b - 476*sin(e + f*x)**3*a + 953*sin(e + f*x)**3*b
 - 1260*sin(e + f*x)**2*a*f*x - 3460*sin(e + f*x)**2*a + 2205*sin(e + f*x)*
*2*b*f*x + 5943*sin(e + f*x)**2*b - 2520*sin(e + f*x)*a*f*x - 1564*sin(e +
 f*x)*a + 4410*sin(e + f*x)*b*f*x + 2729*sin(e + f*x)*b - 1260*a*f*x + 824
*a + 2205*b*f*x - 1470*b))/(24*a**2*f*(cos(e + f*x)*sin(e + f*x) + cos(e +
 f*x) - sin(e + f*x)**2 - 2*sin(e + f*x) - 1))
```

3.61 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 180

$$\begin{aligned}\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^2}\,dx &= \frac{35(A-2B)c^4x}{2a^2}+\frac{35(A-2B)c^4\cos^3(e+fx)}{3a^2f}\\&\quad+\frac{35(A-2B)c^4\cos(e+fx)\sin(e+fx)}{2a^2f}-\frac{a^4(A-B)c^4\cos^9(e+fx)}{3f(a+a\sin(e+fx))^6}\\&\quad+\frac{2a^2(A-2B)c^4\cos^7(e+fx)}{f(a+a\sin(e+fx))^4}+\frac{14(A-2B)c^4\cos^5(e+fx)}{f(a+a\sin(e+fx))^2}\end{aligned}$$

output
```
35/2*(A-2*B)*c^4*x/a^2+35/3*(A-2*B)*c^4*cos(f*x+e)^3/a^2/f+35/2*(A-2*B)*c^
4*cos(f*x+e)*sin(f*x+e)/a^2/f-1/3*a^4*(A-B)*c^4*cos(f*x+e)^9/f/(a+a*sin(f*
x+e))^6+2*a^2*(A-2*B)*c^4*cos(f*x+e)^7/f/(a+a*sin(f*x+e))^4+14*(A-2*B)*c^4
*cos(f*x+e)^5/f/(a+a*sin(f*x+e))^2
```

Mathematica [A] (verified)

Time = 11.87 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.73

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^2}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c-c\sin(e+fx))^4\left(128(A-B)\sin\left(\frac{1}{2}(e+fx)\right)-64(A-B)\left(\cos\right.\right.}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4)/(a + a*Sin[e + f*x
])^2,x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^4*(128*(A - B)
*Sin[(e + f*x)/2] - 64*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 128
*(5*A - 8*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 21
0*(A - 2*B)*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 3*(24*A -
71*B)*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + B*Cos[3*(e +
f*x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 3*(A - 6*B)*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^3*Sin[2*(e + f*x)]))/(12*a^2*f*(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2])^8*(1 + Sin[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.92, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.361$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3159, 3042, 3161, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^4(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^2}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{(c-c\sin(e+fx))^4(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3446}$$

$$a^4c^4\int \frac{\cos^8(e+fx)(A+B\sin(e+fx))}{(\sin(e+fx)a+a)^6}dx$$

$$\Big\downarrow \text{3042}$$

$$a^4c^4\int \frac{\cos(e+fx)^8(A+B\sin(e+fx))}{(\sin(e+fx)a+a)^6}dx$$

$$\Big\downarrow \text{3338}$$

$$a^4c^4\left(-\frac{(A-2B)\int \frac{\cos^8(e+fx)}{(\sin(e+fx)a+a)^5}dx}{a}-\frac{(A-B)\cos^9(e+fx)}{3f(a\sin(e+fx)+a)^6}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^4c^4\left(-\frac{(A-2B)\int \frac{\cos(e+fx)^8}{(\sin(e+fx)a+a)^5}dx}{a}-\frac{(A-B)\cos^9(e+fx)}{3f(a\sin(e+fx)+a)^6}\right)$$

$$\Big\downarrow \text{3159}$$

$$a^4c^4\left(-\frac{(A-2B)\left(-\frac{7\int \frac{\cos^6(e+fx)}{(\sin(e+fx)a+a)^3}dx}{a^2}-\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^4}\right)}{a}-\frac{(A-B)\cos^9(e+fx)}{3f(a\sin(e+fx)+a)^6}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^4c^4\left(-\frac{(A-2B)\left(-\frac{7\int \frac{\cos(e+fx)^6}{(\sin(e+fx)a+a)^3}dx}{a^2}-\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^4}\right)}{a}-\frac{(A-B)\cos^9(e+fx)}{3f(a\sin(e+fx)+a)^6}\right)$$

$$\Big\downarrow \text{3159}$$

$$a^4c^4\left(-\frac{(A-2B)\left(-\frac{7\left(\frac{5\int \frac{\cos^4(e+fx)}{\sin(e+fx)a+a}dx}{a^2}+\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{a^2}-\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^4}\right)}{a}-\frac{(A-B)\cos^9(e+fx)}{3f(a\sin(e+fx)+a)^6}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^4c^4\left(-\frac{(A-2B)\left(-\frac{7\left(\frac{5\int\frac{\cos(e+fx)^4}{\sin(e+fx)a+a}dx}{a^2}+\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{a^2}-\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^4}\right)}{a}-\frac{(A-B)\cos^9(e+fx)}{3f(a\sin(e+fx)+a)^6}\right)$$

$$\Big\downarrow \text{3161}$$

$$a^4c^4\left(-\frac{(A-2B)\left(-\frac{7\left(\frac{5\left(\frac{\int\cos^2(e+fx)dx}{a}+\frac{\cos^3(e+fx)}{3af}\right)}{a^2}+\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{a^2}-\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^4}\right)}{a}-\frac{(A-B)\cos^9(e+f\ldots}{3f(a\sin(e+fx)+\ldots}\right.$$

$$\Big\downarrow \text{3042}$$

$$a^4c^4\left(-\frac{(A-2B)\left(-\frac{7\left(\frac{5\left(\frac{\int\sin\left(e+fx+\frac{\pi}{2}\right)^2dx}{a}+\frac{\cos^3(e+fx)}{3af}\right)}{a^2}+\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{a^2}-\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^4}\right)}{a}-\frac{(A-B)\cos^9(e\ldots}{3f(a\sin(e+fx\ldots}\right.$$

$$\Big\downarrow \text{3115}$$

$$a^4c^4\left(-\frac{(A-2B)\left(-\frac{7\left(\frac{5\left(\frac{\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}}{a}+\frac{\cos^3(e+fx)}{3af}\right)}{a^2}+\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{a^2}-\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^4}\right)}{a}-\frac{(A-B)\,c}{3f(a\sin(e}\right.$$

$\downarrow$ 24

$$a^4c^4\left(-\frac{(A-2B)\left(-\frac{7\left(\frac{5\left(\frac{\cos^3(e+fx)}{3af}+\frac{\frac{\sin(e+fx)\cos(e+fx)}{2f}+\frac{x}{2}}{a}\right)}{a^2}+\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{a^2}-\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^4}\right)}{a}-\frac{(A-B)\cos}{3f(a\sin(e+}\right.$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4)/(a + a*Sin[e + f*x])^2,x
]
```

output
```
a^4*c^4*(-1/3*((A - B)*Cos[e + f*x]^9)/(f*(a + a*Sin[e + f*x])^6) - ((A -
2*B)*((-2*Cos[e + f*x]^7)/(a*f*(a + a*Sin[e + f*x])^4) - (7*((2*Cos[e + f*
x]^5)/(a*f*(a + a*Sin[e + f*x])^2) + (5*(Cos[e + f*x]^3/(3*a*f) + (x/2 + (
Cos[e + f*x]*Sin[e + f*x])/(2*f))/a))/a^2))/a^2))/a)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 11.17 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{2c^4\left(\frac{\left(\frac{A}{2}-3B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5+(6A-17B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+(12A-36B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\left(-\frac{A}{2}+3B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+6A-\frac{53B}{3}}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^3}+\frac{35(A}{}\right.}{f\,a^2}$
default	$\frac{2c^4\left(\frac{\left(\frac{A}{2}-3B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5+(6A-17B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+(12A-36B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\left(-\frac{A}{2}+3B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+6A-\frac{53B}{3}}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^3}+\frac{35(A}{}\right.}{f\,a^2}$
parallelrisch	$-\frac{21\left(\left(-\frac{277}{7}A+\frac{1712}{21}B-20fxA+40fxB\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{\left(-\frac{191}{21}A+\frac{290}{21}B+20fxA-40fxB\right)\cos\left(\frac{3fx}{2}+\frac{3e}{2}\right)}{3}+(40fxB-20\right.}{}$
risch	$\frac{35c^4xA}{2a^2}-\frac{35c^4xB}{a^2}+\frac{ic^4\mathrm{e}^{2i(fx+e)}A}{8a^2f}-\frac{3ic^4\mathrm{e}^{2i(fx+e)}B}{4a^2f}+\frac{3c^4\mathrm{e}^{i(fx+e)}A}{a^2f}-\frac{71c^4\mathrm{e}^{i(fx+e)}B}{8a^2f}+\frac{3c^4\mathrm{e}^{-i(fx+e)}A}{a^2f}$
norman	$\frac{\frac{35c^4(A-2B)x}{2a}+\frac{164A\,c^4-330B\,c^4}{3af}+\frac{\left(131A\,c^4-260B\,c^4\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{\left(979A\,c^4-2004B\,c^4\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{3af}+\frac{\left(1893A\,c^4-3700B\right.}{3a}}{}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x,method=_RETUR
NVERBOSE)
```

output
```
2/f*c^4/a^2*(((1/2*A-3*B)*tan(1/2*f*x+1/2*e)^5+(6*A-17*B)*tan(1/2*f*x+1/2*
e)^4+(12*A-36*B)*tan(1/2*f*x+1/2*e)^2+(-1/2*A+3*B)*tan(1/2*f*x+1/2*e)+6*A-
53/3*B)/(1+tan(1/2*f*x+1/2*e)^2)^3+35/2*(A-2*B)*arctan(tan(1/2*f*x+1/2*e))
-1/2*(-32*A+32*B)/(tan(1/2*f*x+1/2*e)+1)^2-(-16*A+32*B)/(tan(1/2*f*x+1/2*e
)+1)-1/3*(32*A-32*B)/(tan(1/2*f*x+1/2*e)+1)^3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.79

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^2}\,dx$$
$$= \frac{2\,Bc^4\cos(fx+e)^5-(3\,A-16\,B)c^4\cos(fx+e)^4+2\,(15\,A-38\,B)c^4\cos(fx+e)^3-210\,(A-2\,B)c^4 \ldots}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
1/6*(2*B*c^4*cos(f*x + e)^5 - (3*A - 16*B)*c^4*cos(f*x + e)^4 + 2*(15*A -
38*B)*c^4*cos(f*x + e)^3 - 210*(A - 2*B)*c^4*f*x + 32*(A - B)*c^4 + (105*(
A - 2*B)*c^4*f*x - (193*A - 346*B)*c^4)*cos(f*x + e)^2 - (105*(A - 2*B)*c^
4*f*x + 2*(97*A - 202*B)*c^4)*cos(f*x + e) - (2*B*c^4*cos(f*x + e)^4 + (3*
A - 14*B)*c^4*cos(f*x + e)^3 + 210*(A - 2*B)*c^4*f*x + 3*(11*A - 30*B)*c^4
*cos(f*x + e)^2 + 32*(A - B)*c^4 + (105*(A - 2*B)*c^4*f*x + 2*(113*A - 218
*B)*c^4)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x
 + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7337 vs. 2(175) = 350.

Time = 13.37 (sec) , antiderivative size = 7337, normalized size of antiderivative = 40.76

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**4/(a+a*sin(f*x+e))**2,x)
```

output
```
Piecewise((105*A*c**4*f*x*tan(e/2 + f*x/2)**9/(6*a**2*f*tan(e/2 + f*x/2)**
9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**
2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/
2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)
**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 315*A*c**4*f*x*tan(e/2 + f*
x/2)**8/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36
*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*ta
n(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*
x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a
**2*f) + 630*A*c**4*f*x*tan(e/2 + f*x/2)**7/(6*a**2*f*tan(e/2 + f*x/2)**9
+ 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*
f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2
+ f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**
2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 1050*A*c**4*f*x*tan(e/2 + f*x
/2)**6/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*
a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan
(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x
/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a*
*2*f) + 1260*A*c**4*f*x*tan(e/2 + f*x/2)**5/(6*a**2*f*tan(e/2 + f*x/2)**9
+ 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2094 vs. $2(172) = 344$.

Time = 0.16 (sec) , antiderivative size = 2094, normalized size of antiderivative = 11.63

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^4}{(a + a\sin(e + fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x, algori
thm="maxima")
```

output
```
1/3*(A*c^4*((75*sin(f*x + e)/(cos(f*x + e) + 1) + 97*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 98*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 21*sin(
f*x + e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x +
 e) + 1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*a^2*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*a^2
*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin(f*x + e)^6/(cos(f*x + e)
+ 1)^6 + a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f*x + e)
/(cos(f*x + e) + 1))/a^2) - 4*B*c^4*((75*sin(f*x + e)/(cos(f*x + e) + 1) +
 97*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 98*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(
f*x + e) + 1)^5 + 21*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^
2*sin(f*x + e)/(cos(f*x + e) + 1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + 7*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 5*a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin(
f*x + e)^6/(cos(f*x + e) + 1)^6 + a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7)
 + 21*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - 2*B*c^4*((57*sin(f*x
+ e)/(cos(f*x + e) + 1) + 99*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 155*sin
(f*x + e)^3/(cos(f*x + e) + 1)^3 + 153*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
 + 135*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 85*sin(f*x + e)^6/(cos(f*x...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(172) = 344$.

Time = 0.25 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.94

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^2}\,dx$$
$$= \frac{\frac{105\,(Ac^4-2\,Bc^4)(fx+e)}{a^2} + \frac{2\left(99\,Ac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8-210\,Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8+333\,Ac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7-636\,Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7+533\,\ldots}{}}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
1/6*(105*(A*c^4 - 2*B*c^4)*(f*x + e)/a^2 + 2*(99*A*c^4*tan(1/2*f*x + 1/2*e
)^8 - 210*B*c^4*tan(1/2*f*x + 1/2*e)^8 + 333*A*c^4*tan(1/2*f*x + 1/2*e)^7
- 636*B*c^4*tan(1/2*f*x + 1/2*e)^7 + 533*A*c^4*tan(1/2*f*x + 1/2*e)^6 - 11
60*B*c^4*tan(1/2*f*x + 1/2*e)^6 + 1047*A*c^4*tan(1/2*f*x + 1/2*e)^5 - 1980
*B*c^4*tan(1/2*f*x + 1/2*e)^5 + 921*A*c^4*tan(1/2*f*x + 1/2*e)^4 - 1980*B*
c^4*tan(1/2*f*x + 1/2*e)^4 + 1107*A*c^4*tan(1/2*f*x + 1/2*e)^3 - 2140*B*c^
4*tan(1/2*f*x + 1/2*e)^3 + 651*A*c^4*tan(1/2*f*x + 1/2*e)^2 - 1344*B*c^4*t
an(1/2*f*x + 1/2*e)^2 + 393*A*c^4*tan(1/2*f*x + 1/2*e) - 780*B*c^4*tan(1/2
*f*x + 1/2*e) + 164*A*c^4 - 330*B*c^4)/((tan(1/2*f*x + 1/2*e)^3 + tan(1/2*
f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e) + 1)^3*a^2))/f
```

Mupad [B] (verification not implemented)

Time = 39.07 (sec) , antiderivative size = 414, normalized size of antiderivative = 2.30

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^2}\,dx = \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(131\,A\,c^4-260\,B\,c^4\right)+\frac{164\,A\,c^4}{3}-110\,B\,c^4+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8\left(33\,A\,c^4-70\,B\,c^4\right)+\tan\left(\frac{e}{2}+\frac{f}{}\right.}{f\,\left(a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^9+3\,a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8+\right.} + \frac{35\,c^4\,\mathrm{atan}\left(\frac{35\,c^4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\,(A-2\,B)}{35\,A\,c^4-70\,B\,c^4}\right)\,(A-2\,B)}{a^2\,f}$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^4)/(a + a*sin(e + f*x))^2,x
)
```

output
```
(tan(e/2 + (f*x)/2)*(131*A*c^4 - 260*B*c^4) + (164*A*c^4)/3 - 110*B*c^4 +
tan(e/2 + (f*x)/2)^8*(33*A*c^4 - 70*B*c^4) + tan(e/2 + (f*x)/2)^7*(111*A*c
^4 - 212*B*c^4) + tan(e/2 + (f*x)/2)^2*(217*A*c^4 - 448*B*c^4) + tan(e/2 +
 (f*x)/2)^4*(307*A*c^4 - 660*B*c^4) + tan(e/2 + (f*x)/2)^5*(349*A*c^4 - 66
0*B*c^4) + tan(e/2 + (f*x)/2)^6*((533*A*c^4)/3 - (1160*B*c^4)/3) + tan(e/2
 + (f*x)/2)^3*(369*A*c^4 - (2140*B*c^4)/3))/(f*(6*a^2*tan(e/2 + (f*x)/2)^2
 + 10*a^2*tan(e/2 + (f*x)/2)^3 + 12*a^2*tan(e/2 + (f*x)/2)^4 + 12*a^2*tan(
e/2 + (f*x)/2)^5 + 10*a^2*tan(e/2 + (f*x)/2)^6 + 6*a^2*tan(e/2 + (f*x)/2)^
7 + 3*a^2*tan(e/2 + (f*x)/2)^8 + a^2*tan(e/2 + (f*x)/2)^9 + a^2 + 3*a^2*ta
n(e/2 + (f*x)/2))) + (35*c^4*atan((35*c^4*tan(e/2 + (f*x)/2)*(A - 2*B))/(3
5*A*c^4 - 70*B*c^4))*(A - 2*B))/(a^2*f)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.23

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^2}\,dx$$
$$= \frac{c^4\left(66a-140b+105\cos\left(fx+e\right)\sin\left(fx+e\right)afx-210\cos\left(fx+e\right)\sin\left(fx+e\right)bfx+140\cos\left(fx+e\right.\right.}{}$$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x)
```

output
```
(c**4*(2*cos(e + f*x)*sin(e + f*x)**4*b + 3*cos(e + f*x)*sin(e + f*x)**3*a
 - 14*cos(e + f*x)*sin(e + f*x)**3*b - 30*cos(e + f*x)*sin(e + f*x)**2*a +
 72*cos(e + f*x)*sin(e + f*x)**2*b + 105*cos(e + f*x)*sin(e + f*x)*a*f*x -
 131*cos(e + f*x)*sin(e + f*x)*a - 210*cos(e + f*x)*sin(e + f*x)*b*f*x + 2
60*cos(e + f*x)*sin(e + f*x)*b + 105*cos(e + f*x)*a*f*x - 66*cos(e + f*x)*
a - 210*cos(e + f*x)*b*f*x + 140*cos(e + f*x)*b + 2*sin(e + f*x)**5*b + 3*
sin(e + f*x)**4*a - 16*sin(e + f*x)**4*b - 33*sin(e + f*x)**3*a + 86*sin(e
 + f*x)**3*b - 105*sin(e + f*x)**2*a*f*x - 297*sin(e + f*x)**2*a + 210*sin
(e + f*x)**2*b*f*x + 568*sin(e + f*x)**2*b - 210*sin(e + f*x)*a*f*x - 131*
sin(e + f*x)*a + 420*sin(e + f*x)*b*f*x + 260*sin(e + f*x)*b - 105*a*f*x +
 66*a + 210*b*f*x - 140*b))/(6*a**2*f*(cos(e + f*x)*sin(e + f*x) + cos(e +
 f*x) - sin(e + f*x)**2 - 2*sin(e + f*x) - 1))
```

3.62 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 162

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx = \frac{5(2A-5B)c^3x}{2a^2}+\frac{5(2A-5B)c^3\cos(e+fx)}{2a^2f}-\frac{a^3(A-B)c^3\cos^7(e+fx)}{3f(a+a\sin(e+fx))^5}+\frac{2a(2A-5B)c^3\cos^5(e+fx)}{3f(a+a\sin(e+fx))^3}+\frac{5(2A-5B)c^3\cos^3(e+fx)}{6f\left(a^2+a^2\sin(e+fx)\right)}$$

output
```
5/2*(2*A-5*B)*c^3*x/a^2+5/2*(2*A-5*B)*c^3*cos(f*x+e)/a^2/f-1/3*a^3*(A-B)*c
^3*cos(f*x+e)^7/f/(a+a*sin(f*x+e))^5+2/3*a*(2*A-5*B)*c^3*cos(f*x+e)^5/f/(a
+a*sin(f*x+e))^3+5/6*(2*A-5*B)*c^3*cos(f*x+e)^3/f/(a^2+a^2*sin(f*x+e))
```

Mathematica [A] (verified)

Time = 11.53 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.69

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c-c\sin(e+fx))^3\left(64(A-B)\sin\left(\frac{1}{2}(e+fx)\right)-32(A-B)\left(\cos\right.\right.}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3)/(a + a*Sin[e + f*x
])^2,x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^3*(64*(A - B)*
Sin[(e + f*x)/2] - 32*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 32*(
7*A - 13*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 30*
(2*A - 5*B)*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 12*(A - 5*
B)*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 3*B*(Cos[(e + f*
x)/2] + Sin[(e + f*x)/2])^3*Sin[2*(e + f*x)]))/(12*a^2*f*(Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2])^6*(1 + Sin[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.90, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.306, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3158, 3042, 3161, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^3(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(c-c\sin(e+fx))^3(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{ 3446}$$

$$a^3c^3\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(\sin(e+fx)a+a)^5}dx$$

$$\Big\downarrow \text{ 3042}$$

$$a^3c^3\int \frac{\cos(e+fx)^6(A+B\sin(e+fx))}{(\sin(e+fx)a+a)^5}dx$$

$$\Big\downarrow \text{ 3338}$$

$$a^3c^3\left(-\frac{(2A-5B)\int \frac{\cos^6(e+fx)}{(\sin(e+fx)a+a)^4}dx}{3a}-\frac{(A-B)\cos^7(e+fx)}{3f(a\sin(e+fx)+a)^5}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left(-\frac{(2A-5B)\int\frac{\cos(e+fx)^6}{(\sin(e+fx)a+a)^4}dx}{3a}-\frac{(A-B)\cos^7(e+fx)}{3f(a\sin(e+fx)+a)^5}\right)$$

$$\Big\downarrow \text{3159}$$

$$a^3c^3\left(-\frac{(2A-5B)\left(-\frac{5\int\frac{\cos^4(e+fx)}{(\sin(e+fx)a+a)^2}dx}{a^2}-\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{3a}-\frac{(A-B)\cos^7(e+fx)}{3f(a\sin(e+fx)+a)^5}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left(-\frac{(2A-5B)\left(-\frac{5\int\frac{\cos(e+fx)^4}{(\sin(e+fx)a+a)^2}dx}{a^2}-\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{3a}-\frac{(A-B)\cos^7(e+fx)}{3f(a\sin(e+fx)+a)^5}\right)$$

$$\Big\downarrow \text{3158}$$

$$a^3c^3\left(-\frac{(2A-5B)\left(-\frac{5\left(\frac{3\int\frac{\cos^2(e+fx)}{\sin(e+fx)a+a}dx}{2a}+\frac{\cos^3(e+fx)}{2f\left(a^2\sin(e+fx)+a^2\right)}\right)}{a^2}-\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{3a}-\frac{(A-B)\cos^7(e+fx)}{3f(a\sin(e+fx)+a)^5}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left(-\frac{(2A-5B)\left(-\frac{5\left(\frac{3\int\frac{\cos(e+fx)^2}{\sin(e+fx)a+a}dx}{2a}+\frac{\cos^3(e+fx)}{2f\left(a^2\sin(e+fx)+a^2\right)}\right)}{a^2}-\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{3a}-\frac{(A-B)\cos^7(e+fx)}{3f(a\sin(e+fx)+a)^5}\right)$$

$$\Big\downarrow \text{3161}$$

$$a^3c^3\left(-\frac{(2A-5B)\left(-\frac{5\left(\frac{3\left(\frac{\int 1dx}{a}+\frac{\cos(e+fx)}{af}\right)}{2a}+\frac{\cos^3(e+fx)}{2f\left(a^2\sin(e+fx)+a^2\right)}\right)}{a^2}-\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{3a}-\frac{(A-B)\cos^7(e+fx)}{3f(a\sin(e+fx)+a)^5}\right)$$

$$\Big\downarrow \; 24$$

$$a^3c^3\left(-\frac{(2A-5B)\left(-\frac{5\left(\frac{\cos^3(e+fx)}{2f\left(a^2\sin(e+fx)+a^2\right)}+\frac{3\left(\frac{\cos(e+fx)}{af}+\frac{x}{a}\right)}{2a}\right)}{a^2}-\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{3a}-\frac{(A-B)\cos^7(e+fx)}{3f(a\sin(e+fx)+a)^5}\right)$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^2,x
]
```

output
```
a^3*c^3*(-1/3*((A - B)*Cos[e + f*x]^7)/(f*(a + a*Sin[e + f*x])^5) - ((2*A
- 5*B)*((-2*Cos[e + f*x]^5)/(a*f*(a + a*Sin[e + f*x])^3) - (5*((3*(x/a + C
os[e + f*x]/(a*f)))/(2*a) + Cos[e + f*x]^3/(2*f*(a^2 + a^2*Sin[e + f*x]))))
)/a^2))/(3*a))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3158
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(a*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
 EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && In
tegersQ[2*m, 2*p]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 1.95 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{2c^3\left(-\frac{-16A+16B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{-4A+12B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{16A-16B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{-\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2}+(A-5B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\frac{B\tan\left(\frac{fx}{2}+\right.}{2}}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^2}\right.}{f\,a^2}$
default	$\frac{2c^3\left(-\frac{-16A+16B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{-4A+12B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{16A-16B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{-\frac{B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2}+(A-5B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\frac{B\tan\left(\frac{fx}{2}+\right.}{2}}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^2}\right.}{f\,a^2}$
parallelrisch	$-\frac{c^3\left(\left(-30fxA+75fxB-56A+\frac{607}{4}B\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\left(10fxA-25fxB-\frac{19}{3}A+\frac{115}{12}B\right)\cos\left(\frac{3fx}{2}+\frac{3e}{2}\right)+\left(-30fxA+75\right.\right.}{2f\,a^2}$
risch	$\frac{5c^3xA}{a^2}-\frac{25c^3xB}{2a^2}-\frac{iB\,c^3e^{2i(fx+e)}}{8a^2f}+\frac{c^3e^{i(fx+e)}A}{2a^2f}-\frac{5c^3e^{i(fx+e)}B}{2a^2f}+\frac{c^3e^{-i(fx+e)}A}{2a^2f}-\frac{5c^3e^{-i(fx+e)}B}{2a^2f}+$
norman	$\frac{\left(8A\,c^3-25B\,c^3\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{af}+\frac{\left(34A\,c^3-77B\,c^3\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{af}+\frac{\left(38A\,c^3-93B\,c^3\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{\left(148A\,c^3-352B\,c^3\right)\tan\left(\right.}{af}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x,method=_RETUR
NVERBOSE)
```

output
```
2/f*c^3/a^2*(-1/2*(-16*A+16*B)/(tan(1/2*f*x+1/2*e)+1)^2-(-4*A+12*B)/(tan(1
/2*f*x+1/2*e)+1)-1/3*(16*A-16*B)/(tan(1/2*f*x+1/2*e)+1)^3+(-1/2*B*tan(1/2*
f*x+1/2*e)^3+(A-5*B)*tan(1/2*f*x+1/2*e)^2+1/2*B*tan(1/2*f*x+1/2*e)+A-5*B)/
(1+tan(1/2*f*x+1/2*e)^2)^2+5/2*(2*A-5*B)*arctan(tan(1/2*f*x+1/2*e)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.80

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx$$
$$=\frac{3\,Bc^3\cos(fx+e)^4+6\,(A-4\,B)c^3\cos(fx+e)^3-30\,(2\,A-5\,B)c^3fx+16\,(A-B)c^3+(15\,(2\,A-5\,B\right.}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
1/6*(3*B*c^3*cos(f*x + e)^4 + 6*(A - 4*B)*c^3*cos(f*x + e)^3 - 30*(2*A - 5
*B)*c^3*f*x + 16*(A - B)*c^3 + (15*(2*A - 5*B)*c^3*f*x - (62*A - 131*B)*c^
3)*cos(f*x + e)^2 - (15*(2*A - 5*B)*c^3*f*x + 2*(26*A - 71*B)*c^3)*cos(f*x
 + e) + (3*B*c^3*cos(f*x + e)^3 - 30*(2*A - 5*B)*c^3*f*x - 3*(2*A - 9*B)*c
^3*cos(f*x + e)^2 - 16*(A - B)*c^3 - (15*(2*A - 5*B)*c^3*f*x + 2*(34*A - 7
9*B)*c^3)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*
x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4665 vs. $2(148) = 296$.

Time = 7.78 (sec) , antiderivative size = 4665, normalized size of antiderivative = 28.80

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^3}{(a + a\sin(e + fx))^2} \, dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3/(a+a*sin(f*x+e))**2,x)
```

output
```
Piecewise((30*A*c**3*f*x*tan(e/2 + f*x/2)**7/(6*a**2*f*tan(e/2 + f*x/2)**7
 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2
*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2
 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 90*A*c**3*f*x*tan(
e/2 + f*x/2)**6/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)
**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a
**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(
e/2 + f*x/2) + 6*a**2*f) + 150*A*c**3*f*x*tan(e/2 + f*x/2)**5/(6*a**2*f*ta
n(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*
x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 +
30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 2
10*A*c**3*f*x*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*
f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2
+ f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**
2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 210*A*c**3*f*x*tan(e/2 + f*x/
2)**3/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a
**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(
e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/
2) + 6*a**2*f) + 150*A*c**3*f*x*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(e/2 + f*
x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1378 vs. $2(152) = 304$.

Time = 0.15 (sec) , antiderivative size = 1378, normalized size of antiderivative = 8.51

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algori
thm="maxima")
```

output
```
-1/3*(B*c^3*((75*sin(f*x + e)/(cos(f*x + e) + 1) + 97*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 98*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 21*sin
(f*x + e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x
+ e) + 1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*a^2*sin(f*x + e)
^3/(cos(f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*a^
2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin(f*x + e)^6/(cos(f*x + e)
 + 1)^6 + a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f*x + e
)/(cos(f*x + e) + 1))/a^2) - 4*A*c^3*((12*sin(f*x + e)/(cos(f*x + e) + 1)
+ 11*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x +
 e)/(cos(f*x + e) + 1) + 4*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e)
+ 1)^4 + a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/
(cos(f*x + e) + 1))/a^2) + 12*B*c^3*((12*sin(f*x + e)/(cos(f*x + e) + 1) +
 11*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x +
e)/(cos(f*x + e) + 1) + 4*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*
sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) +
 1)^4 + a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e...
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.37

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx$$
$$= \frac{\frac{15\,(2\,Ac^3-5\,Bc^3)(fx+e)}{a^2} - \frac{6\left(Bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-2\,Ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+10\,Bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-Bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-2\,Ac^3+10\,Bc^3\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2a^2}}{6\,f}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
1/6*(15*(2*A*c^3 - 5*B*c^3)*(f*x + e)/a^2 - 6*(B*c^3*tan(1/2*f*x + 1/2*e)^
3 - 2*A*c^3*tan(1/2*f*x + 1/2*e)^2 + 10*B*c^3*tan(1/2*f*x + 1/2*e)^2 - B*c
^3*tan(1/2*f*x + 1/2*e) - 2*A*c^3 + 10*B*c^3)/((tan(1/2*f*x + 1/2*e)^2 + 1
)^2*a^2) + 16*(3*A*c^3*tan(1/2*f*x + 1/2*e)^2 - 9*B*c^3*tan(1/2*f*x + 1/2*
e)^2 + 12*A*c^3*tan(1/2*f*x + 1/2*e) - 24*B*c^3*tan(1/2*f*x + 1/2*e) + 5*A
*c^3 - 11*B*c^3)/(a^2*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

Mupad [B] (verification not implemented)

Time = 38.71 (sec) , antiderivative size = 336, normalized size of antiderivative = 2.07

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx$$
$$= \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(38\,A\,c^3-93\,B\,c^3\right)+\frac{46\,A\,c^3}{3}-\frac{118\,B\,c^3}{3}+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6\left(8\,A\,c^3-25\,B\,c^3\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5(3}{f\,\left(a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7+3\,a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6+5\,a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5+\right.}$$
$$+\frac{5\,c^3\,\mathrm{atan}\left(\frac{5\,c^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(2\,A-5\,B)}{10\,A\,c^3-25\,B\,c^3}\right)(2\,A-5\,B)}{a^2\,f}$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^3)/(a + a*sin(e + f*x))^2,x
)
```

output
```
(tan(e/2 + (f*x)/2)*(38*A*c^3 - 93*B*c^3) + (46*A*c^3)/3 - (118*B*c^3)/3 +
 tan(e/2 + (f*x)/2)^6*(8*A*c^3 - 25*B*c^3) + tan(e/2 + (f*x)/2)^5*(34*A*c^
3 - 77*B*c^3) + tan(e/2 + (f*x)/2)^3*(72*A*c^3 - 166*B*c^3) + tan(e/2 + (f
*x)/2)^4*((106*A*c^3)/3 - (328*B*c^3)/3) + tan(e/2 + (f*x)/2)^2*((128*A*c^
3)/3 - (359*B*c^3)/3))/(f*(5*a^2*tan(e/2 + (f*x)/2)^2 + 7*a^2*tan(e/2 + (f
*x)/2)^3 + 7*a^2*tan(e/2 + (f*x)/2)^4 + 5*a^2*tan(e/2 + (f*x)/2)^5 + 3*a^2
*tan(e/2 + (f*x)/2)^6 + a^2*tan(e/2 + (f*x)/2)^7 + a^2 + 3*a^2*tan(e/2 + (
f*x)/2))) + (5*c^3*atan((5*c^3*tan(e/2 + (f*x)/2)*(2*A - 5*B))/(10*A*c^3 -
 25*B*c^3))*(2*A - 5*B))/(a^2*f)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 345, normalized size of antiderivative = 2.13

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx$$
$$= \frac{c^3\left(16a-50b+30\cos\left(fx+e\right)\sin\left(fx+e\right)afx-75\cos\left(fx+e\right)\sin\left(fx+e\right)bfx+50\cos\left(fx+e\right)b+\right.}{}$$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x)
```

output
```
(c**3*( - 3*cos(e + f*x)*sin(e + f*x)**3*b - 6*cos(e + f*x)*sin(e + f*x)**
2*a + 24*cos(e + f*x)*sin(e + f*x)**2*b + 30*cos(e + f*x)*sin(e + f*x)*a*f
*x - 38*cos(e + f*x)*sin(e + f*x)*a - 75*cos(e + f*x)*sin(e + f*x)*b*f*x +
 93*cos(e + f*x)*sin(e + f*x)*b + 30*cos(e + f*x)*a*f*x - 16*cos(e + f*x)*
a - 75*cos(e + f*x)*b*f*x + 50*cos(e + f*x)*b - 3*sin(e + f*x)**4*b - 6*si
n(e + f*x)**3*a + 27*sin(e + f*x)**3*b - 30*sin(e + f*x)**2*a*f*x - 92*sin
(e + f*x)**2*a + 75*sin(e + f*x)**2*b*f*x + 205*sin(e + f*x)**2*b - 60*sin
(e + f*x)*a*f*x - 38*sin(e + f*x)*a + 150*sin(e + f*x)*b*f*x + 93*sin(e +
f*x)*b - 30*a*f*x + 16*a + 75*b*f*x - 50*b))/(6*a**2*f*(cos(e + f*x)*sin(e
 + f*x) + cos(e + f*x) - sin(e + f*x)**2 - 2*sin(e + f*x) - 1))
```

3.63 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 108

$$\begin{aligned}\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx &= \frac{(A-4B)c^2x}{a^2}+\frac{(A-4B)c^2\cos(e+fx)}{a^2f}\\ &\quad-\frac{a^2(A-B)c^2\cos^5(e+fx)}{3f(a+a\sin(e+fx))^4}+\frac{2(A-4B)c^2\cos^3(e+fx)}{3f(a+a\sin(e+fx))^2}\end{aligned}$$

output
```
(A-4*B)*c^2*x/a^2+(A-4*B)*c^2*cos(f*x+e)/a^2/f-1/3*a^2*(A-B)*c^2*cos(f*x+e
)^5/f/(a+a*sin(f*x+e))^4+2/3*(A-4*B)*c^2*cos(f*x+e)^3/f/(a+a*sin(f*x+e))^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 234 vs. 2(108) = 216.

Time = 11.41 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.17

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(8(A-B)\sin\left(\frac{1}{2}(e+fx)\right)-4(A-B)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e\right.\right.\right.}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2)/(a + a*Sin[e + f*x
])^2,x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(A - B)*Sin[(e + f*x)/2] - 4*(A
- B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 8*(2*A - 5*B)*Sin[(e + f*x)/2
]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 3*(A - 4*B)*(e + f*x)*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^3 - 3*B*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^3)*(c - c*Sin[e + f*x])^2)/(3*a^2*f*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^4*(1 + Sin[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3161, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^2(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-c\sin(e+fx))^2(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3446}$$

$$a^2c^2\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(\sin(e+fx)a+a)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\int \frac{\cos(e+fx)^4(A+B\sin(e+fx))}{(\sin(e+fx)a+a)^4}dx$$

$$\Big\downarrow \text{3338}$$

$$a^2c^2\left(-\frac{(A-4B)\int \frac{\cos^4(e+fx)}{(\sin(e+fx)a+a)^3}dx}{3a}-\frac{(A-B)\cos^5(e+fx)}{3f(a\sin(e+fx)+a)^4}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left(-\frac{(A-4B)\int \frac{\cos(e+fx)^4}{(\sin(e+fx)a+a)^3}dx}{3a}-\frac{(A-B)\cos^5(e+fx)}{3f(a\sin(e+fx)+a)^4}\right)$$

$$\Big\downarrow \text{3159}$$

$$a^2c^2\left(-\frac{(A-4B)\left(-\frac{3\int \frac{\cos^2(e+fx)}{\sin(e+fx)a+a}dx}{a^2}-\frac{2\cos^3(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{3a}-\frac{(A-B)\cos^5(e+fx)}{3f(a\sin(e+fx)+a)^4}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left(-\frac{(A-4B)\left(-\frac{3\int \frac{\cos(e+fx)^2}{\sin(e+fx)a+a}dx}{a^2}-\frac{2\cos^3(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{3a}-\frac{(A-B)\cos^5(e+fx)}{3f(a\sin(e+fx)+a)^4}\right)$$

$$\Big\downarrow \text{3161}$$

$$a^2c^2\left(-\frac{(A-4B)\left(-\frac{3\left(\frac{\int 1dx}{a}+\frac{\cos(e+fx)}{af}\right)}{a^2}-\frac{2\cos^3(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{3a}-\frac{(A-B)\cos^5(e+fx)}{3f(a\sin(e+fx)+a)^4}\right)$$

$$\Big\downarrow \text{24}$$

$$a^2c^2\left(-\frac{(A-4B)\left(-\frac{3\left(\frac{\cos(e+fx)}{af}+\frac{x}{a}\right)}{a^2}-\frac{2\cos^3(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{3a}-\frac{(A-B)\cos^5(e+fx)}{3f(a\sin(e+fx)+a)^4}\right)$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])^2,x
]
```

output
```
a^2*c^2*(-1/3*((A - B)*Cos[e + f*x]^5)/(f*(a + a*Sin[e + f*x])^4) - ((A -
4*B)*((-3*(x/a + Cos[e + f*x]/(a*f)))/a^2 - (2*Cos[e + f*x]^3)/(a*f*(a + a
*Sin[e + f*x])^2)))/(3*a))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{2c^2\left(-\frac{-8A+8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{8A-8B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{4B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{B}{1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}+(A-4B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,a^2}$
default	$\frac{2c^2\left(-\frac{-8A+8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{8A-8B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{4B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{B}{1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}+(A-4B)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,a^2}$
risch	$\frac{c^2xA}{a^2}-\frac{4c^2xB}{a^2}-\frac{B\,c^2\mathrm{e}^{i(fx+e)}}{2a^2f}-\frac{B\,c^2\mathrm{e}^{-i(fx+e)}}{2a^2f}+\frac{8iA\,c^2\mathrm{e}^{i(fx+e)}+8A\,c^2\mathrm{e}^{2i(fx+e)}-24iB\,c^2\mathrm{e}^{i(fx+e)}-16B\,c^2\mathrm{e}}{f\,a^2\left(\mathrm{e}^{i(fx+e)}+i\right)^3}$
parallelrisch	$\frac{3\left(\left(fxA-4fxB+\frac{4}{3}A-8B\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{\left(\left(-\frac{11}{6}+4fx\right)B-fxA+\frac{4A}{3}\right)\cos\left(\frac{3fx}{2}+\frac{3e}{2}\right)}{3}+\left(fxA-4fxB+\frac{4}{3}A-\frac{14}{3}B\right)\sin\left(\frac{fx}{2}\right.\right.}{f\,a^2\left(\sin\left(\frac{3fx}{2}+\frac{3e}{2}\right)+3\sin\left(\frac{fx}{2}+\frac{e}{2}\right)+3\cos\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}$
norman	$\frac{\frac{c^2(A-4B)x}{a}+\frac{\left(8A\,c^2-30B\,c^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{c^2(A-4B)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{a}+\frac{8A\,c^2-38B\,c^2}{3af}-\frac{8B\,c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{af}+\frac{2\left(4A\,c^2-61B\right.}{}}{}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x,method=_RETUR
NVERBOSE)
```

output
```
2/f*c^2/a^2*(-1/2*(-8*A+8*B)/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(8*A-8*B)/(tan(1
/2*f*x+1/2*e)+1)^3-4*B/(tan(1/2*f*x+1/2*e)+1)-B/(1+tan(1/2*f*x+1/2*e)^2)+(
A-4*B)*arctan(tan(1/2*f*x+1/2*e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(104) = 208$.

Time = 0.12 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.24

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx =$$
$$-\frac{3\,Bc^2\cos\left(fx+e\right)^3+6\,(A-4\,B)c^2fx-4\,(A-B)c^2-(3\,(A-4\,B)c^2fx-(8\,A-23\,B)c^2)\cos\left(fx\right.}{3\,(a^2}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
-1/3*(3*B*c^2*cos(f*x + e)^3 + 6*(A - 4*B)*c^2*f*x - 4*(A - B)*c^2 - (3*(A
 - 4*B)*c^2*f*x - (8*A - 23*B)*c^2)*cos(f*x + e)^2 + (3*(A - 4*B)*c^2*f*x
+ 2*(2*A - 11*B)*c^2)*cos(f*x + e) + (6*(A - 4*B)*c^2*f*x - 3*B*c^2*cos(f*
x + e)^2 + 4*(A - B)*c^2 + (3*(A - 4*B)*c^2*f*x + 2*(4*A - 13*B)*c^2)*cos(
f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2
*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2474 vs. $2(102) = 204$.

Time = 4.22 (sec) , antiderivative size = 2474, normalized size of antiderivative = 22.91

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^2}{(a + a\sin(e + fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2/(a+a*sin(f*x+e))**2,x)
```

output
```
Piecewise((3*A*c**2*f*x*tan(e/2 + f*x/2)**5/(3*a**2*f*tan(e/2 + f*x/2)**5
+ 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f
*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 9*A*c**2*f*
x*tan(e/2 + f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f
*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 +
 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 12*A*c**2*f*x*tan(e/2 + f*x/2)**3
/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*
tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f
*x/2) + 3*a**2*f) + 12*A*c**2*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 +
f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 +
 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 9
*A*c**2*f*x*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(
e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/
2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 3*A*c**2*f*x/(3*a**2*f*tan
(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/
2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2
*f) + 24*A*c**2*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2
*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2
 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 8*A*c**2*tan(e/2 +
f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 ...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 833 vs. $2(104) = 208$.

Time = 0.13 (sec) , antiderivative size = 833, normalized size of antiderivative = 7.71

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algori
thm="maxima")
```

output
```
-2/3*(2*B*c^2*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1)
+ 4*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x +
e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2
) - A*c^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)
 + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + 2*B*c^2*((9*sin(f*x +
e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4)/(a^2 +
3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)
/(cos(f*x + e) + 1))/a^2) + A*c^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x
 + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3) - 2*A*c^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/
(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + B*c^2*(3*sin(f
*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e)...
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.20

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx = \frac{\frac{3(Ac^2-4Bc^2)(fx+e)}{a^2} - \frac{6Bc^2}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)a^2} - \frac{8\left(3Bc^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-3Ac^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+9Bc^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-Ac^2+4Bc^2\right)}{a^2\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}}{3f}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
1/3*(3*(A*c^2 - 4*B*c^2)*(f*x + e)/a^2 - 6*B*c^2/((tan(1/2*f*x + 1/2*e)^2
+ 1)*a^2) - 8*(3*B*c^2*tan(1/2*f*x + 1/2*e)^2 - 3*A*c^2*tan(1/2*f*x + 1/2*
e) + 9*B*c^2*tan(1/2*f*x + 1/2*e) - A*c^2 + 4*B*c^2)/(a^2*(tan(1/2*f*x + 1
/2*e) + 1)^3))/f
```

Mupad [B] (verification not implemented)

Time = 38.57 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.24

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx$$
$$=\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(8Ac^2-30Bc^2\right)+\frac{8Ac^2}{3}-\frac{38Bc^2}{3}+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(8Ac^2-26Bc^2\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(\frac{8Ac}{3}\right.}{f\left(a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5+3a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4+4a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3+4a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+3a^2\right.}$$
$$+\frac{2c^2\,\mathrm{atan}\left(\frac{2c^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(A-4B)}{2Ac^2-8Bc^2}\right)(A-4B)}{a^2f}$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^2)/(a + a*sin(e + f*x))^2,x
)
```

output
```
(tan(e/2 + (f*x)/2)*(8*A*c^2 - 30*B*c^2) + (8*A*c^2)/3 - (38*B*c^2)/3 + ta
n(e/2 + (f*x)/2)^3*(8*A*c^2 - 26*B*c^2) + tan(e/2 + (f*x)/2)^2*((8*A*c^2)/
3 - (74*B*c^2)/3) - 8*B*c^2*tan(e/2 + (f*x)/2)^4)/(f*(4*a^2*tan(e/2 + (f*x
)/2)^2 + 4*a^2*tan(e/2 + (f*x)/2)^3 + 3*a^2*tan(e/2 + (f*x)/2)^4 + a^2*tan
(e/2 + (f*x)/2)^5 + a^2 + 3*a^2*tan(e/2 + (f*x)/2))) + (2*c^2*atan((2*c^2*
tan(e/2 + (f*x)/2)*(A - 4*B))/(2*A*c^2 - 8*B*c^2))*(A - 4*B))/(a^2*f)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.56

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx$$
$$=\frac{c^2\left(3\cos(fx+e)\sin(fx+e)^2b+3\cos(fx+e)\sin(fx+e)afx-4\cos(fx+e)\sin(fx+e)a-12\cos\right.}{}$$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x)
```

output
```
(c**2*(3*cos(e + f*x)*sin(e + f*x)**2*b + 3*cos(e + f*x)*sin(e + f*x)*a*f*
x - 4*cos(e + f*x)*sin(e + f*x)*a - 12*cos(e + f*x)*sin(e + f*x)*b*f*x + 1
5*cos(e + f*x)*sin(e + f*x)*b + 3*cos(e + f*x)*a*f*x - 12*cos(e + f*x)*b*f
*x + 8*cos(e + f*x)*b + 3*sin(e + f*x)**3*b - 3*sin(e + f*x)**2*a*f*x - 12
*sin(e + f*x)**2*a + 12*sin(e + f*x)**2*b*f*x + 34*sin(e + f*x)**2*b - 6*s
in(e + f*x)*a*f*x - 4*sin(e + f*x)*a + 24*sin(e + f*x)*b*f*x + 15*sin(e +
f*x)*b - 3*a*f*x + 12*b*f*x - 8*b))/(3*a**2*f*(cos(e + f*x)*sin(e + f*x) +
 cos(e + f*x) - sin(e + f*x)**2 - 2*sin(e + f*x) - 1))
```

3.64 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 72

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = -\frac{Bcx}{a^2}+\frac{(A-7B)c\cos(e+fx)}{3a^2f(1+\sin(e+fx))}-\frac{2(A-B)c\cos(e+fx)}{3f(a+a\sin(e+fx))^2}$$

output
```
-B*c*x/a^2+1/3*(A-7*B)*c*cos(f*x+e)/a^2/f/(1+sin(f*x+e))-2/3*(A-B)*c*cos(f
*x+e)/f/(a+a*sin(f*x+e))^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 156 vs. 2(72) = 144.

Time = 6.52 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.17

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = \frac{c\left(-9Bfx\cos\left(\frac{fx}{2}\right)-6(A-3B)\cos\left(e+\frac{fx}{2}\right)+2A\cos\left(e+\frac{3fx}{2}\right)-14B\cos\left(e+\frac{3fx}{2}\right)+3Bfx\cos\left(2e+\right.\right.}{6a^2f\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}\right.\right.}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))/(a + a*Sin[e + f*x])
^2,x]
```

output
```
(c*(-9*B*f*x*Cos[(f*x)/2] - 6*(A - 3*B)*Cos[e + (f*x)/2] + 2*A*Cos[e + (3*
f*x)/2] - 14*B*Cos[e + (3*f*x)/2] + 3*B*f*x*Cos[2*e + (3*f*x)/2] + 24*B*Si
n[(f*x)/2] - 9*B*f*x*Sin[e + (f*x)/2] - 3*B*f*x*Sin[e + (3*f*x)/2]))/(6*a^
2*f*(Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 3446, 3042, 3336, 3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-c\sin(e+fx))(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3446}$$

$$ac\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(\sin(e+fx)a+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$ac\int \frac{\cos(e+fx)^2(A+B\sin(e+fx))}{(\sin(e+fx)a+a)^3}dx$$

$$\Big\downarrow \text{3336}$$

$$ac\left(-\frac{\int \frac{a(A-4B)+3aB\sin(e+fx)}{\sin(e+fx)a+a}dx}{3a^3}-\frac{2(A-B)\cos(e+fx)}{3af(a\sin(e+fx)+a)^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$ac\left(-\frac{\int \frac{a(A-4B)+3aB\sin(e+fx)}{\sin(e+fx)a+a}dx}{3a^3}-\frac{2(A-B)\cos(e+fx)}{3af(a\sin(e+fx)+a)^2}\right)$$

$$\Big\downarrow \ 3214$$

$$ac\left(-\frac{a(A-7B)\int \frac{1}{\sin(e+fx)a+a}dx+3Bx}{3a^3}-\frac{2(A-B)\cos(e+fx)}{3af(a\sin(e+fx)+a)^2}\right)$$

$$\Big\downarrow \ 3042$$

$$ac\left(-\frac{a(A-7B)\int \frac{1}{\sin(e+fx)a+a}dx+3Bx}{3a^3}-\frac{2(A-B)\cos(e+fx)}{3af(a\sin(e+fx)+a)^2}\right)$$

$$\Big\downarrow \ 3127$$

$$ac\left(-\frac{3Bx-\frac{a(A-7B)\cos(e+fx)}{f(a\sin(e+fx)+a)}}{3a^3}-\frac{2(A-B)\cos(e+fx)}{3af(a\sin(e+fx)+a)^2}\right)$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))/(a + a*Sin[e + f*x])^2,x]
```

output
```
a*c*((-2*(A - B)*Cos[e + f*x])/(3*a*f*(a + a*Sin[e + f*x])^2) - (3*B*x - (
a*(A - 7*B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])))/(3*a^3))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3336
```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Simp[1/(b^
3*(2*m + 3))   Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*
(2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2
- b^2, 0] && LtQ[m, -3/2]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.12

method	result
risch	$-\frac{Bcx}{a^{2}}+\frac{2Ac\,{\mathrm e}^{2i(fx+e)}-8iBc\,{\mathrm e}^{i(fx+e)}-6Bc\,{\mathrm e}^{2i(fx+e)}-\frac{2Ac}{3}+\frac{14Bc}{3}}{f\,a^{2}\left({\mathrm e}^{i(fx+e)}+i\right)^{3}}$
derivativedivides	$\frac{2c\left(-\frac{-4A+4B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{A+B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{4A-4B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,a^{2}}$
default	$\frac{2c\left(-\frac{-4A+4B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{A+B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{4A-4B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\right)}{f\,a^{2}}$
parallelrisch	$-\frac{2\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}xfB}{2}+\left(\frac{3}{2}fxB+A+B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}+B\left(\frac{3fx}{2}+4\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{fxB}{2}+\frac{A}{3}+\frac{5B}{3}\right)c}{f\,a^{2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}$
norman	$\frac{-\frac{2Ac+10Bc}{3af}-\frac{Bcx}{a}-\frac{16Bc\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{af}-\frac{8Bc\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{af}-\frac{8Bc\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{(14Ac+22Bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{3af}-\frac{(10Ac+26Bc)}{3}}{}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^2,x,method=_RETURNV
ERBOSE)
```

output
```
-B*c*x/a^2+2/3*(3*A*c*exp(2*I*(f*x+e))-12*I*B*c*exp(I*(f*x+e))-9*B*c*exp(2
*I*(f*x+e))-A*c+7*B*c)/f/a^2/(exp(I*(f*x+e))+I)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(68) = 136$.

Time = 0.12 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.31

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx$$
$$= \frac{6\,Bcfx-(3\,Bcfx+(A-7\,B)c)\cos{(fx+e)}^2+2\,(A-B)c+(3\,Bcfx+(A+5\,B)c)\cos{(fx+e)}+(6}{3\left(a^2f\cos{(fx+e)}^2-a^2f\cos{(fx+e)}-2\,a^2f-(a^2f\cos{(j}\right.}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorith
m="fricas")
```

output
```
1/3*(6*B*c*f*x - (3*B*c*f*x + (A - 7*B)*c)*cos(f*x + e)^2 + 2*(A - B)*c +
(3*B*c*f*x + (A + 5*B)*c)*cos(f*x + e) + (6*B*c*f*x - 2*(A - B)*c + (3*B*c
*f*x - (A - 7*B)*c)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^
2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 702 vs. $2(70) = 140$.

Time = 2.16 (sec) , antiderivative size = 702, normalized size of antiderivative = 9.75

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))**2,x)
```

output
```
Piecewise((-6*A*c*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a*
*2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 2*A*c/(
3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan
(e/2 + f*x/2) + 3*a**2*f) - 3*B*c*f*x*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/
2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) +
 3*a**2*f) - 9*B*c*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 +
 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 9*
B*c*f*x*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2
+ f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 3*B*c*f*x/(3*a**2*f*
tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*
x/2) + 3*a**2*f) - 6*B*c*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3
 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) -
24*B*c*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 +
 f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 10*B*c/(3*a**2*f*tan(
e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2)
 + 3*a**2*f), Ne(f, 0)), (x*(A + B*sin(e))*(-c*sin(e) + c)/(a*sin(e) + a)*
*2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(68) = 136$.

Time = 0.12 (sec) , antiderivative size = 452, normalized size of antiderivative = 6.28

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx =$$

$$-\frac{2\left(Bc\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+4}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2}\right)+\frac{Ac\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\cdots\right.}{a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\cdots}\cdots}{3f}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorith
m="maxima")
```

output
```
-2/3*(B*c*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)
 + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + A*c*(3*sin(f*x + e)/(c
os(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2
*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) - A*c*(3*sin(f*x + e)/(cos(f
*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
) + B*c*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/
(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.21

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx$$
$$= -\frac{\frac{3(fx+e)Bc}{a^2} + \frac{2\left(3\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,Bc\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+12\,Bc\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+Ac+5\,Bc\right)}{a^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}}{3\,f}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorith
m="giac")
```

output
```
-1/3*(3*(f*x + e)*B*c/a^2 + 2*(3*A*c*tan(1/2*f*x + 1/2*e)^2 + 3*B*c*tan(1/
2*f*x + 1/2*e)^2 + 12*B*c*tan(1/2*f*x + 1/2*e) + A*c + 5*B*c)/(a^2*(tan(1/
2*f*x + 1/2*e) + 1)^3))/f
```

Mupad [B] (verification not implemented)

Time = 37.11 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.85

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = -\frac{B\,c\,x}{a^2} - \frac{\left(\frac{c\,(6\,A+6\,B+9\,B\,(e+f\,x))}{3}-3\,B\,c\,(e+f\,x)\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+\left(\frac{c\,(24\,B+9\,B\,(e+f\,x))}{3}-3\,B\,c\,(e+f\,x)\right)\,\tan\left(\frac{e}{2}+\right.}{a^2\,f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+1\right)^3}$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x)))/(a + a*sin(e + f*x))^2,x)
```

output
```
- (B*c*x)/a^2 - (tan(e/2 + (f*x)/2)^2*((c*(6*A + 6*B + 9*B*(e + f*x)))/3 -
 3*B*c*(e + f*x)) + tan(e/2 + (f*x)/2)*((c*(24*B + 9*B*(e + f*x)))/3 - 3*B
*c*(e + f*x)) + (c*(2*A + 10*B + 3*B*(e + f*x)))/3 - B*c*(e + f*x))/(a^2*f
*(tan(e/2 + (f*x)/2) + 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.15

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = \frac{c\left(2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3a-3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3bfx+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3b-9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2bfx+6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)a\right.}{3a^2f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3+3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+3\tan\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}$$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^2,x)
```

output
```
(c*(2*tan((e + f*x)/2)**3*a - 3*tan((e + f*x)/2)**3*b*f*x + 2*tan((e + f*x
)/2)**3*b - 9*tan((e + f*x)/2)**2*b*f*x + 6*tan((e + f*x)/2)*a - 9*tan((e
+ f*x)/2)*b*f*x - 18*tan((e + f*x)/2)*b - 3*b*f*x - 8*b))/(3*a**2*f*(tan((
e + f*x)/2)**3 + 3*tan((e + f*x)/2)**2 + 3*tan((e + f*x)/2) + 1))
```

3.65 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 62

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))}\,dx = -\frac{(A-B)\sec(e+fx)}{3cf\,(a^2+a^2\sin(e+fx))}+\frac{(2A+B)\tan(e+fx)}{3a^2cf}$$

output
```
-1/3*(A-B)*sec(f*x+e)/c/f/(a^2+a^2*sin(f*x+e))+1/3*(2*A+B)*tan(f*x+e)/a^2/
c/f
```

Mathematica [A] (verified)

Time = 3.19 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.77

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))}\,dx = \frac{\cos(e+fx)(-6B-2(A-B)\cos(e+fx)+2(2A+B)\cos(2(e+fx))-8A\sin(e+fx)-4B\sin(e+}{12a^2cf(-1+\sin(e+fx))(1+\sin(e+fx))^2}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]
)),x]
```

output
```
(Cos[e + f*x]*(-6*B - 2*(A - B)*Cos[e + f*x] + 2*(2*A + B)*Cos[2*(e + f*x)
] - 8*A*Sin[e + f*x] - 4*B*Sin[e + f*x] - A*Sin[2*(e + f*x)] + B*Sin[2*(e
+ f*x)]))/(12*a^2*c*f*(-1 + Sin[e + f*x])*(1 + Sin[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3042, 3446, 3042, 3338, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^2(c-c\sin(e+fx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^2(c-c\sin(e+fx))}dx$$

$$\Big\downarrow \text{3446}$$

$$\frac{\int \frac{\sec^2(e+fx)(A+B\sin(e+fx))}{\sin(e+fx)a+a}dx}{ac}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{A+B\sin(e+fx)}{\cos(e+fx)^2(\sin(e+fx)a+a)}dx}{ac}$$

$$\Big\downarrow \text{3338}$$

$$\frac{\frac{(2A+B)\int \sec^2(e+fx)dx}{3a}-\frac{(A-B)\sec(e+fx)}{3f(a\sin(e+fx)+a)}}{ac}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(2A+B)\int \csc\left(e+fx+\frac{\pi}{2}\right)^2dx}{3a}-\frac{(A-B)\sec(e+fx)}{3f(a\sin(e+fx)+a)}}{ac}$$

$$\Big\downarrow \text{4254}$$

$$\frac{-\frac{(2A+B)\int 1d(-\tan(e+fx))}{3af}-\frac{(A-B)\sec(e+fx)}{3f(a\sin(e+fx)+a)}}{ac}$$

$$\downarrow 24$$

$$\frac{\frac{(2A+B)\tan(e+fx)}{3af}-\frac{(A-B)\sec(e+fx)}{3f(a\sin(e+fx)+a)}}{ac}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])),x]
```

output
```
(-1/3*((A - B)*Sec[e + f*x])/(f*(a + a*Sin[e + f*x])) + ((2*A + B)*Tan[e +
 f*x])/(3*a*f))/(a*c)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.39

method	result
risch	$\frac{2i\left(4iA\,e^{i(fx+e)}+2iB\,e^{i(fx+e)}+3B\,e^{2i(fx+e)}-2A-B\right)}{3\left(e^{i(fx+e)}+i\right)^{3}\left(e^{i(fx+e)}-i\right)f\,a^{2}c}$
parallelrisch	$\frac{-6A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+(-6A-6B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}+(-2A-4B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2A-2B}{3f\,a^{2}c\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}$
derivativedivides	$\frac{-\frac{-A+B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2(A-B)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2\left(\frac{3A}{4}-\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2\left(\frac{A}{4}+\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}}{f\,a^{2}c}$
default	$\frac{-\frac{-A+B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2(A-B)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2\left(\frac{3A}{4}-\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2\left(\frac{A}{4}+\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}}{f\,a^{2}c}$
norman	$\frac{-\frac{2A+4B}{6acf}-\frac{4(2A+B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3acf}+\frac{A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{acf}-\frac{(2A+4B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{2acf}-\frac{(8A+4B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3acf}-\frac{(14A+16B)\tan\left(\frac{fx}{2}\right.}{6acf}}{a\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x,method=_RETURNV
ERBOSE)
```

output
```
2/3*I*(4*I*A*exp(I*(f*x+e))+2*I*B*exp(I*(f*x+e))+3*B*exp(2*I*(f*x+e))-2*A-
B)/(exp(I*(f*x+e))+I)^3/(exp(I*(f*x+e))-I)/f/a^2/c
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))}\,dx$$
$$= -\frac{(2\,A+B)\cos\left(fx+e\right)^2-(2\,A+B)\sin\left(fx+e\right)-A-2\,B}{3\left(a^2cf\cos\left(fx+e\right)\sin\left(fx+e\right)+a^2cf\cos\left(fx+e\right)\right)}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
-1/3*((2*A + B)*cos(f*x + e)^2 - (2*A + B)*sin(f*x + e) - A - 2*B)/(a^2*c*
f*cos(f*x + e)*sin(f*x + e) + a^2*c*f*cos(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 578 vs. 2(51) = 102.

Time = 2.31 (sec) , antiderivative size = 578, normalized size of antiderivative = 9.32

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e)),x)
```

output
```
Piecewise((-6*A*tan(e/2 + f*x/2)**3/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a*
*2*c*f*tan(e/2 + f*x/2)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f) - 6
*A*tan(e/2 + f*x/2)**2/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**2*c*f*tan(e/
2 + f*x/2)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f) - 2*A*tan(e/2 +
f*x/2)/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**2*c*f*tan(e/2 + f*x/2)**3 -
6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f) + 2*A/(3*a**2*c*f*tan(e/2 + f*x/
2)**4 + 6*a**2*c*f*tan(e/2 + f*x/2)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a
**2*c*f) - 6*B*tan(e/2 + f*x/2)**2/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**
2*c*f*tan(e/2 + f*x/2)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f) - 4*
B*tan(e/2 + f*x/2)/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**2*c*f*tan(e/2 +
f*x/2)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f) - 2*B/(3*a**2*c*f*ta
n(e/2 + f*x/2)**4 + 6*a**2*c*f*tan(e/2 + f*x/2)**3 - 6*a**2*c*f*tan(e/2 +
f*x/2) - 3*a**2*c*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(e) + a)**2*(-c*
sin(e) + c)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(58) = 116$.

Time = 0.04 (sec) , antiderivative size = 265, normalized size of antiderivative = 4.27

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))}\,dx = \frac{2\left(\frac{B\left(\frac{2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{a^2c+\frac{2a^2c\sin(fx+e)}{\cos(fx+e)+1}-\frac{2a^2c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{a^2c\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}+\frac{A\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-1\right)}{a^2c+\frac{2a^2c\sin(fx+e)}{\cos(fx+e)+1}-\frac{2a^2c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{a^2c\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}\right)}{3f}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x, algorith
m="maxima")
```

output
```
2/3*(B*(2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + 1)/(a^2*c + 2*a^2*c*sin(f*x + e)/(cos(f*x + e) + 1) - 2*a^2*c*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3 - a^2*c*sin(f*x + e)^4/(cos(f*x + e) + 1
)^4) + A*(sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/(a^2*c + 2*a^2*c*sin(
f*x + e)/(cos(f*x + e) + 1) - 2*a^2*c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
- a^2*c*sin(f*x + e)^4/(cos(f*x + e) + 1)^4))/f
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.56

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))}\,dx$$
$$= -\frac{\frac{3\,(A+B)}{a^2c\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)} + \frac{9\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-3\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+12\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+7\,A-B}{a^2c\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}}{6\,f}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x, algorith
m="giac")
```

output
```
-1/6*(3*(A + B)/(a^2*c*(tan(1/2*f*x + 1/2*e) - 1)) + (9*A*tan(1/2*f*x + 1/
2*e)^2 - 3*B*tan(1/2*f*x + 1/2*e)^2 + 12*A*tan(1/2*f*x + 1/2*e) + 7*A - B)
/(a^2*c*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

Mupad [B] (verification not implemented)

Time = 37.15 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.89

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))}\,dx$$
$$= \frac{2\left(\frac{3\,B}{2} - A\,\cos\left(e+f\,x\right) + B\,\cos\left(e+f\,x\right) + 2\,A\,\sin\left(e+f\,x\right) + B\,\sin\left(e+f\,x\right) - A\,\cos\left(2\,e+2\,f\,x\right) - \right.}{3\,a^2\,c\,f\,\left(2\,\cos\left(e+f\,x\right)+\sin\left(2\,e+2\,f\,x\right)\right)}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))),x)
```

output
```
(2*((3*B)/2 - A*cos(e + f*x) + B*cos(e + f*x) + 2*A*sin(e + f*x) + B*sin(e
 + f*x) - A*cos(2*e + 2*f*x) - (B*cos(2*e + 2*f*x))/2 - (A*sin(2*e + 2*f*x
))/2 + (B*sin(2*e + 2*f*x))/2))/(3*a^2*c*f*(2*cos(e + f*x) + sin(2*e + 2*f
*x)))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.90

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))}\,dx$$
$$= \frac{2\cos\left(fx+e\right)\sin\left(fx+e\right)a+\cos\left(fx+e\right)\sin\left(fx+e\right)b+2\cos\left(fx+e\right)a+\cos\left(fx+e\right)b+2\sin\left(fx+\right.}{3\cos\left(fx+e\right)a^{2}cf\left(\sin\left(fx+e\right)+1\right)}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x)
```

output
```
(2*cos(e + f*x)*sin(e + f*x)*a + cos(e + f*x)*sin(e + f*x)*b + 2*cos(e + f
*x)*a + cos(e + f*x)*b + 2*sin(e + f*x)**2*a + sin(e + f*x)**2*b + 2*sin(e
 + f*x)*a + sin(e + f*x)*b - a + b)/(3*cos(e + f*x)*a**2*c*f*(sin(e + f*x)
 + 1))
```

3.66 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 62

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^2}\,dx = \frac{B\sec^3(e+fx)}{3a^2c^2f}+\frac{A\tan(e+fx)}{a^2c^2f}+\frac{A\tan^3(e+fx)}{3a^2c^2f}$$

output
```
1/3*B*sec(f*x+e)^3/a^2/c^2/f+A*tan(f*x+e)/a^2/c^2/f+1/3*A*tan(f*x+e)^3/a^2
/c^2/f
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.85

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^2}\,dx = \frac{B\sec^3(e+fx)}{3a^2c^2f}+\frac{A\left(\tan(e+fx)+\frac{1}{3}\tan^3(e+fx)\right)}{a^2c^2f}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]
)^2),x]
```

output
```
(B*Sec[e + f*x]^3)/(3*a^2*c^2*f) + (A*(Tan[e + f*x] + Tan[e + f*x]^3/3))/(
a^2*c^2*f)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.82, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3042, 3446, 3042, 3148, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^2(c-c\sin(e+fx))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^2(c-c\sin(e+fx))^2}dx$$

$$\Big\downarrow \text{ 3446}$$

$$\frac{\int \sec^4(e+fx)(A+B\sin(e+fx))dx}{a^2c^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{A+B\sin(e+fx)}{\cos(e+fx)^4}dx}{a^2c^2}$$

$$\Big\downarrow \text{ 3148}$$

$$\frac{A\int \sec^4(e+fx)dx+\frac{B\sec^3(e+fx)}{3f}}{a^2c^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{A\int \csc\left(e+fx+\frac{\pi}{2}\right)^4dx+\frac{B\sec^3(e+fx)}{3f}}{a^2c^2}$$

$$\Big\downarrow \text{ 4254}$$

$$\frac{\frac{B\sec^3(e+fx)}{3f}-\frac{A\int (\tan^2(e+fx)+1)d(-\tan(e+fx))}{f}}{a^2c^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{B\sec^3(e+fx)}{3f}-\frac{A\left(-\frac{1}{3}\tan^3(e+fx)-\tan(e+fx)\right)}{f}}{a^2c^2}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^2),x
]
```

output
```
((B*Sec[e + f*x]^3)/(3*f) - (A*(-Tan[e + f*x] - Tan[e + f*x]^3/3))/f)/(a^2
*c^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

method	result
risch	$\frac{4iA\,\mathrm{e}^{2i(fx+e)}+\frac{8B\,\mathrm{e}^{3i(fx+e)}}{3}+\frac{4iA}{3}}{\left(\mathrm{e}^{i(fx+e)}-i\right)^{3}\left(\mathrm{e}^{i(fx+e)}+i\right)^{3}f\,a^{2}c^{2}}$
parallelrisch	$\frac{2A\sin(3fx+3e)+6A\sin(fx+e)+3\cos(fx+e)B+\cos(3fx+3e)B+4B}{3f\,a^{2}c^{2}(\cos(3fx+3e)+3\cos(fx+e))}$
derivativedivides	$\frac{-\frac{2\left(\frac{A}{2}+\frac{B}{2}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}-\frac{\frac{A}{2}+\frac{B}{2}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{2\left(\frac{A}{2}+\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{-\frac{A}{2}+\frac{B}{2}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2\left(\frac{A}{2}-\frac{B}{2}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2\left(\frac{A}{2}-\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{f\,a^{2}c^{2}}$
default	$\frac{-\frac{2\left(\frac{A}{2}+\frac{B}{2}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}-\frac{\frac{A}{2}+\frac{B}{2}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{2\left(\frac{A}{2}+\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{-\frac{A}{2}+\frac{B}{2}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2\left(\frac{A}{2}-\frac{B}{2}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2\left(\frac{A}{2}-\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{f\,a^{2}c^{2}}$
norman	$\frac{-\frac{2B}{3fac}-\frac{2A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}{afc}-\frac{2A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{fac}-\frac{2A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3fac}-\frac{2A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{3fac}-\frac{2B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{fac}-\frac{2B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{3fac}-2}{a\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}c\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x,method=_RETUR
NVERBOSE)
```

output
```
4/3*(3*I*A*exp(2*I*(f*x+e))+2*B*exp(3*I*(f*x+e))+I*A)/(exp(I*(f*x+e))-I)^3
/(exp(I*(f*x+e))+I)^3/f/a^2/c^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.66

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^2}\,dx = \frac{\left(2\,A\cos\left(fx+e\right)^{2}+A\right)\sin\left(fx+e\right)+B}{3\,a^{2}c^{2}f\cos\left(fx+e\right)^{3}}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
1/3*((2*A*cos(f*x + e)^2 + A)*sin(f*x + e) + B)/(a^2*c^2*f*cos(f*x + e)^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(56) = 112$.

Time = 2.14 (sec) , antiderivative size = 469, normalized size of antiderivative = 7.56

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2(c - c\sin(e + fx))^2}\,dx$$
$$= \begin{cases} -\frac{6A\tan^5\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2c^2f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)-9a^2c^2f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2c^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)-3a^2c^2f} + \frac{4A\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2c^2f\tan^6\left(\frac{e}{2}+\frac{fx}{2}\right)-9a^2c^2f\tan^4\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2c^2f} \\ \frac{x(A+B\sin{(e)})}{(a\sin{(e)}+a)^2(-c\sin{(e)}+c)^2} \end{cases}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**2,x)
```

output
```
Piecewise((-6*A*tan(e/2 + f*x/2)**5/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 - 9
*a**2*c**2*f*tan(e/2 + f*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3*a
**2*c**2*f) + 4*A*tan(e/2 + f*x/2)**3/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 -
 9*a**2*c**2*f*tan(e/2 + f*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3
*a**2*c**2*f) - 6*A*tan(e/2 + f*x/2)/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 -
9*a**2*c**2*f*tan(e/2 + f*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3*
a**2*c**2*f) - 6*B*tan(e/2 + f*x/2)**4/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6
- 9*a**2*c**2*f*tan(e/2 + f*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 -
3*a**2*c**2*f) - 2*B/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 - 9*a**2*c**2*f*ta
n(e/2 + f*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3*a**2*c**2*f), Ne
(f, 0)), (x*(A + B*sin(e))/((a*sin(e) + a)**2*(-c*sin(e) + c)**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.76

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2(c - c\sin(e + fx))^2}\,dx = \frac{\frac{\left(\tan(fx+e)^3+3\,\tan(fx+e)\right)A}{a^2c^2} + \frac{B}{a^2c^2\cos(fx+e)^3}}{3\,f}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x, algori
thm="maxima")
```

output
```
1/3*((tan(f*x + e)^3 + 3*tan(f*x + e))*A/(a^2*c^2) + B/(a^2*c^2*cos(f*x +
e)^3))/f
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.32

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2(c - c\sin(e + fx))^2}\,dx = -\frac{2\left(3\,A\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^5 + 3\,B\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^4 - 2\,A\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^3 + 3\,A\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right) + B\right)}{3\left(\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right)^2 - 1\right)^3 a^2c^2f}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
-2/3*(3*A*tan(1/2*f*x + 1/2*e)^5 + 3*B*tan(1/2*f*x + 1/2*e)^4 - 2*A*tan(1/
2*f*x + 1/2*e)^3 + 3*A*tan(1/2*f*x + 1/2*e) + B)/((tan(1/2*f*x + 1/2*e)^2
- 1)^3*a^2*c^2*f)
```

Mupad [B] (verification not implemented)

Time = 37.50 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.32

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^2}\,dx = -\frac{2\left(3\,A\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+3\,B\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-2\,A\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+3\,A\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+B\right)}{3\,a^2\,c^2\,f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)^3}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^2),x
)
```

output
```
-(2*(B + 3*A*tan(e/2 + (f*x)/2) - 2*A*tan(e/2 + (f*x)/2)^3 + 3*A*tan(e/2 +
 (f*x)/2)^5 + 3*B*tan(e/2 + (f*x)/2)^4))/(3*a^2*c^2*f*(tan(e/2 + (f*x)/2)^
2 - 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.29

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^2}\,dx = \frac{-\cos(fx+e)\sin(fx+e)^2 b+\cos(fx+e)\,b+2\sin(fx+e)^3 a-3a\sin(fx+e)-b}{3\cos(fx+e)\,a^2c^2f\left(\sin(fx+e)^2-1\right)}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x)
```

output
```
( - cos(e + f*x)*sin(e + f*x)**2*b + cos(e + f*x)*b + 2*sin(e + f*x)**3*a
- 3*sin(e + f*x)*a - b)/(3*cos(e + f*x)*a**2*c**2*f*(sin(e + f*x)**2 - 1))
```

3.67 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 93

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^3}\,dx = \frac{(A+B)\sec^3(e+fx)}{5a^2f\left(c^3-c^3\sin(e+fx)\right)}+\frac{(4A-B)\tan(e+fx)}{5a^2c^3f}+\frac{(4A-B)\tan^3(e+fx)}{15a^2c^3f}$$

output
```
1/5*(A+B)*sec(f*x+e)^3/a^2/f/(c^3-c^3*sin(f*x+e))+1/5*(4*A-B)*tan(f*x+e)/a
^2/c^3/f+1/15*(4*A-B)*tan(f*x+e)^3/a^2/c^3/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 237 vs. 2(93) = 186.

Time = 4.98 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.55

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^3}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-240B+54(A+B)\cos(e+$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]
)^3),x]
```

output
```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])*(-240*B + 54*(A + B)*Cos[e + f*x] - 32*(4*A - B)*Cos[2*(e + f*x)] + 18*
A*Cos[3*(e + f*x)] + 18*B*Cos[3*(e + f*x)] - 64*A*Cos[4*(e + f*x)] + 16*B*
Cos[4*(e + f*x)] - 384*A*Sin[e + f*x] + 96*B*Sin[e + f*x] - 18*A*Sin[2*(e
+ f*x)] - 18*B*Sin[2*(e + f*x)] - 128*A*Sin[3*(e + f*x)] + 32*B*Sin[3*(e +
 f*x)] - 9*A*Sin[4*(e + f*x)] - 9*B*Sin[4*(e + f*x)]))/(960*a^2*c^3*f*(-1
+ Sin[e + f*x])^3*(1 + Sin[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.83, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3042, 3446, 3042, 3338, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^2(c-c\sin(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^2(c-c\sin(e+fx))^3}dx$$

$$\Big\downarrow \text{3446}$$

$$\frac{\int \frac{\sec^4(e+fx)(A+B\sin(e+fx))}{c-c\sin(e+fx)}dx}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{A+B\sin(e+fx)}{\cos(e+fx)^4(c-c\sin(e+fx))}dx}{a^2c^2}$$

$$\Big\downarrow \text{3338}$$

$$\frac{\frac{(4A-B)\int \sec^4(e+fx)dx}{5c}+\frac{(A+B)\sec^3(e+fx)}{5f(c-c\sin(e+fx))}}{a^2c^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{(4A-B)\int \csc\left(e+fx+\frac{\pi}{2}\right)^4dx}{5c}+\frac{(A+B)\sec^3(e+fx)}{5f(c-c\sin(e+fx))}}{a^2c^2}$$

$$\Big\downarrow \; 4254$$

$$\frac{\frac{(A+B)\sec^3(e+fx)}{5f(c-c\sin(e+fx))}-\frac{(4A-B)\int (\tan^2(e+fx)+1)d(-\tan(e+fx))}{5cf}}{a^2c^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{(A+B)\sec^3(e+fx)}{5f(c-c\sin(e+fx))}-\frac{(4A-B)\left(-\frac{1}{3}\tan^3(e+fx)-\tan(e+fx)\right)}{5cf}}{a^2c^2}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^3),x
]
```

output
```
(((A + B)*Sec[e + f*x]^3)/(5*f*(c - c*Sin[e + f*x])) - ((4*A - B)*(-Tan[e
+ f*x] - Tan[e + f*x]^3/3))/(5*c*f))/(a^2*c^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.01 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.46

method	result
risch	$-\frac{4i(24iA\,e^{3i(fx+e)}-6iB\,e^{3i(fx+e)}+15B\,e^{4i(fx+e)}+8iA\,e^{i(fx+e)}+8A\,e^{2i(fx+e)}-2iB\,e^{i(fx+e)}-2B\,e^{2i(fx+e)}+4A-}{15(e^{i(fx+e)}+i)^3(e^{i(fx+e)}-i)^5 f\,a^2c^3}$
parallelrisch	$\frac{-30A\tan(\frac{fx}{2}+\frac{e}{2})^7+(30A-30B)\tan(\frac{fx}{2}+\frac{e}{2})^6+(10A+20B)\tan(\frac{fx}{2}+\frac{e}{2})^5+(-50A-10B)\tan(\frac{fx}{2}+\frac{e}{2})^4+(-26A-}{15f\,a^2c^3(\tan(\frac{fx}{2}+\frac{e}{2})+1)^3(\tan(\frac{fx}{2}+}$
derivativedivides	$\frac{-\frac{2(A+B)}{5(\tan(\frac{fx}{2}+\frac{e}{2})-1)^5}-\frac{2A+2B}{2(\tan(\frac{fx}{2}+\frac{e}{2})-1)^4}-\frac{\frac{3A}{2}+B}{(\tan(\frac{fx}{2}+\frac{e}{2})-1)^2}-\frac{2(\frac{5A}{2}+2B)}{3(\tan(\frac{fx}{2}+\frac{e}{2})-1)^3}-\frac{2(\frac{11A}{16}+\frac{3B}{16})}{\tan(\frac{fx}{2}+\frac{e}{2})-1}-\frac{-\frac{A}{4}+\frac{B}{4}}{(\tan(\frac{fx}{2}+\frac{e}{2})+1}}{f\,a^2c^3}$
default	$\frac{-\frac{2(A+B)}{5(\tan(\frac{fx}{2}+\frac{e}{2})-1)^5}-\frac{2A+2B}{2(\tan(\frac{fx}{2}+\frac{e}{2})-1)^4}-\frac{\frac{3A}{2}+B}{(\tan(\frac{fx}{2}+\frac{e}{2})-1)^2}-\frac{2(\frac{5A}{2}+2B)}{3(\tan(\frac{fx}{2}+\frac{e}{2})-1)^3}-\frac{2(\frac{11A}{16}+\frac{3B}{16})}{\tan(\frac{fx}{2}+\frac{e}{2})-1}-\frac{-\frac{A}{4}+\frac{B}{4}}{(\tan(\frac{fx}{2}+\frac{e}{2})+1}}{f\,a^2c^3}$
norman	$\frac{\frac{6A-4B}{10afc}-\frac{4(4A-B)\tan(\frac{fx}{2}+\frac{e}{2})^5}{15afc}-\frac{A\tan(\frac{fx}{2}+\frac{e}{2})^{10}}{afc}+\frac{(14A-16B)\tan(\frac{fx}{2}+\frac{e}{2})^2}{10afc}+\frac{(6A-4B)\tan(\frac{fx}{2}+\frac{e}{2})^8}{2afc}+\frac{(2A-8B)\tan(\frac{fx}{2}}{3afc}}{a(1+\tan(\frac{fx}{2}+\frac{e}{2})^2)(\tan(\frac{fx}{2}+\frac{e}{2})+}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x,method=_RETUR
NVERBOSE)
```

output
```
-4/15*I*(24*I*A*exp(3*I*(f*x+e))-6*I*B*exp(3*I*(f*x+e))+15*B*exp(4*I*(f*x+
e))+8*I*A*exp(I*(f*x+e))+8*A*exp(2*I*(f*x+e))-2*I*B*exp(I*(f*x+e))-2*B*exp
(2*I*(f*x+e))+4*A-B)/(exp(I*(f*x+e))+I)^3/(exp(I*(f*x+e))-I)^5/f/a^2/c^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.25

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^3}\,dx = -\frac{2\,(4\,A-B)\cos\left(fx+e\right)^4-(4\,A-B)\cos\left(fx+e\right)^2+\left(2\,(4\,A-B)\cos\left(fx+e\right)^2+4\,A-B\right)\sin\left(fx\right.}{15\,\left(a^2c^3f\cos\left(fx+e\right)^3\sin\left(fx+e\right)-a^2c^3f\cos\left(fx+e\right)^3\right)}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x, algori
thm="fricas")
```

output
```
-1/15*(2*(4*A - B)*cos(f*x + e)^4 - (4*A - B)*cos(f*x + e)^2 + (2*(4*A - B
)*cos(f*x + e)^2 + 4*A - B)*sin(f*x + e) - A + 4*B)/(a^2*c^3*f*cos(f*x + e
)^3*sin(f*x + e) - a^2*c^3*f*cos(f*x + e)^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2674 vs. 2(82) = 164.

Time = 9.52 (sec) , antiderivative size = 2674, normalized size of antiderivative = 28.75

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**3,x)
```

output
```
Piecewise((-30*A*tan(e/2 + f*x/2)**7/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 -
 30*a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 +
 90*a**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 +
 30*a**2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15
*a**2*c**3*f) + 30*A*tan(e/2 + f*x/2)**6/(15*a**2*c**3*f*tan(e/2 + f*x/2)*
*8 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)*
*6 + 90*a**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)*
*3 + 30*a**2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2)
- 15*a**2*c**3*f) + 10*A*tan(e/2 + f*x/2)**5/(15*a**2*c**3*f*tan(e/2 + f*x
/2)**8 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x
/2)**6 + 90*a**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x
/2)**3 + 30*a**2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x
/2) - 15*a**2*c**3*f) - 50*A*tan(e/2 + f*x/2)**4/(15*a**2*c**3*f*tan(e/2 +
 f*x/2)**8 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 +
 f*x/2)**6 + 90*a**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 +
 f*x/2)**3 + 30*a**2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 +
 f*x/2) - 15*a**2*c**3*f) - 26*A*tan(e/2 + f*x/2)**3/(15*a**2*c**3*f*tan(e
/2 + f*x/2)**8 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e
/2 + f*x/2)**6 + 90*a**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e
/2 + f*x/2)**3 + 30*a**2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*ta...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 651 vs. $2(88) = 176$.

Time = 0.06 (sec) , antiderivative size = 651, normalized size of antiderivative = 7.00

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2(c - c\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x, algori
thm="maxima")
```

output
```
2/15*(A*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 21*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 13*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 25*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 - 5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 15*sin(f*x + e
)^6/(cos(f*x + e) + 1)^6 + 15*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 3)/(a^
2*c^3 - 2*a^2*c^3*sin(f*x + e)/(cos(f*x + e) + 1) - 2*a^2*c^3*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 + 6*a^2*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 -
6*a^2*c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 2*a^2*c^3*sin(f*x + e)^6/(
cos(f*x + e) + 1)^6 + 2*a^2*c^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - a^2*
c^3*sin(f*x + e)^8/(cos(f*x + e) + 1)^8) - B*(6*sin(f*x + e)/(cos(f*x + e)
 + 1) - 9*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 8*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 - 5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 10*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5 - 15*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 3)/(a^2*c^3
- 2*a^2*c^3*sin(f*x + e)/(cos(f*x + e) + 1) - 2*a^2*c^3*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 6*a^2*c^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 6*a^2*
c^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 2*a^2*c^3*sin(f*x + e)^6/(cos(f*
x + e) + 1)^6 + 2*a^2*c^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - a^2*c^3*si
n(f*x + e)^8/(cos(f*x + e) + 1)^8))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(88) = 176$.

Time = 0.23 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.38

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2(c - c\sin(e + fx))^3}\,dx =$$

$$-\frac{\frac{5\left(15\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-9\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+24\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-12\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+13\,A-7\,B\right)}{a^2c^3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}+\frac{165\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+45\,B\,\mathrm{t}}{}}{}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x, algori
thm="giac")
```

output
```
-1/120*(5*(15*A*tan(1/2*f*x + 1/2*e)^2 - 9*B*tan(1/2*f*x + 1/2*e)^2 + 24*A
*tan(1/2*f*x + 1/2*e) - 12*B*tan(1/2*f*x + 1/2*e) + 13*A - 7*B)/(a^2*c^3*(
tan(1/2*f*x + 1/2*e) + 1)^3) + (165*A*tan(1/2*f*x + 1/2*e)^4 + 45*B*tan(1/
2*f*x + 1/2*e)^4 - 480*A*tan(1/2*f*x + 1/2*e)^3 - 60*B*tan(1/2*f*x + 1/2*e
)^3 + 650*A*tan(1/2*f*x + 1/2*e)^2 + 70*B*tan(1/2*f*x + 1/2*e)^2 - 400*A*t
an(1/2*f*x + 1/2*e) - 20*B*tan(1/2*f*x + 1/2*e) + 113*A + 13*B)/(a^2*c^3*(
tan(1/2*f*x + 1/2*e) - 1)^5))/f
```

Mupad [B] (verification not implemented)

Time = 36.99 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.97

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^3}\,dx$$
$$=\frac{\left(\frac{8\,A}{15}-\frac{2\,B}{15}-\frac{16\,A\,\sin(e+f\,x)}{15}+\frac{4\,B\,\sin(e+f\,x)}{15}\right)\,\cos\left(e+f\,x\right)^2+\frac{2\,A}{15}-\frac{8\,B}{15}-\frac{8\,A\,\sin(e+f\,x)}{15}+\frac{2\,B\,\sin(e+f\,x)}{15}}{a^2\,c^3\,f\,\left(2\cos\left(e+f\,x\right)^3\,\sin\left(e+f\,x\right)-2\cos\left(e+f\,x\right)^3\right)}$$
$$-\frac{\frac{2\,A}{5}+\frac{2\,B}{5}-\frac{2\,A\,\sin(e+f\,x)}{5}-\frac{2\,B\,\sin(e+f\,x)}{5}}{a^2\,c^3\,f\,\left(2\,\sin\left(e+f\,x\right)-2\right)}-\frac{\cos\left(e+f\,x\right)\,\left(\frac{16\,A}{15}-\frac{4\,B}{15}\right)}{a^2\,c^3\,f\,\left(2\,\sin\left(e+f\,x\right)-2\right)}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^3),x
)
```

output
```
((2*A)/15 - (8*B)/15 - (8*A*sin(e + f*x))/15 + (2*B*sin(e + f*x))/15 + cos
(e + f*x)^2*((8*A)/15 - (2*B)/15 - (16*A*sin(e + f*x))/15 + (4*B*sin(e + f
*x))/15))/(a^2*c^3*f*(2*cos(e + f*x)^3*sin(e + f*x) - 2*cos(e + f*x)^3)) -
 ((2*A)/5 + (2*B)/5 - (2*A*sin(e + f*x))/5 - (2*B*sin(e + f*x))/5)/(a^2*c^
3*f*(2*sin(e + f*x) - 2)) - (cos(e + f*x)*((16*A)/15 - (4*B)/15))/(a^2*c^3
*f*(2*sin(e + f*x) - 2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.75

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^3}\,dx$$
$$= \frac{-12\cos(fx+e)\sin(fx+e)^3 a+3\cos(fx+e)\sin(fx+e)^3 b+12\cos(fx+e)\sin(fx+e)^2 a-3\cos}{}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x)
```

output
```
( - 12*cos(e + f*x)*sin(e + f*x)**3*a + 3*cos(e + f*x)*sin(e + f*x)**3*b +
 12*cos(e + f*x)*sin(e + f*x)**2*a - 3*cos(e + f*x)*sin(e + f*x)**2*b + 12
*cos(e + f*x)*sin(e + f*x)*a - 3*cos(e + f*x)*sin(e + f*x)*b - 12*cos(e +
f*x)*a + 3*cos(e + f*x)*b + 8*sin(e + f*x)**4*a - 2*sin(e + f*x)**4*b - 8*
sin(e + f*x)**3*a + 2*sin(e + f*x)**3*b - 12*sin(e + f*x)**2*a + 3*sin(e +
 f*x)**2*b + 12*sin(e + f*x)*a - 3*sin(e + f*x)*b + 3*a + 3*b)/(15*cos(e +
 f*x)*a**2*c**3*f*(sin(e + f*x)**3 - sin(e + f*x)**2 - sin(e + f*x) + 1))
```

3.68 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^4}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 135

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^4}\,dx &= \frac{(A+B)\sec^3(e+fx)}{7a^2f\,(c^2-c^2\sin(e+fx))^2}+\frac{(5A-2B)\sec^3(e+fx)}{35a^2f\,(c^4-c^4\sin(e+fx))}\\ &\quad+\frac{4(5A-2B)\tan(e+fx)}{35a^2c^4f}+\frac{4(5A-2B)\tan^3(e+fx)}{105a^2c^4f}\end{aligned}$$

output
```
1/7*(A+B)*sec(f*x+e)^3/a^2/f/(c^2-c^2*sin(f*x+e))^2+1/35*(5*A-2*B)*sec(f*x
+e)^3/a^2/f/(c^4-c^4*sin(f*x+e))+4/35*(5*A-2*B)*tan(f*x+e)/a^2/c^4/f+4/105
*(5*A-2*B)*tan(f*x+e)^3/a^2/c^4/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 285 vs. 2(135) = 270.

Time = 5.74 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.11

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^4}\,dx =$$

$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-2688B+42(25A+4B)\,c}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]
)^4),x]
```

output
```
-1/13440*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2])*(-2688*B + 42*(25*A + 4*B)*Cos[e + f*x] - 512*(5*A - 2*B)*Cos[
2*(e + f*x)] + 225*A*Cos[3*(e + f*x)] + 36*B*Cos[3*(e + f*x)] - 1280*A*Cos
[4*(e + f*x)] + 512*B*Cos[4*(e + f*x)] - 75*A*Cos[5*(e + f*x)] - 12*B*Cos[
5*(e + f*x)] - 4480*A*Sin[e + f*x] + 1792*B*Sin[e + f*x] - 600*A*Sin[2*(e
+ f*x)] - 96*B*Sin[2*(e + f*x)] - 960*A*Sin[3*(e + f*x)] + 384*B*Sin[3*(e
+ f*x)] - 300*A*Sin[4*(e + f*x)] - 48*B*Sin[4*(e + f*x)] + 320*A*Sin[5*(e
+ f*x)] - 128*B*Sin[5*(e + f*x)]))/(a^2*c^4*f*(-1 + Sin[e + f*x])^4*(1 + S
in[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.84, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3446, 3042, 3338, 3042, 3151, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^2(c-c\sin(e+fx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^2(c-c\sin(e+fx))^4}dx$$

$$\Big\downarrow \text{3446}$$

$$\frac{\int \frac{\sec^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}dx}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{A+B\sin(e+fx)}{\cos(e+fx)^4(c-c\sin(e+fx))^2}dx}{a^2c^2}$$

$$\Big\downarrow \text{3338}$$

$$\frac{\frac{(5A-2B)\int \frac{\sec^4(e+fx)}{c-c\sin(e+fx)}dx}{7c}+\frac{(A+B)\sec^3(e+fx)}{7f(c-c\sin(e+fx))^2}}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(5A-2B)\int \frac{1}{\cos(e+fx)^4(c-c\sin(e+fx))}dx}{7c}+\frac{(A+B)\sec^3(e+fx)}{7f(c-c\sin(e+fx))^2}}{a^2c^2}$$

$$\Big\downarrow \text{3151}$$

$$\frac{\frac{(5A-2B)\left(\frac{4\int \sec^4(e+fx)dx}{5c}+\frac{\sec^3(e+fx)}{5f(c-c\sin(e+fx))}\right)}{7c}+\frac{(A+B)\sec^3(e+fx)}{7f(c-c\sin(e+fx))^2}}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(5A-2B)\left(\frac{4\int \csc\left(e+fx+\frac{\pi}{2}\right)^4dx}{5c}+\frac{\sec^3(e+fx)}{5f(c-c\sin(e+fx))}\right)}{7c}+\frac{(A+B)\sec^3(e+fx)}{7f(c-c\sin(e+fx))^2}}{a^2c^2}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\frac{(5A-2B)\left(\frac{\sec^3(e+fx)}{5f(c-c\sin(e+fx))}-\frac{4\int \left(\tan^2(e+fx)+1\right)d(-\tan(e+fx))}{5cf}\right)}{7c}+\frac{(A+B)\sec^3(e+fx)}{7f(c-c\sin(e+fx))^2}}{a^2c^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(A+B)\sec^3(e+fx)}{7f(c-c\sin(e+fx))^2}+\frac{(5A-2B)\left(\frac{\sec^3(e+fx)}{5f(c-c\sin(e+fx))}-\frac{4\left(-\frac{1}{3}\tan^3(e+fx)-\tan(e+fx)\right)}{5cf}\right)}{7c}}{a^2c^2}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^4),x
]
```

output
```
(((A + B)*Sec[e + f*x]^3)/(7*f*(c - c*Sin[e + f*x])^2) + ((5*A - 2*B)*(Sec
[e + f*x]^3/(5*f*(c - c*Sin[e + f*x])) - (4*(-Tan[e + f*x] - Tan[e + f*x]^
3/3))/(5*c*f)))/(7*c))/(a^2*c^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3151
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Simp[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1])   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simpli
fy[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.54 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.19

method	result
risch	$-\frac{16\left(-8B\,e^{i(fx+e)}-28iB\,e^{4i(fx+e)}+70iA\,e^{4i(fx+e)}-5iA+2iB+15iA\,e^{2i(fx+e)}-16B\,e^{3i(fx+e)}+20A\,e^{i(fx+e)}-6iB\right.}{105\left(e^{i(fx+e)}-i\right)^{7}\left(e^{i(fx+e)}+i\right)^{3}f\,a^{2}c^{4}}$
parallelrisch	$\frac{-210A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}+(420A-210B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}+(-280A+280B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}+(-560A-280B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{10}$
derivativedivides	$\frac{-\frac{-\frac{A}{8}+\frac{B}{8}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2\left(\frac{A}{8}-\frac{B}{8}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2\left(\frac{3A}{16}-\frac{B}{8}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(2A+2B)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}-\frac{6A+6B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{6}}-\frac{10A+8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\right.}}{f\,a^{2}c^{4}}$
default	$\frac{-\frac{-\frac{A}{8}+\frac{B}{8}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2\left(\frac{A}{8}-\frac{B}{8}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2\left(\frac{3A}{16}-\frac{B}{8}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(2A+2B)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}-\frac{6A+6B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{6}}-\frac{10A+8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\right.}}{f\,a^{2}c^{4}}$
norman	$\frac{-\frac{20A+6B}{35afc}-\frac{4(10A+11B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{15afc}-\frac{2A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}{afc}+\frac{2(30A-47B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{35afc}-\frac{2(7A-4B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}{3afc}+\frac{2(2A-}{}}{a\left(1\right.}$

input

```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x,method=_RETUR
NVERBOSE)
```

output

```
-16/105*(-8*B*exp(I*(f*x+e))-28*I*B*exp(4*I*(f*x+e))+70*I*A*exp(4*I*(f*x+e
))-5*I*A+2*I*B+15*I*A*exp(2*I*(f*x+e))-16*B*exp(3*I*(f*x+e))+20*A*exp(I*(f
*x+e))-6*I*B*exp(2*I*(f*x+e))+40*A*exp(3*I*(f*x+e))+42*B*exp(5*I*(f*x+e)))
/(exp(I*(f*x+e))-I)^7/(exp(I*(f*x+e))+I)^3/f/a^2/c^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.12

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^4}\,dx = -\frac{16\,(5\,A-2\,B)\cos\left(fx+e\right)^{4}-8\,(5\,A-2\,B)\cos\left(fx+e\right)^{2}-\left(8\,(5\,A-2\,B)\cos\left(fx+e\right)^{4}-12\,(5\,A-\right.}{105\,\left(a^{2}c^{4}f\cos\left(fx+e\right)^{5}+2\,a^{2}c^{4}f\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)-\right.}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x, algori
thm="fricas")
```

output
```
-1/105*(16*(5*A - 2*B)*cos(f*x + e)^4 - 8*(5*A - 2*B)*cos(f*x + e)^2 - (8*
(5*A - 2*B)*cos(f*x + e)^4 - 12*(5*A - 2*B)*cos(f*x + e)^2 - 25*A + 10*B)*
sin(f*x + e) - 10*A + 25*B)/(a^2*c^4*f*cos(f*x + e)^5 + 2*a^2*c^4*f*cos(f*
x + e)^3*sin(f*x + e) - 2*a^2*c^4*f*cos(f*x + e)^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4228 vs. $2(122) = 244$.

Time = 18.28 (sec) , antiderivative size = 4228, normalized size of antiderivative = 31.32

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2(c - c\sin(e + fx))^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**4,x)
```

output
```
Piecewise((-210*A*tan(e/2 + f*x/2)**9/(105*a**2*c**4*f*tan(e/2 + f*x/2)**1
0 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 315*a**2*c**4*f*tan(e/2 + f*x/2)
**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 1470*a**2*c**4*f*tan(e/2 + f*x
/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 840*a**2*c**4*f*tan(e/2 +
f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420*a**2*c**4*f*tan(e/2
+ f*x/2) - 105*a**2*c**4*f) + 420*A*tan(e/2 + f*x/2)**8/(105*a**2*c**4*f*t
an(e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 315*a**2*c**4*
f*tan(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 1470*a**2*c*
*4*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 840*a**2
*c**4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420*a*
*2*c**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) - 280*A*tan(e/2 + f*x/2)**7/
(105*a**2*c**4*f*tan(e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**
9 + 315*a**2*c**4*f*tan(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)
**7 - 1470*a**2*c**4*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*
x/2)**4 - 840*a**2*c**4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 +
f*x/2)**2 + 420*a**2*c**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) - 560*A*ta
n(e/2 + f*x/2)**6/(105*a**2*c**4*f*tan(e/2 + f*x/2)**10 - 420*a**2*c**4*f*
tan(e/2 + f*x/2)**9 + 315*a**2*c**4*f*tan(e/2 + f*x/2)**8 + 840*a**2*c**4*
f*tan(e/2 + f*x/2)**7 - 1470*a**2*c**4*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c
**4*f*tan(e/2 + f*x/2)**4 - 840*a**2*c**4*f*tan(e/2 + f*x/2)**3 - 315*a...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 835 vs. $2(129) = 258$.

Time = 0.07 (sec) , antiderivative size = 835, normalized size of antiderivative = 6.19

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2(c - c\sin(e + fx))^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x, algori
thm="maxima")
```

output
```
-2/105*(B*(36*sin(f*x + e)/(cos(f*x + e) + 1) - 132*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 68*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 14*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 - 84*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 140*sin(f
*x + e)^6/(cos(f*x + e) + 1)^6 + 140*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 -
 105*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 9)/(a^2*c^4 - 4*a^2*c^4*sin(f*x
 + e)/(cos(f*x + e) + 1) + 3*a^2*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 8*a^2*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 14*a^2*c^4*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 14*a^2*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 8
*a^2*c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3*a^2*c^4*sin(f*x + e)^8/(c
os(f*x + e) + 1)^8 + 4*a^2*c^4*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - a^2*c
^4*sin(f*x + e)^10/(cos(f*x + e) + 1)^10) + 5*A*(3*sin(f*x + e)/(cos(f*x +
 e) + 1) + 24*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 76*sin(f*x + e)^3/(cos
(f*x + e) + 1)^3 + 28*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 42*sin(f*x + e
)^5/(cos(f*x + e) + 1)^5 - 56*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 28*sin
(f*x + e)^7/(cos(f*x + e) + 1)^7 + 42*sin(f*x + e)^8/(cos(f*x + e) + 1)^8
- 21*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 6)/(a^2*c^4 - 4*a^2*c^4*sin(f*x
 + e)/(cos(f*x + e) + 1) + 3*a^2*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 8*a^2*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 14*a^2*c^4*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 14*a^2*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 8
*a^2*c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3*a^2*c^4*sin(f*x + e)^8...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(129) = 258$.

Time = 0.23 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.05

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2(c - c\sin(e + fx))^4}\,dx = -\frac{\frac{35\left(9\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-6\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+15\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-9\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+8\,A-5\,B\right)}{a^2c^4\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3} + \frac{1365\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6+210\,B\,\mathrm{t}}{}}{}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x, algori
thm="giac")
```

output
```
-1/840*(35*(9*A*tan(1/2*f*x + 1/2*e)^2 - 6*B*tan(1/2*f*x + 1/2*e)^2 + 15*A
*tan(1/2*f*x + 1/2*e) - 9*B*tan(1/2*f*x + 1/2*e) + 8*A - 5*B)/(a^2*c^4*(ta
n(1/2*f*x + 1/2*e) + 1)^3) + (1365*A*tan(1/2*f*x + 1/2*e)^6 + 210*B*tan(1/
2*f*x + 1/2*e)^6 - 5775*A*tan(1/2*f*x + 1/2*e)^5 - 105*B*tan(1/2*f*x + 1/2
*e)^5 + 12250*A*tan(1/2*f*x + 1/2*e)^4 - 175*B*tan(1/2*f*x + 1/2*e)^4 - 14
350*A*tan(1/2*f*x + 1/2*e)^3 + 910*B*tan(1/2*f*x + 1/2*e)^3 + 10185*A*tan(
1/2*f*x + 1/2*e)^2 - 756*B*tan(1/2*f*x + 1/2*e)^2 - 3955*A*tan(1/2*f*x + 1
/2*e) + 427*B*tan(1/2*f*x + 1/2*e) + 760*A - 31*B)/(a^2*c^4*(tan(1/2*f*x +
 1/2*e) - 1)^7))/f
```

Mupad [B] (verification not implemented)

Time = 36.98 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.46

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^4}\,dx =$$

$$-\frac{\left(\frac{32\,A}{21}-\frac{64\,B}{105}-\frac{16\,A\,\sin(e+f\,x)}{21}+\frac{32\,B\,\sin(e+f\,x)}{105}\right)\,\cos\left(e+f\,x\right)^4+\left(\frac{8\,A}{7}+\frac{12\,B}{35}-\frac{8\,A\,\sin(e+f\,x)}{7}-\frac{12\,B\,\sin(e+f\,x)}{35}\right.}{a^2\,c^4\,f\,\left(4\cos\left(e\right.\right.}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^4),x
)
```

output
```
-((10*B)/21 - (4*A)/21 + (10*A*sin(e + f*x))/21 - (4*B*sin(e + f*x))/21 +
cos(e + f*x)^3*((8*A)/7 + (12*B)/35 - (8*A*sin(e + f*x))/7 - (12*B*sin(e +
 f*x))/35 + ((4*sin(e + f*x) - 4)*((4*A)/7 + (6*B)/35))/2) - cos(e + f*x)^
2*((16*A)/21 - (32*B)/105 - (8*A*sin(e + f*x))/7 + (16*B*sin(e + f*x))/35)
 + cos(e + f*x)^4*((32*A)/21 - (64*B)/105 - (16*A*sin(e + f*x))/21 + (32*B
*sin(e + f*x))/105))/(a^2*c^4*f*(4*cos(e + f*x)^3*sin(e + f*x) - 4*cos(e +
 f*x)^3 + 2*cos(e + f*x)^5))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.06

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^4}\,dx$$
$$= \frac{-45\cos(fx+e)\sin(fx+e)^4 a+18\cos(fx+e)\sin(fx+e)^4 b+90\cos(fx+e)\sin(fx+e)^3 a-36\,\mathrm{c}}{}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x)
```

output
```
( - 45*cos(e + f*x)*sin(e + f*x)**4*a + 18*cos(e + f*x)*sin(e + f*x)**4*b
+ 90*cos(e + f*x)*sin(e + f*x)**3*a - 36*cos(e + f*x)*sin(e + f*x)**3*b -
90*cos(e + f*x)*sin(e + f*x)*a + 36*cos(e + f*x)*sin(e + f*x)*b + 45*cos(e
 + f*x)*a - 18*cos(e + f*x)*b + 80*sin(e + f*x)**5*a - 32*sin(e + f*x)**5*
b - 160*sin(e + f*x)**4*a + 64*sin(e + f*x)**4*b - 40*sin(e + f*x)**3*a +
16*sin(e + f*x)**3*b + 240*sin(e + f*x)**2*a - 96*sin(e + f*x)**2*b - 90*s
in(e + f*x)*a + 36*sin(e + f*x)*b - 60*a - 18*b)/(210*cos(e + f*x)*a**2*c*
*4*f*(sin(e + f*x)**4 - 2*sin(e + f*x)**3 + 2*sin(e + f*x) - 1))
```

3.69 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 175

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5}\,dx = \frac{(A+B)\sec^3(e+fx)}{9a^2c^2f(c-c\sin(e+fx))^3} + \frac{(2A-B)\sec^3(e+fx)}{21a^2c^3f(c-c\sin(e+fx))^2} + \frac{(2A-B)\sec^3(e+fx)}{21a^2f\left(c^5-c^5\sin(e+fx)\right)} + \frac{4(2A-B)\tan(e+fx)}{21a^2c^5f} + \frac{4(2A-B)\tan^3(e+fx)}{63a^2c^5f}$$

output
```
1/9*(A+B)*sec(f*x+e)^3/a^2/c^2/f/(c-c*sin(f*x+e))^3+1/21*(2*A-B)*sec(f*x+e
)^3/a^2/c^3/f/(c-c*sin(f*x+e))^2+1/21*(2*A-B)*sec(f*x+e)^3/a^2/f/(c^5-c^5*
sin(f*x+e))+4/21*(2*A-B)*tan(f*x+e)/a^2/c^5/f+4/63*(2*A-B)*tan(f*x+e)^3/a^
2/c^5/f
```

Mathematica [A] (verified)

Time = 7.68 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.88

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-10752B+180(31A-5B)c}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]
)^5),x]
```

output
```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])*(-10752*B + 180*(31*A - 5*B)*Cos[e + f*x] - 6912*(2*A - B)*Cos[2*(e + f
*x)] + 310*A*Cos[3*(e + f*x)] - 50*B*Cos[3*(e + f*x)] - 6144*A*Cos[4*(e +
f*x)] + 3072*B*Cos[4*(e + f*x)] - 930*A*Cos[5*(e + f*x)] + 150*B*Cos[5*(e
+ f*x)] + 512*A*Cos[6*(e + f*x)] - 256*B*Cos[6*(e + f*x)] - 18432*A*Sin[e
+ f*x] + 9216*B*Sin[e + f*x] - 4185*A*Sin[2*(e + f*x)] + 675*B*Sin[2*(e +
f*x)] - 1024*A*Sin[3*(e + f*x)] + 512*B*Sin[3*(e + f*x)] - 1860*A*Sin[4*(e
 + f*x)] + 300*B*Sin[4*(e + f*x)] + 3072*A*Sin[5*(e + f*x)] - 1536*B*Sin[5
*(e + f*x)] + 155*A*Sin[6*(e + f*x)] - 25*B*Sin[6*(e + f*x)]))/(64512*a^2*
c^5*f*(-1 + Sin[e + f*x])^5*(1 + Sin[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.85, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.306$, Rules used = {3042, 3446, 3042, 3338, 3042, 3151, 3042, 3151, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^2(c-c\sin(e+fx))^5}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^2(c-c\sin(e+fx))^5}dx$$

$$\Big\downarrow \text{3446}$$

$$\frac{\int \frac{\sec^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}dx}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{A+B\sin(e+fx)}{\cos(e+fx)^4(c-c\sin(e+fx))^3}dx}{a^2c^2}$$

$$\Big\downarrow \text{3338}$$

$$\frac{\frac{(2A-B)\int \frac{\sec^4(e+fx)}{(c-c\sin(e+fx))^2}dx}{3c}+\frac{(A+B)\sec^3(e+fx)}{9f(c-c\sin(e+fx))^3}}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(2A-B)\int \frac{1}{\cos(e+fx)^4(c-c\sin(e+fx))^2}dx}{3c}+\frac{(A+B)\sec^3(e+fx)}{9f(c-c\sin(e+fx))^3}}{a^2c^2}$$

$$\Big\downarrow \text{3151}$$

$$\frac{\frac{(2A-B)\left(\frac{5\int \frac{\sec^4(e+fx)}{c-c\sin(e+fx)}dx}{7c}+\frac{\sec^3(e+fx)}{7f(c-c\sin(e+fx))^2}\right)}{3c}+\frac{(A+B)\sec^3(e+fx)}{9f(c-c\sin(e+fx))^3}}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(2A-B)\left(\frac{5\int \frac{1}{\cos(e+fx)^4(c-c\sin(e+fx))}dx}{7c}+\frac{\sec^3(e+fx)}{7f(c-c\sin(e+fx))^2}\right)}{3c}+\frac{(A+B)\sec^3(e+fx)}{9f(c-c\sin(e+fx))^3}}{a^2c^2}$$

$$\Big\downarrow \text{3151}$$

$$\frac{\frac{(2A-B)\left(\frac{5\left(\frac{4\int \sec^4(e+fx)dx}{5c}+\frac{\sec^3(e+fx)}{5f(c-c\sin(e+fx))}\right)}{7c}+\frac{\sec^3(e+fx)}{7f(c-c\sin(e+fx))^2}\right)}{3c}+\frac{(A+B)\sec^3(e+fx)}{9f(c-c\sin(e+fx))^3}}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(2A-B)\left(\frac{5\left(\frac{4\int \csc\left(e+fx+\frac{\pi}{2}\right)^4dx}{5c}+\frac{\sec^3(e+fx)}{5f(c-c\sin(e+fx))}\right)}{7c}+\frac{\sec^3(e+fx)}{7f(c-c\sin(e+fx))^2}\right)}{3c}+\frac{(A+B)\sec^3(e+fx)}{9f(c-c\sin(e+fx))^3}}{a^2c^2}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\frac{(2A-B)\left(\frac{5\left(\frac{\sec^3(e+fx)}{5f(c-c\sin(e+fx))}-\frac{4\int\left(\tan^2(e+fx)+1\right)d(-\tan(e+fx))}{5cf}\right)}{7c}+\frac{\sec^3(e+fx)}{7f(c-c\sin(e+fx))^2}\right)}{3c}+\frac{(A+B)\sec^3(e+fx)}{9f(c-c\sin(e+fx))^3}}{a^2c^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(A+B)\sec^3(e+fx)}{9f(c-c\sin(e+fx))^3}+\frac{(2A-B)\left(\frac{\sec^3(e+fx)}{7f(c-c\sin(e+fx))^2}+\frac{5\left(\frac{\sec^3(e+fx)}{5f(c-c\sin(e+fx))}-\frac{4\left(-\frac{1}{3}\tan^3(e+fx)-\tan(e+fx)\right)}{5cf}\right)}{7c}\right)}{3c}}{a^2c^2}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^5),x
]
```

output
```
(((A + B)*Sec[e + f*x]^3)/(9*f*(c - c*Sin[e + f*x])^3) + ((2*A - B)*(Sec[e
 + f*x]^3/(7*f*(c - c*Sin[e + f*x])^2) + (5*(Sec[e + f*x]^3/(5*f*(c - c*Si
n[e + f*x])) - (4*(-Tan[e + f*x] - Tan[e + f*x]^3/3))/(5*c*f)))/(7*c)))/(3
*c))/(a^2*c^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3151
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Simp[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1])   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simpli
fy[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.24 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.05

method	result
risch	$\frac{16i\left(72iA\,\mathrm{e}^{5i(fx+e)}-36iB\,\mathrm{e}^{5i(fx+e)}+42B\,\mathrm{e}^{6i(fx+e)}+4iA\,\mathrm{e}^{3i(fx+e)}+54A\,\mathrm{e}^{4i(fx+e)}-2iB\,\mathrm{e}^{3i(fx+e)}-27B\,\mathrm{e}^{4i(fx+e)}-1\right.}{63\left(\mathrm{e}^{i(fx+e)}-i\right)^{9}\left(\mathrm{e}^{i(fx+e)}+i\right)^{3}f\,a^{2}c^{5}}$
parallelrisch	$\frac{-126A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}+(378A-126B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}+(-546A+252B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}+(-126A-378B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{}$
derivativedivides	$\frac{-\frac{2(4A+4B)}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{9}}-\frac{16A+16B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{8}}-\frac{2(34A+32B)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}-\frac{46A+40B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{6}}-\frac{\frac{9A}{2}+\frac{13B}{8}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{2\left(\frac{57A}{64}+\frac{5}{6}\right.}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right.}}{}$
default	$\frac{-\frac{2(4A+4B)}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{9}}-\frac{16A+16B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{8}}-\frac{2(34A+32B)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}-\frac{46A+40B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{6}}-\frac{\frac{9A}{2}+\frac{13B}{8}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}}-\frac{2\left(\frac{57A}{64}+\frac{5}{6}\right.}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right.}}{}$
norman	$\frac{\frac{(4A-8B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{afc}+\frac{(6A-2B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{12}}{afc}-\frac{38A+2B}{63afc}+\frac{8(2A-B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}{7acf}-\frac{2A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{13}}{afc}-\frac{4(92A-67B)\,\mathrm{ta}}{63af}}{}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x,method=_RETUR
NVERBOSE)
```

output
```
16/63*I*(72*I*A*exp(5*I*(f*x+e))-36*I*B*exp(5*I*(f*x+e))+42*B*exp(6*I*(f*x
+e))+4*I*A*exp(3*I*(f*x+e))+54*A*exp(4*I*(f*x+e))-2*I*B*exp(3*I*(f*x+e))-2
7*B*exp(4*I*(f*x+e))-12*I*A*exp(I*(f*x+e))+24*A*exp(2*I*(f*x+e))+6*I*B*exp
(I*(f*x+e))-12*B*exp(2*I*(f*x+e))-2*A+B)/(exp(I*(f*x+e))-I)^9/(exp(I*(f*x+
e))+I)^3/f/a^2/c^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.07

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5}\,dx$$
$$= \frac{8\,(2\,A-B)\cos\left(fx+e\right)^{6}-36\,(2\,A-B)\cos\left(fx+e\right)^{4}+15\,(2\,A-B)\cos\left(fx+e\right)^{2}+\left(24\,(2\,A-B)\cos\right.}{63\left(3\,a^{2}c^{5}f\cos\left(fx+e\right)^{5}-4\,a^{2}c^{5}f\cos\left(fx+e\right)^{3}-\left(a^{2}c^{5}f\cos\left(fx\right.\right.\right.}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x, algori
thm="fricas")
```

output
```
1/63*(8*(2*A - B)*cos(f*x + e)^6 - 36*(2*A - B)*cos(f*x + e)^4 + 15*(2*A -
 B)*cos(f*x + e)^2 + (24*(2*A - B)*cos(f*x + e)^4 - 20*(2*A - B)*cos(f*x +
 e)^2 - 14*A + 7*B)*sin(f*x + e) + 7*A - 14*B)/(3*a^2*c^5*f*cos(f*x + e)^5
 - 4*a^2*c^5*f*cos(f*x + e)^3 - (a^2*c^5*f*cos(f*x + e)^5 - 4*a^2*c^5*f*co
s(f*x + e)^3)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5868 vs. $2(160) = 320$.

Time = 34.90 (sec) , antiderivative size = 5868, normalized size of antiderivative = 33.53

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**5,x)
```

output
```
Piecewise((-126*A*tan(e/2 + f*x/2)**11/(63*a**2*c**5*f*tan(e/2 + f*x/2)**1
2 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2
)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f
*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2
 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan(
e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*ta
n(e/2 + f*x/2) - 63*a**2*c**5*f) + 378*A*tan(e/2 + f*x/2)**10/(63*a**2*c**
5*f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2
*c**5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*
a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2
268*a**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4
 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)*
*2 + 378*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c**5*f) - 546*A*tan(e/2 +
f*x/2)**9/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*tan(e/2 +
 f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f*tan(e
/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**5*f*t
an(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*c**5
*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2*c*
*5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c**5
*f) - 126*A*tan(e/2 + f*x/2)**8/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - ...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 998 vs. $2(168) = 336$.

Time = 0.07 (sec) , antiderivative size = 998, normalized size of antiderivative = 5.70

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2(c - c\sin(e + fx))^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x, algori
thm="maxima")
```

output
```
-2/63*(A*(51*sin(f*x + e)/(cos(f*x + e) + 1) - 39*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 - 235*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 450*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 - 306*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 294*sin(
f*x + e)^6/(cos(f*x + e) + 1)^6 + 378*sin(f*x + e)^7/(cos(f*x + e) + 1)^7
- 63*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 273*sin(f*x + e)^9/(cos(f*x + e
) + 1)^9 + 189*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 63*sin(f*x + e)^11/
(cos(f*x + e) + 1)^11 - 19)/(a^2*c^5 - 6*a^2*c^5*sin(f*x + e)/(cos(f*x + e
) + 1) + 12*a^2*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2*a^2*c^5*sin(f*
x + e)^3/(cos(f*x + e) + 1)^3 - 27*a^2*c^5*sin(f*x + e)^4/(cos(f*x + e) +
1)^4 + 36*a^2*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 36*a^2*c^5*sin(f*x
 + e)^7/(cos(f*x + e) + 1)^7 + 27*a^2*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1
)^8 + 2*a^2*c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 12*a^2*c^5*sin(f*x +
 e)^10/(cos(f*x + e) + 1)^10 + 6*a^2*c^5*sin(f*x + e)^11/(cos(f*x + e) + 1
)^11 - a^2*c^5*sin(f*x + e)^12/(cos(f*x + e) + 1)^12) + B*(6*sin(f*x + e)/
(cos(f*x + e) + 1) - 75*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 128*sin(f*x
+ e)^3/(cos(f*x + e) + 1)^3 - 162*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 36
*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 42*sin(f*x + e)^6/(cos(f*x + e) + 1
)^6 - 189*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 126*sin(f*x + e)^9/(cos(f*
x + e) + 1)^9 - 63*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 1)/(a^2*c^5 - 6
*a^2*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 12*a^2*c^5*sin(f*x + e)^2/(c...
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.90

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5}\,dx = -\frac{\frac{21\left(21\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-15\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+36\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-24\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+19\,A-13\,B\right)}{a^2c^5\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}+\frac{3591\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8+3}{}}{}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x, algori
thm="giac")
```

output
```
-1/2016*(21*(21*A*tan(1/2*f*x + 1/2*e)^2 - 15*B*tan(1/2*f*x + 1/2*e)^2 + 3
6*A*tan(1/2*f*x + 1/2*e) - 24*B*tan(1/2*f*x + 1/2*e) + 19*A - 13*B)/(a^2*c
^5*(tan(1/2*f*x + 1/2*e) + 1)^3) + (3591*A*tan(1/2*f*x + 1/2*e)^8 + 315*B*
tan(1/2*f*x + 1/2*e)^8 - 19656*A*tan(1/2*f*x + 1/2*e)^7 + 756*B*tan(1/2*f*
x + 1/2*e)^7 + 56196*A*tan(1/2*f*x + 1/2*e)^6 - 4200*B*tan(1/2*f*x + 1/2*e
)^6 - 95760*A*tan(1/2*f*x + 1/2*e)^5 + 11340*B*tan(1/2*f*x + 1/2*e)^5 + 10
7730*A*tan(1/2*f*x + 1/2*e)^4 - 14994*B*tan(1/2*f*x + 1/2*e)^4 - 79464*A*t
an(1/2*f*x + 1/2*e)^3 + 13356*B*tan(1/2*f*x + 1/2*e)^3 + 38484*A*tan(1/2*f
*x + 1/2*e)^2 - 6768*B*tan(1/2*f*x + 1/2*e)^2 - 10944*A*tan(1/2*f*x + 1/2*
e) + 2196*B*tan(1/2*f*x + 1/2*e) + 1615*A - 209*B)/(a^2*c^5*(tan(1/2*f*x +
 1/2*e) - 1)^9))/f
```

Mupad [B] (verification not implemented)

Time = 36.80 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.93

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5}\,dx = \frac{2\left(7\,A-14\,B-14\,A\,\sin\left(e+f\,x\right)+7\,B\,\sin\left(e+f\,x\right)+30\,A\cos\left(e+f\,x\right)^2-76\,A\cos\left(e+f\,x\right)^3-72\,A\right.}{}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^5),x
)
```

output
```
(2*(7*A - 14*B - 14*A*sin(e + f*x) + 7*B*sin(e + f*x) + 30*A*cos(e + f*x)^
2 - 76*A*cos(e + f*x)^3 - 72*A*cos(e + f*x)^4 + 57*A*cos(e + f*x)^5 + 16*A
*cos(e + f*x)^6 - 15*B*cos(e + f*x)^2 - 4*B*cos(e + f*x)^3 + 36*B*cos(e +
f*x)^4 + 3*B*cos(e + f*x)^5 - 8*B*cos(e + f*x)^6 - 40*A*cos(e + f*x)^2*sin
(e + f*x) + 76*A*cos(e + f*x)^3*sin(e + f*x) + 48*A*cos(e + f*x)^4*sin(e +
 f*x) - 19*A*cos(e + f*x)^5*sin(e + f*x) + 20*B*cos(e + f*x)^2*sin(e + f*x
) + 4*B*cos(e + f*x)^3*sin(e + f*x) - 24*B*cos(e + f*x)^4*sin(e + f*x) - B
*cos(e + f*x)^5*sin(e + f*x)))/(63*a^2*c^5*f*(8*cos(e + f*x)^3*sin(e + f*x
) - 2*cos(e + f*x)^5*sin(e + f*x) - 8*cos(e + f*x)^3 + 6*cos(e + f*x)^5))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 384, normalized size of antiderivative = 2.19

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^5}\,dx$$
$$= \frac{19a+b+16\sin(fx+e)^6a+\cos(fx+e)b-8\sin(fx+e)^6b-48\sin(fx+e)^5a-2\cos(fx+e)\sin}{}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x)
```

output
```
( - 2*cos(e + f*x)*sin(e + f*x)**5*a + cos(e + f*x)*sin(e + f*x)**5*b + 6*
cos(e + f*x)*sin(e + f*x)**4*a - 3*cos(e + f*x)*sin(e + f*x)**4*b - 4*cos(
e + f*x)*sin(e + f*x)**3*a + 2*cos(e + f*x)*sin(e + f*x)**3*b - 4*cos(e +
f*x)*sin(e + f*x)**2*a + 2*cos(e + f*x)*sin(e + f*x)**2*b + 6*cos(e + f*x)
*sin(e + f*x)*a - 3*cos(e + f*x)*sin(e + f*x)*b - 2*cos(e + f*x)*a + cos(e
 + f*x)*b + 16*sin(e + f*x)**6*a - 8*sin(e + f*x)**6*b - 48*sin(e + f*x)**
5*a + 24*sin(e + f*x)**5*b + 24*sin(e + f*x)**4*a - 12*sin(e + f*x)**4*b +
 56*sin(e + f*x)**3*a - 28*sin(e + f*x)**3*b - 66*sin(e + f*x)**2*a + 33*s
in(e + f*x)**2*b + 6*sin(e + f*x)*a - 3*sin(e + f*x)*b + 19*a + b)/(63*cos
(e + f*x)*a**2*c**5*f*(sin(e + f*x)**5 - 3*sin(e + f*x)**4 + 2*sin(e + f*x
)**3 + 2*sin(e + f*x)**2 - 3*sin(e + f*x) + 1))
```

3.70 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 243

$$\begin{aligned}
&\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^3}\,dx \\
&= -\frac{21(3A-8B)c^5x}{2a^3} - \frac{7(3A-8B)c^5\cos^3(e+fx)}{a^3f} \\
&\quad - \frac{21(3A-8B)c^5\cos(e+fx)\sin(e+fx)}{2a^3f} \\
&\quad - \frac{a^5(A-B)c^5\cos^{11}(e+fx)}{5f(a+a\sin(e+fx))^8} + \frac{2a^3(3A-8B)c^5\cos^9(e+fx)}{15f(a+a\sin(e+fx))^6} \\
&\quad - \frac{6a^5(3A-8B)c^5\cos^7(e+fx)}{5f\left(a^2+a^2\sin(e+fx)\right)^4} - \frac{42a^5(3A-8B)c^5\cos^5(e+fx)}{5f\left(a^4+a^4\sin(e+fx)\right)^2}
\end{aligned}$$

output
```
-21/2*(3*A-8*B)*c^5*x/a^3-7*(3*A-8*B)*c^5*cos(f*x+e)^3/a^3/f-21/2*(3*A-8*B
)*c^5*cos(f*x+e)*sin(f*x+e)/a^3/f-1/5*a^5*(A-B)*c^5*cos(f*x+e)^11/f/(a+a*s
in(f*x+e))^8+2/15*a^3*(3*A-8*B)*c^5*cos(f*x+e)^9/f/(a+a*sin(f*x+e))^6-6/5*
a^5*(3*A-8*B)*c^5*cos(f*x+e)^7/f/(a^2+a^2*sin(f*x+e))^4-42/5*a^5*(3*A-8*B)
*c^5*cos(f*x+e)^5/f/(a^4+a^4*sin(f*x+e))^2
```

Mathematica [A] (verified)

Time = 13.09 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.60

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^3}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c-c\sin(e+fx))^5\left(768(A-B)\sin\left(\frac{1}{2}(e+fx)\right)-384(A-B)\left(c\right.\right.}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5)/(a + a*Sin[e + f*x
])^3,x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^5*(768*(A - B)
*Sin[(e + f*x)/2] - 384*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 12
8*(21*A - 31*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 +
 64*(21*A - 31*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 128*(54*A - 11
9*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 630*(3*A -
 8*B)*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 - 15*(32*A - 127*B
)*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 - 5*B*Cos[3*(e + f*
x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 + 15*(A - 8*B)*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^5*Sin[2*(e + f*x)]))/(60*a^3*f*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2])^10*(1 + Sin[e + f*x])^3)
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.85, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3159, 3042, 3159, 3042, 3161, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^5(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^3}\,dx$$
$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-c\sin(e+fx))^5(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^3}dx$$

$$\Big\downarrow \text{3446}$$

$$a^5c^5\int \frac{\cos^{10}(e+fx)(A+B\sin(e+fx))}{(\sin(e+fx)a+a)^8}dx$$

$$\Big\downarrow \text{3042}$$

$$a^5c^5\int \frac{\cos(e+fx)^{10}(A+B\sin(e+fx))}{(\sin(e+fx)a+a)^8}dx$$

$$\Big\downarrow \text{3338}$$

$$a^5c^5\left(-\frac{(3A-8B)\int \frac{\cos^{10}(e+fx)}{(\sin(e+fx)a+a)^7}dx}{5a}-\frac{(A-B)\cos^{11}(e+fx)}{5f(a\sin(e+fx)+a)^8}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^5c^5\left(-\frac{(3A-8B)\int \frac{\cos(e+fx)^{10}}{(\sin(e+fx)a+a)^7}dx}{5a}-\frac{(A-B)\cos^{11}(e+fx)}{5f(a\sin(e+fx)+a)^8}\right)$$

$$\Big\downarrow \text{3159}$$

$$a^5c^5\left(-\frac{(3A-8B)\left(-\frac{3\int \frac{\cos^8(e+fx)}{(\sin(e+fx)a+a)^5}dx}{a^2}-\frac{2\cos^9(e+fx)}{3af(a\sin(e+fx)+a)^6}\right)}{5a}-\frac{(A-B)\cos^{11}(e+fx)}{5f(a\sin(e+fx)+a)^8}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^5c^5\left(-\frac{(3A-8B)\left(-\frac{3\int \frac{\cos(e+fx)^8}{(\sin(e+fx)a+a)^5}dx}{a^2}-\frac{2\cos^9(e+fx)}{3af(a\sin(e+fx)+a)^6}\right)}{5a}-\frac{(A-B)\cos^{11}(e+fx)}{5f(a\sin(e+fx)+a)^8}\right)$$

$$\Big\downarrow \text{3159}$$

$$a^5c^5\left(-\frac{(3A-8B)\left(-\frac{3\left(-\frac{7\int \frac{\cos^6(e+fx)}{(\sin(e+fx)a+a)^3}dx}{a^2}-\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^4}\right)}{a^2}-\frac{2\cos^9(e+fx)}{3af(a\sin(e+fx)+a)^6}\right)}{5a}-\frac{(A-B)\cos^{11}(e+fx)}{5f(a\sin(e+fx)+a)^8}\right.$$

$$\Big\downarrow \text{3042}$$

$$a^5c^5\left(-\frac{(3A-8B)\left(-\frac{3\left(-\frac{7\int \frac{\cos(e+fx)^6}{(\sin(e+fx)a+a)^3}dx}{a^2}-\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^4}\right)}{a^2}-\frac{2\cos^9(e+fx)}{3af(a\sin(e+fx)+a)^6}\right)}{5a}-\frac{(A-B)\cos^{11}(e+fx)}{5f(a\sin(e+fx)+a)^8}\right.$$

$$\Big\downarrow \text{3159}$$

$$a^5c^5\left(-\frac{(3A-8B)\left(-\frac{3\left(-\frac{7\left(\frac{5\int \frac{\cos^4(e+fx)}{\sin(e+fx)a+a}dx}{a^2}+\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{a^2}-\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^4}\right)}{a^2}-\frac{2\cos^9(e+fx)}{3af(a\sin(e+fx)+a)^6}\right)}{5a}-\frac{(A-\ldots}{5f(a\,\mathrm{s}\ldots}\right.$$

$\Big\downarrow$ 3042

$$a^5c^5\left(-\frac{(3A-8B)\left(-\frac{3\left(-\frac{7\left(\frac{5\int \frac{\cos(e+fx)^4}{\sin(e+fx)a+a}dx}{a^2}+\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{a^2}-\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^4}\right)}{a^2}-\frac{2\cos^9(e+fx)}{3af(a\sin(e+fx)+a)^6}\right)}{5a}-\frac{(A-}{5f(a\,\mathrm{s}}\right.$$

$\Big\downarrow$ 3161

$$a^5c^5\left(-\frac{(3A-8B)\left(-\frac{3\left(-\frac{7\left(\frac{5\left(\frac{\int\cos^2(e+fx)dx}{a}+\frac{\cos^3(e+fx)}{3af}\right)}{a^2}+\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{a^2}-\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^4}\right)}{a^2}-\frac{2\cos^9(e+fx)}{3af(a\sin(e+fx)+a)^6}\right)}{5a}\right.$$

$$\Big\downarrow \text{3042}$$

$$a^5c^5\left(-\frac{(3A-8B)\left(-\frac{3\left(-\frac{7\left(\frac{5\left(\frac{\int \sin\left(e+fx+\frac{\pi}{2}\right)^2dx}{a}+\frac{\cos^3(e+fx)}{3af}\right)}{a^2}+\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{a^2}-\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^4}\right)}{a^2}-\frac{2\cos^9(e+fx)}{3af(a\sin(e+fx)+a}\right.}{5a}\right.$$

$$\Big\downarrow \text{3115}$$

$$a^5c^5\left(-\frac{(3A-8B)\left(-\frac{3\left(-\frac{7\left(\frac{5\left(\frac{\frac{\int 1dx}{2}+\frac{\sin(e+fx)\cos(e+fx)}{2f}}{a}+\frac{\cos^3(e+fx)}{3af}\right)}{a^2}+\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{a^2}-\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^4}\right)}{a^2}-\frac{2\cos^9(e}{3af(a\sin(e}\right.}{5a}\right.$$

$\downarrow$ 24

$$a^5c^5\left(-\frac{(3A-8B)\left(-\frac{3\left(-\frac{7\left(\frac{5\left(\frac{\cos^3(e+fx)}{3af}+\frac{\frac{\sin(e+fx)\cos(e+fx)}{2f}+\frac{x}{2}}{a}\right)}{a^2}+\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{a^2}-\frac{2\cos^7(e+fx)}{af(a\sin(e+fx)+a)^4}\right)}{a^2}-\frac{2\cos^9(e+f}{3af(a\sin(e+fx}\right.}{5a}\right.$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5)/(a + a*Sin[e + f*x])^3,x
]
```

output
```
a^5*c^5*(-1/5*((A - B)*Cos[e + f*x]^11)/(f*(a + a*Sin[e + f*x])^8) - ((3*A
 - 8*B)*((-2*Cos[e + f*x]^9)/(3*a*f*(a + a*Sin[e + f*x])^6) - (3*((-2*Cos[
e + f*x]^7)/(a*f*(a + a*Sin[e + f*x])^4) - (7*((2*Cos[e + f*x]^5)/(a*f*(a
+ a*Sin[e + f*x])^2) + (5*(Cos[e + f*x]^3/(3*a*f) + (x/2 + (Cos[e + f*x]*S
in[e + f*x])/(2*f))/a))/a^2))/a^2))/a^2))/(5*a))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 87.96 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{2c^5\left(-\frac{-256A+256B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{32A-96B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{32A-80B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{96A-32B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{128A-128B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{\left(\frac{A}{2}-4B\right)}{}\right.}{}$
default	$\frac{2c^5\left(-\frac{-256A+256B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{32A-96B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{32A-80B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{96A-32B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{128A-128B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{\left(\frac{A}{2}-4B\right)}{}\right.}{}$
parallelrisch	$-\frac{63\left(\left(-\frac{1223}{63}A+\frac{19297}{378}B+\frac{80}{3}fxB-10fxA\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\left(-\frac{2543}{378}B+\frac{341}{126}A+5fxA-\frac{40}{3}fxB\right)\cos\left(\frac{3fx}{2}+\frac{3e}{2}\right)+\left(-\frac{394}{378}\right.\right.}{}$
risch	$-\frac{63c^5xA}{2a^3}+\frac{84c^5xB}{a^3}-\frac{B\,c^5\mathrm{e}^{3i(fx+e)}}{24a^3f}-\frac{ic^5\mathrm{e}^{2i(fx+e)}A}{8a^3f}+\frac{ic^5\mathrm{e}^{2i(fx+e)}B}{a^3f}-\frac{4c^5\mathrm{e}^{i(fx+e)}A}{a^3f}+\frac{127c^5\mathrm{e}^{i(fx+e)}B}{8a^3f}$
norman	Expression too large to display

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x,method=_RETUR
NVERBOSE)
```

output
```
2/f*c^5/a^3*(-1/4*(-256*A+256*B)/(tan(1/2*f*x+1/2*e)+1)^4-1/2*(32*A-96*B)/
(tan(1/2*f*x+1/2*e)+1)^2-(32*A-80*B)/(tan(1/2*f*x+1/2*e)+1)-1/3*(96*A-32*B
)/(tan(1/2*f*x+1/2*e)+1)^3-1/5*(128*A-128*B)/(tan(1/2*f*x+1/2*e)+1)^5-((1/
2*A-4*B)*tan(1/2*f*x+1/2*e)^5+(8*A-31*B)*tan(1/2*f*x+1/2*e)^4+(16*A-64*B)*
tan(1/2*f*x+1/2*e)^2+(-1/2*A+4*B)*tan(1/2*f*x+1/2*e)+8*A-95/3*B)/(1+tan(1/
2*f*x+1/2*e)^2)^3-21/2*(3*A-8*B)*arctan(tan(1/2*f*x+1/2*e)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.77

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^5}{(a + a\sin(e + fx))^3}\,dx =$$
$$-\frac{10\,Bc^5\cos(fx+e)^6 + 15\,(A-6\,B)c^5\cos(fx+e)^5 + 10\,(21\,A-74\,B)c^5\cos(fx+e)^4 - 1260\,(3\,A-$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x, algori
thm="fricas")
```

output
```
-1/30*(10*B*c^5*cos(f*x + e)^6 + 15*(A - 6*B)*c^5*cos(f*x + e)^5 + 10*(21*
A - 74*B)*c^5*cos(f*x + e)^4 - 1260*(3*A - 8*B)*c^5*f*x - 192*(A - B)*c^5
+ (315*(3*A - 8*B)*c^5*f*x + (2373*A - 6128*B)*c^5)*cos(f*x + e)^3 + (945*
(3*A - 8*B)*c^5*f*x - 2*(753*A - 2248*B)*c^5)*cos(f*x + e)^2 - 6*(105*(3*A
 - 8*B)*c^5*f*x + 2*(323*A - 848*B)*c^5)*cos(f*x + e) + (10*B*c^5*cos(f*x
+ e)^5 - 5*(3*A - 20*B)*c^5*cos(f*x + e)^4 + 5*(39*A - 128*B)*c^5*cos(f*x
+ e)^3 - 1260*(3*A - 8*B)*c^5*f*x + 192*(A - B)*c^5 + (315*(3*A - 8*B)*c^5
*f*x - 2*(1089*A - 2744*B)*c^5)*cos(f*x + e)^2 - 6*(105*(3*A - 8*B)*c^5*f*
x + 2*(307*A - 832*B)*c^5)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)
^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(
f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10608 vs. 2(228) = 456.

Time = 39.74 (sec) , antiderivative size = 10608, normalized size of antiderivative = 43.65

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^5}{(a + a\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**5/(a+a*sin(f*x+e))**3,x)
```

output
```
Piecewise((-945*A*c**5*f*x*tan(e/2 + f*x/2)**11/(30*a**3*f*tan(e/2 + f*x/2
)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 +
 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a
**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*
tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2
+ f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 4725*A*c**5*f*x*t
an(e/2 + f*x/2)**10/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 +
 f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)
**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 +
1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a*
*3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan
(e/2 + f*x/2) + 30*a**3*f) - 12285*A*c**5*f*x*tan(e/2 + f*x/2)**9/(30*a**3
*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan
(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 +
f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)
**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 3
90*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) -
 23625*A*c**5*f*x*tan(e/2 + f*x/2)**8/(30*a**3*f*tan(e/2 + f*x/2)**11 + 15
0*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*
f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*t...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3282 vs. $2(231) = 462$.

Time = 0.21 (sec) , antiderivative size = 3282, normalized size of antiderivative = 13.51

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^5}{(a + a\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x, algori
thm="maxima")
```

output
```
1/15*(B*c^5*((2375*sin(f*x + e)/(cos(f*x + e) + 1) + 5347*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 9230*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 12622*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 + 13340*sin(f*x + e)^5/(cos(f*x + e) + 1)
^5 + 11684*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 8050*sin(f*x + e)^7/(cos(
f*x + e) + 1)^7 + 4370*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 1725*sin(f*x
+ e)^9/(cos(f*x + e) + 1)^9 + 345*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 +
544)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 13*a^3*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 25*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 38*a^3
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 46*a^3*sin(f*x + e)^5/(cos(f*x + e)
 + 1)^5 + 46*a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 38*a^3*sin(f*x + e)
^7/(cos(f*x + e) + 1)^7 + 25*a^3*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 13*
a^3*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 5*a^3*sin(f*x + e)^10/(cos(f*x +
 e) + 1)^10 + a^3*sin(f*x + e)^11/(cos(f*x + e) + 1)^11) + 345*arctan(sin(
f*x + e)/(cos(f*x + e) + 1))/a^3) - A*c^5*((1325*sin(f*x + e)/(cos(f*x + e
) + 1) + 2673*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3805*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + 4329*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3575*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5 + 2275*sin(f*x + e)^6/(cos(f*x + e) + 1)^6
+ 975*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 195*sin(f*x + e)^8/(cos(f*x +
e) + 1)^8 + 304)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 12*a^3*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 + 20*a^3*sin(f*x + e)^3/(cos(f*x + e) ...
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.47

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^3}\,dx =$$

$$-\frac{\frac{315\,(3\,Ac^5-8\,Bc^5)(fx+e)}{a^3}+\frac{10\left(3\,Ac^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-24\,Bc^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+48\,Ac^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-186\,Bc^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+96\cdots}{\left(\tan\left(\frac{1}{2}\,f\cdots\right.\right.}}{\cdots}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x, algori
thm="giac")
```

output
```
-1/30*(315*(3*A*c^5 - 8*B*c^5)*(f*x + e)/a^3 + 10*(3*A*c^5*tan(1/2*f*x + 1
/2*e)^5 - 24*B*c^5*tan(1/2*f*x + 1/2*e)^5 + 48*A*c^5*tan(1/2*f*x + 1/2*e)^
4 - 186*B*c^5*tan(1/2*f*x + 1/2*e)^4 + 96*A*c^5*tan(1/2*f*x + 1/2*e)^2 - 3
84*B*c^5*tan(1/2*f*x + 1/2*e)^2 - 3*A*c^5*tan(1/2*f*x + 1/2*e) + 24*B*c^5*
tan(1/2*f*x + 1/2*e) + 48*A*c^5 - 190*B*c^5)/((tan(1/2*f*x + 1/2*e)^2 + 1)
^3*a^3) + 64*(30*A*c^5*tan(1/2*f*x + 1/2*e)^4 - 75*B*c^5*tan(1/2*f*x + 1/2
*e)^4 + 135*A*c^5*tan(1/2*f*x + 1/2*e)^3 - 345*B*c^5*tan(1/2*f*x + 1/2*e)^
3 + 255*A*c^5*tan(1/2*f*x + 1/2*e)^2 - 595*B*c^5*tan(1/2*f*x + 1/2*e)^2 +
165*A*c^5*tan(1/2*f*x + 1/2*e) - 395*B*c^5*tan(1/2*f*x + 1/2*e) + 39*A*c^5
 - 94*B*c^5)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

Mupad [B] (verification not implemented)

Time = 39.66 (sec) , antiderivative size = 501, normalized size of antiderivative = 2.06

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^5)/(a + a*sin(e + f*x))^3,x
)
```

output
```
- (tan(e/2 + (f*x)/2)*(431*A*c^5 - (3454*B*c^5)/3) + (496*A*c^5)/5 - (3958
*B*c^5)/15 + tan(e/2 + (f*x)/2)^10*(65*A*c^5 - 168*B*c^5) + tan(e/2 + (f*x
)/2)^9*(309*A*c^5 - 838*B*c^5) + tan(e/2 + (f*x)/2)^8*(826*A*c^5 - (6418*B
*c^5)/3) + tan(e/2 + (f*x)/2)^7*(1418*A*c^5 - (11636*B*c^5)/3) + tan(e/2 +
 (f*x)/2)^3*(1654*A*c^5 - (13372*B*c^5)/3) + tan(e/2 + (f*x)/2)^5*(2332*A*
c^5 - (19072*B*c^5)/3) + tan(e/2 + (f*x)/2)^2*((4903*A*c^5)/5 - (38884*B*c
^5)/15) + tan(e/2 + (f*x)/2)^6*((11156*A*c^5)/5 - (86708*B*c^5)/15) + tan(
e/2 + (f*x)/2)^4*((11758*A*c^5)/5 - (92224*B*c^5)/15))/(f*(13*a^3*tan(e/2
+ (f*x)/2)^2 + 25*a^3*tan(e/2 + (f*x)/2)^3 + 38*a^3*tan(e/2 + (f*x)/2)^4 +
 46*a^3*tan(e/2 + (f*x)/2)^5 + 46*a^3*tan(e/2 + (f*x)/2)^6 + 38*a^3*tan(e/
2 + (f*x)/2)^7 + 25*a^3*tan(e/2 + (f*x)/2)^8 + 13*a^3*tan(e/2 + (f*x)/2)^9
 + 5*a^3*tan(e/2 + (f*x)/2)^10 + a^3*tan(e/2 + (f*x)/2)^11 + a^3 + 5*a^3*t
an(e/2 + (f*x)/2))) - (21*c^5*atan((21*c^5*tan(e/2 + (f*x)/2)*(3*A - 8*B))
/(63*A*c^5 - 168*B*c^5))*(3*A - 8*B))/(a^3*f)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 547, normalized size of antiderivative = 2.25

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^5}{(a+a\sin(e+fx))^3}\,dx$$
$$= \frac{c^5(-390a+1008b-1890\cos(fx+e)\sin(fx+e)afx+5040\cos(fx+e)\sin(fx+e)bfx-1008\cos}{}$$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x)
```

output
```
(c**5*( - 10*cos(e + f*x)*sin(e + f*x)**5*b - 15*cos(e + f*x)*sin(e + f*x)
**4*a + 90*cos(e + f*x)*sin(e + f*x)**4*b + 195*cos(e + f*x)*sin(e + f*x)*
*3*a - 620*cos(e + f*x)*sin(e + f*x)**3*b - 945*cos(e + f*x)*sin(e + f*x)*
*2*a*f*x + 1305*cos(e + f*x)*sin(e + f*x)**2*a + 2520*cos(e + f*x)*sin(e +
 f*x)**2*b*f*x - 3358*cos(e + f*x)*sin(e + f*x)**2*b - 1890*cos(e + f*x)*s
in(e + f*x)*a*f*x + 1293*cos(e + f*x)*sin(e + f*x)*a + 5040*cos(e + f*x)*s
in(e + f*x)*b*f*x - 3454*cos(e + f*x)*sin(e + f*x)*b - 945*cos(e + f*x)*a*
f*x + 390*cos(e + f*x)*a + 2520*cos(e + f*x)*b*f*x - 1008*cos(e + f*x)*b -
 10*sin(e + f*x)**6*b - 15*sin(e + f*x)**5*a + 100*sin(e + f*x)**5*b + 210
*sin(e + f*x)**4*a - 710*sin(e + f*x)**4*b + 945*sin(e + f*x)**3*a*f*x + 3
306*sin(e + f*x)**3*a - 2520*sin(e + f*x)**3*b*f*x - 8638*sin(e + f*x)**3*
b + 2835*sin(e + f*x)**2*a*f*x + 4380*sin(e + f*x)**2*a - 7560*sin(e + f*x
)**2*b*f*x - 11896*sin(e + f*x)**2*b + 2835*sin(e + f*x)*a*f*x + 1293*sin(
e + f*x)*a - 7560*sin(e + f*x)*b*f*x - 3454*sin(e + f*x)*b + 945*a*f*x - 3
90*a - 2520*b*f*x + 1008*b))/(30*a**3*f*(cos(e + f*x)*sin(e + f*x)**2 + 2*
cos(e + f*x)*sin(e + f*x) + cos(e + f*x) - sin(e + f*x)**3 - 3*sin(e + f*x
)**2 - 3*sin(e + f*x) - 1))
```

3.71 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 201

$$\begin{aligned}\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^3}\,dx &= -\frac{7(2A-7B)c^4x}{2a^3}-\frac{7(2A-7B)c^4\cos(e+fx)}{2a^3f} \\ &\quad -\frac{a^4(A-B)c^4\cos^9(e+fx)}{5f(a+a\sin(e+fx))^7}+\frac{2a^2(2A-7B)c^4\cos^7(e+fx)}{15f(a+a\sin(e+fx))^5} \\ &\quad -\frac{14(2A-7B)c^4\cos^5(e+fx)}{15f(a+a\sin(e+fx))^3}-\frac{7(2A-7B)c^4\cos^3(e+fx)}{6f\,(a^3+a^3\sin(e+fx))}\end{aligned}$$

output
```
-7/2*(2*A-7*B)*c^4*x/a^3-7/2*(2*A-7*B)*c^4*cos(f*x+e)/a^3/f-1/5*a^4*(A-B)*
c^4*cos(f*x+e)^9/f/(a+a*sin(f*x+e))^7+2/15*a^2*(2*A-7*B)*c^4*cos(f*x+e)^7/
f/(a+a*sin(f*x+e))^5-14/15*(2*A-7*B)*c^4*cos(f*x+e)^5/f/(a+a*sin(f*x+e))^3
-7/6*(2*A-7*B)*c^4*cos(f*x+e)^3/f/(a^3+a^3*sin(f*x+e))
```

Mathematica [A] (verified)

Time = 12.17 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.73

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^3}\,dx$$

$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c-c\sin(e+fx))^4\left(384(A-B)\sin\left(\frac{1}{2}(e+fx)\right)-192(A-B)\,(c\right.}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4)/(a + a*Sin[e + f*x
])^3,x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^4*(384*(A - B)
*Sin[(e + f*x)/2] - 192*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 12
8*(8*A - 13*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 +
64*(8*A - 13*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 64*(29*A - 79*B)
*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 210*(2*A - 7*B
)*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 - 60*(A - 7*B)*Cos[e +
 f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 - 15*B*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2])^5*Sin[2*(e + f*x)]))/(60*a^3*f*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2])^8*(1 + Sin[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.92, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.361$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3159, 3042, 3158, 3042, 3161, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^4(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-c\sin(e+fx))^4(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^3}dx$$

$$\Big\downarrow \text{3446}$$

$$a^4c^4\int \frac{\cos^8(e+fx)(A+B\sin(e+fx))}{(\sin(e+fx)a+a)^7}dx$$

$$\Big\downarrow \text{3042}$$

$$a^4c^4\int \frac{\cos(e+fx)^8(A+B\sin(e+fx))}{(\sin(e+fx)a+a)^7}dx$$

$$\Big\downarrow \text{3338}$$

$$a^4c^4\left(-\frac{(2A-7B)\int \frac{\cos^8(e+fx)}{(\sin(e+fx)a+a)^6}dx}{5a}-\frac{(A-B)\cos^9(e+fx)}{5f(a\sin(e+fx)+a)^7}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^4c^4\left(-\frac{(2A-7B)\int \frac{\cos(e+fx)^8}{(\sin(e+fx)a+a)^6}dx}{5a}-\frac{(A-B)\cos^9(e+fx)}{5f(a\sin(e+fx)+a)^7}\right)$$

$$\Big\downarrow \text{3159}$$

$$a^4c^4\left(-\frac{(2A-7B)\left(-\frac{7\int \frac{\cos^6(e+fx)}{(\sin(e+fx)a+a)^4}dx}{3a^2}-\frac{2\cos^7(e+fx)}{3af(a\sin(e+fx)+a)^5}\right)}{5a}-\frac{(A-B)\cos^9(e+fx)}{5f(a\sin(e+fx)+a)^7}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^4c^4\left(-\frac{(2A-7B)\left(-\frac{7\int \frac{\cos(e+fx)^6}{(\sin(e+fx)a+a)^4}dx}{3a^2}-\frac{2\cos^7(e+fx)}{3af(a\sin(e+fx)+a)^5}\right)}{5a}-\frac{(A-B)\cos^9(e+fx)}{5f(a\sin(e+fx)+a)^7}\right)$$

$$\Big\downarrow \text{3159}$$

$$a^4c^4\left(-\frac{(2A-7B)\left(-\frac{7\left(-\frac{5\int\frac{\cos^4(e+fx)}{(\sin(e+fx)a+a)^2}dx}{a^2}-\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{3a^2}-\frac{2\cos^7(e+fx)}{3af(a\sin(e+fx)+a)^5}\right)}{5a}-\frac{(A-B)\cos^9(e+fx)}{5f(a\sin(e+fx)+a)^7}\right.$$

$\big\downarrow$ 3042

$$a^4c^4\left(-\frac{(2A-7B)\left(-\frac{7\left(-\frac{5\int\frac{\cos(e+fx)^4}{(\sin(e+fx)a+a)^2}dx}{a^2}-\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{3a^2}-\frac{2\cos^7(e+fx)}{3af(a\sin(e+fx)+a)^5}\right)}{5a}-\frac{(A-B)\cos^9(e+fx)}{5f(a\sin(e+fx)+a)^7}\right.$$

$\big\downarrow$ 3158

$$a^4c^4\left(-\frac{(2A-7B)\left(-\frac{7\left(-\frac{5\left(\frac{3\int\frac{\cos^2(e+fx)}{\sin(e+fx)a+a}dx}{2a}+\frac{\cos^3(e+fx)}{2f\left(a^2\sin(e+fx)+a^2\right)}\right)}{a^2}-\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{3a^2}-\frac{2\cos^7(e+fx)}{3af(a\sin(e+fx)+a)^5}\right)}{5a}-\right.$$ (A − … / 5f(a …

$\downarrow$ 3042

$$a^4c^4\left(-\frac{(2A-7B)\left(-\frac{7\left(-\frac{5\left(\frac{3\int\frac{\cos(e+fx)^2}{\sin(e+fx)a+a}dx}{2a}+\frac{\cos^3(e+fx)}{2f\left(a^2\sin(e+fx)+a^2\right)}\right)}{a^2}-\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{3a^2}-\frac{2\cos^7(e+fx)}{3af(a\sin(e+fx)+a)^5}\right)}{5a}-\frac{(A-}{5f(a}\right.$$

$\downarrow$ 3161

$$a^4c^4\left(-\frac{(2A-7B)\left(-\frac{7\left(-\frac{5\left(\frac{3\left(\frac{\int 1dx}{a}+\frac{\cos(e+fx)}{af}\right)}{2a}+\frac{\cos^3(e+fx)}{2f\left(a^2\sin(e+fx)+a^2\right)}\right)}{a^2}-\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{3a^2}-\frac{2\cos^7(e+fx)}{3af(a\sin(e+fx)+a)^5}\right)}{5a}-\frac{(A}{5f(}\right.$$

$\downarrow$ 24

$$a^4c^4\left(-\frac{(2A-7B)\left(-\frac{7\left(-\frac{5\left(\frac{\cos^3(e+fx)}{2f\left(a^2\sin(e+fx)+a^2\right)}+\frac{3\left(\frac{\cos(e+fx)}{af}+\frac{x}{a}\right)}{2a}\right)}{a^2}-\frac{2\cos^5(e+fx)}{af(a\sin(e+fx)+a)^3}\right)}{3a^2}-\frac{2\cos^7(e+fx)}{3af(a\sin(e+fx)+a)^5}\right)}{5a}-\frac{(A-}{5f(a\,s}\right.$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4)/(a + a*Sin[e + f*x])^3,x
]
```

output
```
a^4*c^4*(-1/5*((A - B)*Cos[e + f*x]^9)/(f*(a + a*Sin[e + f*x])^7) - ((2*A
- 7*B)*((-2*Cos[e + f*x]^7)/(3*a*f*(a + a*Sin[e + f*x])^5) - (7*((-2*Cos[e
 + f*x]^5)/(a*f*(a + a*Sin[e + f*x])^3) - (5*((3*(x/a + Cos[e + f*x]/(a*f)
))/(2*a) + Cos[e + f*x]^3/(2*f*(a^2 + a^2*Sin[e + f*x]))))/a^2))/(3*a^2)))
/(5*a))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3158
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(a*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
 EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && In
tegersQ[2*m, 2*p]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 11.06 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{2c^4\left(-\frac{-128A+128B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{8A-24B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{64A-64B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{64A-32B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{16B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{-\frac{B\tan\left(\frac{f}{2}\right.}{2}}{}\right.}{f\,a^3}$
default	$\frac{2c^4\left(-\frac{-128A+128B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{8A-24B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{64A-64B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{64A-32B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{16B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{-\frac{B\tan\left(\frac{f}{2}\right.}{2}}{}\right.}{f\,a^3}$
parallelrisch	$-\frac{7\left(\left(-\frac{281}{14}A+\frac{471}{7}B+35fxB-10fxA\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\left(-\frac{761}{84}B+\frac{131}{42}A+5fxA-\frac{35}{2}fxB\right)\cos\left(\frac{3fx}{2}+\frac{3e}{2}\right)+\left(-\frac{1937}{140}B+\right.\right.}{}$
risch	$-\frac{7c^4xA}{a^3}+\frac{49c^4xB}{2a^3}+\frac{iB\,c^4e^{2i(fx+e)}}{8a^3f}-\frac{c^4e^{i(fx+e)}A}{2a^3f}+\frac{7c^4e^{i(fx+e)}B}{2a^3f}-\frac{c^4e^{-i(fx+e)}A}{2a^3f}+\frac{7c^4e^{-i(fx+e)}B}{2a^3f}-$
norman	Expression too large to display

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x,method=_RETUR
NVERBOSE)
```

output
```
2/f*c^4/a^3*(-1/4*(-128*A+128*B)/(tan(1/2*f*x+1/2*e)+1)^4-(8*A-24*B)/(tan(
1/2*f*x+1/2*e)+1)-1/5*(64*A-64*B)/(tan(1/2*f*x+1/2*e)+1)^5-1/3*(64*A-32*B)
/(tan(1/2*f*x+1/2*e)+1)^3+16*B/(tan(1/2*f*x+1/2*e)+1)^2-(-1/2*B*tan(1/2*f*
x+1/2*e)^3+(A-7*B)*tan(1/2*f*x+1/2*e)^2+1/2*B*tan(1/2*f*x+1/2*e)+A-7*B)/(1
+tan(1/2*f*x+1/2*e)^2)^2-7/2*(2*A-7*B)*arctan(tan(1/2*f*x+1/2*e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 392 vs. 2(189) = 378.

Time = 0.10 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.95

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^3}\,dx$$

$$=\frac{15\,Bc^4\cos\left(fx+e\right)^5-30\,(A-6\,B)c^4\cos\left(fx+e\right)^4+420\,(2\,A-7\,B)c^4fx+96\,(A-B)c^4-(105\,(2\,A}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x, algori
thm="fricas")
```

output
```
1/30*(15*B*c^4*cos(f*x + e)^5 - 30*(A - 6*B)*c^4*cos(f*x + e)^4 + 420*(2*A
 - 7*B)*c^4*f*x + 96*(A - B)*c^4 - (105*(2*A - 7*B)*c^4*f*x + (554*A - 181
9*B)*c^4)*cos(f*x + e)^3 - (315*(2*A - 7*B)*c^4*f*x - 2*(134*A - 619*B)*c^
4)*cos(f*x + e)^2 + 6*(35*(2*A - 7*B)*c^4*f*x + 2*(74*A - 249*B)*c^4)*cos(
f*x + e) - (15*B*c^4*cos(f*x + e)^4 + 15*(2*A - 11*B)*c^4*cos(f*x + e)^3 -
 420*(2*A - 7*B)*c^4*f*x + 96*(A - B)*c^4 + (105*(2*A - 7*B)*c^4*f*x - 2*(
262*A - 827*B)*c^4)*cos(f*x + e)^2 - 6*(35*(2*A - 7*B)*c^4*f*x + 2*(66*A -
 241*B)*c^4)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*c
os(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2
*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7337 vs. $2(185) = 370$.

Time = 24.79 (sec) , antiderivative size = 7337, normalized size of antiderivative = 36.50

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^4}{(a + a\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**4/(a+a*sin(f*x+e))**3,x)
```

output
```
Piecewise((-210*A*c**4*f*x*tan(e/2 + f*x/2)**9/(30*a**3*f*tan(e/2 + f*x/2)
**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 60
0*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f
*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2
 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 1050*A*c**4*f*x*
tan(e/2 + f*x/2)**8/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 +
f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**
6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*
a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*t
an(e/2 + f*x/2) + 30*a**3*f) - 2520*A*c**4*f*x*tan(e/2 + f*x/2)**7/(30*a**
3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(
e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*
x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3
+ 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f
) - 4200*A*c**4*f*x*tan(e/2 + f*x/2)**6/(30*a**3*f*tan(e/2 + f*x/2)**9 + 1
50*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*
f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/
2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/
2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 5460*A*c**4*f*x*tan(e/2
 + f*x/2)**5/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2394 vs. $2(189) = 378$.

Time = 0.19 (sec) , antiderivative size = 2394, normalized size of antiderivative = 11.91

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^4}{(a + a\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x, algori
thm="maxima")
```

output
```
1/15*(B*c^4*((1325*sin(f*x + e)/(cos(f*x + e) + 1) + 2673*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 3805*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 4329*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 + 3575*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
 + 2275*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 975*sin(f*x + e)^7/(cos(f*x
+ e) + 1)^7 + 195*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 304)/(a^3 + 5*a^3*
sin(f*x + e)/(cos(f*x + e) + 1) + 12*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + 20*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 26*a^3*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 + 26*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 20*a^3*s
in(f*x + e)^6/(cos(f*x + e) + 1)^6 + 12*a^3*sin(f*x + e)^7/(cos(f*x + e) +
 1)^7 + 5*a^3*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + a^3*sin(f*x + e)^9/(co
s(f*x + e) + 1)^9) + 195*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) - 6*
A*c^4*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15*sin(f*
x + e)^6/(cos(f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e
) + 1) + 11*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 11*a
^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(cos(f*x + e
) + 1)^6 + a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(sin(f*x +
e)/(cos(f*x + e) + 1))/a^3) + 24*B*c^4*((105*sin(f*x + e)/(cos(f*x + e)...
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.44

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^3}\,dx =$$

$$-\frac{\frac{105\,(2\,Ac^4-7\,Bc^4)(fx+e)}{a^3}-\frac{30\left(Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-2\,Ac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+14\,Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-Bc^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-2\,Ac^4+14\right.}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2a^3}}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x, algori
thm="giac")
```

output
```
-1/30*(105*(2*A*c^4 - 7*B*c^4)*(f*x + e)/a^3 - 30*(B*c^4*tan(1/2*f*x + 1/2
*e)^3 - 2*A*c^4*tan(1/2*f*x + 1/2*e)^2 + 14*B*c^4*tan(1/2*f*x + 1/2*e)^2 -
 B*c^4*tan(1/2*f*x + 1/2*e) - 2*A*c^4 + 14*B*c^4)/((tan(1/2*f*x + 1/2*e)^2
 + 1)^2*a^3) + 32*(15*A*c^4*tan(1/2*f*x + 1/2*e)^4 - 45*B*c^4*tan(1/2*f*x
+ 1/2*e)^4 + 60*A*c^4*tan(1/2*f*x + 1/2*e)^3 - 210*B*c^4*tan(1/2*f*x + 1/2
*e)^3 + 130*A*c^4*tan(1/2*f*x + 1/2*e)^2 - 380*B*c^4*tan(1/2*f*x + 1/2*e)^
2 + 80*A*c^4*tan(1/2*f*x + 1/2*e) - 250*B*c^4*tan(1/2*f*x + 1/2*e) + 19*A*
c^4 - 59*B*c^4)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

Mupad [B] (verification not implemented)

Time = 40.06 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.08

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^3}\,dx = -\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{286\,A\,c^4}{3}-\frac{1007\,B\,c^4}{3}\right)+\frac{334\,A\,c^4}{15}-\frac{1154\,B\,c^4}{15}+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8\left(16\,A\,c^4-49\,B\,c^4\right)+\tan\left(\frac{e}{2}+\frac{f}{}\right.}{f\,\left(a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^9+5\,a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8+12\right.} - \frac{7\,c^4\,\mathrm{atan}\left(\frac{7\,c^4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(2\,A-7\,B)}{14\,A\,c^4-49\,B\,c^4}\right)(2\,A-7\,B)}{a^3\,f}$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^4)/(a + a*sin(e + f*x))^3,x
)
```

output
```
- (tan(e/2 + (f*x)/2)*((286*A*c^4)/3 - (1007*B*c^4)/3) + (334*A*c^4)/15 -
(1154*B*c^4)/15 + tan(e/2 + (f*x)/2)^8*(16*A*c^4 - 49*B*c^4) + tan(e/2 + (
f*x)/2)^7*(66*A*c^4 - 243*B*c^4) + tan(e/2 + (f*x)/2)^6*((542*A*c^4)/3 - (
1741*B*c^4)/3) + tan(e/2 + (f*x)/2)^5*((706*A*c^4)/3 - (2621*B*c^4)/3) + t
an(e/2 + (f*x)/2)^3*((794*A*c^4)/3 - (2875*B*c^4)/3) + tan(e/2 + (f*x)/2)^
2*((1006*A*c^4)/5 - (3401*B*c^4)/5) + tan(e/2 + (f*x)/2)^4*((1718*A*c^4)/5
 - (5633*B*c^4)/5))/(f*(12*a^3*tan(e/2 + (f*x)/2)^2 + 20*a^3*tan(e/2 + (f*
x)/2)^3 + 26*a^3*tan(e/2 + (f*x)/2)^4 + 26*a^3*tan(e/2 + (f*x)/2)^5 + 20*a
^3*tan(e/2 + (f*x)/2)^6 + 12*a^3*tan(e/2 + (f*x)/2)^7 + 5*a^3*tan(e/2 + (f
*x)/2)^8 + a^3*tan(e/2 + (f*x)/2)^9 + a^3 + 5*a^3*tan(e/2 + (f*x)/2))) - (
7*c^4*atan((7*c^4*tan(e/2 + (f*x)/2)*(2*A - 7*B))/(14*A*c^4 - 49*B*c^4))*(
2*A - 7*B))/(a^3*f)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 491, normalized size of antiderivative = 2.44

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^4}{(a+a\sin(e+fx))^3}\,dx$$
$$= \frac{c^4\left(-96a+294b-420\cos\left(fx+e\right)\sin\left(fx+e\right)afx+1470\cos\left(fx+e\right)\sin\left(fx+e\right)bfx-294\cos\left(fx+\right.\right.}{}$$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x)
```

output
```
(c**4*(15*cos(e + f*x)*sin(e + f*x)**4*b + 30*cos(e + f*x)*sin(e + f*x)**3
*a - 165*cos(e + f*x)*sin(e + f*x)**3*b - 210*cos(e + f*x)*sin(e + f*x)**2
*a*f*x + 316*cos(e + f*x)*sin(e + f*x)**2*a + 735*cos(e + f*x)*sin(e + f*x
)**2*b*f*x - 989*cos(e + f*x)*sin(e + f*x)**2*b - 420*cos(e + f*x)*sin(e +
 f*x)*a*f*x + 286*cos(e + f*x)*sin(e + f*x)*a + 1470*cos(e + f*x)*sin(e +
f*x)*b*f*x - 1007*cos(e + f*x)*sin(e + f*x)*b - 210*cos(e + f*x)*a*f*x + 9
6*cos(e + f*x)*a + 735*cos(e + f*x)*b*f*x - 294*cos(e + f*x)*b + 15*sin(e
+ f*x)**5*b + 30*sin(e + f*x)**4*a - 180*sin(e + f*x)**4*b + 210*sin(e + f
*x)**3*a*f*x + 762*sin(e + f*x)**3*a - 735*sin(e + f*x)**3*b*f*x - 2544*si
n(e + f*x)**3*b + 630*sin(e + f*x)**2*a*f*x + 922*sin(e + f*x)**2*a - 2205
*sin(e + f*x)**2*b*f*x - 3458*sin(e + f*x)**2*b + 630*sin(e + f*x)*a*f*x +
 286*sin(e + f*x)*a - 2205*sin(e + f*x)*b*f*x - 1007*sin(e + f*x)*b + 210*
a*f*x - 96*a - 735*b*f*x + 294*b))/(30*a**3*f*(cos(e + f*x)*sin(e + f*x)**
2 + 2*cos(e + f*x)*sin(e + f*x) + cos(e + f*x) - sin(e + f*x)**3 - 3*sin(e
 + f*x)**2 - 3*sin(e + f*x) - 1))
```

3.72 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 153

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx = -\frac{(A-6B)c^3x}{a^3}-\frac{(A-6B)c^3\cos(e+fx)}{a^3f}-\frac{a^3(A-B)c^3\cos^7(e+fx)}{5f(a+a\sin(e+fx))^6}+\frac{2a(A-6B)c^3\cos^5(e+fx)}{15f(a+a\sin(e+fx))^4}-\frac{2a^3(A-6B)c^3\cos^3(e+fx)}{3f\left(a^3+a^3\sin(e+fx)\right)^2}$$

output
```
-(A-6*B)*c^3*x/a^3-(A-6*B)*c^3*cos(f*x+e)/a^3/f-1/5*a^3*(A-B)*c^3*cos(f*x+
e)^7/f/(a+a*sin(f*x+e))^6+2/15*a*(A-6*B)*c^3*cos(f*x+e)^5/f/(a+a*sin(f*x+e
))^4-2/3*a^3*(A-6*B)*c^3*cos(f*x+e)^3/f/(a^3+a^3*sin(f*x+e))^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 308 vs. 2(153) = 306.

Time = 11.80 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.01

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(48(A-B)\sin\left(\frac{1}{2}(e+fx)\right)-24(A-B)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}\right.\right.\right.}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3)/(a + a*Sin[e + f*x
])^3,x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(48*(A - B)*Sin[(e + f*x)/2] - 24*(
A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 8*(11*A - 21*B)*Sin[(e + f*
x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 4*(11*A - 21*B)*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^3 + 4*(23*A - 93*B)*Sin[(e + f*x)/2]*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^4 - 15*(A - 6*B)*(e + f*x)*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])^5 + 15*B*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x
)/2])^5)*(c - c*Sin[e + f*x])^3)/(15*a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*
x)/2])^6*(1 + Sin[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.93, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.306$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3159, 3042, 3161, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^3(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-c\sin(e+fx))^3(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^3}dx$$

$$\Big\downarrow \text{3446}$$

$$a^3c^3\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(\sin(e+fx)a+a)^6}dx$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\int \frac{\cos(e+fx)^6(A+B\sin(e+fx))}{(\sin(e+fx)a+a)^6}dx$$

$$\Big\downarrow \text{3338}$$

$$a^3c^3\left(-\frac{(A-6B)\int \frac{\cos^6(e+fx)}{(\sin(e+fx)a+a)^5}dx}{5a}-\frac{(A-B)\cos^7(e+fx)}{5f(a\sin(e+fx)+a)^6}\right)$$

$\Big\downarrow$ 3042

$$a^3c^3\left(-\frac{(A-6B)\int \frac{\cos(e+fx)^6}{(\sin(e+fx)a+a)^5}dx}{5a}-\frac{(A-B)\cos^7(e+fx)}{5f(a\sin(e+fx)+a)^6}\right)$$

$\Big\downarrow$ 3159

$$a^3c^3\left(-\frac{(A-6B)\left(-\frac{5\int \frac{\cos^4(e+fx)}{(\sin(e+fx)a+a)^3}dx}{3a^2}-\frac{2\cos^5(e+fx)}{3af(a\sin(e+fx)+a)^4}\right)}{5a}-\frac{(A-B)\cos^7(e+fx)}{5f(a\sin(e+fx)+a)^6}\right)$$

$\Big\downarrow$ 3042

$$a^3c^3\left(-\frac{(A-6B)\left(-\frac{5\int \frac{\cos(e+fx)^4}{(\sin(e+fx)a+a)^3}dx}{3a^2}-\frac{2\cos^5(e+fx)}{3af(a\sin(e+fx)+a)^4}\right)}{5a}-\frac{(A-B)\cos^7(e+fx)}{5f(a\sin(e+fx)+a)^6}\right)$$

$\Big\downarrow$ 3159

$$a^3c^3\left(-\frac{(A-6B)\left(-\frac{5\left(-\frac{3\int \frac{\cos^2(e+fx)}{\sin(e+fx)a+a}dx}{a^2}-\frac{2\cos^3(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{3a^2}-\frac{2\cos^5(e+fx)}{3af(a\sin(e+fx)+a)^4}\right)}{5a}-\frac{(A-B)\cos^7(e+fx)}{5f(a\sin(e+fx)+a)^6}\right)$$

$\Big\downarrow$ 3042

$$a^3c^3\left(-\frac{(A-6B)\left(-\frac{5\left(-\frac{3\int\frac{\cos(e+fx)^2}{\sin(e+fx)a+a}dx}{a^2}-\frac{2\cos^3(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{3a^2}-\frac{2\cos^5(e+fx)}{3af(a\sin(e+fx)+a)^4}\right)}{5a}-\frac{(A-B)\cos^7(e+fx)}{5f(a\sin(e+fx)+a)^6}\right)$$

$\Big\downarrow$ 3161

$$a^3c^3\left(-\frac{(A-6B)\left(-\frac{5\left(-\frac{3\left(\frac{\int 1dx}{a}+\frac{\cos(e+fx)}{af}\right)}{a^2}-\frac{2\cos^3(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{3a^2}-\frac{2\cos^5(e+fx)}{3af(a\sin(e+fx)+a)^4}\right)}{5a}-\frac{(A-B)\cos^7(e+fx)}{5f(a\sin(e+fx)+a)^6}\right)$$

$\Big\downarrow$ 24

$$a^3c^3\left(-\frac{(A-6B)\left(-\frac{5\left(-\frac{3\left(\frac{\cos(e+fx)}{af}+\frac{x}{a}\right)}{a^2}-\frac{2\cos^3(e+fx)}{af(a\sin(e+fx)+a)^2}\right)}{3a^2}-\frac{2\cos^5(e+fx)}{3af(a\sin(e+fx)+a)^4}\right)}{5a}-\frac{(A-B)\cos^7(e+fx)}{5f(a\sin(e+fx)+a)^6}\right)$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^3,x
]
```

output
```
a^3*c^3*(-1/5*((A - B)*Cos[e + f*x]^7)/(f*(a + a*Sin[e + f*x])^6) - ((A -
6*B)*((-2*Cos[e + f*x]^5)/(3*a*f*(a + a*Sin[e + f*x])^4) - (5*((-3*(x/a +
Cos[e + f*x]/(a*f)))/a^2 - (2*Cos[e + f*x]^3)/(a*f*(a + a*Sin[e + f*x])^2)
))/(3*a^2)))/(5*a))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{2c^3\left(-\frac{-64A+64B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{-8A-8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2A-6B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{32A-32B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{40A-24B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{B}{1+\tan\left(\frac{fx}{2}\right.}\right.}{f a^3}$
default	$\frac{2c^3\left(-\frac{-64A+64B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{-8A-8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2A-6B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{32A-32B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{40A-24B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{B}{1+\tan\left(\frac{fx}{2}\right.}\right.}{f a^3}$
risch	$-\frac{c^3xA}{a^3}+\frac{6c^3xB}{a^3}+\frac{B\,c^3\mathrm{e}^{i(fx+e)}}{2a^3f}+\frac{B\,c^3\mathrm{e}^{-i(fx+e)}}{2a^3f}+\frac{\frac{112A\,c^3\mathrm{e}^{2i(fx+e)}}{3}-24iA\,c^3\mathrm{e}^{3i(fx+e)}+\frac{56iA\,c^3\mathrm{e}^{i(fx+e)}}{3}-1}{}$
parallelrisch	$-\frac{c^3\left(\left(\left(\frac{233}{2}+60fx\right)B-10fxA-24A\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\left(\left(-\frac{33}{2}-30fx\right)B+\frac{16A}{3}+5fxA\right)\cos\left(\frac{3fx}{2}+\frac{3e}{2}\right)+\left(-\frac{243}{10}B+\frac{24}{5}A\right.\right.}{f\,a^3\left(-5\sin\right.}$
norman	$\frac{-\frac{52A\,c^3-282B\,c^3}{15af}-\frac{\left(2012A\,c^3-8802B\,c^3\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{15af}-\frac{\left(136A\,c^3-966B\,c^3\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{3af}-\frac{2\left(64A\,c^3-255B\,c^3\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right.}{3af}}{}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x,method=_RETUR
NVERBOSE)
```

output
```
2/f*c^3/a^3*(-1/4*(-64*A+64*B)/(tan(1/2*f*x+1/2*e)+1)^4-1/2*(-8*A-8*B)/(ta
n(1/2*f*x+1/2*e)+1)^2-(2*A-6*B)/(tan(1/2*f*x+1/2*e)+1)-1/5*(32*A-32*B)/(ta
n(1/2*f*x+1/2*e)+1)^5-1/3*(40*A-24*B)/(tan(1/2*f*x+1/2*e)+1)^3+B/(1+tan(1/
2*f*x+1/2*e)^2)-(A-6*B)*arctan(tan(1/2*f*x+1/2*e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. 2(147) = 294.

Time = 0.11 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.21

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx$$
$$=\frac{15\,Bc^3\cos(fx+e)^4+60\,(A-6\,B)c^3fx+24\,(A-B)c^3-(15\,(A-6\,B)c^3fx+(46\,A-231\,B)c^3)\cos(}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algori
thm="fricas")
```

output
```
1/15*(15*B*c^3*cos(f*x + e)^4 + 60*(A - 6*B)*c^3*f*x + 24*(A - B)*c^3 - (1
5*(A - 6*B)*c^3*f*x + (46*A - 231*B)*c^3)*cos(f*x + e)^3 - (45*(A - 6*B)*c
^3*f*x - 2*(A - 66*B)*c^3)*cos(f*x + e)^2 + 6*(5*(A - 6*B)*c^3*f*x + 2*(6*
A - 31*B)*c^3)*cos(f*x + e) + (15*B*c^3*cos(f*x + e)^3 + 60*(A - 6*B)*c^3*
f*x - 24*(A - B)*c^3 - (15*(A - 6*B)*c^3*f*x - 2*(23*A - 108*B)*c^3)*cos(f
*x + e)^2 + 6*(5*(A - 6*B)*c^3*f*x + 2*(4*A - 29*B)*c^3)*cos(f*x + e))*sin
(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*
x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)
*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4665 vs. $2(143) = 286$.

Time = 14.58 (sec) , antiderivative size = 4665, normalized size of antiderivative = 30.49

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^3}{(a + a\sin(e + fx))^3}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3/(a+a*sin(f*x+e))**3,x)
```

output
```
Piecewise((-15*A*c**3*f*x*tan(e/2 + f*x/2)**7/(15*a**3*f*tan(e/2 + f*x/2)*
*7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*
a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*t
an(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 75*A*c**3*f
*x*tan(e/2 + f*x/2)**6/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2
+ f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)
**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75
*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 165*A*c**3*f*x*tan(e/2 + f*x/2)**5
/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3
*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e
/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/
2) + 15*a**3*f) - 225*A*c**3*f*x*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 +
f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5
 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a
**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 225*
A*c**3*f*x*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*
tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2
+ f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)
**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 165*A*c**3*f*x*tan(e/2 + f
*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1679 vs. $2(147) = 294$.

Time = 0.16 (sec) , antiderivative size = 1679, normalized size of antiderivative = 10.97

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^3}{(a + a\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algori
thm="maxima")
```

output
```
2/15*(3*B*c^3*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1
5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(cos
(f*x + e) + 1) + 11*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^
4 + 11*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(cos
(f*x + e) + 1)^6 + a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(si
n(f*x + e)/(cos(f*x + e) + 1))/a^3) - A*c^3*((95*sin(f*x + e)/(cos(f*x + e
) + 1) + 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^
3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan
(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) + 3*B*c^3*((95*sin(f*x + e)/(cos(f*
x + e) + 1) + 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 +
 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 1...
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.41

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx$$
$$= \frac{\frac{30\,Bc^3}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)a^3} - \frac{15\,(Ac^3-6\,Bc^3)(fx+e)}{a^3} - \frac{4\left(15\,Ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-45\,Bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+30\,Ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-210\,B}{}}{15}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algori
thm="giac")
```

output
```
1/15*(30*B*c^3/((tan(1/2*f*x + 1/2*e)^2 + 1)*a^3) - 15*(A*c^3 - 6*B*c^3)*(
f*x + e)/a^3 - 4*(15*A*c^3*tan(1/2*f*x + 1/2*e)^4 - 45*B*c^3*tan(1/2*f*x +
 1/2*e)^4 + 30*A*c^3*tan(1/2*f*x + 1/2*e)^3 - 210*B*c^3*tan(1/2*f*x + 1/2*
e)^3 + 100*A*c^3*tan(1/2*f*x + 1/2*e)^2 - 420*B*c^3*tan(1/2*f*x + 1/2*e)^2
 + 50*A*c^3*tan(1/2*f*x + 1/2*e) - 270*B*c^3*tan(1/2*f*x + 1/2*e) + 13*A*c
^3 - 63*B*c^3)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

Mupad [B] (verification not implemented)

Time = 38.88 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.18

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx = -\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{40\,A\,c^3}{3}-82\,B\,c^3\right)+\frac{52\,A\,c^3}{15}-\frac{94\,B\,c^3}{5}+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6\left(4\,A\,c^3-12\,B\,c^3\right)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5\left(\right.}{f\,\left(a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7+5\,a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6+11\,a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5+\right.} - \frac{2\,c^3\,\mathrm{atan}\left(\frac{2\,c^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(A-6\,B)}{2\,A\,c^3-12\,B\,c^3}\right)(A-6\,B)}{a^3\,f}$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^3)/(a + a*sin(e + f*x))^3,x
)
```

output
```
- (tan(e/2 + (f*x)/2)*((40*A*c^3)/3 - 82*B*c^3) + (52*A*c^3)/15 - (94*B*c^
3)/5 + tan(e/2 + (f*x)/2)^6*(4*A*c^3 - 12*B*c^3) + tan(e/2 + (f*x)/2)^5*(8
*A*c^3 - 58*B*c^3) + tan(e/2 + (f*x)/2)^3*((64*A*c^3)/3 - 148*B*c^3) + tan
(e/2 + (f*x)/2)^4*((92*A*c^3)/3 - 134*B*c^3) + tan(e/2 + (f*x)/2)^2*((452*
A*c^3)/15 - (744*B*c^3)/5))/(f*(11*a^3*tan(e/2 + (f*x)/2)^2 + 15*a^3*tan(e
/2 + (f*x)/2)^3 + 15*a^3*tan(e/2 + (f*x)/2)^4 + 11*a^3*tan(e/2 + (f*x)/2)^
5 + 5*a^3*tan(e/2 + (f*x)/2)^6 + a^3*tan(e/2 + (f*x)/2)^7 + a^3 + 5*a^3*ta
n(e/2 + (f*x)/2))) - (2*c^3*atan((2*c^3*tan(e/2 + (f*x)/2)*(A - 6*B))/(2*A
*c^3 - 12*B*c^3))*(A - 6*B))/(a^3*f)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 435, normalized size of antiderivative = 2.84

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx$$
$$= \frac{c^3\left(-12a+36b-30\cos\left(fx+e\right)\sin\left(fx+e\right)afx+180\cos\left(fx+e\right)\sin\left(fx+e\right)bfx-36\cos\left(fx+e\right)b\right.}{}$$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x)
```

output
```
(c**3*( - 15*cos(e + f*x)*sin(e + f*x)**3*b - 15*cos(e + f*x)*sin(e + f*x)
**2*a*f*x + 32*cos(e + f*x)*sin(e + f*x)**2*a + 90*cos(e + f*x)*sin(e + f*
x)**2*b*f*x - 126*cos(e + f*x)*sin(e + f*x)**2*b - 30*cos(e + f*x)*sin(e +
 f*x)*a*f*x + 20*cos(e + f*x)*sin(e + f*x)*a + 180*cos(e + f*x)*sin(e + f*
x)*b*f*x - 123*cos(e + f*x)*sin(e + f*x)*b - 15*cos(e + f*x)*a*f*x + 12*co
s(e + f*x)*a + 90*cos(e + f*x)*b*f*x - 36*cos(e + f*x)*b - 15*sin(e + f*x)
**4*b + 15*sin(e + f*x)**3*a*f*x + 60*sin(e + f*x)**3*a - 90*sin(e + f*x)*
*3*b*f*x - 321*sin(e + f*x)**3*b + 45*sin(e + f*x)**2*a*f*x + 44*sin(e + f
*x)**2*a - 270*sin(e + f*x)**2*b*f*x - 417*sin(e + f*x)**2*b + 45*sin(e +
f*x)*a*f*x + 20*sin(e + f*x)*a - 270*sin(e + f*x)*b*f*x - 123*sin(e + f*x)
*b + 15*a*f*x - 12*a - 90*b*f*x + 36*b))/(15*a**3*f*(cos(e + f*x)*sin(e +
f*x)**2 + 2*cos(e + f*x)*sin(e + f*x) + cos(e + f*x) - sin(e + f*x)**3 - 3
*sin(e + f*x)**2 - 3*sin(e + f*x) - 1))
```

3.73 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 110

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx = \frac{Bc^2x}{a^3}-\frac{a^2(A-B)c^2\cos^5(e+fx)}{5f(a+a\sin(e+fx))^5} - \frac{2Bc^2\cos^3(e+fx)}{3f(a+a\sin(e+fx))^3}+\frac{2Bc^2\cos(e+fx)}{f(a^3+a^3\sin(e+fx))}$$

output
```
B*c^2*x/a^3-1/5*a^2*(A-B)*c^2*cos(f*x+e)^5/f/(a+a*sin(f*x+e))^5-2/3*B*c^2*
cos(f*x+e)^3/f/(a+a*sin(f*x+e))^3+2*B*c^2*cos(f*x+e)/f/(a^3+a^3*sin(f*x+e)
)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 272 vs. 2(110) = 220.

Time = 11.50 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.47

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(24(A-B)\sin\left(\frac{1}{2}(e+fx)\right)-12(A-B)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\right.\right.\right.}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2)/(a + a*Sin[e + f*x
])^3,x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(24*(A - B)*Sin[(e + f*x)/2] - 12*(
A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 8*(3*A - 8*B)*Sin[(e + f*x)
/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 4*(3*A - 8*B)*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^3 + 2*(3*A - 43*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^4 + 15*B*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2])^5)*(c - c*Sin[e + f*x])^2)/(15*a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f
*x)/2])^4*(1 + Sin[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3159, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^2(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-c\sin(e+fx))^2(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^3}dx$$

$$\Big\downarrow \text{3446}$$

$$a^2c^2\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(\sin(e+fx)a+a)^5}dx$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\int \frac{\cos(e+fx)^4(A+B\sin(e+fx))}{(\sin(e+fx)a+a)^5}dx$$

$$\Big\downarrow \text{3338}$$

$$a^2c^2\left(\frac{B\int \frac{\cos^4(e+fx)}{(\sin(e+fx)a+a)^4}dx}{a}-\frac{(A-B)\cos^5(e+fx)}{5f(a\sin(e+fx)+a)^5}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left(\frac{B\int \frac{\cos(e+fx)^4}{(\sin(e+fx)a+a)^4}dx}{a}-\frac{(A-B)\cos^5(e+fx)}{5f(a\sin(e+fx)+a)^5}\right)$$

$$\Big\downarrow \text{3159}$$

$$a^2c^2\left(\frac{B\left(-\frac{\int \frac{\cos^2(e+fx)}{(\sin(e+fx)a+a)^2}dx}{a^2}-\frac{2\cos^3(e+fx)}{3af(a\sin(e+fx)+a)^3}\right)}{a}-\frac{(A-B)\cos^5(e+fx)}{5f(a\sin(e+fx)+a)^5}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left(\frac{B\left(-\frac{\int \frac{\cos(e+fx)^2}{(\sin(e+fx)a+a)^2}dx}{a^2}-\frac{2\cos^3(e+fx)}{3af(a\sin(e+fx)+a)^3}\right)}{a}-\frac{(A-B)\cos^5(e+fx)}{5f(a\sin(e+fx)+a)^5}\right)$$

$$\Big\downarrow \text{3159}$$

$$a^2c^2\left(\frac{B\left(-\frac{-\frac{\int 1dx}{a^2}-\frac{2\cos(e+fx)}{f\left(a^2\sin(e+fx)+a^2\right)}}{a^2}-\frac{2\cos^3(e+fx)}{3af(a\sin(e+fx)+a)^3}\right)}{a}-\frac{(A-B)\cos^5(e+fx)}{5f(a\sin(e+fx)+a)^5}\right)$$

$$\Big\downarrow \text{24}$$

$$a^2c^2\left(\frac{B\left(-\frac{-\frac{2\cos(e+fx)}{f\left(a^2\sin(e+fx)+a^2\right)}-\frac{x}{a^2}}{a^2}-\frac{2\cos^3(e+fx)}{3af(a\sin(e+fx)+a)^3}\right)}{a}-\frac{(A-B)\cos^5(e+fx)}{5f(a\sin(e+fx)+a)^5}\right)$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])^3,x
]
```

output
```
a^2*c^2*(-1/5*((A - B)*Cos[e + f*x]^5)/(f*(a + a*Sin[e + f*x])^5) + (B*((-
2*Cos[e + f*x]^3)/(3*a*f*(a + a*Sin[e + f*x])^3) - (-(x/a^2) - (2*Cos[e +
f*x])/(f*(a^2 + a^2*Sin[e + f*x])))/a^2))/a)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{2c^2\left(B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{-32A+32B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{A-B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{16A-16B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{24A-16B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{}{\left(\tan\left(\right.\right.}\right.}{f\,a^3}$
default	$\frac{2c^2\left(B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{-32A+32B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{A-B}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{16A-16B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{24A-16B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{}{\left(\tan\left(\right.\right.}\right.}{f\,a^3}$
parallelrisch	$-\frac{2\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5 xfB}{2}+\left(-\frac{5}{2}fxB+A-B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+\left(-5fx-4\right)B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3+\left(-5fxB+2A-\frac{34}{3}B\right)\tan\left(\frac{f}{2}\right.\right.}{f\,a^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}$
risch	$\frac{B\,c^2x}{a^3}-\frac{2\left(-30A\,c^2e^{2i(fx+e)}+15A\,c^2e^{4i(fx+e)}+250B\,c^2e^{2i(fx+e)}-180iB\,c^2e^{3i(fx+e)}+140iB\,c^2e^{i(fx+e)}-75B\,c^2e\right.}{15f\,a^3\left(e^{i(fx+e)}+i\right)^5}$
norman	$\frac{\frac{B\,c^2x}{a}+\frac{8B\,c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{af}+\frac{48B\,c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{af}+\frac{B\,c^2x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}{a}-\frac{6A\,c^2-46B\,c^2}{15af}+\frac{40B\,c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3af}+\frac{64B\,c^2\tan}{a}}{}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x,method=_RETUR
NVERBOSE)
```

output
```
2/f*c^2/a^3*(B*arctan(tan(1/2*f*x+1/2*e))-1/4*(-32*A+32*B)/(tan(1/2*f*x+1/
2*e)+1)^4-(A-B)/(tan(1/2*f*x+1/2*e)+1)-1/5*(16*A-16*B)/(tan(1/2*f*x+1/2*e)
+1)^5-1/3*(24*A-16*B)/(tan(1/2*f*x+1/2*e)+1)^3+4*A/(tan(1/2*f*x+1/2*e)+1)^
2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. 2(106) = 212.

Time = 0.09 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.54

$$\int\frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx =$$
$$-\frac{60\,Bc^2fx-(15\,Bc^2fx-(3\,A-43\,B)c^2)\cos(fx+e)^3-12\,(A-B)c^2-(45\,Bc^2fx-(9\,A+11\,B)c}{15\left(a^3f\cos(fx+e)^3+\right.}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algori
thm="fricas")
```

output
```
-1/15*(60*B*c^2*f*x - (15*B*c^2*f*x - (3*A - 43*B)*c^2)*cos(f*x + e)^3 - 1
2*(A - B)*c^2 - (45*B*c^2*f*x - (9*A + 11*B)*c^2)*cos(f*x + e)^2 + 6*(5*B*
c^2*f*x - (A - 11*B)*c^2)*cos(f*x + e) + (60*B*c^2*f*x + 12*(A - B)*c^2 -
(15*B*c^2*f*x + (3*A - 43*B)*c^2)*cos(f*x + e)^2 + 6*(5*B*c^2*f*x + (A + 9
*B)*c^2)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f
*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3
*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1647 vs. $2(102) = 204$.

Time = 8.08 (sec) , antiderivative size = 1647, normalized size of antiderivative = 14.97

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^2}{(a + a\sin(e + fx))^3}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2/(a+a*sin(f*x+e))**3,x)
```

output
```
Piecewise((-30*A*c**2*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 +
 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3
*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*A*c*
*2*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2
+ f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)
**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 6*A*c**2/(15*a**3*f*tan(e/
2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2
)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a*
*3*f) + 15*B*c**2*f*x*tan(e/2 + f*x/2)**5/(15*a**3*f*tan(e/2 + f*x/2)**5 +
 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3
*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 75*B*c*
*2*f*x*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(
e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*
x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 150*B*c**2*f*x*tan(e/2
 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**
4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a
**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 150*B*c**2*f*x*tan(e/2 + f*x/2)**2/(
15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f
*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2
+ f*x/2) + 15*a**3*f) + 75*B*c**2*f*x*tan(e/2 + f*x/2)/(15*a**3*f*tan(e...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1134 vs. $2(106) = 212$.

Time = 0.14 (sec) , antiderivative size = 1134, normalized size of antiderivative = 10.31

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^2}{(a + a\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algori
thm="maxima")
```

output
```
2/15*(B*c^2*((95*sin(f*x + e)/(cos(f*x + e) + 1) + 145*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1
) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1)
)/a^3) - A*c^2*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1
) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5) - 2*A*c^2*(5*sin(f*x + e)/(cos(f*x + e) + 1
) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/
(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*s
in(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) +
1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 4*B*c^2*(5*sin(f*x + e)/
(cos(f*x + e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*
a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 6*A*...
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.37

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx = \frac{\frac{15\,(fx+e)Bc^2}{a^3} - \frac{2\left(15\,Ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4 - 15\,Bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4 - 60\,Bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 + 30\,Ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2 - 170\,Bc^2\tan\left(\frac{1}{2}\,fx+\right.\right.}{a^3\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^5}}{15\,f}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algori
thm="giac")
```

output
```
1/15*(15*(f*x + e)*B*c^2/a^3 - 2*(15*A*c^2*tan(1/2*f*x + 1/2*e)^4 - 15*B*c
^2*tan(1/2*f*x + 1/2*e)^4 - 60*B*c^2*tan(1/2*f*x + 1/2*e)^3 + 30*A*c^2*tan
(1/2*f*x + 1/2*e)^2 - 170*B*c^2*tan(1/2*f*x + 1/2*e)^2 - 100*B*c^2*tan(1/2
*f*x + 1/2*e) + 3*A*c^2 - 23*B*c^2)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

Mupad [B] (verification not implemented)

Time = 39.75 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.09

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx$$
$$= \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\left(\frac{c^2\,(120\,B+150\,B\,(e+f\,x))}{15}-10\,B\,c^2\,(e+f\,x)\right)+\frac{c^2\,(46\,B-6\,A+15\,B\,(e+f\,x))}{15}+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\left(\frac{c^2\,(30\,B}{}\right.}{}$$
$$+\frac{B\,c^2\,x}{a^3}$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^2)/(a + a*sin(e + f*x))^3,x
)
```

output
```
(tan(e/2 + (f*x)/2)^3*((c^2*(120*B + 150*B*(e + f*x)))/15 - 10*B*c^2*(e +
f*x)) + (c^2*(46*B - 6*A + 15*B*(e + f*x)))/15 + tan(e/2 + (f*x)/2)^4*((c^
2*(30*B - 30*A + 75*B*(e + f*x)))/15 - 5*B*c^2*(e + f*x)) + tan(e/2 + (f*x
)/2)^2*((c^2*(340*B - 60*A + 150*B*(e + f*x)))/15 - 10*B*c^2*(e + f*x)) +
tan(e/2 + (f*x)/2)*((c^2*(200*B + 75*B*(e + f*x)))/15 - 5*B*c^2*(e + f*x))
 - B*c^2*(e + f*x))/(a^3*f*(tan(e/2 + (f*x)/2) + 1)^5) + (B*c^2*x)/a^3
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.34

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx$$
$$= \frac{c^2\left(6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5a+15\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5bfx-6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5b+75\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4bfx+60\tan\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}{15a^3f\left(\tan\left(\frac{f}{2}\right.\right.}$$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x)
```

output
```
(c**2*(6*tan((e + f*x)/2)**5*a + 15*tan((e + f*x)/2)**5*b*f*x - 6*tan((e +
 f*x)/2)**5*b + 75*tan((e + f*x)/2)**4*b*f*x + 60*tan((e + f*x)/2)**3*a +
150*tan((e + f*x)/2)**3*b*f*x + 60*tan((e + f*x)/2)**3*b + 150*tan((e + f*
x)/2)**2*b*f*x + 280*tan((e + f*x)/2)**2*b + 30*tan((e + f*x)/2)*a + 75*ta
n((e + f*x)/2)*b*f*x + 170*tan((e + f*x)/2)*b + 15*b*f*x + 40*b))/(15*a**3
*f*(tan((e + f*x)/2)**5 + 5*tan((e + f*x)/2)**4 + 10*tan((e + f*x)/2)**3 +
 10*tan((e + f*x)/2)**2 + 5*tan((e + f*x)/2) + 1))
```

3.74 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 103

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = -\frac{2(A-B)c\cos(e+fx)}{5f(a+a\sin(e+fx))^3}+\frac{a(A-11B)c\cos(e+fx)}{15f\left(a^2+a^2\sin(e+fx)\right)^2}+\frac{(A+4B)c\cos(e+fx)}{15f\left(a^3+a^3\sin(e+fx)\right)}$$

output
```
-2/5*(A-B)*c*cos(f*x+e)/f/(a+a*sin(f*x+e))^3+1/15*a*(A-11*B)*c*cos(f*x+e)/
f/(a^2+a^2*sin(f*x+e))^2+1/15*(A+4*B)*c*cos(f*x+e)/f/(a^3+a^3*sin(f*x+e))
```

Mathematica [A] (verified)

Time = 6.92 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.35

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = \frac{c\left(-15(A+B)\cos\left(e+\frac{fx}{2}\right)+5(A+B)\cos\left(e+\frac{3fx}{2}\right)+5A\sin\left(\frac{fx}{2}\right)-25B\sin\left(\frac{fx}{2}\right)-15B\sin\left(2e+\frac{3fx}{2}\right.\right.}{30a^3f\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))/(a + a*Sin[e + f*x])
^3,x]
```

output
```
(c*(-15*(A + B)*Cos[e + (f*x)/2] + 5*(A + B)*Cos[e + (3*f*x)/2] + 5*A*Sin[
(f*x)/2] - 25*B*Sin[(f*x)/2] - 15*B*Sin[2*e + (3*f*x)/2] + A*Sin[2*e + (5*
f*x)/2] + 4*B*Sin[2*e + (5*f*x)/2]))/(30*a^3*f*(Cos[e/2] + Sin[e/2])*(Cos[
(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 3446, 3042, 3336, 3042, 3229, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-c\sin(e+fx))(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^3}dx$$

$$\Big\downarrow \text{3446}$$

$$ac\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(\sin(e+fx)a+a)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$ac\int \frac{\cos(e+fx)^2(A+B\sin(e+fx))}{(\sin(e+fx)a+a)^4}dx$$

$$\Big\downarrow \text{3336}$$

$$ac\left(-\frac{\int \frac{a(A-6B)+5aB\sin(e+fx)}{(\sin(e+fx)a+a)^2}dx}{5a^3}-\frac{2(A-B)\cos(e+fx)}{5af(a\sin(e+fx)+a)^3}\right)$$

$$\Big\downarrow \text{3042}$$

$$ac\left(-\frac{\int \frac{a(A-6B)+5aB\sin(e+fx)}{(\sin(e+fx)a+a)^2}dx}{5a^3}-\frac{2(A-B)\cos(e+fx)}{5af(a\sin(e+fx)+a)^3}\right)$$

$$\Big\downarrow \text{3229}$$

$$ac\left(-\frac{\frac{1}{3}(A+4B)\int \frac{1}{\sin(e+fx)a+a}dx-\frac{a(A-11B)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}}{5a^3}-\frac{2(A-B)\cos(e+fx)}{5af(a\sin(e+fx)+a)^3}\right)$$

$$\Big\downarrow \text{3042}$$

$$ac\left(-\frac{\frac{1}{3}(A+4B)\int \frac{1}{\sin(e+fx)a+a}dx-\frac{a(A-11B)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}}{5a^3}-\frac{2(A-B)\cos(e+fx)}{5af(a\sin(e+fx)+a)^3}\right)$$

$$\Big\downarrow \text{3127}$$

$$ac\left(-\frac{-\frac{(A+4B)\cos(e+fx)}{3f(a\sin(e+fx)+a)}-\frac{a(A-11B)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}}{5a^3}-\frac{2(A-B)\cos(e+fx)}{5af(a\sin(e+fx)+a)^3}\right)$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))/(a + a*Sin[e + f*x])^3,x]
```

output
```
a*c*((-2*(A - B)*Cos[e + f*x])/(5*a*f*(a + a*Sin[e + f*x])^3) - (-1/3*(a*(
A - 11*B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^2) - ((A + 4*B)*Cos[e + f*
x])/(3*f*(a + a*Sin[e + f*x])))/(5*a^3))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 3336
```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Simp[1/(b^
3*(2*m + 3))   Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*
(2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2
- b^2, 0] && LtQ[m, -3/2]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.89

method	result
parallelrisch	$-\frac{2\left(A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}+(A+B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+\frac{(5A-B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{3}+\frac{(A+B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{3}+\frac{4A}{15}+\frac{B}{15}\right)c}{f\,a^{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}$
derivativedivides	$\frac{2c\left(-\frac{8A-8B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{14A-10B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{-16A+16B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{4}}-\frac{-6A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)}{f\,a^{3}}$
default	$\frac{2c\left(-\frac{8A-8B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{14A-10B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{-16A+16B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{4}}-\frac{-6A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)}{f\,a^{3}}$
risch	$\frac{\frac{2Ac\,e^{2i(fx+e)}}{3}-\frac{2iAc\,e^{i(fx+e)}}{3}-\frac{10Bc\,e^{2i(fx+e)}}{3}+2iBc\,e^{3i(fx+e)}-\frac{2iBc\,e^{i(fx+e)}}{3}+2Bc\,e^{4i(fx+e)}+\frac{2Ac}{15}+\frac{8Bc}{15}+2iAc\,e^{3i(f}}{f\,a^{3}\left(e^{i(fx+e)}+i\right)^{5}}$
norman	$\frac{-\frac{8Ac+2Bc}{15af}-\frac{2Ac\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}}{af}-\frac{2(23Ac-3Bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{5af}-\frac{2(7Ac+7Bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{3af}-\frac{2(11Ac-Bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{5af}-\frac{2(11}{}}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\right)^{2}a^{2}\left(\tan\left(\frac{f}{2}\right.\right.}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^3,x,method=_RETURNV
ERBOSE)
```

output
```
-2*(A*tan(1/2*f*x+1/2*e)^4+(A+B)*tan(1/2*f*x+1/2*e)^3+1/3*(5*A-B)*tan(1/2*
f*x+1/2*e)^2+1/3*(A+B)*tan(1/2*f*x+1/2*e)+4/15*A+1/15*B)*c/f/a^3/(tan(1/2*
f*x+1/2*e)+1)^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.85

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = \frac{(A+4B)c\cos(fx+e)^3-(2A-7B)c\cos(fx+e)^2+3(A-B)c\cos(fx+e)+6(A-B)c-((A+}{15\left(a^3f\cos(fx+e)^3+3a^3f\cos(fx+e)^2-2a^3f\cos(fx+e)-4a^3f+\left(a^3f\cos\right.\right.}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorith
m="fricas")
```

output
```
1/15*((A + 4*B)*c*cos(f*x + e)^3 - (2*A - 7*B)*c*cos(f*x + e)^2 + 3*(A - B
)*c*cos(f*x + e) + 6*(A - B)*c - ((A + 4*B)*c*cos(f*x + e)^2 + 3*(A - B)*c
*cos(f*x + e) + 6*(A - B)*c)*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f
*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 -
 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1035 vs. 2(97) = 194.

Time = 4.85 (sec) , antiderivative size = 1035, normalized size of antiderivative = 10.05

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))**3,x)
```

output
```
Piecewise((-30*A*c*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75
*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*
tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*A*c*tan
(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/
2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 +
75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 50*A*c*tan(e/2 + f*x/2)**2/(15*a
**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan
(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*
x/2) + 15*a**3*f) - 10*A*c*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5
 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a*
*3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 8*A*c
/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3
*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/
2 + f*x/2) + 15*a**3*f) - 30*B*c*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 +
f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3
 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f
) + 10*B*c*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*
tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2
+ f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 10*B*c*tan(e/2 + f
*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + ...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 733 vs. $2(97) = 194$.

Time = 0.05 (sec) , antiderivative size = 733, normalized size of antiderivative = 7.12

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))}{(a + a\sin(e + fx))^3} \, dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorith
m="maxima")
```

output
```
-2/15*(A*c*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) +
10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x +
e)^5/(cos(f*x + e) + 1)^5) - 2*B*c*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f
*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x
 + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 +
 a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 3*A*c*(5*sin(f*x + e)/(cos(f*x
 + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos
(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^
3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5) + 3*B*c*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1
)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
))/f
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.26

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = -\frac{2\left(15\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+15\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+15\,Bc\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+25\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\right.\right.}{15\,a^3f\left(\tan\left(\frac{1}{2}\,fx+\right.\right.}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorith
m="giac")
```

output
```
-2/15*(15*A*c*tan(1/2*f*x + 1/2*e)^4 + 15*A*c*tan(1/2*f*x + 1/2*e)^3 + 15*
B*c*tan(1/2*f*x + 1/2*e)^3 + 25*A*c*tan(1/2*f*x + 1/2*e)^2 - 5*B*c*tan(1/2
*f*x + 1/2*e)^2 + 5*A*c*tan(1/2*f*x + 1/2*e) + 5*B*c*tan(1/2*f*x + 1/2*e)
+ 4*A*c + B*c)/(a^3*f*(tan(1/2*f*x + 1/2*e) + 1)^5)
```

Mupad [B] (verification not implemented)

Time = 37.29 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.67

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx$$
$$= \frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{41\,A\,c}{4}-\frac{B\,c}{4}-\frac{11\,A\,c\,\cos(e+f\,x)}{2}+\frac{B\,c\,\cos(e+f\,x)}{2}+5\,A\,c\,\sin\left(e+f\,x\right)+5\,B\,c\,\sin\left(e+f\,x\right)-\right.}{15\,a^3\,f\,\left(\frac{5\,\sqrt{2}\,\cos\left(\frac{3\,e}{2}+\frac{\pi}{4}+\frac{3\,f\,x}{2}\right)}{4}-\frac{5\,\sqrt{2}\,\cos\left(\frac{e}{2}-\frac{\pi}{4}+\frac{f\,x}{2}\right)}{2}+\right.}$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x)))/(a + a*sin(e + f*x))^3,x)
```

output
```
(2*cos(e/2 + (f*x)/2)*((41*A*c)/4 - (B*c)/4 - (11*A*c*cos(e + f*x))/2 + (B
*c*cos(e + f*x))/2 + 5*A*c*sin(e + f*x) + 5*B*c*sin(e + f*x) - (3*A*c*cos(
2*e + 2*f*x))/4 + (3*B*c*cos(2*e + 2*f*x))/4 - (5*A*c*sin(2*e + 2*f*x))/4
- (5*B*c*sin(2*e + 2*f*x))/4))/(15*a^3*f*((5*2^(1/2)*cos((3*e)/2 + pi/4 +
(3*f*x)/2))/4 - (5*2^(1/2)*cos(e/2 - pi/4 + (f*x)/2))/2 + (2^(1/2)*cos((5*
e)/2 - pi/4 + (5*f*x)/2))/4))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.70

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx$$
$$= \frac{2c\left(3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5a+15\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3a-15\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3b+5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2a+5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2b+\right.}{15a^3f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5+5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+10\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3+10\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right.}$$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^3,x)
```

output
```
(2*c*(3*tan((e + f*x)/2)**5*a + 15*tan((e + f*x)/2)**3*a - 15*tan((e + f*x
)/2)**3*b + 5*tan((e + f*x)/2)**2*a + 5*tan((e + f*x)/2)**2*b + 10*tan((e
+ f*x)/2)*a - 5*tan((e + f*x)/2)*b - a - b))/(15*a**3*f*(tan((e + f*x)/2)*
*5 + 5*tan((e + f*x)/2)**4 + 10*tan((e + f*x)/2)**3 + 10*tan((e + f*x)/2)*
*2 + 5*tan((e + f*x)/2) + 1))
```

3.75 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 102

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))}\,dx = -\frac{(A-B)\sec(e+fx)}{5acf(a+a\sin(e+fx))^2} - \frac{(3A+2B)\sec(e+fx)}{15cf\,(a^3+a^3\sin(e+fx))} + \frac{2(3A+2B)\tan(e+fx)}{15a^3cf}$$

output
```
-1/5*(A-B)*sec(f*x+e)/a/c/f/(a+a*sin(f*x+e))^2-1/15*(3*A+2*B)*sec(f*x+e)/c
/f/(a^3+a^3*sin(f*x+e))+2/15*(3*A+2*B)*tan(f*x+e)/a^3/c/f
```

Mathematica [A] (verified)

Time = 3.93 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.53

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))}\,dx = \frac{\cos(e+fx)(-80B-5(9A+B)\cos(e+fx)+32(3A+2B)\cos(2(e+fx))+9A\cos(3(e+fx))+B\,\mathrm{c}}{240a}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x]
)),x]
```

output
```
(Cos[e + f*x]*(-80*B - 5*(9*A + B)*Cos[e + f*x] + 32*(3*A + 2*B)*Cos[2*(e
+ f*x)] + 9*A*Cos[3*(e + f*x)] + B*Cos[3*(e + f*x)] - 120*A*Sin[e + f*x] -
 80*B*Sin[e + f*x] - 36*A*Sin[2*(e + f*x)] - 4*B*Sin[2*(e + f*x)] + 24*A*S
in[3*(e + f*x)] + 16*B*Sin[3*(e + f*x)]))/(240*a^3*c*f*(-1 + Sin[e + f*x])
*(1 + Sin[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.92, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3446, 3042, 3338, 3042, 3151, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c-c\sin(e+fx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c-c\sin(e+fx))}dx$$

$$\Big\downarrow \text{3446}$$

$$\frac{\int \frac{\sec^2(e+fx)(A+B\sin(e+fx))}{(\sin(e+fx)a+a)^2}dx}{ac}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{A+B\sin(e+fx)}{\cos(e+fx)^2(\sin(e+fx)a+a)^2}dx}{ac}$$

$$\Big\downarrow \text{3338}$$

$$\frac{\frac{(3A+2B)\int \frac{\sec^2(e+fx)}{\sin(e+fx)a+a}dx}{5a}-\frac{(A-B)\sec(e+fx)}{5f(a\sin(e+fx)+a)^2}}{ac}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(3A+2B)\int \frac{1}{\cos(e+fx)^2(\sin(e+fx)a+a)}dx}{5a}-\frac{(A-B)\sec(e+fx)}{5f(a\sin(e+fx)+a)^2}}{ac}$$

$$\Big\downarrow \text{3151}$$

$$\frac{\frac{(3A+2B)\left(\frac{2\int \sec^2(e+fx)dx}{3a}-\frac{\sec(e+fx)}{3f(a\sin(e+fx)+a)}\right)}{5a}-\frac{(A-B)\sec(e+fx)}{5f(a\sin(e+fx)+a)^2}}{ac}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{(3A+2B)\left(\frac{2\int \csc\left(e+fx+\frac{\pi}{2}\right)^2dx}{3a}-\frac{\sec(e+fx)}{3f(a\sin(e+fx)+a)}\right)}{5a}-\frac{(A-B)\sec(e+fx)}{5f(a\sin(e+fx)+a)^2}}{ac}$$

$$\Big\downarrow \; 4254$$

$$\frac{\frac{(3A+2B)\left(-\frac{2\int 1d(-\tan(e+fx))}{3af}-\frac{\sec(e+fx)}{3f(a\sin(e+fx)+a)}\right)}{5a}-\frac{(A-B)\sec(e+fx)}{5f(a\sin(e+fx)+a)^2}}{ac}$$

$$\Big\downarrow \; 24$$

$$\frac{\frac{(3A+2B)\left(\frac{2\tan(e+fx)}{3af}-\frac{\sec(e+fx)}{3f(a\sin(e+fx)+a)}\right)}{5a}-\frac{(A-B)\sec(e+fx)}{5f(a\sin(e+fx)+a)^2}}{ac}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])),x]
```

output
```
(-1/5*((A - B)*Sec[e + f*x])/(f*(a + a*Sin[e + f*x])^2) + ((3*A + 2*B)*(-1
/3*Sec[e + f*x]/(f*(a + a*Sin[e + f*x])) + (2*Tan[e + f*x])/(3*a*f)))/(5*a
))/(a*c)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3151
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Simp[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1])   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simpli
fy[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.09

method	result
risch	$-\frac{4\left(15iA\,\mathrm{e}^{2i(fx+e)}-12A\,\mathrm{e}^{i(fx+e)}+10B\,\mathrm{e}^{3i(fx+e)}-8B\,\mathrm{e}^{i(fx+e)}-2iB+10iB\,\mathrm{e}^{2i(fx+e)}-3iA\right)}{15\left(\mathrm{e}^{i(fx+e)}+i\right)^{5}\left(\mathrm{e}^{i(fx+e)}-i\right)f\,a^{3}c}$
parallelrisch	$\frac{-30A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}+(-60A-30B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}+(-60A-40B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}-40B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}+(18A-8B)\tan}{15f\,a^{3}c\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}$
derivativedivides	$\frac{-\frac{2\left(\frac{A}{8}+\frac{B}{8}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{-4A+4B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{4}}-\frac{2(2A-2B)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{-\frac{5A}{2}+\frac{3B}{2}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2\left(\frac{7A}{8}-\frac{B}{8}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2\left(\frac{9A}{2}-\frac{7B}{2}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}}{f\,a^{3}c}$
default	$\frac{-\frac{2\left(\frac{A}{8}+\frac{B}{8}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{-4A+4B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{4}}-\frac{2(2A-2B)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{-\frac{5A}{2}+\frac{3B}{2}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2\left(\frac{7A}{8}-\frac{B}{8}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2\left(\frac{9A}{2}-\frac{7B}{2}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}}{f\,a^{3}c}$
norman	$\frac{\frac{12A-2B}{15cfa}-\frac{2(6A+7B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{3cfa}-\frac{2A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}{afc}+\frac{2(9A-4B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{15cfa}+\frac{2(2A-7B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{5cfa}-\frac{2(9A+4B)\tan(}{3cfa}}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\right)a^{2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x,method=_RETURNV
ERBOSE)
```

output
```
-4/15*(15*I*A*exp(2*I*(f*x+e))-12*A*exp(I*(f*x+e))+10*B*exp(3*I*(f*x+e))-8
*B*exp(I*(f*x+e))-2*I*B+10*I*B*exp(2*I*(f*x+e))-3*I*A)/(exp(I*(f*x+e))+I)^
5/(exp(I*(f*x+e))-I)/f/a^3/c
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.04

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))}\,dx$$
$$=\frac{4\,(3\,A+2\,B)\cos\left(fx+e\right)^{2}+\left(2\,(3\,A+2\,B)\cos\left(fx+e\right)^{2}-9\,A-6\,B\right)\sin\left(fx+e\right)-6\,A-9\,B}{15\,\left(a^{3}cf\cos\left(fx+e\right)^{3}-2\,a^{3}cf\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^{3}cf\cos\left(fx+e\right)\right)}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
1/15*(4*(3*A + 2*B)*cos(f*x + e)^2 + (2*(3*A + 2*B)*cos(f*x + e)^2 - 9*A -
 6*B)*sin(f*x + e) - 6*A - 9*B)/(a^3*c*f*cos(f*x + e)^3 - 2*a^3*c*f*cos(f*
x + e)*sin(f*x + e) - 2*a^3*c*f*cos(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1236 vs. $2(85) = 170$.

Time = 4.95 (sec) , antiderivative size = 1236, normalized size of antiderivative = 12.12

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c - c\sin(e + fx))}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e)),x)
```

output
```
Piecewise((-30*A*tan(e/2 + f*x/2)**5/(15*a**3*c*f*tan(e/2 + f*x/2)**6 + 60
*a**3*c*f*tan(e/2 + f*x/2)**5 + 75*a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a**3*
c*f*tan(e/2 + f*x/2)**2 - 60*a**3*c*f*tan(e/2 + f*x/2) - 15*a**3*c*f) - 60
*A*tan(e/2 + f*x/2)**4/(15*a**3*c*f*tan(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(
e/2 + f*x/2)**5 + 75*a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a**3*c*f*tan(e/2 +
f*x/2)**2 - 60*a**3*c*f*tan(e/2 + f*x/2) - 15*a**3*c*f) - 60*A*tan(e/2 + f
*x/2)**3/(15*a**3*c*f*tan(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/2 + f*x/2)**
5 + 75*a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x/2)**2 - 60
*a**3*c*f*tan(e/2 + f*x/2) - 15*a**3*c*f) + 18*A*tan(e/2 + f*x/2)/(15*a**3
*c*f*tan(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/2 + f*x/2)**5 + 75*a**3*c*f*t
an(e/2 + f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x/2)**2 - 60*a**3*c*f*tan(e/2
 + f*x/2) - 15*a**3*c*f) + 12*A/(15*a**3*c*f*tan(e/2 + f*x/2)**6 + 60*a**3
*c*f*tan(e/2 + f*x/2)**5 + 75*a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a**3*c*f*t
an(e/2 + f*x/2)**2 - 60*a**3*c*f*tan(e/2 + f*x/2) - 15*a**3*c*f) - 30*B*ta
n(e/2 + f*x/2)**4/(15*a**3*c*f*tan(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/2 +
 f*x/2)**5 + 75*a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x/2
)**2 - 60*a**3*c*f*tan(e/2 + f*x/2) - 15*a**3*c*f) - 40*B*tan(e/2 + f*x/2)
**3/(15*a**3*c*f*tan(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/2 + f*x/2)**5 + 7
5*a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x/2)**2 - 60*a**3
*c*f*tan(e/2 + f*x/2) - 15*a**3*c*f) - 40*B*tan(e/2 + f*x/2)**2/(15*a**...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(96) = 192$.

Time = 0.05 (sec) , antiderivative size = 423, normalized size of antiderivative = 4.15

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))}\,dx = \frac{2\left(\frac{B\left(\frac{4\sin(fx+e)}{\cos(fx+e)+1}+\frac{20\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{20\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+1\right)}{a^3c+\frac{4a^3c\sin(fx+e)}{\cos(fx+e)+1}+\frac{5a^3c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{5a^3c\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{4a^3c\sin(fx+e)^5}{(\cos(fx+e)+1)^5}-\frac{a^3c\sin(fx+e)^6}{(\cos(fx+e)+1)^6}} - \frac{3A\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\cdots\right.}{a^3c+\frac{4a^3c\sin(fx+e)}{\cos(fx+e)+1}+\frac{5a^3c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{5a^3\cdots}{(\cos\cdots}}\right.}{15f}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x, algorith
m="maxima")
```

output
```
2/15*(B*(4*sin(f*x + e)/(cos(f*x + e) + 1) + 20*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 1)/(a^3*c + 4*a^3*c*sin(f*x + e)/(cos(f*x + e) + 1) +
5*a^3*c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 5*a^3*c*sin(f*x + e)^4/(cos(
f*x + e) + 1)^4 - 4*a^3*c*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - a^3*c*sin(
f*x + e)^6/(cos(f*x + e) + 1)^6) - 3*A*(3*sin(f*x + e)/(cos(f*x + e) + 1)
- 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 10*sin(f*x + e)^4/(cos(f*x + e)
 + 1)^4 - 5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 2)/(a^3*c + 4*a^3*c*sin(
f*x + e)/(cos(f*x + e) + 1) + 5*a^3*c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
- 5*a^3*c*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 4*a^3*c*sin(f*x + e)^5/(co
s(f*x + e) + 1)^5 - a^3*c*sin(f*x + e)^6/(cos(f*x + e) + 1)^6))/f
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.62

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))}\,dx =$$

$$-\frac{\frac{15\,(A+B)}{a^3c\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)}+\frac{105\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-15\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+270\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+30\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+360\,A\tan\left(\frac{1}{2}\,fx\right.}{a^3c\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}}{60\,f}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x, algorith
m="giac")
```

output
```
-1/60*(15*(A + B)/(a^3*c*(tan(1/2*f*x + 1/2*e) - 1)) + (105*A*tan(1/2*f*x
+ 1/2*e)^4 - 15*B*tan(1/2*f*x + 1/2*e)^4 + 270*A*tan(1/2*f*x + 1/2*e)^3 +
30*B*tan(1/2*f*x + 1/2*e)^3 + 360*A*tan(1/2*f*x + 1/2*e)^2 + 40*B*tan(1/2*
f*x + 1/2*e)^2 + 210*A*tan(1/2*f*x + 1/2*e) + 50*B*tan(1/2*f*x + 1/2*e) +
63*A + 7*B)/(a^3*c*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

Mupad [B] (verification not implemented)

Time = 36.65 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.75

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))}\,dx = -\frac{2\left(\frac{15\,A\,\cos(e+f\,x)}{4}-\frac{5\,B}{2}-\frac{5\,B\,\cos(e+f\,x)}{8}-\frac{15\,A\,\sin(e+f\,x)}{4}-\frac{5\,B\,\sin(e+f\,x)}{2}+3\,A\,\cos\left(2\,e+2\,f\,x\right)-\frac{3\,A\,\cos(3\,e+3}{4}\right.}{15\,a^3\,c\,f\,\left(\frac{5\,\cos(e+f\,x)}{4}\right.}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))),x)
```

output
```
-(2*((15*A*cos(e + f*x))/4 - (5*B)/2 - (5*B*cos(e + f*x))/8 - (15*A*sin(e
+ f*x))/4 - (5*B*sin(e + f*x))/2 + 3*A*cos(2*e + 2*f*x) - (3*A*cos(3*e + 3
*f*x))/4 + 2*B*cos(2*e + 2*f*x) + (B*cos(3*e + 3*f*x))/8 + 3*A*sin(2*e + 2
*f*x) + (3*A*sin(3*e + 3*f*x))/4 - (B*sin(2*e + 2*f*x))/2 + (B*sin(3*e + 3
*f*x))/2))/(15*a^3*c*f*((5*cos(e + f*x))/4 - cos(3*e + 3*f*x)/4 + sin(2*e
+ 2*f*x)))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.86

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))}\,dx = \frac{3\cos\left(fx+e\right)\sin\left(fx+e\right)^2a+2\cos\left(fx+e\right)\sin\left(fx+e\right)^2b+6\cos\left(fx+e\right)\sin\left(fx+e\right)a+4\cos\left(fx-\right.}{}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x)
```

output
```
(3*cos(e + f*x)*sin(e + f*x)**2*a + 2*cos(e + f*x)*sin(e + f*x)**2*b + 6*c
os(e + f*x)*sin(e + f*x)*a + 4*cos(e + f*x)*sin(e + f*x)*b + 3*cos(e + f*x
)*a + 2*cos(e + f*x)*b + 12*sin(e + f*x)**3*a + 8*sin(e + f*x)**3*b + 24*s
in(e + f*x)**2*a + 16*sin(e + f*x)**2*b + 6*sin(e + f*x)*a + 4*sin(e + f*x
)*b - 12*a + 2*b)/(30*cos(e + f*x)*a**3*c*f*(sin(e + f*x)**2 + 2*sin(e + f
*x) + 1))
```

3.76 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 90

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^2}\,dx = -\frac{(A-B)\sec^3(e+fx)}{5c^2f\,(a^3+a^3\sin(e+fx))}+\frac{(4A+B)\tan(e+fx)}{5a^3c^2f}+\frac{(4A+B)\tan^3(e+fx)}{15a^3c^2f}$$

output
```
-1/5*(A-B)*sec(f*x+e)^3/c^2/f/(a^3+a^3*sin(f*x+e))+1/5*(4*A+B)*tan(f*x+e)/
a^3/c^2/f+1/15*(4*A+B)*tan(f*x+e)^3/a^3/c^2/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 237 vs. 2(90) = 180.

Time = 4.79 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.63

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^2}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(240B+54(A-B)\cos(e+f}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x]
)^2),x]
```

output
```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])*(240*B + 54*(A - B)*Cos[e + f*x] - 32*(4*A + B)*Cos[2*(e + f*x)] + 18*A
*Cos[3*(e + f*x)] - 18*B*Cos[3*(e + f*x)] - 64*A*Cos[4*(e + f*x)] - 16*B*C
os[4*(e + f*x)] + 384*A*Sin[e + f*x] + 96*B*Sin[e + f*x] + 18*A*Sin[2*(e +
 f*x)] - 18*B*Sin[2*(e + f*x)] + 128*A*Sin[3*(e + f*x)] + 32*B*Sin[3*(e +
f*x)] + 9*A*Sin[4*(e + f*x)] - 9*B*Sin[4*(e + f*x)]))/(960*a^3*c^2*f*(-1 +
 Sin[e + f*x])^2*(1 + Sin[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3042, 3446, 3042, 3338, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c-c\sin(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c-c\sin(e+fx))^2}dx$$

$$\Big\downarrow \text{3446}$$

$$\frac{\int \frac{\sec^4(e+fx)(A+B\sin(e+fx))}{\sin(e+fx)a+a}dx}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{A+B\sin(e+fx)}{\cos(e+fx)^4(\sin(e+fx)a+a)}dx}{a^2c^2}$$

$$\Big\downarrow \text{3338}$$

$$\frac{\frac{(4A+B)\int \sec^4(e+fx)dx}{5a}-\frac{(A-B)\sec^3(e+fx)}{5f(a\sin(e+fx)+a)}}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(4A+B)\int \csc\left(e+fx+\frac{\pi}{2}\right)^4dx}{5a}-\frac{(A-B)\sec^3(e+fx)}{5f(a\sin(e+fx)+a)}}{a^2c^2}$$

$$\Big\downarrow \text{4254}$$

$$\frac{-\frac{(4A+B)\int (\tan^2(e+fx)+1)d(-\tan(e+fx))}{5af}-\frac{(A-B)\sec^3(e+fx)}{5f(a\sin(e+fx)+a)}}{a^2c^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{(4A+B)\left(-\frac{1}{3}\tan^3(e+fx)-\tan(e+fx)\right)}{5af}-\frac{(A-B)\sec^3(e+fx)}{5f(a\sin(e+fx)+a)}}{a^2c^2}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^2),x
]
```

output
```
(-1/5*((A - B)*Sec[e + f*x]^3)/(f*(a + a*Sin[e + f*x])) - ((4*A + B)*(-Tan
[e + f*x] - Tan[e + f*x]^3/3))/(5*a*f))/(a^2*c^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.51

method	result
risch	$\frac{4i\left(24iA\,e^{3i(fx+e)}+6iB\,e^{3i(fx+e)}+15B\,e^{4i(fx+e)}+8iA\,e^{i(fx+e)}-8A\,e^{2i(fx+e)}+2iB\,e^{i(fx+e)}-2B\,e^{2i(fx+e)}-4A-B\right)}{15\left(e^{i(fx+e)}+i\right)^{5}\left(e^{i(fx+e)}-i\right)^{3}f\,a^{3}c^{2}}$
parallelrisch	$\frac{-30A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}+(-30A-30B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}+(10A-20B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}+(50A-10B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}+(-26A+}{15f\,a^{3}c^{2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}\left(\tan\left(\frac{fx}{2}+\right.\right.}$
derivativedivides	$\frac{-\frac{-2A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{4}}-\frac{2(A-B)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{-\frac{3A}{2}+B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2\left(\frac{5A}{2}-2B\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2\left(\frac{11A}{16}-\frac{3B}{16}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2\left(\frac{A}{4}+\frac{B}{4}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\right.}}{f\,a^{3}c^{2}}$
default	$\frac{-\frac{-2A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{4}}-\frac{2(A-B)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{-\frac{3A}{2}+B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2\left(\frac{5A}{2}-2B\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2\left(\frac{11A}{16}-\frac{3B}{16}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2\left(\frac{A}{4}+\frac{B}{4}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\right.}}{f\,a^{3}c^{2}}$
norman	$\frac{-\frac{6A+4B}{10cfa}-\frac{4(4A+B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{15cfa}+\frac{A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{afc}-\frac{(14A+16B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{10cfa}-\frac{(6A+4B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}}{2cfa}-\frac{(2A+8B)\tan\left(\right.}{3cfa}}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\right)a^{2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x,method=_RETUR
NVERBOSE)
```

output
```
4/15*I*(24*I*A*exp(3*I*(f*x+e))+6*I*B*exp(3*I*(f*x+e))+15*B*exp(4*I*(f*x+e
))+8*I*A*exp(I*(f*x+e))-8*A*exp(2*I*(f*x+e))+2*I*B*exp(I*(f*x+e))-2*B*exp(
2*I*(f*x+e))-4*A-B)/(exp(I*(f*x+e))+I)^5/(exp(I*(f*x+e))-I)^3/f/a^3/c^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.20

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^2}\,dx = -\frac{2\,(4\,A+B)\cos\left(fx+e\right)^4-(4\,A+B)\cos\left(fx+e\right)^2-\left(2\,(4\,A+B)\cos\left(fx+e\right)^2+4\,A+B\right)\sin\left(fx+\right.}{15\,\left(a^3c^2f\cos\left(fx+e\right)^3\sin\left(fx+e\right)+a^3c^2f\cos\left(fx+e\right)^3\right)}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
-1/15*(2*(4*A + B)*cos(f*x + e)^4 - (4*A + B)*cos(f*x + e)^2 - (2*(4*A + B
)*cos(f*x + e)^2 + 4*A + B)*sin(f*x + e) - A - 4*B)/(a^3*c^2*f*cos(f*x + e
)^3*sin(f*x + e) + a^3*c^2*f*cos(f*x + e)^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2674 vs. 2(82) = 164.

Time = 9.82 (sec) , antiderivative size = 2674, normalized size of antiderivative = 29.71

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**2,x)
```

output
```
Piecewise((-30*A*tan(e/2 + f*x/2)**7/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 +
 30*a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 -
 90*a**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 +
 30*a**3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15
*a**3*c**2*f) - 30*A*tan(e/2 + f*x/2)**6/(15*a**3*c**2*f*tan(e/2 + f*x/2)*
*8 + 30*a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)*
*6 - 90*a**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)*
*3 + 30*a**3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2)
- 15*a**3*c**2*f) + 10*A*tan(e/2 + f*x/2)**5/(15*a**3*c**2*f*tan(e/2 + f*x
/2)**8 + 30*a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x
/2)**6 - 90*a**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x
/2)**3 + 30*a**3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x
/2) - 15*a**3*c**2*f) + 50*A*tan(e/2 + f*x/2)**4/(15*a**3*c**2*f*tan(e/2 +
 f*x/2)**8 + 30*a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 +
 f*x/2)**6 - 90*a**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 +
 f*x/2)**3 + 30*a**3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 +
 f*x/2) - 15*a**3*c**2*f) - 26*A*tan(e/2 + f*x/2)**3/(15*a**3*c**2*f*tan(e
/2 + f*x/2)**8 + 30*a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e
/2 + f*x/2)**6 - 90*a**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e
/2 + f*x/2)**3 + 30*a**3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*ta...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 650 vs. $2(84) = 168$.

Time = 0.06 (sec) , antiderivative size = 650, normalized size of antiderivative = 7.22

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c - c\sin(e + fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x, algori
thm="maxima")
```

output
```
2/15*(A*(9*sin(f*x + e)/(cos(f*x + e) + 1) + 21*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 13*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 25*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 - 5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 15*sin(f*x + e
)^6/(cos(f*x + e) + 1)^6 + 15*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3)/(a^
3*c^2 + 2*a^3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) - 2*a^3*c^2*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 - 6*a^3*c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
6*a^3*c^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 2*a^3*c^2*sin(f*x + e)^6/(
cos(f*x + e) + 1)^6 - 2*a^3*c^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - a^3*
c^2*sin(f*x + e)^8/(cos(f*x + e) + 1)^8) + B*(6*sin(f*x + e)/(cos(f*x + e)
 + 1) + 9*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 8*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 10*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5 + 15*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 3)/(a^3*c^2
+ 2*a^3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) - 2*a^3*c^2*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 - 6*a^3*c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 6*a^3*
c^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 2*a^3*c^2*sin(f*x + e)^6/(cos(f*
x + e) + 1)^6 - 2*a^3*c^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - a^3*c^2*si
n(f*x + e)^8/(cos(f*x + e) + 1)^8))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(84) = 168$.

Time = 0.27 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.46

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c - c\sin(e + fx))^2}\,dx =$$

$$-\frac{\frac{5\left(15\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+9\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-24\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-12\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+13\,A+7\,B\right)}{a^3c^2\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right)^3}+\frac{165\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-45\,B\,\mathrm{t}}{}}{}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
-1/120*(5*(15*A*tan(1/2*f*x + 1/2*e)^2 + 9*B*tan(1/2*f*x + 1/2*e)^2 - 24*A
*tan(1/2*f*x + 1/2*e) - 12*B*tan(1/2*f*x + 1/2*e) + 13*A + 7*B)/(a^3*c^2*(
tan(1/2*f*x + 1/2*e) - 1)^3) + (165*A*tan(1/2*f*x + 1/2*e)^4 - 45*B*tan(1/
2*f*x + 1/2*e)^4 + 480*A*tan(1/2*f*x + 1/2*e)^3 - 60*B*tan(1/2*f*x + 1/2*e
)^3 + 650*A*tan(1/2*f*x + 1/2*e)^2 - 70*B*tan(1/2*f*x + 1/2*e)^2 + 400*A*t
an(1/2*f*x + 1/2*e) - 20*B*tan(1/2*f*x + 1/2*e) + 113*A - 13*B)/(a^3*c^2*(
tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

Mupad [B] (verification not implemented)

Time = 36.71 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.03

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^2}\,dx = \frac{\left(\frac{8A}{15}+\frac{2B}{15}+\frac{16A\sin(e+fx)}{15}+\frac{4B\sin(e+fx)}{15}\right)\cos(e+fx)^2+\frac{2A}{15}+\frac{8B}{15}+\frac{8A\sin(e+fx)}{15}+\frac{2B\sin(e+fx)}{15}}{a^3c^2f\left(2\cos(e+fx)^3\sin(e+fx)+2\cos(e+fx)^3\right)} - \frac{\frac{2A}{5}-\frac{2B}{5}+\frac{2A\sin(e+fx)}{5}-\frac{2B\sin(e+fx)}{5}}{a^3c^2f\,(2\sin(e+fx)+2)} - \frac{\cos(e+fx)\left(\frac{16A}{15}+\frac{4B}{15}\right)}{a^3c^2f\,(2\sin(e+fx)+2)}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^2),x
)
```

output
```
((2*A)/15 + (8*B)/15 + (8*A*sin(e + f*x))/15 + (2*B*sin(e + f*x))/15 + cos
(e + f*x)^2*((8*A)/15 + (2*B)/15 + (16*A*sin(e + f*x))/15 + (4*B*sin(e + f
*x))/15))/(a^3*c^2*f*(2*cos(e + f*x)^3*sin(e + f*x) + 2*cos(e + f*x)^3)) -
 ((2*A)/5 - (2*B)/5 + (2*A*sin(e + f*x))/5 - (2*B*sin(e + f*x))/5)/(a^3*c^
2*f*(2*sin(e + f*x) + 2)) - (cos(e + f*x)*((16*A)/15 + (4*B)/15))/(a^3*c^2
*f*(2*sin(e + f*x) + 2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.82

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c - c\sin(e + fx))^2}\,dx$$
$$= \frac{12\cos(fx+e)\sin(fx+e)^3 a + 3\cos(fx+e)\sin(fx+e)^3 b + 12\cos(fx+e)\sin(fx+e)^2 a + 3\cos(j}{}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x)
```

output
```
(12*cos(e + f*x)*sin(e + f*x)**3*a + 3*cos(e + f*x)*sin(e + f*x)**3*b + 12
*cos(e + f*x)*sin(e + f*x)**2*a + 3*cos(e + f*x)*sin(e + f*x)**2*b - 12*co
s(e + f*x)*sin(e + f*x)*a - 3*cos(e + f*x)*sin(e + f*x)*b - 12*cos(e + f*x
)*a - 3*cos(e + f*x)*b + 8*sin(e + f*x)**4*a + 2*sin(e + f*x)**4*b + 8*sin
(e + f*x)**3*a + 2*sin(e + f*x)**3*b - 12*sin(e + f*x)**2*a - 3*sin(e + f*
x)**2*b - 12*sin(e + f*x)*a - 3*sin(e + f*x)*b + 3*a - 3*b)/(15*cos(e + f*
x)*a**3*c**2*f*(sin(e + f*x)**3 + sin(e + f*x)**2 - sin(e + f*x) - 1))
```

3.77 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 84

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^3}\,dx = \frac{B\sec^5(e+fx)}{5a^3c^3f}+\frac{A\tan(e+fx)}{a^3c^3f}+\frac{2A\tan^3(e+fx)}{3a^3c^3f}+\frac{A\tan^5(e+fx)}{5a^3c^3f}$$

output
```
1/5*B*sec(f*x+e)^5/a^3/c^3/f+A*tan(f*x+e)/a^3/c^3/f+2/3*A*tan(f*x+e)^3/a^3
/c^3/f+1/5*A*tan(f*x+e)^5/a^3/c^3/f
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.77

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^3}\,dx = \frac{B\sec^5(e+fx)}{5a^3c^3f}+\frac{A\left(\tan(e+fx)+\frac{2}{3}\tan^3(e+fx)+\frac{1}{5}\tan^5(e+fx)\right)}{a^3c^3f}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x]
)^3),x]
```

output
```
(B*Sec[e + f*x]^5)/(5*a^3*c^3*f) + (A*(Tan[e + f*x] + (2*Tan[e + f*x]^3)/3
 + Tan[e + f*x]^5/5))/(a^3*c^3*f)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.75, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3042, 3446, 3042, 3148, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c-c\sin(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c-c\sin(e+fx))^3}dx$$

$$\Big\downarrow \text{3446}$$

$$\frac{\int \sec^6(e+fx)(A+B\sin(e+fx))dx}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{A+B\sin(e+fx)}{\cos(e+fx)^6}dx}{a^3c^3}$$

$$\Big\downarrow \text{3148}$$

$$\frac{A\int \sec^6(e+fx)dx+\frac{B\sec^5(e+fx)}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\int \csc\left(e+fx+\frac{\pi}{2}\right)^6dx+\frac{B\sec^5(e+fx)}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\frac{B\sec^5(e+fx)}{5f}-\frac{A\int \left(\tan^4(e+fx)+2\tan^2(e+fx)+1\right)d(-\tan(e+fx))}{f}}{a^3c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{B\sec^5(e+fx)}{5f}-\frac{A\left(-\frac{1}{5}\tan^5(e+fx)-\frac{2}{3}\tan^3(e+fx)-\tan(e+fx)\right)}{f}}{a^3c^3}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^3),x
]
```

output
```
((B*Sec[e + f*x]^5)/(5*f) - (A*(-Tan[e + f*x] - (2*Tan[e + f*x]^3)/3 - Tan
[e + f*x]^5/5))/f)/(a^3*c^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99

method	result
risch	$\frac{\frac{16iA\,e^{2i(fx+e)}}{3}+\frac{32iA\,e^{4i(fx+e)}}{3}+\frac{16iA}{15}+\frac{32B\,e^{5i(fx+e)}}{5}}{\left(e^{i(fx+e)}+i\right)^{5}\left(e^{i(fx+e)}-i\right)^{5}f\,a^{3}c^{3}}$
parallelrisch	$\frac{-\frac{2B}{5}-\frac{116A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{15}-4B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}+\frac{8A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-2A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{8A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}{3}-2B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}}{f\,a^{3}c^{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{5}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}$
derivativedivides	$\frac{-\frac{-A+B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{4}}-\frac{-\frac{7A}{8}+\frac{5B}{8}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2\left(\frac{A}{2}-\frac{B}{2}\right)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{2\left(\frac{A}{2}-\frac{3B}{16}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2\left(\frac{11A}{8}-\frac{9B}{8}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{A+B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\right.}}{f\,a^{3}c^{3}}$
default	$\frac{-\frac{-A+B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{4}}-\frac{-\frac{7A}{8}+\frac{5B}{8}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2\left(\frac{A}{2}-\frac{B}{2}\right)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{2\left(\frac{A}{2}-\frac{3B}{16}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2\left(\frac{11A}{8}-\frac{9B}{8}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{A+B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\right.}}{f\,a^{3}c^{3}}$
norman	$\frac{-\frac{2B}{5fac}-\frac{76A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}{15afc}-\frac{2A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}{afc}-\frac{2A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{fac}+\frac{2A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3fac}-\frac{76A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{15fac}+\frac{2A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}{3fac}}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\right)a^{2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}c^{2}}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x,method=_RETUR
NVERBOSE)
```

output
```
16/15*(5*I*A*exp(2*I*(f*x+e))+10*I*A*exp(4*I*(f*x+e))+I*A+6*B*exp(5*I*(f*x
+e)))/(exp(I*(f*x+e))+I)^5/(exp(I*(f*x+e))-I)^5/f/a^3/c^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.67

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^3}\,dx$$
$$=\frac{\left(8\,A\cos\left(fx+e\right)^{4}+4\,A\cos\left(fx+e\right)^{2}+3\,A\right)\sin\left(fx+e\right)+3\,B}{15\,a^{3}c^{3}f\cos\left(fx+e\right)^{5}}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x, algori
thm="fricas")
```

output
```
1/15*((8*A*cos(f*x + e)^4 + 4*A*cos(f*x + e)^2 + 3*A)*sin(f*x + e) + 3*B)/
(a^3*c^3*f*cos(f*x + e)^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1098 vs. $2(78) = 156$.

Time = 6.21 (sec) , antiderivative size = 1098, normalized size of antiderivative = 13.07

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c - c\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**3,x)
```

output
```
Piecewise((-30*A*tan(e/2 + f*x/2)**9/(15*a**3*c**3*f*tan(e/2 + f*x/2)**10
- 75*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*a**3*c**3*f*tan(e/2 + f*x/2)**6
 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75*a**3*c**3*f*tan(e/2 + f*x/2)**
2 - 15*a**3*c**3*f) + 40*A*tan(e/2 + f*x/2)**7/(15*a**3*c**3*f*tan(e/2 + f
*x/2)**10 - 75*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*a**3*c**3*f*tan(e/2 +
 f*x/2)**6 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75*a**3*c**3*f*tan(e/2
+ f*x/2)**2 - 15*a**3*c**3*f) - 116*A*tan(e/2 + f*x/2)**5/(15*a**3*c**3*f*
tan(e/2 + f*x/2)**10 - 75*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*a**3*c**3*
f*tan(e/2 + f*x/2)**6 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75*a**3*c**3
*f*tan(e/2 + f*x/2)**2 - 15*a**3*c**3*f) + 40*A*tan(e/2 + f*x/2)**3/(15*a*
*3*c**3*f*tan(e/2 + f*x/2)**10 - 75*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*
a**3*c**3*f*tan(e/2 + f*x/2)**6 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75
*a**3*c**3*f*tan(e/2 + f*x/2)**2 - 15*a**3*c**3*f) - 30*A*tan(e/2 + f*x/2)
/(15*a**3*c**3*f*tan(e/2 + f*x/2)**10 - 75*a**3*c**3*f*tan(e/2 + f*x/2)**8
 + 150*a**3*c**3*f*tan(e/2 + f*x/2)**6 - 150*a**3*c**3*f*tan(e/2 + f*x/2)*
*4 + 75*a**3*c**3*f*tan(e/2 + f*x/2)**2 - 15*a**3*c**3*f) - 30*B*tan(e/2 +
 f*x/2)**8/(15*a**3*c**3*f*tan(e/2 + f*x/2)**10 - 75*a**3*c**3*f*tan(e/2 +
 f*x/2)**8 + 150*a**3*c**3*f*tan(e/2 + f*x/2)**6 - 150*a**3*c**3*f*tan(e/2
 + f*x/2)**4 + 75*a**3*c**3*f*tan(e/2 + f*x/2)**2 - 15*a**3*c**3*f) - 60*B
*tan(e/2 + f*x/2)**4/(15*a**3*c**3*f*tan(e/2 + f*x/2)**10 - 75*a**3*c**...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.71

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^3}\,dx = \frac{\frac{\left(3\tan(fx+e)^5+10\tan(fx+e)^3+15\tan(fx+e)\right)A}{a^3c^3}+\frac{3B}{a^3c^3\cos(fx+e)^5}}{15f}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x, algori
thm="maxima")
```

output
```
1/15*((3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*A/(a^3*c^3)
 + 3*B/(a^3*c^3*cos(f*x + e)^5))/f
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.50

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^3}\,dx = -\frac{2\left(15A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^9+15B\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^8-20A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7+58A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5\right.}{15\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x, algori
thm="giac")
```

output
```
-2/15*(15*A*tan(1/2*f*x + 1/2*e)^9 + 15*B*tan(1/2*f*x + 1/2*e)^8 - 20*A*ta
n(1/2*f*x + 1/2*e)^7 + 58*A*tan(1/2*f*x + 1/2*e)^5 + 30*B*tan(1/2*f*x + 1/
2*e)^4 - 20*A*tan(1/2*f*x + 1/2*e)^3 + 15*A*tan(1/2*f*x + 1/2*e) + 3*B)/((
tan(1/2*f*x + 1/2*e)^2 - 1)^5*a^3*c^3*f)
```

Mupad [B] (verification not implemented)

Time = 38.83 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.50

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^3}\,dx = -\frac{2\left(15\,A\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9+15\,B\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8-20\,A\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7+58\,A\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+30\,B\tan\left(\frac{e}{2}\right.\right.}{15\,a^3\,c^3\,f\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)^5}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^3),x
)
```

output
```
-(2*(3*B + 15*A*tan(e/2 + (f*x)/2) - 20*A*tan(e/2 + (f*x)/2)^3 + 58*A*tan(
e/2 + (f*x)/2)^5 - 20*A*tan(e/2 + (f*x)/2)^7 + 15*A*tan(e/2 + (f*x)/2)^9 +
 30*B*tan(e/2 + (f*x)/2)^4 + 15*B*tan(e/2 + (f*x)/2)^8))/(15*a^3*c^3*f*(ta
n(e/2 + (f*x)/2)^2 - 1)^5)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.42

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^3}\,dx = \frac{-3\cos(fx+e)\sin(fx+e)^4 b+6\cos(fx+e)\sin(fx+e)^2 b-3\cos(fx+e)\,b+8\sin(fx+e)^5 a-20}{15\cos(fx+e)\,a^3c^3f\left(\sin(fx+e)^4-2\sin(fx+e)^2+1\right)}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x)
```

output
```
( - 3*cos(e + f*x)*sin(e + f*x)**4*b + 6*cos(e + f*x)*sin(e + f*x)**2*b -
3*cos(e + f*x)*b + 8*sin(e + f*x)**5*a - 20*sin(e + f*x)**3*a + 15*sin(e +
 f*x)*a + 3*b)/(15*cos(e + f*x)*a**3*c**3*f*(sin(e + f*x)**4 - 2*sin(e + f
*x)**2 + 1))
```

3.78 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^4}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 121

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^4}\,dx &= \frac{(A+B)\sec^5(e+fx)}{7a^3f\,(c^4-c^4\sin(e+fx))}+\frac{(6A-B)\tan(e+fx)}{7a^3c^4f}\\ &\quad+\frac{2(6A-B)\tan^3(e+fx)}{21a^3c^4f}+\frac{(6A-B)\tan^5(e+fx)}{35a^3c^4f}\end{aligned}$$

output
```
1/7*(A+B)*sec(f*x+e)^5/a^3/f/(c^4-c^4*sin(f*x+e))+1/7*(6*A-B)*tan(f*x+e)/a
^3/c^4/f+2/21*(6*A-B)*tan(f*x+e)^3/a^3/c^4/f+1/35*(6*A-B)*tan(f*x+e)^5/a^3
/c^4/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 325 vs. 2(121) = 242.

Time = 7.38 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.69

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^4}\,dx =$$

$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-8960B+1500(A+B)\mathrm{co}\right.}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x]
)^4),x]
```

output
```
-1/53760*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2])*(-8960*B + 1500*(A + B)*Cos[e + f*x] - 640*(6*A - B)*Cos[2*(e
+ f*x)] + 750*A*Cos[3*(e + f*x)] + 750*B*Cos[3*(e + f*x)] - 3072*A*Cos[4*(
e + f*x)] + 512*B*Cos[4*(e + f*x)] + 150*A*Cos[5*(e + f*x)] + 150*B*Cos[5*
(e + f*x)] - 768*A*Cos[6*(e + f*x)] + 128*B*Cos[6*(e + f*x)] - 15360*A*Sin
[e + f*x] + 2560*B*Sin[e + f*x] - 375*A*Sin[2*(e + f*x)] - 375*B*Sin[2*(e
+ f*x)] - 7680*A*Sin[3*(e + f*x)] + 1280*B*Sin[3*(e + f*x)] - 300*A*Sin[4*
(e + f*x)] - 300*B*Sin[4*(e + f*x)] - 1536*A*Sin[5*(e + f*x)] + 256*B*Sin[
5*(e + f*x)] - 75*A*Sin[6*(e + f*x)] - 75*B*Sin[6*(e + f*x)]))/(a^3*c^4*f*
(-1 + Sin[e + f*x])^4*(1 + Sin[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.74, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3042, 3446, 3042, 3338, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c-c\sin(e+fx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c-c\sin(e+fx))^4}dx$$

$$\Big\downarrow \text{3446}$$

$$\frac{\int \frac{\sec^6(e+fx)(A+B\sin(e+fx))}{c-c\sin(e+fx)}dx}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{A+B\sin(e+fx)}{\cos(e+fx)^6(c-c\sin(e+fx))}dx}{a^3c^3}$$

$$\Big\downarrow \text{3338}$$

$$\frac{\frac{(6A-B)\int \sec^6(e+fx)dx}{7c}+\frac{(A+B)\sec^5(e+fx)}{7f(c-c\sin(e+fx))}}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(6A-B)\int \csc\left(e+fx+\frac{\pi}{2}\right)^6dx}{7c}+\frac{(A+B)\sec^5(e+fx)}{7f(c-c\sin(e+fx))}}{a^3c^3}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\frac{(A+B)\sec^5(e+fx)}{7f(c-c\sin(e+fx))}-\frac{(6A-B)\int (\tan^4(e+fx)+2\tan^2(e+fx)+1)d(-\tan(e+fx))}{7cf}}{a^3c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(A+B)\sec^5(e+fx)}{7f(c-c\sin(e+fx))}-\frac{(6A-B)\left(-\frac{1}{5}\tan^5(e+fx)-\frac{2}{3}\tan^3(e+fx)-\tan(e+fx)\right)}{7cf}}{a^3c^3}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^4),x
]
```

output
```
(((A + B)*Sec[e + f*x]^5)/(7*f*(c - c*Sin[e + f*x])) - ((6*A - B)*(-Tan[e
+ f*x] - (2*Tan[e + f*x]^3)/3 - Tan[e + f*x]^5/5))/(7*c*f))/(a^3*c^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.21 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.54

method	result
risch	$-\frac{16i(120iA\,e^{5i(fx+e)}-20iB\,e^{5i(fx+e)}+70B\,e^{6i(fx+e)}+60iA\,e^{3i(fx+e)}+30A\,e^{4i(fx+e)}-10iB\,e^{3i(fx+e)}-5B\,e^{4i(fx+}}{105\left(e^{i(fx+e)}+i\right)^{5}\left(e^{i(fx+e)}-i\right)^{7} f\,a^{3} c^{4}}$
parallelrisch	$\frac{-210A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}+(210A-210B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}+(210A+140B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}+(-630A-70B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}+}{}$
derivativedivides	$-\frac{-\frac{A}{2}+\frac{3B}{8}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{-\frac{A}{2}+\frac{B}{2}}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{4}}-\frac{2\left(\frac{A}{4}-\frac{B}{4}\right)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{2\left(\frac{3A}{4}-\frac{5B}{8}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2\left(\frac{11A}{32}-\frac{5B}{32}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(A+B)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\right.}$
default	$-\frac{-\frac{A}{2}+\frac{3B}{8}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{-\frac{A}{2}+\frac{B}{2}}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{4}}-\frac{2\left(\frac{A}{4}-\frac{B}{4}\right)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{2\left(\frac{3A}{4}-\frac{5B}{8}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2\left(\frac{11A}{32}-\frac{5B}{32}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2(A+B)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\right.}$
norman	$\frac{(2A-2B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{12}}{fca}-\frac{2A+2B}{7fca}-\frac{152(6A-B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}{105fca}-\frac{2A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{13}}{afc}+\frac{4B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}{3fca}+\frac{(312A-752B)\tan\left(\frac{f}{}\right.}{105fca}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x,method=_RETUR
NVERBOSE)
```

output
```
-16/105*I*(120*I*A*exp(5*I*(f*x+e))-20*I*B*exp(5*I*(f*x+e))+70*B*exp(6*I*(
f*x+e))+60*I*A*exp(3*I*(f*x+e))+30*A*exp(4*I*(f*x+e))-10*I*B*exp(3*I*(f*x+
e))-5*B*exp(4*I*(f*x+e))+12*I*A*exp(I*(f*x+e))+24*A*exp(2*I*(f*x+e))-2*I*B
*exp(I*(f*x+e))-4*B*exp(2*I*(f*x+e))+6*A-B)/(exp(I*(f*x+e))+I)^5/(exp(I*(f
*x+e))-I)^7/f/a^3/c^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.24

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^4}\,dx = -\frac{8\,(6\,A-B)\cos\left(fx+e\right)^{6}-4\,(6\,A-B)\cos\left(fx+e\right)^{4}-(6\,A-B)\cos\left(fx+e\right)^{2}+\left(8\,(6\,A-B)\cos\left(f\right.\right.}{105\,\left(a^{3}c^{4}f\cos\left(fx+e\right)^{5}\sin\left(fx+e\right)-a^{3}\right.}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x, algori
thm="fricas")
```

output
```
-1/105*(8*(6*A - B)*cos(f*x + e)^6 - 4*(6*A - B)*cos(f*x + e)^4 - (6*A - B
)*cos(f*x + e)^2 + (8*(6*A - B)*cos(f*x + e)^4 + 4*(6*A - B)*cos(f*x + e)^
2 + 18*A - 3*B)*sin(f*x + e) - 3*A + 18*B)/(a^3*c^4*f*cos(f*x + e)^5*sin(f
*x + e) - a^3*c^4*f*cos(f*x + e)^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6135 vs. $2(109) = 218$.

Time = 35.50 (sec) , antiderivative size = 6135, normalized size of antiderivative = 50.70

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**4,x)
```

output
```
Piecewise((-210*A*tan(e/2 + f*x/2)**11/(105*a**3*c**4*f*tan(e/2 + f*x/2)**
12 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 420*a**3*c**4*f*tan(e/2 + f*x/
2)**10 + 1050*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 525*a**3*c**4*f*tan(e/2 +
f*x/2)**8 - 2100*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 2100*a**3*c**4*f*tan(e/
2 + f*x/2)**5 - 525*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 1050*a**3*c**4*f*tan
(e/2 + f*x/2)**3 + 420*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 210*a**3*c**4*f*t
an(e/2 + f*x/2) - 105*a**3*c**4*f) + 210*A*tan(e/2 + f*x/2)**10/(105*a**3*
c**4*f*tan(e/2 + f*x/2)**12 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 420*a
**3*c**4*f*tan(e/2 + f*x/2)**10 + 1050*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 5
25*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c**4*f*tan(e/2 + f*x/2)**7
+ 2100*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*c**4*f*tan(e/2 + f*x/2)*
*4 - 1050*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a**3*c**4*f*tan(e/2 + f*x/
2)**2 + 210*a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**3*c**4*f) + 210*A*tan(e/
2 + f*x/2)**9/(105*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 210*a**3*c**4*f*tan(
e/2 + f*x/2)**11 - 420*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 1050*a**3*c**4*f
*tan(e/2 + f*x/2)**9 + 525*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c**
4*f*tan(e/2 + f*x/2)**7 + 2100*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*
c**4*f*tan(e/2 + f*x/2)**4 - 1050*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a*
*3*c**4*f*tan(e/2 + f*x/2)**2 + 210*a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**
3*c**4*f) - 630*A*tan(e/2 + f*x/2)**8/(105*a**3*c**4*f*tan(e/2 + f*x/2)...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1019 vs. $2(114) = 228$.

Time = 0.08 (sec) , antiderivative size = 1019, normalized size of antiderivative = 8.42

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c - c\sin(e + fx))^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x, algori
thm="maxima")
```

output
```
-2/105*(B*(30*sin(f*x + e)/(cos(f*x + e) + 1) - 45*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 - 80*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 110*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 188*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 266*sin(
f*x + e)^6/(cos(f*x + e) + 1)^6 - 112*sin(f*x + e)^7/(cos(f*x + e) + 1)^7
- 35*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 70*sin(f*x + e)^9/(cos(f*x + e)
 + 1)^9 - 105*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 15)/(a^3*c^4 - 2*a^3
*c^4*sin(f*x + e)/(cos(f*x + e) + 1) - 4*a^3*c^4*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 10*a^3*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*c^4*si
n(f*x + e)^4/(cos(f*x + e) + 1)^4 - 20*a^3*c^4*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 + 20*a^3*c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 5*a^3*c^4*sin(
f*x + e)^8/(cos(f*x + e) + 1)^8 - 10*a^3*c^4*sin(f*x + e)^9/(cos(f*x + e)
+ 1)^9 + 4*a^3*c^4*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 2*a^3*c^4*sin(f
*x + e)^11/(cos(f*x + e) + 1)^11 - a^3*c^4*sin(f*x + e)^12/(cos(f*x + e) +
 1)^12) - 3*A*(25*sin(f*x + e)/(cos(f*x + e) + 1) - 55*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 15*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 130*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 + 26*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 182*s
in(f*x + e)^6/(cos(f*x + e) + 1)^6 + 126*sin(f*x + e)^7/(cos(f*x + e) + 1)
^7 + 105*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 35*sin(f*x + e)^9/(cos(f*x
+ e) + 1)^9 - 35*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 35*sin(f*x + e)^1
1/(cos(f*x + e) + 1)^11 + 5)/(a^3*c^4 - 2*a^3*c^4*sin(f*x + e)/(cos(f*x...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(114) = 228$.

Time = 0.30 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.75

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^4}\,dx = -\frac{7\left(165\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-75\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+540\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-210\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+750\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-280\,B\tan\left(\frac{1}{2}\,fx\right.}{a^3c^4\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^5}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x, algori
thm="giac")
```

output
```
-1/1680*(7*(165*A*tan(1/2*f*x + 1/2*e)^4 - 75*B*tan(1/2*f*x + 1/2*e)^4 + 5
40*A*tan(1/2*f*x + 1/2*e)^3 - 210*B*tan(1/2*f*x + 1/2*e)^3 + 750*A*tan(1/2
*f*x + 1/2*e)^2 - 280*B*tan(1/2*f*x + 1/2*e)^2 + 480*A*tan(1/2*f*x + 1/2*e
) - 170*B*tan(1/2*f*x + 1/2*e) + 129*A - 49*B)/(a^3*c^4*(tan(1/2*f*x + 1/2
*e) + 1)^5) + (2205*A*tan(1/2*f*x + 1/2*e)^6 + 525*B*tan(1/2*f*x + 1/2*e)^
6 - 10080*A*tan(1/2*f*x + 1/2*e)^5 - 1470*B*tan(1/2*f*x + 1/2*e)^5 + 21945
*A*tan(1/2*f*x + 1/2*e)^4 + 2555*B*tan(1/2*f*x + 1/2*e)^4 - 26460*A*tan(1/
2*f*x + 1/2*e)^3 - 2240*B*tan(1/2*f*x + 1/2*e)^3 + 18963*A*tan(1/2*f*x + 1
/2*e)^2 + 1407*B*tan(1/2*f*x + 1/2*e)^2 - 7476*A*tan(1/2*f*x + 1/2*e) - 43
4*B*tan(1/2*f*x + 1/2*e) + 1383*A + 137*B)/(a^3*c^4*(tan(1/2*f*x + 1/2*e)
- 1)^7))/f
```

Mupad [B] (verification not implemented)

Time = 37.45 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.79

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^4}\,dx = \frac{\left(\frac{16\,A}{35}-\frac{8\,B}{105}-\frac{32\,A\,\sin(e+f\,x)}{35}+\frac{16\,B\,\sin(e+f\,x)}{105}\right)\,\cos\left(e+f\,x\right)^4+\left(\frac{4\,A}{35}-\frac{2\,B}{105}-\frac{16\,A\,\sin(e+f\,x)}{35}+\frac{8\,B\,\sin(e+f\,x)}{105}\right)\,\mathrm{co}}{a^3\,c^4\,f\,\left(2\cos\left(e+f\,x\right)^5\,\sin\left(e+f\,x\right)-2\cos\left(e+f\,x\right.\right.} - \frac{\frac{2\,A}{7}+\frac{2\,B}{7}-\frac{2\,A\,\sin(e+f\,x)}{7}-\frac{2\,B\,\sin(e+f\,x)}{7}}{a^3\,c^4\,f\,\left(2\,\sin\left(e+f\,x\right)-2\right)} - \frac{\cos\left(e+f\,x\right)\,\left(\frac{32\,A}{35}-\frac{16\,B}{105}\right)}{a^3\,c^4\,f\,\left(2\,\sin\left(e+f\,x\right)-2\right)}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^4),x
)
```

output
```
((2*A)/35 - (12*B)/35 - (12*A*sin(e + f*x))/35 + (2*B*sin(e + f*x))/35 + c
os(e + f*x)^2*((4*A)/35 - (2*B)/105 - (16*A*sin(e + f*x))/35 + (8*B*sin(e
+ f*x))/105) + cos(e + f*x)^4*((16*A)/35 - (8*B)/105 - (32*A*sin(e + f*x))
/35 + (16*B*sin(e + f*x))/105))/(a^3*c^4*f*(2*cos(e + f*x)^5*sin(e + f*x)
- 2*cos(e + f*x)^5)) - ((2*A)/7 + (2*B)/7 - (2*A*sin(e + f*x))/7 - (2*B*si
n(e + f*x))/7)/(a^3*c^4*f*(2*sin(e + f*x) - 2)) - (cos(e + f*x)*((32*A)/35
 - (16*B)/105))/(a^3*c^4*f*(2*sin(e + f*x) - 2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 386, normalized size of antiderivative = 3.19

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c - c\sin(e + fx))^4}\,dx = \frac{-15a - 15b + 48\sin(fx+e)^6 a - 15\cos(fx+e)\,b - 8\sin(fx+e)^6 b - 48\sin(fx+e)^5 a - 90\cos(fx$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x)
```

output
```
( - 90*cos(e + f*x)*sin(e + f*x)**5*a + 15*cos(e + f*x)*sin(e + f*x)**5*b
+ 90*cos(e + f*x)*sin(e + f*x)**4*a - 15*cos(e + f*x)*sin(e + f*x)**4*b +
180*cos(e + f*x)*sin(e + f*x)**3*a - 30*cos(e + f*x)*sin(e + f*x)**3*b - 1
80*cos(e + f*x)*sin(e + f*x)**2*a + 30*cos(e + f*x)*sin(e + f*x)**2*b - 90
*cos(e + f*x)*sin(e + f*x)*a + 15*cos(e + f*x)*sin(e + f*x)*b + 90*cos(e +
 f*x)*a - 15*cos(e + f*x)*b + 48*sin(e + f*x)**6*a - 8*sin(e + f*x)**6*b -
 48*sin(e + f*x)**5*a + 8*sin(e + f*x)**5*b - 120*sin(e + f*x)**4*a + 20*s
in(e + f*x)**4*b + 120*sin(e + f*x)**3*a - 20*sin(e + f*x)**3*b + 90*sin(e
 + f*x)**2*a - 15*sin(e + f*x)**2*b - 90*sin(e + f*x)*a + 15*sin(e + f*x)*
b - 15*a - 15*b)/(105*cos(e + f*x)*a**3*c**4*f*(sin(e + f*x)**5 - sin(e +
f*x)**4 - 2*sin(e + f*x)**3 + 2*sin(e + f*x)**2 + sin(e + f*x) - 1))
```

3.79 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^5}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 162

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^5}\,dx = \frac{(A+B)\sec^5(e+fx)}{9a^3c^3f(c-c\sin(e+fx))^2}+\frac{(7A-2B)\sec^5(e+fx)}{63a^3f(c^5-c^5\sin(e+fx))}+\frac{2(7A-2B)\tan(e+fx)}{21a^3c^5f}+\frac{4(7A-2B)\tan^3(e+fx)}{63a^3c^5f}+\frac{2(7A-2B)\tan^5(e+fx)}{105a^3c^5f}$$

output
```
1/9*(A+B)*sec(f*x+e)^5/a^3/c^3/f/(c-c*sin(f*x+e))^2+1/63*(7*A-2*B)*sec(f*x
+e)^5/a^3/f/(c^5-c^5*sin(f*x+e))+2/21*(7*A-2*B)*tan(f*x+e)/a^3/c^5/f+4/63*
(7*A-2*B)*tan(f*x+e)^3/a^3/c^5/f+2/105*(7*A-2*B)*tan(f*x+e)^5/a^3/c^5/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 373 vs. 2(162) = 324.

Time = 9.89 (sec) , antiderivative size = 373, normalized size of antiderivative = 2.30

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^5}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-184320B+1125(49A+13.}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x]
)^5),x]
```

output
```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])*(-184320*B + 1125*(49*A + 13*B)*Cos[e + f*x] - 20480*(7*A - 2*B)*Cos[2*
(e + f*x)] + 23275*A*Cos[3*(e + f*x)] + 6175*B*Cos[3*(e + f*x)] - 114688*A
*Cos[4*(e + f*x)] + 32768*B*Cos[4*(e + f*x)] + 1225*A*Cos[5*(e + f*x)] + 3
25*B*Cos[5*(e + f*x)] - 28672*A*Cos[6*(e + f*x)] + 8192*B*Cos[6*(e + f*x)]
 - 1225*A*Cos[7*(e + f*x)] - 325*B*Cos[7*(e + f*x)] - 322560*A*Sin[e + f*x
] + 92160*B*Sin[e + f*x] - 24500*A*Sin[2*(e + f*x)] - 6500*B*Sin[2*(e + f*
x)] - 136192*A*Sin[3*(e + f*x)] + 38912*B*Sin[3*(e + f*x)] - 19600*A*Sin[4
*(e + f*x)] - 5200*B*Sin[4*(e + f*x)] - 7168*A*Sin[5*(e + f*x)] + 2048*B*S
in[5*(e + f*x)] - 4900*A*Sin[6*(e + f*x)] - 1300*B*Sin[6*(e + f*x)] + 7168
*A*Sin[7*(e + f*x)] - 2048*B*Sin[7*(e + f*x)]))/(1290240*a^3*c^5*f*(-1 + S
in[e + f*x])^5*(1 + Sin[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.77, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3446, 3042, 3338, 3042, 3151, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c-c\sin(e+fx))^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c-c\sin(e+fx))^5}dx$$

$$\Big\downarrow \text{3446}$$

$$\frac{\int \frac{\sec^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}dx}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{A+B\sin(e+fx)}{\cos(e+fx)^6(c-c\sin(e+fx))^2}dx}{a^3c^3}$$

$\Big\downarrow$ 3338

$$\frac{\frac{(7A-2B)\int \frac{\sec^6(e+fx)}{c-c\sin(e+fx)}dx}{9c}+\frac{(A+B)\sec^5(e+fx)}{9f(c-c\sin(e+fx))^2}}{a^3c^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{(7A-2B)\int \frac{1}{\cos(e+fx)^6(c-c\sin(e+fx))}dx}{9c}+\frac{(A+B)\sec^5(e+fx)}{9f(c-c\sin(e+fx))^2}}{a^3c^3}$$

$\Big\downarrow$ 3151

$$\frac{\frac{(7A-2B)\left(\frac{6\int \sec^6(e+fx)dx}{7c}+\frac{\sec^5(e+fx)}{7f(c-c\sin(e+fx))}\right)}{9c}+\frac{(A+B)\sec^5(e+fx)}{9f(c-c\sin(e+fx))^2}}{a^3c^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{(7A-2B)\left(\frac{6\int \csc\left(e+fx+\frac{\pi}{2}\right)^6dx}{7c}+\frac{\sec^5(e+fx)}{7f(c-c\sin(e+fx))}\right)}{9c}+\frac{(A+B)\sec^5(e+fx)}{9f(c-c\sin(e+fx))^2}}{a^3c^3}$$

$\Big\downarrow$ 4254

$$\frac{\frac{(7A-2B)\left(\frac{\sec^5(e+fx)}{7f(c-c\sin(e+fx))}-\frac{6\int \left(\tan^4(e+fx)+2\tan^2(e+fx)+1\right)d(-\tan(e+fx))}{7cf}\right)}{9c}+\frac{(A+B)\sec^5(e+fx)}{9f(c-c\sin(e+fx))^2}}{a^3c^3}$$

$\Big\downarrow$ 2009

$$\frac{\frac{(A+B)\sec^5(e+fx)}{9f(c-c\sin(e+fx))^2}+\frac{(7A-2B)\left(\frac{\sec^5(e+fx)}{7f(c-c\sin(e+fx))}-\frac{6\left(-\frac{1}{5}\tan^5(e+fx)-\frac{2}{3}\tan^3(e+fx)-\tan(e+fx)\right)}{7cf}\right)}{9c}}{a^3c^3}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^5),x
]
```

output
```
(((A + B)*Sec[e + f*x]^5)/(9*f*(c - c*Sin[e + f*x])^2) + ((7*A - 2*B)*(Sec
[e + f*x]^5/(7*f*(c - c*Sin[e + f*x])) - (6*(-Tan[e + f*x] - (2*Tan[e + f*
x]^3)/3 - Tan[e + f*x]^5/5))/(7*c*f)))/(9*c))/(a^3*c^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3151
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Simp[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1])   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simpli
fy[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.06 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.30

method	result
risch	$-\frac{32\left(140A\,\mathrm{e}^{5i(fx+e)}-40B\,\mathrm{e}^{5i(fx+e)}-32B\,\mathrm{e}^{3i(fx+e)}-7iA+2iB+315iA\,\mathrm{e}^{6i(fx+e)}+133iA\,\mathrm{e}^{4i(fx+e)}-38iB\,\mathrm{e}^{4i(fx+e)}\right.}{315\left(\mathrm{e}^{i(fx+e)}+i\right)^{5}\left(\mathrm{e}^{i(\right.}}$
parallelrisch	$\frac{-630A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{13}+(1260A-630B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{12}+(-420A+840B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}+(-3360A-840B)\tan\left(\frac{fx}{2}+\right.}{}$
derivativedivides	$-\frac{2(2A+2B)}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{9}}-\frac{8A+8B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{8}}-\frac{\frac{35A}{2}+12B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{2\left(\frac{35A}{2}+\frac{33B}{2}\right)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}-\frac{\frac{49A}{2}+\frac{43B}{2}}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{6}}-\frac{\frac{51A}{16}+}{\left(\tan\left(\frac{fx}{2}+\right.\right.}$
default	$-\frac{2(2A+2B)}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{9}}-\frac{8A+8B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{8}}-\frac{\frac{35A}{2}+12B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{4}}-\frac{2\left(\frac{35A}{2}+\frac{33B}{2}\right)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{7}}-\frac{\frac{49A}{2}+\frac{43B}{2}}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{6}}-\frac{\frac{51A}{16}+}{\left(\tan\left(\frac{fx}{2}+\right.\right.}$
norman	$\frac{14A-40B}{252fca}+\frac{2(602A-697B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}}{105fca}-\frac{A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{16}}{2fca}-\frac{(1036A-296B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}{105fca}+\frac{(476A-136B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{315fca}-\frac{(5}{}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x,method=_RETUR
NVERBOSE)
```

output
```
-32/315*(140*A*exp(5*I*(f*x+e))-40*B*exp(5*I*(f*x+e))-32*B*exp(3*I*(f*x+e)
)-7*I*A+2*I*B+315*I*A*exp(6*I*(f*x+e))+133*I*A*exp(4*I*(f*x+e))-38*I*B*exp
(4*I*(f*x+e))-90*I*B*exp(6*I*(f*x+e))+112*A*exp(3*I*(f*x+e))+28*A*exp(I*(f
*x+e))+7*I*A*exp(2*I*(f*x+e))-2*I*B*exp(2*I*(f*x+e))+180*B*exp(7*I*(f*x+e)
)-8*B*exp(I*(f*x+e)))/(exp(I*(f*x+e))+I)^5/(exp(I*(f*x+e))-I)^9/f/a^3/c^5
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.14

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^5}\,dx = -\frac{32\,(7\,A-2\,B)\cos\left(fx+e\right)^{6}-16\,(7\,A-2\,B)\cos\left(fx+e\right)^{4}-4\,(7\,A-2\,B)\cos\left(fx+e\right)^{2}-\left(16\,(7\,A-\right.}{315\,\left(a^{3}c^{5}f\cos\left(fx+e\right)^{7}+2\,a^{3}\right.}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x, algori
thm="fricas")
```

output
```
-1/315*(32*(7*A - 2*B)*cos(f*x + e)^6 - 16*(7*A - 2*B)*cos(f*x + e)^4 - 4*
(7*A - 2*B)*cos(f*x + e)^2 - (16*(7*A - 2*B)*cos(f*x + e)^6 - 24*(7*A - 2*
B)*cos(f*x + e)^4 - 10*(7*A - 2*B)*cos(f*x + e)^2 - 49*A + 14*B)*sin(f*x +
 e) - 14*A + 49*B)/(a^3*c^5*f*cos(f*x + e)^7 + 2*a^3*c^5*f*cos(f*x + e)^5*
sin(f*x + e) - 2*a^3*c^5*f*cos(f*x + e)^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8396 vs. $2(150) = 300$.

Time = 60.20 (sec) , antiderivative size = 8396, normalized size of antiderivative = 51.83

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3 (c - c\sin(e + fx))^5}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**5,x)
```

output
```
Piecewise((-630*A*tan(e/2 + f*x/2)**13/(315*a**3*c**5*f*tan(e/2 + f*x/2)**
14 - 1260*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 315*a**3*c**5*f*tan(e/2 + f*x
/2)**12 + 5040*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 5985*a**3*c**5*f*tan(e/2
 + f*x/2)**10 - 6300*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*t
an(e/2 + f*x/2)**8 - 14175*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 6300*a**3*c**
5*f*tan(e/2 + f*x/2)**5 + 5985*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 5040*a**3
*c**5*f*tan(e/2 + f*x/2)**3 - 315*a**3*c**5*f*tan(e/2 + f*x/2)**2 + 1260*a
**3*c**5*f*tan(e/2 + f*x/2) - 315*a**3*c**5*f) + 1260*A*tan(e/2 + f*x/2)**
12/(315*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 1260*a**3*c**5*f*tan(e/2 + f*x/
2)**13 + 315*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*tan(e/2 +
 f*x/2)**11 - 5985*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 6300*a**3*c**5*f*tan
(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 14175*a**3*c**5
*f*tan(e/2 + f*x/2)**6 + 6300*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 5985*a**3*
c**5*f*tan(e/2 + f*x/2)**4 - 5040*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 315*a*
*3*c**5*f*tan(e/2 + f*x/2)**2 + 1260*a**3*c**5*f*tan(e/2 + f*x/2) - 315*a*
*3*c**5*f) - 420*A*tan(e/2 + f*x/2)**11/(315*a**3*c**5*f*tan(e/2 + f*x/2)*
*14 - 1260*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 315*a**3*c**5*f*tan(e/2 + f*
x/2)**12 + 5040*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 5985*a**3*c**5*f*tan(e/
2 + f*x/2)**10 - 6300*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*
tan(e/2 + f*x/2)**8 - 14175*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 6300*a**3...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1201 vs. $2(154) = 308$.

Time = 0.08 (sec) , antiderivative size = 1201, normalized size of antiderivative = 7.41

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c - c\sin(e + fx))^5}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x, algori
thm="maxima")
```

output
```
-2/315*(B*(100*sin(f*x + e)/(cos(f*x + e) + 1) - 340*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 55*sin(f*x + e)^
4/(cos(f*x + e) + 1)^4 - 88*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 1608*sin
(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1032*sin(f*x + e)^7/(cos(f*x + e) + 1)^
7 - 483*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 588*sin(f*x + e)^9/(cos(f*x
+ e) + 1)^9 - 420*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 420*sin(f*x + e)
^11/(cos(f*x + e) + 1)^11 - 315*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 25
)/(a^3*c^5 - 4*a^3*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + a^3*c^5*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + 16*a^3*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^
3 - 19*a^3*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 20*a^3*c^5*sin(f*x +
e)^5/(cos(f*x + e) + 1)^5 + 45*a^3*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6
 - 45*a^3*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 20*a^3*c^5*sin(f*x + e
)^9/(cos(f*x + e) + 1)^9 + 19*a^3*c^5*sin(f*x + e)^10/(cos(f*x + e) + 1)^1
0 - 16*a^3*c^5*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 - a^3*c^5*sin(f*x + e
)^12/(cos(f*x + e) + 1)^12 + 4*a^3*c^5*sin(f*x + e)^13/(cos(f*x + e) + 1)^
13 - a^3*c^5*sin(f*x + e)^14/(cos(f*x + e) + 1)^14) - 7*A*(5*sin(f*x + e)/
(cos(f*x + e) + 1) - 80*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 190*sin(f*x
+ e)^3/(cos(f*x + e) + 1)^3 + 50*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 269
*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 96*sin(f*x + e)^6/(cos(f*x + e) + 1
)^6 + 516*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 354*sin(f*x + e)^8/(cos...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(154) = 308$.

Time = 0.32 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.40

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c - c\sin(e + fx))^5}\,dx = -\frac{\frac{21\left(435\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-225\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+1470\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-690\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+2060\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-940\,B\tan(}{a^3c^5\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^5}}{}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x, algori
thm="giac")
```

output
```
-1/20160*(21*(435*A*tan(1/2*f*x + 1/2*e)^4 - 225*B*tan(1/2*f*x + 1/2*e)^4
+ 1470*A*tan(1/2*f*x + 1/2*e)^3 - 690*B*tan(1/2*f*x + 1/2*e)^3 + 2060*A*ta
n(1/2*f*x + 1/2*e)^2 - 940*B*tan(1/2*f*x + 1/2*e)^2 + 1330*A*tan(1/2*f*x +
 1/2*e) - 590*B*tan(1/2*f*x + 1/2*e) + 353*A - 163*B)/(a^3*c^5*(tan(1/2*f*
x + 1/2*e) + 1)^5) + (31185*A*tan(1/2*f*x + 1/2*e)^8 + 4725*B*tan(1/2*f*x
+ 1/2*e)^8 - 185220*A*tan(1/2*f*x + 1/2*e)^7 - 11340*B*tan(1/2*f*x + 1/2*e
)^7 + 546840*A*tan(1/2*f*x + 1/2*e)^6 + 15120*B*tan(1/2*f*x + 1/2*e)^6 - 9
61380*A*tan(1/2*f*x + 1/2*e)^5 + 3780*B*tan(1/2*f*x + 1/2*e)^5 + 1101618*A
*tan(1/2*f*x + 1/2*e)^4 - 24318*B*tan(1/2*f*x + 1/2*e)^4 - 828492*A*tan(1/
2*f*x + 1/2*e)^3 + 33852*B*tan(1/2*f*x + 1/2*e)^3 + 404208*A*tan(1/2*f*x +
 1/2*e)^2 - 19368*B*tan(1/2*f*x + 1/2*e)^2 - 116172*A*tan(1/2*f*x + 1/2*e)
 + 6732*B*tan(1/2*f*x + 1/2*e) + 16373*A - 223*B)/(a^3*c^5*(tan(1/2*f*x +
1/2*e) - 1)^9))/f
```

Mupad [B] (verification not implemented)

Time = 38.30 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.43

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^5}\,dx = \frac{\left(\frac{128\,B}{315}-\frac{64\,A}{45}+\frac{32\,A\,\sin(e+f\,x)}{45}-\frac{64\,B\,\sin(e+f\,x)}{315}\right)\cos\left(e+f\,x\right)^6+\left(\frac{8\,A\,\sin(e+f\,x)}{9}-\frac{20\,B}{63}-\frac{8\,A}{9}+\frac{20\,B\,\sin(e+f\,x)}{63}\right.}{}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^5),x
)
```

output
```
((4*A)/45 - (14*B)/45 - (14*A*sin(e + f*x))/45 + (4*B*sin(e + f*x))/45 - c
os(e + f*x)^5*((8*A)/9 + (20*B)/63 - (8*A*sin(e + f*x))/9 - (20*B*sin(e +
f*x))/63 + ((4*sin(e + f*x) - 4)*((4*A)/9 + (10*B)/63))/2) + cos(e + f*x)^
2*((8*A)/45 - (16*B)/315 - (4*A*sin(e + f*x))/9 + (8*B*sin(e + f*x))/63) +
 cos(e + f*x)^4*((32*A)/45 - (64*B)/315 - (16*A*sin(e + f*x))/15 + (32*B*s
in(e + f*x))/105) - cos(e + f*x)^6*((64*A)/45 - (128*B)/315 - (32*A*sin(e
+ f*x))/45 + (64*B*sin(e + f*x))/315))/(a^3*c^5*f*(4*cos(e + f*x)^5*sin(e
+ f*x) - 4*cos(e + f*x)^5 + 2*cos(e + f*x)^7))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 454, normalized size of antiderivative = 2.80

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^5}\,dx$$
$$= \frac{140a+50b-64\sin\left(fx+e\right)^7 b-448\sin\left(fx+e\right)^6 a+50\cos\left(fx+e\right)b+128\sin\left(fx+e\right)^6 b-336\sin\left(\right.}{}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x)
```

output
```
( - 175*cos(e + f*x)*sin(e + f*x)**6*a + 50*cos(e + f*x)*sin(e + f*x)**6*b
 + 350*cos(e + f*x)*sin(e + f*x)**5*a - 100*cos(e + f*x)*sin(e + f*x)**5*b
 + 175*cos(e + f*x)*sin(e + f*x)**4*a - 50*cos(e + f*x)*sin(e + f*x)**4*b
- 700*cos(e + f*x)*sin(e + f*x)**3*a + 200*cos(e + f*x)*sin(e + f*x)**3*b
+ 175*cos(e + f*x)*sin(e + f*x)**2*a - 50*cos(e + f*x)*sin(e + f*x)**2*b +
 350*cos(e + f*x)*sin(e + f*x)*a - 100*cos(e + f*x)*sin(e + f*x)*b - 175*c
os(e + f*x)*a + 50*cos(e + f*x)*b + 224*sin(e + f*x)**7*a - 64*sin(e + f*x
)**7*b - 448*sin(e + f*x)**6*a + 128*sin(e + f*x)**6*b - 336*sin(e + f*x)*
*5*a + 96*sin(e + f*x)**5*b + 1120*sin(e + f*x)**4*a - 320*sin(e + f*x)**4
*b - 140*sin(e + f*x)**3*a + 40*sin(e + f*x)**3*b - 840*sin(e + f*x)**2*a
+ 240*sin(e + f*x)**2*b + 350*sin(e + f*x)*a - 100*sin(e + f*x)*b + 140*a
+ 50*b)/(630*cos(e + f*x)*a**3*c**5*f*(sin(e + f*x)**6 - 2*sin(e + f*x)**5
 - sin(e + f*x)**4 + 4*sin(e + f*x)**3 - sin(e + f*x)**2 - 2*sin(e + f*x)
+ 1))
```

3.80 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^6}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 205

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^6}\,dx &= \frac{(A+B)\sec^5(e+fx)}{11a^3f\left(c^2-c^2\sin(e+fx)\right)^3}+\frac{(8A-3B)\sec^5(e+fx)}{99a^3f\left(c^3-c^3\sin(e+fx)\right)^2}\\ &+\frac{(8A-3B)\sec^5(e+fx)}{99a^3f\left(c^6-c^6\sin(e+fx)\right)}+\frac{2(8A-3B)\tan(e+fx)}{33a^3c^6f}\\ &+\frac{4(8A-3B)\tan^3(e+fx)}{99a^3c^6f}+\frac{2(8A-3B)\tan^5(e+fx)}{165a^3c^6f}\end{aligned}$$

output
```
1/11*(A+B)*sec(f*x+e)^5/a^3/f/(c^2-c^2*sin(f*x+e))^3+1/99*(8*A-3*B)*sec(f*
x+e)^5/a^3/f/(c^3-c^3*sin(f*x+e))^2+1/99*(8*A-3*B)*sec(f*x+e)^5/a^3/f/(c^6
-c^6*sin(f*x+e))+2/33*(8*A-3*B)*tan(f*x+e)/a^3/c^6/f+4/99*(8*A-3*B)*tan(f*
x+e)^3/a^3/c^6/f+2/165*(8*A-3*B)*tan(f*x+e)^5/a^3/c^6/f
```

Mathematica [A] (verified)

Time = 12.85 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.96

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^6}\,dx$$
$$= \frac{1013760B-3850(107A-3B)\cos(e+fx)+135168(8A-3B)\cos(2(e+fx))-127330A\cos(3(e+fx)}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x]
)^6),x]
```

output
```
(1013760*B - 3850*(107*A - 3*B)*Cos[e + f*x] + 135168*(8*A - 3*B)*Cos[2*(e
 + f*x)] - 127330*A*Cos[3*(e + f*x)] + 3570*B*Cos[3*(e + f*x)] + 819200*A*
Cos[4*(e + f*x)] - 307200*B*Cos[4*(e + f*x)] + 37450*A*Cos[5*(e + f*x)] -
1050*B*Cos[5*(e + f*x)] + 163840*A*Cos[6*(e + f*x)] - 61440*B*Cos[6*(e + f
*x)] + 22470*A*Cos[7*(e + f*x)] - 630*B*Cos[7*(e + f*x)] - 16384*A*Cos[8*(
e + f*x)] + 6144*B*Cos[8*(e + f*x)] + 1802240*A*Sin[e + f*x] - 675840*B*Si
n[e + f*x] + 247170*A*Sin[2*(e + f*x)] - 6930*B*Sin[2*(e + f*x)] + 557056*
A*Sin[3*(e + f*x)] - 208896*B*Sin[3*(e + f*x)] + 187250*A*Sin[4*(e + f*x)]
 - 5250*B*Sin[4*(e + f*x)] - 163840*A*Sin[5*(e + f*x)] + 61440*B*Sin[5*(e
+ f*x)] + 37450*A*Sin[6*(e + f*x)] - 1050*B*Sin[6*(e + f*x)] - 98304*A*Sin
[7*(e + f*x)] + 36864*B*Sin[7*(e + f*x)] - 3745*A*Sin[8*(e + f*x)] + 105*B
*Sin[8*(e + f*x)])/(8110080*a^3*c^6*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
)^11*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.79, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.306$, Rules used = {3042, 3446, 3042, 3338, 3042, 3151, 3042, 3151, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c-c\sin(e+fx))^6}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c-c\sin(e+fx))^6}dx$$

$$\Big\downarrow \text{3446}$$

$$\frac{\int \frac{\sec^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}dx}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{A+B\sin(e+fx)}{\cos(e+fx)^6(c-c\sin(e+fx))^3}dx}{a^3c^3}$$

$$\Big\downarrow \text{3338}$$

$$\frac{\frac{(8A-3B)\int \frac{\sec^6(e+fx)}{(c-c\sin(e+fx))^2}dx}{11c}+\frac{(A+B)\sec^5(e+fx)}{11f(c-c\sin(e+fx))^3}}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(8A-3B)\int \frac{1}{\cos(e+fx)^6(c-c\sin(e+fx))^2}dx}{11c}+\frac{(A+B)\sec^5(e+fx)}{11f(c-c\sin(e+fx))^3}}{a^3c^3}$$

$$\Big\downarrow \text{3151}$$

$$\frac{\frac{(8A-3B)\left(\frac{7\int \frac{\sec^6(e+fx)}{c-c\sin(e+fx)}dx}{9c}+\frac{\sec^5(e+fx)}{9f(c-c\sin(e+fx))^2}\right)}{11c}+\frac{(A+B)\sec^5(e+fx)}{11f(c-c\sin(e+fx))^3}}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(8A-3B)\left(\frac{7\int \frac{1}{\cos(e+fx)^6(c-c\sin(e+fx))}dx}{9c}+\frac{\sec^5(e+fx)}{9f(c-c\sin(e+fx))^2}\right)}{11c}+\frac{(A+B)\sec^5(e+fx)}{11f(c-c\sin(e+fx))^3}}{a^3c^3}$$

$$\Big\downarrow \text{3151}$$

$$\frac{\frac{(8A-3B)\left(\frac{7\left(\frac{6\int \sec^6(e+fx)dx}{7c}+\frac{\sec^5(e+fx)}{7f(c-c\sin(e+fx))}\right)}{9c}+\frac{\sec^5(e+fx)}{9f(c-c\sin(e+fx))^2}\right)}{11c}+\frac{(A+B)\sec^5(e+fx)}{11f(c-c\sin(e+fx))^3}}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(8A-3B)\left(\frac{7\left(\frac{6\int \csc\left(e+fx+\frac{\pi}{2}\right)^6dx}{7c}+\frac{\sec^5(e+fx)}{7f(c-c\sin(e+fx))}\right)}{9c}+\frac{\sec^5(e+fx)}{9f(c-c\sin(e+fx))^2}\right)}{11c}+\frac{(A+B)\sec^5(e+fx)}{11f(c-c\sin(e+fx))^3}}{a^3c^3}$$

$$\Big\downarrow \; 4254$$

$$\frac{\frac{(8A-3B)\left(\frac{7\left(\frac{\sec^5(e+fx)}{7f(c-c\sin(e+fx))}-\frac{6\int\left(\tan^4(e+fx)+2\tan^2(e+fx)+1\right)d(-\tan(e+fx))}{7cf}\right)}{9c}+\frac{\sec^5(e+fx)}{9f(c-c\sin(e+fx))^2}\right)}{11c}+\frac{(A+B)\sec^5(e+fx)}{11f(c-c\sin(e+fx))^3}}{a^3c^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{(A+B)\sec^5(e+fx)}{11f(c-c\sin(e+fx))^3}+\frac{(8A-3B)\left(\frac{\sec^5(e+fx)}{9f(c-c\sin(e+fx))^2}+\frac{7\left(\frac{\sec^5(e+fx)}{7f(c-c\sin(e+fx))}-\frac{6\left(-\frac{1}{5}\tan^5(e+fx)-\frac{2}{3}\tan^3(e+fx)-\tan(e+fx)\right)}{7cf}\right)}{9c}\right)}{11c}}{a^3c^3}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^6),x
]
```

output
```
(((A + B)*Sec[e + f*x]^5)/(11*f*(c - c*Sin[e + f*x])^3) + ((8*A - 3*B)*(Se
c[e + f*x]^5/(9*f*(c - c*Sin[e + f*x])^2) + (7*(Sec[e + f*x]^5/(7*f*(c - c
*Sin[e + f*x])) - (6*(-Tan[e + f*x] - (2*Tan[e + f*x]^3)/3 - Tan[e + f*x]^
5/5))/(7*c*f)))/(9*c)))/(11*c))/(a^3*c^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3151
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Simp[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1])   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simpli
fy[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.31 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.15

method	result
risch	$\frac{32i\left(18iB\,e^{i(fx+e)}-80iA\,e^{3i(fx+e)}+495B\,e^{8i(fx+e)}+880iA\,e^{7i(fx+e)}+528A\,e^{6i(fx+e)}+30iB\,e^{3i(fx+e)}-198B\,e^{6i(fx\cdots}{495\cdots}$
parallelrisch	$\frac{-990A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{15}+(2970A-990B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{14}+(-3630A+1980B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{13}+(-4950A-2970B)\tan\left(\cdots\right.}{}$
derivativedivides	$\frac{-\frac{-\frac{5A}{32}+\frac{B}{8}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{-\frac{A}{8}+\frac{B}{8}}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{4}}-\frac{2\left(\frac{A}{16}-\frac{B}{16}\right)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{2\left(\frac{37A}{256}-\frac{21B}{256}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2\left(\frac{7A}{32}-\frac{3B}{16}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2(4A+4B)}{11\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\cdots\right.}}{}$
default	$\frac{-\frac{-\frac{5A}{32}+\frac{B}{8}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{-\frac{A}{8}+\frac{B}{8}}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{4}}-\frac{2\left(\frac{A}{16}-\frac{B}{16}\right)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{2\left(\frac{37A}{256}-\frac{21B}{256}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2\left(\frac{7A}{32}-\frac{3B}{16}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2(4A+4B)}{11\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\cdots\right.}}{}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x,method=_RETUR
NVERBOSE)
```

output
```
32/495*I*(18*I*B*exp(I*(f*x+e))-80*I*A*exp(3*I*(f*x+e))+495*B*exp(8*I*(f*x
+e))+880*I*A*exp(7*I*(f*x+e))+528*A*exp(6*I*(f*x+e))+30*I*B*exp(3*I*(f*x+e
))-198*B*exp(6*I*(f*x+e))-102*I*B*exp(5*I*(f*x+e))+400*A*exp(4*I*(f*x+e))+
272*I*A*exp(5*I*(f*x+e))-150*B*exp(4*I*(f*x+e))-48*I*A*exp(I*(f*x+e))+80*A
*exp(2*I*(f*x+e))-330*I*B*exp(7*I*(f*x+e))-30*B*exp(2*I*(f*x+e))-8*A+3*B)/
(exp(I*(f*x+e))+I)^5/(exp(I*(f*x+e))-I)^11/f/a^3/c^6
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.08

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^6}\,dx$$
$$= \frac{16\,(8\,A-3\,B)\cos\left(fx+e\right)^{8}-72\,(8\,A-3\,B)\cos\left(fx+e\right)^{6}+30\,(8\,A-3\,B)\cos\left(fx+e\right)^{4}+7\,(8\,A-3\,\cdots}{495\,\left(3\,a^{3}c^{6}f\cos\left(fx+e\right)^{7}-4\,\cdots\right.}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x, algori
thm="fricas")
```

output
```
1/495*(16*(8*A - 3*B)*cos(f*x + e)^8 - 72*(8*A - 3*B)*cos(f*x + e)^6 + 30*
(8*A - 3*B)*cos(f*x + e)^4 + 7*(8*A - 3*B)*cos(f*x + e)^2 + (48*(8*A - 3*B
)*cos(f*x + e)^6 - 40*(8*A - 3*B)*cos(f*x + e)^4 - 14*(8*A - 3*B)*cos(f*x
+ e)^2 - 72*A + 27*B)*sin(f*x + e) + 27*A - 72*B)/(3*a^3*c^6*f*cos(f*x + e
)^7 - 4*a^3*c^6*f*cos(f*x + e)^5 - (a^3*c^6*f*cos(f*x + e)^7 - 4*a^3*c^6*f
*cos(f*x + e)^5)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11011 vs. $2(187) = 374$.

Time = 103.02 (sec) , antiderivative size = 11011, normalized size of antiderivative = 53.71

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c - c\sin(e + fx))^6}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**6,x)
```

output
```
Piecewise((-990*A*tan(e/2 + f*x/2)**15/(495*a**3*c**6*f*tan(e/2 + f*x/2)**
16 - 2970*a**3*c**6*f*tan(e/2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2 + f*
x/2)**14 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*tan(e
/2 + f*x/2)**12 + 16830*a**3*c**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c**6
*f*tan(e/2 + f*x/2)**10 - 54450*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a*
*3*c**6*f*tan(e/2 + f*x/2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16
830*a**3*c**6*f*tan(e/2 + f*x/2)**5 + 24750*a**3*c**6*f*tan(e/2 + f*x/2)**
4 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**3 - 4950*a**3*c**6*f*tan(e/2 + f*x/
2)**2 + 2970*a**3*c**6*f*tan(e/2 + f*x/2) - 495*a**3*c**6*f) + 2970*A*tan(
e/2 + f*x/2)**14/(495*a**3*c**6*f*tan(e/2 + f*x/2)**16 - 2970*a**3*c**6*f*
tan(e/2 + f*x/2)**15 + 4950*a**3*c**6*f*tan(e/2 + f*x/2)**14 + 4950*a**3*c
**6*f*tan(e/2 + f*x/2)**13 - 24750*a**3*c**6*f*tan(e/2 + f*x/2)**12 + 1683
0*a**3*c**6*f*tan(e/2 + f*x/2)**11 + 32670*a**3*c**6*f*tan(e/2 + f*x/2)**1
0 - 54450*a**3*c**6*f*tan(e/2 + f*x/2)**9 + 54450*a**3*c**6*f*tan(e/2 + f*
x/2)**7 - 32670*a**3*c**6*f*tan(e/2 + f*x/2)**6 - 16830*a**3*c**6*f*tan(e/
2 + f*x/2)**5 + 24750*a**3*c**6*f*tan(e/2 + f*x/2)**4 - 4950*a**3*c**6*f*t
an(e/2 + f*x/2)**3 - 4950*a**3*c**6*f*tan(e/2 + f*x/2)**2 + 2970*a**3*c**6
*f*tan(e/2 + f*x/2) - 495*a**3*c**6*f) - 3630*A*tan(e/2 + f*x/2)**13/(495*
a**3*c**6*f*tan(e/2 + f*x/2)**16 - 2970*a**3*c**6*f*tan(e/2 + f*x/2)**15 +
 4950*a**3*c**6*f*tan(e/2 + f*x/2)**14 + 4950*a**3*c**6*f*tan(e/2 + f*x...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1387 vs. $2(196) = 392$.

Time = 0.09 (sec) , antiderivative size = 1387, normalized size of antiderivative = 6.77

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3 (c - c\sin(e + fx))^6}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x, algori
thm="maxima")
```

output
```
-2/495*(A*(255*sin(f*x + e)/(cos(f*x + e) + 1) + 235*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 - 3065*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3775*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 + 667*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 821
7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 2035*sin(f*x + e)^7/(cos(f*x + e)
+ 1)^7 + 8745*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 11715*sin(f*x + e)^9/(
cos(f*x + e) + 1)^9 + 33*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 4917*sin(
f*x + e)^11/(cos(f*x + e) + 1)^11 - 2475*sin(f*x + e)^12/(cos(f*x + e) + 1
)^12 - 1815*sin(f*x + e)^13/(cos(f*x + e) + 1)^13 + 1485*sin(f*x + e)^14/(
cos(f*x + e) + 1)^14 - 495*sin(f*x + e)^15/(cos(f*x + e) + 1)^15 - 125)/(a
^3*c^6 - 6*a^3*c^6*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*c^6*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*c^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
 - 50*a^3*c^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 34*a^3*c^6*sin(f*x + e
)^5/(cos(f*x + e) + 1)^5 + 66*a^3*c^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6
- 110*a^3*c^6*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 110*a^3*c^6*sin(f*x +
e)^9/(cos(f*x + e) + 1)^9 - 66*a^3*c^6*sin(f*x + e)^10/(cos(f*x + e) + 1)^
10 - 34*a^3*c^6*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 50*a^3*c^6*sin(f*x
 + e)^12/(cos(f*x + e) + 1)^12 - 10*a^3*c^6*sin(f*x + e)^13/(cos(f*x + e)
+ 1)^13 - 10*a^3*c^6*sin(f*x + e)^14/(cos(f*x + e) + 1)^14 + 6*a^3*c^6*sin
(f*x + e)^15/(cos(f*x + e) + 1)^15 - a^3*c^6*sin(f*x + e)^16/(cos(f*x + e)
 + 1)^16) + 3*B*(30*sin(f*x + e)/(cos(f*x + e) + 1) - 215*sin(f*x + e)^...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(196) = 392$.

Time = 0.28 (sec) , antiderivative size = 445, normalized size of antiderivative = 2.17

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c - c\sin(e + fx))^6}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x, algori
thm="giac")
```

output
```
-1/63360*(33*(555*A*tan(1/2*f*x + 1/2*e)^4 - 315*B*tan(1/2*f*x + 1/2*e)^4
+ 1920*A*tan(1/2*f*x + 1/2*e)^3 - 1020*B*tan(1/2*f*x + 1/2*e)^3 + 2710*A*t
an(1/2*f*x + 1/2*e)^2 - 1410*B*tan(1/2*f*x + 1/2*e)^2 + 1760*A*tan(1/2*f*x
 + 1/2*e) - 900*B*tan(1/2*f*x + 1/2*e) + 463*A - 243*B)/(a^3*c^6*(tan(1/2*
f*x + 1/2*e) + 1)^5) + (108405*A*tan(1/2*f*x + 1/2*e)^10 + 10395*B*tan(1/2
*f*x + 1/2*e)^10 - 784080*A*tan(1/2*f*x + 1/2*e)^9 - 5940*B*tan(1/2*f*x +
1/2*e)^9 + 2901195*A*tan(1/2*f*x + 1/2*e)^8 - 79695*B*tan(1/2*f*x + 1/2*e)
^8 - 6652800*A*tan(1/2*f*x + 1/2*e)^7 + 388080*B*tan(1/2*f*x + 1/2*e)^7 +
10407474*A*tan(1/2*f*x + 1/2*e)^6 - 816354*B*tan(1/2*f*x + 1/2*e)^6 - 1143
5424*A*tan(1/2*f*x + 1/2*e)^5 + 1114344*B*tan(1/2*f*x + 1/2*e)^5 + 8949270
*A*tan(1/2*f*x + 1/2*e)^4 - 990990*B*tan(1/2*f*x + 1/2*e)^4 - 4899840*A*ta
n(1/2*f*x + 1/2*e)^3 + 609840*B*tan(1/2*f*x + 1/2*e)^3 + 1816265*A*tan(1/2
*f*x + 1/2*e)^2 - 235785*B*tan(1/2*f*x + 1/2*e)^2 - 411664*A*tan(1/2*f*x +
 1/2*e) + 56364*B*tan(1/2*f*x + 1/2*e) + 47279*A - 4179*B)/(a^3*c^6*(tan(1
/2*f*x + 1/2*e) - 1)^11))/f
```

Mupad [B] (verification not implemented)

Time = 39.29 (sec) , antiderivative size = 474, normalized size of antiderivative = 2.31

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^6}\,dx$$
$$= \frac{2\left(\frac{165\,B\,\sin(e+f\,x)}{4}-\frac{6875\,A\,\cos(e+f\,x)}{64}-\frac{825\,B\,\cos(e+f\,x)}{64}-110\,A\,\sin\left(e+f\,x\right)-\frac{495\,B}{8}-66\,A\,\cos\left(2\,e+2\,f\,x\right)\right.}{}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^6),x
)
```

output
```
(2*((165*B*sin(e + f*x))/4 - (6875*A*cos(e + f*x))/64 - (825*B*cos(e + f*x
))/64 - 110*A*sin(e + f*x) - (495*B)/8 - 66*A*cos(2*e + 2*f*x) - (2125*A*c
os(3*e + 3*f*x))/64 - 50*A*cos(4*e + 4*f*x) + (625*A*cos(5*e + 5*f*x))/64
- 10*A*cos(6*e + 6*f*x) + (375*A*cos(7*e + 7*f*x))/64 + A*cos(8*e + 8*f*x)
 + (99*B*cos(2*e + 2*f*x))/4 - (255*B*cos(3*e + 3*f*x))/64 + (75*B*cos(4*e
 + 4*f*x))/4 + (75*B*cos(5*e + 5*f*x))/64 + (15*B*cos(6*e + 6*f*x))/4 + (4
5*B*cos(7*e + 7*f*x))/64 - (3*B*cos(8*e + 8*f*x))/8 + (4125*A*sin(2*e + 2*
f*x))/64 - 34*A*sin(3*e + 3*f*x) + (3125*A*sin(4*e + 4*f*x))/64 + 10*A*sin
(5*e + 5*f*x) + (625*A*sin(6*e + 6*f*x))/64 + 6*A*sin(7*e + 7*f*x) - (125*
A*sin(8*e + 8*f*x))/128 + (495*B*sin(2*e + 2*f*x))/64 + (51*B*sin(3*e + 3*
f*x))/4 + (375*B*sin(4*e + 4*f*x))/64 - (15*B*sin(5*e + 5*f*x))/4 + (75*B*
sin(6*e + 6*f*x))/64 - (9*B*sin(7*e + 7*f*x))/4 - (15*B*sin(8*e + 8*f*x))/
128))/(495*a^3*c^6*f*((5*cos(5*e + 5*f*x))/32 - (17*cos(3*e + 3*f*x))/32 -
 (55*cos(e + f*x))/32 + (3*cos(7*e + 7*f*x))/32 + (33*sin(2*e + 2*f*x))/32
 + (25*sin(4*e + 4*f*x))/32 + (5*sin(6*e + 6*f*x))/32 - sin(8*e + 8*f*x)/6
4))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 518, normalized size of antiderivative = 2.53

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^6}\,dx$$
$$= \frac{-125a-15b+144\sin(fx+e)^7 b+64\sin(fx+e)^6 a-15\cos(fx+e)b-24\sin(fx+e)^6 b+832\sin}{}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x)
```

output

```
( - 40*cos(e + f*x)*sin(e + f*x)**7*a + 15*cos(e + f*x)*sin(e + f*x)**7*b
+ 120*cos(e + f*x)*sin(e + f*x)**6*a - 45*cos(e + f*x)*sin(e + f*x)**6*b -
 40*cos(e + f*x)*sin(e + f*x)**5*a + 15*cos(e + f*x)*sin(e + f*x)**5*b - 2
00*cos(e + f*x)*sin(e + f*x)**4*a + 75*cos(e + f*x)*sin(e + f*x)**4*b + 20
0*cos(e + f*x)*sin(e + f*x)**3*a - 75*cos(e + f*x)*sin(e + f*x)**3*b + 40*
cos(e + f*x)*sin(e + f*x)**2*a - 15*cos(e + f*x)*sin(e + f*x)**2*b - 120*c
os(e + f*x)*sin(e + f*x)*a + 45*cos(e + f*x)*sin(e + f*x)*b + 40*cos(e + f
*x)*a - 15*cos(e + f*x)*b + 128*sin(e + f*x)**8*a - 48*sin(e + f*x)**8*b -
 384*sin(e + f*x)**7*a + 144*sin(e + f*x)**7*b + 64*sin(e + f*x)**6*a - 24
*sin(e + f*x)**6*b + 832*sin(e + f*x)**5*a - 312*sin(e + f*x)**5*b - 720*s
in(e + f*x)**4*a + 270*sin(e + f*x)**4*b - 400*sin(e + f*x)**3*a + 150*sin
(e + f*x)**3*b + 680*sin(e + f*x)**2*a - 255*sin(e + f*x)**2*b - 120*sin(e
 + f*x)*a + 45*sin(e + f*x)*b - 125*a - 15*b)/(495*cos(e + f*x)*a**3*c**6*
f*(sin(e + f*x)**7 - 3*sin(e + f*x)**6 + sin(e + f*x)**5 + 5*sin(e + f*x)*
*4 - 5*sin(e + f*x)**3 - sin(e + f*x)**2 + 3*sin(e + f*x) - 1))
```

3.81 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 198

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \frac{256a(11A-5B)c^5\cos^3(e+fx)}{3465f(c-c\sin(e+fx))^{3/2}} + \frac{64a(11A-5B)c^4\cos^3(e+fx)}{1155f\sqrt{c-c\sin(e+fx)}} + \frac{8a(11A-5B)c^3\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{231f} + \frac{2a(11A-5B)c^2\cos^3(e+fx)(c-c\sin(e+fx))^{3/2}}{99f} - \frac{2aBc\cos^3(e+fx)(c-c\sin(e+fx))^{5/2}}{11f}$$

output
```
256/3465*a*(11*A-5*B)*c^5*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(3/2)+64/1155*a*
(11*A-5*B)*c^4*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(1/2)+8/231*a*(11*A-5*B)*c^
3*cos(f*x+e)^3*(c-c*sin(f*x+e))^(1/2)/f+2/99*a*(11*A-5*B)*c^2*cos(f*x+e)^3
*(c-c*sin(f*x+e))^(3/2)/f-2/11*a*B*c*cos(f*x+e)^3*(c-c*sin(f*x+e))^(5/2)/f
```

Mathematica [A] (verified)

Time = 6.76 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.75

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^{7/2}\,dx =$$

$$-\frac{ac^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sqrt{c-c\sin(e+fx)}(-35332A+27085B+60(121A-202B)\mathrm{c}}{13860f\,(}$$

input
```
Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(
7/2),x]
```

output
```
-1/13860*(a*c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e +
 f*x]]*(-35332*A + 27085*B + 60*(121*A - 202*B)*Cos[2*(e + f*x)] + 315*B*C
os[4*(e + f*x)] + 30558*A*Sin[e + f*x] - 31530*B*Sin[e + f*x] - 770*A*Sin[
3*(e + f*x)] + 2870*B*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f
*x)/2]))
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.93, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3446, 3042, 3335, 3042, 3153, 3042, 3153, 3042, 3153, 3042, 3152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3446}$$

$$ac\int \cos^2(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$ac\int \cos(e+fx)^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}dx$$

$$\Big\downarrow \text{3335}$$

$$ac\left(\frac{1}{11}(11A-5B)\int \cos^2(e+fx)(c-c\sin(e+fx))^{5/2}dx-\frac{2B\cos^3(e+fx)(c-c\sin(e+fx))^{5/2}}{11f}\right)$$

$$\Big\downarrow \text{3042}$$

$$ac\left(\frac{1}{11}(11A-5B)\int \cos(e+fx)^2(c-c\sin(e+fx))^{5/2}dx-\frac{2B\cos^3(e+fx)(c-c\sin(e+fx))^{5/2}}{11f}\right)$$

$$\Big\downarrow \text{3153}$$

$$ac\left(\frac{1}{11}(11A-5B)\left(\frac{4}{3}c\int \cos^2(e+fx)(c-c\sin(e+fx))^{3/2}dx+\frac{2c\cos^3(e+fx)(c-c\sin(e+fx))^{3/2}}{9f}\right)-\frac{2B}{\quad}\right.$$

$$\Big\downarrow \text{3042}$$

$$ac\left(\frac{1}{11}(11A-5B)\left(\frac{4}{3}c\int \cos(e+fx)^2(c-c\sin(e+fx))^{3/2}dx+\frac{2c\cos^3(e+fx)(c-c\sin(e+fx))^{3/2}}{9f}\right)-\frac{2B}{\quad}\right.$$

$$\Big\downarrow \text{3153}$$

$$ac\left(\frac{1}{11}(11A-5B)\left(\frac{4}{3}c\left(\frac{8}{7}c\int \cos^2(e+fx)\sqrt{c-c\sin(e+fx)}dx+\frac{2c\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f}\right)+\frac{2c\,c}{\quad}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$ac\left(\frac{1}{11}(11A-5B)\left(\frac{4}{3}c\left(\frac{8}{7}c\int \cos(e+fx)^2\sqrt{c-c\sin(e+fx)}dx+\frac{2c\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f}\right)+\frac{2c\,c}{\quad}\right.\right.$$

$$\Big\downarrow \text{3153}$$

$$ac\left(\frac{1}{11}(11A-5B)\left(\frac{4}{3}c\left(\frac{8}{7}c\left(\frac{4}{5}c\int \frac{\cos^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}dx+\frac{2c\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}\right)+\frac{2c\cos^3(e+fx)\sqrt{c-c\,s}}{7f}\right.\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$ac\left(\frac{1}{11}(11A-5B)\left(\frac{4}{3}c\left(\frac{8}{7}c\left(\frac{4}{5}c\int\frac{\cos(e+fx)^2}{\sqrt{c-c\sin(e+fx)}}dx+\frac{2c\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}\right)+\frac{2c\cos^3(e+fx)\sqrt{c-cs}}{7f}\right.\right.\right.$$

$$\Big\downarrow 3152$$

$$ac\left(\frac{1}{11}(11A-5B)\left(\frac{4}{3}c\left(\frac{8}{7}c\left(\frac{8c^2\cos^3(e+fx)}{15f(c-c\sin(e+fx))^{3/2}}+\frac{2c\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}\right)+\frac{2c\cos^3(e+fx)\sqrt{c-c\sin}}{7f}\right.\right.\right.$$

input
```
Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2),x
]
```

output
```
a*c*((-2*B*Cos[e + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(11*f) + ((11*A - 5*
B)*((2*c*Cos[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(9*f) + (4*c*((2*c*Cos
[e + f*x]^3*Sqrt[c - c*Sin[e + f*x]])/(7*f) + (8*c*((8*c^2*Cos[e + f*x]^3)
/(15*f*(c - c*Sin[e + f*x])^(3/2)) + (2*c*Cos[e + f*x]^3)/(5*f*Sqrt[c - c*
Sin[e + f*x]])))/7))/3))/11)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3153
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] &&
NeQ[m + p, 0]
```

rule 3335
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 7.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.60

method	result
default	$\frac{2(\sin(fx+e)-1)c^4(1+\sin(fx+e))^2 a\left(315B\cos(fx+e)^4+(-385A+1435B)\cos(fx+e)^2\sin(fx+e)+(1815A-3345B)\cos(fx+e)^2+\right.}{3465\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$
parts	$\frac{2Aa(\sin(fx+e)-1)c^4(1+\sin(fx+e))\left(5\sin(fx+e)^3-27\sin(fx+e)^2+71\sin(fx+e)-177\right)}{35\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}+\frac{2Ba(\sin(fx+e)-1)c^4(1+\sin(fx+e))}{}$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,method=_RET
URNVERBOSE)
```

output
```
2/3465*(sin(f*x+e)-1)*c^4*(1+sin(f*x+e))^2*a*(315*B*cos(f*x+e)^4+(-385*A+1
435*B)*cos(f*x+e)^2*sin(f*x+e)+(1815*A-3345*B)*cos(f*x+e)^2+(3916*A-4300*B
)*sin(f*x+e)-5324*A+4940*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.45

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^{7/2}\,dx = \frac{2\left(315\,Bac^3\cos\left(fx + e\right)^6 - 35\,(11\,A - 32\,B)ac^3\cos\left(fx + e\right)^5 + 5\,(209\,A - 221\,B)ac^3\cos\left(f\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, algo
rithm="fricas")
```

output
```
2/3465*(315*B*a*c^3*cos(f*x + e)^6 - 35*(11*A - 32*B)*a*c^3*cos(f*x + e)^5
 + 5*(209*A - 221*B)*a*c^3*cos(f*x + e)^4 + 2*(1243*A - 1195*B)*a*c^3*cos(
f*x + e)^3 - 32*(11*A - 5*B)*a*c^3*cos(f*x + e)^2 + 128*(11*A - 5*B)*a*c^3
*cos(f*x + e) + 256*(11*A - 5*B)*a*c^3 - (315*B*a*c^3*cos(f*x + e)^5 + 35*
(11*A - 23*B)*a*c^3*cos(f*x + e)^4 + 10*(143*A - 191*B)*a*c^3*cos(f*x + e)
^3 - 96*(11*A - 5*B)*a*c^3*cos(f*x + e)^2 - 128*(11*A - 5*B)*a*c^3*cos(f*x
 + e) - 256*(11*A - 5*B)*a*c^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f
*cos(f*x + e) - f*sin(f*x + e) + f)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)(-c\sin(fx+e)+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^
(7/2), x)
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.49

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx= -\frac{\sqrt{2}\left(6930\,Bac^3\cos\left(-\frac{3}{4}\pi+\frac{3}{2}fx+\frac{3}{2}e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-315\,Bac^3\cos\left(-\frac{11}{4}\pi+\frac{11}{2}fx+\frac{1}{2}\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, algo
rithm="giac")
```

output
```
-1/55440*sqrt(2)*(6930*B*a*c^3*cos(-3/4*pi + 3/2*f*x + 3/2*e)*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e)) - 315*B*a*c^3*cos(-11/4*pi + 11/2*f*x + 11/2*e)*sg
n(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 48510*(2*A*a*c^3*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e)) - B*a*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi
+ 1/2*f*x + 1/2*e) - 693*(16*A*a*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) -
 5*B*a*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-5/4*pi + 5/2*f*x + 5/
2*e) + 495*(10*A*a*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 9*B*a*c^3*sgn
(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-7/4*pi + 7/2*f*x + 7/2*e) - 385*(2*
A*a*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 5*B*a*c^3*sgn(sin(-1/4*pi +
1/2*f*x + 1/2*e)))*cos(-9/4*pi + 9/2*f*x + 9/2*e))*sqrt(c)/f
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^{7/2}\,dx = \int (A + B\,\sin(e + f\,x))\,(a + a\,\sin(e + f\,x))\,(c - c\,\sin(e + f\,x))^{7/2}\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2),x
)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2),
x)
```

Reduce [F]

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^{7/2}\,dx = \sqrt{c}\,a\,c^3\left(\left(\int \sqrt{-\sin(fx + e) + 1}dx\right)a - \left(\int \sqrt{-\sin(fx + e) + 1}\,\sin(fx + e)^5\,dx\right)b - \left(\int \sqrt{-\sin(fx + e) + 1}\,\sin(fx + e)^4\,dx\right)a + 2\left(\int \sqrt{-\sin(fx + e) + 1}\,\sin(fx + e)^4\,dx\right)b + 2\left(\int \sqrt{-\sin(fx + e) + 1}\,\sin(fx + e)^3\,dx\right)a - 2\left(\int \sqrt{-\sin(fx + e) + 1}\,\sin(fx + e)^2\,dx\right)b - 2\left(\int \sqrt{-\sin(fx + e) + 1}\,\sin(fx + e)\,dx\right)a + \left(\int \sqrt{-\sin(fx + e) + 1}\,\sin(fx + e)\,dx\right)b\right)$$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x)
```

output
```
sqrt(c)*a*c**3*(int(sqrt( - sin(e + f*x) + 1),x)*a - int(sqrt( - sin(e + f
*x) + 1)*sin(e + f*x)**5,x)*b - int(sqrt( - sin(e + f*x) + 1)*sin(e + f*x)
**4,x)*a + 2*int(sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**4,x)*b + 2*int(sq
rt( - sin(e + f*x) + 1)*sin(e + f*x)**3,x)*a - 2*int(sqrt( - sin(e + f*x)
+ 1)*sin(e + f*x)**2,x)*b - 2*int(sqrt( - sin(e + f*x) + 1)*sin(e + f*x),x
)*a + int(sqrt( - sin(e + f*x) + 1)*sin(e + f*x),x)*b)
```

3.82 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 157

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \frac{64a(3A-B)c^4\cos^3(e+fx)}{315f(c-c\sin(e+fx))^{3/2}}+\frac{16a(3A-B)c^3\cos^3(e+fx)}{105f\sqrt{c-c\sin(e+fx)}}+\frac{2a(3A-B)c^2\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{21f}-\frac{2aBc\cos^3(e+fx)(c-c\sin(e+fx))^{3/2}}{9f}$$

output
```
64/315*a*(3*A-B)*c^4*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(3/2)+16/105*a*(3*A-B
)*c^3*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(1/2)+2/21*a*(3*A-B)*c^2*cos(f*x+e)^
3*(c-c*sin(f*x+e))^(1/2)/f-2/9*a*B*c*cos(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/f
```

Mathematica [A] (verified)

Time = 4.71 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.78

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx =$$
$$-\frac{ac^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sqrt{c-c\sin(e+fx)}(-942A+664B+30(3A-8B)\cos(2(e+}{630f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

input
```
Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(
5/2),x]
```

output
```
-1/630*(a*c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e + f
*x]]*(-942*A + 664*B + 30*(3*A - 8*B)*Cos[2*(e + f*x)] + (648*A - 741*B)*S
in[e + f*x] + 35*B*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)
/2]))
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3042, 3446, 3042, 3335, 3042, 3153, 3042, 3153, 3042, 3152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3446}$$

$$ac\int \cos^2(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$ac\int \cos(e+fx)^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}dx$$

$$\Big\downarrow \text{3335}$$

$$ac\left(\frac{1}{3}(3A-B)\int \cos^2(e+fx)(c-c\sin(e+fx))^{3/2}dx-\frac{2B\cos^3(e+fx)(c-c\sin(e+fx))^{3/2}}{9f}\right)$$

$$\Big\downarrow \text{3042}$$

$$ac\left(\frac{1}{3}(3A-B)\int \cos(e+fx)^2(c-c\sin(e+fx))^{3/2}dx-\frac{2B\cos^3(e+fx)(c-c\sin(e+fx))^{3/2}}{9f}\right)$$

$$\Big\downarrow \text{3153}$$

$$ac\left(\frac{1}{3}(3A-B)\left(\frac{8}{7}c\int \cos^2(e+fx)\sqrt{c-c\sin(e+fx)}dx+\frac{2c\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f}\right)-\frac{2B\cos^3(e+\right.$$

$$\Big\downarrow \text{3042}$$

$$ac\left(\frac{1}{3}(3A-B)\left(\frac{8}{7}c\int \cos(e+fx)^2\sqrt{c-c\sin(e+fx)}dx+\frac{2c\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f}\right)-\frac{2B\cos^3(e+\right.$$

$$\Big\downarrow \text{3153}$$

$$ac\left(\frac{1}{3}(3A-B)\left(\frac{8}{7}c\left(\frac{4}{5}c\int \frac{\cos^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}dx+\frac{2c\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}\right)+\frac{2c\cos^3(e+fx)\sqrt{c-c\sin(e+f}}{7f}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$ac\left(\frac{1}{3}(3A-B)\left(\frac{8}{7}c\left(\frac{4}{5}c\int \frac{\cos(e+fx)^2}{\sqrt{c-c\sin(e+fx)}}dx+\frac{2c\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}\right)+\frac{2c\cos^3(e+fx)\sqrt{c-c\sin(e+f}}{7f}\right.\right.$$

$$\Big\downarrow \text{3152}$$

$$ac\left(\frac{1}{3}(3A-B)\left(\frac{8}{7}c\left(\frac{8c^2\cos^3(e+fx)}{15f(c-c\sin(e+fx))^{3/2}}+\frac{2c\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}\right)+\frac{2c\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f}\right.\right.$$

input
```
Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2),x
]
```

output
```
a*c*((-2*B*Cos[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(9*f) + ((3*A - B)*(
(2*c*Cos[e + f*x]^3*Sqrt[c - c*Sin[e + f*x]])/(7*f) + (8*c*((8*c^2*Cos[e +
 f*x]^3)/(15*f*(c - c*Sin[e + f*x])^(3/2)) + (2*c*Cos[e + f*x]^3)/(5*f*Sqr
t[c - c*Sin[e + f*x]])))/7))/3)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3153
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] &&
NeQ[m + p, 0]
```

rule 3335
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 6.96 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.66

method	result
default	$-\frac{2(\sin(fx+e)-1)c^3(1+\sin(fx+e))^2 a\left(-35B\cos(fx+e)^2\sin(fx+e)+(-45A+120B)\cos(fx+e)^2+(-162A+194B)\sin(fx+e)+2\right.}{315\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$
parts	$-\frac{2Aa(\sin(fx+e)-1)c^3(1+\sin(fx+e))\left(3\sin(fx+e)^2-14\sin(fx+e)+43\right)}{15\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}-\frac{2Ba(\sin(fx+e)-1)c^3(1+\sin(fx+e))\left(35\sin(fx+e\right.}{315\cos(fx+}$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
-2/315*(sin(f*x+e)-1)*c^3*(1+sin(f*x+e))^2*a*(-35*B*cos(f*x+e)^2*sin(f*x+e
)+(-45*A+120*B)*cos(f*x+e)^2+(-162*A+194*B)*sin(f*x+e)+258*A-226*B)/cos(f*
x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.55

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx=\frac{2\left(35\,Bac^2\cos\left(fx+e\right)^5+5\left(9\,A-10\,B\right)ac^2\cos\left(fx+e\right)^4+\left(117\,A-109\,B\right)ac^2\cos\left(fx+e\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algo
rithm="fricas")
```

output
```
2/315*(35*B*a*c^2*cos(f*x + e)^5 + 5*(9*A - 10*B)*a*c^2*cos(f*x + e)^4 + (
117*A - 109*B)*a*c^2*cos(f*x + e)^3 - 8*(3*A - B)*a*c^2*cos(f*x + e)^2 + 3
2*(3*A - B)*a*c^2*cos(f*x + e) + 64*(3*A - B)*a*c^2 + (35*B*a*c^2*cos(f*x
+ e)^4 - 5*(9*A - 17*B)*a*c^2*cos(f*x + e)^3 + 24*(3*A - B)*a*c^2*cos(f*x
+ e)^2 + 32*(3*A - B)*a*c^2*cos(f*x + e) + 64*(3*A - B)*a*c^2)*sin(f*x + e
))*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

Sympy [F]

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = a\left(\int Ac^2\sqrt{-c\sin(e+fx)+c}\,dx + \int\left(-Ac^2\sqrt{-c\sin(e+fx)+c}\sin(e+fx)\right)dx + \int\left(-Ac^2\sqrt{-c\sin(e+fx)+c}\sin^2(e+fx)\right)dx + \int Ac^2\sqrt{-c\sin(e+fx)+c}\sin^3(e+fx)\,dx + \int Bc^2\sqrt{-c\sin(e+fx)+c}\sin(e+fx)\,dx + \int\left(-Bc^2\sqrt{-c\sin(e+fx)+c}\sin^2(e+fx)\right)dx + \int\left(-Bc^2\sqrt{-c\sin(e+fx)+c}\sin^3(e+fx)\right)dx + \int Bc^2\sqrt{-c\sin(e+fx)+c}\sin^4(e+fx)\,dx\right)$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),x)
```

output
```
a*(Integral(A*c**2*sqrt(-c*sin(e + f*x) + c), x) + Integral(-A*c**2*sqrt(-
c*sin(e + f*x) + c)*sin(e + f*x), x) + Integral(-A*c**2*sqrt(-c*sin(e + f*
x) + c)*sin(e + f*x)**2, x) + Integral(A*c**2*sqrt(-c*sin(e + f*x) + c)*si
n(e + f*x)**3, x) + Integral(B*c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)
, x) + Integral(-B*c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x) + In
tegral(-B*c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**3, x) + Integral(B*
c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**4, x))
```

Maxima [F]

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)(-c\sin(fx+e)+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^
(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.67

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = -\frac{\sqrt{2}\left(35\,Bac^2\cos\left(-\frac{9}{4}\,\pi+\frac{9}{2}\,fx+\frac{9}{2}\,e\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+630\left(5\,Aac^2\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\right.\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algo
rithm="giac")
```

output
```
-1/2520*sqrt(2)*(35*B*a*c^2*cos(-9/4*pi + 9/2*f*x + 9/2*e)*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e)) + 630*(5*A*a*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))
- 2*B*a*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi + 1/2*f*x + 1
/2*e) + 210*(A*a*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + B*a*c^2*sgn(sin
(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-3/4*pi + 3/2*f*x + 3/2*e) - 126*(3*A*a*
c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - B*a*c^2*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e)))*cos(-5/4*pi + 5/2*f*x + 5/2*e) + 45*(2*A*a*c^2*sgn(sin(-1/4*p
i + 1/2*f*x + 1/2*e)) - 3*B*a*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos
(-7/4*pi + 7/2*f*x + 7/2*e))*sqrt(c)/f
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \int (A + B\,\sin(e+f\,x))\,\left(a+a\,\sin(e+f\,x)\right)\,\left(c-c\,\sin(e+f\,x)\right)^{5/2}\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2),x
)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2),
x)
```

Reduce [F]

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \sqrt{c}\,a\,c^2\Bigg(\left(\left(\int\sqrt{-\sin(fx+e)+1}dx\right)a + \left(\int\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^4\,dx\right)b + \left(\int\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^3\,dx\right)a - \left(\int\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^3\,dx\right)b - \left(\int\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)a - \left(\int\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)b - \left(\int\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)a + \left(\int\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)b\right)$$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x)
```

output
```
sqrt(c)*a*c**2*(int(sqrt( - sin(e + f*x) + 1),x)*a + int(sqrt( - sin(e + f
*x) + 1)*sin(e + f*x)**4,x)*b + int(sqrt( - sin(e + f*x) + 1)*sin(e + f*x)
**3,x)*a - int(sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**3,x)*b - int(sqrt(
- sin(e + f*x) + 1)*sin(e + f*x)**2,x)*a - int(sqrt( - sin(e + f*x) + 1)*s
in(e + f*x)**2,x)*b - int(sqrt( - sin(e + f*x) + 1)*sin(e + f*x),x)*a + in
t(sqrt( - sin(e + f*x) + 1)*sin(e + f*x),x)*b)
```

3.83 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 116

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx=\frac{8a(7A-B)c^3\cos^3(e+fx)}{105f(c-c\sin(e+fx))^{3/2}}+\frac{2a(7A-B)c^2\cos^3(e+fx)}{35f\sqrt{c-c\sin(e+fx)}}-\frac{2aBc\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f}$$

output
```
8/105*a*(7*A-B)*c^3*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(3/2)+2/35*a*(7*A-B)*c
^2*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(1/2)-2/7*a*B*c*cos(f*x+e)^3*(c-c*sin(f
*x+e))^(1/2)/f
```

Mathematica [A] (verified)

Time = 3.77 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx=\frac{ac\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(98A-59B+15B\cos(2(e+fx))+(-42A+66B}{105f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

input
```
Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(
3/2),x]
```

output
```
(a*c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(98*A - 59*B + 15*B*Cos[2*(e
+ f*x)] + (-42*A + 66*B)*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(105*f*(C
os[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}=0.222$, Rules used = {3042, 3446, 3042, 3335, 3042, 3153, 3042, 3152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3446}$$

$$ac\int \cos^2(e+fx)(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$ac\int \cos(e+fx)^2(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}dx$$

$$\Big\downarrow \text{3335}$$

$$ac\left(\frac{1}{7}(7A-B)\int \cos^2(e+fx)\sqrt{c-c\sin(e+fx)}dx-\frac{2B\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f}\right)$$

$$\Big\downarrow \text{3042}$$

$$ac\left(\frac{1}{7}(7A-B)\int \cos(e+fx)^2\sqrt{c-c\sin(e+fx)}dx-\frac{2B\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f}\right)$$

$$\Big\downarrow \text{3153}$$

$$ac\left(\frac{1}{7}(7A-B)\left(\frac{4}{5}c\int \frac{\cos^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}dx+\frac{2c\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}\right)-\frac{2B\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f}\right)$$

$$\Big\downarrow \text{3042}$$

$$ac\left(\frac{1}{7}(7A-B)\left(\frac{4}{5}c\int \frac{\cos(e+fx)^2}{\sqrt{c-c\sin(e+fx)}}dx+\frac{2c\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}\right)-\frac{2B\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f}\right)$$

$$\Big\downarrow \text{3152}$$

$$ac\left(\frac{1}{7}(7A-B)\left(\frac{8c^2\cos^3(e+fx)}{15f(c-c\sin(e+fx))^{3/2}}+\frac{2c\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}\right)-\frac{2B\cos^3(e+fx)\sqrt{c-c\sin(e+fx)}}{7f}\right)$$

input
```
Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2),x
]
```

output
```
a*c*((-2*B*Cos[e + f*x]^3*Sqrt[c - c*Sin[e + f*x]])/(7*f) + ((7*A - B)*((8
*c^2*Cos[e + f*x]^3)/(15*f*(c - c*Sin[e + f*x])^(3/2)) + (2*c*Cos[e + f*x]
^3)/(5*f*Sqrt[c - c*Sin[e + f*x]])))/7)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3153
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] &&
NeQ[m + p, 0]
```

rule 3335
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

method	result
default	$\frac{2(\sin(fx+e)-1)c^2(1+\sin(fx+e))^2 a\left(-15B\cos(fx+e)^2+\sin(fx+e)(21A-33B)-49A+37B\right)}{105\cos(fx+e)\sqrt{c-c\sin(fx+e)}\, f}$
parts	$\frac{2Aa(\sin(fx+e)-1)c^2(1+\sin(fx+e))(\sin(fx+e)-5)}{3\cos(fx+e)\sqrt{c-c\sin(fx+e)}\, f}+\frac{2Ba(\sin(fx+e)-1)c^2(1+\sin(fx+e))\left(15\sin(fx+e)^3-39\sin(fx+e)^2+52\right.}{105\cos(fx+e)\sqrt{c-c\sin(fx+e)}\, f}$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
2/105*(sin(f*x+e)-1)*c^2*(1+sin(f*x+e))^2*a*(-15*B*cos(f*x+e)^2+sin(f*x+e)
*(21*A-33*B)-49*A+37*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.59

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx =$$
$$-\frac{2\left(15\,Bac\cos\left(fx+e\right)^4-3\,(7\,A-6\,B)ac\cos\left(fx+e\right)^3+(7\,A-B)ac\cos\left(fx+e\right)^2-4\,(7\,A-B)ac\cos\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algo
rithm="fricas")
```

output
```
-2/105*(15*B*a*c*cos(f*x + e)^4 - 3*(7*A - 6*B)*a*c*cos(f*x + e)^3 + (7*A
- B)*a*c*cos(f*x + e)^2 - 4*(7*A - B)*a*c*cos(f*x + e) - 8*(7*A - B)*a*c -
 (15*B*a*c*cos(f*x + e)^3 + 3*(7*A - B)*a*c*cos(f*x + e)^2 + 4*(7*A - B)*a
*c*cos(f*x + e) + 8*(7*A - B)*a*c)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)
/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

Sympy [F]

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^{3/2}\,dx = a\left(\int Ac\sqrt{-c\sin(e + fx) + c}\,dx + \int \left(-Ac\sqrt{-c\sin(e + fx) + c}\sin^2(e + fx)\right)\,dx + \int Bc\sqrt{-c\sin(e + fx) + c}\sin(e + fx)\,dx + \int \left(-Bc\sqrt{-c\sin(e + fx) + c}\sin^3(e + fx)\right)\,dx\right)$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2),x)
```

output
```
a*(Integral(A*c*sqrt(-c*sin(e + f*x) + c), x) + Integral(-A*c*sqrt(-c*sin(
e + f*x) + c)*sin(e + f*x)**2, x) + Integral(B*c*sqrt(-c*sin(e + f*x) + c)
*sin(e + f*x), x) + Integral(-B*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**
3, x))
```

Maxima [F]

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^{3/2}\,dx = \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)(-c\sin(fx + e) + c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^
(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.67

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^{3/2}\,dx = \frac{\sqrt{2}\left(15\,Bac\cos\left(-\frac{7}{4}\,\pi + \frac{7}{2}\,fx + \frac{7}{2}\,e\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,fx + \frac{1}{2}\,e\right)\right) - 105\left(4\,Aac\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\right.\right.\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algo
rithm="giac")
```

output
```
1/420*sqrt(2)*(15*B*a*c*cos(-7/4*pi + 7/2*f*x + 7/2*e)*sgn(sin(-1/4*pi + 1
/2*f*x + 1/2*e)) - 105*(4*A*a*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - B*a*
c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi + 1/2*f*x + 1/2*e) - 35
*(2*A*a*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + B*a*c*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e)))*cos(-3/4*pi + 3/2*f*x + 3/2*e) + 21*(2*A*a*c*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e)) - B*a*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-
5/4*pi + 5/2*f*x + 5/2*e))*sqrt(c)/f
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^{3/2}\,dx = \int (A + B\,\sin(e + f\,x))\,(a + a\,\sin(e + f\,x))\,(c - c\,\sin(e + f\,x))^{3/2}\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2),x
)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2),
x)
```

Reduce [F]

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c - c\sin(e + fx))^{3/2}\,dx = \sqrt{c}\,ac\left(\left(\int \sqrt{-\sin(fx + e) + 1}dx\right)a - \left(\int \sqrt{-\sin(fx + e) + 1}\,\sin(fx + e)^3\,dx\right)b - \left(\int \sqrt{-\sin(fx + e) + 1}\,\sin(fx + e)^2\,dx\right)a + \left(\int \sqrt{-\sin(fx + e) + 1}\,\sin(fx + e)\,dx\right)b\right)$$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x)
```

output
```
sqrt(c)*a*c*(int(sqrt( - sin(e + f*x) + 1),x)*a - int(sqrt( - sin(e + f*x)
 + 1)*sin(e + f*x)**3,x)*b - int(sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2
,x)*a + int(sqrt( - sin(e + f*x) + 1)*sin(e + f*x),x)*b)
```

3.84 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))\sqrt{c-c\sin(e+$

Optimal result

Integrand size = 36, antiderivative size = 73

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \frac{2a(5A+B)c^2\cos^3(e+fx)}{15f(c-c\sin(e+fx))^{3/2}}-\frac{2aBc\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}$$

output
```
2/15*a*(5*A+B)*c^2*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(3/2)-2/5*a*B*c*cos(f*x
+e)^3/f/(c-c*sin(f*x+e))^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 191 vs. 2(73) = 146.

Time = 2.21 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.62

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = -\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)\left(32B-30\sqrt{2}A\sqrt{1+\cos(e+fx)}\right)+\sqrt{2}\sqrt{1+\cos(e+fx)}(5(2A+B)\cos\left(\frac{3}{2}(e+\right.\right.}{30\sqrt{2}f\sqrt{1+\cos(}$$

input
```
Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x
]],x]
```

output
```
-1/30*(a*(Cos[(e + f*x)/2]*(32*B - 30*Sqrt[2]*A*Sqrt[1 + Cos[e + f*x]]) +
Sqrt[2]*Sqrt[1 + Cos[e + f*x]]*(5*(2*A + B)*Cos[(3*(e + f*x))/2] + 3*B*Cos
[(5*(e + f*x))/2] - 2*(20*A + B + 2*(5*A + B)*Cos[e + f*x] - 3*B*Cos[2*(e
+ f*x)])*Sin[(e + f*x)/2]))*Sqrt[c - c*Sin[e + f*x]])/(Sqrt[2]*f*Sqrt[1 +
Cos[e + f*x]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3446, 3042, 3335, 3042, 3152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)\sqrt{c-c\sin(e+fx)}(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)\sqrt{c-c\sin(e+fx)}(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3446}$$

$$ac\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$ac\int \frac{\cos(e+fx)^2(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3335}$$

$$ac\left(\frac{1}{5}(5A+B)\int \frac{\cos^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}dx-\frac{2B\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$ac\left(\frac{1}{5}(5A+B)\int \frac{\cos(e+fx)^2}{\sqrt{c-c\sin(e+fx)}}dx-\frac{2B\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}\right)$$

$$\Big\downarrow \text{3152}$$

$$ac\left(\frac{2c(5A+B)\cos^3(e+fx)}{15f(c-c\sin(e+fx))^{3/2}}-\frac{2B\cos^3(e+fx)}{5f\sqrt{c-c\sin(e+fx)}}\right)$$

input
```
Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]],x]
```

output
```
a*c*((2*(5*A + B)*c*Cos[e + f*x]^3)/(15*f*(c - c*Sin[e + f*x])^(3/2)) - (2
*B*Cos[e + f*x]^3)/(5*f*Sqrt[c - c*Sin[e + f*x]]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3335
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

method	result
default	$-\frac{2(\sin(fx+e)-1)c(1+\sin(fx+e))^2 a(3B\sin(fx+e)+5A-2B)}{15\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$
parts	$-\frac{2Aa(\sin(fx+e)-1)(1+\sin(fx+e))c}{\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}-\frac{2Ba(\sin(fx+e)-1)c(1+\sin(fx+e))\left(3\sin(fx+e)^2-4\sin(fx+e)+8\right)}{15\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}-\frac{2a(A+B)(\sin(}{3}$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-2/15*(sin(f*x+e)-1)*c*(1+sin(f*x+e))^2*a*(3*B*sin(f*x+e)+5*A-2*B)/cos(f*x
+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.78

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx =$$

$$-\frac{2\left(3\,Ba\cos\left(fx+e\right)^3+(5\,A+4\,B)a\cos\left(fx+e\right)^2-(5\,A+B)a\cos\left(fx+e\right)-2\,(5\,A+B)a+(3\,Ba}{15\,(f\cos\left(fx+e\right)-f\,\mathrm{si}}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, algo
rithm="fricas")
```

output
```
-2/15*(3*B*a*cos(f*x + e)^3 + (5*A + 4*B)*a*cos(f*x + e)^2 - (5*A + B)*a*c
os(f*x + e) - 2*(5*A + B)*a + (3*B*a*cos(f*x + e)^2 - (5*A + B)*a*cos(f*x
+ e) - 2*(5*A + B)*a)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x +
 e) - f*sin(f*x + e) + f)
```

Sympy [F]

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = a\left(\int A\sqrt{-c\sin(e+fx)+c}\,dx + \int A\sqrt{-c\sin(e+fx)+c}\sin(e+fx)\,dx + \int B\sqrt{-c\sin(e+fx)+c}\sin(e+fx)\,dx + \int B\sqrt{-c\sin(e+fx)+c}\sin^2(e+fx)\,dx\right)$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2),x)
```

output
```
a*(Integral(A*sqrt(-c*sin(e + f*x) + c), x) + Integral(A*sqrt(-c*sin(e + f
*x) + c)*sin(e + f*x), x) + Integral(B*sqrt(-c*sin(e + f*x) + c)*sin(e + f
*x), x) + Integral(B*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x))
```

Maxima [F]

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)\sqrt{-c\sin(fx+e)+c}\,dx$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) +
 c), x)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.62

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))\sqrt{c - c\sin(e + fx)}\,dx =$$

$$-\frac{\sqrt{2}\left(30\,Aa\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) + 3\,Ba\cos\left(-\frac{5}{4}\pi + \frac{5}{2}fx + \frac{5}{2}e\right)\operatorname{sgn}(\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, algo
rithm="giac")
```

output
```
-1/30*sqrt(2)*(30*A*a*cos(-1/4*pi + 1/2*f*x + 1/2*e)*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e)) + 3*B*a*cos(-5/4*pi + 5/2*f*x + 5/2*e)*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e)) + 5*(2*A*a*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + B*a*sgn(s
in(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-3/4*pi + 3/2*f*x + 3/2*e))*sqrt(c)/f
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))\sqrt{c - c\sin(e + fx)}\,dx$$
$$= \int (A + B\,\sin(e + f\,x))\,(a + a\,\sin(e + f\,x))\,\sqrt{c - c\,\sin(e + f\,x)}\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2),x
)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2),
x)
```

Reduce [F]

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$$
$$=\sqrt{c}\,a\left(\left(\int \sqrt{-\sin(fx+e)+1}dx\right)a+\left(\int \sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)b + \left(\int \sqrt{-\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)a + \left(\int \sqrt{-\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)b\right)$$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)
```

output
```
sqrt(c)*a*(int(sqrt( - sin(e + f*x) + 1),x)*a + int(sqrt( - sin(e + f*x) +
 1)*sin(e + f*x)**2,x)*b + int(sqrt( - sin(e + f*x) + 1)*sin(e + f*x),x)*a
 + int(sqrt( - sin(e + f*x) + 1)*sin(e + f*x),x)*b)
```

3.85 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 122

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{2\sqrt{2}a(A+B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}f} - \frac{2a(3A+5B)\cos(e+fx)}{3f\sqrt{c-c\sin(e+fx)}} + \frac{2aB\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3cf}$$

output
```
2*2^(1/2)*a*(A+B)*arctanh(1/2*c^(1/2)*cos(f*x+e)*2^(1/2)/(c-c*sin(f*x+e))^
(1/2))/c^(1/2)/f-2/3*a*(3*A+5*B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(1/2)+2/3*a
*B*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/c/f
```

Mathematica [A] (verified)

Time = 4.68 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.36

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx =$$
$$-\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(6\sqrt{2}(A+B)\arctan\left(\frac{\sqrt{-c(1+\sin(e+fx))}}{\sqrt{2}\sqrt{c}}\right)\sqrt{-c(1+\sin(e+fx))}\right.}{3\sqrt{c}f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-}}$$

input
```
Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f
*x]],x]
```

output
```
-1/3*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(6*Sqrt[2]*(A + B)*ArcTan[Sq
rt[-(c*(1 + Sin[e + f*x]))]/(Sqrt[2]*Sqrt[c])]*Sqrt[-(c*(1 + Sin[e + f*x])
)] + Sqrt[c]*(6*A + 9*B - B*Cos[2*(e + f*x)] + 2*(3*A + 5*B)*Sin[e + f*x])
))/(Sqrt[c]*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x
]])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3042, 3446, 3042, 3337, 27, 3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}dx$$
$$\Big\downarrow \text{3446}$$
$$ac\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}dx$$

$$\Big\downarrow \; 3042$$

$$ac\int \frac{\cos(e+fx)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}dx$$

$$\Big\downarrow \; 3337$$

$$ac\left(\frac{2B\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3c^2f}-\frac{2\int -\frac{(3A+B)c+(3A+5B)\sin(e+fx)c}{2\sqrt{c-c\sin(e+fx)}}dx}{3c^2}\right)$$

$$\Big\downarrow \; 27$$

$$ac\left(\frac{\int \frac{(3A+B)c+(3A+5B)\sin(e+fx)c}{\sqrt{c-c\sin(e+fx)}}dx}{3c^2}+\frac{2B\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3c^2f}\right)$$

$$\Big\downarrow \; 3042$$

$$ac\left(\frac{\int \frac{(3A+B)c+(3A+5B)\sin(e+fx)c}{\sqrt{c-c\sin(e+fx)}}dx}{3c^2}+\frac{2B\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3c^2f}\right)$$

$$\Big\downarrow \; 3230$$

$$ac\left(\frac{6c(A+B)\int \frac{1}{\sqrt{c-c\sin(e+fx)}}dx-\frac{2c(3A+5B)\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}}{3c^2}+\frac{2B\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3c^2f}\right)$$

$$\Big\downarrow \; 3042$$

$$ac\left(\frac{6c(A+B)\int \frac{1}{\sqrt{c-c\sin(e+fx)}}dx-\frac{2c(3A+5B)\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}}{3c^2}+\frac{2B\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3c^2f}\right)$$

$$\Big\downarrow \; 3128$$

$$ac\left(\frac{-\frac{12c(A+B)\int \frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{f}-\frac{2c(3A+5B)\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}}{3c^2}+\frac{2B\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3c^2f}\right)$$

$$\Big\downarrow \; 219$$

$$ac\left(\frac{\frac{6\sqrt{2}\sqrt{c}(A+B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{f}-\frac{2c(3A+5B)\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}}{3c^2}+\frac{2B\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3c^2f}\right)$$

input
```
Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]],x
]
```

output
```
a*c*((2*B*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(3*c^2*f) + ((6*Sqrt[2]*(
A + B)*Sqrt[c]*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e +
f*x]])])/f - (2*(3*A + 5*B)*c*Cos[e + f*x])/(f*Sqrt[c - c*Sin[e + f*x]]))/
(3*c^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3337
```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d*Cos[e + f*x]*((
a + b*Sin[e + f*x])^(m + 2)/(b^2*f*(m + 3))), x] - Simp[1/(b^2*(m + 3))   I
nt[(a + b*Sin[e + f*x])^(m + 1)*(b*d*(m + 2) - a*c*(m + 3) + (b*c*(m + 3) -
 a*d*(m + 4))*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && GeQ[m, -3/2] && LtQ[m, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.30

method	result
default	$\frac{2(\sin(fx+e)-1)\sqrt{c(1+\sin(fx+e))}\,a\left(-3c^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)A-3c^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)B+B(c(}{3c^2\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$
parts	$-\frac{Aa(\sin(fx+e)-1)\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}+\frac{Ba(\sin(fx+e)-1)\sqrt{c(1+\sin(fx+e))}\left(-3c^{\frac{3}{2}}\sqrt{2}\,\operatorname{ar}\right.}{3c^2\cos(fx+e)}$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
2/3*(sin(f*x+e)-1)*(c*(1+sin(f*x+e)))^(1/2)*a*(-3*c^(3/2)*2^(1/2)*arctanh(
1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*A-3*c^(3/2)*2^(1/2)*arctanh(
1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*B+B*(c*(1+sin(f*x+e)))^(3/2)
+3*A*c*(c*(1+sin(f*x+e)))^(1/2)+3*B*c*(c*(1+sin(f*x+e)))^(1/2))/c^2/cos(f*
x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(105) = 210$.

Time = 0.09 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.08

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$

$$= \frac{\frac{3\sqrt{2}((A+B)ac\cos(fx+e)-(A+B)ac\sin(fx+e)+(A+B)ac)\log\left(-\frac{\cos(fx+e)^2+(\cos(fx+e)-2)\sin(fx+e)+\frac{2\sqrt{2}\sqrt{-c\sin(fx+e)+c}(\cos(fx+e)+\sin(fx}{\sqrt{c}}}{\cos(fx+e)^2+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right.}{\sqrt{c}}}{}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algo
rithm="fricas")
```

output
```
1/3*(3*sqrt(2)*((A + B)*a*c*cos(f*x + e) - (A + B)*a*c*sin(f*x + e) + (A +
 B)*a*c)*log(-(cos(f*x + e)^2 + (cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2
)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x + e) + 1)/sqrt(c) + 3*
cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(
f*x + e) - 2))/sqrt(c) + 2*(B*a*cos(f*x + e)^2 - (3*A + 4*B)*a*cos(f*x + e
) - (3*A + 5*B)*a - (B*a*cos(f*x + e) + (3*A + 5*B)*a)*sin(f*x + e))*sqrt(
-c*sin(f*x + e) + c))/(c*f*cos(f*x + e) - c*f*sin(f*x + e) + c*f)
```

Sympy [F]

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{\sqrt{c - c\sin(e + fx)}}\,dx = a\left(\int \frac{A}{\sqrt{-c\sin(e + fx) + c}}\,dx + \int \frac{A\sin(e + fx)}{\sqrt{-c\sin(e + fx) + c}}\,dx + \int \frac{B\sin(e + fx)}{\sqrt{-c\sin(e + fx) + c}}\,dx + \int \frac{B\sin^2(e + fx)}{\sqrt{-c\sin(e + fx) + c}}\,dx\right)$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

output
```
a*(Integral(A/sqrt(-c*sin(e + f*x) + c), x) + Integral(A*sin(e + f*x)/sqrt
(-c*sin(e + f*x) + c), x) + Integral(B*sin(e + f*x)/sqrt(-c*sin(e + f*x) +
 c), x) + Integral(B*sin(e + f*x)**2/sqrt(-c*sin(e + f*x) + c), x))
```

Maxima [F]

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{\sqrt{c - c\sin(e + fx)}}\,dx = \int \frac{(B\sin(fx + e) + A)(a\sin(fx + e) + a)}{\sqrt{-c\sin(fx + e) + c}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)/sqrt(-c*sin(f*x + e) +
 c), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))}{\sqrt{c-c\,\sin(e+f\,x)}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(1/2)
,x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(1/2)
, x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx =$$

$$-\frac{\sqrt{c}\,a\left(\left(\int \frac{\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)-1}dx\right)a+\left(\int \frac{\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^2}{\sin(fx+e)-1}dx\right)b+\left(\int \frac{\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)-1}dx\right)a+\left(\int \right.\right.}{c}$$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

output
```
( - sqrt(c)*a*(int(sqrt( - sin(e + f*x) + 1)/(sin(e + f*x) - 1),x)*a + int
((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x) - 1),x)*b + int
((sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x) - 1),x)*a + int((s
qrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x) - 1),x)*b))/c
```

3.86 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 115

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = -\frac{a(A+5B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}c^{3/2}f} + \frac{a(A+B)\cos(e+fx)}{f(c-c\sin(e+fx))^{3/2}} + \frac{2aB\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}$$

output
```
-1/2*a*(A+5*B)*arctanh(1/2*c^(1/2)*cos(f*x+e)*2^(1/2)/(c-c*sin(f*x+e))^(1/
2))*2^(1/2)/c^(3/2)/f+a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(3/2)+2*a*B*co
s(f*x+e)/c/f/(c-c*sin(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 6.20 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.37

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{a\sec(e+fx)\left(\sqrt{2}(A+5B)\arctan\left(\frac{\sqrt{-c(1+\sin(e+fx))}}{\sqrt{2}\sqrt{c}}\right)(\cos$$

input
```
Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(3/2),x]
```

output
```
(a*Sec[e + f*x]*(Sqrt[2]*(A + 5*B)*ArcTan[Sqrt[-(c*(1 + Sin[e + f*x]))]/(S
qrt[2]*Sqrt[c])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sqrt[-(c*(1 + Sin
[e + f*x]))] + 2*Sqrt[c]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*(A + 3*B
- 2*B*Sin[e + f*x])))/(2*c^(3/2)*f*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3042, 3446, 3042, 3336, 25, 3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}dx$$

$$\Big\downarrow \; 3446$$

$$ac\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$ac\int \frac{\cos(e+fx)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3336}$$

$$ac\left(\frac{\int -\frac{(A+3B)c+2B\sin(e+fx)c}{\sqrt{c-c\sin(e+fx)}}dx}{2c^3}+\frac{(A+B)\cos(e+fx)}{cf(c-c\sin(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \text{ 25}$$

$$ac\left(\frac{(A+B)\cos(e+fx)}{cf(c-c\sin(e+fx))^{3/2}}-\frac{\int \frac{(A+3B)c+2B\sin(e+fx)c}{\sqrt{c-c\sin(e+fx)}}dx}{2c^3}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$ac\left(\frac{(A+B)\cos(e+fx)}{cf(c-c\sin(e+fx))^{3/2}}-\frac{\int \frac{(A+3B)c+2B\sin(e+fx)c}{\sqrt{c-c\sin(e+fx)}}dx}{2c^3}\right)$$

$$\Big\downarrow \text{ 3230}$$

$$ac\left(\frac{(A+B)\cos(e+fx)}{cf(c-c\sin(e+fx))^{3/2}}-\frac{c(A+5B)\int \frac{1}{\sqrt{c-c\sin(e+fx)}}dx-\frac{4Bc\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}}{2c^3}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$ac\left(\frac{(A+B)\cos(e+fx)}{cf(c-c\sin(e+fx))^{3/2}}-\frac{c(A+5B)\int \frac{1}{\sqrt{c-c\sin(e+fx)}}dx-\frac{4Bc\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}}{2c^3}\right)$$

$$\Big\downarrow \text{ 3128}$$

$$ac\left(\frac{(A+B)\cos(e+fx)}{cf(c-c\sin(e+fx))^{3/2}}-\frac{-\frac{2c(A+5B)\int \frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{f}-\frac{4Bc\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}}{2c^3}\right)$$

$$\Big\downarrow \text{ 219}$$

$$ac\left(\frac{(A+B)\cos(e+fx)}{cf(c-c\sin(e+fx))^{3/2}}-\frac{\frac{\sqrt{2}\sqrt{c}(A+5B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{f}-\frac{4Bc\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}}{2c^3}\right)$$

input
```
Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(3/2)
,x]
```

output
```
a*c*(((A + B)*Cos[e + f*x])/(c*f*(c - c*Sin[e + f*x])^(3/2)) - ((Sqrt[2]*(
A + 5*B)*Sqrt[c]*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e
+ f*x]])])/f - (4*B*c*Cos[e + f*x])/(f*Sqrt[c - c*Sin[e + f*x]]))/(2*c^3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3336
```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Simp[1/(b^
3*(2*m + 3))   Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*
(2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2
- b^2, 0] && LtQ[m, -3/2]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(102) = 204$.

Time = 0.57 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.97

method	result
default	$\frac{a\left(A\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)\sin(fx+e)c+5B\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)\sin(fx+e)c-A\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(}{2\sqrt{}}\right.\right.}{}$
parts	$-\frac{Aa\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)c^2\sin(fx+e)-\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)c^2-2\sqrt{c(1+\sin(fx+e))}\,c^{\frac{3}{2}}\right)\sqrt{c(1+\sin}}{4c^{\frac{7}{2}}\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
1/2/c^(5/2)*a*(A*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1
/2))*sin(f*x+e)*c+5*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)
/c^(1/2))*sin(f*x+e)*c-A*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1
/2)/c^(1/2))*c-5*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^
(1/2))*c-4*(c*(1+sin(f*x+e)))^(1/2)*c^(1/2)*B*sin(f*x+e)+2*(c*(1+sin(f*x+e
)))^(1/2)*c^(1/2)*A+6*(c*(1+sin(f*x+e)))^(1/2)*c^(1/2)*B)*(c*(1+sin(f*x+e)
))^(1/2)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(102) = 204$.

Time = 0.09 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.77

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\frac{\sqrt{2}\left((A+5B)ac\cos(fx+e)^2-(A+5B)ac\cos(fx+e)-2(A+5B)ac+((A+5B)a\right.}{}}{}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algo
rithm="fricas")
```

output
```
1/4*(sqrt(2)*((A + 5*B)*a*c*cos(f*x + e)^2 - (A + 5*B)*a*c*cos(f*x + e) -
2*(A + 5*B)*a*c + ((A + 5*B)*a*c*cos(f*x + e) + 2*(A + 5*B)*a*c)*sin(f*x +
 e))*log(-(cos(f*x + e)^2 + (cos(f*x + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sq
rt(-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x + e) + 1)/sqrt(c) + 3*cos(
f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x
+ e) - 2))/sqrt(c) - 4*(2*B*a*cos(f*x + e)^2 + (A + 3*B)*a*cos(f*x + e) +
(A + B)*a - (2*B*a*cos(f*x + e) - (A + B)*a)*sin(f*x + e))*sqrt(-c*sin(f*x
 + e) + c))/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e) - 2*c^2*f + (c^2*f*
cos(f*x + e) + 2*c^2*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = a\left(\int \frac{A}{-c\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+c\sqrt{-c\sin(e}}\right.$$

$$+\int \frac{A\sin(e+fx)}{-c\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+c\sqrt{-c\sin(e+fx)+c}}\,dx$$

$$+\int \frac{B\sin(e+fx)}{-c\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+c\sqrt{-c\sin(e+fx)+c}}\,dx$$

$$\left.+\int \frac{B\sin^2(e+fx)}{-c\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+c\sqrt{-c\sin(e+fx)+c}}\,dx\right)$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),x)
```

output
```
a*(Integral(A/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c*sqrt(-c*sin(e
 + f*x) + c)), x) + Integral(A*sin(e + f*x)/(-c*sqrt(-c*sin(e + f*x) + c)*
sin(e + f*x) + c*sqrt(-c*sin(e + f*x) + c)), x) + Integral(B*sin(e + f*x)/
(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c*sqrt(-c*sin(e + f*x) + c)),
 x) + Integral(B*sin(e + f*x)**2/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x
) + c*sqrt(-c*sin(e + f*x) + c)), x))
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)}{(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^
(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(3/2)
,x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(3/2)
, x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=\frac{\sqrt{c}\,a\left(\left(\int \frac{\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)^2-2\sin(fx+e)+1}dx\right)a+\left(\int \frac{\sqrt{-\sin(fx+e)+1}\,\mathrm{si}}{\sin(fx+e)^2-2\sin(}\right.\right.}{}$$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*a*(int(sqrt( - sin(e + f*x) + 1)/(sin(e + f*x)**2 - 2*sin(e + f*x
) + 1),x)*a + int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x
)**2 - 2*sin(e + f*x) + 1),x)*b + int((sqrt( - sin(e + f*x) + 1)*sin(e + f
*x))/(sin(e + f*x)**2 - 2*sin(e + f*x) + 1),x)*a + int((sqrt( - sin(e + f*
x) + 1)*sin(e + f*x))/(sin(e + f*x)**2 - 2*sin(e + f*x) + 1),x)*b))/c**2
```

3.87 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 126

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = -\frac{a(A-7B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}c^{5/2}f} + \frac{a(A+B)\cos(e+fx)}{2f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+9B)\cos(e+fx)}{8cf(c-c\sin(e+fx))^{3/2}}$$

output
```
-1/16*a*(A-7*B)*arctanh(1/2*c^(1/2)*cos(f*x+e)*2^(1/2)/(c-c*sin(f*x+e))^(1
/2))*2^(1/2)/c^(5/2)/f+1/2*a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(5/2)-1/8
*a*(A+9*B)*cos(f*x+e)/c/f/(c-c*sin(f*x+e))^(3/2)
```

Mathematica [A] (verified)

Time = 8.01 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.58

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = -\frac{a(-1+\sin(e+fx))(1+\sin(e+fx))\left(\sqrt{2}(A-7B)\arctan\left(\frac{\sqrt{-c(1+\sin(e+fx))}}{\sqrt{2}\sqrt{c}}\right)\sec(e+fx)\sqrt{-c(1+\sin}\right.}{16c^{5/2}f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\sqrt{c-c}}$$

input
```
Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(5/2),x]
```

output
```
-1/16*(a*(-1 + Sin[e + f*x])*(1 + Sin[e + f*x])*(Sqrt[2]*(A - 7*B)*ArcTan[
Sqrt[-(c*(1 + Sin[e + f*x]))]/(Sqrt[2]*Sqrt[c])]*Sec[e + f*x]*Sqrt[-(c*(1
+ Sin[e + f*x]))] + (2*Sqrt[c]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(3*A
- 5*B + (A + 9*B)*Sin[e + f*x]))/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5))
/(c^(5/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*Sqrt[c - c*Sin[e + f*x
]])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3042, 3446, 3042, 3336, 25, 3042, 3229, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}dx$$

$$\Big\downarrow \textbf{3446}$$

$$ac\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}dx$$

$$\Big\downarrow \textbf{3042}$$

$$ac\int \frac{\cos(e+fx)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}dx$$

$$\Big\downarrow \textbf{3336}$$

$$ac\left(\frac{\int -\frac{(A+5B)c+4B\sin(e+fx)c}{(c-c\sin(e+fx))^{3/2}}dx}{4c^3}+\frac{(A+B)\cos(e+fx)}{2cf(c-c\sin(e+fx))^{5/2}}\right)$$

$$\Big\downarrow \text{25}$$

$$ac\left(\frac{(A+B)\cos(e+fx)}{2cf(c-c\sin(e+fx))^{5/2}}-\frac{\int\frac{(A+5B)c+4B\sin(e+fx)c}{(c-c\sin(e+fx))^{3/2}}dx}{4c^3}\right)$$

$$\Big\downarrow \text{3042}$$

$$ac\left(\frac{(A+B)\cos(e+fx)}{2cf(c-c\sin(e+fx))^{5/2}}-\frac{\int\frac{(A+5B)c+4B\sin(e+fx)c}{(c-c\sin(e+fx))^{3/2}}dx}{4c^3}\right)$$

$$\Big\downarrow \text{3229}$$

$$ac\left(\frac{(A+B)\cos(e+fx)}{2cf(c-c\sin(e+fx))^{5/2}}-\frac{\frac{1}{4}(A-7B)\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx+\frac{c(A+9B)\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}}{4c^3}\right)$$

$$\Big\downarrow \text{3042}$$

$$ac\left(\frac{(A+B)\cos(e+fx)}{2cf(c-c\sin(e+fx))^{5/2}}-\frac{\frac{1}{4}(A-7B)\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx+\frac{c(A+9B)\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}}{4c^3}\right)$$

$$\Big\downarrow \text{3128}$$

$$ac\left(\frac{(A+B)\cos(e+fx)}{2cf(c-c\sin(e+fx))^{5/2}}-\frac{\frac{c(A+9B)\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(A-7B)\int\frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{2f}}{4c^3}\right)$$

$$\Big\downarrow \text{219}$$

$$ac\left(\frac{(A+B)\cos(e+fx)}{2cf(c-c\sin(e+fx))^{5/2}}-\frac{\frac{(A-7B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}\sqrt{c}f}+\frac{c(A+9B)\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}}{4c^3}\right)$$

input
```
Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(5/2)
,x]
```

output
```
a*c*(((A + B)*Cos[e + f*x])/(2*c*f*(c - c*Sin[e + f*x])^(5/2)) - (((A - 7*
B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(2*
Sqrt[2]*Sqrt[c]*f) + ((A + 9*B)*c*Cos[e + f*x])/(2*f*(c - c*Sin[e + f*x])^
(3/2)))/(4*c^3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 3336
```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Simp[1/(b^
3*(2*m + 3))   Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*
(2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2
- b^2, 0] && LtQ[m, -3/2]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(107) = 214$.

Time = 0.59 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.12

method	result
default	$-\frac{a\left(\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,c^{2}(A-7B)\cos(fx+e)^{2}+2\sin(fx+e)\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,c^{2}(A-7B)+2A(c+c\ldots\right.}{}$
parts	$\frac{Aa\left(-3\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,\sin(fx+e)^{2}c^{2}+6\sqrt{c(1+\sin(fx+e))}\,c^{\frac{3}{2}}\sin(fx+e)+6\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)\ldots\right.}{32c^{\frac{9}{2}}(\sin(fx+e)-1)\cos(fx+e)\sqrt{c-c\sin\ldots}}$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/16/c^(9/2)*a*(arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/
2)*c^2*(A-7*B)*cos(f*x+e)^2+2*sin(f*x+e)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2
)*2^(1/2)/c^(1/2))*2^(1/2)*c^2*(A-7*B)+2*A*(c+c*sin(f*x+e))^(3/2)*c^(1/2)+
4*A*(c+c*sin(f*x+e))^(1/2)*c^(3/2)-2*A*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*
2^(1/2)/c^(1/2))*2^(1/2)*c^2+18*B*(c+c*sin(f*x+e))^(3/2)*c^(1/2)-28*B*(c+c
*sin(f*x+e))^(1/2)*c^(3/2)+14*B*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)
/c^(1/2))*2^(1/2)*c^2)*(c*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)/cos(f*x+e)/
(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(107) = 214$.

Time = 0.14 (sec) , antiderivative size = 394, normalized size of antiderivative = 3.13

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{5/2}} dx =$$
$$-\frac{\sqrt{2}((A - 7B)a\cos(fx + e)^3 + 3(A - 7B)a\cos(fx + e)^2 - 2(A - 7B)a\cos(fx + e) - 4(A - 7B)a - }{}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algo
rithm="fricas")
```

output
```
-1/32*(sqrt(2)*((A - 7*B)*a*cos(f*x + e)^3 + 3*(A - 7*B)*a*cos(f*x + e)^2
- 2*(A - 7*B)*a*cos(f*x + e) - 4*(A - 7*B)*a - ((A - 7*B)*a*cos(f*x + e)^2
 - 2*(A - 7*B)*a*cos(f*x + e) - 4*(A - 7*B)*a)*sin(f*x + e))*sqrt(c)*log(-
(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x +
 e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*
x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x
+ e) - 2)) - 4*((A + 9*B)*a*cos(f*x + e)^2 - (3*A - 5*B)*a*cos(f*x + e) -
4*(A + B)*a - ((A + 9*B)*a*cos(f*x + e) + 4*(A + B)*a)*sin(f*x + e))*sqrt(
-c*sin(f*x + e) + c))/(c^3*f*cos(f*x + e)^3 + 3*c^3*f*cos(f*x + e)^2 - 2*c
^3*f*cos(f*x + e) - 4*c^3*f - (c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e)
 - 4*c^3*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{5/2}} dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)}{(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^
(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx=\int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))}{(c-c\,\sin(e+f\,x))^{5/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(5/2)
,x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(5/2)
, x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx =$$
$$-\frac{\sqrt{c}\,a\left(\left(\int \frac{\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)^3-3\sin(fx+e)^2+3\sin(fx+e)-1}dx\right)a+\left(\int \frac{\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^2}{\sin(fx+e)^3-3\sin(fx+e)^2+3\sin(fx+e)-1}dx\right)b+\left(\int \frac{\sqrt{}}{\sin(fx+e}\right.\right.}{c^3}$$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)
```

output
```
( - sqrt(c)*a*(int(sqrt( - sin(e + f*x) + 1)/(sin(e + f*x)**3 - 3*sin(e +
f*x)**2 + 3*sin(e + f*x) - 1),x)*a + int((sqrt( - sin(e + f*x) + 1)*sin(e
+ f*x)**2)/(sin(e + f*x)**3 - 3*sin(e + f*x)**2 + 3*sin(e + f*x) - 1),x)*b
 + int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**3 - 3*sin(e
 + f*x)**2 + 3*sin(e + f*x) - 1),x)*a + int((sqrt( - sin(e + f*x) + 1)*sin
(e + f*x))/(sin(e + f*x)**3 - 3*sin(e + f*x)**2 + 3*sin(e + f*x) - 1),x)*b
))/c**3
```

3.88 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 163

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = -\frac{a(A-3B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}c^{7/2}f}+\frac{a(A+B)\cos(e+fx)}{3f(c-c\sin(e+fx))^{7/2}}-\frac{a(A+13B)\cos(e+fx)}{24cf(c-c\sin(e+fx))^{5/2}}-\frac{a(A-3B)\cos(e+fx)}{32c^2f(c-c\sin(e+fx))^{3/2}}$$

output
```
-1/64*a*(A-3*B)*arctanh(1/2*c^(1/2)*cos(f*x+e)*2^(1/2)/(c-c*sin(f*x+e))^(1
/2))*2^(1/2)/c^(7/2)/f+1/3*a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(7/2)-1/2
4*a*(A+13*B)*cos(f*x+e)/c/f/(c-c*sin(f*x+e))^(5/2)-1/32*a*(A-3*B)*cos(f*x+
e)/c^2/f/(c-c*sin(f*x+e))^(3/2)
```

Mathematica [A] (warning: unable to verify)

Time = 9.44 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.33

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx =$$

$$-\frac{a(-1+\sin(e+fx))(1+\sin(e+fx))\left(3\sqrt{2}(A-3B)\arctan\left(\frac{\sqrt{-c(1+\sin(e+fx))}}{\sqrt{2}\sqrt{c}}\right)\sec(e+fx)\sqrt{-c(1+\sin}\right.}{192c^{7/2}f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx\right.\right.}$$

input
```
Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(7/2),x]
```

output
```
-1/192*(a*(-1 + Sin[e + f*x])*(1 + Sin[e + f*x])*(3*Sqrt[2]*(A - 3*B)*ArcT
an[Sqrt[-(c*(1 + Sin[e + f*x]))]/(Sqrt[2]*Sqrt[c])]*Sec[e + f*x]*Sqrt[-(c*
(1 + Sin[e + f*x]))] + (Sqrt[c]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(47*
A - 13*B + 3*(A - 3*B)*Cos[2*(e + f*x)] + 4*(5*A + 17*B)*Sin[e + f*x]))/(C
os[(e + f*x)/2] - Sin[(e + f*x)/2])^7))/(c^(7/2)*f*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^2*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3446, 3042, 3336, 25, 3042, 3229, 3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{ 3446}$$

$$ac\int \frac{\cos^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$ac\int \frac{\cos(e+fx)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}dx$$

$$\Big\downarrow \text{3336}$$

$$ac\left(\frac{\int -\frac{(A+7B)c+6B\sin(e+fx)c}{(c-c\sin(e+fx))^{5/2}}dx}{6c^3}+\frac{(A+B)\cos(e+fx)}{3cf(c-c\sin(e+fx))^{7/2}}\right)$$

$$\Big\downarrow \text{25}$$

$$ac\left(\frac{(A+B)\cos(e+fx)}{3cf(c-c\sin(e+fx))^{7/2}}-\frac{\int \frac{(A+7B)c+6B\sin(e+fx)c}{(c-c\sin(e+fx))^{5/2}}dx}{6c^3}\right)$$

$$\Big\downarrow \text{3042}$$

$$ac\left(\frac{(A+B)\cos(e+fx)}{3cf(c-c\sin(e+fx))^{7/2}}-\frac{\int \frac{(A+7B)c+6B\sin(e+fx)c}{(c-c\sin(e+fx))^{5/2}}dx}{6c^3}\right)$$

$$\Big\downarrow \text{3229}$$

$$ac\left(\frac{(A+B)\cos(e+fx)}{3cf(c-c\sin(e+fx))^{7/2}}-\frac{\frac{3}{8}(A-3B)\int \frac{1}{(c-c\sin(e+fx))^{3/2}}dx+\frac{c(A+13B)\cos(e+fx)}{4f(c-c\sin(e+fx))^{5/2}}}{6c^3}\right)$$

$$\Big\downarrow \text{3042}$$

$$ac\left(\frac{(A+B)\cos(e+fx)}{3cf(c-c\sin(e+fx))^{7/2}}-\frac{\frac{3}{8}(A-3B)\int \frac{1}{(c-c\sin(e+fx))^{3/2}}dx+\frac{c(A+13B)\cos(e+fx)}{4f(c-c\sin(e+fx))^{5/2}}}{6c^3}\right)$$

$$\Big\downarrow \text{3129}$$

$$ac\left(\frac{(A+B)\cos(e+fx)}{3cf(c-c\sin(e+fx))^{7/2}}-\frac{\frac{3}{8}(A-3B)\left(\frac{\int \frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{4c}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}\right)+\frac{c(A+13B)\cos(e+fx)}{4f(c-c\sin(e+fx))^{5/2}}}{6c^3}\right)$$

$$\Big\downarrow \text{3042}$$

$$ac\left(\frac{(A+B)\cos(e+fx)}{3cf(c-c\sin(e+fx))^{7/2}}-\frac{\frac{3}{8}(A-3B)\left(\frac{\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{4c}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}\right)+\frac{c(A+13B)\cos(e+fx)}{4f(c-c\sin(e+fx))^{5/2}}}{6c^3}\right)$$

$\big\downarrow$ 3128

$$ac\left(\frac{(A+B)\cos(e+fx)}{3cf(c-c\sin(e+fx))^{7/2}}-\frac{\frac{3}{8}(A-3B)\left(\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}-\frac{\int\frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{2cf}\right)+\frac{c(A+13B)\cos(e+fx)}{4f(c-c\sin(e+fx))^{5/2}}}{6c^3}\right)$$

$\big\downarrow$ 219

$$ac\left(\frac{(A+B)\cos(e+fx)}{3cf(c-c\sin(e+fx))^{7/2}}-\frac{\frac{3}{8}(A-3B)\left(\frac{\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}c^{3/2}f}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}\right)+\frac{c(A+13B)\cos(e+fx)}{4f(c-c\sin(e+fx))^{5/2}}}{6c^3}\right)$$

input
```
Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(7/2)
,x]
```

output
```
a*c*(((A + B)*Cos[e + f*x])/(3*c*f*(c - c*Sin[e + f*x])^(7/2)) - (((A + 13
*B)*c*Cos[e + f*x])/(4*f*(c - c*Sin[e + f*x])^(5/2)) + (3*(A - 3*B)*(ArcTa
nh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]/(2*Sqrt[2]*c
^(3/2)*f) + Cos[e + f*x]/(2*f*(c - c*Sin[e + f*x])^(3/2))))/8)/(6*c^3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 3336
```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Simp[1/(b^
3*(2*m + 3))   Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*
(2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2
- b^2, 0] && LtQ[m, -3/2]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(140) = 280$.

Time = 0.60 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.16

method	result
default	$-\frac{a\left(3\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,c^{4}(A-3B)\cos(fx+e)^{2}\sin(fx+e)-9\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,c^{4}(A-3B)\cos(fx-$
parts	Expression too large to display

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/192*a*(3*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^
4*(A-3*B)*cos(f*x+e)^2*sin(f*x+e)-9*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(
1/2)/c^(1/2))*2^(1/2)*c^4*(A-3*B)*cos(f*x+e)^2-12*arctanh(1/2*(c+c*sin(f*x
+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^4*(A-3*B)*sin(f*x+e)+6*A*(c+c*sin(f*
x+e))^(5/2)*c^(3/2)-32*A*(c+c*sin(f*x+e))^(3/2)*c^(5/2)-24*A*(c+c*sin(f*x+
e))^(1/2)*c^(7/2)-18*B*(c+c*sin(f*x+e))^(5/2)*c^(3/2)-32*B*(c+c*sin(f*x+e)
)^(3/2)*c^(5/2)+72*B*(c+c*sin(f*x+e))^(1/2)*c^(7/2)+12*A*2^(1/2)*arctanh(1
/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^4-36*B*2^(1/2)*arctanh(1/2*(c
+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^4)*(c*(1+sin(f*x+e)))^(1/2)/c^(15/
2)/(sin(f*x+e)-1)^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(140) = 280$.

Time = 0.11 (sec) , antiderivative size = 490, normalized size of antiderivative = 3.01

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{7/2}}\,dx =$$
$$-\frac{3\sqrt{2}((A - 3B)a\cos(fx + e)^4 - 3(A - 3B)a\cos(fx + e)^3 - 8(A - 3B)a\cos(fx + e)^2 + 4(A - 3B)}{}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algo
rithm="fricas")
```

output
```
-1/384*(3*sqrt(2)*((A - 3*B)*a*cos(f*x + e)^4 - 3*(A - 3*B)*a*cos(f*x + e)
^3 - 8*(A - 3*B)*a*cos(f*x + e)^2 + 4*(A - 3*B)*a*cos(f*x + e) + 8*(A - 3*
B)*a + ((A - 3*B)*a*cos(f*x + e)^3 + 4*(A - 3*B)*a*cos(f*x + e)^2 - 4*(A -
 3*B)*a*cos(f*x + e) - 8*(A - 3*B)*a)*sin(f*x + e))*sqrt(c)*log(-(c*cos(f*
x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin
(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) +
2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)
) - 4*(3*(A - 3*B)*a*cos(f*x + e)^3 - (7*A + 43*B)*a*cos(f*x + e)^2 + 2*(1
1*A - B)*a*cos(f*x + e) + 32*(A + B)*a + (3*(A - 3*B)*a*cos(f*x + e)^2 + 2
*(5*A + 17*B)*a*cos(f*x + e) + 32*(A + B)*a)*sin(f*x + e))*sqrt(-c*sin(f*x
 + e) + c))/(c^4*f*cos(f*x + e)^4 - 3*c^4*f*cos(f*x + e)^3 - 8*c^4*f*cos(f
*x + e)^2 + 4*c^4*f*cos(f*x + e) + 8*c^4*f + (c^4*f*cos(f*x + e)^3 + 4*c^4
*f*cos(f*x + e)^2 - 4*c^4*f*cos(f*x + e) - 8*c^4*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx=\int \frac{(B\sin{(fx+e)}+A)(a\sin{(fx+e)}+a)}{(-c\sin{(fx+e)}+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^
(7/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx=\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{7/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(7/2)
,x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(7/2)
, x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx=\frac{\sqrt{c}\,a\left(\left(\int \frac{\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)^4-4\sin(fx+e)^3+6\sin(fx+e)^2-4\sin(fx+e)+1}dx\right)a$$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x)
```

output
```
(sqrt(c)*a*(int(sqrt( - sin(e + f*x) + 1)/(sin(e + f*x)**4 - 4*sin(e + f*x
)**3 + 6*sin(e + f*x)**2 - 4*sin(e + f*x) + 1),x)*a + int((sqrt( - sin(e +
 f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x)**4 - 4*sin(e + f*x)**3 + 6*sin(e
 + f*x)**2 - 4*sin(e + f*x) + 1),x)*b + int((sqrt( - sin(e + f*x) + 1)*sin
(e + f*x))/(sin(e + f*x)**4 - 4*sin(e + f*x)**3 + 6*sin(e + f*x)**2 - 4*si
n(e + f*x) + 1),x)*a + int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e
 + f*x)**4 - 4*sin(e + f*x)**3 + 6*sin(e + f*x)**2 - 4*sin(e + f*x) + 1),x
)*b))/c**4
```

3.89 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 210

$$\begin{aligned}\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c\\ -c\sin(e+fx))^{7/2}\,dx &= \frac{256a^2(13A-3B)c^6\cos^5(e+fx)}{15015f(c-c\sin(e+fx))^{5/2}}\\ &+\frac{64a^2(13A-3B)c^5\cos^5(e+fx)}{3003f(c-c\sin(e+fx))^{3/2}}+\frac{8a^2(13A-3B)c^4\cos^5(e+fx)}{429f\sqrt{c-c\sin(e+fx)}}\\ &+\frac{2a^2(13A-3B)c^3\cos^5(e+fx)\sqrt{c-c\sin(e+fx)}}{143f}\\ &-\frac{2a^2Bc^2\cos^5(e+fx)(c-c\sin(e+fx))^{3/2}}{13f}\end{aligned}$$

output
```
256/15015*a^2*(13*A-3*B)*c^6*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)+64/3003
*a^2*(13*A-3*B)*c^5*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(3/2)+8/429*a^2*(13*A-
3*B)*c^4*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(1/2)+2/143*a^2*(13*A-3*B)*c^3*co
s(f*x+e)^5*(c-c*sin(f*x+e))^(1/2)/f-2/13*a^2*B*c^2*cos(f*x+e)^5*(c-c*sin(f
*x+e))^(3/2)/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1355 vs. $2(210) = 420$.

Time = 13.36 (sec) , antiderivative size = 1355, normalized size of antiderivative = 6.45

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^(7/2),x]
```

output
```
((7*A - 2*B)*Cos[(e + f*x)/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^
(7/2))/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2])^4) - ((4*A + B)*Cos[(3*(e + f*x))/2]*(a + a*Sin[e + f*x])^
2*(c - c*Sin[e + f*x])^(7/2))/(32*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((22*A - 7*B)*Cos[(5*(e + f*x
))/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2))/(160*f*(Cos[(e +
f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + (
(A - 4*B)*Cos[(7*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])
^(7/2))/(112*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^4) + (A*Cos[(9*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c
- c*Sin[e + f*x])^(7/2))/(48*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((2*A - 3*B)*Cos[(11*(e + f*x))/2]
*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2))/(352*f*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + (B*Cos
[(13*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2))/(416
*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2])^4) + ((7*A - 2*B)*Sin[(e + f*x)/2]*(a + a*Sin[e + f*x])^2*(c - c*Si
n[e + f*x])^(7/2))/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^4) + ((4*A + B)*(a + a*Sin[e + f*x])^2*(c - c*
Sin[e + f*x])^(7/2)*Sin[(3*(e + f*x))/2])/(32*f*(Cos[(e + f*x)/2] - Sin...
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.90, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 3446, 3042, 3335, 3042, 3153, 3042, 3153, 3042, 3153, 3042, 3152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^2(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^2(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3446}$$

$$a^2c^2\int \cos^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\int \cos(e+fx)^4(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}dx$$

$$\Big\downarrow \text{3335}$$

$$a^2c^2\left(\frac{1}{13}(13A-3B)\int \cos^4(e+fx)(c-c\sin(e+fx))^{3/2}dx-\frac{2B\cos^5(e+fx)(c-c\sin(e+fx))^{3/2}}{13f}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left(\frac{1}{13}(13A-3B)\int \cos(e+fx)^4(c-c\sin(e+fx))^{3/2}dx-\frac{2B\cos^5(e+fx)(c-c\sin(e+fx))^{3/2}}{13f}\right)$$

$$\Big\downarrow \text{3153}$$

$$a^2c^2\left(\frac{1}{13}(13A-3B)\left(\frac{12}{11}c\int \cos^4(e+fx)\sqrt{c-c\sin(e+fx)}dx+\frac{2c\cos^5(e+fx)\sqrt{c-c\sin(e+fx)}}{11f}\right)-\frac{2B\,c}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left(\frac{1}{13}(13A-3B)\left(\frac{12}{11}c\int\cos(e+fx)^4\sqrt{c-c\sin(e+fx)}dx+\frac{2c\cos^5(e+fx)\sqrt{c-c\sin(e+fx)}}{11f}\right)-\frac{2Bc}{}\right.$$

$\Big\downarrow$ 3153

$$a^2c^2\left(\frac{1}{13}(13A-3B)\left(\frac{12}{11}c\left(\frac{8}{9}c\int\frac{\cos^4(e+fx)}{\sqrt{c-c\sin(e+fx)}}dx+\frac{2c\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}\right)+\frac{2c\cos^5(e+fx)\sqrt{c-c\sin}}{11f}\right.\right.$$

$\Big\downarrow$ 3042

$$a^2c^2\left(\frac{1}{13}(13A-3B)\left(\frac{12}{11}c\left(\frac{8}{9}c\int\frac{\cos(e+fx)^4}{\sqrt{c-c\sin(e+fx)}}dx+\frac{2c\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}\right)+\frac{2c\cos^5(e+fx)\sqrt{c-c\sin}}{11f}\right.\right.$$

$\Big\downarrow$ 3153

$$a^2c^2\left(\frac{1}{13}(13A-3B)\left(\frac{12}{11}c\left(\frac{8}{9}c\left(\frac{4}{7}c\int\frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{3/2}}dx+\frac{2c\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}\right)+\frac{2c\cos^5(e+f}{9f\sqrt{c-c\sin(e}}\right.\right.\right.$$

$\Big\downarrow$ 3042

$$a^2c^2\left(\frac{1}{13}(13A-3B)\left(\frac{12}{11}c\left(\frac{8}{9}c\left(\frac{4}{7}c\int\frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^{3/2}}dx+\frac{2c\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}\right)+\frac{2c\cos^5(e+f}{9f\sqrt{c-c\sin(e}}\right.\right.\right.$$

$\Big\downarrow$ 3152

$$a^2c^2\left(\frac{1}{13}(13A-3B)\left(\frac{12}{11}c\left(\frac{8}{9}c\left(\frac{8c^2\cos^5(e+fx)}{35f(c-c\sin(e+fx))^{5/2}}+\frac{2c\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}\right)+\frac{2c\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx}}\right.\right.\right.$$

input
```
Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2)
,x]
```

output
```
a^2*c^2*((-2*B*Cos[e + f*x]^5*(c - c*Sin[e + f*x])^(3/2))/(13*f) + ((13*A
- 3*B)*((2*c*Cos[e + f*x]^5*Sqrt[c - c*Sin[e + f*x]])/(11*f) + (12*c*((2*c
*Cos[e + f*x]^5)/(9*f*Sqrt[c - c*Sin[e + f*x]]) + (8*c*((8*c^2*Cos[e + f*x
]^5)/(35*f*(c - c*Sin[e + f*x])^(5/2)) + (2*c*Cos[e + f*x]^5)/(7*f*(c - c*
Sin[e + f*x])^(3/2))))/9))/11))/13)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3153
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] &&
NeQ[m + p, 0]
```

rule 3335
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 37.61 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.58

method	result
default	$\frac{2(\sin(fx+e)-1)c^4(1+\sin(fx+e))^3a^2\left(1155B\cos(fx+e)^4+(-1365A+4935B)\cos(fx+e)^2\sin(fx+e)+(5915A-10605B)\cos(fx+e)\right.}{15015\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$
parts	$\frac{2a^2A(\sin(fx+e)-1)c^4(1+\sin(fx+e))\left(5\sin(fx+e)^3-27\sin(fx+e)^2+71\sin(fx+e)-177\right)}{35\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}+\frac{2a^2B(\sin(fx+e)-1)c^4(1+\sin(fx+e)}{}$

input

```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,method=_R
ETURNVERBOSE)
```

output

```
2/15015*(sin(f*x+e)-1)*c^4*(1+sin(f*x+e))^3*a^2*(1155*B*cos(f*x+e)^4+(-136
5*A+4935*B)*cos(f*x+e)^2*sin(f*x+e)+(5915*A-10605*B)*cos(f*x+e)^2+(11180*A
-11820*B)*sin(f*x+e)-12844*A+12204*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.70

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \frac{2\left(1155\,Ba^2c^3\cos(fx+e)^7+105\,(13\,A-14\,B)a^2c^3\cos(fx+e)^6+35\,(91\,A-87\,B)a^2c^3\cos\right.}{}$$

input

```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, al
gorithm="fricas")
```

output
```
2/15015*(1155*B*a^2*c^3*cos(f*x + e)^7 + 105*(13*A - 14*B)*a^2*c^3*cos(f*x
 + e)^6 + 35*(91*A - 87*B)*a^2*c^3*cos(f*x + e)^5 - 20*(13*A - 3*B)*a^2*c^
3*cos(f*x + e)^4 + 32*(13*A - 3*B)*a^2*c^3*cos(f*x + e)^3 - 64*(13*A - 3*B
)*a^2*c^3*cos(f*x + e)^2 + 256*(13*A - 3*B)*a^2*c^3*cos(f*x + e) + 512*(13
*A - 3*B)*a^2*c^3 + (1155*B*a^2*c^3*cos(f*x + e)^6 - 105*(13*A - 25*B)*a^2
*c^3*cos(f*x + e)^5 + 140*(13*A - 3*B)*a^2*c^3*cos(f*x + e)^4 + 160*(13*A
- 3*B)*a^2*c^3*cos(f*x + e)^3 + 192*(13*A - 3*B)*a^2*c^3*cos(f*x + e)^2 +
256*(13*A - 3*B)*a^2*c^3*cos(f*x + e) + 512*(13*A - 3*B)*a^2*c^3)*sin(f*x
+ e))*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^2(-c\sin(fx+e)+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(-c*sin(f*x + e) + c
)^(7/2), x)
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.78

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, al
gorithm="giac")
```

output
```
-1/480480*sqrt(2)*(10010*A*a^2*c^3*cos(-9/4*pi + 9/2*f*x + 9/2*e)*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e)) - 1155*B*a^2*c^3*cos(-13/4*pi + 13/2*f*x + 13/
2*e)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 60060*(7*A*a^2*c^3*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e)) - 2*B*a^2*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
*cos(-1/4*pi + 1/2*f*x + 1/2*e) + 15015*(4*A*a^2*c^3*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e)) + B*a^2*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-3/4*p
i + 3/2*f*x + 3/2*e) - 3003*(22*A*a^2*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*
e)) - 7*B*a^2*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-5/4*pi + 5/2*f
*x + 5/2*e) + 4290*(A*a^2*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*B*a^
2*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-7/4*pi + 7/2*f*x + 7/2*e)
- 1365*(2*A*a^2*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*B*a^2*c^3*sgn(
sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-11/4*pi + 11/2*f*x + 11/2*e))*sqrt(c
)/f
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \int (A + B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^2\,(c-c\,\sin(e+f\,x))^{7/2}\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(7/2)
,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(7/2)
, x)
```

Reduce [F]

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \sqrt{c}\,a^2c^3\left(\left(\int\sqrt{-\sin(fx+e)+1}dx\right)a - \left(\int\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^6\,dx\right)b - \left(\int\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^5\,dx\right)a + \left(\int\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^5\,dx\right)b + \left(\int\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^4\,dx\right)a + 2\left(\int\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^4\,dx\right)b + 2\left(\int\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^3\,dx\right)a - 2\left(\int\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^3\,dx\right)b - 2\left(\int\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)a - \left(\int\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)b - \left(\int\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)a + \left(\int\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)b\right)$$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x)
```

output

```
sqrt(c)*a**2*c**3*(int(sqrt( - sin(e + f*x) + 1),x)*a - int(sqrt( - sin(e
+ f*x) + 1)*sin(e + f*x)**6,x)*b - int(sqrt( - sin(e + f*x) + 1)*sin(e + f
*x)**5,x)*a + int(sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**5,x)*b + int(sqr
t( - sin(e + f*x) + 1)*sin(e + f*x)**4,x)*a + 2*int(sqrt( - sin(e + f*x) +
 1)*sin(e + f*x)**4,x)*b + 2*int(sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**3
,x)*a - 2*int(sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**3,x)*b - 2*int(sqrt(
 - sin(e + f*x) + 1)*sin(e + f*x)**2,x)*a - int(sqrt( - sin(e + f*x) + 1)*
sin(e + f*x)**2,x)*b - int(sqrt( - sin(e + f*x) + 1)*sin(e + f*x),x)*a + i
nt(sqrt( - sin(e + f*x) + 1)*sin(e + f*x),x)*b)
```

3.90 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 167

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \frac{64a^2(11A-B)c^5\cos^5(e+fx)}{3465f(c-c\sin(e+fx))^{5/2}}+\frac{16a^2(11A-B)c^4\cos^5(e+fx)}{693f(c-c\sin(e+fx))^{3/2}}+\frac{2a^2(11A-B)c^3\cos^5(e+fx)}{99f\sqrt{c-c\sin(e+fx)}}-\frac{2a^2Bc^2\cos^5(e+fx)\sqrt{c-c\sin(e+fx)}}{11f}$$

output

```
64/3465*a^2*(11*A-B)*c^5*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)+16/693*a^2*
(11*A-B)*c^4*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(3/2)+2/99*a^2*(11*A-B)*c^3*c
os(f*x+e)^5/f/(c-c*sin(f*x+e))^(1/2)-2/11*a^2*B*c^2*cos(f*x+e)^5*(c-c*sin(
f*x+e))^(1/2)/f
```

Mathematica [A] (verified)

Time = 12.77 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.79

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx =$$
$$-\frac{a^2c^2\cos^4(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}(-5478A+3648B+70(11A}{6930f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e\right.\right.}$$

input
```
Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^(5/2),x]
```

output
```
-1/6930*(a^2*c^2*Cos[e + f*x]^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt
[c - c*Sin[e + f*x]]*(-5478*A + 3648*B + 70*(11*A - 28*B)*Cos[2*(e + f*x)]
 + 5*(968*A - 1033*B)*Sin[e + f*x] + 315*B*Sin[3*(e + f*x)]))/(f*(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2])^5)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.91, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3042, 3446, 3042, 3335, 3042, 3153, 3042, 3153, 3042, 3152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^2(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^2(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3446}$$

$$a^2c^2\int \cos^4(e+fx)(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\int \cos(e+fx)^4(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}dx$$

$$\Big\downarrow \text{3335}$$

$$a^2c^2\left(\frac{1}{11}(11A-B)\int \cos^4(e+fx)\sqrt{c-c\sin(e+fx)}dx-\frac{2B\cos^5(e+fx)\sqrt{c-c\sin(e+fx)}}{11f}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left(\frac{1}{11}(11A-B)\int \cos(e+fx)^4\sqrt{c-c\sin(e+fx)}dx-\frac{2B\cos^5(e+fx)\sqrt{c-c\sin(e+fx)}}{11f}\right)$$

$$\Big\downarrow \text{3153}$$

$$a^2c^2\left(\frac{1}{11}(11A-B)\left(\frac{8}{9}c\int \frac{\cos^4(e+fx)}{\sqrt{c-c\sin(e+fx)}}dx+\frac{2c\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}\right)-\frac{2B\cos^5(e+fx)\sqrt{c-c\sin(e+}}{11f}\right.$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left(\frac{1}{11}(11A-B)\left(\frac{8}{9}c\int \frac{\cos(e+fx)^4}{\sqrt{c-c\sin(e+fx)}}dx+\frac{2c\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}\right)-\frac{2B\cos^5(e+fx)\sqrt{c-c\sin(e+}}{11f}\right.$$

$$\Big\downarrow \text{3153}$$

$$a^2c^2\left(\frac{1}{11}(11A-B)\left(\frac{8}{9}c\left(\frac{4}{7}c\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{3/2}}dx+\frac{2c\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}\right)+\frac{2c\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}\right)\right.$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left(\frac{1}{11}(11A-B)\left(\frac{8}{9}c\left(\frac{4}{7}c\int \frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^{3/2}}dx+\frac{2c\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}\right)+\frac{2c\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}\right)\right.$$

$$\Big\downarrow \text{3152}$$

$$a^2c^2\left(\frac{1}{11}(11A-B)\left(\frac{8}{9}c\left(\frac{8c^2\cos^5(e+fx)}{35f(c-c\sin(e+fx))^{5/2}}+\frac{2c\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}\right)+\frac{2c\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}\right)-\right.$$

input
```
Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2)
,x]
```

output
```
a^2*c^2*((-2*B*Cos[e + f*x]^5*Sqrt[c - c*Sin[e + f*x]])/(11*f) + ((11*A -
B)*((2*c*Cos[e + f*x]^5)/(9*f*Sqrt[c - c*Sin[e + f*x]]) + (8*c*((8*c^2*Cos
[e + f*x]^5)/(35*f*(c - c*Sin[e + f*x])^(5/2)) + (2*c*Cos[e + f*x]^5)/(7*f
*(c - c*Sin[e + f*x])^(3/2))))/9))/11)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3153
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] &&
NeQ[m + p, 0]
```

rule 3335
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 37.69 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.63

method	result
default	$-\frac{2(\sin(fx+e)-1)c^3(1+\sin(fx+e))^3a^2\left(-315B\cos(fx+e)^2\sin(fx+e)+(-385A+980B)\cos(fx+e)^2+(-1210A+1370B)\sin(fx+e)\right.}{3465\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$
parts	$-\frac{2a^2A(\sin(fx+e)-1)c^3(1+\sin(fx+e))\left(3\sin(fx+e)^2-14\sin(fx+e)+43\right)}{15\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}-\frac{2a^2B(\sin(fx+e)-1)c^3(1+\sin(fx+e))\left(63\sin(fx\right.}{69}$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,method=_R
ETURNVERBOSE)
```

output
```
-2/3465*(sin(f*x+e)-1)*c^3*(1+sin(f*x+e))^3*a^2*(-315*B*cos(f*x+e)^2*sin(f
*x+e)+(-385*A+980*B)*cos(f*x+e)^2+(-1210*A+1370*B)*sin(f*x+e)+1562*A-1402*
B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(151) = 302$.

Time = 0.09 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.87

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx =$$

$$-\frac{2\left(315\,Ba^2c^2\cos\left(fx+e\right)^6-35\,(11\,A-10\,B)a^2c^2\cos\left(fx+e\right)^5+5\,(11\,A-B)a^2c^2\cos\left(fx+e\right)^4-8\,(1\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, al
gorithm="fricas")
```

output
```
-2/3465*(315*B*a^2*c^2*cos(f*x + e)^6 - 35*(11*A - 10*B)*a^2*c^2*cos(f*x +
 e)^5 + 5*(11*A - B)*a^2*c^2*cos(f*x + e)^4 - 8*(11*A - B)*a^2*c^2*cos(f*x
 + e)^3 + 16*(11*A - B)*a^2*c^2*cos(f*x + e)^2 - 64*(11*A - B)*a^2*c^2*cos
(f*x + e) - 128*(11*A - B)*a^2*c^2 - (315*B*a^2*c^2*cos(f*x + e)^5 + 35*(1
1*A - B)*a^2*c^2*cos(f*x + e)^4 + 40*(11*A - B)*a^2*c^2*cos(f*x + e)^3 + 4
8*(11*A - B)*a^2*c^2*cos(f*x + e)^2 + 64*(11*A - B)*a^2*c^2*cos(f*x + e) +
 128*(11*A - B)*a^2*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*
x + e) - f*sin(f*x + e) + f)
```

Sympy [F]

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = a^2\left(\int Ac^2\sqrt{-c\sin(e+fx)+c}\,dx + \int\left(-2Ac^2\sqrt{-c\sin(e+fx)+c}\sin^2(e+fx)\right)dx + \int Ac^2\sqrt{-c\sin(e+fx)+c}\sin^4(e+fx)\,dx + \int Bc^2\sqrt{-c\sin(e+fx)+c}\sin(e+fx)\,dx + \int\left(-2Bc^2\sqrt{-c\sin(e+fx)+c}\sin^3(e+fx)\right)dx + \int Bc^2\sqrt{-c\sin(e+fx)+c}\sin^5(e+fx)\,dx\right)$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),x)
```

output
```
a**2*(Integral(A*c**2*sqrt(-c*sin(e + f*x) + c), x) + Integral(-2*A*c**2*s
qrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x) + Integral(A*c**2*sqrt(-c*sin
(e + f*x) + c)*sin(e + f*x)**4, x) + Integral(B*c**2*sqrt(-c*sin(e + f*x)
+ c)*sin(e + f*x), x) + Integral(-2*B*c**2*sqrt(-c*sin(e + f*x) + c)*sin(e
 + f*x)**3, x) + Integral(B*c**2*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**5
, x))
```

Maxima [F]

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^2(-c\sin(fx+e)+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(-c*sin(f*x + e) + c
)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(151) = 302$.

Time = 0.38 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.04

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = -\frac{\sqrt{2}\left(315\,Ba^2c^2\cos\left(-\frac{11}{4}\pi+\frac{11}{2}fx+\frac{11}{2}e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+6930\left(6\,Aa^2c^2\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\right.\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, al
gorithm="giac")
```

output
```
-1/55440*sqrt(2)*(315*B*a^2*c^2*cos(-11/4*pi + 11/2*f*x + 11/2*e)*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e)) + 6930*(6*A*a^2*c^2*sgn(sin(-1/4*pi + 1/2*f*x
+ 1/2*e)) - B*a^2*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi + 1
/2*f*x + 1/2*e) + 2310*(4*A*a^2*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) +
B*a^2*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-3/4*pi + 3/2*f*x + 3/2
*e) - 693*(8*A*a^2*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*B*a^2*c^2*s
gn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-5/4*pi + 5/2*f*x + 5/2*e) - 495*(
2*A*a^2*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*a^2*c^2*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e)))*cos(-7/4*pi + 7/2*f*x + 7/2*e) + 385*(2*A*a^2*c^2
*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - B*a^2*c^2*sgn(sin(-1/4*pi + 1/2*f*x
 + 1/2*e)))*cos(-9/4*pi + 9/2*f*x + 9/2*e))*sqrt(c)/f
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx=\int (A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)^2\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(5/2)
,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(5/2)
, x)
```

Reduce [F]

$$\int (a + a\sin(e + fx))^2 (A + B\sin(e + fx))(c - c\sin(e + fx))^{5/2}\,dx = \sqrt{c}\,a^2c^2\left(\left(\left(\int \sqrt{-\sin(fx+e)+1}dx\right)a + \left(\int \sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^5\,dx\right)b + \left(\int \sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^4\,dx\right)a - 2\left(\int \sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^3\,dx\right)b - 2\left(\int \sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)a + \left(\int \sqrt{-\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)b\right)$$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x)
```

output
```
sqrt(c)*a**2*c**2*(int(sqrt( - sin(e + f*x) + 1),x)*a + int(sqrt( - sin(e
+ f*x) + 1)*sin(e + f*x)**5,x)*b + int(sqrt( - sin(e + f*x) + 1)*sin(e + f
*x)**4,x)*a - 2*int(sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**3,x)*b - 2*int
(sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2,x)*a + int(sqrt( - sin(e + f*x)
 + 1)*sin(e + f*x),x)*b)
```

3.91 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 120

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx=\frac{8a^2(9A+B)c^4\cos^5(e+fx)}{315f(c-c\sin(e+fx))^{5/2}}+\frac{2a^2(9A+B)c^3\cos^5(e+fx)}{63f(c-c\sin(e+fx))^{3/2}}-\frac{2a^2Bc^2\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}$$

output
```
8/315*a^2*(9*A+B)*c^4*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)+2/63*a^2*(9*A+
B)*c^3*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(3/2)-2/9*a^2*B*c^2*cos(f*x+e)^5/f/
(c-c*sin(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 11.80 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.88

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \frac{a^2c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(162A-87B+35B\cos(2(e+fx))+(-90A+13}{315f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

input
```
Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^(3/2),x]
```

output
```
(a^2*c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(162*A - 87*B + 35*B*Cos[2*
(e + f*x)] + (-90*A + 130*B)*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(315*
f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3446, 3042, 3335, 3042, 3153, 3042, 3152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^2(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (a\sin(e+fx)+a)^2(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))dx$$

$$\Big\downarrow \; 3446$$

$$a^2c^2\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}dx$$

$$\Big\downarrow \; 3042$$

$$a^2c^2\int \frac{\cos(e+fx)^4(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}dx$$

$\downarrow$ 3335

$$a^2c^2\left(\frac{1}{9}(9A+B)\int \frac{\cos^4(e+fx)}{\sqrt{c-c\sin(e+fx)}}dx-\frac{2B\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}\right)$$

$\downarrow$ 3042

$$a^2c^2\left(\frac{1}{9}(9A+B)\int \frac{\cos(e+fx)^4}{\sqrt{c-c\sin(e+fx)}}dx-\frac{2B\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}\right)$$

$\downarrow$ 3153

$$a^2c^2\left(\frac{1}{9}(9A+B)\left(\frac{4}{7}c\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{3/2}}dx+\frac{2c\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}\right)-\frac{2B\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}\right)$$

$\downarrow$ 3042

$$a^2c^2\left(\frac{1}{9}(9A+B)\left(\frac{4}{7}c\int \frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^{3/2}}dx+\frac{2c\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}\right)-\frac{2B\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}\right)$$

$\downarrow$ 3152

$$a^2c^2\left(\frac{1}{9}(9A+B)\left(\frac{8c^2\cos^5(e+fx)}{35f(c-c\sin(e+fx))^{5/2}}+\frac{2c\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}\right)-\frac{2B\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}\right)$$

input
```
Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2)
,x]
```

output
```
a^2*c^2*((-2*B*Cos[e + f*x]^5)/(9*f*Sqrt[c - c*Sin[e + f*x]]) + ((9*A + B)
*((8*c^2*Cos[e + f*x]^5)/(35*f*(c - c*Sin[e + f*x])^(5/2)) + (2*c*Cos[e +
f*x]^5)/(7*f*(c - c*Sin[e + f*x])^(3/2))))/9)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3153
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] &&
NeQ[m + p, 0]
```

rule 3335
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.69

method	result
default	$\frac{2(\sin(fx+e)-1)c^2(1+\sin(fx+e))^3a^2\left(-35B\cos(fx+e)^2+\sin(fx+e)(45A-65B)-81A+61B\right)}{315\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$
parts	$\frac{2a^2A(\sin(fx+e)-1)c^2(1+\sin(fx+e))(\sin(fx+e)-5)}{3\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}+\frac{2a^2B(\sin(fx+e)-1)c^2(1+\sin(fx+e))\left(35\sin(fx+e)^4-85\sin(fx+e)^3+1\right.}{315\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,method=_R
ETURNVERBOSE)
```

output
```
2/315*(sin(f*x+e)-1)*c^2*(1+sin(f*x+e))^3*a^2*(-35*B*cos(f*x+e)^2+sin(f*x+
e)*(45*A-65*B)-81*A+61*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(108) = 216.

Time = 0.11 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.90

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx =$$
$$-\frac{2\left(35\,Ba^2c\cos(fx+e)^5+5\,(9\,A+8\,B)a^2c\cos(fx+e)^4-(9\,A+B)a^2c\cos(fx+e)^3+2\,(9\,A+B)a^2c\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, al
gorithm="fricas")
```

output
```
-2/315*(35*B*a^2*c*cos(f*x + e)^5 + 5*(9*A + 8*B)*a^2*c*cos(f*x + e)^4 - (
9*A + B)*a^2*c*cos(f*x + e)^3 + 2*(9*A + B)*a^2*c*cos(f*x + e)^2 - 8*(9*A
+ B)*a^2*c*cos(f*x + e) - 16*(9*A + B)*a^2*c + (35*B*a^2*c*cos(f*x + e)^4
- 5*(9*A + B)*a^2*c*cos(f*x + e)^3 - 6*(9*A + B)*a^2*c*cos(f*x + e)^2 - 8*
(9*A + B)*a^2*c*cos(f*x + e) - 16*(9*A + B)*a^2*c)*sin(f*x + e))*sqrt(-c*s
in(f*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

Sympy [F]

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = a^2\left(\int Ac\sqrt{-c\sin(e+fx)+c}\,dx + \int Ac\sqrt{-c\sin(e+fx)+c}\sin(e+fx)\,dx + \int \left(-Ac\sqrt{-c\sin(e+fx)+c}\sin^2(e+fx)\right)dx + \int \left(-Ac\sqrt{-c\sin(e+fx)+c}\sin^3(e+fx)\right)dx + \int Bc\sqrt{-c\sin(e+fx)+c}\sin(e+fx)\,dx + \int Bc\sqrt{-c\sin(e+fx)+c}\sin^2(e+fx)\,dx + \int \left(-Bc\sqrt{-c\sin(e+fx)+c}\sin^3(e+fx)\right)dx + \int \left(-Bc\sqrt{-c\sin(e+fx)+c}\sin^4(e+fx)\right)dx\right)$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2),x)
```

output
```
a**2*(Integral(A*c*sqrt(-c*sin(e + f*x) + c), x) + Integral(A*c*sqrt(-c*si
n(e + f*x) + c)*sin(e + f*x), x) + Integral(-A*c*sqrt(-c*sin(e + f*x) + c)
*sin(e + f*x)**2, x) + Integral(-A*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x
)**3, x) + Integral(B*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x), x) + Integ
ral(B*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x) + Integral(-B*c*sqrt
(-c*sin(e + f*x) + c)*sin(e + f*x)**3, x) + Integral(-B*c*sqrt(-c*sin(e +
f*x) + c)*sin(e + f*x)**4, x))
```

Maxima [F]

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^2(-c\sin(fx+e)+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(-c*sin(f*x + e) + c
)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(108) = 216$.

Time = 0.33 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.98

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = -\frac{\sqrt{2}\left(1890\,Aa^2c\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-35\,Ba^2c\cos\left(-\frac{9}{4}\pi+\frac{9}{2}fx+\frac{9}{2}e\right)\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, al
gorithm="giac")
```

output
```
-1/2520*sqrt(2)*(1890*A*a^2*c*cos(-1/4*pi + 1/2*f*x + 1/2*e)*sgn(sin(-1/4*
pi + 1/2*f*x + 1/2*e)) - 35*B*a^2*c*cos(-9/4*pi + 9/2*f*x + 9/2*e)*sgn(sin
(-1/4*pi + 1/2*f*x + 1/2*e)) + 210*(3*A*a^2*c*sgn(sin(-1/4*pi + 1/2*f*x +
1/2*e)) + B*a^2*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-3/4*pi + 3/2*f
*x + 3/2*e) - 126*(A*a^2*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - B*a^2*c*s
gn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-5/4*pi + 5/2*f*x + 5/2*e) - 45*(2
*A*a^2*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + B*a^2*c*sgn(sin(-1/4*pi + 1
/2*f*x + 1/2*e)))*cos(-7/4*pi + 7/2*f*x + 7/2*e))*sqrt(c)/f
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx=\int (A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)^2\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{3/2}\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(3/2)
,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(3/2)
, x)
```

Reduce [F]

$$\begin{aligned}\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c&-c\sin(e+fx))^{3/2}\,dx=\sqrt{c}\,a^2c\Bigg(\left(\left(\int\sqrt{-\sin\left(fx+e\right)+1}dx\right)a\right.\\&-\left(\int\sqrt{-\sin\left(fx+e\right)+1}\,\sin\left(fx+e\right)^4dx\right)b\\&-\left(\int\sqrt{-\sin\left(fx+e\right)+1}\,\sin\left(fx+e\right)^3dx\right)a\\&-\left(\int\sqrt{-\sin\left(fx+e\right)+1}\,\sin\left(fx+e\right)^3dx\right)b\\&-\left(\int\sqrt{-\sin\left(fx+e\right)+1}\,\sin\left(fx+e\right)^2dx\right)a\\&+\left(\int\sqrt{-\sin\left(fx+e\right)+1}\,\sin\left(fx+e\right)^2dx\right)b\\&+\left(\int\sqrt{-\sin\left(fx+e\right)+1}\,\sin\left(fx+e\right)dx\right)a\\&+\left(\int\sqrt{-\sin\left(fx+e\right)+1}\,\sin\left(fx+e\right)dx\right)b\Bigg)\end{aligned}$$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x)
```

output
```
sqrt(c)*a**2*c*(int(sqrt( - sin(e + f*x) + 1),x)*a - int(sqrt( - sin(e + f
*x) + 1)*sin(e + f*x)**4,x)*b - int(sqrt( - sin(e + f*x) + 1)*sin(e + f*x)
**3,x)*a - int(sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**3,x)*b - int(sqrt(
- sin(e + f*x) + 1)*sin(e + f*x)**2,x)*a + int(sqrt( - sin(e + f*x) + 1)*s
in(e + f*x)**2,x)*b + int(sqrt( - sin(e + f*x) + 1)*sin(e + f*x),x)*a + in
t(sqrt( - sin(e + f*x) + 1)*sin(e + f*x),x)*b)
```

3.92 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\sqrt{c-c\sin(e+}$

Optimal result

Integrand size = 38, antiderivative size = 81

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \frac{2a^2(7A+3B)c^3\cos^5(e+fx)}{35f(c-c\sin(e+fx))^{5/2}} - \frac{2a^2Bc^2\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}$$

output
```
2/35*a^2*(7*A+3*B)*c^3*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)-2/7*a^2*B*c^2
*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(3/2)
```

Mathematica [A] (verified)

Time = 1.63 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.10

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \frac{2a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(7A-2B+5B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{35f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

input
```
Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f
*x]],x]
```

output
```
(2*a^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(7*A - 2*B + 5*B*Sin[e + f*
x])*Sqrt[c - c*Sin[e + f*x]])/(35*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 3446, 3042, 3335, 3042, 3152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^2\sqrt{c-c\sin(e+fx)}(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^2\sqrt{c-c\sin(e+fx)}(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3446}$$

$$a^2c^2\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\int \frac{\cos(e+fx)^4(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3335}$$

$$a^2c^2\left(\frac{1}{7}(7A+3B)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{3/2}}dx-\frac{2B\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left(\frac{1}{7}(7A+3B)\int \frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^{3/2}}dx-\frac{2B\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \text{3152}$$

$$a^2c^2\left(\frac{2c(7A+3B)\cos^5(e+fx)}{35f(c-c\sin(e+fx))^{5/2}}-\frac{2B\cos^5(e+fx)}{7f(c-c\sin(e+fx))^{3/2}}\right)$$

input
```
Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]],x
]
```

output
```
a^2*c^2*((2*(7*A + 3*B)*c*Cos[e + f*x]^5)/(35*f*(c - c*Sin[e + f*x])^(5/2)
) - (2*B*Cos[e + f*x]^5)/(7*f*(c - c*Sin[e + f*x])^(3/2)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3335
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.80

method	result
default	$-\frac{2(\sin(fx+e)-1)c(1+\sin(fx+e))^3a^2(5B\sin(fx+e)+7A-2B)}{35\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$
parts	$-\frac{2a^2A(\sin(fx+e)-1)(1+\sin(fx+e))c}{\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}-\frac{2a^2B(\sin(fx+e)-1)c(1+\sin(fx+e))\left(5\sin(fx+e)^3-6\sin(fx+e)^2+8\sin(fx+e)-16\right)}{35\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
-2/35*(sin(f*x+e)-1)*c*(1+sin(f*x+e))^3*a^2*(5*B*sin(f*x+e)+7*A-2*B)/cos(f
*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(73) = 146.

Time = 0.09 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.38

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$$
$$=\frac{2\left(5\,Ba^2\cos(fx+e)^4-(7\,A+8\,B)a^2\cos(fx+e)^3-(21\,A+19\,B)a^2\cos(fx+e)^2+2\,(7\,A+3\,B)a^2\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, al
gorithm="fricas")
```

output
```
2/35*(5*B*a^2*cos(f*x + e)^4 - (7*A + 8*B)*a^2*cos(f*x + e)^3 - (21*A + 19
*B)*a^2*cos(f*x + e)^2 + 2*(7*A + 3*B)*a^2*cos(f*x + e) + 4*(7*A + 3*B)*a^
2 - (5*B*a^2*cos(f*x + e)^3 + (7*A + 13*B)*a^2*cos(f*x + e)^2 - 2*(7*A + 3
*B)*a^2*cos(f*x + e) - 4*(7*A + 3*B)*a^2)*sin(f*x + e))*sqrt(-c*sin(f*x +
e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

Sympy [F]

$$\int (a + a\sin(e + fx))^2(A + B\sin(e + fx))\sqrt{c - c\sin(e + fx)}\,dx = a^2\left(\int A\sqrt{-c\sin(e + fx) + c}\,dx + \int 2A\sqrt{-c\sin(e + fx) + c}\sin(e + fx)\,dx + \int A\sqrt{-c\sin(e + fx) + c}\sin^2(e + fx)\,dx + \int B\sqrt{-c\sin(e + fx) + c}\sin(e + fx)\,dx + \int 2B\sqrt{-c\sin(e + fx) + c}\sin^2(e + fx)\,dx + \int B\sqrt{-c\sin(e + fx) + c}\sin^3(e + fx)\,dx\right)$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2),x)
```

output
```
a**2*(Integral(A*sqrt(-c*sin(e + f*x) + c), x) + Integral(2*A*sqrt(-c*sin(
e + f*x) + c)*sin(e + f*x), x) + Integral(A*sqrt(-c*sin(e + f*x) + c)*sin(
e + f*x)**2, x) + Integral(B*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x), x) +
Integral(2*B*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x) + Integral(B*sq
rt(-c*sin(e + f*x) + c)*sin(e + f*x)**3, x))
```

Maxima [F]

$$\int (a + a\sin(e + fx))^2(A + B\sin(e + fx))\sqrt{c - c\sin(e + fx)}\,dx = \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^2\sqrt{-c\sin(fx + e) + c}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*sqrt(-c*sin(f*x + e)
 + c), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(73) = 146$.

Time = 0.30 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.47

$$\int (a + a\sin(e + fx))^2(A + B\sin(e + fx))\sqrt{c - c\sin(e + fx)}\, dx =$$
$$-\frac{\sqrt{2}\left(5\,Ba^2\cos\left(-\frac{7}{4}\,\pi + \frac{7}{2}\,fx + \frac{7}{2}\,e\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,fx + \frac{1}{2}\,e\right)\right) + 35\,\left(4\,Aa^2\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,fx + \right.\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, al
gorithm="giac")
```

output
```
-1/140*sqrt(2)*(5*B*a^2*cos(-7/4*pi + 7/2*f*x + 7/2*e)*sgn(sin(-1/4*pi + 1
/2*f*x + 1/2*e)) + 35*(4*A*a^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + B*a^2
*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi + 1/2*f*x + 1/2*e) + 35*
(2*A*a^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + B*a^2*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e)))*cos(-3/4*pi + 3/2*f*x + 3/2*e) + 7*(2*A*a^2*sgn(sin(-1/4*p
i + 1/2*f*x + 1/2*e)) + 3*B*a^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-
5/4*pi + 5/2*f*x + 5/2*e))*sqrt(c)/f
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^2(A + B\sin(e + fx))\sqrt{c - c\sin(e + fx)}\, dx$$
$$= \int (A + B\,\sin(e + f\,x))\,(a + a\,\sin(e + f\,x))^2\,\sqrt{c - c\,\sin(e + f\,x)}\, dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(1/2)
,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(1/2)
, x)
```

Reduce [F]

$$\int (a + a\sin(e + fx))^2(A + B\sin(e + fx))\sqrt{c - c\sin(e + fx)}\,dx$$
$$= \sqrt{c}\,a^2\left(\left(\int \sqrt{-\sin(fx+e)+1}dx\right)a + \left(\int \sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^3\,dx\right)b + \left(\int \sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)a + 2\left(\int \sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)b + 2\left(\int \sqrt{-\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)a + \left(\int \sqrt{-\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)b\right)$$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)
```

output
```
sqrt(c)*a**2*(int(sqrt( - sin(e + f*x) + 1),x)*a + int(sqrt( - sin(e + f*x
) + 1)*sin(e + f*x)**3,x)*b + int(sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**
2,x)*a + 2*int(sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2,x)*b + 2*int(sqrt
( - sin(e + f*x) + 1)*sin(e + f*x),x)*a + int(sqrt( - sin(e + f*x) + 1)*si
n(e + f*x),x)*b)
```

3.93 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 161

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{4\sqrt{2}a^2(A+B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}f} - \frac{2a^2Bc^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} - \frac{2a^2(A+B)c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} - \frac{4a^2(A+B)\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}$$

output
```
4*2^(1/2)*a^2*(A+B)*arctanh(1/2*c^(1/2)*cos(f*x+e)*2^(1/2)/(c-c*sin(f*x+e)
)^(1/2))/c^(1/2)/f-2/5*a^2*B*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)-2/3
*a^2*(A+B)*c*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(3/2)-4*a^2*(A+B)*cos(f*x+e)/
f/(c-c*sin(f*x+e))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.05 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.09

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx =$$
$$-\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^2\left((120+120i)\sqrt[4]{-1}(A+B)\arctan\left(\left(\frac{1}{2}+\frac{i}{2}\right)\right.\right.}{15f\left(\cos\left(\frac{1}{2}(e+\right.\right.}$$

input
```
Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e +
 f*x]],x]
```

output
```
-1/15*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*((12
0 + 120*I)*(-1)^(1/4)*(A + B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e +
f*x)/4])] + (Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(70*A + 79*B - 3*B*Cos[2
*(e + f*x)] + 2*(5*A + 11*B)*Sin[e + f*x])))/(f*(Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2])^4*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.289$, Rules used = {3042, 3446, 3042, 3339, 3042, 3158, 3042, 3158, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3446}$$

$$a^2c^2\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\int \frac{\cos(e+fx)^4(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3339}$$

$$a^2c^2\left((A+B)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{5/2}}dx-\frac{2B\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left((A+B)\int \frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^{5/2}}dx-\frac{2B\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}\right)$$

$$\Big\downarrow \text{3158}$$

$$a^2c^2\left((A+B)\left(\frac{2\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}dx}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)-\frac{2B\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left((A+B)\left(\frac{2\int \frac{\cos(e+fx)^2}{(c-c\sin(e+fx))^{3/2}}dx}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)-\frac{2B\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}\right)$$

$$\Big\downarrow \text{3158}$$

$$a^2c^2\left((A+B)\left(\frac{2\left(\frac{2\int \frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{c}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)-\frac{2B\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left((A+B)\left(\frac{2\left(\frac{2\int \frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{c}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)-\frac{2B\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}}\right)$$

$$\Big\downarrow \text{3128}$$

$$a^2c^2\left((A+B)\left(\frac{2\left(-\frac{4\int \frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{cf}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)-\frac{5}{}\right.$$

$$\Big\downarrow 219$$

$$a^2c^2\left((A+B)\left(\frac{2\left(\frac{2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)-\frac{2Bc}{5f(c-c}\right.$$

input
```
Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]]
,x]
```

output
```
a^2*c^2*((-2*B*Cos[e + f*x]^5)/(5*f*(c - c*Sin[e + f*x])^(5/2)) + (A + B)*
((-2*Cos[e + f*x]^3)/(3*c*f*(c - c*Sin[e + f*x])^(3/2)) + (2*((2*Sqrt[2]*A
rcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(c^(3/2
)*f) - (2*Cos[e + f*x])/(c*f*Sqrt[c - c*Sin[e + f*x]])))/c))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3158
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(a*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
 EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && In
tegersQ[2*m, 2*p]
```

rule 3339
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.22

method	result
default	$-\frac{2(\sin(fx+e)-1)\sqrt{c(1+\sin(fx+e))}\,a^2\left(30c^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)A+30c^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)B-}{15c^3\cos(fx+e)}$
parts	$-\frac{a^2A(\sin(fx+e)-1)\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}\,\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}-\frac{a^2B(\sin(fx+e)-1)\sqrt{c(1+\sin(fx+e))}\left(15c^{\frac{5}{2}}\sqrt{2}\right.}{}$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
-2/15*(sin(f*x+e)-1)*(c*(1+sin(f*x+e)))^(1/2)*a^2*(30*c^(5/2)*2^(1/2)*arct
anh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*A+30*c^(5/2)*2^(1/2)*arc
tanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*B-3*B*(c*(1+sin(f*x+e))
)^(5/2)-5*A*(c*(1+sin(f*x+e)))^(3/2)*c-5*B*(c*(1+sin(f*x+e)))^(3/2)*c-30*A
*c^2*(c*(1+sin(f*x+e)))^(1/2)-30*B*c^2*(c*(1+sin(f*x+e)))^(1/2))/c^3/cos(f
*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(142) = 284$.

Time = 0.10 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.93

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{\sqrt{c - c\sin(e + fx)}}\,dx$$

$$= \frac{2\left(\frac{15\sqrt{2}((A+B)a^2c\cos(fx+e)-(A+B)a^2c\sin(fx+e)+(A+B)a^2c)\log\left(-\frac{\cos(fx+e)^2+(\cos(fx+e)-2)\sin(fx+e)+\frac{2\sqrt{2}\sqrt{-c\sin(fx+e)+c}(\cos(fx-}{\sqrt{c}}}{\cos(fx+e)^2+(\cos(fx+e)+2)\sin(fx+e)-\cos(}\right.}{\sqrt{c}}\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, al
gorithm="fricas")
```

output
```
2/15*(15*sqrt(2)*((A + B)*a^2*c*cos(f*x + e) - (A + B)*a^2*c*sin(f*x + e)
+ (A + B)*a^2*c)*log(-(cos(f*x + e)^2 + (cos(f*x + e) - 2)*sin(f*x + e) +
2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x + e) + 1)/sqrt
(c) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e
) - cos(f*x + e) - 2))/sqrt(c) + (3*B*a^2*cos(f*x + e)^3 + (5*A + 14*B)*a^
2*cos(f*x + e)^2 - (35*A + 41*B)*a^2*cos(f*x + e) - 4*(10*A + 13*B)*a^2 +
(3*B*a^2*cos(f*x + e)^2 - (5*A + 11*B)*a^2*cos(f*x + e) - 4*(10*A + 13*B)*
a^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c*f*cos(f*x + e) - c*f*sin(
f*x + e) + c*f)
```

Sympy [F]

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = a^2\left(\int \frac{A}{\sqrt{-c\sin(e+fx)+c}}\,dx + \int \frac{2A\sin(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx + \int \frac{A\sin^2(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx + \int \frac{B\sin(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx + \int \frac{2B\sin^2(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx + \int \frac{B\sin^3(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx\right)$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

output
```
a**2*(Integral(A/sqrt(-c*sin(e + f*x) + c), x) + Integral(2*A*sin(e + f*x)
/sqrt(-c*sin(e + f*x) + c), x) + Integral(A*sin(e + f*x)**2/sqrt(-c*sin(e
+ f*x) + c), x) + Integral(B*sin(e + f*x)/sqrt(-c*sin(e + f*x) + c), x) +
Integral(2*B*sin(e + f*x)**2/sqrt(-c*sin(e + f*x) + c), x) + Integral(B*si
n(e + f*x)**3/sqrt(-c*sin(e + f*x) + c), x))
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^2}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2/sqrt(-c*sin(f*x + e)
 + c), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^2}{\sqrt{c-c\,\sin(e+f\,x)}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(1/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(1/
2), x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= \frac{\sqrt{c}\,a^2\left(-\left(\int \frac{\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)-1}dx\right)a-\left(\int \frac{\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^3}{\sin(fx+e)-1}dx\right)b-\left(\int \frac{\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^2}{\sin(fx+e)-1}dx\right)a-2\cdots}{c}$$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

output
```
(sqrt(c)*a**2*( - int(sqrt( - sin(e + f*x) + 1)/(sin(e + f*x) - 1),x)*a -
int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**3)/(sin(e + f*x) - 1),x)*b -
int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x) - 1),x)*a -
2*int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x) - 1),x)*b
- 2*int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x) - 1),x)*a -
 int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x) - 1),x)*b))/c
```

3.94 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 176

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = -\frac{\sqrt{2}a^2(3A+7B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f}+\frac{a^2(A+B)c^2\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}}+\frac{a^2(3A+7B)\cos^3(e+fx)}{6f(c-c\sin(e+fx))^{3/2}}+\frac{a^2(3A+7B)\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}$$

output
```
-2^(1/2)*a^2*(3*A+7*B)*arctanh(1/2*c^(1/2)*cos(f*x+e)*2^(1/2)/(c-c*sin(f*x
+e))^(1/2))/c^(3/2)/f+1/2*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(7
/2)+1/6*a^2*(3*A+7*B)*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(3/2)+a^2*(3*A+7*B)*
cos(f*x+e)/c/f/(c-c*sin(f*x+e))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.59 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.02

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))}{}$$

input
```
Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^(3/2),x]
```

output
```
(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*(6*(A + B)
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) + (6 + 6*I)*(-1)^(1/4)*(3*A + 7*B)*
ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2])^2 + 3*(2*A + 7*B)*Cos[(e + f*x)/2]*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])^2 - B*Cos[(3*(e + f*x))/2]*(Cos[(e + f*x)/2] - Sin[(e + f
*x)/2])^2 + 12*(A + B)*Sin[(e + f*x)/2] + 3*(2*A + 7*B)*(Cos[(e + f*x)/2]
- Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] + B*(Cos[(e + f*x)/2] - Sin[(e + f*
x)/2])^2*Sin[(3*(e + f*x))/2]))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
^4*(c - c*Sin[e + f*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.289$, Rules used = {3042, 3446, 3042, 3338, 3042, 3158, 3042, 3158, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}dx$$

$$\Big\downarrow \; 3446$$

$$a^2c^2\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}dx$$

$$\Big\downarrow \; 3042$$

$$a^2c^2\int \frac{\cos(e+fx)^4(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}dx$$

$$\Big\downarrow \; 3338$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}}-\frac{(3A+7B)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{5/2}}dx}{4c}\right)$$

$$\Big\downarrow \; 3042$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}}-\frac{(3A+7B)\int \frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^{5/2}}dx}{4c}\right)$$

$$\Big\downarrow \; 3158$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}}-\frac{(3A+7B)\left(\frac{2\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}dx}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)}{4c}\right)$$

$$\Big\downarrow \; 3042$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}}-\frac{(3A+7B)\left(\frac{2\int \frac{\cos(e+fx)^2}{(c-c\sin(e+fx))^{3/2}}dx}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)}{4c}\right)$$

$$\Big\downarrow \; 3158$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}}-\frac{(3A+7B)\left(\frac{2\left(\frac{2\int \frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{c}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)}{4c}\right)$$

$\Big\downarrow$ 3042

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}}-\frac{(3A+7B)\left(\frac{2\left(\frac{2\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{c}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)}{4c}\right)$$

$\Big\downarrow$ 3128

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}}-\frac{(3A+7B)\left(\frac{2\left(-\frac{4\int\frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{cf}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{c}-\frac{2\cos^3}{3cf(c-c\sin}\right.}{4c}\right.$$

$\Big\downarrow$ 219

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{2f(c-c\sin(e+fx))^{7/2}}-\frac{(3A+7B)\left(\frac{2\left(\frac{2\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx)}\right.}{4c}\right.$$

input
```
Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(3/
2),x]
```

output
```
a^2*c^2*(((A + B)*Cos[e + f*x]^5)/(2*f*(c - c*Sin[e + f*x])^(7/2)) - ((3*A
 + 7*B)*((-2*Cos[e + f*x]^3)/(3*c*f*(c - c*Sin[e + f*x])^(3/2)) + (2*((2*S
qrt[2]*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])
/(c^(3/2)*f) - (2*Cos[e + f*x])/(c*f*Sqrt[c - c*Sin[e + f*x]])))/c))/(4*c)
)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3158
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(a*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
 EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && In
tegersQ[2*m, 2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.60

method	result
default	$-\frac{a^{2}\left(\sin(fx+e)\left(6A\sqrt{c+c\sin(fx+e)}\,c^{\frac{3}{2}}-9A\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,c^{2}+18B\sqrt{c+c\sin(fx+e)}\,c^{\frac{3}{2}}+2B(c+c\sin(fx+e)\right.\right.}{}$
parts	$\frac{a^{2}A\left(-\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)c^{2}\sin(fx+e)+2\sqrt{c(1+\sin(fx+e))}\,c^{\frac{3}{2}}+\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)c^{2}\right)\sqrt{c(1+\sin}}{4c^{\frac{7}{2}}\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/3/c^(7/2)*a^2*(sin(f*x+e)*(6*A*(c+c*sin(f*x+e))^(1/2)*c^(3/2)-9*A*arcta
nh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^2+18*B*(c+c*sin(f
*x+e))^(1/2)*c^(3/2)+2*B*(c+c*sin(f*x+e))^(3/2)*c^(1/2)-21*B*arctanh(1/2*(
c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^2)-12*A*(c+c*sin(f*x+e))^
(1/2)*c^(3/2)+9*A*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1
/2)*c^2-24*B*(c+c*sin(f*x+e))^(1/2)*c^(3/2)-2*B*(c+c*sin(f*x+e))^(3/2)*c^(
1/2)+21*B*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^2)
*(c*(1+sin(f*x+e)))^(1/2)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(157) = 314$.

Time = 0.10 (sec) , antiderivative size = 385, normalized size of antiderivative = 2.19

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{3/2}}\,dx = \frac{3\sqrt{2}\Big((3A+7B)a^2c\cos(fx+e)^2 - (3A+7B)a^2c\cos(fx+e) - 2\,(3A+7B)a^2c + \cdots}{\cdots}$$

input

```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, al
gorithm="fricas")
```

output

```
1/6*(3*sqrt(2)*((3*A + 7*B)*a^2*c*cos(f*x + e)^2 - (3*A + 7*B)*a^2*c*cos(f
*x + e) - 2*(3*A + 7*B)*a^2*c + ((3*A + 7*B)*a^2*c*cos(f*x + e) + 2*(3*A +
 7*B)*a^2*c)*sin(f*x + e))*log(-(cos(f*x + e)^2 + (cos(f*x + e) - 2)*sin(f
*x + e) - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x + e)
 + 1)/sqrt(c) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*s
in(f*x + e) - cos(f*x + e) - 2))/sqrt(c) - 4*(B*a^2*cos(f*x + e)^3 + (3*A
+ 10*B)*a^2*cos(f*x + e)^2 + 6*(A + 2*B)*a^2*cos(f*x + e) + 3*(A + B)*a^2
+ (B*a^2*cos(f*x + e)^2 - 3*(A + 3*B)*a^2*cos(f*x + e) + 3*(A + B)*a^2)*si
n(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f
*x + e) - 2*c^2*f + (c^2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = a^2\left(\int \frac{A}{-c\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+c\sqrt{-c\sin(e+fx)+c}}\,dx\right.$$
$$+\int \frac{2A\sin(e+fx)}{-c\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+c\sqrt{-c\sin(e+fx)+c}}\,dx$$
$$+\int \frac{A\sin^2(e+fx)}{-c\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+c\sqrt{-c\sin(e+fx)+c}}\,dx$$
$$+\int \frac{B\sin(e+fx)}{-c\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+c\sqrt{-c\sin(e+fx)+c}}\,dx$$
$$+\int \frac{2B\sin^2(e+fx)}{-c\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+c\sqrt{-c\sin(e+fx)+c}}\,dx$$
$$\left.+\int \frac{B\sin^3(e+fx)}{-c\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+c\sqrt{-c\sin(e+fx)+c}}\,dx\right)$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),x)
```

output
```
a**2*(Integral(A/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c*sqrt(-c*si
n(e + f*x) + c)), x) + Integral(2*A*sin(e + f*x)/(-c*sqrt(-c*sin(e + f*x)
+ c)*sin(e + f*x) + c*sqrt(-c*sin(e + f*x) + c)), x) + Integral(A*sin(e +
f*x)**2/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c*sqrt(-c*sin(e + f*x
) + c)), x) + Integral(B*sin(e + f*x)/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e
+ f*x) + c*sqrt(-c*sin(e + f*x) + c)), x) + Integral(2*B*sin(e + f*x)**2/(
-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c*sqrt(-c*sin(e + f*x) + c)),
x) + Integral(B*sin(e + f*x)**3/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)
 + c*sqrt(-c*sin(e + f*x) + c)), x))
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^2}{(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2/(-c*sin(f*x + e) + c
)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^2}{(c-c\,\sin(e+f\,x))^{3/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(3/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(3/
2), x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\sqrt{c}\,a^2\left(\left(\int \frac{\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)^2-2\sin(fx+e)+1}dx\right)a+\left(\int \frac{\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)^2-2\sin}\right.\right.}{}$$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*a**2*(int(sqrt( - sin(e + f*x) + 1)/(sin(e + f*x)**2 - 2*sin(e +
f*x) + 1),x)*a + int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**3)/(sin(e +
f*x)**2 - 2*sin(e + f*x) + 1),x)*b + int((sqrt( - sin(e + f*x) + 1)*sin(e
+ f*x)**2)/(sin(e + f*x)**2 - 2*sin(e + f*x) + 1),x)*a + 2*int((sqrt( - si
n(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x)**2 - 2*sin(e + f*x) + 1),x)
*b + 2*int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**2 - 2*s
in(e + f*x) + 1),x)*a + int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(
e + f*x)**2 - 2*sin(e + f*x) + 1),x)*b))/c**2
```

3.95 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 175

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{3a^2(A+9B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}c^{5/2}f} + \frac{a^2(A+B)c^2\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}} - \frac{a^2(A+9B)\cos^3(e+fx)}{8f(c-c\sin(e+fx))^{5/2}} - \frac{3a^2(A+9B)\cos(e+fx)}{8c^2f\sqrt{c-c\sin(e+fx)}}$$

output
```
3/8*a^2*(A+9*B)*arctanh(1/2*c^(1/2)*cos(f*x+e)*2^(1/2)/(c-c*sin(f*x+e))^(1
/2))*2^(1/2)/c^(5/2)/f+1/4*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(
9/2)-1/8*a^2*(A+9*B)*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(5/2)-3/8*a^2*(A+9*B)
*cos(f*x+e)/c^2/f/(c-c*sin(f*x+e))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.92 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.97

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(4(A+B)\left(\cos\left(\right.\right.\right.}{}$$

input
```
Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^(5/2),x]
```

output
```
(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(4*(A + B)*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]) - (5*A + 13*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 -
 (3 + 3*I)*(-1)^(1/4)*(A + 9*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e
+ f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 - 8*B*Cos[(e + f*x)/2]
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + 8*(A + B)*Sin[(e + f*x)/2] - 2*
(5*A + 13*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] - 8*
B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(e + f*x)/2])*(1 + Sin[e + f
*x])^2)/(4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(c - c*Sin[e + f*x])^
(5/2))
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.97, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.289$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3158, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3446}$$

$$a^2c^2\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\int \frac{\cos(e+fx)^4(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}dx$$

$$\Big\downarrow \text{3338}$$

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}}-\frac{(A+9B)\int\frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{7/2}}dx}{8c}\right)$$

$\Big\downarrow$ 3042

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}}-\frac{(A+9B)\int\frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^{7/2}}dx}{8c}\right)$$

$\Big\downarrow$ 3159

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}}-\frac{(A+9B)\left(\frac{\cos^3(e+fx)}{cf(c-c\sin(e+fx))^{5/2}}-\frac{3\int\frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}dx}{2c^2}\right)}{8c}\right)$$

$\Big\downarrow$ 3042

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}}-\frac{(A+9B)\left(\frac{\cos^3(e+fx)}{cf(c-c\sin(e+fx))^{5/2}}-\frac{3\int\frac{\cos(e+fx)^2}{(c-c\sin(e+fx))^{3/2}}dx}{2c^2}\right)}{8c}\right)$$

$\Big\downarrow$ 3158

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}}-\frac{(A+9B)\left(\frac{\cos^3(e+fx)}{cf(c-c\sin(e+fx))^{5/2}}-\frac{3\left(\frac{2\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{c}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{2c^2}\right)}{8c}\right)$$

$\Big\downarrow$ 3042

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}}-\frac{(A+9B)\left(\frac{\cos^3(e+fx)}{cf(c-c\sin(e+fx))^{5/2}}-\frac{3\left(\frac{2\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{c}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{2c^2}\right)}{8c}\right)$$

$\downarrow$ 3128

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}}-\frac{(A+9B)\left(\frac{\cos^3(e+fx)}{cf(c-c\sin(e+fx))^{5/2}}-\frac{3\left(-\frac{4\int\frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{cf}-\frac{2\cos(e}{cf\sqrt{c-c\sin}}\right.}{2c^2}\right.}{8c}\right.$$

$\downarrow$ 219

$$a^2c^2\left(\frac{(A+B)\cos^5(e+fx)}{4f(c-c\sin(e+fx))^{9/2}}-\frac{(A+9B)\left(\frac{\cos^3(e+fx)}{cf(c-c\sin(e+fx))^{5/2}}-\frac{3\left(\frac{2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{2c^2}\right.}{8c}\right.$$

input
```
Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(5/
2),x]
```

output
```
a^2*c^2*(((A + B)*Cos[e + f*x]^5)/(4*f*(c - c*Sin[e + f*x])^(9/2)) - ((A +
 9*B)*(Cos[e + f*x]^3/(c*f*(c - c*Sin[e + f*x])^(5/2)) - (3*((2*Sqrt[2]*Ar
cTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(c^(3/2)
*f) - (2*Cos[e + f*x])/(c*f*Sqrt[c - c*Sin[e + f*x]])))/(2*c^2)))/(8*c))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3158
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(a*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
 EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && In
tegersQ[2*m, 2*p]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(152) = 304$.

Time = 0.77 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.21

method	result
default	$-\frac{a^{2}\left(3A\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)\sin(fx+e)^{2}c^{2}+27B\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)\sin(fx+e)^{2}c^{2}-6A\,\operatorname{arctanh}\left(\right.\right.}{}$
parts	Expression too large to display

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/8/c^(9/2)*a^2*(3*A*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)
/c^(1/2))*sin(f*x+e)^2*c^2+27*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/
2)*2^(1/2)/c^(1/2))*sin(f*x+e)^2*c^2-6*A*arctanh(1/2*(c*(1+sin(f*x+e)))^(1
/2)*2^(1/2)/c^(1/2))*2^(1/2)*sin(f*x+e)*c^2-54*B*arctanh(1/2*(c*(1+sin(f*x
+e)))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*sin(f*x+e)*c^2-16*B*(c*(1+sin(f*x+e))
)^(1/2)*c^(3/2)*sin(f*x+e)^2+3*A*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1
/2)/c^(1/2))*2^(1/2)*c^2+10*A*(c*(1+sin(f*x+e)))^(3/2)*c^(1/2)+27*B*arctan
h(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^2+26*B*(c*(1+sin
(f*x+e)))^(3/2)*c^(1/2)+32*B*c^(3/2)*(c*(1+sin(f*x+e)))^(1/2)*sin(f*x+e)-1
2*A*(c*(1+sin(f*x+e)))^(1/2)*c^(3/2)-60*B*(c*(1+sin(f*x+e)))^(1/2)*c^(3/2)
)*(c*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2
)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 449 vs. $2(152) = 304$.

Time = 0.16 (sec) , antiderivative size = 449, normalized size of antiderivative = 2.57

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{5/2}}\, dx = \frac{3\sqrt{2}\left((A + 9B)a^2\cos(fx + e)^3 + 3(A + 9B)a^2\cos(fx\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, al
gorithm="fricas")
```

output
```
1/16*(3*sqrt(2)*((A + 9*B)*a^2*cos(f*x + e)^3 + 3*(A + 9*B)*a^2*cos(f*x +
e)^2 - 2*(A + 9*B)*a^2*cos(f*x + e) - 4*(A + 9*B)*a^2 - ((A + 9*B)*a^2*cos
(f*x + e)^2 - 2*(A + 9*B)*a^2*cos(f*x + e) - 4*(A + 9*B)*a^2)*sin(f*x + e)
)*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqr
t(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e
) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x
+ e) - cos(f*x + e) - 2)) - 4*(8*B*a^2*cos(f*x + e)^3 - (5*A + 21*B)*a^2*c
os(f*x + e)^2 - (A + 25*B)*a^2*cos(f*x + e) + 4*(A + B)*a^2 + (8*B*a^2*cos
(f*x + e)^2 + (5*A + 29*B)*a^2*cos(f*x + e) + 4*(A + B)*a^2)*sin(f*x + e))
*sqrt(-c*sin(f*x + e) + c))/(c^3*f*cos(f*x + e)^3 + 3*c^3*f*cos(f*x + e)^2
 - 2*c^3*f*cos(f*x + e) - 4*c^3*f - (c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*
x + e) - 4*c^3*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{5/2}}\, dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^2}{(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2/(-c*sin(f*x + e) + c
)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^2}{(c-c\,\sin(e+f\,x))^{5/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(5/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(5/
2), x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\sqrt{c}\,a^2\left(-\left(\int \frac{\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)^3-3\sin(fx+e)^2+3\sin(fx+e)-1}dx\right)a-\left(\int\right.\right.}{}$$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)
```

output
```
(sqrt(c)*a**2*( - int(sqrt( - sin(e + f*x) + 1)/(sin(e + f*x)**3 - 3*sin(e
 + f*x)**2 + 3*sin(e + f*x) - 1),x)*a - int((sqrt( - sin(e + f*x) + 1)*sin
(e + f*x)**3)/(sin(e + f*x)**3 - 3*sin(e + f*x)**2 + 3*sin(e + f*x) - 1),x
)*b - int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x)**3 - 3
*sin(e + f*x)**2 + 3*sin(e + f*x) - 1),x)*a - 2*int((sqrt( - sin(e + f*x)
+ 1)*sin(e + f*x)**2)/(sin(e + f*x)**3 - 3*sin(e + f*x)**2 + 3*sin(e + f*x
) - 1),x)*b - 2*int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)
**3 - 3*sin(e + f*x)**2 + 3*sin(e + f*x) - 1),x)*a - int((sqrt( - sin(e +
f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**3 - 3*sin(e + f*x)**2 + 3*sin(e + f
*x) - 1),x)*b))/c**3
```

3.96 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 175

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{a^2(A-11B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{16\sqrt{2}c^{7/2}f} + \frac{a^2(A+B)c^2\cos^5(e+fx)}{6f(c-c\sin(e+fx))^{11/2}} + \frac{a^2(A-11B)\cos^3(e+fx)}{24f(c-c\sin(e+fx))^{7/2}} - \frac{a^2(A-11B)\cos(e+fx)}{16c^2f(c-c\sin(e+fx))^{3/2}}$$

output
```
1/32*a^2*(A-11*B)*arctanh(1/2*c^(1/2)*cos(f*x+e)*2^(1/2)/(c-c*sin(f*x+e))^
(1/2))*2^(1/2)/c^(7/2)/f+1/6*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))
^(11/2)+1/24*a^2*(A-11*B)*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(7/2)-1/16*a^2*(
A-11*B)*cos(f*x+e)/c^2/f/(c-c*sin(f*x+e))^(3/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 12.59 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.95

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(32(A+B)\left(\cos\right.\right.}{}$$

input
```
Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^(7/2),x]
```

output
```
(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(32*(A + B)*(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2]) - 4*(7*A + 19*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
3 + 3*(A + 21*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 - (3 + 3*I)*(-1)^
(1/4)*(A - 11*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^6 + 64*(A + B)*Sin[(e + f*x)/2] - 8*(7*
A + 19*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] + 6*(A
+ 21*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(e + f*x)/2])*(1 + Sin
[e + f*x])^2)/(48*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(c - c*Sin[e +
 f*x])^(7/2))
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.289, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3159, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{3446}$$

$$a^2c^2\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\int \frac{\cos(e+fx)^4(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}dx$$

$$\Big\downarrow \text{3338}$$

$$a^2c^2\left(\frac{(A-11B)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{9/2}}dx}{12c}+\frac{(A+B)\cos^5(e+fx)}{6f(c-c\sin(e+fx))^{11/2}}\right)$$

$\Big\downarrow$ 3042

$$a^2c^2\left(\frac{(A-11B)\int \frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^{9/2}}dx}{12c}+\frac{(A+B)\cos^5(e+fx)}{6f(c-c\sin(e+fx))^{11/2}}\right)$$

$\Big\downarrow$ 3159

$$a^2c^2\left(\frac{(A-11B)\left(\frac{\cos^3(e+fx)}{2cf(c-c\sin(e+fx))^{7/2}}-\frac{3\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{5/2}}dx}{4c^2}\right)}{12c}+\frac{(A+B)\cos^5(e+fx)}{6f(c-c\sin(e+fx))^{11/2}}\right)$$

$\Big\downarrow$ 3042

$$a^2c^2\left(\frac{(A-11B)\left(\frac{\cos^3(e+fx)}{2cf(c-c\sin(e+fx))^{7/2}}-\frac{3\int \frac{\cos(e+fx)^2}{(c-c\sin(e+fx))^{5/2}}dx}{4c^2}\right)}{12c}+\frac{(A+B)\cos^5(e+fx)}{6f(c-c\sin(e+fx))^{11/2}}\right)$$

$\Big\downarrow$ 3159

$$a^2c^2\left(\frac{(A-11B)\left(\frac{\cos^3(e+fx)}{2cf(c-c\sin(e+fx))^{7/2}}-\frac{3\left(\frac{\cos(e+fx)}{cf(c-c\sin(e+fx))^{3/2}}-\frac{\int \frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{2c^2}\right)}{4c^2}\right)}{12c}+\frac{(A+B)\cos^5(e+fx)}{6f(c-c\sin(e+fx))^{11/2}}\right)$$

$\Big\downarrow$ 3042

$$a^2c^2\left(\frac{(A-11B)\left(\frac{\cos^3(e+fx)}{2cf(c-c\sin(e+fx))^{7/2}}-\frac{3\left(\frac{\cos(e+fx)}{cf(c-c\sin(e+fx))^{3/2}}-\frac{\int \frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{2c^2}\right)}{4c^2}\right)}{12c}+\frac{(A+B)\cos^5(e+fx)}{6f(c-c\sin(e+fx))^{11/2}}\right)$$

$$\Big\downarrow \text{3128}$$

$$a^2c^2\left(\frac{(A-11B)\left(\frac{\cos^3(e+fx)}{2cf(c-c\sin(e+fx))^{7/2}}-\frac{3\left(\frac{\int \frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{c^2f}+\frac{\cos(e+fx)}{cf(c-c\sin(e+fx))^{3/2}}\right)}{4c^2}\right)}{12c}+\frac{(A+B)\,\mathrm{c}}{6f(c-c\sin}\right.$$

$$\Big\downarrow \text{219}$$

$$a^2c^2\left(\frac{(A-11B)\left(\frac{\cos^3(e+fx)}{2cf(c-c\sin(e+fx))^{7/2}}-\frac{3\left(\frac{\cos(e+fx)}{cf(c-c\sin(e+fx))^{3/2}}-\frac{\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}c^{5/2}f}\right)}{4c^2}\right)}{12c}+\frac{(A+B)\cos^5(e+}{6f(c-c\sin(e+fx}\right.$$

input
```
Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(7/
2),x]
```

output
```
a^2*c^2*(((A + B)*Cos[e + f*x]^5)/(6*f*(c - c*Sin[e + f*x])^(11/2)) + ((A
- 11*B)*(Cos[e + f*x]^3/(2*c*f*(c - c*Sin[e + f*x])^(7/2)) - (3*(-(ArcTanh
[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]/(Sqrt[2]*c^(5/
2)*f)) + Cos[e + f*x]/(c*f*(c - c*Sin[e + f*x])^(3/2))))/(4*c^2)))/(12*c))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(152) = 304$.

Time = 0.76 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.02

method	result
default	$-\frac{a^{2}\left(-3\sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)c^{3}(A-11B)\cos(fx+e)^{2}\sin(fx+e)+9\sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)c^{3}(A-11B)\cos\right.}{}$
parts	Expression too large to display

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/96*a^2*(-3*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*
c^3*(A-11*B)*cos(f*x+e)^2*sin(f*x+e)+9*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e)
)^(1/2)*2^(1/2)/c^(1/2))*c^3*(A-11*B)*cos(f*x+e)^2+12*2^(1/2)*arctanh(1/2*
(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^3*(A-11*B)*sin(f*x+e)+24*A*(c+c*
sin(f*x+e))^(1/2)*c^(5/2)-32*A*(c+c*sin(f*x+e))^(3/2)*c^(3/2)-6*A*(c+c*sin
(f*x+e))^(5/2)*c^(1/2)-264*B*(c+c*sin(f*x+e))^(1/2)*c^(5/2)+352*B*(c+c*sin
(f*x+e))^(3/2)*c^(3/2)-126*B*(c+c*sin(f*x+e))^(5/2)*c^(1/2)-12*A*arctanh(1
/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^3+132*B*arctanh(1/2*(
c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^3)*(c*(1+sin(f*x+e)))^(1/
2)/c^(13/2)/(sin(f*x+e)-1)^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 521 vs. $2(152) = 304$.

Time = 0.10 (sec) , antiderivative size = 521, normalized size of antiderivative = 2.98

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx =$$

$$-\frac{3\sqrt{2}\left((A-11\,B)a^{2}\cos\left(fx+e\right)^{4}-3\,(A-11\,B)a^{2}\cos\left(fx+e\right)^{3}-8\,(A-11\,B)a^{2}\cos\left(fx+e\right)^{2}+4\,(A\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, al
gorithm="fricas")
```

output
```
-1/192*(3*sqrt(2)*((A - 11*B)*a^2*cos(f*x + e)^4 - 3*(A - 11*B)*a^2*cos(f*
x + e)^3 - 8*(A - 11*B)*a^2*cos(f*x + e)^2 + 4*(A - 11*B)*a^2*cos(f*x + e)
 + 8*(A - 11*B)*a^2 + ((A - 11*B)*a^2*cos(f*x + e)^3 + 4*(A - 11*B)*a^2*co
s(f*x + e)^2 - 4*(A - 11*B)*a^2*cos(f*x + e) - 8*(A - 11*B)*a^2)*sin(f*x +
 e))*sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*
sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x
+ e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f
*x + e) - cos(f*x + e) - 2)) + 4*(3*(A + 21*B)*a^2*cos(f*x + e)^3 + (25*A
+ 13*B)*a^2*cos(f*x + e)^2 - 2*(5*A + 41*B)*a^2*cos(f*x + e) - 32*(A + B)*
a^2 + (3*(A + 21*B)*a^2*cos(f*x + e)^2 - 2*(11*A - 25*B)*a^2*cos(f*x + e)
- 32*(A + B)*a^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c^4*f*cos(f*x
+ e)^4 - 3*c^4*f*cos(f*x + e)^3 - 8*c^4*f*cos(f*x + e)^2 + 4*c^4*f*cos(f*x
 + e) + 8*c^4*f + (c^4*f*cos(f*x + e)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*c^4*f
*cos(f*x + e) - 8*c^4*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^2}{(-c\sin(fx+e)+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2/(-c*sin(f*x + e) + c
)^(7/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \int \frac{(A+B\sin(e+fx))\,(a+a\sin(e+fx))^2}{(c-c\sin(e+fx))^{7/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(7/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(7/
2), x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{\sqrt{c}\,a^2\left(\left(\int \frac{\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)^4-4\sin(fx+e)^3+6\sin(fx+e)^2-4\sin(fx+e)+1}dx\right)\right.}{}$$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x)
```

output
```
(sqrt(c)*a**2*(int(sqrt( - sin(e + f*x) + 1)/(sin(e + f*x)**4 - 4*sin(e +
f*x)**3 + 6*sin(e + f*x)**2 - 4*sin(e + f*x) + 1),x)*a + int((sqrt( - sin(
e + f*x) + 1)*sin(e + f*x)**3)/(sin(e + f*x)**4 - 4*sin(e + f*x)**3 + 6*si
n(e + f*x)**2 - 4*sin(e + f*x) + 1),x)*b + int((sqrt( - sin(e + f*x) + 1)*
sin(e + f*x)**2)/(sin(e + f*x)**4 - 4*sin(e + f*x)**3 + 6*sin(e + f*x)**2
 - 4*sin(e + f*x) + 1),x)*a + 2*int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)
**2)/(sin(e + f*x)**4 - 4*sin(e + f*x)**3 + 6*sin(e + f*x)**2 - 4*sin(e +
f*x) + 1),x)*b + 2*int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f
*x)**4 - 4*sin(e + f*x)**3 + 6*sin(e + f*x)**2 - 4*sin(e + f*x) + 1),x)*a
 + int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**4 - 4*sin(e
 + f*x)**3 + 6*sin(e + f*x)**2 - 4*sin(e + f*x) + 1),x)*b))/c**4
```

3.97 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 222

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{a^2(3A-13B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{256\sqrt{2}c^{9/2}f} + \frac{a^2(A+B)c^2\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}} + \frac{a^2(3A-13B)\cos^3(e+fx)}{48f(c-c\sin(e+fx))^{9/2}} - \frac{a^2(3A-13B)\cos(e+fx)}{64c^2f(c-c\sin(e+fx))^{5/2}} + \frac{a^2(3A-13B)\cos(e+fx)}{256c^3f(c-c\sin(e+fx))^{3/2}}$$

output
```
1/512*a^2*(3*A-13*B)*arctanh(1/2*c^(1/2)*cos(f*x+e)*2^(1/2)/(c-c*sin(f*x+e
))^(1/2))*2^(1/2)/c^(9/2)/f+1/8*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+
e))^(13/2)+1/48*a^2*(3*A-13*B)*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(9/2)-1/64*
a^2*(3*A-13*B)*cos(f*x+e)/c^2/f/(c-c*sin(f*x+e))^(5/2)+1/256*a^2*(3*A-13*B
)*cos(f*x+e)/c^3/f/(c-c*sin(f*x+e))^(3/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 13.50 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.61

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))}{}$$

input
```
Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^(9/2),x]
```

output
```
(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*(2013*A*Co
s[(e + f*x)/2] + 1517*B*Cos[(e + f*x)/2] - 999*A*Cos[(3*(e + f*x))/2] - 79
1*B*Cos[(3*(e + f*x))/2] - 69*A*Cos[(5*(e + f*x))/2] - 725*B*Cos[(5*(e + f
*x))/2] - 9*A*Cos[(7*(e + f*x))/2] + 39*B*Cos[(7*(e + f*x))/2] - (24 + 24*
I)*(-1)^(1/4)*(3*A - 13*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x
)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^8 + 2013*A*Sin[(e + f*x)/2] +
 1517*B*Sin[(e + f*x)/2] + 999*A*Sin[(3*(e + f*x))/2] + 791*B*Sin[(3*(e +
f*x))/2] - 69*A*Sin[(5*(e + f*x))/2] - 725*B*Sin[(5*(e + f*x))/2] + 9*A*Si
n[(7*(e + f*x))/2] - 39*B*Sin[(7*(e + f*x))/2]))/(6144*f*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^4*(c - c*Sin[e + f*x])^(9/2))
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.96, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.342$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3159, 3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}dx$$

$$\Big\downarrow \text{3446}$$

$$a^2c^2\int \frac{\cos^4(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\int \frac{\cos(e+fx)^4(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}dx$$

$$\Big\downarrow \text{3338}$$

$$a^2c^2\left(\frac{(3A-13B)\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{11/2}}dx}{16c}+\frac{(A+B)\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left(\frac{(3A-13B)\int \frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^{11/2}}dx}{16c}+\frac{(A+B)\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}}\right)$$

$$\Big\downarrow \text{3159}$$

$$a^2c^2\left(\frac{(3A-13B)\left(\frac{\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{9/2}}-\frac{\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{7/2}}dx}{2c^2}\right)}{16c}+\frac{(A+B)\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left(\frac{(3A-13B)\left(\frac{\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{9/2}}-\frac{\int \frac{\cos(e+fx)^2}{(c-c\sin(e+fx))^{7/2}}dx}{2c^2}\right)}{16c}+\frac{(A+B)\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}}\right)$$

$$\Big\downarrow \text{3159}$$

$$a^2c^2\left(\frac{(3A-13B)\left(\frac{\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{9/2}}-\frac{\frac{\cos(e+fx)}{2cf(c-c\sin(e+fx))^{5/2}}-\frac{\int\frac{1}{(c-c\sin(e+fx))^{3/2}}dx}{4c^2}}{2c^2}\right)}{16c}+\frac{(A+B)\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}}\right)$$

$\downarrow$ 3042

$$a^2c^2\left(\frac{(3A-13B)\left(\frac{\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{9/2}}-\frac{\frac{\cos(e+fx)}{2cf(c-c\sin(e+fx))^{5/2}}-\frac{\int\frac{1}{(c-c\sin(e+fx))^{3/2}}dx}{4c^2}}{2c^2}\right)}{16c}+\frac{(A+B)\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{13/2}}\right)$$

$\downarrow$ 3129

$$a^2c^2\left(\frac{(3A-13B)\left(\frac{\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{9/2}}-\frac{\frac{\cos(e+fx)}{2cf(c-c\sin(e+fx))^{5/2}}-\frac{\frac{\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{4c}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}}{4c^2}}{2c^2}\right)}{16c}+\frac{(A+B)\,\mathrm{co}}{8f(c-c\sin(}\right.$$

$\downarrow$ 3042

$$a^2c^2\left(\frac{(3A-13B)\left(\frac{\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{9/2}}-\frac{\frac{\cos(e+fx)}{2cf(c-c\sin(e+fx))^{5/2}}-\frac{\frac{\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{4c}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}}{4c^2}}{2c^2}\right)}{16c}+\frac{(A+B)\,\mathrm{co}}{8f(c-c\sin(}\right.$$

$\downarrow$ 3128

$$a^2c^2\left(\frac{(3A-13B)\left(\frac{\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{9/2}}-\frac{\frac{\cos(e+fx)}{2cf(c-c\sin(e+fx))^{5/2}}-\frac{\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}-\frac{\int\frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{2cf}}{4c^2}}{2c^2}\right)}{16c}\right.$$

$$\Big\downarrow \; 219$$

$$a^2c^2\left(\frac{(3A-13B)\left(\frac{\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{9/2}}-\frac{\frac{\cos(e+fx)}{2cf(c-c\sin(e+fx))^{5/2}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}c^{3/2}f}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}}{4c^2}}{2c^2}\right)}{16c}+\frac{(A}{8f(}\right.$$

input
```
Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(9/
2),x]
```

output
```
a^2*c^2*(((A + B)*Cos[e + f*x]^5)/(8*f*(c - c*Sin[e + f*x])^(13/2)) + ((3*
A - 13*B)*(Cos[e + f*x]^3/(3*c*f*(c - c*Sin[e + f*x])^(9/2)) - (Cos[e + f*
x]/(2*c*f*(c - c*Sin[e + f*x])^(5/2)) - (ArcTanh[(Sqrt[c]*Cos[e + f*x])/(S
qrt[2]*Sqrt[c - c*Sin[e + f*x]])]/(2*Sqrt[2]*c^(3/2)*f) + Cos[e + f*x]/(2*
f*(c - c*Sin[e + f*x])^(3/2)))/(4*c^2))/(2*c^2)))/(16*c))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(195) = 390$.

Time = 0.79 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.98

method	result
default	$-\frac{a^2\left(3\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,c^5(3A-13B)\cos(fx+e)^4+12\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,c^5(3A-13B)\cos(fx+e)^2\right.}{}$
parts	Expression too large to display

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/1536/c^(19/2)*a^2*(3*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2)
)*2^(1/2)*c^5*(3*A-13*B)*cos(f*x+e)^4+12*arctanh(1/2*(c+c*sin(f*x+e))^(1/2
)*2^(1/2)/c^(1/2))*2^(1/2)*c^5*(3*A-13*B)*cos(f*x+e)^2*sin(f*x+e)-24*arcta
nh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^5*(3*A-13*B)*cos(
f*x+e)^2-24*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^
5*(3*A-13*B)*sin(f*x+e)-144*A*(c+c*sin(f*x+e))^(1/2)*c^(9/2)+264*A*(c+c*si
n(f*x+e))^(3/2)*c^(7/2)+132*A*(c+c*sin(f*x+e))^(5/2)*c^(5/2)-18*A*(c+c*sin
(f*x+e))^(7/2)*c^(3/2)+624*B*(c+c*sin(f*x+e))^(1/2)*c^(9/2)-1144*B*(c+c*si
n(f*x+e))^(3/2)*c^(7/2)+452*B*(c+c*sin(f*x+e))^(5/2)*c^(5/2)+78*B*(c+c*sin
(f*x+e))^(7/2)*c^(3/2)+72*A*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(
1/2)/c^(1/2))*c^5-312*B*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)
/c^(1/2))*c^5)*(c*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)^3/cos(f*x+e)/(c-c*s
in(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 654 vs. $2(195) = 390$.

Time = 0.10 (sec) , antiderivative size = 654, normalized size of antiderivative = 2.95

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, al
gorithm="fricas")
```

output
```
-1/3072*(3*sqrt(2)*((3*A - 13*B)*a^2*cos(f*x + e)^5 + 5*(3*A - 13*B)*a^2*c
os(f*x + e)^4 - 8*(3*A - 13*B)*a^2*cos(f*x + e)^3 - 20*(3*A - 13*B)*a^2*co
s(f*x + e)^2 + 8*(3*A - 13*B)*a^2*cos(f*x + e) + 16*(3*A - 13*B)*a^2 - ((3
*A - 13*B)*a^2*cos(f*x + e)^4 - 4*(3*A - 13*B)*a^2*cos(f*x + e)^3 - 12*(3*
A - 13*B)*a^2*cos(f*x + e)^2 + 8*(3*A - 13*B)*a^2*cos(f*x + e) + 16*(3*A -
 13*B)*a^2)*sin(f*x + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(
-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f
*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (co
s(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*(3*(3*A - 13*B)*a^2*
cos(f*x + e)^4 + (39*A + 343*B)*a^2*cos(f*x + e)^3 + 2*(129*A + 209*B)*a^2
*cos(f*x + e)^2 - 12*(13*A + 29*B)*a^2*cos(f*x + e) - 384*(A + B)*a^2 - (3
*(3*A - 13*B)*a^2*cos(f*x + e)^3 - 2*(15*A + 191*B)*a^2*cos(f*x + e)^2 + 1
2*(19*A + 3*B)*a^2*cos(f*x + e) + 384*(A + B)*a^2)*sin(f*x + e))*sqrt(-c*s
in(f*x + e) + c))/(c^5*f*cos(f*x + e)^5 + 5*c^5*f*cos(f*x + e)^4 - 8*c^5*f
*cos(f*x + e)^3 - 20*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*
f - (c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*x + e)^3 - 12*c^5*f*cos(f*x + e)
^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^2}{(-c\sin(fx+e)+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2/(-c*sin(f*x + e) + c
)^(9/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx=\int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)^2}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{9/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(9/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(9/
2), x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx=\frac{\sqrt{c}\,a^2\left(-\left(\int \frac{\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)^5-5\sin(fx+e)^4+10\sin(fx+e)^3-10\sin(fx+e)^2+\cdots}\right.\right.}{\cdots}$$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x)
```

output
```
(sqrt(c)*a**2*( - int(sqrt( - sin(e + f*x) + 1)/(sin(e + f*x)**5 - 5*sin(e
 + f*x)**4 + 10*sin(e + f*x)**3 - 10*sin(e + f*x)**2 + 5*sin(e + f*x) - 1)
,x)*a - int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**3)/(sin(e + f*x)**5 -
 5*sin(e + f*x)**4 + 10*sin(e + f*x)**3 - 10*sin(e + f*x)**2 + 5*sin(e + f
*x) - 1),x)*b - int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f
*x)**5 - 5*sin(e + f*x)**4 + 10*sin(e + f*x)**3 - 10*sin(e + f*x)**2 + 5*s
in(e + f*x) - 1),x)*a - 2*int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/
(sin(e + f*x)**5 - 5*sin(e + f*x)**4 + 10*sin(e + f*x)**3 - 10*sin(e + f*x
)**2 + 5*sin(e + f*x) - 1),x)*b - 2*int((sqrt( - sin(e + f*x) + 1)*sin(e +
 f*x))/(sin(e + f*x)**5 - 5*sin(e + f*x)**4 + 10*sin(e + f*x)**3 - 10*sin(
e + f*x)**2 + 5*sin(e + f*x) - 1),x)*a - int((sqrt( - sin(e + f*x) + 1)*si
n(e + f*x))/(sin(e + f*x)**5 - 5*sin(e + f*x)**4 + 10*sin(e + f*x)**3 - 10
*sin(e + f*x)**2 + 5*sin(e + f*x) - 1),x)*b))/c**5
```

3.98 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 210

$$\begin{aligned}\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c&-c\sin(e+fx))^{7/2}\,dx = \frac{256a^3(15A-B)c^7\cos^7(e+fx)}{45045f(c-c\sin(e+fx))^{7/2}}\\ &+\frac{64a^3(15A-B)c^6\cos^7(e+fx)}{6435f(c-c\sin(e+fx))^{5/2}}+\frac{8a^3(15A-B)c^5\cos^7(e+fx)}{715f(c-c\sin(e+fx))^{3/2}}\\ &+\frac{2a^3(15A-B)c^4\cos^7(e+fx)}{195f\sqrt{c-c\sin(e+fx)}}-\frac{2a^3Bc^3\cos^7(e+fx)\sqrt{c-c\sin(e+fx)}}{15f}\end{aligned}$$

output
```
256/45045*a^3*(15*A-B)*c^7*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(7/2)+64/6435*a
^3*(15*A-B)*c^6*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(5/2)+8/715*a^3*(15*A-B)*c
^5*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(3/2)+2/195*a^3*(15*A-B)*c^4*cos(f*x+e)
^7/f/(c-c*sin(f*x+e))^(1/2)-2/15*a^3*B*c^3*cos(f*x+e)^7*(c-c*sin(f*x+e))^(
1/2)/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1569 vs. $2(210) = 420$.

Time = 13.42 (sec) , antiderivative size = 1569, normalized size of antiderivative = 7.47

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^(7/2),x]
```

output
```
(5*(8*A - B)*Cos[(e + f*x)/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^
(7/2))/(64*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2])^6) - (5*(6*A + B)*Cos[(3*(e + f*x))/2]*(a + a*Sin[e + f*x
])^3*(c - c*Sin[e + f*x])^(7/2))/(192*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/
2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + (3*(10*A - 3*B)*Cos[(5*(e
 + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2))/(320*f*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
6) - (3*(4*A + 3*B)*Cos[(7*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin
[e + f*x])^(7/2))/(448*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^6) + ((12*A - 5*B)*Cos[(9*(e + f*x))/2]*(a +
a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2))/(576*f*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - ((2*A + 5*B)
*Cos[(11*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2))/
(704*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])^6) + ((2*A - B)*Cos[(13*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c
 - c*Sin[e + f*x])^(7/2))/(832*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - (B*Cos[(15*(e + f*x))/2]*(a + a*
Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2))/(960*f*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + (5*(8*A - B)*S
in[(e + f*x)/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2))/(64*...
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.90, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 3446, 3042, 3335, 3042, 3153, 3042, 3153, 3042, 3153, 3042, 3152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^3(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^3(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3446}$$

$$a^3c^3\int \cos^6(e+fx)(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\int \cos(e+fx)^6(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}dx$$

$$\Big\downarrow \text{3335}$$

$$a^3c^3\left(\frac{1}{15}(15A-B)\int \cos^6(e+fx)\sqrt{c-c\sin(e+fx)}dx-\frac{2B\cos^7(e+fx)\sqrt{c-c\sin(e+fx)}}{15f}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left(\frac{1}{15}(15A-B)\int \cos(e+fx)^6\sqrt{c-c\sin(e+fx)}dx-\frac{2B\cos^7(e+fx)\sqrt{c-c\sin(e+fx)}}{15f}\right)$$

$$\Big\downarrow \text{3153}$$

$$a^3c^3\left(\frac{1}{15}(15A-B)\left(\frac{12}{13}c\int \frac{\cos^6(e+fx)}{\sqrt{c-c\sin(e+fx)}}dx+\frac{2c\cos^7(e+fx)}{13f\sqrt{c-c\sin(e+fx)}}\right)-\frac{2B\cos^7(e+fx)\sqrt{c-c\sin(e+fx)}}{15f}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left(\frac{1}{15}(15A-B)\left(\frac{12}{13}c\int\frac{\cos(e+fx)^6}{\sqrt{c-c\sin(e+fx)}}dx+\frac{2c\cos^7(e+fx)}{13f\sqrt{c-c\sin(e+fx)}}\right)-\frac{2B\cos^7(e+fx)\sqrt{c-c\sin(e+fx)}}{15f}\right.$$

$\downarrow$ 3153

$$a^3c^3\left(\frac{1}{15}(15A-B)\left(\frac{12}{13}c\left(\frac{8}{11}c\int\frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{3/2}}dx+\frac{2c\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}\right)+\frac{2c\cos^7(e+fx)}{13f\sqrt{c-c\sin(e+fx)}}\right.\right.$$

$\downarrow$ 3042

$$a^3c^3\left(\frac{1}{15}(15A-B)\left(\frac{12}{13}c\left(\frac{8}{11}c\int\frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^{3/2}}dx+\frac{2c\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}\right)+\frac{2c\cos^7(e+fx)}{13f\sqrt{c-c\sin(e+fx)}}\right.\right.$$

$\downarrow$ 3153

$$a^3c^3\left(\frac{1}{15}(15A-B)\left(\frac{12}{13}c\left(\frac{8}{11}c\left(\frac{4}{9}c\int\frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{5/2}}dx+\frac{2c\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}\right)+\frac{2c\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}\right.\right.\right.$$

$\downarrow$ 3042

$$a^3c^3\left(\frac{1}{15}(15A-B)\left(\frac{12}{13}c\left(\frac{8}{11}c\left(\frac{4}{9}c\int\frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^{5/2}}dx+\frac{2c\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}\right)+\frac{2c\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}\right.\right.\right.$$

$\downarrow$ 3152

$$a^3c^3\left(\frac{1}{15}(15A-B)\left(\frac{12}{13}c\left(\frac{8}{11}c\left(\frac{8c^2\cos^7(e+fx)}{63f(c-c\sin(e+fx))^{7/2}}+\frac{2c\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}\right)+\frac{2c\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}\right.\right.\right.$$

input
```
Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2)
,x]
```

output
```
a^3*c^3*((-2*B*Cos[e + f*x]^7*Sqrt[c - c*Sin[e + f*x]])/(15*f) + ((15*A -
B)*((2*c*Cos[e + f*x]^7)/(13*f*Sqrt[c - c*Sin[e + f*x]]) + (12*c*((2*c*Cos
[e + f*x]^7)/(11*f*(c - c*Sin[e + f*x])^(3/2)) + (8*c*((8*c^2*Cos[e + f*x]
^7)/(63*f*(c - c*Sin[e + f*x])^(7/2)) + (2*c*Cos[e + f*x]^7)/(9*f*(c - c*S
in[e + f*x])^(5/2))))/11))/13))/15)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3153
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] &&
NeQ[m + p, 0]
```

rule 3335
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 189.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.58

method	result
default	$\frac{2(\sin(fx+e)-1)c^4(1+\sin(fx+e))^4a^3\left(3003B\cos(fx+e)^4+(-3465A+12243B)\cos(fx+e)^2\sin(fx+e)+(14175A-24969B)\cos(fx\right.}{45045\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$
parts	$\frac{2a^3A(\sin(fx+e)-1)c^4(1+\sin(fx+e))\left(5\sin(fx+e)^3-27\sin(fx+e)^2+71\sin(fx+e)-177\right)}{35\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}+\frac{2a^3B(\sin(fx+e)-1)c^4(1+\sin(fx+\right.}{}$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,method=_R
ETURNVERBOSE)
```

output
```
2/45045*(sin(f*x+e)-1)*c^4*(1+sin(f*x+e))^4*a^3*(3003*B*cos(f*x+e)^4+(-346
5*A+12243*B)*cos(f*x+e)^2*sin(f*x+e)+(14175*A-24969*B)*cos(f*x+e)^2+(24780
*A-25676*B)*sin(f*x+e)-26700*A+25804*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/
f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. 2(190) = 380.

Time = 0.10 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.93

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx =$$
$$-\frac{2\,(3003\,Ba^3c^3\cos(fx+e)^8-231\,(15\,A-14\,B)a^3c^3\cos(fx+e)^7+21\,(15\,A-B)a^3c^3\cos(fx+e)^6-2}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, al
gorithm="fricas")
```

output
```
-2/45045*(3003*B*a^3*c^3*cos(f*x + e)^8 - 231*(15*A - 14*B)*a^3*c^3*cos(f*
x + e)^7 + 21*(15*A - B)*a^3*c^3*cos(f*x + e)^6 - 28*(15*A - B)*a^3*c^3*co
s(f*x + e)^5 + 40*(15*A - B)*a^3*c^3*cos(f*x + e)^4 - 64*(15*A - B)*a^3*c^
3*cos(f*x + e)^3 + 128*(15*A - B)*a^3*c^3*cos(f*x + e)^2 - 512*(15*A - B)*
a^3*c^3*cos(f*x + e) - 1024*(15*A - B)*a^3*c^3 - (3003*B*a^3*c^3*cos(f*x +
 e)^7 + 231*(15*A - B)*a^3*c^3*cos(f*x + e)^6 + 252*(15*A - B)*a^3*c^3*cos
(f*x + e)^5 + 280*(15*A - B)*a^3*c^3*cos(f*x + e)^4 + 320*(15*A - B)*a^3*c
^3*cos(f*x + e)^3 + 384*(15*A - B)*a^3*c^3*cos(f*x + e)^2 + 512*(15*A - B)
*a^3*c^3*cos(f*x + e) + 1024*(15*A - B)*a^3*c^3)*sin(f*x + e))*sqrt(-c*sin
(f*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^3(-c\sin(fx+e)+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*(-c*sin(f*x + e) + c
)^(7/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(190) = 380$.

Time = 0.37 (sec) , antiderivative size = 458, normalized size of antiderivative = 2.18

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, al
gorithm="giac")
```

output
```
1/2882880*sqrt(2)*(3003*B*a^3*c^3*cos(-15/4*pi + 15/2*f*x + 15/2*e)*sgn(si
n(-1/4*pi + 1/2*f*x + 1/2*e)) - 225225*(8*A*a^3*c^3*sgn(sin(-1/4*pi + 1/2*
f*x + 1/2*e)) - B*a^3*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi
 + 1/2*f*x + 1/2*e) - 75075*(6*A*a^3*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e
)) + B*a^3*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-3/4*pi + 3/2*f*x
+ 3/2*e) + 27027*(10*A*a^3*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*B*a
^3*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-5/4*pi + 5/2*f*x + 5/2*e)
 + 19305*(4*A*a^3*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*a^3*c^3*sg
n(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-7/4*pi + 7/2*f*x + 7/2*e) - 5005*(
12*A*a^3*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 5*B*a^3*c^3*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e)))*cos(-9/4*pi + 9/2*f*x + 9/2*e) - 4095*(2*A*a^3*c
^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*B*a^3*c^3*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e)))*cos(-11/4*pi + 11/2*f*x + 11/2*e) + 3465*(2*A*a^3*c^3*sgn(
sin(-1/4*pi + 1/2*f*x + 1/2*e)) - B*a^3*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/
2*e)))*cos(-13/4*pi + 13/2*f*x + 13/2*e))*sqrt(c)/f
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \int (A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^3\,(c-c\,\sin(e+f\,x))^{7/2}\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(7/2)
,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(7/2)
, x)
```

Reduce [F]

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx=\sqrt{c}\,a^3c^3\left(\left(\int\sqrt{-\sin(fx+e)+1}dx\right)a-\left(\int\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^7\,dx\right)b-\left(\int\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^6\,dx\right)a+3\left(\int\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^5\,dx\right)b+3\left(\int\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^4\,dx\right)a-3\left(\int\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^3\,dx\right)b-3\left(\int\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)a+\left(\int\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)b\right)$$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x)
```

output
```
sqrt(c)*a**3*c**3*(int(sqrt( - sin(e + f*x) + 1),x)*a - int(sqrt( - sin(e
+ f*x) + 1)*sin(e + f*x)**7,x)*b - int(sqrt( - sin(e + f*x) + 1)*sin(e + f
*x)**6,x)*a + 3*int(sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**5,x)*b + 3*int
(sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**4,x)*a - 3*int(sqrt( - sin(e + f*
x) + 1)*sin(e + f*x)**3,x)*b - 3*int(sqrt( - sin(e + f*x) + 1)*sin(e + f*x
)**2,x)*a + int(sqrt( - sin(e + f*x) + 1)*sin(e + f*x),x)*b)
```

3.99 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 161

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \frac{64a^3(13A+B)c^6\cos^7(e+fx)}{9009f(c-c\sin(e+fx))^{7/2}} + \frac{16a^3(13A+B)c^5\cos^7(e+fx)}{1287f(c-c\sin(e+fx))^{5/2}} + \frac{2a^3(13A+B)c^4\cos^7(e+fx)}{143f(c-c\sin(e+fx))^{3/2}} - \frac{2a^3Bc^3\cos^7(e+fx)}{13f\sqrt{c-c\sin(e+fx)}}$$

output
```
64/9009*a^3*(13*A+B)*c^6*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(7/2)+16/1287*a^3
*(13*A+B)*c^5*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(5/2)+2/143*a^3*(13*A+B)*c^4
*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(3/2)-2/13*a^3*B*c^3*cos(f*x+e)^7/f/(c-c*
sin(f*x+e))^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1351 vs. $2(161) = 322$.

Time = 13.54 (sec) , antiderivative size = 1351, normalized size of antiderivative = 8.39

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^(5/2),x]
```

output
```
(5*A*Cos[(e + f*x)/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2))/(
8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^6) - (5*(4*A + B)*Cos[(3*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c -
 c*Sin[e + f*x])^(5/2))/(96*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + ((2*A - B)*Cos[(5*(e + f*x))/2]*(a
+ a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2))/(32*f*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - ((5*A + 2*B
)*Cos[(7*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2))/
(112*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])^6) + ((A - 2*B)*Cos[(9*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c
- c*Sin[e + f*x])^(5/2))/(144*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - ((2*A + B)*Cos[(11*(e + f*x))/2]*
(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2))/(352*f*(Cos[(e + f*x)/2
] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - (B*Cos[
(13*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2))/(416*
f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x
)/2])^6) + (5*A*Sin[(e + f*x)/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x
])^(5/2))/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^6) + (5*(4*A + B)*(a + a*Sin[e + f*x])^3*(c - c*Sin[e +
 f*x])^(5/2)*Sin[(3*(e + f*x))/2])/(96*f*(Cos[(e + f*x)/2] - Sin[(e + f...
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 3446, 3042, 3335, 3042, 3153, 3042, 3153, 3042, 3152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^3(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^3(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3446}$$

$$a^3c^3\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\int \frac{\cos(e+fx)^6(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3335}$$

$$a^3c^3\left(\frac{1}{13}(13A+B)\int \frac{\cos^6(e+fx)}{\sqrt{c-c\sin(e+fx)}}dx-\frac{2B\cos^7(e+fx)}{13f\sqrt{c-c\sin(e+fx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left(\frac{1}{13}(13A+B)\int \frac{\cos(e+fx)^6}{\sqrt{c-c\sin(e+fx)}}dx-\frac{2B\cos^7(e+fx)}{13f\sqrt{c-c\sin(e+fx)}}\right)$$

$$\Big\downarrow \text{3153}$$

$$a^3c^3\left(\frac{1}{13}(13A+B)\left(\frac{8}{11}c\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{3/2}}dx+\frac{2c\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}\right)-\frac{2B\cos^7(e+fx)}{13f\sqrt{c-c\sin(e+fx)}}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left(\frac{1}{13}(13A+B)\left(\frac{8}{11}c\int\frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^{3/2}}dx+\frac{2c\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}\right)-\frac{2B\cos^7(e+fx)}{13f\sqrt{c-c\sin(e+fx)}}\right)$$

$\Big\downarrow$ 3153

$$a^3c^3\left(\frac{1}{13}(13A+B)\left(\frac{8}{11}c\left(\frac{4}{9}c\int\frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{5/2}}dx+\frac{2c\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}\right)+\frac{2c\cos^7(e+fx)}{11f(c-c\sin(e+fx))}\right.\right.$$

$\Big\downarrow$ 3042

$$a^3c^3\left(\frac{1}{13}(13A+B)\left(\frac{8}{11}c\left(\frac{4}{9}c\int\frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^{5/2}}dx+\frac{2c\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}\right)+\frac{2c\cos^7(e+fx)}{11f(c-c\sin(e+fx))}\right.\right.$$

$\Big\downarrow$ 3152

$$a^3c^3\left(\frac{1}{13}(13A+B)\left(\frac{8}{11}c\left(\frac{8c^2\cos^7(e+fx)}{63f(c-c\sin(e+fx))^{7/2}}+\frac{2c\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}\right)+\frac{2c\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}\right)\right.$$

input
```
Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2)
,x]
```

output
```
a^3*c^3*((-2*B*Cos[e + f*x]^7)/(13*f*Sqrt[c - c*Sin[e + f*x]]) + ((13*A +
B)*((2*c*Cos[e + f*x]^7)/(11*f*(c - c*Sin[e + f*x])^(3/2)) + (8*c*((8*c^2*
Cos[e + f*x]^7)/(63*f*(c - c*Sin[e + f*x])^(7/2)) + (2*c*Cos[e + f*x]^7)/(
9*f*(c - c*Sin[e + f*x])^(5/2))))/11))/13)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3153
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] &&
NeQ[m + p, 0]
```

rule 3335
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 191.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.65

method	result
default	$-\frac{2(\sin(fx+e)-1)c^3(1+\sin(fx+e))^4a^3\left(-693B\cos(fx+e)^2\sin(fx+e)+(-819A+2016B)\cos(fx+e)^2+(-2366A+2590B)\sin(fx\right.}{9009\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$
parts	$-\frac{2a^3A(\sin(fx+e)-1)c^3(1+\sin(fx+e))\left(3\sin(fx+e)^2-14\sin(fx+e)+43\right)}{15\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}-\frac{2a^3B(\sin(fx+e)-1)c^3(1+\sin(fx+e))\left(3465\sin(f\right.}{}$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,method=_R
ETURNVERBOSE)
```

output
```
-2/9009*(sin(f*x+e)-1)*c^3*(1+sin(f*x+e))^4*a^3*(-693*B*cos(f*x+e)^2*sin(f
*x+e)+(-819*A+2016*B)*cos(f*x+e)^2+(-2366*A+2590*B)*sin(f*x+e)+2782*A-2558
*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(145) = 290$.

Time = 0.09 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.07

$$\int (a + a\sin(e + fx))^3 (A + B\sin(e + fx))(c - c\sin(e + fx))^{5/2} dx =$$
$$-\frac{2\left(693\,Ba^3c^2\cos\left(fx+e\right)^7 + 63\,(13\,A + 12\,B)a^3c^2\cos\left(fx+e\right)^6 - 7\,(13\,A + B)a^3c^2\cos\left(fx+e\right)^5 + 10\,(\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, al
gorithm="fricas")
```

output
```
-2/9009*(693*B*a^3*c^2*cos(f*x + e)^7 + 63*(13*A + 12*B)*a^3*c^2*cos(f*x +
 e)^6 - 7*(13*A + B)*a^3*c^2*cos(f*x + e)^5 + 10*(13*A + B)*a^3*c^2*cos(f*
x + e)^4 - 16*(13*A + B)*a^3*c^2*cos(f*x + e)^3 + 32*(13*A + B)*a^3*c^2*co
s(f*x + e)^2 - 128*(13*A + B)*a^3*c^2*cos(f*x + e) - 256*(13*A + B)*a^3*c^
2 + (693*B*a^3*c^2*cos(f*x + e)^6 - 63*(13*A + B)*a^3*c^2*cos(f*x + e)^5 -
 70*(13*A + B)*a^3*c^2*cos(f*x + e)^4 - 80*(13*A + B)*a^3*c^2*cos(f*x + e)
^3 - 96*(13*A + B)*a^3*c^2*cos(f*x + e)^2 - 128*(13*A + B)*a^3*c^2*cos(f*x
 + e) - 256*(13*A + B)*a^3*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f
*cos(f*x + e) - f*sin(f*x + e) + f)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^3 (A + B\sin(e + fx))(c - c\sin(e + fx))^{5/2} dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))**(c-c*sin(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^3(-c\sin(fx+e)+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*(-c*sin(f*x + e) + c
)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. 2(145) = 290.

Time = 0.40 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.31

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, al
gorithm="giac")
```

output
```
-1/288288*sqrt(2)*(180180*A*a^3*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)*sgn(sin
(-1/4*pi + 1/2*f*x + 1/2*e)) + 693*B*a^3*c^2*cos(-13/4*pi + 13/2*f*x + 13/
2*e)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 15015*(4*A*a^3*c^2*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e)) + B*a^3*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*c
os(-3/4*pi + 3/2*f*x + 3/2*e) - 9009*(2*A*a^3*c^2*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e)) - B*a^3*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-5/4*pi +
 5/2*f*x + 5/2*e) - 2574*(5*A*a^3*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))
+ 2*B*a^3*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-7/4*pi + 7/2*f*x +
 7/2*e) + 2002*(A*a^3*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*B*a^3*c^
2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-9/4*pi + 9/2*f*x + 9/2*e) + 81
9*(2*A*a^3*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + B*a^3*c^2*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e)))*cos(-11/4*pi + 11/2*f*x + 11/2*e))*sqrt(c)/f
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \int (A + B\sin(e+fx))\,(a+a\sin(e+fx))^3\,(c-c\sin(e+fx))^{5/2}\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(5/2)
,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(5/2)
, x)
```

Reduce [F]

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \sqrt{c}\,a^3c^2\left(\left(\int \sqrt{-\sin(fx+e)+1}dx\right)a + \left(\int \sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^6\,dx\right)b + \left(\int \sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^5\,dx\right)a + \left(\int \sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^5\,dx\right)b + \left(\int \sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^4\,dx\right)a - 2\left(\int \sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^4\,dx\right)b - 2\left(\int \sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^3\,dx\right)a - 2\left(\int \sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^3\,dx\right)b - 2\left(\int \sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)a + \left(\int \sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)b + \left(\int \sqrt{-\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)a + \left(\int \sqrt{-\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)b\right)$$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x)
```

output
```
sqrt(c)*a**3*c**2*(int(sqrt( - sin(e + f*x) + 1),x)*a + int(sqrt( - sin(e
+ f*x) + 1)*sin(e + f*x)**6,x)*b + int(sqrt( - sin(e + f*x) + 1)*sin(e + f
*x)**5,x)*a + int(sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**5,x)*b + int(sqr
t( - sin(e + f*x) + 1)*sin(e + f*x)**4,x)*a - 2*int(sqrt( - sin(e + f*x) +
 1)*sin(e + f*x)**4,x)*b - 2*int(sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**3
,x)*a - 2*int(sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**3,x)*b - 2*int(sqrt(
 - sin(e + f*x) + 1)*sin(e + f*x)**2,x)*a + int(sqrt( - sin(e + f*x) + 1)*
sin(e + f*x)**2,x)*b + int(sqrt( - sin(e + f*x) + 1)*sin(e + f*x),x)*a + i
nt(sqrt( - sin(e + f*x) + 1)*sin(e + f*x),x)*b)
```

3.100 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 124

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx=\frac{8a^3(11A+3B)c^5\cos^7(e+fx)}{693f(c-c\sin(e+fx))^{7/2}}+\frac{2a^3(11A+3B)c^4\cos^7(e+fx)}{99f(c-c\sin(e+fx))^{5/2}}-\frac{2a^3Bc^3\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}$$

output
```
8/693*a^3*(11*A+3*B)*c^5*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(7/2)+2/99*a^3*(1
1*A+3*B)*c^4*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(5/2)-2/11*a^3*B*c^3*cos(f*x+
e)^7/f/(c-c*sin(f*x+e))^(3/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1157 vs. 2(124) = 248.

Time = 13.37 (sec) , antiderivative size = 1157, normalized size of antiderivative = 9.33

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^(3/2),x]
```

output
```
((6*A + B)*Cos[(e + f*x)/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3
/2))/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[
(e + f*x)/2])^6) - ((8*A + 3*B)*Cos[(3*(e + f*x))/2]*(a + a*Sin[e + f*x])^
3*(c - c*Sin[e + f*x])^(3/2))/(24*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - (B*Cos[(5*(e + f*x))/2]*(a +
a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2))/(16*f*(Cos[(e + f*x)/2] - Si
n[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - ((6*A + B)*Co
s[(7*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2))/(112
*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2])^6) - ((2*A + 3*B)*Cos[(9*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c -
c*Sin[e + f*x])^(3/2))/(144*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + (B*Cos[(11*(e + f*x))/2]*(a + a*Sin
[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2))/(176*f*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + ((6*A + B)*Sin[(e
 + f*x)/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2))/(8*f*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
+ ((8*A + 3*B)*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2)*Sin[(3*(e
 + f*x))/2])/(24*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^6) - (B*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])
^(3/2)*Sin[(5*(e + f*x))/2])/(16*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]...
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3446, 3042, 3335, 3042, 3153, 3042, 3152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^3(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^3(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3446}$$

$$a^3c^3\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\int \frac{\cos(e+fx)^6(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3335}$$

$$a^3c^3\left(\frac{1}{11}(11A+3B)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{3/2}}dx-\frac{2B\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left(\frac{1}{11}(11A+3B)\int \frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^{3/2}}dx-\frac{2B\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \text{3153}$$

$$a^3c^3\left(\frac{1}{11}(11A+3B)\left(\frac{4}{9}c\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{5/2}}dx+\frac{2c\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}\right)-\frac{2B\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left(\frac{1}{11}(11A+3B)\left(\frac{4}{9}c\int \frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^{5/2}}dx+\frac{2c\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}\right)-\frac{2B\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}\right)$$

$$\Big\downarrow \text{3152}$$

$$a^3c^3\left(\frac{1}{11}(11A+3B)\left(\frac{8c^2\cos^7(e+fx)}{63f(c-c\sin(e+fx))^{7/2}}+\frac{2c\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}\right)-\frac{2B\cos^7(e+fx)}{11f(c-c\sin(e+fx))^{3/2}}\right)$$

input
```
Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2)
,x]
```

output
```
a^3*c^3*((-2*B*Cos[e + f*x]^7)/(11*f*(c - c*Sin[e + f*x])^(3/2)) + ((11*A
+ 3*B)*((8*c^2*Cos[e + f*x]^7)/(63*f*(c - c*Sin[e + f*x])^(7/2)) + (2*c*Co
s[e + f*x]^7)/(9*f*(c - c*Sin[e + f*x])^(5/2))))/11)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3153
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] &&
NeQ[m + p, 0]
```

rule 3335
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 2.01 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.67

method	result
default	$\frac{2(\sin(fx+e)-1)c^2(1+\sin(fx+e))^4a^3\left(-63B\cos(fx+e)^2+\sin(fx+e)(77A-105B)-121A+93B\right)}{693\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$
parts	$\frac{2a^3A(\sin(fx+e)-1)c^2(1+\sin(fx+e))(\sin(fx+e)-5)}{3\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}+\frac{2a^3B(\sin(fx+e)-1)c^2(1+\sin(fx+e))\left(15\sin(fx+e)^5-35\sin(fx+e)^4+4\right.}{165\cos(fx+e)\sqrt{c-c\sin(f\right.}$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,method=_R
ETURNVERBOSE)
```

output
```
2/693*(sin(f*x+e)-1)*c^2*(1+sin(f*x+e))^4*a^3*(-63*B*cos(f*x+e)^2+sin(f*x+
e)*(77*A-105*B)-121*A+93*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(112) = 224$.

Time = 0.09 (sec) , antiderivative size = 287, normalized size of antiderivative = 2.31

$$\int (a + a\sin(e + fx))^3 (A + B\sin(e + fx))(c - c\sin(e + fx))^{3/2}\, dx = \frac{2\left(63\, Ba^3c\cos(fx + e)^6 - 7\,(11\,A + 12\,B)a^3c\cos(fx + e)^5 - (187\,A + 177\,B)a^3c\cos(fx + e)^4 + \cdots\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, al
gorithm="fricas")
```

output
```
2/693*(63*B*a^3*c*cos(f*x + e)^6 - 7*(11*A + 12*B)*a^3*c*cos(f*x + e)^5 -
(187*A + 177*B)*a^3*c*cos(f*x + e)^4 + 2*(11*A + 3*B)*a^3*c*cos(f*x + e)^3
 - 4*(11*A + 3*B)*a^3*c*cos(f*x + e)^2 + 16*(11*A + 3*B)*a^3*c*cos(f*x + e
) + 32*(11*A + 3*B)*a^3*c - (63*B*a^3*c*cos(f*x + e)^5 + 7*(11*A + 21*B)*a
^3*c*cos(f*x + e)^4 - 10*(11*A + 3*B)*a^3*c*cos(f*x + e)^3 - 12*(11*A + 3*
B)*a^3*c*cos(f*x + e)^2 - 16*(11*A + 3*B)*a^3*c*cos(f*x + e) - 32*(11*A +
3*B)*a^3*c)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e) - f*si
n(f*x + e) + f)
```

Sympy [F]

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = a^3\left(\int Ac\sqrt{-c\sin(e+fx)+c}\,dx + \int 2Ac\sqrt{-c\sin(e+fx)+c}\sin(e+fx)\,dx + \int \left(-2Ac\sqrt{-c\sin(e+fx)+c}\sin^3(e+fx)\right)dx + \int \left(-Ac\sqrt{-c\sin(e+fx)+c}\sin^4(e+fx)\right)dx + \int Bc\sqrt{-c\sin(e+fx)+c}\sin(e+fx)\,dx + \int 2Bc\sqrt{-c\sin(e+fx)+c}\sin^2(e+fx)\,dx + \int \left(-2Bc\sqrt{-c\sin(e+fx)+c}\sin^4(e+fx)\right)dx + \int \left(-Bc\sqrt{-c\sin(e+fx)+c}\sin^5(e+fx)\right)dx\right)$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2),x)
```

output
```
a**3*(Integral(A*c*sqrt(-c*sin(e + f*x) + c), x) + Integral(2*A*c*sqrt(-c*
sin(e + f*x) + c)*sin(e + f*x), x) + Integral(-2*A*c*sqrt(-c*sin(e + f*x)
+ c)*sin(e + f*x)**3, x) + Integral(-A*c*sqrt(-c*sin(e + f*x) + c)*sin(e +
 f*x)**4, x) + Integral(B*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x), x) + I
ntegral(2*B*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x) + Integral(-2*
B*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**4, x) + Integral(-B*c*sqrt(-c*
sin(e + f*x) + c)*sin(e + f*x)**5, x))
```

Maxima [F]

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^3(-c\sin(fx+e)+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*(-c*sin(f*x + e) + c
)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(112) = 224$.

Time = 0.37 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.37

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = -\frac{\sqrt{2}\left(693\,Ba^3c\cos\left(-\frac{5}{4}\pi+\frac{5}{2}fx+\frac{5}{2}e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-63\,Ba^3c\cos\left(-\frac{11}{4}\pi+\frac{11}{2}fx+\frac{11}{2}e\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, al
gorithm="giac")
```

output
```
-1/11088*sqrt(2)*(693*B*a^3*c*cos(-5/4*pi + 5/2*f*x + 5/2*e)*sgn(sin(-1/4*
pi + 1/2*f*x + 1/2*e)) - 63*B*a^3*c*cos(-11/4*pi + 11/2*f*x + 11/2*e)*sgn(
sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 1386*(6*A*a^3*c*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e)) + B*a^3*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi + 1
/2*f*x + 1/2*e) + 462*(8*A*a^3*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B
*a^3*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-3/4*pi + 3/2*f*x + 3/2*e)
 - 99*(6*A*a^3*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + B*a^3*c*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e)))*cos(-7/4*pi + 7/2*f*x + 7/2*e) - 77*(2*A*a^3*c*s
gn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*a^3*c*sgn(sin(-1/4*pi + 1/2*f*x +
 1/2*e)))*cos(-9/4*pi + 9/2*f*x + 9/2*e))*sqrt(c)/f
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx=\int (A+B\sin(e+fx))\,(a+a\sin(e+fx))^3\,(c-c\sin(e+fx))^{3/2}\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(3/2)
,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(3/2)
, x)
```

Reduce [F]

$$\begin{aligned}\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c&-c\sin(e+fx))^{3/2}\,dx=\sqrt{c}\,a^3c\Bigg(\left(\left(\int\sqrt{-\sin(fx+e)+1}dx\right)a\\&-\left(\int\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^5\,dx\right)b\\&-\left(\int\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^4\,dx\right)a\\&-2\left(\int\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^4\,dx\right)b\\&-2\left(\int\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^3\,dx\right)a\\&+2\left(\int\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)b\\&+2\left(\int\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)a\\&+\left(\int\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)b\Bigg)\end{aligned}$$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x)
```

output
```
sqrt(c)*a**3*c*(int(sqrt( - sin(e + f*x) + 1),x)*a - int(sqrt( - sin(e + f
*x) + 1)*sin(e + f*x)**5,x)*b - int(sqrt( - sin(e + f*x) + 1)*sin(e + f*x)
**4,x)*a - 2*int(sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**4,x)*b - 2*int(sq
rt( - sin(e + f*x) + 1)*sin(e + f*x)**3,x)*a + 2*int(sqrt( - sin(e + f*x)
+ 1)*sin(e + f*x)**2,x)*b + 2*int(sqrt( - sin(e + f*x) + 1)*sin(e + f*x),x
)*a + int(sqrt( - sin(e + f*x) + 1)*sin(e + f*x),x)*b)
```

3.101 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\sqrt{c-c\sin(e+}$

Optimal result

Integrand size = 38, antiderivative size = 81

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \frac{2a^3(9A+5B)c^4\cos^7(e+fx)}{63f(c-c\sin(e+fx))^{7/2}} - \frac{2a^3Bc^3\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}$$

output
```
2/63*a^3*(9*A+5*B)*c^4*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(7/2)-2/9*a^3*B*c^3
*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(5/2)
```

Mathematica [A] (verified)

Time = 2.35 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.10

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \frac{2a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(9A-2B+7B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{63f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

input
```
Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f
*x]],x]
```

output
```
(2*a^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(9*A - 2*B + 7*B*Sin[e + f*
x])*Sqrt[c - c*Sin[e + f*x]])/(63*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 3446, 3042, 3335, 3042, 3152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^3\sqrt{c-c\sin(e+fx)}(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^3\sqrt{c-c\sin(e+fx)}(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3446}$$

$$a^3c^3\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\int \frac{\cos(e+fx)^6(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3335}$$

$$a^3c^3\left(\frac{1}{9}(9A+5B)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{5/2}}dx-\frac{2B\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left(\frac{1}{9}(9A+5B)\int \frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^{5/2}}dx-\frac{2B\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}\right)$$

$$\Big\downarrow \text{3152}$$

$$a^3c^3\left(\frac{2c(9A+5B)\cos^7(e+fx)}{63f(c-c\sin(e+fx))^{7/2}}-\frac{2B\cos^7(e+fx)}{9f(c-c\sin(e+fx))^{5/2}}\right)$$

input
```
Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]],x
]
```

output
```
a^3*c^3*((2*(9*A + 5*B)*c*Cos[e + f*x]^7)/(63*f*(c - c*Sin[e + f*x])^(7/2)
) - (2*B*Cos[e + f*x]^7)/(9*f*(c - c*Sin[e + f*x])^(5/2)))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3335
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 1.98 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.80

method	result
default	$-\frac{2(\sin(fx+e)-1)c(1+\sin(fx+e))^4 a^3(7B\sin(fx+e)+9A-2B)}{63\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$
parts	$-\frac{2a^3A(\sin(fx+e)-1)(1+\sin(fx+e))c}{\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}-\frac{2a^3B(\sin(fx+e)-1)c(1+\sin(fx+e))\left(35\sin(fx+e)^4-40\sin(fx+e)^3+48\sin(fx+e)^2-\right.}{315\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
-2/63*(sin(f*x+e)-1)*c*(1+sin(f*x+e))^4*a^3*(7*B*sin(f*x+e)+9*A-2*B)/cos(f
*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(73) = 146$.

Time = 0.08 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.86

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$$
$$=\frac{2\left(7\,Ba^3\cos\left(fx+e\right)^5+(9\,A+26\,B)a^3\cos\left(fx+e\right)^4-(27\,A+29\,B)a^3\cos\left(fx+e\right)^3-4\,(18\,A+17\,B\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, al
gorithm="fricas")
```

output
```
2/63*(7*B*a^3*cos(f*x + e)^5 + (9*A + 26*B)*a^3*cos(f*x + e)^4 - (27*A + 2
9*B)*a^3*cos(f*x + e)^3 - 4*(18*A + 17*B)*a^3*cos(f*x + e)^2 + 4*(9*A + 5*
B)*a^3*cos(f*x + e) + 8*(9*A + 5*B)*a^3 + (7*B*a^3*cos(f*x + e)^4 - (9*A +
 19*B)*a^3*cos(f*x + e)^3 - 12*(3*A + 4*B)*a^3*cos(f*x + e)^2 + 4*(9*A + 5
*B)*a^3*cos(f*x + e) + 8*(9*A + 5*B)*a^3)*sin(f*x + e))*sqrt(-c*sin(f*x +
e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)
```

Sympy [F]

$$\int (a + a\sin(e + fx))^3 (A + B\sin(e + fx))\sqrt{c - c\sin(e + fx)}\, dx$$
$$= a^3 \left(\int A\sqrt{-c\sin(e + fx) + c}\, dx + \int 3A\sqrt{-c\sin(e + fx) + c}\sin(e + fx)\, dx + \int 3A\sqrt{-c\sin(e + fx) + c}\sin^2(e + fx)\, dx + \int A\sqrt{-c\sin(e + fx) + c}\sin^3(e + fx)\, dx + \int B\sqrt{-c\sin(e + fx) + c}\sin(e + fx)\, dx + \int 3B\sqrt{-c\sin(e + fx) + c}\sin^2(e + fx)\, dx + \int 3B\sqrt{-c\sin(e + fx) + c}\sin^3(e + fx)\, dx + \int B\sqrt{-c\sin(e + fx) + c}\sin^4(e + fx)\, dx \right)$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2),x)
```

output
```
a**3*(Integral(A*sqrt(-c*sin(e + f*x) + c), x) + Integral(3*A*sqrt(-c*sin(
e + f*x) + c)*sin(e + f*x), x) + Integral(3*A*sqrt(-c*sin(e + f*x) + c)*si
n(e + f*x)**2, x) + Integral(A*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**3,
x) + Integral(B*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x), x) + Integral(3*B*
sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x) + Integral(3*B*sqrt(-c*sin(e
 + f*x) + c)*sin(e + f*x)**3, x) + Integral(B*sqrt(-c*sin(e + f*x) + c)*si
n(e + f*x)**4, x))
```

Maxima [F]

$$\int (a + a\sin(e+fx))^3 (A + B\sin(e+fx))\sqrt{c - c\sin(e+fx)}\,dx$$
$$= \int (B\sin(fx+e) + A)(a\sin(fx+e) + a)^3 \sqrt{-c\sin(fx+e) + c}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*sqrt(-c*sin(f*x + e)
 + c), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(73) = 146$.

Time = 0.34 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.12

$$\int (a + a\sin(e+fx))^3 (A + B\sin(e+fx))\sqrt{c - c\sin(e+fx)}\,dx =$$
$$-\frac{\sqrt{2}\left(7\,Ba^3\cos\left(-\frac{9}{4}\,\pi + \frac{9}{2}\,fx + \frac{9}{2}\,e\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,fx + \frac{1}{2}\,e\right)\right) + 126\left(5\,Aa^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,fx + \right.\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, al
gorithm="giac")
```

output
```
-1/504*sqrt(2)*(7*B*a^3*cos(-9/4*pi + 9/2*f*x + 9/2*e)*sgn(sin(-1/4*pi + 1
/2*f*x + 1/2*e)) + 126*(5*A*a^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*B*
a^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-1/4*pi + 1/2*f*x + 1/2*e) +
42*(9*A*a^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*B*a^3*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e)))*cos(-3/4*pi + 3/2*f*x + 3/2*e) + 126*(A*a^3*sgn(sin(-
1/4*pi + 1/2*f*x + 1/2*e)) + B*a^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*co
s(-5/4*pi + 5/2*f*x + 5/2*e) + 9*(2*A*a^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*
e)) + 5*B*a^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(-7/4*pi + 7/2*f*x +
 7/2*e))*sqrt(c)/f
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$$
$$=\int (A+B\sin(e+f\,x))\,\left(a+a\sin(e+f\,x)\right)^3\,\sqrt{c-c\sin(e+f\,x)}\,dx$$

input

```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(1/2)
,x)
```

output

```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(1/2)
, x)
```

Reduce [F]

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$$
$$=\sqrt{c}\,a^3\Bigg(\left(\int\sqrt{-\sin(fx+e)+1}dx\right)a$$
$$+\left(\int\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^4\,dx\right)b$$
$$+\left(\int\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^3\,dx\right)a$$
$$+3\left(\int\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^3\,dx\right)b$$
$$+3\left(\int\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)a$$
$$+3\left(\int\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)b$$
$$+3\left(\int\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)a$$
$$+\left(\int\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)b\Bigg)$$

input

```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)
```

output
```
sqrt(c)*a**3*(int(sqrt( - sin(e + f*x) + 1),x)*a + int(sqrt( - sin(e + f*x
) + 1)*sin(e + f*x)**4,x)*b + int(sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**
3,x)*a + 3*int(sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**3,x)*b + 3*int(sqrt
( - sin(e + f*x) + 1)*sin(e + f*x)**2,x)*a + 3*int(sqrt( - sin(e + f*x) +
1)*sin(e + f*x)**2,x)*b + 3*int(sqrt( - sin(e + f*x) + 1)*sin(e + f*x),x)*
a + int(sqrt( - sin(e + f*x) + 1)*sin(e + f*x),x)*b)
```

3.102 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 200

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{8\sqrt{2}a^3(A+B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{c}f} - \frac{2a^3Bc^3\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}} - \frac{2a^3(A+B)c^2\cos^5(e+fx)}{5f(c-c\sin(e+fx))^{5/2}} - \frac{4a^3(A+B)c\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} - \frac{8a^3(A+B)\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}$$

output
```
8*2^(1/2)*a^3*(A+B)*arctanh(1/2*c^(1/2)*cos(f*x+e)*2^(1/2)/(c-c*sin(f*x+e)
)^(1/2))/c^(1/2)/f-2/7*a^3*B*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(7/2)-2/5
*a^3*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)-4/3*a^3*(A+B)*c*cos(f
*x+e)^3/f/(c-c*sin(f*x+e))^(3/2)-8*a^3*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))
^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 12.03 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.96

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3\left((6720+6720i)\sqrt[4]{-1}(A+B)\arctan\left(\left(\frac{1}{2}+\right.\right.\right.}{}$$

input
```
Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e +
 f*x]],x]
```

output
```
-1/420*(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*((6
720 + 6720*I)*(-1)^(1/4)*(A + B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e
 + f*x)/4])] - 2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-2086*A - 2236*B +
 6*(7*A + 22*B)*Cos[2*(e + f*x)] - (448*A + 673*B)*Sin[e + f*x] + 15*B*Sin
[3*(e + f*x)])))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*Sqrt[c - c*Sin
[e + f*x]])
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.342$, Rules used = {3042, 3446, 3042, 3339, 3042, 3158, 3042, 3158, 3042, 3158, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3446}$$

$$a^3c^3\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\int \frac{\cos(e+fx)^6(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{3339}$$

$$a^3c^3\left((A+B)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{7/2}}dx-\frac{2B\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left((A+B)\int \frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^{7/2}}dx-\frac{2B\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}}\right)$$

$$\Big\downarrow \text{3158}$$

$$a^3c^3\left((A+B)\left(\frac{2\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{5/2}}dx}{c}-\frac{2\cos^5(e+fx)}{5cf(c-c\sin(e+fx))^{5/2}}\right)-\frac{2B\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left((A+B)\left(\frac{2\int \frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^{5/2}}dx}{c}-\frac{2\cos^5(e+fx)}{5cf(c-c\sin(e+fx))^{5/2}}\right)-\frac{2B\cos^7(e+fx)}{7f(c-c\sin(e+fx))^{7/2}}\right)$$

$$\Big\downarrow \text{3158}$$

$$a^3c^3\left((A+B)\left(\frac{2\left(\frac{2\int \frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}dx}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)}{c}-\frac{2\cos^5(e+fx)}{5cf(c-c\sin(e+fx))^{5/2}}\right)-\frac{2B\cos^7(e+}{7f(c-c\sin(e-}\right.$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left((A+B)\left(\frac{2\left(\frac{2\int \frac{\cos(e+fx)^2}{(c-c\sin(e+fx))^{3/2}}dx}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)}{c}-\frac{2\cos^5(e+fx)}{5cf(c-c\sin(e+fx))^{5/2}}\right)-\frac{2B\cos^7(e+}{7f(c-c\sin(e-}\right.$$

$\downarrow$ 3158

$$a^3c^3\left((A+B)\left(\frac{2\left(\frac{2\left(\frac{2\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{c}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)}{c}-\frac{2\cos^5(e+fx)}{5cf(c-c\sin(e+fx))^{5/2}}\right)\right.$$

$\downarrow$ 3042

$$a^3c^3\left((A+B)\left(\frac{2\left(\frac{2\left(\frac{2\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{c}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)}{c}-\frac{2\cos^5(e+fx)}{5cf(c-c\sin(e+fx))^{5/2}}\right)\right.$$

$\downarrow$ 3128

$$a^3c^3\left((A+B)\left(\frac{2\left(\frac{2\left(-\frac{4\int\frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{cf}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)}{c}-\frac{2\cos^5}{5cf(c-cs}\right.\right.$$

$\downarrow$ 219

$$a^3c^3\left((A+B)\left(\frac{2\left(\frac{2\left(\frac{2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)}{c}-\frac{2\cos^5(e+f}{5cf(c-c\sin(e+}\right.\right.$$

input
```
Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]]
,x]
```

output
```
a^3*c^3*((-2*B*Cos[e + f*x]^7)/(7*f*(c - c*Sin[e + f*x])^(7/2)) + (A + B)*
((-2*Cos[e + f*x]^5)/(5*c*f*(c - c*Sin[e + f*x])^(5/2)) + (2*((-2*Cos[e +
f*x]^3)/(3*c*f*(c - c*Sin[e + f*x])^(3/2)) + (2*((2*Sqrt[2]*ArcTanh[(Sqrt[
c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(c^(3/2)*f) - (2*Cos
[e + f*x])/(c*f*Sqrt[c - c*Sin[e + f*x]])))/c))/c))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3158
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(a*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
 EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && In
tegersQ[2*m, 2*p]
```

rule 3339
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.16

method	result
default	$\frac{2(\sin(fx+e)-1)\sqrt{c(1+\sin(fx+e))}\,a^3\left(-420c^{\frac{7}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)A-420c^{\frac{7}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)B-\right.}{}$
parts	$-\frac{a^3A(\sin(fx+e)-1)\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)}{\sqrt{c}\,\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}-\frac{a^3B(\sin(fx+e)-1)\sqrt{c(1+\sin(fx+e))}\left(105c^{\frac{7}{2}}\sqrt{2}\right.}{}$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
2/105*(sin(f*x+e)-1)*(c*(1+sin(f*x+e)))^(1/2)*a^3*(-420*c^(7/2)*2^(1/2)*ar
ctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*A-420*c^(7/2)*2^(1/2)*
arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*B+15*B*(c*(1+sin(f*x
+e)))^(7/2)+21*A*(c*(1+sin(f*x+e)))^(5/2)*c+21*B*(c*(1+sin(f*x+e)))^(5/2)*
c+70*A*(c*(1+sin(f*x+e)))^(3/2)*c^2+70*B*(c*(1+sin(f*x+e)))^(3/2)*c^2+420*
A*c^3*(c*(1+sin(f*x+e)))^(1/2)+420*B*c^3*(c*(1+sin(f*x+e)))^(1/2))/c^4/cos
(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.76

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$

$$= \frac{2\left(\frac{210\sqrt{2}((A+B)a^3c\cos(fx+e)-(A+B)a^3c\sin(fx+e)+(A+B)a^3c)\log\left(-\frac{\cos(fx+e)^2+(\cos(fx+e)-2)\sin(fx+e)+\frac{2\sqrt{2}\sqrt{-c\sin(fx+e)+c}(\cos(f\cdots}{\sqrt{c}}}{\cos(fx+e)^2+(\cos(fx+e)+2)\sin(fx+e)-\cos\cdots}\right.}{\sqrt{c}}\right.}{\cdots}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, al
gorithm="fricas")
```

output
```
2/105*(210*sqrt(2)*((A + B)*a^3*c*cos(f*x + e) - (A + B)*a^3*c*sin(f*x + e
) + (A + B)*a^3*c)*log(-(cos(f*x + e)^2 + (cos(f*x + e) - 2)*sin(f*x + e)
+ 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x + e) + 1)/sq
rt(c) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x +
 e) - cos(f*x + e) - 2))/sqrt(c) - (15*B*a^3*cos(f*x + e)^4 - 3*(7*A + 22*
B)*a^3*cos(f*x + e)^3 - (133*A + 253*B)*a^3*cos(f*x + e)^2 + 4*(133*A + 14
8*B)*a^3*cos(f*x + e) + 4*(161*A + 191*B)*a^3 - (15*B*a^3*cos(f*x + e)^3 +
 3*(7*A + 27*B)*a^3*cos(f*x + e)^2 - 4*(28*A + 43*B)*a^3*cos(f*x + e) - 4*
(161*A + 191*B)*a^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c*f*cos(f*x
 + e) - c*f*sin(f*x + e) + c*f)
```

Sympy [F]

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = a^3\left(\int \frac{A}{\sqrt{-c\sin(e+fx)+c}}\,dx + \int \frac{3A\sin(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx + \int \frac{3A\sin^2(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx + \int \frac{A\sin^3(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx + \int \frac{B\sin(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx + \int \frac{3B\sin^2(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx + \int \frac{3B\sin^3(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx + \int \frac{B\sin^4(e+fx)}{\sqrt{-c\sin(e+fx)+c}}\,dx\right)$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

output
```
a**3*(Integral(A/sqrt(-c*sin(e + f*x) + c), x) + Integral(3*A*sin(e + f*x)
/sqrt(-c*sin(e + f*x) + c), x) + Integral(3*A*sin(e + f*x)**2/sqrt(-c*sin(
e + f*x) + c), x) + Integral(A*sin(e + f*x)**3/sqrt(-c*sin(e + f*x) + c),
x) + Integral(B*sin(e + f*x)/sqrt(-c*sin(e + f*x) + c), x) + Integral(3*B*
sin(e + f*x)**2/sqrt(-c*sin(e + f*x) + c), x) + Integral(3*B*sin(e + f*x)*
*3/sqrt(-c*sin(e + f*x) + c), x) + Integral(B*sin(e + f*x)**4/sqrt(-c*sin(
e + f*x) + c), x))
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^3}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3/sqrt(-c*sin(f*x + e)
 + c), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{\sqrt{c - c\sin(e + fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{\sqrt{c - c\sin(e + fx)}}\,dx$$
$$= \int \frac{(A + B\,\sin(e + f\,x))\,(a + a\,\sin(e + f\,x))^3}{\sqrt{c - c\,\sin(e + f\,x)}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(1/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(1/
2), x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= \frac{\sqrt{c}\,a^3\left(-\left(\int \frac{\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)-1}dx\right)a-\left(\int \frac{\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^4}{\sin(fx+e)-1}dx\right)b-\left(\int \frac{\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^3}{\sin(fx+e)-1}dx\right)a-3$$

input

```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

output

```
(sqrt(c)*a**3*( - int(sqrt( - sin(e + f*x) + 1)/(sin(e + f*x) - 1),x)*a -
int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**4)/(sin(e + f*x) - 1),x)*b -
int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**3)/(sin(e + f*x) - 1),x)*a -
3*int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**3)/(sin(e + f*x) - 1),x)*b
- 3*int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x) - 1),x)*
a - 3*int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x) - 1),x
)*b - 3*int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x) - 1),x)
*a - int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x) - 1),x)*b)
)/c
```

3.103 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 218

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = -\frac{2\sqrt{2}a^3(5A+9B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f} + \frac{a^3(A+B)c^3\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}} + \frac{a^3(5A+9B)c\cos^5(e+fx)}{10f(c-c\sin(e+fx))^{5/2}} + \frac{a^3(5A+9B)\cos^3(e+fx)}{3f(c-c\sin(e+fx))^{3/2}} + \frac{2a^3(5A+9B)\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}$$

output

```
-2*2^(1/2)*a^3*(5*A+9*B)*arctanh(1/2*c^(1/2)*cos(f*x+e)*2^(1/2)/(c-c*sin(f
*x+e))^(1/2))/c^(3/2)/f+1/2*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^
(9/2)+1/10*a^3*(5*A+9*B)*c*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)+1/3*a^3*(
5*A+9*B)*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(3/2)+2*a^3*(5*A+9*B)*cos(f*x+e)/
c/f/(c-c*sin(f*x+e))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 12.39 (sec) , antiderivative size = 444, normalized size of antiderivative = 2.04

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))}{}$$

input
```
Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^(3/2),x]
```

output
```
(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(120*(A +
B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) + (120 + 120*I)*(-1)^(1/4)*(5*A +
 9*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])^2 + 30*(9*A + 20*B)*Cos[(e + f*x)/2]*(Cos[(e + f*x
)/2] - Sin[(e + f*x)/2])^2 - 5*(2*A + 9*B)*Cos[(3*(e + f*x))/2]*(Cos[(e +
f*x)/2] - Sin[(e + f*x)/2])^2 - 3*B*Cos[(5*(e + f*x))/2]*(Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2])^2 + 240*(A + B)*Sin[(e + f*x)/2] + 30*(9*A + 20*B)*(C
os[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] + 5*(2*A + 9*B)*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(3*(e + f*x))/2] - 3*B*(Cos[(e +
f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(5*(e + f*x))/2]))/(30*f*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^6*(c - c*Sin[e + f*x])^(3/2))
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.97, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.342$, Rules used = {3042, 3446, 3042, 3338, 3042, 3158, 3042, 3158, 3042, 3158, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3446}$$

$$a^3c^3\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\int \frac{\cos(e+fx)^6(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}dx$$

$$\Big\downarrow \text{3338}$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}}-\frac{(5A+9B)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{7/2}}dx}{4c}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}}-\frac{(5A+9B)\int \frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^{7/2}}dx}{4c}\right)$$

$$\Big\downarrow \text{3158}$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}}-\frac{(5A+9B)\left(\frac{2\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{5/2}}dx}{c}-\frac{2\cos^5(e+fx)}{5cf(c-c\sin(e+fx))^{5/2}}\right)}{4c}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}}-\frac{(5A+9B)\left(\frac{2\int \frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^{5/2}}dx}{c}-\frac{2\cos^5(e+fx)}{5cf(c-c\sin(e+fx))^{5/2}}\right)}{4c}\right)$$

$$\Big\downarrow \text{3158}$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}}-\frac{(5A+9B)\left(\frac{2\left(\frac{2\int\frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}dx}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)}{c}-\frac{2\cos^5(e+fx)}{5cf(c-c\sin(e+fx))^{5/2}}\right)}{4c}\right.$$

$\downarrow$ 3042

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}}-\frac{(5A+9B)\left(\frac{2\left(\frac{2\int\frac{\cos(e+fx)^2}{(c-c\sin(e+fx))^{3/2}}dx}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)}{c}-\frac{2\cos^5(e+fx)}{5cf(c-c\sin(e+fx))^{5/2}}\right)}{4c}\right.$$

$\downarrow$ 3158

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}}-\frac{(5A+9B)\left(\frac{2\left(\frac{2\left(\frac{2\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{c}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)}{c}-\frac{}{5cf(c}\right.}{4c}\right.$$

$\downarrow$ 3042

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}}-\frac{(5A+9B)\left(\frac{2\left(\frac{2\left(\frac{2\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{c}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)}{c}-\frac{\cdots}{5cf(\cdots}\right.}{4c}\right.$$

$\downarrow$ 3128

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}}-\frac{(5A+9B)\left(\frac{2\left(\frac{2\left(-\frac{4\int\frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{cf}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{c}-\frac{2\cos^3(\cdots}{3cf(c-c\sin\cdots}\right.}{c}\right.}{4c}\right.$$

$$\Big\downarrow \; 219$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{2f(c-c\sin(e+fx))^{9/2}}-\frac{(5A+9B)\left(\frac{2\left(\frac{2\left(\frac{2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^3}\right)}{c}\right)}{4c}\right)$$

input
```
Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(3/
2),x]
```

output
```
a^3*c^3*(((A + B)*Cos[e + f*x]^7)/(2*f*(c - c*Sin[e + f*x])^(9/2)) - ((5*A
 + 9*B)*((-2*Cos[e + f*x]^5)/(5*c*f*(c - c*Sin[e + f*x])^(5/2)) + (2*((-2*
Cos[e + f*x]^3)/(3*c*f*(c - c*Sin[e + f*x])^(3/2)) + (2*((2*Sqrt[2]*ArcTan
h[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(c^(3/2)*f)
- (2*Cos[e + f*x])/(c*f*Sqrt[c - c*Sin[e + f*x]])))/c))/c))/(4*c))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3158
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(a*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
 EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && In
tegersQ[2*m, 2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.62

method	result
default	$-\frac{2a^3\left(\sin(fx+e)\left(60A\sqrt{c+c\sin(fx+e)}\,c^{\frac{5}{2}}+5A(c+c\sin(fx+e))^{\frac{3}{2}}c^{\frac{3}{2}}+120B\sqrt{c+c\sin(fx+e)}\,c^{\frac{5}{2}}+15B(c+c\sin(fx+e))^{\frac{3}{2}}c^{\frac{3}{2}}+3B\right.\right.}{}$
parts	Expression too large to display

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,method=_R
ETURNVERBOSE)
```

output
```
-2/15/c^(9/2)*a^3*(sin(f*x+e)*(60*A*(c+c*sin(f*x+e))^(1/2)*c^(5/2)+5*A*(c+
c*sin(f*x+e))^(3/2)*c^(3/2)+120*B*(c+c*sin(f*x+e))^(1/2)*c^(5/2)+15*B*(c+c
*sin(f*x+e))^(3/2)*c^(3/2)+3*B*(c+c*sin(f*x+e))^(5/2)*c^(1/2)-75*A*arctanh
(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^3-135*B*arctanh(1/2
*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^3)-90*A*(c+c*sin(f*x+e)
)^(1/2)*c^(5/2)-5*A*(c+c*sin(f*x+e))^(3/2)*c^(3/2)-150*B*(c+c*sin(f*x+e))^
(1/2)*c^(5/2)-15*B*(c+c*sin(f*x+e))^(3/2)*c^(3/2)-3*B*(c+c*sin(f*x+e))^(5/
2)*c^(1/2)+75*A*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2
)*c^3+135*B*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^
3)*(c*(1+sin(f*x+e)))^(1/2)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(195) = 390$.

Time = 0.10 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.97

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\frac{15\sqrt{2}\left((5A+9B)a^3c\cos(fx+e)^2-(5A+9B)a^3c\cos(fx+e)-2(5A+9B)a^3c \cdots\right.}{\cdots}}{\cdots}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, al
gorithm="fricas")
```

output
```
1/15*(15*sqrt(2)*((5*A + 9*B)*a^3*c*cos(f*x + e)^2 - (5*A + 9*B)*a^3*c*cos
(f*x + e) - 2*(5*A + 9*B)*a^3*c + ((5*A + 9*B)*a^3*c*cos(f*x + e) + 2*(5*A
 + 9*B)*a^3*c)*sin(f*x + e))*log(-(cos(f*x + e)^2 + (cos(f*x + e) - 2)*sin
(f*x + e) - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x +
e) + 1)/sqrt(c) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)
*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(c) + 2*(3*B*a^3*cos(f*x + e)^4 - (
5*A + 18*B)*a^3*cos(f*x + e)^3 - (65*A + 141*B)*a^3*cos(f*x + e)^2 - 30*(3
*A + 5*B)*a^3*cos(f*x + e) - 30*(A + B)*a^3 - (3*B*a^3*cos(f*x + e)^3 + (5
*A + 21*B)*a^3*cos(f*x + e)^2 - 60*(A + 2*B)*a^3*cos(f*x + e) + 30*(A + B)
*a^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c^2*f*cos(f*x + e)^2 - c^2
*f*cos(f*x + e) - 2*c^2*f + (c^2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^3}{(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c
)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^3}{(c-c\,\sin(e+f\,x))^{3/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(3/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(3/
2), x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\sqrt{c}\,a^3\left(\left(\int \frac{\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)^2-2\sin(fx+e)+1}dx\right)a+\left(\int \frac{\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)^2-2\,\mathrm{si}}\right.\right.}{}$$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*a**3*(int(sqrt( - sin(e + f*x) + 1)/(sin(e + f*x)**2 - 2*sin(e +
f*x) + 1),x)*a + int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**4)/(sin(e +
f*x)**2 - 2*sin(e + f*x) + 1),x)*b + int((sqrt( - sin(e + f*x) + 1)*sin(e
+ f*x)**3)/(sin(e + f*x)**2 - 2*sin(e + f*x) + 1),x)*a + 3*int((sqrt( - si
n(e + f*x) + 1)*sin(e + f*x)**3)/(sin(e + f*x)**2 - 2*sin(e + f*x) + 1),x)
*b + 3*int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x)**2 -
2*sin(e + f*x) + 1),x)*a + 3*int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**
2)/(sin(e + f*x)**2 - 2*sin(e + f*x) + 1),x)*b + 3*int((sqrt( - sin(e + f*
x) + 1)*sin(e + f*x))/(sin(e + f*x)**2 - 2*sin(e + f*x) + 1),x)*a + int((s
qrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**2 - 2*sin(e + f*x) +
 1),x)*b))/c**2
```

3.104 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 225

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{5a^3(3A+11B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}c^{5/2}f} + \frac{a^3(A+B)c^3\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}} - \frac{a^3(3A+11B)c\cos^5(e+fx)}{8f(c-c\sin(e+fx))^{7/2}} - \frac{5a^3(3A+11B)\cos^3(e+fx)}{24cf(c-c\sin(e+fx))^{3/2}} - \frac{5a^3(3A+11B)\cos(e+fx)}{4c^2f\sqrt{c-c\sin(e+fx)}}$$

output
```
5/4*a^3*(3*A+11*B)*arctanh(1/2*c^(1/2)*cos(f*x+e)*2^(1/2)/(c-c*sin(f*x+e))
^(1/2))*2^(1/2)/c^(5/2)/f+1/4*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e)
)^(11/2)-1/8*a^3*(3*A+11*B)*c*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(7/2)-5/24*a
^3*(3*A+11*B)*cos(f*x+e)^3/c/f/(c-c*sin(f*x+e))^(3/2)-5/4*a^3*(3*A+11*B)*c
os(f*x+e)/c^2/f/(c-c*sin(f*x+e))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 13.09 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.93

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))}{}$$

input
```
Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^(5/2),x]
```

output
```
(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(12*(A + B
)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) - 3*(9*A + 17*B)*(Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2])^3 - (15 + 15*I)*(-1)^(1/4)*(3*A + 11*B)*ArcTan[(1/2 +
 I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)
/2])^4 - 6*(2*A + 11*B)*Cos[(e + f*x)/2]*(Cos[(e + f*x)/2] - Sin[(e + f*x)
/2])^4 + 2*B*Cos[(3*(e + f*x))/2]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4
+ 24*(A + B)*Sin[(e + f*x)/2] - 6*(9*A + 17*B)*(Cos[(e + f*x)/2] - Sin[(e
+ f*x)/2])^2*Sin[(e + f*x)/2] - 6*(2*A + 11*B)*(Cos[(e + f*x)/2] - Sin[(e
+ f*x)/2])^4*Sin[(e + f*x)/2] - 2*B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
4*Sin[(3*(e + f*x))/2]))/(6*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(c -
 c*Sin[e + f*x])^(5/2))
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.93, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.342$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3158, 3042, 3158, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3446}$$

$$a^3c^3\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\int \frac{\cos(e+fx)^6(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}dx$$

$$\Big\downarrow \text{3338}$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}}-\frac{(3A+11B)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{9/2}}dx}{8c}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}}-\frac{(3A+11B)\int \frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^{9/2}}dx}{8c}\right)$$

$$\Big\downarrow \text{3159}$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}}-\frac{(3A+11B)\left(\frac{\cos^5(e+fx)}{cf(c-c\sin(e+fx))^{7/2}}-\frac{5\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{5/2}}dx}{2c^2}\right)}{8c}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}}-\frac{(3A+11B)\left(\frac{\cos^5(e+fx)}{cf(c-c\sin(e+fx))^{7/2}}-\frac{5\int \frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^{5/2}}dx}{2c^2}\right)}{8c}\right)$$

$$\Big\downarrow \text{3158}$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}}-\frac{(3A+11B)\left(\frac{\cos^5(e+fx)}{cf(c-c\sin(e+fx))^{7/2}}-\frac{5\left(\frac{2\int\frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}dx}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)}{2c^2}\right.}{8c}\right.$$

$\downarrow$ 3042

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}}-\frac{(3A+11B)\left(\frac{\cos^5(e+fx)}{cf(c-c\sin(e+fx))^{7/2}}-\frac{5\left(\frac{2\int\frac{\cos(e+fx)^2}{(c-c\sin(e+fx))^{3/2}}dx}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)}{2c^2}\right.}{8c}\right.$$

$\downarrow$ 3158

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}}-\frac{(3A+11B)\left(\frac{\cos^5(e+fx)}{cf(c-c\sin(e+fx))^{7/2}}-\frac{5\left(\frac{2\left(\frac{2\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{c}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{c}-\frac{}{3cf(}\right.}{2c^2}\right.}{8c}\right.$$

$\downarrow$ 3042

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}}-\frac{(3A+11B)\left(\frac{\cos^5(e+fx)}{cf(c-c\sin(e+fx))^{7/2}}-\frac{5\left(\frac{2\left(\frac{2\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{c}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{c}-\frac{}{3cf(}\right.}{2c^2}\right.}{8c}\right.$$

$$\downarrow 3128$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}}-\frac{(3A+11B)\left(\frac{\cos^5(e+fx)}{cf(c-c\sin(e+fx))^{7/2}}-\frac{5\left(\frac{2\left(-\frac{4\int\frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{cf}-\frac{}{cf}\right.}{c}\right.}{2c^2}\right.}{8c}\right.$$

$$\Big\downarrow \text{219}$$

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{4f(c-c\sin(e+fx))^{11/2}}-\frac{(3A+11B)\left(\frac{\cos^5(e+fx)}{cf(c-c\sin(e+fx))^{7/2}}-\frac{5\left(\frac{2\left(\frac{2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{c^{3/2}f}-\frac{2\cos(e+fx)}{cf\sqrt{c-c\sin(e+fx)}}\right)}{c}-\frac{2\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{3/2}}\right)}{2c^2}\right)}{8c}\right)$$

input
```
Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(5/
2),x]
```

output
```
a^3*c^3*(((A + B)*Cos[e + f*x]^7)/(4*f*(c - c*Sin[e + f*x])^(11/2)) - ((3*
A + 11*B)*(Cos[e + f*x]^5/(c*f*(c - c*Sin[e + f*x])^(7/2)) - (5*((-2*Cos[e
 + f*x]^3)/(3*c*f*(c - c*Sin[e + f*x])^(3/2)) + (2*((2*Sqrt[2]*ArcTanh[(Sq
rt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(c^(3/2)*f) - (2*
Cos[e + f*x])/(c*f*Sqrt[c - c*Sin[e + f*x]])))/c))/(2*c^2)))/(8*c))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3158
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(a*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
 EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && In
tegersQ[2*m, 2*p]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(198) = 396$.

Time = 1.86 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.93

method	result
default	$\frac{a^3\left(\left(-24A\sqrt{c+c\sin(fx+e)}\,c^{\frac{3}{2}}+45A\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,c^2-120B\sqrt{c+c\sin(fx+e)}\,c^{\frac{3}{2}}-8B(c+c\sin(fx+e))^{\frac{3}{2}}\sqrt{c+}\right.\right.}{}$
parts	Expression too large to display

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,method=_R
ETURNVERBOSE)
```

output
```
1/12*a^3*((-24*A*(c+c*sin(f*x+e))^(1/2)*c^(3/2)+45*A*arctanh(1/2*(c+c*sin(
f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^2-120*B*(c+c*sin(f*x+e))^(1/2)*c^
(3/2)-8*B*(c+c*sin(f*x+e))^(3/2)*c^(1/2)+165*B*arctanh(1/2*(c+c*sin(f*x+e)
)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^2)*cos(f*x+e)^2+sin(f*x+e)*(-48*A*(c+c*
sin(f*x+e))^(1/2)*c^(3/2)+90*A*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/
c^(1/2))*2^(1/2)*c^2-240*B*(c+c*sin(f*x+e))^(1/2)*c^(3/2)-16*B*(c+c*sin(f*
x+e))^(3/2)*c^(1/2)+330*B*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/
2))*2^(1/2)*c^2)+132*A*(c+c*sin(f*x+e))^(1/2)*c^(3/2)-54*A*(c+c*sin(f*x+e)
)^(3/2)*c^(1/2)-90*A*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2
^(1/2)*c^2+420*B*(c+c*sin(f*x+e))^(1/2)*c^(3/2)-86*B*(c+c*sin(f*x+e))^(3/2
)*c^(1/2)-330*B*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2
)*c^2)*(c*(1+sin(f*x+e)))^(1/2)/c^(9/2)/(sin(f*x+e)-1)/cos(f*x+e)/(c-c*sin
(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 505 vs. $2(198) = 396$.

Time = 0.10 (sec) , antiderivative size = 505, normalized size of antiderivative = 2.24

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{15\sqrt{2}\left((3A+11B)a^3\cos(fx+e)^3+3(3A+11B)a^3 c\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, al
gorithm="fricas")
```

output
```
1/24*(15*sqrt(2)*((3*A + 11*B)*a^3*cos(f*x + e)^3 + 3*(3*A + 11*B)*a^3*cos
(f*x + e)^2 - 2*(3*A + 11*B)*a^3*cos(f*x + e) - 4*(3*A + 11*B)*a^3 - ((3*A
 + 11*B)*a^3*cos(f*x + e)^2 - 2*(3*A + 11*B)*a^3*cos(f*x + e) - 4*(3*A + 1
1*B)*a^3)*sin(f*x + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c
*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x
 + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(
f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*(4*B*a^3*cos(f*x + e)^
4 - 4*(3*A + 14*B)*a^3*cos(f*x + e)^3 + 3*(13*A + 37*B)*a^3*cos(f*x + e)^2
 + 3*(13*A + 53*B)*a^3*cos(f*x + e) - 12*(A + B)*a^3 - (4*B*a^3*cos(f*x +
e)^3 + 12*(A + 5*B)*a^3*cos(f*x + e)^2 + 3*(17*A + 57*B)*a^3*cos(f*x + e)
+ 12*(A + B)*a^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c^3*f*cos(f*x
+ e)^3 + 3*c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f - (c^3*f*
cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^3}{(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c
)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^3}{(c-c\,\sin(e+f\,x))^{5/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(5/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(5/
2), x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\sqrt{c}\,a^3\left(-\left(\int \frac{\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)^3-3\sin(fx+e)^2+3\sin(fx+e)-1}dx\right)a-\left(\int\right.\right.}{}$$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)
```

output
```
(sqrt(c)*a**3*( - int(sqrt( - sin(e + f*x) + 1)/(sin(e + f*x)**3 - 3*sin(e
 + f*x)**2 + 3*sin(e + f*x) - 1),x)*a - int((sqrt( - sin(e + f*x) + 1)*sin
(e + f*x)**4)/(sin(e + f*x)**3 - 3*sin(e + f*x)**2 + 3*sin(e + f*x) - 1),x
)*b - int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**3)/(sin(e + f*x)**3 - 3
*sin(e + f*x)**2 + 3*sin(e + f*x) - 1),x)*a - 3*int((sqrt( - sin(e + f*x)
+ 1)*sin(e + f*x)**3)/(sin(e + f*x)**3 - 3*sin(e + f*x)**2 + 3*sin(e + f*x
) - 1),x)*b - 3*int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f
*x)**3 - 3*sin(e + f*x)**2 + 3*sin(e + f*x) - 1),x)*a - 3*int((sqrt( - sin
(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x)**3 - 3*sin(e + f*x)**2 + 3*s
in(e + f*x) - 1),x)*b - 3*int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(si
n(e + f*x)**3 - 3*sin(e + f*x)**2 + 3*sin(e + f*x) - 1),x)*a - int((sqrt(
- sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**3 - 3*sin(e + f*x)**2 + 3
*sin(e + f*x) - 1),x)*b))/c**3
```

3.105 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 217

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx &= -\frac{5a^3(A+13B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}c^{7/2}f} \\ &+ \frac{a^3(A+B)c^3\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}} - \frac{a^3(A+13B)c\cos^5(e+fx)}{24f(c-c\sin(e+fx))^{9/2}} \\ &+ \frac{5a^3(A+13B)\cos^3(e+fx)}{48cf(c-c\sin(e+fx))^{5/2}} + \frac{5a^3(A+13B)\cos(e+fx)}{16c^3f\sqrt{c-c\sin(e+fx)}}\end{aligned}$$

output
```
-5/16*a^3*(A+13*B)*arctanh(1/2*c^(1/2)*cos(f*x+e)*2^(1/2)/(c-c*sin(f*x+e))
^(1/2))*2^(1/2)/c^(7/2)/f+1/6*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e)
)^(13/2)-1/24*a^3*(A+13*B)*c*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(9/2)+5/48*a^
3*(A+13*B)*cos(f*x+e)^3/c/f/(c-c*sin(f*x+e))^(5/2)+5/16*a^3*(A+13*B)*cos(f
*x+e)/c^3/f/(c-c*sin(f*x+e))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 14.08 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.94

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(32(A+B)\left(\cos\right.\right.}{}$$

input
```
Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^(7/2),x]
```

output
```
(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(32*(A + B)*(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2]) - 4*(13*A + 25*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^3 + 3*(11*A + 47*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 + (15 + 15*I)
*(-1)^(1/4)*(A + 13*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4]
)]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6 + 48*B*Cos[(e + f*x)/2]*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])^6 + 64*(A + B)*Sin[(e + f*x)/2] - 8*(13*A +
 25*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] + 6*(11*A
+ 47*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(e + f*x)/2] + 48*B*(C
os[(e + f*x)/2] - Sin[(e + f*x)/2])^6*Sin[(e + f*x)/2])*(1 + Sin[e + f*x])
^3)/(24*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(c - c*Sin[e + f*x])^(7/
2))
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.97, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.342$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3159, 3042, 3158, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}dx$$

$\Big\downarrow$ 3446

$$a^3c^3\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}dx$$

$\Big\downarrow$ 3042

$$a^3c^3\int \frac{\cos(e+fx)^6(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}dx$$

$\Big\downarrow$ 3338

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}}-\frac{(A+13B)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{11/2}}dx}{12c}\right)$$

$\Big\downarrow$ 3042

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}}-\frac{(A+13B)\int \frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^{11/2}}dx}{12c}\right)$$

$\Big\downarrow$ 3159

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}}-\frac{(A+13B)\left(\frac{\cos^5(e+fx)}{2cf(c-c\sin(e+fx))^{9/2}}-\frac{5\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{7/2}}dx}{4c^2}\right)}{12c}\right)$$

$\Big\downarrow$ 3042

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}}-\frac{(A+13B)\left(\frac{\cos^5(e+fx)}{2cf(c-c\sin(e+fx))^{9/2}}-\frac{5\int \frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^{7/2}}dx}{4c^2}\right)}{12c}\right)$$

$\Big\downarrow$ 3159

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}}-\frac{(A+13B)\left(\frac{\cos^5(e+fx)}{2cf(c-c\sin(e+fx))^{9/2}}-\frac{5\left(\frac{\cos^3(e+fx)}{cf(c-c\sin(e+fx))^{5/2}}-\frac{3\int\frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}dx}{2c^2}\right)}{4c^2}\right)}{12c}\right.$$

$\downarrow$ 3042

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}}-\frac{(A+13B)\left(\frac{\cos^5(e+fx)}{2cf(c-c\sin(e+fx))^{9/2}}-\frac{5\left(\frac{\cos^3(e+fx)}{cf(c-c\sin(e+fx))^{5/2}}-\frac{3\int\frac{\cos(e+fx)^2}{(c-c\sin(e+fx))^{3/2}}dx}{2c^2}\right)}{4c^2}\right)}{12c}\right.$$

$\downarrow$ 3158

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}}-\frac{(A+13B)\left(\frac{\cos^5(e+fx)}{2cf(c-c\sin(e+fx))^{9/2}}-\frac{5\left(\frac{\cos^3(e+fx)}{cf(c-c\sin(e+fx))^{5/2}}-\frac{3\left(\frac{2\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{c}-\frac{}{cf\sqrt{}}\right.}{2c^2}\right.}{4c^2}\right.}{12c}\right.$$

$\downarrow$ 3042

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}}-\frac{(A+13B)\left(\frac{\cos^5(e+fx)}{2cf(c-c\sin(e+fx))^{9/2}}-\frac{5\left(\frac{\cos^3(e+fx)}{cf(c-c\sin(e+fx))^{5/2}}-\frac{3\left(\frac{2\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{c}-\frac{}{cf\sqrt{}}\right.}{2c^2}\right.}{4c^2}\right.}{12c}\right.$$

$\downarrow$ 3128

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}}-\frac{(A+13B)\left(\frac{\cos^5(e+fx)}{2cf(c-c\sin(e+fx))^{9/2}}-\frac{5\left(\frac{\cos^3(e+fx)}{cf(c-c\sin(e+fx))^{5/2}}-\frac{3\left(-\frac{4\int\frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\right.}{cf}\right.}{}\right.}{4c^2}\right.}{12c}\right.$$

$\Big\downarrow$ 219

$$a^3c^3\left(\frac{(A+B)\cos^7(e+fx)}{6f(c-c\sin(e+fx))^{13/2}}-\frac{(A+13B)\left(\frac{\cos^5(e+fx)}{2cf(c-c\sin(e+fx))^{9/2}}-\frac{5\left(\frac{\cos^3(e+fx)}{cf(c-c\sin(e+fx))^{5/2}}-\frac{3\left(\frac{2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c}\cos(e}{\sqrt{2}\sqrt{c-c\sin}}\right.}{c^{3/2}f}\right.}{2}\right.}{4c^2}\right.}{12c}\right.$$

input
```
Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(7/
2),x]
```

output
```
a^3*c^3*(((A + B)*Cos[e + f*x]^7)/(6*f*(c - c*Sin[e + f*x])^(13/2)) - ((A
+ 13*B)*(Cos[e + f*x]^5/(2*c*f*(c - c*Sin[e + f*x])^(9/2)) - (5*(Cos[e + f
*x]^3/(c*f*(c - c*Sin[e + f*x])^(5/2)) - (3*((2*Sqrt[2]*ArcTanh[(Sqrt[c]*C
os[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(c^(3/2)*f) - (2*Cos[e +
 f*x])/(c*f*Sqrt[c - c*Sin[e + f*x]])))/(2*c^2)))/(4*c^2)))/(12*c))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3158
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(a*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
 EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && In
tegersQ[2*m, 2*p]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 523 vs. $2(190) = 380$.

Time = 1.85 (sec) , antiderivative size = 524, normalized size of antiderivative = 2.41

method	result
default	$\frac{a^3\left(15A \operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)\sin(fx+e)^3\sqrt{2}\,c^3+195B \operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)\sin(fx+e)^3\sqrt{2}\,c^3-45A \operatorname{arctanh}\left(\right.\right.}{}$
parts	Expression too large to display

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,method=_R
ETURNVERBOSE)
```

output
```
1/48/c^(13/2)*a^3*(15*A*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/
2))*sin(f*x+e)^3*2^(1/2)*c^3+195*B*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^
(1/2)/c^(1/2))*sin(f*x+e)^3*2^(1/2)*c^3-45*A*arctanh(1/2*(c*(1+sin(f*x+e))
)^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^2*2^(1/2)*c^3-96*B*(c*(1+sin(f*x+e)))^
(1/2)*c^(5/2)*sin(f*x+e)^3-585*B*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1
/2)/c^(1/2))*sin(f*x+e)^2*2^(1/2)*c^3+66*A*(c*(1+sin(f*x+e)))^(5/2)*c^(1/2
)+45*A*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*2^
(1/2)*c^3+282*B*(c*(1+sin(f*x+e)))^(5/2)*c^(1/2)+288*B*(c*(1+sin(f*x+e)))^
(1/2)*c^(5/2)*sin(f*x+e)^2+585*B*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1
/2)/c^(1/2))*sin(f*x+e)*2^(1/2)*c^3-160*A*(c*(1+sin(f*x+e)))^(3/2)*c^(3/2)
-15*A*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^3-92
8*B*(c*(1+sin(f*x+e)))^(3/2)*c^(3/2)-288*B*c^(5/2)*(c*(1+sin(f*x+e)))^(1/2
)*sin(f*x+e)-195*B*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*2
^(1/2)*c^3+120*A*c^(5/2)*(c*(1+sin(f*x+e)))^(1/2)+888*B*c^(5/2)*(c*(1+sin(
f*x+e)))^(1/2))*(c*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)^2/cos(f*x+e)/(c-c*
sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 554 vs. $2(190) = 380$.

Time = 0.10 (sec) , antiderivative size = 554, normalized size of antiderivative = 2.55

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, al
gorithm="fricas")
```

output
```
1/96*(15*sqrt(2)*((A + 13*B)*a^3*cos(f*x + e)^4 - 3*(A + 13*B)*a^3*cos(f*x
 + e)^3 - 8*(A + 13*B)*a^3*cos(f*x + e)^2 + 4*(A + 13*B)*a^3*cos(f*x + e)
+ 8*(A + 13*B)*a^3 + ((A + 13*B)*a^3*cos(f*x + e)^3 + 4*(A + 13*B)*a^3*cos
(f*x + e)^2 - 4*(A + 13*B)*a^3*cos(f*x + e) - 8*(A + 13*B)*a^3)*sin(f*x +
e))*sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*s
qrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x +
 e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*
x + e) - cos(f*x + e) - 2)) - 4*(48*B*a^3*cos(f*x + e)^4 + 3*(11*A + 95*B)
*a^3*cos(f*x + e)^3 + (19*A - 137*B)*a^3*cos(f*x + e)^2 - 2*(23*A + 203*B)
*a^3*cos(f*x + e) - 32*(A + B)*a^3 - (48*B*a^3*cos(f*x + e)^3 - 3*(11*A +
79*B)*a^3*cos(f*x + e)^2 - 2*(7*A + 187*B)*a^3*cos(f*x + e) + 32*(A + B)*a
^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c^4*f*cos(f*x + e)^4 - 3*c^4
*f*cos(f*x + e)^3 - 8*c^4*f*cos(f*x + e)^2 + 4*c^4*f*cos(f*x + e) + 8*c^4*
f + (c^4*f*cos(f*x + e)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*c^4*f*cos(f*x + e)
- 8*c^4*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^3}{(-c\sin(fx+e)+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c
)^(7/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx=\int \frac{(A+B\sin(e+fx))\,(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^{7/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(7/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(7/
2), x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{\sqrt{c}\,a^3\left(\left(\int \frac{\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)^4-4\sin(fx+e)^3+6\sin(fx+e)^2-4\sin(fx+e)+1}dx\right)\right.}{}$$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x)
```

output
```
(sqrt(c)*a**3*(int(sqrt( - sin(e + f*x) + 1)/(sin(e + f*x)**4 - 4*sin(e +
f*x)**3 + 6*sin(e + f*x)**2 - 4*sin(e + f*x) + 1),x)*a + int((sqrt( - sin(
e + f*x) + 1)*sin(e + f*x)**4)/(sin(e + f*x)**4 - 4*sin(e + f*x)**3 + 6*si
n(e + f*x)**2 - 4*sin(e + f*x) + 1),x)*b + int((sqrt( - sin(e + f*x) + 1)*
sin(e + f*x)**3)/(sin(e + f*x)**4 - 4*sin(e + f*x)**3 + 6*sin(e + f*x)**2
- 4*sin(e + f*x) + 1),x)*a + 3*int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)
**3)/(sin(e + f*x)**4 - 4*sin(e + f*x)**3 + 6*sin(e + f*x)**2 - 4*sin(e +
f*x) + 1),x)*b + 3*int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e
+ f*x)**4 - 4*sin(e + f*x)**3 + 6*sin(e + f*x)**2 - 4*sin(e + f*x) + 1),x)
*a + 3*int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x)**4 -
4*sin(e + f*x)**3 + 6*sin(e + f*x)**2 - 4*sin(e + f*x) + 1),x)*b + 3*int((
sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**4 - 4*sin(e + f*x)*
*3 + 6*sin(e + f*x)**2 - 4*sin(e + f*x) + 1),x)*a + int((sqrt( - sin(e + f
*x) + 1)*sin(e + f*x))/(sin(e + f*x)**4 - 4*sin(e + f*x)**3 + 6*sin(e + f*
x)**2 - 4*sin(e + f*x) + 1),x)*b))/c**4
```

3.106 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 217

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = -\frac{5a^3(A-15B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{128\sqrt{2}c^{9/2}f} + \frac{a^3(A+B)c^3\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15/2}} + \frac{a^3(A-15B)c\cos^5(e+fx)}{48f(c-c\sin(e+fx))^{11/2}} - \frac{5a^3(A-15B)\cos^3(e+fx)}{192cf(c-c\sin(e+fx))^{7/2}} + \frac{5a^3(A-15B)\cos(e+fx)}{128c^3f(c-c\sin(e+fx))^{3/2}}$$

output

```
-5/256*a^3*(A-15*B)*arctanh(1/2*c^(1/2)*cos(f*x+e)*2^(1/2)/(c-c*sin(f*x+e)
)^(1/2))*2^(1/2)/c^(9/2)/f+1/8*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e
))^(15/2)+1/48*a^3*(A-15*B)*c*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(11/2)-5/192
*a^3*(A-15*B)*cos(f*x+e)^3/c/f/(c-c*sin(f*x+e))^(7/2)+5/128*a^3*(A-15*B)*c
os(f*x+e)/c^3/f/(c-c*sin(f*x+e))^(3/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 15.46 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.64

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))}{}$$

input
```
Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^(9/2),x]
```

output
```
(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(1765*A*Co
s[(e + f*x)/2] + 405*B*Cos[(e + f*x)/2] - 895*A*Cos[(3*(e + f*x))/2] - 270
3*B*Cos[(3*(e + f*x))/2] - 397*A*Cos[(5*(e + f*x))/2] + 579*B*Cos[(5*(e +
f*x))/2] + 15*A*Cos[(7*(e + f*x))/2] + 543*B*Cos[(7*(e + f*x))/2] + (120 +
 120*I)*(-1)^(1/4)*(A - 15*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e +
f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^8 + 1765*A*Sin[(e + f*x)/2
] + 405*B*Sin[(e + f*x)/2] + 895*A*Sin[(3*(e + f*x))/2] + 2703*B*Sin[(3*(e
 + f*x))/2] - 397*A*Sin[(5*(e + f*x))/2] + 579*B*Sin[(5*(e + f*x))/2] - 15
*A*Sin[(7*(e + f*x))/2] - 543*B*Sin[(7*(e + f*x))/2]))/(3072*f*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2])^6*(c - c*Sin[e + f*x])^(9/2))
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.98, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.342$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3159, 3042, 3159, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}dx$$

$$\Big\downarrow \text{3446}$$

$$a^3c^3\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{15/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\int \frac{\cos(e+fx)^6(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{15/2}}dx$$

$$\Big\downarrow \text{3338}$$

$$a^3c^3\left(\frac{(A-15B)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{13/2}}dx}{16c}+\frac{(A+B)\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left(\frac{(A-15B)\int \frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^{13/2}}dx}{16c}+\frac{(A+B)\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15/2}}\right)$$

$$\Big\downarrow \text{3159}$$

$$a^3c^3\left(\frac{(A-15B)\left(\frac{\cos^5(e+fx)}{3cf(c-c\sin(e+fx))^{11/2}}-\frac{5\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{9/2}}dx}{6c^2}\right)}{16c}+\frac{(A+B)\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left(\frac{(A-15B)\left(\frac{\cos^5(e+fx)}{3cf(c-c\sin(e+fx))^{11/2}}-\frac{5\int \frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^{9/2}}dx}{6c^2}\right)}{16c}+\frac{(A+B)\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15/2}}\right)$$

$$\Big\downarrow \text{3159}$$

$$a^3c^3\left(\frac{(A-15B)\left(\frac{\cos^5(e+fx)}{3cf(c-c\sin(e+fx))^{11/2}}-\frac{5\left(\frac{\cos^3(e+fx)}{2cf(c-c\sin(e+fx))^{7/2}}-\frac{3\int\frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{5/2}}dx}{4c^2}\right)}{6c^2}\right)}{16c}+\frac{(A+B)\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15}}\right.$$

$\downarrow$ 3042

$$a^3c^3\left(\frac{(A-15B)\left(\frac{\cos^5(e+fx)}{3cf(c-c\sin(e+fx))^{11/2}}-\frac{5\left(\frac{\cos^3(e+fx)}{2cf(c-c\sin(e+fx))^{7/2}}-\frac{3\int\frac{\cos(e+fx)^2}{(c-c\sin(e+fx))^{5/2}}dx}{4c^2}\right)}{6c^2}\right)}{16c}+\frac{(A+B)\cos^7(e+fx)}{8f(c-c\sin(e+fx))^{15}}\right.$$

$\downarrow$ 3159

$$a^3c^3\left(\frac{(A-15B)\left(\frac{\cos^5(e+fx)}{3cf(c-c\sin(e+fx))^{11/2}}-\frac{5\left(\frac{\cos^3(e+fx)}{2cf(c-c\sin(e+fx))^{7/2}}-\frac{3\left(\frac{\cos(e+fx)}{cf(c-c\sin(e+fx))^{3/2}}-\frac{\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{2c^2}\right)}{4c^2}\right)}{6c^2}\right)}{16c}+\frac{(A-}{8f(c-}\right.$$

$\downarrow$ 3042

$$a^3c^3\left(\frac{(A-15B)\left(\frac{\cos^5(e+fx)}{3cf(c-c\sin(e+fx))^{11/2}}-\frac{5\left(\frac{\cos^3(e+fx)}{2cf(c-c\sin(e+fx))^{7/2}}-\frac{3\left(\frac{\cos(e+fx)}{cf(c-c\sin(e+fx))^{3/2}}-\frac{\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{2c^2}\right)}{4c^2}\right)}{6c^2}\right)}{16c}+\frac{(A-}{8f(c-}\right.$$

$$\Big\downarrow \text{ 3128}$$

$$a^3c^3\left(\frac{(A-15B)\left(\frac{\cos^5(e+fx)}{3cf(c-c\sin(e+fx))^{11/2}}-\frac{5\left(\frac{\cos^3(e+fx)}{2cf(c-c\sin(e+fx))^{7/2}}-\frac{3\left(\frac{\int\frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{c^2f}+\frac{\cos(e+fx)}{cf(c-c\sin(e+fx))^{3}}\right.}{4c^2}\right.}{6c^2}\right.}{16c}\right.$$

$$\Big\downarrow \text{219}$$

$$a^3c^3\left(\frac{(A-15B)\left(\frac{\cos^5(e+fx)}{3cf(c-c\sin(e+fx))^{11/2}}-\frac{5\left(\frac{\cos^3(e+fx)}{2cf(c-c\sin(e+fx))^{7/2}}-\frac{3\left(\frac{\cos(e+fx)}{cf(c-c\sin(e+fx))^{3/2}}-\frac{\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}c^{5/2}f}\right)}{4c^2}\right)}{6c^2}\right)}{16c}\right.$$

input
```
Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(9/
2),x]
```

output
```
a^3*c^3*(((A + B)*Cos[e + f*x]^7)/(8*f*(c - c*Sin[e + f*x])^(15/2)) + ((A
 - 15*B)*(Cos[e + f*x]^5/(3*c*f*(c - c*Sin[e + f*x])^(11/2)) - (5*(Cos[e +
f*x]^3/(2*c*f*(c - c*Sin[e + f*x])^(7/2)) - (3*(-(ArcTanh[(Sqrt[c]*Cos[e +
 f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]/(Sqrt[2]*c^(5/2)*f)) + Cos[e +
f*x]/(c*f*(c - c*Sin[e + f*x])^(3/2))))/(4*c^2)))/(6*c^2)))/(16*c))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. 2(190) = 380.

Time = 1.89 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.99

method	result
default	$-\frac{a^3\left(-15\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)c^4(A-15B)\cos(fx+e)^4-60\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)c^4(A-15B)\cos(fx+e)^2}{}$
parts	Expression too large to display

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/768/c^(17/2)*a^3*(-15*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2
)/c^(1/2))*c^4*(A-15*B)*cos(f*x+e)^4-60*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e
))^(1/2)*2^(1/2)/c^(1/2))*c^4*(A-15*B)*cos(f*x+e)^2*sin(f*x+e)+120*2^(1/2)
*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^4*(A-15*B)*cos(f*x+
e)^2+120*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^4*(
A-15*B)*sin(f*x+e)+240*A*(c+c*sin(f*x+e))^(1/2)*c^(7/2)-440*A*(c+c*sin(f*x
+e))^(3/2)*c^(5/2)+292*A*(c+c*sin(f*x+e))^(5/2)*c^(3/2)+30*A*(c+c*sin(f*x+
e))^(7/2)*c^(1/2)-3600*B*(c+c*sin(f*x+e))^(1/2)*c^(7/2)+6600*B*(c+c*sin(f*
x+e))^(3/2)*c^(5/2)-4380*B*(c+c*sin(f*x+e))^(5/2)*c^(3/2)+1086*B*(c+c*sin(
f*x+e))^(7/2)*c^(1/2)-120*A*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(
1/2)/c^(1/2))*c^4+1800*B*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2
)/c^(1/2))*c^4)*(c*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)^3/cos(f*x+e)/(c-c*
sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 633 vs. $2(190) = 380$.

Time = 0.14 (sec) , antiderivative size = 633, normalized size of antiderivative = 2.92

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, al
gorithm="fricas")
```

output
```
-1/1536*(15*sqrt(2)*((A - 15*B)*a^3*cos(f*x + e)^5 + 5*(A - 15*B)*a^3*cos(
f*x + e)^4 - 8*(A - 15*B)*a^3*cos(f*x + e)^3 - 20*(A - 15*B)*a^3*cos(f*x +
 e)^2 + 8*(A - 15*B)*a^3*cos(f*x + e) + 16*(A - 15*B)*a^3 - ((A - 15*B)*a^
3*cos(f*x + e)^4 - 4*(A - 15*B)*a^3*cos(f*x + e)^3 - 12*(A - 15*B)*a^3*cos
(f*x + e)^2 + 8*(A - 15*B)*a^3*cos(f*x + e) + 16*(A - 15*B)*a^3)*sin(f*x +
 e))*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*
sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x
+ e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f
*x + e) - cos(f*x + e) - 2)) - 4*(3*(5*A + 181*B)*a^3*cos(f*x + e)^4 - (19
1*A - 561*B)*a^3*cos(f*x + e)^3 - 2*(169*A + 537*B)*a^3*cos(f*x + e)^2 + 1
2*(21*A - 59*B)*a^3*cos(f*x + e) + 384*(A + B)*a^3 - (3*(5*A + 181*B)*a^3*
cos(f*x + e)^3 + 2*(103*A - 9*B)*a^3*cos(f*x + e)^2 - 12*(11*A + 91*B)*a^3
*cos(f*x + e) - 384*(A + B)*a^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/
(c^5*f*cos(f*x + e)^5 + 5*c^5*f*cos(f*x + e)^4 - 8*c^5*f*cos(f*x + e)^3 -
20*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f - (c^5*f*cos(f*x
 + e)^4 - 4*c^5*f*cos(f*x + e)^3 - 12*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f
*x + e) + 16*c^5*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^3}{(-c\sin(fx+e)+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c
)^(9/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \int \frac{(A+B\sin(e+fx))\,(a+a\sin(e+fx))^3}{(c-c\sin(e+fx))^{9/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(9/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(9/
2), x)
```

Reduce [F]

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{9/2}}\,dx = \text{Too large to display}$$

input

```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x)
```

output

```
(sqrt(c)*a**3*( - int(sqrt( - sin(e + f*x) + 1)/(sin(e + f*x)**5 - 5*sin(e
 + f*x)**4 + 10*sin(e + f*x)**3 - 10*sin(e + f*x)**2 + 5*sin(e + f*x) - 1)
,x)*a - int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**4)/(sin(e + f*x)**5 -
 5*sin(e + f*x)**4 + 10*sin(e + f*x)**3 - 10*sin(e + f*x)**2 + 5*sin(e + f
*x) - 1),x)*b - int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**3)/(sin(e + f
*x)**5 - 5*sin(e + f*x)**4 + 10*sin(e + f*x)**3 - 10*sin(e + f*x)**2 + 5*s
in(e + f*x) - 1),x)*a - 3*int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**3)/
(sin(e + f*x)**5 - 5*sin(e + f*x)**4 + 10*sin(e + f*x)**3 - 10*sin(e + f*x
)**2 + 5*sin(e + f*x) - 1),x)*b - 3*int((sqrt( - sin(e + f*x) + 1)*sin(e +
 f*x)**2)/(sin(e + f*x)**5 - 5*sin(e + f*x)**4 + 10*sin(e + f*x)**3 - 10*s
in(e + f*x)**2 + 5*sin(e + f*x) - 1),x)*a - 3*int((sqrt( - sin(e + f*x) +
1)*sin(e + f*x)**2)/(sin(e + f*x)**5 - 5*sin(e + f*x)**4 + 10*sin(e + f*x)
**3 - 10*sin(e + f*x)**2 + 5*sin(e + f*x) - 1),x)*b - 3*int((sqrt( - sin(e
 + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**5 - 5*sin(e + f*x)**4 + 10*sin(e
 + f*x)**3 - 10*sin(e + f*x)**2 + 5*sin(e + f*x) - 1),x)*a - int((sqrt( -
sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**5 - 5*sin(e + f*x)**4 + 10*
sin(e + f*x)**3 - 10*sin(e + f*x)**2 + 5*sin(e + f*x) - 1),x)*b))/c**5
```

3.107 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 266

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = -\frac{a^3(3A-17B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{512\sqrt{2}c^{11/2}f}+\frac{a^3(A+B)c^3\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}}+\frac{a^3(3A-17B)c\cos^5(e+fx)}{80f(c-c\sin(e+fx))^{13/2}}-\frac{a^3(3A-17B)\cos^3(e+fx)}{96cf(c-c\sin(e+fx))^{9/2}}+\frac{a^3(3A-17B)\cos(e+fx)}{128c^3f(c-c\sin(e+fx))^{5/2}}-\frac{a^3(3A-17B)\cos(e+fx)}{512c^4f(c-c\sin(e+fx))^{3/2}}$$

output
```
-1/1024*a^3*(3*A-17*B)*arctanh(1/2*c^(1/2)*cos(f*x+e)*2^(1/2)/(c-c*sin(f*x
+e))^(1/2))*2^(1/2)/c^(11/2)/f+1/10*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(
f*x+e))^(17/2)+1/80*a^3*(3*A-17*B)*c*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(13/2
)-1/96*a^3*(3*A-17*B)*cos(f*x+e)^3/c/f/(c-c*sin(f*x+e))^(9/2)+1/128*a^3*(3
*A-17*B)*cos(f*x+e)/c^3/f/(c-c*sin(f*x+e))^(5/2)-1/512*a^3*(3*A-17*B)*cos(
f*x+e)/c^4/f/(c-c*sin(f*x+e))^(3/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 17.25 (sec) , antiderivative size = 485, normalized size of antiderivative = 1.82

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{\left(\frac{1}{512}+\frac{i}{512}\right)\sqrt[4]{-1}(3A-17B)\arctan\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\sec\left(\frac{1}{4}\right.\right.}{f\left(\cos\right.}$$
$$+\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a+a\sin(e+fx))^3\left(56370A\cos\left(\frac{1}{2}(e+fx)\right)+38970B\cos\left(\frac{1}{2}(e+\right.\right.}{}$$

input
```
Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^(11/2),x]
```

output
```
((1/512 + I/512)*(-1)^(1/4)*(3*A - 17*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*Sec
[(e + f*x)/4]*(Cos[(e + f*x)/4] + Sin[(e + f*x)/4])]*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])^11*(a + a*Sin[e + f*x])^3)/(f*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])^6*(c - c*Sin[e + f*x])^(11/2)) + ((Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])*(a + a*Sin[e + f*x])^3*(56370*A*Cos[(e + f*x)/2] + 38970*B*Cos[(e
 + f*x)/2] - 31140*A*Cos[(3*(e + f*x))/2] - 38580*B*Cos[(3*(e + f*x))/2] -
 10404*A*Cos[(5*(e + f*x))/2] - 12724*B*Cos[(5*(e + f*x))/2] + 435*A*Cos[(
7*(e + f*x))/2] + 7775*B*Cos[(7*(e + f*x))/2] - 45*A*Cos[(9*(e + f*x))/2]
+ 255*B*Cos[(9*(e + f*x))/2] + 56370*A*Sin[(e + f*x)/2] + 38970*B*Sin[(e +
 f*x)/2] + 31140*A*Sin[(3*(e + f*x))/2] + 38580*B*Sin[(3*(e + f*x))/2] - 1
0404*A*Sin[(5*(e + f*x))/2] - 12724*B*Sin[(5*(e + f*x))/2] - 435*A*Sin[(7*
(e + f*x))/2] - 7775*B*Sin[(7*(e + f*x))/2] - 45*A*Sin[(9*(e + f*x))/2] +
255*B*Sin[(9*(e + f*x))/2]))/(122880*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])^6*(c - c*Sin[e + f*x])^(11/2))
```

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.96, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.395$, Rules used = {3042, 3446, 3042, 3338, 3042, 3159, 3042, 3159, 3042, 3159, 3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}dx$$

$$\Big\downarrow \text{3446}$$

$$a^3c^3\int \frac{\cos^6(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{17/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\int \frac{\cos(e+fx)^6(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{17/2}}dx$$

$$\Big\downarrow \text{3338}$$

$$a^3c^3\left(\frac{(3A-17B)\int \frac{\cos^6(e+fx)}{(c-c\sin(e+fx))^{15/2}}dx}{20c}+\frac{(A+B)\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left(\frac{(3A-17B)\int \frac{\cos(e+fx)^6}{(c-c\sin(e+fx))^{15/2}}dx}{20c}+\frac{(A+B)\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}}\right)$$

$$\Big\downarrow \text{3159}$$

$$a^3c^3\left(\frac{(3A-17B)\left(\frac{\cos^5(e+fx)}{4cf(c-c\sin(e+fx))^{13/2}}-\frac{5\int \frac{\cos^4(e+fx)}{(c-c\sin(e+fx))^{11/2}}dx}{8c^2}\right)}{20c}+\frac{(A+B)\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left(\frac{(3A-17B)\left(\frac{\cos^5(e+fx)}{4cf(c-c\sin(e+fx))^{13/2}}-\frac{5\int \frac{\cos(e+fx)^4}{(c-c\sin(e+fx))^{11/2}}dx}{8c^2}\right)}{20c}+\frac{(A+B)\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{17/2}}\right)$$

$$\Big\downarrow \text{3159}$$

$$a^3c^3\left(\frac{(3A-17B)\left(\frac{\cos^5(e+fx)}{4cf(c-c\sin(e+fx))^{13/2}}-\frac{5\left(\frac{\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{9/2}}-\frac{\int\frac{\cos^2(e+fx)}{(c-c\sin(e+fx))^{7/2}}dx}{2c^2}\right)}{8c^2}\right)}{20c}+\frac{(A+B)\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{1}}\right.$$

$\Big\downarrow$ 3042

$$a^3c^3\left(\frac{(3A-17B)\left(\frac{\cos^5(e+fx)}{4cf(c-c\sin(e+fx))^{13/2}}-\frac{5\left(\frac{\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{9/2}}-\frac{\int\frac{\cos(e+fx)^2}{(c-c\sin(e+fx))^{7/2}}dx}{2c^2}\right)}{8c^2}\right)}{20c}+\frac{(A+B)\cos^7(e+fx)}{10f(c-c\sin(e+fx))^{1}}\right.$$

$\Big\downarrow$ 3159

$$a^3c^3\left(\frac{(3A-17B)\left(\frac{\cos^5(e+fx)}{4cf(c-c\sin(e+fx))^{13/2}}-\frac{5\left(\frac{\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{9/2}}-\frac{\frac{\cos(e+fx)}{2cf(c-c\sin(e+fx))^{5/2}}-\frac{\int\frac{1}{(c-c\sin(e+fx))^{3/2}}dx}{4c^2}}{2c^2}\right)}{8c^2}\right)}{20c}+\frac{(A}{10f(}\right.$$

$\Big\downarrow$ 3042

$$a^3c^3\left(\frac{(3A-17B)\left(\frac{\cos^5(e+fx)}{4cf(c-c\sin(e+fx))^{13/2}}-\frac{5\left(\frac{\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{9/2}}-\frac{\frac{\cos(e+fx)}{2cf(c-c\sin(e+fx))^{5/2}}-\frac{\int\frac{1}{(c-c\sin(e+fx))^{3/2}}dx}{4c^2}}{2c^2}\right)}{8c^2}\right)}{20c}+\frac{(A}{10f(}\right.$$

$$\Big\downarrow \ 3129$$

$$a^3c^3\left(\frac{(3A-17B)\left(\frac{\cos^5(e+fx)}{4cf(c-c\sin(e+fx))^{13/2}}-\frac{5\left(\frac{\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{9/2}}-\frac{\frac{\cos(e+fx)}{2cf(c-c\sin(e+fx))^{5/2}}-\frac{\frac{\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{4c}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))}}{4c^2}}{2c^2}\right.}{8c^2}\right.}{20c}\right.$$

$$\Big\downarrow \ 3042$$

$$a^3c^3\left(\frac{(3A-17B)\left(\frac{\cos^5(e+fx)}{4cf(c-c\sin(e+fx))^{13/2}}-\frac{5\left(\frac{\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{9/2}}-\frac{\frac{\cos(e+fx)}{2cf(c-c\sin(e+fx))^{5/2}}-\frac{\frac{\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{4c}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))}}{4c^2}}{2c^2}\right.}{8c^2}\right.}{20c}\right.$$

$$\Big\downarrow \text{3128}$$

$$a^3c^3\left(\frac{(3A-17B)\left(\frac{\cos^5(e+fx)}{4cf(c-c\sin(e+fx))^{13/2}}-\frac{5\left(\frac{\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{9/2}}-\frac{\frac{\cos(e+fx)}{2cf(c-c\sin(e+fx))^{5/2}}-\frac{\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}-\frac{\int\frac{1}{2c-\frac{c^2\cos^2(e+}{c-c\sin(e+}}}{4c^2}}{2c^2}\right.}{8c^2}\right.}{20c}\right.$$

$$\Big\downarrow \text{219}$$

$$a^3c^3\left(\frac{(3A-17B)\left(\frac{\cos^5(e+fx)}{4cf(c-c\sin(e+fx))^{13/2}}-\frac{5\left(\frac{\cos^3(e+fx)}{3cf(c-c\sin(e+fx))^{9/2}}-\frac{\frac{\cos(e+fx)}{2cf(c-c\sin(e+fx))^{5/2}}-\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}c^{3/2}f}+\frac{\cdots}{2f(c-\cdots}}{4c^2}}{2c^2}\right)}{8c^2}\right)}{20c}\cdots\right.$$

input
```
Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(11
/2),x]
```

output
```
a^3*c^3*(((A + B)*Cos[e + f*x]^7)/(10*f*(c - c*Sin[e + f*x])^(17/2)) + ((3
*A - 17*B)*(Cos[e + f*x]^5/(4*c*f*(c - c*Sin[e + f*x])^(13/2)) - (5*(Cos[e
 + f*x]^3/(3*c*f*(c - c*Sin[e + f*x])^(9/2)) - (Cos[e + f*x]/(2*c*f*(c - c
*Sin[e + f*x])^(5/2)) - (ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c -
c*Sin[e + f*x]])]/(2*Sqrt[2]*c^(3/2)*f) + Cos[e + f*x]/(2*f*(c - c*Sin[e +
 f*x])^(3/2)))/(4*c^2))/(2*c^2)))/(8*c^2)))/(20*c))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 525 vs. $2(235) = 470$.

Time = 1.89 (sec) , antiderivative size = 526, normalized size of antiderivative = 1.98

method	result
default	$\frac{a^3\left(15\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)c^6(3A-17B)\cos(fx+e)^4\sin(fx+e)-75\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)c^6(3A-17B)\cos\right.}{}$
parts	Expression too large to display

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x,method=_
RETURNVERBOSE)
```

output
```
1/15360/c^(23/2)*a^3*(15*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2
)/c^(1/2))*c^6*(3*A-17*B)*cos(f*x+e)^4*sin(f*x+e)-75*2^(1/2)*arctanh(1/2*(
c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^6*(3*A-17*B)*cos(f*x+e)^4-180*2^(
1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^6*(3*A-17*B)*co
s(f*x+e)^2*sin(f*x+e)+300*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/
2)/c^(1/2))*c^6*(3*A-17*B)*cos(f*x+e)^2+240*2^(1/2)*arctanh(1/2*(c+c*sin(f
*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^6*(3*A-17*B)*sin(f*x+e)-90*A*(c+c*sin(f*x+
e))^(9/2)*c^(3/2)+840*A*(c+c*sin(f*x+e))^(7/2)*c^(5/2)+3072*A*(c+c*sin(f*x
+e))^(5/2)*c^(7/2)-3360*A*(c+c*sin(f*x+e))^(3/2)*c^(9/2)+1440*A*(c+c*sin(f
*x+e))^(1/2)*c^(11/2)+510*B*(c+c*sin(f*x+e))^(9/2)*c^(3/2)+5480*B*(c+c*sin
(f*x+e))^(7/2)*c^(5/2)-17408*B*(c+c*sin(f*x+e))^(5/2)*c^(7/2)+19040*B*(c+c
*sin(f*x+e))^(3/2)*c^(9/2)-8160*B*(c+c*sin(f*x+e))^(1/2)*c^(11/2)-720*A*2^
(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^6+4080*B*2^(1/
2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^6)*(c*(1+sin(f*x+
e)))^(1/2)/(sin(f*x+e)-1)^4/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 760 vs. $2(235) = 470$.

Time = 0.15 (sec) , antiderivative size = 760, normalized size of antiderivative = 2.86

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x, a
lgorithm="fricas")
```

output
```
-1/30720*(15*sqrt(2)*((3*A - 17*B)*a^3*cos(f*x + e)^6 - 5*(3*A - 17*B)*a^3
*cos(f*x + e)^5 - 18*(3*A - 17*B)*a^3*cos(f*x + e)^4 + 20*(3*A - 17*B)*a^3
*cos(f*x + e)^3 + 48*(3*A - 17*B)*a^3*cos(f*x + e)^2 - 16*(3*A - 17*B)*a^3
*cos(f*x + e) - 32*(3*A - 17*B)*a^3 + ((3*A - 17*B)*a^3*cos(f*x + e)^5 + 6
*(3*A - 17*B)*a^3*cos(f*x + e)^4 - 12*(3*A - 17*B)*a^3*cos(f*x + e)^3 - 32
*(3*A - 17*B)*a^3*cos(f*x + e)^2 + 16*(3*A - 17*B)*a^3*cos(f*x + e) + 32*(
3*A - 17*B)*a^3)*sin(f*x + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*
sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*
cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2
+ (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(15*(3*A - 17*B
)*a^3*cos(f*x + e)^5 - 5*(39*A + 803*B)*a^3*cos(f*x + e)^4 + 4*(609*A + 38
9*B)*a^3*cos(f*x + e)^3 + 12*(449*A + 869*B)*a^3*cos(f*x + e)^2 - 24*(143*
A + 43*B)*a^3*cos(f*x + e) - 6144*(A + B)*a^3 + (15*(3*A - 17*B)*a^3*cos(f
*x + e)^4 + 80*(3*A + 47*B)*a^3*cos(f*x + e)^3 + 12*(223*A + 443*B)*a^3*co
s(f*x + e)^2 - 24*(113*A + 213*B)*a^3*cos(f*x + e) - 6144*(A + B)*a^3)*sin
(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c^6*f*cos(f*x + e)^6 - 5*c^6*f*cos(
f*x + e)^5 - 18*c^6*f*cos(f*x + e)^4 + 20*c^6*f*cos(f*x + e)^3 + 48*c^6*f*
cos(f*x + e)^2 - 16*c^6*f*cos(f*x + e) - 32*c^6*f + (c^6*f*cos(f*x + e)^5
+ 6*c^6*f*cos(f*x + e)^4 - 12*c^6*f*cos(f*x + e)^3 - 32*c^6*f*cos(f*x + e)
^2 + 16*c^6*f*cos(f*x + e) + 32*c^6*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(11/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^3}{(-c\sin(fx+e)+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x, a
lgorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c
)^(11/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x, a
lgorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^3}{(c-c\,\sin(e+f\,x))^{11/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(11
/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(11
/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \text{Too large to display}$$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x)
```

output

```
(sqrt(c)*a**3*(int(sqrt( - sin(e + f*x) + 1)/(sin(e + f*x)**6 - 6*sin(e +
f*x)**5 + 15*sin(e + f*x)**4 - 20*sin(e + f*x)**3 + 15*sin(e + f*x)**2 - 6
*sin(e + f*x) + 1),x)*a + int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**4)/
(sin(e + f*x)**6 - 6*sin(e + f*x)**5 + 15*sin(e + f*x)**4 - 20*sin(e + f*x
)**3 + 15*sin(e + f*x)**2 - 6*sin(e + f*x) + 1),x)*b + int((sqrt( - sin(e
+ f*x) + 1)*sin(e + f*x)**3)/(sin(e + f*x)**6 - 6*sin(e + f*x)**5 + 15*sin
(e + f*x)**4 - 20*sin(e + f*x)**3 + 15*sin(e + f*x)**2 - 6*sin(e + f*x) +
1),x)*a + 3*int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**3)/(sin(e + f*x)*
*6 - 6*sin(e + f*x)**5 + 15*sin(e + f*x)**4 - 20*sin(e + f*x)**3 + 15*sin(
e + f*x)**2 - 6*sin(e + f*x) + 1),x)*b + 3*int((sqrt( - sin(e + f*x) + 1)*
sin(e + f*x)**2)/(sin(e + f*x)**6 - 6*sin(e + f*x)**5 + 15*sin(e + f*x)**4
 - 20*sin(e + f*x)**3 + 15*sin(e + f*x)**2 - 6*sin(e + f*x) + 1),x)*a + 3*
int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x)**6 - 6*sin(e
 + f*x)**5 + 15*sin(e + f*x)**4 - 20*sin(e + f*x)**3 + 15*sin(e + f*x)**2
- 6*sin(e + f*x) + 1),x)*b + 3*int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)
)/(sin(e + f*x)**6 - 6*sin(e + f*x)**5 + 15*sin(e + f*x)**4 - 20*sin(e + f
*x)**3 + 15*sin(e + f*x)**2 - 6*sin(e + f*x) + 1),x)*a + int((sqrt( - sin(
e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**6 - 6*sin(e + f*x)**5 + 15*sin(
e + f*x)**4 - 20*sin(e + f*x)**3 + 15*sin(e + f*x)**2 - 6*sin(e + f*x) + 1
),x)*b))/c**6
```

3.108 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{a+a\sin(e+fx)}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 200

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{a+a\sin(e+fx)}\,dx = -\frac{128(7A-9B)c^4\cos(e+fx)}{35af\sqrt{c-c\sin(e+fx)}} - \frac{32(7A-9B)c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{35af} - \frac{12(7A-9B)c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{35af} - \frac{(7A-9B)c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{7af} - \frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{9/2}}{acf}$$

output
```
-128/35*(7*A-9*B)*c^4*cos(f*x+e)/a/f/(c-c*sin(f*x+e))^(1/2)-32/35*(7*A-9*B
)*c^3*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f-12/35*(7*A-9*B)*c^2*cos(f*x+e)
*(c-c*sin(f*x+e))^(3/2)/a/f-1/7*(7*A-9*B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(5
/2)/a/f-(A-B)*sec(f*x+e)*(c-c*sin(f*x+e))^(9/2)/a/c/f
```

Mathematica [A] (verified)

Time = 16.54 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.78

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{a+a\sin(e+fx)}\,dx =$$
$$-\frac{c^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}(4900A-6125B+196(A-2B)\cos(2(e+fx}{140af\left(\cos\left(\frac{1}{2}(e+fx)\right)-\right.}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e +
 f*x]),x]
```

output
```
-1/140*(c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]
*(4900*A - 6125*B + 196*(A - 2*B)*Cos[2*(e + f*x)] + 5*B*Cos[4*(e + f*x)]
+ 2450*A*Sin[e + f*x] - 3430*B*Sin[e + f*x] - 14*A*Sin[3*(e + f*x)] + 58*B
*Sin[3*(e + f*x)]))/(a*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e
+ f*x]))
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.91, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 3446, 3042, 3334, 3042, 3126, 3042, 3126, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{a\sin(e+fx)+a}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{a\sin(e+fx)+a}dx$$
$$\Big\downarrow \text{3446}$$
$$\frac{\int \sec^2(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}dx}{ac}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{\cos(e+fx)^2}dx}{ac}$$

$$\Big\downarrow \text{3334}$$

$$\frac{-\frac{1}{2}c(7A-9B)\int (c-c\sin(e+fx))^{7/2}dx-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{9/2}}{f}}{ac}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{1}{2}c(7A-9B)\int (c-c\sin(e+fx))^{7/2}dx-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{9/2}}{f}}{ac}$$

$$\Big\downarrow \text{3126}$$

$$\frac{-\frac{1}{2}c(7A-9B)\left(\frac{12}{7}c\int (c-c\sin(e+fx))^{5/2}dx+\frac{2c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{7f}\right)-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{9/2}}{f}}{ac}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{1}{2}c(7A-9B)\left(\frac{12}{7}c\int (c-c\sin(e+fx))^{5/2}dx+\frac{2c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{7f}\right)-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{9/2}}{f}}{ac}$$

$$\Big\downarrow \text{3126}$$

$$\frac{-\frac{1}{2}c(7A-9B)\left(\frac{12}{7}c\left(\frac{8}{5}c\int (c-c\sin(e+fx))^{3/2}dx+\frac{2c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{5f}\right)+\frac{2c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{7f}\right)-}{ac}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{1}{2}c(7A-9B)\left(\frac{12}{7}c\left(\frac{8}{5}c\int (c-c\sin(e+fx))^{3/2}dx+\frac{2c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{5f}\right)+\frac{2c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{7f}\right)-}{ac}$$

$$\Big\downarrow \text{3126}$$

$$\frac{-\frac{1}{2}c(7A-9B)\left(\frac{12}{7}c\left(\frac{8}{5}c\left(\frac{4}{3}c\int \sqrt{c-c\sin(e+fx)}dx+\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3f}\right)+\frac{2c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{5f}\right)+}{ac}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}c(7A-9B)\left(\frac{12}{7}c\left(\frac{8}{5}c\left(\frac{4}{3}c\int\sqrt{c-c\sin(e+fx)}dx+\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3f}\right)+\frac{2c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{5f}\right)+ \right.$$
$$\overline{\qquad ac \qquad}$$

$\downarrow$ 3125

$$-\frac{1}{2}c(7A-9B)\left(\frac{12}{7}c\left(\frac{8}{5}c\left(\frac{8c^2\cos(e+fx)}{3f\sqrt{c-c\sin(e+fx)}}+\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3f}\right)+\frac{2c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{5f}\right)+\frac{2c\cos(e+f}{} \right.$$
$$\overline{\qquad ac \qquad}$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])
,x]
```

output
```
(-(((A - B)*Sec[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/f) - ((7*A - 9*B)*c*(
(2*c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(7*f) + (12*c*((2*c*Cos[e +
f*x]*(c - c*Sin[e + f*x])^(3/2))/(5*f) + (8*c*((8*c^2*Cos[e + f*x])/(3*f*S
qrt[c - c*Sin[e + f*x]]) + (2*c*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(3*
f)))/5))/7))/2)/(a*c)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 2.80 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.56

method	result
default	$\frac{2c^4(\sin(fx+e)-1)\left(5B\cos(fx+e)^4+(-7A+29B)\cos(fx+e)^2\sin(fx+e)+(49A-103B)\cos(fx+e)^2+(308A-436B)\sin(fx+e)+5\right.}{35a\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)),x,method=_RET
URNVERBOSE)
```

output
```
2/35*c^4/a*(sin(f*x+e)-1)*(5*B*cos(f*x+e)^4+(-7*A+29*B)*cos(f*x+e)^2*sin(f
*x+e)+(49*A-103*B)*cos(f*x+e)^2+(308*A-436*B)*sin(f*x+e)+588*A-716*B)/cos(
f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.58

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{a+a\sin(e+fx)}\,dx = -\frac{2\left(5\,Bc^3\cos\left(fx+e\right)^4+(49\,A-103\,B)c^3\cos\left(fx+e\right)^2+4\,(147\,A-179\,B)c^3-\left((7\,A-29\,B)c^3\cos\left(fx\right.\right.}{35\,af\cos\left(fx+e\right)}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)),x, algo
rithm="fricas")
```

output
```
-2/35*(5*B*c^3*cos(f*x + e)^4 + (49*A - 103*B)*c^3*cos(f*x + e)^2 + 4*(147
*A - 179*B)*c^3 - ((7*A - 29*B)*c^3*cos(f*x + e)^2 - 4*(77*A - 109*B)*c^3)
*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(a*f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{a+a\sin(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. 2(182) = 364.

Time = 0.14 (sec) , antiderivative size = 478, normalized size of antiderivative = 2.39

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{a+a\sin(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)),x, algo
rithm="maxima")
```

output
```
2/35*(7*(91*c^(7/2) + 86*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 336*c^(
7/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 266*c^(7/2)*sin(f*x + e)^3/(cos
(f*x + e) + 1)^3 + 490*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 266*c
^(7/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 336*c^(7/2)*sin(f*x + e)^6/(c
os(f*x + e) + 1)^6 + 86*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 91*c
^(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8)*A/((a + a*sin(f*x + e)/(cos(f*
x + e) + 1))*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(7/2)) - 2*(407*c^(
7/2) + 407*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 1442*c^(7/2)*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 + 1337*c^(7/2)*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 + 2030*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1337*c^(7/2)*sin
(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1442*c^(7/2)*sin(f*x + e)^6/(cos(f*x +
e) + 1)^6 + 407*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 407*c^(7/2)*
sin(f*x + e)^8/(cos(f*x + e) + 1)^8)*B/((a + a*sin(f*x + e)/(cos(f*x + e)
+ 1))*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(7/2)))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 777 vs. $2(182) = 364$.

Time = 0.43 (sec) , antiderivative size = 777, normalized size of antiderivative = 3.88

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^{7/2}}{a + a\sin(e + fx)}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)),x, algo
rithm="giac")
```

output
```
16/35*sqrt(2)*sqrt(c)*(35*(A*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - B*c
^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))/(a*((cos(-1/4*pi + 1/2*f*x + 1/2*e
) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1)) - (77*A*c^3*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e)) - 109*B*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) -
504*A*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x +
 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 728*B*c^3*(cos(-1/4*pi + 1
/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/
2*f*x + 1/2*e) + 1) + 1337*A*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sg
n(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 -
 2009*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f
*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 1680*A*c^3*(cos(-1/4
*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/
4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 2800*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e
) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*
e) + 1)^3 + 1015*A*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4 - 1015*B*c^
3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e
))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4 - 280*A*c^3*(cos(-1/4*pi + 1/2*f
*x + 1/2*e) - 1)^5*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*
f*x + 1/2*e) + 1)^5 + 280*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^5*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{a+a\sin(e+fx)}\,dx=\int \frac{(A+B\,\sin(e+f\,x))\,(c-c\,\sin(e+f\,x))^{7/2}}{a+a\,\sin(e+f\,x)}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))
,x)
```

output
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))
, x)
```

Reduce [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{a+a\sin(e+fx)}\,dx = \frac{\sqrt{c}\,c^3\left(\left(\int \frac{\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)+1}dx\right)a-\left(\int \frac{\sqrt{-\sin(fx+e)+1}\,\sin(fx+}{\sin(fx+e)+1}\right.\right.}{}$$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)),x)
```

output
```
(sqrt(c)*c**3*(int(sqrt( - sin(e + f*x) + 1)/(sin(e + f*x) + 1),x)*a - int
((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**4)/(sin(e + f*x) + 1),x)*b - int
((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**3)/(sin(e + f*x) + 1),x)*a + 3*i
nt((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**3)/(sin(e + f*x) + 1),x)*b + 3
*int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x) + 1),x)*a -
 3*int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x) + 1),x)*b
 - 3*int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x) + 1),x)*a
+ int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x) + 1),x)*b))/a
```

3.109 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 159

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx = -\frac{32(5A-7B)c^3\cos(e+fx)}{15af\sqrt{c-c\sin(e+fx)}} - \frac{8(5A-7B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{15af} - \frac{(5A-7B)c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{5af} - \frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{acf}$$

output
```
-32/15*(5*A-7*B)*c^3*cos(f*x+e)/a/f/(c-c*sin(f*x+e))^(1/2)-8/15*(5*A-7*B)*
c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f-1/5*(5*A-7*B)*c*cos(f*x+e)*(c-c*
sin(f*x+e))^(3/2)/a/f-(A-B)*sec(f*x+e)*(c-c*sin(f*x+e))^(7/2)/a/c/f
```

Mathematica [A] (verified)

Time = 12.56 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.84

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx = -\frac{c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}(450A-600B+2(5A-16B)\cos(2(e+fx))}{30af\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+f}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e +
 f*x]),x]
```

output
```
-1/30*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*
(450*A - 600*B + 2*(5*A - 16*B)*Cos[2*(e + f*x)] + 25*(8*A - 13*B)*Sin[e +
 f*x] + 3*B*Sin[3*(e + f*x)]))/(a*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*
(1 + Sin[e + f*x]))
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.92, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 3446, 3042, 3334, 3042, 3126, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{a\sin(e+fx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{a\sin(e+fx)+a}dx$$

$$\Big\downarrow \text{3446}$$

$$\frac{\int \sec^2(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}dx}{ac}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{\cos(e+fx)^2}dx}{ac}$$

$\Big\downarrow$ 3334

$$\frac{-\frac{1}{2}c(5A-7B)\int (c-c\sin(e+fx))^{5/2}dx-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{f}}{ac}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{1}{2}c(5A-7B)\int (c-c\sin(e+fx))^{5/2}dx-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{f}}{ac}$$

$\Big\downarrow$ 3126

$$\frac{-\frac{1}{2}c(5A-7B)\left(\frac{8}{5}c\int (c-c\sin(e+fx))^{3/2}dx+\frac{2c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{5f}\right)-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{f}}{ac}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{1}{2}c(5A-7B)\left(\frac{8}{5}c\int (c-c\sin(e+fx))^{3/2}dx+\frac{2c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{5f}\right)-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{f}}{ac}$$

$\Big\downarrow$ 3126

$$\frac{-\frac{1}{2}c(5A-7B)\left(\frac{8}{5}c\left(\frac{4}{3}c\int \sqrt{c-c\sin(e+fx)}dx+\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3f}\right)+\frac{2c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{5f}\right)-\frac{(A-}{}}{ac}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{1}{2}c(5A-7B)\left(\frac{8}{5}c\left(\frac{4}{3}c\int \sqrt{c-c\sin(e+fx)}dx+\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3f}\right)+\frac{2c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{5f}\right)-\frac{(A-}{}}{ac}$$

$\Big\downarrow$ 3125

$$\frac{-\frac{1}{2}c(5A-7B)\left(\frac{8}{5}c\left(\frac{8c^2\cos(e+fx)}{3f\sqrt{c-c\sin(e+fx)}}+\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3f}\right)+\frac{2c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{5f}\right)-\frac{(A-B)\sec(e+fx)}{}}{ac}$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])
,x]
```

output
```
(-(((A - B)*Sec[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/f) - ((5*A - 7*B)*c*(
(2*c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(5*f) + (8*c*((8*c^2*Cos[e +
 f*x])/(3*f*Sqrt[c - c*Sin[e + f*x]]) + (2*c*Cos[e + f*x]*Sqrt[c - c*Sin[e
 + f*x]])/(3*f)))/5))/2)/(a*c)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 2.80 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.60

method	result	size
default	$-\frac{2c^{3}(\sin(fx+e)-1)\left(-3B\cos(fx+e)^{2}\sin(fx+e)+(-5A+16B)\cos(fx+e)^{2}+(-50A+82B)\sin(fx+e)-110A+142B\right)}{15a\cos(fx+e)\sqrt{c-c\sin(fx+e)}\, f}$	95

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x,method=_RET
URNVERBOSE)
```

output
```
-2/15*c^3/a*(sin(f*x+e)-1)*(-3*B*cos(f*x+e)^2*sin(f*x+e)+(-5*A+16*B)*cos(f
*x+e)^2+(-50*A+82*B)*sin(f*x+e)-110*A+142*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(
1/2)/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.60

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx = -\frac{2\left((5A-16B)c^{2}\cos(fx+e)^{2}+2(55A-71B)c^{2}+\left(3Bc^{2}\cos(fx+e)^{2}+2(25A-41B)c^{2}\right)\sin(fx}{15af\cos(fx+e)}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x, algo
rithm="fricas")
```

output
```
-2/15*((5*A - 16*B)*c^2*cos(f*x + e)^2 + 2*(55*A - 71*B)*c^2 + (3*B*c^2*co
s(f*x + e)^2 + 2*(25*A - 41*B)*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c
)/(a*f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(145) = 290$.

Time = 0.14 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.43

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx = \frac{2\left(\frac{5\left(23c^{\frac{5}{2}}+\frac{20c^{\frac{5}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{65c^{\frac{5}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{40c^{\frac{5}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{65c^{\frac{5}{2}}\,\mathrm{s}\cdots}{(\cos(f\cdots}\cdots\right.}{\left(a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}\right)\left(\frac{\sin(fx+e)\cdots}{(\cos(fx+e)+\cdots}\cdots\right.}\cdots\right.}{\cdots}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x, algo
rithm="maxima")
```

output
```
2/15*(5*(23*c^(5/2) + 20*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 65*c^(5
/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 40*c^(5/2)*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 65*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 20*c^(5/
2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 23*c^(5/2)*sin(f*x + e)^6/(cos(f*
x + e) + 1)^6)*A/((a + a*sin(f*x + e)/(cos(f*x + e) + 1))*(sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 1)^(5/2)) - 2*(79*c^(5/2) + 79*c^(5/2)*sin(f*x + e)
/(cos(f*x + e) + 1) + 205*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 17
0*c^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 205*c^(5/2)*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 79*c^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7
9*c^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)*B/((a + a*sin(f*x + e)/(cos
(f*x + e) + 1))*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(5/2)))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(145) = 290$.

Time = 0.36 (sec) , antiderivative size = 573, normalized size of antiderivative = 3.60

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x, algo
rithm="giac")
```

output
```
8/15*sqrt(2)*sqrt(c)*(15*(A*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - B*c^
2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))/(a*((cos(-1/4*pi + 1/2*f*x + 1/2*e)
 - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1)) - (25*A*c^2*sgn(sin(-1/4*
pi + 1/2*f*x + 1/2*e)) - 41*B*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 11
0*A*c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1
/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 190*B*c^2*(cos(-1/4*pi + 1/2
*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*
f*x + 1/2*e) + 1) + 160*A*c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(s
in(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 32
0*B*c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x +
 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 90*A*c^2*(cos(-1/4*pi +
1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi +
 1/2*f*x + 1/2*e) + 1)^3 + 90*B*c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3
*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^
3 + 15*A*c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*
f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4 - 15*B*c^2*(cos(-1/4*
pi + 1/2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4
*pi + 1/2*f*x + 1/2*e) + 1)^4)/(a*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(c
os(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 1)^5))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx=\int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}}{a+a\,\sin\left(e+f\,x\right)}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))
,x)
```

output
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))
, x)
```

Reduce [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{a+a\sin(e+fx)}\,dx=\frac{\sqrt{c}\,c^2\left(\left(\int\frac{\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)+1}dx\right)a+\left(\int\frac{\sqrt{-\sin(fx+e)+1}\,\sin(fx+}{\sin(fx+e)+1}\right.\right.}{}$$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x)
```

output
```
(sqrt(c)*c**2*(int(sqrt( - sin(e + f*x) + 1)/(sin(e + f*x) + 1),x)*a + int
((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**3)/(sin(e + f*x) + 1),x)*b + int
((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x) + 1),x)*a - 2*i
nt((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x) + 1),x)*b - 2
*int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x) + 1),x)*a + in
t((sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x) + 1),x)*b))/a
```

3.110 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{a+a\sin(e+fx)}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 118

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{a+a\sin(e+fx)}\,dx = -\frac{4(3A-5B)c^2\cos(e+fx)}{3af\sqrt{c-c\sin(e+fx)}}-\frac{(3A-5B)c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3af}-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{acf}$$

output
```
-4/3*(3*A-5*B)*c^2*cos(f*x+e)/a/f/(c-c*sin(f*x+e))^(1/2)-1/3*(3*A-5*B)*c*c
os(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f-(A-B)*sec(f*x+e)*(c-c*sin(f*x+e))^(5/
2)/a/c/f
```

Mathematica [A] (verified)

Time = 11.43 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.96

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{a+a\sin(e+fx)}\,dx = \frac{c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-18A+27B+\right.}{3af\left(\cos\left(\frac{1}{2}(e+fx)\right)-\right.}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e +
 f*x]),x]
```

output
```
(c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-18*A + 27*B + B*Cos[2*(e + f*x)
] + (-6*A + 14*B)*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(3*a*f*(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x]))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3446, 3042, 3334, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{a\sin(e+fx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{a\sin(e+fx)+a}dx$$

$$\Big\downarrow \text{3446}$$

$$\frac{\int \sec^2(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}dx}{ac}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{\cos(e+fx)^2}dx}{ac}$$

$$\Big\downarrow \text{3334}$$

$$\frac{-\frac{1}{2}c(3A-5B)\int (c-c\sin(e+fx))^{3/2}dx-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{f}}{ac}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{1}{2}c(3A-5B)\int (c-c\sin(e+fx))^{3/2}dx-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{f}}{ac}$$

$$\Big\downarrow \, 3126$$

$$\frac{-\frac{1}{2}c(3A-5B)\left(\frac{4}{3}c\int\sqrt{c-c\sin(e+fx)}dx+\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3f}\right)-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{f}}{ac}$$

$$\Big\downarrow \, 3042$$

$$\frac{-\frac{1}{2}c(3A-5B)\left(\frac{4}{3}c\int\sqrt{c-c\sin(e+fx)}dx+\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3f}\right)-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{f}}{ac}$$

$$\Big\downarrow \, 3125$$

$$\frac{-\frac{1}{2}c(3A-5B)\left(\frac{8c^2\cos(e+fx)}{3f\sqrt{c-c\sin(e+fx)}}+\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{3f}\right)-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{f}}{ac}$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])
,x]
```

output
```
(-(((A - B)*Sec[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/f) - ((3*A - 5*B)*c*(
(8*c^2*Cos[e + f*x])/(3*f*Sqrt[c - c*Sin[e + f*x]]) + (2*c*Cos[e + f*x]*Sq
rt[c - c*Sin[e + f*x]])/(3*f)))/2)/(a*c)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.62

method	result	size
default	$\frac{2c^{2}\left(\sin\left(fx+e\right)-1\right)\left(-B\cos\left(fx+e\right)^{2}+\sin\left(fx+e\right)\left(3A-7B\right)+9A-13B\right)}{3a\cos\left(fx+e\right)\sqrt{c-c\sin\left(fx+e\right)}\,f}$	73

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x,method=_RET
URNVERBOSE)
```

output
```
2/3*c^2/a*(sin(f*x+e)-1)*(-B*cos(f*x+e)^2+sin(f*x+e)*(3*A-7*B)+9*A-13*B)/c
os(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.57

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{a+a\sin(e+fx)}\,dx = \frac{2\left(Bc\cos\left(fx+e\right)^2-(3\,A-7\,B)c\sin\left(fx+e\right)-(9\,A\cdots}{3\,af\cos\left(fx+e\right)}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x, algo
rithm="fricas")
```

output
```
2/3*(B*c*cos(f*x + e)^2 - (3*A - 7*B)*c*sin(f*x + e) - (9*A - 13*B)*c)*sqr
t(-c*sin(f*x + e) + c)/(a*f*cos(f*x + e))
```

Sympy [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{a+a\sin(e+fx)}\,dx = \frac{\int \frac{Ac\sqrt{-c\sin\left(e+fx\right)+c}}{\sin\left(e+fx\right)+1}\,dx+\int \left(-\frac{Ac\sqrt{-c\sin\left(e+fx\right)+c}\sin\left(e+fx\right)}{\sin\left(e+fx\right)+1}\right)\cdots}{\cdots}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e)),x)
```

output
```
(Integral(A*c*sqrt(-c*sin(e + f*x) + c)/(sin(e + f*x) + 1), x) + Integral(
-A*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)/(sin(e + f*x) + 1), x) + Integ
ral(B*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)/(sin(e + f*x) + 1), x) + In
tegral(-B*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2/(sin(e + f*x) + 1),
x))/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(108) = 216$.

Time = 0.13 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.49

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^{3/2}}{a + a\sin(e + fx)}\,dx = \frac{2\left(\frac{3\left(3\,c^{\frac{3}{2}} + \frac{2\,c^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1} + \frac{6\,c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{2\,c^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{3\,c^{\frac{3}{2}}\sin(fx\cdots}{(\cos(fx+e)\cdots}\right.}{\left(a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}\right)\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{3}{2}}}\cdots}{\cdots}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x, algo
rithm="maxima")
```

output
```
2/3*(3*(3*c^(3/2) + 2*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 6*c^(3/2)*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2*c^(3/2)*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 3*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)*A/((a + a*sin(f
*x + e)/(cos(f*x + e) + 1))*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2
)) - 2*(7*c^(3/2) + 7*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 12*c^(3/2)
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*c^(3/2)*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 7*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)*B/((a + a*sin(
f*x + e)/(cos(f*x + e) + 1))*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/
2)))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(108) = 216$.

Time = 0.29 (sec) , antiderivative size = 353, normalized size of antiderivative = 2.99

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^{3/2}}{a + a\sin(e + fx)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x, algo
rithm="giac")
```

output
```
4/3*sqrt(2)*sqrt(c)*(3*(A*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - B*c*sgn(
sin(-1/4*pi + 1/2*f*x + 1/2*e)))/(a*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/
(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1)) - (3*A*c*sgn(sin(-1/4*pi + 1/2*
f*x + 1/2*e)) - 7*B*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 6*A*c*(cos(-1/
4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4
*pi + 1/2*f*x + 1/2*e) + 1) + 18*B*c*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*
sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) +
 3*A*c*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x +
1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 3*B*c*(cos(-1/4*pi + 1/2*
f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2
*f*x + 1/2*e) + 1)^2)/(a*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*p
i + 1/2*f*x + 1/2*e) + 1) - 1)^3))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{a+a\sin(e+fx)}\,dx=\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}{a+a\,\sin\left(e+f\,x\right)}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))
,x)
```

output
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))
, x)
```

Reduce [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{a+a\sin(e+fx)}\,dx=\frac{\sqrt{c}\,c\left(\left(\int \frac{\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)+1}dx\right)a-\left(\int \frac{\sqrt{-\sin(fx+e)+1}\,\sin(fx+e}{\sin(fx+e)+1}\right.\right.}{}$$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x)
```

output
```
(sqrt(c)*c*(int(sqrt( - sin(e + f*x) + 1)/(sin(e + f*x) + 1),x)*a - int((s
qrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x) + 1),x)*b - int((s
qrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x) + 1),x)*a + int((sqrt
( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x) + 1),x)*b))/a
```

3.111 $\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{a+a\sin(e+fx)}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 73

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{a+a\sin(e+fx)}\,dx = -\frac{(A-3B)c\cos(e+fx)}{af\sqrt{c-c\sin(e+fx)}}-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{acf}$$

output
```
-(A-3*B)*c*cos(f*x+e)/a/f/(c-c*sin(f*x+e))^(1/2)-(A-B)*sec(f*x+e)*(c-c*sin
(f*x+e))^(3/2)/a/c/f
```

Mathematica [A] (verified)

Time = 3.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.60

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{a+a\sin(e+fx)}\,dx = \frac{2\sec(e+fx)(-A+2B+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{af}$$

input
```
Integrate[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f
*x]),x]
```

output
```
(2*Sec[e + f*x]*(-A + 2*B + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a*f
)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 3446, 3042, 3334, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-c\sin(e+fx)}(A+B\sin(e+fx))}{a\sin(e+fx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{c-c\sin(e+fx)}(A+B\sin(e+fx))}{a\sin(e+fx)+a}dx$$

$$\Big\downarrow \text{3446}$$

$$\frac{\int \sec^2(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}dx}{ac}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{\cos(e+fx)^2}dx}{ac}$$

$$\Big\downarrow \text{3334}$$

$$\frac{-\frac{1}{2}c(A-3B)\int \sqrt{c-c\sin(e+fx)}dx-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{f}}{ac}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{1}{2}c(A-3B)\int \sqrt{c-c\sin(e+fx)}dx-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{f}}{ac}$$

$$\Big\downarrow \text{3125}$$

$$\frac{-\frac{c^2(A-3B)\cos(e+fx)}{f\sqrt{c-c\sin(e+fx)}}-\frac{(A-B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{f}}{ac}$$

input
```
Int[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x]),x
]
```

output
```
(-(((A - 3*B)*c^2*Cos[e + f*x])/(f*Sqrt[c - c*Sin[e + f*x]])) - ((A - B)*S
ec[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/f)/(a*c)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{2c(\sin(fx+e)-1)(-B\sin(fx+e)+A-2B)}{a\cos(fx+e)\sqrt{c-c\sin(fx+e)}\, f}$	53

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x,method=_RET
URNVERBOSE)
```

output
```
2*c/a*(sin(f*x+e)-1)*(-B*sin(f*x+e)+A-2*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/
2)/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.60

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{a+a\sin(e+fx)}\,dx = \frac{2\,(B\sin\left(fx+e\right)-A+2\,B)\sqrt{-c\sin\left(fx+e\right)+c}}{af\cos\left(fx+e\right)}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x, algo
rithm="fricas")
```

output
```
2*(B*sin(f*x + e) - A + 2*B)*sqrt(-c*sin(f*x + e) + c)/(a*f*cos(f*x + e))
```

Sympy [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{a+a\sin(e+fx)}\,dx = \frac{\int \frac{A\sqrt{-c\sin(e+fx)+c}}{\sin(e+fx)+1}\,dx + \int \frac{B\sqrt{-c\sin(e+fx)+c}\sin(e+fx)}{\sin(e+fx)+1}\,dx}{a}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e)),x)
```

output
```
(Integral(A*sqrt(-c*sin(e + f*x) + c)/(sin(e + f*x) + 1), x) + Integral(B*
sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)/(sin(e + f*x) + 1), x))/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(69) = 138.

Time = 0.13 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.38

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{a+a\sin(e+fx)}\,dx = -\frac{2\left(\frac{2B\left(\sqrt{c}+\frac{\sqrt{c}\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)}{\left(a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}\right)\sqrt{\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1}} - \frac{A\left(\sqrt{c}+\frac{\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)}{\left(a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}\right)\sqrt{\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1}}\right)}{f}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x, algo
rithm="maxima")
```

output
```
-2*(2*B*(sqrt(c) + sqrt(c)*sin(f*x + e)/(cos(f*x + e) + 1) + sqrt(c)*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2)/((a + a*sin(f*x + e)/(cos(f*x + e) + 1))*s
qrt(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)) - A*(sqrt(c) + sqrt(c)*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2)/((a + a*sin(f*x + e)/(cos(f*x + e) + 1))*s
qrt(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(69) = 138$.

Time = 0.26 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.40

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{a+a\sin(e+fx)}\,dx = -\frac{2\sqrt{2}\left(A\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-3B\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-\frac{A\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\mathrm{sgn}(\mathrm{s}}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\right.}}{af\left(\frac{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^2}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2}-1\right)}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x, algo
rithm="giac")
```

output
```
-2*sqrt(2)*(A*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*B*sgn(sin(-1/4*pi +
1/2*f*x + 1/2*e)) - A*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - B*(cos(-1/4*pi
 + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi
+ 1/2*f*x + 1/2*e) + 1))*sqrt(c)/(a*f*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1
)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 38.59 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.75

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{a+a\sin(e+fx)}\,dx = \frac{2\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(2\,B\,\sin\left(2\,e+2\,f\,x\right)-2\,A\,\sin\left(2\,e+2\,f\,x\right)-4\,A\,\left(2\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)+7\right.}{a\,f\,\left(4\sin\left(e+f\,x\right)^2+\sin\left(e+f\,x\right)+\sin\left(3\,e+3\,f\,x\right)\right.}$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))
,x)
```

output
```
(2*(-c*(sin(e + f*x) - 1))^(1/2)*(2*B*sin(2*e + 2*f*x) - 2*A*sin(2*e + 2*f
*x) - 4*A*(2*sin(e/2 + (f*x)/2)^2 - 1) + 7*B*(2*sin(e/2 + (f*x)/2)^2 - 1)
+ B*(2*sin((3*e)/2 + (3*f*x)/2)^2 - 1)))/(a*f*(sin(e + f*x) + sin(3*e + 3*
f*x) + 4*sin(e + f*x)^2 - 4))
```

Reduce [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{a+a\sin(e+fx)}\,dx$$
$$=\frac{\sqrt{c}\left(\left(\int\frac{\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)+1}dx\right)a+\left(\int\frac{\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)+1}dx\right)b\right)}{a}$$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x)
```

output
```
(sqrt(c)*(int(sqrt( - sin(e + f*x) + 1)/(sin(e + f*x) + 1),x)*a + int((sqr
t( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x) + 1),x)*b))/a
```

3.112 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 91

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}}\,dx = \frac{(A+B)\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}a\sqrt{c}f} - \frac{(A-B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{acf}$$

output
```
1/2*(A+B)*arctanh(1/2*c^(1/2)*cos(f*x+e)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))*2
^(1/2)/a/c^(1/2)/f-(A-B)*sec(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/c/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.83 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.54

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-A+B-(1+i)\sqrt[4]{-1}(A+\right.}{af(1+\sin(e+fx))\sqrt{c-c\sin}}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*
x]]),x]
```

output
```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])*(-A + B - (1 + I)*(-1)^(1/4)*(A + B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 +
 Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])))/(a*f*(1 + Sin[
e + f*x])*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {3042, 3446, 3042, 3334, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)\sqrt{c-c\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)\sqrt{c-c\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3446}$$

$$\frac{\int \sec^2(e+fx)(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}dx}{ac}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{\cos(e+fx)^2}dx}{ac}$$

$$\Big\downarrow \text{3334}$$

$$\frac{\frac{1}{2}c(A+B)\int \frac{1}{\sqrt{c-c\sin(e+fx)}}dx-\frac{(A-B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{f}}{ac}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{2}c(A+B)\int \frac{1}{\sqrt{c-c\sin(e+fx)}}dx - \frac{(A-B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{f}}{ac}$$

$\big\downarrow$ 3128

$$\frac{-\frac{c(A+B)\int \frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{f} - \frac{(A-B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{f}}{ac}$$

$\big\downarrow$ 219

$$\frac{\frac{\sqrt{c}(A+B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}f} - \frac{(A-B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{f}}{ac}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]]),x
]
```

output
```
(((A + B)*Sqrt[c]*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e
 + f*x]])])/(Sqrt[2]*f) - ((A - B)*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/
f)/(a*c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.43

method	result
default	$-\frac{(\sin(fx+e)-1)\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{c(1+\sin(fx+e))}\,A+\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{c(1+\sin(fx+e))}}{2a\sqrt{c}\,\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/2/a*(sin(f*x+e)-1)*(2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2
)/c^(1/2))*(c*(1+sin(f*x+e)))^(1/2)*A+2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)
))^(1/2)*2^(1/2)/c^(1/2))*(c*(1+sin(f*x+e)))^(1/2)*B-2*c^(1/2)*A+2*c^(1/2)
*B)/c^(1/2)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.78

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\sqrt{2}(A+B)\sqrt{c}\cos(fx+e)\log\left(-\frac{\cos(fx+e)^2+(\cos(fx+e)-2)\sin(fx+e)+\frac{2\sqrt{2}\sqrt{-c\sin(fx+e)+c}(\cos(fx+e)+\sin(fx+e)+1)}{\sqrt{c}}+3\cos(}{\cos(fx+e)^2+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right.}{4\,acf\cos(fx+e)}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algo
rithm="fricas")
```

output
```
1/4*(sqrt(2)*(A + B)*sqrt(c)*cos(f*x + e)*log(-(cos(f*x + e)^2 + (cos(f*x
+ e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e)
 + sin(f*x + e) + 1)/sqrt(c) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(
f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*sqrt(-c*sin(f*x + e) +
 c)*(A - B))/(a*c*f*cos(f*x + e))
```

Sympy [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\int \frac{A}{\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+\sqrt{-c\sin(e+fx)+c}}\,dx + \int \frac{B\sin(e+fx)}{\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+\sqrt{-c\sin(e+fx)+c}}\,dx}{a}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

output
```
(Integral(A/(sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + sqrt(-c*sin(e + f*x)
 + c)), x) + Integral(B*sin(e + f*x)/(sqrt(-c*sin(e + f*x) + c)*sin(e + f*
x) + sqrt(-c*sin(e + f*x) + c)), x))/a
```

Maxima [F]

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))\sqrt{c - c\sin(e + fx)}}\,dx = \int \frac{B\sin(fx + e) + A}{(a\sin(fx + e) + a)\sqrt{-c\sin(fx + e) + c}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
+ c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))\sqrt{c - c\sin(e + fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{A+B\,\sin\left(e+f\,x\right)}{\left(a+a\,\sin\left(e+f\,x\right)\right)\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))
,x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))
, x)
```

Reduce [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{\sqrt{c}\left(\left(\int \frac{\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)^2-1}dx\right)a+\left(\int \frac{\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)^2-1}dx\right)b\right)}{ac}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

output
```
( - sqrt(c)*(int(sqrt( - sin(e + f*x) + 1)/(sin(e + f*x)**2 - 1),x)*a + in
t((sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**2 - 1),x)*b))/(a
*c)
```

3.113 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 136

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}}\,dx=\frac{(3A-B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}ac^{3/2}f}+\frac{(3A-B)\cos(e+fx)}{4af(c-c\sin(e+fx))^{3/2}}-\frac{(A-B)\sec(e+fx)}{acf\sqrt{c-c\sin(e+fx)}}$$

output

```
1/8*(3*A-B)*arctanh(1/2*c^(1/2)*cos(f*x+e)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))
*2^(1/2)/a/c^(3/2)/f+1/4*(3*A-B)*cos(f*x+e)/a/f/(c-c*sin(f*x+e))^(3/2)-(A-
B)*sec(f*x+e)/a/c/f/(c-c*sin(f*x+e))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.56 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.09

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}}\,dx=\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\right.}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(3/2)),x]
```

output
```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])*(2*(-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + (A + B)*(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - (1 +
I)*(-1)^(1/4)*(3*A - B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4
])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2]) + 2*(A + B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
))/(4*a*f*(1 + Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$, Rules used = {3042, 3446, 3042, 3334, 3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)(c-c\sin(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)(c-c\sin(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3446}$$

$$\frac{\int \frac{\sec^2(e+fx)(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}dx}{ac}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{A+B\sin(e+fx)}{\cos(e+fx)^2\sqrt{c-c\sin(e+fx)}}dx}{ac}$$

$$\Big\downarrow \text{ 3334}$$

$$\frac{\frac{1}{2}c(3A-B)\int \frac{1}{(c-c\sin(e+fx))^{3/2}}dx-\frac{(A-B)\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}}{ac}$$

$$\downarrow \text{3042}$$

$$\frac{\frac{1}{2}c(3A-B)\int \frac{1}{(c-c\sin(e+fx))^{3/2}}dx - \frac{(A-B)\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}}{ac}$$

$$\downarrow \text{3129}$$

$$\frac{\frac{1}{2}c(3A-B)\left(\frac{\int \frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{4c}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}\right)-\frac{(A-B)\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}}{ac}$$

$$\downarrow \text{3042}$$

$$\frac{\frac{1}{2}c(3A-B)\left(\frac{\int \frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{4c}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}\right)-\frac{(A-B)\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}}{ac}$$

$$\downarrow \text{3128}$$

$$\frac{\frac{1}{2}c(3A-B)\left(\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}-\frac{\int \frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{2cf}\right)-\frac{(A-B)\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}}{ac}$$

$$\downarrow \text{219}$$

$$\frac{\frac{1}{2}c(3A-B)\left(\frac{\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}c^{3/2}f}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}\right)-\frac{(A-B)\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}}{ac}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))
,x]
```

output
```
(-(((A - B)*Sec[e + f*x])/(f*Sqrt[c - c*Sin[e + f*x]])) + ((3*A - B)*c*(Ar
cTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]/(2*Sqrt[2
]*c^(3/2)*f) + Cos[e + f*x]/(2*f*(c - c*Sin[e + f*x])^(3/2))))/2)/(a*c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.65

method	result
default	$-\frac{\sin(fx+e)\left(3A\sqrt{c+c\sin(fx+e)}\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,c-B\sqrt{c+c\sin(fx+e)}\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,c-6A}{8c^{\frac{5}{2}}\,a\cos(}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/8/c^(5/2)/a*(sin(f*x+e)*(3*A*(c+c*sin(f*x+e))^(1/2)*arctanh(1/2*(c+c*si
n(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c-B*(c+c*sin(f*x+e))^(1/2)*arctan
h(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c-6*A*c^(3/2)+2*B*c^
(3/2))-3*A*(c+c*sin(f*x+e))^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/
2)/c^(1/2))*2^(1/2)*c+B*(c+c*sin(f*x+e))^(1/2)*arctanh(1/2*(c+c*sin(f*x+e)
)^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c+2*A*c^(3/2)-6*B*c^(3/2))/cos(f*x+e)/(c-
c*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.70

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}}\,dx =$$
$$-\frac{\sqrt{2}((3\,A-B)\cos{(fx+e)}\sin{(fx+e)}-(3\,A-B)\cos{(fx+e)})\sqrt{c}\log\left(-\frac{c\cos(fx+e)^2-2\,\sqrt{2}\sqrt{-c\sin(fx+e)+c}}{\cos(fx}\right.}{16\,(ac^2f\cos{(fx+}}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algo
rithm="fricas")
```

output
```
-1/16*(sqrt(2)*((3*A - B)*cos(f*x + e)*sin(f*x + e) - (3*A - B)*cos(f*x +
e))*sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*s
qrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x +
 e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*
x + e) - cos(f*x + e) - 2)) + 4*((3*A - B)*sin(f*x + e) - A + 3*B)*sqrt(-c
*sin(f*x + e) + c))/(a*c^2*f*cos(f*x + e)*sin(f*x + e) - a*c^2*f*cos(f*x +
 e))
```

Sympy [F]

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\int \frac{A}{-c\sqrt{-c\sin(e+fx)+c}\sin^2(e+fx)+c\sqrt{-c\sin(e+fx)+c}}\,dx + \int \frac{}{-c\sqrt{-}}}{a}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),x)
```

output
```
(Integral(A/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2 + c*sqrt(-c*sin(
e + f*x) + c)), x) + Integral(B*sin(e + f*x)/(-c*sqrt(-c*sin(e + f*x) + c)
*sin(e + f*x)**2 + c*sqrt(-c*sin(e + f*x) + c)), x))/a
```

Maxima [F]

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{B\sin(fx+e)+A}{(a\sin(fx+e)+a)(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)
^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{A+B\,\sin(e+f\,x)}{(a+a\,\sin(e+f\,x))\,(c-c\,\sin(e+f\,x))^{3/2}}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))
,x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))
, x)
```

Reduce [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\sqrt{c}\left(\left(\int \frac{\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)^3-\sin(fx+e)^2-\sin(fx+e)+1}dx\right)a+\left(\int \frac{\sqrt{-}}{\sin(fx+e}\right.\right.}{a\,c^2}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)
```

output

```
(sqrt(c)*(int(sqrt( - sin(e + f*x) + 1)/(sin(e + f*x)**3 - sin(e + f*x)**2
 - sin(e + f*x) + 1),x)*a + int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(
sin(e + f*x)**3 - sin(e + f*x)**2 - sin(e + f*x) + 1),x)*b))/(a*c**2)
```

3.114 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 180

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}}\,dx=\frac{3(5A-3B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{32\sqrt{2}ac^{5/2}f}+\frac{3(5A-3B)\cos(e+fx)}{32acf(c-c\sin(e+fx))^{3/2}}+\frac{(A+B)\sec(e+fx)}{4acf(c-c\sin(e+fx))^{3/2}}-\frac{(5A-3B)\sec(e+fx)}{8ac^2f\sqrt{c-c\sin(e+fx)}}$$

output
```
3/64*(5*A-3*B)*arctanh(1/2*c^(1/2)*cos(f*x+e)*2^(1/2)/(c-c*sin(f*x+e))^(1/
2))*2^(1/2)/a/c^(5/2)/f+3/32*(5*A-3*B)*cos(f*x+e)/a/c/f/(c-c*sin(f*x+e))^(
3/2)+1/4*(A+B)*sec(f*x+e)/a/c/f/(c-c*sin(f*x+e))^(3/2)-1/8*(5*A-3*B)*sec(f
*x+e)/a/c^2/f/(c-c*sin(f*x+e))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.29 (sec) , antiderivative size = 404, normalized size of antiderivative = 2.24

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}}\,dx=\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\right.}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^
(5/2)),x]
```

output
```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])*(8*(-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + 4*(A + B)*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (7*
A - B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2]) - (3 + 3*I)*(-1)^(1/4)*(5*A - 3*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4
)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2]) + 8*(A + B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2]) + 2*(7*A - B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^2*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])))/(32*a*f*(1 + S
in[e + f*x])*(c - c*Sin[e + f*x])^(5/2))
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.91, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.289$, Rules used = {3042, 3446, 3042, 3338, 3042, 3166, 3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)(c-c\sin(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)(c-c\sin(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3446}$$

$$\frac{\int \frac{\sec^2(e+fx)(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}dx}{ac}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{A+B\sin(e+fx)}{\cos(e+fx)^2(c-c\sin(e+fx))^{3/2}}dx}{ac}$$

$$\Big\downarrow \ 3338$$

$$\frac{\frac{(5A-3B)\int \frac{\sec^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}dx}{8c}+\frac{(A+B)\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}}{ac}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{(5A-3B)\int \frac{1}{\cos(e+fx)^2\sqrt{c-c\sin(e+fx)}}dx}{8c}+\frac{(A+B)\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}}{ac}$$

$$\Big\downarrow \ 3166$$

$$\frac{\frac{(5A-3B)\left(\frac{3}{2}c\int \frac{1}{(c-c\sin(e+fx))^{3/2}}dx-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)}{8c}+\frac{(A+B)\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}}{ac}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{(5A-3B)\left(\frac{3}{2}c\int \frac{1}{(c-c\sin(e+fx))^{3/2}}dx-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)}{8c}+\frac{(A+B)\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}}{ac}$$

$$\Big\downarrow \ 3129$$

$$\frac{\frac{(5A-3B)\left(\frac{3}{2}c\left(\frac{\int \frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{4c}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}\right)-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)}{8c}+\frac{(A+B)\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}}{ac}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{(5A-3B)\left(\frac{3}{2}c\left(\frac{\int \frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{4c}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}\right)-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)}{8c}+\frac{(A+B)\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}}{ac}$$

$$\Big\downarrow \ 3128$$

$$\frac{\frac{(5A-3B)\left(\frac{3}{2}c\left(\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}-\frac{\int \frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{2cf}\right)-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)}{8c}+\frac{(A+B)\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}}{ac}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{(5A-3B)\left(\frac{3}{2}c\left(\frac{\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}c^{3/2}f}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}\right)-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)}{8c}+\frac{(A+B)\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}}{ac}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))
,x]
```

output
```
(((A + B)*Sec[e + f*x])/(4*f*(c - c*Sin[e + f*x])^(3/2)) + ((5*A - 3*B)*(-
(Sec[e + f*x]/(f*Sqrt[c - c*Sin[e + f*x]])) + (3*c*(ArcTanh[(Sqrt[c]*Cos[e
 + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]/(2*Sqrt[2]*c^(3/2)*f) + Cos[e
 + f*x]/(2*f*(c - c*Sin[e + f*x])^(3/2))))/2))/(8*c))/(a*c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3166
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Simp[a*((2*p + 1)/(2*g^2*(p + 1)))   Int[(g
*Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e
, f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(157) = 314$.

Time = 0.35 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.94

method	result
default	$\frac{\left(-30A\,c^{\frac{5}{2}}+15A\sqrt{c+c\sin(fx+e)}\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,c^{2}+18B\,c^{\frac{5}{2}}-9B\sqrt{c+c\sin(fx+e)}\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right.}{}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
1/64*((-30*A*c^(5/2)+15*A*(c+c*sin(f*x+e))^(1/2)*arctanh(1/2*(c+c*sin(f*x+
e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^2+18*B*c^(5/2)-9*B*(c+c*sin(f*x+e))^(
1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^2)*cos(
f*x+e)^2+sin(f*x+e)*(-40*A*c^(5/2)+30*A*(c+c*sin(f*x+e))^(1/2)*arctanh(1/2
*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^2+24*B*c^(5/2)-18*B*(c+
c*sin(f*x+e))^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^
(1/2)*c^2)+24*A*c^(5/2)-30*A*(c+c*sin(f*x+e))^(1/2)*arctanh(1/2*(c+c*sin(f
*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^2-40*B*c^(5/2)+18*B*(c+c*sin(f*x+e
))^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^2)/
c^(9/2)/a/(sin(f*x+e)-1)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.57

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}}\,dx =$$
$$-\frac{3\sqrt{2}\left((5A-3B)\cos(fx+e)^3+2(5A-3B)\cos(fx+e)\sin(fx+e)-2(5A-3B)\cos(fx+e)\right)\sqrt{}}{}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algo
rithm="fricas")
```

output
```
-1/128*(3*sqrt(2)*((5*A - 3*B)*cos(f*x + e)^3 + 2*(5*A - 3*B)*cos(f*x + e)
*sin(f*x + e) - 2*(5*A - 3*B)*cos(f*x + e))*sqrt(c)*log(-(c*cos(f*x + e)^2
 - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e
) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(co
s(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*(3
*(5*A - 3*B)*cos(f*x + e)^2 + 4*(5*A - 3*B)*sin(f*x + e) - 12*A + 20*B)*sq
rt(-c*sin(f*x + e) + c))/(a*c^3*f*cos(f*x + e)^3 + 2*a*c^3*f*cos(f*x + e)*
sin(f*x + e) - 2*a*c^3*f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{B\sin(fx+e)+A}{(a\sin(fx+e)+a)(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)
^(5/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}}\,dx=\int \frac{A+B\,\sin\left(e+f\,x\right)}{\left(a+a\,\sin\left(e+f\,x\right)\right)\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))
,x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))
, x)
```

Reduce [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c-c\sin(e+fx))^{5/2}}\,dx=$$
$$-\frac{\sqrt{c}\left(\left(\int \frac{\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)^4-2\sin(fx+e)^3+2\sin(fx+e)-1}dx\right)a+\left(\int \frac{\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)^4-2\sin(fx+e)^3+2\sin(fx+e)-1}dx\right)b\right)}{a\,c^3}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)
```

output
```
( - sqrt(c)*(int(sqrt( - sin(e + f*x) + 1)/(sin(e + f*x)**4 - 2*sin(e + f*
x)**3 + 2*sin(e + f*x) - 1),x)*a + int((sqrt( - sin(e + f*x) + 1)*sin(e +
f*x))/(sin(e + f*x)**4 - 2*sin(e + f*x)**3 + 2*sin(e + f*x) - 1),x)*b))/(a
*c**3)
```

3.115 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 242

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{2048(7A-13B)c^4\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{105a^2f} - \frac{512(7A-13B)c^3\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{105a^2f} - \frac{64(7A-13B)c^2\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{105a^2f} - \frac{16(7A-13B)c\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{105a^2f} - \frac{(7A-13B)\sec(e+fx)(c-c\sin(e+fx))^{9/2}}{21a^2f} - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{13/2}}{3a^2c^2f}$$

output
```
2048/105*(7*A-13*B)*c^4*sec(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f-512/105*(7
*A-13*B)*c^3*sec(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a^2/f-64/105*(7*A-13*B)*c^2
*sec(f*x+e)*(c-c*sin(f*x+e))^(5/2)/a^2/f-16/105*(7*A-13*B)*c*sec(f*x+e)*(c
-c*sin(f*x+e))^(7/2)/a^2/f-1/21*(7*A-13*B)*sec(f*x+e)*(c-c*sin(f*x+e))^(9/
2)/a^2/f-1/3*(A-B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(13/2)/a^2/c^2/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 953 vs. $2(242) = 484$.

Time = 16.46 (sec) , antiderivative size = 953, normalized size of antiderivative = 3.94

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e +
 f*x])^2,x]
```

output
```
(-32*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^(9
/2))/(3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a + a*Sin[e + f*x])^2)
+ (32*(2*A - 3*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(c - c*Sin[e + f
*x])^(9/2))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a + a*Sin[e + f*x]
)^2) + ((164*A - 351*B)*Cos[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)
/2])^4*(c - c*Sin[e + f*x])^(9/2))/(4*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/
2])^9*(a + a*Sin[e + f*x])^2) + ((26*A - 83*B)*Cos[(3*(e + f*x))/2]*(Cos[(
e + f*x)/2] + Sin[(e + f*x)/2])^4*(c - c*Sin[e + f*x])^(9/2))/(12*f*(Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])^9*(a + a*Sin[e + f*x])^2) - ((2*A - 13*B)*
Cos[(5*(e + f*x))/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(c - c*Sin[e
+ f*x])^(9/2))/(20*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a + a*Sin[e
+ f*x])^2) + (B*Cos[(7*(e + f*x))/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
^4*(c - c*Sin[e + f*x])^(9/2))/(28*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^9*(a + a*Sin[e + f*x])^2) + ((164*A - 351*B)*Sin[(e + f*x)/2]*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2])^4*(c - c*Sin[e + f*x])^(9/2))/(4*f*(Cos[(e + f*
x)/2] - Sin[(e + f*x)/2])^9*(a + a*Sin[e + f*x])^2) - ((26*A - 83*B)*(Cos[
(e + f*x)/2] + Sin[(e + f*x)/2])^4*(c - c*Sin[e + f*x])^(9/2)*Sin[(3*(e +
f*x))/2])/(12*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a + a*Sin[e + f*x
])^2) - ((2*A - 13*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(c - c*Sin[e
 + f*x])^(9/2)*Sin[(5*(e + f*x))/2])/(20*f*(Cos[(e + f*x)/2] - Sin[(e +...
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.90, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3042, 3446, 3042, 3334, 3042, 3153, 3042, 3153, 3042, 3153, 3042, 3153, 3042, 3152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^{9/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-c\sin(e+fx))^{9/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3446}$$

$$\frac{\int \sec^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{13/2}dx}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{13/2}}{\cos(e+fx)^4}dx}{a^2c^2}$$

$$\Big\downarrow \text{3334}$$

$$\frac{-\frac{1}{6}c(7A-13B)\int \sec^2(e+fx)(c-c\sin(e+fx))^{11/2}dx-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{13/2}}{3f}}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{1}{6}c(7A-13B)\int \frac{(c-c\sin(e+fx))^{11/2}}{\cos(e+fx)^2}dx-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{13/2}}{3f}}{a^2c^2}$$

$$\Big\downarrow \text{3153}$$

$$\frac{-\frac{1}{6}c(7A-13B)\left(\frac{16}{7}c\int \sec^2(e+fx)(c-c\sin(e+fx))^{9/2}dx+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{9/2}}{7f}\right)-\frac{(A-B)\sec^3(e+fx)(c-}{3f}}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{1}{6}c(7A-13B)\left(\frac{16}{7}c\int \frac{(c-c\sin(e+fx))^{9/2}}{\cos(e+fx)^2}dx+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{9/2}}{7f}\right)-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{13/2}}{3f}}{a^2c^2}$$

$\downarrow$ 3153

$$\frac{-\frac{1}{6}c(7A-13B)\left(\frac{16}{7}c\left(\frac{12}{5}c\int \sec^2(e+fx)(c-c\sin(e+fx))^{7/2}dx+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{5f}\right)+\frac{2c\sec(e+fx)(c-c}{7f}\right.}{a^2c^2}$$

$\downarrow$ 3042

$$\frac{-\frac{1}{6}c(7A-13B)\left(\frac{16}{7}c\left(\frac{12}{5}c\int \frac{(c-c\sin(e+fx))^{7/2}}{\cos(e+fx)^2}dx+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{5f}\right)+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{9/2}}{7f}\right)-\frac{(A}{}}{a^2c^2}$$

$\downarrow$ 3153

$$\frac{-\frac{1}{6}c(7A-13B)\left(\frac{16}{7}c\left(\frac{12}{5}c\left(\frac{8}{3}c\int \sec^2(e+fx)(c-c\sin(e+fx))^{5/2}dx+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3f}\right)+\frac{2c\sec(e+fx}{}\right.\right.}{a^2c^2}$$

$\downarrow$ 3042

$$\frac{-\frac{1}{6}c(7A-13B)\left(\frac{16}{7}c\left(\frac{12}{5}c\left(\frac{8}{3}c\int \frac{(c-c\sin(e+fx))^{5/2}}{\cos(e+fx)^2}dx+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3f}\right)+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{5f}\right)\right.}{a^2c^2}$$

$\downarrow$ 3153

$$-\frac{1}{6}c(7A-13B)\left(\frac{16}{7}c\left(\frac{12}{5}c\left(\frac{8}{3}c\left(4c\int \sec^2(e+fx)(c-c\sin(e+fx))^{3/2}dx+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{f}\right)+\frac{2c\sec(e}{}\right.\right.\right.$$

$\downarrow$ 3042

$$\frac{-\frac{1}{6}c(7A-13B)\left(\frac{16}{7}c\left(\frac{12}{5}c\left(\frac{8}{3}c\left(4c\int \frac{(c-c\sin(e+fx))^{3/2}}{\cos(e+fx)^2}dx+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{f}\right)+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{5/}}{3f}\right.\right.\right.}{a^2c^2}$$

$\downarrow$ 3152

$$\frac{-\frac{1}{6}c(7A-13B)\left(\frac{16}{7}c\left(\frac{12}{5}c\left(\frac{8}{3}c\left(\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{f}-\frac{8c^2\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{f}\right)+\frac{2c\sec(e+fx)(c-c\sin(e+fx)}{3f}\right.\right.\right.}{a^2c^2}$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e + f*x])
^2,x]
```

output
```
(-1/3*((A - B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(13/2))/f - ((7*A - 13*
B)*c*((2*c*Sec[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(7*f) + (16*c*((2*c*Se
c[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(5*f) + (12*c*((2*c*Sec[e + f*x]*(c
 - c*Sin[e + f*x])^(5/2))/(3*f) + (8*c*((-8*c^2*Sec[e + f*x]*Sqrt[c - c*Si
n[e + f*x]])/f + (2*c*Sec[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/f))/3))/5))
/7))/6)/(a^2*c^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3153
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] &&
NeQ[m + p, 0]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 20.97 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.59

method	result
default	$-\frac{2c^5(\sin(fx+e)-1)\left(15B\sin(fx+e)\cos(fx+e)^4+(21A-114B)\cos(fx+e)^4+(196A-544B)\cos(fx+e)^2\sin(fx+e)+(-1848A+3\right.}{105a^2(1+\sin(fx+e))\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^2,x,method=_R
ETURNVERBOSE)
```

output
```
-2/105*c^5/a^2*(sin(f*x+e)-1)/(1+sin(f*x+e))*(15*B*sin(f*x+e)*cos(f*x+e)^4
+(21*A-114*B)*cos(f*x+e)^4+(196*A-544*B)*cos(f*x+e)^2*sin(f*x+e)+(-1848*A+
3732*B)*cos(f*x+e)^2+(7448*A-13592*B)*sin(f*x+e)+6888*A-13032*B)/cos(f*x+e
)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.63

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{2\left(3\left(7A-38B\right)c^4\cos\left(fx+e\right)^4-12\left(154A-311B\right)c\right.}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^2,x, al
gorithm="fricas")
```

output
```
2/105*(3*(7*A - 38*B)*c^4*cos(f*x + e)^4 - 12*(154*A - 311*B)*c^4*cos(f*x
+ e)^2 + 24*(287*A - 543*B)*c^4 + (15*B*c^4*cos(f*x + e)^4 + 4*(49*A - 136
*B)*c^4*cos(f*x + e)^2 + 8*(931*A - 1699*B)*c^4)*sin(f*x + e))*sqrt(-c*sin
(f*x + e) + c)/(a^2*f*cos(f*x + e)*sin(f*x + e) + a^2*f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^2}\,dx=\text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(9/2)/(a+a*sin(f*x+e))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 762 vs. $2(218) = 436$.

Time = 0.15 (sec) , antiderivative size = 762, normalized size of antiderivative = 3.15

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^2}\,dx=\text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^2,x, al
gorithm="maxima")
```

output
```
-2/105*(7*(723*c^(9/2) + 2184*c^(9/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 53
70*c^(9/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10696*c^(9/2)*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3 + 15021*c^(9/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4 + 21168*c^(9/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 20748*c^(9/2)*sin
(f*x + e)^6/(cos(f*x + e) + 1)^6 + 21168*c^(9/2)*sin(f*x + e)^7/(cos(f*x +
 e) + 1)^7 + 15021*c^(9/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 10696*c^(
9/2)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 5370*c^(9/2)*sin(f*x + e)^10/(c
os(f*x + e) + 1)^10 + 2184*c^(9/2)*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 +
 723*c^(9/2)*sin(f*x + e)^12/(cos(f*x + e) + 1)^12)*A/((a^2 + 3*a^2*sin(f*
x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^
2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 1)^(9/2)) - 2*(4707*c^(9/2) + 14121*c^(9/2)*sin(f*x + e)/(cos(f*x + e)
 + 1) + 35250*c^(9/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 68549*c^(9/2)*
sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 99549*c^(9/2)*sin(f*x + e)^4/(cos(f*
x + e) + 1)^4 + 134802*c^(9/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 13801
2*c^(9/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 134802*c^(9/2)*sin(f*x + e
)^7/(cos(f*x + e) + 1)^7 + 99549*c^(9/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)
^8 + 68549*c^(9/2)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 35250*c^(9/2)*sin
(f*x + e)^10/(cos(f*x + e) + 1)^10 + 14121*c^(9/2)*sin(f*x + e)^11/(cos(f*
x + e) + 1)^11 + 4707*c^(9/2)*sin(f*x + e)^12/(cos(f*x + e) + 1)^12)*B/...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 978 vs. 2(218) = 436.

Time = 0.45 (sec) , antiderivative size = 978, normalized size of antiderivative = 4.04

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^2,x, al
gorithm="giac")
```

output
```
-16/105*sqrt(2)*sqrt(c)*(35*(11*A*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))
- 17*B*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 24*A*c^4*(cos(-1/4*pi + 1
/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/
2*f*x + 1/2*e) + 1) - 36*B*c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(si
n(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 9*A*c
^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*
e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 15*B*c^4*(cos(-1/4*pi + 1/2*f
*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*
f*x + 1/2*e) + 1)^2)/(a^2*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*
pi + 1/2*f*x + 1/2*e) + 1) + 1)^3) - (511*A*c^4*sgn(sin(-1/4*pi + 1/2*f*x
+ 1/2*e)) - 1069*B*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 3262*A*c^4*(c
os(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(co
s(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 6958*B*c^4*(cos(-1/4*pi + 1/2*f*x + 1/
2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2
*e) + 1) + 8421*A*c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*
pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 18459*B*c^
4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e
))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 10780*A*c^4*(cos(-1/4*pi + 1/2
*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/
2*f*x + 1/2*e) + 1)^3 + 24220*B*c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^2}\,dx=\int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{9/2}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^2}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x))
^2,x)
```

output
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x))
^2, x)
```

Reduce [F]

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^{9/2}}{(a + a\sin(e + fx))^2}\,dx = \frac{\sqrt{c}\,c^4\left(\left(\int \frac{\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)^2+2\sin(fx+e)+1}dx\right)a + \left(\int \frac{\sqrt{-\sin(fx+e)+}}{\sin(fx+e)^2+2\,}\right.\right.}{}$$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^2,x)
```

output
```
(sqrt(c)*c**4*(int(sqrt( - sin(e + f*x) + 1)/(sin(e + f*x)**2 + 2*sin(e +
f*x) + 1),x)*a + int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**5)/(sin(e +
f*x)**2 + 2*sin(e + f*x) + 1),x)*b + int((sqrt( - sin(e + f*x) + 1)*sin(e
+ f*x)**4)/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),x)*a - 4*int((sqrt( - si
n(e + f*x) + 1)*sin(e + f*x)**4)/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),x)
*b - 4*int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**3)/(sin(e + f*x)**2 +
2*sin(e + f*x) + 1),x)*a + 6*int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**
3)/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),x)*b + 6*int((sqrt( - sin(e + f*
x) + 1)*sin(e + f*x)**2)/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),x)*a - 4*i
nt((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x)**2 + 2*sin(e
+ f*x) + 1),x)*b - 4*int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e +
 f*x)**2 + 2*sin(e + f*x) + 1),x)*a + int((sqrt( - sin(e + f*x) + 1)*sin(e
 + f*x))/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),x)*b))/a**2
```

3.116 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 201

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{128(5A-11B)c^3\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{15a^2f} - \frac{32(5A-11B)c^2\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^2f} - \frac{4(5A-11B)c\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{15a^2f} - \frac{(5A-11B)\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{15a^2f} - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{3a^2c^2f}$$

output
```
128/15*(5*A-11*B)*c^3*sec(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f-32/15*(5*A-1
1*B)*c^2*sec(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a^2/f-4/15*(5*A-11*B)*c*sec(f*x
+e)*(c-c*sin(f*x+e))^(5/2)/a^2/f-1/15*(5*A-11*B)*sec(f*x+e)*(c-c*sin(f*x+e
))^(7/2)/a^2/f-1/3*(A-B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(11/2)/a^2/c^2/f
```

Mathematica [A] (verified)

Time = 10.92 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.79

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^2}\,dx =$$
$$-\frac{c^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}(-2100A+4725B+12(25A-62B)\cos(2(e\cdots}{60a^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\cdots}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e +
 f*x])^2,x]
```

output
```
-1/60*(c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*
(-2100*A + 4725*B + 12*(25*A - 62*B)*Cos[2*(e + f*x)] + 3*B*Cos[4*(e + f*x
)] - 2730*A*Sin[e + f*x] + 5838*B*Sin[e + f*x] - 10*A*Sin[3*(e + f*x)] + 4
6*B*Sin[3*(e + f*x)]))/(a^2*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + S
in[e + f*x])^2)
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.91, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 3446, 3042, 3334, 3042, 3153, 3042, 3153, 3042, 3153, 3042, 3152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3446}$$

$$\frac{\int \sec^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{11/2}dx}{a^2c^2}$$

$\big\downarrow$ 3042

$$\frac{\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{11/2}}{\cos(e+fx)^4}dx}{a^2c^2}$$

$\big\downarrow$ 3334

$$\frac{-\frac{1}{6}c(5A-11B)\int \sec^2(e+fx)(c-c\sin(e+fx))^{9/2}dx-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{3f}}{a^2c^2}$$

$\big\downarrow$ 3042

$$\frac{-\frac{1}{6}c(5A-11B)\int \frac{(c-c\sin(e+fx))^{9/2}}{\cos(e+fx)^2}dx-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{3f}}{a^2c^2}$$

$\big\downarrow$ 3153

$$\frac{-\frac{1}{6}c(5A-11B)\left(\frac{12}{5}c\int \sec^2(e+fx)(c-c\sin(e+fx))^{7/2}dx+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{5f}\right)-\frac{(A-B)\sec^3(e+fx)(c-}{3f}}{a^2c^2}$$

$\big\downarrow$ 3042

$$\frac{-\frac{1}{6}c(5A-11B)\left(\frac{12}{5}c\int \frac{(c-c\sin(e+fx))^{7/2}}{\cos(e+fx)^2}dx+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{5f}\right)-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{3f}}{a^2c^2}$$

$\big\downarrow$ 3153

$$\frac{-\frac{1}{6}c(5A-11B)\left(\frac{12}{5}c\left(\frac{8}{3}c\int \sec^2(e+fx)(c-c\sin(e+fx))^{5/2}dx+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3f}\right)+\frac{2c\sec(e+fx)(c-c}{5f}\right.}{a^2c^2}$$

$\big\downarrow$ 3042

$$\frac{-\frac{1}{6}c(5A-11B)\left(\frac{12}{5}c\left(\frac{8}{3}c\int \frac{(c-c\sin(e+fx))^{5/2}}{\cos(e+fx)^2}dx+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3f}\right)+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{7/2}}{5f}\right)-(A}{a^2c^2}$$

$\big\downarrow$ 3153

$$\frac{-\frac{1}{6}c(5A-11B)\left(\frac{12}{5}c\left(\frac{8}{3}c\left(4c\int \sec^2(e+fx)(c-c\sin(e+fx))^{3/2}dx+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{f}\right)+\frac{2c\sec(e+fx)(}{}\right.\right.}{a^2c^2}$$

$\big\downarrow$ 3042

$$\frac{-\frac{1}{6}c(5A-11B)\left(\frac{12}{5}c\left(\frac{8}{3}c\left(4c\int\frac{(c-c\sin(e+fx))^{3/2}}{\cos(e+fx)^2}dx+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{f}\right)+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3f}\right)+}{a^2c^2}$$

$$\Big\downarrow \text{3152}$$

$$\frac{-\frac{1}{6}c(5A-11B)\left(\frac{12}{5}c\left(\frac{8}{3}c\left(\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{f}-\frac{8c^2\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{f}\right)+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3f}\right)\right.}{a^2c^2}$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])
^2,x]
```

output
```
(-1/3*((A - B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(11/2))/f - ((5*A - 11*
B)*c*((2*c*Sec[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(5*f) + (12*c*((2*c*Se
c[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(3*f) + (8*c*((-8*c^2*Sec[e + f*x]*
Sqrt[c - c*Sin[e + f*x]])/f + (2*c*Sec[e + f*x]*(c - c*Sin[e + f*x])^(3/2)
)/f))/3))/5))/6)/(a^2*c^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3153
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] &&
NeQ[m + p, 0]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 20.73 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.60

method	result
default	$\frac{2c^4(\sin(fx+e)-1)\left(3B\cos(fx+e)^4+(-5A+23B)\cos(fx+e)^2\sin(fx+e)+(75A-189B)\cos(fx+e)^2+(-340A+724B)\sin(fx+e)-\right.}{15a^2(1+\sin(fx+e))\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^2,x,method=_R
ETURNVERBOSE)
```

output
```
2/15*c^4/a^2*(sin(f*x+e)-1)/(1+sin(f*x+e))*(3*B*cos(f*x+e)^4+(-5*A+23*B)*c
os(f*x+e)^2*sin(f*x+e)+(75*A-189*B)*cos(f*x+e)^2+(-340*A+724*B)*sin(f*x+e)
-300*A+684*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.66

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^2}\,dx = -\frac{2\left(3\,Bc^3\cos\left(fx+e\right)^4+3\,(25\,A-63\,B)c^3\cos\left(fx+e\right)^2-12\,(25\,A-57\,B)c^3-\left((5\,A-23\,B)c^3\cos\left(fx\right.\right.\right.}{15\left(a^2f\cos\left(fx+e\right)\sin\left(fx+e\right)+a^2f\cos\left(fx\right.\right.}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^2,x, al
gorithm="fricas")
```

output
```
-2/15*(3*B*c^3*cos(f*x + e)^4 + 3*(25*A - 63*B)*c^3*cos(f*x + e)^2 - 12*(2
5*A - 57*B)*c^3 - ((5*A - 23*B)*c^3*cos(f*x + e)^2 + 4*(85*A - 181*B)*c^3)
*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(a^2*f*cos(f*x + e)*sin(f*x + e)
+ a^2*f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 670 vs. $2(181) = 362$.

Time = 0.15 (sec) , antiderivative size = 670, normalized size of antiderivative = 3.33

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^2,x, al
gorithm="maxima")
```

output
```
-2/15*(5*(45*c^(7/2) + 138*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 285*c
^(7/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 544*c^(7/2)*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + 630*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 812
*c^(7/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 630*c^(7/2)*sin(f*x + e)^6/
(cos(f*x + e) + 1)^6 + 544*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 2
85*c^(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 138*c^(7/2)*sin(f*x + e)^
9/(cos(f*x + e) + 1)^9 + 45*c^(7/2)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10)
*A/((a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*(sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 1)^(7/2)) - 2*(249*c^(7/2) + 747*c^(7/2)*sin(f*
x + e)/(cos(f*x + e) + 1) + 1611*c^(7/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + 2896*c^(7/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3612*c^(7/2)*sin(f
*x + e)^4/(cos(f*x + e) + 1)^4 + 4298*c^(7/2)*sin(f*x + e)^5/(cos(f*x + e)
 + 1)^5 + 3612*c^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 2896*c^(7/2)*
sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1611*c^(7/2)*sin(f*x + e)^8/(cos(f*x
 + e) + 1)^8 + 747*c^(7/2)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 249*c^(7/
2)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10)*B/((a^2 + 3*a^2*sin(f*x + e)/(co
s(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x
+ e)^3/(cos(f*x + e) + 1)^3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(7/
2)))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 774 vs. $2(181) = 362$.

Time = 0.40 (sec) , antiderivative size = 774, normalized size of antiderivative = 3.85

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^2,x, al
gorithm="giac")
```

output
```
-16/15*sqrt(2)*sqrt(c)*(5*(4*A*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 7
*B*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 9*A*c^3*(cos(-1/4*pi + 1/2*f*
x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) + 1) - 15*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 3*A*c^3*(c
os(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(
cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 6*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1
/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x +
1/2*e) + 1)^2)/(a^2*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1
/2*f*x + 1/2*e) + 1) + 1)^3) - (20*A*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e
)) - 53*B*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 85*A*c^3*(cos(-1/4*pi
+ 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi +
 1/2*f*x + 1/2*e) + 1) + 235*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sg
n(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1
25*A*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x
+ 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 365*B*c^3*(cos(-1/4*pi
+ 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi
 + 1/2*f*x + 1/2*e) + 1)^2 - 75*A*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)
^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1
)^3 + 165*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^2}\,dx = \int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{7/2}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^2}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))
^2,x)
```

output
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))
^2, x)
```

Reduce [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{\sqrt{c}\,c^3\left(\left(\int \frac{\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)^2+2\sin(fx+e)+1}dx\right)a-\left(\int \frac{\sqrt{-\sin(fx+e)+}}{\sin(fx+e)^2+2}\right.\right.}{}$$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^2,x)
```

output
```
(sqrt(c)*c**3*(int(sqrt( - sin(e + f*x) + 1)/(sin(e + f*x)**2 + 2*sin(e +
f*x) + 1),x)*a - int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**4)/(sin(e +
f*x)**2 + 2*sin(e + f*x) + 1),x)*b - int((sqrt( - sin(e + f*x) + 1)*sin(e
+ f*x)**3)/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),x)*a + 3*int((sqrt( - si
n(e + f*x) + 1)*sin(e + f*x)**3)/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),x)
*b + 3*int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x)**2 +
2*sin(e + f*x) + 1),x)*a - 3*int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**
2)/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),x)*b - 3*int((sqrt( - sin(e + f*
x) + 1)*sin(e + f*x))/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),x)*a + int((s
qrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**2 + 2*sin(e + f*x) +
 1),x)*b))/a**2
```

3.117 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 154

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{32(A-3B)c^2\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2f} - \frac{8(A-3B)c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f} - \frac{(A-3B)\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2f} - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3a^2c^2f}$$

output
```
32/3*(A-3*B)*c^2*sec(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f-8/3*(A-3*B)*c*sec
(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a^2/f-1/3*(A-3*B)*sec(f*x+e)*(c-c*sin(f*x+e
))^(5/2)/a^2/f-1/3*(A-B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(9/2)/a^2/c^2/f
```

Mathematica [A] (verified)

Time = 9.19 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.84

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2}\,dx =$$

$$-\frac{c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}(-50A+160B+6(A-4B)\cos(2(e+fx))+}{6a^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e +
 f*x])^2,x]
```

output
```
-1/6*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(
-50*A + 160*B + 6*(A - 4*B)*Cos[2*(e + f*x)] + (-72*A + 201*B)*Sin[e + f*x
] + B*Sin[3*(e + f*x)]))/(a^2*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 +
 Sin[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 3446, 3042, 3334, 3042, 3153, 3042, 3153, 3042, 3152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3446}$$

$$\frac{\int \sec^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}dx}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{\cos(e+fx)^4}dx}{a^2c^2}$$

$\Big\downarrow$ 3334

$$\frac{-\frac{1}{2}c(A-3B)\int \sec^2(e+fx)(c-c\sin(e+fx))^{7/2}dx-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3f}}{a^2c^2}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{1}{2}c(A-3B)\int \frac{(c-c\sin(e+fx))^{7/2}}{\cos(e+fx)^2}dx-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3f}}{a^2c^2}$$

$\Big\downarrow$ 3153

$$\frac{-\frac{1}{2}c(A-3B)\left(\frac{8}{3}c\int \sec^2(e+fx)(c-c\sin(e+fx))^{5/2}dx+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3f}\right)-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3f}}{a^2c^2}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{1}{2}c(A-3B)\left(\frac{8}{3}c\int \frac{(c-c\sin(e+fx))^{5/2}}{\cos(e+fx)^2}dx+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3f}\right)-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3f}}{a^2c^2}$$

$\Big\downarrow$ 3153

$$\frac{-\frac{1}{2}c(A-3B)\left(\frac{8}{3}c\left(4c\int \sec^2(e+fx)(c-c\sin(e+fx))^{3/2}dx+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{f}\right)+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3f}\right)-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3f}}{a^2c^2}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{1}{2}c(A-3B)\left(\frac{8}{3}c\left(4c\int \frac{(c-c\sin(e+fx))^{3/2}}{\cos(e+fx)^2}dx+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{f}\right)+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3f}\right)-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3f}}{a^2c^2}$$

$\Big\downarrow$ 3152

$$\frac{-\frac{1}{2}c(A-3B)\left(\frac{8}{3}c\left(\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{f}-\frac{8c^2\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{f}\right)+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{5/2}}{3f}\right)-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3f}}{a^2c^2}$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])
^2,x]
```

output
```
(-1/3*((A - B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(9/2))/f - ((A - 3*B)*c
*((2*c*Sec[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(3*f) + (8*c*((-8*c^2*Sec[
e + f*x]*Sqrt[c - c*Sin[e + f*x]])/f + (2*c*Sec[e + f*x]*(c - c*Sin[e + f*
x])^(3/2))/f))/3))/2)/(a^2*c^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3153
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] &&
NeQ[m + p, 0]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 20.71 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.68

method	result	size
default	$-\frac{2 c^{3} \left(\sin \left(f x +e \right)-1\right) \left(-B \cos \left(f x +e \right)^{2} \sin \left(f x +e \right)+\left(-3 A +12 B \right) \cos \left(f x +e \right)^{2}+\left(18 A -50 B \right) \sin \left(f x +e \right)+14 A -46 B \right)}{3 a^{2} \left(1+\sin \left(f x +e \right)\right) \cos \left(f x +e \right) \sqrt{c -c \sin \left(f x +e \right)}\, f}$	105

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x,method=_R
ETURNVERBOSE)
```

output
```
-2/3*c^3/a^2*(sin(f*x+e)-1)/(1+sin(f*x+e))*(-B*cos(f*x+e)^2*sin(f*x+e)+(-3
*A+12*B)*cos(f*x+e)^2+(18*A-50*B)*sin(f*x+e)+14*A-46*B)/cos(f*x+e)/(c-c*si
n(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.71

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2}\,dx =$$
$$-\frac{2\left(3\,(A-4\,B)c^2\cos\left(fx+e\right)^2-2\,(7\,A-23\,B)c^2+\left(Bc^2\cos\left(fx+e\right)^2-2\,(9\,A-25\,B)c^2\right)\sin\left(fx+e\right)\right)}{3\left(a^2f\cos\left(fx+e\right)\sin\left(fx+e\right)+a^2f\cos\left(fx+e\right)\right)}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x, al
gorithm="fricas")
```

output
```
-2/3*(3*(A - 4*B)*c^2*cos(f*x + e)^2 - 2*(7*A - 23*B)*c^2 + (B*c^2*cos(f*x
 + e)^2 - 2*(9*A - 25*B)*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(a^2
*f*cos(f*x + e)*sin(f*x + e) + a^2*f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 577 vs. $2(138) = 276$.

Time $= 0.14$ (sec) , antiderivative size $= 577$, normalized size of antiderivative $= 3.75$

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x, al
gorithm="maxima")
```

output
```
-2/3*((11*c^(5/2) + 36*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 56*c^(5/2
)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 108*c^(5/2)*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 90*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 108*c^(5/
2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 56*c^(5/2)*sin(f*x + e)^6/(cos(f*
x + e) + 1)^6 + 36*c^(5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 11*c^(5/2
)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8)*A/((a^2 + 3*a^2*sin(f*x + e)/(cos(f
*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(5/2))
 - 2*(17*c^(5/2) + 51*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 92*c^(5/2)
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 149*c^(5/2)*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 150*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 149*c^(5/
2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 92*c^(5/2)*sin(f*x + e)^6/(cos(f*
x + e) + 1)^6 + 51*c^(5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 17*c^(5/2
)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8)*B/((a^2 + 3*a^2*sin(f*x + e)/(cos(f
*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(5/2))
)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(138) = 276$.

Time = 0.34 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.90

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{32\sqrt{2}\Big(Ac^2\mathrm{sgn}\big(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\big)-3Bc^2\mathrm{sgn}\big(\mathrm{si}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x, al
gorithm="giac")
```

output
```
32/3*sqrt(2)*(A*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*B*c^2*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e)) - 3*A*c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)
^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1
)^2 + 9*B*c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 2*A*c^2*(cos(-1/4*
pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4
*pi + 1/2*f*x + 1/2*e) + 1)^3 + 2*B*c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) +
 1)^3)*sqrt(c)/(a^2*f*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi
 + 1/2*f*x + 1/2*e) + 1)^2 - 1)^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2}\,dx=\int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^2}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))
^2,x)
```

output
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))
^2, x)
```

Reduce [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^2}\,dx=\frac{\sqrt{c}\,c^2\left(\left(\int \frac{\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)^2+2\sin(fx+e)+1}dx\right)a+\left(\int \frac{\sqrt{-\sin(fx+e)+}}{\sin(fx+e)^2+2}\right.\right.}{}$$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x)
```

output
```
(sqrt(c)*c**2*(int(sqrt( - sin(e + f*x) + 1)/(sin(e + f*x)**2 + 2*sin(e +
f*x) + 1),x)*a + int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**3)/(sin(e +
f*x)**2 + 2*sin(e + f*x) + 1),x)*b + int((sqrt( - sin(e + f*x) + 1)*sin(e
+ f*x)**2)/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),x)*a - 2*int((sqrt( - si
n(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),x)
*b - 2*int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**2 + 2*s
in(e + f*x) + 1),x)*a + int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(
e + f*x)**2 + 2*sin(e + f*x) + 1),x)*b))/a**2
```

3.118 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 115

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{4(A-7B)c\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2f} - \frac{(A-7B)\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2f} - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{3a^2c^2f}$$

output
```
4/3*(A-7*B)*c*sec(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f-1/3*(A-7*B)*sec(f*x+
e)*(c-c*sin(f*x+e))^(3/2)/a^2/f-1/3*(A-B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(7
/2)/a^2/c^2/f
```

Mathematica [A] (verified)

Time = 8.47 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.98

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2A-23B+3B\right.}{3a^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\right.}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e +
 f*x])^2,x]
```

output
```
(c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*A - 23*B + 3*B*Cos[2*(e + f*x)
] + 6*(A - 5*B)*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(3*a^2*f*(Cos[(e +
 f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3446, 3042, 3334, 3042, 3153, 3042, 3152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3446}$$

$$\frac{\int \sec^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}dx}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{\cos(e+fx)^4}dx}{a^2c^2}$$

$$\Big\downarrow \text{3334}$$

$$\frac{-\frac{1}{6}c(A-7B)\int \sec^2(e+fx)(c-c\sin(e+fx))^{5/2}dx-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{3f}}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{1}{6}c(A-7B)\int \frac{(c-c\sin(e+fx))^{5/2}}{\cos(e+fx)^2}dx-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{3f}}{a^2c^2}$$

$$\Big\downarrow \text{ 3153}$$

$$\frac{-\frac{1}{6}c(A-7B)\left(4c\int \sec^2(e+fx)(c-c\sin(e+fx))^{3/2}dx+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{f}\right)-\frac{(A-B)\sec^3(e+fx)(c-c\sin(}{3f}}{a^2c^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-\frac{1}{6}c(A-7B)\left(4c\int \frac{(c-c\sin(e+fx))^{3/2}}{\cos(e+fx)^2}dx+\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{f}\right)-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{3f}}{a^2c^2}$$

$$\Big\downarrow \text{ 3152}$$

$$\frac{-\frac{1}{6}c(A-7B)\left(\frac{2c\sec(e+fx)(c-c\sin(e+fx))^{3/2}}{f}-\frac{8c^2\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{f}\right)-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{3f}}{a^2c^2}$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])
^2,x]
```

output
```
(-1/3*((A - B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(7/2))/f - ((A - 7*B)*c
*((-8*c^2*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/f + (2*c*Sec[e + f*x]*(c
- c*Sin[e + f*x])^(3/2))/f))/6)/(a^2*c^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3153
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] &&
NeQ[m + p, 0]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

method	result	size
default	$-\frac{2c^{2}(\sin(fx+e)-1)\left(3B\cos(fx+e)^{2}+\sin(fx+e)(3A-15B)+A-13B\right)}{3a^{2}(1+\sin(fx+e))\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$	81

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x,method=_R
ETURNVERBOSE)
```

output
```
-2/3*c^2/a^2*(sin(f*x+e)-1)/(1+sin(f*x+e))*(3*B*cos(f*x+e)^2+sin(f*x+e)*(3
*A-15*B)+A-13*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.70

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{2\left(3\,Bc\cos\left(fx+e\right)^2+3\,(A-5\,B)c\sin\left(fx+e\right)+(A\cdots\right.}{3\left(a^2f\cos\left(fx+e\right)\sin\left(fx+e\right)+a^2f\cdots\right.}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x, al
gorithm="fricas")
```

output
```
2/3*(3*B*c*cos(f*x + e)^2 + 3*(A - 5*B)*c*sin(f*x + e) + (A - 13*B)*c)*sqr
t(-c*sin(f*x + e) + c)/(a^2*f*cos(f*x + e)*sin(f*x + e) + a^2*f*cos(f*x +
e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 482 vs. 2(103) = 206.

Time = 0.16 (sec) , antiderivative size = 482, normalized size of antiderivative = 4.19

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x, al
gorithm="maxima")
```

output
```
-2/3*((c^(3/2) + 6*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^(3/2)*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 + 12*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 3*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 6*c^(3/2)*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5 + c^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^
6)*A/((a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*(sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2)) - 2*(5*c^(3/2) + 15*c^(3/2)*sin(f*x
 + e)/(cos(f*x + e) + 1) + 21*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
+ 30*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 21*c^(3/2)*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 + 15*c^(3/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 +
 5*c^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)*B/((a^2 + 3*a^2*sin(f*x +
e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
1)^(3/2)))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(103) = 206$.

Time = 0.31 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.46

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx =$$

$$-\frac{4\sqrt{2}\sqrt{c}\left(\frac{3\,Bc\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^2\left(\frac{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1}-1\right)}+\frac{Ac\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-4\,Bc\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+\frac{3\,Ac\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\cdots}{\cos\left(-\frac{1}{4}\,\pi\cdots\right.}}{\cdots}\cdots\right.}{\cdots}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x, al
gorithm="giac")
```

output
```
-4/3*sqrt(2)*sqrt(c)*(3*B*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(a^2*((cos
(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 1)
) + (A*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*B*c*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e)) + 3*A*c*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*p
i + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 9*B*c*(cos(-1
/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/
4*pi + 1/2*f*x + 1/2*e) + 1) - 3*B*c*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^
2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)
^2)/(a^2*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/
2*e) + 1) + 1)^3))/f
```

Mupad [B] (verification not implemented)

Time = 40.64 (sec) , antiderivative size = 492, normalized size of antiderivative = 4.28

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))
^2,x)
```

output
```
(exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*
1i)*1i)/2))^(1/2)*((2*B*c)/(3*a^2*f) - (c*(2*A - 3*B))/(3*a^2*f) - (2*c*(3
*A - 2*B))/(3*a^2*f) + (c*(A*2i - B*3i)*1i)/(3*a^2*f) + (c*(A*3i - B*2i)*2
i)/(3*a^2*f)))/((exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^3) - (
(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((
2*B*c)/(a^2*f) - (B*c*exp(e*1i + f*x*1i)*2i)/(a^2*f)))/(exp(e*1i + f*x*1i)
 - 1i) - (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*
1i + f*x*1i)*1i)/2))^(1/2)*((c*(A - B)*4i)/(a^2*f) + (c*(A*1i - B*2i))/(a^
2*f) + (c*(A*1i + B*2i))/(3*a^2*f)))/((exp(e*1i + f*x*1i) - 1i)*(exp(e*1i
+ f*x*1i) + 1i)^2) - (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)
/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((4*B*c)/(a^2*f) + (c*(A*1i - B*2i)
*4i)/(a^2*f)))/((exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i))
```

Reduce [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^2}\,dx = \frac{\sqrt{c}\,c\Big(\Big(\int \frac{\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)^2+2\sin(fx+e)+1}dx\Big)a-\Big(\int \frac{\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)^2+2\sin}}{}$$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x)
```

output
```
(sqrt(c)*c*(int(sqrt( - sin(e + f*x) + 1)/(sin(e + f*x)**2 + 2*sin(e + f*x
) + 1),x)*a - int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x
)**2 + 2*sin(e + f*x) + 1),x)*b - int((sqrt( - sin(e + f*x) + 1)*sin(e + f
*x))/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),x)*a + int((sqrt( - sin(e + f*
x) + 1)*sin(e + f*x))/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),x)*b))/a**2
```

3.119 $\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 78

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx = -\frac{(A+5B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2f} - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2c^2f}$$

output
```
-1/3*(A+5*B)*sec(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f-1/3*(A-B)*sec(f*x+e)^
3*(c-c*sin(f*x+e))^(5/2)/a^2/c^2/f
```

Mathematica [A] (verified)

Time = 2.86 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.12

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx = -\frac{2(A+2B+3B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{3a^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

input
```
Integrate[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f
*x])^2,x]
```

output
```
(-2*(A + 2*B + 3*B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(3*a^2*f*(Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 3446, 3042, 3334, 3042, 3152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-c\sin(e+fx)}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{c-c\sin(e+fx)}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3446}$$

$$\frac{\int \sec^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}dx}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{\cos(e+fx)^4}dx}{a^2c^2}$$

$$\Big\downarrow \text{3334}$$

$$\frac{\frac{1}{6}c(A+5B)\int \sec^2(e+fx)(c-c\sin(e+fx))^{3/2}dx-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{3f}}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{6}c(A+5B)\int \frac{(c-c\sin(e+fx))^{3/2}}{\cos(e+fx)^2}dx-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{3f}}{a^2c^2}$$

$$\Big\downarrow \text{ 3152}$$

$$\frac{-\frac{c^2(A+5B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{3f}-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{3f}}{a^2c^2}$$

input
```
Int[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^2
,x]
```

output
```
(-1/3*((A + 5*B)*c^2*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/f - ((A - B)*S
ec[e + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(3*f))/(a^2*c^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{2c(\sin(fx+e)-1)(3B\sin(fx+e)+A+2B)}{3a^2(1+\sin(fx+e))\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$	63

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x,method=_R
ETURNVERBOSE)
```

output
```
2/3*c/a^2*(sin(f*x+e)-1)/(1+sin(f*x+e))*(3*B*sin(f*x+e)+A+2*B)/cos(f*x+e)/
(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.77

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx$$
$$= -\frac{2\,(3\,B\sin\left(fx+e\right)+A+2\,B)\sqrt{-c\sin\left(fx+e\right)+c}}{3\,(a^2f\cos\left(fx+e\right)\sin\left(fx+e\right)+a^2f\cos\left(fx+e\right))}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x, al
gorithm="fricas")
```

output
```
-2/3*(3*B*sin(f*x + e) + A + 2*B)*sqrt(-c*sin(f*x + e) + c)/(a^2*f*cos(f*x
 + e)*sin(f*x + e) + a^2*f*cos(f*x + e))
```

Sympy [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx = \frac{\int \frac{A\sqrt{-c\sin{(e+fx)}+c}}{\sin^2{(e+fx)}+2\sin{(e+fx)}+1}\,dx + \int \frac{B\sqrt{-c\sin{(e+fx)}+c}\sin{(e+fx)}}{\sin^2{(e+fx)}+2\sin{(e+fx)}+1}\,dx}{a^2}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**2,x)
```

output
```
(Integral(A*sqrt(-c*sin(e + f*x) + c)/(sin(e + f*x)**2 + 2*sin(e + f*x) +
1), x) + Integral(B*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)/(sin(e + f*x)**
2 + 2*sin(e + f*x) + 1), x))/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(70) = 140$.

Time = 0.15 (sec) , antiderivative size = 343, normalized size of antiderivative = 4.40

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx = \frac{2\left(\frac{2B\left(\sqrt{c}+\frac{3\sqrt{c}\sin(fx+e)}{\cos(fx+e)+1}+\frac{2\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\sqrt{c}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{\sqrt{c}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}\right)}{\left(a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)\sqrt{\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1}} + \frac{A\left(\sqrt{c}+\frac{2\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{\sqrt{c}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}\cdots\right.}{\left(a^2+\frac{3a^2\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)\cdots}\cdots\right.}{3f}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x, al
gorithm="maxima")
```

output
```
2/3*(2*B*(sqrt(c) + 3*sqrt(c)*sin(f*x + e)/(cos(f*x + e) + 1) + 2*sqrt(c)*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sqrt(c)*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + sqrt(c)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)/((a^2 + 3*a^2*sin
(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*sqrt(sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 + 1)) + A*(sqrt(c) + 2*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + sqrt(c)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)/((a^2 + 3*a^2*sin(f*x + e
)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin
(f*x + e)^3/(cos(f*x + e) + 1)^3)*sqrt(sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 1)))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(70) = 140$.

Time = 0.24 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.86

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx = \frac{\sqrt{2}\left(A\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+5B\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+\frac{12B\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)\mathrm{sgn}(\sin}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}\right.}{3a^2f\left(\frac{\cos(-}{\cos(-}\right.}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x, al
gorithm="giac")
```

output
```
1/3*sqrt(2)*(A*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*B*sgn(sin(-1/4*pi +
 1/2*f*x + 1/2*e)) + 12*B*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 3*A*(cos(-
1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(
-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 3*B*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) +
 1)^2)*sqrt(c)/(a^2*f*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi +
 1/2*f*x + 1/2*e) + 1) + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 49.24 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.76

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx = \frac{4\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,\sqrt{c-c\,\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\,\left(B\,3\mathrm{i}+2\,A\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+4\,B\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-B\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,3\mathrm{i}\right)}{3\,a^2\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\mathrm{i}\right)^3\,\left(1+\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}\right)}$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))
^2,x)
```

output
```
(4*exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*
x*1i)*1i)/2))^(1/2)*(B*3i + 2*A*exp(e*1i + f*x*1i) + 4*B*exp(e*1i + f*x*1i
) - B*exp(e*2i + f*x*2i)*3i))/(3*a^2*f*(exp(e*1i + f*x*1i) + 1i)^3*(exp(e*
1i + f*x*1i)*1i + 1))
```

Reduce [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^2}\,dx = \frac{\sqrt{c}\left(\left(\int \frac{\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)^2+2\sin(fx+e)+1}dx\right)a+\left(\int \frac{\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)^2+2\sin(fx+e)+1}dx\right)b\right)}{a^2}$$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x)
```

output
```
(sqrt(c)*(int(sqrt( - sin(e + f*x) + 1)/(sin(e + f*x)**2 + 2*sin(e + f*x)
+ 1),x)*a + int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**2
+ 2*sin(e + f*x) + 1),x)*b))/a**2
```

3.120 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 135

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}\,dx &= \frac{(A+B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}a^2\sqrt{c}f} - \frac{(A+B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2cf} \\ &\quad - \frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3a^2c^2f}\end{aligned}$$

output
```
1/4*(A+B)*arctanh(1/2*c^(1/2)*cos(f*x+e)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))*2
^(1/2)/a^2/c^(1/2)/f-1/2*(A+B)*sec(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/c/f-1
/3*(A-B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^2/c^2/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.66 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.30

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2(-A+B)-3(A+B)\left(\cos\right.\right.}{6a}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*Sqrt[c - c*Sin[e +
f*x]]),x]
```

output
```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])*(2*(-A + B) - 3*(A + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (3 +
3*I)*(-1)^(1/4)*(A + B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4
])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3))/(6*a^2*f*(1 + Sin[e + f*x])^
2*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$, Rules used = {3042, 3446, 3042, 3334, 3042, 3154, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^2\sqrt{c-c\sin(e+fx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^2\sqrt{c-c\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3446}$$

$$\frac{\int \sec^4(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}dx}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{\cos(e+fx)^4}dx}{a^2c^2}$$

$$\Big\downarrow \text{3334}$$

$$\frac{\frac{1}{2}c(A+B)\int \sec^2(e+fx)\sqrt{c-c\sin(e+fx)}dx-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3f}}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{2}c(A+B)\int \frac{\sqrt{c-c\sin(e+fx)}}{\cos(e+fx)^2}dx-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3f}}{a^2c^2}$$

$$\Big\downarrow \text{3154}$$

$$\frac{\frac{1}{2}c(A+B)\left(\frac{1}{2}c\int \frac{1}{\sqrt{c-c\sin(e+fx)}}dx-\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{f}\right)-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3f}}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{2}c(A+B)\left(\frac{1}{2}c\int \frac{1}{\sqrt{c-c\sin(e+fx)}}dx-\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{f}\right)-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3f}}{a^2c^2}$$

$$\Big\downarrow \text{3128}$$

$$\frac{\frac{1}{2}c(A+B)\left(-\frac{c\int \frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{f}-\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{f}\right)-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3f}}{a^2c^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{2}c(A+B)\left(\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}f}-\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{f}\right)-\frac{(A-B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3f}}{a^2c^2}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]])
,x]
```

output
```
(-1/3*((A - B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/f + ((A + B)*c*(
(Sqrt[c]*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])
])/(Sqrt[2]*f) - (Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/f))/2)/(a^2*c^2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3154
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^m/(a*f*g*(p + 1))), x] + Simp[a*((m + p + 1)/(g^2*(p + 1)))   Int[(g*
Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b,
e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ
[m + 1/2, 2*p]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.24

method	result
default	$-\frac{(\sin(fx+e)-1)\left(3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)(c(1+\sin(fx+e)))^{\frac{3}{2}}cA-6A\,c^{\frac{5}{2}}\sin(fx+e)+3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}}{2\sqrt{c}}\right.}{12a^{2}c^{\frac{5}{2}}(1+\sin(fx+e))\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/12*(sin(f*x+e)-1)*(3*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/
2)/c^(1/2))*(c*(1+sin(f*x+e)))^(3/2)*c*A-6*A*c^(5/2)*sin(f*x+e)+3*2^(1/2)*
arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*(c*(1+sin(f*x+e)))^(
3/2)*c*B-6*B*c^(5/2)*sin(f*x+e)-10*A*c^(5/2)-2*B*c^(5/2))/a^2/c^(5/2)/(1+s
in(f*x+e))/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.61

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}\,dx = \frac{3\sqrt{2}((A+B)\cos(fx+e)\sin(fx+e)+(A+B)\cos(fx+e))\sqrt{c}\log\left(-\frac{c\cos(fx+e)^2+2\sqrt{2}\sqrt{-c\sin(fx+e)+c}\sqrt{}}{\cos(fx+}\right.}{24\left(a^2cf\cos(fx+\right.}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x, al
gorithm="fricas")
```

output
```
1/24*(3*sqrt(2)*((A + B)*cos(f*x + e)*sin(f*x + e) + (A + B)*cos(f*x + e))
*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt
(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e)
 - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x +
 e) - cos(f*x + e) - 2)) - 4*(3*(A + B)*sin(f*x + e) + 5*A + B)*sqrt(-c*si
n(f*x + e) + c))/(a^2*c*f*cos(f*x + e)*sin(f*x + e) + a^2*c*f*cos(f*x + e)
)
```

Sympy [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= \frac{\int \frac{A}{\sqrt{-c\sin(e+fx)+c}\sin^2(e+fx)+2\sqrt{-c\sin(e+fx)+c}\sin(e+fx)+\sqrt{-c\sin(e+fx)+c}}\,dx + \int \frac{B\sin(}{\sqrt{-c\sin(e+fx)+c}\sin^2(e+fx)+2\sqrt{-c\sin}}}{a^2}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(1/2),x)
```

output
```
(Integral(A/(sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2 + 2*sqrt(-c*sin(e +
 f*x) + c)*sin(e + f*x) + sqrt(-c*sin(e + f*x) + c)), x) + Integral(B*sin(
e + f*x)/(sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2 + 2*sqrt(-c*sin(e + f*
x) + c)*sin(e + f*x) + sqrt(-c*sin(e + f*x) + c)), x))/a**2
```

Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{B\sin(fx+e)+A}{(a\sin(fx+e)+a)^2\sqrt{-c\sin(fx+e)+c}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^2*sqrt(-c*sin(f*x + e
) + c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(1/2
)),x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(1/2
)), x)
```

Reduce [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{\sqrt{c}\left(\left(\int \frac{\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)^3+\sin(fx+e)^2-\sin(fx+e)-1}dx\right)a+\left(\int \frac{\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)^3+\sin(fx+e)^2-\sin(fx+e)-1}dx\right)b\right)}{a^2c}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x)
```

output
```
( - sqrt(c)*(int(sqrt( - sin(e + f*x) + 1)/(sin(e + f*x)**3 + sin(e + f*x)
**2 - sin(e + f*x) - 1),x)*a + int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)
)/(sin(e + f*x)**3 + sin(e + f*x)**2 - sin(e + f*x) - 1),x)*b))/(a**2*c)
```

3.121 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 175

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{3/2}}\,dx = \frac{(5A+B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{8\sqrt{2}a^2c^{3/2}f} + \frac{(5A+B)\cos(e+fx)}{8a^2f(c-c\sin(e+fx))^{3/2}} - \frac{(5A+B)\sec(e+fx)}{6a^2cf\sqrt{c-c\sin(e+fx)}} - \frac{(A-B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3a^2c^2f}$$

output
```
1/16*(5*A+B)*arctanh(1/2*c^(1/2)*cos(f*x+e)*2^(1/2)/(c-c*sin(f*x+e))^(1/2)
)*2^(1/2)/a^2/c^(3/2)/f+1/8*(5*A+B)*cos(f*x+e)/a^2/f/(c-c*sin(f*x+e))^(3/2
)-1/6*(5*A+B)*sec(f*x+e)/a^2/c/f/(c-c*sin(f*x+e))^(1/2)-1/3*(A-B)*sec(f*x+
e)^3*(c-c*sin(f*x+e))^(1/2)/a^2/c^2/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.89 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.71

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\right.}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]
)^(3/2)),x]
```

output
```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])*(-12*A*Cos[e + f*x]^2 + 4*(-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
)^2 + 3*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^3 - (3 + 3*I)*(-1)^(1/4)*(5*A + B)*ArcTan[(1/2 + I/2)*(-
1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 6*(A + B)*Sin[(e + f*x)/2]*(Cos[(
e + f*x)/2] + Sin[(e + f*x)/2])^3))/(24*a^2*f*(1 + Sin[e + f*x])^2*(c - c*
Sin[e + f*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.289, Rules used = {3042, 3446, 3042, 3334, 3042, 3166, 3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^2(c-c\sin(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^2(c-c\sin(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3446}$$

$$\frac{\int \sec^4(e+fx)(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}dx}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{\cos(e+fx)^4}dx}{a^2c^2}$$

$$\Big\downarrow \text{3334}$$

$$\frac{\frac{1}{6}c(5A+B)\int \frac{\sec^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}dx-\frac{(A-B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3f}}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{6}c(5A+B)\int \frac{1}{\cos(e+fx)^2\sqrt{c-c\sin(e+fx)}}dx-\frac{(A-B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3f}}{a^2c^2}$$

$$\Big\downarrow \text{3166}$$

$$\frac{\frac{1}{6}c(5A+B)\left(\frac{3}{2}c\int \frac{1}{(c-c\sin(e+fx))^{3/2}}dx-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{(A-B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3f}}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{6}c(5A+B)\left(\frac{3}{2}c\int \frac{1}{(c-c\sin(e+fx))^{3/2}}dx-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{(A-B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3f}}{a^2c^2}$$

$$\Big\downarrow \text{3129}$$

$$\frac{\frac{1}{6}c(5A+B)\left(\frac{3}{2}c\left(\frac{\int \frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{4c}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}\right)-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{(A-B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3f}}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{6}c(5A+B)\left(\frac{3}{2}c\left(\frac{\int \frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{4c}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}\right)-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{(A-B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3f}}{a^2c^2}$$

$$\Big\downarrow \text{3128}$$

$$\frac{\frac{1}{6}c(5A+B)\left(\frac{3}{2}c\left(\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}-\frac{\int \frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{2cf}\right)-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{(A-B)\sec^3(e+fx}{3j}}{a^2c^2}$$

$\downarrow$ 219

$$\frac{\frac{1}{6}c(5A+B)\left(\frac{3}{2}c\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}c^{3/2}f}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}\right)-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{(A-B)\sec^3(e+fx)\sqrt{c-c\sin}}{3f}}{a^2c^2}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(3/2
)),x]
```

output
```
(-1/3*((A - B)*Sec[e + f*x]^3*Sqrt[c - c*Sin[e + f*x]])/f + ((5*A + B)*c*(
-(Sec[e + f*x]/(f*Sqrt[c - c*Sin[e + f*x]])) + (3*c*(ArcTanh[(Sqrt[c]*Cos[
e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]/(2*Sqrt[2]*c^(3/2)*f) + Cos[
e + f*x]/(2*f*(c - c*Sin[e + f*x])^(3/2))))/2))/6)/(a^2*c^2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3166
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Simp[a*((2*p + 1)/(2*g^2*(p + 1)))   Int[(g
*Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e
, f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.60

method	result
default	$\frac{-15A(c+c\sin(fx+e))^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,\sin(fx+e)c-30c^{\frac{5}{2}}\cos(fx+e)^{2}A-3B(c+c\sin(fx+e))^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{c+c\ldots}}{\ldots}\right.}{\ldots}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x,method=_R
ETURNVERBOSE)
```

output
```
1/48*(-15*A*(c+c*sin(f*x+e))^(3/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1
/2)/c^(1/2))*2^(1/2)*sin(f*x+e)*c-30*c^(5/2)*cos(f*x+e)^2*A-3*B*(c+c*sin(f
*x+e))^(3/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*s
in(f*x+e)*c-6*c^(5/2)*cos(f*x+e)^2*B+15*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e
))^(1/2)*2^(1/2)/c^(1/2))*(c+c*sin(f*x+e))^(3/2)*c*A+20*A*c^(5/2)*sin(f*x+
e)+3*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*(c+c*sin(
f*x+e))^(3/2)*c*B+4*B*c^(5/2)*sin(f*x+e)+4*A*c^(5/2)+20*B*c^(5/2))/c^(7/2)
/a^2/(1+sin(f*x+e))/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.18

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{3/2}}\,dx = \frac{3\sqrt{2}(5A+B)\sqrt{c}\cos\left(fx+e\right)^3\log\left(-\frac{c\cos(fx+e)^2+2\sqrt{2}\sqrt{-}}{}\right.}{}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x, al
gorithm="fricas")
```

output
```
1/96*(3*sqrt(2)*(5*A + B)*sqrt(c)*cos(f*x + e)^3*log(-(c*cos(f*x + e)^2 +
2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) +
 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f
*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(3*(5
*A + B)*cos(f*x + e)^2 - 2*(5*A + B)*sin(f*x + e) - 2*A - 10*B)*sqrt(-c*si
n(f*x + e) + c))/(a^2*c^2*f*cos(f*x + e)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2(c - c\sin(e + fx))^{3/2}}\, dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x, al
gorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2(c - c\sin(e + fx))^{3/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{3/2}}\,dx=\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(3/2
)),x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(3/2
)), x)
```

Reduce [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{3/2}}\,dx=\frac{\sqrt{c}\left(\left(\int \frac{\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)^4-2\sin(fx+e)^2+1}dx\right)a+\left(\int \frac{\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)^4-2\sin}\right.\right.}{a^2c^2}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*(int(sqrt( - sin(e + f*x) + 1)/(sin(e + f*x)**4 - 2*sin(e + f*x)*
*2 + 1),x)*a + int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)*
*4 - 2*sin(e + f*x)**2 + 1),x)*b))/(a**2*c**2)
```

3.122 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 225

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}\,dx = \frac{5(7A-B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{64\sqrt{2}a^2c^{5/2}f} + \frac{5(7A-B)\cos(e+fx)}{64a^2cf(c-c\sin(e+fx))^{3/2}} + \frac{(7A-B)\sec(e+fx)}{24a^2cf(c-c\sin(e+fx))^{3/2}} - \frac{5(7A-B)\sec(e+fx)}{48a^2c^2f\sqrt{c-c\sin(e+fx)}} - \frac{(A-B)\sec^3(e+fx)}{3a^2c^2f\sqrt{c-c\sin(e+fx)}}$$

output

```
5/128*(7*A-B)*arctanh(1/2*c^(1/2)*cos(f*x+e)*2^(1/2)/(c-c*sin(f*x+e))^(1/2
))*2^(1/2)/a^2/c^(5/2)/f+5/64*(7*A-B)*cos(f*x+e)/a^2/c/f/(c-c*sin(f*x+e))^
(3/2)+1/24*(7*A-B)*sec(f*x+e)/a^2/c/f/(c-c*sin(f*x+e))^(3/2)-5/48*(7*A-B)*
sec(f*x+e)/a^2/c^2/f/(c-c*sin(f*x+e))^(1/2)-1/3*(A-B)*sec(f*x+e)^3/a^2/c^2
/f/(c-c*sin(f*x+e))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.20 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.91

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\right.}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x]
)^(5/2)),x]
```

output
```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])*(3*(11*A + 3*B)*Cos[e + f*x]^3 + 16*(-A + B)*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2])^4 + 24*(-3*A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 12*(A + B)*(Cos[(e + f*x)/2] - Sin[(
e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - (15 + 15*I)*(-1)^(1
/4)*(7*A - B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3
+ 24*(A + B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 6*
(11*A + 3*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2]*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^3))/(192*a^2*f*(1 + Sin[e + f*x])^2*(c -
 c*Sin[e + f*x])^(5/2))
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.90, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.342$, Rules used = {3042, 3446, 3042, 3334, 3042, 3160, 3042, 3166, 3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^2(c-c\sin(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^2(c-c\sin(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{ 3446}$$

$$\frac{\int \frac{\sec^4(e+fx)(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}dx}{a^2c^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{A+B\sin(e+fx)}{\cos(e+fx)^4\sqrt{c-c\sin(e+fx)}}dx}{a^2c^2}$$

$$\Big\downarrow \text{ 3334}$$

$$\frac{\frac{1}{6}c(7A-B)\int \frac{\sec^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}dx-\frac{(A-B)\sec^3(e+fx)}{3f\sqrt{c-c\sin(e+fx)}}}{a^2c^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{1}{6}c(7A-B)\int \frac{1}{\cos(e+fx)^2(c-c\sin(e+fx))^{3/2}}dx-\frac{(A-B)\sec^3(e+fx)}{3f\sqrt{c-c\sin(e+fx)}}}{a^2c^2}$$

$$\Big\downarrow \text{ 3160}$$

$$\frac{\frac{1}{6}c(7A-B)\left(\frac{5\int \frac{\sec^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}dx}{8c}+\frac{\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}\right)-\frac{(A-B)\sec^3(e+fx)}{3f\sqrt{c-c\sin(e+fx)}}}{a^2c^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{1}{6}c(7A-B)\left(\frac{5\int \frac{1}{\cos(e+fx)^2\sqrt{c-c\sin(e+fx)}}dx}{8c}+\frac{\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}\right)-\frac{(A-B)\sec^3(e+fx)}{3f\sqrt{c-c\sin(e+fx)}}}{a^2c^2}$$

$$\Big\downarrow \text{ 3166}$$

$$\frac{\frac{1}{6}c(7A-B)\left(\frac{5\left(\frac{3}{2}c\int \frac{1}{(c-c\sin(e+fx))^{3/2}}dx-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)}{8c}+\frac{\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}\right)-\frac{(A-B)\sec^3(e+fx)}{3f\sqrt{c-c\sin(e+fx)}}}{a^2c^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{1}{6}c(7A-B)\left(\frac{5\left(\frac{3}{2}c\int \frac{1}{(c-c\sin(e+fx))^{3/2}}dx-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)}{8c}+\frac{\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}\right)-\frac{(A-B)\sec^3(e+fx)}{3f\sqrt{c-c\sin(e+fx)}}}{a^2c^2}$$

$$\Big\downarrow \text{ 3129}$$

$$\frac{\frac{1}{6}c(7A-B)\left(\frac{5\left(\frac{3}{2}c\left(\frac{\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{4c}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}\right)-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)}{8c}+\frac{\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}\right)-\frac{(A-B)\sec^3(e+fx)}{3f\sqrt{c-c\sin(e+fx)}}}{a^2c^2}$$

$\downarrow$ 3042

$$\frac{\frac{1}{6}c(7A-B)\left(\frac{5\left(\frac{3}{2}c\left(\frac{\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{4c}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}\right)-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)}{8c}+\frac{\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}\right)-\frac{(A-B)\sec^3(e+fx)}{3f\sqrt{c-c\sin(e+fx)}}}{a^2c^2}$$

$\downarrow$ 3128

$$\frac{\frac{1}{6}c(7A-B)\left(\frac{5\left(\frac{3}{2}c\left(\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}-\frac{\int\frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{2cf}\right)-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)}{8c}+\frac{\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}\right)-}{a^2c^2}$$

$\downarrow$ 219

$$\frac{\frac{1}{6}c(7A-B)\left(\frac{5\left(\frac{3}{2}c\left(\frac{\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}c^{3/2}f}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}\right)-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)}{8c}+\frac{\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}\right)-\frac{(A-B)}{3f\sqrt{c-}}}{a^2c^2}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2
)),x]
```

output
```
(-1/3*((A - B)*Sec[e + f*x]^3)/(f*Sqrt[c - c*Sin[e + f*x]]) + ((7*A - B)*c
*(Sec[e + f*x]/(4*f*(c - c*Sin[e + f*x])^(3/2)) + (5*(-(Sec[e + f*x]/(f*Sq
rt[c - c*Sin[e + f*x]])) + (3*c*(ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*S
qrt[c - c*Sin[e + f*x]])]/(2*Sqrt[2]*c^(3/2)*f) + Cos[e + f*x]/(2*f*(c - c
*Sin[e + f*x])^(3/2))))/2))/(8*c)))/6)/(a^2*c^2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3160
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*(2*m + p + 1))), x] + Simp[(m + p + 1)/(a*(2*m + p + 1))   Int[
(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e,
f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &
& IntegersQ[2*m, 2*p]
```

rule 3166
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Simp[a*((2*p + 1)/(2*g^2*(p + 1)))   Int[(g
*Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e
, f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 425 vs. $2(198) = 396$.

Time = 0.37 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.89

method	result
default	$-\frac{-86A\,c^{\frac{7}{2}}+122B\,c^{\frac{7}{2}}+105A(c(1+\sin(fx+e)))^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,c^{2}-15B(c(1+\sin(fx+e)))^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin}}{}\right.}{}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/384/c^(11/2)/a^2*(-86*A*c^(7/2)+122*B*c^(7/2)+105*A*(c*(1+sin(f*x+e)))^
(3/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^2-15
*B*(c*(1+sin(f*x+e)))^(3/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c
^(1/2))*2^(1/2)*c^2+322*A*sin(f*x+e)*c^(7/2)-46*B*sin(f*x+e)*c^(7/2)-210*A
*sin(f*x+e)^3*c^(7/2)+30*B*sin(f*x+e)^3*c^(7/2)+105*A*sin(f*x+e)^2*arctanh
(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*(c*(1+sin(f*x+e)))^(3/2)*2^
(1/2)*c^2-15*B*sin(f*x+e)^2*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c
^(1/2))*(c*(1+sin(f*x+e)))^(3/2)*2^(1/2)*c^2-210*A*sin(f*x+e)*arctanh(1/2*
(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*(c*(1+sin(f*x+e)))^(3/2)*2^(1/2)
*c^2+30*B*sin(f*x+e)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))
*(c*(1+sin(f*x+e)))^(3/2)*2^(1/2)*c^2+70*A*sin(f*x+e)^2*c^(7/2)-10*B*sin(f
*x+e)^2*c^(7/2))/(1+sin(f*x+e))/(sin(f*x+e)-1)/cos(f*x+e)/(c-c*sin(f*x+e))
^(1/2)/f
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.24

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2(c - c\sin(e + fx))^{5/2}}\,dx = -\frac{15\sqrt{2}\left((7A - B)\cos(fx + e)^3\sin(fx + e) - (7A - B)\cos(fx + e)^3\right)\sqrt{c}\log\left(-\frac{c\cos(fx+e)^2 - 2\sqrt{2}\sqrt{-c\sin(fx}}{}\right.}{}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x, al
gorithm="fricas")
```

output
```
-1/768*(15*sqrt(2)*((7*A - B)*cos(f*x + e)^3*sin(f*x + e) - (7*A - B)*cos(
f*x + e)^3)*sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(-c*sin(f*x + e
) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*c
os(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2
)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(5*(7*A - B)*cos(f*x + e)^2 - (15*
(7*A - B)*cos(f*x + e)^2 + 56*A - 8*B)*sin(f*x + e) + 8*A - 56*B)*sqrt(-c*
sin(f*x + e) + c))/(a^2*c^3*f*cos(f*x + e)^3*sin(f*x + e) - a^2*c^3*f*cos(
f*x + e)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x, al
gorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}\,dx=\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(5/2
)),x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(5/2
)), x)
```

Reduce [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c-c\sin(e+fx))^{5/2}}\,dx=$$
$$-\frac{\sqrt{c}\left(\left(\int \frac{\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)^5-\sin(fx+e)^4-2\sin(fx+e)^3+2\sin(fx+e)^2+\sin(fx+e)-1}dx\right)a+\left(\int \frac{\sqrt{-\sin(fx+e)+1}\,\sin(f}{\sin(fx+e)^5-\sin(fx+e)^4-2\sin(fx+e)^3+2\sin}\right.\right.}{a^2c^3}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2),x)
```

output
```
( - sqrt(c)*(int(sqrt( - sin(e + f*x) + 1)/(sin(e + f*x)**5 - sin(e + f*x)
**4 - 2*sin(e + f*x)**3 + 2*sin(e + f*x)**2 + sin(e + f*x) - 1),x)*a + int
((sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**5 - sin(e + f*x)*
*4 - 2*sin(e + f*x)**3 + 2*sin(e + f*x)**2 + sin(e + f*x) - 1),x)*b))/(a**
2*c**3)
```

3.123 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 242

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^3}\,dx = -\frac{2048(A-3B)c^3\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f} + \frac{512(A-3B)c^2\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3f} - \frac{64(A-3B)c\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3f} - \frac{16(A-3B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{15a^3f} - \frac{(A-3B)\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{5a^3cf} - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{15/2}}{5a^3c^3f}$$

output

```
-2048/15*(A-3*B)*c^3*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^3/f+512/5*(A-3*
B)*c^2*sec(f*x+e)^3*(c-c*sin(f*x+e))^(5/2)/a^3/f-64/5*(A-3*B)*c*sec(f*x+e)
^3*(c-c*sin(f*x+e))^(7/2)/a^3/f-16/15*(A-3*B)*sec(f*x+e)^3*(c-c*sin(f*x+e)
)^(9/2)/a^3/f-1/5*(A-3*B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(11/2)/a^3/c/f-1/5
*(A-B)*sec(f*x+e)^5*(c-c*sin(f*x+e))^(15/2)/a^3/c^3/f
```

Mathematica [A] (verified)

Time = 13.76 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.73

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^3}\,dx =$$
$$-\frac{c^4(-1+\sin(e+fx))^4\sqrt{c-c\sin(e+fx)}(11298A-33516B-40(137A-402B)\cos(2(e+fx))-10(A}{120a^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)\right.}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e +
 f*x])^3,x]
```

output
```
-1/120*(c^4*(-1 + Sin[e + f*x])^4*Sqrt[c - c*Sin[e + f*x]]*(11298*A - 3351
6*B - 40*(137*A - 402*B)*Cos[2*(e + f*x)] - 10*(A - 6*B)*Cos[4*(e + f*x)]
+ 15600*A*Sin[e + f*x] - 47430*B*Sin[e + f*x] - 400*A*Sin[3*(e + f*x)] + 1
335*B*Sin[3*(e + f*x)] - 3*B*Sin[5*(e + f*x)]))/(a^3*f*(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2])^9*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.91, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 3446, 3042, 3334, 3042, 3153, 3042, 3153, 3042, 3153, 3042, 3153, 3042, 3152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^{9/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^3}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(c-c\sin(e+fx))^{9/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^3}dx$$

$$\Big\downarrow \; 3446$$

$$\frac{\int \sec^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{15/2}dx}{a^3c^3}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{15/2}}{\cos(e+fx)^6}dx}{a^3c^3}$$

$\Big\downarrow$ 3334

$$\frac{-\frac{1}{2}c(A-3B)\int \sec^4(e+fx)(c-c\sin(e+fx))^{13/2}dx-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{15/2}}{5f}}{a^3c^3}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{1}{2}c(A-3B)\int \frac{(c-c\sin(e+fx))^{13/2}}{\cos(e+fx)^4}dx-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{15/2}}{5f}}{a^3c^3}$$

$\Big\downarrow$ 3153

$$\frac{-\frac{1}{2}c(A-3B)\left(\frac{16}{5}c\int \sec^4(e+fx)(c-c\sin(e+fx))^{11/2}dx+\frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{5f}\right)-\frac{(A-B)\sec^5(e+fx)(c-}{5f}}{a^3c^3}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{1}{2}c(A-3B)\left(\frac{16}{5}c\int \frac{(c-c\sin(e+fx))^{11/2}}{\cos(e+fx)^4}dx+\frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{5f}\right)-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{15/2}}{5f}}{a^3c^3}$$

$\Big\downarrow$ 3153

$$\frac{-\frac{1}{2}c(A-3B)\left(\frac{16}{5}c\left(4c\int \sec^4(e+fx)(c-c\sin(e+fx))^{9/2}dx+\frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3f}\right)+\frac{2c\sec^3(e+fx)(c-c\,\mathrm{si}}{5f}\right.}{a^3c^3}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{1}{2}c(A-3B)\left(\frac{16}{5}c\left(4c\int \frac{(c-c\sin(e+fx))^{9/2}}{\cos(e+fx)^4}dx+\frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3f}\right)+\frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{11/2}}{5f}\right)-(A-}{a^3c^3}$$

$\Big\downarrow$ 3153

$$\frac{-\frac{1}{2}c(A-3B)\left(\frac{16}{5}c\left(4c\left(8c\int \sec^4(e+fx)(c-c\sin(e+fx))^{7/2}dx+\frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{f}\right)+\frac{2c\sec^3(e+fx)(c}{3}\right.\right.}{a^3c^3}$$

$\Big\downarrow$ 3042

$$-\frac{1}{2}c(A-3B)\left(\frac{16}{5}c\left(4c\left(8c\int\frac{(c-c\sin(e+fx))^{7/2}}{\cos(e+fx)^4}dx+\frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{f}\right)+\frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3f}\right)+\right.$$
$$\overline{\qquad a^3c^3}$$

$\big\downarrow$ 3153

$$-\frac{1}{2}c(A-3B)\left(\frac{16}{5}c\left(4c\left(8c\left(-4c\int\sec^4(e+fx)(c-c\sin(e+fx))^{5/2}dx-\frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{f}\right)+\frac{2c\sec^3(e\right.\right.\right.$$

$\big\downarrow$ 3042

$$-\frac{1}{2}c(A-3B)\left(\frac{16}{5}c\left(4c\left(8c\left(-4c\int\frac{(c-c\sin(e+fx))^{5/2}}{\cos(e+fx)^4}dx-\frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{f}\right)+\frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{7/}}{f}\right.\right.\right.$$
$$\overline{\qquad a^3c^3}$$

$\big\downarrow$ 3152

$$-\frac{1}{2}c(A-3B)\left(\frac{16}{5}c\left(4c\left(8c\left(\frac{8c^2\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3f}-\frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{f}\right)+\frac{2c\sec^3(e+fx)(c-c\sin(e+fx}{f}\right.\right.\right.$$
$$\overline{\qquad a^3c^3}$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e + f*x])
^3,x]
```

output
```
(-1/5*((A - B)*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(15/2))/f - ((A - 3*B)*
c*((2*c*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(11/2))/(5*f) + (16*c*((2*c*Se
c[e + f*x]^3*(c - c*Sin[e + f*x])^(9/2))/(3*f) + 4*c*((2*c*Sec[e + f*x]^3*
(c - c*Sin[e + f*x])^(7/2))/f + 8*c*((8*c^2*Sec[e + f*x]^3*(c - c*Sin[e +
f*x])^(3/2))/(3*f) - (2*c*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/f))))
/5))/2)/(a^3*c^3)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3153
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] &&
NeQ[m + p, 0]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 126.20 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.59

method	result
default	$-\frac{2c^5(\sin(fx+e)-1)\left(3B\sin(fx+e)\cos(fx+e)^4+(5A-30B)\cos(fx+e)^4+(100A-336B)\cos(fx+e)^2\sin(fx+e)+(680A-1980B)\right.}{15a^3(1+\sin(fx+e))^2\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^3,x,method=_R
ETURNVERBOSE)
```

output
```
-2/15*c^5/a^3*(sin(f*x+e)-1)/(1+sin(f*x+e))^2*(3*B*sin(f*x+e)*cos(f*x+e)^4
+(5*A-30*B)*cos(f*x+e)^4+(100*A-336*B)*cos(f*x+e)^2*sin(f*x+e)+(680*A-1980
*B)*cos(f*x+e)^2+(-1000*A+3048*B)*sin(f*x+e)-1048*A+3096*B)/cos(f*x+e)/(c-
c*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.69

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^3}\,dx =$$
$$-\frac{2\left(5\,(A-6\,B)c^4\cos\left(fx+e\right)^4+20\,(34\,A-99\,B)c^4\cos\left(fx+e\right)^2-8\,(131\,A-387\,B)c^4+\left(3\,Bc^4\cos\left(f\right.\right.}{15\left(a^3f\cos\left(fx+e\right)^3-2\,a^3f\cos\left(fx+\right.\right.}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^3,x, al
gorithm="fricas")
```

output
```
-2/15*(5*(A - 6*B)*c^4*cos(f*x + e)^4 + 20*(34*A - 99*B)*c^4*cos(f*x + e)^
2 - 8*(131*A - 387*B)*c^4 + (3*B*c^4*cos(f*x + e)^4 + 4*(25*A - 84*B)*c^4*
cos(f*x + e)^2 - 8*(125*A - 381*B)*c^4)*sin(f*x + e))*sqrt(-c*sin(f*x + e)
 + c)/(a^3*f*cos(f*x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*f*
cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(9/2)/(a+a*sin(f*x+e))**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 945 vs. 2(218) = 436.

Time = 0.15 (sec) , antiderivative size = 945, normalized size of antiderivative = 3.90

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^3,x, al
gorithm="maxima")
```

output
```
2/15*((363*c^(9/2) + 1800*c^(9/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 5301*c
^(9/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 11600*c^(9/2)*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 21343*c^(9/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 +
 30200*c^(9/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 40065*c^(9/2)*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 + 40800*c^(9/2)*sin(f*x + e)^7/(cos(f*x + e)
+ 1)^7 + 40065*c^(9/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 30200*c^(9/2)
*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 21343*c^(9/2)*sin(f*x + e)^10/(cos(
f*x + e) + 1)^10 + 11600*c^(9/2)*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 5
301*c^(9/2)*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 + 1800*c^(9/2)*sin(f*x +
 e)^13/(cos(f*x + e) + 1)^13 + 363*c^(9/2)*sin(f*x + e)^14/(cos(f*x + e) +
 1)^14)*A/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x +
e) + 1)^5)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(9/2)) - 6*(181*c^(9/
2) + 905*c^(9/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 2627*c^(9/2)*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 5870*c^(9/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)
^3 + 10521*c^(9/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 15351*c^(9/2)*sin
(f*x + e)^5/(cos(f*x + e) + 1)^5 + 19695*c^(9/2)*sin(f*x + e)^6/(cos(f*x +
 e) + 1)^6 + 20772*c^(9/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 19695*c^(
9/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 15351*c^(9/2)*sin(f*x + e)^9...
```

Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.63

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^3,x, al
gorithm="giac")
```

output
```
-1024/15*sqrt(2)*(A*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*B*c^4*sgn(
sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 5*A*c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e)
- 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e)
 + 1)^2 + 15*B*c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 10*A*c^4*(cos
(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(co
s(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^4 - 30*B*c^4*(cos(-1/4*pi + 1/2*f*x + 1/
2*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1
/2*e) + 1)^4 - 6*A*c^4*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^5*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^5 - 6*B*c^4*(
cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^5*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/
(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^5)*sqrt(c)/(a^3*f*((cos(-1/4*pi + 1/2
*f*x + 1/2*e) - 1)^2/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 1)^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^3}\,dx=\int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{9/2}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^3}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x))
^3,x)
```

output
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x))
^3, x)
```

Reduce [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^3}\,dx=\frac{\sqrt{c}\,c^4\left(\left(\int \frac{\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)^3+3\sin(fx+e)^2+3\sin(fx+e)+1}dx\right)a+\left(\int \frac{}{\mathrm{s}}\right.\right.}{}$$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^3,x)
```

output

```
(sqrt(c)*c**4*(int(sqrt( - sin(e + f*x) + 1)/(sin(e + f*x)**3 + 3*sin(e +
f*x)**2 + 3*sin(e + f*x) + 1),x)*a + int((sqrt( - sin(e + f*x) + 1)*sin(e
+ f*x)**5)/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x) + 1),x)*b
 + int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**4)/(sin(e + f*x)**3 + 3*si
n(e + f*x)**2 + 3*sin(e + f*x) + 1),x)*a - 4*int((sqrt( - sin(e + f*x) + 1
)*sin(e + f*x)**4)/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x) +
 1),x)*b - 4*int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**3)/(sin(e + f*x)
**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x) + 1),x)*a + 6*int((sqrt( - sin(e
+ f*x) + 1)*sin(e + f*x)**3)/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(
e + f*x) + 1),x)*b + 6*int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(si
n(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x) + 1),x)*a - 4*int((sqrt
( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x)**3 + 3*sin(e + f*x)**
2 + 3*sin(e + f*x) + 1),x)*b - 4*int((sqrt( - sin(e + f*x) + 1)*sin(e + f*
x))/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x) + 1),x)*a + int(
(sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**3 + 3*sin(e + f*x)
**2 + 3*sin(e + f*x) + 1),x)*b))/a**3
```

3.124 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 209

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^3}\,dx = -\frac{128(3A-13B)c^2\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f} + \frac{32(3A-13B)c\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3f} - \frac{4(3A-13B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3f} - \frac{(3A-13B)\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{15a^3cf} - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{13/2}}{5a^3c^3f}$$

output
```
-128/15*(3*A-13*B)*c^2*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^3/f+32/5*(3*A
-13*B)*c*sec(f*x+e)^3*(c-c*sin(f*x+e))^(5/2)/a^3/f-4/5*(3*A-13*B)*sec(f*x+
e)^3*(c-c*sin(f*x+e))^(7/2)/a^3/f-1/15*(3*A-13*B)*sec(f*x+e)^3*(c-c*sin(f*
x+e))^(9/2)/a^3/c/f-1/5*(A-B)*sec(f*x+e)^5*(c-c*sin(f*x+e))^(13/2)/a^3/c^3
/f
```

Mathematica [A] (verified)

Time = 12.18 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.76

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^3}\,dx =$$
$$-\frac{c^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}(1092A-4557B+(-540A+2200B)\cos(2(e}{60a^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)\right.}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e +
 f*x])^3,x]
```

output
```
-1/60*(c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*
(1092*A - 4557*B + (-540*A + 2200*B)*Cos[2*(e + f*x)] + 5*B*Cos[4*(e + f*x
)] + 1410*A*Sin[e + f*x] - 6390*B*Sin[e + f*x] - 30*A*Sin[3*(e + f*x)] + 1
70*B*Sin[3*(e + f*x)]))/(a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 +
Sin[e + f*x])^3)
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.89, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 3446, 3042, 3334, 3042, 3153, 3042, 3153, 3042, 3153, 3042, 3152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^3}dx$$
$$\Big\downarrow \text{3446}$$
$$\frac{\int \sec^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{13/2}dx}{a^3c^3}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{13/2}}{\cos(e+fx)^6}dx}{a^3c^3}$$

$$\Big\downarrow \textbf{3334}$$

$$\frac{-\frac{1}{10}c(3A-13B)\int \sec^4(e+fx)(c-c\sin(e+fx))^{11/2}dx-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{13/2}}{5f}}{a^3c^3}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{-\frac{1}{10}c(3A-13B)\int \frac{(c-c\sin(e+fx))^{11/2}}{\cos(e+fx)^4}dx-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{13/2}}{5f}}{a^3c^3}$$

$$\Big\downarrow \textbf{3153}$$

$$\frac{-\frac{1}{10}c(3A-13B)\left(4c\int \sec^4(e+fx)(c-c\sin(e+fx))^{9/2}dx+\frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3f}\right)-\frac{(A-B)\sec^5(e+fx)(c-}{5f}}{a^3c^3}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{-\frac{1}{10}c(3A-13B)\left(4c\int \frac{(c-c\sin(e+fx))^{9/2}}{\cos(e+fx)^4}dx+\frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3f}\right)-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{13/2}}{5f}}{a^3c^3}$$

$$\Big\downarrow \textbf{3153}$$

$$\frac{-\frac{1}{10}c(3A-13B)\left(4c\left(8c\int \sec^4(e+fx)(c-c\sin(e+fx))^{7/2}dx+\frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{f}\right)+\frac{2c\sec^3(e+fx)(c-}{3f}}{a^3c^3}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{-\frac{1}{10}c(3A-13B)\left(4c\left(8c\int \frac{(c-c\sin(e+fx))^{7/2}}{\cos(e+fx)^4}dx+\frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{f}\right)+\frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{9/2}}{3f}\right)-\frac{(A}{}}{a^3c^3}$$

$$\Big\downarrow \textbf{3153}$$

$$\frac{-\frac{1}{10}c(3A-13B)\left(4c\left(8c\left(-4c\int \sec^4(e+fx)(c-c\sin(e+fx))^{5/2}dx-\frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{f}\right)+\frac{2c\sec^3(e+}{}}{a^3c^3}$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{\frac{1}{10}c(3A-13B)\left(4c\left(8c\left(-4c\int\frac{(c-c\sin(e+fx))^{5/2}}{\cos(e+fx)^4}dx-\frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{f}\right)+\frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{f}\right.\right.}{a^3c^3}$$

$\downarrow$ 3152

$$-\frac{\frac{1}{10}c(3A-13B)\left(4c\left(8c\left(\frac{8c^2\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3f}-\frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{f}\right)+\frac{2c\sec^3(e+fx)(c-c\sin(e+fx))}{f}\right.\right.}{a^3c^3}$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])
^3,x]
```

output
```
(-1/5*((A - B)*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(13/2))/f - ((3*A - 13*
B)*c*((2*c*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(9/2))/(3*f) + 4*c*((2*c*Se
c[e + f*x]^3*(c - c*Sin[e + f*x])^(7/2))/f + 8*c*((8*c^2*Sec[e + f*x]^3*(c
 - c*Sin[e + f*x])^(3/2))/(3*f) - (2*c*Sec[e + f*x]^3*(c - c*Sin[e + f*x])
^(5/2))/f))))/10)/(a^3*c^3)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3153
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] &&
NeQ[m + p, 0]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 125.86 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.58

method	result
default	$\frac{2c^4(\sin(fx+e)-1)\left(5B\cos(fx+e)^4+(-15A+85B)\cos(fx+e)^2\sin(fx+e)+(-135A+545B)\cos(fx+e)^2+(180A-820B)\sin(fx+e\right)}{15a^3(1+\sin(fx+e))^2\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^3,x,method=_R
ETURNVERBOSE)
```

output
```
2/15*c^4/a^3*(sin(f*x+e)-1)/(1+sin(f*x+e))^2*(5*B*cos(f*x+e)^4+(-15*A+85*B
)*cos(f*x+e)^2*sin(f*x+e)+(-135*A+545*B)*cos(f*x+e)^2+(180*A-820*B)*sin(f*
x+e)+204*A-844*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.71

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^3}\,dx = \frac{2\,(5\,Bc^3\cos{(fx+e)}^4 - 5\,(27\,A-109\,B)c^3\cos{(fx+e}}{15\,(a^3f\,c}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^3,x, al
gorithm="fricas")
```

output
```
2/15*(5*B*c^3*cos(f*x + e)^4 - 5*(27*A - 109*B)*c^3*cos(f*x + e)^2 + 4*(51
*A - 211*B)*c^3 - 5*((3*A - 17*B)*c^3*cos(f*x + e)^2 - 4*(9*A - 41*B)*c^3)
*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(a^3*f*cos(f*x + e)^3 - 2*a^3*f*c
os(f*x + e)*sin(f*x + e) - 2*a^3*f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 854 vs. 2(189) = 378.

Time = 0.15 (sec) , antiderivative size = 854, normalized size of antiderivative = 4.09

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^3,x, al
gorithm="maxima")
```

output
```
2/15*(3*(23*c^(7/2) + 110*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 318*c^
(7/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 590*c^(7/2)*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 1065*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 122
0*c^(7/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1540*c^(7/2)*sin(f*x + e)^
6/(cos(f*x + e) + 1)^6 + 1220*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7
+ 1065*c^(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 590*c^(7/2)*sin(f*x +
 e)^9/(cos(f*x + e) + 1)^9 + 318*c^(7/2)*sin(f*x + e)^10/(cos(f*x + e) + 1
)^10 + 110*c^(7/2)*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 23*c^(7/2)*sin(
f*x + e)^12/(cos(f*x + e) + 1)^12)*A/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x +
 e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3
*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 1)^(7/2)) - 2*(147*c^(7/2) + 735*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1
) + 1992*c^(7/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4015*c^(7/2)*sin(f*
x + e)^3/(cos(f*x + e) + 1)^3 + 6605*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e)
+ 1)^4 + 8370*c^(7/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 9520*c^(7/2)*s
in(f*x + e)^6/(cos(f*x + e) + 1)^6 + 8370*c^(7/2)*sin(f*x + e)^7/(cos(f*x
+ e) + 1)^7 + 6605*c^(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 4015*c^(7
/2)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 1992*c^(7/2)*sin(f*x + e)^10/(co
s(f*x + e) + 1)^10 + 735*c^(7/2)*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 774 vs. $2(189) = 378$.

Time = 0.36 (sec) , antiderivative size = 774, normalized size of antiderivative = 3.70

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^{7/2}}{(a + a\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^3,x, al
gorithm="giac")
```

output
```
-4/15*sqrt(2)*sqrt(c)*(5*(3*A*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 19
*B*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 6*A*c^3*(cos(-1/4*pi + 1/2*f*
x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x
 + 1/2*e) + 1) + 42*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 3*A*c^3*(c
os(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(
cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 15*B*c^3*(cos(-1/4*pi + 1/2*f*x +
1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x +
 1/2*e) + 1)^2)/(a^3*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi +
1/2*f*x + 1/2*e) + 1) - 1)^3) - (33*A*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*
e)) - 113*B*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 150*A*c^3*(cos(-1/4*
pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*p
i + 1/2*f*x + 1/2*e) + 1) - 490*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)
*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)
+ 240*A*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f
*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 - 740*B*c^3*(cos(-1/4*
pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4
*pi + 1/2*f*x + 1/2*e) + 1)^2 + 90*A*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) -
 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e)
+ 1)^3 - 390*B*c^3*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^3}\,dx=\int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{7/2}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^3}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))
^3,x)
```

output
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))
^3, x)
```

Reduce [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^3}\,dx = \frac{\sqrt{c}\,c^3\left(\left(\int \frac{\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)^3+3\sin(fx+e)^2+3\sin(fx+e)+1}dx\right)a-\left(\int \frac{}{\mathrm{s}}\right.\right.}{}$$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^3,x)
```

output
```
(sqrt(c)*c**3*(int(sqrt( - sin(e + f*x) + 1)/(sin(e + f*x)**3 + 3*sin(e +
f*x)**2 + 3*sin(e + f*x) + 1),x)*a - int((sqrt( - sin(e + f*x) + 1)*sin(e
+ f*x)**4)/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x) + 1),x)*b
 - int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**3)/(sin(e + f*x)**3 + 3*si
n(e + f*x)**2 + 3*sin(e + f*x) + 1),x)*a + 3*int((sqrt( - sin(e + f*x) + 1
)*sin(e + f*x)**3)/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x) +
 1),x)*b + 3*int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x)
**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x) + 1),x)*a - 3*int((sqrt( - sin(e
+ f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(
e + f*x) + 1),x)*b - 3*int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e
 + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x) + 1),x)*a + int((sqrt( - s
in(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*si
n(e + f*x) + 1),x)*b))/a**3
```

3.125 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 160

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^3}\,dx = -\frac{32(A-11B)c\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f} + \frac{8(A-11B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3f} - \frac{(A-11B)\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3cf} - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{11/2}}{5a^3c^3f}$$

output
```
-32/15*(A-11*B)*c*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^3/f+8/5*(A-11*B)*s
ec(f*x+e)^3*(c-c*sin(f*x+e))^(5/2)/a^3/f-1/5*(A-11*B)*sec(f*x+e)^3*(c-c*si
n(f*x+e))^(7/2)/a^3/c/f-1/5*(A-B)*sec(f*x+e)^5*(c-c*sin(f*x+e))^(11/2)/a^3
/c^3/f
```

Mathematica [A] (verified)

Time = 10.33 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.82

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^3}\,dx =$$
$$-\frac{c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}(58A-488B-30(A-8B)\cos(2(e+fx))+}{30a^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+f}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e +
 f*x])^3,x]
```

output
```
-1/30*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*
(58*A - 488*B - 30*(A - 8*B)*Cos[2*(e + f*x)] + 5*(8*A - 133*B)*Sin[e + f*
x] + 15*B*Sin[3*(e + f*x)]))/(a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*
(1 + Sin[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 3446, 3042, 3334, 3042, 3153, 3042, 3153, 3042, 3152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^3}dx$$
$$\Big\downarrow \text{3446}$$
$$\frac{\int \sec^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{11/2}dx}{a^3c^3}$$
$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{11/2}}{\cos(e+fx)^6}dx}{a^3c^3}$$

$\downarrow$ 3334

$$\frac{-\frac{1}{10}c(A-11B)\int \sec^4(e+fx)(c-c\sin(e+fx))^{9/2}dx-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{11/2}}{5f}}{a^3c^3}$$

$\downarrow$ 3042

$$\frac{-\frac{1}{10}c(A-11B)\int \frac{(c-c\sin(e+fx))^{9/2}}{\cos(e+fx)^4}dx-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{11/2}}{5f}}{a^3c^3}$$

$\downarrow$ 3153

$$\frac{-\frac{1}{10}c(A-11B)\left(8c\int \sec^4(e+fx)(c-c\sin(e+fx))^{7/2}dx+\frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{f}\right)-\frac{(A-B)\sec^5(e+fx)(c-c}{5f}}{a^3c^3}$$

$\downarrow$ 3042

$$\frac{-\frac{1}{10}c(A-11B)\left(8c\int \frac{(c-c\sin(e+fx))^{7/2}}{\cos(e+fx)^4}dx+\frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{f}\right)-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{11/2}}{5f}}{a^3c^3}$$

$\downarrow$ 3153

$$\frac{-\frac{1}{10}c(A-11B)\left(8c\left(-4c\int \sec^4(e+fx)(c-c\sin(e+fx))^{5/2}dx-\frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{f}\right)+\frac{2c\sec^3(e+fx)(c-}{f}\right.}{a^3c^3}$$

$\downarrow$ 3042

$$\frac{-\frac{1}{10}c(A-11B)\left(8c\left(-4c\int \frac{(c-c\sin(e+fx))^{5/2}}{\cos(e+fx)^4}dx-\frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{f}\right)+\frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{f}\right)-}{a^3c^3}$$

$\downarrow$ 3152

$$\frac{-\frac{1}{10}c(A-11B)\left(8c\left(\frac{8c^2\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3f}-\frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{f}\right)+\frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{7/2}}{f}\right)}{a^3c^3}$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])
^3,x]
```

output
```
(-1/5*((A - B)*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(11/2))/f - ((A - 11*B)
*c*((2*c*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(7/2))/f + 8*c*((8*c^2*Sec[e
+ f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(3*f) - (2*c*Sec[e + f*x]^3*(c - c*Si
n[e + f*x])^(5/2))/f)))/10)/(a^3*c^3)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3153
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] &&
NeQ[m + p, 0]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 125.75 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.66

method	result	size
default	$\frac{2c^{3}(\sin(fx+e)-1)\left(15B\cos(fx+e)^{2}\sin(fx+e)+(-15A+120B)\cos(fx+e)^{2}+(10A-170B)\sin(fx+e)+22A-182B\right)}{15a^{3}(1+\sin(fx+e))^{2}\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$	105

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x,method=_R
ETURNVERBOSE)
```

output
```
2/15*c^3/a^3*(sin(f*x+e)-1)/(1+sin(f*x+e))^2*(15*B*cos(f*x+e)^2*sin(f*x+e)
+(-15*A+120*B)*cos(f*x+e)^2+(10*A-170*B)*sin(f*x+e)+22*A-182*B)/cos(f*x+e)
/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.78

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^3}\,dx = -\frac{2\left(15\,(A-8\,B)c^2\cos\left(fx+e\right)^2-2\,(11\,A-91\,B)c^2-5\left(3\,Bc^2\cos\left(fx+e\right)^2+2\,(A-17\,B)c^2\right)\sin\left(fx+\right.}{15\left(a^3f\cos\left(fx+e\right)^3-2\,a^3f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,a^3f\cos\left(fx+e\right)\right)}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x, al
gorithm="fricas")
```

output
```
-2/15*(15*(A - 8*B)*c^2*cos(f*x + e)^2 - 2*(11*A - 91*B)*c^2 - 5*(3*B*c^2*
cos(f*x + e)^2 + 2*(A - 17*B)*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)
/(a^3*f*cos(f*x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*f*cos(f
*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 761 vs. $2(144) = 288$.

Time = 0.15 (sec) , antiderivative size = 761, normalized size of antiderivative = 4.76

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x, al
gorithm="maxima")
```

output
```
2/15*((7*c^(5/2) + 20*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 95*c^(5/2)
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 80*c^(5/2)*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 250*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 120*c^(5/2
)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 250*c^(5/2)*sin(f*x + e)^6/(cos(f*
x + e) + 1)^6 + 80*c^(5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 95*c^(5/2
)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 20*c^(5/2)*sin(f*x + e)^9/(cos(f*x
 + e) + 1)^9 + 7*c^(5/2)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10)*A/((a^3 +
5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)*(sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(5/2)) - 2*(31*c^(5/2) + 155*c^(5/2)*si
n(f*x + e)/(cos(f*x + e) + 1) + 395*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + 680*c^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1030*c^(5/2)*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1050*c^(5/2)*sin(f*x + e)^5/(cos(f*x +
e) + 1)^5 + 1030*c^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 680*c^(5/2)
*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 395*c^(5/2)*sin(f*x + e)^8/(cos(f*x
 + e) + 1)^8 + 155*c^(5/2)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 31*c^(5/2
)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10)*B/((a^3 + 5*a^3*sin(f*x + e)/(cos
(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 449 vs. $2(144) = 288$.

Time = 0.31 (sec) , antiderivative size = 449, normalized size of antiderivative = 2.81

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^{5/2}}{(a + a\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x, al
gorithm="giac")
```

output
```
4/15*sqrt(2)*sqrt(c)*(15*B*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(a^3*((
cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) -
 1)) + (4*A*c^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 29*B*c^2*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e)) + 20*A*c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*s
gn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) -
130*B*c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x +
 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 40*A*c^2*(cos(-1/4*pi + 1/
2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1
/2*f*x + 1/2*e) + 1)^2 - 200*B*c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*
sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2
 - 90*B*c^2*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi + 1/2*f
*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 - 15*B*c^2*(cos(-1/4*p
i + 1/2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*
pi + 1/2*f*x + 1/2*e) + 1)^4)/(a^3*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(
cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 1)^5))/f
```

Mupad [B] (verification not implemented)

Time = 53.52 (sec) , antiderivative size = 904, normalized size of antiderivative = 5.65

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))
^3,x)
```

output
```
((c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(
(2*B*c^2)/(a^3*f) - (B*c^2*exp(e*1i + f*x*1i)*2i)/(a^3*f)))/(exp(e*1i + f*
x*1i) - 1i) - (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (e
xp(e*1i + f*x*1i)*1i)/2))^(1/2)*((c^2*(A*2i - B*7i)*1i)/(3*a^3*f) - (2*c^2
*(7*A - 12*B))/(3*a^3*f) + (c^2*(A*23i - B*28i)*2i)/(3*a^3*f) - (c^2*(42*A
 - 67*B))/(15*a^3*f) + (2*B*c^2)/(3*a^3*f)))/((exp(e*1i + f*x*1i) - 1i)*(e
xp(e*1i + f*x*1i) + 1i)^3) + (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x
*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((c^2*(A*1i - B*4i)*4i)/(a^
3*f) + (4*B*c^2)/(a^3*f)))/((exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i)
+ 1i)) - (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*
1i + f*x*1i)*1i)/2))^(1/2)*((8*c^2*(A*1i - B*1i))/(a^3*f) + (c^2*(A*1i - B
*3i))/(2*a^3*f) + (c^2*(A*11i - B*1i))/(10*a^3*f) + (c^2*(12*A - 17*B)*1i)
/(4*a^3*f) + (c^2*(52*A - 47*B)*1i)/(4*a^3*f)))/((exp(e*1i + f*x*1i) - 1i)
*(exp(e*1i + f*x*1i) + 1i)^4) + (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i -
f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((c^2*(A*1i - B*4i))/(a^
3*f) + (c^2*(A*5i - B*4i))/(3*a^3*f) + (c^2*(A - 2*B)*8i)/(a^3*f)))/((exp(
e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^2) + (exp(e*1i + f*x*1i)*(c
 - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((c^
2*(A*2i - B*5i)*1i)/(5*a^3*f) - (c^2*(4*A - 3*B))/(a^3*f) - (c^2*(2*A - 5*
B))/(5*a^3*f) + (c^2*(A*4i - B*3i)*1i)/(a^3*f) - (c^2*(10*A - 11*B))/(5...
```

Reduce [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^3}\,dx = \frac{\sqrt{c}\,c^2\left(\left(\int \frac{\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)^3+3\sin(fx+e)^2+3\sin(fx+e)+1}dx\right)a+\left(\int \frac{}{\mathrm{s}}\right.\right.}{}$$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x)
```

output

```
(sqrt(c)*c**2*(int(sqrt( - sin(e + f*x) + 1)/(sin(e + f*x)**3 + 3*sin(e +
f*x)**2 + 3*sin(e + f*x) + 1),x)*a + int((sqrt( - sin(e + f*x) + 1)*sin(e
+ f*x)**3)/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x) + 1),x)*b
 + int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x)**3 + 3*si
n(e + f*x)**2 + 3*sin(e + f*x) + 1),x)*a - 2*int((sqrt( - sin(e + f*x) + 1
)*sin(e + f*x)**2)/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x) +
 1),x)*b - 2*int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**3
 + 3*sin(e + f*x)**2 + 3*sin(e + f*x) + 1),x)*a + int((sqrt( - sin(e + f*x
) + 1)*sin(e + f*x))/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x)
 + 1),x)*b))/a**3
```

3.126 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 121

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx = \frac{4(A+9B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3f} - \frac{(A+9B)\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3cf} - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{5a^3c^3f}$$

output
```
4/15*(A+9*B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^3/f-1/5*(A+9*B)*sec(f*x
+e)^3*(c-c*sin(f*x+e))^(5/2)/a^3/c/f-1/5*(A-B)*sec(f*x+e)^5*(c-c*sin(f*x+e
))^(9/2)/a^3/c^3/f
```

Mathematica [A] (verified)

Time = 8.97 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.93

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx = \frac{c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-2A+27B-15\cdots}{15a^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\cdots\right.}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e +
 f*x])^3,x]
```

output
```
(c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-2*A + 27*B - 15*B*Cos[2*(e + f*
x)] + 10*(A + 3*B)*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(15*a^3*f*(Cos[
(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3446, 3042, 3334, 3042, 3153, 3042, 3152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^3}dx$$

$$\Big\downarrow \text{3446}$$

$$\frac{\int \sec^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}dx}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{\cos(e+fx)^6}dx}{a^3c^3}$$

$$\Big\downarrow \text{3334}$$

$$\frac{\frac{1}{10}c(A+9B)\int \sec^4(e+fx)(c-c\sin(e+fx))^{7/2}dx-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{10}c(A+9B)\int \frac{(c-c\sin(e+fx))^{7/2}}{\cos(e+fx)^4}dx-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{3153}$$

$$\frac{\frac{1}{10}c(A+9B)\left(-4c\int \sec^4(e+fx)(c-c\sin(e+fx))^{5/2}dx-\frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{f}\right)-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{10}c(A+9B)\left(-4c\int \frac{(c-c\sin(e+fx))^{5/2}}{\cos(e+fx)^4}dx-\frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{f}\right)-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{3152}$$

$$\frac{\frac{1}{10}c(A+9B)\left(\frac{8c^2\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3f}-\frac{2c\sec^3(e+fx)(c-c\sin(e+fx))^{5/2}}{f}\right)-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{9/2}}{5f}}{a^3c^3}$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])
^3,x]
```

output
```
(-1/5*((A - B)*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(9/2))/f + ((A + 9*B)*c
*((8*c^2*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(3*f) - (2*c*Sec[e + f
*x]^3*(c - c*Sin[e + f*x])^(5/2))/f))/10)/(a^3*c^3)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3153
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] &&
NeQ[m + p, 0]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.69

method	result	size
default	$-\frac{2 c^{2} \left(\sin \left(f x +e \right)-1\right) \left(-15 B \cos \left(f x +e \right)^{2}+\sin \left(f x +e \right) \left(5 A +15 B \right)-A +21 B \right)}{15 a^{3} \left(1+\sin \left(f x +e \right)\right)^{2} \cos \left(f x +e \right) \sqrt{c -c \sin \left(f x +e \right)}\, f}$	83

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x,method=_R
ETURNVERBOSE)
```

output
```
-2/15*c^2/a^3*(sin(f*x+e)-1)/(1+sin(f*x+e))^2*(-15*B*cos(f*x+e)^2+sin(f*x+
e)*(5*A+15*B)-A+21*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.79

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx = \frac{2\left(15\,Bc\cos\left(fx+e\right)^2-5\left(A+3\,B\right)c\sin\left(fx+e\right)+(A}{15\left(a^3f\cos\left(fx+e\right)^3-2\,a^3f\cos\left(fx+e\right)\sin\left(fx\right.\right.}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x, al
gorithm="fricas")
```

output
```
2/15*(15*B*c*cos(f*x + e)^2 - 5*(A + 3*B)*c*sin(f*x + e) + (A - 21*B)*c)*s
qrt(-c*sin(f*x + e) + c)/(a^3*f*cos(f*x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(
f*x + e) - 2*a^3*f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 663 vs. 2(109) = 218.

Time = 0.15 (sec) , antiderivative size = 663, normalized size of antiderivative = 5.48

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x, al
gorithm="maxima")
```

output
```
2/15*((c^(3/2) - 10*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 4*c^(3/2)*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 - 30*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 6*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 30*c^(3/2)*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5 + 4*c^(3/2)*sin(f*x + e)^6/(cos(f*x + e) +
 1)^6 - 10*c^(3/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + c^(3/2)*sin(f*x +
 e)^8/(cos(f*x + e) + 1)^8)*A/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1
) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(
3/2)) - 6*(c^(3/2) + 5*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 14*c^(3/2
)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*c^(3/2)*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 26*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 15*c^(3/2)
*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 14*c^(3/2)*sin(f*x + e)^6/(cos(f*x
+ e) + 1)^6 + 5*c^(3/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + c^(3/2)*sin(
f*x + e)^8/(cos(f*x + e) + 1)^8)*B/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e
) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*s
in(f*x + e)^5/(cos(f*x + e) + 1)^5)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 1)^(3/2)))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(109) = 218$.

Time = 0.29 (sec) , antiderivative size = 373, normalized size of antiderivative = 3.08

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x, al
gorithm="giac")
```

output
```
-2/15*sqrt(2)*(A*c*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 9*B*c*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e)) + 5*A*c*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(
sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 45*
B*c*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e
))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) - 5*A*c*(cos(-1/4*pi + 1/2*f*x + 1
/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x +
1/2*e) + 1)^2 + 75*B*c*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 15*A*c*(c
os(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(
cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 15*B*c*(cos(-1/4*pi + 1/2*f*x + 1/
2*e) - 1)^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1
/2*e) + 1)^3)*sqrt(c)/(a^3*f*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1
/4*pi + 1/2*f*x + 1/2*e) + 1) + 1)^5)
```

Mupad [B] (verification not implemented)

Time = 42.48 (sec) , antiderivative size = 683, normalized size of antiderivative = 5.64

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))
^3,x)
```

output
```
(exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*
1i)*1i)/2))^(1/2)*((4*B*c)/(5*a^3*f) - (2*c*(2*A - 3*B))/(5*a^3*f) - (4*c*
(3*A - 2*B))/(5*a^3*f) + (c*(A*2i - B*3i)*2i)/(5*a^3*f) + (c*(A*3i - B*2i)
*4i)/(5*a^3*f)))/((exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^5) -
 (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x
*1i)*1i)/2))^(1/2)*((2*B*c)/(3*a^3*f) + (c*(A*2i - B*5i)*2i)/(3*a^3*f) - (
2*c*(10*A - 13*B))/(3*a^3*f) + (c*(A*8i - B*13i)*2i)/(15*a^3*f)))/((exp(e*
1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^3) + (exp(e*1i + f*x*1i)*(c -
 c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((c*(2
*A - 3*B)*1i)/(3*a^3*f) - (B*c*1i)/(a^3*f) + (2*c*(A*1i - B*3i))/(a^3*f)))
/((exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^2) - (exp(e*1i + f*x
*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/
2)*((c*(A - B)*4i)/(a^3*f) - (B*c*1i)/(2*a^3*f) + (c*(8*A - 3*B)*1i)/(10*a
^3*f) + (c*(A*1i - B*2i))/(a^3*f) + (c*(A*7i - B*6i))/(a^3*f)))/((exp(e*1i
 + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^4) + (4*B*c*exp(e*1i + f*x*1i)*
(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2))/(
a^3*f*(exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i))
```

Reduce [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^3}\,dx = \frac{\sqrt{c}\,c\left(\left(\int \frac{\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)^3+3\sin(fx+e)^2+3\sin(fx+e)+1}dx\right)a-\left(\int \frac{}{\sin}\right.\right.}{}$$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x)
```

output
```
(sqrt(c)*c*(int(sqrt( - sin(e + f*x) + 1)/(sin(e + f*x)**3 + 3*sin(e + f*x
)**2 + 3*sin(e + f*x) + 1),x)*a - int((sqrt( - sin(e + f*x) + 1)*sin(e + f
*x)**2)/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x) + 1),x)*b -
int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**3 + 3*sin(e +
f*x)**2 + 3*sin(e + f*x) + 1),x)*a + int((sqrt( - sin(e + f*x) + 1)*sin(e
+ f*x))/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x) + 1),x)*b))/
a**3
```

3.127 $\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 85

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^3}\,dx = -\frac{(3A+7B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15a^3cf} - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{5a^3c^3f}$$

output
```
-1/15*(3*A+7*B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^3/c/f-1/5*(A-B)*sec(
f*x+e)^5*(c-c*sin(f*x+e))^(7/2)/a^3/c^3/f
```

Mathematica [A] (verified)

Time = 6.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.05

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^3}\,dx = -\frac{2(3A+2B+5B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{15a^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

input
```
Integrate[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f
*x])^3,x]
```

output
```
(-2*(3*A + 2*B + 5*B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(15*a^3*f*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5
)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 3446, 3042, 3334, 3042, 3152}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-c\sin(e+fx)}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{c-c\sin(e+fx)}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^3}dx$$

$$\Big\downarrow \text{3446}$$

$$\frac{\int \sec^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}dx}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{\cos(e+fx)^6}dx}{a^3c^3}$$

$$\Big\downarrow \text{3334}$$

$$\frac{\frac{1}{10}c(3A+7B)\int \sec^4(e+fx)(c-c\sin(e+fx))^{5/2}dx-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{10}c(3A+7B)\int \frac{(c-c\sin(e+fx))^{5/2}}{\cos(e+fx)^4}dx-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{3152}$$

$$\frac{-\frac{c^2(3A+7B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{15f}-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{7/2}}{5f}}{a^3c^3}$$

input
```
Int[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^3
,x]
```

output
```
(-1/15*((3*A + 7*B)*c^2*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/f - ((A
 - B)*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(7/2))/(5*f))/(a^3*c^3)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3152
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{2c(\sin(fx+e)-1)(5B\sin(fx+e)+3A+2B)}{15a^3(1+\sin(fx+e))^2\cos(fx+e)\sqrt{c-c\sin(fx+e)}\,f}$	65

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x,method=_R
ETURNVERBOSE)
```

output
```
2/15*c/a^3*(sin(f*x+e)-1)/(1+sin(f*x+e))^2*(5*B*sin(f*x+e)+3*A+2*B)/cos(f*
x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^3}\,dx = \frac{2\,(5\,B\sin(fx+e)+3\,A+2\,B)\sqrt{-c\sin(fx+e)+c}}{15\,\left(a^3f\cos(fx+e)^3-2\,a^3f\cos(fx+e)\sin(fx+e)-2\,a^3f\cos(fx+e)\right)}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x, al
gorithm="fricas")
```

output
```
2/15*(5*B*sin(f*x + e) + 3*A + 2*B)*sqrt(-c*sin(f*x + e) + c)/(a^3*f*cos(f
*x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 505 vs. $2(77) = 154$.

Time = 0.14 (sec) , antiderivative size = 505, normalized size of antiderivative = 5.94

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x, al
gorithm="maxima")
```

output
```
2/15*(2*B*(sqrt(c) + 5*sqrt(c)*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sqrt(c)
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*sqrt(c)*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 3*sqrt(c)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*sqrt(c)*si
n(f*x + e)^5/(cos(f*x + e) + 1)^5 + sqrt(c)*sin(f*x + e)^6/(cos(f*x + e) +
 1)^6)/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)
^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a
^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5)*sqrt(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)) + 3*A*(sqrt(c) + 3*
sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sqrt(c)*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + sqrt(c)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)/((a^3 + 5
*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^
4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)*sqrt(sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 368 vs. $2(77) = 154$.

Time = 0.31 (sec) , antiderivative size = 368, normalized size of antiderivative = 4.33

$$\int \frac{(A + B\sin(e + fx))\sqrt{c - c\sin(e + fx)}}{(a + a\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x, al
gorithm="giac")
```

output
```
1/30*sqrt(2)*(3*A*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 7*B*sgn(sin(-1/4*p
i + 1/2*f*x + 1/2*e)) + 20*B*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1) + 30*A*(c
os(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(
cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^2 + 10*B*(cos(-1/4*pi + 1/2*f*x + 1/2*
e) - 1)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2
*e) + 1)^2 + 60*B*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)^3*sgn(sin(-1/4*pi +
 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1)^3 + 15*A*(cos(-1/4
*pi + 1/2*f*x + 1/2*e) - 1)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/
4*pi + 1/2*f*x + 1/2*e) + 1)^4 + 15*B*(cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)
^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/(cos(-1/4*pi + 1/2*f*x + 1/2*e) + 1
)^4)*sqrt(c)/(a^3*f*((cos(-1/4*pi + 1/2*f*x + 1/2*e) - 1)/(cos(-1/4*pi + 1
/2*f*x + 1/2*e) + 1) + 1)^5)
```

Mupad [B] (verification not implemented)

Time = 41.56 (sec) , antiderivative size = 479, normalized size of antiderivative = 5.64

$$\int \frac{(A + B\sin(e + fx))\sqrt{c - c\sin(e + fx)}}{(a + a\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))
^3,x)
```

output
```
(exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*
1i)*1i)/2))^(1/2)*((8*B)/(5*a^3*f) - (16*A - 8*B)/(10*a^3*f) + ((A*16i - B
*8i)*1i)/(10*a^3*f)))/((exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)
^5) - (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i
+ f*x*1i)*1i)/2))^(1/2)*((4*B)/(3*a^3*f) - (16*A - 16*B)/(30*a^3*f) + ((A*
80i - B*120i)*1i)/(30*a^3*f)))/((exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*
1i) + 1i)^3) - (exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (
exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((A*16i - B*16i)/(40*a^3*f) - (B*1i)/(a^3
*f) + (A*80i - B*80i)/(40*a^3*f) + ((160*A - 120*B)*1i)/(40*a^3*f)))/((exp
(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^4) - (B*exp(e*1i + f*x*1i)
*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*8
i)/(3*a^3*f*(exp(e*1i + f*x*1i) - 1i)*(exp(e*1i + f*x*1i) + 1i)^2)
```

Reduce [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^3}\,dx = \frac{\sqrt{c}\left(\left(\int \frac{\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)^3+3\sin(fx+e)^2+3\sin(fx+e)+1}dx\right)a+\left(\int \frac{\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)^3+3\sin(fx+e)^2+3\sin(fx+e)+1}dx\right)b\right)}{a^3}$$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x)
```

output
```
(sqrt(c)*(int(sqrt( - sin(e + f*x) + 1)/(sin(e + f*x)**3 + 3*sin(e + f*x)*
*2 + 3*sin(e + f*x) + 1),x)*a + int((sqrt( - sin(e + f*x) + 1)*sin(e + f*x
))/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x) + 1),x)*b))/a**3
```

3.128 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 174

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}\,dx = \frac{(A+B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{4\sqrt{2}a^3\sqrt{c}f} - \frac{(A+B)\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{4a^3cf} - \frac{(A+B)\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{6a^3c^2f} - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5a^3c^3f}$$

output
```
1/8*(A+B)*arctanh(1/2*c^(1/2)*cos(f*x+e)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))*2
^(1/2)/a^3/c^(1/2)/f-1/4*(A+B)*sec(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^3/c/f-1
/6*(A+B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^3/c^2/f-1/5*(A-B)*sec(f*x+e
)^5*(c-c*sin(f*x+e))^(5/2)/a^3/c^3/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.30 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.17

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(12(-A+B)-10(A+B)\left(c\right.\right.}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*Sqrt[c - c*Sin[e +
f*x]]),x]
```

output
```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])*(12*(-A + B) - 10*(A + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 15*
(A + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - (15 + 15*I)*(-1)^(1/4)*(
A + B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])^5))/(60*a^3*f*(1 + Sin[e + f*x])^3*Sqrt[c - c*Sin
[e + f*x]])
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.93, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.289$, Rules used = {3042, 3446, 3042, 3334, 3042, 3154, 3042, 3154, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^3\sqrt{c-c\sin(e+fx)}}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^3\sqrt{c-c\sin(e+fx)}}dx$$
$$\Big\downarrow \text{ 3446}$$

$$\frac{\int \sec^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}dx}{a^3c^3}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{\cos(e+fx)^6}dx}{a^3c^3}$$

$\Big\downarrow$ 3334

$$\frac{\frac{1}{2}c(A+B)\int \sec^4(e+fx)(c-c\sin(e+fx))^{3/2}dx-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5f}}{a^3c^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{1}{2}c(A+B)\int \frac{(c-c\sin(e+fx))^{3/2}}{\cos(e+fx)^4}dx-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5f}}{a^3c^3}$$

$\Big\downarrow$ 3154

$$\frac{\frac{1}{2}c(A+B)\left(\frac{1}{2}c\int \sec^2(e+fx)\sqrt{c-c\sin(e+fx)}dx-\frac{\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3f}\right)-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5f}}{a^3c^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{1}{2}c(A+B)\left(\frac{1}{2}c\int \frac{\sqrt{c-c\sin(e+fx)}}{\cos(e+fx)^2}dx-\frac{\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3f}\right)-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5f}}{a^3c^3}$$

$\Big\downarrow$ 3154

$$\frac{\frac{1}{2}c(A+B)\left(\frac{1}{2}c\left(\frac{1}{2}c\int \frac{1}{\sqrt{c-c\sin(e+fx)}}dx-\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{f}\right)-\frac{\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3f}\right)-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5f}}{a^3c^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{1}{2}c(A+B)\left(\frac{1}{2}c\left(\frac{1}{2}c\int \frac{1}{\sqrt{c-c\sin(e+fx)}}dx-\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{f}\right)-\frac{\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3f}\right)-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{5/2}}{5f}}{a^3c^3}$$

$\Big\downarrow$ 3128

$$\frac{\frac{1}{2}c(A+B)\left(\frac{1}{2}c\left(-\frac{c\int \frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{f}-\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{f}\right)-\frac{\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3f}\right)-}{a^3c^3}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{2}c(A+B)\left(\frac{1}{2}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{\sqrt{2}f}-\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}}{f}\right)-\frac{\sec^3(e+fx)(c-c\sin(e+fx))^{3/2}}{3f}\right)-\frac{(A-B)\text{se}}{}}{a^3c^3}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]])
,x]
```

output
```
(-1/5*((A - B)*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(5/2))/f + ((A + B)*c*(
-1/3*(Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/f + (c*((Sqrt[c]*ArcTanh[
(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(Sqrt[2]*f) -
(Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/f))/2))/2)/(a^3*c^3)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3154
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^m/(a*f*g*(p + 1))), x] + Simp[a*((m + p + 1)/(g^2*(p + 1)))   Int[(g*
Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b,
e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ
[m + 1/2, 2*p]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.15

method	result
default	$-\frac{(\sin(fx+e)-1)\left(-30Ac^{\frac{9}{2}}\sin(fx+e)^2-30Bc^{\frac{9}{2}}\sin(fx+e)^2-80Ac^{\frac{9}{2}}\sin(fx+e)-80Bc^{\frac{9}{2}}\sin(fx+e)+15\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(}}{2\sqrt{}}\right.\right.}{120a^3c^{\frac{9}{2}}(1+\sin(fx+e))^2\cos(fx}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/120*(sin(f*x+e)-1)*(-30*A*c^(9/2)*sin(f*x+e)^2-30*B*c^(9/2)*sin(f*x+e)^
2-80*A*c^(9/2)*sin(f*x+e)-80*B*c^(9/2)*sin(f*x+e)+15*2^(1/2)*arctanh(1/2*(
c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*(c*(1+sin(f*x+e)))^(5/2)*c^2*A-74
*c^(9/2)*A+15*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2)
)*(c*(1+sin(f*x+e)))^(5/2)*c^2*B-26*c^(9/2)*B)/a^3/c^(9/2)/(1+sin(f*x+e))^
2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.51

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3\sqrt{c - c\sin(e + fx)}}\,dx$$
$$= \frac{15\sqrt{2}\left((A+B)\cos\left(fx+e\right)^3 - 2\left(A+B\right)\cos\left(fx+e\right)\sin\left(fx+e\right) - 2\left(A+B\right)\cos\left(fx+e\right)\right)\sqrt{c}\log\left(-\right.}{}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x, al
gorithm="fricas")
```

output
```
1/240*(15*sqrt(2)*((A + B)*cos(f*x + e)^3 - 2*(A + B)*cos(f*x + e)*sin(f*x
 + e) - 2*(A + B)*cos(f*x + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)
*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c
*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2
 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(15*(A + B)*co
s(f*x + e)^2 - 40*(A + B)*sin(f*x + e) - 52*A - 28*B)*sqrt(-c*sin(f*x + e)
 + c))/(a^3*c*f*cos(f*x + e)^3 - 2*a^3*c*f*cos(f*x + e)*sin(f*x + e) - 2*a
^3*c*f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3\sqrt{c - c\sin(e + fx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{B\sin{(fx+e)}+A}{(a\sin{(fx+e)}+a)^3\sqrt{-c\sin{(fx+e)}+c}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^3*sqrt(-c*sin(f*x + e
) + c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}\,dx$$
$$=\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^3\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(1/2
)),x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(1/2
)), x)
```

Reduce [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}\,dx =$$
$$-\frac{\sqrt{c}\left(\left(\int \frac{\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)^4+2\sin(fx+e)^3-2\sin(fx+e)-1}dx\right)a+\left(\int \frac{\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)^4+2\sin(fx+e)^3-2\sin(fx+e)-1}dx\right)b\right)}{a^3c}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x)
```

output
```
( - sqrt(c)*(int(sqrt( - sin(e + f*x) + 1)/(sin(e + f*x)**4 + 2*sin(e + f*
x)**3 - 2*sin(e + f*x) - 1),x)*a + int((sqrt( - sin(e + f*x) + 1)*sin(e +
f*x))/(sin(e + f*x)**4 + 2*sin(e + f*x)**3 - 2*sin(e + f*x) - 1),x)*b))/(a
**3*c)
```

3.129 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 224

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}\,dx = \frac{(7A+3B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{16\sqrt{2}a^3c^{3/2}f} + \frac{(7A+3B)\cos(e+fx)}{16a^3f(c-c\sin(e+fx))^{3/2}} - \frac{(7A+3B)\sec(e+fx)}{12a^3cf\sqrt{c-c\sin(e+fx)}} - \frac{(7A+3B)\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{30a^3c^2f} - \frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{3/2}}{5a^3c^3f}$$

output
```
1/32*(7*A+3*B)*arctanh(1/2*c^(1/2)*cos(f*x+e)*2^(1/2)/(c-c*sin(f*x+e))^(1/
2))*2^(1/2)/a^3/c^(3/2)/f+1/16*(7*A+3*B)*cos(f*x+e)/a^3/f/(c-c*sin(f*x+e))
^(3/2)-1/12*(7*A+3*B)*sec(f*x+e)/a^3/c/f/(c-c*sin(f*x+e))^(1/2)-1/30*(7*A+
3*B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(1/2)/a^3/c^2/f-1/5*(A-B)*sec(f*x+e)^5*
(c-c*sin(f*x+e))^(3/2)/a^3/c^3/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.02 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.59

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\right.}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x]
)^(3/2)),x]
```

output
```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])*(-40*A*Cos[e + f*x]^2 + 24*(-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2
])^2 - 30*(3*A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^4 + 15*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2
])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 - (15 + 15*I)*(-1)^(1/4)*(7*A +
 3*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 + 30*(A
+ B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5))/(240*a^3*f
*(1 + Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2))
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.90, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.342$, Rules used = {3042, 3446, 3042, 3334, 3042, 3154, 3042, 3166, 3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c-c\sin(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c-c\sin(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3446}$$

$$\frac{\int \sec^6(e+fx)(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}dx}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{\cos(e+fx)^6}dx}{a^3c^3}$$

$$\Big\downarrow \text{3334}$$

$$\frac{\frac{1}{10}c(7A+3B)\int \sec^4(e+fx)\sqrt{c-c\sin(e+fx)}dx-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{3/2}}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{10}c(7A+3B)\int \frac{\sqrt{c-c\sin(e+fx)}}{\cos(e+fx)^4}dx-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{3/2}}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{3154}$$

$$\frac{\frac{1}{10}c(7A+3B)\left(\frac{5}{6}c\int \frac{\sec^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}dx-\frac{\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3f}\right)-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{3/2}}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{10}c(7A+3B)\left(\frac{5}{6}c\int \frac{1}{\cos(e+fx)^2\sqrt{c-c\sin(e+fx)}}dx-\frac{\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3f}\right)-\frac{(A-B)\sec^5(e+fx)(c-c\sin(e+fx))^{3/2}}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{3166}$$

$$\frac{\frac{1}{10}c(7A+3B)\left(\frac{5}{6}c\left(\frac{3}{2}c\int \frac{1}{(c-c\sin(e+fx))^{3/2}}dx-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3f}\right)-\frac{(A-B)\sec^5(e+fx)(c-c\text{s}}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{10}c(7A+3B)\left(\frac{5}{6}c\left(\frac{3}{2}c\int \frac{1}{(c-c\sin(e+fx))^{3/2}}dx-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3f}\right)-\frac{(A-B)\sec^5(e+fx)(c-c\text{s}}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{3129}$$

$$\frac{\frac{1}{10}c(7A+3B)\left(\frac{5}{6}c\left(\frac{3}{2}c\left(\frac{\int \frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{4c}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}\right)-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3f}\right)-}{a^3c^3}$$

$\downarrow$ 3042

$$\frac{\frac{1}{10}c(7A+3B)\left(\frac{5}{6}c\left(\frac{3}{2}c\left(\frac{\int \frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{4c}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}\right)-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{\sec^3(e+fx)\sqrt{c-c\sin(e+fx)}}{3f}\right)-}{a^3c^3}$$

$\downarrow$ 3128

$$\frac{\frac{1}{10}c(7A+3B)\left(\frac{5}{6}c\left(\frac{3}{2}c\left(\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}-\frac{\int \frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{2cf}\right)-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{\sec^3(e+fx)}{}\right.}{a^3c^3}$$

$\downarrow$ 219

$$\frac{\frac{1}{10}c(7A+3B)\left(\frac{5}{6}c\left(\frac{3}{2}c\left(\frac{\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}c^{3/2}f}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}\right)-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{\sec^3(e+fx)\sqrt{c-c\sin}}{3f}\right.}{a^3c^3}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2
)),x]
```

output
```
(-1/5*((A - B)*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(3/2))/f + ((7*A + 3*B)
*c*(-1/3*(Sec[e + f*x]^3*Sqrt[c - c*Sin[e + f*x]])/f + (5*c*(-(Sec[e + f*x
]/(f*Sqrt[c - c*Sin[e + f*x]])) + (3*c*(ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sq
rt[2]*Sqrt[c - c*Sin[e + f*x]])]/(2*Sqrt[2]*c^(3/2)*f) + Cos[e + f*x]/(2*f
*(c - c*Sin[e + f*x])^(3/2))))/2))/6))/10)/(a^3*c^3)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3154
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^m/(a*f*g*(p + 1))), x] + Simp[a*((m + p + 1)/(g^2*(p + 1)))   Int[(g*
Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b,
e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ
[m + 1/2, 2*p]
```

rule 3166
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Simp[a*((2*p + 1)/(2*g^2*(p + 1)))   Int[(g
*Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e
, f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.38

method	result
default	$-\frac{278 A c^{\frac{7}{2}}-18 B c^{\frac{7}{2}}+105 A (c(1+\sin(fx+e)))^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\sqrt{c(1+\sin(fx+e))}\,\sqrt{2}}{2\sqrt{c}}\right)\sqrt{2}\,\sin(fx+e)c+45 B (c(1+\sin(fx+e)))^{\frac{5}{2}} \operatorname{arctanh}\left(\right.}{}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/480/c^(9/2)/a^3*(278*A*c^(7/2)-18*B*c^(7/2)+105*A*(c*(1+sin(f*x+e)))^(5
/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*sin(f*x+
e)*c+45*B*(c*(1+sin(f*x+e)))^(5/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^
(1/2)/c^(1/2))*2^(1/2)*sin(f*x+e)*c-105*A*(c*(1+sin(f*x+e)))^(5/2)*arctanh
(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c-45*B*(c*(1+sin(f*
x+e)))^(5/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)
*c+42*A*sin(f*x+e)*c^(7/2)+18*B*sin(f*x+e)*c^(7/2)-210*A*sin(f*x+e)^3*c^(7
/2)-90*B*sin(f*x+e)^3*c^(7/2)-350*A*sin(f*x+e)^2*c^(7/2)-150*B*sin(f*x+e)^
2*c^(7/2))/(1+sin(f*x+e))^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.24

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}\,dx = \frac{15\sqrt{2}((7A+3B)\cos{(fx+e)}^3\sin{(fx+e)}+(7A+3$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x, al
gorithm="fricas")
```

output
```
1/960*(15*sqrt(2)*((7*A + 3*B)*cos(f*x + e)^3*sin(f*x + e) + (7*A + 3*B)*c
os(f*x + e)^3)*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x
+ e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (
c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e)
+ 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(25*(7*A + 3*B)*cos(f*x + e)^2
+ 3*(5*(7*A + 3*B)*cos(f*x + e)^2 - 28*A - 12*B)*sin(f*x + e) - 36*A - 84*
B)*sqrt(-c*sin(f*x + e) + c))/(a^3*c^2*f*cos(f*x + e)^3*sin(f*x + e) + a^3
*c^2*f*cos(f*x + e)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c - c\sin(e + fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x, al
gorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c - c\sin(e + fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c - c\sin(e + fx))^{3/2}}\,dx = \int \frac{A + B\,\sin\left(e + f\,x\right)}{\left(a + a\,\sin\left(e + f\,x\right)\right)^3\,\left(c - c\,\sin\left(e + f\,x\right)\right)^{3/2}}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(3/2
)),x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(3/2
)), x)
```

Reduce [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\sqrt{c}\left(\left(\int \frac{\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)^5+\sin(fx+e)^4-2\sin(fx+e)^3-2\sin(fx+e)^2+\sin(fx+}\right.\right.}{}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*(int(sqrt( - sin(e + f*x) + 1)/(sin(e + f*x)**5 + sin(e + f*x)**4
 - 2*sin(e + f*x)**3 - 2*sin(e + f*x)**2 + sin(e + f*x) + 1),x)*a + int((s
qrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**5 + sin(e + f*x)**4
- 2*sin(e + f*x)**3 - 2*sin(e + f*x)**2 + sin(e + f*x) + 1),x)*b))/(a**3*c
**2)
```

3.130 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 258

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{5/2}}\,dx=\frac{7(9A+B)\text{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{128\sqrt{2}a^3c^{5/2}f}$$
$$+\frac{7(9A+B)\cos(e+fx)}{128a^3cf(c-c\sin(e+fx))^{3/2}}+\frac{7(9A+B)\sec(e+fx)}{240a^3cf(c-c\sin(e+fx))^{3/2}}$$
$$-\frac{7(9A+B)\sec(e+fx)}{96a^3c^2f\sqrt{c-c\sin(e+fx)}}-\frac{(9A+B)\sec^3(e+fx)}{30a^3c^2f\sqrt{c-c\sin(e+fx)}}$$
$$-\frac{(A-B)\sec^5(e+fx)\sqrt{c-c\sin(e+fx)}}{5a^3c^3f}$$

output
```
7/256*(9*A+B)*arctanh(1/2*c^(1/2)*cos(f*x+e)*2^(1/2)/(c-c*sin(f*x+e))^(1/2
))*2^(1/2)/a^3/c^(5/2)/f+7/128*(9*A+B)*cos(f*x+e)/a^3/c/f/(c-c*sin(f*x+e))
^(3/2)+7/240*(9*A+B)*sec(f*x+e)/a^3/c/f/(c-c*sin(f*x+e))^(3/2)-7/96*(9*A+B
)*sec(f*x+e)/a^3/c^2/f/(c-c*sin(f*x+e))^(1/2)-1/30*(9*A+B)*sec(f*x+e)^3/a^
3/c^2/f/(c-c*sin(f*x+e))^(1/2)-1/5*(A-B)*sec(f*x+e)^5*(c-c*sin(f*x+e))^(1/
2)/a^3/c^3/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.02 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.86

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\right.}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x]
)^(5/2)),x]
```

output
```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])*(-720*A*Cos[e + f*x]^4 + 96*(-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/
2])^4 + 80*(-3*A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e + f*
x)/2] + Sin[(e + f*x)/2])^2 + 60*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)
/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 + 15*(15*A + 7*B)*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 - (10
5 + 105*I)*(-1)^(1/4)*(9*A + B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e
+ f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2])^5 + 120*(A + B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^5 + 30*(15*A + 7*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*S
in[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5))/(1920*a^3*f*(1 +
 Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2))
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.91, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.395$, Rules used = {3042, 3446, 3042, 3334, 3042, 3166, 3042, 3160, 3042, 3166, 3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c-c\sin(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c-c\sin(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3446}$$

$$\frac{\int \sec^6(e+fx)(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}dx}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{\cos(e+fx)^6}dx}{a^3c^3}$$

$$\Big\downarrow \text{3334}$$

$$\frac{\frac{1}{10}c(9A+B)\int \frac{\sec^4(e+fx)}{\sqrt{c-c\sin(e+fx)}}dx-\frac{(A-B)\sec^5(e+fx)\sqrt{c-c\sin(e+fx)}}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{10}c(9A+B)\int \frac{1}{\cos(e+fx)^4\sqrt{c-c\sin(e+fx)}}dx-\frac{(A-B)\sec^5(e+fx)\sqrt{c-c\sin(e+fx)}}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{3166}$$

$$\frac{\frac{1}{10}c(9A+B)\left(\frac{7}{6}c\int \frac{\sec^2(e+fx)}{(c-c\sin(e+fx))^{3/2}}dx-\frac{\sec^3(e+fx)}{3f\sqrt{c-c\sin(e+fx)}}\right)-\frac{(A-B)\sec^5(e+fx)\sqrt{c-c\sin(e+fx)}}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{10}c(9A+B)\left(\frac{7}{6}c\int \frac{1}{\cos(e+fx)^2(c-c\sin(e+fx))^{3/2}}dx-\frac{\sec^3(e+fx)}{3f\sqrt{c-c\sin(e+fx)}}\right)-\frac{(A-B)\sec^5(e+fx)\sqrt{c-c\sin(e+fx)}}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{3160}$$

$$\frac{\frac{1}{10}c(9A+B)\left(\frac{7}{6}c\left(\frac{5\int \frac{\sec^2(e+fx)}{\sqrt{c-c\sin(e+fx)}}dx}{8c}+\frac{\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}\right)-\frac{\sec^3(e+fx)}{3f\sqrt{c-c\sin(e+fx)}}\right)-\frac{(A-B)\sec^5(e+fx)\sqrt{c-c\sin(e+fx)}}{5f}}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{10}c(9A+B)\left(\frac{7}{6}c\left(\frac{5\int \frac{1}{\cos(e+fx)^2\sqrt{c-c\sin(e+fx)}}dx}{8c}+\frac{\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}\right)-\frac{\sec^3(e+fx)}{3f\sqrt{c-c\sin(e+fx)}}\right)-\frac{(A-B)\sec^5(e+fx)\sqrt{c-c\,\mathrm{si}}}{5f}}{a^3c^3}$$

$$\Big\downarrow \ 3166$$

$$\frac{\frac{1}{10}c(9A+B)\left(\frac{7}{6}c\left(\frac{5\left(\frac{3}{2}c\int\frac{1}{(c-c\sin(e+fx))^{3/2}}dx-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)}{8c}+\frac{\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}\right)-\frac{\sec^3(e+fx)}{3f\sqrt{c-c\sin(e+fx)}}\right)-\frac{(A-B)\,\mathrm{se}}{}}{a^3c^3}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{1}{10}c(9A+B)\left(\frac{7}{6}c\left(\frac{5\left(\frac{3}{2}c\int\frac{1}{(c-c\sin(e+fx))^{3/2}}dx-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)}{8c}+\frac{\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}\right)-\frac{\sec^3(e+fx)}{3f\sqrt{c-c\sin(e+fx)}}\right)-\frac{(A-B)\,\mathrm{se}}{}}{a^3c^3}$$

$$\Big\downarrow \ 3129$$

$$\frac{\frac{1}{10}c(9A+B)\left(\frac{7}{6}c\left(\frac{5\left(\frac{3}{2}c\left(\frac{\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{4c}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}\right)-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)}{8c}+\frac{\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}\right)-\frac{\sec^3(e+j}{3f\sqrt{c-c\sin(}}\right.}{a^3c^3}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{1}{10}c(9A+B)\left(\frac{7}{6}c\left(\frac{5\left(\frac{3}{2}c\left(\frac{\int\frac{1}{\sqrt{c-c\sin(e+fx)}}dx}{4c}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}\right)-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)}{8c}+\frac{\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}\right)-\frac{\sec^3(e+j}{3f\sqrt{c-c\sin(}}\right.}{a^3c^3}$$

$$\Big\downarrow \ 3128$$

$$\frac{\frac{1}{10}c(9A+B)\left(\frac{7}{6}c\left(\frac{5\left(\frac{3}{2}c\left(\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}-\frac{\int\frac{1}{2c-\frac{c^2\cos^2(e+fx)}{c-c\sin(e+fx)}}d\left(-\frac{c\cos(e+fx)}{\sqrt{c-c\sin(e+fx)}}\right)}{2cf}\right)-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)}{8c}+\frac{\sec(e+fx)}{4f(c-c\sin(e+fx))^3}\right.\right.}{a^3c^3}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{1}{10}c(9A+B)\left(\frac{7}{6}c\left(\frac{5\left(\frac{3}{2}c\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\cos(e+fx)}{\sqrt{2}\sqrt{c-c\sin(e+fx)}}\right)}{2\sqrt{2}c^{3/2}f}+\frac{\cos(e+fx)}{2f(c-c\sin(e+fx))^{3/2}}\right)-\frac{\sec(e+fx)}{f\sqrt{c-c\sin(e+fx)}}\right)}{8c}+\frac{\sec(e+fx)}{4f(c-c\sin(e+fx))^{3/2}}\right)-\right.}{a^3c^3}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2
)),x]
```

output
```
(-1/5*((A - B)*Sec[e + f*x]^5*Sqrt[c - c*Sin[e + f*x]])/f + ((9*A + B)*c*(
-1/3*Sec[e + f*x]^3/(f*Sqrt[c - c*Sin[e + f*x]]) + (7*c*(Sec[e + f*x]/(4*f
*(c - c*Sin[e + f*x])^(3/2)) + (5*(-(Sec[e + f*x]/(f*Sqrt[c - c*Sin[e + f*
x]])) + (3*c*(ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f
*x]])]/(2*Sqrt[2]*c^(3/2)*f) + Cos[e + f*x]/(2*f*(c - c*Sin[e + f*x])^(3/2
))))/2))/(8*c)))/6))/10)/(a^3*c^3)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3160

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*(2*m + p + 1))), x] + Simp[(m + p + 1)/(a*(2*m + p + 1))   Int[
(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e,
f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &
& IntegersQ[2*m, 2*p]
```

rule 3166

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Simp[a*((2*p + 1)/(2*g^2*(p + 1)))   Int[(g
*Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e
, f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 3334

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

rule 3446

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.59

method	result
default	$\frac{\left(1890c^{\frac{9}{2}}A+210c^{\frac{9}{2}}B\right)\cos(fx+e)^4+\left(-1260c^{\frac{9}{2}}A-140c^{\frac{9}{2}}B\right)\cos(fx+e)^2\sin(fx+e)+\left(-252c^{\frac{9}{2}}A+945\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e}}{2\sqrt{c}}\right.\right.}{}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x,method=_R
ETURNVERBOSE)
```

output
```
1/3840*((1890*c^(9/2)*A+210*c^(9/2)*B)*cos(f*x+e)^4+(-1260*c^(9/2)*A-140*c
^(9/2)*B)*cos(f*x+e)^2*sin(f*x+e)+(-252*c^(9/2)*A+945*2^(1/2)*arctanh(1/2*
(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*(c+c*sin(f*x+e))^(5/2)*c^2*A-28*c^
(9/2)*B+105*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*(c
+c*sin(f*x+e))^(5/2)*c^2*B)*cos(f*x+e)^2+(-864*c^(9/2)*A+1890*2^(1/2)*arct
anh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*(c+c*sin(f*x+e))^(5/2)*c^2
*A-96*c^(9/2)*B+210*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(
1/2))*(c+c*sin(f*x+e))^(5/2)*c^2*B)*sin(f*x+e)-96*c^(9/2)*A-1890*2^(1/2)*a
rctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*(c+c*sin(f*x+e))^(5/2)*
c^2*A-864*c^(9/2)*B-210*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)
/c^(1/2))*(c+c*sin(f*x+e))^(5/2)*c^2*B)/c^(13/2)/a^3/(1+sin(f*x+e))^2/(sin
(f*x+e)-1)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.92

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c-c\sin(e+fx))^{5/2}}\,dx = \frac{105\sqrt{2}(9A+B)\sqrt{c}\cos(fx+e)^5\log\left(-\frac{c\cos(fx+e)^2+2\sqrt{2}}{}\right.}{}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x, al
gorithm="fricas")
```

output
```
1/7680*(105*sqrt(2)*(9*A + B)*sqrt(c)*cos(f*x + e)^5*log(-(c*cos(f*x + e)^
2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x +
e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(c
os(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(
105*(9*A + B)*cos(f*x + e)^4 - 14*(9*A + B)*cos(f*x + e)^2 - 2*(35*(9*A +
B)*cos(f*x + e)^2 + 216*A + 24*B)*sin(f*x + e) - 48*A - 432*B)*sqrt(-c*sin
(f*x + e) + c))/(a^3*c^3*f*cos(f*x + e)^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c - c\sin(e + fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c - c\sin(e + fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x, al
gorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c - c\sin(e + fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c - c\sin(e + fx))^{5/2}}\,dx = \int \frac{A + B\,\sin(e + f\,x)}{(a + a\,\sin(e + f\,x))^3\,(c - c\,\sin(e + f\,x))^{5/2}}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(5/2
)),x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(5/2
)), x)
```

Reduce [F]

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c - c\sin(e + fx))^{5/2}}\,dx = -\frac{\sqrt{c}\left(\left(\int \frac{\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)^6-3\sin(fx+e)^4+3\sin(fx+e)^2-1}dx\right)a + \left(\int \frac{\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)^6-3\sin(fx+e)^4+3\sin(fx+e)^2-1}dx\right)b\right)}{a^3c^3}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x)
```

output
```
( - sqrt(c)*(int(sqrt( - sin(e + f*x) + 1)/(sin(e + f*x)**6 - 3*sin(e + f*
x)**4 + 3*sin(e + f*x)**2 - 1),x)*a + int((sqrt( - sin(e + f*x) + 1)*sin(e
 + f*x))/(sin(e + f*x)**6 - 3*sin(e + f*x)**4 + 3*sin(e + f*x)**2 - 1),x)*
b))/(a**3*c**3)
```

3.131 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 94

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = -\frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f\sqrt{a+a\sin(e+fx)}} + \frac{aB\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{5cf\sqrt{a+a\sin(e+fx)}}$$

output
```
-1/4*a*(A+B)*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/f/(a+a*sin(f*x+e))^(1/2)+1/
5*a*B*cos(f*x+e)*(c-c*sin(f*x+e))^(9/2)/c/f/(a+a*sin(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 3.73 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.26

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = -\frac{c^3\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}(4(-60A+23B)\sin(e+fx)+4\cos(2(e+fx))(\cdots}{160f}$$

input
```
Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x
])^(7/2),x]
```

output
```
-1/160*(c^3*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x
]]*(4*(-60*A + 23*B)*Sin[e + f*x] + 4*Cos[2*(e + f*x)]*(-35*A + 25*B + 4*(
5*A - 6*B)*Sin[e + f*x]) + Cos[4*(e + f*x)]*(5*A - 15*B + 4*B*Sin[e + f*x]
)))/f
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3042, 3450, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3450}$$

$$(A+B)\int \sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{7/2}dx-\frac{B\int \sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{9/2}dx}{c}$$

$$\Big\downarrow \text{3042}$$

$$(A+B)\int \sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{7/2}dx-\frac{B\int \sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{9/2}dx}{c}$$

$$\Big\downarrow \text{3217}$$

$$\frac{aB\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{5cf\sqrt{a\sin(e+fx)+a}}-\frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f\sqrt{a\sin(e+fx)+a}}$$

input
```
Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/
2),x]
```

output
```
-1/4*(a*(A + B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(f*Sqrt[a + a*Sin
[e + f*x]]) + (a*B*Cos[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(5*c*f*Sqrt[a
+ a*Sin[e + f*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3450
```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[B/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] -
Simp[(B*c - A*d)/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[
a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(82) = 164$.

Time = 8.32 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.53

method	result
default	$\frac{40c^3\left(A\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\frac{1}{2}\right)\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^4+1\right)\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\frac{8\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^2B\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2\left(\cos\left(\right.\right.}{}\right.}{}$
parts	$\frac{2A\sqrt{4}\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^4+1\right)\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2}\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2}c^3\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{2Bc^3\cos}{}$

input

```
int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,metho
d=_RETURNVERBOSE)
```

output

```
40*c^3*(A*(cos(1/4*Pi+1/2*f*x+1/2*e)^2+1)*(cos(1/2*f*x+1/2*e)^2-1/2)*(cos(
1/4*Pi+1/2*f*x+1/2*e)^4+1)*tan(1/4*Pi+1/2*f*x+1/2*e)+8/5*sin(1/2*f*x+1/2*e
)^2*B*cos(1/2*f*x+1/2*e)^2*(cos(1/2*f*x+1/2*e)^5*sin(1/2*f*x+1/2*e)-cos(1/
2*f*x+1/2*e)^3*sin(1/2*f*x+1/2*e)-15/8*cos(1/2*f*x+1/2*e)^4-5/4*sin(1/2*f*
x+1/2*e)*cos(1/2*f*x+1/2*e)+15/8*cos(1/2*f*x+1/2*e)^2+5/16))*(c*cos(1/4*Pi
+1/2*f*x+1/2*e)^2)^(1/2)*(a*sin(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)/(10*f*cos(1
/2*f*x+1/2*e)^2-5*f)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.49

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx =$$
$$-\frac{\left(5\,(A-3\,B)c^3\cos\left(fx+e\right)^4-40\,(A-B)c^3\cos\left(fx+e\right)^2+5\,(7\,A-5\,B)c^3+4\,\left(Bc^3\cos\left(fx+e\right)^4+(5\right.\right.}{20\,f\cos\left(f\right.}$$

input

```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x
, algorithm="fricas")
```

output
```
-1/20*(5*(A - 3*B)*c^3*cos(f*x + e)^4 - 40*(A - B)*c^3*cos(f*x + e)^2 + 5*
(7*A - 5*B)*c^3 + 4*(B*c^3*cos(f*x + e)^4 + (5*A - 7*B)*c^3*cos(f*x + e)^2
 - 2*(5*A - 3*B)*c^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f
*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx=\text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(1/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx=\int (B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}(-c\sin(fx+e)+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) +
 c)^(7/2), x)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.60

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx =$$

$$-\frac{4\left(8\,Bc^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}-5\,Ac^3\mathrm{sg}\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x
, algorithm="giac")
```

output
```
-4/5*(8*B*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^10 - 5*A*c^3*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*
x + 1/2*e)^8 - 5*B*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8)*sqrt(a)*sqrt(c)/f
```

Mupad [B] (verification not implemented)

Time = 40.07 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.84

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx =$$

$$-\frac{c^3\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(100\,B\,\cos\left(e+f\,x\right)-140\,A\,\cos\left(e+f\,x\right)-135\,A\,\mathrm{co}\right.}{}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(
7/2),x)
```

output
```
-(c^3*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(100*B*co
s(e + f*x) - 140*A*cos(e + f*x) - 135*A*cos(3*e + 3*f*x) + 5*A*cos(5*e + 5
*f*x) + 85*B*cos(3*e + 3*f*x) - 15*B*cos(5*e + 5*f*x) - 240*A*sin(2*e + 2*
f*x) + 40*A*sin(4*e + 4*f*x) + 90*B*sin(2*e + 2*f*x) - 48*B*sin(4*e + 4*f*
x) + 2*B*sin(6*e + 6*f*x)))/(160*f*(cos(2*e + 2*f*x) + 1))
```

Reduce [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \sqrt{c}\sqrt{a}\,c^3\left(-\left(\int \sqrt{\sin(fx+e)+1}\sqrt{-\sin(fx+e)+1}\sin(fx+e)^4\,dx\right)b - \left(\int \sqrt{\sin(fx+e)+1}\sqrt{-\sin(fx+e)+1}\sin(fx+e)^3\,dx\right)a + 3\left(\int \sqrt{\sin(fx+e)+1}\sqrt{-\sin(fx+e)+1}\sin(fx+e)^3\,dx\right)b + 3\left(\int \sqrt{\sin(fx+e)+1}\sqrt{-\sin(fx+e)+1}\sin(fx+e)^2\,dx\right)a - 3\left(\int \sqrt{\sin(fx+e)+1}\sqrt{-\sin(fx+e)+1}\sin(fx+e)^2\,dx\right)b - 3\left(\int \sqrt{\sin(fx+e)+1}\sqrt{-\sin(fx+e)+1}\sin(fx+e)\,dx\right)a + \left(\int \sqrt{\sin(fx+e)+1}\sqrt{-\sin(fx+e)+1}\sin(fx+e)\,dx\right)b + \left(\int \sqrt{\sin(fx+e)+1}\sqrt{-\sin(fx+e)+1}dx\right)a\right)$$

input
```
int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x)
```

output
```
sqrt(c)*sqrt(a)*c**3*( - int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) +
 1)*sin(e + f*x)**4,x)*b - int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x)
 + 1)*sin(e + f*x)**3,x)*a + 3*int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e +
f*x) + 1)*sin(e + f*x)**3,x)*b + 3*int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(
e + f*x) + 1)*sin(e + f*x)**2,x)*a - 3*int(sqrt(sin(e + f*x) + 1)*sqrt( -
sin(e + f*x) + 1)*sin(e + f*x)**2,x)*b - 3*int(sqrt(sin(e + f*x) + 1)*sqrt
( - sin(e + f*x) + 1)*sin(e + f*x),x)*a + int(sqrt(sin(e + f*x) + 1)*sqrt(
 - sin(e + f*x) + 1)*sin(e + f*x),x)*b + int(sqrt(sin(e + f*x) + 1)*sqrt(
- sin(e + f*x) + 1),x)*a)
```

3.132 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 94

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = -\frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a+a\sin(e+fx)}} + \frac{aB\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4cf\sqrt{a+a\sin(e+fx)}}$$

output
```
-1/3*a*(A+B)*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/f/(a+a*sin(f*x+e))^(1/2)+1/
4*a*B*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/c/f/(a+a*sin(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 3.22 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.09

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \frac{c^2\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}(3B\cos(4(e+fx))+16(7A-2B)\sin}{96f}$$

input
```
Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x
])^(5/2),x]
```

output
```
(c^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(3*B
*Cos[4*(e + f*x)] + 16*(7*A - 2*B)*Sin[e + f*x] - 4*Cos[2*(e + f*x)]*(-12*
A + 9*B + 4*(A - 2*B)*Sin[e + f*x])))/(96*f)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3042, 3450, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3450}$$

$$(A+B)\int \sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{5/2}dx-\frac{B\int \sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{7/2}dx}{c}$$

$$\Big\downarrow \text{3042}$$

$$(A+B)\int \sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{5/2}dx-\frac{B\int \sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{7/2}dx}{c}$$

$$\Big\downarrow \text{ 3217}$$

$$\frac{aB\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4cf\sqrt{a\sin(e+fx)+a}}-\frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}$$

input
```
Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/
2),x]
```

output
```
-1/3*(a*(A + B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(f*Sqrt[a + a*Sin
[e + f*x]]) + (a*B*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(4*c*f*Sqrt[a
+ a*Sin[e + f*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3450
```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[B/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] -
Simp[(B*c - A*d)/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[
a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(82) = 164$.

Time = 7.24 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.34

method	result
default	$\frac{16c^2\left(\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2-\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1\right)A\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2+\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1\right)\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\frac{1}{2}\right)\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)}{6f\,\mathrm{c}}$
parts	$\frac{4A\sqrt{4}\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^4+\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2}\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2}\,c^2\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)}{3f}-\frac{2B\,c^2\cos\left(\frac{fx}{2}\right.}{}$

input
```
int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,metho
d=_RETURNVERBOSE)
```

output
```
16*c^2*((cos(1/4*Pi+1/2*f*x+1/2*e)^2-cos(1/4*Pi+1/2*f*x+1/2*e)+1)*A*(cos(1
/4*Pi+1/2*f*x+1/2*e)^2+cos(1/4*Pi+1/2*f*x+1/2*e)+1)*(cos(1/2*f*x+1/2*e)^2-
1/2)*tan(1/4*Pi+1/2*f*x+1/2*e)-2*sin(1/2*f*x+1/2*e)^2*(3/4*cos(1/2*f*x+1/2
*e)^4+sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-3/4*cos(1/2*f*x+1/2*e)^2-3/8)*
B*cos(1/2*f*x+1/2*e)^2)*(c*cos(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)*(a*sin(1/4*P
i+1/2*f*x+1/2*e)^2)^(1/2)/(6*f*cos(1/2*f*x+1/2*e)^2-3*f)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.29

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \frac{(3\,Bc^2\cos(fx+e)^4+12\,(A-B)c^2\cos(fx+e)^2-3\,(4\,A-3\,B)c^2-4\,((A-2\,B)c^2\cos(f}{12\,f\cos(fx+}$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x
, algorithm="fricas")
```

output
```
1/12*(3*B*c^2*cos(f*x + e)^4 + 12*(A - B)*c^2*cos(f*x + e)^2 - 3*(4*A - 3*
B)*c^2 - 4*((A - 2*B)*c^2*cos(f*x + e)^2 - 2*(2*A - B)*c^2)*sin(f*x + e))*
sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx=\text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(1/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx=\int (B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}(-c\sin(fx+e)+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) +
 c)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.60

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx =$$
$$-\frac{4\left(3\,Bc^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^8-2\,Ac^2\mathrm{sgn}\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x
, algorithm="giac")
```

output
```
-4/3*(3*B*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8 - 2*A*c^2*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x
 + 1/2*e)^6 - 2*B*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^6)*sqrt(a)*sqrt(c)/f
```

Mupad [B] (verification not implemented)

Time = 38.34 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.59

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e$$
$$+fx))^{5/2}\,dx = \frac{c^2\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(48\,A\,\cos\left(e+f\,x\right)-36\,B\,\cos\left(e+f\,x\right)+\right.}{}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(
5/2),x)
```

output
```
(c^2*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(48*A*cos(
e + f*x) - 36*B*cos(e + f*x) + 48*A*cos(3*e + 3*f*x) - 33*B*cos(3*e + 3*f*
x) + 3*B*cos(5*e + 5*f*x) + 112*A*sin(2*e + 2*f*x) - 8*A*sin(4*e + 4*f*x)
- 32*B*sin(2*e + 2*f*x) + 16*B*sin(4*e + 4*f*x)))/(96*f*(cos(2*e + 2*f*x)
+ 1))
```

Reduce [F]

$$
\begin{aligned}
&\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e \\
&+fx))^{5/2}\,dx=\sqrt{c}\,\sqrt{a}\,c^2\Bigg(\left(\left(\int \sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^3\,dx\right)b\right. \\
&+\left(\int \sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)a \\
&-2\left(\int \sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)b \\
&-2\left(\int \sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)a \\
&+\left(\int \sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)b \\
&+\left(\int \sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}dx\right)a\Bigg)
\end{aligned}
$$

input
```
int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x)
```

output
```
sqrt(c)*sqrt(a)*c**2*(int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)
*sin(e + f*x)**3,x)*b + int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) +
1)*sin(e + f*x)**2,x)*a - 2*int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x
) + 1)*sin(e + f*x)**2,x)*b - 2*int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e +
 f*x) + 1)*sin(e + f*x),x)*a + int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e +
f*x) + 1)*sin(e + f*x),x)*b + int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f
*x) + 1),x)*a)
```

3.133 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 94

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = -\frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}} + \frac{aB\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3cf\sqrt{a+a\sin(e+fx)}}$$

output
```
-1/2*a*(A+B)*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^(1/2)+1/
3*a*B*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/c/f/(a+a*sin(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 2.52 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.89

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx=\frac{c\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}(2(6A-B)\sin(e+fx)+\cos(2(e+f$$

$$12f$$

input
```
Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x
])^(3/2),x]
```

output
```
(c*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(2*(6*
A - B)*Sin[e + f*x] + Cos[2*(e + f*x)]*(3*A - 3*B + 2*B*Sin[e + f*x])))/(1
2*f)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3042, 3450, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3450}$$

$$(A+B)\int \sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{3/2}dx-\frac{B\int \sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{5/2}dx}{c}$$

$$\Big\downarrow \text{3042}$$

$$(A+B)\int \sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{3/2}dx - \frac{B\int \sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{5/2}dx}{c}$$

$$\Big\downarrow \text{ 3217}$$

$$\frac{aB\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3cf\sqrt{a\sin(e+fx)+a}} - \frac{a(A+B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}$$

input
```
Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/
2),x]
```

output
```
-1/2*(a*(A + B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(f*Sqrt[a + a*Sin
[e + f*x]]) + (a*B*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(3*c*f*Sqrt[a
+ a*Sin[e + f*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3450
```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[B/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] -
Simp[(B*c - A*d)/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[
a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 7.05 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.63

method	result
default	$\frac{12\left(A\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}+1\right)\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-\frac{1}{2}\right)\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{4B\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+B\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{6f\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-3f}$
parts	$\frac{A\sqrt{4}\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}+1\right)\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,c\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{2Bc\sqrt{-\left(2\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}}{}$

input
```
int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,metho
d=_RETURNVERBOSE)
```

output
```
12*(A*(cos(1/4*Pi+1/2*f*x+1/2*e)^2+1)*(cos(1/2*f*x+1/2*e)^2-1/2)*tan(1/4*P
i+1/2*f*x+1/2*e)-4/3*B*sin(1/2*f*x+1/2*e)^3*cos(1/2*f*x+1/2*e)^3+B*sin(1/2
*f*x+1/2*e)^2*cos(1/2*f*x+1/2*e)^2)*c*(c*cos(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2
)*(a*sin(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)/(6*f*cos(1/2*f*x+1/2*e)^2-3*f)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.98

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx=\frac{\left(3\,(A-B)c\cos\left(fx+e\right)^{2}-3\,(A-B)c+2\left(Bc\cos\left(fx+e\right)^{2}+(3\,A-B)c\right)\sin\left(fx+e\right)\right)\sqrt{}}{6\,f\cos\left(fx+e\right)}$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x
, algorithm="fricas")
```

output
```
1/6*(3*(A - B)*c*cos(f*x + e)^2 - 3*(A - B)*c + 2*(B*c*cos(f*x + e)^2 + (3
*A - B)*c)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c
)/(f*cos(f*x + e))
```

Sympy [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(-c(\sin\left(e+fx\right)-1)\right)^{\frac{3}{2}}\left(A+B\sin\left(e+fx\right)\right)\,dx$$

input
```
integrate((a+a*sin(f*x+e))**(1/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)
,x)
```

output
```
Integral(sqrt(a*(sin(e + f*x) + 1))*(-c*(sin(e + f*x) - 1))**(3/2)*(A + B*
sin(e + f*x)), x)
```

Maxima [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \int \left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) +
 c)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.53

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx =$$
$$-\frac{2\left(4\,Bc\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}-3\,Ac\mathrm{sgn}(}{}$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x
, algorithm="giac")
```

output
```
-2/3*(4*B*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x
+ 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^6 - 3*A*c*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1
/2*e)^4 - 3*B*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*
f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^4)*sqrt(a)*sqrt(c)/f
```

Mupad [B] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.30

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e$$
$$+fx))^{3/2}\,dx = \frac{c\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(3\,A\,\cos\left(e+f\,x\right)-3\,B\,\cos\left(e+f\,x\right)+3\right.}{12\,f}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(
3/2),x)
```

output
```
(c*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(3*A*cos(e +
 f*x) - 3*B*cos(e + f*x) + 3*A*cos(3*e + 3*f*x) - 3*B*cos(3*e + 3*f*x) + 1
2*A*sin(2*e + 2*f*x) - 2*B*sin(2*e + 2*f*x) + B*sin(4*e + 4*f*x)))/(12*f*(
cos(2*e + 2*f*x) + 1))
```

Reduce [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx=\sqrt{c}\,\sqrt{a}\,c\left(-\left(\int \sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)b-\left(\int \sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)a+\left(\int \sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)b+\left(\int \sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}dx\right)a\right)$$

input
```
int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x)
```

output
```
sqrt(c)*sqrt(a)*c*( - int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)
*sin(e + f*x)**2,x)*b - int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) +
1)*sin(e + f*x),x)*a + int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1
)*sin(e + f*x),x)*b + int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)
,x)*a)
```

3.134 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\sqrt{c-c\sin($

Optimal result

Integrand size = 40, antiderivative size = 92

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = -\frac{a(A+B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}}+\frac{aB\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2cf\sqrt{a+a\sin(e+fx)}}$$

output
```
-a*(A+B)*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(1/2)+1/2*a*
B*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/c/f/(a+a*sin(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.62

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \frac{\sec(e+fx)\sqrt{a(1+\sin(e+fx))}(A+B\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}{2Bf}$$

input
```
Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e +
 f*x]],x]
```

output
```
(Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*(A + B*Sin[e + f*x])^2*Sqrt[c - c
*Sin[e + f*x]])/(2*B*f)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3042, 3450, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3450}$$

$$(A+B)\int \sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}dx-\frac{B\int \sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{3/2}dx}{c}$$

$$\Big\downarrow \text{3042}$$

$$(A+B)\int \sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}dx-\frac{B\int \sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{3/2}dx}{c}$$

$$\Big\downarrow \text{3217}$$

$$\frac{aB\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2cf\sqrt{a\sin(e+fx)+a}}-\frac{a(A+B)\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}$$

input
```
Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]]
,x]
```

output
```
-((a*(A + B)*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e +
f*x]])) + (a*B*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(2*c*f*Sqrt[a + a*
Sin[e + f*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3450
```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[B/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] -
Simp[(B*c - A*d)/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[
a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 5.31 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.21

method	result
default	$\frac{4\left(A\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-\frac{1}{2}\right)\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+B\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\right)\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}}{2f\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-f}$
parts	$\frac{A\sqrt{4}\,\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{2B\sqrt{\left(2\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)a}\,\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\sqrt{-(2}}{f\left(-1+2\cos\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}$

input
```
int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
4*(A*(cos(1/2*f*x+1/2*e)^2-1/2)*tan(1/4*Pi+1/2*f*x+1/2*e)+B*sin(1/2*f*x+1/
2*e)^2*cos(1/2*f*x+1/2*e)^2)*(c*cos(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)*(a*sin(
1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)/(2*f*cos(1/2*f*x+1/2*e)^2-f)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.66

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = -\frac{\left(B\cos\left(fx+e\right)^2-2\,A\sin\left(fx+e\right)-B\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{2\,f\cos\left(fx+e\right)}$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x
, algorithm="fricas")
```

output
```
-1/2*(B*cos(f*x + e)^2 - 2*A*sin(f*x + e) - B)*sqrt(a*sin(f*x + e) + a)*sq
rt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\left(A+B\sin\left(e+fx\right)\right)\,dx$$

input
```
integrate((a+a*sin(f*x+e))**(1/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)
,x)
```

output
```
Integral(sqrt(a*(sin(e + f*x) + 1))*sqrt(-c*(sin(e + f*x) - 1))*(A + B*sin
(e + f*x)), x)
```

Maxima [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$$
$$=\int (B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
e) + c), x)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.52

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx =$$
$$-\frac{2\left(B\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4-A\mathrm{sgn}(\cos\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x
, algorithm="giac")
```

output
```
-2*(B*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*
e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^4 - A*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 -
B*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*
sin(-1/4*pi + 1/2*f*x + 1/2*e)^2)*sqrt(a)*sqrt(c)/f
```

Mupad [B] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.82

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = -\frac{\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(B\,\cos\left(e+f\,x\right)+B\,\cos\left(3\,e+3\,f\,x\right)-4\,A\,\sin\left(2\,e+\right.\right.}{4\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(
1/2),x)
```

output
```
-((a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(B*cos(e + f*
x) + B*cos(3*e + 3*f*x) - 4*A*sin(2*e + 2*f*x)))/(4*f*(cos(2*e + 2*f*x) +
1))
```

Reduce [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \sqrt{c}\,\sqrt{a}\left(\left(\int \sqrt{\sin\left(fx+e\right)+1}\,\sqrt{-\sin\left(fx+e\right)+1}\,\sin\left(fx+e\right)dx\right)b + \left(\int \sqrt{\sin\left(fx+e\right)+1}\,\sqrt{-\sin\left(fx+e\right)+1}dx\right)a\right)$$

input
```
int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)
```

output
```
sqrt(c)*sqrt(a)*(int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(
e + f*x),x)*b + int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1),x)*a)
```

3.135 $\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 100

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{a(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{aB\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{cf\sqrt{a+a\sin(e+fx)}}$$

output
```
-a*(A+B)*cos(f*x+e)*ln(1-sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x
+e))^(1/2)+a*B*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/c/f/(a+a*sin(f*x+e))^(1/2
)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.47 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\left((A+B)\left(-ifx+2\log\left(i-e^{i(e+fx)}\right)\right)+B\right.}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}}$$

input
```
Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e
 + f*x]],x]
```

output
```
-(((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*((A +
B)*((-I)*f*x + 2*Log[I - E^(I*(e + f*x))]) + B*Sin[e + f*x]))/(f*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]))
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3450, 3042, 3216, 3042, 3146, 16, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a \sin(e+fx)+a}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a \sin(e+fx)+a}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3450}$$

$$(A+B)\int \frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{B\int \sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}dx}{c}$$

$$\Big\downarrow \text{3042}$$

$$(A+B)\int \frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{B\int \sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}dx}{c}$$

$$\Big\downarrow \text{3216}$$

$$\frac{ac(A+B)\cos(e+fx)\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{B\int \sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}dx}{c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{ac(A+B)\cos(e+fx)\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{B\int \sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}dx}{c}$$

$$\Big\downarrow \text{3146}$$

$$-\frac{a(A+B)\cos(e+fx)\int \frac{1}{c-c\sin(e+fx)}d(-c\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{B\int \sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}dx}{c}$$

$$\Big\downarrow \text{16}$$

$$-\frac{B\int \sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}dx}{c}-\frac{a(A+B)\cos(e+fx)\log(c-c\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{3217}$$

$$\frac{aB\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{cf\sqrt{a\sin(e+fx)+a}}-\frac{a(A+B)\cos(e+fx)\log(c-c\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

input
```
Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x
]],x]
```

output
```
-((a*(A + B)*Cos[e + f*x]*Log[c - c*Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f
*x]]*Sqrt[c - c*Sin[e + f*x]])) + (a*B*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x
]])/(c*f*Sqrt[a + a*Sin[e + f*x]])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3450
```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[B/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] -
Simp[(B*c - A*d)/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[
a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(92) = 184$.

Time = 4.67 (sec) , antiderivative size = 315, normalized size of antiderivative = 3.15

method	result
default	$-\frac{A\sqrt{4}\left(\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1\right)+\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)-1\right)-\ln\left(\frac{2}{\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\right)\sqrt{}}{f\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}}$
parts	$-\frac{A\sqrt{4}\left(\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1\right)+\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)-1\right)-\ln\left(\frac{2}{\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\right)\sqrt{}}{f\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}}$

input
```
int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
-A/f*4^(1/2)*(ln(-cot(1/4*Pi+1/2*f*x+1/2*e)+csc(1/4*Pi+1/2*f*x+1/2*e)+1)+l
n(-cot(1/4*Pi+1/2*f*x+1/2*e)+csc(1/4*Pi+1/2*f*x+1/2*e)-1)-ln(2/(cos(1/4*Pi
+1/2*f*x+1/2*e)+1)))*(a*sin(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)/(c*cos(1/4*Pi+1
/2*f*x+1/2*e)^2)^(1/2)*cot(1/4*Pi+1/2*f*x+1/2*e)+2*B/f*(sin(1/2*f*x+1/2*e)
-cos(1/2*f*x+1/2*e))*(sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+ln(2*(sin(1/2*
f*x+1/2*e)-cos(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1))-ln(2/(cos(1/2*f*x+1
/2*e)+1)))*((2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+1)*a)^(1/2)/(cos(1/2*
f*x+1/2*e)+sin(1/2*f*x+1/2*e))/(-(2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-
1)*c)^(1/2)
```

Fricas [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x
, algorithm="fricas")
```

output
```
integral(-(B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
e) + c)/(c*sin(f*x + e) - c), x)
```

Sympy [F]

$$\int \frac{\sqrt{a + a\sin(e + fx)}(A + B\sin(e + fx))}{\sqrt{c - c\sin(e + fx)}}\,dx = \int \frac{\sqrt{a\left(\sin\left(e + fx\right) + 1\right)}(A + B\sin\left(e + fx\right))}{\sqrt{-c\left(\sin\left(e + fx\right) - 1\right)}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))**(1/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2)
,x)
```

output
```
Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))/sqrt(-c*(sin(e +
f*x) - 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.75

$$\int \frac{\sqrt{a + a\sin(e + fx)}(A + B\sin(e + fx))}{\sqrt{c - c\sin(e + fx)}}\,dx = \frac{B\left(\frac{2\sqrt{a}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{\sqrt{c}} - \frac{\sqrt{a}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{\sqrt{c}} + \frac{2\sqrt{a}\sqrt{c}\sin(fx+e)}{\left(c+\frac{c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}\right) + A\left(\frac{2\sqrt{a}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-\right.}{\sqrt{c}}\right.}{f}$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x
, algorithm="maxima")
```

output
```
(B*(2*sqrt(a)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/sqrt(c) - sqrt(a)*l
og(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/sqrt(c) + 2*sqrt(a)*sqrt(c)*si
n(f*x + e)/((c + c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)
)) + A*(2*sqrt(a)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/sqrt(c) - sqrt(
a)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/sqrt(c)))/f
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,\sqrt{a+a\,\sin(e+f\,x)}}{\sqrt{c-c\,\sin(e+f\,x)}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))
^(1/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))
^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{\sqrt{c}\,\sqrt{a}\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)-1}dx\right)b+\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)-1}dx\right)a\right)}{c}$$

input
```
int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

output
```
( - sqrt(c)*sqrt(a)*(int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)
*sin(e + f*x))/(sin(e + f*x) - 1),x)*b + int((sqrt(sin(e + f*x) + 1)*sqrt(
 - sin(e + f*x) + 1))/(sin(e + f*x) - 1),x)*a))/c
```

3.136 $\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 99

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{a(A+B)\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} + \frac{aB\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

output
```
a*(A+B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2)+a*B*cos
(f*x+e)*ln(1-sin(f*x+e))/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.31 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.48

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx)}}{f\left(\cos\left(\frac{1}{2}(e+f\right.\right.}$$

input
```
Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f
*x])^(3/2),x]
```

output
```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(A + B -
 I*B*f*x + 2*B*Log[I - E^(I*(e + f*x))] + I*B*(f*x + (2*I)*Log[I - E^(I*(e
 + f*x))])*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*
Sin[e + f*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3450, 3042, 3216, 3042, 3146, 16, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3450}$$

$$(A+B)\int \frac{\sqrt{\sin(e+fx)a+a}}{(c-c\sin(e+fx))^{3/2}}dx-\frac{B\int \frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx}{c}$$

$$\Big\downarrow \text{3042}$$

$$(A+B)\int \frac{\sqrt{\sin(e+fx)a+a}}{(c-c\sin(e+fx))^{3/2}}dx-\frac{B\int \frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx}{c}$$

$$\Big\downarrow \text{3216}$$

$$(A+B)\int \frac{\sqrt{\sin(e+fx)a+a}}{(c-c\sin(e+fx))^{3/2}}dx-\frac{aB\cos(e+fx)\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$(A+B)\int \frac{\sqrt{\sin(e+fx)a+a}}{(c-c\sin(e+fx))^{3/2}}dx-\frac{aB\cos(e+fx)\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{3146}$$

$$(A+B)\int \frac{\sqrt{\sin(e+fx)a+a}}{(c-c\sin(e+fx))^{3/2}}dx+\frac{aB\cos(e+fx)\int \frac{1}{c-c\sin(e+fx)}d(-c\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{16}$$

$$(A+B)\int \frac{\sqrt{\sin(e+fx)a+a}}{(c-c\sin(e+fx))^{3/2}}dx+\frac{aB\cos(e+fx)\log(c-c\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{3217}$$

$$\frac{a(A+B)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{aB\cos(e+fx)\log(c-c\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

input
```
Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(
3/2),x]
```

output
```
(a*(A + B)*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^
(3/2)) + (a*B*Cos[e + f*x]*Log[c - c*Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e
+ f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3450
```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[B/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] -
Simp[(B*c - A*d)/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[
a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(91) = 182$.

Time = 7.80 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.63

method	result
parts	$\frac{A\sqrt{4}\,\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)}{4fc\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}}+\frac{2B\left(\left(-2\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\ln\left(\frac{2\sin\left(\frac{fx}{2}+\frac{e}{2}\right)-2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)+\left(2\sin\left(\right.\right.\right.}{cf\sqrt{-\left(2\sin\left(\frac{fx}{2}\right.\right.}}$
default	$\frac{A\sqrt{4}\,\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)}{4fc\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}}-\frac{2B\left(\left(2\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\ln\left(\frac{2\sin\left(\frac{fx}{2}+\frac{e}{2}\right)-2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)+\left(-2\sin\left(\right.\right.\right.}{cf\sqrt{-\left(2\sin\left(\frac{fx}{2}\right.\right.}}$

input
```
int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/4*A/f*4^(1/2)*(a*sin(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)/c/(c*cos(1/4*Pi+1/2*
f*x+1/2*e)^2)^(1/2)*tan(1/4*Pi+1/2*f*x+1/2*e)+2*B/c/f*((-2*sin(1/2*f*x+1/2
*e)*cos(1/2*f*x+1/2*e)+1)*ln(2*(sin(1/2*f*x+1/2*e)-cos(1/2*f*x+1/2*e))/(co
s(1/2*f*x+1/2*e)+1))+(2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-1)*ln(2/(cos
(1/2*f*x+1/2*e)+1))+sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e))*((2*sin(1/2*f*x
+1/2*e)*cos(1/2*f*x+1/2*e)+1)*a)^(1/2)/(-(2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x
+1/2*e)-1)*c)^(1/2)/(-1+2*cos(1/2*f*x+1/2*e)^2)
```

Fricas [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}}{(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x
, algorithm="fricas")
```

output
```
integral(-(B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
e) + c)/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{a(\sin(e+fx)+1)}(A+B\sin(e+fx))}{(-c(\sin(e+fx)-1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))**(1/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2)
,x)
```

output
```
Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))/(-c*(sin(e + f*x)
 - 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=\int \frac{(B\sin{(fx+e)}+A)\sqrt{a\sin{(fx+e)}+a}}{(-c\sin{(fx+e)}+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e) +
 c)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=\int \frac{(A+B\,\sin{(e+f\,x)})\,\sqrt{a+a\,\sin{(e+f\,x)}}}{(c-c\,\sin{(e+f\,x)})^{3/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))
^(3/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))
^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)^2-2\sin(fx+e)+1}dx\right)b+\left(\int\right.\right.}{c^2}$$

input
```
int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*(int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*si
n(e + f*x))/(sin(e + f*x)**2 - 2*sin(e + f*x) + 1),x)*b + int((sqrt(sin(e
+ f*x) + 1)*sqrt( - sin(e + f*x) + 1))/(sin(e + f*x)**2 - 2*sin(e + f*x) +
 1),x)*a))/c**2
```

3.137 $\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 92

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx=\frac{a(A+B)\cos(e+fx)}{2f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}-\frac{aB\cos(e+fx)}{cf\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}$$

output
```
1/2*a*(A+B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2)-a*B
*cos(f*x+e)/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2)
```

Mathematica [A] (verified)

Time = 4.16 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx=\frac{\sqrt{a(1+\sin(e+fx))}(A-B+2B\sin(e+fx))\sqrt{c}}{2c^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\left(\cos\left(\frac{1}{2}(e+f\right.\right.}$$

input
```
Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f
*x])^(5/2),x]
```

output
```
(Sqrt[a*(1 + Sin[e + f*x])]*(A - B + 2*B*Sin[e + f*x])*Sqrt[c - c*Sin[e +
f*x]])/(2*c^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2]))
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3042, 3450, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3450}$$

$$(A+B)\int \frac{\sqrt{\sin(e+fx)a+a}}{(c-c\sin(e+fx))^{5/2}}dx-\frac{B\int \frac{\sqrt{\sin(e+fx)a+a}}{(c-c\sin(e+fx))^{3/2}}dx}{c}$$

$$\Big\downarrow \text{3042}$$

$$(A+B)\int \frac{\sqrt{\sin(e+fx)a+a}}{(c-c\sin(e+fx))^{5/2}}dx-\frac{B\int \frac{\sqrt{\sin(e+fx)a+a}}{(c-c\sin(e+fx))^{3/2}}dx}{c}$$

$$\Big\downarrow \text{3217}$$

$$\frac{a(A+B)\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}-\frac{aB\cos(e+fx)}{cf\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}$$

input
```
Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(
5/2),x]
```

output
```
(a*(A + B)*Cos[e + f*x])/(2*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x]
)^(5/2)) - (a*B*Cos[e + f*x])/(c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e +
 f*x])^(3/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3450
```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[B/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] -
Simp[(B*c - A*d)/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[
a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(82) = 164$.

Time = 8.05 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.91

method	result
default	$\frac{\left(\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-\frac{1}{2}\right)A\left(\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{2}\right)\left(1+\sec\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}\right)\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)-4B\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\right)}{2\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,f\,c^{2}\left(-1+2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\right)\left(2\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}$
parts	$\frac{A\sqrt{4}\,\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\left(\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\sec\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}\right)}{16f\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,c^{2}}-\frac{2B\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\sin\left(\frac{fx}{2}+\right.}{c^{2}f\left(-1+2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\right)\left(2\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}$

input
```
int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/2/(c*cos(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)*((cos(1/2*f*x+1/2*e)^2-1/2)*A*(s
in(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-1/2)*(1+sec(1/4*Pi+1/2*f*x+1/2*e)^2)*
tan(1/4*Pi+1/2*f*x+1/2*e)-4*B*sin(1/2*f*x+1/2*e)^2*cos(1/2*f*x+1/2*e)^2)*(
a*sin(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)/f/c^2/(-1+2*cos(1/2*f*x+1/2*e)^2)/(2*
sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = -\frac{(2\,B\sin(fx+e)+A-B)\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{2\left(c^3f\cos(fx+e)^3+2\,c^3f\cos(fx+e)\sin(fx+e)-2\,c^3f\cos(fx+e)\right)}$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x
, algorithm="fricas")
```

output
```
-1/2*(2*B*sin(f*x + e) + A - B)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
 e) + c)/(c^3*f*cos(f*x + e)^3 + 2*c^3*f*cos(f*x + e)*sin(f*x + e) - 2*c^3
*f*cos(f*x + e))
```

Sympy [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{a(\sin(e+fx)+1)}(A+B\sin(e+fx))}{(-c(\sin(e+fx)-1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))**(1/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2)
,x)
```

output
```
Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))/(-c*(sin(e + f*x)
 - 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx=\int \frac{(B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}}{(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e) +
 c)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx=\int \frac{(A+B\,\sin\left(e+f\,x\right))\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))
^(5/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))
^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx=-\frac{\sqrt{c}\,\sqrt{a}\,\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)^{3}-3\sin(fx+e)^{2}+3\sin(fx+e)-1}dx\right)b+\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)^{3}-3\sin(fx+e)^{2}+3\sin(fx+e)-1}dx\right)a\right)}{c^{3}}$$

input
```
int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)
```

output
```
( - sqrt(c)*sqrt(a)*(int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)
*sin(e + f*x))/(sin(e + f*x)**3 - 3*sin(e + f*x)**2 + 3*sin(e + f*x) - 1),
x)*b + int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1))/(sin(e + f*x
)**3 - 3*sin(e + f*x)**2 + 3*sin(e + f*x) - 1),x)*a))/c**3
```

3.138 $\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 94

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx=\frac{a(A+B)\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}}-\frac{aB\cos(e+fx)}{2cf\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}$$

output
```
1/3*a*(A+B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2)-1/2
*a*B*cos(f*x+e)/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2)
```

Mathematica [A] (verified)

Time = 5.88 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx=\frac{\sqrt{a(1+\sin(e+fx))}(2A-B+3B\sin(e+fx))\sqrt{c}}{6c^4f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx\right.\right.}$$

input
```
Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f
*x])^(7/2),x]
```

output
```
(Sqrt[a*(1 + Sin[e + f*x])]*(2*A - B + 3*B*Sin[e + f*x])*Sqrt[c - c*Sin[e
+ f*x]])/(6*c^4*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2]))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3042, 3450, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{3450}$$

$$(A+B)\int \frac{\sqrt{\sin(e+fx)a+a}}{(c-c\sin(e+fx))^{7/2}}dx-\frac{B\int \frac{\sqrt{\sin(e+fx)a+a}}{(c-c\sin(e+fx))^{5/2}}dx}{c}$$

$$\Big\downarrow \text{3042}$$

$$(A+B)\int \frac{\sqrt{\sin(e+fx)a+a}}{(c-c\sin(e+fx))^{7/2}}dx-\frac{B\int \frac{\sqrt{\sin(e+fx)a+a}}{(c-c\sin(e+fx))^{5/2}}dx}{c}$$

$$\Big\downarrow \text{3217}$$

$$\frac{a(A+B)\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}}-\frac{aB\cos(e+fx)}{2cf\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}$$

input
```
Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(
7/2),x]
```

output
```
(a*(A + B)*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x]
)^(7/2)) - (a*B*Cos[e + f*x])/(2*c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e
 + f*x])^(5/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3450
```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[B/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] -
Simp[(B*c - A*d)/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[
a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. 2(82) = 164.

Time = 8.71 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.97

method	result
parts	$\frac{A\sqrt{4}\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}+\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}+1\right)\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\sec\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}}{48f\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,c^{3}}+\frac{2B\sqrt{}}{3c^{3}f\left(-1+2\cos\left(\frac{fx}{2}\right.\right.}$
default	$\frac{\left(\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}+\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-\frac{1}{4}\right)A\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}-\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1\right)\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}+\right.\right.}{3\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,f\,c^{3}\left(-1+2\cos\left(\frac{fx}{2}\right.\right.}$

input
```
int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/48*A/f*4^(1/2)*(cos(1/4*Pi+1/2*f*x+1/2*e)^4+cos(1/4*Pi+1/2*f*x+1/2*e)^2+
1)*(a*sin(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)/(c*cos(1/4*Pi+1/2*f*x+1/2*e)^2)^(
1/2)/c^3*tan(1/4*Pi+1/2*f*x+1/2*e)*sec(1/4*Pi+1/2*f*x+1/2*e)^4+2/3*B/c^3/f
*((2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+1)*a)^(1/2)*cos(1/2*f*x+1/2*e)^
2*(2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-3)*sin(1/2*f*x+1/2*e)^2/(-1+2*c
os(1/2*f*x+1/2*e)^2)/(4*cos(1/2*f*x+1/2*e)^4+4*sin(1/2*f*x+1/2*e)*cos(1/2*
f*x+1/2*e)-4*cos(1/2*f*x+1/2*e)^2-1)/(-(2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1
/2*e)-1)*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = -\frac{(3\,B\sin(fx+e)+2\,A-B)\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{6\,\left(3\,c^4f\cos(fx+e)^3-4\,c^4f\cos(fx+e)-\left(c^4f\cos(fx+e)^3-4\,c^4f\cos(fx+e)\right)\sin(fx+e)\right)}$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x
, algorithm="fricas")
```

output
```
-1/6*(3*B*sin(f*x + e) + 2*A - B)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
 + e) + c)/(3*c^4*f*cos(f*x + e)^3 - 4*c^4*f*cos(f*x + e) - (c^4*f*cos(f*x
 + e)^3 - 4*c^4*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(1/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2)
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \int \frac{(B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}}{(-c\sin(fx+e)+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e) +
 c)^(7/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 43.00 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.63

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx =$$
$$-\frac{2\,A\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}-B\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}+3\,B\,\sin\left(\right.}{\frac{9\,c^4\,f\,\cos\left(3\,e+3\,f\,x\right)}{2}+\frac{21\,c^4\,f\,\sin\left(2\,e+2\,f\,x\right)}{2}-\frac{3\,c^4\,f\,\sin\left(4\,e+4\,f\,x\right)}{4}-\frac{21\,c^4\,f\,}{}}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))
^(7/2),x)
```

output
```
-(2*A*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2) - B*(a + a*sin
(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2) + 3*B*sin(e + f*x)*(a + a*sin(
e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2))/((9*c^4*f*cos(3*e + 3*f*x))/2
+ (21*c^4*f*sin(2*e + 2*f*x))/2 - (3*c^4*f*sin(4*e + 4*f*x))/4 - (21*c^4*f
*cos(e + f*x))/2)
```

Reduce [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)^4-4\sin(fx+e)^3+6\sin(fx+e)^2-4\sin(fx+e)+1}dx\right.\right.}{}$$

input
```
int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x)
```

output
```
(sqrt(c)*sqrt(a)*(int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*si
n(e + f*x))/(sin(e + f*x)**4 - 4*sin(e + f*x)**3 + 6*sin(e + f*x)**2 - 4*s
in(e + f*x) + 1),x)*b + int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) +
 1))/(sin(e + f*x)**4 - 4*sin(e + f*x)**3 + 6*sin(e + f*x)**2 - 4*sin(e +
f*x) + 1),x)*a))/c**4
```

3.139 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 146

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = -\frac{a^2(3A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{30f\sqrt{a+a\sin(e+fx)}} - \frac{a(3A-B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}}{15f} - \frac{B\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}{6f}$$

output
```
-1/30*a^2*(3*A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/f/(a+a*sin(f*x+e))^(1/
2)-1/15*a*(3*A-B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(7/2)
/f-1/6*B*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/f
```

Mathematica [A] (verified)

Time = 9.49 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.40

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx =$$
$$-\frac{c^3(-1+\sin(e+fx))^3(a(1+\sin(e+fx)))^{3/2}\sqrt{c-c\sin(e+fx)}(15(16A-11B)\cos(2(e+fx))+30(2\ldots}{960f\ldots}$$

input
```
Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f
*x])^(7/2),x]
```

output
```
-1/960*(c^3*(-1 + Sin[e + f*x])^3*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c - c*
Sin[e + f*x]]*(15*(16*A - 11*B)*Cos[2*(e + f*x)] + 30*(2*A - B)*Cos[4*(e +
 f*x)] + 5*B*Cos[6*(e + f*x)] + 840*A*Sin[e + f*x] - 240*B*Sin[e + f*x] +
60*A*Sin[3*(e + f*x)] + 40*B*Sin[3*(e + f*x)] - 12*A*Sin[5*(e + f*x)] + 24
*B*Sin[5*(e + f*x)]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^3)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3452, 3042, 3219, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3452}$$

$$\frac{1}{3}(3A-B)\int (\sin(e+fx)a+a)^{3/2}(c-c\sin(e+fx))^{7/2}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{7/2}}{6f}$$

$\big\downarrow$ 3042

$$\frac{1}{3}(3A-B)\int (\sin(e+fx)a+a)^{3/2}(c-c\sin(e+fx))^{7/2}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{7/2}}{6f}$$

$\big\downarrow$ 3219

$$\frac{1}{3}(3A-B)\left(\frac{2}{5}a\int \sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{7/2}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}}{5f}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{7/2}}{6f}$$

$\big\downarrow$ 3042

$$\frac{1}{3}(3A-B)\left(\frac{2}{5}a\int \sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{7/2}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}}{5f}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{7/2}}{6f}$$

$\big\downarrow$ 3217

$$\frac{1}{3}(3A-B)\left(-\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{10f\sqrt{a\sin(e+fx)+a}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}}{5f}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{7/2}}{6f}$$

input
```
Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(
7/2),x]
```

output
```
-1/6*(B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(7/2)
)/f + ((3*A - B)*(-1/10*(a^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(f*S
qrt[a + a*Sin[e + f*x]]) - (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c
*Sin[e + f*x])^(7/2))/(5*f)))/3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3452
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. 2(128) = 256.

Time = 7.95 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.93

method	result
default	$\frac{192c^3a\left(A\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^6+\frac{3\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^4}{4}+\frac{\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2}{2}+\frac{1}{4}\right)\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\frac{1}{2}\right)\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2-}{}$
parts	$\frac{4A\sqrt{4}\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2}\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2}\left(4\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^6+3\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^4+2\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)ac^3\sin\left(\frac{\pi}{4}+\right.}{5f}$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,metho
d=_RETURNVERBOSE)
```

output
```
192*c^3*a*(A*(cos(1/4*Pi+1/2*f*x+1/2*e)^6+3/4*cos(1/4*Pi+1/2*f*x+1/2*e)^4+
1/2*cos(1/4*Pi+1/2*f*x+1/2*e)^2+1/4)*tan(1/4*Pi+1/2*f*x+1/2*e)*(cos(1/2*f*
x+1/2*e)^2-1/2)*sin(1/4*Pi+1/2*f*x+1/2*e)^2-2*sin(1/2*f*x+1/2*e)^2*(5/6*co
s(1/2*f*x+1/2*e)^8+cos(1/2*f*x+1/2*e)^5*sin(1/2*f*x+1/2*e)-5/3*cos(1/2*f*x
+1/2*e)^6-cos(1/2*f*x+1/2*e)^3*sin(1/2*f*x+1/2*e)+5/6*cos(1/2*f*x+1/2*e)^4
+5/12*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-5/32)*B*cos(1/2*f*x+1/2*e)^2)*
(c*cos(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)*(a*sin(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2
)/(30*f*cos(1/2*f*x+1/2*e)^2-15*f)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.01

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \frac{\left(5\,Bac^3\cos\left(fx+e\right)^6+15\,(A-B)ac^3\cos\left(fx+e\right)^4-5\,(3\,A-2\,B)ac^3-2\,(3\,(A-\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x
, algorithm="fricas")
```

output
```
1/30*(5*B*a*c^3*cos(f*x + e)^6 + 15*(A - B)*a*c^3*cos(f*x + e)^4 - 5*(3*A
- 2*B)*a*c^3 - 2*(3*(A - 2*B)*a*c^3*cos(f*x + e)^4 - 2*(3*A - B)*a*c^3*cos
(f*x + e)^2 - 4*(3*A - B)*a*c^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sq
rt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx=\text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}(-c\sin(fx+e)+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e)
 + c)^(7/2), x)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.69

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \frac{8\left(20\,Bac^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x
, algorithm="giac")
```

output
```
8/15*(20*B*a*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^12 - 12*A*a*c^3*sgn(cos(-1/4
*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi +
1/2*f*x + 1/2*e)^10 - 36*B*a*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(s
in(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^10 + 15*A*a*
c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)
)*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8 + 15*B*a*c^3*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*
e)^8)*sqrt(a)*sqrt(c)/f
```

Mupad [B] (verification not implemented)

Time = 42.47 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.21

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \frac{\mathrm{e}^{-e\,6\mathrm{i}-f\,x\,6\mathrm{i}}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{B\,a\,c^3\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\cos(6\,e+6\,f\,x)\,\sqrt{a+a\,\sin(e+f\,x)}}{96\,f}-\frac{a\,c^3\,\mathrm{e}^{e\,6\mathrm{i}+f\,x}}{}\right.}{}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(
7/2),x)
```

output
```
(exp(- e*6i - f*x*6i)*(c - c*sin(e + f*x))^(1/2)*((B*a*c^3*exp(e*6i + f*x*
6i)*cos(6*e + 6*f*x)*(a + a*sin(e + f*x))^(1/2))/(96*f) - (a*c^3*exp(e*6i
+ f*x*6i)*sin(e + f*x)*(A*7i - B*2i)*(a + a*sin(e + f*x))^(1/2)*1i)/(4*f)
+ (a*c^3*exp(e*6i + f*x*6i)*cos(4*e + 4*f*x)*(2*A - B)*(a + a*sin(e + f*x)
)^(1/2))/(16*f) + (a*c^3*exp(e*6i + f*x*6i)*cos(2*e + 2*f*x)*(16*A - 11*B)
*(a + a*sin(e + f*x))^(1/2))/(32*f) - (a*c^3*exp(e*6i + f*x*6i)*sin(3*e +
3*f*x)*(A*3i + B*2i)*(a + a*sin(e + f*x))^(1/2)*1i)/(24*f) + (a*c^3*exp(e*
6i + f*x*6i)*sin(5*e + 5*f*x)*(A*1i - B*2i)*(a + a*sin(e + f*x))^(1/2)*1i)
/(40*f)))/(2*cos(e + f*x))
```

Reduce [F]

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c -c\sin(e+fx))^{7/2}\,dx = \sqrt{c}\,\sqrt{a}\,a\,c^3\left(-\left(\int \sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^5\,dx\right)b-\left(\int\right.\right.$$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x)
```

output
```
sqrt(c)*sqrt(a)*a*c**3*( - int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x)
 + 1)*sin(e + f*x)**5,x)*b - int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*
x) + 1)*sin(e + f*x)**4,x)*a + 2*int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e
+ f*x) + 1)*sin(e + f*x)**4,x)*b + 2*int(sqrt(sin(e + f*x) + 1)*sqrt( - si
n(e + f*x) + 1)*sin(e + f*x)**3,x)*a - 2*int(sqrt(sin(e + f*x) + 1)*sqrt(
- sin(e + f*x) + 1)*sin(e + f*x)**2,x)*b - 2*int(sqrt(sin(e + f*x) + 1)*sq
rt( - sin(e + f*x) + 1)*sin(e + f*x),x)*a + int(sqrt(sin(e + f*x) + 1)*sqr
t( - sin(e + f*x) + 1)*sin(e + f*x),x)*b + int(sqrt(sin(e + f*x) + 1)*sqrt
( - sin(e + f*x) + 1),x)*a)
```

3.140 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 146

$$\begin{aligned}\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = &-\frac{a^2(5A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{30f\sqrt{a+a\sin(e+fx)}}\\&-\frac{a(5A-B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}{20f}\\&-\frac{B\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}{5f}\end{aligned}$$

output
```
-1/30*a^2*(5*A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/f/(a+a*sin(f*x+e))^(1/
2)-1/20*a*(5*A-B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(5/2)
/f-1/5*B*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/f
```

Mathematica [A] (verified)

Time = 8.44 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.18

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx=\frac{c^2(-1+\sin(e+fx))^2(a(1+\sin(e+fx)))^{3/2}\sqrt{c-c\sin(e+fx)}(4(100A-11B)\text{s}}{480f\left(\cos\left(\frac{1}{2}(e+fx)\right)\right.}$$

input
```
Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f
*x])^(5/2),x]
```

output
```
(c^2*(-1 + Sin[e + f*x])^2*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c - c*Sin[e +
 f*x]]*(4*(100*A - 11*B)*Sin[e + f*x] + 3*Cos[4*(e + f*x)]*(5*A - 5*B + 4*
B*Sin[e + f*x]) + 4*Cos[2*(e + f*x)]*(15*(A - B) + 4*(5*A + 2*B)*Sin[e + f
*x])))/(480*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^3)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3042, 3452, 3042, 3219, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3452}$$

$$\frac{1}{5}(5A-B)\int (\sin(e+fx)a+a)^{3/2}(c-c\sin(e+fx))^{5/2}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}{5f}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}(5A-B)\int (\sin(e+fx)a+a)^{3/2}(c-c\sin(e+fx))^{5/2}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}{5f}$$

$\Big\downarrow$ 3219

$$\frac{1}{5}(5A-B)\left(\frac{1}{2}a\int \sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{5/2}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}{4f}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}{5f}$$

$\Big\downarrow$ 3042

$$\frac{1}{5}(5A-B)\left(\frac{1}{2}a\int \sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{5/2}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}{4f}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}{5f}$$

$\Big\downarrow$ 3217

$$\frac{1}{5}(5A-B)\left(-\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{6f\sqrt{a\sin(e+fx)+a}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}{4f}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}{5f}$$

input
```
Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(
5/2),x]
```

output
```
-1/5*(B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2)
)/f + ((5*A - B)*(-1/6*(a^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(f*Sq
rt[a + a*Sin[e + f*x]]) - (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*
Sin[e + f*x])^(5/2))/(4*f)))/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3452
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

Maple [A] (verified)

Time = 7.42 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.73

method	result
default	$\frac{120\left(A\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}+\frac{2\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}{3}+\frac{1}{3}\right)\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-\frac{1}{2}\right)\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}-\frac{8\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\left(\cos\left(\frac{f}{}\right.\right.}{}\right.\right.}{}$
parts	$\frac{2A\sqrt{4}\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\left(3\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}+2\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}+1\right)a\,c^{2}\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}\tan\left(\frac{\pi}{4}+\frac{fx}{2}\right.}{3f}$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,metho
d=_RETURNVERBOSE)
```

output
```
120*(A*(cos(1/4*Pi+1/2*f*x+1/2*e)^4+2/3*cos(1/4*Pi+1/2*f*x+1/2*e)^2+1/3)*t
an(1/4*Pi+1/2*f*x+1/2*e)*(cos(1/2*f*x+1/2*e)^2-1/2)*sin(1/4*Pi+1/2*f*x+1/2
*e)^2-8/5*sin(1/2*f*x+1/2*e)^2*(cos(1/2*f*x+1/2*e)^5*sin(1/2*f*x+1/2*e)-co
s(1/2*f*x+1/2*e)^3*sin(1/2*f*x+1/2*e)-5/8*cos(1/2*f*x+1/2*e)^4+5/12*sin(1/
2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+5/8*cos(1/2*f*x+1/2*e)^2-5/16)*B*cos(1/2*f
*x+1/2*e)^2)*c^2*a*(c*cos(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)*(a*sin(1/4*Pi+1/2
*f*x+1/2*e)^2)^(1/2)/(30*f*cos(1/2*f*x+1/2*e)^2-15*f)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.86

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \frac{\left(15\,(A-B)ac^{2}\cos\left(fx+e\right)^{4}-15\,(A-B)ac^{2}+4\left(3\,Bac^{2}\cos\left(fx+e\right)^{4}+(5\,A-\right.\right.}{60\,}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x
, algorithm="fricas")
```

output
```
1/60*(15*(A - B)*a*c^2*cos(f*x + e)^4 - 15*(A - B)*a*c^2 + 4*(3*B*a*c^2*co
s(f*x + e)^4 + (5*A - B)*a*c^2*cos(f*x + e)^2 + 2*(5*A - B)*a*c^2)*sin(f*x
 + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}(-c\sin(fx+e)+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e)
 + c)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.69

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \frac{4\left(24\,Bac^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x
, algorithm="giac")
```

output
```
4/15*(24*B*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^10 - 15*A*a*c^2*sgn(cos(-1/4
*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi +
1/2*f*x + 1/2*e)^8 - 45*B*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(si
n(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8 + 20*A*a*c^
2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*
sin(-1/4*pi + 1/2*f*x + 1/2*e)^6 + 20*B*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)
^6)*sqrt(a)*sqrt(c)/f
```

Mupad [B] (verification not implemented)

Time = 41.06 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.19

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \frac{a\,c^2\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(60\,A\,\cos\left(e+f\,x\right)-60\,B\,\cos\right.}{}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(
5/2),x)
```

output
```
(a*c^2*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(60*A*co
s(e + f*x) - 60*B*cos(e + f*x) + 75*A*cos(3*e + 3*f*x) + 15*A*cos(5*e + 5*
f*x) - 75*B*cos(3*e + 3*f*x) - 15*B*cos(5*e + 5*f*x) + 400*A*sin(2*e + 2*f
*x) + 40*A*sin(4*e + 4*f*x) - 50*B*sin(2*e + 2*f*x) + 16*B*sin(4*e + 4*f*x
) + 6*B*sin(6*e + 6*f*x)))/(480*f*(cos(2*e + 2*f*x) + 1))
```

Reduce [F]

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx=\sqrt{c}\,\sqrt{a}\,a\,c^2\left(\left(\left(\int\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^4\,dx\right)b+\left(\int\right.\right.\right.$$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x)
```

output
```
sqrt(c)*sqrt(a)*a*c**2*(int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) +
1)*sin(e + f*x)**4,x)*b + int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x)
+ 1)*sin(e + f*x)**3,x)*a - int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x
) + 1)*sin(e + f*x)**3,x)*b - int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f
*x) + 1)*sin(e + f*x)**2,x)*a - int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e +
 f*x) + 1)*sin(e + f*x)**2,x)*b - int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e
 + f*x) + 1)*sin(e + f*x),x)*a + int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e
+ f*x) + 1)*sin(e + f*x),x)*b + int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e +
 f*x) + 1),x)*a)
```

3.141 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 134

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = -\frac{a^2A\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{3f\sqrt{a+a\sin(e+fx)}} - \frac{aA\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}{3f} - \frac{B\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}{4f}$$

output
```
-1/3*a^2*A*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^(1/2)-1/3*
a*A*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(3/2)/f-1/4*B*cos(f
*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/f
```

Mathematica [A] (verified)

Time = 2.86 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.74

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx=\frac{c\sec^3(e+fx)(-1+\sin(e+fx))(a(1+\sin(e+fx)))^{3/2}\sqrt{c-c\sin(e+fx)}(9B+}{96}$$

input
```
Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f
*x])^(3/2),x]
```

output
```
(c*Sec[e + f*x]^3*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c
- c*Sin[e + f*x]]*(9*B + 12*B*Cos[2*(e + f*x)] + 3*B*Cos[4*(e + f*x)] - 72
*A*Sin[e + f*x] - 8*A*Sin[3*(e + f*x)]))/(96*f)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3042, 3452, 3042, 3219, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3452}$$

$$A\int (\sin(e+fx)a+a)^{3/2}(c-c\sin(e+fx))^{3/2}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}{4f}$$

$$\Big\downarrow \text{3042}$$

$$A\int(\sin(e+fx)a+a)^{3/2}(c-c\sin(e+fx))^{3/2}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}{4f}$$

$\Big\downarrow$ 3219

$$A\left(\frac{2}{3}a\int\sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{3/2}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}{3f}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}{4f}$$

$\Big\downarrow$ 3042

$$A\left(\frac{2}{3}a\int\sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{3/2}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}{3f}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}{4f}$$

$\Big\downarrow$ 3217

$$A\left(-\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{3f\sqrt{a\sin(e+fx)+a}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}{3f}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}{4f}$$

input
```
Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(
3/2),x]
```

output
```
-1/4*(B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2)
)/f + A*(-1/3*(a^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(f*Sqrt[a + a*
Sin[e + f*x]]) - (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f
*x])^(3/2))/(3*f))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3452
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

Maple [A] (verified)

Time = 6.46 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.27

method	result
default	$\frac{16ca\left(A\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\frac{1}{2}\right)\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2+\frac{1}{2}\right)\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2+\frac{3\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^2\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^4-\cos\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}}{6f\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-3f}$
parts	$\frac{2A\sqrt{4}\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2}\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2}\left(2\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)ac\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)}{3f}+\frac{2Bac\sqrt{\left(2\sin\right.}}{}$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,metho
d=_RETURNVERBOSE)
```

output
```
16*c*a*(A*(cos(1/2*f*x+1/2*e)^2-1/2)*(cos(1/4*Pi+1/2*f*x+1/2*e)^2+1/2)*tan
(1/4*Pi+1/2*f*x+1/2*e)*sin(1/4*Pi+1/2*f*x+1/2*e)^2+3/2*sin(1/2*f*x+1/2*e)^
2*(cos(1/2*f*x+1/2*e)^4-cos(1/2*f*x+1/2*e)^2+1/2)*B*cos(1/2*f*x+1/2*e)^2)*
(c*cos(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)*(a*sin(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2
)/(6*f*cos(1/2*f*x+1/2*e)^2-3*f)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.62

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = -\frac{\left(3\,Bac\cos\left(fx+e\right)^4-3\,Bac-4\left(Aac\cos\left(fx+e\right)^2+2\,Aac\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin}}{12\,f\cos\left(fx+e\right)}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x
, algorithm="fricas")
```

output
```
-1/12*(3*B*a*c*cos(f*x + e)^4 - 3*B*a*c - 4*(A*a*c*cos(f*x + e)^2 + 2*A*a*
c)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos
(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}(-c\sin(fx+e)+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e)
 + c)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(116) = 232.

Time = 0.26 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.77

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \frac{4\left(3\,Bac\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x
, algorithm="giac")
```

output
```
4/3*(3*B*a*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x
 + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8 - 2*A*a*c*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x
+ 1/2*e)^6 - 6*B*a*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi +
 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^6 + 3*A*a*c*sgn(cos(-1/4
*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi +
1/2*f*x + 1/2*e)^4 + 3*B*a*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-
1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^4)*sqrt(a)*sqrt(
c)/f
```

Mupad [B] (verification not implemented)

Time = 2.17 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.77

$$\int (a + a\sin(e + fx))^{3/2}(A + B\sin(e + fx))(c - c\sin(e + fx))^{3/2}\,dx =$$
$$-\frac{a\,c\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(12\,B\,\cos\left(e+f\,x\right)+15\,B\,\cos\left(3\,e+3\,f\,x\right)+3\,B\,\cos\right.}{96\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(
3/2),x)
```

output
```
-(a*c*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(12*B*cos
(e + f*x) + 15*B*cos(3*e + 3*f*x) + 3*B*cos(5*e + 5*f*x) - 80*A*sin(2*e +
2*f*x) - 8*A*sin(4*e + 4*f*x)))/(96*f*(cos(2*e + 2*f*x) + 1))
```

Reduce [F]

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \sqrt{c}\,\sqrt{a}\,ac\left(-\left(\int \sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^3\,dx\right)b-\left(\int\right.\right.$$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x)
```

output
```
sqrt(c)*sqrt(a)*a*c*( - int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) +
1)*sin(e + f*x)**3,x)*b - int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x)
+ 1)*sin(e + f*x)**2,x)*a + int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x
) + 1)*sin(e + f*x),x)*b + int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x)
 + 1),x)*a)
```

3.142 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e}$

Optimal result

Integrand size = 40, antiderivative size = 96

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx=\frac{(A-B)c\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}+\frac{Bc\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3af\sqrt{c-c\sin(e+fx)}}$$

output
```
1/2*(A-B)*c*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(1/2)+1/3
*B*c*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/a/f/(c-c*sin(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 2.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.84

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx=$$
$$-\frac{a\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}(-2(6A+B)\sin(e+fx)+\cos(2(e+fx))(3(A+}{12f}$$

input
```
Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e
 + f*x]],x]
```

output
```
-1/12*(a*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*
(-2*(6*A + B)*Sin[e + f*x] + Cos[2*(e + f*x)]*(3*(A + B) + 2*B*Sin[e + f*x
])))/f
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3042, 3450, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3450}$$

$$(A-B)\int (\sin(e+fx)a+a)^{3/2}\sqrt{c-c\sin(e+fx)}dx+\frac{B\int (\sin(e+fx)a+a)^{5/2}\sqrt{c-c\sin(e+fx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$(A-B)\int (\sin(e+fx)a+a)^{3/2}\sqrt{c-c\sin(e+fx)}dx+\frac{B\int (\sin(e+fx)a+a)^{5/2}\sqrt{c-c\sin(e+fx)}dx}{a}$$

$$\Big\downarrow \text{3217}$$

$$\frac{c(A-B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}+\frac{Bc\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3af\sqrt{c-c\sin(e+fx)}}$$

input
```
Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x
]],x]
```

output
```
((A - B)*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*f*Sqrt[c - c*Sin[e
+ f*x]]) + (B*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*a*f*Sqrt[c - c
*Sin[e + f*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3450
```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[B/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] -
Simp[(B*c - A*d)/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[
a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 7.50 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.57

method	result
default	$\frac{12\left(A\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-\frac{1}{2}\right)\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}+\frac{4B\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+B\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\right)a\sqrt{c}}{6f\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-3f}$
parts	$\frac{A\sqrt{4}\,a\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{2Ba\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\sqrt{-\left(2\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\cos\right.}}{}$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
12*(A*tan(1/4*Pi+1/2*f*x+1/2*e)*(cos(1/2*f*x+1/2*e)^2-1/2)*sin(1/4*Pi+1/2*
f*x+1/2*e)^2+4/3*B*sin(1/2*f*x+1/2*e)^3*cos(1/2*f*x+1/2*e)^3+B*sin(1/2*f*x
+1/2*e)^2*cos(1/2*f*x+1/2*e)^2)*a*(c*cos(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)*(a
*sin(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)/(6*f*cos(1/2*f*x+1/2*e)^2-3*f)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.91

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = -\frac{\left(3\,(A+B)a\cos\left(fx+e\right)^2-3\,(A+B)a+2\left(Ba\cos\left(fx+e\right)^2-(3\,A+B)a\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+\right.}}{6\,f\cos\left(fx+e\right)}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x
, algorithm="fricas")
```

output
```
-1/6*(3*(A + B)*a*cos(f*x + e)^2 - 3*(A + B)*a + 2*(B*a*cos(f*x + e)^2 - (
3*A + B)*a)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) +
c)/(f*cos(f*x + e))
```

Sympy [F]

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \int \left(a(\sin\left(e+fx\right)+1)\right)^{\frac{3}{2}}\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\left(A+B\sin\left(e+fx\right)\right)\,dx$$

input
```
integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)
,x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**(3/2)*sqrt(-c*(sin(e + f*x) - 1))*(A + B*
sin(e + f*x)), x)
```

Maxima [F]

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}\sqrt{-c\sin(fx+e)+c}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x
+ e) + c), x)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.50

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx=$$
$$-\frac{2\left(4\,Ba\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+3\,Aa\cos\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x
, algorithm="giac")
```

output
```
-2/3*(4*B*a*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*A*a*cos(-1/4*pi + 1/2*f*x +
 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x +
1/2*e)) - 3*B*a*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)*sqrt(c)/f
```

Mupad [B] (verification not implemented)

Time = 37.17 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.27

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx =$$
$$-\frac{a\sqrt{a\,(\sin(e+f\,x)+1)}\,\sqrt{-c\,(\sin(e+f\,x)-1)}\,(3\,A\,\cos(e+f\,x)+3\,B\,\cos(e+f\,x)+3\,A\,\cos(3\,e+}{12\,f\,(\cos(2\,e+2}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(
1/2),x)
```

output
```
-(a*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(3*A*cos(e
+ f*x) + 3*B*cos(e + f*x) + 3*A*cos(3*e + 3*f*x) + 3*B*cos(3*e + 3*f*x) -
12*A*sin(2*e + 2*f*x) - 2*B*sin(2*e + 2*f*x) + B*sin(4*e + 4*f*x)))/(12*f*
(cos(2*e + 2*f*x) + 1))
```

Reduce [F]

$$\int (a+a\sin(e+fx))^{3/2}(A$$
$$+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx=\sqrt{c}\,\sqrt{a}\,a\left(\left(\left(\int\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)\right.\right.\right.$$
$$+\left(\int\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)a$$
$$+\left(\int\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)b$$
$$+\left(\int\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}dx\right)a\Bigg)$$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)
```

output
```
sqrt(c)*sqrt(a)*a*(int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*si
n(e + f*x)**2,x)*b + int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*
sin(e + f*x),x)*a + int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*s
in(e + f*x),x)*b + int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1),x)
*a)
```

3.143 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 145

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{2a^2(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{a(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{B\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}$$

output
```
-2*a^2*(A+B)*cos(f*x+e)*ln(1-sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin
(f*x+e))^(1/2)-a*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f/(c-c*sin(f*x+e)
)^(1/2)-1/2*B*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 6.94 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.94

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx =$$

$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{3/2}\,(-B\cos(2(e+fx))+16(A+B)\log(\cos(}{4f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sqrt{c-c\sin(e}}$$

input
```
Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin
[e + f*x]],x]
```

output
```
-1/4*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(3/2)*(
-(B*Cos[2*(e + f*x)]) + 16*(A + B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
] + 4*(A + 2*B)*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*
Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.225$, Rules used = {3042, 3452, 3042, 3219, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3452}$$

$$(A+B)\int \frac{(\sin(e+fx)a+a)^{3/2}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$(A+B)\int \frac{(\sin(e+fx)a+a)^{3/2}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{3219}$$

$$(A+B)\left(2a\int \frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$(A+B)\left(2a\int \frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{3216}$$

$$(A+B)\left(\frac{2a^2c\cos(e+fx)\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$(A+B)\left(\frac{2a^2c\cos(e+fx)\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{3146}$$

$$(A+B)\left(-\frac{2a^2\cos(e+fx)\int \frac{1}{c-c\sin(e+fx)}d(-c\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{16}$$

$$(A+B)\left(-\frac{2a^2\cos(e+fx)\log(c-c\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}$$

input
```
Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f
*x]],x]
```

output
```
-1/2*(B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(f*Sqrt[c - c*Sin[e + f*x
]]) + (A + B)*((-2*a^2*Cos[e + f*x]*Log[c - c*Sin[e + f*x]])/(f*Sqrt[a + a
*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (a*Cos[e + f*x]*Sqrt[a + a*Sin[
e + f*x]])/(f*Sqrt[c - c*Sin[e + f*x]]))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3452
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(131) = 262$.

Time = 6.54 (sec) , antiderivative size = 361, normalized size of antiderivative = 2.49

method	result
default	$-\frac{A\sqrt{4}\left(\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}+2\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)-1\right)-2\ln\left(\frac{2}{\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1}\right)+2\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+c\right.\right.}{f\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}}$
parts	$\frac{A\sqrt{4}\left(-\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}+2\ln\left(\frac{2}{\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1}\right)-2\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1\right)-2\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+cs\right.\right.}{f\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}}$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
-A/f*4^(1/2)*(sin(1/4*Pi+1/2*f*x+1/2*e)^2+2*ln(-cot(1/4*Pi+1/2*f*x+1/2*e)+
csc(1/4*Pi+1/2*f*x+1/2*e)-1)-2*ln(2/(cos(1/4*Pi+1/2*f*x+1/2*e)+1))+2*ln(-c
ot(1/4*Pi+1/2*f*x+1/2*e)+csc(1/4*Pi+1/2*f*x+1/2*e)+1))*(a*sin(1/4*Pi+1/2*f
*x+1/2*e)^2)^(1/2)*a/(c*cos(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)*cot(1/4*Pi+1/2*
f*x+1/2*e)-2*B*a/f*(cos(1/2*f*x+1/2*e)-sin(1/2*f*x+1/2*e))*(cos(1/2*f*x+1/
2*e)^2*sin(1/2*f*x+1/2*e)^2+2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+2*ln(-
2*(cos(1/2*f*x+1/2*e)-sin(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1))-2*ln(2/(
cos(1/2*f*x+1/2*e)+1)))*((2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+1)*a)^(1
/2)/(cos(1/2*f*x+1/2*e)+sin(1/2*f*x+1/2*e))/(-(2*sin(1/2*f*x+1/2*e)*cos(1/
2*f*x+1/2*e)-1)*c)^(1/2)
```

Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx=\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x
, algorithm="fricas")
```

output
```
integral((B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*sqrt(a*
sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c*sin(f*x + e) - c), x)
```

Sympy [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx=\int \frac{(a(\sin(e+fx)+1))^{\frac{3}{2}}(A+B\sin(e+fx))}{\sqrt{-c(\sin(e+fx)-1)}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2)
,x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**(3/2)*(A + B*sin(e + f*x))/sqrt(-c*(sin(e
 + f*x) - 1)), x)
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx=\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/sqrt(-c*sin(f*x
+ e) + c), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx=\int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{3/2}}{\sqrt{c-c\,\sin(e+f\,x)}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))
^(1/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))
^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx =$$
$$-\frac{\sqrt{c}\,\sqrt{a}\,a\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^2}{\sin(fx+e)-1}dx\right)b+\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)-1}dx\right)a+\left(\int\right.\right.}{c}$$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

output
```
( - sqrt(c)*sqrt(a)*a*(int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) +
1)*sin(e + f*x)**2)/(sin(e + f*x) - 1),x)*b + int((sqrt(sin(e + f*x) + 1)*
sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x) - 1),x)*a + int((sqr
t(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)
- 1),x)*b + int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1))/(sin(e
+ f*x) - 1),x)*a))/c
```

3.144 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 158

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f(c-c\sin(e+fx))^{3/2}} + \frac{a^2(A+3B)\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{a(A+3B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2cf\sqrt{c-c\sin(e+fx)}}$$

output
```
1/2*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(3/2)+a^2*(
A+3*B)*cos(f*x+e)*ln(1-sin(f*x+e))/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x
+e))^(1/2)+1/2*a*(A+3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*sin(f*
x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 11.62 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.33

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx =$$
$$-\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\left(4A+3B+B\cos(2(e+fx))+4A\log\left(\cos\right.\right.}{2cf\left(\cos\left(\frac{1}{2}(e\right.\right.}$$

input
```
Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
 f*x])^(3/2),x]
```

output
```
-1/2*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x]]*(
4*A + 3*B + B*Cos[2*(e + f*x)] + 4*A*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/
2]] + 12*B*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 2*(-B + 2*(A + 3*B)*
Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]])*Sin[e + f*x]))/(c*f*(Cos[(e + f*
x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.225$, Rules used = {3042, 3451, 3042, 3219, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{(a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}dx$$
$$\Big\downarrow \ 3451$$
$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(A+3B)\int \frac{(\sin(e+fx)a+a)^{3/2}}{\sqrt{c-c\sin(e+fx)}}dx}{2c}$$

$$\Big\downarrow \ 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(A+3B)\int\frac{(\sin(e+fx)a+a)^{3/2}}{\sqrt{c-c\sin(e+fx)}}dx}{2c}$$

$$\Big\downarrow \ 3219$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(A+3B)\left(2a\int\frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)}{2c}$$

$$\Big\downarrow \ 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(A+3B)\left(2a\int\frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)}{2c}$$

$$\Big\downarrow \ 3216$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(A+3B)\left(\frac{2a^2c\cos(e+fx)\int\frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)}{2c}$$

$$\Big\downarrow \ 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(A+3B)\left(\frac{2a^2c\cos(e+fx)\int\frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)}{2c}$$

$$\Big\downarrow \ 3146$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(A+3B)\left(-\frac{2a^2\cos(e+fx)\int\frac{1}{c-c\sin(e+fx)}d(-c\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)}{2c}$$

$$\Big\downarrow \ 16$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(A+3B)\left(-\frac{2a^2\cos(e+fx)\log(c-c\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)}{2c}$$

input
```
Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(3/2),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*f*(c - c*Sin[e + f*x]
)^(3/2)) - ((A + 3*B)*((-2*a^2*Cos[e + f*x]*Log[c - c*Sin[e + f*x]])/(f*Sq
rt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (a*Cos[e + f*x]*Sqrt[a
+ a*Sin[e + f*x]])/(f*Sqrt[c - c*Sin[e + f*x]])))/(2*c)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(142) = 284$.

Time = 6.44 (sec) , antiderivative size = 407, normalized size of antiderivative = 2.58

method	result
default	$\frac{A\sqrt{4}\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2}\,a\left(2\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1\right)\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+2\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{f}{}\right.\right.\right.}{2f\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2}\,c}$
parts	$-\frac{A\sqrt{4}\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2}\,a\left(2\ln\left(\frac{2}{\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)-2\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1\right)\cot\left(\frac{\pi}{4}+\right.\right.}{2f\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2}\,c}$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/2*A/f*4^(1/2)*(a*sin(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)*a/(c*cos(1/4*Pi+1/2*
f*x+1/2*e)^2)^(1/2)/c*(2*ln(-cot(1/4*Pi+1/2*f*x+1/2*e)+csc(1/4*Pi+1/2*f*x+
1/2*e)+1)*cot(1/4*Pi+1/2*f*x+1/2*e)+2*ln(-cot(1/4*Pi+1/2*f*x+1/2*e)+csc(1/
4*Pi+1/2*f*x+1/2*e)-1)*cot(1/4*Pi+1/2*f*x+1/2*e)-2*ln(2/(cos(1/4*Pi+1/2*f*
x+1/2*e)+1))*cot(1/4*Pi+1/2*f*x+1/2*e)+tan(1/4*Pi+1/2*f*x+1/2*e))-2*B/f*((
6*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-3)*ln(2*(sin(1/2*f*x+1/2*e)-cos(1/
2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1))+(-6*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1
/2*e)+3)*ln(2/(cos(1/2*f*x+1/2*e)+1))+cos(1/2*f*x+1/2*e)*(2*cos(1/2*f*x+1/
2*e)*sin(1/2*f*x+1/2*e)^2-3*sin(1/2*f*x+1/2*e)))*((2*sin(1/2*f*x+1/2*e)*co
s(1/2*f*x+1/2*e)+1)*a)^(1/2)*a/(-1+2*cos(1/2*f*x+1/2*e)^2)/(-(2*sin(1/2*f*
x+1/2*e)*cos(1/2*f*x+1/2*e)-1)*c)^(1/2)/c
```

Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}}{(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x
, algorithm="fricas")
```

output
```
integral((B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*sqrt(a*
sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^2*cos(f*x + e)^2 + 2*c^2*si
n(f*x + e) - 2*c^2), x)
```

Sympy [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=\int \frac{(a(\sin(e+fx)+1))^{\frac{3}{2}}(A+B\sin(e+fx))}{(-c(\sin(e+fx)-1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2)
,x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**(3/2)*(A + B*sin(e + f*x))/(-c*(sin(e + f
*x) - 1))**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(142) = 284$.

Time = 0.15 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.32

$$\int \frac{(a + a\sin(e + fx))^{3/2}(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{3/2}}\,dx = -\frac{B\left(\frac{6\,a^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^{\frac{3}{2}}} - \frac{3\,a^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{c^{\frac{3}{2}}} + \frac{2\left(\frac{3\,a^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1} - \frac{2\,a^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{3\,a^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{c^{\frac{3}{2}} - \frac{2\,c^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1} + \frac{2\,c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{2\,c^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{c^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}\right)}{f}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x
, algorithm="maxima")
```

output
```
-(B*(6*a^(3/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(3/2) - 3*a^(3/2
)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(3/2) + 2*(3*a^(3/2)*sin(
f*x + e)/(cos(f*x + e) + 1) - 2*a^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 3*a^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/(c^(3/2) - 2*c^(3/2)*si
n(f*x + e)/(cos(f*x + e) + 1) + 2*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 - 2*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + c^(3/2)*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4)) + A*(2*a^(3/2)*log(sin(f*x + e)/(cos(f*x + e) +
1) - 1)/c^(3/2) - a^(3/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(
3/2) + 4*a^(3/2)*sqrt(c)*sin(f*x + e)/((c^2 - 2*c^2*sin(f*x + e)/(cos(f*x
+ e) + 1) + c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1))))
/f
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{3/2}}{(c-c\,\sin(e+f\,x))^{3/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))
^(3/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))
^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \sqrt{c}\,\sqrt{a}\,a\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^2}{\sin(fx+e)^2-2\sin(fx+e)+1}dx\right)b+\right.$$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a*(int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*
sin(e + f*x)**2)/(sin(e + f*x)**2 - 2*sin(e + f*x) + 1),x)*b + int((sqrt(s
in(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**2
- 2*sin(e + f*x) + 1),x)*a + int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f
*x) + 1)*sin(e + f*x))/(sin(e + f*x)**2 - 2*sin(e + f*x) + 1),x)*b + int((
sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1))/(sin(e + f*x)**2 - 2*sin
(e + f*x) + 1),x)*a))/c**2
```

3.145 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 149

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf(c-c\sin(e+fx))^{3/2}} - \frac{a^2B\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

output
```
1/4*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(5/2)-a*B*c
os(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*sin(f*x+e))^(3/2)-a^2*B*cos(f*x+
e)*ln(1-sin(f*x+e))/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 11.68 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.33

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+}}{}$$

input
```
Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
 f*x])^(5/2),x]
```

output
```
(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(B*Cos
[2*(e + f*x)]*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - B*(2 + 3*Log[Cos[
(e + f*x)/2] - Sin[(e + f*x)/2]]) + (A + 3*B + 4*B*Log[Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]])*Sin[e + f*x]))/(c^2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)
/2])*(-1 + Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.225$, Rules used = {3042, 3451, 3042, 3218, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3451}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{B\int \frac{(\sin(e+fx)a+a)^{3/2}}{(c-c\sin(e+fx))^{3/2}}dx}{c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{B\int \frac{(\sin(e+fx)a+a)^{3/2}}{(c-c\sin(e+fx))^{3/2}}dx}{c}$$

$$\Big\downarrow \text{3218}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{B\left(\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f(c-c\sin(e+fx))^{3/2}}-\frac{a\int \frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx}{c}\right)}{c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{B\left(\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f(c-c\sin(e+fx))^{3/2}}-\frac{a\int\frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx}{c}\right)}{c}$$

$$\Big\downarrow \text{3216}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{B\left(\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f(c-c\sin(e+fx))^{3/2}}-\frac{a^2\cos(e+fx)\int\frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}\right)}{c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{B\left(\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f(c-c\sin(e+fx))^{3/2}}-\frac{a^2\cos(e+fx)\int\frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}\right)}{c}$$

$$\Big\downarrow \text{3146}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{B\left(\frac{a^2\cos(e+fx)\int\frac{1}{c-c\sin(e+fx)}d(-c\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f(c-c\sin(e+fx))^{3/2}}\right)}{c}$$

$$\Big\downarrow \text{16}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{B\left(\frac{a^2\cos(e+fx)\log(c-c\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f(c-c\sin(e+fx))^{3/2}}\right)}{c}$$

input

```
Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(5/2),x]
```

output

```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(4*f*(c - c*Sin[e + f*x]
)^(5/2)) - (B*((a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(f*(c - c*Sin[e +
 f*x])^(3/2)) + (a^2*Cos[e + f*x]*Log[c - c*Sin[e + f*x]])/(c*f*Sqrt[a + a
*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])))/c
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3218
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Simp[b*((2*m - 1)/(d*(
2*n + 1)))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m +
n + 1, 0])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(135) = 270$.

Time = 6.85 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.56

method	result
parts	$\frac{A\sqrt{4}\,a\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2}\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^3}{8fc^2\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2}}+\frac{2B\left(\left(-4\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^4+4\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-4\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\ln\left(\right.}{}$
default	$\frac{A\sqrt{4}\,a\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2}\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^3}{8fc^2\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2}}-\frac{2B\left(\left(4\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^4+4\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-4\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)\ln\left(-\right.}{}$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/8*A/f*4^(1/2)*a*(a*sin(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)/c^2/(c*cos(1/4*Pi+
1/2*f*x+1/2*e)^2)^(1/2)*tan(1/4*Pi+1/2*f*x+1/2*e)^3+2*B/f*((-4*cos(1/2*f*x
+1/2*e)^4+4*cos(1/2*f*x+1/2*e)^2-4*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+1
)*ln(2*(sin(1/2*f*x+1/2*e)-cos(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1))+(4*
cos(1/2*f*x+1/2*e)^4+4*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-4*cos(1/2*f*x
+1/2*e)^2-1)*ln(2/(cos(1/2*f*x+1/2*e)+1))+cos(1/2*f*x+1/2*e)*(-4*cos(1/2*f
*x+1/2*e)*sin(1/2*f*x+1/2*e)^2+sin(1/2*f*x+1/2*e)))*((2*sin(1/2*f*x+1/2*e)
*cos(1/2*f*x+1/2*e)+1)*a)^(1/2)*a/(-(2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*
e)-1)*c)^(1/2)/c^2/(4*cos(1/2*f*x+1/2*e)^3*sin(1/2*f*x+1/2*e)-2*cos(1/2*f*
x+1/2*e)^2-2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+1)
```

Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}}{(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x
, algorithm="fricas")
```

output
```
integral((B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*sqrt(a*
sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*c^3*cos(f*x + e)^2 - 4*c^3
- (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

Sympy [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx=\int \frac{(a(\sin(e+fx)+1))^{\frac{3}{2}}(A+B\sin(e+fx))}{(-c(\sin(e+fx)-1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2)
,x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**(3/2)*(A + B*sin(e + f*x))/(-c*(sin(e + f
*x) - 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}}{(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e)
 + c)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{3/2}}{(c-c\,\sin(e+f\,x))^{5/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))
^(5/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))
^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = -\frac{\sqrt{c}\,\sqrt{a}\,a\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^2}{\sin(fx+e)^3-3\sin(fx+e)^2+3\sin(fx+e)-1}dx\right)b+\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)^3-3\sin(fx+e)^2+3\sin(fx+e)-1}dx\right)a+\left(\int\right.\right.}{c^3}$$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)
```

output
```
( - sqrt(c)*sqrt(a)*a*(int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) +
1)*sin(e + f*x)**2)/(sin(e + f*x)**3 - 3*sin(e + f*x)**2 + 3*sin(e + f*x)
- 1),x)*b + int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e +
f*x))/(sin(e + f*x)**3 - 3*sin(e + f*x)**2 + 3*sin(e + f*x) - 1),x)*a + in
t((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e +
 f*x)**3 - 3*sin(e + f*x)**2 + 3*sin(e + f*x) - 1),x)*b + int((sqrt(sin(e
+ f*x) + 1)*sqrt( - sin(e + f*x) + 1))/(sin(e + f*x)**3 - 3*sin(e + f*x)**
2 + 3*sin(e + f*x) - 1),x)*a))/c**3
```

3.146 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 96

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{6f(c-c\sin(e+fx))^{7/2}} + \frac{(A-5B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{24cf(c-c\sin(e+fx))^{5/2}}$$

output
```
1/6*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(7/2)+1/24*
(A-5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(5/2)
```

Mathematica [A] (verified)

Time = 11.71 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.30

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = -\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}(A+4B-3B\cos(2(e+fx))+3(A-B)\mathrm{si}}{6c^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}$$

input
```
Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
 f*x])^(7/2),x]
```

output
```
-1/6*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(
A + 4*B - 3*B*Cos[2*(e + f*x)] + 3*(A - B)*Sin[e + f*x]))/(c^3*f*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]
])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3042, 3451, 3042, 3221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{3451}$$

$$\frac{(A-5B)\int \frac{(\sin(e+fx)a+a)^{3/2}}{(c-c\sin(e+fx))^{5/2}}dx}{6c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{6f(c-c\sin(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-5B)\int \frac{(\sin(e+fx)a+a)^{3/2}}{(c-c\sin(e+fx))^{5/2}}dx}{6c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{6f(c-c\sin(e+fx))^{7/2}}$$

$$\Big\downarrow \text{3221}$$

$$\frac{(A-5B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{24cf(c-c\sin(e+fx))^{5/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{6f(c-c\sin(e+fx))^{7/2}}$$

input
```
Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(7/2),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(6*f*(c - c*Sin[e + f*x]
)^(7/2)) + ((A - 5*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(24*c*f*(c
- c*Sin[e + f*x])^(5/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3221
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(84) = 168$.

Time = 7.12 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.65

method	result
default	$\frac{2a\left(\left(\sec\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2+\frac{1}{2}\right)\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\frac{1}{2}\right)A\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^4+\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\frac{1}{4}\right)\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right.}{3\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2}\,c^3 f\left(-1+2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)\left(4\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^4+4\sin\left(\frac{fx}{2}-\right.\right.}$
parts	$\frac{A\sqrt{4}\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2}\,a\left(\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^3+2\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^3\sec\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2\right)}{48f\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2}\,c^3} - \frac{2B\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2\sqrt{(}}{3f\left(-1+2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)\left(4\cos\left(\frac{fx}{2}\right.\right.}$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,metho
d=_RETURNVERBOSE)
```

output
```
2/3/(c*cos(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)*a*((sec(1/4*Pi+1/2*f*x+1/2*e)^2+
1/2)*(cos(1/2*f*x+1/2*e)^2-1/2)*A*(cos(1/2*f*x+1/2*e)^4+sin(1/2*f*x+1/2*e)
*cos(1/2*f*x+1/2*e)-cos(1/2*f*x+1/2*e)^2-1/4)*tan(1/4*Pi+1/2*f*x+1/2*e)^3-
2*B*sin(1/2*f*x+1/2*e)^3*cos(1/2*f*x+1/2*e)^3-3*B*sin(1/2*f*x+1/2*e)^2*cos
(1/2*f*x+1/2*e)^2)*(a*sin(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)/c^3/f/(-1+2*cos(1
/2*f*x+1/2*e)^2)/(4*cos(1/2*f*x+1/2*e)^4+4*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+
1/2*e)-4*cos(1/2*f*x+1/2*e)^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.30

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{\left(6\,Ba\cos\left(fx+e\right)^2-3\,(A-B)a\sin\left(fx+e\right)-(A+\right.}{6\left(3\,c^4f\cos\left(fx+e\right)^3-4\,c^4f\cos\left(fx+e\right)-\left(c^4f\cos\right.\right.}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x
, algorithm="fricas")
```

output
```
1/6*(6*B*a*cos(f*x + e)^2 - 3*(A - B)*a*sin(f*x + e) - (A + 7*B)*a)*sqrt(a
*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*c^4*f*cos(f*x + e)^3 - 4*c
^4*f*cos(f*x + e) - (c^4*f*cos(f*x + e)^3 - 4*c^4*f*cos(f*x + e))*sin(f*x
+ e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2)
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}}{(-c\sin(fx+e)+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e)
 + c)^(7/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx=\int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{7/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))
^(7/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))
^(7/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx=\frac{\sqrt{c}\,\sqrt{a}\,a\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^2}{\sin(fx+e)^4-4\sin(fx+e)^3+6\sin(fx+e)^2-4\sin(fx+e)+}\right.\right.}{}$$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a*(int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*
sin(e + f*x)**2)/(sin(e + f*x)**4 - 4*sin(e + f*x)**3 + 6*sin(e + f*x)**2
- 4*sin(e + f*x) + 1),x)*b + int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f
*x) + 1)*sin(e + f*x))/(sin(e + f*x)**4 - 4*sin(e + f*x)**3 + 6*sin(e + f*
x)**2 - 4*sin(e + f*x) + 1),x)*a + int((sqrt(sin(e + f*x) + 1)*sqrt( - sin
(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**4 - 4*sin(e + f*x)**3 + 6*sin(
e + f*x)**2 - 4*sin(e + f*x) + 1),x)*b + int((sqrt(sin(e + f*x) + 1)*sqrt(
 - sin(e + f*x) + 1))/(sin(e + f*x)**4 - 4*sin(e + f*x)**3 + 6*sin(e + f*x
)**2 - 4*sin(e + f*x) + 1),x)*a))/c**4
```

3.147 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 146

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{8f(c-c\sin(e+fx))^{9/2}} + \frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{24cf(c-c\sin(e+fx))^{7/2}} + \frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{96c^2f(c-c\sin(e+fx))^{5/2}}$$

output
```
1/8*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(9/2)+1/24*
(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(7/2)+1/96*
(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c^2/f/(c-c*sin(f*x+e))^(5/2)
```

Mathematica [A] (verified)

Time = 12.06 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.84

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+}}{12c^4f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right.}$$

input
```
Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
 f*x])^(9/2),x]
```

output
```
(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(2*A +
 3*B - 3*B*Cos[2*(e + f*x)] + 4*A*Sin[e + f*x]))/(12*c^4*f*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^4*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3042, 3451, 3042, 3222, 3042, 3221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}dx$$

$$\Big\downarrow \text{3451}$$

$$\frac{(A-3B)\int \frac{(\sin(e+fx)a+a)^{3/2}}{(c-c\sin(e+fx))^{7/2}}dx}{4c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{8f(c-c\sin(e+fx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-3B)\int \frac{(\sin(e+fx)a+a)^{3/2}}{(c-c\sin(e+fx))^{7/2}}dx}{4c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{8f(c-c\sin(e+fx))^{9/2}}$$

$$\Big\downarrow \text{3222}$$

$$\frac{(A-3B)\left(\frac{\int \frac{(\sin(e+fx)a+a)^{3/2}}{(c-c\sin(e+fx))^{5/2}}dx}{6c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{6f(c-c\sin(e+fx))^{7/2}}\right)}{4c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{8f(c-c\sin(e+fx))^{9/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{(A-3B)\left(\frac{\int \frac{(\sin(e+fx)a+a)^{3/2}}{(c-c\sin(e+fx))^{5/2}}dx}{6c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{6f(c-c\sin(e+fx))^{7/2}}\right)}{4c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{8f(c-c\sin(e+fx))^{9/2}}$$

$$\Big\downarrow \; 3221$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{8f(c-c\sin(e+fx))^{9/2}}+\frac{(A-3B)\left(\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{24cf(c-c\sin(e+fx))^{5/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{6f(c-c\sin(e+fx))^{7/2}}\right)}{4c}$$

input
```
Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(9/2),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(8*f*(c - c*Sin[e + f*x]
)^(9/2)) + ((A - 3*B)*((Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(6*f*(c -
 c*Sin[e + f*x])^(7/2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(24*c*
f*(c - c*Sin[e + f*x])^(5/2))))/(4*c)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3221
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

rule 3222
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp[(m + n + 1)/(a*(2*m + 1)
)   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
 ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||
!SumSimplerQ[n, 1])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(128) = 256$.

Time = 7.40 (sec) , antiderivative size = 307, normalized size of antiderivative = 2.10

method	result
parts	$\frac{A\sqrt{4}\,\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}+2\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}+3\right)a\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{3}\sec\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}}{192f\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,c^{4}}+\frac{\cdots}{f\left(-1+2\cos\left(\frac{fx}{2}\cdots\right.\right.}$
default	$\frac{\left(\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}\sin\left(\frac{fx}{2}+\frac{e}{2}\right)-\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}\sin\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{3\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{2}-\frac{3\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)}{4}+\frac{3\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2}+\frac{1}{8}\right)\left(\cos\left(\cdots\right.\right.\right.}{6\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,c^{4}f\left(-1+2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\right)\left(8\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}\sin\left(\frac{fx}{2}+\frac{e}{2}\right.\cdots\right.}$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/192*A/f*4^(1/2)*(a*sin(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)*(cos(1/4*Pi+1/2*f*
x+1/2*e)^4+2*cos(1/4*Pi+1/2*f*x+1/2*e)^2+3)*a/(c*cos(1/4*Pi+1/2*f*x+1/2*e)
^2)^(1/2)/c^4*tan(1/4*Pi+1/2*f*x+1/2*e)^3*sec(1/4*Pi+1/2*f*x+1/2*e)^4+2*B/
f*((2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+1)*a)^(1/2)*sin(1/2*f*x+1/2*e)
^2*cos(1/2*f*x+1/2*e)^2*a/(-1+2*cos(1/2*f*x+1/2*e)^2)/(8*cos(1/2*f*x+1/2*e
)^5*sin(1/2*f*x+1/2*e)-8*cos(1/2*f*x+1/2*e)^3*sin(1/2*f*x+1/2*e)-12*cos(1/
2*f*x+1/2*e)^4-6*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+12*cos(1/2*f*x+1/2*
e)^2+1)/(-(2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-1)*c)^(1/2)/c^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.92

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = -\frac{\left(3\,Ba\cos\left(fx+e\right)^{2}-2\,Aa\sin\left(fx+e\right)-(A+3\,B)a\right)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)}}{6\left(c^{5}f\cos\left(fx+e\right)^{5}-8\,c^{5}f\cos\left(fx+e\right)^{3}+8\,c^{5}f\cos\left(fx+e\right)+4\left(c^{5}f\cos\left(fx+e\right)^{3}-2\,c^{5}f\cos\left(fx+e\right)\right.\right.}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x
, algorithm="fricas")
```

output
```
-1/6*(3*B*a*cos(f*x + e)^2 - 2*A*a*sin(f*x + e) - (A + 3*B)*a)*sqrt(a*sin(
f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^5*f*cos(f*x + e)^5 - 8*c^5*f*co
s(f*x + e)^3 + 8*c^5*f*cos(f*x + e) + 4*(c^5*f*cos(f*x + e)^3 - 2*c^5*f*co
s(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(9/2)
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a + a\sin(e + fx))^{3/2}(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{9/2}}\,dx = \int \frac{(B\sin(fx + e) + A)(a\sin(fx + e) + a)^{\frac{3}{2}}}{(-c\sin(fx + e) + c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e)
 + c)^(9/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + a\sin(e + fx))^{3/2}(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{9/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 45.21 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.68

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx=\frac{\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{8\,a\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\left(2\,A+3\,B\right)\,\sqrt{a+a\,\mathrm{si}}}{3\,c^5\,f}\right.}{84\,\cos\left(e+f\,x\right)\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}-54\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right.}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))
^(9/2),x)
```

output
```
((c - c*sin(e + f*x))^(1/2)*((8*a*exp(e*5i + f*x*5i)*(2*A + 3*B)*(a + a*si
n(e + f*x))^(1/2))/(3*c^5*f) + (32*A*a*exp(e*5i + f*x*5i)*sin(e + f*x)*(a
+ a*sin(e + f*x))^(1/2))/(3*c^5*f) - (8*B*a*exp(e*5i + f*x*5i)*cos(2*e + 2
*f*x)*(a + a*sin(e + f*x))^(1/2))/(c^5*f)))/(84*cos(e + f*x)*exp(e*5i + f*
x*5i) - 54*exp(e*5i + f*x*5i)*cos(3*e + 3*f*x) + 2*exp(e*5i + f*x*5i)*cos(
5*e + 5*f*x) - 96*exp(e*5i + f*x*5i)*sin(2*e + 2*f*x) + 16*exp(e*5i + f*x*
5i)*sin(4*e + 4*f*x))
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx=$$

$$-\frac{\sqrt{c}\,\sqrt{a}\,a\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^2}{\sin(fx+e)^5-5\sin(fx+e)^4+10\sin(fx+e)^3-10\sin(fx+e)^2+5\sin(fx+e)-1}dx\right)b+\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{}}{\sin(fx+e)^5-5\sin(fx+e)^4+10\,\mathrm{si}}\right.\right.}{}$$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x)
```

output
```
( - sqrt(c)*sqrt(a)*a*(int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) +
1)*sin(e + f*x)**2)/(sin(e + f*x)**5 - 5*sin(e + f*x)**4 + 10*sin(e + f*x)
**3 - 10*sin(e + f*x)**2 + 5*sin(e + f*x) - 1),x)*b + int((sqrt(sin(e + f*
x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**5 - 5*sin(e
 + f*x)**4 + 10*sin(e + f*x)**3 - 10*sin(e + f*x)**2 + 5*sin(e + f*x) - 1)
,x)*a + int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)
)/(sin(e + f*x)**5 - 5*sin(e + f*x)**4 + 10*sin(e + f*x)**3 - 10*sin(e + f
*x)**2 + 5*sin(e + f*x) - 1),x)*b + int((sqrt(sin(e + f*x) + 1)*sqrt( - si
n(e + f*x) + 1))/(sin(e + f*x)**5 - 5*sin(e + f*x)**4 + 10*sin(e + f*x)**3
 - 10*sin(e + f*x)**2 + 5*sin(e + f*x) - 1),x)*a))/c**5
```

3.148 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}} \, dx$

Optimal result

Integrand size = 40, antiderivative size = 154

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}} \, dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{a(3A-7B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{40cf(c-c\sin(e+fx))^{9/2}} - \frac{a^2(3A-7B)\cos(e+fx)}{120c^2f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}}$$

output
```
1/10*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(11/2)+1/4
0*a*(3*A-7*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*sin(f*x+e))^(9/2)
-1/120*a^2*(3*A-7*B)*cos(f*x+e)/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+
e))^(7/2)
```

Mathematica [A] (verified)

Time = 12.74 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.82

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx =$$
$$-\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}(9(A+B)-10B\cos(2(e+fx))+5(3A+B}{60c^5f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^5\sqrt{c-c\sin(e+fx)}}$$

input
```
Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
 f*x])^(11/2),x]
```

output
```
-1/60*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*
(9*(A + B) - 10*B*Cos[2*(e + f*x)] + 5*(3*A + B)*Sin[e + f*x]))/(c^5*f*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^5*Sqrt[c - c*Sin[e
+ f*x]])
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3451, 3042, 3218, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}dx$$
$$\Big\downarrow \text{3451}$$
$$\frac{(3A-7B)\int \frac{(\sin(e+fx)a+a)^{3/2}}{(c-c\sin(e+fx))^{9/2}}dx}{10c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(3A-7B)\int \frac{(\sin(e+fx)a+a)^{3/2}}{(c-c\sin(e+fx))^{9/2}}dx}{10c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

$$\Big\downarrow \text{3218}$$

$$\frac{(3A-7B)\left(\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{4f(c-c\sin(e+fx))^{9/2}}-\frac{a\int \frac{\sqrt{\sin(e+fx)a+a}}{(c-c\sin(e+fx))^{7/2}}dx}{4c}\right)}{10c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(3A-7B)\left(\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{4f(c-c\sin(e+fx))^{9/2}}-\frac{a\int \frac{\sqrt{\sin(e+fx)a+a}}{(c-c\sin(e+fx))^{7/2}}dx}{4c}\right)}{10c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

$$\Big\downarrow \text{3217}$$

$$\frac{(3A-7B)\left(\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{4f(c-c\sin(e+fx))^{9/2}}-\frac{a^2\cos(e+fx)}{12cf\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}}\right)}{10c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

input
```
Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(11/2),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(10*f*(c - c*Sin[e + f*x
])^(11/2)) + ((3*A - 7*B)*((a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(4*f*
(c - c*Sin[e + f*x])^(9/2)) - (a^2*Cos[e + f*x])/(12*c*f*Sqrt[a + a*Sin[e
+ f*x]]*(c - c*Sin[e + f*x])^(7/2))))/(10*c)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3218
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Simp[b*((2*m - 1)/(d*(
2*n + 1)))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m +
n + 1, 0])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 440 vs. $2(136) = 272$.

Time = 8.64 (sec) , antiderivative size = 441, normalized size of antiderivative = 2.86

method	result
parts	$\frac{A\sqrt{4}\,\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{6}+2\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}+3\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}+4\right)a\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{3}\sec\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{6}}{640f\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,c^{5}}$
default	$\frac{a\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\left(\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{6}+2\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}+3\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}+4\right)A\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-\frac{1}{2}\right)\left(\frac{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}}{2}+\cdots\right.\right.}{5\sqrt{c\cos\left(\frac{\pi}{4}+\frac{f\cdots}{2}\right.}}$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x,meth
od=_RETURNVERBOSE)
```

output
```
1/640*A/f*4^(1/2)*(a*sin(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)*(cos(1/4*Pi+1/2*f*
x+1/2*e)^6+2*cos(1/4*Pi+1/2*f*x+1/2*e)^4+3*cos(1/4*Pi+1/2*f*x+1/2*e)^2+4)*
a/(c*cos(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)/c^5*tan(1/4*Pi+1/2*f*x+1/2*e)^3*se
c(1/4*Pi+1/2*f*x+1/2*e)^6+2/15*B/f*cos(1/2*f*x+1/2*e)^2*((2*sin(1/2*f*x+1/
2*e)*cos(1/2*f*x+1/2*e)+1)*a)^(1/2)*(4*cos(1/2*f*x+1/2*e)^5*sin(1/2*f*x+1/
2*e)-4*cos(1/2*f*x+1/2*e)^3*sin(1/2*f*x+1/2*e)-10*cos(1/2*f*x+1/2*e)^4-10*
sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+10*cos(1/2*f*x+1/2*e)^2+15)*sin(1/2*
f*x+1/2*e)^2*a/(-1+2*cos(1/2*f*x+1/2*e)^2)/(16*cos(1/2*f*x+1/2*e)^8+32*cos
(1/2*f*x+1/2*e)^5*sin(1/2*f*x+1/2*e)-32*cos(1/2*f*x+1/2*e)^6-32*cos(1/2*f*
x+1/2*e)^3*sin(1/2*f*x+1/2*e)-8*cos(1/2*f*x+1/2*e)^4-8*sin(1/2*f*x+1/2*e)*
cos(1/2*f*x+1/2*e)+24*cos(1/2*f*x+1/2*e)^2+1)/(-(2*sin(1/2*f*x+1/2*e)*cos(
1/2*f*x+1/2*e)-1)*c)^(1/2)/c^5
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.01

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx =$$
$$-\frac{\left(20\,Ba\cos\left(fx+e\right)^{2}-5\,(3\,A+B)a\sin\left(fx+e\right)-(9\,A+19\,B)a\right)\sqrt{a\sin\left(fx+e\right)+}}{60\left(5\,c^{6}f\cos\left(fx+e\right)^{5}-20\,c^{6}f\cos\left(fx+e\right)^{3}+16\,c^{6}f\cos\left(fx+e\right)-\left(c^{6}f\cos\left(fx+e\right)^{5}-12\,c^{6}f\cos\left(fx\right.\right.\right.}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),
x, algorithm="fricas")
```

output
```
-1/60*(20*B*a*cos(f*x + e)^2 - 5*(3*A + B)*a*sin(f*x + e) - (9*A + 19*B)*a
)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(5*c^6*f*cos(f*x + e)
^5 - 20*c^6*f*cos(f*x + e)^3 + 16*c^6*f*cos(f*x + e) - (c^6*f*cos(f*x + e)
^5 - 12*c^6*f*cos(f*x + e)^3 + 16*c^6*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(11/2
),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}}{(-c\sin(fx+e)+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),
x, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e)
 + c)^(11/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),
x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 48.69 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.81

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{\sqrt{c-c\sin(e+fx)}\left(\frac{a\,e^{e\,6i+f\,x\,6i}}{\cdots}\right.}{\cos(e+fx)\,e^{e\,6i+f\,x\,6i}\,264i-e^{e\,6i+f\,x\,6i}\,\cos(3\,e+3\,f\,x)\cdots}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))
^(11/2),x)
```

output
```
((c - c*sin(e + f*x))^(1/2)*((a*exp(e*6i + f*x*6i)*(A + B)*(a + a*sin(e +
f*x))^(1/2)*48i)/(5*c^6*f) - (B*a*exp(e*6i + f*x*6i)*cos(2*e + 2*f*x)*(a +
 a*sin(e + f*x))^(1/2)*32i)/(3*c^6*f) + (16*a*exp(e*6i + f*x*6i)*sin(e + f
*x)*(A*3i + B*1i)*(a + a*sin(e + f*x))^(1/2))/(3*c^6*f)))/(cos(e + f*x)*ex
p(e*6i + f*x*6i)*264i - exp(e*6i + f*x*6i)*cos(3*e + 3*f*x)*220i + exp(e*6
i + f*x*6i)*cos(5*e + 5*f*x)*20i - exp(e*6i + f*x*6i)*sin(2*e + 2*f*x)*330
i + exp(e*6i + f*x*6i)*sin(4*e + 4*f*x)*88i - exp(e*6i + f*x*6i)*sin(6*e +
 6*f*x)*2i)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,a\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)^6-6\sin(fx+e)^5+15\sin(fx+e)^4-20\sin(fx+e)}\right.\right.}{}$$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a*(int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*
sin(e + f*x)**2)/(sin(e + f*x)**6 - 6*sin(e + f*x)**5 + 15*sin(e + f*x)**4
 - 20*sin(e + f*x)**3 + 15*sin(e + f*x)**2 - 6*sin(e + f*x) + 1),x)*b + in
t((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e +
 f*x)**6 - 6*sin(e + f*x)**5 + 15*sin(e + f*x)**4 - 20*sin(e + f*x)**3 + 1
5*sin(e + f*x)**2 - 6*sin(e + f*x) + 1),x)*a + int((sqrt(sin(e + f*x) + 1)
*sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**6 - 6*sin(e + f*x)
**5 + 15*sin(e + f*x)**4 - 20*sin(e + f*x)**3 + 15*sin(e + f*x)**2 - 6*sin
(e + f*x) + 1),x)*b + int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1
))/(sin(e + f*x)**6 - 6*sin(e + f*x)**5 + 15*sin(e + f*x)**4 - 20*sin(e +
f*x)**3 + 15*sin(e + f*x)**2 - 6*sin(e + f*x) + 1),x)*a))/c**6
```

3.149 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 198

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = -\frac{a^3(7A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{105f\sqrt{a+a\sin(e+fx)}} - \frac{2a^2(7A-B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}}{105f} - \frac{a(7A-B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}{42f} - \frac{B\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}}{7f}$$

output
```
-1/105*a^3*(7*A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/f/(a+a*sin(f*x+e))^(1
/2)-2/105*a^2*(7*A-B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(
7/2)/f-1/42*a*(7*A-B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(
7/2)/f-1/7*B*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2)/f
```

Mathematica [A] (verified)

Time = 13.42 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.13

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx =$$
$$-\frac{c^3(-1+\sin(e+fx))^3(a(1+\sin(e+fx)))^{5/2}\sqrt{c-c\sin(e+fx)}(525(A-B)\cos(2(e+fx))+210(A-$$

input
```
Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f
*x])^(7/2),x]
```

output
```
-1/6720*(c^3*(-1 + Sin[e + f*x])^3*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c - c
*Sin[e + f*x]]*(525*(A - B)*Cos[2*(e + f*x)] + 210*(A - B)*Cos[4*(e + f*x)
] + 35*A*Cos[6*(e + f*x)] - 35*B*Cos[6*(e + f*x)] + 4200*A*Sin[e + f*x] -
525*B*Sin[e + f*x] + 700*A*Sin[3*(e + f*x)] + 35*B*Sin[3*(e + f*x)] + 84*A
*Sin[5*(e + f*x)] + 63*B*Sin[5*(e + f*x)] + 15*B*Sin[7*(e + f*x)]))/(f*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
^5)
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3452, 3042, 3219, 3042, 3219, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{ 3452}$$

$$\frac{1}{7}(7A-B)\int (\sin(e+fx)a+a)^{5/2}(c-c\sin(e+fx))^{7/2}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{7/2}}{7f}$$

$\downarrow$ 3042

$$\frac{1}{7}(7A-B)\int (\sin(e+fx)a+a)^{5/2}(c-c\sin(e+fx))^{7/2}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{7/2}}{7f}$$

$\downarrow$ 3219

$$\frac{1}{7}(7A-B)\left(\frac{2}{3}a\int (\sin(e+fx)a+a)^{3/2}(c-c\sin(e+fx))^{7/2}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx)}{6f}\right.$$
$$-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{7/2}}{7f}$$

$\downarrow$ 3042

$$\frac{1}{7}(7A-B)\left(\frac{2}{3}a\int (\sin(e+fx)a+a)^{3/2}(c-c\sin(e+fx))^{7/2}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx)}{6f}\right.$$
$$-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{7/2}}{7f}$$

$\downarrow$ 3219

$$\frac{1}{7}(7A-B)\left(\frac{2}{3}a\left(\frac{2}{5}a\int \sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{7/2}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx}{5f}\right.\right.$$
$$-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{7/2}}{7f}$$

$\downarrow$ 3042

$$\frac{1}{7}(7A-B)\left(\frac{2}{3}a\left(\frac{2}{5}a\int \sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{7/2}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx}{5f}\right.\right.$$
$$-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{7/2}}{7f}$$

$$\Big\downarrow \text{3217}$$

$$\frac{1}{7}(7A-B)\left(\frac{2}{3}a\left(-\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{10f\sqrt{a\sin(e+fx)+a}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}}{5f}\right)-a\right.$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{7/2}}{7f}$$

input
```
Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(
7/2),x]
```

output
```
-1/7*(B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(7/2)
)/f + ((7*A - B)*(-1/6*(a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*S
in[e + f*x])^(7/2))/f + (2*a*(-1/10*(a^2*Cos[e + f*x]*(c - c*Sin[e + f*x])
^(7/2))/(f*Sqrt[a + a*Sin[e + f*x]]) - (a*Cos[e + f*x]*Sqrt[a + a*Sin[e +
f*x]]*(c - c*Sin[e + f*x])^(7/2))/(5*f)))/3))/7
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3452
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

Maple [A] (verified)

Time = 79.27 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.72

method	result
default	$\frac{2240c^3a^2\left(\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^6+\frac{3\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^4}{5}+\frac{3\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2}{10}+\frac{1}{10}\right)A\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\frac{1}{2}\right)\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\right.\right.}{}$
parts	$\frac{8A\sqrt{4}\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2}\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2}\left(10\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^6+6\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^4+3\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)a^2c^3\sin\left(\frac{\pi}{4}\right.}{15f}$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,metho
d=_RETURNVERBOSE)
```

output
```
2240*c^3*a^2*((cos(1/4*Pi+1/2*f*x+1/2*e)^6+3/5*cos(1/4*Pi+1/2*f*x+1/2*e)^4
+3/10*cos(1/4*Pi+1/2*f*x+1/2*e)^2+1/10)*A*(cos(1/2*f*x+1/2*e)^2-1/2)*tan(1
/4*Pi+1/2*f*x+1/2*e)*sin(1/4*Pi+1/2*f*x+1/2*e)^4-12/7*sin(1/2*f*x+1/2*e)^2
*B*cos(1/2*f*x+1/2*e)^2*(sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)^9-2*sin(1/2
*f*x+1/2*e)*cos(1/2*f*x+1/2*e)^7-7/12*cos(1/2*f*x+1/2*e)^8+17/10*cos(1/2*f
*x+1/2*e)^5*sin(1/2*f*x+1/2*e)+7/6*cos(1/2*f*x+1/2*e)^6-7/10*cos(1/2*f*x+1
/2*e)^3*sin(1/2*f*x+1/2*e)-49/48*cos(1/2*f*x+1/2*e)^4+7/48*sin(1/2*f*x+1/2
*e)*cos(1/2*f*x+1/2*e)+7/16*cos(1/2*f*x+1/2*e)^2-7/64))*(c*cos(1/4*Pi+1/2*
f*x+1/2*e)^2)^(1/2)*(a*sin(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)/(210*f*cos(1/2*f
*x+1/2*e)^2-105*f)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.81

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx=\frac{\left(35\,(A-B)a^2c^3\cos\left(fx+e\right)^6-35\,(A-B)a^2c^3+2\left(15\,Ba^2c^3\cos\left(fx+e\right)^6+3\,(7\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x
, algorithm="fricas")
```

output
```
1/210*(35*(A - B)*a^2*c^3*cos(f*x + e)^6 - 35*(A - B)*a^2*c^3 + 2*(15*B*a^
2*c^3*cos(f*x + e)^6 + 3*(7*A - B)*a^2*c^3*cos(f*x + e)^4 + 4*(7*A - B)*a^
2*c^3*cos(f*x + e)^2 + 8*(7*A - B)*a^2*c^3)*sin(f*x + e))*sqrt(a*sin(f*x +
 e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx=\text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}(-c\sin(fx+e)+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e)
 + c)^(7/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(174) = 348$.

Time = 0.30 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.79

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx=\text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x
, algorithm="giac")
```

output
```
-16/105*(120*B*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^14 - 70*A*a^2*c^3*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1
/4*pi + 1/2*f*x + 1/2*e)^12 - 350*B*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^1
2 + 168*A*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^10 + 336*B*a^2*c^3*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*p
i + 1/2*f*x + 1/2*e)^10 - 105*A*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8 - 1
05*B*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x
 + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8)*sqrt(a)*sqrt(c)/f
```

Mupad [B] (verification not implemented)

Time = 41.89 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.93

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \frac{e^{-e\,7i-f\,x\,7i}\sqrt{c-c\,\sin\left(e+f\,x\right)}\left(-\frac{a^2\,c^3\,e^{e\,7i+f\,x\,7i}\,\cos(2\,e+2\,f\,x)\,(A\,1i-B\,1i)\,\sqrt{a+a\,\sin(e+f\,x)}\,5i}{32\,f}\right.}{}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(
7/2),x)
```

output
```
(exp(- e*7i - f*x*7i)*(c - c*sin(e + f*x))^(1/2)*((a^2*c^3*exp(e*7i + f*x*
7i)*sin(5*e + 5*f*x)*(4*A + 3*B)*(a + a*sin(e + f*x))^(1/2))/(160*f) - (a^
2*c^3*exp(e*7i + f*x*7i)*cos(4*e + 4*f*x)*(A*1i - B*1i)*(a + a*sin(e + f*x
))^(1/2)*1i)/(16*f) - (a^2*c^3*exp(e*7i + f*x*7i)*cos(6*e + 6*f*x)*(A*1i -
 B*1i)*(a + a*sin(e + f*x))^(1/2)*1i)/(96*f) - (a^2*c^3*exp(e*7i + f*x*7i)
*cos(2*e + 2*f*x)*(A*1i - B*1i)*(a + a*sin(e + f*x))^(1/2)*5i)/(32*f) + (5
*a^2*c^3*exp(e*7i + f*x*7i)*sin(e + f*x)*(8*A - B)*(a + a*sin(e + f*x))^(1
/2))/(32*f) + (a^2*c^3*exp(e*7i + f*x*7i)*sin(3*e + 3*f*x)*(20*A + B)*(a +
 a*sin(e + f*x))^(1/2))/(96*f) + (B*a^2*c^3*exp(e*7i + f*x*7i)*sin(7*e + 7
*f*x)*(a + a*sin(e + f*x))^(1/2))/(224*f)))/(2*cos(e + f*x))
```

Reduce [F]

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \sqrt{c}\,\sqrt{a}\,a^2c^3\left(-\left(\int\sqrt{\sin\left(fx+e\right)+1}\,\sqrt{-\sin\left(fx+e\right)+1}\,\sin\left(fx+e\right)^6dx\right)b-\left(\int\right.\right.$$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x)
```

output

```
sqrt(c)*sqrt(a)*a**2*c**3*( - int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f
*x) + 1)*sin(e + f*x)**6,x)*b - int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e +
 f*x) + 1)*sin(e + f*x)**5,x)*a + int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e
 + f*x) + 1)*sin(e + f*x)**5,x)*b + int(sqrt(sin(e + f*x) + 1)*sqrt( - sin
(e + f*x) + 1)*sin(e + f*x)**4,x)*a + 2*int(sqrt(sin(e + f*x) + 1)*sqrt( -
 sin(e + f*x) + 1)*sin(e + f*x)**4,x)*b + 2*int(sqrt(sin(e + f*x) + 1)*sqr
t( - sin(e + f*x) + 1)*sin(e + f*x)**3,x)*a - 2*int(sqrt(sin(e + f*x) + 1)
*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**3,x)*b - 2*int(sqrt(sin(e + f*x)
+ 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2,x)*a - int(sqrt(sin(e + f*x
) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2,x)*b - int(sqrt(sin(e + f
*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x),x)*a + int(sqrt(sin(e + f*
x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x),x)*b + int(sqrt(sin(e + f*x
) + 1)*sqrt( - sin(e + f*x) + 1),x)*a)
```

3.150 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 180

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = -\frac{2a^3A\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{15f\sqrt{a+a\sin(e+fx)}} - \frac{a^2A\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}{5f} - \frac{aA\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}{5f} - \frac{B\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}{6f}$$

output
```
-2/15*a^3*A*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/f/(a+a*sin(f*x+e))^(1/2)-1/5
*a^2*A*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(5/2)/f-1/5*a*A*
cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/f-1/6*B*cos(f*x+e
)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2)/f
```

Mathematica [A] (verified)

Time = 5.40 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.64

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx =$$
$$-\frac{a^2c^2\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}(50B+75B\cos(2(e+fx))+30B\cos(4(e+fx$$
$$960f$$

input
```
Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f
*x])^(5/2),x]
```

output
```
-1/960*(a^2*c^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e +
 f*x]]*(50*B + 75*B*Cos[2*(e + f*x)] + 30*B*Cos[4*(e + f*x)] + 5*B*Cos[6*(
e + f*x)] - 600*A*Sin[e + f*x] - 100*A*Sin[3*(e + f*x)] - 12*A*Sin[5*(e +
f*x)]))/f
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3452, 3042, 3219, 3042, 3219, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3452}$$

$$A\int (\sin(e+fx)a+a)^{5/2}(c-c\sin(e+fx))^{5/2}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}{6f}$$

$$\Big\downarrow \text{3042}$$

$$A\int (\sin(e+fx)a+a)^{5/2}(c-c\sin(e+fx))^{5/2}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}{6f}$$

$$\Big\downarrow \text{3219}$$

$$A\left(\frac{4}{5}a\int (\sin(e+fx)a+a)^{3/2}(c-c\sin(e+fx))^{5/2}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5}}{5f}\right.$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}{6f}$$

$$\Big\downarrow \text{3042}$$

$$A\left(\frac{4}{5}a\int (\sin(e+fx)a+a)^{3/2}(c-c\sin(e+fx))^{5/2}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5}}{5f}\right.$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}{6f}$$

$$\Big\downarrow \text{3219}$$

$$A\left(\frac{4}{5}a\left(\frac{1}{2}a\int \sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{5/2}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))}{4f}\right.\right.$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}{6f}$$

$$\Big\downarrow \text{3042}$$

$$A\left(\frac{4}{5}a\left(\frac{1}{2}a\int \sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{5/2}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))}{4f}\right.\right.$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}{6f}$$

$$\Big\downarrow \text{3217}$$

$$A\left(\frac{4}{5}a\left(-\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{6f\sqrt{a\sin(e+fx)+a}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}{4f}\right)-\frac{a\,c}{}\right.$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}{6f}$$

input
```
Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(
5/2),x]
```

output
```
-1/6*(B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(5/2)
)/f + A*(-1/5*(a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*
x])^(5/2))/f + (4*a*(-1/6*(a^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(f
*Sqrt[a + a*Sin[e + f*x]]) - (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c -
 c*Sin[e + f*x])^(5/2))/(4*f)))/5)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3452
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

Maple [A] (verified)

Time = 79.08 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.20

method	result
default	$\frac{192c^2\left(A\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\frac{1}{2}\right)\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^4+\frac{\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2}{2}+\frac{1}{6}\right)\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^4+\frac{5\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^2\left(\cos\left(\right.\right.}{}\right.}{30f\cos\left(\frac{fx}{2}+\frac{e}{2}\right.}$
parts	$\frac{8A\sqrt{4}\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2}\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2}\left(6\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^4+3\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)a^2c^2\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^4\tan\left(\frac{\pi}{4}+\frac{fx}{2}\right.}{15f}$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,metho
d=_RETURNVERBOSE)
```

output
```
192*c^2*(A*(cos(1/2*f*x+1/2*e)^2-1/2)*(cos(1/4*Pi+1/2*f*x+1/2*e)^4+1/2*cos
(1/4*Pi+1/2*f*x+1/2*e)^2+1/6)*tan(1/4*Pi+1/2*f*x+1/2*e)*sin(1/4*Pi+1/2*f*x
+1/2*e)^4+5/3*sin(1/2*f*x+1/2*e)^2*(cos(1/2*f*x+1/2*e)^4-1/2*cos(1/2*f*x+1
/2*e)^2+1/4)*B*cos(1/2*f*x+1/2*e)^2*(cos(1/2*f*x+1/2*e)^4-3/2*cos(1/2*f*x+
1/2*e)^2+3/4))*a^2*(c*cos(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)*(a*sin(1/4*Pi+1/2
*f*x+1/2*e)^2)^(1/2)/(30*f*cos(1/2*f*x+1/2*e)^2-15*f)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.65

$$\int(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx =$$
$$-\frac{\left(5\,Ba^2c^2\cos\left(fx+e\right)^6-5\,Ba^2c^2-2\left(3\,Aa^2c^2\cos\left(fx+e\right)^4+4\,Aa^2c^2\cos\left(fx+e\right)^2+8\,Aa^2c^2\right)\sin\left(fx+\right.}{30\,f\cos\left(fx+e\right)}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x
, algorithm="fricas")
```

output
```
-1/30*(5*B*a^2*c^2*cos(f*x + e)^6 - 5*B*a^2*c^2 - 2*(3*A*a^2*c^2*cos(f*x +
 e)^4 + 4*A*a^2*c^2*cos(f*x + e)^2 + 8*A*a^2*c^2)*sin(f*x + e))*sqrt(a*sin
(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}(-c\sin(fx+e)+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e)
 + c)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(156) = 312$.

Time = 0.25 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.97

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x
, algorithm="giac")
```

output
```
-16/15*(10*B*a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi +
 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^12 - 6*A*a^2*c^2*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*
pi + 1/2*f*x + 1/2*e)^10 - 30*B*a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^10 +
15*A*a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x
 + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8 + 30*B*a^2*c^2*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2
*f*x + 1/2*e)^8 - 10*A*a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin
(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^6 - 10*B*a^2*c
^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))
*sin(-1/4*pi + 1/2*f*x + 1/2*e)^6)*sqrt(a)*sqrt(c)/f
```

Mupad [B] (verification not implemented)

Time = 38.45 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.73

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx =$$
$$-\frac{a^2\,c^2\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(75\,B\,\cos\left(e+f\,x\right)+105\,B\,\cos\left(3\,e+3\,f\,x\right)+35\,B\right.}{960\,f\,\left(\mathrm{co}\right.}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(
5/2),x)
```

output
```
-(a^2*c^2*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(75*B
*cos(e + f*x) + 105*B*cos(3*e + 3*f*x) + 35*B*cos(5*e + 5*f*x) + 5*B*cos(7
*e + 7*f*x) - 700*A*sin(2*e + 2*f*x) - 112*A*sin(4*e + 4*f*x) - 12*A*sin(6
*e + 6*f*x)))/(960*f*(cos(2*e + 2*f*x) + 1))
```

Reduce [F]

$$\int (a + a\sin(e + fx))^{5/2}(A + B\sin(e + fx))(c - c\sin(e+fx))^{5/2}\,dx = \sqrt{c}\,\sqrt{a}\,a^2c^2\left(\left(\left(\int\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^5\,dx\right)b + \left(\int\right.\right.\right.$$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x)
```

output
```
sqrt(c)*sqrt(a)*a**2*c**2*(int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x)
 + 1)*sin(e + f*x)**5,x)*b + int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*
x) + 1)*sin(e + f*x)**4,x)*a - 2*int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e
+ f*x) + 1)*sin(e + f*x)**3,x)*b - 2*int(sqrt(sin(e + f*x) + 1)*sqrt( - si
n(e + f*x) + 1)*sin(e + f*x)**2,x)*a + int(sqrt(sin(e + f*x) + 1)*sqrt( -
sin(e + f*x) + 1)*sin(e + f*x),x)*b + int(sqrt(sin(e + f*x) + 1)*sqrt( - s
in(e + f*x) + 1),x)*a)
```

3.151 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 142

$$\begin{aligned}\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c&-c\sin(e+fx))^{3/2}\,dx=\frac{(5A+B)c^2\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{30f\sqrt{c-c\sin(e+fx)}}\\&+\frac{(5A+B)c\cos(e+fx)(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{20f}\\&-\frac{B\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}{5f}\end{aligned}$$

output
```
1/30*(5*A+B)*c^2*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(1/2
)+1/20*(5*A+B)*c*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2)/
f-1/5*B*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2)/f
```

Mathematica [A] (verified)

Time = 9.12 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.16

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx =$$
$$-\frac{c(-1+\sin(e+fx))(a(1+\sin(e+fx)))^{5/2}\sqrt{c-c\sin(e+fx)}(4(100A+11B)\sin(e+fx)+4\cos(2(e}{480f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

input
```
Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f
*x])^(3/2),x]
```

output
```
-1/480*(c*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c - c*Sin[
e + f*x]]*(4*(100*A + 11*B)*Sin[e + f*x] + 4*Cos[2*(e + f*x)]*(-15*(A + B)
 + 4*(5*A - 2*B)*Sin[e + f*x]) - 3*Cos[4*(e + f*x)]*(5*(A + B) + 4*B*Sin[e
 + f*x])))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^5)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3452, 3042, 3219, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))dx$$

$$\Big\downarrow \; 3452$$

$$\frac{1}{5}(5A+B)\int(\sin(e+fx)a+a)^{5/2}(c-c\sin(e+fx))^{3/2}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{3/2}}{5f}$$

$\big\downarrow$ 3042

$$\frac{1}{5}(5A+B)\int(\sin(e+fx)a+a)^{5/2}(c-c\sin(e+fx))^{3/2}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{3/2}}{5f}$$

$\big\downarrow$ 3219

$$\frac{1}{5}(5A+B)\left(\frac{1}{2}c\int(\sin(e+fx)a+a)^{5/2}\sqrt{c-c\sin(e+fx)}dx+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}{4f}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{3/2}}{5f}$$

$\big\downarrow$ 3042

$$\frac{1}{5}(5A+B)\left(\frac{1}{2}c\int(\sin(e+fx)a+a)^{5/2}\sqrt{c-c\sin(e+fx)}dx+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}{4f}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{3/2}}{5f}$$

$\big\downarrow$ 3217

$$\frac{1}{5}(5A+B)\left(\frac{c^2\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6f\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}{4f}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{3/2}}{5f}$$

input
```
Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(
3/2),x]
```

output
```
-1/5*(B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(3/2)
)/f + ((5*A + B)*((c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(6*f*Sqrt[
c - c*Sin[e + f*x]]) + (c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c -
 c*Sin[e + f*x]])/(4*f)))/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3452
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

Maple [A] (verified)

Time = 7.54 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.67

method	result
default	$\frac{120\left(\left(\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-\frac{1}{2}\right)A\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}+\frac{1}{3}\right)\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}+\frac{8\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}B\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}{}\right.\right.}{3}$
parts	$\frac{2A\sqrt{4}\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\left(3\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}+1\right)a^{2}c\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)}{3f}+\frac{2Bca^{2}\cos\left(\right.}{}$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,metho
d=_RETURNVERBOSE)
```

output
```
120*((cos(1/2*f*x+1/2*e)^2-1/2)*A*(cos(1/4*Pi+1/2*f*x+1/2*e)^2+1/3)*tan(1/
4*Pi+1/2*f*x+1/2*e)*sin(1/4*Pi+1/2*f*x+1/2*e)^4+8/5*sin(1/2*f*x+1/2*e)^2*B
*cos(1/2*f*x+1/2*e)^2*(cos(1/2*f*x+1/2*e)^5*sin(1/2*f*x+1/2*e)-cos(1/2*f*x
+1/2*e)^3*sin(1/2*f*x+1/2*e)+5/8*cos(1/2*f*x+1/2*e)^4+5/12*sin(1/2*f*x+1/2
*e)*cos(1/2*f*x+1/2*e)-5/8*cos(1/2*f*x+1/2*e)^2+5/16))*c*a^2*(c*cos(1/4*Pi
+1/2*f*x+1/2*e)^2)^(1/2)*(a*sin(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)/(30*f*cos(1
/2*f*x+1/2*e)^2-15*f)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.84

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx =$$
$$-\frac{(15\,(A+B)a^{2}c\cos\left(fx+e\right)^{4}-15\,(A+B)a^{2}c+4\,(3\,Ba^{2}c\cos\left(fx+e\right)^{4}-(5\,A+B)a^{2}c\cos\left(fx+e\right)^{2}-}{60\,f\cos\left(fx+e\right)}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x
, algorithm="fricas")
```

output
```
-1/60*(15*(A + B)*a^2*c*cos(f*x + e)^4 - 15*(A + B)*a^2*c + 4*(3*B*a^2*c*c
os(f*x + e)^4 - (5*A + B)*a^2*c*cos(f*x + e)^2 - 2*(5*A + B)*a^2*c)*sin(f*
x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e)
)
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}(-c\sin(fx+e)+c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e)
 + c)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(124) = 248$.

Time = 0.32 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.40

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x
, algorithm="giac")
```

output
```
-4/15*(24*B*a^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^10 - 15*A*a^2*c*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi +
 1/2*f*x + 1/2*e)^8 - 75*B*a^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(s
in(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8 + 40*A*a^2
*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))
*sin(-1/4*pi + 1/2*f*x + 1/2*e)^6 + 80*B*a^2*c*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e
)^6 - 30*A*a^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^4 - 30*B*a^2*c*sgn(cos(-1/4*
pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1
/2*f*x + 1/2*e)^4)*sqrt(a)*sqrt(c)/f
```

Mupad [B] (verification not implemented)

Time = 38.58 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.23

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx =$$
$$-\frac{a^2\,c\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(60\,A\,\cos\left(e+f\,x\right)+60\,B\,\cos\left(e+f\,x\right)+75\,A\,\cos\left(\right.\right.}{}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(
3/2),x)
```

output
```
-(a^2*c*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(60*A*c
os(e + f*x) + 60*B*cos(e + f*x) + 75*A*cos(3*e + 3*f*x) + 15*A*cos(5*e + 5
*f*x) + 75*B*cos(3*e + 3*f*x) + 15*B*cos(5*e + 5*f*x) - 400*A*sin(2*e + 2*
f*x) - 40*A*sin(4*e + 4*f*x) - 50*B*sin(2*e + 2*f*x) + 16*B*sin(4*e + 4*f*
x) + 6*B*sin(6*e + 6*f*x)))/(480*f*(cos(2*e + 2*f*x) + 1))
```

Reduce [F]

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx=\sqrt{c}\,\sqrt{a}\,a^2c\left(-\left(\int\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^4\,dx\right)b-\left(\int\right.\right.$$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x)
```

output
```
sqrt(c)*sqrt(a)*a**2*c*( - int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x)
 + 1)*sin(e + f*x)**4,x)*b - int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*
x) + 1)*sin(e + f*x)**3,x)*a - int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e +
f*x) + 1)*sin(e + f*x)**3,x)*b - int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e
+ f*x) + 1)*sin(e + f*x)**2,x)*a + int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(
e + f*x) + 1)*sin(e + f*x)**2,x)*b + int(sqrt(sin(e + f*x) + 1)*sqrt( - si
n(e + f*x) + 1)*sin(e + f*x),x)*a + int(sqrt(sin(e + f*x) + 1)*sqrt( - sin
(e + f*x) + 1)*sin(e + f*x),x)*b + int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(
e + f*x) + 1),x)*a)
```

3.152 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e$

Optimal result

Integrand size = 40, antiderivative size = 96

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx=\frac{(A-B)c\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}+\frac{Bc\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4af\sqrt{c-c\sin(e+fx)}}$$

output
```
1/3*(A-B)*c*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(1/2)+1/4
*B*c*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/a/f/(c-c*sin(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 2.84 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.06

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx=\frac{a^2\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}(3B\cos(4(e$$

input
```
Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e
 + f*x]],x]
```

output
```
(a^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(3*B
*Cos[4*(e + f*x)] + 16*(7*A + 2*B)*Sin[e + f*x] - 4*Cos[2*(e + f*x)]*(12*A
 + 9*B + 4*(A + 2*B)*Sin[e + f*x])))/(96*f)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3042, 3450, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3450}$$

$$(A-B)\int (\sin(e+fx)a+a)^{5/2}\sqrt{c-c\sin(e+fx)}dx+\frac{B\int (\sin(e+fx)a+a)^{7/2}\sqrt{c-c\sin(e+fx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$(A-B)\int (\sin(e+fx)a+a)^{5/2}\sqrt{c-c\sin(e+fx)}dx+\frac{B\int (\sin(e+fx)a+a)^{7/2}\sqrt{c-c\sin(e+fx)}dx}{a}$$

$$\Big\downarrow \text{3217}$$

$$\frac{c(A-B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}+\frac{Bc\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4af\sqrt{c-c\sin(e+fx)}}$$

input
```
Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x
]],x]
```

output
```
((A - B)*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*f*Sqrt[c - c*Sin[e
+ f*x]]) + (B*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(4*a*f*Sqrt[c - c
*Sin[e + f*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3450
```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[B/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] -
Simp[(B*c - A*d)/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[
a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(84) = 168$.

Time = 8.51 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.83

method	result
default	$\frac{16a^2\left(A\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\frac{1}{2}\right)\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^4+2\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^2\left(-\frac{3\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{4}+\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{3c}{}\right.\right.}{6f\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-3f}$
parts	$\frac{4A\sqrt{4}\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2a^2\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2}\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2}\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)}{3f}+\frac{2Ba^2\sqrt{-}}{}$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
16*a^2*(A*tan(1/4*Pi+1/2*f*x+1/2*e)*(cos(1/2*f*x+1/2*e)^2-1/2)*sin(1/4*Pi+
1/2*f*x+1/2*e)^4+2*sin(1/2*f*x+1/2*e)^2*(-3/4*cos(1/2*f*x+1/2*e)^4+sin(1/2
*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+3/4*cos(1/2*f*x+1/2*e)^2+3/8)*B*cos(1/2*f*x
+1/2*e)^2)*(c*cos(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)*(a*sin(1/4*Pi+1/2*f*x+1/2
*e)^2)^(1/2)/(6*f*cos(1/2*f*x+1/2*e)^2-3*f)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.22

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \frac{(3\,Ba^2\cos(fx+e)^4-12\,(A+B)a^2\cos(fx+e)^2+3\,(4\,A+3\,B)}{}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x
, algorithm="fricas")
```

output
```
1/12*(3*B*a^2*cos(f*x + e)^4 - 12*(A + B)*a^2*cos(f*x + e)^2 + 3*(4*A + 3*
B)*a^2 - 4*((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(2*A + B)*a^2)*sin(f*x + e))*
sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^{5/2}(A + B\sin(e + fx))\sqrt{c - c\sin(e + fx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a + a\sin(e + fx))^{5/2}(A + B\sin(e+fx))\sqrt{c - c\sin(e + fx)}\,dx = \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^{\frac{5}{2}}\sqrt{-c\sin(fx + e) + c}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*sqrt(-c*sin(f*x
+ e) + c), x)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.56

$$\int (a + a\sin(e + fx))^{5/2}(A + B\sin(e + fx))\sqrt{c - c\sin(e + fx)}\,dx =$$

$$-\frac{4\left(3\,Ba^2\cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,fx + \frac{1}{2}\,e\right)^8\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,fx + \frac{1}{2}\,e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi + \frac{1}{2}\,fx + \frac{1}{2}\,e\right)\right) + 2\,Aa^2\,\mathrm{c}\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x
, algorithm="giac")
```

output
```
-4/3*(3*B*a^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*A*a^2*cos(-1/4*pi + 1/2*f
*x + 1/2*e)^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e)) - 2*B*a^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)*sqrt(c)/f
```

Mupad [B] (verification not implemented)

Time = 3.15 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.55

$$\int (a + a\sin(e + fx))^{5/2}(A + B\sin(e + fx))\sqrt{c - c\sin(e + fx)}\,dx =$$

$$-\frac{a^2\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(48\,A\,\cos\left(e+f\,x\right)+36\,B\,\cos\left(e+f\,x\right)+48\,A\,\cos\left(3\right.\right.}{}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(
1/2),x)
```

output
```
-(a^2*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(48*A*cos
(e + f*x) + 36*B*cos(e + f*x) + 48*A*cos(3*e + 3*f*x) + 33*B*cos(3*e + 3*f
*x) - 3*B*cos(5*e + 5*f*x) - 112*A*sin(2*e + 2*f*x) + 8*A*sin(4*e + 4*f*x)
 - 32*B*sin(2*e + 2*f*x) + 16*B*sin(4*e + 4*f*x)))/(96*f*(cos(2*e + 2*f*x)
 + 1))
```

Reduce [F]

$$\int (a + a\sin(e + fx))^{5/2}(A + B\sin(e+fx))\sqrt{c - c\sin(e + fx)}\,dx = \sqrt{c}\,\sqrt{a}\,a^2\Bigg(\bigg(\bigg(\int \sqrt{\sin(fx + e) + 1}\,\sqrt{-\sin(fx + e) + 1}\,\sin(fx+e$$

$$+ \left(\int \sqrt{\sin(fx + e) + 1}\,\sqrt{-\sin(fx + e) + 1}\,\sin(fx + e)^2\,dx\right) a$$

$$+ 2\left(\int \sqrt{\sin(fx + e) + 1}\,\sqrt{-\sin(fx + e) + 1}\,\sin(fx + e)^2\,dx\right) b$$

$$+ 2\left(\int \sqrt{\sin(fx + e) + 1}\,\sqrt{-\sin(fx + e) + 1}\,\sin(fx + e)\,dx\right) a$$

$$+ \left(\int \sqrt{\sin(fx + e) + 1}\,\sqrt{-\sin(fx + e) + 1}\,\sin(fx + e)\,dx\right) b$$

$$+ \left(\int \sqrt{\sin(fx + e) + 1}\,\sqrt{-\sin(fx + e) + 1}dx\right) a\Bigg)$$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)
```

output
```
sqrt(c)*sqrt(a)*a**2*(int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)
*sin(e + f*x)**3,x)*b + int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) +
1)*sin(e + f*x)**2,x)*a + 2*int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x
) + 1)*sin(e + f*x)**2,x)*b + 2*int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e +
 f*x) + 1)*sin(e + f*x),x)*a + int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e +
f*x) + 1)*sin(e + f*x),x)*b + int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f
*x) + 1),x)*a)
```

3.153 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 193

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{4a^3(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{2a^2(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}} - \frac{a(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2f\sqrt{c-c\sin(e+fx)}} - \frac{B\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}$$

output
```
-4*a^3*(A+B)*cos(f*x+e)*ln(1-sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin
(f*x+e))^(1/2)-2*a^2*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f/(c-c*sin(f*
x+e))^(1/2)-1/2*a*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e
))^(1/2)-1/3*B*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 11.15 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.92

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{5/2}\left(-3(A+3B)\cos(2(e+fx))+96A\log\left(\cos\right.\right.}{12f\left(\cos\left(\frac{1}{2}(e+f\right.\right.}$$

input
```
Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin
[e + f*x]],x]
```

output
```
-1/12*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2)*
(-3*(A + 3*B)*Cos[2*(e + f*x)] + 96*A*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)
/2]] + 96*B*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + (36*A + 51*B)*Sin[e
 + f*x] - B*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*
Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.275$, Rules used = {3042, 3452, 3042, 3219, 3042, 3219, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3452}$$

$$(A+B)\int \frac{(\sin(e+fx)a+a)^{5/2}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$(A+B)\int \frac{(\sin(e+fx)a+a)^{5/2}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{3219}$$

$$(A+B)\left(2a\int \frac{(\sin(e+fx)a+a)^{3/2}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$(A+B)\left(2a\int \frac{(\sin(e+fx)a+a)^{3/2}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{3219}$$

$$(A+B)\left(2a\left(2a\int \frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$(A+B)\left(2a\left(2a\int \frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{3216}$$

$$(A+B)\left(2a\left(\frac{2a^2c\cos(e+fx)\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$(A+B)\left(2a\left(\frac{2a^2c\cos(e+fx)\int\frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin(e+\ldots}{2f\sqrt{c-c\sin(e\ldots}}\right.$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}$$

$\Big\downarrow$ 3146

$$(A+B)\left(2a\left(-\frac{2a^2\cos(e+fx)\int\frac{1}{c-c\sin(e+fx)}d(-c\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+f\ldots}{2f\sqrt{\ldots}}\right.$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}$$

$\Big\downarrow$ 16

$$(A+B)\left(2a\left(-\frac{2a^2\cos(e+fx)\log(c-c\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin\ldots}{2f\sqrt{c-c\sin\ldots}}\right.$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}$$

input
```
Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f
*x]],x]
```

output
```
-1/3*(B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(f*Sqrt[c - c*Sin[e + f*x
]]) + (A + B)*(-1/2*(a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(f*Sqrt[c
- c*Sin[e + f*x]]) + 2*a*((-2*a^2*Cos[e + f*x]*Log[c - c*Sin[e + f*x]])/(f
*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (a*Cos[e + f*x]*Sqrt
[a + a*Sin[e + f*x]])/(f*Sqrt[c - c*Sin[e + f*x]])))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3452
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 475 vs. 2(173) = 346.

Time = 6.98 (sec) , antiderivative size = 476, normalized size of antiderivative = 2.47

method	result
default	$\frac{A\sqrt{4}\left(-\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}+4\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}+4\ln\left(\frac{2}{\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1}\right)-4\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1\right)-4\ln\left(-\right.\right.}{f\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}}$
parts	$\frac{A\sqrt{4}\left(-\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}+4\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}+4\ln\left(\frac{2}{\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1}\right)-4\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1\right)-4\ln\left(-\right.\right.}{f\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}}$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
A/f*4^(1/2)*(-cos(1/4*Pi+1/2*f*x+1/2*e)^4+4*cos(1/4*Pi+1/2*f*x+1/2*e)^2+4*
ln(2/(cos(1/4*Pi+1/2*f*x+1/2*e)+1))-4*ln(-cot(1/4*Pi+1/2*f*x+1/2*e)+csc(1/
4*Pi+1/2*f*x+1/2*e)+1)-4*ln(-cot(1/4*Pi+1/2*f*x+1/2*e)+csc(1/4*Pi+1/2*f*x+
1/2*e)-1)-3)*(a*sin(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)*a^2/(c*cos(1/4*Pi+1/2*f
*x+1/2*e)^2)^(1/2)*cot(1/4*Pi+1/2*f*x+1/2*e)+2/3*B*a^2/f*((12*sin(1/2*f*x+
1/2*e)-12*cos(1/2*f*x+1/2*e))*ln(-2*(cos(1/2*f*x+1/2*e)-sin(1/2*f*x+1/2*e)
)/(cos(1/2*f*x+1/2*e)+1))+(-12*sin(1/2*f*x+1/2*e)+12*cos(1/2*f*x+1/2*e))*l
n(2/(cos(1/2*f*x+1/2*e)+1))+cos(1/2*f*x+1/2*e)*((-4*cos(1/2*f*x+1/2*e)^4-4
*cos(1/2*f*x+1/2*e)^3*sin(1/2*f*x+1/2*e)-5*cos(1/2*f*x+1/2*e)^2+9*sin(1/2*
f*x+1/2*e)*cos(1/2*f*x+1/2*e)+12)*sin(1/2*f*x+1/2*e)^2-12*sin(1/2*f*x+1/2*
e)*cos(1/2*f*x+1/2*e)))*((2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+1)*a)^(1
/2)/(cos(1/2*f*x+1/2*e)+sin(1/2*f*x+1/2*e))/(-(2*sin(1/2*f*x+1/2*e)*cos(1/
2*f*x+1/2*e)-1)*c)^(1/2)
```

Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x
, algorithm="fricas")
```

output
```
integral(((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x +
e)^2 - 2*(A + B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f
*x + e) + c)/(c*sin(f*x + e) - c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2)
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/sqrt(-c*sin(f*x
+ e) + c), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + a\sin(e + fx))^{5/2}(A + B\sin(e + fx))}{\sqrt{c - c\sin(e + fx)}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a\sin(e + fx))^{5/2}(A + B\sin(e + fx))}{\sqrt{c - c\sin(e + fx)}}\, dx = \int \frac{(A + B\,\sin(e + f\,x))\,(a + a\,\sin(e + f\,x))^{5/2}}{\sqrt{c - c\,\sin(e + f\,x)}}\, dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))
^(1/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))
^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,a^2\left(-\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^3}{\sin(fx+e)-1}dx\right)\right.}{}$$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a**2*( - int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x)
 + 1)*sin(e + f*x)**3)/(sin(e + f*x) - 1),x)*b - int((sqrt(sin(e + f*x) +
1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x) - 1),x)*a - 2*
int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(si
n(e + f*x) - 1),x)*b - 2*int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x)
+ 1)*sin(e + f*x))/(sin(e + f*x) - 1),x)*a - int((sqrt(sin(e + f*x) + 1)*s
qrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x) - 1),x)*b - int((sqrt
(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1))/(sin(e + f*x) - 1),x)*a))/c
```

3.154 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 210

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2f(c-c\sin(e+fx))^{3/2}}$$
$$+\frac{4a^3(A+2B)\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$
$$+\frac{2a^2(A+2B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}}$$
$$+\frac{a(A+2B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf\sqrt{c-c\sin(e+fx)}}$$

output
```
1/2*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(3/2)+4*a^3
*(A+2*B)*cos(f*x+e)*ln(1-sin(f*x+e))/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f
*x+e))^(1/2)+2*a^2*(A+2*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*sin(
f*x+e))^(1/2)+1/2*a*(A+2*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin
(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 12.41 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.10

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx =$$
$$-\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}(28A+16B+2(2A+7B)\cos(2(e+fx))+$$

input
```
Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
 f*x])^(3/2),x]
```

output
```
-1/8*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]
*(28*A + 16*B + 2*(2*A + 7*B)*Cos[2*(e + f*x)] + 64*A*Log[Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2]] + 128*B*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + (8
*A + 31*B - 64*(A + 2*B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]])*Sin[e +
 f*x] + B*Sin[3*(e + f*x)]))/(c*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-
1 + Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.95, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.275$, Rules used = {3042, 3451, 3042, 3219, 3042, 3219, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}dx$$
$$\Big\downarrow \text{3451}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(A+2B)\int\frac{(\sin(e+fx)a+a)^{5/2}}{\sqrt{c-c\sin(e+fx)}}dx}{c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(A+2B)\int\frac{(\sin(e+fx)a+a)^{5/2}}{\sqrt{c-c\sin(e+fx)}}dx}{c}$$

$$\Big\downarrow \text{3219}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(A+2B)\left(2a\int\frac{(\sin(e+fx)a+a)^{3/2}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right)}{c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(A+2B)\left(2a\int\frac{(\sin(e+fx)a+a)^{3/2}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right)}{c}$$

$$\Big\downarrow \text{3219}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(A+2B)\left(2a\left(2a\int\frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right)}{c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(A+2B)\left(2a\left(2a\int\frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right)}{c}$$

$$\Big\downarrow \text{3216}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(A+2B)\left(2a\left(\frac{2a^2c\cos(e+fx)\int\frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right)}{c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(A+2B)\left(2a\left(\frac{2a^2c\cos(e+fx)\int\frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right)}{c}$$

$$\Big\downarrow \text{ 3146}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(A+2B)\left(2a\left(-\frac{2a^2\cos(e+fx)\int\frac{1}{c-c\sin(e+fx)}d(-c\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right)}{c}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(A+2B)\left(2a\left(-\frac{2a^2\cos(e+fx)\log(c-c\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right)}{c}$$

input
```
Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(3/2),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(2*f*(c - c*Sin[e + f*x]
)^(3/2)) - ((A + 2*B)*(-1/2*(a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(f
*Sqrt[c - c*Sin[e + f*x]]) + 2*a*((-2*a^2*Cos[e + f*x]*Log[c - c*Sin[e + f
*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (a*Cos[e + f
*x]*Sqrt[a + a*Sin[e + f*x]])/(f*Sqrt[c - c*Sin[e + f*x]]))))/c
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(190) = 380$.

Time = 6.96 (sec) , antiderivative size = 470, normalized size of antiderivative = 2.24

method	result
default	$-\frac{A\sqrt{4}\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^4-4\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)-1\right)\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2+4\ln\left(\frac{2}{\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\cos\left(\frac{\pi}{4}+\frac{f}{2}\right.\right.}{f\sqrt{c\cos}}$
parts	$-\frac{A\sqrt{4}\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^4-4\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)-1\right)\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2+4\ln\left(\frac{2}{\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\cos\left(\frac{\pi}{4}+\frac{f}{2}\right.\right.}{f\sqrt{c\cos}}$

input

```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,metho
d=_RETURNVERBOSE)
```

output

```
-A/f*4^(1/2)*(cos(1/4*Pi+1/2*f*x+1/2*e)^4-4*ln(-cot(1/4*Pi+1/2*f*x+1/2*e)+
csc(1/4*Pi+1/2*f*x+1/2*e)-1)*cos(1/4*Pi+1/2*f*x+1/2*e)^2+4*ln(2/(cos(1/4*P
i+1/2*f*x+1/2*e)+1))*cos(1/4*Pi+1/2*f*x+1/2*e)^2-4*ln(-cot(1/4*Pi+1/2*f*x+
1/2*e)+csc(1/4*Pi+1/2*f*x+1/2*e)+1)*cos(1/4*Pi+1/2*f*x+1/2*e)^2-1)*a^2*(a*
sin(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)/(c*cos(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)/c
*sec(1/4*Pi+1/2*f*x+1/2*e)*csc(1/4*Pi+1/2*f*x+1/2*e)+2*B/f*a^2/c*(8*(-2*si
n(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+1)*ln(2*(sin(1/2*f*x+1/2*e)-cos(1/2*f*
x+1/2*e))/(cos(1/2*f*x+1/2*e)+1))+8*(2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*
e)-1)*ln(2/(cos(1/2*f*x+1/2*e)+1))+cos(1/2*f*x+1/2*e)*((-2*sin(1/2*f*x+1/2
*e)*cos(1/2*f*x+1/2*e)^2-7*cos(1/2*f*x+1/2*e))*sin(1/2*f*x+1/2*e)^2+8*sin(
1/2*f*x+1/2*e)))*((2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+1)*a)^(1/2)/(-1
+2*cos(1/2*f*x+1/2*e)^2)/(-(2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-1)*c)^
(1/2)
```

Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}}{(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x
, algorithm="fricas")
```

output
```
integral(((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x +
e)^2 - 2*(A + B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f
*x + e) + c)/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2)
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}}{(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e)
 + c)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,\left(a+a\,\sin(e+f\,x)\right)^{5/2}}{\left(c-c\,\sin(e+f\,x)\right)^{3/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))
^(3/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))
^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,a^2\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^3}{\sin(fx+e)^2-2\sin(fx+e)+1}dx\right)b\right.}{}$$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a**2*(int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) +
1)*sin(e + f*x)**3)/(sin(e + f*x)**2 - 2*sin(e + f*x) + 1),x)*b + int((sqr
t(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*
x)**2 - 2*sin(e + f*x) + 1),x)*a + 2*int((sqrt(sin(e + f*x) + 1)*sqrt( - s
in(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x)**2 - 2*sin(e + f*x) + 1),x
)*b + 2*int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)
)/(sin(e + f*x)**2 - 2*sin(e + f*x) + 1),x)*a + int((sqrt(sin(e + f*x) + 1
)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**2 - 2*sin(e + f*x
) + 1),x)*b + int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1))/(sin(
e + f*x)**2 - 2*sin(e + f*x) + 1),x)*a))/c**2
```

3.155 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 212

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+5B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4cf(c-c\sin(e+fx))^{3/2}} - \frac{a^3(A+5B)\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{a^2(A+5B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2c^2f\sqrt{c-c\sin(e+fx)}}$$

output

```
1/4*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(5/2)-1/4*a
*(A+5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(3/2)-a^3*
(A+5*B)*cos(f*x+e)*ln(1-sin(f*x+e))/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(
f*x+e))^(1/2)-1/2*a^2*(A+5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c^2/f/(c-c
*sin(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 11.79 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.98

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx$$

input
```
Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
 f*x])^(5/2),x]
```

output
```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2)*(2*(A
+ B) - 4*(A + 2*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 - 2*(A + 5*B)*L
og[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/
2])^4 - B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[e + f*x]))/(f*(Cos[(
e + f*x)/2] + Sin[(e + f*x)/2])^5*(c - c*Sin[e + f*x])^(5/2))
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.95, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.275$, Rules used = {3042, 3451, 3042, 3218, 3042, 3219, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3451}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{(A+5B)\int \frac{(\sin(e+fx)a+a)^{5/2}}{(c-c\sin(e+fx))^{3/2}}dx}{4c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{(A+5B)\int\frac{(\sin(e+fx)a+a)^{5/2}}{(c-c\sin(e+fx))^{3/2}}dx}{4c}$$

$\Big\downarrow$ 3218

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{(A+5B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f(c-c\sin(e+fx))^{3/2}}-\frac{2a\int\frac{(\sin(e+fx)a+a)^{3/2}}{\sqrt{c-c\sin(e+fx)}}dx}{c}\right)}{4c}$$

$\Big\downarrow$ 3042

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{(A+5B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f(c-c\sin(e+fx))^{3/2}}-\frac{2a\int\frac{(\sin(e+fx)a+a)^{3/2}}{\sqrt{c-c\sin(e+fx)}}dx}{c}\right)}{4c}$$

$\Big\downarrow$ 3219

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{(A+5B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f(c-c\sin(e+fx))^{3/2}}-\frac{2a\left(2a\int\frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)}{c}\right)}{4c}$$

$\Big\downarrow$ 3042

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{(A+5B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f(c-c\sin(e+fx))^{3/2}}-\frac{2a\left(2a\int\frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)}{c}\right)}{4c}$$

$\Big\downarrow$ 3216

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{(A+5B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f(c-c\sin(e+fx))^{3/2}}-\frac{2a\left(\frac{2a^2c\cos(e+fx)\int\frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)}{c}\right)}{4c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{(A+5B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f(c-c\sin(e+fx))^{3/2}}-\frac{2a\left(\frac{2a^2c\cos(e+fx)\int\frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)}{c}\right)}{4c}$$

$$\Big\downarrow \text{3146}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{(A+5B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f(c-c\sin(e+fx))^{3/2}}-\frac{2a\left(-\frac{2a^2\cos(e+fx)\int\frac{1}{c-c\sin(e+fx)}d(-c\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)}{c}\right)}{4c}$$

$$\Big\downarrow \text{16}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{(A+5B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f(c-c\sin(e+fx))^{3/2}}-\frac{2a\left(-\frac{2a^2\cos(e+fx)\log(c-c\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)}{c}\right)}{4c}$$

input
```
Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(5/2),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(4*f*(c - c*Sin[e + f*x]
)^(5/2)) - ((A + 5*B)*((a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(f*(c -
 c*Sin[e + f*x])^(3/2)) - (2*a*((-2*a^2*Cos[e + f*x]*Log[c - c*Sin[e + f*x
]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (a*Cos[e + f*x
]*Sqrt[a + a*Sin[e + f*x]])/(f*Sqrt[c - c*Sin[e + f*x]])))/c))/(4*c)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3218
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Simp[b*((2*m - 1)/(d*(
2*n + 1)))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m +
n + 1, 0])
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 581 vs. $2(190) = 380$.

Time = 7.28 (sec) , antiderivative size = 582, normalized size of antiderivative = 2.75

method	result
default	$-\frac{A\sqrt{4}\left(4\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1\right)-4\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}\ln\left(\frac{2}{\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1}\right)+4\cos\left(\frac{\pi}{4}+\right.\right.}{}$
parts	$\frac{A\sqrt{4}\left(4\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}\ln\left(\frac{2}{\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1}\right)-4\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1\right)-4\cos\left(\frac{\pi}{4}+\frac{fx}{2}\right.\right.}{}$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,metho
d=_RETURNVERBOSE)
```

output
```
-1/4*A/f*4^(1/2)*(4*cos(1/4*Pi+1/2*f*x+1/2*e)^4*ln(-cot(1/4*Pi+1/2*f*x+1/2
*e)+csc(1/4*Pi+1/2*f*x+1/2*e)+1)-4*cos(1/4*Pi+1/2*f*x+1/2*e)^4*ln(2/(cos(1
/4*Pi+1/2*f*x+1/2*e)+1))+4*cos(1/4*Pi+1/2*f*x+1/2*e)^4*ln(-cot(1/4*Pi+1/2*
f*x+1/2*e)+csc(1/4*Pi+1/2*f*x+1/2*e)-1)-3*cos(1/4*Pi+1/2*f*x+1/2*e)^4+4*co
s(1/4*Pi+1/2*f*x+1/2*e)^2-1)*a^2*(a*sin(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)/(c*
cos(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)/c^2*sec(1/4*Pi+1/2*f*x+1/2*e)^3*csc(1/4
*Pi+1/2*f*x+1/2*e)-2*B/f*((20*cos(1/2*f*x+1/2*e)^4-20*cos(1/2*f*x+1/2*e)^2
+20*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-5)*ln(2*(sin(1/2*f*x+1/2*e)-cos(
1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1))+(-20*cos(1/2*f*x+1/2*e)^4+20*cos(1
/2*f*x+1/2*e)^2-20*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+5)*ln(2/(cos(1/2*
f*x+1/2*e)+1))+cos(1/2*f*x+1/2*e)*((-4*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*
e)^2+16*cos(1/2*f*x+1/2*e))*sin(1/2*f*x+1/2*e)^2-5*sin(1/2*f*x+1/2*e)))*((
2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+1)*a)^(1/2)*a^2/(-(2*sin(1/2*f*x+1
/2*e)*cos(1/2*f*x+1/2*e)-1)*c)^(1/2)/c^2/(4*cos(1/2*f*x+1/2*e)^3*sin(1/2*f
*x+1/2*e)-2*cos(1/2*f*x+1/2*e)^2-2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+1
)
```

Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}}{(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x
, algorithm="fricas")
```

output
```
integral(((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x +
e)^2 - 2*(A + B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f
*x + e) + c)/(3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*
sin(f*x + e)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2)
,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 506 vs. $2(190) = 380$.

Time = 0.16 (sec) , antiderivative size = 506, normalized size of antiderivative = 2.39

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x
, algorithm="maxima")
```

output
```
-((8*a^(5/2)*sqrt(c)*sin(f*x + e)^2/((c^3 - 4*c^3*sin(f*x + e)/(cos(f*x +
e) + 1) + 6*c^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4*c^3*sin(f*x + e)^3
/(cos(f*x + e) + 1)^3 + c^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)*(cos(f*x
+ e) + 1)^2) - 2*a^(5/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(5/2)
+ a^(5/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(5/2))*A - B*(10*
a^(5/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(5/2) - 5*a^(5/2)*log(s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(5/2) + 2*(5*a^(5/2)*sin(f*x + e
)/(cos(f*x + e) + 1) - 16*a^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 14
*a^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 16*a^(5/2)*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 5*a^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(c^(5
/2) - 4*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 7*c^(5/2)*sin(f*x + e)^2
/(cos(f*x + e) + 1)^2 - 8*c^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 7*
c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 4*c^(5/2)*sin(f*x + e)^5/(co
s(f*x + e) + 1)^5 + c^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)))/f
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{5/2}}{(c-c\,\sin(e+f\,x))^{5/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))
^(5/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))
^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,a^2\left(-\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^3}{\sin(fx+e)^3-3\sin(fx+e)^2+3\sin(fx+e)-1}dx\right)\right.}{}$$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)
```

output

```
(sqrt(c)*sqrt(a)*a**2*( - int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x)
 + 1)*sin(e + f*x)**3)/(sin(e + f*x)**3 - 3*sin(e + f*x)**2 + 3*sin(e + f*
x) - 1),x)*b - int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e
 + f*x)**2)/(sin(e + f*x)**3 - 3*sin(e + f*x)**2 + 3*sin(e + f*x) - 1),x)*
a - 2*int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**
2)/(sin(e + f*x)**3 - 3*sin(e + f*x)**2 + 3*sin(e + f*x) - 1),x)*b - 2*int
((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e +
f*x)**3 - 3*sin(e + f*x)**2 + 3*sin(e + f*x) - 1),x)*a - int((sqrt(sin(e +
 f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**3 - 3*si
n(e + f*x)**2 + 3*sin(e + f*x) - 1),x)*b - int((sqrt(sin(e + f*x) + 1)*sqr
t( - sin(e + f*x) + 1))/(sin(e + f*x)**3 - 3*sin(e + f*x)**2 + 3*sin(e + f
*x) - 1),x)*a))/c**3
```

3.156 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 196

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6f(c-c\sin(e+fx))^{7/2}} - \frac{aB\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}} + \frac{a^2B\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{c^2f(c-c\sin(e+fx))^{3/2}} + \frac{a^3B\cos(e+fx)\log(1-\sin(e+fx))}{c^3f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

output
```
1/6*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(7/2)-1/2*a
*B*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(5/2)+a^2*B*cos(
f*x+e)*(a+a*sin(f*x+e))^(1/2)/c^2/f/(c-c*sin(f*x+e))^(3/2)+a^3*B*cos(f*x+e
)*ln(1-sin(f*x+e))/c^3/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 11.74 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.04

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{\left(4(A+B)-6(A+2B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+\right.\right.\right.}{}$$

input
```
Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
 f*x])^(7/2),x]
```

output
```
((4*(A + B) - 6*(A + 2*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + 3*(A +
 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + 6*B*Log[Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6)*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2))/(3*f*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^5*(c - c*Sin[e + f*x])^(7/2))
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.275$, Rules used = {3042, 3451, 3042, 3218, 3042, 3218, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}dx$$

$$\Big\downarrow \text{3451}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6f(c-c\sin(e+fx))^{7/2}}-\frac{B\int \frac{(\sin(e+fx)a+a)^{5/2}}{(c-c\sin(e+fx))^{5/2}}dx}{c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6f(c-c\sin(e+fx))^{7/2}}-\frac{B\int\frac{(\sin(e+fx)a+a)^{5/2}}{(c-c\sin(e+fx))^{5/2}}dx}{c}$$

$\downarrow$ 3218

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6f(c-c\sin(e+fx))^{7/2}}-\frac{B\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{5/2}}-\frac{a\int\frac{(\sin(e+fx)a+a)^{3/2}}{(c-c\sin(e+fx))^{3/2}}dx}{c}\right)}{c}$$

$\downarrow$ 3042

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6f(c-c\sin(e+fx))^{7/2}}-\frac{B\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{5/2}}-\frac{a\int\frac{(\sin(e+fx)a+a)^{3/2}}{(c-c\sin(e+fx))^{3/2}}dx}{c}\right)}{c}$$

$\downarrow$ 3218

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6f(c-c\sin(e+fx))^{7/2}}-\frac{B\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{5/2}}-\frac{a\left(\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f(c-c\sin(e+fx))^{3/2}}-\frac{a\int\frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx}{c}\right)}{c}\right)}{c}$$

$\downarrow$ 3042

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6f(c-c\sin(e+fx))^{7/2}}-\frac{B\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{5/2}}-\frac{a\left(\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f(c-c\sin(e+fx))^{3/2}}-\frac{a\int\frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx}{c}\right)}{c}\right)}{c}$$

$\downarrow$ 3216

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6f(c-c\sin(e+fx))^{7/2}}-\frac{B\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{5/2}}-\frac{a\left(\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f(c-c\sin(e+fx))^{3/2}}-\frac{a^2\cos(e+fx)\int\frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}\right)}{c}\right)}{c}$$

$\Big\downarrow$ 3042

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6f(c-c\sin(e+fx))^{7/2}}-\frac{B\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{5/2}}-\frac{a\left(\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f(c-c\sin(e+fx))^{3/2}}-\frac{a^2\cos(e+fx)\int\frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}\right)}{c}\right)}{c}$$

$\Big\downarrow$ 3146

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6f(c-c\sin(e+fx))^{7/2}}-\frac{B\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{5/2}}-\frac{a\left(\frac{a^2\cos(e+fx)\int\frac{1}{c-c\sin(e+fx)}d(-c\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f(c-c\sin(e+fx))^{3/2}}\right)}{c}\right)}{c}$$

$\Big\downarrow$ 16

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6f(c-c\sin(e+fx))^{7/2}}-\frac{B\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{5/2}}-\frac{a\left(\frac{a^2\cos(e+fx)\log(c-c\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f(c-c\sin(e+fx))^{3/2}}\right)}{c}\right)}{c}$$

input

```
Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(7/2),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(6*f*(c - c*Sin[e + f*x]
)^(7/2)) - (B*((a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*f*(c - c*Sin
[e + f*x])^(5/2)) - (a*((a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(f*(c -
c*Sin[e + f*x])^(3/2)) + (a^2*Cos[e + f*x]*Log[c - c*Sin[e + f*x]])/(c*f*S
qrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])))/c))/c
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3218
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Simp[b*((2*m - 1)/(d*(
2*n + 1)))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m +
n + 1, 0])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 536 vs. $2(176) = 352$.

Time = 7.32 (sec) , antiderivative size = 537, normalized size of antiderivative = 2.74

method	result
default	$\frac{A\sqrt{4}\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}a^{2}\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\sec\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}}{12f\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,c^{3}}+\frac{2B\left(\left(\left(-24\,\mathrm{co}\right.\right.\right.}{}$
parts	$\frac{A\sqrt{4}\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{2}a^{2}\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\sec\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}}{12f\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,c^{3}}+\frac{2B\left(\left(\left(-24\,\mathrm{co}\right.\right.\right.}{}$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/12*A/f*4^(1/2)*(cos(1/4*Pi+1/2*f*x+1/2*e)-1)^2*a^2*(a*sin(1/4*Pi+1/2*f*x
+1/2*e)^2)^(1/2)*(cos(1/4*Pi+1/2*f*x+1/2*e)+1)^2/(c*cos(1/4*Pi+1/2*f*x+1/2
*e)^2)^(1/2)/c^3*tan(1/4*Pi+1/2*f*x+1/2*e)*sec(1/4*Pi+1/2*f*x+1/2*e)^4+2/3
*B/f*(((-24*cos(1/2*f*x+1/2*e)^5+24*cos(1/2*f*x+1/2*e)^3+18*cos(1/2*f*x+1/
2*e))*sin(1/2*f*x+1/2*e)+36*cos(1/2*f*x+1/2*e)^4-36*cos(1/2*f*x+1/2*e)^2-3
)*ln(-2*(cos(1/2*f*x+1/2*e)-sin(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1))+((
24*cos(1/2*f*x+1/2*e)^5-24*cos(1/2*f*x+1/2*e)^3-18*cos(1/2*f*x+1/2*e))*sin
(1/2*f*x+1/2*e)-36*cos(1/2*f*x+1/2*e)^4+36*cos(1/2*f*x+1/2*e)^2+3)*ln(2/(c
os(1/2*f*x+1/2*e)+1))+cos(1/2*f*x+1/2*e)*((-28*sin(1/2*f*x+1/2*e)*cos(1/2*
f*x+1/2*e)^2+12*cos(1/2*f*x+1/2*e))*sin(1/2*f*x+1/2*e)^2-3*sin(1/2*f*x+1/2
*e)))*((2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+1)*a)^(1/2)*a^2/(8*cos(1/2
*f*x+1/2*e)^6-12*cos(1/2*f*x+1/2*e)^4+8*cos(1/2*f*x+1/2*e)^3*sin(1/2*f*x+1
/2*e)+2*cos(1/2*f*x+1/2*e)^2-4*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+1)/(-
(2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-1)*c)^(1/2)/c^3
```

Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}}{(-c\sin(fx+e)+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x
, algorithm="fricas")
```

output
```
integral(-((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x +
 e)^2 - 2*(A + B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(
f*x + e) + c)/(c^4*cos(f*x + e)^4 - 8*c^4*cos(f*x + e)^2 + 8*c^4 + 4*(c^4*
cos(f*x + e)^2 - 2*c^4)*sin(f*x + e)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2)
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}}{(-c\sin(fx+e)+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e)
 + c)^(7/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{5/2}}{(c-c\,\sin(e+f\,x))^{7/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))
^(7/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))
^(7/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,a^2\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^3}{\sin(fx+e)^4-4\sin(fx+e)^3+6\sin(fx+e)^2-4\sin(fx+e)\cdot}\right.\right.}{}$$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a**2*(int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) +
1)*sin(e + f*x)**3)/(sin(e + f*x)**4 - 4*sin(e + f*x)**3 + 6*sin(e + f*x)*
*2 - 4*sin(e + f*x) + 1),x)*b + int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e
+ f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x)**4 - 4*sin(e + f*x)**3 + 6*sin(
e + f*x)**2 - 4*sin(e + f*x) + 1),x)*a + 2*int((sqrt(sin(e + f*x) + 1)*sqr
t( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x)**4 - 4*sin(e + f*x)*
*3 + 6*sin(e + f*x)**2 - 4*sin(e + f*x) + 1),x)*b + 2*int((sqrt(sin(e + f*
x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**4 - 4*sin(e
 + f*x)**3 + 6*sin(e + f*x)**2 - 4*sin(e + f*x) + 1),x)*a + int((sqrt(sin(
e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**4 - 4
*sin(e + f*x)**3 + 6*sin(e + f*x)**2 - 4*sin(e + f*x) + 1),x)*b + int((sqr
t(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1))/(sin(e + f*x)**4 - 4*sin(e
+ f*x)**3 + 6*sin(e + f*x)**2 - 4*sin(e + f*x) + 1),x)*a))/c**4
```

3.157 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 96

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{8f(c-c\sin(e+fx))^{9/2}} + \frac{(A-7B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{48cf(c-c\sin(e+fx))^{7/2}}$$

output
```
1/8*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(9/2)+1/48*
(A-7*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(7/2)
```

Mathematica [A] (verified)

Time = 13.78 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.51

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+}}{12c^4f\left(\cos\left(\frac{1}{2}(e+fx)\right)\right.}$$

input
```
Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
 f*x])^(9/2),x]
```

output
```
(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(5*A
 - 5*B - 3*(A - B)*Cos[2*(e + f*x)] + (4*A + 17*B)*Sin[e + f*x] - 3*B*Sin[
3*(e + f*x)]))/(12*c^4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e
 + f*x])^4*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3042, 3451, 3042, 3221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}dx$$

$$\Big\downarrow \text{3451}$$

$$\frac{(A-7B)\int \frac{(\sin(e+fx)a+a)^{5/2}}{(c-c\sin(e+fx))^{7/2}}dx}{8c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{8f(c-c\sin(e+fx))^{9/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-7B)\int \frac{(\sin(e+fx)a+a)^{5/2}}{(c-c\sin(e+fx))^{7/2}}dx}{8c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{8f(c-c\sin(e+fx))^{9/2}}$$

$$\Big\downarrow \text{3221}$$

$$\frac{(A-7B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{48cf(c-c\sin(e+fx))^{7/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{8f(c-c\sin(e+fx))^{9/2}}$$

input
```
Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(9/2),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(8*f*(c - c*Sin[e + f*x]
)^(9/2)) + ((A - 7*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(48*c*f*(c
- c*Sin[e + f*x])^(7/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3221
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(84) = 168$.

Time = 8.06 (sec) , antiderivative size = 344, normalized size of antiderivative = 3.58

method	result
parts	$\frac{A\sqrt{4}\,\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,a^{2}\left(\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{5}+3\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{5}\sec\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}\right)}{96f\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,c^{4}}+\frac{2B\sqrt{(2}}{3f\left(-1+2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\right)\left(8\cos\left(\frac{fx}{2}\right.\right.}$
default	$\frac{a^{2}\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\left(\left(\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}\sin\left(\frac{fx}{2}+\frac{e}{2}\right)-\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}\sin\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{3\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{2}-\frac{3\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)}{4}+\frac{3\,\mathrm{c}}{}\right.\right.\right.}{\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,c^{4}f\left(-1+2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\right)\left(8\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}\mathrm{si}\right.}$

input

```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,metho
d=_RETURNVERBOSE)
```

output

```
1/96*A/f*4^(1/2)*(a*sin(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)*a^2/(c*cos(1/4*Pi+1
/2*f*x+1/2*e)^2)^(1/2)/c^4*(tan(1/4*Pi+1/2*f*x+1/2*e)^5+3*tan(1/4*Pi+1/2*f
*x+1/2*e)^5*sec(1/4*Pi+1/2*f*x+1/2*e)^2)+2/3*B/f*((2*sin(1/2*f*x+1/2*e)*co
s(1/2*f*x+1/2*e)+1)*a)^(1/2)*cos(1/2*f*x+1/2*e)^2*(-4*cos(1/2*f*x+1/2*e)^4
+4*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+4*cos(1/2*f*x+1/2*e)^2+3)*sin(1/2
*f*x+1/2*e)^2*a^2/(-1+2*cos(1/2*f*x+1/2*e)^2)/(8*cos(1/2*f*x+1/2*e)^5*sin(
1/2*f*x+1/2*e)-8*cos(1/2*f*x+1/2*e)^3*sin(1/2*f*x+1/2*e)-12*cos(1/2*f*x+1/
2*e)^4-6*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+12*cos(1/2*f*x+1/2*e)^2+1)/
(-(2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-1)*c)^(1/2)/c^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.72

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx =$$
$$-\frac{\left(3\,(A-B)a^{2}\cos\left(fx+e\right)^{2}-4\,(A-B)a^{2}+2\left(3\,Ba^{2}\cos\left(fx+e\right)^{2}-(A+5\,B)a^{2}\right)\sin\left(fx+e\right)\right)\sqrt{a\sin}}{6\left(c^{5}f\cos\left(fx+e\right)^{5}-8\,c^{5}f\cos\left(fx+e\right)^{3}+8\,c^{5}f\cos\left(fx+e\right)+4\left(c^{5}f\cos\left(fx+e\right)^{3}-2\,c^{5}f\,\mathrm{co}\right.\right.}$$

input

```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x
, algorithm="fricas")
```

output
```
-1/6*(3*(A - B)*a^2*cos(f*x + e)^2 - 4*(A - B)*a^2 + 2*(3*B*a^2*cos(f*x +
e)^2 - (A + 5*B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f
*x + e) + c)/(c^5*f*cos(f*x + e)^5 - 8*c^5*f*cos(f*x + e)^3 + 8*c^5*f*cos(
f*x + e) + 4*(c^5*f*cos(f*x + e)^3 - 2*c^5*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(9/2)
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}}{(-c\sin(fx+e)+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e)
 + c)^(9/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{5/2}}{(c-c\,\sin(e+f\,x))^{9/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))
^(9/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))
^(9/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,a^2\Bigl(-\Bigl(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(}{\sin(fx+e)^5-5\sin(fx+e)^4+10\sin(fx+e)^3-10\sin(fx}}{}$$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x)
```

output

```
(sqrt(c)*sqrt(a)*a**2*( - int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x)
 + 1)*sin(e + f*x)**3)/(sin(e + f*x)**5 - 5*sin(e + f*x)**4 + 10*sin(e + f
*x)**3 - 10*sin(e + f*x)**2 + 5*sin(e + f*x) - 1),x)*b - int((sqrt(sin(e +
 f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x)**5 - 5
*sin(e + f*x)**4 + 10*sin(e + f*x)**3 - 10*sin(e + f*x)**2 + 5*sin(e + f*x
) - 1),x)*a - 2*int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(
e + f*x)**2)/(sin(e + f*x)**5 - 5*sin(e + f*x)**4 + 10*sin(e + f*x)**3 - 1
0*sin(e + f*x)**2 + 5*sin(e + f*x) - 1),x)*b - 2*int((sqrt(sin(e + f*x) +
1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**5 - 5*sin(e + f*
x)**4 + 10*sin(e + f*x)**3 - 10*sin(e + f*x)**2 + 5*sin(e + f*x) - 1),x)*a
 - int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(si
n(e + f*x)**5 - 5*sin(e + f*x)**4 + 10*sin(e + f*x)**3 - 10*sin(e + f*x)**
2 + 5*sin(e + f*x) - 1),x)*b - int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e +
 f*x) + 1))/(sin(e + f*x)**5 - 5*sin(e + f*x)**4 + 10*sin(e + f*x)**3 - 10
*sin(e + f*x)**2 + 5*sin(e + f*x) - 1),x)*a))/c**5
```

3.158 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 146

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{10f(c-c\sin(e+fx))^{11/2}} + \frac{(A-4B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{40cf(c-c\sin(e+fx))^{9/2}} + \frac{(A-4B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{240c^2f(c-c\sin(e+fx))^{7/2}}$$

output
```
1/10*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(11/2)+1/4
0*(A-4*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(9/2)+1/2
40*(A-4*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c^2/f/(c-c*sin(f*x+e))^(7/2)
```

Mathematica [A] (verified)

Time = 14.83 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+}}{120c^5f\left(\cos\left(\frac{1}{2}(e+\right.\right.}$$

input
```
Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
 f*x])^(11/2),x]
```

output
```
(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(-36
*A - 6*B + 10*(2*A + B)*Cos[2*(e + f*x)] - 5*(8*A + 13*B)*Sin[e + f*x] + 1
5*B*Sin[3*(e + f*x)]))/(120*c^5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-
1 + Sin[e + f*x])^5*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3451, 3042, 3222, 3042, 3221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}dx$$

$$\Big\downarrow \text{ 3451}$$

$$\frac{(A-4B)\int \frac{(\sin(e+fx)a+a)^{5/2}}{(c-c\sin(e+fx))^{9/2}}dx}{5c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(A-4B)\int \frac{(\sin(e+fx)a+a)^{5/2}}{(c-c\sin(e+fx))^{9/2}}dx}{5c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

$$\Big\downarrow \text{ 3222}$$

$$\frac{(A-4B)\left(\frac{\int \frac{(\sin(e+fx)a+a)^{5/2}}{(c-c\sin(e+fx))^{7/2}}dx}{8c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{8f(c-c\sin(e+fx))^{9/2}}\right)}{5c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-4B)\left(\frac{\int \frac{(\sin(e+fx)a+a)^{5/2}}{(c-c\sin(e+fx))^{7/2}}dx}{8c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{8f(c-c\sin(e+fx))^{9/2}}\right)}{5c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

$$\Big\downarrow \text{3221}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{10f(c-c\sin(e+fx))^{11/2}}+\frac{(A-4B)\left(\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{48cf(c-c\sin(e+fx))^{7/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{8f(c-c\sin(e+fx))^{9/2}}\right)}{5c}$$

input
```
Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(11/2),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(10*f*(c - c*Sin[e + f*x
])^(11/2)) + ((A - 4*B)*((Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(8*f*(c
 - c*Sin[e + f*x])^(9/2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(48*
c*f*(c - c*Sin[e + f*x])^(7/2))))/(5*c)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3221
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

rule 3222
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp[(m + n + 1)/(a*(2*m + 1)
)   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
 ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||
!SumSimplerQ[n, 1])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(128) = 256$.

Time = 9.00 (sec) , antiderivative size = 429, normalized size of antiderivative = 2.94

method	result
parts	$\frac{A\sqrt{4}\,\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}+3\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}+6\right)a^{2}\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{5}\sec\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}}{480f\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,c^{5}}+\frac{}{15f\left(-1+2\cos\right.}$
default	$\frac{4\left(\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}+3\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}+6\right)A\left(\frac{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}}{2}+\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}\sin\left(\frac{fx}{2}+\frac{e}{2}\right)-\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}-\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}\sin\left(\frac{f}{2}\right.\right.\right.}{15\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right.}}$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x,meth
od=_RETURNVERBOSE)
```

output
```
1/480*A/f*4^(1/2)*(a*sin(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)*(cos(1/4*Pi+1/2*f*
x+1/2*e)^4+3*cos(1/4*Pi+1/2*f*x+1/2*e)^2+6)*a^2/(c*cos(1/4*Pi+1/2*f*x+1/2*
e)^2)^(1/2)/c^5*tan(1/4*Pi+1/2*f*x+1/2*e)^5*sec(1/4*Pi+1/2*f*x+1/2*e)^4+2/
15*B/f*((2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+1)*a)^(1/2)*cos(1/2*f*x+1
/2*e)^2*(8*cos(1/2*f*x+1/2*e)^5*sin(1/2*f*x+1/2*e)-8*cos(1/2*f*x+1/2*e)^3*
sin(1/2*f*x+1/2*e)-20*cos(1/2*f*x+1/2*e)^4+10*sin(1/2*f*x+1/2*e)*cos(1/2*f
*x+1/2*e)+20*cos(1/2*f*x+1/2*e)^2+15)*sin(1/2*f*x+1/2*e)^2*a^2/(-1+2*cos(1
/2*f*x+1/2*e)^2)/(16*cos(1/2*f*x+1/2*e)^8+32*cos(1/2*f*x+1/2*e)^5*sin(1/2*
f*x+1/2*e)-32*cos(1/2*f*x+1/2*e)^6-32*cos(1/2*f*x+1/2*e)^3*sin(1/2*f*x+1/2
*e)-8*cos(1/2*f*x+1/2*e)^4-8*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+24*cos(
1/2*f*x+1/2*e)^2+1)/(-(2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-1)*c)^(1/2)
/c^5
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.25

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx =$$

$$-\frac{\left(5\,(2\,A+B)a^{2}\cos\left(fx+e\right)^{2}-2\,(7\,A+2\,B)a^{2}+5\,\left(3\,Ba^{2}\cos\left(fx+e\right)^{2}-2\,(A+2\,B)a^{2}\right)\sin\left(fx+e\right)\right)}{30\,\left(5\,c^{6}f\cos\left(fx+e\right)^{5}-20\,c^{6}f\cos\left(fx+e\right)^{3}+16\,c^{6}f\cos\left(fx+e\right)-\left(c^{6}f\cos\left(fx+e\right)^{5}-12\,c^{6}f\cos\left(fx\right.\right.\right.}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),
x, algorithm="fricas")
```

output
```
-1/30*(5*(2*A + B)*a^2*cos(f*x + e)^2 - 2*(7*A + 2*B)*a^2 + 5*(3*B*a^2*cos
(f*x + e)^2 - 2*(A + 2*B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt
(-c*sin(f*x + e) + c)/(5*c^6*f*cos(f*x + e)^5 - 20*c^6*f*cos(f*x + e)^3 +
16*c^6*f*cos(f*x + e) - (c^6*f*cos(f*x + e)^5 - 12*c^6*f*cos(f*x + e)^3 +
16*c^6*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(11/2
),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}}{(-c\sin(fx+e)+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),
x, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e)
 + c)^(11/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),
x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 49.11 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.34

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{16\,a^2\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\left(A\,6\mathrm{i}+B\,1\mathrm{i}\right)\,\sqrt{a+a\,\sin\left(e+f\right.}}{5\,c^6\,f}\right.}{\cos\left(e+f\,x\right)\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,264\mathrm{i}-\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\cos\left(3\,e+3\,f\right.}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))
^(11/2),x)
```

output
```
((c - c*sin(e + f*x))^(1/2)*((16*a^2*exp(e*6i + f*x*6i)*(A*6i + B*1i)*(a +
 a*sin(e + f*x))^(1/2))/(5*c^6*f) - (16*a^2*exp(e*6i + f*x*6i)*cos(2*e + 2
*f*x)*(A*2i + B*1i)*(a + a*sin(e + f*x))^(1/2))/(3*c^6*f) + (a^2*exp(e*6i
+ f*x*6i)*sin(e + f*x)*(8*A + 13*B)*(a + a*sin(e + f*x))^(1/2)*8i)/(3*c^6*
f) - (B*a^2*exp(e*6i + f*x*6i)*sin(3*e + 3*f*x)*(a + a*sin(e + f*x))^(1/2)
*8i)/(c^6*f)))/(cos(e + f*x)*exp(e*6i + f*x*6i)*264i - exp(e*6i + f*x*6i)*
cos(3*e + 3*f*x)*220i + exp(e*6i + f*x*6i)*cos(5*e + 5*f*x)*20i - exp(e*6i
 + f*x*6i)*sin(2*e + 2*f*x)*330i + exp(e*6i + f*x*6i)*sin(4*e + 4*f*x)*88i
 - exp(e*6i + f*x*6i)*sin(6*e + 6*f*x)*2i)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,a^2\Big(\Big(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+}}{\sin(fx+e)^6-6\sin(fx+e)^5+15\sin(fx+e)^4-20\sin(fx+}}{}$$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a**2*(int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) +
1)*sin(e + f*x)**3)/(sin(e + f*x)**6 - 6*sin(e + f*x)**5 + 15*sin(e + f*x)
**4 - 20*sin(e + f*x)**3 + 15*sin(e + f*x)**2 - 6*sin(e + f*x) + 1),x)*b +
 int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(s
in(e + f*x)**6 - 6*sin(e + f*x)**5 + 15*sin(e + f*x)**4 - 20*sin(e + f*x)*
*3 + 15*sin(e + f*x)**2 - 6*sin(e + f*x) + 1),x)*a + 2*int((sqrt(sin(e + f
*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x)**6 - 6*s
in(e + f*x)**5 + 15*sin(e + f*x)**4 - 20*sin(e + f*x)**3 + 15*sin(e + f*x)
**2 - 6*sin(e + f*x) + 1),x)*b + 2*int((sqrt(sin(e + f*x) + 1)*sqrt( - sin
(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**6 - 6*sin(e + f*x)**5 + 15*sin
(e + f*x)**4 - 20*sin(e + f*x)**3 + 15*sin(e + f*x)**2 - 6*sin(e + f*x) +
1),x)*a + int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*
x))/(sin(e + f*x)**6 - 6*sin(e + f*x)**5 + 15*sin(e + f*x)**4 - 20*sin(e +
 f*x)**3 + 15*sin(e + f*x)**2 - 6*sin(e + f*x) + 1),x)*b + int((sqrt(sin(e
 + f*x) + 1)*sqrt( - sin(e + f*x) + 1))/(sin(e + f*x)**6 - 6*sin(e + f*x)*
*5 + 15*sin(e + f*x)**4 - 20*sin(e + f*x)**3 + 15*sin(e + f*x)**2 - 6*sin(
e + f*x) + 1),x)*a))/c**6
```

3.159 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 196

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{12f(c-c\sin(e+fx))^{13/2}} + \frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{40cf(c-c\sin(e+fx))^{11/2}} + \frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{160c^2f(c-c\sin(e+fx))^{9/2}} + \frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{960c^3f(c-c\sin(e+fx))^{7/2}}$$

output
```
1/12*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(13/2)+1/4
0*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(11/2)+1/
160*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c^2/f/(c-c*sin(f*x+e))^(9/2)
+1/960*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c^3/f/(c-c*sin(f*x+e))^(7
/2)
```

Mathematica [A] (verified)

Time = 16.56 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.73

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx = \frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+}}{120c^6f\left(\cos\left(\frac{1}{2}(e+f\right.\right.}$$

input
```
Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
 f*x])^(13/2),x]
```

output
```
(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(29*
A + 13*B - 15*(A + B)*Cos[2*(e + f*x)] + 6*(6*A + 7*B)*Sin[e + f*x] - 10*B
*Sin[3*(e + f*x)]))/(120*c^6*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 +
 Sin[e + f*x])^6*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3451, 3042, 3222, 3042, 3222, 3042, 3221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}dx$$

$$\Big\downarrow \text{ 3451}$$

$$\frac{(A-3B)\int \frac{(\sin(e+fx)a+a)^{5/2}}{(c-c\sin(e+fx))^{11/2}}dx}{4c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{12f(c-c\sin(e+fx))^{13/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(A-3B)\int \frac{(\sin(e+fx)a+a)^{5/2}}{(c-c\sin(e+fx))^{11/2}}dx}{4c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{12f(c-c\sin(e+fx))^{13/2}}$$

$\Big\downarrow$ 3222

$$\frac{(A-3B)\left(\frac{\int \frac{(\sin(e+fx)a+a)^{5/2}}{(c-c\sin(e+fx))^{9/2}}dx}{5c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{10f(c-c\sin(e+fx))^{11/2}}\right)}{4c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{12f(c-c\sin(e+fx))^{13/2}}$$

$\Big\downarrow$ 3042

$$\frac{(A-3B)\left(\frac{\int \frac{(\sin(e+fx)a+a)^{5/2}}{(c-c\sin(e+fx))^{9/2}}dx}{5c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{10f(c-c\sin(e+fx))^{11/2}}\right)}{4c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{12f(c-c\sin(e+fx))^{13/2}}$$

$\Big\downarrow$ 3222

$$\frac{(A-3B)\left(\frac{\frac{\int \frac{(\sin(e+fx)a+a)^{5/2}}{(c-c\sin(e+fx))^{7/2}}dx}{8c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{8f(c-c\sin(e+fx))^{9/2}}}{5c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{10f(c-c\sin(e+fx))^{11/2}}\right)}{4c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{12f(c-c\sin(e+fx))^{13/2}}$$

$\Big\downarrow$ 3042

$$\frac{(A-3B)\left(\frac{\frac{\int \frac{(\sin(e+fx)a+a)^{5/2}}{(c-c\sin(e+fx))^{7/2}}dx}{8c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{8f(c-c\sin(e+fx))^{9/2}}}{5c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{10f(c-c\sin(e+fx))^{11/2}}\right)}{4c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{12f(c-c\sin(e+fx))^{13/2}}$$

$\Big\downarrow$ 3221

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{12f(c-c\sin(e+fx))^{13/2}}+\frac{(A-3B)\left(\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{10f(c-c\sin(e+fx))^{11/2}}+\frac{\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{48cf(c-c\sin(e+fx))^{7/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{8f(c-c\sin(e+fx))^{9/2}}}{5c}\right)}{4c}$$

input
```
Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(13/2),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(12*f*(c - c*Sin[e + f*x
])^(13/2)) + ((A - 3*B)*((Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(10*f*(
c - c*Sin[e + f*x])^(11/2)) + ((Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(
8*f*(c - c*Sin[e + f*x])^(9/2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)
)/(48*c*f*(c - c*Sin[e + f*x])^(7/2)))/(5*c)))/(4*c)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3221
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

rule 3222
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp[(m + n + 1)/(a*(2*m + 1)
)   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
 ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||
!SumSimplerQ[n, 1])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 494 vs. $2(172) = 344$.

Time = 10.49 (sec) , antiderivative size = 495, normalized size of antiderivative = 2.53

method	result
parts	$\frac{A\sqrt{4}\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{6}+3\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}+6\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}+10\right)a^{2}\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{5}\sec\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)}{1920f\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,c^{6}}$
default	$\frac{a^{2}\left(A\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-\frac{1}{2}\right)\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{6}+3\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}+6\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}+10\right)\left(\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}-2\sin\left(\right.\right.\right.}{15\sqrt{c\cos}}$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),x,meth
od=_RETURNVERBOSE)
```

output
```
1/1920*A/f*4^(1/2)*(a*sin(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)*(cos(1/4*Pi+1/2*f
*x+1/2*e)^6+3*cos(1/4*Pi+1/2*f*x+1/2*e)^4+6*cos(1/4*Pi+1/2*f*x+1/2*e)^2+10
)*a^2/(c*cos(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)/c^6*tan(1/4*Pi+1/2*f*x+1/2*e)^
5*sec(1/4*Pi+1/2*f*x+1/2*e)^6-2/15*B/f*cos(1/2*f*x+1/2*e)^2*((2*sin(1/2*f*
x+1/2*e)*cos(1/2*f*x+1/2*e)+1)*a)^(1/2)*(8*cos(1/2*f*x+1/2*e)^8-16*cos(1/2
*f*x+1/2*e)^6+24*cos(1/2*f*x+1/2*e)^5*sin(1/2*f*x+1/2*e)-22*cos(1/2*f*x+1/
2*e)^4-24*cos(1/2*f*x+1/2*e)^3*sin(1/2*f*x+1/2*e)+30*cos(1/2*f*x+1/2*e)^2+
15)*sin(1/2*f*x+1/2*e)^2*a^2/(-1+2*cos(1/2*f*x+1/2*e)^2)/(32*sin(1/2*f*x+1
/2*e)*cos(1/2*f*x+1/2*e)^9-64*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)^7-80*c
os(1/2*f*x+1/2*e)^8-48*cos(1/2*f*x+1/2*e)^5*sin(1/2*f*x+1/2*e)+160*cos(1/2
*f*x+1/2*e)^6+80*cos(1/2*f*x+1/2*e)^3*sin(1/2*f*x+1/2*e)-40*cos(1/2*f*x+1/
2*e)^4+10*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-40*cos(1/2*f*x+1/2*e)^2-1)
/(-(2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-1)*c)^(1/2)/c^6
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx = \frac{\left(15\,(A+B)a^2\cos\left(fx+e\right)^2-2\,(11\,A+7\,B\right.}{60\,\left(c^7f\cos\left(fx+e\right)^7-18\,c^7f\cos\left(fx+e\right)^5+48\,c^7f\,\mathrm{co}\right.}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),
x, algorithm="fricas")
```

output
```
1/60*(15*(A + B)*a^2*cos(f*x + e)^2 - 2*(11*A + 7*B)*a^2 + 2*(10*B*a^2*cos
(f*x + e)^2 - (9*A + 13*B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqr
t(-c*sin(f*x + e) + c)/(c^7*f*cos(f*x + e)^7 - 18*c^7*f*cos(f*x + e)^5 + 4
8*c^7*f*cos(f*x + e)^3 - 32*c^7*f*cos(f*x + e) + 2*(3*c^7*f*cos(f*x + e)^5
 - 16*c^7*f*cos(f*x + e)^3 + 16*c^7*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(13/2
),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}}{(-c\sin(fx+e)+c)^{\frac{13}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),
x, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e)
 + c)^(13/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),
x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 45.08 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.82

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx = \frac{\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{a^2\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\left(A\,29\mathrm{i}+B\,13\mathrm{i}\right)}{15\,c^7\,f}\right.}{-858\,\cos\left(e+f\,x\right)\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}+858\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\cos\left(3\,e+\right.}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))
^(13/2),x)
```

output
```
((c - c*sin(e + f*x))^(1/2)*((a^2*exp(e*7i + f*x*7i)*(A*29i + B*13i)*(a +
a*sin(e + f*x))^(1/2)*16i)/(15*c^7*f) - (a^2*exp(e*7i + f*x*7i)*cos(2*e +
2*f*x)*(A*1i + B*1i)*(a + a*sin(e + f*x))^(1/2)*16i)/(c^7*f) - (32*a^2*exp
(e*7i + f*x*7i)*sin(e + f*x)*(6*A + 7*B)*(a + a*sin(e + f*x))^(1/2))/(5*c^
7*f) + (32*B*a^2*exp(e*7i + f*x*7i)*sin(3*e + 3*f*x)*(a + a*sin(e + f*x))^
(1/2))/(3*c^7*f)))/(858*exp(e*7i + f*x*7i)*cos(3*e + 3*f*x) - 858*cos(e +
f*x)*exp(e*7i + f*x*7i) - 130*exp(e*7i + f*x*7i)*cos(5*e + 5*f*x) + 2*exp(
e*7i + f*x*7i)*cos(7*e + 7*f*x) + 1144*exp(e*7i + f*x*7i)*sin(2*e + 2*f*x)
 - 416*exp(e*7i + f*x*7i)*sin(4*e + 4*f*x) + 24*exp(e*7i + f*x*7i)*sin(6*e
 + 6*f*x))
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx = \text{Too large to display}$$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a**2*( - int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x)
 + 1)*sin(e + f*x)**3)/(sin(e + f*x)**7 - 7*sin(e + f*x)**6 + 21*sin(e + f
*x)**5 - 35*sin(e + f*x)**4 + 35*sin(e + f*x)**3 - 21*sin(e + f*x)**2 + 7*
sin(e + f*x) - 1),x)*b - int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x)
+ 1)*sin(e + f*x)**2)/(sin(e + f*x)**7 - 7*sin(e + f*x)**6 + 21*sin(e + f*
x)**5 - 35*sin(e + f*x)**4 + 35*sin(e + f*x)**3 - 21*sin(e + f*x)**2 + 7*s
in(e + f*x) - 1),x)*a - 2*int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x)
 + 1)*sin(e + f*x)**2)/(sin(e + f*x)**7 - 7*sin(e + f*x)**6 + 21*sin(e + f
*x)**5 - 35*sin(e + f*x)**4 + 35*sin(e + f*x)**3 - 21*sin(e + f*x)**2 + 7*
sin(e + f*x) - 1),x)*b - 2*int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x
) + 1)*sin(e + f*x))/(sin(e + f*x)**7 - 7*sin(e + f*x)**6 + 21*sin(e + f*x
)**5 - 35*sin(e + f*x)**4 + 35*sin(e + f*x)**3 - 21*sin(e + f*x)**2 + 7*si
n(e + f*x) - 1),x)*a - int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) +
1)*sin(e + f*x))/(sin(e + f*x)**7 - 7*sin(e + f*x)**6 + 21*sin(e + f*x)**5
 - 35*sin(e + f*x)**4 + 35*sin(e + f*x)**3 - 21*sin(e + f*x)**2 + 7*sin(e
+ f*x) - 1),x)*b - int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1))/
(sin(e + f*x)**7 - 7*sin(e + f*x)**6 + 21*sin(e + f*x)**5 - 35*sin(e + f*x
)**4 + 35*sin(e + f*x)**3 - 21*sin(e + f*x)**2 + 7*sin(e + f*x) - 1),x)*a)
)/c**7
```

3.160 $\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 250

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}\,dx = -\frac{a^4(9A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{315f\sqrt{a+a\sin(e+fx)}} - \frac{a^3(9A-B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{9/2}}{126f} - \frac{a^2(9A-B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{9/2}}{84f} - \frac{a(9A-B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{9/2}}{72f} - \frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{9/2}}{9f}$$

output
```
-1/315*a^4*(9*A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(9/2)/f/(a+a*sin(f*x+e))^(1
/2)-1/126*a^3*(9*A-B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(
9/2)/f-1/84*a^2*(9*A-B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))
^(9/2)/f-1/72*a*(9*A-B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))
^(9/2)/f-1/9*B*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(9/2)/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 870 vs. $2(250) = 500$.

Time = 16.32 (sec) , antiderivative size = 870, normalized size of antiderivative = 3.48

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f
*x])^(9/2),x]
```

output
```
(7*(A - B)*Cos[2*(e + f*x)]*(a*(1 + Sin[e + f*x]))^(7/2)*(c - c*Sin[e + f*
x])^(9/2))/(128*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])^7) + (7*(A - B)*Cos[4*(e + f*x)]*(a*(1 + Sin[e + f*x
]))^(7/2)*(c - c*Sin[e + f*x])^(9/2))/(256*f*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])^9*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7) + ((A - B)*Cos[6*(e +
f*x)]*(a*(1 + Sin[e + f*x]))^(7/2)*(c - c*Sin[e + f*x])^(9/2))/(128*f*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
7) + ((A - B)*Cos[8*(e + f*x)]*(a*(1 + Sin[e + f*x]))^(7/2)*(c - c*Sin[e +
 f*x])^(9/2))/(1024*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(Cos[(e + f*
x)/2] + Sin[(e + f*x)/2])^7) + (7*(10*A - B)*Sin[e + f*x]*(a*(1 + Sin[e +
f*x]))^(7/2)*(c - c*Sin[e + f*x])^(9/2))/(128*f*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2])^9*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7) + (7*A*(a*(1 + Sin[
e + f*x]))^(7/2)*(c - c*Sin[e + f*x])^(9/2)*Sin[3*(e + f*x)])/(64*f*(Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])^9*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7)
 + ((7*A + 2*B)*(a*(1 + Sin[e + f*x]))^(7/2)*(c - c*Sin[e + f*x])^(9/2)*Si
n[5*(e + f*x)])/(320*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2])^7) + ((4*A + 5*B)*(a*(1 + Sin[e + f*x]))^(7/2)*
(c - c*Sin[e + f*x])^(9/2)*Sin[7*(e + f*x)])/(1792*f*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])^9*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7) + (B*(a*(1 + S
in[e + f*x]))^(7/2)*(c - c*Sin[e + f*x])^(9/2)*Sin[9*(e + f*x)])/(2304*...
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3452, 3042, 3219, 3042, 3219, 3042, 3219, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{9/2}(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{9/2}(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{ 3452}$$

$$\frac{1}{9}(9A-B)\int (\sin(e+fx)a+a)^{7/2}(c-c\sin(e+fx))^{9/2}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{9/2}}{9f}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{9}(9A-B)\int (\sin(e+fx)a+a)^{7/2}(c-c\sin(e+fx))^{9/2}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{9/2}}{9f}$$

$$\Big\downarrow \text{ 3219}$$

$$\frac{1}{9}(9A-B)\left(\frac{3}{4}a\int (\sin(e+fx)a+a)^{5/2}(c-c\sin(e+fx))^{9/2}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx)}{8f}\right.$$
$$-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{9/2}}{9f}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{9}(9A-B)\left(\frac{3}{4}a\int (\sin(e+fx)a+a)^{5/2}(c-c\sin(e+fx))^{9/2}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx)}{8f}\right.$$
$$-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{9/2}}{9f}$$

$\Big\downarrow$ 3219

$$\frac{1}{9}(9A-B)\left(\frac{3}{4}a\left(\frac{4}{7}a\int(\sin(e+fx)a+a)^{3/2}(c-c\sin(e+fx))^{9/2}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+\right.\right.$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{9/2}}{9f}$$

$\Big\downarrow$ 3042

$$\frac{1}{9}(9A-B)\left(\frac{3}{4}a\left(\frac{4}{7}a\int(\sin(e+fx)a+a)^{3/2}(c-c\sin(e+fx))^{9/2}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+\right.\right.$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{9/2}}{9f}$$

$\Big\downarrow$ 3219

$$\frac{1}{9}(9A-B)\left(\frac{3}{4}a\left(\frac{4}{7}a\left(\frac{1}{3}a\int\sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{9/2}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e\right.\right.\right.$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{9/2}}{9f}$$

$\Big\downarrow$ 3042

$$\frac{1}{9}(9A-B)\left(\frac{3}{4}a\left(\frac{4}{7}a\left(\frac{1}{3}a\int\sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{9/2}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e\right.\right.\right.$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{9/2}}{9f}$$

$\Big\downarrow$ 3217

$$\frac{1}{9}(9A-B)\left(\frac{3}{4}a\left(\frac{4}{7}a\left(-\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{15f\sqrt{a\sin(e+fx)+a}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{9/2}}{6f}\right)\right.\right.$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{9/2}}{9f}$$

input
```
Int[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(
9/2),x]
```

output
```
-1/9*(B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(9/2)
)/f + ((9*A - B)*(-1/8*(a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(c - c*S
in[e + f*x])^(9/2))/f + (3*a*(-1/7*(a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3
/2)*(c - c*Sin[e + f*x])^(9/2))/f + (4*a*(-1/15*(a^2*Cos[e + f*x]*(c - c*S
in[e + f*x])^(9/2))/(f*Sqrt[a + a*Sin[e + f*x]]) - (a*Cos[e + f*x]*Sqrt[a
+ a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(9/2))/(6*f)))/7))/4))/9
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3452
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

Maple [A] (verified)

Time = 90.08 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.71

method	result
default	$\frac{20160\left(\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-\frac{1}{2}\right)A\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{8}+\frac{4\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{6}}{7}+\frac{2\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}}{7}+\frac{4\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}{35}+\frac{1}{35}\right)\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\cdots\right.\right.}{}$
parts	$\frac{16A\sqrt{4}\,\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,\left(35\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{8}+20\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{6}+10\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}+4\cos\left(\frac{\pi}{4}+\frac{f}{\cdots}\right.\right.}{35f}$

input
```
int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2),x,metho
d=_RETURNVERBOSE)
```

output
```
20160*((cos(1/2*f*x+1/2*e)^2-1/2)*A*(cos(1/4*Pi+1/2*f*x+1/2*e)^8+4/7*cos(1
/4*Pi+1/2*f*x+1/2*e)^6+2/7*cos(1/4*Pi+1/2*f*x+1/2*e)^4+4/35*cos(1/4*Pi+1/2
*f*x+1/2*e)^2+1/35)*tan(1/4*Pi+1/2*f*x+1/2*e)*sin(1/4*Pi+1/2*f*x+1/2*e)^6-
16/9*sin(1/2*f*x+1/2*e)^2*(sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)^13-3*sin(
1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)^11-9/16*cos(1/2*f*x+1/2*e)^12+111/28*sin
(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)^9+27/16*cos(1/2*f*x+1/2*e)^10-41/14*sin
(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)^7-9/4*cos(1/2*f*x+1/2*e)^8+729/560*cos(
1/2*f*x+1/2*e)^5*sin(1/2*f*x+1/2*e)+27/16*cos(1/2*f*x+1/2*e)^6-27/80*cos(1
/2*f*x+1/2*e)^3*sin(1/2*f*x+1/2*e)-99/128*cos(1/2*f*x+1/2*e)^4+3/64*sin(1/
2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+27/128*cos(1/2*f*x+1/2*e)^2-9/256)*B*cos(1
/2*f*x+1/2*e)^2)*c^4*a^3*(c*cos(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)*(a*sin(1/4*
Pi+1/2*f*x+1/2*e)^2)^(1/2)/(630*f*cos(1/2*f*x+1/2*e)^2-315*f)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.73

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}\,dx = \frac{\left(315\,(A-B)a^3c^4\cos\left(fx+e\right)^8-315\,(A-B)a^3c^4+8\left(35\,Ba^3c^4\cos\left(fx+e\right)^8+5\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2),x
, algorithm="fricas")
```

output
```
1/2520*(315*(A - B)*a^3*c^4*cos(f*x + e)^8 - 315*(A - B)*a^3*c^4 + 8*(35*B
*a^3*c^4*cos(f*x + e)^8 + 5*(9*A - B)*a^3*c^4*cos(f*x + e)^6 + 6*(9*A - B)
*a^3*c^4*cos(f*x + e)^4 + 8*(9*A - B)*a^3*c^4*cos(f*x + e)^2 + 16*(9*A - B
)*a^3*c^4)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c
)/(f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(9/2)
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}\,dx=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}(-c\sin(fx+e)+c)^{\frac{9}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e)
 + c)^(9/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 453 vs. $2(220) = 440$.

Time = 0.30 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.81

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}\,dx=\text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2),x
, algorithm="giac")
```

output
```
32/315*(560*B*a^3*c^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^18 - 315*A*a^3*c^4*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1
/4*pi + 1/2*f*x + 1/2*e)^16 - 2205*B*a^3*c^4*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^
16 + 1080*A*a^3*c^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi +
1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^14 + 3240*B*a^3*c^4*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/
4*pi + 1/2*f*x + 1/2*e)^14 - 1260*A*a^3*c^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^1
2 - 2100*B*a^3*c^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1
/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^12 + 504*A*a^3*c^4*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*
pi + 1/2*f*x + 1/2*e)^10 + 504*B*a^3*c^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^10)*
sqrt(a)*sqrt(c)/f
```

Mupad [B] (verification not implemented)

Time = 45.85 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.93

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}\,dx=\frac{e^{-e\,9i-f\,x\,9i}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(-\frac{a^3\,c^4\,e^{e\,9i+f\,x\,9i}\,\cos(2\,e+2\,f\,x)\,(A\,1i-B\,1i)\,\sqrt{a+a\,\sin(e+f\,x)}\,7i}{64\,f}\right.}{}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(
9/2),x)
```

output
```
(exp(- e*9i - f*x*9i)*(c - c*sin(e + f*x))^(1/2)*((a^3*c^4*exp(e*9i + f*x*
9i)*sin(5*e + 5*f*x)*(7*A + 2*B)*(a + a*sin(e + f*x))^(1/2))/(160*f) - (a^
3*c^4*exp(e*9i + f*x*9i)*cos(4*e + 4*f*x)*(A*1i - B*1i)*(a + a*sin(e + f*x
))^(1/2)*7i)/(128*f) - (a^3*c^4*exp(e*9i + f*x*9i)*cos(6*e + 6*f*x)*(A*1i
- B*1i)*(a + a*sin(e + f*x))^(1/2)*1i)/(64*f) - (a^3*c^4*exp(e*9i + f*x*9i
)*cos(8*e + 8*f*x)*(A*1i - B*1i)*(a + a*sin(e + f*x))^(1/2)*1i)/(512*f) -
(a^3*c^4*exp(e*9i + f*x*9i)*cos(2*e + 2*f*x)*(A*1i - B*1i)*(a + a*sin(e +
f*x))^(1/2)*7i)/(64*f) + (a^3*c^4*exp(e*9i + f*x*9i)*sin(7*e + 7*f*x)*(4*A
 + 5*B)*(a + a*sin(e + f*x))^(1/2))/(896*f) + (7*A*a^3*c^4*exp(e*9i + f*x*
9i)*sin(3*e + 3*f*x)*(a + a*sin(e + f*x))^(1/2))/(32*f) + (7*a^3*c^4*exp(e
*9i + f*x*9i)*sin(e + f*x)*(10*A - B)*(a + a*sin(e + f*x))^(1/2))/(64*f) +
 (B*a^3*c^4*exp(e*9i + f*x*9i)*sin(9*e + 9*f*x)*(a + a*sin(e + f*x))^(1/2)
)/(1152*f)))/(2*cos(e + f*x))
```

Reduce [F]

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}\,dx = \text{Too large to display}$$

input
```
int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2),x)
```

output

```
sqrt(c)*sqrt(a)*a**3*c**4*(int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x)
 + 1)*sin(e + f*x)**8,x)*b + int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*
x) + 1)*sin(e + f*x)**7,x)*a - int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e +
f*x) + 1)*sin(e + f*x)**7,x)*b - int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e
+ f*x) + 1)*sin(e + f*x)**6,x)*a - 3*int(sqrt(sin(e + f*x) + 1)*sqrt( - si
n(e + f*x) + 1)*sin(e + f*x)**6,x)*b - 3*int(sqrt(sin(e + f*x) + 1)*sqrt(
- sin(e + f*x) + 1)*sin(e + f*x)**5,x)*a + 3*int(sqrt(sin(e + f*x) + 1)*sq
rt( - sin(e + f*x) + 1)*sin(e + f*x)**5,x)*b + 3*int(sqrt(sin(e + f*x) + 1
)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**4,x)*a + 3*int(sqrt(sin(e + f*x)
 + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**4,x)*b + 3*int(sqrt(sin(e +
f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**3,x)*a - 3*int(sqrt(sin(
e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**3,x)*b - 3*int(sqrt(
sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2,x)*a - int(sqr
t(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2,x)*b - int(s
qrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x),x)*a + int(sq
rt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x),x)*b + int(sqr
t(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1),x)*a)
```

3.161 $\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 226

$$\begin{aligned}\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = & -\frac{2a^4A\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{35f\sqrt{a+a\sin(e+fx)}} \\ & -\frac{4a^3A\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{7/2}}{35f} \\ & -\frac{a^2A\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}{7f} \\ & -\frac{aA\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}}{7f} \\ & -\frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{7/2}}{8f}\end{aligned}$$

output

```
-2/35*a^4*A*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/f/(a+a*sin(f*x+e))^(1/2)-4/3
5*a^3*A*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(7/2)/f-1/7*a^2
*A*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/f-1/7*a*A*cos(
f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2)/f-1/8*B*cos(f*x+e)*(a
+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(7/2)/f
```

Mathematica [A] (verified)

Time = 7.03 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.61

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx =$$
$$-\frac{a^3c^3\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}(1225B+1960B\cos(2(e+fx))+980B\cos(4(}{}$$

input
```
Integrate[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f
*x])^(7/2),x]
```

output
```
-1/35840*(a^3*c^3*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e
 + f*x]]*(1225*B + 1960*B*Cos[2*(e + f*x)] + 980*B*Cos[4*(e + f*x)] + 280*
B*Cos[6*(e + f*x)] + 35*B*Cos[8*(e + f*x)] - 19600*A*Sin[e + f*x] - 3920*A
*Sin[3*(e + f*x)] - 784*A*Sin[5*(e + f*x)] - 80*A*Sin[7*(e + f*x)]))/f
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3452, 3042, 3219, 3042, 3219, 3042, 3219, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3452}$$

$$A\int (\sin(e+fx)a+a)^{7/2}(c-c\sin(e+fx))^{7/2}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{7/2}}{8f}$$

$$\Big\downarrow \text{3042}$$

$$A\int (\sin(e+fx)a+a)^{7/2}(c-c\sin(e+fx))^{7/2}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{7/2}}{8f}$$

$$\Big\downarrow \text{3219}$$

$$A\left(\frac{6}{7}a\int (\sin(e+fx)a+a)^{5/2}(c-c\sin(e+fx))^{7/2}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{7/2}}{7f}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{7/2}}{8f}$$

$$\Big\downarrow \text{3042}$$

$$A\left(\frac{6}{7}a\int (\sin(e+fx)a+a)^{5/2}(c-c\sin(e+fx))^{7/2}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{7/2}}{7f}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{7/2}}{8f}$$

$$\Big\downarrow \text{3219}$$

$$A\left(\frac{6}{7}a\left(\frac{2}{3}a\int (\sin(e+fx)a+a)^{3/2}(c-c\sin(e+fx))^{7/2}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{7/2}}{6f}\right)-\ldots\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{7/2}}{8f}$$

$$\Big\downarrow \text{3042}$$

$$A\left(\frac{6}{7}a\left(\frac{2}{3}a\int (\sin(e+fx)a+a)^{3/2}(c-c\sin(e+fx))^{7/2}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{7/2}}{6f}\right)-\ldots\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{7/2}}{8f}$$

$$\Big\downarrow \text{3219}$$

$$A\left(\frac{6}{7}a\left(\frac{2}{3}a\left(\frac{2}{5}a\int \sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{7/2}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}}{5f}\right)-\ldots\right)-\ldots\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{7/2}}{8f}$$

$$\Big\downarrow \text{3042}$$

$$A\left(\frac{6}{7}a\left(\frac{2}{3}a\left(\frac{2}{5}a\int\sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{7/2}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+\right.\right.\right.$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{7/2}}{8f}$$

$$\Big\downarrow \text{3217}$$

$$A\left(\frac{6}{7}a\left(\frac{2}{3}a\left(-\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{10f\sqrt{a\sin(e+fx)+a}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}}{5f}\right)\right.\right.$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{7/2}}{8f}$$

input
```
Int[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(
7/2),x]
```

output
```
-1/8*(B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(7/2)
)/f + A*(-1/7*(a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*
x])^(7/2))/f + (6*a*(-1/6*(a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c -
c*Sin[e + f*x])^(7/2))/f + (2*a*(-1/10*(a^2*Cos[e + f*x]*(c - c*Sin[e + f*
x])^(7/2))/(f*Sqrt[a + a*Sin[e + f*x]]) - (a*Cos[e + f*x]*Sqrt[a + a*Sin[e
 + f*x]]*(c - c*Sin[e + f*x])^(7/2))/(5*f)))/3))/7)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3452
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

Maple [A] (verified)

Time = 90.03 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.14

method	result
default	$\frac{1280c^3a^3\left(A\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^6+\frac{\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^4}{2}+\frac{\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2}{5}+\frac{1}{20}\right)\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\frac{1}{2}\right)\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\right.}{}$
parts	$\frac{16A\sqrt{4}\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2}\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2}\left(20\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^6+10\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^4+4\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2+1\right)a^3c^3\sin\left(\right.}{35f}$

input
```
int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1280*c^3*a^3*(A*(cos(1/4*Pi+1/2*f*x+1/2*e)^6+1/2*cos(1/4*Pi+1/2*f*x+1/2*e)
^4+1/5*cos(1/4*Pi+1/2*f*x+1/2*e)^2+1/20)*(cos(1/2*f*x+1/2*e)^2-1/2)*tan(1/
4*Pi+1/2*f*x+1/2*e)*sin(1/4*Pi+1/2*f*x+1/2*e)^6+7/4*(cos(1/2*f*x+1/2*e)^4-
cos(1/2*f*x+1/2*e)^2+1/2)*sin(1/2*f*x+1/2*e)^2*B*cos(1/2*f*x+1/2*e)^2*(cos
(1/2*f*x+1/2*e)^8-2*cos(1/2*f*x+1/2*e)^6+3/2*cos(1/2*f*x+1/2*e)^4-1/2*cos(
1/2*f*x+1/2*e)^2+1/8))*(c*cos(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)*(a*sin(1/4*Pi
+1/2*f*x+1/2*e)^2)^(1/2)/(70*f*cos(1/2*f*x+1/2*e)^2-35*f)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.59

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = -\frac{\left(35\,Ba^3c^3\cos\left(fx+e\right)^8-35\,Ba^3c^3-8\,\left(5\,Aa^3c^3\cos\left(fx+e\right)^6+6\,Aa^3c^3\cos\left(fx+e\right)^4+8\,Aa^3c^3\cos\left(fx\right.\right.\right.}{280\,f\cos\left(fx+e\right)}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x
, algorithm="fricas")
```

output
```
-1/280*(35*B*a^3*c^3*cos(f*x + e)^8 - 35*B*a^3*c^3 - 8*(5*A*a^3*c^3*cos(f*
x + e)^6 + 6*A*a^3*c^3*cos(f*x + e)^4 + 8*A*a^3*c^3*cos(f*x + e)^2 + 16*A*
a^3*c^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/
(f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}(-c\sin(fx+e)+c)^{\frac{7}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e)
 + c)^(7/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 453 vs. $2(196) = 392$.

Time = 0.29 (sec) , antiderivative size = 453, normalized size of antiderivative = 2.00

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx=\text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x
, algorithm="giac")
```

output
```
32/35*(35*B*a^3*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi +
1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^16 - 20*A*a^3*c^3*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*
pi + 1/2*f*x + 1/2*e)^14 - 140*B*a^3*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^14 +
 70*A*a^3*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^12 + 210*B*a^3*c^3*sgn(cos(-1/4
*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi +
1/2*f*x + 1/2*e)^12 - 84*A*a^3*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn
(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^10 - 140*B
*a^3*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1
/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^10 + 35*A*a^3*c^3*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*
x + 1/2*e)^8 + 35*B*a^3*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1
/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8)*sqrt(a)*sqrt(c
)/f
```

Mupad [B] (verification not implemented)

Time = 40.76 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.70

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx = \frac{e^{-e\,8i-f\,x\,8i}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{35\,A\,a^3\,c^3\,e^{e\,8i+f\,x\,8i}\,\sin(e+f\,x)\,\sqrt{a+a\,\sin(e+f\,x)}}{32\,f}-\frac{7\,B\,a^3\,c^3\,e^{e\,8}\cdots}{\cdots}\right.\cdots}{\cdots}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(
7/2),x)
```

output
```
(exp(- e*8i - f*x*8i)*(c - c*sin(e + f*x))^(1/2)*((35*A*a^3*c^3*exp(e*8i +
 f*x*8i)*sin(e + f*x)*(a + a*sin(e + f*x))^(1/2))/(32*f) - (7*B*a^3*c^3*ex
p(e*8i + f*x*8i)*cos(2*e + 2*f*x)*(a + a*sin(e + f*x))^(1/2))/(64*f) - (7*
B*a^3*c^3*exp(e*8i + f*x*8i)*cos(4*e + 4*f*x)*(a + a*sin(e + f*x))^(1/2))/
(128*f) - (B*a^3*c^3*exp(e*8i + f*x*8i)*cos(6*e + 6*f*x)*(a + a*sin(e + f*
x))^(1/2))/(64*f) - (B*a^3*c^3*exp(e*8i + f*x*8i)*cos(8*e + 8*f*x)*(a + a*
sin(e + f*x))^(1/2))/(512*f) + (7*A*a^3*c^3*exp(e*8i + f*x*8i)*sin(3*e + 3
*f*x)*(a + a*sin(e + f*x))^(1/2))/(32*f) + (7*A*a^3*c^3*exp(e*8i + f*x*8i)
*sin(5*e + 5*f*x)*(a + a*sin(e + f*x))^(1/2))/(160*f) + (A*a^3*c^3*exp(e*8
i + f*x*8i)*sin(7*e + 7*f*x)*(a + a*sin(e + f*x))^(1/2))/(224*f)))/(2*cos(
e + f*x))
```

Reduce [F]

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}\,dx=\sqrt{c}\,\sqrt{a}\,a^3c^3\left(-\left(\int\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^7\,dx\right)b-\left(\right.\right.$$

input
```
int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x)
```

output
```
sqrt(c)*sqrt(a)*a**3*c**3*( - int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f
*x) + 1)*sin(e + f*x)**7,x)*b - int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e +
 f*x) + 1)*sin(e + f*x)**6,x)*a + 3*int(sqrt(sin(e + f*x) + 1)*sqrt( - sin
(e + f*x) + 1)*sin(e + f*x)**5,x)*b + 3*int(sqrt(sin(e + f*x) + 1)*sqrt( -
 sin(e + f*x) + 1)*sin(e + f*x)**4,x)*a - 3*int(sqrt(sin(e + f*x) + 1)*sqr
t( - sin(e + f*x) + 1)*sin(e + f*x)**3,x)*b - 3*int(sqrt(sin(e + f*x) + 1)
*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2,x)*a + int(sqrt(sin(e + f*x) +
1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x),x)*b + int(sqrt(sin(e + f*x) + 1
)*sqrt( - sin(e + f*x) + 1),x)*a)
```

3.162 $\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 192

$$\begin{aligned}\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c&-c\sin(e+fx))^{5/2}\,dx = \frac{(7A+B)c^3\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{105f\sqrt{c-c\sin(e+fx)}}\\&+\frac{2(7A+B)c^2\cos(e+fx)(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}}{105f}\\&+\frac{(7A+B)c\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{3/2}}{42f}\\&-\frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{5/2}}{7f}\end{aligned}$$

output
```
1/105*(7*A+B)*c^3*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(1/
2)+2/105*(7*A+B)*c^2*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1
/2)/f+1/42*(7*A+B)*c*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3
/2)/f-1/7*B*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2)/f
```

Mathematica [A] (warning: unable to verify)

Time = 10.07 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.21

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \frac{a^3c^2(-1+\sin(e+fx))^2(1+\sin(e+fx))^3\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}$$

input
```
Integrate[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f
*x])^(5/2),x]
```

output
```
(a^3*c^2*(-1 + Sin[e + f*x])^2*(1 + Sin[e + f*x])^3*Sqrt[a*(1 + Sin[e + f*
x])]*Sqrt[c - c*Sin[e + f*x]]*(-525*(A + B)*Cos[2*(e + f*x)] - 210*(A + B)
*Cos[4*(e + f*x)] - 35*A*Cos[6*(e + f*x)] - 35*B*Cos[6*(e + f*x)] + 4200*A
*Sin[e + f*x] + 525*B*Sin[e + f*x] + 700*A*Sin[3*(e + f*x)] - 35*B*Sin[3*(
e + f*x)] + 84*A*Sin[5*(e + f*x)] - 63*B*Sin[5*(e + f*x)] - 15*B*Sin[7*(e
+ f*x)]))/(6720*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])^7)
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3452, 3042, 3219, 3042, 3219, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3452}$$

$$\frac{1}{7}(7A+B)\int(\sin(e+fx)a+a)^{7/2}(c-c\sin(e+fx))^{5/2}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{5/2}}{7f}$$

$\downarrow$ 3042

$$\frac{1}{7}(7A+B)\int(\sin(e+fx)a+a)^{7/2}(c-c\sin(e+fx))^{5/2}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{5/2}}{7f}$$

$\downarrow$ 3219

$$\frac{1}{7}(7A+B)\left(\frac{2}{3}c\int(\sin(e+fx)a+a)^{7/2}(c-c\sin(e+fx))^{3/2}dx+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx)}{6f}\right.$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{5/2}}{7f}$$

$\downarrow$ 3042

$$\frac{1}{7}(7A+B)\left(\frac{2}{3}c\int(\sin(e+fx)a+a)^{7/2}(c-c\sin(e+fx))^{3/2}dx+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx)}{6f}\right.$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{5/2}}{7f}$$

$\downarrow$ 3219

$$\frac{1}{7}(7A+B)\left(\frac{2}{3}c\left(\frac{2}{5}c\int(\sin(e+fx)a+a)^{7/2}\sqrt{c-c\sin(e+fx)}dx+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{7/2}\sqrt{c-c\sin(e+}}{5f}\right.\right.$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{5/2}}{7f}$$

$\downarrow$ 3042

$$\frac{1}{7}(7A+B)\left(\frac{2}{3}c\left(\frac{2}{5}c\int(\sin(e+fx)a+a)^{7/2}\sqrt{c-c\sin(e+fx)}dx+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{7/2}\sqrt{c-c\sin(e+}}{5f}\right.\right.$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{5/2}}{7f}$$

$\downarrow$ 3217

$$\frac{1}{7}(7A+B)\left(\frac{2}{3}c\left(\frac{c^2\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{7/2}\sqrt{c-c\sin(e+fx)}}{5f}\right)+\frac{c\cos\right.$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{5/2}}{7f}$$

input
```
Int[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(
5/2),x]
```

output
```
-1/7*(B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(5/2)
)/f + ((7*A + B)*((c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e
+ f*x])^(3/2))/(6*f) + (2*c*((c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))
/(10*f*Sqrt[c - c*Sin[e + f*x]]) + (c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7
/2)*Sqrt[c - c*Sin[e + f*x]])/(5*f)))/3))/7
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3452
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

Maple [A] (verified)

Time = 89.15 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.69

method	result
default	$\frac{2240\left(A\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-\frac{1}{2}\right)\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}+\frac{2\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}{5}+\frac{1}{10}\right)\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{6}+\frac{12\sin\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}B\cos\cdots}{\cdots}\right.}{\cdots}$
parts	$\frac{8A\sqrt{4}\,\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\left(10\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}+4\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}+1\right)a^{3}c^{2}\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{6}\tan\left(\frac{\pi}{4}+\cdots\right)}{15f}$

input
```
int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x,metho
d=_RETURNVERBOSE)
```

output
```
2240*(A*(cos(1/2*f*x+1/2*e)^2-1/2)*(cos(1/4*Pi+1/2*f*x+1/2*e)^4+2/5*cos(1/
4*Pi+1/2*f*x+1/2*e)^2+1/10)*tan(1/4*Pi+1/2*f*x+1/2*e)*sin(1/4*Pi+1/2*f*x+1
/2*e)^6+12/7*sin(1/2*f*x+1/2*e)^2*B*cos(1/2*f*x+1/2*e)^2*(sin(1/2*f*x+1/2*
e)*cos(1/2*f*x+1/2*e)^9-2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)^7+7/12*cos
(1/2*f*x+1/2*e)^8+17/10*cos(1/2*f*x+1/2*e)^5*sin(1/2*f*x+1/2*e)-7/6*cos(1/
2*f*x+1/2*e)^6-7/10*cos(1/2*f*x+1/2*e)^3*sin(1/2*f*x+1/2*e)+49/48*cos(1/2*
f*x+1/2*e)^4+7/48*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-7/16*cos(1/2*f*x+1
/2*e)^2+7/64))*c^2*a^3*(c*cos(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)*(a*sin(1/4*Pi
+1/2*f*x+1/2*e)^2)^(1/2)/(210*f*cos(1/2*f*x+1/2*e)^2-105*f)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.78

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx =$$
$$-\frac{(35\,(A+B)a^3c^2\cos(fx+e)^6-35\,(A+B)a^3c^2+2\,(15\,Ba^3c^2\cos(fx+e)^6-3\,(7\,A+B)a^3c^2\cos(fx-}{210\,f}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x
, algorithm="fricas")
```

output
```
-1/210*(35*(A + B)*a^3*c^2*cos(f*x + e)^6 - 35*(A + B)*a^3*c^2 + 2*(15*B*a
^3*c^2*cos(f*x + e)^6 - 3*(7*A + B)*a^3*c^2*cos(f*x + e)^4 - 4*(7*A + B)*a
^3*c^2*cos(f*x + e)^2 - 8*(7*A + B)*a^3*c^2)*sin(f*x + e))*sqrt(a*sin(f*x
+ e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}(-c\sin(fx+e)+c)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e)
 + c)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 453 vs. $2(168) = 336$.

Time = 0.31 (sec) , antiderivative size = 453, normalized size of antiderivative = 2.36

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx=\text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x
, algorithm="giac")
```

output
```
16/105*(120*B*a^3*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^14 - 70*A*a^3*c^2*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/
4*pi + 1/2*f*x + 1/2*e)^12 - 490*B*a^3*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^12
 + 252*A*a^3*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^10 + 756*B*a^3*c^2*sgn(cos(-
1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi
 + 1/2*f*x + 1/2*e)^10 - 315*A*a^3*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8 - 52
5*B*a^3*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x
+ 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8 + 140*A*a^3*c^2*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2
*f*x + 1/2*e)^6 + 140*B*a^3*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(si
n(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^6)*sqrt(a)*sq
rt(c)/f
```

Mupad [B] (verification not implemented)

Time = 41.70 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.99

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \frac{e^{-e\,7i-f\,x\,7i}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{a^3\,c^2\,e^{e\,7i+f\,x\,7i}\,\cos(2\,e+2\,f\,x)\,(A\,1i+B\,1i)\,\sqrt{a+a\,\sin(e+f\,x)}\,5i}{32\,f}+\right.}{}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(
5/2),x)
```

output
```
(exp(- e*7i - f*x*7i)*(c - c*sin(e + f*x))^(1/2)*((a^3*c^2*exp(e*7i + f*x*
7i)*cos(2*e + 2*f*x)*(A*1i + B*1i)*(a + a*sin(e + f*x))^(1/2)*5i)/(32*f) +
 (a^3*c^2*exp(e*7i + f*x*7i)*cos(4*e + 4*f*x)*(A*1i + B*1i)*(a + a*sin(e +
 f*x))^(1/2)*1i)/(16*f) + (a^3*c^2*exp(e*7i + f*x*7i)*cos(6*e + 6*f*x)*(A*
1i + B*1i)*(a + a*sin(e + f*x))^(1/2)*1i)/(96*f) + (a^3*c^2*exp(e*7i + f*x
*7i)*sin(5*e + 5*f*x)*(4*A - 3*B)*(a + a*sin(e + f*x))^(1/2))/(160*f) + (a
^3*c^2*exp(e*7i + f*x*7i)*sin(3*e + 3*f*x)*(20*A - B)*(a + a*sin(e + f*x))
^(1/2))/(96*f) + (5*a^3*c^2*exp(e*7i + f*x*7i)*sin(e + f*x)*(8*A + B)*(a +
 a*sin(e + f*x))^(1/2))/(32*f) - (B*a^3*c^2*exp(e*7i + f*x*7i)*sin(7*e + 7
*f*x)*(a + a*sin(e + f*x))^(1/2))/(224*f)))/(2*cos(e + f*x))
```

Reduce [F]

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx=\sqrt{c}\,\sqrt{a}\,a^3c^2\left(\left(\int\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^6\,dx\right)b+\left(\int\right.\right.$$

input
```
int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x)
```

output
```
sqrt(c)*sqrt(a)*a**3*c**2*(int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x)
 + 1)*sin(e + f*x)**6,x)*b + int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*
x) + 1)*sin(e + f*x)**5,x)*a + int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e +
f*x) + 1)*sin(e + f*x)**5,x)*b + int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e
+ f*x) + 1)*sin(e + f*x)**4,x)*a - 2*int(sqrt(sin(e + f*x) + 1)*sqrt( - si
n(e + f*x) + 1)*sin(e + f*x)**4,x)*b - 2*int(sqrt(sin(e + f*x) + 1)*sqrt(
- sin(e + f*x) + 1)*sin(e + f*x)**3,x)*a - 2*int(sqrt(sin(e + f*x) + 1)*sq
rt( - sin(e + f*x) + 1)*sin(e + f*x)**3,x)*b - 2*int(sqrt(sin(e + f*x) + 1
)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2,x)*a + int(sqrt(sin(e + f*x) +
 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2,x)*b + int(sqrt(sin(e + f*x)
 + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x),x)*a + int(sqrt(sin(e + f*x)
+ 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x),x)*b + int(sqrt(sin(e + f*x) +
 1)*sqrt( - sin(e + f*x) + 1),x)*a)
```

3.163 $\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 142

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx=\frac{(3A+B)c^2\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{30f\sqrt{c-c\sin(e+fx)}}+\frac{(3A+B)c\cos(e+fx)(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}}{15f}-\frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{3/2}}{6f}$$

output
```
1/30*(3*A+B)*c^2*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(1/2
)+1/15*(3*A+B)*c*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2)/
f-1/6*B*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2)/f
```

Mathematica [A] (verified)

Time = 10.34 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.49

$$\int (a + a\sin(e + fx))^{7/2}(A + B\sin(e + fx))(c - c\sin(e + fx))^{3/2} dx =$$
$$-\frac{a^3c(-1 + \sin(e + fx))(1 + \sin(e + fx))^3\sqrt{a(1 + \sin(e + fx))}\sqrt{c - c\sin(e + fx)}(-15(16A + 11B)\cos(\ldots}{}$$

input
```
Integrate[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f
*x])^(3/2),x]
```

output
```
-1/960*(a^3*c*(-1 + Sin[e + f*x])*(1 + Sin[e + f*x])^3*Sqrt[a*(1 + Sin[e +
 f*x])]*Sqrt[c - c*Sin[e + f*x]]*(-15*(16*A + 11*B)*Cos[2*(e + f*x)] - 30*
(2*A + B)*Cos[4*(e + f*x)] + 5*B*Cos[6*(e + f*x)] + 840*A*Sin[e + f*x] + 2
40*B*Sin[e + f*x] + 60*A*Sin[3*(e + f*x)] - 40*B*Sin[3*(e + f*x)] - 12*A*S
in[5*(e + f*x)] - 24*B*Sin[5*(e + f*x)]))/(f*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3452, 3042, 3219, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e + fx) + a)^{7/2}(c - c\sin(e + fx))^{3/2}(A + B\sin(e + fx)) dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a\sin(e + fx) + a)^{7/2}(c - c\sin(e + fx))^{3/2}(A + B\sin(e + fx))dx$$

$$\Big\downarrow \text{ 3452}$$

$$\frac{1}{3}(3A+B)\int(\sin(e+fx)a+a)^{7/2}(c-c\sin(e+fx))^{3/2}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{3/2}}{6f}$$

$\big\downarrow$ 3042

$$\frac{1}{3}(3A+B)\int(\sin(e+fx)a+a)^{7/2}(c-c\sin(e+fx))^{3/2}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{3/2}}{6f}$$

$\big\downarrow$ 3219

$$\frac{1}{3}(3A+B)\left(\frac{2}{5}c\int(\sin(e+fx)a+a)^{7/2}\sqrt{c-c\sin(e+fx)}dx+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{7/2}\sqrt{c-c\sin(e+fx)}}{5f}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{3/2}}{6f}$$

$\big\downarrow$ 3042

$$\frac{1}{3}(3A+B)\left(\frac{2}{5}c\int(\sin(e+fx)a+a)^{7/2}\sqrt{c-c\sin(e+fx)}dx+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{7/2}\sqrt{c-c\sin(e+fx)}}{5f}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{3/2}}{6f}$$

$\big\downarrow$ 3217

$$\frac{1}{3}(3A+B)\left(\frac{c^2\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{7/2}\sqrt{c-c\sin(e+fx)}}{5f}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{3/2}}{6f}$$

input
```
Int[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(
3/2),x]
```

output

```
-1/6*(B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(3/2)
)/f + ((3*A + B)*((c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(10*f*Sqrt
[c - c*Sin[e + f*x]]) + (c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*Sqrt[c
- c*Sin[e + f*x]])/(5*f)))/3
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3219

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3452

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. 2(124) = 248.

Time = 8.22 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.76

method	result
default	$\frac{192 c\,a^{3} \left(A \left(\cos\left(\frac{\pi}{4}+\frac{f x}{2}+\frac{e}{2}\right)^{2}+\frac{1}{4}\right) \left(\cos\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-\frac{1}{2}\right) \tan\left(\frac{\pi}{4}+\frac{f x}{2}+\frac{e}{2}\right) \sin\left(\frac{\pi}{4}+\frac{f x}{2}+\frac{e}{2}\right)^{6}+2 \sin\left(\frac{f x}{2}+\frac{e}{2}\right)^{2} B \cos\left(\frac{f x}{2}+\frac{e}{2}\right)^{2} \left(-\frac{5 \mathrm{co}}{}\right.\right.}{}$
parts	$\frac{4 A \sqrt{4}\, \sqrt{a \sin\left(\frac{\pi}{4}+\frac{f x}{2}+\frac{e}{2}\right)^{2}}\, \sqrt{c \cos\left(\frac{\pi}{4}+\frac{f x}{2}+\frac{e}{2}\right)^{2}}\, \left(4 \cos\left(\frac{\pi}{4}+\frac{f x}{2}+\frac{e}{2}\right)^{2}+1\right) a^{3} c \sin\left(\frac{\pi}{4}+\frac{f x}{2}+\frac{e}{2}\right)^{6} \tan\left(\frac{\pi}{4}+\frac{f x}{2}+\frac{e}{2}\right)}{5 f}+\frac{2 B \sqrt{-\left(2 \sin\right.}}{}$

input
```
int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x,metho
d=_RETURNVERBOSE)
```

output
```
192*c*a^3*(A*(cos(1/4*Pi+1/2*f*x+1/2*e)^2+1/4)*(cos(1/2*f*x+1/2*e)^2-1/2)*
tan(1/4*Pi+1/2*f*x+1/2*e)*sin(1/4*Pi+1/2*f*x+1/2*e)^6+2*sin(1/2*f*x+1/2*e)
^2*B*cos(1/2*f*x+1/2*e)^2*(-5/6*cos(1/2*f*x+1/2*e)^8+cos(1/2*f*x+1/2*e)^5*
sin(1/2*f*x+1/2*e)+5/3*cos(1/2*f*x+1/2*e)^6-cos(1/2*f*x+1/2*e)^3*sin(1/2*f
*x+1/2*e)-5/6*cos(1/2*f*x+1/2*e)^4+5/12*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2
*e)+5/32))*(c*cos(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)*(a*sin(1/4*Pi+1/2*f*x+1/2
*e)^2)^(1/2)/(30*f*cos(1/2*f*x+1/2*e)^2-15*f)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \frac{(5\,Ba^3c\cos(fx+e)^6-15\,(A+B)a^3c\cos(fx+e)^4+5\,(3\,A+2\,B)a^3c-2\,(3\,(A}{}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x
, algorithm="fricas")
```

output
```
1/30*(5*B*a^3*c*cos(f*x + e)^6 - 15*(A + B)*a^3*c*cos(f*x + e)^4 + 5*(3*A
+ 2*B)*a^3*c - 2*(3*(A + 2*B)*a^3*c*cos(f*x + e)^4 - 2*(3*A + B)*a^3*c*cos
(f*x + e)^2 - 4*(3*A + B)*a^3*c)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sq
rt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^{7/2}(A + B\sin(e + fx))(c - c\sin(e + fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a + a\sin(e + fx))^{7/2}(A + B\sin(e + fx))(c - c\sin(e+fx))^{3/2}\,dx = \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^{\frac{7}{2}}(-c\sin(fx + e) + c)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e)
 + c)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.74

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \frac{8\left(20\,Ba^3c\cos\left(-\frac{1}{4}\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{12}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi\right.\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x
, algorithm="giac")
```

output
```
8/15*(20*B*a^3*c*cos(-1/4*pi + 1/2*f*x + 1/2*e)^12*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*A*a^3*c*cos(-1/4*pi
+ 1/2*f*x + 1/2*e)^10*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e)) - 36*B*a^3*c*cos(-1/4*pi + 1/2*f*x + 1/2*e)^10*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 15*A*a^
3*c*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*s
gn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 15*B*a^3*c*cos(-1/4*pi + 1/2*f*x + 1/
2*e)^8*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2
*e)))*sqrt(a)*sqrt(c)/f
```

Mupad [B] (verification not implemented)

Time = 41.28 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.26

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = -\frac{e^{-e\,6i-f\,x\,6i}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{a^3\,c\,e^{e\,6i+f\,x\,6i}\,\cos(4\,e+4\,f\,x)\,(2\,A+B)\,\sqrt{a+a\,\sin(e+f\,x)}}{16\,f}-\frac{B\,a^3\,c\,e^{e\,6i+f\,x\,6i}\,\cos(6\,e+6\,f\,x)\,\sqrt{a}}{96\,f}\right.}{}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(
3/2),x)
```

output
```
-(exp(- e*6i - f*x*6i)*(c - c*sin(e + f*x))^(1/2)*((a^3*c*exp(e*6i + f*x*6
i)*cos(4*e + 4*f*x)*(2*A + B)*(a + a*sin(e + f*x))^(1/2))/(16*f) - (B*a^3*
c*exp(e*6i + f*x*6i)*cos(6*e + 6*f*x)*(a + a*sin(e + f*x))^(1/2))/(96*f) +
 (a^3*c*exp(e*6i + f*x*6i)*sin(e + f*x)*(A*7i + B*2i)*(a + a*sin(e + f*x))
^(1/2)*1i)/(4*f) + (a^3*c*exp(e*6i + f*x*6i)*cos(2*e + 2*f*x)*(16*A + 11*B
)*(a + a*sin(e + f*x))^(1/2))/(32*f) + (a^3*c*exp(e*6i + f*x*6i)*sin(3*e +
 3*f*x)*(A*3i - B*2i)*(a + a*sin(e + f*x))^(1/2)*1i)/(24*f) - (a^3*c*exp(e
*6i + f*x*6i)*sin(5*e + 5*f*x)*(A*1i + B*2i)*(a + a*sin(e + f*x))^(1/2)*1i
)/(40*f)))/(2*cos(e + f*x))
```

Reduce [F]

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx=\sqrt{c}\,\sqrt{a}\,a^3c\left(-\left(\int\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^5\,dx\right)b-\left(\int\right.\right.$$

input
```
int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x)
```

output
```
sqrt(c)*sqrt(a)*a**3*c*( - int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x)
 + 1)*sin(e + f*x)**5,x)*b - int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*
x) + 1)*sin(e + f*x)**4,x)*a - 2*int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e
+ f*x) + 1)*sin(e + f*x)**4,x)*b - 2*int(sqrt(sin(e + f*x) + 1)*sqrt( - si
n(e + f*x) + 1)*sin(e + f*x)**3,x)*a + 2*int(sqrt(sin(e + f*x) + 1)*sqrt(
- sin(e + f*x) + 1)*sin(e + f*x)**2,x)*b + 2*int(sqrt(sin(e + f*x) + 1)*sq
rt( - sin(e + f*x) + 1)*sin(e + f*x),x)*a + int(sqrt(sin(e + f*x) + 1)*sqr
t( - sin(e + f*x) + 1)*sin(e + f*x),x)*b + int(sqrt(sin(e + f*x) + 1)*sqrt
( - sin(e + f*x) + 1),x)*a)
```

3.164 $\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e$

Optimal result

Integrand size = 40, antiderivative size = 96

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx=\frac{(A-B)c\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}+\frac{Bc\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{5af\sqrt{c-c\sin(e+fx)}}$$

output
```
1/4*(A-B)*c*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(1/2)+1/5
*B*c*cos(f*x+e)*(a+a*sin(f*x+e))^(9/2)/a/f/(c-c*sin(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 3.31 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.26

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx=\frac{a^3\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}(4(60A+2$$

input
```
Integrate[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e
 + f*x]],x]
```

output
```
(a^3*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(4*(
60*A + 23*B)*Sin[e + f*x] + Cos[4*(e + f*x)]*(5*A + 15*B + 4*B*Sin[e + f*x
]) - 4*Cos[2*(e + f*x)]*(5*(7*A + 5*B) + 4*(5*A + 6*B)*Sin[e + f*x])))/(16
0*f)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3042, 3450, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{7/2}\sqrt{c-c\sin(e+fx)}(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^{7/2}\sqrt{c-c\sin(e+fx)}(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3450}$$

$$(A-B)\int (\sin(e+fx)a+a)^{7/2}\sqrt{c-c\sin(e+fx)}dx+\frac{B\int (\sin(e+fx)a+a)^{9/2}\sqrt{c-c\sin(e+fx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$(A-B)\int (\sin(e+fx)a+a)^{7/2}\sqrt{c-c\sin(e+fx)}dx+\frac{B\int (\sin(e+fx)a+a)^{9/2}\sqrt{c-c\sin(e+fx)}dx}{a}$$

$$\Big\downarrow \text{3217}$$

$$\frac{c(A-B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}+\frac{Bc\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{5af\sqrt{c-c\sin(e+fx)}}$$

input
```
Int[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x
]],x]
```

output
```
((A - B)*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(4*f*Sqrt[c - c*Sin[e
+ f*x]]) + (B*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(5*a*f*Sqrt[c - c
*Sin[e + f*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3450
```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[B/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] -
Simp[(B*c - A*d)/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[
a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(84) = 168.

Time = 9.57 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.48

method	result
default	$-\frac{40\left(\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\frac{1}{2}\right)A\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\frac{8\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^5\sin\left(\frac{fx}{2}+\frac{e}{2}\right)-\cos\left(\frac{fx}{2}\right.\right.}{}\right.}{}$
parts	$-\frac{2A\sqrt{4}\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3a^3\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2}\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2}\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{2B\,a^3\,c}{}$

input
```
int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
-40*((cos(1/2*f*x+1/2*e)^2-1/2)*A*(cos(1/4*Pi+1/2*f*x+1/2*e)+1)^3*(cos(1/4
*Pi+1/2*f*x+1/2*e)-1)^3*tan(1/4*Pi+1/2*f*x+1/2*e)+8/5*(cos(1/2*f*x+1/2*e)^
5*sin(1/2*f*x+1/2*e)-cos(1/2*f*x+1/2*e)^3*sin(1/2*f*x+1/2*e)+15/8*cos(1/2*
f*x+1/2*e)^4-5/4*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-15/8*cos(1/2*f*x+1/
2*e)^2-5/16)*sin(1/2*f*x+1/2*e)^2*B*cos(1/2*f*x+1/2*e)^2)*a^3*(c*cos(1/4*P
i+1/2*f*x+1/2*e)^2)^(1/2)*(a*sin(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)/(10*f*cos(
1/2*f*x+1/2*e)^2-5*f)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.45

$$\int(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx=\frac{(5\,(A+3\,B)a^3\cos\left(fx+e\right)^4-40\,(A+B)a^3\cos\left(fx+e\right)^2+5\,(7}{}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x
, algorithm="fricas")
```

output
```
1/20*(5*(A + 3*B)*a^3*cos(f*x + e)^4 - 40*(A + B)*a^3*cos(f*x + e)^2 + 5*(
7*A + 5*B)*a^3 + 4*(B*a^3*cos(f*x + e)^4 - (5*A + 7*B)*a^3*cos(f*x + e)^2
+ 2*(5*A + 3*B)*a^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*
x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}\sqrt{-c\sin(fx+e)+c}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)*sqrt(-c*sin(f*x
+ e) + c), x)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.56

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx =$$

$$-\frac{4\left(8\,Ba^3\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+5\,Aa^3\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x
, algorithm="giac")
```

output
```
-4/5*(8*B*a^3*cos(-1/4*pi + 1/2*f*x + 1/2*e)^10*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*A*a^3*cos(-1/4*pi + 1/2*
f*x + 1/2*e)^8*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f
*x + 1/2*e)) - 5*B*a^3*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)*sqrt(c)/f
```

Mupad [B] (verification not implemented)

Time = 38.81 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.80

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx =$$

$$-\frac{a^3\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(140\,A\,\cos\left(e+f\,x\right)+100\,B\,\cos\left(e+f\,x\right)+135\,A\,\right.}{}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(
1/2),x)
```

output
```
-(a^3*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(140*A*co
s(e + f*x) + 100*B*cos(e + f*x) + 135*A*cos(3*e + 3*f*x) - 5*A*cos(5*e + 5
*f*x) + 85*B*cos(3*e + 3*f*x) - 15*B*cos(5*e + 5*f*x) - 240*A*sin(2*e + 2*
f*x) + 40*A*sin(4*e + 4*f*x) - 90*B*sin(2*e + 2*f*x) + 48*B*sin(4*e + 4*f*
x) - 2*B*sin(6*e + 6*f*x)))/(160*f*(cos(2*e + 2*f*x) + 1))
```

Reduce [F]

$$\int (a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx=\sqrt{c}\,\sqrt{a}\,a^3\Bigg(\Bigg(\int\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)$$
$$+\left(\int\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^3\,dx\right)a$$
$$+3\left(\int\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^3\,dx\right)b$$
$$+3\left(\int\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)a$$
$$+3\left(\int\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)b$$
$$+3\left(\int\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)a$$
$$+\left(\int\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)b$$
$$+\left(\int\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}dx\right)a\Bigg)$$

input
```
int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)
```

output
```
sqrt(c)*sqrt(a)*a**3*(int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)
*sin(e + f*x)**4,x)*b + int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) +
1)*sin(e + f*x)**3,x)*a + 3*int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x
) + 1)*sin(e + f*x)**3,x)*b + 3*int(sqrt(sin(e + f*x) + 1)*sqrt( - sin(e +
 f*x) + 1)*sin(e + f*x)**2,x)*a + 3*int(sqrt(sin(e + f*x) + 1)*sqrt( - sin
(e + f*x) + 1)*sin(e + f*x)**2,x)*b + 3*int(sqrt(sin(e + f*x) + 1)*sqrt( -
 sin(e + f*x) + 1)*sin(e + f*x),x)*a + int(sqrt(sin(e + f*x) + 1)*sqrt( -
sin(e + f*x) + 1)*sin(e + f*x),x)*b + int(sqrt(sin(e + f*x) + 1)*sqrt( - s
in(e + f*x) + 1),x)*a)
```

3.165 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 239

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx =& -\frac{8a^4(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\\&-\frac{4a^3(A+B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{f\sqrt{c-c\sin(e+fx)}}\\&-\frac{a^2(A+B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{f\sqrt{c-c\sin(e+fx)}}\\&-\frac{a(A+B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}\\&-\frac{B\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}\end{aligned}$$

output

```
-8*a^4*(A+B)*cos(f*x+e)*ln(1-sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin
(f*x+e))^(1/2)-4*a^3*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f/(c-c*sin(f*
x+e))^(1/2)-a^2*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))
^(1/2)-1/3*a*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(1
/2)-1/4*B*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 13.90 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.77

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx =$$
$$-\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3\sqrt{a(1+\sin(e+fx))}\,(-12(8A+15B)\cos(2(e}{96f\left(\cos\left(\right.\right.}$$

input
```
Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin
[e + f*x]],x]
```

output
```
-1/96*(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*Sqrt
[a*(1 + Sin[e + f*x])]*(-12*(8*A + 15*B)*Cos[2*(e + f*x)] + 3*B*Cos[4*(e +
 f*x)] + 1536*(A + B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 24*(29*A
+ 36*B)*Sin[e + f*x] - 8*(A + 4*B)*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^7*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.325$, Rules used = {3042, 3452, 3042, 3219, 3042, 3219, 3042, 3219, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{7/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^{7/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3452}$$

$$(A+B)\int \frac{(\sin(e+fx)a+a)^{7/2}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$(A+B)\int \frac{(\sin(e+fx)a+a)^{7/2}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{3219}$$

$$(A+B)\left(2a\int \frac{(\sin(e+fx)a+a)^{5/2}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$(A+B)\left(2a\int \frac{(\sin(e+fx)a+a)^{5/2}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{3219}$$

$$(A+B)\left(2a\left(2a\int \frac{(\sin(e+fx)a+a)^{3/2}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin(e+fx)+}{3f\sqrt{c-c\sin(e+fx)}}\right.$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$(A+B)\left(2a\left(2a\int \frac{(\sin(e+fx)a+a)^{3/2}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin(e+fx)+}{3f\sqrt{c-c\sin(e+fx)}}\right.$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{3219}$$

$$(A+B)\left(2a\left(2a\left(2a\int \frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin(e+fx)+}{2f\sqrt{c-c\sin(e+fx)}}\right.\right.$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$(A+B)\left(2a\left(2a\left(2a\int \frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin(e+fx)+\right.\right.$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}$$

$\Big\downarrow$ 3216

$$(A+B)\left(2a\left(2a\left(\frac{2a^2c\cos(e+fx)\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin}{2f\sqrt{c-c\sin}}\right.\right.$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}$$

$\Big\downarrow$ 3042

$$(A+B)\left(2a\left(2a\left(\frac{2a^2c\cos(e+fx)\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin}{2f\sqrt{c-c\sin}}\right.\right.$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}$$

$\Big\downarrow$ 3146

$$(A+B)\left(2a\left(2a\left(-\frac{2a^2\cos(e+fx)\int \frac{1}{c-c\sin(e+fx)}d(-c\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e}{2}\right.\right.$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}$$

$\Big\downarrow$ 16

$$(A+B)\left(2a\left(2a\left(-\frac{2a^2\cos(e+fx)\log(c-c\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a}{2f\sqrt{c-}}\right.\right.$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f\sqrt{c-c\sin(e+fx)}}$$

input
```
Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f
*x]],x]
```

output
```
-1/4*(B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(f*Sqrt[c - c*Sin[e + f*x
]]) + (A + B)*(-1/3*(a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(f*Sqrt[c
- c*Sin[e + f*x]]) + 2*a*(-1/2*(a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))
/(f*Sqrt[c - c*Sin[e + f*x]]) + 2*a*((-2*a^2*Cos[e + f*x]*Log[c - c*Sin[e
+ f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (a*Cos[e
+ f*x]*Sqrt[a + a*Sin[e + f*x]])/(f*Sqrt[c - c*Sin[e + f*x]]))))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3452
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(215) = 430$.

Time = 7.50 (sec) , antiderivative size = 528, normalized size of antiderivative = 2.21

method	result
default	$\frac{2A\sqrt{4}\left(2\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{6}-9\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}+18\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}+12\ln\left(\frac{2}{\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1}\right)-12\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\right.}{3f\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right.}}$
parts	$-\frac{2A\sqrt{4}\left(-2\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{6}+9\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}-18\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}+12\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)-1\right)-12\ln\right.}{3f\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}\right.}}$

input
```
int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
2/3*A/f*4^(1/2)*(2*cos(1/4*Pi+1/2*f*x+1/2*e)^6-9*cos(1/4*Pi+1/2*f*x+1/2*e)
^4+18*cos(1/4*Pi+1/2*f*x+1/2*e)^2+12*ln(2/(cos(1/4*Pi+1/2*f*x+1/2*e)+1))-1
2*ln(-cot(1/4*Pi+1/2*f*x+1/2*e)+csc(1/4*Pi+1/2*f*x+1/2*e)-1)-12*ln(-cot(1/
4*Pi+1/2*f*x+1/2*e)+csc(1/4*Pi+1/2*f*x+1/2*e)+1)-11)*(a*sin(1/4*Pi+1/2*f*x
+1/2*e)^2)^(1/2)*a^3/(c*cos(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)*cot(1/4*Pi+1/2*
f*x+1/2*e)-2/3*B*a^3/f*((-24*sin(1/2*f*x+1/2*e)+24*cos(1/2*f*x+1/2*e))*ln(
-2*(cos(1/2*f*x+1/2*e)-sin(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1))+(24*sin
(1/2*f*x+1/2*e)-24*cos(1/2*f*x+1/2*e))*ln(2/(cos(1/2*f*x+1/2*e)+1))+cos(1/
2*f*x+1/2*e)*((-6*cos(1/2*f*x+1/2*e)^6+6*cos(1/2*f*x+1/2*e)^5*sin(1/2*f*x+
1/2*e)+22*cos(1/2*f*x+1/2*e)^4+10*cos(1/2*f*x+1/2*e)^3*sin(1/2*f*x+1/2*e)+
5*cos(1/2*f*x+1/2*e)^2-21*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-24)*sin(1/
2*f*x+1/2*e)^2+24*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)))*((2*sin(1/2*f*x+
1/2*e)*cos(1/2*f*x+1/2*e)+1)*a)^(1/2)/(cos(1/2*f*x+1/2*e)+sin(1/2*f*x+1/2*
e))/(-(2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-1)*c)^(1/2)
```

Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x
, algorithm="fricas")
```

output
```
integral(-(B*a^3*cos(f*x + e)^4 - (3*A + 5*B)*a^3*cos(f*x + e)^2 + 4*(A +
B)*a^3 - ((A + 3*B)*a^3*cos(f*x + e)^2 - 4*(A + B)*a^3)*sin(f*x + e))*sqrt
(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c*sin(f*x + e) - c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2)
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx=\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/sqrt(-c*sin(f*x
+ e) + c), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx=\int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{7/2}}{\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))
^(1/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))
^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx=\frac{\sqrt{c}\,\sqrt{a}\,a^3\left(-\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^4}{\sin(fx+e)-1}dx\right)\right.}{}$$

input
```
int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a**3*( - int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x)
 + 1)*sin(e + f*x)**4)/(sin(e + f*x) - 1),x)*b - int((sqrt(sin(e + f*x) +
1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**3)/(sin(e + f*x) - 1),x)*a - 3*
int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**3)/(si
n(e + f*x) - 1),x)*b - 3*int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x)
+ 1)*sin(e + f*x)**2)/(sin(e + f*x) - 1),x)*a - 3*int((sqrt(sin(e + f*x) +
 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x) - 1),x)*b - 3
*int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(
e + f*x) - 1),x)*a - int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)
*sin(e + f*x))/(sin(e + f*x) - 1),x)*b - int((sqrt(sin(e + f*x) + 1)*sqrt(
 - sin(e + f*x) + 1))/(sin(e + f*x) - 1),x)*a))/c
```

3.166 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 271

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2f(c-c\sin(e+fx))^{3/2}}+\frac{4a^4(3A+5B)\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{2a^3(3A+5B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}}+\frac{a^2(3A+5B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf\sqrt{c-c\sin(e+fx)}}+\frac{a(3A+5B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6cf\sqrt{c-c\sin(e+fx)}}$$

output
```
1/2*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(3/2)+4*a^4
*(3*A+5*B)*cos(f*x+e)*ln(1-sin(f*x+e))/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin
(f*x+e))^(1/2)+2*a^3*(3*A+5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*
sin(f*x+e))^(1/2)+1/2*a^2*(3*A+5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/
(c-c*sin(f*x+e))^(1/2)+1/6*a*(3*A+5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c
/f/(c-c*sin(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 14.78 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.08

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+$$

input
```
Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
 f*x])^(3/2),x]
```

output
```
(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(-13
2*A - 45*B - 2*(27*A + 59*B)*Cos[2*(e + f*x)] + B*Cos[4*(e + f*x)] - 576*A
*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 960*B*Log[Cos[(e + f*x)/2] - S
in[(e + f*x)/2]] - 117*A*Sin[e + f*x] - 279*B*Sin[e + f*x] + 576*A*Log[Cos
[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sin[e + f*x] + 960*B*Log[Cos[(e + f*x)/2
] - Sin[(e + f*x)/2]]*Sin[e + f*x] - 3*A*Sin[3*(e + f*x)] - 13*B*Sin[3*(e
+ f*x)]))/(24*c*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x]
)*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.93, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.325$, Rules used = {3042, 3451, 3042, 3219, 3042, 3219, 3042, 3219, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3451}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(3A+5B)\int\frac{(\sin(e+fx)a+a)^{7/2}}{\sqrt{c-c\sin(e+fx)}}dx}{2c}$$

$\Big\downarrow$ 3042

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(3A+5B)\int\frac{(\sin(e+fx)a+a)^{7/2}}{\sqrt{c-c\sin(e+fx)}}dx}{2c}$$

$\Big\downarrow$ 3219

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(3A+5B)\left(2a\int\frac{(\sin(e+fx)a+a)^{5/2}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}\right)}{2c}$$

$\Big\downarrow$ 3042

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(3A+5B)\left(2a\int\frac{(\sin(e+fx)a+a)^{5/2}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}\right)}{2c}$$

$\Big\downarrow$ 3219

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(3A+5B)\left(2a\left(2a\int\frac{(\sin(e+fx)a+a)^{3/2}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}\right)}{2c}$$

$\Big\downarrow$ 3042

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(3A+5B)\left(2a\left(2a\int\frac{(\sin(e+fx)a+a)^{3/2}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sin(e+fx)}}\right)}{2c}$$

$\Big\downarrow$ 3219

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(3A+5B)\left(2a\left(2a\left(2a\int\frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(}{3f\sqrt{c}}\right.}{2c}$$

$\Big\downarrow$ 3042

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(3A+5B)\left(2a\left(2a\left(2a\int\frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(}{3f\sqrt{c}}\right.}{2c}$$

↓ 3216

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(3A+5B)\left(2a\left(2a\left(\frac{2a^2c\cos(e+fx)\int\frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\,c}{}\right.}{2c}$$

↓ 3042

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(3A+5B)\left(2a\left(2a\left(\frac{2a^2c\cos(e+fx)\int\frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\,c}{}\right.}{2c}$$

↓ 3146

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(3A+5B)\left(2a\left(2a\left(-\frac{2a^2\cos(e+fx)\int\frac{1}{c-c\sin(e+fx)}d(-c\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin(e+fx)+}{2f\sqrt{c-c\sin(e+fx)}}\right.\right.}{2c}$$

↓ 16

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{2f(c-c\sin(e+fx))^{3/2}}-\frac{(3A+5B)\left(2a\left(2a\left(-\frac{2a^2\cos(e+fx)\log(c-c\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a}{}\right.}{2c}$$

input

```
Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(3/2),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(2*f*(c - c*Sin[e + f*x]
)^(3/2)) - ((3*A + 5*B)*(-1/3*(a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/
(f*Sqrt[c - c*Sin[e + f*x]]) + 2*a*(-1/2*(a*Cos[e + f*x]*(a + a*Sin[e + f*
x])^(3/2))/(f*Sqrt[c - c*Sin[e + f*x]]) + 2*a*((-2*a^2*Cos[e + f*x]*Log[c
- c*Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) -
 (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(f*Sqrt[c - c*Sin[e + f*x]]))))
)/(2*c)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 526 vs. $2(245) = 490$.

Time = 7.51 (sec) , antiderivative size = 527, normalized size of antiderivative = 1.94

method	result
default	$\frac{A\sqrt{4}\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{6}-6\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}+12\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)-1\right)\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}-12\ln\left(\frac{2}{\cos\left(\frac{\pi}{4}+\frac{f}{}\right.}\right.}{}$
parts	$-\frac{A\sqrt{4}\left(-\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{6}+6\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}+12\ln\left(\frac{2}{\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}-12\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\right.\right.}{}$

input
```
int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,metho
d=_RETURNVERBOSE)
```

output
```
A/f*4^(1/2)*(cos(1/4*Pi+1/2*f*x+1/2*e)^6-6*cos(1/4*Pi+1/2*f*x+1/2*e)^4+12*
ln(-cot(1/4*Pi+1/2*f*x+1/2*e)+csc(1/4*Pi+1/2*f*x+1/2*e)-1)*cos(1/4*Pi+1/2*
f*x+1/2*e)^2-12*ln(2/(cos(1/4*Pi+1/2*f*x+1/2*e)+1))*cos(1/4*Pi+1/2*f*x+1/2
*e)^2+12*ln(-cot(1/4*Pi+1/2*f*x+1/2*e)+csc(1/4*Pi+1/2*f*x+1/2*e)+1)*cos(1/
4*Pi+1/2*f*x+1/2*e)^2+3*cos(1/4*Pi+1/2*f*x+1/2*e)^2+2)*a^3*(a*sin(1/4*Pi+1
/2*f*x+1/2*e)^2)^(1/2)/(c*cos(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)/c*sec(1/4*Pi+
1/2*f*x+1/2*e)*csc(1/4*Pi+1/2*f*x+1/2*e)-2/3*B/f*a^3/c*(60*(2*sin(1/2*f*x+
1/2*e)*cos(1/2*f*x+1/2*e)-1)*ln(-2*(cos(1/2*f*x+1/2*e)-sin(1/2*f*x+1/2*e))
/(cos(1/2*f*x+1/2*e)+1))+60*(-2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+1)*l
n(2/(cos(1/2*f*x+1/2*e)+1))+cos(1/2*f*x+1/2*e)*((-8*cos(1/2*f*x+1/2*e)^5+8
*cos(1/2*f*x+1/2*e)^3+26*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)^2+57*cos(1/
2*f*x+1/2*e))*sin(1/2*f*x+1/2*e)^2-60*sin(1/2*f*x+1/2*e)))*((2*sin(1/2*f*x
+1/2*e)*cos(1/2*f*x+1/2*e)+1)*a)^(1/2)/(-1+2*cos(1/2*f*x+1/2*e)^2)/(-(2*si
n(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-1)*c)^(1/2)
```

Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}}{(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x
, algorithm="fricas")
```

output
```
integral(-(B*a^3*cos(f*x + e)^4 - (3*A + 5*B)*a^3*cos(f*x + e)^2 + 4*(A +
B)*a^3 - ((A + 3*B)*a^3*cos(f*x + e)^2 - 4*(A + B)*a^3)*sin(f*x + e))*sqrt
(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^2*cos(f*x + e)^2 + 2*c^2
*sin(f*x + e) - 2*c^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2)
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}}{(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e)
 + c)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=\int \frac{(A+B\,\sin(e+f\,x))\,\left(a+a\,\sin(e+f\,x)\right)^{7/2}}{\left(c-c\,\sin(e+f\,x)\right)^{3/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))
^(3/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))
^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=\frac{\sqrt{c}\,\sqrt{a}\,a^{3}\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^{4}}{\sin(fx+e)^{2}-2\sin(fx+e)+1}dx\right)b\right.}{}$$

input
```
int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)
```

output

```
(sqrt(c)*sqrt(a)*a**3*(int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) +
1)*sin(e + f*x)**4)/(sin(e + f*x)**2 - 2*sin(e + f*x) + 1),x)*b + int((sqr
t(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**3)/(sin(e + f*
x)**2 - 2*sin(e + f*x) + 1),x)*a + 3*int((sqrt(sin(e + f*x) + 1)*sqrt( - s
in(e + f*x) + 1)*sin(e + f*x)**3)/(sin(e + f*x)**2 - 2*sin(e + f*x) + 1),x
)*b + 3*int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)
**2)/(sin(e + f*x)**2 - 2*sin(e + f*x) + 1),x)*a + 3*int((sqrt(sin(e + f*x
) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x)**2 - 2*sin
(e + f*x) + 1),x)*b + 3*int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) +
 1)*sin(e + f*x))/(sin(e + f*x)**2 - 2*sin(e + f*x) + 1),x)*a + int((sqrt(
sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**2
 - 2*sin(e + f*x) + 1),x)*b + int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e +
f*x) + 1))/(sin(e + f*x)**2 - 2*sin(e + f*x) + 1),x)*a))/c**2
```

3.167 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 263

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4f(c-c\sin(e+fx))^{5/2}} - \frac{a(A+3B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2cf(c-c\sin(e+fx))^{3/2}} - \frac{6a^4(A+3B)\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{3a^3(A+3B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{c^2f\sqrt{c-c\sin(e+fx)}} - \frac{3a^2(A+3B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4c^2f\sqrt{c-c\sin(e+fx)}}$$

output
```
1/4*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(5/2)-1/2*a
*(A+3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(3/2)-6*a^
4*(A+3*B)*cos(f*x+e)*ln(1-sin(f*x+e))/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*si
n(f*x+e))^(1/2)-3*a^3*(A+3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c^2/f/(c-c
*sin(f*x+e))^(1/2)-3/4*a^2*(A+3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c^2/f
/(c-c*sin(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 13.51 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.95

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx$$

input
```
Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
 f*x])^(5/2),x]
```

output
```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/2)*(16*(A
 + B) - 16*(3*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + B*Cos[2*(
e + f*x)]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 - 48*(A + 3*B)*Log[Cos[(
e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 -
4*(A + 6*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[e + f*x]))/(4*f*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(5/2))
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.95, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.325$, Rules used = {3042, 3451, 3042, 3218, 3042, 3219, 3042, 3219, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3451}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{(A+3B)\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{3/2}}dx}{2c}$$

$$\Big\downarrow \; 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{(A+3B)\int\frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{3/2}}dx}{2c}$$

$$\Big\downarrow \; 3218$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{(A+3B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{f(c-c\sin(e+fx))^{3/2}}-\frac{3a\int\frac{(\sin(e+fx)a+a)^{5/2}}{\sqrt{c-c\sin(e+fx)}}dx}{c}\right)}{2c}$$

$$\Big\downarrow \; 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{(A+3B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{f(c-c\sin(e+fx))^{3/2}}-\frac{3a\int\frac{(\sin(e+fx)a+a)^{5/2}}{\sqrt{c-c\sin(e+fx)}}dx}{c}\right)}{2c}$$

$$\Big\downarrow \; 3219$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{(A+3B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{f(c-c\sin(e+fx))^{3/2}}-\frac{3a\left(2a\int\frac{(\sin(e+fx)a+a)^{3/2}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right)}{c}\right)}{2c}$$

$$\Big\downarrow \; 3042$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{(A+3B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{f(c-c\sin(e+fx))^{3/2}}-\frac{3a\left(2a\int\frac{(\sin(e+fx)a+a)^{3/2}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}\right)}{c}\right)}{2c}$$

$$\Big\downarrow \; 3219$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{(A+3B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{f(c-c\sin(e+fx))^{3/2}}-\frac{3a\left(2a\left(2a\int\frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}}{c}\right)}{2c}$$

$\downarrow$ 3042

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{(A+3B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{f(c-c\sin(e+fx))^{3/2}}-\frac{3a\left(2a\left(2a\int\frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sin(e+fx)}}}{c}\right.}{2c}$$

$\downarrow$ 3216

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{(A+3B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{f(c-c\sin(e+fx))^{3/2}}-\frac{3a\left(2a\left(\frac{2a^2c\cos(e+fx)\int\frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin(e+}{2f\sqrt{c-c\sin(e}}{c}\right.}{2c}$$

$\downarrow$ 3042

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{(A+3B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{f(c-c\sin(e+fx))^{3/2}}-\frac{3a\left(2a\left(\frac{2a^2c\cos(e+fx)\int\frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin(e+}{2f\sqrt{c-c\sin(e}}{c}\right.}{2c}$$

$\downarrow$ 3146

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{(A+3B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{f(c-c\sin(e+fx))^{3/2}}-\frac{3a\left(2a\left(-\frac{2a^2\cos(e+fx)\int\frac{1}{c-c\sin(e+fx)}d(-c\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(}{}}{c}\right.}{2c}$$

$\downarrow$ 16

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4f(c-c\sin(e+fx))^{5/2}}-\frac{(A+3B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{f(c-c\sin(e+fx))^{3/2}}-\frac{3a\left(2a\left(-\frac{2a^2\cos(e+fx)\log(c-c\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)-\frac{a\cos(e+fx)(a\sin(e-}{2f\sqrt{c-c\sin(e}}{c}\right.}{2c}$$

input
```
Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(5/2),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(4*f*(c - c*Sin[e + f*x]
)^(5/2)) - ((A + 3*B)*((a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(f*(c -
 c*Sin[e + f*x])^(3/2)) - (3*a*(-1/2*(a*Cos[e + f*x]*(a + a*Sin[e + f*x])^
(3/2))/(f*Sqrt[c - c*Sin[e + f*x]]) + 2*a*((-2*a^2*Cos[e + f*x]*Log[c - c*
Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (a*
Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(f*Sqrt[c - c*Sin[e + f*x]]))))/c))
/(2*c)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3218
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Simp[b*((2*m - 1)/(d*(
2*n + 1)))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m +
n + 1, 0])
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 625 vs. $2(237) = 474$.

Time = 7.80 (sec) , antiderivative size = 626, normalized size of antiderivative = 2.38

method	result
default	$\frac{A\sqrt{4}\left(2\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{6}+12\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}\ln\left(\frac{2}{\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1}\right)-12\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}\right.\right.}{}$
parts	$\frac{A\sqrt{4}\left(2\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{6}+12\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}\ln\left(\frac{2}{\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1}\right)-12\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}\right.\right.}{}$

input
```
int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/2*A/f*4^(1/2)*(2*cos(1/4*Pi+1/2*f*x+1/2*e)^6+12*cos(1/4*Pi+1/2*f*x+1/2*e
)^4*ln(2/(cos(1/4*Pi+1/2*f*x+1/2*e)+1))-12*cos(1/4*Pi+1/2*f*x+1/2*e)^4*ln(
-cot(1/4*Pi+1/2*f*x+1/2*e)+csc(1/4*Pi+1/2*f*x+1/2*e)+1)-12*cos(1/4*Pi+1/2*
f*x+1/2*e)^4*ln(-cot(1/4*Pi+1/2*f*x+1/2*e)+csc(1/4*Pi+1/2*f*x+1/2*e)-1)+3*
cos(1/4*Pi+1/2*f*x+1/2*e)^4-6*cos(1/4*Pi+1/2*f*x+1/2*e)^2+1)*a^3*(a*sin(1/
4*Pi+1/2*f*x+1/2*e)^2)^(1/2)/(c*cos(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)/c^2*sec
(1/4*Pi+1/2*f*x+1/2*e)^3*csc(1/4*Pi+1/2*f*x+1/2*e)-2*B/f*a^3/c^2*(18*(4*co
s(1/2*f*x+1/2*e)^4+4*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-4*cos(1/2*f*x+1
/2*e)^2-1)*ln(2*(sin(1/2*f*x+1/2*e)-cos(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e
)+1))+18*(-4*cos(1/2*f*x+1/2*e)^4+4*cos(1/2*f*x+1/2*e)^2-4*sin(1/2*f*x+1/2
*e)*cos(1/2*f*x+1/2*e)+1)*ln(2/(cos(1/2*f*x+1/2*e)+1))+cos(1/2*f*x+1/2*e)*
((4*cos(1/2*f*x+1/2*e)^5-4*cos(1/2*f*x+1/2*e)^3-20*sin(1/2*f*x+1/2*e)*cos(
1/2*f*x+1/2*e)^2+55*cos(1/2*f*x+1/2*e))*sin(1/2*f*x+1/2*e)^2-18*sin(1/2*f*
x+1/2*e)))*((2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+1)*a)^(1/2)/(-(2*sin(
1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-1)*c)^(1/2)/(4*cos(1/2*f*x+1/2*e)^3*sin(
1/2*f*x+1/2*e)-2*cos(1/2*f*x+1/2*e)^2-2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2
*e)+1)
```

Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx=\int \frac{(B\sin\left(fx+e\right)+A)(a\sin\left(fx+e\right)+a)^{\frac{7}{2}}}{(-c\sin\left(fx+e\right)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x
, algorithm="fricas")
```

output
```
integral(-(B*a^3*cos(f*x + e)^4 - (3*A + 5*B)*a^3*cos(f*x + e)^2 + 4*(A +
B)*a^3 - ((A + 3*B)*a^3*cos(f*x + e)^2 - 4*(A + B)*a^3)*sin(f*x + e))*sqrt
(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*c^3*cos(f*x + e)^2 - 4*c
^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2)
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}}{(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e)
 + c)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{7/2}}{(c-c\,\sin(e+f\,x))^{5/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))
^(5/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))
^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,a^3\left(-\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^4}{\sin(fx+e)^3-3\sin(fx+e)^2+3\sin(fx+e)-1}dx\right)\right.}{}$$

input
```
int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)
```

output

```
(sqrt(c)*sqrt(a)*a**3*( - int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x)
 + 1)*sin(e + f*x)**4)/(sin(e + f*x)**3 - 3*sin(e + f*x)**2 + 3*sin(e + f*
x) - 1),x)*b - int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e
 + f*x)**3)/(sin(e + f*x)**3 - 3*sin(e + f*x)**2 + 3*sin(e + f*x) - 1),x)*
a - 3*int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**
3)/(sin(e + f*x)**3 - 3*sin(e + f*x)**2 + 3*sin(e + f*x) - 1),x)*b - 3*int
((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e
 + f*x)**3 - 3*sin(e + f*x)**2 + 3*sin(e + f*x) - 1),x)*a - 3*int((sqrt(si
n(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x)**
3 - 3*sin(e + f*x)**2 + 3*sin(e + f*x) - 1),x)*b - 3*int((sqrt(sin(e + f*x
) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**3 - 3*sin(e
+ f*x)**2 + 3*sin(e + f*x) - 1),x)*a - int((sqrt(sin(e + f*x) + 1)*sqrt( -
 sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**3 - 3*sin(e + f*x)**2 + 3*
sin(e + f*x) - 1),x)*b - int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x)
+ 1))/(sin(e + f*x)**3 - 3*sin(e + f*x)**2 + 3*sin(e + f*x) - 1),x)*a))/c*
*3
```

3.168 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 264

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx &= \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{6f(c-c\sin(e+fx))^{7/2}} \\ &- \frac{a(A+7B)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{12cf(c-c\sin(e+fx))^{5/2}} \\ &+ \frac{a^2(A+7B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4c^2f(c-c\sin(e+fx))^{3/2}} \\ &+ \frac{a^4(A+7B)\cos(e+fx)\log(1-\sin(e+fx))}{c^3f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} \\ &+ \frac{a^3(A+7B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2c^3f\sqrt{c-c\sin(e+fx)}}\end{aligned}$$

output

```
1/6*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(7/2)-1/12*
a*(A+7*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(5/2)+1/4
*a^2*(A+7*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c^2/f/(c-c*sin(f*x+e))^(3/2
)+a^4*(A+7*B)*cos(f*x+e)*ln(1-sin(f*x+e))/c^3/f/(a+a*sin(f*x+e))^(1/2)/(c-
c*sin(f*x+e))^(1/2)+1/2*a^3*(A+7*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c^3/
f/(c-c*sin(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 14.03 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.92

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(a(1+\sin(e+fx\right.}{}$$

input
```
Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
 f*x])^(7/2),x]
```

output
```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/2)*(8*(A
+ B) - 6*(3*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + 18*(A + 3*B
)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + 6*(A + 7*B)*Log[Cos[(e + f*x)/
2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6 + 3*B*(Cos[
(e + f*x)/2] - Sin[(e + f*x)/2])^6*Sin[e + f*x]))/(3*f*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(7/2))
```

Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.96, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.325$, Rules used = {3042, 3451, 3042, 3218, 3042, 3218, 3042, 3219, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a\sin(e+fx)+a)^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}dx$$

$$\Big\downarrow \; 3451$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{6f(c-c\sin(e+fx))^{7/2}}-\frac{(A+7B)\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{5/2}}dx}{6c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{6f(c-c\sin(e+fx))^{7/2}}-\frac{(A+7B)\int\frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{5/2}}dx}{6c}$$

$$\Big\downarrow \text{3218}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{6f(c-c\sin(e+fx))^{7/2}}-\frac{(A+7B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f(c-c\sin(e+fx))^{5/2}}-\frac{3a\int\frac{(\sin(e+fx)a+a)^{5/2}}{(c-c\sin(e+fx))^{3/2}}dx}{2c}\right)}{6c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{6f(c-c\sin(e+fx))^{7/2}}-\frac{(A+7B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f(c-c\sin(e+fx))^{5/2}}-\frac{3a\int\frac{(\sin(e+fx)a+a)^{5/2}}{(c-c\sin(e+fx))^{3/2}}dx}{2c}\right)}{6c}$$

$$\Big\downarrow \text{3218}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{6f(c-c\sin(e+fx))^{7/2}}-\frac{(A+7B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f(c-c\sin(e+fx))^{5/2}}-\frac{3a\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f(c-c\sin(e+fx))^{3/2}}-\frac{2a\int\frac{(\sin(e+fx)a+a)^{3/2}}{\sqrt{c-c\sin(e+fx)}}dx}{c}\right)}{2c}\right)}{6c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{6f(c-c\sin(e+fx))^{7/2}}-\frac{(A+7B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f(c-c\sin(e+fx))^{5/2}}-\frac{3a\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f(c-c\sin(e+fx))^{3/2}}-\frac{2a\int\frac{(\sin(e+fx)a+a)^{3/2}}{\sqrt{c-c\sin(e+fx)}}dx}{c}\right)}{2c}\right)}{6c}$$

$$\Big\downarrow \text{3219}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{6f(c-c\sin(e+fx))^{7/2}}-\frac{(A+7B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f(c-c\sin(e+fx))^{5/2}}-\frac{3a\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f(c-c\sin(e+fx))^{3/2}}-\frac{2a\left(2a\int\frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)}{c}\right)}{2c}\right)}{6c}$$

$\Big\downarrow$ 3042

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{6f(c-c\sin(e+fx))^{7/2}}-\frac{(A+7B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f(c-c\sin(e+fx))^{5/2}}-\frac{3a\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f(c-c\sin(e+fx))^{3/2}}-\frac{2a\left(2a\int\frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)}{c}\right)}{2c}\right)}{6c}$$

$\Big\downarrow$ 3216

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{6f(c-c\sin(e+fx))^{7/2}}-\frac{(A+7B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f(c-c\sin(e+fx))^{5/2}}-\frac{3a\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f(c-c\sin(e+fx))^{3/2}}-\frac{2a\left(\frac{2a^2c\cos(e+fx)\int\frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)}{c}\right)}{2c}\right)}{6c}$$

$\Big\downarrow$ 3042

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{6f(c-c\sin(e+fx))^{7/2}}-\frac{(A+7B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f(c-c\sin(e+fx))^{5/2}}-\frac{3a\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f(c-c\sin(e+fx))^{3/2}}-\frac{2a\left(\frac{2a^2c\cos(e+fx)\int\frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f\sqrt{c-c\sin(e+fx)}}\right)}{c}\right)}{2c}\right)}{6c}$$

$\Big\downarrow$ 3146

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{6f(c-c\sin(e+fx))^{7/2}}-$$

$$\frac{(A+7B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f(c-c\sin(e+fx))^{5/2}}-\frac{3a\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f(c-c\sin(e+fx))^{3/2}}-\frac{2a\left(-\frac{2a^2\cos(e+fx)\int\frac{1}{c-c\sin(e+fx)}d(-c\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(\ldots}{\ldots}\right.}{c}\right.}{2c}\right.}{6c}$$

$\Big\downarrow$ 16

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{6f(c-c\sin(e+fx))^{7/2}}-$$

$$\frac{(A+7B)\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2f(c-c\sin(e+fx))^{5/2}}-\frac{3a\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{f(c-c\sin(e+fx))^{3/2}}-\frac{2a\left(-\frac{2a^2\cos(e+fx)\log(c-c\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(\ldots}}{f\sqrt{c-c\sin(e+\ldots}}\right.}{c}\right.}{2c}\right.}{6c}$$

input
```
Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(7/2),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(6*f*(c - c*Sin[e + f*x]
)^(7/2)) - ((A + 7*B)*((a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(2*f*(c
 - c*Sin[e + f*x])^(5/2)) - (3*a*((a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/
2))/(f*(c - c*Sin[e + f*x])^(3/2)) - (2*a*((-2*a^2*Cos[e + f*x]*Log[c - c*
Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (a*
Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(f*Sqrt[c - c*Sin[e + f*x]])))/c))/
(2*c)))/(6*c)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3218
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Simp[b*((2*m - 1)/(d*(
2*n + 1)))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m +
n + 1, 0])
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 705 vs. $2(236) = 472$.

Time = 7.94 (sec) , antiderivative size = 706, normalized size of antiderivative = 2.67

method	result
default	Expression too large to display
parts	$-\frac{A\sqrt{4}\left(12\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{6}\ln\left(\frac{2}{\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1}\right)-12\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{6}\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)-1\right)-12\cos\left(\right.}{}$

input
```
int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,metho
d=_RETURNVERBOSE)
```

output
```
-1/12*A/f*4^(1/2)*(12*cos(1/4*Pi+1/2*f*x+1/2*e)^6*ln(2/(cos(1/4*Pi+1/2*f*x
+1/2*e)+1))-12*cos(1/4*Pi+1/2*f*x+1/2*e)^6*ln(-cot(1/4*Pi+1/2*f*x+1/2*e)+c
sc(1/4*Pi+1/2*f*x+1/2*e)-1)-12*cos(1/4*Pi+1/2*f*x+1/2*e)^6*ln(-cot(1/4*Pi+
1/2*f*x+1/2*e)+csc(1/4*Pi+1/2*f*x+1/2*e)+1)+11*cos(1/4*Pi+1/2*f*x+1/2*e)^6
-18*cos(1/4*Pi+1/2*f*x+1/2*e)^4+9*cos(1/4*Pi+1/2*f*x+1/2*e)^2-2)*a^3*(a*si
n(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)/(c*cos(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)/c^3
*sec(1/4*Pi+1/2*f*x+1/2*e)^5*csc(1/4*Pi+1/2*f*x+1/2*e)-2/3*B/f*(((168*cos(
1/2*f*x+1/2*e)^5-168*cos(1/2*f*x+1/2*e)^3-126*cos(1/2*f*x+1/2*e))*sin(1/2*
f*x+1/2*e)-252*cos(1/2*f*x+1/2*e)^4+252*cos(1/2*f*x+1/2*e)^2+21)*ln(-2*(co
s(1/2*f*x+1/2*e)-sin(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1))+((-168*cos(1/
2*f*x+1/2*e)^5+168*cos(1/2*f*x+1/2*e)^3+126*cos(1/2*f*x+1/2*e))*sin(1/2*f*
x+1/2*e)+252*cos(1/2*f*x+1/2*e)^4-252*cos(1/2*f*x+1/2*e)^2-21)*ln(2/(cos(1
/2*f*x+1/2*e)+1))+cos(1/2*f*x+1/2*e)*((24*cos(1/2*f*x+1/2*e)^5-24*cos(1/2*
f*x+1/2*e)^3+164*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)^2-102*cos(1/2*f*x+1
/2*e))*sin(1/2*f*x+1/2*e)^2+21*sin(1/2*f*x+1/2*e)))*((2*sin(1/2*f*x+1/2*e)
*cos(1/2*f*x+1/2*e)+1)*a)^(1/2)*a^3/(8*cos(1/2*f*x+1/2*e)^6-12*cos(1/2*f*x
+1/2*e)^4+8*cos(1/2*f*x+1/2*e)^3*sin(1/2*f*x+1/2*e)+2*cos(1/2*f*x+1/2*e)^2
-4*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+1)/(-(2*sin(1/2*f*x+1/2*e)*cos(1/
2*f*x+1/2*e)-1)*c)^(1/2)/c^3
```

Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}}{(-c\sin(fx+e)+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x
, algorithm="fricas")
```

output
```
integral((B*a^3*cos(f*x + e)^4 - (3*A + 5*B)*a^3*cos(f*x + e)^2 + 4*(A + B
)*a^3 - ((A + 3*B)*a^3*cos(f*x + e)^2 - 4*(A + B)*a^3)*sin(f*x + e))*sqrt(
a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^4*cos(f*x + e)^4 - 8*c^4*
cos(f*x + e)^2 + 8*c^4 + 4*(c^4*cos(f*x + e)^2 - 2*c^4)*sin(f*x + e)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2)
,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 749 vs. $2(236) = 472$.

Time = 0.15 (sec) , antiderivative size = 749, normalized size of antiderivative = 2.84

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x
, algorithm="maxima")
```

output
```
-1/3*(B*(42*a^(7/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(7/2) - 21*
a^(7/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(7/2) + 2*(21*a^(7/
2)*sin(f*x + e)/(cos(f*x + e) + 1) - 102*a^(7/2)*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 227*a^(7/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 228*a^(7/2)
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 227*a^(7/2)*sin(f*x + e)^5/(cos(f*x
 + e) + 1)^5 - 102*a^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 21*a^(7/2
)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7)/(c^(7/2) - 6*c^(7/2)*sin(f*x + e)/(
cos(f*x + e) + 1) + 16*c^(7/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 26*c^
(7/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 30*c^(7/2)*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 - 26*c^(7/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 16*c^(
7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 6*c^(7/2)*sin(f*x + e)^7/(cos(f
*x + e) + 1)^7 + c^(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8)) + A*(6*a^(7
/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(7/2) - 3*a^(7/2)*log(sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(7/2) + 4*(3*a^(7/2)*sqrt(c)*sin(f*x
 + e)/(cos(f*x + e) + 1) - 6*a^(7/2)*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 22*a^(7/2)*sqrt(c)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 6*a^(7/2
)*sqrt(c)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3*a^(7/2)*sqrt(c)*sin(f*x
+ e)^5/(cos(f*x + e) + 1)^5)/(c^4 - 6*c^4*sin(f*x + e)/(cos(f*x + e) + 1)
+ 15*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 20*c^4*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 15*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 6*c^4*si...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx=\int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{7/2}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{7/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))
^(7/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))
^(7/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{7/2}}\,dx=\text{Too large to display}$$

input
```
int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x)
```

output

```
(sqrt(c)*sqrt(a)*a**3*(int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) +
1)*sin(e + f*x)**4)/(sin(e + f*x)**4 - 4*sin(e + f*x)**3 + 6*sin(e + f*x)*
*2 - 4*sin(e + f*x) + 1),x)*b + int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e
+ f*x) + 1)*sin(e + f*x)**3)/(sin(e + f*x)**4 - 4*sin(e + f*x)**3 + 6*sin(
e + f*x)**2 - 4*sin(e + f*x) + 1),x)*a + 3*int((sqrt(sin(e + f*x) + 1)*sqr
t( - sin(e + f*x) + 1)*sin(e + f*x)**3)/(sin(e + f*x)**4 - 4*sin(e + f*x)*
*3 + 6*sin(e + f*x)**2 - 4*sin(e + f*x) + 1),x)*b + 3*int((sqrt(sin(e + f*
x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x)**4 - 4*si
n(e + f*x)**3 + 6*sin(e + f*x)**2 - 4*sin(e + f*x) + 1),x)*a + 3*int((sqrt
(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x
)**4 - 4*sin(e + f*x)**3 + 6*sin(e + f*x)**2 - 4*sin(e + f*x) + 1),x)*b +
3*int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin
(e + f*x)**4 - 4*sin(e + f*x)**3 + 6*sin(e + f*x)**2 - 4*sin(e + f*x) + 1)
,x)*a + int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)
)/(sin(e + f*x)**4 - 4*sin(e + f*x)**3 + 6*sin(e + f*x)**2 - 4*sin(e + f*x
) + 1),x)*b + int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1))/(sin(
e + f*x)**4 - 4*sin(e + f*x)**3 + 6*sin(e + f*x)**2 - 4*sin(e + f*x) + 1),
x)*a))/c**4
```

3.169 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}} dx$

Optimal result

Integrand size = 40, antiderivative size = 247

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}} dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8f(c-c\sin(e+fx))^{9/2}} - \frac{aB\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3cf(c-c\sin(e+fx))^{7/2}} + \frac{a^2B\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2c^2f(c-c\sin(e+fx))^{5/2}} - \frac{a^3B\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{c^3f(c-c\sin(e+fx))^{3/2}} - \frac{a^4B\cos(e+fx)\log(1-\sin(e+fx))}{c^4f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

output
```
1/8*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(9/2)-1/3*a
*B*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(7/2)+1/2*a^2*B*
cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c^2/f/(c-c*sin(f*x+e))^(5/2)-a^3*B*cos(f
*x+e)*(a+a*sin(f*x+e))^(1/2)/c^3/f/(c-c*sin(f*x+e))^(3/2)-a^4*B*cos(f*x+e)
*ln(1-sin(f*x+e))/c^4/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 13.58 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.96

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{\left(6(A+B)-4(3A+5B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e\right.\right.\right.}{}$$

input
```
Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
 f*x])^(9/2),x]
```

output
```
((6*(A + B) - 4*(3*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + 9*(A
 + 3*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 - 3*(A + 7*B)*(Cos[(e + f*
x)/2] - Sin[(e + f*x)/2])^6 - 6*B*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^8)*(Cos[(e + f*x)/2] - Sin[(e + f*x
)/2])*(a*(1 + Sin[e + f*x]))^(7/2))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)
/2])^7*(c - c*Sin[e + f*x])^(9/2))
```

Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.325, Rules used = {3042, 3451, 3042, 3218, 3042, 3218, 3042, 3218, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}dx$$

$$\Big\downarrow \text{3451}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{8f(c-c\sin(e+fx))^{9/2}}-\frac{B\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{7/2}}dx}{c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{8f(c-c\sin(e+fx))^{9/2}}-\frac{B\int\frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{7/2}}dx}{c}$$

$$\Big\downarrow \text{3218}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{8f(c-c\sin(e+fx))^{9/2}}-\frac{B\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f(c-c\sin(e+fx))^{7/2}}-\frac{a\int\frac{(\sin(e+fx)a+a)^{5/2}}{(c-c\sin(e+fx))^{5/2}}dx}{c}\right)}{c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{8f(c-c\sin(e+fx))^{9/2}}-\frac{B\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f(c-c\sin(e+fx))^{7/2}}-\frac{a\int\frac{(\sin(e+fx)a+a)^{5/2}}{(c-c\sin(e+fx))^{5/2}}dx}{c}\right)}{c}$$

$$\Big\downarrow \text{3218}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{8f(c-c\sin(e+fx))^{9/2}}-\frac{B\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f(c-c\sin(e+fx))^{7/2}}-\frac{a\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{5/2}}-\frac{a\int\frac{(\sin(e+fx)a+a)^{3/2}}{(c-c\sin(e+fx))^{3/2}}dx}{c}\right)}{c}\right)}{c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{8f(c-c\sin(e+fx))^{9/2}}-\frac{B\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f(c-c\sin(e+fx))^{7/2}}-\frac{a\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{5/2}}-\frac{a\int\frac{(\sin(e+fx)a+a)^{3/2}}{(c-c\sin(e+fx))^{3/2}}dx}{c}\right)}{c}\right)}{c}$$

$$\Big\downarrow \text{3218}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{8f(c-c\sin(e+fx))^{9/2}}-\frac{B\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f(c-c\sin(e+fx))^{7/2}}-\frac{a\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{5/2}}-\frac{a\left(\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f(c-c\sin(e+fx))^{3/2}}-\frac{a\int\frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx}{c}\right)}{c}\right)}{c}\right)}{c}$$

$\downarrow$ 3042

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{8f(c-c\sin(e+fx))^{9/2}}-\frac{B\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f(c-c\sin(e+fx))^{7/2}}-\frac{a\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{5/2}}-\frac{a\left(\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f(c-c\sin(e+fx))^{3/2}}-\frac{a\int\frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx}{c}\right)}{c}\right)}{c}\right)}{c}$$

$\downarrow$ 3216

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{8f(c-c\sin(e+fx))^{9/2}}-\frac{B\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f(c-c\sin(e+fx))^{7/2}}-\frac{a\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{5/2}}-\frac{a\left(\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f(c-c\sin(e+fx))^{3/2}}-\frac{a^2\cos(e+fx)\int\frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}\right)}{c}\right)}{c}\right)}{c}$$

$\downarrow$ 3042

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{8f(c-c\sin(e+fx))^{9/2}}-\frac{B\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f(c-c\sin(e+fx))^{7/2}}-\frac{a\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{5/2}}-\frac{a\left(\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f(c-c\sin(e+fx))^{3/2}}-\frac{a^2\cos(e+fx)\int\frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}\right)}{c}\right)}{c}\right)}{c}$$

$\downarrow$ 3146

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{8f(c-c\sin(e+fx))^{9/2}}-\frac{B\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f(c-c\sin(e+fx))^{7/2}}-\frac{a\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{5/2}}-\frac{a\left(\frac{a^2\cos(e+fx)\int\frac{1}{c-c\sin(e+fx)}d(-c\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f(c-c\sin(e+fx))^{3/2}}\right)}{c}\right)}{c}\right)}{c}$$

$\downarrow$ 16

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{8f(c-c\sin(e+fx))^{9/2}}-\frac{B\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f(c-c\sin(e+fx))^{7/2}}-\frac{a\left(\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f(c-c\sin(e+fx))^{5/2}}-\frac{a\left(\frac{a^2\cos(e+fx)\log(c-c\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{f(c-c\sin(e+fx))^{3/2}}\right)}{c}\right)}{c}\right)}{c}$$

input
```
Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(9/2),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(8*f*(c - c*Sin[e + f*x]
)^(9/2)) - (B*((a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*f*(c - c*Sin
[e + f*x])^(7/2)) - (a*((a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*f*(
c - c*Sin[e + f*x])^(5/2)) - (a*((a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])
/(f*(c - c*Sin[e + f*x])^(3/2)) + (a^2*Cos[e + f*x]*Log[c - c*Sin[e + f*x]
])/(c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])))/c))/c))/c
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3218
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Simp[b*((2*m - 1)/(d*(
2*n + 1)))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m +
n + 1, 0])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 658 vs. $2(221) = 442$.

Time = 7.90 (sec) , antiderivative size = 659, normalized size of antiderivative = 2.67

method	result
default	$-\frac{A\sqrt{4}\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}a^{3}\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\sec\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{6}}{16f\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,c^{4}}+\frac{2B\left(\left(\left(-96\right.\right.\right.}{}$
parts	$-\frac{A\sqrt{4}\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}a^{3}\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\sec\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{6}}{16f\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,c^{4}}+\frac{2B\left(\left(\left(-96\right.\right.\right.}{}$

input
```
int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,metho
d=_RETURNVERBOSE)
```

output
```
-1/16*A/f*4^(1/2)*(cos(1/4*Pi+1/2*f*x+1/2*e)-1)^3*a^3*(a*sin(1/4*Pi+1/2*f*
x+1/2*e)^2)^(1/2)*(cos(1/4*Pi+1/2*f*x+1/2*e)+1)^3/(c*cos(1/4*Pi+1/2*f*x+1/
2*e)^2)^(1/2)/c^4*tan(1/4*Pi+1/2*f*x+1/2*e)*sec(1/4*Pi+1/2*f*x+1/2*e)^6+2/
3*B/f*(((-96*cos(1/2*f*x+1/2*e)^5+96*cos(1/2*f*x+1/2*e)^3+24*cos(1/2*f*x+1
/2*e))*sin(1/2*f*x+1/2*e)-48*cos(1/2*f*x+1/2*e)^8+96*cos(1/2*f*x+1/2*e)^6+
24*cos(1/2*f*x+1/2*e)^4-72*cos(1/2*f*x+1/2*e)^2-3)*ln(2*(sin(1/2*f*x+1/2*e
)-cos(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1))+((96*cos(1/2*f*x+1/2*e)^5-96
*cos(1/2*f*x+1/2*e)^3-24*cos(1/2*f*x+1/2*e))*sin(1/2*f*x+1/2*e)+48*cos(1/2
*f*x+1/2*e)^8-96*cos(1/2*f*x+1/2*e)^6-24*cos(1/2*f*x+1/2*e)^4+72*cos(1/2*f
*x+1/2*e)^2+3)*ln(2/(cos(1/2*f*x+1/2*e)+1))+cos(1/2*f*x+1/2*e)*((-64*cos(1
/2*f*x+1/2*e)^5+64*cos(1/2*f*x+1/2*e)^3-44*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+
1/2*e)^2+24*cos(1/2*f*x+1/2*e))*sin(1/2*f*x+1/2*e)^2-3*sin(1/2*f*x+1/2*e))
)*((2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+1)*a)^(1/2)*a^3/(16*sin(1/2*f*
x+1/2*e)*cos(1/2*f*x+1/2*e)^7-24*cos(1/2*f*x+1/2*e)^6-24*cos(1/2*f*x+1/2*e
)^5*sin(1/2*f*x+1/2*e)+36*cos(1/2*f*x+1/2*e)^4-4*cos(1/2*f*x+1/2*e)^3*sin(
1/2*f*x+1/2*e)-10*cos(1/2*f*x+1/2*e)^2+6*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/
2*e)-1)/(-(2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-1)*c)^(1/2)/c^4
```

Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}}{(-c\sin(fx+e)+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x
, algorithm="fricas")
```

output
```
integral((B*a^3*cos(f*x + e)^4 - (3*A + 5*B)*a^3*cos(f*x + e)^2 + 4*(A + B
)*a^3 - ((A + 3*B)*a^3*cos(f*x + e)^2 - 4*(A + B)*a^3)*sin(f*x + e))*sqrt(
a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(5*c^5*cos(f*x + e)^4 - 20*c
^5*cos(f*x + e)^2 + 16*c^5 - (c^5*cos(f*x + e)^4 - 12*c^5*cos(f*x + e)^2 +
 16*c^5)*sin(f*x + e)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(9/2)
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}}{(-c\sin(fx+e)+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e)
 + c)^(9/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{7/2}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{9/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))
^(9/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))
^(9/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{9/2}}\,dx = \text{Too large to display}$$

input
```
int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x)
```

output

```
(sqrt(c)*sqrt(a)*a**3*( - int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x)
 + 1)*sin(e + f*x)**4)/(sin(e + f*x)**5 - 5*sin(e + f*x)**4 + 10*sin(e + f
*x)**3 - 10*sin(e + f*x)**2 + 5*sin(e + f*x) - 1),x)*b - int((sqrt(sin(e +
 f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**3)/(sin(e + f*x)**5 - 5
*sin(e + f*x)**4 + 10*sin(e + f*x)**3 - 10*sin(e + f*x)**2 + 5*sin(e + f*x
) - 1),x)*a - 3*int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(
e + f*x)**3)/(sin(e + f*x)**5 - 5*sin(e + f*x)**4 + 10*sin(e + f*x)**3 - 1
0*sin(e + f*x)**2 + 5*sin(e + f*x) - 1),x)*b - 3*int((sqrt(sin(e + f*x) +
1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x)**5 - 5*sin(e +
 f*x)**4 + 10*sin(e + f*x)**3 - 10*sin(e + f*x)**2 + 5*sin(e + f*x) - 1),x
)*a - 3*int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)
**2)/(sin(e + f*x)**5 - 5*sin(e + f*x)**4 + 10*sin(e + f*x)**3 - 10*sin(e
+ f*x)**2 + 5*sin(e + f*x) - 1),x)*b - 3*int((sqrt(sin(e + f*x) + 1)*sqrt(
 - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**5 - 5*sin(e + f*x)**4 +
10*sin(e + f*x)**3 - 10*sin(e + f*x)**2 + 5*sin(e + f*x) - 1),x)*a - int((
sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*
x)**5 - 5*sin(e + f*x)**4 + 10*sin(e + f*x)**3 - 10*sin(e + f*x)**2 + 5*si
n(e + f*x) - 1),x)*b - int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) +
1))/(sin(e + f*x)**5 - 5*sin(e + f*x)**4 + 10*sin(e + f*x)**3 - 10*sin(e +
 f*x)**2 + 5*sin(e + f*x) - 1),x)*a))/c**5
```

3.170 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 96

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx=\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{10f(c-c\sin(e+fx))^{11/2}}+\frac{(A-9B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{80cf(c-c\sin(e+fx))^{9/2}}$$

output
```
1/10*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(11/2)+1/8
0*(A-9*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c/f/(c-c*sin(f*x+e))^(9/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 434 vs. $2(96) = 192$.

Time = 17.29 (sec) , antiderivative size = 434, normalized size of antiderivative = 4.52

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{8(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\text{s}}{5f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(\text{e}}$$
$$+\frac{(-3A-5B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(a(1+\sin(e+fx)))^{7/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{11/2}}$$
$$+\frac{2(A+3B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(a(1+\sin(e+fx)))^{7/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{11/2}}$$
$$+\frac{(-A-7B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(a(1+\sin(e+fx)))^{7/2}}{2f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{11/2}}$$
$$+\frac{B\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9(a(1+\sin(e+fx)))^{7/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{11/2}}$$

input
```
Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
 f*x])^(11/2),x]
```

output
```
(8*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7
/2))/(5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(11
/2)) + ((-3*A - 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e
 + f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e +
 f*x])^(11/2)) + (2*(A + 3*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(
1 + Sin[e + f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c -
c*Sin[e + f*x])^(11/2)) + ((-A - 7*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
)^7*(a*(1 + Sin[e + f*x]))^(7/2))/(2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])^7*(c - c*Sin[e + f*x])^(11/2)) + (B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2
])^9*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^7*(c - c*Sin[e + f*x])^(11/2))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3042, 3451, 3042, 3221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}dx$$

$$\Big\downarrow \text{3451}$$

$$\frac{(A-9B)\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{9/2}}dx}{10c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-9B)\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{9/2}}dx}{10c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

$$\Big\downarrow \text{3221}$$

$$\frac{(A-9B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{80cf(c-c\sin(e+fx))^{9/2}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f(c-c\sin(e+fx))^{11/2}}$$

input
```
Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(11/2),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(10*f*(c - c*Sin[e + f*x
])^(11/2)) + ((A - 9*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(80*c*f*(
c - c*Sin[e + f*x])^(9/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3221
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(84) = 168$.

Time = 9.45 (sec), antiderivative size = 414, normalized size of antiderivative = 4.31

method	result
parts	$\frac{A\sqrt{4}\,\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,a^{3}\left(\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{7}+4\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{7}\sec\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}\right)}{160f\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,c^{5}}-\frac{2B\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\sqrt{}}{5f\left(-1+2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\right)\left(16\cos\left(\right.\right.}$
default	$\frac{16\left(\left(\sec\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}+\frac{1}{4}\right)A\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-\frac{1}{2}\right)\left(\frac{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}}{2}+\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}\sin\left(\frac{fx}{2}+\frac{e}{2}\right)-\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}-\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}\sin\left(\right.\right.\right.}{5\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,c^{5}f\left(-1+2\cos\left(\right.\right.}$

input
```
int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x,meth
od=_RETURNVERBOSE)
```

output
```
1/160*A/f*4^(1/2)*(a*sin(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)*a^3/(c*cos(1/4*Pi+
1/2*f*x+1/2*e)^2)^(1/2)/c^5*(tan(1/4*Pi+1/2*f*x+1/2*e)^7+4*tan(1/4*Pi+1/2*
f*x+1/2*e)^7*sec(1/4*Pi+1/2*f*x+1/2*e)^2)-2/5*B/f*cos(1/2*f*x+1/2*e)^2*((2
*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+1)*a)^(1/2)*(8*cos(1/2*f*x+1/2*e)^5
*sin(1/2*f*x+1/2*e)-8*cos(1/2*f*x+1/2*e)^3*sin(1/2*f*x+1/2*e)+20*cos(1/2*f
*x+1/2*e)^4-10*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-20*cos(1/2*f*x+1/2*e)
^2-5)*sin(1/2*f*x+1/2*e)^2*a^3/(-1+2*cos(1/2*f*x+1/2*e)^2)/(16*cos(1/2*f*x
+1/2*e)^8+32*cos(1/2*f*x+1/2*e)^5*sin(1/2*f*x+1/2*e)-32*cos(1/2*f*x+1/2*e)
^6-32*cos(1/2*f*x+1/2*e)^3*sin(1/2*f*x+1/2*e)-8*cos(1/2*f*x+1/2*e)^4-8*sin
(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+24*cos(1/2*f*x+1/2*e)^2+1)/(-(2*sin(1/2
*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-1)*c)^(1/2)/c^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(84) = 168$.

Time = 0.10 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.07

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{\left(10\,Ba^3\cos\left(fx+e\right)^4-5\,(A+7\,B)a^3\cos\left(fx+e\right)^2+\right.}{10\,\left(5\,c^6f\cos\left(fx+e\right)^5-20\,c^6f\cos\left(fx+\right.\right.}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),
x, algorithm="fricas")
```

output
```
1/10*(10*B*a^3*cos(f*x + e)^4 - 5*(A + 7*B)*a^3*cos(f*x + e)^2 + 2*(3*A +
13*B)*a^3 - 5*((A - B)*a^3*cos(f*x + e)^2 - 2*(A - B)*a^3)*sin(f*x + e))*s
qrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(5*c^6*f*cos(f*x + e)^5
- 20*c^6*f*cos(f*x + e)^3 + 16*c^6*f*cos(f*x + e) - (c^6*f*cos(f*x + e)^5
- 12*c^6*f*cos(f*x + e)^3 + 16*c^6*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(11/2
),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}}{(-c\sin(fx+e)+c)^{\frac{11}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),
x, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e)
 + c)^(11/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),
x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx=\int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{7/2}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{11/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))
^(11/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))
^(11/2), x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{11/2}}\,dx=\text{Too large to display}$$

input
```
int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a**3*(int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) +
1)*sin(e + f*x)**4)/(sin(e + f*x)**6 - 6*sin(e + f*x)**5 + 15*sin(e + f*x)
**4 - 20*sin(e + f*x)**3 + 15*sin(e + f*x)**2 - 6*sin(e + f*x) + 1),x)*b +
 int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**3)/(s
in(e + f*x)**6 - 6*sin(e + f*x)**5 + 15*sin(e + f*x)**4 - 20*sin(e + f*x)*
*3 + 15*sin(e + f*x)**2 - 6*sin(e + f*x) + 1),x)*a + 3*int((sqrt(sin(e + f
*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**3)/(sin(e + f*x)**6 - 6*s
in(e + f*x)**5 + 15*sin(e + f*x)**4 - 20*sin(e + f*x)**3 + 15*sin(e + f*x)
**2 - 6*sin(e + f*x) + 1),x)*b + 3*int((sqrt(sin(e + f*x) + 1)*sqrt( - sin
(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x)**6 - 6*sin(e + f*x)**5 + 15*
sin(e + f*x)**4 - 20*sin(e + f*x)**3 + 15*sin(e + f*x)**2 - 6*sin(e + f*x)
 + 1),x)*a + 3*int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e
 + f*x)**2)/(sin(e + f*x)**6 - 6*sin(e + f*x)**5 + 15*sin(e + f*x)**4 - 20
*sin(e + f*x)**3 + 15*sin(e + f*x)**2 - 6*sin(e + f*x) + 1),x)*b + 3*int((
sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*
x)**6 - 6*sin(e + f*x)**5 + 15*sin(e + f*x)**4 - 20*sin(e + f*x)**3 + 15*s
in(e + f*x)**2 - 6*sin(e + f*x) + 1),x)*a + int((sqrt(sin(e + f*x) + 1)*sq
rt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**6 - 6*sin(e + f*x)**5
 + 15*sin(e + f*x)**4 - 20*sin(e + f*x)**3 + 15*sin(e + f*x)**2 - 6*sin(e
+ f*x) + 1),x)*b + int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1...
```

3.171 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 146

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{12f(c-c\sin(e+fx))^{13/2}} + \frac{(A-5B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{60cf(c-c\sin(e+fx))^{11/2}} + \frac{(A-5B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{480c^2f(c-c\sin(e+fx))^{9/2}}$$

output
```
1/12*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(13/2)+1/6
0*(A-5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c/f/(c-c*sin(f*x+e))^(11/2)+1/
480*(A-5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c^2/f/(c-c*sin(f*x+e))^(9/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 442 vs. 2(146) = 292.

Time = 17.28 (sec) , antiderivative size = 442, normalized size of antiderivative = 3.03

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx = \frac{4(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+s}{3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(}$$

$$-\frac{4(3A+5B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(a(1+\sin(e+fx)))^{7/2}}{5f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{13/2}}$$

$$+\frac{3(A+3B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(a(1+\sin(e+fx)))^{7/2}}{2f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{13/2}}$$

$$+\frac{(-A-7B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(a(1+\sin(e+fx)))^{7/2}}{3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{13/2}}$$

$$+\frac{B\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9(a(1+\sin(e+fx)))^{7/2}}{2f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{13/2}}$$

input
```
Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
 f*x])^(13/2),x]
```

output
```
(4*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7
/2))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(13
/2)) - (4*(3*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[
e + f*x]))^(7/2))/(5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[
e + f*x])^(13/2)) + (3*(A + 3*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(
a*(1 + Sin[e + f*x]))^(7/2))/(2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*
(c - c*Sin[e + f*x])^(13/2)) + ((-A - 7*B)*(Cos[(e + f*x)/2] - Sin[(e + f*
x)/2])^7*(a*(1 + Sin[e + f*x]))^(7/2))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^7*(c - c*Sin[e + f*x])^(13/2)) + (B*(Cos[(e + f*x)/2] - Sin[(e + f
*x)/2])^9*(a*(1 + Sin[e + f*x]))^(7/2))/(2*f*(Cos[(e + f*x)/2] + Sin[(e +
f*x)/2])^7*(c - c*Sin[e + f*x])^(13/2))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3042, 3451, 3042, 3222, 3042, 3221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}dx$$

$$\Big\downarrow \text{3451}$$

$$\frac{(A-5B)\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{11/2}}dx}{6c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{12f(c-c\sin(e+fx))^{13/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-5B)\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{11/2}}dx}{6c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{12f(c-c\sin(e+fx))^{13/2}}$$

$$\Big\downarrow \text{3222}$$

$$\frac{(A-5B)\left(\frac{\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{9/2}}dx}{10c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f(c-c\sin(e+fx))^{11/2}}\right)}{6c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{12f(c-c\sin(e+fx))^{13/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-5B)\left(\frac{\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{9/2}}dx}{10c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f(c-c\sin(e+fx))^{11/2}}\right)}{6c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{12f(c-c\sin(e+fx))^{13/2}}$$

$$\Big\downarrow \text{3221}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{12f(c-c\sin(e+fx))^{13/2}}+\frac{(A-5B)\left(\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{80cf(c-c\sin(e+fx))^{9/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f(c-c\sin(e+fx))^{11/2}}\right)}{6c}$$

input
```
Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(13/2),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(12*f*(c - c*Sin[e + f*x
])^(13/2)) + ((A - 5*B)*((Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(10*f*(
c - c*Sin[e + f*x])^(11/2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(8
0*c*f*(c - c*Sin[e + f*x])^(9/2))))/(6*c)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3221
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

rule 3222
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp[(m + n + 1)/(a*(2*m + 1)
)   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
 ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||
!SumSimplerQ[n, 1])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(128) = 256$.

Time = 9.86 (sec) , antiderivative size = 429, normalized size of antiderivative = 2.94

method	result
parts	$\frac{A\sqrt{4}\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}+4\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}+10\right)a^{3}\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{7}\sec\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}}{960f\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,c^{6}} - \frac{}{3f\left(-1+2\cos\right.}$
default	$\frac{2a^{3}\left(A\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}+4\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}+10\right)\left(\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}-2\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}-\frac{5\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}}{2}-\right.\right.}{15\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,c^{6}f\left(-1+2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}$

input
```
int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),x,meth
od=_RETURNVERBOSE)
```

output
```
1/960*A/f*4^(1/2)*(a*sin(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)*(cos(1/4*Pi+1/2*f*
x+1/2*e)^4+4*cos(1/4*Pi+1/2*f*x+1/2*e)^2+10)*a^3/(c*cos(1/4*Pi+1/2*f*x+1/2
*e)^2)^(1/2)/c^6*tan(1/4*Pi+1/2*f*x+1/2*e)^7*sec(1/4*Pi+1/2*f*x+1/2*e)^4-2
/3*B/f*((2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+1)*a)^(1/2)*cos(1/2*f*x+1
/2*e)^2*(-12*cos(1/2*f*x+1/2*e)^4+4*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+
12*cos(1/2*f*x+1/2*e)^2+3)*sin(1/2*f*x+1/2*e)^2*a^3/(-1+2*cos(1/2*f*x+1/2*
e)^2)/(32*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)^9-64*sin(1/2*f*x+1/2*e)*co
s(1/2*f*x+1/2*e)^7-80*cos(1/2*f*x+1/2*e)^8-48*cos(1/2*f*x+1/2*e)^5*sin(1/2
*f*x+1/2*e)+160*cos(1/2*f*x+1/2*e)^6+80*cos(1/2*f*x+1/2*e)^3*sin(1/2*f*x+1
/2*e)-40*cos(1/2*f*x+1/2*e)^4+10*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-40*
cos(1/2*f*x+1/2*e)^2-1)/(-(2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-1)*c)^(
1/2)/c^6
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.47

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx =$$
$$-\frac{\left(15\,Ba^3\cos\left(fx+e\right)^4-15\,(A+3\,B)a^3\cos\left(fx+e\right)^2+6\,(3\,A+5\,B)a^3-2\left(5\,(A+B)a^3\cos\left(fx+e\right)^2-\right.}{30\left(c^7f\cos\left(fx+e\right)^7-18\,c^7f\cos\left(fx+e\right)^5+48\,c^7f\cos\left(fx+e\right)^3-32\,c^7f\cos\left(fx+e\right)+2\left(3\,c^7f\cos\right.\right.}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),
x, algorithm="fricas")
```

output
```
-1/30*(15*B*a^3*cos(f*x + e)^4 - 15*(A + 3*B)*a^3*cos(f*x + e)^2 + 6*(3*A
+ 5*B)*a^3 - 2*(5*(A + B)*a^3*cos(f*x + e)^2 - (11*A + 5*B)*a^3)*sin(f*x +
 e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^7*f*cos(f*x + e
)^7 - 18*c^7*f*cos(f*x + e)^5 + 48*c^7*f*cos(f*x + e)^3 - 32*c^7*f*cos(f*x
 + e) + 2*(3*c^7*f*cos(f*x + e)^5 - 16*c^7*f*cos(f*x + e)^3 + 16*c^7*f*cos
(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(13/2
),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{7}{2}}}{(-c\sin(fx+e)+c)^{\frac{13}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),
x, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e)
 + c)^(13/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),
x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 46.67 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.78

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx = -\frac{\sqrt{c-c\,\sin(e+f\,x)}\left(\frac{56\,a^3\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,(4\,A+5\,B)\,\sqrt{a+a\,\sin(e+f\,x)}}{5\,c^7\,f}+\frac{a^3\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\sin(3\,e+3\,f\,x)\,(A\,1\mathrm{i}+B\,1\mathrm{i})\,\sqrt{a+a\,\sin(e+f\,x)}\,32\mathrm{i}}{3\,c^7\,f}\right.}{-858\,\cos(e+f\,x)\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}+858\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\cos(3\,e+3\,f\,x)-130\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\cos(5\,e+5\,f\,x)}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))
^(13/2),x)
```

output
```
-((c - c*sin(e + f*x))^(1/2)*((56*a^3*exp(e*7i + f*x*7i)*(4*A + 5*B)*(a +
a*sin(e + f*x))^(1/2))/(5*c^7*f) + (a^3*exp(e*7i + f*x*7i)*sin(3*e + 3*f*x
)*(A*1i + B*1i)*(a + a*sin(e + f*x))^(1/2)*32i)/(3*c^7*f) - (32*a^3*exp(e*
7i + f*x*7i)*cos(2*e + 2*f*x)*(A + 2*B)*(a + a*sin(e + f*x))^(1/2))/(c^7*f
) + (8*B*a^3*exp(e*7i + f*x*7i)*cos(4*e + 4*f*x)*(a + a*sin(e + f*x))^(1/2
))/(c^7*f) - (a^3*exp(e*7i + f*x*7i)*sin(e + f*x)*(A*13i + B*5i)*(a + a*si
n(e + f*x))^(1/2)*32i)/(5*c^7*f)))/(858*exp(e*7i + f*x*7i)*cos(3*e + 3*f*x
) - 858*cos(e + f*x)*exp(e*7i + f*x*7i) - 130*exp(e*7i + f*x*7i)*cos(5*e +
 5*f*x) + 2*exp(e*7i + f*x*7i)*cos(7*e + 7*f*x) + 1144*exp(e*7i + f*x*7i)*
sin(2*e + 2*f*x) - 416*exp(e*7i + f*x*7i)*sin(4*e + 4*f*x) + 24*exp(e*7i +
 f*x*7i)*sin(6*e + 6*f*x))
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{13/2}}\,dx = \text{Too large to display}$$

input
```
int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a**3*( - int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x)
 + 1)*sin(e + f*x)**4)/(sin(e + f*x)**7 - 7*sin(e + f*x)**6 + 21*sin(e + f
*x)**5 - 35*sin(e + f*x)**4 + 35*sin(e + f*x)**3 - 21*sin(e + f*x)**2 + 7*
sin(e + f*x) - 1),x)*b - int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x)
+ 1)*sin(e + f*x)**3)/(sin(e + f*x)**7 - 7*sin(e + f*x)**6 + 21*sin(e + f*
x)**5 - 35*sin(e + f*x)**4 + 35*sin(e + f*x)**3 - 21*sin(e + f*x)**2 + 7*s
in(e + f*x) - 1),x)*a - 3*int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x)
 + 1)*sin(e + f*x)**3)/(sin(e + f*x)**7 - 7*sin(e + f*x)**6 + 21*sin(e + f
*x)**5 - 35*sin(e + f*x)**4 + 35*sin(e + f*x)**3 - 21*sin(e + f*x)**2 + 7*
sin(e + f*x) - 1),x)*b - 3*int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x
) + 1)*sin(e + f*x)**2)/(sin(e + f*x)**7 - 7*sin(e + f*x)**6 + 21*sin(e +
f*x)**5 - 35*sin(e + f*x)**4 + 35*sin(e + f*x)**3 - 21*sin(e + f*x)**2 + 7
*sin(e + f*x) - 1),x)*a - 3*int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*
x) + 1)*sin(e + f*x)**2)/(sin(e + f*x)**7 - 7*sin(e + f*x)**6 + 21*sin(e +
 f*x)**5 - 35*sin(e + f*x)**4 + 35*sin(e + f*x)**3 - 21*sin(e + f*x)**2 +
7*sin(e + f*x) - 1),x)*b - 3*int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f
*x) + 1)*sin(e + f*x))/(sin(e + f*x)**7 - 7*sin(e + f*x)**6 + 21*sin(e + f
*x)**5 - 35*sin(e + f*x)**4 + 35*sin(e + f*x)**3 - 21*sin(e + f*x)**2 + 7*
sin(e + f*x) - 1),x)*a - int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x)
+ 1)*sin(e + f*x))/(sin(e + f*x)**7 - 7*sin(e + f*x)**6 + 21*sin(e + f*...
```

3.172 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{15/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 202

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{15/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{14f(c-c\sin(e+fx))^{15/2}} + \frac{(3A-11B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{168cf(c-c\sin(e+fx))^{13/2}} + \frac{(3A-11B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{840c^2f(c-c\sin(e+fx))^{11/2}} + \frac{(3A-11B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{6720c^3f(c-c\sin(e+fx))^{9/2}}$$

output
```
1/14*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(15/2)+1/1
68*(3*A-11*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c/f/(c-c*sin(f*x+e))^(13/2
)+1/840*(3*A-11*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c^2/f/(c-c*sin(f*x+e)
)^(11/2)+1/6720*(3*A-11*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c^3/f/(c-c*si
n(f*x+e))^(9/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 442 vs. $2(202) = 404$.

Time = 17.37 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.19

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{15/2}}\,dx = \frac{8(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\mathrm{s}}{7f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e}$$
$$-\frac{2(3A+5B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(a(1+\sin(e+fx)))^{7/2}}{3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{15/2}}$$
$$+\frac{6(A+3B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(a(1+\sin(e+fx)))^{7/2}}{5f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{15/2}}$$
$$+\frac{(-A-7B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(a(1+\sin(e+fx)))^{7/2}}{4f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{15/2}}$$
$$+\frac{B\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9(a(1+\sin(e+fx)))^{7/2}}{3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{15/2}}$$

input

```
Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
 f*x])^(15/2),x]
```

output

```
(8*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7
/2))/(7*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(15
/2)) - (2*(3*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[
e + f*x]))^(7/2))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[
e + f*x])^(15/2)) + (6*(A + 3*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(
a*(1 + Sin[e + f*x]))^(7/2))/(5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*
(c - c*Sin[e + f*x])^(15/2)) + ((-A - 7*B)*(Cos[(e + f*x)/2] - Sin[(e + f*
x)/2])^7*(a*(1 + Sin[e + f*x]))^(7/2))/(4*f*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^7*(c - c*Sin[e + f*x])^(15/2)) + (B*(Cos[(e + f*x)/2] - Sin[(e + f
*x)/2])^9*(a*(1 + Sin[e + f*x]))^(7/2))/(3*f*(Cos[(e + f*x)/2] + Sin[(e +
f*x)/2])^7*(c - c*Sin[e + f*x])^(15/2))
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3451, 3042, 3222, 3042, 3222, 3042, 3221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{15/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{15/2}}dx$$

$$\Big\downarrow \text{3451}$$

$$\frac{(3A-11B)\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{13/2}}dx}{14c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{14f(c-c\sin(e+fx))^{15/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(3A-11B)\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{13/2}}dx}{14c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{14f(c-c\sin(e+fx))^{15/2}}$$

$$\Big\downarrow \text{3222}$$

$$\frac{(3A-11B)\left(\frac{\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{11/2}}dx}{6c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{12f(c-c\sin(e+fx))^{13/2}}\right)}{14c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{14f(c-c\sin(e+fx))^{15/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(3A-11B)\left(\frac{\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{11/2}}dx}{6c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{12f(c-c\sin(e+fx))^{13/2}}\right)}{14c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{14f(c-c\sin(e+fx))^{15/2}}$$

$$\Big\downarrow \text{3222}$$

$$\frac{(3A-11B)\left(\frac{\frac{\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{9/2}}dx}{10c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f(c-c\sin(e+fx))^{11/2}}}{6c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{12f(c-c\sin(e+fx))^{13/2}}\right)}{14c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{14f(c-c\sin(e+fx))^{15/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(3A-11B)\left(\frac{\frac{\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{9/2}}dx}{10c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f(c-c\sin(e+fx))^{11/2}}}{6c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{12f(c-c\sin(e+fx))^{13/2}}\right)}{14c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{14f(c-c\sin(e+fx))^{15/2}}$$

$$\Big\downarrow \text{3221}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{14f(c-c\sin(e+fx))^{15/2}}+\frac{(3A-11B)\left(\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{12f(c-c\sin(e+fx))^{13/2}}+\frac{\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{80cf(c-c\sin(e+fx))^{9/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f(c-c\sin(e+fx))^{11/2}}}{6c}\right)}{14c}$$

input
```
Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(15/2),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(14*f*(c - c*Sin[e + f*x
])^(15/2)) + ((3*A - 11*B)*((Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(12*
f*(c - c*Sin[e + f*x])^(13/2)) + ((Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)
)/(10*f*(c - c*Sin[e + f*x])^(11/2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^
(7/2))/(80*c*f*(c - c*Sin[e + f*x])^(9/2)))/(6*c)))/(14*c)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3221
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

rule 3222
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp[(m + n + 1)/(a*(2*m + 1)
)   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
 ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||
!SumSimplerQ[n, 1])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 516 vs. $2(178) = 356$.

Time = 12.12 (sec) , antiderivative size = 517, normalized size of antiderivative = 2.56

method	result
parts	$\frac{A\sqrt{4}\,\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{6}+4\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}+10\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}+20\right)a^{3}\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{7}\sec\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right.}{4480f\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,c^{7}}$
default	Expression too large to display

input
```
int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(15/2),x,meth
od=_RETURNVERBOSE)
```

output
```
1/4480*A/f*4^(1/2)*(a*sin(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)*(cos(1/4*Pi+1/2*f
*x+1/2*e)^6+4*cos(1/4*Pi+1/2*f*x+1/2*e)^4+10*cos(1/4*Pi+1/2*f*x+1/2*e)^2+2
0)*a^3/(c*cos(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)/c^7*tan(1/4*Pi+1/2*f*x+1/2*e)
^7*sec(1/4*Pi+1/2*f*x+1/2*e)^6+2/105*B/f*((2*sin(1/2*f*x+1/2*e)*cos(1/2*f*
x+1/2*e)+1)*a)^(1/2)*cos(1/2*f*x+1/2*e)^2*((48*cos(1/2*f*x+1/2*e)^9-96*cos
(1/2*f*x+1/2*e)^7-204*cos(1/2*f*x+1/2*e)^5+252*cos(1/2*f*x+1/2*e)^3-70*cos
(1/2*f*x+1/2*e))*sin(1/2*f*x+1/2*e)-168*cos(1/2*f*x+1/2*e)^8+336*cos(1/2*f
*x+1/2*e)^6+322*cos(1/2*f*x+1/2*e)^4-490*cos(1/2*f*x+1/2*e)^2-105)*sin(1/2
*f*x+1/2*e)^2*a^3/(-1+2*cos(1/2*f*x+1/2*e)^2)/((192*cos(1/2*f*x+1/2*e)^9-3
84*cos(1/2*f*x+1/2*e)^7+32*cos(1/2*f*x+1/2*e)^5+160*cos(1/2*f*x+1/2*e)^3+1
2*cos(1/2*f*x+1/2*e))*sin(1/2*f*x+1/2*e)+64*cos(1/2*f*x+1/2*e)^12-192*cos(
1/2*f*x+1/2*e)^10-48*cos(1/2*f*x+1/2*e)^8+416*cos(1/2*f*x+1/2*e)^6-180*cos
(1/2*f*x+1/2*e)^4-60*cos(1/2*f*x+1/2*e)^2-1)/(-(2*sin(1/2*f*x+1/2*e)*cos(1
/2*f*x+1/2*e)-1)*c)^(1/2)/c^7
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.16

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{15/2}}\,dx =$$

$$-\frac{\left(140\,Ba^{3}\cos\left(fx+e\right)^{4}-7\left(27\,A+61\,B\right)a^{3}\cos\left(fx+e\right)^{2}+4\left(57\,A+71\,B\right)a^{3}-7\left(5\left(3\,A+5\,B\right)a^{3}\cos\right.\right.}{420\left(7\,c^{8}f\cos\left(fx+e\right)^{7}-56\,c^{8}f\cos\left(fx+e\right)^{5}+112\,c^{8}f\cos\left(fx+e\right)^{3}-64\,c^{8}f\cos\left(fx+e\right)-\left(c^{8}f\cos\left(\right.\right.\right.}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(15/2),
x, algorithm="fricas")
```

output
```
-1/420*(140*B*a^3*cos(f*x + e)^4 - 7*(27*A + 61*B)*a^3*cos(f*x + e)^2 + 4*
(57*A + 71*B)*a^3 - 7*(5*(3*A + 5*B)*a^3*cos(f*x + e)^2 - 4*(9*A + 7*B)*a^
3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(7*c^8
*f*cos(f*x + e)^7 - 56*c^8*f*cos(f*x + e)^5 + 112*c^8*f*cos(f*x + e)^3 - 6
4*c^8*f*cos(f*x + e) - (c^8*f*cos(f*x + e)^7 - 24*c^8*f*cos(f*x + e)^5 + 8
0*c^8*f*cos(f*x + e)^3 - 64*c^8*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{15/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(15/2
),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{15/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(15/2),
x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{15/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(15/2),
x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 52.46 (sec) , antiderivative size = 827, normalized size of antiderivative = 4.09

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{15/2}}\,dx = \text{Too large to display}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))
^(15/2),x)
```

output
```
-((c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*
((B*a^3*exp(e*4i + f*x*4i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i
 + f*x*1i)*1i)/2))^(1/2)*16i)/(3*c^8*f) + (B*a^3*exp(e*12i + f*x*12i)*(a +
 a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*16i)/(
3*c^8*f) - (a^3*exp(e*5i + f*x*5i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (
exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(A*3i + B*5i)*8i)/(3*c^8*f) + (a^3*exp(e*
11i + f*x*11i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1
i)/2))^(1/2)*(A*3i + B*5i)*8i)/(3*c^8*f) - (a^3*exp(e*6i + f*x*6i)*(a + a*
((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(27*A + 4
1*B)*16i)/(15*c^8*f) - (a^3*exp(e*10i + f*x*10i)*(a + a*((exp(- e*1i - f*x
*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(27*A + 41*B)*16i)/(15*c^8*
f) + (a^3*exp(e*7i + f*x*7i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*
1i + f*x*1i)*1i)/2))^(1/2)*(A*43i + B*29i)*8i)/(5*c^8*f) - (a^3*exp(e*9i +
 f*x*9i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))
^(1/2)*(A*43i + B*29i)*8i)/(5*c^8*f) + (a^3*exp(e*8i + f*x*8i)*(a + a*((ex
p(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(89*A + 82*B)
*32i)/(35*c^8*f)))/(exp(e*1i + f*x*1i)*14i - 90*exp(e*2i + f*x*2i) - exp(e
*3i + f*x*3i)*350i + 910*exp(e*4i + f*x*4i) + exp(e*5i + f*x*5i)*1638i - 2
002*exp(e*6i + f*x*6i) - exp(e*7i + f*x*7i)*1430i - exp(e*9i + f*x*9i)*143
0i + 2002*exp(e*10i + f*x*10i) + exp(e*11i + f*x*11i)*1638i - 910*exp(e...
```

Reduce [F]

$$\int \frac{(a + a\sin(e + fx))^{7/2}(A + B\sin(e + fx))}{(c - c\sin(e + fx))^{15/2}}\,dx = \text{Too large to display}$$

input
```
int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(15/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a**3*(int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) +
1)*sin(e + f*x)**4)/(sin(e + f*x)**8 - 8*sin(e + f*x)**7 + 28*sin(e + f*x)
**6 - 56*sin(e + f*x)**5 + 70*sin(e + f*x)**4 - 56*sin(e + f*x)**3 + 28*si
n(e + f*x)**2 - 8*sin(e + f*x) + 1),x)*b + int((sqrt(sin(e + f*x) + 1)*sqr
t( - sin(e + f*x) + 1)*sin(e + f*x)**3)/(sin(e + f*x)**8 - 8*sin(e + f*x)*
*7 + 28*sin(e + f*x)**6 - 56*sin(e + f*x)**5 + 70*sin(e + f*x)**4 - 56*sin
(e + f*x)**3 + 28*sin(e + f*x)**2 - 8*sin(e + f*x) + 1),x)*a + 3*int((sqrt
(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**3)/(sin(e + f*x
)**8 - 8*sin(e + f*x)**7 + 28*sin(e + f*x)**6 - 56*sin(e + f*x)**5 + 70*si
n(e + f*x)**4 - 56*sin(e + f*x)**3 + 28*sin(e + f*x)**2 - 8*sin(e + f*x) +
 1),x)*b + 3*int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e +
 f*x)**2)/(sin(e + f*x)**8 - 8*sin(e + f*x)**7 + 28*sin(e + f*x)**6 - 56*s
in(e + f*x)**5 + 70*sin(e + f*x)**4 - 56*sin(e + f*x)**3 + 28*sin(e + f*x)
**2 - 8*sin(e + f*x) + 1),x)*a + 3*int((sqrt(sin(e + f*x) + 1)*sqrt( - sin
(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x)**8 - 8*sin(e + f*x)**7 + 28*
sin(e + f*x)**6 - 56*sin(e + f*x)**5 + 70*sin(e + f*x)**4 - 56*sin(e + f*x
)**3 + 28*sin(e + f*x)**2 - 8*sin(e + f*x) + 1),x)*b + 3*int((sqrt(sin(e +
 f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**8 - 8*si
n(e + f*x)**7 + 28*sin(e + f*x)**6 - 56*sin(e + f*x)**5 + 70*sin(e + f*x)*
*4 - 56*sin(e + f*x)**3 + 28*sin(e + f*x)**2 - 8*sin(e + f*x) + 1),x)*a...
```

3.173 $\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{17/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 246

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{17/2}}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{16f(c-c\sin(e+fx))^{17/2}} + \frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{56cf(c-c\sin(e+fx))^{15/2}} + \frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{224c^2f(c-c\sin(e+fx))^{13/2}} + \frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{1120c^3f(c-c\sin(e+fx))^{11/2}} + \frac{(A-3B)\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8960c^4f(c-c\sin(e+fx))^{9/2}}$$

output
```
1/16*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(17/2)+1/5
6*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c/f/(c-c*sin(f*x+e))^(15/2)+1/
224*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c^2/f/(c-c*sin(f*x+e))^(13/2
)+1/1120*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c^3/f/(c-c*sin(f*x+e))^
(11/2)+1/8960*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c^4/f/(c-c*sin(f*x
+e))^(9/2)
```

Mathematica [A] (verified)

Time = 17.41 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.77

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{17/2}}\,dx = \frac{(A+B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\mathrm{si}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e}$$

$$-\frac{4(3A+5B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(a(1+\sin(e+fx)))^{7/2}}{7f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{17/2}}$$

$$+\frac{(A+3B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(a(1+\sin(e+fx)))^{7/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{17/2}}$$

$$+\frac{(-A-7B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(a(1+\sin(e+fx)))^{7/2}}{5f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{17/2}}$$

$$+\frac{B\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9(a(1+\sin(e+fx)))^{7/2}}{4f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{17/2}}$$

input
```
Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
 f*x])^(17/2),x]
```

output
```
((A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/2
))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(17/2))
 - (4*(3*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e +
f*x]))^(7/2))/(7*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e +
f*x])^(17/2)) + ((A + 3*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 +
 Sin[e + f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*S
in[e + f*x])^(17/2)) + ((-A - 7*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7
*(a*(1 + Sin[e + f*x]))^(7/2))/(5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
7*(c - c*Sin[e + f*x])^(17/2)) + (B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
9*(a*(1 + Sin[e + f*x]))^(7/2))/(4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
^7*(c - c*Sin[e + f*x])^(17/2))
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3451, 3042, 3222, 3042, 3222, 3042, 3222, 3042, 3221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{17/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{17/2}}dx$$

$$\Big\downarrow \text{3451}$$

$$\frac{(A-3B)\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{15/2}}dx}{4c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{16f(c-c\sin(e+fx))^{17/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-3B)\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{15/2}}dx}{4c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{16f(c-c\sin(e+fx))^{17/2}}$$

$$\Big\downarrow \text{3222}$$

$$\frac{(A-3B)\left(\frac{3\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{13/2}}dx}{14c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{14f(c-c\sin(e+fx))^{15/2}}\right)}{4c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{16f(c-c\sin(e+fx))^{17/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-3B)\left(\frac{3\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{13/2}}dx}{14c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{14f(c-c\sin(e+fx))^{15/2}}\right)}{4c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{16f(c-c\sin(e+fx))^{17/2}}$$

$$\Big\downarrow \text{3222}$$

$$\frac{(A-3B)\left(\frac{3\left(\frac{\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{11/2}}dx}{6c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{12f(c-c\sin(e+fx))^{13/2}}\right)}{14c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{14f(c-c\sin(e+fx))^{15/2}}\right)}{4c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{16f(c-c\sin(e+fx))^{17/2}}$$

$\Big\downarrow$ 3042

$$\frac{(A-3B)\left(\frac{3\left(\frac{\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{11/2}}dx}{6c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{12f(c-c\sin(e+fx))^{13/2}}\right)}{14c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{14f(c-c\sin(e+fx))^{15/2}}\right)}{4c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{16f(c-c\sin(e+fx))^{17/2}}$$

$\Big\downarrow$ 3222

$$\frac{(A-3B)\left(\frac{3\left(\frac{\frac{\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{9/2}}dx}{10c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f(c-c\sin(e+fx))^{11/2}}}{6c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{12f(c-c\sin(e+fx))^{13/2}}\right)}{14c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{14f(c-c\sin(e+fx))^{15/2}}\right)}{4c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{16f(c-c\sin(e+fx))^{17/2}}$$

$\Big\downarrow$ 3042

$$\frac{(A-3B)\left(\frac{3\left(\frac{\frac{\int \frac{(\sin(e+fx)a+a)^{7/2}}{(c-c\sin(e+fx))^{9/2}}dx}{10c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f(c-c\sin(e+fx))^{11/2}}}{6c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{12f(c-c\sin(e+fx))^{13/2}}\right)}{14c}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{14f(c-c\sin(e+fx))^{15/2}}\right)}{4c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{16f(c-c\sin(e+fx))^{17/2}}$$

$\downarrow$ 3221

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{16f(c-c\sin(e+fx))^{17/2}}+\frac{(A-3B)\left(\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{14f(c-c\sin(e+fx))^{15/2}}+\frac{3\left(\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{12f(c-c\sin(e+fx))^{13/2}}+\frac{\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{80cf(c-c\sin(e+fx))^{9/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10f(c-c\sin(e+fx))^{11/2}}}{6c}\right)}{14c}\right)}{4c}$$

input
```
Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(17/2),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(16*f*(c - c*Sin[e + f*x
])^(17/2)) + ((A - 3*B)*((Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(14*f*(
c - c*Sin[e + f*x])^(15/2)) + (3*((Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)
)/(12*f*(c - c*Sin[e + f*x])^(13/2)) + ((Cos[e + f*x]*(a + a*Sin[e + f*x])
^(7/2))/(10*f*(c - c*Sin[e + f*x])^(11/2)) + (Cos[e + f*x]*(a + a*Sin[e +
f*x])^(7/2))/(80*c*f*(c - c*Sin[e + f*x])^(9/2)))/(6*c)))/(14*c)))/(4*c)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3221
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

rule 3222
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp[(m + n + 1)/(a*(2*m + 1)
)   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
 ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||
!SumSimplerQ[n, 1])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. 2(216) = 432.

Time = 15.05 (sec) , antiderivative size = 574, normalized size of antiderivative = 2.33

method	result
parts	$\frac{A\sqrt{4}\,\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{8}+4\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{6}+10\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}+20\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}+35\right)a^{3}\tan\left(\frac{\pi}{4}+\frac{f}{2}\right.}{17920f\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,c^{8}}$
default	Expression too large to display

input
```
int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(17/2),x,meth
od=_RETURNVERBOSE)
```

output
```
1/17920*A/f*4^(1/2)*(a*sin(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)*(cos(1/4*Pi+1/2*
f*x+1/2*e)^8+4*cos(1/4*Pi+1/2*f*x+1/2*e)^6+10*cos(1/4*Pi+1/2*f*x+1/2*e)^4+
20*cos(1/4*Pi+1/2*f*x+1/2*e)^2+35)*a^3/(c*cos(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/
2)/c^8*tan(1/4*Pi+1/2*f*x+1/2*e)^7*sec(1/4*Pi+1/2*f*x+1/2*e)^8-2/35*B/f*((
2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+1)*a)^(1/2)*cos(1/2*f*x+1/2*e)^2*(
(128*cos(1/2*f*x+1/2*e)^9-256*cos(1/2*f*x+1/2*e)^7-96*cos(1/2*f*x+1/2*e)^5
+224*cos(1/2*f*x+1/2*e)^3)*sin(1/2*f*x+1/2*e)+32*cos(1/2*f*x+1/2*e)^12-96*
cos(1/2*f*x+1/2*e)^10-128*cos(1/2*f*x+1/2*e)^8+416*cos(1/2*f*x+1/2*e)^6-14
*cos(1/2*f*x+1/2*e)^4-210*cos(1/2*f*x+1/2*e)^2-35)*sin(1/2*f*x+1/2*e)^2*a^
3/(-1+2*cos(1/2*f*x+1/2*e)^2)/((128*cos(1/2*f*x+1/2*e)^13-384*cos(1/2*f*x+
1/2*e)^11-288*cos(1/2*f*x+1/2*e)^9+1216*cos(1/2*f*x+1/2*e)^7-392*cos(1/2*f
*x+1/2*e)^5-280*cos(1/2*f*x+1/2*e)^3-14*cos(1/2*f*x+1/2*e))*sin(1/2*f*x+1/
2*e)-448*cos(1/2*f*x+1/2*e)^12+1344*cos(1/2*f*x+1/2*e)^10-784*cos(1/2*f*x+
1/2*e)^8-672*cos(1/2*f*x+1/2*e)^6+476*cos(1/2*f*x+1/2*e)^4+84*cos(1/2*f*x+
1/2*e)^2+1)/(-(2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-1)*c)^(1/2)/c^8
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.99

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{17/2}}\,dx = \frac{\left(35\,Ba^3\cos\left(fx+e\right)^4-56\,(A+2\,B\right.}{140\,\left(c^9 f\cos\left(fx+e\right)^9-32\,c^9 f\cos\left(fx+e\right)^7+160\,c^9 f\right.}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(17/2),
x, algorithm="fricas")
```

output
```
1/140*(35*B*a^3*cos(f*x + e)^4 - 56*(A + 2*B)*a^3*cos(f*x + e)^2 + 4*(17*A
 + 19*B)*a^3 - 4*(7*(A + 2*B)*a^3*cos(f*x + e)^2 - 2*(9*A + 8*B)*a^3)*sin(
f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^9*f*cos(f*
x + e)^9 - 32*c^9*f*cos(f*x + e)^7 + 160*c^9*f*cos(f*x + e)^5 - 256*c^9*f*
cos(f*x + e)^3 + 128*c^9*f*cos(f*x + e) + 8*(c^9*f*cos(f*x + e)^7 - 10*c^9
*f*cos(f*x + e)^5 + 24*c^9*f*cos(f*x + e)^3 - 16*c^9*f*cos(f*x + e))*sin(f
*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{17/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(17/2
),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{17/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(17/2),
x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{17/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(17/2),
x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 54.77 (sec) , antiderivative size = 841, normalized size of antiderivative = 3.42

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{17/2}}\,dx = \text{Too large to display}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))
^(17/2),x)
```

output
```
((c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(
(8*B*a^3*exp(e*5i + f*x*5i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1
i + f*x*1i)*1i)/2))^(1/2))/(c^9*f) + (8*B*a^3*exp(e*13i + f*x*13i)*(a + a*
((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2))/(c^9*f)
- (64*a^3*exp(e*6i + f*x*6i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*
1i + f*x*1i)*1i)/2))^(1/2)*(A*1i + B*2i))/(5*c^9*f) - (32*a^3*exp(e*7i + f
*x*7i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(
1/2)*(8*A + 11*B))/(5*c^9*f) + (64*a^3*exp(e*12i + f*x*12i)*(a + a*((exp(-
 e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(A*1i + B*2i))/(
5*c^9*f) - (32*a^3*exp(e*11i + f*x*11i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/
2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(8*A + 11*B))/(5*c^9*f) + (64*a^3*ex
p(e*8i + f*x*8i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)
*1i)/2))^(1/2)*(A*13i + B*10i))/(7*c^9*f) - (64*a^3*exp(e*10i + f*x*10i)*(
a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(A*
13i + B*10i))/(7*c^9*f) + (16*a^3*exp(e*9i + f*x*9i)*(a + a*((exp(- e*1i -
 f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(64*A + 53*B))/(7*c^9*f
)))/(exp(e*1i + f*x*1i)*16i - 119*exp(e*2i + f*x*2i) - exp(e*3i + f*x*3i)*
544i + 1700*exp(e*4i + f*x*4i) + exp(e*5i + f*x*5i)*3808i - 6188*exp(e*6i
+ f*x*6i) - exp(e*7i + f*x*7i)*7072i + 4862*exp(e*8i + f*x*8i) + 4862*exp(
e*10i + f*x*10i) + exp(e*11i + f*x*11i)*7072i - 6188*exp(e*12i + f*x*12...
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{17/2}}\,dx = \text{Too large to display}$$

input
```
int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(17/2),x)
```

output
```
(sqrt(c)*sqrt(a)*a**3*( - int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x)
 + 1)*sin(e + f*x)**4)/(sin(e + f*x)**9 - 9*sin(e + f*x)**8 + 36*sin(e + f
*x)**7 - 84*sin(e + f*x)**6 + 126*sin(e + f*x)**5 - 126*sin(e + f*x)**4 +
84*sin(e + f*x)**3 - 36*sin(e + f*x)**2 + 9*sin(e + f*x) - 1),x)*b - int((
sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**3)/(sin(e +
 f*x)**9 - 9*sin(e + f*x)**8 + 36*sin(e + f*x)**7 - 84*sin(e + f*x)**6 + 1
26*sin(e + f*x)**5 - 126*sin(e + f*x)**4 + 84*sin(e + f*x)**3 - 36*sin(e +
 f*x)**2 + 9*sin(e + f*x) - 1),x)*a - 3*int((sqrt(sin(e + f*x) + 1)*sqrt(
- sin(e + f*x) + 1)*sin(e + f*x)**3)/(sin(e + f*x)**9 - 9*sin(e + f*x)**8
+ 36*sin(e + f*x)**7 - 84*sin(e + f*x)**6 + 126*sin(e + f*x)**5 - 126*sin(
e + f*x)**4 + 84*sin(e + f*x)**3 - 36*sin(e + f*x)**2 + 9*sin(e + f*x) - 1
),x)*b - 3*int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f
*x)**2)/(sin(e + f*x)**9 - 9*sin(e + f*x)**8 + 36*sin(e + f*x)**7 - 84*sin
(e + f*x)**6 + 126*sin(e + f*x)**5 - 126*sin(e + f*x)**4 + 84*sin(e + f*x)
**3 - 36*sin(e + f*x)**2 + 9*sin(e + f*x) - 1),x)*a - 3*int((sqrt(sin(e +
f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x)**9 - 9*
sin(e + f*x)**8 + 36*sin(e + f*x)**7 - 84*sin(e + f*x)**6 + 126*sin(e + f*
x)**5 - 126*sin(e + f*x)**4 + 84*sin(e + f*x)**3 - 36*sin(e + f*x)**2 + 9*
sin(e + f*x) - 1),x)*b - 3*int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x
) + 1)*sin(e + f*x))/(sin(e + f*x)**9 - 9*sin(e + f*x)**8 + 36*sin(e + ...
```

3.174 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 197

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx=\frac{4(A-B)c^3\cos(e+fx)\log(1+\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{2(A-B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}}+\frac{(A-B)c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}}-\frac{B\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a+a\sin(e+fx)}}$$

output
```
4*(A-B)*c^3*cos(f*x+e)*ln(1+sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(
f*x+e))^(1/2)+2*(A-B)*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x
+e))^(1/2)+1/2*(A-B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e)
)^(1/2)-1/3*B*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/f/(a+a*sin(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 11.97 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.94

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}\,(3(A-3B)\cos(2(e+fx)}{12f\left(\cos\left(\frac{1}{2}\right.\right.}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/Sqrt[a + a*Sin
[e + f*x]],x]
```

output
```
-1/12*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^2*Sqr
t[c - c*Sin[e + f*x]]*(3*(A - 3*B)*Cos[2*(e + f*x)] - 96*A*Log[Cos[(e + f*
x)/2] + Sin[(e + f*x)/2]] + 96*B*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]
+ (36*A - 51*B)*Sin[e + f*x] + B*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2])^5*Sqrt[a*(1 + Sin[e + f*x])])
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.97, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.275$, Rules used = {3042, 3452, 3042, 3219, 3042, 3219, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{\sqrt{a\sin(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{\sqrt{a\sin(e+fx)+a}}dx$$

$$\Big\downarrow \text{3452}$$

$$(A-B)\int \frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{\sin(e+fx)a+a}}dx-\frac{B\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$(A-B)\int \frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{\sin(e+fx)a+a}}dx-\frac{B\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3219}$$

$$(A-B)\left(2c\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)-\frac{B\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$(A-B)\left(2c\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)-\frac{B\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3219}$$

$$(A-B)\left(2c\left(2c\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right.-\frac{B\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$(A-B)\left(2c\left(2c\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right.-\frac{B\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3216}$$

$$(A-B)\left(2c\left(\frac{2ac^2\cos(e+fx)\int \frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx)}{2f\sqrt{a\sin(e+f}}\right.-\frac{B\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$(A-B)\left(2c\left(\frac{2ac^2\cos(e+fx)\int \frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(}{2f\sqrt{a\sin(e+fx}\right.$$
$$\frac{B\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}$$

$\Big\downarrow$ 3146

$$(A-B)\left(2c\left(\frac{2c^2\cos(e+fx)\int \frac{1}{\sin(e+fx)a+a}d(a\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-}{2f\sqrt{a\sin(}\right.$$
$$\frac{B\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}$$

$\Big\downarrow$ 16

$$(A-B)\left(2c\left(\frac{2c^2\cos(e+fx)\log(a\sin(e+fx)+a)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin}{2f\sqrt{a\sin(e+f}\right.$$
$$\frac{B\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/Sqrt[a + a*Sin[e + f
*x]],x]
```

output
```
-1/3*(B*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(f*Sqrt[a + a*Sin[e + f*x
]]) + (A - B)*((c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(2*f*Sqrt[a + a
*Sin[e + f*x]]) + 2*c*((2*c^2*Cos[e + f*x]*Log[a + a*Sin[e + f*x]])/(f*Sqr
t[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (c*Cos[e + f*x]*Sqrt[c -
 c*Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]])))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3452
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 441 vs. $2(177) = 354$.

Time = 7.30 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.24

method	result
parts	$\frac{A\sqrt{4}\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^4+2\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2-4\ln\left(\frac{2}{\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1}\right)+4\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\right)-3\right)\sqrt{c\cos(}}{f\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2}}$
default	$-\frac{A\sqrt{4}\left(-\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^4-2\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2+4\ln\left(\frac{2}{\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1}\right)-4\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\right)+3\right)\sqrt{c}}{f\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2}}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
A/f*4^(1/2)*(cos(1/4*Pi+1/2*f*x+1/2*e)^4+2*cos(1/4*Pi+1/2*f*x+1/2*e)^2-4*l
n(2/(cos(1/4*Pi+1/2*f*x+1/2*e)+1))+4*ln(-cot(1/4*Pi+1/2*f*x+1/2*e)+csc(1/4
*Pi+1/2*f*x+1/2*e))-3)*(c*cos(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)*c^2/(a*sin(1/
4*Pi+1/2*f*x+1/2*e)^2)^(1/2)*tan(1/4*Pi+1/2*f*x+1/2*e)+2/3*B*c^2/f*((12*si
n(1/2*f*x+1/2*e)+12*cos(1/2*f*x+1/2*e))*ln(-2*(cos(1/2*f*x+1/2*e)+sin(1/2*
f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1))+(-12*sin(1/2*f*x+1/2*e)-12*cos(1/2*f*x
+1/2*e))*ln(2/(cos(1/2*f*x+1/2*e)+1))+cos(1/2*f*x+1/2*e)*((4*cos(1/2*f*x+1
/2*e)^4-4*cos(1/2*f*x+1/2*e)^3*sin(1/2*f*x+1/2*e)+5*cos(1/2*f*x+1/2*e)^2+9
*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-12)*sin(1/2*f*x+1/2*e)^2-12*sin(1/2
*f*x+1/2*e)*cos(1/2*f*x+1/2*e)))*(-(2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e
)-1)*c)^(1/2)/(sin(1/2*f*x+1/2*e)-cos(1/2*f*x+1/2*e))/((2*sin(1/2*f*x+1/2*
e)*cos(1/2*f*x+1/2*e)+1)*a)^(1/2)
```

Fricas [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx=\int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{5}{2}}}{\sqrt{a\sin(fx+e)+a}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x
, algorithm="fricas")
```

output
```
integral(-((A - 2*B)*c^2*cos(f*x + e)^2 - 2*(A - B)*c^2 + (B*c^2*cos(f*x +
 e)^2 + 2*(A - B)*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/sqrt(a*sin(
f*x + e) + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx=\text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(1/2)
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx=\int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{5}{2}}}{\sqrt{a\sin(fx+e)+a}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(5/2)/sqrt(a*sin(f*x
+ e) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))
^(1/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))
^(1/2), x)
```

Reduce [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,c^2\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^3}{\sin(fx+e)+1}dx\right)b+\right.}{}$$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x)
```

output
```
(sqrt(c)*sqrt(a)*c**2*(int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) +
1)*sin(e + f*x)**3)/(sin(e + f*x) + 1),x)*b + int((sqrt(sin(e + f*x) + 1)*
sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x) + 1),x)*a - 2*int
((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e
 + f*x) + 1),x)*b - 2*int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1
)*sin(e + f*x))/(sin(e + f*x) + 1),x)*a + int((sqrt(sin(e + f*x) + 1)*sqrt
( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x) + 1),x)*b + int((sqrt(si
n(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1))/(sin(e + f*x) + 1),x)*a))/a
```

3.175 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 146

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx=\frac{2(A-B)c^2\cos(e+fx)\log(1+\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{(A-B)c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}}-\frac{B\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a+a\sin(e+fx)}}$$

output
```
2*(A-B)*c^2*cos(f*x+e)*ln(1+sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(
f*x+e))^(1/2)+(A-B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))
^(1/2)-1/2*B*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 7.87 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx=$$

$$-\frac{c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))\sqrt{c-c\sin(e+fx)}\left(B\cos(2(e+fx))-4(4(-\right.}{4f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sqrt{a(}}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/Sqrt[a + a*Sin
[e + f*x]],x]
```

output
```
-1/4*(c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])*Sqrt[c -
 c*Sin[e + f*x]]*(B*Cos[2*(e + f*x)] - 4*(4*(-A + B)*Log[Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2]] + (A - 2*B)*Sin[e + f*x])))/(f*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])])
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.225$, Rules used = {3042, 3452, 3042, 3219, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{\sqrt{a\sin(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{\sqrt{a\sin(e+fx)+a}}dx$$

$$\Big\downarrow \text{3452}$$

$$(A-B)\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{\sin(e+fx)a+a}}dx-\frac{B\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$(A-B)\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{\sin(e+fx)a+a}}dx-\frac{B\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3219}$$

$$(A-B)\left(2c\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)-\frac{B\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$(A-B)\left(2c\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)-\frac{B\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3216}$$

$$(A-B)\left(\frac{2ac^2\cos(e+fx)\int \frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)-\frac{B\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$(A-B)\left(\frac{2ac^2\cos(e+fx)\int \frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)-\frac{B\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3146}$$

$$(A-B)\left(\frac{2c^2\cos(e+fx)\int \frac{1}{\sin(e+fx)a+a}d(a\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)-\frac{B\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{16}$$

$$(A-B)\left(\frac{2c^2\cos(e+fx)\log(a\sin(e+fx)+a)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)-\frac{B\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/Sqrt[a + a*Sin[e + f
*x]],x]
```

output
```
-1/2*(B*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(f*Sqrt[a + a*Sin[e + f*x
]]) + (A - B)*((2*c^2*Cos[e + f*x]*Log[a + a*Sin[e + f*x]])/(f*Sqrt[a + a*
Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (c*Cos[e + f*x]*Sqrt[c - c*Sin[e
 + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3452
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(132) = 264$.

Time = 7.09 (sec) , antiderivative size = 327, normalized size of antiderivative = 2.24

method	result
default	$-\frac{A\sqrt{4}\left(\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}+2\ln\left(\frac{2}{\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1}\right)-2\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\right)\right)\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,c\tan\left(\frac{\pi}{4}\right.}{f\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}}$
parts	$-\frac{A\sqrt{4}\left(\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}+2\ln\left(\frac{2}{\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1}\right)-2\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\right)\right)\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,c\tan\left(\frac{\pi}{4}\right.}{f\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
-A/f*4^(1/2)*(sin(1/4*Pi+1/2*f*x+1/2*e)^2+2*ln(2/(cos(1/4*Pi+1/2*f*x+1/2*e
)+1))-2*ln(-cot(1/4*Pi+1/2*f*x+1/2*e)+csc(1/4*Pi+1/2*f*x+1/2*e)))*(c*cos(1
/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)*c/(a*sin(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)*tan(
1/4*Pi+1/2*f*x+1/2*e)-2*B*c/f*(cos(1/2*f*x+1/2*e)+sin(1/2*f*x+1/2*e))*(cos
(1/2*f*x+1/2*e)^2*sin(1/2*f*x+1/2*e)^2-2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/
2*e)+2*ln(-2*(cos(1/2*f*x+1/2*e)+sin(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1
))-2*ln(2/(cos(1/2*f*x+1/2*e)+1)))*(-(2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2
*e)-1)*c)^(1/2)/(cos(1/2*f*x+1/2*e)-sin(1/2*f*x+1/2*e))/((2*sin(1/2*f*x+1/
2*e)*cos(1/2*f*x+1/2*e)+1)*a)^(1/2)
```

Fricas [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx=\int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{3}{2}}}{\sqrt{a\sin(fx+e)+a}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x
, algorithm="fricas")
```

output
```
integral((B*c*cos(f*x + e)^2 - (A - B)*c*sin(f*x + e) + (A - B)*c)*sqrt(-c
*sin(f*x + e) + c)/sqrt(a*sin(f*x + e) + a), x)
```

Sympy [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx=\int \frac{(-c(\sin(e+fx)-1))^{\frac{3}{2}}(A+B\sin(e+fx))}{\sqrt{a(\sin(e+fx)+1)}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(1/2)
,x)
```

output
```
Integral((-c*(sin(e + f*x) - 1))**(3/2)*(A + B*sin(e + f*x))/sqrt(a*(sin(e
 + f*x) + 1)), x)
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx=\int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{3}{2}}}{\sqrt{a\sin(fx+e)+a}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(3/2)/sqrt(a*sin(f*x
+ e) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,\left(c-c\,\sin(e+f\,x)\right)^{3/2}}{\sqrt{a+a\,\sin(e+f\,x)}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))
^(1/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))
^(1/2), x)
```

Reduce [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,c\left(-\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^2}{\sin(fx+e)+1}dx\right)b$$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x)
```

output
```
(sqrt(c)*sqrt(a)*c*( - int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) +
1)*sin(e + f*x)**2)/(sin(e + f*x) + 1),x)*b - int((sqrt(sin(e + f*x) + 1)*
sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x) + 1),x)*a + int((sqr
t(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)
+ 1),x)*b + int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1))/(sin(e
+ f*x) + 1),x)*a))/a
```

3.176 $\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 96

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{(A-B)c\cos(e+fx)\log(1+\sin(e+fx))}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{B\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a+a\sin(e+fx)}}$$

output
```
(A-B)*c*cos(f*x+e)*ln(1+sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+
e))^(1/2)-B*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.32 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.24

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left((A-B)\left(-ifx+2\log\left(i+e^{i(e+fx)}\right)\right)+B\sin(e+fx)\right)\sqrt{c-c\sin}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}}$$

input
```
Integrate[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/Sqrt[a + a*Sin[e
 + f*x]],x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((A - B)*((-I)*f*x + 2*Log[I + E^(I
*(e + f*x))]) + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(f*(Cos[(e + f*x
)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])])
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3450, 3042, 3216, 3042, 3146, 16, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-c\sin(e+fx)}(A+B\sin(e+fx))}{\sqrt{a\sin(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{c-c\sin(e+fx)}(A+B\sin(e+fx))}{\sqrt{a\sin(e+fx)+a}}dx$$

$$\Big\downarrow \text{3450}$$

$$(A-B)\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{B\int \sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$(A-B)\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{B\int \sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}dx}{a}$$

$$\Big\downarrow \text{3216}$$

$$\frac{ac(A-B)\cos(e+fx)\int \frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{B\int \sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{ac(A-B)\cos(e+fx)\int\frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{B\int\sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}dx}{a}$$

$$\Big\downarrow \text{3146}$$

$$\frac{c(A-B)\cos(e+fx)\int\frac{1}{\sin(e+fx)a+a}d(a\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{B\int\sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}dx}{a}$$

$$\Big\downarrow \text{16}$$

$$\frac{B\int\sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}dx}{a}+\frac{c(A-B)\cos(e+fx)\log(a\sin(e+fx)+a)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{3217}$$

$$\frac{c(A-B)\cos(e+fx)\log(a\sin(e+fx)+a)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{B\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}$$

input
```
Int[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/Sqrt[a + a*Sin[e + f*x
]],x]
```

output
```
((A - B)*c*Cos[e + f*x]*Log[a + a*Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x
]]*Sqrt[c - c*Sin[e + f*x]]) - (B*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(
f*Sqrt[a + a*Sin[e + f*x]])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3450
```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[B/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] -
Simp[(B*c - A*d)/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[
a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(88) = 176$.

Time = 5.34 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.95

method	result
default	$-\frac{A\sqrt{4}\left(\ln\left(\frac{2}{\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1}\right)-\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\right)\right)\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)}{f\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}}-\frac{2B\left(\cos\left(\right.\right.}{}$
parts	$-\frac{A\sqrt{4}\left(\ln\left(\frac{2}{\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1}\right)-\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\right)\right)\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)}{f\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}}-\frac{2B\left(\cos\left(\right.\right.}{}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
-A/f*4^(1/2)*(ln(2/(cos(1/4*Pi+1/2*f*x+1/2*e)+1))-ln(-cot(1/4*Pi+1/2*f*x+1
/2*e)+csc(1/4*Pi+1/2*f*x+1/2*e)))*(c*cos(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)/(a
*sin(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)*tan(1/4*Pi+1/2*f*x+1/2*e)-2*B/f*(cos(1
/2*f*x+1/2*e)+sin(1/2*f*x+1/2*e))*(sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+l
n(2/(cos(1/2*f*x+1/2*e)+1))-ln(-2*(cos(1/2*f*x+1/2*e)+sin(1/2*f*x+1/2*e))/
(cos(1/2*f*x+1/2*e)+1)))*(-(2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-1)*c)^
(1/2)/(sin(1/2*f*x+1/2*e)-cos(1/2*f*x+1/2*e))/((2*sin(1/2*f*x+1/2*e)*cos(1
/2*f*x+1/2*e)+1)*a)^(1/2)
```

Fricas [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)\sqrt{-c\sin(fx+e)+c}}{\sqrt{a\sin(fx+e)+a}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x
, algorithm="fricas")
```

output
```
integral((B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)/sqrt(a*sin(f*x + e
) + a), x)
```

Sympy [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{\sqrt{-c(\sin(e+fx)-1)}(A+B\sin(e+fx))}{\sqrt{a(\sin(e+fx)+1)}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(1/2)
,x)
```

output
```
Integral(sqrt(-c*(sin(e + f*x) - 1))*(A + B*sin(e + f*x))/sqrt(a*(sin(e +
f*x) + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.83

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{B\left(\frac{2\sqrt{c}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{\sqrt{a}} - \frac{\sqrt{c}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{\sqrt{a}} - \frac{2\sqrt{a}\sqrt{c}\sin(fx+e)}{\left(a+\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}\right) - A\left(\frac{2\sqrt{c}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{\sqrt{a}}\right.}{f}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x
, algorithm="maxima")
```

output
```
(B*(2*sqrt(c)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/sqrt(a) - sqrt(c)*l
og(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/sqrt(a) - 2*sqrt(a)*sqrt(c)*si
n(f*x + e)/((a + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)
)) - A*(2*sqrt(c)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/sqrt(a) - sqrt(
c)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/sqrt(a)))/f
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,\sqrt{c-c\,\sin(e+f\,x)}}{\sqrt{a+a\,\sin(e+f\,x)}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))
^(1/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))
^(1/2), x)
```

Reduce [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx$$
$$=\frac{\sqrt{c}\,\sqrt{a}\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)+1}dx\right)b+\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)+1}dx\right)a\right)}{a}$$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x)
```

output
```
(sqrt(c)*sqrt(a)*(int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*si
n(e + f*x))/(sin(e + f*x) + 1),x)*b + int((sqrt(sin(e + f*x) + 1)*sqrt( -
sin(e + f*x) + 1))/(sin(e + f*x) + 1),x)*a))/a
```

3.177 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 113

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{(A+B)\cos(e+fx)\log(1-\sin(e+fx))}{2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{(A-B)\cos(e+fx)\log(1+\sin(e+fx))}{2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

output
```
-1/2*(A+B)*cos(f*x+e)*ln(1-sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f
*x+e))^(1/2)+1/2*(A-B)*cos(f*x+e)*ln(1+sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2
)/(c-c*sin(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 1.87 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.73

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{\cos(e+fx)\left(B\log(\cos(e+fx)) + A\left(\log\left(1-\tan\left(\frac{1}{2}(e+fx)\right)\right) - \log\left(1+\tan\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)}{f\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}}$$

input
```
Integrate[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e
+ f*x]]),x]
```

output
```
-((Cos[e + f*x]*(B*Log[Cos[e + f*x]] + A*(Log[1 - Tan[(e + f*x)/2]] - Log[
1 + Tan[(e + f*x)/2]])))/(f*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e +
f*x]]))
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {3042, 3448, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3448}$$

$$\frac{(A+B)\int \frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx}{2a}+\frac{(A-B)\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx}{2c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A+B)\int \frac{\sqrt{\sin(e+fx)a+a}}{\sqrt{c-c\sin(e+fx)}}dx}{2a}+\frac{(A-B)\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx}{2c}$$

$$\Big\downarrow \text{3216}$$

$$\frac{a(A-B)\cos(e+fx)\int \frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{2\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c(A+B)\cos(e+fx)\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{2\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(A-B)\cos(e+fx)\int \frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{2\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c(A+B)\cos(e+fx)\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}dx}{2\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{3146}$$

$$\frac{(A-B)\cos(e+fx)\int \frac{1}{\sin(e+fx)a+a}d(a\sin(e+fx))}{2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{(A+B)\cos(e+fx)\int \frac{1}{c-c\sin(e+fx)}d(-c\sin(e+fx))}{2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{16}$$

$$\frac{(A-B)\cos(e+fx)\log(a\sin(e+fx)+a)}{2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{(A+B)\cos(e+fx)\log(c-c\sin(e+fx))}{2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

input
```
Int[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]
]),x]
```

output
```
((A - B)*Cos[e + f*x]*Log[a + a*Sin[e + f*x]])/(2*f*Sqrt[a + a*Sin[e + f*x
]]*Sqrt[c - c*Sin[e + f*x]]) - ((A + B)*Cos[e + f*x]*Log[c - c*Sin[e + f*x
]])/(2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3448
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp
[(A*b + a*B)/(2*a*b)   Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]
], x], x] + Simp[(B*c + A*d)/(2*c*d)   Int[Sqrt[c + d*Sin[e + f*x]]/Sqrt[a
+ b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(101) = 202$.

Time = 4.67 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.88

method	result
default	$-\frac{A\sqrt{4}\,\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\left(\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)-1\right)-\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\right)+\ln\left(-\cot\left(\frac{\pi}{4}+\right.\right.\right.}{2f\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}}$
parts	$-\frac{A\sqrt{4}\,\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\left(\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)-1\right)-\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\right)+\ln\left(-\cot\left(\frac{\pi}{4}+\right.\right.\right.}{2f\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
-1/2*A/f*4^(1/2)*sin(1/4*Pi+1/2*f*x+1/2*e)*(ln(-cot(1/4*Pi+1/2*f*x+1/2*e)+
csc(1/4*Pi+1/2*f*x+1/2*e)-1)-ln(-cot(1/4*Pi+1/2*f*x+1/2*e)+csc(1/4*Pi+1/2*
f*x+1/2*e))+ln(-cot(1/4*Pi+1/2*f*x+1/2*e)+csc(1/4*Pi+1/2*f*x+1/2*e)+1))*co
s(1/4*Pi+1/2*f*x+1/2*e)/(a*sin(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)/(c*cos(1/4*P
i+1/2*f*x+1/2*e)^2)^(1/2)-B/f*(ln(-2*(cos(1/2*f*x+1/2*e)-sin(1/2*f*x+1/2*e
))/(cos(1/2*f*x+1/2*e)+1))-2*ln(2/(cos(1/2*f*x+1/2*e)+1))+ln(-2*(cos(1/2*f
*x+1/2*e)+sin(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1)))*(-1+2*cos(1/2*f*x+1
/2*e)^2)/((2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+1)*a)^(1/2)/(-(2*sin(1/
2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-1)*c)^(1/2)
```

Fricas [F]

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{B\sin\left(fx+e\right)+A}{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x
, algorithm="fricas")
```

output
```
integral((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)/(a*c*cos(f*x + e)^2), x)
```

Sympy [F]

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{A+B\sin\left(e+fx\right)}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(1/2)
,x)
```

output
```
Integral((A + B*sin(e + f*x))/(sqrt(a*(sin(e + f*x) + 1))*sqrt(-c*(sin(e +
 f*x) - 1))), x)
```

Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx$$
$$=\int \frac{B\sin\left(fx+e\right)+A}{\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/(sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
 e) + c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{A+B\,\sin\left(e+f\,x\right)}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^
(1/2)),x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^
(1/2)), x)
```

Reduce [F]

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{\sqrt{c}\,\sqrt{a}\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)^2-1}dx\right)b+\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)^2-1}dx\right)a\right)}{ac}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x)
```

output
```
( - sqrt(c)*sqrt(a)*(int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)
*sin(e + f*x))/(sin(e + f*x)**2 - 1),x)*b + int((sqrt(sin(e + f*x) + 1)*sq
rt( - sin(e + f*x) + 1))/(sin(e + f*x)**2 - 1),x)*a))/(a*c)
```

3.178 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 103

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}\,dx = \frac{(A+B)\cos(e+fx)}{2f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} + \frac{(A-B)\operatorname{arctanh}(\sin(e+fx))\cos(e+fx)}{2cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

output
```
1/2*(A+B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2)+1/2*(
A-B)*arctanh(sin(f*x+e))*cos(f*x+e)/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*
x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 3.83 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.85

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\left(A+B+(-A+B)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+\right.\right.\right.}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*
x])^(3/2)),x]
```

output
```
((A + B + (-A + B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])^2 + (A - B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2
]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2)*(Cos[(e + f*x)/2] - Sin[(e + f
*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/(2*f*Sqrt[a*(1 + Sin[e + f*
x])]*(c - c*Sin[e + f*x])^(3/2))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3042, 3451, 3042, 3220, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{ 3451}$$

$$\frac{(A-B)\int \frac{1}{\sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}}dx}{2c}+\frac{(A+B)\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(A-B)\int \frac{1}{\sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}}dx}{2c}+\frac{(A+B)\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 3220}$$

$$\frac{(A-B)\cos(e+fx)\int \sec(e+fx)dx}{2c\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(A-B)\cos(e+fx)\int \csc\left(e+fx+\frac{\pi}{2}\right)dx}{2c\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{(A-B)\cos(e+fx)\text{arctanh}(\sin(e+fx))}{2cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}$$

input
```
Int[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3
/2)),x]
```

output
```
((A + B)*Cos[e + f*x])/(2*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^
(3/2)) + ((A - B)*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*c*f*Sqrt[a + a*Si
n[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3220
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]])   Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 517 vs. $2(91) = 182$.

Time = 7.97 (sec) , antiderivative size = 518, normalized size of antiderivative = 5.03

method	result
default	$\frac{A\sqrt{4}\left(2\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\right)\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}-2\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)-1\right)\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right.}{8f\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\sqrt{c\cos\left(\frac{\pi}{4}\right.}}$
parts	$\frac{A\sqrt{4}\left(2\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\right)\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}-2\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)-1\right)\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right.}{8f\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\sqrt{c\cos\left(\frac{\pi}{4}\right.}}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/8*A/f*4^(1/2)*(2*ln(-cot(1/4*Pi+1/2*f*x+1/2*e)+csc(1/4*Pi+1/2*f*x+1/2*e)
)*cos(1/4*Pi+1/2*f*x+1/2*e)^2-2*ln(-cot(1/4*Pi+1/2*f*x+1/2*e)+csc(1/4*Pi+1
/2*f*x+1/2*e)-1)*cos(1/4*Pi+1/2*f*x+1/2*e)^2-2*ln(-cot(1/4*Pi+1/2*f*x+1/2*
e)+csc(1/4*Pi+1/2*f*x+1/2*e)+1)*cos(1/4*Pi+1/2*f*x+1/2*e)^2+sin(1/4*Pi+1/2
*f*x+1/2*e)^2)/(a*sin(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)/(c*cos(1/4*Pi+1/2*f*x
+1/2*e)^2)^(1/2)/c*tan(1/4*Pi+1/2*f*x+1/2*e)+1/2*B/c/f*(((1-2*cos(1/2*f*x+
1/2*e)^2)*sin(1/2*f*x+1/2*e)+cos(1/2*f*x+1/2*e)*(-1+2*cos(1/2*f*x+1/2*e)^2
))*ln(-2*(cos(1/2*f*x+1/2*e)+sin(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1))+(
(-1+2*cos(1/2*f*x+1/2*e)^2)*sin(1/2*f*x+1/2*e)-cos(1/2*f*x+1/2*e)*(-1+2*co
s(1/2*f*x+1/2*e)^2))*ln(2*(sin(1/2*f*x+1/2*e)-cos(1/2*f*x+1/2*e))/(cos(1/2
*f*x+1/2*e)+1))+cos(1/2*f*x+1/2*e)*(-2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*
e)-2*sin(1/2*f*x+1/2*e)^2))/(sin(1/2*f*x+1/2*e)-cos(1/2*f*x+1/2*e))/((2*si
n(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+1)*a)^(1/2)/(-(2*sin(1/2*f*x+1/2*e)*co
s(1/2*f*x+1/2*e)-1)*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 335, normalized size of antiderivative = 3.25

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}\,dx = \left[-\frac{((A-B)\cos(fx+e)\sin(fx+e)-(A-B)\cos(f\dots}{\dots}\right.$$

$$-\frac{((A-B)\cos(fx+e)\sin(fx+e)-(A-B)\cos(fx+e))\sqrt{-ac}\arctan\left(\frac{\sqrt{-ac}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+\dots}}{ac\cos(fx+e)}\right.}{2\,(ac^2f\cos(fx+e)\sin(fx+e)-ac^2f\cos(f\dots}$$

input

```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x
, algorithm="fricas")
```

output

```
[-1/4*(((A - B)*cos(f*x + e)*sin(f*x + e) - (A - B)*cos(f*x + e))*sqrt(a*c
)*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) + 2*sqrt(a*c)*sqrt(a*sin(f
*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3) + 2*s
qrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*(A + B))/(a*c^2*f*cos(f*
x + e)*sin(f*x + e) - a*c^2*f*cos(f*x + e)), -1/2*(((A - B)*cos(f*x + e)*s
in(f*x + e) - (A - B)*cos(f*x + e))*sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt(a*si
n(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e)/(a*c*cos(f*x + e)))
 + sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*(A + B))/(a*c^2*f*co
s(f*x + e)*sin(f*x + e) - a*c^2*f*cos(f*x + e))]
```

Sympy [F]

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{A+B\sin(e+fx)}{\sqrt{a(\sin(e+fx)+1)}\,(-c(\sin(e+fx)-1))^{\frac{3}{2}}}\,dx$$

input

```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(3/2)
,x)
```

output

```
Integral((A + B*sin(e + f*x))/(sqrt(a*(sin(e + f*x) + 1))*(-c*(sin(e + f*x
) - 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{B\sin(fx+e)+A}{\sqrt{a\sin(fx+e)+a}(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e)
+ c)^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{A+B\,\sin(e+f\,x)}{\sqrt{a+a\,\sin(e+f\,x)}\,(c-c\,\sin(e+f\,x))^{3/2}}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^
(3/2)),x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^
(3/2)), x)
```

Reduce [F]

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)^3-\sin(fx+e)^2-\sin(fx+e)+1}dx\right)b+\left(\int\right.\right.}{a\,c^2}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*(int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*si
n(e + f*x))/(sin(e + f*x)**3 - sin(e + f*x)**2 - sin(e + f*x) + 1),x)*b +
int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1))/(sin(e + f*x)**3 -
sin(e + f*x)**2 - sin(e + f*x) + 1),x)*a))/(a*c**2)
```

3.179 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 153

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}\,dx=\frac{(A+B)\cos(e+fx)}{4f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}+\frac{(A-B)\cos(e+fx)}{4cf\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}}+\frac{(A-B)\text{arctanh}(\sin(e+fx))\cos(e+fx)}{4c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

output
```
1/4*(A+B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2)+1/4*(
A-B)*cos(f*x+e)/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2)+1/4*(A-B
)*arctanh(sin(f*x+e))*cos(f*x+e)/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x
+e))^(1/2)
```

Mathematica [A] (verified)

Time = 4.14 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.45

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\left(A+B+(A-B)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\right.}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*
x])^(5/2)),x]
```

output
```
((A + B + (A - B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + (-A + B)*Log[C
os[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
4 + (A - B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])^4)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2]))/(4*f*Sqrt[a*(1 + Sin[e + f*x])]*(c - c*Sin[e + f*x]
)^(5/2))
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3451, 3042, 3222, 3042, 3220, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3451}$$

$$\frac{(A-B)\int \frac{1}{\sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{3/2}}dx}{2c}+\frac{(A+B)\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{(A-B)\int \frac{1}{\sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{3/2}}dx}{2c}+\frac{(A+B)\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}$$

$$\Big\downarrow \; 3222$$

$$\frac{(A-B)\left(\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}}dx}{2c}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}\right)}{2c}+\frac{(A+B)\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{(A-B)\left(\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}}dx}{2c}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}\right)}{2c}+\frac{(A+B)\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}$$

$$\Big\downarrow \; 3220$$

$$\frac{(A-B)\left(\frac{\cos(e+fx)\int \sec(e+fx)dx}{2c\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}\right)}{2c}+\frac{(A+B)\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{(A-B)\left(\frac{\cos(e+fx)\int \csc\left(e+fx+\frac{\pi}{2}\right)dx}{2c\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}\right)}{2c}+\frac{(A+B)\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}$$

$$\Big\downarrow \; 4257$$

$$\frac{(A-B)\left(\frac{\cos(e+fx)\text{arctanh}(\sin(e+fx))}{2cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}\right)}{2c}+\frac{(A+B)\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}$$

input
```
Int[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5
/2)),x]
```

output
```
((A + B)*Cos[e + f*x])/(4*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^
(5/2)) + ((A - B)*(Cos[e + f*x]/(2*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e
 + f*x])^(3/2)) + (ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*c*f*Sqrt[a + a*S
in[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])))/(2*c)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3220
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]])   Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

rule 3222
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp[(m + n + 1)/(a*(2*m + 1)
)   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
 ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||
!SumSimplerQ[n, 1])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 646 vs. $2(135) = 270$.

Time = 8.24 (sec) , antiderivative size = 647, normalized size of antiderivative = 4.23

method	result
default	$-\frac{A\sqrt{4}\,(4\cos(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2})^4\ln(-\cot(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2})+\csc(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2})+1)+4\cos(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2})^4\ln(-\cot(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2})+\csc(\frac{\pi}{4}+\frac{fx}{2}}{32f\sqrt{a\sin(}}$
parts	$-\frac{A\sqrt{4}\,(4\cos(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2})^4\ln(-\cot(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2})+\csc(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2})+1)+4\cos(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2})^4\ln(-\cot(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2})+\csc(\frac{\pi}{4}+\frac{fx}{2}}{32f\sqrt{a\sin(}}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x,metho
d=_RETURNVERBOSE)
```

output
```
-1/32*A/f*4^(1/2)*(4*cos(1/4*Pi+1/2*f*x+1/2*e)^4*ln(-cot(1/4*Pi+1/2*f*x+1/
2*e)+csc(1/4*Pi+1/2*f*x+1/2*e)+1)+4*cos(1/4*Pi+1/2*f*x+1/2*e)^4*ln(-cot(1/
4*Pi+1/2*f*x+1/2*e)+csc(1/4*Pi+1/2*f*x+1/2*e)-1)-4*ln(-cot(1/4*Pi+1/2*f*x+
1/2*e)+csc(1/4*Pi+1/2*f*x+1/2*e))*cos(1/4*Pi+1/2*f*x+1/2*e)^4+3*cos(1/4*Pi
+1/2*f*x+1/2*e)^4-2*cos(1/4*Pi+1/2*f*x+1/2*e)^2-1)/(a*sin(1/4*Pi+1/2*f*x+1
/2*e)^2)^(1/2)/(c*cos(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)/c^2*tan(1/4*Pi+1/2*f*
x+1/2*e)*sec(1/4*Pi+1/2*f*x+1/2*e)^2+1/4*B/c^2/f*(((2*cos(1/2*f*x+1/2*e)^2
+1)*(-1+2*cos(1/2*f*x+1/2*e)^2)*sin(1/2*f*x+1/2*e)+cos(1/2*f*x+1/2*e)*(2*c
os(1/2*f*x+1/2*e)^2-3)*(-1+2*cos(1/2*f*x+1/2*e)^2))*ln(2*(sin(1/2*f*x+1/2*
e)-cos(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1))+(-(2*cos(1/2*f*x+1/2*e)^2+1
)*(-1+2*cos(1/2*f*x+1/2*e)^2)*sin(1/2*f*x+1/2*e)-cos(1/2*f*x+1/2*e)*(2*cos
(1/2*f*x+1/2*e)^2-3)*(-1+2*cos(1/2*f*x+1/2*e)^2))*ln(-2*(cos(1/2*f*x+1/2*e
)+sin(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1))+cos(1/2*f*x+1/2*e)*(-2*sin(1
/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-2*sin(1/2*f*x+1/2*e)^2))/(2*cos(1/2*f*x+1
/2*e)^3+2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)^2-3*cos(1/2*f*x+1/2*e)+sin
(1/2*f*x+1/2*e))/((2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+1)*a)^(1/2)/(-(
2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-1)*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 422, normalized size of antiderivative = 2.76

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}\,dx = \left[-\frac{\left((A-B)\cos\left(fx+e\right)^3+2\,(A-B)\cos\left(fx+e\right)\sin\right.}{}\right.$$

$$-\frac{\left((A-B)\cos\left(fx+e\right)^3+2\,(A-B)\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,(A-B)\cos\left(fx+e\right)\right)\sqrt{-ac}\arctan\left(\right.}{4\left(ac^3f\cos\left(fx+e\right)^3+2\,ac^3f\cos\left(\right.\right.}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x
, algorithm="fricas")
```

output
```
[-1/8*(((A - B)*cos(f*x + e)^3 + 2*(A - B)*cos(f*x + e)*sin(f*x + e) - 2*(
A - B)*cos(f*x + e))*sqrt(a*c)*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x +
e) + 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*
x + e))/cos(f*x + e)^3) - 2*((A - B)*sin(f*x + e) - 2*A)*sqrt(a*sin(f*x +
e) + a)*sqrt(-c*sin(f*x + e) + c))/(a*c^3*f*cos(f*x + e)^3 + 2*a*c^3*f*cos
(f*x + e)*sin(f*x + e) - 2*a*c^3*f*cos(f*x + e)), -1/4*(((A - B)*cos(f*x +
 e)^3 + 2*(A - B)*cos(f*x + e)*sin(f*x + e) - 2*(A - B)*cos(f*x + e))*sqrt
(-a*c)*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c
)*sin(f*x + e)/(a*c*cos(f*x + e))) - ((A - B)*sin(f*x + e) - 2*A)*sqrt(a*s
in(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a*c^3*f*cos(f*x + e)^3 + 2*a*
c^3*f*cos(f*x + e)*sin(f*x + e) - 2*a*c^3*f*cos(f*x + e))]
```

Sympy [F]

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{A+B\sin\left(e+fx\right)}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(5/2)
,x)
```

output
```
Integral((A + B*sin(e + f*x))/(sqrt(a*(sin(e + f*x) + 1))*(-c*(sin(e + f*x
) - 1))**(5/2)), x)
```

Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{B\sin\left(fx+e\right)+A}{\sqrt{a\sin\left(fx+e\right)+a}(-c\sin\left(fx+e\right)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e)
+ c)^(5/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{A+B\,\sin\left(e+f\,x\right)}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^
(5/2)),x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^
(5/2)), x)
```

Reduce [F]

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}}\,dx = -\frac{\sqrt{c}\,\sqrt{a}\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)^4-2\sin(fx+e)^3+2\sin(fx+e)-1}dx\right)b+\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)^4-2\sin(fx+e)^3+2\sin(fx+e)-1}dx\right)a\right)}{a\,c^3}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x)
```

output
```
( - sqrt(c)*sqrt(a)*(int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)
*sin(e + f*x))/(sin(e + f*x)**4 - 2*sin(e + f*x)**3 + 2*sin(e + f*x) - 1),
x)*b + int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1))/(sin(e + f*x
)**4 - 2*sin(e + f*x)**3 + 2*sin(e + f*x) - 1),x)*a))/(a*c**3)
```

3.180 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 271

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{4(3A-5B)c^4\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{2(3A-5B)c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{(3A-5B)c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(3A-5B)c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{6af\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a+a\sin(e+fx))^{3/2}}$$

output
```
-4*(3*A-5*B)*c^4*cos(f*x+e)*ln(1+sin(f*x+e))/a/f/(a+a*sin(f*x+e))^(1/2)/(c
-c*sin(f*x+e))^(1/2)-2*(3*A-5*B)*c^3*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f
/(a+a*sin(f*x+e))^(1/2)-1/2*(3*A-5*B)*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2
)/a/f/(a+a*sin(f*x+e))^(1/2)-1/6*(3*A-5*B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(
5/2)/a/f/(a+a*sin(f*x+e))^(1/2)-1/2*(A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2
)/f/(a+a*sin(f*x+e))^(3/2)
```

Mathematica [A] (verified)

Time = 14.28 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx =$$
$$-\frac{c^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}(132A-45B+2(27A-59B)\cos(2(e+fx))$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e +
 f*x])^(3/2),x]
```

output
```
-1/24*(c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*
(132*A - 45*B + 2*(27*A - 59*B)*Cos[2*(e + f*x)] + B*Cos[4*(e + f*x)] + 57
6*A*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] - 960*B*Log[Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2]] - 117*A*Sin[e + f*x] + 279*B*Sin[e + f*x] + 576*A*Log[
Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[e + f*x] - 960*B*Log[Cos[(e + f*x
)/2] + Sin[(e + f*x)/2]]*Sin[e + f*x] - 3*A*Sin[3*(e + f*x)] + 13*B*Sin[3*
(e + f*x)]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]
))^(3/2))
```

Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.93, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.325$, Rules used = {3042, 3451, 3042, 3219, 3042, 3219, 3042, 3219, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3451}$$

$$-\frac{(3A-5B)\int \frac{(c-c\sin(e+fx))^{7/2}}{\sqrt{\sin(e+fx)a+a}}dx}{2a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{(3A-5B)\int \frac{(c-c\sin(e+fx))^{7/2}}{\sqrt{\sin(e+fx)a+a}}dx}{2a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \ 3219$$

$$-\frac{(3A-5B)\left(2c\int \frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}\right)}{2a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{(3A-5B)\left(2c\int \frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}\right)}{2a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \ 3219$$

$$-\frac{(3A-5B)\left(2c\left(2c\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}\right)}{2a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{(3A-5B)\left(2c\left(2c\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}\right)}{2a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \ 3219$$

$$-\frac{(3A-5B)\left(2c\left(2c\left(2c\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(}{3f\sqrt{a}}\right.}{2a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$\big\downarrow$ 3042

$$-\frac{(3A-5B)\left(2c\left(2c\left(2c\int\frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(}{3f\sqrt{a}}\right.}{2a}$$
$$\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$\big\downarrow$ 3216

$$-\frac{(3A-5B)\left(2c\left(2c\left(\frac{2ac^2\cos(e+fx)\int\frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\,c}{}\right.}{2a}$$
$$\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$\big\downarrow$ 3042

$$-\frac{(3A-5B)\left(2c\left(2c\left(\frac{2ac^2\cos(e+fx)\int\frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\,c}{}\right.}{2a}$$
$$\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$\big\downarrow$ 3146

$$-\frac{(3A-5B)\left(2c\left(2c\left(\frac{2c^2\cos(e+fx)\int\frac{1}{\sin(e+fx)a+a}d(a\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3}}{2f\sqrt{a\sin(e+fx)+a}}\right.\right.}{2a}$$
$$\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$\big\downarrow$ 16

$$-\frac{(3A-5B)\left(2c\left(2c\left(\frac{2c^2\cos(e+fx)\log(a\sin(e+fx)+a)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\,c}{}\right.}{2a}$$
$$\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])
^(3/2),x]
```

output
```
-1/2*((A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(f*(a + a*Sin[e + f
*x])^(3/2)) - ((3*A - 5*B)*((c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(3
*f*Sqrt[a + a*Sin[e + f*x]]) + 2*c*((c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(
3/2))/(2*f*Sqrt[a + a*Sin[e + f*x]]) + 2*c*((2*c^2*Cos[e + f*x]*Log[a + a*
Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (c*
Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]])))))/(2
*a)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

Maple [A] (verified)

Time = 7.97 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.77

method	result
default	$\frac{A\sqrt{4}\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{6}+3\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}+12\ln\left(\frac{2}{\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}-12\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}\right.\right.\right.}{f\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}}$
parts	$\frac{A\sqrt{4}\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{6}+3\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}+12\ln\left(\frac{2}{\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}-12\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}\right.\right.\right.}{f\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x,metho
d=_RETURNVERBOSE)
```

output
```
A/f*4^(1/2)*(cos(1/4*Pi+1/2*f*x+1/2*e)^6+3*cos(1/4*Pi+1/2*f*x+1/2*e)^4+12*
ln(2/(cos(1/4*Pi+1/2*f*x+1/2*e)+1))*sin(1/4*Pi+1/2*f*x+1/2*e)^2-12*ln(-cot
(1/4*Pi+1/2*f*x+1/2*e)+csc(1/4*Pi+1/2*f*x+1/2*e))*sin(1/4*Pi+1/2*f*x+1/2*e
)^2-10*cos(1/4*Pi+1/2*f*x+1/2*e)^2+4)*(c*cos(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2
)*c^3/(a*sin(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)/a*sec(1/4*Pi+1/2*f*x+1/2*e)*cs
c(1/4*Pi+1/2*f*x+1/2*e)+2/3*B/f*c^3/a*(60*(2*sin(1/2*f*x+1/2*e)*cos(1/2*f*
x+1/2*e)+1)*ln(-2*(cos(1/2*f*x+1/2*e)+sin(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2
*e)+1))+60*(-2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-1)*ln(2/(cos(1/2*f*x+
1/2*e)+1))+cos(1/2*f*x+1/2*e)*((8*cos(1/2*f*x+1/2*e)^5-8*cos(1/2*f*x+1/2*e
)^3+26*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)^2-57*cos(1/2*f*x+1/2*e))*sin(
1/2*f*x+1/2*e)^2-60*sin(1/2*f*x+1/2*e)))*(-(2*sin(1/2*f*x+1/2*e)*cos(1/2*f
*x+1/2*e)-1)*c)^(1/2)/(-1+2*cos(1/2*f*x+1/2*e)^2)/((2*sin(1/2*f*x+1/2*e)*c
os(1/2*f*x+1/2*e)+1)*a)^(1/2)
```

Fricas [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{7}{2}}}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x
, algorithm="fricas")
```

output
```
integral((B*c^3*cos(f*x + e)^4 + (3*A - 5*B)*c^3*cos(f*x + e)^2 - 4*(A - B
)*c^3 - ((A - 3*B)*c^3*cos(f*x + e)^2 - 4*(A - B)*c^3)*sin(f*x + e))*sqrt(
a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*
sin(f*x + e) - 2*a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**(3/2)
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{7}{2}}}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(7/2)/(a*sin(f*x + e)
 + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(c-c\,\sin(e+f\,x))^{7/2}}{(a+a\,\sin(e+f\,x))^{3/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))
^(3/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))
^(3/2), x)
```

Reduce [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,c^3\left(-\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^4}{\sin(fx+e)^2+2\sin(fx+e)+1}dx\right)b\right.}{}$$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*c**3*( - int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x)
 + 1)*sin(e + f*x)**4)/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),x)*b - int((
sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**3)/(sin(e +
 f*x)**2 + 2*sin(e + f*x) + 1),x)*a + 3*int((sqrt(sin(e + f*x) + 1)*sqrt(
- sin(e + f*x) + 1)*sin(e + f*x)**3)/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1
),x)*b + 3*int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f
*x)**2)/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),x)*a - 3*int((sqrt(sin(e +
f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x)**2 + 2*
sin(e + f*x) + 1),x)*b - 3*int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x
) + 1)*sin(e + f*x))/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),x)*a + int((sq
rt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)
**2 + 2*sin(e + f*x) + 1),x)*b + int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e
 + f*x) + 1))/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),x)*a))/a**2
```

3.181 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 210

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{4(A-2B)c^3\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{2(A-2B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} - \frac{(A-2B)c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2f(a+a\sin(e+fx))^{3/2}}$$

output
```
-4*(A-2*B)*c^3*cos(f*x+e)*ln(1+sin(f*x+e))/a/f/(a+a*sin(f*x+e))^(1/2)/(c-c
*sin(f*x+e))^(1/2)-2*(A-2*B)*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f/(a+
a*sin(f*x+e))^(1/2)-1/2*(A-2*B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a/f/(a
+a*sin(f*x+e))^(1/2)-1/2*(A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/f/(a+a*si
n(f*x+e))^(3/2)
```

Mathematica [A] (verified)

Time = 12.27 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.01

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx =$$
$$-\frac{c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}(28A-16B+2(2A-7B)\cos(2(e+fx))+6$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e +
 f*x])^(3/2),x]
```

output
```
-1/8*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(
28*A - 16*B + 2*(2*A - 7*B)*Cos[2*(e + f*x)] + 64*A*Log[Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2]] - 128*B*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (-8*
A + 31*B + 64*(A - 2*B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])*Sin[e +
f*x] + B*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1
 + Sin[e + f*x]))^(3/2))
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.95, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.275$, Rules used = {3042, 3451, 3042, 3219, 3042, 3219, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3451}$$

$$-\frac{(A-2B)\int \frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{\sin(e+fx)a+a}}dx}{a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$\downarrow$ 3042

$$-\frac{(A-2B)\int \frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{\sin(e+fx)a+a}}dx}{a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$\downarrow$ 3219

$$-\frac{(A-2B)\left(2c\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$\downarrow$ 3042

$$-\frac{(A-2B)\left(2c\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$\downarrow$ 3219

$$-\frac{(A-2B)\left(2c\left(2c\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$\downarrow$ 3042

$$-\frac{(A-2B)\left(2c\left(2c\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$\downarrow$ 3216

$$-\frac{(A-2B)\left(2c\left(\frac{2ac^2\cos(e+fx)\int \frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{(A-2B)\left(2c\left(\frac{2ac^2\cos(e+fx)\int\frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \; 3146$$

$$-\frac{(A-2B)\left(2c\left(\frac{2c^2\cos(e+fx)\int\frac{1}{\sin(e+fx)a+a}d(a\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \; 16$$

$$-\frac{(A-2B)\left(2c\left(\frac{2c^2\cos(e+fx)\log(a\sin(e+fx)+a)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])
^(3/2),x]
```

output
```
-1/2*((A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(f*(a + a*Sin[e + f
*x])^(3/2)) - ((A - 2*B)*((c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(2*f
*Sqrt[a + a*Sin[e + f*x]]) + 2*c*((2*c^2*Cos[e + f*x]*Log[a + a*Sin[e + f*
x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (c*Cos[e + f*
x]*Sqrt[c - c*Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]))))/a
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(190) = 380$.

Time = 7.21 (sec) , antiderivative size = 440, normalized size of antiderivative = 2.10

method	result
default	$\frac{A\sqrt{4}\left(2\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}+8\ln\left(\frac{2}{\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}-8\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\right)\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right.}{2f\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,a}$
parts	$-\frac{A\sqrt{4}\left(-2\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}+8\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\right)\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}-8\ln\left(\frac{2}{\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\sin\left(\frac{\pi}{4}+\right.}{2f\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,a}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/2*A/f*4^(1/2)*(2*cos(1/4*Pi+1/2*f*x+1/2*e)^4+8*ln(2/(cos(1/4*Pi+1/2*f*x+
1/2*e)+1))*sin(1/4*Pi+1/2*f*x+1/2*e)^2-8*ln(-cot(1/4*Pi+1/2*f*x+1/2*e)+csc
(1/4*Pi+1/2*f*x+1/2*e))*sin(1/4*Pi+1/2*f*x+1/2*e)^2-5*cos(1/4*Pi+1/2*f*x+1
/2*e)^2+1)*(c*cos(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)*c^2/(a*sin(1/4*Pi+1/2*f*x
+1/2*e)^2)^(1/2)/a*sec(1/4*Pi+1/2*f*x+1/2*e)*csc(1/4*Pi+1/2*f*x+1/2*e)-2*B
/f*c^2/a*(8*(-2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-1)*ln(-2*(cos(1/2*f*
x+1/2*e)+sin(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1))+8*(2*sin(1/2*f*x+1/2*
e)*cos(1/2*f*x+1/2*e)+1)*ln(2/(cos(1/2*f*x+1/2*e)+1))+cos(1/2*f*x+1/2*e)*(
(-2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)^2+7*cos(1/2*f*x+1/2*e))*sin(1/2*
f*x+1/2*e)^2+8*sin(1/2*f*x+1/2*e)))*(-(2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/
2*e)-1)*c)^(1/2)/(-1+2*cos(1/2*f*x+1/2*e)^2)/((2*sin(1/2*f*x+1/2*e)*cos(1/
2*f*x+1/2*e)+1)*a)^(1/2)
```

Fricas [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{5}{2}}}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x
, algorithm="fricas")
```

output
```
integral(((A - 2*B)*c^2*cos(f*x + e)^2 - 2*(A - B)*c^2 + (B*c^2*cos(f*x +
e)^2 + 2*(A - B)*c^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f
*x + e) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(3/2)
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{5}{2}}}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e)
 + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx=\int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))
^(3/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))
^(3/2), x)
```

Reduce [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx=\frac{\sqrt{c}\,\sqrt{a}\,c^{2}\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^{3}}{\sin(fx+e)^{2}+2\sin(fx+e)+1}dx\right)b+\right.}{}$$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*c**2*(int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) +
1)*sin(e + f*x)**3)/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),x)*b + int((sqr
t(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*
x)**2 + 2*sin(e + f*x) + 1),x)*a - 2*int((sqrt(sin(e + f*x) + 1)*sqrt( - s
in(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),x
)*b - 2*int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)
)/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),x)*a + int((sqrt(sin(e + f*x) + 1
)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**2 + 2*sin(e + f*x
) + 1),x)*b + int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1))/(sin(
e + f*x)**2 + 2*sin(e + f*x) + 1),x)*a))/a**2
```

3.182 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 159

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(A-3B)c^2\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{(A-3B)c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2af\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a+a\sin(e+fx))^{3/2}}$$

output
```
-(A-3*B)*c^2*cos(f*x+e)*ln(1+sin(f*x+e))/a/f/(a+a*sin(f*x+e))^(1/2)/(c-c*s
in(f*x+e))^(1/2)-1/2*(A-3*B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f/(a+a*
sin(f*x+e))^(1/2)-1/2*(A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/f/(a+a*sin(f
*x+e))^(3/2)
```

Mathematica [A] (verified)

Time = 11.61 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.19

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx =$$
$$-\frac{c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}\left(4A-3B-B\cos(2(e+fx))+4A\log\left(\cos\left(\frac{1}{2}\right.\right.\right.}{2f\left(\cos\right.}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e +
 f*x])^(3/2),x]
```

output
```
-1/2*(c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(4*
A - 3*B - B*Cos[2*(e + f*x)] + 4*A*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
] - 12*B*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + 2*(B + 2*(A - 3*B)*Log
[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(3/2))
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.225$, Rules used = {3042, 3451, 3042, 3219, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^{3/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^{3/2}}dx$$
$$\Big\downarrow \text{3451}$$
$$-\frac{(A-3B)\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{\sin(e+fx)a+a}}dx}{2a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(A-3B)\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{\sin(e+fx)a+a}}dx}{2a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{3219}$$

$$-\frac{(A-3B)\left(2c\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)}{2a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(A-3B)\left(2c\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)}{2a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{3216}$$

$$-\frac{(A-3B)\left(\frac{2ac^2\cos(e+fx)\int \frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)}{2a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(A-3B)\left(\frac{2ac^2\cos(e+fx)\int \frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)}{2a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{3146}$$

$$-\frac{(A-3B)\left(\frac{2c^2\cos(e+fx)\int \frac{1}{\sin(e+fx)a+a}d(a\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)}{2a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{16}$$

$$-\frac{(A-3B)\left(\frac{2c^2\cos(e+fx)\log(a\sin(e+fx)+a)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)}{2a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f(a\sin(e+fx)+a)^{3/2}}$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])
^(3/2),x]
```

output
```
-1/2*((A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(f*(a + a*Sin[e + f
*x])^(3/2)) - ((A - 3*B)*((2*c^2*Cos[e + f*x]*Log[a + a*Sin[e + f*x]])/(f*
Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (c*Cos[e + f*x]*Sqrt[
c - c*Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]])))/(2*a)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(143) = 286$.

Time = 6.93 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.45

method	result
default	$\frac{A\sqrt{4}\,\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,c\left(4\ln\left(\frac{2}{\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)-4\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\right)\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\right.\right.}{4f\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,a}$
parts	$\frac{A\sqrt{4}\,\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,c\left(4\ln\left(\frac{2}{\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)-4\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\right)\tan\left(\frac{\pi}{4}+\frac{fx}{2}+\right.\right.}{4f\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,a}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/4*A/f*4^(1/2)*(c*cos(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)*c/(a*sin(1/4*Pi+1/2*
f*x+1/2*e)^2)^(1/2)/a*(4*ln(2/(cos(1/4*Pi+1/2*f*x+1/2*e)+1))*tan(1/4*Pi+1/
2*f*x+1/2*e)-4*ln(-cot(1/4*Pi+1/2*f*x+1/2*e)+csc(1/4*Pi+1/2*f*x+1/2*e))*ta
n(1/4*Pi+1/2*f*x+1/2*e)-cot(1/4*Pi+1/2*f*x+1/2*e)-sec(1/4*Pi+1/2*f*x+1/2*e
)*csc(1/4*Pi+1/2*f*x+1/2*e))-2*B/f*((-6*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2
*e)-3)*ln(-2*(cos(1/2*f*x+1/2*e)+sin(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1
))+(6*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+3)*ln(2/(cos(1/2*f*x+1/2*e)+1)
)+cos(1/2*f*x+1/2*e)*(2*cos(1/2*f*x+1/2*e)*sin(1/2*f*x+1/2*e)^2+3*sin(1/2*
f*x+1/2*e)))*(-(2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-1)*c)^(1/2)*c/(-1+
2*cos(1/2*f*x+1/2*e)^2)/((2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+1)*a)^(1
/2)/a
```

Fricas [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{3}{2}}}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x
, algorithm="fricas")
```

output
```
integral(-(B*c*cos(f*x + e)^2 - (A - B)*c*sin(f*x + e) + (A - B)*c)*sqrt(a
*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*s
in(f*x + e) - 2*a^2), x)
```

Sympy [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx=\int \frac{(-c(\sin(e+fx)-1))^{\frac{3}{2}}(A+B\sin(e+fx))}{(a(\sin(e+fx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(3/2)
,x)
```

output
```
Integral((-c*(sin(e + f*x) - 1))**(3/2)*(A + B*sin(e + f*x))/(a*(sin(e + f
*x) + 1))**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(143) = 286$.

Time = 0.15 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.31

$$\int \frac{(A + B\sin(e + fx))(c - c\sin(e + fx))^{3/2}}{(a + a\sin(e + fx))^{3/2}}\,dx = -\frac{B\left(\frac{6\,c^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^{\frac{3}{2}}} - \frac{3\,c^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{a^{\frac{3}{2}}} - \frac{2\left(\frac{3\,c^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{2\,c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{3\,c^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^{\frac{3}{2}}+\frac{2\,a^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{2\,a^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{2\,a^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{a^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}\right)}{f}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x
, algorithm="maxima")
```

output
```
-(B*(6*c^(3/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^(3/2) - 3*c^(3/2
)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/a^(3/2) - 2*(3*c^(3/2)*sin(
f*x + e)/(cos(f*x + e) + 1) + 2*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 3*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/(a^(3/2) + 2*a^(3/2)*si
n(f*x + e)/(cos(f*x + e) + 1) + 2*a^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + 2*a^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a^(3/2)*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4)) - A*(2*c^(3/2)*log(sin(f*x + e)/(cos(f*x + e) +
1) + 1)/a^(3/2) - c^(3/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/a^(
3/2) - 4*sqrt(a)*c^(3/2)*sin(f*x + e)/((a^2 + 2*a^2*sin(f*x + e)/(cos(f*x
+ e) + 1) + a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1))))
/f
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(c-c\,\sin(e+f\,x))^{3/2}}{(a+a\,\sin(e+f\,x))^{3/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))
^(3/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))
^(3/2), x)
```

Reduce [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,c\left(-\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^2}{\sin(fx+e)^2+2\sin(fx+e)+1}dx\right)b\right.}{}$$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*c*( - int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) +
1)*sin(e + f*x)**2)/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),x)*b - int((sqr
t(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)*
*2 + 2*sin(e + f*x) + 1),x)*a + int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e
+ f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),x)*b + in
t((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1))/(sin(e + f*x)**2 + 2*
sin(e + f*x) + 1),x)*a))/a**2
```

3.183 $\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 100

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(A-B)c\cos(e+fx)}{f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{Bc\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

output
```
-(A-B)*c*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2)+B*c*co
s(f*x+e)*ln(1+sin(f*x+e))/a/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2
)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.65 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.43

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx=\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}(}{f\left(\cos\left(\frac{1}{2}(e+fx\right.\right.}$$

input
```
Integrate[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f
*x])^(3/2),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(-A + B -
I*B*f*x + 2*B*Log[I + E^(I*(e + f*x))] + B*((-I)*f*x + 2*Log[I + E^(I*(e +
 f*x))])*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + S
in[e + f*x]))^(3/2))
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3450, 3042, 3216, 3042, 3146, 16, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-c\sin(e+fx)}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^{3/2}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sqrt{c-c\sin(e+fx)}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \ 3450$$

$$(A-B)\int \frac{\sqrt{c-c\sin(e+fx)}}{(\sin(e+fx)a+a)^{3/2}}dx+\frac{B\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx}{a}$$

$$\Big\downarrow \ 3042$$

$$(A-B)\int \frac{\sqrt{c-c\sin(e+fx)}}{(\sin(e+fx)a+a)^{3/2}}dx+\frac{B\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx}{a}$$

$$\Big\downarrow \text{3216}$$

$$(A-B)\int \frac{\sqrt{c-c\sin(e+fx)}}{(\sin(e+fx)a+a)^{3/2}}dx+\frac{Bc\cos(e+fx)\int \frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$(A-B)\int \frac{\sqrt{c-c\sin(e+fx)}}{(\sin(e+fx)a+a)^{3/2}}dx+\frac{Bc\cos(e+fx)\int \frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{3146}$$

$$(A-B)\int \frac{\sqrt{c-c\sin(e+fx)}}{(\sin(e+fx)a+a)^{3/2}}dx+\frac{Bc\cos(e+fx)\int \frac{1}{\sin(e+fx)a+a}d(a\sin(e+fx))}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{16}$$

$$(A-B)\int \frac{\sqrt{c-c\sin(e+fx)}}{(\sin(e+fx)a+a)^{3/2}}dx+\frac{Bc\cos(e+fx)\log(a\sin(e+fx)+a)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{3217}$$

$$\frac{Bc\cos(e+fx)\log(a\sin(e+fx)+a)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{c(A-B)\cos(e+fx)}{f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

input
```
Int[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^(
3/2),x]
```

output
```
-(((A - B)*c*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e
+ f*x]])) + (B*c*Cos[e + f*x]*Log[a + a*Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin
[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3450
```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[B/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] -
Simp[(B*c - A*d)/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[
a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(92) = 184$.

Time = 8.80 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.84

method	result
parts	$-\frac{A\sqrt{4}\,\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,\left(\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\sec\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\right)}{8f\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,a}-\frac{2B\left(\left(-2\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\ln$
default	$-\frac{A\sqrt{4}\,\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,\left(\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\sec\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\right)}{8f\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,a}+\frac{2B\left(\left(2\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\ln\left(\right.$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x,metho
d=_RETURNVERBOSE)
```

output
```
-1/8*A/f*4^(1/2)*(c*cos(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)/(a*sin(1/4*Pi+1/2*f
*x+1/2*e)^2)^(1/2)/a*(cot(1/4*Pi+1/2*f*x+1/2*e)+sec(1/4*Pi+1/2*f*x+1/2*e)*
csc(1/4*Pi+1/2*f*x+1/2*e))-2*B/a/f*((-2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2
*e)-1)*ln(-2*(cos(1/2*f*x+1/2*e)+sin(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1
))+(2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+1)*ln(2/(cos(1/2*f*x+1/2*e)+1)
)+sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e))*(-(2*sin(1/2*f*x+1/2*e)*cos(1/2*f
*x+1/2*e)-1)*c)^(1/2)/(-1+2*cos(1/2*f*x+1/2*e)^2)/((2*sin(1/2*f*x+1/2*e)*c
os(1/2*f*x+1/2*e)+1)*a)^(1/2)
```

Fricas [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx=\int \frac{(B\sin(fx+e)+A)\sqrt{-c\sin(fx+e)+c}}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x
, algorithm="fricas")
```

output
```
integral(-(B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
e) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

Sympy [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{-c(\sin(e+fx)-1)}(A+B\sin(e+fx))}{(a(\sin(e+fx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(3/2)
,x)
```

output
```
Integral(sqrt(-c*(sin(e + f*x) - 1))*(A + B*sin(e + f*x))/(a*(sin(e + f*x)
 + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)\sqrt{-c\sin(fx+e)+c}}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)/(a*sin(f*x + e) +
 a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,\sqrt{c-c\,\sin(e+f\,x)}}{(a+a\,\sin(e+f\,x))^{3/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))
^(3/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))
^(3/2), x)
```

Reduce [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\sqrt{c}\sqrt{a}\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)^2+2\sin(fx+e)+1}dx\right)b+\left(\int\right.\right.}{a^2}$$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*(int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*si
n(e + f*x))/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),x)*b + int((sqrt(sin(e
+ f*x) + 1)*sqrt( - sin(e + f*x) + 1))/(sin(e + f*x)**2 + 2*sin(e + f*x) +
 1),x)*a))/a**2
```

3.184 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 103

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{(A+B)\operatorname{arctanh}(\sin(e+fx))\cos(e+fx)}{2af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

output
```
-1/2*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2)+1/2*
(A+B)*arctanh(sin(f*x+e))*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f
*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 3.85 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.81

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\right.}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[
e + f*x]]),x]
```

output
```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])*(-A + B - (A + B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*
x)/2] + Sin[(e + f*x)/2])^2 + (A + B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)
/2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2))/(2*f*(a*(1 + Sin[e + f*x]))
^(3/2)*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3451, 3042, 3220, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3451}$$

$$\frac{(A+B)\int \frac{1}{\sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}}dx}{2a}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A+B)\int \frac{1}{\sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}}dx}{2a}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{3220}$$

$$\frac{(A+B)\cos(e+fx)\int \sec(e+fx)dx}{2a\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A+B)\cos(e+fx)\int\csc\left(e+fx+\frac{\pi}{2}\right)dx}{2a\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{(A+B)\cos(e+fx)\text{arctanh}(\sin(e+fx))}{2af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*
x]]),x]
```

output
```
-1/2*((A - B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e
 + f*x]]) + ((A + B)*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*a*f*Sqrt[a + a
*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3220
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]])   Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 516 vs. $2(91) = 182$.

Time = 8.02 (sec) , antiderivative size = 517, normalized size of antiderivative = 5.02

method	result
parts	$-\frac{A\sqrt{4}\left(4\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1\right)\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2+4\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin\left(\frac{\pi}{4}+\right.}{16fa\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2}\sqrt{c\,co}}$
default	$\frac{A\sqrt{4}\left(4\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\right)\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2-4\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1\right)\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right.\right.}{16fa\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^2}\sqrt{c\cos\left(\right.}}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
-1/16*A/f*4^(1/2)*(4*ln(-cot(1/4*Pi+1/2*f*x+1/2*e)+csc(1/4*Pi+1/2*f*x+1/2*
e)+1)*sin(1/4*Pi+1/2*f*x+1/2*e)^2+4*ln(-cot(1/4*Pi+1/2*f*x+1/2*e)+csc(1/4*
Pi+1/2*f*x+1/2*e)-1)*sin(1/4*Pi+1/2*f*x+1/2*e)^2-4*ln(-cot(1/4*Pi+1/2*f*x+
1/2*e)+csc(1/4*Pi+1/2*f*x+1/2*e))*sin(1/4*Pi+1/2*f*x+1/2*e)^2+cos(1/4*Pi+1
/2*f*x+1/2*e)^2+1)/a/(a*sin(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)/(c*cos(1/4*Pi+1
/2*f*x+1/2*e)^2)^(1/2)*cot(1/4*Pi+1/2*f*x+1/2*e)+1/2*B/a/f*(((1-2*cos(1/2*
f*x+1/2*e)^2)*sin(1/2*f*x+1/2*e)-cos(1/2*f*x+1/2*e)*(-1+2*cos(1/2*f*x+1/2*
e)^2))*ln(2*(sin(1/2*f*x+1/2*e)-cos(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1)
)+((-1+2*cos(1/2*f*x+1/2*e)^2)*sin(1/2*f*x+1/2*e)+cos(1/2*f*x+1/2*e)*(-1+2
*cos(1/2*f*x+1/2*e)^2))*ln(-2*(cos(1/2*f*x+1/2*e)+sin(1/2*f*x+1/2*e))/(cos
(1/2*f*x+1/2*e)+1))+cos(1/2*f*x+1/2*e)*(-2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+
1/2*e)+2*sin(1/2*f*x+1/2*e)^2))/(cos(1/2*f*x+1/2*e)+sin(1/2*f*x+1/2*e))/((
2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+1)*a)^(1/2)/(-(2*sin(1/2*f*x+1/2*e
)*cos(1/2*f*x+1/2*e)-1)*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 327, normalized size of antiderivative = 3.17

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx = \left[\frac{((A+B)\cos(fx+e)\sin(fx+e)+(A+B)\cos(fx\right.$$

$$-\frac{((A+B)\cos(fx+e)\sin(fx+e)+(A+B)\cos(fx+e))\sqrt{-ac}\arctan\left(\frac{\sqrt{-ac}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+}}{ac\cos(fx+e)}\right.}{2\,(a^2cf\cos(fx+e)\sin(fx+e)+a^2cf\cos(fx}$$

input

```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x
, algorithm="fricas")
```

output

```
[1/4*(((A + B)*cos(f*x + e)*sin(f*x + e) + (A + B)*cos(f*x + e))*sqrt(a*c)
*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*sqrt(a*sin(f*
x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3) - 2*sq
rt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*(A - B))/(a^2*c*f*cos(f*x
 + e)*sin(f*x + e) + a^2*c*f*cos(f*x + e)), -1/2*(((A + B)*cos(f*x + e)*si
n(f*x + e) + (A + B)*cos(f*x + e))*sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt(a*sin
(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e)/(a*c*cos(f*x + e)))
+ sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*(A - B))/(a^2*c*f*cos
(f*x + e)*sin(f*x + e) + a^2*c*f*cos(f*x + e))]
```

Sympy [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{A+B\sin(e+fx)}{(a(\sin(e+fx)+1))^{\frac{3}{2}}\sqrt{-c(\sin(e+fx)-1)}}\,dx$$

input

```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(1/2)
,x)
```

output

```
Integral((A + B*sin(e + f*x))/((a*(sin(e + f*x) + 1))**(3/2)*sqrt(-c*(sin(
e + f*x) - 1))), x)
```

Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{B\sin(fx+e)+A}{(a\sin(fx+e)+a)^{\frac{3}{2}}\sqrt{-c\sin(fx+e)+c}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x
 + e) + c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{A+B\,\sin(e+f\,x)}{(a+a\,\sin(e+f\,x))^{3/2}\,\sqrt{c-c\,\sin(e+f\,x)}}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^
(1/2)),x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^
(1/2)), x)
```

Reduce [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{\sqrt{c}\,\sqrt{a}\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)^3+\sin(fx+e)^2-\sin(fx+e)-1}dx\right)b+\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)^3+\sin(fx+e)^2-\sin(fx+e)-1}dx\right)a\right)}{a^2c}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x)
```

output
```
( - sqrt(c)*sqrt(a)*(int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)
*sin(e + f*x))/(sin(e + f*x)**3 + sin(e + f*x)**2 - sin(e + f*x) - 1),x)*b
 + int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1))/(sin(e + f*x)**3
 + sin(e + f*x)**2 - sin(e + f*x) - 1),x)*a))/(a**2*c)
```

3.185 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 150

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx = -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{A\cos(e+fx)}{2af\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} + \frac{A\text{arctanh}(\sin(e+fx))\cos(e+fx)}{2acf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

output
```
-1/2*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2)+1/2*
A*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2)+1/2*A*arcta
nh(sin(f*x+e))*cos(f*x+e)/a/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1
/2)
```

Mathematica [A] (verified)

Time = 4.01 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.95

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx =$$
$$-\frac{\cos(e+fx)\left(2B-A\log\left(1-\tan\left(\frac{1}{2}(e+fx)\right)\right)+A\log\left(1+\tan\left(\frac{1}{2}(e+fx)\right)\right)+A\cos(2(e+fx))\left(-\log\right.\right.}{4cf(-1+\sin(e+fx))(a(1+\sin(e+fx)))^{3/2}\sqrt{}}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e +
f*x])^(3/2)),x]
```

output
```
-1/4*(Cos[e + f*x]*(2*B - A*Log[1 - Tan[(e + f*x)/2]] + A*Log[1 + Tan[(e +
 f*x)/2]] + A*Cos[2*(e + f*x)]*(-Log[1 - Tan[(e + f*x)/2]] + Log[1 + Tan[(
e + f*x)/2]]) + 2*A*Sin[e + f*x]))/(c*f*(-1 + Sin[e + f*x])*(a*(1 + Sin[e
+ f*x]))^(3/2)*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3451, 3042, 3222, 3042, 3220, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}dx$$
$$\Big\downarrow \text{ 3451}$$
$$\frac{A\int \frac{1}{\sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{3/2}}dx}{a}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}$$
$$\Big\downarrow \text{ 3042}$$

$$\frac{A\int \frac{1}{\sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{3/2}}dx}{a}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}$$

$\downarrow$ 3222

$$\frac{A\left(\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}}dx}{2c}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}\right)}{a}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{A\left(\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}}dx}{2c}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}\right)}{a}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}$$

$\downarrow$ 3220

$$\frac{A\left(\frac{\cos(e+fx)\int \sec(e+fx)dx}{2c\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}\right)}{a}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{A\left(\frac{\cos(e+fx)\int \csc\left(e+fx+\frac{\pi}{2}\right)dx}{2c\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}\right)}{a}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}$$

$\downarrow$ 4257

$$\frac{A\left(\frac{\cos(e+fx)\text{arctanh}(\sin(e+fx))}{2cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}\right)}{a}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^
(3/2)),x]
```

output
```
-1/2*((A - B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f
*x])^(3/2)) + (A*(Cos[e + f*x]/(2*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e
+ f*x])^(3/2)) + (ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*c*f*Sqrt[a + a*Si
n[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])))/a
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3220
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]])   Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

rule 3222
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp[(m + n + 1)/(a*(2*m + 1)
)   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
 ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||
!SumSimplerQ[n, 1])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. 2(132) = 264.

Time = 7.70 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.59

method	result
default	$\frac{A\sqrt{4}\left(-8\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}-8\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\right.\right.\right.}{}$
parts	$-\frac{A\sqrt{4}\left(8\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1\right)\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}+8\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\right.\right.\right.}{}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/32*A/f*4^(1/2)*(-8*cos(1/4*Pi+1/2*f*x+1/2*e)^2*ln(-cot(1/4*Pi+1/2*f*x+1/
2*e)+csc(1/4*Pi+1/2*f*x+1/2*e)-1)*sin(1/4*Pi+1/2*f*x+1/2*e)^2-8*cos(1/4*Pi
+1/2*f*x+1/2*e)^2*ln(-cot(1/4*Pi+1/2*f*x+1/2*e)+csc(1/4*Pi+1/2*f*x+1/2*e)+
1)*sin(1/4*Pi+1/2*f*x+1/2*e)^2+8*cos(1/4*Pi+1/2*f*x+1/2*e)^2*ln(-cot(1/4*P
i+1/2*f*x+1/2*e)+csc(1/4*Pi+1/2*f*x+1/2*e))*sin(1/4*Pi+1/2*f*x+1/2*e)^2+co
s(1/4*Pi+1/2*f*x+1/2*e)^4-5*cos(1/4*Pi+1/2*f*x+1/2*e)^2+2)/(a*sin(1/4*Pi+1
/2*f*x+1/2*e)^2)^(1/2)/(c*cos(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)/a/c*sec(1/4*P
i+1/2*f*x+1/2*e)*csc(1/4*Pi+1/2*f*x+1/2*e)+2*B/a/c/f*cos(1/2*f*x+1/2*e)^2*
sin(1/2*f*x+1/2*e)^2/(-1+2*cos(1/2*f*x+1/2*e)^2)/((2*sin(1/2*f*x+1/2*e)*co
s(1/2*f*x+1/2*e)+1)*a)^(1/2)/(-(2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-1)
*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.80

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx = \left[\frac{\sqrt{ac}A\cos(fx+e)^{3}\log\left(-\frac{ac\cos(fx+e)^{3}-2\,ac\cos(fx+e)-2}{}\right.}{}\right.$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x
, algorithm="fricas")
```

output
```
[1/4*(sqrt(a*c)*A*cos(f*x + e)^3*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x
+ e) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(
f*x + e))/cos(f*x + e)^3) + 2*(A*sin(f*x + e) + B)*sqrt(a*sin(f*x + e) + a
)*sqrt(-c*sin(f*x + e) + c))/(a^2*c^2*f*cos(f*x + e)^3), -1/2*(sqrt(-a*c)*
A*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin
(f*x + e)/(a*c*cos(f*x + e)))*cos(f*x + e)^3 - (A*sin(f*x + e) + B)*sqrt(a
*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a^2*c^2*f*cos(f*x + e)^3)]
```

Sympy [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx=\int \frac{A+B\sin\left(e+fx\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(3/2)
,x)
```

output
```
Integral((A + B*sin(e + f*x))/((a*(sin(e + f*x) + 1))**(3/2)*(-c*(sin(e +
f*x) - 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx=\int \frac{B\sin\left(fx+e\right)+A}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e
) + c)^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^
(3/2)),x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^
(3/2)), x)
```

Reduce [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)^4-2\sin(fx+e)^2+1}dx\right)b+\right.}{a^2c^2}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*(int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*si
n(e + f*x))/(sin(e + f*x)**4 - 2*sin(e + f*x)**2 + 1),x)*b + int((sqrt(sin
(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1))/(sin(e + f*x)**4 - 2*sin(e + f*x
)**2 + 1),x)*a))/(a**2*c**2)
```

3.186 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 217

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx = & -\frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} \\ & +\frac{(3A-B)\cos(e+fx)}{8af\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}} \\ & +\frac{(3A-B)\cos(e+fx)}{8acf\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} \\ & +\frac{(3A-B)\text{arctanh}(\sin(e+fx))\cos(e+fx)}{8ac^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\end{aligned}$$

output
```
-1/2*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2)+1/8*
(3*A-B)*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2)+1/8*(
3*A-B)*cos(f*x+e)/a/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2)+1/8*
(3*A-B)*arctanh(sin(f*x+e))*cos(f*x+e)/a/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c
*sin(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 5.35 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.41

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)\right.}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e +
f*x])^(5/2)),x]
```

output
```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])*(2*A*Cos[e + f*x]^2 + (-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4
+ (A + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (-3*A + B)*Log[Cos[(e
+ f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (3*A - B)*Log[Cos[(e + f*x)/2] + Sin
[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^2))/(8*f*(a*(1 + Sin[e + f*x]))^(3/2)*(c - c*Sin[e + f*
x])^(5/2))
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3451, 3042, 3222, 3042, 3222, 3042, 3220, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}dx$$

$$\Big\downarrow \; 3451$$

$$\frac{(3A-B)\int \frac{1}{\sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{5/2}}dx}{2a}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{(3A-B)\int \frac{1}{\sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{5/2}}dx}{2a}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}$$

$\Big\downarrow$ 3222

$$\frac{(3A-B)\left(\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{3/2}}dx}{2c}+\frac{\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}\right)}{2a}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{(3A-B)\left(\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{3/2}}dx}{2c}+\frac{\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}\right)}{2a}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}$$

$\Big\downarrow$ 3222

$$\frac{(3A-B)\left(\frac{\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}}dx}{2c}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}}{2c}+\frac{\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}\right)}{2a}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{(3A-B)\left(\frac{\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}}dx}{2c}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}}{2c}+\frac{\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}\right)}{2a}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}$$

$\Big\downarrow$ 3220

$$\frac{(3A-B)\left(\frac{\frac{\cos(e+fx)\int\sec(e+fx)dx}{2c\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}}{2c}+\frac{\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}\right)}{2a}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(3A-B)\left(\frac{\frac{\cos(e+fx)\int\csc\left(e+fx+\frac{\pi}{2}\right)dx}{2c\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}}{2c}+\frac{\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}\right)}{2a}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{(3A-B)\left(\frac{\frac{\cos(e+fx)\text{arctanh}(\sin(e+fx))}{2cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}}{2c}+\frac{\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}\right)}{2a}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^
(5/2)),x]
```

output
```
-1/2*((A - B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f
*x])^(5/2)) + ((3*A - B)*(Cos[e + f*x]/(4*f*Sqrt[a + a*Sin[e + f*x]]*(c -
c*Sin[e + f*x])^(5/2)) + (Cos[e + f*x]/(2*f*Sqrt[a + a*Sin[e + f*x]]*(c -
c*Sin[e + f*x])^(3/2)) + (ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*c*f*Sqrt[
a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]))/(2*c)))/(2*a)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3220
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]])   Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

rule 3222
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp[(m + n + 1)/(a*(2*m + 1)
)   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
 ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||
!SumSimplerQ[n, 1])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 723 vs. $2(193) = 386$.

Time = 8.42 (sec) , antiderivative size = 724, normalized size of antiderivative = 3.34

method	result	size
default	Expression too large to display	724
parts	Expression too large to display	724

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/64*A/f*4^(1/2)*(12*ln(-cot(1/4*Pi+1/2*f*x+1/2*e)+csc(1/4*Pi+1/2*f*x+1/2*
e))*cos(1/4*Pi+1/2*f*x+1/2*e)^4*sin(1/4*Pi+1/2*f*x+1/2*e)^2-12*ln(-cot(1/4
*Pi+1/2*f*x+1/2*e)+csc(1/4*Pi+1/2*f*x+1/2*e)-1)*cos(1/4*Pi+1/2*f*x+1/2*e)^
4*sin(1/4*Pi+1/2*f*x+1/2*e)^2-12*ln(-cot(1/4*Pi+1/2*f*x+1/2*e)+csc(1/4*Pi+
1/2*f*x+1/2*e)+1)*cos(1/4*Pi+1/2*f*x+1/2*e)^4*sin(1/4*Pi+1/2*f*x+1/2*e)^2+
4*cos(1/4*Pi+1/2*f*x+1/2*e)^6-10*cos(1/4*Pi+1/2*f*x+1/2*e)^4+3*cos(1/4*Pi+
1/2*f*x+1/2*e)^2+1)/(a*sin(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)/(c*cos(1/4*Pi+1/
2*f*x+1/2*e)^2)^(1/2)/a/c^2*sec(1/4*Pi+1/2*f*x+1/2*e)^3*csc(1/4*Pi+1/2*f*x
+1/2*e)+1/8*B/a/c^2/f*((2*cos(1/2*f*x+1/2*e)*(4*cos(1/2*f*x+1/2*e)^4-4*cos
(1/2*f*x+1/2*e)^2+1)*sin(1/2*f*x+1/2*e)-4*cos(1/2*f*x+1/2*e)^4+4*cos(1/2*f
*x+1/2*e)^2-1)*ln(-2*(cos(1/2*f*x+1/2*e)-sin(1/2*f*x+1/2*e))/(cos(1/2*f*x+
1/2*e)+1))+(-2*cos(1/2*f*x+1/2*e)*(4*cos(1/2*f*x+1/2*e)^4-4*cos(1/2*f*x+1/
2*e)^2+1)*sin(1/2*f*x+1/2*e)+4*cos(1/2*f*x+1/2*e)^4-4*cos(1/2*f*x+1/2*e)^2
+1)*ln(-2*(cos(1/2*f*x+1/2*e)+sin(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1))+
cos(1/2*f*x+1/2*e)*((16*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)^2-12*cos(1/2
*f*x+1/2*e))*sin(1/2*f*x+1/2*e)^2-2*sin(1/2*f*x+1/2*e)))/(4*cos(1/2*f*x+1/
2*e)^3*sin(1/2*f*x+1/2*e)-2*cos(1/2*f*x+1/2*e)^2-2*sin(1/2*f*x+1/2*e)*cos(
1/2*f*x+1/2*e)+1)/((2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+1)*a)^(1/2)/(-
(2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-1)*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 435, normalized size of antiderivative = 2.00

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx = \left[-\frac{((3A-B)\cos(fx+e)^3\sin(fx+e)-(3A-B)\right.$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x
, algorithm="fricas")
```

output
```
[-1/16*(((3*A - B)*cos(f*x + e)^3*sin(f*x + e) - (3*A - B)*cos(f*x + e)^3)
*sqrt(a*c)*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) + 2*sqrt(a*c)*sqr
t(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)
^3) + 2*((3*A - B)*cos(f*x + e)^2 + (3*A - B)*sin(f*x + e) - A + 3*B)*sqrt
(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a^2*c^3*f*cos(f*x + e)^3*
sin(f*x + e) - a^2*c^3*f*cos(f*x + e)^3), -1/8*(((3*A - B)*cos(f*x + e)^3*
sin(f*x + e) - (3*A - B)*cos(f*x + e)^3)*sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt
(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e)/(a*c*cos(f*x +
 e))) + ((3*A - B)*cos(f*x + e)^2 + (3*A - B)*sin(f*x + e) - A + 3*B)*sqrt
(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a^2*c^3*f*cos(f*x + e)^3*
sin(f*x + e) - a^2*c^3*f*cos(f*x + e)^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(5/2)
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx=\int \frac{B\sin\left(fx+e\right)+A}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e
) + c)^(5/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx=\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}} \,d x$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^
(5/2)),x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^
(5/2)), x)
```

Reduce [F]

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^{3/2}(c - c\sin(e + fx))^{5/2}}\,dx = -\frac{\sqrt{c}\,\sqrt{a}\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)^5-\sin(fx+e)^4-2\sin(fx+e)^3+2\sin(fx+e)^2+\sin(fx+e)-1}dx\right)b + \left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)^5-\sin(fx+e)^4-2\sin(fx+e)^3\right.\right.}{a^2c^3}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x)
```

output
```
( - sqrt(c)*sqrt(a)*(int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)
*sin(e + f*x))/(sin(e + f*x)**5 - sin(e + f*x)**4 - 2*sin(e + f*x)**3 + 2*
sin(e + f*x)**2 + sin(e + f*x) - 1),x)*b + int((sqrt(sin(e + f*x) + 1)*sqr
t( - sin(e + f*x) + 1))/(sin(e + f*x)**5 - sin(e + f*x)**4 - 2*sin(e + f*x
)**3 + 2*sin(e + f*x)**2 + sin(e + f*x) - 1),x)*a))/(a**2*c**3)
```

3.187 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 323

$$\begin{aligned}\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx &= \frac{8(3A-7B)c^5\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} \\ &+ \frac{4(3A-7B)c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} \\ &+ \frac{(3A-7B)c^3\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a+a\sin(e+fx)}} \\ &+ \frac{(3A-7B)c^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2f\sqrt{a+a\sin(e+fx)}} \\ &+ \frac{(3A-7B)c\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4af(a+a\sin(e+fx))^{3/2}} \\ &- \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a+a\sin(e+fx))^{5/2}}\end{aligned}$$

output
```
8*(3*A-7*B)*c^5*cos(f*x+e)*ln(1+sin(f*x+e))/a^2/f/(a+a*sin(f*x+e))^(1/2)/(
c-c*sin(f*x+e))^(1/2)+4*(3*A-7*B)*c^4*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^
2/f/(a+a*sin(f*x+e))^(1/2)+(3*A-7*B)*c^3*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)
/a^2/f/(a+a*sin(f*x+e))^(1/2)+1/3*(3*A-7*B)*c^2*cos(f*x+e)*(c-c*sin(f*x+e)
)^(5/2)/a^2/f/(a+a*sin(f*x+e))^(1/2)+1/4*(3*A-7*B)*c*cos(f*x+e)*(c-c*sin(f
*x+e))^(7/2)/a/f/(a+a*sin(f*x+e))^(3/2)-1/4*(A-B)*cos(f*x+e)*(c-c*sin(f*x+
e))^(9/2)/f/(a+a*sin(f*x+e))^(5/2)
```

Mathematica [A] (verified)

Time = 17.27 (sec) , antiderivative size = 573, normalized size of antiderivative = 1.77

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx =$$
$$-\frac{8(A-B)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c-c\sin(e+fx))^{9/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9(a(1+\sin(e+fx)))^{5/2}}$$
$$+\frac{16(2A-3B)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(c-c\sin(e+fx))^{9/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9(a(1+\sin(e+fx)))^{5/2}}$$
$$-\frac{(A-7B)\cos(2(e+fx))\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(c-c\sin(e+fx))^{9/2}}{4f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9(a(1+\sin(e+fx)))^{5/2}}$$
$$+\frac{16(3A-7B)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(c-c\sin(e+}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9(a(1+\sin(e+fx)))^{5/2}}$$
$$-\frac{(28A-97B)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\sin(e+fx)(c-c\sin(e+fx))^{9/2}}{4f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9(a(1+\sin(e+fx)))^{5/2}}$$
$$-\frac{B\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(c-c\sin(e+fx))^{9/2}\sin(3(e+fx))}{12f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9(a(1+\sin(e+fx)))^{5/2}}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e +
 f*x])^(5/2),x]
```

output
```
(-8*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^(9/
2))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(5/2
)) + (16*(2*A - 3*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(c - c*Sin[e
+ f*x])^(9/2))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e +
f*x]))^(5/2)) - ((A - 7*B)*Cos[2*(e + f*x)]*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^5*(c - c*Sin[e + f*x])^(9/2))/(4*f*(Cos[(e + f*x)/2] - Sin[(e + f*
x)/2])^9*(a*(1 + Sin[e + f*x]))^(5/2)) + (16*(3*A - 7*B)*Log[Cos[(e + f*x)
/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(c - c*Sin
[e + f*x])^(9/2))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e
 + f*x]))^(5/2)) - ((28*A - 97*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*
Sin[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(4*f*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(5/2)) - (B*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])^5*(c - c*Sin[e + f*x])^(9/2)*Sin[3*(e + f*x)])/(12*f*(Cos[(e +
f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(5/2))
```

Rubi [A] (verified)

Time = 1.72 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.93, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3451, 3042, 3218, 3042, 3219, 3042, 3219, 3042, 3219, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^{9/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-c\sin(e+fx))^{9/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3451}$$

$$-\frac{(3A-7B)\int \frac{(c-c\sin(e+fx))^{9/2}}{(\sin(e+fx)a+a)^{3/2}}dx}{4a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(3A-7B)\int \frac{(c-c\sin(e+fx))^{9/2}}{(\sin(e+fx)a+a)^{3/2}}dx}{4a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$\Big\downarrow$ 3218

$$-\frac{(3A-7B)\left(-\frac{4c\int \frac{(c-c\sin(e+fx))^{7/2}}{\sqrt{\sin(e+fx)a+a}}dx}{a}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{4a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$-\frac{(3A-7B)\left(-\frac{4c\int \frac{(c-c\sin(e+fx))^{7/2}}{\sqrt{\sin(e+fx)a+a}}dx}{a}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{4a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$\Big\downarrow$ 3219

$$-\frac{(3A-7B)\left(-\frac{4c\left(2c\int \frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{4a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$-\frac{(3A-7B)\left(-\frac{4c\left(2c\int \frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{4a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$\Big\downarrow$ 3219

$$-\frac{(3A-7B)\left(-\frac{4c\left(2c\left(2c\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{c\cos(e+fx)(c-c\sin}{f(a\sin(e+fx)}\cdots\right.}{4a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$-\frac{(3A-7B)\left(-\frac{4c\left(2c\left(2c\int\frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{c\cos(e+fx)(c-c\sin\dots}{f(a\sin(e+fx)\dots}\right.}{4a}\dots$$
$$\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$\Big\downarrow$ 3219

$$-\frac{(3A-7B)\left(-\frac{4c\left(2c\left(2c\left(2c\int\frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx)\dots}{3f\sqrt{a\sin(e+fx)+a}}\dots}{a}\right.}{4a}\dots$$
$$\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$-\frac{(3A-7B)\left(-\frac{4c\left(2c\left(2c\left(2c\int\frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx)\dots}{3f\sqrt{a\sin(e+fx)+a}}\dots}{a}\right.}{4a}\dots$$
$$\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$\Big\downarrow$ 3216

$$-\frac{(3A-7B)\left(-\frac{4c\left(2c\left(2c\left(\frac{2ac^2\cos(e+fx)\int\frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c\dots}{3f\sqrt{a\,\mathrm{si}\dots}}\dots}{a}\right.}{4a}\dots$$
$$\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$-\frac{(3A-7B)\left(-\frac{4c\left(2c\left(2c\left(\frac{2ac^2\cos(e+fx)\int \frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-}{3f\sqrt{a\sin}}}{a}\right.}{4a}$$

$$\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$\Big\downarrow$ 3146

$$-\frac{(3A-7B)\left(-\frac{4c\left(2c\left(2c\left(\frac{2c^2\cos(e+fx)\int \frac{1}{\sin(e+fx)a+a}d(a\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos}{}}{a}\right.}{4a}$$

$$\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$\Big\downarrow$ 16

$$-\frac{(3A-7B)\left(-\frac{4c\left(2c\left(2c\left(\frac{2c^2\cos(e+fx)\log(a\sin(e+fx)+a)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c}{3f\sqrt{a\sin(}}}{a}\right.}{4a}$$

$$\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e + f*x])
^(5/2),x]
```

output
```
-1/4*((A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(f*(a + a*Sin[e + f
*x])^(5/2)) - ((3*A - 7*B)*(-((c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/
(f*(a + a*Sin[e + f*x])^(3/2))) - (4*c*((c*Cos[e + f*x]*(c - c*Sin[e + f*x
])^(5/2))/(3*f*Sqrt[a + a*Sin[e + f*x]]) + 2*c*((c*Cos[e + f*x]*(c - c*Sin
[e + f*x])^(3/2))/(2*f*Sqrt[a + a*Sin[e + f*x]]) + 2*c*((2*c^2*Cos[e + f*x
]*Log[a + a*Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e +
f*x]]) + (c*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f
*x]])))))/a))/(4*a)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3218
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Simp[b*((2*m - 1)/(d*(
2*n + 1)))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m +
n + 1, 0])
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 637 vs. $2(293) = 586$.

Time = 8.76 (sec) , antiderivative size = 638, normalized size of antiderivative = 1.98

method	result
default	$\frac{A\sqrt{4}\left(8\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{8}+32\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{6}+192\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\right)\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}-192\ln\left(\frac{\cdots}{\cos\left(\frac{\pi}{4}+\cdots\right.}\right.\right.}{8f\sqrt{a\,\mathrm{si}\cdots}}$
parts	$-\frac{A\sqrt{4}\left(-8\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{8}-32\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{6}+192\ln\left(\frac{2}{\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}-192\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\mathrm{c}\cdots\right.\right.}{8f\sqrt{\cdots}}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/8*A/f*4^(1/2)*(8*cos(1/4*Pi+1/2*f*x+1/2*e)^8+32*cos(1/4*Pi+1/2*f*x+1/2*e
)^6+192*ln(-cot(1/4*Pi+1/2*f*x+1/2*e)+csc(1/4*Pi+1/2*f*x+1/2*e))*sin(1/4*P
i+1/2*f*x+1/2*e)^4-192*ln(2/(cos(1/4*Pi+1/2*f*x+1/2*e)+1))*sin(1/4*Pi+1/2*
f*x+1/2*e)^4-173*cos(1/4*Pi+1/2*f*x+1/2*e)^4+154*cos(1/4*Pi+1/2*f*x+1/2*e)
^2-29)*(c*cos(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)*c^4/(a*sin(1/4*Pi+1/2*f*x+1/2
*e)^2)^(1/2)/a^2*sec(1/4*Pi+1/2*f*x+1/2*e)*csc(1/4*Pi+1/2*f*x+1/2*e)^3+2/3
*B/f*c^4/a^2*(168*(4*cos(1/2*f*x+1/2*e)^4-4*cos(1/2*f*x+1/2*e)^2-4*sin(1/2
*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-1)*ln(-2*(cos(1/2*f*x+1/2*e)+sin(1/2*f*x+1/
2*e))/(cos(1/2*f*x+1/2*e)+1))+168*(-4*cos(1/2*f*x+1/2*e)^4+4*sin(1/2*f*x+1
/2*e)*cos(1/2*f*x+1/2*e)+4*cos(1/2*f*x+1/2*e)^2+1)*ln(2/(cos(1/2*f*x+1/2*e
)+1))+cos(1/2*f*x+1/2*e)*((-16*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)^6+68*
cos(1/2*f*x+1/2*e)^5+16*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)^4-68*cos(1/2
*f*x+1/2*e)^3+208*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)^2+507*cos(1/2*f*x+
1/2*e))*sin(1/2*f*x+1/2*e)^2+168*sin(1/2*f*x+1/2*e)))*(-(2*sin(1/2*f*x+1/2
*e)*cos(1/2*f*x+1/2*e)-1)*c)^(1/2)/((2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*
e)+1)*a)^(1/2)/(4*cos(1/2*f*x+1/2*e)^3*sin(1/2*f*x+1/2*e)+2*cos(1/2*f*x+1/
2*e)^2-2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-1)
```

Fricas [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx=\int \frac{(B\sin\left(fx+e\right)+A)(-c\sin\left(fx+e\right)+c)^{\frac{9}{2}}}{(a\sin\left(fx+e\right)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x
, algorithm="fricas")
```

output
```
integral(-((A - 4*B)*c^4*cos(f*x + e)^4 - 4*(2*A - 3*B)*c^4*cos(f*x + e)^2
 + 8*(A - B)*c^4 + (B*c^4*cos(f*x + e)^4 + 4*(A - 2*B)*c^4*cos(f*x + e)^2
- 8*(A - B)*c^4)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f
*x + e)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(9/2)/(a+a*sin(f*x+e))**(5/2)
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{9}{2}}}{(a\sin(fx+e)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(9/2)/(a*sin(f*x + e)
 + a)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{9/2}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x))
^(5/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x))
^(5/2), x)
```

Reduce [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x)
```

output

```
(sqrt(c)*sqrt(a)*c**4*(int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) +
1)*sin(e + f*x)**5)/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x)
+ 1),x)*b + int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e +
f*x)**4)/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x) + 1),x)*a -
 4*int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**4)/
(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x) + 1),x)*b - 4*int((s
qrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**3)/(sin(e +
f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x) + 1),x)*a + 6*int((sqrt(sin(e
 + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**3)/(sin(e + f*x)**3 +
 3*sin(e + f*x)**2 + 3*sin(e + f*x) + 1),x)*b + 6*int((sqrt(sin(e + f*x) +
 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x)**3 + 3*sin(e
+ f*x)**2 + 3*sin(e + f*x) + 1),x)*a - 4*int((sqrt(sin(e + f*x) + 1)*sqrt(
 - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x)**3 + 3*sin(e + f*x)**2
 + 3*sin(e + f*x) + 1),x)*b - 4*int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e
+ f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e +
 f*x) + 1),x)*a + int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*si
n(e + f*x))/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x) + 1),x)*
b + int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1))/(sin(e + f*x)**
3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x) + 1),x)*a))/a**3
```

3.188 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 263

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{6(A-3B)c^4\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{3(A-3B)c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} + \frac{3(A-3B)c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(A-3B)c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{2af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f(a+a\sin(e+fx))^{5/2}}$$

output

```
6*(A-3*B)*c^4*cos(f*x+e)*ln(1+sin(f*x+e))/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-
c*sin(f*x+e))^(1/2)+3*(A-3*B)*c^3*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f/
(a+a*sin(f*x+e))^(1/2)+3/4*(A-3*B)*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a
^2/f/(a+a*sin(f*x+e))^(1/2)+1/2*(A-3*B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2
)/a/f/(a+a*sin(f*x+e))^(3/2)-1/4*(A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/f
/(a+a*sin(f*x+e))^(5/2)
```

Mathematica [A] (verified)

Time = 13.13 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.92

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c-c\sin(e+fx))}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e +
 f*x])^(5/2),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^(7/2)*(-16*(A
- B) + 16*(3*A - 5*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + B*Cos[2*(e
 + f*x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 + 48*(A - 3*B)*Log[Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 4
*(A - 6*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*Sin[e + f*x]))/(4*f*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(5/2))
```

Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.95, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.325$, Rules used = {3042, 3451, 3042, 3218, 3042, 3219, 3042, 3219, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-c\sin(e+fx))^{7/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3451}$$

$$-\frac{(A-3B)\int \frac{(c-c\sin(e+fx))^{7/2}}{(\sin(e+fx)a+a)^{3/2}}dx}{2a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(A-3B)\int \frac{(c-c\sin(e+fx))^{7/2}}{(\sin(e+fx)a+a)^{3/2}}dx}{2a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{3218}$$

$$-\frac{(A-3B)\left(-\frac{3c\int \frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{\sin(e+fx)a+a}}dx}{a}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{2a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(A-3B)\left(-\frac{3c\int \frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{\sin(e+fx)a+a}}dx}{a}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{2a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{3219}$$

$$-\frac{(A-3B)\left(-\frac{3c\left(2c\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{2a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(A-3B)\left(-\frac{3c\left(2c\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{2a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{3219}$$

$$-\frac{(A-3B)\left(-\frac{3c\left(2c\left(2c\int\frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{c\cos(e+fx)(c-c\sin(e+fx)}{f(a\sin(e+fx)+a)^{3/2}}}{2a}$$

$$\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$-\frac{(A-3B)\left(-\frac{3c\left(2c\left(2c\int\frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{c\cos(e+fx)(c-c\sin(e+fx)}{f(a\sin(e+fx)+a)^{3/2}}}{2a}$$

$$\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$\Big\downarrow$ 3216

$$-\frac{(A-3B)\left(-\frac{3c\left(2c\left(\frac{2ac^2\cos(e+fx)\int\frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{c\cos(e+fx)(c-}{f(a\sin(e+}}{2a}$$

$$\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$-\frac{(A-3B)\left(-\frac{3c\left(2c\left(\frac{2ac^2\cos(e+fx)\int\frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{c\cos(e+fx)(c-}{f(a\sin(e+}}{2a}$$

$$\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$\Big\downarrow$ 3146

$$-\frac{(A-3B)\left(-\frac{3c\left(2c\left(\frac{2c^2\cos(e+fx)\int\frac{1}{\sin(e+fx)a+a}d(a\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{c\cos(e\cdots}{f(\cdots}\right.}{2a}$$
$$\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$\downarrow$ 16

$$-\frac{(A-3B)\left(-\frac{3c\left(2c\left(\frac{2c^2\cos(e+fx)\log(a\sin(e+fx)+a)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{c\cos(e+fx)(c-c\,s\cdots}{f(a\sin(e+fx\cdots}\right.}{2a}$$
$$\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])
^(5/2),x]
```

output
```
-1/4*((A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(f*(a + a*Sin[e + f
*x])^(5/2)) - ((A - 3*B)*(-((c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(f
*(a + a*Sin[e + f*x])^(3/2))) - (3*c*((c*Cos[e + f*x]*(c - c*Sin[e + f*x])
^(3/2))/(2*f*Sqrt[a + a*Sin[e + f*x]]) + 2*c*((2*c^2*Cos[e + f*x]*Log[a +
a*Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (
c*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]))))/a
))/(2*a)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3218
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Simp[b*((2*m - 1)/(d*(
2*n + 1)))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m +
n + 1, 0])
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 577 vs. 2(237) = 474.

Time = 8.13 (sec) , antiderivative size = 578, normalized size of antiderivative = 2.20

method	result
default	$\frac{A\sqrt{4}\left(16\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{6}+96\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\right)\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}-96\ln\left(\frac{2}{\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\sin\left(\frac{\pi}{4}+\frac{fx}{2}\right.}{16f\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right.}}$
parts	$-\frac{A\sqrt{4}\left(-16\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{6}+96\ln\left(\frac{2}{\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}-96\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\right)\sin\left(\frac{\pi}{4}\right.}{16f\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}\right.}}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/16*A/f*4^(1/2)*(16*cos(1/4*Pi+1/2*f*x+1/2*e)^6+96*ln(-cot(1/4*Pi+1/2*f*x
+1/2*e)+csc(1/4*Pi+1/2*f*x+1/2*e))*sin(1/4*Pi+1/2*f*x+1/2*e)^4-96*ln(2/(co
s(1/4*Pi+1/2*f*x+1/2*e)+1))*sin(1/4*Pi+1/2*f*x+1/2*e)^4-69*cos(1/4*Pi+1/2*
f*x+1/2*e)^4+42*cos(1/4*Pi+1/2*f*x+1/2*e)^2+3)*(c*cos(1/4*Pi+1/2*f*x+1/2*e
)^2)^(1/2)*c^3/(a*sin(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)/a^2*sec(1/4*Pi+1/2*f*
x+1/2*e)*csc(1/4*Pi+1/2*f*x+1/2*e)^3-2*B/f*c^3/a^2*(18*(-4*cos(1/2*f*x+1/2
*e)^4+4*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+4*cos(1/2*f*x+1/2*e)^2+1)*ln
(-2*(cos(1/2*f*x+1/2*e)+sin(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1))+18*(4*
cos(1/2*f*x+1/2*e)^4-4*cos(1/2*f*x+1/2*e)^2-4*sin(1/2*f*x+1/2*e)*cos(1/2*f
*x+1/2*e)-1)*ln(2/(cos(1/2*f*x+1/2*e)+1))+cos(1/2*f*x+1/2*e)*((-4*cos(1/2*
f*x+1/2*e)^5+4*cos(1/2*f*x+1/2*e)^3-20*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*
e)^2-55*cos(1/2*f*x+1/2*e))*sin(1/2*f*x+1/2*e)^2-18*sin(1/2*f*x+1/2*e)))*(
-(2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-1)*c)^(1/2)/((2*sin(1/2*f*x+1/2*
e)*cos(1/2*f*x+1/2*e)+1)*a)^(1/2)/(4*cos(1/2*f*x+1/2*e)^3*sin(1/2*f*x+1/2*
e)+2*cos(1/2*f*x+1/2*e)^2-2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-1)
```

Fricas [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{7}{2}}}{(a\sin(fx+e)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x
, algorithm="fricas")
```

output
```
integral((B*c^3*cos(f*x + e)^4 + (3*A - 5*B)*c^3*cos(f*x + e)^2 - 4*(A - B
)*c^3 - ((A - 3*B)*c^3*cos(f*x + e)^2 - 4*(A - B)*c^3)*sin(f*x + e))*sqrt(
a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^
3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**(5/2)
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{7}{2}}}{(a\sin(fx+e)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(7/2)/(a*sin(f*x + e)
 + a)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(c-c\,\sin(e+f\,x))^{7/2}}{(a+a\,\sin(e+f\,x))^{5/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))
^(5/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))
^(5/2), x)
```

Reduce [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,c^3\left(-\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^4}{\sin(fx+e)^3+3\sin(fx+e)^2+3\sin(fx+e)+1}dx\right) b\right.}{}$$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x)
```

output
```
(sqrt(c)*sqrt(a)*c**3*( - int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x)
 + 1)*sin(e + f*x)**4)/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*
x) + 1),x)*b - int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e
 + f*x)**3)/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x) + 1),x)*
a + 3*int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**
3)/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x) + 1),x)*b + 3*int
((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e
 + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x) + 1),x)*a - 3*int((sqrt(si
n(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x)**
3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x) + 1),x)*b - 3*int((sqrt(sin(e + f*x
) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**3 + 3*sin(e
+ f*x)**2 + 3*sin(e + f*x) + 1),x)*a + int((sqrt(sin(e + f*x) + 1)*sqrt( -
 sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*
sin(e + f*x) + 1),x)*b + int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x)
+ 1))/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x) + 1),x)*a))/a*
*3
```

3.189 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 211

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{(A-5B)c^3\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{(A-5B)c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{2a^2f\sqrt{a+a\sin(e+fx)}} + \frac{(A-5B)c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{4f(a+a\sin(e+fx))^{5/2}}$$

output
```
(A-5*B)*c^3*cos(f*x+e)*ln(1+sin(f*x+e))/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*
sin(f*x+e))^(1/2)+1/2*(A-5*B)*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f/
(a+a*sin(f*x+e))^(1/2)+1/4*(A-5*B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a/f
/(a+a*sin(f*x+e))^(3/2)-1/4*(A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/f/(a+a
*sin(f*x+e))^(5/2)
```

Mathematica [A] (verified)

Time = 11.77 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.94

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c-c\sin(e+fx))}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e +
 f*x])^(5/2),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^(5/2)*(-2*A +
2*B + 4*(A - 2*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 2*(A - 5*B)*Lo
g[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])^4 + B*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*Sin[e + f*x]))/(f*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + Sin[e + f*x]))^(5/2))
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.96, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.275$, Rules used = {3042, 3451, 3042, 3218, 3042, 3219, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-c\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3451}$$

$$-\frac{(A-5B)\int \frac{(c-c\sin(e+fx))^{5/2}}{(\sin(e+fx)a+a)^{3/2}}dx}{4a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(A-5B)\int \frac{(c-c\sin(e+fx))^{5/2}}{(\sin(e+fx)a+a)^{3/2}}dx}{4a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$\Big\downarrow$ 3218

$$-\frac{(A-5B)\left(-\frac{2c\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{\sin(e+fx)a+a}}dx}{a}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{4a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$-\frac{(A-5B)\left(-\frac{2c\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{\sin(e+fx)a+a}}dx}{a}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{4a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$\Big\downarrow$ 3219

$$-\frac{(A-5B)\left(-\frac{2c\left(2c\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{4a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$-\frac{(A-5B)\left(-\frac{2c\left(2c\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{4a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$\Big\downarrow$ 3216

$$-\frac{(A-5B)\left(-\frac{2c\left(\frac{2ac^2\cos(e+fx)\int \frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{4a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(A-5B)\left(-\frac{2c\left(\frac{2ac^2\cos(e+fx)\int\frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{4a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{3146}$$

$$-\frac{(A-5B)\left(-\frac{2c\left(\frac{2c^2\cos(e+fx)\int\frac{1}{\sin(e+fx)a+a}d(a\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{4a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{16}$$

$$-\frac{(A-5B)\left(-\frac{2c\left(\frac{2c^2\cos(e+fx)\log(a\sin(e+fx)+a)}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f\sqrt{a\sin(e+fx)+a}}\right)}{a}-\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{4a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

input

```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])
^(5/2),x]
```

output

```
-1/4*((A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(f*(a + a*Sin[e + f
*x])^(5/2)) - ((A - 5*B)*(-((c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(f
*(a + a*Sin[e + f*x])^(3/2))) - (2*c*((2*c^2*Cos[e + f*x]*Log[a + a*Sin[e
+ f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (c*Cos[e
+ f*x]*Sqrt[c - c*Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]])))/a))/(4*a)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3218
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Simp[b*((2*m - 1)/(d*(
2*n + 1)))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m +
n + 1, 0])
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 533 vs. $2(189) = 378$.

Time = 7.60 (sec) , antiderivative size = 534, normalized size of antiderivative = 2.53

method	result
default	$\frac{A\sqrt{4}\left(32\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\right)\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}-32\ln\left(\frac{2}{\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}-13\cos\left(\frac{\pi}{4}+\frac{fx}{2}\right.\right.}{32f\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,a^{2}}$
parts	$-\frac{A\sqrt{4}\left(32\ln\left(\frac{2}{\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}-32\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\right)\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}+13\cos\left(\frac{\pi}{4}+\right.\right.}{32f\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,a^{2}}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/32*A/f*4^(1/2)*(32*ln(-cot(1/4*Pi+1/2*f*x+1/2*e)+csc(1/4*Pi+1/2*f*x+1/2*
e))*sin(1/4*Pi+1/2*f*x+1/2*e)^4-32*ln(2/(cos(1/4*Pi+1/2*f*x+1/2*e)+1))*sin
(1/4*Pi+1/2*f*x+1/2*e)^4-13*cos(1/4*Pi+1/2*f*x+1/2*e)^4-6*cos(1/4*Pi+1/2*f
*x+1/2*e)^2+11)*(c*cos(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)*c^2/(a*sin(1/4*Pi+1/
2*f*x+1/2*e)^2)^(1/2)/a^2*sec(1/4*Pi+1/2*f*x+1/2*e)*csc(1/4*Pi+1/2*f*x+1/2
*e)^3-2*B/f*((-20*cos(1/2*f*x+1/2*e)^4+20*cos(1/2*f*x+1/2*e)^2+20*sin(1/2*
f*x+1/2*e)*cos(1/2*f*x+1/2*e)+5)*ln(-2*(cos(1/2*f*x+1/2*e)+sin(1/2*f*x+1/2
*e))/(cos(1/2*f*x+1/2*e)+1))+(20*cos(1/2*f*x+1/2*e)^4-20*cos(1/2*f*x+1/2*e
)^2-20*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-5)*ln(2/(cos(1/2*f*x+1/2*e)+1
))+cos(1/2*f*x+1/2*e)*((-4*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)^2-16*cos(
1/2*f*x+1/2*e))*sin(1/2*f*x+1/2*e)^2-5*sin(1/2*f*x+1/2*e)))*(-(2*sin(1/2*f
*x+1/2*e)*cos(1/2*f*x+1/2*e)-1)*c)^(1/2)*c^2/((2*sin(1/2*f*x+1/2*e)*cos(1/
2*f*x+1/2*e)+1)*a)^(1/2)/a^2/(4*cos(1/2*f*x+1/2*e)^3*sin(1/2*f*x+1/2*e)+2*
cos(1/2*f*x+1/2*e)^2-2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-1)
```

Fricas [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx=\int \frac{(B\sin\left(fx+e\right)+A)(-c\sin\left(fx+e\right)+c)^{\frac{5}{2}}}{(a\sin\left(fx+e\right)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x
, algorithm="fricas")
```

output
```
integral(((A - 2*B)*c^2*cos(f*x + e)^2 - 2*(A - B)*c^2 + (B*c^2*cos(f*x +
e)^2 + 2*(A - B)*c^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f
*x + e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*
sin(f*x + e)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(5/2)
,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 504 vs. $2(189) = 378$.

Time = 0.16 (sec) , antiderivative size = 504, normalized size of antiderivative = 2.39

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x
, algorithm="maxima")
```

output
```
((8*sqrt(a)*c^(5/2)*sin(f*x + e)^2/((a^3 + 4*a^3*sin(f*x + e)/(cos(f*x + e
) + 1) + 6*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^3*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)*(cos(f*x +
 e) + 1)^2) - 2*c^(5/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^(5/2) +
 c^(5/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/a^(5/2))*A + B*(10*c
^(5/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^(5/2) - 5*c^(5/2)*log(si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/a^(5/2) - 2*(5*c^(5/2)*sin(f*x + e)
/(cos(f*x + e) + 1) + 16*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 14*
c^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 16*c^(5/2)*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 + 5*c^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^(5/
2) + 4*a^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 7*a^(5/2)*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 8*a^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 7*a
^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 4*a^(5/2)*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5 + a^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)))/f
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,\left(c-c\,\sin(e+f\,x)\right)^{5/2}}{\left(a+a\,\sin(e+f\,x)\right)^{5/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))
^(5/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))
^(5/2), x)
```

Reduce [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,c^2\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^3}{\sin(fx+e)^3+3\sin(fx+e)^2+3\sin(fx+e)+1}dx\right)b+\right.}{}$$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x)
```

output

```
(sqrt(c)*sqrt(a)*c**2*(int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) +
1)*sin(e + f*x)**3)/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x)
+ 1),x)*b + int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e +
f*x)**2)/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x) + 1),x)*a -
 2*int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x)**2)/
(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x) + 1),x)*b - 2*int((s
qrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x
)**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x) + 1),x)*a + int((sqrt(sin(e + f*
x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**3 + 3*sin(e
 + f*x)**2 + 3*sin(e + f*x) + 1),x)*b + int((sqrt(sin(e + f*x) + 1)*sqrt(
- sin(e + f*x) + 1))/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x)
 + 1),x)*a))/a**3
```

3.190 $\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 149

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{Bc^2\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{Bc\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4f(a+a\sin(e+fx))^{5/2}}$$

output
```
-B*c^2*cos(f*x+e)*ln(1+sin(f*x+e))/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f
*x+e))^(1/2)-B*c*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f/(a+a*sin(f*x+e))^(3
/2)-1/4*(A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^(5/2)
```

Mathematica [A] (verified)

Time = 11.75 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.20

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+f}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e +
 f*x])^(5/2),x]
```

output
```
(c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(B*Cos[2
*(e + f*x)]*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] - B*(2 + 3*Log[Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2]]) + (A - 3*B - 4*B*Log[Cos[(e + f*x)/2] + Si
n[(e + f*x)/2]])*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(
a*(1 + Sin[e + f*x]))^(5/2))
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.225$, Rules used = {3042, 3451, 3042, 3218, 3042, 3216, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c-c\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3451}$$

$$\frac{B\int \frac{(c-c\sin(e+fx))^{3/2}}{(\sin(e+fx)a+a)^{3/2}}dx}{a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{B\int \frac{(c-c\sin(e+fx))^{3/2}}{(\sin(e+fx)a+a)^{3/2}}dx}{a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{3218}$$

$$\frac{B\left(-\frac{c\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx}{a}-\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{B\left(-\frac{c\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{\sin(e+fx)a+a}}dx}{a}-\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{3216}$$

$$\frac{B\left(-\frac{c^2\cos(e+fx)\int \frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{B\left(-\frac{c^2\cos(e+fx)\int \frac{\cos(e+fx)}{\sin(e+fx)a+a}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{3146}$$

$$\frac{B\left(-\frac{c^2\cos(e+fx)\int \frac{1}{\sin(e+fx)a+a}d(a\sin(e+fx))}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{16}$$

$$\frac{B\left(-\frac{c^2\cos(e+fx)\log(a\sin(e+fx)+a)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{f(a\sin(e+fx)+a)^{3/2}}\right)}{a}-\frac{(A-B)\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{4f(a\sin(e+fx)+a)^{5/2}}$$

input
```
Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])
^(5/2),x]
```

output
```
-1/4*((A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(f*(a + a*Sin[e + f
*x])^(5/2)) + (B*(-((c^2*Cos[e + f*x]*Log[a + a*Sin[e + f*x]])/(a*f*Sqrt[a
 + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])) - (c*Cos[e + f*x]*Sqrt[c - c
*Sin[e + f*x]])/(f*(a + a*Sin[e + f*x])^(3/2))))/a
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3216
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Simp[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]]))   Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
]
```

rule 3218
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Simp[b*((2*m - 1)/(d*(
2*n + 1)))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m +
n + 1, 0])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(135) = 270$.

Time = 7.42 (sec) , antiderivative size = 425, normalized size of antiderivative = 2.85

method	result
parts	$-\frac{A\sqrt{4}\left(5\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}+6\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}-3\right)c\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\sec\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{3}}{64f\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,a^{2}}+\frac{2B\left(\left(4\cos\left(\frac{f}{}\right.\right.\right.}{}$
default	$-\frac{A\sqrt{4}\left(5\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}+6\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}-3\right)c\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\sec\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{3}}{64f\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,a^{2}}-\frac{2B\left(\left(-4\cos\left(\right.\right.\right.}{}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x,metho
d=_RETURNVERBOSE)
```

output
```
-1/64*A/f*4^(1/2)*(5*cos(1/4*Pi+1/2*f*x+1/2*e)^4+6*cos(1/4*Pi+1/2*f*x+1/2*
e)^2-3)*c*(c*cos(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)/(a*sin(1/4*Pi+1/2*f*x+1/2*
e)^2)^(1/2)/a^2*sec(1/4*Pi+1/2*f*x+1/2*e)*csc(1/4*Pi+1/2*f*x+1/2*e)^3+2*B/
f*((4*cos(1/2*f*x+1/2*e)^4-4*cos(1/2*f*x+1/2*e)^2-4*sin(1/2*f*x+1/2*e)*cos
(1/2*f*x+1/2*e)-1)*ln(-2*(cos(1/2*f*x+1/2*e)+sin(1/2*f*x+1/2*e))/(cos(1/2*
f*x+1/2*e)+1))+(-4*cos(1/2*f*x+1/2*e)^4+4*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1
/2*e)+4*cos(1/2*f*x+1/2*e)^2+1)*ln(2/(cos(1/2*f*x+1/2*e)+1))+cos(1/2*f*x+1
/2*e)*(4*cos(1/2*f*x+1/2*e)*sin(1/2*f*x+1/2*e)^2+sin(1/2*f*x+1/2*e)))*(-(2
*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-1)*c)^(1/2)*c/((2*sin(1/2*f*x+1/2*e
)*cos(1/2*f*x+1/2*e)+1)*a)^(1/2)/a^2/(4*cos(1/2*f*x+1/2*e)^3*sin(1/2*f*x+1
/2*e)+2*cos(1/2*f*x+1/2*e)^2-2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-1)
```

Fricas [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{3}{2}}}{(a\sin(fx+e)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x
, algorithm="fricas")
```

output
```
integral(-(B*c*cos(f*x + e)^2 - (A - B)*c*sin(f*x + e) + (A - B)*c)*sqrt(a
*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3
 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

Sympy [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{(-c(\sin(e+fx)-1))^{\frac{3}{2}}(A+B\sin(e+fx))}{(a(\sin(e+fx)+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(5/2)
,x)
```

output
```
Integral((-c*(sin(e + f*x) - 1))**(3/2)*(A + B*sin(e + f*x))/(a*(sin(e + f
*x) + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{3}{2}}}{(a\sin(fx+e)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e)
 + a)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,\left(c-c\,\sin(e+f\,x)\right)^{3/2}}{\left(a+a\,\sin(e+f\,x)\right)^{5/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))
^(5/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))
^(5/2), x)
```

Reduce [F]

$$\int \frac{(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,c\left(-\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^{2}}{\sin(fx+e)^{3}+3\sin(fx+e)^{2}+3\sin(fx+e)+1}dx\right)b\right.}{}$$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x)
```

output
```
(sqrt(c)*sqrt(a)*c*( - int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) +
1)*sin(e + f*x)**2)/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x)
+ 1),x)*b - int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e +
f*x))/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x) + 1),x)*a + in
t((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*sin(e + f*x))/(sin(e +
 f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x) + 1),x)*b + int((sqrt(sin(e
+ f*x) + 1)*sqrt( - sin(e + f*x) + 1))/(sin(e + f*x)**3 + 3*sin(e + f*x)**
2 + 3*sin(e + f*x) + 1),x)*a))/a**3
```

3.191 $\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 94

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(A-B)c\cos(e+fx)}{2f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{Bc\cos(e+fx)}{af(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}$$

output
```
-1/2*(A-B)*c*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2)-B*
c*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 3.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.05

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{\sqrt{a(1+\sin(e+fx))}(A+B+2B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{2a^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

input
```
Integrate[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f
*x])^(5/2),x]
```

output
```
-1/2*(Sqrt[a*(1 + Sin[e + f*x])]*(A + B + 2*B*Sin[e + f*x])*Sqrt[c - c*Sin
[e + f*x]])/(a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^5)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3042, 3450, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-c\sin(e+fx)}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^{5/2}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sqrt{c-c\sin(e+fx)}(A+B\sin(e+fx))}{(a\sin(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \ 3450$$

$$(A-B)\int \frac{\sqrt{c-c\sin(e+fx)}}{(\sin(e+fx)a+a)^{5/2}}dx+\frac{B\int \frac{\sqrt{c-c\sin(e+fx)}}{(\sin(e+fx)a+a)^{3/2}}dx}{a}$$

$$\Big\downarrow \ 3042$$

$$(A-B)\int \frac{\sqrt{c-c\sin(e+fx)}}{(\sin(e+fx)a+a)^{5/2}}dx+\frac{B\int \frac{\sqrt{c-c\sin(e+fx)}}{(\sin(e+fx)a+a)^{3/2}}dx}{a}$$

$$\Big\downarrow \ 3217$$

$$-\frac{c(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}-\frac{Bc\cos(e+fx)}{af(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

input
```
Int[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^(
5/2),x]
```

output
```
-1/2*((A - B)*c*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin
[e + f*x]]) - (B*c*Cos[e + f*x])/(a*f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c -
c*Sin[e + f*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3450
```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[B/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] -
Simp[(B*c - A*d)/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[
a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(84) = 168$.

Time = 9.23 (sec), antiderivative size = 206, normalized size of antiderivative = 2.19

method	result
default	$\frac{3\left(A\left(\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{2}\right)\left(\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}-2\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}-\frac{5}{3}\right)\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-\frac{1}{2}\right)\sec\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\csc\left(\frac{\pi}{4}+\frac{fx}{2}\right.}{16\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,f\,a^{2}\left(-1+2\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\right)\left(2\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\cos\left(\frac{fx}{2}+\right.\right.}$
parts	$\frac{A\sqrt{4}\left(3\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}-6\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}-5\right)\sqrt{c\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\sec\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{3}}{128f\sqrt{a\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{2}}\,a^{2}}+\frac{2B\sqrt{}}{a^{2}f\left(-1+2\cos\left(\right.\right.}$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x,metho
d=_RETURNVERBOSE)
```

output
```
3/16/(a*sin(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)*(A*(sin(1/2*f*x+1/2*e)*cos(1/2*
f*x+1/2*e)+1/2)*(cos(1/4*Pi+1/2*f*x+1/2*e)^4-2*cos(1/4*Pi+1/2*f*x+1/2*e)^2
-5/3)*(cos(1/2*f*x+1/2*e)^2-1/2)*sec(1/4*Pi+1/2*f*x+1/2*e)*csc(1/4*Pi+1/2*
f*x+1/2*e)^3+32/3*B*sin(1/2*f*x+1/2*e)^2*cos(1/2*f*x+1/2*e)^2)*(c*cos(1/4*
Pi+1/2*f*x+1/2*e)^2)^(1/2)/f/a^2/(-1+2*cos(1/2*f*x+1/2*e)^2)/(2*sin(1/2*f*
x+1/2*e)*cos(1/2*f*x+1/2*e)+1)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec), antiderivative size = 85, normalized size of antiderivative = 0.90

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{(2\,B\sin\left(fx+e\right)+A+B)\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\,\mathrm{si}}}{2\left(a^{3}f\cos\left(fx+e\right)^{3}-2\,a^{3}f\cos\left(fx+e\right)\sin\left(fx+e\right)-2\right.}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x
, algorithm="fricas")
```

output
```
1/2*(2*B*sin(f*x + e) + A + B)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
e) + c)/(a^3*f*cos(f*x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*
f*cos(f*x + e))
```

Sympy [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\left(A+B\sin\left(e+fx\right)\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(5/2)
,x)
```

output
```
Integral(sqrt(-c*(sin(e + f*x) - 1))*(A + B*sin(e + f*x))/(a*(sin(e + f*x)
 + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{\left(B\sin\left(fx+e\right)+A\right)\sqrt{-c\sin\left(fx+e\right)+c}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)/(a*sin(f*x + e) +
 a)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 37.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.66

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx =$$
$$-\frac{2\sqrt{-c\,(\sin(e+f\,x)-1)}\,\left(A\,\sin(2\,e+2\,f\,x)+3\,B\,\sin(2\,e+2\,f\,x)-2\,A\,\left(2\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)-3\,B}{a^2\,f\,\sqrt{a\,(\sin(e+f\,x)+1)}\,\left(-8\sin(e+f\,x)^2+4\,\sin(e+f\,x)+2\sin(2\,e+2\,f}$$

input
```
int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))
^(5/2),x)
```

output
```
-(2*(-c*(sin(e + f*x) - 1))^(1/2)*(A*sin(2*e + 2*f*x) + 3*B*sin(2*e + 2*f*
x) - 2*A*(2*sin(e/2 + (f*x)/2)^2 - 1) - 3*B*(2*sin(e/2 + (f*x)/2)^2 - 1) +
 B*(2*sin((3*e)/2 + (3*f*x)/2)^2 - 1)))/(a^2*f*(a*(sin(e + f*x) + 1))^(1/2
)*(4*sin(e + f*x) + 4*sin(3*e + 3*f*x) + 2*sin(2*e + 2*f*x)^2 - 8*sin(e +
f*x)^2 + 8))
```

Reduce [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)^3+3\sin(fx+e)^2+3\sin(fx+e)+1}dx\right)b+\left(\int\right.\right.}{a^3}$$

input
```
int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x)
```

output
```
(sqrt(c)*sqrt(a)*(int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*si
n(e + f*x))/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x) + 1),x)*
b + int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1))/(sin(e + f*x)**
3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x) + 1),x)*a))/a**3
```

3.192 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 151

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{(A+B)\cos(e+fx)}{4af(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{(A+B)\text{arctanh}(\sin(e+fx))\cos(e+fx)}{4a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

output
```
-1/4*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2)-1/4*
(A+B)*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2)+1/4*(A+
B)*arctanh(sin(f*x+e))*cos(f*x+e)/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*
x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 4.14 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.42

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\right.}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[
e + f*x]]),x]
```

output
```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])*(-A + B - (A + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (A + B)*Log
[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^4 + (A + B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^4))/(4*f*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c - c*Sin[e
+ f*x]])
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3451, 3042, 3222, 3042, 3220, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3451}$$

$$\frac{(A+B)\int \frac{1}{(\sin(e+fx)a+a)^{3/2}\sqrt{c-c\sin(e+fx)}}dx}{2a}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{(A+B)\int \frac{1}{(\sin(e+fx)a+a)^{3/2}\sqrt{c-c\sin(e+fx)}}dx}{2a}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \; 3222$$

$$\frac{(A+B)\left(\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}}dx}{2a}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}\right)}{2a}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{(A+B)\left(\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}}dx}{2a}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}\right)}{2a}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \; 3220$$

$$\frac{(A+B)\left(\frac{\cos(e+fx)\int \sec(e+fx)dx}{2a\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}\right)}{2a}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{(A+B)\left(\frac{\cos(e+fx)\int \csc\left(e+fx+\frac{\pi}{2}\right)dx}{2a\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}\right)}{2a}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \; 4257$$

$$\frac{(A+B)\left(\frac{\cos(e+fx)\text{arctanh}(\sin(e+fx))}{2af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}\right)}{2a}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*
x]]),x]
```

output
```
-1/4*((A - B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e
 + f*x]]) + ((A + B)*(-1/2*Cos[e + f*x]/(f*(a + a*Sin[e + f*x])^(3/2)*Sqrt
[c - c*Sin[e + f*x]]) + (ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*a*f*Sqrt[a
 + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])))/(2*a)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3220
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]])   Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

rule 3222
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp[(m + n + 1)/(a*(2*m + 1)
)   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
 ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||
!SumSimplerQ[n, 1])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 646 vs. $2(133) = 266$.

Time = 8.33 (sec) , antiderivative size = 647, normalized size of antiderivative = 4.28

method	result
default	$\frac{A\sqrt{4}\left(32\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\right)\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}-32\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\right.\right.}{256f\sqrt{a\sin\left(\right.}}$
parts	$\frac{A\sqrt{4}\left(32\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)\right)\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^{4}-32\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)-1\right)\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\right.\right.}{256f\sqrt{a\sin\left(\right.}}$

input

```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output

```
1/256*A/f*4^(1/2)*(32*ln(-cot(1/4*Pi+1/2*f*x+1/2*e)+csc(1/4*Pi+1/2*f*x+1/2
*e))*sin(1/4*Pi+1/2*f*x+1/2*e)^4-32*ln(-cot(1/4*Pi+1/2*f*x+1/2*e)+csc(1/4*
Pi+1/2*f*x+1/2*e)-1)*sin(1/4*Pi+1/2*f*x+1/2*e)^4-32*ln(-cot(1/4*Pi+1/2*f*x
+1/2*e)+csc(1/4*Pi+1/2*f*x+1/2*e)+1)*sin(1/4*Pi+1/2*f*x+1/2*e)^4+11*cos(1/
4*Pi+1/2*f*x+1/2*e)^4-6*cos(1/4*Pi+1/2*f*x+1/2*e)^2-13)/(a*sin(1/4*Pi+1/2*
f*x+1/2*e)^2)^(1/2)/(c*cos(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)/a^2*cot(1/4*Pi+1
/2*f*x+1/2*e)*csc(1/4*Pi+1/2*f*x+1/2*e)^2+1/4*B/a^2/f*(((2*cos(1/2*f*x+1/2
*e)^2+1)*(-1+2*cos(1/2*f*x+1/2*e)^2)*sin(1/2*f*x+1/2*e)-cos(1/2*f*x+1/2*e)
*(2*cos(1/2*f*x+1/2*e)^2-3)*(-1+2*cos(1/2*f*x+1/2*e)^2))*ln(-2*(cos(1/2*f*
x+1/2*e)+sin(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1))+(-(2*cos(1/2*f*x+1/2*
e)^2+1)*(-1+2*cos(1/2*f*x+1/2*e)^2)*sin(1/2*f*x+1/2*e)+cos(1/2*f*x+1/2*e)*
(2*cos(1/2*f*x+1/2*e)^2-3)*(-1+2*cos(1/2*f*x+1/2*e)^2))*ln(2*(sin(1/2*f*x+
1/2*e)-cos(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1))+cos(1/2*f*x+1/2*e)*(-2*
sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+2*sin(1/2*f*x+1/2*e)^2))/(2*sin(1/2*
f*x+1/2*e)*cos(1/2*f*x+1/2*e)^2-2*cos(1/2*f*x+1/2*e)^3+sin(1/2*f*x+1/2*e)+
3*cos(1/2*f*x+1/2*e))/((2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+1)*a)^(1/2
)/(-(2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-1)*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.69

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx = \left[\frac{\left((A+B)\cos\left(fx+e\right)^3-2\left(A+B\right)\cos\left(fx+e\right)\sin\left(f\right.\right.}{}\right.$$

$$-\frac{\left((A+B)\cos\left(fx+e\right)^3-2\left(A+B\right)\cos\left(fx+e\right)\sin\left(fx+e\right)-2\left(A+B\right)\cos\left(fx+e\right)\right)\sqrt{-ac}\arctan\left(\frac{\sqrt{}}{}\right.}{4\left(a^3cf\cos\left(fx+e\right)^3-2\,a^3cf\cos\left(\right.\right.}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x
, algorithm="fricas")
```

output
```
[1/8*(((A + B)*cos(f*x + e)^3 - 2*(A + B)*cos(f*x + e)*sin(f*x + e) - 2*(A
 + B)*cos(f*x + e))*sqrt(a*c)*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e
) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x
 + e))/cos(f*x + e)^3) + 2*((A + B)*sin(f*x + e) + 2*A)*sqrt(a*sin(f*x + e
) + a)*sqrt(-c*sin(f*x + e) + c))/(a^3*c*f*cos(f*x + e)^3 - 2*a^3*c*f*cos(
f*x + e)*sin(f*x + e) - 2*a^3*c*f*cos(f*x + e)), -1/4*(((A + B)*cos(f*x +
e)^3 - 2*(A + B)*cos(f*x + e)*sin(f*x + e) - 2*(A + B)*cos(f*x + e))*sqrt(
-a*c)*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)
*sin(f*x + e)/(a*c*cos(f*x + e))) - ((A + B)*sin(f*x + e) + 2*A)*sqrt(a*si
n(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a^3*c*f*cos(f*x + e)^3 - 2*a^3
*c*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*c*f*cos(f*x + e))]
```

Sympy [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{A+B\sin\left(e+fx\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(1/2)
,x)
```

output
```
Integral((A + B*sin(e + f*x))/((a*(sin(e + f*x) + 1))**(5/2)*sqrt(-c*(sin(
e + f*x) - 1))), x)
```

Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx=\int \frac{B\sin{(fx+e)}+A}{(a\sin{(fx+e)}+a)^{\frac{5}{2}}\sqrt{-c\sin{(fx+e)}+c}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(5/2)*sqrt(-c*sin(f*x
 + e) + c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx=\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^
(1/2)),x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^
(1/2)), x)
```

Reduce [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{\sqrt{c}\,\sqrt{a}\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)^4+2\sin(fx+e)^3-2\sin(fx+e)-1}dx\right)b+\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)^4+2\sin(fx+e)^3-2\sin(fx+e)-1}dx\right)a\right)}{a^3c}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x)
```

output
```
( - sqrt(c)*sqrt(a)*(int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)
*sin(e + f*x))/(sin(e + f*x)**4 + 2*sin(e + f*x)**3 - 2*sin(e + f*x) - 1),
x)*b + int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1))/(sin(e + f*x
)**4 + 2*sin(e + f*x)**3 - 2*sin(e + f*x) - 1),x)*a))/(a**3*c)
```

3.193 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 208

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx = -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} - \frac{(3A+B)\cos(e+fx)}{8af(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{(3A+B)\cos(e+fx)}{8a^2f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} + \frac{(3A+B)\text{arctanh}(\sin(e+fx))\cos(e+fx)}{8a^2cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

output
```
-1/4*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2)-1/8*
(3*A+B)*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2)+1/8*(
3*A+B)*cos(f*x+e)/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2)+1/8*
(3*A+B)*arctanh(sin(f*x+e))*cos(f*x+e)/a^2/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c
*sin(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 5.41 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.47

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)\right.}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e +
f*x])^(3/2)),x]
```

output
```
((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])*(-2*A*Cos[e + f*x]^2 + (-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2
 + (A + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - (3*A + B)*Log[Cos[(e
+ f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^4 + (3*A + B)*Log[Cos[(e + f*x)/2] + Sin
[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^4))/(8*f*(a*(1 + Sin[e + f*x]))^(5/2)*(c - c*Sin[e + f*
x])^(3/2))
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3451, 3042, 3222, 3042, 3222, 3042, 3220, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3451}$$

$$\frac{(3A+B)\int \frac{1}{(\sin(e+fx)a+a)^{3/2}(c-c\sin(e+fx))^{3/2}}dx}{4a}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(3A+B)\int \frac{1}{(\sin(e+fx)a+a)^{3/2}(c-c\sin(e+fx))^{3/2}}dx}{4a}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3222}$$

$$\frac{(3A+B)\left(\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{3/2}}dx}{a}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}\right)}{4a}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(3A+B)\left(\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{3/2}}dx}{a}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}\right)}{4a}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3222}$$

$$\frac{(3A+B)\left(\frac{\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}}dx}{2c}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}}{a}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}\right)}{4a}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(3A+B)\left(\frac{\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}}dx}{2c}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}}{a}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}\right)}{4a}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3220}$$

$$\frac{(3A+B)\left(\frac{\frac{\cos(e+fx)\int\sec(e+fx)dx}{2c\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}}{a}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}\right)}{4a}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{(3A+B)\left(\frac{\frac{\cos(e+fx)\int\csc\left(e+fx+\frac{\pi}{2}\right)dx}{2c\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}}{a}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}\right)}{4a}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{3/2}}$$

$$\Big\downarrow \; 4257$$

$$\frac{(3A+B)\left(\frac{\frac{\cos(e+fx)\text{arctanh}(\sin(e+fx))}{2cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}}{a}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}\right)}{4a}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{3/2}}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^
(3/2)),x]
```

output
```
-1/4*((A - B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f
*x])^(3/2)) + ((3*A + B)*(-1/2*Cos[e + f*x]/(f*(a + a*Sin[e + f*x])^(3/2)*
(c - c*Sin[e + f*x])^(3/2)) + (Cos[e + f*x]/(2*f*Sqrt[a + a*Sin[e + f*x]]*
(c - c*Sin[e + f*x])^(3/2)) + (ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*c*f*
Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]))/a))/(4*a)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3220
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]])   Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

rule 3222
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp[(m + n + 1)/(a*(2*m + 1)
)   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
 ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||
!SumSimplerQ[n, 1])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 723 vs. 2(184) = 368.

Time = 8.66 (sec) , antiderivative size = 724, normalized size of antiderivative = 3.48

method	result	size
default	Expression too large to display	724
parts	Expression too large to display	724

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/512*A/f*4^(1/2)*(96*cos(1/4*Pi+1/2*f*x+1/2*e)^2*sin(1/4*Pi+1/2*f*x+1/2*e
)^4*ln(-cot(1/4*Pi+1/2*f*x+1/2*e)+csc(1/4*Pi+1/2*f*x+1/2*e))-96*cos(1/4*Pi
+1/2*f*x+1/2*e)^2*sin(1/4*Pi+1/2*f*x+1/2*e)^4*ln(-cot(1/4*Pi+1/2*f*x+1/2*e
)+csc(1/4*Pi+1/2*f*x+1/2*e)-1)-96*cos(1/4*Pi+1/2*f*x+1/2*e)^2*sin(1/4*Pi+1
/2*f*x+1/2*e)^4*ln(-cot(1/4*Pi+1/2*f*x+1/2*e)+csc(1/4*Pi+1/2*f*x+1/2*e)+1)
+3*cos(1/4*Pi+1/2*f*x+1/2*e)^6+42*cos(1/4*Pi+1/2*f*x+1/2*e)^4-69*cos(1/4*P
i+1/2*f*x+1/2*e)^2+16)/(a*sin(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)/(c*cos(1/4*Pi
+1/2*f*x+1/2*e)^2)^(1/2)/a^2/c*sec(1/4*Pi+1/2*f*x+1/2*e)*csc(1/4*Pi+1/2*f*
x+1/2*e)^3-1/8*B/a^2/c/f*((2*cos(1/2*f*x+1/2*e)*(4*cos(1/2*f*x+1/2*e)^4-4*
cos(1/2*f*x+1/2*e)^2+1)*sin(1/2*f*x+1/2*e)+4*cos(1/2*f*x+1/2*e)^4-4*cos(1/
2*f*x+1/2*e)^2+1)*ln(-2*(cos(1/2*f*x+1/2*e)-sin(1/2*f*x+1/2*e))/(cos(1/2*f
*x+1/2*e)+1))+(-2*cos(1/2*f*x+1/2*e)*(4*cos(1/2*f*x+1/2*e)^4-4*cos(1/2*f*x
+1/2*e)^2+1)*sin(1/2*f*x+1/2*e)-4*cos(1/2*f*x+1/2*e)^4+4*cos(1/2*f*x+1/2*e
)^2-1)*ln(-2*(cos(1/2*f*x+1/2*e)+sin(1/2*f*x+1/2*e))/(cos(1/2*f*x+1/2*e)+1
))+cos(1/2*f*x+1/2*e)*((-16*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)^2-12*cos
(1/2*f*x+1/2*e))*sin(1/2*f*x+1/2*e)^2+2*sin(1/2*f*x+1/2*e)))/(4*cos(1/2*f*
x+1/2*e)^3*sin(1/2*f*x+1/2*e)+2*cos(1/2*f*x+1/2*e)^2-2*sin(1/2*f*x+1/2*e)*
cos(1/2*f*x+1/2*e)-1)/((2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)+1)*a)^(1/2
)/(-(2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-1)*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 417, normalized size of antiderivative = 2.00

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx = \left[\frac{\left((3A+B)\cos\left(fx+e\right)^3\sin\left(fx+e\right)+(3A+B)\,\mathrm{co}\right.}{}\right.$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x
, algorithm="fricas")
```

output
```
[1/16*(((3*A + B)*cos(f*x + e)^3*sin(f*x + e) + (3*A + B)*cos(f*x + e)^3)*
sqrt(a*c)*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*sqrt
(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^
3) - 2*((3*A + B)*cos(f*x + e)^2 - (3*A + B)*sin(f*x + e) - A - 3*B)*sqrt(
a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a^3*c^2*f*cos(f*x + e)^3*s
in(f*x + e) + a^3*c^2*f*cos(f*x + e)^3), -1/8*(((3*A + B)*cos(f*x + e)^3*s
in(f*x + e) + (3*A + B)*cos(f*x + e)^3)*sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt(
a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e)/(a*c*cos(f*x +
e))) + ((3*A + B)*cos(f*x + e)^2 - (3*A + B)*sin(f*x + e) - A - 3*B)*sqrt(
a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a^3*c^2*f*cos(f*x + e)^3*s
in(f*x + e) + a^3*c^2*f*cos(f*x + e)^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(3/2)
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{B\sin\left(fx+e\right)+A}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e
) + c)^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^
(3/2)),x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^
(3/2)), x)
```

Reduce [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)^5+\sin(fx+e)^4-2\sin(fx+e)^3-2\sin(fx+e)^2+\mathrm{s}}\right.\right.}{}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*(int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)*si
n(e + f*x))/(sin(e + f*x)**5 + sin(e + f*x)**4 - 2*sin(e + f*x)**3 - 2*sin
(e + f*x)**2 + sin(e + f*x) + 1),x)*b + int((sqrt(sin(e + f*x) + 1)*sqrt(
- sin(e + f*x) + 1))/(sin(e + f*x)**5 + sin(e + f*x)**4 - 2*sin(e + f*x)**
3 - 2*sin(e + f*x)**2 + sin(e + f*x) + 1),x)*a))/(a**3*c**2)
```

3.194 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 245

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx = & -\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} \\ & -\frac{A\cos(e+fx)}{2af(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} \\ & +\frac{3A\cos(e+fx)}{8a^2f\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{5/2}} \\ & +\frac{3A\cos(e+fx)}{8a^2cf\sqrt{a+a\sin(e+fx)}(c-c\sin(e+fx))^{3/2}} \\ & +\frac{3A\operatorname{arctanh}(\sin(e+fx))\cos(e+fx)}{8a^2c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\end{aligned}$$

output
```
-1/4*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2)-1/2*
A*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2)+3/8*A*cos(f
*x+e)/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2)+3/8*A*cos(f*x+e)
/a^2/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2)+3/8*A*arctanh(sin(f
*x+e))*cos(f*x+e)/a^2/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 4.96 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.37

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx=\frac{\cos^5(e+fx)\left(3A\text{arctanh}(\sin(e+fx))+\frac{1}{4}\sec^4(e+f\right.}{8f(a(1+\sin(e+fx)))^{5/2}(}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e +
f*x])^(5/2)),x]
```

output
```
(Cos[e + f*x]^5*(3*A*ArcTanh[Sin[e + f*x]] + (Sec[e + f*x]^4*(8*B + 11*A*S
in[e + f*x] + 3*A*Sin[3*(e + f*x)]))/4))/(8*f*(a*(1 + Sin[e + f*x]))^(5/2)
*(c - c*Sin[e + f*x])^(5/2))
```

Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3451, 3042, 3222, 3042, 3222, 3042, 3222, 3042, 3220, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3451}$$

$$\frac{A\int \frac{1}{(\sin(e+fx)a+a)^{3/2}(c-c\sin(e+fx))^{5/2}}dx}{a}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\int \frac{1}{(\sin(e+fx)a+a)^{3/2}(c-c\sin(e+fx))^{5/2}}dx}{a}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}$$

$$\Big\downarrow \; 3222$$

$$\frac{A\left(\frac{3\int \frac{1}{\sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{5/2}}dx}{2a}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}\right)}{a}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{A\left(\frac{3\int \frac{1}{\sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{5/2}}dx}{2a}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}\right)}{a}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}$$

$$\Big\downarrow \; 3222$$

$$\frac{A\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{3/2}}dx}{2c}+\frac{\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}\right)}{2a}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}\right)}{a}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{A\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}(c-c\sin(e+fx))^{3/2}}dx}{2c}+\frac{\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}\right)}{2a}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}\right)}{a}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}$$

$$\Big\downarrow \; 3222$$

$$\frac{A\left(\frac{3\left(\frac{\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}}dx}{2c}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}}{2c}+\frac{\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}\right)}{2a}-\frac{\cos(\cdots}{2f(a\sin(e+fx)+a)\cdots}\right.}{a}$$

$$\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{A\left(\frac{3\left(\frac{\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}\sqrt{c-c\sin(e+fx)}}dx}{2c}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}}{2c}+\frac{\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}\right)}{2a}-\frac{\cos(\cdots}{2f(a\sin(e+fx)+a)\cdots}\right.}{a}$$

$$\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}$$

$$\Big\downarrow \text{ 3220}$$

$$\frac{A\left(\frac{3\left(\frac{\frac{\cos(e+fx)\int \sec(e+fx)dx}{2c\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}}{2c}+\frac{\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}\right)}{2a}-\frac{\cos(e\cdots}{2f(a\sin(e+fx)+a)^{3/\cdots}}\right.}{a}$$

$$\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{A\left(\frac{3\left(\frac{\frac{\cos(e+fx)\int \csc\left(e+fx+\frac{\pi}{2}\right)dx}{2c\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}}{2c}+\frac{\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}\right)}{2a}-\frac{\cos(e\cdots}{2f(a\sin(e+fx)+a)^{3/\cdots}}\right.}{a}$$

$$\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}$$

$$\Big\downarrow \ 4257$$

$$\frac{A\left(\frac{3\left(\frac{\frac{\cos(e+fx)\text{arctanh}(\sin(e+fx))}{2cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{2f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}}{2c}+\frac{\cos(e+fx)}{4f\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}\right)}{2a}-\frac{\cos(e}{2f(a\sin(e+fx)+a)^3}\right.}{a}$$

$$\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^
(5/2)),x]
```

output
```
-1/4*((A - B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f
*x])^(5/2)) + (A*(-1/2*Cos[e + f*x]/(f*(a + a*Sin[e + f*x])^(3/2)*(c - c*S
in[e + f*x])^(5/2)) + (3*(Cos[e + f*x]/(4*f*Sqrt[a + a*Sin[e + f*x]]*(c -
c*Sin[e + f*x])^(5/2)) + (Cos[e + f*x]/(2*f*Sqrt[a + a*Sin[e + f*x]]*(c -
c*Sin[e + f*x])^(3/2)) + (ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*c*f*Sqrt[
a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]))/(2*c)))/(2*a)))/a
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3220
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]])   Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

rule 3222
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp[(m + n + 1)/(a*(2*m + 1)
)   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
 ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||
!SumSimplerQ[n, 1])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(215) = 430$.

Time = 7.93 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.86

method	result
default	$-\frac{A\sqrt{4}\left(192\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1\right)\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^4\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^4+192\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\right.\right.\right.}{}$
parts	$-\frac{A\sqrt{4}\left(192\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+1\right)\sin\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^4\cos\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)^4+192\ln\left(-\cot\left(\frac{\pi}{4}+\frac{fx}{2}+\frac{e}{2}\right)+\csc\left(\frac{\pi}{4}+\right.\right.\right.}{}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x,metho
d=_RETURNVERBOSE)
```

output
```
-1/1024*A/f*4^(1/2)*(192*ln(-cot(1/4*Pi+1/2*f*x+1/2*e)+csc(1/4*Pi+1/2*f*x+
1/2*e)+1)*sin(1/4*Pi+1/2*f*x+1/2*e)^4*cos(1/4*Pi+1/2*f*x+1/2*e)^4+192*ln(-
cot(1/4*Pi+1/2*f*x+1/2*e)+csc(1/4*Pi+1/2*f*x+1/2*e)-1)*sin(1/4*Pi+1/2*f*x+
1/2*e)^4*cos(1/4*Pi+1/2*f*x+1/2*e)^4-192*ln(-cot(1/4*Pi+1/2*f*x+1/2*e)+csc
(1/4*Pi+1/2*f*x+1/2*e))*sin(1/4*Pi+1/2*f*x+1/2*e)^4*cos(1/4*Pi+1/2*f*x+1/2
*e)^4+29*cos(1/4*Pi+1/2*f*x+1/2*e)^8-154*cos(1/4*Pi+1/2*f*x+1/2*e)^6+173*c
os(1/4*Pi+1/2*f*x+1/2*e)^4-32*cos(1/4*Pi+1/2*f*x+1/2*e)^2-8)/(a*sin(1/4*Pi
+1/2*f*x+1/2*e)^2)^(1/2)/(c*cos(1/4*Pi+1/2*f*x+1/2*e)^2)^(1/2)/a^2/c^2*sec
(1/4*Pi+1/2*f*x+1/2*e)^3*csc(1/4*Pi+1/2*f*x+1/2*e)^3+2*B/a^2/c^2/f*cos(1/2
*f*x+1/2*e)^2*(2*cos(1/2*f*x+1/2*e)^4-2*cos(1/2*f*x+1/2*e)^2+1)*sin(1/2*f*
x+1/2*e)^2/(-1+2*cos(1/2*f*x+1/2*e)^2)^3/((2*sin(1/2*f*x+1/2*e)*cos(1/2*f*
x+1/2*e)+1)*a)^(1/2)/(-(2*sin(1/2*f*x+1/2*e)*cos(1/2*f*x+1/2*e)-1)*c)^(1/2
)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.24

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx = \left[\frac{3\sqrt{ac}A\cos(fx+e)^5\log\left(-\frac{ac\cos(fx+e)^3-2\,ac\cos(fx+e)-\right.}{}\right.$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x
, algorithm="fricas")
```

output
```
[1/16*(3*sqrt(a*c)*A*cos(f*x + e)^5*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f
*x + e) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*s
in(f*x + e))/cos(f*x + e)^3) + 2*((3*A*cos(f*x + e)^2 + 2*A)*sin(f*x + e)
+ 2*B)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a^3*c^3*f*cos(
f*x + e)^5), -1/8*(3*sqrt(-a*c)*A*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) +
a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e)/(a*c*cos(f*x + e)))*cos(f*x + e)
^5 - ((3*A*cos(f*x + e)^2 + 2*A)*sin(f*x + e) + 2*B)*sqrt(a*sin(f*x + e) +
 a)*sqrt(-c*sin(f*x + e) + c))/(a^3*c^3*f*cos(f*x + e)^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(5/2)
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx=\int \frac{B\sin\left(fx+e\right)+A}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\left(-c\sin\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e
) + c)^(5/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x
, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx=\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^
(5/2)),x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^
(5/2)), x)
```

Reduce [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx=-\frac{\sqrt{c}\,\sqrt{a}\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)^6-3\sin(fx+e)^4+3\sin(fx+e)^2-1}dx\right)b+\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)^6-3\sin(fx+e)^4+3\sin(fx+e)^2-1}dx\right)a\right)}{a^3c^3}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x)
```

output
```
( - sqrt(c)*sqrt(a)*(int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1)
*sin(e + f*x))/(sin(e + f*x)**6 - 3*sin(e + f*x)**4 + 3*sin(e + f*x)**2 -
1),x)*b + int((sqrt(sin(e + f*x) + 1)*sqrt( - sin(e + f*x) + 1))/(sin(e +
f*x)**6 - 3*sin(e + f*x)**4 + 3*sin(e + f*x)**2 - 1),x)*a))/(a**3*c**3)
```

3.195 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^n\,dx$

Optimal result

Integrand size = 36, antiderivative size = 173

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^n\,dx$$
$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n}{f(1+m+n)}$$
$$+\frac{2^{\frac{1}{2}+n}(B(m-n)+A(1+m+n))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2}(1+2m),\frac{1}{2}(1-2n),\frac{1}{2}(3+2m)\right.}{f(1+2m)(1+m}$$

output
```
-B*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n/f/(1+m+n)+2^(1/2+n)*(B
*(m-n)+A*(1+m+n))*cos(f*x+e)*hypergeom([1/2-n, 1/2+m],[3/2+m],1/2+1/2*sin(
f*x+e))*(1-sin(f*x+e))^(-1/2-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n/f/(1
+2*m)/(1+m+n)
```

Mathematica [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^n\,dx$$
$$=\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^n\,dx$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^n,x]
```

output
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^n, x]
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.22, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3042, 3452, 3042, 3224, 3042, 3168, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c-c\sin(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c-c\sin(e+fx))^ndx$$

$$\Big\downarrow \text{3452}$$

$$\left(A+\frac{B(m-n)}{m+n+1}\right)\int (\sin(e+fx)a+a)^m(c-c\sin(e+fx))^ndx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n}{f(m+n+1)}$$

$$\Big\downarrow \text{3042}$$

$$\left(A+\frac{B(m-n)}{m+n+1}\right)\int (\sin(e+fx)a+a)^m(c-c\sin(e+fx))^ndx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n}{f(m+n+1)}$$

$\big\downarrow$ 3224

$$\left(A+\frac{B(m-n)}{m+n+1}\right)\cos^{-2m}(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^m\int \cos^{2m}(e+fx)(c-c\sin(e+fx))^{n-m}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n}{f(m+n+1)}$$

$\big\downarrow$ 3042

$$\left(A+\frac{B(m-n)}{m+n+1}\right)\cos^{-2m}(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^m\int \cos(e+fx)^{2m}(c-c\sin(e+fx))^{n-m}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n}{f(m+n+1)}$$

$\big\downarrow$ 3168

$$\frac{c^2\left(A+\frac{B(m-n)}{m+n+1}\right)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{\frac{1}{2}(-2m-1)+m}(c\sin(e+fx)+c)^{\frac{1}{2}(-2m-1)}\int (c}{f}$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n}{f(m+n+1)}$$

$\big\downarrow$ 80

$$\frac{c^2 2^{n-\frac{1}{2}}\left(A+\frac{B(m-n)}{m+n+1}\right)\cos(e+fx)(1-\sin(e+fx))^{\frac{1}{2}-n}(a\sin(e+fx)+a)^m(c\sin(e+fx)+c)^{\frac{1}{2}(-2m-1)}(c-c\,\mathrm{si}}{f}$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n}{f(m+n+1)}$$

$\big\downarrow$ 79

$$\frac{c2^{n+\frac{1}{2}}\left(A+\frac{B(m-n)}{m+n+1}\right)\cos(e+fx)(1-\sin(e+fx))^{\frac{1}{2}-n}(a\sin(e+fx)+a)^m(c\sin(e+fx)+c)^{\frac{1}{2}(-2m-1)+\frac{1}{2}(2m+1)}}{f(2m}$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n}{f(m+n+1)}$$

input
```
Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^n,x]
```

output
```
-((B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n)/(f*(1 + m
 + n))) + (2^(1/2 + n)*c*(A + (B*(m - n))/(1 + m + n))*Cos[e + f*x]*Hyperg
eometric2F1[(1 + 2*m)/2, (1 - 2*n)/2, (3 + 2*m)/2, (1 + Sin[e + f*x])/2]*(
1 - Sin[e + f*x])^(1/2 - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-
1/2 + (-1 - 2*m)/2 + m + n)*(c + c*Sin[e + f*x])^((-1 - 2*m)/2 + (1 + 2*m)
/2))/(f*(1 + 2*m))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3168
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Simp[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)))   Subst[Int[(a +
b*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Fre
eQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

rule 3224
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m]))   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (FractionQ[m] ||  !FractionQ[n])
```

rule 3452
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

Maple [F]

$$\int (a+a\sin(fx+e))^m (A+B\sin(fx+e)) (c-c\sin(fx+e))^n dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^n,x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^n,x)
```

Fricas [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^n dx$$
$$= \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m(-c\sin(fx+e)+c)^n dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^n,x, algori
thm="fricas")
```

output
```
integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^n, x)
```

Sympy [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^n \, dx$$
$$= \int (a(\sin(e + fx) + 1))^m (-c(\sin(e + fx) - 1))^n (A + B\sin(e + fx)) \, dx$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**n,x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*(-c*(sin(e + f*x) - 1))**n*(A + B*sin(e
 + f*x)), x)
```

Maxima [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^n \, dx$$
$$= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^m (-c\sin(fx + e) + c)^n \, dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^n,x, algori
thm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c
)^n, x)
```

Giac [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^n\,dx$$
$$=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m(-c\sin(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^n,x, algori
thm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c
)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^n\,dx$$
$$=\int (A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m\,(c-c\,\sin(e+f\,x))^n\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n, x)
```

Reduce [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^n\,dx$$
$$=\left(\int (a+a\sin(fx+e))^m\,(-\sin(fx+e)\,c+c)^n\sin(fx+e)\,dx\right)b$$
$$+\left(\int (a+a\sin(fx+e))^m\,(-\sin(fx+e)\,c+c)^n\,dx\right)a$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^n,x)
```

output
```
int((sin(e + f*x)*a + a)**m*( - sin(e + f*x)*c + c)**n*sin(e + f*x),x)*b +
 int((sin(e + f*x)*a + a)**m*( - sin(e + f*x)*c + c)**n,x)*a
```

3.196 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$

Optimal result

Integrand size = 36, antiderivative size = 145

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$$
$$= -\frac{a^3Bc^3\cos^7(e+fx)(a+a\sin(e+fx))^{-3+m}}{f(4+m)}$$
$$+\frac{2^{\frac{1}{2}+m}a^3c^3(B(3-m)-A(4+m))\cos^7(e+fx)\,\text{Hypergeometric2F1}\left(\frac{7}{2},\frac{1}{2}-m,\frac{9}{2},\frac{1}{2}(1-\sin(e+fx))\right)}{7f(4+m)}$$

output
```
-a^3*B*c^3*cos(f*x+e)^7*(a+a*sin(f*x+e))^(-3+m)/f/(4+m)+1/7*2^(1/2+m)*a^3*
c^3*(B*(3-m)-A*(4+m))*cos(f*x+e)^7*hypergeom([7/2, 1/2-m],[9/2],1/2-1/2*si
n(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^(-3+m)/f/(4+m)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 32.50 (sec) , antiderivative size = 604, normalized size of antiderivative = 4.17

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx =$$
$$-\frac{(a(1+\sin(e+fx)))^m(c-c\sin(e+fx))^3(\cos(e+fx)+i(1+\sin(e+fx)))\left(-\frac{10(4A-3B)\,\text{Hypergeometric}}{}\right.}{}$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^3,x]
```

output
```
-1/16*((a*(1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^3*(Cos[e + f*x] + I*(
1 + Sin[e + f*x]))*((-10*(4*A - 3*B)*Hypergeometric2F1[1, 1 + m, 1 - m, I*
Cos[e + f*x] - Sin[e + f*x]])/m + (2*(15*A - 13*B)*Hypergeometric2F1[1, 2
+ m, 2 - m, I*Cos[e + f*x] - Sin[e + f*x]]*((-I)*Cos[e + f*x] + Sin[e + f*
x]))/(-1 + m) + (2*(15*A - 13*B)*Hypergeometric2F1[1, m, -m, I*Cos[e + f*x
] - Sin[e + f*x]]*(I*Cos[e + f*x] + Sin[e + f*x]))/(1 + m) + (4*(3*A - 4*B
)*Hypergeometric2F1[1, -1 + m, -1 - m, I*Cos[e + f*x] - Sin[e + f*x]]*(Cos
[2*(e + f*x)] - I*Sin[2*(e + f*x)]))/(2 + m) + (4*(3*A - 4*B)*Hypergeometr
ic2F1[1, 3 + m, 3 - m, I*Cos[e + f*x] - Sin[e + f*x]]*(Cos[2*(e + f*x)] +
I*Sin[2*(e + f*x)]))/(-2 + m) - ((2*I)*(A - 3*B)*Hypergeometric2F1[1, -2 +
 m, -2 - m, I*Cos[e + f*x] - Sin[e + f*x]]*(Cos[3*(e + f*x)] - I*Sin[3*(e
+ f*x)]))/(3 + m) + ((2*I)*(A - 3*B)*Hypergeometric2F1[1, 4 + m, 4 - m, I*
Cos[e + f*x] - Sin[e + f*x]]*(Cos[3*(e + f*x)] + I*Sin[3*(e + f*x)]))/(-3
+ m) + (B*Hypergeometric2F1[1, -3 + m, -3 - m, I*Cos[e + f*x] - Sin[e + f*
x]]*(Cos[4*(e + f*x)] - I*Sin[4*(e + f*x)]))/(4 + m) + (B*Hypergeometric2F
1[1, 5 + m, 5 - m, I*Cos[e + f*x] - Sin[e + f*x]]*(Cos[4*(e + f*x)] + I*Si
n[4*(e + f*x)]))/(-4 + m)))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 3446, 3042, 3339, 3042, 3168, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c - c\sin(e + fx))^3(a\sin(e + fx) + a)^m(A + B\sin(e + fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c - c\sin(e + fx))^3(a\sin(e + fx) + a)^m(A + B\sin(e + fx))dx$$

$$\Big\downarrow \text{3446}$$

$$a^3c^3 \int \cos^6(e + fx)(\sin(e + fx)a + a)^{m-3}(A + B\sin(e + fx))dx$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3 \int \cos(e + fx)^6(\sin(e + fx)a + a)^{m-3}(A + B\sin(e + fx))dx$$

$$\Big\downarrow \text{3339}$$

$$a^3c^3\left(\left(A - \frac{B(3 - m)}{m + 4}\right)\int \cos^6(e + fx)(\sin(e + fx)a + a)^{m-3}dx - \frac{B\cos^7(e + fx)(a\sin(e + fx) + a)^{m-3}}{f(m + 4)}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\left(\left(A - \frac{B(3 - m)}{m + 4}\right)\int \cos(e + fx)^6(\sin(e + fx)a + a)^{m-3}dx - \frac{B\cos^7(e + fx)(a\sin(e + fx) + a)^{m-3}}{f(m + 4)}\right)$$

$$\Big\downarrow \text{3168}$$

$$a^3c^3\left(\frac{a^2\left(A - \frac{B(3-m)}{m+4}\right)\cos^7(e + fx)\int (a - a\sin(e + fx))^{5/2}(\sin(e + fx)a + a)^{m-\frac{1}{2}}d\sin(e + fx)}{f(a - a\sin(e + fx))^{7/2}(a\sin(e + fx) + a)^{7/2}} - \frac{B\cos^7(e +

$$a^3c^3\left(\frac{a^2 2^{m-\frac{1}{2}}\left(A-\frac{B(3-m)}{m+4}\right)\cos^7(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-4}\int\left(\frac{1}{2}\sin(e+fx)+\frac{1}{2}\right)^{m-}}{f(a-a\sin(e+fx))^{7/2}}\right.$$

$$\Big\downarrow 79$$

$$a^3c^3\left(-\frac{a2^{m+\frac{1}{2}}\left(A-\frac{B(3-m)}{m+4}\right)\cos^7(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-4}\,\text{Hypergeometric2F1}\left(\frac{7}{2},\right.}{7f}\right.$$

input
```
Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3,x]
```

output
```
a^3*c^3*(-1/7*(2^(1/2 + m)*a*(A - (B*(3 - m))/(4 + m))*Cos[e + f*x]^7*Hype
rgeometric2F1[7/2, 1/2 - m, 9/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^
(1/2 - m)*(a + a*Sin[e + f*x])^(-4 + m))/f - (B*Cos[e + f*x]^7*(a + a*Sin[
e + f*x])^(-3 + m))/(f*(4 + m)))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3168
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Simp[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)))   Subst[Int[(a +
b*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Fre
eQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

rule 3339
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [F]

$$\int (a + a\sin(fx + e))^m (A + B\sin(fx + e)) (c - c\sin(fx + e))^3 \, dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x)
```

Fricas [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx = \int -(B\sin(fx+e)+A)(c\sin(fx+e)-c)^3(a\sin(fx+e)+a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algori
thm="fricas")
```

output
```
integral(-(B*c^3*cos(f*x + e)^4 + (3*A - 5*B)*c^3*cos(f*x + e)^2 - 4*(A -
B)*c^3 - ((A - 3*B)*c^3*cos(f*x + e)^2 - 4*(A - B)*c^3)*sin(f*x + e))*(a*s
in(f*x + e) + a)^m, x)
```

Sympy [F]

$$\begin{aligned}\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx &= -c^3\bigg(\int \left(-A(a\sin(e+fx)+a)^m\right)dx + \int 3A(a\sin(e+fx)+a)^m\sin(e+fx)\,dx \\ &+ \int \left(-3A(a\sin(e+fx)+a)^m\sin^2(e+fx)\right)dx \\ &+ \int A(a\sin(e+fx)+a)^m\sin^3(e+fx)\,dx \\ &+ \int \left(-B(a\sin(e+fx)+a)^m\sin(e+fx)\right)dx \\ &+ \int 3B(a\sin(e+fx)+a)^m\sin^2(e+fx)\,dx \\ &+ \int \left(-3B(a\sin(e+fx)+a)^m\sin^3(e+fx)\right)dx \\ &+ \int B(a\sin(e+fx)+a)^m\sin^4(e+fx)\,dx\bigg)\end{aligned}$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3,x)
```

output
```
-c**3*(Integral(-A*(a*sin(e + f*x) + a)**m, x) + Integral(3*A*(a*sin(e + f
*x) + a)**m*sin(e + f*x), x) + Integral(-3*A*(a*sin(e + f*x) + a)**m*sin(e
 + f*x)**2, x) + Integral(A*(a*sin(e + f*x) + a)**m*sin(e + f*x)**3, x) +
Integral(-B*(a*sin(e + f*x) + a)**m*sin(e + f*x), x) + Integral(3*B*(a*sin
(e + f*x) + a)**m*sin(e + f*x)**2, x) + Integral(-3*B*(a*sin(e + f*x) + a)
**m*sin(e + f*x)**3, x) + Integral(B*(a*sin(e + f*x) + a)**m*sin(e + f*x)*
*4, x))
```

Maxima [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$$
$$=\int -(B\sin(fx+e)+A)(c\sin(fx+e)-c)^3(a\sin(fx+e)+a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algori
thm="maxima")
```

output
```
-integrate((B*sin(f*x + e) + A)*(c*sin(f*x + e) - c)^3*(a*sin(f*x + e) + a
)^m, x)
```

Giac [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx$$
$$=\int -(B\sin(fx+e)+A)(c\sin(fx+e)-c)^3(a\sin(fx+e)+a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algori
thm="giac")
```

output
```
integrate(-(B*sin(f*x + e) + A)*(c*sin(f*x + e) - c)^3*(a*sin(f*x + e) + a
)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx = \int (A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m\,(c-c\,\sin(e+f\,x))^3\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^3,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^3, x)
```

Reduce [F]

$$\begin{aligned}
&\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^3\,dx \\
&= c^3\Bigg(\left(\left(\int (a+a\sin(fx+e))^m\,dx\right)a-\left(\int (a+a\sin(fx+e))^m\sin(fx+e)^4\,dx\right)b\right. \\
&\quad-\left(\int (a+a\sin(fx+e))^m\sin(fx+e)^3\,dx\right)a \\
&\quad+3\left(\int (a+a\sin(fx+e))^m\sin(fx+e)^3\,dx\right)b \\
&\quad+3\left(\int (a+a\sin(fx+e))^m\sin(fx+e)^2\,dx\right)a \\
&\quad-3\left(\int (a+a\sin(fx+e))^m\sin(fx+e)^2\,dx\right)b \\
&\quad-3\left(\int (a+a\sin(fx+e))^m\sin(fx+e)\,dx\right)a \\
&\quad\left.+\left(\int (a+a\sin(fx+e))^m\sin(fx+e)\,dx\right)b\right)\Bigg)
\end{aligned}$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x)
```

output
```
c**3*(int((sin(e + f*x)*a + a)**m,x)*a - int((sin(e + f*x)*a + a)**m*sin(e
 + f*x)**4,x)*b - int((sin(e + f*x)*a + a)**m*sin(e + f*x)**3,x)*a + 3*int
((sin(e + f*x)*a + a)**m*sin(e + f*x)**3,x)*b + 3*int((sin(e + f*x)*a + a)
**m*sin(e + f*x)**2,x)*a - 3*int((sin(e + f*x)*a + a)**m*sin(e + f*x)**2,x
)*b - 3*int((sin(e + f*x)*a + a)**m*sin(e + f*x),x)*a + int((sin(e + f*x)*
a + a)**m*sin(e + f*x),x)*b)
```

3.197 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$

Optimal result

Integrand size = 36, antiderivative size = 145

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx = -\frac{a^2Bc^2\cos^5(e+fx)(a+a\sin(e+fx))^{-2+m}}{f(3+m)} + \frac{2^{\frac{1}{2}+m}a^2c^2(B(2-m)-A(3+m))\cos^5(e+fx)\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{2}-m,\frac{7}{2},\frac{1}{2}(1-\sin(e+fx))\right)}{5f(3+m)}$$

output
```
-a^2*B*c^2*cos(f*x+e)^5*(a+a*sin(f*x+e))^(-2+m)/f/(3+m)+1/5*2^(1/2+m)*a^2*
c^2*(B*(2-m)-A*(3+m))*cos(f*x+e)^5*hypergeom([5/2, 1/2-m],[7/2],1/2-1/2*si
n(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^(-2+m)/f/(3+m)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 16.15 (sec) , antiderivative size = 443, normalized size of antiderivative = 3.06

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$$
$$= \frac{ic^2(a(1+\sin(e+fx)))^m(\cos(e+fx)+i(1+\sin(e+fx)))\left(-\frac{4i(3A-2B)\,\text{Hypergeometric2F1}(1,1+m,1-m,i\cos(e+}{m}\right.}{}$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^2,x]
```

output
```
((I/8)*c^2*(a*(1 + Sin[e + f*x]))^m*(Cos[e + f*x] + I*(1 + Sin[e + f*x]))*
(((-4*I)*(3*A - 2*B)*Hypergeometric2F1[1, 1 + m, 1 - m, I*Cos[e + f*x] - S
in[e + f*x]])/m - ((8*A - 7*B)*Hypergeometric2F1[1, m, -m, I*Cos[e + f*x]
- Sin[e + f*x]]*(Cos[e + f*x] - I*Sin[e + f*x]))/(1 + m) + ((8*A - 7*B)*Hy
pergeometric2F1[1, 2 + m, 2 - m, I*Cos[e + f*x] - Sin[e + f*x]]*(Cos[e + f
*x] + I*Sin[e + f*x]))/(-1 + m) + ((2*I)*(A - 2*B)*Hypergeometric2F1[1, 3
+ m, 3 - m, I*Cos[e + f*x] - Sin[e + f*x]]*(Cos[2*(e + f*x)] + I*Sin[2*(e
+ f*x)]))/(-2 + m) + (2*(A - 2*B)*Hypergeometric2F1[1, -1 + m, -1 - m, I*C
os[e + f*x] - Sin[e + f*x]]*(I*Cos[2*(e + f*x)] + Sin[2*(e + f*x)]))/(2 +
m) - (B*Hypergeometric2F1[1, -2 + m, -2 - m, I*Cos[e + f*x] - Sin[e + f*x]
]*(Cos[3*(e + f*x)] - I*Sin[3*(e + f*x)]))/(3 + m) + (B*Hypergeometric2F1[
1, 4 + m, 4 - m, I*Cos[e + f*x] - Sin[e + f*x]]*(Cos[3*(e + f*x)] + I*Sin[
3*(e + f*x)]))/(-3 + m)))/f
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 3446, 3042, 3339, 3042, 3168, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-c\sin(e+fx))^2(a\sin(e+fx)+a)^m(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c-c\sin(e+fx))^2(a\sin(e+fx)+a)^m(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3446}$$

$$a^2c^2\int \cos^4(e+fx)(\sin(e+fx)a+a)^{m-2}(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\int \cos(e+fx)^4(\sin(e+fx)a+a)^{m-2}(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3339}$$

$$a^2c^2\left(\left(A-\frac{B(2-m)}{m+3}\right)\int \cos^4(e+fx)(\sin(e+fx)a+a)^{m-2}dx-\frac{B\cos^5(e+fx)(a\sin(e+fx)+a)^{m-2}}{f(m+3)}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c^2\left(\left(A-\frac{B(2-m)}{m+3}\right)\int \cos(e+fx)^4(\sin(e+fx)a+a)^{m-2}dx-\frac{B\cos^5(e+fx)(a\sin(e+fx)+a)^{m-2}}{f(m+3)}\right)$$

$$\Big\downarrow \text{3168}$$

$$a^2c^2\left(\frac{a^2\left(A-\frac{B(2-m)}{m+3}\right)\cos^5(e+fx)\int (a-a\sin(e+fx))^{3/2}(\sin(e+fx)a+a)^{m-\frac{1}{2}}d\sin(e+fx)}{f(a-a\sin(e+fx))^{5/2}(a\sin(e+fx)+a)^{5/2}}-\frac{B\cos^5(e+\right.$$

$$\Big\downarrow \text{80}$$

$$a^2c^2\left(\frac{a^22^{m-\frac{1}{2}}\left(A-\frac{B(2-m)}{m+3}\right)\cos^5(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-3}\int \left(\frac{1}{2}\sin(e+fx)+\frac{1}{2}\right)^{m-}}{f(a-a\sin(e+fx))^{5/2}}\right.$$

$$\Big\downarrow \text{79}$$

$$a^2c^2\left(-\frac{a2^{m+\frac{1}{2}}\left(A-\frac{B(2-m)}{m+3}\right)\cos^5(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-3}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\right.}{5f}\right.$$

input
```
Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2,x]
```

output
```
a^2*c^2*(-1/5*(2^(1/2 + m)*a*(A - (B*(2 - m))/(3 + m))*Cos[e + f*x]^5*Hype
rgeometric2F1[5/2, 1/2 - m, 7/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^
(1/2 - m)*(a + a*Sin[e + f*x])^(-3 + m))/f - (B*Cos[e + f*x]^5*(a + a*Sin[
e + f*x])^(-2 + m))/(f*(3 + m)))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3168
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Simp[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)))   Subst[Int[(a +
b*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Fre
eQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

rule 3339
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [F]

$$\int (a+a\sin(fx+e))^m (A+B\sin(fx+e)) (c-c\sin(fx+e))^2 dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x)
```

Fricas [F]

$$\begin{aligned}&\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx\\&=\int (B\sin(fx+e)+A)(c\sin(fx+e)-c)^2(a\sin(fx+e)+a)^m\,dx\end{aligned}$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
integral(-((A - 2*B)*c^2*cos(f*x + e)^2 - 2*(A - B)*c^2 + (B*c^2*cos(f*x +
 e)^2 + 2*(A - B)*c^2)*sin(f*x + e))*(a*sin(f*x + e) + a)^m, x)
```

Sympy [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$$
$$=c^2\left(\int A(a\sin(e+fx)+a)^m\,dx+\int\left(-2A(a\sin(e+fx)+a)^m\sin(e+fx)\right)dx\right.$$
$$+\int A(a\sin(e+fx)+a)^m\sin^2(e+fx)\,dx$$
$$+\int B(a\sin(e+fx)+a)^m\sin(e+fx)\,dx$$
$$+\int\left(-2B(a\sin(e+fx)+a)^m\sin^2(e+fx)\right)dx$$
$$\left.+\int B(a\sin(e+fx)+a)^m\sin^3(e+fx)\,dx\right)$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2,x)
```

output
```
c**2*(Integral(A*(a*sin(e + f*x) + a)**m, x) + Integral(-2*A*(a*sin(e + f*
x) + a)**m*sin(e + f*x), x) + Integral(A*(a*sin(e + f*x) + a)**m*sin(e + f
*x)**2, x) + Integral(B*(a*sin(e + f*x) + a)**m*sin(e + f*x), x) + Integra
l(-2*B*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2, x) + Integral(B*(a*sin(e +
 f*x) + a)**m*sin(e + f*x)**3, x))
```

Maxima [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$$
$$=\int (B\sin(fx+e)+A)(c\sin(fx+e)-c)^2(a\sin(fx+e)+a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algori
thm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(c*sin(f*x + e) - c)^2*(a*sin(f*x + e) + a)
^m, x)
```

Giac [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$$
$$=\int (B\sin(fx+e)+A)(c\sin(fx+e)-c)^2(a\sin(fx+e)+a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(c*sin(f*x + e) - c)^2*(a*sin(f*x + e) + a)
^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$$
$$=\int (A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m\,(c-c\,\sin(e+f\,x))^2\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^2,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^2, x)
```

Reduce [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^2\,dx$$
$$= c^2\left(\left(\left(\int (a+a\sin(fx+e))^m\,dx\right)a+\left(\int (a+a\sin(fx+e))^m\sin(fx+e)^3\,dx\right)b\right.\right.$$
$$+\left(\int (a+a\sin(fx+e))^m\sin(fx+e)^2\,dx\right)a$$
$$-2\left(\int (a+a\sin(fx+e))^m\sin(fx+e)^2\,dx\right)b$$
$$-2\left(\int (a+a\sin(fx+e))^m\sin(fx+e)\,dx\right)a$$
$$\left.+\left(\int (a+a\sin(fx+e))^m\sin(fx+e)\,dx\right)b\right)$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x)
```

output
```
c**2*(int((sin(e + f*x)*a + a)**m,x)*a + int((sin(e + f*x)*a + a)**m*sin(e
 + f*x)**3,x)*b + int((sin(e + f*x)*a + a)**m*sin(e + f*x)**2,x)*a - 2*int
((sin(e + f*x)*a + a)**m*sin(e + f*x)**2,x)*b - 2*int((sin(e + f*x)*a + a)
**m*sin(e + f*x),x)*a + int((sin(e + f*x)*a + a)**m*sin(e + f*x),x)*b)
```

3.198 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 137

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$$
$$=-\frac{aBc\cos^3(e+fx)(a+a\sin(e+fx))^{-1+m}}{f(2+m)}$$
$$+\frac{2^{\frac{1}{2}+m}ac(B(1-m)-A(2+m))\cos^3(e+fx)\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}-m,\frac{5}{2},\frac{1}{2}(1-\sin(e+fx))\right)(1}{3f(2+m)}$$

output
```
-a*B*c*cos(f*x+e)^3*(a+a*sin(f*x+e))^(-1+m)/f/(2+m)+1/3*2^(1/2+m)*a*c*(B*(
1-m)-A*(2+m))*cos(f*x+e)^3*hypergeom([3/2, 1/2-m],[5/2],1/2-1/2*sin(f*x+e)
)*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^(-1+m)/f/(2+m)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.48 (sec) , antiderivative size = 468, normalized size of antiderivative = 3.42

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx =$$
$$-\frac{2^{-2-m}ce^{ifmx}\left(1-ie^{i(e+fx)}\right)^{-2m}\left(-iae^{-i(e+fx)}\left(i+e^{i(e+fx)}\right)^2\right)^m\left(\frac{iBe^{-i(2e+f(2+m)x)}\text{Hypergeometric2F1}(-2-m,-}{2+m}\right.}{}$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
,x]
```

output
```
-((2^(-2 - m)*c*E^(I*f*m*x)*(((-I)*a*(I + E^(I*(e + f*x)))^2)/E^(I*(e + f*
x)))^m*((I*B*Hypergeometric2F1[-2 - m, -2*m, -1 - m, I*E^(I*(e + f*x))])/(
E^(I*(2*e + f*(2 + m)*x))*(2 + m)) + (2*(A - B)*Hypergeometric2F1[-1 - m,
-2*m, -m, I*E^(I*(e + f*x))])/(E^(I*(e + f*(1 + m)*x))*(1 + m)) - (2*A*E^(
I*(e - f*(-1 + m)*x))*Hypergeometric2F1[1 - m, -2*m, 2 - m, I*E^(I*(e + f*
x))])/(-1 + m) + (2*B*E^(I*(e - f*(-1 + m)*x))*Hypergeometric2F1[1 - m, -2
*m, 2 - m, I*E^(I*(e + f*x))])/(-1 + m) + (I*B*E^((2*I)*e - I*f*(-2 + m)*x
)*Hypergeometric2F1[2 - m, -2*m, 3 - m, I*E^(I*(e + f*x))])/(-2 + m) + ((4
*I)*A*Hypergeometric2F1[-2*m, -m, 1 - m, I*E^(I*(e + f*x))])/(E^(I*f*m*x)*
m) - ((2*I)*B*Hypergeometric2F1[-2*m, -m, 1 - m, I*E^(I*(e + f*x))])/(E^(I
*f*m*x)*m))*(-1 + Sin[e + f*x]))/((1 - I*E^(I*(e + f*x)))^(2*m)*f*(Cos[(e
+ f*x)/2] - Sin[(e + f*x)/2])^2))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3042, 3446, 3042, 3339, 3042, 3168, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c - c\sin(e+fx))(a\sin(e+fx)+a)^m(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (c - c\sin(e+fx))(a\sin(e+fx)+a)^m(A+B\sin(e+fx))dx$$

$$\Big\downarrow \; 3446$$

$$ac\int \cos^2(e+fx)(\sin(e+fx)a+a)^{m-1}(A+B\sin(e+fx))dx$$

$$\Big\downarrow \; 3042$$

$$ac\int \cos(e+fx)^2(\sin(e+fx)a+a)^{m-1}(A+B\sin(e+fx))dx$$

$$\Big\downarrow \; 3339$$

$$ac\left(\left(A-\frac{B(1-m)}{m+2}\right)\int \cos^2(e+fx)(\sin(e+fx)a+a)^{m-1}dx-\frac{B\cos^3(e+fx)(a\sin(e+fx)+a)^{m-1}}{f(m+2)}\right)$$

$$\Big\downarrow \; 3042$$

$$ac\left(\left(A-\frac{B(1-m)}{m+2}\right)\int \cos(e+fx)^2(\sin(e+fx)a+a)^{m-1}dx-\frac{B\cos^3(e+fx)(a\sin(e+fx)+a)^{m-1}}{f(m+2)}\right)$$

$$\Big\downarrow \; 3168$$

$$ac\left(\frac{a^2\left(A-\frac{B(1-m)}{m+2}\right)\cos^3(e+fx)\int \sqrt{a-a\sin(e+fx)}(\sin(e+fx)a+a)^{m-\frac{1}{2}}d\sin(e+fx)}{f(a-a\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^{3/2}}-\frac{B\cos^3(e+fx)(}{f}\right.$$

$$\Big\downarrow \; 80$$

$$ac\left(\frac{a^2 2^{m-\frac{1}{2}}\left(A-\frac{B(1-m)}{m+2}\right)\cos^3(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-2}\int\left(\frac{1}{2}\sin(e+fx)+\frac{1}{2}\right)^{m-\frac{1}{2}}}{f(a-a\sin(e+fx))^{3/2}}\right.$$

$$\Big\downarrow \; 79$$

$$ac\left(-\frac{a2^{m+\frac{1}{2}}\left(A-\frac{B(1-m)}{m+2}\right)\cos^3(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m-2}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}\right.}{3f}\right.$$

input
```
Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),x]
```

output
```
a*c*(-1/3*(2^(1/2 + m)*a*(A - (B*(1 - m))/(2 + m))*Cos[e + f*x]^3*Hypergeo
metric2F1[3/2, 1/2 - m, 5/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(1/2
 - m)*(a + a*Sin[e + f*x])^(-2 + m))/f - (B*Cos[e + f*x]^3*(a + a*Sin[e +
f*x])^(-1 + m))/(f*(2 + m)))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3168
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Simp[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)))   Subst[Int[(a +
b*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Fre
eQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

rule 3339
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [F]

$$\int (a+a\sin(fx+e))^m (A+B\sin(fx+e)) (c-c\sin(fx+e)) \, dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

Fricas [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx)) \, dx = \int -(B\sin(fx+e)+A)(c\sin(fx+e)-c)(a\sin(fx+e)+a)^m \, dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
integral((B*c*cos(f*x + e)^2 - (A - B)*c*sin(f*x + e) + (A - B)*c)*(a*sin(
f*x + e) + a)^m, x)
```

Sympy [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$$
$$= -c\left(\int \left(-A(a\sin(e+fx)+a)^m\right)\,dx + \int A(a\sin(e+fx)+a)^m\sin(e+fx)\,dx + \int \left(-B(a\sin(e+fx)+a)^m\sin(e+fx)\right)\,dx + \int B(a\sin(e+fx)+a)^m\sin^2(e+fx)\,dx\right)$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

output
```
-c*(Integral(-A*(a*sin(e + f*x) + a)**m, x) + Integral(A*(a*sin(e + f*x) +
 a)**m*sin(e + f*x), x) + Integral(-B*(a*sin(e + f*x) + a)**m*sin(e + f*x)
, x) + Integral(B*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2, x))
```

Maxima [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx$$
$$= \int -(B\sin(fx+e)+A)(c\sin(fx+e)-c)(a\sin(fx+e)+a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorith
m="maxima")
```

output
```
-integrate((B*sin(f*x + e) + A)*(c*sin(f*x + e) - c)*(a*sin(f*x + e) + a)^
m, x)
```

Giac [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx = \int -(B\sin(fx+e)+A)(c\sin(fx+e)-c)(a\sin(fx+e)+a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorith
m="giac")
```

output
```
integrate(-(B*sin(f*x + e) + A)*(c*sin(f*x + e) - c)*(a*sin(f*x + e) + a)^
m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx = \int (A+B\sin(e+fx))\,(a+a\sin(e+fx))^m\,(c-c\sin(e+fx))\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x)),x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x)), x)
```

Reduce [F]

$$\begin{aligned}\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))\,dx &= c\Bigg(\left(\left(\int (a+a\sin(fx+e))^m\,dx\right)a-\left(\int (a+a\sin(fx+e))^m\sin(fx+e)^2\,dx\right)b\right.\\ &\quad-\left(\int (a+a\sin(fx+e))^m\sin(fx+e)\,dx\right)a\\ &\quad+\left(\int (a+a\sin(fx+e))^m\sin(fx+e)\,dx\right)b\Bigg)\end{aligned}$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

output
```
c*(int((sin(e + f*x)*a + a)**m,x)*a - int((sin(e + f*x)*a + a)**m*sin(e +
f*x)**2,x)*b - int((sin(e + f*x)*a + a)**m*sin(e + f*x),x)*a + int((sin(e
+ f*x)*a + a)**m*sin(e + f*x),x)*b)
```

3.199 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 117

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)} - \frac{2^{\frac{1}{2}+m}(A+Am+Bm)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+ $$
$$f(1+m)$$

output
```
-B*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+m)-2^(1/2+m)*(A*m+B*m+A)*cos(f*x+e)*
hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(
a+a*sin(f*x+e))^m/f/(1+m)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.60 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.86

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$
$$= \frac{2^m\left((A-B)B_{\frac{1}{2}(1+\sin(e+fx))}\left(\frac{1}{2}+m,\frac{1}{2}\right)+2BB_{\frac{1}{2}(1+\sin(e+fx))}\left(\frac{3}{2}+m,\frac{1}{2}\right)\right)\sqrt{\cos^2(e+fx)}\sec(e+fx)(1+s}{f}$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

output
```
(2^m*((A - B)*Beta[(1 + Sin[e + f*x])/2, 1/2 + m, 1/2] + 2*B*Beta[(1 + Sin
[e + f*x])/2, 3/2 + m, 1/2])*Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*(a*(1 + Sin
[e + f*x]))^m)/(f*(1 + Sin[e + f*x])^m)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3230, 3042, 3131, 3042, 3130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{ 3230}$$

$$\frac{(Am+A+Bm)\int(\sin(e+fx)a+a)^mdx}{m+1}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(m+1)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(Am+A+Bm)\int(\sin(e+fx)a+a)^mdx}{m+1}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(m+1)}$$

$$\Big\downarrow \text{ 3131}$$

$$\frac{(Am+A+Bm)(\sin(e+fx)+1)^{-m}(a\sin(e+fx)+a)^m\int(\sin(e+fx)+1)^mdx}{m+1}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(m+1)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(Am+A+Bm)(\sin(e+fx)+1)^{-m}(a\sin(e+fx)+a)^m\int(\sin(e+fx)+1)^m dx}{m+1}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(m+1)}$$

$$\Big\downarrow \text{3130}$$

$$-\frac{2^{m+\frac{1}{2}}(Am+A+Bm)\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m\right.}{f(m+1)}\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(m+1)}$$

input
```
Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

output
```
-((B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + m))) - (2^(1/2 + m)*(A +
 A*m + B*m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e +
 f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 + m)
)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3130
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

rule 3131
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Maple [F]

$$\int (a + a\sin(fx + e))^m (A + B\sin(fx + e))\, dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Fricas [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))\, dx = \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^m\, dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fricas")
```

output
```
integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m, x)
```

Sympy [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))\,dx$$
$$= \int (a(\sin(e + fx) + 1))^m (A + B\sin(e + fx))\;dx$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x)), x)
```

Maxima [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))\,dx$$
$$= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m, x)
```

Giac [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))\,dx$$
$$= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$
$$=\int (A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m, x)
```

Reduce [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$
$$=\left(\int (a+a\sin(fx+e))^m\,dx\right)a+\left(\int (a+a\sin(fx+e))^m\sin(fx+e)\,dx\right)b$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

output
```
int((sin(e + f*x)*a + a)**m,x)*a + int((sin(e + f*x)*a + a)**m*sin(e + f*x
),x)*b
```

3.200 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 128

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx=-\frac{B\sec(e+fx)(a+a\sin(e+fx))^{1+m}}{acfm}+\frac{2^{\frac{1}{2}+m}(B+Am+Bm)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}-m,\frac{1}{2},\frac{1}{2}(1-\sin(e+fx))\right)\sec(e+fx)(1+\sin(e}{acfm}$$

output
```
-B*sec(f*x+e)*(a+a*sin(f*x+e))^(1+m)/a/c/f/m+2^(1/2+m)*(A*m+B*m+B)*hyperge
om([-1/2, 1/2-m],[1/2],1/2-1/2*sin(f*x+e))*sec(f*x+e)*(1+sin(f*x+e))^(-1/2
-m)*(a+a*sin(f*x+e))^(1+m)/a/c/f/m
```

Mathematica [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = \int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx$$

input
```
Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
]),x]
```

output
```
Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
]), x]
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 3446, 3042, 3339, 3042, 3168, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^m(A+B\sin(e+fx))}{c-c\sin(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^m(A+B\sin(e+fx))}{c-c\sin(e+fx)}dx$$

$$\Big\downarrow \text{3446}$$

$$\frac{\int \sec^2(e+fx)(\sin(e+fx)a+a)^{m+1}(A+B\sin(e+fx))dx}{ac}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(\sin(e+fx)a+a)^{m+1}(A+B\sin(e+fx))}{\cos(e+fx)^2}dx}{ac}$$

$$\Big\downarrow \text{3339}$$

$$\frac{\frac{(Am+Bm+B)\int \sec^2(e+fx)(\sin(e+fx)a+a)^{m+1}dx}{m}-\frac{B\sec(e+fx)(a\sin(e+fx)+a)^{m+1}}{fm}}{ac}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(Am+Bm+B)\int \frac{(\sin(e+fx)a+a)^{m+1}}{\cos(e+fx)^2}dx}{m}-\frac{B\sec(e+fx)(a\sin(e+fx)+a)^{m+1}}{fm}}{ac}$$

$$\Big\downarrow \text{3168}$$

$$\frac{\frac{a^2(Am+Bm+B)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\int\frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}}{(a-a\sin(e+fx))^{3/2}}d\sin(e+fx)}{fm}-\frac{B\sec(e+fx)(a\sin(e+fx)+a)^{m+1}}{fm}}{ac}$$

$\Big\downarrow$ 80

$$\frac{\frac{a^2 2^{m-\frac{1}{2}}(Am+Bm+B)\sec(e+fx)\sqrt{a-a\sin(e+fx)}(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m\int\frac{\left(\frac{1}{2}\sin(e+fx)+\frac{1}{2}\right)^{m-\frac{1}{2}}}{(a-a\sin(e+fx))^{3/2}}d\sin(e+fx)}{fm}-B\sec(e+f}{ac}$$

$\Big\downarrow$ 79

$$\frac{\frac{a2^{m+\frac{1}{2}}(Am+Bm+B)\sec(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}-m,\frac{1}{2},\frac{1}{2}(1-\sin(e+fx))\right)}{fm}-B\sec(e+fx)(}{ac}$$

input
```
Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]),x]
```

output
```
((2^(1/2 + m)*a*(B + A*m + B*m)*Hypergeometric2F1[-1/2, 1/2 - m, 1/2, (1 -
 Sin[e + f*x])/2]*Sec[e + f*x]*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e +
 f*x])^m)/(f*m) - (B*Sec[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(f*m))/(a*
c)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3168
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Simp[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)))   Subst[Int[(a +
b*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Fre
eQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

rule 3339
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [F]

$$\int \frac{(a+a\sin(fx+e))^m (A+B\sin(fx+e))}{c-c\sin(fx+e)}\,dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = \int -\frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{c\sin(fx+e)-c}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
integral(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)
, x)
```

Sympy [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = -\frac{\int \frac{A(a\sin(e+fx)+a)^m}{\sin(e+fx)-1}\,dx + \int \frac{B(a\sin(e+fx)+a)^m\sin(e+fx)}{\sin(e+fx)-1}\,dx}{c}$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

output
```
-(Integral(A*(a*sin(e + f*x) + a)**m/(sin(e + f*x) - 1), x) + Integral(B*(
a*sin(e + f*x) + a)**m*sin(e + f*x)/(sin(e + f*x) - 1), x))/c
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = \int -\frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{c\sin(fx+e)-c}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorith
m="maxima")
```

output
```
-integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c
), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[0,1,0]%%%} / %%%{1,[0,0,1]%%%} Error: Bad Argument Val
ue
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m}{c-c\,\sin(e+f\,x)}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x)),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x)), x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c-c\sin(e+fx)}\,dx = \frac{-\left(\int \frac{(a+a\sin(fx+e))^m}{\sin(fx+e)-1}dx\right)a-\left(\int \frac{(a+a\sin(fx+e))^m\sin(fx+e)}{\sin(fx+e)-1}dx\right)b}{c}$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

output
```
( - (int((sin(e + f*x)*a + a)**m/(sin(e + f*x) - 1),x)*a + int(((sin(e + f
*x)*a + a)**m*sin(e + f*x))/(sin(e + f*x) - 1),x)*b))/c
```

3.201 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 148

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \frac{B\sec^3(e+fx)(a+a\sin(e+fx))^{2+m}}{a^2c^2f(1-m)} + \frac{2^{\frac{1}{2}+m}(A(1-m)-B(2+m))\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}-m,-\frac{1}{2},\frac{1}{2}(1-\sin(e+fx))\right)\sec^3(e+fx)(}{3a^2c^2f(1-m)}$$

output
```
B*sec(f*x+e)^3*(a+a*sin(f*x+e))^(2+m)/a^2/c^2/f/(1-m)+1/3*2^(1/2+m)*(A*(1-
m)-B*(2+m))*hypergeom([-3/2, 1/2-m],[-1/2],1/2-1/2*sin(f*x+e))*sec(f*x+e)^
3*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^(2+m)/a^2/c^2/f/(1-m)
```

Mathematica [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx$$

input
```
Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^2,x]
```

output
```
Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^2, x]
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.93, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 3446, 3042, 3339, 3042, 3168, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}dx$$

$$\Big\downarrow \text{3446}$$

$$\frac{\int \sec^4(e+fx)(\sin(e+fx)a+a)^{m+2}(A+B\sin(e+fx))dx}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(\sin(e+fx)a+a)^{m+2}(A+B\sin(e+fx))}{\cos(e+fx)^4}dx}{a^2c^2}$$

$$\Big\downarrow \text{3339}$$

$$\frac{\left(A-\frac{B(m+2)}{1-m}\right)\int \sec^4(e+fx)(\sin(e+fx)a+a)^{m+2}dx+\frac{B\sec^3(e+fx)(a\sin(e+fx)+a)^{m+2}}{f(1-m)}}{a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(A-\frac{B(m+2)}{1-m}\right)\int \frac{(\sin(e+fx)a+a)^{m+2}}{\cos(e+fx)^4}dx+\frac{B\sec^3(e+fx)(a\sin(e+fx)+a)^{m+2}}{f(1-m)}}{a^2c^2}$$

$$\Big\downarrow \text{3168}$$

$$\frac{\frac{a^2\left(A-\frac{B(m+2)}{1-m}\right)\sec^3(e+fx)(a-a\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^{3/2}\int\frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}}{(a-a\sin(e+fx))^{5/2}}d\sin(e+fx)}{f}+\frac{B\sec^3(e+fx)(a\sin(e+fx)+a)^{m+2}}{f(1-m)}}{a^2c^2}$$

$$\Big\downarrow \text{80}$$

$$\frac{\frac{a^2 2^{m-\frac{1}{2}}\left(A-\frac{B(m+2)}{1-m}\right)\sec^3(e+fx)(a-a\sin(e+fx))^{3/2}(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m+1}\int\frac{\left(\frac{1}{2}\sin(e+fx)+\frac{1}{2}\right)^{m-\frac{1}{2}}}{(a-a\sin(e+fx))^{5/2}}d\sin(e+fx)}{f}+\frac{B\sec}{}}{a^2c^2}$$

$$\Big\downarrow \text{79}$$

$$\frac{\frac{a2^{m+\frac{1}{2}}\left(A-\frac{B(m+2)}{1-m}\right)\sec^3(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m+1}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}-m,-\frac{1}{2},\frac{1}{2}(1-\sin(e+fx))\right)}{3f}+\frac{B\sec^3(e}{}}{a^2c^2}$$

input
```
Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^2,x
]
```

output
```
((2^(1/2 + m)*a*(A - (B*(2 + m))/(1 - m))*Hypergeometric2F1[-3/2, 1/2 - m,
 -1/2, (1 - Sin[e + f*x])/2]*Sec[e + f*x]^3*(1 + Sin[e + f*x])^(1/2 - m)*(
a + a*Sin[e + f*x])^(1 + m))/(3*f) + (B*Sec[e + f*x]^3*(a + a*Sin[e + f*x]
)^(2 + m))/(f*(1 - m)))/(a^2*c^2)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3168

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Simp[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)))   Subst[Int[(a +
b*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Fre
eQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

rule 3339

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

rule 3446

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [F]

$$\int \frac{(a+a\sin(fx+e))^m (A+B\sin(fx+e))}{(c-c\sin(fx+e))^2}dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x)
```

Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(c\sin(fx+e)-c)^2}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
integral(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c^2*cos(f*x + e)^2
+ 2*c^2*sin(f*x + e) - 2*c^2), x)
```

Sympy [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \frac{\int \frac{A(a\sin(e+fx)+a)^m}{\sin^2(e+fx)-2\sin(e+fx)+1}\,dx + \int \frac{B(a\sin(e+fx)+a)^m\sin(e+fx)}{\sin^2(e+fx)-2\sin(e+fx)+1}\,dx}{c^2}$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**2,x)
```

output
```
(Integral(A*(a*sin(e + f*x) + a)**m/(sin(e + f*x)**2 - 2*sin(e + f*x) + 1)
, x) + Integral(B*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(sin(e + f*x)**2 -
2*sin(e + f*x) + 1), x))/c**2
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(c\sin(fx+e)-c)^2}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algori
thm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)
^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,0]%%%} / %%%{1,[0,0,2]%%%} Error: Bad Argument Valu
e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m}{\left(c-c\,\sin\left(e+f\,x\right)\right)^2}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^2,x
)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^2,
x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^2}\,dx = \frac{\left(\int \frac{(a+a\sin(fx+e))^m}{\sin(fx+e)^2-2\sin(fx+e)+1}dx\right)a+\left(\int \frac{(a+a\sin(fx+e))^m\sin(fx+e)}{\sin(fx+e)^2-2\sin(fx+e)+1}dx\right)b}{c^2}$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x)
```

output
```
(int((sin(e + f*x)*a + a)**m/(sin(e + f*x)**2 - 2*sin(e + f*x) + 1),x)*a +
 int(((sin(e + f*x)*a + a)**m*sin(e + f*x))/(sin(e + f*x)**2 - 2*sin(e + f
*x) + 1),x)*b)/c**2
```

3.202 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 148

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = \frac{B\sec^5(e+fx)(a+a\sin(e+fx))^{3+m}}{a^3c^3f(2-m)} + \frac{2^{\frac{1}{2}+m}(A(2-m)-B(3+m))\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{2}-m,-\frac{3}{2},\frac{1}{2}(1-\sin(e+fx))\right)\sec^5(e+fx)(}{5a^3c^3f(2-m)}$$

output
```
B*sec(f*x+e)^5*(a+a*sin(f*x+e))^(3+m)/a^3/c^3/f/(2-m)+1/5*2^(1/2+m)*(A*(2-
m)-B*(3+m))*hypergeom([-5/2, 1/2-m],[-3/2],1/2-1/2*sin(f*x+e))*sec(f*x+e)^
5*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^(3+m)/a^3/c^3/f/(2-m)
```

Mathematica [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = \int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx$$

input
```
Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^3,x]
```

output
```
Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^3, x]
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.93, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3042, 3446, 3042, 3339, 3042, 3168, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}dx$$

$$\Big\downarrow \text{3446}$$

$$\frac{\int \sec^6(e+fx)(\sin(e+fx)a+a)^{m+3}(A+B\sin(e+fx))dx}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(\sin(e+fx)a+a)^{m+3}(A+B\sin(e+fx))}{\cos(e+fx)^6}dx}{a^3c^3}$$

$$\Big\downarrow \text{3339}$$

$$\frac{\left(A-\frac{B(m+3)}{2-m}\right)\int \sec^6(e+fx)(\sin(e+fx)a+a)^{m+3}dx+\frac{B\sec^5(e+fx)(a\sin(e+fx)+a)^{m+3}}{f(2-m)}}{a^3c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(A-\frac{B(m+3)}{2-m}\right)\int \frac{(\sin(e+fx)a+a)^{m+3}}{\cos(e+fx)^6}dx+\frac{B\sec^5(e+fx)(a\sin(e+fx)+a)^{m+3}}{f(2-m)}}{a^3c^3}$$

$$\Big\downarrow \text{3168}$$

$$\frac{\frac{a^2\left(A-\frac{B(m+3)}{2-m}\right)\sec^5(e+fx)(a-a\sin(e+fx))^{5/2}(a\sin(e+fx)+a)^{5/2}\int\frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}}{(a-a\sin(e+fx))^{7/2}}d\sin(e+fx)}{f}+\frac{B\sec^5(e+fx)(a\sin(e+fx)+a)^{m+3}}{f(2-m)}}{a^3c^3}$$

$\Big\downarrow$ 80

$$\frac{\frac{a^2 2^{m-\frac{1}{2}}\left(A-\frac{B(m+3)}{2-m}\right)\sec^5(e+fx)(a-a\sin(e+fx))^{5/2}(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m+2}\int\frac{\left(\frac{1}{2}\sin(e+fx)+\frac{1}{2}\right)^{m-\frac{1}{2}}}{(a-a\sin(e+fx))^{7/2}}d\sin(e+fx)}{f}+\frac{B\sec}{}}{a^3c^3}$$

$\Big\downarrow$ 79

$$\frac{\frac{a2^{m+\frac{1}{2}}\left(A-\frac{B(m+3)}{2-m}\right)\sec^5(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m+2}\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{2}-m,-\frac{3}{2},\frac{1}{2}(1-\sin(e+fx))\right)}{5f}+\frac{B\sec^5(e\cdot}{}}{a^3c^3}$$

input
```
Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^3,x
]
```

output
```
((2^(1/2 + m)*a*(A - (B*(3 + m))/(2 - m))*Hypergeometric2F1[-5/2, 1/2 - m,
 -3/2, (1 - Sin[e + f*x])/2]*Sec[e + f*x]^5*(1 + Sin[e + f*x])^(1/2 - m)*(
a + a*Sin[e + f*x])^(2 + m))/(5*f) + (B*Sec[e + f*x]^5*(a + a*Sin[e + f*x]
)^(3 + m))/(f*(2 - m)))/(a^3*c^3)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && !IntegerQ[m] && !Integ
erQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3168
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Simp[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)))   Subst[Int[(a +
b*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Fre
eQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

rule 3339
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [F]

$$\int \frac{(a+a\sin(fx+e))^m (A+B\sin(fx+e))}{(c-c\sin(fx+e))^3}dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x)
```

Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = \int -\frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(c\sin(fx+e)-c)^3}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algori
thm="fricas")
```

output
```
integral(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(3*c^3*cos(f*x + e)^
2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a + a\sin(e + fx))^m (A + B\sin(e + fx))}{(c - c\sin(e + fx))^3}\, dx = \int -\frac{(B\sin(fx + e) + A)(a\sin(fx + e) + a)^m}{(c\sin(fx + e) - c)^3}\, dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algori
thm="maxima")
```

output
```
-integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c
)^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a + a\sin(e + fx))^m (A + B\sin(e + fx))}{(c - c\sin(e + fx))^3}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[0,1,0]%%%} / %%%{1,[0,0,3]%%%} Error: Bad Argument Val
ue
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m}{(c-c\,\sin(e+f\,x))^3}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^3,x
)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^3,
x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^3}\,dx = \frac{-\left(\int \frac{(a+a\sin(fx+e))^m}{\sin(fx+e)^3-3\sin(fx+e)^2+3\sin(fx+e)-1}dx\right)a-\left(\int \frac{(a+a\sin(fx+e))^m\sin(fx+e)}{\sin(fx+e)^3-3\sin(fx+e)^2+3\sin(fx+e)-1}dx\right)b}{c^3}$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x)
```

output
```
( - (int((sin(e + f*x)*a + a)**m/(sin(e + f*x)**3 - 3*sin(e + f*x)**2 + 3*
sin(e + f*x) - 1),x)*a + int(((sin(e + f*x)*a + a)**m*sin(e + f*x))/(sin(e
 + f*x)**3 - 3*sin(e + f*x)**2 + 3*sin(e + f*x) - 1),x)*b))/c**3
```

3.203 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 118

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx=-\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}}$$

output
```
-2*B*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)+(A+B)*
cos(f*x+e)*hypergeom([1, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+e))*(a+a*sin(f*x+e
))^m/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 22.41 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.12

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$
$$=\frac{\cos(e+fx)(a(1+\sin(e+fx)))^m\left(2A(3+2m)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx\right.\right.}{2f(1+2m)(3+}$$

input
```
Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e +
 f*x]],x]
```

output
```
(Cos[e + f*x]*(a*(1 + Sin[e + f*x]))^m*(2*A*(3 + 2*m)*Hypergeometric2F1[1,
 1/2 + m, 3/2 + m, (1 + Sin[e + f*x])/2] + B*(-6 - 4*m + (1 + 2*m)*Hyperge
ometric2F1[1, 3/2 + m, 5/2 + m, (1 + Sin[e + f*x])/2]*(1 + Sin[e + f*x])))
)/(2*f*(1 + 2*m)*(3 + 2*m)*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {3042, 3452, 3042, 3224, 3042, 3146, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3452}$$

$$(A+B)\int \frac{(\sin(e+fx)a+a)^m}{\sqrt{c-c\sin(e+fx)}}dx-\frac{2B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$(A+B)\int \frac{(\sin(e+fx)a+a)^m}{\sqrt{c-c\sin(e+fx)}}dx-\frac{2B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{3224}$$

$$\frac{(A+B)\cos(e+fx)\int \sec(e+fx)(\sin(e+fx)a+a)^{m+\frac{1}{2}}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A+B)\cos(e+fx)\int \frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}}{\cos(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{3146}$$

$$\frac{a(A+B)\cos(e+fx)\int \frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}}{a-a\sin(e+fx)}d(a\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{78}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m \text{Hypergeometric2F1}\left(1,m+\frac{1}{2},m+\frac{3}{2},\frac{\sin(e+fx)a+a}{2a}\right)}{f(2m+1)\sqrt{c-c\sin(e+fx)}}-\frac{2B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

input
```
Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]]
,x]
```

output
```
(-2*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*Sin[e +
 f*x]]) + ((A + B)*Cos[e + f*x]*Hypergeometric2F1[1, 1/2 + m, 3/2 + m, (a
+ a*Sin[e + f*x])/(2*a)]*(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*S
in[e + f*x]])
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3224
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m]))   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (FractionQ[m] ||  !FractionQ[n])
```

rule 3452
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

Maple [F]

$$\int \frac{(a+a\sin(fx+e))^m (A+B\sin(fx+e))}{\sqrt{c-c\sin(fx+e)}}dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, al
gorithm="fricas")
```

output
```
integral(-(B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) +
 a)^m/(c*sin(f*x + e) - c), x)
```

Sympy [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(a(\sin(e+fx)+1))^m\,(A+B\sin(e+fx))}{\sqrt{-c(\sin(e+fx)-1)}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))/sqrt(-c*(sin(e + f
*x) - 1)), x)
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$
$$=\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/sqrt(-c*sin(f*x + e)
 + c), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx=\text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, al
gorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$$
$$=\int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m}{\sqrt{c-c\,\sin(e+f\,x)}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(1/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(1/
2), x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{\sqrt{c}\left(\left(\int \frac{(a+a\sin(fx+e))^m\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)-1}dx\right)b+\left(\int \frac{(a+a\sin(fx+e))^m\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)-1}dx\right)a\right)}{c}$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

output
```
( - sqrt(c)*(int(((sin(e + f*x)*a + a)**m*sqrt( - sin(e + f*x) + 1)*sin(e
+ f*x))/(sin(e + f*x) - 1),x)*b + int(((sin(e + f*x)*a + a)**m*sqrt( - sin
(e + f*x) + 1))/(sin(e + f*x) - 1),x)*a))/c
```

3.204 $\int \frac{(A+B\sin(e+fx))(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 118

$$\int \frac{(A+B\sin(e+fx))(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx = -\frac{2B\cos(e+fx)(c+c\sin(e+fx))^m}{f(1+2m)\sqrt{a-a\sin(e+fx)}} + \frac{(A+B)\cos(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx))\right)(c+c\sin(e+fx))^m}{f(1+2m)\sqrt{a-a\sin(e+fx)}}$$

output
```
-2*B*cos(f*x+e)*(c+c*sin(f*x+e))^m/f/(1+2*m)/(a-a*sin(f*x+e))^(1/2)+(A+B)*
cos(f*x+e)*hypergeom([1, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+e))*(c+c*sin(f*x+e
))^m/f/(1+2*m)/(a-a*sin(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 22.27 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.12

$$\int \frac{(A+B\sin(e+fx))(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx = \frac{\cos(e+fx)(c(1+\sin(e+fx)))^m\left(2A(3+2m)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx\right.\right.}{2f(1+2m)(3+2}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + c*Sin[e + f*x])^m)/Sqrt[a - a*Sin[e +
 f*x]],x]
```

output
```
(Cos[e + f*x]*(c*(1 + Sin[e + f*x]))^m*(2*A*(3 + 2*m)*Hypergeometric2F1[1,
 1/2 + m, 3/2 + m, (1 + Sin[e + f*x])/2] + B*(-6 - 4*m + (1 + 2*m)*Hyperge
ometric2F1[1, 3/2 + m, 5/2 + m, (1 + Sin[e + f*x])/2]*(1 + Sin[e + f*x])))
)/(2*f*(1 + 2*m)*(3 + 2*m)*Sqrt[a - a*Sin[e + f*x]])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {3042, 3452, 3042, 3224, 3042, 3146, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c\sin(e+fx)+c)^m}{\sqrt{a-a\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(A+B\sin(e+fx))(c\sin(e+fx)+c)^m}{\sqrt{a-a\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3452}$$

$$(A+B)\int \frac{(\sin(e+fx)c+c)^m}{\sqrt{a-a\sin(e+fx)}}dx-\frac{2B\cos(e+fx)(c\sin(e+fx)+c)^m}{f(2m+1)\sqrt{a-a\sin(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$(A+B)\int \frac{(\sin(e+fx)c+c)^m}{\sqrt{a-a\sin(e+fx)}}dx-\frac{2B\cos(e+fx)(c\sin(e+fx)+c)^m}{f(2m+1)\sqrt{a-a\sin(e+fx)}}$$

$$\Big\downarrow \text{3224}$$

$$\frac{(A+B)\cos(e+fx)\int \sec(e+fx)(\sin(e+fx)c+c)^{m+\frac{1}{2}}dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}-\frac{2B\cos(e+fx)(c\sin(e+fx)+c)^m}{f(2m+1)\sqrt{a-a\sin(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A+B)\cos(e+fx)\int \frac{(\sin(e+fx)c+c)^{m+\frac{1}{2}}}{\cos(e+fx)}dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}-\frac{2B\cos(e+fx)(c\sin(e+fx)+c)^m}{f(2m+1)\sqrt{a-a\sin(e+fx)}}$$

$$\Big\downarrow \text{3146}$$

$$\frac{c(A+B)\cos(e+fx)\int \frac{(\sin(e+fx)c+c)^{m-\frac{1}{2}}}{c-c\sin(e+fx)}d(c\sin(e+fx))}{f\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}-\frac{2B\cos(e+fx)(c\sin(e+fx)+c)^m}{f(2m+1)\sqrt{a-a\sin(e+fx)}}$$

$$\Big\downarrow \text{78}$$

$$\frac{(A+B)\cos(e+fx)(c\sin(e+fx)+c)^m \text{Hypergeometric2F1}\left(1,m+\frac{1}{2},m+\frac{3}{2},\frac{\sin(e+fx)c+c}{2c}\right)}{f(2m+1)\sqrt{a-a\sin(e+fx)}}-\frac{2B\cos(e+fx)(c\sin(e+fx)+c)^m}{f(2m+1)\sqrt{a-a\sin(e+fx)}}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + c*Sin[e + f*x])^m)/Sqrt[a - a*Sin[e + f*x]]
,x]
```

output
```
(-2*B*Cos[e + f*x]*(c + c*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[a - a*Sin[e +
 f*x]]) + ((A + B)*Cos[e + f*x]*Hypergeometric2F1[1, 1/2 + m, 3/2 + m, (c
+ c*Sin[e + f*x])/(2*c)]*(c + c*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[a - a*S
in[e + f*x]])
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3224
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m]))   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (FractionQ[m] ||  !FractionQ[n])
```

rule 3452
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

Maple [F]

$$\int \frac{(A+B\sin(fx+e))\,(c+c\sin(fx+e))^m}{\sqrt{a-a\sin(fx+e)}}dx$$

input
```
int((A+B*sin(f*x+e))*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x)
```

output
```
int((A+B*sin(f*x+e))*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int \frac{(A+B\sin(e+fx))(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(c\sin(fx+e)+c)^m}{\sqrt{-a\sin(fx+e)+a}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x, al
gorithm="fricas")
```

output
```
integral(-(B*sin(f*x + e) + A)*sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) +
 c)^m/(a*sin(f*x + e) - a), x)
```

Sympy [F]

$$\int \frac{(A+B\sin(e+fx))(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx = \int \frac{(c(\sin(e+fx)+1))^m(A+B\sin(e+fx))}{\sqrt{-a(\sin(e+fx)-1)}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+c*sin(f*x+e))**m/(a-a*sin(f*x+e))**(1/2),x)
```

output
```
Integral((c*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))/sqrt(-a*(sin(e + f
*x) - 1)), x)
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(c\sin(fx+e)+c)^m}{\sqrt{-a\sin(fx+e)+a}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(c*sin(f*x + e) + c)^m/sqrt(-a*sin(f*x + e)
 + a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x, al
gorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(c+c\,\sin(e+f\,x))^m}{\sqrt{a-a\,\sin(e+f\,x)}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c + c*sin(e + f*x))^m)/(a - a*sin(e + f*x))^(1/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c + c*sin(e + f*x))^m)/(a - a*sin(e + f*x))^(1/
2), x)
```

Reduce [F]

$$\int \frac{(A+B\sin(e+fx))(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx = -\frac{\sqrt{a}\left(\left(\int \frac{(\sin(fx+e)c+c)^m\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)-1}dx\right)b+\left(\int \frac{(\sin(fx+e)c+c)^m\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)-1}dx\right)a\right)}{a}$$

input
```
int((A+B*sin(f*x+e))*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x)
```

output
```
( - sqrt(a)*(int(((sin(e + f*x)*c + c)**m*sqrt( - sin(e + f*x) + 1)*sin(e
+ f*x))/(sin(e + f*x) - 1),x)*b + int(((sin(e + f*x)*c + c)**m*sqrt( - sin
(e + f*x) + 1))/(sin(e + f*x) - 1),x)*a))/a
```

3.205 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 275

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = -\frac{64c^3(B(5-2m)-A(7+2m))\cos(e+fx)(a+a\sin(e+fx))^m}{f(5+2m)(7+2m)\left(3+8m+4m^2\right)\sqrt{c-c\sin(e+fx)}} - \frac{16c^2(B(5-2m)-A(7+2m))\cos(e+fx)(a+a\sin(e+fx))^m\sqrt{c-c\sin(e+fx)}}{f(7+2m)\left(15+16m+4m^2\right)} - \frac{2c(B(5-2m)-A(7+2m))\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{3/2}}{f(5+2m)(7+2m)} - \frac{2B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{5/2}}{f(7+2m)}$$

output
```
-64*c^3*(B*(5-2*m)-A*(7+2*m))*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(5+2*m)/(7+2
*m)/(4*m^2+8*m+3)/(c-c*sin(f*x+e))^(1/2)-16*c^2*(B*(5-2*m)-A*(7+2*m))*cos(
f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2)/f/(7+2*m)/(4*m^2+16*m+15)
-2*c*(B*(5-2*m)-A*(7+2*m))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^
(3/2)/f/(5+2*m)/(7+2*m)-2*B*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))
^(5/2)/f/(7+2*m)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 14.94 (sec) , antiderivative size = 667, normalized size of antiderivative = 2.43

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \frac{(a(1+\sin(e+fx)))^m(c-c\sin(e+fx))^{5/2}\left(\frac{(2100A-1575B+1272Am-110Bm+304Am^2-68Bm^2+32Am}{(1+2m)(3+2m)(5}\right.}{}$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^(5/2),x]
```

output
```
((a*(1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^(5/2)*(((2100*A - 1575*B +
1272*A*m - 110*B*m + 304*A*m^2 - 68*B*m^2 + 32*A*m^3 - 8*B*m^3)*((1/8 + I/
8)*Cos[(e + f*x)/2] + (1/8 - I/8)*Sin[(e + f*x)/2]))/((1 + 2*m)*(3 + 2*m)*
(5 + 2*m)*(7 + 2*m)) + ((2100*A - 1575*B + 1272*A*m - 110*B*m + 304*A*m^2
- 68*B*m^2 + 32*A*m^3 - 8*B*m^3)*((1/8 - I/8)*Cos[(e + f*x)/2] + (1/8 + I/
8)*Sin[(e + f*x)/2]))/((1 + 2*m)*(3 + 2*m)*(5 + 2*m)*(7 + 2*m)) + ((350*A
- 385*B + 184*A*m - 104*B*m + 24*A*m^2 - 12*B*m^2)*((1/8 - I/8)*Cos[(3*(e
+ f*x))/2] - (1/8 + I/8)*Sin[(3*(e + f*x))/2]))/((3 + 2*m)*(5 + 2*m)*(7 +
2*m)) + ((350*A - 385*B + 184*A*m - 104*B*m + 24*A*m^2 - 12*B*m^2)*((1/8 +
 I/8)*Cos[(3*(e + f*x))/2] - (1/8 - I/8)*Sin[(3*(e + f*x))/2]))/((3 + 2*m)
*(5 + 2*m)*(7 + 2*m)) + ((14*A - 35*B + 4*A*m - 6*B*m)*((-1/8 + I/8)*Cos[(
5*(e + f*x))/2] - (1/8 + I/8)*Sin[(5*(e + f*x))/2]))/((5 + 2*m)*(7 + 2*m))
 + ((14*A - 35*B + 4*A*m - 6*B*m)*((-1/8 - I/8)*Cos[(5*(e + f*x))/2] - (1/
8 - I/8)*Sin[(5*(e + f*x))/2]))/((5 + 2*m)*(7 + 2*m)) + ((1/8 - I/8)*B*Cos
[(7*(e + f*x))/2] - (1/8 + I/8)*B*Sin[(7*(e + f*x))/2])/(7 + 2*m) + ((1/8
+ I/8)*B*Cos[(7*(e + f*x))/2] - (1/8 - I/8)*B*Sin[(7*(e + f*x))/2])/(7 + 2
*m)))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5)
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.84, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3452, 3042, 3219, 3042, 3219, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-c\sin(e+fx))^{5/2}(a\sin(e+fx)+a)^m(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c-c\sin(e+fx))^{5/2}(a\sin(e+fx)+a)^m(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3452}$$

$$-\frac{(B(5-2m)-A(2m+7))\int(\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{5/2}dx}{2m+7}-\frac{2B\cos(e+fx)(c-c\sin(e+fx))^{5/2}(a\sin(e+fx)+a)^m}{f(2m+7)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(B(5-2m)-A(2m+7))\int(\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{5/2}dx}{2m+7}-\frac{2B\cos(e+fx)(c-c\sin(e+fx))^{5/2}(a\sin(e+fx)+a)^m}{f(2m+7)}$$

$$\Big\downarrow \text{3219}$$

$$-\frac{(B(5-2m)-A(2m+7))\left(\frac{8c\int(\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{3/2}dx}{2m+5}+\frac{2c\cos(e+fx)(c-c\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^m}{f(2m+5)}\right)}{2m+7}-\frac{2B\cos(e+fx)(c-c\sin(e+fx))^{5/2}(a\sin(e+fx)+a)^m}{f(2m+7)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(B(5-2m)-A(2m+7))\left(\frac{8c\int(\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{3/2}dx}{2m+5}+\frac{2c\cos(e+fx)(c-c\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^m}{f(2m+5)}\right)}{2m+7}-\frac{2B\cos(e+fx)(c-c\sin(e+fx))^{5/2}(a\sin(e+fx)+a)^m}{f(2m+7)}$$

$$\Big\downarrow \text{3219}$$

$$-\frac{(B(5-2m)-A(2m+7))\left(\frac{8c\left(\frac{4c\int (\sin(e+fx)a+a)^m\sqrt{c-c\sin(e+fx)}dx}{2m+3}+\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}(a\sin(e+fx)+a)^m}{f(2m+3)}\right)}{2m+5}+\frac{2c\cos(e+f}{\cdots}\right.}{2m+7}$$
$$\frac{2B\cos(e+fx)(c-c\sin(e+fx))^{5/2}(a\sin(e+fx)+a)^m}{f(2m+7)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(B(5-2m)-A(2m+7))\left(\frac{8c\left(\frac{4c\int (\sin(e+fx)a+a)^m\sqrt{c-c\sin(e+fx)}dx}{2m+3}+\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}(a\sin(e+fx)+a)^m}{f(2m+3)}\right)}{2m+5}+\frac{2c\cos(e+f}{\cdots}\right.}{2m+7}$$
$$\frac{2B\cos(e+fx)(c-c\sin(e+fx))^{5/2}(a\sin(e+fx)+a)^m}{f(2m+7)}$$

$$\Big\downarrow \text{3217}$$

$$-\frac{(B(5-2m)-A(2m+7))\left(\frac{8c\left(\frac{8c^2\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)(2m+3)\sqrt{c-c\sin(e+fx)}}+\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}(a\sin(e+fx)+a)^m}{f(2m+3)}\right)}{2m+5}+\frac{2c\cos(e+fx)(c-c}{\cdots}\right.}{2m+7}$$
$$\frac{2B\cos(e+fx)(c-c\sin(e+fx))^{5/2}(a\sin(e+fx)+a)^m}{f(2m+7)}$$

input
```
Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2)
,x]
```

output
```
(-2*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(5/2))/(f*(
7 + 2*m)) - ((B*(5 - 2*m) - A*(7 + 2*m))*((2*c*Cos[e + f*x]*(a + a*Sin[e +
 f*x])^m*(c - c*Sin[e + f*x])^(3/2))/(f*(5 + 2*m)) + (8*c*((8*c^2*Cos[e +
f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*(3 + 2*m)*Sqrt[c - c*Sin[e + f*x
]]) + (2*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqrt[c - c*Sin[e + f*x]])/(
f*(3 + 2*m))))/(5 + 2*m)))/(7 + 2*m)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3452
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

Maple [F]

$$\int (a + a\sin(fx + e))^m (A + B\sin(fx + e)) (c - c\sin(fx + e))^{\frac{5}{2}} dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 562 vs. $2(264) = 528$.

Time = 0.13 (sec) , antiderivative size = 562, normalized size of antiderivative = 2.04

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \frac{2\left((8\,Bc^2m^3+36\,Bc^2m^2+46\,Bc^2m+15\,Bc^2)\cos\left(fx+e\right)^4+64\,(A+B)c^2m-(8\,(A-2\,\ldots\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, al
gorithm="fricas")
```

output
```
2*((8*B*c^2*m^3 + 36*B*c^2*m^2 + 46*B*c^2*m + 15*B*c^2)*cos(f*x + e)^4 + 6
4*(A + B)*c^2*m - (8*(A - 2*B)*c^2*m^3 + 4*(11*A - 28*B)*c^2*m^2 + 2*(31*A
 - 86*B)*c^2*m + 3*(7*A - 20*B)*c^2)*cos(f*x + e)^3 + 32*(7*A - 5*B)*c^2 +
 (8*(A - B)*c^2*m^3 + 4*(19*A - 11*B)*c^2*m^2 + 190*(A - B)*c^2*m + (77*A
- 85*B)*c^2)*cos(f*x + e)^2 + 2*(8*(A - B)*c^2*m^3 + 60*(A - B)*c^2*m^2 +
2*(79*A - 63*B)*c^2*m + (161*A - 145*B)*c^2)*cos(f*x + e) + (64*(A + B)*c^
2*m - (8*B*c^2*m^3 + 36*B*c^2*m^2 + 46*B*c^2*m + 15*B*c^2)*cos(f*x + e)^3
+ 32*(7*A - 5*B)*c^2 - (8*(A - B)*c^2*m^3 + 4*(11*A - 19*B)*c^2*m^2 + 2*(3
1*A - 63*B)*c^2*m + 3*(7*A - 15*B)*c^2)*cos(f*x + e)^2 - 2*(8*(A - B)*c^2*
m^3 + 60*(A - B)*c^2*m^2 + 2*(63*A - 79*B)*c^2*m + (49*A - 65*B)*c^2)*cos(
f*x + e))*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(
16*f*m^4 + 128*f*m^3 + 344*f*m^2 + 352*f*m + (16*f*m^4 + 128*f*m^3 + 344*f
*m^2 + 352*f*m + 105*f)*cos(f*x + e) - (16*f*m^4 + 128*f*m^3 + 344*f*m^2 +
 352*f*m + 105*f)*sin(f*x + e) + 105*f)
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 725 vs. 2(264) = 528.

Time = 0.17 (sec) , antiderivative size = 725, normalized size of antiderivative = 2.64

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, al
gorithm="maxima")
```

output
```
-2*(((4*m^2 + 24*m + 43)*a^m*c^(5/2) - (12*m^2 + 40*m - 15)*a^m*c^(5/2)*si
n(f*x + e)/(cos(f*x + e) + 1) + 2*(4*m^2 + 8*m + 35)*a^m*c^(5/2)*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 + 2*(4*m^2 + 8*m + 35)*a^m*c^(5/2)*sin(f*x + e)
^3/(cos(f*x + e) + 1)^3 - (12*m^2 + 40*m - 15)*a^m*c^(5/2)*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 + (4*m^2 + 24*m + 43)*a^m*c^(5/2)*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5)*A*e^(2*m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) - m*lo
g(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))/((8*m^3 + 36*m^2 + 46*m + 15)*
(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(5/2)) - 2*((4*m^2 + 40*m + 115)
*a^m*c^(5/2) - 2*(4*m^3 + 40*m^2 + 115*m)*a^m*c^(5/2)*sin(f*x + e)/(cos(f*
x + e) + 1) + 2*(12*m^3 + 76*m^2 + 97*m + 175)*a^m*c^(5/2)*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 - (16*m^3 + 76*m^2 + 260*m - 175)*a^m*c^(5/2)*sin(f*x
 + e)^3/(cos(f*x + e) + 1)^3 - (16*m^3 + 76*m^2 + 260*m - 175)*a^m*c^(5/2)
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 2*(12*m^3 + 76*m^2 + 97*m + 175)*a^
m*c^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 2*(4*m^3 + 40*m^2 + 115*m)
*a^m*c^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + (4*m^2 + 40*m + 115)*a^
m*c^(5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7)*B*e^(2*m*log(sin(f*x + e)/(
cos(f*x + e) + 1) + 1) - m*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))/(
(16*m^4 + 128*m^3 + 344*m^2 + 352*m + (16*m^4 + 128*m^3 + 344*m^2 + 352*m
+ 105)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 105)*(sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 1)^(5/2)))/f
```

Giac [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx=\int (B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{5}{2}}(a\sin(fx+e)+a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, al
gorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(5/2)*(a*sin(f*x + e)
 + a)^m, x)
```

Mupad [B] (verification not implemented)

Time = 48.17 (sec) , antiderivative size = 749, normalized size of antiderivative = 2.72

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx=\text{Too large to display}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(5/2)
,x)
```

output
```
-((c - c*sin(e + f*x))^(1/2)*((B*c^2*(a + a*sin(e + f*x))^m*(m*46i + m^2*3
6i + m^3*8i + 15i))/(4*f*(352*m + 344*m^2 + 128*m^3 + 16*m^4 + 105)) - (c^
2*exp(e*3i + f*x*3i)*(a + a*sin(e + f*x))^m*(2100*A - 1575*B + 1272*A*m -
110*B*m + 304*A*m^2 + 32*A*m^3 - 68*B*m^2 - 8*B*m^3))/(4*f*(352*m + 344*m^
2 + 128*m^3 + 16*m^4 + 105)) + (c^2*exp(e*4i + f*x*4i)*(a + a*sin(e + f*x)
)^m*(A*2100i - B*1575i + A*m*1272i - B*m*110i + A*m^2*304i + A*m^3*32i - B
*m^2*68i - B*m^3*8i))/(4*f*(352*m + 344*m^2 + 128*m^3 + 16*m^4 + 105)) - (
c^2*exp(e*5i + f*x*5i)*(2*m + 1)*(a + a*sin(e + f*x))^m*(350*A - 385*B + 1
84*A*m - 104*B*m + 24*A*m^2 - 12*B*m^2))/(4*f*(352*m + 344*m^2 + 128*m^3 +
 16*m^4 + 105)) + (c^2*exp(e*2i + f*x*2i)*(2*m + 1)*(a + a*sin(e + f*x))^m
*(A*350i - B*385i + A*m*184i - B*m*104i + A*m^2*24i - B*m^2*12i))/(4*f*(35
2*m + 344*m^2 + 128*m^3 + 16*m^4 + 105)) - (B*c^2*exp(e*7i + f*x*7i)*(a +
a*sin(e + f*x))^m*(46*m + 36*m^2 + 8*m^3 + 15))/(4*f*(352*m + 344*m^2 + 12
8*m^3 + 16*m^4 + 105)) + (c^2*exp(e*1i + f*x*1i)*(a + a*sin(e + f*x))^m*(8
*m + 4*m^2 + 3)*(14*A - 35*B + 4*A*m - 6*B*m))/(4*f*(352*m + 344*m^2 + 128
*m^3 + 16*m^4 + 105)) - (c^2*exp(e*6i + f*x*6i)*(a + a*sin(e + f*x))^m*(8*
m + 4*m^2 + 3)*(A*14i - B*35i + A*m*4i - B*m*6i))/(4*f*(352*m + 344*m^2 +
128*m^3 + 16*m^4 + 105))))/(exp(e*4i + f*x*4i) - (exp(e*3i + f*x*3i)*(m*35
2i + m^2*344i + m^3*128i + m^4*16i + 105i))/(352*m + 344*m^2 + 128*m^3 + 1
6*m^4 + 105))
```

Reduce [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{5/2}\,dx = \sqrt{c}\,c^2\left(\left(\left(\int (a+a\sin(fx+e))^m\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^3\,dx\right)b + \left(\int (a+a\sin(fx+e))^m\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)a - 2\left(\int (a+a\sin(fx+e))^m\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)b - 2\left(\int (a+a\sin(fx+e))^m\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)a + \left(\int (a+a\sin(fx+e))^m\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)b + \left(\int (a+a\sin(fx+e))^m\sqrt{-\sin(fx+e)+1}dx\right)a\right)$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x)
```

output
```
sqrt(c)*c**2*(int((sin(e + f*x)*a + a)**m*sqrt( - sin(e + f*x) + 1)*sin(e
+ f*x)**3,x)*b + int((sin(e + f*x)*a + a)**m*sqrt( - sin(e + f*x) + 1)*sin
(e + f*x)**2,x)*a - 2*int((sin(e + f*x)*a + a)**m*sqrt( - sin(e + f*x) + 1
)*sin(e + f*x)**2,x)*b - 2*int((sin(e + f*x)*a + a)**m*sqrt( - sin(e + f*x
) + 1)*sin(e + f*x),x)*a + int((sin(e + f*x)*a + a)**m*sqrt( - sin(e + f*x
) + 1)*sin(e + f*x),x)*b + int((sin(e + f*x)*a + a)**m*sqrt( - sin(e + f*x
) + 1),x)*a)
```

3.206 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 166

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx=\frac{4(A-B)c^2\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}}-\frac{2(A-3B)c^2\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(3+2m)\sqrt{c-c\sin(e+fx)}}-\frac{2Bc^2\cos(e+fx)(a+a\sin(e+fx))^{2+m}}{a^2f(5+2m)\sqrt{c-c\sin(e+fx)}}$$

output
```
4*(A-B)*c^2*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)
-2*(A-3*B)*c^2*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)/a/f/(3+2*m)/(c-c*sin(f*x+
e))^(1/2)-2*B*c^2*cos(f*x+e)*(a+a*sin(f*x+e))^(2+m)/a^2/f/(5+2*m)/(c-c*sin
(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 6.78 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.05

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx = \frac{c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^m\sqrt{c-c\sin(e+fx)}(50A-39B}{f(1+2m)(3+}$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^(3/2),x]
```

output
```
(c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^m*Sqrt[c -
 c*Sin[e + f*x]]*(50*A - 39*B + 40*A*m - 16*B*m + 8*A*m^2 - 4*B*m^2 + B*(3
 + 8*m + 4*m^2)*Cos[2*(e + f*x)] - 2*(1 + 2*m)*(5*A - 9*B + 2*A*m - 2*B*m)
*Sin[e + f*x]))/(f*(1 + 2*m)*(3 + 2*m)*(5 + 2*m)*(Cos[(e + f*x)/2] - Sin[(
e + f*x)/2]))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 3452, 3042, 3219, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c-c\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^m(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c-c\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^m(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3452}$$

$$-\frac{(B(3-2m)-A(2m+5))\int(\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{3/2}dx}{2m+5}-\frac{2B\cos(e+fx)(c-c\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^m}{f(2m+5)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(B(3-2m)-A(2m+5))\int(\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{3/2}dx}{2m+5}-\frac{2B\cos(e+fx)(c-c\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^m}{f(2m+5)}$$

$$\Big\downarrow \text{3219}$$

$$-\frac{(B(3-2m)-A(2m+5))\left(\frac{4c\int(\sin(e+fx)a+a)^m\sqrt{c-c\sin(e+fx)}dx}{2m+3}+\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}(a\sin(e+fx)+a)^m}{f(2m+3)}\right)}{2m+5}-\frac{2B\cos(e+fx)(c-c\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^m}{f(2m+5)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(B(3-2m)-A(2m+5))\left(\frac{4c\int(\sin(e+fx)a+a)^m\sqrt{c-c\sin(e+fx)}dx}{2m+3}+\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}(a\sin(e+fx)+a)^m}{f(2m+3)}\right)}{2m+5}-\frac{2B\cos(e+fx)(c-c\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^m}{f(2m+5)}$$

$$\Big\downarrow \text{3217}$$

$$-\frac{(B(3-2m)-A(2m+5))\left(\frac{8c^2\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)(2m+3)\sqrt{c-c\sin(e+fx)}}+\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}(a\sin(e+fx)+a)^m}{f(2m+3)}\right)}{2m+5}-\frac{2B\cos(e+fx)(c-c\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^m}{f(2m+5)}$$

input
```
Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2)
,x]
```

output
```
(-2*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(3/2))/(f*(
5 + 2*m)) - ((B*(3 - 2*m) - A*(5 + 2*m))*((8*c^2*Cos[e + f*x]*(a + a*Sin[e
 + f*x])^m)/(f*(1 + 2*m)*(3 + 2*m)*Sqrt[c - c*Sin[e + f*x]]) + (2*c*Cos[e
+ f*x]*(a + a*Sin[e + f*x])^m*Sqrt[c - c*Sin[e + f*x]])/(f*(3 + 2*m))))/(5
 + 2*m)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3219
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Simp[a*((2*m - 1)/(m + n
))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Fre
eQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && I
GtQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(
ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

rule 3452
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

Maple [F]

$$\int (a + a\sin(fx+e))^m (A + B\sin(fx+e)) (c - c\sin(fx+e))^{\frac{3}{2}} dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.89

$$\int (a + a\sin(e+fx))^m (A + B\sin(e+fx))(c - c\sin(e+fx))^{3/2} dx = \frac{2\left((4\,Bcm^2 + 8\,Bcm + 3\,Bc)\cos(fx+e)^3 + 8\,(A+B)cm + (4\,Acm^2 + 12\,(A-B)cm + (5$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, al
gorithm="fricas")
```

output
```
2*((4*B*c*m^2 + 8*B*c*m + 3*B*c)*cos(f*x + e)^3 + 8*(A + B)*c*m + (4*A*c*m
^2 + 12*(A - B)*c*m + (5*A - 6*B)*c)*cos(f*x + e)^2 + 4*(5*A - 3*B)*c + (4
*(A - B)*c*m^2 + 4*(5*A - 3*B)*c*m + (25*A - 21*B)*c)*cos(f*x + e) + (8*(A
 + B)*c*m + (4*B*c*m^2 + 8*B*c*m + 3*B*c)*cos(f*x + e)^2 + 4*(5*A - 3*B)*c
 - (4*(A - B)*c*m^2 + 4*(3*A - 5*B)*c*m + (5*A - 9*B)*c)*cos(f*x + e))*sin
(f*x + e))*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(8*f*m^3 + 36*
f*m^2 + 46*f*m + (8*f*m^3 + 36*f*m^2 + 46*f*m + 15*f)*cos(f*x + e) - (8*f*
m^3 + 36*f*m^2 + 46*f*m + 15*f)*sin(f*x + e) + 15*f)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(160) = 320$.

Time = 0.15 (sec) , antiderivative size = 498, normalized size of antiderivative = 3.00

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, al
gorithm="maxima")
```

output
```
-2*((a^m*c^(3/2)*(2*m + 5) - a^m*c^(3/2)*(2*m - 3)*sin(f*x + e)/(cos(f*x +
 e) + 1) - a^m*c^(3/2)*(2*m - 3)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^m
*c^(3/2)*(2*m + 5)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*A*e^(2*m*log(sin(f
*x + e)/(cos(f*x + e) + 1) + 1) - m*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 1))/((4*m^2 + 8*m + 3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2)
) - 2*(a^m*c^(3/2)*(2*m + 9) - 2*(2*m^2 + 9*m)*a^m*c^(3/2)*sin(f*x + e)/(c
os(f*x + e) + 1) + (4*m^2 + 15)*a^m*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + (4*m^2 + 15)*a^m*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 2*(
2*m^2 + 9*m)*a^m*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^m*c^(3/2)
*(2*m + 9)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)*B*e^(2*m*log(sin(f*x + e)/
(cos(f*x + e) + 1) + 1) - m*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))/
((8*m^3 + 36*m^2 + 46*m + (8*m^3 + 36*m^2 + 46*m + 15)*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 15)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2)))/f
```

Giac [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx=\int (B\sin(fx+e)+A)(-c\sin(fx+e)+c)^{\frac{3}{2}}(a\sin(fx+e)+a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, al
gorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(3/2)*(a*sin(f*x + e)
 + a)^m, x)
```

Mupad [B] (verification not implemented)

Time = 43.21 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.89

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx=\frac{\sqrt{c-c\,\sin(e+f\,x)}\,\left(\frac{c\,\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,(a+a\,\sin(e+f\,x))^m\,(45\,A-30\,B+28\,A\,m+4\,B\,m+4\,A\,m^2)}{f\,(m^3\,8\mathrm{i}+m^2\,36\mathrm{i}+m\,46\mathrm{i}+15\mathrm{i})}+\frac{c\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,(a+a\,\mathrm{si}}{}\right.}{}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(3/2)
,x)
```

output
```
((c - c*sin(e + f*x))^(1/2)*((c*exp(e*3i + f*x*3i)*(a + a*sin(e + f*x))^m*
(45*A - 30*B + 28*A*m + 4*B*m + 4*A*m^2))/(f*(m*46i + m^2*36i + m^3*8i + 1
5i)) + (c*exp(e*2i + f*x*2i)*(a + a*sin(e + f*x))^m*(A*45i - B*30i + A*m*2
8i + B*m*4i + A*m^2*4i))/(f*(m*46i + m^2*36i + m^3*8i + 15i)) + (B*c*(a +
a*sin(e + f*x))^m*(m*8i + m^2*4i + 3i))/(2*f*(m*46i + m^2*36i + m^3*8i + 1
5i)) + (B*c*exp(e*5i + f*x*5i)*(a + a*sin(e + f*x))^m*(8*m + 4*m^2 + 3))/(
2*f*(m*46i + m^2*36i + m^3*8i + 15i)) + (c*exp(e*1i + f*x*1i)*(2*m + 1)*(a
 + a*sin(e + f*x))^m*(10*A - 15*B + 4*A*m - 2*B*m))/(2*f*(m*46i + m^2*36i
+ m^3*8i + 15i)) + (c*exp(e*4i + f*x*4i)*(2*m + 1)*(a + a*sin(e + f*x))^m*
(A*10i - B*15i + A*m*4i - B*m*2i))/(2*f*(m*46i + m^2*36i + m^3*8i + 15i)))
)/(exp(e*3i + f*x*3i) + (exp(e*2i + f*x*2i)*(46*m + 36*m^2 + 8*m^3 + 15))/
(m*46i + m^2*36i + m^3*8i + 15i))
```

Reduce [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{3/2}\,dx=\sqrt{c}\,c\left(-\left(\int (a+a\sin(fx+e))^m\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)b-\left(\int (a+a\sin(fx+e))^m\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)a+\left(\int (a+a\sin(fx+e))^m\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)b+\left(\int (a+a\sin(fx+e))^m\sqrt{-\sin(fx+e)+1}dx\right)a\right)$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x)
```

output
```
sqrt(c)*c*( - int((sin(e + f*x)*a + a)**m*sqrt( - sin(e + f*x) + 1)*sin(e
+ f*x)**2,x)*b - int((sin(e + f*x)*a + a)**m*sqrt( - sin(e + f*x) + 1)*sin
(e + f*x),x)*a + int((sin(e + f*x)*a + a)**m*sqrt( - sin(e + f*x) + 1)*sin
(e + f*x),x)*b + int((sin(e + f*x)*a + a)**m*sqrt( - sin(e + f*x) + 1),x)*
a)
```

3.207 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c-c\sin(e$

Optimal result

Integrand size = 38, antiderivative size = 104

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \frac{2(A-B)c\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}} + \frac{2Bc\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(3+2m)\sqrt{c-c\sin(e+fx)}}$$

output
```
2*(A-B)*c*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)+2
*B*c*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)/a/f/(3+2*m)/(c-c*sin(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 2.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.12

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^m\sqrt{c-c\sin(e+fx)}(-2B+A(3+2m)+B}{f(1+2m)(3+2m)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f
*x]],x]
```

output
```
(2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^m*Sqrt[c -
 c*Sin[e + f*x]]*(-2*B + A*(3 + 2*m) + B*(1 + 2*m)*Sin[e + f*x]))/(f*(1 +
2*m)*(3 + 2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3042, 3450, 3042, 3217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c-c\sin(e+fx)}(a\sin(e+fx)+a)^m(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{c-c\sin(e+fx)}(a\sin(e+fx)+a)^m(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3450}$$

$$(A-B)\int (\sin(e+fx)a+a)^m\sqrt{c-c\sin(e+fx)}dx+\frac{B\int (\sin(e+fx)a+a)^{m+1}\sqrt{c-c\sin(e+fx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$(A-B)\int (\sin(e+fx)a+a)^m\sqrt{c-c\sin(e+fx)}dx+\frac{B\int (\sin(e+fx)a+a)^{m+1}\sqrt{c-c\sin(e+fx)}dx}{a}$$

$$\Big\downarrow \text{3217}$$

$$\frac{2c(A-B)\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sin(e+fx)}}+\frac{2Bc\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(2m+3)\sqrt{c-c\sin(e+fx)}}$$

input
```
Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]],x
]
```

output
```
(2*(A - B)*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*
Sin[e + f*x]]) + (2*B*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(3
 + 2*m)*Sqrt[c - c*Sin[e + f*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3217
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

rule 3450
```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[B/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] -
Simp[(B*c - A*d)/d   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[
a^2 - b^2, 0]
```

Maple [F]

$$\int (a+a\sin(fx+e))^m (A+B\sin(fx+e))\sqrt{c-c\sin(fx+e)}dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.59

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = -\frac{2\left((2\,Bm+B)\cos\left(fx+e\right)^2-2\,(A+B)m-(2\,Am+3\,A-2\,B)\cos\left(fx+e\right)-(2\,(A+B)m+(2\,B\right.}{4\,fm^2+8\,fm+(4\,fm^2+8\,fm+3\,f)\cos\left(fx+\right.}$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, al
gorithm="fricas")
```

output
```
-2*((2*B*m + B)*cos(f*x + e)^2 - 2*(A + B)*m - (2*A*m + 3*A - 2*B)*cos(f*x
 + e) - (2*(A + B)*m + (2*B*m + B)*cos(f*x + e) + 3*A - B)*sin(f*x + e) -
3*A + B)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(4*f*m^2 + 8*f*m
 + (4*f*m^2 + 8*f*m + 3*f)*cos(f*x + e) - (4*f*m^2 + 8*f*m + 3*f)*sin(f*x
+ e) + 3*f)
```

Sympy [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx = \int (a(\sin(e+fx)+1))^m\sqrt{-c(\sin(e+fx)-1)}(A+B\sin(e+fx))\,dx$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*sqrt(-c*(sin(e + f*x) - 1))*(A + B*sin(
e + f*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(100) = 200$.

Time = 0.15 (sec) , antiderivative size = 323, normalized size of antiderivative = 3.11

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx =$$
$$-\frac{2\left(\frac{2\left(\frac{2a^m\sqrt{c}m\sin(fx+e)}{\cos(fx+e)+1}+\frac{2a^m\sqrt{c}m\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-a^m\sqrt{c}-\frac{a^m\sqrt{c}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)Be^{\left(2m\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)-m\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)\right)}}{\left(4m^2+8m+\frac{(4m^2+8m+3)\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+3\right)\sqrt{\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1}}+\right.}{f}$$

input

```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, al
gorithm="maxima")
```

output

```
-2*(2*(2*a^m*sqrt(c)*m*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a^m*sqrt(c)*m*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 - a^m*sqrt(c) - a^m*sqrt(c)*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3)*B*e^(2*m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1
) - m*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))/((4*m^2 + 8*m + (4*m^2
 + 8*m + 3)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3)*sqrt(sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 1)) + (a^m*sqrt(c) + a^m*sqrt(c)*sin(f*x + e)/(cos(f
*x + e) + 1))*A*e^(2*m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) - m*log(si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))/((2*m + 1)*sqrt(sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 1)))/f
```

Giac [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$$
$$=\int (B\sin(fx+e)+A)\sqrt{-c\sin(fx+e)+c}(a\sin(fx+e)+a)^m\,dx$$

input

```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, al
gorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) +
 a)^m, x)
```

Mupad [B] (verification not implemented)

Time = 1.60 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.01

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx =$$
$$-\frac{(a\,(\sin(e+f\,x)+1))^m\,\sqrt{-c\,(\sin(e+f\,x)-1)}\,(6\,A\,\cos(e+f\,x)-4\,B\,\cos(e+f\,x)+B\,\sin(2\,e+}{f\,\left(\sin(e+f\,x)-1\right)\,\left(4\,m^2+8\,m+3\right)}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(1/2)
,x)
```

output
```
-((a*(sin(e + f*x) + 1))^m*(-c*(sin(e + f*x) - 1))^(1/2)*(6*A*cos(e + f*x)
 - 4*B*cos(e + f*x) + B*sin(2*e + 2*f*x) + 4*A*m*cos(e + f*x) + 2*B*m*sin(
2*e + 2*f*x)))/(f*(sin(e + f*x) - 1)*(8*m + 4*m^2 + 3))
```

Reduce [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,dx$$
$$=\sqrt{c}\left(\left(\int (a+a\sin(fx+e))^m\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)b+\left(\int (a+a\sin(fx+e))^m\sqrt{-\sin(fx+e)+1}dx\right)a\right)$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)
```

output
```
sqrt(c)*(int((sin(e + f*x)*a + a)**m*sqrt( - sin(e + f*x) + 1)*sin(e + f*x
),x)*b + int((sin(e + f*x)*a + a)**m*sqrt( - sin(e + f*x) + 1),x)*a)
```

3.208 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 118

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{2B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}} + \frac{(A+B)\cos(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(e+fx))^m}{f(1+2m)\sqrt{c-c\sin(e+fx)}}$$

output
```
-2*B*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)+(A+B)*
cos(f*x+e)*hypergeom([1, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+e))*(a+a*sin(f*x+e
))^m/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.12

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{\cos(e+fx)(a(1+\sin(e+fx)))^m\left(2A(3+2m)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx\right.\right.}{2f(1+2m)(3+2}$$

input
```
Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e +
 f*x]],x]
```

output
```
(Cos[e + f*x]*(a*(1 + Sin[e + f*x]))^m*(2*A*(3 + 2*m)*Hypergeometric2F1[1,
 1/2 + m, 3/2 + m, (1 + Sin[e + f*x])/2] + B*(-6 - 4*m + (1 + 2*m)*Hyperge
ometric2F1[1, 3/2 + m, 5/2 + m, (1 + Sin[e + f*x])/2]*(1 + Sin[e + f*x])))
)/(2*f*(1 + 2*m)*(3 + 2*m)*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {3042, 3452, 3042, 3224, 3042, 3146, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a \sin(e+fx)+a)^m (A+B \sin(e+fx))}{\sqrt{c-c \sin(e+fx)}} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a \sin(e+fx)+a)^m (A+B \sin(e+fx))}{\sqrt{c-c \sin(e+fx)}}dx$$

$$\Big\downarrow \text{3452}$$

$$(A+B) \int \frac{(\sin(e+fx)a+a)^m}{\sqrt{c-c \sin(e+fx)}}dx-\frac{2B \cos(e+fx)(a \sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c \sin(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$(A+B) \int \frac{(\sin(e+fx)a+a)^m}{\sqrt{c-c \sin(e+fx)}}dx-\frac{2B \cos(e+fx)(a \sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c \sin(e+fx)}}$$

$$\Big\downarrow \text{3224}$$

$$\frac{(A+B)\cos(e+fx)\int \sec(e+fx)(\sin(e+fx)a+a)^{m+\frac{1}{2}}dx}{\sqrt{a \sin(e+fx)+a}\sqrt{c-c \sin(e+fx)}}-\frac{2B \cos(e+fx)(a \sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c \sin(e+fx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A+B)\cos(e+fx)\int \frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}}{\cos(e+fx)}dx}{\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{3146}$$

$$\frac{a(A+B)\cos(e+fx)\int \frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}}{a-a\sin(e+fx)}d(a\sin(e+fx))}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

$$\Big\downarrow \text{78}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m \text{Hypergeometric2F1}\left(1,m+\frac{1}{2},m+\frac{3}{2},\frac{\sin(e+fx)a+a}{2a}\right)}{f(2m+1)\sqrt{c-c\sin(e+fx)}}-\frac{2B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sin(e+fx)}}$$

input
```
Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]]
,x]
```

output
```
(-2*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*Sin[e +
 f*x]]) + ((A + B)*Cos[e + f*x]*Hypergeometric2F1[1, 1/2 + m, 3/2 + m, (a
+ a*Sin[e + f*x])/(2*a)]*(a + a*Sin[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*S
in[e + f*x]])
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3224
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m]))   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (FractionQ[m] ||  !FractionQ[n])
```

rule 3452
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

Maple [F]

$$\int \frac{(a + a\sin(fx + e))^m (A + B\sin(fx + e))}{\sqrt{c - c\sin(fx + e)}} dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, al
gorithm="fricas")
```

output
```
integral(-(B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) +
 a)^m/(c*sin(f*x + e) - c), x)
```

Sympy [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(a(\sin(e+fx)+1))^m\,(A+B\sin(e+fx))}{\sqrt{-c\,(\sin(e+fx)-1)}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))/sqrt(-c*(sin(e + f
*x) - 1)), x)
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{\sqrt{-c\sin(fx+e)+c}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/sqrt(-c*sin(f*x + e)
 + c), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, al
gorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m}{\sqrt{c-c\,\sin(e+f\,x)}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(1/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(1/
2), x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{\sqrt{c}\left(\left(\int\frac{(a+a\sin(fx+e))^m\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)-1}dx\right)b+\left(\int\frac{(a+a\sin(fx+e))^m\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)-1}dx\right)a\right)}{c}$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

output
```
( - sqrt(c)*(int(((sin(e + f*x)*a + a)**m*sqrt( - sin(e + f*x) + 1)*sin(e
+ f*x))/(sin(e + f*x) - 1),x)*b + int(((sin(e + f*x)*a + a)**m*sqrt( - sin
(e + f*x) + 1))/(sin(e + f*x) - 1),x)*a))/c
```

3.209 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 134

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m}{2f(c-c\sin(e+fx))^{3/2}}+\frac{(A(1-2m)-B(3+2m))\cos(e+fx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx))\right)(a+a$$
$$\overline{4cf(1+2m)\sqrt{c-c\sin(e+fx)}}$$

output
```
1/2*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(c-c*sin(f*x+e))^(3/2)+1/4*(A*(1
-2*m)-B*(3+2*m))*cos(f*x+e)*hypergeom([1, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+e
))*(a+a*sin(f*x+e))^m/c/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 36.36 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.07

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=$$
$$-\frac{\cos(e+fx)(a(1+\sin(e+fx)))^m\left(B(3+2m)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx)\right)\right.}{4cf(1+2m)(-1}$$

input
```
Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^(3/2),x]
```

output
```
-1/4*(Cos[e + f*x]*(a*(1 + Sin[e + f*x]))^m*(B*(3 + 2*m)*Hypergeometric2F1
[1, 1/2 + m, 3/2 + m, (1 + Sin[e + f*x])/2]*(-1 + Sin[e + f*x]) + 2*(B + 2
*B*m + Hypergeometric2F1[2, 1/2 + m, 3/2 + m, (1 + Sin[e + f*x])/2]*(A - A
*Sin[e + f*x]))))/(c*f*(1 + 2*m)*(-1 + Sin[e + f*x])*Sqrt[c - c*Sin[e + f*
x]])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {3042, 3451, 3042, 3224, 3042, 3146, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3451}$$

$$\frac{(A(1-2m)-B(2m+3))\int \frac{(\sin(e+fx)a+a)^m}{\sqrt{c-c\sin(e+fx)}}dx}{4c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f(c-c\sin(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A(1-2m)-B(2m+3))\int \frac{(\sin(e+fx)a+a)^m}{\sqrt{c-c\sin(e+fx)}}dx}{4c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f(c-c\sin(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3224}$$

$$\frac{(A(1-2m)-B(2m+3))\cos(e+fx)\int\sec(e+fx)(\sin(e+fx)a+a)^{m+\frac{1}{2}}dx}{4c\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f(c-c\sin(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A(1-2m)-B(2m+3))\cos(e+fx)\int\frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}}{\cos(e+fx)}dx}{4c\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f(c-c\sin(e+fx))^{3/2}}$$

$$\Big\downarrow \text{3146}$$

$$\frac{a(A(1-2m)-B(2m+3))\cos(e+fx)\int\frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}}{a-a\sin(e+fx)}d(a\sin(e+fx))}{4cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f(c-c\sin(e+fx))^{3/2}}$$

$$\Big\downarrow \text{78}$$

$$\frac{(A(1-2m)-B(2m+3))\cos(e+fx)(a\sin(e+fx)+a)^m\,\text{Hypergeometric2F1}\left(1,m+\frac{1}{2},m+\frac{3}{2},\frac{\sin(e+fx)a+a}{2a}\right)}{4cf(2m+1)\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f(c-c\sin(e+fx))^{3/2}}$$

input
```
Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(3/
2),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(2*f*(c - c*Sin[e + f*x])^(3
/2)) + ((A*(1 - 2*m) - B*(3 + 2*m))*Cos[e + f*x]*Hypergeometric2F1[1, 1/2
+ m, 3/2 + m, (a + a*Sin[e + f*x])/(2*a)]*(a + a*Sin[e + f*x])^m)/(4*c*f*(
1 + 2*m)*Sqrt[c - c*Sin[e + f*x]])
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3224
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m]))   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (FractionQ[m] ||  !FractionQ[n])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

Maple [F]

$$\int \frac{(a+a\sin(fx+e))^m (A+B\sin(fx+e))}{(c-c\sin(fx+e))^{\frac{3}{2}}}dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, al
gorithm="fricas")
```

output
```
integral(-(B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) +
 a)^m/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

Sympy [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \int \frac{(a(\sin(e+fx)+1))^m\,(A+B\sin(e+fx))}{(-c(\sin(e+fx)-1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))/(-c*(sin(e + f*x)
- 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(-c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c
)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,3,0,0,0,0]%%%}+%%%{3,[0,1,1,0,0,0,0]%%%} / %%%{16,[
0,0,0,1,1
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx=\int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m}{(c-c\,\sin(e+f\,x))^{3/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(3/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(3/
2), x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\sqrt{c}\left(\left(\int \frac{(a+a\sin(fx+e))^m\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)^2-2\sin(fx+e)+1}dx\right)b+\left(\int \right.\right.}{c^2}$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)
```

output
```
(sqrt(c)*(int(((sin(e + f*x)*a + a)**m*sqrt( - sin(e + f*x) + 1)*sin(e + f
*x))/(sin(e + f*x)**2 - 2*sin(e + f*x) + 1),x)*b + int(((sin(e + f*x)*a +
a)**m*sqrt( - sin(e + f*x) + 1))/(sin(e + f*x)**2 - 2*sin(e + f*x) + 1),x)
*a))/c**2
```

3.210 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 134

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx=\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m}{4f(c-c\sin(e+fx))^{5/2}}+\frac{(A(3-2m)-B(5+2m))\cos(e+fx)\,\text{Hypergeometric2F1}\left(2,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx))\right)(a+a}{16c^2f(1+2m)\sqrt{c-c\sin(e+fx)}}$$

output
```
1/4*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(c-c*sin(f*x+e))^(5/2)+1/16*(A*(
3-2*m)-B*(5+2*m))*cos(f*x+e)*hypergeom([2, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+
e))*(a+a*sin(f*x+e))^m/c^2/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 48.21 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.19

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx=-\frac{\cos(e+fx)\left(B(5+2m)\,\text{Hypergeometric2F1}\left(2,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sin(e+fx))\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\right.\right.}{16c^2(f+}$$

input
```
Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^(5/2),x]
```

output
```
-1/16*(Cos[e + f*x]*(B*(5 + 2*m)*Hypergeometric2F1[2, 1/2 + m, 3/2 + m, (1
 + Sin[e + f*x])/2]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 - 4*(B + 2*B*m
 + A*Hypergeometric2F1[3, 1/2 + m, 3/2 + m, (1 + Sin[e + f*x])/2]*(-1 + Si
n[e + f*x])^2))*(a*(1 + Sin[e + f*x]))^m)/(c^2*(f + 2*f*m)*(-1 + Sin[e + f
*x])^2*Sqrt[c - c*Sin[e + f*x]])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {3042, 3451, 3042, 3224, 3042, 3146, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3451}$$

$$\frac{(A(3-2m)-B(2m+5))\int \frac{(\sin(e+fx)a+a)^m}{(c-c\sin(e+fx))^{3/2}}dx}{8c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m}{4f(c-c\sin(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A(3-2m)-B(2m+5))\int \frac{(\sin(e+fx)a+a)^m}{(c-c\sin(e+fx))^{3/2}}dx}{8c}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m}{4f(c-c\sin(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3224}$$

$$\frac{(A(3-2m)-B(2m+5))\cos(e+fx)\int \sec^3(e+fx)(\sin(e+fx)a+a)^{m+\frac{3}{2}}dx}{8ac^2\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m}{4f(c-c\sin(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A(3-2m)-B(2m+5))\cos(e+fx)\int \frac{(\sin(e+fx)a+a)^{m+\frac{3}{2}}}{\cos(e+fx)^3}dx}{8ac^2\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m}{4f(c-c\sin(e+fx))^{5/2}}$$

$$\Big\downarrow \text{3146}$$

$$\frac{a^2(A(3-2m)-B(2m+5))\cos(e+fx)\int \frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}}{(a-a\sin(e+fx))^2}d(a\sin(e+fx))}{8c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m}{4f(c-c\sin(e+fx))^{5/2}}$$

$$\Big\downarrow \text{78}$$

$$\frac{(A(3-2m)-B(2m+5))\cos(e+fx)(a\sin(e+fx)+a)^m\,\text{Hypergeometric2F1}\left(2,m+\frac{1}{2},m+\frac{3}{2},\frac{\sin(e+fx)a+a}{2a}\right)}{16c^2f(2m+1)\sqrt{c-c\sin(e+fx)}}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m}{4f(c-c\sin(e+fx))^{5/2}}$$

input
```
Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(5/
2),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(4*f*(c - c*Sin[e + f*x])^(5
/2)) + ((A*(3 - 2*m) - B*(5 + 2*m))*Cos[e + f*x]*Hypergeometric2F1[2, 1/2
+ m, 3/2 + m, (a + a*Sin[e + f*x])/(2*a)]*(a + a*Sin[e + f*x])^m)/(16*c^2*
f*(1 + 2*m)*Sqrt[c - c*Sin[e + f*x]])
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3224
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m]))   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (FractionQ[m] ||  !FractionQ[n])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

Maple [F]

$$\int \frac{(a + a \sin(fx + e))^m (A + B \sin(fx + e))}{(c - c \sin(fx + e))^{\frac{5}{2}}} dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)
```

Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, al
gorithm="fricas")
```

output
```
integral(-(B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) +
 a)^m/(3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x
 + e)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(-c\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c
)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error index.cc index_gcd Error: Bad
 Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m}{(c-c\,\sin(e+f\,x))^{5/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(5/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(5/
2), x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c-c\sin(e+fx))^{5/2}}\,dx = -\frac{\sqrt{c}\left(\left(\int \frac{(a+a\sin(fx+e))^m\sqrt{-\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)^3-3\sin(fx+e)^2+3\sin(fx+e)-1}dx\right)b+\left(\int \frac{(a+a\sin(fx+e))^m\sqrt{-\sin(fx+e)+1}}{\sin(fx+e)^3-3\sin(fx+e)^2+3\sin(fx+e)-1}dx\right)a\right)}{c^3}$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)
```

output
```
( - sqrt(c)*(int(((sin(e + f*x)*a + a)**m*sqrt( - sin(e + f*x) + 1)*sin(e
+ f*x))/(sin(e + f*x)**3 - 3*sin(e + f*x)**2 + 3*sin(e + f*x) - 1),x)*b +
int(((sin(e + f*x)*a + a)**m*sqrt( - sin(e + f*x) + 1))/(sin(e + f*x)**3 -
 3*sin(e + f*x)**2 + 3*sin(e + f*x) - 1),x)*a))/c**3
```

3.211 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-4-m}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 267

$$\begin{aligned}&\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-4-m}\,dx\\&=\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-4-m}}{f(7+2m)}\\&\quad+\frac{(3A-2B(2+m))\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3-m}}{cf(5+2m)(7+2m)}\\&\quad+\frac{2(3A-2B(2+m))\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2-m}}{c^2f(7+2m)\left(15+16m+4m^2\right)}\\&\quad+\frac{2(3A-2B(2+m))\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}}{c^3f(5+2m)(7+2m)\left(3+8m+4m^2\right)}\end{aligned}$$

output
```
(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-4-m)/f/(7+2*m)+(3*A
-2*B*(2+m))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m)/c/f/(5+2
*m)/(7+2*m)+2*(3*A-2*B*(2+m))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e
))^(-2-m)/c^2/f/(7+2*m)/(4*m^2+16*m+15)+2*(3*A-2*B*(2+m))*cos(f*x+e)*(a+a*
sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m)/c^3/f/(5+2*m)/(7+2*m)/(4*m^2+8*m+3)
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.69

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-4-m}\,dx$$
$$=\frac{\sec(e+fx)(a(1+\sin(e+fx)))^{1+m}(c-c\sin(e+fx))^{-m}\left(B(13+16m+4m^2)-2A(18+41m+24m^{2}\right.}{ac^4f(1+2n}$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^(-4 - m),x]
```

output
```
(Sec[e + f*x]*(a*(1 + Sin[e + f*x]))^(1 + m)*(B*(13 + 16*m + 4*m^2) - 2*A*
(18 + 41*m + 24*m^2 + 4*m^3) + (13 + 16*m + 4*m^2)*(3*A - 2*B*(2 + m))*Sin
[e + f*x] + 4*(2 + m)*(-3*A + 2*B*(2 + m))*Sin[e + f*x]^2 + (6*A - 4*B*(2
+ m))*Sin[e + f*x]^3))/(a*c^4*f*(1 + 2*m)*(3 + 2*m)*(5 + 2*m)*(7 + 2*m)*(-
1 + Sin[e + f*x])^3*(c - c*Sin[e + f*x])^m)
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.87, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3451, 3042, 3222, 3042, 3222, 3042, 3221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m-4}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m-4}dx$$

$$\Big\downarrow \text{3451}$$

$$\frac{(3A-2B(m+2))\int (\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{-m-3}dx}{c(2m+7)}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-4}}{f(2m+7)}$$

$\Big\downarrow$ 3042

$$\frac{(3A-2B(m+2))\int (\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{-m-3}dx}{c(2m+7)}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-4}}{f(2m+7)}$$

$\Big\downarrow$ 3222

$$\frac{(3A-2B(m+2))\left(\frac{2\int (\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{-m-2}dx}{c(2m+5)}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-3}}{f(2m+5)}\right)}{c(2m+7)}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-4}}{f(2m+7)}$$

$\Big\downarrow$ 3042

$$\frac{(3A-2B(m+2))\left(\frac{2\int (\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{-m-2}dx}{c(2m+5)}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-3}}{f(2m+5)}\right)}{c(2m+7)}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-4}}{f(2m+7)}$$

$\Big\downarrow$ 3222

$$\frac{(3A-2B(m+2))\left(\frac{2\left(\frac{\int (\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{-m-1}dx}{c(2m+3)}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}}{f(2m+3)}\right)}{c(2m+5)}+\frac{\cos(e+fx)(a\sin(e\cdots}{\cdots}\right.}{c(2m+7)}$$
$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-4}}{f(2m+7)}$$

$\Big\downarrow$ 3042

$$\frac{(3A-2B(m+2))\left(\frac{2\left(\frac{\int (\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{-m-1}dx}{c(2m+3)}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}}{f(2m+3)}\right)}{c(2m+5)}+\frac{\cos(e+fx)(a\sin(e\cdots}{\cdots}\right.}{c(2m+7)}$$
$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-4}}{f(2m+7)}$$

$\Big\downarrow$ 3221

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-4}}{f(2m+7)}+$$
$$\frac{(3A-2B(m+2))\left(\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-3}}{f(2m+5)}+\frac{2\left(\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}}{f(2m+3)}+\frac{\cos(e+fx}{\cdots}\right.}{c(2m+5)}\right.}{c(2m+7)}$$

input
```
Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-4 -
 m),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-4 - m)
)/(f*(7 + 2*m)) + ((3*A - 2*B*(2 + m))*((Cos[e + f*x]*(a + a*Sin[e + f*x])
^m*(c - c*Sin[e + f*x])^(-3 - m))/(f*(5 + 2*m)) + (2*((Cos[e + f*x]*(a + a
*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m))/(f*(3 + 2*m)) + (Cos[e + f
*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(c*f*(1 + 2*m)*(
3 + 2*m))))/(c*(5 + 2*m))))/(c*(7 + 2*m))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3221
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

rule 3222
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp[(m + n + 1)/(a*(2*m + 1)
)   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
 ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||
!SumSimplerQ[n, 1])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

Maple [F]

$$\int (a + a\sin(fx + e))^m (A + B\sin(fx + e)) (c - c\sin(fx + e))^{-4-m} dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-m),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-m),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.77

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{-4-m} dx$$
$$= \frac{(4 (2 Bm^2 - (3 A - 8 B)m - 6 A + 8 B) \cos(fx + e)^3 + (8 Am^3 + 12 (4 A - B)m^2 + 2 (47 A - 24 B)m}{}$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-m),x, a
lgorithm="fricas")
```

output
```
(4*(2*B*m^2 - (3*A - 8*B)*m - 6*A + 8*B)*cos(f*x + e)^3 + (8*A*m^3 + 12*(4
*A - B)*m^2 + 2*(47*A - 24*B)*m + 60*A - 45*B)*cos(f*x + e) - (2*(2*B*m -
3*A + 4*B)*cos(f*x + e)^3 - (8*B*m^3 - 12*(A - 4*B)*m^2 - 2*(24*A - 47*B)*
m - 45*A + 60*B)*cos(f*x + e))*sin(f*x + e))*(a*sin(f*x + e) + a)^m*(-c*si
n(f*x + e) + c)^(-m - 4)/(16*f*m^4 + 128*f*m^3 + 344*f*m^2 + 352*f*m + 105
*f)
```

Sympy [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-4-m}\,dx$$
$$=\int (a(\sin(e+fx)+1))^m\,(-c(\sin(e+fx)-1))^{-m-4}\,(A+B\sin(e+fx))\,dx$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(-4-m),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*(-c*(sin(e + f*x) - 1))**(-m - 4)*(A +
B*sin(e + f*x)), x)
```

Maxima [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-4-m}\,dx$$
$$=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m(-c\sin(fx+e)+c)^{-m-4}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-m),x, a
lgorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c
)^(-m - 4), x)
```

Giac [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-4-m}\,dx$$
$$=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m(-c\sin(fx+e)+c)^{-m-4}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-m),x, a
lgorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c
)^(-m - 4), x)
```

Mupad [B] (verification not implemented)

Time = 47.02 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.38

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-4-m}\,dx$$

$$= -\frac{\sin(4e+4fx)\,(a+a\sin(e+fx))^m\,(4B-3A+2Bm)\,1\mathrm{i}}{4f\,(c-c\sin(e+fx))^{m+4}\,(m^4\,16\mathrm{i}+m^3\,128\mathrm{i}+m^2\,344\mathrm{i}+m\,352\mathrm{i}+105\mathrm{i})} + \frac{\cos(e+fx)\,(a+a\sin(e+fx))^m\,(A\,168\mathrm{i}-B\,84\mathrm{i}+A\,m\,340\mathrm{i}-B\,m\,96\mathrm{i}+A\,m^2\,192\mathrm{i}+A\,m^3\,32\mathrm{i}- \ldots}{4f\,(c-c\sin(e+fx))^{m+4}\,(m^4\,16\mathrm{i}+m^3\,128\mathrm{i}+m^2\,344\mathrm{i}+m\,352\mathrm{i}+105\mathrm{i})} + \frac{\sin(2e+2fx)\,(a+a\sin(e+fx))^m\,(2m^2+8m+7)\,(4B-3A+2Bm)\,1\mathrm{i}}{f\,(c-c\sin(e+fx))^{m+4}\,(m^4\,16\mathrm{i}+m^3\,128\mathrm{i}+m^2\,344\mathrm{i}+m\,352\mathrm{i}+105\mathrm{i})} + \frac{\cos(3e+3fx)\,(m+2)\,(a+a\sin(e+fx))^m\,(-A\,3\mathrm{i}+B\,4\mathrm{i}+B\,m\,2\mathrm{i})}{f\,(c-c\sin(e+fx))^{m+4}\,(m^4\,16\mathrm{i}+m^3\,128\mathrm{i}+m^2\,344\mathrm{i}+m\,352\mathrm{i}+105\mathrm{i})}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m
+ 4),x)
```

output
```
(cos(e + f*x)*(a + a*sin(e + f*x))^m*(A*168i - B*84i + A*m*340i - B*m*96i
+ A*m^2*192i + A*m^3*32i - B*m^2*24i))/(4*f*(c - c*sin(e + f*x))^(m + 4)*(
m*352i + m^2*344i + m^3*128i + m^4*16i + 105i)) - (sin(4*e + 4*f*x)*(a + a
*sin(e + f*x))^m*(4*B - 3*A + 2*B*m)*1i)/(4*f*(c - c*sin(e + f*x))^(m + 4)
*(m*352i + m^2*344i + m^3*128i + m^4*16i + 105i)) + (sin(2*e + 2*f*x)*(a +
 a*sin(e + f*x))^m*(8*m + 2*m^2 + 7)*(4*B - 3*A + 2*B*m)*1i)/(f*(c - c*sin
(e + f*x))^(m + 4)*(m*352i + m^2*344i + m^3*128i + m^4*16i + 105i)) + (cos
(3*e + 3*f*x)*(m + 2)*(a + a*sin(e + f*x))^m*(B*4i - A*3i + B*m*2i))/(f*(c
 - c*sin(e + f*x))^(m + 4)*(m*352i + m^2*344i + m^3*128i + m^4*16i + 105i)
)
```

Reduce [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-4-m}\,dx$$
$$=\frac{\left(\int\frac{(a+a\sin(fx+e))^m}{(-\sin(fx+e)c+c)^m\sin(fx+e)^4-4(-\sin(fx+e)c+c)^m\sin(fx+e)^3+6(-\sin(fx+e)c+c)^m\sin(fx+e)^2-4(-\sin(fx+e)c+c)^m\sin(fx+e)+$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-m),x)
```

output
```
(int((sin(e + f*x)*a + a)**m/(( - sin(e + f*x)*c + c)**m*sin(e + f*x)**4 -
 4*( - sin(e + f*x)*c + c)**m*sin(e + f*x)**3 + 6*( - sin(e + f*x)*c + c)*
*m*sin(e + f*x)**2 - 4*( - sin(e + f*x)*c + c)**m*sin(e + f*x) + ( - sin(e
 + f*x)*c + c)**m),x)*a + int(((sin(e + f*x)*a + a)**m*sin(e + f*x))/(( -
sin(e + f*x)*c + c)**m*sin(e + f*x)**4 - 4*( - sin(e + f*x)*c + c)**m*sin(
e + f*x)**3 + 6*( - sin(e + f*x)*c + c)**m*sin(e + f*x)**2 - 4*( - sin(e +
 f*x)*c + c)**m*sin(e + f*x) + ( - sin(e + f*x)*c + c)**m),x)*b)/c**4
```

3.212 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-3-m}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 191

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-3-m}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3-m}}{f(5+2m)} + \frac{(2A-B(3+2m))\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2-m}}{cf(3+2m)(5+2m)} + \frac{(2A-B(3+2m))\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}}{c^2f(5+2m)\left(3+8m+4m^2\right)}$$

output

```
(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m)/f/(5+2*m)+(2*A
-B*(3+2*m))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m)/c/f/(3+2
*m)/(5+2*m)+(2*A-B*(3+2*m))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))
^(-1-m)/c^2/f/(5+2*m)/(4*m^2+8*m+3)
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.74

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-3-m}\,dx$$
$$= \frac{\sec(e+fx)(a(1+\sin(e+fx)))^{1+m}(c-c\sin(e+fx))^{-m}\left(-B(3+2m)+A(7+12m+4m^2)+(3+2\right.}{ac^3f(1+2m)(3+2m)(5+2m)(-1+\sin(}$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^(-3 - m),x]
```

output
```
(Sec[e + f*x]*(a*(1 + Sin[e + f*x]))^(1 + m)*(-(B*(3 + 2*m)) + A*(7 + 12*m
 + 4*m^2) + (3 + 2*m)*(-2*A + B*(3 + 2*m))*Sin[e + f*x] + (2*A - B*(3 + 2*
m))*Sin[e + f*x]^2))/(a*c^3*f*(1 + 2*m)*(3 + 2*m)*(5 + 2*m)*(-1 + Sin[e +
f*x])^2*(c - c*Sin[e + f*x])^m)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3451, 3042, 3222, 3042, 3221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m-3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m-3}dx$$

$$\Big\downarrow \text{3451}$$

$$\frac{(2A-B(2m+3))\int (\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{-m-2}dx}{c(2m+5)}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-3}}{f(2m+5)}$$

$$\Big\downarrow \; 3042$$

$$\frac{(2A-B(2m+3))\int (\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{-m-2}dx}{c(2m+5)}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-3}}{f(2m+5)}$$

$$\Big\downarrow \; 3222$$

$$\frac{(2A-B(2m+3))\left(\frac{\int (\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{-m-1}dx}{c(2m+3)}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}}{f(2m+3)}\right)}{c(2m+5)}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-3}}{f(2m+5)}$$

$$\Big\downarrow \; 3042$$

$$\frac{(2A-B(2m+3))\left(\frac{\int (\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{-m-1}dx}{c(2m+3)}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}}{f(2m+3)}\right)}{c(2m+5)}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-3}}{f(2m+5)}$$

$$\Big\downarrow \; 3221$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-3}}{f(2m+5)}+\frac{(2A-B(2m+3))\left(\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}}{f(2m+3)}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{cf(2m+1)(2m+3)}\right)}{c(2m+5)}$$

input
```
Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-3 -
 m),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-3 - m)
)/(f*(5 + 2*m)) + ((2*A - B*(3 + 2*m))*((Cos[e + f*x]*(a + a*Sin[e + f*x])
^m*(c - c*Sin[e + f*x])^(-2 - m))/(f*(3 + 2*m)) + (Cos[e + f*x]*(a + a*Sin
[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(c*f*(1 + 2*m)*(3 + 2*m))))/(c
*(5 + 2*m))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3221
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

rule 3222
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Simp[(m + n + 1)/(a*(2*m + 1)
)   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
 ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||
!SumSimplerQ[n, 1])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

Maple [F]

$$\int (a+a\sin(fx+e))^m (A+B\sin(fx+e)) (c-c\sin(fx+e))^{-3-m} dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-m),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-m),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.72

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-3-m}\,dx$$
$$= \frac{((2\,Bm-2\,A+3\,B)\cos{(fx+e)}^3+(4\,Bm^2-4\,(A-3\,B)m-6\,A+9\,B)\cos{(fx+e)}\sin{(fx+e)}+(}{8\,fm^3+36\,fm^2+}$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-m),x, a
lgorithm="fricas")
```

output
```
((2*B*m - 2*A + 3*B)*cos(f*x + e)^3 + (4*B*m^2 - 4*(A - 3*B)*m - 6*A + 9*B
)*cos(f*x + e)*sin(f*x + e) + (4*A*m^2 + 4*(3*A - B)*m + 9*A - 6*B)*cos(f*
x + e))*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 3)/(8*f*m^3 + 3
6*f*m^2 + 46*f*m + 15*f)
```

Sympy [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-3-m}\,dx$$
$$= \int (a(\sin{(e+fx)}+1))^m\,(-c(\sin{(e+fx)}-1))^{-m-3}\,(A+B\sin{(e+fx)})\,dx$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(-3-m),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*(-c*(sin(e + f*x) - 1))**(-m - 3)*(A +
B*sin(e + f*x)), x)
```

Maxima [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-3-m}\,dx$$
$$=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m(-c\sin(fx+e)+c)^{-m-3}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-m),x, a
lgorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c
)^(-m - 3), x)
```

Giac [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-3-m}\,dx$$
$$=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m(-c\sin(fx+e)+c)^{-m-3}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-m),x, a
lgorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c
)^(-m - 3), x)
```

Mupad [B] (verification not implemented)

Time = 39.01 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.25

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-3-m}\,dx = -\frac{(a\,(\sin(e+f\,x)+1))^m\,(30\,A\,\cos(e+f\,x)-15\,B\,\cos(e+f\,x)-2\,A\,\cos(3\,e+3\,f\,x)+3\,B\,\cos(3\,e$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m
+ 3),x)
```

output
```
-((a*(sin(e + f*x) + 1))^m*(30*A*cos(e + f*x) - 15*B*cos(e + f*x) - 2*A*co
s(3*e + 3*f*x) + 3*B*cos(3*e + 3*f*x) - 12*A*sin(2*e + 2*f*x) + 18*B*sin(2
*e + 2*f*x) + 8*B*m^2*sin(2*e + 2*f*x) + 48*A*m*cos(e + f*x) - 10*B*m*cos(
e + f*x) + 16*A*m^2*cos(e + f*x) + 2*B*m*cos(3*e + 3*f*x) - 8*A*m*sin(2*e
+ 2*f*x) + 24*B*m*sin(2*e + 2*f*x)))/(c^3*f*(-c*(sin(e + f*x) - 1))^m*(46*
m + 36*m^2 + 8*m^3 + 15)*(15*sin(e + f*x) + 6*cos(2*e + 2*f*x) - sin(3*e +
 3*f*x) - 10))
```

Reduce [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-3-m}\,dx = \frac{-\left(\int \frac{(a+a\sin(fx+e))^m}{(-\sin(fx+e)c+c)^m\sin(fx+e)^3-3(-\sin(fx+e)c+c)^m\sin(fx+e)^2+3(-\sin(fx+e)c+c)^m\sin(fx+e)-(-\sin(fx+e)c+c)^m}dx\right)a-}{c^3}$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-m),x)
```

output
```
( - (int((sin(e + f*x)*a + a)**m/(( - sin(e + f*x)*c + c)**m*sin(e + f*x)*
*3 - 3*( - sin(e + f*x)*c + c)**m*sin(e + f*x)**2 + 3*( - sin(e + f*x)*c +
 c)**m*sin(e + f*x) - ( - sin(e + f*x)*c + c)**m),x)*a + int(((sin(e + f*x
)*a + a)**m*sin(e + f*x))/(( - sin(e + f*x)*c + c)**m*sin(e + f*x)**3 - 3*
( - sin(e + f*x)*c + c)**m*sin(e + f*x)**2 + 3*( - sin(e + f*x)*c + c)**m*
sin(e + f*x) - ( - sin(e + f*x)*c + c)**m),x)*b))/c**3
```

3.213 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-2-m}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 114

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-2-m}\,dx = \frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2-m}}{f(3+2m)} + \frac{(A-2B(1+m))\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}}{cf(1+2m)(3+2m)}$$

output
```
(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m)/f/(3+2*m)+(A-2
*B*(1+m))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m)/c/f/(1+2*m
)/(3+2*m)
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.81

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-2-m}\,dx$$
$$=\frac{\sec(e+fx)(a(1+\sin(e+fx)))^{1+m}(c-c\sin(e+fx))^{-m}(B-2A(1+m)+(A-2B(1+m))\sin(e+ \ldots}{ac^2f(1+2m)(3+2m)(-1+\sin(e+fx))}$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^(-2 - m),x]
```

output
```
(Sec[e + f*x]*(a*(1 + Sin[e + f*x]))^(1 + m)*(B - 2*A*(1 + m) + (A - 2*B*(
1 + m))*Sin[e + f*x]))/(a*c^2*f*(1 + 2*m)*(3 + 2*m)*(-1 + Sin[e + f*x])*(c
 - c*Sin[e + f*x])^m)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3042, 3451, 3042, 3221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m-2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m-2}dx$$
$$\Big\downarrow \text{3451}$$
$$\frac{(A-2B(m+1))\int (\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{-m-1}dx}{c(2m+3)}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}}{f(2m+3)}$$
$$\Big\downarrow \text{3042}$$

$$\frac{(A-2B(m+1))\int(\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{-m-1}dx}{c(2m+3)}+\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}}{f(2m+3)}$$

$$\Big\downarrow \text{3221}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}}{f(2m+3)}+\frac{(A-2B(m+1))\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{cf(2m+1)(2m+3)}$$

input
```
Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-2 -
 m),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m)
)/(f*(3 + 2*m)) + ((A - 2*B*(1 + m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(
c - c*Sin[e + f*x])^(-1 - m))/(c*f*(1 + 2*m)*(3 + 2*m))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3221
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

Maple [F]

$$\int (a + a\sin(fx + e))^m (A + B\sin(fx + e)) (c - c\sin(fx + e))^{-2-m} dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-m),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-m),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.78

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{-2-m} dx = \frac{((2Bm - A + 2B)\cos(fx + e)\sin(fx + e) + (2Am + 2A - B)\cos(fx + e))(a\sin(fx + e) + a)^m(-c}{4fm^2 + 8fm + 3f}$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-m),x, a
lgorithm="fricas")
```

output
```
((2*B*m - A + 2*B)*cos(f*x + e)*sin(f*x + e) + (2*A*m + 2*A - B)*cos(f*x +
 e))*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 2)/(4*f*m^2 + 8*f*
m + 3*f)
```

Sympy [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-2-m}\,dx = \int (a(\sin(e+fx)+1))^m\,(-c(\sin(e+fx)-1))^{-m-2}\,(A+B\sin(e+fx))\,dx$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(-2-m),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*(-c*(sin(e + f*x) - 1))**(-m - 2)*(A +
B*sin(e + f*x)), x)
```

Maxima [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-2-m}\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m(-c\sin(fx+e)+c)^{-m-2}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-m),x, a
lgorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c
)^(-m - 2), x)
```

Giac [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-2-m}\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m(-c\sin(fx+e)+c)^{-m-2}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-m),x, a
lgorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c
)^(-m - 2), x)
```

Mupad [B] (verification not implemented)

Time = 37.59 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.18

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-2-m}\,dx = -\frac{(a\,(\sin(e+f\,x)+1))^m\,(4\,A\,\cos(e+f\,x)-2\,B\,\cos(e+f\,x)-A\,\sin(2\,e+2\,f\,x)+2\,B\,\sin(2\,e+2\,}{c^2\,f\,(-c\,(\sin(e+f\,x)-1))^m\,(4\,m^2+8\,m+3)\,(4\,\sin(e+f\,x)+\mathrm{co}}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m
+ 2),x)
```

output
```
-((a*(sin(e + f*x) + 1))^m*(4*A*cos(e + f*x) - 2*B*cos(e + f*x) - A*sin(2*
e + 2*f*x) + 2*B*sin(2*e + 2*f*x) + 4*A*m*cos(e + f*x) + 2*B*m*sin(2*e + 2
*f*x)))/(c^2*f*(-c*(sin(e + f*x) - 1))^m*(8*m + 4*m^2 + 3)*(4*sin(e + f*x)
 + cos(2*e + 2*f*x) - 3))
```

Reduce [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-2-m}\,dx = \frac{\left(\int \frac{(a+a\sin(fx+e))^m}{(-\sin(fx+e)c+c)^m\sin(fx+e)^2-2(-\sin(fx+e)c+c)^m\sin(fx+e)+(-\sin(fx+e)c+c)^m}dx\right)a+\left(\int \frac{(a\cdots}{(-\sin(fx+e)c+c)^m\sin(fx+e)^2\cdots}\right.}{c^2}$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-m),x)
```

output
```
(int((sin(e + f*x)*a + a)**m/(( - sin(e + f*x)*c + c)**m*sin(e + f*x)**2 -
 2*( - sin(e + f*x)*c + c)**m*sin(e + f*x) + ( - sin(e + f*x)*c + c)**m),x
)*a + int(((sin(e + f*x)*a + a)**m*sin(e + f*x))/(( - sin(e + f*x)*c + c)*
*m*sin(e + f*x)**2 - 2*( - sin(e + f*x)*c + c)**m*sin(e + f*x) + ( - sin(e
 + f*x)*c + c)**m),x)*b)/c**2
```

3.214 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-1-m}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 168

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-1-m}\,dx$$
$$=\frac{(A+B)\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}}{f(1+2m)}$$
$$-\frac{2^{\frac{1}{2}-m}B\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2}(1+2m),\frac{1}{2}(1+2m),\frac{1}{2}(3+2m),\frac{1}{2}(1+\sin(e+fx))\right)(1-}{cf(1+2m)}$$

output
```
(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m)/f/(1+2*m)-2^(1
/2-m)*B*cos(f*x+e)*hypergeom([1/2+m, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+e))*(1
-sin(f*x+e))^(-1/2+m)*(a+a*sin(f*x+e))^m/c/f/(1+2*m)/((c-c*sin(f*x+e))^m)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 12.30 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.85

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-1-m}\,dx$$

$$= \frac{2^{-1-m}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{-2(-1-m)-2m}(a(1+\sin(e+fx)))^m(c-c\sin(e+fx))^{-1-m}\Big(}{}$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^(-1 - m),x]
```

output
```
(2^(-1 - m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(-2*(-1 - m) - 2*m)*(a*(
1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^(-1 - m)*((1 - Tan[(e + f*x)/2])
/Sqrt[(1 + Cos[e + f*x])^(-1)])^(2*m)*(I*B*(1 + 2*m)*Hypergeometric2F1[1,
-2*m, 1 - 2*m, ((-I)*(-1 + Tan[(e + f*x)/2]))/(1 + Tan[(e + f*x)/2])]*(-1
+ Tan[(e + f*x)/2]) - I*B*(1 + 2*m)*Hypergeometric2F1[1, -2*m, 1 - 2*m, (I
*(-1 + Tan[(e + f*x)/2]))/(1 + Tan[(e + f*x)/2])]*(-1 + Tan[(e + f*x)/2])
- 2*(A + B)*m*(1 + Tan[(e + f*x)/2])))/(f*m*(1 + 2*m)*((1 - Tan[(e + f*x)/
2])/Sqrt[Sec[(e + f*x)/2]^2])^(2*m)*(-1 + Tan[(e + f*x)/2]))
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.20, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3451, 3042, 3224, 3042, 3168, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m-1}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m-1}dx$$

$$\Big\downarrow \text{3451}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{f(2m+1)}-\frac{B\int(\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{-m}dx}{c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{f(2m+1)}-\frac{B\int(\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{-m}dx}{c}$$

$$\Big\downarrow \text{3224}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{f(2m+1)}-\frac{B\cos^{-2m}(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^m\int\cos^{2m}(e+fx)(c-c\sin(e+fx))^{-2m}dx}{c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{f(2m+1)}-\frac{B\cos^{-2m}(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^m\int\cos(e+fx)^{2m}(c-c\sin(e+fx))^{-2m}dx}{c}$$

$$\Big\downarrow \text{3168}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{f(2m+1)}-\frac{Bc\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{\frac{1}{2}(-2m-1)+m}(c\sin(e+fx)+c)^{\frac{1}{2}(-2m-1)}\int(c-c\sin(e+f}{f}$$

$$\Big\downarrow \text{80}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{f(2m+1)}-\frac{Bc2^{-m-\frac{1}{2}}\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{\frac{1}{2}(-2m-1)-\frac{1}{2}}(c\sin(e+fx)+}{f}$$

$$\Big\downarrow \text{79}$$

$$\frac{(A+B)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}}{f(2m+1)} - \frac{B2^{\frac{1}{2}-m}\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{\frac{1}{2}(-2m-1)-\frac{1}{2}}(c\sin(e+fx)+c)^{\frac{1}{2}}}{f(2m+1)}$$

input
```
Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-1 -
 m),x]
```

output
```
((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m)
)/(f*(1 + 2*m)) - (2^(1/2 - m)*B*Cos[e + f*x]*Hypergeometric2F1[(1 + 2*m)/
2, (1 + 2*m)/2, (3 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2
 + m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1/2 + (-1 - 2*m)/2)*(c
 + c*Sin[e + f*x])^((-1 - 2*m)/2 + (1 + 2*m)/2))/(f*(1 + 2*m))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3168
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Simp[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)))   Subst[Int[(a +
b*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Fre
eQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

rule 3224
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m]))   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (FractionQ[m] ||  !FractionQ[n])
```

rule 3451
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] + Simp[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0
] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] &&  !SumSimplerQ[n, 1])) && NeQ[2*
m + 1, 0]
```

Maple [F]

$$\int (a+a\sin(fx+e))^m (A+B\sin(fx+e)) (c-c\sin(fx+e))^{-1-m}\,dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-m),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-m),x)
```

Fricas [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-1-m}\,dx$$
$$=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m(-c\sin(fx+e)+c)^{-m-1}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-m),x, a
lgorithm="fricas")
```

output
```
integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m - 1), x)
```

Sympy [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-1-m}\,dx$$
$$=\int (a(\sin(e+fx)+1))^m\,(-c(\sin(e+fx)-1))^{-m-1}\,(A+B\sin(e+fx))\,dx$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(-1-m),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*(-c*(sin(e + f*x) - 1))**(-m - 1)*(A +
B*sin(e + f*x)), x)
```

Maxima [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-1-m}\,dx$$
$$=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m(-c\sin(fx+e)+c)^{-m-1}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-m),x, a
lgorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c
)^(-m - 1), x)
```

Giac [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-1-m}\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m(-c\sin(fx+e)+c)^{-m-1}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-m),x, a
lgorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c
)^(-m - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-1-m}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m}{(c-c\,\sin(e+f\,x))^{m+1}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m
+ 1),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m
+ 1), x)
```

Reduce [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-1-m}\,dx$$
$$=\frac{-\left(\int \frac{(a+a\sin(fx+e))^m}{(-\sin(fx+e)c+c)^m\sin(fx+e)-(-\sin(fx+e)c+c)^m}dx\right)a-\left(\int \frac{(a+a\sin(fx+e))^m\sin(fx+e)}{(-\sin(fx+e)c+c)^m\sin(fx+e)-(-\sin(fx+e)c+c)^m}dx\right)b}{c}$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-m),x)
```

output
```
( - (int((sin(e + f*x)*a + a)**m/(( - sin(e + f*x)*c + c)**m*sin(e + f*x)
- ( - sin(e + f*x)*c + c)**m),x)*a + int(((sin(e + f*x)*a + a)**m*sin(e +
f*x))/(( - sin(e + f*x)*c + c)**m*sin(e + f*x) - ( - sin(e + f*x)*c + c)**
m),x)*b))/c
```

3.215 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 159

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m}\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-m}}{f} + \frac{2^{\frac{1}{2}-m}(A+2Bm)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2}(1+2m),\frac{1}{2}(1+2m),\frac{1}{2}(3+2m),\frac{1}{2}(1+\sin(e+f\ldots}{f(1+2m)}$$

output
```
-B*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/((c-c*sin(f*x+e))^m)+2^(1/2-m)*(2*B*m+A
)*cos(f*x+e)*hypergeom([1/2+m, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+e))*(1-sin(f
*x+e))^(-1/2+m)*(a+a*sin(f*x+e))^m/f/(1+2*m)/((c-c*sin(f*x+e))^m)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.30 (sec) , antiderivative size = 402, normalized size of antiderivative = 2.53

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m}\,dx =$$

$$-\frac{i2^{-1-m}\left(a\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\right)^m(c-c\sin(e+fx))^{-m}(\cosh(m\log(2))+\sinh(m\log}{}$$

input
```
Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x
])^m,x]
```

output
```
((-I)*2^(-1 - m)*(a*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)^m*(Cosh[m*Log
[2]] + Sinh[m*Log[2]])*((A*Hypergeometric2F1[1, -2*m, 1 - 2*m, -((Cos[e +
f*x] + I*(-1 + Sin[e + f*x]))/(Cos[e + f*x] + I*(1 + Sin[e + f*x])))])/m -
 (2*B*Hypergeometric2F1[2, 1 - 2*m, 2 - 2*m, -((Cos[e + f*x] + I*(-1 + Sin
[e + f*x]))/(Cos[e + f*x] + I*(1 + Sin[e + f*x])))]*(Cos[e + f*x] + I*(-1
+ Sin[e + f*x])))/((-1 + 2*m)*(Cos[e + f*x] + I*(1 + Sin[e + f*x]))) + ((B
*Hypergeometric2F1[2*m, 1 + 2*m, 2 + 2*m, (1 - I*Cos[e + f*x] + Sin[e + f*
x])/2]*(Cos[e + f*x]*(Cos[e + f*x] + I*Sin[e + f*x]))^(2*m)*(1 - I*Cos[e +
 f*x] + Sin[e + f*x]))/(1 + 2*m) - (A*Hypergeometric2F1[-2*m, -2*m, 1 - 2*
m, (1 + I*Cos[e + f*x] - Sin[e + f*x])/2]*(Cosh[m*Log[16]] + Sinh[m*Log[16
]]))/(4^m*m))/(1 - I*Cos[e + f*x] + Sin[e + f*x])^(2*m)))/(f*(c - c*Sin[e
+ f*x])^m)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.23, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3452, 3042, 3224, 3042, 3168, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m}dx$$

$$\Big\downarrow \text{3452}$$

$$(A+2Bm)\int (\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{-m}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m}}{f}$$

$$\Big\downarrow \text{3042}$$

$$(A+2Bm)\int (\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{-m}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m}}{f}$$

$$\Big\downarrow \text{3224}$$

$$(A+2Bm)\cos^{-2m}(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^m\int \cos^{2m}(e+fx)(c-c\sin(e+fx))^{-2m}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m}}{f}$$

$$\Big\downarrow \text{3042}$$

$$(A+2Bm)\cos^{-2m}(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^m\int \cos(e+fx)^{2m}(c-c\sin(e+fx))^{-2m}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m}}{f}$$

$$\Big\downarrow \text{3168}$$

$$\frac{c^2(A+2Bm)\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{\frac{1}{2}(-2m-1)+m}(c\sin(e+fx)+c)^{\frac{1}{2}(-2m-1)}\int(c-}{f}$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m}}{f}$$

$$\Big\downarrow \text{80}$$

$$\frac{c^2 2^{-m-\frac{1}{2}}(A+2Bm)\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{\frac{1}{2}(-2m-1)-\frac{1}{2}}(c\sin(}{f}$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m}}{f}$$

$$\Big\downarrow \text{ 79}$$

$$\frac{c2^{\frac{1}{2}-m}(A+2Bm)\cos(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{\frac{1}{2}(-2m-1)-\frac{1}{2}}(c\sin(e\;\ldots}{f(2m+\;\ldots}$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m}}{f}$$

input
```
Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^m,x
]
```

output
```
-((B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(c - c*Sin[e + f*x])^m)) + (2
^(1/2 - m)*c*(A + 2*B*m)*Cos[e + f*x]*Hypergeometric2F1[(1 + 2*m)/2, (1 +
2*m)/2, (3 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 + m)*(a
 + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1/2 + (-1 - 2*m)/2)*(c + c*Sin
[e + f*x])^((-1 - 2*m)/2 + (1 + 2*m)/2))/(f*(1 + 2*m))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3168
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Simp[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)))   Subst[Int[(a +
b*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Fre
eQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

rule 3224
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m]))   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (FractionQ[m] ||  !FractionQ[n])
```

rule 3452
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

Maple [F]

$$\int (a+a\sin(fx+e))^m (A+B\sin(fx+e)) (-c(\sin(fx+e)-1))^{-m} dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^m),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^m),x)
```

Fricas [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(-c\sin(fx+e)+c)^m}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^m),x, algo
rithm="fricas")
```

output
```
integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)
^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))**m),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(-c\sin(fx+e)+c)^m}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^m),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c
)^m, x)
```

Giac [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(-c\sin(fx+e)+c)^m}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^m),x, algo
rithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c
)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m}{(c-c\,\sin(e+f\,x))^m}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^m,x
)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^m,
x)
```

Reduce [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{-m}\,dx$$
$$=\left(\int \frac{(a+a\sin(fx+e))^m}{(-\sin(fx+e)\,c+c)^m}dx\right)a+\left(\int \frac{(a+a\sin(fx+e))^m\sin(fx+e)}{(-\sin(fx+e)\,c+c)^m}dx\right)b$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^m),x)
```

output
```
int((sin(e + f*x)*a + a)**m/( - sin(e + f*x)*c + c)**m,x)*a + int(((sin(e
+ f*x)*a + a)**m*sin(e + f*x))/( - sin(e + f*x)*c + c)**m,x)*b
```

3.216 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{1-m}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 171

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{1-m}\,dx$$
$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{1-m}}{2f}$$
$$+\frac{2^{\frac{1}{2}-m}(2A-B(1-2m))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-1+2m),\frac{1}{2}(1+2m),\frac{1}{2}(3+2m),\frac{1}{2}(1+}{f(1+2m)}$$

output

```
-1/2*B*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m)/f+2^(1/2-m)*(2
*A-B*(1-2*m))*cos(f*x+e)*hypergeom([-1/2+m, 1/2+m],[3/2+m],1/2+1/2*sin(f*x
+e))*(1-sin(f*x+e))^(-3/2+m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m)/f/(
1+2*m)
```

Mathematica [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{1-m}\,dx$$
$$=\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{1-m}\,dx$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^(1 - m),x]
```

output
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^(1 - m), x]
```

Rubi [A] (warning: unable to verify)

Time = 0.72 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.22, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3452, 3042, 3224, 3042, 3168, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{1-m}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{1-m}dx$$

$$\Big\downarrow \text{3452}$$

$$\frac{1}{2}(2A-B(1-2m))\int (\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{1-m}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{1-m}}{2f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}(2A-B(1-2m))\int(\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{1-m}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{1-m}}{2f}$$

$$\Big\downarrow \; 3224$$

$$\frac{1}{2}(2A-B(1-2m))\cos^{-2m}(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^m\int\cos^{2m}(e+fx)(c-c\sin(e+fx))^{1-2m}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{1-m}}{2f}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{2}(2A-B(1-2m))\cos^{-2m}(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^m\int\cos(e+fx)^{2m}(c-c\sin(e+fx))^{1-2m}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{1-m}}{2f}$$

$$\Big\downarrow \; 3168$$

$$\frac{c^2(2A-B(1-2m))\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{\frac{1}{2}(-2m-1)+m}(c\sin(e+fx)+c)^{\frac{1}{2}(-2m-1)}}{2f} \frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{1-m}}{2f}$$

$$\Big\downarrow \; 80$$

$$\frac{c^2 2^{m-\frac{3}{2}}(2A-B(1-2m))\cos(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{\frac{1}{2}(-2m-1)+m}}{f} \frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{1-m}}{2f}$$

$$\Big\downarrow \; 79$$

$$-\frac{c2^{m-\frac{1}{2}}(2A-B(1-2m))\cos(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{\frac{1}{2}(-2m-1)+\frac{1}{2}}}{} \frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{1-m}}{2f}$$

input
```
Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(1 -
m),x]
```

output
```
-1/2*(B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(1 - m))/
f - (2^(-1/2 + m)*c*(2*A - B*(1 - 2*m))*Cos[e + f*x]*Hypergeometric2F1[(1
- 2*m)/2, (3 - 2*m)/2, (5 - 2*m)/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x
])^(1/2 - m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^((-1 - 2*m)/2 + (
3 - 2*m)/2 + m)*(c + c*Sin[e + f*x])^(-1/2 + (-1 - 2*m)/2 + m))/(f*(3 - 2*
m))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3168
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Simp[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)))   Subst[Int[(a +
b*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Fre
eQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

rule 3224
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m]))   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (FractionQ[m] ||  !FractionQ[n])
```

rule 3452
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

Maple [F]

$$\int (a+a\sin(fx+e))^m (A+B\sin(fx+e))(c-c\sin(fx+e))^{1-m}\,dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-m),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-m),x)
```

Fricas [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{1-m}\,dx$$
$$=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m(-c\sin(fx+e)+c)^{-m+1}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-m),x, al
gorithm="fricas")
```

output
```
integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m + 1), x)
```

Sympy [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{1-m}\,dx$$
$$= \int (a(\sin(e + fx) + 1))^m (-c(\sin(e + fx) - 1))^{1-m} (A + B\sin(e + fx))\,dx$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1-m),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*(-c*(sin(e + f*x) - 1))**(1 - m)*(A + B
*sin(e + f*x)), x)
```

Maxima [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c - c\sin(e + fx))^{1-m}\,dx$$
$$= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^m (-c\sin(fx + e) + c)^{-m+1}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-m),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c
)^(-m + 1), x)
```

Giac [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{1-m}\,dx$$
$$=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m(-c\sin(fx+e)+c)^{-m+1}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-m),x, al
gorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c
)^(-m + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{1-m}\,dx$$
$$=\int (A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m\,(c-c\,\sin(e+f\,x))^{1-m}\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(1 -
m),x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(1 -
m), x)
```

Reduce [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{1-m}\,dx$$
$$= c\left(\left(\int \frac{(a+a\sin(fx+e))^m}{(-\sin(fx+e)\,c+c)^m}dx\right)a - \left(\int \frac{(a+a\sin(fx+e))^m\sin(fx+e)^2}{(-\sin(fx+e)\,c+c)^m}dx\right)b - \left(\int \frac{(a+a\sin(fx+e))^m\sin(fx+e)}{(-\sin(fx+e)\,c+c)^m}dx\right)a + \left(\int \frac{(a+a\sin(fx+e))^m\sin(fx+e)}{(-\sin(fx+e)\,c+c)^m}dx\right)b\right)$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-m),x)
```

output
```
c*(int((sin(e + f*x)*a + a)**m/( - sin(e + f*x)*c + c)**m,x)*a - int(((sin
(e + f*x)*a + a)**m*sin(e + f*x)**2)/( - sin(e + f*x)*c + c)**m,x)*b - int
(((sin(e + f*x)*a + a)**m*sin(e + f*x))/( - sin(e + f*x)*c + c)**m,x)*a +
int(((sin(e + f*x)*a + a)**m*sin(e + f*x))/( - sin(e + f*x)*c + c)**m,x)*b
)
```

3.217 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{2-m}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 174

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{2-m}\,dx$$
$$= -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{2-m}}{3f}$$
$$+ \frac{2^{\frac{5}{2}-m}(3A-2B(1-m))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-3+2m),\frac{1}{2}(1+2m),\frac{1}{2}(3+2m),\frac{1}{2}(1+\right.}{3f(1+2m)}$$

output
```
-1/3*B*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2-m)/f+1/3*2^(5/2-m
)*(3*A-2*B*(1-m))*cos(f*x+e)*hypergeom([-3/2+m, 1/2+m],[3/2+m],1/2+1/2*sin
(f*x+e))*(1-sin(f*x+e))^(-5/2+m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2-m)
/f/(1+2*m)
```

Mathematica [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{2-m}\,dx$$
$$=\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{2-m}\,dx$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^(2 - m),x]
```

output
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])
^(2 - m), x]
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.21, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3452, 3042, 3224, 3042, 3168, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{2-m}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{2-m}dx$$

$$\Big\downarrow \text{3452}$$

$$\frac{1}{3}(3A-2B(1-m))\int (\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{2-m}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{2-m}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}(3A-2B(1-m))\int(\sin(e+fx)a+a)^m(c-c\sin(e+fx))^{2-m}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{2-m}}{3f}$$

$\downarrow$ 3224

$$\frac{1}{3}(3A-2B(1-m))\cos^{-2m}(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^m\int\cos^{2m}(e+fx)(c-c\sin(e+fx))^{2-2m}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{2-m}}{3f}$$

$\downarrow$ 3042

$$\frac{1}{3}(3A-2B(1-m))\cos^{-2m}(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^m\int\cos(e+fx)^{2m}(c-c\sin(e+fx))^{2-2m}dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{2-m}}{3f}$$

$\downarrow$ 3168

$$\frac{c^2(3A-2B(1-m))\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{\frac{1}{2}(-2m-1)+m}(c\sin(e+fx)+c)^{\frac{1}{2}(-2m-1)}}{3f}$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{2-m}}{3f}$$

$\downarrow$ 80

$$\frac{c^3 2^{\frac{3}{2}-m}(3A-2B(1-m))\cos(e+fx)(1-\sin(e+fx))^{m-\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{\frac{1}{2}(-2m-1)+\frac{1}{2}}}{3f}$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{2-m}}{3f}$$

$\downarrow$ 79

$$\frac{c^2 2^{\frac{5}{2}-m}(3A-2B(1-m))\cos(e+fx)(1-\sin(e+fx))^{m-\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{\frac{1}{2}(-2m-1)+\frac{1}{2}}}{3f(2}$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{2-m}}{3f}$$

input
```
Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(2 -
m),x]
```

output
```
-1/3*(B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(2 - m))/
f + (2^(5/2 - m)*c^2*(3*A - 2*B*(1 - m))*Cos[e + f*x]*Hypergeometric2F1[(-
3 + 2*m)/2, (1 + 2*m)/2, (3 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f
*x])^(-1/2 + m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(1/2 + (-1 - 2
*m)/2)*(c + c*Sin[e + f*x])^((-1 - 2*m)/2 + (1 + 2*m)/2))/(3*f*(1 + 2*m))
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3168
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Simp[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)))   Subst[Int[(a +
b*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Fre
eQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

rule 3224
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m]))   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && (FractionQ[m] ||  !FractionQ[n])
```

rule 3452
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] - Simp[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1))   Int[(
a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&  !LtQ[m,
 -2^(-1)] && NeQ[m + n + 1, 0]
```

Maple [F]

$$\int (a+a\sin(fx+e))^m (A+B\sin(fx+e)) (c-c\sin(fx+e))^{2-m}\, dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-m),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-m),x)
```

Fricas [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{2-m}\, dx$$
$$= \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m(-c\sin(fx+e)+c)^{-m+2}\, dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-m),x, al
gorithm="fricas")
```

output
```
integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)
^(-m + 2), x)
```

Sympy [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{2-m}\,dx$$
$$=\int (a(\sin{(e+fx)}+1))^m\,(-c(\sin{(e+fx)}-1))^{2-m}\,(A+B\sin{(e+fx)})\,\,dx$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(2-m),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*(-c*(sin(e + f*x) - 1))**(2 - m)*(A + B
*sin(e + f*x)), x)
```

Maxima [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{2-m}\,dx$$
$$=\int (B\sin{(fx+e)}+A)(a\sin{(fx+e)}+a)^m(-c\sin{(fx+e)}+c)^{-m+2}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-m),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c
)^(-m + 2), x)
```

Giac [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{2-m}\,dx$$
$$=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m(-c\sin(fx+e)+c)^{-m+2}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-m),x, al
gorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c
)^(-m + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{2-m}\,dx$$
$$=\int (A+B\sin(e+fx))\,(a+a\sin(e+fx))^m\,(c-c\sin(e+fx))^{2-m}\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(2 -
m),x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(2 -
m), x)
```

Reduce [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c-c\sin(e+fx))^{2-m}\,dx$$
$$= c^2\left(\left(\int\frac{(a+a\sin(fx+e))^m}{(-\sin(fx+e)\,c+c)^m}dx\right)a + \left(\int\frac{(a+a\sin(fx+e))^m\sin(fx+e)^3}{(-\sin(fx+e)\,c+c)^m}dx\right)b + \left(\int\frac{(a+a\sin(fx+e))^m\sin(fx+e)^2}{(-\sin(fx+e)\,c+c)^m}dx\right)a - 2\left(\int\frac{(a+a\sin(fx+e))^m\sin(fx+e)^2}{(-\sin(fx+e)\,c+c)^m}dx\right)b - 2\left(\int\frac{(a+a\sin(fx+e))^m\sin(fx+e)}{(-\sin(fx+e)\,c+c)^m}dx\right)a + \left(\int\frac{(a+a\sin(fx+e))^m\sin(fx+e)}{(-\sin(fx+e)\,c+c)^m}dx\right)b\right)$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-m),x)
```

output
```
c**2*(int((sin(e + f*x)*a + a)**m/( - sin(e + f*x)*c + c)**m,x)*a + int(((
sin(e + f*x)*a + a)**m*sin(e + f*x)**3)/( - sin(e + f*x)*c + c)**m,x)*b +
int(((sin(e + f*x)*a + a)**m*sin(e + f*x)**2)/( - sin(e + f*x)*c + c)**m,x
)*a - 2*int(((sin(e + f*x)*a + a)**m*sin(e + f*x)**2)/( - sin(e + f*x)*c +
 c)**m,x)*b - 2*int(((sin(e + f*x)*a + a)**m*sin(e + f*x))/( - sin(e + f*x
)*c + c)**m,x)*a + int(((sin(e + f*x)*a + a)**m*sin(e + f*x))/( - sin(e +
f*x)*c + c)**m,x)*b)
```

3.218 $\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^n(B(3-n)-B(4+n)\sin(e+fx))\,dx$

Optimal result

Integrand size = 46, antiderivative size = 34

$$\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^n(B(3-n)-B(4+n)\sin(e+fx))\,dx = \frac{a^3Bc^3\cos^7(e+fx)(c-c\sin(e+fx))^{-3+n}}{f}$$

output
```
a^3*B*c^3*cos(f*x+e)^7*(c-c*sin(f*x+e))^(-3+n)/f
```

Mathematica [A] (verified)

Time = 2.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.85

$$\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^n(B(3-n)-B(4+n)\sin(e+fx))\,dx = \frac{a^3B(c-c\sin(e+fx))^n(14\cos(e+fx)-6\cos(3(e+fx))+14\sin(2(e+fx))-\sin(4(e+fx)))}{8f}$$

input
```
Integrate[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^n*(B*(3 - n) - B*(4
+ n)*Sin[e + f*x]),x]
```

output
```
(a^3*B*(c - c*Sin[e + f*x])^n*(14*Cos[e + f*x] - 6*Cos[3*(e + f*x)] + 14*S
in[2*(e + f*x)] - Sin[4*(e + f*x)]))/(8*f)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3042, 3446, 3042, 3333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^3(B(3-n)-B(n+4)\sin(e+fx))(c-c\sin(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^3(B(3-n)-B(n+4)\sin(e+fx))(c-c\sin(e+fx))^ndx$$

$$\Big\downarrow \text{3446}$$

$$a^3c^3\int \cos^6(e+fx)(c-c\sin(e+fx))^{n-3}(B(3-n)-B(n+4)\sin(e+fx))dx$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\int \cos(e+fx)^6(c-c\sin(e+fx))^{n-3}(B(3-n)-B(n+4)\sin(e+fx))dx$$

$$\Big\downarrow \text{3333}$$

$$\frac{a^3Bc^3\cos^7(e+fx)(c-c\sin(e+fx))^{n-3}}{f}$$

input
```
Int[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^n*(B*(3 - n) - B*(4 + n)*S
in[e + f*x]),x]
```

output
```
(a^3*B*c^3*Cos[e + f*x]^7*(c - c*Sin[e + f*x])^(-3 + n))/f
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3333
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] /;
FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[a*d*m + b
*c*(m + p + 1), 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 8.72 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.82

method	result	size
parallelrisch	$-\frac{a^{3} B \left(-c \left(\sin \left(f x +e \right)-1\right)\right)^{n} \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)-1\right) \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{7}}{f \left(1+\tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}\right)^{4}}$	62

input
```
int((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^n*(B*(3-n)-B*(4+n)*sin(f*x+e)),x,m
ethod=_RETURNVERBOSE)
```

output
```
-a^3*B*(-c*(sin(f*x+e)-1))^n*(tan(1/2*f*x+1/2*e)-1)*(tan(1/2*f*x+1/2*e)+1)
^7/f/(1+tan(1/2*f*x+1/2*e)^2)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(34) = 68$.

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.29

$$\int (a + a\sin(e + fx))^3(c - c\sin(e + fx))^n(B(3 - n) - B(4 + n)\sin(e + fx))\,dx = -\frac{\left(3\,Ba^3\cos\left(fx + e\right)^3 - 4\,Ba^3\cos\left(fx + e\right) + \left(Ba^3\cos\left(fx + e\right)^3 - 4\,Ba^3\cos\left(fx + e\right)\right)\sin\left(fx + e\right)\right)(-}{f}$$

input
```
integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^n*(B*(3-n)-B*(4+n)*sin(f*x+e
)),x, algorithm="fricas")
```

output
```
-(3*B*a^3*cos(f*x + e)^3 - 4*B*a^3*cos(f*x + e) + (B*a^3*cos(f*x + e)^3 -
4*B*a^3*cos(f*x + e))*sin(f*x + e))*(-c*sin(f*x + e) + c)^n/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 898 vs. $2(31) = 62$.

Time = 73.19 (sec) , antiderivative size = 898, normalized size of antiderivative = 26.41

$$\int (a + a\sin(e + fx))^3(c - c\sin(e + fx))^n(B(3 - n) - B(4 + n)\sin(e + fx))\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**3*(c-c*sin(f*x+e))**n*(B*(3-n)-B*(4+n)*sin(f*x
+e)),x)
```

output
```
Piecewise((-B*a**3*(c - 2*c*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**n
*tan(e/2 + f*x/2)**8/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*
f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) - 6*B*a**3*(c - 2*c*t
an(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**n*tan(e/2 + f*x/2)**7/(f*tan(e
/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*t
an(e/2 + f*x/2)**2 + f) - 14*B*a**3*(c - 2*c*tan(e/2 + f*x/2)/(tan(e/2 + f
*x/2)**2 + 1))**n*tan(e/2 + f*x/2)**6/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2
 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) - 14
*B*a**3*(c - 2*c*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**n*tan(e/2 +
f*x/2)**5/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 +
 f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) + 14*B*a**3*(c - 2*c*tan(e/2 + f
*x/2)/(tan(e/2 + f*x/2)**2 + 1))**n*tan(e/2 + f*x/2)**3/(f*tan(e/2 + f*x/2
)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f
*x/2)**2 + f) + 14*B*a**3*(c - 2*c*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 +
 1))**n*tan(e/2 + f*x/2)**2/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)*
*6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) + 6*B*a**3*(c
- 2*c*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**n*tan(e/2 + f*x/2)/(f*t
an(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4
*f*tan(e/2 + f*x/2)**2 + f) + B*a**3*(c - 2*c*tan(e/2 + f*x/2)/(tan(e/2 +
f*x/2)**2 + 1))**n/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6...
```

Maxima [F]

$$\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^n(B(3-n)-B(4+n)\sin(e+fx))\,dx$$
$$=\int -(B(n+4)\sin(fx+e)+B(n-3))(a\sin(fx+e)+a)^3(-c\sin(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^n*(B*(3-n)-B*(4+n)*sin(f*x+e
)),x, algorithm="maxima")
```

output
```
-integrate((B*(n + 4)*sin(f*x + e) + B*(n - 3))*(a*sin(f*x + e) + a)^3*(-c
*sin(f*x + e) + c)^n, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6973 vs. $2(34) = 68$.

Time = 12.27 (sec) , antiderivative size = 6973, normalized size of antiderivative = 205.09

$$\int (a + a\sin(e + fx))^3(c - c\sin(e + fx))^n(B(3 - n) - B(4 + n)\sin(e + fx))\,dx$$
$$= \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^n*(B*(3-n)-B*(4+n)*sin(f*x+e
)),x, algorithm="giac")
```

output
```
(B*a^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*ta
n(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x
 + e)^2*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) - 8
*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)
^2*tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 - 8*tan(f*x + e)^2*tan(
1/2*f*x + 1/2*e) - 4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2
*f*x + 1/2*e)^2 - 4*tan(1/2*f*x + 1/2*e) + 1)*abs(c)/(tan(f*x + e)^2*tan(1
/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^n*tan(-1
/4*pi*n*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 - 4*c*tan(1/2*f*x + 1/2*e)^3 + 4*c*
tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f
*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/2*pi*n*floor(f*x/pi + e/pi +
 1/2) - 1/4*pi*n*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 - 8*c*tan(1/2*f*x + 1/2*e)
^2 + 4*c*tan(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x + 1/2*e)^8 + 6*B*a^3*(sqrt(2
*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/
2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/
2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) - 8*tan(f*x + e)^
2*tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x
 + 1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*
e) - 4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^
2 - 4*tan(1/2*f*x + 1/2*e) + 1)*abs(c)/(tan(f*x + e)^2*tan(1/2*f*x + 1/...
```

Mupad [B] (verification not implemented)

Time = 37.51 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.88

$$\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^n(B(3-n)-B(4+n)\sin(e+fx))\,dx$$
$$=\frac{B\,a^3\,(-c\,(\sin(e+f\,x)-1))^n\,(14\,\cos(e+f\,x)-6\,\cos(3\,e+3\,f\,x)+14\,\sin(2\,e+2\,f\,x)-\sin(4\,e+4\,f\,x))}{8\,f}$$

input
```
int(-(B*(n - 3) + B*sin(e + f*x)*(n + 4))*(a + a*sin(e + f*x))^3*(c - c*si
n(e + f*x))^n,x)
```

output
```
(B*a^3*(-c*(sin(e + f*x) - 1))^n*(14*cos(e + f*x) - 6*cos(3*e + 3*f*x) + 1
4*sin(2*e + 2*f*x) - sin(4*e + 4*f*x)))/(8*f)
```

Reduce [F]

$$\int (a+a\sin(e+fx))^3(c-c\sin(e+fx))^n(B(3-n)-B(4+n)\sin(e+fx))\,dx$$
$$\begin{aligned}
&= a^3b\bigg(-\left(\int(-\sin(fx+e)\,c+c)^n\,dx\right)n+3\left(\int(-\sin(fx+e)\,c+c)^n\,dx\right) \\
&\quad-\left(\int(-\sin(fx+e)\,c+c)^n\sin(fx+e)^4\,dx\right)n \\
&\quad-4\left(\int(-\sin(fx+e)\,c+c)^n\sin(fx+e)^4\,dx\right) \\
&\quad-4\left(\int(-\sin(fx+e)\,c+c)^n\sin(fx+e)^3\,dx\right)n \\
&\quad-9\left(\int(-\sin(fx+e)\,c+c)^n\sin(fx+e)^3\,dx\right) \\
&\quad-6\left(\int(-\sin(fx+e)\,c+c)^n\sin(fx+e)^2\,dx\right)n \\
&\quad-3\left(\int(-\sin(fx+e)\,c+c)^n\sin(fx+e)^2\,dx\right) \\
&\quad-4\left(\int(-\sin(fx+e)\,c+c)^n\sin(fx+e)\,dx\right)n \\
&\quad+5\left(\int(-\sin(fx+e)\,c+c)^n\sin(fx+e)\,dx\right)\bigg)
\end{aligned}$$

input
```
int((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^n*(B*(3-n)-B*(4+n)*sin(f*x+e)),x)
```

output
```
a**3*b*( - int(( - sin(e + f*x)*c + c)**n,x)*n + 3*int(( - sin(e + f*x)*c
+ c)**n,x) - int(( - sin(e + f*x)*c + c)**n*sin(e + f*x)**4,x)*n - 4*int((
 - sin(e + f*x)*c + c)**n*sin(e + f*x)**4,x) - 4*int(( - sin(e + f*x)*c +
c)**n*sin(e + f*x)**3,x)*n - 9*int(( - sin(e + f*x)*c + c)**n*sin(e + f*x)
**3,x) - 6*int(( - sin(e + f*x)*c + c)**n*sin(e + f*x)**2,x)*n - 3*int(( -
 sin(e + f*x)*c + c)**n*sin(e + f*x)**2,x) - 4*int(( - sin(e + f*x)*c + c)
**n*sin(e + f*x),x)*n + 5*int(( - sin(e + f*x)*c + c)**n*sin(e + f*x),x))
```

3.219 $\int (a-a\sin(e+fx))^3(c+c\sin(e+fx))^n(B(3-n)+B(4+n)\sin(e+fx))\,dx$

Optimal result

Integrand size = 45, antiderivative size = 34

$$\int (a-a\sin(e+fx))^3(c+c\sin(e+fx))^n(B(3-n)+B(4+n)\sin(e+fx))\,dx = -\frac{a^3Bc^3\cos^7(e+fx)(c+c\sin(e+fx))^{-3+n}}{f}$$

output
```
-a^3*B*c^3*cos(f*x+e)^7*(c+c*sin(f*x+e))^(-3+n)/f
```

Mathematica [A] (verified)

Time = 8.99 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.97

$$\int (a-a\sin(e+fx))^3(c+c\sin(e+fx))^n(B(3-n)+B(4+n)\sin(e+fx))\,dx = -\frac{a^3B\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c(1+\sin(e+fx)))^n}{f}$$

input
```
Integrate[(a - a*Sin[e + f*x])^3*(c + c*Sin[e + f*x])^n*(B*(3 - n) + B*(4
+ n)*Sin[e + f*x]),x]
```

output
```
-((a^3*B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])*(c*(1 + Sin[e + f*x]))^n)/f)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.089$, Rules used = {3042, 3446, 3042, 3333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-a\sin(e+fx))^3(B(n+4)\sin(e+fx)+B(3-n))(c\sin(e+fx)+c)^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a-a\sin(e+fx))^3(B(n+4)\sin(e+fx)+B(3-n))(c\sin(e+fx)+c)^ndx$$

$$\Big\downarrow \text{3446}$$

$$a^3c^3\int \cos^6(e+fx)(\sin(e+fx)c+c)^{n-3}(B(3-n)+B(n+4)\sin(e+fx))dx$$

$$\Big\downarrow \text{3042}$$

$$a^3c^3\int \cos(e+fx)^6(\sin(e+fx)c+c)^{n-3}(B(3-n)+B(n+4)\sin(e+fx))dx$$

$$\Big\downarrow \text{3333}$$

$$-\frac{a^3Bc^3\cos^7(e+fx)(c\sin(e+fx)+c)^{n-3}}{f}$$

input
```
Int[(a - a*Sin[e + f*x])^3*(c + c*Sin[e + f*x])^n*(B*(3 - n) + B*(4 + n)*S
in[e + f*x]),x]
```

output
```
-((a^3*B*c^3*Cos[e + f*x]^7*(c + c*Sin[e + f*x])^(-3 + n))/f)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3333
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] /;
FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[a*d*m + b
*c*(m + p + 1), 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 7.87 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.76

method	result	size
parallelrisch	$\frac{a^{3} B \left(c \left(1+\sin \left(f x +e \right)\right)\right)^{n} \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1\right) \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)^{7}}{f \left(1+\tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}\right)^{4}}$	60

input
```
int((a-a*sin(f*x+e))^3*(c+c*sin(f*x+e))^n*(B*(3-n)+B*(4+n)*sin(f*x+e)),x,m
ethod=_RETURNVERBOSE)
```

output
```
a^3*B*(c*(1+sin(f*x+e)))^n*(tan(1/2*f*x+1/2*e)+1)*(tan(1/2*f*x+1/2*e)-1)^7
/f/(1+tan(1/2*f*x+1/2*e)^2)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(34) = 68$.

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.26

$$\int (a - a\sin(e+fx))^3(c+c\sin(e+fx))^n(B(3-n)+B(4+n)\sin(e+fx))\,dx$$
$$= \frac{\left(3\,Ba^3\cos\left(fx+e\right)^3 - 4\,Ba^3\cos\left(fx+e\right) - \left(Ba^3\cos\left(fx+e\right)^3 - 4\,Ba^3\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)(c\sin}{f}$$

input
```
integrate((a-a*sin(f*x+e))^3*(c+c*sin(f*x+e))^n*(B*(3-n)+B*(4+n)*sin(f*x+e
)),x, algorithm="fricas")
```

output
```
(3*B*a^3*cos(f*x + e)^3 - 4*B*a^3*cos(f*x + e) - (B*a^3*cos(f*x + e)^3 - 4
*B*a^3*cos(f*x + e))*sin(f*x + e))*(c*sin(f*x + e) + c)^n/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 898 vs. $2(32) = 64$.

Time = 71.84 (sec) , antiderivative size = 898, normalized size of antiderivative = 26.41

$$\int (a - a\sin(e+fx))^3(c+c\sin(e+fx))^n(B(3-n)+B(4+n)\sin(e+fx))\,dx$$
$$= \text{Too large to display}$$

input
```
integrate((a-a*sin(f*x+e))**3*(c+c*sin(f*x+e))**n*(B*(3-n)+B*(4+n)*sin(f*x
+e)),x)
```

output
```
Piecewise((B*a**3*(c + 2*c*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**n*
tan(e/2 + f*x/2)**8/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f
*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) - 6*B*a**3*(c + 2*c*ta
n(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**n*tan(e/2 + f*x/2)**7/(f*tan(e/
2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*ta
n(e/2 + f*x/2)**2 + f) + 14*B*a**3*(c + 2*c*tan(e/2 + f*x/2)/(tan(e/2 + f*
x/2)**2 + 1))**n*tan(e/2 + f*x/2)**6/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2
+ f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) - 14*
B*a**3*(c + 2*c*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**n*tan(e/2 + f
*x/2)**5/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 +
f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) + 14*B*a**3*(c + 2*c*tan(e/2 + f*
x/2)/(tan(e/2 + f*x/2)**2 + 1))**n*tan(e/2 + f*x/2)**3/(f*tan(e/2 + f*x/2)
**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*
x/2)**2 + f) - 14*B*a**3*(c + 2*c*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 +
1))**n*tan(e/2 + f*x/2)**2/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**
6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) + 6*B*a**3*(c +
 2*c*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**n*tan(e/2 + f*x/2)/(f*ta
n(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*
f*tan(e/2 + f*x/2)**2 + f) - B*a**3*(c + 2*c*tan(e/2 + f*x/2)/(tan(e/2 + f
*x/2)**2 + 1))**n/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*...
```

Maxima [F]

$$\int (a-a\sin(e+fx))^3(c+c\sin(e+fx))^n(B(3-n)+B(4+n)\sin(e+fx))\,dx$$
$$=\int -(B(n+4)\sin(fx+e)-B(n-3))(a\sin(fx+e)-a)^3(c\sin(fx+e)+c)^n\,dx$$

input
```
integrate((a-a*sin(f*x+e))^3*(c+c*sin(f*x+e))^n*(B*(3-n)+B*(4+n)*sin(f*x+e
)),x, algorithm="maxima")
```

output
```
-integrate((B*(n + 4)*sin(f*x + e) - B*(n - 3))*(a*sin(f*x + e) - a)^3*(c*
sin(f*x + e) + c)^n, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6974 vs. $2(34) = 68$.

Time = 11.84 (sec) , antiderivative size = 6974, normalized size of antiderivative = 205.12

$$\int (a - a\sin(e + fx))^3 (c + c\sin(e + fx))^n (B(3 - n) + B(4 + n)\sin(e + fx))\, dx$$
$$= \text{Too large to display}$$

input
```
integrate((a-a*sin(f*x+e))^3*(c+c*sin(f*x+e))^n*(B*(3-n)+B*(4+n)*sin(f*x+e
)),x, algorithm="giac")
```

output
```
-(B*a^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*t
an(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*
x + e)^2*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) +
8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e
)^2*tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 + 8*tan(f*x + e)^2*tan
(1/2*f*x + 1/2*e) + 4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/
2*f*x + 1/2*e)^2 + 4*tan(1/2*f*x + 1/2*e) + 1)*abs(c)/(tan(f*x + e)^2*tan(
1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^n*tan(-
1/4*pi*n*sgn(2*c*tan(1/2*f*x + 1/2*e)^4 + 4*c*tan(1/2*f*x + 1/2*e)^3 - 4*c
*tan(1/2*f*x + 1/2*e) - 2*c)*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*
f*x + 1/2*e)^2 + 4*c*tan(1/2*f*x + 1/2*e)) + 1/2*pi*n*floor(f*x/pi + e/pi
+ 1/2) - 1/4*pi*n*sgn(4*c*tan(1/2*f*x + 1/2*e)^3 + 8*c*tan(1/2*f*x + 1/2*e
)^2 + 4*c*tan(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x + 1/2*e)^8 - 6*B*a^3*(sqrt(
2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1
/2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1
/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) + 8*tan(f*x + e)
^2*tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*
x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2
*e) + 4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)
^2 + 4*tan(1/2*f*x + 1/2*e) + 1)*abs(c)/(tan(f*x + e)^2*tan(1/2*f*x + 1...
```

Mupad [B] (verification not implemented)

Time = 37.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.79

$$\int (a-a\sin(e+fx))^3(c+c\sin(e+fx))^n(B(3-n)+B(4+n)\sin(e+fx))\,dx = -\frac{B\,a^3\,(c\,(\sin(e+f\,x)+1))^n\,(14\,\cos(e+f\,x)-6\,\cos(3\,e+3\,f\,x)-14\,\sin(2\,e+2\,f\,x)+\sin(4\,e+4}{8\,f}$$

input
```
int(-(B*(n - 3) - B*sin(e + f*x)*(n + 4))*(a - a*sin(e + f*x))^3*(c + c*si
n(e + f*x))^n,x)
```

output
```
-(B*a^3*(c*(sin(e + f*x) + 1))^n*(14*cos(e + f*x) - 6*cos(3*e + 3*f*x) - 1
4*sin(2*e + 2*f*x) + sin(4*e + 4*f*x)))/(8*f)
```

Reduce [F]

$$\begin{aligned}\int (a-a\sin(e+fx))^3(c+c\sin(e+fx))^n(B(3-n)+B(4+n)\sin(e+fx))\,dx &= a^3b\Bigg(-\left(\int(\sin(fx+e)c+c)^n\,dx\right)n+3\left(\int(\sin(fx+e)c+c)^n\,dx\right)\\ &\quad-\left(\int(\sin(fx+e)c+c)^n\sin(fx+e)^4\,dx\right)n\\ &\quad-4\left(\int(\sin(fx+e)c+c)^n\sin(fx+e)^4\,dx\right)\\ &\quad+4\left(\int(\sin(fx+e)c+c)^n\sin(fx+e)^3\,dx\right)n\\ &\quad+9\left(\int(\sin(fx+e)c+c)^n\sin(fx+e)^3\,dx\right)\\ &\quad-6\left(\int(\sin(fx+e)c+c)^n\sin(fx+e)^2\,dx\right)n\\ &\quad-3\left(\int(\sin(fx+e)c+c)^n\sin(fx+e)^2\,dx\right)\\ &\quad+4\left(\int(\sin(fx+e)c+c)^n\sin(fx+e)\,dx\right)n\\ &\quad-5\left(\int(\sin(fx+e)c+c)^n\sin(fx+e)\,dx\right)\Bigg)\end{aligned}$$

input
```
int((a-a*sin(f*x+e))^3*(c+c*sin(f*x+e))^n*(B*(3-n)+B*(4+n)*sin(f*x+e)),x)
```

output
```
a**3*b*( - int((sin(e + f*x)*c + c)**n,x)*n + 3*int((sin(e + f*x)*c + c)**
n,x) - int((sin(e + f*x)*c + c)**n*sin(e + f*x)**4,x)*n - 4*int((sin(e + f
*x)*c + c)**n*sin(e + f*x)**4,x) + 4*int((sin(e + f*x)*c + c)**n*sin(e + f
*x)**3,x)*n + 9*int((sin(e + f*x)*c + c)**n*sin(e + f*x)**3,x) - 6*int((si
n(e + f*x)*c + c)**n*sin(e + f*x)**2,x)*n - 3*int((sin(e + f*x)*c + c)**n*
sin(e + f*x)**2,x) + 4*int((sin(e + f*x)*c + c)**n*sin(e + f*x),x)*n - 5*i
nt((sin(e + f*x)*c + c)**n*sin(e + f*x),x))
```

3.220 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^3(B(-3+m)-B(4+m)\sin(e+fx))\,dx$

Optimal result

Integrand size = 44, antiderivative size = 33

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^3(B(-3+m)-B(4+m)\sin(e+fx))\,dx = \frac{a^3Bc^3\cos^7(e+fx)(a+a\sin(e+fx))^{-3+m}}{f}$$

output
```
a^3*B*c^3*cos(f*x+e)^7*(a+a*sin(f*x+e))^(-3+m)/f
```

Mathematica [A] (verified)

Time = 8.73 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.00

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^3(B(-3+m)-B(4+m)\sin(e+fx))\,dx = \frac{Bc^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^m}{f}$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^3*(B*(-3 + m) - B*(4
 + m)*Sin[e + f*x]),x]
```

output
```
(B*c^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])*(a*(1 + Sin[e + f*x]))^m)/f
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3042, 3446, 25, 3042, 3333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c - c\sin(e + fx))^3 (a\sin(e + fx) + a)^m (B(m - 3) - B(m + 4)\sin(e + fx))\, dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c - c\sin(e + fx))^3 (a\sin(e + fx) + a)^m (B(m - 3) - B(m + 4)\sin(e + fx))dx$$

$$\Big\downarrow \text{3446}$$

$$a^3c^3 \int -\cos^6(e + fx)(\sin(e + fx)a + a)^{m-3}(B(3 - m) + B(m + 4)\sin(e + fx))dx$$

$$\Big\downarrow \text{25}$$

$$-a^3c^3 \int \cos^6(e + fx)(\sin(e + fx)a + a)^{m-3}(B(3 - m) + B(m + 4)\sin(e + fx))dx$$

$$\Big\downarrow \text{3042}$$

$$-a^3c^3 \int \cos(e + fx)^6(\sin(e + fx)a + a)^{m-3}(B(3 - m) + B(m + 4)\sin(e + fx))dx$$

$$\Big\downarrow \text{3333}$$

$$\frac{a^3Bc^3\cos^7(e + fx)(a\sin(e + fx) + a)^{m-3}}{f}$$

input
```
Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^3*(B*(-3 + m) - B*(4 + m)*
Sin[e + f*x]),x]
```

output
```
(a^3*B*c^3*Cos[e + f*x]^7*(a + a*Sin[e + f*x])^(-3 + m))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3333
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] /;
FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[a*d*m + b
*c*(m + p + 1), 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 7.45 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.85

method	result	size
parallelrisch	$-\frac{c^{3} B \left(a \left(1+\sin \left(f x +e \right)\right)\right)^{m} \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)+1\right) \left(\tan \left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)^{7}}{f \left(1+\tan \left(\frac{f x}{2}+\frac{e}{2}\right)^{2}\right)^{4}}$	61

input
```
int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3*(B*(-3+m)-B*(4+m)*sin(f*x+e)),x,
method=_RETURNVERBOSE)
```

output
```
-c^3*B*(a*(1+sin(f*x+e)))^m*(tan(1/2*f*x+1/2*e)+1)*(tan(1/2*f*x+1/2*e)-1)^
7/f/(1+tan(1/2*f*x+1/2*e)^2)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(33) = 66$.

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.36

$$\int (a + a\sin(e + fx))^m (c - c\sin(e + fx))^3 (B(-3 + m) - B(4 + m)\sin(e + fx))\,dx = -\frac{\left(3\,Bc^3\cos\left(fx+e\right)^3 - 4\,Bc^3\cos\left(fx+e\right) - \left(Bc^3\cos\left(fx+e\right)^3 - 4\,Bc^3\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)(a\,\mathrm{s}}{f}$$

input
```
integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3*(B*(-3+m)-B*(4+m)*sin(f*x+
e)),x, algorithm="fricas")
```

output
```
-(3*B*c^3*cos(f*x + e)^3 - 4*B*c^3*cos(f*x + e) - (B*c^3*cos(f*x + e)^3 -
4*B*c^3*cos(f*x + e))*sin(f*x + e))*(a*sin(f*x + e) + a)^m/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 898 vs. $2(31) = 62$.

Time = 71.04 (sec) , antiderivative size = 898, normalized size of antiderivative = 27.21

$$\int (a + a\sin(e + fx))^m (c - c\sin(e + fx))^3 (B(-3 + m) - B(4 + m)\sin(e + fx))\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**3*(B*(-3+m)-B*(4+m)*sin(f*
x+e)),x)
```

output
```
Piecewise((-B*c**3*(a + 2*a*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**m
*tan(e/2 + f*x/2)**8/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*
f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) + 6*B*c**3*(a + 2*a*t
an(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**m*tan(e/2 + f*x/2)**7/(f*tan(e
/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*t
an(e/2 + f*x/2)**2 + f) - 14*B*c**3*(a + 2*a*tan(e/2 + f*x/2)/(tan(e/2 + f
*x/2)**2 + 1))**m*tan(e/2 + f*x/2)**6/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2
 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) + 14
*B*c**3*(a + 2*a*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**m*tan(e/2 +
f*x/2)**5/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 +
 f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) - 14*B*c**3*(a + 2*a*tan(e/2 + f
*x/2)/(tan(e/2 + f*x/2)**2 + 1))**m*tan(e/2 + f*x/2)**3/(f*tan(e/2 + f*x/2
)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f
*x/2)**2 + f) + 14*B*c**3*(a + 2*a*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 +
 1))**m*tan(e/2 + f*x/2)**2/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)*
*6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) - 6*B*c**3*(a
+ 2*a*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**m*tan(e/2 + f*x/2)/(f*t
an(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4
*f*tan(e/2 + f*x/2)**2 + f) + B*c**3*(a + 2*a*tan(e/2 + f*x/2)/(tan(e/2 +
f*x/2)**2 + 1))**m/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6...
```

Maxima [F]

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^3(B(-3+m)-B(4+m)\sin(e+fx))\,dx$$
$$=\int (B(m+4)\sin(fx+e)-B(m-3))(c\sin(fx+e)-c)^3(a\sin(fx+e)+a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3*(B*(-3+m)-B*(4+m)*sin(f*x+
e)),x, algorithm="maxima")
```

output
```
integrate((B*(m + 4)*sin(f*x + e) - B*(m - 3))*(c*sin(f*x + e) - c)^3*(a*s
in(f*x + e) + a)^m, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6973 vs. 2(33) = 66.

Time = 11.88 (sec) , antiderivative size = 6973, normalized size of antiderivative = 211.30

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^3(B(-3+m)-B(4+m)\sin(e+fx))\,dx$$
$$= \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3*(B*(-3+m)-B*(4+m)*sin(f*x+
e)),x, algorithm="giac")
```

output
```
(B*c^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*ta
n(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x
 + e)^2*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) + 8
*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)
^2*tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 + 8*tan(f*x + e)^2*tan(
1/2*f*x + 1/2*e) + 4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2
*f*x + 1/2*e)^2 + 4*tan(1/2*f*x + 1/2*e) + 1)*abs(a)/(tan(f*x + e)^2*tan(1
/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^m*tan(-1
/4*pi*m*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 + 4*a*tan(1/2*f*x + 1/2*e)^3 - 4*a*
tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f
*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) + 1/2*pi*m*floor(f*x/pi + e/pi +
 1/2) - 1/4*pi*m*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 + 8*a*tan(1/2*f*x + 1/2*e)
^2 + 4*a*tan(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x + 1/2*e)^8 - 6*B*c^3*(sqrt(2
*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/
2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1/
2*f*x + 1/2*e)^4 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) + 8*tan(f*x + e)^
2*tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*x
 + 1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*
e) + 4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^
2 + 4*tan(1/2*f*x + 1/2*e) + 1)*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1/...
```

Mupad [B] (verification not implemented)

Time = 36.93 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.85

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^3(B(-3+m)-B(4+m)\sin(e+fx))\,dx = \frac{B\,c^3\,\left(a\,\left(\sin\left(e+f\,x\right)+1\right)\right)^m\,\left(14\,\cos\left(e+f\,x\right)-6\,\cos\left(3\,e+3\,f\,x\right)-14\,\sin\left(2\,e+2\,f\,x\right)+\sin\left(4\,e+4\,f\,x\right)\right)}{8\,f}$$

input
```
int((B*(m - 3) - B*sin(e + f*x)*(m + 4))*(a + a*sin(e + f*x))^m*(c - c*sin
(e + f*x))^3,x)
```

output
```
(B*c^3*(a*(sin(e + f*x) + 1))^m*(14*cos(e + f*x) - 6*cos(3*e + 3*f*x) - 14
*sin(2*e + 2*f*x) + sin(4*e + 4*f*x)))/(8*f)
```

Reduce [F]

$$\begin{aligned}\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^3(B(-3+m)-B(4+m)\sin(e+fx))\,dx &= b\,c^3\Bigg(\left(\int (a+a\sin(fx+e))^m\,dx\right)m-3\left(\int (a+a\sin(fx+e))^m\,dx\right)\\ &\quad+\left(\int (a+a\sin(fx+e))^m\sin(fx+e)^4\,dx\right)m\\ &\quad+4\left(\int (a+a\sin(fx+e))^m\sin(fx+e)^4\,dx\right)\\ &\quad-4\left(\int (a+a\sin(fx+e))^m\sin(fx+e)^3\,dx\right)m\\ &\quad-9\left(\int (a+a\sin(fx+e))^m\sin(fx+e)^3\,dx\right)\\ &\quad+6\left(\int (a+a\sin(fx+e))^m\sin(fx+e)^2\,dx\right)m\\ &\quad+3\left(\int (a+a\sin(fx+e))^m\sin(fx+e)^2\,dx\right)\\ &\quad-4\left(\int (a+a\sin(fx+e))^m\sin(fx+e)\,dx\right)m\\ &\quad+5\left(\int (a+a\sin(fx+e))^m\sin(fx+e)\,dx\right)\Bigg)\end{aligned}$$

input
```
int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3*(B*(-3+m)-B*(4+m)*sin(f*x+e)),x)
```

output
```
b*c**3*(int((sin(e + f*x)*a + a)**m,x)*m - 3*int((sin(e + f*x)*a + a)**m,x
) + int((sin(e + f*x)*a + a)**m*sin(e + f*x)**4,x)*m + 4*int((sin(e + f*x)
*a + a)**m*sin(e + f*x)**4,x) - 4*int((sin(e + f*x)*a + a)**m*sin(e + f*x)
**3,x)*m - 9*int((sin(e + f*x)*a + a)**m*sin(e + f*x)**3,x) + 6*int((sin(e
 + f*x)*a + a)**m*sin(e + f*x)**2,x)*m + 3*int((sin(e + f*x)*a + a)**m*sin
(e + f*x)**2,x) - 4*int((sin(e + f*x)*a + a)**m*sin(e + f*x),x)*m + 5*int(
(sin(e + f*x)*a + a)**m*sin(e + f*x),x))
```

3.221 $\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^3(B(-3+m)+B(4+m)\sin(e+fx))\,dx$

Optimal result

Integrand size = 43, antiderivative size = 35

$$\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^3(B(-3+m)+B(4+m)\sin(e+fx))\,dx = -\frac{a^3Bc^3\cos^7(e+fx)(a-a\sin(e+fx))^{-3+m}}{f}$$

output
```
-a^3*B*c^3*cos(f*x+e)^7*(a-a*sin(f*x+e))^(-3+m)/f
```

Mathematica [A] (verified)

Time = 1.66 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.74

$$\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^3(B(-3+m)+B(4+m)\sin(e+fx))\,dx = \frac{Bc^3(a-a\sin(e+fx))^m(-14\cos(e+fx)+6\cos(3(e+fx))-14\sin(2(e+fx))+\sin(4(e+fx)))}{8f}$$

input
```
Integrate[(a - a*Sin[e + f*x])^m*(c + c*Sin[e + f*x])^3*(B*(-3 + m) + B*(4
 + m)*Sin[e + f*x]),x]
```

output
```
(B*c^3*(a - a*Sin[e + f*x])^m*(-14*Cos[e + f*x] + 6*Cos[3*(e + f*x)] - 14*
Sin[2*(e + f*x)] + Sin[4*(e + f*x)]))/(8*f)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {3042, 3446, 25, 3042, 3333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\sin(e+fx)+c)^3(a-a\sin(e+fx))^m(B(m+4)\sin(e+fx)+B(m-3))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c\sin(e+fx)+c)^3(a-a\sin(e+fx))^m(B(m+4)\sin(e+fx)+B(m-3))dx$$

$$\Big\downarrow \text{3446}$$

$$a^3c^3\int -\cos^6(e+fx)(a-a\sin(e+fx))^{m-3}(B(3-m)-B(m+4)\sin(e+fx))dx$$

$$\Big\downarrow \text{25}$$

$$-a^3c^3\int \cos^6(e+fx)(a-a\sin(e+fx))^{m-3}(B(3-m)-B(m+4)\sin(e+fx))dx$$

$$\Big\downarrow \text{3042}$$

$$-a^3c^3\int \cos(e+fx)^6(a-a\sin(e+fx))^{m-3}(B(3-m)-B(m+4)\sin(e+fx))dx$$

$$\Big\downarrow \text{3333}$$

$$-\frac{a^3Bc^3\cos^7(e+fx)(a-a\sin(e+fx))^{m-3}}{f}$$

input
```
Int[(a - a*Sin[e + f*x])^m*(c + c*Sin[e + f*x])^3*(B*(-3 + m) + B*(4 + m)*
Sin[e + f*x]),x]
```

output
```
-((a^3*B*c^3*Cos[e + f*x]^7*(a - a*Sin[e + f*x])^(-3 + m))/f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3333
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] /;
FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[a*d*m + b
*c*(m + p + 1), 0]
```

rule 3446
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Si
mp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*
d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((LtQ[m, 0]
&& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 7.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.74

method	result	size
parallelrisch	$\frac{c^3 B(-a(\sin(fx+e)-1))^m \left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^7}{f\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^4}$	61

input
```
int((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^3*(B*(-3+m)+B*(4+m)*sin(f*x+e)),x,
method=_RETURNVERBOSE)
```

output
```
c^3*B*(-a*(sin(f*x+e)-1))^m*(tan(1/2*f*x+1/2*e)-1)*(tan(1/2*f*x+1/2*e)+1)^
7/f/(1+tan(1/2*f*x+1/2*e)^2)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(35) = 70$.

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.20

$$\int (a - a\sin(e + fx))^m (c + c\sin(e + fx))^3 (B(-3 + m) + B(4 + m)\sin(e + fx))\,dx$$
$$= \frac{\left(3\,Bc^3\cos\left(fx+e\right)^3 - 4\,Bc^3\cos\left(fx+e\right) + \left(Bc^3\cos\left(fx+e\right)^3 - 4\,Bc^3\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)(-a\,\mathrm{s}}{f}$$

input
```
integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^3*(B*(-3+m)+B*(4+m)*sin(f*x+
e)),x, algorithm="fricas")
```

output
```
(3*B*c^3*cos(f*x + e)^3 - 4*B*c^3*cos(f*x + e) + (B*c^3*cos(f*x + e)^3 - 4
*B*c^3*cos(f*x + e))*sin(f*x + e))*(-a*sin(f*x + e) + a)^m/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 898 vs. $2(32) = 64$.

Time = 72.11 (sec) , antiderivative size = 898, normalized size of antiderivative = 25.66

$$\int (a - a\sin(e + fx))^m (c + c\sin(e + fx))^3 (B(-3 + m) + B(4 + m)\sin(e + fx))\,dx$$
$$= \text{Too large to display}$$

input
```
integrate((a-a*sin(f*x+e))**m*(c+c*sin(f*x+e))**3*(B*(-3+m)+B*(4+m)*sin(f*
x+e)),x)
```

output
```
Piecewise((B*c**3*(a - 2*a*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**m*
tan(e/2 + f*x/2)**8/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f
*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) + 6*B*c**3*(a - 2*a*ta
n(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**m*tan(e/2 + f*x/2)**7/(f*tan(e/
2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*ta
n(e/2 + f*x/2)**2 + f) + 14*B*c**3*(a - 2*a*tan(e/2 + f*x/2)/(tan(e/2 + f*
x/2)**2 + 1))**m*tan(e/2 + f*x/2)**6/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2
+ f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) + 14*
B*c**3*(a - 2*a*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**m*tan(e/2 + f
*x/2)**5/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 +
f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) - 14*B*c**3*(a - 2*a*tan(e/2 + f*
x/2)/(tan(e/2 + f*x/2)**2 + 1))**m*tan(e/2 + f*x/2)**3/(f*tan(e/2 + f*x/2)
**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*
x/2)**2 + f) - 14*B*c**3*(a - 2*a*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 +
1))**m*tan(e/2 + f*x/2)**2/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**
6 + 6*f*tan(e/2 + f*x/2)**4 + 4*f*tan(e/2 + f*x/2)**2 + f) - 6*B*c**3*(a -
 2*a*tan(e/2 + f*x/2)/(tan(e/2 + f*x/2)**2 + 1))**m*tan(e/2 + f*x/2)/(f*ta
n(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*f*tan(e/2 + f*x/2)**4 + 4*
f*tan(e/2 + f*x/2)**2 + f) - B*c**3*(a - 2*a*tan(e/2 + f*x/2)/(tan(e/2 + f
*x/2)**2 + 1))**m/(f*tan(e/2 + f*x/2)**8 + 4*f*tan(e/2 + f*x/2)**6 + 6*...
```

Maxima [F]

$$\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^3(B(-3+m)+B(4+m)\sin(e+fx))\,dx$$
$$=\int (B(m+4)\sin(fx+e)+B(m-3))(c\sin(fx+e)+c)^3(-a\sin(fx+e)+a)^m\,dx$$

input
```
integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^3*(B*(-3+m)+B*(4+m)*sin(f*x+
e)),x, algorithm="maxima")
```

output
```
integrate((B*(m + 4)*sin(f*x + e) + B*(m - 3))*(c*sin(f*x + e) + c)^3*(-a*
sin(f*x + e) + a)^m, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6974 vs. $2(35) = 70$.

Time = 12.21 (sec) , antiderivative size = 6974, normalized size of antiderivative = 199.26

$$\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^3(B(-3+m)+B(4+m)\sin(e+fx))\,dx = \text{Too large to display}$$

input
```
integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^3*(B*(-3+m)+B*(4+m)*sin(f*x+
e)),x, algorithm="giac")
```

output
```
-(B*c^3*(sqrt(2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*t
an(1/2*f*x + 1/2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*
x + e)^2*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) -
8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e
)^2*tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 - 8*tan(f*x + e)^2*tan
(1/2*f*x + 1/2*e) - 4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/
2*f*x + 1/2*e)^2 - 4*tan(1/2*f*x + 1/2*e) + 1)*abs(a)/(tan(f*x + e)^2*tan(
1/2*f*x + 1/2*e)^2 + tan(f*x + e)^2 + tan(1/2*f*x + 1/2*e)^2 + 1))^m*tan(-
1/4*pi*m*sgn(2*a*tan(1/2*f*x + 1/2*e)^4 - 4*a*tan(1/2*f*x + 1/2*e)^3 + 4*a
*tan(1/2*f*x + 1/2*e) - 2*a)*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*
f*x + 1/2*e)^2 + 4*a*tan(1/2*f*x + 1/2*e)) + 1/2*pi*m*floor(f*x/pi + e/pi
+ 1/2) - 1/4*pi*m*sgn(4*a*tan(1/2*f*x + 1/2*e)^3 - 8*a*tan(1/2*f*x + 1/2*e
)^2 + 4*a*tan(1/2*f*x + 1/2*e)))^2*tan(1/2*f*x + 1/2*e)^8 + 6*B*c^3*(sqrt(
2*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1
/2*e)^3 + 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e)^2 + 3*tan(f*x + e)^2*tan(1
/2*f*x + 1/2*e)^4 - 4*tan(f*x + e)^4*tan(1/2*f*x + 1/2*e) - 8*tan(f*x + e)
^2*tan(1/2*f*x + 1/2*e)^3 + 2*tan(f*x + e)^4 + 6*tan(f*x + e)^2*tan(1/2*f*
x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e)^4 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2
*e) - 4*tan(1/2*f*x + 1/2*e)^3 + 3*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)
^2 - 4*tan(1/2*f*x + 1/2*e) + 1)*abs(a)/(tan(f*x + e)^2*tan(1/2*f*x + 1...
```

Mupad [B] (verification not implemented)

Time = 36.35 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.83

$$\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^3(B(-3+m)+B(4+m)\sin(e+fx))\,dx =$$
$$-\frac{B\,c^3\,(-a\,(\sin(e+f\,x)-1))^m\,(14\,\cos(e+f\,x)-6\,\cos(3\,e+3\,f\,x)+14\,\sin(2\,e+2\,f\,x)-\sin(4\,e+}{8\,f}$$

input
```
int((B*(m - 3) + B*sin(e + f*x)*(m + 4))*(a - a*sin(e + f*x))^m*(c + c*sin
(e + f*x))^3,x)
```

output
```
-(B*c^3*(-a*(sin(e + f*x) - 1))^m*(14*cos(e + f*x) - 6*cos(3*e + 3*f*x) +
14*sin(2*e + 2*f*x) - sin(4*e + 4*f*x)))/(8*f)
```

Reduce [F]

$$\begin{aligned}
&\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^3(B(-3+m)+B(4+m)\sin(e+fx))\,dx \\
&= b\,c^3\Bigg(\left(\int (-a\sin(fx+e)+a)^m\,dx\right)m-3\left(\int (-a\sin(fx+e)+a)^m\,dx\right) \\
&\quad+\left(\int (-a\sin(fx+e)+a)^m\sin(fx+e)^4\,dx\right)m \\
&\quad+4\left(\int (-a\sin(fx+e)+a)^m\sin(fx+e)^4\,dx\right) \\
&\quad+4\left(\int (-a\sin(fx+e)+a)^m\sin(fx+e)^3\,dx\right)m \\
&\quad+9\left(\int (-a\sin(fx+e)+a)^m\sin(fx+e)^3\,dx\right) \\
&\quad+6\left(\int (-a\sin(fx+e)+a)^m\sin(fx+e)^2\,dx\right)m \\
&\quad+3\left(\int (-a\sin(fx+e)+a)^m\sin(fx+e)^2\,dx\right) \\
&\quad+4\left(\int (-a\sin(fx+e)+a)^m\sin(fx+e)\,dx\right)m \\
&\quad-5\left(\int (-a\sin(fx+e)+a)^m\sin(fx+e)\,dx\right)\Bigg)
\end{aligned}$$

input
```
int((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^3*(B*(-3+m)+B*(4+m)*sin(f*x+e)),x)
```

output
```
b*c**3*(int(( - sin(e + f*x)*a + a)**m,x)*m - 3*int(( - sin(e + f*x)*a + a
)**m,x) + int(( - sin(e + f*x)*a + a)**m*sin(e + f*x)**4,x)*m + 4*int(( -
sin(e + f*x)*a + a)**m*sin(e + f*x)**4,x) + 4*int(( - sin(e + f*x)*a + a)*
*m*sin(e + f*x)**3,x)*m + 9*int(( - sin(e + f*x)*a + a)**m*sin(e + f*x)**3
,x) + 6*int(( - sin(e + f*x)*a + a)**m*sin(e + f*x)**2,x)*m + 3*int(( - si
n(e + f*x)*a + a)**m*sin(e + f*x)**2,x) + 4*int(( - sin(e + f*x)*a + a)**m
*sin(e + f*x),x)*m - 5*int(( - sin(e + f*x)*a + a)**m*sin(e + f*x),x))
```

3.222 $\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^n(B(m-n)-B(1+m+n)\sin(e+fx))\,dx$

Optimal result

Integrand size = 47, antiderivative size = 36

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^n(B(m-n)-B(1+m+n)\sin(e+fx))\,dx = \frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n}{f}$$

output
```
B*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n/f
```

Mathematica [A] (verified)

Time = 3.43 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^n(B(m-n)-B(1+m+n)\sin(e+fx))\,dx = \frac{B\cos(e+fx)(a(1+\sin(e+fx)))^m(c-c\sin(e+fx))^n}{f}$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n*(B*(m - n) - B*(1
+ m + n)*Sin[e + f*x]),x]
```

output
```
(B*Cos[e + f*x]*(a*(1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^n)/f
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {3042, 3449}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(B(m-n)-B(m+n+1)\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(B(m-n)-B(m+n+1)\sin(e+fx))dx$$

$$\Big\downarrow \text{3449}$$

$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n}{f}$$

input
```
Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n*(B*(m - n) - B*(1 + m +
n)*Sin[e + f*x]),x]
```

output
```
(B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n)/f
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3449
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d,
0] && EqQ[a^2 - b^2, 0] && EqQ[A*b*(m + n + 1) + a*B*(m - n), 0] && NeQ[m,
-2^(-1)]
```

Maple [A] (verified)

Time = 5.75 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

method	result	size
parallelrisch	$\frac{(a(1+\sin(fx+e)))^m(-c(\sin(fx+e)-1))^n\cos(fx+e)B}{f}$	37

input
```
int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n*(B*(m-n)-B*(1+m+n)*sin(f*x+e)),x
,method=_RETURNVERBOSE)
```

output
```
1/f*(a*(1+sin(f*x+e)))^m*(-c*(sin(f*x+e)-1))^n*cos(f*x+e)*B
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^n(B(m-n)-B(1+m+n)\sin(e+fx))\,dx = \frac{(a\sin(fx+e)+a)^m(-c\sin(fx+e)+c)^nB\cos(fx+e)}{f}$$

input
```
integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n*(B*(m-n)-B*(1+m+n)*sin(f*x
+e)),x, algorithm="fricas")
```

output
```
(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n*B*cos(f*x + e)/f
```

Sympy [F]

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^n(B(m-n)-B(1+m+n)\sin(e+fx))\,dx$$
$$= -B\left(\int\left(-m(a\sin(e+fx)+a)^m(-c\sin(e+fx)+c)^n\right)dx + \int n(a\sin(e+fx)+a)^m(-c\sin(e+fx)+c)^n\,dx + \int(a\sin(e+fx)+a)^m(-c\sin(e+fx)+c)^n\sin(e+fx)\,dx + \int m(a\sin(e+fx)+a)^m(-c\sin(e+fx)+c)^n\sin(e+fx)\,dx + \int n(a\sin(e+fx)+a)^m(-c\sin(e+fx)+c)^n\sin(e+fx)\,dx\right)$$

input
```
integrate((a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**n*(B*(m-n)-B*(1+m+n)*sin(f
*x+e)),x)
```

output
```
-B*(Integral(-m*(a*sin(e + f*x) + a)**m*(-c*sin(e + f*x) + c)**n, x) + Int
egral(n*(a*sin(e + f*x) + a)**m*(-c*sin(e + f*x) + c)**n, x) + Integral((a
*sin(e + f*x) + a)**m*(-c*sin(e + f*x) + c)**n*sin(e + f*x), x) + Integral
(m*(a*sin(e + f*x) + a)**m*(-c*sin(e + f*x) + c)**n*sin(e + f*x), x) + Int
egral(n*(a*sin(e + f*x) + a)**m*(-c*sin(e + f*x) + c)**n*sin(e + f*x), x))
```

Maxima [F]

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^n(B(m-n)-B(1+m+n)\sin(e+fx))\,dx$$
$$= \int -(B(m+n+1)\sin(fx+e)-B(m-n))(a\sin(fx+e)+a)^m(-c\sin(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n*(B*(m-n)-B*(1+m+n)*sin(f*x
+e)),x, algorithm="maxima")
```

output
```
-integrate((B*(m + n + 1)*sin(f*x + e) - B*(m - n))*(a*sin(f*x + e) + a)^m
*(-c*sin(f*x + e) + c)^n, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8947 vs. $2(36) = 72$.

Time = 50.84 (sec) , antiderivative size = 8947, normalized size of antiderivative = 248.53

$$\int (a + a\sin(e + fx))^m (c - c\sin(e + fx))^n (B(m - n) - B(1 + m + n)\sin(e + fx))\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n*(B*(m-n)-B*(1+m+n)*sin(f*x
+e)),x, algorithm="giac")
```

output
```
(B*cos(2*pi*m*floor(-1/8*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*t
an(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/
2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2) + 5/8) + 2*pi*n*floor(-1/8*sgn(4*tan(
f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)
+ 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2
) + 5/8) + 1/4*pi*m*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*
x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^
2 + 8*tan(1/2*f*x + 1/2*e) + 2) + 1/4*pi*n*sgn(4*tan(f*x + e)^2*tan(1/2*f*
x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 +
2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2) - 1/4*pi*m - 1/4*pi
*n)*e^(-m*log(2) - n*log(2) + m*log(sqrt(2)*sqrt(abs(4*tan(f*x + e)^2*tan(
1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e
)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^
2*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8
*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x +
1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2 + abs(4*tan(f*x + e)
^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(
f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2)*tan(1/
2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f*x
 + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e...
```

Mupad [B] (verification not implemented)

Time = 35.48 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^n(B(m-n)-B(1+m+n)\sin(e+fx))\,dx = \frac{B\,\cos\left(e+f\,x\right)\,{\left(a\,\left(\sin\left(e+f\,x\right)+1\right)\right)}^m\,{\left(-c\,\left(\sin\left(e+f\,x\right)-1\right)\right)}^n}{f}$$

input
```
int((B*(m - n) - B*sin(e + f*x)*(m + n + 1))*(a + a*sin(e + f*x))^m*(c - c
*sin(e + f*x))^n,x)
```

output
```
(B*cos(e + f*x)*(a*(sin(e + f*x) + 1))^m*(-c*(sin(e + f*x) - 1))^n)/f
```

Reduce [F]

$$\begin{aligned}\int (a+a\sin(e+fx))^m(c-c\sin(e+fx))^n(B(m-n)-B(1+m+n)\sin(e+fx))\,dx &= b\Bigg(-\left(\int (a+a\sin(fx+e))^m(-\sin(fx+e)c+c)^n\sin(fx+e)\,dx\right)m\\ &\quad-\left(\int (a+a\sin(fx+e))^m(-\sin(fx+e)c+c)^n\sin(fx+e)\,dx\right)n\\ &\quad-\left(\int (a+a\sin(fx+e))^m(-\sin(fx+e)c+c)^n\sin(fx+e)\,dx\right)\\ &\quad+\left(\int (a+a\sin(fx+e))^m(-\sin(fx+e)c+c)^n\,dx\right)m\\ &\quad-\left(\int (a+a\sin(fx+e))^m(-\sin(fx+e)c+c)^n\,dx\right)n\Bigg)\end{aligned}$$

input
```
int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n*(B*(m-n)-B*(1+m+n)*sin(f*x+e)),x
)
```

output
```
b*( - int((sin(e + f*x)*a + a)**m*( - sin(e + f*x)*c + c)**n*sin(e + f*x),
x)*m - int((sin(e + f*x)*a + a)**m*( - sin(e + f*x)*c + c)**n*sin(e + f*x)
,x)*n - int((sin(e + f*x)*a + a)**m*( - sin(e + f*x)*c + c)**n*sin(e + f*x
),x) + int((sin(e + f*x)*a + a)**m*( - sin(e + f*x)*c + c)**n,x)*m - int((
sin(e + f*x)*a + a)**m*( - sin(e + f*x)*c + c)**n,x)*n)
```

3.223 $\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^n(B(m-n)+B(1+m+n)\sin(e+fx))\,dx$

Optimal result

Integrand size = 46, antiderivative size = 37

$$\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^n(B(m-n)+B(1+m+n)\sin(e+fx))\,dx = -\frac{B\cos(e+fx)(a-a\sin(e+fx))^m(c+c\sin(e+fx))^n}{f}$$

output
```
-B*cos(f*x+e)*(a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^n/f
```

Mathematica [A] (verified)

Time = 2.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^n(B(m-n)+B(1+m+n)\sin(e+fx))\,dx = -\frac{B\cos(e+fx)(c(1+\sin(e+fx)))^n(a-a\sin(e+fx))^m}{f}$$

input
```
Integrate[(a - a*Sin[e + f*x])^m*(c + c*Sin[e + f*x])^n*(B*(m - n) + B*(1
+ m + n)*Sin[e + f*x]),x]
```

output
```
-((B*Cos[e + f*x]*(c*(1 + Sin[e + f*x]))^n*(a - a*Sin[e + f*x])^m)/f)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {3042, 3449}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-a\sin(e+fx))^m(c\sin(e+fx)+c)^n(B(m+n+1)\sin(e+fx)+B(m-n))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a-a\sin(e+fx))^m(c\sin(e+fx)+c)^n(B(m+n+1)\sin(e+fx)+B(m-n))dx$$

$$\Big\downarrow \text{3449}$$

$$-\frac{B\cos(e+fx)(a-a\sin(e+fx))^m(c\sin(e+fx)+c)^n}{f}$$

input
```
Int[(a - a*Sin[e + f*x])^m*(c + c*Sin[e + f*x])^n*(B*(m - n) + B*(1 + m +
n)*Sin[e + f*x]),x]
```

output
```
-((B*Cos[e + f*x]*(a - a*Sin[e + f*x])^m*(c + c*Sin[e + f*x])^n)/f)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3449
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d,
0] && EqQ[a^2 - b^2, 0] && EqQ[A*b*(m + n + 1) + a*B*(m - n), 0] && NeQ[m,
-2^(-1)]
```

Maple [A] (verified)

Time = 5.55 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

method	result	size
parallelrisch	$-\frac{(-a(\sin(fx+e)-1))^m (c(1+\sin(fx+e)))^n \cos(fx+e)B}{f}$	38

input
```
int((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^n*(B*(m-n)+B*(1+m+n)*sin(f*x+e)),x
,method=_RETURNVERBOSE)
```

output
```
-1/f*(-a*(sin(f*x+e)-1))^m*(c*(1+sin(f*x+e)))^n*cos(f*x+e)*B
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^n(B(m-n)+B(1+m+n)\sin(e+fx))\,dx = -\frac{(-a\sin(fx+e)+a)^m(c\sin(fx+e)+c)^nB\cos(fx+e)}{f}$$

input
```
integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^n*(B*(m-n)+B*(1+m+n)*sin(f*x
+e)),x, algorithm="fricas")
```

output
```
-(-a*sin(f*x + e) + a)^m*(c*sin(f*x + e) + c)^n*B*cos(f*x + e)/f
```

Sympy [F]

$$\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^n(B(m-n)+B(1+m+n)\sin(e+fx))\,dx$$
$$= B\left(\int m(-a\sin(e+fx)+a)^m(c\sin(e+fx)+c)^n\,dx + \int(-n(-a\sin(e+fx)+a)^m(c\sin(e+fx)+c)^n)\,dx + \int(-a\sin(e+fx)+a)^m(c\sin(e+fx)+c)^n\sin(e+fx)\,dx + \int m(-a\sin(e+fx)+a)^m(c\sin(e+fx)+c)^n\sin(e+fx)\,dx + \int n(-a\sin(e+fx)+a)^m(c\sin(e+fx)+c)^n\sin(e+fx)\,dx\right)$$

input
```
integrate((a-a*sin(f*x+e))**m*(c+c*sin(f*x+e))**n*(B*(m-n)+B*(1+m+n)*sin(f
*x+e)),x)
```

output
```
B*(Integral(m*(-a*sin(e + f*x) + a)**m*(c*sin(e + f*x) + c)**n, x) + Integ
ral(-n*(-a*sin(e + f*x) + a)**m*(c*sin(e + f*x) + c)**n, x) + Integral((-a
*sin(e + f*x) + a)**m*(c*sin(e + f*x) + c)**n*sin(e + f*x), x) + Integral(
m*(-a*sin(e + f*x) + a)**m*(c*sin(e + f*x) + c)**n*sin(e + f*x), x) + Inte
gral(n*(-a*sin(e + f*x) + a)**m*(c*sin(e + f*x) + c)**n*sin(e + f*x), x))
```

Maxima [F]

$$\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^n(B(m-n)+B(1+m+n)\sin(e+fx))\,dx$$
$$= \int (B(m+n+1)\sin(fx+e)+B(m-n))(-a\sin(fx+e)+a)^m(c\sin(fx+e)+c)^n\,dx$$

input
```
integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^n*(B*(m-n)+B*(1+m+n)*sin(f*x
+e)),x, algorithm="maxima")
```

output
```
integrate((B*(m + n + 1)*sin(f*x + e) + B*(m - n))*(-a*sin(f*x + e) + a)^m
*(c*sin(f*x + e) + c)^n, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8948 vs. $2(37) = 74$.

Time = 50.24 (sec) , antiderivative size = 8948, normalized size of antiderivative = 241.84

$$\int (a-a\sin(e+fx))^m(c+c\sin(e+fx))^n(B(m-n)+B(1+m+n)\sin(e+fx))\,dx = \text{Too large to display}$$

input
```
integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^n*(B*(m-n)+B*(1+m+n)*sin(f*x
+e)),x, algorithm="giac")
```

output
```
-(B*cos(2*pi*n*floor(-1/8*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*
tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1
/2*e)^2 + 8*tan(1/2*f*x + 1/2*e) + 2) + 5/8) + 2*pi*m*floor(-1/8*sgn(4*tan
(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)
 + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) +
2) + 5/8) + 1/4*pi*n*sgn(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 + 8*tan(f
*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)
^2 + 8*tan(1/2*f*x + 1/2*e) + 2) + 1/4*pi*m*sgn(4*tan(f*x + e)^2*tan(1/2*f
*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 +
 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2) - 1/4*pi*m - 1/4*p
i*n)*e^(-m*log(2) - n*log(2) + m*log(sqrt(2)*sqrt(abs(4*tan(f*x + e)^2*tan
(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x +
e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)
^2*tan(1/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 -
8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x +
 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(f*x + e)^2 + abs(4*tan(f*x + e
)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan
(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(1/2*f*x + 1/2*e) + 2)*tan(1
/2*f*x + 1/2*e)^2 + abs(4*tan(f*x + e)^2*tan(1/2*f*x + 1/2*e)^2 - 8*tan(f*
x + e)^2*tan(1/2*f*x + 1/2*e) + 4*tan(f*x + e)^2 + 2*tan(1/2*f*x + 1/2*...
```

Mupad [B] (verification not implemented)

Time = 35.48 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int (a - a\sin(e + fx))^m (c + c\sin(e + fx))^n (B(m - n) + B(1 + m + n)\sin(e + fx))\, dx$$
$$= -\frac{B\,\cos(e + f\,x)\,\left(-a\,(\sin(e + f\,x) - 1)\right)^m\,\left(c\,(\sin(e + f\,x) + 1)\right)^n}{f}$$

input
```
int((B*(m - n) + B*sin(e + f*x)*(m + n + 1))*(a - a*sin(e + f*x))^m*(c + c
*sin(e + f*x))^n,x)
```

output
```
-(B*cos(e + f*x)*(-a*(sin(e + f*x) - 1))^m*(c*(sin(e + f*x) + 1))^n)/f
```

Reduce [F]

$$\int (a - a\sin(e + fx))^m (c + c\sin(e + fx))^n (B(m - n) + B(1 + m + n)\sin(e + fx))\, dx$$
$$= b\Bigg(\left(\int (\sin(fx+e)\,c + c)^n\,(-a\sin(fx+e) + a)^m \sin(fx+e)\,dx\right) m + \left(\int (\sin(fx+e)\,c + c)^n\,(-a\sin(fx+e) + a)^m \sin(fx+e)\,dx\right) n + \int (\sin(fx+e)\,c + c)^n\,(-a\sin(fx+e) + a)^m \sin(fx+e)\,dx + \left(\int (\sin(fx+e)\,c + c)^n\,(-a\sin(fx+e) + a)^m\,dx\right) m - \left(\int (\sin(fx+e)\,c + c)^n\,(-a\sin(fx+e) + a)^m\,dx\right) n\Bigg)$$

input
```
int((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^n*(B*(m-n)+B*(1+m+n)*sin(f*x+e)),x
)
```

output

```
b*(int((sin(e + f*x)*c + c)**n*( - sin(e + f*x)*a + a)**m*sin(e + f*x),x)*
m + int((sin(e + f*x)*c + c)**n*( - sin(e + f*x)*a + a)**m*sin(e + f*x),x)
*n + int((sin(e + f*x)*c + c)**n*( - sin(e + f*x)*a + a)**m*sin(e + f*x),x
) + int((sin(e + f*x)*c + c)**n*( - sin(e + f*x)*a + a)**m,x)*m - int((sin
(e + f*x)*c + c)**n*( - sin(e + f*x)*a + a)**m,x)*n)
```

3.224 $\int \sin^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal result

Integrand size = 32, antiderivative size = 140

$$\begin{aligned}&\int \sin^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx\\&=\frac{1}{8}a^3Ax-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{3a^3A\cos^5(c+dx)}{5d}\\&-\frac{a^3A\cos^7(c+dx)}{7d}-\frac{a^3A\cos(c+dx)\sin(c+dx)}{8d}\\&-\frac{a^3A\cos(c+dx)\sin^3(c+dx)}{12d}+\frac{a^3A\cos(c+dx)\sin^5(c+dx)}{3d}\end{aligned}$$

output
```
1/8*a^3*A*x-2/3*a^3*A*cos(d*x+c)^3/d+3/5*a^3*A*cos(d*x+c)^5/d-1/7*a^3*A*co
s(d*x+c)^7/d-1/8*a^3*A*cos(d*x+c)*sin(d*x+c)/d-1/12*a^3*A*cos(d*x+c)*sin(d
*x+c)^3/d+1/3*a^3*A*cos(d*x+c)*sin(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.62

$$\int \sin^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \frac{a^3A(840c+840dx-1365\cos(c+dx)-175\cos(3(c+dx))+147\cos(5(c+dx))-15\cos(7(c+dx))-}{6720d}$$

input
```
Integrate[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
(a^3*A*(840*c + 840*d*x - 1365*Cos[c + d*x] - 175*Cos[3*(c + d*x)] + 147*C
os[5*(c + d*x)] - 15*Cos[7*(c + d*x)] - 210*Sin[2*(c + d*x)] - 210*Sin[4*(
c + d*x)] + 70*Sin[6*(c + d*x)]))/(6720*d)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3042, 3445, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(c+dx)(a\sin(c+dx)+a)^3(A-A\sin(c+dx))\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \sin(c+dx)^3(a\sin(c+dx)+a)^3(A-A\sin(c+dx))dx$$
$$\Big\downarrow \text{ 3445}$$
$$\int \left(-a^3A\sin^7(c+dx)-2a^3A\sin^6(c+dx)+2a^3A\sin^4(c+dx)+a^3A\sin^3(c+dx)\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^3A\cos^7(c+dx)}{7d}+\frac{3a^3A\cos^5(c+dx)}{5d}-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{a^3A\sin^5(c+dx)\cos(c+dx)}{3d}-\frac{a^3A\sin^3(c+dx)\cos(c+dx)}{12d}-\frac{a^3A\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}a^3Ax$$

input
```
Int[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
(a^3*A*x)/8 - (2*a^3*A*Cos[c + d*x]^3)/(3*d) + (3*a^3*A*Cos[c + d*x]^5)/(5
*d) - (a^3*A*Cos[c + d*x]^7)/(7*d) - (a^3*A*Cos[c + d*x]*Sin[c + d*x])/(8*
d) - (a^3*A*Cos[c + d*x]*Sin[c + d*x]^3)/(12*d) + (a^3*A*Cos[c + d*x]*Sin[
c + d*x]^5)/(3*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3445
```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.13

$$\frac{\frac{a^3A\left(\frac{16}{5}+\sin(dx+c)^6+\frac{6\sin(dx+c)^4}{5}+\frac{8\sin(dx+c)^2}{5}\right)\cos(dx+c)}{7}-2a^3A\left(-\frac{\left(\sin(dx+c)^5+\frac{5\sin(dx+c)^3}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\frac{5dx}{16}+\frac{5}{1}\right.}{d}$$

input
```
int(sin(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)
```

output
```
1/d*(1/7*a^3*A*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d
*x+c)-2*a^3*A*(-1/6*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*
x+c)+5/16*d*x+5/16*c)+2*a^3*A*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+
c)+3/8*d*x+3/8*c)-1/3*a^3*A*(2+sin(d*x+c)^2)*cos(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.75

$$\int \sin^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = -\frac{120\,Aa^3\cos\left(dx+c\right)^7-504\,Aa^3\cos\left(dx+c\right)^5+560\,Aa^3\cos\left(dx+c\right)^3-105\,Aa^3dx-35\,\left(8\,Aa^3\cos\left(d\right.\right.}{840\,d}$$

input
```
integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="f
ricas")
```

output
```
-1/840*(120*A*a^3*cos(d*x + c)^7 - 504*A*a^3*cos(d*x + c)^5 + 560*A*a^3*co
s(d*x + c)^3 - 105*A*a^3*d*x - 35*(8*A*a^3*cos(d*x + c)^5 - 14*A*a^3*cos(d
*x + c)^3 + 3*A*a^3*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 440 vs. $2(131) = 262$.

Time = 0.54 (sec) , antiderivative size = 440, normalized size of antiderivative = 3.14

$$\int \sin^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \begin{cases} -\frac{5Aa^3x\sin^6(c+dx)}{8} - \frac{15Aa^3x\sin^4(c+dx)\cos^2(c+dx)}{8} + \frac{3Aa^3x\sin^4(c+dx)}{4} - \frac{15Aa^3x\sin^2(c+dx)\cos^4(c+dx)}{8} + \frac{3Aa^3x\sin^2(c+d}{2} \\ x(-A\sin(c)+A)\left(a\sin(c)+a\right)^3\sin^3(c) \end{cases}$$

input
```
integrate(sin(d*x+c)**3*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

output
```
Piecewise((-5*A*a**3*x*sin(c + d*x)**6/8 - 15*A*a**3*x*sin(c + d*x)**4*cos
(c + d*x)**2/8 + 3*A*a**3*x*sin(c + d*x)**4/4 - 15*A*a**3*x*sin(c + d*x)**
2*cos(c + d*x)**4/8 + 3*A*a**3*x*sin(c + d*x)**2*cos(c + d*x)**2/2 - 5*A*a
**3*x*cos(c + d*x)**6/8 + 3*A*a**3*x*cos(c + d*x)**4/4 + A*a**3*sin(c + d*
x)**6*cos(c + d*x)/d + 11*A*a**3*sin(c + d*x)**5*cos(c + d*x)/(8*d) + 2*A*
a**3*sin(c + d*x)**4*cos(c + d*x)**3/d + 5*A*a**3*sin(c + d*x)**3*cos(c +
d*x)**3/(3*d) - 5*A*a**3*sin(c + d*x)**3*cos(c + d*x)/(4*d) + 8*A*a**3*sin
(c + d*x)**2*cos(c + d*x)**5/(5*d) - A*a**3*sin(c + d*x)**2*cos(c + d*x)/d
 + 5*A*a**3*sin(c + d*x)*cos(c + d*x)**5/(8*d) - 3*A*a**3*sin(c + d*x)*cos
(c + d*x)**3/(4*d) + 16*A*a**3*cos(c + d*x)**7/(35*d) - 2*A*a**3*cos(c + d
*x)**3/(3*d), Ne(d, 0)), (x*(-A*sin(c) + A)*(a*sin(c) + a)**3*sin(c)**3, T
rue))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.12

$$\int \sin^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx =$$

$$-\frac{96\left(5\cos\left(dx+c\right)^7-21\cos\left(dx+c\right)^5+35\cos\left(dx+c\right)^3-35\cos\left(dx+c\right)\right)Aa^3-1120\left(\cos\left(dx+c\right)^3\right.}{}$$

input
```
integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="m
axima")
```

output
```
-1/3360*(96*(5*cos(d*x + c)^7 - 21*cos(d*x + c)^5 + 35*cos(d*x + c)^3 - 35
*cos(d*x + c))*A*a^3 - 1120*(cos(d*x + c)^3 - 3*cos(d*x + c))*A*a^3 + 35*(
4*sin(2*d*x + 2*c)^3 + 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*sin(2*d*x +
 2*c))*A*a^3 - 210*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x + 2*c))
*A*a^3)/d
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.94

$$\int \sin^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \frac{1}{8}\,Aa^3x-\frac{Aa^3\cos\left(7\,dx+7\,c\right)}{448\,d}+\frac{7\,Aa^3\cos\left(5\,dx+5\,c\right)}{320\,d}-\frac{5\,Aa^3\cos\left(3\,dx+3\,c\right)}{192\,d}-\frac{13\,Aa^3\cos\left(dx+c\right)}{64\,d}+\frac{Aa^3\sin\left(6\,dx+6\,c\right)}{96\,d}-\frac{Aa^3\sin\left(4\,dx+4\,c\right)}{32\,d}-\frac{Aa^3\sin\left(2\,dx+2\,c\right)}{32\,d}$$

input
```
integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="g
iac")
```

output
```
1/8*A*a^3*x - 1/448*A*a^3*cos(7*d*x + 7*c)/d + 7/320*A*a^3*cos(5*d*x + 5*c
)/d - 5/192*A*a^3*cos(3*d*x + 3*c)/d - 13/64*A*a^3*cos(d*x + c)/d + 1/96*A
*a^3*sin(6*d*x + 6*c)/d - 1/32*A*a^3*sin(4*d*x + 4*c)/d - 1/32*A*a^3*sin(2
*d*x + 2*c)/d
```

Mupad [B] (verification not implemented)

Time = 37.81 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.14

$$\int \sin^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \frac{A\,a^3\,\left(105\,c-210\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2464\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1400\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-4032\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+6790\,\mathrm{t}\right.}{}$$

input
```
int(sin(c + d*x)^3*(A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3,x)
```

output
```
(A*a^3*(105*c - 210*tan(c/2 + (d*x)/2) - 2464*tan(c/2 + (d*x)/2)^2 - 1400*
tan(c/2 + (d*x)/2)^3 - 4032*tan(c/2 + (d*x)/2)^4 + 6790*tan(c/2 + (d*x)/2)
^5 + 2240*tan(c/2 + (d*x)/2)^6 - 14560*tan(c/2 + (d*x)/2)^8 - 6790*tan(c/2
 + (d*x)/2)^9 - 3360*tan(c/2 + (d*x)/2)^10 + 1400*tan(c/2 + (d*x)/2)^11 +
210*tan(c/2 + (d*x)/2)^13 + 105*d*x + 735*tan(c/2 + (d*x)/2)^2*(c + d*x) +
 2205*tan(c/2 + (d*x)/2)^4*(c + d*x) + 3675*tan(c/2 + (d*x)/2)^6*(c + d*x)
 + 3675*tan(c/2 + (d*x)/2)^8*(c + d*x) + 2205*tan(c/2 + (d*x)/2)^10*(c + d
*x) + 735*tan(c/2 + (d*x)/2)^12*(c + d*x) + 105*tan(c/2 + (d*x)/2)^14*(c +
 d*x) - 352))/(840*d*(tan(c/2 + (d*x)/2)^2 + 1)^7)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.83

$$\int \sin^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \frac{a^4\left(120\cos\left(dx+c\right)\sin\left(dx+c\right)^6+280\cos\left(dx+c\right)\sin\left(dx+c\right)^5+144\cos\left(dx+c\right)\sin\left(dx+c\right)^4-70\,\mathrm{co}\right.}{}$$

input
```
int(sin(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)
```

output
```
(a**4*(120*cos(c + d*x)*sin(c + d*x)**6 + 280*cos(c + d*x)*sin(c + d*x)**5
 + 144*cos(c + d*x)*sin(c + d*x)**4 - 70*cos(c + d*x)*sin(c + d*x)**3 - 88
*cos(c + d*x)*sin(c + d*x)**2 - 105*cos(c + d*x)*sin(c + d*x) - 176*cos(c
+ d*x) + 105*d*x + 176))/(840*d)
```

3.225 $\int \sin^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal result

Integrand size = 32, antiderivative size = 121

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \frac{3}{16}a^3Ax-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{2a^3A\cos^5(c+dx)}{5d}-\frac{3a^3A\cos(c+dx)\sin(c+dx)}{16d}+\frac{5a^3A\cos(c+dx)\sin^3(c+dx)}{24d}+\frac{a^3A\cos(c+dx)\sin^5(c+dx)}{6d}$$

output
```
3/16*a^3*A*x-2/3*a^3*A*cos(d*x+c)^3/d+2/5*a^3*A*cos(d*x+c)^5/d-3/16*a^3*A*
cos(d*x+c)*sin(d*x+c)/d+5/24*a^3*A*cos(d*x+c)*sin(d*x+c)^3/d+1/6*a^3*A*cos
(d*x+c)*sin(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.64

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$=\frac{a^3A(180c+180dx-240\cos(c+dx)-40\cos(3(c+dx))+24\cos(5(c+dx))-15\sin(2(c+dx))-45\,\mathrm{s}}{960d}$$

input
```
Integrate[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
(a^3*A*(180*c + 180*d*x - 240*Cos[c + d*x] - 40*Cos[3*(c + d*x)] + 24*Cos[
5*(c + d*x)] - 15*Sin[2*(c + d*x)] - 45*Sin[4*(c + d*x)] + 5*Sin[6*(c + d*
x)]))/(960*d)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {3042, 3445, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(c+dx)(a\sin(c+dx)+a)^3(A-A\sin(c+dx))\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \sin(c+dx)^2(a\sin(c+dx)+a)^3(A-A\sin(c+dx))dx$$
$$\Big\downarrow \text{ 3445}$$
$$\int \left(-a^3A\sin^6(c+dx)-2a^3A\sin^5(c+dx)+2a^3A\sin^3(c+dx)+a^3A\sin^2(c+dx)\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{2a^3A\cos^5(c+dx)}{5d}-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{a^3A\sin^5(c+dx)\cos(c+dx)}{6d}+\frac{5a^3A\sin^3(c+dx)\cos(c+dx)}{24d}-\frac{3a^3A\sin(c+dx)\cos(c+dx)}{16d}+\frac{3}{16}a^3Ax$$

input
```
Int[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
(3*a^3*A*x)/16 - (2*a^3*A*Cos[c + d*x]^3)/(3*d) + (2*a^3*A*Cos[c + d*x]^5)
/(5*d) - (3*a^3*A*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (5*a^3*A*Cos[c + d*x
]*Sin[c + d*x]^3)/(24*d) + (a^3*A*Cos[c + d*x]*Sin[c + d*x]^5)/(6*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3445
```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Maple [A] (verified)

Time = 122.64 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.65

method	result
parallelrisch	$-\frac{a^3 A(-180dx+240\cos(dx+c)-5\sin(6dx+6c)-24\cos(5dx+5c)+45\sin(4dx+4c)+40\cos(3dx+3c)+15\sin(2dx+2c)}{960d}$
risch	$\frac{3a^3Ax}{16}-\frac{a^3A\cos(dx+c)}{4d}+\frac{a^3A\sin(6dx+6c)}{192d}+\frac{a^3A\cos(5dx+5c)}{40d}-\frac{3a^3A\sin(4dx+4c)}{64d}-\frac{a^3A\cos(3dx+3c)}{24d}$
derivativedivides	$\frac{-a^3A\left(-\frac{\left(\sin(dx+c)^5+\frac{5\sin(dx+c)^3}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{2a^3A\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+}{5}}{d}$
default	$\frac{-a^3A\left(-\frac{\left(\sin(dx+c)^5+\frac{5\sin(dx+c)^3}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{2a^3A\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+}{5}}{d}$
parts	$\frac{a^3A\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2a^3A\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3d}+\frac{2a^3A\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)}{5d}$
norman	$\frac{-\frac{8a^3A}{15d}+\frac{3a^3Ax}{16}-\frac{16a^3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{3d}-\frac{16a^3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{5d}-\frac{8a^3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{d}-\frac{3a^3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}+\frac{13a^3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{24d}}{}$
orering	Expression too large to display

input
```
int(sin(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
-1/960*a^3*A*(-180*d*x+240*cos(d*x+c)-5*sin(6*d*x+6*c)-24*cos(5*d*x+5*c)+4
5*sin(4*d*x+4*c)+40*cos(3*d*x+3*c)+15*sin(2*d*x+2*c)+256)/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.75

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$=\frac{96\,Aa^3\cos\left(dx+c\right)^5-160\,Aa^3\cos\left(dx+c\right)^3+45\,Aa^3dx+5\left(8\,Aa^3\cos\left(dx+c\right)^5-26\,Aa^3\cos\left(dx+c\right)^3\right.}{240\,d}$$

input
```
integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="f
ricas")
```

output
```
1/240*(96*A*a^3*cos(d*x + c)^5 - 160*A*a^3*cos(d*x + c)^3 + 45*A*a^3*d*x +
 5*(8*A*a^3*cos(d*x + c)^5 - 26*A*a^3*cos(d*x + c)^3 + 9*A*a^3*cos(d*x + c
))*sin(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(119) = 238$.

Time = 0.39 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.97

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \begin{cases} -\frac{5Aa^3x\sin^6(c+dx)}{16} - \frac{15Aa^3x\sin^4(c+dx)\cos^2(c+dx)}{16} - \frac{15Aa^3x\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{Aa^3x\sin^2(c+dx)}{2} - \frac{5Aa^3x\cos^6(c+dx)}{16} \\ x(-A\sin(c)+A)\left(a\sin(c)+a\right)^3\sin^2(c) \end{cases}$$

input
```
integrate(sin(d*x+c)**2*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

output
```
Piecewise((-5*A*a**3*x*sin(c + d*x)**6/16 - 15*A*a**3*x*sin(c + d*x)**4*co
s(c + d*x)**2/16 - 15*A*a**3*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + A*a**3
*x*sin(c + d*x)**2/2 - 5*A*a**3*x*cos(c + d*x)**6/16 + A*a**3*x*cos(c + d*
x)**2/2 + 11*A*a**3*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 2*A*a**3*sin(c +
 d*x)**4*cos(c + d*x)/d + 5*A*a**3*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) +
 8*A*a**3*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 2*A*a**3*sin(c + d*x)**2
*cos(c + d*x)/d + 5*A*a**3*sin(c + d*x)*cos(c + d*x)**5/(16*d) - A*a**3*si
n(c + d*x)*cos(c + d*x)/(2*d) + 16*A*a**3*cos(c + d*x)**5/(15*d) - 4*A*a**
3*cos(c + d*x)**3/(3*d), Ne(d, 0)), (x*(-A*sin(c) + A)*(a*sin(c) + a)**3*s
in(c)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.14

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \frac{128\left(3\,\cos\left(dx+c\right)^5-10\,\cos\left(dx+c\right)^3+15\,\cos\left(dx+c\right)\right)Aa^3+640\left(\cos\left(dx+c\right)^3-3\,\cos\left(dx+c\right)\right)Aa}{}$$

input
```
integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="m
axima")
```

output
```
1/960*(128*(3*cos(d*x + c)^5 - 10*cos(d*x + c)^3 + 15*cos(d*x + c))*A*a^3
+ 640*(cos(d*x + c)^3 - 3*cos(d*x + c))*A*a^3 - 5*(4*sin(2*d*x + 2*c)^3 +
60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*A*a^3 + 240*(2*d
*x + 2*c - sin(2*d*x + 2*c))*A*a^3)/d
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.93

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \frac{3}{16}\,Aa^3x+\frac{Aa^3\cos\left(5\,dx+5\,c\right)}{40\,d}-\frac{Aa^3\cos\left(3\,dx+3\,c\right)}{24\,d}-\frac{Aa^3\cos\left(dx+c\right)}{4\,d}$$
$$+\frac{Aa^3\sin\left(6\,dx+6\,c\right)}{192\,d}-\frac{3\,Aa^3\sin\left(4\,dx+4\,c\right)}{64\,d}-\frac{Aa^3\sin\left(2\,dx+2\,c\right)}{64\,d}$$

input
```
integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="g
iac")
```

output
```
3/16*A*a^3*x + 1/40*A*a^3*cos(5*d*x + 5*c)/d - 1/24*A*a^3*cos(3*d*x + 3*c)
/d - 1/4*A*a^3*cos(d*x + c)/d + 1/192*A*a^3*sin(6*d*x + 6*c)/d - 3/64*A*a^
3*sin(4*d*x + 4*c)/d - 1/64*A*a^3*sin(2*d*x + 2*c)/d
```

Mupad [B] (verification not implemented)

Time = 37.69 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.12

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$=\frac{A\,a^3\left(45\,c-90\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-768\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+130\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+1500\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-1280\tan\left(\frac{c}{2}\right.\right.}{}$$

input
```
int(sin(c + d*x)^2*(A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3,x)
```

output
```
(A*a^3*(45*c - 90*tan(c/2 + (d*x)/2) - 768*tan(c/2 + (d*x)/2)^2 + 130*tan(
c/2 + (d*x)/2)^3 + 1500*tan(c/2 + (d*x)/2)^5 - 1280*tan(c/2 + (d*x)/2)^6 -
 1500*tan(c/2 + (d*x)/2)^7 - 1920*tan(c/2 + (d*x)/2)^8 - 130*tan(c/2 + (d*
x)/2)^9 + 90*tan(c/2 + (d*x)/2)^11 + 45*d*x + 270*tan(c/2 + (d*x)/2)^2*(c
+ d*x) + 675*tan(c/2 + (d*x)/2)^4*(c + d*x) + 900*tan(c/2 + (d*x)/2)^6*(c
+ d*x) + 675*tan(c/2 + (d*x)/2)^8*(c + d*x) + 270*tan(c/2 + (d*x)/2)^10*(c
 + d*x) + 45*tan(c/2 + (d*x)/2)^12*(c + d*x) - 128))/(240*d*(tan(c/2 + (d*
x)/2)^2 + 1)^6)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.83

$$\int \sin^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$=\frac{a^4\left(40\cos\left(dx+c\right)\sin\left(dx+c\right)^5+96\cos\left(dx+c\right)\sin\left(dx+c\right)^4+50\cos\left(dx+c\right)\sin\left(dx+c\right)^3-32\cos\left(d\right.\right.}{240d}$$

input
```
int(sin(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)
```

output
```
(a**4*(40*cos(c + d*x)*sin(c + d*x)**5 + 96*cos(c + d*x)*sin(c + d*x)**4 +
 50*cos(c + d*x)*sin(c + d*x)**3 - 32*cos(c + d*x)*sin(c + d*x)**2 - 45*co
s(c + d*x)*sin(c + d*x) - 64*cos(c + d*x) + 45*d*x + 64))/(240*d)
```

3.226 $\int \sin(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal result

Integrand size = 30, antiderivative size = 96

$$\int \sin(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \frac{1}{4}a^3Ax - \frac{2a^3A\cos^3(c+dx)}{3d} + \frac{a^3A\cos^5(c+dx)}{5d} - \frac{a^3A\cos(c+dx)\sin(c+dx)}{4d} + \frac{a^3A\cos(c+dx)\sin^3(c+dx)}{2d}$$

output
```
1/4*a^3*A*x-2/3*a^3*A*cos(d*x+c)^3/d+1/5*a^3*A*cos(d*x+c)^5/d-1/4*a^3*A*co
s(d*x+c)*sin(d*x+c)/d+1/2*a^3*A*cos(d*x+c)*sin(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.10

$$\int \sin(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \frac{a^3A\cos(c+dx)\left(-30\arcsin\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(c+dx)}(-28-15\sin(c+dx)+16\sin^2(c+dx)+\right.}{60d\sqrt{\cos^2(c+dx)}}$$

input
```
Integrate[Sin[c + d*x]*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
(a^3*A*Cos[c + d*x]*(-30*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]] + Sqrt[Cos
[c + d*x]^2]*(-28 - 15*Sin[c + d*x] + 16*Sin[c + d*x]^2 + 30*Sin[c + d*x]^
3 + 12*Sin[c + d*x]^4)))/(60*d*Sqrt[Cos[c + d*x]^2])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3042, 3445, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(c+dx)(a\sin(c+dx)+a)^3(A-A\sin(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(c+dx)(a\sin(c+dx)+a)^3(A-A\sin(c+dx))dx$$

$$\Big\downarrow \text{ 3445}$$

$$\int \left(-a^3A\sin^5(c+dx)-2a^3A\sin^4(c+dx)+2a^3A\sin^2(c+dx)+a^3A\sin(c+dx)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^3A\cos^5(c+dx)}{5d}-\frac{2a^3A\cos^3(c+dx)}{3d}+\frac{a^3A\sin^3(c+dx)\cos(c+dx)}{2d}-\frac{a^3A\sin(c+dx)\cos(c+dx)}{4d}+\frac{1}{4}a^3Ax$$

input
```
Int[Sin[c + d*x]*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
(a^3*A*x)/4 - (2*a^3*A*Cos[c + d*x]^3)/(3*d) + (a^3*A*Cos[c + d*x]^5)/(5*d
) - (a^3*A*Cos[c + d*x]*Sin[c + d*x])/(4*d) + (a^3*A*Cos[c + d*x]*Sin[c +
d*x]^3)/(2*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3445
```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Maple [A] (verified)

Time = 28.50 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.59

method	result
parallelrisch	$-\frac{a^3A(-60dx+90\cos(dx+c)-3\cos(5dx+5c)+15\sin(4dx+4c)+25\cos(3dx+3c)+112)}{240d}$
risch	$\frac{a^3Ax}{4}-\frac{3a^3A\cos(dx+c)}{8d}+\frac{a^3A\cos(5dx+5c)}{80d}-\frac{a^3A\sin(4dx+4c)}{16d}-\frac{5a^3A\cos(3dx+3c)}{48d}$
derivativedivides	$\frac{\frac{a^3A\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)}{5}-2a^3A\left(-\frac{\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+2a^3A\left(-\frac{\sin(d}{}\right.}{d}$
default	$\frac{\frac{a^3A\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)}{5}-2a^3A\left(-\frac{\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+2a^3A\left(-\frac{\sin(d}{}\right.}{d}$
parts	$-\frac{a^3A\cos(dx+c)}{d}+\frac{2a^3A\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2a^3A\left(-\frac{\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3}{}\right.}{d}$
norman	$\frac{-\frac{14a^3A}{15d}+\frac{a^3Ax}{4}-\frac{4a^3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{3d}-\frac{2a^3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{d}-\frac{8a^3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{d}-\frac{8a^3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{3d}-\frac{a^3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}+}{}$

input
```
int(sin(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x,method=_RETURNVERBOSE
)
```

output
```
-1/240*a^3*A*(-60*d*x+90*cos(d*x+c)-3*cos(5*d*x+5*c)+15*sin(4*d*x+4*c)+25*
cos(3*d*x+3*c)+112)/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.80

$$\int \sin(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$=\frac{12\,Aa^3\cos\left(dx+c\right)^5-40\,Aa^3\cos\left(dx+c\right)^3+15\,Aa^3dx-15\,\left(2\,Aa^3\cos\left(dx+c\right)^3-Aa^3\cos\left(dx+c\right)\right)\sin\ldots}{60\,d}$$

input
```
integrate(sin(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fri
cas")
```

output
```
1/60*(12*A*a^3*cos(d*x + c)^5 - 40*A*a^3*cos(d*x + c)^3 + 15*A*a^3*d*x - 1
5*(2*A*a^3*cos(d*x + c)^3 - A*a^3*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(88) = 176.

Time = 0.27 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.78

$$\int \sin(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$=\begin{cases}-\frac{3Aa^3x\sin^4(c+dx)}{4}-\frac{3Aa^3x\sin^2(c+dx)\cos^2(c+dx)}{2}+Aa^3x\sin^2(c+dx)-\frac{3Aa^3x\cos^4(c+dx)}{4}+Aa^3x\cos^2(c+dx)\ldots\\ x(-A\sin(c)+A)\left(a\sin(c)+a\right)^3\sin(c)\end{cases}$$

input
```
integrate(sin(d*x+c)*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

output
```
Piecewise((-3*A*a**3*x*sin(c + d*x)**4/4 - 3*A*a**3*x*sin(c + d*x)**2*cos(
c + d*x)**2/2 + A*a**3*x*sin(c + d*x)**2 - 3*A*a**3*x*cos(c + d*x)**4/4 +
A*a**3*x*cos(c + d*x)**2 + A*a**3*sin(c + d*x)**4*cos(c + d*x)/d + 5*A*a**
3*sin(c + d*x)**3*cos(c + d*x)/(4*d) + 4*A*a**3*sin(c + d*x)**2*cos(c + d*
x)**3/(3*d) + 3*A*a**3*sin(c + d*x)*cos(c + d*x)**3/(4*d) - A*a**3*sin(c +
 d*x)*cos(c + d*x)/d + 8*A*a**3*cos(c + d*x)**5/(15*d) - A*a**3*cos(c + d*
x)/d, Ne(d, 0)), (x*(-A*sin(c) + A)*(a*sin(c) + a)**3*sin(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.17

$$\int \sin(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \frac{16\left(3\cos\left(dx+c\right)^5-10\cos\left(dx+c\right)^3+15\cos\left(dx+c\right)\right)Aa^3-15\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)-8\,\mathrm{si}\right.}{240\,d}$$

input
```
integrate(sin(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="max
ima")
```

output
```
1/240*(16*(3*cos(d*x + c)^5 - 10*cos(d*x + c)^3 + 15*cos(d*x + c))*A*a^3 -
 15*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*A*a^3 + 120*(2
*d*x + 2*c - sin(2*d*x + 2*c))*A*a^3 - 240*A*a^3*cos(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.80

$$\int \sin(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \frac{1}{4}Aa^3x + \frac{Aa^3\cos\left(5\,dx+5\,c\right)}{80\,d} - \frac{5\,Aa^3\cos\left(3\,dx+3\,c\right)}{48\,d} - \frac{3\,Aa^3\cos\left(dx+c\right)}{8\,d} - \frac{Aa^3\sin\left(4\,dx+4\,c\right)}{16\,d}$$

input
```
integrate(sin(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="gia
c")
```

output
```
1/4*A*a^3*x + 1/80*A*a^3*cos(5*d*x + 5*c)/d - 5/48*A*a^3*cos(3*d*x + 3*c)/
d - 3/8*A*a^3*cos(d*x + c)/d - 1/16*A*a^3*sin(4*d*x + 4*c)/d
```

Mupad [B] (verification not implemented)

Time = 37.47 (sec) , antiderivative size = 292, normalized size of antiderivative = 3.04

$$\int \sin(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \frac{A\,a^3\,x}{4}$$
$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\left(\frac{A\,a^3\,(15\,c+15\,d\,x)}{12}-\frac{A\,a^3\,(75\,c+75\,d\,x-120)}{60}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(\frac{A\,a^3\,(15\,c+15\,d\,x)}{12}-\frac{A\,a^3\,(75\,c+75\,d\,x-16}{60}\right.}{}$$

input
```
int(sin(c + d*x)*(A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3,x)
```

output
```
(A*a^3*x)/4 - (tan(c/2 + (d*x)/2)^8*((A*a^3*(15*c + 15*d*x))/12 - (A*a^3*(
75*c + 75*d*x - 120))/60) + tan(c/2 + (d*x)/2)^2*((A*a^3*(15*c + 15*d*x))/
12 - (A*a^3*(75*c + 75*d*x - 160))/60) + tan(c/2 + (d*x)/2)^4*((A*a^3*(15*
c + 15*d*x))/6 - (A*a^3*(150*c + 150*d*x - 80))/60) + tan(c/2 + (d*x)/2)^6
*((A*a^3*(15*c + 15*d*x))/6 - (A*a^3*(150*c + 150*d*x - 480))/60) + (A*a^3
*tan(c/2 + (d*x)/2))/2 - 3*A*a^3*tan(c/2 + (d*x)/2)^3 + 3*A*a^3*tan(c/2 +
(d*x)/2)^7 - (A*a^3*tan(c/2 + (d*x)/2)^9)/2 + (A*a^3*(15*c + 15*d*x))/60 -
 (A*a^3*(15*c + 15*d*x - 56))/60)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

$$\int \sin(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$=\frac{a^4\left(12\cos(dx+c)\sin(dx+c)^4+30\cos(dx+c)\sin(dx+c)^3+16\cos(dx+c)\sin(dx+c)^2-15\cos(d\right.}{60d}$$

input
```
int(sin(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)
```

output
```
(a**4*(12*cos(c + d*x)*sin(c + d*x)**4 + 30*cos(c + d*x)*sin(c + d*x)**3 +
 16*cos(c + d*x)*sin(c + d*x)**2 - 15*cos(c + d*x)*sin(c + d*x) - 28*cos(c
 + d*x) + 15*d*x + 28))/(60*d)
```

3.227 $\int (a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal result

Integrand size = 24, antiderivative size = 82

$$\int (a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \frac{5}{8}a^3Ax - \frac{5a^3A\cos^3(c+dx)}{12d} + \frac{5a^3A\cos(c+dx)\sin(c+dx)}{8d} - \frac{A\cos^3(c+dx)\,(a^3+a^3\sin(c+dx))}{4d}$$

output
```
5/8*a^3*A*x-5/12*a^3*A*cos(d*x+c)^3/d+5/8*a^3*A*cos(d*x+c)*sin(d*x+c)/d-1/
4*A*cos(d*x+c)^3*(a^3+a^3*sin(d*x+c))/d
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.66

$$\int (a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \frac{a^3A(60dx-48\cos(c+dx)-16\cos(3(c+dx))+24\sin(2(c+dx))-3\sin(4(c+dx)))}{96d}$$

input
```
Integrate[(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
(a^3*A*(60*d*x - 48*Cos[c + d*x] - 16*Cos[3*(c + d*x)] + 24*Sin[2*(c + d*x
)] - 3*Sin[4*(c + d*x)]))/(96*d)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3215, 3042, 3157, 3042, 3148, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(c+dx)+a)^3(A-A\sin(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(c+dx)+a)^3(A-A\sin(c+dx))dx$$

$$\Big\downarrow \text{3215}$$

$$aA\int \cos^2(c+dx)(\sin(c+dx)a+a)^2dx$$

$$\Big\downarrow \text{3042}$$

$$aA\int \cos(c+dx)^2(\sin(c+dx)a+a)^2dx$$

$$\Big\downarrow \text{3157}$$

$$aA\left(\frac{5}{4}a\int \cos^2(c+dx)(\sin(c+dx)a+a)dx-\frac{\cos^3(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{4d}\right)$$

$$\Big\downarrow \text{3042}$$

$$aA\left(\frac{5}{4}a\int \cos(c+dx)^2(\sin(c+dx)a+a)dx-\frac{\cos^3(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{4d}\right)$$

$$\Big\downarrow \text{3148}$$

$$aA\left(\frac{5}{4}a\left(a\int \cos^2(c+dx)dx-\frac{a\cos^3(c+dx)}{3d}\right)-\frac{\cos^3(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{4d}\right)$$

$$\downarrow \text{3042}$$

$$aA\left(\frac{5}{4}a\left(a\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx-\frac{a\cos^3(c+dx)}{3d}\right)-\frac{\cos^3(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{4d}\right)$$

$$\downarrow \text{3115}$$

$$aA\left(\frac{5}{4}a\left(a\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{a\cos^3(c+dx)}{3d}\right)-\frac{\cos^3(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{4d}\right)$$

$$\downarrow \text{24}$$

$$aA\left(\frac{5}{4}a\left(a\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)-\frac{a\cos^3(c+dx)}{3d}\right)-\frac{\cos^3(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{4d}\right)$$

input
```
Int[(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
a*A*(-1/4*(Cos[c + d*x]^3*(a^2 + a^2*Sin[c + d*x]))/d + (5*a*(-1/3*(a*Cos[
c + d*x]^3)/d + a*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d))))/4)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3157
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[a*((2*m + p - 1)/(m + p))   Int[(g*
Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && Integers
Q[2*m, 2*p]
```

rule 3215
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c +
 d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((Lt
Q[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.09

$$\frac{-a^3A\left(-\frac{\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{2a^3A\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3}-2A\cos\left(dx+c\right)a^3+a^3A(}{d}$$

input
```
int((a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)
```

output
```
1/d*(-a^3*A*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)+
2/3*a^3*A*(2+sin(d*x+c)^2)*cos(d*x+c)-2*A*cos(d*x+c)*a^3+a^3*A*(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.77

$$\int (a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = -\frac{16\,Aa^3\cos(dx+c)^3-15\,Aa^3dx+3\left(2\,Aa^3\cos(dx+c)^3-5\,Aa^3\cos(dx+c)\right)\sin(dx+c)}{24\,d}$$

input
```
integrate((a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/24*(16*A*a^3*cos(d*x + c)^3 - 15*A*a^3*d*x + 3*(2*A*a^3*cos(d*x + c)^3
- 5*A*a^3*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(78) = 156$.

Time = 0.19 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.39

$$\int (a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \begin{cases} -\frac{3Aa^3x\sin^4(c+dx)}{8}-\frac{3Aa^3x\sin^2(c+dx)\cos^2(c+dx)}{4}-\frac{3Aa^3x\cos^4(c+dx)}{8}+Aa^3x+\frac{5Aa^3\sin^3(c+dx)\cos(c+dx)}{8d}+\frac{2Aa^3\sin^2}{} \\ x(-A\sin(c)+A)\,(a\sin(c)+a)^3 \end{cases}$$

input
```
integrate((a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

output
```
Piecewise((-3*A*a**3*x*sin(c + d*x)**4/8 - 3*A*a**3*x*sin(c + d*x)**2*cos(
c + d*x)**2/4 - 3*A*a**3*x*cos(c + d*x)**4/8 + A*a**3*x + 5*A*a**3*sin(c +
 d*x)**3*cos(c + d*x)/(8*d) + 2*A*a**3*sin(c + d*x)**2*cos(c + d*x)/d + 3*
A*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 4*A*a**3*cos(c + d*x)**3/(3*d)
 - 2*A*a**3*cos(c + d*x)/d, Ne(d, 0)), (x*(-A*sin(c) + A)*(a*sin(c) + a)**
3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05

$$\int (a + a\sin(c + dx))^3 (A - A\sin(c + dx))\, dx =$$
$$-\frac{64\left(\cos\left(dx+c\right)^3 - 3\cos\left(dx+c\right)\right)Aa^3 + 3\left(12\,dx + 12\,c + \sin\left(4\,dx+4\,c\right) - 8\sin\left(2\,dx+2\,c\right)\right)Aa^3 -}{96\,d}$$

input
```
integrate((a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/96*(64*(cos(d*x + c)^3 - 3*cos(d*x + c))*A*a^3 + 3*(12*d*x + 12*c + sin
(4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*A*a^3 - 96*(d*x + c)*A*a^3 + 192*A*a^3
*cos(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.94

$$\int (a + a\sin(c + dx))^3 (A - A\sin(c + dx))\, dx$$
$$= \frac{5}{8}Aa^3x - \frac{Aa^3\cos\left(3\,dx + 3\,c\right)}{6\,d} - \frac{Aa^3\cos\left(dx + c\right)}{2\,d}$$
$$- \frac{Aa^3\sin\left(4\,dx + 4\,c\right)}{32\,d} + \frac{Aa^3\sin\left(2\,dx + 2\,c\right)}{4\,d}$$

input
```
integrate((a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="giac")
```

output
```
5/8*A*a^3*x - 1/6*A*a^3*cos(3*d*x + 3*c)/d - 1/2*A*a^3*cos(d*x + c)/d - 1/
32*A*a^3*sin(4*d*x + 4*c)/d + 1/4*A*a^3*sin(2*d*x + 2*c)/d
```

Mupad [B] (verification not implemented)

Time = 38.17 (sec) , antiderivative size = 250, normalized size of antiderivative = 3.05

$$\int (a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx=\frac{5\,A\,a^3\,x}{8}$$
$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(\frac{A\,a^3\,(15\,c+15\,d\,x)}{6}-\frac{A\,a^3\,(60\,c+60\,d\,x-32)}{24}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(\frac{A\,a^3\,(15\,c+15\,d\,x)}{6}-\frac{A\,a^3\,(60\,c+60\,d\,x-96)}{24}\right.}{}$$

input
```
int((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3,x)
```

output
```
(5*A*a^3*x)/8 - (tan(c/2 + (d*x)/2)^2*((A*a^3*(15*c + 15*d*x))/6 - (A*a^3*
(60*c + 60*d*x - 32))/24) + tan(c/2 + (d*x)/2)^6*((A*a^3*(15*c + 15*d*x))/
6 - (A*a^3*(60*c + 60*d*x - 96))/24) + tan(c/2 + (d*x)/2)^4*((A*a^3*(15*c
+ 15*d*x))/4 - (A*a^3*(90*c + 90*d*x - 96))/24) - (3*A*a^3*tan(c/2 + (d*x)
/2))/4 - (11*A*a^3*tan(c/2 + (d*x)/2)^3)/4 + (11*A*a^3*tan(c/2 + (d*x)/2)^
5)/4 + (3*A*a^3*tan(c/2 + (d*x)/2)^7)/4 + (A*a^3*(15*c + 15*d*x))/24 - (A*
a^3*(15*c + 15*d*x - 32))/24)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.83

$$\int (a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$=\frac{a^4\left(6\cos(dx+c)\sin(dx+c)^3+16\cos(dx+c)\sin(dx+c)^2+9\cos(dx+c)\sin(dx+c)-16\cos(dx+\right.}{24d}$$

input
```
int((a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)
```

output
```
(a**4*(6*cos(c + d*x)*sin(c + d*x)**3 + 16*cos(c + d*x)*sin(c + d*x)**2 +
9*cos(c + d*x)*sin(c + d*x) - 16*cos(c + d*x) + 15*d*x + 16))/(24*d)
```

3.228 $\int \csc(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal result

Integrand size = 30, antiderivative size = 76

$$\int \csc(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = a^3Ax-\frac{a^3A\text{arctanh}(\cos(c+dx))}{d}+\frac{a^3A\cos(c+dx)}{d} - \frac{a^3A\cos^3(c+dx)}{3d}+\frac{a^3A\cos(c+dx)\sin(c+dx)}{d}$$

output
```
a^3*A*x-a^3*A*arctanh(cos(d*x+c))/d+a^3*A*cos(d*x+c)/d-1/3*a^3*A*cos(d*x+c
)^3/d+a^3*A*cos(d*x+c)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.71

$$\int \csc(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = -\frac{a^3A(12c-12dx+12\text{arctanh}(\cos(c+dx))-9\cos(c+dx)+\cos(3(c+dx))-6\sin(2(c+dx)))}{12d}$$

input
```
Integrate[Csc[c + d*x]*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
-1/12*(a^3*A*(12*c - 12*d*x + 12*ArcTanh[Cos[c + d*x]] - 9*Cos[c + d*x] +
Cos[3*(c + d*x)] - 6*Sin[2*(c + d*x)]))/d
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3042, 3445, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(c+dx)(a\sin(c+dx)+a)^3(A-A\sin(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(c+dx)+a)^3(A-A\sin(c+dx))}{\sin(c+dx)}dx$$

$$\Big\downarrow \text{3445}$$

$$\int \left(a^3(-A)\sin^3(c+dx)-2a^3A\sin^2(c+dx)+a^3A\csc(c+dx)+2a^3A\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3A\text{arctanh}(\cos(c+dx))}{d}-\frac{a^3A\cos^3(c+dx)}{3d}+\frac{a^3A\cos(c+dx)}{d}+\frac{a^3A\sin(c+dx)\cos(c+dx)}{d}+a^3Ax$$

input
```
Int[Csc[c + d*x]*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
a^3*A*x - (a^3*A*ArcTanh[Cos[c + d*x]])/d + (a^3*A*Cos[c + d*x])/d - (a^3*
A*Cos[c + d*x]^3)/(3*d) + (a^3*A*Cos[c + d*x]*Sin[c + d*x])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3445
```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.76

method	result
parallelrisch	$\frac{a^{3} A \left(12 d x+8+12 \ln \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)\right)-\cos \left(3 d x+3 c\right)+9 \cos \left(d x+c\right)+6 \sin \left(2 d x+2 c\right)\right)}{12 d}$
derivativedivides	$\frac{\frac{a^{3} A \left(2+\sin \left(d x+c\right)^{2}\right) \cos \left(d x+c\right)}{3}-2 a^{3} A \left(-\frac{\sin \left(d x+c\right) \cos \left(d x+c\right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)+2 a^{3} A \left(d x+c\right)+a^{3} A \ln \left(\csc \left(d x+c\right)-\cot \left(d x+c\right)\right)}{d}$
default	$\frac{\frac{a^{3} A \left(2+\sin \left(d x+c\right)^{2}\right) \cos \left(d x+c\right)}{3}-2 a^{3} A \left(-\frac{\sin \left(d x+c\right) \cos \left(d x+c\right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)+2 a^{3} A \left(d x+c\right)+a^{3} A \ln \left(\csc \left(d x+c\right)-\cot \left(d x+c\right)\right)}{d}$
risch	$a^{3} A x+\frac{3 a^{3} A \,{\mathrm e}^{i \left(d x+c\right)}}{8 d}+\frac{3 a^{3} A \,{\mathrm e}^{-i \left(d x+c\right)}}{8 d}-\frac{a^{3} A \ln \left({\mathrm e}^{i \left(d x+c\right)}+1\right)}{d}+\frac{a^{3} A \ln \left({\mathrm e}^{i \left(d x+c\right)}-1\right)}{d}-\frac{a^{3} A \cos \left(3 d x+3 c\right)}{12 d}$
norman	$\frac{a^{3} A x+a^{3} A x \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{8}+\frac{4 a^{3} A}{3 d}+\frac{4 a^{3} A \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{4}}{d}+\frac{16 a^{3} A \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}{3 d}+\frac{2 a^{3} A \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{d}+\frac{2 a^{3} A \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{d}-}{\left(1+\tan \left(\frac{d x}{2}+\frac{c}{2}\right.\right.}$

input
```
int(csc(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x,method=_RETURNVERBOSE
)
```

output
```
1/12*a^3*A*(12*d*x+8+12*ln(tan(1/2*d*x+1/2*c))-cos(3*d*x+3*c)+9*cos(d*x+c)
+6*sin(2*d*x+2*c))/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.21

$$\int \csc(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = -\frac{2\,Aa^3\cos\left(dx+c\right)^3-6\,Aa^3dx-6\,Aa^3\cos\left(dx+c\right)\sin\left(dx+c\right)-6\,Aa^3\cos\left(dx+c\right)+3\,Aa^3\log\left(\frac{1}{2}\,\mathrm{co}\right.}{6\,d}$$

input
```
integrate(csc(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fri
cas")
```

output
```
-1/6*(2*A*a^3*cos(d*x + c)^3 - 6*A*a^3*d*x - 6*A*a^3*cos(d*x + c)*sin(d*x
+ c) - 6*A*a^3*cos(d*x + c) + 3*A*a^3*log(1/2*cos(d*x + c) + 1/2) - 3*A*a^
3*log(-1/2*cos(d*x + c) + 1/2))/d
```

Sympy [F]

$$\int \csc(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = -Aa^3\left(\int\left(-2\sin\left(c+dx\right)\csc\left(c+dx\right)\right)\,dx+\int 2\sin^3\left(c+dx\right)\csc\left(c+dx\right)dx + \int\sin^4\left(c+dx\right)\csc\left(c+dx\right)dx+\int\left(-\csc\left(c+dx\right)\right)\,dx\right)$$

input
```
integrate(csc(d*x+c)*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

output
```
-A*a**3*(Integral(-2*sin(c + d*x)*csc(c + d*x), x) + Integral(2*sin(c + d*
x)**3*csc(c + d*x), x) + Integral(sin(c + d*x)**4*csc(c + d*x), x) + Integ
ral(-csc(c + d*x), x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.12

$$\int \csc(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = -\frac{2\left(\cos\left(dx+c\right)^3-3\cos\left(dx+c\right)\right)Aa^3+3\left(2\,dx+2\,c-\sin\left(2\,dx+2\,c\right)\right)Aa^3-12\left(dx+c\right)Aa^3+6\,Aa^3}{6\,d}$$

input
```
integrate(csc(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="max
ima")
```

output
```
-1/6*(2*(cos(d*x + c)^3 - 3*cos(d*x + c))*A*a^3 + 3*(2*d*x + 2*c - sin(2*d
*x + 2*c))*A*a^3 - 12*(d*x + c)*A*a^3 + 6*A*a^3*log(cot(d*x + c) + csc(d*x
 + c)))/d
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.41

$$\int \csc(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \frac{3\left(dx+c\right)Aa^3+3\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\left(3\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{3\,d}$$

input
```
integrate(csc(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="gia
c")
```

output
```
1/3*(3*(d*x + c)*A*a^3 + 3*A*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(3*A*a
^3*tan(1/2*d*x + 1/2*c)^5 - 6*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 3*A*a^3*tan(1
/2*d*x + 1/2*c) - 2*A*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 37.59 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.79

$$\int \csc(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \frac{-2\,A\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+4\,A\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,A\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{4\,A\,a^3}{3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$
$$+\frac{2\,A\,a^3\,\mathrm{atan}\left(\frac{4\,A^2\,a^6}{4\,A^2\,a^6-4\,A^2\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{4\,A^2\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,A^2\,a^6-4\,A^2\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$
$$+\frac{A\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

input
```
int(((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3)/sin(c + d*x),x)
```

output
```
((4*A*a^3)/3 + 2*A*a^3*tan(c/2 + (d*x)/2) + 4*A*a^3*tan(c/2 + (d*x)/2)^2 -
 2*A*a^3*tan(c/2 + (d*x)/2)^5)/(d*(3*tan(c/2 + (d*x)/2)^2 + 3*tan(c/2 + (d
*x)/2)^4 + tan(c/2 + (d*x)/2)^6 + 1)) + (2*A*a^3*atan((4*A^2*a^6)/(4*A^2*a
^6 - 4*A^2*a^6*tan(c/2 + (d*x)/2)) + (4*A^2*a^6*tan(c/2 + (d*x)/2))/(4*A^2
*a^6 - 4*A^2*a^6*tan(c/2 + (d*x)/2))))/d + (A*a^3*log(tan(c/2 + (d*x)/2)))
/d
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

$$\int \csc(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \frac{a^4\left(\cos\left(dx+c\right)\sin\left(dx+c\right)^2+3\cos\left(dx+c\right)\sin\left(dx+c\right)+2\cos\left(dx+c\right)+3\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3c+\right.}{3d}$$

input
```
int(csc(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)
```

output
```
(a**4*(cos(c + d*x)*sin(c + d*x)**2 + 3*cos(c + d*x)*sin(c + d*x) + 2*cos(
c + d*x) + 3*log(tan((c + d*x)/2)) + 3*c + 3*d*x - 2))/(3*d)
```

3.229 $\int \csc^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal result

Integrand size = 32, antiderivative size = 79

$$\int \csc^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = -\frac{1}{2}a^3Ax - \frac{2a^3A\text{arctanh}(\cos(c+dx))}{d} + \frac{2a^3A\cos(c+dx)}{d} - \frac{a^3A\cot(c+dx)}{d} + \frac{a^3A\cos(c+dx)\sin(c+dx)}{2d}$$

output
```
-1/2*a^3*A*x-2*a^3*A*arctanh(cos(d*x+c))/d+2*a^3*A*cos(d*x+c)/d-a^3*A*cot(
d*x+c)/d+1/2*a^3*A*cos(d*x+c)*sin(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.78

$$\int \csc^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = -\frac{a^3A(2c+2dx+8\text{arctanh}(\cos(c+dx)) - 8\cos(c)\cos(dx) + 4\cot(c+dx) + 8\sin(c)\sin(dx) - \sin(2(c$$

$$4d$$

input
```
Integrate[Csc[c + d*x]^2*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
-1/4*(a^3*A*(2*c + 2*d*x + 8*ArcTanh[Cos[c + d*x]] - 8*Cos[c]*Cos[d*x] + 4
*Cot[c + d*x] + 8*Sin[c]*Sin[d*x] - Sin[2*(c + d*x)]))/d
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {3042, 3429, 3042, 3188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(c+dx)(a\sin(c+dx)+a)^3(A-A\sin(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(c+dx)+a)^3(A-A\sin(c+dx))}{\sin(c+dx)^2}dx$$

$$\Big\downarrow \text{3429}$$

$$aA\int \cot^2(c+dx)(\sin(c+dx)a+a)^2dx$$

$$\Big\downarrow \text{3042}$$

$$aA\int \frac{(\sin(c+dx)a+a)^2}{\tan(c+dx)^2}dx$$

$$\Big\downarrow \text{3188}$$

$$\frac{A\int \left(\csc^2(c+dx)a^4-\sin^2(c+dx)a^4+2\csc(c+dx)a^4-2\sin(c+dx)a^4\right)dx}{a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{A\left(-\frac{2a^4\text{arctanh}(\cos(c+dx))}{d}+\frac{2a^4\cos(c+dx)}{d}-\frac{a^4\cot(c+dx)}{d}+\frac{a^4\sin(c+dx)\cos(c+dx)}{2d}-\frac{a^4x}{2}\right)}{a}$$

input
```
Int[Csc[c + d*x]^2*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
(A*(-1/2*(a^4*x) - (2*a^4*ArcTanh[Cos[c + d*x]])/d + (2*a^4*Cos[c + d*x])/
d - (a^4*Cot[c + d*x])/d + (a^4*Cos[c + d*x]*Sin[c + d*x])/(2*d)))/a
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3188
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Simp[a^p   Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
 + f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b,
e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m
- p/2, 0])
```

rule 3429
```
Int[sin[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^n*c^n
  Int[Tan[e + f*x]^p*(a + b*Sin[e + f*x])^(m - n), x], x] /; FreeQ[{a, b, c
, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[p + 2*n,
 0] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{-a^3 A\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2A\cos(dx+c)a^3+2a^3A\ln(\csc(dx+c)-\cot(dx+c))-a^3A\cot(dx+c)}{d}$
default	$\frac{-a^3 A\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2A\cos(dx+c)a^3+2a^3A\ln(\csc(dx+c)-\cot(dx+c))-a^3A\cot(dx+c)}{d}$
parallelrisch	$-\frac{A a^3\left(-4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\cos(dx+c)-\frac{\cos(2dx+2c)}{2}+4\sin(dx+c)-\frac{5}{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)\csc\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}$
risch	$-\frac{a^3Ax}{2}-\frac{ia^3A\,e^{2i(dx+c)}}{8d}+\frac{a^3A\,e^{i(dx+c)}}{d}+\frac{a^3A\,e^{-i(dx+c)}}{d}+\frac{ia^3A\,e^{-2i(dx+c)}}{8d}-\frac{2ia^3A}{d\left(e^{2i(dx+c)}-1\right)}-\frac{2a^3A\ln}{}$
norman	$\frac{\frac{4a^3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{a^3A}{2d}+\frac{4a^3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{d}+\frac{12a^3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}+\frac{12a^3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}-\frac{a^3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2d}+\frac{a^3A\tan\left(\right.}{2d}}{\tan\left(\frac{dx}{2}\right.}$

input
```
int(csc(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
1/d*(-a^3*A*(-1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+2*A*cos(d*x+c)*a^3+
2*a^3*A*ln(csc(d*x+c)-cot(d*x+c))-a^3*A*cot(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.41

$$\int \csc^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx =$$
$$-\frac{Aa^3\cos\left(dx+c\right)^3+2\,Aa^3\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-2\,Aa^3\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(\right.}{2\,d\sin\left(dx+c\right)}$$

input
```
integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="f
ricas")
```

output
```
-1/2*(A*a^3*cos(d*x + c)^3 + 2*A*a^3*log(1/2*cos(d*x + c) + 1/2)*sin(d*x +
 c) - 2*A*a^3*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + A*a^3*cos(d*x +
c) + (A*a^3*d*x - 4*A*a^3*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

Sympy [F]

$$\int \csc^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = -Aa^3\left(\int \left(-2\sin\left(c+dx\right)\csc^2\left(c+dx\right)\right)\,dx + \int 2\sin^3\left(c+dx\right)\csc^2\left(c+dx\right)dx + \int \sin^4\left(c+dx\right)\csc^2\left(c+dx\right)dx + \int \left(-\csc^2\left(c+dx\right)\right)\,dx\right)$$

input
```
integrate(csc(d*x+c)**2*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

output
```
-A*a**3*(Integral(-2*sin(c + d*x)*csc(c + d*x)**2, x) + Integral(2*sin(c +
 d*x)**3*csc(c + d*x)**2, x) + Integral(sin(c + d*x)**4*csc(c + d*x)**2, x
) + Integral(-csc(c + d*x)**2, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.05

$$\int \csc^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = -\frac{(2\,dx+2\,c-\sin\left(2\,dx+2\,c\right))Aa^3+4\,Aa^3(\log\left(\cos\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)-1\right))-8\,Aa^3\cos}{4\,d}$$

input
```
integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="m
axima")
```

output
```
-1/4*((2*d*x + 2*c - sin(2*d*x + 2*c))*A*a^3 + 4*A*a^3*(log(cos(d*x + c) +
 1) - log(cos(d*x + c) - 1)) - 8*A*a^3*cos(d*x + c) + 4*A*a^3/tan(d*x + c)
)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(75) = 150.

Time = 0.20 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.94

$$\int \csc^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx =$$

$$-\frac{(dx+c)Aa^3 - 4\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right) - Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + \frac{4\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+Aa^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)} + \frac{2\left(Aa^3\tan\right.}{}}{2\,d}$$

input
```
integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="g
iac")
```

output
```
-1/2*((d*x + c)*A*a^3 - 4*A*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - A*a^3*tan
(1/2*d*x + 1/2*c) + (4*A*a^3*tan(1/2*d*x + 1/2*c) + A*a^3)/tan(1/2*d*x + 1
/2*c) + 2*(A*a^3*tan(1/2*d*x + 1/2*c)^3 - 4*A*a^3*tan(1/2*d*x + 1/2*c)^2 -
 A*a^3*tan(1/2*d*x + 1/2*c) - 4*A*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 40.43 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.86

$$\int \csc^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \frac{A\,a^3\,\mathrm{atan}\left(\frac{A^2\,a^6}{4\,A^2\,a^6+A^2\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)} - \frac{4\,A^2\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,A^2\,a^6+A^2\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$
$$- \frac{3\,A\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 - 8\,A\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 - 8\,A\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right) + A\,a^3}{d\,\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5 + 4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 + 2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$
$$+ \frac{2\,A\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} + \frac{A\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}$$

input
```
int(((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3)/sin(c + d*x)^2,x)
```

output
```
(A*a^3*atan((A^2*a^6)/(4*A^2*a^6 + A^2*a^6*tan(c/2 + (d*x)/2)) - (4*A^2*a^
6*tan(c/2 + (d*x)/2))/(4*A^2*a^6 + A^2*a^6*tan(c/2 + (d*x)/2))))/d - (A*a^
3 - 8*A*a^3*tan(c/2 + (d*x)/2) - 8*A*a^3*tan(c/2 + (d*x)/2)^3 + 3*A*a^3*ta
n(c/2 + (d*x)/2)^4)/(d*(2*tan(c/2 + (d*x)/2) + 4*tan(c/2 + (d*x)/2)^3 + 2*
tan(c/2 + (d*x)/2)^5)) + (2*A*a^3*log(tan(c/2 + (d*x)/2)))/d + (A*a^3*tan(
c/2 + (d*x)/2))/(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.14

$$\int \csc^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \frac{a^4\left(\cos(dx+c)\sin(dx+c)^2+4\cos(dx+c)\sin(dx+c)-2\cos(dx+c)+4\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sin(dx+ \cdots\right.}{2\sin(dx+c)\,d}$$

input
```
int(csc(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)
```

output
```
(a**4*(cos(c + d*x)*sin(c + d*x)**2 + 4*cos(c + d*x)*sin(c + d*x) - 2*cos(
c + d*x) + 4*log(tan((c + d*x)/2))*sin(c + d*x) - sin(c + d*x)*d*x - 4*sin
(c + d*x)))/(2*sin(c + d*x)*d)
```

3.230 $\int \csc^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal result

Integrand size = 32, antiderivative size = 78

$$\int \csc^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = -2a^3Ax-\frac{a^3A\text{arctanh}(\cos(c+dx))}{2d}+\frac{a^3A\cos(c+dx)}{d}-\frac{2a^3A\cot(c+dx)}{d}-\frac{a^3A\cot(c+dx)\csc(c+dx)}{2d}$$

output
```
-2*a^3*A*x-1/2*a^3*A*arctanh(cos(d*x+c))/d+a^3*A*cos(d*x+c)/d-2*a^3*A*cot(
d*x+c)/d-1/2*a^3*A*cot(d*x+c)*csc(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.82

$$\int \csc^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= -2a^3Ax + \frac{a^3A\cos(c)\cos(dx)}{d} - \frac{2a^3A\cot(c+dx)}{d} - \frac{a^3A\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a^3A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{a^3A\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{a^3A\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a^3A\sin(c)\sin(dx)}{d}$$

input
```
Integrate[Csc[c + d*x]^3*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
-2*a^3*A*x + (a^3*A*Cos[c]*Cos[d*x])/d - (2*a^3*A*Cot[c + d*x])/d - (a^3*A
*Csc[(c + d*x)/2]^2)/(8*d) - (a^3*A*Log[Cos[(c + d*x)/2]])/(2*d) + (a^3*A*
Log[Sin[(c + d*x)/2]])/(2*d) + (a^3*A*Sec[(c + d*x)/2]^2)/(8*d) - (a^3*A*S
in[c]*Sin[d*x])/d
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3042, 3445, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(c+dx)(a\sin(c+dx)+a)^3(A-A\sin(c+dx))\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a\sin(c+dx)+a)^3(A-A\sin(c+dx))}{\sin(c+dx)^3}dx$$
$$\Big\downarrow \text{3445}$$
$$\int \left(-a^3A\sin(c+dx)+a^3A\csc^3(c+dx)+2a^3A\csc^2(c+dx)-2a^3A\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3A\text{arctanh}(\cos(c+dx))}{2d}+\frac{a^3A\cos(c+dx)}{d}-\frac{2a^3A\cot(c+dx)}{d}-\frac{a^3A\cot(c+dx)\csc(c+dx)}{2d}-2a^3Ax$$

input
```
Int[Csc[c + d*x]^3*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
-2*a^3*A*x - (a^3*A*ArcTanh[Cos[c + d*x]])/(2*d) + (a^3*A*Cos[c + d*x])/d
- (2*a^3*A*Cot[c + d*x])/d - (a^3*A*Cot[c + d*x]*Csc[c + d*x])/(2*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3445
```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{A\cos(dx+c)a^3-2a^3A(dx+c)-2a^3A\cot(dx+c)+a^3A\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)}{d}$
default	$\frac{A\cos(dx+c)a^3-2a^3A(dx+c)-2a^3A\cot(dx+c)+a^3A\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)}{d}$
parallelrisch	$\frac{a^3A\left(-16dx+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\csc\left(\frac{dx}{2}+\frac{c}{2}\right)^2-8\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^2\cos(dx+c)+8\sec\left(\frac{dx}{2}+\frac{c}{2}\right)\csc\left(\frac{dx}{2}+\frac{c}{2}\right.}{8d}$
risch	$-2a^3Ax+\frac{a^3A\,\mathrm{e}^{i(dx+c)}}{2d}+\frac{a^3A\,\mathrm{e}^{-i(dx+c)}}{2d}+\frac{a^3A\left(\mathrm{e}^{3i(dx+c)}+\mathrm{e}^{i(dx+c)}-4i\mathrm{e}^{2i(dx+c)}+4i\right)}{d\left(\mathrm{e}^{2i(dx+c)}-1\right)^2}+\frac{a^3A\ln\left(\mathrm{e}^{i(dx+c)}-\right.}{2d}$
norman	$\frac{\frac{a^3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}+\frac{a^3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{d}-\frac{a^3A}{8d}+\frac{3a^3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{d}+\frac{5a^3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{8d}+\frac{27a^3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{8d}-\frac{a^3A\tan\left(\right.}{d}}{}$

input
```
int(csc(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
1/d*(A*cos(d*x+c)*a^3-2*a^3*A*(d*x+c)-2*a^3*A*cot(d*x+c)+a^3*A*(-1/2*csc(d
*x+c)*cot(d*x+c)+1/2*ln(csc(d*x+c)-cot(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(74) = 148$.

Time = 0.09 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.95

$$\int \csc^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx =$$
$$-\frac{8\,Aa^3dx\cos(dx+c)^2-4\,Aa^3\cos(dx+c)^3-8\,Aa^3dx-8\,Aa^3\cos(dx+c)\sin(dx+c)+2\,Aa^3\cos(c}{4\,(d}$$

input
```
integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="f
ricas")
```

output
```
-1/4*(8*A*a^3*d*x*cos(d*x + c)^2 - 4*A*a^3*cos(d*x + c)^3 - 8*A*a^3*d*x -
8*A*a^3*cos(d*x + c)*sin(d*x + c) + 2*A*a^3*cos(d*x + c) + (A*a^3*cos(d*x
+ c)^2 - A*a^3)*log(1/2*cos(d*x + c) + 1/2) - (A*a^3*cos(d*x + c)^2 - A*a^
3)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^2 - d)
```

Sympy [F]

$$\int \csc^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= -Aa^3\left(\int \left(-2\sin\left(c+dx\right)\csc^3\left(c+dx\right)\right)\,dx + \int 2\sin^3\left(c+dx\right)\csc^3\left(c+dx\right)dx + \int \sin^4\left(c+dx\right)\csc^3\left(c+dx\right)dx + \int \left(-\csc^3\left(c+dx\right)\right)\,dx\right)$$

input
```
integrate(csc(d*x+c)**3*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

output
```
-A*a**3*(Integral(-2*sin(c + d*x)*csc(c + d*x)**3, x) + Integral(2*sin(c +
 d*x)**3*csc(c + d*x)**3, x) + Integral(sin(c + d*x)**4*csc(c + d*x)**3, x
) + Integral(-csc(c + d*x)**3, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.15

$$\int \csc^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx =$$
$$-\frac{8\,(dx+c)Aa^3 - Aa^3\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1} - \log\left(\cos\left(dx+c\right)+1\right) + \log\left(\cos\left(dx+c\right)-1\right)\right) - 4\,Aa^3\cos\left(dx+\right.}{4\,d}$$

input
```
integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="m
axima")
```

output
```
-1/4*(8*(d*x + c)*A*a^3 - A*a^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - log
(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 4*A*a^3*cos(d*x + c) + 8*A*a
^3/tan(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.76

$$\int \csc^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$=\frac{Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-16\,(dx+c)Aa^3+4\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+8\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{}{\tan(\frac{1}{2}}}{8\,d}$$

input
```
integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="g
iac")
```

output
```
1/8*(A*a^3*tan(1/2*d*x + 1/2*c)^2 - 16*(d*x + c)*A*a^3 + 4*A*a^3*log(abs(t
an(1/2*d*x + 1/2*c))) + 8*A*a^3*tan(1/2*d*x + 1/2*c) + 16*A*a^3/(tan(1/2*d
*x + 1/2*c)^2 + 1) - (6*A*a^3*tan(1/2*d*x + 1/2*c)^2 + 8*A*a^3*tan(1/2*d*x
 + 1/2*c) + A*a^3)/tan(1/2*d*x + 1/2*c)^2)/d
```

Mupad [B] (verification not implemented)

Time = 32.27 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.82

$$\int \csc^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$=\frac{A\,a^3\left(\frac{\cos(c+d\,x)}{2}-4\,\mathrm{atan}\left(\frac{\sqrt{17}\,\left(4\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{17\,\cos\left(\frac{c}{2}-\mathrm{atan}(4)+\frac{d\,x}{2}\right)}\right)-\frac{\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2}+\cos\left(2\,c+2\,d\,x\right)+\frac{\cos(3\,c+3\,d\,x)}{2}\right.}{2\,d\,\left(\cos\left(\right.\right.}$$

input
```
int(((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3)/sin(c + d*x)^3,x)
```

output
```
(A*a^3*(cos(c + d*x)/2 - 4*atan((17^(1/2)*(4*cos(c/2 + (d*x)/2) - sin(c/2
+ (d*x)/2)))/(17*cos(c/2 - atan(4) + (d*x)/2))) - log(sin(c/2 + (d*x)/2)/c
os(c/2 + (d*x)/2))/2 + cos(2*c + 2*d*x) + cos(3*c + 3*d*x)/2 + 2*sin(2*c +
 2*d*x) + 4*atan((17^(1/2)*(4*cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2)))/(1
7*cos(c/2 - atan(4) + (d*x)/2)))*cos(2*c + 2*d*x) + (log(sin(c/2 + (d*x)/2
)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x))/2 - 1))/(2*d*(cos(c + d*x)^2 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.24

$$\int \csc^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \frac{a^4\left(8\cos\left(dx+c\right)\sin\left(dx+c\right)^2-16\cos\left(dx+c\right)\sin\left(dx+c\right)-4\cos\left(dx+c\right)+4\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sin\left(c\right.\right.}{8\sin\left(dx+c\right)^2d}$$

input
```
int(csc(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)
```

output
```
(a**4*(8*cos(c + d*x)*sin(c + d*x)**2 - 16*cos(c + d*x)*sin(c + d*x) - 4*c
os(c + d*x) + 4*log(tan((c + d*x)/2))*sin(c + d*x)**2 - 16*sin(c + d*x)**2
*d*x - 7*sin(c + d*x)**2))/(8*sin(c + d*x)**2*d)
```

3.231 $\int \csc^4(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal result

Integrand size = 32, antiderivative size = 78

$$\int \csc^4(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = -a^3Ax+\frac{a^3A\text{arctanh}(\cos(c+dx))}{d}-\frac{a^3A\cot(c+dx)}{d}-\frac{a^3A\cot^3(c+dx)}{3d}-\frac{a^3A\cot(c+dx)\csc(c+dx)}{d}$$

output
```
-a^3*A*x+a^3*A*arctanh(cos(d*x+c))/d-a^3*A*cot(d*x+c)/d-1/3*a^3*A*cot(d*x+
c)^3/d-a^3*A*cot(d*x+c)*csc(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.81

$$\int \csc^4(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = -\frac{a^3A\left(24c+24dx+8\cot\left(\frac{1}{2}(c+dx)\right)+6\csc^2\left(\frac{1}{2}(c+dx)\right)-24\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+24\log\left(\sin\left(\frac{1}{2}(c\right.\right.\right.}{}$$

input
```
Integrate[Csc[c + d*x]^4*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
-1/24*(a^3*A*(24*c + 24*d*x + 8*Cot[(c + d*x)/2] + 6*Csc[(c + d*x)/2]^2 -
24*Log[Cos[(c + d*x)/2]] + 24*Log[Sin[(c + d*x)/2]] - 6*Sec[(c + d*x)/2]^2
 - 8*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + (Csc[(c + d*x)/2]^4*Sin[c + d*x])
/2 - 8*Tan[(c + d*x)/2]))/d
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {3042, 3445, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4(c+dx)(a\sin(c+dx)+a)^3(A-A\sin(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(c+dx)+a)^3(A-A\sin(c+dx))}{\sin(c+dx)^4}dx$$

$$\Big\downarrow \text{3445}$$

$$\int \left(a^3A\csc^4(c+dx)+2a^3A\csc^3(c+dx)-2a^3A\csc(c+dx)-a^3A\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3A\text{arctanh}(\cos(c+dx))}{d}-\frac{a^3A\cot^3(c+dx)}{3d}-\frac{a^3A\cot(c+dx)}{d}-\frac{a^3A\cot(c+dx)\csc(c+dx)}{d}-a^3Ax$$

input
```
Int[Csc[c + d*x]^4*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
-(a^3*A*x) + (a^3*A*ArcTanh[Cos[c + d*x]])/d - (a^3*A*Cot[c + d*x])/d - (a
^3*A*Cot[c + d*x]^3)/(3*d) - (a^3*A*Cot[c + d*x]*Csc[c + d*x])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3445
```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.27

method	result
parallelrisch	$\frac{a^3A\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^3+6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-6\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^2-24dx+9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-24\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-9}{24d}$
derivativedivides	$\frac{-a^3A(dx+c)-2a^3A\ln(\csc(dx+c)-\cot(dx+c))+2a^3A\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+a^3A\left(-\frac{2}{3}-\right.}{d}$
default	$\frac{-a^3A(dx+c)-2a^3A\ln(\csc(dx+c)-\cot(dx+c))+2a^3A\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+a^3A\left(-\frac{2}{3}-\right.}{d}$
risch	$-a^3Ax+\frac{2a^3A(3\,e^{5i(dx+c)}+6ie^{2i(dx+c)}-2i-3\,e^{i(dx+c)})}{3d(e^{2i(dx+c)}-1)^3}+\frac{a^3A\ln(e^{i(dx+c)}+1)}{d}-\frac{a^3A\ln(e^{i(dx+c)}-1)}{d}$
norman	$\frac{\frac{2a^3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{d}-\frac{a^3A}{24d}+\frac{6a^3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{d}+\frac{11a^3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{4d}+\frac{21a^3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{4d}-\frac{a^3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{13a^3A\tan}{2}}{}$

input
```
int(csc(d*x+c)^4*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
1/24*a^3*A*(tan(1/2*d*x+1/2*c)^3-cot(1/2*d*x+1/2*c)^3+6*tan(1/2*d*x+1/2*c)
^2-6*cot(1/2*d*x+1/2*c)^2-24*d*x+9*tan(1/2*d*x+1/2*c)-24*ln(tan(1/2*d*x+1/
2*c))-9*cot(1/2*d*x+1/2*c))/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(76) = 152$.

Time = 0.10 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.24

$$\int \csc^4(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx =$$
$$-\frac{4\,Aa^3\cos(dx+c)^3-6\,Aa^3\cos(dx+c)-3\left(Aa^3\cos(dx+c)^2-Aa^3\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx}{}$$

input
```
integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="f
ricas")
```

output
```
-1/6*(4*A*a^3*cos(d*x + c)^3 - 6*A*a^3*cos(d*x + c) - 3*(A*a^3*cos(d*x + c
)^2 - A*a^3)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(A*a^3*cos(d*x +
 c)^2 - A*a^3)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 6*(A*a^3*d*x*co
s(d*x + c)^2 - A*a^3*d*x - A*a^3*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x +
 c)^2 - d)*sin(d*x + c))
```

Sympy [F]

$$\int \csc^4(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= -Aa^3\left(\int\left(-2\sin(c+dx)\csc^4(c+dx)\right)\,dx+\int 2\sin^3(c+dx)\csc^4(c+dx)\,dx+\int\sin^4(c+dx)\csc^4(c+dx)\,dx+\int\left(-\csc^4(c+dx)\right)\,dx\right)$$

input
```
integrate(csc(d*x+c)**4*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

output
```
-A*a**3*(Integral(-2*sin(c + d*x)*csc(c + d*x)**4, x) + Integral(2*sin(c +
 d*x)**3*csc(c + d*x)**4, x) + Integral(sin(c + d*x)**4*csc(c + d*x)**4, x
) + Integral(-csc(c + d*x)**4, x))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.50

$$\int \csc^4(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx =$$

$$-\frac{6\,(dx+c)Aa^3-3\,Aa^3\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)-6\,Aa^3(\log\left(\cos\right.}{6\,d}$$

input
```
integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="m
axima")
```

output
```
-1/6*(6*(d*x + c)*A*a^3 - 3*A*a^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - l
og(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 6*A*a^3*(log(cos(d*x + c)
+ 1) - log(cos(d*x + c) - 1)) + 2*(3*tan(d*x + c)^2 + 1)*A*a^3/tan(d*x + c
)^3)/d
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.92

$$\int \csc^4(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$

$$=\frac{Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-24\,(dx+c)Aa^3-24\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+9\,A}{24\,d}$$

input
```
integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="g
iac")
```

output
```
1/24*(A*a^3*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 24*(
d*x + c)*A*a^3 - 24*A*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 9*A*a^3*tan(1/2
*d*x + 1/2*c) + (44*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 9*A*a^3*tan(1/2*d*x + 1
/2*c)^2 - 6*A*a^3*tan(1/2*d*x + 1/2*c) - A*a^3)/tan(1/2*d*x + 1/2*c)^3)/d
```

Mupad [B] (verification not implemented)

Time = 37.84 (sec) , antiderivative size = 245, normalized size of antiderivative = 3.14

$$\int \csc^4(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = -\frac{\frac{A\,a^3\,\sin(2\,c+2\,d\,x)}{2}-\frac{A\,a^3\,\cos(3\,c+3\,d\,x)}{6}+\frac{A\,a^3\,\cos(c+d\,x)}{2}-\frac{A\,a^3\,\sin(3\,c+3\,d\,x)\,\mathrm{atan}\left(\frac{\sqrt{2}\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{d\,x}{2}\right)}\right)}{2}+\frac{3\,A\,a^3\,\mathrm{s}}{}}{\frac{3\,d\,\sin(c+d\,x)}{4}-\frac{d}{}}$$

input
```
int(((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3)/sin(c + d*x)^4,x)
```

output
```
-((A*a^3*sin(2*c + 2*d*x))/2 - (A*a^3*cos(3*c + 3*d*x))/6 + (A*a^3*cos(c +
 d*x))/2 - (A*a^3*sin(3*c + 3*d*x)*atan((2^(1/2)*(cos(c/2 + (d*x)/2) + sin
(c/2 + (d*x)/2)))/(2*cos(c/2 + pi/4 + (d*x)/2))))/2 + (3*A*a^3*sin(c + d*x
)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/4 + (3*A*a^3*sin(c + d*x)*at
an((2^(1/2)*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2)))/(2*cos(c/2 + pi/4 +
 (d*x)/2))))/2 - (A*a^3*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(3*c
 + 3*d*x))/4)/((3*d*sin(c + d*x))/4 - (d*sin(3*c + 3*d*x))/4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.12

$$\int \csc^4(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \frac{a^4\left(-2\cos(dx+c)\sin(dx+c)^2-3\cos(dx+c)\sin(dx+c)-\cos(dx+c)-3\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sin(dx\right.}{3\sin(dx+c)^3\,d}$$

input
```
int(csc(d*x+c)^4*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)
```

output
```
(a**4*( - 2*cos(c + d*x)*sin(c + d*x)**2 - 3*cos(c + d*x)*sin(c + d*x) - c
os(c + d*x) - 3*log(tan((c + d*x)/2))*sin(c + d*x)**3 - 3*sin(c + d*x)**3*
d*x))/(3*sin(c + d*x)**3*d)
```

3.232 $\int \csc^5(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal result

Integrand size = 32, antiderivative size = 86

$$\int \csc^5(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \frac{5a^3A\text{arctanh}(\cos(c+dx))}{8d}-\frac{2a^3A\cot^3(c+dx)}{3d}-\frac{3a^3A\cot(c+dx)\csc(c+dx)}{8d}-\frac{a^3A\cot(c+dx)\csc^3(c+dx)}{4d}$$

output
```
5/8*a^3*A*arctanh(cos(d*x+c))/d-2/3*a^3*A*cot(d*x+c)^3/d-3/8*a^3*A*cot(d*x
+c)*csc(d*x+c)/d-1/4*a^3*A*cot(d*x+c)*csc(d*x+c)^3/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 210 vs. 2(86) = 172.

Time = 0.40 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.44

$$\int \csc^5(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$=a^3A\left(\frac{\cot\left(\frac{1}{2}(c+dx)\right)}{3d}-\frac{3\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d}-\frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{12d}-\frac{\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{5\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8d}-\frac{5\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d}+\frac{3\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{3d}+\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{12d}\right)$$

input
```
Integrate[Csc[c + d*x]^5*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
a^3*A*(Cot[(c + d*x)/2]/(3*d) - (3*Csc[(c + d*x)/2]^2)/(32*d) - (Cot[(c +
d*x)/2]*Csc[(c + d*x)/2]^2)/(12*d) - Csc[(c + d*x)/2]^4/(64*d) + (5*Log[Co
s[(c + d*x)/2]])/(8*d) - (5*Log[Sin[(c + d*x)/2]])/(8*d) + (3*Sec[(c + d*x
)/2]^2)/(32*d) + Sec[(c + d*x)/2]^4/(64*d) - Tan[(c + d*x)/2]/(3*d) + (Sec
[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(12*d))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}=0.094$, Rules used = {3042, 3445, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^5(c+dx)(a\sin(c+dx)+a)^3(A-A\sin(c+dx))\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a\sin(c+dx)+a)^3(A-A\sin(c+dx))}{\sin(c+dx)^5}dx$$

$$\Big\downarrow \textbf{3445}$$

$$\int \left(a^3 A \csc^5(c+dx) + 2a^3 A \csc^4(c+dx) - 2a^3 A \csc^2(c+dx) - a^3 A \csc(c+dx)\right) dx$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{5a^3 A \operatorname{arctanh}(\cos(c+dx))}{8d} - \frac{2a^3 A \cot^3(c+dx)}{3d} - \frac{a^3 A \cot(c+dx)\csc^3(c+dx)}{4d} - \frac{3a^3 A \cot(c+dx)\csc(c+dx)}{8d}$$

input
```
Int[Csc[c + d*x]^5*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
(5*a^3*A*ArcTanh[Cos[c + d*x]])/(8*d) - (2*a^3*A*Cot[c + d*x]^3)/(3*d) - (
3*a^3*A*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a^3*A*Cot[c + d*x]*Csc[c + d*x
]^3)/(4*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3445
```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.33

method	result
derivativedivides	$\frac{-a^3 A \ln(\csc(dx+c)-\cot(dx+c))+2a^3 A \cot(dx+c)+2a^3 A\left(-\frac{2}{3}-\frac{\csc(dx+c)^2}{3}\right)\cot(dx+c)+a^3 A\left(\left(-\frac{\csc(dx+c)^3}{4}-\frac{3\csc(}{}\right.\right.}{d}$
default	$\frac{-a^3 A \ln(\csc(dx+c)-\cot(dx+c))+2a^3 A \cot(dx+c)+2a^3 A\left(-\frac{2}{3}-\frac{\csc(dx+c)^2}{3}\right)\cot(dx+c)+a^3 A\left(\left(-\frac{\csc(dx+c)^3}{4}-\frac{3\csc(}{}\right.\right.}{d}$
parallelrisch	$-\frac{A a^3\left(\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^4-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\frac{16\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-\frac{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+8\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^2-8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-16\cot\left(\right.\right.}{64d}$
risch	$\frac{a^3 A\left(9\,e^{7i(dx+c)}-33\,e^{5i(dx+c)}+48i e^{6i(dx+c)}-33\,e^{3i(dx+c)}-48i e^{4i(dx+c)}+9\,e^{i(dx+c)}+16i e^{2i(dx+c)}-16i\right)}{12d\left(e^{2i(dx+c)}-1\right)^4}+\frac{5a^3 A \ln}{}$
norman	$\frac{-\frac{a^3 A}{64d}-\frac{57a^3 A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{16d}-\frac{19a^3 A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{16d}-\frac{27a^3 A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{16d}-\frac{49a^3 A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{16d}-\frac{a^3 A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{12d}-\frac{3a^3 A}{}}{}$

input
```
int(csc(d*x+c)^5*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
1/d*(-a^3*A*ln(csc(d*x+c)-cot(d*x+c))+2*a^3*A*cot(d*x+c)+2*a^3*A*(-2/3-1/3
*csc(d*x+c)^2)*cot(d*x+c)+a^3*A*((-1/4*csc(d*x+c)^3-3/8*csc(d*x+c))*cot(d*
x+c)+3/8*ln(csc(d*x+c)-cot(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(78) = 156.

Time = 0.09 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.93

$$\int \csc^5(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = -\frac{32\,Aa^3\cos(dx+c)^3\sin(dx+c)-18\,Aa^3\cos(dx+c)^3+30\,Aa^3\cos(dx+c)-15\left(Aa^3\cos(dx+c)^4\right.}{48\,(d}$$

input
```
integrate(csc(d*x+c)^5*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="f
ricas")
```

output
```
-1/48*(32*A*a^3*cos(d*x + c)^3*sin(d*x + c) - 18*A*a^3*cos(d*x + c)^3 + 30
*A*a^3*cos(d*x + c) - 15*(A*a^3*cos(d*x + c)^4 - 2*A*a^3*cos(d*x + c)^2 +
A*a^3)*log(1/2*cos(d*x + c) + 1/2) + 15*(A*a^3*cos(d*x + c)^4 - 2*A*a^3*co
s(d*x + c)^2 + A*a^3)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^4 - 2*
d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^5(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)**5*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.69

$$\int \csc^5(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \frac{3\,Aa^3\left(\frac{2\,\left(3\,\cos(dx+c)^3-5\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)+24\,Aa^3(\log\left(\cos\right.}{48\,d}$$

input
```
integrate(csc(d*x+c)^5*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="m
axima")
```

output
```
1/48*(3*A*a^3*(2*(3*cos(d*x + c)^3 - 5*cos(d*x + c))/(cos(d*x + c)^4 - 2*c
os(d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) +
24*A*a^3*(log(cos(d*x + c) + 1) - log(cos(d*x + c) - 1)) + 96*A*a^3/tan(d*
x + c) - 32*(3*tan(d*x + c)^2 + 1)*A*a^3/tan(d*x + c)^3)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(78) = 156.

Time = 0.25 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.02

$$\int \csc^5(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \frac{3\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+16\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-120\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx\right.\right.\right.}{}$$

input
```
integrate(csc(d*x+c)^5*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="g
iac")
```

output
```
1/192*(3*A*a^3*tan(1/2*d*x + 1/2*c)^4 + 16*A*a^3*tan(1/2*d*x + 1/2*c)^3 +
24*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 120*A*a^3*log(abs(tan(1/2*d*x + 1/2*c)))
 - 48*A*a^3*tan(1/2*d*x + 1/2*c) + (250*A*a^3*tan(1/2*d*x + 1/2*c)^4 + 48*
A*a^3*tan(1/2*d*x + 1/2*c)^3 - 24*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 16*A*a^3*
tan(1/2*d*x + 1/2*c) - 3*A*a^3)/tan(1/2*d*x + 1/2*c)^4)/d
```

Mupad [B] (verification not implemented)

Time = 37.68 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.84

$$\int \csc^5(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx =$$

$$-\frac{A\,a^3\left(3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-3\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-16\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+16\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\sin\left(\frac{c}{2}+\right.\right.}{}$$

input
```
int(((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3)/sin(c + d*x)^5,x)
```

output
```
-(A*a^3*(3*cos(c/2 + (d*x)/2)^8 - 3*sin(c/2 + (d*x)/2)^8 - 16*cos(c/2 + (d
*x)/2)*sin(c/2 + (d*x)/2)^7 + 16*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2) -
 24*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^6 + 48*cos(c/2 + (d*x)/2)^3*si
n(c/2 + (d*x)/2)^5 - 48*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^3 + 24*cos
(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^2 + 120*log(sin(c/2 + (d*x)/2)/cos(c/
2 + (d*x)/2))*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^4))/(192*d*cos(c/2 +
 (d*x)/2)^4*sin(c/2 + (d*x)/2)^4)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.06

$$\int \csc^5(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \frac{a^4\left(16\cos(dx+c)\sin(dx+c)^3-9\cos(dx+c)\sin(dx+c)^2-16\cos(dx+c)\sin(dx+c)-6\cos(dx+\right.}{24\sin(dx+c)^4 d}$$

input
```
int(csc(d*x+c)^5*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)
```

output
```
(a**4*(16*cos(c + d*x)*sin(c + d*x)**3 - 9*cos(c + d*x)*sin(c + d*x)**2 -
16*cos(c + d*x)*sin(c + d*x) - 6*cos(c + d*x) - 15*log(tan((c + d*x)/2))*s
in(c + d*x)**4))/(24*sin(c + d*x)**4*d)
```

3.233 $\int \csc^6(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal result

Integrand size = 32, antiderivative size = 105

$$\int \csc^6(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \frac{a^3A\text{arctanh}(\cos(c+dx))}{4d}-\frac{2a^3A\cot^3(c+dx)}{3d}-\frac{a^3A\cot^5(c+dx)}{5d}+\frac{a^3A\cot(c+dx)\csc(c+dx)}{4d}-\frac{a^3A\cot(c+dx)\csc^3(c+dx)}{2d}$$

output
```
1/4*a^3*A*arctanh(cos(d*x+c))/d-2/3*a^3*A*cot(d*x+c)^3/d-1/5*a^3*A*cot(d*x
+c)^5/d+1/4*a^3*A*cot(d*x+c)*csc(d*x+c)/d-1/2*a^3*A*cot(d*x+c)*csc(d*x+c)^
3/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 268 vs. $2(105) = 210$.

Time = 0.47 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.55

$$\int \csc^6(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= a^3A\left(\frac{7\cot\left(\frac{1}{2}(c+dx)\right)}{30d}+\frac{\csc^2\left(\frac{1}{2}(c+dx)\right)}{16d}-\frac{19\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{480d}-\frac{\csc^4\left(\frac{1}{2}(c+dx)\right)}{32d}-\frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^4\left(\frac{1}{2}(c+dx)\right)}{160d}+\frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{4d}-\frac{\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d}-\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)}{16d}+\frac{\sec^4\left(\frac{1}{2}(c+dx)\right)}{32d}-\frac{7\tan\left(\frac{1}{2}(c+dx)\right)}{30d}+\frac{19\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{480d}+\frac{\sec^4\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{160d}\right)$$

input
```
Integrate[Csc[c + d*x]^6*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
a^3*A*((7*Cot[(c + d*x)/2])/(30*d) + Csc[(c + d*x)/2]^2/(16*d) - (19*Cot[(
c + d*x)/2]*Csc[(c + d*x)/2]^2)/(480*d) - Csc[(c + d*x)/2]^4/(32*d) - (Cot
[(c + d*x)/2]*Csc[(c + d*x)/2]^4)/(160*d) + Log[Cos[(c + d*x)/2]]/(4*d) -
Log[Sin[(c + d*x)/2]]/(4*d) - Sec[(c + d*x)/2]^2/(16*d) + Sec[(c + d*x)/2]
^4/(32*d) - (7*Tan[(c + d*x)/2])/(30*d) + (19*Sec[(c + d*x)/2]^2*Tan[(c +
d*x)/2])/(480*d) + (Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(160*d))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {3042, 3429, 3042, 3188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^6(c+dx)(a\sin(c+dx)+a)^3(A-A\sin(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(c+dx)+a)^3(A-A\sin(c+dx))}{\sin(c+dx)^6}dx$$

$$\Big\downarrow \text{3429}$$

$$a^3A^3\int \frac{\cot^6(c+dx)}{(A-A\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$a^3A^3\int \frac{1}{(A-A\sin(c+dx))^2\tan(c+dx)^6}dx$$

$$\Big\downarrow \text{3188}$$

$$\frac{a^3\int \left(A^4\csc^6(c+dx)+2A^4\csc^5(c+dx)-2A^4\csc^3(c+dx)-A^4\csc^2(c+dx)\right)dx}{A^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3\left(\frac{A^4\text{arctanh}(\cos(c+dx))}{4d}-\frac{A^4\cot^5(c+dx)}{5d}-\frac{2A^4\cot^3(c+dx)}{3d}-\frac{A^4\cot(c+dx)\csc^3(c+dx)}{2d}+\frac{A^4\cot(c+dx)\csc(c+dx)}{4d}\right)}{A^3}$$

input
```
Int[Csc[c + d*x]^6*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
(a^3*((A^4*ArcTanh[Cos[c + d*x]])/(4*d) - (2*A^4*Cot[c + d*x]^3)/(3*d) - (
A^4*Cot[c + d*x]^5)/(5*d) + (A^4*Cot[c + d*x]*Csc[c + d*x])/(4*d) - (A^4*C
ot[c + d*x]*Csc[c + d*x]^3)/(2*d)))/A^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3188
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Simp[a^p   Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
 + f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b,
e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m
- p/2, 0])
```

rule 3429
```
Int[sin[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^n*c^n
  Int[Tan[e + f*x]^p*(a + b*Sin[e + f*x])^(m - n), x], x] /; FreeQ[{a, b, c
, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[p + 2*n,
 0] && IntegerQ[n]
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.15

method	result
parallelrisch	$-\frac{A\left(\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}+5\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+\frac{25\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-\frac{25\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-30\cot\left(\frac{dx}{2}+\right.\right.}{160d}$
derivativedivides	$\frac{a^{3}A\cot(dx+c)-2a^{3}A\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+2a^{3}A\left(\left(-\frac{\csc(dx+c)^{3}}{4}-\frac{3\csc(dx+c)}{8}\right)\cot(dx+\right.}{d}$
default	$\frac{a^{3}A\cot(dx+c)-2a^{3}A\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+2a^{3}A\left(\left(-\frac{\csc(dx+c)^{3}}{4}-\frac{3\csc(dx+c)}{8}\right)\cot(dx+\right.}{d}$
risch	$-\frac{a^{3}A\left(-60i e^{8i(dx+c)}+15\,e^{9i(dx+c)}+240i e^{6i(dx+c)}+90\,e^{7i(dx+c)}-40i e^{4i(dx+c)}+80i e^{2i(dx+c)}-90\,e^{3i(dx+c)}-28i-15\right.}{30d\left(e^{2i(dx+c)}-1\right)^{5}}$

input
```
int(csc(d*x+c)^6*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
-1/160*A*(cot(1/2*d*x+1/2*c)^5-tan(1/2*d*x+1/2*c)^5+5*cot(1/2*d*x+1/2*c)^4
-5*tan(1/2*d*x+1/2*c)^4+25/3*cot(1/2*d*x+1/2*c)^3-25/3*tan(1/2*d*x+1/2*c)^
3-30*cot(1/2*d*x+1/2*c)+30*tan(1/2*d*x+1/2*c)+40*ln(tan(1/2*d*x+1/2*c)))*a
^3/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(95) = 190$.

Time = 0.09 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.91

$$\int \csc^6(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \frac{56\,Aa^3\cos\left(dx+c\right)^5 - 80\,Aa^3\cos\left(dx+c\right)^3 + 15\left(Aa^3\cos\left(dx+c\right)^4 - 2\,Aa^3\cos\left(dx+c\right)^2 + Aa^3\right)\log\left(\frac{1}{2}\right.}{}$$

input
```
integrate(csc(d*x+c)^6*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="f
ricas")
```

output
```
1/120*(56*A*a^3*cos(d*x + c)^5 - 80*A*a^3*cos(d*x + c)^3 + 15*(A*a^3*cos(d
*x + c)^4 - 2*A*a^3*cos(d*x + c)^2 + A*a^3)*log(1/2*cos(d*x + c) + 1/2)*si
n(d*x + c) - 15*(A*a^3*cos(d*x + c)^4 - 2*A*a^3*cos(d*x + c)^2 + A*a^3)*lo
g(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 30*(A*a^3*cos(d*x + c)^3 + A*a^3
*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*
sin(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \csc^6(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(csc(d*x+c)**6*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.67

$$\int \csc^6(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$=\frac{15\,Aa^3\left(\frac{2\left(3\,\cos(dx+c)^3-5\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)-60\,Aa^3\left(\frac{2\,\cos(d}{\cos(dx+}\right.}{120}$$

input
```
integrate(csc(d*x+c)^6*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="m
axima")
```

output
```
1/120*(15*A*a^3*(2*(3*cos(d*x + c)^3 - 5*cos(d*x + c))/(cos(d*x + c)^4 - 2
*cos(d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1))
- 60*A*a^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - log(cos(d*x + c) + 1) +
log(cos(d*x + c) - 1)) + 120*A*a^3/tan(d*x + c) - 8*(15*tan(d*x + c)^4 + 1
0*tan(d*x + c)^2 + 3)*A*a^3/tan(d*x + c)^5)/d
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.66

$$\int \csc^6(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$=\frac{3\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+15\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+25\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-120\,Aa^3\log\left(\left|\tan\left(\frac{1}{2}\,dx\right.\right.\right.}{}$$

input
```
integrate(csc(d*x+c)^6*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="g
iac")
```

output
```
1/480*(3*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 15*A*a^3*tan(1/2*d*x + 1/2*c)^4 +
25*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 120*A*a^3*log(abs(tan(1/2*d*x + 1/2*c)))
 - 90*A*a^3*tan(1/2*d*x + 1/2*c) + (274*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 90*
A*a^3*tan(1/2*d*x + 1/2*c)^4 - 25*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 15*A*a^3*
tan(1/2*d*x + 1/2*c) - 3*A*a^3)/tan(1/2*d*x + 1/2*c)^5)/d
```

Mupad [B] (verification not implemented)

Time = 34.91 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.32

$$\int \csc^6(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx =$$

$$-\frac{A\,a^3\left(3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-3\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-15\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+15\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\,\sin\left(\frac{c}{2}\right.\right.}{}$$

input
```
int(((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3)/sin(c + d*x)^6,x)
```

output
```
-(A*a^3*(3*cos(c/2 + (d*x)/2)^10 - 3*sin(c/2 + (d*x)/2)^10 - 15*cos(c/2 +
(d*x)/2)*sin(c/2 + (d*x)/2)^9 + 15*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)
 - 25*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^8 + 90*cos(c/2 + (d*x)/2)^4*
sin(c/2 + (d*x)/2)^6 - 90*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^4 + 25*c
os(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^2 + 120*log(sin(c/2 + (d*x)/2)/cos(
c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^5))/(480*d*cos(c/2
 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^5)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.02

$$\int \csc^6(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$

$$=\frac{a^4\left(28\cos\left(dx+c\right)\sin\left(dx+c\right)^4+15\cos\left(dx+c\right)\sin\left(dx+c\right)^3-16\cos\left(dx+c\right)\sin\left(dx+c\right)^2-30\cos\left(d\right.\right.}{60\sin\left(dx+c\right)^5d}$$

input
```
int(csc(d*x+c)^6*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)
```

output
```
(a**4*(28*cos(c + d*x)*sin(c + d*x)**4 + 15*cos(c + d*x)*sin(c + d*x)**3 -
 16*cos(c + d*x)*sin(c + d*x)**2 - 30*cos(c + d*x)*sin(c + d*x) - 12*cos(c
 + d*x) - 15*log(tan((c + d*x)/2))*sin(c + d*x)**5))/(60*sin(c + d*x)**5*d
)
```

3.234 $\int \csc^7(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$

Optimal result

Integrand size = 32, antiderivative size = 130

$$\begin{aligned}\int \csc^7(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx &= \frac{3a^3A\text{arctanh}(\cos(c+dx))}{16d}-\frac{2a^3A\cot^3(c+dx)}{3d} \\ &\quad -\frac{2a^3A\cot^5(c+dx)}{5d}+\frac{3a^3A\cot(c+dx)\csc(c+dx)}{16d} \\ &\quad -\frac{5a^3A\cot(c+dx)\csc^3(c+dx)}{24d}-\frac{a^3A\cot(c+dx)\csc^5(c+dx)}{6d}\end{aligned}$$

output
```
3/16*a^3*A*arctanh(cos(d*x+c))/d-2/3*a^3*A*cot(d*x+c)^3/d-2/5*a^3*A*cot(d*
x+c)^5/d+3/16*a^3*A*cot(d*x+c)*csc(d*x+c)/d-5/24*a^3*A*cot(d*x+c)*csc(d*x+
c)^3/d-1/6*a^3*A*cot(d*x+c)*csc(d*x+c)^5/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 306 vs. $2(130) = 260$.

Time = 0.51 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.35

$$\int \csc^7(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= a^3A\left(\frac{2\cot\left(\frac{1}{2}(c+dx)\right)}{15d}+\frac{3\csc^2\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{240d}-\frac{\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^4\left(\frac{1}{2}(c+dx)\right)}{80d}-\frac{\csc^6\left(\frac{1}{2}(c+dx)\right)}{384d}+\frac{3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{16d}-\frac{3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{16d}-\frac{3\sec^2\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{\sec^6\left(\frac{1}{2}(c+dx)\right)}{384d}-\frac{2\tan\left(\frac{1}{2}(c+dx)\right)}{15d}-\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{240d}+\frac{\sec^4\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{80d}\right)$$

input
```
Integrate[Csc[c + d*x]^7*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
a^3*A*((2*Cot[(c + d*x)/2])/(15*d) + (3*Csc[(c + d*x)/2]^2)/(64*d) + (Cot[
(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(240*d) - Csc[(c + d*x)/2]^4/(64*d) - (Co
t[(c + d*x)/2]*Csc[(c + d*x)/2]^4)/(80*d) - Csc[(c + d*x)/2]^6/(384*d) + (
3*Log[Cos[(c + d*x)/2]])/(16*d) - (3*Log[Sin[(c + d*x)/2]])/(16*d) - (3*Se
c[(c + d*x)/2]^2)/(64*d) + Sec[(c + d*x)/2]^4/(64*d) + Sec[(c + d*x)/2]^6/
(384*d) - (2*Tan[(c + d*x)/2])/(15*d) - (Sec[(c + d*x)/2]^2*Tan[(c + d*x)/
2])/(240*d) + (Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(80*d))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3042, 3445, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^7(c+dx)(a\sin(c+dx)+a)^3(A-A\sin(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(c+dx)+a)^3(A-A\sin(c+dx))}{\sin(c+dx)^7}dx$$

$$\Big\downarrow \text{3445}$$

$$\int \left(a^3A\csc^7(c+dx)+2a^3A\csc^6(c+dx)-2a^3A\csc^4(c+dx)-a^3A\csc^3(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3a^3A\text{arctanh}(\cos(c+dx))}{16d}-\frac{2a^3A\cot^5(c+dx)}{5d}-\frac{2a^3A\cot^3(c+dx)}{3d}-\frac{a^3A\cot(c+dx)\csc^5(c+dx)}{6d}-\frac{5a^3A\cot(c+dx)\csc^3(c+dx)}{24d}+\frac{3a^3A\cot(c+dx)\csc(c+dx)}{16d}$$

input
```
Int[Csc[c + d*x]^7*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
```

output
```
(3*a^3*A*ArcTanh[Cos[c + d*x]])/(16*d) - (2*a^3*A*Cot[c + d*x]^3)/(3*d) -
(2*a^3*A*Cot[c + d*x]^5)/(5*d) + (3*a^3*A*Cot[c + d*x]*Csc[c + d*x])/(16*d
) - (5*a^3*A*Cot[c + d*x]*Csc[c + d*x]^3)/(24*d) - (a^3*A*Cot[c + d*x]*Csc
[c + d*x]^5)/(6*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3445
```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\frac{-a^3A\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)-2a^3A\left(-\frac{2}{3}-\frac{\csc(dx+c)^2}{3}\right)\cot(dx+c)+2a^3A\left(-\frac{8}{15}-\frac{\csc(dx+c)}{5}\right.}{}$
default	$\frac{-a^3A\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)-2a^3A\left(-\frac{2}{3}-\frac{\csc(dx+c)^2}{3}\right)\cot(dx+c)+2a^3A\left(-\frac{8}{15}-\frac{\csc(dx+c)}{5}\right.}{}$
risch	$-\frac{a^3A(45\,e^{11i(dx+c)}+65\,e^{9i(dx+c)}-750\,e^{7i(dx+c)}+960ie^{8i(dx+c)}-750\,e^{5i(dx+c)}-640ie^{6i(dx+c)}+65\,e^{3i(dx+c)}+45\,e^{i(}}{120d\left(e^{2i(dx+c)}-1\right)^6}$
parallelrisch	$-\frac{Aa^3\left(\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^6-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+\frac{24\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}-\frac{24\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+9\cot\left(\frac{dx}{2}+\frac{c}{2}\right)^4-9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+8\cot\left(\frac{dx}{2}\right.\right.}{38}$

input
```
int(csc(d*x+c)^7*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x,method=_RETURNVERBO
SE)
```

output
```
1/d*(-a^3*A*(-1/2*csc(d*x+c)*cot(d*x+c)+1/2*ln(csc(d*x+c)-cot(d*x+c)))-2*a
^3*A*(-2/3-1/3*csc(d*x+c)^2)*cot(d*x+c)+2*a^3*A*(-8/15-1/5*csc(d*x+c)^4-4/
15*csc(d*x+c)^2)*cot(d*x+c)+a^3*A*((-1/6*csc(d*x+c)^5-5/24*csc(d*x+c)^3-5/
16*csc(d*x+c))*cot(d*x+c)+5/16*ln(csc(d*x+c)-cot(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(118) = 236.

Time = 0.09 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.85

$$\int \csc^7(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx = -\frac{90\,Aa^3\cos(dx+c)^5-80\,Aa^3\cos(dx+c)^3-90\,Aa^3\cos(dx+c)-45\left(Aa^3\cos(dx+c)^6-3\,Aa^3\cos\right.}{}$$

input
```
integrate(csc(d*x+c)^7*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="f
ricas")
```

output
```
-1/480*(90*A*a^3*cos(d*x + c)^5 - 80*A*a^3*cos(d*x + c)^3 - 90*A*a^3*cos(d
*x + c) - 45*(A*a^3*cos(d*x + c)^6 - 3*A*a^3*cos(d*x + c)^4 + 3*A*a^3*cos(
d*x + c)^2 - A*a^3)*log(1/2*cos(d*x + c) + 1/2) + 45*(A*a^3*cos(d*x + c)^6
 - 3*A*a^3*cos(d*x + c)^4 + 3*A*a^3*cos(d*x + c)^2 - A*a^3)*log(-1/2*cos(d
*x + c) + 1/2) + 64*(2*A*a^3*cos(d*x + c)^5 - 5*A*a^3*cos(d*x + c)^3)*sin(
d*x + c))/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^7(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx=\text{Timed out}$$

input
```
integrate(csc(d*x+c)**7*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.59

$$\int \csc^7(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$=\frac{5\,Aa^3\left(\frac{2\left(15\,\cos(dx+c)^5-40\,\cos(dx+c)^3+33\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)-}{}$$

input
```
integrate(csc(d*x+c)^7*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="m
axima")
```

output
```
1/480*(5*A*a^3*(2*(15*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 33*cos(d*x + c)
)/(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 15*log(cos(
d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) - 120*A*a^3*(2*cos(d*x + c)/(cos
(d*x + c)^2 - 1) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) + 320*(3
*tan(d*x + c)^2 + 1)*A*a^3/tan(d*x + c)^3 - 64*(15*tan(d*x + c)^4 + 10*tan
(d*x + c)^2 + 3)*A*a^3/tan(d*x + c)^5)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(118) = 236$.

Time = 0.29 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.86

$$\int \csc^7(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$= \frac{5\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+24\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+45\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+40\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}{}$$

input
```
integrate(csc(d*x+c)^7*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="g
iac")
```

output
```
1/1920*(5*A*a^3*tan(1/2*d*x + 1/2*c)^6 + 24*A*a^3*tan(1/2*d*x + 1/2*c)^5 +
 45*A*a^3*tan(1/2*d*x + 1/2*c)^4 + 40*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 15*A*
a^3*tan(1/2*d*x + 1/2*c)^2 - 360*A*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 24
0*A*a^3*tan(1/2*d*x + 1/2*c) + (882*A*a^3*tan(1/2*d*x + 1/2*c)^6 + 240*A*a
^3*tan(1/2*d*x + 1/2*c)^5 + 15*A*a^3*tan(1/2*d*x + 1/2*c)^4 - 40*A*a^3*tan
(1/2*d*x + 1/2*c)^3 - 45*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 24*A*a^3*tan(1/2*d
*x + 1/2*c) - 5*A*a^3)/tan(1/2*d*x + 1/2*c)^6)/d
```

Mupad [B] (verification not implemented)

Time = 34.65 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.62

$$\int \csc^7(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx =$$
$$-\frac{A\,a^3\left(5\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-24\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+24\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\,\sin\left(\frac{c}{2}\right.\right.}{}$$

input
```
int(((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3)/sin(c + d*x)^7,x)
```

output
```
-(A*a^3*(5*cos(c/2 + (d*x)/2)^12 - 5*sin(c/2 + (d*x)/2)^12 - 24*cos(c/2 +
(d*x)/2)*sin(c/2 + (d*x)/2)^11 + 24*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/
2) - 45*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^10 - 40*cos(c/2 + (d*x)/2)
^3*sin(c/2 + (d*x)/2)^9 + 15*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^8 + 2
40*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^7 - 240*cos(c/2 + (d*x)/2)^7*si
n(c/2 + (d*x)/2)^5 - 15*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^4 + 40*cos
(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^3 + 45*cos(c/2 + (d*x)/2)^10*sin(c/2
+ (d*x)/2)^2 + 360*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d
*x)/2)^6*sin(c/2 + (d*x)/2)^6))/(1920*d*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*
x)/2)^6)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.95

$$\int \csc^7(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))\,dx$$
$$=\frac{a^4\left(64\cos\left(dx+c\right)\sin\left(dx+c\right)^5+45\cos\left(dx+c\right)\sin\left(dx+c\right)^4+32\cos\left(dx+c\right)\sin\left(dx+c\right)^3-50\cos\left(d\right.\right.}{240\sin\left(\right.}$$

input
```
int(csc(d*x+c)^7*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)
```

output
```
(a**4*(64*cos(c + d*x)*sin(c + d*x)**5 + 45*cos(c + d*x)*sin(c + d*x)**4 +
 32*cos(c + d*x)*sin(c + d*x)**3 - 50*cos(c + d*x)*sin(c + d*x)**2 - 96*co
s(c + d*x)*sin(c + d*x) - 40*cos(c + d*x) - 45*log(tan((c + d*x)/2))*sin(c
 + d*x)**6))/(240*sin(c + d*x)**6*d)
```

3.235 $\int \frac{\sin^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 129

$$\int \frac{\sin^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{19Ax}{2a^3}-\frac{4A\cos(c+dx)}{a^3d}+\frac{A\cos(c+dx)\sin(c+dx)}{2a^3d}-\frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}+\frac{41A\cos(c+dx)}{15a^3d(1+\sin(c+dx))^2}-\frac{199A\cos(c+dx)}{15a^3d(1+\sin(c+dx))}$$

output
```
-19/2*A*x/a^3-4*A*cos(d*x+c)/a^3/d+1/2*A*cos(d*x+c)*sin(d*x+c)/a^3/d-2/5*A
*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^3+41/15*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))
^2-199/15*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.18 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.64

$$\int \frac{\sin^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$
$$= \frac{A\sec(c+dx)\sqrt{1-\sin(c+dx)}\Big(140\sqrt{2}\sqrt{a}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},\frac{1}{2}(1+\sin(c+dx))\right)(1+\text{s}}{}$$

input
```
Integrate[(Sin[c + d*x]^4*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

output
```
(A*Sec[c + d*x]*Sqrt[1 - Sin[c + d*x]]*(140*Sqrt[2]*Sqrt[a]*Hypergeometric
2F1[-3/2, -3/2, -1/2, (1 + Sin[c + d*x])/2]*(1 + Sin[c + d*x]) - 360*ArcSi
n[Sqrt[a*(1 + Sin[c + d*x])]/(Sqrt[2]*Sqrt[a])]*(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2])^4*Sqrt[a*(1 + Sin[c + d*x])] + Sqrt[a]*Sqrt[1 - Sin[c + d*x]]*
(-308 - 639*Sin[c + d*x] - 433*Sin[c + d*x]^2 - 75*Sin[c + d*x]^3 + 15*Sin
[c + d*x]^4)))/(30*a^(7/2)*d*(1 + Sin[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3042, 3445, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(c+dx)(A-A\sin(c+dx))}{(a\sin(c+dx)+a)^3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin(c+dx)^4(A-A\sin(c+dx))}{(a\sin(c+dx)+a)^3}dx$$
$$\Big\downarrow \text{3445}$$

$$\int \left(-\frac{A\sin^2(c+dx)}{a^3}+\frac{4A\sin(c+dx)}{a^3}+\frac{16A}{a^3(\sin(c+dx)+1)}-\frac{9A}{a^3(\sin(c+dx)+1)^2}+\frac{2A}{a^3(\sin(c+dx)+1)^3}-\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4A\cos(c+dx)}{a^3d}+\frac{A\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{199A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)}+\frac{41A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)^2}-\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}-\frac{19Ax}{2a^3}$$

input
```
Int[(Sin[c + d*x]^4*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

output
```
(-19*A*x)/(2*a^3) - (4*A*Cos[c + d*x])/(a^3*d) + (A*Cos[c + d*x]*Sin[c + d
*x])/(2*a^3*d) - (2*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^3) + (41*A
*Cos[c + d*x])/(15*a^3*d*(1 + Sin[c + d*x])^2) - (199*A*Cos[c + d*x])/(15*
a^3*d*(1 + Sin[c + d*x]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3445
```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.19

method	result
derivativedivides	$\frac{32A\left(-\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{2}+4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+4}{16\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}}-\frac{19\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}-\frac{1}{10\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}+\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}}{d\,a^{3}}$
default	$\frac{32A\left(-\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{2}+4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+4}{16\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}}-\frac{19\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}-\frac{1}{10\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}+\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}}{d\,a^{3}}$
risch	$-\frac{19Ax}{2a^{3}}-\frac{iA\,e^{2i(dx+c)}}{8d\,a^{3}}-\frac{2A\,e^{i(dx+c)}}{a^{3}d}-\frac{2A\,e^{-i(dx+c)}}{a^{3}d}+\frac{iA\,e^{-2i(dx+c)}}{8d\,a^{3}}-\frac{2(825iA\,e^{3i(dx+c)}+240A\,e^{4i(dx+c)}}{15d\,a^{3}}$
parallelrisch	$\frac{A\left((76dx-120)\sin\left(\frac{5dx}{2}+\frac{5c}{2}\right)+(380dx-128)\cos\left(\frac{3dx}{2}+\frac{3c}{2}\right)+\left(76dx+\frac{3484}{15}\right)\cos\left(\frac{5dx}{2}+\frac{5c}{2}\right)+(-760dx-304)\sin\left(\frac{dx}{2}+\right.\right.}{8d\,a^{3}\left(-\sin\left(\frac{5dx}{2}+\frac{5c}{2}\right)+5\sin\left(\frac{3dx}{2}+\right.\right.}$
norman	$\frac{-\frac{95Ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{285Ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{2a}-\frac{665Ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{2a}-\frac{1235Ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{2a}-\frac{1919Ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{2a}-\frac{2565Ax\tan}{2}}{}$

input
```
int(sin(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBO
SE)
```

output
```
32/d*A/a^3*(-1/16*(1/2*tan(1/2*d*x+1/2*c)^3+4*tan(1/2*d*x+1/2*c)^2-1/2*tan
(1/2*d*x+1/2*c)+4)/(1+tan(1/2*d*x+1/2*c)^2)^2-19/32*arctan(tan(1/2*d*x+1/2
*c))-1/10/(tan(1/2*d*x+1/2*c)+1)^5+1/4/(tan(1/2*d*x+1/2*c)+1)^4+1/24/(tan(
1/2*d*x+1/2*c)+1)^3-5/16/(tan(1/2*d*x+1/2*c)+1)^2-9/16/(tan(1/2*d*x+1/2*c)
+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. 2(119) = 238.

Time = 0.08 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.92

$$\int\frac{\sin^{4}(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^{3}}\,dx = -\frac{15\,A\cos(dx+c)^{5}+90\,A\cos(dx+c)^{4}+(285\,Adx+683\,A)\cos(dx+c)^{3}-1140\,Adx+(855\,Adx-}{30\,(a^{3}d\cos(d}$$

input
```
integrate(sin(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="f
ricas")
```

output
```
-1/30*(15*A*cos(d*x + c)^5 + 90*A*cos(d*x + c)^4 + (285*A*d*x + 683*A)*cos
(d*x + c)^3 - 1140*A*d*x + (855*A*d*x - 526*A)*cos(d*x + c)^2 - 6*(95*A*d*
x + 191*A)*cos(d*x + c) - (15*A*cos(d*x + c)^4 - 75*A*cos(d*x + c)^3 + 114
0*A*d*x - 19*(15*A*d*x - 32*A)*cos(d*x + c)^2 + 6*(95*A*d*x + 189*A)*cos(d
*x + c) - 12*A)*sin(d*x + c) - 12*A)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d
*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*x + c)^2 - 2*a^3
*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3614 vs. $2(126) = 252$.

Time = 25.14 (sec) , antiderivative size = 3614, normalized size of antiderivative = 28.02

$$\int \frac{\sin^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)**4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

output
```
Piecewise((-285*A*d*x*tan(c/2 + d*x/2)**9/(30*a**3*d*tan(c/2 + d*x/2)**9 +
 150*a**3*d*tan(c/2 + d*x/2)**8 + 360*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**
3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(
c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**3 + 360*a**3*d*tan(c/2 + d*
x/2)**2 + 150*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d) - 1425*A*d*x*tan(c/2 +
d*x/2)**8/(30*a**3*d*tan(c/2 + d*x/2)**9 + 150*a**3*d*tan(c/2 + d*x/2)**8
+ 360*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3*d*tan(c/2 + d*x/2)**6 + 780*a*
*3*d*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan
(c/2 + d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/2)**2 + 150*a**3*d*tan(c/2 + d
*x/2) + 30*a**3*d) - 3420*A*d*x*tan(c/2 + d*x/2)**7/(30*a**3*d*tan(c/2 + d
*x/2)**9 + 150*a**3*d*tan(c/2 + d*x/2)**8 + 360*a**3*d*tan(c/2 + d*x/2)**7
 + 600*a**3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*tan(c/2 + d*x/2)**5 + 780*a
**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**3 + 360*a**3*d*ta
n(c/2 + d*x/2)**2 + 150*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d) - 5700*A*d*x*
tan(c/2 + d*x/2)**6/(30*a**3*d*tan(c/2 + d*x/2)**9 + 150*a**3*d*tan(c/2 +
d*x/2)**8 + 360*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3*d*tan(c/2 + d*x/2)**
6 + 780*a**3*d*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(c/2 + d*x/2)**4 + 600*
a**3*d*tan(c/2 + d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/2)**2 + 150*a**3*d*t
an(c/2 + d*x/2) + 30*a**3*d) - 7410*A*d*x*tan(c/2 + d*x/2)**5/(30*a**3*d*t
an(c/2 + d*x/2)**9 + 150*a**3*d*tan(c/2 + d*x/2)**8 + 360*a**3*d*tan(c/...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 715 vs. $2(119) = 238$.

Time = 0.13 (sec) , antiderivative size = 715, normalized size of antiderivative = 5.54

$$\int \frac{\sin^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="m
axima")
```

output
```
-1/15*(A*((1325*sin(d*x + c)/(cos(d*x + c) + 1) + 2673*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + 3805*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 4329*sin(d*x
 + c)^4/(cos(d*x + c) + 1)^4 + 3575*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 +
2275*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 975*sin(d*x + c)^7/(cos(d*x + c
) + 1)^7 + 195*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 304)/(a^3 + 5*a^3*sin
(d*x + c)/(cos(d*x + c) + 1) + 12*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
+ 20*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 26*a^3*sin(d*x + c)^4/(cos(
d*x + c) + 1)^4 + 26*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 20*a^3*sin(
d*x + c)^6/(cos(d*x + c) + 1)^6 + 12*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)
^7 + 5*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + a^3*sin(d*x + c)^9/(cos(d
*x + c) + 1)^9) + 195*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) + 6*A*(
(105*sin(d*x + c)/(cos(d*x + c) + 1) + 189*sin(d*x + c)^2/(cos(d*x + c) +
1)^2 + 200*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 160*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4 + 75*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*sin(d*x + c)^
6/(cos(d*x + c) + 1)^6 + 24)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1)
+ 11*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a^3*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 15*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 11*a^3*sin(
d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^
6 + a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7) + 15*arctan(sin(d*x + c)/(cos
(d*x + c) + 1))/a^3))/d
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.21

$$\int \frac{\sin^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{\frac{285\,(dx+c)A}{a^3} + \frac{30\left(A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+8\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+8\,A\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^3} + \frac{4\left(135\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+615\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}{a^3\left(\tan\right.}}{30\,d}$$

input
```
integrate(sin(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="g
iac")
```

output
```
-1/30*(285*(d*x + c)*A/a^3 + 30*(A*tan(1/2*d*x + 1/2*c)^3 + 8*A*tan(1/2*d*
x + 1/2*c)^2 - A*tan(1/2*d*x + 1/2*c) + 8*A)/((tan(1/2*d*x + 1/2*c)^2 + 1)
^2*a^3) + 4*(135*A*tan(1/2*d*x + 1/2*c)^4 + 615*A*tan(1/2*d*x + 1/2*c)^3 +
 1025*A*tan(1/2*d*x + 1/2*c)^2 + 685*A*tan(1/2*d*x + 1/2*c) + 164*A)/(a^3*
(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

Mupad [B] (verification not implemented)

Time = 38.84 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.53

$$\int \frac{\sin^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{\left(\frac{95\,A\,(c+d\,x)}{2}-\frac{A\,(1425\,c+1425\,d\,x+570)}{30}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\left(114\,A\,(c+d\,x)-\frac{A\,(3420\,c+3420\,d\,x+2850)}{30}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{}$$

$$-\frac{19\,A\,x}{2\,a^3}$$

input
```
int((sin(c + d*x)^4*(A - A*sin(c + d*x)))/(a + a*sin(c + d*x))^3,x)
```

output
```
(tan(c/2 + (d*x)/2)*((95*A*(c + d*x))/2 - (A*(1425*c + 1425*d*x + 3910))/3
0) + tan(c/2 + (d*x)/2)^8*((95*A*(c + d*x))/2 - (A*(1425*c + 1425*d*x + 57
0))/30) + tan(c/2 + (d*x)/2)^7*(114*A*(c + d*x) - (A*(3420*c + 3420*d*x +
2850))/30) + tan(c/2 + (d*x)/2)^2*(114*A*(c + d*x) - (A*(3420*c + 3420*d*x
 + 7902))/30) + tan(c/2 + (d*x)/2)^6*(190*A*(c + d*x) - (A*(5700*c + 5700*
d*x + 6650))/30) + tan(c/2 + (d*x)/2)^3*(190*A*(c + d*x) - (A*(5700*c + 57
00*d*x + 11270))/30) + tan(c/2 + (d*x)/2)^5*(247*A*(c + d*x) - (A*(7410*c
+ 7410*d*x + 10450))/30) + tan(c/2 + (d*x)/2)^4*(247*A*(c + d*x) - (A*(741
0*c + 7410*d*x + 12846))/30) + (19*A*(c + d*x))/2 - (A*(285*c + 285*d*x +
896))/30)/(a^3*d*(tan(c/2 + (d*x)/2) + 1)^5*(tan(c/2 + (d*x)/2)^2 + 1)^2)
- (19*A*x)/(2*a^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.15

$$\int \frac{\sin^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$
$$= \frac{-15\cos(dx+c)\sin(dx+c)^4+75\cos(dx+c)\sin(dx+c)^3-285\cos(dx+c)\sin(dx+c)^2dx+379\,\mathrm{co}}{}$$

input
```
int(sin(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x)
```

output
```
( - 15*cos(c + d*x)*sin(c + d*x)**4 + 75*cos(c + d*x)*sin(c + d*x)**3 - 28
5*cos(c + d*x)*sin(c + d*x)**2*d*x + 379*cos(c + d*x)*sin(c + d*x)**2 - 57
0*cos(c + d*x)*sin(c + d*x)*d*x + 391*cos(c + d*x)*sin(c + d*x) - 285*cos(
c + d*x)*d*x + 114*cos(c + d*x) - 15*sin(c + d*x)**5 + 90*sin(c + d*x)**4
+ 285*sin(c + d*x)**3*d*x + 972*sin(c + d*x)**3 + 855*sin(c + d*x)**2*d*x
+ 1348*sin(c + d*x)**2 + 855*sin(c + d*x)*d*x + 391*sin(c + d*x) + 285*d*x
 - 114)/(30*a**2*d*(cos(c + d*x)*sin(c + d*x)**2 + 2*cos(c + d*x)*sin(c +
d*x) + cos(c + d*x) - sin(c + d*x)**3 - 3*sin(c + d*x)**2 - 3*sin(c + d*x)
 - 1))
```

3.236 $\int \frac{\sin^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 103

$$\int \frac{\sin^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{4Ax}{a^3}+\frac{A\cos(c+dx)}{a^3d}+\frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3} - \frac{31A\cos(c+dx)}{15a^3d(1+\sin(c+dx))^2} + \frac{104A\cos(c+dx)}{15a^3d(1+\sin(c+dx))}$$

output
```
4*A*x/a^3+A*cos(d*x+c)/a^3/d+2/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^3-31/15
*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^2+104/15*A*cos(d*x+c)/a^3/d/(1+sin(d*x+
c))
```

Mathematica [A] (verified)

Time = 4.39 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.50

$$\int \frac{\sin^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{A\sec(c+dx)\left(-120\arcsin\left(\frac{\sqrt{a(1+\sin(c+dx))}}{\sqrt{2}\sqrt{a}}\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4\sqrt{1-\sin(c+dx)}\sqrt{}\right.}{15a^{7/2}d(1}$$

input
```
Integrate[(Sin[c + d*x]^3*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

output
```
-1/15*(A*Sec[c + d*x]*(-120*ArcSin[Sqrt[a*(1 + Sin[c + d*x])]/(Sqrt[2]*Sqr
t[a])]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*Sqrt[1 - Sin[c + d*x]]*Sqrt
[a*(1 + Sin[c + d*x])] + Sqrt[a]*(-94 - 128*Sin[c + d*x] + 73*Sin[c + d*x]
^2 + 134*Sin[c + d*x]^3 + 15*Sin[c + d*x]^4)))/(a^(7/2)*d*(1 + Sin[c + d*x
])^2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3042, 3445, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(c+dx)(A-A\sin(c+dx))}{(a\sin(c+dx)+a)^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin(c+dx)^3(A-A\sin(c+dx))}{(a\sin(c+dx)+a)^3}dx$$
$$\Big\downarrow \text{3445}$$
$$\int \left(-\frac{A\sin(c+dx)}{a^3}-\frac{9A}{a^3(\sin(c+dx)+1)}+\frac{7A}{a^3(\sin(c+dx)+1)^2}-\frac{2A}{a^3(\sin(c+dx)+1)^3}+\frac{4A}{a^3}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{A\cos(c+dx)}{a^3d}+\frac{104A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)}-\frac{31A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)^2}+\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}+\frac{4Ax}{a^3}$$

input
```
Int[(Sin[c + d*x]^3*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

output
```
(4*A*x)/a^3 + (A*Cos[c + d*x])/(a^3*d) + (2*A*Cos[c + d*x])/(5*a^3*d*(1 +
Sin[c + d*x])^3) - (31*A*Cos[c + d*x])/(15*a^3*d*(1 + Sin[c + d*x])^2) + (
104*A*Cos[c + d*x])/(15*a^3*d*(1 + Sin[c + d*x]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3445
```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{16A\left(\frac{1}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{1}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{3}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{1}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2}+\frac{}{8+8\tan(}\right.}{d\,a^3}$
default	$\frac{16A\left(\frac{1}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{1}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{3}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{1}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2}+\frac{}{8+8\tan(}\right.}{d\,a^3}$
risch	$\frac{4Ax}{a^3}+\frac{A\,e^{i(dx+c)}}{2a^3d}+\frac{A\,e^{-i(dx+c)}}{2a^3d}+\frac{2A\left(435i\,e^{3i(dx+c)}+135\,e^{4i(dx+c)}-385i\,e^{i(dx+c)}-605\,e^{2i(dx+c)}+104\right)}{15d\,a^3\left(e^{i(dx+c)}+i\right)^5}$
parallelrisch	$\frac{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7xd+(5dx+2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+(11dx+10)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+\left(15dx+\frac{64}{3}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\left(15dx+\frac{77}{3}\right)\tan(\right.}{d\,a^3\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$
norman	$\frac{\frac{20Ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}+\frac{56Ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{a}+\frac{120Ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{a}+\frac{204Ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{a}+\frac{284Ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{a}+\frac{336Ax\tan\left(\frac{dx}{2}+\right.}{a}}{}$

input
```
int(sin(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBO
SE)
```

output
```
16/d*A/a^3*(1/5/(tan(1/2*d*x+1/2*c)+1)^5-1/2/(tan(1/2*d*x+1/2*c)+1)^4+1/12
/(tan(1/2*d*x+1/2*c)+1)^3+3/8/(tan(1/2*d*x+1/2*c)+1)^2+1/2/(tan(1/2*d*x+1/
2*c)+1)+1/8/(1+tan(1/2*d*x+1/2*c)^2)+1/2*arctan(tan(1/2*d*x+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(97) = 194$.

Time = 0.09 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.18

$$\int \frac{\sin^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{15\,A\cos\left(dx+c\right)^4+(60\,Adx+149\,A)\cos\left(dx+c\right)^3-240\,Adx+(180\,Adx-103\,A)\cos\left(dx+c\right)^2-3\,(4}{15\,\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2}$$

input
```
integrate(sin(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="f
ricas")
```

output
```
1/15*(15*A*cos(d*x + c)^4 + (60*A*d*x + 149*A)*cos(d*x + c)^3 - 240*A*d*x
+ (180*A*d*x - 103*A)*cos(d*x + c)^2 - 3*(40*A*d*x + 81*A)*cos(d*x + c) +
(15*A*cos(d*x + c)^3 - 240*A*d*x + 2*(30*A*d*x - 67*A)*cos(d*x + c)^2 - 3*
(40*A*d*x + 79*A)*cos(d*x + c) + 6*A)*sin(d*x + c) - 6*A)/(a^3*d*cos(d*x +
 c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*c
os(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2290 vs. $2(100) = 200$.

Time = 14.48 (sec) , antiderivative size = 2290, normalized size of antiderivative = 22.23

$$\int \frac{\sin^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)**3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

output
```
Piecewise((60*A*d*x*tan(c/2 + d*x/2)**7/(15*a**3*d*tan(c/2 + d*x/2)**7 + 7
5*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d
*tan(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2
 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 300*A*d*x*tan(c/2
 + d*x/2)**6/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**
6 + 165*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*
a**3*d*tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*ta
n(c/2 + d*x/2) + 15*a**3*d) + 660*A*d*x*tan(c/2 + d*x/2)**5/(15*a**3*d*tan
(c/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 + d*
x/2)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3
+ 165*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d)
 + 900*A*d*x*tan(c/2 + d*x/2)**4/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*a**3*
d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*tan(c/
2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d*x/
2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 900*A*d*x*tan(c/2 + d*x/
2)**3/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 165
*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*a**3*d*
tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 +
 d*x/2) + 15*a**3*d) + 660*A*d*x*tan(c/2 + d*x/2)**2/(15*a**3*d*tan(c/2 +
d*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2)...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 543 vs. $2(97) = 194$.

Time = 0.12 (sec) , antiderivative size = 543, normalized size of antiderivative = 5.27

$$\int \frac{\sin^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="m
axima")
```

output
```
2/15*(3*A*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 189*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 200*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 160*sin(d*x +
c)^4/(cos(d*x + c) + 1)^4 + 75*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*si
n(d*x + c)^6/(cos(d*x + c) + 1)^6 + 24)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x
 + c) + 1) + 11*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a^3*sin(d*x +
 c)^3/(cos(d*x + c) + 1)^3 + 15*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 +
11*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*a^3*sin(d*x + c)^6/(cos(d*x
 + c) + 1)^6 + a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7) + 15*arctan(sin(d*
x + c)/(cos(d*x + c) + 1))/a^3) + A*((95*sin(d*x + c)/(cos(d*x + c) + 1) +
 145*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 75*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 15*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 22)/(a^3 + 5*a^3*sin(d*
x + c)/(cos(d*x + c) + 1) + 10*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1
0*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x
+ c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5) + 15*arctan(sin(d*x
 + c)/(cos(d*x + c) + 1))/a^3))/d
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.10

$$\int \frac{\sin^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$

$$= \frac{2\left(\frac{30\,(dx+c)A}{a^3} + \frac{15\,A}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^3} + \frac{60\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+285\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+505\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+335\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}\right.}{15\,d}$$

input
```
integrate(sin(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="g
iac")
```

output
```
2/15*(30*(d*x + c)*A/a^3 + 15*A/((tan(1/2*d*x + 1/2*c)^2 + 1)*a^3) + (60*A
*tan(1/2*d*x + 1/2*c)^4 + 285*A*tan(1/2*d*x + 1/2*c)^3 + 505*A*tan(1/2*d*x
 + 1/2*c)^2 + 335*A*tan(1/2*d*x + 1/2*c) + 79*A)/(a^3*(tan(1/2*d*x + 1/2*c
) + 1)^5))/d
```

Mupad [B] (verification not implemented)

Time = 38.48 (sec) , antiderivative size = 261, normalized size of antiderivative = 2.53

$$\int \frac{\sin^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{4\,A\,x}{a^3} - \frac{\left(20\,A\,(c+d\,x)-\frac{4\,A\,(75\,c+75\,d\,x+30)}{15}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\left(44\,A\,(c+d\,x)-\frac{4\,A\,(165\,c+165\,d\,x+150)}{15}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)$$

input
```
int((sin(c + d*x)^3*(A - A*sin(c + d*x)))/(a + a*sin(c + d*x))^3,x)
```

output
```
(4*A*x)/a^3 - (tan(c/2 + (d*x)/2)*(20*A*(c + d*x) - (4*A*(75*c + 75*d*x +
205))/15) + tan(c/2 + (d*x)/2)^6*(20*A*(c + d*x) - (4*A*(75*c + 75*d*x + 3
0))/15) + tan(c/2 + (d*x)/2)^5*(44*A*(c + d*x) - (4*A*(165*c + 165*d*x + 1
50))/15) + tan(c/2 + (d*x)/2)^2*(44*A*(c + d*x) - (4*A*(165*c + 165*d*x +
367))/15) + tan(c/2 + (d*x)/2)^4*(60*A*(c + d*x) - (4*A*(225*c + 225*d*x +
 320))/15) + tan(c/2 + (d*x)/2)^3*(60*A*(c + d*x) - (4*A*(225*c + 225*d*x
+ 385))/15) + 4*A*(c + d*x) - (4*A*(15*c + 15*d*x + 47))/15)/(a^3*d*(tan(c
/2 + (d*x)/2) + 1)^5*(tan(c/2 + (d*x)/2)^2 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.44

$$\int \frac{\sin^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$
$$= \frac{-15\cos(dx+c)\sin(dx+c)^3+60\cos(dx+c)\sin(dx+c)^2dx-79\cos(dx+c)\sin(dx+c)^2+120\cos}{}$$

input
```
int(sin(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x)
```

output
```
( - 15*cos(c + d*x)*sin(c + d*x)**3 + 60*cos(c + d*x)*sin(c + d*x)**2*d*x
- 79*cos(c + d*x)*sin(c + d*x)**2 + 120*cos(c + d*x)*sin(c + d*x)*d*x - 82
*cos(c + d*x)*sin(c + d*x) + 60*cos(c + d*x)*d*x - 24*cos(c + d*x) - 15*si
n(c + d*x)**4 - 60*sin(c + d*x)**3*d*x - 204*sin(c + d*x)**3 - 180*sin(c +
 d*x)**2*d*x - 283*sin(c + d*x)**2 - 180*sin(c + d*x)*d*x - 82*sin(c + d*x
) - 60*d*x + 24)/(15*a**2*d*(cos(c + d*x)*sin(c + d*x)**2 + 2*cos(c + d*x)
*sin(c + d*x) + cos(c + d*x) - sin(c + d*x)**3 - 3*sin(c + d*x)**2 - 3*sin
(c + d*x) - 1))
```

3.237 $\int \frac{\sin^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 89

$$\int \frac{\sin^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{Ax}{a^3}-\frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}+\frac{7A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^2}-\frac{13A\cos(c+dx)}{5a^3d(1+\sin(c+dx))}$$

output
```
-A*x/a^3-2/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^3+7/5*A*cos(d*x+c)/a^3/d/(1
+sin(d*x+c))^2-13/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 4.16 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.26

$$\int \frac{\sin^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{A\sec(c+dx)\sqrt{1-\sin(c+dx)}\left(20\sqrt{2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2},-\frac{1}{2},\frac{1}{2}(1+\sin(c+dx))\right)(1+\sin(c\right.}{15a^3d(1+\sin(c+dx))^2}$$

input
```
Integrate[(Sin[c + d*x]^2*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

output
```
(A*Sec[c + d*x]*Sqrt[1 - Sin[c + d*x]]*(20*Sqrt[2]*Hypergeometric2F1[-3/2,
 -3/2, -1/2, (1 + Sin[c + d*x])/2]*(1 + Sin[c + d*x]) + Sqrt[1 - Sin[c + d
*x]]*(-4 + 3*Sin[c + d*x] + Sin[c + d*x]^2)))/(15*a^3*d*(1 + Sin[c + d*x])
^2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3042, 3445, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(c+dx)(A-A\sin(c+dx))}{(a\sin(c+dx)+a)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)^2(A-A\sin(c+dx))}{(a\sin(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3445}$$

$$\int \left(\frac{4A}{a^3(\sin(c+dx)+1)}-\frac{5A}{a^3(\sin(c+dx)+1)^2}+\frac{2A}{a^3(\sin(c+dx)+1)^3}-\frac{A}{a^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{13A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)}+\frac{7A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^2}-\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}-\frac{Ax}{a^3}$$

input
```
Int[(Sin[c + d*x]^2*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

output
```
-((A*x)/a^3) - (2*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^3) + (7*A*Co
s[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^2) - (13*A*Cos[c + d*x])/(5*a^3*d*
(1 + Sin[c + d*x]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3445
```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96

method	result
risch	$-\frac{Ax}{a^3}-\frac{2\left(-75A\,e^{2i(dx+c)}+55iA\,e^{3i(dx+c)}+20A\,e^{4i(dx+c)}-45iA\,e^{i(dx+c)}+13A\right)}{5d\,a^3\left(e^{i(dx+c)}+i\right)^5}$
derivativedivides	$\frac{8A\left(-\frac{2}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{\arctan(t}{}\right.}{d\,a^3}$
default	$\frac{8A\left(-\frac{2}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{\arctan(t}{}\right.}{d\,a^3}$
parallelrisch	$-\frac{A\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5dx+(5dx+2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+(10dx+10)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+(10dx+22)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+(5dx+14)\tan(\right.}{d\,a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}$
norman	$\frac{-\frac{Ax}{a}-\frac{16A}{5ad}-\frac{5Ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}-\frac{13Ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{a}-\frac{25Ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{a}-\frac{38Ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{a}-\frac{46Ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{a}-\frac{46Ax}{}}{}$

input
```
int(sin(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBO
SE)
```

output
```
-A*x/a^3-2/5*(-75*A*exp(2*I*(d*x+c))+55*I*A*exp(3*I*(d*x+c))+20*A*exp(4*I*
(d*x+c))-45*I*A*exp(I*(d*x+c))+13*A)/d/a^3/(exp(I*(d*x+c))+I)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(83) = 166$.

Time = 0.08 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.29

$$\int \frac{\sin^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{(5\,Adx+13\,A)\cos(dx+c)^3-20\,Adx+3\,(5\,Adx-2\,A)\cos(dx+c)^2-(10\,Adx+21\,A)\cos(dx+c}{5\,(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx+c)^2-2\,a^3d\cos(dx+c)-4\,a}$$

input
```
integrate(sin(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="f
ricas")
```

output
```
-1/5*((5*A*d*x + 13*A)*cos(d*x + c)^3 - 20*A*d*x + 3*(5*A*d*x - 2*A)*cos(d
*x + c)^2 - (10*A*d*x + 21*A)*cos(d*x + c) - (20*A*d*x - (5*A*d*x - 13*A)*
cos(d*x + c)^2 + (10*A*d*x + 19*A)*cos(d*x + c) - 2*A)*sin(d*x + c) - 2*A)
/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4
*a^3*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x +
 c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1268 vs. $2(85) = 170$.

Time = 8.27 (sec) , antiderivative size = 1268, normalized size of antiderivative = 14.25

$$\int \frac{\sin^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(d*x+c)**2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

output
```
Piecewise((-5*A*d*x*tan(c/2 + d*x/2)**5/(5*a**3*d*tan(c/2 + d*x/2)**5 + 25
*a**3*d*tan(c/2 + d*x/2)**4 + 50*a**3*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*ta
n(c/2 + d*x/2)**2 + 25*a**3*d*tan(c/2 + d*x/2) + 5*a**3*d) - 25*A*d*x*tan(
c/2 + d*x/2)**4/(5*a**3*d*tan(c/2 + d*x/2)**5 + 25*a**3*d*tan(c/2 + d*x/2)
**4 + 50*a**3*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*tan(c/2 + d*x/2)**2 + 25*a
**3*d*tan(c/2 + d*x/2) + 5*a**3*d) - 50*A*d*x*tan(c/2 + d*x/2)**3/(5*a**3*
d*tan(c/2 + d*x/2)**5 + 25*a**3*d*tan(c/2 + d*x/2)**4 + 50*a**3*d*tan(c/2
+ d*x/2)**3 + 50*a**3*d*tan(c/2 + d*x/2)**2 + 25*a**3*d*tan(c/2 + d*x/2) +
 5*a**3*d) - 50*A*d*x*tan(c/2 + d*x/2)**2/(5*a**3*d*tan(c/2 + d*x/2)**5 +
25*a**3*d*tan(c/2 + d*x/2)**4 + 50*a**3*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*
tan(c/2 + d*x/2)**2 + 25*a**3*d*tan(c/2 + d*x/2) + 5*a**3*d) - 25*A*d*x*ta
n(c/2 + d*x/2)/(5*a**3*d*tan(c/2 + d*x/2)**5 + 25*a**3*d*tan(c/2 + d*x/2)*
*4 + 50*a**3*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*tan(c/2 + d*x/2)**2 + 25*a*
*3*d*tan(c/2 + d*x/2) + 5*a**3*d) - 5*A*d*x/(5*a**3*d*tan(c/2 + d*x/2)**5
+ 25*a**3*d*tan(c/2 + d*x/2)**4 + 50*a**3*d*tan(c/2 + d*x/2)**3 + 50*a**3*
d*tan(c/2 + d*x/2)**2 + 25*a**3*d*tan(c/2 + d*x/2) + 5*a**3*d) - 10*A*tan(
c/2 + d*x/2)**4/(5*a**3*d*tan(c/2 + d*x/2)**5 + 25*a**3*d*tan(c/2 + d*x/2)
**4 + 50*a**3*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*tan(c/2 + d*x/2)**2 + 25*a
**3*d*tan(c/2 + d*x/2) + 5*a**3*d) - 50*A*tan(c/2 + d*x/2)**3/(5*a**3*d*ta
n(c/2 + d*x/2)**5 + 25*a**3*d*tan(c/2 + d*x/2)**4 + 50*a**3*d*tan(c/2 +...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 392 vs. $2(83) = 166$.

Time = 0.12 (sec) , antiderivative size = 392, normalized size of antiderivative = 4.40

$$\int \frac{\sin^2(c+dx)(A - A\sin(c+dx))}{(a + a\sin(c+dx))^3}\,dx =$$

$$-\frac{2\left(A\left(\frac{\frac{95\sin(dx+c)}{\cos(dx+c)+1}+\frac{145\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{75\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+22}{a^3+\frac{5a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{10a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}+\frac{15\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)+\frac{\cdots}{a^3+\frac{5\cdots}{\cdots}}\cdots\right)}{15\,d}$$

input
```
integrate(sin(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="m
axima")
```

output
```
-2/15*(A*((95*sin(d*x + c)/(cos(d*x + c) + 1) + 145*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 + 75*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 + 22)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) +
 10*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5) + 15*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a
^3) + 2*A*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 1)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^3*sin
(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1
)^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(
d*x + c) + 1)^5))/d
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04

$$\int \frac{\sin^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{\frac{5\,(dx+c)A}{a^3}+\frac{2\left(5\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+25\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+55\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+35\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+8\,A\right)}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}}{5\,d}$$

input
```
integrate(sin(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="g
iac")
```

output
```
-1/5*(5*(d*x + c)*A/a^3 + 2*(5*A*tan(1/2*d*x + 1/2*c)^4 + 25*A*tan(1/2*d*x
 + 1/2*c)^3 + 55*A*tan(1/2*d*x + 1/2*c)^2 + 35*A*tan(1/2*d*x + 1/2*c) + 8*
A)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

Mupad [B] (verification not implemented)

Time = 36.36 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.00

$$\int \frac{\sin^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$
$$= \frac{\left(5\,A\,(c+d\,x)-\frac{A\,(25\,c+25\,d\,x+10)}{5}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\left(10\,A\,(c+d\,x)-\frac{A\,(50\,c+50\,d\,x+50)}{5}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\Big(1}{a}$$
$$-\frac{A\,x}{a^3}$$

input
```
int((sin(c + d*x)^2*(A - A*sin(c + d*x)))/(a + a*sin(c + d*x))^3,x)
```

output
```
(tan(c/2 + (d*x)/2)*(5*A*(c + d*x) - (A*(25*c + 25*d*x + 70))/5) + tan(c/2
 + (d*x)/2)^4*(5*A*(c + d*x) - (A*(25*c + 25*d*x + 10))/5) + tan(c/2 + (d*
x)/2)^3*(10*A*(c + d*x) - (A*(50*c + 50*d*x + 50))/5) + tan(c/2 + (d*x)/2)
^2*(10*A*(c + d*x) - (A*(50*c + 50*d*x + 110))/5) + A*(c + d*x) - (A*(5*c
+ 5*d*x + 16))/5)/(a^3*d*(tan(c/2 + (d*x)/2) + 1)^5) - (A*x)/a^3
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.27

$$\int \frac{\sin^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$
$$= \frac{-5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5dx+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5-25\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4dx-50\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3dx-30\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-}{5a^2d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}$$

input
```
int(sin(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x)
```

output
```
( - 5*tan((c + d*x)/2)**5*d*x + 2*tan((c + d*x)/2)**5 - 25*tan((c + d*x)/2
)**4*d*x - 50*tan((c + d*x)/2)**3*d*x - 30*tan((c + d*x)/2)**3 - 50*tan((c
 + d*x)/2)**2*d*x - 90*tan((c + d*x)/2)**2 - 25*tan((c + d*x)/2)*d*x - 60*
tan((c + d*x)/2) - 5*d*x - 14)/(5*a**2*d*(tan((c + d*x)/2)**5 + 5*tan((c +
 d*x)/2)**4 + 10*tan((c + d*x)/2)**3 + 10*tan((c + d*x)/2)**2 + 5*tan((c +
 d*x)/2) + 1))
```

3.238 $\int \frac{\sin(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 82

$$\int \frac{\sin(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3} - \frac{11A\cos(c+dx)}{15a^3d(1+\sin(c+dx))^2} + \frac{4A\cos(c+dx)}{15a^3d(1+\sin(c+dx))}$$

output
```
2/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^3-11/15*A*cos(d*x+c)/a^3/d/(1+sin(d*
x+c))^2+4/15*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))
```

Mathematica [A] (verified)

Time = 4.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.30

$$\int \frac{\sin(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{A\left(15\cos\left(c+\frac{dx}{2}\right)-5\cos\left(c+\frac{3dx}{2}\right)+25\sin\left(\frac{dx}{2}\right)+15\sin\left(2c+\frac{3dx}{2}\right)-4\sin\left(2c+\frac{5dx}{2}\right)\right)}{30a^3d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}$$

input
```
Integrate[(Sin[c + d*x]*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

output
```
-1/30*(A*(15*Cos[c + (d*x)/2] - 5*Cos[c + (3*d*x)/2] + 25*Sin[(d*x)/2] + 1
5*Sin[2*c + (3*d*x)/2] - 4*Sin[2*c + (5*d*x)/2]))/(a^3*d*(Cos[c/2] + Sin[c
/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3042, 3445, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)(A-A\sin(c+dx))}{(a\sin(c+dx)+a)^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin(c+dx)(A-A\sin(c+dx))}{(a\sin(c+dx)+a)^3}dx$$
$$\Big\downarrow \text{3445}$$
$$\int \left(-\frac{A}{a^3(\sin(c+dx)+1)}+\frac{3A}{a^3(\sin(c+dx)+1)^2}-\frac{2A}{a^3(\sin(c+dx)+1)^3}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{4A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)}-\frac{11A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)^2}+\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}$$

input
```
Int[(Sin[c + d*x]*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

output
```
(2*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^3) - (11*A*Cos[c + d*x])/(1
5*a^3*d*(1 + Sin[c + d*x])^2) + (4*A*Cos[c + d*x])/(15*a^3*d*(1 + Sin[c +
d*x]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3445
```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.76

method	result
parallelrisch	$-\frac{2 A \left(15 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{3}-5 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}+5 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)}{15 d \,a^{3} \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^{5}}$
derivativedivides	$\frac{4 A \left(-\frac{1}{2 \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{2}{\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^{4}}+\frac{4}{5 \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^{5}}+\frac{5}{3 \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^{3}}\right)}{d \,a^{3}}$
default	$\frac{4 A \left(-\frac{1}{2 \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{2}{\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^{4}}+\frac{4}{5 \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^{5}}+\frac{5}{3 \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^{3}}\right)}{d \,a^{3}}$
risch	$\frac{2 A \left(15 i {\mathrm e}^{3 i \left(d x +c \right)}+15 \,{\mathrm e}^{4 i \left(d x +c \right)}-5 i {\mathrm e}^{i \left(d x +c \right)}-25 \,{\mathrm e}^{2 i \left(d x +c \right)}+4\right)}{15 d \,a^{3} \left({\mathrm e}^{i \left(d x +c \right)}+i \right)^{5}}$
norman	$\frac{-\frac{2 A}{15 a d}-\frac{14 A \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{5}}{3 a d}-\frac{2 A \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{3 a d}-\frac{10 A \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{3 a d}-\frac{2 A \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{7}}{a d}+\frac{2 A \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}{5 a d}+\frac{2 A \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{6}}{3 a d}+}{\left(1+\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}\right)^{2} a^{2} \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^{5}}$

input
```
int(sin(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE
)
```

output
```
-2/15*A*(15*tan(1/2*d*x+1/2*c)^3-5*tan(1/2*d*x+1/2*c)^2+5*tan(1/2*d*x+1/2*
c)+1)/d/a^3/(tan(1/2*d*x+1/2*c)+1)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(76) = 152$.

Time = 0.12 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.90

$$\int \frac{\sin(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{4\,A\cos\left(dx+c\right)^3+7\,A\cos\left(dx+c\right)^2-3\,A\cos\left(dx+c\right)-\left(4\,A\cos\left(dx+c\right)^2-3\,A\cos\left(dx+c\right)-\right.}{15\,\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2-2\,a^3d\cos\left(dx+c\right)-4\,a^3d+\left(a^3d\cos\left(dx+c\right)^2-2\,a^3d\cos\left(\right.\right.\right.}$$

input
```
integrate(sin(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fri
cas")
```

output
```
1/15*(4*A*cos(d*x + c)^3 + 7*A*cos(d*x + c)^2 - 3*A*cos(d*x + c) - (4*A*co
s(d*x + c)^2 - 3*A*cos(d*x + c) - 6*A)*sin(d*x + c) - 6*A)/(a^3*d*cos(d*x
+ c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*
cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(78) = 156$.

Time = 4.71 (sec) , antiderivative size = 461, normalized size of antiderivative = 5.62

$$\int \frac{\sin(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \begin{cases} -\frac{30A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{15a^3d\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^3d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^3d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^3d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+15a^3d}+\frac{}{15a^3d\tan^5\left(\frac{c}{2}\right.} \\ \frac{x(-A\sin(c)+A)\sin(c)}{(a\sin(c)+a)^3} \end{cases}$$

input
```
integrate(sin(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

output
```
Piecewise((-30*A*tan(c/2 + d*x/2)**3/(15*a**3*d*tan(c/2 + d*x/2)**5 + 75*a
**3*d*tan(c/2 + d*x/2)**4 + 150*a**3*d*tan(c/2 + d*x/2)**3 + 150*a**3*d*ta
n(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 10*A*tan(c/2
 + d*x/2)**2/(15*a**3*d*tan(c/2 + d*x/2)**5 + 75*a**3*d*tan(c/2 + d*x/2)**
4 + 150*a**3*d*tan(c/2 + d*x/2)**3 + 150*a**3*d*tan(c/2 + d*x/2)**2 + 75*a
**3*d*tan(c/2 + d*x/2) + 15*a**3*d) - 10*A*tan(c/2 + d*x/2)/(15*a**3*d*tan
(c/2 + d*x/2)**5 + 75*a**3*d*tan(c/2 + d*x/2)**4 + 150*a**3*d*tan(c/2 + d*
x/2)**3 + 150*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15
*a**3*d) - 2*A/(15*a**3*d*tan(c/2 + d*x/2)**5 + 75*a**3*d*tan(c/2 + d*x/2)
**4 + 150*a**3*d*tan(c/2 + d*x/2)**3 + 150*a**3*d*tan(c/2 + d*x/2)**2 + 75
*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d), Ne(d, 0)), (x*(-A*sin(c) + A)*sin(c
)/(a*sin(c) + a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(76) = 152$.

Time = 0.04 (sec) , antiderivative size = 348, normalized size of antiderivative = 4.24

$$\int \frac{\sin(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{2\left(\frac{2A\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)}{a^3+\frac{5a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{10a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}} - \frac{3A\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\sin(dx+c)}{(\cos(dx+c)+}\right.}{a^3+\frac{5a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{10a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^3\sin(d}{(\cos(dx+c}}\right.}{15d}$$

input
```
integrate(sin(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="max
ima")
```

output
```
2/15*(2*A*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 1)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^3*sin
(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1
)^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(
d*x + c) + 1)^5) - 3*A*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)
^2/(cos(d*x + c) + 1)^2 + 5*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1)/(a^3
+ 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^3*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*a^3*sin(d*x +
c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5))/d
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.77

$$\int \frac{\sin(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{2\left(15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+5\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+A\right)}{15\,a^3d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}$$

input
```
integrate(sin(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="gia
c")
```

output
```
-2/15*(15*A*tan(1/2*d*x + 1/2*c)^3 - 5*A*tan(1/2*d*x + 1/2*c)^2 + 5*A*tan(
1/2*d*x + 1/2*c) + A)/(a^3*d*(tan(1/2*d*x + 1/2*c) + 1)^5)
```

Mupad [B] (verification not implemented)

Time = 35.00 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.34

$$\int \frac{\sin(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{2\,A\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+5\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-5\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+15}{15\,a^3\,d\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)^5}$$

input
```
int((sin(c + d*x)*(A - A*sin(c + d*x)))/(a + a*sin(c + d*x))^3,x)
```

output
```
-(2*A*cos(c/2 + (d*x)/2)^2*(cos(c/2 + (d*x)/2)^3 + 15*sin(c/2 + (d*x)/2)^3
 - 5*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^2 + 5*cos(c/2 + (d*x)/2)^2*sin(
c/2 + (d*x)/2)))/(15*a^3*d*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))^5)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.37

$$\int \frac{\sin(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$
$$=\frac{-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{3}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3}-\frac{2}{15}}{a^2d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$$

input
```
int(sin(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x)
```

output
```
(2*( - 15*tan((c + d*x)/2)**3 + 5*tan((c + d*x)/2)**2 - 5*tan((c + d*x)/2)
 - 1))/(15*a**2*d*(tan((c + d*x)/2)**5 + 5*tan((c + d*x)/2)**4 + 10*tan((c
 + d*x)/2)**3 + 10*tan((c + d*x)/2)**2 + 5*tan((c + d*x)/2) + 1))
```

3.239 $\int \frac{A-A\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 58

$$\int \frac{A-A\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx=-\frac{aA\cos^3(c+dx)}{5d(a+a\sin(c+dx))^4}-\frac{A\cos^3(c+dx)}{15d(a+a\sin(c+dx))^3}$$

output
```
-1/5*a*A*cos(d*x+c)^3/d/(a+a*sin(d*x+c))^4-1/15*A*cos(d*x+c)^3/d/(a+a*sin(
d*x+c))^3
```

Mathematica [A] (verified)

Time = 3.45 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.59

$$\int \frac{A-A\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx =\frac{A\left(-15\cos\left(c+\frac{dx}{2}\right)+5\cos\left(c+\frac{3dx}{2}\right)+5\sin\left(\frac{dx}{2}\right)+\sin\left(2c+\frac{5dx}{2}\right)\right)}{30a^3d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}$$

input
```
Integrate[(A - A*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

output
```
(A*(-15*Cos[c + (d*x)/2] + 5*Cos[c + (3*d*x)/2] + 5*Sin[(d*x)/2] + Sin[2*c
 + (5*d*x)/2]))/(30*a^3*d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2])^5)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3215, 3042, 3151, 3042, 3150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A-A\sin(c+dx)}{(a\sin(c+dx)+a)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A-A\sin(c+dx)}{(a\sin(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3215}$$

$$aA\int \frac{\cos^2(c+dx)}{(\sin(c+dx)a+a)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$aA\int \frac{\cos(c+dx)^2}{(\sin(c+dx)a+a)^4}dx$$

$$\Big\downarrow \text{3151}$$

$$aA\left(\frac{\int \frac{\cos^2(c+dx)}{(\sin(c+dx)a+a)^3}dx}{5a}-\frac{\cos^3(c+dx)}{5d(a\sin(c+dx)+a)^4}\right)$$

$$\Big\downarrow \text{3042}$$

$$aA\left(\frac{\int \frac{\cos(c+dx)^2}{(\sin(c+dx)a+a)^3}dx}{5a}-\frac{\cos^3(c+dx)}{5d(a\sin(c+dx)+a)^4}\right)$$

$$\Big\downarrow \text{3150}$$

$$aA\left(-\frac{\cos^3(c+dx)}{15ad(a\sin(c+dx)+a)^3}-\frac{\cos^3(c+dx)}{5d(a\sin(c+dx)+a)^4}\right)$$

input
```
Int[(A - A*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

output
```
a*A*(-1/5*Cos[c + d*x]^3/(d*(a + a*Sin[c + d*x])^4) - Cos[c + d*x]^3/(15*a
*d*(a + a*Sin[c + d*x])^3))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3150
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

rule 3151
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Simp[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1])   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simpli
fy[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

rule 3215
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^m*c^m   Int[Cos[e + f*x]^(2*m)*(c +
 d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && ((Lt
Q[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.07

method	result
risch	$\frac{2iA\left(-5i\,\mathrm{e}^{2i(dx+c)}+15\,\mathrm{e}^{3i(dx+c)}-i-5\,\mathrm{e}^{i(dx+c)}\right)}{15d\,a^{3}\left(\mathrm{e}^{i(dx+c)}+i\right)^{5}}$
parallelrisch	$-\frac{2A\left(15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+25\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+4\right)}{15d\,a^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}$
derivativedivides	$\frac{2A\left(\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}+\frac{3}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{14}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{8}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}\right)}{d\,a^{3}}$
default	$\frac{2A\left(\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}+\frac{3}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{14}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{8}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}\right)}{d\,a^{3}}$
norman	$\frac{-\frac{8A}{15ad}-\frac{58A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{15ad}-\frac{8A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3ad}-\frac{16A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{3ad}-\frac{2A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3ad}-\frac{2A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{ad}-\frac{2A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}}{ad}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)a^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}$

input
```
int((A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
2/15*I*A*(-5*I*exp(2*I*(d*x+c))+15*exp(3*I*(d*x+c))-I-5*exp(I*(d*x+c)))/d/
a^3/(exp(I*(d*x+c))+I)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(54) = 108$.

Time = 0.07 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.66

$$\int \frac{A-A\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx$$
$$=\frac{A\cos(dx+c)^{3}-2\,A\cos(dx+c)^{2}+3\,A\cos(dx+c)-\left(A\cos(dx+c)^{2}+3\,A\cos(dx+c)+6\right.}{15\left(a^{3}d\cos(dx+c)^{3}+3\,a^{3}d\cos(dx+c)^{2}-2\,a^{3}d\cos(dx+c)-4\,a^{3}d+\left(a^{3}d\cos(dx+c)^{2}-2\,a^{3}d\cos(\right.\right.}$$

input
```
integrate((A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/15*(A*cos(d*x + c)^3 - 2*A*cos(d*x + c)^2 + 3*A*cos(d*x + c) - (A*cos(d*
x + c)^2 + 3*A*cos(d*x + c) + 6*A)*sin(d*x + c) + 6*A)/(a^3*d*cos(d*x + c)
^3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos(
d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(53) = 106$.

Time = 2.56 (sec) , antiderivative size = 573, normalized size of antiderivative = 9.88

$$\int \frac{A - A\sin(c + dx)}{(a + a\sin(c + dx))^3}\, dx = \text{Too large to display}$$

input
```
integrate((A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

output
```
Piecewise((-30*A*tan(c/2 + d*x/2)**4/(15*a**3*d*tan(c/2 + d*x/2)**5 + 75*a
**3*d*tan(c/2 + d*x/2)**4 + 150*a**3*d*tan(c/2 + d*x/2)**3 + 150*a**3*d*ta
n(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) - 30*A*tan(c/2
 + d*x/2)**3/(15*a**3*d*tan(c/2 + d*x/2)**5 + 75*a**3*d*tan(c/2 + d*x/2)**
4 + 150*a**3*d*tan(c/2 + d*x/2)**3 + 150*a**3*d*tan(c/2 + d*x/2)**2 + 75*a
**3*d*tan(c/2 + d*x/2) + 15*a**3*d) - 50*A*tan(c/2 + d*x/2)**2/(15*a**3*d*
tan(c/2 + d*x/2)**5 + 75*a**3*d*tan(c/2 + d*x/2)**4 + 150*a**3*d*tan(c/2 +
 d*x/2)**3 + 150*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) +
 15*a**3*d) - 10*A*tan(c/2 + d*x/2)/(15*a**3*d*tan(c/2 + d*x/2)**5 + 75*a*
*3*d*tan(c/2 + d*x/2)**4 + 150*a**3*d*tan(c/2 + d*x/2)**3 + 150*a**3*d*tan
(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) - 8*A/(15*a**3*
d*tan(c/2 + d*x/2)**5 + 75*a**3*d*tan(c/2 + d*x/2)**4 + 150*a**3*d*tan(c/2
 + d*x/2)**3 + 150*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2)
 + 15*a**3*d), Ne(d, 0)), (x*(-A*sin(c) + A)/(a*sin(c) + a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(54) = 108$.

Time = 0.04 (sec) , antiderivative size = 387, normalized size of antiderivative = 6.67

$$\int \frac{A - A\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx = -\frac{2\left(\frac{A\left(\frac{20\sin(dx+c)}{\cos(dx+c)+1}+\frac{40\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{30\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+7\right)}{a^3+\frac{5a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{10a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}} - \frac{3A\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\cdots\right)}{a^3+\frac{5a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{10a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\cdots}\right)}{15d}$$

input
```
integrate((A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
-2/15*(A*(20*sin(d*x + c)/(cos(d*x + c) + 1) + 40*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 30*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 + 7)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10
*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)
^5/(cos(d*x + c) + 1)^5) - 3*A*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(
d*x + c)^2/(cos(d*x + c) + 1)^2 + 5*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 +
1)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^3*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*a^3*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^
5))/d
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.36

$$\int \frac{A - A\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx = -\frac{2\left(15A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+15A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+25A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+5A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+4\cdots\right)}{15a^3d\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^5}$$

input
```
integrate((A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
-2/15*(15*A*tan(1/2*d*x + 1/2*c)^4 + 15*A*tan(1/2*d*x + 1/2*c)^3 + 25*A*ta
n(1/2*d*x + 1/2*c)^2 + 5*A*tan(1/2*d*x + 1/2*c) + 4*A)/(a^3*d*(tan(1/2*d*x
 + 1/2*c) + 1)^5)
```

Mupad [B] (verification not implemented)

Time = 35.15 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.31

$$\int \frac{A-A\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx = -\frac{2\,A\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(4\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+25\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\right.}{15\,a^3\,d\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)^5}$$

input
```
int((A - A*sin(c + d*x))/(a + a*sin(c + d*x))^3,x)
```

output
```
-(2*A*cos(c/2 + (d*x)/2)*(4*cos(c/2 + (d*x)/2)^4 + 15*sin(c/2 + (d*x)/2)^4
 + 15*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^3 + 5*cos(c/2 + (d*x)/2)^3*sin
(c/2 + (d*x)/2) + 25*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2))/(15*a^3*d
*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))^5)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.16

$$\int \frac{A-A\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{5}+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{3}+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3}-\frac{2}{15}}{a^2d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$$

input
```
int((A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x)
```

output
```
(2*(3*tan((c + d*x)/2)**5 + 15*tan((c + d*x)/2)**3 + 5*tan((c + d*x)/2)**2
 + 10*tan((c + d*x)/2) - 1))/(15*a**2*d*(tan((c + d*x)/2)**5 + 5*tan((c +
d*x)/2)**4 + 10*tan((c + d*x)/2)**3 + 10*tan((c + d*x)/2)**2 + 5*tan((c +
d*x)/2) + 1))
```

3.240 $\int \frac{\csc(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3} dx$

Optimal result

Integrand size = 30, antiderivative size = 98

$$\int \frac{\csc(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3} dx = -\frac{A\text{arctanh}(\cos(c+dx))}{a^3d} + \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3} + \frac{3A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^2} + \frac{8A\cos(c+dx)}{5a^3d(1+\sin(c+dx))}$$

output

```
-A*arctanh(cos(d*x+c))/a^3/d+2/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^3+3/5*A
*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^2+8/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))
```

Mathematica [A] (verified)

Time = 5.31 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.72

$$\int \frac{\csc(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{A\left(15\cos(c+dx)-2\cos(3(c+dx))-5\text{arctanh}(\cos(c+dx))(1+\sin(c+dx))^3+\frac{19}{2}\sin(2(c+dx))\right)}{5a^3d(1+\sin(c+dx))^3}$$

input
```
Integrate[(Csc[c + d*x]*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

output
```
(A*(15*Cos[c + d*x] - 2*Cos[3*(c + d*x)] - 5*ArcTanh[Cos[c + d*x]]*(1 + Si
n[c + d*x])^3 + (19*Sin[2*(c + d*x)])/2))/(5*a^3*d*(1 + Sin[c + d*x])^3)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3042, 3445, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(c+dx)(A-A\sin(c+dx))}{(a\sin(c+dx)+a)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A-A\sin(c+dx)}{\sin(c+dx)(a\sin(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3445}$$

$$\int \left(-\frac{A}{a^3(\sin(c+dx)+1)}-\frac{A}{a^3(\sin(c+dx)+1)^2}-\frac{2A}{a^3(\sin(c+dx)+1)^3}+\frac{A\csc(c+dx)}{a^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{A\text{arctanh}(\cos(c+dx))}{a^3d}+\frac{8A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)}+\frac{3A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^2}+\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}$$

input
```
Int[(Csc[c + d*x]*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

output
```
-((A*ArcTanh[Cos[c + d*x]])/(a^3*d)) + (2*A*Cos[c + d*x])/(5*a^3*d*(1 + Si
n[c + d*x])^3) + (3*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^2) + (8*A*
Cos[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3445
```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{A\left(\frac{16}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}-\frac{8}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}+\frac{12}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{10}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{8}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{d\,a^{3}}$
default	$\frac{A\left(\frac{16}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}-\frac{8}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}+\frac{12}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{10}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{8}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{d\,a^{3}}$
parallelrisch	$\frac{A\left(\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+22\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+30\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+18\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{26}{5}\right.}{d\,a^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}$
risch	$\frac{2A\left(25i\,\mathrm{e}^{3i(dx+c)}+5\,\mathrm{e}^{4i(dx+c)}-35i\,\mathrm{e}^{i(dx+c)}-55\,\mathrm{e}^{2i(dx+c)}+8\right)}{5d\,a^{3}\left(\mathrm{e}^{i(dx+c)}+i\right)^{5}}-\frac{A\ln\left(\mathrm{e}^{i(dx+c)}+1\right)}{d\,a^{3}}+\frac{A\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{d\,a^{3}}$
norman	$\frac{\frac{8A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}}{ad}+\frac{18A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{38A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{ad}+\frac{26A}{5ad}+\frac{22A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{ad}+\frac{40A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{ad}+\frac{176A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{5ad}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)a^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}$

input
```
int(csc(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE
)
```

output
```
1/d*A/a^3*(16/5/(tan(1/2*d*x+1/2*c)+1)^5-8/(tan(1/2*d*x+1/2*c)+1)^4+12/(ta
n(1/2*d*x+1/2*c)+1)^3-10/(tan(1/2*d*x+1/2*c)+1)^2+8/(tan(1/2*d*x+1/2*c)+1)
+ln(tan(1/2*d*x+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(92) = 184.

Time = 0.09 (sec) , antiderivative size = 310, normalized size of antiderivative = 3.16

$$\int \frac{\csc(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$
$$= \frac{16\,A\cos\left(dx+c\right)^{3}-22\,A\cos\left(dx+c\right)^{2}-42\,A\cos\left(dx+c\right)-5\,\left(A\cos\left(dx+c\right)^{3}+3\,A\cos\left(dx+c\right)^{2}-2\,A\right.}{}$$

input
```
integrate(csc(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fri
cas")
```

output
```
1/10*(16*A*cos(d*x + c)^3 - 22*A*cos(d*x + c)^2 - 42*A*cos(d*x + c) - 5*(A
*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 - 2*A*cos(d*x + c) + (A*cos(d*x + c)^
2 - 2*A*cos(d*x + c) - 4*A)*sin(d*x + c) - 4*A)*log(1/2*cos(d*x + c) + 1/2
) + 5*(A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 - 2*A*cos(d*x + c) + (A*cos(d
*x + c)^2 - 2*A*cos(d*x + c) - 4*A)*sin(d*x + c) - 4*A)*log(-1/2*cos(d*x +
 c) + 1/2) - 2*(8*A*cos(d*x + c)^2 + 19*A*cos(d*x + c) - 2*A)*sin(d*x + c)
 - 4*A)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x +
 c) - 4*a^3*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d)*si
n(d*x + c))
```

Sympy [F]

$$\int \frac{\csc(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{A\left(\int\left(-\frac{\csc(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\right)dx+\int\frac{\sin(c+dx)\csc(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx\right)}{a^3}$$

input
```
integrate(csc(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

output
```
-A*(Integral(-csc(c + d*x)/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c
+ d*x) + 1), x) + Integral(sin(c + d*x)*csc(c + d*x)/(sin(c + d*x)**3 + 3*
sin(c + d*x)**2 + 3*sin(c + d*x) + 1), x))/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(92) = 184$.

Time = 0.04 (sec) , antiderivative size = 433, normalized size of antiderivative = 4.42

$$\int \frac{\csc(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{A\left(\frac{2\left(\frac{115\sin(dx+c)}{\cos(dx+c)+1}+\frac{185\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{135\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{45\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+32\right)}{a^3+\frac{5a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{10a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}+\frac{15\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)+\frac{2A\left(20\sin\ldots\right)}{a^3+\frac{5a^3\sin(dx+c)}{\cos(dx+c)+1}\ldots}}{15d}$$

input
```
integrate(csc(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="max
ima")
```

output
```
1/15*(A*(2*(115*sin(d*x + c)/(cos(d*x + c) + 1) + 185*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 135*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 45*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 + 32)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1
) + 10*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(co
s(d*x + c) + 1)^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*
x + c)^5/(cos(d*x + c) + 1)^5) + 15*log(sin(d*x + c)/(cos(d*x + c) + 1))/a
^3) + 2*A*(20*sin(d*x + c)/(cos(d*x + c) + 1) + 40*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 30*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^4/
(cos(d*x + c) + 1)^4 + 7)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 1
0*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c
)^5/(cos(d*x + c) + 1)^5))/d
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01

$$\int \frac{\csc(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{\frac{5A\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3}+\frac{2\left(20A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+55A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+75A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+45A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+13A\right)}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^5}}{5d}$$

input
```
integrate(csc(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="gia
c")
```

output
```
1/5*(5*A*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + 2*(20*A*tan(1/2*d*x + 1/2*c)
^4 + 55*A*tan(1/2*d*x + 1/2*c)^3 + 75*A*tan(1/2*d*x + 1/2*c)^2 + 45*A*tan(
1/2*d*x + 1/2*c) + 13*A)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

Mupad [B] (verification not implemented)

Time = 37.13 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.03

$$\int \frac{\csc(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{A\left(5\ \ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)+90\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+150\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+110\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+40\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\right.}{}$$

input
```
int((A - A*sin(c + d*x))/(sin(c + d*x)*(a + a*sin(c + d*x))^3),x)
```

output
```
(A*(5*log(tan(c/2 + (d*x)/2)) + 90*tan(c/2 + (d*x)/2) + 150*tan(c/2 + (d*x
)/2)^2 + 110*tan(c/2 + (d*x)/2)^3 + 40*tan(c/2 + (d*x)/2)^4 + 25*log(tan(c
/2 + (d*x)/2))*tan(c/2 + (d*x)/2) + 50*log(tan(c/2 + (d*x)/2))*tan(c/2 + (
d*x)/2)^2 + 50*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^3 + 25*log(tan(c
/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^4 + 5*log(tan(c/2 + (d*x)/2))*tan(c/2 +
(d*x)/2)^5 + 26))/(5*a^3*d*(tan(c/2 + (d*x)/2) + 1)^5)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.55

$$\int \frac{\csc(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{5\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+25\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+50\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\tan\left(\frac{dx}{2}+\right.}{5a^2d\left(\tan\right.}$$

input
```
int(csc(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x)
```

output
```
(5*log(tan((c + d*x)/2))*tan((c + d*x)/2)**5 + 25*log(tan((c + d*x)/2))*ta
n((c + d*x)/2)**4 + 50*log(tan((c + d*x)/2))*tan((c + d*x)/2)**3 + 50*log(
tan((c + d*x)/2))*tan((c + d*x)/2)**2 + 25*log(tan((c + d*x)/2))*tan((c +
d*x)/2) + 5*log(tan((c + d*x)/2)) - 8*tan((c + d*x)/2)**5 + 30*tan((c + d*
x)/2)**3 + 70*tan((c + d*x)/2)**2 + 50*tan((c + d*x)/2) + 18)/(5*a**2*d*(t
an((c + d*x)/2)**5 + 5*tan((c + d*x)/2)**4 + 10*tan((c + d*x)/2)**3 + 10*t
an((c + d*x)/2)**2 + 5*tan((c + d*x)/2) + 1))
```

3.241 $\int \frac{\csc^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3} dx$

Optimal result

Integrand size = 32, antiderivative size = 113

$$\int \frac{\csc^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3} dx = \frac{4A\text{arctanh}(\cos(c+dx))}{a^3d} - \frac{A\cot(c+dx)}{a^3d} - \frac{2A\cot(c+dx)}{5a^3d(1+\csc(c+dx))^3} + \frac{31A\cot(c+dx)}{15a^3d(1+\csc(c+dx))^2} - \frac{104A\cot(c+dx)}{15a^3d(1+\csc(c+dx))}$$

output
```
4*A*arctanh(cos(d*x+c))/a^3/d-A*cot(d*x+c)/a^3/d-2/5*A*cot(d*x+c)/a^3/d/(1
+csc(d*x+c))^3+31/15*A*cot(d*x+c)/a^3/d/(1+csc(d*x+c))^2-104/15*A*cot(d*x+
c)/a^3/d/(1+csc(d*x+c))
```

Mathematica [A] (verified)

Time = 4.27 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.48

$$\int \frac{\csc^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{A\left(15\cot\left(\frac{1}{2}(c+dx)\right)-120\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+120\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{12}{(\cos(\frac{1}{2}(c+dx))+\sin(\frac{1}{2}(c+dx$$

$$30a$$

input
```
Integrate[(Csc[c + d*x]^2*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

output
```
-1/30*(A*(15*Cot[(c + d*x)/2] - 120*Log[Cos[(c + d*x)/2]] + 120*Log[Sin[(c
 + d*x)/2]] + 12/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 + 38/(Cos[(c + d*
x)/2] + Sin[(c + d*x)/2])^2 + (2*Sin[(c + d*x)/2]*(-287 + 79*Cos[2*(c + d*
x)] - 354*Sin[c + d*x]))/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5 - 15*Tan[
(c + d*x)/2]))/(a^3*d)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3042, 3429, 3042, 3188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(c+dx)(A-A\sin(c+dx))}{(a\sin(c+dx)+a)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A-A\sin(c+dx)}{\sin(c+dx)^2(a\sin(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3429}$$

$$aA\int \frac{\cot^2(c+dx)}{(\sin(c+dx)a+a)^4}dx$$

$$\downarrow \text{3042}$$

$$aA\int \frac{1}{(\sin(c+dx)a+a)^4\tan(c+dx)^2}dx$$

$$\downarrow \text{3188}$$

$$\frac{A\int \left(\frac{\csc^2(c+dx)}{a^2}-\frac{4\csc(c+dx)}{a^2}-\frac{16}{a^2(\csc(c+dx)+1)}+\frac{9}{a^2}+\frac{9}{a^2(\csc(c+dx)+1)^2}-\frac{2}{a^2(\csc(c+dx)+1)^3}\right)dx}{a}$$

$$\downarrow \text{2009}$$

$$\frac{A\left(\frac{4\text{arctanh}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{a^2d}-\frac{104\cot(c+dx)}{15a^2d(\csc(c+dx)+1)}+\frac{31\cot(c+dx)}{15a^2d(\csc(c+dx)+1)^2}-\frac{2\cot(c+dx)}{5a^2d(\csc(c+dx)+1)^3}\right)}{a}$$

input
```
Int[(Csc[c + d*x]^2*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

output
```
(A*((4*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cot[c + d*x]/(a^2*d) - (2*Cot[c +
d*x])/(5*a^2*d*(1 + Csc[c + d*x])^3) + (31*Cot[c + d*x])/(15*a^2*d*(1 + Cs
c[c + d*x])^2) - (104*Cot[c + d*x])/(15*a^2*d*(1 + Csc[c + d*x]))))/a
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3188
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Simp[a^p   Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
 + f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b,
e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m
- p/2, 0])
```

rule 3429
```
Int[sin[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^n*c^n
  Int[Tan[e + f*x]^p*(a + b*Sin[e + f*x])^(m - n), x], x] /; FreeQ[{a, b, c
, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[p + 2*n,
 0] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{A\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-8\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{32}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{16}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{88}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\cdots\right.}{2 d a^3}$
default	$\frac{A\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-8\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{32}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{16}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{88}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\cdots\right.}{2 d a^3}$
parallelrisch	$-\frac{\left(8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+52\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+161\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\frac{649\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{3}+\cdots\right.}{2 d a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}$
risch	$-\frac{4\left(-320A\,e^{4i(dx+c)}+150iA\,e^{5i(dx+c)}+367A\,e^{2i(dx+c)}-385iA\,e^{3i(dx+c)}-47A+205iA\,e^{i(dx+c)}+30A\,e^{6i(dx+c)}\right)}{15\left(e^{2i(dx+c)}-1\right)\left(e^{i(dx+c)}+i\right)^5 d\,a^3}-\cdots$
norman	$\frac{-\frac{A}{2ad}+\frac{A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{2ad}-\frac{3811A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{30ad}-\frac{893A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{6ad}-\frac{413A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{6ad}-\frac{161A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{2ad}-\frac{805A\tan\left(\frac{dx}{2}\cdots\right.}{6ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}$

input
```
int(csc(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBO
SE)
```

output
```
1/2/d*A/a^3*(tan(1/2*d*x+1/2*c)-1/tan(1/2*d*x+1/2*c)-8*ln(tan(1/2*d*x+1/2*
c))-32/5/(tan(1/2*d*x+1/2*c)+1)^5+16/(tan(1/2*d*x+1/2*c)+1)^4-88/3/(tan(1/
2*d*x+1/2*c)+1)^3+28/(tan(1/2*d*x+1/2*c)+1)^2-36/(tan(1/2*d*x+1/2*c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(107) = 214$.

Time = 0.09 (sec) , antiderivative size = 406, normalized size of antiderivative = 3.59

$$\int \frac{\csc^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$
$$= \frac{94\,A\cos(dx+c)^4 + 222\,A\cos(dx+c)^3 - 115\,A\cos(dx+c)^2 - 237\,A\cos(dx+c) + 30\,\big(A\cos(dx+c)^4}{}$$

input
```
integrate(csc(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="f
ricas")
```

output
```
1/15*(94*A*cos(d*x + c)^4 + 222*A*cos(d*x + c)^3 - 115*A*cos(d*x + c)^2 -
237*A*cos(d*x + c) + 30*(A*cos(d*x + c)^4 - 2*A*cos(d*x + c)^3 - 5*A*cos(d
*x + c)^2 + 2*A*cos(d*x + c) - (A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 - 2*
A*cos(d*x + c) - 4*A)*sin(d*x + c) + 4*A)*log(1/2*cos(d*x + c) + 1/2) - 30
*(A*cos(d*x + c)^4 - 2*A*cos(d*x + c)^3 - 5*A*cos(d*x + c)^2 + 2*A*cos(d*x
 + c) - (A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 - 2*A*cos(d*x + c) - 4*A)*s
in(d*x + c) + 4*A)*log(-1/2*cos(d*x + c) + 1/2) + (94*A*cos(d*x + c)^3 - 1
28*A*cos(d*x + c)^2 - 243*A*cos(d*x + c) - 6*A)*sin(d*x + c) + 6*A)/(a^3*d
*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^3 - 5*a^3*d*cos(d*x + c)^2 + 2*a^3*
d*cos(d*x + c) + 4*a^3*d - (a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2
- 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\csc^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{A\left(\int\left(-\frac{\csc^2(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\right)dx + \int\frac{\sin(c+dx)\csc^2(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx\right)}{a^3}$$

input
```
integrate(csc(d*x+c)**2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

output
```
-A*(Integral(-csc(c + d*x)**2/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin
(c + d*x) + 1), x) + Integral(sin(c + d*x)*csc(c + d*x)**2/(sin(c + d*x)**
3 + 3*sin(c + d*x)**2 + 3*sin(c + d*x) + 1), x))/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 519 vs. $2(107) = 214$.

Time = 0.05 (sec) , antiderivative size = 519, normalized size of antiderivative = 4.59

$$\int \frac{\csc^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(csc(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="m
axima")
```

output
```
-1/30*(3*A*((121*sin(d*x + c)/(cos(d*x + c) + 1) + 410*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + 610*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 425*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4 + 125*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5)/
(a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*a^3*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 10*a^3*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 + 5*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + a^3*s
in(d*x + c)^6/(cos(d*x + c) + 1)^6) + 30*log(sin(d*x + c)/(cos(d*x + c) +
1))/a^3 - 5*sin(d*x + c)/(a^3*(cos(d*x + c) + 1))) + 2*A*(2*(115*sin(d*x +
 c)/(cos(d*x + c) + 1) + 185*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 135*sin
(d*x + c)^3/(cos(d*x + c) + 1)^3 + 45*sin(d*x + c)^4/(cos(d*x + c) + 1)^4
+ 32)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^3*sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*a^3
*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(d*x + c) +
1)^5) + 15*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3))/d
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.29

$$\int \frac{\csc^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx =$$

$$-\frac{\frac{120\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}-\frac{15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3}-\frac{15\,\left(8\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-A\right)}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{4\,\left(135\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+435\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3\right.}{a^3\left(\tan\left(\right.\right.}}{30\,d}$$

input
```
integrate(csc(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="g
iac")
```

output
```
-1/30*(120*A*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 15*A*tan(1/2*d*x + 1/2*c
)/a^3 - 15*(8*A*tan(1/2*d*x + 1/2*c) - A)/(a^3*tan(1/2*d*x + 1/2*c)) + 4*(
135*A*tan(1/2*d*x + 1/2*c)^4 + 435*A*tan(1/2*d*x + 1/2*c)^3 + 605*A*tan(1/
2*d*x + 1/2*c)^2 + 385*A*tan(1/2*d*x + 1/2*c) + 104*A)/(a^3*(tan(1/2*d*x +
 1/2*c) + 1)^5))/d
```

Mupad [B] (verification not implemented)

Time = 37.96 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.86

$$\int \frac{\csc^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,a^3\,d}-\frac{4\,A\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^3\,d}$$

$$-\frac{37\,A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+121\,A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+\frac{514\,A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3}+\frac{338\,A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{3}+\frac{491\,A\,\mathrm{t}}{}}{d\,\left(2\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+10\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+20\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+20\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+10\,a^3\tan\left(\right.\right.}$$

input
```
int((A - A*sin(c + d*x))/(sin(c + d*x)^2*(a + a*sin(c + d*x))^3),x)
```

output
```
(A*tan(c/2 + (d*x)/2))/(2*a^3*d) - (4*A*log(tan(c/2 + (d*x)/2)))/(a^3*d) -
 (A + (491*A*tan(c/2 + (d*x)/2))/15 + (338*A*tan(c/2 + (d*x)/2)^2)/3 + (51
4*A*tan(c/2 + (d*x)/2)^3)/3 + 121*A*tan(c/2 + (d*x)/2)^4 + 37*A*tan(c/2 +
(d*x)/2)^5)/(d*(10*a^3*tan(c/2 + (d*x)/2)^2 + 20*a^3*tan(c/2 + (d*x)/2)^3
+ 20*a^3*tan(c/2 + (d*x)/2)^4 + 10*a^3*tan(c/2 + (d*x)/2)^5 + 2*a^3*tan(c/
2 + (d*x)/2)^6 + 2*a^3*tan(c/2 + (d*x)/2)))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.64

$$\int \frac{\csc^2(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$
$$= \frac{-120\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6-600\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5-1200\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\mathrm{t}}{}$$

input
```
int(csc(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x)
```

output
```
( - 120*log(tan((c + d*x)/2))*tan((c + d*x)/2)**6 - 600*log(tan((c + d*x)/
2))*tan((c + d*x)/2)**5 - 1200*log(tan((c + d*x)/2))*tan((c + d*x)/2)**4 -
 1200*log(tan((c + d*x)/2))*tan((c + d*x)/2)**3 - 600*log(tan((c + d*x)/2)
)*tan((c + d*x)/2)**2 - 120*log(tan((c + d*x)/2))*tan((c + d*x)/2) + 15*ta
n((c + d*x)/2)**7 + 156*tan((c + d*x)/2)**6 - 855*tan((c + d*x)/2)**4 - 16
85*tan((c + d*x)/2)**3 - 1270*tan((c + d*x)/2)**2 - 410*tan((c + d*x)/2) -
 15)/(30*tan((c + d*x)/2)*a**2*d*(tan((c + d*x)/2)**5 + 5*tan((c + d*x)/2)
**4 + 10*tan((c + d*x)/2)**3 + 10*tan((c + d*x)/2)**2 + 5*tan((c + d*x)/2)
 + 1))
```

3.242 $\int \frac{\csc^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 138

$$\int \frac{\csc^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{19A\text{arctanh}(\cos(c+dx))}{2a^3d}+\frac{4A\cot(c+dx)}{a^3d}-\frac{A\cot(c+dx)\csc(c+dx)}{2a^3d}+\frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3}+\frac{29A\cos(c+dx)}{15a^3d(1+\sin(c+dx))^2}+\frac{164A\cos(c+dx)}{15a^3d(1+\sin(c+dx))}$$

output

```
-19/2*A*arctanh(cos(d*x+c))/a^3/d+4*A*cot(d*x+c)/a^3/d-1/2*A*cot(d*x+c)*cs
c(d*x+c)/a^3/d+2/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^3+29/15*A*cos(d*x+c)/
a^3/d/(1+sin(d*x+c))^2+164/15*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))
```

Mathematica [A] (verified)

Time = 5.29 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.78

$$\int \frac{\csc^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{A\left(240\cot\left(\frac{1}{2}(c+dx)\right)-15\csc^2\left(\frac{1}{2}(c+dx)\right)-1140\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+1140\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right.}{}$$

input
```
Integrate[(Csc[c + d*x]^3*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

output
```
(A*(240*Cot[(c + d*x)/2] - 15*Csc[(c + d*x)/2]^2 - 1140*Log[Cos[(c + d*x)/
2]] + 1140*Log[Sin[(c + d*x)/2]] + 15*Sec[(c + d*x)/2]^2 - (96*Sin[(c + d*
x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5 + 48/(Cos[(c + d*x)/2] + Si
n[(c + d*x)/2])^4 - (464*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2])^3 + 232/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - (2624*Sin[(c + d*
x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) - 240*Tan[(c + d*x)/2]))/(120
*a^3*d)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3042, 3445, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(c+dx)(A-A\sin(c+dx))}{(a\sin(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A-A\sin(c+dx)}{\sin(c+dx)^3(a\sin(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3445}$$

$$\int \left(-\frac{9A}{a^3(\sin(c+dx)+1)}-\frac{5A}{a^3(\sin(c+dx)+1)^2}-\frac{2A}{a^3(\sin(c+dx)+1)^3}+\frac{A\csc^3(c+dx)}{a^3}-\frac{4A\csc^2(c+dx)}{a^3}+\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{19A\text{arctanh}(\cos(c+dx))}{2a^3d}+\frac{4A\cot(c+dx)}{a^3d}+\frac{164A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)}+\frac{29A\cos(c+dx)}{15a^3d(\sin(c+dx)+1)^2}+\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}-\frac{A\cot(c+dx)\csc(c+dx)}{2a^3d}$$

input
```
Int[(Csc[c + d*x]^3*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

output
```
(-19*A*ArcTanh[Cos[c + d*x]])/(2*a^3*d) + (4*A*Cot[c + d*x])/(a^3*d) - (A*
Cot[c + d*x]*Csc[c + d*x])/(2*a^3*d) + (2*A*Cos[c + d*x])/(5*a^3*d*(1 + Si
n[c + d*x])^3) + (29*A*Cos[c + d*x])/(15*a^3*d*(1 + Sin[c + d*x])^2) + (16
4*A*Cos[c + d*x])/(15*a^3*d*(1 + Sin[c + d*x]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3445
```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{A\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{2}-8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{64}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}-\frac{32}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}+\frac{208}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{72}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\right.}{4d\,a^{3}}$
default	$\frac{A\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{2}-8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{64}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}-\frac{32}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}+\frac{208}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{72}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\right.}{4d\,a^{3}}$
parallelrisch	$-\frac{A\left(-76\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}+11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-472\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-1504\tan\left(\frac{dx}{2}+\right.\right.}{8d\,a^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}$
risch	$\frac{A\left(1425i\,e^{7i(dx+c)}+285\,e^{8i(dx+c)}-5225i\,e^{5i(dx+c)}-3325\,e^{6i(dx+c)}+5635i\,e^{3i(dx+c)}+6423\,e^{4i(dx+c)}-1955i\,e^{i(dx+c)}-3\right.}{15\left(e^{2i(dx+c)}-1\right)^{2}\left(e^{i(dx+c)}+i\right)^{5}d\,a^{3}}$
norman	$\frac{\frac{57A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}}{ad}-\frac{A}{8ad}+\frac{11A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{11A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{8ad}+\frac{A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{8ad}+\frac{1943A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{12ad}+\frac{2627A\tan\left(\frac{dx}{2}+\right.}{60ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)a^{2}\left(\tan\left(\right.\right.}$

input
```
int(csc(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBO
SE)
```

output
```
1/4/d*A/a^3*(1/2*tan(1/2*d*x+1/2*c)^2-8*tan(1/2*d*x+1/2*c)+64/5/(tan(1/2*d
*x+1/2*c)+1)^5-32/(tan(1/2*d*x+1/2*c)+1)^4+208/3/(tan(1/2*d*x+1/2*c)+1)^3-
72/(tan(1/2*d*x+1/2*c)+1)^2+128/(tan(1/2*d*x+1/2*c)+1)-1/2/tan(1/2*d*x+1/2
*c)^2+8/tan(1/2*d*x+1/2*c)+38*ln(tan(1/2*d*x+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(128) = 256$.

Time = 0.09 (sec) , antiderivative size = 498, normalized size of antiderivative = 3.61

$$\int \frac{\csc^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{896\,A\cos(dx+c)^5 - 1222\,A\cos(dx+c)^4 - 3218\,A\cos(dx+c)^3 + 1168\,A\cos(dx+c)^2 + 2292\,A\cos(d}{}$$

input
```
integrate(csc(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="f
ricas")
```

output
```
1/60*(896*A*cos(d*x + c)^5 - 1222*A*cos(d*x + c)^4 - 3218*A*cos(d*x + c)^3
 + 1168*A*cos(d*x + c)^2 + 2292*A*cos(d*x + c) - 285*(A*cos(d*x + c)^5 + 3
*A*cos(d*x + c)^4 - 3*A*cos(d*x + c)^3 - 7*A*cos(d*x + c)^2 + 2*A*cos(d*x
+ c) + (A*cos(d*x + c)^4 - 2*A*cos(d*x + c)^3 - 5*A*cos(d*x + c)^2 + 2*A*c
os(d*x + c) + 4*A)*sin(d*x + c) + 4*A)*log(1/2*cos(d*x + c) + 1/2) + 285*(
A*cos(d*x + c)^5 + 3*A*cos(d*x + c)^4 - 3*A*cos(d*x + c)^3 - 7*A*cos(d*x +
 c)^2 + 2*A*cos(d*x + c) + (A*cos(d*x + c)^4 - 2*A*cos(d*x + c)^3 - 5*A*co
s(d*x + c)^2 + 2*A*cos(d*x + c) + 4*A)*sin(d*x + c) + 4*A)*log(-1/2*cos(d*
x + c) + 1/2) - 2*(448*A*cos(d*x + c)^4 + 1059*A*cos(d*x + c)^3 - 550*A*co
s(d*x + c)^2 - 1134*A*cos(d*x + c) + 12*A)*sin(d*x + c) + 24*A)/(a^3*d*cos
(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 - 3*a^3*d*cos(d*x + c)^3 - 7*a^3*d*co
s(d*x + c)^2 + 2*a^3*d*cos(d*x + c) + 4*a^3*d + (a^3*d*cos(d*x + c)^4 - 2*
a^3*d*cos(d*x + c)^3 - 5*a^3*d*cos(d*x + c)^2 + 2*a^3*d*cos(d*x + c) + 4*a
^3*d)*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\csc^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{A\left(\int\left(-\frac{\csc^3(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\right)dx + \int\frac{\sin(c+dx)\csc^3(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx\right)}{a^3}$$

input
```
integrate(csc(d*x+c)**3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

output
```
-A*(Integral(-csc(c + d*x)**3/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin
(c + d*x) + 1), x) + Integral(sin(c + d*x)*csc(c + d*x)**3/(sin(c + d*x)**
3 + 3*sin(c + d*x)**2 + 3*sin(c + d*x) + 1), x))/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 622 vs. $2(128) = 256$.

Time = 0.05 (sec) , antiderivative size = 622, normalized size of antiderivative = 4.51

$$\int \frac{\csc^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(csc(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="m
axima")
```

output
```
1/120*(12*A*((121*sin(d*x + c)/(cos(d*x + c) + 1) + 410*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 + 610*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 425*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4 + 125*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5)
/(a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*a^3*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 10*a^3*sin(d*x + c)
^4/(cos(d*x + c) + 1)^4 + 5*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + a^3*
sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + 30*log(sin(d*x + c)/(cos(d*x + c) +
 1))/a^3 - 5*sin(d*x + c)/(a^3*(cos(d*x + c) + 1))) + A*((105*sin(d*x + c)
/(cos(d*x + c) + 1) + 2782*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 9410*sin(
d*x + c)^3/(cos(d*x + c) + 1)^3 + 13645*sin(d*x + c)^4/(cos(d*x + c) + 1)^
4 + 9285*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2580*sin(d*x + c)^6/(cos(d*
x + c) + 1)^6 - 15)/(a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 5*a^3*sin(d
*x + c)^3/(cos(d*x + c) + 1)^3 + 10*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^
4 + 10*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*a^3*sin(d*x + c)^6/(cos
(d*x + c) + 1)^6 + a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7) - 15*(12*sin(d
*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/a^3 + 78
0*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3))/d
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.30

$$\int \frac{\csc^3(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{\frac{1140\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3} - \frac{15\left(114\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-16\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+A\right)}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2} + \frac{15\left(Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-16\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^6}}{120\,d}$$

input
```
integrate(csc(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="g
iac")
```

output
```
1/120*(1140*A*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 15*(114*A*tan(1/2*d*x +
 1/2*c)^2 - 16*A*tan(1/2*d*x + 1/2*c) + A)/(a^3*tan(1/2*d*x + 1/2*c)^2) +
15*(A*a^3*tan(1/2*d*x + 1/2*c)^2 - 16*A*a^3*tan(1/2*d*x + 1/2*c))/a^6 + 16
*(240*A*tan(1/2*d*x + 1/2*c)^4 + 825*A*tan(1/2*d*x + 1/2*c)^3 + 1165*A*tan
(1/2*d*x + 1/2*c)^2 + 755*A*tan(1/2*d*x + 1/2*c) + 199*A)/(a^3*(tan(1/2*d*
x + 1/2*c) + 1)^5))/d
```

Mupad [B] (verification not implemented)

Time = 37.25 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.09

$$\int \frac{\csc^3(c+dx)(A - A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$
$$= \frac{A\left(165\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+4234\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+14090\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+19780\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+12060\tan\left(\frac{c}{2}+\right.\right.}{}$$

input
```
int((A - A*sin(c + d*x))/(sin(c + d*x)^3*(a + a*sin(c + d*x))^3),x)
```

output
```
(A*(165*tan(c/2 + (d*x)/2) + 4234*tan(c/2 + (d*x)/2)^2 + 14090*tan(c/2 + (
d*x)/2)^3 + 19780*tan(c/2 + (d*x)/2)^4 + 12060*tan(c/2 + (d*x)/2)^5 + 1830
*tan(c/2 + (d*x)/2)^6 - 1050*tan(c/2 + (d*x)/2)^7 - 165*tan(c/2 + (d*x)/2)
^8 + 15*tan(c/2 + (d*x)/2)^9 + 1140*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x
)/2)^2 + 5700*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^3 + 11400*log(tan
(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^4 + 11400*log(tan(c/2 + (d*x)/2))*tan(
c/2 + (d*x)/2)^5 + 5700*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^6 + 114
0*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^7 - 15))/(120*a^3*d*tan(c/2 +
 (d*x)/2)^2*(tan(c/2 + (d*x)/2) + 1)^5)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.36

$$\int \frac{\csc^3(c+dx)(A - A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$
$$= \frac{1140\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7+5700\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+11400\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{}$$

input
```
int(csc(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x)
```

output
```
(1140*log(tan((c + d*x)/2))*tan((c + d*x)/2)**7 + 5700*log(tan((c + d*x)/2
))*tan((c + d*x)/2)**6 + 11400*log(tan((c + d*x)/2))*tan((c + d*x)/2)**5 +
 11400*log(tan((c + d*x)/2))*tan((c + d*x)/2)**4 + 5700*log(tan((c + d*x)/
2))*tan((c + d*x)/2)**3 + 1140*log(tan((c + d*x)/2))*tan((c + d*x)/2)**2 +
 15*tan((c + d*x)/2)**9 - 165*tan((c + d*x)/2)**8 - 1416*tan((c + d*x)/2)*
*7 + 8400*tan((c + d*x)/2)**5 + 16120*tan((c + d*x)/2)**4 + 12260*tan((c +
 d*x)/2)**3 + 3868*tan((c + d*x)/2)**2 + 165*tan((c + d*x)/2) - 15)/(120*t
an((c + d*x)/2)**2*a**2*d*(tan((c + d*x)/2)**5 + 5*tan((c + d*x)/2)**4 + 1
0*tan((c + d*x)/2)**3 + 10*tan((c + d*x)/2)**2 + 5*tan((c + d*x)/2) + 1))
```

3.243 $\int \frac{\csc^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 153

$$\int \frac{\csc^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{18A\text{arctanh}(\cos(c+dx))}{a^3d} - \frac{10A\cot(c+dx)}{a^3d} - \frac{A\cot^3(c+dx)}{3a^3d} + \frac{2A\cot(c+dx)\csc(c+dx)}{a^3d} - \frac{2A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^3} - \frac{13A\cos(c+dx)}{5a^3d(1+\sin(c+dx))^2} - \frac{93A\cos(c+dx)}{5a^3d(1+\sin(c+dx))}$$

output
```
18*A*arctanh(cos(d*x+c))/a^3/d-10*A*cot(d*x+c)/a^3/d-1/3*A*cot(d*x+c)^3/a^
3/d+2*A*cot(d*x+c)*csc(d*x+c)/a^3/d-2/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^
3-13/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^2-93/5*A*cos(d*x+c)/a^3/d/(1+sin(
d*x+c))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 348 vs. $2(153) = 306$.

Time = 7.09 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.27

$$\int \frac{\csc^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx$$
$$= \frac{A\left(-\frac{29\cot\left(\frac{1}{2}(c+dx)\right)}{6d}+\frac{\csc^2\left(\frac{1}{2}(c+dx)\right)}{2d}-\frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{24d}+\frac{18\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d}-\frac{18\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}-\frac{\sec^2(\ldots}{}\right.}{}$$

input
```
Integrate[(Csc[c + d*x]^4*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

output
```
(A*((-29*Cot[(c + d*x)/2])/(6*d) + Csc[(c + d*x)/2]^2/(2*d) - (Cot[(c + d*
x)/2]*Csc[(c + d*x)/2]^2)/(24*d) + (18*Log[Cos[(c + d*x)/2]])/d - (18*Log[
Sin[(c + d*x)/2]])/d - Sec[(c + d*x)/2]^2/(2*d) + (4*Sin[(c + d*x)/2])/(5*
d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5) - 2/(5*d*(Cos[(c + d*x)/2] + Si
n[(c + d*x)/2])^4) + (26*Sin[(c + d*x)/2])/(5*d*(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2])^3) - 13/(5*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (186*S
in[(c + d*x)/2])/(5*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (29*Tan[(c
+ d*x)/2])/(6*d) + (Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*d)))/a^3
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {3042, 3445, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(c+dx)(A-A\sin(c+dx))}{(a\sin(c+dx)+a)^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A-A\sin(c+dx)}{\sin(c+dx)^4(a\sin(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3445}$$

$$\int \left(\frac{16A}{a^3(\sin(c+dx)+1)}+\frac{7A}{a^3(\sin(c+dx)+1)^2}+\frac{2A}{a^3(\sin(c+dx)+1)^3}+\frac{A\csc^4(c+dx)}{a^3}-\frac{4A\csc^3(c+dx)}{a^3}+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{18A\text{arctanh}(\cos(c+dx))}{a^3d}-\frac{A\cot^3(c+dx)}{3a^3d}-\frac{10A\cot(c+dx)}{a^3d}-\frac{93A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)}-\frac{13A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^2}-\frac{2A\cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3}+\frac{2A\cot(c+dx)\csc(c+dx)}{a^3d}$$

input
```
Int[(Csc[c + d*x]^4*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

output
```
(18*A*ArcTanh[Cos[c + d*x]])/(a^3*d) - (10*A*Cot[c + d*x])/(a^3*d) - (A*Co
t[c + d*x]^3)/(3*a^3*d) + (2*A*Cot[c + d*x]*Csc[c + d*x])/(a^3*d) - (2*A*C
os[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^3) - (13*A*Cos[c + d*x])/(5*a^3*d
*(1 + Sin[c + d*x])^2) - (93*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3445
```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{A\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+39\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{39}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-144\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{8d\,a^3}$
default	$\frac{A\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+39\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{39}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-144\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{8d\,a^3}$
parallelrisch	$-\frac{A\left(432\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8+7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7-67\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+2637\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{24d\,a^3\left(\mathrm{ta}\right.}$
risch	$-\frac{4\left(675iA\,e^{9i(dx+c)}+135\,e^{10i(dx+c)}A-3150iA\,e^{7i(dx+c)}-1710A\,e^{8i(dx+c)}+5180iA\,e^{5i(dx+c)}+4572A\,e^{6i(dx+c)}-359\right.}{15\left(e^{2i(dx+c)}-1\right)^3\left(e^{i(dx+c)}+i\right)^5d\,a^3}$
norman	$\frac{-\frac{A}{24ad}+\frac{7A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{24ad}-\frac{17A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{6ad}+\frac{17A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{6ad}-\frac{7A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{24ad}+\frac{A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}}{24ad}-\frac{66469A\tan\left(\frac{dx}{2}+\right.}{120ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3\left(1+\tan\right.}$

input
```
int(csc(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBO
SE)
```

output
```
1/8/d*A/a^3*(1/3*tan(1/2*d*x+1/2*c)^3-4*tan(1/2*d*x+1/2*c)^2+39*tan(1/2*d*
x+1/2*c)-1/3/tan(1/2*d*x+1/2*c)^3+4/tan(1/2*d*x+1/2*c)^2-39/tan(1/2*d*x+1/
2*c)-144*ln(tan(1/2*d*x+1/2*c))-128/5/(tan(1/2*d*x+1/2*c)+1)^5+64/(tan(1/2
*d*x+1/2*c)+1)^4-160/(tan(1/2*d*x+1/2*c)+1)^3+176/(tan(1/2*d*x+1/2*c)+1)^2
-400/(tan(1/2*d*x+1/2*c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 594 vs. 2(145) = 290.

Time = 0.10 (sec) , antiderivative size = 594, normalized size of antiderivative = 3.88

$$\int\frac{\csc^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx=\text{Too large to display}$$

input
```
integrate(csc(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="f
ricas")
```

output
```
1/15*(424*A*cos(d*x + c)^6 + 1002*A*cos(d*x + c)^5 - 944*A*cos(d*x + c)^4
- 2074*A*cos(d*x + c)^3 + 531*A*cos(d*x + c)^2 + 1077*A*cos(d*x + c) + 135
*(A*cos(d*x + c)^6 - 2*A*cos(d*x + c)^5 - 6*A*cos(d*x + c)^4 + 4*A*cos(d*x
 + c)^3 + 9*A*cos(d*x + c)^2 - 2*A*cos(d*x + c) - (A*cos(d*x + c)^5 + 3*A*
cos(d*x + c)^4 - 3*A*cos(d*x + c)^3 - 7*A*cos(d*x + c)^2 + 2*A*cos(d*x + c
) + 4*A)*sin(d*x + c) - 4*A)*log(1/2*cos(d*x + c) + 1/2) - 135*(A*cos(d*x
+ c)^6 - 2*A*cos(d*x + c)^5 - 6*A*cos(d*x + c)^4 + 4*A*cos(d*x + c)^3 + 9*
A*cos(d*x + c)^2 - 2*A*cos(d*x + c) - (A*cos(d*x + c)^5 + 3*A*cos(d*x + c)
^4 - 3*A*cos(d*x + c)^3 - 7*A*cos(d*x + c)^2 + 2*A*cos(d*x + c) + 4*A)*sin
(d*x + c) - 4*A)*log(-1/2*cos(d*x + c) + 1/2) + (424*A*cos(d*x + c)^5 - 57
8*A*cos(d*x + c)^4 - 1522*A*cos(d*x + c)^3 + 552*A*cos(d*x + c)^2 + 1083*A
*cos(d*x + c) + 6*A)*sin(d*x + c) - 6*A)/(a^3*d*cos(d*x + c)^6 - 2*a^3*d*c
os(d*x + c)^5 - 6*a^3*d*cos(d*x + c)^4 + 4*a^3*d*cos(d*x + c)^3 + 9*a^3*d*
cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d - (a^3*d*cos(d*x + c)^5 +
3*a^3*d*cos(d*x + c)^4 - 3*a^3*d*cos(d*x + c)^3 - 7*a^3*d*cos(d*x + c)^2 +
 2*a^3*d*cos(d*x + c) + 4*a^3*d)*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\csc^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = -\frac{A\left(\int\left(-\frac{\csc^4(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\right)dx+\int\frac{\sin(c+dx)\csc^4(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx\right)}{a^3}$$

input
```
integrate(csc(d*x+c)**4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

output
```
-A*(Integral(-csc(c + d*x)**4/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin
(c + d*x) + 1), x) + Integral(sin(c + d*x)*csc(c + d*x)**4/(sin(c + d*x)**
3 + 3*sin(c + d*x)**2 + 3*sin(c + d*x) + 1), x))/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 706 vs. $2(145) = 290$.

Time = 0.05 (sec) , antiderivative size = 706, normalized size of antiderivative = 4.61

$$\int \frac{\csc^4(c+dx)(A - A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(csc(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="m
axima")
```

output
```
-1/120*(A*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 2782*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + 9410*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 13645*sin(d*
x + c)^4/(cos(d*x + c) + 1)^4 + 9285*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 +
 2580*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 15)/(a^3*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + 5*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 10*a^3*sin(d*
x + c)^4/(cos(d*x + c) + 1)^4 + 10*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5
 + 5*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^3*sin(d*x + c)^7/(cos(d*x
 + c) + 1)^7) - 15*(12*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(c
os(d*x + c) + 1)^2)/a^3 + 780*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) -
A*((20*sin(d*x + c)/(cos(d*x + c) + 1) - 230*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - 4777*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 15785*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 - 22390*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 14940*si
n(d*x + c)^6/(cos(d*x + c) + 1)^6 - 4005*sin(d*x + c)^7/(cos(d*x + c) + 1)
^7 - 5)/(a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*a^3*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 + 10*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 10*a^3*s
in(d*x + c)^6/(cos(d*x + c) + 1)^6 + 5*a^3*sin(d*x + c)^7/(cos(d*x + c) +
1)^7 + a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) + 5*(81*sin(d*x + c)/(cos(
d*x + c) + 1) - 9*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^3/(co
s(d*x + c) + 1)^3)/a^3 - 1380*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3))/d
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.39

$$\int \frac{\csc^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx =$$
$$-\frac{\frac{2160\,A\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}-\frac{5\left(792\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-117\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-A\right)}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}+\frac{48\left(125\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}{}}{}$$

input
```
integrate(csc(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="g
iac")
```

output
```
-1/120*(2160*A*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 5*(792*A*tan(1/2*d*x +
 1/2*c)^3 - 117*A*tan(1/2*d*x + 1/2*c)^2 + 12*A*tan(1/2*d*x + 1/2*c) - A)/
(a^3*tan(1/2*d*x + 1/2*c)^3) + 48*(125*A*tan(1/2*d*x + 1/2*c)^4 + 445*A*ta
n(1/2*d*x + 1/2*c)^3 + 635*A*tan(1/2*d*x + 1/2*c)^2 + 415*A*tan(1/2*d*x +
1/2*c) + 108*A)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^5) - 5*(A*a^6*tan(1/2*d*x
+ 1/2*c)^3 - 12*A*a^6*tan(1/2*d*x + 1/2*c)^2 + 117*A*a^6*tan(1/2*d*x + 1/2
*c))/a^9)/d
```

Mupad [B] (verification not implemented)

Time = 37.06 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.05

$$\int \frac{\csc^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx =$$
$$-\frac{A\left(335\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+7559\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+24610\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+33170\tan\left(\frac{c}{2}+\right.\right.}{}$$

input
```
int((A - A*sin(c + d*x))/(sin(c + d*x)^4*(a + a*sin(c + d*x))^3),x)
```

output
```
-(A*(335*tan(c/2 + (d*x)/2)^2 - 35*tan(c/2 + (d*x)/2) + 7559*tan(c/2 + (d*
x)/2)^3 + 24610*tan(c/2 + (d*x)/2)^4 + 33170*tan(c/2 + (d*x)/2)^5 + 18670*
tan(c/2 + (d*x)/2)^6 + 1310*tan(c/2 + (d*x)/2)^7 - 2375*tan(c/2 + (d*x)/2)
^8 - 335*tan(c/2 + (d*x)/2)^9 + 35*tan(c/2 + (d*x)/2)^10 - 5*tan(c/2 + (d*
x)/2)^11 + 2160*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^3 + 10800*log(t
an(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^4 + 21600*log(tan(c/2 + (d*x)/2))*ta
n(c/2 + (d*x)/2)^5 + 21600*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^6 +
10800*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^7 + 2160*log(tan(c/2 + (d
*x)/2))*tan(c/2 + (d*x)/2)^8 + 5))/(120*a^3*d*tan(c/2 + (d*x)/2)^3*(tan(c/
2 + (d*x)/2) + 1)^5)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.30

$$\int \frac{\csc^4(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3}\,dx = \frac{-2160\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8-10800\log\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7-21600\log\left(\tan\left(\frac{dx}{2}+\right.\right.}{}$$

input
```
int(csc(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x)
```

output
```
( - 2160*log(tan((c + d*x)/2))*tan((c + d*x)/2)**8 - 10800*log(tan((c + d*
x)/2))*tan((c + d*x)/2)**7 - 21600*log(tan((c + d*x)/2))*tan((c + d*x)/2)*
*6 - 21600*log(tan((c + d*x)/2))*tan((c + d*x)/2)**5 - 10800*log(tan((c +
d*x)/2))*tan((c + d*x)/2)**4 - 2160*log(tan((c + d*x)/2))*tan((c + d*x)/2)
**3 + 5*tan((c + d*x)/2)**11 - 35*tan((c + d*x)/2)**10 + 335*tan((c + d*x)
/2)**9 + 2637*tan((c + d*x)/2)**8 - 16050*tan((c + d*x)/2)**6 - 30550*tan(
(c + d*x)/2)**5 - 23300*tan((c + d*x)/2)**4 - 7297*tan((c + d*x)/2)**3 - 3
35*tan((c + d*x)/2)**2 + 35*tan((c + d*x)/2) - 5)/(120*tan((c + d*x)/2)**3
*a**2*d*(tan((c + d*x)/2)**5 + 5*tan((c + d*x)/2)**4 + 10*tan((c + d*x)/2)
**3 + 10*tan((c + d*x)/2)**2 + 5*tan((c + d*x)/2) + 1))
```

3.244 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

Optimal result

Integrand size = 33, antiderivative size = 327

$$\begin{aligned}
&\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx \\
&= \frac{1}{8}a\left(B\left(4c^3+12c^2d+9cd^2+3d^3\right)+A\left(8c^3+12c^2d+12cd^2+3d^3\right)\right)x \\
&\quad - \frac{a(5Ad(3c^3+16c^2d+12cd^2+4d^3)-B(3c^4-15c^3d-52c^2d^2-60cd^3-16d^4))\cos(e+fx)}{30df} \\
&\quad - \frac{a(5Ad(6c^2+20cd+9d^2)-B(6c^3-30c^2d-71cd^2-45d^3))\cos(e+fx)\sin(e+fx)}{120f} \\
&\quad - \frac{a(4(5A+4B)d^2-3c(Bc-5(A+B)d))\cos(e+fx)(c+d\sin(e+fx))^2}{60df} \\
&\quad + \frac{a(Bc-5(A+B)d)\cos(e+fx)(c+d\sin(e+fx))^3}{20df} \\
&\quad - \frac{aB\cos(e+fx)(c+d\sin(e+fx))^4}{5df}
\end{aligned}$$

output
```
1/8*a*(B*(4*c^3+12*c^2*d+9*c*d^2+3*d^3)+A*(8*c^3+12*c^2*d+12*c*d^2+3*d^3))
*x-1/30*a*(5*A*d*(3*c^3+16*c^2*d+12*c*d^2+4*d^3)-B*(3*c^4-15*c^3*d-52*c^2*
d^2-60*c*d^3-16*d^4))*cos(f*x+e)/d/f-1/120*a*(5*A*d*(6*c^2+20*c*d+9*d^2)-B
*(6*c^3-30*c^2*d-71*c*d^2-45*d^3))*cos(f*x+e)*sin(f*x+e)/f-1/60*a*(4*(5*A+
4*B)*d^2-3*c*(B*c-5*(A+B)*d))*cos(f*x+e)*(c+d*sin(f*x+e))^2/d/f+1/20*a*(B*
c-5*(A+B)*d)*cos(f*x+e)*(c+d*sin(f*x+e))^3/d/f-1/5*a*B*cos(f*x+e)*(c+d*sin
(f*x+e))^4/d/f
```

Mathematica [A] (verified)

Time = 3.97 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.82

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$$
$$= \frac{a(1+\sin(e+fx))\left(-60(2A(4c^3+12c^2d+9cd^2+3d^3)+B(8c^3+18c^2d+18cd^2+5d^3))\cos(e+fx)+\right.}{}$$

input
```
Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3
,x]
```

output
```
(a*(1 + Sin[e + f*x])*(-60*(2*A*(4*c^3 + 12*c^2*d + 9*c*d^2 + 3*d^3) + B*(
8*c^3 + 18*c^2*d + 18*c*d^2 + 5*d^3))*Cos[e + f*x] + 10*d*(4*A*d*(3*c + d)
 + B*(12*c^2 + 12*c*d + 5*d^2))*Cos[3*(e + f*x)] - 6*B*d^3*Cos[5*(e + f*x)
] + 15*(4*(B*(4*c^3 + 12*c^2*d + 9*c*d^2 + 3*d^3) + A*(8*c^3 + 12*c^2*d +
12*c*d^2 + 3*d^3))*f*x - 8*(B*(c + d)^3 + A*d*(3*c^2 + 3*c*d + d^2))*Sin[2
*(e + f*x)] + d^2*(A*d + B*(3*c + d))*Sin[4*(e + f*x)])))/(480*f*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^2)
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {3042, 3447, 3042, 3502, 3042, 3232, 3042, 3232, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)(A+B\sin(e+fx))(c+d\sin(e+fx))^3dx$$

$$\Big\downarrow \text{3447}$$

$$\int (c+d\sin(e+fx))^3\left((aA+aB)\sin(e+fx)+aA+aB\sin^2(e+fx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+d\sin(e+fx))^3\left((aA+aB)\sin(e+fx)+aA+aB\sin(e+fx)^2\right)dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{\int (c+d\sin(e+fx))^3(a(5A+4B)d-a(Bc-5(A+B)d)\sin(e+fx))dx}{5d}-\frac{aB\cos(e+fx)(c+d\sin(e+fx))^4}{5df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (c+d\sin(e+fx))^3(a(5A+4B)d-a(Bc-5(A+B)d)\sin(e+fx))dx}{5d}-\frac{aB\cos(e+fx)(c+d\sin(e+fx))^4}{5df}$$

$$\Big\downarrow \text{3232}$$

$$\frac{\frac{1}{4}\int (c+d\sin(e+fx))^2\left(ad(20Ac+13Bc+15Ad+15Bd)+a\left(4(5A+4B)d^2-3c(Bc-5(A+B)d)\right)\sin(e+f\right.}{5d}$$
$$\frac{aB\cos(e+fx)(c+d\sin(e+fx))^4}{5df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{4}\int (c+d\sin(e+fx))^2\left(ad(20Ac+13Bc+15Ad+15Bd)+a\left(4(5A+4B)d^2-3c(Bc-5(A+B)d)\right)\sin(e+f\right.}{5d}$$
$$\frac{aB\cos(e+fx)(c+d\sin(e+fx))^4}{5df}$$

$$\Big\downarrow \text{3232}$$

$$\frac{\frac{1}{4}\left(\frac{1}{3}\int (c+d\sin(e+fx))\left(ad\left(60Ac^2+33Bc^2+75Adc+75Bdc+40Ad^2+32Bd^2\right)+a\left(5Ad\left(6c^2+20dc+9d^2\right)\right.\right.\right.}{\frac{aB\cos(e+fx)(c+d\sin(e+fx))^4}{5df}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{1}{4}\left(\frac{1}{3}\int (c+d\sin(e+fx))\left(ad\left(60Ac^2+33Bc^2+75Adc+75Bdc+40Ad^2+32Bd^2\right)+a\left(5Ad\left(6c^2+20dc+9d^2\right)\right.\right.\right.}{\frac{aB\cos(e+fx)(c+d\sin(e+fx))^4}{5df}}$$

$\Big\downarrow$ 3213

$$\frac{\frac{1}{4}\left(\frac{1}{3}\left(-\frac{ad\left(5Ad\left(6c^2+20cd+9d^2\right)-B\left(6c^3-30c^2d-71cd^2-45d^3\right)\right)\sin(e+fx)\cos(e+fx)}{2f}+\frac{15}{2}adx\left(A\left(8c^3+12c^2d+12cd^2+3d^3\right)+\right.\right.\right.}{\frac{aB\cos(e+fx)(c+d\sin(e+fx))^4}{5df}}$$

input
```
Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x]
```

output
```
-1/5*(a*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(d*f) + ((a*(B*c - 5*(A + B
)*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(4*f) + (-1/3*(a*(4*(5*A + 4*B)*
d^2 - 3*c*(B*c - 5*(A + B)*d))*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/f + ((
15*a*d*(B*(4*c^3 + 12*c^2*d + 9*c*d^2 + 3*d^3) + A*(8*c^3 + 12*c^2*d + 12*
c*d^2 + 3*d^3))*x)/2 - (2*a*(5*A*d*(3*c^3 + 16*c^2*d + 12*c*d^2 + 4*d^3) -
 B*(3*c^4 - 15*c^3*d - 52*c^2*d^2 - 60*c*d^3 - 16*d^4))*Cos[e + f*x])/f -
(a*d*(5*A*d*(6*c^2 + 20*c*d + 9*d^2) - B*(6*c^3 - 30*c^2*d - 71*c*d^2 - 45
*d^3))*Cos[e + f*x]*Sin[e + f*x])/(2*f))/3)/4)/(5*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 98.55 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.75

method	result
parts	$\frac{(Aa\,d^3+3Bac\,d^2+Ba\,d^3)\left(-\frac{\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)}{f}-\frac{(Aa\,c^3+3Aa\,c^2d+Ba\,c^3)\cos(fx+e}{f}$
parallelrisch	$\frac{\left(\left((-A-B)d^3-3c(A+B)d^2-3c^2(A+B)d-B\,c^3\right)\sin(2fx+2e)+\left(\frac{\left(A+\frac{5B}{4}\right)d^2}{3}+c(A+B)d+B\,c^2\right)d\cos(3fx+3e)+\frac{((A}{}\right.}{}$
derivativedivides	$\frac{-Aa\,c^3\cos(fx+e)+3Aa\,c^2d\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-Aac\,d^2\left(2+\sin(fx+e)^2\right)\cos(fx+e)+Aa\,d^3\left(-\frac{\left(\sin(fx-\right.}{}\right.}{}$
default	$\frac{-Aa\,c^3\cos(fx+e)+3Aa\,c^2d\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-Aac\,d^2\left(2+\sin(fx+e)^2\right)\cos(fx+e)+Aa\,d^3\left(-\frac{\left(\sin(fx-\right.}{}\right.}{}$
risch	$Aa\,c^3x+\frac{3Aa\,d^3x}{8}+\frac{Ba\,c^3x}{2}+\frac{3Ba\,d^3x}{8}+\frac{a\,d^2\cos(3fx+3e)Ac}{4f}+\frac{ad\cos(3fx+3e)B\,c^2}{4f}+\frac{a\,d^2\cos(3fx+3e}{4f}$
norman	Expression too large to display
orering	Expression too large to display

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x,method=_RETURNV
ERBOSE)
```

output
```
(A*a*d^3+3*B*a*c*d^2+B*a*d^3)/f*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*
x+e)+3/8*f*x+3/8*e)-(A*a*c^3+3*A*a*c^2*d+B*a*c^3)/f*cos(f*x+e)-1/3*(3*A*a*
c*d^2+A*a*d^3+3*B*a*c^2*d+3*B*a*c*d^2)/f*(2+sin(f*x+e)^2)*cos(f*x+e)+(3*A*
a*c^2*d+3*A*a*c*d^2+B*a*c^3+3*B*a*c^2*d)/f*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2
*f*x+1/2*e)+A*a*c^3*x-1/5*B*a*d^3/f*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*co
s(f*x+e)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.73

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = -\frac{24\,Bad^3\cos{(fx+e)}^5-40\,(3\,Bac^2d+3\,(A+B)acd^2+(A+2\,B)ad^3)\cos{(fx+e)}^3-15\,(4\,(2\,A+B}{}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorith
m="fricas")
```

output
```
-1/120*(24*B*a*d^3*cos(f*x + e)^5 - 40*(3*B*a*c^2*d + 3*(A + B)*a*c*d^2 +
(A + 2*B)*a*d^3)*cos(f*x + e)^3 - 15*(4*(2*A + B)*a*c^3 + 12*(A + B)*a*c^2
*d + 3*(4*A + 3*B)*a*c*d^2 + 3*(A + B)*a*d^3)*f*x + 120*((A + B)*a*c^3 + 3
*(A + B)*a*c^2*d + 3*(A + B)*a*c*d^2 + (A + B)*a*d^3)*cos(f*x + e) - 15*(2
*(3*B*a*c*d^2 + (A + B)*a*d^3)*cos(f*x + e)^3 - (4*B*a*c^3 + 12*(A + B)*a*
c^2*d + 3*(4*A + 5*B)*a*c*d^2 + 5*(A + B)*a*d^3)*cos(f*x + e))*sin(f*x + e
))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 996 vs. 2(311) = 622.

Time = 0.38 (sec) , antiderivative size = 996, normalized size of antiderivative = 3.05

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3,x)
```

output
```
Piecewise((A*a*c**3*x - A*a*c**3*cos(e + f*x)/f + 3*A*a*c**2*d*x*sin(e + f
*x)**2/2 + 3*A*a*c**2*d*x*cos(e + f*x)**2/2 - 3*A*a*c**2*d*sin(e + f*x)*co
s(e + f*x)/(2*f) - 3*A*a*c**2*d*cos(e + f*x)/f + 3*A*a*c*d**2*x*sin(e + f*
x)**2/2 + 3*A*a*c*d**2*x*cos(e + f*x)**2/2 - 3*A*a*c*d**2*sin(e + f*x)**2*
cos(e + f*x)/f - 3*A*a*c*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a*c*d*
*2*cos(e + f*x)**3/f + 3*A*a*d**3*x*sin(e + f*x)**4/8 + 3*A*a*d**3*x*sin(e
 + f*x)**2*cos(e + f*x)**2/4 + 3*A*a*d**3*x*cos(e + f*x)**4/8 - 5*A*a*d**3
*sin(e + f*x)**3*cos(e + f*x)/(8*f) - A*a*d**3*sin(e + f*x)**2*cos(e + f*x
)/f - 3*A*a*d**3*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 2*A*a*d**3*cos(e + f
*x)**3/(3*f) + B*a*c**3*x*sin(e + f*x)**2/2 + B*a*c**3*x*cos(e + f*x)**2/2
 - B*a*c**3*sin(e + f*x)*cos(e + f*x)/(2*f) - B*a*c**3*cos(e + f*x)/f + 3*
B*a*c**2*d*x*sin(e + f*x)**2/2 + 3*B*a*c**2*d*x*cos(e + f*x)**2/2 - 3*B*a*
c**2*d*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a*c**2*d*sin(e + f*x)*cos(e +
f*x)/(2*f) - 2*B*a*c**2*d*cos(e + f*x)**3/f + 9*B*a*c*d**2*x*sin(e + f*x)*
*4/8 + 9*B*a*c*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 9*B*a*c*d**2*x*c
os(e + f*x)**4/8 - 15*B*a*c*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*B*
a*c*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 9*B*a*c*d**2*sin(e + f*x)*cos(e
+ f*x)**3/(8*f) - 2*B*a*c*d**2*cos(e + f*x)**3/f + 3*B*a*d**3*x*sin(e + f*
x)**4/8 + 3*B*a*d**3*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*B*a*d**3*x*co
s(e + f*x)**4/8 - B*a*d**3*sin(e + f*x)**4*cos(e + f*x)/f - 5*B*a*d**3*...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.24

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c + d\sin(e + fx))^3\, dx$$
$$= \frac{480\,(fx + e)Aac^3 + 120\,(2\,fx + 2\,e - \sin(2\,fx + 2\,e))Bac^3 + 360\,(2\,fx + 2\,e - \sin(2\,fx + 2\,e))Aac^2d + \cdots}{}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorith
m="maxima")
```

output
```
1/480*(480*(f*x + e)*A*a*c^3 + 120*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c^
3 + 360*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a*c^2*d + 480*(cos(f*x + e)^3 -
 3*cos(f*x + e))*B*a*c^2*d + 360*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c^2*
d + 480*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a*c*d^2 + 360*(2*f*x + 2*e - s
in(2*f*x + 2*e))*A*a*c*d^2 + 480*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a*c*d
^2 + 45*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a*c*d^2
+ 160*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a*d^3 + 15*(12*f*x + 12*e + sin(
4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a*d^3 - 32*(3*cos(f*x + e)^5 - 10*cos
(f*x + e)^3 + 15*cos(f*x + e))*B*a*d^3 + 15*(12*f*x + 12*e + sin(4*f*x + 4
*e) - 8*sin(2*f*x + 2*e))*B*a*d^3 - 480*A*a*c^3*cos(f*x + e) - 480*B*a*c^3
*cos(f*x + e) - 1440*A*a*c^2*d*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.94

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = -\frac{Bad^3\cos(5fx+5e)}{80f} + \frac{1}{8}\left(8Aac^3+4Bac^3+12Aac^2d+12Bac^2d+12Aacd^2+9Bacd^2+3Aad^3+3Bad^3\right)x + \frac{\left(12Bac^2d+12Aacd^2+12Bacd^2+4Aad^3+5Bad^3\right)\cos(3fx+3e)}{48f} - \frac{\left(8Aac^3+8Bac^3+24Aac^2d+18Bac^2d+18Aacd^2+18Bacd^2+6Aad^3+5Bad^3\right)\cos(fx+e)}{8f} + \frac{\left(3Bacd^2+Aad^3+Bad^3\right)\sin(4fx+4e)}{32f} - \frac{\left(Bac^3+3Aac^2d+3Bac^2d+3Aacd^2+3Bacd^2+Aad^3+Bad^3\right)\sin(2fx+2e)}{4f}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorith
m="giac")
```

output
```
-1/80*B*a*d^3*cos(5*f*x + 5*e)/f + 1/8*(8*A*a*c^3 + 4*B*a*c^3 + 12*A*a*c^2
*d + 12*B*a*c^2*d + 12*A*a*c*d^2 + 9*B*a*c*d^2 + 3*A*a*d^3 + 3*B*a*d^3)*x
+ 1/48*(12*B*a*c^2*d + 12*A*a*c*d^2 + 12*B*a*c*d^2 + 4*A*a*d^3 + 5*B*a*d^3
)*cos(3*f*x + 3*e)/f - 1/8*(8*A*a*c^3 + 8*B*a*c^3 + 24*A*a*c^2*d + 18*B*a*
c^2*d + 18*A*a*c*d^2 + 18*B*a*c*d^2 + 6*A*a*d^3 + 5*B*a*d^3)*cos(f*x + e)/
f + 1/32*(3*B*a*c*d^2 + A*a*d^3 + B*a*d^3)*sin(4*f*x + 4*e)/f - 1/4*(B*a*c
^3 + 3*A*a*c^2*d + 3*B*a*c^2*d + 3*A*a*c*d^2 + 3*B*a*c*d^2 + A*a*d^3 + B*a
*d^3)*sin(2*f*x + 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 37.39 (sec) , antiderivative size = 830, normalized size of antiderivative = 2.54

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c + d\sin(e + fx))^3\,dx = \text{Too large to display}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c + d*sin(e + f*x))^3,x)
```

output
```
(a*atan((a*tan(e/2 + (f*x)/2)*(8*A*c^3 + 3*A*d^3 + 4*B*c^3 + 3*B*d^3 + 12*
A*c*d^2 + 12*A*c^2*d + 9*B*c*d^2 + 12*B*c^2*d))/(4*(2*A*a*c^3 + (3*A*a*d^3
)/4 + B*a*c^3 + (3*B*a*d^3)/4 + 3*A*a*c*d^2 + 3*A*a*c^2*d + (9*B*a*c*d^2)/
4 + 3*B*a*c^2*d)))*(8*A*c^3 + 3*A*d^3 + 4*B*c^3 + 3*B*d^3 + 12*A*c*d^2 + 1
2*A*c^2*d + 9*B*c*d^2 + 12*B*c^2*d))/(4*f) - (tan(e/2 + (f*x)/2)*((3*A*a*d
^3)/4 + B*a*c^3 + (3*B*a*d^3)/4 + 3*A*a*c*d^2 + 3*A*a*c^2*d + (9*B*a*c*d^2
)/4 + 3*B*a*c^2*d) + tan(e/2 + (f*x)/2)^8*(2*A*a*c^3 + 2*B*a*c^3 + 6*A*a*c
^2*d) + tan(e/2 + (f*x)/2)^2*(8*A*a*c^3 + (20*A*a*d^3)/3 + 8*B*a*c^3 + (16
*B*a*d^3)/3 + 20*A*a*c*d^2 + 24*A*a*c^2*d + 20*B*a*c*d^2 + 20*B*a*c^2*d) +
 tan(e/2 + (f*x)/2)^4*(12*A*a*c^3 + (28*A*a*d^3)/3 + 12*B*a*c^3 + (32*B*a*
d^3)/3 + 28*A*a*c*d^2 + 36*A*a*c^2*d + 28*B*a*c*d^2 + 28*B*a*c^2*d) + tan(
e/2 + (f*x)/2)^6*(8*A*a*c^3 + 4*A*a*d^3 + 8*B*a*c^3 + 12*A*a*c*d^2 + 24*A*
a*c^2*d + 12*B*a*c*d^2 + 12*B*a*c^2*d) - tan(e/2 + (f*x)/2)^9*((3*A*a*d^3)
/4 + B*a*c^3 + (3*B*a*d^3)/4 + 3*A*a*c*d^2 + 3*A*a*c^2*d + (9*B*a*c*d^2)/4
 + 3*B*a*c^2*d) + tan(e/2 + (f*x)/2)^3*((7*A*a*d^3)/2 + 2*B*a*c^3 + (7*B*a
*d^3)/2 + 6*A*a*c*d^2 + 6*A*a*c^2*d + (21*B*a*c*d^2)/2 + 6*B*a*c^2*d) - ta
n(e/2 + (f*x)/2)^7*((7*A*a*d^3)/2 + 2*B*a*c^3 + (7*B*a*d^3)/2 + 6*A*a*c*d^
2 + 6*A*a*c^2*d + (21*B*a*c*d^2)/2 + 6*B*a*c^2*d) + 2*A*a*c^3 + (4*A*a*d^3
)/3 + 2*B*a*c^3 + (16*B*a*d^3)/15 + 4*A*a*c*d^2 + 6*A*a*c^2*d + 4*B*a*c*d^
2 + 4*B*a*c^2*d)/(f*(5*tan(e/2 + (f*x)/2)^2 + 10*tan(e/2 + (f*x)/2)^4 +...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.65

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c + d\sin(e + fx))^3\,dx$$
$$= \frac{a\left(-90\cos(fx + e)\sin(fx + e)^3 bc\,d^2 - 120\cos(fx + e)\sin(fx + e)^2 ac\,d^2 - 120\cos(fx + e)\sin(fx + \right.}{}$$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x)
```

output
```
(a*( - 24*cos(e + f*x)*sin(e + f*x)**4*b*d**3 - 30*cos(e + f*x)*sin(e + f*
x)**3*a*d**3 - 90*cos(e + f*x)*sin(e + f*x)**3*b*c*d**2 - 30*cos(e + f*x)*
sin(e + f*x)**3*b*d**3 - 120*cos(e + f*x)*sin(e + f*x)**2*a*c*d**2 - 40*co
s(e + f*x)*sin(e + f*x)**2*a*d**3 - 120*cos(e + f*x)*sin(e + f*x)**2*b*c**
2*d - 120*cos(e + f*x)*sin(e + f*x)**2*b*c*d**2 - 32*cos(e + f*x)*sin(e +
f*x)**2*b*d**3 - 180*cos(e + f*x)*sin(e + f*x)*a*c**2*d - 180*cos(e + f*x)
*sin(e + f*x)*a*c*d**2 - 45*cos(e + f*x)*sin(e + f*x)*a*d**3 - 60*cos(e +
f*x)*sin(e + f*x)*b*c**3 - 180*cos(e + f*x)*sin(e + f*x)*b*c**2*d - 135*co
s(e + f*x)*sin(e + f*x)*b*c*d**2 - 45*cos(e + f*x)*sin(e + f*x)*b*d**3 - 1
20*cos(e + f*x)*a*c**3 - 360*cos(e + f*x)*a*c**2*d - 240*cos(e + f*x)*a*c*
d**2 - 80*cos(e + f*x)*a*d**3 - 120*cos(e + f*x)*b*c**3 - 240*cos(e + f*x)
*b*c**2*d - 240*cos(e + f*x)*b*c*d**2 - 64*cos(e + f*x)*b*d**3 + 120*a*c**
3*f*x + 120*a*c**3 + 180*a*c**2*d*f*x + 360*a*c**2*d + 180*a*c*d**2*f*x +
240*a*c*d**2 + 45*a*d**3*f*x + 80*a*d**3 + 60*b*c**3*f*x + 120*b*c**3 + 18
0*b*c**2*d*f*x + 240*b*c**2*d + 135*b*c*d**2*f*x + 240*b*c*d**2 + 45*b*d**
3*f*x + 64*b*d**3))/(120*f)
```

3.245 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

Optimal result

Integrand size = 33, antiderivative size = 213

$$\begin{aligned}\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx &= \frac{1}{8}a\left(4A\left(2c^2+2cd+d^2\right)+B\left(4c^2+8cd+3d^2\right)\right)x \\ &\quad-\frac{a(4Ad(c^2+3cd+d^2)-B(c^3-4c^2d-8cd^2-4d^3))\cos(e+fx)}{6df} \\ &\quad-\frac{a(3(4A+3B)d^2-2c(Bc-4(A+B)d))\cos(e+fx)\sin(e+fx)}{24f} \\ &\quad+\frac{a(Bc-4(A+B)d)\cos(e+fx)(c+d\sin(e+fx))^2}{12df} \\ &\quad-\frac{aB\cos(e+fx)(c+d\sin(e+fx))^3}{4df}\end{aligned}$$

output
```
1/8*a*(4*A*(2*c^2+2*c*d+d^2)+B*(4*c^2+8*c*d+3*d^2))*x-1/6*a*(4*A*d*(c^2+3*
c*d+d^2)-B*(c^3-4*c^2*d-8*c*d^2-4*d^3))*cos(f*x+e)/d/f-1/24*a*(3*(4*A+3*B)
*d^2-2*c*(B*c-4*(A+B)*d))*cos(f*x+e)*sin(f*x+e)/f+1/12*a*(B*c-4*(A+B)*d)*c
os(f*x+e)*(c+d*sin(f*x+e))^2/d/f-1/4*a*B*cos(f*x+e)*(c+d*sin(f*x+e))^3/d/f
```

Mathematica [A] (verified)

Time = 2.69 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.87

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$
$$=\frac{a(1+\sin(e+fx))\left(-24(B(4c^2+6cd+3d^2)+A(4c^2+8cd+3d^2))\cos(e+fx)+8d(Ad+B(2c+d))\right.}{96f}$$

input
```
Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2
,x]
```

output
```
(a*(1 + Sin[e + f*x])*(-24*(B*(4*c^2 + 6*c*d + 3*d^2) + A*(4*c^2 + 8*c*d +
 3*d^2))*Cos[e + f*x] + 8*d*(A*d + B*(2*c + d))*Cos[3*(e + f*x)] + 3*(4*(4
*A*(2*c^2 + 2*c*d + d^2) + B*(4*c^2 + 8*c*d + 3*d^2))*f*x - 8*(B*(c + d)^2
 + A*d*(2*c + d))*Sin[2*(e + f*x)] + B*d^2*Sin[4*(e + f*x)])))/(96*f*(Cos[
(e + f*x)/2] + Sin[(e + f*x)/2])^2)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3042, 3447, 3042, 3502, 3042, 3232, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)(A+B\sin(e+fx))(c+d\sin(e+fx))^2dx$$

$$\Big\downarrow \text{3447}$$

$$\int (c+d\sin(e+fx))^2\left((aA+aB)\sin(e+fx)+aA+aB\sin^2(e+fx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+d\sin(e+fx))^2\left((aA+aB)\sin(e+fx)+aA+aB\sin(e+fx)^2\right)dx$$

$\downarrow$ 3502

$$\frac{\int (c+d\sin(e+fx))^2(a(4A+3B)d-a(Bc-4(A+B)d)\sin(e+fx))dx}{4d}-\frac{aB\cos(e+fx)(c+d\sin(e+fx))^3}{4df}$$

$\downarrow$ 3042

$$\frac{\int (c+d\sin(e+fx))^2(a(4A+3B)d-a(Bc-4(A+B)d)\sin(e+fx))dx}{4d}-\frac{aB\cos(e+fx)(c+d\sin(e+fx))^3}{4df}$$

$\downarrow$ 3232

$$\frac{\frac{1}{3}\int (c+d\sin(e+fx))\left(ad(12Ac+7Bc+8Ad+8Bd)-a\left(2Bc^2-8(A+B)dc-3(4A+3B)d^2\right)\sin(e+fx)\right) d}{4d}$$
$$\frac{aB\cos(e+fx)(c+d\sin(e+fx))^3}{4df}$$

$\downarrow$ 3042

$$\frac{\frac{1}{3}\int (c+d\sin(e+fx))\left(ad(12Ac+7Bc+8Ad+8Bd)-a\left(2Bc^2-8(A+B)dc-3(4A+3B)d^2\right)\sin(e+fx)\right) d}{4d}$$
$$\frac{aB\cos(e+fx)(c+d\sin(e+fx))^3}{4df}$$

$\downarrow$ 3213

$$\frac{\frac{1}{3}\left(\frac{ad\left(-8cd(A+B)-3d^2(4A+3B)+2Bc^2\right)\sin(e+fx)\cos(e+fx)}{2f}+\frac{3}{2}adx\left(4A\left(2c^2+2cd+d^2\right)+B\left(4c^2+8cd+3d^2\right)\right)-\frac{2a(4Ac}{}\right.}{4d}$$
$$\frac{aB\cos(e+fx)(c+d\sin(e+fx))^3}{4df}$$

input
```
Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
```

output
```
-1/4*(a*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(d*f) + ((a*(B*c - 4*(A + B
)*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(3*f) + ((3*a*d*(4*A*(2*c^2 + 2*
c*d + d^2) + B*(4*c^2 + 8*c*d + 3*d^2))*x)/2 - (2*a*(4*A*d*(c^2 + 3*c*d +
d^2) - B*(c^3 - 4*c^2*d - 8*c*d^2 - 4*d^3))*Cos[e + f*x])/f + (a*d*(2*B*c^
2 - 8*(A + B)*c*d - 3*(4*A + 3*B)*d^2)*Cos[e + f*x]*Sin[e + f*x])/(2*f))/3
)/(4*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 13.11 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.81

method	result
parts	$-\frac{\left(A a \,d^{2}+2 B a c d +B a \,d^{2}\right) \left(2+\sin \left(f x +e \right)^{2}\right) \cos \left(f x +e \right)}{3 f}-\frac{\left(A a \,c^{2}+2 A a c d +B a \,c^{2}\right) \cos \left(f x +e \right)}{f}+\frac{\left(2 A a c d +A a \,d^{2}+B\right.}{}$
parallelrisch	$\frac{a \left(\left(\left(-3 A -3 B \right) d^{2}-6 c \left(A +B \right) d -3 B \,c^{2}\right) \sin \left(2 f x +2 e \right)+d \left(\left(A +B \right) d +2 B c \right) \cos \left(3 f x +3 e \right)+\frac{3 B \,d^{2} \sin \left(4 f x +4 e \right)}{8}+\left(\left(-9 A -9 B\right.\right.\right.}{}$
derivativedivides	$\frac{-A a \,c^{2} \cos \left(f x +e \right)+2 A a c d \left(-\frac{\sin \left(f x +e \right) \cos \left(f x +e \right)}{2}+\frac{f x}{2}+\frac{e}{2}\right)-\frac{A a \,d^{2} \left(2+\sin \left(f x +e \right)^{2}\right) \cos \left(f x +e \right)}{3}+B a \,c^{2} \left(-\frac{\sin \left(f x +e \right) \cos \left(f x\right.}{2}\right.}{}$
default	$\frac{-A a \,c^{2} \cos \left(f x +e \right)+2 A a c d \left(-\frac{\sin \left(f x +e \right) \cos \left(f x +e \right)}{2}+\frac{f x}{2}+\frac{e}{2}\right)-\frac{A a \,d^{2} \left(2+\sin \left(f x +e \right)^{2}\right) \cos \left(f x +e \right)}{3}+B a \,c^{2} \left(-\frac{\sin \left(f x +e \right) \cos \left(f x\right.}{2}\right.}{}$
risch	$A a \,c^{2} x +A a c d x +\frac{A a \,d^{2} x}{2}+\frac{B a \,c^{2} x}{2}+B a c d x +\frac{3 B a \,d^{2} x}{8}-\frac{a \cos \left(f x +e \right) A \,c^{2}}{f}-\frac{2 a \cos \left(f x +e \right) A c d}{f}$
norman	$\frac{\left(A a \,c^{2}+A a c d +\frac{1}{2} A a \,d^{2}+\frac{1}{2} B a \,c^{2}+B a c d +\frac{3}{8} B a \,d^{2}\right) x +\left(A a \,c^{2}+A a c d +\frac{1}{2} A a \,d^{2}+\frac{1}{2} B a \,c^{2}+B a c d +\frac{3}{8} B a \,d^{2}\right) x \tan \left(\frac{f x}{2}+\frac{e}{2}\right)}{}$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x,method=_RETURNV
ERBOSE)
```

output
```
-1/3*(A*a*d^2+2*B*a*c*d+B*a*d^2)/f*(2+sin(f*x+e)^2)*cos(f*x+e)-(A*a*c^2+2*
A*a*c*d+B*a*c^2)/f*cos(f*x+e)+(2*A*a*c*d+A*a*d^2+B*a*c^2+2*B*a*c*d)/f*(-1/
2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+A*a*c^2*x+B*a*d^2/f*(-1/4*(sin(f*x+
e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.75

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c + d\sin(e + fx))^2\,dx$$
$$= \frac{8\,(2\,Bacd + (A + B)ad^2)\cos{(fx + e)}^3 + 3\,(4\,(2\,A + B)ac^2 + 8\,(A + B)acd + (4\,A + 3\,B)ad^2)fx - 24\,(}{}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorith
m="fricas")
```

output
```
1/24*(8*(2*B*a*c*d + (A + B)*a*d^2)*cos(f*x + e)^3 + 3*(4*(2*A + B)*a*c^2
+ 8*(A + B)*a*c*d + (4*A + 3*B)*a*d^2)*f*x - 24*((A + B)*a*c^2 + 2*(A + B)
*a*c*d + (A + B)*a*d^2)*cos(f*x + e) + 3*(2*B*a*d^2*cos(f*x + e)^3 - (4*B*
a*c^2 + 8*(A + B)*a*c*d + (4*A + 5*B)*a*d^2)*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. 2(201) = 402.

Time = 0.26 (sec) , antiderivative size = 571, normalized size of antiderivative = 2.68

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c + d\sin(e + fx))^2\,dx$$
$$= \begin{cases} Aac^2x - \frac{Aac^2\cos{(e+fx)}}{f} + Aacdx\sin^2{(e + fx)} + Aacdx\cos^2{(e + fx)} - \frac{Aacd\sin{(e+fx)}\cos{(e+fx)}}{f} - \frac{2Aacd\cos}{f} \\ x(A + B\sin{(e)})\,(c + d\sin{(e)})^2\,(a\sin{(e)} + a) \end{cases}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
```

output
```
Piecewise((A*a*c**2*x - A*a*c**2*cos(e + f*x)/f + A*a*c*d*x*sin(e + f*x)**
2 + A*a*c*d*x*cos(e + f*x)**2 - A*a*c*d*sin(e + f*x)*cos(e + f*x)/f - 2*A*
a*c*d*cos(e + f*x)/f + A*a*d**2*x*sin(e + f*x)**2/2 + A*a*d**2*x*cos(e + f
*x)**2/2 - A*a*d**2*sin(e + f*x)**2*cos(e + f*x)/f - A*a*d**2*sin(e + f*x)
*cos(e + f*x)/(2*f) - 2*A*a*d**2*cos(e + f*x)**3/(3*f) + B*a*c**2*x*sin(e
+ f*x)**2/2 + B*a*c**2*x*cos(e + f*x)**2/2 - B*a*c**2*sin(e + f*x)*cos(e +
 f*x)/(2*f) - B*a*c**2*cos(e + f*x)/f + B*a*c*d*x*sin(e + f*x)**2 + B*a*c*
d*x*cos(e + f*x)**2 - 2*B*a*c*d*sin(e + f*x)**2*cos(e + f*x)/f - B*a*c*d*s
in(e + f*x)*cos(e + f*x)/f - 4*B*a*c*d*cos(e + f*x)**3/(3*f) + 3*B*a*d**2*
x*sin(e + f*x)**4/8 + 3*B*a*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*B
*a*d**2*x*cos(e + f*x)**4/8 - 5*B*a*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f
) - B*a*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a*d**2*sin(e + f*x)*cos(
e + f*x)**3/(8*f) - 2*B*a*d**2*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(A + B
*sin(e))*(c + d*sin(e))**2*(a*sin(e) + a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.24

$$\int (a + a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = \frac{96\,(fx+e)Aac^2 + 24\,(2\,fx+2\,e-\sin(2\,fx+2\,e))Bac^2 + 48\,(2\,fx+2\,e-\sin(2\,fx+2\,e))Aacd + 64}{}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorith
m="maxima")
```

output
```
1/96*(96*(f*x + e)*A*a*c^2 + 24*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c^2 +
 48*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a*c*d + 64*(cos(f*x + e)^3 - 3*cos(
f*x + e))*B*a*c*d + 48*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c*d + 32*(cos(
f*x + e)^3 - 3*cos(f*x + e))*A*a*d^2 + 24*(2*f*x + 2*e - sin(2*f*x + 2*e))
*A*a*d^2 + 32*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a*d^2 + 3*(12*f*x + 12*e
 + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a*d^2 - 96*A*a*c^2*cos(f*x + e
) - 96*B*a*c^2*cos(f*x + e) - 192*A*a*c*d*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.91

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$
$$=\frac{Bad^2\sin(4fx+4e)}{32f}+\frac{1}{8}\left(8Aac^2+4Bac^2+8Aacd+8Bacd+4Aad^2+3Bad^2\right)x+\frac{(2Bacd+Aad^2+Bad^2)\cos(3fx+3e)}{12f}-\frac{(4Aac^2+4Bac^2+8Aacd+6Bacd+3Aad^2+3Bad^2)\cos(fx+e)}{4f}-\frac{(Bac^2+2Aacd+2Bacd+Aad^2+Bad^2)\sin(2fx+2e)}{4f}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorith
m="giac")
```

output
```
1/32*B*a*d^2*sin(4*f*x + 4*e)/f + 1/8*(8*A*a*c^2 + 4*B*a*c^2 + 8*A*a*c*d +
 8*B*a*c*d + 4*A*a*d^2 + 3*B*a*d^2)*x + 1/12*(2*B*a*c*d + A*a*d^2 + B*a*d^
2)*cos(3*f*x + 3*e)/f - 1/4*(4*A*a*c^2 + 4*B*a*c^2 + 8*A*a*c*d + 6*B*a*c*d
 + 3*A*a*d^2 + 3*B*a*d^2)*cos(f*x + e)/f - 1/4*(B*a*c^2 + 2*A*a*c*d + 2*B*
a*c*d + A*a*d^2 + B*a*d^2)*sin(2*f*x + 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 36.90 (sec) , antiderivative size = 547, normalized size of antiderivative = 2.57

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$
$$=\frac{a\,\mathrm{atan}\left(\frac{a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(8Ac^2+4Ad^2+4Bc^2+3Bd^2+8Acd+8Bcd)}{4\left(2Aac^2+Aad^2+Bac^2+\frac{3Bad^2}{4}+2Aacd+2Bacd\right)}\right)(8Ac^2+4Ad^2+4Bc^2+3Bd^2+8Acd+8}{4f}$$
$$-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6(2Aac^2+2Bac^2+4Aacd)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(Aad^2+Bac^2+\frac{3Bad^2}{4}+2Aacd+2\right.}{}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c + d*sin(e + f*x))^2,x)
```

output
```
(a*atan((a*tan(e/2 + (f*x)/2)*(8*A*c^2 + 4*A*d^2 + 4*B*c^2 + 3*B*d^2 + 8*A
*c*d + 8*B*c*d))/(4*(2*A*a*c^2 + A*a*d^2 + B*a*c^2 + (3*B*a*d^2)/4 + 2*A*a
*c*d + 2*B*a*c*d)))*(8*A*c^2 + 4*A*d^2 + 4*B*c^2 + 3*B*d^2 + 8*A*c*d + 8*B
*c*d))/(4*f) - (tan(e/2 + (f*x)/2)^6*(2*A*a*c^2 + 2*B*a*c^2 + 4*A*a*c*d) +
 tan(e/2 + (f*x)/2)*(A*a*d^2 + B*a*c^2 + (3*B*a*d^2)/4 + 2*A*a*c*d + 2*B*a
*c*d) + tan(e/2 + (f*x)/2)^4*(6*A*a*c^2 + 4*A*a*d^2 + 6*B*a*c^2 + 4*B*a*d^
2 + 12*A*a*c*d + 8*B*a*c*d) + tan(e/2 + (f*x)/2)^2*(6*A*a*c^2 + (16*A*a*d^
2)/3 + 6*B*a*c^2 + (16*B*a*d^2)/3 + 12*A*a*c*d + (32*B*a*c*d)/3) - tan(e/2
 + (f*x)/2)^7*(A*a*d^2 + B*a*c^2 + (3*B*a*d^2)/4 + 2*A*a*c*d + 2*B*a*c*d)
+ tan(e/2 + (f*x)/2)^3*(A*a*d^2 + B*a*c^2 + (11*B*a*d^2)/4 + 2*A*a*c*d + 2
*B*a*c*d) - tan(e/2 + (f*x)/2)^5*(A*a*d^2 + B*a*c^2 + (11*B*a*d^2)/4 + 2*A
*a*c*d + 2*B*a*c*d) + 2*A*a*c^2 + (4*A*a*d^2)/3 + 2*B*a*c^2 + (4*B*a*d^2)/
3 + 4*A*a*c*d + (8*B*a*c*d)/3)/(f*(4*tan(e/2 + (f*x)/2)^2 + 6*tan(e/2 + (f
*x)/2)^4 + 4*tan(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.52

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c + d\sin(e + fx))^2 \, dx$$
$$= \frac{a\big(-6\cos(fx+e)\sin(fx+e)^3 b d^2 - 8\cos(fx+e)\sin(fx+e)^2 a d^2 - 8\cos(fx+e)\sin(fx+e)^2 b d^2}{}$$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x)
```

output
```
(a*( - 6*cos(e + f*x)*sin(e + f*x)**3*b*d**2 - 8*cos(e + f*x)*sin(e + f*x)
**2*a*d**2 - 16*cos(e + f*x)*sin(e + f*x)**2*b*c*d - 8*cos(e + f*x)*sin(e
+ f*x)**2*b*d**2 - 24*cos(e + f*x)*sin(e + f*x)*a*c*d - 12*cos(e + f*x)*si
n(e + f*x)*a*d**2 - 12*cos(e + f*x)*sin(e + f*x)*b*c**2 - 24*cos(e + f*x)*
sin(e + f*x)*b*c*d - 9*cos(e + f*x)*sin(e + f*x)*b*d**2 - 24*cos(e + f*x)*
a*c**2 - 48*cos(e + f*x)*a*c*d - 16*cos(e + f*x)*a*d**2 - 24*cos(e + f*x)*
b*c**2 - 32*cos(e + f*x)*b*c*d - 16*cos(e + f*x)*b*d**2 + 24*a*c**2*f*x +
24*a*c**2 + 24*a*c*d*f*x + 48*a*c*d + 12*a*d**2*f*x + 16*a*d**2 + 12*b*c**
2*f*x + 24*b*c**2 + 24*b*c*d*f*x + 32*b*c*d + 9*b*d**2*f*x + 16*b*d**2))/(
24*f)
```

3.246 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$

Optimal result

Integrand size = 31, antiderivative size = 111

$$\begin{aligned}\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx &= \frac{1}{2}a(B(c+d)+A(2c+d))x-\frac{a(3A(c+d)+B(3c+d))\cos(e+fx)}{3f}\\ &\quad-\frac{a(3Bc+3Ad-Bd)\cos(e+fx)\sin(e+fx)}{6f}\\ &\quad-\frac{Bd\cos(e+fx)(a+a\sin(e+fx))^2}{3af}\end{aligned}$$

output
```
1/2*a*(B*(c+d)+A*(2*c+d))*x-1/3*a*(3*A*(c+d)+B*(3*c+d))*cos(f*x+e)/f-1/6*a
*(3*A*d+3*B*c-B*d)*cos(f*x+e)*sin(f*x+e)/f-1/3*B*d*cos(f*x+e)*(a+a*sin(f*x
+e))^2/a/f
```

Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.94

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$=\frac{a(12Acfx+6Bcfx+6Adfx+6Bdfx-3(4A(c+d)+B(4c+3d))\cos(e+fx)+Bd\cos(3(e+fx))}{12f}$$

input
```
Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x
]
```

output
```
(a*(12*A*c*f*x + 6*B*c*f*x + 6*A*d*f*x + 6*B*d*f*x - 3*(4*A*(c + d) + B*(4
*c + 3*d))*Cos[e + f*x] + B*d*Cos[3*(e + f*x)] - 3*B*c*Sin[2*(e + f*x)] -
3*A*d*Sin[2*(e + f*x)] - 3*B*d*Sin[2*(e + f*x)]))/(12*f)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3042, 3447, 3042, 3502, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (a\sin(e+fx)+a)(A+B\sin(e+fx))(c+d\sin(e+fx))dx$$
$$\Big\downarrow \text{3447}$$
$$\int (a\sin(e+fx)+a)\left((Ad+Bc)\sin(e+fx)+Ac+Bd\sin^2(e+fx)\right)dx$$
$$\Big\downarrow \text{3042}$$
$$\int (a\sin(e+fx)+a)\left((Ad+Bc)\sin(e+fx)+Ac+Bd\sin(e+fx)^2\right)dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{\int (\sin(e+fx)a+a)(a(3Ac+2Bd)+a(3Bc+3Ad-Bd)\sin(e+fx))dx}{3a}-\frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^2}{3af}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (\sin(e+fx)a+a)(a(3Ac+2Bd)+a(3Bc+3Ad-Bd)\sin(e+fx))dx}{3a}-\frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^2}{3af}$$

$$\Big\downarrow \text{3213}$$

$$\frac{-\frac{a^2(3A(c+d)+B(3c+d))\cos(e+fx)}{f}-\frac{a^2(3Ad+3Bc-Bd)\sin(e+fx)\cos(e+fx)}{2f}+\frac{3}{2}a^2x(A(2c+d)+B(c+d))}{3a}-\frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^2}{3af}$$

input
```
Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

output
```
-1/3*(B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^2)/(a*f) + ((3*a^2*(B*(c + d)
+ A*(2*c + d))*x)/2 - (a^2*(3*A*(c + d) + B*(3*c + d))*Cos[e + f*x])/f - (
a^2*(3*B*c + 3*A*d - B*d)*Cos[e + f*x]*Sin[e + f*x])/(2*f))/(3*a)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.32

$$\frac{-\frac{Bad\left(2+\sin(fx+e)^2\right)\cos(fx+e)}{3}+Aad\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+Bac\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+B}{}$$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

output
```
1/f*(-1/3*B*a*d*(2+sin(f*x+e)^2)*cos(f*x+e)+A*a*d*(-1/2*sin(f*x+e)*cos(f*x
+e)+1/2*f*x+1/2*e)+B*a*c*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+B*a*d*
(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-A*a*c*cos(f*x+e)-A*a*d*cos(f*x+
e)-B*a*c*cos(f*x+e)+A*a*c*(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.76

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$=\frac{2\,Bad\cos{(fx+e)}^3+3\,((2\,A+B)ac+(A+B)ad)fx-3\,(Bac+(A+B)ad)\cos{(fx+e)}\sin{(fx+e)}}{6\,f}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm=
"fricas")
```

output
```
1/6*(2*B*a*d*cos(f*x + e)^3 + 3*((2*A + B)*a*c + (A + B)*a*d)*f*x - 3*(B*a
*c + (A + B)*a*d)*cos(f*x + e)*sin(f*x + e) - 6*((A + B)*a*c + (A + B)*a*d
)*cos(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(100) = 200$.

Time = 0.15 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.50

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c + d\sin(e + fx))\,dx$$
$$= \begin{cases} Aacx - \frac{Aac\cos(e+fx)}{f} + \frac{Aadx\sin^2(e+fx)}{2} + \frac{Aadx\cos^2(e+fx)}{2} - \frac{Aad\sin(e+fx)\cos(e+fx)}{2f} - \frac{Aad\cos(e+fx)}{f} + \frac{Bacx\sin^2(}{2} \\ x(A + B\sin(e))(c + d\sin(e))(a\sin(e) + a) \end{cases}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

output
```
Piecewise((A*a*c*x - A*a*c*cos(e + f*x)/f + A*a*d*x*sin(e + f*x)**2/2 + A*
a*d*x*cos(e + f*x)**2/2 - A*a*d*sin(e + f*x)*cos(e + f*x)/(2*f) - A*a*d*co
s(e + f*x)/f + B*a*c*x*sin(e + f*x)**2/2 + B*a*c*x*cos(e + f*x)**2/2 - B*a
*c*sin(e + f*x)*cos(e + f*x)/(2*f) - B*a*c*cos(e + f*x)/f + B*a*d*x*sin(e
+ f*x)**2/2 + B*a*d*x*cos(e + f*x)**2/2 - B*a*d*sin(e + f*x)**2*cos(e + f*
x)/f - B*a*d*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*B*a*d*cos(e + f*x)**3/(3*
f), Ne(f, 0)), (x*(A + B*sin(e))*(c + d*sin(e))*(a*sin(e) + a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.29

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$=\frac{12\,(fx+e)Aac+3\,(2\,fx+2\,e-\sin{(2\,fx+2\,e)})Bac+3\,(2\,fx+2\,e-\sin{(2\,fx+2\,e)})Aad+4\,(\cos{(f}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm=
"maxima")
```

output
```
1/12*(12*(f*x + e)*A*a*c + 3*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c + 3*(2
*f*x + 2*e - sin(2*f*x + 2*e))*A*a*d + 4*(cos(f*x + e)^3 - 3*cos(f*x + e))
*B*a*d + 3*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*d - 12*A*a*c*cos(f*x + e)
- 12*B*a*c*cos(f*x + e) - 12*A*a*d*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.88

$$\begin{aligned}\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx\\ &=\frac{Bad\cos{(3\,fx+3\,e)}}{12\,f}+\frac{1}{2}\,(2\,Aac+Bac+Aad+Bad)x\\ &-\frac{(4\,Aac+4\,Bac+4\,Aad+3\,Bad)\cos{(fx+e)}}{4\,f}\\ &-\frac{(Bac+Aad+Bad)\sin{(2\,fx+2\,e)}}{4\,f}\end{aligned}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm=
"giac")
```

output
```
1/12*B*a*d*cos(3*f*x + 3*e)/f + 1/2*(2*A*a*c + B*a*c + A*a*d + B*a*d)*x -
1/4*(4*A*a*c + 4*B*a*c + 4*A*a*d + 3*B*a*d)*cos(f*x + e)/f - 1/4*(B*a*c +
A*a*d + B*a*d)*sin(2*f*x + 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 34.59 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.21

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c + d\sin(e + fx))\,dx =$$
$$-\frac{\frac{3\,A\,a\,d\,\sin(2\,e+2\,f\,x)}{2} - \frac{B\,a\,d\,\cos(3\,e+3\,f\,x)}{2} + \frac{3\,B\,a\,c\,\sin(2\,e+2\,f\,x)}{2} + \frac{3\,B\,a\,d\,\sin(2\,e+2\,f\,x)}{2} + 6\,A\,a\,c\,\cos(e + f\,x) + 6\,A}{}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c + d*sin(e + f*x)),x)
```

output
```
-((3*A*a*d*sin(2*e + 2*f*x))/2 - (B*a*d*cos(3*e + 3*f*x))/2 + (3*B*a*c*sin
(2*e + 2*f*x))/2 + (3*B*a*d*sin(2*e + 2*f*x))/2 + 6*A*a*c*cos(e + f*x) + 6
*A*a*d*cos(e + f*x) + 6*B*a*c*cos(e + f*x) + (9*B*a*d*cos(e + f*x))/2 - 6*
A*a*c*f*x - 3*A*a*d*f*x - 3*B*a*c*f*x - 3*B*a*d*f*x)/(6*f)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.38

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))(c + d\sin(e + fx))\,dx$$
$$= \frac{a\left(-2\cos\left(fx+e\right)\sin\left(fx+e\right)^{2}bd - 3\cos\left(fx+e\right)\sin\left(fx+e\right)ad - 3\cos\left(fx+e\right)\sin\left(fx+e\right)bc - 3\cos\right.}{}$$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

output
```
(a*( - 2*cos(e + f*x)*sin(e + f*x)**2*b*d - 3*cos(e + f*x)*sin(e + f*x)*a*
d - 3*cos(e + f*x)*sin(e + f*x)*b*c - 3*cos(e + f*x)*sin(e + f*x)*b*d - 6*
cos(e + f*x)*a*c - 6*cos(e + f*x)*a*d - 6*cos(e + f*x)*b*c - 4*cos(e + f*x
)*b*d + 6*a*c*f*x + 6*a*c + 3*a*d*f*x + 6*a*d + 3*b*c*f*x + 6*b*c + 3*b*d*
f*x + 4*b*d))/(6*f)
```

3.247 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 48

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))\,dx = \frac{1}{2}a(2A+B)x - \frac{a(A+B)\cos(e+fx)}{f} - \frac{aB\cos(e+fx)\sin(e+fx)}{2f}$$

output
```
1/2*a*(2*A+B)*x-a*(A+B)*cos(f*x+e)/f-1/2*a*B*cos(f*x+e)*sin(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))\,dx = \frac{a(2Be+4Afx+2Bfx-4(A+B)\cos(e+fx)-B\sin(2(e+fx)))}{4f}$$

input
```
Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]),x]
```

output
```
(a*(2*B*e + 4*A*f*x + 2*B*f*x - 4*(A + B)*Cos[e + f*x] - B*Sin[2*(e + f*x)
]))/(4*f)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3213}$$

$$-\frac{a(A+B)\cos(e+fx)}{f}+\frac{1}{2}ax(2A+B)-\frac{aB\sin(e+fx)\cos(e+fx)}{2f}$$

input
```
Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]),x]
```

output
```
(a*(2*A + B)*x)/2 - (a*(A + B)*Cos[e + f*x])/f - (a*B*Cos[e + f*x]*Sin[e +
 f*x])/(2*f)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

method	result
parallelrisch	$\frac{\left(-\frac{\sin(2fx+2e)B}{4}+(-A-B)\cos(fx+e)+fxA+\frac{fxB}{2}+A+B\right)a}{f}$
parts	$axA-\frac{(Aa+Ba)\cos(fx+e)}{f}+\frac{Ba\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}$
risch	$axA+\frac{axB}{2}-\frac{a\cos(fx+e)A}{f}-\frac{a\cos(fx+e)B}{f}-\frac{Ba\sin(2fx+2e)}{4f}$
derivativedivides	$\frac{Ba\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-Aa\cos(fx+e)-Ba\cos(fx+e)+Aa(fx+e)}{f}$
default	$\frac{Ba\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-Aa\cos(fx+e)-Ba\cos(fx+e)+Aa(fx+e)}{f}$
norman	$\frac{(Aa+\frac{1}{2}Ba)x+(Aa+\frac{1}{2}Ba)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+(2Aa+Ba)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\frac{(2Aa+2Ba)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{f}+\frac{Ba\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{f}+}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^2}$
orering	$x(a+a\sin(fx+e))(A+B\sin(fx+e))-\frac{5(af\cos(fx+e)(A+B\sin(fx+e))+(a+a\sin(fx+e))B}{4f^2}$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
(-1/4*sin(2*f*x+2*e)*B+(-A-B)*cos(f*x+e)+f*x*A+1/2*f*x*B+A+B)*a/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))\,dx = \frac{(2\,A+B)afx-Ba\cos(fx+e)\sin(fx+e)-2\,(A+B)a\cos(fx+e)}{2\,f}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x, algorithm="fricas")
```

output
```
1/2*((2*A + B)*a*f*x - B*a*cos(f*x + e)*sin(f*x + e) - 2*(A + B)*a*cos(f*x
 + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(42) = 84$.

Time = 0.10 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.96

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))\,dx = \begin{cases} Aax - \frac{Aa\cos(e+fx)}{f} + \frac{Bax\sin^2(e+fx)}{2} + \frac{Bax\cos^2(e+fx)}{2} - \frac{Ba\sin(e+fx)\cos(e+fx)}{2f} - \frac{Ba\cos(e+fx)}{f} & \text{for } f \neq 0 \\ x(A + B\sin(e))(a\sin(e) + a) & \text{otherwise} \end{cases}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x)
```

output
```
Piecewise((A*a*x - A*a*cos(e + f*x)/f + B*a*x*sin(e + f*x)**2/2 + B*a*x*co
s(e + f*x)**2/2 - B*a*sin(e + f*x)*cos(e + f*x)/(2*f) - B*a*cos(e + f*x)/f
, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int (a + a\sin(e + fx))(A + B\sin(e + fx))\,dx = \frac{4(fx + e)Aa + (2fx + 2e - \sin(2fx + 2e))Ba - 4Aa\cos(fx + e) - 4Ba\cos(fx + e)}{4f}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x, algorithm="maxima")
```

output
```
1/4*(4*(f*x + e)*A*a + (2*f*x + 2*e - sin(2*f*x + 2*e))*B*a - 4*A*a*cos(f*
x + e) - 4*B*a*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))\,dx = \frac{1}{2}\,(2\,Aa+Ba)x-\frac{Ba\sin\left(2\,fx+2\,e\right)}{4\,f}-\frac{(Aa+Ba)\cos\left(fx+e\right)}{f}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x, algorithm="giac")
```

output
```
1/2*(2*A*a + B*a)*x - 1/4*B*a*sin(2*f*x + 2*e)/f - (A*a + B*a)*cos(f*x + e
)/f
```

Mupad [B] (verification not implemented)

Time = 34.57 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.08

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))\,dx = A\,a\,x - \frac{-B\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+(2\,A\,a+2\,B\,a)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+B\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+2\,A\,a+2\,B\,a}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)} + \frac{B\,a\,x}{2}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x)),x)
```

output
```
A*a*x - (2*A*a + 2*B*a + tan(e/2 + (f*x)/2)^2*(2*A*a + 2*B*a) - B*a*tan(e/
2 + (f*x)/2)^3 + B*a*tan(e/2 + (f*x)/2))/(f*(2*tan(e/2 + (f*x)/2)^2 + tan(
e/2 + (f*x)/2)^4 + 1)) + (B*a*x)/2
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.15

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))\,dx$$
$$=\frac{a(-\cos(fx+e)\sin(fx+e)\,b-2\cos(fx+e)\,a-2\cos(fx+e)\,b+2afx+2a+bfx+2b)}{2f}$$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x)
```

output
```
(a*( - cos(e + f*x)*sin(e + f*x)*b - 2*cos(e + f*x)*a - 2*cos(e + f*x)*b +
 2*a*f*x + 2*a + b*f*x + 2*b))/(2*f)
```

3.248 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 98

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = -\frac{a(Bc-(A+B)d)x}{d^2} + \frac{2a(c-d)(Bc-Ad)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^2\sqrt{c^2-d^2}\,f} - \frac{aB\cos(e+fx)}{df}$$

output
```
-a*(B*c-(A+B)*d)*x/d^2+2*a*(c-d)*(-A*d+B*c)*arctan((d+c*tan(1/2*f*x+1/2*e)
)/(c^2-d^2)^(1/2))/d^2/(c^2-d^2)^(1/2)/f-a*B*cos(f*x+e)/d/f
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.94 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.00

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$
$$= \frac{a\left(Adx+B(-c+d)x-\frac{Bd\cos(e)\cos(fx)}{f}+\frac{2(c-d)(Bc-Ad)\arctan\left(\frac{\sec\left(\frac{fx}{2}\right)(\cos(e)-i\sin(e))\left(d\cos\left(e+\frac{fx}{2}\right)+c\sin\left(\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right)(\cos(e)-}{\sqrt{c^2-d^2}f\sqrt{(\cos(e)-i\sin(e))^2}}\right.}{d^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}$$

input
```
Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])
,x]
```

output
```
(a*(A*d*x + B*(-c + d)*x - (B*d*Cos[e]*Cos[f*x])/f + (2*(c - d)*(B*c - A*d
)*ArcTan[(Sec[(f*x)/2]*(Cos[e] - I*Sin[e])*(d*Cos[e + (f*x)/2] + c*Sin[(f*
x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*(Cos[e] - I*Sin[e])
)/(Sqrt[c^2 - d^2]*f*Sqrt[(Cos[e] - I*Sin[e])^2]) + (B*d*Sin[e]*Sin[f*x])/
f)*(1 + Sin[e + f*x]))/(d^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {3042, 3447, 3042, 3502, 3042, 3214, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{c+d\sin(e+fx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{c+d\sin(e+fx)}dx$$
$$\Big\downarrow \text{3447}$$
$$\int \frac{(aA+aB)\sin(e+fx)+aA+aB\sin^2(e+fx)}{c+d\sin(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(aA+aB)\sin(e+fx)+aA+aB\sin(e+fx)^2}{c+d\sin(e+fx)}dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{\int \frac{aAd-a(Bc-(A+B)d)\sin(e+fx)}{c+d\sin(e+fx)}dx}{d}-\frac{aB\cos(e+fx)}{df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{aAd-a(Bc-(A+B)d)\sin(e+fx)}{c+d\sin(e+fx)}dx}{d}-\frac{aB\cos(e+fx)}{df}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\frac{a(c-d)(Bc-Ad)\int \frac{1}{c+d\sin(e+fx)}dx}{d}-\frac{ax(Bc-d(A+B))}{d}}{d}-\frac{aB\cos(e+fx)}{df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{a(c-d)(Bc-Ad)\int \frac{1}{c+d\sin(e+fx)}dx}{d}-\frac{ax(Bc-d(A+B))}{d}}{d}-\frac{aB\cos(e+fx)}{df}$$

$$\Big\downarrow \text{3139}$$

$$\frac{\frac{2a(c-d)(Bc-Ad)\int \frac{1}{c\tan^2\left(\frac{1}{2}(e+fx)\right)+2d\tan\left(\frac{1}{2}(e+fx)\right)+c}d\tan\left(\frac{1}{2}(e+fx)\right)}{df}-\frac{ax(Bc-d(A+B))}{d}}{d}-\frac{aB\cos(e+fx)}{df}$$

$$\Big\downarrow \text{1083}$$

$$\frac{-\frac{4a(c-d)(Bc-Ad)\int \frac{1}{-\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)^2-4\left(c^2-d^2\right)}d\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{df}-\frac{ax(Bc-d(A+B))}{d}}{d}-\frac{aB\cos(e+fx)}{df}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{2a(c-d)(Bc-Ad)\arctan\left(\frac{2c\tan\left(\frac{1}{2}(e+fx)\right)+2d}{2\sqrt{c^2-d^2}}\right)}{df\sqrt{c^2-d^2}}-\frac{ax(Bc-d(A+B))}{d}}{d}-\frac{aB\cos(e+fx)}{df}$$

input

```
Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),x]
```

output
```
(-((a*(B*c - (A + B)*d)*x)/d) + (2*a*(c - d)*(B*c - A*d)*ArcTan[(2*d + 2*c
*Tan[(e + f*x)/2])/(2*Sqrt[c^2 - d^2])])/(d*Sqrt[c^2 - d^2]*f))/d - (a*B*C
os[e + f*x])/(d*f)
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{2a\left(\frac{-\frac{Bd}{1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}+(Ad-Bc+Bd)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d^{2}}+\frac{\left(-Acd+A\,d^{2}+B\,c^{2}-Bcd\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^{2}-d^{2}}}\right)}{d^{2}\sqrt{c^{2}-d^{2}}}\right)}{f}$
default	$\frac{2a\left(\frac{-\frac{Bd}{1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}+(Ad-Bc+Bd)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d^{2}}+\frac{\left(-Acd+A\,d^{2}+B\,c^{2}-Bcd\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^{2}-d^{2}}}\right)}{d^{2}\sqrt{c^{2}-d^{2}}}\right)}{f}$
risch	$\frac{axA}{d}-\frac{axBc}{d^{2}}+\frac{axB}{d}-\frac{Ba\,e^{i(fx+e)}}{2df}-\frac{Ba\,e^{-i(fx+e)}}{2df}+\frac{\sqrt{-(c+d)(c-d)}\,a\ln\left(e^{i(fx+e)}+\frac{ic-\sqrt{-(c+d)(c-d)}}{d}\right)A}{(c+d)fd}$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x,method=_RETURNVER
BOSE)
```

output
```
2/f*a*(1/d^2*(-B*d/(1+tan(1/2*f*x+1/2*e)^2)+(A*d-B*c+B*d)*arctan(tan(1/2*f
*x+1/2*e)))+(-A*c*d+A*d^2+B*c^2-B*c*d)/d^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c
*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.96

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$
$$= \left[-\frac{2\,Bad\cos\left(fx+e\right)+2\left(Bac-(A+B)ad\right)fx+\left(Bac-Aad\right)\sqrt{-\frac{c-d}{c+d}}\log\left(\frac{\left(2\,c^{2}-d^{2}\right)\cos\left(fx+e\right)^{2}-2\,cd\sin(fx}{}\right.}{2\,d^{2}f},\right.$$
$$\left.-\frac{Bad\cos\left(fx+e\right)+\left(Bac-(A+B)ad\right)fx+\left(Bac-Aad\right)\sqrt{\frac{c-d}{c+d}}\arctan\left(-\frac{\left(c\sin\left(fx+e\right)+d\right)\sqrt{\frac{c-d}{c+d}}}{\left(c-d\right)\cos\left(fx+e\right)}\right)}{d^{2}f}\right]$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm=
"fricas")
```

output
```
[-1/2*(2*B*a*d*cos(f*x + e) + 2*(B*a*c - (A + B)*a*d)*f*x + (B*a*c - A*a*d
)*sqrt(-(c - d)/(c + d))*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x
 + e) - c^2 - d^2 + 2*((c^2 + c*d)*cos(f*x + e)*sin(f*x + e) + (c*d + d^2)
*cos(f*x + e))*sqrt(-(c - d)/(c + d)))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x
 + e) - c^2 - d^2)))/(d^2*f), -(B*a*d*cos(f*x + e) + (B*a*c - (A + B)*a*d)
*f*x + (B*a*c - A*a*d)*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x + e) + d)*
sqrt((c - d)/(c + d))/((c - d)*cos(f*x + e))))/(d^2*f)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5508 vs. $2(82) = 164$.

Time = 125.83 (sec) , antiderivative size = 5508, normalized size of antiderivative = 56.20

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

output
```
Piecewise((zoo*x*(A + B*sin(e))*(a*sin(e) + a)/sin(e), Eq(c, 0) & Eq(d, 0)
 & Eq(f, 0)), ((A*a*f*x*tan(e/2 + f*x/2)**2/(f*tan(e/2 + f*x/2)**2 + f) +
A*a*f*x/(f*tan(e/2 + f*x/2)**2 + f) + A*a*log(tan(e/2 + f*x/2))*tan(e/2 +
f*x/2)**2/(f*tan(e/2 + f*x/2)**2 + f) + A*a*log(tan(e/2 + f*x/2))/(f*tan(e
/2 + f*x/2)**2 + f) + B*a*f*x*tan(e/2 + f*x/2)**2/(f*tan(e/2 + f*x/2)**2 +
 f) + B*a*f*x/(f*tan(e/2 + f*x/2)**2 + f) - 2*B*a/(f*tan(e/2 + f*x/2)**2 +
 f))/d, Eq(c, 0)), (A*a*d**2*f*x*tan(e/2 + f*x/2)**3/(d**3*f*tan(e/2 + f*x
/2)**3 + d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f
*(d**2)**(3/2)) + A*a*d**2*f*x*tan(e/2 + f*x/2)/(d**3*f*tan(e/2 + f*x/2)**
3 + d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**
2)**(3/2)) + 2*A*a*d**2*tan(e/2 + f*x/2)**2/(d**3*f*tan(e/2 + f*x/2)**3 +
d**3*f*tan(e/2 + f*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**
(3/2)) + 2*A*a*d**2/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2)
- f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(3/2)) - A*a*d*f*x*sqrt(
d**2)*tan(e/2 + f*x/2)**2/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f
*x/2) - f*(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(3/2)) - A*a*d*f*x
*sqrt(d**2)/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) - f*(d**
2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(3/2)) + 2*A*a*d*sqrt(d**2)*tan(
e/2 + f*x/2)**2/(d**3*f*tan(e/2 + f*x/2)**3 + d**3*f*tan(e/2 + f*x/2) - f*
(d**2)**(3/2)*tan(e/2 + f*x/2)**2 - f*(d**2)**(3/2)) + 2*A*a*d*sqrt(d**...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{c + d\sin(e + fx)}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm=
"maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.40

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = -\frac{\frac{(Bac-Aad-Bad)(fx+e)}{d^2}+\frac{2\,Ba}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)d}-\frac{2\,(Bac^2-Aacd-Bacd+Aad^2)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\sqrt{c^2-d^2}\,d^2}}{f}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm=
"giac")
```

output
```
-((B*a*c - A*a*d - B*a*d)*(f*x + e)/d^2 + 2*B*a/((tan(1/2*f*x + 1/2*e)^2 +
 1)*d) - 2*(B*a*c^2 - A*a*c*d - B*a*c*d + A*a*d^2)*(pi*floor(1/2*(f*x + e)
/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/
(sqrt(c^2 - d^2)*d^2))/f
```

Mupad [B] (verification not implemented)

Time = 38.14 (sec) , antiderivative size = 3074, normalized size of antiderivative = 31.37

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \text{Too large to display}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c + d*sin(e + f*x)),x)
```

output
```
(2*A*a*atan(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(f*(c + d)) + (2*B*a*a
tan(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(f*(c + d)) - (B*a*cos(e + f*x
))/(f*(c + d)) + (2*A*a*c*atan(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(d*
f*(c + d)) - (A*a*atan((A^2*d^4*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*3i +
A^2*d^6*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i - B^2*c^4*sin(e/2 + (f*x)/
2)*(d^2 - c^2)^(3/2)*2i - B^2*c^6*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i
+ B^2*d^6*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i + A*B*d^6*sin(e/2 + (f*x
)/2)*(d^2 - c^2)^(1/2)*4i + A^2*c*d^3*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)
*1i + A^2*c*d^5*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i + B^2*c*d^5*cos(e/
2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i + B^2*c^3*d*cos(e/2 + (f*x)/2)*(d^2 - c^
2)^(3/2)*1i + B^2*c^5*d*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i + A^2*c*d^
5*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*4i + A^2*c^2*d^4*cos(e/2 + (f*x)/2)
*(d^2 - c^2)^(1/2)*2i + A^2*c^3*d^3*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1
i - B^2*c^3*d^3*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i - A^2*c^2*d^2*sin(
e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*2i + A^2*c^2*d^4*sin(e/2 + (f*x)/2)*(d^2
- c^2)^(1/2)*3i - A^2*c^3*d^3*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i - A^
2*c^4*d^2*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i + B^2*c^2*d^2*sin(e/2 +
(f*x)/2)*(d^2 - c^2)^(3/2)*3i - B^2*c^2*d^4*sin(e/2 + (f*x)/2)*(d^2 - c^2)
^(1/2)*6i + B^2*c^4*d^2*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*6i + A*B*c*d^
5*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i - A*B*c*d^3*sin(e/2 + (f*x)/2...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.50

$$\int \frac{(a + a\sin(e+fx))(A + B\sin(e+fx))}{c + d\sin(e+fx)}\,dx$$
$$= \frac{a\left(-2\sqrt{c^2-d^2}\,atan\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c+d}{\sqrt{c^2-d^2}}\right)ad + 2\sqrt{c^2-d^2}\,atan\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c+d}{\sqrt{c^2-d^2}}\right)bc - \cos(fx+e)\,bcd - \cos(f\right.}{d^2 f\,(c+d)}$$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

output
```
(a*( - 2*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2)
)*a*d + 2*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2
))*b*c - cos(e + f*x)*b*c*d - cos(e + f*x)*b*d**2 + a*c*d*f*x + a*d**2*f*x
 - b*c**2*f*x + b*d**2*f*x))/(d**2*f*(c + d))
```

3.249 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 124

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \frac{aBx}{d^2}+\frac{2a((A+B)(c-d)d^2-Bc(c^2-d^2))\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^2(c^2-d^2)^{3/2}f} + \frac{a(Bc-Ad)\cos(e+fx)}{d(c+d)f(c+d\sin(e+fx))}$$

output

```
a*B*x/d^2+2*a*((A+B)*(c-d)*d^2-B*c*(c^2-d^2))*arctan((d+c*tan(1/2*f*x+1/2*
e))/(c^2-d^2)^(1/2))/d^2/(c^2-d^2)^(3/2)/f+a*(-A*d+B*c)*cos(f*x+e)/d/(c+d)
/f/(c+d*sin(f*x+e))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.63 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.75

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$$

$$= \frac{a(1+\sin(e+fx))\left(Bx+\frac{2(Ad^2-B(c^2+cd-d^2))\arctan\left(\frac{\sec\left(\frac{fx}{2}\right)(\cos(e)-i\sin(e))\left(d\cos\left(e+\frac{fx}{2}\right)+c\sin\left(\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right)(\cos(e)-i\sin(e))}{(c+d)\sqrt{c^2-d^2}f\sqrt{(\cos(e)-i\sin(e))^2}}+\cdots\right.}{d^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}$$

input
```
Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])
^2,x]
```

output
```
(a*(1 + Sin[e + f*x])*(B*x + (2*(A*d^2 - B*(c^2 + c*d - d^2))*ArcTan[(Sec[
(f*x)/2]*(Cos[e] - I*Sin[e])*(d*Cos[e + (f*x)/2] + c*Sin[(f*x)/2]))/(Sqrt[
c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*(Cos[e] - I*Sin[e]))/((c + d)*Sqr
t[c^2 - d^2]*f*Sqrt[(Cos[e] - I*Sin[e])^2]) + ((-(B*c) + A*d)*Csc[e]*(c*Co
s[e] + d*Sin[f*x]))/((c + d)*f*(c + d*Sin[e + f*x]))))/(d^2*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^2)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.25, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3447, 3042, 3500, 25, 3042, 3214, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}dx$$

$$\Big\downarrow \text{3447}$$

$$\int \frac{(aA+aB)\sin(e+fx)+aA+aB\sin^2(e+fx)}{(c+d\sin(e+fx))^2}dx$$

$\Big\downarrow$ 3042

$$\int \frac{(aA+aB)\sin(e+fx)+aA+aB\sin(e+fx)^2}{(c+d\sin(e+fx))^2}dx$$

$\Big\downarrow$ 3500

$$\frac{a(Bc-Ad)\cos(e+fx)}{df(c+d)(c+d\sin(e+fx))}-\frac{\int -\frac{a(A+B)(c-d)d+aB(c^2-d^2)\sin(e+fx)}{c+d\sin(e+fx)}dx}{d(c^2-d^2)}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{a(A+B)(c-d)d+aB(c^2-d^2)\sin(e+fx)}{c+d\sin(e+fx)}dx}{d(c^2-d^2)}+\frac{a(Bc-Ad)\cos(e+fx)}{df(c+d)(c+d\sin(e+fx))}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{a(A+B)(c-d)d+aB(c^2-d^2)\sin(e+fx)}{c+d\sin(e+fx)}dx}{d(c^2-d^2)}+\frac{a(Bc-Ad)\cos(e+fx)}{df(c+d)(c+d\sin(e+fx))}$$

$\Big\downarrow$ 3214

$$\frac{\frac{a(c-d)(Ad^2-B(c^2+cd-d^2))\int \frac{1}{c+d\sin(e+fx)}dx}{d}+\frac{aBx(c^2-d^2)}{d}}{d(c^2-d^2)}+\frac{a(Bc-Ad)\cos(e+fx)}{df(c+d)(c+d\sin(e+fx))}$$

$\Big\downarrow$ 3042

$$\frac{\frac{a(c-d)(Ad^2-B(c^2+cd-d^2))\int \frac{1}{c+d\sin(e+fx)}dx}{d}+\frac{aBx(c^2-d^2)}{d}}{d(c^2-d^2)}+\frac{a(Bc-Ad)\cos(e+fx)}{df(c+d)(c+d\sin(e+fx))}$$

$\Big\downarrow$ 3139

$$\frac{\frac{2a(c-d)(Ad^2-B(c^2+cd-d^2))\int \frac{1}{c\tan^2\left(\frac{1}{2}(e+fx)\right)+2d\tan\left(\frac{1}{2}(e+fx)\right)+c}d\tan\left(\frac{1}{2}(e+fx)\right)}{df}+\frac{aBx(c^2-d^2)}{d}}{d(c^2-d^2)}+\frac{a(Bc-Ad)\cos(e+fx)}{df(c+d)(c+d\sin(e+fx))}$$

$\Big\downarrow$ 1083

$$\frac{\frac{aBx(c^2-d^2)}{d}-\frac{4a(c-d)(Ad^2-B(c^2+cd-d^2))\int \frac{1}{-\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)^2-4(c^2-d^2)}d\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{df}}{d(c^2-d^2)}+\frac{a(Bc-Ad)\cos(e+fx)}{df(c+d)(c+d\sin(e+fx))}$$

$$\Big\downarrow \ 217$$

$$\frac{\frac{2a(c-d)(Ad^2-B(c^2+cd-d^2))\arctan\left(\frac{2c\tan\left(\frac{1}{2}(e+fx)\right)+2d}{2\sqrt{c^2-d^2}}\right)}{df\sqrt{c^2-d^2}}+\frac{aBx(c^2-d^2)}{d}}{d\left(c^2-d^2\right)}+\frac{a(Bc-Ad)\cos(e+fx)}{df(c+d)(c+d\sin(e+fx))}$$

input
```
Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2,x]
```

output
```
((a*B*(c^2 - d^2)*x)/d + (2*a*(c - d)*(A*d^2 - B*(c^2 + c*d - d^2))*ArcTan
[(2*d + 2*c*Tan[(e + f*x)/2])/(2*Sqrt[c^2 - d^2])])/(d*Sqrt[c^2 - d^2]*f))
/(d*(c^2 - d^2)) + (a*(B*c - A*d)*Cos[e + f*x])/(d*(c + d)*f*(c + d*Sin[e
 + f*x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.40

method	result
derivativedivides	$\frac{2a\left(\frac{\frac{-\frac{d^2(Ad-Bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}-\frac{d(Ad-Bc)}{c+d}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}+\frac{\left(A\,d^2-B\,c^2-Bcd+B\,d^2\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}}}{d^2}+\frac{B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\cdots\right)}{d^2}\cdots\right)}{f}$
default	$\frac{2a\left(\frac{\frac{-\frac{d^2(Ad-Bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}-\frac{d(Ad-Bc)}{c+d}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}+\frac{\left(A\,d^2-B\,c^2-Bcd+B\,d^2\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}}}{d^2}+\frac{B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\cdots\right)}{d^2}\cdots\right)}{f}$
risch	$\frac{aBx}{d^2}-\frac{2ia(-Ad+Bc)\left(id+c\,e^{i(fx+e)}\right)}{d^2(c+d)f\left(-id\,e^{2i(fx+e)}+id+2c\,e^{i(fx+e)}\right)}-\frac{a\ln\left(e^{i(fx+e)}+\frac{i\sqrt{-c^2+d^2}\,c-c^2+d^2}{\sqrt{-c^2+d^2}\,d}\right)A}{\sqrt{-c^2+d^2}\,(c+d)f}+\frac{a\ln\left(e^{i(fx+e)}+\frac{i\cdots}{\cdots}\right)\cdots}{\sqrt{-c^2+\cdots}}\cdots$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x,method=_RETURNV
ERBOSE)
```

output
```
2/f*a*(1/d^2*((-d^2*(A*d-B*c)/(c+d)/c*tan(1/2*f*x+1/2*e)-d*(A*d-B*c)/(c+d)
)/(tan(1/2*f*x+1/2*e)^2*c+2*d*tan(1/2*f*x+1/2*e)+c)+(A*d^2-B*c^2-B*c*d+B*d
^2)/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2
)^(1/2)))+B/d^2*arctan(tan(1/2*f*x+1/2*e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. 2(119) = 238.

Time = 0.12 (sec) , antiderivative size = 655, normalized size of antiderivative = 5.28

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$$
$$= \left[\frac{2\,(Bac^3d+Bac^2d^2-Bacd^3-Bad^4)fx\sin\,(fx+e)+2\,(Bac^4+Bac^3d-Bac^2d^2-Bacd^3)fx+(Ba\right.$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorith
m="fricas")
```

output
```
[1/2*(2*(B*a*c^3*d + B*a*c^2*d^2 - B*a*c*d^3 - B*a*d^4)*f*x*sin(f*x + e) +
 2*(B*a*c^4 + B*a*c^3*d - B*a*c^2*d^2 - B*a*c*d^3)*f*x + (B*a*c^3 + B*a*c^
2*d - (A + B)*a*c*d^2 + (B*a*c^2*d + B*a*c*d^2 - (A + B)*a*d^3)*sin(f*x +
e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e
) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2
 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(B*a*c
^3*d - A*a*c^2*d^2 - B*a*c*d^3 + A*a*d^4)*cos(f*x + e))/((c^3*d^3 + c^2*d^
4 - c*d^5 - d^6)*f*sin(f*x + e) + (c^4*d^2 + c^3*d^3 - c^2*d^4 - c*d^5)*f)
, ((B*a*c^3*d + B*a*c^2*d^2 - B*a*c*d^3 - B*a*d^4)*f*x*sin(f*x + e) + (B*a
*c^4 + B*a*c^3*d - B*a*c^2*d^2 - B*a*c*d^3)*f*x + (B*a*c^3 + B*a*c^2*d - (
A + B)*a*c*d^2 + (B*a*c^2*d + B*a*c*d^2 - (A + B)*a*d^3)*sin(f*x + e))*sqr
t(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e)))
+ (B*a*c^3*d - A*a*c^2*d^2 - B*a*c*d^3 + A*a*d^4)*cos(f*x + e))/((c^3*d^3
+ c^2*d^4 - c*d^5 - d^6)*f*sin(f*x + e) + (c^4*d^2 + c^3*d^3 - c^2*d^4 - c
*d^5)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx=\text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.59

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \frac{\frac{(fx+e)Ba}{d^2} - \frac{2\,(Bac^2+Bacd-Aad^2-Bad^2)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(cd^2+d^3)\sqrt{c^2-d^2}} + \frac{2\,(Bacd\tan(\frac{1}{2}\,fx+\frac{1}{2}\,e)-Aad^2\tan(\frac{1}{2}}{(c^2d+cd^2)\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,a\right.}}{f}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorith
m="giac")
```

output
```
((f*x + e)*B*a/d^2 - 2*(B*a*c^2 + B*a*c*d - A*a*d^2 - B*a*d^2)*(pi*floor(1
/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c
^2 - d^2)))/((c*d^2 + d^3)*sqrt(c^2 - d^2)) + 2*(B*a*c*d*tan(1/2*f*x + 1/2
*e) - A*a*d^2*tan(1/2*f*x + 1/2*e) + B*a*c^2 - A*a*c*d)/((c^2*d + c*d^2)*(
c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c))/f
```

Mupad [B] (verification not implemented)

Time = 43.62 (sec) , antiderivative size = 5102, normalized size of antiderivative = 41.15

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c + d*sin(e + f*x))^2,x)
```

output
```
(2*B*a*atan(((B*a*((32*(B^2*a^2*c^2*d^3 + 2*B^2*a^2*c^3*d^2 + B^2*a^2*c^4*
d))/(2*c*d^3 + d^4 + c^2*d^2) + (32*tan(e/2 + (f*x)/2)*(6*B^2*a^2*c^2*d^4
+ 2*B^2*a^2*c^3*d^3 - 4*B^2*a^2*c^4*d^2 - A^2*a^2*c*d^5 + B^2*a^2*c*d^5 -
2*B^2*a^2*c^5*d + 2*A*B*a^2*c^2*d^4 + 2*A*B*a^2*c^3*d^3 - 2*A*B*a^2*c*d^5)
)/(2*c*d^4 + d^5 + c^2*d^3) + (B*a*((32*tan(e/2 + (f*x)/2)*(2*A*a*c*d^7 +
2*B*a*c*d^7 + 2*A*a*c^2*d^6 - 4*B*a*c^3*d^5 - 2*B*a*c^4*d^4))/(2*c*d^4 + d
^5 + c^2*d^3) - (32*(B*a*c*d^6 - A*a*c^2*d^5 - A*a*c^3*d^4 + B*a*c^2*d^5))
/(2*c*d^3 + d^4 + c^2*d^2) + (B*a*((32*(c^2*d^7 + 2*c^3*d^6 + c^4*d^5))/(2
*c*d^3 + d^4 + c^2*d^2) + (32*tan(e/2 + (f*x)/2)*(3*c*d^9 + 6*c^2*d^8 + c^
3*d^7 - 4*c^4*d^6 - 2*c^5*d^5))/(2*c*d^4 + d^5 + c^2*d^3))*1i)/d^2)*1i)/d^
2))/d^2 + (B*a*((32*(B^2*a^2*c^2*d^3 + 2*B^2*a^2*c^3*d^2 + B^2*a^2*c^4*d))
/(2*c*d^3 + d^4 + c^2*d^2) + (32*tan(e/2 + (f*x)/2)*(6*B^2*a^2*c^2*d^4 + 2
*B^2*a^2*c^3*d^3 - 4*B^2*a^2*c^4*d^2 - A^2*a^2*c*d^5 + B^2*a^2*c*d^5 - 2*B
^2*a^2*c^5*d + 2*A*B*a^2*c^2*d^4 + 2*A*B*a^2*c^3*d^3 - 2*A*B*a^2*c*d^5))/(
2*c*d^4 + d^5 + c^2*d^3) + (B*a*((32*(B*a*c*d^6 - A*a*c^2*d^5 - A*a*c^3*d^
4 + B*a*c^2*d^5))/(2*c*d^3 + d^4 + c^2*d^2) - (32*tan(e/2 + (f*x)/2)*(2*A*
a*c*d^7 + 2*B*a*c*d^7 + 2*A*a*c^2*d^6 - 4*B*a*c^3*d^5 - 2*B*a*c^4*d^4))/(2
*c*d^4 + d^5 + c^2*d^3) + (B*a*((32*(c^2*d^7 + 2*c^3*d^6 + c^4*d^5))/(2*c*
d^3 + d^4 + c^2*d^2) + (32*tan(e/2 + (f*x)/2)*(3*c*d^9 + 6*c^2*d^8 + c^3*d
^7 - 4*c^4*d^6 - 2*c^5*d^5))/(2*c*d^4 + d^5 + c^2*d^3))*1i)/d^2)*1i)/d^...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 599, normalized size of antiderivative = 4.83

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \frac{a\left(2\sqrt{c^2-d^2}\,atan\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c+d}{\sqrt{c^2-d^2}}\right)\sin(fx+e)\,a\,d^3-2\sqrt{c^2-d^2}\,atan\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c+d}{\sqrt{c^2-d^2}}\right)\sin(fx+e)\,b\,c^2d}{}$$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

output
```
(a*(2*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*s
in(e + f*x)*a*d**3 - 2*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqr
t(c**2 - d**2))*sin(e + f*x)*b*c**2*d - 2*sqrt(c**2 - d**2)*atan((tan((e +
 f*x)/2)*c + d)/sqrt(c**2 - d**2))*sin(e + f*x)*b*c*d**2 + 2*sqrt(c**2 - d
**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*sin(e + f*x)*b*d**3
+ 2*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*a*c
*d**2 - 2*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2
))*b*c**3 - 2*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 -
d**2))*b*c**2*d + 2*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c
**2 - d**2))*b*c*d**2 - cos(e + f*x)*a*c**2*d**2 + cos(e + f*x)*a*d**4 + c
os(e + f*x)*b*c**3*d - cos(e + f*x)*b*c*d**3 + sin(e + f*x)*b*c**3*d*f*x +
 sin(e + f*x)*b*c**2*d**2*f*x - sin(e + f*x)*b*c*d**3*f*x - sin(e + f*x)*b
*d**4*f*x + b*c**4*f*x + b*c**3*d*f*x - b*c**2*d**2*f*x - b*c*d**3*f*x))/(
d**2*f*(sin(e + f*x)*c**3*d + sin(e + f*x)*c**2*d**2 - sin(e + f*x)*c*d**3
 - sin(e + f*x)*d**4 + c**4 + c**3*d - c**2*d**2 - c*d**3))
```

3.250 $\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 176

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \frac{a(2Ac+Bc-Ad-2Bd)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c+d)\left(c^2-d^2\right)^{3/2}f} + \frac{a(Bc-Ad)\cos(e+fx)}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a(A(c-2d)d+B(c^2+2cd-2d^2))\cos(e+fx)}{2(c-d)d(c+d)^2f(c+d\sin(e+fx))}$$

output
```
a*(2*A*c-A*d+B*c-2*B*d)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/(
c+d)/(c^2-d^2)^(3/2)/f+1/2*a*(-A*d+B*c)*cos(f*x+e)/d/(c+d)/f/(c+d*sin(f*x+
e))^2-1/2*a*(A*(c-2*d)*d+B*(c^2+2*c*d-2*d^2))*cos(f*x+e)/(c-d)/d/(c+d)^2/f
/(c+d*sin(f*x+e))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.41 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.96

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{(c + d\sin(e + fx))^3} dx$$

$$= \frac{a(1+\sin(e+fx))\left(\frac{4(2Ac+Bc-Ad-2Bd)\arctan\left(\frac{\sec\left(\frac{fx}{2}\right)(\cos(e)-i\sin(e))\left(d\cos\left(e+\frac{fx}{2}\right)+c\sin\left(\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right)(\cos(e)-i\sin(e))}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}} + \frac{(2c^2+d^2)(}{}\right.}{4(c-d}$$

input
```
Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])
^3,x]
```

output
```
(a*(1 + Sin[e + f*x])*((4*(2*A*c + B*c - A*d - 2*B*d)*ArcTan[(Sec[(f*x)/2]
*(Cos[e] - I*Sin[e])*(d*Cos[e + (f*x)/2] + c*Sin[(f*x)/2]))/(Sqrt[c^2 - d^
2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*(Cos[e] - I*Sin[e]))/(Sqrt[c^2 - d^2]*Sqr
t[(Cos[e] - I*Sin[e])^2]) + ((2*c^2 + d^2)*(A*(c - 2*d)*d + B*(c^2 + 2*c*d
 - 2*d^2))*Cot[e] + d*Csc[e]*(-(d*(A*(c - 2*d)*d + B*(c^2 + 2*c*d - 2*d^2)
)*Cos[e + 2*f*x]) + (B*c*(2*c^2 + 6*c*d - 5*d^2) - A*d*(-4*c^2 + 6*c*d + d
^2))*Sin[f*x] + (A*d^2*(-2*c + d) + B*c*(2*c^2 + 2*c*d - 3*d^2))*Sin[2*e +
 f*x]))/(d^2*(c + d*Sin[e + f*x])^2)))/(4*(c - d)*(c + d)^2*f*(Cos[(e + f*
x)/2] + Sin[(e + f*x)/2])^2)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.09, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$, Rules used = {3042, 3447, 3042, 3500, 25, 3042, 3233, 25, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e + fx) + a)(A + B\sin(e + fx))}{(c + d\sin(e + fx))^3} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}dx$$

$$\Big\downarrow \text{3447}$$

$$\int \frac{(aA+aB)\sin(e+fx)+aA+aB\sin^2(e+fx)}{(c+d\sin(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(aA+aB)\sin(e+fx)+aA+aB\sin(e+fx)^2}{(c+d\sin(e+fx))^3}dx$$

$$\Big\downarrow \text{3500}$$

$$\frac{a(Bc-Ad)\cos(e+fx)}{2df(c+d)(c+d\sin(e+fx))^2}-\frac{\int -\frac{2a(A+B)(c-d)d+a(c-d)(Ad+B(c+2d))\sin(e+fx)}{(c+d\sin(e+fx))^2}dx}{2d\left(c^2-d^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2a(A+B)(c-d)d+a(c-d)(Ad+B(c+2d))\sin(e+fx)}{(c+d\sin(e+fx))^2}dx}{2d\left(c^2-d^2\right)}+\frac{a(Bc-Ad)\cos(e+fx)}{2df(c+d)(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{2a(A+B)(c-d)d+a(c-d)(Ad+B(c+2d))\sin(e+fx)}{(c+d\sin(e+fx))^2}dx}{2d\left(c^2-d^2\right)}+\frac{a(Bc-Ad)\cos(e+fx)}{2df(c+d)(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \text{3233}$$

$$-\frac{\frac{\int -\frac{a(c-d)d(2Ac+Bc-Ad-2Bd)}{c+d\sin(e+fx)}dx}{c^2-d^2}-\frac{a\left(Ad(c-2d)+B\left(c^2+2cd-2d^2\right)\right)\cos(e+fx)}{f(c+d)(c+d\sin(e+fx))}}{2d\left(c^2-d^2\right)}+\frac{a(Bc-Ad)\cos(e+fx)}{2df(c+d)(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{a(c-d)d(2Ac+Bc-Ad-2Bd)}{c+d\sin(e+fx)}dx}{c^2-d^2}-\frac{a\left(Ad(c-2d)+B\left(c^2+2cd-2d^2\right)\right)\cos(e+fx)}{f(c+d)(c+d\sin(e+fx))}}{2d\left(c^2-d^2\right)}+\frac{a(Bc-Ad)\cos(e+fx)}{2df(c+d)(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{ad(c-d)(2Ac-Ad+Bc-2Bd)\int \frac{1}{c+d\sin(e+fx)}dx}{c^2-d^2}-\frac{a\left(Ad(c-2d)+B\left(c^2+2cd-2d^2\right)\right)\cos(e+fx)}{f(c+d)(c+d\sin(e+fx))}}{2d\left(c^2-d^2\right)}+\frac{a(Bc-Ad)\cos(e+fx)}{2df(c+d)(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{ad(c-d)(2Ac-Ad+Bc-2Bd)\int \frac{1}{c+d\sin(e+fx)}dx}{c^2-d^2}-\frac{a\left(Ad(c-2d)+B\left(c^2+2cd-2d^2\right)\right)\cos(e+fx)}{f(c+d)(c+d\sin(e+fx))}}{2d\left(c^2-d^2\right)}+\frac{a(Bc-Ad)\cos(e+fx)}{2df(c+d)(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \text{3139}$$

$$\frac{\frac{2ad(c-d)(2Ac-Ad+Bc-2Bd)\int \frac{1}{c\tan^2\left(\frac{1}{2}(e+fx)\right)+2d\tan\left(\frac{1}{2}(e+fx)\right)+c}d\tan\left(\frac{1}{2}(e+fx)\right)}{f\left(c^2-d^2\right)}-\frac{a\left(Ad(c-2d)+B\left(c^2+2cd-2d^2\right)\right)\cos(e+fx)}{f(c+d)(c+d\sin(e+fx))}}{2d\left(c^2-d^2\right)}+\frac{a(Bc-Ad)\cos(e+fx)}{2df(c+d)(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \text{1083}$$

$$\frac{-\frac{4ad(c-d)(2Ac-Ad+Bc-2Bd)\int \frac{1}{-\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)^2-4\left(c^2-d^2\right)}d\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{f\left(c^2-d^2\right)}-\frac{a\left(Ad(c-2d)+B\left(c^2+2cd-2d^2\right)\right)\cos(e+fx)}{f(c+d)(c+d\sin(e+fx))}}{2d\left(c^2-d^2\right)}+\frac{a(Bc-Ad)\cos(e+fx)}{2df(c+d)(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{2ad(c-d)(2Ac-Ad+Bc-2Bd)\arctan\left(\frac{2c\tan\left(\frac{1}{2}(e+fx)\right)+2d}{2\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{3/2}}-\frac{a\left(Ad(c-2d)+B\left(c^2+2cd-2d^2\right)\right)\cos(e+fx)}{f(c+d)(c+d\sin(e+fx))}}{2d\left(c^2-d^2\right)}+\frac{a(Bc-Ad)\cos(e+fx)}{2df(c+d)(c+d\sin(e+fx))^2}$$

input

```
Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^3,x]
```

output

```
(a*(B*c - A*d)*Cos[e + f*x])/(2*d*(c + d)*f*(c + d*Sin[e + f*x])^2) + ((2*
a*(c - d)*d*(2*A*c + B*c - A*d - 2*B*d)*ArcTan[(2*d + 2*c*Tan[(e + f*x)/2]
)/(2*Sqrt[c^2 - d^2])])/((c^2 - d^2)^(3/2)*f) - (a*(A*(c - 2*d)*d + B*(c^2
 + 2*c*d - 2*d^2))*Cos[e + f*x])/((c + d)*f*(c + d*Sin[e + f*x])))/(2*d*(c
^2 - d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(167) = 334$.

Time = 0.73 (sec) , antiderivative size = 424, normalized size of antiderivative = 2.41

method	result
derivativedivides	$2a\left(\frac{-\frac{\left(3A\,c^2d-2Ac\,d^2-2A\,d^3-B\,c^3+2B\,c^2d\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2c\left(c^3+c^2d-c\,d^2-d^3\right)}-\frac{\left(2A\,c^4-2A\,c^3d+3A\,c^2d^2-4Ac\,d^3-2A\,d^4+2B\,c^4-B\,c^3d+4B\,c^2d^2\right.}{2\left(c^3+c^2d-c\,d^2-d^3\right)c^2}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c+2c\right.}\right.$
default	$2a\left(\frac{-\frac{\left(3A\,c^2d-2Ac\,d^2-2A\,d^3-B\,c^3+2B\,c^2d\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2c\left(c^3+c^2d-c\,d^2-d^3\right)}-\frac{\left(2A\,c^4-2A\,c^3d+3A\,c^2d^2-4Ac\,d^3-2A\,d^4+2B\,c^4-B\,c^3d+4B\,c^2d^2\right.}{2\left(c^3+c^2d-c\,d^2-d^3\right)c^2}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c+2c\right.}\right.$
risch	Expression too large to display

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x,method=_RETURNV
ERBOSE)
```

output
```
2/f*a*((-1/2*(3*A*c^2*d-2*A*c*d^2-2*A*d^3-B*c^3+2*B*c^2*d)/c/(c^3+c^2*d-c*
d^2-d^3)*tan(1/2*f*x+1/2*e)^3-1/2*(2*A*c^4-2*A*c^3*d+3*A*c^2*d^2-4*A*c*d^3
-2*A*d^4+2*B*c^4-B*c^3*d+4*B*c^2*d^2-2*B*c*d^3)/(c^3+c^2*d-c*d^2-d^3)/c^2*
tan(1/2*f*x+1/2*e)^2-1/2*(5*A*c^2*d-6*A*c*d^2-2*A*d^3+B*c^3+6*B*c^2*d-4*B*
c*d^2)/c/(c^3+c^2*d-c*d^2-d^3)*tan(1/2*f*x+1/2*e)-1/2*(2*A*c^2-2*A*c*d-A*d
^2+2*B*c^2-B*c*d)/(c^3+c^2*d-c*d^2-d^3))/(tan(1/2*f*x+1/2*e)^2*c+2*d*tan(1
/2*f*x+1/2*e)+c)^2+1/2*(2*A*c-A*d+B*c-2*B*d)/(c^3+c^2*d-c*d^2-d^3)/(c^2-d^
2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 441 vs. $2(167) = 334$.

Time = 0.13 (sec) , antiderivative size = 967, normalized size of antiderivative = 5.49

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{(c + d\sin(e + fx))^3} \, dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorith
m="fricas")
```

output
```
[1/4*(2*(B*a*c^4 + (A + 2*B)*a*c^3*d - (2*A + 3*B)*a*c^2*d^2 - (A + 2*B)*a
*c*d^3 + 2*(A + B)*a*d^4)*cos(f*x + e)*sin(f*x + e) + ((2*A + B)*a*c^3 - (
A + 2*B)*a*c^2*d + (2*A + B)*a*c*d^2 - (A + 2*B)*a*d^3 - ((2*A + B)*a*c*d^
2 - (A + 2*B)*a*d^3)*cos(f*x + e)^2 + 2*((2*A + B)*a*c^2*d - (A + 2*B)*a*c
*d^2)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2
*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x
 + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 -
d^2)) + 2*(2*(A + B)*a*c^4 - (2*A + B)*a*c^3*d - (3*A + 2*B)*a*c^2*d^2 + (
2*A + B)*a*c*d^3 + A*a*d^4)*cos(f*x + e))/((c^5*d^2 + c^4*d^3 - 2*c^3*d^4
- 2*c^2*d^5 + c*d^6 + d^7)*f*cos(f*x + e)^2 - 2*(c^6*d + c^5*d^2 - 2*c^4*d
^3 - 2*c^3*d^4 + c^2*d^5 + c*d^6)*f*sin(f*x + e) - (c^7 + c^6*d - c^5*d^2
- c^4*d^3 - c^3*d^4 - c^2*d^5 + c*d^6 + d^7)*f), 1/2*((B*a*c^4 + (A + 2*B)
*a*c^3*d - (2*A + 3*B)*a*c^2*d^2 - (A + 2*B)*a*c*d^3 + 2*(A + B)*a*d^4)*co
s(f*x + e)*sin(f*x + e) + ((2*A + B)*a*c^3 - (A + 2*B)*a*c^2*d + (2*A + B)
*a*c*d^2 - (A + 2*B)*a*d^3 - ((2*A + B)*a*c*d^2 - (A + 2*B)*a*d^3)*cos(f*x
 + e)^2 + 2*((2*A + B)*a*c^2*d - (A + 2*B)*a*c*d^2)*sin(f*x + e))*sqrt(c^2
 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (2*
(A + B)*a*c^4 - (2*A + B)*a*c^3*d - (3*A + 2*B)*a*c^2*d^2 + (2*A + B)*a*c*
d^3 + A*a*d^4)*cos(f*x + e))/((c^5*d^2 + c^4*d^3 - 2*c^3*d^4 - 2*c^2*d^5 +
 c*d^6 + d^7)*f*cos(f*x + e)^2 - 2*(c^6*d + c^5*d^2 - 2*c^4*d^3 - 2*c^3...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 570 vs. $2(167) = 334$.

Time = 0.29 (sec) , antiderivative size = 570, normalized size of antiderivative = 3.24

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorith
m="giac")
```

output
```
((2*A*a*c + B*a*c - A*a*d - 2*B*a*d)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn
(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c^3 + c^2*d
- c*d^2 - d^3)*sqrt(c^2 - d^2)) + (B*a*c^4*tan(1/2*f*x + 1/2*e)^3 - 3*A*a*
c^3*d*tan(1/2*f*x + 1/2*e)^3 - 2*B*a*c^3*d*tan(1/2*f*x + 1/2*e)^3 + 2*A*a*
c^2*d^2*tan(1/2*f*x + 1/2*e)^3 + 2*A*a*c*d^3*tan(1/2*f*x + 1/2*e)^3 - 2*A*
a*c^4*tan(1/2*f*x + 1/2*e)^2 - 2*B*a*c^4*tan(1/2*f*x + 1/2*e)^2 + 2*A*a*c^
3*d*tan(1/2*f*x + 1/2*e)^2 + B*a*c^3*d*tan(1/2*f*x + 1/2*e)^2 - 3*A*a*c^2*
d^2*tan(1/2*f*x + 1/2*e)^2 - 4*B*a*c^2*d^2*tan(1/2*f*x + 1/2*e)^2 + 4*A*a*
c*d^3*tan(1/2*f*x + 1/2*e)^2 + 2*B*a*c*d^3*tan(1/2*f*x + 1/2*e)^2 + 2*A*a*
d^4*tan(1/2*f*x + 1/2*e)^2 - B*a*c^4*tan(1/2*f*x + 1/2*e) - 5*A*a*c^3*d*ta
n(1/2*f*x + 1/2*e) - 6*B*a*c^3*d*tan(1/2*f*x + 1/2*e) + 6*A*a*c^2*d^2*tan(
1/2*f*x + 1/2*e) + 4*B*a*c^2*d^2*tan(1/2*f*x + 1/2*e) + 2*A*a*c*d^3*tan(1/
2*f*x + 1/2*e) - 2*A*a*c^4 - 2*B*a*c^4 + 2*A*a*c^3*d + B*a*c^3*d + A*a*c^2
*d^2)/((c^5 + c^4*d - c^3*d^2 - c^2*d^3)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*t
an(1/2*f*x + 1/2*e) + c)^2))/f
```

Mupad [B] (verification not implemented)

Time = 37.53 (sec) , antiderivative size = 554, normalized size of antiderivative = 3.15

$$\int \frac{(a+a\sin(e+fx))(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = -\frac{\frac{A\,a\,d^2-2\,A\,a\,c^2-2\,B\,a\,c^2+2\,A\,a\,c\,d+B\,a\,c\,d}{-c^3-c^2\,d+c\,d^2+d^3}+\frac{a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,A\,d^3-B\,c^3+6\,A\,c\,d^2-5\,A\,c^2\,d+4\,B\,c\,d^2-6\,B\,c^2\,d\right)}{c\,\left(-c^3-c^2\,d+c\,d^2+d^3\right)}+\frac{a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,(2\,\ldots}{c\ldots}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(2\,c^2+4\,d^2\right)+c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+c^2+4\,c\ldots\right)} - \frac{a\,\mathrm{atan}\left(\frac{\left(\frac{a\,(2\,A\,c-A\,d+B\,c-2\,B\,d)\,\left(-2\,c^3\,d-2\,c^2\,d^2+2\,c\,d^3+2\,d^4\right)}{2\,(c+d)^{5/2}\,(c-d)^{3/2}\,\left(-c^3-c^2\,d+c\,d^2+d^3\right)}+\frac{a\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(2\,A\,c-A\,d+B\,c-2\,B\,d)}{(c+d)^{5/2}\,(c-d)^{3/2}}\right)\,\left(-c^3-c^2\,d+c\,d^2+d^3\right)}{2\,A\,a\,c-A\,a\,d+B\,a\,c-2\,B\,a\,d}\right)\,(2\,A\,c\ldots}{f\,(c+d)^{5/2}\,(c-d)^{3/2}}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c + d*sin(e + f*x))^3,x)
```

output
```
- ((A*a*d^2 - 2*A*a*c^2 - 2*B*a*c^2 + 2*A*a*c*d + B*a*c*d)/(c*d^2 - c^2*d
- c^3 + d^3) + (a*tan(e/2 + (f*x)/2)*(2*A*d^3 - B*c^3 + 6*A*c*d^2 - 5*A*c^
2*d + 4*B*c*d^2 - 6*B*c^2*d))/(c*(c*d^2 - c^2*d - c^3 + d^3)) + (a*tan(e/2
 + (f*x)/2)^3*(2*A*d^3 + B*c^3 + 2*A*c*d^2 - 3*A*c^2*d - 2*B*c^2*d))/(c*(c
*d^2 - c^2*d - c^3 + d^3)) + (a*tan(e/2 + (f*x)/2)^2*(c^2 + 2*d^2)*(A*d^2
- 2*A*c^2 - 2*B*c^2 + 2*A*c*d + B*c*d))/(c^2*(c*d^2 - c^2*d - c^3 + d^3)))
/(f*(tan(e/2 + (f*x)/2)^2*(2*c^2 + 4*d^2) + c^2*tan(e/2 + (f*x)/2)^4 + c^2
 + 4*c*d*tan(e/2 + (f*x)/2)^3 + 4*c*d*tan(e/2 + (f*x)/2))) - (a*atan((((a*
(2*A*c - A*d + B*c - 2*B*d)*(2*c*d^3 - 2*c^3*d + 2*d^4 - 2*c^2*d^2))/(2*(c
 + d)^(5/2)*(c - d)^(3/2)*(c*d^2 - c^2*d - c^3 + d^3)) + (a*c*tan(e/2 + (f
*x)/2)*(2*A*c - A*d + B*c - 2*B*d))/((c + d)^(5/2)*(c - d)^(3/2)))*(c*d^2
- c^2*d - c^3 + d^3))/(2*A*a*c - A*a*d + B*a*c - 2*B*a*d))*(2*A*c - A*d +
B*c - 2*B*d))/(f*(c + d)^(5/2)*(c - d)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1477, normalized size of antiderivative = 8.39

$$\int \frac{(a + a\sin(e + fx))(A + B\sin(e + fx))}{(c + d\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)
```

output

```
(a*(8*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*s
in(e + f*x)**2*a*c**2*d**3 - 4*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c
+ d)/sqrt(c**2 - d**2))*sin(e + f*x)**2*a*c*d**4 + 4*sqrt(c**2 - d**2)*ata
n((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*sin(e + f*x)**2*b*c**2*d**3
- 8*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*sin
(e + f*x)**2*b*c*d**4 + 16*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)
/sqrt(c**2 - d**2))*sin(e + f*x)*a*c**3*d**2 - 8*sqrt(c**2 - d**2)*atan((t
an((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*sin(e + f*x)*a*c**2*d**3 + 8*sqr
t(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*sin(e + f*
x)*b*c**3*d**2 - 16*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c
**2 - d**2))*sin(e + f*x)*b*c**2*d**3 + 8*sqrt(c**2 - d**2)*atan((tan((e +
 f*x)/2)*c + d)/sqrt(c**2 - d**2))*a*c**4*d - 4*sqrt(c**2 - d**2)*atan((ta
n((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*a*c**3*d**2 + 4*sqrt(c**2 - d**2)
*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*b*c**4*d - 8*sqrt(c**2 -
 d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*b*c**3*d**2 - 2*co
s(e + f*x)*sin(e + f*x)*a*c**4*d**2 + 4*cos(e + f*x)*sin(e + f*x)*a*c**3*d
**3 + 2*cos(e + f*x)*sin(e + f*x)*a*c**2*d**4 - 4*cos(e + f*x)*sin(e + f*x
)*a*c*d**5 - 2*cos(e + f*x)*sin(e + f*x)*b*c**5*d - 4*cos(e + f*x)*sin(e +
 f*x)*b*c**4*d**2 + 6*cos(e + f*x)*sin(e + f*x)*b*c**3*d**3 + 4*cos(e + f*
x)*sin(e + f*x)*b*c**2*d**4 - 4*cos(e + f*x)*sin(e + f*x)*b*c*d**5 - 4*...
```

3.251 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

Optimal result

Integrand size = 35, antiderivative size = 464

$$
\begin{aligned}
&\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx \\
&= \frac{1}{16}a^2\big(6A\big(4c^3+8c^2d+7cd^2+2d^3\big)+B\big(16c^3+42c^2d+36cd^2+11d^3\big)\big)\,x \\
&\quad + \frac{a^2(6Ad(c^4-10c^3d-44c^2d^2-40cd^3-12d^4)-B(2c^5-12c^4d+47c^3d^2+208c^2d^3+216cd^4+64d^5))}{60d^2f} \\
&\quad + \frac{a^2(6Ad(2c^3-20c^2d-57cd^2-30d^3)-B(4c^4-24c^3d+96c^2d^2+284cd^3+165d^4))\cos(e+fx)\sin(e}{240df} \\
&\quad + \frac{a^2(6Ad(c^2-10cd-12d^2)-B(2c^3-12c^2d+51cd^2+64d^3))\cos(e+fx)(c+d\sin(e+fx))^2}{120d^2f} \\
&\quad + \frac{a^2(6A(c-10d)d-B(2c^2-12cd+55d^2))\cos(e+fx)(c+d\sin(e+fx))^3}{120d^2f} \\
&\quad + \frac{a^2(2Bc-6Ad-7Bd)\cos(e+fx)(c+d\sin(e+fx))^4}{30d^2f} \\
&\quad - \frac{B\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)(c+d\sin(e+fx))^4}{6df}
\end{aligned}
$$

output
```
1/16*a^2*(6*A*(4*c^3+8*c^2*d+7*c*d^2+2*d^3)+B*(16*c^3+42*c^2*d+36*c*d^2+11
*d^3))*x+1/60*a^2*(6*A*d*(c^4-10*c^3*d-44*c^2*d^2-40*c*d^3-12*d^4)-B*(2*c^
5-12*c^4*d+47*c^3*d^2+208*c^2*d^3+216*c*d^4+64*d^5))*cos(f*x+e)/d^2/f+1/24
0*a^2*(6*A*d*(2*c^3-20*c^2*d-57*c*d^2-30*d^3)-B*(4*c^4-24*c^3*d+96*c^2*d^2
+284*c*d^3+165*d^4))*cos(f*x+e)*sin(f*x+e)/d/f+1/120*a^2*(6*A*d*(c^2-10*c*
d-12*d^2)-B*(2*c^3-12*c^2*d+51*c*d^2+64*d^3))*cos(f*x+e)*(c+d*sin(f*x+e))^
2/d^2/f+1/120*a^2*(6*A*(c-10*d)*d-B*(2*c^2-12*c*d+55*d^2))*cos(f*x+e)*(c+d
*sin(f*x+e))^3/d^2/f+1/30*a^2*(-6*A*d+2*B*c-7*B*d)*cos(f*x+e)*(c+d*sin(f*x
+e))^4/d^2/f-1/6*B*cos(f*x+e)*(a^2+a^2*sin(f*x+e))*(c+d*sin(f*x+e))^4/d/f
```

Mathematica [A] (warning: unable to verify)

Time = 3.71 (sec) , antiderivative size = 437, normalized size of antiderivative = 0.94

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = -\frac{a^2\cos(e+fx)\left(60(6A(4c^3+8c^2d+7cd^2+2d^3)+B(16c^3+42c^2d+36cd^2+11d^3))\arcsin\left(\frac{\sqrt{1-\sin(e-}}{\sqrt{2}}\right.\right.}{}$$

input
```
Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])
^3,x]
```

output
```
-1/480*(a^2*Cos[e + f*x]*(60*(6*A*(4*c^3 + 8*c^2*d + 7*c*d^2 + 2*d^3) + B*
(16*c^3 + 42*c^2*d + 36*c*d^2 + 11*d^3))*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqr
t[2]] + Sqrt[Cos[e + f*x]^2]*(960*A*c^3 + 880*B*c^3 + 2640*A*c^2*d + 2400*
B*c^2*d + 2400*A*c*d^2 + 2268*B*c*d^2 + 756*A*d^3 + 712*B*d^3 - 16*(3*A*d*
(5*c^2 + 10*c*d + 4*d^2) + B*(5*c^3 + 30*c^2*d + 36*c*d^2 + 14*d^3))*Cos[2
*(e + f*x)] + 12*d^2*(3*B*c + A*d + 2*B*d)*Cos[4*(e + f*x)] + 240*A*c^3*Si
n[e + f*x] + 480*B*c^3*Sin[e + f*x] + 1440*A*c^2*d*Sin[e + f*x] + 1530*B*c
^2*d*Sin[e + f*x] + 1530*A*c*d^2*Sin[e + f*x] + 1620*B*c*d^2*Sin[e + f*x]
+ 540*A*d^3*Sin[e + f*x] + 545*B*d^3*Sin[e + f*x] - 90*B*c^2*d*Sin[3*(e +
f*x)] - 90*A*c*d^2*Sin[3*(e + f*x)] - 180*B*c*d^2*Sin[3*(e + f*x)] - 60*A*
d^3*Sin[3*(e + f*x)] - 80*B*d^3*Sin[3*(e + f*x)] + 5*B*d^3*Sin[5*(e + f*x)
])))/(f*Sqrt[Cos[e + f*x]^2])
```

Rubi [A] (verified)

Time = 1.79 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3455, 3042, 3447, 3042, 3502, 25, 3042, 3232, 27, 3042, 3232, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^2(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^2(A+B\sin(e+fx))(c+d\sin(e+fx))^3dx$$

$$\Big\downarrow \text{3455}$$

$$\frac{\int (\sin(e+fx)a+a)(c+d\sin(e+fx))^3(a(6Ad+B(c+4d))-a(2Bc-6Ad-7Bd)\sin(e+fx))dx}{6d}-\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^4}{6df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (\sin(e+fx)a+a)(c+d\sin(e+fx))^3(a(6Ad+B(c+4d))-a(2Bc-6Ad-7Bd)\sin(e+fx))dx}{6d}-\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^4}{6df}$$

$$\Big\downarrow \text{3447}$$

$$\frac{\int (c+d\sin(e+fx))^3\left(-\left((2Bc-6Ad-7Bd)\sin^2(e+fx)a^2\right)+(6Ad+B(c+4d))a^2+\left(a^2(6Ad+B(c+4d))\right.\right.}{6d}$$
$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^4}{6df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (c+d\sin(e+fx))^3\left(-\left((2Bc-6Ad-7Bd)\sin(e+fx)^2a^2\right)+(6Ad+B(c+4d))a^2+\left(a^2(6Ad+B(c+4d))\right.\right.}{6d}$$
$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^4}{6df}$$

$\Big\downarrow$ 3502

$$\frac{\frac{\int -(c+d\sin(e+fx))^3\left(3d(Bc-18Ad-16Bd)a^2+\left(6A(c-10d)d-B\left(2c^2-12dc+55d^2\right)\right)\sin(e+fx)a^2\right)dx}{5d}+\frac{a^2(-6Ad+2Bc-7Bd)\cos(e+fx)(c+}{5df}}{6d}$$
$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^4}{6df}$$

$\Big\downarrow$ 25

$$\frac{\frac{a^2(-6Ad+2Bc-7Bd)\cos(e+fx)(c+d\sin(e+fx))^4}{5df}-\frac{\int(c+d\sin(e+fx))^3\left(3d(B(c-16d)-18Ad)a^2+\left(6A(c-10d)d-B\left(2c^2-12dc+55d^2\right)\right)\sin(e}{5d}}{6d}$$
$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^4}{6df}$$

$\Big\downarrow$ 3042

$$\frac{\frac{a^2(-6Ad+2Bc-7Bd)\cos(e+fx)(c+d\sin(e+fx))^4}{5df}-\frac{\int(c+d\sin(e+fx))^3\left(3d(B(c-16d)-18Ad)a^2+\left(6A(c-10d)d-B\left(2c^2-12dc+55d^2\right)\right)\sin(e}{5d}}{6d}$$
$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^4}{6df}$$

$\Big\downarrow$ 3232

$$\frac{\frac{a^2(-6Ad+2Bc-7Bd)\cos(e+fx)(c+d\sin(e+fx))^4}{5df}-\frac{\frac{1}{4}\int -3(c+d\sin(e+fx))^2\left(a^2d\left(6Ad(11c+10d)-B\left(2c^2-52dc-55d^2\right)\right)-a^2\left(6Ad\left(c^2-10dc-\right.\right.\right.}{}}{6d}$$
$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^4}{6df}$$

$\Big\downarrow$ 27

$$\frac{\frac{a^2(-6Ad+2Bc-7Bd)\cos(e+fx)(c+d\sin(e+fx))^4}{5df}-\frac{-\frac{3}{4}\int(c+d\sin(e+fx))^2\left(a^2d\left(6Ad(11c+10d)-B\left(2c^2-52dc-55d^2\right)\right)-a^2\left(6Ad\left(c^2-10dc-\right.\right.\right.}{}}{6d}$$
$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^4}{6df}$$

$\Big\downarrow$ 3042

$$\frac{a^2(-6Ad+2Bc-7Bd)\cos(e+fx)(c+d\sin(e+fx))^4}{5df}-\frac{-\frac{3}{4}\int(c+d\sin(e+fx))^2\left(a^2d\left(6Ad(11c+10d)-B\left(2c^2-52dc-55d^2\right)\right)-a^2\left(6Ad\left(c^2-10dc-\cdots\right.\right.\right.}{6d}\cdots$$

$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^4}{6df}$$

$$\Big\downarrow \text{3232}$$

$$\frac{a^2(-6Ad+2Bc-7Bd)\cos(e+fx)(c+d\sin(e+fx))^4}{5df}-\frac{-\frac{3}{4}\left(\frac{1}{3}\int(c+d\sin(e+fx))\left(a^2d\left(6Ad\left(31c^2+50dc+24d^2\right)-B\left(2c^3-132dc^2-267d^2c-128d^3\cdots\right.\right.\right.\right.}{\cdots}\cdots$$

$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^4}{6df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2(-6Ad+2Bc-7Bd)\cos(e+fx)(c+d\sin(e+fx))^4}{5df}-\frac{-\frac{3}{4}\left(\frac{1}{3}\int(c+d\sin(e+fx))\left(a^2d\left(6Ad\left(31c^2+50dc+24d^2\right)-B\left(2c^3-132dc^2-267d^2c-128d^3\cdots\right.\right.\right.\right.}{\cdots}\cdots$$

$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^4}{6df}$$

$$\Big\downarrow \text{3213}$$

$$\frac{a^2(-6Ad+2Bc-7Bd)\cos(e+fx)(c+d\sin(e+fx))^4}{5df}-\frac{-\frac{a^2\left(6Ad(c-10d)-B\left(2c^2-12cd+55d^2\right)\right)\cos(e+fx)(c+d\sin(e+fx))^3}{4f}-\frac{3}{4}\left(\frac{a^2\left(6Ad\left(c^2-10cd-12\cdots\right.\right.}{\cdots}\right.}{\cdots}\cdots$$

$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^4}{6df}$$

input
```
Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x]
```

output
```
-1/6*(B*Cos[e + f*x]*(a^2 + a^2*Sin[e + f*x])*(c + d*Sin[e + f*x])^4)/(d*f
) + ((a^2*(2*B*c - 6*A*d - 7*B*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(5*
d*f) - (-1/4*(a^2*(6*A*(c - 10*d)*d - B*(2*c^2 - 12*c*d + 55*d^2))*Cos[e +
 f*x]*(c + d*Sin[e + f*x])^3)/f - (3*((a^2*(6*A*d*(c^2 - 10*c*d - 12*d^2)
- B*(2*c^3 - 12*c^2*d + 51*c*d^2 + 64*d^3))*Cos[e + f*x]*(c + d*Sin[e + f*
x])^2)/(3*f) + ((15*a^2*d^2*(6*A*(4*c^3 + 8*c^2*d + 7*c*d^2 + 2*d^3) + B*(
16*c^3 + 42*c^2*d + 36*c*d^2 + 11*d^3))*x)/2 + (2*a^2*(6*A*d*(c^4 - 10*c^3
*d - 44*c^2*d^2 - 40*c*d^3 - 12*d^4) - B*(2*c^5 - 12*c^4*d + 47*c^3*d^2 +
208*c^2*d^3 + 216*c*d^4 + 64*d^5))*Cos[e + f*x])/f + (a^2*d*(6*A*d*(2*c^3
- 20*c^2*d - 57*c*d^2 - 30*d^3) - B*(4*c^4 - 24*c^3*d + 96*c^2*d^2 + 284*c
*d^3 + 165*d^4))*Cos[e + f*x]*Sin[e + f*x])/(2*f))/3))/4)/(5*d))/(6*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 745, normalized size of antiderivative = 1.61

$$a^2 A\,c^3\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-a^2 A\,c^2 d\left(2+\sin(fx+e)^2\right)\cos(fx+e)+3a^2 A c\,d^2\left(-\frac{\left(\sin(fx+e)^3+\right.}{}\right.$$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x)
```

output
```
1/f*(a^2*A*c^3*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-a^2*A*c^2*d*(2+s
in(f*x+e)^2)*cos(f*x+e)+3*a^2*A*c*d^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*
cos(f*x+e)+3/8*f*x+3/8*e)-1/5*a^2*A*d^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2
)*cos(f*x+e)-1/3*a^2*B*c^3*(2+sin(f*x+e)^2)*cos(f*x+e)+3*a^2*B*c^2*d*(-1/4
*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-3/5*a^2*B*c*d^2*(
8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+a^2*B*d^3*(-1/6*(sin(f*x+e)^
5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)-2*a^2*A*c^
3*cos(f*x+e)+6*a^2*A*c^2*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-2*a^
2*A*c*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)+2*a^2*A*d^3*(-1/4*(sin(f*x+e)^3+3/2*
sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+2*a^2*B*c^3*(-1/2*sin(f*x+e)*cos(f*x
+e)+1/2*f*x+1/2*e)-2*a^2*B*c^2*d*(2+sin(f*x+e)^2)*cos(f*x+e)+6*a^2*B*c*d^2
*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2/5*a^2*B*d
^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+a^2*A*c^3*(f*x+e)-3*a^2*
A*c^2*d*cos(f*x+e)+3*a^2*A*c*d^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e
)-1/3*a^2*A*d^3*(2+sin(f*x+e)^2)*cos(f*x+e)-a^2*B*c^3*cos(f*x+e)+3*a^2*B*c
^2*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-a^2*B*c*d^2*(2+sin(f*x+e)^
2)*cos(f*x+e)+a^2*B*d^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8
*f*x+3/8*e))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.78

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx =$$
$$-\frac{48\,(3\,Ba^2cd^2+(A+2\,B)a^2d^3)\cos{(fx+e)}^5-80\,(Ba^2c^3+3\,(A+2\,B)a^2c^2d+3\,(2\,A+3\,B)a^2cd^2+}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algori
thm="fricas")
```

output
```
-1/240*(48*(3*B*a^2*c*d^2 + (A + 2*B)*a^2*d^3)*cos(f*x + e)^5 - 80*(B*a^2*
c^3 + 3*(A + 2*B)*a^2*c^2*d + 3*(2*A + 3*B)*a^2*c*d^2 + (3*A + 4*B)*a^2*d^
3)*cos(f*x + e)^3 - 15*(8*(3*A + 2*B)*a^2*c^3 + 6*(8*A + 7*B)*a^2*c^2*d +
6*(7*A + 6*B)*a^2*c*d^2 + (12*A + 11*B)*a^2*d^3)*f*x + 480*((A + B)*a^2*c^
3 + 3*(A + B)*a^2*c^2*d + 3*(A + B)*a^2*c*d^2 + (A + B)*a^2*d^3)*cos(f*x +
 e) + 5*(8*B*a^2*d^3*cos(f*x + e)^5 - 2*(18*B*a^2*c^2*d + 18*(A + 2*B)*a^2
*c*d^2 + (12*A + 19*B)*a^2*d^3)*cos(f*x + e)^3 + 3*(8*(A + 2*B)*a^2*c^3 +
6*(8*A + 9*B)*a^2*c^2*d + 6*(9*A + 10*B)*a^2*c*d^2 + (20*A + 21*B)*a^2*d^3
)*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1865 vs. $2(450) = 900$.

Time = 0.55 (sec) , antiderivative size = 1865, normalized size of antiderivative = 4.02

$$\int (a + a\sin(e + fx))^2 (A + B\sin(e + fx))(c + d\sin(e + fx))^3 \, dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3,x)
```

output
```
Piecewise((A*a**2*c**3*x*sin(e + f*x)**2/2 + A*a**2*c**3*x*cos(e + f*x)**2
/2 + A*a**2*c**3*x - A*a**2*c**3*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a**
2*c**3*cos(e + f*x)/f + 3*A*a**2*c**2*d*x*sin(e + f*x)**2 + 3*A*a**2*c**2*
d*x*cos(e + f*x)**2 - 3*A*a**2*c**2*d*sin(e + f*x)**2*cos(e + f*x)/f - 3*A
*a**2*c**2*d*sin(e + f*x)*cos(e + f*x)/f - 2*A*a**2*c**2*d*cos(e + f*x)**3
/f - 3*A*a**2*c**2*d*cos(e + f*x)/f + 9*A*a**2*c*d**2*x*sin(e + f*x)**4/8
+ 9*A*a**2*c*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*A*a**2*c*d**2*x*
sin(e + f*x)**2/2 + 9*A*a**2*c*d**2*x*cos(e + f*x)**4/8 + 3*A*a**2*c*d**2*
x*cos(e + f*x)**2/2 - 15*A*a**2*c*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f)
- 6*A*a**2*c*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 9*A*a**2*c*d**2*sin(e +
 f*x)*cos(e + f*x)**3/(8*f) - 3*A*a**2*c*d**2*sin(e + f*x)*cos(e + f*x)/(2
*f) - 4*A*a**2*c*d**2*cos(e + f*x)**3/f + 3*A*a**2*d**3*x*sin(e + f*x)**4/
4 + 3*A*a**2*d**3*x*sin(e + f*x)**2*cos(e + f*x)**2/2 + 3*A*a**2*d**3*x*co
s(e + f*x)**4/4 - A*a**2*d**3*sin(e + f*x)**4*cos(e + f*x)/f - 5*A*a**2*d*
*3*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 4*A*a**2*d**3*sin(e + f*x)**2*cos(
e + f*x)**3/(3*f) - A*a**2*d**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**2*
d**3*sin(e + f*x)*cos(e + f*x)**3/(4*f) - 8*A*a**2*d**3*cos(e + f*x)**5/(1
5*f) - 2*A*a**2*d**3*cos(e + f*x)**3/(3*f) + B*a**2*c**3*x*sin(e + f*x)**2
 + B*a**2*c**3*x*cos(e + f*x)**2 - B*a**2*c**3*sin(e + f*x)**2*cos(e + f*x
)/f - B*a**2*c**3*sin(e + f*x)*cos(e + f*x)/f - 2*B*a**2*c**3*cos(e + f...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 724, normalized size of antiderivative = 1.56

$$\int (a + a\sin(e + fx))^2 (A + B\sin(e + fx))(c + d\sin(e + fx))^3 \, dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algori
thm="maxima")
```

output
```
1/960*(240*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c^3 + 960*(f*x + e)*A*a^
2*c^3 + 320*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*c^3 + 480*(2*f*x + 2*e
 - sin(2*f*x + 2*e))*B*a^2*c^3 + 960*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a
^2*c^2*d + 1440*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c^2*d + 1920*(cos(f
*x + e)^3 - 3*cos(f*x + e))*B*a^2*c^2*d + 90*(12*f*x + 12*e + sin(4*f*x +
4*e) - 8*sin(2*f*x + 2*e))*B*a^2*c^2*d + 720*(2*f*x + 2*e - sin(2*f*x + 2*
e))*B*a^2*c^2*d + 1920*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*c*d^2 + 90*
(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^2*c*d^2 + 720*
(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c*d^2 - 192*(3*cos(f*x + e)^5 - 10*
cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^2*c*d^2 + 960*(cos(f*x + e)^3 - 3*co
s(f*x + e))*B*a^2*c*d^2 + 180*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*
f*x + 2*e))*B*a^2*c*d^2 - 64*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*co
s(f*x + e))*A*a^2*d^3 + 320*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*d^3 +
60*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^2*d^3 - 128
*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^2*d^3 + 5*(4
*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x +
2*e))*B*a^2*d^3 + 30*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e
))*B*a^2*d^3 - 1920*A*a^2*c^3*cos(f*x + e) - 960*B*a^2*c^3*cos(f*x + e) -
2880*A*a^2*c^2*d*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 468, normalized size of antiderivative = 1.01

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = -\frac{Ba^2d^3\sin(6fx+6e)}{192f}$$
$$+\frac{1}{16}\left(24Aa^2c^3+16Ba^2c^3+48Aa^2c^2d+42Ba^2c^2d+42Aa^2cd^2+36Ba^2cd^2+12Aa^2d^3+11Ba^2d^3\right)$$
$$-\frac{(3Ba^2cd^2+Aa^2d^3+2Ba^2d^3)\cos(5fx+5e)}{80f}$$
$$+\frac{(4Ba^2c^3+12Aa^2c^2d+24Ba^2c^2d+24Aa^2cd^2+27Ba^2cd^2+9Aa^2d^3+10Ba^2d^3)\cos(3fx+3e)}{48f}$$
$$-\frac{(16Aa^2c^3+14Ba^2c^3+42Aa^2c^2d+36Ba^2c^2d+36Aa^2cd^2+33Ba^2cd^2+11Aa^2d^3+10Ba^2d^3)\cos}{8f}$$
$$+\frac{(6Ba^2c^2d+6Aa^2cd^2+12Ba^2cd^2+4Aa^2d^3+5Ba^2d^3)\sin(4fx+4e)}{64f}$$
$$-\frac{(16Aa^2c^3+32Ba^2c^3+96Aa^2c^2d+96Ba^2c^2d+96Aa^2cd^2+96Ba^2cd^2+32Aa^2d^3+31Ba^2d^3)\sin}{64f}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algori
thm="giac")
```

output
```
-1/192*B*a^2*d^3*sin(6*f*x + 6*e)/f + 1/16*(24*A*a^2*c^3 + 16*B*a^2*c^3 +
48*A*a^2*c^2*d + 42*B*a^2*c^2*d + 42*A*a^2*c*d^2 + 36*B*a^2*c*d^2 + 12*A*a
^2*d^3 + 11*B*a^2*d^3)*x - 1/80*(3*B*a^2*c*d^2 + A*a^2*d^3 + 2*B*a^2*d^3)*
cos(5*f*x + 5*e)/f + 1/48*(4*B*a^2*c^3 + 12*A*a^2*c^2*d + 24*B*a^2*c^2*d +
 24*A*a^2*c*d^2 + 27*B*a^2*c*d^2 + 9*A*a^2*d^3 + 10*B*a^2*d^3)*cos(3*f*x +
 3*e)/f - 1/8*(16*A*a^2*c^3 + 14*B*a^2*c^3 + 42*A*a^2*c^2*d + 36*B*a^2*c^2
*d + 36*A*a^2*c*d^2 + 33*B*a^2*c*d^2 + 11*A*a^2*d^3 + 10*B*a^2*d^3)*cos(f*
x + e)/f + 1/64*(6*B*a^2*c^2*d + 6*A*a^2*c*d^2 + 12*B*a^2*c*d^2 + 4*A*a^2*
d^3 + 5*B*a^2*d^3)*sin(4*f*x + 4*e)/f - 1/64*(16*A*a^2*c^3 + 32*B*a^2*c^3
+ 96*A*a^2*c^2*d + 96*B*a^2*c^2*d + 96*A*a^2*c*d^2 + 96*B*a^2*c*d^2 + 32*A
*a^2*d^3 + 31*B*a^2*d^3)*sin(2*f*x + 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 38.37 (sec) , antiderivative size = 1291, normalized size of antiderivative = 2.78

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = \text{Too large to display}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^3,x)
```

output
```
(a^2*atan((a^2*tan(e/2 + (f*x)/2)*(24*A*c^3 + 12*A*d^3 + 16*B*c^3 + 11*B*d
^3 + 42*A*c*d^2 + 48*A*c^2*d + 36*B*c*d^2 + 42*B*c^2*d))/(8*(3*A*a^2*c^3 +
 (3*A*a^2*d^3)/2 + 2*B*a^2*c^3 + (11*B*a^2*d^3)/8 + (21*A*a^2*c*d^2)/4 + 6
*A*a^2*c^2*d + (9*B*a^2*c*d^2)/2 + (21*B*a^2*c^2*d)/4)))*(24*A*c^3 + 12*A*
d^3 + 16*B*c^3 + 11*B*d^3 + 42*A*c*d^2 + 48*A*c^2*d + 36*B*c*d^2 + 42*B*c^
2*d))/(8*f) - (tan(e/2 + (f*x)/2)*(A*a^2*c^3 + (3*A*a^2*d^3)/2 + 2*B*a^2*c
^3 + (11*B*a^2*d^3)/8 + (21*A*a^2*c*d^2)/4 + 6*A*a^2*c^2*d + (9*B*a^2*c*d^
2)/2 + (21*B*a^2*c^2*d)/4) + tan(e/2 + (f*x)/2)^8*(20*A*a^2*c^3 + 4*A*a^2*
d^3 + 14*B*a^2*c^3 + 24*A*a^2*c*d^2 + 42*A*a^2*c^2*d + 12*B*a^2*c*d^2 + 24
*B*a^2*c^2*d) - tan(e/2 + (f*x)/2)^11*(A*a^2*c^3 + (3*A*a^2*d^3)/2 + 2*B*a
^2*c^3 + (11*B*a^2*d^3)/8 + (21*A*a^2*c*d^2)/4 + 6*A*a^2*c^2*d + (9*B*a^2*
c*d^2)/2 + (21*B*a^2*c^2*d)/4) + tan(e/2 + (f*x)/2)^5*(2*A*a^2*c^3 + 7*A*a
^2*d^3 + 4*B*a^2*c^3 + (47*B*a^2*d^3)/4 + (33*A*a^2*c*d^2)/2 + 12*A*a^2*c^
2*d + 21*B*a^2*c*d^2 + (33*B*a^2*c^2*d)/2) - tan(e/2 + (f*x)/2)^7*(2*A*a^2
*c^3 + 7*A*a^2*d^3 + 4*B*a^2*c^3 + (47*B*a^2*d^3)/4 + (33*A*a^2*c*d^2)/2 +
 12*A*a^2*c^2*d + 21*B*a^2*c*d^2 + (33*B*a^2*c^2*d)/2) + tan(e/2 + (f*x)/2
)^3*(3*A*a^2*c^3 + (17*A*a^2*d^3)/2 + 6*B*a^2*c^3 + (187*B*a^2*d^3)/24 + (
87*A*a^2*c*d^2)/4 + 18*A*a^2*c^2*d + (51*B*a^2*c*d^2)/2 + (87*B*a^2*c^2*d)
/4) - tan(e/2 + (f*x)/2)^9*(3*A*a^2*c^3 + (17*A*a^2*d^3)/2 + 6*B*a^2*c^3 +
 (187*B*a^2*d^3)/24 + (87*A*a^2*c*d^2)/4 + 18*A*a^2*c^2*d + (51*B*a^2*c...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 705, normalized size of antiderivative = 1.52

$$\int (a + a\sin(e + fx))^2 (A + B\sin(e + fx))(c + d\sin(e + fx))^3 \, dx = \text{Too large to display}$$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x)
```

output
```
(a**2*( - 40*cos(e + f*x)*sin(e + f*x)**5*b*d**3 - 48*cos(e + f*x)*sin(e +
 f*x)**4*a*d**3 - 144*cos(e + f*x)*sin(e + f*x)**4*b*c*d**2 - 96*cos(e + f
*x)*sin(e + f*x)**4*b*d**3 - 180*cos(e + f*x)*sin(e + f*x)**3*a*c*d**2 - 1
20*cos(e + f*x)*sin(e + f*x)**3*a*d**3 - 180*cos(e + f*x)*sin(e + f*x)**3*
b*c**2*d - 360*cos(e + f*x)*sin(e + f*x)**3*b*c*d**2 - 110*cos(e + f*x)*si
n(e + f*x)**3*b*d**3 - 240*cos(e + f*x)*sin(e + f*x)**2*a*c**2*d - 480*cos
(e + f*x)*sin(e + f*x)**2*a*c*d**2 - 144*cos(e + f*x)*sin(e + f*x)**2*a*d*
*3 - 80*cos(e + f*x)*sin(e + f*x)**2*b*c**3 - 480*cos(e + f*x)*sin(e + f*x
)**2*b*c**2*d - 432*cos(e + f*x)*sin(e + f*x)**2*b*c*d**2 - 128*cos(e + f*
x)*sin(e + f*x)**2*b*d**3 - 120*cos(e + f*x)*sin(e + f*x)*a*c**3 - 720*cos
(e + f*x)*sin(e + f*x)*a*c**2*d - 630*cos(e + f*x)*sin(e + f*x)*a*c*d**2 -
 180*cos(e + f*x)*sin(e + f*x)*a*d**3 - 240*cos(e + f*x)*sin(e + f*x)*b*c*
*3 - 630*cos(e + f*x)*sin(e + f*x)*b*c**2*d - 540*cos(e + f*x)*sin(e + f*x
)*b*c*d**2 - 165*cos(e + f*x)*sin(e + f*x)*b*d**3 - 480*cos(e + f*x)*a*c**
3 - 1200*cos(e + f*x)*a*c**2*d - 960*cos(e + f*x)*a*c*d**2 - 288*cos(e + f
*x)*a*d**3 - 400*cos(e + f*x)*b*c**3 - 960*cos(e + f*x)*b*c**2*d - 864*cos
(e + f*x)*b*c*d**2 - 256*cos(e + f*x)*b*d**3 + 360*a*c**3*f*x + 480*a*c**3
 + 720*a*c**2*d*f*x + 1200*a*c**2*d + 630*a*c*d**2*f*x + 960*a*c*d**2 + 18
0*a*d**3*f*x + 288*a*d**3 + 240*b*c**3*f*x + 400*b*c**3 + 630*b*c**2*d*f*x
 + 960*b*c**2*d + 540*b*c*d**2*f*x + 864*b*c*d**2 + 165*b*d**3*f*x + 25...
```

3.252 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

Optimal result

Integrand size = 35, antiderivative size = 336

$$\begin{aligned}
&\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx \\
&= \frac{1}{8}a^2\left(12Ac^2+8Bc^2+16Acd+14Bcd+7Ad^2+6Bd^2\right)x \\
&\quad+\frac{a^2(5Ad(c^3-8c^2d-20cd^2-8d^3)-2B(c^4-5c^3d+16c^2d^2+40cd^3+18d^4))\cos(e+fx)}{30d^2f} \\
&\quad+\frac{a^2(5Ad(2c^2-16cd-21d^2)-B(4c^3-20c^2d+66cd^2+90d^3))\cos(e+fx)\sin(e+fx)}{120df} \\
&\quad+\frac{a^2(5A(c-8d)d-2B(c^2-5cd+18d^2))\cos(e+fx)(c+d\sin(e+fx))^2}{60d^2f} \\
&\quad+\frac{a^2(2B(c-3d)-5Ad)\cos(e+fx)(c+d\sin(e+fx))^3}{20d^2f} \\
&\quad-\frac{B\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)(c+d\sin(e+fx))^3}{5df}
\end{aligned}$$

output
```
1/8*a^2*(12*A*c^2+16*A*c*d+7*A*d^2+8*B*c^2+14*B*c*d+6*B*d^2)*x+1/30*a^2*(5
*A*d*(c^3-8*c^2*d-20*c*d^2-8*d^3)-2*B*(c^4-5*c^3*d+16*c^2*d^2+40*c*d^3+18*
d^4))*cos(f*x+e)/d^2/f+1/120*a^2*(5*A*d*(2*c^2-16*c*d-21*d^2)-B*(4*c^3-20*
c^2*d+66*c*d^2+90*d^3))*cos(f*x+e)*sin(f*x+e)/d/f+1/60*a^2*(5*A*(c-8*d)*d-
2*B*(c^2-5*c*d+18*d^2))*cos(f*x+e)*(c+d*sin(f*x+e))^2/d^2/f+1/20*a^2*(2*B*
(c-3*d)-5*A*d)*cos(f*x+e)*(c+d*sin(f*x+e))^3/d^2/f-1/5*B*cos(f*x+e)*(a^2+a
^2*sin(f*x+e))*(c+d*sin(f*x+e))^3/d/f
```

Mathematica [A] (verified)

Time = 1.87 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.88

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx =$$

$$-\frac{a^2\cos(e+fx)\left(60(2B(4c^2+7cd+3d^2)+A(12c^2+16cd+7d^2))\arcsin\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+\right.}{}$$

input
```
Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])
^2,x]
```

output
```
-1/240*(a^2*Cos[e + f*x]*(60*(2*B*(4*c^2 + 7*c*d + 3*d^2) + A*(12*c^2 + 16
*c*d + 7*d^2))*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]] + Sqrt[Cos[e + f*x]^
2]*(480*A*c^2 + 440*B*c^2 + 880*A*c*d + 800*B*c*d + 400*A*d^2 + 378*B*d^2
- 8*(10*A*d*(c + d) + B*(5*c^2 + 20*c*d + 12*d^2))*Cos[2*(e + f*x)] + 6*B*
d^2*Cos[4*(e + f*x)] + 120*A*c^2*Sin[e + f*x] + 240*B*c^2*Sin[e + f*x] + 4
80*A*c*d*Sin[e + f*x] + 510*B*c*d*Sin[e + f*x] + 255*A*d^2*Sin[e + f*x] +
270*B*d^2*Sin[e + f*x] - 30*B*c*d*Sin[3*(e + f*x)] - 15*A*d^2*Sin[3*(e + f
*x)] - 30*B*d^2*Sin[3*(e + f*x)])))/(f*Sqrt[Cos[e + f*x]^2])
```

Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {3042, 3455, 3042, 3447, 3042, 3502, 25, 3042, 3232, 25, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^2(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^2(A+B\sin(e+fx))(c+d\sin(e+fx))^2dx$$

$$\Big\downarrow \text{3455}$$

$$\frac{\int (\sin(e+fx)a+a)(c+d\sin(e+fx))^2(a(5Ad+B(c+3d))-a(2B(c-3d)-5Ad)\sin(e+fx))dx}{5d}-\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^3}{5df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (\sin(e+fx)a+a)(c+d\sin(e+fx))^2(a(5Ad+B(c+3d))-a(2B(c-3d)-5Ad)\sin(e+fx))dx}{5d}-\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^3}{5df}$$

$$\Big\downarrow \text{3447}$$

$$\frac{\int (c+d\sin(e+fx))^2\left(-\left((2B(c-3d)-5Ad)\sin^2(e+fx)a^2\right)+(5Ad+B(c+3d))a^2+\left(a^2(5Ad+B(c+3d))\right.\right.}{5d} \frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^3}{5df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (c+d\sin(e+fx))^2\left(-\left((2B(c-3d)-5Ad)\sin(e+fx)^2a^2\right)+(5Ad+B(c+3d))a^2+\left(a^2(5Ad+B(c+3d))\right.\right.}{5d} \frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^3}{5df}$$

$$\Big\downarrow \ 3502$$

$$\frac{\frac{\int -(c+d\sin(e+fx))^2\left(d(2Bc-35Ad-30Bd)a^2+\left(5A(c-8d)d-2B\left(c^2-5dc+18d^2\right)\right)\sin(e+fx)a^2\right)dx}{4d}+\frac{a^2(2B(c-3d)-5Ad)\cos(e+fx)(c+d\sin}{4df}}{5d}$$

$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^3}{5df}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{a^2(2B(c-3d)-5Ad)\cos(e+fx)(c+d\sin(e+fx))^3}{4df}-\frac{\int (c+d\sin(e+fx))^2\left(d(2B(c-15d)-35Ad)a^2+\left(5A(c-8d)d-2B\left(c^2-5dc+18d^2\right)\right)\sin(e+fx\right.}{4d}}{5d}$$

$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^3}{5df}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{a^2(2B(c-3d)-5Ad)\cos(e+fx)(c+d\sin(e+fx))^3}{4df}-\frac{\int (c+d\sin(e+fx))^2\left(d(2B(c-15d)-35Ad)a^2+\left(5A(c-8d)d-2B\left(c^2-5dc+18d^2\right)\right)\sin(e+fx\right.}{4d}}{5d}$$

$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^3}{5df}$$

$$\Big\downarrow \ 3232$$

$$\frac{\frac{a^2(2B(c-3d)-5Ad)\cos(e+fx)(c+d\sin(e+fx))^3}{4df}-\frac{\frac{1}{3}\int -\left((c+d\sin(e+fx))\left(a^2d\left(5Ad(19c+16d)-B\left(2c^2-70dc-72d^2\right)\right)-a^2\left(5Ad\left(2c^2-16dc-2\right.\right.\right.\right.}{}}{5d}$$

$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^3}{5df}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{a^2(2B(c-3d)-5Ad)\cos(e+fx)(c+d\sin(e+fx))^3}{4df}-\frac{-\frac{1}{3}\int (c+d\sin(e+fx))\left(a^2d\left(5Ad(19c+16d)-B\left(2c^2-70dc-72d^2\right)\right)-a^2\left(5Ad\left(2c^2-16dc-21\right.\right.\right.}{}}{5d}$$

$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^3}{5df}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{a^2(2B(c-3d)-5Ad)\cos(e+fx)(c+d\sin(e+fx))^3}{4df}-\frac{-\frac{1}{3}\int(c+d\sin(e+fx))\left(a^2d(5Ad(19c+16d)-B\left(2c^2-70dc-72d^2\right))-a^2\left(5Ad\left(2c^2-16dc-21\right.\right.\right.}{}}{5d}$$

$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^3}{5df}$$

$\Big\downarrow$ 3213

$$\frac{\frac{a^2(2B(c-3d)-5Ad)\cos(e+fx)(c+d\sin(e+fx))^3}{4df}-\frac{\frac{1}{3}\left(-\frac{15}{2}a^2d^2x\left(12Ac^2+16Acd+7Ad^2+8Bc^2+14Bcd+6Bd^2\right)-\frac{a^2d\left(5Ad\left(2c^2-16cd-21d^2\right)-B\right.}{}\right.}{}}{}$$

$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)(c+d\sin(e+fx))^3}{5df}$$

input
```
Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
```

output
```
-1/5*(B*Cos[e + f*x]*(a^2 + a^2*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(d*f
) + ((a^2*(2*B*(c - 3*d) - 5*A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(4*
d*f) - (-1/3*(a^2*(5*A*(c - 8*d)*d - 2*B*(c^2 - 5*c*d + 18*d^2))*Cos[e + f
*x]*(c + d*Sin[e + f*x])^2)/f + ((-15*a^2*d^2*(12*A*c^2 + 8*B*c^2 + 16*A*c
*d + 14*B*c*d + 7*A*d^2 + 6*B*d^2)*x)/2 - (2*a^2*(5*A*d*(c^3 - 8*c^2*d - 2
0*c*d^2 - 8*d^3) - 2*B*(c^4 - 5*c^3*d + 16*c^2*d^2 + 40*c*d^3 + 18*d^4))*C
os[e + f*x])/f - (a^2*d*(5*A*d*(2*c^2 - 16*c*d - 21*d^2) - B*(4*c^3 - 20*c
^2*d + 66*c*d^2 + 90*d^3))*Cos[e + f*x]*Sin[e + f*x])/(2*f))/3)/(4*d))/(5*
d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 296.27 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.66

method	result
parallelrisch	$\frac{a^{2}\left(\left(\left(-3A-3B\right)d^{2}-6c\left(A+B\right)d-\frac{3c^{2}\left(A+2B\right)}{2}\right)\sin\left(2fx+2e\right)+\left(\left(A+\frac{9B}{8}\right)d^{2}+c\left(A+2B\right)d+\frac{B\,c^{2}}{2}\right)\cos\left(3fx+3e\right)+\frac{3((A+}{}}{}$
parts	$\frac{\left(a^{2}A\,d^{2}+2a^{2}Bcd+2a^{2}B\,d^{2}\right)\left(-\frac{\left(\sin\left(fx+e\right)^{3}+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(fx+e\right)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)}{f}-\frac{\left(2a^{2}A\,c^{2}+2a^{2}Acd+a^{2}B\,c^{2}\right)\cos}{f}$
risch	$-\frac{B\,a^{2}d^{2}\cos\left(5fx+5e\right)}{80f}+\frac{a^{2}\cos\left(3fx+3e\right)Bcd}{3f}+\frac{3a^{2}A\,c^{2}x}{2}-\frac{7a^{2}\cos\left(fx+e\right)Acd}{2f}+B\,a^{2}c^{2}x+\frac{3B\,a^{2}d^{2}x}{4}-$
derivativedivides	$\frac{a^{2}A\,c^{2}\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{2a^{2}Acd\left(2+\sin\left(fx+e\right)^{2}\right)\cos\left(fx+e\right)}{3}+a^{2}A\,d^{2}\left(-\frac{\left(\sin\left(fx+e\right)^{3}+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(f\right.}{4}\right.}{}$
default	$\frac{a^{2}A\,c^{2}\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{2a^{2}Acd\left(2+\sin\left(fx+e\right)^{2}\right)\cos\left(fx+e\right)}{3}+a^{2}A\,d^{2}\left(-\frac{\left(\sin\left(fx+e\right)^{3}+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(f\right.}{4}\right.}{}$
norman	$\frac{\left(\frac{3}{2}a^{2}A\,c^{2}+2a^{2}Acd+\frac{7}{8}a^{2}A\,d^{2}+a^{2}B\,c^{2}+\frac{7}{4}a^{2}Bcd+\frac{3}{4}a^{2}B\,d^{2}\right)x+\left(15a^{2}A\,c^{2}+20a^{2}Acd+\frac{35}{4}a^{2}A\,d^{2}+10a^{2}B\,c^{2}+\frac{35}{2}a^{2}Bcd\right.}{}$
orering	Expression too large to display

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x,method=_RETUR
NVERBOSE)
```

output
```
1/6*a^2*(((-3*A-3*B)*d^2-6*c*(A+B)*d-3/2*c^2*(A+2*B))*sin(2*f*x+2*e)+((A+9
/8*B)*d^2+c*(A+2*B)*d+1/2*B*c^2)*cos(3*f*x+3*e)+3/16*((A+2*B)*d+2*B*c)*d*s
in(4*f*x+4*e)-3/40*B*d^2*cos(5*f*x+5*e)+((-33/4*B-9*A)*d^2-21*(A+6/7*B)*c*
d-12*c^2*(A+7/8*B))*cos(f*x+e)+(-36/5*B+21/4*f*x*A+9/2*f*x*B-8*A)*d^2+12*(
f*x*A+7/8*f*x*B-5/3*A-4/3*B)*c*d+9*c^2*(f*x*A+2/3*f*x*B-4/3*A-10/9*B))/f
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.73

$$\int (a + a\sin(e + fx))^2(A + B\sin(e + fx))(c + d\sin(e + fx))^2\,dx =$$
$$-\frac{24\,Ba^2d^2\cos\left(fx+e\right)^5 - 40\left(Ba^2c^2 + 2\left(A+2\,B\right)a^2cd + \left(2\,A+3\,B\right)a^2d^2\right)\cos\left(fx+e\right)^3 - 15\left(4\left(3\,A\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
-1/120*(24*B*a^2*d^2*cos(f*x + e)^5 - 40*(B*a^2*c^2 + 2*(A + 2*B)*a^2*c*d
+ (2*A + 3*B)*a^2*d^2)*cos(f*x + e)^3 - 15*(4*(3*A + 2*B)*a^2*c^2 + 2*(8*A
 + 7*B)*a^2*c*d + (7*A + 6*B)*a^2*d^2)*f*x + 240*((A + B)*a^2*c^2 + 2*(A +
 B)*a^2*c*d + (A + B)*a^2*d^2)*cos(f*x + e) - 15*(2*(2*B*a^2*c*d + (A + 2*
B)*a^2*d^2)*cos(f*x + e)^3 - (4*(A + 2*B)*a^2*c^2 + 2*(8*A + 9*B)*a^2*c*d
+ (9*A + 10*B)*a^2*d^2)*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1129 vs. 2(330) = 660.

Time = 0.37 (sec) , antiderivative size = 1129, normalized size of antiderivative = 3.36

$$\int (a + a\sin(e + fx))^2(A + B\sin(e + fx))(c + d\sin(e + fx))^2\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
```

output
```
Piecewise((A*a**2*c**2*x*sin(e + f*x)**2/2 + A*a**2*c**2*x*cos(e + f*x)**2
/2 + A*a**2*c**2*x - A*a**2*c**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a**
2*c**2*cos(e + f*x)/f + 2*A*a**2*c*d*x*sin(e + f*x)**2 + 2*A*a**2*c*d*x*co
s(e + f*x)**2 - 2*A*a**2*c*d*sin(e + f*x)**2*cos(e + f*x)/f - 2*A*a**2*c*d
*sin(e + f*x)*cos(e + f*x)/f - 4*A*a**2*c*d*cos(e + f*x)**3/(3*f) - 2*A*a*
*2*c*d*cos(e + f*x)/f + 3*A*a**2*d**2*x*sin(e + f*x)**4/8 + 3*A*a**2*d**2*
x*sin(e + f*x)**2*cos(e + f*x)**2/4 + A*a**2*d**2*x*sin(e + f*x)**2/2 + 3*
A*a**2*d**2*x*cos(e + f*x)**4/8 + A*a**2*d**2*x*cos(e + f*x)**2/2 - 5*A*a*
*2*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 2*A*a**2*d**2*sin(e + f*x)**2
*cos(e + f*x)/f - 3*A*a**2*d**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) - A*a**
2*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 4*A*a**2*d**2*cos(e + f*x)**3/(3*
f) + B*a**2*c**2*x*sin(e + f*x)**2 + B*a**2*c**2*x*cos(e + f*x)**2 - B*a**
2*c**2*sin(e + f*x)**2*cos(e + f*x)/f - B*a**2*c**2*sin(e + f*x)*cos(e + f
*x)/f - 2*B*a**2*c**2*cos(e + f*x)**3/(3*f) - B*a**2*c**2*cos(e + f*x)/f +
 3*B*a**2*c*d*x*sin(e + f*x)**4/4 + 3*B*a**2*c*d*x*sin(e + f*x)**2*cos(e +
 f*x)**2/2 + B*a**2*c*d*x*sin(e + f*x)**2 + 3*B*a**2*c*d*x*cos(e + f*x)**4
/4 + B*a**2*c*d*x*cos(e + f*x)**2 - 5*B*a**2*c*d*sin(e + f*x)**3*cos(e + f
*x)/(4*f) - 4*B*a**2*c*d*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a**2*c*d*sin
(e + f*x)*cos(e + f*x)**3/(4*f) - B*a**2*c*d*sin(e + f*x)*cos(e + f*x)/f -
 8*B*a**2*c*d*cos(e + f*x)**3/(3*f) + 3*B*a**2*d**2*x*sin(e + f*x)**4/4...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.42

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$
$$= \frac{120\,(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right))Aa^2c^2+480\,(fx+e)Aa^2c^2+160\,\left(\cos\left(fx+e\right)^3-3\,\cos\left(fx+e\right)\right)Ba}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algori
thm="maxima")
```

output
```
1/480*(120*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c^2 + 480*(f*x + e)*A*a^
2*c^2 + 160*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*c^2 + 240*(2*f*x + 2*e
 - sin(2*f*x + 2*e))*B*a^2*c^2 + 320*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a
^2*c*d + 480*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c*d + 640*(cos(f*x + e
)^3 - 3*cos(f*x + e))*B*a^2*c*d + 30*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8
*sin(2*f*x + 2*e))*B*a^2*c*d + 240*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^2*
c*d + 320*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*d^2 + 15*(12*f*x + 12*e
+ sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^2*d^2 + 120*(2*f*x + 2*e - si
n(2*f*x + 2*e))*A*a^2*d^2 - 32*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*
cos(f*x + e))*B*a^2*d^2 + 160*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*d^2
+ 30*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^2*d^2 - 9
60*A*a^2*c^2*cos(f*x + e) - 480*B*a^2*c^2*cos(f*x + e) - 960*A*a^2*c*d*cos
(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.91

$$
\begin{aligned}
&\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx \\
&= -\frac{Ba^2d^2\cos(5\,fx+5\,e)}{80\,f} \\
&\quad+\frac{1}{8}\left(12\,Aa^2c^2+8\,Ba^2c^2+16\,Aa^2cd+14\,Ba^2cd+7\,Aa^2d^2+6\,Ba^2d^2\right)x \\
&\quad+\frac{(4\,Ba^2c^2+8\,Aa^2cd+16\,Ba^2cd+8\,Aa^2d^2+9\,Ba^2d^2)\cos(3\,fx+3\,e)}{48\,f} \\
&\quad-\frac{(16\,Aa^2c^2+14\,Ba^2c^2+28\,Aa^2cd+24\,Ba^2cd+12\,Aa^2d^2+11\,Ba^2d^2)\cos(fx+e)}{8\,f} \\
&\quad+\frac{(2\,Ba^2cd+Aa^2d^2+2\,Ba^2d^2)\sin(4\,fx+4\,e)}{32\,f} \\
&\quad-\frac{(Aa^2c^2+2\,Ba^2c^2+4\,Aa^2cd+4\,Ba^2cd+2\,Aa^2d^2+2\,Ba^2d^2)\sin(2\,fx+2\,e)}{4\,f}
\end{aligned}
$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
-1/80*B*a^2*d^2*cos(5*f*x + 5*e)/f + 1/8*(12*A*a^2*c^2 + 8*B*a^2*c^2 + 16*
A*a^2*c*d + 14*B*a^2*c*d + 7*A*a^2*d^2 + 6*B*a^2*d^2)*x + 1/48*(4*B*a^2*c^
2 + 8*A*a^2*c*d + 16*B*a^2*c*d + 8*A*a^2*d^2 + 9*B*a^2*d^2)*cos(3*f*x + 3*
e)/f - 1/8*(16*A*a^2*c^2 + 14*B*a^2*c^2 + 28*A*a^2*c*d + 24*B*a^2*c*d + 12
*A*a^2*d^2 + 11*B*a^2*d^2)*cos(f*x + e)/f + 1/32*(2*B*a^2*c*d + A*a^2*d^2
+ 2*B*a^2*d^2)*sin(4*f*x + 4*e)/f - 1/4*(A*a^2*c^2 + 2*B*a^2*c^2 + 4*A*a^2
*c*d + 4*B*a^2*c*d + 2*A*a^2*d^2 + 2*B*a^2*d^2)*sin(2*f*x + 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 37.56 (sec) , antiderivative size = 765, normalized size of antiderivative = 2.28

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = \text{Too large to display}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^2,x)
```

output
```
(a^2*atan((a^2*tan(e/2 + (f*x)/2)*(12*A*c^2 + 7*A*d^2 + 8*B*c^2 + 6*B*d^2
+ 16*A*c*d + 14*B*c*d))/(4*(3*A*a^2*c^2 + (7*A*a^2*d^2)/4 + 2*B*a^2*c^2 +
(3*B*a^2*d^2)/2 + 4*A*a^2*c*d + (7*B*a^2*c*d)/2)))*(12*A*c^2 + 7*A*d^2 + 8
*B*c^2 + 6*B*d^2 + 16*A*c*d + 14*B*c*d))/(4*f) - (tan(e/2 + (f*x)/2)^8*(4*
A*a^2*c^2 + 2*B*a^2*c^2 + 4*A*a^2*c*d) + tan(e/2 + (f*x)/2)*(A*a^2*c^2 + (
7*A*a^2*d^2)/4 + 2*B*a^2*c^2 + (3*B*a^2*d^2)/2 + 4*A*a^2*c*d + (7*B*a^2*c*
d)/2) - tan(e/2 + (f*x)/2)^9*(A*a^2*c^2 + (7*A*a^2*d^2)/4 + 2*B*a^2*c^2 +
(3*B*a^2*d^2)/2 + 4*A*a^2*c*d + (7*B*a^2*c*d)/2) + tan(e/2 + (f*x)/2)^3*(2
*A*a^2*c^2 + (11*A*a^2*d^2)/2 + 4*B*a^2*c^2 + 7*B*a^2*d^2 + 8*A*a^2*c*d +
11*B*a^2*c*d) - tan(e/2 + (f*x)/2)^7*(2*A*a^2*c^2 + (11*A*a^2*d^2)/2 + 4*B
*a^2*c^2 + 7*B*a^2*d^2 + 8*A*a^2*c*d + 11*B*a^2*c*d) + tan(e/2 + (f*x)/2)^
6*(16*A*a^2*c^2 + 8*A*a^2*d^2 + 12*B*a^2*c^2 + 4*B*a^2*d^2 + 24*A*a^2*c*d
+ 16*B*a^2*c*d) + tan(e/2 + (f*x)/2)^2*(16*A*a^2*c^2 + (40*A*a^2*d^2)/3 +
(44*B*a^2*c^2)/3 + 12*B*a^2*d^2 + (88*A*a^2*c*d)/3 + (80*B*a^2*c*d)/3) + t
an(e/2 + (f*x)/2)^4*(24*A*a^2*c^2 + (56*A*a^2*d^2)/3 + (64*B*a^2*c^2)/3 +
20*B*a^2*d^2 + (128*A*a^2*c*d)/3 + (112*B*a^2*c*d)/3) + 4*A*a^2*c^2 + (8*A
*a^2*d^2)/3 + (10*B*a^2*c^2)/3 + (12*B*a^2*d^2)/5 + (20*A*a^2*c*d)/3 + (16
*B*a^2*c*d)/3)/(f*(5*tan(e/2 + (f*x)/2)^2 + 10*tan(e/2 + (f*x)/2)^4 + 10*t
an(e/2 + (f*x)/2)^6 + 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^10 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.32

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$
$$= \frac{a^2\left(-60\cos(fx+e)\sin(fx+e)^3bd^2-80\cos(fx+e)\sin(fx+e)^2ad^2-72\cos(fx+e)\sin(fx+e)^2\right.}{}$$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x)
```

output
```
(a**2*( - 24*cos(e + f*x)*sin(e + f*x)**4*b*d**2 - 30*cos(e + f*x)*sin(e +
 f*x)**3*a*d**2 - 60*cos(e + f*x)*sin(e + f*x)**3*b*c*d - 60*cos(e + f*x)*
sin(e + f*x)**3*b*d**2 - 80*cos(e + f*x)*sin(e + f*x)**2*a*c*d - 80*cos(e
+ f*x)*sin(e + f*x)**2*a*d**2 - 40*cos(e + f*x)*sin(e + f*x)**2*b*c**2 - 1
60*cos(e + f*x)*sin(e + f*x)**2*b*c*d - 72*cos(e + f*x)*sin(e + f*x)**2*b*
d**2 - 60*cos(e + f*x)*sin(e + f*x)*a*c**2 - 240*cos(e + f*x)*sin(e + f*x)
*a*c*d - 105*cos(e + f*x)*sin(e + f*x)*a*d**2 - 120*cos(e + f*x)*sin(e + f
*x)*b*c**2 - 210*cos(e + f*x)*sin(e + f*x)*b*c*d - 90*cos(e + f*x)*sin(e +
 f*x)*b*d**2 - 240*cos(e + f*x)*a*c**2 - 400*cos(e + f*x)*a*c*d - 160*cos(
e + f*x)*a*d**2 - 200*cos(e + f*x)*b*c**2 - 320*cos(e + f*x)*b*c*d - 144*c
os(e + f*x)*b*d**2 + 180*a*c**2*f*x + 240*a*c**2 + 240*a*c*d*f*x + 400*a*c
*d + 105*a*d**2*f*x + 160*a*d**2 + 120*b*c**2*f*x + 200*b*c**2 + 210*b*c*d
*f*x + 320*b*c*d + 90*b*d**2*f*x + 144*b*d**2))/(120*f)
```

3.253 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 166

$$
\begin{aligned}
\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx
&= \frac{1}{8}a^2(12Ac+8Bc+8Ad+7Bd)x-\frac{a^2(12Ac+8Bc+8Ad+7Bd)\cos(e+fx)}{6f} \\
&\quad -\frac{a^2(12Ac+8Bc+8Ad+7Bd)\cos(e+fx)\sin(e+fx)}{24f} \\
&\quad -\frac{(4Bc+4Ad-Bd)\cos(e+fx)(a+a\sin(e+fx))^2}{12f} \\
&\quad -\frac{Bd\cos(e+fx)(a+a\sin(e+fx))^3}{4af}
\end{aligned}
$$

output
```
1/8*a^2*(12*A*c+8*A*d+8*B*c+7*B*d)*x-1/6*a^2*(12*A*c+8*A*d+8*B*c+7*B*d)*co
s(f*x+e)/f-1/24*a^2*(12*A*c+8*A*d+8*B*c+7*B*d)*cos(f*x+e)*sin(f*x+e)/f-1/1
2*(4*A*d+4*B*c-B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^2/f-1/4*B*d*cos(f*x+e)*(a+
a*sin(f*x+e))^3/a/f
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.87

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$=\frac{\cos(e+fx)\left(-\frac{1}{3}a^3(4Bc+4Ad-Bd)(1+\sin(e+fx))^2-Bd(a+a\sin(e+fx))^3-\frac{a^3(12Ac+8Bc+8Ad+7\ldots}{\ldots}\right.}{4af}$$

input
```
Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])
,x]
```

output
```
(Cos[e + f*x]*(-1/3*(a^3*(4*B*c + 4*A*d - B*d)*(1 + Sin[e + f*x])^2) - B*d
*(a + a*Sin[e + f*x])^3 - (a^3*(12*A*c + 8*B*c + 8*A*d + 7*B*d)*(6*ArcSin[
Sqrt[1 - Sin[e + f*x]]/Sqrt[2]] + Sqrt[Cos[e + f*x]^2]*(4 + Sin[e + f*x]))
)/(6*Sqrt[Cos[e + f*x]^2])))/(4*a*f)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.87, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {3042, 3447, 3042, 3502, 3042, 3230, 3042, 3123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^2(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^2(A+B\sin(e+fx))(c+d\sin(e+fx))dx$$

$$\Big\downarrow \text{3447}$$

$$\int (a\sin(e+fx)+a)^2\left((Ad+Bc)\sin(e+fx)+Ac+Bd\sin^2(e+fx)\right)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a\sin(e+fx)+a)^2\left((Ad+Bc)\sin(e+fx)+Ac+Bd\sin(e+fx)^2\right)dx$$

$$\Big\downarrow \text{ 3502}$$

$$\frac{\int (\sin(e+fx)a+a)^2(a(4Ac+3Bd)+a(4Bc+4Ad-Bd)\sin(e+fx))dx}{4a}-\frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^3}{4af}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int (\sin(e+fx)a+a)^2(a(4Ac+3Bd)+a(4Bc+4Ad-Bd)\sin(e+fx))dx}{4a}-\frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^3}{4af}$$

$$\Big\downarrow \text{ 3230}$$

$$\frac{\frac{1}{3}a(12Ac+8Ad+8Bc+7Bd)\int (\sin(e+fx)a+a)^2dx-\frac{a(4Ad+4Bc-Bd)\cos(e+fx)(a\sin(e+fx)+a)^2}{3f}}{4a}-\frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^3}{4af}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{1}{3}a(12Ac+8Ad+8Bc+7Bd)\int (\sin(e+fx)a+a)^2dx-\frac{a(4Ad+4Bc-Bd)\cos(e+fx)(a\sin(e+fx)+a)^2}{3f}}{4a}-\frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^3}{4af}$$

$$\Big\downarrow \text{ 3123}$$

$$\frac{\frac{1}{3}a(12Ac+8Ad+8Bc+7Bd)\left(-\frac{2a^2\cos(e+fx)}{f}-\frac{a^2\sin(e+fx)\cos(e+fx)}{2f}+\frac{3a^2x}{2}\right)-\frac{a(4Ad+4Bc-Bd)\cos(e+fx)(a\sin(e+fx)+a)^2}{3f}}{4a}-\frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^3}{4af}$$

input
```
Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

output
```
-1/4*(B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^3)/(a*f) + (-1/3*(a*(4*B*c + 4
*A*d - B*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^2)/f + (a*(12*A*c + 8*B*c +
8*A*d + 7*B*d)*((3*a^2*x)/2 - (2*a^2*Cos[e + f*x])/f - (a^2*Cos[e + f*x]*S
in[e + f*x])/(2*f)))/3)/(4*a)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3123
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.67

$$a^2Ac\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{a^2Ad\left(2+\sin(fx+e)^2\right)\cos(fx+e)}{3}-\frac{a^2Bc\left(2+\sin(fx+e)^2\right)\cos(fx+e)}{3}+a^2Bd\left(-\frac{(\sin$$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

output
```
1/f*(a^2*A*c*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/3*a^2*A*d*(2+sin
(f*x+e)^2)*cos(f*x+e)-1/3*a^2*B*c*(2+sin(f*x+e)^2)*cos(f*x+e)+a^2*B*d*(-1/
4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2*A*cos(f*x+e)*a
^2*c+2*a^2*A*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+2*a^2*B*c*(-1/2*
sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-2/3*a^2*B*d*(2+sin(f*x+e)^2)*cos(f*x+
e)+a^2*A*c*(f*x+e)-a^2*A*d*cos(f*x+e)-B*cos(f*x+e)*a^2*c+a^2*B*d*(-1/2*sin
(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.87

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$=\frac{8\left(Ba^2c+(A+2B)a^2d\right)\cos\left(fx+e\right)^3+3\left(4\left(3A+2B\right)a^2c+(8A+7B)a^2d\right)fx-48\left((A+B)a^2c+\left(\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
1/24*(8*(B*a^2*c + (A + 2*B)*a^2*d)*cos(f*x + e)^3 + 3*(4*(3*A + 2*B)*a^2*
c + (8*A + 7*B)*a^2*d)*f*x - 48*((A + B)*a^2*c + (A + B)*a^2*d)*cos(f*x +
e) + 3*(2*B*a^2*d*cos(f*x + e)^3 - (4*(A + 2*B)*a^2*c + (8*A + 9*B)*a^2*d)
*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(163) = 326$.

Time = 0.24 (sec) , antiderivative size = 571, normalized size of antiderivative = 3.44

$$\int (a + a\sin(e + fx))^2 (A + B\sin(e + fx))(c + d\sin(e + fx))\,dx$$
$$= \begin{cases} \frac{Aa^2cx\sin^2(e+fx)}{2} + \frac{Aa^2cx\cos^2(e+fx)}{2} + Aa^2cx - \frac{Aa^2c\sin(e+fx)\cos(e+fx)}{2f} - \frac{2Aa^2c\cos(e+fx)}{f} + Aa^2dx\sin^2(e+fx \\ x(A + B\sin(e))(c + d\sin(e))(a\sin(e) + a)^2 \end{cases}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

output
```
Piecewise((A*a**2*c*x*sin(e + f*x)**2/2 + A*a**2*c*x*cos(e + f*x)**2/2 + A
*a**2*c*x - A*a**2*c*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a**2*c*cos(e +
f*x)/f + A*a**2*d*x*sin(e + f*x)**2 + A*a**2*d*x*cos(e + f*x)**2 - A*a**2*
d*sin(e + f*x)**2*cos(e + f*x)/f - A*a**2*d*sin(e + f*x)*cos(e + f*x)/f -
2*A*a**2*d*cos(e + f*x)**3/(3*f) - A*a**2*d*cos(e + f*x)/f + B*a**2*c*x*si
n(e + f*x)**2 + B*a**2*c*x*cos(e + f*x)**2 - B*a**2*c*sin(e + f*x)**2*cos(
e + f*x)/f - B*a**2*c*sin(e + f*x)*cos(e + f*x)/f - 2*B*a**2*c*cos(e + f*x
)**3/(3*f) - B*a**2*c*cos(e + f*x)/f + 3*B*a**2*d*x*sin(e + f*x)**4/8 + 3*
B*a**2*d*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + B*a**2*d*x*sin(e + f*x)**2/
2 + 3*B*a**2*d*x*cos(e + f*x)**4/8 + B*a**2*d*x*cos(e + f*x)**2/2 - 5*B*a*
*2*d*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 2*B*a**2*d*sin(e + f*x)**2*cos(e
 + f*x)/f - 3*B*a**2*d*sin(e + f*x)*cos(e + f*x)**3/(8*f) - B*a**2*d*sin(e
 + f*x)*cos(e + f*x)/(2*f) - 4*B*a**2*d*cos(e + f*x)**3/(3*f), Ne(f, 0)),
(x*(A + B*sin(e))*(c + d*sin(e))*(a*sin(e) + a)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.61

$$\int (a + a\sin(e + fx))^2 (A + B\sin(e + fx))(c + d\sin(e + fx))\,dx$$
$$= \frac{24(2fx + 2e - \sin(2fx + 2e))Aa^2c + 96(fx + e)Aa^2c + 32(\cos(fx + e)^3 - 3\cos(fx + e))Ba^2c + 4}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorith
m="maxima")
```

output
```
1/96*(24*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c + 96*(f*x + e)*A*a^2*c +
 32*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*c + 48*(2*f*x + 2*e - sin(2*f*
x + 2*e))*B*a^2*c + 32*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*d + 48*(2*f
*x + 2*e - sin(2*f*x + 2*e))*A*a^2*d + 64*(cos(f*x + e)^3 - 3*cos(f*x + e)
)*B*a^2*d + 3*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^
2*d + 24*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^2*d - 192*A*a^2*c*cos(f*x +
e) - 96*B*a^2*c*cos(f*x + e) - 96*A*a^2*d*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.01

$$\begin{aligned}\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx &= \frac{Ba^2d\sin(4fx+4e)}{32f}+\frac{1}{8}\left(12Aa^2c+8Ba^2c+8Aa^2d+7Ba^2d\right)x\\&\quad+\frac{(Ba^2c+Aa^2d+2Ba^2d)\cos(3fx+3e)}{12f}\\&\quad-\frac{(8Aa^2c+7Ba^2c+7Aa^2d+6Ba^2d)\cos(fx+e)}{4f}\\&\quad-\frac{(Aa^2c+2Ba^2c+2Aa^2d+2Ba^2d)\sin(2fx+2e)}{4f}\end{aligned}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorith
m="giac")
```

output
```
1/32*B*a^2*d*sin(4*f*x + 4*e)/f + 1/8*(12*A*a^2*c + 8*B*a^2*c + 8*A*a^2*d
+ 7*B*a^2*d)*x + 1/12*(B*a^2*c + A*a^2*d + 2*B*a^2*d)*cos(3*f*x + 3*e)/f -
 1/4*(8*A*a^2*c + 7*B*a^2*c + 7*A*a^2*d + 6*B*a^2*d)*cos(f*x + e)/f - 1/4*
(A*a^2*c + 2*B*a^2*c + 2*A*a^2*d + 2*B*a^2*d)*sin(2*f*x + 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 36.73 (sec) , antiderivative size = 492, normalized size of antiderivative = 2.96

$$\int (a + a\sin(e + fx))^2(A + B\sin(e + fx))(c + d\sin(e + fx))\,dx$$
$$= \frac{a^2\,\mathrm{atan}\left(\frac{a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)(12\,A\,c+8\,A\,d+8\,B\,c+7\,B\,d)}{4\left(3\,A\,a^2\,c+2\,A\,a^2\,d+2\,B\,a^2\,c+\frac{7\,B\,a^2\,d}{4}\right)}\right)\,(12\,A\,c+8\,A\,d+8\,B\,c+7\,B\,d)}{4\,f}$$
$$- \frac{a^2\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-\frac{f\,x}{2}\right)\,(12\,A\,c+8\,A\,d+8\,B\,c+7\,B\,d)}{4\,f}$$
$$- \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\left(A\,a^2\,c+2\,A\,a^2\,d+2\,B\,a^2\,c+\frac{15\,B\,a^2\,d}{4}\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7\left(A\,a^2\,c+2\,A\,a^2\,d+2\,B\,a^2\,c+\right.}{}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c + d*sin(e + f*x)),x)
```

output
```
(a^2*atan((a^2*tan(e/2 + (f*x)/2)*(12*A*c + 8*A*d + 8*B*c + 7*B*d))/(4*(3*
A*a^2*c + 2*A*a^2*d + 2*B*a^2*c + (7*B*a^2*d)/4)))*(12*A*c + 8*A*d + 8*B*c
 + 7*B*d))/(4*f) - (a^2*(atan(tan(e/2 + (f*x)/2)) - (f*x)/2)*(12*A*c + 8*A
*d + 8*B*c + 7*B*d))/(4*f) - (tan(e/2 + (f*x)/2)^3*(A*a^2*c + 2*A*a^2*d +
2*B*a^2*c + (15*B*a^2*d)/4) - tan(e/2 + (f*x)/2)^7*(A*a^2*c + 2*A*a^2*d +
2*B*a^2*c + (7*B*a^2*d)/4) - tan(e/2 + (f*x)/2)^5*(A*a^2*c + 2*A*a^2*d + 2
*B*a^2*c + (15*B*a^2*d)/4) + tan(e/2 + (f*x)/2)^4*(12*A*a^2*c + 10*A*a^2*d
 + 10*B*a^2*c + 8*B*a^2*d) + tan(e/2 + (f*x)/2)^2*(12*A*a^2*c + (34*A*a^2*
d)/3 + (34*B*a^2*c)/3 + (32*B*a^2*d)/3) + tan(e/2 + (f*x)/2)^6*(4*A*a^2*c
+ 2*A*a^2*d + 2*B*a^2*c) + tan(e/2 + (f*x)/2)*(A*a^2*c + 2*A*a^2*d + 2*B*a
^2*c + (7*B*a^2*d)/4) + 4*A*a^2*c + (10*A*a^2*d)/3 + (10*B*a^2*c)/3 + (8*B
*a^2*d)/3)/(f*(4*tan(e/2 + (f*x)/2)^2 + 6*tan(e/2 + (f*x)/2)^4 + 4*tan(e/2
 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.36

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$=\frac{a^2\left(-6\cos\left(fx+e\right)\sin\left(fx+e\right)^3bd-8\cos\left(fx+e\right)\sin\left(fx+e\right)^2ad-8\cos\left(fx+e\right)\sin\left(fx+e\right)^2bc-1\right.}{}$$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

output
```
(a**2*( - 6*cos(e + f*x)*sin(e + f*x)**3*b*d - 8*cos(e + f*x)*sin(e + f*x)
**2*a*d - 8*cos(e + f*x)*sin(e + f*x)**2*b*c - 16*cos(e + f*x)*sin(e + f*x
)**2*b*d - 12*cos(e + f*x)*sin(e + f*x)*a*c - 24*cos(e + f*x)*sin(e + f*x)
*a*d - 24*cos(e + f*x)*sin(e + f*x)*b*c - 21*cos(e + f*x)*sin(e + f*x)*b*d
 - 48*cos(e + f*x)*a*c - 40*cos(e + f*x)*a*d - 40*cos(e + f*x)*b*c - 32*co
s(e + f*x)*b*d + 36*a*c*f*x + 48*a*c + 24*a*d*f*x + 40*a*d + 24*b*c*f*x +
40*b*c + 21*b*d*f*x + 32*b*d))/(24*f)
```

3.254 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 94

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx = \frac{1}{2}a^2(3A+2B)x - \frac{2a^2(3A+2B)\cos(e+fx)}{3f} - \frac{a^2(3A+2B)\cos(e+fx)\sin(e+fx)}{6f} - \frac{B\cos(e+fx)(a+a\sin(e+fx))^2}{3f}$$

output
```
1/2*a^2*(3*A+2*B)*x-2/3*a^2*(3*A+2*B)*cos(f*x+e)/f-1/6*a^2*(3*A+2*B)*cos(f
*x+e)*sin(f*x+e)/f-1/3*B*cos(f*x+e)*(a+a*sin(f*x+e))^2/f
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.13

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx = -\frac{a^2\cos(e+fx)\left(6(3A+2B)\arcsin\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}(2(6A+5B)+3(A+2B)\sin(e-}{6f\sqrt{\cos^2(e+fx)}}$$

input
```
Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]),x]
```

output
```
-1/6*(a^2*Cos[e + f*x]*(6*(3*A + 2*B)*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2
]] + Sqrt[Cos[e + f*x]^2]*(2*(6*A + 5*B) + 3*(A + 2*B)*Sin[e + f*x] + 2*B*
Sin[e + f*x]^2)))/(f*Sqrt[Cos[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3230, 3042, 3123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^2(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^2(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3230}$$

$$\frac{1}{3}(3A+2B)\int (\sin(e+fx)a+a)^2dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^2}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}(3A+2B)\int (\sin(e+fx)a+a)^2dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^2}{3f}$$

$$\Big\downarrow \text{3123}$$

$$\frac{1}{3}(3A+2B)\left(-\frac{2a^2\cos(e+fx)}{f}-\frac{a^2\sin(e+fx)\cos(e+fx)}{2f}+\frac{3a^2x}{2}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^2}{3f}$$

input
```
Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]),x]
```

output
```
-1/3*(B*Cos[e + f*x]*(a + a*Sin[e + f*x])^2)/f + ((3*A + 2*B)*((3*a^2*x)/2
 - (2*a^2*Cos[e + f*x])/f - (a^2*Cos[e + f*x]*Sin[e + f*x])/(2*f)))/3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3123
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 233.87 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

method	result
parallelrisch	$\frac{a^{2}\left(3\left(-A-2B\right)\sin\left(2fx+2e\right)+\cos\left(3fx+3e\right)B+3\left(-8A-7B\right)\cos\left(fx+e\right)+18fxA+12fxB-24A-20B\right)}{12f}$
parts	$a^{2}xA+\frac{\left(a^{2}A+2a^{2}B\right)\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{\left(2a^{2}A+a^{2}B\right)\cos\left(fx+e\right)}{f}-\frac{a^{2}B\left(2+\sin\left(fx+e\right)^{2}\right)\cos(}{3f}$
risch	$\frac{3a^{2}xA}{2}+a^{2}Bx-\frac{2a^{2}\cos\left(fx+e\right)A}{f}-\frac{7a^{2}\cos\left(fx+e\right)B}{4f}+\frac{B\,a^{2}\cos\left(3fx+3e\right)}{12f}-\frac{\sin\left(2fx+2e\right)a^{2}A}{4f}-\sin(2fx$
derivativedivides	$\frac{a^{2}A\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{a^{2}B\left(2+\sin\left(fx+e\right)^{2}\right)\cos\left(fx+e\right)}{3}-2a^{2}A\cos\left(fx+e\right)+2a^{2}B\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\right.}{f}$
default	$\frac{a^{2}A\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{a^{2}B\left(2+\sin\left(fx+e\right)^{2}\right)\cos\left(fx+e\right)}{3}-2a^{2}A\cos\left(fx+e\right)+2a^{2}B\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\right.}{f}$
norman	$\frac{\left(\frac{3}{2}a^{2}A+a^{2}B\right)x+\left(\frac{3}{2}a^{2}A+a^{2}B\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}+\left(\frac{9}{2}a^{2}A+3a^{2}B\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}+\left(\frac{9}{2}a^{2}A+3a^{2}B\right)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}+a^{2}}{\left(1+\tan(\right.}$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/12*a^2*(3*(-A-2*B)*sin(2*f*x+2*e)+cos(3*f*x+3*e)*B+3*(-8*A-7*B)*cos(f*x+
e)+18*f*x*A+12*f*x*B-24*A-20*B)/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx$$
$$= \frac{2\,Ba^2\cos{(fx+e)}^3+3\,(3\,A+2\,B)a^2fx-3\,(A+2\,B)a^2\cos{(fx+e)}\sin{(fx+e)}-12\,(A+B)a^2\cos{(}}{6\,f}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x, algorithm="fricas")
```

output
```
1/6*(2*B*a^2*cos(f*x + e)^3 + 3*(3*A + 2*B)*a^2*f*x - 3*(A + 2*B)*a^2*cos(
f*x + e)*sin(f*x + e) - 12*(A + B)*a^2*cos(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(85) = 170.

Time = 0.14 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.12

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx$$
$$= \begin{cases} \frac{Aa^2x\sin^2{(e+fx)}}{2}+\frac{Aa^2x\cos^2{(e+fx)}}{2}+Aa^2x-\frac{Aa^2\sin{(e+fx)}\cos{(e+fx)}}{2f}-\frac{2Aa^2\cos{(e+fx)}}{f}+Ba^2x\sin^2{(e+fx)}+I \\ x(A+B\sin{(e)})\,(a\sin{(e)}+a)^2 \end{cases}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e)),x)
```

output
```
Piecewise((A*a**2*x*sin(e + f*x)**2/2 + A*a**2*x*cos(e + f*x)**2/2 + A*a**
2*x - A*a**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a**2*cos(e + f*x)/f + B
*a**2*x*sin(e + f*x)**2 + B*a**2*x*cos(e + f*x)**2 - B*a**2*sin(e + f*x)**
2*cos(e + f*x)/f - B*a**2*sin(e + f*x)*cos(e + f*x)/f - 2*B*a**2*cos(e + f
*x)**3/(3*f) - B*a**2*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(
e) + a)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.21

$$\int (a + a\sin(e+fx))^2(A + B\sin(e+fx))\,dx$$
$$= \frac{3\,(2\,fx + 2\,e - \sin{(2\,fx + 2\,e)})Aa^2 + 12\,(fx + e)Aa^2 + 4\left(\cos{(fx + e)}^3 - 3\,\cos{(fx + e)}\right)Ba^2 + 6\,(2\,f\ldots}{12\,f}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x, algorithm="maxima")
```

output
```
1/12*(3*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2 + 12*(f*x + e)*A*a^2 + 4*(c
os(f*x + e)^3 - 3*cos(f*x + e))*B*a^2 + 6*(2*f*x + 2*e - sin(2*f*x + 2*e))
*B*a^2 - 24*A*a^2*cos(f*x + e) - 12*B*a^2*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.90

$$\int (a + a\sin(e+fx))^2(A + B\sin(e+fx))\,dx$$
$$= \frac{Ba^2\cos{(3\,fx + 3\,e)}}{12\,f} + \frac{1}{2}\,(3\,Aa^2 + 2\,Ba^2)x - \frac{(8\,Aa^2 + 7\,Ba^2)\cos{(fx + e)}}{4\,f} - \frac{(Aa^2 + 2\,Ba^2)\sin{(2\,fx + 2\,e)}}{4\,f}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x, algorithm="giac")
```

output
```
1/12*B*a^2*cos(3*f*x + 3*e)/f + 1/2*(3*A*a^2 + 2*B*a^2)*x - 1/4*(8*A*a^2 +
 7*B*a^2)*cos(f*x + e)/f - 1/4*(A*a^2 + 2*B*a^2)*sin(2*f*x + 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 35.42 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.97

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx = -\frac{\frac{3\,A\,a^2\,\sin(2\,e+2\,f\,x)}{2}-\frac{B\,a^2\,\cos(3\,e+3\,f\,x)}{2}+3\,B\,a^2\,\sin\left(2\,e+2\,f\,x\right)+12\,A\,a^2\,\cos\left(e+f\,x\right)+\frac{21\,B\,a^2\,\cos(e+f\,x)}{2}-}{6\,f}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2,x)
```

output
```
-((3*A*a^2*sin(2*e + 2*f*x))/2 - (B*a^2*cos(3*e + 3*f*x))/2 + 3*B*a^2*sin(
2*e + 2*f*x) + 12*A*a^2*cos(e + f*x) + (21*B*a^2*cos(e + f*x))/2 - 9*A*a^2
*f*x - 6*B*a^2*f*x)/(6*f)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.96

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))\,dx = \frac{a^2\left(-2\cos\left(fx+e\right)\sin\left(fx+e\right)^2b-3\cos\left(fx+e\right)\sin\left(fx+e\right)a-6\cos\left(fx+e\right)\sin\left(fx+e\right)b-12\cos\right.}{6f}$$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x)
```

output
```
(a**2*( - 2*cos(e + f*x)*sin(e + f*x)**2*b - 3*cos(e + f*x)*sin(e + f*x)*a
 - 6*cos(e + f*x)*sin(e + f*x)*b - 12*cos(e + f*x)*a - 10*cos(e + f*x)*b +
 9*a*f*x + 12*a + 6*b*f*x + 10*b))/(6*f)
```

3.255 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 171

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx &= -\frac{a^2(2A(c-2d)d-B(2c^2-4cd+3d^2))\,x}{2d^3} \\ &\quad -\frac{2a^2(c-d)^2(Bc-Ad)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^3\sqrt{c^2-d^2}\,f} \\ &\quad +\frac{a^2(2Bc-2Ad-3Bd)\cos(e+fx)}{2d^2f}-\frac{B\cos(e+fx)\,(a^2+a^2\sin(e+fx))}{2df}\end{aligned}$$

output
```
-1/2*a^2*(2*A*(c-2*d)*d-B*(2*c^2-4*c*d+3*d^2))*x/d^3-2*a^2*(c-d)^2*(-A*d+B
*c)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/d^3/(c^2-d^2)^(1/2)/f
+1/2*a^2*(-2*A*d+2*B*c-3*B*d)*cos(f*x+e)/d^2/f-1/2*B*cos(f*x+e)*(a^2+a^2*s
in(f*x+e))/d/f
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.04

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \frac{a^2(1+\sin(e+fx))^2\left(2(2Ad(-c+2d)+B(2c^2-4cd+3d^2))(e+fx)-\frac{8(c-d)^2(Bc-Ad)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}\right.}{4d^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right.}$$

input
```
Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x
]),x]
```

output
```
(a^2*(1 + Sin[e + f*x])^2*(2*(2*A*d*(-c + 2*d) + B*(2*c^2 - 4*c*d + 3*d^2)
)*(e + f*x) - (8*(c - d)^2*(B*c - A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqr
t[c^2 - d^2]])/Sqrt[c^2 - d^2] - 4*d*(-(B*c) + A*d + 2*B*d)*Cos[e + f*x] -
 B*d^2*Sin[2*(e + f*x)]))/(4*d^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4
)
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {3042, 3455, 3042, 3447, 3042, 3502, 3042, 3214, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{c+d\sin(e+fx)}dx$$

$$\Big\downarrow \text{3455}$$

$$\frac{\int \frac{(\sin(e+fx)a+a)(a(Bc+2Ad)-a(2Bc-2Ad-3Bd)\sin(e+fx))}{c+d\sin(e+fx)}dx}{2d}-\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{(\sin(e+fx)a+a)(a(Bc+2Ad)-a(2Bc-2Ad-3Bd)\sin(e+fx))}{c+d\sin(e+fx)}dx}{2d}-\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df}$$

$\Big\downarrow$ 3447

$$\frac{\int \frac{-((2Bc-2Ad-3Bd)\sin^2(e+fx)a^2)+(Bc+2Ad)a^2+(a^2(Bc+2Ad)-a^2(2Bc-2Ad-3Bd))\sin(e+fx)}{c+d\sin(e+fx)}dx}{2d}-\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{-((2Bc-2Ad-3Bd)\sin(e+fx)^2a^2)+(Bc+2Ad)a^2+(a^2(Bc+2Ad)-a^2(2Bc-2Ad-3Bd))\sin(e+fx)}{c+d\sin(e+fx)}dx}{2d}-\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df}$$

$\Big\downarrow$ 3502

$$\frac{\frac{\int \frac{a^2d(Bc+2Ad)-a^2\left(2A(c-2d)d-B\left(2c^2-4dc+3d^2\right)\right)\sin(e+fx)}{c+d\sin(e+fx)}dx}{d}+\frac{a^2(-2Ad+2Bc-3Bd)\cos(e+fx)}{df}}{2d}-\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{a^2d(Bc+2Ad)-a^2\left(2A(c-2d)d-B\left(2c^2-4dc+3d^2\right)\right)\sin(e+fx)}{c+d\sin(e+fx)}dx}{d}+\frac{a^2(-2Ad+2Bc-3Bd)\cos(e+fx)}{df}}{2d}-\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df}$$

$\Big\downarrow$ 3214

$$\frac{-\frac{\frac{2a^2(c-d)^2(Bc-Ad)\int\frac{1}{c+d\sin(e+fx)}dx}{d}-\frac{a^2x\left(2Ad(c-2d)-B\left(2c^2-4cd+3d^2\right)\right)}{d}}{d}+\frac{a^2(-2Ad+2Bc-3Bd)\cos(e+fx)}{df}}{2d}-\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df}$$

$\Big\downarrow$ 3042

$$\frac{\frac{-\frac{2a^2(c-d)^2(Bc-Ad)\int \frac{1}{c+d\sin(e+fx)}dx}{d}-\frac{a^2x\left(2Ad(c-2d)-B\left(2c^2-4cd+3d^2\right)\right)}{d}}{d}+\frac{a^2(-2Ad+2Bc-3Bd)\cos(e+fx)}{df}}{2d}-\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df}$$

$\Big\downarrow$ 3139

$$\frac{\frac{-\frac{4a^2(c-d)^2(Bc-Ad)\int \frac{1}{c\tan^2\left(\frac{1}{2}(e+fx)\right)+2d\tan\left(\frac{1}{2}(e+fx)\right)+c}d\tan\left(\frac{1}{2}(e+fx)\right)}{df}-\frac{a^2x\left(2Ad(c-2d)-B\left(2c^2-4cd+3d^2\right)\right)}{d}}{d}+\frac{a^2(-2Ad+2Bc-3Bd)\cos(e+}{df}}{2d}$$
$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df}$$

$\Big\downarrow$ 1083

$$\frac{\frac{\frac{8a^2(c-d)^2(Bc-Ad)\int \frac{1}{-\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)^2-4\left(c^2-d^2\right)}d\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{df}-\frac{a^2x\left(2Ad(c-2d)-B\left(2c^2-4cd+3d^2\right)\right)}{d}}{d}+\frac{a^2(-2Ad+2Bc-3Bd)\,\mathrm{co}}{df}}{2d}$$
$$\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df}$$

$\Big\downarrow$ 217

$$\frac{\frac{-\frac{4a^2(c-d)^2(Bc-Ad)\arctan\left(\frac{2c\tan\left(\frac{1}{2}(e+fx)\right)+2d}{2\sqrt{c^2-d^2}}\right)}{df\sqrt{c^2-d^2}}-\frac{a^2x\left(2Ad(c-2d)-B\left(2c^2-4cd+3d^2\right)\right)}{d}}{d}+\frac{a^2(-2Ad+2Bc-3Bd)\cos(e+fx)}{df}}{2d}-\frac{B\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df}$$

input
```
Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),x]
```

output
```
((-((a^2*(2*A*(c - 2*d)*d - B*(2*c^2 - 4*c*d + 3*d^2))*x)/d) - (4*a^2*(c -
 d)^2*(B*c - A*d)*ArcTan[(2*d + 2*c*Tan[(e + f*x)/2])/(2*Sqrt[c^2 - d^2])]
)/(d*Sqrt[c^2 - d^2]*f))/d + (a^2*(2*B*c - 2*A*d - 3*B*d)*Cos[e + f*x])/(d
*f))/(2*d) - (B*Cos[e + f*x]*(a^2 + a^2*Sin[e + f*x]))/(2*d*f)
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.37

method	result
derivativedivides	$\frac{2a^2\left(\frac{\left(Ac^2d-2Acd^2+Ad^3-Bc^3+2Bc^2d-Bcd^2\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{d^3\sqrt{c^2-d^2}}-\frac{-\frac{Bd^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2}+\left(Ad^2-Bcd+2Bd^2\right)\cdots}{\left(1+\mathrm{ta}\cdots\right.}\right.}{f}$
default	$\frac{2a^2\left(\frac{\left(Ac^2d-2Acd^2+Ad^3-Bc^3+2Bc^2d-Bcd^2\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{d^3\sqrt{c^2-d^2}}-\frac{-\frac{Bd^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2}+\left(Ad^2-Bcd+2Bd^2\right)\cdots}{\left(1+\mathrm{ta}\cdots\right.}\right.}{f}$
risch	$-\frac{a^2xAc}{d^2}+\frac{2a^2xA}{d}+\frac{a^2xBc^2}{d^3}-\frac{2a^2xBc}{d^2}+\frac{3a^2xB}{2d}-\frac{a^2e^{i(fx+e)}A}{2df}+\frac{a^2e^{i(fx+e)}Bc}{2d^2f}-\frac{a^2e^{i(fx+e)}B}{df}-\frac{a^2}{}\cdots$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x,method=_RETURNV
ERBOSE)
```

output
```
2/f*a^2*((A*c^2*d-2*A*c*d^2+A*d^3-B*c^3+2*B*c^2*d-B*c*d^2)/d^3/(c^2-d^2)^(
1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))-1/d^3*((-1/2
*B*d^2*tan(1/2*f*x+1/2*e)^3+(A*d^2-B*c*d+2*B*d^2)*tan(1/2*f*x+1/2*e)^2+1/2
*B*d^2*tan(1/2*f*x+1/2*e)+A*d^2-B*c*d+2*B*d^2)/(1+tan(1/2*f*x+1/2*e)^2)^2+
1/2*(2*A*c*d-4*A*d^2-2*B*c^2+4*B*c*d-3*B*d^2)*arctan(tan(1/2*f*x+1/2*e))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 452, normalized size of antiderivative = 2.64

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$

$$= \left[-\frac{Ba^2d^2\cos(fx+e)\sin(fx+e) - (2\,Ba^2c^2 - 2\,(A+2\,B)a^2cd + (4\,A+3\,B)a^2d^2)fx - (Ba^2c^2 - (A\right.$$

$$-\frac{Ba^2d^2\cos(fx+e)\sin(fx+e) - (2\,Ba^2c^2 - 2\,(A+2\,B)a^2cd + (4\,A+3\,B)a^2d^2)fx - 2\,(Ba^2c^2 - (A}{2\,d^3}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
[-1/2*(B*a^2*d^2*cos(f*x + e)*sin(f*x + e) - (2*B*a^2*c^2 - 2*(A + 2*B)*a^
2*c*d + (4*A + 3*B)*a^2*d^2)*f*x - (B*a^2*c^2 - (A + B)*a^2*c*d + A*a^2*d^
2)*sqrt(-(c - d)/(c + d))*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*
x + e) - c^2 - d^2 + 2*((c^2 + c*d)*cos(f*x + e)*sin(f*x + e) + (c*d + d^2
)*cos(f*x + e))*sqrt(-(c - d)/(c + d)))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*
x + e) - c^2 - d^2)) - 2*(B*a^2*c*d - (A + 2*B)*a^2*d^2)*cos(f*x + e))/(d^
3*f), -1/2*(B*a^2*d^2*cos(f*x + e)*sin(f*x + e) - (2*B*a^2*c^2 - 2*(A + 2*
B)*a^2*c*d + (4*A + 3*B)*a^2*d^2)*f*x - 2*(B*a^2*c^2 - (A + B)*a^2*c*d + A
*a^2*d^2)*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x + e) + d)*sqrt((c - d)/
(c + d))/((c - d)*cos(f*x + e))) - 2*(B*a^2*c*d - (A + 2*B)*a^2*d^2)*cos(f
*x + e))/(d^3*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.78

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$

$$= \frac{\frac{(2\,Ba^2c^2-2\,Aa^2cd-4\,Ba^2cd+4\,Aa^2d^2+3\,Ba^2d^2)(fx+e)}{d^3} - \frac{4\,(Ba^2c^3-Aa^2c^2d-2\,Ba^2c^2d+2\,Aa^2cd^2+Ba^2cd^2-Aa^2d^3)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)\right.}{\sqrt{c^2-d^2}\,d^3}}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorith
m="giac")
```

output
```
1/2*((2*B*a^2*c^2 - 2*A*a^2*c*d - 4*B*a^2*c*d + 4*A*a^2*d^2 + 3*B*a^2*d^2)
*(f*x + e)/d^3 - 4*(B*a^2*c^3 - A*a^2*c^2*d - 2*B*a^2*c^2*d + 2*A*a^2*c*d^
2 + B*a^2*c*d^2 - A*a^2*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + ar
ctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/(sqrt(c^2 - d^2)*d^3)
+ 2*(B*a^2*d*tan(1/2*f*x + 1/2*e)^3 + 2*B*a^2*c*tan(1/2*f*x + 1/2*e)^2 - 2
*A*a^2*d*tan(1/2*f*x + 1/2*e)^2 - 4*B*a^2*d*tan(1/2*f*x + 1/2*e)^2 - B*a^2
*d*tan(1/2*f*x + 1/2*e) + 2*B*a^2*c - 2*A*a^2*d - 4*B*a^2*d)/((tan(1/2*f*x
 + 1/2*e)^2 + 1)^2*d^2))/f
```

Mupad [B] (verification not implemented)

Time = 42.97 (sec) , antiderivative size = 7371, normalized size of antiderivative = 43.11

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \text{Too large to display}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c + d*sin(e + f*x)),x)
```

output
```
(atan(((((8*(16*A^2*a^4*c^2*d^6 - 16*A^2*a^4*c^3*d^5 + 4*A^2*a^4*c^4*d^4 +
 9*B^2*a^4*c^2*d^6 - 24*B^2*a^4*c^3*d^5 + 28*B^2*a^4*c^4*d^4 - 16*B^2*a^4*
c^5*d^3 + 4*B^2*a^4*c^6*d^2 + 24*A*B*a^4*c^2*d^6 - 44*A*B*a^4*c^3*d^5 + 32
*A*B*a^4*c^4*d^4 - 8*A*B*a^4*c^5*d^3))/d^5 + ((((32*c^2*d^3 + (8*tan(e/2 +
 (f*x)/2)*(12*c*d^10 - 8*c^3*d^8))/d^6)*(B*a^2*c^2*1i + (a^2*d^2*(4*A + 3*
B)*1i)/2 - (a^2*d*(2*A*c + 4*B*c)*1i)/2))/d^3 - (8*(8*A*a^2*c*d^8 + 6*B*a^
2*c*d^8 - 8*A*a^2*c^2*d^7 - 8*B*a^2*c^2*d^7 + 2*B*a^2*c^3*d^6))/d^5 + (8*t
an(e/2 + (f*x)/2)*(8*A*a^2*c*d^9 - 16*A*a^2*c^2*d^8 + 8*A*a^2*c^3*d^7 - 8*
B*a^2*c^2*d^8 + 16*B*a^2*c^3*d^7 - 8*B*a^2*c^4*d^6))/d^6)*(B*a^2*c^2*1i +
(a^2*d^2*(4*A + 3*B)*1i)/2 - (a^2*d*(2*A*c + 4*B*c)*1i)/2))/d^3 + (8*tan(e
/2 + (f*x)/2)*(32*A^2*a^4*c^4*d^5 - 32*A^2*a^4*c^3*d^6 - 16*A^2*a^4*c^2*d^
7 - 8*A^2*a^4*c^5*d^4 - 48*B^2*a^4*c^2*d^7 + 43*B^2*a^4*c^3*d^6 + 8*B^2*a^
4*c^4*d^5 - 44*B^2*a^4*c^5*d^4 + 32*B^2*a^4*c^6*d^3 - 8*B^2*a^4*c^7*d^2 +
28*A^2*a^4*c*d^8 + 18*B^2*a^4*c*d^8 - 80*A*B*a^4*c^2*d^7 + 8*A*B*a^4*c^3*d
^6 + 76*A*B*a^4*c^4*d^5 - 64*A*B*a^4*c^5*d^4 + 16*A*B*a^4*c^6*d^3 + 48*A*B
*a^4*c*d^8))/d^6)*(B*a^2*c^2*1i + (a^2*d^2*(4*A + 3*B)*1i)/2 - (a^2*d*(2*A
*c + 4*B*c)*1i)/2)*1i)/d^3 + (((8*(16*A^2*a^4*c^2*d^6 - 16*A^2*a^4*c^3*d^5
 + 4*A^2*a^4*c^4*d^4 + 9*B^2*a^4*c^2*d^6 - 24*B^2*a^4*c^3*d^5 + 28*B^2*a^4
*c^4*d^4 - 16*B^2*a^4*c^5*d^3 + 4*B^2*a^4*c^6*d^2 + 24*A*B*a^4*c^2*d^6 - 4
4*A*B*a^4*c^3*d^5 + 32*A*B*a^4*c^4*d^4 - 8*A*B*a^4*c^5*d^3))/d^5 + (((8...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.34

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{c + d\sin(e + fx)}\,dx$$

$$= \frac{a^2\left(4a\,d^3e + 2b\,c^3e + 3b\,d^3e - \cos(fx + e)\sin(fx + e)\,bc\,d^2 - 2a\,c^2dfx + 2ac\,d^2fx - 2b\,c^2dfx - bc\,d^2fx\right.}{}$$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

output
```
(a**2*(4*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2)
)*a*c*d - 4*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d*
*2))*a*d**2 - 4*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2
- d**2))*b*c**2 + 4*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c
**2 - d**2))*b*c*d - cos(e + f*x)*sin(e + f*x)*b*c*d**2 - cos(e + f*x)*sin
(e + f*x)*b*d**3 - 2*cos(e + f*x)*a*c*d**2 - 2*cos(e + f*x)*a*d**3 + 2*cos
(e + f*x)*b*c**2*d - 2*cos(e + f*x)*b*c*d**2 - 4*cos(e + f*x)*b*d**3 - 2*a
*c**2*d*e - 2*a*c**2*d*f*x + 2*a*c*d**2*e + 2*a*c*d**2*f*x + 4*a*d**3*e +
4*a*d**3*f*x + 2*b*c**3*e + 2*b*c**3*f*x - 2*b*c**2*d*e - 2*b*c**2*d*f*x -
 b*c*d**2*e - b*c*d**2*f*x + 3*b*d**3*e + 3*b*d**3*f*x))/(2*d**3*f*(c + d)
)
```

3.256 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 198

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx &= -\frac{a^2(2Bc-Ad-2Bd)x}{d^3} \\ &\quad - \frac{2a^2(c-d)\left(Ad(c+2d)-B(2c^2+2cd-d^2)\right)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^3(c+d)\sqrt{c^2-d^2}f} \\ &\quad + \frac{a^2(Ad-B(2c+d))\cos(e+fx)}{d^2(c+d)f} + \frac{(Bc-Ad)\cos(e+fx)\left(a^2+a^2\sin(e+fx)\right)}{d(c+d)f(c+d\sin(e+fx))}\end{aligned}$$

output
```
-a^2*(-A*d+2*B*c-2*B*d)*x/d^3-2*a^2*(c-d)*(A*d*(c+2*d)-B*(2*c^2+2*c*d-d^2)
)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/d^3/(c+d)/(c^2-d^2)^(1/
2)/f+a^2*(A*d-B*(2*c+d))*cos(f*x+e)/d^2/(c+d)/f+(-A*d+B*c)*cos(f*x+e)*(a^2
+a^2*sin(f*x+e))/d/(c+d)/f/(c+d*sin(f*x+e))
```

Mathematica [A] (verified)

Time = 6.16 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.97

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c + d\sin(e + fx))^2}\,dx = \frac{a^2(1+\sin(e+fx))^2\left((-2Bc+Ad+2Bd)(e+fx)+\frac{2(c-d)(-Ad(c+2d)+B(2c^2+2cd-d^2))\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}}\right.}{d^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}$$

input
```
Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x
])^2,x]
```

output
```
(a^2*(1 + Sin[e + f*x])^2*((-2*B*c + A*d + 2*B*d)*(e + f*x) + (2*(c - d)*(
-(A*d*(c + 2*d)) + B*(2*c^2 + 2*c*d - d^2))*ArcTan[(d + c*Tan[(e + f*x)/2]
)/Sqrt[c^2 - d^2]])/((c + d)*Sqrt[c^2 - d^2]) - B*d*Cos[e + f*x] - (d*(-c
+ d)*(-(B*c) + A*d)*Cos[e + f*x])/((c + d)*(c + d*Sin[e + f*x]))))/(d^3*f*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.08, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 3454, 25, 3042, 3447, 3042, 3502, 3042, 3214, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}dx$$

$$\Big\downarrow \; 3454$$

$$\frac{\int -\frac{(\sin(e+fx)a+a)(a(B(c-d)-2Ad)+a(Ad-B(2c+d))\sin(e+fx))}{c+d\sin(e+fx)}dx}{d(c+d)}+\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{df(c+d)(c+d\sin(e+fx))}$$

$\Big\downarrow$ 25

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{df(c+d)(c+d\sin(e+fx))}-\frac{\int \frac{(\sin(e+fx)a+a)(a(B(c-d)-2Ad)+a(Ad-B(2c+d))\sin(e+fx))}{c+d\sin(e+fx)}dx}{d(c+d)}$$

$\Big\downarrow$ 3042

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{df(c+d)(c+d\sin(e+fx))}-\frac{\int \frac{(\sin(e+fx)a+a)(a(B(c-d)-2Ad)+a(Ad-B(2c+d))\sin(e+fx))}{c+d\sin(e+fx)}dx}{d(c+d)}$$

$\Big\downarrow$ 3447

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{df(c+d)(c+d\sin(e+fx))}-\frac{\int \frac{(Ad-B(2c+d))\sin^2(e+fx)a^2+(B(c-d)-2Ad)a^2+\left((B(c-d)-2Ad)a^2+(Ad-B(2c+d))a^2\right)\sin(e+fx)}{c+d\sin(e+fx)}dx}{d(c+d)}$$

$\Big\downarrow$ 3042

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{df(c+d)(c+d\sin(e+fx))}-\frac{\int \frac{(Ad-B(2c+d))\sin(e+fx)^2a^2+(B(c-d)-2Ad)a^2+\left((B(c-d)-2Ad)a^2+(Ad-B(2c+d))a^2\right)\sin(e+fx)}{c+d\sin(e+fx)}dx}{d(c+d)}$$

$\Big\downarrow$ 3502

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{df(c+d)(c+d\sin(e+fx))}-\frac{\frac{\int \frac{d(B(c-d)-2Ad)a^2+(c+d)(2B(c-d)-Ad)\sin(e+fx)a^2}{c+d\sin(e+fx)}dx}{d}-\frac{a^2(Ad-B(2c+d))\cos(e+fx)}{df}}{d(c+d)}$$

$\Big\downarrow$ 3042

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{df(c+d)(c+d\sin(e+fx))}-\frac{\frac{\int\frac{d(B(c-d)-2Ad)a^2+(c+d)(2B(c-d)-Ad)\sin(e+fx)a^2}{c+d\sin(e+fx)}dx}{d}-\frac{a^2(Ad-B(2c+d))\cos(e+fx)}{df}}{d(c+d)}$$

$\Big\downarrow$ 3214

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{df(c+d)(c+d\sin(e+fx))}-\frac{\frac{\frac{a^2(c-d)\left(Ad(c+2d)-B\left(2c^2+2cd-d^2\right)\right)\int\frac{1}{c+d\sin(e+fx)}dx}{d}+\frac{a^2x(c+d)(-Ad+2Bc-2Bd)}{d}}{d}-\frac{a^2(Ad-B(2c+d))\cos(e+fx)}{df}}{d(c+d)}$$

$\Big\downarrow$ 3042

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{df(c+d)(c+d\sin(e+fx))}-\frac{\frac{\frac{a^2(c-d)\left(Ad(c+2d)-B\left(2c^2+2cd-d^2\right)\right)\int\frac{1}{c+d\sin(e+fx)}dx}{d}+\frac{a^2x(c+d)(-Ad+2Bc-2Bd)}{d}}{d}-\frac{a^2(Ad-B(2c+d))\cos(e+fx)}{df}}{d(c+d)}$$

$\Big\downarrow$ 3139

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{df(c+d)(c+d\sin(e+fx))}-\frac{\frac{\frac{2a^2(c-d)\left(Ad(c+2d)-B\left(2c^2+2cd-d^2\right)\right)\int\frac{1}{c\tan^2\left(\frac{1}{2}(e+fx)\right)+2d\tan\left(\frac{1}{2}(e+fx)\right)+c}d\tan\left(\frac{1}{2}(e+fx)\right)}{df}+\frac{a^2x(c+d)(-Ad+2Bc-2Bd)}{d}}{d}-\frac{a^2(Ad-B(2c+d))\cos}{df}}{d(c+d)}$$

$\Big\downarrow$ 1083

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{df(c+d)(c+d\sin(e+fx))}-\frac{\frac{\frac{a^2x(c+d)(-Ad+2Bc-2Bd)}{d}-\frac{4a^2(c-d)\left(Ad(c+2d)-B\left(2c^2+2cd-d^2\right)\right)\int\frac{1}{-\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)^2-4\left(c^2-d^2\right)}d\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{df}}{d}-\frac{a^2(Ad-B(2c+}{d}}{d(c+d)}$$

$\Big\downarrow$ 217

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{df(c+d)(c+d\sin(e+fx))}-\frac{\frac{\frac{2a^2(c-d)\left(Ad(c+2d)-B\left(2c^2+2cd-d^2\right)\right)\arctan\left(\frac{2c\tan\left(\frac{1}{2}(e+fx)\right)+2d}{2\sqrt{c^2-d^2}}\right)}{df\sqrt{c^2-d^2}}+\frac{a^2x(c+d)(-Ad+2Bc-2Bd)}{d}}{d}-\frac{a^2(Ad-B(2c+d))\cos(e+fx)}{df}}{d(c+d)}$$

input
```
Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2,x
]
```

output
```
-((((a^2*(c + d)*(2*B*c - A*d - 2*B*d)*x)/d + (2*a^2*(c - d)*(A*d*(c + 2*d
) - B*(2*c^2 + 2*c*d - d^2))*ArcTan[(2*d + 2*c*Tan[(e + f*x)/2])/(2*Sqrt[c
^2 - d^2])])/(d*Sqrt[c^2 - d^2]*f))/d - (a^2*(A*d - B*(2*c + d))*Cos[e + f
*x])/(d*f))/(d*(c + d))) + ((B*c - A*d)*Cos[e + f*x]*(a^2 + a^2*Sin[e + f*
x]))/(d*(c + d)*f*(c + d*Sin[e + f*x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.27

method	result
derivativedivides	$\frac{2a^2\left(\frac{-\frac{Bd}{1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}+(Ad-2Bc+2Bd)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d^3}-\frac{-\frac{d^2\left(Acd-A\,d^2-B\,c^2+Bcd\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}-\frac{d\left(Acd-A\,d^2-B\cdots\right.}{c+d}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}\cdots\right.}{f}$
default	$\frac{2a^2\left(\frac{-\frac{Bd}{1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}+(Ad-2Bc+2Bd)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d^3}-\frac{-\frac{d^2\left(Acd-A\,d^2-B\,c^2+Bcd\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}-\frac{d\left(Acd-A\,d^2-B\cdots\right.}{c+d}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}\cdots\right.}{f}$
risch	$\frac{a^2xA}{d^2}-\frac{2a^2xBc}{d^3}+\frac{2a^2xB}{d^2}-\frac{B\,a^2\mathrm{e}^{i(fx+e)}}{2d^2f}-\frac{B\,a^2\mathrm{e}^{-i(fx+e)}}{2d^2f}+\frac{2ia^2\left(-Acd+A\,d^2+B\,c^2-Bcd\right)\left(id+c\,\mathrm{e}^{i(fx+e)}\cdots\right.}{d^3(c+d)f\left(-id\,\mathrm{e}^{2i(fx+e)}+id+2c\,\mathrm{e}^{i(fx+e)}\right)}$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x,method=_RETUR
NVERBOSE)
```

output
```
2/f*a^2*(1/d^3*(-B*d/(1+tan(1/2*f*x+1/2*e)^2)+(A*d-2*B*c+2*B*d)*arctan(tan
(1/2*f*x+1/2*e)))-1/d^3*((-d^2*(A*c*d-A*d^2-B*c^2+B*c*d)/(c+d)/c*tan(1/2*f
*x+1/2*e)-d*(A*c*d-A*d^2-B*c^2+B*c*d)/(c+d))/(tan(1/2*f*x+1/2*e)^2*c+2*d*t
an(1/2*f*x+1/2*e)+c)+(A*c^2*d+A*c*d^2-2*A*d^3-2*B*c^3+3*B*c*d^2-B*d^3)/(c+
d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)
)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 731, normalized size of antiderivative = 3.69

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
[-1/2*(2*(2*B*a^2*c^3 - A*a^2*c^2*d - (A + 2*B)*a^2*c*d^2)*f*x + (2*B*a^2*
c^3 - (A - 2*B)*a^2*c^2*d - (2*A + B)*a^2*c*d^2 + (2*B*a^2*c^2*d - (A - 2*
B)*a^2*c*d^2 - (2*A + B)*a^2*d^3)*sin(f*x + e))*sqrt(-(c - d)/(c + d))*log
(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*((c^2
+ c*d)*cos(f*x + e)*sin(f*x + e) + (c*d + d^2)*cos(f*x + e))*sqrt(-(c - d)
/(c + d)))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(2*B
*a^2*c^2*d - A*a^2*c*d^2 + A*a^2*d^3)*cos(f*x + e) + 2*((2*B*a^2*c^2*d - A
*a^2*c*d^2 - (A + 2*B)*a^2*d^3)*f*x + (B*a^2*c*d^2 + B*a^2*d^3)*cos(f*x +
e))*sin(f*x + e))/((c*d^4 + d^5)*f*sin(f*x + e) + (c^2*d^3 + c*d^4)*f), -(
(2*B*a^2*c^3 - A*a^2*c^2*d - (A + 2*B)*a^2*c*d^2)*f*x + (2*B*a^2*c^3 - (A
- 2*B)*a^2*c^2*d - (2*A + B)*a^2*c*d^2 + (2*B*a^2*c^2*d - (A - 2*B)*a^2*c*
d^2 - (2*A + B)*a^2*d^3)*sin(f*x + e))*sqrt((c - d)/(c + d))*arctan(-(c*si
n(f*x + e) + d)*sqrt((c - d)/(c + d))/((c - d)*cos(f*x + e))) + (2*B*a^2*c
^2*d - A*a^2*c*d^2 + A*a^2*d^3)*cos(f*x + e) + ((2*B*a^2*c^2*d - A*a^2*c*d
^2 - (A + 2*B)*a^2*d^3)*f*x + (B*a^2*c*d^2 + B*a^2*d^3)*cos(f*x + e))*sin(
f*x + e))/((c*d^4 + d^5)*f*sin(f*x + e) + (c^2*d^3 + c*d^4)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algori
thm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(194) = 388$.

Time = 0.23 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.42

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$$

$$= \frac{\frac{2\,(2\,Ba^2c^3-Aa^2c^2d-Aa^2cd^2-3\,Ba^2cd^2+2\,Aa^2d^3+Ba^2d^3)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(cd^3+d^4)\sqrt{c^2-d^2}} - \frac{2\left(Ba^2c^2d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\right.\right.}{}}{}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
(2*(2*B*a^2*c^3 - A*a^2*c^2*d - A*a^2*c*d^2 - 3*B*a^2*c*d^2 + 2*A*a^2*d^3
+ B*a^2*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*
f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c*d^3 + d^4)*sqrt(c^2 - d^2)) - 2*(B
*a^2*c^2*d*tan(1/2*f*x + 1/2*e)^3 - A*a^2*c*d^2*tan(1/2*f*x + 1/2*e)^3 - B
*a^2*c*d^2*tan(1/2*f*x + 1/2*e)^3 + A*a^2*d^3*tan(1/2*f*x + 1/2*e)^3 + 2*B
*a^2*c^3*tan(1/2*f*x + 1/2*e)^2 - A*a^2*c^2*d*tan(1/2*f*x + 1/2*e)^2 + A*a
^2*c*d^2*tan(1/2*f*x + 1/2*e)^2 + 3*B*a^2*c^2*d*tan(1/2*f*x + 1/2*e) - A*a
^2*c*d^2*tan(1/2*f*x + 1/2*e) + B*a^2*c*d^2*tan(1/2*f*x + 1/2*e) + A*a^2*d
^3*tan(1/2*f*x + 1/2*e) + 2*B*a^2*c^3 - A*a^2*c^2*d + A*a^2*c*d^2)/((c*tan
(1/2*f*x + 1/2*e)^4 + 2*d*tan(1/2*f*x + 1/2*e)^3 + 2*c*tan(1/2*f*x + 1/2*e
)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)*(c^2*d^2 + c*d^3)) - (2*B*a^2*c - A*a^
2*d - 2*B*a^2*d)*(f*x + e)/d^3)/f
```

Mupad [B] (verification not implemented)

Time = 44.03 (sec) , antiderivative size = 8706, normalized size of antiderivative = 43.97

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c + d*sin(e + f*x))^2,x
)
```

output
```
- ((2*(A*a^2*d^2 + 2*B*a^2*c^2 - A*a^2*c*d))/(d^2*(c + d)) + (2*tan(e/2 +
(f*x)/2)^2*(A*a^2*d^2 + 2*B*a^2*c^2 - A*a^2*c*d))/(d^2*(c + d)) + (2*tan(e
/2 + (f*x)/2)*(A*a^2*d^2 + 3*B*a^2*c^2 - A*a^2*c*d + B*a^2*c*d))/(c*d*(c +
 d)) + (2*tan(e/2 + (f*x)/2)^3*(A*a^2*d^2 + B*a^2*c^2 - A*a^2*c*d - B*a^2*
c*d))/(c*d*(c + d)))/(f*(c + 2*d*tan(e/2 + (f*x)/2) + 2*c*tan(e/2 + (f*x)/
2)^2 + c*tan(e/2 + (f*x)/2)^4 + 2*d*tan(e/2 + (f*x)/2)^3)) - (atan((((B*a^
2*c*2i - a^2*d*(A + 2*B)*1i)*((32*(A^2*a^4*c^2*d^6 + 2*A^2*a^4*c^3*d^5 + A
^2*a^4*c^4*d^4 + 4*B^2*a^4*c^2*d^6 - 8*B^2*a^4*c^4*d^4 + 4*B^2*a^4*c^6*d^2
 + 4*A*B*a^4*c^2*d^6 + 4*A*B*a^4*c^3*d^5 - 4*A*B*a^4*c^4*d^4 - 4*A*B*a^4*c
^5*d^3))/(2*c*d^6 + d^7 + c^2*d^5) + ((B*a^2*c*2i - a^2*d*(A + 2*B)*1i)*((
((32*(c^2*d^10 + 2*c^3*d^9 + c^4*d^8))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan
(e/2 + (f*x)/2)*(3*c*d^12 + 6*c^2*d^11 + c^3*d^10 - 4*c^4*d^9 - 2*c^5*d^8)
)/(2*c*d^7 + d^8 + c^2*d^6))*(B*a^2*c*2i - a^2*d*(A + 2*B)*1i))/d^3 - (32*
(A*a^2*c*d^9 + 2*B*a^2*c*d^9 - A*a^2*c^3*d^7 + B*a^2*c^2*d^8 - 2*B*a^2*c^3
*d^7 - B*a^2*c^4*d^6))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*
(4*A*a^2*c*d^10 + 2*B*a^2*c*d^10 + 2*A*a^2*c^2*d^9 - 4*A*a^2*c^3*d^8 - 2*A
*a^2*c^4*d^7 - 4*B*a^2*c^2*d^9 - 6*B*a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 4*B*a
^2*c^5*d^6))/(2*c*d^7 + d^8 + c^2*d^6)))/d^3 + (32*tan(e/2 + (f*x)/2)*(8*A
^2*a^4*c^2*d^7 + 4*A^2*a^4*c^3*d^6 - 4*A^2*a^4*c^4*d^5 - 2*A^2*a^4*c^5*d^4
 + 6*B^2*a^4*c^2*d^7 - 29*B^2*a^4*c^3*d^6 - 4*B^2*a^4*c^4*d^5 + 28*B^2*...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 872, normalized size of antiderivative = 4.40

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

output
```
(a**2*( - 2*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d*
*2))*sin(e + f*x)*a*c*d**2 - 4*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c
+ d)/sqrt(c**2 - d**2))*sin(e + f*x)*a*d**3 + 4*sqrt(c**2 - d**2)*atan((ta
n((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*sin(e + f*x)*b*c**2*d + 4*sqrt(c*
*2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*sin(e + f*x)*b
*c*d**2 - 2*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d*
*2))*sin(e + f*x)*b*d**3 - 2*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c +
d)/sqrt(c**2 - d**2))*a*c**2*d - 4*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2
)*c + d)/sqrt(c**2 - d**2))*a*c*d**2 + 4*sqrt(c**2 - d**2)*atan((tan((e +
f*x)/2)*c + d)/sqrt(c**2 - d**2))*b*c**3 + 4*sqrt(c**2 - d**2)*atan((tan((
e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*b*c**2*d - 2*sqrt(c**2 - d**2)*atan(
(tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*b*c*d**2 - cos(e + f*x)*sin(e
+ f*x)*b*c**2*d**2 - 2*cos(e + f*x)*sin(e + f*x)*b*c*d**3 - cos(e + f*x)*s
in(e + f*x)*b*d**4 + cos(e + f*x)*a*c**2*d**2 - cos(e + f*x)*a*d**4 - 2*co
s(e + f*x)*b*c**3*d - 2*cos(e + f*x)*b*c**2*d**2 + sin(e + f*x)*a*c**2*d**
2*f*x + 2*sin(e + f*x)*a*c*d**3*f*x + sin(e + f*x)*a*d**4*f*x - 2*sin(e +
f*x)*b*c**3*d*f*x - 2*sin(e + f*x)*b*c**2*d**2*f*x - sin(e + f*x)*b*c**2*d
**2 + 2*sin(e + f*x)*b*c*d**3*f*x - 2*sin(e + f*x)*b*c*d**3 + 2*sin(e + f*
x)*b*d**4*f*x - sin(e + f*x)*b*d**4 + a*c**3*d*f*x + 2*a*c**2*d**2*f*x + a
*c*d**3*f*x - 2*b*c**4*f*x - 2*b*c**3*d*f*x - b*c**3*d + 2*b*c**2*d**2*...
```

3.257 $\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 215

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx &= \frac{a^2Bx}{d^3} + \frac{a^2(3Ad^3-B(2c^3+4c^2d+cd^2-4d^3))\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^3(c+d)^2\sqrt{c^2-d^2}f} \\ &+ \frac{(Bc-Ad)\cos(e+fx)\,(a^2+a^2\sin(e+fx))}{2d(c+d)f(c+d\sin(e+fx))^2} \\ &- \frac{a^2(3Ad^2-B(2c^2+3cd-2d^2))\cos(e+fx)}{2d^2(c+d)^2f(c+d\sin(e+fx))}\end{aligned}$$

output

```
a^2*B*x/d^3+a^2*(3*A*d^3-B*(2*c^3+4*c^2*d+c*d^2-4*d^3))*arctan((d+c*tan(1/
2*f*x+1/2*e))/(c^2-d^2)^(1/2))/d^3/(c+d)^2/(c^2-d^2)^(1/2)/f+1/2*(-A*d+B*c
)*cos(f*x+e)*(a^2+a^2*sin(f*x+e))/d/(c+d)/f/(c+d*sin(f*x+e))^2-1/2*a^2*(3*
A*d^2-B*(2*c^2+3*c*d-2*d^2))*cos(f*x+e)/d^2/(c+d)^2/f/(c+d*sin(f*x+e))
```

Mathematica [A] (verified)

Time = 6.61 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.05

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \frac{a^2(1+\sin(e+fx))^2\left(2B(e+fx)-\frac{2(-3Ad^3+B(2c^3+4c^2d+cd^2-4d^3))\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c+d)^2\sqrt{c^2-d^2}}-\frac{d(-c+d)(-Bc+Ad)}{(c+d)(c+d\sin(e+fx))}\right.}{2d^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}$$

input
```
Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x
])^3,x]
```

output
```
(a^2*(1 + Sin[e + f*x])^2*(2*B*(e + f*x) - (2*(-3*A*d^3 + B*(2*c^3 + 4*c^2
*d + c*d^2 - 4*d^3))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c
 + d)^2*Sqrt[c^2 - d^2]) - (d*(-c + d)*(-(B*c) + A*d)*Cos[e + f*x])/((c +
d)*(c + d*Sin[e + f*x])^2) - (d*(A*d*(c + 4*d) + B*(-3*c^2 - 4*c*d + 2*d^2
))*Cos[e + f*x])/((c + d)^2*(c + d*Sin[e + f*x]))))/(2*d^3*f*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])^4)
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.19, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 3454, 25, 3042, 3447, 3042, 3500, 3042, 3214, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a\sin(e+fx)+a)^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}dx$$

$$\Big\downarrow \ 3454$$

$$\frac{\int -\frac{(\sin(e+fx)a+a)(a(Bc-3Ad-2Bd)-2aB(c+d)\sin(e+fx))}{(c+d\sin(e+fx))^2}dx}{2d(c+d)}+\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df(c+d)(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \ 25$$

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df(c+d)(c+d\sin(e+fx))^2}-\frac{\int \frac{(\sin(e+fx)a+a)(a(B(c-2d)-3Ad)-2aB(c+d)\sin(e+fx))}{(c+d\sin(e+fx))^2}dx}{2d(c+d)}$$

$$\Big\downarrow \ 3042$$

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df(c+d)(c+d\sin(e+fx))^2}-\frac{\int \frac{(\sin(e+fx)a+a)(a(B(c-2d)-3Ad)-2aB(c+d)\sin(e+fx))}{(c+d\sin(e+fx))^2}dx}{2d(c+d)}$$

$$\Big\downarrow \ 3447$$

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df(c+d)(c+d\sin(e+fx))^2}-\frac{\int \frac{-2B(c+d)\sin^2(e+fx)a^2+(B(c-2d)-3Ad)a^2+\left(a^2(B(c-2d)-3Ad)-2a^2B(c+d)\right)\sin(e+fx)}{(c+d\sin(e+fx))^2}dx}{2d(c+d)}$$

$$\Big\downarrow \ 3042$$

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df(c+d)(c+d\sin(e+fx))^2}-\frac{\int \frac{-2B(c+d)\sin(e+fx)^2a^2+(B(c-2d)-3Ad)a^2+\left(a^2(B(c-2d)-3Ad)-2a^2B(c+d)\right)\sin(e+fx)}{(c+d\sin(e+fx))^2}dx}{2d(c+d)}$$

$$\Big\downarrow \ 3500$$

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df(c+d)(c+d\sin(e+fx))^2}-\frac{\frac{a^2\left(3Ad^2-B\left(2c^2+3cd-2d^2\right)\right)\cos(e+fx)}{df(c+d)(c+d\sin(e+fx))}-\frac{\int \frac{(c-d)d(3Ad+B(c+4d))a^2+2B(c-d)(c+d)^2\sin(e+fx)a^2}{c+d\sin(e+fx)}dx}{d(c^2-d^2)}}{2d(c+d)}$$

$$\Big\downarrow \ 3042$$

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df(c+d)(c+d\sin(e+fx))^2}-\frac{\frac{a^2\left(3Ad^2-B\left(2c^2+3cd-2d^2\right)\right)\cos(e+fx)}{df(c+d)(c+d\sin(e+fx))}-\frac{\int\frac{(c-d)d(3Ad+B(c+4d))a^2+2B(c-d)(c+d)^2\sin(e+fx)a^2}{c+d\sin(e+fx)}dx}{d(c^2-d^2)}}{2d(c+d)}$$

$\downarrow$ 3214

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df(c+d)(c+d\sin(e+fx))^2}-\frac{\frac{a^2\left(3Ad^2-B\left(2c^2+3cd-2d^2\right)\right)\cos(e+fx)}{df(c+d)(c+d\sin(e+fx))}-\frac{\frac{2a^2Bx(c-d)(c+d)^2}{d}-\frac{a^2(c-d)\left(2Bc(c+d)^2-d^2(3Ad+B(c+4d))\right)\int\frac{1}{c+d\sin(e+fx)}dx}{d}}{d(c^2-d^2)}}{2d(c+d)}$$

$\downarrow$ 3042

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df(c+d)(c+d\sin(e+fx))^2}-\frac{\frac{a^2\left(3Ad^2-B\left(2c^2+3cd-2d^2\right)\right)\cos(e+fx)}{df(c+d)(c+d\sin(e+fx))}-\frac{\frac{2a^2Bx(c-d)(c+d)^2}{d}-\frac{a^2(c-d)\left(2Bc(c+d)^2-d^2(3Ad+B(c+4d))\right)\int\frac{1}{c+d\sin(e+fx)}dx}{d}}{d(c^2-d^2)}}{2d(c+d)}$$

$\downarrow$ 3139

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df(c+d)(c+d\sin(e+fx))^2}-\frac{\frac{a^2\left(3Ad^2-B\left(2c^2+3cd-2d^2\right)\right)\cos(e+fx)}{df(c+d)(c+d\sin(e+fx))}-\frac{\frac{2a^2Bx(c-d)(c+d)^2}{d}-\frac{2a^2(c-d)\left(2Bc(c+d)^2-d^2(3Ad+B(c+4d))\right)\int\frac{1}{c\tan^2\left(\frac{1}{2}(e+fx)\right)+2d\tan\left(\frac{1}{2}(e+fx)\right)+c}d}{df}}{d(c^2-d^2)}}{2d(c+d)}$$

$\downarrow$ 1083

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df(c+d)(c+d\sin(e+fx))^2}-\frac{\frac{a^2\left(3Ad^2-B\left(2c^2+3cd-2d^2\right)\right)\cos(e+fx)}{df(c+d)(c+d\sin(e+fx))}-\frac{\frac{4a^2(c-d)\left(2Bc(c+d)^2-d^2(3Ad+B(c+4d))\right)\int\frac{1}{-\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)^2-4\left(c^2-d^2\right)}d\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right.}{df}}{d(c^2-d^2)}}{2d(c+d)}$$

$\downarrow$ 217

$$\frac{(Bc-Ad)\cos(e+fx)\left(a^2\sin(e+fx)+a^2\right)}{2df(c+d)(c+d\sin(e+fx))^2}-\frac{\frac{a^2\left(3Ad^2-B\left(2c^2+3cd-2d^2\right)\right)\cos(e+fx)}{df(c+d)(c+d\sin(e+fx))}-\frac{\frac{2a^2Bx(c-d)(c+d)^2}{d}-\frac{2a^2(c-d)\left(2Bc(c+d)^2-d^2(3Ad+B(c+4d))\right)\arctan\left(\frac{2c\tan\left(\frac{1}{2}(e+fx)\right)+2d}{2\sqrt{c^2-d^2}}\right)}{df\sqrt{c^2-d^2}}}{d(c^2-d^2)}}{2d(c+d)}$$

input
```
Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^3,x
]
```

output
```
((B*c - A*d)*Cos[e + f*x]*(a^2 + a^2*Sin[e + f*x]))/(2*d*(c + d)*f*(c + d*
Sin[e + f*x])^2) - (-(((2*a^2*B*(c - d)*(c + d)^2*x)/d - (2*a^2*(c - d)*(2
*B*c*(c + d)^2 - d^2*(3*A*d + B*(c + 4*d)))*ArcTan[(2*d + 2*c*Tan[(e + f*x
)/2])/(2*Sqrt[c^2 - d^2])])/(d*Sqrt[c^2 - d^2]*f))/(d*(c^2 - d^2))) + (a^2
*(3*A*d^2 - B*(2*c^2 + 3*c*d - 2*d^2))*Cos[e + f*x])/(d*(c + d)*f*(c + d*S
in[e + f*x]))))/(2*d*(c + d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. 2(206) = 412.

Time = 0.84 (sec) , antiderivative size = 431, normalized size of antiderivative = 2.00

method	result
derivativedivides	$2a^2\left(\frac{B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d^3}+\frac{\frac{d^2\left(A\,c^2d-4Ac\,d^2-2A\,d^3+B\,c^3+4B\,c^2d\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2\left(c^2+2cd+d^2\right)c}-\frac{d\left(4A\,c^3d^2+A\,c^2d^3+8Ac\,d^4+2A\,d^5-2\right.}{}}{}\right.$
default	$2a^2\left(\frac{B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d^3}+\frac{\frac{d^2\left(A\,c^2d-4Ac\,d^2-2A\,d^3+B\,c^3+4B\,c^2d\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2\left(c^2+2cd+d^2\right)c}-\frac{d\left(4A\,c^3d^2+A\,c^2d^3+8Ac\,d^4+2A\,d^5-2\right.}{}}{}\right.$
risch	Expression too large to display

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x,method=_RETUR
NVERBOSE)
```

output
```
2/f*a^2*(B/d^3*arctan(tan(1/2*f*x+1/2*e))+1/d^3*((1/2*d^2*(A*c^2*d-4*A*c*d
^2-2*A*d^3+B*c^3+4*B*c^2*d)/(c^2+2*c*d+d^2)/c*tan(1/2*f*x+1/2*e)^3-1/2*d*(
4*A*c^3*d^2+A*c^2*d^3+8*A*c*d^4+2*A*d^5-2*B*c^5-4*B*c^4*d-3*B*c^3*d^2-8*B*
c^2*d^3+2*B*c*d^4)/(c^2+2*c*d+d^2)/c^2*tan(1/2*f*x+1/2*e)^2-1/2*d^2*(A*c^2
*d+12*A*c*d^2+2*A*d^3-7*B*c^3-12*B*c^2*d+4*B*c*d^2)/c/(c^2+2*c*d+d^2)*tan(
1/2*f*x+1/2*e)-1/2*d*(4*A*c*d^2+A*d^3-2*B*c^3-4*B*c^2*d+B*c*d^2)/(c^2+2*c*
d+d^2))/(tan(1/2*f*x+1/2*e)^2*c+2*d*tan(1/2*f*x+1/2*e)+c)^2+1/2*(3*A*d^3-2
*B*c^3-4*B*c^2*d-B*c*d^2+4*B*d^3)/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1
/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 700 vs. $2(206) = 412$.

Time = 0.15 (sec) , antiderivative size = 1483, normalized size of antiderivative = 6.90

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c + d\sin(e + fx))^3} dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algori
thm="fricas")
```

output
```
[1/4*(4*(B*a^2*c^4*d^2 + 2*B*a^2*c^3*d^3 - 2*B*a^2*c*d^5 - B*a^2*d^6)*f*x*
cos(f*x + e)^2 - 4*(B*a^2*c^6 + 2*B*a^2*c^5*d + B*a^2*c^4*d^2 - B*a^2*c^2*
d^4 - 2*B*a^2*c*d^5 - B*a^2*d^6)*f*x - (2*B*a^2*c^5 + 4*B*a^2*c^4*d + 3*B*
a^2*c^3*d^2 - 3*A*a^2*c^2*d^3 + B*a^2*c*d^4 - (3*A + 4*B)*a^2*d^5 - (2*B*a
^2*c^3*d^2 + 4*B*a^2*c^2*d^3 + B*a^2*c*d^4 - (3*A + 4*B)*a^2*d^5)*cos(f*x
+ e)^2 + 2*(2*B*a^2*c^4*d + 4*B*a^2*c^3*d^2 + B*a^2*c^2*d^3 - (3*A + 4*B)*
a^2*c*d^4)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^
2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*co
s(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c
^2 - d^2)) - 2*(2*B*a^2*c^5*d + 4*B*a^2*c^4*d^2 - (4*A + 3*B)*a^2*c^3*d^3
- (A + 4*B)*a^2*c^2*d^4 + (4*A + B)*a^2*c*d^5 + A*a^2*d^6)*cos(f*x + e) -
2*(4*(B*a^2*c^5*d + 2*B*a^2*c^4*d^2 - 2*B*a^2*c^2*d^4 - B*a^2*c*d^5)*f*x +
 (3*B*a^2*c^4*d^2 - (A - 4*B)*a^2*c^3*d^3 - (4*A + 5*B)*a^2*c^2*d^4 + (A -
 4*B)*a^2*c*d^5 + 2*(2*A + B)*a^2*d^6)*cos(f*x + e))*sin(f*x + e))/((c^4*d
^5 + 2*c^3*d^6 - 2*c*d^8 - d^9)*f*cos(f*x + e)^2 - 2*(c^5*d^4 + 2*c^4*d^5
- 2*c^2*d^7 - c*d^8)*f*sin(f*x + e) - (c^6*d^3 + 2*c^5*d^4 + c^4*d^5 - c^2
*d^7 - 2*c*d^8 - d^9)*f), 1/2*(2*(B*a^2*c^4*d^2 + 2*B*a^2*c^3*d^3 - 2*B*a^
2*c*d^5 - B*a^2*d^6)*f*x*cos(f*x + e)^2 - 2*(B*a^2*c^6 + 2*B*a^2*c^5*d + B
*a^2*c^4*d^2 - B*a^2*c^2*d^4 - 2*B*a^2*c*d^5 - B*a^2*d^6)*f*x - (2*B*a^2*c
^5 + 4*B*a^2*c^4*d + 3*B*a^2*c^3*d^2 - 3*A*a^2*c^2*d^3 + B*a^2*c*d^4 - ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algori
thm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 678 vs. $2(206) = 412$.

Time = 0.25 (sec) , antiderivative size = 678, normalized size of antiderivative = 3.15

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algori
thm="giac")
```

output
```
((f*x + e)*B*a^2/d^3 - (2*B*a^2*c^3 + 4*B*a^2*c^2*d + B*a^2*c*d^2 - 3*A*a^
2*d^3 - 4*B*a^2*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*
tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c^2*d^3 + 2*c*d^4 + d^5)*sqr
t(c^2 - d^2)) + (B*a^2*c^4*d*tan(1/2*f*x + 1/2*e)^3 + A*a^2*c^3*d^2*tan(1/
2*f*x + 1/2*e)^3 + 4*B*a^2*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 - 4*A*a^2*c^2*d^
3*tan(1/2*f*x + 1/2*e)^3 - 2*A*a^2*c*d^4*tan(1/2*f*x + 1/2*e)^3 + 2*B*a^2*
c^5*tan(1/2*f*x + 1/2*e)^2 + 4*B*a^2*c^4*d*tan(1/2*f*x + 1/2*e)^2 - 4*A*a^
2*c^3*d^2*tan(1/2*f*x + 1/2*e)^2 + 3*B*a^2*c^3*d^2*tan(1/2*f*x + 1/2*e)^2
- A*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^2 + 8*B*a^2*c^2*d^3*tan(1/2*f*x + 1/2
*e)^2 - 8*A*a^2*c*d^4*tan(1/2*f*x + 1/2*e)^2 - 2*B*a^2*c*d^4*tan(1/2*f*x +
 1/2*e)^2 - 2*A*a^2*d^5*tan(1/2*f*x + 1/2*e)^2 + 7*B*a^2*c^4*d*tan(1/2*f*x
 + 1/2*e) - A*a^2*c^3*d^2*tan(1/2*f*x + 1/2*e) + 12*B*a^2*c^3*d^2*tan(1/2*
f*x + 1/2*e) - 12*A*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e) - 4*B*a^2*c^2*d^3*tan
(1/2*f*x + 1/2*e) - 2*A*a^2*c*d^4*tan(1/2*f*x + 1/2*e) + 2*B*a^2*c^5 + 4*B
*a^2*c^4*d - 4*A*a^2*c^3*d^2 - B*a^2*c^3*d^2 - A*a^2*c^2*d^3)/((c^4*d^2 +
2*c^3*d^3 + c^2*d^4)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e)
+ c)^2))/f
```

Mupad [B] (verification not implemented)

Time = 44.46 (sec) , antiderivative size = 8632, normalized size of antiderivative = 40.15

$$\int \frac{(a + a\sin(e + fx))^2(A + B\sin(e + fx))}{(c + d\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c + d*sin(e + f*x))^3,x
)
```

output
```
(2*B*a^2*atan(((B*a^2*((8*(4*B^2*a^4*c^2*d^6 + 16*B^2*a^4*c^3*d^5 + 24*B^2
*a^4*c^4*d^4 + 16*B^2*a^4*c^5*d^3 + 4*B^2*a^4*c^6*d^2))/(4*c*d^8 + d^9 + 6
*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(40*B^2*a^4*c^2*d^
7 + 75*B^2*a^4*c^3*d^6 + 24*B^2*a^4*c^4*d^5 - 36*B^2*a^4*c^5*d^4 - 32*B^2*
a^4*c^6*d^3 - 8*B^2*a^4*c^7*d^2 - 9*A^2*a^4*c*d^8 - 8*B^2*a^4*c*d^8 + 6*A*
B*a^4*c^2*d^7 + 24*A*B*a^4*c^3*d^6 + 12*A*B*a^4*c^4*d^5 - 24*A*B*a^4*c*d^8
))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (B*a^2*((8*tan(e/2
 + (f*x)/2)*(12*A*a^2*c*d^11 + 16*B*a^2*c*d^11 + 24*A*a^2*c^2*d^10 + 12*A*
a^2*c^3*d^9 + 28*B*a^2*c^2*d^10 - 8*B*a^2*c^3*d^9 - 44*B*a^2*c^4*d^8 - 32*
B*a^2*c^5*d^7 - 8*B*a^2*c^6*d^6))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7
+ c^4*d^6) - (8*(4*B*a^2*c*d^10 - 6*A*a^2*c^2*d^9 - 12*A*a^2*c^3*d^8 - 6*A
*a^2*c^4*d^7 + 8*B*a^2*c^2*d^9 + 6*B*a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 2*B*a
^2*c^5*d^6))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (B*a^2*((
8*(4*c^2*d^12 + 16*c^3*d^11 + 24*c^4*d^10 + 16*c^5*d^9 + 4*c^6*d^8))/(4*c*
d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(12*c
*d^14 + 48*c^2*d^13 + 64*c^3*d^12 + 16*c^4*d^11 - 36*c^5*d^10 - 32*c^6*d^9
 - 8*c^7*d^8))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*1i)/d^3
)*1i)/d^3))/d^3 + (B*a^2*((8*(4*B^2*a^4*c^2*d^6 + 16*B^2*a^4*c^3*d^5 + 24*
B^2*a^4*c^4*d^4 + 16*B^2*a^4*c^5*d^3 + 4*B^2*a^4*c^6*d^2))/(4*c*d^8 + d^9
+ 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*tan(e/2 + (f*x)/2)*(40*B^2*a^4*...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1748, normalized size of antiderivative = 8.13

$$\int \frac{(a+a\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)
```

output

```
(a**2*(12*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2
))*sin(e + f*x)**2*a*c*d**5 - 8*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c
 + d)/sqrt(c**2 - d**2))*sin(e + f*x)**2*b*c**4*d**2 - 16*sqrt(c**2 - d**2
)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*sin(e + f*x)**2*b*c**3*
d**3 - 4*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2)
)*sin(e + f*x)**2*b*c**2*d**4 + 16*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2
)*c + d)/sqrt(c**2 - d**2))*sin(e + f*x)**2*b*c*d**5 + 24*sqrt(c**2 - d**2
)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*sin(e + f*x)*a*c**2*d**
4 - 16*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*
sin(e + f*x)*b*c**5*d - 32*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)
/sqrt(c**2 - d**2))*sin(e + f*x)*b*c**4*d**2 - 8*sqrt(c**2 - d**2)*atan((t
an((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*sin(e + f*x)*b*c**3*d**3 + 32*sq
rt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*sin(e + f
*x)*b*c**2*d**4 + 12*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(
c**2 - d**2))*a*c**3*d**3 - 8*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c +
 d)/sqrt(c**2 - d**2))*b*c**6 - 16*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2
)*c + d)/sqrt(c**2 - d**2))*b*c**5*d - 4*sqrt(c**2 - d**2)*atan((tan((e +
f*x)/2)*c + d)/sqrt(c**2 - d**2))*b*c**4*d**2 + 16*sqrt(c**2 - d**2)*atan(
(tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*b*c**3*d**3 - 2*cos(e + f*x)*s
in(e + f*x)*a*c**4*d**3 - 8*cos(e + f*x)*sin(e + f*x)*a*c**3*d**4 + 2*c...
```

3.258 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

Optimal result

Integrand size = 35, antiderivative size = 604

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$$
$$= \frac{1}{16}a^3\left(3B\left(10c^3+26c^2d+23cd^2+7d^3\right)+A\left(40c^3+90c^2d+78cd^2+23d^3\right)\right)x$$
$$-\frac{a^3(7Ad(2c^5-18c^4d+107c^3d^2+472c^2d^3+456cd^4+136d^5)-3B(2c^6-14c^5d+51c^4d^2-189c^3d^3-}{420d^3f}$$
$$-\frac{a^3(7Ad(4c^4-36c^3d+216c^2d^2+626cd^3+345d^4)-3B(4c^5-28c^4d+104c^3d^2-392c^2d^3-1263cd^4}{1680d^2f}$$
$$-\frac{a^3(7Ad(2c^3-18c^2d+111cd^2+136d^3)-B(6c^4-42c^3d+165c^2d^2-651cd^3-864d^4))\cos(e+fx)(c}{840d^3f}$$
$$-\frac{a^3(7Ad(2c^2-18cd+115d^2)-B(6c^3-42c^2d+177cd^2-735d^3))\cos(e+fx)(c+d\sin(e+fx))^3}{840d^3f}$$
$$-\frac{a^3(6Bc^2-14Acd-27Bcd+91Ad^2+87Bd^2)\cos(e+fx)(c+d\sin(e+fx))^4}{210d^3f}$$
$$-\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx))^4}{7df}$$
$$+\frac{(3B(c-3d)-7Ad)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))^4}{42d^2f}$$

output
```
1/16*a^3*(3*B*(10*c^3+26*c^2*d+23*c*d^2+7*d^3)+A*(40*c^3+90*c^2*d+78*c*d^2
+23*d^3))*x-1/420*a^3*(7*A*d*(2*c^5-18*c^4*d+107*c^3*d^2+472*c^2*d^3+456*c
*d^4+136*d^5)-3*B*(2*c^6-14*c^5*d+51*c^4*d^2-189*c^3*d^3-920*c^2*d^4-952*c
*d^5-288*d^6))*cos(f*x+e)/d^3/f-1/1680*a^3*(7*A*d*(4*c^4-36*c^3*d+216*c^2*
d^2+626*c*d^3+345*d^4)-3*B*(4*c^5-28*c^4*d+104*c^3*d^2-392*c^2*d^3-1263*c*
d^4-735*d^5))*cos(f*x+e)*sin(f*x+e)/d^2/f-1/840*a^3*(7*A*d*(2*c^3-18*c^2*d
+111*c*d^2+136*d^3)-B*(6*c^4-42*c^3*d+165*c^2*d^2-651*c*d^3-864*d^4))*cos(
f*x+e)*(c+d*sin(f*x+e))^2/d^3/f-1/840*a^3*(7*A*d*(2*c^2-18*c*d+115*d^2)-B*
(6*c^3-42*c^2*d+177*c*d^2-735*d^3))*cos(f*x+e)*(c+d*sin(f*x+e))^3/d^3/f-1/
210*a^3*(-14*A*c*d+91*A*d^2+6*B*c^2-27*B*c*d+87*B*d^2)*cos(f*x+e)*(c+d*sin
(f*x+e))^4/d^3/f-1/7*a*B*cos(f*x+e)*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^4/
d/f+1/42*(3*B*(c-3*d)-7*A*d)*cos(f*x+e)*(a^3+a^3*sin(f*x+e))*(c+d*sin(f*x+
e))^4/d^2/f
```

Mathematica [A] (warning: unable to verify)

Time = 5.65 (sec) , antiderivative size = 528, normalized size of antiderivative = 0.87

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx =$$

$$-\frac{a^3\cos(e+fx)\left(420(3B(10c^3+26c^2d+23cd^2+7d^3)+A(40c^3+90c^2d+78cd^2+23d^3))\arcsin\left(\frac{\sqrt{1-}\cdots}{\cdots}\right)\cdots\right)}{\cdots}$$

input
```
Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])
^3,x]
```

output
```
-1/3360*(a^3*Cos[e + f*x]*(420*(3*B*(10*c^3 + 26*c^2*d + 23*c*d^2 + 7*d^3)
 + A*(40*c^3 + 90*c^2*d + 78*c*d^2 + 23*d^3))*ArcSin[Sqrt[1 - Sin[e + f*x]
]/Sqrt[2]] + Sqrt[Cos[e + f*x]^2]*(12880*A*c^3 + 11760*B*c^3 + 35280*A*c^2
*d + 32676*B*c^2*d + 32676*A*c*d^2 + 30828*B*c*d^2 + 10276*A*d^3 + 9762*B*
d^3 - (112*A*(5*c^3 + 45*c^2*d + 66*c*d^2 + 26*d^3) + 3*B*(560*c^3 + 2464*
c^2*d + 2912*c*d^2 + 1083*d^3))*Cos[2*(e + f*x)] + 18*d*(14*A*d*(c + d) +
B*(14*c^2 + 42*c*d + 23*d^2))*Cos[4*(e + f*x)] - 15*B*d^3*Cos[6*(e + f*x)]
 + 5040*A*c^3*Sin[e + f*x] + 6930*B*c^3*Sin[e + f*x] + 20790*A*c^2*d*Sin[e
 + f*x] + 22050*B*c^2*d*Sin[e + f*x] + 22050*A*c*d^2*Sin[e + f*x] + 22785*
B*c*d^2*Sin[e + f*x] + 7595*A*d^3*Sin[e + f*x] + 7665*B*d^3*Sin[e + f*x] -
 210*B*c^3*Sin[3*(e + f*x)] - 630*A*c^2*d*Sin[3*(e + f*x)] - 1890*B*c^2*d*
Sin[3*(e + f*x)] - 1890*A*c*d^2*Sin[3*(e + f*x)] - 2940*B*c*d^2*Sin[3*(e +
 f*x)] - 980*A*d^3*Sin[3*(e + f*x)] - 1260*B*d^3*Sin[3*(e + f*x)] + 105*B*
c*d^2*Sin[5*(e + f*x)] + 35*A*d^3*Sin[5*(e + f*x)] + 105*B*d^3*Sin[5*(e +
f*x)])))/(f*Sqrt[Cos[e + f*x]^2])
```

Rubi [A] (verified)

Time = 2.63 (sec) , antiderivative size = 628, normalized size of antiderivative = 1.04, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.457$, Rules used = {3042, 3455, 3042, 3455, 3042, 3447, 3042, 3502, 25, 3042, 3232, 27, 3042, 3232, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^3(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^3(A+B\sin(e+fx))(c+d\sin(e+fx))^3dx$$

$$\Big\downarrow \text{3455}$$

$$\frac{\int (\sin(e+fx)a+a)^2(c+d\sin(e+fx))^3(a(7Ad+2B(c+2d))-a(3B(c-3d)-7Ad)\sin(e+fx))dx}{7d}-\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^4}{7df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (\sin(e+fx)a+a)^2(c+d\sin(e+fx))^3(a(7Ad+2B(c+2d))-a(3B(c-3d)-7Ad)\sin(e+fx))dx}{7d}-\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^4}{7df}$$

$\Big\downarrow$ 3455

$$\frac{\frac{\int (\sin(e+fx)a+a)(c+d\sin(e+fx))^3\left(\left(7Ad(c+10d)-B\left(3c^2-9dc-60d^2\right)\right)a^2+\left(6Bc^2-14Adc-27Bdc+91Ad^2+87Bd^2\right)\sin(e+fx)a^2\right)dx}{6d}+\frac{(3B(c}{}}{7d}-\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^4}{7df}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int (\sin(e+fx)a+a)(c+d\sin(e+fx))^3\left(\left(7Ad(c+10d)-B\left(3c^2-9dc-60d^2\right)\right)a^2+\left(6Bc^2-14Adc-27Bdc+91Ad^2+87Bd^2\right)\sin(e+fx)a^2\right)dx}{6d}+\frac{(3B(c}{}}{7d}-\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^4}{7df}$$

$\Big\downarrow$ 3447

$$\frac{\frac{\int (c+d\sin(e+fx))^3\left(\left(6Bc^2-14Adc-27Bdc+91Ad^2+87Bd^2\right)\sin^2(e+fx)a^3+\left(7Ad(c+10d)-B\left(3c^2-9dc-60d^2\right)\right)a^3+\left(\left(6Bc^2-14Adc-27Bdc+9\right.\right.}{6d}}{7d}-\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^4}{7df}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int (c+d\sin(e+fx))^3\left(\left(6Bc^2-14Adc-27Bdc+91Ad^2+87Bd^2\right)\sin(e+fx)^2a^3+\left(7Ad(c+10d)-B\left(3c^2-9dc-60d^2\right)\right)a^3+\left(\left(6Bc^2-14Adc-27Bdc+9\right.\right.}{6d}}{7d}-\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^4}{7df}$$

$\Big\downarrow$ 3502

$$\frac{\frac{\frac{\int -(c+d\sin(e+fx))^3\left(3a^3d\left(7A(c-34d)d-3B\left(c^2-7dc+72d^2\right)\right)-a^3\left(7Ad\left(2c^2-18dc+115d^2\right)-B\left(6c^3-42dc^2+177d^2c-735d^3\right)\right)\sin(e+fx)\right)dx}{5d}-\frac{a^3\left(-14Acd+91\right.}{}}{6d}}{7d}-\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^4}{7df}$$

$\Big\downarrow$ 25

$$\frac{\frac{-\frac{\int (c+d\sin(e+fx))^3\left(3a^3d\left(7A(c-34d)d-3B\left(c^2-7dc+72d^2\right)\right)-a^3\left(7Ad\left(2c^2-18dc+115d^2\right)-B\left(6c^3-42dc^2+177d^2c-735d^3\right)\right)\sin(e+fx)\right)dx}{5d}-\frac{a^3\left(-14Acd+91\cdots\right.}{\cdots}}{6d}\cdots}{7d}$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^4}{7df}$$

$\downarrow$ 3042

$$\frac{\frac{-\frac{\int (c+d\sin(e+fx))^3\left(3a^3d\left(7A(c-34d)d-3B\left(c^2-7dc+72d^2\right)\right)-a^3\left(7Ad\left(2c^2-18dc+115d^2\right)-B\left(6c^3-42dc^2+177d^2c-735d^3\right)\right)\sin(e+fx)\right)dx}{5d}-\frac{a^3\left(-14Acd+91\cdots\right.}{\cdots}}{6d}\cdots}{7d}$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^4}{7df}$$

$\downarrow$ 3232

$$\frac{\frac{-\frac{\frac{1}{4}\int 3(c+d\sin(e+fx))^2\left(a^3d\left(7Ad\left(2c^2-118dc-115d^2\right)-3B\left(2c^3-14dc^2+229d^2c+245d^3\right)\right)-a^3\left(7Ad\left(2c^3-18dc^2+111d^2c+136d^3\right)-B\left(6c^4-42dc^3+165d^2c^2-6\cdots\right.\right.\right.}{5d}\cdots}{\cdots}}{\cdots}$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^4}{7df}$$

$\downarrow$ 27

$$\frac{\frac{-\frac{\frac{3}{4}\int (c+d\sin(e+fx))^2\left(a^3d\left(7Ad\left(2c^2-118dc-115d^2\right)-3B\left(2c^3-14dc^2+229d^2c+245d^3\right)\right)-a^3\left(7Ad\left(2c^3-18dc^2+111d^2c+136d^3\right)-B\left(6c^4-42dc^3+165d^2c^2-65\cdots\right.\right.\right.}{5d}\cdots}{\cdots}}{\cdots}$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^4}{7df}$$

$\downarrow$ 3042

$$\frac{\frac{-\frac{\frac{3}{4}\int (c+d\sin(e+fx))^2\left(a^3d\left(7Ad\left(2c^2-118dc-115d^2\right)-3B\left(2c^3-14dc^2+229d^2c+245d^3\right)\right)-a^3\left(7Ad\left(2c^3-18dc^2+111d^2c+136d^3\right)-B\left(6c^4-42dc^3+165d^2c^2-65\cdots\right.\right.\right.}{5d}\cdots}{\cdots}}{\cdots}$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^4}{7df}$$

$\downarrow$ 3232

$$-\frac{\frac{3}{4}\left(\frac{1}{3}\int(c+d\sin(e+fx))\left(a^3d\left(7Ad\left(2c^3-318dc^2-567d^2c-272d^3\right)-3B\left(2c^4-14dc^3+577d^2c^2+1169d^3c+576d^4\right)\right)-a^3\left(7Ad\left(4c^4-36dc^3+216d^2c^2+626d^3c+345d\right.\right.\right.\right.}{}$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^4}{7df}$$

$\downarrow$ 3042

$$-\frac{\frac{3}{4}\left(\frac{1}{3}\int(c+d\sin(e+fx))\left(a^3d\left(7Ad\left(2c^3-318dc^2-567d^2c-272d^3\right)-3B\left(2c^4-14dc^3+577d^2c^2+1169d^3c+576d^4\right)\right)-a^3\left(7Ad\left(4c^4-36dc^3+216d^2c^2+626d^3c+345d\right.\right.\right.\right.}{}$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^4}{7df}$$

$\downarrow$ 3213

$$-\frac{a^3\left(-14Acd+91Ad^2+6Bc^2-27Bcd+87Bd^2\right)\cos(e+fx)(c+d\sin(e+fx))^4}{5df}-\frac{\frac{a^3\left(7Ad\left(2c^2-18cd+115d^2\right)-B\left(6c^3-42c^2d+177cd^2-735d^3\right)\right)\cos(e+fx)(c+d\sin(}{4f}}{}$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^4}{7df}$$

input
```
Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x]
```

output
```
-1/7*(a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^4)/(d*f
) + (((3*B*(c - 3*d) - 7*A*d)*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x])*(c + d
*Sin[e + f*x])^4)/(6*d*f) + (-1/5*(a^3*(6*B*c^2 - 14*A*c*d - 27*B*c*d + 91
*A*d^2 + 87*B*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(d*f) - ((a^3*(7*A
*d*(2*c^2 - 18*c*d + 115*d^2) - B*(6*c^3 - 42*c^2*d + 177*c*d^2 - 735*d^3)
)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(4*f) + (3*((a^3*(7*A*d*(2*c^3 - 18
*c^2*d + 111*c*d^2 + 136*d^3) - B*(6*c^4 - 42*c^3*d + 165*c^2*d^2 - 651*c*
d^3 - 864*d^4))*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(3*f) + ((-105*a^3*d^
3*(3*B*(10*c^3 + 26*c^2*d + 23*c*d^2 + 7*d^3) + A*(40*c^3 + 90*c^2*d + 78*
c*d^2 + 23*d^3))*x)/2 + (2*a^3*(7*A*d*(2*c^5 - 18*c^4*d + 107*c^3*d^2 + 47
2*c^2*d^3 + 456*c*d^4 + 136*d^5) - 3*B*(2*c^6 - 14*c^5*d + 51*c^4*d^2 - 18
9*c^3*d^3 - 920*c^2*d^4 - 952*c*d^5 - 288*d^6))*Cos[e + f*x])/f + (a^3*d*(
7*A*d*(4*c^4 - 36*c^3*d + 216*c^2*d^2 + 626*c*d^3 + 345*d^4) - 3*B*(4*c^5
- 28*c^4*d + 104*c^3*d^2 - 392*c^2*d^3 - 1263*c*d^4 - 735*d^5))*Cos[e + f*
x]*Sin[e + f*x])/(2*f))/3))/4)/(5*d))/(6*d))/(7*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 1077, normalized size of antiderivative = 1.78

Expression too large to display

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x)
```

output
```
1/f*(-3/5*a^3*A*c*d^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-3/5*a
^3*B*c^2*d*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-3*a^3*A*c^2*d*(2
+sin(f*x+e)^2)*cos(f*x+e)-9/5*a^3*B*c*d^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)
^2)*cos(f*x+e)-3*a^3*A*c*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)-3*a^3*B*c^2*d*(2+
sin(f*x+e)^2)*cos(f*x+e)-3/5*a^3*B*d^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)
*cos(f*x+e)-1/3*a^3*A*d^3*(2+sin(f*x+e)^2)*cos(f*x+e)+3*a^3*A*c^2*d*(-1/4*
(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+3*a^3*B*c*d^2*(-1/
6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16
*e)-1/3*a^3*A*c^3*(2+sin(f*x+e)^2)*cos(f*x+e)+a^3*A*c^3*(f*x+e)-a^3*B*c^3*
cos(f*x+e)+a^3*B*d^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*
x+3/8*e)+3*a^3*A*c^3*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+a^3*A*d^3*
(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+
5/16*e)+3*a^3*B*d^3*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*
cos(f*x+e)+5/16*f*x+5/16*e)-3*a^3*A*c^3*cos(f*x+e)+3*a^3*A*d^3*(-1/4*(sin(
f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+3*a^3*B*c^3*(-1/2*sin(f
*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+a^3*B*c^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+
e))*cos(f*x+e)+3/8*f*x+3/8*e)+9*a^3*A*c*d^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*
x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+9*a^3*B*c^2*d*(-1/4*(sin(f*x+e)^3+3/2*sin(
f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+9*a^3*A*c^2*d*(-1/2*sin(f*x+e)*cos(f*x+e
)+1/2*f*x+1/2*e)+3*a^3*A*c*d^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*...
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 432, normalized size of antiderivative = 0.72

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$$
$$=\frac{240\,Ba^3d^3\cos{(fx+e)}^7-1008\,(Ba^3c^2d+(A+3\,B)a^3cd^2+(A+2\,B)a^3d^3)\cos{(fx+e)}^5+560\,((A+\cdots}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algori
thm="fricas")
```

output
```
1/1680*(240*B*a^3*d^3*cos(f*x + e)^7 - 1008*(B*a^3*c^2*d + (A + 3*B)*a^3*c
*d^2 + (A + 2*B)*a^3*d^3)*cos(f*x + e)^5 + 560*((A + 3*B)*a^3*c^3 + 3*(3*A
 + 5*B)*a^3*c^2*d + 3*(5*A + 7*B)*a^3*c*d^2 + (7*A + 9*B)*a^3*d^3)*cos(f*x
 + e)^3 + 105*(10*(4*A + 3*B)*a^3*c^3 + 6*(15*A + 13*B)*a^3*c^2*d + 3*(26*
A + 23*B)*a^3*c*d^2 + (23*A + 21*B)*a^3*d^3)*f*x - 6720*((A + B)*a^3*c^3 +
 3*(A + B)*a^3*c^2*d + 3*(A + B)*a^3*c*d^2 + (A + B)*a^3*d^3)*cos(f*x + e)
 - 35*(8*(3*B*a^3*c*d^2 + (A + 3*B)*a^3*d^3)*cos(f*x + e)^5 - 2*(6*B*a^3*c
^3 + 18*(A + 3*B)*a^3*c^2*d + 3*(18*A + 31*B)*a^3*c*d^2 + (31*A + 45*B)*a^
3*d^3)*cos(f*x + e)^3 + 3*(2*(12*A + 17*B)*a^3*c^3 + 6*(17*A + 19*B)*a^3*c
^2*d + 3*(38*A + 41*B)*a^3*c*d^2 + (41*A + 43*B)*a^3*d^3)*cos(f*x + e))*si
n(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2878 vs. $2(598) = 1196$.

Time = 0.76 (sec) , antiderivative size = 2878, normalized size of antiderivative = 4.76

$$\int (a + a\sin(e + fx))^3 (A + B\sin(e + fx))(c + d\sin(e + fx))^3\, dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3,x)
```

output
```
Piecewise((3*A*a**3*c**3*x*sin(e + f*x)**2/2 + 3*A*a**3*c**3*x*cos(e + f*x
)**2/2 + A*a**3*c**3*x - A*a**3*c**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*
a**3*c**3*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a**3*c**3*cos(e + f*x)**3/
(3*f) - 3*A*a**3*c**3*cos(e + f*x)/f + 9*A*a**3*c**2*d*x*sin(e + f*x)**4/8
 + 9*A*a**3*c**2*d*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 9*A*a**3*c**2*d*x
*sin(e + f*x)**2/2 + 9*A*a**3*c**2*d*x*cos(e + f*x)**4/8 + 9*A*a**3*c**2*d
*x*cos(e + f*x)**2/2 - 15*A*a**3*c**2*d*sin(e + f*x)**3*cos(e + f*x)/(8*f)
 - 9*A*a**3*c**2*d*sin(e + f*x)**2*cos(e + f*x)/f - 9*A*a**3*c**2*d*sin(e
+ f*x)*cos(e + f*x)**3/(8*f) - 9*A*a**3*c**2*d*sin(e + f*x)*cos(e + f*x)/(
2*f) - 6*A*a**3*c**2*d*cos(e + f*x)**3/f - 3*A*a**3*c**2*d*cos(e + f*x)/f
+ 27*A*a**3*c*d**2*x*sin(e + f*x)**4/8 + 27*A*a**3*c*d**2*x*sin(e + f*x)**
2*cos(e + f*x)**2/4 + 3*A*a**3*c*d**2*x*sin(e + f*x)**2/2 + 27*A*a**3*c*d*
*2*x*cos(e + f*x)**4/8 + 3*A*a**3*c*d**2*x*cos(e + f*x)**2/2 - 3*A*a**3*c*
d**2*sin(e + f*x)**4*cos(e + f*x)/f - 45*A*a**3*c*d**2*sin(e + f*x)**3*cos
(e + f*x)/(8*f) - 4*A*a**3*c*d**2*sin(e + f*x)**2*cos(e + f*x)**3/f - 9*A*
a**3*c*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 27*A*a**3*c*d**2*sin(e + f*x)
*cos(e + f*x)**3/(8*f) - 3*A*a**3*c*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) -
 8*A*a**3*c*d**2*cos(e + f*x)**5/(5*f) - 6*A*a**3*c*d**2*cos(e + f*x)**3/f
 + 5*A*a**3*d**3*x*sin(e + f*x)**6/16 + 15*A*a**3*d**3*x*sin(e + f*x)**4*c
os(e + f*x)**2/16 + 9*A*a**3*d**3*x*sin(e + f*x)**4/8 + 15*A*a**3*d**3*...
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 1056, normalized size of antiderivative = 1.75

$$\int (a + a\sin(e + fx))^3 (A + B\sin(e + fx))(c + d\sin(e + fx))^3 \, dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algori
thm="maxima")
```

output
```
1/6720*(2240*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c^3 + 5040*(2*f*x + 2
*e - sin(2*f*x + 2*e))*A*a^3*c^3 + 6720*(f*x + e)*A*a^3*c^3 + 6720*(cos(f*
x + e)^3 - 3*cos(f*x + e))*B*a^3*c^3 + 210*(12*f*x + 12*e + sin(4*f*x + 4*
e) - 8*sin(2*f*x + 2*e))*B*a^3*c^3 + 5040*(2*f*x + 2*e - sin(2*f*x + 2*e))
*B*a^3*c^3 + 20160*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c^2*d + 630*(12
*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^3*c^2*d + 15120*(
2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^3*c^2*d - 1344*(3*cos(f*x + e)^5 - 10*
cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*c^2*d + 20160*(cos(f*x + e)^3 - 3*
cos(f*x + e))*B*a^3*c^2*d + 1890*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin
(2*f*x + 2*e))*B*a^3*c^2*d + 5040*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3*c
^2*d - 1344*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*A*a^3
*c*d^2 + 20160*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c*d^2 + 1890*(12*f*
x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^3*c*d^2 + 5040*(2*f*
x + 2*e - sin(2*f*x + 2*e))*A*a^3*c*d^2 - 4032*(3*cos(f*x + e)^5 - 10*cos(
f*x + e)^3 + 15*cos(f*x + e))*B*a^3*c*d^2 + 6720*(cos(f*x + e)^3 - 3*cos(f
*x + e))*B*a^3*c*d^2 + 105*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4
*f*x + 4*e) - 48*sin(2*f*x + 2*e))*B*a^3*c*d^2 + 1890*(12*f*x + 12*e + sin
(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*c*d^2 - 1344*(3*cos(f*x + e)^5 -
 10*cos(f*x + e)^3 + 15*cos(f*x + e))*A*a^3*d^3 + 2240*(cos(f*x + e)^3 - 3
*cos(f*x + e))*A*a^3*d^3 + 35*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9...
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 559, normalized size of antiderivative = 0.93

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx=\frac{Ba^3d^3\cos(7\,fx+7\,e)}{448\,f}$$
$$+\frac{1}{16}\left(40\,Aa^3c^3+30\,Ba^3c^3+90\,Aa^3c^2d+78\,Ba^3c^2d+78\,Aa^3cd^2+69\,Ba^3cd^2+23\,Aa^3d^3+21\,Ba^3d^3\right)x$$
$$-\frac{(12\,Ba^3c^2d+12\,Aa^3cd^2+36\,Ba^3cd^2+12\,Aa^3d^3+19\,Ba^3d^3)\cos(5\,fx+5\,e)}{320\,f}$$
$$+\frac{(16\,Aa^3c^3+48\,Ba^3c^3+144\,Aa^3c^2d+204\,Ba^3c^2d+204\,Aa^3cd^2+228\,Ba^3cd^2+76\,Aa^3d^3+81\,Ba^3d^3)\cos(3\,fx+3\,e)}{192\,f}$$
$$-\frac{(240\,Aa^3c^3+208\,Ba^3c^3+624\,Aa^3c^2d+552\,Ba^3c^2d+552\,Aa^3cd^2+504\,Ba^3cd^2+168\,Aa^3d^3+155\,Ba^3d^3)\cos(fx+e)}{64\,f}$$
$$-\frac{(3\,Ba^3cd^2+Aa^3d^3+3\,Ba^3d^3)\sin(6\,fx+6\,e)}{192\,f}$$
$$+\frac{(2\,Ba^3c^3+6\,Aa^3c^2d+18\,Ba^3c^2d+18\,Aa^3cd^2+27\,Ba^3cd^2+9\,Aa^3d^3+11\,Ba^3d^3)\sin(4\,fx+4\,e)}{64\,f}$$
$$-\frac{(48\,Aa^3c^3+64\,Ba^3c^3+192\,Aa^3c^2d+192\,Ba^3c^2d+192\,Aa^3cd^2+189\,Ba^3cd^2+63\,Aa^3d^3+61\,Ba^3d^3)\sin(2\,fx+2\,e)}{64\,f}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algori
thm="giac")
```

output
```
1/448*B*a^3*d^3*cos(7*f*x + 7*e)/f + 1/16*(40*A*a^3*c^3 + 30*B*a^3*c^3 + 9
0*A*a^3*c^2*d + 78*B*a^3*c^2*d + 78*A*a^3*c*d^2 + 69*B*a^3*c*d^2 + 23*A*a^
3*d^3 + 21*B*a^3*d^3)*x - 1/320*(12*B*a^3*c^2*d + 12*A*a^3*c*d^2 + 36*B*a^
3*c*d^2 + 12*A*a^3*d^3 + 19*B*a^3*d^3)*cos(5*f*x + 5*e)/f + 1/192*(16*A*a^
3*c^3 + 48*B*a^3*c^3 + 144*A*a^3*c^2*d + 204*B*a^3*c^2*d + 204*A*a^3*c*d^2
 + 228*B*a^3*c*d^2 + 76*A*a^3*d^3 + 81*B*a^3*d^3)*cos(3*f*x + 3*e)/f - 1/6
4*(240*A*a^3*c^3 + 208*B*a^3*c^3 + 624*A*a^3*c^2*d + 552*B*a^3*c^2*d + 552
*A*a^3*c*d^2 + 504*B*a^3*c*d^2 + 168*A*a^3*d^3 + 155*B*a^3*d^3)*cos(f*x +
e)/f - 1/192*(3*B*a^3*c*d^2 + A*a^3*d^3 + 3*B*a^3*d^3)*sin(6*f*x + 6*e)/f
+ 1/64*(2*B*a^3*c^3 + 6*A*a^3*c^2*d + 18*B*a^3*c^2*d + 18*A*a^3*c*d^2 + 27
*B*a^3*c*d^2 + 9*A*a^3*d^3 + 11*B*a^3*d^3)*sin(4*f*x + 4*e)/f - 1/64*(48*A
*a^3*c^3 + 64*B*a^3*c^3 + 192*A*a^3*c^2*d + 192*B*a^3*c^2*d + 192*A*a^3*c*
d^2 + 189*B*a^3*c*d^2 + 63*A*a^3*d^3 + 61*B*a^3*d^3)*sin(2*f*x + 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 38.13 (sec) , antiderivative size = 1395, normalized size of antiderivative = 2.31

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = \text{Too large to display}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^3,x)
```

output
```
(a^3*atan((a^3*tan(e/2 + (f*x)/2)*(40*A*c^3 + 23*A*d^3 + 30*B*c^3 + 21*B*d
^3 + 78*A*c*d^2 + 90*A*c^2*d + 69*B*c*d^2 + 78*B*c^2*d))/(8*(5*A*a^3*c^3 +
 (23*A*a^3*d^3)/8 + (15*B*a^3*c^3)/4 + (21*B*a^3*d^3)/8 + (39*A*a^3*c*d^2)
/4 + (45*A*a^3*c^2*d)/4 + (69*B*a^3*c*d^2)/8 + (39*B*a^3*c^2*d)/4)))*(40*A
*c^3 + 23*A*d^3 + 30*B*c^3 + 21*B*d^3 + 78*A*c*d^2 + 90*A*c^2*d + 69*B*c*d
^2 + 78*B*c^2*d))/(8*f) - (tan(e/2 + (f*x)/2)*(3*A*a^3*c^3 + (23*A*a^3*d^3
)/8 + (15*B*a^3*c^3)/4 + (21*B*a^3*d^3)/8 + (39*A*a^3*c*d^2)/4 + (45*A*a^3
*c^2*d)/4 + (69*B*a^3*c*d^2)/8 + (39*B*a^3*c^2*d)/4) + tan(e/2 + (f*x)/2)^
10*(40*A*a^3*c^3 + 4*A*a^3*d^3 + 24*B*a^3*c^3 + 36*A*a^3*c*d^2 + 72*A*a^3*
c^2*d + 12*B*a^3*c*d^2 + 36*B*a^3*c^2*d) - tan(e/2 + (f*x)/2)^13*(3*A*a^3*
c^3 + (23*A*a^3*d^3)/8 + (15*B*a^3*c^3)/4 + (21*B*a^3*d^3)/8 + (39*A*a^3*c
*d^2)/4 + (45*A*a^3*c^2*d)/4 + (69*B*a^3*c*d^2)/8 + (39*B*a^3*c^2*d)/4) +
tan(e/2 + (f*x)/2)^3*(12*A*a^3*c^3 + (115*A*a^3*d^3)/6 + 17*B*a^3*c^3 + (3
5*B*a^3*d^3)/2 + 57*A*a^3*c*d^2 + 51*A*a^3*c^2*d + (115*B*a^3*c*d^2)/2 + 5
7*B*a^3*c^2*d) - tan(e/2 + (f*x)/2)^11*(12*A*a^3*c^3 + (115*A*a^3*d^3)/6 +
 17*B*a^3*c^3 + (35*B*a^3*d^3)/2 + 57*A*a^3*c*d^2 + 51*A*a^3*c^2*d + (115*
B*a^3*c*d^2)/2 + 57*B*a^3*c^2*d) + tan(e/2 + (f*x)/2)^8*((322*A*a^3*c^3)/3
 + (148*A*a^3*d^3)/3 + 82*B*a^3*c^3 + 32*B*a^3*d^3 + 188*A*a^3*c*d^2 + 246
*A*a^3*c^2*d + 148*B*a^3*c*d^2 + 188*B*a^3*c^2*d) + tan(e/2 + (f*x)/2)^6*(
(448*A*a^3*c^3)/3 + (328*A*a^3*d^3)/3 + 128*B*a^3*c^3 + 112*B*a^3*d^3 +...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 869, normalized size of antiderivative = 1.44

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = \text{Too large to display}$$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x)
```

output
```
(a**3*( - 240*cos(e + f*x)*sin(e + f*x)**6*b*d**3 - 280*cos(e + f*x)*sin(e
 + f*x)**5*a*d**3 - 840*cos(e + f*x)*sin(e + f*x)**5*b*c*d**2 - 840*cos(e
+ f*x)*sin(e + f*x)**5*b*d**3 - 1008*cos(e + f*x)*sin(e + f*x)**4*a*c*d**2
 - 1008*cos(e + f*x)*sin(e + f*x)**4*a*d**3 - 1008*cos(e + f*x)*sin(e + f*
x)**4*b*c**2*d - 3024*cos(e + f*x)*sin(e + f*x)**4*b*c*d**2 - 1296*cos(e +
 f*x)*sin(e + f*x)**4*b*d**3 - 1260*cos(e + f*x)*sin(e + f*x)**3*a*c**2*d
- 3780*cos(e + f*x)*sin(e + f*x)**3*a*c*d**2 - 1610*cos(e + f*x)*sin(e + f
*x)**3*a*d**3 - 420*cos(e + f*x)*sin(e + f*x)**3*b*c**3 - 3780*cos(e + f*x
)*sin(e + f*x)**3*b*c**2*d - 4830*cos(e + f*x)*sin(e + f*x)**3*b*c*d**2 -
1470*cos(e + f*x)*sin(e + f*x)**3*b*d**3 - 560*cos(e + f*x)*sin(e + f*x)**
2*a*c**3 - 5040*cos(e + f*x)*sin(e + f*x)**2*a*c**2*d - 6384*cos(e + f*x)*
sin(e + f*x)**2*a*c*d**2 - 1904*cos(e + f*x)*sin(e + f*x)**2*a*d**3 - 1680
*cos(e + f*x)*sin(e + f*x)**2*b*c**3 - 6384*cos(e + f*x)*sin(e + f*x)**2*b
*c**2*d - 5712*cos(e + f*x)*sin(e + f*x)**2*b*c*d**2 - 1728*cos(e + f*x)*s
in(e + f*x)**2*b*d**3 - 2520*cos(e + f*x)*sin(e + f*x)*a*c**3 - 9450*cos(e
 + f*x)*sin(e + f*x)*a*c**2*d - 8190*cos(e + f*x)*sin(e + f*x)*a*c*d**2 -
2415*cos(e + f*x)*sin(e + f*x)*a*d**3 - 3150*cos(e + f*x)*sin(e + f*x)*b*c
**3 - 8190*cos(e + f*x)*sin(e + f*x)*b*c**2*d - 7245*cos(e + f*x)*sin(e +
f*x)*b*c*d**2 - 2205*cos(e + f*x)*sin(e + f*x)*b*d**3 - 6160*cos(e + f*x)*
a*c**3 - 15120*cos(e + f*x)*a*c**2*d - 12768*cos(e + f*x)*a*c*d**2 - 38...
```

3.259 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

Optimal result

Integrand size = 35, antiderivative size = 463

$$\begin{aligned}
&\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx \\
&= \frac{1}{16}a^3\left(B\left(30c^2+52cd+23d^2\right)+A\left(40c^2+60cd+26d^2\right)\right)x \\
&\quad -\frac{a^3(2Ad(2c^4-15c^3d+72c^2d^2+180cd^3+76d^4)-B(2c^5-12c^4d+37c^3d^2-112c^2d^3-304cd^4-136}{60d^3f} \\
&\quad -\frac{a^3(2Ad(4c^3-30c^2d+146cd^2+195d^3)-B(4c^4-24c^3d+76c^2d^2-236cd^3-345d^4))\cos(e+fx)\sin}{240d^2f} \\
&\quad -\frac{a^3(2Ad(2c^2-15cd+76d^2)-B(2c^3-12c^2d+41cd^2-136d^3))\cos(e+fx)(c+d\sin(e+fx))^2}{120d^3f} \\
&\quad +\frac{a^3(2A(2c-11d)d-B(2c^2-8cd+21d^2))\cos(e+fx)(c+d\sin(e+fx))^3}{40d^3f} \\
&\quad -\frac{aB\cos(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}{6df} \\
&\quad +\frac{(3Bc-6Ad-8Bd)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)(c+d\sin(e+fx))^3}{30d^2f}
\end{aligned}$$

output
```
1/16*a^3*(B*(30*c^2+52*c*d+23*d^2)+A*(40*c^2+60*c*d+26*d^2))*x-1/60*a^3*(2
*A*d*(2*c^4-15*c^3*d+72*c^2*d^2+180*c*d^3+76*d^4)-B*(2*c^5-12*c^4*d+37*c^3
*d^2-112*c^2*d^3-304*c*d^4-136*d^5))*cos(f*x+e)/d^3/f-1/240*a^3*(2*A*d*(4*
c^3-30*c^2*d+146*c*d^2+195*d^3)-B*(4*c^4-24*c^3*d+76*c^2*d^2-236*c*d^3-345
*d^4))*cos(f*x+e)*sin(f*x+e)/d^2/f-1/120*a^3*(2*A*d*(2*c^2-15*c*d+76*d^2)-
B*(2*c^3-12*c^2*d+41*c*d^2-136*d^3))*cos(f*x+e)*(c+d*sin(f*x+e))^2/d^3/f+1
/40*a^3*(2*A*(2*c-11*d)*d-B*(2*c^2-8*c*d+21*d^2))*cos(f*x+e)*(c+d*sin(f*x+
e))^3/d^3/f-1/6*a*B*cos(f*x+e)*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^3/d/f+1
/30*(-6*A*d+3*B*c-8*B*d)*cos(f*x+e)*(a^3+a^3*sin(f*x+e))*(c+d*sin(f*x+e))^
3/d^2/f
```

Mathematica [A] (warning: unable to verify)

Time = 2.98 (sec) , antiderivative size = 355, normalized size of antiderivative = 0.77

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx =$$

$$-\frac{a^3\cos(e+fx)\left(60(B(30c^2+52cd+23d^2)+A(40c^2+60cd+26d^2))\arcsin\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(}\right.}{}$$

input
```
Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])
^2,x]
```

output
```
-1/480*(a^3*Cos[e + f*x]*(60*(B*(30*c^2 + 52*c*d + 23*d^2) + A*(40*c^2 + 6
0*c*d + 26*d^2))*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]] + Sqrt[Cos[e + f*x
]^2]*(1840*A*c^2 + 1680*B*c^2 + 3360*A*c*d + 3112*B*c*d + 1556*A*d^2 + 146
8*B*d^2 - 16*(A*(5*c^2 + 30*c*d + 22*d^2) + B*(15*c^2 + 44*c*d + 26*d^2))*
Cos[2*(e + f*x)] + 12*d*(2*B*c + A*d + 3*B*d)*Cos[4*(e + f*x)] + 720*A*c^2
*Sin[e + f*x] + 990*B*c^2*Sin[e + f*x] + 1980*A*c*d*Sin[e + f*x] + 2100*B*
c*d*Sin[e + f*x] + 1050*A*d^2*Sin[e + f*x] + 1085*B*d^2*Sin[e + f*x] - 30*
B*c^2*Sin[3*(e + f*x)] - 60*A*c*d*Sin[3*(e + f*x)] - 180*B*c*d*Sin[3*(e +
f*x)] - 90*A*d^2*Sin[3*(e + f*x)] - 140*B*d^2*Sin[3*(e + f*x)] + 5*B*d^2*S
in[5*(e + f*x)])))/(f*Sqrt[Cos[e + f*x]^2])
```

Rubi [A] (verified)

Time = 2.04 (sec) , antiderivative size = 485, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3455, 3042, 3455, 27, 3042, 3447, 3042, 3502, 25, 3042, 3232, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^3(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (a\sin(e+fx)+a)^3(A+B\sin(e+fx))(c+d\sin(e+fx))^2dx$$

$$\Big\downarrow \; 3455$$

$$\frac{\int (\sin(e+fx)a+a)^2(c+d\sin(e+fx))^2(a(2Bc+6Ad+3Bd)-a(3Bc-6Ad-8Bd)\sin(e+fx))dx}{6d}-\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^3}{6df}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int (\sin(e+fx)a+a)^2(c+d\sin(e+fx))^2(a(2Bc+6Ad+3Bd)-a(3Bc-6Ad-8Bd)\sin(e+fx))dx}{6d}-\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^3}{6df}$$

$$\Big\downarrow \; 3455$$

$$\frac{\frac{\int 3(\sin(e+fx)a+a)(c+d\sin(e+fx))^2\left(a^2\left(2Ad(c+8d)-B\left(c^2-3dc-13d^2\right)\right)-a^2\left(2A(2c-11d)d-B\left(2c^2-8dc+21d^2\right)\right)\sin(e+fx)\right)dx}{5d}+\frac{(-6Ad+3}{}}{6d}-\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^3}{6df}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3\int (\sin(e+fx)a+a)(c+d\sin(e+fx))^2\left(a^2\left(2Ad(c+8d)-B\left(c^2-3dc-13d^2\right)\right)-a^2\left(2A(2c-11d)d-B\left(2c^2-8dc+21d^2\right)\right)\sin(e+fx)\right)dx}{5d}+\frac{(-6Ad+3}{}}{6d}-\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^3}{6df}$$

$\downarrow$ 3042

$$\frac{\frac{3\int(\sin(e+fx)a+a)(c+d\sin(e+fx))^2\left(a^2\left(2Ad(c+8d)-B\left(c^2-3dc-13d^2\right)\right)-a^2\left(2A(2c-11d)d-B\left(2c^2-8dc+21d^2\right)\right)\sin(e+fx)\right)dx}{5d}+\frac{(-6Ad+3}{}}{6d}$$
$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^3}{6df}$$

$\downarrow$ 3447

$$\frac{\frac{3\int(c+d\sin(e+fx))^2\left(-\left(\left(2A(2c-11d)d-B\left(2c^2-8dc+21d^2\right)\right)\sin^2(e+fx)a^3\right)+\left(2Ad(c+8d)-B\left(c^2-3dc-13d^2\right)\right)a^3+\left(a^3\left(2Ad(c+8d)-B\left(c^2-3dc\right.\right.\right.}{5d}}{6d}$$
$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^3}{6df}$$

$\downarrow$ 3042

$$\frac{\frac{3\int(c+d\sin(e+fx))^2\left(-\left(\left(2A(2c-11d)d-B\left(2c^2-8dc+21d^2\right)\right)\sin(e+fx)^2a^3\right)+\left(2Ad(c+8d)-B\left(c^2-3dc-13d^2\right)\right)a^3+\left(a^3\left(2Ad(c+8d)-B\left(c^2-3dc\right.\right.\right.}{5d}}{6d}$$
$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^3}{6df}$$

$\downarrow$ 3502

$$\frac{\frac{3\left(\frac{\int-(c+d\sin(e+fx))^2\left(a^3d\left(2A(2c-65d)d-B\left(2c^2-12dc+115d^2\right)\right)-a^3\left(2Ad\left(2c^2-15dc+76d^2\right)-B\left(2c^3-12dc^2+41d^2c-136d^3\right)\right)\sin(e+fx)\right)dx}{4d}+\frac{a^3\left(2Ad(2c-\right.}{}\right.}{5d}}{6d}$$
$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^3}{6df}$$

$\downarrow$ 25

$$\frac{\frac{3\left(\frac{a^3\left(2Ad(2c-11d)-B\left(2c^2-8cd+21d^2\right)\right)\cos(e+fx)(c+d\sin(e+fx))^3}{4df}-\frac{\int(c+d\sin(e+fx))^2\left(a^3d\left(2A(2c-65d)d-B\left(2c^2-12dc+115d^2\right)\right)-a^3\left(2Ad\left(2c^2-15dc+7\right.\right.\right.}{4d}\right.}{5d}}{6d}$$
$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^3}{6df}$$

$\downarrow$ 3042

$$\frac{3\left(\frac{a^3\left(2Ad(2c-11d)-B\left(2c^2-8cd+21d^2\right)\right)\cos(e+fx)(c+d\sin(e+fx))^3}{4df}-\frac{\int(c+d\sin(e+fx))^2\left(a^3d\left(2A(2c-65d)d-B\left(2c^2-12dc+115d^2\right)\right)-a^3\left(2Ad\left(2c^2-15dc+7\right.\right.\right.}{4d}}{5d}}{6d}$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^3}{6df}$$

$\downarrow$ 3232

$$3\left(\frac{a^3\left(2Ad(2c-11d)-B\left(2c^2-8cd+21d^2\right)\right)\cos(e+fx)(c+d\sin(e+fx))^3}{4df}-\frac{\frac{1}{3}\int(c+d\sin(e+fx))\left(a^3d\left(2Ad\left(2c^2-165dc-152d^2\right)-B\left(2c^3-12dc^2+263d^2c+272d^3\right)\right)\right.}{}\right.$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^3}{6df}$$

$\downarrow$ 3042

$$3\left(\frac{a^3\left(2Ad(2c-11d)-B\left(2c^2-8cd+21d^2\right)\right)\cos(e+fx)(c+d\sin(e+fx))^3}{4df}-\frac{\frac{1}{3}\int(c+d\sin(e+fx))\left(a^3d\left(2Ad\left(2c^2-165dc-152d^2\right)-B\left(2c^3-12dc^2+263d^2c+272d^3\right)\right)\right.}{}\right.$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^3}{6df}$$

$\downarrow$ 3213

$$3\left(\frac{a^3\left(2Ad(2c-11d)-B\left(2c^2-8cd+21d^2\right)\right)\cos(e+fx)(c+d\sin(e+fx))^3}{4df}-\frac{\frac{a^3\left(2Ad\left(2c^2-15cd+76d^2\right)-B\left(2c^3-12c^2d+41cd^2-136d^3\right)\right)\cos(e+fx)(c+d\sin(e+fx)}{3f}}{}\right.$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^3}{6df}$$

input
```
Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
```

output
```
-1/6*(a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^3)/(d*f
) + (((3*B*c - 6*A*d - 8*B*d)*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x])*(c + d
*Sin[e + f*x])^3)/(5*d*f) + (3*((a^3*(2*A*(2*c - 11*d)*d - B*(2*c^2 - 8*c*
d + 21*d^2))*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(4*d*f) - ((a^3*(2*A*d*(
2*c^2 - 15*c*d + 76*d^2) - B*(2*c^3 - 12*c^2*d + 41*c*d^2 - 136*d^3))*Cos[
e + f*x]*(c + d*Sin[e + f*x])^2)/(3*f) + ((-15*a^3*d^3*(B*(30*c^2 + 52*c*d
 + 23*d^2) + A*(40*c^2 + 60*c*d + 26*d^2))*x)/2 + (2*a^3*(2*A*d*(2*c^4 - 1
5*c^3*d + 72*c^2*d^2 + 180*c*d^3 + 76*d^4) - B*(2*c^5 - 12*c^4*d + 37*c^3*
d^2 - 112*c^2*d^3 - 304*c*d^4 - 136*d^5))*Cos[e + f*x])/f + (a^3*d*(2*A*d*
(4*c^3 - 30*c^2*d + 146*c*d^2 + 195*d^3) - B*(4*c^4 - 24*c^3*d + 76*c^2*d^
2 - 236*c*d^3 - 345*d^4))*Cos[e + f*x]*Sin[e + f*x])/(2*f))/3)/(4*d)))/(5*
d))/(6*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 725, normalized size of antiderivative = 1.57

$$-\frac{a^3 A c^2 \left(2+\sin(fx+e)^2\right) \cos(fx+e)}{3}+2a^3 Acd\left(-\frac{\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)-\frac{a^3 A d^2\left(\frac{8}{3}+\sin(fx+e)^4+\frac{4}{\ldots}\right.}{5}$$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x)
```

output
```
1/f*(-1/3*a^3*A*c^2*(2+sin(f*x+e)^2)*cos(f*x+e)+2*a^3*A*c*d*(-1/4*(sin(f*x
+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/5*a^3*A*d^2*(8/3+sin(f*x
+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+a^3*B*c^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f
*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2/5*a^3*B*c*d*(8/3+sin(f*x+e)^4+4/3*sin(f
*x+e)^2)*cos(f*x+e)+a^3*B*d^2*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*si
n(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)+3*a^3*A*c^2*(-1/2*sin(f*x+e)*cos(f*x
+e)+1/2*f*x+1/2*e)-2*a^3*A*c*d*(2+sin(f*x+e)^2)*cos(f*x+e)+3*a^3*A*d^2*(-1
/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-a^3*B*c^2*(2+si
n(f*x+e)^2)*cos(f*x+e)+6*a^3*B*c*d*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos
(f*x+e)+3/8*f*x+3/8*e)-3/5*a^3*B*d^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*c
os(f*x+e)-3*a^3*A*c^2*cos(f*x+e)+6*a^3*A*c*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1
/2*f*x+1/2*e)-a^3*A*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)+3*a^3*B*c^2*(-1/2*sin(
f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-2*a^3*B*c*d*(2+sin(f*x+e)^2)*cos(f*x+e)+3
*a^3*B*d^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+a
^3*A*c^2*(f*x+e)-2*a^3*A*c*d*cos(f*x+e)+a^3*A*d^2*(-1/2*sin(f*x+e)*cos(f*x
+e)+1/2*f*x+1/2*e)-a^3*B*c^2*cos(f*x+e)+2*a^3*B*c*d*(-1/2*sin(f*x+e)*cos(f
*x+e)+1/2*f*x+1/2*e)-1/3*a^3*B*d^2*(2+sin(f*x+e)^2)*cos(f*x+e))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.65

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx =$$
$$-\frac{48\,(2\,Ba^3cd+(A+3\,B)a^3d^2)\cos{(fx+e)}^5-80\,((A+3\,B)a^3c^2+2\,(3\,A+5\,B)a^3cd+(5\,A+7\,B)a^3}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
-1/240*(48*(2*B*a^3*c*d + (A + 3*B)*a^3*d^2)*cos(f*x + e)^5 - 80*((A + 3*B
)*a^3*c^2 + 2*(3*A + 5*B)*a^3*c*d + (5*A + 7*B)*a^3*d^2)*cos(f*x + e)^3 -
15*(10*(4*A + 3*B)*a^3*c^2 + 4*(15*A + 13*B)*a^3*c*d + (26*A + 23*B)*a^3*d
^2)*f*x + 960*((A + B)*a^3*c^2 + 2*(A + B)*a^3*c*d + (A + B)*a^3*d^2)*cos(
f*x + e) + 5*(8*B*a^3*d^2*cos(f*x + e)^5 - 2*(6*B*a^3*c^2 + 12*(A + 3*B)*a
^3*c*d + (18*A + 31*B)*a^3*d^2)*cos(f*x + e)^3 + 3*(2*(12*A + 17*B)*a^3*c^
2 + 4*(17*A + 19*B)*a^3*c*d + (38*A + 41*B)*a^3*d^2)*cos(f*x + e))*sin(f*x
 + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1804 vs. $2(449) = 898$.

Time = 0.53 (sec) , antiderivative size = 1804, normalized size of antiderivative = 3.90

$$\int (a + a\sin(e + fx))^3 (A + B\sin(e + fx))(c + d\sin(e + fx))^2\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
```

output
```
Piecewise((3*A*a**3*c**2*x*sin(e + f*x)**2/2 + 3*A*a**3*c**2*x*cos(e + f*x
)**2/2 + A*a**3*c**2*x - A*a**3*c**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*
a**3*c**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a**3*c**2*cos(e + f*x)**3/
(3*f) - 3*A*a**3*c**2*cos(e + f*x)/f + 3*A*a**3*c*d*x*sin(e + f*x)**4/4 +
3*A*a**3*c*d*x*sin(e + f*x)**2*cos(e + f*x)**2/2 + 3*A*a**3*c*d*x*sin(e +
f*x)**2 + 3*A*a**3*c*d*x*cos(e + f*x)**4/4 + 3*A*a**3*c*d*x*cos(e + f*x)**
2 - 5*A*a**3*c*d*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 6*A*a**3*c*d*sin(e +
 f*x)**2*cos(e + f*x)/f - 3*A*a**3*c*d*sin(e + f*x)*cos(e + f*x)**3/(4*f)
- 3*A*a**3*c*d*sin(e + f*x)*cos(e + f*x)/f - 4*A*a**3*c*d*cos(e + f*x)**3/
f - 2*A*a**3*c*d*cos(e + f*x)/f + 9*A*a**3*d**2*x*sin(e + f*x)**4/8 + 9*A*
a**3*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + A*a**3*d**2*x*sin(e + f*x)
**2/2 + 9*A*a**3*d**2*x*cos(e + f*x)**4/8 + A*a**3*d**2*x*cos(e + f*x)**2/
2 - A*a**3*d**2*sin(e + f*x)**4*cos(e + f*x)/f - 15*A*a**3*d**2*sin(e + f*
x)**3*cos(e + f*x)/(8*f) - 4*A*a**3*d**2*sin(e + f*x)**2*cos(e + f*x)**3/(
3*f) - 3*A*a**3*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 9*A*a**3*d**2*sin(e
+ f*x)*cos(e + f*x)**3/(8*f) - A*a**3*d**2*sin(e + f*x)*cos(e + f*x)/(2*f)
 - 8*A*a**3*d**2*cos(e + f*x)**5/(15*f) - 2*A*a**3*d**2*cos(e + f*x)**3/f
+ 3*B*a**3*c**2*x*sin(e + f*x)**4/8 + 3*B*a**3*c**2*x*sin(e + f*x)**2*cos(
e + f*x)**2/4 + 3*B*a**3*c**2*x*sin(e + f*x)**2/2 + 3*B*a**3*c**2*x*cos(e
+ f*x)**4/8 + 3*B*a**3*c**2*x*cos(e + f*x)**2/2 - 5*B*a**3*c**2*sin(e +...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 704, normalized size of antiderivative = 1.52

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algori
thm="maxima")
```

output
```
1/960*(320*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c^2 + 720*(2*f*x + 2*e
- sin(2*f*x + 2*e))*A*a^3*c^2 + 960*(f*x + e)*A*a^3*c^2 + 960*(cos(f*x + e
)^3 - 3*cos(f*x + e))*B*a^3*c^2 + 30*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8
*sin(2*f*x + 2*e))*B*a^3*c^2 + 720*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3*
c^2 + 1920*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c*d + 60*(12*f*x + 12*e
 + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^3*c*d + 1440*(2*f*x + 2*e -
sin(2*f*x + 2*e))*A*a^3*c*d - 128*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 +
15*cos(f*x + e))*B*a^3*c*d + 1920*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^3*
c*d + 180*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*c*
d + 480*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3*c*d - 64*(3*cos(f*x + e)^5
- 10*cos(f*x + e)^3 + 15*cos(f*x + e))*A*a^3*d^2 + 960*(cos(f*x + e)^3 - 3
*cos(f*x + e))*A*a^3*d^2 + 90*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*
f*x + 2*e))*A*a^3*d^2 + 240*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^3*d^2 - 1
92*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*d^2 + 32
0*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^3*d^2 + 5*(4*sin(2*f*x + 2*e)^3 +
60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*B*a^3*d^2 + 90*(
12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*d^2 - 2880*A*
a^3*c^2*cos(f*x + e) - 960*B*a^3*c^2*cos(f*x + e) - 1920*A*a^3*c*d*cos(f*x
 + e))/f
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.81

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = -\frac{Ba^3d^2\sin(6fx+6e)}{192f}$$
$$+\frac{1}{16}\left(40Aa^3c^2+30Ba^3c^2+60Aa^3cd+52Ba^3cd+26Aa^3d^2+23Ba^3d^2\right)x$$
$$-\frac{(2Ba^3cd+Aa^3d^2+3Ba^3d^2)\cos(5fx+5e)}{80f}$$
$$+\frac{(4Aa^3c^2+12Ba^3c^2+24Aa^3cd+34Ba^3cd+17Aa^3d^2+19Ba^3d^2)\cos(3fx+3e)}{48f}$$
$$-\frac{(30Aa^3c^2+26Ba^3c^2+52Aa^3cd+46Ba^3cd+23Aa^3d^2+21Ba^3d^2)\cos(fx+e)}{8f}$$
$$+\frac{(2Ba^3c^2+4Aa^3cd+12Ba^3cd+6Aa^3d^2+9Ba^3d^2)\sin(4fx+4e)}{64f}$$
$$-\frac{(48Aa^3c^2+64Ba^3c^2+128Aa^3cd+128Ba^3cd+64Aa^3d^2+63Ba^3d^2)\sin(2fx+2e)}{64f}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
-1/192*B*a^3*d^2*sin(6*f*x + 6*e)/f + 1/16*(40*A*a^3*c^2 + 30*B*a^3*c^2 +
60*A*a^3*c*d + 52*B*a^3*c*d + 26*A*a^3*d^2 + 23*B*a^3*d^2)*x - 1/80*(2*B*a
^3*c*d + A*a^3*d^2 + 3*B*a^3*d^2)*cos(5*f*x + 5*e)/f + 1/48*(4*A*a^3*c^2 +
 12*B*a^3*c^2 + 24*A*a^3*c*d + 34*B*a^3*c*d + 17*A*a^3*d^2 + 19*B*a^3*d^2)
*cos(3*f*x + 3*e)/f - 1/8*(30*A*a^3*c^2 + 26*B*a^3*c^2 + 52*A*a^3*c*d + 46
*B*a^3*c*d + 23*A*a^3*d^2 + 21*B*a^3*d^2)*cos(f*x + e)/f + 1/64*(2*B*a^3*c
^2 + 4*A*a^3*c*d + 12*B*a^3*c*d + 6*A*a^3*d^2 + 9*B*a^3*d^2)*sin(4*f*x + 4
*e)/f - 1/64*(48*A*a^3*c^2 + 64*B*a^3*c^2 + 128*A*a^3*c*d + 128*B*a^3*c*d
+ 64*A*a^3*d^2 + 63*B*a^3*d^2)*sin(2*f*x + 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 37.39 (sec) , antiderivative size = 976, normalized size of antiderivative = 2.11

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = \text{Too large to display}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^2,x)
```

output
```
(a^3*atan((a^3*tan(e/2 + (f*x)/2)*(40*A*c^2 + 26*A*d^2 + 30*B*c^2 + 23*B*d
^2 + 60*A*c*d + 52*B*c*d))/(8*(5*A*a^3*c^2 + (13*A*a^3*d^2)/4 + (15*B*a^3*
c^2)/4 + (23*B*a^3*d^2)/8 + (15*A*a^3*c*d)/2 + (13*B*a^3*c*d)/2)))*(40*A*c
^2 + 26*A*d^2 + 30*B*c^2 + 23*B*d^2 + 60*A*c*d + 52*B*c*d))/(8*f) - (tan(e
/2 + (f*x)/2)^10*(6*A*a^3*c^2 + 2*B*a^3*c^2 + 4*A*a^3*c*d) + tan(e/2 + (f*
x)/2)*(3*A*a^3*c^2 + (13*A*a^3*d^2)/4 + (15*B*a^3*c^2)/4 + (23*B*a^3*d^2)/
8 + (15*A*a^3*c*d)/2 + (13*B*a^3*c*d)/2) - tan(e/2 + (f*x)/2)^11*(3*A*a^3*
c^2 + (13*A*a^3*d^2)/4 + (15*B*a^3*c^2)/4 + (23*B*a^3*d^2)/8 + (15*A*a^3*c
*d)/2 + (13*B*a^3*c*d)/2) + tan(e/2 + (f*x)/2)^8*(34*A*a^3*c^2 + 12*A*a^3*
d^2 + 22*B*a^3*c^2 + 4*B*a^3*d^2 + 44*A*a^3*c*d + 24*B*a^3*c*d) + tan(e/2
+ (f*x)/2)^5*(6*A*a^3*c^2 + (25*A*a^3*d^2)/2 + (19*B*a^3*c^2)/2 + (75*B*a^
3*d^2)/4 + 19*A*a^3*c*d + 25*B*a^3*c*d) - tan(e/2 + (f*x)/2)^7*(6*A*a^3*c^
2 + (25*A*a^3*d^2)/2 + (19*B*a^3*c^2)/2 + (75*B*a^3*d^2)/4 + 19*A*a^3*c*d
+ 25*B*a^3*c*d) + tan(e/2 + (f*x)/2)^4*(76*A*a^3*c^2 + 64*A*a^3*d^2 + 68*B
*a^3*c^2 + 64*B*a^3*d^2 + 136*A*a^3*c*d + 128*B*a^3*c*d) + tan(e/2 + (f*x)
/2)^3*(9*A*a^3*c^2 + (63*A*a^3*d^2)/4 + (53*B*a^3*c^2)/4 + (391*B*a^3*d^2)
/24 + (53*A*a^3*c*d)/2 + (63*B*a^3*c*d)/2) - tan(e/2 + (f*x)/2)^9*(9*A*a^3
*c^2 + (63*A*a^3*d^2)/4 + (53*B*a^3*c^2)/4 + (391*B*a^3*d^2)/24 + (53*A*a^
3*c*d)/2 + (63*B*a^3*c*d)/2) + tan(e/2 + (f*x)/2)^2*(38*A*a^3*c^2 + (152*A
*a^3*d^2)/5 + 34*B*a^3*c^2 + (136*B*a^3*d^2)/5 + 68*A*a^3*c*d + (304*B*...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 560, normalized size of antiderivative = 1.21

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$
$$= \frac{a^3\left(-48\cos(fx+e)\sin(fx+e)^4 a d^2 - 60\cos(fx+e)\sin(fx+e)^3 b c^2 - 80\cos(fx+e)\sin(fx+e)^2\right.}{}$$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x)
```

output
```
(a**3*( - 40*cos(e + f*x)*sin(e + f*x)**5*b*d**2 - 48*cos(e + f*x)*sin(e +
 f*x)**4*a*d**2 - 96*cos(e + f*x)*sin(e + f*x)**4*b*c*d - 144*cos(e + f*x)
*sin(e + f*x)**4*b*d**2 - 120*cos(e + f*x)*sin(e + f*x)**3*a*c*d - 180*cos
(e + f*x)*sin(e + f*x)**3*a*d**2 - 60*cos(e + f*x)*sin(e + f*x)**3*b*c**2
- 360*cos(e + f*x)*sin(e + f*x)**3*b*c*d - 230*cos(e + f*x)*sin(e + f*x)**
3*b*d**2 - 80*cos(e + f*x)*sin(e + f*x)**2*a*c**2 - 480*cos(e + f*x)*sin(e
 + f*x)**2*a*c*d - 304*cos(e + f*x)*sin(e + f*x)**2*a*d**2 - 240*cos(e + f
*x)*sin(e + f*x)**2*b*c**2 - 608*cos(e + f*x)*sin(e + f*x)**2*b*c*d - 272*
cos(e + f*x)*sin(e + f*x)**2*b*d**2 - 360*cos(e + f*x)*sin(e + f*x)*a*c**2
 - 900*cos(e + f*x)*sin(e + f*x)*a*c*d - 390*cos(e + f*x)*sin(e + f*x)*a*d
**2 - 450*cos(e + f*x)*sin(e + f*x)*b*c**2 - 780*cos(e + f*x)*sin(e + f*x)
*b*c*d - 345*cos(e + f*x)*sin(e + f*x)*b*d**2 - 880*cos(e + f*x)*a*c**2 -
1440*cos(e + f*x)*a*c*d - 608*cos(e + f*x)*a*d**2 - 720*cos(e + f*x)*b*c**
2 - 1216*cos(e + f*x)*b*c*d - 544*cos(e + f*x)*b*d**2 + 600*a*c**2*f*x + 8
80*a*c**2 + 900*a*c*d*f*x + 1440*a*c*d + 390*a*d**2*f*x + 608*a*d**2 + 450
*b*c**2*f*x + 720*b*c**2 + 780*b*c*d*f*x + 1216*b*c*d + 345*b*d**2*f*x + 5
44*b*d**2))/(240*f)
```

3.260 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 201

$$\begin{aligned}
&\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx \\
&= \frac{1}{8}a^3(20Ac+15Bc+15Ad+13Bd)x - \frac{a^3(20Ac+15Bc+15Ad+13Bd)\cos(e+fx)}{5f} \\
&\quad + \frac{a^3(20Ac+15Bc+15Ad+13Bd)\cos^3(e+fx)}{60f} \\
&\quad - \frac{3a^3(20Ac+15Bc+15Ad+13Bd)\cos(e+fx)\sin(e+fx)}{40f} \\
&\quad - \frac{(5Bc+5Ad-Bd)\cos(e+fx)(a+a\sin(e+fx))^3}{20f} \\
&\quad - \frac{Bd\cos(e+fx)(a+a\sin(e+fx))^4}{5af}
\end{aligned}$$

output
```
1/8*a^3*(20*A*c+15*A*d+15*B*c+13*B*d)*x-1/5*a^3*(20*A*c+15*A*d+15*B*c+13*B
*d)*cos(f*x+e)/f+1/60*a^3*(20*A*c+15*A*d+15*B*c+13*B*d)*cos(f*x+e)^3/f-3/4
0*a^3*(20*A*c+15*A*d+15*B*c+13*B*d)*cos(f*x+e)*sin(f*x+e)/f-1/20*(5*A*d+5*
B*c-B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^3/f-1/5*B*d*cos(f*x+e)*(a+a*sin(f*x+e
))^4/a/f
```

Mathematica [A] (warning: unable to verify)

Time = 1.18 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.78

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$

$$= \frac{\cos(e+fx)\left(-\frac{1}{4}a^4(5Bc+5Ad-Bd)(1+\sin(e+fx))^3-Bd(a+a\sin(e+fx))^4-\frac{a^4(20Ac+15Bc+15Ad+\cdots}{\cdots}\right.}{5af}$$

input
```
Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])
,x]
```

output
```
(Cos[e + f*x]*(-1/4*(a^4*(5*B*c + 5*A*d - B*d)*(1 + Sin[e + f*x])^3) - B*d
*(a + a*Sin[e + f*x])^4 - (a^4*(20*A*c + 15*B*c + 15*A*d + 13*B*d)*(30*Arc
Sin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]] + Sqrt[Cos[e + f*x]^2]*(22 + 9*Sin[e +
 f*x] + 2*Sin[e + f*x]^2)))/(24*Sqrt[Cos[e + f*x]^2])))/(5*a*f)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.81, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 3447, 3042, 3502, 3042, 3230, 3042, 3124, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^3(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^3(A+B\sin(e+fx))(c+d\sin(e+fx))dx$$

$$\Big\downarrow \text{3447}$$

$$\int (a\sin(e+fx)+a)^3\left((Ad+Bc)\sin(e+fx)+Ac+Bd\sin^2(e+fx)\right)dx$$

$\big\downarrow$ 3042

$$\int (a\sin(e+fx)+a)^3\left((Ad+Bc)\sin(e+fx)+Ac+Bd\sin(e+fx)^2\right)dx$$

$\big\downarrow$ 3502

$$\frac{\int (\sin(e+fx)a+a)^3(a(5Ac+4Bd)+a(5Bc+5Ad-Bd)\sin(e+fx))dx}{5a}-\frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^4}{5af}$$

$\big\downarrow$ 3042

$$\frac{\int (\sin(e+fx)a+a)^3(a(5Ac+4Bd)+a(5Bc+5Ad-Bd)\sin(e+fx))dx}{5a}-\frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^4}{5af}$$

$\big\downarrow$ 3230

$$\frac{\frac{1}{4}a(20Ac+15Ad+15Bc+13Bd)\int (\sin(e+fx)a+a)^3dx-\frac{a(5Ad+5Bc-Bd)\cos(e+fx)(a\sin(e+fx)+a)^3}{4f}}{5a}-\frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^4}{5af}$$

$\big\downarrow$ 3042

$$\frac{\frac{1}{4}a(20Ac+15Ad+15Bc+13Bd)\int (\sin(e+fx)a+a)^3dx-\frac{a(5Ad+5Bc-Bd)\cos(e+fx)(a\sin(e+fx)+a)^3}{4f}}{5a}-\frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^4}{5af}$$

$\big\downarrow$ 3124

$$\frac{\frac{1}{4}a(20Ac+15Ad+15Bc+13Bd)\int \left(\sin^3(e+fx)a^3+3\sin^2(e+fx)a^3+3\sin(e+fx)a^3+a^3\right)dx-\frac{a(5Ad+5Bc}{}}{5a}$$
$$\frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^4}{5af}$$

$\big\downarrow$ 2009

$$\frac{\frac{1}{4}a(20Ac+15Ad+15Bc+13Bd)\left(\frac{a^3\cos^3(e+fx)}{3f}-\frac{4a^3\cos(e+fx)}{f}-\frac{3a^3\sin(e+fx)\cos(e+fx)}{2f}+\frac{5a^3x}{2}\right)-\frac{a(5Ad+5Bc-Bd)}{}}{5a}$$
$$\frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^4}{5af}$$

input
```
Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

output
```
-1/5*(B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^4)/(a*f) + (-1/4*(a*(5*B*c + 5
*A*d - B*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^3)/f + (a*(20*A*c + 15*B*c +
 15*A*d + 13*B*d)*((5*a^3*x)/2 - (4*a^3*Cos[e + f*x])/f + (a^3*Cos[e + f*x
]^3)/(3*f) - (3*a^3*Cos[e + f*x]*Sin[e + f*x])/(2*f)))/4)/(5*a)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3124
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(189) = 378$.

Time = 0.67 (sec) , antiderivative size = 414, normalized size of antiderivative = 2.06

$$\frac{-\frac{a^3Ac\left(2+\sin(fx+e)^2\right)\cos(fx+e)}{3}+a^3Ad\left(-\frac{\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+a^3Bc\left(-\frac{\left(\sin(fx+e)^3+\frac{3\sin(\right.}{4}\right.}{}$$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

output
```
1/f*(-1/3*a^3*A*c*(2+sin(f*x+e)^2)*cos(f*x+e)+a^3*A*d*(-1/4*(sin(f*x+e)^3+
3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+a^3*B*c*(-1/4*(sin(f*x+e)^3+3/2*
sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/5*a^3*B*d*(8/3+sin(f*x+e)^4+4/3*si
n(f*x+e)^2)*cos(f*x+e)+3*a^3*A*c*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e
)-a^3*A*d*(2+sin(f*x+e)^2)*cos(f*x+e)-a^3*B*c*(2+sin(f*x+e)^2)*cos(f*x+e)+
3*a^3*B*d*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-3*
a^3*A*c*cos(f*x+e)+3*a^3*A*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+3*
a^3*B*c*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-a^3*B*d*(2+sin(f*x+e)^2
)*cos(f*x+e)+a^3*A*c*(f*x+e)-a^3*A*d*cos(f*x+e)-a^3*B*c*cos(f*x+e)+a^3*B*d
*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.89

$$\int (a + a\sin(e + fx))^3 (A + B\sin(e + fx))(c + d\sin(e + fx))\, dx = -\frac{24\, Ba^3 d\cos(fx + e)^5 - 40\,((A + 3\,B)a^3 c + (3\,A + 5\,B)a^3 d)\cos(fx + e)^3 - 15\,(5\,(4\,A + 3\,B)a^3 c + (\cdots}{\cdots}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
-1/120*(24*B*a^3*d*cos(f*x + e)^5 - 40*((A + 3*B)*a^3*c + (3*A + 5*B)*a^3*
d)*cos(f*x + e)^3 - 15*(5*(4*A + 3*B)*a^3*c + (15*A + 13*B)*a^3*d)*f*x + 4
80*((A + B)*a^3*c + (A + B)*a^3*d)*cos(f*x + e) - 15*(2*(B*a^3*c + (A + 3*
B)*a^3*d)*cos(f*x + e)^3 - ((12*A + 17*B)*a^3*c + (17*A + 19*B)*a^3*d)*cos
(f*x + e))*sin(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 960 vs. 2(201) = 402.

Time = 0.35 (sec) , antiderivative size = 960, normalized size of antiderivative = 4.78

$$\int (a + a\sin(e + fx))^3 (A + B\sin(e + fx))(c + d\sin(e + fx))\, dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

output
```
Piecewise((3*A*a**3*c*x*sin(e + f*x)**2/2 + 3*A*a**3*c*x*cos(e + f*x)**2/2
 + A*a**3*c*x - A*a**3*c*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**3*c*sin(e
 + f*x)*cos(e + f*x)/(2*f) - 2*A*a**3*c*cos(e + f*x)**3/(3*f) - 3*A*a**3*c
*cos(e + f*x)/f + 3*A*a**3*d*x*sin(e + f*x)**4/8 + 3*A*a**3*d*x*sin(e + f*
x)**2*cos(e + f*x)**2/4 + 3*A*a**3*d*x*sin(e + f*x)**2/2 + 3*A*a**3*d*x*co
s(e + f*x)**4/8 + 3*A*a**3*d*x*cos(e + f*x)**2/2 - 5*A*a**3*d*sin(e + f*x)
**3*cos(e + f*x)/(8*f) - 3*A*a**3*d*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a
**3*d*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 3*A*a**3*d*sin(e + f*x)*cos(e +
 f*x)/(2*f) - 2*A*a**3*d*cos(e + f*x)**3/f - A*a**3*d*cos(e + f*x)/f + 3*B
*a**3*c*x*sin(e + f*x)**4/8 + 3*B*a**3*c*x*sin(e + f*x)**2*cos(e + f*x)**2
/4 + 3*B*a**3*c*x*sin(e + f*x)**2/2 + 3*B*a**3*c*x*cos(e + f*x)**4/8 + 3*B
*a**3*c*x*cos(e + f*x)**2/2 - 5*B*a**3*c*sin(e + f*x)**3*cos(e + f*x)/(8*f
) - 3*B*a**3*c*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a**3*c*sin(e + f*x)*co
s(e + f*x)**3/(8*f) - 3*B*a**3*c*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*B*a**
3*c*cos(e + f*x)**3/f - B*a**3*c*cos(e + f*x)/f + 9*B*a**3*d*x*sin(e + f*x
)**4/8 + 9*B*a**3*d*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + B*a**3*d*x*sin(e
 + f*x)**2/2 + 9*B*a**3*d*x*cos(e + f*x)**4/8 + B*a**3*d*x*cos(e + f*x)**2
/2 - B*a**3*d*sin(e + f*x)**4*cos(e + f*x)/f - 15*B*a**3*d*sin(e + f*x)**3
*cos(e + f*x)/(8*f) - 4*B*a**3*d*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 3
*B*a**3*d*sin(e + f*x)**2*cos(e + f*x)/f - 9*B*a**3*d*sin(e + f*x)*cos(...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(189) = 378$.

Time = 0.04 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.98

$$\int (a + a\sin(e + fx))^3(A + B\sin(e + fx))(c + d\sin(e + fx))\,dx$$
$$= \frac{160\left(\cos(fx + e)^3 - 3\cos(fx + e)\right)Aa^3c + 360\left(2fx + 2e - \sin(2fx + 2e)\right)Aa^3c + 480(fx + e)Aa^3c}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorith
m="maxima")
```

output
```
1/480*(160*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c + 360*(2*f*x + 2*e -
sin(2*f*x + 2*e))*A*a^3*c + 480*(f*x + e)*A*a^3*c + 480*(cos(f*x + e)^3 -
3*cos(f*x + e))*B*a^3*c + 15*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f
*x + 2*e))*B*a^3*c + 360*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3*c + 480*(c
os(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*d + 15*(12*f*x + 12*e + sin(4*f*x +
4*e) - 8*sin(2*f*x + 2*e))*A*a^3*d + 360*(2*f*x + 2*e - sin(2*f*x + 2*e))*
A*a^3*d - 32*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^
3*d + 480*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^3*d + 45*(12*f*x + 12*e +
sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*d + 120*(2*f*x + 2*e - sin(2*
f*x + 2*e))*B*a^3*d - 1440*A*a^3*c*cos(f*x + e) - 480*B*a^3*c*cos(f*x + e)
 - 480*A*a^3*d*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.05

$$\begin{aligned}\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx &= -\frac{Ba^3d\cos(5\,fx+5\,e)}{80\,f}+\frac{1}{8}\left(20\,Aa^3c+15\,Ba^3c+15\,Aa^3d+13\,Ba^3d\right)x\\&\quad+\frac{\left(4\,Aa^3c+12\,Ba^3c+12\,Aa^3d+17\,Ba^3d\right)\cos(3\,fx+3\,e)}{48\,f}\\&\quad-\frac{\left(30\,Aa^3c+26\,Ba^3c+26\,Aa^3d+23\,Ba^3d\right)\cos(fx+e)}{8\,f}\\&\quad+\frac{\left(Ba^3c+Aa^3d+3\,Ba^3d\right)\sin(4\,fx+4\,e)}{32\,f}\\&\quad-\frac{\left(3\,Aa^3c+4\,Ba^3c+4\,Aa^3d+4\,Ba^3d\right)\sin(2\,fx+2\,e)}{4\,f}\end{aligned}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorith
m="giac")
```

output
```
-1/80*B*a^3*d*cos(5*f*x + 5*e)/f + 1/8*(20*A*a^3*c + 15*B*a^3*c + 15*A*a^3
*d + 13*B*a^3*d)*x + 1/48*(4*A*a^3*c + 12*B*a^3*c + 12*A*a^3*d + 17*B*a^3*
d)*cos(3*f*x + 3*e)/f - 1/8*(30*A*a^3*c + 26*B*a^3*c + 26*A*a^3*d + 23*B*a
^3*d)*cos(f*x + e)/f + 1/32*(B*a^3*c + A*a^3*d + 3*B*a^3*d)*sin(4*f*x + 4*
e)/f - 1/4*(3*A*a^3*c + 4*B*a^3*c + 4*A*a^3*d + 4*B*a^3*d)*sin(2*f*x + 2*e
)/f
```

Mupad [B] (verification not implemented)

Time = 36.06 (sec) , antiderivative size = 550, normalized size of antiderivative = 2.74

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = \text{Too large to display}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c + d*sin(e + f*x)),x)
```

output
```
(a^3*atan((a^3*tan(e/2 + (f*x)/2)*(20*A*c + 15*A*d + 15*B*c + 13*B*d))/(4*
(5*A*a^3*c + (15*A*a^3*d)/4 + (15*B*a^3*c)/4 + (13*B*a^3*d)/4)))*(20*A*c +
 15*A*d + 15*B*c + 13*B*d))/(4*f) - (a^3*(atan(tan(e/2 + (f*x)/2)) - (f*x)
/2)*(20*A*c + 15*A*d + 15*B*c + 13*B*d))/(4*f) - (tan(e/2 + (f*x)/2)^3*(6*
A*a^3*c + (19*A*a^3*d)/2 + (19*B*a^3*c)/2 + (25*B*a^3*d)/2) - tan(e/2 + (f
*x)/2)^9*(3*A*a^3*c + (15*A*a^3*d)/4 + (15*B*a^3*c)/4 + (13*B*a^3*d)/4) -
tan(e/2 + (f*x)/2)^7*(6*A*a^3*c + (19*A*a^3*d)/2 + (19*B*a^3*c)/2 + (25*B*
a^3*d)/2) + tan(e/2 + (f*x)/2)^6*(28*A*a^3*c + 20*A*a^3*d + 20*B*a^3*c + 1
2*B*a^3*d) + tan(e/2 + (f*x)/2)^2*((92*A*a^3*c)/3 + 28*A*a^3*d + 28*B*a^3*
c + (76*B*a^3*d)/3) + tan(e/2 + (f*x)/2)^4*((136*A*a^3*c)/3 + 40*A*a^3*d +
 40*B*a^3*c + (116*B*a^3*d)/3) + tan(e/2 + (f*x)/2)^8*(6*A*a^3*c + 2*A*a^3
*d + 2*B*a^3*c) + tan(e/2 + (f*x)/2)*(3*A*a^3*c + (15*A*a^3*d)/4 + (15*B*a
^3*c)/4 + (13*B*a^3*d)/4) + (22*A*a^3*c)/3 + 6*A*a^3*d + 6*B*a^3*c + (76*B
*a^3*d)/15)/(f*(5*tan(e/2 + (f*x)/2)^2 + 10*tan(e/2 + (f*x)/2)^4 + 10*tan(
e/2 + (f*x)/2)^6 + 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^10 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.48

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = \frac{a^3\left(-24\cos(fx+e)\sin(fx+e)^4bd-30\cos(fx+e)\sin(fx+e)^3ad-30\cos(fx+e)\sin(fx+e)^3bc\right.}{}$$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

output
```
(a**3*( - 24*cos(e + f*x)*sin(e + f*x)**4*b*d - 30*cos(e + f*x)*sin(e + f*
x)**3*a*d - 30*cos(e + f*x)*sin(e + f*x)**3*b*c - 90*cos(e + f*x)*sin(e +
f*x)**3*b*d - 40*cos(e + f*x)*sin(e + f*x)**2*a*c - 120*cos(e + f*x)*sin(e
 + f*x)**2*a*d - 120*cos(e + f*x)*sin(e + f*x)**2*b*c - 152*cos(e + f*x)*s
in(e + f*x)**2*b*d - 180*cos(e + f*x)*sin(e + f*x)*a*c - 225*cos(e + f*x)*
sin(e + f*x)*a*d - 225*cos(e + f*x)*sin(e + f*x)*b*c - 195*cos(e + f*x)*si
n(e + f*x)*b*d - 440*cos(e + f*x)*a*c - 360*cos(e + f*x)*a*d - 360*cos(e +
 f*x)*b*c - 304*cos(e + f*x)*b*d + 300*a*c*f*x + 440*a*c + 225*a*d*f*x + 3
60*a*d + 225*b*c*f*x + 360*b*c + 195*b*d*f*x + 304*b*d))/(120*f)
```

3.261 $\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 127

$$\begin{aligned}
&\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx \\
&= \frac{5}{8}a^3(4A+3B)x - \frac{5a^3(4A+3B)\cos(e+fx)}{6f} \\
&\quad - \frac{5a^3(4A+3B)\cos(e+fx)\sin(e+fx)}{24f} \\
&\quad - \frac{a(4A+3B)\cos(e+fx)(a+a\sin(e+fx))^2}{12f} - \frac{B\cos(e+fx)(a+a\sin(e+fx))^3}{4f}
\end{aligned}$$

output
```
5/8*a^3*(4*A+3*B)*x-5/6*a^3*(4*A+3*B)*cos(f*x+e)/f-5/24*a^3*(4*A+3*B)*cos(
f*x+e)*sin(f*x+e)/f-1/12*a*(4*A+3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^2/f-1/4*B
*cos(f*x+e)*(a+a*sin(f*x+e))^3/f
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.94

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx = -\frac{a^3\cos(e+fx)\left(30(4A+3B)\arcsin\left(\frac{\sqrt{1-\sin(e+fx)}}{\sqrt{2}}\right)+\sqrt{\cos^2(e+fx)}(88A+72B+9(4A+5B)\sin(e}{24f\sqrt{\cos^2(e+fx)}}$$

input
```
Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]),x]
```

output
```
-1/24*(a^3*Cos[e + f*x]*(30*(4*A + 3*B)*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt
[2]] + Sqrt[Cos[e + f*x]^2]*(88*A + 72*B + 9*(4*A + 5*B)*Sin[e + f*x] + 8*
(A + 3*B)*Sin[e + f*x]^2 + 6*B*Sin[e + f*x]^3)))/(f*Sqrt[Cos[e + f*x]^2])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.80, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 3230, 3042, 3124, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^3(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^3(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3230}$$

$$\frac{1}{4}(4A+3B)\int (\sin(e+fx)a+a)^3dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^3}{4f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}(4A+3B)\int (\sin(e+fx)a+a)^3dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^3}{4f}$$

$$\Big\downarrow \; 3124$$

$$\frac{1}{4}(4A+3B)\int \left(\sin^3(e+fx)a^3+3\sin^2(e+fx)a^3+3\sin(e+fx)a^3+a^3\right)dx-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^3}{4f}$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{4}(4A+3B)\left(\frac{a^3\cos^3(e+fx)}{3f}-\frac{4a^3\cos(e+fx)}{f}-\frac{3a^3\sin(e+fx)\cos(e+fx)}{2f}+\frac{5a^3x}{2}\right)-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^3}{4f}$$

input
```
Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]),x]
```

output
```
-1/4*(B*Cos[e + f*x]*(a + a*Sin[e + f*x])^3)/f + ((4*A + 3*B)*((5*a^3*x)/2
 - (4*a^3*Cos[e + f*x])/f + (a^3*Cos[e + f*x]^3)/(3*f) - (3*a^3*Cos[e + f*
x]*Sin[e + f*x])/(2*f)))/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3124
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.40

$$\frac{-\frac{a^3A\left(2+\sin(fx+e)^2\right)\cos(fx+e)}{3}+a^3B\left(-\frac{\left(\sin(fx+e)^3+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4}+\frac{3fx}{8}+\frac{3e}{8}\right)+3a^3A\left(-\frac{\sin(fx+e)\cos(fx+e)}{2}\right.}{}$$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x)
```

output
```
1/f*(-1/3*a^3*A*(2+sin(f*x+e)^2)*cos(f*x+e)+a^3*B*(-1/4*(sin(f*x+e)^3+3/2*
sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+3*a^3*A*(-1/2*sin(f*x+e)*cos(f*x+e)+
1/2*f*x+1/2*e)-a^3*B*(2+sin(f*x+e)^2)*cos(f*x+e)-3*a^3*A*cos(f*x+e)+3*a^3*
B*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+a^3*A*(f*x+e)-a^3*B*cos(f*x+e
))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.73

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx$$
$$=\frac{8\,(A+3\,B)a^3\cos{(fx+e)}^3+15\,(4\,A+3\,B)a^3fx-96\,(A+B)a^3\cos{(fx+e)}+3\,\left(2\,Ba^3\cos{(fx+e)}^3\right.}{24\,f}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x, algorithm="fricas")
```

output
```
1/24*(8*(A + 3*B)*a^3*cos(f*x + e)^3 + 15*(4*A + 3*B)*a^3*f*x - 96*(A + B)
*a^3*cos(f*x + e) + 3*(2*B*a^3*cos(f*x + e)^3 - (12*A + 17*B)*a^3*cos(f*x
+ e))*sin(f*x + e))/f
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(119) = 238$.

Time = 0.21 (sec) , antiderivative size = 371, normalized size of antiderivative = 2.92

$$\int (a + a\sin(e + fx))^3 (A + B\sin(e + fx))\,dx$$
$$= \begin{cases} \frac{3Aa^3x\sin^2(e+fx)}{2} + \frac{3Aa^3x\cos^2(e+fx)}{2} + Aa^3x - \frac{Aa^3\sin^2(e+fx)\cos(e+fx)}{f} - \frac{3Aa^3\sin(e+fx)\cos(e+fx)}{2f} - \frac{2Aa^3\cos^3(e+f}{3f} \\ x(A + B\sin(e))(a\sin(e) + a)^3 \end{cases}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e)),x)
```

output
```
Piecewise((3*A*a**3*x*sin(e + f*x)**2/2 + 3*A*a**3*x*cos(e + f*x)**2/2 + A
*a**3*x - A*a**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**3*sin(e + f*x)*co
s(e + f*x)/(2*f) - 2*A*a**3*cos(e + f*x)**3/(3*f) - 3*A*a**3*cos(e + f*x)/
f + 3*B*a**3*x*sin(e + f*x)**4/8 + 3*B*a**3*x*sin(e + f*x)**2*cos(e + f*x)
**2/4 + 3*B*a**3*x*sin(e + f*x)**2/2 + 3*B*a**3*x*cos(e + f*x)**4/8 + 3*B*
a**3*x*cos(e + f*x)**2/2 - 5*B*a**3*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3
*B*a**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a**3*sin(e + f*x)*cos(e + f*x
)**3/(8*f) - 3*B*a**3*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*B*a**3*cos(e + f
*x)**3/f - B*a**3*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) +
 a)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.35

$$\int (a + a\sin(e + fx))^3 (A + B\sin(e + fx))\,dx$$
$$= \frac{32\left(\cos(fx+e)^3 - 3\cos(fx+e)\right)Aa^3 + 72\left(2fx + 2e - \sin(2fx+2e)\right)Aa^3 + 96(fx+e)Aa^3 + 96\left(\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x, algorithm="maxima")
```

output
```
1/96*(32*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3 + 72*(2*f*x + 2*e - sin(2
*f*x + 2*e))*A*a^3 + 96*(f*x + e)*A*a^3 + 96*(cos(f*x + e)^3 - 3*cos(f*x +
 e))*B*a^3 + 3*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a
^3 + 72*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3 - 288*A*a^3*cos(f*x + e) -
96*B*a^3*cos(f*x + e))/f
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.88

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx = \frac{Ba^3\sin(4fx+4e)}{32f} + \frac{5}{8}(4Aa^3+3Ba^3)x + \frac{(Aa^3+3Ba^3)\cos(3fx+3e)}{12f} - \frac{(15Aa^3+13Ba^3)\cos(fx+e)}{4f} - \frac{(3Aa^3+4Ba^3)\sin(2fx+2e)}{4f}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x, algorithm="giac")
```

output
```
1/32*B*a^3*sin(4*f*x + 4*e)/f + 5/8*(4*A*a^3 + 3*B*a^3)*x + 1/12*(A*a^3 +
3*B*a^3)*cos(3*f*x + 3*e)/f - 1/4*(15*A*a^3 + 13*B*a^3)*cos(f*x + e)/f - 1
/4*(3*A*a^3 + 4*B*a^3)*sin(2*f*x + 2*e)/f
```

Mupad [B] (verification not implemented)

Time = 35.80 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.60

$$\int (a+a\sin(e+fx))^3(A+B\sin(e+fx))\,dx = \frac{5a^3\,\mathrm{atan}\left(\frac{5a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(4A+3B)}{4\left(5Aa^3+\frac{15Ba^3}{4}\right)}\right)(4A+3B)}{4f} - \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(3Aa^3+\frac{15Ba^3}{4}\right)+\frac{22Aa^3}{3}+6Ba^3+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6(6Aa^3+2Ba^3)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7\left(3\right.}{f\left(\tan\left(\frac{e}{2}+\right.\right.} - \frac{5a^3(4A+3B)\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)-\frac{fx}{2}\right)}{4f}$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3,x)
```

output
```
(5*a^3*atan((5*a^3*tan(e/2 + (f*x)/2)*(4*A + 3*B))/(4*(5*A*a^3 + (15*B*a^3
)/4)))*(4*A + 3*B))/(4*f) - (tan(e/2 + (f*x)/2)*(3*A*a^3 + (15*B*a^3)/4) +
 (22*A*a^3)/3 + 6*B*a^3 + tan(e/2 + (f*x)/2)^6*(6*A*a^3 + 2*B*a^3) - tan(e
/2 + (f*x)/2)^7*(3*A*a^3 + (15*B*a^3)/4) + tan(e/2 + (f*x)/2)^3*(3*A*a^3 +
 (23*B*a^3)/4) - tan(e/2 + (f*x)/2)^5*(3*A*a^3 + (23*B*a^3)/4) + tan(e/2 +
 (f*x)/2)^4*(22*A*a^3 + 18*B*a^3) + tan(e/2 + (f*x)/2)^2*((70*A*a^3)/3 + 2
2*B*a^3))/(f*(4*tan(e/2 + (f*x)/2)^2 + 6*tan(e/2 + (f*x)/2)^4 + 4*tan(e/2
+ (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1)) - (5*a^3*(4*A + 3*B)*(atan(tan(e
/2 + (f*x)/2)) - (f*x)/2))/(4*f)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.98

$$\int (a + a\sin(e + fx))^3(A + B\sin(e + fx))\,dx$$
$$= \frac{a^3\left(-6\cos(fx+e)\sin(fx+e)^3 b - 8\cos(fx+e)\sin(fx+e)^2 a - 24\cos(fx+e)\sin(fx+e)^2 b - 36\right.}{}$$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x)
```

output
```
(a**3*( - 6*cos(e + f*x)*sin(e + f*x)**3*b - 8*cos(e + f*x)*sin(e + f*x)**
2*a - 24*cos(e + f*x)*sin(e + f*x)**2*b - 36*cos(e + f*x)*sin(e + f*x)*a -
 45*cos(e + f*x)*sin(e + f*x)*b - 88*cos(e + f*x)*a - 72*cos(e + f*x)*b +
60*a*f*x + 88*a + 45*b*f*x + 72*b))/(24*f)
```

3.262 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 246

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx &= \frac{a^3(Ad(2c^2-6cd+7d^2)-B(2c^3-6c^2d+7cd^2-5d^3))\,x}{2d^4} \\ &+ \frac{2a^3(c-d)^3(Bc-Ad)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^4\sqrt{c^2-d^2}\,f} \\ &+ \frac{a^3(A(2c-5d)d-B(2c^2-5cd+5d^2))\cos(e+fx)}{2d^3f} \\ &- \frac{aB\cos(e+fx)(a+a\sin(e+fx))^2}{3df} \\ &+ \frac{(3Bc-3Ad-5Bd)\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{6d^2f}\end{aligned}$$

output
```
1/2*a^3*(A*d*(2*c^2-6*c*d+7*d^2)-B*(2*c^3-6*c^2*d+7*c*d^2-5*d^3))*x/d^4+2*
a^3*(c-d)^3*(-A*d+B*c)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/d^
4/(c^2-d^2)^(1/2)/f+1/2*a^3*(A*(2*c-5*d)*d-B*(2*c^2-5*c*d+5*d^2))*cos(f*x+
e)/d^3/f-1/3*a*B*cos(f*x+e)*(a+a*sin(f*x+e))^2/d/f+1/6*(-3*A*d+3*B*c-5*B*d
)*cos(f*x+e)*(a^3+a^3*sin(f*x+e))/d^2/f
```

Mathematica [A] (verified)

Time = 3.87 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.95

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$

$$= \frac{a^3(1+\sin(e+fx))^3\left(6(Ad(2c^2-6cd+7d^2)+B(-2c^3+6c^2d-7cd^2+5d^3))(e+fx)+\frac{24(c-d)^3(Bc-A\ldots}{\ldots}\right.}{\ldots}$$

input
```
Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x
]),x]
```

output
```
(a^3*(1 + Sin[e + f*x])^3*(6*(A*d*(2*c^2 - 6*c*d + 7*d^2) + B*(-2*c^3 + 6*
c^2*d - 7*c*d^2 + 5*d^3))*(e + f*x) + (24*(c - d)^3*(B*c - A*d)*ArcTan[(d
+ c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/Sqrt[c^2 - d^2] - 3*d*(4*A*d*(-c +
 3*d) + B*(4*c^2 - 12*c*d + 15*d^2))*Cos[e + f*x] + B*d^3*Cos[3*(e + f*x)]
 - 3*d^2*(-(B*c) + A*d + 3*B*d)*Sin[2*(e + f*x)]))/(12*d^4*f*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])^6)
```

Rubi [A] (verified)

Time = 1.71 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.09, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3455, 3042, 3455, 27, 3042, 3447, 3042, 3502, 3042, 3214, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{c+d\sin(e+fx)}dx$$

$$\Big\downarrow \text{3455}$$

$$\frac{\int \frac{(\sin(e+fx)a+a)^2(a(2Bc+3Ad)-a(3Bc-3Ad-5Bd)\sin(e+fx))}{c+d\sin(e+fx)}dx}{3d}-\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2}{3df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(\sin(e+fx)a+a)^2(a(2Bc+3Ad)-a(3Bc-3Ad-5Bd)\sin(e+fx))}{c+d\sin(e+fx)}dx}{3d}-\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2}{3df}$$

$$\Big\downarrow \text{3455}$$

$$\frac{\frac{\int -\frac{3(\sin(e+fx)a+a)\left((Bc(c-3d)-Ad(c+2d))a^2+\left(A(2c-5d)d-B\left(2c^2-5dc+5d^2\right)\right)\sin(e+fx)a^2\right)}{c+d\sin(e+fx)}dx}{2d}+\frac{(-3Ad+3Bc-5Bd)\cos(e+fx)(a^3\sin(e+fx)+a^3)}{2df}}{3d}$$
$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2}{3df}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{(-3Ad+3Bc-5Bd)\cos(e+fx)(a^3\sin(e+fx)+a^3)}{2df}-\frac{3\int \frac{(\sin(e+fx)a+a)\left((Bc(c-3d)-Ad(c+2d))a^2+\left(A(2c-5d)d-B\left(2c^2-5dc+5d^2\right)\right)\sin(e+fx)a^2\right)}{c+d\sin(e+fx)}dx}{2d}}{3d}$$
$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2}{3df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(-3Ad+3Bc-5Bd)\cos(e+fx)(a^3\sin(e+fx)+a^3)}{2df}-\frac{3\int \frac{(\sin(e+fx)a+a)\left((Bc(c-3d)-Ad(c+2d))a^2+\left(A(2c-5d)d-B\left(2c^2-5dc+5d^2\right)\right)\sin(e+fx)a^2\right)}{c+d\sin(e+fx)}dx}{2d}}{3d}$$
$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2}{3df}$$

$$\Big\downarrow \text{3447}$$

$$\frac{\frac{(-3Ad+3Bc-5Bd)\cos(e+fx)(a^3\sin(e+fx)+a^3)}{2df}-\frac{3\int \frac{\left(A(2c-5d)d-B\left(2c^2-5dc+5d^2\right)\right)\sin^2(e+fx)a^3+(Bc(c-3d)-Ad(c+2d))a^3+\left((Bc(c-3d)-Ad(\right.}{c+d\sin(e+fx)}}{2d}}{3d}$$
$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2}{3df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{(-3Ad+3Bc-5Bd)\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{2df}-\frac{3\int\frac{\left(A(2c-5d)d-B\left(2c^2-5dc+5d^2\right)\right)\sin(e+fx)^2a^3+(Bc(c-3d)-Ad(c+2d))a^3+\left((Bc(c-3d)-Ad(c\cdots\right.}{c+d\sin(e+fx)}\cdots}{2d}\cdots}{3d}\cdots$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2}{3df}$$

$\Big\downarrow$ 3502

$$\frac{\frac{(-3Ad+3Bc-5Bd)\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{2df}-\frac{3\left(\frac{\int\frac{a^3d(Bc(c-3d)-Ad(c+2d))-a^3\left(Ad\left(2c^2-6dc+7d^2\right)-B\left(2c^3-6dc^2+7d^2c-5d^3\right)\right)\sin(e+fx)}{c+d\sin(e+fx)}d\cdots}{d}\cdots\right.}{2d}\cdots}{3d}\cdots$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2}{3df}$$

$\Big\downarrow$ 3042

$$\frac{\frac{(-3Ad+3Bc-5Bd)\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{2df}-\frac{3\left(\frac{\int\frac{a^3d(Bc(c-3d)-Ad(c+2d))-a^3\left(Ad\left(2c^2-6dc+7d^2\right)-B\left(2c^3-6dc^2+7d^2c-5d^3\right)\right)\sin(e+fx)}{c+d\sin(e+fx)}d\cdots}{d}\cdots\right.}{2d}\cdots}{3d}\cdots$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2}{3df}$$

$\Big\downarrow$ 3214

$$\frac{\frac{(-3Ad+3Bc-5Bd)\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{2df}-\frac{3\left(\frac{-\frac{2a^3(c-d)^3(Bc-Ad)\int\frac{1}{c+d\sin(e+fx)}dx}{d}-\frac{a^3x\left(Ad\left(2c^2-6cd+7d^2\right)-B\left(2c^3-6c^2d+7cd^2-5d^3\cdots\right.\right.}{d}}{d}\cdots\right.}{2d}\cdots}{3d}\cdots$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2}{3df}$$

$\Big\downarrow$ 3042

$$\frac{\frac{(-3Ad+3Bc-5Bd)\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{2df}-\frac{3\left(\frac{-\frac{2a^3(c-d)^3(Bc-Ad)\int\frac{1}{c+d\sin(e+fx)}dx}{d}-\frac{a^3x\left(Ad\left(2c^2-6cd+7d^2\right)-B\left(2c^3-6c^2d+7cd^2-5d^3\cdots\right.\right.}{d}}{d}\cdots\right.}{2d}\cdots}{3d}\cdots$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2}{3df}$$

$\Big\downarrow$ 3139

$$\frac{\frac{(-3Ad+3Bc-5Bd)\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{2df}-\frac{3\left(\frac{-\frac{4a^3(c-d)^3(Bc-Ad)\int\frac{1}{c\tan^2\left(\frac{1}{2}(e+fx)\right)+2d\tan\left(\frac{1}{2}(e+fx)\right)+c}d\tan\left(\frac{1}{2}(e+fx)\right)}{df}}{d}-\frac{a^3x\left(Ad\left(2c\right.\right.}{}\cdots\right)}{2d}}{3d}$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2}{3df}$$

$\Big\downarrow$ 1083

$$\frac{\frac{(-3Ad+3Bc-5Bd)\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{2df}-\frac{3\left(\frac{\frac{8a^3(c-d)^3(Bc-Ad)\int\frac{1}{-\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)^2-4\left(c^2-d^2\right)}d\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{df}}{d}-\frac{a^3x\left(\right.}{}\cdots\right)}{\cdots}}{3d}$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2}{3df}$$

$\Big\downarrow$ 217

$$\frac{\frac{(-3Ad+3Bc-5Bd)\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{2df}-\frac{3\left(\frac{-\frac{4a^3(c-d)^3(Bc-Ad)\arctan\left(\frac{2c\tan\left(\frac{1}{2}(e+fx)\right)+2d}{2\sqrt{c^2-d^2}}\right)}{df\sqrt{c^2-d^2}}-\frac{a^3x\left(Ad\left(2c^2-6cd+7d^2\right)-B\left(2c^3-6c^2\right.\right.}{d}}{d}\cdots\right)}{2d}}{3d}$$

$$\frac{aB\cos(e+fx)(a\sin(e+fx)+a)^2}{3df}$$

input
```
Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),x]
```

output
```
-1/3*(a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^2)/(d*f) + ((-3*((-((a^3*(A*d*
(2*c^2 - 6*c*d + 7*d^2) - B*(2*c^3 - 6*c^2*d + 7*c*d^2 - 5*d^3))*x)/d) - (
4*a^3*(c - d)^3*(B*c - A*d)*ArcTan[(2*d + 2*c*Tan[(e + f*x)/2])/(2*Sqrt[c^
2 - d^2])])/(d*Sqrt[c^2 - d^2]*f))/d - (a^3*(A*(2*c - 5*d)*d - B*(2*c^2 -
5*c*d + 5*d^2))*Cos[e + f*x])/(d*f)))/(2*d) + ((3*B*c - 3*A*d - 5*B*d)*Cos
[e + f*x]*(a^3 + a^3*Sin[e + f*x]))/(2*d*f))/(3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.54

method	result
derivativedivides	$2a^3\left(\frac{\left(-A\,c^3d+3A\,c^2d^2-3Ac\,d^3+A\,d^4+B\,c^4-3B\,c^3d+3B\,c^2d^2-Bc\,d^3\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{d^4\sqrt{c^2-d^2}}+\frac{\left(\frac{1}{2}A\,d^3-\frac{1}{2}Bc\,d^2+\frac{3}{2}B\right.}{}\right.$
default	$2a^3\left(\frac{\left(-A\,c^3d+3A\,c^2d^2-3Ac\,d^3+A\,d^4+B\,c^4-3B\,c^3d+3B\,c^2d^2-Bc\,d^3\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{d^4\sqrt{c^2-d^2}}+\frac{\left(\frac{1}{2}A\,d^3-\frac{1}{2}Bc\,d^2+\frac{3}{2}B\right.}{}\right.$
risch	Expression too large to display

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x,method=_RETURNV
ERBOSE)
```

output
```
2/f*a^3*((-A*c^3*d+3*A*c^2*d^2-3*A*c*d^3+A*d^4+B*c^4-3*B*c^3*d+3*B*c^2*d^2
-B*c*d^3)/d^4/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2
-d^2)^(1/2))+1/d^4*(((1/2*A*d^3-1/2*B*c*d^2+3/2*B*d^3)*tan(1/2*f*x+1/2*e)^
5+(A*c*d^2-3*A*d^3-B*c^2*d+3*B*c*d^2-3*B*d^3)*tan(1/2*f*x+1/2*e)^4+(2*A*c*
d^2-6*A*d^3-2*B*c^2*d+6*B*c*d^2-8*B*d^3)*tan(1/2*f*x+1/2*e)^2+(-1/2*A*d^3+
1/2*B*c*d^2-3/2*B*d^3)*tan(1/2*f*x+1/2*e)+A*c*d^2-3*A*d^3-B*c^2*d+3*B*c*d^
2-11/3*B*d^3)/(1+tan(1/2*f*x+1/2*e)^2)^3+1/2*(2*A*c^2*d-6*A*c*d^2+7*A*d^3-
2*B*c^3+6*B*c^2*d-7*B*c*d^2+5*B*d^3)*arctan(tan(1/2*f*x+1/2*e))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 626, normalized size of antiderivative = 2.54

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$

$$= \left[\frac{2\,Ba^3d^3\cos{(fx+e)}^3 - 3\,(2\,Ba^3c^3 - 2\,(A+3\,B)a^3c^2d + (6\,A+7\,B)a^3cd^2 - (7\,A+5\,B)a^3d^3)fx + 3\,(}{}\right.$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
[1/6*(2*B*a^3*d^3*cos(f*x + e)^3 - 3*(2*B*a^3*c^3 - 2*(A + 3*B)*a^3*c^2*d
+ (6*A + 7*B)*a^3*c*d^2 - (7*A + 5*B)*a^3*d^3)*f*x + 3*(B*a^3*c*d^2 - (A +
 3*B)*a^3*d^3)*cos(f*x + e)*sin(f*x + e) - 3*(B*a^3*c^3 - (A + 2*B)*a^3*c^
2*d + (2*A + B)*a^3*c*d^2 - A*a^3*d^3)*sqrt(-(c - d)/(c + d))*log(((2*c^2
- d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*((c^2 + c*d)*co
s(f*x + e)*sin(f*x + e) + (c*d + d^2)*cos(f*x + e))*sqrt(-(c - d)/(c + d))
)/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) - 6*(B*a^3*c^2*d
- (A + 3*B)*a^3*c*d^2 + (3*A + 4*B)*a^3*d^3)*cos(f*x + e))/(d^4*f), 1/6*(2
*B*a^3*d^3*cos(f*x + e)^3 - 3*(2*B*a^3*c^3 - 2*(A + 3*B)*a^3*c^2*d + (6*A
+ 7*B)*a^3*c*d^2 - (7*A + 5*B)*a^3*d^3)*f*x + 3*(B*a^3*c*d^2 - (A + 3*B)*a
^3*d^3)*cos(f*x + e)*sin(f*x + e) - 6*(B*a^3*c^3 - (A + 2*B)*a^3*c^2*d + (
2*A + B)*a^3*c*d^2 - A*a^3*d^3)*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x +
 e) + d)*sqrt((c - d)/(c + d))/((c - d)*cos(f*x + e))) - 6*(B*a^3*c^2*d -
(A + 3*B)*a^3*c*d^2 + (3*A + 4*B)*a^3*d^3)*cos(f*x + e))/(d^4*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 597 vs. 2(233) = 466.

Time = 0.24 (sec) , antiderivative size = 597, normalized size of antiderivative = 2.43

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorith
m="giac")
```

output
```
-1/6*(3*(2*B*a^3*c^3 - 2*A*a^3*c^2*d - 6*B*a^3*c^2*d + 6*A*a^3*c*d^2 + 7*B
*a^3*c*d^2 - 7*A*a^3*d^3 - 5*B*a^3*d^3)*(f*x + e)/d^4 - 12*(B*a^3*c^4 - A*
a^3*c^3*d - 3*B*a^3*c^3*d + 3*A*a^3*c^2*d^2 + 3*B*a^3*c^2*d^2 - 3*A*a^3*c*
d^3 - B*a^3*c*d^3 + A*a^3*d^4)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) +
arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/(sqrt(c^2 - d^2)*d^4
) + 2*(3*B*a^3*c*d*tan(1/2*f*x + 1/2*e)^5 - 3*A*a^3*d^2*tan(1/2*f*x + 1/2*
e)^5 - 9*B*a^3*d^2*tan(1/2*f*x + 1/2*e)^5 + 6*B*a^3*c^2*tan(1/2*f*x + 1/2*
e)^4 - 6*A*a^3*c*d*tan(1/2*f*x + 1/2*e)^4 - 18*B*a^3*c*d*tan(1/2*f*x + 1/2
*e)^4 + 18*A*a^3*d^2*tan(1/2*f*x + 1/2*e)^4 + 18*B*a^3*d^2*tan(1/2*f*x + 1
/2*e)^4 + 12*B*a^3*c^2*tan(1/2*f*x + 1/2*e)^2 - 12*A*a^3*c*d*tan(1/2*f*x +
 1/2*e)^2 - 36*B*a^3*c*d*tan(1/2*f*x + 1/2*e)^2 + 36*A*a^3*d^2*tan(1/2*f*x
 + 1/2*e)^2 + 48*B*a^3*d^2*tan(1/2*f*x + 1/2*e)^2 - 3*B*a^3*c*d*tan(1/2*f*
x + 1/2*e) + 3*A*a^3*d^2*tan(1/2*f*x + 1/2*e) + 9*B*a^3*d^2*tan(1/2*f*x +
1/2*e) + 6*B*a^3*c^2 - 6*A*a^3*c*d - 18*B*a^3*c*d + 18*A*a^3*d^2 + 22*B*a^
3*d^2)/((tan(1/2*f*x + 1/2*e)^2 + 1)^3*d^3))/f
```

Mupad [B] (verification not implemented)

Time = 43.81 (sec) , antiderivative size = 10256, normalized size of antiderivative = 41.69

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \text{Too large to display}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c + d*sin(e + f*x)),x)
```

output
```
- ((2*(9*A*a^3*d^2 + 3*B*a^3*c^2 + 11*B*a^3*d^2 - 3*A*a^3*c*d - 9*B*a^3*c*
d))/(3*d^3) - (tan(e/2 + (f*x)/2)^5*(A*a^3*d - B*a^3*c + 3*B*a^3*d))/d^2 +
 (4*tan(e/2 + (f*x)/2)^2*(3*A*a^3*d^2 + B*a^3*c^2 + 4*B*a^3*d^2 - A*a^3*c*
d - 3*B*a^3*c*d))/d^3 + (2*tan(e/2 + (f*x)/2)^4*(3*A*a^3*d^2 + B*a^3*c^2 +
 3*B*a^3*d^2 - A*a^3*c*d - 3*B*a^3*c*d))/d^3 + (tan(e/2 + (f*x)/2)*(A*a^3*
d - B*a^3*c + 3*B*a^3*d))/d^2)/(f*(3*tan(e/2 + (f*x)/2)^2 + 3*tan(e/2 + (f
*x)/2)^4 + tan(e/2 + (f*x)/2)^6 + 1)) - (atan(((((8*(49*A^2*a^6*c^2*d^9 -
84*A^2*a^6*c^3*d^8 + 64*A^2*a^6*c^4*d^7 - 24*A^2*a^6*c^5*d^6 + 4*A^2*a^6*c
^6*d^5 + 25*B^2*a^6*c^2*d^9 - 70*B^2*a^6*c^3*d^8 + 109*B^2*a^6*c^4*d^7 - 1
04*B^2*a^6*c^5*d^6 + 64*B^2*a^6*c^6*d^5 - 24*B^2*a^6*c^7*d^4 + 4*B^2*a^6*c
^8*d^3 + 70*A*B*a^6*c^2*d^9 - 158*A*B*a^6*c^3*d^8 + 188*A*B*a^6*c^4*d^7 -
128*A*B*a^6*c^5*d^6 + 48*A*B*a^6*c^6*d^5 - 8*A*B*a^6*c^7*d^4))/d^8 + (8*ta
n(e/2 + (f*x)/2)*(19*A^2*a^6*c^3*d^9 - 144*A^2*a^6*c^2*d^10 + 116*A^2*a^6*
c^4*d^8 - 116*A^2*a^6*c^5*d^7 + 48*A^2*a^6*c^6*d^6 - 8*A^2*a^6*c^7*d^5 - 1
40*B^2*a^6*c^2*d^10 + 189*B^2*a^6*c^3*d^9 - 114*B^2*a^6*c^4*d^8 - 41*B^2*a
^6*c^5*d^7 + 136*B^2*a^6*c^6*d^6 - 116*B^2*a^6*c^7*d^5 + 48*B^2*a^6*c^8*d^
4 - 8*B^2*a^6*c^9*d^3 + 94*A^2*a^6*c*d^11 + 50*B^2*a^6*c*d^11 - 308*A*B*a^
6*c^2*d^10 + 258*A*B*a^6*c^3*d^9 + 22*A*B*a^6*c^4*d^8 - 252*A*B*a^6*c^5*d^
7 + 232*A*B*a^6*c^6*d^6 - 96*A*B*a^6*c^7*d^5 + 16*A*B*a^6*c^8*d^4 + 140*A*
B*a^6*c*d^11))/d^9 + ((((32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^13 ...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 714, normalized size of antiderivative = 2.90

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{c + d\sin(e + fx)}\,dx = \text{Too large to display}$$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

output

```
(a**3*( - 12*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d
**2))*a*c**2*d + 24*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c
**2 - d**2))*a*c*d**2 - 12*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)
/sqrt(c**2 - d**2))*a*d**3 + 12*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c
 + d)/sqrt(c**2 - d**2))*b*c**3 - 24*sqrt(c**2 - d**2)*atan((tan((e + f*x)
/2)*c + d)/sqrt(c**2 - d**2))*b*c**2*d + 12*sqrt(c**2 - d**2)*atan((tan((e
 + f*x)/2)*c + d)/sqrt(c**2 - d**2))*b*c*d**2 - 2*cos(e + f*x)*sin(e + f*x
)**2*b*c*d**3 - 2*cos(e + f*x)*sin(e + f*x)**2*b*d**4 - 3*cos(e + f*x)*sin
(e + f*x)*a*c*d**3 - 3*cos(e + f*x)*sin(e + f*x)*a*d**4 + 3*cos(e + f*x)*s
in(e + f*x)*b*c**2*d**2 - 6*cos(e + f*x)*sin(e + f*x)*b*c*d**3 - 9*cos(e +
 f*x)*sin(e + f*x)*b*d**4 + 6*cos(e + f*x)*a*c**2*d**2 - 12*cos(e + f*x)*a
*c*d**3 - 18*cos(e + f*x)*a*d**4 - 6*cos(e + f*x)*b*c**3*d + 12*cos(e + f*
x)*b*c**2*d**2 - 4*cos(e + f*x)*b*c*d**3 - 22*cos(e + f*x)*b*d**4 + 6*a*c*
*3*d*e + 6*a*c**3*d*f*x - 12*a*c**2*d**2*e - 12*a*c**2*d**2*f*x + 2*a*c**2
*d**2 + 3*a*c*d**3*e + 3*a*c*d**3*f*x - 4*a*c*d**3 + 21*a*d**4*e + 21*a*d*
*4*f*x - 6*a*d**4 - 6*b*c**4*e - 6*b*c**4*f*x + 12*b*c**3*d*e + 12*b*c**3*
d*f*x - 2*b*c**3*d - 3*b*c**2*d**2*e - 3*b*c**2*d**2*f*x + 4*b*c**2*d**2 -
 6*b*c*d**3*e - 6*b*c*d**3*f*x - 4*b*c*d**3 + 15*b*d**4*e + 15*b*d**4*f*x
- 10*b*d**4))/(6*d**4*f*(c + d))
```

3.263 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 283

$$\begin{aligned}\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx &= -\frac{a^3(2A(2c-3d)d-B(6c^2-12cd+7d^2))\,x}{2d^4}\\
&\quad+\frac{2a^3(c-d)^2\,(Ad(2c+3d)-B(3c^2+3cd-d^2))\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^4(c+d)\sqrt{c^2-d^2}\,f}\\
&\quad-\frac{a^3(4Acd-B(6c^2-3cd-5d^2))\cos(e+fx)}{2d^3(c+d)f}\\
&\quad+\frac{(2Ad-B(3c+d))\cos(e+fx)\,(a^3+a^3\sin(e+fx))}{2d^2(c+d)f}\\
&\quad+\frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^2}{d(c+d)f(c+d\sin(e+fx))}\end{aligned}$$

output
```
-1/2*a^3*(2*A*(2*c-3*d)*d-B*(6*c^2-12*c*d+7*d^2))*x/d^4+2*a^3*(c-d)^2*(A*d
*(2*c+3*d)-B*(3*c^2+3*c*d-d^2))*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^
(1/2))/d^4/(c+d)/(c^2-d^2)^(1/2)/f-1/2*a^3*(4*A*c*d-B*(6*c^2-3*c*d-5*d^2))
*cos(f*x+e)/d^3/(c+d)/f+1/2*(2*A*d-B*(3*c+d))*cos(f*x+e)*(a^3+a^3*sin(f*x+
e))/d^2/(c+d)/f+a*(-A*d+B*c)*cos(f*x+e)*(a+a*sin(f*x+e))^2/d/(c+d)/f/(c+d*
sin(f*x+e))
```

Mathematica [A] (verified)

Time = 6.76 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.86

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$$

$$= \frac{a^3(1+\sin(e+fx))^3\left(2(2Ad(-2c+3d)+B(6c^2-12cd+7d^2))(e+fx)-\frac{8(c-d)^2(-Ad(2c+3d)+B(3c^2+3cd\cdots}{(c+d)\sqrt{\cdots}}\right.}{4d^4f\left(\cos\left(\frac{1}{2}(e+\cdots\right.\right.}$$

input
```
Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x
])^2,x]
```

output
```
(a^3*(1 + Sin[e + f*x])^3*(2*(2*A*d*(-2*c + 3*d) + B*(6*c^2 - 12*c*d + 7*d
^2))*(e + f*x) - (8*(c - d)^2*(-(A*d*(2*c + 3*d)) + B*(3*c^2 + 3*c*d - d^2
))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c + d)*Sqrt[c^2 - d
^2]) - 4*d*(-2*B*c + A*d + 3*B*d)*Cos[e + f*x] + (4*(c - d)^2*d*(B*c - A*d
)*Cos[e + f*x])/((c + d)*(c + d*Sin[e + f*x])) - B*d^2*Sin[2*(e + f*x)]))/
(4*d^4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
```

Rubi [A] (verified)

Time = 1.80 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.04, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3454, 25, 3042, 3455, 3042, 3447, 3042, 3502, 3042, 3214, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}dx$$

$$\Big\downarrow \text{3454}$$

$$\frac{\int -\frac{(\sin(e+fx)a+a)^2(a(B(2c-d)-3Ad)+a(2Ad-B(3c+d))\sin(e+fx))}{c+d\sin(e+fx)}dx}{d(c+d)}+\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{df(c+d)(c+d\sin(e+fx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{df(c+d)(c+d\sin(e+fx))}-\frac{\int \frac{(\sin(e+fx)a+a)^2(a(2Bc-3Ad-Bd)+a(2Ad-B(3c+d))\sin(e+fx))}{c+d\sin(e+fx)}dx}{d(c+d)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{df(c+d)(c+d\sin(e+fx))}-\frac{\int \frac{(\sin(e+fx)a+a)^2(a(2Bc-3Ad-Bd)+a(2Ad-B(3c+d))\sin(e+fx))}{c+d\sin(e+fx)}dx}{d(c+d)}$$

$$\Big\downarrow \text{3455}$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{df(c+d)(c+d\sin(e+fx))}-\frac{\frac{\int \frac{(\sin(e+fx)a+a)\left(a^2\left(2A(c-3d)d-B\left(3c^2-3dc+2d^2\right)\right)-a^2\left(4Acd-B\left(6c^2-3dc-5d^2\right)\right)\sin(e+fx)\right)}{c+d\sin(e+fx)}dx}{2d}-\frac{(2Ad-B(3c+d))\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{2df}}{d(c+d)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{df(c+d)(c+d\sin(e+fx))}-\frac{\frac{\int \frac{(\sin(e+fx)a+a)\left(a^2\left(2A(c-3d)d-B\left(3c^2-3dc+2d^2\right)\right)-a^2\left(4Acd-B\left(6c^2-3dc-5d^2\right)\right)\sin(e+fx)\right)}{c+d\sin(e+fx)}dx}{2d}-\frac{(2Ad-B(3c+d))\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{2df}}{d(c+d)}$$

$$\Big\downarrow \text{3447}$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{df(c+d)(c+d\sin(e+fx))}-\frac{\frac{\int \frac{-\left(\left(4Acd-B\left(6c^2-3dc-5d^2\right)\right)\sin^2(e+fx)a^3\right)+\left(2A(c-3d)d-B\left(3c^2-3dc+2d^2\right)\right)a^3+\left(a^3\left(2A(c-3d)d-B\left(3c^2-3dc+2d^2\right)\right)-a^3\left(4Acd-B\left(6c^2-3dc-5d^2\right)\right)\right)\sin}{c+d\sin(e+fx)}}{2d}}{d(c+d)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{df(c+d)(c+d\sin(e+fx))}-\frac{\frac{\int \frac{-\left(\left(4Acd-B\left(6c^2-3dc-5d^2\right)\right)\sin(e+fx)^2a^3\right)+\left(2A(c-3d)d-B\left(3c^2-3dc+2d^2\right)\right)a^3+\left(a^3\left(2A(c-3d)d-B\left(3c^2-3dc+2d^2\right)\right)-a^3\left(4Acd-B\left(6c^2-3dc-5d^2\right)\right)\right)s}{c+d\sin(e+fx)}}{2d}}{d(c+d)}$$

$\downarrow$ 3502

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{df(c+d)(c+d\sin(e+fx))}-\frac{\frac{\frac{\int \frac{d\left(2A(c-3d)d-B\left(3c^2-3dc+2d^2\right)\right)a^3+(c+d)\left(2A(2c-3d)d-B\left(6c^2-12dc+7d^2\right)\right)\sin(e+fx)a^3}{c+d\sin(e+fx)}dx}{d}+\frac{a^3\left(4Acd-B\left(6c^2-3cd-5d^2\right)\right)\cos(e+fx)}{df}}{2d}-\frac{(2Ad-B(3}{}}{d(c+d)}$$

$\downarrow$ 3042

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{df(c+d)(c+d\sin(e+fx))}-\frac{\frac{\frac{\int \frac{d\left(2A(c-3d)d-B\left(3c^2-3dc+2d^2\right)\right)a^3+(c+d)\left(2A(2c-3d)d-B\left(6c^2-12dc+7d^2\right)\right)\sin(e+fx)a^3}{c+d\sin(e+fx)}dx}{d}+\frac{a^3\left(4Acd-B\left(6c^2-3cd-5d^2\right)\right)\cos(e+fx)}{df}}{2d}-\frac{(2Ad-B(3}{}}{d(c+d)}$$

$\downarrow$ 3214

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{df(c+d)(c+d\sin(e+fx))}-\frac{\frac{\frac{a^3x(c+d)\left(2Ad(2c-3d)-B\left(6c^2-12cd+7d^2\right)\right)}{d}-\frac{2a^3(c-d)^2\left(Ad(2c+3d)-B\left(3c^2+3cd-d^2\right)\right)\int\frac{1}{c+d\sin(e+fx)}dx}{d}}{d}+\frac{a^3\left(4Acd-B\left(6c^2-3cd-5d^2\right)\right)\cos(e+fx)}{df}}{2d}-}{d(c+d)}$$

$\downarrow$ 3042

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{df(c+d)(c+d\sin(e+fx))}-\frac{\frac{\frac{a^3x(c+d)\left(2Ad(2c-3d)-B\left(6c^2-12cd+7d^2\right)\right)}{d}-\frac{2a^3(c-d)^2\left(Ad(2c+3d)-B\left(3c^2+3cd-d^2\right)\right)\int\frac{1}{c+d\sin(e+fx)}dx}{d}}{d}+\frac{a^3\left(4Acd-B\left(6c^2-3cd-5d^2\right)\right)\cos(e+fx)}{df}}{2d}-}{d(c+d)}$$

$\downarrow$ 3139

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{df(c+d)(c+d\sin(e+fx))}-\frac{\frac{\frac{a^3x(c+d)\left(2Ad(2c-3d)-B\left(6c^2-12cd+7d^2\right)\right)}{d}-\frac{4a^3(c-d)^2\left(Ad(2c+3d)-B\left(3c^2+3cd-d^2\right)\right)\int\frac{1}{c\tan^2\left(\frac{1}{2}(e+fx)\right)+2d\tan\left(\frac{1}{2}(e+fx)\right)+c}d\tan\left(\frac{1}{2}(e+fx)\right)}{df}}{d}+\frac{a^3(4A}{}}{2d}}{d(c+d)}$$

$$\Big\downarrow \text{1083}$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{df(c+d)(c+d\sin(e+fx))}-\frac{\frac{\frac{\frac{8a^3(c-d)^2\left(Ad(2c+3d)-B\left(3c^2+3cd-d^2\right)\right)\int\frac{1}{-\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)^2-4\left(c^2-d^2\right)}d\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{df}}{d}+\frac{a^3x(c+d)\left(2Ad(2c-3d)-B\left(6c^2-12cd+7d^2\right)\right)}{d}}{2d}}{d(c+d)}$$

$$\Big\downarrow \text{217}$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{df(c+d)(c+d\sin(e+fx))}-\frac{\frac{\frac{\frac{a^3x(c+d)\left(2Ad(2c-3d)-B\left(6c^2-12cd+7d^2\right)\right)}{d}-\frac{4a^3(c-d)^2\left(Ad(2c+3d)-B\left(3c^2+3cd-d^2\right)\right)\arctan\left(\frac{2c\tan\left(\frac{1}{2}(e+fx)\right)+2d}{2\sqrt{c^2-d^2}}\right)}{df\sqrt{c^2-d^2}}}{d}+\frac{a^3\left(4Acd-B\left(6c^2-3cd-5d^2\right)\right)\cos}{df}}{2d}}{d(c+d)}$$

input
```
Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2,x
]
```

output
```
(a*(B*c - A*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^2)/(d*(c + d)*f*(c + d*Si
n[e + f*x])) - ((((a^3*(c + d)*(2*A*(2*c - 3*d)*d - B*(6*c^2 - 12*c*d + 7*
d^2))*x)/d - (4*a^3*(c - d)^2*(A*d*(2*c + 3*d) - B*(3*c^2 + 3*c*d - d^2))*
ArcTan[(2*d + 2*c*Tan[(e + f*x)/2])/(2*Sqrt[c^2 - d^2])])/(d*Sqrt[c^2 - d^
2]*f))/d + (a^3*(4*A*c*d - B*(6*c^2 - 3*c*d - 5*d^2))*Cos[e + f*x])/(d*f))
/(2*d) - ((2*A*d - B*(3*c + d))*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x]))/(2*
d*f))/(d*(c + d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 2.52 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.43

method	result
derivativedivides	$2a^3\left(-\frac{\frac{-\frac{B\,d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2}+\left(A\,d^2-2Bcd+3B\,d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\frac{B\,d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}+A\,d^2-2Bcd+3B\,d^2}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^2}+\frac{(4Acd-6A\,d^2-6B}{}}{d^4}\right.$
default	$2a^3\left(-\frac{\frac{-\frac{B\,d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2}+\left(A\,d^2-2Bcd+3B\,d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+\frac{B\,d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}+A\,d^2-2Bcd+3B\,d^2}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^2}+\frac{(4Acd-6A\,d^2-6B}{}}{d^4}\right.$
risch	Expression too large to display

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x,method=_RETUR
NVERBOSE)
```

output
```
2/f*a^3*(-1/d^4*((-1/2*B*d^2*tan(1/2*f*x+1/2*e)^3+(A*d^2-2*B*c*d+3*B*d^2)*
tan(1/2*f*x+1/2*e)^2+1/2*B*d^2*tan(1/2*f*x+1/2*e)+A*d^2-2*B*c*d+3*B*d^2)/(
1+tan(1/2*f*x+1/2*e)^2)^2+1/2*(4*A*c*d-6*A*d^2-6*B*c^2+12*B*c*d-7*B*d^2)*a
rctan(tan(1/2*f*x+1/2*e)))+1/d^4*((-d^2*(A*c^2*d-2*A*c*d^2+A*d^3-B*c^3+2*B
*c^2*d-B*c*d^2)/(c+d)/c*tan(1/2*f*x+1/2*e)-d*(A*c^2*d-2*A*c*d^2+A*d^3-B*c^
3+2*B*c^2*d-B*c*d^2)/(c+d))/(tan(1/2*f*x+1/2*e)^2*c+2*d*tan(1/2*f*x+1/2*e)
+c)+(2*A*c^3*d-A*c^2*d^2-4*A*c*d^3+3*A*d^4-3*B*c^4+3*B*c^3*d+4*B*c^2*d^2-5
*B*c*d^3+B*d^4)/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2
*d)/(c^2-d^2)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 1027, normalized size of antiderivative = 3.63

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
[1/2*((B*a^3*c*d^3 + B*a^3*d^4)*cos(f*x + e)^3 + (6*B*a^3*c^4 - 2*(2*A + 3
*B)*a^3*c^3*d + (2*A - 5*B)*a^3*c^2*d^2 + (6*A + 7*B)*a^3*c*d^3)*f*x + (3*
B*a^3*c^4 - 2*A*a^3*c^3*d - (A + 4*B)*a^3*c^2*d^2 + (3*A + B)*a^3*c*d^3 +
(3*B*a^3*c^3*d - 2*A*a^3*c^2*d^2 - (A + 4*B)*a^3*c*d^3 + (3*A + B)*a^3*d^4
)*sin(f*x + e))*sqrt(-(c - d)/(c + d))*log(((2*c^2 - d^2)*cos(f*x + e)^2 -
 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*((c^2 + c*d)*cos(f*x + e)*sin(f*x + e)
 + (c*d + d^2)*cos(f*x + e))*sqrt(-(c - d)/(c + d)))/(d^2*cos(f*x + e)^2 -
 2*c*d*sin(f*x + e) - c^2 - d^2)) + (6*B*a^3*c^3*d - 2*(2*A + 3*B)*a^3*c^2
*d^2 + (2*A - 5*B)*a^3*c*d^3 - (2*A + B)*a^3*d^4)*cos(f*x + e) + ((6*B*a^3
*c^3*d - 2*(2*A + 3*B)*a^3*c^2*d^2 + (2*A - 5*B)*a^3*c*d^3 + (6*A + 7*B)*a
^3*d^4)*f*x + (3*B*a^3*c^2*d^2 - (2*A + 3*B)*a^3*c*d^3 - 2*(A + 3*B)*a^3*d
^4)*cos(f*x + e))*sin(f*x + e))/((c*d^5 + d^6)*f*sin(f*x + e) + (c^2*d^4 +
 c*d^5)*f), 1/2*((B*a^3*c*d^3 + B*a^3*d^4)*cos(f*x + e)^3 + (6*B*a^3*c^4 -
 2*(2*A + 3*B)*a^3*c^3*d + (2*A - 5*B)*a^3*c^2*d^2 + (6*A + 7*B)*a^3*c*d^3
)*f*x + 2*(3*B*a^3*c^4 - 2*A*a^3*c^3*d - (A + 4*B)*a^3*c^2*d^2 + (3*A + B)
*a^3*c*d^3 + (3*B*a^3*c^3*d - 2*A*a^3*c^2*d^2 - (A + 4*B)*a^3*c*d^3 + (3*A
 + B)*a^3*d^4)*sin(f*x + e))*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x + e)
 + d)*sqrt((c - d)/(c + d))/((c - d)*cos(f*x + e))) + (6*B*a^3*c^3*d - 2*(
2*A + 3*B)*a^3*c^2*d^2 + (2*A - 5*B)*a^3*c*d^3 - (2*A + B)*a^3*d^4)*cos(f*
x + e) + ((6*B*a^3*c^3*d - 2*(2*A + 3*B)*a^3*c^2*d^2 + (2*A - 5*B)*a^3*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c + d\sin(e + fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algori
thm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. 2(271) = 542.

Time = 0.25 (sec) , antiderivative size = 571, normalized size of antiderivative = 2.02

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx =$$

$$-\frac{\frac{4\left(3Ba^3c^4-2Aa^3c^3d-3Ba^3c^3d+Aa^3c^2d^2-4Ba^3c^2d^2+4Aa^3cd^3+5Ba^3cd^3-3Aa^3d^4-Ba^3d^4\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx-\cdots}{\sqrt{c^2-\cdots}}\right.\right.}{(cd^4+d^5)\sqrt{c^2-d^2}}}{\cdots}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
-1/2*(4*(3*B*a^3*c^4 - 2*A*a^3*c^3*d - 3*B*a^3*c^3*d + A*a^3*c^2*d^2 - 4*B
*a^3*c^2*d^2 + 4*A*a^3*c*d^3 + 5*B*a^3*c*d^3 - 3*A*a^3*d^4 - B*a^3*d^4)*(p
i*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) +
d)/sqrt(c^2 - d^2)))/((c*d^4 + d^5)*sqrt(c^2 - d^2)) - 4*(B*a^3*c^3*d*tan(
1/2*f*x + 1/2*e) - A*a^3*c^2*d^2*tan(1/2*f*x + 1/2*e) - 2*B*a^3*c^2*d^2*ta
n(1/2*f*x + 1/2*e) + 2*A*a^3*c*d^3*tan(1/2*f*x + 1/2*e) + B*a^3*c*d^3*tan(
1/2*f*x + 1/2*e) - A*a^3*d^4*tan(1/2*f*x + 1/2*e) + B*a^3*c^4 - A*a^3*c^3*
d - 2*B*a^3*c^3*d + 2*A*a^3*c^2*d^2 + B*a^3*c^2*d^2 - A*a^3*c*d^3)/((c^2*d
^3 + c*d^4)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)) - (
6*B*a^3*c^2 - 4*A*a^3*c*d - 12*B*a^3*c*d + 6*A*a^3*d^2 + 7*B*a^3*d^2)*(f*x
 + e)/d^4 - 2*(B*a^3*d*tan(1/2*f*x + 1/2*e)^3 + 4*B*a^3*c*tan(1/2*f*x + 1/
2*e)^2 - 2*A*a^3*d*tan(1/2*f*x + 1/2*e)^2 - 6*B*a^3*d*tan(1/2*f*x + 1/2*e)
^2 - B*a^3*d*tan(1/2*f*x + 1/2*e) + 4*B*a^3*c - 2*A*a^3*d - 6*B*a^3*d)/((t
an(1/2*f*x + 1/2*e)^2 + 1)^2*d^3))/f
```

Mupad [B] (verification not implemented)

Time = 47.63 (sec) , antiderivative size = 11993, normalized size of antiderivative = 42.38

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c + d\sin(e + fx))^2} \, dx = \text{Too large to display}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c + d*sin(e + f*x))^2,x
)
```

output

```
- ((2*(A*a^3*d^3 - 3*B*a^3*c^3 - A*a^3*c*d^2 + 2*A*a^3*c^2*d + 2*B*a^3*c*d
^2 + 3*B*a^3*c^2*d))/(d^3*(c + d)) + (2*tan(e/2 + (f*x)/2)^4*(A*a^3*d^3 -
3*B*a^3*c^3 - B*a^3*d^3 - A*a^3*c*d^2 + 2*A*a^3*c^2*d + B*a^3*c*d^2 + 3*B*
a^3*c^2*d))/(d^3*(c + d)) + (2*tan(e/2 + (f*x)/2)^2*(2*A*a^3*d^3 - 6*B*a^3
*c^3 + B*a^3*d^3 - 2*A*a^3*c*d^2 + 4*A*a^3*c^2*d + 5*B*a^3*c*d^2 + 6*B*a^3
*c^2*d))/(d^3*(c + d)) + (4*tan(e/2 + (f*x)/2)^3*(A*a^3*d^3 - 3*B*a^3*c^3
- A*a^3*c*d^2 + 2*A*a^3*c^2*d + 2*B*a^3*c*d^2 + 3*B*a^3*c^2*d))/(c*d^2*(c
+ d)) + (tan(e/2 + (f*x)/2)^5*(2*A*a^3*d^3 - 3*B*a^3*c^3 - 4*A*a^3*c*d^2 +
 2*A*a^3*c^2*d - 2*B*a^3*c*d^2 + 3*B*a^3*c^2*d))/(c*d^2*(c + d)) + (tan(e/
2 + (f*x)/2)*(2*A*a^3*d^3 - 9*B*a^3*c^3 + 6*A*a^3*c^2*d + 10*B*a^3*c*d^2 +
 9*B*a^3*c^2*d))/(c*d^2*(c + d)))/(f*(c + 2*d*tan(e/2 + (f*x)/2) + 3*c*tan
(e/2 + (f*x)/2)^2 + 3*c*tan(e/2 + (f*x)/2)^4 + c*tan(e/2 + (f*x)/2)^6 + 4*
d*tan(e/2 + (f*x)/2)^3 + 2*d*tan(e/2 + (f*x)/2)^5)) - (atan(((((8*(36*A^2*
a^6*c^2*d^9 + 24*A^2*a^6*c^3*d^8 - 44*A^2*a^6*c^4*d^7 - 16*A^2*a^6*c^5*d^6
 + 16*A^2*a^6*c^6*d^5 + 49*B^2*a^6*c^2*d^9 - 70*B^2*a^6*c^3*d^8 - 59*B^2*a
^6*c^4*d^7 + 144*B^2*a^6*c^5*d^6 - 24*B^2*a^6*c^6*d^5 - 72*B^2*a^6*c^7*d^4
 + 36*B^2*a^6*c^8*d^3 + 84*A*B*a^6*c^2*d^9 - 32*A*B*a^6*c^3*d^8 - 148*A*B*
a^6*c^4*d^7 + 88*A*B*a^6*c^5*d^6 + 72*A*B*a^6*c^6*d^5 - 48*A*B*a^6*c^7*d^4
))/(2*c*d^9 + d^10 + c^2*d^8) + (8*tan(e/2 + (f*x)/2)*(144*A^2*a^6*c^2*d^1
0 - 164*A^2*a^6*c^3*d^9 - 136*A^2*a^6*c^4*d^8 + 136*A^2*a^6*c^5*d^7 + 3...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1223, normalized size of antiderivative = 4.32

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c + d\sin(e + fx))^2}\,dx = \text{Too large to display}$$

input

```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

output

```
(a**3*(8*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2)
)*sin(e + f*x)*a*c**2*d**2 + 4*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c
+ d)/sqrt(c**2 - d**2))*sin(e + f*x)*a*c*d**3 - 12*sqrt(c**2 - d**2)*atan(
(tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*sin(e + f*x)*a*d**4 - 12*sqrt(
c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*sin(e + f*x)
*b*c**3*d + 16*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 -
 d**2))*sin(e + f*x)*b*c*d**3 - 4*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)
*c + d)/sqrt(c**2 - d**2))*sin(e + f*x)*b*d**4 + 8*sqrt(c**2 - d**2)*atan(
(tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*a*c**3*d + 4*sqrt(c**2 - d**2)
*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*a*c**2*d**2 - 12*sqrt(c*
*2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*a*c*d**3 - 12*
sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*b*c**4
+ 16*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*b*
c**2*d**2 - 4*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 -
d**2))*b*c*d**3 - cos(e + f*x)*sin(e + f*x)**2*b*c**2*d**3 - 2*cos(e + f*x
)*sin(e + f*x)**2*b*c*d**4 - cos(e + f*x)*sin(e + f*x)**2*b*d**5 - 2*cos(e
 + f*x)*sin(e + f*x)*a*c**2*d**3 - 4*cos(e + f*x)*sin(e + f*x)*a*c*d**4 -
2*cos(e + f*x)*sin(e + f*x)*a*d**5 + 3*cos(e + f*x)*sin(e + f*x)*b*c**3*d*
*2 - 9*cos(e + f*x)*sin(e + f*x)*b*c*d**4 - 6*cos(e + f*x)*sin(e + f*x)*b*
d**5 - 4*cos(e + f*x)*a*c**3*d**2 - 2*cos(e + f*x)*a*c**2*d**3 - 2*cos(...
```

3.264 $\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 305

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = -\frac{a^3(3Bc-Ad-3Bd)x}{d^4} - \frac{a^3(c-d)\left(Ad(2c^2+6cd+7d^2)-3B(2c^3+4c^2d+cd^2-2d^3)\right)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^4(c+d)^2\sqrt{c^2-d^2}\,f} - \frac{a^3(3Bc(2c+3d)-Ad(2c+5d))\cos(e+fx)}{2d^3(c+d)^2f} + \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^2}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{(Ad(c+4d)-B(3c^2+4cd-2d^2))\cos(e+fx)\left(a^3+a^3\sin(e+fx)\right)}{2d^2(c+d)^2f(c+d\sin(e+fx))}$$

output

```
-a^3*(-A*d+3*B*c-3*B*d)*x/d^4-a^3*(c-d)*(A*d*(2*c^2+6*c*d+7*d^2)-3*B*(2*c^
3+4*c^2*d+c*d^2-2*d^3))*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/d
^4/(c+d)^2/(c^2-d^2)^(1/2)/f-1/2*a^3*(3*B*c*(2*c+3*d)-A*d*(2*c+5*d))*cos(f
*x+e)/d^3/(c+d)^2/f+1/2*a*(-A*d+B*c)*cos(f*x+e)*(a+a*sin(f*x+e))^2/d/(c+d)
/f/(c+d*sin(f*x+e))^2-1/2*(A*d*(c+4*d)-B*(3*c^2+4*c*d-2*d^2))*cos(f*x+e)*(
a^3+a^3*sin(f*x+e))/d^2/(c+d)^2/f/(c+d*sin(f*x+e))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 830 vs. $2(305) = 610$.

Time = 9.03 (sec) , antiderivative size = 830, normalized size of antiderivative = 2.72

$$\int \frac{(a + a\sin(e + fx))^3 (A + B\sin(e + fx))}{(c + d\sin(e + fx))^3}\,dx$$

$$= \frac{a^3(1+\sin(e+fx))^3 \left(\frac{4(c-d)(-Ad(2c^2+6cd+7d^2)+3B(2c^3+4c^2d+cd^2-2d^3))\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} + \frac{-12Bc^5e+4Ac^4de-}{}\right.}{}$$

input
```
Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x
])^3,x]
```

output
```
(a^3*(1 + Sin[e + f*x])^3*((4*(c - d)*(-(A*d*(2*c^2 + 6*c*d + 7*d^2)) + 3*
B*(2*c^3 + 4*c^2*d + c*d^2 - 2*d^3))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[
c^2 - d^2]])/Sqrt[c^2 - d^2] + (-12*B*c^5*e + 4*A*c^4*d*e - 12*B*c^4*d*e +
 8*A*c^3*d^2*e + 6*B*c^3*d^2*e + 6*A*c^2*d^3*e + 6*B*c^2*d^3*e + 4*A*c*d^4
*e + 6*B*c*d^4*e + 2*A*d^5*e + 6*B*d^5*e - 12*B*c^5*f*x + 4*A*c^4*d*f*x -
12*B*c^4*d*f*x + 8*A*c^3*d^2*f*x + 6*B*c^3*d^2*f*x + 6*A*c^2*d^3*f*x + 6*B
*c^2*d^3*f*x + 4*A*c*d^4*f*x + 6*B*c*d^4*f*x + 2*A*d^5*f*x + 6*B*d^5*f*x -
 d*(2*A*d*(-2*c^3 - 4*c^2*d + 5*c*d^2 + d^3) + B*(12*c^4 + 12*c^3*d - 9*c^
2*d^2 + 4*c*d^3 + d^4))*Cos[e + f*x] - 2*d^2*(c + d)^2*(-3*B*c + A*d + 3*B
*d)*(e + f*x)*Cos[2*(e + f*x)] + B*c^2*d^3*Cos[3*(e + f*x)] + 2*B*c*d^4*Co
s[3*(e + f*x)] + B*d^5*Cos[3*(e + f*x)] - 24*B*c^4*d*e*Sin[e + f*x] + 8*A*
c^3*d^2*e*Sin[e + f*x] - 24*B*c^3*d^2*e*Sin[e + f*x] + 16*A*c^2*d^3*e*Sin[
e + f*x] + 24*B*c^2*d^3*e*Sin[e + f*x] + 8*A*c*d^4*e*Sin[e + f*x] + 24*B*c
*d^4*e*Sin[e + f*x] - 24*B*c^4*d*f*x*Sin[e + f*x] + 8*A*c^3*d^2*f*x*Sin[e
+ f*x] - 24*B*c^3*d^2*f*x*Sin[e + f*x] + 16*A*c^2*d^3*f*x*Sin[e + f*x] + 2
4*B*c^2*d^3*f*x*Sin[e + f*x] + 8*A*c*d^4*f*x*Sin[e + f*x] + 24*B*c*d^4*f*x
*Sin[e + f*x] - 9*B*c^3*d^2*Sin[2*(e + f*x)] + 3*A*c^2*d^3*Sin[2*(e + f*x)
] - 9*B*c^2*d^3*Sin[2*(e + f*x)] + 3*A*c*d^4*Sin[2*(e + f*x)] + 4*B*c*d^4*
Sin[2*(e + f*x)] - 6*A*d^5*Sin[2*(e + f*x)] - 2*B*d^5*Sin[2*(e + f*x)])/(c
 + d*Sin[e + f*x])^2))/(4*d^4*(c + d)^2*f*(Cos[(e + f*x)/2] + Sin[(e + ...
```

Rubi [A] (verified)

Time = 1.92 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.08, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.457$, Rules used = {3042, 3454, 25, 3042, 3454, 25, 3042, 3447, 3042, 3502, 3042, 3214, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}dx$$

$$\Big\downarrow \text{3454}$$

$$\frac{\int -\frac{(\sin(e+fx)a+a)^2(2a(B(c-d)-2Ad)-a(3Bc-Ad+2Bd)\sin(e+fx))}{(c+d\sin(e+fx))^2}dx}{2d(c+d)}+\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{2df(c+d)(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{2df(c+d)(c+d\sin(e+fx))^2}-\frac{\int \frac{(\sin(e+fx)a+a)^2(2a(B(c-d)-2Ad)-a(3Bc-Ad+2Bd)\sin(e+fx))}{(c+d\sin(e+fx))^2}dx}{2d(c+d)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{2df(c+d)(c+d\sin(e+fx))^2}-\frac{\int \frac{(\sin(e+fx)a+a)^2(2a(B(c-d)-2Ad)-a(3Bc-Ad+2Bd)\sin(e+fx))}{(c+d\sin(e+fx))^2}dx}{2d(c+d)}$$

$$\Big\downarrow \text{3454}$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{2df(c+d)(c+d\sin(e+fx))^2}-\frac{\frac{\int -\frac{(\sin(e+fx)a+a)\left(\left(Ad(c+7d)-3B\left(c^2+dc-2d^2\right)\right)a^2+(3Bc(2c+3d)-Ad(2c+5d))\sin(e+fx)a^2\right)}{c+d\sin(e+fx)}dx}{d(c+d)}+\frac{\left(Ad(c+4d)-B\left(3c^2+4cd-2d^2\right)\right)\cos(e+fx)(a^3 \text{si}}{df(c+d)(c+d\sin(e+fx))}}{2d(c+d)}$$

$\Big\downarrow$ 25

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{2df(c+d)(c+d\sin(e+fx))^2}-\frac{\frac{\left(Ad(c+4d)-B\left(3c^2+4cd-2d^2\right)\right)\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{df(c+d)(c+d\sin(e+fx))}-\frac{\int\frac{(\sin(e+fx)a+a)\left(\left(Ad(c+7d)-3B\left(c^2+dc-2d^2\right)\right)a^2+(3Bc(2c+3d)-Ad(2c+5d))\sin(\cdots}{c+d\sin(e+fx)}}{d(c+d)}}{2d(c+d)}$$

$\Big\downarrow$ 3042

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{2df(c+d)(c+d\sin(e+fx))^2}-\frac{\frac{\left(Ad(c+4d)-B\left(3c^2+4cd-2d^2\right)\right)\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{df(c+d)(c+d\sin(e+fx))}-\frac{\int\frac{(\sin(e+fx)a+a)\left(\left(Ad(c+7d)-3B\left(c^2+dc-2d^2\right)\right)a^2+(3Bc(2c+3d)-Ad(2c+5d))\sin(\cdots}{c+d\sin(e+fx)}}{d(c+d)}}{2d(c+d)}$$

$\Big\downarrow$ 3447

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{2df(c+d)(c+d\sin(e+fx))^2}-\frac{\frac{\left(Ad(c+4d)-B\left(3c^2+4cd-2d^2\right)\right)\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{df(c+d)(c+d\sin(e+fx))}-\frac{\int\frac{(3Bc(2c+3d)-Ad(2c+5d))\sin^2(e+fx)a^3+\left(Ad(c+7d)-3B\left(c^2+dc-2d^2\right)\right)a^3+\left((3B\cdots}{c+d\sin(e+f\cdots}}{d(c+d)}}{2d(c+d)}$$

$\Big\downarrow$ 3042

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{2df(c+d)(c+d\sin(e+fx))^2}-\frac{\frac{\left(Ad(c+4d)-B\left(3c^2+4cd-2d^2\right)\right)\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{df(c+d)(c+d\sin(e+fx))}-\frac{\int\frac{(3Bc(2c+3d)-Ad(2c+5d))\sin(e+fx)^2a^3+\left(Ad(c+7d)-3B\left(c^2+dc-2d^2\right)\right)a^3+\left((3B\cdots}{c+d\sin(e+f\cdots}}{d(c+d)}}{2d(c+d)}$$

$\Big\downarrow$ 3502

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{2df(c+d)(c+d\sin(e+fx))^2}-\frac{\frac{\left(Ad(c+4d)-B\left(3c^2+4cd-2d^2\right)\right)\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{df(c+d)(c+d\sin(e+fx))}-\frac{\frac{\int\frac{a^3d\left(Ad(c+7d)-3B\left(c^2+dc-2d^2\right)\right)-2a^3(c+d)^2(3B(c-d)-Ad)\sin(e+fx)}{c+d\sin(e+fx)}dx}{d}-\frac{a^3(3\cdots}{\cdots}}{d(c+d)}}{2d(c+d)}$$

$\Big\downarrow$ 3042

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{2df(c+d)(c+d\sin(e+fx))^2}-\frac{\frac{\left(Ad(c+4d)-B\left(3c^2+4cd-2d^2\right)\right)\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{df(c+d)(c+d\sin(e+fx))}-\frac{\frac{\int\frac{a^3d\left(Ad(c+7d)-3B\left(c^2+dc-2d^2\right)\right)-2a^3(c+d)^2(3B(c-d)-Ad)\sin(e+fx)}{c+d\sin(e+fx)}dx}{d}-\frac{a^3(3\cdots}{\cdots}}{d(c+d)}}{2d(c+d)}$$

$\downarrow$ 3214

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{2df(c+d)(c+d\sin(e+fx))^2}-\frac{\frac{\left(Ad(c+4d)-B\left(3c^2+4cd-2d^2\right)\right)\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{df(c+d)(c+d\sin(e+fx))}-\frac{-\frac{\frac{a^3(c-d)\left(Ad\left(2c^2+6cd+7d^2\right)-3B\left(2c^3+4c^2d+cd^2-2d^3\right)\right)\int\frac{1}{c+d\sin(e+fx)}dx}{d}-2a^3}{d}}{d(c+d)}}{2d(c+d)}$$

$\downarrow$ 3042

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{2df(c+d)(c+d\sin(e+fx))^2}-\frac{\frac{\left(Ad(c+4d)-B\left(3c^2+4cd-2d^2\right)\right)\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{df(c+d)(c+d\sin(e+fx))}-\frac{-\frac{\frac{a^3(c-d)\left(Ad\left(2c^2+6cd+7d^2\right)-3B\left(2c^3+4c^2d+cd^2-2d^3\right)\right)\int\frac{1}{c+d\sin(e+fx)}dx}{d}-2a^3}{d}}{d(c+d)}}{2d(c+d)}$$

$\downarrow$ 3139

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{2df(c+d)(c+d\sin(e+fx))^2}-\frac{\frac{\left(Ad(c+4d)-B\left(3c^2+4cd-2d^2\right)\right)\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{df(c+d)(c+d\sin(e+fx))}-\frac{-\frac{2a^3(c-d)\left(Ad\left(2c^2+6cd+7d^2\right)-3B\left(2c^3+4c^2d+cd^2-2d^3\right)\right)\int\frac{1}{c\tan^2\left(\frac{1}{2}(e+fx)\right)+2d}}{df}}{d}}{2d(c+d)}$$

$\downarrow$ 1083

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{2df(c+d)(c+d\sin(e+fx))^2}-\frac{\frac{\left(Ad(c+4d)-B\left(3c^2+4cd-2d^2\right)\right)\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{df(c+d)(c+d\sin(e+fx))}-\frac{\frac{4a^3(c-d)\left(Ad\left(2c^2+6cd+7d^2\right)-3B\left(2c^3+4c^2d+cd^2-2d^3\right)\right)\int\frac{1}{-\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right.}}{df}}{d}}{2d(c+d)}$$

$\downarrow$ 217

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^2}{2df(c+d)(c+d\sin(e+fx))^2}-\frac{\frac{\left(Ad(c+4d)-B\left(3c^2+4cd-2d^2\right)\right)\cos(e+fx)\left(a^3\sin(e+fx)+a^3\right)}{df(c+d)(c+d\sin(e+fx))}-\frac{-\frac{\frac{2a^3(c-d)\left(Ad\left(2c^2+6cd+7d^2\right)-3B\left(2c^3+4c^2d+cd^2-2d^3\right)\right)\arctan\left(\frac{2c\tan\left(\frac{1}{2}(e+fx\right.}{2\sqrt{c^2-d^2}}\right.}{df\sqrt{c^2-d^2}}}{d}}{d(c+}}{2d(c+d)}$$

input
```
Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^3,x
]
```

output
```
(a*(B*c - A*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^2)/(2*d*(c + d)*f*(c + d*
Sin[e + f*x])^2) - (-((((-2*a^3*(c + d)^2*(3*B*c - A*d - 3*B*d)*x)/d - (2*
a^3*(c - d)*(A*d*(2*c^2 + 6*c*d + 7*d^2) - 3*B*(2*c^3 + 4*c^2*d + c*d^2 -
2*d^3))*ArcTan[(2*d + 2*c*Tan[(e + f*x)/2])/(2*Sqrt[c^2 - d^2])])/(d*Sqrt[
c^2 - d^2]*f))/d - (a^3*(3*B*c*(2*c + 3*d) - A*d*(2*c + 5*d))*Cos[e + f*x]
)/(d*f))/(d*(c + d))) + ((A*d*(c + 4*d) - B*(3*c^2 + 4*c*d - 2*d^2))*Cos[e
 + f*x]*(a^3 + a^3*Sin[e + f*x]))/(d*(c + d)*f*(c + d*Sin[e + f*x])))/(2*d
*(c + d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 2.95 (sec) , antiderivative size = 586, normalized size of antiderivative = 1.92

method	result
derivativedivides	$2a^3\left(\frac{-\frac{Bd}{1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}+(Ad-3Bc+3Bd)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d^4}-\frac{-\frac{d^2\left(A\,c^3d+5A\,c^2d^2-4Ac\,d^3-2A\,d^4-3B\,c^4-3B\,c^3d+6B\,c^2\right.}{2\left(c^2+2cd+d^2\right)c}}{}\right.$
default	$2a^3\left(\frac{-\frac{Bd}{1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}+(Ad-3Bc+3Bd)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{d^4}-\frac{-\frac{d^2\left(A\,c^3d+5A\,c^2d^2-4Ac\,d^3-2A\,d^4-3B\,c^4-3B\,c^3d+6B\,c^2\right.}{2\left(c^2+2cd+d^2\right)c}}{}\right.$
risch	Expression too large to display

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x,method=_RETUR
NVERBOSE)
```

output
```
2/f*a^3*(1/d^4*(-B*d/(1+tan(1/2*f*x+1/2*e)^2)+(A*d-3*B*c+3*B*d)*arctan(tan
(1/2*f*x+1/2*e)))-1/d^4*((-1/2*d^2*(A*c^3*d+5*A*c^2*d^2-4*A*c*d^3-2*A*d^4-
3*B*c^4-3*B*c^3*d+6*B*c^2*d^2)/(c^2+2*c*d+d^2)/c*tan(1/2*f*x+1/2*e)^3-1/2*
d*(2*A*c^5*d+4*A*c^4*d^2-A*c^3*d^3+7*A*c^2*d^4-10*A*c*d^5-2*A*d^6-4*B*c^6-
2*B*c^5*d-B*c^4*d^2-5*B*c^3*d^3+14*B*c^2*d^4-2*B*c*d^5)/(c^2+2*c*d+d^2)/c^
2*tan(1/2*f*x+1/2*e)^2-1/2*d^2*(7*A*c^3*d+11*A*c^2*d^2-16*A*c*d^3-2*A*d^4-
13*B*c^4-5*B*c^3*d+22*B*c^2*d^2-4*B*c*d^3)/c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1
/2*e)-1/2*d*(2*A*c^3*d+4*A*c^2*d^2-5*A*c*d^3-A*d^4-4*B*c^4-2*B*c^3*d+7*B*c
^2*d^2-B*c*d^3)/(c^2+2*c*d+d^2))/(tan(1/2*f*x+1/2*e)^2*c+2*d*tan(1/2*f*x+1
/2*e)+c)^2+1/2*(2*A*c^3*d+4*A*c^2*d^2+A*c*d^3-7*A*d^4-6*B*c^4-6*B*c^3*d+9*
B*c^2*d^2+9*B*c*d^3-6*B*d^4)/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2
*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 790 vs. $2(294) = 588$.

Time = 0.17 (sec) , antiderivative size = 1670, normalized size of antiderivative = 5.48

$$\int \frac{(a + a\sin(e + fx))^3(A + B\sin(e + fx))}{(c + d\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algori
thm="fricas")
```

output
```
[-1/4*(4*(3*B*a^3*c^3*d^2 - (A - 3*B)*a^3*c^2*d^3 - (2*A + 3*B)*a^3*c*d^4
- (A + 3*B)*a^3*d^5)*f*x*cos(f*x + e)^2 + 4*(B*a^3*c^2*d^3 + 2*B*a^3*c*d^4
 + B*a^3*d^5)*cos(f*x + e)^3 - 4*(3*B*a^3*c^5 - (A - 3*B)*a^3*c^4*d - 2*A*
a^3*c^3*d^2 - 2*A*a^3*c^2*d^3 - (2*A + 3*B)*a^3*c*d^4 - (A + 3*B)*a^3*d^5)
*f*x - (6*B*a^3*c^5 - 2*(A - 6*B)*a^3*c^4*d - 3*(2*A - 3*B)*a^3*c^3*d^2 -
3*(3*A - 2*B)*a^3*c^2*d^3 - 3*(2*A - B)*a^3*c*d^4 - (7*A + 6*B)*a^3*d^5 -
(6*B*a^3*c^3*d^2 - 2*(A - 6*B)*a^3*c^2*d^3 - 3*(2*A - B)*a^3*c*d^4 - (7*A
+ 6*B)*a^3*d^5)*cos(f*x + e)^2 + 2*(6*B*a^3*c^4*d - 2*(A - 6*B)*a^3*c^3*d^
2 - 3*(2*A - B)*a^3*c^2*d^3 - (7*A + 6*B)*a^3*c*d^4)*sin(f*x + e))*sqrt(-(
c - d)/(c + d))*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c
^2 - d^2 + 2*((c^2 + c*d)*cos(f*x + e)*sin(f*x + e) + (c*d + d^2)*cos(f*x
+ e))*sqrt(-(c - d)/(c + d)))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c
^2 - d^2)) - 2*(6*B*a^3*c^4*d - 2*(A - 3*B)*a^3*c^3*d^2 - (4*A + 3*B)*a^3*
c^2*d^3 + 5*(A + B)*a^3*c*d^4 + (A + 2*B)*a^3*d^5)*cos(f*x + e) - 2*(4*(3*
B*a^3*c^4*d - (A - 3*B)*a^3*c^3*d^2 - (2*A + 3*B)*a^3*c^2*d^3 - (A + 3*B)*
a^3*c*d^4)*f*x + (9*B*a^3*c^3*d^2 - 3*(A - 3*B)*a^3*c^2*d^3 - (3*A + 4*B)*
a^3*c*d^4 + 2*(3*A + B)*a^3*d^5)*cos(f*x + e))*sin(f*x + e))/((c^2*d^6 + 2
*c*d^7 + d^8)*f*cos(f*x + e)^2 - 2*(c^3*d^5 + 2*c^2*d^6 + c*d^7)*f*sin(f*x
 + e) - (c^4*d^4 + 2*c^3*d^5 + 2*c^2*d^6 + 2*c*d^7 + d^8)*f), -1/2*(2*(3*B
*a^3*c^3*d^2 - (A - 3*B)*a^3*c^2*d^3 - (2*A + 3*B)*a^3*c*d^4 - (A + 3*B...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algori
thm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 953 vs. 2(294) = 588.

Time = 0.28 (sec) , antiderivative size = 953, normalized size of antiderivative = 3.12

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algori
thm="giac")
```

output
```
((6*B*a^3*c^4 - 2*A*a^3*c^3*d + 6*B*a^3*c^3*d - 4*A*a^3*c^2*d^2 - 9*B*a^3*
c^2*d^2 - A*a^3*c*d^3 - 9*B*a^3*c*d^3 + 7*A*a^3*d^4 + 6*B*a^3*d^4)*(pi*flo
or(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sq
rt(c^2 - d^2)))/((c^2*d^4 + 2*c*d^5 + d^6)*sqrt(c^2 - d^2)) - 2*B*a^3/((ta
n(1/2*f*x + 1/2*e)^2 + 1)*d^3) - (3*B*a^3*c^5*d*tan(1/2*f*x + 1/2*e)^3 - A
*a^3*c^4*d^2*tan(1/2*f*x + 1/2*e)^3 + 3*B*a^3*c^4*d^2*tan(1/2*f*x + 1/2*e)
^3 - 5*A*a^3*c^3*d^3*tan(1/2*f*x + 1/2*e)^3 - 6*B*a^3*c^3*d^3*tan(1/2*f*x
+ 1/2*e)^3 + 4*A*a^3*c^2*d^4*tan(1/2*f*x + 1/2*e)^3 + 2*A*a^3*c*d^5*tan(1/
2*f*x + 1/2*e)^3 + 4*B*a^3*c^6*tan(1/2*f*x + 1/2*e)^2 - 2*A*a^3*c^5*d*tan(
1/2*f*x + 1/2*e)^2 + 2*B*a^3*c^5*d*tan(1/2*f*x + 1/2*e)^2 - 4*A*a^3*c^4*d^
2*tan(1/2*f*x + 1/2*e)^2 + B*a^3*c^4*d^2*tan(1/2*f*x + 1/2*e)^2 + A*a^3*c^
3*d^3*tan(1/2*f*x + 1/2*e)^2 + 5*B*a^3*c^3*d^3*tan(1/2*f*x + 1/2*e)^2 - 7*
A*a^3*c^2*d^4*tan(1/2*f*x + 1/2*e)^2 - 14*B*a^3*c^2*d^4*tan(1/2*f*x + 1/2*
e)^2 + 10*A*a^3*c*d^5*tan(1/2*f*x + 1/2*e)^2 + 2*B*a^3*c*d^5*tan(1/2*f*x +
 1/2*e)^2 + 2*A*a^3*d^6*tan(1/2*f*x + 1/2*e)^2 + 13*B*a^3*c^5*d*tan(1/2*f*
x + 1/2*e) - 7*A*a^3*c^4*d^2*tan(1/2*f*x + 1/2*e) + 5*B*a^3*c^4*d^2*tan(1/
2*f*x + 1/2*e) - 11*A*a^3*c^3*d^3*tan(1/2*f*x + 1/2*e) - 22*B*a^3*c^3*d^3*
tan(1/2*f*x + 1/2*e) + 16*A*a^3*c^2*d^4*tan(1/2*f*x + 1/2*e) + 4*B*a^3*c^2
*d^4*tan(1/2*f*x + 1/2*e) + 2*A*a^3*c*d^5*tan(1/2*f*x + 1/2*e) + 4*B*a^3*c
^6 - 2*A*a^3*c^5*d + 2*B*a^3*c^5*d - 4*A*a^3*c^4*d^2 - 7*B*a^3*c^4*d^2 ...
```

Mupad [B] (verification not implemented)

Time = 45.75 (sec) , antiderivative size = 13891, normalized size of antiderivative = 45.54

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c + d*sin(e + f*x))^3,x
)
```

output
```
- ((A*a^3*d^4 + 6*B*a^3*c^4 + 5*A*a^3*c*d^3 - 2*A*a^3*c^3*d + B*a^3*c*d^3
+ 6*B*a^3*c^3*d - 4*A*a^3*c^2*d^2 - 5*B*a^3*c^2*d^2)/(d^3*(c + d)^2) + (4*
tan(e/2 + (f*x)/2)^3*(A*a^3*d^4 + 6*B*a^3*c^4 + 5*A*a^3*c*d^3 - 2*A*a^3*c^
3*d + B*a^3*c*d^3 + 6*B*a^3*c^3*d - 4*A*a^3*c^2*d^2 - 5*B*a^3*c^2*d^2))/(c
*d^2*(c + d)^2) + (tan(e/2 + (f*x)/2)^5*(2*A*a^3*d^4 + 3*B*a^3*c^4 + 4*A*a
^3*c*d^3 - A*a^3*c^3*d + 3*B*a^3*c^3*d - 5*A*a^3*c^2*d^2 - 6*B*a^3*c^2*d^2
))/(c*d^2*(c + d)^2) + (2*tan(e/2 + (f*x)/2)^2*(A*a^3*d^6 + 6*B*a^3*c^6 +
5*A*a^3*c*d^5 - 2*A*a^3*c^5*d + B*a^3*c*d^5 + 6*B*a^3*c^5*d - 3*A*a^3*c^2*
d^4 + 3*A*a^3*c^3*d^3 - 4*A*a^3*c^4*d^2 - 3*B*a^3*c^2*d^4 + 11*B*a^3*c^3*d
^3 + 3*B*a^3*c^4*d^2))/(c^2*d^3*(c + d)^2) + (tan(e/2 + (f*x)/2)^4*(2*A*a^
3*d^6 + 6*B*a^3*c^6 + 10*A*a^3*c*d^5 - 2*A*a^3*c^5*d + 2*B*a^3*c*d^5 + 6*B
*a^3*c^5*d - 7*A*a^3*c^2*d^4 + A*a^3*c^3*d^3 - 4*A*a^3*c^4*d^2 - 14*B*a^3*
c^2*d^4 + 5*B*a^3*c^3*d^3 + 3*B*a^3*c^4*d^2))/(c^2*d^3*(c + d)^2) + (tan(e
/2 + (f*x)/2)*(2*A*a^3*d^4 + 21*B*a^3*c^4 + 16*A*a^3*c*d^3 - 7*A*a^3*c^3*d
 + 4*B*a^3*c*d^3 + 21*B*a^3*c^3*d - 11*A*a^3*c^2*d^2 - 14*B*a^3*c^2*d^2))/
(c*d^2*(c + d)^2))/(f*(tan(e/2 + (f*x)/2)^2*(3*c^2 + 4*d^2) + tan(e/2 + (f
*x)/2)^4*(3*c^2 + 4*d^2) + c^2*tan(e/2 + (f*x)/2)^6 + c^2 + 8*c*d*tan(e/2
+ (f*x)/2)^3 + 4*c*d*tan(e/2 + (f*x)/2)^5 + 4*c*d*tan(e/2 + (f*x)/2))) - (
atan((((B*a^3*c*3i - a^3*d*(A + 3*B)*1i)*((8*(4*A^2*a^6*c^2*d^9 + 16*A^2*a
^6*c^3*d^8 + 24*A^2*a^6*c^4*d^7 + 16*A^2*a^6*c^5*d^6 + 4*A^2*a^6*c^6*d^...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 2339, normalized size of antiderivative = 7.67

$$\int \frac{(a+a\sin(e+fx))^3(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)
```

output
```
(a**3*( - 8*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d*
*2))*sin(e + f*x)**2*a*c**3*d**3 - 24*sqrt(c**2 - d**2)*atan((tan((e + f*x
)/2)*c + d)/sqrt(c**2 - d**2))*sin(e + f*x)**2*a*c**2*d**4 - 28*sqrt(c**2
- d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*sin(e + f*x)**2*a
*c*d**5 + 24*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d
**2))*sin(e + f*x)**2*b*c**4*d**2 + 48*sqrt(c**2 - d**2)*atan((tan((e + f*
x)/2)*c + d)/sqrt(c**2 - d**2))*sin(e + f*x)**2*b*c**3*d**3 + 12*sqrt(c**2
 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*sin(e + f*x)**2*
b*c**2*d**4 - 24*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2
 - d**2))*sin(e + f*x)**2*b*c*d**5 - 16*sqrt(c**2 - d**2)*atan((tan((e + f
*x)/2)*c + d)/sqrt(c**2 - d**2))*sin(e + f*x)*a*c**4*d**2 - 48*sqrt(c**2 -
 d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*sin(e + f*x)*a*c**
3*d**3 - 56*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d*
*2))*sin(e + f*x)*a*c**2*d**4 + 48*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2
)*c + d)/sqrt(c**2 - d**2))*sin(e + f*x)*b*c**5*d + 96*sqrt(c**2 - d**2)*a
tan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*sin(e + f*x)*b*c**4*d**2 +
 24*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*sin
(e + f*x)*b*c**3*d**3 - 48*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)
/sqrt(c**2 - d**2))*sin(e + f*x)*b*c**2*d**4 - 8*sqrt(c**2 - d**2)*atan((t
an((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*a*c**5*d - 24*sqrt(c**2 - d**...
```

3.265 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 220

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx = \frac{(3Ad(2c^2-2cd+d^2)+B(2c^3-6c^2d+9cd^2-3d^3))\,x}{2a} + \frac{2d(3A(c^2-3cd+d^2)-B(7c^2-9cd+4d^2))\cos(e+fx)}{3af} + \frac{d^2(6Ac-11Bc-9Ad+9Bd)\cos(e+fx)\sin(e+fx)}{6af} + \frac{(3A-4B)d\cos(e+fx)(c+d\sin(e+fx))^2}{3af} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{f(a+a\sin(e+fx))}$$

output
```
1/2*(3*A*d*(2*c^2-2*c*d+d^2)+B*(2*c^3-6*c^2*d+9*c*d^2-3*d^3))*x/a+2/3*d*(3
*A*(c^2-3*c*d+d^2)-B*(7*c^2-9*c*d+4*d^2))*cos(f*x+e)/a/f+1/6*d^2*(6*A*c-9*
A*d-11*B*c+9*B*d)*cos(f*x+e)*sin(f*x+e)/a/f+1/3*(3*A-4*B)*d*cos(f*x+e)*(c+
d*sin(f*x+e))^2/a/f-(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))^3/f/(a+a*sin(f*x+e))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 788 vs. $2(220) = 440$.

Time = 7.89 (sec) , antiderivative size = 788, normalized size of antiderivative = 3.58

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x
]),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(3*(4*A*d*(6*c^2*(e + f*x) - 3*c*d*
(1 + 2*e + 2*f*x) + d^2*(1 + 3*e + 3*f*x)) + B*(8*c^3*(e + f*x) - 12*c^2*d
*(1 + 2*e + 2*f*x) + 12*c*d^2*(1 + 3*e + 3*f*x) - d^3*(7 + 12*e + 12*f*x))
)*Cos[(e + f*x)/2] + 9*d*(A*d*(-4*c + d) + B*(-4*c^2 + 3*c*d - 2*d^2))*Cos
[(3*(e + f*x))/2] + 9*B*c*d^2*Cos[(5*(e + f*x))/2] + 3*A*d^3*Cos[(5*(e + f
*x))/2] - 2*B*d^3*Cos[(5*(e + f*x))/2] + B*d^3*Cos[(7*(e + f*x))/2] + 48*A
*c^3*Sin[(e + f*x)/2] - 48*B*c^3*Sin[(e + f*x)/2] - 144*A*c^2*d*Sin[(e + f
*x)/2] + 180*B*c^2*d*Sin[(e + f*x)/2] + 180*A*c*d^2*Sin[(e + f*x)/2] - 180
*B*c*d^2*Sin[(e + f*x)/2] - 60*A*d^3*Sin[(e + f*x)/2] + 69*B*d^3*Sin[(e +
f*x)/2] + 24*B*c^3*e*Sin[(e + f*x)/2] + 72*A*c^2*d*e*Sin[(e + f*x)/2] - 72
*B*c^2*d*e*Sin[(e + f*x)/2] - 72*A*c*d^2*e*Sin[(e + f*x)/2] + 108*B*c*d^2*
e*Sin[(e + f*x)/2] + 36*A*d^3*e*Sin[(e + f*x)/2] - 36*B*d^3*e*Sin[(e + f*x
)/2] + 24*B*c^3*f*x*Sin[(e + f*x)/2] + 72*A*c^2*d*f*x*Sin[(e + f*x)/2] - 7
2*B*c^2*d*f*x*Sin[(e + f*x)/2] - 72*A*c*d^2*f*x*Sin[(e + f*x)/2] + 108*B*c
*d^2*f*x*Sin[(e + f*x)/2] + 36*A*d^3*f*x*Sin[(e + f*x)/2] - 36*B*d^3*f*x*S
in[(e + f*x)/2] - 36*B*c^2*d*Sin[(3*(e + f*x))/2] - 36*A*c*d^2*Sin[(3*(e +
 f*x))/2] + 27*B*c*d^2*Sin[(3*(e + f*x))/2] + 9*A*d^3*Sin[(3*(e + f*x))/2]
 - 18*B*d^3*Sin[(3*(e + f*x))/2] - 9*B*c*d^2*Sin[(5*(e + f*x))/2] - 3*A*d^
3*Sin[(5*(e + f*x))/2] + 2*B*d^3*Sin[(5*(e + f*x))/2] + B*d^3*Sin[(7*(e +
f*x))/2]))/(24*a*f*(1 + Sin[e + f*x]))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3042, 3456, 3042, 3232, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{a\sin(e+fx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{a\sin(e+fx)+a}dx$$

$$\Big\downarrow \text{3456}$$

$$\frac{\int (c+d\sin(e+fx))^2(a(B(c-3d)+3Ad)-a(3A-4B)d\sin(e+fx))dx}{a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{f(a\sin(e+fx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (c+d\sin(e+fx))^2(a(B(c-3d)+3Ad)-a(3A-4B)d\sin(e+fx))dx}{a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{f(a\sin(e+fx)+a)}$$

$$\Big\downarrow \text{3232}$$

$$\frac{\frac{1}{3}\int (c+d\sin(e+fx))\left(a\left(3A(3c-2d)d+B\left(3c^2-9dc+8d^2\right)\right)-ad(6Ac-11Bc-9Ad+9Bd)\sin(e+fx)\right)dx}{a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{f(a\sin(e+fx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{3}\int (c+d\sin(e+fx))\left(a\left(3A(3c-2d)d+B\left(3c^2-9dc+8d^2\right)\right)-ad(6Ac-11Bc-9Ad+9Bd)\sin(e+fx)\right)dx}{a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{f(a\sin(e+fx)+a)}$$

$$\Big\downarrow \text{3213}$$

$$\frac{\frac{1}{3}\left(\frac{2ad(3A(c^2-3cd+d^2)-B(7c^2-9cd+4d^2))\cos(e+fx)}{f}+\frac{3}{2}ax\left(3Ad\left(2c^2-2cd+d^2\right)+B\left(2c^3-6c^2d+9cd^2-3d^3\right)\right)+\frac{ad^2}{}\right.}{a^2}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{f(a\sin(e+fx)+a)}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]),x]
```

output
```
-(((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(f*(a + a*Sin[e + f*x])))
+ ((a*(3*A - 4*B)*d*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(3*f) + ((3*a*(3*
A*d*(2*c^2 - 2*c*d + d^2) + B*(2*c^3 - 6*c^2*d + 9*c*d^2 - 3*d^3))*x)/2 +
(2*a*d*(3*A*(c^2 - 3*c*d + d^2) - B*(7*c^2 - 9*c*d + 4*d^2))*Cos[e + f*x])
/f + (a*d^2*(6*A*c - 11*B*c - 9*A*d + 9*B*d)*Cos[e + f*x]*Sin[e + f*x])/(2
*f))/3)/a^2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.53

method	result
derivativedivides	$\frac{2\left(\left(\frac{1}{2}A\,d^{3}+\frac{3}{2}Bc\,d^{2}-\frac{1}{2}B\,d^{3}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}+\left(-3Ac\,d^{2}+A\,d^{3}-3B\,c^{2}d+3Bc\,d^{2}-B\,d^{3}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}+\left(-6Ac\,d^{2}+2A\,d^{3}-6B\,c^{2}d+\right.\right.}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\right.}$
default	$\frac{2\left(\left(\frac{1}{2}A\,d^{3}+\frac{3}{2}Bc\,d^{2}-\frac{1}{2}B\,d^{3}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}+\left(-3Ac\,d^{2}+A\,d^{3}-3B\,c^{2}d+3Bc\,d^{2}-B\,d^{3}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}+\left(-6Ac\,d^{2}+2A\,d^{3}-6B\,c^{2}d+\right.\right.}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\right.}$
parallelrisch	$\left((36fxA-36fxB+84A-109B)d^{3}-72c\left(\frac{(-3fx-7)B}{2}+A\left(fx+\frac{7}{2}\right)\right)d^{2}+72c^{2}\left(\left(-fx-\frac{7}{2}\right)B+A(fx+2)\right)d-48c^{3}\left(\left(-\frac{fx}{2}-\right.\right.\right.$
risch	$\frac{3xA\,d^{3}}{2a}+\frac{xB\,c^{3}}{a}-\frac{3xB\,d^{3}}{2a}-\frac{3d^{2}\sin(2fx+2e)Bc}{4af}-\frac{d^{3}\sin(2fx+2e)A}{4af}+\frac{d^{3}\sin(2fx+2e)B}{4af}+\frac{B\,d^{3}\cos(3fx+}{12af}$
norman	Expression too large to display

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e)),x,method=_RETURNV
ERBOSE)
```

output
```
2/f/a*(((1/2*A*d^3+3/2*B*c*d^2-1/2*B*d^3)*tan(1/2*f*x+1/2*e)^5+(-3*A*c*d^2
+A*d^3-3*B*c^2*d+3*B*c*d^2-B*d^3)*tan(1/2*f*x+1/2*e)^4+(-6*A*c*d^2+2*A*d^3
-6*B*c^2*d+6*B*c*d^2-4*B*d^3)*tan(1/2*f*x+1/2*e)^2+(-1/2*A*d^3-3/2*B*c*d^2
+1/2*B*d^3)*tan(1/2*f*x+1/2*e)-3*A*c*d^2+A*d^3-3*B*c^2*d+3*B*c*d^2-5/3*B*d
^3)/(1+tan(1/2*f*x+1/2*e)^2)^3+1/2*(6*A*c^2*d-6*A*c*d^2+3*A*d^3+2*B*c^3-6*
B*c^2*d+9*B*c*d^2-3*B*d^3)*arctan(tan(1/2*f*x+1/2*e))-(A*c^3-3*A*c^2*d+3*A
*c*d^2-A*d^3-B*c^3+3*B*c^2*d-3*B*c*d^2+B*d^3)/(tan(1/2*f*x+1/2*e)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 470 vs. $2(212) = 424$.

Time = 0.15 (sec) , antiderivative size = 470, normalized size of antiderivative = 2.14

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx$$
$$= \frac{2\,Bd^3\cos{(fx+e)}^4 - 6\,(A-B)c^3 + 18\,(A-B)c^2d - 18\,(A-B)cd^2 + 6\,(A-B)d^3 + (9\,Bcd^2 + (3\,A-}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
1/6*(2*B*d^3*cos(f*x + e)^4 - 6*(A - B)*c^3 + 18*(A - B)*c^2*d - 18*(A - B
)*c*d^2 + 6*(A - B)*d^3 + (9*B*c*d^2 + (3*A - B)*d^3)*cos(f*x + e)^3 + 3*(
2*B*c^3 + 6*(A - B)*c^2*d - 3*(2*A - 3*B)*c*d^2 + 3*(A - B)*d^3)*f*x - 6*(
3*B*c^2*d + 3*(A - B)*c*d^2 - (A - 2*B)*d^3)*cos(f*x + e)^2 - 3*(2*(A - B)
*c^3 - 6*(A - 2*B)*c^2*d + 3*(4*A - 3*B)*c*d^2 - (3*A - 5*B)*d^3 - (2*B*c^
3 + 6*(A - B)*c^2*d - 3*(2*A - 3*B)*c*d^2 + 3*(A - B)*d^3)*f*x)*cos(f*x +
e) + (2*B*d^3*cos(f*x + e)^3 + 6*(A - B)*c^3 - 18*(A - B)*c^2*d + 18*(A -
B)*c*d^2 - 6*(A - B)*d^3 + 3*(2*B*c^3 + 6*(A - B)*c^2*d - 3*(2*A - 3*B)*c*
d^2 + 3*(A - B)*d^3)*f*x - 3*(3*B*c*d^2 + (A - B)*d^3)*cos(f*x + e)^2 - 3*
(6*B*c^2*d + 3*(2*A - B)*c*d^2 - (A - 3*B)*d^3)*cos(f*x + e))*sin(f*x + e)
)/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14644 vs. $2(204) = 408$.

Time = 4.31 (sec) , antiderivative size = 14644, normalized size of antiderivative = 66.56

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^3}{a + a\sin(e + fx)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e)),x)
```

output
```
Piecewise((-12*A*c**3*tan(e/2 + f*x/2)**6/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a
*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x
/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*t
an(e/2 + f*x/2) + 6*a*f) - 36*A*c**3*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 +
f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*
f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/
2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 36*A*c**3*tan(e/2 + f*x/2)**2/(6
*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*
x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f
*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) - 12*A*c**3/(6*a*f*
tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)*
*5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(
e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 18*A*c**2*d*f*x*tan(e/
2 + f*x/2)**7/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*
a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*
x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 1
8*A*c**2*d*f*x*tan(e/2 + f*x/2)**6/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(
e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4
+ 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2
+ f*x/2) + 6*a*f) + 54*A*c**2*d*f*x*tan(e/2 + f*x/2)**5/(6*a*f*tan(e/2 ...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1124 vs. 2(212) = 424.

Time = 0.14 (sec) , antiderivative size = 1124, normalized size of antiderivative = 5.11

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorith
m="maxima")
```

output
```
-1/3*(B*d^3*((7*sin(f*x + e)/(cos(f*x + e) + 1) + 39*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 24*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 24*sin(f*x + e)^
4/(cos(f*x + e) + 1)^4 + 9*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 9*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6 + 16)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1)
+ 3*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*a*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 3*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3*a*sin(f*x + e)^5/(
cos(f*x + e) + 1)^5 + a*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a*sin(f*x +
e)^7/(cos(f*x + e) + 1)^7) + 9*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a)
- 9*B*c*d^2*((sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 4)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1
)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 3*A*d^3*((sin(f*x +
e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sin(f*x
+ e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 4)/(
a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x + e)^4/(cos(f*
x + e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x
+ e)/(cos(f*x + e) + 1))/a) + 18*B*c^2*d*((sin(f*x + e)/(cos(f*x + e) +...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 460 vs. $2(212) = 424$.

Time = 0.22 (sec) , antiderivative size = 460, normalized size of antiderivative = 2.09

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx$$

$$= \frac{\frac{3\left(2\,Bc^3+6\,Ac^2d-6\,Bc^2d-6\,Acd^2+9\,Bcd^2+3\,Ad^3-3\,Bd^3\right)(fx+e)}{a} - \frac{12\left(Ac^3-Bc^3-3\,Ac^2d+3\,Bc^2d+3\,Acd^2-3\,Bcd^2-Ad^3+Bd^3\right)}{a\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)} + \frac{2\left(9\,B\right.}{}}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorith
m="giac")
```

output
```
1/6*(3*(2*B*c^3 + 6*A*c^2*d - 6*B*c^2*d - 6*A*c*d^2 + 9*B*c*d^2 + 3*A*d^3
- 3*B*d^3)*(f*x + e)/a - 12*(A*c^3 - B*c^3 - 3*A*c^2*d + 3*B*c^2*d + 3*A*c
*d^2 - 3*B*c*d^2 - A*d^3 + B*d^3)/(a*(tan(1/2*f*x + 1/2*e) + 1)) + 2*(9*B*
c*d^2*tan(1/2*f*x + 1/2*e)^5 + 3*A*d^3*tan(1/2*f*x + 1/2*e)^5 - 3*B*d^3*ta
n(1/2*f*x + 1/2*e)^5 - 18*B*c^2*d*tan(1/2*f*x + 1/2*e)^4 - 18*A*c*d^2*tan(
1/2*f*x + 1/2*e)^4 + 18*B*c*d^2*tan(1/2*f*x + 1/2*e)^4 + 6*A*d^3*tan(1/2*f
*x + 1/2*e)^4 - 6*B*d^3*tan(1/2*f*x + 1/2*e)^4 - 36*B*c^2*d*tan(1/2*f*x +
1/2*e)^2 - 36*A*c*d^2*tan(1/2*f*x + 1/2*e)^2 + 36*B*c*d^2*tan(1/2*f*x + 1/
2*e)^2 + 12*A*d^3*tan(1/2*f*x + 1/2*e)^2 - 24*B*d^3*tan(1/2*f*x + 1/2*e)^2
 - 9*B*c*d^2*tan(1/2*f*x + 1/2*e) - 3*A*d^3*tan(1/2*f*x + 1/2*e) + 3*B*d^3
*tan(1/2*f*x + 1/2*e) - 18*B*c^2*d - 18*A*c*d^2 + 18*B*c*d^2 + 6*A*d^3 - 1
0*B*d^3)/((tan(1/2*f*x + 1/2*e)^2 + 1)^3*a))/f
```

Mupad [B] (verification not implemented)

Time = 38.14 (sec) , antiderivative size = 839, normalized size of antiderivative = 3.81

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{a+a\sin(e+fx)}\,dx = \text{Too large to display}$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x)),x)
```

output
```
-(12*A*c^3*cos(e/2 + (f*x)/2) - 18*A*d^3*cos(e/2 + (f*x)/2) - 12*B*c^3*cos
(e/2 + (f*x)/2) + 18*B*d^3*cos(e/2 + (f*x)/2) + 6*A*d^3*cos(e/2 + (f*x)/2)
^3 - 12*A*d^3*cos(e/2 + (f*x)/2)^5 - 6*B*d^3*cos(e/2 + (f*x)/2)^3 + 36*B*d
^3*cos(e/2 + (f*x)/2)^5 - 16*B*d^3*cos(e/2 + (f*x)/2)^7 - 9*A*d^3*cos(e/2
+ (f*x)/2)*(e + f*x) - 6*B*c^3*cos(e/2 + (f*x)/2)*(e + f*x) + 9*B*d^3*cos(
e/2 + (f*x)/2)*(e + f*x) - 9*A*d^3*sin(e/2 + (f*x)/2)*(e + f*x) - 6*B*c^3*
sin(e/2 + (f*x)/2)*(e + f*x) + 9*B*d^3*sin(e/2 + (f*x)/2)*(e + f*x) - 18*A
*d^3*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2) + 12*A*d^3*cos(e/2 + (f*x)/2)
^4*sin(e/2 + (f*x)/2) + 18*B*d^3*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2) +
 12*B*d^3*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x)/2) - 16*B*d^3*cos(e/2 + (f*
x)/2)^6*sin(e/2 + (f*x)/2) + 36*A*c*d^2*cos(e/2 + (f*x)/2) - 36*A*c^2*d*co
s(e/2 + (f*x)/2) - 54*B*c*d^2*cos(e/2 + (f*x)/2) + 36*B*c^2*d*cos(e/2 + (f
*x)/2) + 36*A*c*d^2*cos(e/2 + (f*x)/2)^3 + 18*B*c*d^2*cos(e/2 + (f*x)/2)^3
 + 36*B*c^2*d*cos(e/2 + (f*x)/2)^3 - 36*B*c*d^2*cos(e/2 + (f*x)/2)^5 + 18*
A*c*d^2*cos(e/2 + (f*x)/2)*(e + f*x) - 18*A*c^2*d*cos(e/2 + (f*x)/2)*(e +
f*x) - 27*B*c*d^2*cos(e/2 + (f*x)/2)*(e + f*x) + 18*B*c^2*d*cos(e/2 + (f*x
)/2)*(e + f*x) + 18*A*c*d^2*sin(e/2 + (f*x)/2)*(e + f*x) - 18*A*c^2*d*sin(
e/2 + (f*x)/2)*(e + f*x) - 27*B*c*d^2*sin(e/2 + (f*x)/2)*(e + f*x) + 18*B*
c^2*d*sin(e/2 + (f*x)/2)*(e + f*x) + 36*A*c*d^2*cos(e/2 + (f*x)/2)^2*sin(e
/2 + (f*x)/2) - 54*B*c*d^2*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2) + 36...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 810, normalized size of antiderivative = 3.68

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^3}{a + a\sin(e + fx)}\,dx = \text{Too large to display}$$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e)),x)
```

output

```
(2*cos(e + f*x)*sin(e + f*x)**3*b*d**3 + 3*cos(e + f*x)*sin(e + f*x)**2*a*
d**3 + 9*cos(e + f*x)*sin(e + f*x)**2*b*c*d**2 - cos(e + f*x)*sin(e + f*x)
**2*b*d**3 + 18*cos(e + f*x)*sin(e + f*x)*a*c*d**2 - 3*cos(e + f*x)*sin(e
+ f*x)*a*d**3 + 18*cos(e + f*x)*sin(e + f*x)*b*c**2*d - 9*cos(e + f*x)*sin
(e + f*x)*b*c*d**2 + 7*cos(e + f*x)*sin(e + f*x)*b*d**3 + 12*cos(e + f*x)*
a*c**3 + 18*cos(e + f*x)*a*c**2*d*f*x - 36*cos(e + f*x)*a*c**2*d - 18*cos(
e + f*x)*a*c*d**2*f*x + 36*cos(e + f*x)*a*c*d**2 + 9*cos(e + f*x)*a*d**3*f
*x - 18*cos(e + f*x)*a*d**3 + 6*cos(e + f*x)*b*c**3*f*x - 12*cos(e + f*x)*
b*c**3 - 18*cos(e + f*x)*b*c**2*d*f*x + 36*cos(e + f*x)*b*c**2*d + 27*cos(
e + f*x)*b*c*d**2*f*x - 54*cos(e + f*x)*b*c*d**2 - 9*cos(e + f*x)*b*d**3*f
*x + 18*cos(e + f*x)*b*d**3 + 2*sin(e + f*x)**4*b*d**3 + 3*sin(e + f*x)**3
*a*d**3 + 9*sin(e + f*x)**3*b*c*d**2 - 3*sin(e + f*x)**3*b*d**3 + 18*sin(e
 + f*x)**2*a*c*d**2 - 6*sin(e + f*x)**2*a*d**3 + 18*sin(e + f*x)**2*b*c**2
*d - 18*sin(e + f*x)**2*b*c*d**2 + 8*sin(e + f*x)**2*b*d**3 - 18*sin(e + f
*x)*a*c**2*d*f*x + 18*sin(e + f*x)*a*c*d**2*f*x + 18*sin(e + f*x)*a*c*d**2
 - 9*sin(e + f*x)*a*d**3*f*x - 3*sin(e + f*x)*a*d**3 - 6*sin(e + f*x)*b*c*
*3*f*x + 18*sin(e + f*x)*b*c**2*d*f*x + 18*sin(e + f*x)*b*c**2*d - 27*sin(
e + f*x)*b*c*d**2*f*x - 9*sin(e + f*x)*b*c*d**2 + 9*sin(e + f*x)*b*d**3*f*
x + 7*sin(e + f*x)*b*d**3 - 12*a*c**3 - 18*a*c**2*d*f*x + 36*a*c**2*d + 18
*a*c*d**2*f*x - 36*a*c*d**2 - 9*a*d**3*f*x + 18*a*d**3 - 6*b*c**3*f*x +...
```

3.266 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 143

$$\begin{aligned}\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx &= \frac{(2A(2c-d)d+B(2c^2-4cd+3d^2))\,x}{2a}+\frac{2(A(c-d)-B(2c-d))d\cos(e+fx)}{af}\\ &\quad+\frac{(2A-3B)d^2\cos(e+fx)\sin(e+fx)}{2af}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{f(a+a\sin(e+fx))}\end{aligned}$$

output
```
1/2*(2*A*(2*c-d)*d+B*(2*c^2-4*c*d+3*d^2))*x/a+2*(A*(c-d)-B*(2*c-d))*d*cos(
f*x+e)/a/f+1/2*(2*A-3*B)*d^2*cos(f*x+e)*sin(f*x+e)/a/f-(A-B)*cos(f*x+e)*(c
+d*sin(f*x+e))^2/f/(a+a*sin(f*x+e))
```

Mathematica [A] (verified)

Time = 6.86 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.40

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(8(A-B)(c-d)^2\sin\left(\frac{1}{2}(e+fx)\right)+2(2A(2c-d)d+B(2c^2-4cd\right.}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x
]),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(A - B)*(c - d)^2*Sin[(e + f*x)/
2] + 2*(2*A*(2*c - d)*d + B*(2*c^2 - 4*c*d + 3*d^2))*(e + f*x)*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2]) + 4*d*(-(A*d) + B*(-2*c + d))*Cos[e + f*x]*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2]) - B*d^2*(Cos[(e + f*x)/2] + Sin[(e + f*x
)/2])*Sin[2*(e + f*x)]))/(4*a*f*(1 + Sin[e + f*x]))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {3042, 3456, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{a\sin(e+fx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{a\sin(e+fx)+a}dx$$

$$\Big\downarrow \text{3456}$$

$$\frac{\int (c+d\sin(e+fx))(a(B(c-2d)+2Ad)-a(2A-3B)d\sin(e+fx))dx}{a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{f(a\sin(e+fx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (c+d\sin(e+fx))(a(B(c-2d)+2Ad)-a(2A-3B)d\sin(e+fx))dx}{a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{f(a\sin(e+fx)+a)}$$

$$\Big\downarrow \text{3213}$$

$$\frac{\frac{1}{2}ax\left(-\left(d^2(2A-3B)\right)+4Acd+2Bc(c-2d)\right)+\frac{2ad(Ac-Ad-2Bc+Bd)\cos(e+fx)}{f}+\frac{ad^2(2A-3B)\sin(e+fx)\cos(e+fx)}{2f}}{a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{f(a\sin(e+fx)+a)}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]),x]
```

output
```
-(((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(f*(a + a*Sin[e + f*x])))
+ ((a*(2*B*c*(c - 2*d) + 4*A*c*d - (2*A - 3*B)*d^2)*x)/2 + (2*a*d*(A*c - 2
*B*c - A*d + B*d)*Cos[e + f*x])/f + (a*(2*A - 3*B)*d^2*Cos[e + f*x]*Sin[e
+ f*x])/(2*f))/a^2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.35

method	result
derivativedivides	$\frac{-\frac{2\left(A c^2-2Acd+A d^2-B c^2+2Bcd-B d^2\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}+\frac{2\left(\frac{B d^2 \tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2}+\left(-A d^2-2Bcd+B d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\frac{B d^2 \tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}-\cdots\right)}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^2}\cdots}{af}$
default	$\frac{-\frac{2\left(A c^2-2Acd+A d^2-B c^2+2Bcd-B d^2\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}+\frac{2\left(\frac{B d^2 \tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2}+\left(-A d^2-2Bcd+B d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-\frac{B d^2 \tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}-\cdots\right)}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right)^2}\cdots}{af}$
parallelrisch	$\frac{\left((-2fxA+3fxB-7A+7B)d^2+4\left(\left(-fx-\frac{7}{2}\right)B+A(fx+2)\right)cd-4\left(\left(-\frac{fx}{2}-1\right)B+A\right)c^2\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\left((-2fxA+3fx\cdots\right.}{\cdots}$
risch	$\frac{2xAcd}{a}-\frac{xA\,d^2}{a}+\frac{xB\,c^2}{a}-\frac{2xBcd}{a}+\frac{3xB\,d^2}{2a}-\frac{d^2 e^{i(fx+e)}A}{2af}-\frac{d\,e^{i(fx+e)}Bc}{af}+\frac{d^2 e^{i(fx+e)}B}{2af}-\frac{d^2 e^{-i(fx\cdots}}{2af}\cdots$
norman	$\frac{\frac{\left(2A c^2-4Acd-2B c^2-2B d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{\left(2A c^2-4Acd+2A d^2-2B c^2+4Bcd-3B d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{af}+\frac{\left(6A c^2-12Acd+2A d^2\cdots\right.}{\cdots}}{\cdots}$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e)),x,method=_RETURNV
ERBOSE)
```

output
```
2/f/a*(-(A*c^2-2*A*c*d+A*d^2-B*c^2+2*B*c*d-B*d^2)/(tan(1/2*f*x+1/2*e)+1)+(
1/2*B*d^2*tan(1/2*f*x+1/2*e)^3+(-A*d^2-2*B*c*d+B*d^2)*tan(1/2*f*x+1/2*e)^2
-1/2*B*d^2*tan(1/2*f*x+1/2*e)-A*d^2-2*B*c*d+B*d^2)/(1+tan(1/2*f*x+1/2*e)^2
)^2+1/2*(4*A*c*d-2*A*d^2+2*B*c^2-4*B*c*d+3*B*d^2)*arctan(tan(1/2*f*x+1/2*e
)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(139) = 278$.

Time = 0.09 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.12

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx$$
$$= \frac{Bd^2\cos{(fx+e)}^3 - 2\,(A-B)c^2 + 4\,(A-B)cd - 2\,(A-B)d^2 + (2\,Bc^2 + 4\,(A-B)cd - (2\,A-3\,B)d^2}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
1/2*(B*d^2*cos(f*x + e)^3 - 2*(A - B)*c^2 + 4*(A - B)*c*d - 2*(A - B)*d^2
+ (2*B*c^2 + 4*(A - B)*c*d - (2*A - 3*B)*d^2)*f*x - 2*(2*B*c*d + (A - B)*d
^2)*cos(f*x + e)^2 - (2*(A - B)*c^2 - 4*(A - 2*B)*c*d + (4*A - 3*B)*d^2 -
(2*B*c^2 + 4*(A - B)*c*d - (2*A - 3*B)*d^2)*f*x)*cos(f*x + e) - (B*d^2*cos
(f*x + e)^2 - 2*(A - B)*c^2 + 4*(A - B)*c*d - 2*(A - B)*d^2 - (2*B*c^2 + 4
*(A - B)*c*d - (2*A - 3*B)*d^2)*f*x + (4*B*c*d + (2*A - B)*d^2)*cos(f*x +
e))*sin(f*x + e))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5763 vs. $2(117) = 234$.

Time = 2.28 (sec) , antiderivative size = 5763, normalized size of antiderivative = 40.30

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e)),x)
```

output
```
Piecewise((-4*A*c**2*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*
f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)
**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 8*A*c**2*tan(e/2 + f*x/2)**2/(2*a*
f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)
**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) - 4*A*c*
*2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2
+ f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f)
+ 4*A*c*d*f*x*tan(e/2 + f*x/2)**5/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e
/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2
*a*f*tan(e/2 + f*x/2) + 2*a*f) + 4*A*c*d*f*x*tan(e/2 + f*x/2)**4/(2*a*f*ta
n(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3
+ 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 8*A*c*d*f*
x*tan(e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)*
*4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2
 + f*x/2) + 2*a*f) + 8*A*c*d*f*x*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x/
2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(
e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 4*A*c*d*f*x*tan(e/2 +
f*x/2)/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(
e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a
*f) + 4*A*c*d*f*x/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 606 vs. $2(139) = 278$.

Time = 0.13 (sec) , antiderivative size = 606, normalized size of antiderivative = 4.24

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^2}{a + a\sin(e + fx)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorith
m="maxima")
```

output
```
(B*d^2*((sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x
 + e) + 1)^4 + 4)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)
+ 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 4*B*c*d*((sin(f*x + e)/(c
os(f*x + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x
 + e)/(cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/
a) - 2*A*d^2*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin
(f*x + e)/(cos(f*x + e) + 1))/a) + 2*B*c^2*(arctan(sin(f*x + e)/(cos(f*x +
 e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) + 4*A*c*d*(arctan
(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f*x + e)
+ 1))) - 2*A*c^2/(a + a*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.50

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx = \frac{\frac{(2\,Bc^2+4\,Acd-4\,Bcd-2\,Ad^2+3\,Bd^2)(fx+e)}{a} - \frac{4\,(Ac^2-Bc^2-2\,Acd+2\,Bcd+Ad^2-Bd^2)}{a\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)} + \frac{2\left(Bd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-4\,Bcd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right.}{}}{2\,f}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorith
m="giac")
```

output
```
1/2*((2*B*c^2 + 4*A*c*d - 4*B*c*d - 2*A*d^2 + 3*B*d^2)*(f*x + e)/a - 4*(A*
c^2 - B*c^2 - 2*A*c*d + 2*B*c*d + A*d^2 - B*d^2)/(a*(tan(1/2*f*x + 1/2*e)
+ 1)) + 2*(B*d^2*tan(1/2*f*x + 1/2*e)^3 - 4*B*c*d*tan(1/2*f*x + 1/2*e)^2 -
 2*A*d^2*tan(1/2*f*x + 1/2*e)^2 + 2*B*d^2*tan(1/2*f*x + 1/2*e)^2 - B*d^2*t
an(1/2*f*x + 1/2*e) - 4*B*c*d - 2*A*d^2 + 2*B*d^2)/((tan(1/2*f*x + 1/2*e)^
2 + 1)^2*a))/f
```

Mupad [B] (verification not implemented)

Time = 40.11 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.08

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx$$
$$= \frac{x\,(2\,B\,c^2-2\,A\,d^2+3\,B\,d^2+4\,A\,c\,d-4\,B\,c\,d)}{2\,a}$$
$$-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3(2\,A\,d^2-3\,B\,d^2+4\,B\,c\,d)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4(2\,A\,c^2+2\,A\,d^2-2\,B\,c^2-3\,B\,d^2-4\,A\,c\,d+}{f\,\left(a\tan\left(\frac{e}{2}+\right.\right.}$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x)),x)
```

output
```
(x*(2*B*c^2 - 2*A*d^2 + 3*B*d^2 + 4*A*c*d - 4*B*c*d))/(2*a) - (tan(e/2 + (
f*x)/2)^3*(2*A*d^2 - 3*B*d^2 + 4*B*c*d) + tan(e/2 + (f*x)/2)^4*(2*A*c^2 +
2*A*d^2 - 2*B*c^2 - 3*B*d^2 - 4*A*c*d + 4*B*c*d) + tan(e/2 + (f*x)/2)^2*(4
*A*c^2 + 6*A*d^2 - 4*B*c^2 - 5*B*d^2 - 8*A*c*d + 12*B*c*d) + 2*A*c^2 + 4*A
*d^2 - 2*B*c^2 - 4*B*d^2 + tan(e/2 + (f*x)/2)*(2*A*d^2 - B*d^2 + 4*B*c*d)
- 4*A*c*d + 8*B*c*d)/(f*(a + a*tan(e/2 + (f*x)/2) + 2*a*tan(e/2 + (f*x)/2)
^2 + 2*a*tan(e/2 + (f*x)/2)^3 + a*tan(e/2 + (f*x)/2)^4 + a*tan(e/2 + (f*x)
/2)^5))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 466, normalized size of antiderivative = 3.26

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{a+a\sin(e+fx)}\,dx$$
$$= \frac{-2\sin(fx+e)^2\,b\,d^2+2\sin(fx+e)\,a\,d^2+\cos(fx+e)\sin(fx+e)^2\,b\,d^2+2\cos(fx+e)\sin(fx+e)\,a}{}$$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e)),x)
```

output
```
(cos(e + f*x)*sin(e + f*x)**2*b*d**2 + 2*cos(e + f*x)*sin(e + f*x)*a*d**2
+ 4*cos(e + f*x)*sin(e + f*x)*b*c*d - cos(e + f*x)*sin(e + f*x)*b*d**2 + 4
*cos(e + f*x)*a*c**2 + 4*cos(e + f*x)*a*c*d*f*x - 8*cos(e + f*x)*a*c*d - 2
*cos(e + f*x)*a*d**2*f*x + 4*cos(e + f*x)*a*d**2 + 2*cos(e + f*x)*b*c**2*f
*x - 4*cos(e + f*x)*b*c**2 - 4*cos(e + f*x)*b*c*d*f*x + 8*cos(e + f*x)*b*c
*d + 3*cos(e + f*x)*b*d**2*f*x - 6*cos(e + f*x)*b*d**2 + sin(e + f*x)**3*b
*d**2 + 2*sin(e + f*x)**2*a*d**2 + 4*sin(e + f*x)**2*b*c*d - 2*sin(e + f*x
)**2*b*d**2 - 4*sin(e + f*x)*a*c*d*f*x + 2*sin(e + f*x)*a*d**2*f*x + 2*sin
(e + f*x)*a*d**2 - 2*sin(e + f*x)*b*c**2*f*x + 4*sin(e + f*x)*b*c*d*f*x +
4*sin(e + f*x)*b*c*d - 3*sin(e + f*x)*b*d**2*f*x - sin(e + f*x)*b*d**2 - 4
*a*c**2 - 4*a*c*d*f*x + 8*a*c*d + 2*a*d**2*f*x - 4*a*d**2 - 2*b*c**2*f*x +
 4*b*c**2 + 4*b*c*d*f*x - 8*b*c*d - 3*b*d**2*f*x + 6*b*d**2)/(2*a*f*(cos(e
 + f*x) - sin(e + f*x) - 1))
```

3.267 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{a+a\sin(e+fx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 67

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{a+a\sin(e+fx)}\,dx = \frac{(B(c-d)+Ad)x}{a}-\frac{Bd\cos(e+fx)}{af}-\frac{(A-B)(c-d)\cos(e+fx)}{af(1+\sin(e+fx))}$$

output
```
(B*(c-d)+A*d)*x/a-B*d*cos(f*x+e)/a/f-(A-B)*(c-d)*cos(f*x+e)/a/f/(1+sin(f*x
+e))
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.88

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{a+a\sin(e+fx)}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)\left((B(c-d)+Ad)(e+fx)-Bd\cos(e+fx)\right)+(2\cdots}{af(1+\sin(e+fx)\cdots}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])
,x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(Cos[(e + f*x)/2]*((B*(c - d) + A*d
)*(e + f*x) - B*d*Cos[e + f*x]) + (2*A*c + B*(c - d)*(-2 + e + f*x) + A*d*
(-2 + e + f*x) - B*d*Cos[e + f*x])*Sin[(e + f*x)/2]))/(a*f*(1 + Sin[e + f*
x]))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3042, 3447, 3042, 3502, 3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{a\sin(e+fx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{a\sin(e+fx)+a}dx$$

$$\Big\downarrow \text{3447}$$

$$\int \frac{(Ad+Bc)\sin(e+fx)+Ac+Bd\sin^2(e+fx)}{a\sin(e+fx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(Ad+Bc)\sin(e+fx)+Ac+Bd\sin(e+fx)^2}{a\sin(e+fx)+a}dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{\int \frac{aAc+a(B(c-d)+Ad)\sin(e+fx)}{\sin(e+fx)a+a}dx}{a}-\frac{Bd\cos(e+fx)}{af}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{aAc+a(B(c-d)+Ad)\sin(e+fx)}{\sin(e+fx)a+a}dx}{a}-\frac{Bd\cos(e+fx)}{af}$$

$$\Big\downarrow \text{3214}$$

$$\frac{a(A-B)(c-d)\int \frac{1}{\sin(e+fx)a+a}dx + x(Ad+B(c-d))}{a} - \frac{Bd\cos(e+fx)}{af}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(A-B)(c-d)\int \frac{1}{\sin(e+fx)a+a}dx + x(Ad+B(c-d))}{a} - \frac{Bd\cos(e+fx)}{af}$$

$$\Big\downarrow \text{3127}$$

$$\frac{x(Ad+B(c-d)) - \frac{a(A-B)(c-d)\cos(e+fx)}{f(a\sin(e+fx)+a)}}{a} - \frac{Bd\cos(e+fx)}{af}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x]),x]
```

output
```
-((B*d*Cos[e + f*x])/(a*f)) + ((B*(c - d) + A*d)*x - (a*(A - B)*(c - d)*Co
s[e + f*x])/(f*(a + a*Sin[e + f*x])))/a
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{-\frac{2(Ac-Ad-Bc+Bd)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2Bd}{1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}+2(Ad+Bc-Bd)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}$
default	$\frac{-\frac{2(Ac-Ad-Bc+Bd)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{2Bd}{1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}+2(Ad+Bc-Bd)\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}$
parallelrisch	$\frac{\left((2cfx-2dfx-4c+3d)B+4A\left(\left(\frac{fx}{2}-1\right)d+c\right)\right)\sin\left(\frac{fx}{2}+\frac{e}{2}\right)+((2cfx-2dfx-3d)B+2Adfx)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-Bd\left(\cos\left(\frac{3f}{2}\right.\right.}{2af\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\sin\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}$
risch	$\frac{xAd}{a}+\frac{xBc}{a}-\frac{xBd}{a}-\frac{Bd\,e^{i(fx+e)}}{2af}-\frac{Bd\,e^{-i(fx+e)}}{2af}-\frac{2Ac}{fa\left(e^{i(fx+e)}+i\right)}+\frac{2Ad}{fa\left(e^{i(fx+e)}+i\right)}+\frac{2Bc}{fa\left(e^{i(fx+e)}+\right.}$
norman	$\frac{\frac{(Ad+Bc-Bd)x}{a}+\frac{(Ad+Bc-Bd)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a}+\frac{(Ad+Bc-Bd)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{a}+\frac{(Ad+Bc-Bd)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{a}-\frac{2Ac-2Ad-2Bc}{af}}{}$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x,method=_RETURNVER
BOSE)
```

output
```
2/f/a*(-(A*c-A*d-B*c+B*d)/(tan(1/2*f*x+1/2*e)+1)-B*d/(1+tan(1/2*f*x+1/2*e)
^2)+(A*d+B*c-B*d)*arctan(tan(1/2*f*x+1/2*e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(67) = 134$.

Time = 0.08 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.30

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))}{a + a\sin(e + fx)}\,dx = -\frac{Bd\cos(fx + e)^2 - (Bc + (A - B)d)fx + (A - B)c - (A - B)d - ((Bc + (A - B)d)fx - (A - B)c}{af\cos(fx + e)}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm=
"fricas")
```

output
```
-(B*d*cos(f*x + e)^2 - (B*c + (A - B)*d)*f*x + (A - B)*c - (A - B)*d - ((B
*c + (A - B)*d)*f*x - (A - B)*c + (A - 2*B)*d)*cos(f*x + e) - ((B*c + (A -
 B)*d)*f*x - B*d*cos(f*x + e) + (A - B)*c - (A - B)*d)*sin(f*x + e))/(a*f*
cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1307 vs. $2(49) = 98$.

Time = 1.15 (sec) , antiderivative size = 1307, normalized size of antiderivative = 19.51

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))}{a + a\sin(e + fx)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

output
```
Piecewise((-2*A*c*tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e
/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - 2*A*c/(a*f*tan(e/2 + f*x/2)
**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + A*d*f*x*tan(
e/2 + f*x/2)**3/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*t
an(e/2 + f*x/2) + a*f) + A*d*f*x*tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2)
**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + A*d*f*x*tan(
e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(
e/2 + f*x/2) + a*f) + A*d*f*x/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x
/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*A*d*tan(e/2 + f*x/2)**2/(a*f*tan(
e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) +
2*A*d/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f
*x/2) + a*f) + B*c*f*x*tan(e/2 + f*x/2)**3/(a*f*tan(e/2 + f*x/2)**3 + a*f*
tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + B*c*f*x*tan(e/2 + f*x/
2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f
*x/2) + a*f) + B*c*f*x*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan
(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + B*c*f*x/(a*f*tan(e/2 + f*
x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*B*c*ta
n(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f
*tan(e/2 + f*x/2) + a*f) + 2*B*c/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 +
f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - B*d*f*x*tan(e/2 + f*x/2)**3/(...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(67) = 134$.

Time = 0.11 (sec) , antiderivative size = 256, normalized size of antiderivative = 3.82

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{a+a\sin(e+fx)}\,dx =$$

$$-\frac{2\left(Bd\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+2}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}+\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}\right)-Bc\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}+\frac{1}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right.}{f}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm=
"maxima")
```

output
```
-2*(B*d*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x
+ e)/(cos(f*x + e) + 1))/a) - B*c*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))
/a + 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) - A*d*(arctan(sin(f*x + e)
/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) + A*c/
(a + a*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(67) = 134$.

Time = 0.20 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.25

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{a+a\sin(e+fx)}\,dx$$
$$= \frac{\frac{(Bc+Ad-Bd)(fx+e)}{a} - \frac{2\left(Ac\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 - Bc\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 - Ad\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 + Bd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 + Bd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right) + Ac - B}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3 + \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 + \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right) + 1\right)a}}{f}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm=
"giac")
```

output
```
((B*c + A*d - B*d)*(f*x + e)/a - 2*(A*c*tan(1/2*f*x + 1/2*e)^2 - B*c*tan(1
/2*f*x + 1/2*e)^2 - A*d*tan(1/2*f*x + 1/2*e)^2 + B*d*tan(1/2*f*x + 1/2*e)^
2 + B*d*tan(1/2*f*x + 1/2*e) + A*c - B*c - A*d + 2*B*d)/((tan(1/2*f*x + 1/
2*e)^3 + tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e) + 1)*a))/f
```

Mupad [B] (verification not implemented)

Time = 36.45 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.82

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{a+a\sin(e+fx)}\,dx = \frac{x\,(A\,d+B\,c-B\,d)}{a}$$
$$-\frac{(2\,A\,c-2\,A\,d-2\,B\,c+2\,B\,d)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+2\,B\,d\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+2\,A\,c-2\,A\,d-2\,B\,c+4\,B\,d}{f\,\left(a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a\right)}$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x)),x)
```

output
```
(x*(A*d + B*c - B*d))/a - (2*A*c - 2*A*d - 2*B*c + 4*B*d + tan(e/2 + (f*x)
/2)^2*(2*A*c - 2*A*d - 2*B*c + 2*B*d) + 2*B*d*tan(e/2 + (f*x)/2))/(f*(a +
a*tan(e/2 + (f*x)/2) + a*tan(e/2 + (f*x)/2)^2 + a*tan(e/2 + (f*x)/2)^3))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.40

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{a+a\sin(e+fx)}\,dx = \frac{-\cos(fx+e)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)bd-\cos(fx+e)\,bd+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)ac+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)adfx-2\tan\left(\frac{fx}{2}+\right.}{af\left(\tan\right.}$$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

output
```
( - cos(e + f*x)*tan((e + f*x)/2)*b*d - cos(e + f*x)*b*d + 2*tan((e + f*x)
/2)*a*c + tan((e + f*x)/2)*a*d*f*x - 2*tan((e + f*x)/2)*a*d + tan((e + f*x
)/2)*b*c*f*x - 2*tan((e + f*x)/2)*b*c - tan((e + f*x)/2)*b*d*f*x + 2*tan((
e + f*x)/2)*b*d + a*d*f*x + b*c*f*x - b*d*f*x)/(a*f*(tan((e + f*x)/2) + 1)
)
```

3.268 $\int \frac{A+B\sin(e+fx)}{a+a\sin(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 35

$$\int \frac{A+B\sin(e+fx)}{a+a\sin(e+fx)}\,dx=\frac{Bx}{a}-\frac{(A-B)\cos(e+fx)}{f(a+a\sin(e+fx))}$$

output
```
B*x/a-(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 79 vs. 2(35) = 70.

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.26

$$\int \frac{A+B\sin(e+fx)}{a+a\sin(e+fx)}\,dx =\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(B(e+fx)\cos\left(\frac{1}{2}(e+fx)\right)+(2A+B(-2+e+fx))\sin\left(\frac{1}{2}(e+f\right.\right.}{af(1+\sin(e+fx))}$$

input
```
Integrate[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x]),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(B*(e + f*x)*Cos[(e + f*x)/2] + (2*
A + B*(-2 + e + f*x))*Sin[(e + f*x)/2]))/(a*f*(1 + Sin[e + f*x]))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{a\sin(e+fx)+a}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\sin(e+fx)}{a\sin(e+fx)+a}dx$$

$$\Big\downarrow \text{ 3214}$$

$$(A-B)\int \frac{1}{\sin(e+fx)a+a}dx+\frac{Bx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$(A-B)\int \frac{1}{\sin(e+fx)a+a}dx+\frac{Bx}{a}$$

$$\Big\downarrow \text{ 3127}$$

$$\frac{Bx}{a}-\frac{(A-B)\cos(e+fx)}{f(a\sin(e+fx)+a)}$$

input
```
Int[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x]),x]
```

output
```
(B*x)/a - ((A - B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$\frac{-\frac{2(A-B)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}+2B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}$	42
default	$\frac{-\frac{2(A-B)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}+2B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af}$	42
parallelrisch	$\frac{fxB+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)(fxB+2A-2B)}{fa\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}$	47
risch	$\frac{Bx}{a}-\frac{2A}{fa\left(\mathrm{e}^{i(fx+e)}+i\right)}+\frac{2B}{fa\left(\mathrm{e}^{i(fx+e)}+i\right)}$	54
norman	$\frac{\frac{Bx}{a}+\frac{Bx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{a}+\frac{Bx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{a}+\frac{Bx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{a}+\frac{(2A-2B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{(2A-2B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{af}}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}$	134

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
2/f/a*(-(A-B)/(tan(1/2*f*x+1/2*e)+1)+B*arctan(tan(1/2*f*x+1/2*e)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.89

$$\int \frac{A+B\sin(e+fx)}{a+a\sin(e+fx)}\,dx = \frac{Bfx+(Bfx-A+B)\cos(fx+e)+(Bfx+A-B)\sin(fx+e)-A+B}{af\cos(fx+e)+af\sin(fx+e)+af}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

output
```
(B*f*x + (B*f*x - A + B)*cos(f*x + e) + (B*f*x + A - B)*sin(f*x + e) - A +
 B)/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(26) = 52.

Time = 0.60 (sec) , antiderivative size = 109, normalized size of antiderivative = 3.11

$$\int \frac{A+B\sin(e+fx)}{a+a\sin(e+fx)}\,dx = \begin{cases} -\frac{2A}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af}+\frac{Bfx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af}+\frac{Bfx}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af}+\frac{2B}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+af} & \text{for } f\neq 0 \\ \frac{x(A+B\sin(e))}{a\sin(e)+a} & \text{otherwise} \end{cases}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

output
```
Piecewise((-2*A/(a*f*tan(e/2 + f*x/2) + a*f) + B*f*x*tan(e/2 + f*x/2)/(a*f
*tan(e/2 + f*x/2) + a*f) + B*f*x/(a*f*tan(e/2 + f*x/2) + a*f) + 2*B/(a*f*t
an(e/2 + f*x/2) + a*f), Ne(f, 0)), (x*(A + B*sin(e))/(a*sin(e) + a), True)
)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(35) = 70$.

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.23

$$\int \frac{A + B\sin(e + fx)}{a + a\sin(e + fx)}\,dx = \frac{2\left(B\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a} + \frac{1}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right) - \frac{A}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right)}{f}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

output
```
2*(B*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(c
os(f*x + e) + 1))) - A/(a + a*sin(f*x + e)/(cos(f*x + e) + 1)))/f
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int \frac{A + B\sin(e + fx)}{a + a\sin(e + fx)}\,dx = \frac{\frac{(fx+e)B}{a} - \frac{2(A-B)}{a\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}}{f}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="giac")
```

output
```
((f*x + e)*B/a - 2*(A - B)/(a*(tan(1/2*f*x + 1/2*e) + 1)))/f
```

Mupad [B] (verification not implemented)

Time = 35.79 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{A + B\sin(e + fx)}{a + a\sin(e + fx)}\,dx = \frac{B\,x}{a} - \frac{2\,A - 2\,B}{a\,f\,\left(\tan\left(\frac{e}{2} + \frac{f\,x}{2}\right) + 1\right)}$$

input
```
int((A + B*sin(e + f*x))/(a + a*sin(e + f*x)),x)
```

output
```
(B*x)/a - (2*A - 2*B)/(a*f*(tan(e/2 + (f*x)/2) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.77

$$\int \frac{A+B\sin(e+fx)}{a+a\sin(e+fx)}\,dx = \frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)a+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)bfx-2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)b+bfx}{af\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

output
```
(2*tan((e + f*x)/2)*a + tan((e + f*x)/2)*b*f*x - 2*tan((e + f*x)/2)*b + b*
f*x)/(a*f*(tan((e + f*x)/2) + 1))
```

3.269 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 101

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))}\,dx = \frac{2(Bc-Ad)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a(c-d)\sqrt{c^2-d^2}\,f} - \frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))}$$

output
```
2*(-A*d+B*c)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a/(c-d)/(c^2
-d^2)^(1/2)/f-(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))
```

Mathematica [A] (verified)

Time = 6.97 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.47

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))}\,dx = \frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left((A-B)\sqrt{c^2-d^2}\sin\left(\frac{1}{2}(e+fx)\right)+(Bc-Ad)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(\cdots}{\sqrt{c^2-\cdots}}\right.\right.}{a(c-d)\sqrt{c^2-d^2}\,f(1+\sin(e+fx))}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x]))
,x]
```

output
```
(2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((A - B)*Sqrt[c^2 - d^2]*Sin[(e +
 f*x)/2] + (B*c - A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])))/(a*(c - d)*Sqrt[c^2 - d^2]*f*(1 + Si
n[e + f*x]))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {3042, 3457, 25, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)(c+d\sin(e+fx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)(c+d\sin(e+fx))}dx$$

$$\Big\downarrow \text{3457}$$

$$-\frac{\int -\frac{a(Bc-Ad)}{c+d\sin(e+fx)}dx}{a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{a(Bc-Ad)}{c+d\sin(e+fx)}dx}{a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)}$$

$$\Big\downarrow \text{27}$$

$$\frac{(Bc-Ad)\int \frac{1}{c+d\sin(e+fx)}dx}{a(c-d)}-\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(Bc-Ad)\int \frac{1}{c+d\sin(e+fx)}dx}{a(c-d)}-\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)}$$

$$\Big\downarrow \text{3139}$$

$$\frac{2(Bc-Ad)\int \frac{1}{c\tan^2\left(\frac{1}{2}(e+fx)\right)+2d\tan\left(\frac{1}{2}(e+fx)\right)+c}d\tan\left(\frac{1}{2}(e+fx)\right)}{af(c-d)}-\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{4(Bc-Ad)\int \frac{1}{-\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)^2-4(c^2-d^2)}d\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{af(c-d)}-\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)}$$

$$\Big\downarrow \text{217}$$

$$\frac{2(Bc-Ad)\arctan\left(\frac{2c\tan\left(\frac{1}{2}(e+fx)\right)+2d}{2\sqrt{c^2-d^2}}\right)}{af(c-d)\sqrt{c^2-d^2}}-\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])),x]
```

output
```
(2*(B*c - A*d)*ArcTan[(2*d + 2*c*Tan[(e + f*x)/2])/(2*Sqrt[c^2 - d^2])])/(
a*(c - d)*Sqrt[c^2 - d^2]*f) - ((A - B)*Cos[e + f*x])/((c - d)*f*(a + a*Si
n[e + f*x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{-\frac{2(A-B)}{(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{2(-Ad+Bc)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c-d)\sqrt{c^2-d^2}}}{af}$
default	$\frac{-\frac{2(A-B)}{(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{2(-Ad+Bc)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c-d)\sqrt{c^2-d^2}}}{af}$
risch	$-\frac{2A}{fa(c-d)\left(e^{i(fx+e)}+i\right)}+\frac{2B}{fa(c-d)\left(e^{i(fx+e)}+i\right)}-\frac{\ln\left(e^{i(fx+e)}+\frac{i\sqrt{-c^2+d^2}\,c+c^2-d^2}{\sqrt{-c^2+d^2}\,d}\right)Ad}{\sqrt{-c^2+d^2}\,(c-d)fa}+\frac{\ln\left(e^{i(fx+e)}+\frac{i\sqrt{-}}{}\right.}{\sqrt{-c^2+d}}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x,method=_RETURNVER
BOSE)
```

output
```
2/f/a*(-(A-B)/(c-d)/(tan(1/2*f*x+1/2*e)+1)+1/(c-d)*(-A*d+B*c)/(c^2-d^2)^(1
/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(96) = 192$.

Time = 0.11 (sec) , antiderivative size = 595, normalized size of antiderivative = 5.89

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))(c + d\sin(e + fx))}\,dx$$
$$= \left[-\frac{2\,(A - B)c^2 - 2\,(A - B)d^2 + (Bc - Ad + (Bc - Ad)\cos(fx + e) + (Bc - Ad)\sin(fx + e))\sqrt{-c^2 -}}{2\,((ac^3 - ac^2d - acd^2 +} \right.$$
$$-\frac{(A - B)c^2 - (A - B)d^2 + (Bc - Ad + (Bc - Ad)\cos(fx + e) + (Bc - Ad)\sin(fx + e))\sqrt{c^2 - d^2}\,\mathrm{ar}}{(ac^3 - ac^2d - acd^2 + ad^3)f\cos(fx + e) + (ac^3 - ac^2}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm=
"fricas")
```

output
```
[-1/2*(2*(A - B)*c^2 - 2*(A - B)*d^2 + (B*c - A*d + (B*c - A*d)*cos(f*x +
e) + (B*c - A*d)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x
 + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e)
+ d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x +
e) - c^2 - d^2)) + 2*((A - B)*c^2 - (A - B)*d^2)*cos(f*x + e) - 2*((A - B)
*c^2 - (A - B)*d^2)*sin(f*x + e))/((a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f*c
os(f*x + e) + (a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f*sin(f*x + e) + (a*c^3
- a*c^2*d - a*c*d^2 + a*d^3)*f), -((A - B)*c^2 - (A - B)*d^2 + (B*c - A*d
+ (B*c - A*d)*cos(f*x + e) + (B*c - A*d)*sin(f*x + e))*sqrt(c^2 - d^2)*arc
tan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + ((A - B)*c^2 -
 (A - B)*d^2)*cos(f*x + e) - ((A - B)*c^2 - (A - B)*d^2)*sin(f*x + e))/((a
*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f*cos(f*x + e) + (a*c^3 - a*c^2*d - a*c*
d^2 + a*d^3)*f*sin(f*x + e) + (a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm=
"maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.09

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))}\,dx = \frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)(Bc-Ad)}{(ac-ad)\sqrt{c^2-d^2}}-\frac{A-B}{(ac-ad)\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}\right)}{f}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm=
"giac")
```

output
```
2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*
e) + d)/sqrt(c^2 - d^2)))*(B*c - A*d)/((a*c - a*d)*sqrt(c^2 - d^2)) - (A -
 B)/((a*c - a*d)*(tan(1/2*f*x + 1/2*e) + 1)))/f
```

Mupad [B] (verification not implemented)

Time = 36.24 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.52

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))}\,dx = \frac{2\,\mathrm{atan}\left(\frac{\frac{(A\,d-B\,c)\,\left(2\,a\,d^2-2\,a\,c\,d\right)}{a\,\sqrt{c+d}\,(c-d)^{3/2}}-\frac{2\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(A\,d-B\,c)\,(a\,c-a\,d)}{a\,\sqrt{c+d}\,(c-d)^{3/2}}}{2\,A\,d-2\,B\,c}\right)\,(A\,d-B\,c)}{a\,f\,\sqrt{c+d}\,(c-d)^{3/2}} - \frac{2\,(A-B)}{f\,\left(a+a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,(c-d)}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c + d*sin(e + f*x))),x)
```

output
```
(2*atan((((A*d - B*c)*(2*a*d^2 - 2*a*c*d))/(a*(c + d)^(1/2)*(c - d)^(3/2))
 - (2*c*tan(e/2 + (f*x)/2)*(A*d - B*c)*(a*c - a*d))/(a*(c + d)^(1/2)*(c -
d)^(3/2)))/(2*A*d - 2*B*c))*(A*d - B*c))/(a*f*(c + d)^(1/2)*(c - d)^(3/2))
 - (2*(A - B))/(f*(a + a*tan(e/2 + (f*x)/2))*(c - d))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 324, normalized size of antiderivative = 3.21

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))(c + d\sin(e + fx))}\,dx$$
$$= \frac{-2\sqrt{c^2-d^2}\,atan\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c+d}{\sqrt{c^2-d^2}}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)ad + 2\sqrt{c^2-d^2}\,atan\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c+d}{\sqrt{c^2-d^2}}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)bc - 2}{af\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c^3 - \mathrm{ta}\right.}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

output
```
(2*( - sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*
tan((e + f*x)/2)*a*d + sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqr
t(c**2 - d**2))*tan((e + f*x)/2)*b*c - sqrt(c**2 - d**2)*atan((tan((e + f*
x)/2)*c + d)/sqrt(c**2 - d**2))*a*d + sqrt(c**2 - d**2)*atan((tan((e + f*x
)/2)*c + d)/sqrt(c**2 - d**2))*b*c + tan((e + f*x)/2)*a*c**2 - tan((e + f*
x)/2)*a*d**2 - tan((e + f*x)/2)*b*c**2 + tan((e + f*x)/2)*b*d**2))/(a*f*(t
an((e + f*x)/2)*c**3 - tan((e + f*x)/2)*c**2*d - tan((e + f*x)/2)*c*d**2 +
 tan((e + f*x)/2)*d**3 + c**3 - c**2*d - c*d**2 + d**3))
```

3.270 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 181

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^2}\,dx = -\frac{2(Ad(2c+d)-B(c^2+cd+d^2))\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a(c-d)\left(c^2-d^2\right)^{3/2}f} + \frac{d(B(2c+d)-A(c+2d))\cos(e+fx)}{a(c-d)^2(c+d)f(c+d\sin(e+fx))} - \frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+d\sin(e+fx))}$$

output

```
-2*(A*d*(2*c+d)-B*(c^2+c*d+d^2))*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)
^(1/2))/a/(c-d)/(c^2-d^2)^(3/2)/f+d*(B*(2*c+d)-A*(c+2*d))*cos(f*x+e)/a/(c-
d)^2/(c+d)/f/(c+d*sin(f*x+e))-(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))/(c
+d*sin(f*x+e))
```

Mathematica [A] (verified)

Time = 4.81 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.15

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^2}\,dx$$

$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2(A-B)\sin\left(\frac{1}{2}(e+fx)\right)+\frac{2(-Ad(2c+d)+B(c^2+cd+d^2))\arctan\left(\frac{d+c\tan\left(\frac{1}{2}\right.}{\sqrt{c^2\cdots}}\right.}{(c+d)\sqrt{c^2\cdots}}\right.}{a(c-d)^2f(1+\sin(e+fx))\cdots}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^
2),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*Sin[(e + f*x)/2] + (2*(-
(A*d*(2*c + d)) + B*(c^2 + c*d + d^2))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqr
t[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/((c + d)*Sqrt[c^2 - d
^2]) + (d*(B*c - A*d)*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/
((c + d)*(c + d*Sin[e + f*x]))))/(a*(c - d)^2*f*(1 + Sin[e + f*x]))
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3457, 3042, 3233, 25, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)(c+d\sin(e+fx))^2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)(c+d\sin(e+fx))^2}dx$$

$$\Big\downarrow \ 3457$$

$$-\frac{\int \frac{a(2Ad-B(c+d))-a(A-B)d\sin(e+fx)}{(c+d\sin(e+fx))^2}dx}{a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))}$$

$\Big\downarrow$ 3042

$$-\frac{\int \frac{a(2Ad-B(c+d))-a(A-B)d\sin(e+fx)}{(c+d\sin(e+fx))^2}dx}{a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))}$$

$\Big\downarrow$ 3233

$$-\frac{-\frac{\int -\frac{a\left(Ad(2c+d)-B\left(c^2+dc+d^2\right)\right)}{c+d\sin(e+fx)}dx}{c^2-d^2}-\frac{ad(B(2c+d)-A(c+2d))\cos(e+fx)}{f(c^2-d^2)(c+d\sin(e+fx))}}{a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\int \frac{a\left(Ad(2c+d)-B\left(c^2+dc+d^2\right)\right)}{c+d\sin(e+fx)}dx}{c^2-d^2}-\frac{ad(B(2c+d)-A(c+2d))\cos(e+fx)}{f(c^2-d^2)(c+d\sin(e+fx))}}{a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))}$$

$\Big\downarrow$ 27

$$-\frac{\frac{a\left(Ad(2c+d)-B\left(c^2+cd+d^2\right)\right)\int \frac{1}{c+d\sin(e+fx)}dx}{c^2-d^2}-\frac{ad(B(2c+d)-A(c+2d))\cos(e+fx)}{f(c^2-d^2)(c+d\sin(e+fx))}}{a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{a\left(Ad(2c+d)-B\left(c^2+cd+d^2\right)\right)\int \frac{1}{c+d\sin(e+fx)}dx}{c^2-d^2}-\frac{ad(B(2c+d)-A(c+2d))\cos(e+fx)}{f(c^2-d^2)(c+d\sin(e+fx))}}{a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))}$$

$\Big\downarrow$ 3139

$$-\frac{\frac{2a\left(Ad(2c+d)-B\left(c^2+cd+d^2\right)\right)\int \frac{1}{c\tan^2\left(\frac{1}{2}(e+fx)\right)+2d\tan\left(\frac{1}{2}(e+fx)\right)+c}d\tan\left(\frac{1}{2}(e+fx)\right)}{f(c^2-d^2)}-\frac{ad(B(2c+d)-A(c+2d))\cos(e+fx)}{f(c^2-d^2)(c+d\sin(e+fx))}}{a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))}$$

$$\Big\downarrow \ 1083$$

$$-\frac{-\frac{4a\left(Ad(2c+d)-B\left(c^2+cd+d^2\right)\right)\int \frac{1}{-\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)^2-4\left(c^2-d^2\right)}d\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{f\left(c^2-d^2\right)}-\frac{ad(B(2c+d)-A(c+2d))\cos(e+fx)}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}}{a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))}$$

$$\Big\downarrow \ 217$$

$$-\frac{\frac{2a\left(Ad(2c+d)-B\left(c^2+cd+d^2\right)\right)\arctan\left(\frac{2c\tan\left(\frac{1}{2}(e+fx)\right)+2d}{2\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{3/2}}-\frac{ad(B(2c+d)-A(c+2d))\cos(e+fx)}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}}{a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^2),x]
```

output
```
-(((A - B)*Cos[e + f*x])/((c - d)*f*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*
x]))) - ((2*a*(A*d*(2*c + d) - B*(c^2 + c*d + d^2))*ArcTan[(2*d + 2*c*Tan[
(e + f*x)/2])/(2*Sqrt[c^2 - d^2])])/((c^2 - d^2)^(3/2)*f) - (a*d*(B*(2*c +
 d) - A*(c + 2*d))*Cos[e + f*x])/((c^2 - d^2)*f*(c + d*Sin[e + f*x])))/(a^
2*(c - d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{-\frac{2\left(\frac{\frac{d^2(Ad-Bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}+\frac{d(Ad-Bc)}{c+d}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2 c+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}+\frac{\left(2Acd+A\,d^2-B\,c^2-Bcd-B\,d^2\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}}\right)}{(c-d)^2}-\frac{2(A-B}{(c-d)^2\left(\tan\left(\frac{f}{2}\right.\right.}}{af}$
default	$\frac{-\frac{2\left(\frac{\frac{d^2(Ad-Bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}+\frac{d(Ad-Bc)}{c+d}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2 c+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}+\frac{\left(2Acd+A\,d^2-B\,c^2-Bcd-B\,d^2\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}}\right)}{(c-d)^2}-\frac{2(A-B}{(c-d)^2\left(\tan\left(\frac{f}{2}\right.\right.}}{af}$
risch	Expression too large to display

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x,method=_RETURNV
ERBOSE)
```

output
```
2/f/a*(-1/(c-d)^2*((d^2*(A*d-B*c)/(c+d)/c*tan(1/2*f*x+1/2*e)+d*(A*d-B*c)/(
c+d))/(tan(1/2*f*x+1/2*e)^2*c+2*d*tan(1/2*f*x+1/2*e)+c)+(2*A*c*d+A*d^2-B*c
^2-B*c*d-B*d^2)/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2
*d)/(c^2-d^2)^(1/2)))-(A-B)/(c-d)^2/(tan(1/2*f*x+1/2*e)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 725 vs. $2(176) = 352$.

Time = 0.13 (sec) , antiderivative size = 1538, normalized size of antiderivative = 8.50

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorith
m="fricas")
```

output
```
[1/2*(2*(A - B)*c^4 - 4*(A - B)*c^2*d^2 + 2*(A - B)*d^4 + 2*((A - 2*B)*c^3
*d + (2*A - B)*c^2*d^2 - (A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e)^2 +
 (B*c^3 - 2*(A - B)*c^2*d - (3*A - 2*B)*c*d^2 - (A - B)*d^3 - (B*c^2*d - (
2*A - B)*c*d^2 - (A - B)*d^3)*cos(f*x + e)^2 + (B*c^3 - (2*A - B)*c^2*d -
(A - B)*c*d^2)*cos(f*x + e) + (B*c^3 - 2*(A - B)*c^2*d - (3*A - 2*B)*c*d^2
 - (A - B)*d^3 + (B*c^2*d - (2*A - B)*c*d^2 - (A - B)*d^3)*cos(f*x + e))*s
in(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*si
n(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*
sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) +
 2*((A - B)*c^4 + (A - 2*B)*c^3*d + B*c^2*d^2 - (A - 2*B)*c*d^3 - A*d^4)*c
os(f*x + e) - 2*((A - B)*c^4 - 2*(A - B)*c^2*d^2 + (A - B)*d^4 - ((A - 2*B
)*c^3*d + (2*A - B)*c^2*d^2 - (A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e
))*sin(f*x + e))/((a*c^5*d - a*c^4*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d
^5 - a*d^6)*f*cos(f*x + e)^2 - (a*c^6 - a*c^5*d - 2*a*c^4*d^2 + 2*a*c^3*d^
3 + a*c^2*d^4 - a*c*d^5)*f*cos(f*x + e) - (a*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d
^4 - a*d^6)*f - ((a*c^5*d - a*c^4*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^
5 - a*d^6)*f*cos(f*x + e) + (a*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*f)
*sin(f*x + e)), ((A - B)*c^4 - 2*(A - B)*c^2*d^2 + (A - B)*d^4 + ((A - 2*B
)*c^3*d + (2*A - B)*c^2*d^2 - (A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e
)^2 + (B*c^3 - 2*(A - B)*c^2*d - (3*A - 2*B)*c*d^2 - (A - B)*d^3 - (B*c...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))(c + d\sin(e + fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 425 vs. $2(176) = 352$.

Time = 0.24 (sec) , antiderivative size = 425, normalized size of antiderivative = 2.35

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^2}\,dx = \frac{2\left(\frac{(Bc^2-2\,Acd+Bcd-Ad^2+Bd^2)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(ac^3-ac^2d-acd^2+ad^3)\sqrt{c^2-d^2}} - \frac{Ac^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-Bc^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+A}{}\right.}{}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorith
m="giac")
```

output
```
2*((B*c^2 - 2*A*c*d + B*c*d - A*d^2 + B*d^2)*(pi*floor(1/2*(f*x + e)/pi +
1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a*c^
3 - a*c^2*d - a*c*d^2 + a*d^3)*sqrt(c^2 - d^2)) - (A*c^3*tan(1/2*f*x + 1/2
*e)^2 - B*c^3*tan(1/2*f*x + 1/2*e)^2 + A*c^2*d*tan(1/2*f*x + 1/2*e)^2 - B*
c^2*d*tan(1/2*f*x + 1/2*e)^2 - B*c*d^2*tan(1/2*f*x + 1/2*e)^2 + A*d^3*tan(
1/2*f*x + 1/2*e)^2 + 2*A*c^2*d*tan(1/2*f*x + 1/2*e) - 3*B*c^2*d*tan(1/2*f*
x + 1/2*e) + 3*A*c*d^2*tan(1/2*f*x + 1/2*e) - 3*B*c*d^2*tan(1/2*f*x + 1/2*
e) + A*d^3*tan(1/2*f*x + 1/2*e) + A*c^3 - B*c^3 + A*c^2*d - 2*B*c^2*d + A*
c*d^2)/((a*c^4 - a*c^3*d - a*c^2*d^2 + a*c*d^3)*(c*tan(1/2*f*x + 1/2*e)^3
+ c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e)^2 + c*tan(1/2*f*x +
1/2*e) + 2*d*tan(1/2*f*x + 1/2*e) + c)))/f
```

Mupad [B] (verification not implemented)

Time = 37.78 (sec) , antiderivative size = 437, normalized size of antiderivative = 2.41

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^2}\,dx = \frac{2\,\mathrm{atan}\left(\frac{\frac{\left(2\,a\,c^3\,d-2\,a\,c^2\,d^2-2\,a\,c\,d^3+2\,a\,d^4\right)\,\left(B\,c^2-A\,d^2+B\,d^2-2\,A\,c\,d+B\,c\,d\right)}{a\,{\left(c+d\right)}^{3/2}\,{\left(c-d\right)}^{5/2}}+\frac{2\,c\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(a\,c^3-a\,c^2\,d-a\,c\,d^2+a\,d^3\right)\,\left(B\,c^2-A\,d^2+B\,d^2-2\,A\,c\,d+\cdots\right)}{a\,{\left(c+d\right)}^{3/2}\,{\left(c-d\right)}^{5/2}}}{2\,B\,c^2-2\,A\,d^2+2\,B\,d^2-4\,A\,c\,d+2\,B\,c\,d}\cdots\right)\cdots}{a\,f\,{\left(c+d\right)}^{3/2}\,{\left(c-d\right)}^{5/2}} - \frac{\frac{2\,\left(A\,c^2+A\,d^2-B\,c^2+A\,c\,d-2\,B\,c\,d\right)}{\left(c+d\right)\,{\left(c-d\right)}^2}+\frac{2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(A\,d^2+2\,A\,c\,d-3\,B\,c\,d\right)}{c\,{\left(c-d\right)}^2}+\frac{2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2\,\left(A\,c^3+A\,d^3-B\,c^3+A\,c^2\,d-B\,c\,d^2-B\,c^2\cdots\right)}{c\,\left(c+d\right)\,{\left(c-d\right)}^2}}{f\,\left(a\,c\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3+\left(a\,c+2\,a\,d\right)\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+\left(a\,c+2\,a\,d\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a\,c\right)}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^2),x)
```

output
```
(2*atan((((2*a*d^4 - 2*a*c^2*d^2 - 2*a*c*d^3 + 2*a*c^3*d)*(B*c^2 - A*d^2 +
 B*d^2 - 2*A*c*d + B*c*d))/(a*(c + d)^(3/2)*(c - d)^(5/2)) + (2*c*tan(e/2
+ (f*x)/2)*(a*c^3 + a*d^3 - a*c*d^2 - a*c^2*d)*(B*c^2 - A*d^2 + B*d^2 - 2*
A*c*d + B*c*d))/(a*(c + d)^(3/2)*(c - d)^(5/2)))/(2*B*c^2 - 2*A*d^2 + 2*B*
d^2 - 4*A*c*d + 2*B*c*d))*(B*c^2 - A*d^2 + B*d^2 - 2*A*c*d + B*c*d))/(a*f*
(c + d)^(3/2)*(c - d)^(5/2)) - ((2*(A*c^2 + A*d^2 - B*c^2 + A*c*d - 2*B*c*
d))/((c + d)*(c - d)^2) + (2*tan(e/2 + (f*x)/2)*(A*d^2 + 2*A*c*d - 3*B*c*d
))/(c*(c - d)^2) + (2*tan(e/2 + (f*x)/2)^2*(A*c^3 + A*d^3 - B*c^3 + A*c^2*
d - B*c*d^2 - B*c^2*d))/(c*(c + d)*(c - d)^2))/(f*(a*c + tan(e/2 + (f*x)/2
)^2*(a*c + 2*a*d) + tan(e/2 + (f*x)/2)*(a*c + 2*a*d) + a*c*tan(e/2 + (f*x)
/2)^3))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 2468, normalized size of antiderivative = 13.64

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

output

```
(2*( - 2*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2)
)*tan((e + f*x)/2)**3*a*c**3*d - 5*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2
)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)**3*a*c**2*d**2 - 2*sqrt(c**2
- d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)*
*3*a*c*d**3 + sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 -
d**2))*tan((e + f*x)/2)**3*b*c**4 + 3*sqrt(c**2 - d**2)*atan((tan((e + f*x
)/2)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)**3*b*c**3*d + 3*sqrt(c**2
- d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)*
*3*b*c**2*d**2 + 2*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c*
*2 - d**2))*tan((e + f*x)/2)**3*b*c*d**3 - 2*sqrt(c**2 - d**2)*atan((tan((
e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)**2*a*c**3*d - 9*sqr
t(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e + f
*x)/2)**2*a*c**2*d**2 - 12*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)
/sqrt(c**2 - d**2))*tan((e + f*x)/2)**2*a*c*d**3 - 4*sqrt(c**2 - d**2)*ata
n((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)**2*a*d**4 +
 sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e
 + f*x)/2)**2*b*c**4 + 5*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/s
qrt(c**2 - d**2))*tan((e + f*x)/2)**2*b*c**3*d + 9*sqrt(c**2 - d**2)*atan(
(tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)**2*b*c**2*d**
2 + 8*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2)...
```

3.271 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 283

$$\begin{aligned}
&\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^3}\,dx \\
&= -\frac{(3Ad(2c^2+2cd+d^2)-B(2c^3+4c^2d+7cd^2+2d^3))\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a(c-d)\left(c^2-d^2\right)^{5/2}f} \\
&\quad -\frac{d(2Ac-3Bc+3Ad-2Bd)\cos(e+fx)}{2a(c-d)^2(c+d)f(c+d\sin(e+fx))^2} \\
&\quad -\frac{(A-B)\cos(e+fx)}{(c-d)f(a+a\sin(e+fx))(c+d\sin(e+fx))^2} \\
&\quad -\frac{d(2Ac^2-5Bc^2+9Acd-6Bcd+4Ad^2-4Bd^2)\cos(e+fx)}{2a(c-d)^3(c+d)^2f(c+d\sin(e+fx))}
\end{aligned}$$

output
```
-(3*A*d*(2*c^2+2*c*d+d^2)-B*(2*c^3+4*c^2*d+7*c*d^2+2*d^3))*arctan((d+c*tan
(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a/(c-d)/(c^2-d^2)^(5/2)/f-1/2*d*(2*A*c+3
*A*d-3*B*c-2*B*d)*cos(f*x+e)/a/(c-d)^2/(c+d)/f/(c+d*sin(f*x+e))^2-(A-B)*co
s(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2-1/2*d*(2*A*c^2+9*A*c*
d+4*A*d^2-5*B*c^2-6*B*c*d-4*B*d^2)*cos(f*x+e)/a/(c-d)^3/(c+d)^2/f/(c+d*sin
(f*x+e))
```

Mathematica [A] (verified)

Time = 5.51 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.11

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^3}\,dx$$

$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(4(A-B)\sin\left(\frac{1}{2}(e+fx)\right)+\frac{2(-3Ad(2c^2+2cd+d^2)+B(2c^3+4c^2d+7cd^2+2d^3)}{(}\right.}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^
3),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(4*(A - B)*Sin[(e + f*x)/2] + (2*(-
3*A*d*(2*c^2 + 2*c*d + d^2) + B*(2*c^3 + 4*c^2*d + 7*c*d^2 + 2*d^3))*ArcTa
n[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2]))/((c + d)^2*Sqrt[c^2 - d^2]) + ((c - d)*d*(B*c - A*d)*Cos[e + f*x]
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/((c + d)*(c + d*Sin[e + f*x])^2) +
 (d*(-(A*d*(5*c + 2*d)) + B*(3*c^2 + 2*c*d + 2*d^2))*Cos[e + f*x]*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2]))/((c + d)^2*(c + d*Sin[e + f*x]))))/(2*a*(c
- d)^3*f*(1 + Sin[e + f*x]))
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 3457, 3042, 3233, 25, 3042, 3233, 25, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)(c+d\sin(e+fx))^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)(c+d\sin(e+fx))^3}dx$$

$$\Big\downarrow \; 3457$$

$$-\frac{\int \frac{a(3Ad-B(c+2d))-2a(A-B)d\sin(e+fx)}{(c+d\sin(e+fx))^3}dx}{a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \frac{a(3Ad-B(c+2d))-2a(A-B)d\sin(e+fx)}{(c+d\sin(e+fx))^3}dx}{a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \; 3233$$

$$-\frac{\frac{ad(2Ac+3Ad-3Bc-2Bd)\cos(e+fx)}{2f(c^2-d^2)(c+d\sin(e+fx))^2}-\frac{\int -\frac{2a\left(2(A-B)d^2+3Acd-Bc(c+2d)\right)-ad(2Ac-3Bc+3Ad-2Bd)\sin(e+fx)}{(c+d\sin(e+fx))^2}dx}{2(c^2-d^2)}}{a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\int \frac{2a\left(2(A-B)d^2+3Acd-Bc(c+2d)\right)-ad(2Ac-3Bc+3Ad-2Bd)\sin(e+fx)}{(c+d\sin(e+fx))^2}dx}{2(c^2-d^2)}+\frac{ad(2Ac+3Ad-3Bc-2Bd)\cos(e+fx)}{2f(c^2-d^2)(c+d\sin(e+fx))^2}}{a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\int \frac{2a\left(2(A-B)d^2+3Acd-Bc(c+2d)\right)-ad(2Ac-3Bc+3Ad-2Bd)\sin(e+fx)}{(c+d\sin(e+fx))^2}dx}{2(c^2-d^2)}+\frac{ad(2Ac+3Ad-3Bc-2Bd)\cos(e+fx)}{2f(c^2-d^2)(c+d\sin(e+fx))^2}}{a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \; 3233$$

$$-\frac{\frac{-\frac{\int -\frac{a\left(3Ad\left(2c^2+2dc+d^2\right)-B\left(2c^3+4dc^2+7d^2c+2d^3\right)\right)}{c+d\sin(e+fx)}dx}{c^2-d^2}-\frac{ad\left(B\left(5c^2+6cd+4d^2\right)-A\left(2c^2+9cd+4d^2\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}}{2(c^2-d^2)}+\frac{ad(2Ac+3Ad-3Bc-2Bd)\cos(e+fx)}{2f(c^2-d^2)(c+d\sin(e+fx))^2}}{a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\frac{\int \frac{a\left(3Ad\left(2c^2+2dc+d^2\right)-B\left(2c^3+4dc^2+7d^2c+2d^3\right)\right)}{c+d\sin(e+fx)}dx}{c^2-d^2}-\frac{ad\left(B\left(5c^2+6cd+4d^2\right)-A\left(2c^2+9cd+4d^2\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}}{2(c^2-d^2)}+\frac{ad(2Ac+3Ad-3Bc-2Bd)\cos(e+f}{2f(c^2-d^2)(c+d\sin(e+fx))^2}}{a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))^2}$$

$\Big\downarrow$ 27

$$-\frac{\frac{\frac{a\left(3Ad\left(2c^2+2cd+d^2\right)-B\left(2c^3+4c^2d+7cd^2+2d^3\right)\right)\int\frac{1}{c+d\sin(e+fx)}dx}{c^2-d^2}-\frac{ad\left(B\left(5c^2+6cd+4d^2\right)-A\left(2c^2+9cd+4d^2\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}}{2(c^2-d^2)}+\frac{ad(2Ac+3Ad-3Bc-2}{2f(c^2-d^2)(c+d\sin}}{a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))^2}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\frac{a\left(3Ad\left(2c^2+2cd+d^2\right)-B\left(2c^3+4c^2d+7cd^2+2d^3\right)\right)\int\frac{1}{c+d\sin(e+fx)}dx}{c^2-d^2}-\frac{ad\left(B\left(5c^2+6cd+4d^2\right)-A\left(2c^2+9cd+4d^2\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}}{2(c^2-d^2)}+\frac{ad(2Ac+3Ad-3Bc-2}{2f(c^2-d^2)(c+d\sin}}{a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))^2}$$

$\Big\downarrow$ 3139

$$-\frac{\frac{\frac{2a\left(3Ad\left(2c^2+2cd+d^2\right)-B\left(2c^3+4c^2d+7cd^2+2d^3\right)\right)\int\frac{1}{c\tan^2\left(\frac{1}{2}(e+fx)\right)+2d\tan\left(\frac{1}{2}(e+fx)\right)+c}d\tan\left(\frac{1}{2}(e+fx)\right)}{f\left(c^2-d^2\right)}-\frac{ad\left(B\left(5c^2+6cd+4d^2\right)-A\left(2c^2+9cd+4d^2\right)\right)}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}}{2(c^2-d^2)}}{a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))^2}$$

$\Big\downarrow$ 1083

$$-\frac{\frac{-\frac{4a\left(3Ad\left(2c^2+2cd+d^2\right)-B\left(2c^3+4c^2d+7cd^2+2d^3\right)\right)\int\frac{1}{-\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)^2-4\left(c^2-d^2\right)}d\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{f\left(c^2-d^2\right)}-\frac{ad\left(B\left(5c^2+6cd+4d^2\right)-A\left(2c^2+9\right.\right.}{f\left(c^2-d^2\right)(c+d\sin(}}{2(c^2-d^2)}}{a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))^2}$$

$\Big\downarrow$ 217

$$-\frac{\frac{\frac{2a\left(3Ad\left(2c^2+2cd+d^2\right)-B\left(2c^3+4c^2d+7cd^2+2d^3\right)\right)\arctan\left(\frac{2c\tan\left(\frac{1}{2}(e+fx)\right)+2d}{2\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{3/2}}-\frac{ad\left(B\left(5c^2+6cd+4d^2\right)-A\left(2c^2+9cd+4d^2\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}}{2(c^2-d^2)}+\frac{ad(2Ac+3}{2f(c^2}}{a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))^2}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^3),x]
```

output
```
-(((A - B)*Cos[e + f*x])/((c - d)*f*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*
x])^2)) - ((a*d*(2*A*c - 3*B*c + 3*A*d - 2*B*d)*Cos[e + f*x])/(2*(c^2 - d^
2)*f*(c + d*Sin[e + f*x])^2) + ((2*a*(3*A*d*(2*c^2 + 2*c*d + d^2) - B*(2*c
^3 + 4*c^2*d + 7*c*d^2 + 2*d^3))*ArcTan[(2*d + 2*c*Tan[(e + f*x)/2])/(2*Sq
rt[c^2 - d^2])])/((c^2 - d^2)^(3/2)*f) - (a*d*(B*(5*c^2 + 6*c*d + 4*d^2) -
 A*(2*c^2 + 9*c*d + 4*d^2))*Cos[e + f*x])/((c^2 - d^2)*f*(c + d*Sin[e + f*
x])))/(2*(c^2 - d^2)))/(a^2*(c - d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.70

method	result
derivativedivides	$-\frac{2(A-B)}{(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{2\left(\frac{d^2\left(7A\,c^2d+2Ac\,d^2-2A\,d^3-5B\,c^3-2B\,c^2d\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2c\left(c^2+2cd+d^2\right)}+\frac{d\left(6A\,c^4d+2A\,c^3d^2+11A\,c^2d^3+4Ac\cdots\right.}{\cdots}\cdots\right.}{\cdots}$
default	$-\frac{2(A-B)}{(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{2\left(\frac{d^2\left(7A\,c^2d+2Ac\,d^2-2A\,d^3-5B\,c^3-2B\,c^2d\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2c\left(c^2+2cd+d^2\right)}+\frac{d\left(6A\,c^4d+2A\,c^3d^2+11A\,c^2d^3+4Ac\cdots\right.}{\cdots}\cdots\right.}{\cdots}$
risch	Expression too large to display

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x,method=_RETURNV
ERBOSE)
```

output
```
2/f/a*(-(A-B)/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)-1/(c-d)^3*((1/2*d^2*(7*A*c^2*
d+2*A*c*d^2-2*A*d^3-5*B*c^3-2*B*c^2*d)/c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e
)^3+1/2*d*(6*A*c^4*d+2*A*c^3*d^2+11*A*c^2*d^3+4*A*c*d^4-2*A*d^5-4*B*c^5-2*
B*c^4*d-9*B*c^3*d^2-4*B*c^2*d^3-2*B*c*d^4)/(c^2+2*c*d+d^2)/c^2*tan(1/2*f*x
+1/2*e)^2+1/2*d^2*(17*A*c^2*d+6*A*c*d^2-2*A*d^3-11*B*c^3-6*B*c^2*d-4*B*c*d
^2)/c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)+1/2*d*(6*A*c^2*d+2*A*c*d^2-A*d^3-
4*B*c^3-2*B*c^2*d-B*c*d^2)/(c^2+2*c*d+d^2))/(tan(1/2*f*x+1/2*e)^2*c+2*d*ta
n(1/2*f*x+1/2*e)+c)^2+1/2*(6*A*c^2*d+6*A*c*d^2+3*A*d^3-2*B*c^3-4*B*c^2*d-7
*B*c*d^2-2*B*d^3)/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*
f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1608 vs. $2(274) = 548$.

Time = 0.21 (sec) , antiderivative size = 3303, normalized size of antiderivative = 11.67

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorith
m="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^3}\,dx=\text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))(c+d\sin(e+fx))^3}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 727 vs. $2(274) = 548$.

Time = 0.26 (sec) , antiderivative size = 727, normalized size of antiderivative = 2.57

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))(c + d\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorith
m="giac")
```

output
```
((2*B*c^3 - 6*A*c^2*d + 4*B*c^2*d - 6*A*c*d^2 + 7*B*c*d^2 - 3*A*d^3 + 2*B*
d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/
2*e) + d)/sqrt(c^2 - d^2)))/((a*c^5 - a*c^4*d - 2*a*c^3*d^2 + 2*a*c^2*d^3
+ a*c*d^4 - a*d^5)*sqrt(c^2 - d^2)) - 2*(A - B)/((a*c^3 - 3*a*c^2*d + 3*a*
c*d^2 - a*d^3)*(tan(1/2*f*x + 1/2*e) + 1)) + (5*B*c^4*d^2*tan(1/2*f*x + 1/
2*e)^3 - 7*A*c^3*d^3*tan(1/2*f*x + 1/2*e)^3 + 2*B*c^3*d^3*tan(1/2*f*x + 1/
2*e)^3 - 2*A*c^2*d^4*tan(1/2*f*x + 1/2*e)^3 + 2*A*c*d^5*tan(1/2*f*x + 1/2*
e)^3 + 4*B*c^5*d*tan(1/2*f*x + 1/2*e)^2 - 6*A*c^4*d^2*tan(1/2*f*x + 1/2*e)
^2 + 2*B*c^4*d^2*tan(1/2*f*x + 1/2*e)^2 - 2*A*c^3*d^3*tan(1/2*f*x + 1/2*e)
^2 + 9*B*c^3*d^3*tan(1/2*f*x + 1/2*e)^2 - 11*A*c^2*d^4*tan(1/2*f*x + 1/2*e
)^2 + 4*B*c^2*d^4*tan(1/2*f*x + 1/2*e)^2 - 4*A*c*d^5*tan(1/2*f*x + 1/2*e)^
2 + 2*B*c*d^5*tan(1/2*f*x + 1/2*e)^2 + 2*A*d^6*tan(1/2*f*x + 1/2*e)^2 + 11
*B*c^4*d^2*tan(1/2*f*x + 1/2*e) - 17*A*c^3*d^3*tan(1/2*f*x + 1/2*e) + 6*B*
c^3*d^3*tan(1/2*f*x + 1/2*e) - 6*A*c^2*d^4*tan(1/2*f*x + 1/2*e) + 4*B*c^2*
d^4*tan(1/2*f*x + 1/2*e) + 2*A*c*d^5*tan(1/2*f*x + 1/2*e) + 4*B*c^5*d - 6*
A*c^4*d^2 + 2*B*c^4*d^2 - 2*A*c^3*d^3 + B*c^3*d^3 + A*c^2*d^4)/((a*c^7 - a
*c^6*d - 2*a*c^5*d^2 + 2*a*c^4*d^3 + a*c^3*d^4 - a*c^2*d^5)*(c*tan(1/2*f*x
 + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^2))/f
```

Mupad [B] (verification not implemented)

Time = 40.75 (sec) , antiderivative size = 1076, normalized size of antiderivative = 3.80

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))(c + d\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^3),x)
```

output
```
((A*d^4 - 2*A*c^4 + 2*B*c^4 - 8*A*c^2*d^2 + 4*B*c^2*d^2 - 2*A*c*d^3 - 4*A*
c^3*d + B*c*d^3 + 8*B*c^3*d)/((c + d)*(c^2 - d^2)*(c^2 - 2*c*d + d^2)) - (
tan(e/2 + (f*x)/2)^3*(2*A*d^6 - 13*A*c^2*d^4 - 17*A*c^3*d^3 - 22*A*c^4*d^2
 + 4*B*c^2*d^4 + 19*B*c^3*d^3 + 23*B*c^4*d^2 - 2*A*c*d^5 - 8*A*c^5*d + 2*B
*c*d^5 + 12*B*c^5*d))/(c^2*(c^2 - 2*c*d + d^2)*(c*d^2 - c^2*d - c^3 + d^3)
) + (tan(e/2 + (f*x)/2)^2*(2*A*d^5 - 4*A*c^5 + 4*B*c^5 - 21*A*c^2*d^3 - 14
*A*c^3*d^2 + 14*B*c^2*d^3 + 17*B*c^3*d^2 - 4*A*c*d^4 - 4*A*c^4*d + 2*B*c*d
^4 + 8*B*c^4*d))/(c^2*(c^2 - d^2)*(c^2 - 2*c*d + d^2)) + (tan(e/2 + (f*x)/
2)^4*(2*A*c^5 - 2*A*d^5 - 2*B*c^5 + 7*A*c^2*d^3 + 2*A*c^3*d^2 - 2*B*c^2*d^
3 - 7*B*c^3*d^2 + 2*A*c*d^4 + 4*A*c^4*d - 4*B*c^4*d))/(c*(c^2 - 2*c*d + d^
2)*(c*d^2 - c^2*d - c^3 + d^3)) + (tan(e/2 + (f*x)/2)*(2*A*d^5 - 27*A*c^2*
d^3 - 22*A*c^3*d^2 + 15*B*c^2*d^3 + 29*B*c^3*d^2 - 5*A*c*d^4 - 8*A*c^4*d +
 4*B*c*d^4 + 12*B*c^4*d))/(c*(c + d)*(c^2 - d^2)*(c^2 - 2*c*d + d^2)))/(f*
(tan(e/2 + (f*x)/2)^2*(2*a*c^2 + 4*a*d^2 + 4*a*c*d) + tan(e/2 + (f*x)/2)^3
*(2*a*c^2 + 4*a*d^2 + 4*a*c*d) + a*c^2 + tan(e/2 + (f*x)/2)*(a*c^2 + 4*a*c
*d) + tan(e/2 + (f*x)/2)^4*(a*c^2 + 4*a*c*d) + a*c^2*tan(e/2 + (f*x)/2)^5)
) - (atan((((2*a*d^6 - 4*a*c^2*d^4 + 4*a*c^3*d^3 + 2*a*c^4*d^2 - 2*a*c*d^5
 - 2*a*c^5*d)*(2*B*c^3 - 3*A*d^3 + 2*B*d^3 - 6*A*c*d^2 - 6*A*c^2*d + 7*B*c
*d^2 + 4*B*c^2*d))/(2*a*(c + d)^(5/2)*(c - d)^(7/2)) - (c*tan(e/2 + (f*x)/
2)*(a*c^5 - a*d^5 + 2*a*c^2*d^3 - 2*a*c^3*d^2 + a*c*d^4 - a*c^4*d)*(2*B...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 5820, normalized size of antiderivative = 20.57

$$\int \frac{A + B\sin(e+fx)}{(a + a\sin(e+fx))(c + d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)
```

output
```
( - 6*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*t
an((e + f*x)/2)**5*a*c**6*d - 30*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*
c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)**5*a*c**5*d**2 - 27*sqrt(c**2 -
 d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)**
5*a*c**4*d**3 - 12*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c*
*2 - d**2))*tan((e + f*x)/2)**5*a*c**3*d**4 + 2*sqrt(c**2 - d**2)*atan((ta
n((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)**5*b*c**7 + 12*s
qrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e +
 f*x)/2)**5*b*c**6*d + 23*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/
sqrt(c**2 - d**2))*tan((e + f*x)/2)**5*b*c**5*d**2 + 30*sqrt(c**2 - d**2)*
atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)**5*b*c**
4*d**3 + 8*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**
2))*tan((e + f*x)/2)**5*b*c**3*d**4 - 6*sqrt(c**2 - d**2)*atan((tan((e + f
*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)**4*a*c**6*d - 54*sqrt(c*
*2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/
2)**4*a*c**5*d**2 - 147*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sq
rt(c**2 - d**2))*tan((e + f*x)/2)**4*a*c**4*d**3 - 120*sqrt(c**2 - d**2)*a
tan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)**4*a*c**3
*d**4 - 48*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**
2))*tan((e + f*x)/2)**4*a*c**2*d**5 + 2*sqrt(c**2 - d**2)*atan((tan((e ...
```

3.272 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 228

$$\begin{aligned}
&\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx \\
&= \frac{d(2A(3c-2d)d+B(6c^2-12cd+7d^2))\,x}{2a^2} \\
&\quad+\frac{2d(A(c^2+6cd-5d^2)+B(2c^2-15cd+8d^2))\cos(e+fx)}{3a^2f} \\
&\quad+\frac{d^2(B(4c-21d)+2A(c+6d))\cos(e+fx)\sin(e+fx)}{6a^2f} \\
&\quad-\frac{(2B(c-4d)+A(c+5d))\cos(e+fx)(c+d\sin(e+fx))^2}{3a^2f(1+\sin(e+fx))} \\
&\quad-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{3f(a+a\sin(e+fx))^2}
\end{aligned}$$

output
```
1/2*d*(2*A*(3*c-2*d)*d+B*(6*c^2-12*c*d+7*d^2))*x/a^2+2/3*d*(A*(c^2+6*c*d-5
*d^2)+B*(2*c^2-15*c*d+8*d^2))*cos(f*x+e)/a^2/f+1/6*d^2*(B*(4*c-21*d)+2*A*(
c+6*d))*cos(f*x+e)*sin(f*x+e)/a^2/f-1/3*(2*B*(c-4*d)+A*(c+5*d))*cos(f*x+e)
*(c+d*sin(f*x+e))^2/a^2/f/(1+sin(f*x+e))-1/3*(A-B)*cos(f*x+e)*(c+d*sin(f*x
+e))^3/f/(a+a*sin(f*x+e))^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 547 vs. $2(228) = 456$.

Time = 4.81 (sec) , antiderivative size = 547, normalized size of antiderivative = 2.40

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(3(8Ad(6c^2+d^2(5-6e-6fx)+3cd(-4+3e+3fx))+B(16c^3\right.}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x
])^2,x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(3*(8*A*d*(6*c^2 + d^2*(5 - 6*e - 6
*f*x) + 3*c*d*(-4 + 3*e + 3*f*x)) + B*(16*c^3 + 24*c^2*d*(-4 + 3*e + 3*f*x
) - 24*c*d^2*(-5 + 6*e + 6*f*x) + 7*d^3*(-7 + 12*e + 12*f*x)))*Cos[(e + f*
x)/2] - (4*A*(4*c^3 + 24*c^2*d + d^3*(41 - 12*e - 12*f*x) + 6*c*d^2*(-10 +
 3*e + 3*f*x)) + B*(32*c^3 + 24*c^2*d*(-10 + 3*e + 3*f*x) - 12*c*d^2*(-41
+ 12*e + 12*f*x) + d^3*(-239 + 84*e + 84*f*x)))*Cos[(3*(e + f*x))/2] + 3*(
d^2*(12*B*c + 4*A*d - 5*B*d)*Cos[(5*(e + f*x))/2] + B*d^3*Cos[(7*(e + f*x)
)/2] + 2*(8*A*c^3 + 8*B*c^3 + 24*A*c^2*d - 72*B*c^2*d - 72*A*c*d^2 + 108*B
*c*d^2 + 36*A*d^3 - 50*B*d^3 + 48*B*c^2*d*e + 48*A*c*d^2*e - 96*B*c*d^2*e
- 32*A*d^3*e + 56*B*d^3*e + 48*B*c^2*d*f*x + 48*A*c*d^2*f*x - 96*B*c*d^2*f
*x - 32*A*d^3*f*x + 56*B*d^3*f*x + d*(8*A*d*(3*c*(e + f*x) - 2*d*(1 + e +
f*x)) + B*(24*c^2*(e + f*x) - 48*c*d*(1 + e + f*x) + d^2*(27 + 28*e + 28*f
*x)))*Cos[e + f*x] + 2*d^2*(-6*B*c - 2*A*d + 3*B*d)*Cos[2*(e + f*x)] + B*d
^3*Cos[3*(e + f*x)])*Sin[(e + f*x)/2])))/(48*a^2*f*(1 + Sin[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3042, 3456, 3042, 3456, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a\sin(e+fx)+a)^2}dx$$

$\Big\downarrow$ 3042

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a\sin(e+fx)+a)^2}dx$$

$\Big\downarrow$ 3456

$$\frac{\int \frac{(c+d\sin(e+fx))^2(a(Ac+2Bc+3Ad-3Bd)-a(2A-5B)d\sin(e+fx))}{\sin(e+fx)a+a}dx}{3a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{3f(a\sin(e+fx)+a)^2}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{(c+d\sin(e+fx))^2(a(Ac+2Bc+3Ad-3Bd)-a(2A-5B)d\sin(e+fx))}{\sin(e+fx)a+a}dx}{3a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{3f(a\sin(e+fx)+a)^2}$$

$\Big\downarrow$ 3456

$$\frac{\frac{\int (c+d\sin(e+fx))\left(a^2d(9Bc+10Ad-16Bd)-a^2d(B(4c-21d)+2A(c+6d))\sin(e+fx)\right)dx}{a^2}-\frac{(A(c+5d)+2B(c-4d))\cos(e+fx)(c+d\sin(e+fx))^2}{f(\sin(e+fx)+1)}}{3a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{3f(a\sin(e+fx)+a)^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int (c+d\sin(e+fx))\left(a^2d(9Bc+10Ad-16Bd)-a^2d(B(4c-21d)+2A(c+6d))\sin(e+fx)\right)dx}{a^2}-\frac{(A(c+5d)+2B(c-4d))\cos(e+fx)(c+d\sin(e+fx))^2}{f(\sin(e+fx)+1)}}{3a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{3f(a\sin(e+fx)+a)^2}$$

$\Big\downarrow$ 3213

$$\frac{\frac{\frac{2a^2d\left(A\left(c^2+6cd-5d^2\right)+B\left(2c^2-15cd+8d^2\right)\right)\cos(e+fx)}{f}+\frac{3}{2}a^2dx\left(2Ad(3c-2d)+B\left(6c^2-12cd+7d^2\right)\right)+\frac{a^2d^2(2A(c+6d)+B(4c-21d))\sin(e+fx)\cos(e+fx)}{2f}}{a^2}}{3a^2}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{3f(a\sin(e+fx)+a)^2}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^2,x
]
```

output
```
-1/3*((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(f*(a + a*Sin[e + f*x])
^2) + (-(((2*B*(c - 4*d) + A*(c + 5*d))*Cos[e + f*x]*(c + d*Sin[e + f*x])^
2)/(f*(1 + Sin[e + f*x]))) + ((3*a^2*d*(2*A*(3*c - 2*d)*d + B*(6*c^2 - 12*
c*d + 7*d^2))*x)/2 + (2*a^2*d*(A*(c^2 + 6*c*d - 5*d^2) + B*(2*c^2 - 15*c*d
 + 8*d^2))*Cos[e + f*x])/f + (a^2*d^2*(B*(4*c - 21*d) + 2*A*(c + 6*d))*Cos
[e + f*x]*Sin[e + f*x])/(2*f))/a^2)/(3*a^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

Maple [A] (verified)

Time = 2.58 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.49

method	result
derivativedivides	$\frac{2d\left(\frac{\frac{B\,d^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{2}+\left(-A\,d^{2}-3Bcd+2B\,d^{2}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-\frac{B\,d^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}-A\,d^{2}-3Bcd+2B\,d^{2}}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\right)^{2}}+\frac{\left(6Acd-4A\,d^{2}+6B\,c^{2}\right.}{}\right.}{}$
default	$\frac{2d\left(\frac{\frac{B\,d^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{2}+\left(-A\,d^{2}-3Bcd+2B\,d^{2}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-\frac{B\,d^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}-A\,d^{2}-3Bcd+2B\,d^{2}}{\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\right)^{2}}+\frac{\left(6Acd-4A\,d^{2}+6B\,c^{2}\right.}{}\right.}{}$
parallelrisch	$\frac{\left(\left(720fxA-1260fxB+696A-993B\right)d^{3}-1080\left(fxA-2fxB+\frac{7}{15}A-\frac{29}{15}B\right)c\,d^{2}-72c^{2}\left(15fxB+A+7B\right)d+216c^{3}\left(A-\frac{B}{9}\right)\right.}{}$
risch	$\frac{3d^{2}xAc}{a^{2}}-\frac{2d^{3}xA}{a^{2}}+\frac{3dxB\,c^{2}}{a^{2}}-\frac{6d^{2}xBc}{a^{2}}+\frac{7d^{3}xB}{2a^{2}}+\frac{iB\,d^{3}{\mathrm e}^{2i(fx+e)}}{8a^{2}f}-\frac{d^{3}{\mathrm e}^{i(fx+e)}A}{2a^{2}f}-\frac{3d^{2}{\mathrm e}^{i(fx+e)}Bc}{2a^{2}f}+$
norman	Expression too large to display

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x,method=_RETUR
NVERBOSE)
```

output
```
2/f/a^2*(d*((1/2*B*d^2*tan(1/2*f*x+1/2*e)^3+(-A*d^2-3*B*c*d+2*B*d^2)*tan(1
/2*f*x+1/2*e)^2-1/2*B*d^2*tan(1/2*f*x+1/2*e)-A*d^2-3*B*c*d+2*B*d^2)/(1+tan
(1/2*f*x+1/2*e)^2)^2+1/2*(6*A*c*d-4*A*d^2+6*B*c^2-12*B*c*d+7*B*d^2)*arctan
(tan(1/2*f*x+1/2*e)))-(A*c^3-3*A*c*d^2+2*A*d^3-3*B*c^2*d+6*B*c*d^2-3*B*d^3
)/(tan(1/2*f*x+1/2*e)+1)-1/2*(-2*A*c^3+6*A*c^2*d-6*A*c*d^2+2*A*d^3+2*B*c^3
-6*B*c^2*d+6*B*c*d^2-2*B*d^3)/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(2*A*c^3-6*A*c^
2*d+6*A*c*d^2-2*A*d^3-2*B*c^3+6*B*c^2*d-6*B*c*d^2+2*B*d^3)/(tan(1/2*f*x+1/
2*e)+1)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 584 vs. 2(218) = 436.

Time = 0.10 (sec) , antiderivative size = 584, normalized size of antiderivative = 2.56

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
-1/6*(3*B*d^3*cos(f*x + e)^4 - 2*(A - B)*c^3 + 6*(A - B)*c^2*d - 6*(A - B)
*c*d^2 + 2*(A - B)*d^3 + 6*(3*B*c*d^2 + (A - B)*d^3)*cos(f*x + e)^3 + 6*(6
*B*c^2*d + 6*(A - 2*B)*c*d^2 - (4*A - 7*B)*d^3)*f*x - (2*(A + 2*B)*c^3 + 6
*(2*A - 5*B)*c^2*d - 6*(5*A - 11*B)*c*d^2 + (22*A - 31*B)*d^3 + 3*(6*B*c^2
*d + 6*(A - 2*B)*c*d^2 - (4*A - 7*B)*d^3)*f*x)*cos(f*x + e)^2 - (2*(2*A +
B)*c^3 + 6*(A - 4*B)*c^2*d - 6*(4*A - 13*B)*c*d^2 + 2*(13*A - 19*B)*d^3 -
3*(6*B*c^2*d + 6*(A - 2*B)*c*d^2 - (4*A - 7*B)*d^3)*f*x)*cos(f*x + e) + (3
*B*d^3*cos(f*x + e)^3 + 2*(A - B)*c^3 - 6*(A - B)*c^2*d + 6*(A - B)*c*d^2
- 2*(A - B)*d^3 + 6*(6*B*c^2*d + 6*(A - 2*B)*c*d^2 - (4*A - 7*B)*d^3)*f*x
- 3*(6*B*c*d^2 + (2*A - 3*B)*d^3)*cos(f*x + e)^2 - (2*(A + 2*B)*c^3 + 6*(2
*A - 5*B)*c^2*d - 6*(5*A - 14*B)*c*d^2 + 4*(7*A - 10*B)*d^3 - 3*(6*B*c^2*d
 + 6*(A - 2*B)*c*d^2 - (4*A - 7*B)*d^3)*f*x)*cos(f*x + e))*sin(f*x + e))/(
a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e)
+ 2*a^2*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14612 vs. 2(216) = 432.

Time = 8.52 (sec) , antiderivative size = 14612, normalized size of antiderivative = 64.09

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**2,x)
```

output
```
Piecewise((-12*A*c**3*tan(e/2 + f*x/2)**6/(6*a**2*f*tan(e/2 + f*x/2)**7 +
18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*
tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 +
f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 12*A*c**3*tan(e/2 + f
*x/2)**5/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 3
0*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*t
an(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f
*x/2) + 6*a**2*f) - 32*A*c**3*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 + f*x/
2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42
*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*ta
n(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 24*A*c**3*tan
(e/2 + f*x/2)**3/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2
)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*
a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan
(e/2 + f*x/2) + 6*a**2*f) - 28*A*c**3*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(e/
2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)
**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a
**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 12*A*
c**3*tan(e/2 + f*x/2)/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 +
f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1382 vs. $2(218) = 436$.

Time = 0.15 (sec) , antiderivative size = 1382, normalized size of antiderivative = 6.06

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algori
thm="maxima")
```

output
```
1/3*(B*d^3*((75*sin(f*x + e)/(cos(f*x + e) + 1) + 97*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 98*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 21*sin(
f*x + e)^6/(cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x +
 e) + 1) + 5*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 7*a^2*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 7*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*a^2
*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 3*a^2*sin(f*x + e)^6/(cos(f*x + e)
+ 1)^6 + a^2*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f*x + e)
/(cos(f*x + e) + 1))/a^2) - 12*B*c*d^2*((12*sin(f*x + e)/(cos(f*x + e) + 1
) + 11*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x
 + e)/(cos(f*x + e) + 1) + 4*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a
^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e
) + 1)^4 + a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e
)/(cos(f*x + e) + 1))/a^2) - 4*A*d^3*((12*sin(f*x + e)/(cos(f*x + e) + 1)
+ 11*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x +
 e)/(cos(f*x + e) + 1) + 4*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e)
+ 1)^4 + a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(218) = 436$.

Time = 0.24 (sec) , antiderivative size = 472, normalized size of antiderivative = 2.07

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx$$
$$= \frac{\frac{3\,(6\,Bc^2d+6\,Acd^2-12\,Bcd^2-4\,Ad^3+7\,Bd^3)(fx+e)}{a^2} + \frac{6\left(Bd^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-6\,Bcd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-2\,Ad^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+4\,Bd^3\tan(\cdots}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)^2a^2 \cdots}}{\cdots}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
1/6*(3*(6*B*c^2*d + 6*A*c*d^2 - 12*B*c*d^2 - 4*A*d^3 + 7*B*d^3)*(f*x + e)/
a^2 + 6*(B*d^3*tan(1/2*f*x + 1/2*e)^3 - 6*B*c*d^2*tan(1/2*f*x + 1/2*e)^2 -
 2*A*d^3*tan(1/2*f*x + 1/2*e)^2 + 4*B*d^3*tan(1/2*f*x + 1/2*e)^2 - B*d^3*t
an(1/2*f*x + 1/2*e) - 6*B*c*d^2 - 2*A*d^3 + 4*B*d^3)/((tan(1/2*f*x + 1/2*e
)^2 + 1)^2*a^2) - 4*(3*A*c^3*tan(1/2*f*x + 1/2*e)^2 - 9*B*c^2*d*tan(1/2*f*
x + 1/2*e)^2 - 9*A*c*d^2*tan(1/2*f*x + 1/2*e)^2 + 18*B*c*d^2*tan(1/2*f*x +
 1/2*e)^2 + 6*A*d^3*tan(1/2*f*x + 1/2*e)^2 - 9*B*d^3*tan(1/2*f*x + 1/2*e)^
2 + 3*A*c^3*tan(1/2*f*x + 1/2*e) + 3*B*c^3*tan(1/2*f*x + 1/2*e) + 9*A*c^2*
d*tan(1/2*f*x + 1/2*e) - 27*B*c^2*d*tan(1/2*f*x + 1/2*e) - 27*A*c*d^2*tan(
1/2*f*x + 1/2*e) + 45*B*c*d^2*tan(1/2*f*x + 1/2*e) + 15*A*d^3*tan(1/2*f*x
+ 1/2*e) - 21*B*d^3*tan(1/2*f*x + 1/2*e) + 2*A*c^3 + B*c^3 + 3*A*c^2*d - 1
2*B*c^2*d - 12*A*c*d^2 + 21*B*c*d^2 + 7*A*d^3 - 10*B*d^3)/(a^2*(tan(1/2*f*
x + 1/2*e) + 1)^3))/f
```

Mupad [B] (verification not implemented)

Time = 39.54 (sec) , antiderivative size = 663, normalized size of antiderivative = 2.91

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^2}\,dx$$

$$= \frac{d\,\mathrm{atan}\left(\frac{d\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(6\,B\,c^2-4\,A\,d^2+7\,B\,d^2+6\,A\,c\,d-12\,B\,c\,d)}{7\,B\,d^3-4\,A\,d^3+6\,A\,c\,d^2-12\,B\,c\,d^2+6\,B\,c^2\,d}\right)\,(6\,B\,c^2-4\,A\,d^2+7\,B\,d^2+6\,A\,c\,d-12\,B\,c\,d)}{a^2\,f}$$

$$- \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(2\,A\,c^3+16\,A\,d^3+2\,B\,c^3-25\,B\,d^3-18\,A\,c\,d^2+6\,A\,c^2\,d+48\,B\,c\,d^2-18\,B\,c^2\,d)+\frac{4\,A\,c}{3}\cdots}{\cdots}$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^2,x
)
```

output
```
(d*atan((d*tan(e/2 + (f*x)/2)*(6*B*c^2 - 4*A*d^2 + 7*B*d^2 + 6*A*c*d - 12*
B*c*d))/(7*B*d^3 - 4*A*d^3 + 6*A*c*d^2 - 12*B*c*d^2 + 6*B*c^2*d))*(6*B*c^2
 - 4*A*d^2 + 7*B*d^2 + 6*A*c*d - 12*B*c*d))/(a^2*f) - (tan(e/2 + (f*x)/2)*
(2*A*c^3 + 16*A*d^3 + 2*B*c^3 - 25*B*d^3 - 18*A*c*d^2 + 6*A*c^2*d + 48*B*c
*d^2 - 18*B*c^2*d) + (4*A*c^3)/3 + (20*A*d^3)/3 + (2*B*c^3)/3 - (32*B*d^3)
/3 + tan(e/2 + (f*x)/2)^6*(2*A*c^3 + 4*A*d^3 - 7*B*d^3 - 6*A*c*d^2 + 12*B*
c*d^2 - 6*B*c^2*d) + tan(e/2 + (f*x)/2)^5*(2*A*c^3 + 12*A*d^3 + 2*B*c^3 -
21*B*d^3 - 18*A*c*d^2 + 6*A*c^2*d + 36*B*c*d^2 - 18*B*c^2*d) + tan(e/2 + (
f*x)/2)^3*(4*A*c^3 + 28*A*d^3 + 4*B*c^3 - 42*B*d^3 - 36*A*c*d^2 + 12*A*c^2
*d + 84*B*c*d^2 - 36*B*c^2*d) + tan(e/2 + (f*x)/2)^4*((16*A*c^3)/3 + (56*A
*d^3)/3 + (2*B*c^3)/3 - (98*B*d^3)/3 - 20*A*c*d^2 + 2*A*c^2*d + 56*B*c*d^2
 - 20*B*c^2*d) + tan(e/2 + (f*x)/2)^2*((14*A*c^3)/3 + (64*A*d^3)/3 + (4*B*
c^3)/3 - (97*B*d^3)/3 - 22*A*c*d^2 + 4*A*c^2*d + 64*B*c*d^2 - 22*B*c^2*d)
- 8*A*c*d^2 + 2*A*c^2*d + 20*B*c*d^2 - 8*B*c^2*d)/(f*(5*a^2*tan(e/2 + (f*x
)/2)^2 + 7*a^2*tan(e/2 + (f*x)/2)^3 + 7*a^2*tan(e/2 + (f*x)/2)^4 + 5*a^2*t
an(e/2 + (f*x)/2)^5 + 3*a^2*tan(e/2 + (f*x)/2)^6 + a^2*tan(e/2 + (f*x)/2)^
7 + a^2 + 3*a^2*tan(e/2 + (f*x)/2)))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1041, normalized size of antiderivative = 4.57

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^3}{(a + a\sin(e + fx))^2}\,dx = \text{Too large to display}$$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x)
```

output
```
(3*cos(e + f*x)*sin(e + f*x)**3*b*d**3 + 6*cos(e + f*x)*sin(e + f*x)**2*a*
d**3 + 18*cos(e + f*x)*sin(e + f*x)**2*b*c*d**2 - 6*cos(e + f*x)*sin(e + f
*x)**2*b*d**3 + 2*cos(e + f*x)*sin(e + f*x)*a*c**3 + 6*cos(e + f*x)*sin(e
+ f*x)*a*c**2*d + 18*cos(e + f*x)*sin(e + f*x)*a*c*d**2*f*x - 18*cos(e + f
*x)*sin(e + f*x)*a*c*d**2 - 12*cos(e + f*x)*sin(e + f*x)*a*d**3*f*x + 16*c
os(e + f*x)*sin(e + f*x)*a*d**3 + 2*cos(e + f*x)*sin(e + f*x)*b*c**3 + 18*
cos(e + f*x)*sin(e + f*x)*b*c**2*d*f*x - 18*cos(e + f*x)*sin(e + f*x)*b*c*
*2*d - 36*cos(e + f*x)*sin(e + f*x)*b*c*d**2*f*x + 48*cos(e + f*x)*sin(e +
 f*x)*b*c*d**2 + 21*cos(e + f*x)*sin(e + f*x)*b*d**3*f*x - 25*cos(e + f*x)
*sin(e + f*x)*b*d**3 + 4*cos(e + f*x)*a*c**3 + 18*cos(e + f*x)*a*c*d**2*f*
x - 12*cos(e + f*x)*a*c*d**2 - 12*cos(e + f*x)*a*d**3*f*x + 8*cos(e + f*x)
*a*d**3 + 18*cos(e + f*x)*b*c**2*d*f*x - 12*cos(e + f*x)*b*c**2*d - 36*cos
(e + f*x)*b*c*d**2*f*x + 24*cos(e + f*x)*b*c*d**2 + 21*cos(e + f*x)*b*d**3
*f*x - 14*cos(e + f*x)*b*d**3 + 3*sin(e + f*x)**4*b*d**3 + 6*sin(e + f*x)*
*3*a*d**3 + 18*sin(e + f*x)**3*b*c*d**2 - 9*sin(e + f*x)**3*b*d**3 + 2*sin
(e + f*x)**2*a*c**3 + 18*sin(e + f*x)**2*a*c**2*d - 18*sin(e + f*x)**2*a*c
*d**2*f*x - 42*sin(e + f*x)**2*a*c*d**2 + 12*sin(e + f*x)**2*a*d**3*f*x +
34*sin(e + f*x)**2*a*d**3 + 6*sin(e + f*x)**2*b*c**3 - 18*sin(e + f*x)**2*
b*c**2*d*f*x - 42*sin(e + f*x)**2*b*c**2*d + 36*sin(e + f*x)**2*b*c*d**2*f
*x + 102*sin(e + f*x)**2*b*c*d**2 - 21*sin(e + f*x)**2*b*d**3*f*x - 55*...
```

3.273 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 132

$$\begin{aligned}\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx &= \frac{d(2B(c-d)+Ad)x}{a^2}+\frac{(A-4B)d^2\cos(e+fx)}{3a^2f}\\ &\quad-\frac{(c-d)(2B(c-3d)+A(c+3d))\cos(e+fx)}{3a^2f(1+\sin(e+fx))}\\ &\quad-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a+a\sin(e+fx))^2}\end{aligned}$$

output
```
d*(2*B*(c-d)+A*d)*x/a^2+1/3*(A-4*B)*d^2*cos(f*x+e)/a^2/f-1/3*(c-d)*(2*B*(c
-3*d)+A*(c+3*d))*cos(f*x+e)/a^2/f/(1+sin(f*x+e))-1/3*(A-B)*cos(f*x+e)*(c+d
*sin(f*x+e))^2/f/(a+a*sin(f*x+e))^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 338 vs. $2(132) = 264$.

Time = 2.38 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.56

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(6(Ad(4c+d(-4+3e+3fx))+B(2c^2+d^2(5-6e-6fx)+2cd\right.}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x
])^2,x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(6*(A*d*(4*c + d*(-4 + 3*e + 3*f*x)
) + B*(2*c^2 + d^2*(5 - 6*e - 6*f*x) + 2*c*d*(-4 + 3*e + 3*f*x)))*Cos[(e +
 f*x)/2] - (B*(8*c^2 + d^2*(41 - 12*e - 12*f*x) + 4*c*d*(-10 + 3*e + 3*f*x
)) + 2*A*(2*c^2 + 8*c*d + d^2*(-10 + 3*e + 3*f*x)))*Cos[(3*(e + f*x))/2] +
 3*B*d^2*Cos[(5*(e + f*x))/2] + 6*(2*A*c^2 + 2*B*c^2 + 4*A*c*d - 12*B*c*d
- 6*A*d^2 + 9*B*d^2 + 8*B*c*d*e + 4*A*d^2*e - 8*B*d^2*e + 8*B*c*d*f*x + 4*
A*d^2*f*x - 8*B*d^2*f*x - 2*d*(-2*B*c*(e + f*x) - A*d*(e + f*x) + 2*B*d*(1
 + e + f*x))*Cos[e + f*x] - B*d^2*Cos[2*(e + f*x)])*Sin[(e + f*x)/2]))/(12
*a^2*f*(1 + Sin[e + f*x])^2)
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3456, 3042, 3447, 3042, 3502, 3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a\sin(e+fx)+a)^2}\,dx$$
$$\Big\downarrow \text{3042}$$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a\sin(e+fx)+a)^2}dx$$

$\downarrow$ 3456

$$\frac{\int \frac{(c+d\sin(e+fx))(a(2B(c-d)+A(c+2d))-a(A-4B)d\sin(e+fx))}{\sin(e+fx)a+a}dx}{3a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a\sin(e+fx)+a)^2}$$

$\downarrow$ 3042

$$\frac{\int \frac{(c+d\sin(e+fx))(a(2B(c-d)+A(c+2d))-a(A-4B)d\sin(e+fx))}{\sin(e+fx)a+a}dx}{3a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a\sin(e+fx)+a)^2}$$

$\downarrow$ 3447

$$\frac{\int \frac{-a(A-4B)d^2\sin^2(e+fx)+(ad(2B(c-d)+A(c+2d))-a(A-4B)cd)\sin(e+fx)+ac(2B(c-d)+A(c+2d))}{\sin(e+fx)a+a}dx}{3a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a\sin(e+fx)+a)^2}$$

$\downarrow$ 3042

$$\frac{\int \frac{-a(A-4B)d^2\sin(e+fx)^2+(ad(2B(c-d)+A(c+2d))-a(A-4B)cd)\sin(e+fx)+ac(2B(c-d)+A(c+2d))}{\sin(e+fx)a+a}dx}{3a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a\sin(e+fx)+a)^2}$$

$\downarrow$ 3502

$$\frac{\frac{\int \frac{c(2B(c-d)+A(c+2d))a^2+3d(2B(c-d)+Ad)\sin(e+fx)a^2}{\sin(e+fx)a+a}dx}{a}+\frac{d^2(A-4B)\cos(e+fx)}{f}}{3a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a\sin(e+fx)+a)^2}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{c(2B(c-d)+A(c+2d))a^2+3d(2B(c-d)+Ad)\sin(e+fx)a^2}{\sin(e+fx)a+a}dx}{a}+\frac{d^2(A-4B)\cos(e+fx)}{f}}{3a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a\sin(e+fx)+a)^2}$$

$\downarrow$ 3214

$$\frac{\frac{a^2(c-d)(A(c+3d)+2B(c-3d))\int \frac{1}{\sin(e+fx)a+a}dx+3adx(Ad+2B(c-d))}{a}+\frac{d^2(A-4B)\cos(e+fx)}{f}}{3a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a\sin(e+fx)+a)^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{a^2(c-d)(A(c+3d)+2B(c-3d))\int \frac{1}{\sin(e+fx)a+a}dx+3adx(Ad+2B(c-d))}{a}+\frac{d^2(A-4B)\cos(e+fx)}{f}}{3a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a\sin(e+fx)+a)^2}$$

$\Big\downarrow$ 3127

$$\frac{\frac{3adx(Ad+2B(c-d))-\frac{a^2(c-d)(A(c+3d)+2B(c-3d))\cos(e+fx)}{f(a\sin(e+fx)+a)}}{a}+\frac{d^2(A-4B)\cos(e+fx)}{f}}{3a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{3f(a\sin(e+fx)+a)^2}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])^2,x
]
```

output
```
-1/3*((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(f*(a + a*Sin[e + f*x])
^2) + (((A - 4*B)*d^2*Cos[e + f*x])/f + (3*a*d*(2*B*(c - d) + A*d)*x - (a^
2*(c - d)*(2*B*(c - 3*d) + A*(c + 3*d))*Cos[e + f*x])/(f*(a + a*Sin[e + f*
x])))/a)/(3*a^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.46

method	result
derivativedivides	$\frac{-\frac{2\left(A c^2-A d^2-2Bcd+2B d^2\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-2A c^2+4Acd-2A d^2+2B c^2-4Bcd+2B d^2}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(2A c^2-4Acd+2A d^2-2B c^2+4Bcd-2B d^2\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+2c}{a^2 f}$
default	$\frac{-\frac{2\left(A c^2-A d^2-2Bcd+2B d^2\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-2A c^2+4Acd-2A d^2+2B c^2-4Bcd+2B d^2}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(2A c^2-4Acd+2A d^2-2B c^2+4Bcd-2B d^2\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+2c}{a^2 f}$
risch	$\frac{d^2xA}{a^2}+\frac{2dxBc}{a^2}-\frac{2d^2xB}{a^2}-\frac{B d^2 e^{i(fx+e)}}{2a^2 f}-\frac{B d^2 e^{-i(fx+e)}}{2a^2 f}-\frac{2\left(3B c^2 e^{2i(fx+e)}-A c^2-2B c^2+5A d^2-4Acd-\right.}{}$
parallelrisch	$\frac{\left((-18fxA+36fxB-30A+78B)d^2+12((-3fx-5)B+A)cd+18c^2\left(A+\frac{B}{3}\right)\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+((6fxA-12fxB-2A+5B)}{}$
norman	$\frac{\frac{d(Ad+2Bc-2Bd)x}{a}+\frac{d(Ad+2Bc-2Bd)x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{a}-\frac{4A c^2+4Acd-8A d^2+2B c^2-16Bcd+20B d^2}{3af}-\frac{\left(2A c^2-2A d^2-4Bcd+4B\right.}{af}}{}$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x,method=_RETUR
NVERBOSE)
```

output
```
2/f/a^2*(-(A*c^2-A*d^2-2*B*c*d+2*B*d^2)/(tan(1/2*f*x+1/2*e)+1)-1/2*(-2*A*c
^2+4*A*c*d-2*A*d^2+2*B*c^2-4*B*c*d+2*B*d^2)/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(
2*A*c^2-4*A*c*d+2*A*d^2-2*B*c^2+4*B*c*d-2*B*d^2)/(tan(1/2*f*x+1/2*e)+1)^3+
d*(-B*d/(1+tan(1/2*f*x+1/2*e)^2)+(A*d+2*B*c-2*B*d)*arctan(tan(1/2*f*x+1/2*
e))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. 2(126) = 252.

Time = 0.09 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.84

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx =$$
$$-\frac{3\,Bd^2\cos\left(fx+e\right)^3-(A-B)c^2+2\,(A-B)cd-(A-B)d^2+6\,(2\,Bcd+(A-2\,B)d^2)fx-((A+2\right.}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
-1/3*(3*B*d^2*cos(f*x + e)^3 - (A - B)*c^2 + 2*(A - B)*c*d - (A - B)*d^2 +
 6*(2*B*c*d + (A - 2*B)*d^2)*f*x - ((A + 2*B)*c^2 + 2*(2*A - 5*B)*c*d - (5
*A - 11*B)*d^2 + 3*(2*B*c*d + (A - 2*B)*d^2)*f*x)*cos(f*x + e)^2 - ((2*A +
 B)*c^2 + 2*(A - 4*B)*c*d - (4*A - 13*B)*d^2 - 3*(2*B*c*d + (A - 2*B)*d^2)
*f*x)*cos(f*x + e) - (3*B*d^2*cos(f*x + e)^2 - (A - B)*c^2 + 2*(A - B)*c*d
 - (A - B)*d^2 - 6*(2*B*c*d + (A - 2*B)*d^2)*f*x + ((A + 2*B)*c^2 + 2*(2*A
 - 5*B)*c*d - (5*A - 14*B)*d^2 - 3*(2*B*c*d + (A - 2*B)*d^2)*f*x)*cos(f*x
+ e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f -
 (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5358 vs. $2(121) = 242$.

Time = 4.61 (sec) , antiderivative size = 5358, normalized size of antiderivative = 40.59

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^2}{(a + a\sin(e + fx))^2}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**2,x)
```

output
```
Piecewise((-6*A*c**2*tan(e/2 + f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9
*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*ta
n(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*A*c**2*tan(e
/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**
4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2
*f*tan(e/2 + f*x/2) + 3*a**2*f) - 10*A*c**2*tan(e/2 + f*x/2)**2/(3*a**2*f*
tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f
*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a
**2*f) - 6*A*c**2*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*
f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2
+ f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 4*A*c**2/(3*a**2*f*t
an(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*
x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a*
*2*f) - 12*A*c*d*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**
2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/
2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 4*A*c*d*tan(e/2 +
f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 1
2*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*ta
n(e/2 + f*x/2) + 3*a**2*f) - 12*A*c*d*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 +
 f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 831 vs. $2(126) = 252$.

Time = 0.13 (sec) , antiderivative size = 831, normalized size of antiderivative = 6.30

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^2}{(a + a\sin(e + fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algori
thm="maxima")
```

output
```
-2/3*(2*B*d^2*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1)
+ 4*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x +
e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2
) - 2*B*c*d*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^
3) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - A*d^2*((9*sin(f*x +
e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4)/(a^2 +
3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e)
 + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)
/(cos(f*x + e) + 1))/a^2) + A*c^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x
 + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3) + B*c^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a
^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 2*A*c*d*(3*sin(f
*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e)...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(126) = 252$.

Time = 0.23 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.00

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx$$

$$= \frac{\frac{3(2Bcd+Ad^2-2Bd^2)(fx+e)}{a^2} - \frac{6Bd^2}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)a^2} - \frac{2\left(3Ac^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 - 6Bcd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 - 3Ad^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 + 6}{}}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
1/3*(3*(2*B*c*d + A*d^2 - 2*B*d^2)*(f*x + e)/a^2 - 6*B*d^2/((tan(1/2*f*x +
 1/2*e)^2 + 1)*a^2) - 2*(3*A*c^2*tan(1/2*f*x + 1/2*e)^2 - 6*B*c*d*tan(1/2*
f*x + 1/2*e)^2 - 3*A*d^2*tan(1/2*f*x + 1/2*e)^2 + 6*B*d^2*tan(1/2*f*x + 1/
2*e)^2 + 3*A*c^2*tan(1/2*f*x + 1/2*e) + 3*B*c^2*tan(1/2*f*x + 1/2*e) + 6*A
*c*d*tan(1/2*f*x + 1/2*e) - 18*B*c*d*tan(1/2*f*x + 1/2*e) - 9*A*d^2*tan(1/
2*f*x + 1/2*e) + 15*B*d^2*tan(1/2*f*x + 1/2*e) + 2*A*c^2 + B*c^2 + 2*A*c*d
 - 8*B*c*d - 4*A*d^2 + 7*B*d^2)/(a^2*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

Mupad [B] (verification not implemented)

Time = 39.33 (sec) , antiderivative size = 365, normalized size of antiderivative = 2.77

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^2}\,dx$$
$$= \frac{2\,d\,\mathrm{atan}\left(\frac{2\,d\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)(A\,d+2\,B\,c-2\,B\,d)}{2\,A\,d^2-4\,B\,d^2+4\,B\,c\,d}\right)\,(A\,d+2\,B\,c-2\,B\,d)}{a^2\,f}$$
$$- \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,(2\,A\,c^2-6\,A\,d^2+2\,B\,c^2+12\,B\,d^2+4\,A\,c\,d-12\,B\,c\,d)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(\frac{10\,A\,c^2}{3}-\frac{14\,A\,d^2}{3}\right.}{f}$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^2,x
)
```

output
```
(2*d*atan((2*d*tan(e/2 + (f*x)/2)*(A*d + 2*B*c - 2*B*d))/(2*A*d^2 - 4*B*d^
2 + 4*B*c*d))*(A*d + 2*B*c - 2*B*d))/(a^2*f) - (tan(e/2 + (f*x)/2)^3*(2*A*
c^2 - 6*A*d^2 + 2*B*c^2 + 12*B*d^2 + 4*A*c*d - 12*B*c*d) + tan(e/2 + (f*x)
/2)^2*((10*A*c^2)/3 - (14*A*d^2)/3 + (2*B*c^2)/3 + (44*B*d^2)/3 + (4*A*c*d
)/3 - (28*B*c*d)/3) + (4*A*c^2)/3 - (8*A*d^2)/3 + (2*B*c^2)/3 + (20*B*d^2)
/3 + tan(e/2 + (f*x)/2)^4*(2*A*c^2 - 2*A*d^2 + 4*B*d^2 - 4*B*c*d) + tan(e/
2 + (f*x)/2)*(2*A*c^2 - 6*A*d^2 + 2*B*c^2 + 16*B*d^2 + 4*A*c*d - 12*B*c*d)
 + (4*A*c*d)/3 - (16*B*c*d)/3)/(f*(4*a^2*tan(e/2 + (f*x)/2)^2 + 4*a^2*tan(
e/2 + (f*x)/2)^3 + 3*a^2*tan(e/2 + (f*x)/2)^4 + a^2*tan(e/2 + (f*x)/2)^5 +
 a^2 + 3*a^2*tan(e/2 + (f*x)/2)))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 618, normalized size of antiderivative = 4.68

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^2}{(a + a\sin(e + fx))^2}\,dx$$
$$= \frac{17\sin(fx + e)^2\,b\,d^2 - 3\sin(fx + e)\,a\,d^2 + 3\sin(fx + e)^2\,b\,c^2 + 6\cos(fx + e)\sin(fx + e)\,bcdfx + 3\cos}{}$$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x)
```

output
```
(3*cos(e + f*x)*sin(e + f*x)**2*b*d**2 + cos(e + f*x)*sin(e + f*x)*a*c**2
+ 2*cos(e + f*x)*sin(e + f*x)*a*c*d + 3*cos(e + f*x)*sin(e + f*x)*a*d**2*f
*x - 3*cos(e + f*x)*sin(e + f*x)*a*d**2 + cos(e + f*x)*sin(e + f*x)*b*c**2
 + 6*cos(e + f*x)*sin(e + f*x)*b*c*d*f*x - 6*cos(e + f*x)*sin(e + f*x)*b*c
*d - 6*cos(e + f*x)*sin(e + f*x)*b*d**2*f*x + 8*cos(e + f*x)*sin(e + f*x)*
b*d**2 + 2*cos(e + f*x)*a*c**2 + 3*cos(e + f*x)*a*d**2*f*x - 2*cos(e + f*x
)*a*d**2 + 6*cos(e + f*x)*b*c*d*f*x - 4*cos(e + f*x)*b*c*d - 6*cos(e + f*x
)*b*d**2*f*x + 4*cos(e + f*x)*b*d**2 + 3*sin(e + f*x)**3*b*d**2 + sin(e +
f*x)**2*a*c**2 + 6*sin(e + f*x)**2*a*c*d - 3*sin(e + f*x)**2*a*d**2*f*x -
7*sin(e + f*x)**2*a*d**2 + 3*sin(e + f*x)**2*b*c**2 - 6*sin(e + f*x)**2*b*
c*d*f*x - 14*sin(e + f*x)**2*b*c*d + 6*sin(e + f*x)**2*b*d**2*f*x + 17*sin
(e + f*x)**2*b*d**2 + sin(e + f*x)*a*c**2 + 2*sin(e + f*x)*a*c*d - 6*sin(e
 + f*x)*a*d**2*f*x - 3*sin(e + f*x)*a*d**2 + sin(e + f*x)*b*c**2 - 12*sin(
e + f*x)*b*c*d*f*x - 6*sin(e + f*x)*b*c*d + 12*sin(e + f*x)*b*d**2*f*x + 8
*sin(e + f*x)*b*d**2 - 2*a*c**2 - 3*a*d**2*f*x + 2*a*d**2 - 6*b*c*d*f*x +
4*b*c*d + 6*b*d**2*f*x - 4*b*d**2)/(3*a**2*f*(cos(e + f*x)*sin(e + f*x) +
cos(e + f*x) - sin(e + f*x)**2 - 2*sin(e + f*x) - 1))
```

3.274 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^2} dx$

Optimal result

Integrand size = 33, antiderivative size = 85

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^2} dx = \frac{Bdx}{a^2} - \frac{(Ac+2Bc+2Ad-5Bd)\cos(e+fx)}{3a^2f(1+\sin(e+fx))} - \frac{(A-B)(c-d)\cos(e+fx)}{3f(a+a\sin(e+fx))^2}$$

output
```
B*d*x/a^2-1/3*(A*c+2*A*d+2*B*c-5*B*d)*cos(f*x+e)/a^2/f/(1+sin(f*x+e))-1/3*
(A-B)*(c-d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 180 vs. 2(85) = 170.

Time = 2.73 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.12

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^2} dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2(A-B)(c-d)\sin\left(\frac{1}{2}(e+fx)\right)-(A-B)(c-d)\left(\cos\left(\frac{1}{2}(e+fx\right.\right.\right.}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])
^2,x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*(c - d)*Sin[(e + f*x)/2]
 - (A - B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(A*c + 2*B*c
+ 2*A*d - 5*B*d)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2
+ 3*B*d*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3))/(3*a^2*f*(1 +
Sin[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 3447, 3042, 3498, 25, 3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a\sin(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a\sin(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3447}$$

$$\int \frac{(Ad+Bc)\sin(e+fx)+Ac+Bd\sin^2(e+fx)}{(a\sin(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(Ad+Bc)\sin(e+fx)+Ac+Bd\sin(e+fx)^2}{(a\sin(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3498}$$

$$-\frac{\int -\frac{a(2B(c-d)+A(c+2d))+3aBd\sin(e+fx)}{\sin(e+fx)a+a}dx}{3a^2}-\frac{(A-B)(c-d)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{a(2B(c-d)+A(c+2d))+3aBd\sin(e+fx)}{\sin(e+fx)a+a}dx}{3a^2} - \frac{(A-B)(c-d)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(2B(c-d)+A(c+2d))+3aBd\sin(e+fx)}{\sin(e+fx)a+a}dx}{3a^2} - \frac{(A-B)(c-d)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

$$\Big\downarrow \text{3214}$$

$$\frac{a(Ac+2Ad+2Bc-5Bd)\int \frac{1}{\sin(e+fx)a+a}dx + 3Bdx}{3a^2} - \frac{(A-B)(c-d)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(Ac+2Ad+2Bc-5Bd)\int \frac{1}{\sin(e+fx)a+a}dx + 3Bdx}{3a^2} - \frac{(A-B)(c-d)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

$$\Big\downarrow \text{3127}$$

$$\frac{3Bdx - \frac{a(Ac+2Ad+2Bc-5Bd)\cos(e+fx)}{f(a\sin(e+fx)+a)}}{3a^2} - \frac{(A-B)(c-d)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^2,x]
```

output
```
-1/3*((A - B)*(c - d)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^2) + (3*B*d*x
- (a*(A*c + 2*B*c + 2*A*d - 5*B*d)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])))
/(3*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3498
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[1
/(a^2*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b
*B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{-\frac{2(Ac-Bd)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-2Ac+2Ad+2Bc-2Bd}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(2Ac-2Ad-2Bc+2Bd)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+2Bd\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f\,a^2}$
default	$\frac{-\frac{2(Ac-Bd)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-2Ac+2Ad+2Bc-2Bd}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(2Ac-2Ad-2Bc+2Bd)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+2Bd\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f\,a^2}$
parallelrisch	$\frac{3Bd\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3xf+((9fx+6)dB-6Ac)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+((9dfx-6c+18d)B-6A(c+d))\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+(3dfx-2c+8d}{3f\,a^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}$
risch	$\frac{Bdx}{a^2}-\frac{2\left(-Ac-2Ad+3iAc\,e^{i(fx+e)}+3iAd\,e^{i(fx+e)}-2Bc+5Bd+3iBc\,e^{i(fx+e)}-9iBd\,e^{i(fx+e)}+3Ad\,e^{2i(fx+e)}+3B\right.}{3f\,a^2\left(e^{i(fx+e)}+i\right)^3}$
norman	$\frac{\frac{xBd}{a}+\frac{xBd\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{a}-\frac{4Ac+2Ad+2Bc-8Bd}{3af}-\frac{(2Ac-2Bd)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{af}-\frac{(16Ac+2Ad+2Bc-20Bd)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{3af}-\frac{(14Ac}{}}{}$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^2,x,method=_RETURNV
ERBOSE)
```

output
```
2/f/a^2*(-(A*c-B*d)/(tan(1/2*f*x+1/2*e)+1)-1/2*(-2*A*c+2*A*d+2*B*c-2*B*d)/
(tan(1/2*f*x+1/2*e)+1)^2-1/3*(2*A*c-2*A*d-2*B*c+2*B*d)/(tan(1/2*f*x+1/2*e)
+1)^3+B*d*arctan(tan(1/2*f*x+1/2*e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(81) = 162$.

Time = 0.08 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.45

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = -\frac{6\,Bdfx-(3\,Bdfx+(A+2\,B)c+(2\,A-5\,B)d)\cos{(fx+e)}^2-(A-B)c+(A-B)d+(3\,Bdfx-}{3\left(a^2f\cos{(fx+e)}^2-a^2\right.}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorith
m="fricas")
```

output
```
-1/3*(6*B*d*f*x - (3*B*d*f*x + (A + 2*B)*c + (2*A - 5*B)*d)*cos(f*x + e)^2
 - (A - B)*c + (A - B)*d + (3*B*d*f*x - (2*A + B)*c - (A - 4*B)*d)*cos(f*x
 + e) + (6*B*d*f*x + (A - B)*c - (A - B)*d + (3*B*d*f*x - (A + 2*B)*c - (2
*A - 5*B)*d)*cos(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos
(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1062 vs. $2(83) = 166$.

Time = 2.32 (sec) , antiderivative size = 1062, normalized size of antiderivative = 12.49

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))}{(a + a\sin(e + fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))**2,x)
```

output
```
Piecewise((-6*A*c*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a*
*2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*A*c*t
an(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)*
*2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 4*A*c/(3*a**2*f*tan(e/2 + f*x
/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2
*f) - 6*A*d*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(
e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 2*A*d/(3*a**2*f*
tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*
x/2) + 3*a**2*f) - 6*B*c*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 +
9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 2*B
*c/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f
*tan(e/2 + f*x/2) + 3*a**2*f) + 3*B*d*f*x*tan(e/2 + f*x/2)**3/(3*a**2*f*ta
n(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/
2) + 3*a**2*f) + 9*B*d*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)*
*3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f)
 + 9*B*d*f*x*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(
e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 3*B*d*f*x/(3*a**
2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2
 + f*x/2) + 3*a**2*f) + 6*B*d*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2
)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(81) = 162$.

Time = 0.12 (sec) , antiderivative size = 454, normalized size of antiderivative = 5.34

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))}{(a + a\sin(e + fx))^2}\,dx$$
$$= \frac{2\left(Bd\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1} + \frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + 4}{a^2 + \frac{3a^2\sin(fx+e)}{\cos(fx+e)+1} + \frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}} + \frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2}\right) - \frac{Ac\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1} + \frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + 2\right)}{a^2 + \frac{3a^2\sin(fx+e)}{\cos(fx+e)+1} + \frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}\right.}{3f}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorith
m="maxima")
```

output
```
2/3*(B*d*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)
+ 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) - A*c*(3*sin(f*x + e)/(co
s(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*
sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) - B*c*(3*sin(f*x + e)/(cos(f*
x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)
 - A*d*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(
cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*
x + e)^3/(cos(f*x + e) + 1)^3))/f
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.56

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))}{(a + a\sin(e + fx))^2}\,dx$$
$$= \frac{\frac{3(fx+e)Bd}{a^2} - \frac{2\left(3Ac\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 - 3Bd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 + 3Ac\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right) + 3Bc\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right) + 3Ad\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right) - 9Bd\tan\left(\right.\right.}{a^2\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^3}}{3f}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorith
m="giac")
```

output
```
1/3*(3*(f*x + e)*B*d/a^2 - 2*(3*A*c*tan(1/2*f*x + 1/2*e)^2 - 3*B*d*tan(1/2
*f*x + 1/2*e)^2 + 3*A*c*tan(1/2*f*x + 1/2*e) + 3*B*c*tan(1/2*f*x + 1/2*e)
+ 3*A*d*tan(1/2*f*x + 1/2*e) - 9*B*d*tan(1/2*f*x + 1/2*e) + 2*A*c + B*c +
A*d - 4*B*d)/(a^2*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

Mupad [B] (verification not implemented)

Time = 36.59 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.11

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = \frac{B\,d\,x}{a^2} - \frac{(2\,A\,c-2\,B\,d)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+(2\,A\,c+2\,A\,d+2\,B\,c-6\,B\,d)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\frac{4\,A\,c}{3}+\frac{2\,A\,d}{3}+\frac{2\,B\,c}{3}-\frac{8\,B\,d}{3}}{a^2\,f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+1\right)^3}$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^2,x)
```

output
```
(B*d*x)/a^2 - ((4*A*c)/3 + (2*A*d)/3 + (2*B*c)/3 - (8*B*d)/3 + tan(e/2 + (
f*x)/2)*(2*A*c + 2*A*d + 2*B*c - 6*B*d) + tan(e/2 + (f*x)/2)^2*(2*A*c - 2*
B*d))/(a^2*f*(tan(e/2 + (f*x)/2) + 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.21

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^2}\,dx = \frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3ac+3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3bdfx-2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3bd+9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2bdfx-6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)a\cdots}{3a^2f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3+3\tan\left(\frac{fx}{2}\cdots\right.\right.}$$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^2,x)
```

output
```
(2*tan((e + f*x)/2)**3*a*c + 3*tan((e + f*x)/2)**3*b*d*f*x - 2*tan((e + f*
x)/2)**3*b*d + 9*tan((e + f*x)/2)**2*b*d*f*x - 6*tan((e + f*x)/2)*a*d - 6*
tan((e + f*x)/2)*b*c + 9*tan((e + f*x)/2)*b*d*f*x + 12*tan((e + f*x)/2)*b*
d - 2*a*c - 2*a*d - 2*b*c + 3*b*d*f*x + 6*b*d)/(3*a**2*f*(tan((e + f*x)/2)
**3 + 3*tan((e + f*x)/2)**2 + 3*tan((e + f*x)/2) + 1))
```

3.275 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 65

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx=-\frac{(A-B)\cos(e+fx)}{3f(a+a\sin(e+fx))^2}-\frac{(A+2B)\cos(e+fx)}{3f\,(a^2+a^2\sin(e+fx))}$$

output
```
-1/3*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^2-1/3*(A+2*B)*cos(f*x+e)/f/(a^2+a
^2*sin(f*x+e))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.66

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx=-\frac{\cos(e+fx)(2A+B+(A+2B)\sin(e+fx))}{3a^2f(1+\sin(e+fx))^2}$$

input
```
Integrate[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^2,x]
```

output
```
-1/3*(Cos[e + f*x]*(2*A + B + (A + 2*B)*Sin[e + f*x]))/(a^2*f*(1 + Sin[e +
 f*x])^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 3229, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{ 3229}$$

$$\frac{(A+2B)\int \frac{1}{\sin(e+fx)a+a}dx}{3a}-\frac{(A-B)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(A+2B)\int \frac{1}{\sin(e+fx)a+a}dx}{3a}-\frac{(A-B)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

$$\Big\downarrow \text{ 3127}$$

$$-\frac{(A+2B)\cos(e+fx)}{3af(a\sin(e+fx)+a)}-\frac{(A-B)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}$$

input
```
Int[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^2,x]
```

output
```
-1/3*((A - B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^2) - ((A + 2*B)*Cos[e
+ f*x])/(3*a*f*(a + a*Sin[e + f*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

method	result	size
parallelrisch	$\frac{-6A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}+(-6A-6B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-4A-2B}{3f\,a^{2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}$	60
risch	$-\frac{2\left(-A+3iA\,e^{i(fx+e)}+3iB\,e^{i(fx+e)}+3B\,e^{2i(fx+e)}-2B\right)}{3f\,a^{2}\left(e^{i(fx+e)}+i\right)^{3}}$	68
derivativedivides	$\frac{-\frac{2(2A-2B)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-2A+2B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}}{a^{2}f}$	70
default	$\frac{-\frac{2(2A-2B)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-2A+2B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}}{a^{2}f}$	70
norman	$\frac{-\frac{4A+2B}{3af}-\frac{2A\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{af}-\frac{2(5A+B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{3af}-\frac{(2A+2B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{2(A+B)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{af}}{a\left(1+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}$	139

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(-6*A*tan(1/2*f*x+1/2*e)^2+(-6*A-6*B)*tan(1/2*f*x+1/2*e)-4*A-2*B)/f/a^
2/(tan(1/2*f*x+1/2*e)+1)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.80

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2}\,dx$$
$$= \frac{(A + 2\,B)\cos\left(fx + e\right)^2 + (2\,A + B)\cos\left(fx + e\right) + ((A + 2\,B)\cos\left(fx + e\right) - A + B)\sin\left(fx + e\right) + A}{3\left(a^2 f\cos\left(fx + e\right)^2 - a^2 f\cos\left(fx + e\right) - 2\,a^2 f - (a^2 f\cos\left(fx + e\right) + 2\,a^2 f)\sin\left(fx + e\right)\right)}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/3*((A + 2*B)*cos(f*x + e)^2 + (2*A + B)*cos(f*x + e) + ((A + 2*B)*cos(f*
x + e) - A + B)*sin(f*x + e) + A - B)/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*
x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(56) = 112$.

Time = 1.26 (sec) , antiderivative size = 372, normalized size of antiderivative = 5.72

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2}\,dx$$
$$= \begin{cases} -\frac{6A\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3a^2f} - \frac{6A\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+9a^2f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+3} \\ \frac{x(A+B\sin(e))}{(a\sin(e)+a)^2} \end{cases}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2,x)
```

output
```
Piecewise((-6*A*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2
*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*A*tan(e
/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 +
 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 4*A/(3*a**2*f*tan(e/2 + f*x/2)**3
 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) -
6*B*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*
x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 2*B/(3*a**2*f*tan(e/2 +
f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a
**2*f), Ne(f, 0)), (x*(A + B*sin(e))/(a*sin(e) + a)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(61) = 122$.

Time = 0.04 (sec) , antiderivative size = 214, normalized size of antiderivative = 3.29

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2}\, dx = -\frac{2\left(\frac{A\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1} + \frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + 2\right)}{a^2 + \frac{3a^2\sin(fx+e)}{\cos(fx+e)+1} + \frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}} + \frac{B\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1} + 1\right)}{a^2 + \frac{3a^2\sin(fx+e)}{\cos(fx+e)+1} + \frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}\right)}{3f}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

output
```
-2/3*(A*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) +
B*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f
*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3))/f
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx = -\frac{2\left(3\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,A\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+3\,B\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+2\,A+B\right)}{3\,a^2f\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)^3}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

output
```
-2/3*(3*A*tan(1/2*f*x + 1/2*e)^2 + 3*A*tan(1/2*f*x + 1/2*e) + 3*B*tan(1/2*
f*x + 1/2*e) + 2*A + B)/(a^2*f*(tan(1/2*f*x + 1/2*e) + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 36.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.49

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx = -\frac{2\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{5\,A}{2}+\frac{B}{2}-\frac{A\,\cos(e+f\,x)}{2}+\frac{B\,\cos(e+f\,x)}{2}+\frac{3\,A\,\sin(e+f\,x)}{2}+\frac{3\,B\,\sin(e+f\,x)}{2}\right)}{3\,a^2\,f\,\left(\frac{3\,\sqrt{2}\,\cos\left(\frac{e}{2}-\frac{\pi}{4}+\frac{f\,x}{2}\right)}{2}-\frac{\sqrt{2}\,\cos\left(\frac{3\,e}{2}+\frac{\pi}{4}+\frac{3\,f\,x}{2}\right)}{2}\right)}$$

input
```
int((A + B*sin(e + f*x))/(a + a*sin(e + f*x))^2,x)
```

output
```
-(2*cos(e/2 + (f*x)/2)*((5*A)/2 + B/2 - (A*cos(e + f*x))/2 + (B*cos(e + f*
x))/2 + (3*A*sin(e + f*x))/2 + (3*B*sin(e + f*x))/2))/(3*a^2*f*((3*2^(1/2)
*cos(e/2 - pi/4 + (f*x)/2))/2 - (2^(1/2)*cos((3*e)/2 + pi/4 + (3*f*x)/2))/
2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.22

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2}\,dx = \frac{\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3 a}{3}-2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)b-\frac{2a}{3}-\frac{2b}{3}}{a^2f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3+3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x)
```

output
```
(2*(tan((e + f*x)/2)**3*a - 3*tan((e + f*x)/2)*b - a - b))/(3*a**2*f*(tan(
(e + f*x)/2)**3 + 3*tan((e + f*x)/2)**2 + 3*tan((e + f*x)/2) + 1))
```

3.276 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 152

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))}\,dx = -\frac{2d(Bc-Ad)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^2(c-d)^2\sqrt{c^2-d^2}f} - \frac{(A(c-4d)+B(2c+d))\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))} - \frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2}$$

output
```
-2*d*(-A*d+B*c)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a^2/(c-d)
^2/(c^2-d^2)^(1/2)/f-1/3*(A*(c-4*d)+B*(2*c+d))*cos(f*x+e)/a^2/(c-d)^2/f/(1
+sin(f*x+e))-1/3*(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^2
```

Mathematica [A] (verified)

Time = 4.17 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.51

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2(A-B)(c-d)\sin\left(\frac{1}{2}(e+fx)\right)+(-A+B)(c-d)\left(\cos\left(\frac{1}{2}(e+\right.\right.}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x]
)),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*(c - d)*Sin[(e + f*x)/2]
 + (-A + B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(A*(c - 4*d)
 + B*(2*c + d))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 +
 (6*d*(-(B*c) + A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/Sqrt[c^2 - d^2]))/(3*a^2*(c - d)^2*f*
(1 + Sin[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3457, 25, 3042, 3457, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^2(c+d\sin(e+fx))}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^2(c+d\sin(e+fx))}dx$$
$$\Big\downarrow \text{3457}$$

$$-\frac{\int -\frac{a(2Bc+A(c-3d))+a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)(c+d\sin(e+fx))}dx}{3a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{a(2Bc+A(c-3d))+a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)(c+d\sin(e+fx))}dx}{3a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{a(2Bc+A(c-3d))+a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)(c+d\sin(e+fx))}dx}{3a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2}$$

$$\Big\downarrow \; 3457$$

$$\frac{-\frac{\int \frac{3a^2d(Bc-Ad)}{c+d\sin(e+fx)}dx}{a^2(c-d)}-\frac{(A(c-4d)+B(2c+d))\cos(e+fx)}{f(c-d)(\sin(e+fx)+1)}}{3a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{3d(Bc-Ad)\int \frac{1}{c+d\sin(e+fx)}dx}{c-d}-\frac{(A(c-4d)+B(2c+d))\cos(e+fx)}{f(c-d)(\sin(e+fx)+1)}}{3a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{3d(Bc-Ad)\int \frac{1}{c+d\sin(e+fx)}dx}{c-d}-\frac{(A(c-4d)+B(2c+d))\cos(e+fx)}{f(c-d)(\sin(e+fx)+1)}}{3a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2}$$

$$\Big\downarrow \; 3139$$

$$\frac{-\frac{6d(Bc-Ad)\int \frac{1}{c\tan^2\left(\frac{1}{2}(e+fx)\right)+2d\tan\left(\frac{1}{2}(e+fx)\right)+c}d\tan\left(\frac{1}{2}(e+fx)\right)}{f(c-d)}-\frac{(A(c-4d)+B(2c+d))\cos(e+fx)}{f(c-d)(\sin(e+fx)+1)}}{3a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2}$$

$$\Big\downarrow \; 1083$$

$$\frac{\frac{12d(Bc-Ad)\int \frac{1}{-\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)^2-4\left(c^2-d^2\right)}d\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{f(c-d)}-\frac{(A(c-4d)+B(2c+d))\cos(e+fx)}{f(c-d)(\sin(e+fx)+1)}}{3a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2}$$

$$\Big\downarrow \; 217$$

$$\frac{-\frac{6d(Bc-Ad)\arctan\left(\frac{2c\tan\left(\frac{1}{2}(e+fx)\right)+2d}{2\sqrt{c^2-d^2}}\right)}{f(c-d)\sqrt{c^2-d^2}}-\frac{(A(c-4d)+B(2c+d))\cos(e+fx)}{f(c-d)(\sin(e+fx)+1)}}{3a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])),x]
```

output
```
-1/3*((A - B)*Cos[e + f*x])/((c - d)*f*(a + a*Sin[e + f*x])^2) + ((-6*d*(B
*c - A*d)*ArcTan[(2*d + 2*c*Tan[(e + f*x)/2])/(2*Sqrt[c^2 - d^2])])/((c -
d)*Sqrt[c^2 - d^2]*f) - ((A*(c - 4*d) + B*(2*c + d))*Cos[e + f*x])/((c - d
)*f*(1 + Sin[e + f*x])))/(3*a^2*(c - d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{-\frac{-2A+2B}{(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2(2A-2B)}{3(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2(Ac-2Ad+Bd)}{(c-d)^{2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{2d(Ad-Bc)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^{2}-d^{2}}}\right)}{(c-d)^{2}\sqrt{c^{2}-d^{2}}}}{a^{2}f}$
default	$\frac{-\frac{-2A+2B}{(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2(2A-2B)}{3(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2(Ac-2Ad+Bd)}{(c-d)^{2}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{2d(Ad-Bc)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^{2}-d^{2}}}\right)}{(c-d)^{2}\sqrt{c^{2}-d^{2}}}}{a^{2}f}$
risch	$\frac{\frac{2Ac}{3}-\frac{8Ad}{3}-2iAc\,e^{i(fx+e)}+6iAd\,e^{i(fx+e)}+2Ad\,e^{2i(fx+e)}+\frac{4Bc}{3}+\frac{2Bd}{3}-2iBc\,e^{i(fx+e)}-2iBd\,e^{i(fx+e)}-2Bc\,e^{2i(fx+e)}}{\left(e^{i(fx+e)}+i\right)^{3}(c-d)^{2}f\,a^{2}}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x,method=_RETURNV
ERBOSE)
```

output
```
2/f/a^2*(-1/2*(-2*A+2*B)/(c-d)/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(2*A-2*B)/(c-d
)/(tan(1/2*f*x+1/2*e)+1)^3-(A*c-2*A*d+B*d)/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)+
d*(A*d-B*c)/(c-d)^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d
)/(c^2-d^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 599 vs. $2(143) = 286$.

Time = 0.11 (sec) , antiderivative size = 1285, normalized size of antiderivative = 8.45

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2(c + d\sin(e + fx))}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
[1/6*(2*(A - B)*c^3 - 2*(A - B)*c^2*d - 2*(A - B)*c*d^2 + 2*(A - B)*d^3 +
2*((A + 2*B)*c^3 - (4*A - B)*c^2*d - (A + 2*B)*c*d^2 + (4*A - B)*d^3)*cos(
f*x + e)^2 - 3*(2*B*c*d - 2*A*d^2 - (B*c*d - A*d^2)*cos(f*x + e)^2 + (B*c*
d - A*d^2)*cos(f*x + e) + (2*B*c*d - 2*A*d^2 + (B*c*d - A*d^2)*cos(f*x + e
))*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*
d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x +
e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2
)) + 2*((2*A + B)*c^3 - (5*A - 2*B)*c^2*d - (2*A + B)*c*d^2 + (5*A - 2*B)*
d^3)*cos(f*x + e) - 2*((A - B)*c^3 - (A - B)*c^2*d - (A - B)*c*d^2 + (A -
B)*d^3 - ((A + 2*B)*c^3 - (4*A - B)*c^2*d - (A + 2*B)*c*d^2 + (4*A - B)*d^
3)*cos(f*x + e))*sin(f*x + e))/((a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2
*d^4)*f*cos(f*x + e)^2 - (a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f
*cos(f*x + e) - 2*(a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f - ((a^
2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f*cos(f*x + e) + 2*(a^2*c^4 -
 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f)*sin(f*x + e)), 1/3*((A - B)*c^3 -
 (A - B)*c^2*d - (A - B)*c*d^2 + (A - B)*d^3 + ((A + 2*B)*c^3 - (4*A - B)*
c^2*d - (A + 2*B)*c*d^2 + (4*A - B)*d^3)*cos(f*x + e)^2 - 3*(2*B*c*d - 2*A
*d^2 - (B*c*d - A*d^2)*cos(f*x + e)^2 + (B*c*d - A*d^2)*cos(f*x + e) + (2*
B*c*d - 2*A*d^2 + (B*c*d - A*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(c^2 - d
^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + ((2*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))}\,dx=\text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c+d*sin(f*x+e)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.64

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))}\,dx=$$

$$-\frac{2\left(\frac{3\,(Bcd-Ad^2)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(a^2c^2-2\,a^2cd+a^2d^2)\sqrt{c^2-d^2}}+\frac{3\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-6\,Ad\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+3\,Bd\tan\left(\frac{1}{2}\,fx+\right.}{}\right.}{3\,f}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorith
m="giac")
```

output
```
-2/3*(3*(B*c*d - A*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan(
(c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a^2*c^2 - 2*a^2*c*d + a^2
*d^2)*sqrt(c^2 - d^2)) + (3*A*c*tan(1/2*f*x + 1/2*e)^2 - 6*A*d*tan(1/2*f*x
 + 1/2*e)^2 + 3*B*d*tan(1/2*f*x + 1/2*e)^2 + 3*A*c*tan(1/2*f*x + 1/2*e) +
3*B*c*tan(1/2*f*x + 1/2*e) - 9*A*d*tan(1/2*f*x + 1/2*e) + 3*B*d*tan(1/2*f*
x + 1/2*e) + 2*A*c + B*c - 5*A*d + 2*B*d)/((a^2*c^2 - 2*a^2*c*d + a^2*d^2)
*(tan(1/2*f*x + 1/2*e) + 1)^3))/f
```

Mupad [B] (verification not implemented)

Time = 37.97 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.99

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))}\,dx = \frac{2\,d\,\mathrm{atan}\left(\frac{\frac{d\,(A\,d-B\,c)\,\left(2\,a^2\,c^2\,d-4\,a^2\,c\,d^2+2\,a^2\,d^3\right)}{a^2\,\sqrt{c+d}\,(c-d)^{5/2}}+\frac{2\,c\,d\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(A\,d-B\,c)\,\left(a^2\,c^2-2\,a^2\,c\,d+a^2\,d^2\right)}{a^2\,\sqrt{c+d}\,(c-d)^{5/2}}}{2\,A\,d^2-2\,B\,c\,d}\right)\,(A\,d-B\,c)}{a^2\,f\,\sqrt{c+d}\,(c-d)^{5/2}} - \frac{\frac{2\,(2\,A\,c-5\,A\,d+B\,c+2\,B\,d)}{3\,(c-d)^2}+\frac{2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(A\,c-3\,A\,d+B\,c+B\,d)}{(c-d)^2}+\frac{2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,(A\,c-2\,A\,d+B\,d)}{(c-d)^2}}{f\,\left(a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+3\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+3\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)+a^2\right)}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))),x)
```

output
```
(2*d*atan(((d*(A*d - B*c)*(2*a^2*d^3 - 4*a^2*c*d^2 + 2*a^2*c^2*d))/(a^2*(c
 + d)^(1/2)*(c - d)^(5/2)) + (2*c*d*tan(e/2 + (f*x)/2)*(A*d - B*c)*(a^2*c^
2 + a^2*d^2 - 2*a^2*c*d))/(a^2*(c + d)^(1/2)*(c - d)^(5/2)))/(2*A*d^2 - 2*
B*c*d))*(A*d - B*c))/(a^2*f*(c + d)^(1/2)*(c - d)^(5/2)) - ((2*(2*A*c - 5*
A*d + B*c + 2*B*d))/(3*(c - d)^2) + (2*tan(e/2 + (f*x)/2)*(A*c - 3*A*d + B
*c + B*d))/(c - d)^2 + (2*tan(e/2 + (f*x)/2)^2*(A*c - 2*A*d + B*d))/(c - d
)^2)/(f*(3*a^2*tan(e/2 + (f*x)/2)^2 + a^2*tan(e/2 + (f*x)/2)^3 + a^2 + 3*a
^2*tan(e/2 + (f*x)/2)))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 837, normalized size of antiderivative = 5.51

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))}\,dx = \text{Too large to display}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x)
```

output
```
(2*(3*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*t
an((e + f*x)/2)**3*a*d**2 - 3*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c +
 d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)**3*b*c*d + 9*sqrt(c**2 - d**2)*ata
n((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)**2*a*d**2 -
 9*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan(
(e + f*x)/2)**2*b*c*d + 9*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/
sqrt(c**2 - d**2))*tan((e + f*x)/2)*a*d**2 - 9*sqrt(c**2 - d**2)*atan((tan
((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)*b*c*d + 3*sqrt(c*
*2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*a*d**2 - 3*sqr
t(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*b*c*d + ta
n((e + f*x)/2)**3*a*c**3 - 2*tan((e + f*x)/2)**3*a*c**2*d - tan((e + f*x)/
2)**3*a*c*d**2 + 2*tan((e + f*x)/2)**3*a*d**3 + tan((e + f*x)/2)**3*b*c**2
*d - tan((e + f*x)/2)**3*b*d**3 + 3*tan((e + f*x)/2)*a*c**2*d - 3*tan((e +
 f*x)/2)*a*d**3 - 3*tan((e + f*x)/2)*b*c**3 + 3*tan((e + f*x)/2)*b*c*d**2
- a*c**3 + 3*a*c**2*d + a*c*d**2 - 3*a*d**3 - b*c**3 - b*c**2*d + b*c*d**2
 + b*d**3))/(3*a**2*f*(tan((e + f*x)/2)**3*c**4 - 2*tan((e + f*x)/2)**3*c*
*3*d + 2*tan((e + f*x)/2)**3*c*d**3 - tan((e + f*x)/2)**3*d**4 + 3*tan((e
+ f*x)/2)**2*c**4 - 6*tan((e + f*x)/2)**2*c**3*d + 6*tan((e + f*x)/2)**2*c
*d**3 - 3*tan((e + f*x)/2)**2*d**4 + 3*tan((e + f*x)/2)*c**4 - 6*tan((e +
f*x)/2)*c**3*d + 6*tan((e + f*x)/2)*c*d**3 - 3*tan((e + f*x)/2)*d**4 + ...
```

3.277 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 275

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx = \frac{2d(Ad(3c+2d)-B(2c^2+2cd+d^2))\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^2(c-d)^3(c+d)\sqrt{c^2-d^2}f} - \frac{d(A(c^2-6cd-10d^2)+B(2c^2+9cd+4d^2))\cos(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sin(e+fx))} - \frac{(Ac+2Bc-6Ad+3Bd)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))(c+d\sin(e+fx))} - \frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2(c+d\sin(e+fx))}$$

output
```
2*d*(A*d*(3*c+2*d)-B*(2*c^2+2*c*d+d^2))*arctan((d+c*tan(1/2*f*x+1/2*e))/(c
^2-d^2)^(1/2))/a^2/(c-d)^3/(c+d)/(c^2-d^2)^(1/2)/f-1/3*d*(A*(c^2-6*c*d-10*
d^2)+B*(2*c^2+9*c*d+4*d^2))*cos(f*x+e)/a^2/(c-d)^3/(c+d)/f/(c+d*sin(f*x+e)
)-1/3*(A*c-6*A*d+2*B*c+3*B*d)*cos(f*x+e)/a^2/(c-d)^2/f/(1+sin(f*x+e))/(c+d
*sin(f*x+e))-1/3*(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+
e))
```

Mathematica [A] (verified)

Time = 9.43 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.14

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx$$
$$=\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2(A-B)(c-d)\sin\left(\frac{1}{2}(e+fx)\right)+(-A+B)(c-d)\left(\cos\left(\frac{1}{2}(e+\right.\right.}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x]
)^2),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*(c - d)*Sin[(e + f*x)/2]
 + (-A + B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(A*(c - 7*d)
 + 2*B*(c + 2*d))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2
 - (6*d*(-(A*d*(3*c + 2*d)) + B*(2*c^2 + 2*c*d + d^2))*ArcTan[(d + c*Tan[(
e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/((c
 + d)*Sqrt[c^2 - d^2]) + (3*d^2*(-(B*c) + A*d)*Cos[e + f*x]*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^3)/((c + d)*(c + d*Sin[e + f*x]))))/(3*a^2*(c - d)
^3*f*(1 + Sin[e + f*x])^2)
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {3042, 3457, 25, 3042, 3457, 3042, 3233, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^2}\,dx$$
$$\Big\downarrow \; 3042$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^2}dx$$

$$\Big\downarrow \; 3457$$

$$-\frac{\int -\frac{a(A(c-4d)+B(2c+d))+2a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)(c+d\sin(e+fx))^2}dx}{3a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{a(A(c-4d)+B(2c+d))+2a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)(c+d\sin(e+fx))^2}dx}{3a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{a(A(c-4d)+B(2c+d))+2a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)(c+d\sin(e+fx))^2}dx}{3a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))}$$

$$\Big\downarrow \; 3457$$

$$\frac{-\frac{\int \frac{2a^2d(3Bc-5Ad+2Bd)-a^2d(Ac+2Bc-6Ad+3Bd)\sin(e+fx)}{(c+d\sin(e+fx))^2}dx}{a^2(c-d)}-\frac{(Ac-6Ad+2Bc+3Bd)\cos(e+fx)}{f(c-d)(\sin(e+fx)+1)(c+d\sin(e+fx))}}{3a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{\int \frac{2a^2d(3Bc-5Ad+2Bd)-a^2d(Ac+2Bc-6Ad+3Bd)\sin(e+fx)}{(c+d\sin(e+fx))^2}dx}{a^2(c-d)}-\frac{(Ac-6Ad+2Bc+3Bd)\cos(e+fx)}{f(c-d)(\sin(e+fx)+1)(c+d\sin(e+fx))}}{3a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))}$$

$$\Big\downarrow \; 3233$$

$$\frac{-\frac{\frac{a^2d\left(A\left(c^2-6cd-10d^2\right)+B\left(2c^2+9cd+4d^2\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}-\frac{\int \frac{3a^2d\left(Ad(3c+2d)-B\left(2c^2+2dc+d^2\right)\right)}{c+d\sin(e+fx)}dx}{c^2-d^2}}{a^2(c-d)}-\frac{(Ac-6Ad+2Bc+3Bd)\cos(e+fx)}{f(c-d)(\sin(e+fx)+1)(c+d\sin(e+fx))}}{3a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{a^2 d\left(A\left(c^2-6cd-10d^2\right)+B\left(2c^2+9cd+4d^2\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}-\frac{3a^2 d\left(Ad(3c+2d)-B\left(2c^2+2cd+d^2\right)\right)\int\frac{1}{c+d\sin(e+fx)}dx}{c^2-d^2}}{a^2(c-d)}-\frac{(Ac-6Ad+2Bc+3Bd)\cos(e+\ldots}{f(c-d)(\sin(e+fx)+1)(c+d\sin(e\ldots}}{3a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))}$$

$\downarrow$ 3042

$$-\frac{\frac{\frac{a^2 d\left(A\left(c^2-6cd-10d^2\right)+B\left(2c^2+9cd+4d^2\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}-\frac{3a^2 d\left(Ad(3c+2d)-B\left(2c^2+2cd+d^2\right)\right)\int\frac{1}{c+d\sin(e+fx)}dx}{c^2-d^2}}{a^2(c-d)}-\frac{(Ac-6Ad+2Bc+3Bd)\cos(e+\ldots}{f(c-d)(\sin(e+fx)+1)(c+d\sin(e\ldots}}{3a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))}$$

$\downarrow$ 3139

$$-\frac{\frac{\frac{a^2 d\left(A\left(c^2-6cd-10d^2\right)+B\left(2c^2+9cd+4d^2\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}-\frac{6a^2 d\left(Ad(3c+2d)-B\left(2c^2+2cd+d^2\right)\right)\int\frac{1}{c\tan^2\left(\frac{1}{2}(e+fx)\right)+2d\tan\left(\frac{1}{2}(e+fx)\right)+c}d\tan\left(\frac{1}{2}(e+fx)\right)}{f\left(c^2-d^2\right)}}{a^2(c-d)}-\ldots}{3a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))}$$

$\downarrow$ 1083

$$-\frac{\frac{\frac{12a^2 d\left(Ad(3c+2d)-B\left(2c^2+2cd+d^2\right)\right)\int\frac{1}{-\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)^2-4\left(c^2-d^2\right)}d\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{f\left(c^2-d^2\right)}+\frac{a^2 d\left(A\left(c^2-6cd-10d^2\right)+B\left(2c^2+9cd+4d^2\right)\right)\cos(\ldots}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}}{a^2(c-d)}\ldots}{3a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))}$$

$\downarrow$ 217

$$-\frac{\frac{\frac{a^2 d\left(A\left(c^2-6cd-10d^2\right)+B\left(2c^2+9cd+4d^2\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}-\frac{6a^2 d\left(Ad(3c+2d)-B\left(2c^2+2cd+d^2\right)\right)\arctan\left(\frac{2c\tan\left(\frac{1}{2}(e+fx)\right)+2d}{2\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{3/2}}}{a^2(c-d)}-\frac{(Ac-6Ad+2Bc+3\ldots}{f(c-d)(\sin(e+fx)+1\ldots}}{3a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^2),x
]
```

output
```
-1/3*((A - B)*Cos[e + f*x])/((c - d)*f*(a + a*Sin[e + f*x])^2*(c + d*Sin[e
 + f*x])) + (-(((A*c + 2*B*c - 6*A*d + 3*B*d)*Cos[e + f*x])/((c - d)*f*(1
+ Sin[e + f*x])*(c + d*Sin[e + f*x]))) - ((-6*a^2*d*(A*d*(3*c + 2*d) - B*(
2*c^2 + 2*c*d + d^2))*ArcTan[(2*d + 2*c*Tan[(e + f*x)/2])/(2*Sqrt[c^2 - d^
2])])/((c^2 - d^2)^(3/2)*f) + (a^2*d*(A*(c^2 - 6*c*d - 10*d^2) + B*(2*c^2
+ 9*c*d + 4*d^2))*Cos[e + f*x])/((c^2 - d^2)*f*(c + d*Sin[e + f*x])))/(a^2
*(c - d)))/(3*a^2*(c - d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{\frac{2d\left(\frac{\frac{d^2(Ad-Bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}+\frac{d(Ad-Bc)}{c+d}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}+\frac{(3Acd+2A\,d^2-2B\,c^2-2Bcd-B\,d^2)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}}\right)}{(c-d)^3}-\frac{-2A}{(c-d)^2\left(\tan\left(\right.\right.}}{a^2f}$
default	$\frac{\frac{2d\left(\frac{\frac{d^2(Ad-Bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}+\frac{d(Ad-Bc)}{c+d}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}+\frac{(3Acd+2A\,d^2-2B\,c^2-2Bcd-B\,d^2)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}}\right)}{(c-d)^3}-\frac{-2A}{(c-d)^2\left(\tan\left(\right.\right.}}{a^2f}$
risch	Expression too large to display

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x,method=_RETUR
NVERBOSE)
```

output
```
2/f/a^2*(1/(c-d)^3*d*((d^2*(A*d-B*c)/(c+d)/c*tan(1/2*f*x+1/2*e)+d*(A*d-B*c
)/(c+d))/(tan(1/2*f*x+1/2*e)^2*c+2*d*tan(1/2*f*x+1/2*e)+c)+(3*A*c*d+2*A*d^
2-2*B*c^2-2*B*c*d-B*d^2)/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x
+1/2*e)+2*d)/(c^2-d^2)^(1/2)))-1/2*(-2*A+2*B)/(c-d)^2/(tan(1/2*f*x+1/2*e)+
1)^2-1/3*(2*A-2*B)/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^3-(A*c-3*A*d+2*B*d)/(c-d
)^3/(tan(1/2*f*x+1/2*e)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1518 vs. $2(264) = 528$.

Time = 0.18 (sec) , antiderivative size = 3123, normalized size of antiderivative = 11.36

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algori
thm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.49

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx = -\frac{2\left(\frac{3\left(2Bc^2d-3Acd^2+2Bcd^2-2Ad^3+Bd^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\left(a^2c^4-2a^2c^3d+2a^2cd^3-a^2d^4\right)\sqrt{c^2-d^2}}+\frac{3\left(Bcd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\right.}{\left(a^2c^5-2a^2c^4d+2a^2c^2d^3-a^2cd^4\right)}\right.}{}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
-2/3*(3*(2*B*c^2*d - 3*A*c*d^2 + 2*B*c*d^2 - 2*A*d^3 + B*d^3)*(pi*floor(1/
2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^
2 - d^2)))/((a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*sqrt(c^2 - d^2
)) + 3*(B*c*d^3*tan(1/2*f*x + 1/2*e) - A*d^4*tan(1/2*f*x + 1/2*e) + B*c^2*
d^2 - A*c*d^3)/((a^2*c^5 - 2*a^2*c^4*d + 2*a^2*c^2*d^3 - a^2*c*d^4)*(c*tan
(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)) + (3*A*c*tan(1/2*f*x
+ 1/2*e)^2 - 9*A*d*tan(1/2*f*x + 1/2*e)^2 + 6*B*d*tan(1/2*f*x + 1/2*e)^2 +
 3*A*c*tan(1/2*f*x + 1/2*e) + 3*B*c*tan(1/2*f*x + 1/2*e) - 15*A*d*tan(1/2*
f*x + 1/2*e) + 9*B*d*tan(1/2*f*x + 1/2*e) + 2*A*c + B*c - 8*A*d + 5*B*d)/(
(a^2*c^3 - 3*a^2*c^2*d + 3*a^2*c*d^2 - a^2*d^3)*(tan(1/2*f*x + 1/2*e) + 1)
^3))/f
```

Mupad [B] (verification not implemented)

Time = 40.45 (sec) , antiderivative size = 844, normalized size of antiderivative = 3.07

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^2),x
)
```

output
```
(2*d*atan(((d*(2*a^2*d^5 - 4*a^2*c*d^4 - 2*a^2*c^4*d + 4*a^2*c^3*d^2)*(2*B
*c^2 - 2*A*d^2 + B*d^2 - 3*A*c*d + 2*B*c*d))/(a^2*(c + d)^(3/2)*(c - d)^(7
/2)) - (2*c*d*tan(e/2 + (f*x)/2)*(a^2*c^4 - a^2*d^4 + 2*a^2*c*d^3 - 2*a^2*
c^3*d)*(2*B*c^2 - 2*A*d^2 + B*d^2 - 3*A*c*d + 2*B*c*d))/(a^2*(c + d)^(3/2)
*(c - d)^(7/2)))/(2*B*d^3 - 4*A*d^3 - 6*A*c*d^2 + 4*B*c*d^2 + 4*B*c^2*d))*
(2*B*c^2 - 2*A*d^2 + B*d^2 - 3*A*c*d + 2*B*c*d))/(a^2*f*(c + d)^(3/2)*(c -
 d)^(7/2)) - ((2*(2*A*c^3 - 3*A*d^3 + B*c^3 - 8*A*c*d^2 - 6*A*c^2*d + 8*B*
c*d^2 + 6*B*c^2*d))/(3*(c + d)*(c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 +
 (f*x)/2)^2*(5*A*c^3 - 9*A*d^3 + B*c^3 - 30*A*c*d^2 - 11*A*c^2*d + 27*B*c*
d^2 + 17*B*c^2*d))/(3*c*(c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/
2)^3*(A*c^4 - 3*A*d^4 + B*c^4 - 9*A*c^2*d^2 + 8*B*c^2*d^2 - 7*A*c*d^3 - 2*
A*c^3*d + 7*B*c*d^3 + 4*B*c^3*d))/(c*(c + d)*(c - d)*(c^2 - 2*c*d + d^2))
+ (2*tan(e/2 + (f*x)/2)*(3*A*c^4 - 3*A*d^4 + 3*B*c^4 - 27*A*c^2*d^2 + 30*B
*c^2*d^2 - 25*A*c*d^3 - 8*A*c^3*d + 13*B*c*d^3 + 14*B*c^3*d))/(3*c*(c + d)
*(c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2)^4*(A*c^4 - A*d^4 - 3
*A*c^2*d^2 + 2*B*c^2*d^2 - 2*A*c^3*d + B*c*d^3 + 2*B*c^3*d))/(c*(c + d)*(c
 - d)*(c^2 - 2*c*d + d^2)))/(f*(a^2*c + tan(e/2 + (f*x)/2)*(3*a^2*c + 2*a^
2*d) + tan(e/2 + (f*x)/2)^4*(3*a^2*c + 2*a^2*d) + tan(e/2 + (f*x)/2)^2*(4*
a^2*c + 6*a^2*d) + tan(e/2 + (f*x)/2)^3*(4*a^2*c + 6*a^2*d) + a^2*c*tan(e/
2 + (f*x)/2)^5))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 4579, normalized size of antiderivative = 16.65

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2(c + d\sin(e + fx))^2} dx = \text{Too large to display}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x)
```

output
```
(2*(27*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*
tan((e + f*x)/2)**5*a*c**3*d**2 + 36*sqrt(c**2 - d**2)*atan((tan((e + f*x)
/2)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)**5*a*c**2*d**3 + 12*sqrt(c*
*2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/
2)**5*a*c*d**4 - 18*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c
**2 - d**2))*tan((e + f*x)/2)**5*b*c**4*d - 30*sqrt(c**2 - d**2)*atan((tan
((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)**5*b*c**3*d**2 -
21*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan(
(e + f*x)/2)**5*b*c**2*d**3 - 6*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c
 + d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)**5*b*c*d**4 + 81*sqrt(c**2 - d**
2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)**4*a*
c**3*d**2 + 162*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2
- d**2))*tan((e + f*x)/2)**4*a*c**2*d**3 + 108*sqrt(c**2 - d**2)*atan((tan
((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)**4*a*c*d**4 + 24*
sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e
+ f*x)/2)**4*a*d**5 - 54*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/s
qrt(c**2 - d**2))*tan((e + f*x)/2)**4*b*c**4*d - 126*sqrt(c**2 - d**2)*ata
n((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)**4*b*c**3*d
**2 - 123*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2
))*tan((e + f*x)/2)**4*b*c**2*d**3 - 60*sqrt(c**2 - d**2)*atan((tan((e ...
```

3.278 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 386

$$\begin{aligned}
&\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}\,dx \\
&= \frac{d(Ad(12c^2+16cd+7d^2)-B(6c^3+12c^2d+13cd^2+4d^3))\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^2(c-d)^4(c+d)^2\sqrt{c^2-d^2}\,f} \\
&\quad - \frac{d(A(2c^2-16cd-21d^2)+B(4c^2+19cd+12d^2))\cos(e+fx)}{6a^2(c-d)^3(c+d)f(c+d\sin(e+fx))^2} \\
&\quad - \frac{(Ac+2Bc-8Ad+5Bd)\cos(e+fx)}{3a^2(c-d)^2f(1+\sin(e+fx))(c+d\sin(e+fx))^2} \\
&\quad - \frac{(A-B)\cos(e+fx)}{3(c-d)f(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2} \\
&\quad - \frac{d(A(2c^3-16c^2d-59cd^2-32d^3)+B(4c^3+37c^2d+44cd^2+20d^3))\cos(e+fx)}{6a^2(c-d)^4(c+d)^2f(c+d\sin(e+fx))}
\end{aligned}$$

output
```
d*(A*d*(12*c^2+16*c*d+7*d^2)-B*(6*c^3+12*c^2*d+13*c*d^2+4*d^3))*arctan((d+
c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a^2/(c-d)^4/(c+d)^2/(c^2-d^2)^(1/2)
/f-1/6*d*(A*(2*c^2-16*c*d-21*d^2)+B*(4*c^2+19*c*d+12*d^2))*cos(f*x+e)/a^2/
(c-d)^3/(c+d)/f/(c+d*sin(f*x+e))^2-1/3*(A*c-8*A*d+2*B*c+5*B*d)*cos(f*x+e)/
a^2/(c-d)^2/f/(1+sin(f*x+e))/(c+d*sin(f*x+e))^2-1/3*(A-B)*cos(f*x+e)/(c-d)
/f/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2-1/6*d*(A*(2*c^3-16*c^2*d-59*c*d^2
-32*d^3)+B*(4*c^3+37*c^2*d+44*c*d^2+20*d^3))*cos(f*x+e)/a^2/(c-d)^4/(c+d)^
2/f/(c+d*sin(f*x+e))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1522 vs. $2(386) = 772$.

Time = 12.55 (sec) , antiderivative size = 1522, normalized size of antiderivative = 3.94

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x]
)^3),x]
```

output
```
-((d*(6*B*c^3 - 12*A*c^2*d + 12*B*c^2*d - 16*A*c*d^2 + 13*B*c*d^2 - 7*A*d^
3 + 4*B*d^3)*ArcTan[(Sec[(e + f*x)/2]*(d*Cos[(e + f*x)/2] + c*Sin[(e + f*x
)/2]))/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)/((c - d)^
4*(c + d)^2*Sqrt[c^2 - d^2]*f*(a + a*Sin[e + f*x])^2)) + ((Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])*(48*B*c^5*Cos[(e + f*x)/2] - 96*A*c^4*d*Cos[(e + f*x
)/2] + 240*B*c^4*d*Cos[(e + f*x)/2] - 524*A*c^3*d^2*Cos[(e + f*x)/2] + 536
*B*c^3*d^2*Cos[(e + f*x)/2] - 776*A*c^2*d^3*Cos[(e + f*x)/2] + 701*B*c^2*d
^3*Cos[(e + f*x)/2] - 487*A*c*d^4*Cos[(e + f*x)/2] + 400*B*c*d^4*Cos[(e +
f*x)/2] - 112*A*d^5*Cos[(e + f*x)/2] + 70*B*d^5*Cos[(e + f*x)/2] - 16*A*c^
5*Cos[(3*(e + f*x))/2] - 32*B*c^5*Cos[(3*(e + f*x))/2] + 80*A*c^4*d*Cos[(3
*(e + f*x))/2] - 224*B*c^4*d*Cos[(3*(e + f*x))/2] + 536*A*c^3*d^2*Cos[(3*(
e + f*x))/2] - 728*B*c^3*d^2*Cos[(3*(e + f*x))/2] + 1028*A*c^2*d^3*Cos[(3*
(e + f*x))/2] - 893*B*c^2*d^3*Cos[(3*(e + f*x))/2] + 695*A*c*d^4*Cos[(3*(e
 + f*x))/2] - 482*B*c*d^4*Cos[(3*(e + f*x))/2] + 134*A*d^5*Cos[(3*(e + f*x
))/2] - 98*B*d^5*Cos[(3*(e + f*x))/2] + 24*B*c^3*d^2*Cos[(5*(e + f*x))/2]
- 12*A*c^2*d^3*Cos[(5*(e + f*x))/2] + 21*B*c^2*d^3*Cos[(5*(e + f*x))/2] -
15*A*c*d^4*Cos[(5*(e + f*x))/2] - 18*B*c*d^4*Cos[(5*(e + f*x))/2] + 6*A*d^
5*Cos[(5*(e + f*x))/2] - 6*B*d^5*Cos[(5*(e + f*x))/2] + 4*A*c^3*d^2*Cos[(7
*(e + f*x))/2] + 8*B*c^3*d^2*Cos[(7*(e + f*x))/2] - 32*A*c^2*d^3*Cos[(7*(e
 + f*x))/2] + 59*B*c^2*d^3*Cos[(7*(e + f*x))/2] - 97*A*c*d^4*Cos[(7*(e ...
```

Rubi [A] (verified)

Time = 1.70 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.08, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3457, 25, 3042, 3457, 3042, 3233, 3042, 3233, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^3}dx$$

$$\Big\downarrow \text{3457}$$

$$-\frac{\int -\frac{a(A(c-5d)+2B(c+d))+3a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)(c+d\sin(e+fx))^3}dx}{3a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{a(A(c-5d)+2B(c+d))+3a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)(c+d\sin(e+fx))^3}dx}{3a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{a(A(c-5d)+2B(c+d))+3a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)(c+d\sin(e+fx))^3}dx}{3a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \text{ 3457}$$

$$\frac{-\frac{\int \frac{3a^2d(3Bc-7Ad+4Bd)-2a^2d(Ac+2Bc-8Ad+5Bd)\sin(e+fx)}{(c+d\sin(e+fx))^3}dx}{a^2(c-d)}-\frac{(Ac-8Ad+2Bc+5Bd)\cos(e+fx)}{f(c-d)(\sin(e+fx)+1)(c+d\sin(e+fx))^2}}{3a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-\frac{\int \frac{3a^2d(3Bc-7Ad+4Bd)-2a^2d(Ac+2Bc-8Ad+5Bd)\sin(e+fx)}{(c+d\sin(e+fx))^3}dx}{a^2(c-d)}-\frac{(Ac-8Ad+2Bc+5Bd)\cos(e+fx)}{f(c-d)(\sin(e+fx)+1)(c+d\sin(e+fx))^2}}{3a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \text{ 3233}$$

$$-\frac{\frac{a^2d(3d(-7Ad+3Bc+4Bd)+2c(Ac-8Ad+2Bc+5Bd))\cos(e+fx)}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}-\frac{\int \frac{2d\left(Ad(19c+16d)-B\left(9c^2+16dc+10d^2\right)\right)a^2+d\left(A\left(2c^2-16dc-21d^2\right)+B\left(4c^2+19dc+12d^2\cdots\right.\right.}{(c+d\sin(e+fx))^2}}{2\left(c^2-d^2\right)}}{a^2(c-d)}\cdots}{3a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\frac{a^2d(3d(-7Ad+3Bc+4Bd)+2c(Ac-8Ad+2Bc+5Bd))\cos(e+fx)}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}-\frac{\int\frac{2d\left(Ad(19c+16d)-B\left(9c^2+16dc+10d^2\right)\right)a^2+d\left(A\left(2c^2-16dc-21d^2\right)+B\left(4c^2+19dc+12d^2\right.\right.\cdots}{(c+d\sin(e+fx))^2}\cdots}{2\left(c^2-d^2\right)}}{a^2(c-d)}\cdots$$

$$3a^2(c-d)$$

$$\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \; 3233$$

$$-\frac{\frac{a^2d(3d(-7Ad+3Bc+4Bd)+2c(Ac-8Ad+2Bc+5Bd))\cos(e+fx)}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}-\frac{-\frac{\int-\frac{3a^2d\left(Ad\left(12c^2+16dc+7d^2\right)-B\left(6c^3+12dc^2+13d^2c+4d^3\right)\right)}{c+d\sin(e+fx)}dx}{c^2-d^2}-\frac{a^2d\left(A\left(2c^3-16c^2d-\cdots\right.\right.}{\cdots}}{2\left(c^2-d^2\right)}}{a^2(c-d)}\cdots$$

$$3a^2(c-d)$$

$$\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{a^2d(3d(-7Ad+3Bc+4Bd)+2c(Ac-8Ad+2Bc+5Bd))\cos(e+fx)}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}-\frac{\frac{3a^2d\left(Ad\left(12c^2+16cd+7d^2\right)-B\left(6c^3+12c^2d+13cd^2+4d^3\right)\right)\int\frac{1}{c+d\sin(e+fx)}dx}{c^2-d^2}-\frac{a^2d\left(A\left(2\cdots\right.\right.}{\cdots}}{2\left(c^2-d^2\right)}}{a^2(c-d)}\cdots$$

$$3a^2(c-d)$$

$$\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{a^2d(3d(-7Ad+3Bc+4Bd)+2c(Ac-8Ad+2Bc+5Bd))\cos(e+fx)}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}-\frac{\frac{3a^2d\left(Ad\left(12c^2+16cd+7d^2\right)-B\left(6c^3+12c^2d+13cd^2+4d^3\right)\right)\int\frac{1}{c+d\sin(e+fx)}dx}{c^2-d^2}-\frac{a^2d\left(A\left(2\cdots\right.\right.}{\cdots}}{2\left(c^2-d^2\right)}}{a^2(c-d)}\cdots$$

$$3a^2(c-d)$$

$$\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \; 3139$$

$$-\frac{\frac{a^2d(3d(-7Ad+3Bc+4Bd)+2c(Ac-8Ad+2Bc+5Bd))\cos(e+fx)}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}-\frac{\frac{6a^2d\left(Ad\left(12c^2+16cd+7d^2\right)-B\left(6c^3+12c^2d+13cd^2+4d^3\right)\right)\int\frac{1}{c\tan^2\left(\frac{1}{2}(e+fx)\right)+2d\tan\left(\frac{1}{2}\cdots\right)\cdots}}{f\left(c^2-d^2\right)}\cdots}{\cdots}}{a^2(c-d)}\cdots$$

$$3a^2(c\cdots$$

$$\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^2}$$

$\downarrow$ 1083

$$-\frac{\frac{a^2d(3d(-7Ad+3Bc+4Bd)+2c(Ac-8Ad+2Bc+5Bd))\cos(e+fx)}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}-\frac{-\frac{12a^2d\left(Ad\left(12c^2+16cd+7d^2\right)-B\left(6c^3+12c^2d+13cd^2+4d^3\right)\right)\int\frac{1}{-\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}}{f\left(c^2-d^2\right)}}{}}{a^2(c-d)}$$

$3a$

$$\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^2}$$

$\downarrow$ 217

$$-\frac{\frac{a^2d(3d(-7Ad+3Bc+4Bd)+2c(Ac-8Ad+2Bc+5Bd))\cos(e+fx)}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}-\frac{\frac{6a^2d\left(Ad\left(12c^2+16cd+7d^2\right)-B\left(6c^3+12c^2d+13cd^2+4d^3\right)\right)\arctan\left(\frac{2c\tan\left(\frac{1}{2}(e+fx)\right)+2d}{2\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{3/2}}}{2\left(c^2-d^2\right)}}{a^2(c-d)}$$

$3a^2(c-d)$

$$\frac{(A-B)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^2}$$

input

```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^3),x
]
```

output

```
-1/3*((A - B)*Cos[e + f*x])/((c - d)*f*(a + a*Sin[e + f*x])^2*(c + d*Sin[e
 + f*x])^2) + (-(((A*c + 2*B*c - 8*A*d + 5*B*d)*Cos[e + f*x])/((c - d)*f*(
1 + Sin[e + f*x])*(c + d*Sin[e + f*x])^2)) - ((a^2*d*(3*d*(3*B*c - 7*A*d +
 4*B*d) + 2*c*(A*c + 2*B*c - 8*A*d + 5*B*d))*Cos[e + f*x])/(2*(c^2 - d^2)*
f*(c + d*Sin[e + f*x])^2) - ((6*a^2*d*(A*d*(12*c^2 + 16*c*d + 7*d^2) - B*(
6*c^3 + 12*c^2*d + 13*c*d^2 + 4*d^3))*ArcTan[(2*d + 2*c*Tan[(e + f*x)/2])/
(2*Sqrt[c^2 - d^2])])/((c^2 - d^2)^(3/2)*f) - (a^2*d*(A*(2*c^3 - 16*c^2*d
 - 59*c*d^2 - 32*d^3) + B*(4*c^3 + 37*c^2*d + 44*c*d^2 + 20*d^3))*Cos[e + f
*x])/((c^2 - d^2)*f*(c + d*Sin[e + f*x])))/(2*(c^2 - d^2)))/(a^2*(c - d)))
/(3*a^2*(c - d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Maple [A] (verified)

Time = 3.47 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.42

method	result
derivativedivides	$\frac{-\frac{-2A+2B}{(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(2A-2B)}{3(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2(Ac-4Ad+3Bd)}{(c-d)^4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{2d\left(\frac{d^2\left(9A\,c^2d+4Ac\,d^2-2A\,d^3-7B\,c^3-\right.}{2c\left(c^2+2cd+d\right.}\right.}{}}{}$
default	$\frac{-\frac{-2A+2B}{(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2(2A-2B)}{3(c-d)^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2(Ac-4Ad+3Bd)}{(c-d)^4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{2d\left(\frac{d^2\left(9A\,c^2d+4Ac\,d^2-2A\,d^3-7B\,c^3-\right.}{2c\left(c^2+2cd+d\right.}\right.}{}}{}$
risch	Expression too large to display

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x,method=_RETUR
NVERBOSE)
```

output
```
2/f/a^2*(-1/2*(-2*A+2*B)/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(2*A-2*B)/(c
-d)^3/(tan(1/2*f*x+1/2*e)+1)^3-(A*c-4*A*d+3*B*d)/(c-d)^4/(tan(1/2*f*x+1/2*
e)+1)+d/(c-d)^4*((1/2*d^2*(9*A*c^2*d+4*A*c*d^2-2*A*d^3-7*B*c^3-4*B*c^2*d)/
c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3+1/2*d*(8*A*c^4*d+4*A*c^3*d^2+15*A*c
^2*d^3+8*A*c*d^4-2*A*d^5-6*B*c^5-4*B*c^4*d-13*B*c^3*d^2-8*B*c^2*d^3-2*B*c*
d^4)/(c^2+2*c*d+d^2)/c^2*tan(1/2*f*x+1/2*e)^2+1/2*d^2*(23*A*c^2*d+12*A*c*d
^2-2*A*d^3-17*B*c^3-12*B*c^2*d-4*B*c*d^2)/c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/
2*e)+1/2*d*(8*A*c^2*d+4*A*c*d^2-A*d^3-6*B*c^3-4*B*c^2*d-B*c*d^2)/(c^2+2*c*
d+d^2))/(tan(1/2*f*x+1/2*e)^2*c+2*d*tan(1/2*f*x+1/2*e)+c)^2+1/2*(12*A*c^2*
d+16*A*c*d^2+7*A*d^3-6*B*c^3-12*B*c^2*d-13*B*c*d^2-4*B*d^3)/(c^2+2*c*d+d^2
)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2456 vs. $2(373) = 746$.

Time = 0.26 (sec) , antiderivative size = 4997, normalized size of antiderivative = 12.95

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algori
thm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2(c + d\sin(e + fx))^3} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algori
thm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 911 vs. $2(373) = 746$.

Time = 0.29 (sec) , antiderivative size = 911, normalized size of antiderivative = 2.36

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2(c + d\sin(e + fx))^3} \, dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algori
thm="giac")
```

output
```
-1/3*(3*(6*B*c^3*d - 12*A*c^2*d^2 + 12*B*c^2*d^2 - 16*A*c*d^3 + 13*B*c*d^3
 - 7*A*d^4 + 4*B*d^4)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c
*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a^2*c^6 - 2*a^2*c^5*d - a^2
*c^4*d^2 + 4*a^2*c^3*d^3 - a^2*c^2*d^4 - 2*a^2*c*d^5 + a^2*d^6)*sqrt(c^2 -
 d^2)) + 3*(7*B*c^4*d^3*tan(1/2*f*x + 1/2*e)^3 - 9*A*c^3*d^4*tan(1/2*f*x +
 1/2*e)^3 + 4*B*c^3*d^4*tan(1/2*f*x + 1/2*e)^3 - 4*A*c^2*d^5*tan(1/2*f*x +
 1/2*e)^3 + 2*A*c*d^6*tan(1/2*f*x + 1/2*e)^3 + 6*B*c^5*d^2*tan(1/2*f*x + 1
/2*e)^2 - 8*A*c^4*d^3*tan(1/2*f*x + 1/2*e)^2 + 4*B*c^4*d^3*tan(1/2*f*x + 1
/2*e)^2 - 4*A*c^3*d^4*tan(1/2*f*x + 1/2*e)^2 + 13*B*c^3*d^4*tan(1/2*f*x +
1/2*e)^2 - 15*A*c^2*d^5*tan(1/2*f*x + 1/2*e)^2 + 8*B*c^2*d^5*tan(1/2*f*x +
 1/2*e)^2 - 8*A*c*d^6*tan(1/2*f*x + 1/2*e)^2 + 2*B*c*d^6*tan(1/2*f*x + 1/2
*e)^2 + 2*A*d^7*tan(1/2*f*x + 1/2*e)^2 + 17*B*c^4*d^3*tan(1/2*f*x + 1/2*e)
 - 23*A*c^3*d^4*tan(1/2*f*x + 1/2*e) + 12*B*c^3*d^4*tan(1/2*f*x + 1/2*e) -
 12*A*c^2*d^5*tan(1/2*f*x + 1/2*e) + 4*B*c^2*d^5*tan(1/2*f*x + 1/2*e) + 2*
A*c*d^6*tan(1/2*f*x + 1/2*e) + 6*B*c^5*d^2 - 8*A*c^4*d^3 + 4*B*c^4*d^3 - 4
*A*c^3*d^4 + B*c^3*d^4 + A*c^2*d^5)/((a^2*c^8 - 2*a^2*c^7*d - a^2*c^6*d^2
+ 4*a^2*c^5*d^3 - a^2*c^4*d^4 - 2*a^2*c^3*d^5 + a^2*c^2*d^6)*(c*tan(1/2*f*
x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^2) + 2*(3*A*c*tan(1/2*f*x + 1
/2*e)^2 - 12*A*d*tan(1/2*f*x + 1/2*e)^2 + 9*B*d*tan(1/2*f*x + 1/2*e)^2 + 3
*A*c*tan(1/2*f*x + 1/2*e) + 3*B*c*tan(1/2*f*x + 1/2*e) - 21*A*d*tan(1/2...
```

Mupad [B] (verification not implemented)

Time = 42.29 (sec) , antiderivative size = 1686, normalized size of antiderivative = 4.37

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^2(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^3),x
)
```

output
```
(d*atan(((d*(4*a^2*c*d^6 - 2*a^2*d^7 - 2*a^2*c^6*d + 2*a^2*c^2*d^5 - 8*a^2
*c^3*d^4 + 2*a^2*c^4*d^3 + 4*a^2*c^5*d^2)*(6*B*c^3 - 7*A*d^3 + 4*B*d^3 - 1
6*A*c*d^2 - 12*A*c^2*d + 13*B*c*d^2 + 12*B*c^2*d))/(2*a^2*(c + d)^(5/2)*(c
 - d)^(9/2)) + (c*d*tan(e/2 + (f*x)/2)*(2*a^2*c*d^5 - a^2*d^6 - a^2*c^6 +
2*a^2*c^5*d + a^2*c^2*d^4 - 4*a^2*c^3*d^3 + a^2*c^4*d^2)*(6*B*c^3 - 7*A*d^
3 + 4*B*d^3 - 16*A*c*d^2 - 12*A*c^2*d + 13*B*c*d^2 + 12*B*c^2*d))/(a^2*(c
+ d)^(5/2)*(c - d)^(9/2)))/(4*B*d^4 - 7*A*d^4 - 12*A*c^2*d^2 + 12*B*c^2*d^
2 - 16*A*c*d^3 + 13*B*c*d^3 + 6*B*c^3*d))*(6*B*c^3 - 7*A*d^3 + 4*B*d^3 - 1
6*A*c*d^2 - 12*A*c^2*d + 13*B*c*d^2 + 12*B*c^2*d))/(a^2*f*(c + d)^(5/2)*(c
 - d)^(9/2)) - ((tan(e/2 + (f*x)/2)^5*(2*A*c^6 + 2*A*d^6 + 2*B*c^6 - 23*A*
c^2*d^4 - 40*A*c^3*d^3 - 38*A*c^4*d^2 + 6*B*c^2*d^4 + 43*B*c^3*d^3 + 40*B*
c^4*d^2 - 4*A*c*d^5 - 4*A*c^5*d + 2*B*c*d^5 + 12*B*c^5*d))/(c^2*(c^5 - 3*c
^4*d - 3*c*d^4 + d^5 + 2*c^2*d^3 + 2*c^3*d^2)) + (4*A*c^5 + 3*A*d^5 + 2*B*
c^5 - 46*A*c^2*d^3 - 40*A*c^3*d^2 + 28*B*c^2*d^3 + 52*B*c^3*d^2 - 12*A*c*d
^4 - 14*A*c^4*d + 3*B*c*d^4 + 20*B*c^4*d)/(3*(c + d)*(c^2 - d^2)*(3*c*d^2
- 3*c^2*d + c^3 - d^3)) + (2*tan(e/2 + (f*x)/2)^3*(6*A*c^6 + 9*A*d^6 + 6*B
*c^6 - 177*A*c^2*d^4 - 212*A*c^3*d^3 - 102*A*c^4*d^2 + 105*B*c^2*d^4 + 215
*B*c^3*d^3 + 150*B*c^4*d^2 - 33*A*c*d^5 - 16*A*c^5*d + 9*B*c*d^5 + 40*B*c^
5*d))/(3*c^2*(c^2 - d^2)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (tan(e/2 + (f*
x)/2)*(6*A*c^5 + 6*A*d^5 + 6*B*c^5 - 160*A*c^2*d^3 - 114*A*c^3*d^2 + 97...
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 8867, normalized size of antiderivative = 22.97

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^2(c + d\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x)
```

output
```
(108*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*ta
n((e + f*x)/2)**7*a*c**6*d**2 + 288*sqrt(c**2 - d**2)*atan((tan((e + f*x)/
2)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)**7*a*c**5*d**3 + 255*sqrt(c*
*2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/
2)**7*a*c**4*d**4 + 84*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqr
t(c**2 - d**2))*tan((e + f*x)/2)**7*a*c**3*d**5 - 54*sqrt(c**2 - d**2)*ata
n((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)**7*b*c**7*d
 - 180*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*
tan((e + f*x)/2)**7*b*c**6*d**2 - 261*sqrt(c**2 - d**2)*atan((tan((e + f*x
)/2)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)**7*b*c**5*d**3 - 192*sqrt(
c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x
)/2)**7*b*c**4*d**4 - 48*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/s
qrt(c**2 - d**2))*tan((e + f*x)/2)**7*b*c**3*d**5 + 324*sqrt(c**2 - d**2)*
atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)**6*a*c**
6*d**2 + 1296*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 -
d**2))*tan((e + f*x)/2)**6*a*c**5*d**3 + 1917*sqrt(c**2 - d**2)*atan((tan(
(e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)**6*a*c**4*d**4 + 1
272*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan
((e + f*x)/2)**6*a*c**3*d**5 + 336*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2
)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)**6*a*c**2*d**6 - 162*sqrt(...
```

3.279 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 225

$$\begin{aligned}\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx &= \frac{d^2(3B(c-d)+Ad)x}{a^3}+\frac{d^2(3B(c-9d)+A(2c+7d))\cos(e+fx)}{15a^3f}\\ &\quad-\frac{(c-d)\left(3B(c^2+6cd-15d^2)+A(2c^2+7cd+15d^2)\right)\cos(e+fx)}{15f\left(a^3+a^3\sin(e+fx)\right)}\\ &\quad-\frac{(3B(c-3d)+2A(c+2d))\cos(e+fx)(c+d\sin(e+fx))^2}{15af(a+a\sin(e+fx))^2}\\ &\quad-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{5f(a+a\sin(e+fx))^3}\end{aligned}$$

output
```
d^2*(3*B*(c-d)+A*d)*x/a^3+1/15*d^2*(3*B*(c-9*d)+A*(2*c+7*d))*cos(f*x+e)/a^
3/f-1/15*(c-d)*(3*B*(c^2+6*c*d-15*d^2)+A*(2*c^2+7*c*d+15*d^2))*cos(f*x+e)/
f/(a^3+a^3*sin(f*x+e))-1/15*(3*B*(c-3*d)+2*A*(c+2*d))*cos(f*x+e)*(c+d*sin(
f*x+e))^2/a/f/(a+a*sin(f*x+e))^2-1/5*(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))^3/f
/(a+a*sin(f*x+e))^3
```

Mathematica [A] (verified)

Time = 5.46 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.63

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(6(A-B)(c-d)^3\sin\left(\frac{1}{2}(e+fx)\right)-3(A-B)(c-d)^3\left(\cos\left(\frac{1}{2}(e+\right.\right.}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x
])^3,x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(6*(A - B)*(c - d)^3*Sin[(e + f*x)/
2] - 3*(A - B)*(c - d)^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(c - d)
^2*(3*B*(c - 6*d) + A*(2*c + 13*d))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2])^2 - (c - d)^2*(3*B*(c - 6*d) + A*(2*c + 13*d))*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^3 + 2*(c - d)*(3*B*(c^2 + 8*c*d - 24*d^2) + A*
(2*c^2 + 11*c*d + 32*d^2))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^4 - 15*d^2*(-3*B*c - A*d + 3*B*d)*(e + f*x)*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2])^5 - 15*B*d^3*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x
)/2])^5))/(15*a^3*f*(1 + Sin[e + f*x])^3)
```

Rubi [A] (verified)

Time = 1.62 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {3042, 3456, 3042, 3456, 3042, 3447, 3042, 3502, 3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a\sin(e+fx)+a)^3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a\sin(e+fx)+a)^3}dx$$

$$\Big\downarrow \text{3456}$$

$$\frac{\int \frac{(c+d\sin(e+fx))^2(a(2Ac+3Bc+3Ad-3Bd)-a(A-6B)d\sin(e+fx))}{(\sin(e+fx)a+a)^2}dx}{5a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{5f(a\sin(e+fx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(c+d\sin(e+fx))^2(a(2Ac+3Bc+3Ad-3Bd)-a(A-6B)d\sin(e+fx))}{(\sin(e+fx)a+a)^2}dx}{5a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{5f(a\sin(e+fx)+a)^3}$$

$$\Big\downarrow \text{3456}$$

$$\frac{\frac{\int \frac{(c+d\sin(e+fx))\left(a^2\left(3B\left(c^2+5dc-6d^2\right)+A\left(2c^2+5dc+8d^2\right)\right)-a^2d(3B(c-9d)+A(2c+7d))\sin(e+fx)\right)}{\sin(e+fx)a+a}dx}{3a^2}-\frac{a(2A(c+2d)+3B(c-3d))\cos(e+fx)(c+d\sin}{3f(a\sin(e+fx)+a)^2}}{5a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{5f(a\sin(e+fx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{(c+d\sin(e+fx))\left(a^2\left(3B\left(c^2+5dc-6d^2\right)+A\left(2c^2+5dc+8d^2\right)\right)-a^2d(3B(c-9d)+A(2c+7d))\sin(e+fx)\right)}{\sin(e+fx)a+a}dx}{3a^2}-\frac{a(2A(c+2d)+3B(c-3d))\cos(e+fx)(c+d\sin}{3f(a\sin(e+fx)+a)^2}}{5a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{5f(a\sin(e+fx)+a)^3}$$

$$\Big\downarrow \text{3447}$$

$$\frac{\frac{\int \frac{-d^2(3B(c-9d)+A(2c+7d))\sin^2(e+fx)a^2+c\left(3B\left(c^2+5dc-6d^2\right)+A\left(2c^2+5dc+8d^2\right)\right)a^2+\left(a^2d\left(3B\left(c^2+5dc-6d^2\right)+A\left(2c^2+5dc+8d^2\right)\right)-a^2cd(3B(c-9d)+A(2}{\sin(e+fx)a+a}}{3a^2}}{5a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{5f(a\sin(e+fx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{-d^2(3B(c-9d)+A(2c+7d))\sin(e+fx)^2a^2+c\left(3B\left(c^2+5dc-6d^2\right)+A\left(2c^2+5dc+8d^2\right)\right)a^2+\left(a^2d\left(3B\left(c^2+5dc-6d^2\right)+A\left(2c^2+5dc+8d^2\right)\right)-a^2cd(3B(c-9d)+A(2}{\sin(e+fx)a+a}}{3a^2}}{5a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{5f(a\sin(e+fx)+a)^3}$$

$$\Big\downarrow \ 3502$$

$$\frac{\frac{\frac{\int \frac{c\left(3B\left(c^2+5dc-6d^2\right)+A\left(2c^2+5dc+8d^2\right)\right)a^3+15d^2(3B(c-d)+Ad)\sin(e+fx)a^3}{\sin(e+fx)a+a}dx}{a}+\frac{ad^2(A(2c+7d)+3B(c-9d))\cos(e+fx)}{f}}{3a^2}-\frac{a(2A(c+2d)+3B(c-3d))\,\mathrm{co}\cdots}{3f(a\sin(e\cdots}}{5a^2}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{5f(a\sin(e+fx)+a)^3}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\frac{\int \frac{c\left(3B\left(c^2+5dc-6d^2\right)+A\left(2c^2+5dc+8d^2\right)\right)a^3+15d^2(3B(c-d)+Ad)\sin(e+fx)a^3}{\sin(e+fx)a+a}dx}{a}+\frac{ad^2(A(2c+7d)+3B(c-9d))\cos(e+fx)}{f}}{3a^2}-\frac{a(2A(c+2d)+3B(c-3d))\,\mathrm{co}\cdots}{3f(a\sin(e\cdots}}{5a^2}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{5f(a\sin(e+fx)+a)^3}$$

$$\Big\downarrow \ 3214$$

$$\frac{\frac{\frac{a^3(c-d)\left(A\left(2c^2+7cd+15d^2\right)+3B\left(c^2+6cd-15d^2\right)\right)\int \frac{1}{\sin(e+fx)a+a}dx+15a^2d^2x(Ad+3B(c-d))}{a}+\frac{ad^2(A(2c+7d)+3B(c-9d))\cos(e+fx)}{f}}{3a^2}-\frac{a(2A(c+2d)+3\cdots}{\cdots}}{5a^2}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{5f(a\sin(e+fx)+a)^3}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{\frac{a^3(c-d)\left(A\left(2c^2+7cd+15d^2\right)+3B\left(c^2+6cd-15d^2\right)\right)\int \frac{1}{\sin(e+fx)a+a}dx+15a^2d^2x(Ad+3B(c-d))}{a}+\frac{ad^2(A(2c+7d)+3B(c-9d))\cos(e+fx)}{f}}{3a^2}-\frac{a(2A(c+2d)+3\cdots}{\cdots}}{5a^2}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{5f(a\sin(e+fx)+a)^3}$$

$$\Big\downarrow \ 3127$$

$$\frac{\frac{\frac{15a^2d^2x(Ad+3B(c-d))-\frac{a^3(c-d)\left(A\left(2c^2+7cd+15d^2\right)+3B\left(c^2+6cd-15d^2\right)\right)\cos(e+fx)}{f(a\sin(e+fx)+a)}}{a}+\frac{ad^2(A(2c+7d)+3B(c-9d))\cos(e+fx)}{f}}{3a^2}-\frac{a(2A(c+2d)+3B(c-3d\cdots}{3f(a\,\mathrm{si}\cdots}}{5a^2}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{5f(a\sin(e+fx)+a)^3}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^3,x
]
```

output
```
-1/5*((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(f*(a + a*Sin[e + f*x])
^3) + (-1/3*(a*(3*B*(c - 3*d) + 2*A*(c + 2*d))*Cos[e + f*x]*(c + d*Sin[e +
 f*x])^2)/(f*(a + a*Sin[e + f*x])^2) + ((a*d^2*(3*B*(c - 9*d) + A*(2*c + 7
*d))*Cos[e + f*x])/f + (15*a^2*d^2*(3*B*(c - d) + A*d)*x - (a^3*(c - d)*(3
*B*(c^2 + 6*c*d - 15*d^2) + A*(2*c^2 + 7*c*d + 15*d^2))*Cos[e + f*x])/(f*(
a + a*Sin[e + f*x])))/a)/(3*a^2))/(5*a^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 2.45 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.55

method	result
derivativedivides	$\frac{-\frac{2\left(A c^3-A d^3-3Bc d^2+3B d^3\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-4A c^3+6A c^2d-2A d^3+2B c^3-6Bc d^2+4B d^3}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(8A c^3-18A c^2d+12Ac d^2-2A d^3-6B c^3+1\ldots\right.}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}\ldots}{\ldots}$
default	$\frac{-\frac{2\left(A c^3-A d^3-3Bc d^2+3B d^3\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-4A c^3+6A c^2d-2A d^3+2B c^3-6Bc d^2+4B d^3}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(8A c^3-18A c^2d+12Ac d^2-2A d^3-6B c^3+1\ldots\right.}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}\ldots}{\ldots}$
parallelrisch	$\frac{\left((-300fxA+900fxB-540A+1755B)d^3+180c(-5fxB+A-9B)d^2+180c^2(A+B)d+180\left(A+\frac{B}{3}\right)c^3\right)\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+\ldots}{\ldots}$
risch	$\frac{d^3xA}{a^3}+\frac{3d^2xBc}{a^3}-\frac{3d^3xB}{a^3}-\frac{B\,d^3e^{i(fx+e)}}{2a^3f}-\frac{B\,d^3e^{-i(fx+e)}}{2a^3f}-\frac{2\left(2A c^3+3B c^3-96Bc d^2+9A c^2d+21Ac d^2+\ldots\right.}{\ldots}$
norman	Expression too large to display

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x,method=_RETUR
NVERBOSE)
```

output
```
2/f/a^3*(-(A*c^3-A*d^3-3*B*c*d^2+3*B*d^3)/(tan(1/2*f*x+1/2*e)+1)-1/2*(-4*A
*c^3+6*A*c^2*d-2*A*d^3+2*B*c^3-6*B*c*d^2+4*B*d^3)/(tan(1/2*f*x+1/2*e)+1)^2
-1/3*(8*A*c^3-18*A*c^2*d+12*A*c*d^2-2*A*d^3-6*B*c^3+12*B*c^2*d-6*B*c*d^2)/
(tan(1/2*f*x+1/2*e)+1)^3-1/4*(-8*A*c^3+24*A*c^2*d-24*A*c*d^2+8*A*d^3+8*B*c
^3-24*B*c^2*d+24*B*c*d^2-8*B*d^3)/(tan(1/2*f*x+1/2*e)+1)^4-1/5*(4*A*c^3-12
*A*c^2*d+12*A*c*d^2-4*A*d^3-4*B*c^3+12*B*c^2*d-12*B*c*d^2+4*B*d^3)/(tan(1/
2*f*x+1/2*e)+1)^5+d^2*(-B*d/(1+tan(1/2*f*x+1/2*e)^2)+(A*d+3*B*c-3*B*d)*arc
tan(tan(1/2*f*x+1/2*e))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 649 vs. $2(217) = 434$.

Time = 0.11 (sec) , antiderivative size = 649, normalized size of antiderivative = 2.88

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algori
thm="fricas")
```

output
```
-1/15*(15*B*d^3*cos(f*x + e)^4 - 3*(A - B)*c^3 + 9*(A - B)*c^2*d - 9*(A -
B)*c*d^2 + 3*(A - B)*d^3 + ((2*A + 3*B)*c^3 + 3*(3*A + 7*B)*c^2*d + 3*(7*A
 - 32*B)*c*d^2 - (32*A - 117*B)*d^3 - 15*(3*B*c*d^2 + (A - 3*B)*d^3)*f*x)*
cos(f*x + e)^3 + 60*(3*B*c*d^2 + (A - 3*B)*d^3)*f*x - (2*(2*A + 3*B)*c^3 +
 3*(6*A - B)*c^2*d - 3*(A + 19*B)*c*d^2 - (19*A - 84*B)*d^3 + 45*(3*B*c*d^
2 + (A - 3*B)*d^3)*f*x)*cos(f*x + e)^2 - 3*((3*A + 2*B)*c^3 + 3*(2*A + 3*B
)*c^2*d + 9*(A - 6*B)*c*d^2 - 9*(2*A - 7*B)*d^3 - 10*(3*B*c*d^2 + (A - 3*B
)*d^3)*f*x)*cos(f*x + e) + (15*B*d^3*cos(f*x + e)^3 + 3*(A - B)*c^3 - 9*(A
 - B)*c^2*d + 9*(A - B)*c*d^2 - 3*(A - B)*d^3 + 60*(3*B*c*d^2 + (A - 3*B)*
d^3)*f*x - ((2*A + 3*B)*c^3 + 3*(3*A + 7*B)*c^2*d + 3*(7*A - 32*B)*c*d^2 -
 2*(16*A - 51*B)*d^3 + 15*(3*B*c*d^2 + (A - 3*B)*d^3)*f*x)*cos(f*x + e)^2
 - 3*((2*A + 3*B)*c^3 + 3*(3*A + 2*B)*c^2*d + 3*(2*A - 17*B)*c*d^2 - (17*A
 - 62*B)*d^3 - 10*(3*B*c*d^2 + (A - 3*B)*d^3)*f*x)*cos(f*x + e))*sin(f*x +
 e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e)
 - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*
x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11456 vs. 2(206) = 412.

Time = 15.43 (sec) , antiderivative size = 11456, normalized size of antiderivative = 50.92

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**3,x)
```

output
```
Piecewise((-30*A*c**3*tan(e/2 + f*x/2)**6/(15*a**3*f*tan(e/2 + f*x/2)**7 +
 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3
*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e
/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*A*c**3*tan(e
/2 + f*x/2)**5/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)
**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 22
5*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*
tan(e/2 + f*x/2) + 15*a**3*f) - 110*A*c**3*tan(e/2 + f*x/2)**4/(15*a**3*f*
tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 +
 f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)*
*3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3
*f) - 100*A*c**3*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a
**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*ta
n(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 +
f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 94*A*c**3*tan(e/2 +
f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 +
 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**
3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e
/2 + f*x/2) + 15*a**3*f) - 40*A*c**3*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 +
 f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1682 vs. $2(217) = 434$.

Time = 0.16 (sec) , antiderivative size = 1682, normalized size of antiderivative = 7.48

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^3}{(a + a\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algori
thm="maxima")
```

output
```
-2/15*(3*B*d^3*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 189*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 200*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 160*sin(f
*x + e)^4/(cos(f*x + e) + 1)^4 + 75*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 +
15*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*sin(f*x + e)/(co
s(f*x + e) + 1) + 11*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*a^3*sin(
f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4 + 11*a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 5*a^3*sin(f*x + e)^6/(co
s(f*x + e) + 1)^6 + a^3*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 15*arctan(s
in(f*x + e)/(cos(f*x + e) + 1))/a^3) - 3*B*c*d^2*((95*sin(f*x + e)/(cos(f*
x + e) + 1) + 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 +
 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 15*a
rctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^3) - A*d^3*((95*sin(f*x + e)/(cos
(f*x + e) + 1) + 145*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 75*sin(f*x + e)
^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 22)/(a^
3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x
+ e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 567 vs. $2(217) = 434$.

Time = 0.21 (sec) , antiderivative size = 567, normalized size of antiderivative = 2.52

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algori
thm="giac")
```

output
```
-1/15*(30*B*d^3/((tan(1/2*f*x + 1/2*e)^2 + 1)*a^3) - 15*(3*B*c*d^2 + A*d^3
 - 3*B*d^3)*(f*x + e)/a^3 + 2*(15*A*c^3*tan(1/2*f*x + 1/2*e)^4 - 45*B*c*d^
2*tan(1/2*f*x + 1/2*e)^4 - 15*A*d^3*tan(1/2*f*x + 1/2*e)^4 + 45*B*d^3*tan(
1/2*f*x + 1/2*e)^4 + 30*A*c^3*tan(1/2*f*x + 1/2*e)^3 + 15*B*c^3*tan(1/2*f*
x + 1/2*e)^3 + 45*A*c^2*d*tan(1/2*f*x + 1/2*e)^3 - 225*B*c*d^2*tan(1/2*f*x
 + 1/2*e)^3 - 75*A*d^3*tan(1/2*f*x + 1/2*e)^3 + 210*B*d^3*tan(1/2*f*x + 1/
2*e)^3 + 40*A*c^3*tan(1/2*f*x + 1/2*e)^2 + 15*B*c^3*tan(1/2*f*x + 1/2*e)^2
 + 45*A*c^2*d*tan(1/2*f*x + 1/2*e)^2 + 60*B*c^2*d*tan(1/2*f*x + 1/2*e)^2 +
 60*A*c*d^2*tan(1/2*f*x + 1/2*e)^2 - 435*B*c*d^2*tan(1/2*f*x + 1/2*e)^2 -
145*A*d^3*tan(1/2*f*x + 1/2*e)^2 + 360*B*d^3*tan(1/2*f*x + 1/2*e)^2 + 20*A
*c^3*tan(1/2*f*x + 1/2*e) + 15*B*c^3*tan(1/2*f*x + 1/2*e) + 45*A*c^2*d*tan
(1/2*f*x + 1/2*e) + 30*B*c^2*d*tan(1/2*f*x + 1/2*e) + 30*A*c*d^2*tan(1/2*f
*x + 1/2*e) - 285*B*c*d^2*tan(1/2*f*x + 1/2*e) - 95*A*d^3*tan(1/2*f*x + 1/
2*e) + 240*B*d^3*tan(1/2*f*x + 1/2*e) + 7*A*c^3 + 3*B*c^3 + 9*A*c^2*d + 6*
B*c^2*d + 6*A*c*d^2 - 66*B*c*d^2 - 22*A*d^3 + 57*B*d^3)/(a^3*(tan(1/2*f*x
+ 1/2*e) + 1)^5))/f
```

Mupad [B] (verification not implemented)

Time = 38.55 (sec) , antiderivative size = 593, normalized size of antiderivative = 2.64

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx$$
$$= \frac{2\,d^2\,\mathrm{atan}\left(\frac{2\,d^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(A\,d+3\,B\,c-3\,B\,d)}{2\,A\,d^3-6\,B\,d^3+6\,B\,c\,d^2}\right)\,(A\,d+3\,B\,c-3\,B\,d)}{a^3\,f}$$
$$-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,(4\,A\,c^3-10\,A\,d^3+2\,B\,c^3+30\,B\,d^3+6\,A\,c^2\,d-30\,B\,c\,d^2)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{8\,A\,c^3}{3}-\frac{38\,A}{3}\right.}{}$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^3,x
)
```

output
```
(2*d^2*atan((2*d^2*tan(e/2 + (f*x)/2)*(A*d + 3*B*c - 3*B*d))/(2*A*d^3 - 6*
B*d^3 + 6*B*c*d^2))*(A*d + 3*B*c - 3*B*d))/(a^3*f) - (tan(e/2 + (f*x)/2)^5
*(4*A*c^3 - 10*A*d^3 + 2*B*c^3 + 30*B*d^3 + 6*A*c^2*d - 30*B*c*d^2) + tan(
e/2 + (f*x)/2)*((8*A*c^3)/3 - (38*A*d^3)/3 + 2*B*c^3 + 42*B*d^3 + 4*A*c*d^
2 + 6*A*c^2*d - 38*B*c*d^2 + 4*B*c^2*d) + (14*A*c^3)/15 - (44*A*d^3)/15 +
(2*B*c^3)/5 + (48*B*d^3)/5 + tan(e/2 + (f*x)/2)^4*((22*A*c^3)/3 - (64*A*d^
3)/3 + 2*B*c^3 + 64*B*d^3 + 8*A*c*d^2 + 6*A*c^2*d - 64*B*c*d^2 + 8*B*c^2*d
) + tan(e/2 + (f*x)/2)^3*((20*A*c^3)/3 - (68*A*d^3)/3 + 4*B*c^3 + 80*B*d^3
 + 4*A*c*d^2 + 12*A*c^2*d - 68*B*c*d^2 + 4*B*c^2*d) + tan(e/2 + (f*x)/2)^2
*((94*A*c^3)/15 - (334*A*d^3)/15 + (12*B*c^3)/5 + (378*B*d^3)/5 + (44*A*c*
d^2)/5 + (36*A*c^2*d)/5 - (334*B*c*d^2)/5 + (44*B*c^2*d)/5) + tan(e/2 + (f
*x)/2)^6*(2*A*c^3 - 2*A*d^3 + 6*B*d^3 - 6*B*c*d^2) + (4*A*c*d^2)/5 + (6*A*
c^2*d)/5 - (44*B*c*d^2)/5 + (4*B*c^2*d)/5)/(f*(11*a^3*tan(e/2 + (f*x)/2)^2
 + 15*a^3*tan(e/2 + (f*x)/2)^3 + 15*a^3*tan(e/2 + (f*x)/2)^4 + 11*a^3*tan(
e/2 + (f*x)/2)^5 + 5*a^3*tan(e/2 + (f*x)/2)^6 + a^3*tan(e/2 + (f*x)/2)^7 +
 a^3 + 5*a^3*tan(e/2 + (f*x)/2)))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1123, normalized size of antiderivative = 4.99

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x)
```

output
```
(15*cos(e + f*x)*sin(e + f*x)**3*b*d**3 + cos(e + f*x)*sin(e + f*x)**2*a*c
**3 + 15*cos(e + f*x)*sin(e + f*x)**2*a*c*d**2 + 15*cos(e + f*x)*sin(e + f
*x)**2*a*d**3*f*x - 16*cos(e + f*x)*sin(e + f*x)**2*a*d**3 + 15*cos(e + f*
x)*sin(e + f*x)**2*b*c**2*d + 45*cos(e + f*x)*sin(e + f*x)**2*b*c*d**2*f*x
 - 48*cos(e + f*x)*sin(e + f*x)**2*b*c*d**2 - 45*cos(e + f*x)*sin(e + f*x)
**2*b*d**3*f*x + 63*cos(e + f*x)*sin(e + f*x)**2*b*d**3 + 4*cos(e + f*x)*s
in(e + f*x)*a*c**3 + 9*cos(e + f*x)*sin(e + f*x)*a*c**2*d + 6*cos(e + f*x)
*sin(e + f*x)*a*c*d**2 + 30*cos(e + f*x)*sin(e + f*x)*a*d**3*f*x - 19*cos(
e + f*x)*sin(e + f*x)*a*d**3 + 3*cos(e + f*x)*sin(e + f*x)*b*c**3 + 6*cos(
e + f*x)*sin(e + f*x)*b*c**2*d + 90*cos(e + f*x)*sin(e + f*x)*b*c*d**2*f*x
 - 57*cos(e + f*x)*sin(e + f*x)*b*c*d**2 - 90*cos(e + f*x)*sin(e + f*x)*b*
d**3*f*x + 63*cos(e + f*x)*sin(e + f*x)*b*d**3 + 6*cos(e + f*x)*a*c**3 + 1
5*cos(e + f*x)*a*d**3*f*x - 6*cos(e + f*x)*a*d**3 + 45*cos(e + f*x)*b*c*d*
*2*f*x - 18*cos(e + f*x)*b*c*d**2 - 45*cos(e + f*x)*b*d**3*f*x + 18*cos(e
+ f*x)*b*d**3 + 15*sin(e + f*x)**4*b*d**3 + 3*sin(e + f*x)**3*a*c**3 + 18*
sin(e + f*x)**3*a*c**2*d + 27*sin(e + f*x)**3*a*c*d**2 - 15*sin(e + f*x)**
3*a*d**3*f*x - 48*sin(e + f*x)**3*a*d**3 + 6*sin(e + f*x)**3*b*c**3 + 27*s
in(e + f*x)**3*b*c**2*d - 45*sin(e + f*x)**3*b*c*d**2*f*x - 144*sin(e + f*
x)**3*b*c*d**2 + 45*sin(e + f*x)**3*b*d**3*f*x + 156*sin(e + f*x)**3*b*d**
3 + 7*sin(e + f*x)**2*a*c**3 + 45*sin(e + f*x)**2*a*c**2*d + 15*sin(e +...
```

3.280 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 164

$$\begin{aligned}\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx &= \frac{Bd^2x}{a^3}-\frac{(c-d)(B(3c-7d)+2A(c+d))\cos(e+fx)}{15af(a+a\sin(e+fx))^2}\\ &\quad-\frac{(B(3c^2+14cd-29d^2)+2A(c^2+3cd+2d^2))\cos(e+fx)}{15f\,(a^3+a^3\sin(e+fx))}\\ &\quad-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a+a\sin(e+fx))^3}\end{aligned}$$

output
```
B*d^2*x/a^3-1/15*(c-d)*(B*(3*c-7*d)+2*A*(c+d))*cos(f*x+e)/a/f/(a+a*sin(f*x
+e))^2-1/15*(B*(3*c^2+14*c*d-29*d^2)+2*A*(c^2+3*c*d+2*d^2))*cos(f*x+e)/f/(
a^3+a^3*sin(f*x+e))-1/5*(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))^2/f/(a+a*sin(f*x
+e))^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 514 vs. $2(164) = 328$.

Time = 1.70 (sec) , antiderivative size = 514, normalized size of antiderivative = 3.13

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(30(2Ad(c+d)+B(c^2+4cd+d^2(-9+5e+5fx)))\cos\left(\frac{1}{2}(e+fx\right.\right.}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x
])^3,x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(30*(2*A*d*(c + d) + B*(c^2 + 4*c*d
 + d^2*(-9 + 5*e + 5*f*x)))*Cos[(e + f*x)/2] - 5*(4*A*(c^2 + 3*c*d + 2*d^2
) + B*(6*c^2 + 16*c*d + d^2*(-46 + 15*e + 15*f*x)))*Cos[(3*(e + f*x))/2] -
 15*B*d^2*e*Cos[(5*(e + f*x))/2] - 15*B*d^2*f*x*Cos[(5*(e + f*x))/2] + 40*
A*c^2*Sin[(e + f*x)/2] + 30*B*c^2*Sin[(e + f*x)/2] + 60*A*c*d*Sin[(e + f*x
)/2] + 160*B*c*d*Sin[(e + f*x)/2] + 80*A*d^2*Sin[(e + f*x)/2] - 370*B*d^2*
Sin[(e + f*x)/2] + 150*B*d^2*e*Sin[(e + f*x)/2] + 150*B*d^2*f*x*Sin[(e + f
*x)/2] + 60*B*c*d*Sin[(3*(e + f*x))/2] + 30*A*d^2*Sin[(3*(e + f*x))/2] - 9
0*B*d^2*Sin[(3*(e + f*x))/2] + 75*B*d^2*e*Sin[(3*(e + f*x))/2] + 75*B*d^2*
f*x*Sin[(3*(e + f*x))/2] - 4*A*c^2*Sin[(5*(e + f*x))/2] - 6*B*c^2*Sin[(5*(
e + f*x))/2] - 12*A*c*d*Sin[(5*(e + f*x))/2] - 28*B*c*d*Sin[(5*(e + f*x))/
2] - 14*A*d^2*Sin[(5*(e + f*x))/2] + 64*B*d^2*Sin[(5*(e + f*x))/2] - 15*B*
d^2*e*Sin[(5*(e + f*x))/2] - 15*B*d^2*f*x*Sin[(5*(e + f*x))/2]))/(60*a^3*f
*(1 + Sin[e + f*x])^3)
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {3042, 3456, 3042, 3447, 3042, 3498, 25, 3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a\sin(e+fx)+a)^3}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a\sin(e+fx)+a)^3}dx$$

$$\Big\downarrow \ 3456$$

$$\frac{\int \frac{(c+d\sin(e+fx))(a(B(3c-2d)+2A(c+d))+5aBd\sin(e+fx))}{(\sin(e+fx)a+a)^2}dx}{5a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a\sin(e+fx)+a)^3}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{(c+d\sin(e+fx))(a(B(3c-2d)+2A(c+d))+5aBd\sin(e+fx))}{(\sin(e+fx)a+a)^2}dx}{5a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a\sin(e+fx)+a)^3}$$

$$\Big\downarrow \ 3447$$

$$\frac{\int \frac{5aBd^2\sin^2(e+fx)+(5aBcd+a(B(3c-2d)+2A(c+d))d)\sin(e+fx)+ac(B(3c-2d)+2A(c+d))}{(\sin(e+fx)a+a)^2}dx}{5a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a\sin(e+fx)+a)^3}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{5aBd^2\sin(e+fx)^2+(5aBcd+a(B(3c-2d)+2A(c+d))d)\sin(e+fx)+ac(B(3c-2d)+2A(c+d))}{(\sin(e+fx)a+a)^2}dx}{5a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a\sin(e+fx)+a)^3}$$

$$\Big\downarrow \ 3498$$

$$\frac{-\frac{\int -\frac{\left(B\left(3c^2+14dc-14d^2\right)+2A\left(c^2+3dc+2d^2\right)\right)a^2+15Bd^2\sin(e+fx)a^2}{\sin(e+fx)a+a}dx}{3a^2}-\frac{a(c-d)(2A(c+d)+B(3c-7d))\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}}{5a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a\sin(e+fx)+a)^3}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{\left(B\left(3c^2+14dc-14d^2\right)+2A\left(c^2+3dc+2d^2\right)\right)a^2+15Bd^2\sin(e+fx)a^2}{\sin(e+fx)a+a}dx}{3a^2}-\frac{a(c-d)(2A(c+d)+B(3c-7d))\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}}{5a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a\sin(e+fx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{\left(B\left(3c^2+14dc-14d^2\right)+2A\left(c^2+3dc+2d^2\right)\right)a^2+15Bd^2\sin(e+fx)a^2}{\sin(e+fx)a+a}dx}{3a^2}-\frac{a(c-d)(2A(c+d)+B(3c-7d))\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}}{5a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a\sin(e+fx)+a)^3}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\frac{a^2(2A(c^2+3cd+2d^2)+B(3c^2+14cd-29d^2))\int \frac{1}{\sin(e+fx)a+a}dx+15aBd^2x}{3a^2}-\frac{a(c-d)(2A(c+d)+B(3c-7d))\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}}{5a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a\sin(e+fx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{a^2(2A(c^2+3cd+2d^2)+B(3c^2+14cd-29d^2))\int \frac{1}{\sin(e+fx)a+a}dx+15aBd^2x}{3a^2}-\frac{a(c-d)(2A(c+d)+B(3c-7d))\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}}{5a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a\sin(e+fx)+a)^3}$$

$$\Big\downarrow \text{3127}$$

$$\frac{\frac{15aBd^2x-\frac{a^2\left(2A\left(c^2+3cd+2d^2\right)+B\left(3c^2+14cd-29d^2\right)\right)\cos(e+fx)}{f(a\sin(e+fx)+a)}}{3a^2}-\frac{a(c-d)(2A(c+d)+B(3c-7d))\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}}{5a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{5f(a\sin(e+fx)+a)^3}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])^3,x
]
```

output
```
-1/5*((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(f*(a + a*Sin[e + f*x])
^3) + (-1/3*(a*(c - d)*(B*(3*c - 7*d) + 2*A*(c + d))*Cos[e + f*x])/(f*(a +
 a*Sin[e + f*x])^2) + (15*a*B*d^2*x - (a^2*(B*(3*c^2 + 14*c*d - 29*d^2) +
2*A*(c^2 + 3*c*d + 2*d^2))*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])))/(3*a^2)
)/(5*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3498
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[1
/(a^2*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b
*B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.46

method	result
parallelrisch	$\frac{15Bx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}d^{2}f+\left(\left(75fx+30\right)B\,d^{2}-30A\,c^{2}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}+\left(\left(150fx+150\right)B\,d^{2}-60Acd-60c^{2}\left(A+\frac{B}{2}\right)\right)\tan(}{}$
derivativedivides	$\frac{2B\,d^{2}\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{2\left(A\,c^{2}-B\,d^{2}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-4A\,c^{2}+4Acd+2B\,c^{2}-2B\,d^{2}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{-8A\,c^{2}+16Acd-8A\,d^{2}+8B\,c^{2}-16Bcd+8B\,d^{2}}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{4}}}{a^{3}f}$
default	$\frac{2B\,d^{2}\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\frac{2\left(A\,c^{2}-B\,d^{2}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-4A\,c^{2}+4Acd+2B\,c^{2}-2B\,d^{2}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{-8A\,c^{2}+16Acd-8A\,d^{2}+8B\,c^{2}-16Bcd+8B\,d^{2}}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{4}}}{a^{3}f}$
risch	$\frac{B\,d^{2}x}{a^{3}}-\frac{2\left(2A\,c^{2}+3B\,c^{2}+7A\,d^{2}+6Acd-32B\,d^{2}-10iA\,c^{2}e^{i(fx+e)}-15iB\,c^{2}e^{i(fx+e)}+14Bcd-20A\,c^{2}e^{2i(fx+e)}-15B}{}$
norman	$\frac{\frac{B\,d^{2}x}{a}+\frac{B\,d^{2}x\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}{a}-\frac{14A\,c^{2}+12Acd+4A\,d^{2}+6B\,c^{2}+8Bcd-44B\,d^{2}}{15af}-\frac{\left(2A\,c^{2}-2B\,d^{2}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{af}-\frac{(4A\,c^{2}+4Acd+}{}}{}$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x,method=_RETUR
NVERBOSE)
```

output
```
1/15*(15*B*x*tan(1/2*f*x+1/2*e)^5*d^2*f+((75*f*x+30)*B*d^2-30*A*c^2)*tan(1
/2*f*x+1/2*e)^4+((150*f*x+150)*B*d^2-60*A*c*d-60*c^2*(A+1/2*B))*tan(1/2*f*
x+1/2*e)^3+((150*B*f*x-40*A+290*B)*d^2-60*c*(A+4/3*B)*d-80*c^2*(A+3/8*B))*
tan(1/2*f*x+1/2*e)^2+((75*B*f*x-20*A+190*B)*d^2-60*c*(A+2/3*B)*d-40*c^2*(A
+3/4*B))*tan(1/2*f*x+1/2*e)+(15*B*f*x-4*A+44*B)*d^2-12*c*(A+2/3*B)*d-14*c^
2*(A+3/7*B))/f/a^3/(tan(1/2*f*x+1/2*e)+1)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 432 vs. $2(158) = 316$.

Time = 0.10 (sec) , antiderivative size = 432, normalized size of antiderivative = 2.63

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx = -\frac{60\,Bd^2fx-(15\,Bd^2fx-(2\,A+3\,B)c^2-2\,(3\,A+7\,B)cd-(7\,A-32\,B)d^2)\cos{(fx+e)}^3-3\,(A-}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algori
thm="fricas")
```

output
```
-1/15*(60*B*d^2*f*x - (15*B*d^2*f*x - (2*A + 3*B)*c^2 - 2*(3*A + 7*B)*c*d
- (7*A - 32*B)*d^2)*cos(f*x + e)^3 - 3*(A - B)*c^2 + 6*(A - B)*c*d - 3*(A
- B)*d^2 - (45*B*d^2*f*x + 2*(2*A + 3*B)*c^2 + 2*(6*A - B)*c*d - (A + 19*B
)*d^2)*cos(f*x + e)^2 + 3*(10*B*d^2*f*x - (3*A + 2*B)*c^2 - 2*(2*A + 3*B)*
c*d - 3*(A - 6*B)*d^2)*cos(f*x + e) + (60*B*d^2*f*x + 3*(A - B)*c^2 - 6*(A
 - B)*c*d + 3*(A - B)*d^2 - (15*B*d^2*f*x + (2*A + 3*B)*c^2 + 2*(3*A + 7*B
)*c*d + (7*A - 32*B)*d^2)*cos(f*x + e)^2 + 3*(10*B*d^2*f*x - (2*A + 3*B)*c
^2 - 2*(3*A + 2*B)*c*d - (2*A - 17*B)*d^2)*cos(f*x + e))*sin(f*x + e))/(a^
3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3
*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3468 vs. 2(151) = 302.

Time = 8.82 (sec) , antiderivative size = 3468, normalized size of antiderivative = 21.15

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**3,x)
```

output
```
Piecewise((-30*A*c**2*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 +
 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3
*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*A*c*
*2*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2
+ f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)
**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 80*A*c**2*tan(e/2 + f*x/2)
**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a
**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan
(e/2 + f*x/2) + 15*a**3*f) - 40*A*c**2*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2
 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)
**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**
3*f) - 14*A*c**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/
2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 +
75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*A*c*d*tan(e/2 + f*x/2)**3/(15
*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*t
an(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 +
f*x/2) + 15*a**3*f) - 60*A*c*d*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*
x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 +
 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f)
- 60*A*c*d*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1132 vs. $2(158) = 316$.

Time = 0.14 (sec) , antiderivative size = 1132, normalized size of antiderivative = 6.90

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^2}{(a + a\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algori
thm="maxima")
```

output
```
2/15*(B*d^2*((95*sin(f*x + e)/(cos(f*x + e) + 1) + 145*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 75*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1
) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5) + 15*arctan(sin(f*x + e)/(cos(f*x + e) + 1)
)/a^3) - A*c^2*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1
) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5) - 4*B*c*d*(5*sin(f*x + e)/(cos(f*x + e) + 1
) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/
(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*s
in(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) +
1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 2*A*d^2*(5*sin(f*x + e)/
(cos(f*x + e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*
a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 3*B*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(158) = 316$.

Time = 0.27 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.21

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx$$
$$= \frac{\frac{15\,(fx+e)Bd^2}{a^3} - \frac{2\left(15\,Ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4 - 15\,Bd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4 + 30\,Ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 + 15\,Bc^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 + 30\,Acd\tan\left(\frac{1}{2}\,fx+\right.}{}}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algori
thm="giac")
```

output
```
1/15*(15*(f*x + e)*B*d^2/a^3 - 2*(15*A*c^2*tan(1/2*f*x + 1/2*e)^4 - 15*B*d
^2*tan(1/2*f*x + 1/2*e)^4 + 30*A*c^2*tan(1/2*f*x + 1/2*e)^3 + 15*B*c^2*tan
(1/2*f*x + 1/2*e)^3 + 30*A*c*d*tan(1/2*f*x + 1/2*e)^3 - 75*B*d^2*tan(1/2*f
*x + 1/2*e)^3 + 40*A*c^2*tan(1/2*f*x + 1/2*e)^2 + 15*B*c^2*tan(1/2*f*x + 1
/2*e)^2 + 30*A*c*d*tan(1/2*f*x + 1/2*e)^2 + 40*B*c*d*tan(1/2*f*x + 1/2*e)^
2 + 20*A*d^2*tan(1/2*f*x + 1/2*e)^2 - 145*B*d^2*tan(1/2*f*x + 1/2*e)^2 + 2
0*A*c^2*tan(1/2*f*x + 1/2*e) + 15*B*c^2*tan(1/2*f*x + 1/2*e) + 30*A*c*d*ta
n(1/2*f*x + 1/2*e) + 20*B*c*d*tan(1/2*f*x + 1/2*e) + 10*A*d^2*tan(1/2*f*x
+ 1/2*e) - 95*B*d^2*tan(1/2*f*x + 1/2*e) + 7*A*c^2 + 3*B*c^2 + 6*A*c*d + 4
*B*c*d + 2*A*d^2 - 22*B*d^2)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

Mupad [B] (verification not implemented)

Time = 40.05 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.74

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx = \frac{B\,d^2\,x}{a^3}$$
$$- \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\left(\frac{16\,A\,c^2}{3}+\frac{8\,A\,d^2}{3}+2\,B\,c^2-\frac{58\,B\,d^2}{3}+4\,A\,c\,d+\frac{16\,B\,c\,d}{3}\right)+\frac{14\,A\,c^2}{15}+\frac{4\,A\,d^2}{15}+\frac{2\,B\,c^2}{5}-\frac{44\,B\,d^2}{15}+}{f\left(a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+5\right.}$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^3,x
)
```

output
```
(B*d^2*x)/a^3 - (tan(e/2 + (f*x)/2)^2*((16*A*c^2)/3 + (8*A*d^2)/3 + 2*B*c^
2 - (58*B*d^2)/3 + 4*A*c*d + (16*B*c*d)/3) + (14*A*c^2)/15 + (4*A*d^2)/15
+ (2*B*c^2)/5 - (44*B*d^2)/15 + tan(e/2 + (f*x)/2)^3*(4*A*c^2 + 2*B*c^2 -
10*B*d^2 + 4*A*c*d) + tan(e/2 + (f*x)/2)^4*(2*A*c^2 - 2*B*d^2) + tan(e/2 +
 (f*x)/2)*((8*A*c^2)/3 + (4*A*d^2)/3 + 2*B*c^2 - (38*B*d^2)/3 + 4*A*c*d +
(8*B*c*d)/3) + (4*A*c*d)/5 + (8*B*c*d)/15)/(f*(10*a^3*tan(e/2 + (f*x)/2)^2
 + 10*a^3*tan(e/2 + (f*x)/2)^3 + 5*a^3*tan(e/2 + (f*x)/2)^4 + a^3*tan(e/2
+ (f*x)/2)^5 + a^3 + 5*a^3*tan(e/2 + (f*x)/2)))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 481, normalized size of antiderivative = 2.93

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^3}\,dx$$
$$= \frac{6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5 a\,c^2 - 6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5 b\,d^2 - 30\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3 b\,c^2 + 90\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3 b\,d^2 - 20\tan\left(\frac{fx}{2}+\right.}{}$$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x)
```

output
```
(6*tan((e + f*x)/2)**5*a*c**2 + 15*tan((e + f*x)/2)**5*b*d**2*f*x - 6*tan(
(e + f*x)/2)**5*b*d**2 + 75*tan((e + f*x)/2)**4*b*d**2*f*x - 60*tan((e + f
*x)/2)**3*a*c*d - 30*tan((e + f*x)/2)**3*b*c**2 + 150*tan((e + f*x)/2)**3*
b*d**2*f*x + 90*tan((e + f*x)/2)**3*b*d**2 - 20*tan((e + f*x)/2)**2*a*c**2
 - 60*tan((e + f*x)/2)**2*a*c*d - 40*tan((e + f*x)/2)**2*a*d**2 - 30*tan((
e + f*x)/2)**2*b*c**2 - 80*tan((e + f*x)/2)**2*b*c*d + 150*tan((e + f*x)/2
)**2*b*d**2*f*x + 230*tan((e + f*x)/2)**2*b*d**2 - 10*tan((e + f*x)/2)*a*c
**2 - 60*tan((e + f*x)/2)*a*c*d - 20*tan((e + f*x)/2)*a*d**2 - 30*tan((e +
 f*x)/2)*b*c**2 - 40*tan((e + f*x)/2)*b*c*d + 75*tan((e + f*x)/2)*b*d**2*f
*x + 160*tan((e + f*x)/2)*b*d**2 - 8*a*c**2 - 12*a*c*d - 4*a*d**2 - 6*b*c*
*2 - 8*b*c*d + 15*b*d**2*f*x + 38*b*d**2)/(15*a**3*f*(tan((e + f*x)/2)**5
+ 5*tan((e + f*x)/2)**4 + 10*tan((e + f*x)/2)**3 + 10*tan((e + f*x)/2)**2
+ 5*tan((e + f*x)/2) + 1))
```

3.281 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 127

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = -\frac{(A-B)(c-d)\cos(e+fx)}{5f(a+a\sin(e+fx))^3} - \frac{(2Ac+3Bc+3Ad-8Bd)\cos(e+fx)}{15af(a+a\sin(e+fx))^2} - \frac{(2Ac+3Bc+3Ad+7Bd)\cos(e+fx)}{15f\left(a^3+a^3\sin(e+fx)\right)}$$

output
```
-1/5*(A-B)*(c-d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^3-1/15*(2*A*c+3*A*d+3*B*c-8
*B*d)*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^2-1/15*(2*A*c+3*A*d+3*B*c+7*B*d)*cos
(f*x+e)/f/(a^3+a^3*sin(f*x+e))
```

Mathematica [A] (verified)

Time = 3.39 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.39

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(15(Ad+B(c+2d))\cos\left(\frac{1}{2}(e+fx)\right)-5(2Ac+3Bc+3Ad+4B\right.}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])
^3,x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(15*(A*d + B*(c + 2*d))*Cos[(e + f*
x)/2] - 5*(2*A*c + 3*B*c + 3*A*d + 4*B*d)*Cos[(3*(e + f*x))/2] - 2*(-3*(3*
A*c + 2*B*c + 2*A*d + 8*B*d) + (2*A*c + 3*B*c + 3*A*d - 8*B*d)*Cos[e + f*x
] + (2*A*c + 3*B*c + 3*A*d + 7*B*d)*Cos[2*(e + f*x)])*Sin[(e + f*x)/2]))/(
30*a^3*f*(1 + Sin[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 3447, 3042, 3498, 25, 3042, 3229, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a\sin(e+fx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a\sin(e+fx)+a)^3}dx$$

$$\Big\downarrow \text{3447}$$

$$\int \frac{(Ad+Bc)\sin(e+fx)+Ac+Bd\sin^2(e+fx)}{(a\sin(e+fx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(Ad+Bc)\sin(e+fx)+Ac+Bd\sin(e+fx)^2}{(a\sin(e+fx)+a)^3}dx$$

$$\Big\downarrow \text{3498}$$

$$-\frac{\int -\frac{a(2Ac+3Bc+3Ad-3Bd)+5aBd\sin(e+fx)}{(\sin(e+fx)a+a)^2}dx}{5a^2}-\frac{(A-B)(c-d)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{a(2Ac+3Bc+3Ad-3Bd)+5aBd\sin(e+fx)}{(\sin(e+fx)a+a)^2}dx}{5a^2}-\frac{(A-B)(c-d)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{a(2Ac+3Bc+3Ad-3Bd)+5aBd\sin(e+fx)}{(\sin(e+fx)a+a)^2}dx}{5a^2}-\frac{(A-B)(c-d)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

$\Big\downarrow$ 3229

$$\frac{\frac{1}{3}(2Ac+3Ad+3Bc+7Bd)\int \frac{1}{\sin(e+fx)a+a}dx-\frac{a(2Ac+3Ad+3Bc-8Bd)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}}{5a^2}-\frac{(A-B)(c-d)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{1}{3}(2Ac+3Ad+3Bc+7Bd)\int \frac{1}{\sin(e+fx)a+a}dx-\frac{a(2Ac+3Ad+3Bc-8Bd)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}}{5a^2}-\frac{(A-B)(c-d)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

$\Big\downarrow$ 3127

$$\frac{-\frac{(2Ac+3Ad+3Bc+7Bd)\cos(e+fx)}{3f(a\sin(e+fx)+a)}-\frac{a(2Ac+3Ad+3Bc-8Bd)\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}}{5a^2}-\frac{(A-B)(c-d)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^3,x]
```

output
```
-1/5*((A - B)*(c - d)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^3) + (-1/3*(a*
(2*A*c + 3*B*c + 3*A*d - 8*B*d)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^2) -
 ((2*A*c + 3*B*c + 3*A*d + 7*B*d)*Cos[e + f*x])/(3*f*(a + a*Sin[e + f*x]))
)/(5*a^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3498
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[1
/(a^2*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b
*B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.09

method	result
parallelrisch	$\frac{-30Ac\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}+((-60c-30d)A-30Bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+\left((-80c-30d)A-30B\left(c+\frac{4d}{3}\right)\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}+\left((-40c\right.}{15fa^{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}$
derivativedivides	$\frac{-\frac{-8Ac+8Ad+8Bc-8Bd}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{4}}-\frac{2(4Ac-4Ad-4Bc+4Bd)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{2(8Ac-6Ad-6Bc+4Bd)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2Ac}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-4Ac+2Ad+2Bc}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}}{a^{3}f}$
default	$\frac{-\frac{-8Ac+8Ad+8Bc-8Bd}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{4}}-\frac{2(4Ac-4Ad-4Bc+4Bd)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{2(8Ac-6Ad-6Bc+4Bd)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{2Ac}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}-\frac{-4Ac+2Ad+2Bc}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{2}}}{a^{3}f}$
risch	$-\frac{2\left(2Ac+3Ad+7Bd+3Bc+30iBd\,e^{3i(fx+e)}+15iBc\,e^{3i(fx+e)}-20iBd\,e^{i(fx+e)}-10iAc\,e^{i(fx+e)}+15iAd\,e^{3i(fx+e)}-1\right.}{15fa^{3}\left(e^{i\right.}}$
norman	$-\frac{14Ac+6Ad+6Bc+4Bd}{15af}-\frac{2Ac\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}}{af}-\frac{2(2Ac+Ad+Bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}{af}-\frac{2(34Ac+11Ad+11Bc+14Bd)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{5af}-\frac{2(}{}$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^3,x,method=_RETURNV
ERBOSE)
```

output
```
1/15*(-30*A*c*tan(1/2*f*x+1/2*e)^4+((-60*c-30*d)*A-30*B*c)*tan(1/2*f*x+1/2
*e)^3+((-80*c-30*d)*A-30*B*(c+4/3*d))*tan(1/2*f*x+1/2*e)^2+((-40*c-30*d)*A
-30*(c+2/3*d)*B)*tan(1/2*f*x+1/2*e)+(-14*c-6*d)*A-6*(c+2/3*d)*B)/f/a^3/(ta
n(1/2*f*x+1/2*e)+1)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(121) = 242.

Time = 0.07 (sec) , antiderivative size = 271, normalized size of antiderivative = 2.13

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx = -\frac{((2A+3B)c+(3A+7B)d)\cos(fx+e)^3-(2(2A+3B)c+(6A-B)d)\cos(fx+e)^2-3(A-}{15\left(a^3f\cos(fx+\right.}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorith
m="fricas")
```

output
```
-1/15*(((2*A + 3*B)*c + (3*A + 7*B)*d)*cos(f*x + e)^3 - (2*(2*A + 3*B)*c +
 (6*A - B)*d)*cos(f*x + e)^2 - 3*(A - B)*c + 3*(A - B)*d - 3*((3*A + 2*B)*
c + (2*A + 3*B)*d)*cos(f*x + e) - (((2*A + 3*B)*c + (3*A + 7*B)*d)*cos(f*x
 + e)^2 - 3*(A - B)*c + 3*(A - B)*d + 3*((2*A + 3*B)*c + (3*A + 2*B)*d)*co
s(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 -
 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x
+ e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1819 vs. $2(121) = 242$.

Time = 4.94 (sec) , antiderivative size = 1819, normalized size of antiderivative = 14.32

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))}{(a + a\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))**3,x)
```

output
```
Piecewise((-30*A*c*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75
*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*
tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*A*c*tan
(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/
2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 +
75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 80*A*c*tan(e/2 + f*x/2)**2/(15*a
**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan
(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*
x/2) + 15*a**3*f) - 40*A*c*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5
 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a*
*3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 14*A*
c/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**
3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e
/2 + f*x/2) + 15*a**3*f) - 30*A*d*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 +
 f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**
3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*
f) - 30*A*d*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f
*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2
 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*A*d*tan(e/2 +
f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 +...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 733 vs. $2(121) = 242$.

Time = 0.06 (sec) , antiderivative size = 733, normalized size of antiderivative = 5.77

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))}{(a + a\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorith
m="maxima")
```

output
```
-2/15*(A*c*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) +
10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x +
e)^5/(cos(f*x + e) + 1)^5) + 2*B*d*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f
*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x
 + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 +
 a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*B*c*(5*sin(f*x + e)/(cos(f*x
 + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos
(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^
3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/
(cos(f*x + e) + 1)^5) + 3*A*d*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1
)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
))/f
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.65

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx =$$
$$-\frac{2\left(15\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+30\,Ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+15\,Bc\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+15\,Ad\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\right.\right.}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorith
m="giac")
```

output
```
-2/15*(15*A*c*tan(1/2*f*x + 1/2*e)^4 + 30*A*c*tan(1/2*f*x + 1/2*e)^3 + 15*
B*c*tan(1/2*f*x + 1/2*e)^3 + 15*A*d*tan(1/2*f*x + 1/2*e)^3 + 40*A*c*tan(1/
2*f*x + 1/2*e)^2 + 15*B*c*tan(1/2*f*x + 1/2*e)^2 + 15*A*d*tan(1/2*f*x + 1/
2*e)^2 + 20*B*d*tan(1/2*f*x + 1/2*e)^2 + 20*A*c*tan(1/2*f*x + 1/2*e) + 15*
B*c*tan(1/2*f*x + 1/2*e) + 15*A*d*tan(1/2*f*x + 1/2*e) + 10*B*d*tan(1/2*f*
x + 1/2*e) + 7*A*c + 3*B*c + 3*A*d + 2*B*d)/(a^3*f*(tan(1/2*f*x + 1/2*e) +
 1)^5)
```

Mupad [B] (verification not implemented)

Time = 37.42 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.93

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx$$
$$= \frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{53Ac}{4}+3Ad+3Bc+\frac{13Bd}{4}-4Ac\cos(e+fx)+\frac{3Ad\cos(e+fx)}{2}+\frac{3Bc\cos(e+fx)}{2}+Bd\right.}{}$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^3,x)
```

output
```
(2*cos(e/2 + (f*x)/2)*((53*A*c)/4 + 3*A*d + 3*B*c + (13*B*d)/4 - 4*A*c*cos
(e + f*x) + (3*A*d*cos(e + f*x))/2 + (3*B*c*cos(e + f*x))/2 + B*d*cos(e +
f*x) + (25*A*c*sin(e + f*x))/2 + (15*A*d*sin(e + f*x))/2 + (15*B*c*sin(e +
 f*x))/2 + (5*B*d*sin(e + f*x))/2 - (9*A*c*cos(2*e + 2*f*x))/4 - (3*A*d*co
s(2*e + 2*f*x))/2 - (3*B*c*cos(2*e + 2*f*x))/2 - (9*B*d*cos(2*e + 2*f*x))/
4 - (5*A*c*sin(2*e + 2*f*x))/4 + (5*B*d*sin(2*e + 2*f*x))/4))/(15*a^3*f*((
5*2^(1/2)*cos((3*e)/2 + pi/4 + (3*f*x)/2))/4 - (5*2^(1/2)*cos(e/2 - pi/4 +
 (f*x)/2))/2 + (2^(1/2)*cos((5*e)/2 - pi/4 + (5*f*x)/2))/4))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.94

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^3}\,dx$$
$$= \frac{\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5 ac}{5} - 2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3 ad - 2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3 bc - \frac{4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2 ac}{3} - 2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2 ad - 2\tan\left(\frac{f}{2}\right.}{a^3 f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5 + 5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4 + 1\right.}$$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^3,x)
```

output
```
(2*(3*tan((e + f*x)/2)**5*a*c - 15*tan((e + f*x)/2)**3*a*d - 15*tan((e + f
*x)/2)**3*b*c - 10*tan((e + f*x)/2)**2*a*c - 15*tan((e + f*x)/2)**2*a*d -
15*tan((e + f*x)/2)**2*b*c - 20*tan((e + f*x)/2)**2*b*d - 5*tan((e + f*x)/
2)*a*c - 15*tan((e + f*x)/2)*a*d - 15*tan((e + f*x)/2)*b*c - 10*tan((e + f
*x)/2)*b*d - 4*a*c - 3*a*d - 3*b*c - 2*b*d))/(15*a**3*f*(tan((e + f*x)/2)*
*5 + 5*tan((e + f*x)/2)**4 + 10*tan((e + f*x)/2)**3 + 10*tan((e + f*x)/2)*
*2 + 5*tan((e + f*x)/2) + 1))
```

3.282 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 102

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3}\,dx = -\frac{(A-B)\cos(e+fx)}{5f(a+a\sin(e+fx))^3}-\frac{(2A+3B)\cos(e+fx)}{15af(a+a\sin(e+fx))^2} -\frac{(2A+3B)\cos(e+fx)}{15f\left(a^3+a^3\sin(e+fx)\right)}$$

output
```
-1/5*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^3-1/15*(2*A+3*B)*cos(f*x+e)/a/f/(
a+a*sin(f*x+e))^2-1/15*(2*A+3*B)*cos(f*x+e)/f/(a^3+a^3*sin(f*x+e))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.62

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3}\,dx = -\frac{\cos(e+fx)\left(7A+3B+(6A+9B)\sin(e+fx)+(2A+3B)\sin^2(e+fx)\right)}{15a^3f(1+\sin(e+fx))^3}$$

input
```
Integrate[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^3,x]
```

output
```
-1/15*(Cos[e + f*x]*(7*A + 3*B + (6*A + 9*B)*Sin[e + f*x] + (2*A + 3*B)*Si
n[e + f*x]^2))/(a^3*f*(1 + Sin[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 3229, 3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^3}dx$$

$$\Big\downarrow \text{3229}$$

$$\frac{(2A+3B)\int \frac{1}{(\sin(e+fx)a+a)^2}dx}{5a}-\frac{(A-B)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(2A+3B)\int \frac{1}{(\sin(e+fx)a+a)^2}dx}{5a}-\frac{(A-B)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

$$\Big\downarrow \text{3129}$$

$$\frac{(2A+3B)\left(\frac{\int \frac{1}{\sin(e+fx)a+a}dx}{3a}-\frac{\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}\right)}{5a}-\frac{(A-B)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(2A+3B)\left(\frac{\int \frac{1}{\sin(e+fx)a+a}dx}{3a}-\frac{\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}\right)}{5a}-\frac{(A-B)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

$$\Big\downarrow \text{3127}$$

$$\frac{(2A+3B)\left(-\frac{\cos(e+fx)}{3af(a\sin(e+fx)+a)}-\frac{\cos(e+fx)}{3f(a\sin(e+fx)+a)^2}\right)}{5a}-\frac{(A-B)\cos(e+fx)}{5f(a\sin(e+fx)+a)^3}$$

input
```
Int[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^3,x]
```

output
```
-1/5*((A - B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^3) + ((2*A + 3*B)*(-1/
3*Cos[e + f*x]/(f*(a + a*Sin[e + f*x])^2) - Cos[e + f*x]/(3*a*f*(a + a*Sin
[e + f*x]))))/(5*a)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.93

method	result
risch	$-\frac{2 i \left(20 i A \,{\mathrm e}^{2 i \left(f x +e \right)}+15 i B \,{\mathrm e}^{2 i \left(f x +e \right)}+15 B \,{\mathrm e}^{3 i \left(f x +e \right)}-2 i A -10 A \,{\mathrm e}^{i \left(f x +e \right)}-3 i B -15 B \,{\mathrm e}^{i \left(f x +e \right)}\right)}{15 f \,a^{3} \left({\mathrm e}^{i \left(f x +e \right)}+i \right)^{5}}$
parallelrisch	$\frac{-30 A \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{4}+\left(-60 A -30 B \right) \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}+\left(-80 A -30 B \right) \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}+\left(-40 A -30 B \right) \tan\left(\frac{f x}{2}+\frac{e}{2}\right)-14 A -}{15 f \,a^{3} \left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{5}}$
derivativedivides	$\frac{-\frac{2 \left(4 A -4 B \right)}{5 \left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{-8 A +8 B}{2 \left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{4}}-\frac{-4 A +2 B}{\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2 A}{\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+1}-\frac{2 \left(8 A -6 B \right)}{3 \left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{3}}}{a^{3} f}$
default	$\frac{-\frac{2 \left(4 A -4 B \right)}{5 \left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{-8 A +8 B}{2 \left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{4}}-\frac{-4 A +2 B}{\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{2}}-\frac{2 A}{\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+1}-\frac{2 \left(8 A -6 B \right)}{3 \left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{3}}}{a^{3} f}$
norman	$\frac{-\frac{14 A +6 B}{15 a f}-\frac{2 A \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{6}}{a f}-\frac{\left(94 A +36 B \right) \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{15 a f}-\frac{\left(20 A +12 B \right) \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}{3 a f}-\frac{\left(22 A +6 B \right) \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{4}}{3 a f}-\frac{\left(8 A +6 B \right) \mathrm{ta}}{3 a}}{\left(1+\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}\right) a^{2} \left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)^{5}}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
-2/15*I*(20*I*A*exp(2*I*(f*x+e))+15*I*B*exp(2*I*(f*x+e))+15*B*exp(3*I*(f*x
+e))-2*I*A-10*A*exp(I*(f*x+e))-3*I*B-15*B*exp(I*(f*x+e)))/f/a^3/(exp(I*(f*
x+e))+I)^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.86

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3}\,dx =$$
$$-\frac{(2\,A+3\,B)\cos\left(fx+e\right)^{3}-2\,(2\,A+3\,B)\cos\left(fx+e\right)^{2}-3\,(3\,A+2\,B)\cos\left(fx+e\right)-\left((2\,A+3\,B)\,\mathrm{co}\right.}{15\,\left(a^{3}f\cos\left(fx+e\right)^{3}+3\,a^{3}f\cos\left(fx+e\right)^{2}-2\,a^{3}f\cos\left(fx+e\right)-4\,a^{3}f+\left(a^{3}f\right.\right.}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

output
```
-1/15*((2*A + 3*B)*cos(f*x + e)^3 - 2*(2*A + 3*B)*cos(f*x + e)^2 - 3*(3*A
+ 2*B)*cos(f*x + e) - ((2*A + 3*B)*cos(f*x + e)^2 + 3*(2*A + 3*B)*cos(f*x
+ e) - 3*A + 3*B)*sin(f*x + e) - 3*A + 3*B)/(a^3*f*cos(f*x + e)^3 + 3*a^3*
f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2
- 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1015 vs. $2(87) = 174$.

Time = 2.72 (sec) , antiderivative size = 1015, normalized size of antiderivative = 9.95

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3,x)
```

output
```
Piecewise((-30*A*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a
**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*ta
n(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*A*tan(e/2
 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**
4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a
**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 80*A*tan(e/2 + f*x/2)**2/(15*a**3*f*
tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 +
 f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) +
 15*a**3*f) - 40*A*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a*
*3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan
(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 14*A/(15*a**3
*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/
2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2
) + 15*a**3*f) - 30*B*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 +
 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3
*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*B*ta
n(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x
/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 +
 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*B*tan(e/2 + f*x/2)/(15*a**3*
f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(96) = 192$.

Time = 0.04 (sec) , antiderivative size = 387, normalized size of antiderivative = 3.79

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3}\,dx = -\frac{2\left(\frac{A\left(\frac{20\sin(fx+e)}{\cos(fx+e)+1} + \frac{40\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{30\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4} + 7\right)}{a^3 + \frac{5a^3\sin(fx+e)}{\cos(fx+e)+1} + \frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}} + \frac{3B\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1} + \frac{5\sin(\ldots}{(\cos(fx\ldots}\right.}{a^3 + \frac{5a^3\sin(fx+e)}{\cos(fx+e)+1} + \frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{10a^3\ldots}{(\cos(\ldots}}\right.}{15f}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

output
```
-2/15*(A*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10
*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)
^5/(cos(f*x + e) + 1)^5) + 3*B*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*si
n(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^
5))/f
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.20

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3}\,dx = -\frac{2\left(15A\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 + 30A\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + 15B\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + 40A\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 \ldots\right.}{15a^3f\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}\ldots\right.\right.}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

output
```
-2/15*(15*A*tan(1/2*f*x + 1/2*e)^4 + 30*A*tan(1/2*f*x + 1/2*e)^3 + 15*B*ta
n(1/2*f*x + 1/2*e)^3 + 40*A*tan(1/2*f*x + 1/2*e)^2 + 15*B*tan(1/2*f*x + 1/
2*e)^2 + 20*A*tan(1/2*f*x + 1/2*e) + 15*B*tan(1/2*f*x + 1/2*e) + 7*A + 3*B
)/(a^3*f*(tan(1/2*f*x + 1/2*e) + 1)^5)
```

Mupad [B] (verification not implemented)

Time = 36.80 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.47

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3}\,dx = \frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{53A}{4}+3B-4A\cos(e+fx)+\frac{3B\cos(e+fx)}{2}+\frac{25A\sin(e+fx)}{2}+\frac{15B\sin(e+fx)}{2}-\frac{9A\cos(2e+\cdots}{4}\right.}{15a^3f\left(\frac{5\sqrt{2}\cos\left(\frac{3e}{2}+\frac{\pi}{4}+\frac{3fx}{2}\right)}{4}-\frac{5\sqrt{2}\cos\left(\frac{e}{2}-\frac{\pi}{4}+\frac{fx}{2}\right)}{2}+\frac{\sqrt{2}\cos\left(\frac{5e}{2}-\frac{\pi}{4}+\frac{5fx}{2}\right)}{4}\right.}$$

input
```
int((A + B*sin(e + f*x))/(a + a*sin(e + f*x))^3,x)
```

output
```
(2*cos(e/2 + (f*x)/2)*((53*A)/4 + 3*B - 4*A*cos(e + f*x) + (3*B*cos(e + f*
x))/2 + (25*A*sin(e + f*x))/2 + (15*B*sin(e + f*x))/2 - (9*A*cos(2*e + 2*f
*x))/4 - (3*B*cos(2*e + 2*f*x))/2 - (5*A*sin(2*e + 2*f*x))/4))/(15*a^3*f*(
(5*2^(1/2)*cos((3*e)/2 + pi/4 + (3*f*x)/2))/4 - (5*2^(1/2)*cos(e/2 - pi/4
+ (f*x)/2))/2 + (2^(1/2)*cos((5*e)/2 - pi/4 + (5*f*x)/2))/4))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.57

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3}\,dx = \frac{\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5 a}{5}-2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3 b-\frac{4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2 a}{3}-2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2 b-\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)a}{3}-2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)b-\cdots}{a^3f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5+5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+10\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3+10\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2+5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\cdots\right.}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x)
```

output
```
(2*(3*tan((e + f*x)/2)**5*a - 15*tan((e + f*x)/2)**3*b - 10*tan((e + f*x)/
2)**2*a - 15*tan((e + f*x)/2)**2*b - 5*tan((e + f*x)/2)*a - 15*tan((e + f*
x)/2)*b - 4*a - 3*b))/(15*a**3*f*(tan((e + f*x)/2)**5 + 5*tan((e + f*x)/2)
**4 + 10*tan((e + f*x)/2)**3 + 10*tan((e + f*x)/2)**2 + 5*tan((e + f*x)/2)
 + 1))
```

3.283 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 229

$$\begin{aligned}
&\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))}\,dx \\
&= \frac{2d^2(Bc-Ad)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^3(c-d)^3\sqrt{c^2-d^2}f} - \frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3} \\
&\quad - \frac{(2Ac+3Bc-7Ad+2Bd)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2} \\
&\quad - \frac{(B(3c^2-16cd-2d^2)+A(2c^2-9cd+22d^2))\cos(e+fx)}{15(c-d)^3f\,(a^3+a^3\sin(e+fx))}
\end{aligned}$$

output
```
2*d^2*(-A*d+B*c)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a^3/(c-d
)^3/(c^2-d^2)^(1/2)/f-1/5*(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^3-1/15
*(2*A*c-7*A*d+3*B*c+2*B*d)*cos(f*x+e)/a/(c-d)^2/f/(a+a*sin(f*x+e))^2-1/15*
(B*(3*c^2-16*c*d-2*d^2)+A*(2*c^2-9*c*d+22*d^2))*cos(f*x+e)/(c-d)^3/f/(a^3+
a^3*sin(f*x+e))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 502 vs. $2(229) = 458$.

Time = 7.62 (sec) , antiderivative size = 502, normalized size of antiderivative = 2.19

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c + d\sin(e + fx))}\,dx$$

$$= \frac{\left(\cos\left(\frac{1}{2}(e + fx)\right) + \sin\left(\frac{1}{2}(e + fx)\right)\right)\left(15Bc^2\cos\left(\frac{1}{2}(e + fx)\right) - 15Acd\cos\left(\frac{1}{2}(e + fx)\right) - 75Bcd\cos\left(\frac{1}{2}(\right.\right.}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x]
)),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(15*B*c^2*Cos[(e + f*x)/2] - 15*A*c
*d*Cos[(e + f*x)/2] - 75*B*c*d*Cos[(e + f*x)/2] + 75*A*d^2*Cos[(e + f*x)/2
] - 10*A*c^2*Cos[(3*(e + f*x))/2] - 15*B*c^2*Cos[(3*(e + f*x))/2] + 45*A*c
*d*Cos[(3*(e + f*x))/2] + 65*B*c*d*Cos[(3*(e + f*x))/2] - 95*A*d^2*Cos[(3*
(e + f*x))/2] + 10*B*d^2*Cos[(3*(e + f*x))/2] + 20*A*c^2*Sin[(e + f*x)/2]
+ 15*B*c^2*Sin[(e + f*x)/2] - 75*A*c*d*Sin[(e + f*x)/2] - 85*B*c*d*Sin[(e
+ f*x)/2] + 145*A*d^2*Sin[(e + f*x)/2] - 20*B*d^2*Sin[(e + f*x)/2] - (60*d
^2*(-(B*c) + A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^5)/Sqrt[c^2 - d^2] - 15*B*c*d*Sin[(3*(e + f
*x))/2] + 15*A*d^2*Sin[(3*(e + f*x))/2] - 2*A*c^2*Sin[(5*(e + f*x))/2] - 3
*B*c^2*Sin[(5*(e + f*x))/2] + 9*A*c*d*Sin[(5*(e + f*x))/2] + 16*B*c*d*Sin[
(5*(e + f*x))/2] - 22*A*d^2*Sin[(5*(e + f*x))/2] + 2*B*d^2*Sin[(5*(e + f*x
))/2]))/(30*a^3*(c - d)^3*f*(1 + Sin[e + f*x])^3)
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.13, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 3457, 25, 3042, 3457, 25, 3042, 3457, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c+d\sin(e+fx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c+d\sin(e+fx))}dx$$

$$\Big\downarrow \text{3457}$$

$$-\frac{\int -\frac{a(2Ac+3Bc-5Ad)+2a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)^2(c+d\sin(e+fx))}dx}{5a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{a(2Ac+3Bc-5Ad)+2a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)^2(c+d\sin(e+fx))}dx}{5a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(2Ac+3Bc-5Ad)+2a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)^2(c+d\sin(e+fx))}dx}{5a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3}$$

$$\Big\downarrow \text{3457}$$

$$\frac{-\frac{\int -\frac{\left(Bc(3c-13d)+A\left(2c^2-7dc+15d^2\right)\right)a^2+d(2Ac+3Bc-7Ad+2Bd)\sin(e+fx)a^2}{(\sin(e+fx)a+a)(c+d\sin(e+fx))}dx}{3a^2(c-d)}-\frac{a(2Ac-7Ad+3Bc+2Bd)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2}}{5a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{\left(Bc(3c-13d)+A\left(2c^2-7dc+15d^2\right)\right)a^2+d(2Ac+3Bc-7Ad+2Bd)\sin(e+fx)a^2}{(\sin(e+fx)a+a)(c+d\sin(e+fx))}dx}{3a^2(c-d)}-\frac{a(2Ac-7Ad+3Bc+2Bd)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2}}{5a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{\left(Bc(3c-13d)+A\left(2c^2-7dc+15d^2\right)\right)a^2+d(2Ac+3Bc-7Ad+2Bd)\sin(e+fx)a^2}{(\sin(e+fx)a+a)(c+d\sin(e+fx))}dx}{3a^2(c-d)}-\frac{a(2Ac-7Ad+3Bc+2Bd)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2}}{5a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{-\frac{\int -\frac{15a^3d^2(Bc-Ad)}{c+d\sin(e+fx)}dx}{a^2(c-d)}-\frac{a^2\left(A\left(2c^2-9cd+22d^2\right)+B\left(3c^2-16cd-2d^2\right)\right)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)}}{3a^2(c-d)}-\frac{a(2Ac-7Ad+3Bc+2Bd)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2}}{5a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3}$$

$\downarrow$ 27

$$\frac{\frac{\frac{15ad^2(Bc-Ad)\int\frac{1}{c+d\sin(e+fx)}dx}{c-d}-\frac{a^2\left(A\left(2c^2-9cd+22d^2\right)+B\left(3c^2-16cd-2d^2\right)\right)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)}}{3a^2(c-d)}-\frac{a(2Ac-7Ad+3Bc+2Bd)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2}}{5a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{15ad^2(Bc-Ad)\int\frac{1}{c+d\sin(e+fx)}dx}{c-d}-\frac{a^2\left(A\left(2c^2-9cd+22d^2\right)+B\left(3c^2-16cd-2d^2\right)\right)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)}}{3a^2(c-d)}-\frac{a(2Ac-7Ad+3Bc+2Bd)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2}}{5a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3}$$

$\downarrow$ 3139

$$\frac{\frac{\frac{30ad^2(Bc-Ad)\int\frac{1}{c\tan^2\left(\frac{1}{2}(e+fx)\right)+2d\tan\left(\frac{1}{2}(e+fx)\right)+c}d\tan\left(\frac{1}{2}(e+fx)\right)}{f(c-d)}-\frac{a^2\left(A\left(2c^2-9cd+22d^2\right)+B\left(3c^2-16cd-2d^2\right)\right)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)}}{3a^2(c-d)}-\frac{a(2Ac-7Ad+3Bc-\cdots}{3f(c-d)(a\sin\cdots}}{5a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3}$$

$\downarrow$ 1083

$$\frac{\frac{-\frac{60ad^2(Bc-Ad)\int\frac{1}{-\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)^2-4\left(c^2-d^2\right)}d\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{f(c-d)}-\frac{a^2\left(A\left(2c^2-9cd+22d^2\right)+B\left(3c^2-16cd-2d^2\right)\right)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)}}{3a^2(c-d)}-\frac{a(2Ac-7\cdots}{3f(c\cdots}}{5a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3}$$

$\downarrow$ 217

$$\frac{\frac{\frac{30ad^2(Bc-Ad)\arctan\left(\frac{2c\tan\left(\frac{1}{2}(e+fx)\right)+2d}{2\sqrt{c^2-d^2}}\right)}{f(c-d)\sqrt{c^2-d^2}}-\frac{a^2\left(A\left(2c^2-9cd+22d^2\right)+B\left(3c^2-16cd-2d^2\right)\right)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)}}{3a^2(c-d)}-\frac{a(2Ac-7Ad+3Bc+2Bd)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2}}{5a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])),x]
```

output
```
-1/5*((A - B)*Cos[e + f*x])/((c - d)*f*(a + a*Sin[e + f*x])^3) + (-1/3*(a*
(2*A*c + 3*B*c - 7*A*d + 2*B*d)*Cos[e + f*x])/((c - d)*f*(a + a*Sin[e + f*
x])^2) + ((30*a*d^2*(B*c - A*d)*ArcTan[(2*d + 2*c*Tan[(e + f*x)/2])/(2*Sqr
t[c^2 - d^2])])/((c - d)*Sqrt[c^2 - d^2]*f) - (a^2*(B*(3*c^2 - 16*c*d - 2*
d^2) + A*(2*c^2 - 9*c*d + 22*d^2))*Cos[e + f*x])/((c - d)*f*(a + a*Sin[e +
 f*x])))/(3*a^2*(c - d)))/(5*a^2*(c - d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{-\frac{2d^2(Ad-Bc)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c-d)^3\sqrt{c^2-d^2}}-\frac{-8A+8B}{2(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{2(4A-4B)}{5(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{-4Ac+6Ad+2Bc-4B}{(c-d)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\right.}}{a^3f}$
default	$\frac{-\frac{2d^2(Ad-Bc)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c-d)^3\sqrt{c^2-d^2}}-\frac{-8A+8B}{2(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}-\frac{2(4A-4B)}{5(c-d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}-\frac{-4Ac+6Ad+2Bc-4B}{(c-d)^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\right.}}{a^3f}$
risch	$-\frac{4A\,c^2}{15}-\frac{2B\,c^2}{5}-\frac{44A\,d^2}{15}+\frac{6Acd}{5}+\frac{4B\,d^2}{15}+\frac{32Bcd}{15}-10Acd\,e^{2i(fx+e)}-\frac{34Bcd\,e^{2i(fx+e)}}{3}+2Bcd\,e^{4i(fx+e)}+2iB\,c^2e^{i(fx+e)}-$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x,method=_RETURNV
ERBOSE)
```

output
```
2/f/a^3*(-d^2*(A*d-B*c)/(c-d)^3/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*
x+1/2*e)+2*d)/(c^2-d^2)^(1/2))-1/4*(-8*A+8*B)/(c-d)/(tan(1/2*f*x+1/2*e)+1)
^4-1/5*(4*A-4*B)/(c-d)/(tan(1/2*f*x+1/2*e)+1)^5-1/2*(-4*A*c+6*A*d+2*B*c-4*
B*d)/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(8*A*c-10*A*d-6*B*c+8*B*d)/(c-d)
^2/(tan(1/2*f*x+1/2*e)+1)^3-(A*c^2-3*A*c*d+3*A*d^2-B*d^2)/(c-d)^3/(tan(1/2
*f*x+1/2*e)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1104 vs. $2(218) = 436$.

Time = 0.15 (sec) , antiderivative size = 2292, normalized size of antiderivative = 10.01

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c + d\sin(e + fx))}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
[1/30*(6*(A - B)*c^4 - 12*(A - B)*c^3*d + 12*(A - B)*c*d^3 - 6*(A - B)*d^4
 - 2*((2*A + 3*B)*c^4 - (9*A + 16*B)*c^3*d + 5*(4*A - B)*c^2*d^2 + (9*A +
16*B)*c*d^3 - 2*(11*A - B)*d^4)*cos(f*x + e)^3 + 2*(2*(2*A + 3*B)*c^4 - (1
8*A + 17*B)*c^3*d + 5*(5*A - 2*B)*c^2*d^2 + (18*A + 17*B)*c*d^3 - (29*A -
4*B)*d^4)*cos(f*x + e)^2 + 15*(4*B*c*d^2 - 4*A*d^3 - (B*c*d^2 - A*d^3)*cos
(f*x + e)^3 - 3*(B*c*d^2 - A*d^3)*cos(f*x + e)^2 + 2*(B*c*d^2 - A*d^3)*cos
(f*x + e) + (4*B*c*d^2 - 4*A*d^3 - (B*c*d^2 - A*d^3)*cos(f*x + e)^2 + 2*(B
*c*d^2 - A*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 -
 d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*
sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c
*d*sin(f*x + e) - c^2 - d^2)) + 6*((3*A + 2*B)*c^4 - (11*A + 9*B)*c^3*d +
5*(3*A - B)*c^2*d^2 + (11*A + 9*B)*c*d^3 - 3*(6*A - B)*d^4)*cos(f*x + e) -
 2*(3*(A - B)*c^4 - 6*(A - B)*c^3*d + 6*(A - B)*c*d^3 - 3*(A - B)*d^4 - ((
2*A + 3*B)*c^4 - (9*A + 16*B)*c^3*d + 5*(4*A - B)*c^2*d^2 + (9*A + 16*B)*c
*d^3 - 2*(11*A - B)*d^4)*cos(f*x + e)^2 - 3*((2*A + 3*B)*c^4 - (9*A + 11*B
)*c^3*d + 5*(3*A - B)*c^2*d^2 + (9*A + 11*B)*c*d^3 - (17*A - 2*B)*d^4)*cos
(f*x + e))*sin(f*x + e))/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c
^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^3 + 3*(a^3*c^5 - 3*a^3*c^4*
d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^
2 - 2*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c + d\sin(e + fx))}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c+d*sin(f*x+e)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 553 vs. 2(218) = 436.

Time = 0.28 (sec) , antiderivative size = 553, normalized size of antiderivative = 2.41

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorith
m="giac")
```

output
```
2/15*(15*(B*c*d^2 - A*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arct
an((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a^3*c^3 - 3*a^3*c^2*d
+ 3*a^3*c*d^2 - a^3*d^3)*sqrt(c^2 - d^2)) - (15*A*c^2*tan(1/2*f*x + 1/2*e)
^4 - 45*A*c*d*tan(1/2*f*x + 1/2*e)^4 + 45*A*d^2*tan(1/2*f*x + 1/2*e)^4 - 1
5*B*d^2*tan(1/2*f*x + 1/2*e)^4 + 30*A*c^2*tan(1/2*f*x + 1/2*e)^3 + 15*B*c^
2*tan(1/2*f*x + 1/2*e)^3 - 105*A*c*d*tan(1/2*f*x + 1/2*e)^3 - 45*B*c*d*tan
(1/2*f*x + 1/2*e)^3 + 135*A*d^2*tan(1/2*f*x + 1/2*e)^3 - 30*B*d^2*tan(1/2*
f*x + 1/2*e)^3 + 40*A*c^2*tan(1/2*f*x + 1/2*e)^2 + 15*B*c^2*tan(1/2*f*x +
1/2*e)^2 - 135*A*c*d*tan(1/2*f*x + 1/2*e)^2 - 65*B*c*d*tan(1/2*f*x + 1/2*e
)^2 + 185*A*d^2*tan(1/2*f*x + 1/2*e)^2 - 40*B*d^2*tan(1/2*f*x + 1/2*e)^2 +
 20*A*c^2*tan(1/2*f*x + 1/2*e) + 15*B*c^2*tan(1/2*f*x + 1/2*e) - 75*A*c*d*
tan(1/2*f*x + 1/2*e) - 55*B*c*d*tan(1/2*f*x + 1/2*e) + 115*A*d^2*tan(1/2*f
*x + 1/2*e) - 20*B*d^2*tan(1/2*f*x + 1/2*e) + 7*A*c^2 + 3*B*c^2 - 24*A*c*d
 - 11*B*c*d + 32*A*d^2 - 7*B*d^2)/((a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 -
a^3*d^3)*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

Mupad [B] (verification not implemented)

Time = 39.95 (sec) , antiderivative size = 591, normalized size of antiderivative = 2.58

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))}\,dx$$

$$= \frac{2\,d^2\,\mathrm{atan}\left(\frac{\frac{d^2\,(A\,d-B\,c)\,\left(-2\,a^3\,c^3\,d+6\,a^3\,c^2\,d^2-6\,a^3\,c\,d^3+2\,a^3\,d^4\right)}{a^3\,\sqrt{c+d}\,(c-d)^{7/2}}-\frac{2\,c\,d^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(A\,d-B\,c)\,\left(a^3\,c^3-3\,a^3\,c^2\,d+3\,a^3\,c\,d^2-a^3\,d^3\right)}{a^3\,\sqrt{c+d}\,(c-d)^{7/2}}}{2\,A\,d^3-2\,B\,c\,d^2}\right)\,(A\,d-}{a^3\,f\,\sqrt{c+d}\,(c-d)^{7/2}}$$

$$-\frac{\frac{2\,(7\,A\,c^2+32\,A\,d^2+3\,B\,c^2-7\,B\,d^2-24\,A\,c\,d-11\,B\,c\,d)}{15\,(c-d)\,(c^2-2\,c\,d+d^2)}+\frac{2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(4\,A\,c^2+23\,A\,d^2+3\,B\,c^2-4\,B\,d^2-15\,A\,c\,d-11\,B\,c\,d)}{3\,(c-d)\,(c^2-2\,c\,d+d^2)}+\frac{2\,\tan\left(\frac{e}{2}+\right.}{}}{f\,\left(a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+5\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+\right.}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))),x)
```

output
```
(2*d^2*atan(((d^2*(A*d - B*c)*(2*a^3*d^4 - 6*a^3*c*d^3 - 2*a^3*c^3*d + 6*a
^3*c^2*d^2))/(a^3*(c + d)^(1/2)*(c - d)^(7/2)) - (2*c*d^2*tan(e/2 + (f*x)/
2)*(A*d - B*c)*(a^3*c^3 - a^3*d^3 + 3*a^3*c*d^2 - 3*a^3*c^2*d))/(a^3*(c +
d)^(1/2)*(c - d)^(7/2)))/(2*A*d^3 - 2*B*c*d^2))*(A*d - B*c))/(a^3*f*(c + d
)^(1/2)*(c - d)^(7/2)) - ((2*(7*A*c^2 + 32*A*d^2 + 3*B*c^2 - 7*B*d^2 - 24*
A*c*d - 11*B*c*d))/(15*(c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2
)*(4*A*c^2 + 23*A*d^2 + 3*B*c^2 - 4*B*d^2 - 15*A*c*d - 11*B*c*d))/(3*(c -
d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2)^3*(2*A*c^2 + 9*A*d^2 + B*c
^2 - 2*B*d^2 - 7*A*c*d - 3*B*c*d))/((c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(
e/2 + (f*x)/2)^2*(8*A*c^2 + 37*A*d^2 + 3*B*c^2 - 8*B*d^2 - 27*A*c*d - 13*B
*c*d))/(3*(c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2)^4*(A*c^2 +
3*A*d^2 - B*d^2 - 3*A*c*d))/((c - d)*(c^2 - 2*c*d + d^2)))/(f*(10*a^3*tan(
e/2 + (f*x)/2)^2 + 10*a^3*tan(e/2 + (f*x)/2)^3 + 5*a^3*tan(e/2 + (f*x)/2)^
4 + a^3*tan(e/2 + (f*x)/2)^5 + a^3 + 5*a^3*tan(e/2 + (f*x)/2)))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1863, normalized size of antiderivative = 8.14

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))}\,dx = \text{Too large to display}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x)
```

output
```
(2*( - 15*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2
))*tan((e + f*x)/2)**5*a*d**3 + 15*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2
)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)**5*b*c*d**2 - 75*sqrt(c**2 -
d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)**4
*a*d**3 + 75*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d
**2))*tan((e + f*x)/2)**4*b*c*d**2 - 150*sqrt(c**2 - d**2)*atan((tan((e +
f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)**3*a*d**3 + 150*sqrt(c*
*2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/
2)**3*b*c*d**2 - 150*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(
c**2 - d**2))*tan((e + f*x)/2)**2*a*d**3 + 150*sqrt(c**2 - d**2)*atan((tan
((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)**2*b*c*d**2 - 75*
sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e
+ f*x)/2)*a*d**3 + 75*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt
(c**2 - d**2))*tan((e + f*x)/2)*b*c*d**2 - 15*sqrt(c**2 - d**2)*atan((tan(
(e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*a*d**3 + 15*sqrt(c**2 - d**2)*atan(
(tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*b*c*d**2 + 3*tan((e + f*x)/2)*
*5*a*c**4 - 9*tan((e + f*x)/2)**5*a*c**3*d + 6*tan((e + f*x)/2)**5*a*c**2*
d**2 + 9*tan((e + f*x)/2)**5*a*c*d**3 - 9*tan((e + f*x)/2)**5*a*d**4 - 3*t
an((e + f*x)/2)**5*b*c**2*d**2 + 3*tan((e + f*x)/2)**5*b*d**4 + 15*tan((e
+ f*x)/2)**3*a*c**3*d - 45*tan((e + f*x)/2)**3*a*c**2*d**2 - 15*tan((e ...
```

3.284 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 381

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2}\,dx &= -\frac{2d^2(Ad(4c+3d)-B(3c^2+3cd+d^2))\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^3(c-d)^4(c+d)\sqrt{c^2-d^2}\,f}\\ &\quad -\frac{d(B(3c^3-23c^2d-63cd^2-22d^3)+A(2c^3-12c^2d+43cd^2+72d^3))\cos(e+fx)}{15a^3(c-d)^4(c+d)f(c+d\sin(e+fx))}\\ &\quad -\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))}\\ &\quad -\frac{(2Ac+3Bc-9Ad+4Bd)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2(c+d\sin(e+fx))}\\ &\quad -\frac{(B(3c^2-23cd-15d^2)+A(2c^2-12cd+45d^2))\cos(e+fx)}{15(c-d)^3f\,(a^3+a^3\sin(e+fx))\,(c+d\sin(e+fx))}\end{aligned}$$

output
```
-2*d^2*(A*d*(4*c+3*d)-B*(3*c^2+3*c*d+d^2))*arctan((d+c*tan(1/2*f*x+1/2*e))
/(c^2-d^2)^(1/2))/a^3/(c-d)^4/(c+d)/(c^2-d^2)^(1/2)/f-1/15*d*(B*(3*c^3-23*
c^2*d-63*c*d^2-22*d^3)+A*(2*c^3-12*c^2*d+43*c*d^2+72*d^3))*cos(f*x+e)/a^3/
(c-d)^4/(c+d)/f/(c+d*sin(f*x+e))-1/5*(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x
+e))^3/(c+d*sin(f*x+e))-1/15*(2*A*c-9*A*d+3*B*c+4*B*d)*cos(f*x+e)/a/(c-d)^
2/f/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))-1/15*(B*(3*c^2-23*c*d-15*d^2)+A*(2
*c^2-12*c*d+45*d^2))*cos(f*x+e)/(c-d)^3/f/(a^3+a^3*sin(f*x+e))/(c+d*sin(f*
x+e))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1253 vs. $2(381) = 762$.

Time = 12.52 (sec) , antiderivative size = 1253, normalized size of antiderivative = 3.29

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c + d\sin(e + fx))^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x]
)^2),x]
```

output
```
(2*d^2*(3*B*c^2 - 4*A*c*d + 3*B*c*d - 3*A*d^2 + B*d^2)*ArcTan[(Sec[(e + f*
x)/2]*(d*Cos[(e + f*x)/2] + c*Sin[(e + f*x)/2]))/Sqrt[c^2 - d^2]]*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^6)/((c - d)^4*(c + d)*Sqrt[c^2 - d^2]*f*(a +
 a*Sin[e + f*x])^3) + ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(60*B*c^4*Cos
[(e + f*x)/2] - 80*A*c^3*d*Cos[(e + f*x)/2] - 390*B*c^3*d*Cos[(e + f*x)/2]
 + 540*A*c^2*d^2*Cos[(e + f*x)/2] - 1090*B*c^2*d^2*Cos[(e + f*x)/2] + 1430
*A*c*d^3*Cos[(e + f*x)/2] - 885*B*c*d^3*Cos[(e + f*x)/2] + 735*A*d^4*Cos[(
e + f*x)/2] - 320*B*d^4*Cos[(e + f*x)/2] - 40*A*c^4*Cos[(3*(e + f*x))/2] -
 60*B*c^4*Cos[(3*(e + f*x))/2] + 196*A*c^3*d*Cos[(3*(e + f*x))/2] + 304*B*
c^3*d*Cos[(3*(e + f*x))/2] - 476*A*c^2*d^2*Cos[(3*(e + f*x))/2] + 1076*B*c
^2*d^2*Cos[(3*(e + f*x))/2] - 1546*A*c*d^3*Cos[(3*(e + f*x))/2] + 1181*B*c
*d^3*Cos[(3*(e + f*x))/2] - 969*A*d^4*Cos[(3*(e + f*x))/2] + 334*B*d^4*Cos
[(3*(e + f*x))/2] + 60*B*c^2*d^2*Cos[(5*(e + f*x))/2] - 90*A*c*d^3*Cos[(5*
(e + f*x))/2] + 15*B*c*d^3*Cos[(5*(e + f*x))/2] - 15*A*d^4*Cos[(5*(e + f*x
))/2] + 30*B*d^4*Cos[(5*(e + f*x))/2] + 4*A*c^3*d*Cos[(7*(e + f*x))/2] + 6
*B*c^3*d*Cos[(7*(e + f*x))/2] - 24*A*c^2*d^2*Cos[(7*(e + f*x))/2] - 46*B*c
^2*d^2*Cos[(7*(e + f*x))/2] + 86*A*c*d^3*Cos[(7*(e + f*x))/2] - 111*B*c*d^
3*Cos[(7*(e + f*x))/2] + 129*A*d^4*Cos[(7*(e + f*x))/2] - 44*B*d^4*Cos[(7*
(e + f*x))/2] + 80*A*c^4*Sin[(e + f*x)/2] + 60*B*c^4*Sin[(e + f*x)/2] - 34
0*A*c^3*d*Sin[(e + f*x)/2] - 440*B*c^3*d*Sin[(e + f*x)/2] + 820*A*c^2*d...
```

Rubi [A] (verified)

Time = 1.80 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.07, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.457$, Rules used = {3042, 3457, 25, 3042, 3457, 25, 3042, 3457, 25, 3042, 3233, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^2}dx$$

$$\Big\downarrow \text{3457}$$

$$-\frac{\int -\frac{a(2A(c-3d)+B(3c+d))+3a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)^2(c+d\sin(e+fx))^2}dx}{5a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))}$$

$\downarrow$ 25

$$\frac{\int \frac{a(2A(c-3d)+B(3c+d))+3a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)^2(c+d\sin(e+fx))^2}dx}{5a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))}$$

$\downarrow$ 3042

$$\frac{\int \frac{a(2A(c-3d)+B(3c+d))+3a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)^2(c+d\sin(e+fx))^2}dx}{5a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))}$$

$\downarrow$ 3457

$$\frac{-\frac{\int -\frac{\left(B\left(3c^2-17dc-7d^2\right)+A\left(2c^2-8dc+27d^2\right)\right)a^2+2d(2Ac+3Bc-9Ad+4Bd)\sin(e+fx)a^2}{(\sin(e+fx)a+a)(c+d\sin(e+fx))^2}dx}{3a^2(c-d)}-\frac{a(2Ac-9Ad+3Bc+4Bd)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))}}{5a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))}$$

$\downarrow$ 25

$$\frac{\frac{\int \frac{\left(B\left(3c^2-17dc-7d^2\right)+A\left(2c^2-8dc+27d^2\right)\right)a^2+2d(2Ac+3Bc-9Ad+4Bd)\sin(e+fx)a^2}{(\sin(e+fx)a+a)(c+d\sin(e+fx))^2}dx}{3a^2(c-d)}-\frac{a(2Ac-9Ad+3Bc+4Bd)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))}}{5a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{\left(B\left(3c^2-17dc-7d^2\right)+A\left(2c^2-8dc+27d^2\right)\right)a^2+2d(2Ac+3Bc-9Ad+4Bd)\sin(e+fx)a^2}{(\sin(e+fx)a+a)(c+d\sin(e+fx))^2}dx}{3a^2(c-d)}-\frac{a(2Ac-9Ad+3Bc+4Bd)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))}}{5a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))}$$

$\downarrow$ 3457

$$-\frac{\frac{\int -\frac{2d^2(Ac+24Bc-36Ad+11Bd)a^3+d\left(B\left(3c^2-23dc-15d^2\right)+A\left(2c^2-12dc+45d^2\right)\right)\sin(e+fx)a^3}{(c+d\sin(e+fx))^2}dx}{a^2(c-d)}-\frac{a^2\left(A\left(2c^2-12cd+45d^2\right)+B\left(3c^2-23cd-15d^2\right)\right)\cos(e+\cdots}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))}}{3a^2(c-d)}$$

$$5a^2(c-d)$$

$$\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))}$$

$\downarrow$ 25

$$\frac{\frac{\int \frac{2d^2(Ac+24Bc-36Ad+11Bd)a^3+d\left(B\left(3c^2-23dc-15d^2\right)+A\left(2c^2-12dc+45d^2\right)\right)\sin(e+fx)a^3}{(c+d\sin(e+fx))^2}dx}{a^2(c-d)}-\frac{a^2\left(A\left(2c^2-12cd+45d^2\right)+B\left(3c^2-23cd-15d^2\right)\right)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))}}{3a^2(c-d)}$$

$$5a^2(c-d)$$

$$\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{2d^2(Ac+24Bc-36Ad+11Bd)a^3+d\left(B\left(3c^2-23dc-15d^2\right)+A\left(2c^2-12dc+45d^2\right)\right)\sin(e+fx)a^3}{(c+d\sin(e+fx))^2}dx}{a^2(c-d)}-\frac{a^2\left(A\left(2c^2-12cd+45d^2\right)+B\left(3c^2-23cd-15d^2\right)\right)\cos(e+fx)}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))}}{3a^2(c-d)}$$

$$5a^2(c-d)$$

$$\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))}$$

$\downarrow$ 3233

$$\frac{\frac{-\frac{\int \frac{15a^3d^2\left(Ad(4c+3d)-B\left(3c^2+3dc+d^2\right)\right)}{c+d\sin(e+fx)}dx}{c^2-d^2}-\frac{a^3d\left(A\left(2c^3-12c^2d+43cd^2+72d^3\right)+B\left(3c^3-23c^2d-63cd^2-22d^3\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}}{a^2(c-d)}-\frac{a^2\left(A\left(2c^2-12cd+45d^2\right)+B\left(3\cdots\right.\right.}{f(c-d)(a\sin(e+fx)\cdots}}{3a^2(c-d)}$$

$$5a^2(c-d)$$

$$\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))}$$

$\downarrow$ 27

$$\frac{\frac{-\frac{15a^3d^2\left(Ad(4c+3d)-B\left(3c^2+3cd+d^2\right)\right)\int \frac{1}{c+d\sin(e+fx)}dx}{c^2-d^2}-\frac{a^3d\left(A\left(2c^3-12c^2d+43cd^2+72d^3\right)+B\left(3c^3-23c^2d-63cd^2-22d^3\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}}{a^2(c-d)}-\frac{a^2\left(A\left(2c^2-12c\cdots\right.\right.}{f(c-\cdots}}{3a^2(c-d)}$$

$$5a^2(c-d)$$

$$\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))}$$

$\downarrow$ 3042

$$\frac{\frac{-\frac{15a^3d^2\left(Ad(4c+3d)-B\left(3c^2+3cd+d^2\right)\right)\int\frac{1}{c+d\sin(e+fx)}dx}{c^2-d^2}-\frac{a^3d\left(A\left(2c^3-12c^2d+43cd^2+72d^3\right)+B\left(3c^3-23c^2d-63cd^2-22d^3\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}}{a^2(c-d)}-\frac{a^2\left(A\left(2c^2-12c\cdots\right.\right.}{f(c-\cdots}}{3a^2(c-d)}\cdots}{5a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))}$$

$$\Big\downarrow \; 3139$$

$$\frac{\frac{-\frac{30a^3d^2\left(Ad(4c+3d)-B\left(3c^2+3cd+d^2\right)\right)\int\frac{1}{c\tan^2\left(\frac{1}{2}(e+fx)\right)+2d\tan\left(\frac{1}{2}(e+fx)\right)+c}d\tan\left(\frac{1}{2}(e+fx)\right)}{f\left(c^2-d^2\right)}-\frac{a^3d\left(A\left(2c^3-12c^2d+43cd^2+72d^3\right)+B\left(3c^3-23c^2d-63cd\cdots\right.\right.}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}}{a^2(c-d)}\cdots}{3a^2(c-d)}\cdots}{5a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))}$$

$$\Big\downarrow \; 1083$$

$$\frac{\frac{\frac{60a^3d^2\left(Ad(4c+3d)-B\left(3c^2+3cd+d^2\right)\right)\int\frac{1}{-\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)^2-4\left(c^2-d^2\right)}d\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{f\left(c^2-d^2\right)}-\frac{a^3d\left(A\left(2c^3-12c^2d+43cd^2+72d^3\right)+B\left(3c^3-23c^2d\cdots\right.\right.}{f\left(c^2-d^2\right)(c+d\sin(e+fx\cdots}}{a^2(c-d)}\cdots}{3a^2(c-d)}\cdots}{5a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{\frac{-\frac{30a^3d^2\left(Ad(4c+3d)-B\left(3c^2+3cd+d^2\right)\right)\arctan\left(\frac{2c\tan\left(\frac{1}{2}(e+fx)\right)+2d}{2\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{3/2}}-\frac{a^3d\left(A\left(2c^3-12c^2d+43cd^2+72d^3\right)+B\left(3c^3-23c^2d-63cd^2-22d^3\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}}{a^2(c-d)}-\frac{a^2\cdots}{\cdots}}{3a^2(c-d)}\cdots}{5a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))}$$

input

```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^2),x
]
```

output
```
-1/5*((A - B)*Cos[e + f*x])/((c - d)*f*(a + a*Sin[e + f*x])^3*(c + d*Sin[e
 + f*x])) + (-1/3*(a*(2*A*c + 3*B*c - 9*A*d + 4*B*d)*Cos[e + f*x])/((c - d
)*f*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])) + (-((a^2*(B*(3*c^2 - 23*
c*d - 15*d^2) + A*(2*c^2 - 12*c*d + 45*d^2))*Cos[e + f*x])/((c - d)*f*(a +
 a*Sin[e + f*x])*(c + d*Sin[e + f*x]))) + ((-30*a^3*d^2*(A*d*(4*c + 3*d) -
 B*(3*c^2 + 3*c*d + d^2))*ArcTan[(2*d + 2*c*Tan[(e + f*x)/2])/(2*Sqrt[c^2
- d^2])])/((c^2 - d^2)^(3/2)*f) - (a^3*d*(B*(3*c^3 - 23*c^2*d - 63*c*d^2 -
 22*d^3) + A*(2*c^3 - 12*c^2*d + 43*c*d^2 + 72*d^3))*Cos[e + f*x])/((c^2 -
 d^2)*f*(c + d*Sin[e + f*x])))/(a^2*(c - d)))/(3*a^2*(c - d)))/(5*a^2*(c -
 d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Maple [A] (verified)

Time = 2.51 (sec) , antiderivative size = 355, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{-\frac{2d^2\left(\frac{\frac{d^2(Ad-Bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}+\frac{d(Ad-Bc)}{c+d}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}+\frac{\left(4Acd+3A\,d^2-3B\,c^2-3Bcd-B\,d^2\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}}\right)}{(c-d)^4}-\frac{}{2(c-d)^2(}}{}$
default	$\frac{-\frac{2d^2\left(\frac{\frac{d^2(Ad-Bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)c}+\frac{d(Ad-Bc)}{c+d}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}+\frac{\left(4Acd+3A\,d^2-3B\,c^2-3Bcd-B\,d^2\right)\arctan\left(\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}}\right)}{(c-d)^4}-\frac{}{2(c-d)^2(}}{}$
risch	Expression too large to display

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x,method=_RETUR
NVERBOSE)
```

output
```
2/f/a^3*(-d^2/(c-d)^4*((d^2*(A*d-B*c)/(c+d)/c*tan(1/2*f*x+1/2*e)+d*(A*d-B*
c)/(c+d))/(tan(1/2*f*x+1/2*e)^2*c+2*d*tan(1/2*f*x+1/2*e)+c)+(4*A*c*d+3*A*d
^2-3*B*c^2-3*B*c*d-B*d^2)/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*
x+1/2*e)+2*d)/(c^2-d^2)^(1/2)))-1/4*(-8*A+8*B)/(c-d)^2/(tan(1/2*f*x+1/2*e)
+1)^4-1/5*(4*A-4*B)/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^5-1/2*(-4*A*c+8*A*d+2*B
*c-6*B*d)/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(8*A*c-12*A*d-6*B*c+10*B*d)
/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^3-(A*c^2-4*A*c*d+6*A*d^2-3*B*d^2)/(c-d)^4/
(tan(1/2*f*x+1/2*e)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2201 vs. $2(368) = 736$.

Time = 0.23 (sec) , antiderivative size = 4486, normalized size of antiderivative = 11.77

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c + d\sin(e + fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c + d\sin(e + fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c + d\sin(e + fx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algori
thm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 743 vs. $2(368) = 736$.

Time = 0.32 (sec) , antiderivative size = 743, normalized size of antiderivative = 1.95

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c + d\sin(e + fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
2/15*(15*(3*B*c^2*d^2 - 4*A*c*d^3 + 3*B*c*d^3 - 3*A*d^4 + B*d^4)*(pi*floor
(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt
(c^2 - d^2)))/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*
a^3*c*d^4 + a^3*d^5)*sqrt(c^2 - d^2)) + 15*(B*c*d^4*tan(1/2*f*x + 1/2*e) -
 A*d^5*tan(1/2*f*x + 1/2*e) + B*c^2*d^3 - A*c*d^4)/((a^3*c^6 - 3*a^3*c^5*d
 + 2*a^3*c^4*d^2 + 2*a^3*c^3*d^3 - 3*a^3*c^2*d^4 + a^3*c*d^5)*(c*tan(1/2*f
*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)) - (15*A*c^2*tan(1/2*f*x + 1
/2*e)^4 - 60*A*c*d*tan(1/2*f*x + 1/2*e)^4 + 90*A*d^2*tan(1/2*f*x + 1/2*e)^
4 - 45*B*d^2*tan(1/2*f*x + 1/2*e)^4 + 30*A*c^2*tan(1/2*f*x + 1/2*e)^3 + 15
*B*c^2*tan(1/2*f*x + 1/2*e)^3 - 150*A*c*d*tan(1/2*f*x + 1/2*e)^3 - 60*B*c*
d*tan(1/2*f*x + 1/2*e)^3 + 300*A*d^2*tan(1/2*f*x + 1/2*e)^3 - 135*B*d^2*ta
n(1/2*f*x + 1/2*e)^3 + 40*A*c^2*tan(1/2*f*x + 1/2*e)^2 + 15*B*c^2*tan(1/2*
f*x + 1/2*e)^2 - 190*A*c*d*tan(1/2*f*x + 1/2*e)^2 - 100*B*c*d*tan(1/2*f*x
+ 1/2*e)^2 + 420*A*d^2*tan(1/2*f*x + 1/2*e)^2 - 185*B*d^2*tan(1/2*f*x + 1/
2*e)^2 + 20*A*c^2*tan(1/2*f*x + 1/2*e) + 15*B*c^2*tan(1/2*f*x + 1/2*e) - 1
10*A*c*d*tan(1/2*f*x + 1/2*e) - 80*B*c*d*tan(1/2*f*x + 1/2*e) + 270*A*d^2*
tan(1/2*f*x + 1/2*e) - 115*B*d^2*tan(1/2*f*x + 1/2*e) + 7*A*c^2 + 3*B*c^2
- 34*A*c*d - 16*B*c*d + 72*A*d^2 - 32*B*d^2)/((a^3*c^4 - 4*a^3*c^3*d + 6*a
^3*c^2*d^2 - 4*a^3*c*d^3 + a^3*d^4)*(tan(1/2*f*x + 1/2*e) + 1)^5))/f
```

Mupad [B] (verification not implemented)

Time = 41.92 (sec) , antiderivative size = 1349, normalized size of antiderivative = 3.54

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^2),x
)
```

output
```
(2*d^2*atan(((d^2*(3*B*c^2 - 3*A*d^2 + B*d^2 - 4*A*c*d + 3*B*c*d)*(2*a^3*d
^6 - 6*a^3*c*d^5 + 2*a^3*c^5*d + 4*a^3*c^2*d^4 + 4*a^3*c^3*d^3 - 6*a^3*c^4
*d^2))/(a^3*(c + d)^(3/2)*(c - d)^(9/2)) + (2*c*d^2*tan(e/2 + (f*x)/2)*(3*
B*c^2 - 3*A*d^2 + B*d^2 - 4*A*c*d + 3*B*c*d)*(a^3*c^5 + a^3*d^5 - 3*a^3*c*
d^4 - 3*a^3*c^4*d + 2*a^3*c^2*d^3 + 2*a^3*c^3*d^2))/(a^3*(c + d)^(3/2)*(c
- d)^(9/2)))/(2*B*d^4 - 6*A*d^4 + 6*B*c^2*d^2 - 8*A*c*d^3 + 6*B*c*d^3))*(3
*B*c^2 - 3*A*d^2 + B*d^2 - 4*A*c*d + 3*B*c*d))/(a^3*f*(c + d)^(3/2)*(c - d
)^(9/2)) - ((2*(7*A*c^4 + 15*A*d^4 + 3*B*c^4 + 38*A*c^2*d^2 - 48*B*c^2*d^2
 + 72*A*c*d^3 - 27*A*c^3*d - 47*B*c*d^3 - 13*B*c^3*d))/(15*(c + d)*(c - d)
*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (4*tan(e/2 + (f*x)/2)^3*(5*A*c^4 + 15*
A*d^4 + 3*B*c^4 + 19*A*c^2*d^2 - 45*B*c^2*d^2 + 84*A*c*d^3 - 18*A*c^3*d -
52*B*c*d^3 - 11*B*c^3*d))/(3*c*(c - d)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) +
(2*tan(e/2 + (f*x)/2)*(20*A*c^5 + 15*A*d^5 + 15*B*c^5 + 346*A*c^2*d^3 + 10
6*A*c^3*d^2 - 286*B*c^2*d^3 - 221*B*c^3*d^2 + 219*A*c*d^4 - 76*A*c^4*d - 7
9*B*c*d^4 - 59*B*c^4*d))/(15*c*(c + d)*(c - d)*(3*c*d^2 - 3*c^2*d + c^3 -
d^3)) + (2*tan(e/2 + (f*x)/2)^5*(2*A*c^5 + 5*A*d^5 + B*c^5 + 24*A*c^2*d^3
+ 4*A*c^3*d^2 - 16*B*c^2*d^3 - 13*B*c^3*d^2 + 13*A*c*d^4 - 6*A*c^4*d - 11*
B*c*d^4 - 3*B*c^4*d))/(c*(c + d)*(c - d)*(3*c*d^2 - 3*c^2*d + c^3 - d^3))
+ (2*tan(e/2 + (f*x)/2)^4*(11*A*c^5 + 30*A*d^5 + 3*B*c^5 + 162*A*c^2*d^3 +
 4*A*c^3*d^2 - 139*B*c^2*d^3 - 84*B*c^3*d^2 + 135*A*c*d^4 - 27*A*c^4*d ...
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 6828, normalized size of antiderivative = 17.92

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c + d\sin(e + fx))^2}\,dx = \text{Too large to display}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x)
```

output
```
(2*( - 300*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**
2))*tan((e + f*x)/2)**7*a*c**3*d**3 - 345*sqrt(c**2 - d**2)*atan((tan((e +
 f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)**7*a*c**2*d**4 - 90*sq
rt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e +
f*x)/2)**7*a*c*d**5 + 225*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/
sqrt(c**2 - d**2))*tan((e + f*x)/2)**7*b*c**4*d**2 + 315*sqrt(c**2 - d**2)
*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)**7*b*c*
*3*d**3 + 165*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 -
d**2))*tan((e + f*x)/2)**7*b*c**2*d**4 + 30*sqrt(c**2 - d**2)*atan((tan((e
 + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)**7*b*c*d**5 - 1500*s
qrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e +
 f*x)/2)**6*a*c**3*d**3 - 2325*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c
+ d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)**6*a*c**2*d**4 - 1140*sqrt(c**2 -
 d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)**
6*a*c*d**5 - 180*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2
 - d**2))*tan((e + f*x)/2)**6*a*d**6 + 1125*sqrt(c**2 - d**2)*atan((tan((e
 + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)**6*b*c**4*d**2 + 202
5*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((
e + f*x)/2)**6*b*c**3*d**3 + 1455*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)
*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)**6*b*c**2*d**4 + 480*sqrt(c...
```

3.285 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 508

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx = -\frac{d^2(Ad(20c^2+30cd+13d^2)-3B(4c^3+8c^2d+7cd^2+2d^3))\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^3(c-d)^5(c+d)^2\sqrt{c^2-d^2}\,f}$$
$$-\frac{d(3B(2c^3-20c^2d-57cd^2-30d^3)+A(4c^3-30c^2d+146cd^2+195d^3))\cos(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sin(e+fx))^2}$$
$$-\frac{(A-B)\cos(e+fx)}{5(c-d)f(a+a\sin(e+fx))^3(c+d\sin(e+fx))^2}$$
$$-\frac{(2Ac+3Bc-11Ad+6Bd)\cos(e+fx)}{15a(c-d)^2f(a+a\sin(e+fx))^2(c+d\sin(e+fx))^2}$$
$$-\frac{(3B(c^2-10cd-12d^2)+A(2c^2-15cd+76d^2))\cos(e+fx)}{15(c-d)^3f\,(a^3+a^3\sin(e+fx))\,(c+d\sin(e+fx))^2}$$
$$-\frac{d(3B(2c^4-20c^3d-119c^2d^2-130cd^3-48d^4)+A(4c^4-30c^3d+142c^2d^2+525cd^3+304d^4))\cos(e-}{30a^3(c-d)^5(c+d)^2f(c+d\sin(e+fx))}$$

output
```
-d^2*(A*d*(20*c^2+30*c*d+13*d^2)-3*B*(4*c^3+8*c^2*d+7*c*d^2+2*d^3))*arctan
((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a^3/(c-d)^5/(c+d)^2/(c^2-d^2)^(
1/2)/f-1/30*d*(3*B*(2*c^3-20*c^2*d-57*c*d^2-30*d^3)+A*(4*c^3-30*c^2*d+146*
c*d^2+195*d^3))*cos(f*x+e)/a^3/(c-d)^4/(c+d)/f/(c+d*sin(f*x+e))^2-1/5*(A-B
)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2-1/15*(2*A*c-11*
A*d+3*B*c+6*B*d)*cos(f*x+e)/a/(c-d)^2/f/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)
)^2-1/15*(3*B*(c^2-10*c*d-12*d^2)+A*(2*c^2-15*c*d+76*d^2))*cos(f*x+e)/(c-d
)^3/f/(a^3+a^3*sin(f*x+e))/(c+d*sin(f*x+e))^2-1/30*d*(3*B*(2*c^4-20*c^3*d-
119*c^2*d^2-130*c*d^3-48*d^4)+A*(4*c^4-30*c^3*d+142*c^2*d^2+525*c*d^3+304*
d^4))*cos(f*x+e)/a^3/(c-d)^5/(c+d)^2/f/(c+d*sin(f*x+e))
```

Mathematica [A] (verified)

Time = 11.78 (sec) , antiderivative size = 548, normalized size of antiderivative = 1.08

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx$$

$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(12(A-B)(c-d)^2\sin\left(\frac{1}{2}(e+fx)\right)+6(-A+B)(c-d)^2\left(\cos\left(\frac{1}{2}\right.\right.\right.}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x]
)^3),x]
```

output

```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(12*(A - B)*(c - d)^2*Sin[(e + f*x)
/2] + 6*(-A + B)*(c - d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 4*(c -
d)*(A*(2*c - 17*d) + 3*B*(c + 4*d))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2])^2 - 2*(c - d)*(A*(2*c - 17*d) + 3*B*(c + 4*d))*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^3 + 4*(3*B*(c^2 - 12*c*d - 19*d^2) + A*(2*c^2
- 19*c*d + 107*d^2))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^4 + (30*d^2*(-(A*d*(20*c^2 + 30*c*d + 13*d^2)) + 3*B*(4*c^3 + 8*c^2*d +
7*c*d^2 + 2*d^3))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^5)/((c + d)^2*Sqrt[c^2 - d^2]) + (15*(c - d
)*d^3*(B*c - A*d)*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/((
c + d)*(c + d*Sin[e + f*x])^2) + (15*d^3*(-3*A*d*(3*c + 2*d) + B*(7*c^2 +
6*c*d + 2*d^2))*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/((c
+ d)^2*(c + d*Sin[e + f*x]))))/(30*a^3*(c - d)^5*f*(1 + Sin[e + f*x])^3)
```

Rubi [A] (verified)

Time = 2.35 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.08, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.543$, Rules used = {3042, 3457, 25, 3042, 3457, 25, 3042, 3457, 25, 3042, 3233, 25, 3042, 3233, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^3}dx$$

$$\Big\downarrow \text{3457}$$

$$-\frac{\int -\frac{a(2Ac+3Bc-7Ad+2Bd)+4a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)^2(c+d\sin(e+fx))^3}dx}{5a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{a(2Ac+3Bc-7Ad+2Bd)+4a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)^2(c+d\sin(e+fx))^3}dx}{5a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^2}$$

$\downarrow$ 3042

$$\frac{\int \frac{a(2Ac+3Bc-7Ad+2Bd)+4a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)^2(c+d\sin(e+fx))^3}dx}{5a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^2}$$

$\downarrow$ 3457

$$\frac{-\frac{\int -\frac{\left(3B\left(c^2-7dc-6d^2\right)+A\left(2c^2-9dc+43d^2\right)\right)a^2+3d(2Ac+3Bc-11Ad+6Bd)\sin(e+fx)a^2}{(\sin(e+fx)a+a)(c+d\sin(e+fx))^3}dx}{3a^2(c-d)} - \frac{a(2Ac-11Ad+3Bc+6Bd)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^2}}{5a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^2}$$

$\downarrow$ 25

$$\frac{\frac{\int \frac{\left(3B\left(c^2-7dc-6d^2\right)+A\left(2c^2-9dc+43d^2\right)\right)a^2+3d(2Ac+3Bc-11Ad+6Bd)\sin(e+fx)a^2}{(\sin(e+fx)a+a)(c+d\sin(e+fx))^3}dx}{3a^2(c-d)} - \frac{a(2Ac-11Ad+3Bc+6Bd)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^2}}{5a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{\left(3B\left(c^2-7dc-6d^2\right)+A\left(2c^2-9dc+43d^2\right)\right)a^2+3d(2Ac+3Bc-11Ad+6Bd)\sin(e+fx)a^2}{(\sin(e+fx)a+a)(c+d\sin(e+fx))^3}dx}{3a^2(c-d)} - \frac{a(2Ac-11Ad+3Bc+6Bd)\cos(e+fx)}{3f(c-d)(a\sin(e+fx)+a)^2(c+d\sin(e+fx))^2}}{5a^2(c-d)} - \frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^2}$$

$\downarrow$ 3457

$$\frac{\frac{-\frac{\int -\frac{3d^2(2Ac+33Bc-65Ad+30Bd)a^3+2d\left(3B\left(c^2-10dc-12d^2\right)+A\left(2c^2-15dc+76d^2\right)\right)\sin(e+fx)a^3}{(c+d\sin(e+fx))^3}dx}{a^2(c-d)} - \frac{a^2\left(A\left(2c^2-15cd+76d^2\right)+3B\left(c^2-10cd-12d^2\right)\right)\cos(e}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))^2}}{3a^2(c-d)}}{5a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{\int \frac{3d^2(2Ac+33Bc-65Ad+30Bd)a^3+2d\left(3B\left(c^2-10dc-12d^2\right)+A\left(2c^2-15dc+76d^2\right)\right)\sin(e+fx)a^3}{(c+d\sin(e+fx))^3}dx}{a^2(c-d)}-\frac{a^2\left(A\left(2c^2-15cd+76d^2\right)+3B\left(c^2-10cd-12d^2\right)\right)\cos(e+fx\cdots}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))^2}}{3a^2(c-d)}\cdots}{5a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{\int \frac{3d^2(2Ac+33Bc-65Ad+30Bd)a^3+2d\left(3B\left(c^2-10dc-12d^2\right)+A\left(2c^2-15dc+76d^2\right)\right)\sin(e+fx)a^3}{(c+d\sin(e+fx))^3}dx}{a^2(c-d)}-\frac{a^2\left(A\left(2c^2-15cd+76d^2\right)+3B\left(c^2-10cd-12d^2\right)\right)\cos(e+fx\cdots}{f(c-d)(a\sin(e+fx)+a)(c+d\sin(e+fx))^2}}{3a^2(c-d)}\cdots}{5a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \; 3233$$

$$\frac{\frac{-\frac{\int -\frac{2d^2\left(2Ac^2+93Bc^2-165Adc+150Bdc-152Ad^2+72Bd^2\right)a^3+d\left(3B\left(2c^3-20dc^2-57d^2c-30d^3\right)+A\left(4c^3-30dc^2+146d^2c+195d^3\right)\right)\sin(e+fx)a^3}{(c+d\sin(e+fx))^2}dx}{2\left(c^2-d^2\right)}-\frac{a^3d\left(A\left(\cdots\right.\right.}{\cdots}}{a^2(c-d)}\cdots}{3a^2(c-d)}\cdots$$

$$\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{\int \frac{2d^2\left(2Ac^2+93Bc^2-165Adc+150Bdc-152Ad^2+72Bd^2\right)a^3+d\left(3B\left(2c^3-20dc^2-57d^2c-30d^3\right)+A\left(4c^3-30dc^2+146d^2c+195d^3\right)\right)\sin(e+fx)a^3}{(c+d\sin(e+fx))^2}dx}{2\left(c^2-d^2\right)}-\frac{a^3d\left(A\left(4c^3\cdots\right.\right.}{\cdots}}{a^2(c-d)}\cdots}{3a^2(c-d)}\cdots$$

$$\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{\int \frac{2d^2\left(2Ac^2+93Bc^2-165Adc+150Bdc-152Ad^2+72Bd^2\right)a^3+d\left(3B\left(2c^3-20dc^2-57d^2c-30d^3\right)+A\left(4c^3-30dc^2+146d^2c+195d^3\right)\right)\sin(e+fx)a^3}{(c+d\sin(e+fx))^2}dx}{2\left(c^2-d^2\right)}-\frac{a^3d\left(A\left(4c^3\cdots\right.\right.}{\cdots}}{a^2(c-d)}\cdots}{3a^2(c-d)}\cdots$$

$$\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \; 3233$$

$$\frac{\frac{\frac{-\frac{\int \frac{15a^3d^2\left(Ad\left(20c^2+30dc+13d^2\right)-3B\left(4c^3+8dc^2+7d^2c+2d^3\right)\right)}{c+d\sin(e+fx)}dx}{c^2-d^2}-\frac{a^3d\left(A\left(4c^4-30c^3d+142c^2d^2+525cd^3+304d^4\right)+3B\left(2c^4-20c^3d-119c^2d^2-130cd^3-48d^4\cdots\right.\right.}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}}{2\left(c^2-d^2\right)}\cdots}{a^2(c-d)}\cdots}{3a^2(c\cdots}$$

$$\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{-\frac{15a^3d^2\left(Ad\left(20c^2+30cd+13d^2\right)-3B\left(4c^3+8c^2d+7cd^2+2d^3\right)\right)\int \frac{1}{c+d\sin(e+fx)}dx}{c^2-d^2}-\frac{a^3d\left(A\left(4c^4-30c^3d+142c^2d^2+525cd^3+304d^4\right)+3B\left(2c^4-20c^3d-119c^2d^2\cdots\right.\right.}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}}{2\left(c^2-d^2\right)}\cdots}{a^2(c-d)}\cdots$$

$$\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{-\frac{15a^3d^2\left(Ad\left(20c^2+30cd+13d^2\right)-3B\left(4c^3+8c^2d+7cd^2+2d^3\right)\right)\int \frac{1}{c+d\sin(e+fx)}dx}{c^2-d^2}-\frac{a^3d\left(A\left(4c^4-30c^3d+142c^2d^2+525cd^3+304d^4\right)+3B\left(2c^4-20c^3d-119c^2d^2\cdots\right.\right.}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}}{2\left(c^2-d^2\right)}\cdots}{a^2(c-d)}\cdots$$

$$\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \; 3139$$

$$-\frac{30a^3d^2\left(Ad\left(20c^2+30cd+13d^2\right)-3B\left(4c^3+8c^2d+7cd^2+2d^3\right)\right)\int \frac{1}{c\tan^2\left(\frac{1}{2}(e+fx)\right)+2d\tan\left(\frac{1}{2}(e+fx)\right)+c}d\tan\left(\frac{1}{2}(e+fx)\right)}{f\left(c^2-d^2\right)} - \frac{a^3d\left(A\left(4c^4-30c^3d+142c^2d^2+525\right.\right.}{}$$

$$2\left(c^2-d^2\right)$$

$$a^2(c-d)$$

$$\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^2}$$

$\downarrow$ 1083

$$\frac{60a^3d^2\left(Ad\left(20c^2+30cd+13d^2\right)-3B\left(4c^3+8c^2d+7cd^2+2d^3\right)\right)\int \frac{1}{-\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)^2-4\left(c^2-d^2\right)}d\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{f\left(c^2-d^2\right)} - \frac{a^3d\left(A\left(4c^4-30c^3d+142c^2d\right.\right.}{}$$

$$2\left(c^2-d^2\right)$$

$$a^2(c-$$

$$\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^2}$$

$\downarrow$ 217

$$-\frac{30a^3d^2\left(Ad\left(20c^2+30cd+13d^2\right)-3B\left(4c^3+8c^2d+7cd^2+2d^3\right)\right)\arctan\left(\frac{2c\tan\left(\frac{1}{2}(e+fx)\right)+2d}{2\sqrt{c^2-d^2}}\right)}{f\left(c^2-d^2\right)^{3/2}} - \frac{a^3d\left(A\left(4c^4-30c^3d+142c^2d^2+525cd^3+304d^4\right)+3B\left(2c^4-20c\right.\right.}{f\left(c^2-d^2\right)(c+d\sin(e+f}$$

$$2\left(c^2-d^2\right)$$

$$a^2(c-d)$$

$$\frac{(A-B)\cos(e+fx)}{5f(c-d)(a\sin(e+fx)+a)^3(c+d\sin(e+fx))^2}$$

input

```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^3),x
]
```

output
```
-1/5*((A - B)*Cos[e + f*x])/((c - d)*f*(a + a*Sin[e + f*x])^3*(c + d*Sin[e
 + f*x])^2) + (-1/3*(a*(2*A*c + 3*B*c - 11*A*d + 6*B*d)*Cos[e + f*x])/((c
- d)*f*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^2) + (-((a^2*(3*B*(c^2
- 10*c*d - 12*d^2) + A*(2*c^2 - 15*c*d + 76*d^2))*Cos[e + f*x])/((c - d)*f
*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)) + (-1/2*(a^3*d*(3*B*(2*c^3
- 20*c^2*d - 57*c*d^2 - 30*d^3) + A*(4*c^3 - 30*c^2*d + 146*c*d^2 + 195*d^
3))*Cos[e + f*x])/((c^2 - d^2)*f*(c + d*Sin[e + f*x])^2) + ((-30*a^3*d^2*(
A*d*(20*c^2 + 30*c*d + 13*d^2) - 3*B*(4*c^3 + 8*c^2*d + 7*c*d^2 + 2*d^3))*
ArcTan[(2*d + 2*c*Tan[(e + f*x)/2])/(2*Sqrt[c^2 - d^2])])/((c^2 - d^2)^(3/
2)*f) - (a^3*d*(3*B*(2*c^4 - 20*c^3*d - 119*c^2*d^2 - 130*c*d^3 - 48*d^4)
+ A*(4*c^4 - 30*c^3*d + 142*c^2*d^2 + 525*c*d^3 + 304*d^4))*Cos[e + f*x])/
((c^2 - d^2)*f*(c + d*Sin[e + f*x])))/(2*(c^2 - d^2)))/(a^2*(c - d)))/(3*a
^2*(c - d)))/(5*a^2*(c - d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Maple [A] (verified)

Time = 5.00 (sec) , antiderivative size = 639, normalized size of antiderivative = 1.26

method	result
derivativedivides	$-\frac{2d^2\left(\frac{d^2\left(11A\,c^2d+6Ac\,d^2-2A\,d^3-9B\,c^3-6B\,c^2d\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2c\left(c^2+2cd+d^2\right)}+\frac{d\left(10A\,c^4d+6A\,c^3d^2+19A\,c^2d^3+12Ac\,d^4-2A\,d^5-8B\,c^5-6B\right.}{2\left(c^2+2cd+d^2\right.}\right.}{}$
default	$-\frac{2d^2\left(\frac{d^2\left(11A\,c^2d+6Ac\,d^2-2A\,d^3-9B\,c^3-6B\,c^2d\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2c\left(c^2+2cd+d^2\right)}+\frac{d\left(10A\,c^4d+6A\,c^3d^2+19A\,c^2d^3+12Ac\,d^4-2A\,d^5-8B\,c^5-6B\right.}{2\left(c^2+2cd+d^2\right.}\right.}{}$
risch	Expression too large to display

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x,method=_RETUR
NVERBOSE)
```

output
```
2/f/a^3*(-d^2/(c-d)^5*((1/2*d^2*(11*A*c^2*d+6*A*c*d^2-2*A*d^3-9*B*c^3-6*B*
c^2*d)/c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3+1/2*d*(10*A*c^4*d+6*A*c^3*d^
2+19*A*c^2*d^3+12*A*c*d^4-2*A*d^5-8*B*c^5-6*B*c^4*d-17*B*c^3*d^2-12*B*c^2*
d^3-2*B*c*d^4)/(c^2+2*c*d+d^2)/c^2*tan(1/2*f*x+1/2*e)^2+1/2*d^2*(29*A*c^2*
d+18*A*c*d^2-2*A*d^3-23*B*c^3-18*B*c^2*d-4*B*c*d^2)/c/(c^2+2*c*d+d^2)*tan(
1/2*f*x+1/2*e)+1/2*d*(10*A*c^2*d+6*A*c*d^2-A*d^3-8*B*c^3-6*B*c^2*d-B*c*d^2
)/(c^2+2*c*d+d^2))/(tan(1/2*f*x+1/2*e)^2*c+2*d*tan(1/2*f*x+1/2*e)+c)^2+1/2
*(20*A*c^2*d+30*A*c*d^2+13*A*d^3-12*B*c^3-24*B*c^2*d-21*B*c*d^2-6*B*d^3)/(
c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^
2-d^2)^(1/2)))-1/4*(-8*A+8*B)/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^4-1/5*(4*A-4*
B)/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)^5-1/2*(-4*A*c+10*A*d+2*B*c-8*B*d)/(c-d)^
4/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(8*A*c-14*A*d-6*B*c+12*B*d)/(c-d)^4/(tan(1/
2*f*x+1/2*e)+1)^3-(A*c^2-5*A*c*d+10*A*d^2-6*B*d^2)/(c-d)^5/(tan(1/2*f*x+1/
2*e)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3599 vs. $2(493) = 986$.

Time = 0.37 (sec) , antiderivative size = 7283, normalized size of antiderivative = 14.34

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c + d\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algori
thm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c + d\sin(e + fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^3(c + d\sin(e + fx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algori
thm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1224 vs. 2(493) = 986.

Time = 0.36 (sec) , antiderivative size = 1224, normalized size of antiderivative = 2.41

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algori
thm="giac")
```

output
```
1/15*(15*(12*B*c^3*d^2 - 20*A*c^2*d^3 + 24*B*c^2*d^3 - 30*A*c*d^4 + 21*B*c
*d^4 - 13*A*d^5 + 6*B*d^5)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arct
an((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a^3*c^7 - 3*a^3*c^6*d
+ a^3*c^5*d^2 + 5*a^3*c^4*d^3 - 5*a^3*c^3*d^4 - a^3*c^2*d^5 + 3*a^3*c*d^6
- a^3*d^7)*sqrt(c^2 - d^2)) + 15*(9*B*c^4*d^4*tan(1/2*f*x + 1/2*e)^3 - 11*
A*c^3*d^5*tan(1/2*f*x + 1/2*e)^3 + 6*B*c^3*d^5*tan(1/2*f*x + 1/2*e)^3 - 6*
A*c^2*d^6*tan(1/2*f*x + 1/2*e)^3 + 2*A*c*d^7*tan(1/2*f*x + 1/2*e)^3 + 8*B*
c^5*d^3*tan(1/2*f*x + 1/2*e)^2 - 10*A*c^4*d^4*tan(1/2*f*x + 1/2*e)^2 + 6*B
*c^4*d^4*tan(1/2*f*x + 1/2*e)^2 - 6*A*c^3*d^5*tan(1/2*f*x + 1/2*e)^2 + 17*
B*c^3*d^5*tan(1/2*f*x + 1/2*e)^2 - 19*A*c^2*d^6*tan(1/2*f*x + 1/2*e)^2 + 1
2*B*c^2*d^6*tan(1/2*f*x + 1/2*e)^2 - 12*A*c*d^7*tan(1/2*f*x + 1/2*e)^2 + 2
*B*c*d^7*tan(1/2*f*x + 1/2*e)^2 + 2*A*d^8*tan(1/2*f*x + 1/2*e)^2 + 23*B*c^
4*d^4*tan(1/2*f*x + 1/2*e) - 29*A*c^3*d^5*tan(1/2*f*x + 1/2*e) + 18*B*c^3*
d^5*tan(1/2*f*x + 1/2*e) - 18*A*c^2*d^6*tan(1/2*f*x + 1/2*e) + 4*B*c^2*d^6
*tan(1/2*f*x + 1/2*e) + 2*A*c*d^7*tan(1/2*f*x + 1/2*e) + 8*B*c^5*d^3 - 10*
A*c^4*d^4 + 6*B*c^4*d^4 - 6*A*c^3*d^5 + B*c^3*d^5 + A*c^2*d^6)/((a^3*c^9 -
 3*a^3*c^8*d + a^3*c^7*d^2 + 5*a^3*c^6*d^3 - 5*a^3*c^5*d^4 - a^3*c^4*d^5 +
 3*a^3*c^3*d^6 - a^3*c^2*d^7)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x
+ 1/2*e) + c)^2) - 2*(15*A*c^2*tan(1/2*f*x + 1/2*e)^4 - 75*A*c*d*tan(1/2*f
*x + 1/2*e)^4 + 150*A*d^2*tan(1/2*f*x + 1/2*e)^4 - 90*B*d^2*tan(1/2*f*x...
```

Mupad [B] (verification not implemented)

Time = 45.30 (sec) , antiderivative size = 2387, normalized size of antiderivative = 4.70

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^3),x
)
```

output
```
((15*A*d^6 - 14*A*c^6 - 6*B*c^6 - 404*A*c^2*d^4 - 420*A*c^3*d^3 - 92*A*c^4
*d^2 + 234*B*c^2*d^4 + 450*B*c^3*d^3 + 222*B*c^4*d^2 - 90*A*c*d^5 + 60*A*c
^5*d + 15*B*c*d^5 + 30*B*c^5*d)/(15*(c + d)^2*(c - d)*(c^4 - 4*c^3*d - 4*c
*d^3 + d^4 + 6*c^2*d^2)) + (tan(e/2 + (f*x)/2)^7*(2*A*d^8 - 4*A*c^8 - 2*B*
c^8 - 49*A*c^2*d^6 - 141*A*c^3*d^5 - 200*A*c^4*d^4 - 122*A*c^5*d^3 + 2*A*c
^6*d^2 + 12*B*c^2*d^6 + 95*B*c^3*d^5 + 187*B*c^4*d^4 + 146*B*c^5*d^3 + 58*
B*c^6*d^2 - 2*A*c*d^7 + 10*A*c^7*d + 2*B*c*d^7 + 6*B*c^7*d))/(c^2*(c - d)*
(2*c*d + c^2 + d^2)*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)) + (tan(e/
2 + (f*x)/2)^6*(30*A*d^8 - 28*A*c^8 - 6*B*c^8 - 759*A*c^2*d^6 - 1707*A*c^3
*d^5 - 1960*A*c^4*d^4 - 870*A*c^5*d^3 + 62*A*c^6*d^2 + 336*B*c^2*d^6 + 125
7*B*c^3*d^5 + 1893*B*c^4*d^4 + 1350*B*c^5*d^3 + 414*B*c^6*d^2 - 114*A*c*d^
7 + 54*A*c^7*d + 30*B*c*d^7 + 18*B*c^7*d))/(3*c^2*(c - d)*(2*c*d + c^2 + d
^2)*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)) + (tan(e/2 + (f*x)/2)^5*(
60*A*d^8 - 32*A*c^8 - 18*B*c^8 - 1857*A*c^2*d^6 - 3763*A*c^3*d^5 - 3560*A*
c^4*d^4 - 1294*A*c^5*d^3 + 70*A*c^6*d^2 + 900*B*c^2*d^6 + 2859*B*c^3*d^5 +
 3705*B*c^4*d^4 + 2358*B*c^5*d^3 + 678*B*c^6*d^2 - 270*A*c*d^7 + 62*A*c^7*
d + 60*B*c*d^7 + 42*B*c^7*d))/(3*c^2*(c - d)*(2*c*d + c^2 + d^2)*(c^4 - 4*
c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)) + (tan(e/2 + (f*x)/2)^2*(30*A*d^8 - 10
8*A*c^8 - 42*B*c^8 - 2501*A*c^2*d^6 - 8725*A*c^3*d^5 - 10616*A*c^4*d^4 - 4
810*A*c^5*d^3 + 10*A*c^6*d^2 + 1056*B*c^2*d^6 + 5235*B*c^3*d^5 + 9891*B...
```

Reduce [B] (verification not implemented)

Time = 3.21 (sec) , antiderivative size = 12007, normalized size of antiderivative = 23.64

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^3(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x)
```

output
```
( - 1500*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2)
)*tan((e + f*x)/2)**9*a*c**6*d**3 - 3450*sqrt(c**2 - d**2)*atan((tan((e +
f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)**9*a*c**5*d**4 - 2775*s
qrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e +
 f*x)/2)**9*a*c**4*d**5 - 780*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c +
 d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)**9*a*c**3*d**6 + 900*sqrt(c**2 - d
**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)**9*
b*c**7*d**2 + 2520*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c*
*2 - d**2))*tan((e + f*x)/2)**9*b*c**6*d**3 + 3015*sqrt(c**2 - d**2)*atan(
(tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)**9*b*c**5*d**
4 + 1710*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2)
)*tan((e + f*x)/2)**9*b*c**4*d**5 + 360*sqrt(c**2 - d**2)*atan((tan((e + f
*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)**9*b*c**3*d**6 - 7500*sq
rt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e +
f*x)/2)**8*a*c**6*d**3 - 23250*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c
+ d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)**8*a*c**5*d**4 - 27675*sqrt(c**2
- d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)*
*8*a*c**4*d**5 - 15000*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqr
t(c**2 - d**2))*tan((e + f*x)/2)**8*a*c**3*d**6 - 3120*sqrt(c**2 - d**2)*a
tan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*tan((e + f*x)/2)**8*a*c...
```

3.286 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

Optimal result

Integrand size = 37, antiderivative size = 256

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = \frac{4a(c+d)(Bc-9Ad-8Bd)\left(15c^2+10cd+7d^2\right)\cos(e+fx)}{315df\sqrt{a+a\sin(e+fx)}} + \frac{8(5c-d)(c+d)(Bc-9Ad-8Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{315f} + \frac{4d(c+d)(Bc-9Ad-8Bd)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{105af} + \frac{2a(Bc-9Ad-8Bd)\cos(e+fx)(c+d\sin(e+fx))^3}{63df\sqrt{a+a\sin(e+fx)}} - \frac{2aB\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a+a\sin(e+fx)}}$$

output

```
4/315*a*(c+d)*(-9*A*d+B*c-8*B*d)*(15*c^2+10*c*d+7*d^2)*cos(f*x+e)/d/f/(a+a
*sin(f*x+e))^(1/2)+8/315*(5*c-d)*(c+d)*(-9*A*d+B*c-8*B*d)*cos(f*x+e)*(a+a*
sin(f*x+e))^(1/2)/f+4/105*d*(c+d)*(-9*A*d+B*c-8*B*d)*cos(f*x+e)*(a+a*sin(f
*x+e))^(3/2)/a/f+2/63*a*(-9*A*d+B*c-8*B*d)*cos(f*x+e)*(c+d*sin(f*x+e))^3/d
/f/(a+a*sin(f*x+e))^(1/2)-2/9*a*B*cos(f*x+e)*(c+d*sin(f*x+e))^4/d/f/(a+a*s
in(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 2.47 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.19

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx =$$
$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}(2520Ac^3+1680Bc^3+5040Ac^2d+4788Bc}{}$$

input
```
Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x
])^3,x]
```

output
```
-1/1260*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*
(2520*A*c^3 + 1680*B*c^3 + 5040*A*c^2*d + 4788*B*c^2*d + 4788*A*c*d^2 + 41
04*B*c*d^2 + 1368*A*d^3 + 1321*B*d^3 - 4*d*(27*A*d*(7*c + 2*d) + B*(189*c^
2 + 162*c*d + 83*d^2))*Cos[2*(e + f*x)] + 35*B*d^3*Cos[4*(e + f*x)] + 840*
B*c^3*Sin[e + f*x] + 2520*A*c^2*d*Sin[e + f*x] + 2016*B*c^2*d*Sin[e + f*x]
 + 2016*A*c*d^2*Sin[e + f*x] + 2538*B*c*d^2*Sin[e + f*x] + 846*A*d^3*Sin[e
 + f*x] + 752*B*d^3*Sin[e + f*x] - 270*B*c*d^2*Sin[3*(e + f*x)] - 90*A*d^3
*Sin[3*(e + f*x)] - 80*B*d^3*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2]))
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.92, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.297$, Rules used = {3042, 3460, 3042, 3249, 3042, 3240, 27, 3042, 3230, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))(c+d\sin(e+fx))^3dx$$

$$\Big\downarrow \text{3460}$$

$$-\frac{(-9Ad+Bc-8Bd)\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^3dx}{9d}-\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(-9Ad+Bc-8Bd)\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^3dx}{9d}-\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3249}$$

$$-\frac{(-9Ad+Bc-8Bd)\left(\frac{6}{7}(c+d)\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2dx-\frac{2a\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}\right)}{9d}-\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(-9Ad+Bc-8Bd)\left(\frac{6}{7}(c+d)\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2dx-\frac{2a\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}\right)}{9d}-\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3240}$$

$$-\frac{(-9Ad+Bc-8Bd)\left(\frac{6}{7}(c+d)\left(\frac{2\int \frac{1}{2}\sqrt{\sin(e+fx)a+a}(a(5c^2+3d^2)+2a(5c-d)d\sin(e+fx))dx}{5a}-\frac{2d^2\cos(e+fx)(a\sin(e+fx)+a)}{5af}\right.\right.}{9d}-\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{(-9Ad+Bc-8Bd)\left(\frac{6}{7}(c+d)\left(\frac{\int \sqrt{\sin(e+fx)a+a}(a(5c^2+3d^2)+2a(5c-d)d\sin(e+fx))dx}{5a}-\frac{2d^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5af}\right.\right.}{9d}-\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(-9Ad+Bc-8Bd)\left(\frac{6}{7}(c+d)\left(\frac{\int\sqrt{\sin(e+fx)a+a}\left(a(5c^2+3d^2)+2a(5c-d)d\sin(e+fx)\right)dx}{5a}-\frac{2d^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5af}\right.\right.}{9d}$$

$$\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3230}$$

$$-\frac{(-9Ad+Bc-8Bd)\left(\frac{6}{7}(c+d)\left(\frac{\frac{1}{3}a(15c^2+10cd+7d^2)\int\sqrt{\sin(e+fx)a+a}dx-\frac{4ad(5c-d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{5a}-\frac{2d^2\cos(e+fx)\right.\right.}{9d}$$

$$\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(-9Ad+Bc-8Bd)\left(\frac{6}{7}(c+d)\left(\frac{\frac{1}{3}a(15c^2+10cd+7d^2)\int\sqrt{\sin(e+fx)a+a}dx-\frac{4ad(5c-d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{5a}-\frac{2d^2\cos(e+fx)\right.\right.}{9d}$$

$$\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3125}$$

$$-\frac{(-9Ad+Bc-8Bd)\left(\frac{6}{7}(c+d)\left(\frac{-\frac{2a^2\left(15c^2+10cd+7d^2\right)\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{4ad(5c-d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{5a}-\frac{2d^2\cos(e+fx)(a\sin(e+}{5af}\right.\right.}{9d}$$

$$\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}}$$

input
```
Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x
]
```

output
```
(-2*a*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(9*d*f*Sqrt[a + a*Sin[e + f*x
]]) - ((B*c - 9*A*d - 8*B*d)*((-2*a*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(
7*f*Sqrt[a + a*Sin[e + f*x]]) + (6*(c + d)*((-2*d^2*Cos[e + f*x]*(a + a*Si
n[e + f*x])^(3/2))/(5*a*f) + ((-2*a^2*(15*c^2 + 10*c*d + 7*d^2)*Cos[e + f*
x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) - (4*a*(5*c - d)*d*Cos[e + f*x]*Sqrt[a
+ a*Sin[e + f*x]])/(3*f))/(5*a)))/7))/(9*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3240
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])^
m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

rule 3249
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[2*n*((b*c + a*d)/(b*(
2*n + 1)))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.95

method	result
default	$\frac{2(1+\sin(fx+e))a(\sin(fx+e)-1)\left(35B\cos(fx+e)^4d^3+(-45A\,d^3-135Bc\,d^2-40B\,d^3)\cos(fx+e)^2\sin(fx+e)+(-189Ac\,d^2-54A\,d \cdots\right.}{\cdots}$
parts	$\frac{2(1+\sin(fx+e))a(\sin(fx+e)-1)d^2(Ad+3Bc)\left(5\sin(fx+e)^3+6\sin(fx+e)^2+8\sin(fx+e)+16\right)}{35\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}+\frac{2(1+\sin(fx+e))a(\sin(fx+e)- \cdots}{5\cos(f \cdots}$

input
```
int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x,method=_R
ETURNVERBOSE)
```

output
```
2/315*(1+sin(f*x+e))*a*(sin(f*x+e)-1)*(35*B*cos(f*x+e)^4*d^3+(-45*A*d^3-13
5*B*c*d^2-40*B*d^3)*cos(f*x+e)^2*sin(f*x+e)+(-189*A*c*d^2-54*A*d^3-189*B*c
^2*d-162*B*c*d^2-118*B*d^3)*cos(f*x+e)^2+(315*A*c^2*d+252*A*c*d^2+117*A*d^
3+105*B*c^3+252*B*c^2*d+351*B*c*d^2+104*B*d^3)*sin(f*x+e)+315*A*c^3+630*A*
c^2*d+693*A*c*d^2+198*A*d^3+210*B*c^3+693*B*c^2*d+594*B*c*d^2+211*B*d^3)/c
os(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.82

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx =$$
$$-\frac{2\,(35\,Bd^3\cos{(fx+e)}^5-5\,(27\,Bcd^2+(9\,A+B)d^3)\cos{(fx+e)}^4+105\,(3\,A+B)c^3+63\,(5\,A+7\,B$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, al
gorithm="fricas")
```

output
```
-2/315*(35*B*d^3*cos(f*x + e)^5 - 5*(27*B*c*d^2 + (9*A + B)*d^3)*cos(f*x +
 e)^4 + 105*(3*A + B)*c^3 + 63*(5*A + 7*B)*c^2*d + 9*(49*A + 27*B)*c*d^2 +
 (81*A + 107*B)*d^3 - (189*B*c^2*d + 27*(7*A + 6*B)*c*d^2 + 2*(27*A + 59*B
)*d^3)*cos(f*x + e)^3 + (105*B*c^3 + 63*(5*A + B)*c^2*d + 9*(7*A + 36*B)*c
*d^2 + 2*(54*A + 13*B)*d^3)*cos(f*x + e)^2 + (105*(3*A + 2*B)*c^3 + 63*(10
*A + 11*B)*c^2*d + 99*(7*A + 6*B)*c*d^2 + (198*A + 211*B)*d^3)*cos(f*x + e
) - (35*B*d^3*cos(f*x + e)^4 + 105*(3*A + B)*c^3 + 63*(5*A + 7*B)*c^2*d +
9*(49*A + 27*B)*c*d^2 + (81*A + 107*B)*d^3 + 5*(27*B*c*d^2 + (9*A + 8*B)*d
^3)*cos(f*x + e)^3 - 3*(63*B*c^2*d + 9*(7*A + B)*c*d^2 + (3*A + 26*B)*d^3)
*cos(f*x + e)^2 - (105*B*c^3 + 63*(5*A + 4*B)*c^2*d + 9*(28*A + 39*B)*c*d^
2 + 13*(9*A + 8*B)*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) +
a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

Sympy [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$$
$$= \int \sqrt{a\,(\sin{(e+fx)}+1)}(A+B\sin{(e+fx)})\,(c+d\sin{(e+fx)})^3\;dx$$

input
```
integrate((a+a*sin(f*x+e))**(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3,x)
```

output
```
Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))*(c + d*sin(e + f*
x))**3, x)
```

Maxima [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$$
$$=\int (B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}(d\sin(fx+e)+c)^3\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) +
c)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. 2(236) = 472.

Time = 0.39 (sec) , antiderivative size = 550, normalized size of antiderivative = 2.15

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx=\text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, al
gorithm="giac")
```

output
```
1/2520*sqrt(2)*(35*B*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-9/4*pi +
 9/2*f*x + 9/2*e) + 630*(8*A*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 4*B
*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*A*c^2*d*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e)) + 12*B*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*A*c*
d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 9*B*c*d^2*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e)) + 3*A*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*d^3*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 210*(4*B
*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*A*c^2*d*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e)) + 6*B*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*A*c*d^
2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 9*B*c*d^2*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e)) + 3*A*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*B*d^3*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/2*f*x + 3/2*e) + 126*(6*B*c
^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*A*c*d^2*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e)) + 3*B*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + A*d^3*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*B*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e)))*sin(-5/4*pi + 5/2*f*x + 5/2*e) + 45*(6*B*c*d^2*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e)) + 2*A*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + B*d^3*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-7/4*pi + 7/2*f*x + 7/2*e))*sqrt(a)/f
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$$
$$=\int (A+B\,\sin(e+f\,x))\,\sqrt{a+a\,\sin(e+f\,x)}\,(c+d\,\sin(e+f\,x))^3\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^3
,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^3
, x)
```

Reduce [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$$
$$= \sqrt{a}\left(\left(\int \sqrt{\sin(fx+e)+1}dx\right)a\,c^3+\left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)^4\,dx\right)b\,d^3 + \left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)^3\,dx\right)a\,d^3 + 3\left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)^3\,dx\right)bc\,d^2 + 3\left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)ac\,d^2 + 3\left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)b\,c^2d + 3\left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)a\,c^2d + \left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)b\,c^3\right)$$

input
```
int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x)
```

output
```
sqrt(a)*(int(sqrt(sin(e + f*x) + 1),x)*a*c**3 + int(sqrt(sin(e + f*x) + 1)
*sin(e + f*x)**4,x)*b*d**3 + int(sqrt(sin(e + f*x) + 1)*sin(e + f*x)**3,x)
*a*d**3 + 3*int(sqrt(sin(e + f*x) + 1)*sin(e + f*x)**3,x)*b*c*d**2 + 3*int
(sqrt(sin(e + f*x) + 1)*sin(e + f*x)**2,x)*a*c*d**2 + 3*int(sqrt(sin(e + f
*x) + 1)*sin(e + f*x)**2,x)*b*c**2*d + 3*int(sqrt(sin(e + f*x) + 1)*sin(e
+ f*x),x)*a*c**2*d + int(sqrt(sin(e + f*x) + 1)*sin(e + f*x),x)*b*c**3)
```

3.287 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

Optimal result

Integrand size = 37, antiderivative size = 192

$$\begin{aligned}\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx &= \frac{2a(Bc-7Ad-6Bd)\left(15c^2+10cd+7d^2\right)\cos(e+fx)}{105df\sqrt{a+a\sin(e+fx)}} \\ &+ \frac{4(5c-d)(Bc-7Ad-6Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{105f} \\ &+ \frac{2d(Bc-7Ad-6Bd)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{35af} \\ &- \frac{2aB\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a+a\sin(e+fx)}}\end{aligned}$$

output
```
2/105*a*(-7*A*d+B*c-6*B*d)*(15*c^2+10*c*d+7*d^2)*cos(f*x+e)/d/f/(a+a*sin(f
*x+e))^(1/2)+4/105*(5*c-d)*(-7*A*d+B*c-6*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^
(1/2)/f+2/35*d*(-7*A*d+B*c-6*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/a/f-2/
7*a*B*cos(f*x+e)*(c+d*sin(f*x+e))^3/d/f/(a+a*sin(f*x+e))^(1/2)
```

Mathematica [A] (verified)

Time = 1.63 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.92

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx =$$
$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}(420Ac^2+280Bc^2+560Acd+532Bcd+2}{}$$

input
```
Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x
])^2,x]
```

output
```
-1/210*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(
420*A*c^2 + 280*B*c^2 + 560*A*c*d + 532*B*c*d + 266*A*d^2 + 228*B*d^2 - 6*
d*(14*B*c + 7*A*d + 6*B*d)*Cos[2*(e + f*x)] + (56*A*d*(5*c + 2*d) + B*(140
*c^2 + 224*c*d + 141*d^2))*Sin[e + f*x] - 15*B*d^2*Sin[3*(e + f*x)]))/(f*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {3042, 3460, 3042, 3240, 27, 3042, 3230, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))(c+d\sin(e+fx))^2dx$$

$$\Big\downarrow \text{ 3460}$$

$$-\frac{(-7Ad+Bc-6Bd)\int\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2dx}{7d}-\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{(-7Ad+Bc-6Bd)\int\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2dx}{7d}-\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \textbf{3240}$$

$$-\frac{(-7Ad+Bc-6Bd)\left(\frac{2\int\frac{1}{2}\sqrt{\sin(e+fx)a+a}(a(5c^2+3d^2)+2a(5c-d)d\sin(e+fx))dx}{5a}-\frac{2d^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5af}\right)}{7d}-\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{(-7Ad+Bc-6Bd)\left(\frac{\int\sqrt{\sin(e+fx)a+a}(a(5c^2+3d^2)+2a(5c-d)d\sin(e+fx))dx}{5a}-\frac{2d^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5af}\right)}{7d}-\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{(-7Ad+Bc-6Bd)\left(\frac{\int\sqrt{\sin(e+fx)a+a}(a(5c^2+3d^2)+2a(5c-d)d\sin(e+fx))dx}{5a}-\frac{2d^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5af}\right)}{7d}-\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \textbf{3230}$$

$$-\frac{(-7Ad+Bc-6Bd)\left(\frac{\frac{1}{3}a(15c^2+10cd+7d^2)\int\sqrt{\sin(e+fx)a+a}dx-\frac{4ad(5c-d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{5a}-\frac{2d^2\cos(e+fx)(a\sin(e+fx)}{5af}\right.}{7d}-\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{(-7Ad+Bc-6Bd)\left(\frac{\frac{1}{3}a(15c^2+10cd+7d^2)\int\sqrt{\sin(e+fx)a+a}dx-\frac{4ad(5c-d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{5a}-\frac{2d^2\cos(e+fx)(a\sin(e+fx)}{5af}\right.}{7d}$$
$$\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a\sin(e+fx)+a}}$$

$$\downarrow \text{3125}$$

$$-\frac{(-7Ad+Bc-6Bd)\left(\frac{-\frac{2a^2(15c^2+10cd+7d^2)\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{4ad(5c-d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{5a}-\frac{2d^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5af}\right)}{7d}$$
$$\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a\sin(e+fx)+a}}$$

input
```
Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x
]
```

output
```
(-2*a*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(7*d*f*Sqrt[a + a*Sin[e + f*x
]]) - ((B*c - 7*A*d - 6*B*d)*((-2*d^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3
/2))/(5*a*f) + ((-2*a^2*(15*c^2 + 10*c*d + 7*d^2)*Cos[e + f*x])/(3*f*Sqrt[
a + a*Sin[e + f*x]]) - (4*a*(5*c - d)*d*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*
x]])/(3*f))/(5*a)))/(7*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3240
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])^
m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.84

method	result
default	$\frac{2(1+\sin(fx+e))a(\sin(fx+e)-1)\left(-15B\cos(fx+e)^2\sin(fx+e)d^2+\left(-21A\,d^2-42Bcd-18B\,d^2\right)\cos(fx+e)^2+\left(70Acd+28A\,d^2+35\right.}{105\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$
parts	$\frac{2(1+\sin(fx+e))a(\sin(fx+e)-1)d(Ad+2Bc)\left(3\sin(fx+e)^2+4\sin(fx+e)+8\right)}{15\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}+\frac{2(1+\sin(fx+e))a(\sin(fx+e)-1)c(2Ad+Bc)(\sin}{3\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$

input
```
int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x,method=_R
ETURNVERBOSE)
```

output
```
2/105*(1+sin(f*x+e))*a*(sin(f*x+e)-1)*(-15*B*cos(f*x+e)^2*sin(f*x+e)*d^2+(
-21*A*d^2-42*B*c*d-18*B*d^2)*cos(f*x+e)^2+(70*A*c*d+28*A*d^2+35*B*c^2+56*B
*c*d+39*B*d^2)*sin(f*x+e)+105*A*c^2+140*A*c*d+77*A*d^2+70*B*c^2+154*B*c*d+
66*B*d^2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.59

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = \frac{2\left(15\,Bd^2\cos\left(fx+e\right)^4+3\left(14\,Bcd+(7\,A+6\,B)d^2\right)\cos\left(fx+e\right)^3-35\,(3\,A+B)c^2-14\,(5\,A+7\,B)cd}{}$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, al
gorithm="fricas")
```

output
```
2/105*(15*B*d^2*cos(f*x + e)^4 + 3*(14*B*c*d + (7*A + 6*B)*d^2)*cos(f*x +
e)^3 - 35*(3*A + B)*c^2 - 14*(5*A + 7*B)*c*d - (49*A + 27*B)*d^2 - (35*B*c
^2 + 14*(5*A + B)*c*d + (7*A + 36*B)*d^2)*cos(f*x + e)^2 - (35*(3*A + 2*B)
*c^2 + 14*(10*A + 11*B)*c*d + 11*(7*A + 6*B)*d^2)*cos(f*x + e) + (15*B*d^2
*cos(f*x + e)^3 + 35*(3*A + B)*c^2 + 14*(5*A + 7*B)*c*d + (49*A + 27*B)*d^
2 - 3*(14*B*c*d + (7*A + B)*d^2)*cos(f*x + e)^2 - (35*B*c^2 + 14*(5*A + 4*
B)*c*d + (28*A + 39*B)*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e
) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

Sympy [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = \int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}(A+B\sin\left(e+fx\right))\left(c+d\sin\left(e+fx\right)\right)^2\,dx$$

input
```
integrate((a+a*sin(f*x+e))**(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
```

output
```
Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))*(c + d*sin(e + f*
x))**2, x)
```

Maxima [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$
$$=\int (B\sin{(fx+e)}+A)\sqrt{a\sin{(fx+e)}+a}(d\sin{(fx+e)}+c)^2\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) +
c)^2, x)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.81

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, al
gorithm="giac")
```

output
```
1/420*sqrt(2)*(15*B*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-7/4*pi +
7/2*f*x + 7/2*e) + 105*(8*A*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 4*B*
c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 8*A*c*d*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e)) + 8*B*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 4*A*d^2*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 35*(4*B*c^2*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e)) + 8*A*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 4*B*c*d*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e)) + 2*A*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
+ 3*B*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/2*f*x + 3/2
*e) + 21*(4*B*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*A*d^2*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e)) + B*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin
(-5/4*pi + 5/2*f*x + 5/2*e))*sqrt(a)/f
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$
$$=\int (A+B\,\sin(e+f\,x))\,\sqrt{a+a\,\sin(e+f\,x)}\,(c+d\,\sin(e+f\,x))^2\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^2
,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^2
, x)
```

Reduce [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$
$$= \sqrt{a}\left(\left(\left(\int \sqrt{\sin(fx+e)+1}dx\right)a\,c^2+\left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)^3\,dx\right)b\,d^2 + \left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)a\,d^2 + 2\left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)bcd + 2\left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)acd + \left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)b\,c^2\right)\right)$$

input
```
int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x)
```

output
```
sqrt(a)*(int(sqrt(sin(e + f*x) + 1),x)*a*c**2 + int(sqrt(sin(e + f*x) + 1)
*sin(e + f*x)**3,x)*b*d**2 + int(sqrt(sin(e + f*x) + 1)*sin(e + f*x)**2,x)
*a*d**2 + 2*int(sqrt(sin(e + f*x) + 1)*sin(e + f*x)**2,x)*b*c*d + 2*int(sq
rt(sin(e + f*x) + 1)*sin(e + f*x),x)*a*c*d + int(sqrt(sin(e + f*x) + 1)*si
n(e + f*x),x)*b*c**2)
```

3.288 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 118

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = -\frac{2a(15Ac+5Bc+5Ad+7Bd)\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} - \frac{2(5Bc+5Ad-2Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} - \frac{2Bd\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5af}$$

output
```
-2/15*a*(15*A*c+5*A*d+5*B*c+7*B*d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-2/1
5*(5*A*d+5*B*c-2*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f-2/5*B*d*cos(f*x+
e)*(a+a*sin(f*x+e))^(3/2)/a/f
```

Mathematica [A] (verified)

Time = 1.37 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.99

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx =$$

$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}(30Ac+20Bc+20Ad+19Bd-3Bd\cos(2}{15f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

input
```
Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x
]),x]
```

output
```
-1/15*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(3
0*A*c + 20*B*c + 20*A*d + 19*B*d - 3*B*d*Cos[2*(e + f*x)] + 2*(5*B*c + 5*A
*d + 4*B*d)*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {3042, 3447, 3042, 3502, 27, 3042, 3230, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))(c+d\sin(e+fx))dx$$

$$\Big\downarrow \text{3447}$$

$$\int \sqrt{a\sin(e+fx)+a}\left((Ad+Bc)\sin(e+fx)+Ac+Bd\sin^2(e+fx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\sin(e+fx)+a}\left((Ad+Bc)\sin(e+fx)+Ac+Bd\sin(e+fx)^2\right)dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{2\int \frac{1}{2}\sqrt{\sin(e+fx)a+a}(a(5Ac+3Bd)+a(5Bc+5Ad-2Bd)\sin(e+fx))dx}{5a}-\frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5af}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sqrt{\sin(e+fx)a+a}(a(5Ac+3Bd)+a(5Bc+5Ad-2Bd)\sin(e+fx))dx}{5a}-\frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5af}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{\sin(e+fx)a+a}(a(5Ac+3Bd)+a(5Bc+5Ad-2Bd)\sin(e+fx))dx}{5a}-\frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5af}$$

$$\Big\downarrow \text{3230}$$

$$\frac{\frac{1}{3}a(15Ac+5Ad+5Bc+7Bd)\int \sqrt{\sin(e+fx)a+a}dx-\frac{2a(5Ad+5Bc-2Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{5a}-\frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5af}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{3}a(15Ac+5Ad+5Bc+7Bd)\int \sqrt{\sin(e+fx)a+a}dx-\frac{2a(5Ad+5Bc-2Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{5a}-\frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5af}$$

$$\Big\downarrow \text{3125}$$

$$\frac{-\frac{2a^2(15Ac+5Ad+5Bc+7Bd)\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a(5Ad+5Bc-2Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{5a}-\frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5af}$$

input
```
Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

output
```
(-2*B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(5*a*f) + ((-2*a^2*(15*A*
c + 5*B*c + 5*A*d + 7*B*d)*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) -
(2*a*(5*B*c + 5*A*d - 2*B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*f))
/(5*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.86

method	result
default	$\frac{2(1+\sin(fx+e))a(\sin(fx+e)-1)\left(3B\sin(fx+e)^2d+5A\sin(fx+e)d+5B\sin(fx+e)c+4B\sin(fx+e)d+15Ac+10Ad+10Bc+8Bd\right)}{15\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$
parts	$\frac{2(1+\sin(fx+e))a(\sin(fx+e)-1)(Ad+Bc)(\sin(fx+e)+2)}{3\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}+\frac{2Ac(1+\sin(fx+e))(\sin(fx+e)-1)a}{\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}+\frac{2Bd(1+\sin(fx+e))a(\sin(f}{15\cos(fx}$

input
```
int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x,method=_RET
URNVERBOSE)
```

output
```
2/15*(1+sin(f*x+e))*a*(sin(f*x+e)-1)*(3*B*sin(f*x+e)^2*d+5*A*sin(f*x+e)*d+
5*B*sin(f*x+e)*c+4*B*sin(f*x+e)*d+15*A*c+10*A*d+10*B*c+8*B*d)/cos(f*x+e)/(
a+a*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.48

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$=\frac{2\left(3\,Bd\cos\left(fx+e\right)^3-(5\,Bc+(5\,A+B)d)\cos\left(fx+e\right)^2-5\,(3\,A+B)c-(5\,A+7\,B)d-(5\,(3\,A+2$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algo
rithm="fricas")
```

output
```
2/15*(3*B*d*cos(f*x + e)^3 - (5*B*c + (5*A + B)*d)*cos(f*x + e)^2 - 5*(3*A
 + B)*c - (5*A + 7*B)*d - (5*(3*A + 2*B)*c + (10*A + 11*B)*d)*cos(f*x + e)
 - (3*B*d*cos(f*x + e)^2 - 5*(3*A + B)*c - (5*A + 7*B)*d + (5*B*c + (5*A +
 4*B)*d)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x +
 e) + f*sin(f*x + e) + f)
```

Sympy [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$=\int \sqrt{a(\sin(e+fx)+1)}(A+B\sin(e+fx))\,(c+d\sin(e+fx))\,dx$$

input
```
integrate((a+a*sin(f*x+e))**(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

output
```
Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))*(c + d*sin(e + f*
x)), x)
```

Maxima [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$=\int (B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}(d\sin(fx+e)+c)\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) +
c), x)
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.58

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$= \frac{\sqrt{2}\left(3\,Bd\text{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{5}{4}\,\pi+\frac{5}{2}\,fx+\frac{5}{2}\,e\right)+30\,\left(2\,Ac\text{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algo
rithm="giac")
```

output
```
1/30*sqrt(2)*(3*B*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-5/4*pi + 5/2*
f*x + 5/2*e) + 30*(2*A*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + B*c*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e)) + A*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + B
*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 5
*(2*B*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*A*d*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e)) + B*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/2
*f*x + 3/2*e))*sqrt(a)/f
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$= \int (A+B\,\sin(e+f\,x))\,\sqrt{a+a\,\sin(e+f\,x)}\,(c+d\,\sin(e+f\,x))\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x)),x
)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x)),
x)
```

Reduce [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$= \sqrt{a}\left(\left(\int \sqrt{\sin(fx+e)+1}dx\right)ac+\left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)bd + \left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)ad + \left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)bc\right)$$

input
```
int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

output
```
sqrt(a)*(int(sqrt(sin(e + f*x) + 1),x)*a*c + int(sqrt(sin(e + f*x) + 1)*si
n(e + f*x)**2,x)*b*d + int(sqrt(sin(e + f*x) + 1)*sin(e + f*x),x)*a*d + in
t(sqrt(sin(e + f*x) + 1)*sin(e + f*x),x)*b*c)
```

3.289 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 62

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx = -\frac{2a(3A+B)\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}}-\frac{2B\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f}$$

output
```
-2/3*a*(3*A+B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-2/3*B*cos(f*x+e)*(a+a*s
in(f*x+e))^(1/2)/f
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.32

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx = -\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}(3A+2B+B\sin(e+fx))}{3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

input
```
Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]),x]
```

output
```
(-2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(3*A
+ 2*B + B*Sin[e + f*x]))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3042, 3230, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{ 3230}$$

$$\frac{1}{3}(3A+B)\int \sqrt{\sin(e+fx)a+a}dx-\frac{2B\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3}(3A+B)\int \sqrt{\sin(e+fx)a+a}dx-\frac{2B\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}$$

$$\Big\downarrow \text{ 3125}$$

$$-\frac{2a(3A+B)\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2B\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}$$

input
```
Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]),x]
```

output
```
(-2*a*(3*A + B)*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) - (2*B*Cos[e
+ f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*f)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{2(1+\sin(fx+e))a(\sin(fx+e)-1)(B\sin(fx+e)+3A+2B)}{3\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$	58
parts	$\frac{2A(1+\sin(fx+e))(\sin(fx+e)-1)a}{\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}+\frac{2B(1+\sin(fx+e))a(\sin(fx+e)-1)(\sin(fx+e)+2)}{3\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$	96

input
```
int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
2/3*(1+sin(f*x+e))*a*(sin(f*x+e)-1)*(B*sin(f*x+e)+3*A+2*B)/cos(f*x+e)/(a+a
*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.37

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx = -\frac{2\left(B\cos\left(fx+e\right)^2+\left(3A+2B\right)\cos\left(fx+e\right)+\left(B\cos\left(fx+e\right)-3A-B\right)\sin\left(fx+e\right)+3A+B\right)\sqrt{}}{3\left(f\cos\left(fx+e\right)+f\sin\left(fx+e\right)+f\right)}$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e)),x, algorithm="fricas")
```

output
```
-2/3*(B*cos(f*x + e)^2 + (3*A + 2*B)*cos(f*x + e) + (B*cos(f*x + e) - 3*A
- B)*sin(f*x + e) + 3*A + B)*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*
sin(f*x + e) + f)
```

Sympy [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx = \int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(A+B\sin\left(e+fx\right)\right)\,dx$$

input
```
integrate((a+a*sin(f*x+e))**(1/2)*(A+B*sin(f*x+e)),x)
```

output
```
Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x)), x)
```

Maxima [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx = \int \left(B\sin\left(fx+e\right)+A\right)\sqrt{a\sin\left(fx+e\right)+a}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a), x)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.37

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx = \frac{\sqrt{2}\left(B\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{3}{4}\pi+\frac{3}{2}fx+\frac{3}{2}e\right)+3\left(2A\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+B\mathrm{s}\right.\right.}{3f}$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e)),x, algorithm="giac")
```

output
```
1/3*sqrt(2)*(B*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-3/4*pi + 3/2*f*x +
 3/2*e) + 3*(2*A*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + B*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e))*sqrt(a)/f
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx = \int (A+B\,\sin(e+f\,x))\,\sqrt{a+a\,\sin(e+f\,x)}\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2),x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2), x)
```

Reduce [F]

$$
\begin{aligned}
&\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))\,dx \\
&= \sqrt{a}\left(\left(\int \sqrt{\sin(fx+e)+1}dx\right)a+\left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)b\right)
\end{aligned}
$$

input
```
int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e)),x)
```

output
```
sqrt(a)*(int(sqrt(sin(e + f*x) + 1),x)*a + int(sqrt(sin(e + f*x) + 1)*sin(
e + f*x),x)*b)
```

3.290 $\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 100

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \frac{2\sqrt{a}(Bc-Ad)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{d^{3/2}\sqrt{c+d}f} - \frac{2aB\cos(e+fx)}{df\sqrt{a+a\sin(e+fx)}}$$

output
```
2*a^(1/2)*(-A*d+B*c)*arctanh(a^(1/2)*d^(1/2)*cos(f*x+e)/(c+d)^(1/2)/(a+a*s
in(f*x+e))^(1/2))/d^(3/2)/(c+d)^(1/2)/f-2*a*B*cos(f*x+e)/d/f/(a+a*sin(f*x+
e))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 11.79 (sec) , antiderivative size = 903, normalized size of antiderivative = 9.03

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \text{Too large to display}$$

input
```
Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f
*x]),x]
```

output
```
((1/2 + I/2)*(((-2 + 2*I)*B*Sqrt[d]*Cos[(f*x)/2]*(Cos[e/2] - Sin[e/2]))/f
+ ((-(B*c) + A*d)*(Cos[e/2] + I*Sin[e/2])*((-1 + I)*x*Cos[e] + (RootSum[-d
 + (2*I)*c*E^(I*e)*#1^2 + d*E^((2*I)*e)*#1^4 & , ((1 + I)*d*Sqrt[E^((-I)*e
)]*f*x - (2 - 2*I)*d*Sqrt[E^((-I)*e)]*Log[E^((I/2)*f*x) - #1] - I*Sqrt[d]*
Sqrt[c + d]*f*x*#1 + 2*Sqrt[d]*Sqrt[c + d]*Log[E^((I/2)*f*x) - #1]*#1 + ((
1 - I)*c*f*x*#1^2)/Sqrt[E^((-I)*e)] + ((2 + 2*I)*c*Log[E^((I/2)*f*x) - #1]
*#1^2)/Sqrt[E^((-I)*e)] - Sqrt[d]*Sqrt[c + d]*E^(I*e)*f*x*#1^3 - (2*I)*Sqr
t[d]*Sqrt[c + d]*E^(I*e)*Log[E^((I/2)*f*x) - #1]*#1^3)/(d - I*c*E^(I*e)*#1
^2) & ]*(Cos[e] + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]])/(4*f) + (1 + I
)*x*Sin[e]))/(Sqrt[c + d]*(Cos[e] + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e
]]) + ((-(B*c) + A*d)*(Cos[e/2] + I*Sin[e/2])*((1 - I)*x*Cos[e] - (1 + I)*
x*Sin[e] + (RootSum[-d + (2*I)*c*E^(I*e)*#1^2 + d*E^((2*I)*e)*#1^4 & , ((1
 - I)*d*Sqrt[E^((-I)*e)]*f*x + (2 + 2*I)*d*Sqrt[E^((-I)*e)]*Log[E^((I/2)*f
*x) - #1] + Sqrt[d]*Sqrt[c + d]*f*x*#1 + (2*I)*Sqrt[d]*Sqrt[c + d]*Log[E^(
(I/2)*f*x) - #1]*#1 - ((1 + I)*c*f*x*#1^2)/Sqrt[E^((-I)*e)] + ((2 - 2*I)*c
*Log[E^((I/2)*f*x) - #1]*#1^2)/Sqrt[E^((-I)*e)] - I*Sqrt[d]*Sqrt[c + d]*E^
(I*e)*f*x*#1^3 + 2*Sqrt[d]*Sqrt[c + d]*E^(I*e)*Log[E^((I/2)*f*x) - #1]*#1^
3)/(d - I*c*E^(I*e)*#1^2) & ]*Sqrt[Cos[e] - I*Sin[e]]*(-1 - I*Cos[e] + Sin
[e]))/(4*f)))/(Sqrt[c + d]*(Cos[e] + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[
e]]) + ((2 - 2*I)*B*Sqrt[d]*(Cos[e/2] + Sin[e/2])*Sin[(f*x)/2])/f)*Sqrt...
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {3042, 3460, 3042, 3252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a \sin(e+fx)+a}(A+B \sin(e+fx))}{c+d \sin(e+fx)} \, dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sqrt{a \sin(e+fx)+a}(A+B\sin(e+fx))}{c+d\sin(e+fx)}dx$$

$\Big\downarrow$ 3460

$$-\frac{(Bc-Ad)\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{d}-\frac{2aB\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}}$$

$\Big\downarrow$ 3042

$$-\frac{(Bc-Ad)\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{d}-\frac{2aB\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}}$$

$\Big\downarrow$ 3252

$$\frac{2a(Bc-Ad)\int \frac{1}{a(c+d)-\frac{a^2d\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{df}-\frac{2aB\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}}$$

$\Big\downarrow$ 221

$$\frac{2\sqrt{a}(Bc-Ad)\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{3/2}f\sqrt{c+d}}-\frac{2aB\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}}$$

input
```
Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),x
]
```

output
```
(2*Sqrt[a]*(B*c - A*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]
*Sqrt[a + a*Sin[e + f*x]])])/(d^(3/2)*Sqrt[c + d]*f) - (2*a*B*Cos[e + f*x]
)/(d*f*Sqrt[a + a*Sin[e + f*x]])
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.39

method	result
default	$-\frac{2(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(A\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}\,d}{\sqrt{(c+d)ad}}\right)ad-B\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}\,d}{\sqrt{(c+d)ad}}\right)ac+B\sqrt{-a(\sin(}{d\sqrt{(c+d)ad}\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$

input
```
int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x,method=_RET
URNVERBOSE)
```

output
```
-2*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(A*arctanh((-a*(sin(f*x+e)-1))
^(1/2)*d/((c+d)*a*d)^(1/2))*a*d-B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/((c+
d)*a*d)^(1/2))*a*c+B*(-a*(sin(f*x+e)-1))^(1/2)*((c+d)*a*d)^(1/2))/d/((c+d)
*a*d)^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 651, normalized size of antiderivative = 6.51

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algo
rithm="fricas")
```

output
```
[-1/2*((B*c - A*d + (B*c - A*d)*cos(f*x + e) + (B*c - A*d)*sin(f*x + e))*s
qrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (
6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 +
d^3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*
c*d^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x +
 e) + a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) +
(a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*co
s(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)
^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 -
2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(B*cos(f*x + e)
 - B*sin(f*x + e) + B)*sqrt(a*sin(f*x + e) + a))/(d*f*cos(f*x + e) + d*f*s
in(f*x + e) + d*f), ((B*c - A*d + (B*c - A*d)*cos(f*x + e) + (B*c - A*d)*s
in(f*x + e))*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*s
in(f*x + e) - c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) - 2*(B*cos(f
*x + e) - B*sin(f*x + e) + B)*sqrt(a*sin(f*x + e) + a))/(d*f*cos(f*x + e)
+ d*f*sin(f*x + e) + d*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(1/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \int \frac{(B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}}{d\sin(fx+e)+c}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(d*sin(f*x + e) +
c), x)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \frac{\sqrt{2}\left(\frac{2\,B\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{d}+\frac{\sqrt{2}\left(Bc\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-Ad\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\arctan\left(\right.}{\sqrt{-cd-d^2}d}\right.}{f}$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algo
rithm="giac")
```

output
```
sqrt(2)*(2*B*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1
/2*e)/d + sqrt(2)*(B*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - A*d*sgn(cos(-
1/4*pi + 1/2*f*x + 1/2*e)))*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e
)/sqrt(-c*d - d^2))/(sqrt(-c*d - d^2)*d))*sqrt(a)/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \int \frac{(A+B\,\sin\left(e+f\,x\right))\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{c+d\,\sin\left(e+f\,x\right)}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c + d*sin(e + f*x))
,x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c + d*sin(e + f*x))
, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \sqrt{a}\left(\left(\int \frac{\sqrt{\sin\left(fx+e\right)+1}}{\sin\left(fx+e\right)d+c}dx\right)a+\left(\int \frac{\sqrt{\sin\left(fx+e\right)+1}\,\sin\left(fx+e\right)}{\sin\left(fx+e\right)d+c}dx\right)b\right)$$

input
```
int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

output
```
sqrt(a)*(int(sqrt(sin(e + f*x) + 1)/(sin(e + f*x)*d + c),x)*a + int((sqrt(
sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)*d + c),x)*b)
```

3.291 $\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 126

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = -\frac{\sqrt{a}(Ad+B(c+2d))\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{d^{3/2}(c+d)^{3/2}f} + \frac{a(Bc-Ad)\cos(e+fx)}{d(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}$$

output
```
-a^(1/2)*(A*d+B*(c+2*d))*arctanh(a^(1/2)*d^(1/2)*cos(f*x+e)/(c+d)^(1/2)/(a
+a*sin(f*x+e))^(1/2))/d^(3/2)/(c+d)^(3/2)/f+a*(-A*d+B*c)*cos(f*x+e)/d/(c+d
)/f/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 12.54 (sec) , antiderivative size = 901, normalized size of antiderivative = 7.15

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f
*x])^2,x]
```

output
```
((1/4 + I/4)*Sqrt[a*(1 + Sin[e + f*x])]*(((A*d + B*(c + 2*d))*(Cos[e/2] +
I*Sin[e/2])*((-1 + I)*x*Cos[e] + (RootSum[-d + (2*I)*c*E^(I*e)*#1^2 + d*E^
((2*I)*e)*#1^4 & , ((1 + I)*d*Sqrt[E^((-I)*e)]*f*x - (2 - 2*I)*d*Sqrt[E^((
-I)*e)]*Log[E^((I/2)*f*x) - #1] - I*Sqrt[d]*Sqrt[c + d]*f*x*#1 + 2*Sqrt[d]
*Sqrt[c + d]*Log[E^((I/2)*f*x) - #1]*#1 + ((1 - I)*c*f*x*#1^2)/Sqrt[E^((-I
)*e)] + ((2 + 2*I)*c*Log[E^((I/2)*f*x) - #1]*#1^2)/Sqrt[E^((-I)*e)] - Sqrt
[d]*Sqrt[c + d]*E^(I*e)*f*x*#1^3 - (2*I)*Sqrt[d]*Sqrt[c + d]*E^(I*e)*Log[E
^((I/2)*f*x) - #1]*#1^3)/(d - I*c*E^(I*e)*#1^2) & ]*(Cos[e] + I*(-1 + Sin[
e]))*Sqrt[Cos[e] - I*Sin[e]])/(4*f) + (1 + I)*x*Sin[e]))/((c + d)^(3/2)*(C
os[e] + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]]) + ((A*d + B*(c + 2*d))*(
Cos[e/2] + I*Sin[e/2])*((1 - I)*x*Cos[e] - (1 + I)*x*Sin[e] + (RootSum[-d
+ (2*I)*c*E^(I*e)*#1^2 + d*E^((2*I)*e)*#1^4 & , ((1 - I)*d*Sqrt[E^((-I)*e)
]*f*x + (2 + 2*I)*d*Sqrt[E^((-I)*e)]*Log[E^((I/2)*f*x) - #1] + Sqrt[d]*Sqr
t[c + d]*f*x*#1 + (2*I)*Sqrt[d]*Sqrt[c + d]*Log[E^((I/2)*f*x) - #1]*#1 - (
(1 + I)*c*f*x*#1^2)/Sqrt[E^((-I)*e)] + ((2 - 2*I)*c*Log[E^((I/2)*f*x) - #1
]*#1^2)/Sqrt[E^((-I)*e)] - I*Sqrt[d]*Sqrt[c + d]*E^(I*e)*f*x*#1^3 + 2*Sqrt
[d]*Sqrt[c + d]*E^(I*e)*Log[E^((I/2)*f*x) - #1]*#1^3)/(d - I*c*E^(I*e)*#1^
2) & ]*Sqrt[Cos[e] - I*Sin[e]]*(-1 - I*Cos[e] + Sin[e]))/(4*f)))/((c + d)^
(3/2)*(Cos[e] + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]]) - ((2 - 2*I)*Sqr
t[d]*(-(B*c) + A*d)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c + d)*f*(...
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {3042, 3459, 3042, 3252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}dx$$

$$\Big\downarrow \text{3459}$$

$$\frac{(Ad+B(c+2d))\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{2d(c+d)}+\frac{a(Bc-Ad)\cos(e+fx)}{df(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(Ad+B(c+2d))\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{2d(c+d)}+\frac{a(Bc-Ad)\cos(e+fx)}{df(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}$$

$$\Big\downarrow \text{3252}$$

$$\frac{a(Bc-Ad)\cos(e+fx)}{df(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{a(Ad+B(c+2d))\int \frac{1}{a(c+d)-\frac{a^2d\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{df(c+d)}$$

$$\Big\downarrow \text{221}$$

$$\frac{a(Bc-Ad)\cos(e+fx)}{df(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{\sqrt{a}(Ad+B(c+2d))\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{3/2}f(c+d)^{3/2}}$$

input
```
Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2
,x]
```

output
```
-((Sqrt[a]*(A*d + B*(c + 2*d))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqr
t[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(d^(3/2)*(c + d)^(3/2)*f)) + (a*(B*c
- A*d)*Cos[e + f*x])/(d*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e +
f*x]))
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(110) = 220$.

Time = 0.48 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.17

method	result
default	$-\frac{(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(\sin(fx+e)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,d}{\sqrt{cda+d^2a}}\right)ad(Ad+Bc+2Bd)+A\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,d}{\sqrt{cda+d^2a}}\right.\right.}{d(c+d)(c+d\sin(fx+e)}$

input
```
int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x,method=_R
ETURNVERBOSE)
```

output
```
-(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(sin(f*x+e)*arctanh((a-a*sin(f*x
+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*d*(A*d+B*c+2*B*d)+A*arctanh((a-a*sin(f
*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c*d+B*arctanh((a-a*sin(f*x+e))^(1/2)
*d/(a*c*d+a*d^2)^(1/2))*a*c^2+2*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+
a*d^2)^(1/2))*a*c*d+A*(a-a*sin(f*x+e))^(1/2)*((c+d)*a*d)^(1/2)*d-B*(a-a*si
n(f*x+e))^(1/2)*((c+d)*a*d)^(1/2)*c)/d/(c+d)/(c+d*sin(f*x+e))/((c+d)*a*d)^
(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(110) = 220$.

Time = 0.78 (sec) , antiderivative size = 1012, normalized size of antiderivative = 8.03

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, al
gorithm="fricas")
```

output
```
[-1/4*((B*c^2 + (A + 3*B)*c*d + (A + 2*B)*d^2 - (B*c*d + (A + 2*B)*d^2)*co
s(f*x + e)^2 + (B*c^2 + (A + 2*B)*c*d)*cos(f*x + e) + (B*c^2 + (A + 3*B)*c
*d + (A + 2*B)*d^2 + (B*c*d + (A + 2*B)*d^2)*cos(f*x + e))*sin(f*x + e))*s
qrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (
6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 +
d^3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*
c*d^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x +
 e) + a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) +
(a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*co
s(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)
^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 -
2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(B*c - A*d + (B
*c - A*d)*cos(f*x + e) - (B*c - A*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a
))/((c*d^2 + d^3)*f*cos(f*x + e)^2 - (c^2*d + c*d^2)*f*cos(f*x + e) - (c^2
*d + 2*c*d^2 + d^3)*f - ((c*d^2 + d^3)*f*cos(f*x + e) + (c^2*d + 2*c*d^2 +
 d^3)*f)*sin(f*x + e)), 1/2*((B*c^2 + (A + 3*B)*c*d + (A + 2*B)*d^2 - (B*c
*d + (A + 2*B)*d^2)*cos(f*x + e)^2 + (B*c^2 + (A + 2*B)*c*d)*cos(f*x + e)
+ (B*c^2 + (A + 3*B)*c*d + (A + 2*B)*d^2 + (B*c*d + (A + 2*B)*d^2)*cos(f*x
 + e))*sin(f*x + e))*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e) +
 a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) -...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx=\text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(1/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \int \frac{(B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}}{(d\sin(fx+e)+c)^2}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(d*sin(f*x + e) +
c)^2, x)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.68

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = -\frac{\sqrt{2}\sqrt{a}\left(\frac{\sqrt{2}(Bc\,\mathrm{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))+Ad\,\mathrm{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))+2Bd\,\mathrm{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e)))\arctan\left(\frac{\sqrt{2}d\sin(-\frac{1}{4}\pi+\frac{1}{2}\right.}{\sqrt{-cd-c}}}{(cd+d^2)\sqrt{-cd-d^2}}\right.}{2f}$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, al
gorithm="giac")
```

output
```
-1/2*sqrt(2)*sqrt(a)*(sqrt(2)*(B*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + A
*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*B*d*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e)))*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/sqrt(-c*d - d^2)
)/((c*d + d^2)*sqrt(-c*d - d^2)) - 2*(B*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*
e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - A*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
))*sin(-1/4*pi + 1/2*f*x + 1/2*e))/((2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2
- c - d)*(c*d + d^2)))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$$
$$=\int \frac{(A+B\,\sin\left(e+f\,x\right))\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^2}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c + d*sin(e + f*x))
^2,x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c + d*sin(e + f*x))
^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$$
$$=\sqrt{a}\left(\left(\int \frac{\sqrt{\sin\left(fx+e\right)+1}}{\sin\left(fx+e\right)^2d^2+2\sin\left(fx+e\right)cd+c^2}dx\right)a+\left(\int \frac{\sqrt{\sin\left(fx+e\right)+1}\,\sin\left(fx+e\right)}{\sin\left(fx+e\right)^2d^2+2\sin\left(fx+e\right)cd+c^2}dx\right)b\right)$$

input
```
int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

output
```
sqrt(a)*(int(sqrt(sin(e + f*x) + 1)/(sin(e + f*x)**2*d**2 + 2*sin(e + f*x)
*c*d + c**2),x)*a + int((sqrt(sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x
)**2*d**2 + 2*sin(e + f*x)*c*d + c**2),x)*b)
```

3.292 $\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 192

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = -\frac{\sqrt{a}(3Ad+B(c+4d))\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{4d^{3/2}(c+d)^{5/2}f} + \frac{a(Bc-Ad)\cos(e+fx)}{2d(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} - \frac{a(3Ad+B(c+4d))\cos(e+fx)}{4d(c+d)^2f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}$$

output
```
-1/4*a^(1/2)*(3*A*d+B*(c+4*d))*arctanh(a^(1/2)*d^(1/2)*cos(f*x+e)/(c+d)^(1
/2)/(a+a*sin(f*x+e))^(1/2))/d^(3/2)/(c+d)^(5/2)/f+1/2*a*(-A*d+B*c)*cos(f*x
+e)/d/(c+d)/f/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2-1/4*a*(3*A*d+B*(c+
4*d))*cos(f*x+e)/d/(c+d)^2/f/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 15.17 (sec) , antiderivative size = 967, normalized size of antiderivative = 5.04

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f
*x])^3,x]
```

output
```
((1/16 + I/16)*Sqrt[a*(1 + Sin[e + f*x])]*(((3*A*d + B*(c + 4*d))*(Cos[e/2
] + I*Sin[e/2])*((-1 + I)*x*Cos[e] + (RootSum[-d + (2*I)*c*E^(I*e)*#1^2 +
d*E^((2*I)*e)*#1^4 & , ((1 + I)*d*Sqrt[E^((-I)*e)]*f*x - (2 - 2*I)*d*Sqrt[
E^((-I)*e)]*Log[E^((I/2)*f*x) - #1] - I*Sqrt[d]*Sqrt[c + d]*f*x*#1 + 2*Sqr
t[d]*Sqrt[c + d]*Log[E^((I/2)*f*x) - #1]*#1 + ((1 - I)*c*f*x*#1^2)/Sqrt[E^
((-I)*e)] + ((2 + 2*I)*c*Log[E^((I/2)*f*x) - #1]*#1^2)/Sqrt[E^((-I)*e)] -
Sqrt[d]*Sqrt[c + d]*E^(I*e)*f*x*#1^3 - (2*I)*Sqrt[d]*Sqrt[c + d]*E^(I*e)*L
og[E^((I/2)*f*x) - #1]*#1^3)/(d - I*c*E^(I*e)*#1^2) & ]*(Cos[e] + I*(-1 +
Sin[e]))*Sqrt[Cos[e] - I*Sin[e]])/(4*f) + (1 + I)*x*Sin[e]))/((c + d)^(5/2
)*(Cos[e] + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]]) + ((3*A*d + B*(c + 4
*d))*(Cos[e/2] + I*Sin[e/2])*((1 - I)*x*Cos[e] - (1 + I)*x*Sin[e] + (RootS
um[-d + (2*I)*c*E^(I*e)*#1^2 + d*E^((2*I)*e)*#1^4 & , ((1 - I)*d*Sqrt[E^((
-I)*e)]*f*x + (2 + 2*I)*d*Sqrt[E^((-I)*e)]*Log[E^((I/2)*f*x) - #1] + Sqrt[
d]*Sqrt[c + d]*f*x*#1 + (2*I)*Sqrt[d]*Sqrt[c + d]*Log[E^((I/2)*f*x) - #1]*
#1 - ((1 + I)*c*f*x*#1^2)/Sqrt[E^((-I)*e)] + ((2 - 2*I)*c*Log[E^((I/2)*f*x
) - #1]*#1^2)/Sqrt[E^((-I)*e)] - I*Sqrt[d]*Sqrt[c + d]*E^(I*e)*f*x*#1^3 +
2*Sqrt[d]*Sqrt[c + d]*E^(I*e)*Log[E^((I/2)*f*x) - #1]*#1^3)/(d - I*c*E^(I*
e)*#1^2) & ]*Sqrt[Cos[e] - I*Sin[e]]*(-1 - I*Cos[e] + Sin[e]))/(4*f)))/((c
 + d)^(5/2)*(Cos[e] + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]]) - ((4 - 4*
I)*Sqrt[d]*(-(B*c) + A*d)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c + ...
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {3042, 3459, 3042, 3251, 3042, 3252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\sin(e+fx)+a}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}dx$$

$$\Big\downarrow \text{3459}$$

$$\frac{(3Ad+B(c+4d))\int \frac{\sqrt{\sin(e+fx)a+a}}{(c+d\sin(e+fx))^2}dx}{4d(c+d)}+\frac{a(Bc-Ad)\cos(e+fx)}{2df(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(3Ad+B(c+4d))\int \frac{\sqrt{\sin(e+fx)a+a}}{(c+d\sin(e+fx))^2}dx}{4d(c+d)}+\frac{a(Bc-Ad)\cos(e+fx)}{2df(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \text{3251}$$

$$\frac{(3Ad+B(c+4d))\left(\frac{\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{2(c+d)}-\frac{a\cos(e+fx)}{f(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}\right)}{4d(c+d)}+\frac{a(Bc-Ad)\cos(e+fx)}{2df(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(3Ad+B(c+4d))\left(\frac{\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{2(c+d)}-\frac{a\cos(e+fx)}{f(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}\right)}{4d(c+d)}+\frac{a(Bc-Ad)\cos(e+fx)}{2df(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \text{3252}$$

$$\frac{(3Ad+B(c+4d))\left(-\frac{a\int \frac{1}{a(c+d)-\frac{a^2d\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f(c+d)}-\frac{a\cos(e+fx)}{f(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}\right)}{4d(c+d)}+\frac{a(Bc-Ad)\cos(e+fx)}{2df(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow 221$$

$$\frac{(3Ad+B(c+4d))\left(-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{d}f(c+d)^{3/2}}-\frac{a\cos(e+fx)}{f(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}\right)}{4d(c+d)}+\frac{a(Bc-Ad)\cos(e+fx)}{2df(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

input
```
Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^3
,x]
```

output
```
(a*(B*c - A*d)*Cos[e + f*x])/(2*d*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c +
d*Sin[e + f*x])^2) + ((3*A*d + B*(c + 4*d))*(-((Sqrt[a]*ArcTanh[(Sqrt[a]*S
qrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])]])/(Sqrt[d]*(c
 + d)^(3/2)*f)) - (a*Cos[e + f*x])/((c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c +
 d*Sin[e + f*x]))))/(4*d*(c + d))
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 627 vs. $2(168) = 336$.

Time = 0.51 (sec) , antiderivative size = 628, normalized size of antiderivative = 3.27

method	result
default	$\frac{\left(\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,d}{\sqrt{cda+d^2a}}\right)a^2d^2(3Ad+Bc+4Bd)\cos(fx+e)^2-2\sin(fx+e)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,d}{\sqrt{cda+d^2a}}\right)a^2cd(3Ad+Bc+4Bd)+\right.}{}$

input
```
int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x,method=_R
ETURNVERBOSE)
```

output
```
1/4/a*(arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^2*(3*A*
d+B*c+4*B*d)*cos(f*x+e)^2-2*sin(f*x+e)*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a
*c*d+a*d^2)^(1/2))*a^2*c*d*(3*A*d+B*c+4*B*d)+3*A*(a-a*sin(f*x+e))^(3/2)*((
c+d)*a*d)^(1/2)*d^2-3*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/
2))*a^2*c^2*d-3*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^
2*d^3+B*(a-a*sin(f*x+e))^(3/2)*((c+d)*a*d)^(1/2)*c*d+4*B*(a-a*sin(f*x+e))^
(3/2)*((c+d)*a*d)^(1/2)*d^2-a^2*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*
d^2)^(1/2))*B*c^3-4*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2)
)*a^2*c^2*d-B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*
d^2-4*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^3-5*A*
(a-a*sin(f*x+e))^(1/2)*((c+d)*a*d)^(1/2)*a*c*d-5*A*(a-a*sin(f*x+e))^(1/2)*
((c+d)*a*d)^(1/2)*a*d^2+B*(a-a*sin(f*x+e))^(1/2)*((c+d)*a*d)^(1/2)*a*c^2-3
*B*(a-a*sin(f*x+e))^(1/2)*((c+d)*a*d)^(1/2)*a*c*d-4*B*(a-a*sin(f*x+e))^(1/
2)*((c+d)*a*d)^(1/2)*a*d^2)*(-a*(sin(f*x+e)-1))^(1/2)*(1+sin(f*x+e))/((c+d
)*a*d)^(1/2)/(c+d*sin(f*x+e))^2/(c+d)^2/d/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2
)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 717 vs. $2(168) = 336$.

Time = 1.14 (sec) , antiderivative size = 1750, normalized size of antiderivative = 9.11

$$\int \frac{\sqrt{a + a\sin(e + fx)}(A + B\sin(e + fx))}{(c + d\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, al
gorithm="fricas")
```

output
```
[-1/16*((B*c^3 + 3*(A + 2*B)*c^2*d + 3*(2*A + 3*B)*c*d^2 + (3*A + 4*B)*d^3
 - (B*c*d^2 + (3*A + 4*B)*d^3)*cos(f*x + e)^3 - (2*B*c^2*d + 3*(2*A + 3*B)
*c*d^2 + (3*A + 4*B)*d^3)*cos(f*x + e)^2 + (B*c^3 + (3*A + 4*B)*c^2*d + B*
c*d^2 + (3*A + 4*B)*d^3)*cos(f*x + e) + (B*c^3 + 3*(A + 2*B)*c^2*d + 3*(2*
A + 3*B)*c*d^2 + (3*A + 4*B)*d^3 - (B*c*d^2 + (3*A + 4*B)*d^3)*cos(f*x + e
)^2 + 2*(B*c^2*d + (3*A + 4*B)*c*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a/(
c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d
 + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3)*co
s(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^2 +
 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a
)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*
cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x +
 e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^
2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*c
os(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(B*c^2 - (5*A + B)*c*d
 + (A + 4*B)*d^2 - (B*c*d + (3*A + 4*B)*d^2)*cos(f*x + e)^2 + (B*c^2 - (5*
A + 2*B)*c*d - 2*A*d^2)*cos(f*x + e) - (B*c^2 - (5*A + B)*c*d + (A + 4*B)*
d^2 + (B*c*d + (3*A + 4*B)*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x
 + e) + a))/((c^2*d^3 + 2*c*d^4 + d^5)*f*cos(f*x + e)^3 + (2*c^3*d^2 + 5*c
^2*d^3 + 4*c*d^4 + d^5)*f*cos(f*x + e)^2 - (c^4*d + 2*c^3*d^2 + 2*c^2*d...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx=\text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(1/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \int \frac{(B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}}{(d\sin(fx+e)+c)^3}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(d*sin(f*x + e) +
c)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. 2(168) = 336.

Time = 0.27 (sec) , antiderivative size = 422, normalized size of antiderivative = 2.20

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, al
gorithm="giac")
```

output
```
-1/8*sqrt(2)*sqrt(a)*(sqrt(2)*(B*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3
*A*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 4*B*d*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e)))*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/sqrt(-c*d - d^
2))/((c^2*d + 2*c*d^2 + d^3)*sqrt(-c*d - d^2)) + 2*(2*B*c*d*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 6*A*d^2*sgn(cos(-
1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 8*B*d^2*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + B*c^2*s
gn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 5*A*c*
d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 3*B
*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) -
5*A*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)
 - 4*B*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2
*e))/((c^2*d + 2*c*d^2 + d^3)*(2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - c -
d)^2))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$$
$$=\int \frac{(A+B\,\sin(e+f\,x))\,\sqrt{a+a\,\sin(e+f\,x)}}{(c+d\,\sin(e+f\,x))^3}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c + d*sin(e + f*x))
^3,x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c + d*sin(e + f*x))
^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$$
$$=\sqrt{a}\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}}{\sin(fx+e)^3 d^3+3\sin(fx+e)^2 c\,d^2+3\sin(fx+e)\,c^2d+c^3}dx\right)a + \left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)^3 d^3+3\sin(fx+e)^2 c\,d^2+3\sin(fx+e)\,c^2d+c^3}dx\right)b\right)$$

input
```
int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)
```

output
```
sqrt(a)*(int(sqrt(sin(e + f*x) + 1)/(sin(e + f*x)**3*d**3 + 3*sin(e + f*x)
**2*c*d**2 + 3*sin(e + f*x)*c**2*d + c**3),x)*a + int((sqrt(sin(e + f*x) +
 1)*sin(e + f*x))/(sin(e + f*x)**3*d**3 + 3*sin(e + f*x)**2*c*d**2 + 3*sin
(e + f*x)*c**2*d + c**3),x)*b)
```

3.293 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

Optimal result

Integrand size = 37, antiderivative size = 374

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = \frac{4a^2(c+d)\left(15c^2+10cd+7d^2\right)\left(11A(c-17d)d-3B\left(c^2-9cd+56d^2\right)\right)\cos(e+fx)}{3465d^2f\sqrt{a+a\sin(e+fx)}}$$
$$+\frac{8a(5c-d)(c+d)\left(11A(c-17d)d-3B\left(c^2-9cd+56d^2\right)\right)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3465df}$$
$$+\frac{4(c+d)\left(11A(c-17d)d-3B\left(c^2-9cd+56d^2\right)\right)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{1155f}$$
$$+\frac{2a^2\left(11A(c-17d)d-3B\left(c^2-9cd+56d^2\right)\right)\cos(e+fx)(c+d\sin(e+fx))^3}{693d^2f\sqrt{a+a\sin(e+fx)}}$$
$$+\frac{2a^2(3B(c-4d)-11Ad)\cos(e+fx)(c+d\sin(e+fx))^4}{99d^2f\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{2aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^4}{11df}$$

output
```
4/3465*a^2*(c+d)*(15*c^2+10*c*d+7*d^2)*(11*A*(c-17*d)*d-3*B*(c^2-9*c*d+56*
d^2))*cos(f*x+e)/d^2/f/(a+a*sin(f*x+e))^(1/2)+8/3465*a*(5*c-d)*(c+d)*(11*A
*(c-17*d)*d-3*B*(c^2-9*c*d+56*d^2))*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/d/f+
4/1155*(c+d)*(11*A*(c-17*d)*d-3*B*(c^2-9*c*d+56*d^2))*cos(f*x+e)*(a+a*sin(
f*x+e))^(3/2)/f+2/693*a^2*(11*A*(c-17*d)*d-3*B*(c^2-9*c*d+56*d^2))*cos(f*x
+e)*(c+d*sin(f*x+e))^3/d^2/f/(a+a*sin(f*x+e))^(1/2)+2/99*a^2*(3*B*(c-4*d)-
11*A*d)*cos(f*x+e)*(c+d*sin(f*x+e))^4/d^2/f/(a+a*sin(f*x+e))^(1/2)-2/11*a*
B*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^4/d/f
```

Mathematica [A] (verified)

Time = 6.97 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.04

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = -\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}(92400Ac^3+72072Bc^3+216216Ac^2d+195\ldots}{}$$

input
```
Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f
*x])^3,x]
```

output
```
-1/27720*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x]
)]*(92400*A*c^3 + 72072*B*c^3 + 216216*A*c^2*d + 195624*B*c^2*d + 195624*A
*c*d^2 + 177474*B*c*d^2 + 59158*A*d^3 + 55482*B*d^3 - 8*(11*A*d*(189*c^2 +
 351*c*d + 137*d^2) + 3*B*(231*c^3 + 1287*c^2*d + 1507*c*d^2 + 581*d^3))*C
os[2*(e + f*x)] + 70*d^2*(33*B*c + 11*A*d + 21*B*d)*Cos[4*(e + f*x)] + 184
80*A*c^3*Sin[e + f*x] + 33264*B*c^3*Sin[e + f*x] + 99792*A*c^2*d*Sin[e + f
*x] + 100188*B*c^2*d*Sin[e + f*x] + 100188*A*c*d^2*Sin[e + f*x] + 105468*B
*c*d^2*Sin[e + f*x] + 35156*A*d^3*Sin[e + f*x] + 34734*B*d^3*Sin[e + f*x]
- 5940*B*c^2*d*Sin[3*(e + f*x)] - 5940*A*c*d^2*Sin[3*(e + f*x)] - 11220*B*
c*d^2*Sin[3*(e + f*x)] - 3740*A*d^3*Sin[3*(e + f*x)] - 4935*B*d^3*Sin[3*(e
 + f*x)] + 315*B*d^3*Sin[5*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2]))
```

Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.84, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.378, Rules used = {3042, 3455, 27, 3042, 3460, 3042, 3249, 3042, 3240, 27, 3042, 3230, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3dx$$

$$\Big\downarrow \text{3455}$$

$$\frac{2\int \frac{1}{2}\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^3(a(11Ad+B(c+8d))-a(3B(c-4d)-11Ad)\sin(e+fx))dx}{11d}-\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^4}{11df}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^3(a(11Ad+B(c+8d))-a(3B(c-4d)-11Ad)\sin(e+fx))dx}{11d}-\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^4}{11df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^3(a(11Ad+B(c+8d))-a(3B(c-4d)-11Ad)\sin(e+fx))dx}{11d}-\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^4}{11df}$$

$$\Big\downarrow \text{3460}$$

$$\frac{\frac{2a^2(3B(c-4d)-11Ad)\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}}-\frac{a(11Ad(c-17d)-3B(c^2-9cd+56d^2))\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^3dx}{9d}}{11d}-\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^4}{11df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2a^2(3B(c-4d)-11Ad)\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}}-\frac{a\left(11Ad(c-17d)-3B\left(c^2-9cd+56d^2\right)\right)\int\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^3dx}{9d}}{11d}-\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^4}{11df}$$

$$\Big\downarrow \text{3249}$$

$$\frac{\frac{2a^2(3B(c-4d)-11Ad)\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}}-\frac{a\left(11Ad(c-17d)-3B\left(c^2-9cd+56d^2\right)\right)\left(\frac{6}{7}(c+d)\int\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2dx-\right.}{9d}}{11d}$$

$$\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^4}{11df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2a^2(3B(c-4d)-11Ad)\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}}-\frac{a\left(11Ad(c-17d)-3B\left(c^2-9cd+56d^2\right)\right)\left(\frac{6}{7}(c+d)\int\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2dx-\right.}{9d}}{11d}$$

$$\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^4}{11df}$$

$$\Big\downarrow \text{3240}$$

$$\frac{\frac{2a^2(3B(c-4d)-11Ad)\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}}-\frac{a\left(11Ad(c-17d)-3B\left(c^2-9cd+56d^2\right)\right)\left(\frac{6}{7}(c+d)\left(\frac{2\int\frac{1}{2}\sqrt{\sin(e+fx)a+a}\left(a\left(5c^2+3d^2\right)+2a(5c\right.}{5a}\right.\right.}{9d}}{11d}$$

$$\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^4}{11df}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2a^2(3B(c-4d)-11Ad)\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}}-\frac{a\left(11Ad(c-17d)-3B\left(c^2-9cd+56d^2\right)\right)\left(\frac{6}{7}(c+d)\left(\frac{\int\sqrt{\sin(e+fx)a+a}\left(a\left(5c^2+3d^2\right)+2a(5c-d\right.}{5a}\right.\right.}{9d}}{11d}$$

$$\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^4}{11df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2a^2(3B(c-4d)-11Ad)\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}}-\frac{a(11Ad(c-17d)-3B(c^2-9cd+56d^2))\left(\frac{6}{7}(c+d)\left(\frac{\int\sqrt{\sin(e+fx)a+a}\left(a\left(5c^2+3d^2\right)+2a(5c-d)\right.}{5a}\right.\right.}{9d}}{11d}$$

$$\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^4}{11df}$$

$\big\downarrow$ 3230

$$\frac{\frac{2a^2(3B(c-4d)-11Ad)\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}}-\frac{a(11Ad(c-17d)-3B(c^2-9cd+56d^2))\left(\frac{6}{7}(c+d)\left(\frac{\frac{1}{3}a\left(15c^2+10cd+7d^2\right)\int\sqrt{\sin(e+fx)a+a}dx}{5a}\right.\right.}{}}{11d}$$

$$\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^4}{11df}$$

$\big\downarrow$ 3042

$$\frac{\frac{2a^2(3B(c-4d)-11Ad)\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}}-\frac{a(11Ad(c-17d)-3B(c^2-9cd+56d^2))\left(\frac{6}{7}(c+d)\left(\frac{\frac{1}{3}a\left(15c^2+10cd+7d^2\right)\int\sqrt{\sin(e+fx)a+a}dx}{5a}\right.\right.}{}}{11d}$$

$$\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^4}{11df}$$

$\big\downarrow$ 3125

$$\frac{\frac{2a^2(3B(c-4d)-11Ad)\cos(e+fx)(c+d\sin(e+fx))^4}{9df\sqrt{a\sin(e+fx)+a}}-\frac{a(11Ad(c-17d)-3B(c^2-9cd+56d^2))\left(\frac{6}{7}(c+d)\left(\frac{-\frac{2a^2\left(15c^2+10cd+7d^2\right)\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{4ad(}{}}{5a}\right.\right.}{}}{11d}$$

$$\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^4}{11df}$$

input
```
Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3
,x]
```

output
```
(-2*a*B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^4)/(11*
d*f) + ((2*a^2*(3*B*(c - 4*d) - 11*A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^
4)/(9*d*f*Sqrt[a + a*Sin[e + f*x]]) - (a*(11*A*(c - 17*d)*d - 3*B*(c^2 - 9
*c*d + 56*d^2))*((-2*a*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(7*f*Sqrt[a +
a*Sin[e + f*x]]) + (6*(c + d)*((-2*d^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(
3/2))/(5*a*f) + ((-2*a^2*(15*c^2 + 10*c*d + 7*d^2)*Cos[e + f*x])/(3*f*Sqrt
[a + a*Sin[e + f*x]]) - (4*a*(5*c - d)*d*Cos[e + f*x]*Sqrt[a + a*Sin[e + f
*x]])/(3*f))/(5*a)))/7))/(9*d))/(11*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3240
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])^
m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

rule 3249
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[2*n*((b*c + a*d)/(b*(
2*n + 1)))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 1.43 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.83

method	result
default	$\frac{2(1+\sin(fx+e))a^2(\sin(fx+e)-1)\left(315B\cos(fx+e)^4\sin(fx+e)d^3+\left(385A\,d^3+1155Bc\,d^2+735B\,d^3\right)\cos(fx+e)^4+\left(-1485Ac\,d^2-9\right.}{}$
parts	$\frac{2(1+\sin(fx+e))a^2(\sin(fx+e)-1)d^2(Ad+3Bc)\left(35\sin(fx+e)^4+85\sin(fx+e)^3+102\sin(fx+e)^2+136\sin(fx+e)+272\right)}{315\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}+\frac{2(1+\sin}{}$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x,method=_R
ETURNVERBOSE)
```

output
```
2/3465*(1+sin(f*x+e))*a^2*(sin(f*x+e)-1)*(315*B*cos(f*x+e)^4*sin(f*x+e)*d^
3+(385*A*d^3+1155*B*c*d^2+735*B*d^3)*cos(f*x+e)^4+(-1485*A*c*d^2-935*A*d^3
-1485*B*c^2*d-2805*B*c*d^2-1470*B*d^3)*cos(f*x+e)^2*sin(f*x+e)+(-2079*A*c^
2*d-3861*A*c*d^2-1892*A*d^3-693*B*c^3-3861*B*c^2*d-5676*B*c*d^2-2478*B*d^3
)*cos(f*x+e)^2+(1155*A*c^3+6237*A*c^2*d+6633*A*c*d^2+2431*A*d^3+2079*B*c^3
+6633*B*c^2*d+7293*B*c*d^2+2499*B*d^3)*sin(f*x+e)+5775*A*c^3+14553*A*c^2*d
+14157*A*c*d^2+4499*A*d^3+4851*B*c^3+14157*B*c^2*d+13497*B*c*d^2+4431*B*d^
3)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 637, normalized size of antiderivative = 1.70

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, al
gorithm="fricas")
```

output
```
-2/3465*(315*B*a*d^3*cos(f*x + e)^6 + 35*(33*B*a*c*d^2 + (11*A + 21*B)*a*d
^3)*cos(f*x + e)^5 + 924*(5*A + 3*B)*a*c^3 + 396*(21*A + 19*B)*a*c^2*d + 1
32*(57*A + 47*B)*a*c*d^2 + 4*(517*A + 483*B)*a*d^3 - 5*(297*B*a*c^2*d + 33
*(9*A + 10*B)*a*c*d^2 + 10*(11*A + 21*B)*a*d^3)*cos(f*x + e)^4 - (693*B*a*
c^3 + 297*(7*A + 13*B)*a*c^2*d + 33*(117*A + 172*B)*a*c*d^2 + 2*(946*A + 1
239*B)*a*d^3)*cos(f*x + e)^3 + (231*(5*A + 6*B)*a*c^3 + 99*(42*A + 43*B)*a
*c^2*d + 33*(129*A + 134*B)*a*c*d^2 + (1474*A + 1491*B)*a*d^3)*cos(f*x + e
)^2 + (231*(25*A + 21*B)*a*c^3 + 99*(147*A + 143*B)*a*c^2*d + 33*(429*A +
409*B)*a*c*d^2 + (4499*A + 4431*B)*a*d^3)*cos(f*x + e) + (315*B*a*d^3*cos(
f*x + e)^5 - 924*(5*A + 3*B)*a*c^3 - 396*(21*A + 19*B)*a*c^2*d - 132*(57*A
 + 47*B)*a*c*d^2 - 4*(517*A + 483*B)*a*d^3 - 35*(33*B*a*c*d^2 + (11*A + 12
*B)*a*d^3)*cos(f*x + e)^4 - 5*(297*B*a*c^2*d + 33*(9*A + 17*B)*a*c*d^2 + (
187*A + 294*B)*a*d^3)*cos(f*x + e)^3 + 3*(231*B*a*c^3 + 99*(7*A + 8*B)*a*c
^2*d + 33*(24*A + 29*B)*a*c*d^2 + (319*A + 336*B)*a*d^3)*cos(f*x + e)^2 +
(231*(5*A + 9*B)*a*c^3 + 99*(63*A + 67*B)*a*c^2*d + 33*(201*A + 221*B)*a*c
*d^2 + 17*(143*A + 147*B)*a*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*
x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

Sympy [F]

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx=\int \left(a(\sin{(e+fx)}+1)\right)^{\frac{3}{2}}(A+B\sin{(e+fx)})\,(c+d\sin{(e+fx)})^3\,dx$$

input
```
integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3,x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**(3/2)*(A + B*sin(e + f*x))*(c + d*sin(e +
 f*x))**3, x)
```

Maxima [F]

$$\int (a + a\sin(e + fx))^{3/2}(A + B\sin(e + fx))(c + d\sin(e+fx))^3\,dx = \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^{\frac{3}{2}}(d\sin(fx + e) + c)^3\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e)
+ c)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 755 vs. $2(350) = 700$.

Time = 0.37 (sec) , antiderivative size = 755, normalized size of antiderivative = 2.02

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, al
gorithm="giac")
```

output
```
1/55440*sqrt(2)*(315*B*a*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-11/4
*pi + 11/2*f*x + 11/2*e) + 6930*(24*A*a*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e)) + 16*B*a*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 48*A*a*c^2*d*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 42*B*a*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e)) + 42*A*a*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 36*B*a*c*d^2
*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*A*a*d^3*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e)) + 11*B*a*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-1/4*pi
+ 1/2*f*x + 1/2*e) + 2310*(8*A*a*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) +
 12*B*a*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 36*A*a*c^2*d*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e)) + 30*B*a*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
) + 30*A*a*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 30*B*a*c*d^2*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e)) + 10*A*a*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e)) + 9*B*a*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/2*f*
x + 3/2*e) + 693*(8*B*a*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 24*A*a*c
^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 36*B*a*c^2*d*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e)) + 36*A*a*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 36*
B*a*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*A*a*d^3*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e)) + 13*B*a*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin
(-5/4*pi + 5/2*f*x + 5/2*e) + 495*(12*B*a*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e)) + 12*A*a*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 18*B*a*c...
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx=\int (A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}\,\left(c+d\,\sin\left(e+f\,x\right)\right)^3\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^3
,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^3
, x)
```

Reduce [F]

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = \sqrt{a}\,a\left(\left(\int \sqrt{\sin(fx+e)+1}dx\right)a\,c^3 + \left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)^5\,dx\right)b\,d^3 + \left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)^4\,dx\right)a\,d^3 + 3\left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)^4\,dx\right)bc\,d^2 + \left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)^4\,dx\right)b\,d^3 + 3\left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)^3\,dx\right)ac\,d^2 + \left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)^3\,dx\right)a\,d^3 + 3\left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)^3\,dx\right)b\,c^2d + 3\left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)^3\,dx\right)bc\,d^2 + 3\left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)a\,c^2d + 3\left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)ac\,d^2 + \left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)b\,c^3 + 3\left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)b\,c^2d + \left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)a\,c^3 + 3\left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)a\,c^2d + \left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)b\,c^3\right)$$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x)
```

output

```
sqrt(a)*a*(int(sqrt(sin(e + f*x) + 1),x)*a*c**3 + int(sqrt(sin(e + f*x) +
1)*sin(e + f*x)**5,x)*b*d**3 + int(sqrt(sin(e + f*x) + 1)*sin(e + f*x)**4,
x)*a*d**3 + 3*int(sqrt(sin(e + f*x) + 1)*sin(e + f*x)**4,x)*b*c*d**2 + int
(sqrt(sin(e + f*x) + 1)*sin(e + f*x)**4,x)*b*d**3 + 3*int(sqrt(sin(e + f*x
) + 1)*sin(e + f*x)**3,x)*a*c*d**2 + int(sqrt(sin(e + f*x) + 1)*sin(e + f*
x)**3,x)*a*d**3 + 3*int(sqrt(sin(e + f*x) + 1)*sin(e + f*x)**3,x)*b*c**2*d
 + 3*int(sqrt(sin(e + f*x) + 1)*sin(e + f*x)**3,x)*b*c*d**2 + 3*int(sqrt(s
in(e + f*x) + 1)*sin(e + f*x)**2,x)*a*c**2*d + 3*int(sqrt(sin(e + f*x) + 1
)*sin(e + f*x)**2,x)*a*c*d**2 + int(sqrt(sin(e + f*x) + 1)*sin(e + f*x)**2
,x)*b*c**3 + 3*int(sqrt(sin(e + f*x) + 1)*sin(e + f*x)**2,x)*b*c**2*d + in
t(sqrt(sin(e + f*x) + 1)*sin(e + f*x),x)*a*c**3 + 3*int(sqrt(sin(e + f*x)
+ 1)*sin(e + f*x),x)*a*c**2*d + int(sqrt(sin(e + f*x) + 1)*sin(e + f*x),x)
*b*c**3)
```

3.294 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

Optimal result

Integrand size = 37, antiderivative size = 294

$$\begin{aligned}\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx &= \frac{2a^2(15c^2+10cd+7d^2)\left(3A(c-13d)d-B(c^2-7cd+34d^2)\right)\cos(e+fx)}{315d^2f\sqrt{a+a\sin(e+fx)}}\\ &+\frac{4a(5c-d)\left(3A(c-13d)d-B(c^2-7cd+34d^2)\right)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{315df}\\ &+\frac{2(3A(c-13d)d-B(c^2-7cd+34d^2))\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{105f}\\ &+\frac{2a^2(3Bc-9Ad-10Bd)\cos(e+fx)(c+d\sin(e+fx))^3}{63d^2f\sqrt{a+a\sin(e+fx)}}\\ &-\frac{2aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3}{9df}\end{aligned}$$

output

```
2/315*a^2*(15*c^2+10*c*d+7*d^2)*(3*A*(c-13*d)*d-B*(c^2-7*c*d+34*d^2))*cos(
f*x+e)/d^2/f/(a+a*sin(f*x+e))^(1/2)+4/315*a*(5*c-d)*(3*A*(c-13*d)*d-B*(c^2
-7*c*d+34*d^2))*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/d/f+2/105*(3*A*(c-13*d)*
d-B*(c^2-7*c*d+34*d^2))*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f+2/63*a^2*(-9*A
*d+3*B*c-10*B*d)*cos(f*x+e)*(c+d*sin(f*x+e))^3/d^2/f/(a+a*sin(f*x+e))^(1/2
)-2/9*a*B*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^3/d/f
```

Mathematica [A] (verified)

Time = 4.67 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.91

$$\int (a + a\sin(e + fx))^{3/2}(A + B\sin(e + fx))(c + d\sin(e + fx))^2 \, dx =$$

$$-\frac{a\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right)\sqrt{a(1 + \sin(e + fx))}(4200Ac^2 + 3276Bc^2 + 6552Acd + 5928Bcd}{}$$

input
```
Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f
*x])^2,x]
```

output
```
-1/1260*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])
]*(4200*A*c^2 + 3276*B*c^2 + 6552*A*c*d + 5928*B*c*d + 2964*A*d^2 + 2689*B
*d^2 - 4*(9*A*d*(14*c + 13*d) + B*(63*c^2 + 234*c*d + 137*d^2))*Cos[2*(e +
 f*x)] + 35*B*d^2*Cos[4*(e + f*x)] + 840*A*c^2*Sin[e + f*x] + 1512*B*c^2*S
in[e + f*x] + 3024*A*c*d*Sin[e + f*x] + 3036*B*c*d*Sin[e + f*x] + 1518*A*d
^2*Sin[e + f*x] + 1598*B*d^2*Sin[e + f*x] - 180*B*c*d*Sin[3*(e + f*x)] - 9
0*A*d^2*Sin[3*(e + f*x)] - 170*B*d^2*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2]))
```

Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.91, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {3042, 3455, 27, 3042, 3460, 3042, 3240, 27, 3042, 3230, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e + fx) + a)^{3/2}(A + B\sin(e + fx))(c + d\sin(e + fx))^2 \, dx$$

$$\Big\downarrow \; 3042$$

$$\int (a\sin(e + fx) + a)^{3/2}(A + B\sin(e + fx))(c + d\sin(e + fx))^2 dx$$

$$\Big\downarrow \; 3455$$

$$\frac{2\int \frac{1}{2}\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2(a(9Ad+B(c+6d))-a(3Bc-9Ad-10Bd)\sin(e+fx))dx}{9d}-\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^3}{9df}$$

$\Big\downarrow$ 27

$$\frac{\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2(a(9Ad+B(c+6d))-a(3Bc-9Ad-10Bd)\sin(e+fx))dx}{9d}-\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^3}{9df}$$

$\Big\downarrow$ 3042

$$\frac{\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2(a(9Ad+B(c+6d))-a(3Bc-9Ad-10Bd)\sin(e+fx))dx}{9d}-\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^3}{9df}$$

$\Big\downarrow$ 3460

$$\frac{\frac{2a^2(-9Ad+3Bc-10Bd)\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a\sin(e+fx)+a}}-\frac{3a(3Ad(c-13d)-B(c^2-7cd+34d^2))\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2dx}{7d}}{9d}-\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^3}{9df}$$

$\Big\downarrow$ 3042

$$\frac{\frac{2a^2(-9Ad+3Bc-10Bd)\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a\sin(e+fx)+a}}-\frac{3a(3Ad(c-13d)-B(c^2-7cd+34d^2))\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2dx}{7d}}{9d}-\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^3}{9df}$$

$\Big\downarrow$ 3240

$$\frac{\frac{2a^2(-9Ad+3Bc-10Bd)\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a\sin(e+fx)+a}}-\frac{3a(3Ad(c-13d)-B(c^2-7cd+34d^2))\left(\frac{2\int \frac{1}{2}\sqrt{\sin(e+fx)a+a}\left(a\left(5c^2+3d^2\right)+2a(5c-d)d\sin(e\right.}{5a}\right.}{7d}}{9d}-\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^3}{9df}$$

$\Big\downarrow$ 27

$$\frac{\frac{2a^2(-9Ad+3Bc-10Bd)\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a\sin(e+fx)+a}}-\frac{3a\left(3Ad(c-13d)-B\left(c^2-7cd+34d^2\right)\right)\left(\frac{\int\sqrt{\sin(e+fx)a+a}\left(a\left(5c^2+3d^2\right)+2a(5c-d)d\sin(e+f\cdots\right.}{5a}\cdots\right.}{7d}\cdots}{9d}$$

$$\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^3}{9df}$$

$\downarrow$ 3042

$$\frac{\frac{2a^2(-9Ad+3Bc-10Bd)\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a\sin(e+fx)+a}}-\frac{3a\left(3Ad(c-13d)-B\left(c^2-7cd+34d^2\right)\right)\left(\frac{\int\sqrt{\sin(e+fx)a+a}\left(a\left(5c^2+3d^2\right)+2a(5c-d)d\sin(e+f\cdots\right.}{5a}\cdots\right.}{7d}\cdots}{9d}$$

$$\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^3}{9df}$$

$\downarrow$ 3230

$$\frac{\frac{2a^2(-9Ad+3Bc-10Bd)\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a\sin(e+fx)+a}}-\frac{3a\left(3Ad(c-13d)-B\left(c^2-7cd+34d^2\right)\right)\left(\frac{\frac{1}{3}a\left(15c^2+10cd+7d^2\right)\int\sqrt{\sin(e+fx)a+a}dx-\frac{4ad(5c\cdots}{\cdots}}{5a}\cdots\right.}{7d}\cdots}{9d}$$

$$\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^3}{9df}$$

$\downarrow$ 3042

$$\frac{\frac{2a^2(-9Ad+3Bc-10Bd)\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a\sin(e+fx)+a}}-\frac{3a\left(3Ad(c-13d)-B\left(c^2-7cd+34d^2\right)\right)\left(\frac{\frac{1}{3}a\left(15c^2+10cd+7d^2\right)\int\sqrt{\sin(e+fx)a+a}dx-\frac{4ad(5c\cdots}{\cdots}}{5a}\cdots\right.}{7d}\cdots}{9d}$$

$$\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^3}{9df}$$

$\downarrow$ 3125

$$\frac{\frac{2a^2(-9Ad+3Bc-10Bd)\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a\sin(e+fx)+a}}-\frac{3a\left(3Ad(c-13d)-B\left(c^2-7cd+34d^2\right)\right)\left(\frac{-\frac{2a^2\left(15c^2+10cd+7d^2\right)\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{4ad(5c-d)\cos(\cdots}{\cdots}}{5a}\cdots\right.}{7d}\cdots}{9d}$$

$$\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^3}{9df}$$

input
```
Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2
,x]
```

output
```
(-2*a*B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^3)/(9*d
*f) + ((2*a^2*(3*B*c - 9*A*d - 10*B*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3
)/(7*d*f*Sqrt[a + a*Sin[e + f*x]]) - (3*a*(3*A*(c - 13*d)*d - B*(c^2 - 7*c
*d + 34*d^2))*((-2*d^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(5*a*f) +
((-2*a^2*(15*c^2 + 10*c*d + 7*d^2)*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f
*x]]) - (4*a*(5*c - d)*d*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*f))/(5*
a)))/(7*d))/(9*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3240
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])^
m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.70

method	result
default	$\frac{2(1+\sin(fx+e))a^2(\sin(fx+e)-1)\left(35B\cos(fx+e)^4d^2+(-45A\,d^2-90Bcd-85B\,d^2)\cos(fx+e)^2\sin(fx+e)+(-126Acd-117A\,d^2-\right.}{}$
parts	$\frac{2(1+\sin(fx+e))a^2(\sin(fx+e)-1)d(Ad+2Bc)\left(15\sin(fx+e)^3+39\sin(fx+e)^2+52\sin(fx+e)+104\right)}{105\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}+\frac{2(1+\sin(fx+e))a^2(\sin(f}{5\,\mathrm{co}}$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x,method=_R
ETURNVERBOSE)
```

output
```
2/315*(1+sin(f*x+e))*a^2*(sin(f*x+e)-1)*(35*B*cos(f*x+e)^4*d^2+(-45*A*d^2-
90*B*c*d-85*B*d^2)*cos(f*x+e)^2*sin(f*x+e)+(-126*A*c*d-117*A*d^2-63*B*c^2-
234*B*c*d-172*B*d^2)*cos(f*x+e)^2+(105*A*c^2+378*A*c*d+201*A*d^2+189*B*c^2
+402*B*c*d+221*B*d^2)*sin(f*x+e)+525*A*c^2+882*A*c*d+429*A*d^2+441*B*c^2+8
58*B*c*d+409*B*d^2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.46

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx =$$
$$-\frac{2\left(35\,Bad^2\cos\left(fx+e\right)^5-5\left(18\,Bacd+(9\,A+10\,B)ad^2\right)\cos\left(fx+e\right)^4+84\,(5\,A+3\,B)ac^2+24\,(21\,A\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, al
gorithm="fricas")
```

output
```
-2/315*(35*B*a*d^2*cos(f*x + e)^5 - 5*(18*B*a*c*d + (9*A + 10*B)*a*d^2)*co
s(f*x + e)^4 + 84*(5*A + 3*B)*a*c^2 + 24*(21*A + 19*B)*a*c*d + 4*(57*A + 4
7*B)*a*d^2 - (63*B*a*c^2 + 18*(7*A + 13*B)*a*c*d + (117*A + 172*B)*a*d^2)*
cos(f*x + e)^3 + (21*(5*A + 6*B)*a*c^2 + 6*(42*A + 43*B)*a*c*d + (129*A +
134*B)*a*d^2)*cos(f*x + e)^2 + (21*(25*A + 21*B)*a*c^2 + 6*(147*A + 143*B)
*a*c*d + (429*A + 409*B)*a*d^2)*cos(f*x + e) - (35*B*a*d^2*cos(f*x + e)^4
+ 84*(5*A + 3*B)*a*c^2 + 24*(21*A + 19*B)*a*c*d + 4*(57*A + 47*B)*a*d^2 +
5*(18*B*a*c*d + (9*A + 17*B)*a*d^2)*cos(f*x + e)^3 - 3*(21*B*a*c^2 + 6*(7*
A + 8*B)*a*c*d + (24*A + 29*B)*a*d^2)*cos(f*x + e)^2 - (21*(5*A + 9*B)*a*c
^2 + 6*(63*A + 67*B)*a*c*d + (201*A + 221*B)*a*d^2)*cos(f*x + e))*sin(f*x
+ e))*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

Sympy [F]

$$\int (a + a\sin(e + fx))^{3/2}(A + B\sin(e + fx))(c + d\sin(e + fx))^2 \, dx = \int (a(\sin(e + fx) + 1))^{\frac{3}{2}} (A + B\sin(e + fx)) (c + d\sin(e + fx))^2 \, dx$$

input
```
integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**(3/2)*(A + B*sin(e + f*x))*(c + d*sin(e +
 f*x))**2, x)
```

Maxima [F]

$$\int (a + a\sin(e + fx))^{3/2}(A + B\sin(e + fx))(c + d\sin(e + fx))^2 \, dx = \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^{\frac{3}{2}}(d\sin(fx + e) + c)^2 \, dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e)
+ c)^2, x)
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.69

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, al
gorithm="giac")
```

output
```
1/2520*sqrt(2)*(35*B*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-9/4*pi
 + 9/2*f*x + 9/2*e) + 630*(12*A*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
+ 8*B*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 16*A*a*c*d*sgn(cos(-1/4*
pi + 1/2*f*x + 1/2*e)) + 14*B*a*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) +
7*A*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*B*a*d^2*sgn(cos(-1/4*pi
+ 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 210*(4*A*a*c^2*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*B*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e)) + 12*A*a*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 10*B*a*c*d*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*A*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
)) + 5*B*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/2*f*x
+ 3/2*e) + 126*(2*B*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 4*A*a*c*d*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*B*a*c*d*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e)) + 3*A*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*a*d^2*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-5/4*pi + 5/2*f*x + 5/2*e) + 45*(4*B*
a*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*A*a*d^2*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e)) + 3*B*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-7/4*
pi + 7/2*f*x + 7/2*e))*sqrt(a)/f
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = \int (A + B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{3/2}\,(c+d\,\sin(e+f\,x))^2\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^2
,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^2
, x)
```

Reduce [F]

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx=\sqrt{a}\,a\left(\left(\int\sqrt{\sin(fx+e)+1}dx\right)a\,c^2+\left(\int\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^4\,dx\right)b\,d^2+\left(\int\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^3\,dx\right)a\,d^2+2\left(\int\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^3\,dx\right)bcd+\left(\int\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^3\,dx\right)b\,d^2+2\left(\int\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)acd+\left(\int\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)a\,d^2+\left(\int\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)b\,c^2+2\left(\int\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)bcd+\left(\int\sqrt{\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)a\,c^2+2\left(\int\sqrt{\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)acd+\left(\int\sqrt{\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)b\,c^2\right)$$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x)
```

output

```
sqrt(a)*a*(int(sqrt(sin(e + f*x) + 1),x)*a*c**2 + int(sqrt(sin(e + f*x) +
1)*sin(e + f*x)**4,x)*b*d**2 + int(sqrt(sin(e + f*x) + 1)*sin(e + f*x)**3,
x)*a*d**2 + 2*int(sqrt(sin(e + f*x) + 1)*sin(e + f*x)**3,x)*b*c*d + int(sq
rt(sin(e + f*x) + 1)*sin(e + f*x)**3,x)*b*d**2 + 2*int(sqrt(sin(e + f*x) +
 1)*sin(e + f*x)**2,x)*a*c*d + int(sqrt(sin(e + f*x) + 1)*sin(e + f*x)**2,
x)*a*d**2 + int(sqrt(sin(e + f*x) + 1)*sin(e + f*x)**2,x)*b*c**2 + 2*int(s
qrt(sin(e + f*x) + 1)*sin(e + f*x)**2,x)*b*c*d + int(sqrt(sin(e + f*x) + 1
)*sin(e + f*x),x)*a*c**2 + 2*int(sqrt(sin(e + f*x) + 1)*sin(e + f*x),x)*a*
c*d + int(sqrt(sin(e + f*x) + 1)*sin(e + f*x),x)*b*c**2)
```

3.295 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 165

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = -\frac{8a^2(35Ac+21Bc+21Ad+19Bd)\cos(e+fx)}{105f\sqrt{a+a\sin(e+fx)}} - \frac{2a(35Ac+21Bc+21Ad+19Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{105f} - \frac{2(7Bc+7Ad-2Bd)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{35f} - \frac{2Bd\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{7af}$$

output
```
-8/105*a^2*(35*A*c+21*A*d+21*B*c+19*B*d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/
2)-2/105*a*(35*A*c+21*A*d+21*B*c+19*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)
/f-2/35*(7*A*d+7*B*c-2*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f-2/7*B*d*co
s(f*x+e)*(a+a*sin(f*x+e))^(5/2)/a/f
```

Mathematica [A] (verified)

Time = 3.65 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.87

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx =$$
$$-\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}(700Ac+546Bc+546Ad+494Bd-6(7Bc}{210f\left(\cos\left(\frac{1}{2}(e+f\right.\right.}$$

input
```
Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f
*x]),x]
```

output
```
-1/210*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]
*(700*A*c + 546*B*c + 546*A*d + 494*B*d - 6*(7*B*c + 7*A*d + 13*B*d)*Cos[2
*(e + f*x)] + (140*A*c + 252*B*c + 252*A*d + 253*B*d)*Sin[e + f*x] - 15*B*
d*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {3042, 3447, 3042, 3502, 27, 3042, 3230, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))dx$$
$$\Big\downarrow \text{3447}$$
$$\int (a\sin(e+fx)+a)^{3/2}\left((Ad+Bc)\sin(e+fx)+Ac+Bd\sin^2(e+fx)\right)dx$$
$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^{3/2}\left((Ad+Bc)\sin(e+fx)+Ac+Bd\sin(e+fx)^2\right)dx$$

$$\Big\downarrow \; 3502$$

$$\frac{2\int \frac{1}{2}(\sin(e+fx)a+a)^{3/2}(a(7Ac+5Bd)+a(7Bc+7Ad-2Bd)\sin(e+fx))dx}{7a}-\frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{7af}$$

$$\Big\downarrow \; 27$$

$$\frac{\int (\sin(e+fx)a+a)^{3/2}(a(7Ac+5Bd)+a(7Bc+7Ad-2Bd)\sin(e+fx))dx}{7a}-\frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{7af}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int (\sin(e+fx)a+a)^{3/2}(a(7Ac+5Bd)+a(7Bc+7Ad-2Bd)\sin(e+fx))dx}{7a}-\frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{7af}$$

$$\Big\downarrow \; 3230$$

$$\frac{\frac{1}{5}a(35Ac+21Ad+21Bc+19Bd)\int (\sin(e+fx)a+a)^{3/2}dx-\frac{2a(7Ad+7Bc-2Bd)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}}{7a}-\frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{7af}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{1}{5}a(35Ac+21Ad+21Bc+19Bd)\int (\sin(e+fx)a+a)^{3/2}dx-\frac{2a(7Ad+7Bc-2Bd)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}}{7a}-\frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{7af}$$

$$\Big\downarrow \; 3126$$

$$\frac{\frac{1}{5}a(35Ac+21Ad+21Bc+19Bd)\left(\frac{4}{3}a\int \sqrt{\sin(e+fx)a+a}dx-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)-\frac{2a(7Ad+7Bc-2Bd)\,\mathrm{c}}{}}{7a}-\frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{7af}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{1}{5}a(35Ac+21Ad+21Bc+19Bd)\left(\frac{4}{3}a\int\sqrt{\sin(e+fx)a+a}dx-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)-\frac{2a(7Ad+7Bc-2Bd)\cos}{}}{7a}$$
$$\frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{7af}$$

$\Big\downarrow$ 3125

$$\frac{\frac{1}{5}a(35Ac+21Ad+21Bc+19Bd)\left(-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)-\frac{2a(7Ad+7Bc-2Bd)\cos(e+fx)(a}{5f}}{7a}$$
$$\frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{7af}$$

input
```
Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x
]
```

output
```
(-2*B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(7*a*f) + ((-2*a*(7*B*c +
 7*A*d - 2*B*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(5*f) + (a*(35*A*
c + 21*B*c + 21*A*d + 19*B*d)*((-8*a^2*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e
 + f*x]]) - (2*a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*f)))/5)/(7*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.91

method	result
default	$\frac{2(1+\sin(fx+e))a^2(\sin(fx+e)-1)\left(15B\sin(fx+e)^3d+21A\sin(fx+e)^2d+21B\sin(fx+e)^2c+39B\sin(fx+e)^2d+35Ac\sin(fx+e)+6}{105\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$
parts	$\frac{2(1+\sin(fx+e))a^2(\sin(fx+e)-1)(Ad+Bc)\left(\sin(fx+e)^2+3\sin(fx+e)+6\right)}{5\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}+\frac{2Ac(1+\sin(fx+e))a^2(\sin(fx+e)-1)(\sin(fx+e)+5}{3\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x,method=_RET
URNVERBOSE)
```

output
```
2/105*(1+sin(f*x+e))*a^2*(sin(f*x+e)-1)*(15*B*sin(f*x+e)^3*d+21*A*sin(f*x+
e)^2*d+21*B*sin(f*x+e)^2*c+39*B*sin(f*x+e)^2*d+35*A*c*sin(f*x+e)+63*A*sin(
f*x+e)*d+63*B*sin(f*x+e)*c+52*B*sin(f*x+e)*d+175*A*c+126*A*d+126*B*c+104*B
*d)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.56

$$\int (a + a\sin(e + fx))^{3/2}(A + B\sin(e + fx))(c + d\sin(e+fx))\,dx = \frac{2\left(15\,Bad\cos\left(fx+e\right)^4 + 3\left(7\,Bac + (7\,A + 13\,B)ad\right)\cos\left(fx+e\right)^3 - 28\left(5\,A + 3\,B\right)a}{}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algo
rithm="fricas")
```

output
```
2/105*(15*B*a*d*cos(f*x + e)^4 + 3*(7*B*a*c + (7*A + 13*B)*a*d)*cos(f*x +
e)^3 - 28*(5*A + 3*B)*a*c - 4*(21*A + 19*B)*a*d - (7*(5*A + 6*B)*a*c + (42
*A + 43*B)*a*d)*cos(f*x + e)^2 - (7*(25*A + 21*B)*a*c + (147*A + 143*B)*a*
d)*cos(f*x + e) + (15*B*a*d*cos(f*x + e)^3 + 28*(5*A + 3*B)*a*c + 4*(21*A
+ 19*B)*a*d - 3*(7*B*a*c + (7*A + 8*B)*a*d)*cos(f*x + e)^2 - (7*(5*A + 9*B
)*a*c + (63*A + 67*B)*a*d)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e)
 + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

Sympy [F]

$$\int (a + a\sin(e + fx))^{3/2}(A + B\sin(e + fx))(c + d\sin(e + fx))\,dx = \int \left(a(\sin\left(e + fx\right) + 1)\right)^{\frac{3}{2}}\left(A + B\sin\left(e + fx\right)\right)\left(c + d\sin\left(e + fx\right)\right)\,dx$$

input
```
integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**(3/2)*(A + B*sin(e + f*x))*(c + d*sin(e +
 f*x)), x)
```

Maxima [F]

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}(d\sin(fx+e)+c)\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e)
+ c), x)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.73

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx=\frac{\sqrt{2}\left(15\,Bad\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{7}{4}\,\pi+\frac{7}{2}\,fx+\frac{7}{2}\,e\right)+105\,(12\,Aac\mathrm{sgn}(c$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algo
rithm="giac")
```

output
```
1/420*sqrt(2)*(15*B*a*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-7/4*pi +
7/2*f*x + 7/2*e) + 105*(12*A*a*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 8*B
*a*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 8*A*a*d*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e)) + 7*B*a*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-1/4*pi +
1/2*f*x + 1/2*e) + 35*(4*A*a*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*B*a
*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*A*a*d*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e)) + 5*B*a*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/
2*f*x + 3/2*e) + 21*(2*B*a*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*A*a*d
*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*a*d*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e)))*sin(-5/4*pi + 5/2*f*x + 5/2*e))*sqrt(a)/f
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx=\int (A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}\left(c+d\,\sin\left(e+f\,x\right)\right)\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x)),x
)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x)),
x)
```

Reduce [F]

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = \sqrt{a}\,a\left(\left(\int \sqrt{\sin(fx+e)+1}dx\right)ac + \left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)^3\,dx\right)bd + \left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)ad + \left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)bc + \left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)bd + \left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)ac + \left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)ad + \left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)bc\right)$$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

output
```
sqrt(a)*a*(int(sqrt(sin(e + f*x) + 1),x)*a*c + int(sqrt(sin(e + f*x) + 1)*
sin(e + f*x)**3,x)*b*d + int(sqrt(sin(e + f*x) + 1)*sin(e + f*x)**2,x)*a*d
 + int(sqrt(sin(e + f*x) + 1)*sin(e + f*x)**2,x)*b*c + int(sqrt(sin(e + f*
x) + 1)*sin(e + f*x)**2,x)*b*d + int(sqrt(sin(e + f*x) + 1)*sin(e + f*x),x
)*a*c + int(sqrt(sin(e + f*x) + 1)*sin(e + f*x),x)*a*d + int(sqrt(sin(e +
f*x) + 1)*sin(e + f*x),x)*b*c)
```

3.296 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 101

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx = -\frac{8a^2(5A+3B)\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} - \frac{2a(5A+3B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} - \frac{2B\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5f}$$

output
```
-8/15*a^2*(5*A+3*B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-2/15*a*(5*A+3*B)*c
os(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f-2/5*B*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)
/f
```

Mathematica [A] (verified)

Time = 1.99 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx = -\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}(50A+39B-3B\cos(2(e+fx))+2(5A+9}{15f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

input
```
Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]),x]
```

output
```
-1/15*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*
(50*A + 39*B - 3*B*Cos[2*(e + f*x)] + 2*(5*A + 9*B)*Sin[e + f*x]))/(f*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3042, 3230, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3230}$$

$$\frac{1}{5}(5A+3B)\int (\sin(e+fx)a+a)^{3/2}dx-\frac{2B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}(5A+3B)\int (\sin(e+fx)a+a)^{3/2}dx-\frac{2B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}$$

$$\Big\downarrow \text{3126}$$

$$\frac{1}{5}(5A+3B)\left(\frac{4}{3}a\int \sqrt{\sin(e+fx)a+a}dx-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)-\frac{2B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}(5A+3B)\left(\frac{4}{3}a\int\sqrt{\sin(e+fx)a+a}dx-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)-\frac{2B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}$$

$\Big\downarrow$ 3125

$$\frac{1}{5}(5A+3B)\left(-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)-\frac{2B\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}$$

input
```
Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]),x]
```

output
```
(-2*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(5*f) + ((5*A + 3*B)*((-8*a
^2*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a*Cos[e + f*x]*Sqrt[a
 + a*Sin[e + f*x]])/(3*f)))/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.76

method	result	siz
default	$\frac{2(1+\sin(fx+e))a^2(\sin(fx+e)-1)\left(-3B\cos(fx+e)^2+\sin(fx+e)(5A+9B)+25A+21B\right)}{15\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$	77
parts	$\frac{2A(1+\sin(fx+e))a^2(\sin(fx+e)-1)(\sin(fx+e)+5)}{3\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}+\frac{2B(1+\sin(fx+e))a^2(\sin(fx+e)-1)\left(\sin(fx+e)^2+3\sin(fx+e)+6\right)}{5\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$	11

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
2/15*(1+sin(f*x+e))*a^2*(sin(f*x+e)-1)*(-3*B*cos(f*x+e)^2+sin(f*x+e)*(5*A+
9*B)+25*A+21*B)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.36

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx=\frac{2\left(3\,Ba\cos\left(fx+e\right)^3-(5\,A+6\,B)a\cos\left(fx+e\right)^2-(25\,A+21\,B)a\cos\left(fx+e\right)-4}{15\,(}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x, algorithm="fricas")
```

output
```
2/15*(3*B*a*cos(f*x + e)^3 - (5*A + 6*B)*a*cos(f*x + e)^2 - (25*A + 21*B)*
a*cos(f*x + e) - 4*(5*A + 3*B)*a - (3*B*a*cos(f*x + e)^2 + (5*A + 9*B)*a*c
os(f*x + e) - 4*(5*A + 3*B)*a)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*c
os(f*x + e) + f*sin(f*x + e) + f)
```

Sympy [F]

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx = \int (a(\sin(e+fx)+1))^{\frac{3}{2}}(A+B\sin(e+fx))\,dx$$

input
```
integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e)),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**(3/2)*(A + B*sin(e + f*x)), x)
```

Maxima [F]

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.38

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx = \frac{\sqrt{2}\left(3\,Basgn\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{5}{4}\pi+\frac{5}{2}fx+\frac{5}{2}e\right)+30\left(3\,Aasgn\left(\cos\left(-\right.\right.\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x, algorithm="giac")
```

output
```
1/30*sqrt(2)*(3*B*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-5/4*pi + 5/2*
f*x + 5/2*e) + 30*(3*A*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*B*a*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 5*(2*A*a*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e)))*sin(-3/4*pi + 3/2*f*x + 3/2*e))*sqrt(a)/f
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx = \int (A+B\,\sin\left(e+f\,x\right))\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2),x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int (a + a\sin(e+fx))^{3/2}(A+B\sin(e+fx))\,dx = \sqrt{a}\,a\left(\left(\int \sqrt{\sin(fx+e)+1}dx\right)a + \left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)b + \left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)a + \left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)b\right)$$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x)
```

output
```
sqrt(a)*a*(int(sqrt(sin(e + f*x) + 1),x)*a + int(sqrt(sin(e + f*x) + 1)*si
n(e + f*x)**2,x)*b + int(sqrt(sin(e + f*x) + 1)*sin(e + f*x),x)*a + int(sq
rt(sin(e + f*x) + 1)*sin(e + f*x),x)*b)
```

3.297 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 153

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = -\frac{2a^{3/2}(c-d)(Bc-Ad)\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{d^{5/2}\sqrt{c+d}f} + \frac{2a^2(3Bc-3Ad-4Bd)\cos(e+fx)}{3d^2f\sqrt{a+a\sin(e+fx)}} - \frac{2aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3df}$$

output
```
-2*a^(3/2)*(c-d)*(-A*d+B*c)*arctanh(a^(1/2)*d^(1/2)*cos(f*x+e)/(c+d)^(1/2)
/(a+a*sin(f*x+e))^(1/2))/d^(5/2)/(c+d)^(1/2)/f+2/3*a^2*(-3*A*d+3*B*c-4*B*d
)*cos(f*x+e)/d^2/f/(a+a*sin(f*x+e))^(1/2)-2/3*a*B*cos(f*x+e)*(a+a*sin(f*x+
e))^(1/2)/d/f
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 6.03 (sec) , antiderivative size = 898, normalized size of antiderivative = 5.87

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e +
 f*x]),x]
```

output
```
((a*(1 + Sin[e + f*x]))^(3/2)*(-6*Sqrt[d]*(-2*B*c + 2*A*d + 3*B*d)*Cos[(e
+ f*x)/2] - 2*B*d^(3/2)*Cos[(3*(e + f*x))/2] + (3*(c - d)*(B*c - A*d)*((c
+ d)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2]) + Sqrt[c + d]*RootSum[c + 4*d*#
1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/
4]]) - d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]] - d*Sqrt[c + d]*Log[-#1 + Tan[(
e + f*x)/4]] - 2*c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*d^(3/2)*Log[
-#1 + Tan[(e + f*x)/4]]*#1 - c*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1
+ c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + d^(3/2)*Log[-#1 + Tan[(e +
f*x)/4]]*#1^2 + 3*d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - c*Sqrt[
c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) &
 ]))/(c + d)^(3/2) + (3*(c - d)*(B*c - A*d)*(-((c + d)*(e + f*x - 2*Log[Se
c[(e + f*x)/4]^2])) + Sqrt[c + d]*RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3
 + c*#1^4 & , (-(c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]) - d^(3/2)*Log[-#1
+ Tan[(e + f*x)/4]] + d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - 2*c*Sqrt
[d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]]
*#1 + c*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + c*Sqrt[d]*Log[-#1 + T
an[(e + f*x)/4]]*#1^2 + d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - 3*d*Sqr
t[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + c*Sqrt[c + d]*Log[-#1 + Tan[(e
 + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ]))/(c + d)^(3/2) + 6*
Sqrt[d]*(-2*B*c + 2*A*d + 3*B*d)*Sin[(e + f*x)/2] - 2*B*d^(3/2)*Sin[(3*...
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {3042, 3455, 27, 3042, 3460, 3042, 3252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}dx$$

$$\Big\downarrow \text{3455}$$

$$\frac{2\int \frac{\sqrt{\sin(e+fx)a+a}(a(Bc+3Ad)-a(3Bc-3Ad-4Bd)\sin(e+fx))}{2(c+d\sin(e+fx))}dx}{3d}-\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3df}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{\sin(e+fx)a+a}(a(Bc+3Ad)-a(3Bc-3Ad-4Bd)\sin(e+fx))}{c+d\sin(e+fx)}dx}{3d}-\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{\sin(e+fx)a+a}(a(Bc+3Ad)-a(3Bc-3Ad-4Bd)\sin(e+fx))}{c+d\sin(e+fx)}dx}{3d}-\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3df}$$

$$\Big\downarrow \text{3460}$$

$$\frac{\frac{3a(c-d)(Bc-Ad)\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{d}+\frac{2a^2(-3Ad+3Bc-4Bd)\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}}}{3d}-\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3a(c-d)(Bc-Ad)\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{d}+\frac{2a^2(-3Ad+3Bc-4Bd)\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}}}{3d}-\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3df}$$

$$\Big\downarrow \text{3252}$$

$$\frac{\frac{2a^2(-3Ad+3Bc-4Bd)\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}}-\frac{6a^2(c-d)(Bc-Ad)\int\frac{1}{a(c+d)-\frac{a^2d\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{df}}{3d}-\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3df}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{2a^2(-3Ad+3Bc-4Bd)\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}}-\frac{6a^{3/2}(c-d)(Bc-Ad)\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{3/2}f\sqrt{c+d}}}{3d}-\frac{2aB\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3df}$$

input
```
Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])
,x]
```

output
```
(-2*a*B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*d*f) + ((-6*a^(3/2)*(c -
 d)*(B*c - A*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a
 + a*Sin[e + f*x]])])/(d^(3/2)*Sqrt[c + d]*f) + (2*a^2*(3*B*c - 3*A*d - 4*
B*d)*Cos[e + f*x])/(d*f*Sqrt[a + a*Sin[e + f*x]]))/(3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. 2(131) = 262.

Time = 0.48 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.91

method	result
default	$-\frac{2(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(-3A\,\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}\,d}{\sqrt{(c+d)ad}}\right)a^2cd+3A\,\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}\,d}{\sqrt{(c+d)ad}}\right)a^2d^2-B(\cdots\right.}{\cdots}$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x,method=_RET
URNVERBOSE)
```

output
```
-2/3*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(-3*A*arctanh((-a*(sin(f*x+e
)-1))^(1/2)*d/((c+d)*a*d)^(1/2))*a^2*c*d+3*A*arctanh((-a*(sin(f*x+e)-1))^(
1/2)*d/((c+d)*a*d)^(1/2))*a^2*d^2-B*(-a*(sin(f*x+e)-1))^(3/2)*((c+d)*a*d)^
(1/2)*d+3*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/((c+d)*a*d)^(1/2))*a^2*c^2
-3*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/((c+d)*a*d)^(1/2))*a^2*c*d+3*A*(-
a*(sin(f*x+e)-1))^(1/2)*((c+d)*a*d)^(1/2)*a*d-3*B*(-a*(sin(f*x+e)-1))^(1/2
)*((c+d)*a*d)^(1/2)*a*c+6*B*(-a*(sin(f*x+e)-1))^(1/2)*((c+d)*a*d)^(1/2)*a*
d)/d^2/((c+d)*a*d)^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(131) = 262$.

Time = 0.69 (sec) , antiderivative size = 880, normalized size of antiderivative = 5.75

$$\int \frac{(a + a\sin(e + fx))^{3/2}(A + B\sin(e + fx))}{c + d\sin(e + fx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algo
rithm="fricas")
```

output
```
[1/6*(3*(B*a*c^2 - (A + B)*a*c*d + A*a*d^2 + (B*a*c^2 - (A + B)*a*c*d + A*
a*d^2)*cos(f*x + e) + (B*a*c^2 - (A + B)*a*c*d + A*a*d^2)*sin(f*x + e))*sq
rt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6
*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d
^3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c
*d^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x +
e) + a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (
a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos
(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^
2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2
*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) - 4*(B*a*d*cos(f*x +
 e)^2 - 3*B*a*c + (3*A + 4*B)*a*d - (3*B*a*c - (3*A + 5*B)*a*d)*cos(f*x +
e) + (B*a*d*cos(f*x + e) + 3*B*a*c - (3*A + 4*B)*a*d)*sin(f*x + e))*sqrt(a
*sin(f*x + e) + a))/(d^2*f*cos(f*x + e) + d^2*f*sin(f*x + e) + d^2*f), -1/
3*(3*(B*a*c^2 - (A + B)*a*c*d + A*a*d^2 + (B*a*c^2 - (A + B)*a*c*d + A*a*d
^2)*cos(f*x + e) + (B*a*c^2 - (A + B)*a*c*d + A*a*d^2)*sin(f*x + e))*sqrt(
-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c -
2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) + 2*(B*a*d*cos(f*x + e)^2 - 3*
B*a*c + (3*A + 4*B)*a*d - (3*B*a*c - (3*A + 5*B)*a*d)*cos(f*x + e) + (B*a*
d*cos(f*x + e) + 3*B*a*c - (3*A + 4*B)*a*d)*sin(f*x + e))*sqrt(a*sin(f*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}}{d\sin(fx+e)+c}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e)
+ c), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. 2(131) = 262.

Time = 0.21 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.79

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx =$$

$$-\frac{\sqrt{2}\sqrt{a}\left(\frac{3\sqrt{2}\left(Bac^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-Aacd\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-Bacd\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+Aad^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\right.\right.\right.}{\sqrt{-cd-d^2}d^2}\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algo
rithm="giac")
```

output
```
-1/3*sqrt(2)*sqrt(a)*(3*sqrt(2)*(B*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
)) - A*a*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - B*a*c*d*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e)) + A*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*arctan
(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/sqrt(-c*d - d^2))/(sqrt(-c*d - d
^2)*d^2) + 2*(2*B*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi +
1/2*f*x + 1/2*e)^3 + 3*B*a*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/
4*pi + 1/2*f*x + 1/2*e) - 3*A*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*si
n(-1/4*pi + 1/2*f*x + 1/2*e) - 6*B*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
))*sin(-1/4*pi + 1/2*f*x + 1/2*e))/d^3)/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{3/2}}{c+d\,\sin(e+f\,x)}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c + d*sin(e + f*x))
,x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c + d*sin(e + f*x))
, x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \sqrt{a}\,a\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}}{\sin(fx+e)\,d+c}dx\right)a + \left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^2}{\sin(fx+e)\,d+c}dx\right)b + \left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)\,d+c}dx\right)a + \left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)\,d+c}dx\right)b\right)$$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

output
```
sqrt(a)*a*(int(sqrt(sin(e + f*x) + 1)/(sin(e + f*x)*d + c),x)*a + int((sqr
t(sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x)*d + c),x)*b + int((sqrt
(sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)*d + c),x)*a + int((sqrt(sin
(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)*d + c),x)*b)
```

3.298 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 191

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = -\frac{a^{3/2}(Ad(c+3d)-B(3c^2+3cd-2d^2))\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{d^{5/2}(c+d)^{3/2}f} - \frac{a^2(3Bc-Ad+2Bd)\cos(e+fx)}{d^2(c+d)f\sqrt{a+a\sin(e+fx)}} + \frac{a(Bc-Ad)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{d(c+d)f(c+d\sin(e+fx))}$$

output

```
-a^(3/2)*(A*d*(c+3*d)-B*(3*c^2+3*c*d-2*d^2))*arctanh(a^(1/2)*d^(1/2)*cos(f
*x+e)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/d^(5/2)/(c+d)^(3/2)/f-a^2*(-A*d+
3*B*c+2*B*d)*cos(f*x+e)/d^2/(c+d)/f/(a+a*sin(f*x+e))^(1/2)+a*(-A*d+B*c)*co
s(f*x+e)*(a+a*sin(f*x+e))^(1/2)/d/(c+d)/f/(c+d*sin(f*x+e))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 13.43 (sec) , antiderivative size = 922, normalized size of antiderivative = 4.83

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e +
 f*x])^2,x]
```

output
```
((a*(1 + Sin[e + f*x]))^(3/2)*(-8*B*Sqrt[d]*Cos[(e + f*x)/2] + ((A*d*(c +
3*d) + B*(-3*c^2 - 3*c*d + 2*d^2))*((c + d)*(e + f*x - 2*Log[Sec[(e + f*x)
/4]^2]) + Sqrt[c + d]*RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 &
, (-(c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]) - d^(3/2)*Log[-#1 + Tan[(e + f
*x)/4]] - d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - 2*c*Sqrt[d]*Log[-#1
+ Tan[(e + f*x)/4]]*#1 - 2*d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]]*#1 - c*Sqrt
[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)
/4]]*#1^2 + d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + 3*d*Sqrt[c + d]*Log
[-#1 + Tan[(e + f*x)/4]]*#1^2 - c*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*
#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ]))/(c + d)^(5/2) + ((A*d*(c + 3*d
) + B*(-3*c^2 - 3*c*d + 2*d^2))*(-((c + d)*(e + f*x - 2*Log[Sec[(e + f*x)/
4]^2])) + Sqrt[c + d]*RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 &
, (-(c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]) - d^(3/2)*Log[-#1 + Tan[(e + f
*x)/4]] + d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - 2*c*Sqrt[d]*Log[-#1
+ Tan[(e + f*x)/4]]*#1 - 2*d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]]*#1 + c*Sqrt
[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)
/4]]*#1^2 + d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - 3*d*Sqrt[c + d]*Log
[-#1 + Tan[(e + f*x)/4]]*#1^2 + c*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*
#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ]))/(c + d)^(5/2) + 8*B*Sqrt[d]*Si
n[(e + f*x)/2] - (4*Sqrt[d]*(-c + d)*(-(B*c) + A*d)*(Cos[(e + f*x)/2] -...
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.216$, Rules used = {3042, 3454, 27, 3042, 3460, 3042, 3252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}dx$$

$$\Big\downarrow \text{3454}$$

$$\frac{\int -\frac{\sqrt{\sin(e+fx)a+a}(a(B(c-2d)-3Ad)-a(3Bc-Ad+2Bd)\sin(e+fx))}{2(c+d\sin(e+fx))}dx}{d(c+d)}+\frac{a(Bc-Ad)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{df(c+d)(c+d\sin(e+fx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{a(Bc-Ad)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{df(c+d)(c+d\sin(e+fx))}-\frac{\int \frac{\sqrt{\sin(e+fx)a+a}(a(B(c-2d)-3Ad)-a(3Bc-Ad+2Bd)\sin(e+fx))}{c+d\sin(e+fx)}dx}{2d(c+d)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(Bc-Ad)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{df(c+d)(c+d\sin(e+fx))}-\frac{\int \frac{\sqrt{\sin(e+fx)a+a}(a(B(c-2d)-3Ad)-a(3Bc-Ad+2Bd)\sin(e+fx))}{c+d\sin(e+fx)}dx}{2d(c+d)}$$

$$\Big\downarrow \text{3460}$$

$$\frac{a(Bc-Ad)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{df(c+d)(c+d\sin(e+fx))}-\frac{\frac{2a^2(-Ad+3Bc+2Bd)\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}}-\frac{a\left(Ad(c+3d)-B\left(3c^2+3cd-2d^2\right)\right)\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{d}}{2d(c+d)}$$

$$\Big\downarrow \; 3042$$

$$\frac{a(Bc-Ad)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{df(c+d)(c+d\sin(e+fx))}-\frac{\frac{2a^2(-Ad+3Bc+2Bd)\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}}-\frac{a(Ad(c+3d)-B(3c^2+3cd-2d^2))\int\frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{d}}{2d(c+d)}$$

$$\Big\downarrow \; 3252$$

$$\frac{a(Bc-Ad)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{df(c+d)(c+d\sin(e+fx))}-\frac{\frac{2a^2(Ad(c+3d)-B(3c^2+3cd-2d^2))\int\frac{1}{a(c+d)-\frac{a^2d\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{df}+\frac{2a^2(-Ad+3Bc+2Bd)\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}}}{2d(c+d)}$$

$$\Big\downarrow \; 221$$

$$\frac{a(Bc-Ad)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{df(c+d)(c+d\sin(e+fx))}-\frac{\frac{2a^{3/2}(Ad(c+3d)-B(3c^2+3cd-2d^2))\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{3/2}f\sqrt{c+d}}+\frac{2a^2(-Ad+3Bc+2Bd)\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}}}{2d(c+d)}$$

input
```
Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])
^2,x]
```

output
```
(a*(B*c - A*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(d*(c + d)*f*(c + d*
Sin[e + f*x])) - ((2*a^(3/2)*(A*d*(c + 3*d) - B*(3*c^2 + 3*c*d - 2*d^2))*A
rcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]
])])/(d^(3/2)*Sqrt[c + d]*f) + (2*a^2*(3*B*c - A*d + 2*B*d)*Cos[e + f*x])/
(d*f*Sqrt[a + a*Sin[e + f*x]]))/(2*d*(c + d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 591 vs. $2(173) = 346$.

Time = 0.54 (sec) , antiderivative size = 592, normalized size of antiderivative = 3.10

method	result
default	$\frac{a(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(-\sin(fx+e)d\left(A\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,d}{\sqrt{cda+d^2a}}\right)acd+3A\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,d}{\sqrt{cda+d^2a}}\right)a\,d^2-3\right.\right.}{}$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x,method=_R
ETURNVERBOSE)
```

output
```
a*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(-sin(f*x+e)*d*(A*arctanh((a-a*
sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c*d+3*A*arctanh((a-a*sin(f*x+e)
)^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*d^2-3*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/
(a*c*d+a*d^2)^(1/2))*a*c^2-3*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d
^2)^(1/2))*a*c*d+2*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))
*a*d^2+2*B*(a-a*sin(f*x+e))^(1/2)*((c+d)*a*d)^(1/2)*c+2*B*((c+d)*a*d)^(1/2
)*(a-a*sin(f*x+e))^(1/2)*d)-A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^
2)^(1/2))*a*c^2*d-3*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2)
)*a*c*d^2+3*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c^3+
3*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c^2*d-2*B*arct
anh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c*d^2+A*(a-a*sin(f*x+e
))^(1/2)*((c+d)*a*d)^(1/2)*c*d-A*(a-a*sin(f*x+e))^(1/2)*((c+d)*a*d)^(1/2)*
d^2-3*B*(a-a*sin(f*x+e))^(1/2)*((c+d)*a*d)^(1/2)*c^2-B*(a-a*sin(f*x+e))^(1
/2)*((c+d)*a*d)^(1/2)*c*d)/d^2/(c+d)/(c+d*sin(f*x+e))/((c+d)*a*d)^(1/2)/co
s(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 556 vs. $2(173) = 346$.

Time = 0.81 (sec) , antiderivative size = 1428, normalized size of antiderivative = 7.48

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, al
gorithm="fricas")
```

output
```
[1/4*((3*B*a*c^3 - (A - 6*B)*a*c^2*d - (4*A - B)*a*c*d^2 - (3*A + 2*B)*a*d
^3 - (3*B*a*c^2*d - (A - 3*B)*a*c*d^2 - (3*A + 2*B)*a*d^3)*cos(f*x + e)^2
+ (3*B*a*c^3 - (A - 3*B)*a*c^2*d - (3*A + 2*B)*a*c*d^2)*cos(f*x + e) + (3*
B*a*c^3 - (A - 6*B)*a*c^2*d - (4*A - B)*a*c*d^2 - (3*A + 2*B)*a*d^3 + (3*B
*a*c^2*d - (A - 3*B)*a*c*d^2 - (3*A + 2*B)*a*d^3)*cos(f*x + e))*sin(f*x +
e))*sqrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^
2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d
^2 + d^3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d
 + 4*c*d^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(
f*x + e) + a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x +
e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^
2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x
 + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)
^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(3*B*a*c^2
 - (A + B)*a*c*d + (A - 2*B)*a*d^2 + 2*(B*a*c*d + B*a*d^2)*cos(f*x + e)^2
+ (3*B*a*c^2 - (A - B)*a*c*d + A*a*d^2)*cos(f*x + e) - (3*B*a*c^2 - (A + B
)*a*c*d + (A - 2*B)*a*d^2 - 2*(B*a*c*d + B*a*d^2)*cos(f*x + e))*sin(f*x +
e))*sqrt(a*sin(f*x + e) + a))/((c*d^3 + d^4)*f*cos(f*x + e)^2 - (c^2*d^2 +
 c*d^3)*f*cos(f*x + e) - (c^2*d^2 + 2*c*d^3 + d^4)*f - ((c*d^3 + d^4)*f*co
s(f*x + e) + (c^2*d^2 + 2*c*d^3 + d^4)*f)*sin(f*x + e)), -1/2*((3*B*a*c...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx=\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}}{(d\sin(fx+e)+c)^2}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e)
+ c)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(173) = 346$.

Time = 0.24 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.91

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx=\frac{\sqrt{2}\left(\frac{4\,Ba\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{d^2}+\frac{\sqrt{2}(3\,Bc\cdots}{\cdots}\cdots\right)}{\cdots}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, al
gorithm="giac")
```

output
```
1/2*sqrt(2)*(4*B*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f
*x + 1/2*e)/d^2 + sqrt(2)*(3*B*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) -
 A*a*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*a*c*d*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e)) - 3*A*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*B*a
*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*arctan(sqrt(2)*d*sin(-1/4*pi + 1
/2*f*x + 1/2*e)/sqrt(-c*d - d^2))/((c*d^2 + d^3)*sqrt(-c*d - d^2)) - 2*(B*
a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) -
 A*a*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e
) - B*a*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/
2*e) + A*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x +
 1/2*e))/((c*d^2 + d^3)*(2*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - c - d)))*s
qrt(a)/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}{\left(c+d\,\sin\left(e+f\,x\right)\right)^2}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c + d*sin(e + f*x))
^2,x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c + d*sin(e + f*x))
^2, x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \sqrt{a}\,a\left(\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}}{\sin(fx+e)^2 d^2+2\sin(fx+e)\,cd+c^2}dx\right)\right.\right.$$
$$+\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^2}{\sin(fx+e)^2 d^2+2\sin(fx+e)\,cd+c^2}dx\right)b$$
$$+\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)^2 d^2+2\sin(fx+e)\,cd+c^2}dx\right)a$$
$$\left.+\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)^2 d^2+2\sin(fx+e)\,cd+c^2}dx\right)b\right)$$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

output
```
sqrt(a)*a*(int(sqrt(sin(e + f*x) + 1)/(sin(e + f*x)**2*d**2 + 2*sin(e + f*
x)*c*d + c**2),x)*a + int((sqrt(sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e
+ f*x)**2*d**2 + 2*sin(e + f*x)*c*d + c**2),x)*b + int((sqrt(sin(e + f*x)
+ 1)*sin(e + f*x))/(sin(e + f*x)**2*d**2 + 2*sin(e + f*x)*c*d + c**2),x)*a
 + int((sqrt(sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**2*d**2 + 2*sin
(e + f*x)*c*d + c**2),x)*b)
```

3.299 $\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 221

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = -\frac{a^{3/2}(Ad(c+7d)+3B(c^2+3cd+4d^2))\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{4d^{5/2}(c+d)^{5/2}f} + \frac{a(Bc-Ad)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{2d(c+d)f(c+d\sin(e+fx))^2} + \frac{a^2(A(c-5d)d+B(3c^2+5cd-4d^2))\cos(e+fx)}{4d^2(c+d)^2f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}$$

output
```
-1/4*a^(3/2)*(A*d*(c+7*d)+3*B*(c^2+3*c*d+4*d^2))*arctanh(a^(1/2)*d^(1/2)*c
os(f*x+e)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/d^(5/2)/(c+d)^(5/2)/f+1/2*a*
(-A*d+B*c)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/d/(c+d)/f/(c+d*sin(f*x+e))^2+
1/4*a^2*(A*(c-5*d)*d+B*(3*c^2+5*c*d-4*d^2))*cos(f*x+e)/d^2/(c+d)^2/f/(a+a*
sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 16.34 (sec) , antiderivative size = 957, normalized size of antiderivative = 4.33

$$\int \frac{(a + a\sin(e + fx))^{3/2}(A + B\sin(e + fx))}{(c + d\sin(e + fx))^3}\, dx = \text{Too large to display}$$

input
```
Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e +
 f*x])^3,x]
```

output
```
((a*(1 + Sin[e + f*x]))^(3/2)*(((A*d*(c + 7*d) + 3*B*(c^2 + 3*c*d + 4*d^2)
)*((c + d)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2]) + Sqrt[c + d]*RootSum[c +
 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(c*Sqrt[d]*Log[-#1 + Tan[(e +
 f*x)/4]]) - d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]] - d*Sqrt[c + d]*Log[-#1 +
 Tan[(e + f*x)/4]] - 2*c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*d^(3/2
)*Log[-#1 + Tan[(e + f*x)/4]]*#1 - c*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4
]]*#1 + c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + d^(3/2)*Log[-#1 + Tan
[(e + f*x)/4]]*#1^2 + 3*d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - c
*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#
1^3) & ]))/(c + d)^(7/2) + ((A*d*(c + 7*d) + 3*B*(c^2 + 3*c*d + 4*d^2))*(-
((c + d)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2])) + Sqrt[c + d]*RootSum[c +
4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(c*Sqrt[d]*Log[-#1 + Tan[(e +
f*x)/4]]) - d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]] + d*Sqrt[c + d]*Log[-#1 +
Tan[(e + f*x)/4]] - 2*c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*d^(3/2)
*Log[-#1 + Tan[(e + f*x)/4]]*#1 + c*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]
]*#1 + c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + d^(3/2)*Log[-#1 + Tan[
(e + f*x)/4]]*#1^2 - 3*d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + c*
Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1
^3) & ]))/(c + d)^(7/2) - (8*Sqrt[d]*(-c + d)*(-(B*c) + A*d)*(Cos[(e + f*x
)/2] - Sin[(e + f*x)/2]))/((c + d)*(c + d*Sin[e + f*x])^2) - (4*Sqrt[d]...
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.216$, Rules used = {3042, 3454, 27, 3042, 3459, 3042, 3252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}dx$$

$$\Big\downarrow \text{3454}$$

$$\frac{\int -\frac{\sqrt{\sin(e+fx)a+a}(a(B(c-4d)-5Ad)-a(3Bc+Ad+4Bd)\sin(e+fx))}{2(c+d\sin(e+fx))^2}dx}{2d(c+d)}+\frac{a(Bc-Ad)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{2df(c+d)(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{a(Bc-Ad)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{2df(c+d)(c+d\sin(e+fx))^2}-\frac{\int \frac{\sqrt{\sin(e+fx)a+a}(a(B(c-4d)-5Ad)-a(3Bc+Ad+4Bd)\sin(e+fx))}{(c+d\sin(e+fx))^2}dx}{4d(c+d)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(Bc-Ad)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{2df(c+d)(c+d\sin(e+fx))^2}-\frac{\int \frac{\sqrt{\sin(e+fx)a+a}(a(B(c-4d)-5Ad)-a(3Bc+Ad+4Bd)\sin(e+fx))}{(c+d\sin(e+fx))^2}dx}{4d(c+d)}$$

$$\Big\downarrow \text{3459}$$

$$\frac{a(Bc-Ad)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{2df(c+d)(c+d\sin(e+fx))^2}-\frac{-\frac{a\left(Ad(c+7d)+3B\left(c^2+3cd+4d^2\right)\right)\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{2d(c+d)}-\frac{a^2\left(Ad(c-5d)+B\left(3c^2+5cd-4d^2\right)\right)\cos(e+fx)}{df(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}}{4d(c+d)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a(Bc-Ad)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{2df(c+d)(c+d\sin(e+fx))^2}-\frac{-\frac{a(Ad(c+7d)+3B(c^2+3cd+4d^2))\int\frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{2d(c+d)}-\frac{a^2(Ad(c-5d)+B(3c^2+5cd-4d^2))\cos(e+fx)}{df(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}}{4d(c+d)}$$

$$\Big\downarrow \text{ 3252}$$

$$\frac{a(Bc-Ad)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{2df(c+d)(c+d\sin(e+fx))^2}-\frac{\frac{a^2(Ad(c+7d)+3B(c^2+3cd+4d^2))\int\frac{1}{a(c+d)-\frac{a^2d\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{df(c+d)}-\frac{a^2(Ad(c-5d)+B(3c^2+5cd-4d^2))\cos(e+fx)}{df(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}}{4d(c+d)}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{a(Bc-Ad)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{2df(c+d)(c+d\sin(e+fx))^2}-\frac{\frac{a^{3/2}(Ad(c+7d)+3B(c^2+3cd+4d^2))\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{3/2}f(c+d)^{3/2}}-\frac{a^2(Ad(c-5d)+B(3c^2+5cd-4d^2))\cos(e+fx)}{df(c+d)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}}{4d(c+d)}$$

input
```
Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])
^3,x]
```

output
```
(a*(B*c - A*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(2*d*(c + d)*f*(c +
d*Sin[e + f*x])^2) - ((a^(3/2)*(A*d*(c + 7*d) + 3*B*(c^2 + 3*c*d + 4*d^2))
*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*
x]])])/(d^(3/2)*(c + d)^(3/2)*f) - (a^2*(A*(c - 5*d)*d + B*(3*c^2 + 5*c*d
- 4*d^2))*Cos[e + f*x])/(d*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e
 + f*x])))/(4*d*(c + d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 894 vs. 2(197) = 394.

Time = 0.55 (sec) , antiderivative size = 895, normalized size of antiderivative = 4.05

method	result
default	$\frac{\left(\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,d}{\sqrt{cda+d^2a}}\right)a^2d^2\left(Acd+7A\,d^2+3B\,c^2+9Bcd+12B\,d^2\right)\cos(fx+e)^2-2\sin(fx+e)\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,d}{\sqrt{cda+d^2a}}\right)a^2c}{}$

input

```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x,method=_R
ETURNVERBOSE)
```

output

```
1/4*(arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^2*(A*c*d+
7*A*d^2+3*B*c^2+9*B*c*d+12*B*d^2)*cos(f*x+e)^2-2*sin(f*x+e)*arctanh((a-a*s
in(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d*(A*c*d+7*A*d^2+3*B*c^2+9*B
*c*d+12*B*d^2)+A*(a-a*sin(f*x+e))^(3/2)*((c+d)*a*d)^(1/2)*c*d^2+7*A*(a-a*s
in(f*x+e))^(3/2)*((c+d)*a*d)^(1/2)*d^3-A*arctanh((a-a*sin(f*x+e))^(1/2)*d/
(a*c*d+a*d^2)^(1/2))*a^2*c^3*d-7*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d
+a*d^2)^(1/2))*a^2*c^2*d^2-A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2
)^(1/2))*a^2*c*d^3-7*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2
))*a^2*d^4-5*B*(a-a*sin(f*x+e))^(3/2)*((c+d)*a*d)^(1/2)*c^2*d-7*B*(a-a*sin
(f*x+e))^(3/2)*((c+d)*a*d)^(1/2)*c*d^2+4*B*(a-a*sin(f*x+e))^(3/2)*((c+d)*a
*d)^(1/2)*d^3-3*a^2*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*
B*c^4-9*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^3*d-
15*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^2*d^2-9*B
*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d^3-12*B*arct
anh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^4+A*(a-a*sin(f*x+e
))^(1/2)*((c+d)*a*d)^(1/2)*a*c^2*d-8*A*(a-a*sin(f*x+e))^(1/2)*((c+d)*a*d)^
(1/2)*a*c*d^2-9*A*(a-a*sin(f*x+e))^(1/2)*((c+d)*a*d)^(1/2)*a*d^3+3*B*(a-a*
sin(f*x+e))^(1/2)*((c+d)*a*d)^(1/2)*a*c^3+12*B*(a-a*sin(f*x+e))^(1/2)*((c+
d)*a*d)^(1/2)*a*c^2*d+5*B*(a-a*sin(f*x+e))^(1/2)*((c+d)*a*d)^(1/2)*a*c*d^2
-4*B*(a-a*sin(f*x+e))^(1/2)*((c+d)*a*d)^(1/2)*a*d^3)*(-a*(sin(f*x+e)-1)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 946 vs. $2(197) = 394$.

Time = 1.33 (sec) , antiderivative size = 2208, normalized size of antiderivative = 9.99

$$\int \frac{(a + a\sin(e + fx))^{3/2}(A + B\sin(e + fx))}{(c + d\sin(e + fx))^3} dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, al
gorithm="fricas")
```

output
```
[-1/16*((3*B*a*c^4 + (A + 15*B)*a*c^3*d + 3*(3*A + 11*B)*a*c^2*d^2 + 3*(5*
A + 11*B)*a*c*d^3 + (7*A + 12*B)*a*d^4 - (3*B*a*c^2*d^2 + (A + 9*B)*a*c*d^
3 + (7*A + 12*B)*a*d^4)*cos(f*x + e)^3 - (6*B*a*c^3*d + (2*A + 21*B)*a*c^2
*d^2 + 3*(5*A + 11*B)*a*c*d^3 + (7*A + 12*B)*a*d^4)*cos(f*x + e)^2 + (3*B*
a*c^4 + (A + 9*B)*a*c^3*d + (7*A + 15*B)*a*c^2*d^2 + (A + 9*B)*a*c*d^3 + (
7*A + 12*B)*a*d^4)*cos(f*x + e) + (3*B*a*c^4 + (A + 15*B)*a*c^3*d + 3*(3*A
 + 11*B)*a*c^2*d^2 + 3*(5*A + 11*B)*a*c*d^3 + (7*A + 12*B)*a*d^4 - (3*B*a*
c^2*d^2 + (A + 9*B)*a*c*d^3 + (7*A + 12*B)*a*d^4)*cos(f*x + e)^2 + 2*(3*B*
a*c^3*d + (A + 9*B)*a*c^2*d^2 + (7*A + 12*B)*a*c*d^3)*cos(f*x + e))*sin(f*
x + e))*sqrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d -
a*d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 -
(c*d^2 + d^3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c
^2*d + 4*c*d^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*
sin(f*x + e) + a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*
x + e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*
a*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos
(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x
+ e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(3*B*a
*c^3 + (A + 2*B)*a*c^2*d - 3*(2*A + 3*B)*a*c*d^2 + (5*A + 4*B)*a*d^3 + (5*
B*a*c^2*d - (A - 7*B)*a*c*d^2 - (7*A + 4*B)*a*d^3)*cos(f*x + e)^2 + (3*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx=\text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx=\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}}{(d\sin(fx+e)+c)^3}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e)
+ c)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(197) = 394$.

Time = 0.29 (sec) , antiderivative size = 624, normalized size of antiderivative = 2.82

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx=\text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, al
gorithm="giac")
```

output

```
-1/8*sqrt(2)*sqrt(a)*(sqrt(2)*(3*B*a*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
)) + A*a*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 9*B*a*c*d*sgn(cos(-1/4*
pi + 1/2*f*x + 1/2*e)) + 7*A*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 1
2*B*a*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*arctan(sqrt(2)*d*sin(-1/4*p
i + 1/2*f*x + 1/2*e)/sqrt(-c*d - d^2))/((c^2*d^2 + 2*c*d^3 + d^4)*sqrt(-c*
d - d^2)) - 2*(10*B*a*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*p
i + 1/2*f*x + 1/2*e)^3 - 2*A*a*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*s
in(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 14*B*a*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 14*A*a*d^3*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 8*B*a*d^3*sgn(cos(-1/4
*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 3*B*a*c^3*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - A*a*c^2*d*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 12*B*
a*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)
 + 8*A*a*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x +
 1/2*e) - 5*B*a*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/
2*f*x + 1/2*e) + 9*A*a*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi
 + 1/2*f*x + 1/2*e) + 4*B*a*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1
/4*pi + 1/2*f*x + 1/2*e))/((c^2*d^2 + 2*c*d^3 + d^4)*(2*d*sin(-1/4*pi + 1/
2*f*x + 1/2*e)^2 - c - d)^2))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx=\int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}{\left(c+d\,\sin\left(e+f\,x\right)\right)^3}\,dx$$

input

```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c + d*sin(e + f*x))
^3,x)
```

output

```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c + d*sin(e + f*x))
^3, x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \sqrt{a}\,a\left(\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}}{\sin(fx+e)^3d^3+3\sin(fx+e)^2cd^2+3\sin\right.\right.\right.$$

$$+\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^2}{\sin(fx+e)^3d^3+3\sin(fx+e)^2cd^2+3\sin(fx+e)c^2d+c^3}dx\right)b$$

$$+\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)^3d^3+3\sin(fx+e)^2cd^2+3\sin(fx+e)c^2d+c^3}dx\right)a$$

$$+\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)^3d^3+3\sin(fx+e)^2cd^2+3\sin(fx+e)c^2d+c^3}dx\right)b\right)$$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)
```

output
```
sqrt(a)*a*(int(sqrt(sin(e + f*x) + 1)/(sin(e + f*x)**3*d**3 + 3*sin(e + f*
x)**2*c*d**2 + 3*sin(e + f*x)*c**2*d + c**3),x)*a + int((sqrt(sin(e + f*x)
 + 1)*sin(e + f*x)**2)/(sin(e + f*x)**3*d**3 + 3*sin(e + f*x)**2*c*d**2 +
3*sin(e + f*x)*c**2*d + c**3),x)*b + int((sqrt(sin(e + f*x) + 1)*sin(e + f
*x))/(sin(e + f*x)**3*d**3 + 3*sin(e + f*x)**2*c*d**2 + 3*sin(e + f*x)*c**
2*d + c**3),x)*a + int((sqrt(sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)
**3*d**3 + 3*sin(e + f*x)**2*c*d**2 + 3*sin(e + f*x)*c**2*d + c**3),x)*b)
```

3.300 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$

Optimal result

Integrand size = 37, antiderivative size = 534

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = -\frac{4a^3(c+d)\left(15c^2+10cd+7d^2\right)\left(13Ad(3c^2-38cd+355d^2)-B(15c^3-150c^2d+799cd^2-4184d^3)\right)\cos(}{45045d^3f\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{8a^2(5c-d)(c+d)\left(13Ad(3c^2-38cd+355d^2)-B(15c^3-150c^2d+799cd^2-4184d^3)\right)\cos(e+fx)\sqrt{a}}{45045d^2f}$$
$$-\frac{4a(c+d)\left(13Ad(3c^2-38cd+355d^2)-B(15c^3-150c^2d+799cd^2-4184d^3)\right)\cos(e+fx)(a+a\sin(e+}{15015df}$$
$$-\frac{2a^3\left(13Ad(3c^2-38cd+355d^2)-B(15c^3-150c^2d+799cd^2-4184d^3)\right)\cos(e+fx)(c+d\sin(e+fx))^3}{9009d^3f\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{2a^3\left(15Bc^2-39Acd-75Bcd+299Ad^2+280Bd^2\right)\cos(e+fx)(c+d\sin(e+fx))^4}{1287d^3f\sqrt{a+a\sin(e+fx)}}$$
$$+\frac{2a^2(5Bc-13Ad-16Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^4}{143d^2f}$$
$$-\frac{2aB\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^4}{13df}$$

output
```
-4/45045*a^3*(c+d)*(15*c^2+10*c*d+7*d^2)*(13*A*d*(3*c^2-38*c*d+355*d^2)-B*
(15*c^3-150*c^2*d+799*c*d^2-4184*d^3))*cos(f*x+e)/d^3/f/(a+a*sin(f*x+e))^(
1/2)-8/45045*a^2*(5*c-d)*(c+d)*(13*A*d*(3*c^2-38*c*d+355*d^2)-B*(15*c^3-15
0*c^2*d+799*c*d^2-4184*d^3))*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/d^2/f-4/150
15*a*(c+d)*(13*A*d*(3*c^2-38*c*d+355*d^2)-B*(15*c^3-150*c^2*d+799*c*d^2-41
84*d^3))*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/d/f-2/9009*a^3*(13*A*d*(3*c^2-3
8*c*d+355*d^2)-B*(15*c^3-150*c^2*d+799*c*d^2-4184*d^3))*cos(f*x+e)*(c+d*si
n(f*x+e))^3/d^3/f/(a+a*sin(f*x+e))^(1/2)-2/1287*a^3*(-39*A*c*d+299*A*d^2+1
5*B*c^2-75*B*c*d+280*B*d^2)*cos(f*x+e)*(c+d*sin(f*x+e))^4/d^3/f/(a+a*sin(f
*x+e))^(1/2)+2/143*a^2*(-13*A*d+5*B*c-16*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^
(1/2)*(c+d*sin(f*x+e))^4/d^2/f-2/13*a*B*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*
(c+d*sin(f*x+e))^4/d/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.00 (sec) , antiderivative size = 1565, normalized size of antiderivative = 2.93

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = \text{Too large to display}$$

input
```
Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f
*x])^3,x]
```

output
```
(B*d^3*Cos[(13*(e + f*x))/2]*(a*(1 + Sin[e + f*x]))^(5/2))/(416*f*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^5) + ((40*A*c^3 + 30*B*c^3 + 90*A*c^2*d + 78
*B*c^2*d + 78*A*c*d^2 + 69*B*c*d^2 + 23*A*d^3 + 21*B*d^3)*((-1/16 - I/16)*
Cos[(e + f*x)/2] + (1/16 - I/16)*Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^
(5/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) + ((40*A*c^3 + 30*B*c^3
 + 90*A*c^2*d + 78*B*c^2*d + 78*A*c*d^2 + 69*B*c*d^2 + 23*A*d^3 + 21*B*d^3
)*((-1/16 + I/16)*Cos[(e + f*x)/2] + (1/16 + I/16)*Sin[(e + f*x)/2])*(a*(1
 + Sin[e + f*x]))^(5/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) + ((8
0*A*c^3 + 88*B*c^3 + 264*A*c^2*d + 240*B*c^2*d + 240*A*c*d^2 + 228*B*c*d^2
 + 76*A*d^3 + 71*B*d^3)*(a*(1 + Sin[e + f*x]))^(5/2)*((-1/192 + I/192)*Cos
[(3*(e + f*x))/2] - (1/192 + I/192)*Sin[(3*(e + f*x))/2]))/(f*(Cos[(e + f*
x)/2] + Sin[(e + f*x)/2])^5) + ((80*A*c^3 + 88*B*c^3 + 264*A*c^2*d + 240*B
*c^2*d + 240*A*c*d^2 + 228*B*c*d^2 + 76*A*d^3 + 71*B*d^3)*(a*(1 + Sin[e +
f*x]))^(5/2)*((-1/192 - I/192)*Cos[(3*(e + f*x))/2] - (1/192 - I/192)*Sin[
(3*(e + f*x))/2]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) + ((16*A*c^
3 + 40*B*c^3 + 120*A*c^2*d + 144*B*c^2*d + 144*A*c*d^2 + 150*B*c*d^2 + 50*
A*d^3 + 51*B*d^3)*(a*(1 + Sin[e + f*x]))^(5/2)*((1/320 - I/320)*Cos[(5*(e
+ f*x))/2] - (1/320 + I/320)*Sin[(5*(e + f*x))/2]))/(f*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^5) + ((16*A*c^3 + 40*B*c^3 + 120*A*c^2*d + 144*B*c^2*d
+ 144*A*c*d^2 + 150*B*c*d^2 + 50*A*d^3 + 51*B*d^3)*(a*(1 + Sin[e + f*x]...
```

Rubi [A] (verified)

Time = 2.24 (sec) , antiderivative size = 420, normalized size of antiderivative = 0.79, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.459$, Rules used = {3042, 3455, 27, 3042, 3455, 27, 3042, 3460, 3042, 3249, 3042, 3240, 27, 3042, 3230, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3dx$$

$$\Big\downarrow \text{3455}$$

$$\frac{2\int \frac{1}{2}(\sin(e+fx)a+a)^{3/2}(c+d\sin(e+fx))^3(a(3Bc+13Ad+8Bd)-a(5Bc-13Ad-16Bd)\sin(e+fx))dx}{13d}$$
$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^4}{13df}$$

$\Big\downarrow$ 27

$$\frac{\int(\sin(e+fx)a+a)^{3/2}(c+d\sin(e+fx))^3(a(3Bc+13Ad+8Bd)-a(5Bc-13Ad-16Bd)\sin(e+fx))dx}{13d}-$$
$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^4}{13df}$$

$\Big\downarrow$ 3042

$$\frac{\int(\sin(e+fx)a+a)^{3/2}(c+d\sin(e+fx))^3(a(3Bc+13Ad+8Bd)-a(5Bc-13Ad-16Bd)\sin(e+fx))dx}{13d}-$$
$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^4}{13df}$$

$\Big\downarrow$ 3455

$$\frac{\frac{2\int \frac{1}{2}\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^3\left(\left(13Ad(c+19d)-B\left(5c^2-9dc-216d^2\right)\right)a^2+\left(15Bc^2-39Adc-75Bdc+299Ad^2+280Bd^2\right)\sin(e+fx)a^2\right)dx}{11d}}{13d}$$
$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^4}{13df}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^3\left(\left(13Ad(c+19d)-B\left(5c^2-9dc-216d^2\right)\right)a^2+\left(15Bc^2-39Adc-75Bdc+299Ad^2+280Bd^2\right)\sin(e+fx)a^2\right)dx}{11d}+}{13d}$$
$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^4}{13df}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^3\left(\left(13Ad(c+19d)-B\left(5c^2-9dc-216d^2\right)\right)a^2+\left(15Bc^2-39Adc-75Bdc+299Ad^2+280Bd^2\right)\sin(e+fx)a^2\right)dx}{11d}+}{13d}$$
$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^4}{13df}$$

$\Big\downarrow$ 3460

$$\frac{\frac{\frac{a^2\left(13Ad\left(3c^2-38cd+355d^2\right)-B\left(15c^3-150c^2d+799cd^2-4184d^3\right)\right)\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^3dx}{9d}-\frac{2a^3\left(-39Acd+299Ad^2+15Bc^2-75Bcd+280Bd^2\right)\text{co}}{9df\sqrt{a\sin(e+fx)+a}}}{11d}}{13d}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^4}{13df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{a^2\left(13Ad\left(3c^2-38cd+355d^2\right)-B\left(15c^3-150c^2d+799cd^2-4184d^3\right)\right)\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^3dx}{9d}-\frac{2a^3\left(-39Acd+299Ad^2+15Bc^2-75Bcd+280Bd^2\right)\text{co}}{9df\sqrt{a\sin(e+fx)+a}}}{11d}}{13d}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^4}{13df}$$

$$\Big\downarrow \text{3249}$$

$$\frac{\frac{\frac{a^2\left(13Ad\left(3c^2-38cd+355d^2\right)-B\left(15c^3-150c^2d+799cd^2-4184d^3\right)\right)\left(\frac{6}{7}(c+d)\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2dx-\frac{2a\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}\right)}{9d}-\frac{2a^3\left(-3\right.}{}}{11d}}{13d}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^4}{13df}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{a^2\left(13Ad\left(3c^2-38cd+355d^2\right)-B\left(15c^3-150c^2d+799cd^2-4184d^3\right)\right)\left(\frac{6}{7}(c+d)\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2dx-\frac{2a\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}\right)}{9d}-\frac{2a^3\left(-3\right.}{}}{11d}}{13d}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^4}{13df}$$

$$\Big\downarrow \text{3240}$$

$$\frac{\frac{a^2\left(13Ad\left(3c^2-38cd+355d^2\right)-B\left(15c^3-150c^2d+799cd^2-4184d^3\right)\right)\left(\frac{6}{7}(c+d)\left(\frac{2\int \frac{1}{2}\sqrt{\sin(e+fx)a+a}\left(a\left(5c^2+3d^2\right)+2a(5c-d)d\sin(e+fx)\right)dx}{5a}-\frac{2d^2\cos(e+fx)(a}{5}\right.\right.}{9d}}{11d}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^4}{13df}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{a^2\left(13Ad\left(3c^2-38cd+355d^2\right)-B\left(15c^3-150c^2d+799cd^2-4184d^3\right)\right)\left(\frac{6}{7}(c+d)\left(\frac{\int\sqrt{\sin(e+fx)a+a}\left(a\left(5c^2+3d^2\right)+2a(5c-d)d\sin(e+fx)\right)dx}{5a}-\frac{2d^2\cos(e+fx)(a\sin(}{5af}\right.\right.}{9d}}{11d}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^4}{13df}$$

$\downarrow$ 3042

$$\frac{\frac{a^2\left(13Ad\left(3c^2-38cd+355d^2\right)-B\left(15c^3-150c^2d+799cd^2-4184d^3\right)\right)\left(\frac{6}{7}(c+d)\left(\frac{\int\sqrt{\sin(e+fx)a+a}\left(a\left(5c^2+3d^2\right)+2a(5c-d)d\sin(e+fx)\right)dx}{5a}-\frac{2d^2\cos(e+fx)(a\sin(}{5af}\right.\right.}{9d}}{11d}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^4}{13df}$$

$\downarrow$ 3230

$$\frac{\frac{a^2\left(13Ad\left(3c^2-38cd+355d^2\right)-B\left(15c^3-150c^2d+799cd^2-4184d^3\right)\right)\left(\frac{6}{7}(c+d)\left(\frac{\frac{1}{3}a\left(15c^2+10cd+7d^2\right)\int\sqrt{\sin(e+fx)a+a}dx-\frac{4ad(5c-d)\cos(e+fx)\sqrt{a\sin(e+fx)+}}{3f}}{5a}\right.\right.}{9d}}{11d}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^4}{13df}$$

$\downarrow$ 3042

$$\frac{\frac{a^2\left(13Ad\left(3c^2-38cd+355d^2\right)-B\left(15c^3-150c^2d+799cd^2-4184d^3\right)\right)\left(\frac{6}{7}(c+d)\left(\frac{\frac{1}{3}a\left(15c^2+10cd+7d^2\right)\int\sqrt{\sin(e+fx)a+a}dx-\frac{4ad(5c-d)\cos(e+fx)\sqrt{a\sin(e+fx)+}}{3f}}{5a}\right.\right.}{9d}}{11d}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^4}{13df}$$

$\downarrow$ 3125

$$\frac{\frac{2a^2(-13Ad+5Bc-16Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^4}{11df}+\frac{a^2\left(13Ad\left(3c^2-38cd+355d^2\right)-B\left(15c^3-150c^2d+799cd^2-4184d^3\right)\right)\left(\frac{6}{7}(c+\right.}{}}{}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^4}{13df}$$

input
```
Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3
,x]
```

output
```
(-2*a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^4)/(1
3*d*f) + ((2*a^2*(5*B*c - 13*A*d - 16*B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^4)/(11*d*f) + ((-2*a^3*(15*B*c^2 - 39*A*c*d -
75*B*c*d + 299*A*d^2 + 280*B*d^2)*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(9*
d*f*Sqrt[a + a*Sin[e + f*x]]) + (a^2*(13*A*d*(3*c^2 - 38*c*d + 355*d^2) -
B*(15*c^3 - 150*c^2*d + 799*c*d^2 - 4184*d^3))*((-2*a*Cos[e + f*x]*(c + d*
Sin[e + f*x])^3)/(7*f*Sqrt[a + a*Sin[e + f*x]]) + (6*(c + d)*((-2*d^2*Cos[
e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(5*a*f) + ((-2*a^2*(15*c^2 + 10*c*d +
 7*d^2)*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) - (4*a*(5*c - d)*d*Co
s[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*f))/(5*a)))/7))/(9*d))/(11*d))/(13
*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3240
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])^
m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

rule 3249
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[2*n*((b*c + a*d)/(b*(
2*n + 1)))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 208.69 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.70

method	result
default	$\frac{2(1+\sin(fx+e))a^3(\sin(fx+e)-1)\left(-3465B\,d^3\cos(fx+e)^6+(4095A\,d^3+12285Bc\,d^2+11970B\,d^3)\cos(fx+e)^4\sin(fx+e)+(15015A\right.}{}$
parts	$\frac{2(1+\sin(fx+e))a^3(\sin(fx+e)-1)d^2(Ad+3Bc)\left(63\sin(fx+e)^5+224\sin(fx+e)^4+355\sin(fx+e)^3+426\sin(fx+e)^2+568\sin(fx+e)\right.}{693\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x,method=_R
ETURNVERBOSE)
```

output
```
2/45045*(1+sin(f*x+e))*a^3*(sin(f*x+e)-1)*(-3465*B*d^3*cos(f*x+e)^6+(4095*
A*d^3+12285*B*c*d^2+11970*B*d^3)*cos(f*x+e)^4*sin(f*x+e)+(15015*A*c*d^2+14
560*A*d^3+15015*B*c^2*d+43680*B*c*d^2+28700*B*d^3)*cos(f*x+e)^4+(-19305*A*
c^2*d-55770*A*c*d^2-31265*A*d^3-6435*B*c^3-55770*B*c^2*d-93795*B*c*d^2-448
60*B*d^3)*cos(f*x+e)^2*sin(f*x+e)+(-9009*A*c^3-77220*A*c^2*d-123981*A*c*d^
2-56810*A*d^3-25740*B*c^3-123981*B*c^2*d-170430*B*c*d^2-72109*B*d^3)*cos(f
*x+e)^2+(42042*A*c^3+167310*A*c^2*d+181038*A*c*d^2+64090*A*d^3+55770*B*c^3
+181038*B*c^2*d+192270*B*c*d^2+66362*B*d^3)*sin(f*x+e)+138138*A*c^3+373230
*A*c^2*d+359502*A*c*d^2+116090*A*d^3+124410*B*c^3+359502*B*c^2*d+348270*B*
c*d^2+113818*B*d^3)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 863, normalized size of antiderivative = 1.62

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, al
gorithm="fricas")
```

output
```
2/45045*(3465*B*a^2*d^3*cos(f*x + e)^7 - 315*(39*B*a^2*c*d^2 + (13*A + 27*
B)*a^2*d^3)*cos(f*x + e)^6 - 13728*(7*A + 5*B)*a^2*c^3 - 13728*(15*A + 13*
B)*a^2*c^2*d - 1248*(143*A + 125*B)*a^2*c*d^2 - 32*(1625*A + 1483*B)*a^2*d
^3 - 35*(429*B*a^2*c^2*d + 39*(11*A + 32*B)*a^2*c*d^2 + 4*(104*A + 205*B)*
a^2*d^3)*cos(f*x + e)^5 + 5*(1287*B*a^2*c^3 + 429*(9*A + 19*B)*a^2*c^2*d +
 39*(209*A + 320*B)*a^2*c*d^2 + 2*(2080*A + 2813*B)*a^2*d^3)*cos(f*x + e)^
4 + (1287*(7*A + 20*B)*a^2*c^3 + 429*(180*A + 289*B)*a^2*c^2*d + 39*(3179*
A + 4370*B)*a^2*c*d^2 + (56810*A + 72109*B)*a^2*d^3)*cos(f*x + e)^3 - (429
*(77*A + 85*B)*a^2*c^3 + 429*(255*A + 263*B)*a^2*c^2*d + 39*(2893*A + 2965
*B)*a^2*c*d^2 + (38545*A + 39113*B)*a^2*d^3)*cos(f*x + e)^2 - 2*(429*(161*
A + 145*B)*a^2*c^3 + 429*(435*A + 419*B)*a^2*c^2*d + 39*(4609*A + 4465*B)*
a^2*c*d^2 + (58045*A + 56909*B)*a^2*d^3)*cos(f*x + e) - (3465*B*a^2*d^3*co
s(f*x + e)^6 - 13728*(7*A + 5*B)*a^2*c^3 - 13728*(15*A + 13*B)*a^2*c^2*d -
 1248*(143*A + 125*B)*a^2*c*d^2 - 32*(1625*A + 1483*B)*a^2*d^3 + 315*(39*B
*a^2*c*d^2 + (13*A + 38*B)*a^2*d^3)*cos(f*x + e)^5 - 35*(429*B*a^2*c^2*d +
 39*(11*A + 23*B)*a^2*c*d^2 + (299*A + 478*B)*a^2*d^3)*cos(f*x + e)^4 - 5*
(1287*B*a^2*c^3 + 429*(9*A + 26*B)*a^2*c^2*d + 507*(22*A + 37*B)*a^2*c*d^2
 + (6253*A + 8972*B)*a^2*d^3)*cos(f*x + e)^3 + 3*(429*(7*A + 15*B)*a^2*c^3
 + 429*(45*A + 53*B)*a^2*c^2*d + 39*(583*A + 655*B)*a^2*c*d^2 + (8515*A +
9083*B)*a^2*d^3)*cos(f*x + e)^2 + 2*(429*(49*A + 65*B)*a^2*c^3 + 429*(1...
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^{5/2}(A + B\sin(e + fx))(c + d\sin(e + fx))^3 \, dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}(d\sin(fx+e)+c)^3\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e)
+ c)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1014 vs. 2(506) = 1012.

Time = 0.57 (sec) , antiderivative size = 1014, normalized size of antiderivative = 1.90

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx=\text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, al
gorithm="giac")
```

output
```
1/1441440*sqrt(2)*(3465*B*a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(
-13/4*pi + 13/2*f*x + 13/2*e) + 180180*(40*A*a^2*c^3*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e)) + 30*B*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 90*A*a
^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 78*B*a^2*c^2*d*sgn(cos(-1/4
*pi + 1/2*f*x + 1/2*e)) + 78*A*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
)) + 69*B*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 23*A*a^2*d^3*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 21*B*a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 15015*(80*A*a^2*c^3*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e)) + 88*B*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
)) + 264*A*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 240*B*a^2*c^2*d
*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 240*A*a^2*c*d^2*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e)) + 228*B*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 7
6*A*a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 71*B*a^2*d^3*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/2*f*x + 3/2*e) + 9009*(16*A*a^2*
c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 40*B*a^2*c^3*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e)) + 120*A*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 1
44*B*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 144*A*a^2*c*d^2*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e)) + 150*B*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e)) + 50*A*a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 51*B*a^2*d
^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-5/4*pi + 5/2*f*x + 5/2*e) ...
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx=\int (A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{5/2}\,(c+d\,\sin(e+f\,x))^3\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^3
,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^3
, x)
```

Reduce [F]

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^3\,dx = \text{Too large to display}$$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x)
```

output
```
sqrt(a)*a**2*(int(sqrt(sin(e + f*x) + 1),x)*a*c**3 + int(sqrt(sin(e + f*x)
 + 1)*sin(e + f*x)**6,x)*b*d**3 + int(sqrt(sin(e + f*x) + 1)*sin(e + f*x)*
*5,x)*a*d**3 + 3*int(sqrt(sin(e + f*x) + 1)*sin(e + f*x)**5,x)*b*c*d**2 +
2*int(sqrt(sin(e + f*x) + 1)*sin(e + f*x)**5,x)*b*d**3 + 3*int(sqrt(sin(e
+ f*x) + 1)*sin(e + f*x)**4,x)*a*c*d**2 + 2*int(sqrt(sin(e + f*x) + 1)*sin
(e + f*x)**4,x)*a*d**3 + 3*int(sqrt(sin(e + f*x) + 1)*sin(e + f*x)**4,x)*b
*c**2*d + 6*int(sqrt(sin(e + f*x) + 1)*sin(e + f*x)**4,x)*b*c*d**2 + int(s
qrt(sin(e + f*x) + 1)*sin(e + f*x)**4,x)*b*d**3 + 3*int(sqrt(sin(e + f*x)
+ 1)*sin(e + f*x)**3,x)*a*c**2*d + 6*int(sqrt(sin(e + f*x) + 1)*sin(e + f*
x)**3,x)*a*c*d**2 + int(sqrt(sin(e + f*x) + 1)*sin(e + f*x)**3,x)*a*d**3 +
 int(sqrt(sin(e + f*x) + 1)*sin(e + f*x)**3,x)*b*c**3 + 6*int(sqrt(sin(e +
 f*x) + 1)*sin(e + f*x)**3,x)*b*c**2*d + 3*int(sqrt(sin(e + f*x) + 1)*sin(
e + f*x)**3,x)*b*c*d**2 + int(sqrt(sin(e + f*x) + 1)*sin(e + f*x)**2,x)*a*
c**3 + 6*int(sqrt(sin(e + f*x) + 1)*sin(e + f*x)**2,x)*a*c**2*d + 3*int(sq
rt(sin(e + f*x) + 1)*sin(e + f*x)**2,x)*a*c*d**2 + 2*int(sqrt(sin(e + f*x)
 + 1)*sin(e + f*x)**2,x)*b*c**3 + 3*int(sqrt(sin(e + f*x) + 1)*sin(e + f*x
)**2,x)*b*c**2*d + 2*int(sqrt(sin(e + f*x) + 1)*sin(e + f*x),x)*a*c**3 + 3
*int(sqrt(sin(e + f*x) + 1)*sin(e + f*x),x)*a*c**2*d + int(sqrt(sin(e + f*
x) + 1)*sin(e + f*x),x)*b*c**3)
```

3.301 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

Optimal result

Integrand size = 37, antiderivative size = 429

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx =$$
$$-\frac{2a^3(15c^2+10cd+7d^2)\,(11Ad(c^2-10cd+73d^2)-B(5c^3-40c^2d+169cd^2-710d^3))\cos(e+fx)}{3465d^3f\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{4a^2(5c-d)\,(11Ad(c^2-10cd+73d^2)-B(5c^3-40c^2d+169cd^2-710d^3))\cos(e+fx)\sqrt{a+a\sin(e+f}}{3465d^2f}$$
$$-\frac{2a(11Ad(c^2-10cd+73d^2)-B(5c^3-40c^2d+169cd^2-710d^3))\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{1155df}$$
$$+\frac{2a^3(11A(3c-19d)d-B(15c^2-65cd+194d^2))\cos(e+fx)(c+d\sin(e+fx))^3}{693d^3f\sqrt{a+a\sin(e+fx)}}$$
$$+\frac{2a^2(5Bc-11Ad-14Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3}{99d^2f}$$
$$-\frac{2aB\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3}{11df}$$

output
```
-2/3465*a^3*(15*c^2+10*c*d+7*d^2)*(11*A*d*(c^2-10*c*d+73*d^2)-B*(5*c^3-40*
c^2*d+169*c*d^2-710*d^3))*cos(f*x+e)/d^3/f/(a+a*sin(f*x+e))^(1/2)-4/3465*a
^2*(5*c-d)*(11*A*d*(c^2-10*c*d+73*d^2)-B*(5*c^3-40*c^2*d+169*c*d^2-710*d^3
))*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/d^2/f-2/1155*a*(11*A*d*(c^2-10*c*d+73
*d^2)-B*(5*c^3-40*c^2*d+169*c*d^2-710*d^3))*cos(f*x+e)*(a+a*sin(f*x+e))^(3
/2)/d/f+2/693*a^3*(11*A*(3*c-19*d)*d-B*(15*c^2-65*c*d+194*d^2))*cos(f*x+e)
*(c+d*sin(f*x+e))^3/d^3/f/(a+a*sin(f*x+e))^(1/2)+2/99*a^2*(-11*A*d+5*B*c-1
4*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)*(c+d*sin(f*x+e))^3/d^2/f-2/11*a*B
*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^3/d/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 891 vs. $2(429) = 858$.

Time = 9.64 (sec) , antiderivative size = 891, normalized size of antiderivative = 2.08

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = \text{Too large to display}$$

input
```
Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f
*x])^2,x]
```

output
```
((a*(1 + Sin[e + f*x]))^(5/2)*(-277200*A*c^2*Cos[(e + f*x)/2] - 207900*B*c
^2*Cos[(e + f*x)/2] - 415800*A*c*d*Cos[(e + f*x)/2] - 360360*B*c*d*Cos[(e
+ f*x)/2] - 180180*A*d^2*Cos[(e + f*x)/2] - 159390*B*d^2*Cos[(e + f*x)/2]
- 46200*A*c^2*Cos[(3*(e + f*x))/2] - 50820*B*c^2*Cos[(3*(e + f*x))/2] - 10
1640*A*c*d*Cos[(3*(e + f*x))/2] - 92400*B*c*d*Cos[(3*(e + f*x))/2] - 46200
*A*d^2*Cos[(3*(e + f*x))/2] - 43890*B*d^2*Cos[(3*(e + f*x))/2] + 5544*A*c^
2*Cos[(5*(e + f*x))/2] + 13860*B*c^2*Cos[(5*(e + f*x))/2] + 27720*A*c*d*Co
s[(5*(e + f*x))/2] + 33264*B*c*d*Cos[(5*(e + f*x))/2] + 16632*A*d^2*Cos[(5
*(e + f*x))/2] + 17325*B*d^2*Cos[(5*(e + f*x))/2] + 1980*B*c^2*Cos[(7*(e +
 f*x))/2] + 3960*A*c*d*Cos[(7*(e + f*x))/2] + 9900*B*c*d*Cos[(7*(e + f*x))
/2] + 4950*A*d^2*Cos[(7*(e + f*x))/2] + 6435*B*d^2*Cos[(7*(e + f*x))/2] -
1540*B*c*d*Cos[(9*(e + f*x))/2] - 770*A*d^2*Cos[(9*(e + f*x))/2] - 1925*B*
d^2*Cos[(9*(e + f*x))/2] - 315*B*d^2*Cos[(11*(e + f*x))/2] + 277200*A*c^2*
Sin[(e + f*x)/2] + 207900*B*c^2*Sin[(e + f*x)/2] + 415800*A*c*d*Sin[(e + f
*x)/2] + 360360*B*c*d*Sin[(e + f*x)/2] + 180180*A*d^2*Sin[(e + f*x)/2] + 1
59390*B*d^2*Sin[(e + f*x)/2] - 46200*A*c^2*Sin[(3*(e + f*x))/2] - 50820*B*
c^2*Sin[(3*(e + f*x))/2] - 101640*A*c*d*Sin[(3*(e + f*x))/2] - 92400*B*c*d
*Sin[(3*(e + f*x))/2] - 46200*A*d^2*Sin[(3*(e + f*x))/2] - 43890*B*d^2*Sin
[(3*(e + f*x))/2] - 5544*A*c^2*Sin[(5*(e + f*x))/2] - 13860*B*c^2*Sin[(5*(
e + f*x))/2] - 27720*A*c*d*Sin[(5*(e + f*x))/2] - 33264*B*c*d*Sin[(5*(e...
```

Rubi [A] (verified)

Time = 1.93 (sec) , antiderivative size = 371, normalized size of antiderivative = 0.86, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.405$, Rules used = {3042, 3455, 27, 3042, 3455, 27, 3042, 3460, 3042, 3240, 27, 3042, 3230, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2dx$$

$$\Big\downarrow \text{3455}$$

$$\frac{2\int \frac{1}{2}(\sin(e+fx)a+a)^{3/2}(c+d\sin(e+fx))^2(a(11Ad+3B(c+2d))-a(5Bc-11Ad-14Bd)\sin(e+fx))dx}{11d}$$
$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^3}{11df}$$

$\big\downarrow$ 27

$$\frac{\int (\sin(e+fx)a+a)^{3/2}(c+d\sin(e+fx))^2(a(11Ad+3B(c+2d))-a(5Bc-11Ad-14Bd)\sin(e+fx))dx}{11d}-$$
$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^3}{11df}$$

$\big\downarrow$ 3042

$$\frac{\int (\sin(e+fx)a+a)^{3/2}(c+d\sin(e+fx))^2(a(11Ad+3B(c+2d))-a(5Bc-11Ad-14Bd)\sin(e+fx))dx}{11d}-$$
$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^3}{11df}$$

$\big\downarrow$ 3455

$$\frac{\frac{2\int \frac{1}{2}\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2\left(a^2\left(11Ad(c+15d)-B\left(5c^2-11dc-138d^2\right)\right)-a^2\left(11A(3c-19d)d-B\left(15c^2-65dc+194d^2\right)\right)\sin(e+fx)\right)dx}{9d}}{11d}$$
$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^3}{11df}$$

$\big\downarrow$ 27

$$\frac{\frac{\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2\left(a^2\left(11Ad(c+15d)-B\left(5c^2-11dc-138d^2\right)\right)-a^2\left(11A(3c-19d)d-B\left(15c^2-65dc+194d^2\right)\right)\sin(e+fx)\right)dx}{9d}+}{11d}$$
$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^3}{11df}$$

$\big\downarrow$ 3042

$$\frac{\frac{\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2\left(a^2\left(11Ad(c+15d)-B\left(5c^2-11dc-138d^2\right)\right)-a^2\left(11A(3c-19d)d-B\left(15c^2-65dc+194d^2\right)\right)\sin(e+fx)\right)dx}{9d}+}{11d}$$
$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^3}{11df}$$

$\big\downarrow$ 3460

$$\frac{\frac{\frac{3a^2\left(11Ad\left(c^2-10cd+73d^2\right)-B\left(5c^3-40c^2d+169cd^2-710d^3\right)\right)\int\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2dx}{7d}+\frac{2a^3\left(11Ad(3c-19d)-B\left(15c^2-65cd+194d^2\right)\right)\cos(e+fx)\cdots}{7df\sqrt{a\sin(e+fx)+a}}}{9d}\cdots}{11d}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^3}{11df}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{3a^2\left(11Ad\left(c^2-10cd+73d^2\right)-B\left(5c^3-40c^2d+169cd^2-710d^3\right)\right)\int\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2dx}{7d}+\frac{2a^3\left(11Ad(3c-19d)-B\left(15c^2-65cd+194d^2\right)\right)\cos(e+fx)\cdots}{7df\sqrt{a\sin(e+fx)+a}}}{9d}\cdots}{11d}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^3}{11df}$$

$\Big\downarrow$ 3240

$$\frac{\frac{3a^2\left(11Ad\left(c^2-10cd+73d^2\right)-B\left(5c^3-40c^2d+169cd^2-710d^3\right)\right)\left(\frac{2\int\frac{1}{2}\sqrt{\sin(e+fx)a+a}\left(a\left(5c^2+3d^2\right)+2a(5c-d)d\sin(e+fx)\right)dx}{5a}-\frac{2d^2\cos(e+fx)(a\sin(e+fx)+a)\cdots}{5af}\cdots\right.}{7d}\cdots}{9d}\cdots$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^3}{11df}$$

$\Big\downarrow$ 27

$$\frac{\frac{3a^2\left(11Ad\left(c^2-10cd+73d^2\right)-B\left(5c^3-40c^2d+169cd^2-710d^3\right)\right)\left(\frac{\int\sqrt{\sin(e+fx)a+a}\left(a\left(5c^2+3d^2\right)+2a(5c-d)d\sin(e+fx)\right)dx}{5a}-\frac{2d^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5af}\cdots\right.}{7d}\cdots}{9d}\cdots$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^3}{11df}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3a^2\left(11Ad\left(c^2-10cd+73d^2\right)-B\left(5c^3-40c^2d+169cd^2-710d^3\right)\right)\left(\frac{\int\sqrt{\sin(e+fx)a+a}\left(a\left(5c^2+3d^2\right)+2a(5c-d)d\sin(e+fx)\right)dx}{5a}-\frac{2d^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5af}\cdots\right.}{7d}\cdots}{9d}\cdots$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^3}{11df}$$

$\Big\downarrow$ 3230

$$\frac{\frac{3a^2\left(11Ad\left(c^2-10cd+73d^2\right)-B\left(5c^3-40c^2d+169cd^2-710d^3\right)\right)\left(\frac{\frac{1}{3}a\left(15c^2+10cd+7d^2\right)\int\sqrt{\sin(e+fx)a+a}dx-\frac{4ad(5c-d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{5a}-\frac{2d^2\cos(e\cdots}{\cdots}\right)}{7d}}{9d}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^3}{11df}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3a^2\left(11Ad\left(c^2-10cd+73d^2\right)-B\left(5c^3-40c^2d+169cd^2-710d^3\right)\right)\left(\frac{\frac{1}{3}a\left(15c^2+10cd+7d^2\right)\int\sqrt{\sin(e+fx)a+a}dx-\frac{4ad(5c-d)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{5a}-\frac{2d^2\cos(e\cdots}{\cdots}\right)}{7d}}{9d}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^3}{11df}$$

$\Big\downarrow$ 3125

$$\frac{\frac{2a^2(-11Ad+5Bc-14Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^3}{9df}+\frac{\frac{2a^3\left(11Ad(3c-19d)-B\left(15c^2-65cd+194d^2\right)\right)\cos(e+fx)(c+d\sin(e+fx))^3}{7df\sqrt{a\sin(e+fx)+a}}+\cdots}{\cdots}}{\cdots}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^3}{11df}$$

input
```
Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2
,x]
```

output
```
(-2*a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^3)/(1
1*d*f) + ((2*a^2*(5*B*c - 11*A*d - 14*B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^3)/(9*d*f) + ((2*a^3*(11*A*(3*c - 19*d)*d - B*
(15*c^2 - 65*c*d + 194*d^2))*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(7*d*f*S
qrt[a + a*Sin[e + f*x]]) + (3*a^2*(11*A*d*(c^2 - 10*c*d + 73*d^2) - B*(5*c
^3 - 40*c^2*d + 169*c*d^2 - 710*d^3))*((-2*d^2*Cos[e + f*x]*(a + a*Sin[e +
 f*x])^(3/2))/(5*a*f) + ((-2*a^2*(15*c^2 + 10*c*d + 7*d^2)*Cos[e + f*x])/(
3*f*Sqrt[a + a*Sin[e + f*x]]) - (4*a*(5*c - d)*d*Cos[e + f*x]*Sqrt[a + a*S
in[e + f*x]])/(3*f))/(5*a)))/(7*d))/(9*d))/(11*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3240
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^2, x_Symbol] :> Simp[(-d^2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^
(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int[(a + b*Sin[e + f*x])^
m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [A] (verified)

Time = 39.15 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.60

method	result
default	$\frac{2(1+\sin(fx+e))a^3(\sin(fx+e)-1)\left(315B\cos(fx+e)^4\sin(fx+e)d^2+(385A\,d^2+770Bcd+1120B\,d^2)\cos(fx+e)^4+(-990Acd-1430\right.}{}$
parts	$\frac{2(1+\sin(fx+e))a^3(\sin(fx+e)-1)d(Ad+2Bc)\left(35\sin(fx+e)^4+130\sin(fx+e)^3+219\sin(fx+e)^2+292\sin(fx+e)+584\right)}{315\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}+\frac{2(1+\mathrm{si}}{}$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x,method=_R
ETURNVERBOSE)
```

output
```
2/3465*(1+sin(f*x+e))*a^3*(sin(f*x+e)-1)*(315*B*cos(f*x+e)^4*sin(f*x+e)*d^
2+(385*A*d^2+770*B*c*d+1120*B*d^2)*cos(f*x+e)^4+(-990*A*c*d-1430*A*d^2-495
*B*c^2-2860*B*c*d-2405*B*d^2)*cos(f*x+e)^2*sin(f*x+e)+(-693*A*c^2-3960*A*c
*d-3179*A*d^2-1980*B*c^2-6358*B*c*d-4370*B*d^2)*cos(f*x+e)^2+(3234*A*c^2+8
580*A*c*d+4642*A*d^2+4290*B*c^2+9284*B*c*d+4930*B*d^2)*sin(f*x+e)+10626*A*
c^2+19140*A*c*d+9218*A*d^2+9570*B*c^2+18436*B*c*d+8930*B*d^2)/cos(f*x+e)/(
a+a*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 593, normalized size of antiderivative = 1.38

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, al
gorithm="fricas")
```

output
```
-2/3465*(315*B*a^2*d^2*cos(f*x + e)^6 + 35*(22*B*a^2*c*d + (11*A + 32*B)*a
^2*d^2)*cos(f*x + e)^5 + 1056*(7*A + 5*B)*a^2*c^2 + 704*(15*A + 13*B)*a^2*
c*d + 32*(143*A + 125*B)*a^2*d^2 - 5*(99*B*a^2*c^2 + 22*(9*A + 19*B)*a^2*c
*d + (209*A + 320*B)*a^2*d^2)*cos(f*x + e)^4 - (99*(7*A + 20*B)*a^2*c^2 +
22*(180*A + 289*B)*a^2*c*d + (3179*A + 4370*B)*a^2*d^2)*cos(f*x + e)^3 + (
33*(77*A + 85*B)*a^2*c^2 + 22*(255*A + 263*B)*a^2*c*d + (2893*A + 2965*B)*
a^2*d^2)*cos(f*x + e)^2 + 2*(33*(161*A + 145*B)*a^2*c^2 + 22*(435*A + 419*
B)*a^2*c*d + (4609*A + 4465*B)*a^2*d^2)*cos(f*x + e) + (315*B*a^2*d^2*cos(
f*x + e)^5 - 1056*(7*A + 5*B)*a^2*c^2 - 704*(15*A + 13*B)*a^2*c*d - 32*(14
3*A + 125*B)*a^2*d^2 - 35*(22*B*a^2*c*d + (11*A + 23*B)*a^2*d^2)*cos(f*x +
 e)^4 - 5*(99*B*a^2*c^2 + 22*(9*A + 26*B)*a^2*c*d + 13*(22*A + 37*B)*a^2*d
^2)*cos(f*x + e)^3 + 3*(33*(7*A + 15*B)*a^2*c^2 + 22*(45*A + 53*B)*a^2*c*d
 + (583*A + 655*B)*a^2*d^2)*cos(f*x + e)^2 + 2*(33*(49*A + 65*B)*a^2*c^2 +
 22*(195*A + 211*B)*a^2*c*d + (2321*A + 2465*B)*a^2*d^2)*cos(f*x + e))*sin
(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}(d\sin(fx+e)+c)^2\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e)
+ c)^2, x)
```

Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 684, normalized size of antiderivative = 1.59

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx=\text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, al
gorithm="giac")
```

output
```
1/55440*sqrt(2)*(315*B*a^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-11
/4*pi + 11/2*f*x + 11/2*e) + 6930*(40*A*a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e)) + 30*B*a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 60*A*a^2*c*
d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 52*B*a^2*c*d*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e)) + 26*A*a^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 23*B*a
^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e)
 + 2310*(20*A*a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 22*B*a^2*c^2*s
gn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 44*A*a^2*c*d*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e)) + 40*B*a^2*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 20*A*a^2*
d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 19*B*a^2*d^2*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/2*f*x + 3/2*e) + 693*(8*A*a^2*c^2*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e)) + 20*B*a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e)) + 40*A*a^2*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 48*B*a^2*c*d*s
gn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 24*A*a^2*d^2*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e)) + 25*B*a^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-5/4*p
i + 5/2*f*x + 5/2*e) + 495*(4*B*a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
) + 8*A*a^2*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 20*B*a^2*c*d*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e)) + 10*A*a^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e)) + 13*B*a^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-7/4*pi + 7/
2*f*x + 7/2*e) + 385*(4*B*a^2*c*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) +...
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx=\int (A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}\,\left(c+d\,\sin\left(e+f\,x\right)\right)^2\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^2
,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^2
, x)
```

Reduce [F]

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx=\sqrt{a}\,a^2\Bigg(\left(\int\sqrt{\sin(fx+e)+1}dx\right)a\,c^2+\left(\int\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^5\,dx\right)b\,d^2+\left(\int\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^4\,dx\right)a\,d^2+2\left(\int\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^4\,dx\right)bcd+2\left(\int\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^4\,dx\right)b\,d^2+2\left(\int\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^3\,dx\right)acd+2\left(\int\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^3\,dx\right)a\,d^2+\left(\int\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^3\,dx\right)b\,c^2+4\left(\int\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^3\,dx\right)bcd+\left(\int\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^3\,dx\right)b\,d^2+\left(\int\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)a\,c^2+4\left(\int\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)acd+\left(\int\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)a\,d^2+2\left(\int\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)b\,c^2+2\left(\int\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)bcd+2\left(\int\sqrt{\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)a\,c^2+2\left(\int\sqrt{\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)acd+\left(\int\sqrt{\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)b\,c^2\Bigg)$$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x)
```

output

```
sqrt(a)*a**2*(int(sqrt(sin(e + f*x) + 1),x)*a*c**2 + int(sqrt(sin(e + f*x)
 + 1)*sin(e + f*x)**5,x)*b*d**2 + int(sqrt(sin(e + f*x) + 1)*sin(e + f*x)*
*4,x)*a*d**2 + 2*int(sqrt(sin(e + f*x) + 1)*sin(e + f*x)**4,x)*b*c*d + 2*i
nt(sqrt(sin(e + f*x) + 1)*sin(e + f*x)**4,x)*b*d**2 + 2*int(sqrt(sin(e + f
*x) + 1)*sin(e + f*x)**3,x)*a*c*d + 2*int(sqrt(sin(e + f*x) + 1)*sin(e + f
*x)**3,x)*a*d**2 + int(sqrt(sin(e + f*x) + 1)*sin(e + f*x)**3,x)*b*c**2 +
4*int(sqrt(sin(e + f*x) + 1)*sin(e + f*x)**3,x)*b*c*d + int(sqrt(sin(e + f
*x) + 1)*sin(e + f*x)**3,x)*b*d**2 + int(sqrt(sin(e + f*x) + 1)*sin(e + f*
x)**2,x)*a*c**2 + 4*int(sqrt(sin(e + f*x) + 1)*sin(e + f*x)**2,x)*a*c*d +
int(sqrt(sin(e + f*x) + 1)*sin(e + f*x)**2,x)*a*d**2 + 2*int(sqrt(sin(e +
f*x) + 1)*sin(e + f*x)**2,x)*b*c**2 + 2*int(sqrt(sin(e + f*x) + 1)*sin(e +
 f*x)**2,x)*b*c*d + 2*int(sqrt(sin(e + f*x) + 1)*sin(e + f*x),x)*a*c**2 +
2*int(sqrt(sin(e + f*x) + 1)*sin(e + f*x),x)*a*c*d + int(sqrt(sin(e + f*x)
 + 1)*sin(e + f*x),x)*b*c**2)
```

3.302 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$

Optimal result

Integrand size = 35, antiderivative size = 212

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = -\frac{64a^3(21Ac+15Bc+15Ad+13Bd)\cos(e+fx)}{315f\sqrt{a+a\sin(e+fx)}} - \frac{16a^2(21Ac+15Bc+15Ad+13Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{315f} - \frac{2a(21Ac+15Bc+15Ad+13Bd)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{105f} - \frac{2(9Bc+9Ad-2Bd)\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{63f} - \frac{2Bd\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{9af}$$

output
```
-64/315*a^3*(21*A*c+15*A*d+15*B*c+13*B*d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1
/2)-16/315*a^2*(21*A*c+15*A*d+15*B*c+13*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(
1/2)/f-2/105*a*(21*A*c+15*A*d+15*B*c+13*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(
3/2)/f-2/63*(9*A*d+9*B*c-2*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f-2/9*B*
d*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/a/f
```

Mathematica [A] (verified)

Time = 8.01 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.95

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx =$$

$$-\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}(7476Ac+6240Bc+6240Ad+5653Bd-4}{}$$

input
```
Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f
*x]),x]
```

output
```
-1/1260*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x
])]*(7476*A*c + 6240*B*c + 6240*A*d + 5653*B*d - 4*(63*A*c + 180*B*c + 180
*A*d + 254*B*d)*Cos[2*(e + f*x)] + 35*B*d*Cos[4*(e + f*x)] + 2352*A*c*Sin[
e + f*x] + 3030*B*c*Sin[e + f*x] + 3030*A*d*Sin[e + f*x] + 3116*B*d*Sin[e
+ f*x] - 90*B*c*Sin[3*(e + f*x)] - 90*A*d*Sin[3*(e + f*x)] - 260*B*d*Sin[3
*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.93, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 3447, 3042, 3502, 27, 3042, 3230, 3042, 3126, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))dx$$

$$\Big\downarrow \text{3447}$$

$$\int (a\sin(e+fx)+a)^{5/2}\left((Ad+Bc)\sin(e+fx)+Ac+Bd\sin^2(e+fx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^{5/2}\left((Ad+Bc)\sin(e+fx)+Ac+Bd\sin(e+fx)^2\right)dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{2\int \frac{1}{2}(\sin(e+fx)a+a)^{5/2}(a(9Ac+7Bd)+a(9Bc+9Ad-2Bd)\sin(e+fx))dx}{9a}-\frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{9af}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (\sin(e+fx)a+a)^{5/2}(a(9Ac+7Bd)+a(9Bc+9Ad-2Bd)\sin(e+fx))dx}{9a}-\frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{9af}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (\sin(e+fx)a+a)^{5/2}(a(9Ac+7Bd)+a(9Bc+9Ad-2Bd)\sin(e+fx))dx}{9a}-\frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{9af}$$

$$\Big\downarrow \text{3230}$$

$$\frac{\frac{3}{7}a(21Ac+15Ad+15Bc+13Bd)\int (\sin(e+fx)a+a)^{5/2}dx-\frac{2a(9Ad+9Bc-2Bd)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{7f}}{9a}-\frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{9af}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{7}a(21Ac+15Ad+15Bc+13Bd)\int (\sin(e+fx)a+a)^{5/2}dx-\frac{2a(9Ad+9Bc-2Bd)\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{7f}}{9a}-\frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{9af}$$

$$\Big\downarrow \text{3126}$$

$$\frac{\frac{3}{7}a(21Ac+15Ad+15Bc+13Bd)\left(\frac{8}{5}a\int (\sin(e+fx)a+a)^{3/2}dx-\frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}\right)-\frac{2a(9Ad+9Bc-2.}{}}{9a}-\frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{9af}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{3}{7}a(21Ac+15Ad+15Bc+13Bd)\left(\frac{8}{5}a\int (\sin(e+fx)a+a)^{3/2}dx-\frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}\right)-\frac{2a(9Ad+9Bc-2}{}}{9a}$$
$$\frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{9af}$$

$$\Big\downarrow \; 3126$$

$$\frac{\frac{3}{7}a(21Ac+15Ad+15Bc+13Bd)\left(\frac{8}{5}a\left(\frac{4}{3}a\int \sqrt{\sin(e+fx)a+a}dx-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)-\frac{2a\cos(e+fx)(a\,s}{5}\right.}{9a}$$
$$\frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{9af}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{3}{7}a(21Ac+15Ad+15Bc+13Bd)\left(\frac{8}{5}a\left(\frac{4}{3}a\int \sqrt{\sin(e+fx)a+a}dx-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)-\frac{2a\cos(e+fx)(a\,s}{5}\right.}{9a}$$
$$\frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{9af}$$

$$\Big\downarrow \; 3125$$

$$\frac{\frac{3}{7}a(21Ac+15Ad+15Bc+13Bd)\left(\frac{8}{5}a\left(-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)-\frac{2a\cos(e+fx)(a\sin(e+fx)+}{5f}\right.}{9a}$$
$$\frac{2Bd\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{9af}$$

input
```
Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x
]
```

output
```
(-2*B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(9*a*f) + ((-2*a*(9*B*c +
 9*A*d - 2*B*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(7*f) + (3*a*(21*
A*c + 15*B*c + 15*A*d + 13*B*d)*((-2*a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(
3/2))/(5*f) + (8*a*((-8*a^2*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) -
 (2*a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*f)))/5))/7)/(9*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 7.33 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.72

method	result
default	$\frac{2(1+\sin(fx+e))a^3(\sin(fx+e)-1)\left(35Bd\cos(fx+e)^4+(-45Ad-45Bc-130Bd)\cos(fx+e)^2\sin(fx+e)+(-63Ac-180Ad-180Bc-\right.}{315\cos(fx+e)\sqrt{a+a\sin(fx+e)}}$
parts	$\frac{2(1+\sin(fx+e))a^3(\sin(fx+e)-1)(Ad+Bc)\left(3\sin(fx+e)^3+12\sin(fx+e)^2+23\sin(fx+e)+46\right)}{21\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}+\frac{2Ac(1+\sin(fx+e))a^3(\sin(fx+}{15\cos(fx+}$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x,method=_RET
URNVERBOSE)
```

output
```
2/315*(1+sin(f*x+e))*a^3*(sin(f*x+e)-1)*(35*B*d*cos(f*x+e)^4+(-45*A*d-45*B
*c-130*B*d)*cos(f*x+e)^2*sin(f*x+e)+(-63*A*c-180*A*d-180*B*c-289*B*d)*cos(
f*x+e)^2+(294*A*c+390*A*d+390*B*c+422*B*d)*sin(f*x+e)+966*A*c+870*A*d+870*
B*c+838*B*d)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.70

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx =$$
$$-\frac{2\left(35\,Ba^2d\cos\left(fx+e\right)^5-5\left(9\,Ba^2c+(9\,A+19\,B)a^2d\right)\cos\left(fx+e\right)^4+96\,(7\,A+5\,B)a^2c+32\,(15\,A+\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algo
rithm="fricas")
```

output
```
-2/315*(35*B*a^2*d*cos(f*x + e)^5 - 5*(9*B*a^2*c + (9*A + 19*B)*a^2*d)*cos
(f*x + e)^4 + 96*(7*A + 5*B)*a^2*c + 32*(15*A + 13*B)*a^2*d - (9*(7*A + 20
*B)*a^2*c + (180*A + 289*B)*a^2*d)*cos(f*x + e)^3 + (3*(77*A + 85*B)*a^2*c
 + (255*A + 263*B)*a^2*d)*cos(f*x + e)^2 + 2*(3*(161*A + 145*B)*a^2*c + (4
35*A + 419*B)*a^2*d)*cos(f*x + e) - (35*B*a^2*d*cos(f*x + e)^4 + 96*(7*A +
 5*B)*a^2*c + 32*(15*A + 13*B)*a^2*d + 5*(9*B*a^2*c + (9*A + 26*B)*a^2*d)*
cos(f*x + e)^3 - 3*(3*(7*A + 15*B)*a^2*c + (45*A + 53*B)*a^2*d)*cos(f*x +
e)^2 - 2*(3*(49*A + 65*B)*a^2*c + (195*A + 211*B)*a^2*d)*cos(f*x + e))*sin
(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

Sympy [F]

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx=\int (a(\sin(e+fx)+1))^{\frac{5}{2}}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$

input
```
integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**(5/2)*(A + B*sin(e + f*x))*(c + d*sin(e +
 f*x)), x)
```

Maxima [F]

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}(d\sin(fx+e)+c)\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e)
+ c), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(192) = 384$.

Time = 0.30 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.91

$$\int (a + a\sin(e + fx))^{5/2}(A + B\sin(e + fx))(c + d\sin(e + fx))\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algo
rithm="giac")
```

output
```
1/2520*sqrt(2)*(35*B*a^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-9/4*pi
 + 9/2*f*x + 9/2*e) + 630*(20*A*a^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
+ 15*B*a^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 15*A*a^2*d*sgn(cos(-1/4
*pi + 1/2*f*x + 1/2*e)) + 13*B*a^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*
sin(-1/4*pi + 1/2*f*x + 1/2*e) + 210*(10*A*a^2*c*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e)) + 11*B*a^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 11*A*a^2*d*s
gn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 10*B*a^2*d*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e)))*sin(-3/4*pi + 3/2*f*x + 3/2*e) + 126*(2*A*a^2*c*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e)) + 5*B*a^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*A
*a^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*B*a^2*d*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e)))*sin(-5/4*pi + 5/2*f*x + 5/2*e) + 45*(2*B*a^2*c*sgn(cos(-
1/4*pi + 1/2*f*x + 1/2*e)) + 2*A*a^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
 + 5*B*a^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-7/4*pi + 7/2*f*x +
7/2*e))*sqrt(a)/f
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx=\int (A+B\sin(e+fx))\,(a+a\sin(e+fx))^{5/2}\,(c+d\sin(e+fx))\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x)),x
)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x)),
x)
```

Reduce [F]

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = \sqrt{a}\,a^2\left(\left(\int \sqrt{\sin(fx+e)+1}dx\right)ac + \left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)^4\,dx\right)bd + \left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)^3\,dx\right)ad + \left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)^3\,dx\right)bc + 2\left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)^3\,dx\right)bd + \left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)ac + 2\left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)ad + 2\left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)bc + \left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)bd + 2\left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)ac + \left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)ad + \left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)bc\right)$$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

output

```
sqrt(a)*a**2*(int(sqrt(sin(e + f*x) + 1),x)*a*c + int(sqrt(sin(e + f*x) +
1)*sin(e + f*x)**4,x)*b*d + int(sqrt(sin(e + f*x) + 1)*sin(e + f*x)**3,x)*
a*d + int(sqrt(sin(e + f*x) + 1)*sin(e + f*x)**3,x)*b*c + 2*int(sqrt(sin(e
 + f*x) + 1)*sin(e + f*x)**3,x)*b*d + int(sqrt(sin(e + f*x) + 1)*sin(e + f
*x)**2,x)*a*c + 2*int(sqrt(sin(e + f*x) + 1)*sin(e + f*x)**2,x)*a*d + 2*in
t(sqrt(sin(e + f*x) + 1)*sin(e + f*x)**2,x)*b*c + int(sqrt(sin(e + f*x) +
1)*sin(e + f*x)**2,x)*b*d + 2*int(sqrt(sin(e + f*x) + 1)*sin(e + f*x),x)*a
*c + int(sqrt(sin(e + f*x) + 1)*sin(e + f*x),x)*a*d + int(sqrt(sin(e + f*x
) + 1)*sin(e + f*x),x)*b*c)
```

3.303 $\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx$

Optimal result

Integrand size = 25, antiderivative size = 138

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx = -\frac{64a^3(7A+5B)\cos(e+fx)}{105f\sqrt{a+a\sin(e+fx)}}-\frac{16a^2(7A+5B)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{105f} - \frac{2a(7A+5B)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{35f} - \frac{2B\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{7f}$$

output
```
-64/105*a^3*(7*A+5*B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-16/105*a^2*(7*A+
5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f-2/35*a*(7*A+5*B)*cos(f*x+e)*(a+a*
sin(f*x+e))^(3/2)/f-2/7*B*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f
```

Mathematica [A] (verified)

Time = 4.03 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.86

$$\int (a + a\sin(e + fx))^{5/2}(A + B\sin(e + fx))\,dx =$$
$$-\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}(1246A+1040B-6(7A+20B)\cos(2(e+}{210f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

input
```
Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]),x]
```

output
```
-1/210*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x]
)]*(1246*A + 1040*B - 6*(7*A + 20*B)*Cos[2*(e + f*x)] + (392*A + 505*B)*Si
n[e + f*x] - 15*B*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2]))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3042, 3230, 3042, 3126, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3230}$$

$$\frac{1}{7}(7A+5B)\int (\sin(e+fx)a+a)^{5/2}dx-\frac{2B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{7f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}(7A+5B)\int(\sin(e+fx)a+a)^{5/2}dx-\frac{2B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{7f}$$

$\Big\downarrow$ 3126

$$\frac{1}{7}(7A+5B)\left(\frac{8}{5}a\int(\sin(e+fx)a+a)^{3/2}dx-\frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}\right)-\frac{2B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{7f}$$

$\Big\downarrow$ 3042

$$\frac{1}{7}(7A+5B)\left(\frac{8}{5}a\int(\sin(e+fx)a+a)^{3/2}dx-\frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}\right)-\frac{2B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{7f}$$

$\Big\downarrow$ 3126

$$\frac{1}{7}(7A+5B)\left(\frac{8}{5}a\left(\frac{4}{3}a\int\sqrt{\sin(e+fx)a+a}dx-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)-\frac{2a\cos(e+fx)(a\sin(e+fx)+}{5f}\right.$$
$$\frac{2B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{7f}$$

$\Big\downarrow$ 3042

$$\frac{1}{7}(7A+5B)\left(\frac{8}{5}a\left(\frac{4}{3}a\int\sqrt{\sin(e+fx)a+a}dx-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)-\frac{2a\cos(e+fx)(a\sin(e+fx)+}{5f}\right.$$
$$\frac{2B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{7f}$$

$\Big\downarrow$ 3125

$$\frac{1}{7}(7A+5B)\left(\frac{8}{5}a\left(-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)-\frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{3}}{5f}\right.$$
$$\frac{2B\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{7f}$$

input
```
Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]),x]
```

output

```
(-2*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(7*f) + ((7*A + 5*B)*((-2*a
*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(5*f) + (8*a*((-8*a^2*Cos[e + f*
x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a*Cos[e + f*x]*Sqrt[a + a*Sin[e +
f*x]])/(3*f)))/5))/7
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

rule 3230

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.72

method	result
default	$\frac{2(1+\sin(fx+e))a^3(\sin(fx+e)-1)\left(-15B\cos(fx+e)^2\sin(fx+e)+(-21A-60B)\cos(fx+e)^2+(98A+130B)\sin(fx+e)+322A+290}{105\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$
parts	$\frac{2A(1+\sin(fx+e))a^3(\sin(fx+e)-1)\left(3\sin(fx+e)^2+14\sin(fx+e)+43\right)}{15\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}+\frac{2B(1+\sin(fx+e))a^3(\sin(fx+e)-1)\left(3\sin(fx+e)^3+12}{21\cos(fx+e)\sqrt{a+a\sin(fx}$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
2/105*(1+sin(f*x+e))*a^3*(sin(f*x+e)-1)*(-15*B*cos(f*x+e)^2*sin(f*x+e)+(-2
1*A-60*B)*cos(f*x+e)^2+(98*A+130*B)*sin(f*x+e)+322*A+290*B)/cos(f*x+e)/(a+
a*sin(f*x+e))^(1/2)/f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.38

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx = \frac{2\left(15\,Ba^2\cos\left(fx+e\right)^4+3\left(7\,A+20\,B\right)a^2\cos\left(fx+e\right)^3-\left(77\,A+85\,B\right)a^2\cos\left(fx+\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x, algorithm="fricas")
```

output
```
2/105*(15*B*a^2*cos(f*x + e)^4 + 3*(7*A + 20*B)*a^2*cos(f*x + e)^3 - (77*A
 + 85*B)*a^2*cos(f*x + e)^2 - 2*(161*A + 145*B)*a^2*cos(f*x + e) - 32*(7*A
 + 5*B)*a^2 + (15*B*a^2*cos(f*x + e)^3 - 3*(7*A + 15*B)*a^2*cos(f*x + e)^2
 - 2*(49*A + 65*B)*a^2*cos(f*x + e) + 32*(7*A + 5*B)*a^2)*sin(f*x + e))*sq
rt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

Sympy [F]

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx = \int \left(a(\sin\left(e+fx\right)+1)\right)^{\frac{5}{2}}\left(A+B\sin\left(e+fx\right)\right)\,dx$$

input
```
integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e)),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**(5/2)*(A + B*sin(e + f*x)), x)
```

Maxima [F]

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.46

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx = \frac{\sqrt{2}\left(15\,Ba^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{7}{4}\,\pi+\frac{7}{2}\,fx+\frac{7}{2}\,e\right)+525\,\left(4\,Aa^2\mathrm{sgn}(\mathrm{co}\right.\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x, algorithm="giac")
```

output
```
1/420*sqrt(2)*(15*B*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-7/4*pi +
7/2*f*x + 7/2*e) + 525*(4*A*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*B*
a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(-1/4*pi + 1/2*f*x + 1/2*e) +
35*(10*A*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 11*B*a^2*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e)))*sin(-3/4*pi + 3/2*f*x + 3/2*e) + 21*(2*A*a^2*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e)) + 5*B*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
))*sin(-5/4*pi + 5/2*f*x + 5/2*e))*sqrt(a)/f
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx = \int (A+B\sin(e+fx))\,(a+a\sin(e+fx))^{5/2}\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2),x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2), x)
```

Reduce [F]

$$\int (a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))\,dx = \sqrt{a}\,a^2\left(\left(\int \sqrt{\sin(fx+e)+1}dx\right)a + \left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)^3\,dx\right)b + \left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)a + 2\left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)b + 2\left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)a + \left(\int \sqrt{\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)b\right)$$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x)
```

output
```
sqrt(a)*a**2*(int(sqrt(sin(e + f*x) + 1),x)*a + int(sqrt(sin(e + f*x) + 1)
*sin(e + f*x)**3,x)*b + int(sqrt(sin(e + f*x) + 1)*sin(e + f*x)**2,x)*a +
2*int(sqrt(sin(e + f*x) + 1)*sin(e + f*x)**2,x)*b + 2*int(sqrt(sin(e + f*x
) + 1)*sin(e + f*x),x)*a + int(sqrt(sin(e + f*x) + 1)*sin(e + f*x),x)*b)
```

3.304 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 218

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \frac{2a^{5/2}(c-d)^2(Bc-Ad)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{d^{7/2}\sqrt{c+d}f}$$
$$+\frac{2a^3(5A(3c-7d)d-B(15c^2-35cd+32d^2))\cos(e+fx)}{15d^3f\sqrt{a+a\sin(e+fx)}}$$
$$+\frac{2a^2(5Bc-5Ad-8Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15d^2f}$$
$$-\frac{2aB\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5df}$$

output
```
2*a^(5/2)*(c-d)^2*(-A*d+B*c)*arctanh(a^(1/2)*d^(1/2)*cos(f*x+e)/(c+d)^(1/2
)/(a+a*sin(f*x+e))^(1/2))/d^(7/2)/(c+d)^(1/2)/f+2/15*a^3*(5*A*(3*c-7*d)*d-
B*(15*c^2-35*c*d+32*d^2))*cos(f*x+e)/d^3/f/(a+a*sin(f*x+e))^(1/2)+2/15*a^2
*(-5*A*d+5*B*c-8*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/d^2/f-2/5*a*B*cos(
f*x+e)*(a+a*sin(f*x+e))^(3/2)/d/f
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 9.93 (sec) , antiderivative size = 992, normalized size of antiderivative = 4.55

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e +
 f*x]),x]
```

output
```
((a*(1 + Sin[e + f*x]))^(5/2)*(-30*Sqrt[d]*(A*d*(-2*c + 5*d) + B*(2*c^2 -
5*c*d + 5*d^2))*Cos[(e + f*x)/2] - 5*d^(3/2)*(-2*B*c + 2*A*d + 5*B*d)*Cos[
(3*(e + f*x))/2] + 3*B*d^(5/2)*Cos[(5*(e + f*x))/2] - (15*(c - d)^2*(B*c -
 A*d)*((c + d)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2]) + Sqrt[c + d]*RootSum
[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(c*Sqrt[d]*Log[-#1 + Tan[
(e + f*x)/4]]) - d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]] - d*Sqrt[c + d]*Log[-
#1 + Tan[(e + f*x)/4]] - 2*c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*d^
(3/2)*Log[-#1 + Tan[(e + f*x)/4]]*#1 - c*Sqrt[c + d]*Log[-#1 + Tan[(e + f*
x)/4]]*#1 + c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + d^(3/2)*Log[-#1 +
 Tan[(e + f*x)/4]]*#1^2 + 3*d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2
 - c*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 -
 c*#1^3) & ]))/(c + d)^(3/2) + (15*(c - d)^2*(B*c - A*d)*((c + d)*(e + f*x
 - 2*Log[Sec[(e + f*x)/4]^2]) - Sqrt[c + d]*RootSum[c + 4*d*#1 + 2*c*#1^2
- 4*d*#1^3 + c*#1^4 & , (-(c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]) - d^(3/2
)*Log[-#1 + Tan[(e + f*x)/4]] + d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]
- 2*c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*d^(3/2)*Log[-#1 + Tan[(e
+ f*x)/4]]*#1 + c*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + c*Sqrt[d]*L
og[-#1 + Tan[(e + f*x)/4]]*#1^2 + d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]]*#1^2
 - 3*d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + c*Sqrt[c + d]*Log[-#
1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ]))/(c + ...
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.297, Rules used = {3042, 3455, 27, 3042, 3455, 27, 3042, 3460, 3042, 3252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}dx$$

$$\Big\downarrow \; 3455$$

$$\frac{2\int \frac{(\sin(e+fx)a+a)^{3/2}(a(3Bc+5Ad)-a(5Bc-5Ad-8Bd)\sin(e+fx))}{2(c+d\sin(e+fx))}dx}{5d}-\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5df}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(\sin(e+fx)a+a)^{3/2}(a(3Bc+5Ad)-a(5Bc-5Ad-8Bd)\sin(e+fx))}{c+d\sin(e+fx)}dx}{5d}-\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5df}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{(\sin(e+fx)a+a)^{3/2}(a(3Bc+5Ad)-a(5Bc-5Ad-8Bd)\sin(e+fx))}{c+d\sin(e+fx)}dx}{5d}-\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5df}$$

$$\Big\downarrow \; 3455$$

$$\frac{\frac{2\int -\frac{\sqrt{\sin(e+fx)a+a}\left((Bc(5c-17d)-5Ad(c+3d))a^2+\left(5A(3c-7d)d-B\left(15c^2-35dc+32d^2\right)\right)\sin(e+fx)a^2\right)}{2(c+d\sin(e+fx))}dx}{3d}+\frac{2a^2(-5Ad+5Bc-8Bd)\cos(e+fx)\sqrt{a\sin}}{3df}}{5d}-\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5df}$$

$\big\downarrow$ 27

$$\frac{\frac{2a^2(-5Ad+5Bc-8Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3df}-\frac{\int \frac{\sqrt{\sin(e+fx)a+a}\left((Bc(5c-17d)-5Ad(c+3d))a^2+\left(5A(3c-7d)d-B\left(15c^2-35dc+32d^2\right)\right)\sin(e+f}{c+d\sin(e+fx)}}{3d}}{5d}$$
$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5df}$$

$\big\downarrow$ 3042

$$\frac{\frac{2a^2(-5Ad+5Bc-8Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3df}-\frac{\int \frac{\sqrt{\sin(e+fx)a+a}\left((Bc(5c-17d)-5Ad(c+3d))a^2+\left(5A(3c-7d)d-B\left(15c^2-35dc+32d^2\right)\right)\sin(e+f}{c+d\sin(e+fx)}}{3d}}{5d}$$
$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5df}$$

$\big\downarrow$ 3460

$$\frac{\frac{2a^2(-5Ad+5Bc-8Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3df}-\frac{\frac{15a^2(c-d)^2(Bc-Ad)\int\frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{d}-\frac{2a^3\left(5Ad(3c-7d)-B\left(15c^2-35cd+32d^2\right)\right)\cos(e+fx}{df\sqrt{a\sin(e+fx)+a}}}{3d}}{5d}$$
$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5df}$$

$\big\downarrow$ 3042

$$\frac{\frac{2a^2(-5Ad+5Bc-8Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3df}-\frac{\frac{15a^2(c-d)^2(Bc-Ad)\int\frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{d}-\frac{2a^3\left(5Ad(3c-7d)-B\left(15c^2-35cd+32d^2\right)\right)\cos(e+fx}{df\sqrt{a\sin(e+fx)+a}}}{3d}}{5d}$$
$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5df}$$

$\big\downarrow$ 3252

$$\frac{\frac{2a^2(-5Ad+5Bc-8Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3df}-\frac{-\frac{30a^3(c-d)^2(Bc-Ad)\int\frac{1}{a(c+d)-\frac{a^2d\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{df}-\frac{2a^3\left(5Ad(3c-7d)-B\left(15c\right.\right.}{df\sqrt{a\sin}}}{3d}}{5d}$$
$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5df}$$

$\big\downarrow$ 221

$$\frac{\frac{2a^2(-5Ad+5Bc-8Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3df}-\frac{-\frac{30a^{5/2}(c-d)^2(Bc-Ad)\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{d^{3/2}f\sqrt{c+d}}-\frac{2a^3\left(5Ad(3c-7d)-B\left(15c^2-35c\right.\right.}{df\sqrt{a\sin(e+fx}}}{3d}}{5d}$$

$$\frac{2aB\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5df}$$

input
```
Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])
,x]
```

output
```
(-2*a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(5*d*f) + ((2*a^2*(5*B*c
 - 5*A*d - 8*B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*d*f) - ((-30*a^
(5/2)*(c - d)^2*(B*c - A*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c
 + d]*Sqrt[a + a*Sin[e + f*x]])])/(d^(3/2)*Sqrt[c + d]*f) - (2*a^3*(5*A*(3
*c - 7*d)*d - B*(15*c^2 - 35*c*d + 32*d^2))*Cos[e + f*x])/(d*f*Sqrt[a + a*
Sin[e + f*x]]))/(3*d))/(5*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 542 vs. $2(192) = 384$.

Time = 2.60 (sec) , antiderivative size = 543, normalized size of antiderivative = 2.49

method	result
default	$\frac{2(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(-3B(a-a\sin(fx+e))^{\frac{5}{2}}\sqrt{(c+d)ad}\,d^2+5A(a-a\sin(fx+e))^{\frac{3}{2}}\sqrt{(c+d)ad}\,a\,d^2-15A\,\operatorname{arctanh}\right.}{}$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x,method=_RET
URNVERBOSE)
```

output
```
2/15*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(-3*B*(a-a*sin(f*x+e))^(5/2)
*((c+d)*a*d)^(1/2)*d^2+5*A*(a-a*sin(f*x+e))^(3/2)*((c+d)*a*d)^(1/2)*a*d^2-
15*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^3*c^2*d+30*A*
arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^3*c*d^2-15*A*arcta
nh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^3*d^3-5*B*(a-a*sin(f*x+
e))^(3/2)*((c+d)*a*d)^(1/2)*a*c*d+20*B*(a-a*sin(f*x+e))^(3/2)*((c+d)*a*d)^
(1/2)*a*d^2+15*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^3
*c^3-30*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^3*c^2*d+
15*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^3*c*d^2+15*A*
(a-a*sin(f*x+e))^(1/2)*((c+d)*a*d)^(1/2)*a^2*c*d-45*A*(a-a*sin(f*x+e))^(1/
2)*((c+d)*a*d)^(1/2)*a^2*d^2-15*B*(a-a*sin(f*x+e))^(1/2)*((c+d)*a*d)^(1/2)
*a^2*c^2+45*B*(a-a*sin(f*x+e))^(1/2)*((c+d)*a*d)^(1/2)*a^2*c*d-60*B*(a-a*s
in(f*x+e))^(1/2)*((c+d)*a*d)^(1/2)*a^2*d^2)/d^3/((c+d)*a*d)^(1/2)/cos(f*x+
e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 499 vs. $2(192) = 384$.

Time = 1.14 (sec) , antiderivative size = 1314, normalized size of antiderivative = 6.03

$$\int \frac{(a + a\sin(e + fx))^{5/2}(A + B\sin(e + fx))}{c + d\sin(e + fx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algo
rithm="fricas")
```

output
```
[-1/30*(15*(B*a^2*c^3 - (A + 2*B)*a^2*c^2*d + (2*A + B)*a^2*c*d^2 - A*a^2*
d^3 + (B*a^2*c^3 - (A + 2*B)*a^2*c^2*d + (2*A + B)*a^2*c*d^2 - A*a^2*d^3)*
cos(f*x + e) + (B*a^2*c^3 - (A + 2*B)*a^2*c^2*d + (2*A + B)*a^2*c*d^2 - A*
a^2*d^3)*sin(f*x + e))*sqrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c
^2 - 2*a*c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c
*d^2 + 3*d^3 - (c*d^2 + d^3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*co
s(f*x + e) - (c^2*d + 4*c*d^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*
x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d +
9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 +
2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2
*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e)
+ (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x +
e))) - 4*(3*B*a^2*d^2*cos(f*x + e)^3 - 15*B*a^2*c^2 + 5*(3*A + 7*B)*a^2*c*
d - (35*A + 32*B)*a^2*d^2 + (5*B*a^2*c*d - (5*A + 11*B)*a^2*d^2)*cos(f*x +
 e)^2 - (15*B*a^2*c^2 - 5*(3*A + 8*B)*a^2*c*d + 2*(20*A + 23*B)*a^2*d^2)*c
os(f*x + e) - (3*B*a^2*d^2*cos(f*x + e)^2 - 15*B*a^2*c^2 + 5*(3*A + 7*B)*a
^2*c*d - (35*A + 32*B)*a^2*d^2 - (5*B*a^2*c*d - (5*A + 14*B)*a^2*d^2)*cos(
f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(d^3*f*cos(f*x + e) + d^
3*f*sin(f*x + e) + d^3*f), 1/15*(15*(B*a^2*c^3 - (A + 2*B)*a^2*c^2*d + (2*
A + B)*a^2*c*d^2 - A*a^2*d^3 + (B*a^2*c^3 - (A + 2*B)*a^2*c^2*d + (2*A ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}}{d\sin(fx+e)+c}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e)
+ c), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 523 vs. 2(192) = 384.

Time = 0.27 (sec) , antiderivative size = 523, normalized size of antiderivative = 2.40

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algo
rithm="giac")
```

output
```
1/15*sqrt(2)*sqrt(a)*(15*sqrt(2)*(B*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e)) - A*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*B*a^2*c^2*d*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*A*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e)) + B*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - A*a^2*d^3*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*
x + 1/2*e)/sqrt(-c*d - d^2))/(sqrt(-c*d - d^2)*d^3) + 2*(12*B*a^2*d^4*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^5 + 10*B*a^
2*c*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)
^3 - 10*A*a^2*d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*
x + 1/2*e)^3 - 40*B*a^2*d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*p
i + 1/2*f*x + 1/2*e)^3 + 15*B*a^2*c^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*
e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 15*A*a^2*c*d^3*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 45*B*a^2*c*d^3*sgn(cos(-1/4
*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 45*A*a^2*d^4*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 60*B*a^2*
d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e))/d^
5)/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx=\int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{5/2}}{c+d\,\sin(e+f\,x)}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c + d*sin(e + f*x))
,x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c + d*sin(e + f*x))
, x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \sqrt{a}\,a^2\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}}{\sin(fx+e)\,d+c}dx\right)a + \left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^3}{\sin(fx+e)\,d+c}dx\right)b + \left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^2}{\sin(fx+e)\,d+c}dx\right)a + 2\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^2}{\sin(fx+e)\,d+c}dx\right)b + 2\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)\,d+c}dx\right)a + \left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)\,d+c}dx\right)b\right)$$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

output
```
sqrt(a)*a**2*(int(sqrt(sin(e + f*x) + 1)/(sin(e + f*x)*d + c),x)*a + int((
sqrt(sin(e + f*x) + 1)*sin(e + f*x)**3)/(sin(e + f*x)*d + c),x)*b + int((s
qrt(sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x)*d + c),x)*a + 2*int((
sqrt(sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x)*d + c),x)*b + 2*int(
(sqrt(sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)*d + c),x)*a + int((sqr
t(sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)*d + c),x)*b)
```

3.305 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 265

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \frac{a^{5/2}(c-d)\left(Ad(3c+5d)-B(5c^2+5cd-2d^2)\right)\operatorname{arctan}}{d^{7/2}(c+d)^{3/2}f}$$
$$-\frac{a^3(3Ad(3c+d)-B(15c^2-5cd-14d^2))\cos(e+fx)}{3d^3(c+d)f\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{a^2(5Bc-3Ad+2Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3d^2(c+d)f}$$
$$+\frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{d(c+d)f(c+d\sin(e+fx))}$$

output
```
a^(5/2)*(c-d)*(A*d*(3*c+5*d)-B*(5*c^2+5*c*d-2*d^2))*arctanh(a^(1/2)*d^(1/2
)*cos(f*x+e)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/d^(7/2)/(c+d)^(3/2)/f-1/3
*a^3*(3*A*d*(3*c+d)-B*(15*c^2-5*c*d-14*d^2))*cos(f*x+e)/d^3/(c+d)/f/(a+a*s
in(f*x+e))^(1/2)-1/3*a^2*(-3*A*d+5*B*c+2*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^
(1/2)/d^2/(c+d)/f+a*(-A*d+B*c)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/d/(c+d)/f
/(c+d*sin(f*x+e))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 15.29 (sec) , antiderivative size = 1002, normalized size of antiderivative = 3.78

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e +
 f*x])^2,x]
```

output
```
((a*(1 + Sin[e + f*x]))^(5/2)*(-12*Sqrt[d]*(-4*B*c + 2*A*d + 5*B*d)*Cos[(e
 + f*x)/2] - 4*B*d^(3/2)*Cos[(3*(e + f*x))/2] + (3*(c - d)*(-(A*d*(3*c + 5
*d)) + B*(5*c^2 + 5*c*d - 2*d^2))*((c + d)*(e + f*x - 2*Log[Sec[(e + f*x)/
4]^2]) + Sqrt[c + d]*RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & ,
 (-(c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]) - d^(3/2)*Log[-#1 + Tan[(e + f*
x)/4]] - d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - 2*c*Sqrt[d]*Log[-#1 +
 Tan[(e + f*x)/4]]*#1 - 2*d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]]*#1 - c*Sqrt[
c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/
4]]*#1^2 + d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + 3*d*Sqrt[c + d]*Log[
-#1 + Tan[(e + f*x)/4]]*#1^2 - c*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#
1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ]))/(c + d)^(5/2) + (3*(c - d)*(-(A
*d*(3*c + 5*d)) + B*(5*c^2 + 5*c*d - 2*d^2))*(-((c + d)*(e + f*x - 2*Log[S
ec[(e + f*x)/4]^2])) + Sqrt[c + d]*RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^
3 + c*#1^4 & , (-(c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]) - d^(3/2)*Log[-#1
 + Tan[(e + f*x)/4]] + d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - 2*c*Sqr
t[d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]
]*#1 + c*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + c*Sqrt[d]*Log[-#1 +
Tan[(e + f*x)/4]]*#1^2 + d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - 3*d*Sq
rt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + c*Sqrt[c + d]*Log[-#1 + Tan[(
e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ]))/(c + d)^(5/2) ...
```

Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.297, Rules used = {3042, 3454, 27, 3042, 3455, 27, 3042, 3460, 3042, 3252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}dx$$

$$\Big\downarrow \text{3454}$$

$$\frac{\int -\frac{(\sin(e+fx)a+a)^{3/2}(a(3Bc-5Ad-2Bd)-a(5Bc-3Ad+2Bd)\sin(e+fx))}{2(c+d\sin(e+fx))}dx}{d(c+d)}+\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{df(c+d)(c+d\sin(e+fx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{df(c+d)(c+d\sin(e+fx))}-\frac{\int \frac{(\sin(e+fx)a+a)^{3/2}(a(3Bc-5Ad-2Bd)-a(5Bc-3Ad+2Bd)\sin(e+fx))}{c+d\sin(e+fx)}dx}{2d(c+d)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{df(c+d)(c+d\sin(e+fx))}-\frac{\int \frac{(\sin(e+fx)a+a)^{3/2}(a(3Bc-5Ad-2Bd)-a(5Bc-3Ad+2Bd)\sin(e+fx))}{c+d\sin(e+fx)}dx}{2d(c+d)}$$

$$\Big\downarrow \text{3455}$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{df(c+d)(c+d\sin(e+fx))}-\frac{\frac{2\int \frac{\sqrt{\sin(e+fx)a+a}\left(a^2\left(3A(c-5d)d-B\left(5c^2-7dc+6d^2\right)\right)-a^2\left(3Ad(3c+d)-B\left(15c^2-5dc-14d^2\right)\right)\sin(e+fx)\right)}{2(c+d\sin(e+fx))}dx}{3d}+\frac{2a^2(-3Ad+5Bc+2Bd)\cos(e+fx)\sqrt{a}}{3df}}{2d(c+d)}$$

$\Big\downarrow$ 27

$$\frac{\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{df(c+d)(c+d\sin(e+fx))}-\frac{\frac{\int\frac{\sqrt{\sin(e+fx)a+a}\left(a^2\left(3A(c-5d)d-B\left(5c^2-7dc+6d^2\right)\right)-a^2\left(3Ad(3c+d)-B\left(15c^2-5dc-14d^2\right)\right)\sin(e+fx)\right)}{c+d\sin(e+fx)}dx}{3d}+\frac{2a^2(-3Ad+5Bc+2Bd)\cos(e+fx)\sqrt{a\sin\ldots}}{3df}}{2d(c+d)}}{}$$

$\Big\downarrow$ 3042

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{df(c+d)(c+d\sin(e+fx))}-\frac{\frac{\int\frac{\sqrt{\sin(e+fx)a+a}\left(a^2\left(3A(c-5d)d-B\left(5c^2-7dc+6d^2\right)\right)-a^2\left(3Ad(3c+d)-B\left(15c^2-5dc-14d^2\right)\right)\sin(e+fx)\right)}{c+d\sin(e+fx)}dx}{3d}+\frac{2a^2(-3Ad+5Bc+2Bd)\cos(e+fx)\sqrt{a\sin\ldots}}{3df}}{2d(c+d)}$$

$\Big\downarrow$ 3460

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{df(c+d)(c+d\sin(e+fx))}-\frac{\frac{\frac{3a^2(c-d)\left(Ad(3c+5d)-B\left(5c^2+5cd-2d^2\right)\right)\int\frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{d}+\frac{2a^3\left(3Ad(3c+d)-B\left(15c^2-5cd-14d^2\right)\right)\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}}}{3d}+\frac{2a^2(-3Ad+5Bc+2Bd)\cos(e+fx\ldots}{3df}}{2d(c+d)}$$

$\Big\downarrow$ 3042

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{df(c+d)(c+d\sin(e+fx))}-\frac{\frac{\frac{3a^2(c-d)\left(Ad(3c+5d)-B\left(5c^2+5cd-2d^2\right)\right)\int\frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{d}+\frac{2a^3\left(3Ad(3c+d)-B\left(15c^2-5cd-14d^2\right)\right)\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}}}{3d}+\frac{2a^2(-3Ad+5Bc+2Bd)\cos(e+fx\ldots}{3df}}{2d(c+d)}$$

$\Big\downarrow$ 3252

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{df(c+d)(c+d\sin(e+fx))}-\frac{\frac{\frac{2a^3\left(3Ad(3c+d)-B\left(15c^2-5cd-14d^2\right)\right)\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}}-\frac{6a^3(c-d)\left(Ad(3c+5d)-B\left(5c^2+5cd-2d^2\right)\right)\int\frac{1}{a(c+d)-\frac{a^2d\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{df}}{3d}+\frac{2a^2(-3Ad\ldots}{}}{2d(c+d)}$$

$\Big\downarrow$ 221

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{df(c+d)(c+d\sin(e+fx))}-\frac{\frac{2a^2(-3Ad+5Bc+2Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3df}+\frac{\frac{2a^3\left(3Ad(3c+d)-B\left(15c^2-5cd-14d^2\right)\right)\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}}-\frac{6a^{5/2}(c-d)\left(Ad(3c+5d)-B\left(5c^2+5cd-2d^2\right)\cdots\right.}{d^{3/2}f\sqrt{c\cdots}}}{3d}}{2d(c+d)}$$

input
```
Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])
^2,x]
```

output
```
(a*(B*c - A*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(d*(c + d)*f*(c +
d*Sin[e + f*x])) - ((2*a^2*(5*B*c - 3*A*d + 2*B*d)*Cos[e + f*x]*Sqrt[a + a
*Sin[e + f*x]])/(3*d*f) + ((-6*a^(5/2)*(c - d)*(A*d*(3*c + 5*d) - B*(5*c^2
 + 5*c*d - 2*d^2))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqr
t[a + a*Sin[e + f*x]])])/(d^(3/2)*Sqrt[c + d]*f) + (2*a^3*(3*A*d*(3*c + d)
 - B*(15*c^2 - 5*c*d - 14*d^2))*Cos[e + f*x])/(d*f*Sqrt[a + a*Sin[e + f*x]
]))/(3*d))/(2*d*(c + d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 931 vs. $2(241) = 482$.

Time = 10.77 (sec) , antiderivative size = 932, normalized size of antiderivative = 3.52

method	result	size
default	Expression too large to display	932

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x,method=_R
ETURNVERBOSE)
```

output
```
-1/3*a*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(sin(f*x+e)*d*(-9*A*arctan
h((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^2*d-6*A*arctanh((a-a
*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d^2+15*A*arctanh((a-a*sin(
f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^3-2*B*(a-a*sin(f*x+e))^(3/2)*((
c+d)*a*d)^(1/2)*c*d-2*B*(a-a*sin(f*x+e))^(3/2)*((c+d)*a*d)^(1/2)*d^2+15*a^
2*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*B*c^3-21*B*arctanh
((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d^2+6*B*arctanh((a-a*
sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^3+6*A*(a-a*sin(f*x+e))^(1/2
)*((c+d)*a*d)^(1/2)*a*c*d+6*A*(a-a*sin(f*x+e))^(1/2)*((c+d)*a*d)^(1/2)*a*d
^2-12*B*(a-a*sin(f*x+e))^(1/2)*((c+d)*a*d)^(1/2)*a*c^2+6*B*(a-a*sin(f*x+e)
)^(1/2)*((c+d)*a*d)^(1/2)*a*c*d+18*B*(a-a*sin(f*x+e))^(1/2)*((c+d)*a*d)^(1
/2)*a*d^2)-9*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c
^3*d-6*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^2*d^2
+15*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d^3-2*B*
(a-a*sin(f*x+e))^(3/2)*((c+d)*a*d)^(1/2)*c^2*d-2*B*(a-a*sin(f*x+e))^(3/2)*
((c+d)*a*d)^(1/2)*c*d^2+15*a^2*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d
^2)^(1/2))*B*c^4-21*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2)
)*a^2*c^2*d^2+6*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^
2*c*d^3+9*A*(a-a*sin(f*x+e))^(1/2)*((c+d)*a*d)^(1/2)*a*c^2*d+3*A*(a-a*sin(
f*x+e))^(1/2)*((c+d)*a*d)^(1/2)*a*d^3-15*B*(a-a*sin(f*x+e))^(1/2)*((c+d...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 865 vs. $2(241) = 482$.

Time = 1.33 (sec) , antiderivative size = 2046, normalized size of antiderivative = 7.72

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, al
gorithm="fricas")
```

output
```
[-1/12*(3*(5*B*a^2*c^4 - (3*A - 5*B)*a^2*c^3*d - (5*A + 7*B)*a^2*c^2*d^2 +
 (3*A - 5*B)*a^2*c*d^3 + (5*A + 2*B)*a^2*d^4 - (5*B*a^2*c^3*d - 3*A*a^2*c^
2*d^2 - (2*A + 7*B)*a^2*c*d^3 + (5*A + 2*B)*a^2*d^4)*cos(f*x + e)^2 + (5*B
*a^2*c^4 - 3*A*a^2*c^3*d - (2*A + 7*B)*a^2*c^2*d^2 + (5*A + 2*B)*a^2*c*d^3
)*cos(f*x + e) + (5*B*a^2*c^4 - (3*A - 5*B)*a^2*c^3*d - (5*A + 7*B)*a^2*c^
2*d^2 + (3*A - 5*B)*a^2*c*d^3 + (5*A + 2*B)*a^2*d^4 + (5*B*a^2*c^3*d - 3*A
*a^2*c^2*d^2 - (2*A + 7*B)*a^2*c*d^3 + (5*A + 2*B)*a^2*d^4)*cos(f*x + e))*
sin(f*x + e))*sqrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*
c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*
d^3 - (c*d^2 + d^3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e
) - (c^2*d + 4*c*d^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*s
qrt(a*sin(f*x + e) + a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*
cos(f*x + e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*
d + 4*a*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^
2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*co
s(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*
(15*B*a^2*c^3 - (9*A + 20*B)*a^2*c^2*d + 3*(2*A - 3*B)*a^2*c*d^2 + (3*A +
14*B)*a^2*d^3 + 2*(B*a^2*c*d^2 + B*a^2*d^3)*cos(f*x + e)^3 + 2*(5*B*a^2*c^
2*d - (3*A + 2*B)*a^2*c*d^2 - (3*A + 7*B)*a^2*d^3)*cos(f*x + e)^2 + (15*B*
a^2*c^3 - (9*A + 10*B)*a^2*c^2*d - 15*B*a^2*c*d^2 - (3*A + 2*B)*a^2*d^3...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{5}{2}}}{(d\sin(fx+e)+c)^2}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e)
+ c)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 597 vs. $2(241) = 482$.

Time = 0.31 (sec) , antiderivative size = 597, normalized size of antiderivative = 2.25

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, al
gorithm="giac")
```

output
```
-1/6*sqrt(2)*sqrt(a)*(3*sqrt(2)*(5*B*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e)) - 3*A*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*A*a^2*c*d^2
*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 7*B*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e)) + 5*A*a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*B*a^2
*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*arctan(sqrt(2)*d*sin(-1/4*pi + 1
/2*f*x + 1/2*e)/sqrt(-c*d - d^2))/((c*d^3 + d^4)*sqrt(-c*d - d^2)) - 6*(B*
a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)
 - A*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x +
 1/2*e) - 2*B*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi +
1/2*f*x + 1/2*e) + 2*A*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-
1/4*pi + 1/2*f*x + 1/2*e) + B*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
)*sin(-1/4*pi + 1/2*f*x + 1/2*e) - A*a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e))/((c*d^3 + d^4)*(2*d*sin(-1/4*pi + 1
/2*f*x + 1/2*e)^2 - c - d)) + 4*(2*B*a^2*d^4*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 6*B*a^2*c*d^3*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 3*A*a^2*d^4*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 9*B*a^2*d^4*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e))/d^6)/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx=\int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{5/2}}{(c+d\,\sin(e+f\,x))^2}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c + d*sin(e + f*x))
^2,x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c + d*sin(e + f*x))
^2, x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \sqrt{a}\,a^2\Bigg(\Bigg(\left(\int \frac{\sqrt{\sin(fx+e)+1}}{\sin(fx+e)^2 d^2+2\sin(fx+e)\,cd+c^2}dx\right)a$$
$$+\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^3}{\sin(fx+e)^2 d^2+2\sin(fx+e)\,cd+c^2}dx\right)b$$
$$+\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^2}{\sin(fx+e)^2 d^2+2\sin(fx+e)\,cd+c^2}dx\right)a$$
$$+2\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^2}{\sin(fx+e)^2 d^2+2\sin(fx+e)\,cd+c^2}dx\right)b$$
$$+2\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)^2 d^2+2\sin(fx+e)\,cd+c^2}dx\right)a$$
$$+\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)^2 d^2+2\sin(fx+e)\,cd+c^2}dx\right)b\Bigg)$$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

output
```
sqrt(a)*a**2*(int(sqrt(sin(e + f*x) + 1)/(sin(e + f*x)**2*d**2 + 2*sin(e +
 f*x)*c*d + c**2),x)*a + int((sqrt(sin(e + f*x) + 1)*sin(e + f*x)**3)/(sin
(e + f*x)**2*d**2 + 2*sin(e + f*x)*c*d + c**2),x)*b + int((sqrt(sin(e + f*
x) + 1)*sin(e + f*x)**2)/(sin(e + f*x)**2*d**2 + 2*sin(e + f*x)*c*d + c**2
),x)*a + 2*int((sqrt(sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x)**2*d
**2 + 2*sin(e + f*x)*c*d + c**2),x)*b + 2*int((sqrt(sin(e + f*x) + 1)*sin(
e + f*x))/(sin(e + f*x)**2*d**2 + 2*sin(e + f*x)*c*d + c**2),x)*a + int((s
qrt(sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**2*d**2 + 2*sin(e + f*x)
*c*d + c**2),x)*b)
```

3.306 $\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 308

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = -\frac{a^{5/2}(Ad(3c^2+10cd+19d^2)-B(15c^3+30c^2d+7cd^2-20d^3))\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{4d^{7/2}(c+d)^{5/2}f} + \frac{a^3(3Ad(c+3d)-B(15c^2+25cd+4d^2))\cos(e+fx)}{4d^3(c+d)^2f\sqrt{a+a\sin(e+fx)}} + \frac{a(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2d(c+d)f(c+d\sin(e+fx))^2} - \frac{a^2(Ad(c+7d)-B(5c^2+7cd-4d^2))\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{4d^2(c+d)^2f(c+d\sin(e+fx))}$$

output
```
-1/4*a^(5/2)*(A*d*(3*c^2+10*c*d+19*d^2)-B*(15*c^3+30*c^2*d+7*c*d^2-20*d^3)
)*arctanh(a^(1/2)*d^(1/2)*cos(f*x+e)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/d
^(7/2)/(c+d)^(5/2)/f+1/4*a^3*(3*A*d*(c+3*d)-B*(15*c^2+25*c*d+4*d^2))*cos(f
*x+e)/d^3/(c+d)^2/f/(a+a*sin(f*x+e))^(1/2)+1/2*a*(-A*d+B*c)*cos(f*x+e)*(a+
a*sin(f*x+e))^(3/2)/d/(c+d)/f/(c+d*sin(f*x+e))^2-1/4*a^2*(A*d*(c+7*d)-B*(5
*c^2+7*c*d-4*d^2))*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/d^2/(c+d)^2/f/(c+d*si
n(f*x+e))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 19.01 (sec) , antiderivative size = 1046, normalized size of antiderivative = 3.40

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e +
 f*x])^3,x]
```

output
```
((a*(1 + Sin[e + f*x]))^(5/2)*(((A*d*(3*c^2 + 10*c*d + 19*d^2) - B*(15*c^3
 + 30*c^2*d + 7*c*d^2 - 20*d^3))*((c + d)*(e + f*x - 2*Log[Sec[(e + f*x)/4
]^2]) + Sqrt[c + d]*RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & ,
(-(c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]) - d^(3/2)*Log[-#1 + Tan[(e + f*x
)/4]] - d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - 2*c*Sqrt[d]*Log[-#1 +
Tan[(e + f*x)/4]]*#1 - 2*d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]]*#1 - c*Sqrt[c
 + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4
]]*#1^2 + d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + 3*d*Sqrt[c + d]*Log[-
#1 + Tan[(e + f*x)/4]]*#1^2 - c*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1
^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ]))/(c + d)^(7/2) + ((A*d*(3*c^2 + 1
0*c*d + 19*d^2) - B*(15*c^3 + 30*c^2*d + 7*c*d^2 - 20*d^3))*(-((c + d)*(e
+ f*x - 2*Log[Sec[(e + f*x)/4]^2])) + Sqrt[c + d]*RootSum[c + 4*d*#1 + 2*c
*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]) -
d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]] + d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x
)/4]] - 2*c*Sqrt[d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*d^(3/2)*Log[-#1 + T
an[(e + f*x)/4]]*#1 + c*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + c*Sqr
t[d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + d^(3/2)*Log[-#1 + Tan[(e + f*x)/4]
]*#1^2 - 3*d*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + c*Sqrt[c + d]*
Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ]))/(c
 + d)^(7/2) - (4*Sqrt[d]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(15*B*c^...
```

Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.297$, Rules used = {3042, 3454, 27, 3042, 3454, 27, 3042, 3460, 3042, 3252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(a\sin(e+fx)+a)^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}dx$$

$$\Big\downarrow \ 3454$$

$$\frac{\int -\frac{(\sin(e+fx)a+a)^{3/2}(a(3Bc-7Ad-4Bd)-a(5Bc-Ad+4Bd)\sin(e+fx))}{2(c+d\sin(e+fx))^2}dx}{2d(c+d)}+\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2df(c+d)(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \ 27$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2df(c+d)(c+d\sin(e+fx))^2}-\frac{\int \frac{(\sin(e+fx)a+a)^{3/2}(a(3Bc-7Ad-4Bd)-a(5Bc-Ad+4Bd)\sin(e+fx))}{(c+d\sin(e+fx))^2}dx}{4d(c+d)}$$

$$\Big\downarrow \ 3042$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2df(c+d)(c+d\sin(e+fx))^2}-\frac{\int \frac{(\sin(e+fx)a+a)^{3/2}(a(3Bc-7Ad-4Bd)-a(5Bc-Ad+4Bd)\sin(e+fx))}{(c+d\sin(e+fx))^2}dx}{4d(c+d)}$$

$$\Big\downarrow \ 3454$$

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2df(c+d)(c+d\sin(e+fx))^2}-\frac{\frac{\int -\frac{\sqrt{\sin(e+fx)a+a}\left(a^2\left(Ad(c+19d)-B\left(5c^2+3dc-20d^2\right)\right)-a^2\left(3Ad(c+3d)-B\left(15c^2+25dc+4d^2\right)\right)\sin(e+fx)\right)}{2(c+d\sin(e+fx))}dx}{d(c+d)}+\frac{a^2\left(Ad(c+7d)-B\left(5c^2+7cd-4d^2\right)\right)\text{co}}{df(c+d)(c+d\sin(}}{4d(c+d)}$$

$\downarrow$ 27

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2df(c+d)(c+d\sin(e+fx))^2}-$$
$$\frac{\frac{a^2\left(Ad(c+7d)-B\left(5c^2+7cd-4d^2\right)\right)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{df(c+d)(c+d\sin(e+fx))}-\frac{\int\frac{\sqrt{\sin(e+fx)a+a}\left(a^2\left(Ad(c+19d)-B\left(5c^2+3dc-20d^2\right)\right)-a^2\left(3Ad(c+3d)-B\left(15c^2+\right.\right.\right.}{c+d\sin(e+fx)}}{2d(c+d)}}{4d(c+d)}$$

$\downarrow$ 3042

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2df(c+d)(c+d\sin(e+fx))^2}-$$
$$\frac{\frac{a^2\left(Ad(c+7d)-B\left(5c^2+7cd-4d^2\right)\right)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{df(c+d)(c+d\sin(e+fx))}-\frac{\int\frac{\sqrt{\sin(e+fx)a+a}\left(a^2\left(Ad(c+19d)-B\left(5c^2+3dc-20d^2\right)\right)-a^2\left(3Ad(c+3d)-B\left(15c^2+\right.\right.\right.}{c+d\sin(e+fx)}}{2d(c+d)}}{4d(c+d)}$$

$\downarrow$ 3460

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2df(c+d)(c+d\sin(e+fx))^2}-$$
$$\frac{\frac{a^2\left(Ad(c+7d)-B\left(5c^2+7cd-4d^2\right)\right)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{df(c+d)(c+d\sin(e+fx))}-\frac{\frac{a^2\left(Ad\left(3c^2+10cd+19d^2\right)-B\left(15c^3+30c^2d+7cd^2-20d^3\right)\right)\int\frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{d}+\frac{2a}{}}{2d(c+d)}}{4d(c+d)}$$

$\downarrow$ 3042

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2df(c+d)(c+d\sin(e+fx))^2}-$$
$$\frac{\frac{a^2\left(Ad(c+7d)-B\left(5c^2+7cd-4d^2\right)\right)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{df(c+d)(c+d\sin(e+fx))}-\frac{\frac{a^2\left(Ad\left(3c^2+10cd+19d^2\right)-B\left(15c^3+30c^2d+7cd^2-20d^3\right)\right)\int\frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{d}+\frac{2a}{}}{2d(c+d)}}{4d(c+d)}$$

$\downarrow$ 3252

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2df(c+d)(c+d\sin(e+fx))^2}-$$
$$\frac{\frac{a^2\left(Ad(c+7d)-B\left(5c^2+7cd-4d^2\right)\right)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{df(c+d)(c+d\sin(e+fx))}-\frac{\frac{2a^3\left(3Ad(c+3d)-B\left(15c^2+25cd+4d^2\right)\right)\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}}-\frac{2a^3\left(Ad\left(3c^2+10cd+19d^2\right)-B\left(1\right.\right.}{}}{2d(c+d)}}{4d(c+d)}$$

$\downarrow$ 221

$$\frac{a(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2df(c+d)(c+d\sin(e+fx))^2}-$$

$$\frac{\frac{a^2\left(Ad(c+7d)-B\left(5c^2+7cd-4d^2\right)\right)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{df(c+d)(c+d\sin(e+fx))}-\frac{\frac{2a^3\left(3Ad(c+3d)-B\left(15c^2+25cd+4d^2\right)\right)\cos(e+fx)}{df\sqrt{a\sin(e+fx)+a}}-\frac{2a^{5/2}\left(Ad\left(3c^2+10cd+19d^2\right)-B\right.}{}}{2d(c+d)}}{4d(c+d)}$$

input
```
Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])
^3,x]
```

output
```
(a*(B*c - A*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*d*(c + d)*f*(c
+ d*Sin[e + f*x])^2) - ((a^2*(A*d*(c + 7*d) - B*(5*c^2 + 7*c*d - 4*d^2))*C
os[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(d*(c + d)*f*(c + d*Sin[e + f*x])) -
 ((-2*a^(5/2)*(A*d*(3*c^2 + 10*c*d + 19*d^2) - B*(15*c^3 + 30*c^2*d + 7*c*
d^2 - 20*d^3))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a
+ a*Sin[e + f*x]])])/(d^(3/2)*Sqrt[c + d]*f) + (2*a^3*(3*A*d*(c + 3*d) - B
*(15*c^2 + 25*c*d + 4*d^2))*Cos[e + f*x])/(d*f*Sqrt[a + a*Sin[e + f*x]]))/
(2*d*(c + d)))/(4*d*(c + d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1586 vs. $2(280) = 560$.

Time = 70.02 (sec) , antiderivative size = 1587, normalized size of antiderivative = 5.15

method	result	size
default	Expression too large to display	1587

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x,method=_R
ETURNVERBOSE)
```

output
```
-1/4*a*(-11*A*(-a*(sin(f*x+e)-1))^(3/2)*((c+d)*a*d)^(1/2)*d^4-4*B*(-a*(sin
(f*x+e)-1))^(3/2)*((c+d)*a*d)^(1/2)*d^4-15*a^2*arctanh((-a*(sin(f*x+e)-1))
^(1/2)*d/((c+d)*a*d)^(1/2))*B*c^5-13*B*(-a*(sin(f*x+e)-1))^(1/2)*((c+d)*a*
d)^(1/2)*a*c*d^3+3*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/((c+d)*a*d)^(1/2)
)*sin(f*x+e)^2*a^2*c^2*d^3+10*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/((c+d)
*a*d)^(1/2))*sin(f*x+e)^2*a^2*c*d^4-15*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)
*d/((c+d)*a*d)^(1/2))*sin(f*x+e)^2*a^2*c^3*d^2-30*B*arctanh((-a*(sin(f*x+e
)-1))^(1/2)*d/((c+d)*a*d)^(1/2))*sin(f*x+e)^2*a^2*c^2*d^3-7*B*arctanh((-a*
(sin(f*x+e)-1))^(1/2)*d/((c+d)*a*d)^(1/2))*sin(f*x+e)^2*a^2*c*d^4+6*A*arct
anh((-a*(sin(f*x+e)-1))^(1/2)*d/((c+d)*a*d)^(1/2))*sin(f*x+e)*a^2*c^3*d^2+
20*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/((c+d)*a*d)^(1/2))*sin(f*x+e)*a^2
*c^2*d^3+38*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/((c+d)*a*d)^(1/2))*sin(f
*x+e)*a^2*c*d^4+8*B*(-a*(sin(f*x+e)-1))^(1/2)*((c+d)*a*d)^(1/2)*sin(f*x+e)
^2*a*d^4-30*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/((c+d)*a*d)^(1/2))*sin(f
*x+e)*a^2*c^4*d-60*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/((c+d)*a*d)^(1/2)
)*sin(f*x+e)*a^2*c^3*d^2-14*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/((c+d)*a
*d)^(1/2))*sin(f*x+e)*a^2*c^2*d^3+40*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d
/((c+d)*a*d)^(1/2))*sin(f*x+e)*a^2*c*d^4-3*A*(-a*(sin(f*x+e)-1))^(1/2)*((c
+d)*a*d)^(1/2)*a*c^3*d-13*A*(-a*(sin(f*x+e)-1))^(1/2)*((c+d)*a*d)^(1/2)*a*
c^2*d^2+3*A*(-a*(sin(f*x+e)-1))^(1/2)*((c+d)*a*d)^(1/2)*a*c*d^3+29*B*(-...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1365 vs. $2(280) = 560$.

Time = 1.55 (sec) , antiderivative size = 3046, normalized size of antiderivative = 9.89

$$\int \frac{(a + a\sin(e + fx))^{5/2}(A + B\sin(e + fx))}{(c + d\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, al
gorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, al
gorithm="maxima")
```

output
```
Timed out
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 895 vs. $2(280) = 560$.

Time = 0.32 (sec) , antiderivative size = 895, normalized size of antiderivative = 2.91

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, al
gorithm="giac")
```

output
```
1/8*sqrt(2)*(16*B*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/
2*f*x + 1/2*e)/d^3 + sqrt(2)*(15*B*a^2*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e)) - 3*A*a^2*c^2*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 30*B*a^2*c^2*d*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 10*A*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e)) + 7*B*a^2*c*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 19*A*
a^2*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - 20*B*a^2*d^3*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e)))*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1/2*e)/sqrt
(-c*d - d^2))/((c^2*d^3 + 2*c*d^4 + d^5)*sqrt(-c*d - d^2)) - 2*(18*B*a^2*c
^3*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3
- 10*A*a^2*c^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f
*x + 1/2*e)^3 + 4*B*a^2*c^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1
/4*pi + 1/2*f*x + 1/2*e)^3 - 12*A*a^2*c*d^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 30*B*a^2*c*d^3*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 22*A*a^2*d^4*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 8*B*a^2*d^4
*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 7*
B*a^2*c^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*
e) + 3*A*a^2*c^3*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f
*x + 1/2*e) - 13*B*a^2*c^3*d*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*
pi + 1/2*f*x + 1/2*e) + 13*A*a^2*c^2*d^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx=\int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^{5/2}}{(c+d\,\sin(e+f\,x))^3}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c + d*sin(e + f*x))
^3,x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c + d*sin(e + f*x))
^3, x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^{5/2}(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \sqrt{a}\,a^2\left(\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}}{\sin(fx+e)^3 d^3+3\sin(fx+e)^2 c\,d^2+3\sin(fx+e)\,c^2d+c^3}dx\right)a\right.\right.$$
$$+\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^3}{\sin(fx+e)^3 d^3+3\sin(fx+e)^2 c\,d^2+3\sin(fx+e)\,c^2d+c^3}dx\right)b$$
$$+\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^2}{\sin(fx+e)^3 d^3+3\sin(fx+e)^2 c\,d^2+3\sin(fx+e)\,c^2d+c^3}dx\right)a$$
$$+2\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^2}{\sin(fx+e)^3 d^3+3\sin(fx+e)^2 c\,d^2+3\sin(fx+e)\,c^2d+c^3}dx\right)b$$
$$+2\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)^3 d^3+3\sin(fx+e)^2 c\,d^2+3\sin(fx+e)\,c^2d+c^3}dx\right)a$$
$$\left.+\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)^3 d^3+3\sin(fx+e)^2 c\,d^2+3\sin(fx+e)\,c^2d+c^3}dx\right)b\right)$$

input
```
int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)
```

output
```
sqrt(a)*a**2*(int(sqrt(sin(e + f*x) + 1)/(sin(e + f*x)**3*d**3 + 3*sin(e +
 f*x)**2*c*d**2 + 3*sin(e + f*x)*c**2*d + c**3),x)*a + int((sqrt(sin(e + f
*x) + 1)*sin(e + f*x)**3)/(sin(e + f*x)**3*d**3 + 3*sin(e + f*x)**2*c*d**2
 + 3*sin(e + f*x)*c**2*d + c**3),x)*b + int((sqrt(sin(e + f*x) + 1)*sin(e
+ f*x)**2)/(sin(e + f*x)**3*d**3 + 3*sin(e + f*x)**2*c*d**2 + 3*sin(e + f*
x)*c**2*d + c**3),x)*a + 2*int((sqrt(sin(e + f*x) + 1)*sin(e + f*x)**2)/(s
in(e + f*x)**3*d**3 + 3*sin(e + f*x)**2*c*d**2 + 3*sin(e + f*x)*c**2*d + c
**3),x)*b + 2*int((sqrt(sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**3*d
**3 + 3*sin(e + f*x)**2*c*d**2 + 3*sin(e + f*x)*c**2*d + c**3),x)*a + int(
(sqrt(sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**3*d**3 + 3*sin(e + f*
x)**2*c*d**2 + 3*sin(e + f*x)*c**2*d + c**3),x)*b)
```

3.307 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 284

$$\begin{aligned}\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{\sqrt{a+a\sin(e+fx)}}\,dx &= -\frac{\sqrt{2}(A-B)(c-d)^3\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f}\\ &\quad-\frac{4(7Ad(21c^2-12cd+7d^2)+B(36c^3-63c^2d+144cd^2-37d^3))\cos(e+fx)}{105f\sqrt{a+a\sin(e+fx)}}\\ &\quad-\frac{2d(7A(9c-d)d+B(24c^2-15cd+31d^2))\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{105af}\\ &\quad-\frac{2(6Bc+7Ad-Bd)\cos(e+fx)(c+d\sin(e+fx))^2}{35f\sqrt{a+a\sin(e+fx)}}\\ &\quad-\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a+a\sin(e+fx)}}\end{aligned}$$

output
```
-2^(1/2)*(A-B)*(c-d)^3*arctanh(1/2*a^(1/2)*cos(f*x+e)*2^(1/2)/(a+a*sin(f*x
+e))^(1/2))/a^(1/2)/f-4/105*(7*A*d*(21*c^2-12*c*d+7*d^2)+B*(36*c^3-63*c^2*
d+144*c*d^2-37*d^3))*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-2/105*d*(7*A*(9*c
-d)*d+B*(24*c^2-15*c*d+31*d^2))*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/a/f-2/35
*(7*A*d+6*B*c-B*d)*cos(f*x+e)*(c+d*sin(f*x+e))^2/f/(a+a*sin(f*x+e))^(1/2)-
2/7*B*cos(f*x+e)*(c+d*sin(f*x+e))^3/f/(a+a*sin(f*x+e))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.57 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.32

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{\sqrt{a+a\sin(e+fx)}}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left((840+840i)(-1)^{3/4}(A-B)(c-d)^3\text{arctanh}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1\right.\right.\right.}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/Sqrt[a + a*Sin[e +
 f*x]],x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((840 + 840*I)*(-1)^(3/4)*(A - B)*(
c - d)^3*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] - 105*(4*
A*d*(6*c^2 - 3*c*d + 2*d^2) + B*(8*c^3 - 12*c^2*d + 24*c*d^2 - 5*d^3))*Cos
[(e + f*x)/2] - 35*d*(2*A*(6*c - d)*d + B*(12*c^2 - 6*c*d + 5*d^2))*Cos[(3
*(e + f*x))/2] + 21*d^2*(6*B*c + 2*A*d - B*d)*Cos[(5*(e + f*x))/2] + 15*B*
d^3*Cos[(7*(e + f*x))/2] + 105*(4*A*d*(6*c^2 - 3*c*d + 2*d^2) + B*(8*c^3 -
 12*c^2*d + 24*c*d^2 - 5*d^3))*Sin[(e + f*x)/2] - 35*d*(2*A*(6*c - d)*d +
B*(12*c^2 - 6*c*d + 5*d^2))*Sin[(3*(e + f*x))/2] + 21*d^2*(-2*A*d + B*(-6*
c + d))*Sin[(5*(e + f*x))/2] + 15*B*d^3*Sin[(7*(e + f*x))/2]))/(420*f*Sqrt
[a*(1 + Sin[e + f*x])])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{\sqrt{a\sin(e+fx)+a}}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{\sqrt{a\sin(e+fx)+a}}dx$$
$$\Big\downarrow \text{ 3462}$$

$$\frac{2\int \frac{(c+d\sin(e+fx))^2(a(7Ac-Bc+6Bd)+a(6Bc+7Ad-Bd)\sin(e+fx))}{2\sqrt{\sin(e+fx)a+a}}dx}{7a}-\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{(c+d\sin(e+fx))^2(a(7Ac-Bc+6Bd)+a(6Bc+7Ad-Bd)\sin(e+fx))}{\sqrt{\sin(e+fx)a+a}}dx}{7a}-\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{(c+d\sin(e+fx))^2(a(7Ac-Bc+6Bd)+a(6Bc+7Ad-Bd)\sin(e+fx))}{\sqrt{\sin(e+fx)a+a}}dx}{7a}-\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}$$

$\Big\downarrow$ 3462

$$\frac{\frac{2\int \frac{(c+d\sin(e+fx))\left(\left(35Ac^2-11Bc^2-7Adc+55Bdc+28Ad^2-4Bd^2\right)a^2+\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)a^2\right)}{2\sqrt{\sin(e+fx)a+a}}dx}{5a}-\frac{2a(7Ad+6Bc-Bd)\cos(e\cdots}{5f\sqrt{a\sin(\cdots}}}{7a}-\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int -\frac{(c+d\sin(e+fx))\left(a^2\left(B\left(11c^2-55dc+4d^2\right)-7A\left(5c^2-dc+4d^2\right)\right)-a^2\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a}-\frac{2a(7Ad+6Bc-Bd)\cos(e+f\cdots}{5f\sqrt{a\sin(e+f\cdots}}}{7a}-\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}$$

$\Big\downarrow$ 25

$$\frac{-\frac{\int -\frac{(c+d\sin(e+fx))\left(\left(35Ac^2-11Bc^2-7Adc+55Bdc+28Ad^2-4Bd^2\right)a^2+\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)a^2\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a}-\frac{2a(7Ad+6Bc-Bd)\cos\cdots}{5f\sqrt{a\sin\cdots}}}{7a}-\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int -\frac{(c+d\sin(e+fx))\left(a^2\left(B\left(11c^2-55dc+4d^2\right)-7A\left(5c^2-dc+4d^2\right)\right)-a^2\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a}-\frac{2a(7Ad+6Bc-Bd)\cos(e+f}{5f\sqrt{a\sin(e+f}}}{7a}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}$$

$\downarrow$ 25

$$\frac{-\frac{\int -\frac{(c+d\sin(e+fx))\left(\left(35Ac^2-11Bc^2-7Adc+55Bdc+28Ad^2-4Bd^2\right)a^2+\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)a^2\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a}-\frac{2a(7Ad+6Bc-Bd)\cos}{5f\sqrt{a\sin}}}{7a}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}$$

$\downarrow$ 25

$$\frac{\frac{\int -\frac{(c+d\sin(e+fx))\left(a^2\left(B\left(11c^2-55dc+4d^2\right)-7A\left(5c^2-dc+4d^2\right)\right)-a^2\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a}-\frac{2a(7Ad+6Bc-Bd)\cos(e+f}{5f\sqrt{a\sin(e+f}}}{7a}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}$$

$\downarrow$ 25

$$\frac{-\frac{\int -\frac{(c+d\sin(e+fx))\left(\left(35Ac^2-11Bc^2-7Adc+55Bdc+28Ad^2-4Bd^2\right)a^2+\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)a^2\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a}-\frac{2a(7Ad+6Bc-Bd)\cos}{5f\sqrt{a\sin}}}{7a}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}$$

$\downarrow$ 25

$$\frac{\frac{\int -\frac{(c+d\sin(e+fx))\left(a^2\left(B\left(11c^2-55dc+4d^2\right)-7A\left(5c^2-dc+4d^2\right)\right)-a^2\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a}-\frac{2a(7Ad+6Bc-Bd)\cos(e+f}{5f\sqrt{a\sin(e+f}}}{7a}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}$$

$\downarrow$ 25

$$\frac{-\frac{\int -\frac{(c+d\sin(e+fx))\left(\left(35Ac^2-11Bc^2-7Adc+55Bdc+28Ad^2-4Bd^2\right)a^2+\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)a^2\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a}-\frac{2a(7Ad+6Bc-Bd)\cos\cdots}{5f\sqrt{a\sin\cdots}}}{7a}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int -\frac{(c+d\sin(e+fx))\left(a^2\left(B\left(11c^2-55dc+4d^2\right)-7A\left(5c^2-dc+4d^2\right)\right)-a^2\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a}-\frac{2a(7Ad+6Bc-Bd)\cos(e+f\cdots}{5f\sqrt{a\sin(e+f\cdots}}}{7a}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{\int -\frac{(c+d\sin(e+fx))\left(\left(35Ac^2-11Bc^2-7Adc+55Bdc+28Ad^2-4Bd^2\right)a^2+\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)a^2\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a}-\frac{2a(7Ad+6Bc-Bd)\cos\cdots}{5f\sqrt{a\sin\cdots}}}{7a}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int -\frac{(c+d\sin(e+fx))\left(a^2\left(B\left(11c^2-55dc+4d^2\right)-7A\left(5c^2-dc+4d^2\right)\right)-a^2\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a}-\frac{2a(7Ad+6Bc-Bd)\cos(e+f\cdots}{5f\sqrt{a\sin(e+f\cdots}}}{7a}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{\int -\frac{(c+d\sin(e+fx))\left(\left(35Ac^2-11Bc^2-7Adc+55Bdc+28Ad^2-4Bd^2\right)a^2+\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)a^2\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a}-\frac{2a(7Ad+6Bc-Bd)\cos\cdots}{5f\sqrt{a\sin\cdots}}}{7a}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int -\frac{(c+d\sin(e+fx))\left(a^2\left(B\left(11c^2-55dc+4d^2\right)-7A\left(5c^2-dc+4d^2\right)\right)-a^2\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a}-\frac{2a(7Ad+6Bc-Bd)\cos(e+f}{5f\sqrt{a\sin(e+f}}}{7a}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{\int -\frac{(c+d\sin(e+fx))\left(\left(35Ac^2-11Bc^2-7Adc+55Bdc+28Ad^2-4Bd^2\right)a^2+\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)a^2\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a}-\frac{2a(7Ad+6Bc-Bd)\cos}{5f\sqrt{a\sin}}}{7a}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int -\frac{(c+d\sin(e+fx))\left(a^2\left(B\left(11c^2-55dc+4d^2\right)-7A\left(5c^2-dc+4d^2\right)\right)-a^2\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a}-\frac{2a(7Ad+6Bc-Bd)\cos(e+f}{5f\sqrt{a\sin(e+f}}}{7a}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{\int -\frac{(c+d\sin(e+fx))\left(\left(35Ac^2-11Bc^2-7Adc+55Bdc+28Ad^2-4Bd^2\right)a^2+\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)a^2\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a}-\frac{2a(7Ad+6Bc-Bd)\cos}{5f\sqrt{a\sin}}}{7a}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int -\frac{(c+d\sin(e+fx))\left(a^2\left(B\left(11c^2-55dc+4d^2\right)-7A\left(5c^2-dc+4d^2\right)\right)-a^2\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a}-\frac{2a(7Ad+6Bc-Bd)\cos(e+f}{5f\sqrt{a\sin(e+f}}}{7a}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{\int -\frac{(c+d\sin(e+fx))\left(\left(35Ac^2-11Bc^2-7Adc+55Bdc+28Ad^2-4Bd^2\right)a^2+\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)a^2\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a}-\frac{2a(7Ad+6Bc-Bd)\cos\ldots}{5f\sqrt{a\sin\ldots}}}{7a}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}$$

$\downarrow$ 25

$$\frac{\frac{\int -\frac{(c+d\sin(e+fx))\left(a^2\left(B\left(11c^2-55dc+4d^2\right)-7A\left(5c^2-dc+4d^2\right)\right)-a^2\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a}-\frac{2a(7Ad+6Bc-Bd)\cos(e+f\ldots}{5f\sqrt{a\sin(e+f\ldots}}}{7a}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}$$

$\downarrow$ 25

$$\frac{-\frac{\int -\frac{(c+d\sin(e+fx))\left(\left(35Ac^2-11Bc^2-7Adc+55Bdc+28Ad^2-4Bd^2\right)a^2+\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)a^2\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a}-\frac{2a(7Ad+6Bc-Bd)\cos\ldots}{5f\sqrt{a\sin\ldots}}}{7a}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}$$

$\downarrow$ 25

$$\frac{\frac{\int -\frac{(c+d\sin(e+fx))\left(a^2\left(B\left(11c^2-55dc+4d^2\right)-7A\left(5c^2-dc+4d^2\right)\right)-a^2\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a}-\frac{2a(7Ad+6Bc-Bd)\cos(e+f\ldots}{5f\sqrt{a\sin(e+f\ldots}}}{7a}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}$$

$\downarrow$ 25

$$\frac{-\frac{\int -\frac{(c+d\sin(e+fx))\left(\left(35Ac^2-11Bc^2-7Adc+55Bdc+28Ad^2-4Bd^2\right)a^2+\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)a^2\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a}-\frac{2a(7Ad+6Bc-Bd)\cos\ldots}{5f\sqrt{a\sin\ldots}}}{7a}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}$$

$\downarrow$ 25

$$\frac{\frac{\int -\frac{(c+d\sin(e+fx))\left(a^2\left(B\left(11c^2-55dc+4d^2\right)-7A\left(5c^2-dc+4d^2\right)\right)-a^2\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a}-\frac{2a(7Ad+6Bc-Bd)\cos(e+f}{5f\sqrt{a\sin(e+f}}}{7a}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}$$

$\Big\downarrow$ 25

$$\frac{-\frac{\int -\frac{(c+d\sin(e+fx))\left(\left(35Ac^2-11Bc^2-7Adc+55Bdc+28Ad^2-4Bd^2\right)a^2+\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)a^2\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a}-\frac{2a(7Ad+6Bc-Bd)\cos}{5f\sqrt{a\sin}}}{7a}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int -\frac{(c+d\sin(e+fx))\left(a^2\left(B\left(11c^2-55dc+4d^2\right)-7A\left(5c^2-dc+4d^2\right)\right)-a^2\left(7A(9c-d)d+B\left(24c^2-15dc+31d^2\right)\right)\sin(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a}-\frac{2a(7Ad+6Bc-Bd)\cos(e+f}{5f\sqrt{a\sin(e+f}}}{7a}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^3}{7f\sqrt{a\sin(e+fx)+a}}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/Sqrt[a + a*Sin[e + f*x]]
,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3462
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Simp[1/(b*(m + n + 1))   Int[(a + b*Sin[e + f*x])^m*(c + d*S
in[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
 + n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 561 vs. $2(259) = 518$.

Time = 2.17 (sec) , antiderivative size = 562, normalized size of antiderivative = 1.98

method	result
parts	$-\frac{A\,c^{3}(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}\,\sqrt{2}}{2\sqrt{a}}\right)}{\sqrt{a}\,\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}-\frac{B\,d^{3}(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\,(105}{}$
default	$-\frac{(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\,\left(105A\,a^{\frac{7}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)\sqrt{2}\,c^{3}-315A\,a^{\frac{7}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)\sqrt{2}\right.}{}$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
-A*c^3*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2)*arctanh(1/
2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))/cos(f*x+e)/(a+a*sin(f*x+e))^(
1/2)/f-1/105*B*d^3*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(105*a^(7/2)*2
^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))-30*(a-a*sin(f*x
+e))^(7/2)+84*a*(a-a*sin(f*x+e))^(5/2)-140*a^2*(a-a*sin(f*x+e))^(3/2))/a^4
/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f-c^2*(3*A*d+B*c)*(1+sin(f*x+e))*(-a*(s
in(f*x+e)-1))^(1/2)*(-a^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2
^(1/2)/a^(1/2))+2*(a-a*sin(f*x+e))^(1/2))/a/cos(f*x+e)/(a+a*sin(f*x+e))^(1
/2)/f-1/15*d^2*(A*d+3*B*c)*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(-15*a
^(5/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))+6*(a-a*
sin(f*x+e))^(5/2)-10*a*(a-a*sin(f*x+e))^(3/2)+30*a^2*(a-a*sin(f*x+e))^(1/2
))/a^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f+c*d*(A*d+B*c)*(1+sin(f*x+e))*(-
a*(sin(f*x+e)-1))^(1/2)*(-3*a^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(
1/2)*2^(1/2)/a^(1/2))+2*(a-a*sin(f*x+e))^(3/2))/a^2/cos(f*x+e)/(a+a*sin(f*
x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 629 vs. $2(259) = 518$.

Time = 0.11 (sec) , antiderivative size = 629, normalized size of antiderivative = 2.21

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{\sqrt{a+a\sin(e+fx)}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, al
gorithm="fricas")
```

output
```
1/210*(105*sqrt(2)*((A - B)*a*c^3 - 3*(A - B)*a*c^2*d + 3*(A - B)*a*c*d^2
- (A - B)*a*d^3 + ((A - B)*a*c^3 - 3*(A - B)*a*c^2*d + 3*(A - B)*a*c*d^2 -
 (A - B)*a*d^3)*cos(f*x + e) + ((A - B)*a*c^3 - 3*(A - B)*a*c^2*d + 3*(A -
 B)*a*c*d^2 - (A - B)*a*d^3)*sin(f*x + e))*log(-(cos(f*x + e)^2 - (cos(f*x
 + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e)
 - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(
f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) + 4*(15*B*d^3*cos(
f*x + e)^4 - 105*B*c^3 - 105*(3*A - 2*B)*c^2*d + 21*(10*A - 17*B)*c*d^2 -
(119*A - 92*B)*d^3 + 3*(21*B*c*d^2 + (7*A - B)*d^3)*cos(f*x + e)^3 - (105*
B*c^2*d + 21*(5*A - 4*B)*c*d^2 - 4*(7*A - 16*B)*d^3)*cos(f*x + e)^2 - (105
*B*c^3 + 105*(3*A - B)*c^2*d - 21*(5*A - 16*B)*c*d^2 + 2*(56*A - 23*B)*d^3
)*cos(f*x + e) + (15*B*d^3*cos(f*x + e)^3 + 105*B*c^3 + 105*(3*A - 2*B)*c^
2*d - 21*(10*A - 17*B)*c*d^2 + (119*A - 92*B)*d^3 - 3*(21*B*c*d^2 + (7*A -
 6*B)*d^3)*cos(f*x + e)^2 - (105*B*c^2*d + 21*(5*A - B)*c*d^2 - (7*A - 46*
B)*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a*f*cos(f*x
 + e) + a*f*sin(f*x + e) + a*f)
```

Sympy [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{\sqrt{a(\sin(e+fx)+1)}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**(1/2),x)
```

output
```
Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))**3/sqrt(a*(sin(e + f*x)
 + 1)), x)
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^3}{\sqrt{a\sin(fx+e)+a}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^3/sqrt(a*sin(f*x + e)
+ a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{\sqrt{a+a\sin(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{\sqrt{a+a\sin(e+fx)}}\,dx$$
$$=\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^3}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^(1/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^(1/
2), x)
```

Reduce [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{\sqrt{a+a\sin(e+fx)}}\,dx$$
$$=\frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}}{\sin(fx+e)+1}dx\right)a\,c^3+\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^4}{\sin(fx+e)+1}dx\right)b\,d^3+\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^3}{\sin(fx+e)+1}dx\right)a\,d^3+3(\cdots}{}$$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x)
```

output
```
(sqrt(a)*(int(sqrt(sin(e + f*x) + 1)/(sin(e + f*x) + 1),x)*a*c**3 + int((s
qrt(sin(e + f*x) + 1)*sin(e + f*x)**4)/(sin(e + f*x) + 1),x)*b*d**3 + int(
(sqrt(sin(e + f*x) + 1)*sin(e + f*x)**3)/(sin(e + f*x) + 1),x)*a*d**3 + 3*
int((sqrt(sin(e + f*x) + 1)*sin(e + f*x)**3)/(sin(e + f*x) + 1),x)*b*c*d**
2 + 3*int((sqrt(sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x) + 1),x)*a
*c*d**2 + 3*int((sqrt(sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x) + 1
),x)*b*c**2*d + 3*int((sqrt(sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)
+ 1),x)*a*c**2*d + int((sqrt(sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)
 + 1),x)*b*c**3))/a
```

3.308 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 200

$$\begin{aligned}\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{\sqrt{a+a\sin(e+fx)}}\,dx &= -\frac{\sqrt{2}(A-B)(c-d)^2\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} \\ &\quad-\frac{4(5A(3c-d)d+B(6c^2-7cd+7d^2))\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} \\ &\quad-\frac{2d(4Bc+5Ad-Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15af} \\ &\quad-\frac{2B\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a+a\sin(e+fx)}}\end{aligned}$$

output

```
-2^(1/2)*(A-B)*(c-d)^2*arctanh(1/2*a^(1/2)*cos(f*x+e)*2^(1/2)/(a+a*sin(f*x
+e))^(1/2))/a^(1/2)/f-4/15*(5*A*(3*c-d)*d+B*(6*c^2-7*c*d+7*d^2))*cos(f*x+e
)/f/(a+a*sin(f*x+e))^(1/2)-2/15*d*(5*A*d+4*B*c-B*d)*cos(f*x+e)*(a+a*sin(f*
x+e))^(1/2)/a/f-2/5*B*cos(f*x+e)*(c+d*sin(f*x+e))^2/f/(a+a*sin(f*x+e))^(1/
2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.63 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.23

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{\sqrt{a+a\sin(e+fx)}}\,dx$$
$$= \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left((60+60i)(-1)^{3/4}(A-B)(c-d)^2\text{arctanh}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\right.\right.\right.}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/Sqrt[a + a*Sin[e +
 f*x]],x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((60 + 60*I)*(-1)^(3/4)*(A - B)*(c
- d)^2*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] - 30*(A*(4*
c - d)*d + 2*B*(c^2 - c*d + d^2))*Cos[(e + f*x)/2] + 5*d*(-2*A*d + B*(-4*c
 + d))*Cos[(3*(e + f*x))/2] + 3*B*d^2*Cos[(5*(e + f*x))/2] + 30*(A*(4*c -
d)*d + 2*B*(c^2 - c*d + d^2))*Sin[(e + f*x)/2] + 5*d*(-2*A*d + B*(-4*c + d
))*Sin[(3*(e + f*x))/2] - 3*B*d^2*Sin[(5*(e + f*x))/2]))/(30*f*Sqrt[a*(1 +
 Sin[e + f*x])])
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.351$, Rules used = {3042, 3462, 27, 3042, 3447, 3042, 3502, 27, 3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{\sqrt{a\sin(e+fx)+a}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{\sqrt{a\sin(e+fx)+a}}dx$$

$$\Big\downarrow \text{3462}$$

$$\frac{2\int \frac{(c+d\sin(e+fx))(a(5Ac-Bc+4Bd)+a(4Bc+5Ad-Bd)\sin(e+fx))}{2\sqrt{\sin(e+fx)a+a}}dx}{5a}-\frac{2B\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(c+d\sin(e+fx))(a(5Ac-Bc+4Bd)+a(4Bc+5Ad-Bd)\sin(e+fx))}{\sqrt{\sin(e+fx)a+a}}dx}{5a}-\frac{2B\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(c+d\sin(e+fx))(a(5Ac-Bc+4Bd)+a(4Bc+5Ad-Bd)\sin(e+fx))}{\sqrt{\sin(e+fx)a+a}}dx}{5a}-\frac{2B\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3447}$$

$$\frac{\int \frac{ad(4Bc+5Ad-Bd)\sin^2(e+fx)+(ac(4Bc+5Ad-Bd)+ad(5Ac-Bc+4Bd))\sin(e+fx)+ac(5Ac-Bc+4Bd)}{\sqrt{\sin(e+fx)a+a}}dx}{5a}-\frac{2B\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{ad(4Bc+5Ad-Bd)\sin(e+fx)^2+(ac(4Bc+5Ad-Bd)+ad(5Ac-Bc+4Bd))\sin(e+fx)+ac(5Ac-Bc+4Bd)}{\sqrt{\sin(e+fx)a+a}}dx}{5a}-\frac{2B\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\frac{2\int \frac{\left(5A\left(3c^2+d^2\right)-B\left(3c^2-16dc+d^2\right)\right)a^2+2\left(5A(3c-d)d+B\left(6c^2-7dc+7d^2\right)\right)\sin(e+fx)a^2}{2\sqrt{\sin(e+fx)a+a}}dx}{3a}-\frac{2d(5Ad+4Bc-Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{5a}-\frac{2B\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\left(5A\left(3c^2+d^2\right)-B\left(3c^2-16dc+d^2\right)\right)a^2+2\left(5A(3c-d)d+B\left(6c^2-7dc+7d^2\right)\right)\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}}dx}{3a}-\frac{2d(5Ad+4Bc-Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{5a}-\frac{2B\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a\sin(e+fx)+a}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{\left(5A\left(3c^2+d^2\right)-B\left(3c^2-16dc+d^2\right)\right)a^2+2\left(5A(3c-d)d+B\left(6c^2-7dc+7d^2\right)\right)\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}}dx}{3a}-\frac{2d(5Ad+4Bc-Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{5a}-\frac{2B\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a\sin(e+fx)+a}}$$

$\Big\downarrow$ 3230

$$\frac{\frac{15a^2(A-B)(c-d)^2\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx-\frac{4a^2\left(5Ad(3c-d)+B\left(6c^2-7cd+7d^2\right)\right)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{3a}-\frac{2d(5Ad+4Bc-Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{5a}-\frac{2B\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a\sin(e+fx)+a}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{15a^2(A-B)(c-d)^2\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx-\frac{4a^2\left(5Ad(3c-d)+B\left(6c^2-7cd+7d^2\right)\right)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{3a}-\frac{2d(5Ad+4Bc-Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{5a}-\frac{2B\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a\sin(e+fx)+a}}$$

$\Big\downarrow$ 3128

$$\frac{\frac{-\frac{30a^2(A-B)(c-d)^2\int \frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f}-\frac{4a^2\left(5Ad(3c-d)+B\left(6c^2-7cd+7d^2\right)\right)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{3a}-\frac{2d(5Ad+4Bc-Bd)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{5a}-\frac{2B\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a\sin(e+fx)+a}}$$

$\Big\downarrow$ 219

$$\frac{\frac{-\frac{15\sqrt{2}a^{3/2}(A-B)(c-d)^2\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f}-\frac{4a^2\left(5Ad(3c-d)+B\left(6c^2-7cd+7d^2\right)\right)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{3a}-\frac{2d(5Ad+4Bc-Bd)\cos(e+fx)\sqrt{a\sin(e}{3f}}{5a}$$

$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^2}{5f\sqrt{a\sin(e+fx)+a}}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/Sqrt[a + a*Sin[e + f*x]]
,x]
```

output
```
(-2*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(5*f*Sqrt[a + a*Sin[e + f*x]])
+ ((-2*d*(4*B*c + 5*A*d - B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*f
) + ((-15*Sqrt[2]*a^(3/2)*(A - B)*(c - d)^2*ArcTanh[(Sqrt[a]*Cos[e + f*x])
/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/f - (4*a^2*(5*A*(3*c - d)*d + B*(6*c
^2 - 7*c*d + 7*d^2))*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]))/(3*a))/(5
*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3462
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Simp[1/(b*(m + n + 1))   Int[(a + b*Sin[e + f*x])^m*(c + d*S
in[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
 + n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(179) = 358.

Time = 0.85 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.98

method	result
default	$-\frac{(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(15A\sqrt{2}\,a^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)c^{2}-30A\sqrt{2}\,a^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)cd-$
parts	$-\frac{A\,c^{2}(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\,\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}\,\sqrt{2}}{2\sqrt{a}}\right)}{\sqrt{a}\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}+\frac{B\,d^{2}(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\,(15c$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/15*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(15*A*2^(1/2)*a^(5/2)*arcta
nh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c^2-30*A*2^(1/2)*a^(5/2)*ar
ctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c*d+15*A*2^(1/2)*a^(5/2)
*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*d^2-15*B*2^(1/2)*a^(5
/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c^2+30*B*2^(1/2)*a
^(5/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c*d-15*B*2^(1/2
)*a^(5/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*d^2+6*B*(a-a
*sin(f*x+e))^(5/2)*d^2-10*A*(a-a*sin(f*x+e))^(3/2)*a*d^2-20*B*(a-a*sin(f*x
+e))^(3/2)*a*c*d-10*B*(a-a*sin(f*x+e))^(3/2)*a*d^2+60*A*a^2*c*d*(a-a*sin(f
*x+e))^(1/2)+30*a^2*B*c^2*(a-a*sin(f*x+e))^(1/2)+30*B*a^2*d^2*(a-a*sin(f*x
+e))^(1/2))/a^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 448 vs. 2(179) = 358.

Time = 0.14 (sec) , antiderivative size = 448, normalized size of antiderivative = 2.24

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{\sqrt{a+a\sin(e+fx)}}\,dx =$$

$$-\frac{\frac{15\sqrt{2}((A-B)ac^2-2\,(A-B)acd+(A-B)ad^2+((A-B)ac^2-2\,(A-B)acd+(A-B)ad^2)\cos(fx+e)+((A-B)ac^2-2\,(A-B)acd+(A-B)ad^2)}{\sqrt{a}}}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, al
gorithm="fricas")
```

output
```
-1/30*(15*sqrt(2)*((A - B)*a*c^2 - 2*(A - B)*a*c*d + (A - B)*a*d^2 + ((A -
 B)*a*c^2 - 2*(A - B)*a*c*d + (A - B)*a*d^2)*cos(f*x + e) + ((A - B)*a*c^2
 - 2*(A - B)*a*c*d + (A - B)*a*d^2)*sin(f*x + e))*log(-(cos(f*x + e)^2 - (
cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f
*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2
- (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) - 4*(3*B*d^
2*cos(f*x + e)^3 - 15*B*c^2 - 10*(3*A - 2*B)*c*d + (10*A - 17*B)*d^2 - (10
*B*c*d + (5*A - 4*B)*d^2)*cos(f*x + e)^2 - (15*B*c^2 + 10*(3*A - B)*c*d -
(5*A - 16*B)*d^2)*cos(f*x + e) - (3*B*d^2*cos(f*x + e)^2 - 15*B*c^2 - 10*(
3*A - 2*B)*c*d + (10*A - 17*B)*d^2 + (10*B*c*d + (5*A - B)*d^2)*cos(f*x +
e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a*f*cos(f*x + e) + a*f*sin(f*
x + e) + a*f)
```

Sympy [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(A+B\sin{(e+fx)})\,(c+d\sin{(e+fx)})^2}{\sqrt{a\,(\sin{(e+fx)}+1)}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**(1/2),x)
```

output
```
Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))**2/sqrt(a*(sin(e + f*x)
 + 1)), x)
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^2}{\sqrt{a\sin(fx+e)+a}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^2/sqrt(a*sin(f*x + e)
+ a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{\sqrt{a+a\sin(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^2}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^(1/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^(1/
2), x)
```

Reduce [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}}{\sin(fx+e)+1}dx\right)a\,c^2+\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^3}{\sin(fx+e)+1}dx\right)b\,d^2+\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^2}{\sin(fx+e)+1}dx\right)a\,d^2+2(\right.}{a}$$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x)
```

output
```
(sqrt(a)*(int(sqrt(sin(e + f*x) + 1)/(sin(e + f*x) + 1),x)*a*c**2 + int((s
qrt(sin(e + f*x) + 1)*sin(e + f*x)**3)/(sin(e + f*x) + 1),x)*b*d**2 + int(
(sqrt(sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x) + 1),x)*a*d**2 + 2*
int((sqrt(sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x) + 1),x)*b*c*d +
 2*int((sqrt(sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x) + 1),x)*a*c*d +
 int((sqrt(sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x) + 1),x)*b*c**2))/
a
```

3.309 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 130

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{\sqrt{2}(A-B)(c-d)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} - \frac{2(3Bc+3Ad-2Bd)\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} - \frac{2Bd\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3af}$$

output
```
-2^(1/2)*(A-B)*(c-d)*arctanh(1/2*a^(1/2)*cos(f*x+e)*2^(1/2)/(a+a*sin(f*x+e
))^(1/2))/a^(1/2)/f-2/3*(3*A*d+3*B*c-2*B*d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^
(1/2)-2/3*B*d*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/a/f
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.94 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.04

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx =$$
$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left((-6-6i)(-1)^{3/4}(A-B)(c-d)\text{arctanh}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1\right.\right.\right.}{3f\sqrt{a(1+\text{si}}}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/Sqrt[a + a*Sin[e + f
*x]],x]
```

output
```
-1/3*((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((-6 - 6*I)*(-1)^(3/4)*(A - B)
*(c - d)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] + 2*(Cos[
(e + f*x)/2] - Sin[(e + f*x)/2])*(3*B*c + 3*A*d - B*d + B*d*Sin[e + f*x]))
)/(f*Sqrt[a*(1 + Sin[e + f*x])])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3447, 3042, 3502, 27, 3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{\sqrt{a\sin(e+fx)+a}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{\sqrt{a\sin(e+fx)+a}}dx$$
$$\Big\downarrow \text{3447}$$
$$\int \frac{(Ad+Bc)\sin(e+fx)+Ac+Bd\sin^2(e+fx)}{\sqrt{a\sin(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(Ad+Bc)\sin(e+fx)+Ac+Bd\sin(e+fx)^2}{\sqrt{a\sin(e+fx)+a}}dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{2\int \frac{a(3Ac+Bd)+a(3Bc+3Ad-2Bd)\sin(e+fx)}{2\sqrt{\sin(e+fx)a+a}}dx}{3a}-\frac{2Bd\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3af}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a(3Ac+Bd)+a(3Bc+3Ad-2Bd)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}}dx}{3a}-\frac{2Bd\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3af}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(3Ac+Bd)+a(3Bc+3Ad-2Bd)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}}dx}{3a}-\frac{2Bd\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3af}$$

$$\Big\downarrow \text{3230}$$

$$\frac{3a(A-B)(c-d)\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx-\frac{2a(3Ad+3Bc-2Bd)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{3a}-\frac{2Bd\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3af}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3a(A-B)(c-d)\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx-\frac{2a(3Ad+3Bc-2Bd)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{3a}-\frac{2Bd\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3af}$$

$$\Big\downarrow \text{3128}$$

$$\frac{-\frac{6a(A-B)(c-d)\int \frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f}-\frac{2a(3Ad+3Bc-2Bd)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{3a}-\frac{2Bd\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3af}$$

$$\Big\downarrow \text{219}$$

$$-\frac{-\frac{3\sqrt{2}\sqrt{a}(A-B)(c-d)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f}-\frac{2a(3Ad+3Bc-2Bd)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{3a}-\frac{2Bd\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3af}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/Sqrt[a + a*Sin[e + f*x]],x
]
```

output
```
(-2*B*d*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*a*f) + ((-3*Sqrt[2]*Sqrt
[a]*(A - B)*(c - d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin
[e + f*x]])])/f - (2*a*(3*B*c + 3*A*d - 2*B*d)*Cos[e + f*x])/(f*Sqrt[a + a
*Sin[e + f*x]]))/(3*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(113) = 226$.

Time = 0.73 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.78

method	result
default	$-\frac{(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(3A\sqrt{2}\,a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)c-3A\sqrt{2}\,a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)d-3B\cdots\right.}{3a\cdots}$
parts	$-\frac{Ac(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\,\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}\,\sqrt{2}}{2\sqrt{a}}\right)}{\sqrt{a}\,\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}+\frac{(Ad+Bc)(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\,\left(\cdots\right.}{a\cos(fx\cdots}$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/3*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(3*A*2^(1/2)*a^(3/2)*arctanh
(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c-3*A*2^(1/2)*a^(3/2)*arctanh
(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*d-3*B*2^(1/2)*a^(3/2)*arctanh
(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c+3*B*2^(1/2)*a^(3/2)*arctanh
(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*d-2*B*(a-a*sin(f*x+e))^(3/2)*
d+6*A*a*d*(a-a*sin(f*x+e))^(1/2)+6*B*a*c*(a-a*sin(f*x+e))^(1/2))/a^2/cos(f
*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(113) = 226$.

Time = 0.10 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.33

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx$$

$$= \frac{\frac{3\sqrt{2}((A-B)ac-(A-B)ad+((A-B)ac-(A-B)ad)\cos(fx+e)+((A-B)ac-(A-B)ad)\sin(fx+e))\log\left(-\frac{\cos(fx+e)^2-(\cos(fx+e)-2)\sin(fx+e)-}{\cos(fx+e)^2-}\right.}{\sqrt{a}}}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algo
rithm="fricas")
```

output
```
1/6*(3*sqrt(2)*((A - B)*a*c - (A - B)*a*d + ((A - B)*a*c - (A - B)*a*d)*co
s(f*x + e) + ((A - B)*a*c - (A - B)*a*d)*sin(f*x + e))*log(-(cos(f*x + e)^
2 - (cos(f*x + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(
cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x +
e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) - 4*(B
*d*cos(f*x + e)^2 + 3*B*c + (3*A - 2*B)*d + (3*B*c + (3*A - B)*d)*cos(f*x
+ e) + (B*d*cos(f*x + e) - 3*B*c - (3*A - 2*B)*d)*sin(f*x + e))*sqrt(a*sin
(f*x + e) + a))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

Sympy [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(A+B\sin(e+fx))\,(c+d\sin(e+fx))}{\sqrt{a\,(\sin(e+fx)+1)}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))**(1/2),x)
```

output
```
Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))/sqrt(a*(sin(e + f*x) +
1)), x)
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)}{\sqrt{a\sin(fx+e)+a}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)/sqrt(a*sin(f*x + e) +
a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(c+d\,\sin(e+f\,x))}{\sqrt{a+a\,\sin(e+f\,x)}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^(1/2)
,x)
```

output
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^(1/2)
, x)
```

Reduce [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{\sqrt{a+a\sin(e+fx)}}\,dx$$
$$=\frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}}{\sin(fx+e)+1}dx\right)ac+\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^2}{\sin(fx+e)+1}dx\right)bd+\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)+1}dx\right)ad+\left(\int \frac{\sqrt{\mathrm{si}}}{}\right.\right.}{a}$$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x)
```

output
```
(sqrt(a)*(int(sqrt(sin(e + f*x) + 1)/(sin(e + f*x) + 1),x)*a*c + int((sqrt
(sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x) + 1),x)*b*d + int((sqrt(
sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x) + 1),x)*a*d + int((sqrt(sin(
e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x) + 1),x)*b*c))/a
```

3.310 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 79

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{\sqrt{2}(A-B)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} - \frac{2B\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}}$$

output
```
-2^(1/2)*(A-B)*arctanh(1/2*a^(1/2)*cos(f*x+e)*2^(1/2)/(a+a*sin(f*x+e))^(1/
2))/a^(1/2)/f-2*B*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.34

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$$
$$= \frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left((1+i)(-1)^{3/4}(A-B)\text{arctanh}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e-\right.\right.\right.\right.}{f\sqrt{a(1+\sin(e+fx))}}$$

input
```
Integrate[(A + B*Sin[e + f*x])/Sqrt[a + a*Sin[e + f*x]],x]
```

output
```
(2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((1 + I)*(-1)^(3/4)*(A - B)*ArcTa
nh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] + B*(-Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])))/(f*Sqrt[a*(1 + Sin[e + f*x])])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a\sin(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a\sin(e+fx)+a}}dx$$

$$\Big\downarrow \text{3230}$$

$$(A-B)\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx-\frac{2B\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$(A-B)\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx-\frac{2B\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3128}$$

$$-\frac{2(A-B)\int \frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f}-\frac{2B\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\sqrt{2}(A-B)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{2B\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}$$

input
```
Int[(A + B*Sin[e + f*x])/Sqrt[a + a*Sin[e + f*x]],x]
```

output
```
-((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[
e + f*x]])])/(Sqrt[a]*f)) - (2*B*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]
)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.62

method	result
default	$-\frac{(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(\sqrt{a}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)A-\sqrt{a}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)B+2B\sqrt{a}\right.}{a\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$
parts	$-\frac{A(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}\,\sqrt{2}}{2\sqrt{a}}\right)}{\sqrt{a}\,\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}+\frac{B(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(\sqrt{a}\,\sqrt{2}\right.}{a\cos(fx+e)\sqrt{}}$
risch	$-\frac{\left(-2iA+iB+B\,e^{i(fx+e)}\right)\left(e^{i(fx+e)}+i\right)\sqrt{2}\,e^{-i(fx+e)}}{f\sqrt{-a\left(ie^{2i(fx+e)}-i-2\,e^{i(fx+e)}\right)e^{-i(fx+e)}}}-\frac{2i(A-B)\left(e^{i(fx+e)}+i\right)\left(a^{\frac{3}{2}}+\arctan\left(\frac{\sqrt{-ie^{i(fx+e)}a}}{\sqrt{a}}\right)a\sqrt{-ie^{i(fx+e)}a}\right)}{f\,a^{\frac{3}{2}}\sqrt{-a\left(ie^{2i(fx+e)}-i-2\,e^{i(fx+e)}\right)e^{-i(fx+e)}}}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(a^(1/2)*2^(1/2)*arctanh(1/2*(a-
a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*A-a^(1/2)*2^(1/2)*arctanh(1/2*(a-a*si
n(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*B+2*B*(a-a*sin(f*x+e))^(1/2))/a/cos(f*x+e
)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(68) = 136$.

Time = 0.09 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.66

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx =$$

$$-\frac{\frac{\sqrt{2}((A-B)a\cos(fx+e)+(A-B)a\sin(fx+e)+(A-B)a)\log\left(-\frac{\cos(fx+e)^2-(\cos(fx+e)-2)\sin(fx+e)+\frac{2\sqrt{2}\sqrt{a\sin(fx+e)+a}(\cos(fx+e)-\sin(fx+e}{\sqrt{a}}}{\cos(fx+e)^2-(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right.}{\sqrt{a}}}{2\left(af\cos\left(fx+e\right)+af\sin\left(fx\right.\right.}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(2)*((A - B)*a*cos(f*x + e) + (A - B)*a*sin(f*x + e) + (A - B)*a
)*log(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(
a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x
+ e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e)
 - 2))/sqrt(a) + 4*(B*cos(f*x + e) - B*sin(f*x + e) + B)*sqrt(a*sin(f*x +
e) + a))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

Sympy [F]

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{A+B\sin{(e+fx)}}{\sqrt{a\,(\sin{(e+fx)}+1)}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(1/2),x)
```

output
```
Integral((A + B*sin(e + f*x))/sqrt(a*(sin(e + f*x) + 1)), x)
```

Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{B\sin{(fx+e)}+A}{\sqrt{a\sin{(fx+e)}+a}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/sqrt(a*sin(f*x + e) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.91

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{A\,\mathrm{F}\left(\frac{\pi}{4}-\frac{e}{2}-\frac{f\,x}{2}\middle|1\right)\,\sqrt{\frac{2\,(a+a\,\sin(e+f\,x))}{a}}}{f\,\sqrt{a+a\,\sin\left(e+f\,x\right)}} - \frac{B\,\left(4\,\mathrm{E}\left(\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{1-\sin(e+f\,x)}}{2}\right)\middle|1\right)-2\,\mathrm{F}\left(\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{1-\sin(e+f\,x)}}{2}\right)\middle|1\right)\right)\,\sqrt{\cos\left(e+f\,x\right)^2}\,\sqrt{\frac{a+a\,\sin(e+f\,x)}{2\,a}}}{f\,\cos\left(e+f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}$$

input
```
int((A + B*sin(e + f*x))/(a + a*sin(e + f*x))^(1/2),x)
```

output
```
- (A*ellipticF(pi/4 - e/2 - (f*x)/2, 1)*((2*(a + a*sin(e + f*x)))/a)^(1/2)
)/(f*(a + a*sin(e + f*x))^(1/2)) - (B*(4*ellipticE(asin((2^(1/2)*(1 - sin(
e + f*x))^(1/2))/2), 1) - 2*ellipticF(asin((2^(1/2)*(1 - sin(e + f*x))^(1/
2))/2), 1))*(cos(e + f*x)^2)^(1/2)*((a + a*sin(e + f*x))/(2*a))^(1/2))/(f*
cos(e + f*x)*(a + a*sin(e + f*x))^(1/2))
```

Reduce [F]

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}}{\sin(fx+e)+1}dx\right)a+\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)+1}dx\right)b\right)}{a}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x)
```

output
```
(sqrt(a)*(int(sqrt(sin(e + f*x) + 1)/(sin(e + f*x) + 1),x)*a + int((sqrt(s
in(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x) + 1),x)*b))/a
```

3.311 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 136

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx = -\frac{\sqrt{2}(A-B)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}(c-d)f} - \frac{2(Bc-Ad)\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}(c-d)\sqrt{d}\sqrt{c+d}f}$$

output
```
-2^(1/2)*(A-B)*arctanh(1/2*a^(1/2)*cos(f*x+e)*2^(1/2)/(a+a*sin(f*x+e))^(1/
2))/a^(1/2)/(c-d)/f-2*(-A*d+B*c)*arctanh(a^(1/2)*d^(1/2)*cos(f*x+e)/(c+d)^
(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(1/2)/(c-d)/d^(1/2)/(c+d)^(1/2)/f
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.67 (sec) , antiderivative size = 619, normalized size of antiderivative = 4.55

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx = \frac{(-1)^{3/4}\left((4+4i)(A-B)\sqrt{d}\sqrt{c+d}\text{arctan}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)+\sqrt[4]{-1}(Bc-A\right.}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*
x])),x]
```

output
```
((-1)^(3/4)*((4 + 4*I)*(A - B)*Sqrt[d]*Sqrt[c + d]*ArcTanh[(1/2 + I/2)*(-1
)^(3/4)*(-1 + Tan[(e + f*x)/4])] + (-1)^(1/4)*(B*c - A*d)*RootSum[c + 4*d*
#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(d*Log[-#1 + Tan[(e + f*x)/4]]) +
Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - c*Log[-#1 + Tan[(e + f*x
)/4]]*#1 + 2*Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 3*d*Log[
-#1 + Tan[(e + f*x)/4]]*#1^2 - Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)
/4]]*#1^2 - c*Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*
#1^3) & ] - (-1)^(1/4)*(B*c - A*d)*RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^
3 + c*#1^4 & , (-(d*Log[-#1 + Tan[(e + f*x)/4]]) - Sqrt[d]*Sqrt[c + d]*Log
[-#1 + Tan[(e + f*x)/4]] - c*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*Sqrt[d]*Sq
rt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 3*d*Log[-#1 + Tan[(e + f*x)/4]]
*#1^2 + Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - c*Log[-#1 +
 Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ])*(Cos[(e + f*
x)/2] + Sin[(e + f*x)/2]))/(2*(c - d)*Sqrt[d]*Sqrt[c + d]*f*Sqrt[a*(1 + Si
n[e + f*x])])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {3042, 3464, 3042, 3128, 219, 3252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}dx$$

$$\Big\downarrow \text{3464}$$

$$\frac{(A-B)\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{c-d}+\frac{(Bc-Ad)\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{a(c-d)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-B)\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{c-d}+\frac{(Bc-Ad)\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{a(c-d)}$$

$$\Big\downarrow \text{3128}$$

$$\frac{(Bc-Ad)\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{a(c-d)}-\frac{2(A-B)\int \frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f(c-d)}$$

$$\Big\downarrow \text{219}$$

$$\frac{(Bc-Ad)\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{a(c-d)}-\frac{\sqrt{2}(A-B)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f(c-d)}$$

$$\Big\downarrow \text{3252}$$

$$-\frac{2(Bc-Ad)\int \frac{1}{a(c+d)-\frac{a^2d\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f(c-d)}-\frac{\sqrt{2}(A-B)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f(c-d)}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\sqrt{2}(A-B)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f(c-d)}-\frac{2(Bc-Ad)\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}\sqrt{d}f(c-d)\sqrt{c+d}}$$

input
```
Int[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])),x
]
```

output
```
-((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[
e + f*x]])])/(Sqrt[a]*(c - d)*f)) - (2*(B*c - A*d)*ArcTanh[(Sqrt[a]*Sqrt[d
]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[a]*(c - d)*
Sqrt[d]*Sqrt[c + d]*f)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3464
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A
*b - a*B)/(b*c - a*d)   Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Simp[(B*c
- A*d)/(b*c - a*d)   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.46

method	result
default	$-\frac{(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}\,\sqrt{2}}{2\sqrt{a}}\right)\sqrt{(c+d)ad}\,A-2A\,\mathrm{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}\,d}{\sqrt{(c+d)ad}}\right)\sqrt{a}\right.}{(c-d)\sqrt{a}\,\sqrt{(c+d)ad}\,\cos(fx+e)\sqrt{a+a\sin}}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x,method=_RET
URNVERBOSE)
```

output
```
-(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(2^(1/2)*arctanh(1/2*(-a*(sin(f*
x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*((c+d)*a*d)^(1/2)*A-2*A*arctanh((-a*(sin(f
*x+e)-1))^(1/2)*d/((c+d)*a*d)^(1/2))*a^(1/2)*d-2^(1/2)*arctanh(1/2*(-a*(si
n(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*((c+d)*a*d)^(1/2)*B+2*B*arctanh((-a*(s
in(f*x+e)-1))^(1/2)*d/((c+d)*a*d)^(1/2))*a^(1/2)*c)/(c-d)/a^(1/2)/((c+d)*a
*d)^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(113) = 226$.

Time = 0.68 (sec) , antiderivative size = 744, normalized size of antiderivative = 5.47

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx=\text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, algo
rithm="fricas")
```

output
```
[1/2*(sqrt(a*c*d + a*d^2)*(B*c - A*d)*log((a*d^2*cos(f*x + e)^3 - a*c^2 -
2*a*c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 - 4*sqrt(a*c*d + a*d^
2)*(d*cos(f*x + e)^2 - (c + 2*d)*cos(f*x + e) + (d*cos(f*x + e) + c + 3*d)
*sin(f*x + e) - c - 3*d)*sqrt(a*sin(f*x + e) + a) - (a*c^2 + 8*a*c*d + 9*a
*d^2)*cos(f*x + e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(
3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*
d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (
d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))
) + sqrt(2)*((A - B)*a*c*d + (A - B)*a*d^2)*log(-(cos(f*x + e)^2 - (cos(f*
x + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e
) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos
(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a))/((a*c^2*d - a*d^
3)*f), -1/2*(2*sqrt(-a*c*d - a*d^2)*(B*c - A*d)*arctan(1/2*sqrt(-a*c*d - a
*d^2)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)/((a*c*d + a*d^2)
*cos(f*x + e))) - sqrt(2)*((A - B)*a*c*d + (A - B)*a*d^2)*log(-(cos(f*x +
e)^2 - (cos(f*x + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a
)*(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x
 + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a))/((
a*c^2*d - a*d^3)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e + fx)}{\sqrt{a + a\sin(e + fx)}(c + d\sin(e + fx))}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx = \int \frac{B\sin(fx+e)+A}{\sqrt{a\sin(fx+e)+a}(d\sin(fx+e)+c)}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) +
 c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx = \int \frac{A+B\,\sin\left(e+f\,x\right)}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\left(c+d\,\sin\left(e+f\,x\right)\right)}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x)))
,x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x)))
, x)
```

Reduce [F]

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}}{\sin(fx+e)^2d+\sin(fx+e)c+\sin(fx+e)d+c}dx\right)a+\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)^2d+\sin(fx+e)c+\sin(fx+e)d+c}dx\right)b\right)}{a}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x)
```

output
```
(sqrt(a)*(int(sqrt(sin(e + f*x) + 1)/(sin(e + f*x)**2*d + sin(e + f*x)*c +
 sin(e + f*x)*d + c),x)*a + int((sqrt(sin(e + f*x) + 1)*sin(e + f*x))/(sin
(e + f*x)**2*d + sin(e + f*x)*c + sin(e + f*x)*d + c),x)*b))/a
```

3.312 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 207

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2}\,dx &= -\frac{\sqrt{2}(A-B)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}(c-d)^2f} \\ &\quad+\frac{(Ad(3c+d)-B(c^2+cd+2d^2))\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}(c-d)^2\sqrt{d}(c+d)^{3/2}f} \\ &\quad-\frac{(Bc-Ad)\cos(e+fx)}{(c^2-d^2)\,f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\end{aligned}$$

output
```
-2^(1/2)*(A-B)*arctanh(1/2*a^(1/2)*cos(f*x+e)*2^(1/2)/(a+a*sin(f*x+e))^(1/
2))/a^(1/2)/(c-d)^2/f+(A*d*(3*c+d)-B*(c^2+c*d+2*d^2))*arctanh(a^(1/2)*d^(1
/2)*cos(f*x+e)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(1/2)/(c-d)^2/d^(1/2)
/(c+d)^(3/2)/f-(-A*d+B*c)*cos(f*x+e)/(c^2-d^2)/f/(a+a*sin(f*x+e))^(1/2)/(c
+d*sin(f*x+e))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 10.61 (sec) , antiderivative size = 736, normalized size of antiderivative = 3.56

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*
x])^2),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((8 + 8*I)*(-1)^(3/4)*(A - B)*ArcTa
nh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] - ((-(A*d*(3*c + d)) +
B*(c^2 + c*d + 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + RootSum[c +
4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(d*Log[-#1 + Tan[(e + f*x)/4]]
) + Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - c*Log[-#1 + Tan[(e +
 f*x)/4]]*#1 + 2*Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 3*d*
Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e +
f*x)/4]]*#1^2 - c*Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2
- c*#1^3) & ]))/(Sqrt[d]*(c + d)^(3/2)) + ((-(A*d*(3*c + d)) + B*(c^2 + c*
d + 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + RootSum[c + 4*d*#1 + 2*
c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(d*Log[-#1 + Tan[(e + f*x)/4]]) - Sqrt[d]
*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - c*Log[-#1 + Tan[(e + f*x)/4]]*#
1 - 2*Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 3*d*Log[-#1 + T
an[(e + f*x)/4]]*#1^2 + Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1
^2 - c*Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) &
 ]))/(Sqrt[d]*(c + d)^(3/2)) - (4*(c - d)*(B*c - A*d)*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]))/((c + d)*(c + d*Sin[e + f*x]))))/(4*(c - d)^2*f*Sqrt[a*
(1 + Sin[e + f*x])])
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.270$, Rules used = {3042, 3463, 27, 3042, 3464, 3042, 3128, 219, 3252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}dx$$

$$\Big\downarrow \text{3463}$$

$$-\frac{\int -\frac{a(A(2c+d)-B(c+2d))+a(Bc-Ad)\sin(e+fx)}{2\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a(A(2c+d)-B(c+2d))+a(Bc-Ad)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(A(2c+d)-B(c+2d))+a(Bc-Ad)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}$$

$$\Big\downarrow \text{3464}$$

$$\frac{\frac{2a(A-B)(c+d)\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{c-d}-\frac{\left(Ad(3c+d)-B\left(c^2+cd+2d^2\right)\right)\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}}{2a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2a(A-B)(c+d)\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{c-d}-\frac{\left(Ad(3c+d)-B\left(c^2+cd+2d^2\right)\right)\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}}{2a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}$$

$$\Big\downarrow \text{3128}$$

$$\frac{-\frac{4a(A-B)(c+d)\int \frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f(c-d)}-\frac{\left(Ad(3c+d)-B\left(c^2+cd+2d^2\right)\right)\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}}{2a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}$$

$$\Big\downarrow \text{219}$$

$$\frac{-\frac{(Ad(3c+d)-B(c^2+cd+2d^2))\int\frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}-\frac{2\sqrt{2}\sqrt{a}(A-B)(c+d)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f(c-d)}}{2a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}$$

$$\Big\downarrow \text{3252}$$

$$\frac{\frac{2a(Ad(3c+d)-B(c^2+cd+2d^2))\int\frac{1}{a(c+d)-\frac{a^2d\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f(c-d)}-\frac{2\sqrt{2}\sqrt{a}(A-B)(c+d)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f(c-d)}}{2a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{2\sqrt{a}(Ad(3c+d)-B(c^2+cd+2d^2))\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{d}f(c-d)\sqrt{c+d}}-\frac{2\sqrt{2}\sqrt{a}(A-B)(c+d)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f(c-d)}}{2a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}$$

input
```
Int[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2)
,x]
```

output
```
((-2*Sqrt[2]*Sqrt[a]*(A - B)*(c + d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[
2]*Sqrt[a + a*Sin[e + f*x]])]])/((c - d)*f) + (2*Sqrt[a]*(A*d*(3*c + d) - B
*(c^2 + c*d + 2*d^2))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*
Sqrt[a + a*Sin[e + f*x]])])/((c - d)*Sqrt[d]*Sqrt[c + d]*f))/(2*a*(c^2 - d
^2)) - ((B*c - A*d)*Cos[e + f*x])/((c^2 - d^2)*f*Sqrt[a + a*Sin[e + f*x]]*
(c + d*Sin[e + f*x]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

rule 3464
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A
*b - a*B)/(b*c - a*d)   Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Simp[(B*c
- A*d)/(b*c - a*d)   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 898 vs. $2(182) = 364$.

Time = 0.45 (sec) , antiderivative size = 899, normalized size of antiderivative = 4.34

method	result	size
default	Expression too large to display	899

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x,method=_R
ETURNVERBOSE)
```

output
```
(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)/a^(5/2)*(sin(f*x+e)*d*(3*A*arctan
h((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*c*d+A*arctanh((a-a
*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*d^2-B*arctanh((a-a*sin(f
*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*c^2-B*arctanh((a-a*sin(f*x+e))
^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*c*d-2*B*arctanh((a-a*sin(f*x+e))^(1/
2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*d^2-A*((c+d)*a*d)^(1/2)*arctanh(1/2*(a-a
*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^2*c-A*((c+d)*a*d)^(1/2)*arct
anh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^2*d+B*((c+d)*a*d
)^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^2*c+
B*((c+d)*a*d)^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*2^
(1/2)*a^2*d)+3*A*a^(5/2)*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1
/2))*c^2*d+A*a^(5/2)*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))
*c*d^2-B*a^(5/2)*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*c^3
-B*a^(5/2)*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*c^2*d-2*B
*a^(5/2)*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*c*d^2+A*a^(
3/2)*(a-a*sin(f*x+e))^(1/2)*((c+d)*a*d)^(1/2)*c*d-A*a^(3/2)*(a-a*sin(f*x+e
))^(1/2)*((c+d)*a*d)^(1/2)*d^2-A*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2
)/a^(1/2))*((c+d)*a*d)^(1/2)*2^(1/2)*a^2*c^2-A*arctanh(1/2*(a-a*sin(f*x+e)
)^(1/2)*2^(1/2)/a^(1/2))*((c+d)*a*d)^(1/2)*2^(1/2)*a^2*c*d-B*a^(3/2)*(a-a*
sin(f*x+e))^(1/2)*((c+d)*a*d)^(1/2)*c^2+B*a^(3/2)*(a-a*sin(f*x+e))^(1/2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 952 vs. $2(182) = 364$.

Time = 1.96 (sec) , antiderivative size = 2159, normalized size of antiderivative = 10.43

$$\int \frac{A + B\sin(e + fx)}{\sqrt{a + a\sin(e + fx)}(c + d\sin(e + fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x, al
gorithm="fricas")
```

output
```
[-1/4*((B*c^3 - (3*A - 2*B)*c^2*d - (4*A - 3*B)*c*d^2 - (A - 2*B)*d^3 - (B
*c^2*d - (3*A - B)*c*d^2 - (A - 2*B)*d^3)*cos(f*x + e)^2 + (B*c^3 - (3*A -
 B)*c^2*d - (A - 2*B)*c*d^2)*cos(f*x + e) + (B*c^3 - (3*A - 2*B)*c^2*d - (
4*A - 3*B)*c*d^2 - (A - 2*B)*d^3 + (B*c^2*d - (3*A - B)*c*d^2 - (A - 2*B)*
d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*c*d + a*d^2)*log((a*d^2*cos(f*x +
e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 - 4*sq
rt(a*c*d + a*d^2)*(d*cos(f*x + e)^2 - (c + 2*d)*cos(f*x + e) + (d*cos(f*x
+ e) + c + 3*d)*sin(f*x + e) - c - 3*d)*sqrt(a*sin(f*x + e) + a) - (a*c^2
+ 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*
d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x
 + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*c
os(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2
)*sin(f*x + e))) - 2*sqrt(2)*((A - B)*a*c^3*d + 3*(A - B)*a*c^2*d^2 + 3*(A
 - B)*a*c*d^3 + (A - B)*a*d^4 - ((A - B)*a*c^2*d^2 + 2*(A - B)*a*c*d^3 + (
A - B)*a*d^4)*cos(f*x + e)^2 + ((A - B)*a*c^3*d + 2*(A - B)*a*c^2*d^2 + (A
 - B)*a*c*d^3)*cos(f*x + e) + ((A - B)*a*c^3*d + 3*(A - B)*a*c^2*d^2 + 3*(
A - B)*a*c*d^3 + (A - B)*a*d^4 + ((A - B)*a*c^2*d^2 + 2*(A - B)*a*c*d^3 +
(A - B)*a*d^4)*cos(f*x + e))*sin(f*x + e))*log(-(cos(f*x + e)^2 - (cos(f*x
 + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e)
 - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (c...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e + fx)}{\sqrt{a + a\sin(e + fx)}(c + d\sin(e + fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2}\,dx = \int \frac{B\sin(fx+e)+A}{\sqrt{a\sin(fx+e)+a}(d\sin(fx+e)+c)^2}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) +
 c)^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2}\,dx$$
$$=\int \frac{A+B\,\sin\left(e+f\,x\right)}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^2}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^
2),x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^
2), x)
```

Reduce [F]

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2}\,dx$$
$$=\frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}}{\sin(fx+e)^3d^2+2\sin(fx+e)^2cd+\sin(fx+e)^2d^2+\sin(fx+e)c^2+2\sin(fx+e)cd+c^2}dx\right)a+\left(\int \frac{\sqrt{\sin}}{\sin(fx+e)^3d^2+2\sin(fx+e)^2cd+}\right.\right.}{a}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x)
```

output
```
(sqrt(a)*(int(sqrt(sin(e + f*x) + 1)/(sin(e + f*x)**3*d**2 + 2*sin(e + f*x
)**2*c*d + sin(e + f*x)**2*d**2 + sin(e + f*x)*c**2 + 2*sin(e + f*x)*c*d +
 c**2),x)*a + int((sqrt(sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**3*d
**2 + 2*sin(e + f*x)**2*c*d + sin(e + f*x)**2*d**2 + sin(e + f*x)*c**2 + 2
*sin(e + f*x)*c*d + c**2),x)*b))/a
```

3.313 $\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 309

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3}\,dx = -\frac{\sqrt{2}(A-B)\mathrm{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}(c-d)^3f} + \frac{(Ad(15c^2+10cd+7d^2)-B(3c^3+6c^2d+19cd^2+4d^3))\,\mathrm{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{4\sqrt{a}(c-d)^3\sqrt{d}(c+d)^{5/2}f} - \frac{(Bc-Ad)\cos(e+fx)}{2(c^2-d^2)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} + \frac{(Ad(7c+d)-B(3c^2+cd+4d^2))\cos(e+fx)}{4(c^2-d^2)^2f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}$$

output
```
-2^(1/2)*(A-B)*arctanh(1/2*a^(1/2)*cos(f*x+e)*2^(1/2)/(a+a*sin(f*x+e))^(1/
2))/a^(1/2)/(c-d)^3/f+1/4*(A*d*(15*c^2+10*c*d+7*d^2)-B*(3*c^3+6*c^2*d+19*c
*d^2+4*d^3))*arctanh(a^(1/2)*d^(1/2)*cos(f*x+e)/(c+d)^(1/2)/(a+a*sin(f*x+e
))^(1/2))/a^(1/2)/(c-d)^3/d^(1/2)/(c+d)^(5/2)/f-1/2*(-A*d+B*c)*cos(f*x+e)/
(c^2-d^2)/f/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2+1/4*(A*d*(7*c+d)-B*(
3*c^2+c*d+4*d^2))*cos(f*x+e)/(c^2-d^2)^2/f/(a+a*sin(f*x+e))^(1/2)/(c+d*sin
(f*x+e))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 14.46 (sec) , antiderivative size = 1209, normalized size of antiderivative = 3.91

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*
x])^3),x]
```

output
```
((2 + 2*I)*(A - B)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[(e + f*x)/4]*(Cos[(e
 + f*x)/4] - Sin[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/(((
-1)^(1/4)*c^3 - 3*(-1)^(1/4)*c^2*d + 3*(-1)^(1/4)*c*d^2 - (-1)^(1/4)*d^3)*
f*Sqrt[a*(1 + Sin[e + f*x])]) - ((-(A*d*(15*c^2 + 10*c*d + 7*d^2)) + B*(3*
c^3 + 6*c^2*d + 19*c*d^2 + 4*d^3))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] +
RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(d*Log[-#1 + Tan[(
e + f*x)/4]]) + Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - c*Log[-#
1 + Tan[(e + f*x)/4]]*#1 + 2*Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4
]]*#1 + 3*d*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - Sqrt[d]*Sqrt[c + d]*Log[-#1
 + Tan[(e + f*x)/4]]*#1^2 - c*Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1
 + 3*d*#1^2 - c*#1^3) & ])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/(16*(c -
 d)^3*Sqrt[d]*(c + d)^(5/2)*f*Sqrt[a*(1 + Sin[e + f*x])]) + ((-(A*d*(15*c^
2 + 10*c*d + 7*d^2)) + B*(3*c^3 + 6*c^2*d + 19*c*d^2 + 4*d^3))*(e + f*x -
2*Log[Sec[(e + f*x)/4]^2] + RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#
1^4 & , (-(d*Log[-#1 + Tan[(e + f*x)/4]]) - Sqrt[d]*Sqrt[c + d]*Log[-#1 +
Tan[(e + f*x)/4]] - c*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*Sqrt[d]*Sqrt[c +
d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 3*d*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 +
 Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - c*Log[-#1 + Tan[(e
 + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ])*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2]))/(16*(c - d)^3*Sqrt[d]*(c + d)^(5/2)*f*Sqrt[a*(1 + S...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^3}dx$$

$$\Big\downarrow \text{ 3463}$$

$$-\frac{\int -\frac{a(A(4c+d)-B(c+4d))+3a(Bc-Ad)\sin(e+fx)}{2\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2}dx}{2a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{a(A(4c+d)-B(c+4d))+3a(Bc-Ad)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2}dx}{4a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{a(A(4c+d)-B(c+4d))+3a(Bc-Ad)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2}dx}{4a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \text{ 3463}$$

$$\frac{\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{\int -\frac{a^2\left(8Ac^2-5Bc^2+9Adc-15Bdc+7Ad^2-4Bd^2\right)-a^2\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)}{2\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{a(c^2-d^2)}}{4a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int -\frac{\left(B\left(5c^2+15dc+4d^2\right)-A\left(8c^2+9dc+7d^2\right)\right)a^2+\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a(c^2-d^2)}+\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}}{4a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{\int -\frac{a^2\left(8Ac^2-5Bc^2+9Adc-15Bdc+7Ad^2-4Bd^2\right)-a^2\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a(c^2-d^2)}}{4a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int -\frac{\left(B\left(5c^2+15dc+4d^2\right)-A\left(8c^2+9dc+7d^2\right)\right)a^2+\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a(c^2-d^2)}+\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}}{4a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{\int -\frac{a^2\left(8Ac^2-5Bc^2+9Adc-15Bdc+7Ad^2-4Bd^2\right)-a^2\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a(c^2-d^2)}}{4a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int -\frac{\left(B\left(5c^2+15dc+4d^2\right)-A\left(8c^2+9dc+7d^2\right)\right)a^2+\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a(c^2-d^2)}+\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}}{4a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{\int -\frac{a^2\left(8Ac^2-5Bc^2+9Adc-15Bdc+7Ad^2-4Bd^2\right)-a^2\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a(c^2-d^2)}}{4a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int -\frac{\left(B\left(5c^2+15dc+4d^2\right)-A\left(8c^2+9dc+7d^2\right)\right)a^2+\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a(c^2-d^2)}+\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}}{4a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{\int -\frac{a^2\left(8Ac^2-5Bc^2+9Adc-15Bdc+7Ad^2-4Bd^2\right)-a^2\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a(c^2-d^2)}}{4a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int -\frac{\left(B\left(5c^2+15dc+4d^2\right)-A\left(8c^2+9dc+7d^2\right)\right)a^2+\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a(c^2-d^2)}+\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}}{4a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{\int -\frac{a^2\left(8Ac^2-5Bc^2+9Adc-15Bdc+7Ad^2-4Bd^2\right)-a^2\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a(c^2-d^2)}}{4a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int -\frac{\left(B\left(5c^2+15dc+4d^2\right)-A\left(8c^2+9dc+7d^2\right)\right)a^2+\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a(c^2-d^2)}+\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}}{4a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{\int -\frac{a^2\left(8Ac^2-5Bc^2+9Adc-15Bdc+7Ad^2-4Bd^2\right)-a^2\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a(c^2-d^2)}}{4a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int -\frac{\left(B\left(5c^2+15dc+4d^2\right)-A\left(8c^2+9dc+7d^2\right)\right)a^2+\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a(c^2-d^2)}+\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}}{4a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{\int -\frac{a^2\left(8Ac^2-5Bc^2+9Adc-15Bdc+7Ad^2-4Bd^2\right)-a^2\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a(c^2-d^2)}}{4a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int -\frac{\left(B\left(5c^2+15dc+4d^2\right)-A\left(8c^2+9dc+7d^2\right)\right)a^2+\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a(c^2-d^2)}+\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}}{4a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{\int -\frac{a^2\left(8Ac^2-5Bc^2+9Adc-15Bdc+7Ad^2-4Bd^2\right)-a^2\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a(c^2-d^2)}}{4a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int -\frac{\left(B\left(5c^2+15dc+4d^2\right)-A\left(8c^2+9dc+7d^2\right)\right)a^2+\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a(c^2-d^2)}+\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}}{4a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{\int -\frac{a^2\left(8Ac^2-5Bc^2+9Adc-15Bdc+7Ad^2-4Bd^2\right)-a^2\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a(c^2-d^2)}}{4a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int -\frac{\left(B\left(5c^2+15dc+4d^2\right)-A\left(8c^2+9dc+7d^2\right)\right)a^2+\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a(c^2-d^2)}+\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}}{4a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{\int -\frac{a^2\left(8Ac^2-5Bc^2+9Adc-15Bdc+7Ad^2-4Bd^2\right)-a^2\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a(c^2-d^2)}}{4a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int -\frac{\left(B\left(5c^2+15dc+4d^2\right)-A\left(8c^2+9dc+7d^2\right)\right)a^2+\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a(c^2-d^2)}+\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}}{4a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{\int -\frac{a^2\left(8Ac^2-5Bc^2+9Adc-15Bdc+7Ad^2-4Bd^2\right)-a^2\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a(c^2-d^2)}}{4a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int -\frac{\left(B\left(5c^2+15dc+4d^2\right)-A\left(8c^2+9dc+7d^2\right)\right)a^2+\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a(c^2-d^2)}+\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}}{4a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{\int -\frac{a^2\left(8Ac^2-5Bc^2+9Adc-15Bdc+7Ad^2-4Bd^2\right)-a^2\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a(c^2-d^2)}}{4a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int -\frac{\left(B\left(5c^2+15dc+4d^2\right)-A\left(8c^2+9dc+7d^2\right)\right)a^2+\left(Ad(7c+d)-B\left(3c^2+dc+4d^2\right)\right)\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a(c^2-d^2)}+\frac{a\left(Ad(7c+d)-B\left(3c^2+cd+4d^2\right)\right)\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}}{4a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)}{2f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}$$

input
```
Int[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^3)
,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2276 vs. $2(276) = 552$.

Time = 0.51 (sec) , antiderivative size = 2277, normalized size of antiderivative = 7.37

method	result	size
default	Expression too large to display	2277

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x,method=_R
ETURNVERBOSE)
```

output
```
-1/4*(8*A*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*(
(c+d)*a*d)^(1/2)*sin(f*x+e)^2*a^4*c*d^3-4*B*2^(1/2)*arctanh(1/2*(-a*(sin(f
*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*((c+d)*a*d)^(1/2)*sin(f*x+e)^2*a^4*c^2*d^
2-8*B*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*((c+d
)*a*d)^(1/2)*sin(f*x+e)^2*a^4*c*d^3+8*A*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e
)-1))^(1/2)*2^(1/2)/a^(1/2))*((c+d)*a*d)^(1/2)*sin(f*x+e)*a^4*c^3*d+16*A*2
^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*((c+d)*a*d)^
(1/2)*sin(f*x+e)*a^4*c^2*d^2+8*A*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(
1/2)*2^(1/2)/a^(1/2))*((c+d)*a*d)^(1/2)*sin(f*x+e)*a^4*c*d^3-8*B*2^(1/2)*a
rctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*((c+d)*a*d)^(1/2)*si
n(f*x+e)*a^4*c^3*d-16*B*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1
/2)/a^(1/2))*((c+d)*a*d)^(1/2)*sin(f*x+e)*a^4*c^2*d^2-8*B*2^(1/2)*arctanh(
1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*((c+d)*a*d)^(1/2)*sin(f*x+e
)*a^4*c*d^3+4*A*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1
/2))*((c+d)*a*d)^(1/2)*sin(f*x+e)^2*a^4*c^2*d^2+3*B*a^(9/2)*arctanh((-a*(s
in(f*x+e)-1))^(1/2)*d/((c+d)*a*d)^(1/2))*c^5+4*B*a^(9/2)*arctanh((-a*(sin(
f*x+e)-1))^(1/2)*d/((c+d)*a*d)^(1/2))*c^2*d^3-A*a^(7/2)*(-a*(sin(f*x+e)-1)
)^(1/2)*((c+d)*a*d)^(1/2)*d^4+5*B*a^(7/2)*(-a*(sin(f*x+e)-1))^(1/2)*((c+d)
*a*d)^(1/2)*c^4-4*B*a^(7/2)*(-a*(sin(f*x+e)-1))^(1/2)*((c+d)*a*d)^(1/2)*d^
4-A*a^(5/2)*(-a*(sin(f*x+e)-1))^(3/2)*((c+d)*a*d)^(1/2)*d^4+4*B*a^(5/2)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1963 vs. $2(276) = 552$.

Time = 4.24 (sec) , antiderivative size = 4180, normalized size of antiderivative = 13.53

$$\int \frac{A + B\sin(e + fx)}{\sqrt{a + a\sin(e + fx)}(c + d\sin(e + fx))^3}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x, al
gorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e + fx)}{\sqrt{a + a\sin(e + fx)}(c + d\sin(e + fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**3,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e + fx)}{\sqrt{a + a\sin(e + fx)}(c + d\sin(e + fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x, al
gorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A + B\sin(e + fx)}{\sqrt{a + a\sin(e + fx)}(c + d\sin(e + fx))^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3}\,dx = \int \frac{A+B\,\sin\left(e+f\,x\right)}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^3}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^
3),x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^
3), x)
```

Reduce [F]

$$\int \frac{A+B\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^3}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}}{\sin(fx+e)^4d^3+3\sin(fx+e)^3c\,d^2+\sin(fx+e)^3d^3+3\sin(fx+e)^2c^2d+3\sin(fx+e)^2c\,d^2+\sin(fx+e)c^3+3\sin(fx+e)c^2d+c^3}dx\right)a - \right.}{a}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x)
```

output
```
(sqrt(a)*(int(sqrt(sin(e + f*x) + 1)/(sin(e + f*x)**4*d**3 + 3*sin(e + f*x
)**3*c*d**2 + sin(e + f*x)**3*d**3 + 3*sin(e + f*x)**2*c**2*d + 3*sin(e +
f*x)**2*c*d**2 + sin(e + f*x)*c**3 + 3*sin(e + f*x)*c**2*d + c**3),x)*a +
int((sqrt(sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**4*d**3 + 3*sin(e
+ f*x)**3*c*d**2 + sin(e + f*x)**3*d**3 + 3*sin(e + f*x)**2*c**2*d + 3*sin
(e + f*x)**2*c*d**2 + sin(e + f*x)*c**3 + 3*sin(e + f*x)*c**2*d + c**3),x)
*b))/a
```

3.314 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 283

$$\begin{aligned}\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{3/2}}\,dx = &-\frac{(c-d)^2(3B(c-5d)+A(c+11d))\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f}\\&+\frac{d(15Ac^2-99Bc^2-120Acd+168Bcd+65Ad^2-93Bd^2)\cos(e+fx)}{15af\sqrt{a+a\sin(e+fx)}}\\&+\frac{d^2(15Ac-51Bc-35Ad+39Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{30a^2f}\\&+\frac{(5A-9B)d\cos(e+fx)(c+d\sin(e+fx))^2}{10af\sqrt{a+a\sin(e+fx)}}\\&-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a+a\sin(e+fx))^{3/2}}\end{aligned}$$

output
```
-1/4*(c-d)^2*(3*B*(c-5*d)+A*(c+11*d))*arctanh(1/2*a^(1/2)*cos(f*x+e)*2^(1/
2)/(a+a*sin(f*x+e))^(1/2))*2^(1/2)/a^(3/2)/f+1/15*d*(15*A*c^2-120*A*c*d+65
*A*d^2-99*B*c^2+168*B*c*d-93*B*d^2)*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(1/2)+
1/30*d^2*(15*A*c-35*A*d-51*B*c+39*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/a
^2/f+1/10*(5*A-9*B)*d*cos(f*x+e)*(c+d*sin(f*x+e))^2/a/f/(a+a*sin(f*x+e))^(
1/2)-1/2*(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))^3/f/(a+a*sin(f*x+e))^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.14 (sec) , antiderivative size = 684, normalized size of antiderivative = 2.42

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x
])^(3/2),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-30*A*c^3*Cos[(e + f*x)/2] + 30*B*
c^3*Cos[(e + f*x)/2] + 90*A*c^2*d*Cos[(e + f*x)/2] - 270*B*c^2*d*Cos[(e +
f*x)/2] - 270*A*c*d^2*Cos[(e + f*x)/2] + 330*B*c*d^2*Cos[(e + f*x)/2] + 11
0*A*d^3*Cos[(e + f*x)/2] - 165*B*d^3*Cos[(e + f*x)/2] - 180*B*c^2*d*Cos[(3
*(e + f*x))/2] - 180*A*c*d^2*Cos[(3*(e + f*x))/2] + 210*B*c*d^2*Cos[(3*(e
+ f*x))/2] + 70*A*d^3*Cos[(3*(e + f*x))/2] - 123*B*d^3*Cos[(3*(e + f*x))/2
] + 30*B*c*d^2*Cos[(5*(e + f*x))/2] + 10*A*d^3*Cos[(5*(e + f*x))/2] - 9*B*
d^3*Cos[(5*(e + f*x))/2] + 3*B*d^3*Cos[(7*(e + f*x))/2] + 30*A*c^3*Sin[(e
+ f*x)/2] - 30*B*c^3*Sin[(e + f*x)/2] - 90*A*c^2*d*Sin[(e + f*x)/2] + 270*
B*c^2*d*Sin[(e + f*x)/2] + 270*A*c*d^2*Sin[(e + f*x)/2] - 330*B*c*d^2*Sin[
(e + f*x)/2] - 110*A*d^3*Sin[(e + f*x)/2] + 165*B*d^3*Sin[(e + f*x)/2] + (
30 + 30*I)*(-1)^(3/4)*(c - d)^2*(3*B*(c - 5*d) + A*(c + 11*d))*ArcTanh[(1/
2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e +
f*x)/2])^2 - 180*B*c^2*d*Sin[(3*(e + f*x))/2] - 180*A*c*d^2*Sin[(3*(e + f*
x))/2] + 210*B*c*d^2*Sin[(3*(e + f*x))/2] + 70*A*d^3*Sin[(3*(e + f*x))/2]
- 123*B*d^3*Sin[(3*(e + f*x))/2] - 30*B*c*d^2*Sin[(5*(e + f*x))/2] - 10*A*
d^3*Sin[(5*(e + f*x))/2] + 9*B*d^3*Sin[(5*(e + f*x))/2] + 3*B*d^3*Sin[(7*(
e + f*x))/2]))/(60*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

Rubi [A] (verified)

Time = 1.88 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.06, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.432$, Rules used = {3042, 3456, 27, 3042, 3462, 27, 3042, 3447, 3042, 3502, 27, 3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a\sin(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a\sin(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{ 3456}$$

$$\frac{\int \frac{(c+d\sin(e+fx))^2(a(3B(c-2d)+A(c+6d))-a(5A-9B)d\sin(e+fx))}{2\sqrt{\sin(e+fx)a+a}}dx}{2a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{(c+d\sin(e+fx))^2(a(3B(c-2d)+A(c+6d))-a(5A-9B)d\sin(e+fx))}{\sqrt{\sin(e+fx)a+a}}dx}{4a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{(c+d\sin(e+fx))^2(a(3B(c-2d)+A(c+6d))-a(5A-9B)d\sin(e+fx))}{\sqrt{\sin(e+fx)a+a}}dx}{4a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 3462}$$

$$\frac{\frac{2\int \frac{(c+d\sin(e+fx))\left(a^2((5A-9B)(c-4d)d+5c(3B(c-2d)+A(c+6d)))-a^2d(15Ac-51Bc-35Ad+39Bd)\sin(e+fx)\right)}{2\sqrt{\sin(e+fx)a+a}}dx}{5a}+\frac{2ad(5A-9B)\cos(e+fx)(c+d\sin(e}{5f\sqrt{a\sin(e+fx)+a}}}{4a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{(c+d\sin(e+fx))\left(a^2((5A-9B)(c-4d)d+5c(3B(c-2d)+A(c+6d)))-a^2d(15Ac-51Bc-35Ad+39Bd)\sin(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a}+\frac{2ad(5A-9B)\cos(e+fx)(c+d\sin(e+\cdots}{5f\sqrt{a\sin(e+fx)+a}}\cdots}{4a^2}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a\sin(e+fx)+a)^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{(c+d\sin(e+fx))\left(a^2((5A-9B)(c-4d)d+5c(3B(c-2d)+A(c+6d)))-a^2d(15Ac-51Bc-35Ad+39Bd)\sin(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}}dx}{5a}+\frac{2ad(5A-9B)\cos(e+fx)(c+d\sin(e+\cdots}{5f\sqrt{a\sin(e+fx)+a}}\cdots}{4a^2}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a\sin(e+fx)+a)^{3/2}}$$

$\downarrow$ 3447

$$\frac{\frac{\int \frac{-d^2(15Ac-51Bc-35Ad+39Bd)\sin^2(e+fx)a^2+c((5A-9B)(c-4d)d+5c(3B(c-2d)+A(c+6d)))a^2+\left(a^2d((5A-9B)(c-4d)d+5c(3B(c-2d)+A(c+6d)))-a^2cd(15\cdots\right.}{\sqrt{\sin(e+fx)a+a}}\cdots}{5a}\cdots}{4a^2}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a\sin(e+fx)+a)^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{-d^2(15Ac-51Bc-35Ad+39Bd)\sin(e+fx)^2a^2+c((5A-9B)(c-4d)d+5c(3B(c-2d)+A(c+6d)))a^2+\left(a^2d((5A-9B)(c-4d)d+5c(3B(c-2d)+A(c+6d)))-a^2cd(15\cdots\right.}{\sqrt{\sin(e+fx)a+a}}\cdots}{5a}\cdots}{4a^2}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a\sin(e+fx)+a)^{3/2}}$$

$\downarrow$ 3502

$$\frac{\frac{\frac{2\int \frac{a^3\left(3B\left(15c^3-39dc^2+53d^2c-13d^3\right)+5A\left(3c^3+21dc^2-15d^2c+7d^3\right)\right)-2a^3d\left(15Ac^2-99Bc^2-120Adc+168Bdc+65Ad^2-93Bd^2\right)\sin(e+fx)}{2\sqrt{\sin(e+fx)a+a}}dx}{3a}+\frac{2ad^2(15Ac-3\cdots}{\cdots}}{5a}\cdots}{4a^2}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a\sin(e+fx)+a)^{3/2}}$$

$\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{a^3\left(3B\left(15c^3-39dc^2+53d^2c-13d^3\right)+5A\left(3c^3+21dc^2-15d^2c+7d^3\right)\right)-2a^3d\left(15Ac^2-99Bc^2-120Adc+168Bdc+65Ad^2-93Bd^2\right)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}}dx}{3a}+\frac{2ad^2(15Ac-35\cdots}{\cdots}}{5a}\cdots}{4a^2}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a\sin(e+fx)+a)^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{a^3\left(3B\left(15c^3-39dc^2+53d^2c-13d^3\right)+5A\left(3c^3+21dc^2-15d^2c+7d^3\right)\right)-2a^3d\left(15Ac^2-99Bc^2-120Adc+168Bdc+65Ad^2-93Bd^2\right)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}}dx}{3a}+\frac{2ad^2(15Ac-35\cdots}{\cdots}}{5a}\cdots}{4a^2}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a\sin(e+fx)+a)^{3/2}}$$

$\downarrow$ 3230

$$\frac{\frac{\frac{15a^3(c-d)^2(A(c+11d)+3B(c-5d))\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx+\frac{4a^3d\left(15Ac^2-120Acd+65Ad^2-99Bc^2+168Bcd-93Bd^2\right)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{3a}+\frac{2ad^2(15Ac-35Ad-51Bc+39B\cdots}{3f\cdots}}{5a}\cdots}{4a^2}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a\sin(e+fx)+a)^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{15a^3(c-d)^2(A(c+11d)+3B(c-5d))\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx+\frac{4a^3d\left(15Ac^2-120Acd+65Ad^2-99Bc^2+168Bcd-93Bd^2\right)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{3a}+\frac{2ad^2(15Ac-35Ad-51Bc+39B\cdots}{3f\cdots}}{5a}\cdots}{4a^2}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a\sin(e+fx)+a)^{3/2}}$$

$\downarrow$ 3128

$$\frac{\frac{\frac{4a^3d\left(15Ac^2-120Acd+65Ad^2-99Bc^2+168Bcd-93Bd^2\right)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}-\frac{30a^3(c-d)^2(A(c+11d)+3B(c-5d))\int \frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f}}{3a}+\frac{2ad^2(15Ac\cdots}{\cdots}}{5a}\cdots}{4a^2}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a\sin(e+fx)+a)^{3/2}}$$

$\downarrow$ 219

$$\frac{\frac{\frac{\frac{4a^3d\left(15Ac^2-120Acd+65Ad^2-99Bc^2+168Bcd-93Bd^2\right)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}-\frac{15\sqrt{2}a^{5/2}(c-d)^2(A(c+11d)+3B(c-5d))\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f}}{3a}+\frac{2ad^2(15Ac-}{}}{5a}}{4a^2}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{2f(a\sin(e+fx)+a)^{3/2}}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^(3/
2),x]
```

output
```
-1/2*((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(f*(a + a*Sin[e + f*x])
^(3/2)) + ((2*a*(5*A - 9*B)*d*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(5*f*Sq
rt[a + a*Sin[e + f*x]]) + ((2*a*d^2*(15*A*c - 51*B*c - 35*A*d + 39*B*d)*Co
s[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*f) + ((-15*Sqrt[2]*a^(5/2)*(c - d)
^2*(3*B*(c - 5*d) + A*(c + 11*d))*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*
Sqrt[a + a*Sin[e + f*x]])])/f + (4*a^3*d*(15*A*c^2 - 99*B*c^2 - 120*A*c*d
+ 168*B*c*d + 65*A*d^2 - 93*B*d^2)*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x
]]))/(3*a))/(5*a))/(4*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3462
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Simp[1/(b*(m + n + 1))   Int[(a + b*Sin[e + f*x])^m*(c + d*S
in[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
 + n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 816 vs. 2(256) = 512.

Time = 2.32 (sec) , antiderivative size = 817, normalized size of antiderivative = 2.89

method	result	size
parts	Expression too large to display	817
default	Expression too large to display	1031

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(3/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/4*A*c^3/a^(7/2)*(2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(
1/2))*a^2*sin(f*x+e)+2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^
(1/2))*a^2+2*(a-a*sin(f*x+e))^(1/2)*a^(3/2))*(-a*(sin(f*x+e)-1))^(1/2)/cos
(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f+1/20*B*d^3/a^(9/2)*(-8*(a-a*sin(f*x+e))^(
5/2)*a^(1/2)*sin(f*x+e)+75*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1
/2)/a^(1/2))*sin(f*x+e)*a^3-8*(a-a*sin(f*x+e))^(5/2)*a^(1/2)+75*2^(1/2)*ar
ctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3-80*a^(5/2)*(a-a*sin(
f*x+e))^(1/2)*sin(f*x+e)-90*a^(5/2)*(a-a*sin(f*x+e))^(1/2))*(-a*(sin(f*x+e
)-1))^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f-1/4*c^2*(3*A*d+B*c)/a^(5/2
)*(3*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*sin(f*x
+e)+3*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a-2*(a-a
*sin(f*x+e))^(1/2)*a^(1/2))*(-a*(sin(f*x+e)-1))^(1/2)/cos(f*x+e)/(a+a*sin(
f*x+e))^(1/2)/f-1/12*d^2*(A*d+3*B*c)/a^(7/2)*(33*2^(1/2)*arctanh(1/2*(a-a*
sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*sin(f*x+e)-8*(a-a*sin(f*x+e))^(3/2)
*a^(1/2)*sin(f*x+e)+33*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/
a^(1/2))*a^2-8*(a-a*sin(f*x+e))^(3/2)*a^(1/2)-24*a^(3/2)*(a-a*sin(f*x+e))^
(1/2)*sin(f*x+e)-30*(a-a*sin(f*x+e))^(1/2)*a^(3/2))*(-a*(sin(f*x+e)-1))^(1
/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f+3/4*c*d*(A*d+B*c)/a^(5/2)*(7*2^(1/
2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*sin(f*x+e)+7*2^(1
/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a-8*(a-a*sin(f*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 784 vs. $2(256) = 512$.

Time = 0.12 (sec) , antiderivative size = 784, normalized size of antiderivative = 2.77

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(3/2),x, al
gorithm="fricas")
```

output
```
1/120*(15*sqrt(2)*(2*(A + 3*B)*c^3 + 6*(3*A - 7*B)*c^2*d - 6*(7*A - 11*B)*
c*d^2 + 2*(11*A - 15*B)*d^3 - ((A + 3*B)*c^3 + 3*(3*A - 7*B)*c^2*d - 3*(7*
A - 11*B)*c*d^2 + (11*A - 15*B)*d^3)*cos(f*x + e)^2 + ((A + 3*B)*c^3 + 3*(
3*A - 7*B)*c^2*d - 3*(7*A - 11*B)*c*d^2 + (11*A - 15*B)*d^3)*cos(f*x + e)
+ (2*(A + 3*B)*c^3 + 6*(3*A - 7*B)*c^2*d - 6*(7*A - 11*B)*c*d^2 + 2*(11*A
- 15*B)*d^3 + ((A + 3*B)*c^3 + 3*(3*A - 7*B)*c^2*d - 3*(7*A - 11*B)*c*d^2
+ (11*A - 15*B)*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x +
 e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x
 + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)
/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) -
4*(12*B*d^3*cos(f*x + e)^4 - 15*(A - B)*c^3 + 45*(A - B)*c^2*d - 45*(A - B
)*c*d^2 + 15*(A - B)*d^3 + 4*(15*B*c*d^2 + (5*A - 3*B)*d^3)*cos(f*x + e)^3
 - 4*(45*B*c^2*d + 15*(3*A - 4*B)*c*d^2 - 4*(5*A - 9*B)*d^3)*cos(f*x + e)^
2 - 15*((A - B)*c^3 - 3*(A - 5*B)*c^2*d + 15*(A - B)*c*d^2 - (5*A - 9*B)*d
^3)*cos(f*x + e) + (12*B*d^3*cos(f*x + e)^3 + 15*(A - B)*c^3 - 45*(A - B)*
c^2*d + 45*(A - B)*c*d^2 - 15*(A - B)*d^3 - 4*(15*B*c*d^2 + (5*A - 6*B)*d^
3)*cos(f*x + e)^2 - 60*(3*B*c^2*d + 3*(A - B)*c*d^2 - (A - 2*B)*d^3)*cos(f
*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a^2*f*cos(f*x + e)^2 - a
^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^3}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(3/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^3/(a*sin(f*x + e) + a)
^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(3/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{3/2}}\,dx=\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^3}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^(3/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^(3/
2), x)
```

Reduce [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{3/2}}\,dx=\frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}}{\sin(fx+e)^2+2\sin(fx+e)+1}dx\right)a\,c^3+\left(\int \frac{\sqrt{\sin(fx+e)+1}\ \mathrm{s}}{\sin(fx+e)^2+2\sin}\right.\right.}{}$$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(3/2),x)
```

output
```
(sqrt(a)*(int(sqrt(sin(e + f*x) + 1)/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1
),x)*a*c**3 + int((sqrt(sin(e + f*x) + 1)*sin(e + f*x)**4)/(sin(e + f*x)**
2 + 2*sin(e + f*x) + 1),x)*b*d**3 + int((sqrt(sin(e + f*x) + 1)*sin(e + f*
x)**3)/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),x)*a*d**3 + 3*int((sqrt(sin(
e + f*x) + 1)*sin(e + f*x)**3)/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),x)*b
*c*d**2 + 3*int((sqrt(sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x)**2
+ 2*sin(e + f*x) + 1),x)*a*c*d**2 + 3*int((sqrt(sin(e + f*x) + 1)*sin(e +
f*x)**2)/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),x)*b*c**2*d + 3*int((sqrt(
sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),x)*
a*c**2*d + int((sqrt(sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**2 + 2*
sin(e + f*x) + 1),x)*b*c**3))/a**2
```

3.315 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 203

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(c-d)(Ac+3Bc+7Ad-11Bd)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} + \frac{d(3Ac-15Bc-9Ad+13Bd)\cos(e+fx)}{3af\sqrt{a+a\sin(e+fx)}} + \frac{(3A-7B)d^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{6a^2f} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a+a\sin(e+fx))^{3/2}}$$

output
```
-1/4*(c-d)*(A*c+7*A*d+3*B*c-11*B*d)*arctanh(1/2*a^(1/2)*cos(f*x+e)*2^(1/2)
/(a+a*sin(f*x+e))^(1/2))*2^(1/2)/a^(3/2)/f+1/3*d*(3*A*c-9*A*d-15*B*c+13*B*
d)*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(1/2)+1/6*(3*A-7*B)*d^2*cos(f*x+e)*(a+a
*sin(f*x+e))^(1/2)/a^2/f-1/2*(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))^2/f/(a+a*si
n(f*x+e))^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.36 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.76

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(6(A-B)(c-d)^2 \mathrm{s}\right.}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x
])^(3/2),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(6*(A - B)*(c - d)^2*Sin[(e + f*x)/
2] - 3*(A - B)*(c - d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (3 + 3*I)
*(-1)^(3/4)*(c - d)*(A*c + 3*B*c + 7*A*d - 11*B*d)*ArcTanh[(1/2 + I/2)*(-1
)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 +
 6*d*(-4*B*c - 2*A*d + 3*B*d)*Cos[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])^2 - 2*B*d^2*Cos[(3*(e + f*x))/2]*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^2 - 6*d*(-4*B*c - 2*A*d + 3*B*d)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])^2 - 2*B*d^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*
Sin[(3*(e + f*x))/2]))/(6*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.351$, Rules used = {3042, 3456, 27, 3042, 3447, 3042, 3502, 27, 3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a\sin(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a\sin(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3456}$$

$$\frac{\int \frac{(c+d\sin(e+fx))(a(Ac+3Bc+4Ad-4Bd)-a(3A-7B)d\sin(e+fx))}{2\sqrt{\sin(e+fx)a+a}}dx}{2a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(c+d\sin(e+fx))(a(Ac+3Bc+4Ad-4Bd)-a(3A-7B)d\sin(e+fx))}{\sqrt{\sin(e+fx)a+a}}dx}{4a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(c+d\sin(e+fx))(a(Ac+3Bc+4Ad-4Bd)-a(3A-7B)d\sin(e+fx))}{\sqrt{\sin(e+fx)a+a}}dx}{4a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{3447}$$

$$\frac{\int \frac{-a(3A-7B)d^2\sin^2(e+fx)+(ad(Ac+3Bc+4Ad-4Bd)-a(3A-7B)cd)\sin(e+fx)+ac(Ac+3Bc+4Ad-4Bd)}{\sqrt{\sin(e+fx)a+a}}dx}{4a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{-a(3A-7B)d^2\sin(e+fx)^2+(ad(Ac+3Bc+4Ad-4Bd)-a(3A-7B)cd)\sin(e+fx)+ac(Ac+3Bc+4Ad-4Bd)}{\sqrt{\sin(e+fx)a+a}}dx}{4a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\frac{2\int -\frac{\left((3A-7B)d^2-3c(Ac+3Bc+4Ad-4Bd)\right)a^2+2d(3Ac-15Bc-9Ad+13Bd)\sin(e+fx)a^2}{2\sqrt{\sin(e+fx)a+a}}dx}{3a}+\frac{2d^2(3A-7B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{4a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2d^2(3A-7B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}-\frac{\int \frac{\left((3A-7B)d^2-3c(Ac+3Bc+4Ad-4Bd)\right)a^2+2d(3Ac-15Bc-9Ad+13Bd)\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}}dx}{3a}}{4a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a\sin(e+fx)+a)^{3/2}}$$

$\big\downarrow$ 3042

$$\frac{\frac{2d^2(3A-7B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}-\frac{\int \frac{\left((3A-7B)d^2-3c(Ac+3Bc+4Ad-4Bd)\right)a^2+2d(3Ac-15Bc-9Ad+13Bd)\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}}dx}{3a}}{4a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a\sin(e+fx)+a)^{3/2}}$$

$\big\downarrow$ 3230

$$\frac{\frac{2d^2(3A-7B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}-\frac{-3a^2(c-d)(Ac+7Ad+3Bc-11Bd)\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx-\frac{4a^2d(3Ac-9Ad-15Bc+13Bd)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{3a}}{4a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a\sin(e+fx)+a)^{3/2}}$$

$\big\downarrow$ 3042

$$\frac{\frac{2d^2(3A-7B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}-\frac{-3a^2(c-d)(Ac+7Ad+3Bc-11Bd)\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx-\frac{4a^2d(3Ac-9Ad-15Bc+13Bd)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{3a}}{4a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a\sin(e+fx)+a)^{3/2}}$$

$\big\downarrow$ 3128

$$\frac{\frac{2d^2(3A-7B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}-\frac{\frac{6a^2(c-d)(Ac+7Ad+3Bc-11Bd)\int \frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f}-\frac{4a^2d(3Ac-9Ad-15Bc+13Bd)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{3a}}{4a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a\sin(e+fx)+a)^{3/2}}$$

$\big\downarrow$ 219

$$\frac{\frac{2d^2(3A-7B)\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}-\frac{\frac{3\sqrt{2}a^{3/2}(c-d)(Ac+7Ad+3Bc-11Bd)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f}-\frac{4a^2d(3Ac-9Ad-15Bc+13Bd)\cos}{f\sqrt{a\sin(e+fx)+a}}}{3a}}{4a^2}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{2f(a\sin(e+fx)+a)^{3/2}}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])^(3/
2),x]
```

output
```
-1/2*((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(f*(a + a*Sin[e + f*x])
^(3/2)) + ((2*(3*A - 7*B)*d^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*f)
 - ((3*Sqrt[2]*a^(3/2)*(c - d)*(A*c + 3*B*c + 7*A*d - 11*B*d)*ArcTanh[(Sqr
t[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/f - (4*a^2*d*(3*A*
c - 15*B*c - 9*A*d + 13*B*d)*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]))/(
3*a))/(4*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 611 vs. $2(180) = 360$.

Time = 0.98 (sec) , antiderivative size = 612, normalized size of antiderivative = 3.01

method	result
parts	$-\frac{A c^2 \left(\sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right) a^2 \sin(fx+e)+\sqrt{2}\, \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right) a^2+2\sqrt{a-a\sin(fx+e)}\, a^{\frac{3}{2}}\right)\sqrt{-a(\sin(}{4a^{\frac{7}{2}}\cos(fx+e)\sqrt{a+a\sin(fx+e)}\, f}$
default	$-\left(\sin(fx+e)\left(24A\, a^{\frac{3}{2}} d^2 \sqrt{a-a\sin(fx+e)}+3A\, \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)\sqrt{2}\, a^2 c^2+18A\, \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)\sqrt{2}\, a^2\right.\right.$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(3/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/4*A*c^2/a^(7/2)*(2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(
1/2))*a^2*sin(f*x+e)+2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^
(1/2))*a^2+2*(a-a*sin(f*x+e))^(1/2)*a^(3/2))*(-a*(sin(f*x+e)-1))^(1/2)/cos
(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f+1/12*B*d^2/a^(7/2)*(8*(a-a*sin(f*x+e))^(3
/2)*a^(1/2)*sin(f*x+e)-33*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/
2)/a^(1/2))*a^2*sin(f*x+e)+8*(a-a*sin(f*x+e))^(3/2)*a^(1/2)+24*a^(3/2)*(a-
a*sin(f*x+e))^(1/2)*sin(f*x+e)-33*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/
2)*2^(1/2)/a^(1/2))*a^2+30*(a-a*sin(f*x+e))^(1/2)*a^(3/2))*(-a*(sin(f*x+e)
-1))^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f+1/4*c*(2*A*d+B*c)/a^(5/2)*(
-3*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*sin(f*x+e
)-3*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a+2*(a-a*s
in(f*x+e))^(1/2)*a^(1/2))*(-a*(sin(f*x+e)-1))^(1/2)/cos(f*x+e)/(a+a*sin(f*
x+e))^(1/2)/f-1/4*d*(A*d+2*B*c)/a^(5/2)*(-7*2^(1/2)*arctanh(1/2*(a-a*sin(f
*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*sin(f*x+e)+8*(a-a*sin(f*x+e))^(1/2)*a^(1/2
)*sin(f*x+e)-7*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))
*a+10*(a-a*sin(f*x+e))^(1/2)*a^(1/2))*(-a*(sin(f*x+e)-1))^(1/2)/cos(f*x+e)
/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 584 vs. 2(180) = 360.

Time = 0.11 (sec) , antiderivative size = 584, normalized size of antiderivative = 2.88

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(3/2),x, al
gorithm="fricas")
```

output
```
-1/24*(3*sqrt(2)*(2*(A + 3*B)*c^2 + 4*(3*A - 7*B)*c*d - 2*(7*A - 11*B)*d^2
 - ((A + 3*B)*c^2 + 2*(3*A - 7*B)*c*d - (7*A - 11*B)*d^2)*cos(f*x + e)^2 +
 ((A + 3*B)*c^2 + 2*(3*A - 7*B)*c*d - (7*A - 11*B)*d^2)*cos(f*x + e) + (2*
(A + 3*B)*c^2 + 4*(3*A - 7*B)*c*d - 2*(7*A - 11*B)*d^2 + ((A + 3*B)*c^2 +
2*(3*A - 7*B)*c*d - (7*A - 11*B)*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*
log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f
*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*si
n(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(
f*x + e) - 2)) + 4*(4*B*d^2*cos(f*x + e)^3 - 3*(A - B)*c^2 + 6*(A - B)*c*d
 - 3*(A - B)*d^2 - 4*(6*B*c*d + (3*A - 4*B)*d^2)*cos(f*x + e)^2 - 3*((A -
B)*c^2 - 2*(A - 5*B)*c*d + 5*(A - B)*d^2)*cos(f*x + e) - (4*B*d^2*cos(f*x
+ e)^2 - 3*(A - B)*c^2 + 6*(A - B)*c*d - 3*(A - B)*d^2 + 12*(2*B*c*d + (A
- B)*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a^2*f*cos
(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f
)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\sin{(e+fx)})\,(c+d\sin{(e+fx)})^2}{(a\,(\sin{(e+fx)}+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**(3/2),x)
```

output
```
Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))**2/(a*(sin(e + f*x) + 1
))**(3/2), x)
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin{(fx+e)}+A)(d\sin{(fx+e)}+c)^2}{(a\sin{(fx+e)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(3/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^2/(a*sin(f*x + e) + a)
^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(3/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{3/2}}\,dx=\int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(c+d\,\sin\left(e+f\,x\right)\right)^2}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^(3/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^(3/
2), x)
```

Reduce [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{3/2}}\,dx=\frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}}{\sin(fx+e)^2+2\sin(fx+e)+1}dx\right)a\,c^2+\left(\int \frac{\sqrt{\sin(fx+e)+1}\;\mathrm{s}}{\sin(fx+e)^2+2\sin}\right.\right.}{}$$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(3/2),x)
```

output
```
(sqrt(a)*(int(sqrt(sin(e + f*x) + 1)/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1
),x)*a*c**2 + int((sqrt(sin(e + f*x) + 1)*sin(e + f*x)**3)/(sin(e + f*x)**
2 + 2*sin(e + f*x) + 1),x)*b*d**2 + int((sqrt(sin(e + f*x) + 1)*sin(e + f*
x)**2)/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),x)*a*d**2 + 2*int((sqrt(sin(
e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),x)*b
*c*d + 2*int((sqrt(sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**2 + 2*si
n(e + f*x) + 1),x)*a*c*d + int((sqrt(sin(e + f*x) + 1)*sin(e + f*x))/(sin(
e + f*x)**2 + 2*sin(e + f*x) + 1),x)*b*c**2))/a**2
```

3.316 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 133

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(Ac+3Bc+3Ad-7Bd)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} - \frac{(A-B)(c-d)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} - \frac{2Bd\cos(e+fx)}{af\sqrt{a+a\sin(e+fx)}}$$

output
```
-1/4*(A*c+3*A*d+3*B*c-7*B*d)*arctanh(1/2*a^(1/2)*cos(f*x+e)*2^(1/2)/(a+a*s
in(f*x+e))^(1/2))*2^(1/2)/a^(3/2)/f-1/2*(A-B)*(c-d)*cos(f*x+e)/f/(a+a*sin(
f*x+e))^(3/2)-2*B*d*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.48 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.85

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2(A-B)(c-d)\sin\right.}{}$$

input

```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])
^(3/2),x]
```

output

```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*(c - d)*Sin[(e + f*x)/2]
 - (A - B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (1 + I)*(-1)^(3
/4)*(A*c + 3*B*c + 3*A*d - 7*B*d)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan
[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 4*B*d*Cos[(e + f
*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 4*B*d*Sin[(e + f*x)/2]*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])^2))/(2*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3447, 3042, 3498, 27, 3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a\sin(e+fx)+a)^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a\sin(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3447}$$

$$\int \frac{(Ad+Bc)\sin(e+fx)+Ac+Bd\sin^2(e+fx)}{(a\sin(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(Ad+Bc)\sin(e+fx)+Ac+Bd\sin(e+fx)^2}{(a\sin(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3498}$$

$$-\frac{\int -\frac{a(3B(c-d)+A(c+3d))+4aBd\sin(e+fx)}{2\sqrt{\sin(e+fx)a+a}}dx}{2a^2}-\frac{(A-B)(c-d)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a(3B(c-d)+A(c+3d))+4aBd\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}}dx}{4a^2}-\frac{(A-B)(c-d)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(3B(c-d)+A(c+3d))+4aBd\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}}dx}{4a^2}-\frac{(A-B)(c-d)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{3230}$$

$$\frac{a(Ac+3Ad+3Bc-7Bd)\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx-\frac{8aBd\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{4a^2}-\frac{(A-B)(c-d)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(Ac+3Ad+3Bc-7Bd)\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx-\frac{8aBd\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{4a^2}-\frac{(A-B)(c-d)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{3128}$$

$$\frac{-\frac{2a(Ac+3Ad+3Bc-7Bd)\int \frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f}-\frac{8aBd\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{4a^2}-\frac{(A-B)(c-d)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{-\frac{\sqrt{2}\sqrt{a}(Ac+3Ad+3Bc-7Bd)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f}-\frac{8aBd\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{4a^2}-\frac{(A-B)(c-d)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^(3/2)
,x]
```

output
```
-1/2*((A - B)*(c - d)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(3/2)) + (-((S
qrt[2]*Sqrt[a]*(A*c + 3*B*c + 3*A*d - 7*B*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x]
)/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/f) - (8*a*B*d*Cos[e + f*x])/(f*Sqrt
[a + a*Sin[e + f*x]]))/(4*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3498
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[1
/(a^2*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b
*B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(116) = 232$.

Time = 0.83 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.92

method	result
default	$-\frac{\left(\sin(fx+e)\left(A\ \mathrm{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)\sqrt{2}\,ac+3A\ \mathrm{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)\sqrt{2}\,ad+3B\ \mathrm{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right.\right.\right.}{}$
parts	$-\frac{Ac\left(\sqrt{2}\ \mathrm{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^2\sin(fx+e)+\sqrt{2}\ \mathrm{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^2+2\sqrt{a-a\sin(fx+e)}\,a^{\frac{3}{2}}\right)\sqrt{-a(\sin(f}{4a^{\frac{7}{2}}\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/4/a^(5/2)*(sin(f*x+e)*(A*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(
1/2))*2^(1/2)*a*c+3*A*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*
2^(1/2)*a*d+3*B*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2
)*a*c-7*B*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a*d+
8*B*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*d)+A*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)
*2^(1/2)/a^(1/2))*2^(1/2)*a*c+3*A*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/
2)/a^(1/2))*2^(1/2)*a*d+3*B*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(
1/2))*2^(1/2)*a*c-7*B*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*
2^(1/2)*a*d+2*A*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*c-2*A*(a-a*sin(f*x+e))^(1/2
)*a^(1/2)*d-2*B*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*c+10*B*(a-a*sin(f*x+e))^(1/
2)*a^(1/2)*d)*(-a*(sin(f*x+e)-1))^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/
f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(116) = 232$.

Time = 0.11 (sec) , antiderivative size = 407, normalized size of antiderivative = 3.06

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))}{(a + a\sin(e + fx))^{3/2}}\,dx =$$

$$-\frac{\sqrt{2}(((A+3\,B)c+(3\,A-7\,B)d)\cos\left(fx+e\right)^2-2\,(A+3\,B)c-2\,(3\,A-7\,B)d-((A+3\,B)c+(3\,A-}{}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algo
rithm="fricas")
```

output
```
-1/8*(sqrt(2)*(((A + 3*B)*c + (3*A - 7*B)*d)*cos(f*x + e)^2 - 2*(A + 3*B)*
c - 2*(3*A - 7*B)*d - ((A + 3*B)*c + (3*A - 7*B)*d)*cos(f*x + e) - (2*(A +
 3*B)*c + 2*(3*A - 7*B)*d + ((A + 3*B)*c + (3*A - 7*B)*d)*cos(f*x + e))*si
n(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e)
 + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*co
s(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)
*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(4*B*d*cos(f*x + e)^2 + (A - B)*c -
 (A - B)*d + ((A - B)*c - (A - 5*B)*d)*cos(f*x + e) + (4*B*d*cos(f*x + e)
- (A - B)*c + (A - B)*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a^2*f*co
s(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*
f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx=\int \frac{(A+B\sin{(e+fx)})\,(c+d\sin{(e+fx)})}{(a\,(\sin{(e+fx)}+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2),x)
```

output
```
Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))/(a*(sin(e + f*x) + 1))*
*(3/2), x)
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx=\int \frac{(B\sin{(fx+e)}+A)(d\sin{(fx+e)}+c)}{(a\sin{(fx+e)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^(
3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,\,(c+d\,\sin(e+f\,x))}{(a+a\,\sin(e+f\,x))^{3/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^(3/2)
,x)
```

output
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^(3/2)
, x)
```

Reduce [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}}{\sin(fx+e)^2+2\sin(fx+e)+1}dx\right)ac+\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sin(}{\sin(fx+e)^2+2\sin(f}\right.\right.}{}$$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x)
```

output
```
(sqrt(a)*(int(sqrt(sin(e + f*x) + 1)/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1
),x)*a*c + int((sqrt(sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x)**2 +
 2*sin(e + f*x) + 1),x)*b*d + int((sqrt(sin(e + f*x) + 1)*sin(e + f*x))/(s
in(e + f*x)**2 + 2*sin(e + f*x) + 1),x)*a*d + int((sqrt(sin(e + f*x) + 1)*
sin(e + f*x))/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),x)*b*c))/a**2
```

3.317 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 87

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{(A+3B)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} - \frac{(A-B)\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}}$$

output
```
-1/4*(A+3*B)*arctanh(1/2*a^(1/2)*cos(f*x+e)*2^(1/2)/(a+a*sin(f*x+e))^(1/2)
)*2^(1/2)/a^(3/2)/f-1/2*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.72

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(2(A-B)\sin\left(\frac{1}{2}(e+fx)\right)+(-A+B\right.}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^(3/2),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*Sin[(e + f*x)/2] + (-A +
 B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (1 + I)*(-1)^(3/4)*(A + 3*B)*A
rcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^2))/(2*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3229, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3229}$$

$$\frac{(A+3B)\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{4a}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A+3B)\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{4a}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{3128}$$

$$-\frac{(A+3B)\int \frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{2af}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{(A+3B)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f}-\frac{(A-B)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

input
```
Int[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^(3/2),x]
```

output
```
-1/2*((A + 3*B)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e +
 f*x]])])/(Sqrt[2]*a^(3/2)*f) - ((A - B)*Cos[e + f*x])/(2*f*(a + a*Sin[e +
 f*x])^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(72) = 144$.

Time = 0.47 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.02

method	result
default	$-\frac{\left(\sin(fx+e)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a(A+3B)+A\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)\sqrt{2}\,a+3B\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{}}{2\sqrt{a}}\right.\right.}{4a^{\frac{5}{2}}\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$
parts	$-\frac{A\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^2\sin(fx+e)+\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^2+2\sqrt{a-a\sin(fx+e)}\,a^{\frac{3}{2}}\right)\sqrt{-a(\sin(fx}}{4a^{\frac{7}{2}}\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/a^(5/2)*(sin(f*x+e)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2
)/a^(1/2))*a*(A+3*B)+A*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))
*2^(1/2)*a+3*B*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)
*a+2*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*A-2*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*B)*
(-a*(sin(f*x+e)-1))^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(72) = 144$.

Time = 0.09 (sec) , antiderivative size = 293, normalized size of antiderivative = 3.37

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\sqrt{2}\left((A+3\,B)\cos\left(fx+e\right)^2-(A+3\,B)\cos\left(fx+e\right)-((A+3\,B)\cos\left(fx\right.\right.}{}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
1/8*(sqrt(2)*((A + 3*B)*cos(f*x + e)^2 - (A + 3*B)*cos(f*x + e) - ((A + 3*
B)*cos(f*x + e) + 2*A + 6*B)*sin(f*x + e) - 2*A - 6*B)*sqrt(a)*log(-(a*cos
(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - s
in(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e)
+ 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) -
2)) + 4*((A - B)*cos(f*x + e) - (A - B)*sin(f*x + e) + A - B)*sqrt(a*sin(f
*x + e) + a))/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*
f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{A+B\sin{(e+fx)}}{(a(\sin{(e+fx)}+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2),x)
```

output
```
Integral((A + B*sin(e + f*x))/(a*(sin(e + f*x) + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{B\sin{(fx+e)}+A}{(a\sin{(fx+e)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/(a*sin(f*x + e) + a)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx=\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

input
```
int((A + B*sin(e + f*x))/(a + a*sin(e + f*x))^(3/2),x)
```

output
```
int((A + B*sin(e + f*x))/(a + a*sin(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx=\frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}}{\sin(fx+e)^2+2\sin(fx+e)+1}dx\right)a+\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)^2+2\sin(fx+e)+1}dx\right)b\right)}{a^2}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x)
```

output
```
(sqrt(a)*(int(sqrt(sin(e + f*x) + 1)/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1
),x)*a + int((sqrt(sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**2 + 2*si
n(e + f*x) + 1),x)*b))/a**2
```

3.318 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 187

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx = -\frac{(A(c-5d)+B(3c+d))\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}(c-d)^2f} + \frac{2\sqrt{d}(Bc-Ad)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{a^{3/2}(c-d)^2\sqrt{c+d}f} - \frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}}$$

output
```
-1/4*(A*(c-5*d)+B*(3*c+d))*arctanh(1/2*a^(1/2)*cos(f*x+e)*2^(1/2)/(a+a*sin
(f*x+e))^(1/2))*2^(1/2)/a^(3/2)/(c-d)^2/f+2*d^(1/2)*(-A*d+B*c)*arctanh(a^(
1/2)*d^(1/2)*cos(f*x+e)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(3/2)/(c-d)^
2/(c+d)^(1/2)/f-1/2*(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^(3/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 6.85 (sec) , antiderivative size = 781, normalized size of antiderivative = 4.18

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e +
f*x])),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*(c - d)*Sin[(e + f*x)/2]
 + (-A + B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (1 + I)*(-1)^(
3/4)*(A*(c - 5*d) + B*(3*c + d))*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[
(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (Sqrt[d]*(B*c - A
*d)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + RootSum[c + 4*d*#1 + 2*c*#1^2 -
 4*d*#1^3 + c*#1^4 & , (-(d*Log[-#1 + Tan[(e + f*x)/4]]) + Sqrt[d]*Sqrt[c
+ d]*Log[-#1 + Tan[(e + f*x)/4]] - c*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 2*Sq
rt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 3*d*Log[-#1 + Tan[(e +
f*x)/4]]*#1^2 - Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - c*L
og[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ])*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/Sqrt[c + d] + (Sqrt[d]*(-(B*c) + A*d)
*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*
d*#1^3 + c*#1^4 & , (-(d*Log[-#1 + Tan[(e + f*x)/4]]) - Sqrt[d]*Sqrt[c + d
]*Log[-#1 + Tan[(e + f*x)/4]] - c*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*Sqrt[
d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 3*d*Log[-#1 + Tan[(e + f*x
)/4]]*#1^2 + Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - c*Log[
-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ])*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^2)/Sqrt[c + d]))/(2*(c - d)^2*f*(a*(1 + Sin
[e + f*x]))^(3/2))
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.270$, Rules used = {3042, 3457, 27, 3042, 3464, 3042, 3128, 219, 3252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))}dx$$

$$\Big\downarrow \text{3457}$$

$$-\frac{\int -\frac{a(3Bc+A(c-4d))+a(A-B)d\sin(e+fx)}{2\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a(3Bc+A(c-4d))+a(A-B)d\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{4a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(3Bc+A(c-4d))+a(A-B)d\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{4a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{3464}$$

$$\frac{\frac{a(A(c-5d)+B(3c+d))\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{c-d}-\frac{4d(Bc-Ad)\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}}{4a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{a(A(c-5d)+B(3c+d))\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{c-d}-\frac{4d(Bc-Ad)\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}}{4a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{3128}$$

$$\frac{-\frac{2a(A(c-5d)+B(3c+d))\int \frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f(c-d)}-\frac{4d(Bc-Ad)\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}}{4a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{-\frac{4d(Bc-Ad)\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}-\frac{\sqrt{2}\sqrt{a}(A(c-5d)+B(3c+d))\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f(c-d)}}{4a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{3252}$$

$$\frac{\frac{8ad(Bc-Ad)\int \frac{1}{a(c+d)-\frac{a^2d\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f(c-d)}-\frac{\sqrt{2}\sqrt{a}(A(c-5d)+B(3c+d))\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f(c-d)}}{4a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{8\sqrt{a}\sqrt{d}(Bc-Ad)\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{f(c-d)\sqrt{c+d}}-\frac{\sqrt{2}\sqrt{a}(A(c-5d)+B(3c+d))\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f(c-d)}}{4a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x]))
,x]
```

output
```
(-((Sqrt[2]*Sqrt[a]*(A*(c - 5*d) + B*(3*c + d))*ArcTanh[(Sqrt[a]*Cos[e + f
*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/((c - d)*f)) + (8*Sqrt[a]*Sqrt[d
]*(B*c - A*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a +
 a*Sin[e + f*x]])])/((c - d)*Sqrt[c + d]*f))/(4*a^2*(c - d)) - ((A - B)*Co
s[e + f*x])/(2*(c - d)*f*(a + a*Sin[e + f*x])^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3464
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A
*b - a*B)/(b*c - a*d)   Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Simp[(B*c
- A*d)/(b*c - a*d)   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 623 vs. $2(158) = 316$.

Time = 0.45 (sec) , antiderivative size = 624, normalized size of antiderivative = 3.34

method	result
default	$-\frac{\left(\sin(fx+e)\left(8A\,a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,d}{\sqrt{cda+d^2a}}\right)d^2-8B\,a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,d}{\sqrt{cda+d^2a}}\right)cd+A\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)\sqrt{}\right.\right.}{}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x,method=_RET
URNVERBOSE)
```

output
```
-1/4/a^(5/2)*(sin(f*x+e)*(8*A*a^(3/2)*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*
c*d+a*d^2)^(1/2))*d^2-8*B*a^(3/2)*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+
a*d^2)^(1/2))*c*d+A*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*((
c+d)*a*d)^(1/2)*2^(1/2)*a*c-5*A*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)
/a^(1/2))*((c+d)*a*d)^(1/2)*2^(1/2)*a*d+3*B*arctanh(1/2*(a-a*sin(f*x+e))^(
1/2)*2^(1/2)/a^(1/2))*((c+d)*a*d)^(1/2)*2^(1/2)*a*c+B*arctanh(1/2*(a-a*sin
(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*((c+d)*a*d)^(1/2)*2^(1/2)*a*d)+8*A*a^(3/2)
*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*d^2-8*B*a^(3/2)*arc
tanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*c*d+A*arctanh(1/2*(a-a*
sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*((c+d)*a*d)^(1/2)*2^(1/2)*a*c-5*A*arcta
nh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*((c+d)*a*d)^(1/2)*2^(1/2)*a
*d+3*B*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*((c+d)*a*d)^(1/
2)*2^(1/2)*a*c+B*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*((c+d
)*a*d)^(1/2)*2^(1/2)*a*d+2*A*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*((c+d)*a*d)^(1
/2)*c-2*A*(a-a*sin(f*x+e))^(1/2)*a^(1/2)*((c+d)*a*d)^(1/2)*d-2*B*(a-a*sin(
f*x+e))^(1/2)*a^(1/2)*((c+d)*a*d)^(1/2)*c+2*B*(a-a*sin(f*x+e))^(1/2)*a^(1/
2)*((c+d)*a*d)^(1/2)*d)*(-a*(sin(f*x+e)-1))^(1/2)/((c+d)*a*d)^(1/2)/(c-d)^
2/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 638 vs. $2(158) = 316$.

Time = 1.67 (sec) , antiderivative size = 1561, normalized size of antiderivative = 8.35

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^{3/2}(c + d\sin(e + fx))}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algo
rithm="fricas")
```

output
```
[1/8*(sqrt(2)*(((A + 3*B)*c - (5*A - B)*d)*cos(f*x + e)^2 - 2*(A + 3*B)*c
+ 2*(5*A - B)*d - ((A + 3*B)*c - (5*A - B)*d)*cos(f*x + e) - (2*(A + 3*B)*
c - 2*(5*A - B)*d + ((A + 3*B)*c - (5*A - B)*d)*cos(f*x + e))*sin(f*x + e)
)*sqrt(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt
(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e)
 - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x +
 e) - cos(f*x + e) - 2)) + 4*(2*B*a*c - 2*A*a*d - (B*a*c - A*a*d)*cos(f*x
+ e)^2 + (B*a*c - A*a*d)*cos(f*x + e) + (2*B*a*c - 2*A*a*d + (B*a*c - A*a*
d)*cos(f*x + e))*sin(f*x + e))*sqrt(d/(a*c + a*d))*log((d^2*cos(f*x + e)^3
 - (6*c*d + 7*d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - 4*((c*d + d^2)*cos
(f*x + e)^2 - c^2 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (
c^2 + 4*c*d + 3*d^2 + (c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f
*x + e) + a)*sqrt(d/(a*c + a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e) + (d
^2*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*si
n(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c
*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x
+ e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*((A - B)*c - (A - B)*d + ((A
- B)*c - (A - B)*d)*cos(f*x + e) - ((A - B)*c - (A - B)*d)*sin(f*x + e))*s
qrt(a*sin(f*x + e) + a))/((a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f*cos(f*x + e)^2
 - (a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f*cos(f*x + e) - 2*(a^2*c^2 - 2*a^2*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^{3/2}(c + d\sin(e + fx))}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx=\int \frac{B\sin\left(fx+e\right)+A}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e)
 + c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx=\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,\left(c+d\,\sin\left(e+f\,x\right)\right)}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x)))
,x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x)))
, x)
```

Reduce [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}}{\sin(fx+e)^3 d+\sin(fx+e)^2 c+2\sin(fx+e)^2 d+2\sin(fx+e) c+\sin(f}\right.\right.}{}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x)
```

output
```
(sqrt(a)*(int(sqrt(sin(e + f*x) + 1)/(sin(e + f*x)**3*d + sin(e + f*x)**2*
c + 2*sin(e + f*x)**2*d + 2*sin(e + f*x)*c + sin(e + f*x)*d + c),x)*a + in
t((sqrt(sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**3*d + sin(e + f*x)*
*2*c + 2*sin(e + f*x)**2*d + 2*sin(e + f*x)*c + sin(e + f*x)*d + c),x)*b))
/a**2
```

3.319 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 292

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2}\,dx = -\frac{(Ac+3Bc-9Ad+5Bd)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}(c-d)^3f} - \frac{\sqrt{d}(Ad(5c+3d)-B(3c^2+3cd+2d^2))\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{a^{3/2}(c-d)^3(c+d)^{3/2}f} - \frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))} + \frac{d(B(3c+d)-A(c+3d))\cos(e+fx)}{2a(c-d)^2(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}$$

output
```
-1/4*(A*c-9*A*d+3*B*c+5*B*d)*arctanh(1/2*a^(1/2)*cos(f*x+e)*2^(1/2)/(a+a*s
in(f*x+e))^(1/2))*2^(1/2)/a^(3/2)/(c-d)^3/f-d^(1/2)*(A*d*(5*c+3*d)-B*(3*c^
2+3*c*d+2*d^2))*arctanh(a^(1/2)*d^(1/2)*cos(f*x+e)/(c+d)^(1/2)/(a+a*sin(f*
x+e))^(1/2))/a^(3/2)/(c-d)^3/(c+d)^(3/2)/f-1/2*(A-B)*cos(f*x+e)/(c-d)/f/(a
+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))+1/2*d*(B*(3*c+d)-A*(c+3*d))*cos(f*x+
e)/a/(c-d)^2/(c+d)/f/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 15.95 (sec) , antiderivative size = 904, normalized size of antiderivative = 3.10

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e +
f*x])^2),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(4*(A - B)*(c - d)*Sin[(e + f*x)/2]
 + 2*(-A + B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (2 + 2*I)*(-
1)^(3/4)*(A*c + 3*B*c - 9*A*d + 5*B*d)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1
+ Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (Sqrt[d]*(-
(A*d*(5*c + 3*d)) + B*(3*c^2 + 3*c*d + 2*d^2))*(e + f*x - 2*Log[Sec[(e + f
*x)/4]^2] + RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(d*Log
[-#1 + Tan[(e + f*x)/4]]) + Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]
] - c*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 2*Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan
[(e + f*x)/4]]*#1 + 3*d*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - Sqrt[d]*Sqrt[c
+ d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - c*Log[-#1 + Tan[(e + f*x)/4]]*#1^3
)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])^2)/(c + d)^(3/2) + (Sqrt[d]*(A*d*(5*c + 3*d) - B*(3*c^2 + 3*c*d + 2*d^2
))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + RootSum[c + 4*d*#1 + 2*c*#1^2 -
4*d*#1^3 + c*#1^4 & , (-(d*Log[-#1 + Tan[(e + f*x)/4]]) - Sqrt[d]*Sqrt[c +
 d]*Log[-#1 + Tan[(e + f*x)/4]] - c*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*Sqr
t[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 3*d*Log[-#1 + Tan[(e + f
*x)/4]]*#1^2 + Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - c*Lo
g[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ])*(Cos[
(e + f*x)/2] + Sin[(e + f*x)/2])^2)/(c + d)^(3/2) + (4*(c - d)*d*(B*c - A*
d)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f...
```

Rubi [A] (verified)

Time = 1.73 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.09, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.351$, Rules used = {3042, 3457, 27, 3042, 3463, 25, 3042, 3464, 3042, 3128, 219, 3252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}dx$$

$$\Big\downarrow \text{3457}$$

$$-\frac{\int -\frac{a(Ac+3Bc-6Ad+2Bd)+3a(A-B)d\sin(e+fx)}{2\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2}dx}{2a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a(Ac+3Bc-6Ad+2Bd)+3a(A-B)d\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2}dx}{4a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(Ac+3Bc-6Ad+2Bd)+3a(A-B)d\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2}dx}{4a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))}$$

$$\Big\downarrow \text{3463}$$

$$\frac{\frac{2ad(B(3c+d)-A(c+3d))\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}-\frac{\int -\frac{a^2\left(A\left(c^2-7dc-6d^2\right)+B\left(3c^2+5dc+4d^2\right)\right)-a^2d(B(3c+d)-A(c+3d))\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{a(c^2-d^2)}}{4a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{a^2\left(A\left(c^2-7dc-6d^2\right)+B\left(3c^2+5dc+4d^2\right)\right)-a^2d(B(3c+d)-A(c+3d))\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{a(c^2-d^2)}+\frac{2ad(B(3c+d)-A(c+3d))\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}}{4a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{a^2\left(A\left(c^2-7dc-6d^2\right)+B\left(3c^2+5dc+4d^2\right)\right)-a^2d(B(3c+d)-A(c+3d))\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{a(c^2-d^2)}+\frac{2ad(B(3c+d)-A(c+3d))\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))}}{4a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))}$$

$\Big\downarrow$ 3464

$$\frac{\frac{\frac{a^2(c+d)(Ac-9Ad+3Bc+5Bd)\int\frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{c-d}+\frac{2ad\left(Ad(5c+3d)-B\left(3c^2+3cd+2d^2\right)\right)\int\frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}}{a(c^2-d^2)}+\frac{2ad(B(3c+d)-A(c+3d))\cos(e+}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e}}{4a^2(c-d)}$$
$$\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{a^2(c+d)(Ac-9Ad+3Bc+5Bd)\int\frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{c-d}+\frac{2ad\left(Ad(5c+3d)-B\left(3c^2+3cd+2d^2\right)\right)\int\frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}}{a(c^2-d^2)}+\frac{2ad(B(3c+d)-A(c+3d))\cos(e+}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e}}{4a^2(c-d)}$$
$$\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))}$$

$\Big\downarrow$ 3128

$$\frac{\frac{\frac{2ad\left(Ad(5c+3d)-B\left(3c^2+3cd+2d^2\right)\right)\int\frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}-\frac{2a^2(c+d)(Ac-9Ad+3Bc+5Bd)\int\frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f(c-d)}}{a(c^2-d^2)}+\frac{2ad(B(3c+d)-}{f(c^2-d^2)\sqrt{a\sin(e-}}}{4a^2(c-d)}$$
$$\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))}$$

$\Big\downarrow$ 219

$$\frac{\frac{\frac{2ad\left(Ad(5c+3d)-B\left(3c^2+3cd+2d^2\right)\right)\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}-\frac{\sqrt{2}a^{3/2}(c+d)(Ac-9Ad+3Bc+5Bd)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f(c-d)}}{a(c^2-d^2)}+\frac{2ad(B(3c+d)-A}{f(c^2-d^2)\sqrt{a\sin(e+}}}{4a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))}$$

$\Big\downarrow$ 3252

$$\frac{\frac{-\frac{4a^2d\left(Ad(5c+3d)-B\left(3c^2+3cd+2d^2\right)\right)\int \frac{1}{a(c+d)-\frac{a^2d\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f(c-d)}-\frac{\sqrt{2}a^{3/2}(c+d)(Ac-9Ad+3Bc+5Bd)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f(c-d)}}{a(c^2-d^2)}}{4a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))}$$

$\Big\downarrow$ 221

$$\frac{\frac{-\frac{4a^{3/2}\sqrt{d}\left(Ad(5c+3d)-B\left(3c^2+3cd+2d^2\right)\right)\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{f(c-d)\sqrt{c+d}}-\frac{\sqrt{2}a^{3/2}(c+d)(Ac-9Ad+3Bc+5Bd)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f(c-d)}}{a(c^2-d^2)}+}{4a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^
2),x]
```

output
```
-1/2*((A - B)*Cos[e + f*x])/((c - d)*f*(a + a*Sin[e + f*x])^(3/2)*(c + d*S
in[e + f*x])) + ((-((Sqrt[2]*a^(3/2)*(c + d)*(A*c + 3*B*c - 9*A*d + 5*B*d)
*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/((c -
 d)*f)) - (4*a^(3/2)*Sqrt[d]*(A*d*(5*c + 3*d) - B*(3*c^2 + 3*c*d + 2*d^2))
*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*
x]])])/((c - d)*Sqrt[c + d]*f))/(a*(c^2 - d^2)) + (2*a*d*(B*(3*c + d) - A*
(c + 3*d))*Cos[e + f*x])/((c^2 - d^2)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Si
n[e + f*x])))/(4*a^2*(c - d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

rule 3464
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A
*b - a*B)/(b*c - a*d)   Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Simp[(B*c
- A*d)/(b*c - a*d)   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2048 vs. $2(259) = 518$.

Time = 0.52 (sec) , antiderivative size = 2049, normalized size of antiderivative = 7.02

method	result	size
default	Expression too large to display	2049

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x,method=_R
ETURNVERBOSE)
```

output
```
-1/4/a^(5/2)*(2*A*(-a*(sin(f*x+e)-1))^(1/2)*a^(1/2)*((c+d)*a*d)^(1/2)*c*d^
2-4*B*(-a*(sin(f*x+e)-1))^(1/2)*a^(1/2)*((c+d)*a*d)^(1/2)*c^2*d+6*B*(-a*(s
in(f*x+e)-1))^(1/2)*a^(1/2)*((c+d)*a*d)^(1/2)*c*d^2+20*A*a^(3/2)*arctanh((
-a*(sin(f*x+e)-1))^(1/2)*d/((c+d)*a*d)^(1/2))*sin(f*x+e)^2*c*d^3-12*B*a^(3
/2)*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/((c+d)*a*d)^(1/2))*sin(f*x+e)^2*c^
2*d^2-12*B*a^(3/2)*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/((c+d)*a*d)^(1/2))*
sin(f*x+e)^2*c*d^3+A*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)
/a^(1/2))*((c+d)*a*d)^(1/2)*a*c^3-6*A*(-a*(sin(f*x+e)-1))^(1/2)*a^(1/2)*((
c+d)*a*d)^(1/2)*sin(f*x+e)*d^3+20*A*a^(3/2)*arctanh((-a*(sin(f*x+e)-1))^(1
/2)*d/((c+d)*a*d)^(1/2))*sin(f*x+e)*c^2*d^2+32*A*a^(3/2)*arctanh((-a*(sin(
f*x+e)-1))^(1/2)*d/((c+d)*a*d)^(1/2))*sin(f*x+e)*c*d^3-12*B*a^(3/2)*arctan
h((-a*(sin(f*x+e)-1))^(1/2)*d/((c+d)*a*d)^(1/2))*sin(f*x+e)*c^3*d-24*B*a^(
3/2)*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/((c+d)*a*d)^(1/2))*sin(f*x+e)*c^2
*d^2-20*B*a^(3/2)*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/((c+d)*a*d)^(1/2))*s
in(f*x+e)*c*d^3+3*B*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/
a^(1/2))*((c+d)*a*d)^(1/2)*a*c^3+2*B*(-a*(sin(f*x+e)-1))^(1/2)*a^(1/2)*((c
+d)*a*d)^(1/2)*sin(f*x+e)*d^3+8*B*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^
(1/2)*2^(1/2)/a^(1/2))*((c+d)*a*d)^(1/2)*sin(f*x+e)^2*a*c*d^2-7*A*2^(1/2)*
arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*((c+d)*a*d)^(1/2)*s
in(f*x+e)*a*c^2*d+13*B*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1559 vs. $2(259) = 518$.

Time = 4.01 (sec) , antiderivative size = 3403, normalized size of antiderivative = 11.65

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^{3/2}(c + d\sin(e + fx))^2}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x, al
gorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**2,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x, al
gorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2}\,dx=\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^2}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^
2),x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^
2), x)
```

Reduce [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2}\,dx=\frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sin(\cdots}}{\sin(fx+e)^4d^2+2\sin(fx+e)^3cd+2\sin(fx+e)^3d^2+\sin(fx+e)^2c^2\cdots}\right.\right.}{\cdots}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x)
```

output
```
(sqrt(a)*(int(sqrt(sin(e + f*x) + 1)/(sin(e + f*x)**4*d**2 + 2*sin(e + f*x
)**3*c*d + 2*sin(e + f*x)**3*d**2 + sin(e + f*x)**2*c**2 + 4*sin(e + f*x)*
*2*c*d + sin(e + f*x)**2*d**2 + 2*sin(e + f*x)*c**2 + 2*sin(e + f*x)*c*d +
 c**2),x)*a + int((sqrt(sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**4*d
**2 + 2*sin(e + f*x)**3*c*d + 2*sin(e + f*x)**3*d**2 + sin(e + f*x)**2*c**
2 + 4*sin(e + f*x)**2*c*d + sin(e + f*x)**2*d**2 + 2*sin(e + f*x)*c**2 + 2
*sin(e + f*x)*c*d + c**2),x)*b))/a**2
```

3.320 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 402

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3}\,dx = -\frac{(A(c-13d)+3B(c+3d))\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}(c-d)^4f} - \frac{\sqrt{d}(Ad(35c^2+42cd+19d^2)-3B(5c^3+10c^2d+13cd^2+4d^3))\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{4a^{3/2}(c-d)^4(c+d)^{5/2}f} - \frac{(A-B)\cos(e+fx)}{2(c-d)f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} + \frac{d(B(2c+d)-A(c+2d))\cos(e+fx)}{2a(c-d)^2(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} + \frac{d(3B(3c^2+3cd+2d^2)-A(2c^2+15cd+7d^2))\cos(e+fx)}{4a(c-d)^3(c+d)^2f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}$$

output
```
-1/4*(A*(c-13*d)+3*B*(c+3*d))*arctanh(1/2*a^(1/2)*cos(f*x+e)*2^(1/2)/(a+a*
sin(f*x+e))^(1/2))*2^(1/2)/a^(3/2)/(c-d)^4/f-1/4*d^(1/2)*(A*d*(35*c^2+42*c
*d+19*d^2)-3*B*(5*c^3+10*c^2*d+13*c*d^2+4*d^3))*arctanh(a^(1/2)*d^(1/2)*co
s(f*x+e)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(3/2)/(c-d)^4/(c+d)^(5/2)/f
-1/2*(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2+1/
2*d*(B*(2*c+d)-A*(c+2*d))*cos(f*x+e)/a/(c-d)^2/(c+d)/f/(a+a*sin(f*x+e))^(1
/2)/(c+d*sin(f*x+e))^2+1/4*d*(3*B*(3*c^2+3*c*d+2*d^2)-A*(2*c^2+15*c*d+7*d^
2))*cos(f*x+e)/a/(c-d)^3/(c+d)^2/f/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 17.45 (sec) , antiderivative size = 1757, normalized size of antiderivative = 4.37

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e +
f*x])^3),x]
```

output
```
((1 + I)*(A*c + 3*B*c - 13*A*d + 9*B*d)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec
[(e + f*x)/4]*(Cos[(e + f*x)/4] - Sin[(e + f*x)/4])]*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2])^3)/((2*(-1)^(1/4)*c^4 - 8*(-1)^(1/4)*c^3*d + 12*(-1)^(1/4
)*c^2*d^2 - 8*(-1)^(1/4)*c*d^3 + 2*(-1)^(1/4)*d^4)*f*(a*(1 + Sin[e + f*x])
)^(3/2)) + (Sqrt[d]*(-(A*d*(35*c^2 + 42*c*d + 19*d^2)) + 3*B*(5*c^3 + 10*c
^2*d + 13*c*d^2 + 4*d^3))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + RootSum[c
 + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(d*Log[-#1 + Tan[(e + f*x)/
4]]) + Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - c*Log[-#1 + Tan[(
e + f*x)/4]]*#1 + 2*Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 3
*d*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e
 + f*x)/4]]*#1^2 - c*Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1
^2 - c*#1^3) & ])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/(16*(c - d)^4*(
c + d)^(5/2)*f*(a*(1 + Sin[e + f*x]))^(3/2)) - (Sqrt[d]*(-(A*d*(35*c^2 + 4
2*c*d + 19*d^2)) + 3*B*(5*c^3 + 10*c^2*d + 13*c*d^2 + 4*d^3))*(e + f*x - 2
*Log[Sec[(e + f*x)/4]^2] + RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1
^4 & , (-(d*Log[-#1 + Tan[(e + f*x)/4]]) - Sqrt[d]*Sqrt[c + d]*Log[-#1 + T
an[(e + f*x)/4]] - c*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*Sqrt[d]*Sqrt[c + d
]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 3*d*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 +
Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - c*Log[-#1 + Tan[(e
+ f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ])*(Cos[(e + f*x)/2]...
```

Rubi [A] (verified)

Time = 2.50 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.10, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.432$, Rules used = {3042, 3457, 27, 3042, 3463, 27, 3042, 3463, 27, 3042, 3464, 3042, 3128, 219, 3252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^3}dx$$

$$\Big\downarrow \text{3457}$$

$$-\frac{\int -\frac{a(Ac+3Bc-8Ad+4Bd)+5a(A-B)d\sin(e+fx)}{2\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^3}dx}{2a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a(Ac+3Bc-8Ad+4Bd)+5a(A-B)d\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^3}dx}{4a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(Ac+3Bc-8Ad+4Bd)+5a(A-B)d\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^3}dx}{4a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \text{3463}$$

$$\frac{\frac{2ad(B(2c+d)-A(c+2d))\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}-\frac{\int -\frac{2\left(a^2\left(A\left(c^2-9dc-7d^2\right)+3B\left(c^2+2dc+2d^2\right)\right)-3a^2d(B(2c+d)-A(c+2d))\sin(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2}dx}{2a(c^2-d^2)}}{4a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{a^2\left(A\left(c^2-9dc-7d^2\right)+3B\left(c^2+2dc+2d^2\right)\right)-3a^2d(B(2c+d)-A(c+2d))\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2}dx}{a(c^2-d^2)}+\frac{2ad(B(2c+d)-A(c+2d))\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}}{4a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{a^2\left(A\left(c^2-9dc-7d^2\right)+3B\left(c^2+2dc+2d^2\right)\right)-3a^2d(B(2c+d)-A(c+2d))\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2}dx}{a(c^2-d^2)}+\frac{2ad(B(2c+d)-A(c+2d))\cos(e+fx)}{f(c^2-d^2)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^2}}{4a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \text{3463}$$

$$\frac{\frac{\frac{a^2 d\left(3B\left(3c^2+3cd+2d^2\right)-A\left(2c^2+15cd+7d^2\right)\right)\cos(e+fx)}{f\left(c^2-d^2\right)\sqrt{a\sin(e+fx)+a}\,(c+d\sin(e+fx))}-\frac{\int -\frac{a^3\left(A\left(2c^3-20dc^2-35d^2c-19d^3\right)+3B\left(2c^3+7dc^2+11d^2c+4d^3\right)\right)-a^3d\left(3B\left(3c^2+3dc+2d^2\right)-A\left(\cdots\right.\right.}{2\sqrt{\sin(e+fx)a+a}\,(c+d\sin(e+fx))}}{a\left(c^2-d^2\right)}}{a(c^2-d^2)}}{4a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\int \frac{\left(A\left(2c^3-20dc^2-35d^2c-19d^3\right)+3B\left(2c^3+7dc^2+11d^2c+4d^3\right)\right)a^3+d\left(2Ac^2-9Bc^2+15Adc-9Bdc+7Ad^2-6Bd^2\right)\sin(e+fx)a^3}{\sqrt{\sin(e+fx)a+a}\,(c+d\sin(e+fx))}dx}{2a\left(c^2-d^2\right)}+\frac{a^2d\left(3B\left(3c^2+3cd+2d^2\right)-A\cdots\right.}{f\left(c^2-d^2\right)\sqrt{a\sin(e+\cdots}}}{a(c^2-d^2)}}{4a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{\int \frac{\left(A\left(2c^3-20dc^2-35d^2c-19d^3\right)+3B\left(2c^3+7dc^2+11d^2c+4d^3\right)\right)a^3+d\left(2Ac^2-9Bc^2+15Adc-9Bdc+7Ad^2-6Bd^2\right)\sin(e+fx)a^3}{\sqrt{\sin(e+fx)a+a}\,(c+d\sin(e+fx))}dx}{2a\left(c^2-d^2\right)}+\frac{a^2d\left(3B\left(3c^2+3cd+2d^2\right)-A\cdots\right.}{f\left(c^2-d^2\right)\sqrt{a\sin(e+\cdots}}}{a(c^2-d^2)}}{4a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \; 3464$$

$$\frac{\frac{\frac{\frac{2a^3(c+d)^2(A(c-13d)+3B(c+3d))\int\frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{c-d}+\frac{a^2d\left(Ad\left(35c^2+42cd+19d^2\right)-3B\left(5c^3+10c^2d+13cd^2+4d^3\right)\right)\int\frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}}{2a\left(c^2-d^2\right)}+\frac{a^2d\left(3B\left(3c^2+3c\cdots\right.\right.}{f\left(c^2-d^2\cdots\right.}}{a(c^2-d^2)}}{4a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{\frac{2a^3(c+d)^2(A(c-13d)+3B(c+3d))\int\frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{c-d}+\frac{a^2d\left(Ad\left(35c^2+42cd+19d^2\right)-3B\left(5c^3+10c^2d+13cd^2+4d^3\right)\right)\int\frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}}{2a\left(c^2-d^2\right)}+\frac{a^2d\left(3B\left(3c^2+3c\cdots\right.\right.}{f\left(c^2-d^2\cdots\right.}}{a(c^2-d^2)}}{4a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}$$

$\Big\downarrow$ 3128

$$\frac{\frac{\frac{\frac{a^2 d\left(Ad\left(35c^2+42cd+19d^2\right)-3B\left(5c^3+10c^2d+13cd^2+4d^3\right)\right)\int\frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}-\frac{4a^3(c+d)^2(A(c-13d)+3B(c+3d))\int\frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f(c-d)}}{2a\left(c^2-d^2\right)}}{a(c^2-d^2)}}{4a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}$$

$\Big\downarrow$ 219

$$\frac{\frac{\frac{\frac{a^2 d\left(Ad\left(35c^2+42cd+19d^2\right)-3B\left(5c^3+10c^2d+13cd^2+4d^3\right)\right)\int\frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}-\frac{2\sqrt{2}a^{5/2}(c+d)^2(A(c-13d)+3B(c+3d))\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f(c-d)}}{2a\left(c^2-d^2\right)}}{a(c^2-d^2)}}{4a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}$$

$\Big\downarrow$ 3252

$$\frac{\frac{\frac{-\frac{2a^3 d\left(Ad\left(35c^2+42cd+19d^2\right)-3B\left(5c^3+10c^2d+13cd^2+4d^3\right)\right)\int\frac{1}{a(c+d)-\frac{a^2d\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f(c-d)}-\frac{2\sqrt{2}a^{5/2}(c+d)^2(A(c-13d)+3B(c+3d))\text{arctan}}{f(c-d)}}{2a\left(c^2-d^2\right)}}{a(c^2-d^2)}}{4a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}$$

$\Big\downarrow$ 221

$$\frac{\frac{\frac{-\frac{2a^{5/2}\sqrt{d}\left(Ad\left(35c^2+42cd+19d^2\right)-3B\left(5c^3+10c^2d+13cd^2+4d^3\right)\right)\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{f(c-d)\sqrt{c+d}}-\frac{2\sqrt{2}a^{5/2}(c+d)^2(A(c-13d)+3B(c+3d))\text{arctanh}\left(\right.}{f(c-d)}}{2a\left(c^2-d^2\right)}}{a(c^2-d^2)}}{4a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^
3),x]
```

output
```
-1/2*((A - B)*Cos[e + f*x])/((c - d)*f*(a + a*Sin[e + f*x])^(3/2)*(c + d*S
in[e + f*x])^2) + ((2*a*d*(B*(2*c + d) - A*(c + 2*d))*Cos[e + f*x])/((c^2
- d^2)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2) + (((-2*Sqrt[2]*
a^(5/2)*(c + d)^2*(A*(c - 13*d) + 3*B*(c + 3*d))*ArcTanh[(Sqrt[a]*Cos[e +
f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/((c - d)*f) - (2*a^(5/2)*Sqrt[d
]*(A*d*(35*c^2 + 42*c*d + 19*d^2) - 3*B*(5*c^3 + 10*c^2*d + 13*c*d^2 + 4*d
^3))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e
+ f*x]])])/((c - d)*Sqrt[c + d]*f))/(2*a*(c^2 - d^2)) + (a^2*d*(3*B*(3*c^2
 + 3*c*d + 2*d^2) - A*(2*c^2 + 15*c*d + 7*d^2))*Cos[e + f*x])/((c^2 - d^2)
*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])))/(a*(c^2 - d^2)))/(4*a^2
*(c - d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

rule 3464
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A
*b - a*B)/(b*c - a*d)   Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Simp[(B*c
- A*d)/(b*c - a*d)   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4706 vs. $2(363) = 726$.

Time = 0.60 (sec) , antiderivative size = 4707, normalized size of antiderivative = 11.71

method	result	size
default	Expression too large to display	4707

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x,method=_R
ETURNVERBOSE)
```

output
```
1/4/a^(7/2)*(-a*(sin(f*x+e)-1))^(1/2)*(51*A*arctanh(1/2*(-a*(sin(f*x+e)-1)
)^(1/2)*2^(1/2)/a^(1/2))*((c+d)*a*d)^(1/2)*2^(1/2)*sin(f*x+e)^2*a^2*c*d^4-
51*B*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*((c+d)*a*d)^(1
/2)*2^(1/2)*sin(f*x+e)*a^2*c^3*d^2-51*B*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1
/2)*2^(1/2)/a^(1/2))*((c+d)*a*d)^(1/2)*2^(1/2)*sin(f*x+e)*a^2*c^2*d^3-18*B
*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*((c+d)*a*d)^(1/2)*
2^(1/2)*sin(f*x+e)*a^2*c*d^4-6*B*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(
1/2)/a^(1/2))*((c+d)*a*d)^(1/2)*2^(1/2)*sin(f*x+e)^2*a^2*c^4*d-33*B*arctan
h(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*((c+d)*a*d)^(1/2)*2^(1/2)
*sin(f*x+e)^2*a^2*c^3*d^2-57*B*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/
2)/a^(1/2))*((c+d)*a*d)^(1/2)*2^(1/2)*sin(f*x+e)^2*a^2*c^2*d^3+63*A*arctan
h(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*((c+d)*a*d)^(1/2)*2^(1/2)
*sin(f*x+e)*a^2*c^2*d^3+26*A*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)
/a^(1/2))*((c+d)*a*d)^(1/2)*2^(1/2)*sin(f*x+e)*a^2*c*d^4-39*B*arctanh(1/2*
(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*((c+d)*a*d)^(1/2)*2^(1/2)*sin(f
*x+e)^2*a^2*c*d^4+9*A*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2
))*((c+d)*a*d)^(1/2)*2^(1/2)*sin(f*x+e)*a^2*c^4*d+47*A*arctanh(1/2*(-a*(si
n(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*((c+d)*a*d)^(1/2)*2^(1/2)*sin(f*x+e)*a
^2*c^3*d^2-21*B*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*((c
+d)*a*d)^(1/2)*2^(1/2)*sin(f*x+e)*a^2*c^4*d-A*arctanh(1/2*(-a*(sin(f*x+...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2789 vs. $2(362) = 724$.

Time = 8.03 (sec) , antiderivative size = 5864, normalized size of antiderivative = 14.59

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^{3/2}(c + d\sin(e + fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x, al
gorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^{3/2}(c + d\sin(e + fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**3,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^{3/2}(c + d\sin(e + fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x, al
gorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^{3/2}(c + d\sin(e + fx))^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^{3/2}(c + d\sin(e + fx))^3}\,dx = \int \frac{A + B\,\sin(e + f\,x)}{(a + a\,\sin(e + f\,x))^{3/2}\,(c + d\,\sin(e + f\,x))^3}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^
3),x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^
3), x)
```

Reduce [F]

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^{3/2}(c + d\sin(e + fx))^3}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{}{\sin(fx+e)^5 d^3+3\sin(fx+e)^4 c\,d^2+2\sin(fx+e)^4 d^3+3\sin(fx+e)^3}\right.\right.}{}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x)
```

output

```
(sqrt(a)*(int(sqrt(sin(e + f*x) + 1)/(sin(e + f*x)**5*d**3 + 3*sin(e + f*x
)**4*c*d**2 + 2*sin(e + f*x)**4*d**3 + 3*sin(e + f*x)**3*c**2*d + 6*sin(e
+ f*x)**3*c*d**2 + sin(e + f*x)**3*d**3 + sin(e + f*x)**2*c**3 + 6*sin(e +
 f*x)**2*c**2*d + 3*sin(e + f*x)**2*c*d**2 + 2*sin(e + f*x)*c**3 + 3*sin(e
 + f*x)*c**2*d + c**3),x)*a + int((sqrt(sin(e + f*x) + 1)*sin(e + f*x))/(s
in(e + f*x)**5*d**3 + 3*sin(e + f*x)**4*c*d**2 + 2*sin(e + f*x)**4*d**3 +
3*sin(e + f*x)**3*c**2*d + 6*sin(e + f*x)**3*c*d**2 + sin(e + f*x)**3*d**3
 + sin(e + f*x)**2*c**3 + 6*sin(e + f*x)**2*c**2*d + 3*sin(e + f*x)**2*c*d
**2 + 2*sin(e + f*x)*c**3 + 3*sin(e + f*x)*c**2*d + c**3),x)*b))/a**2
```

3.321 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 308

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(c-d)\left(B(5c^2+62cd-163d^2)+3A(c^2+6cd+25d^2)\right)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f} + \frac{d(A(9c^2+36cd-93d^2)+B(15c^2-228cd+197d^2))\cos(e+fx)}{24a^2f\sqrt{a+a\sin(e+fx)}} + \frac{d^2(9Ac+15Bc+39Ad-95Bd)\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{48a^3f} - \frac{(3Ac+5Bc+9Ad-17Bd)\cos(e+fx)(c+d\sin(e+fx))^2}{16af(a+a\sin(e+fx))^{3/2}} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a+a\sin(e+fx))^{5/2}}$$

output
```
-1/32*(c-d)*(B*(5*c^2+62*c*d-163*d^2)+3*A*(c^2+6*c*d+25*d^2))*arctanh(1/2*
a^(1/2)*cos(f*x+e)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))*2^(1/2)/a^(5/2)/f+1/24*
d*(A*(9*c^2+36*c*d-93*d^2)+B*(15*c^2-228*c*d+197*d^2))*cos(f*x+e)/a^2/f/(a
+a*sin(f*x+e))^(1/2)+1/48*d^2*(9*A*c+39*A*d+15*B*c-95*B*d)*cos(f*x+e)*(a+a
*sin(f*x+e))^(1/2)/a^3/f-1/16*(3*A*c+9*A*d+5*B*c-17*B*d)*cos(f*x+e)*(c+d*s
in(f*x+e))^2/a/f/(a+a*sin(f*x+e))^(3/2)-1/4*(A-B)*cos(f*x+e)*(c+d*sin(f*x+
e))^3/f/(a+a*sin(f*x+e))^(5/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.32 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.70

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(24(A-B)(c-d)^3\right.}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x
])^(5/2),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(24*(A - B)*(c - d)^3*Sin[(e + f*x)
/2] - 12*(A - B)*(c - d)^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 6*(c -
d)^2*(B*(5*c - 29*d) + 3*A*(c + 7*d))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^2 - 3*(c - d)^2*(B*(5*c - 29*d) + 3*A*(c + 7*d))*(Cos[(
e + f*x)/2] + Sin[(e + f*x)/2])^3 + (3 + 3*I)*(-1)^(3/4)*(c - d)*(B*(5*c^2
 + 62*c*d - 163*d^2) + 3*A*(c^2 + 6*c*d + 25*d^2))*ArcTanh[(1/2 + I/2)*(-1
)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 -
 16*B*d^3*Cos[(3*(e + f*x))/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 + (
24 + 24*I)*d^2*(-6*B*c - 2*A*d + 5*B*d)*(Cos[(e + f*x)/2] + I*Sin[(e + f*x
)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 + (24 + 24*I)*d^2*(6*B*c + 2
*A*d - 5*B*d)*(I*Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^4 - 16*B*d^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*Sin
[(3*(e + f*x))/2]))/(48*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

Rubi [A] (verified)

Time = 1.97 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.05, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.432$, Rules used = {3042, 3456, 27, 3042, 3456, 27, 3042, 3447, 3042, 3502, 27, 3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a\sin(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a\sin(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3456}$$

$$\frac{\int \frac{(c+d\sin(e+fx))^2(a(3Ac+5Bc+6Ad-6Bd)-a(3A-11B)d\sin(e+fx))}{2(\sin(e+fx)a+a)^{3/2}}dx}{4a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(c+d\sin(e+fx))^2(a(3Ac+5Bc+6Ad-6Bd)-a(3A-11B)d\sin(e+fx))}{(\sin(e+fx)a+a)^{3/2}}dx}{8a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(c+d\sin(e+fx))^2(a(3Ac+5Bc+6Ad-6Bd)-a(3A-11B)d\sin(e+fx))}{(\sin(e+fx)a+a)^{3/2}}dx}{8a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{3456}$$

$$\frac{\frac{\int \frac{(c+d\sin(e+fx))\left(a^2\left(B\left(5c^2+47dc-68d^2\right)+3A\left(c^2+3dc+12d^2\right)\right)-a^2d(9Ac+15Bc+39Ad-95Bd)\sin(e+fx)\right)}{2\sqrt{\sin(e+fx)a+a}}dx}{2a^2}-\frac{a(3Ac+9Ad+5Bc-17Bd)\cos(e+fx)(}{2f(a\sin(e+fx)+a)^{3/}}}{8a^2}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a\sin(e+fx)+a)^{5/2}}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{(c+d\sin(e+fx))\left(a^2\left(B\left(5c^2+47dc-68d^2\right)+3A\left(c^2+3dc+12d^2\right)\right)-a^2d(9Ac+15Bc+39Ad-95Bd)\sin(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}}dx}{4a^2}-\frac{a(3Ac+9Ad+5Bc-17Bd)\cos(e+fx)(}{2f(a\sin(e+fx)+a)^{3/}}}{8a^2}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a\sin(e+fx)+a)^{5/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{(c+d\sin(e+fx))\left(a^2\left(B\left(5c^2+47dc-68d^2\right)+3A\left(c^2+3dc+12d^2\right)\right)-a^2d(9Ac+15Bc+39Ad-95Bd)\sin(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}}dx}{4a^2}-\frac{a(3Ac+9Ad+5Bc-17Bd)\cos(e+fx)(}{2f(a\sin(e+fx)+a)^{3/}}}{8a^2}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a\sin(e+fx)+a)^{5/2}}$$

$\downarrow$ 3447

$$\frac{\frac{\int \frac{-d^2(9Ac+15Bc+39Ad-95Bd)\sin^2(e+fx)a^2+c\left(B\left(5c^2+47dc-68d^2\right)+3A\left(c^2+3dc+12d^2\right)\right)a^2+\left(a^2d\left(B\left(5c^2+47dc-68d^2\right)+3A\left(c^2+3dc+12d^2\right)\right)-a^2cd(9Ac+\right.}{\sqrt{\sin(e+fx)a+a}}}{4a^2}}{8a^2}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a\sin(e+fx)+a)^{5/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{-d^2(9Ac+15Bc+39Ad-95Bd)\sin(e+fx)^2a^2+c\left(B\left(5c^2+47dc-68d^2\right)+3A\left(c^2+3dc+12d^2\right)\right)a^2+\left(a^2d\left(B\left(5c^2+47dc-68d^2\right)+3A\left(c^2+3dc+12d^2\right)\right)-a^2cd(9Ac+\right.}{\sqrt{\sin(e+fx)a+a}}}{4a^2}}{8a^2}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a\sin(e+fx)+a)^{5/2}}$$

$\downarrow$ 3502

$$\frac{\frac{\frac{2\int \frac{a^3\left(3A\left(3c^3+9dc^2+33d^2c-13d^3\right)+B\left(15c^3+141dc^2-219d^2c+95d^3\right)\right)-2a^3d\left(A\left(9c^2+36dc-93d^2\right)+B\left(15c^2-228dc+197d^2\right)\right)\sin(e+fx)}{2\sqrt{\sin(e+fx)a+a}}dx}{3a}+\frac{2ad^2(9Ac+39Ac\cdots}{\cdots}}{4a^2}\cdots}{8a^2}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a\sin(e+fx)+a)^{5/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{a^3\left(3A\left(3c^3+9dc^2+33d^2c-13d^3\right)+B\left(15c^3+141dc^2-219d^2c+95d^3\right)\right)-2a^3d\left(A\left(9c^2+36dc-93d^2\right)+B\left(15c^2-228dc+197d^2\right)\right)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}}dx}{3a}+\frac{2ad^2(9Ac+39Ad+\cdots}{\cdots}}{4a^2}\cdots}{8a^2}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a\sin(e+fx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{a^3\left(3A\left(3c^3+9dc^2+33d^2c-13d^3\right)+B\left(15c^3+141dc^2-219d^2c+95d^3\right)\right)-2a^3d\left(A\left(9c^2+36dc-93d^2\right)+B\left(15c^2-228dc+197d^2\right)\right)\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}}dx}{3a}+\frac{2ad^2(9Ac+39Ad+\cdots}{\cdots}}{4a^2}\cdots}{8a^2}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a\sin(e+fx)+a)^{5/2}}$$

$\Big\downarrow$ 3230

$$\frac{\frac{\frac{3a^3(c-d)\left(3A\left(c^2+6cd+25d^2\right)+B\left(5c^2+62cd-163d^2\right)\right)\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx+\frac{4a^3d\left(A\left(9c^2+36cd-93d^2\right)+B\left(15c^2-228cd+197d^2\right)\right)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{3a}+\frac{2ad^2(9Ac+39\cdots}{\cdots}}{4a^2}\cdots}{8a^2}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a\sin(e+fx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{3a^3(c-d)\left(3A\left(c^2+6cd+25d^2\right)+B\left(5c^2+62cd-163d^2\right)\right)\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx+\frac{4a^3d\left(A\left(9c^2+36cd-93d^2\right)+B\left(15c^2-228cd+197d^2\right)\right)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{3a}+\frac{2ad^2(9Ac+39\cdots}{\cdots}}{4a^2}\cdots}{8a^2}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a\sin(e+fx)+a)^{5/2}}$$

$\Big\downarrow$ 3128

$$\frac{\frac{\frac{\frac{4a^3d\left(A\left(9c^2+36cd-93d^2\right)+B\left(15c^2-228cd+197d^2\right)\right)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}-\frac{6a^3(c-d)\left(3A\left(c^2+6cd+25d^2\right)+B\left(5c^2+62cd-163d^2\right)\right)\int\frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f}}{3a}}{4a^2}}{8a^2}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{\frac{4a^3d\left(A\left(9c^2+36cd-93d^2\right)+B\left(15c^2-228cd+197d^2\right)\right)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}-\frac{3\sqrt{2}a^{5/2}(c-d)\left(3A\left(c^2+6cd+25d^2\right)+B\left(5c^2+62cd-163d^2\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f}}{3a}}{4a^2}}{8a^2}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^3}{4f(a\sin(e+fx)+a)^{5/2}}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^(5/
2),x]
```

output
```
-1/4*((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(f*(a + a*Sin[e + f*x])
^(5/2)) + (-1/2*(a*(3*A*c + 5*B*c + 9*A*d - 17*B*d)*Cos[e + f*x]*(c + d*Si
n[e + f*x])^2)/(f*(a + a*Sin[e + f*x])^(3/2)) + ((2*a*d^2*(9*A*c + 15*B*c
+ 39*A*d - 95*B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*f) + ((-3*Sqr
t[2]*a^(5/2)*(c - d)*(B*(5*c^2 + 62*c*d - 163*d^2) + 3*A*(c^2 + 6*c*d + 25
*d^2))*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])
/f + (4*a^3*d*(A*(9*c^2 + 36*c*d - 93*d^2) + B*(15*c^2 - 228*c*d + 197*d^2
))*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]))/(3*a))/(4*a^2))/(8*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1156 vs. 2(281) = 562.

Time = 2.31 (sec) , antiderivative size = 1157, normalized size of antiderivative = 3.76

method	result	size
parts	Expression too large to display	1157
default	Expression too large to display	1438

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(5/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/32*A*c^3/a^(9/2)*(-3*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)
/a^(1/2))*a^2*cos(f*x+e)^2+6*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^
(1/2)/a^(1/2))*a^2*sin(f*x+e)+6*a^(3/2)*(a-a*sin(f*x+e))^(1/2)*sin(f*x+e)+
6*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2+14*(a-a*
sin(f*x+e))^(1/2)*a^(3/2))*(-a*(sin(f*x+e)-1))^(1/2)/(1+sin(f*x+e))/cos(f*
x+e)/(a+a*sin(f*x+e))^(1/2)/f-1/96*B*d^3/a^(9/2)*(-489*2^(1/2)*arctanh(1/2
*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*cos(f*x+e)^2+384*a^(3/2)*(a-a
*sin(f*x+e))^(1/2)*cos(f*x+e)^2+64*a^(1/2)*(a-a*sin(f*x+e))^(3/2)*cos(f*x+
e)^2+978*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*s
in(f*x+e)-768*a^(3/2)*(a-a*sin(f*x+e))^(1/2)*sin(f*x+e)-128*(a-a*sin(f*x+e
))^(3/2)*a^(1/2)*sin(f*x+e)+978*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)
*2^(1/2)/a^(1/2))*a^2-1092*(a-a*sin(f*x+e))^(1/2)*a^(3/2)+46*(a-a*sin(f*x+
e))^(3/2)*a^(1/2))*(-a*(sin(f*x+e)-1))^(1/2)/(1+sin(f*x+e))/cos(f*x+e)/(a+
a*sin(f*x+e))^(1/2)/f-1/32*c^2*(3*A*d+B*c)/a^(11/2)*(-5*2^(1/2)*arctanh(1/
2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*cos(f*x+e)^2+10*2^(1/2)*arct
anh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^3+12*a^(5/2)*
(a-a*sin(f*x+e))^(1/2)-10*(a-a*sin(f*x+e))^(3/2)*a^(3/2)+10*2^(1/2)*arctan
h(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3)*(-a*(sin(f*x+e)-1))^(1/
2)/(1+sin(f*x+e))/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f+1/32*d^2*(A*d+3*B*c)
/a^(9/2)*(-75*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 980 vs. $2(282) = 564$.

Time = 0.14 (sec) , antiderivative size = 980, normalized size of antiderivative = 3.18

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(5/2),x, al
gorithm="fricas")
```

output
```
-1/192*(3*sqrt(2)*(4*(3*A + 5*B)*c^3 + 12*(5*A + 19*B)*c^2*d + 12*(19*A -
75*B)*c*d^2 - 4*(75*A - 163*B)*d^3 - ((3*A + 5*B)*c^3 + 3*(5*A + 19*B)*c^2
*d + 3*(19*A - 75*B)*c*d^2 - (75*A - 163*B)*d^3)*cos(f*x + e)^3 - 3*((3*A
+ 5*B)*c^3 + 3*(5*A + 19*B)*c^2*d + 3*(19*A - 75*B)*c*d^2 - (75*A - 163*B)
*d^3)*cos(f*x + e)^2 + 2*((3*A + 5*B)*c^3 + 3*(5*A + 19*B)*c^2*d + 3*(19*A
 - 75*B)*c*d^2 - (75*A - 163*B)*d^3)*cos(f*x + e) + (4*(3*A + 5*B)*c^3 + 1
2*(5*A + 19*B)*c^2*d + 12*(19*A - 75*B)*c*d^2 - 4*(75*A - 163*B)*d^3 - ((3
*A + 5*B)*c^3 + 3*(5*A + 19*B)*c^2*d + 3*(19*A - 75*B)*c*d^2 - (75*A - 163
*B)*d^3)*cos(f*x + e)^2 + 2*((3*A + 5*B)*c^3 + 3*(5*A + 19*B)*c^2*d + 3*(1
9*A - 75*B)*c*d^2 - (75*A - 163*B)*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a
)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos
(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*
sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - co
s(f*x + e) - 2)) + 4*(32*B*d^3*cos(f*x + e)^4 - 12*(A - B)*c^3 + 36*(A - B
)*c^2*d - 36*(A - B)*c*d^2 + 12*(A - B)*d^3 + 32*(9*B*c*d^2 + (3*A - 5*B)*
d^3)*cos(f*x + e)^3 - 3*((3*A + 5*B)*c^3 + 3*(5*A - 13*B)*c^2*d - 3*(13*A
- 53*B)*c*d^2 + (53*A - 93*B)*d^3)*cos(f*x + e)^2 - 3*((7*A + B)*c^3 + 3*(
A - 9*B)*c^2*d - 27*(A - 9*B)*c*d^2 + 9*(9*A - 17*B)*d^3)*cos(f*x + e) + (
32*B*d^3*cos(f*x + e)^3 + 12*(A - B)*c^3 - 36*(A - B)*c^2*d + 36*(A - B)*c
*d^2 - 12*(A - B)*d^3 - 96*(3*B*c*d^2 + (A - 2*B)*d^3)*cos(f*x + e)^2 -...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^3}{(a\sin(fx+e)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(5/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^3/(a*sin(f*x + e) + a)
^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(5/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{5/2}}\,dx=\int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(c+d\,\sin\left(e+f\,x\right)\right)^3}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^(5/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^(5/
2), x)
```

Reduce [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^3}{(a+a\sin(e+fx))^{5/2}}\,dx=\frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}}{\sin(fx+e)^3+3\sin(fx+e)^2+3\sin(fx+e)+1}dx\right)a\,c^3+\left(\int \frac{}{\sin}\right.\right.}{}$$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(5/2),x)
```

output
```
(sqrt(a)*(int(sqrt(sin(e + f*x) + 1)/(sin(e + f*x)**3 + 3*sin(e + f*x)**2
+ 3*sin(e + f*x) + 1),x)*a*c**3 + int((sqrt(sin(e + f*x) + 1)*sin(e + f*x)
**4)/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x) + 1),x)*b*d**3
+ int((sqrt(sin(e + f*x) + 1)*sin(e + f*x)**3)/(sin(e + f*x)**3 + 3*sin(e
+ f*x)**2 + 3*sin(e + f*x) + 1),x)*a*d**3 + 3*int((sqrt(sin(e + f*x) + 1)*
sin(e + f*x)**3)/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x) + 1
),x)*b*c*d**2 + 3*int((sqrt(sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*
x)**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x) + 1),x)*a*c*d**2 + 3*int((sqrt(
sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 +
3*sin(e + f*x) + 1),x)*b*c**2*d + 3*int((sqrt(sin(e + f*x) + 1)*sin(e + f*
x))/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x) + 1),x)*a*c**2*d
 + int((sqrt(sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**3 + 3*sin(e +
f*x)**2 + 3*sin(e + f*x) + 1),x)*b*c**3))/a**3
```

3.322 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 219

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(B(5c^2+38cd-75d^2)+A(3c^2+10cd+19d^2))\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f} - \frac{(c-d)(3Ac+5Bc+5Ad-13Bd)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} + \frac{(A-9B)d^2\cos(e+fx)}{4a^2f\sqrt{a+a\sin(e+fx)}} - \frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a+a\sin(e+fx))^{5/2}}$$

output
```
-1/32*(B*(5*c^2+38*c*d-75*d^2)+A*(3*c^2+10*c*d+19*d^2))*arctanh(1/2*a^(1/2
)*cos(f*x+e)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))*2^(1/2)/a^(5/2)/f-1/16*(c-d)*
(3*A*c+5*A*d+5*B*c-13*B*d)*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(3/2)+1/4*(A-9*
B)*d^2*cos(f*x+e)/a^2/f/(a+a*sin(f*x+e))^(1/2)-1/4*(A-B)*cos(f*x+e)*(c+d*s
in(f*x+e))^2/f/(a+a*sin(f*x+e))^(5/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.67 (sec) , antiderivative size = 544, normalized size of antiderivative = 2.48

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-11Ac^2\cos\left(\frac{1}{2}(e+\right.\right.}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x
])^(5/2),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-11*A*c^2*Cos[(e + f*x)/2] + 3*B*c
^2*Cos[(e + f*x)/2] + 6*A*c*d*Cos[(e + f*x)/2] + 10*B*c*d*Cos[(e + f*x)/2]
 + 5*A*d^2*Cos[(e + f*x)/2] - 45*B*d^2*Cos[(e + f*x)/2] - 3*A*c^2*Cos[(3*(
e + f*x))/2] - 5*B*c^2*Cos[(3*(e + f*x))/2] - 10*A*c*d*Cos[(3*(e + f*x))/2
] + 26*B*c*d*Cos[(3*(e + f*x))/2] + 13*A*d^2*Cos[(3*(e + f*x))/2] - 69*B*d
^2*Cos[(3*(e + f*x))/2] + 16*B*d^2*Cos[(5*(e + f*x))/2] + 11*A*c^2*Sin[(e
+ f*x)/2] - 3*B*c^2*Sin[(e + f*x)/2] - 6*A*c*d*Sin[(e + f*x)/2] - 10*B*c*d
*Sin[(e + f*x)/2] - 5*A*d^2*Sin[(e + f*x)/2] + 45*B*d^2*Sin[(e + f*x)/2] +
 (2 + 2*I)*(-1)^(3/4)*(B*(5*c^2 + 38*c*d - 75*d^2) + A*(3*c^2 + 10*c*d + 1
9*d^2))*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^4 - 3*A*c^2*Sin[(3*(e + f*x))/2] - 5*B*c^2*Sin
[(3*(e + f*x))/2] - 10*A*c*d*Sin[(3*(e + f*x))/2] + 26*B*c*d*Sin[(3*(e + f
*x))/2] + 13*A*d^2*Sin[(3*(e + f*x))/2] - 69*B*d^2*Sin[(3*(e + f*x))/2] -
16*B*d^2*Sin[(5*(e + f*x))/2]))/(32*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.351$, Rules used = {3042, 3456, 27, 3042, 3447, 3042, 3498, 27, 3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a\sin(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a\sin(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3456}$$

$$\frac{\int \frac{(c+d\sin(e+fx))(a(3Ac+5Bc+4Ad-4Bd)-a(A-9B)d\sin(e+fx))}{2(\sin(e+fx)a+a)^{3/2}}dx}{4a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(c+d\sin(e+fx))(a(3Ac+5Bc+4Ad-4Bd)-a(A-9B)d\sin(e+fx))}{(\sin(e+fx)a+a)^{3/2}}dx}{8a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(c+d\sin(e+fx))(a(3Ac+5Bc+4Ad-4Bd)-a(A-9B)d\sin(e+fx))}{(\sin(e+fx)a+a)^{3/2}}dx}{8a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{3447}$$

$$\frac{\int \frac{-a(A-9B)d^2\sin^2(e+fx)+(ad(3Ac+5Bc+4Ad-4Bd)-a(A-9B)cd)\sin(e+fx)+ac(3Ac+5Bc+4Ad-4Bd)}{(\sin(e+fx)a+a)^{3/2}}dx}{8a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{-a(A-9B)d^2\sin(e+fx)^2+(ad(3Ac+5Bc+4Ad-4Bd)-a(A-9B)cd)\sin(e+fx)+ac(3Ac+5Bc+4Ad-4Bd)}{(\sin(e+fx)a+a)^{3/2}}dx}{8a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{3498}$$

$$\frac{-\frac{\int -\frac{a^2\left(B\left(5c^2+38dc-39d^2\right)+A\left(3c^2+10dc+15d^2\right)\right)-4a^2(A-9B)d^2\sin(e+fx)}{2\sqrt{\sin(e+fx)a+a}}dx}{2a^2}-\frac{a(c-d)(3Ac+5Ad+5Bc-13Bd)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a\sin(e+fx)+a)^{5/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{a^2\left(B\left(5c^2+38dc-39d^2\right)+A\left(3c^2+10dc+15d^2\right)\right)-4a^2(A-9B)d^2\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}}dx}{4a^2}-\frac{a(c-d)(3Ac+5Ad+5Bc-13Bd)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a\sin(e+fx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{a^2\left(B\left(5c^2+38dc-39d^2\right)+A\left(3c^2+10dc+15d^2\right)\right)-4a^2(A-9B)d^2\sin(e+fx)}{\sqrt{\sin(e+fx)a+a}}dx}{4a^2}-\frac{a(c-d)(3Ac+5Ad+5Bc-13Bd)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a\sin(e+fx)+a)^{5/2}}$$

$\Big\downarrow$ 3230

$$\frac{\frac{a^2(A(3c^2+10cd+19d^2)+B(5c^2+38cd-75d^2))\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx+\frac{8a^2d^2(A-9B)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{4a^2}-\frac{a(c-d)(3Ac+5Ad+5Bc-13Bd)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a\sin(e+fx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{a^2(A(3c^2+10cd+19d^2)+B(5c^2+38cd-75d^2))\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx+\frac{8a^2d^2(A-9B)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{4a^2}-\frac{a(c-d)(3Ac+5Ad+5Bc-13Bd)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a\sin(e+fx)+a)^{5/2}}$$

$\Big\downarrow$ 3128

$$\frac{\frac{\frac{8a^2d^2(A-9B)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}-\frac{2a^2\left(A\left(3c^2+10cd+19d^2\right)+B\left(5c^2+38cd-75d^2\right)\right)\int \frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f}}{4a^2}-\frac{a(c-d)(3Ac+5Ad+5Bc-13Bd)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{8a^2d^2(A-9B)\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}-\frac{\sqrt{2}a^{3/2}\left(A\left(3c^2+10cd+19d^2\right)+B\left(5c^2+38cd-75d^2\right)\right)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f}}{4a^2}-\frac{a(c-d)(3Ac+5Ad+5Bc-13Bd)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}}{8a^2}-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^2}{4f(a\sin(e+fx)+a)^{5/2}}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])^(5/
2),x]
```

output
```
-1/4*((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(f*(a + a*Sin[e + f*x])
^(5/2)) + (-1/2*(a*(c - d)*(3*A*c + 5*B*c + 5*A*d - 13*B*d)*Cos[e + f*x])/
(f*(a + a*Sin[e + f*x])^(3/2)) + (-((Sqrt[2]*a^(3/2)*(B*(5*c^2 + 38*c*d -
75*d^2) + A*(3*c^2 + 10*c*d + 19*d^2))*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqr
t[2]*Sqrt[a + a*Sin[e + f*x]])])/f) + (8*a^2*(A - 9*B)*d^2*Cos[e + f*x])/(
f*Sqrt[a + a*Sin[e + f*x]]))/(4*a^2))/(8*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

rule 3498
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[1
/(a^2*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b
*B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 851 vs. $2(196) = 392$.

Time = 1.00 (sec) , antiderivative size = 852, normalized size of antiderivative = 3.89

method	result	size
parts	Expression too large to display	852
default	Expression too large to display	982

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(5/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/32*A*c^2/a^(9/2)*(-3*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)
/a^(1/2))*a^2*cos(f*x+e)^2+6*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^
(1/2)/a^(1/2))*a^2*sin(f*x+e)+6*a^(3/2)*(a-a*sin(f*x+e))^(1/2)*sin(f*x+e)+
6*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2+14*(a-a*
sin(f*x+e))^(1/2)*a^(3/2))*(-a*(sin(f*x+e)-1))^(1/2)/(1+sin(f*x+e))/cos(f*
x+e)/(a+a*sin(f*x+e))^(1/2)/f+1/32*B*d^2/a^(9/2)*(-75*2^(1/2)*arctanh(1/2*
(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*cos(f*x+e)^2+64*a^(3/2)*(a-a*s
in(f*x+e))^(1/2)*cos(f*x+e)^2+150*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/
2)*2^(1/2)/a^(1/2))*a^2*sin(f*x+e)-128*a^(3/2)*(a-a*sin(f*x+e))^(1/2)*sin(
f*x+e)+150*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2
-204*(a-a*sin(f*x+e))^(1/2)*a^(3/2)+42*(a-a*sin(f*x+e))^(3/2)*a^(1/2))*(-a
*(sin(f*x+e)-1))^(1/2)/(1+sin(f*x+e))/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f-
1/32*c*(2*A*d+B*c)/a^(11/2)*(-5*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)
*2^(1/2)/a^(1/2))*a^3*cos(f*x+e)^2+10*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))
^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^3+12*a^(5/2)*(a-a*sin(f*x+e))^(1/2)-1
0*(a-a*sin(f*x+e))^(3/2)*a^(3/2)+10*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(
1/2)*2^(1/2)/a^(1/2))*a^3)*(-a*(sin(f*x+e)-1))^(1/2)/(1+sin(f*x+e))/cos(f*
x+e)/(a+a*sin(f*x+e))^(1/2)/f+1/32*d*(A*d+2*B*c)*(19*2^(1/2)*arctanh(1/2*(
a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*cos(f*x+e)^2-38*2^(1/2)*arctanh
(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*sin(f*x+e)+44*(a-a*sin...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 744 vs. $2(196) = 392$.

Time = 0.10 (sec) , antiderivative size = 744, normalized size of antiderivative = 3.40

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^2}{(a + a\sin(e + fx))^{5/2}}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(5/2),x, al
gorithm="fricas")
```

output
```
-1/64*(sqrt(2)*(((3*A + 5*B)*c^2 + 2*(5*A + 19*B)*c*d + (19*A - 75*B)*d^2)
*cos(f*x + e)^3 - 4*(3*A + 5*B)*c^2 - 8*(5*A + 19*B)*c*d - 4*(19*A - 75*B)
*d^2 + 3*((3*A + 5*B)*c^2 + 2*(5*A + 19*B)*c*d + (19*A - 75*B)*d^2)*cos(f*
x + e)^2 - 2*((3*A + 5*B)*c^2 + 2*(5*A + 19*B)*c*d + (19*A - 75*B)*d^2)*co
s(f*x + e) - (4*(3*A + 5*B)*c^2 + 8*(5*A + 19*B)*c*d + 4*(19*A - 75*B)*d^2
 - ((3*A + 5*B)*c^2 + 2*(5*A + 19*B)*c*d + (19*A - 75*B)*d^2)*cos(f*x + e)
^2 + 2*((3*A + 5*B)*c^2 + 2*(5*A + 19*B)*c*d + (19*A - 75*B)*d^2)*cos(f*x
+ e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(
f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e)
 - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x +
 e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*(32*B*d^2*cos(f*x + e)^3 -
4*(A - B)*c^2 + 8*(A - B)*c*d - 4*(A - B)*d^2 - ((3*A + 5*B)*c^2 + 2*(5*A
- 13*B)*c*d - (13*A - 53*B)*d^2)*cos(f*x + e)^2 - ((7*A + B)*c^2 + 2*(A -
9*B)*c*d - 9*(A - 9*B)*d^2)*cos(f*x + e) - (32*B*d^2*cos(f*x + e)^2 - 4*(A
 - B)*c^2 + 8*(A - B)*c*d - 4*(A - B)*d^2 + ((3*A + 5*B)*c^2 + 2*(5*A - 13
*B)*c*d - (13*A - 85*B)*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x +
e) + a))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x
+ e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*s
in(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^2}{(a\sin(fx+e)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(5/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^2/(a*sin(f*x + e) + a)
^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(5/2),x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{5/2}}\,dx=\int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(c+d\,\sin\left(e+f\,x\right)\right)^2}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^(5/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^(5/
2), x)
```

Reduce [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^2}{(a+a\sin(e+fx))^{5/2}}\,dx=\frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}}{\sin(fx+e)^3+3\sin(fx+e)^2+3\sin(fx+e)+1}dx\right)a\,c^2+\left(\int \frac{}{\mathrm{sin}}\right.\right.}{}$$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(5/2),x)
```

output
```
(sqrt(a)*(int(sqrt(sin(e + f*x) + 1)/(sin(e + f*x)**3 + 3*sin(e + f*x)**2
+ 3*sin(e + f*x) + 1),x)*a*c**2 + int((sqrt(sin(e + f*x) + 1)*sin(e + f*x)
**3)/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x) + 1),x)*b*d**2
+ int((sqrt(sin(e + f*x) + 1)*sin(e + f*x)**2)/(sin(e + f*x)**3 + 3*sin(e
+ f*x)**2 + 3*sin(e + f*x) + 1),x)*a*d**2 + 2*int((sqrt(sin(e + f*x) + 1)*
sin(e + f*x)**2)/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x) + 1
),x)*b*c*d + 2*int((sqrt(sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**3
+ 3*sin(e + f*x)**2 + 3*sin(e + f*x) + 1),x)*a*c*d + int((sqrt(sin(e + f*x
) + 1)*sin(e + f*x))/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x)
 + 1),x)*b*c**2))/a**3
```

3.323 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 151

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{(3Ac+5Bc+5Ad+19Bd)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f} - \frac{(A-B)(c-d)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{(3Ac+5Bc+5Ad-13Bd)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}}$$

output
```
-1/32*(3*A*c+5*A*d+5*B*c+19*B*d)*arctanh(1/2*a^(1/2)*cos(f*x+e)*2^(1/2)/(a
+a*sin(f*x+e))^(1/2))*2^(1/2)/a^(5/2)/f-1/4*(A-B)*(c-d)*cos(f*x+e)/f/(a+a*
sin(f*x+e))^(5/2)-1/16*(3*A*c+5*A*d+5*B*c-13*B*d)*cos(f*x+e)/a/f/(a+a*sin(
f*x+e))^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.98 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.77

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(8(A-B)(c-d)\sin\right.}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])
^(5/2),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(A - B)*(c - d)*Sin[(e + f*x)/2]
 - 4*(A - B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(3*A*c + 5*
B*c + 5*A*d - 13*B*d)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])^2 - (3*A*c + 5*B*c + 5*A*d - 13*B*d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2])^3 + (1 + I)*(-1)^(3/4)*(3*A*c + 5*B*c + 5*A*d + 19*B*d)*ArcTanh[(1/2 +
 I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x
)/2])^4))/(16*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3447, 3042, 3498, 27, 3042, 3229, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a\sin(e+fx)+a)^{5/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a\sin(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \; 3447$$

$$\int \frac{(Ad+Bc)\sin(e+fx)+Ac+Bd\sin^2(e+fx)}{(a\sin(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(Ad+Bc)\sin(e+fx)+Ac+Bd\sin(e+fx)^2}{(a\sin(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3498}$$

$$-\frac{\int -\frac{a(3Ac+5Bc+5Ad-5Bd)+8aBd\sin(e+fx)}{2(\sin(e+fx)a+a)^{3/2}}dx}{4a^2}-\frac{(A-B)(c-d)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a(3Ac+5Bc+5Ad-5Bd)+8aBd\sin(e+fx)}{(\sin(e+fx)a+a)^{3/2}}dx}{8a^2}-\frac{(A-B)(c-d)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(3Ac+5Bc+5Ad-5Bd)+8aBd\sin(e+fx)}{(\sin(e+fx)a+a)^{3/2}}dx}{8a^2}-\frac{(A-B)(c-d)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{3229}$$

$$\frac{\frac{1}{4}(3Ac+5Ad+5Bc+19Bd)\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx-\frac{a(3Ac+5Ad+5Bc-13Bd)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}}{8a^2}-\frac{(A-B)(c-d)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{4}(3Ac+5Ad+5Bc+19Bd)\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx-\frac{a(3Ac+5Ad+5Bc-13Bd)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}}{8a^2}-\frac{(A-B)(c-d)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{3128}$$

$$\frac{-\frac{(3Ac+5Ad+5Bc+19Bd)\int \frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{2f}-\frac{a(3Ac+5Ad+5Bc-13Bd)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}}{8a^2}-\frac{(A-B)(c-d)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{-\frac{(3Ac+5Ad+5Bc+19Bd)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}\sqrt{a}f}-\frac{a(3Ac+5Ad+5Bc-13Bd)\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}}{8a^2}-\frac{(A-B)(c-d)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^(5/2)
,x]
```

output
```
-1/4*((A - B)*(c - d)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(5/2)) + (-1/2
*((3*A*c + 5*B*c + 5*A*d + 19*B*d)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]
*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[2]*Sqrt[a]*f) - (a*(3*A*c + 5*B*c + 5*A
*d - 13*B*d)*Cos[e + f*x])/(2*f*(a + a*Sin[e + f*x])^(3/2)))/(8*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3498
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b - a*
B + b*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))), x] + Simp[1
/(a^2*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b
*B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(132) = 264$.

Time = 0.84 (sec) , antiderivative size = 448, normalized size of antiderivative = 2.97

method	result
default	$\frac{\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^2(3Ac+5Ad+5Bc+19Bd)\cos(fx+e)^2-2\sin(fx+e)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^2(3A\cdots}{\cdots}$
parts	$-\frac{Ac\left(-3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^2\cos(fx+e)^2+6\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^2\sin(fx+e)+6a^{\frac{3}{2}}\sqrt{a-a\sin(fx+\cdots}}{32a^{\frac{9}{2}}(1+\sin(fx+e))\cos(fx+e)\sqrt{a+a\cdots}}$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
1/32*(2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*(3*A
*c+5*A*d+5*B*c+19*B*d)*cos(f*x+e)^2-2*sin(f*x+e)*2^(1/2)*arctanh(1/2*(a-a*
sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*(3*A*c+5*A*d+5*B*c+19*B*d)+6*A*(a-a
*sin(f*x+e))^(3/2)*a^(1/2)*c+10*A*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*d-20*A*(a
-a*sin(f*x+e))^(1/2)*a^(3/2)*c-12*A*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*d-6*A*2
^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c-10*A*2^(1
/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d+10*B*(a-a*si
n(f*x+e))^(3/2)*a^(1/2)*c-26*B*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*d-12*B*(a-a*
sin(f*x+e))^(1/2)*a^(3/2)*c+44*B*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*d-10*B*2^(
1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c-38*B*2^(1/2
)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d)*(-a*(sin(f*x+
e)-1))^(1/2)/a^(9/2)/(1+sin(f*x+e))/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 536 vs. $2(132) = 264$.

Time = 0.10 (sec) , antiderivative size = 536, normalized size of antiderivative = 3.55

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))}{(a + a\sin(e + fx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algo
rithm="fricas")
```

output
```
1/64*(sqrt(2)*((((3*A + 5*B)*c + (5*A + 19*B)*d)*cos(f*x + e)^3 + 3*((3*A +
 5*B)*c + (5*A + 19*B)*d)*cos(f*x + e)^2 - 4*(3*A + 5*B)*c - 4*(5*A + 19*B
)*d - 2*((3*A + 5*B)*c + (5*A + 19*B)*d)*cos(f*x + e) + (((3*A + 5*B)*c +
(5*A + 19*B)*d)*cos(f*x + e)^2 - 4*(3*A + 5*B)*c - 4*(5*A + 19*B)*d - 2*((
3*A + 5*B)*c + (5*A + 19*B)*d)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a
*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e)
 - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x +
 e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e
) - 2)) + 4*(((3*A + 5*B)*c + (5*A - 13*B)*d)*cos(f*x + e)^2 + 4*(A - B)*c
 - 4*(A - B)*d + ((7*A + B)*c + (A - 9*B)*d)*cos(f*x + e) - (4*(A - B)*c -
 4*(A - B)*d - ((3*A + 5*B)*c + (5*A - 13*B)*d)*cos(f*x + e))*sin(f*x + e)
)*sqrt(a*sin(f*x + e) + a))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2
 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*
x + e) - 4*a^3*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)}{(a\sin(fx+e)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^(
5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(c+d\,\sin(e+f\,x))}{(a+a\,\sin(e+f\,x))^{5/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^(5/2)
,x)
```

output
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^(5/2)
, x)
```

Reduce [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}}{\sin(fx+e)^3+3\sin(fx+e)^2+3\sin(fx+e)+1}dx\right)ac + \left(\int \frac{}{\sin(f}\right.\right.}{}$$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x)
```

output
```
(sqrt(a)*(int(sqrt(sin(e + f*x) + 1)/(sin(e + f*x)**3 + 3*sin(e + f*x)**2
+ 3*sin(e + f*x) + 1),x)*a*c + int((sqrt(sin(e + f*x) + 1)*sin(e + f*x)**2
)/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x) + 1),x)*b*d + int(
(sqrt(sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**3 + 3*sin(e + f*x)**2
 + 3*sin(e + f*x) + 1),x)*a*d + int((sqrt(sin(e + f*x) + 1)*sin(e + f*x))/
(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x) + 1),x)*b*c))/a**3
```

3.324 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 126

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx=-\frac{(3A+5B)\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f}-\frac{(A-B)\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}}-\frac{(3A+5B)\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}}$$

output
```
-1/32*(3*A+5*B)*arctanh(1/2*a^(1/2)*cos(f*x+e)*2^(1/2)/(a+a*sin(f*x+e))^(1
/2))*2^(1/2)/a^(5/2)/f-1/4*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)-1/16*
(3*A+5*B)*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.85 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.80

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx=\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(8(A-B)\sin\left(\frac{1}{2}(e+fx)\right)+4(-A+\right.}{}$$

input
```
Integrate[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^(5/2),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(A - B)*Sin[(e + f*x)/2] + 4*(-A
 + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(3*A + 5*B)*Sin[(e + f*x)/
2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (3*A + 5*B)*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^3 + (1 + I)*(-1)^(3/4)*(3*A + 5*B)*ArcTanh[(1/2 + I/2
)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^4))/(16*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3042, 3229, 3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3229}$$

$$\frac{(3A+5B)\int \frac{1}{(\sin(e+fx)a+a)^{3/2}}dx}{8a}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(3A+5B)\int \frac{1}{(\sin(e+fx)a+a)^{3/2}}dx}{8a}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{3129}$$

$$\frac{(3A+5B)\left(\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{4a}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}\right)}{8a}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(3A+5B)\left(\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{4a}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}\right)}{8a}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{3128}$$

$$\frac{(3A+5B)\left(-\frac{\int \frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{2af}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}\right)}{8a}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{(3A+5B)\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}\right)}{8a}-\frac{(A-B)\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

input
```
Int[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^(5/2),x]
```

output
```
-1/4*((A - B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(5/2)) + ((3*A + 5*B)*
(-1/2*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])]/(
Sqrt[2]*a^(3/2)*f) - Cos[e + f*x]/(2*f*(a + a*Sin[e + f*x])^(3/2))))/(8*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(107) = 214$.

Time = 0.49 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.21

method	result
default	$-\frac{\left(-\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^3(3A+5B)\cos(fx+e)^2+2\sin(fx+e)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^3(3A+5B)+20A}{}$
parts	$-\frac{A\left(-3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^2\cos(fx+e)^2+6\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)a^2\sin(fx+e)+6a^{\frac{3}{2}}\sqrt{a-a\sin(fx+e}\right.}{32a^{\frac{9}{2}}(1+\sin(fx+e))\cos(fx+e)\sqrt{a+a\,s}}$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32/a^(11/2)*(-2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2
))*a^3*(3*A+5*B)*cos(f*x+e)^2+2*sin(f*x+e)*2^(1/2)*arctanh(1/2*(a-a*sin(f*
x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*(3*A+5*B)+20*A*(a-a*sin(f*x+e))^(1/2)*a^(
5/2)-6*A*(a-a*sin(f*x+e))^(3/2)*a^(3/2)+12*B*(a-a*sin(f*x+e))^(1/2)*a^(5/2
)-10*B*(a-a*sin(f*x+e))^(3/2)*a^(3/2)+6*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x
+e))^(1/2)*2^(1/2)/a^(1/2))*a^3+10*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^
(1/2)*2^(1/2)/a^(1/2))*a^3)*(-a*(sin(f*x+e)-1))^(1/2)/(1+sin(f*x+e))/cos(f
*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 392 vs. $2(107) = 214$.

Time = 0.09 (sec) , antiderivative size = 392, normalized size of antiderivative = 3.11

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\sqrt{2}\left((3\,A+5\,B)\cos\left(fx+e\right)^3+3\,(3\,A+5\,B)\cos\left(fx+e\right)^2-2\,(3\,A+5\,B)\right.}{}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
1/64*(sqrt(2)*((3*A + 5*B)*cos(f*x + e)^3 + 3*(3*A + 5*B)*cos(f*x + e)^2 -
 2*(3*A + 5*B)*cos(f*x + e) + ((3*A + 5*B)*cos(f*x + e)^2 - 2*(3*A + 5*B)*
cos(f*x + e) - 12*A - 20*B)*sin(f*x + e) - 12*A - 20*B)*sqrt(a)*log(-(a*co
s(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) -
sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e)
 + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) -
 2)) + 4*((3*A + 5*B)*cos(f*x + e)^2 + (7*A + B)*cos(f*x + e) + ((3*A + 5*
B)*cos(f*x + e) - 4*A + 4*B)*sin(f*x + e) + 4*A - 4*B)*sqrt(a*sin(f*x + e)
 + a))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x +
e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin
(f*x + e))
```

Sympy [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{A+B\sin\left(e+fx\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2),x)
```

output
```
Integral((A + B*sin(e + f*x))/(a*(sin(e + f*x) + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx=\int \frac{B\sin\left(fx+e\right)+A}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/(a*sin(f*x + e) + a)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx=\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

input
```
int((A + B*sin(e + f*x))/(a + a*sin(e + f*x))^(5/2),x)
```

output
```
int((A + B*sin(e + f*x))/(a + a*sin(e + f*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}}{\sin(fx+e)^3+3\sin(fx+e)^2+3\sin(fx+e)+1}dx\right)a+\left(\int \frac{\sqrt{\sin(fx+e)+1}\,\sin(fx}{\sin(fx+e)^3+3\sin(fx+e)^2+3\,\mathrm{si}}\right.\right.}{a^3}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x)
```

output
```
(sqrt(a)*(int(sqrt(sin(e + f*x) + 1)/(sin(e + f*x)**3 + 3*sin(e + f*x)**2
+ 3*sin(e + f*x) + 1),x)*a + int((sqrt(sin(e + f*x) + 1)*sin(e + f*x))/(si
n(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(e + f*x) + 1),x)*b))/a**3
```

3.325 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 261

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))}\,dx = -\frac{(B(5c^2-34cd-3d^2)+A(3c^2-14cd+43d^2))\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}(c-d)^3f} - \frac{2d^{3/2}(Bc-Ad)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{a^{5/2}(c-d)^3\sqrt{c+d}f} - \frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}} - \frac{(3Ac+5Bc-11Ad+3Bd)\cos(e+fx)}{16a(c-d)^2f(a+a\sin(e+fx))^{3/2}}$$

output
```
-1/32*(B*(5*c^2-34*c*d-3*d^2)+A*(3*c^2-14*c*d+43*d^2))*arctanh(1/2*a^(1/2)
*cos(f*x+e)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))*2^(1/2)/a^(5/2)/(c-d)^3/f-2*d^
(3/2)*(-A*d+B*c)*arctanh(a^(1/2)*d^(1/2)*cos(f*x+e)/(c+d)^(1/2)/(a+a*sin(f
*x+e))^(1/2))/a^(5/2)/(c-d)^3/(c+d)^(1/2)/f-1/4*(A-B)*cos(f*x+e)/(c-d)/f/(
a+a*sin(f*x+e))^(5/2)-1/16*(3*A*c-11*A*d+5*B*c+3*B*d)*cos(f*x+e)/a/(c-d)^2
/f/(a+a*sin(f*x+e))^(3/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 11.45 (sec) , antiderivative size = 912, normalized size of antiderivative = 3.49

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e +
f*x])),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(A - B)*(c - d)^2*Sin[(e + f*x)/
2] + 4*(-A + B)*(c - d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(c - d
)*(3*A*c + 5*B*c - 11*A*d + 3*B*d)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2])^2 - (c - d)*(3*A*c + 5*B*c - 11*A*d + 3*B*d)*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])^3 + (1 + I)*(-1)^(3/4)*(B*(5*c^2 - 34*c*d - 3*d^2
) + A*(3*c^2 - 14*c*d + 43*d^2))*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[
(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 + (8*d^(3/2)*(-(B*c
) + A*d)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + RootSum[c + 4*d*#1 + 2*c*#
1^2 - 4*d*#1^3 + c*#1^4 & , (-(d*Log[-#1 + Tan[(e + f*x)/4]]) + Sqrt[d]*Sq
rt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - c*Log[-#1 + Tan[(e + f*x)/4]]*#1 +
 2*Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 3*d*Log[-#1 + Tan[
(e + f*x)/4]]*#1^2 - Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2
- c*Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ])
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)/Sqrt[c + d] + (8*d^(3/2)*(B*c -
A*d)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + RootSum[c + 4*d*#1 + 2*c*#1^2
- 4*d*#1^3 + c*#1^4 & , (-(d*Log[-#1 + Tan[(e + f*x)/4]]) - Sqrt[d]*Sqrt[c
 + d]*Log[-#1 + Tan[(e + f*x)/4]] - c*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*S
qrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 3*d*Log[-#1 + Tan[(e +
 f*x)/4]]*#1^2 + Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - c*
Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1^3) & ])*...
```

Rubi [A] (verified)

Time = 1.64 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.10, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.351$, Rules used = {3042, 3457, 27, 3042, 3457, 27, 3042, 3464, 3042, 3128, 219, 3252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))}dx$$

$$\Big\downarrow \text{3457}$$

$$-\frac{\int -\frac{a(3Ac+5Bc-8Ad)+3a(A-B)d\sin(e+fx)}{2(\sin(e+fx)a+a)^{3/2}(c+d\sin(e+fx))}dx}{4a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a(3Ac+5Bc-8Ad)+3a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)^{3/2}(c+d\sin(e+fx))}dx}{8a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a(3Ac+5Bc-8Ad)+3a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)^{3/2}(c+d\sin(e+fx))}dx}{8a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{3457}$$

$$\frac{-\frac{\int -\frac{\left(Bc(5c-29d)+A\left(3c^2-11dc+32d^2\right)\right)a^2+d(3Ac+5Bc-11Ad+3Bd)\sin(e+fx)a^2}{2\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{2a^2(c-d)}-\frac{a(3Ac-11Ad+5Bc+3Bd)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}}}{8a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\left(Bc(5c-29d)+A\left(3c^2-11dc+32d^2\right)\right)a^2+d(3Ac+5Bc-11Ad+3Bd)\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{4a^2(c-d)}-\frac{a(3Ac-11Ad+5Bc+3Bd)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}}}{8a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{\left(Bc(5c-29d)+A\left(3c^2-11dc+32d^2\right)\right)a^2+d(3Ac+5Bc-11Ad+3Bd)\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{4a^2(c-d)}-\frac{a(3Ac-11Ad+5Bc+3Bd)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}}}{8a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}}$$

$\Big\downarrow$ 3464

$$\frac{\frac{\frac{a^2\left(A\left(3c^2-14cd+43d^2\right)+B\left(5c^2-34cd-3d^2\right)\right)\int\frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{c-d}+\frac{32ad^2(Bc-Ad)\int\frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}}{4a^2(c-d)}-\frac{a(3Ac-11Ad+5Bc+3Bd)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}}}{8a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{a^2\left(A\left(3c^2-14cd+43d^2\right)+B\left(5c^2-34cd-3d^2\right)\right)\int\frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{c-d}+\frac{32ad^2(Bc-Ad)\int\frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}}{4a^2(c-d)}-\frac{a(3Ac-11Ad+5Bc+3Bd)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}}}{8a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}}$$

$\Big\downarrow$ 3128

$$\frac{\frac{\frac{32ad^2(Bc-Ad)\int\frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}-\frac{2a^2\left(A\left(3c^2-14cd+43d^2\right)+B\left(5c^2-34cd-3d^2\right)\right)\int\frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f(c-d)}}{4a^2(c-d)}-\frac{a(3Ac-11Ad+5Bc+3B}{2f(c-d)(a\sin(e+f}}{8a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}}$$

$\Big\downarrow$ 219

$$\frac{\frac{\frac{32ad^2(Bc-Ad)\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}-\frac{\sqrt{2}a^{3/2}\left(A\left(3c^2-14cd+43d^2\right)+B\left(5c^2-34cd-3d^2\right)\right)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f(c-d)}}{4a^2(c-d)}-\frac{a(3Ac-11Ad+5Bc+3Ba}{2f(c-d)(a\sin(e+fx}}{8a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{3252}$$

$$\frac{\frac{-\frac{64a^2d^2(Bc-Ad)\int \frac{1}{a(c+d)-\frac{a^2d\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f(c-d)}-\frac{\sqrt{2}a^{3/2}\left(A\left(3c^2-14cd+43d^2\right)+B\left(5c^2-34cd-3d^2\right)\right)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f(c-d)}}{4a^2(c-d)}-}{8a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{-\frac{\sqrt{2}a^{3/2}\left(A\left(3c^2-14cd+43d^2\right)+B\left(5c^2-34cd-3d^2\right)\right)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f(c-d)}-\frac{64a^{3/2}d^{3/2}(Bc-Ad)\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{f(c-d)\sqrt{c+d}}}{4a^2(c-d)}-\frac{a(3}{}}{8a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x]))
,x]
```

output
```
-1/4*((A - B)*Cos[e + f*x])/((c - d)*f*(a + a*Sin[e + f*x])^(5/2)) + ((-((
Sqrt[2]*a^(3/2)*(B*(5*c^2 - 34*c*d - 3*d^2) + A*(3*c^2 - 14*c*d + 43*d^2))
*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/((c -
 d)*f)) - (64*a^(3/2)*d^(3/2)*(B*c - A*d)*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e +
 f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/((c - d)*Sqrt[c + d]*f))/(
4*a^2*(c - d)) - (a*(3*A*c + 5*B*c - 11*A*d + 3*B*d)*Cos[e + f*x])/(2*(c -
 d)*f*(a + a*Sin[e + f*x])^(3/2)))/(8*a^2*(c - d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3464
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A
*b - a*B)/(b*c - a*d)   Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Simp[(B*c
- A*d)/(b*c - a*d)   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1417 vs. $2(228) = 456$.

Time = 0.46 (sec) , antiderivative size = 1418, normalized size of antiderivative = 5.43

method	result	size
default	Expression too large to display	1418

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x,method=_RET
URNVERBOSE)
```

output
```
1/32/a^(9/2)*((-64*A*a^(5/2)*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2
)^(1/2))*d^3+64*B*a^(5/2)*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(
1/2))*c*d^2+3*A*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*((c+d)
*a*d)^(1/2)*2^(1/2)*a^2*c^2-14*A*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2
)/a^(1/2))*((c+d)*a*d)^(1/2)*2^(1/2)*a^2*c*d+43*A*arctanh(1/2*(a-a*sin(f*x
+e))^(1/2)*2^(1/2)/a^(1/2))*((c+d)*a*d)^(1/2)*2^(1/2)*a^2*d^2+5*B*arctanh(
1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*((c+d)*a*d)^(1/2)*2^(1/2)*a^2*
c^2-34*B*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*((c+d)*a*d)^(
1/2)*2^(1/2)*a^2*c*d-3*B*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2
))*((c+d)*a*d)^(1/2)*2^(1/2)*a^2*d^2)*cos(f*x+e)^2+2*sin(f*x+e)*(64*A*a^(5
/2)*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*d^3-64*B*a^(5/2)
*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*c*d^2-3*A*arctanh(1
/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*((c+d)*a*d)^(1/2)*2^(1/2)*a^2*c
^2+14*A*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*((c+d)*a*d)^(1
/2)*2^(1/2)*a^2*c*d-43*A*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2
))*((c+d)*a*d)^(1/2)*2^(1/2)*a^2*d^2-5*B*arctanh(1/2*(a-a*sin(f*x+e))^(1/2
)*2^(1/2)/a^(1/2))*((c+d)*a*d)^(1/2)*2^(1/2)*a^2*c^2+34*B*arctanh(1/2*(a-a
*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*((c+d)*a*d)^(1/2)*2^(1/2)*a^2*c*d+3*B*
arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*((c+d)*a*d)^(1/2)*2^(1
/2)*a^2*d^2)+128*A*a^(5/2)*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1146 vs. $2(228) = 456$.

Time = 3.52 (sec) , antiderivative size = 2577, normalized size of antiderivative = 9.87

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^{5/2}(c + d\sin(e + fx))}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x, algo
rithm="fricas")
```

output
```
[1/64*(sqrt(2)*(((3*A + 5*B)*c^2 - 2*(7*A + 17*B)*c*d + (43*A - 3*B)*d^2)*
cos(f*x + e)^3 - 4*(3*A + 5*B)*c^2 + 8*(7*A + 17*B)*c*d - 4*(43*A - 3*B)*d
^2 + 3*((3*A + 5*B)*c^2 - 2*(7*A + 17*B)*c*d + (43*A - 3*B)*d^2)*cos(f*x +
 e)^2 - 2*((3*A + 5*B)*c^2 - 2*(7*A + 17*B)*c*d + (43*A - 3*B)*d^2)*cos(f*
x + e) - (4*(3*A + 5*B)*c^2 - 8*(7*A + 17*B)*c*d + 4*(43*A - 3*B)*d^2 - ((
3*A + 5*B)*c^2 - 2*(7*A + 17*B)*c*d + (43*A - 3*B)*d^2)*cos(f*x + e)^2 + 2
*((3*A + 5*B)*c^2 - 2*(7*A + 17*B)*c*d + (43*A - 3*B)*d^2)*cos(f*x + e))*s
in(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e
) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*c
os(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2
)*sin(f*x + e) - cos(f*x + e) - 2)) - 32*(4*B*a*c*d - 4*A*a*d^2 - (B*a*c*d
 - A*a*d^2)*cos(f*x + e)^3 - 3*(B*a*c*d - A*a*d^2)*cos(f*x + e)^2 + 2*(B*a
*c*d - A*a*d^2)*cos(f*x + e) + (4*B*a*c*d - 4*A*a*d^2 - (B*a*c*d - A*a*d^2
)*cos(f*x + e)^2 + 2*(B*a*c*d - A*a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(
d/(a*c + a*d))*log((d^2*cos(f*x + e)^3 - (6*c*d + 7*d^2)*cos(f*x + e)^2 -
c^2 - 2*c*d - d^2 - 4*((c*d + d^2)*cos(f*x + e)^2 - c^2 - 4*c*d - 3*d^2 -
(c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (c^2 + 4*c*d + 3*d^2 + (c*d + d^2)*co
s(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d/(a*c + a*d)) - (
c^2 + 8*c*d + 9*d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - c^2 - 2*c*d - d^
2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 +...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^{5/2}(c + d\sin(e + fx))}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))}\,dx = \int \frac{B\sin(fx+e)+A}{(a\sin(fx+e)+a)^{\frac{5}{2}}(d\sin(fx+e)+c)}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x, algo
rithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e)
 + c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))}\,dx = \int \frac{A+B\,\sin(e+f\,x)}{(a+a\,\sin(e+f\,x))^{5/2}\,(c+d\,\sin(e+f\,x))}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x)))
,x)
```

output

```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x)))
, x)
```

Reduce [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\sin(fx+e)+1}}{\sin(fx+e)^4 d+\sin(fx+e)^3 c+3\sin(fx+e)^3 d+3\sin(fx+e)^2 c+3\sin}\right.\right.}{}$$

input

```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x)
```

output

```
(sqrt(a)*(int(sqrt(sin(e + f*x) + 1)/(sin(e + f*x)**4*d + sin(e + f*x)**3*
c + 3*sin(e + f*x)**3*d + 3*sin(e + f*x)**2*c + 3*sin(e + f*x)**2*d + 3*si
n(e + f*x)*c + sin(e + f*x)*d + c),x)*a + int((sqrt(sin(e + f*x) + 1)*sin(
e + f*x))/(sin(e + f*x)**4*d + sin(e + f*x)**3*c + 3*sin(e + f*x)**3*d + 3
*sin(e + f*x)**2*c + 3*sin(e + f*x)**2*d + 3*sin(e + f*x)*c + sin(e + f*x)
*d + c),x)*b))/a**3
```

3.326 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 395

$$\begin{aligned}\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2}\,dx &= -\frac{(B(5c^2-58cd-43d^2)+A(3c^2-22cd+115d^2))\,\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}(c-d)^4f}\\
&\quad+\frac{d^{3/2}(Ad(7c+5d)-B(5c^2+5cd+2d^2))\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{a^{5/2}(c-d)^4(c+d)^{3/2}f}\\
&\quad-\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))}\\
&\quad-\frac{(3Ac+5Bc-15Ad+7Bd)\cos(e+fx)}{16a(c-d)^2f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\\
&\quad-\frac{d(A(3c^2-16cd-35d^2)+B(5c^2+32cd+11d^2))\cos(e+fx)}{16a^2(c-d)^3(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}\end{aligned}$$

output
```
-1/32*(B*(5*c^2-58*c*d-43*d^2)+A*(3*c^2-22*c*d+115*d^2))*arctanh(1/2*a^(1/
2)*cos(f*x+e)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))*2^(1/2)/a^(5/2)/(c-d)^4/f+d^
(3/2)*(A*d*(7*c+5*d)-B*(5*c^2+5*c*d+2*d^2))*arctanh(a^(1/2)*d^(1/2)*cos(f*
x+e)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/(c-d)^4/(c+d)^(3/2)/f-1/4
*(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))-1/16*(3*
A*c-15*A*d+5*B*c+7*B*d)*cos(f*x+e)/a/(c-d)^2/f/(a+a*sin(f*x+e))^(3/2)/(c+d
*sin(f*x+e))-1/16*d*(A*(3*c^2-16*c*d-35*d^2)+B*(5*c^2+32*c*d+11*d^2))*cos(
f*x+e)/a^2/(c-d)^3/(c+d)/f/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 19.92 (sec) , antiderivative size = 1680, normalized size of antiderivative = 4.25

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^{5/2}(c + d\sin(e + fx))^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e +
f*x])^2),x]
```

output
```
((1 + I)*(3*A*c^2 + 5*B*c^2 - 22*A*c*d - 58*B*c*d + 115*A*d^2 - 43*B*d^2)*
ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[(e + f*x)/4]*(Cos[(e + f*x)/4] - Sin[(e
 + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/((16*(-1)^(1/4)*c^4
- 64*(-1)^(1/4)*c^3*d + 96*(-1)^(1/4)*c^2*d^2 - 64*(-1)^(1/4)*c*d^3 + 16*(
-1)^(1/4)*d^4)*f*(a*(1 + Sin[e + f*x]))^(5/2)) + (d^(3/2)*(A*d*(7*c + 5*d)
 - B*(5*c^2 + 5*c*d + 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + RootS
um[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(d*Log[-#1 + Tan[(e + f
*x)/4]]) + Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - c*Log[-#1 + T
an[(e + f*x)/4]]*#1 + 2*Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1
 + 3*d*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - Sqrt[d]*Sqrt[c + d]*Log[-#1 + Ta
n[(e + f*x)/4]]*#1^2 - c*Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*
d*#1^2 - c*#1^3) & ])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/(4*(c - d)^
4*(c + d)^(3/2)*f*(a*(1 + Sin[e + f*x]))^(5/2)) + (d^(3/2)*(-(A*d*(7*c + 5
*d)) + B*(5*c^2 + 5*c*d + 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + R
ootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(d*Log[-#1 + Tan[(e
 + f*x)/4]]) - Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - c*Log[-#1
 + Tan[(e + f*x)/4]]*#1 - 2*Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]
]*#1 + 3*d*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + Sqrt[d]*Sqrt[c + d]*Log[-#1
+ Tan[(e + f*x)/4]]*#1^2 - c*Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1
+ 3*d*#1^2 - c*#1^3) & ])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/(4*(...
```

Rubi [A] (verified)

Time = 2.52 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.11, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.432$, Rules used = {3042, 3457, 27, 3042, 3457, 27, 3042, 3463, 25, 3042, 3464, 3042, 3128, 219, 3252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^2}dx$$

$$\Big\downarrow \text{ 3457}$$

$$-\frac{\int -\frac{a(3Ac+5Bc-10Ad+2Bd)+5a(A-B)d\sin(e+fx)}{2(\sin(e+fx)a+a)^{3/2}(c+d\sin(e+fx))^2}dx}{4a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{a(3Ac+5Bc-10Ad+2Bd)+5a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)^{3/2}(c+d\sin(e+fx))^2}dx}{8a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{a(3Ac+5Bc-10Ad+2Bd)+5a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)^{3/2}(c+d\sin(e+fx))^2}dx}{8a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))}$$

$\Big\downarrow$ 3457

$$\frac{-\frac{\int -\frac{\left(B\left(5c^2-43dc-22d^2\right)+A\left(3c^2-13dc+70d^2\right)\right)a^2+3d(3A(c-5d)+B(5c+7d))\sin(e+fx)a^2}{2\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2}dx}{2a^2(c-d)}-\frac{a(3Ac-15Ad+5Bc+7Bd)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))}}{8a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{\left(B\left(5c^2-43dc-22d^2\right)+A\left(3c^2-13dc+70d^2\right)\right)a^2+3d(3A(c-5d)+B(5c+7d))\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2}dx}{4a^2(c-d)}-\frac{a(3Ac-15Ad+5Bc+7Bd)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))}}{8a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{\left(B\left(5c^2-43dc-22d^2\right)+A\left(3c^2-13dc+70d^2\right)\right)a^2+3d(3A(c-5d)+B(5c+7d))\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^2}dx}{4a^2(c-d)}-\frac{a(3Ac-15Ad+5Bc+7Bd)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))}}{8a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))}$$

$\Big\downarrow$ 3463

$$\frac{-\frac{\int -\frac{\left(B\left(5c^3-48dc^2-69d^2c-32d^3\right)+A\left(3c^3-16dc^2+77d^2c+80d^3\right)\right)a^3+d\left(A\left(3c^2-16dc-35d^2\right)+B\left(5c^2+32dc+11d^2\right)\right)\sin(e+fx)a^3}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{a\left(c^2-d^2\right)}-\frac{2a^2d\left(A\left(3c^2-16cd-3\cdots\right.\right.}{f\left(c^2-d^2\right)\sqrt{\cdots}}}{4a^2(c-d)}\cdots}{8a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))}$$

$\big\downarrow$ 25

$$\frac{\frac{\int \frac{\left(B\left(5c^3-48dc^2-69d^2c-32d^3\right)+A\left(3c^3-16dc^2+77d^2c+80d^3\right)\right)a^3+d\left(A\left(3c^2-16dc-35d^2\right)+B\left(5c^2+32dc+11d^2\right)\right)\sin(e+fx)a^3}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{a\left(c^2-d^2\right)}-\frac{2a^2d\left(A\left(3c^2-16cd-35d^2\cdots\right.\right.}{f\left(c^2-d^2\right)\sqrt{a\sin\cdots}}}{4a^2(c-d)}\cdots}{8a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))}$$

$\big\downarrow$ 3042

$$\frac{\frac{\int \frac{\left(B\left(5c^3-48dc^2-69d^2c-32d^3\right)+A\left(3c^3-16dc^2+77d^2c+80d^3\right)\right)a^3+d\left(A\left(3c^2-16dc-35d^2\right)+B\left(5c^2+32dc+11d^2\right)\right)\sin(e+fx)a^3}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}dx}{a\left(c^2-d^2\right)}-\frac{2a^2d\left(A\left(3c^2-16cd-35d^2\cdots\right.\right.}{f\left(c^2-d^2\right)\sqrt{a\sin\cdots}}}{4a^2(c-d)}\cdots}{8a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))}$$

$\big\downarrow$ 3464

$$\frac{\frac{\frac{a^3(c+d)\left(A\left(3c^2-22cd+115d^2\right)+B\left(5c^2-58cd-43d^2\right)\right)\int\frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{c-d}-\frac{16a^2d^2\left(Ad(7c+5d)-B\left(5c^2+5cd+2d^2\right)\right)\int\frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}}{a\left(c^2-d^2\right)}-\frac{2a^2d\left(A\left(3c^2-\cdots\right.\right.}{f\left(c^2-\cdots\right.}}{4a^2(c-d)}\cdots}{8a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))}$$

$\big\downarrow$ 3042

$$\frac{\frac{\frac{a^3(c+d)\left(A\left(3c^2-22cd+115d^2\right)+B\left(5c^2-58cd-43d^2\right)\right)\int\frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{c-d}-\frac{16a^2d^2\left(Ad(7c+5d)-B\left(5c^2+5cd+2d^2\right)\right)\int\frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}}{a\left(c^2-d^2\right)}-\frac{2a^2d\left(A\left(3c^2-\cdots\right.\right.}{f\left(c^2-\cdots\right.}}{4a^2(c-d)}\cdots}{8a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))}$$

$\downarrow$ 3128

$$-\frac{\frac{-\frac{16a^2d^2\left(Ad(7c+5d)-B\left(5c^2+5cd+2d^2\right)\right)\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}-\frac{2a^3(c+d)\left(A\left(3c^2-22cd+115d^2\right)+B\left(5c^2-58cd-43d^2\right)\right)\int \frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f(c-d)}}{a\left(c^2-d^2\right)}}{4a^2(c-d)}}{8a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))}$$

$\downarrow$ 219

$$-\frac{\frac{-\frac{16a^2d^2\left(Ad(7c+5d)-B\left(5c^2+5cd+2d^2\right)\right)\int \frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}-\frac{\sqrt{2}a^{5/2}(c+d)\left(A\left(3c^2-22cd+115d^2\right)+B\left(5c^2-58cd-43d^2\right)\right)\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f(c-d)}}{a\left(c^2-d^2\right)}}{4a^2(c-d)}}{8a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))}$$

$\downarrow$ 3252

$$\frac{\frac{\frac{32a^3d^2\left(Ad(7c+5d)-B\left(5c^2+5cd+2d^2\right)\right)\int \frac{1}{a(c+d)-\frac{a^2d\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f(c-d)}-\frac{\sqrt{2}a^{5/2}(c+d)\left(A\left(3c^2-22cd+115d^2\right)+B\left(5c^2-58cd-43d^2\right)\right)\text{arctanh}}{f(c-d)}}{a\left(c^2-d^2\right)}}{4a^2(c-d)}}{8a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))}$$

$\downarrow$ 221

$$\frac{\frac{\frac{32a^{5/2}d^{3/2}\left(Ad(7c+5d)-B\left(5c^2+5cd+2d^2\right)\right)\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{f(c-d)\sqrt{c+d}}-\frac{\sqrt{2}a^{5/2}(c+d)\left(A\left(3c^2-22cd+115d^2\right)+B\left(5c^2-58cd-43d^2\right)\right)\text{arctanh}\left(\right.}{f(c-d)}}{a\left(c^2-d^2\right)}}{4a^2(c-d)}}{8a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^
2),x]
```

output
```
-1/4*((A - B)*Cos[e + f*x])/((c - d)*f*(a + a*Sin[e + f*x])^(5/2)*(c + d*S
in[e + f*x])) + (-1/2*(a*(3*A*c + 5*B*c - 15*A*d + 7*B*d)*Cos[e + f*x])/((
c - d)*f*(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])) + ((-((Sqrt[2]*a
^(5/2)*(c + d)*(B*(5*c^2 - 58*c*d - 43*d^2) + A*(3*c^2 - 22*c*d + 115*d^2)
)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/((c
- d)*f)) + (32*a^(5/2)*d^(3/2)*(A*d*(7*c + 5*d) - B*(5*c^2 + 5*c*d + 2*d^2
))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e +
f*x]])])/((c - d)*Sqrt[c + d]*f))/(a*(c^2 - d^2)) - (2*a^2*d*(A*(3*c^2 - 1
6*c*d - 35*d^2) + B*(5*c^2 + 32*c*d + 11*d^2))*Cos[e + f*x])/((c^2 - d^2)*
f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])))/(4*a^2*(c - d)))/(8*a^2*
(c - d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

rule 3464
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A
*b - a*B)/(b*c - a*d)   Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Simp[(B*c
- A*d)/(b*c - a*d)   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4091 vs. $2(358) = 716$.

Time = 0.56 (sec) , antiderivative size = 4092, normalized size of antiderivative = 10.36

method	result	size
default	Expression too large to display	4092

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x,method=_R
ETURNVERBOSE)
```

output
```
1/32*(6*A*(-a*(sin(f*x+e)-1))^(3/2)*a^(1/2)*((c+d)*a*d)^(1/2)*c^4+101*B*2^
(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*((c+d)*a*d)^(
1/2)*sin(f*x+e)*a^2*c^3*d+255*B*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1
/2)*2^(1/2)/a^(1/2))*((c+d)*a*d)^(1/2)*sin(f*x+e)*a^2*c^2*d^2+43*B*2^(1/2)
*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*((c+d)*a*d)^(1/2)*
sin(f*x+e)^2*a^2*c^3*d-3*A*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2
^(1/2)/a^(1/2))*((c+d)*a*d)^(1/2)*sin(f*x+e)^3*a^2*c^3*d+19*A*2^(1/2)*arct
anh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*((c+d)*a*d)^(1/2)*sin(f
*x+e)^3*a^2*c^2*d^2-93*A*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(
1/2)/a^(1/2))*((c+d)*a*d)^(1/2)*sin(f*x+e)^3*a^2*c*d^3-5*B*2^(1/2)*arctanh
(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*((c+d)*a*d)^(1/2)*sin(f*x+
e)^3*a^2*c^3*d+53*B*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/
a^(1/2))*((c+d)*a*d)^(1/2)*sin(f*x+e)^3*a^2*c^2*d^2+101*B*2^(1/2)*arctanh(
1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*((c+d)*a*d)^(1/2)*sin(f*x+e
)^3*a^2*c*d^3+13*A*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a
^(1/2))*((c+d)*a*d)^(1/2)*sin(f*x+e)^2*a^2*c^3*d-55*A*2^(1/2)*arctanh(1/2*
(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*((c+d)*a*d)^(1/2)*sin(f*x+e)^2*
a^2*c^2*d^2-301*A*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^
(1/2))*((c+d)*a*d)^(1/2)*sin(f*x+e)^2*a^2*c*d^3+187*B*2^(1/2)*arctanh(1/2*
(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*((c+d)*a*d)^(1/2)*sin(f*x+e)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2433 vs. $2(358) = 716$.

Time = 7.95 (sec) , antiderivative size = 5151, normalized size of antiderivative = 13.04

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^{5/2}(c + d\sin(e + fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x, al
gorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^{5/2}(c + d\sin(e + fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**2,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{A + B\sin(e + fx)}{(a + a\sin(e + fx))^{5/2}(c + d\sin(e + fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x, al
gorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2}\,dx = \int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^2}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^
2),x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^
2), x)
```

Reduce [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{}{\sin(fx+e)^5 d^2+2\sin(fx+e)^4 cd+3\sin(fx+e)^4 d^2+\sin(fx+e)^3 c^2}\right.\right.}{}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x)
```

output

```
(sqrt(a)*(int(sqrt(sin(e + f*x) + 1)/(sin(e + f*x)**5*d**2 + 2*sin(e + f*x
)**4*c*d + 3*sin(e + f*x)**4*d**2 + sin(e + f*x)**3*c**2 + 6*sin(e + f*x)*
*3*c*d + 3*sin(e + f*x)**3*d**2 + 3*sin(e + f*x)**2*c**2 + 6*sin(e + f*x)*
*2*c*d + sin(e + f*x)**2*d**2 + 3*sin(e + f*x)*c**2 + 2*sin(e + f*x)*c*d +
 c**2),x)*a + int((sqrt(sin(e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**5*d
**2 + 2*sin(e + f*x)**4*c*d + 3*sin(e + f*x)**4*d**2 + sin(e + f*x)**3*c**
2 + 6*sin(e + f*x)**3*c*d + 3*sin(e + f*x)**3*d**2 + 3*sin(e + f*x)**2*c**
2 + 6*sin(e + f*x)**2*c*d + sin(e + f*x)**2*d**2 + 3*sin(e + f*x)*c**2 + 2
*sin(e + f*x)*c*d + c**2),x)*b))/a**3
```

3.327 $\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 519

$$\begin{aligned}
&\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3}\,dx = \\
&-\frac{(B(5c^2-82cd-115d^2)+3A(c^2-10cd+73d^2))\,\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}(c-d)^5f} \\
&+\frac{d^{3/2}(3Ad(21c^2+30cd+13d^2)-B(35c^3+70c^2d+67cd^2+20d^3))\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a+a\sin(e+fx)}}\right)}{4a^{5/2}(c-d)^5(c+d)^{5/2}f} \\
&-\frac{(A-B)\cos(e+fx)}{4(c-d)f(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^2} \\
&-\frac{(3Ac+5Bc-19Ad+11Bd)\cos(e+fx)}{16a(c-d)^2f(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))^2} \\
&-\frac{d(A(3c^2-20cd-31d^2)+B(5c^2+28cd+15d^2))\cos(e+fx)}{16a^2(c-d)^3(c+d)f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^2} \\
&-\frac{d(3A(c^3-7c^2d-37cd^2-21d^3)+B(5c^3+73c^2d+79cd^2+35d^3))\cos(e+fx)}{16a^2(c-d)^4(c+d)^2f\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))}
\end{aligned}$$

output
```
-1/32*(B*(5*c^2-82*c*d-115*d^2)+3*A*(c^2-10*c*d+73*d^2))*arctanh(1/2*a^(1/
2)*cos(f*x+e)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))*2^(1/2)/a^(5/2)/(c-d)^5/f+1/
4*d^(3/2)*(3*A*d*(21*c^2+30*c*d+13*d^2)-B*(35*c^3+70*c^2*d+67*c*d^2+20*d^3
))*arctanh(a^(1/2)*d^(1/2)*cos(f*x+e)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/
a^(5/2)/(c-d)^5/(c+d)^(5/2)/f-1/4*(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e)
)^(5/2)/(c+d*sin(f*x+e))^2-1/16*(3*A*c-19*A*d+5*B*c+11*B*d)*cos(f*x+e)/a/(
c-d)^2/f/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2-1/16*d*(A*(3*c^2-20*c*d
-31*d^2)+B*(5*c^2+28*c*d+15*d^2))*cos(f*x+e)/a^2/(c-d)^3/(c+d)/f/(a+a*sin(
f*x+e))^(1/2)/(c+d*sin(f*x+e))^2-1/16*d*(3*A*(c^3-7*c^2*d-37*c*d^2-21*d^3)
+B*(5*c^3+73*c^2*d+79*c*d^2+35*d^3))*cos(f*x+e)/a^2/(c-d)^4/(c+d)^2/f/(a+a
*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 19.39 (sec) , antiderivative size = 2465, normalized size of antiderivative = 4.75

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e +
f*x])^3),x]
```

output
```
((1 + I)*(3*A*c^2 + 5*B*c^2 - 30*A*c*d - 82*B*c*d + 219*A*d^2 - 115*B*d^2)
*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[(e + f*x)/4]*(Cos[(e + f*x)/4] - Sin[(
e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/((16*(-1)^(1/4)*c^5
 - 80*(-1)^(1/4)*c^4*d + 160*(-1)^(1/4)*c^3*d^2 - 160*(-1)^(1/4)*c^2*d^3 +
 80*(-1)^(1/4)*c*d^4 - 16*(-1)^(1/4)*d^5)*f*(a*(1 + Sin[e + f*x]))^(5/2))
- (d^(3/2)*(-3*A*d*(21*c^2 + 30*c*d + 13*d^2) + B*(35*c^3 + 70*c^2*d + 67*
c*d^2 + 20*d^3))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + RootSum[c + 4*d*#1
 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-(d*Log[-#1 + Tan[(e + f*x)/4]]) + Sq
rt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]] - c*Log[-#1 + Tan[(e + f*x)/
4]]*#1 + 2*Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1 + 3*d*Log[-#
1 + Tan[(e + f*x)/4]]*#1^2 - Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f*x)/4
]]*#1^2 - c*Log[-#1 + Tan[(e + f*x)/4]]*#1^3)/(-d - c*#1 + 3*d*#1^2 - c*#1
^3) & ])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/(16*(c - d)^5*(c + d)^(5
/2)*f*(a*(1 + Sin[e + f*x]))^(5/2)) + (d^(3/2)*(-3*A*d*(21*c^2 + 30*c*d +
13*d^2) + B*(35*c^3 + 70*c^2*d + 67*c*d^2 + 20*d^3))*(e + f*x - 2*Log[Sec[
(e + f*x)/4]^2] + RootSum[c + 4*d*#1 + 2*c*#1^2 - 4*d*#1^3 + c*#1^4 & , (-
(d*Log[-#1 + Tan[(e + f*x)/4]]) - Sqrt[d]*Sqrt[c + d]*Log[-#1 + Tan[(e + f
*x)/4]] - c*Log[-#1 + Tan[(e + f*x)/4]]*#1 - 2*Sqrt[d]*Sqrt[c + d]*Log[-#1
 + Tan[(e + f*x)/4]]*#1 + 3*d*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 + Sqrt[d]*S
qrt[c + d]*Log[-#1 + Tan[(e + f*x)/4]]*#1^2 - c*Log[-#1 + Tan[(e + f*x)...
```

Rubi [A] (verified)

Time = 3.36 (sec) , antiderivative size = 574, normalized size of antiderivative = 1.11, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.514$, Rules used = {3042, 3457, 27, 3042, 3457, 27, 3042, 3463, 27, 3042, 3463, 25, 3042, 3464, 3042, 3128, 219, 3252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^3}dx$$

$$\Big\downarrow \text{3457}$$

$$-\frac{\int -\frac{a(3Ac+5Bc-12Ad+4Bd)+7a(A-B)d\sin(e+fx)}{2(\sin(e+fx)a+a)^{3/2}(c+d\sin(e+fx))^3}dx}{4a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^2}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{a(3Ac+5Bc-12Ad+4Bd)+7a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)^{3/2}(c+d\sin(e+fx))^3}dx}{8a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{a(3Ac+5Bc-12Ad+4Bd)+7a(A-B)d\sin(e+fx)}{(\sin(e+fx)a+a)^{3/2}(c+d\sin(e+fx))^3}dx}{8a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^2}$$

$\Big\downarrow$ 3457

$$\frac{-\frac{\int -\frac{\left(B\left(5c^2-57dc-60d^2\right)+A\left(3c^2-15dc+124d^2\right)\right)a^2+5d(3Ac+5Bc-19Ad+11Bd)\sin(e+fx)a^2}{2\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^3}dx}{2a^2(c-d)}-\frac{a(3Ac-19Ad+5Bc+11Bd)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}}{8a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{\left(B\left(5c^2-57dc-60d^2\right)+A\left(3c^2-15dc+124d^2\right)\right)a^2+5d(3Ac+5Bc-19Ad+11Bd)\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^3}dx}{4a^2(c-d)}-\frac{a(3Ac-19Ad+5Bc+11Bd)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}}{8a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{\left(B\left(5c^2-57dc-60d^2\right)+A\left(3c^2-15dc+124d^2\right)\right)a^2+5d(3Ac+5Bc-19Ad+11Bd)\sin(e+fx)a^2}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^3}dx}{4a^2(c-d)}-\frac{a(3Ac-19Ad+5Bc+11Bd)\cos(e+fx)}{2f(c-d)(a\sin(e+fx)+a)^{3/2}(c+d\sin(e+fx))^2}}{8a^2(c-d)}-\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^2}$$

$\Big\downarrow$ 3463

$$-\frac{\frac{-\frac{\int -\frac{2\left(\left(B\left(5c^3-62dc^2-113d^2c-70d^3\right)+3A\left(c^3-6dc^2+43d^2c+42d^3\right)\right)a^3+3d\left(A\left(3c^2-20dc-31d^2\right)+B\left(5c^2+28dc+15d^2\right)\right)\sin(e+fx)a^3\right)}{\sqrt{\sin(e+fx)a+a}\,(c+d\sin(e+fx))^2}dx}{2a\left(c^2-d^2\right)}-\frac{2a^2d\left(A\left(3c^2-20\right.\right.}{f\left(c^2-d^2\right.}}{4a^2(c-d)}}{8a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^2}$$

$\big\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{\left(B\left(5c^3-62dc^2-113d^2c-70d^3\right)+3A\left(c^3-6dc^2+43d^2c+42d^3\right)\right)a^3+3d\left(A\left(3c^2-20dc-31d^2\right)+B\left(5c^2+28dc+15d^2\right)\right)\sin(e+fx)a^3}{\sqrt{\sin(e+fx)a+a}\,(c+d\sin(e+fx))^2}dx}{a\left(c^2-d^2\right)}-\frac{2a^2d\left(A\left(3c^2-20cd-31d\right.\right.}{f\left(c^2-d^2\right)\sqrt{a\sin}}}{4a^2(c-d)}}{8a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^2}$$

$\big\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{\left(B\left(5c^3-62dc^2-113d^2c-70d^3\right)+3A\left(c^3-6dc^2+43d^2c+42d^3\right)\right)a^3+3d\left(A\left(3c^2-20dc-31d^2\right)+B\left(5c^2+28dc+15d^2\right)\right)\sin(e+fx)a^3}{\sqrt{\sin(e+fx)a+a}\,(c+d\sin(e+fx))^2}dx}{a\left(c^2-d^2\right)}-\frac{2a^2d\left(A\left(3c^2-20cd-31d\right.\right.}{f\left(c^2-d^2\right)\sqrt{a\sin}}}{4a^2(c-d)}}{8a^2(c-d)}$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^2}$$

$\big\downarrow$ 3463

$$-\frac{\frac{\frac{\int -\frac{\left(B\left(5c^4-67dc^3-201d^2c^2-233d^3c-80d^4\right)+3A\left(c^4-7dc^3+47d^2c^2+99d^3c+52d^4\right)\right)a^4+d\left(3A\left(c^3-7dc^2-37d^2c-21d^3\right)+B\left(5c^3+73dc^2+79d^2c+35d^3\right)\right)\sin(e\ldots}{\sqrt{\sin(e+fx)a+a}\,(c+d\sin(e+fx))}}{a\left(c^2-d^2\right)}}{a\left(c^2-d^2\right)}}{4a^2(c-\ldots}$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^2}$$

$\big\downarrow$ 25

$$\frac{\frac{\frac{\int \frac{\left(B\left(5c^4-67dc^3-201d^2c^2-233d^3c-80d^4\right)+3A\left(c^4-7dc^3+47d^2c^2+99d^3c+52d^4\right)\right)a^4+d\left(3A\left(c^3-7dc^2-37d^2c-21d^3\right)+B\left(5c^3+73dc^2+79d^2c+35d^3\right)\right)\sin(e+f}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}}{a\left(c^2-d^2\right)}}{a\left(c^2-d^2\right)}}{4a^2(c-d}$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\int \frac{\left(B\left(5c^4-67dc^3-201d^2c^2-233d^3c-80d^4\right)+3A\left(c^4-7dc^3+47d^2c^2+99d^3c+52d^4\right)\right)a^4+d\left(3A\left(c^3-7dc^2-37d^2c-21d^3\right)+B\left(5c^3+73dc^2+79d^2c+35d^3\right)\right)\sin(e+f}{\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))}}{a\left(c^2-d^2\right)}}{a\left(c^2-d^2\right)}}{4a^2(c-d}$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \text{3464}$$

$$\frac{\frac{\frac{\frac{a^4(c+d)^2\left(3A\left(c^2-10cd+73d^2\right)+B\left(5c^2-82cd-115d^2\right)\right)\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{c-d}-\frac{4a^3d^2\left(3Ad\left(21c^2+30cd+13d^2\right)-B\left(35c^3+70c^2d+67cd^2+20d^3\right)\right)\int \frac{\sqrt{\sin(e+fx}}{c+d\sin(e+}}{c-d}}{a\left(c^2-d^2\right)}}{a\left(c^2-d^2\right)}}{4a^2(c-d}$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\frac{a^4(c+d)^2\left(3A\left(c^2-10cd+73d^2\right)+B\left(5c^2-82cd-115d^2\right)\right)\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{c-d}-\frac{4a^3d^2\left(3Ad\left(21c^2+30cd+13d^2\right)-B\left(35c^3+70c^2d+67cd^2+20d^3\right)\right)\int \frac{\sqrt{\sin(e+fx}}{c+d\sin(e+}}{c-d}}{a\left(c^2-d^2\right)}}{a\left(c^2-d^2\right)}}{4a^2(c-d}$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \text{3128}$$

$$\frac{\frac{\frac{-\frac{4a^3d^2\left(3Ad\left(21c^2+30cd+13d^2\right)-B\left(35c^3+70c^2d+67cd^2+20d^3\right)\right)\int\frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}-\frac{2a^4(c+d)^2\left(3A\left(c^2-10cd+73d^2\right)+B\left(5c^2-82cd-115d^2\right)\right)\int\frac{1}{2a-\frac{a^2c\cdots}{\sin(\cdots}}}{f(c-d)}}{a\left(c^2-d^2\right)}\cdots}{a\left(c^2-d^2\right)}\cdots}{\cdots}$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\frac{-\frac{4a^3d^2\left(3Ad\left(21c^2+30cd+13d^2\right)-B\left(35c^3+70c^2d+67cd^2+20d^3\right)\right)\int\frac{\sqrt{\sin(e+fx)a+a}}{c+d\sin(e+fx)}dx}{c-d}-\frac{\sqrt{2}a^{7/2}(c+d)^2\left(3A\left(c^2-10cd+73d^2\right)+B\left(5c^2-82cd-115d^2\right)\right)\text{arcta}\cdots}{f(c-d)}}{a\left(c^2-d^2\right)}\cdots}{a\left(c^2-d^2\right)}\cdots}{\cdots}$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \; 3252$$

$$\frac{\frac{\frac{\frac{8a^4d^2\left(3Ad\left(21c^2+30cd+13d^2\right)-B\left(35c^3+70c^2d+67cd^2+20d^3\right)\right)\int\frac{1}{a(c+d)-\frac{a^2d\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f(c-d)}-\frac{\sqrt{2}a^{7/2}(c+d)^2\left(3A\left(c^2-10cd+73d^2\right)+B\left(5c^2\cdots\right.\right.}{f\cdots}}{a\left(c^2-d^2\right)}\cdots}{a\left(c^2-d^2\right)}\cdots}{\cdots}$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{\frac{8a^{7/2}d^{3/2}\left(3Ad\left(21c^2+30cd+13d^2\right)-B\left(35c^3+70c^2d+67cd^2+20d^3\right)\right)\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{c+d}\sqrt{a\sin(e+fx)+a}}\right)}{f(c-d)\sqrt{c+d}}-\frac{\sqrt{2}a^{7/2}(c+d)^2\left(3A\left(c^2-10cd+73d^2\right)+B\left(5c^2-82c\cdots\right.\right.}{f(c-\cdots}}{a\left(c^2-d^2\right)}\cdots}{a\left(c^2-d^2\right)}\cdots}{\cdots}$$

$$\frac{(A-B)\cos(e+fx)}{4f(c-d)(a\sin(e+fx)+a)^{5/2}(c+d\sin(e+fx))^2}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])^
3),x]
```

output
```
-1/4*((A - B)*Cos[e + f*x])/((c - d)*f*(a + a*Sin[e + f*x])^(5/2)*(c + d*S
in[e + f*x])^2) + (-1/2*(a*(3*A*c + 5*B*c - 19*A*d + 11*B*d)*Cos[e + f*x])
/((c - d)*f*(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^2) + ((-2*a^2*
d*(A*(3*c^2 - 20*c*d - 31*d^2) + B*(5*c^2 + 28*c*d + 15*d^2))*Cos[e + f*x]
)/((c^2 - d^2)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2) + ((-((S
qrt[2]*a^(7/2)*(c + d)^2*(B*(5*c^2 - 82*c*d - 115*d^2) + 3*A*(c^2 - 10*c*d
 + 73*d^2))*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x
]])])/((c - d)*f)) + (8*a^(7/2)*d^(3/2)*(3*A*d*(21*c^2 + 30*c*d + 13*d^2)
- B*(35*c^3 + 70*c^2*d + 67*c*d^2 + 20*d^3))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[
e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/((c - d)*Sqrt[c + d]*f)
)/(a*(c^2 - d^2)) - (2*a^3*d*(3*A*(c^3 - 7*c^2*d - 37*c*d^2 - 21*d^3) + B*
(5*c^3 + 73*c^2*d + 79*c*d^2 + 35*d^3))*Cos[e + f*x])/((c^2 - d^2)*f*Sqrt[
a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])))/(a*(c^2 - d^2)))/(4*a^2*(c - d)
))/(8*a^2*(c - d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

rule 3464
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A
*b - a*B)/(b*c - a*d)   Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Simp[(B*c
- A*d)/(b*c - a*d)   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 7321 vs. 2(476) = 952.

Time = 0.64 (sec) , antiderivative size = 7322, normalized size of antiderivative = 14.11

method	result	size
default	Expression too large to display	7322

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x,method=_R
ETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4135 vs. 2(476) = 952.

Time = 24.95 (sec) , antiderivative size = 8555, normalized size of antiderivative = 16.48

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x, al
gorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**3,x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x, al
gorithm="maxima")
```

output
```
Timed out
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x, al
gorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3}\,dx=\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^3}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^
3),x)
```

output
```
int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^
3), x)
```

Reduce [F]

$$\int \frac{A+B\sin(e+fx)}{(a+a\sin(e+fx))^{5/2}(c+d\sin(e+fx))^3}\,dx=\frac{\sqrt{a}\left(\left(\int \frac{}{\sin(fx+e)^6 d^3+3\sin(fx+e)^5 c\,d^2+3\sin(fx+e)^5 d^3+3\sin(fx+e)^4}\right.\right.}{}$$

input
```
int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x)
```

output
```
(sqrt(a)*(int(sqrt(sin(e + f*x) + 1)/(sin(e + f*x)**6*d**3 + 3*sin(e + f*x
)**5*c*d**2 + 3*sin(e + f*x)**5*d**3 + 3*sin(e + f*x)**4*c**2*d + 9*sin(e
+ f*x)**4*c*d**2 + 3*sin(e + f*x)**4*d**3 + sin(e + f*x)**3*c**3 + 9*sin(e
 + f*x)**3*c**2*d + 9*sin(e + f*x)**3*c*d**2 + sin(e + f*x)**3*d**3 + 3*si
n(e + f*x)**2*c**3 + 9*sin(e + f*x)**2*c**2*d + 3*sin(e + f*x)**2*c*d**2 +
 3*sin(e + f*x)*c**3 + 3*sin(e + f*x)*c**2*d + c**3),x)*a + int((sqrt(sin(
e + f*x) + 1)*sin(e + f*x))/(sin(e + f*x)**6*d**3 + 3*sin(e + f*x)**5*c*d*
*2 + 3*sin(e + f*x)**5*d**3 + 3*sin(e + f*x)**4*c**2*d + 9*sin(e + f*x)**4
*c*d**2 + 3*sin(e + f*x)**4*d**3 + sin(e + f*x)**3*c**3 + 9*sin(e + f*x)**
3*c**2*d + 9*sin(e + f*x)**3*c*d**2 + sin(e + f*x)**3*d**3 + 3*sin(e + f*x
)**2*c**3 + 9*sin(e + f*x)**2*c**2*d + 3*sin(e + f*x)**2*c*d**2 + 3*sin(e
+ f*x)*c**3 + 3*sin(e + f*x)*c**2*d + c**3),x)*b))/a**3
```

3.328 $\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal result

Integrand size = 35, antiderivative size = 221

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx = -\frac{8\sqrt{2}a^2B\,\text{AppellF1}\left(\frac{1}{2},-\frac{5}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx))^n\,(}{f\sqrt{1+\sin(e+fx)}} - \frac{4\sqrt{2}a^2(A-B)\,\text{AppellF1}\left(\frac{1}{2},-\frac{3}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+f}{f\sqrt{1+\sin(e+fx)}}$$

output
```
-8*2^(1/2)*a^2*B*AppellF1(1/2,-n,-5/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*s
in(f*x+e))*cos(f*x+e)*(c+d*sin(f*x+e))^n/f/(1+sin(f*x+e))^(1/2)/(((c+d*sin
(f*x+e))/(c+d))^n)-4*2^(1/2)*a^2*(A-B)*AppellF1(1/2,-n,-3/2,3/2,d*(1-sin(f
*x+e))/(c+d),1/2-1/2*sin(f*x+e))*cos(f*x+e)*(c+d*sin(f*x+e))^n/f/(1+sin(f*
x+e))^(1/2)/(((c+d*sin(f*x+e))/(c+d))^n)
```

Mathematica [F]

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$=\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$

input
```
Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])
^n,x]
```

output
```
Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])
^n, x]
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3042, 3466, 3042, 3263, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^2(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int (a\sin(e+fx)+a)^2(A+B\sin(e+fx))(c+d\sin(e+fx))^n dx$$
$$\Big\downarrow \text{ 3466}$$
$$(A-B)\int (\sin(e+fx)a+a)^2(c+d\sin(e+fx))^n dx+\frac{B\int (\sin(e+fx)a+a)^3(c+d\sin(e+fx))^n dx}{a}$$
$$\Big\downarrow \text{ 3042}$$

$$\frac{(A-B)\int (\sin(e+fx)a+a)^2(c+d\sin(e+fx))^n dx+B\int (\sin(e+fx)a+a)^3(c+d\sin(e+fx))^n dx}{a}$$

$\Big\downarrow$ 3263

$$\frac{a^2(A-B)\cos(e+fx)\int \frac{(\sin(e+fx)+1)^{3/2}(c+d\sin(e+fx))^n}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{1-\sin(e+fx)}\sqrt{\sin(e+fx)+1}}+\frac{a^2B\cos(e+fx)\int \frac{(\sin(e+fx)+1)^{5/2}(c+d\sin(e+fx))^n}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{1-\sin(e+fx)}\sqrt{\sin(e+fx)+1}}$$

$\Big\downarrow$ 156

$$\frac{a^2(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\int \frac{(\sin(e+fx)+1)^{3/2}\left(\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}\right)^n}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{1-\sin(e+fx)}\sqrt{\sin(e+fx)+1}}+\frac{a^2B\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\int \frac{(\sin(e+fx)+1)^{5/2}\left(\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}\right)^n}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{1-\sin(e+fx)}\sqrt{\sin(e+fx)+1}}$$

$\Big\downarrow$ 155

$$-\frac{4\sqrt{2}a^2(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},-\frac{3}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\right.}{f\sqrt{\sin(e+fx)+1}}$$
$$\frac{8\sqrt{2}a^2B\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},-\frac{5}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e\right.}{f\sqrt{\sin(e+fx)+1}}$$

input
```
Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

output
```
(-8*Sqrt[2]*a^2*B*AppellF1[1/2, -5/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1
 - Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1
+ Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n) - (4*Sqrt[2]*a^2*(A - B)
*AppellF1[1/2, -3/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))
/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1 + Sin[e + f*x]]*(
(c + d*Sin[e + f*x])/(c + d))^n)
```

Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3263
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^m*(Cos[e + f*x]/(f*Sqrt[1 + Sin[e +
 f*x]]*Sqrt[1 - Sin[e + f*x]]))   Subst[Int[(1 + (b/a)*x)^(m - 1/2)*((c + d
*x)^n/Sqrt[1 - (b/a)*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& IntegerQ[m]
```

rule 3466
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)/b   Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x
] + Simp[B/b   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [F]

$$\int (a + a\sin(fx + e))^2 (A + B\sin(fx + e)) (c + d\sin(fx + e))^n dx$$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

output
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

Fricas [F]

$$\int (a + a\sin(e + fx))^2 (A + B\sin(e + fx))(c + d\sin(e + fx))^n dx = \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^2 (d\sin(fx + e) + c)^n dx$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algori
thm="fricas")
```

output
```
integral(-((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x +
 e)^2 - 2*(A + B)*a^2)*sin(f*x + e))*(d*sin(f*x + e) + c)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^2 (A + B\sin(e + fx))(c + d\sin(e + fx))^n dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^2(d\sin(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algori
thm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)
^n, x)
```

Giac [F]

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^2(d\sin(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algori
thm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)
^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx = \int (A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^2\,(c+d\,\sin(e+f\,x))^n\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^n,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^n, x)
```

Reduce [F]

$$\int (a+a\sin(e+fx))^2(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx = a^2\left(\left(\int (\sin(fx+e)\,d+c)^n\,dx\right)a+\left(\int (\sin(fx+e)\,d+c)^n\sin(fx+e)^3\,dx\right)b + \left(\int (\sin(fx+e)\,d+c)^n\sin(fx+e)^2\,dx\right)a + 2\left(\int (\sin(fx+e)\,d+c)^n\sin(fx+e)^2\,dx\right)b + 2\left(\int (\sin(fx+e)\,d+c)^n\sin(fx+e)\,dx\right)a + \left(\int (\sin(fx+e)\,d+c)^n\sin(fx+e)\,dx\right)b\right)$$

input
```
int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

output
```
a**2*(int((sin(e + f*x)*d + c)**n,x)*a + int((sin(e + f*x)*d + c)**n*sin(e
 + f*x)**3,x)*b + int((sin(e + f*x)*d + c)**n*sin(e + f*x)**2,x)*a + 2*int
((sin(e + f*x)*d + c)**n*sin(e + f*x)**2,x)*b + 2*int((sin(e + f*x)*d + c)
**n*sin(e + f*x),x)*a + int((sin(e + f*x)*d + c)**n*sin(e + f*x),x)*b)
```

3.329 $\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal result

Integrand size = 33, antiderivative size = 217

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx =$$

$$-\frac{4\sqrt{2}aB\operatorname{AppellF1}\left(\frac{1}{2},-\frac{3}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\right.}{f\sqrt{1+\sin(e+fx)}}$$

$$-\frac{2\sqrt{2}a(A-B)\operatorname{AppellF1}\left(\frac{1}{2},-\frac{1}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+f}{f\sqrt{1+\sin(e+fx)}}$$

output
```
-4*2^(1/2)*a*B*AppellF1(1/2,-n,-3/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin
(f*x+e))*cos(f*x+e)*(c+d*sin(f*x+e))^n/f/(1+sin(f*x+e))^(1/2)/(((c+d*sin(f
*x+e))/(c+d))^n)-2*2^(1/2)*a*(A-B)*AppellF1(1/2,-n,-1/2,3/2,d*(1-sin(f*x+e
))/(c+d),1/2-1/2*sin(f*x+e))*cos(f*x+e)*(c+d*sin(f*x+e))^n/f/(1+sin(f*x+e)
)^(1/2)/(((c+d*sin(f*x+e))/(c+d))^n)
```

Mathematica [F]

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$=\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$

input
```
Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n
,x]
```

output
```
Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n
, x]
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3447, 3042, 3496, 3042, 3234, 156, 155, 3263, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)(A+B\sin(e+fx))(c+d\sin(e+fx))^ndx$$

$$\Big\downarrow \text{3447}$$

$$\int \left((aA+aB)\sin(e+fx)+aA+aB\sin^2(e+fx)\right)(c+d\sin(e+fx))^ndx$$

$$\Big\downarrow \text{3042}$$

$$\int \left((aA+aB)\sin(e+fx)+aA+aB\sin(e+fx)^2\right)(c+d\sin(e+fx))^ndx$$

$$\Big\downarrow \text{3496}$$

$$a(A-B)\int(\sin(e+fx)+1)(c+d\sin(e+fx))^n dx+aB\int(\sin(e+fx)+1)^2(c+d\sin(e+fx))^n dx$$

$$\Big\downarrow \text{3042}$$

$$a(A-B)\int(\sin(e+fx)+1)(c+d\sin(e+fx))^n dx+aB\int(\sin(e+fx)+1)^2(c+d\sin(e+fx))^n dx$$

$$\Big\downarrow \text{3234}$$

$$\frac{a(A-B)\cos(e+fx)\int\frac{\sqrt{\sin(e+fx)+1}(c+d\sin(e+fx))^n}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{1-\sin(e+fx)}\sqrt{\sin(e+fx)+1}}+aB\int(\sin(e+fx)+1)^2(c+d\sin(e+fx))^n dx$$

$$\Big\downarrow \text{156}$$

$$\frac{a(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\int\frac{\sqrt{\sin(e+fx)+1}\left(\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}\right)^n}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{1-\sin(e+fx)}\sqrt{\sin(e+fx)+1}}+aB\int(\sin(e+fx)+1)^2(c+d\sin(e+fx))^n dx$$

$$\Big\downarrow \text{155}$$

$$aB\int(\sin(e+fx)+1)^2(c+d\sin(e+fx))^n dx-\frac{2\sqrt{2}a(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},-\frac{1}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1}{}\right.}{f\sqrt{\sin(e+fx)+1}}$$

$$\Big\downarrow \text{3263}$$

$$\frac{aB\cos(e+fx)\int\frac{(\sin(e+fx)+1)^{3/2}(c+d\sin(e+fx))^n}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{1-\sin(e+fx)}\sqrt{\sin(e+fx)+1}}-\frac{2\sqrt{2}a(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},-\frac{1}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1}{}\right.}{f\sqrt{\sin(e+fx)+1}}$$

$$\Big\downarrow \text{156}$$

$$\frac{aB\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\int\frac{(\sin(e+fx)+1)^{3/2}\left(\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}\right)^n}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{1-\sin(e+fx)}\sqrt{\sin(e+fx)+1}}-\frac{2\sqrt{2}a(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},-\frac{1}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1}{}\right.}{f\sqrt{\sin(e+fx)+1}}$$

$$\Big\downarrow \text{155}$$

$$-\frac{2\sqrt{2}a(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},-\frac{1}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d}{}\right.}{f\sqrt{\sin(e+fx)+1}}$$
$$\frac{4\sqrt{2}aB\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},-\frac{3}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+}{c+d}\right.}{f\sqrt{\sin(e+fx)+1}}$$

input
```
Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

output
```
(-4*Sqrt[2]*a*B*AppellF1[1/2, -3/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 -
 Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1 +
Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n) - (2*Sqrt[2]*a*(A - B)*App
ellF1[1/2, -1/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c
+ d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[1 + Sin[e + f*x]]*((c +
 d*Sin[e + f*x])/(c + d))^n)
```

Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3234
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[c*(Cos[e + f*x]/(f*Sqrt[1 + Sin[e + f*x]]*Sq
rt[1 - Sin[e + f*x]]))   Subst[Int[(a + b*x)^m*(Sqrt[1 + (d/c)*x]/Sqrt[1 -
(d/c)*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && N
eQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m] && EqQ[c^2 - d^2,
0]
```

rule 3263
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^m*(Cos[e + f*x]/(f*Sqrt[1 + Sin[e +
 f*x]]*Sqrt[1 - Sin[e + f*x]]))   Subst[Int[(1 + (b/a)*x)^(m - 1/2)*((c + d
*x)^n/Sqrt[1 - (b/a)*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& IntegerQ[m]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3496
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A - C)
  Int[(a + b*Sin[e + f*x])^m*(1 + Sin[e + f*x]), x], x] + Simp[C   Int[(a +
 b*Sin[e + f*x])^m*(1 + Sin[e + f*x])^2, x], x] /; FreeQ[{a, b, e, f, A, B,
 C, m}, x] && EqQ[A - B + C, 0] &&  !IntegerQ[2*m]
```

Maple [F]

$$\int (a + a \sin (fx + e)) (A + B \sin (fx + e)) (c + d \sin (fx + e))^n dx$$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

output
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

Fricas [F]

$$\int (a + a \sin(e + fx))(A + B \sin(e + fx))(c + d \sin(e + fx))^n dx = \int (B \sin (fx + e) + A)(a \sin (fx + e) + a)(d \sin (fx + e) + c)^n dx$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorith
m="fricas")
```

output
```
integral(-(B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*(d*sin
(f*x + e) + c)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a \sin(e + fx))(A + B \sin(e + fx))(c + d \sin(e + fx))^n dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)(d\sin(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorith
m="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n
, x)
```

Giac [F]

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)(d\sin(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorith
m="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n
, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$=\int (A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)\,\left(c+d\,\sin\left(e+f\,x\right)\right)^n\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c + d*sin(e + f*x))^n,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c + d*sin(e + f*x))^n, x)
```

Reduce [F]

$$\int (a+a\sin(e+fx))(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$=a\left(\left(\int (\sin(fx+e)\,d+c)^n\,dx\right)a+\left(\int (\sin(fx+e)\,d+c)^n\sin(fx+e)^2\,dx\right)b+\left(\int (\sin(fx+e)\,d+c)^n\sin(fx+e)\,dx\right)a+\left(\int (\sin(fx+e)\,d+c)^n\sin(fx+e)\,dx\right)b\right)$$

input
```
int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

output
```
a*(int((sin(e + f*x)*d + c)**n,x)*a + int((sin(e + f*x)*d + c)**n*sin(e +
f*x)**2,x)*b + int((sin(e + f*x)*d + c)**n*sin(e + f*x),x)*a + int((sin(e
+ f*x)*d + c)**n*sin(e + f*x),x)*b)
```

3.330 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 221

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx = -\frac{\sqrt{2}B\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin}{c}\right.}{af\sqrt{1+\sin(e+fx)}} - \frac{(A-B)\,\text{AppellF1}\left(\frac{1}{2},\frac{3}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+}{}\right.}{\sqrt{2}af\sqrt{1+\sin(e+fx)}}$$

output
```
-2^(1/2)*B*AppellF1(1/2,-n,1/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+
e))*cos(f*x+e)*(c+d*sin(f*x+e))^n/a/f/(1+sin(f*x+e))^(1/2)/(((c+d*sin(f*x+
e))/(c+d))^n)-1/2*(A-B)*AppellF1(1/2,-n,3/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2
-1/2*sin(f*x+e))*cos(f*x+e)*(c+d*sin(f*x+e))^n*2^(1/2)/a/f/(1+sin(f*x+e))^
(1/2)/(((c+d*sin(f*x+e))/(c+d))^n)
```

Mathematica [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx = \int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x
]),x]
```

output
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x
]), x]
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {3042, 3466, 3042, 3144, 156, 155, 3263, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{a\sin(e+fx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{a\sin(e+fx)+a}dx$$

$$\Big\downarrow \text{3466}$$

$$(A-B)\int \frac{(c+d\sin(e+fx))^n}{\sin(e+fx)a+a}dx+\frac{B\int (c+d\sin(e+fx))^n dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$(A-B)\int \frac{(c+d\sin(e+fx))^n}{\sin(e+fx)a+a}dx+\frac{B\int (c+d\sin(e+fx))^n dx}{a}$$

$$\Big\downarrow \text{3144}$$

$$(A-B)\int \frac{(c+d\sin(e+fx))^n}{\sin(e+fx)a+a}dx+\frac{B\cos(e+fx)\int \frac{(c+d\sin(e+fx))^n}{\sqrt{1-\sin(e+fx)}\sqrt{\sin(e+fx)+1}}d\sin(e+fx)}{af\sqrt{1-\sin(e+fx)}\sqrt{\sin(e+fx)+1}}$$

$$\Big\downarrow \text{156}$$

$$(A-B)\int \frac{(c+d\sin(e+fx))^n}{\sin(e+fx)a+a}dx+\frac{B\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\int \frac{\left(\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}\right)^n}{\sqrt{1-\sin(e+fx)}\sqrt{\sin(e+fx)+1}}d\sin(e+fx)}{af\sqrt{1-\sin(e+fx)}\sqrt{\sin(e+fx)+1}}$$

$$\Big\downarrow \text{155}$$

$$(A-B)\int \frac{(c+d\sin(e+fx))^n}{\sin(e+fx)a+a}dx-\frac{\sqrt{2}B\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{af\sqrt{\sin(e+fx)+1}}$$

$$\Big\downarrow \text{3263}$$

$$\frac{(A-B)\cos(e+fx)\int \frac{(c+d\sin(e+fx))^n}{\sqrt{1-\sin(e+fx)}(\sin(e+fx)+1)^{3/2}}d\sin(e+fx)}{af\sqrt{1-\sin(e+fx)}\sqrt{\sin(e+fx)+1}}-\frac{\sqrt{2}B\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{af\sqrt{\sin(e+fx)+1}}$$

$$\Big\downarrow \text{156}$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\int \frac{\left(\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}\right)^n}{\sqrt{1-\sin(e+fx)}(\sin(e+fx)+1)^{3/2}}d\sin(e+fx)}{af\sqrt{1-\sin(e+fx)}\sqrt{\sin(e+fx)+1}}-\frac{\sqrt{2}B\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{af\sqrt{\sin(e+fx)+1}}$$

$$\Big\downarrow \text{155}$$

$$-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},\frac{3}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{\sqrt{2}af\sqrt{\sin(e+fx)+1}}-\frac{\sqrt{2}B\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{af\sqrt{\sin(e+fx)+1}}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]),x]
```

output
```
-((Sqrt[2]*B*AppellF1[1/2, 1/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin
[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(a*f*Sqrt[1 + Si
n[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)) - ((A - B)*AppellF1[1/2, 3/
2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*Cos[e +
f*x]*(c + d*Sin[e + f*x])^n)/(Sqrt[2]*a*f*Sqrt[1 + Sin[e + f*x]]*((c + d*S
in[e + f*x])/(c + d))^n)
```

Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3144
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]])   Subst[Int[(a + b*x
)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

rule 3263
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^m*(Cos[e + f*x]/(f*Sqrt[1 + Sin[e +
 f*x]]*Sqrt[1 - Sin[e + f*x]]))   Subst[Int[(1 + (b/a)*x)^(m - 1/2)*((c + d
*x)^n/Sqrt[1 - (b/a)*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& IntegerQ[m]
```

rule 3466
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)/b   Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x
] + Simp[B/b   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [F]

$$\int \frac{(A+B\sin(fx+e))\,(c+d\sin(fx+e))^n}{a+a\sin(fx+e)}dx$$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

output
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

Fricas [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^n}{a\sin(fx+e)+a}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
integral((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a),
 x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^n}{a\sin(fx+e)+a}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorith
m="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)
, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,0]%%%} / %%%{1,[0,0,1]%%%} Error: Bad Argument Valu
e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx = \int \frac{(A+B\sin(e+fx))\,(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x)),x)
```

output
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x)), x)
```

Reduce [F]

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^n}{a + a\sin(e + fx)}\,dx$$
$$= \frac{\left(\int \frac{(\sin(fx+e)d+c)^n}{\sin(fx+e)+1}dx\right)a + \left(\int \frac{(\sin(fx+e)d+c)^n\sin(fx+e)}{\sin(fx+e)+1}dx\right)b}{a}$$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

output
```
(int((sin(e + f*x)*d + c)**n/(sin(e + f*x) + 1),x)*a + int(((sin(e + f*x)*
d + c)**n*sin(e + f*x))/(sin(e + f*x) + 1),x)*b)/a
```

3.331 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 223

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx = -\frac{B\,\text{AppellF1}\left(\frac{1}{2},\frac{3}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+}{c+d}\right.}{\sqrt{2}a^2f\sqrt{1+\sin(e+fx)}} - \frac{(A-B)\,\text{AppellF1}\left(\frac{1}{2},\frac{5}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+}{}\right.}{2\sqrt{2}a^2f\sqrt{1+\sin(e+fx)}}$$

output
```
-1/2*B*AppellF1(1/2,-n,3/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*
cos(f*x+e)*(c+d*sin(f*x+e))^n*2^(1/2)/a^2/f/(1+sin(f*x+e))^(1/2)/(((c+d*si
n(f*x+e))/(c+d))^n)-1/4*(A-B)*AppellF1(1/2,-n,5/2,3/2,d*(1-sin(f*x+e))/(c+
d),1/2-1/2*sin(f*x+e))*cos(f*x+e)*(c+d*sin(f*x+e))^n*2^(1/2)/a^2/f/(1+sin(
f*x+e))^(1/2)/(((c+d*sin(f*x+e))/(c+d))^n)
```

Mathematica [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx = \int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x
])^2,x]
```

output
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x
])^2, x]
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3042, 3466, 3042, 3263, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a\sin(e+fx)+a)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a\sin(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3466}$$

$$(A-B)\int \frac{(c+d\sin(e+fx))^n}{(\sin(e+fx)a+a)^2}dx + \frac{B\int \frac{(c+d\sin(e+fx))^n}{\sin(e+fx)a+a}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$(A-B)\int \frac{(c+d\sin(e+fx))^n}{(\sin(e+fx)a+a)^2}dx + \frac{B\int \frac{(c+d\sin(e+fx))^n}{\sin(e+fx)a+a}dx}{a}$$

$$\Big\downarrow \ 3263$$

$$\frac{(A-B)\cos(e+fx)\int \frac{(c+d\sin(e+fx))^n}{\sqrt{1-\sin(e+fx)}(\sin(e+fx)+1)^{5/2}}d\sin(e+fx)}{a^2f\sqrt{1-\sin(e+fx)}\sqrt{\sin(e+fx)+1}}+\frac{B\cos(e+fx)\int \frac{(c+d\sin(e+fx))^n}{\sqrt{1-\sin(e+fx)}(\sin(e+fx)+1)^{3/2}}d\sin(e+fx)}{a^2f\sqrt{1-\sin(e+fx)}\sqrt{\sin(e+fx)+1}}$$

$$\Big\downarrow \ 156$$

$$\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\int \frac{\left(\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}\right)^n}{\sqrt{1-\sin(e+fx)}(\sin(e+fx)+1)^{5/2}}d\sin(e+fx)}{a^2f\sqrt{1-\sin(e+fx)}\sqrt{\sin(e+fx)+1}}+\frac{B\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\int \frac{\left(\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}\right)^n}{\sqrt{1-\sin(e+fx)}(\sin(e+fx)+1)^{3/2}}d\sin(e+fx)}{a^2f\sqrt{1-\sin(e+fx)}\sqrt{\sin(e+fx)+1}}$$

$$\Big\downarrow \ 155$$

$$-\frac{(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\operatorname{AppellF1}\left(\frac{1}{2},\frac{5}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{2\sqrt{2}a^2f\sqrt{\sin(e+fx)+1}}-\frac{B\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\operatorname{AppellF1}\left(\frac{1}{2},\frac{3}{2},-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{\sqrt{2}a^2f\sqrt{\sin(e+fx)+1}}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^2,x
]
```

output
```
-((B*AppellF1[1/2, 3/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x
]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(Sqrt[2]*a^2*f*Sqrt[1 +
Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)) - ((A - B)*AppellF1[1/2,
5/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*Cos[e
+ f*x]*(c + d*Sin[e + f*x])^n)/(2*Sqrt[2]*a^2*f*Sqrt[1 + Sin[e + f*x]]*((c
 + d*Sin[e + f*x])/(c + d))^n)
```

Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3263
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^m*(Cos[e + f*x]/(f*Sqrt[1 + Sin[e +
 f*x]]*Sqrt[1 - Sin[e + f*x]]))   Subst[Int[(1 + (b/a)*x)^(m - 1/2)*((c + d
*x)^n/Sqrt[1 - (b/a)*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
 f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& IntegerQ[m]
```

rule 3466
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)/b   Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x
] + Simp[B/b   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [F]

$$\int \frac{(A + B\sin(fx + e))(c + d\sin(fx + e))^n}{(a + a\sin(fx + e))^2}dx$$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

output
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

Fricas [F]

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^n}{(a + a\sin(e + fx))^2} dx = \int \frac{(B\sin(fx + e) + A)(d\sin(fx + e) + c)^n}{(a\sin(fx + e) + a)^2} dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
integral(-(B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a^2*cos(f*x + e)^2
- 2*a^2*sin(f*x + e) - 2*a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^n}{(a + a\sin(e + fx))^2} dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^n}{(a\sin(fx+e)+a)^2}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algori
thm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)
^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,0]%%%} / %%%{1,[0,0,2]%%%} Error: Bad Argument Valu
e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx = \int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,\left(c+d\,\sin\left(e+f\,x\right)\right)^n}{\left(a+a\,\sin\left(e+f\,x\right)\right)^2}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^2,x
)
```

output
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^2,
x)
```

Reduce [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx = \frac{\left(\int \frac{(\sin(fx+e)d+c)^n}{\sin(fx+e)^2+2\sin(fx+e)+1}dx\right)a+\left(\int \frac{(\sin(fx+e)d+c)^n\sin(fx+e)}{\sin(fx+e)^2+2\sin(fx+e)+1}dx\right)b}{a^2}$$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

output
```
(int((sin(e + f*x)*d + c)**n/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),x)*a +
 int(((sin(e + f*x)*d + c)**n*sin(e + f*x))/(sin(e + f*x)**2 + 2*sin(e + f
*x) + 1),x)*b)/a**2
```

3.332 $\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal result

Integrand size = 37, antiderivative size = 427

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx =$$

$$-\frac{2a^2(A-B)\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}}$$

$$+\frac{2a^2B(3c-d(11+4n))\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{d^2f(3+2n)(5+2n)\sqrt{a+a\sin(e+fx)}}$$

$$-\frac{2aB\cos(e+fx)\sqrt{a+a\sin(e+fx)}(c+d\sin(e+fx))^{1+n}}{df(5+2n)}$$

$$+\frac{2a^2(A-B)(c-d(5+4n))\cos(e+fx)\operatorname{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{d(1-\sin(e+fx))}{c+d}\right)(c+d\sin(e+fx))}{df(3+2n)\sqrt{a+a\sin(e+fx)}}$$

$$-\frac{2a^2B(3c^2-2cd(7+4n)+d^2(43+56n+16n^2))\cos(e+fx)\operatorname{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{d(1-\sin(e+fx}{c+d}\right.}{d^2f(3+2n)(5+2n)\sqrt{a+a\sin(e+fx)}}$$

output
```
-2*a^2*(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))^(1+n)/d/f/(3+2*n)/(a+a*sin(f*x+e)
)^(1/2)+2*a^2*B*(3*c-d*(11+4*n))*cos(f*x+e)*(c+d*sin(f*x+e))^(1+n)/d^2/f/(
3+2*n)/(5+2*n)/(a+a*sin(f*x+e))^(1/2)-2*a*B*cos(f*x+e)*(a+a*sin(f*x+e))^(1
/2)*(c+d*sin(f*x+e))^(1+n)/d/f/(5+2*n)+2*a^2*(A-B)*(c-d*(5+4*n))*cos(f*x+e
)*hypergeom([1/2, -n],[3/2],d*(1-sin(f*x+e))/(c+d))*(c+d*sin(f*x+e))^n/d/f
/(3+2*n)/(a+a*sin(f*x+e))^(1/2)/(((c+d*sin(f*x+e))/(c+d))^n)-2*a^2*B*(3*c^
2-2*c*d*(7+4*n)+d^2*(16*n^2+56*n+43))*cos(f*x+e)*hypergeom([1/2, -n],[3/2]
,d*(1-sin(f*x+e))/(c+d))*(c+d*sin(f*x+e))^n/d^2/f/(3+2*n)/(5+2*n)/(a+a*sin
(f*x+e))^(1/2)/(((c+d*sin(f*x+e))/(c+d))^n)
```

Mathematica [A] (warning: unable to verify)

Time = 15.19 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.57

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx =$$

$$-\frac{a^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\left(-30(A+B)(c-d(5+4n))\,\text{Hypergeometric2F1}\left(\right.\right.}{}$$

input
```
Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f
*x])^n,x]
```

output
```
-1/15*(a^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^n*(-30*(A + B)*(c - d*(5 + 4*
n))*Hypergeometric2F1[1/2, -n, 3/2, -((d*(-1 + Sin[e + f*x]))/(c + d))] +
6*B*d*(3 + 2*n)*Hypergeometric2F1[5/2, -n, 7/2, -((d*(-1 + Sin[e + f*x]))/
(c + d))]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + 20*B*d*(3 + 2*n)*Hyper
geometric2F1[3/2, -n, 5/2, -((d*(-1 + Sin[e + f*x]))/(c + d))]*(-1 + Sin[e
 + f*x]) + 30*(A + B)*(c + d)*((c + d*Sin[e + f*x])/(c + d))^(1 + n)))/(d*
f*(3 + 2*n)*Sqrt[a*(1 + Sin[e + f*x])]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rubi [A] (verified)

Time = 1.88 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.405$, Rules used = {3042, 3466, 3042, 3242, 27, 2011, 3042, 3255, 80, 79, 3460, 3042, 3255, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^n dx$$

$$\Big\downarrow \text{3466}$$

$$(A-B)\int (\sin(e+fx)a+a)^{3/2}(c+d\sin(e+fx))^n dx+\frac{B\int (\sin(e+fx)a+a)^{5/2}(c+d\sin(e+fx))^n dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$(A-B)\int (\sin(e+fx)a+a)^{3/2}(c+d\sin(e+fx))^n dx+\frac{B\int (\sin(e+fx)a+a)^{5/2}(c+d\sin(e+fx))^n dx}{a}$$

$$\Big\downarrow \text{3242}$$

$$(A-B)\left(\frac{2\int -\frac{(c+d\sin(e+fx))^n\left((c-5d-4dn)a^2+(c-5d-4dn)\sin(e+fx)a^2\right)}{2\sqrt{\sin(e+fx)a+a}}dx}{d(2n+3)}-\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}\right)+\frac{B\left(\frac{2\int \frac{1}{2}\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^n\left(a^2(c+d(4n+7))-a^2(3c-11d-4dn)\sin(e+fx)\right)dx}{d(2n+5)}-\frac{2a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+}{df(2n+5)}\right.}{a}$$

$$\Big\downarrow \text{27}$$

$$(A-B)\left(-\frac{\int \frac{(c+d\sin(e+fx))^n\left((c-5d-4dn)a^2+(c-5d-4dn)\sin(e+fx)a^2\right)}{\sqrt{\sin(e+fx)a+a}}dx}{d(2n+3)}-\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}\right)+B\left(\frac{\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^n\left(a^2(c+d(4n+7))-a^2(3c-11d-4dn)\sin(e+fx)\right)dx}{d(2n+5)}-\frac{2a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx)}{df(2n+5)}\right.$$
$$\overline{\qquad a \qquad}$$

$\downarrow$ 2011

$$(A-B)\left(-\frac{a(c-4dn-5d)\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^n dx}{d(2n+3)}-\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}\right)+B\left(\frac{\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^n\left(a^2(c+d(4n+7))-a^2(3c-11d-4dn)\sin(e+fx)\right)dx}{d(2n+5)}-\frac{2a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx)}{df(2n+5)}\right.$$
$$\overline{\qquad a \qquad}$$

$\downarrow$ 3042

$$(A-B)\left(-\frac{a(c-4dn-5d)\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^n dx}{d(2n+3)}-\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}\right)+B\left(\frac{\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^n\left(a^2(c+d(4n+7))-a^2(3c-11d-4dn)\sin(e+fx)\right)dx}{d(2n+5)}-\frac{2a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx)}{df(2n+5)}\right.$$
$$\overline{\qquad a \qquad}$$

$\downarrow$ 3255

$$B\left(\frac{\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^n\left(a^2(c+d(4n+7))-a^2(3c-11d-4dn)\sin(e+fx)\right)dx}{d(2n+5)}-\frac{2a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx)}{df(2n+5)}\right.$$
$$\overline{\qquad a \qquad}$$
$$(A-B)\left(-\frac{a^3(c-4dn-5d)\cos(e+fx)\int \frac{(c+d\sin(e+fx))^n}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{df(2n+3)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}-\frac{2a^2\cos(e+fx)(c+d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}\right)$$

$\downarrow$ 80

$$B\left(\frac{\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^n\left(a^2(c+d(4n+7))-a^2(3c-11d-4dn)\sin(e+fx)\right)dx}{d(2n+5)}-\frac{2a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx)}{df(2n+5)}\right.$$
$$\overline{\qquad a \qquad}$$
$$(A-B)\left(-\frac{a^3(c-4dn-5d)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\int \frac{\left(\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}\right)^n}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{df(2n+3)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}-2a\right.$$

$$\Big\downarrow 79$$

$$B\left(\frac{\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^n\left(a^2(c+d(4n+7))-a^2(3c-11d-4dn)\sin(e+fx)\right)dx}{d(2n+5)}-\frac{2a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx)}{df(2n+5)}\right.$$
$$\overline{\qquad a \qquad}$$
$$(A-B)\left(\frac{2a^2(c-4dn-5d)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{d(1-\sin}{c+}\right.}{df(2n+3)\sqrt{a\sin(e+fx)+a}}\right.$$

$$\Big\downarrow 3460$$

$$B\left(\frac{\frac{a^2\left(3c^2-2cd(4n+7)+d^2\left(16n^2+56n+43\right)\right)\int\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^n dx}{d(2n+3)}+\frac{2a^3(3c-d(4n+11))\cos(e+fx)(c+d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}}{d(2n+5)}-\frac{2a^2\cos(e+fx}{}\right.$$
$$\overline{\qquad a \qquad}$$
$$(A-B)\left(\frac{2a^2(c-4dn-5d)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{d(1-\sin}{c+}\right.}{df(2n+3)\sqrt{a\sin(e+fx)+a}}\right.$$

$$\Big\downarrow 3042$$

$$B\left(\frac{\frac{a^2\left(3c^2-2cd(4n+7)+d^2\left(16n^2+56n+43\right)\right)\int\sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^n dx}{d(2n+3)}+\frac{2a^3(3c-d(4n+11))\cos(e+fx)(c+d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}}{d(2n+5)}-\frac{2a^2\cos(e+fx}{}\right.$$
$$\overline{\qquad a \qquad}$$
$$(A-B)\left(\frac{2a^2(c-4dn-5d)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{d(1-\sin}{c+}\right.}{df(2n+3)\sqrt{a\sin(e+fx)+a}}\right.$$

$$\Big\downarrow 3255$$

$$B\left(\frac{\frac{a^4\left(3c^2-2cd(4n+7)+d^2\left(16n^2+56n+43\right)\right)\cos(e+fx)\int\frac{(c+d\sin(e+fx))^n}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{df(2n+3)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}+\frac{2a^3(3c-d(4n+11))\cos(e+fx)(c+d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}}{d(2n+5)}-\frac{2a^2\cos(e-}{}\right.$$
$$\overline{\qquad a \qquad}$$
$$(A-B)\left(\frac{2a^2(c-4dn-5d)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{d(1-\sin}{c+}\right.}{df(2n+3)\sqrt{a\sin(e+fx)+a}}\right.$$

$\Big\downarrow$ 80

$$B\left(\frac{\frac{a^4\left(3c^2-2cd(4n+7)+d^2\left(16n^2+56n+43\right)\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\int\frac{\left(\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}\right)^n}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{df(2n+3)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}+\frac{2a^3(3c-d(4n+11))\cos}{df(2n+3)}}{d(2n+5)}\right.$$

$$\overline{\qquad\qquad\qquad\qquad\qquad a \qquad\qquad}$$

$$(A-B)\left(\frac{2a^2(c-4dn-5d)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{d(1-\sin}{c+}\right.}{df(2n+3)\sqrt{a\sin(e+fx)+a}}\right.$$

$\Big\downarrow$ 79

$$(A-B)\left(\frac{2a^2(c-4dn-5d)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{d(1-\sin}{c+}\right.}{df(2n+3)\sqrt{a\sin(e+fx)+a}}\right.$$

$$B\left(\frac{\frac{2a^3(3c-d(4n+11))\cos(e+fx)(c+d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}-\frac{2a^3\left(3c^2-2cd(4n+7)+d^2\left(16n^2+56n+43\right)\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{Hyperge}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}}{d(2n+5)}\right.$$

$$\overline{\qquad\qquad\qquad\qquad\qquad a \qquad\qquad}$$

input
```
Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n
,x]
```

output
```
(A - B)*((-2*a^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(1 + n))/(d*f*(3 + 2*n)
*Sqrt[a + a*Sin[e + f*x]]) + (2*a^2*(c - 5*d - 4*d*n)*Cos[e + f*x]*Hyperge
ometric2F1[1/2, -n, 3/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(c + d*Sin[e + f*
x])^n)/(d*f*(3 + 2*n)*Sqrt[a + a*Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c +
d))^n)) + (B*((-2*a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e +
 f*x])^(1 + n))/(d*f*(5 + 2*n)) + ((2*a^3*(3*c - d*(11 + 4*n))*Cos[e + f*x
]*(c + d*Sin[e + f*x])^(1 + n))/(d*f*(3 + 2*n)*Sqrt[a + a*Sin[e + f*x]]) -
 (2*a^3*(3*c^2 - 2*c*d*(7 + 4*n) + d^2*(43 + 56*n + 16*n^2))*Cos[e + f*x]*
Hypergeometric2F1[1/2, -n, 3/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(c + d*Sin
[e + f*x])^n)/(d*f*(3 + 2*n)*Sqrt[a + a*Sin[e + f*x]]*((c + d*Sin[e + f*x]
)/(c + d))^n))/(d*(5 + 2*n))))/a
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3242
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
+ n))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*
(m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
 2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
 - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[
n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[
c, 0]))
```

rule 3255
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x]
, x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

rule 3466
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)/b   Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x
] + Simp[B/b   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [F]

$$\int (a + a\sin(fx + e))^{\frac{3}{2}} (A + B\sin(fx + e)) (c + d\sin(fx + e))^n \, dx$$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

output
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

Fricas [F]

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}(d\sin(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, al
gorithm="fricas")
```

output
```
integral(-(B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*sqrt(a
*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}(d\sin(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e)
+ c)^n, x)
```

Giac [F]

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^{\frac{3}{2}}(d\sin(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, al
gorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e)
+ c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx = \int (A+B\sin(e+fx))\,(a+a\sin(e+fx))^{3/2}\,(c+d\sin(e+fx))^n\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^n
,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^n
, x)
```

Reduce [F]

$$\int (a+a\sin(e+fx))^{3/2}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx=\sqrt{a}\,a\left(\left(\int (\sin(fx+e)\,d+c)^n\sqrt{\sin(fx+e)+1}\,\sin(fx+e)^2\,dx\right)b+\left(\int (\sin(fx+e)\,d+c)^n\sqrt{\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)a+\left(\int (\sin(fx+e)\,d+c)^n\sqrt{\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)b+\left(\int (\sin(fx+e)\,d+c)^n\sqrt{\sin(fx+e)+1}dx\right)a\right)$$

input
```
int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

output
```
sqrt(a)*a*(int((sin(e + f*x)*d + c)**n*sqrt(sin(e + f*x) + 1)*sin(e + f*x)
**2,x)*b + int((sin(e + f*x)*d + c)**n*sqrt(sin(e + f*x) + 1)*sin(e + f*x)
,x)*a + int((sin(e + f*x)*d + c)**n*sqrt(sin(e + f*x) + 1)*sin(e + f*x),x)
*b + int((sin(e + f*x)*d + c)**n*sqrt(sin(e + f*x) + 1),x)*a)
```

3.333 $\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal result

Integrand size = 37, antiderivative size = 167

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$= -\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+2n)\sqrt{a+a\sin(e+fx)}}$$
$$-\frac{2a(Ad(3+2n)-B(c-2d(1+n)))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{d(1-\sin(e+fx))}{c+d}\right)(c+d\sin}{df(3+2n)\sqrt{a+a\sin(e+fx)}}$$

output
```
-2*a*B*cos(f*x+e)*(c+d*sin(f*x+e))^(1+n)/d/f/(3+2*n)/(a+a*sin(f*x+e))^(1/2
)-2*a*(A*d*(3+2*n)-B*(c-2*d*(1+n)))*cos(f*x+e)*hypergeom([1/2, -n],[3/2],d
*(1-sin(f*x+e))/(c+d))*(c+d*sin(f*x+e))^n/d/f/(3+2*n)/(a+a*sin(f*x+e))^(1/
2)/(((c+d*sin(f*x+e))/(c+d))^n)
```

Mathematica [F]

$$\int \sqrt{a + a\sin(e + fx)}(A + B\sin(e + fx))(c + d\sin(e + fx))^n\,dx$$
$$= \int \sqrt{a + a\sin(e + fx)}(A + B\sin(e + fx))(c + d\sin(e + fx))^n\,dx$$

input
```
Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x
])^n,x]
```

output
```
Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x
])^n, x]
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {3042, 3460, 3042, 3255, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\sin(e + fx) + a}(A + B\sin(e + fx))(c + d\sin(e + fx))^n\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{a\sin(e + fx) + a}(A + B\sin(e + fx))(c + d\sin(e + fx))^n dx$$

$$\Big\downarrow \; 3460$$

$$-\frac{(-Ad(2n + 3) + Bc - 2Bd(n + 1))\int \sqrt{\sin(e + fx)a + a}(c + d\sin(e + fx))^n dx}{d(2n + 3)} - \frac{2aB\cos(e + fx)(c + d\sin(e + fx))^{n+1}}{df(2n + 3)\sqrt{a\sin(e + fx) + a}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{(-Ad(2n+3)+Bc-2Bd(n+1))\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^n dx}{d(2n+3)}-\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}$$

$\Big\downarrow$ 3255

$$-\frac{a^2\cos(e+fx)(-Ad(2n+3)+Bc-2Bd(n+1))\int \frac{(c+d\sin(e+fx))^n}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{df(2n+3)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}-\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}$$

$\Big\downarrow$ 80

$$-\frac{a^2\cos(e+fx)(-Ad(2n+3)+Bc-2Bd(n+1))(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\int \frac{\left(\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}\right)^n}{\sqrt{a-a\sin(e+fx)}}d\,\mathrm{si}}{df(2n+3)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}$$
$$\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}$$

$\Big\downarrow$ 79

$$\frac{2a\cos(e+fx)(-Ad(2n+3)+Bc-2Bd(n+1))(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2}\right.}{df(2n+3)\sqrt{a\sin(e+fx)+a}}$$
$$\frac{2aB\cos(e+fx)(c+d\sin(e+fx))^{n+1}}{df(2n+3)\sqrt{a\sin(e+fx)+a}}$$

input
```
Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x
]
```

output
```
(-2*a*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(1 + n))/(d*f*(3 + 2*n)*Sqrt[a +
 a*Sin[e + f*x]]) + (2*a*(B*c - 2*B*d*(1 + n) - A*d*(3 + 2*n))*Cos[e + f*x
]*Hypergeometric2F1[1/2, -n, 3/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(c + d*S
in[e + f*x])^n)/(d*f*(3 + 2*n)*Sqrt[a + a*Sin[e + f*x]]*((c + d*Sin[e + f*
x])/(c + d))^n)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3255
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x]
, x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Maple [F]

$$\int \sqrt{a + a\sin(fx + e)}\,(A + B\sin(fx + e))\,(c + d\sin(fx + e))^n\,dx$$

input
```
int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

output
```
int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

Fricas [F]

$$\int \sqrt{a + a\sin(e + fx)}(A + B\sin(e + fx))(c + d\sin(e + fx))^n\,dx$$
$$= \int (B\sin(fx + e) + A)\sqrt{a\sin(fx + e) + a}(d\sin(fx + e) + c)^n\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, al
gorithm="fricas")
```

output
```
integral((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c
)^n, x)
```

Sympy [F]

$$\int \sqrt{a + a\sin(e + fx)}(A + B\sin(e + fx))(c + d\sin(e + fx))^n\,dx$$
$$= \int \sqrt{a(\sin(e + fx) + 1)}(A + B\sin(e + fx))\,(c + d\sin(e + fx))^n\,dx$$

input
```
integrate((a+a*sin(f*x+e))**(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

output
```
Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))*(c + d*sin(e + f*
x))**n, x)
```

Maxima [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx = \int (B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}(d\sin(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) +
c)^n, x)
```

Giac [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx = \int (B\sin(fx+e)+A)\sqrt{a\sin(fx+e)+a}(d\sin(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, al
gorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) +
c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$=\int (A+B\,\sin(e+f\,x))\,\sqrt{a+a\,\sin(e+f\,x)}\,(c+d\,\sin(e+f\,x))^n\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^n
,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^n
, x)
```

Reduce [F]

$$\int \sqrt{a+a\sin(e+fx)}(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$=\sqrt{a}\left(\left(\int (\sin(fx+e)\,d+c)^n\sqrt{\sin(fx+e)+1}\,\sin(fx+e)\,dx\right)b+\left(\int (\sin(fx+e)\,d+c)^n\sqrt{\sin(fx+e)+1}dx\right)a\right)$$

input
```
int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

output
```
sqrt(a)*(int((sin(e + f*x)*d + c)**n*sqrt(sin(e + f*x) + 1)*sin(e + f*x),x
)*b + int((sin(e + f*x)*d + c)**n*sqrt(sin(e + f*x) + 1),x)*a)
```

3.334 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 190

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx =$$

$$-\frac{(A-B)\,\text{AppellF1}\left(\frac{1}{2},-n,1,\frac{3}{2},\frac{d(1-\sin(e+fx))}{c+d},\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+\cdots}{}\right.}{f\sqrt{a+a\sin(e+fx)}}$$

$$-\frac{2B\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{d(1-\sin(e+fx))}{c+d}\right)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}}{f\sqrt{a+a\sin(e+fx)}}$$

output
```
-(A-B)*AppellF1(1/2,-n,1,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*co
s(f*x+e)*(c+d*sin(f*x+e))^n/f/(a+a*sin(f*x+e))^(1/2)/(((c+d*sin(f*x+e))/(c
+d))^n)-2*B*cos(f*x+e)*hypergeom([1/2, -n],[3/2],d*(1-sin(f*x+e))/(c+d))*(
c+d*sin(f*x+e))^n/f/(a+a*sin(f*x+e))^(1/2)/(((c+d*sin(f*x+e))/(c+d))^n)
```

Mathematica [A] (warning: unable to verify)

Time = 10.07 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.28

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx$$

$$= \frac{\cos(e+fx)\sqrt{a(1+\sin(e+fx))}(c+d\sin(e+fx))^n \left(-\left((A+B)\,\text{AppellF1}\left(1,\frac{1}{2},-n,2,\frac{1}{2}(1+\sin(e+\right.\right.\right.}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/Sqrt[a + a*Sin[e +
 f*x]],x]
```

output
```
(Cos[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*(c + d*Sin[e + f*x])^n*(-(((A + B
)*AppellF1[1, 1/2, -n, 2, (1 + Sin[e + f*x])/2, (d*(1 + Sin[e + f*x]))/(-c
 + d)]*Sqrt[2 - 2*Sin[e + f*x]])/((c + d*Sin[e + f*x])/(c - d))^n) + (4*(A
 - B)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (-c + d)
/(d + d*Sin[e + f*x])]*Sqrt[(-1 + Sin[e + f*x])/(1 + Sin[e + f*x])])/((1 +
 2*n)*(1 + (c - d)/(d + d*Sin[e + f*x]))^n)))/(4*a*f*(-1 + Sin[e + f*x]))
```

Rubi [A] (warning: unable to verify)

Time = 0.74 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.17, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.270$, Rules used = {3042, 3466, 3042, 3255, 80, 79, 3267, 27, 154, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{\sqrt{a\sin(e+fx)+a}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{\sqrt{a\sin(e+fx)+a}}dx$$

$$\Big\downarrow \text{3466}$$

$$(A-B)\int \frac{(c+d\sin(e+fx))^n}{\sqrt{\sin(e+fx)a+a}}dx+\frac{B\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^n dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$(A-B)\int \frac{(c+d\sin(e+fx))^n}{\sqrt{\sin(e+fx)a+a}}dx+\frac{B\int \sqrt{\sin(e+fx)a+a}(c+d\sin(e+fx))^n dx}{a}$$

$$\Big\downarrow \text{ 3255}$$

$$(A-B)\int \frac{(c+d\sin(e+fx))^n}{\sqrt{\sin(e+fx)a+a}}dx+\frac{aB\cos(e+fx)\int \frac{(c+d\sin(e+fx))^n}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{ 80}$$

$$(A-B)\int \frac{(c+d\sin(e+fx))^n}{\sqrt{\sin(e+fx)a+a}}dx+$$
$$\frac{aB\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\int \frac{\left(\frac{c}{c+d}+\frac{d\sin(e+fx)}{c+d}\right)^n}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{ 79}$$

$$(A-B)\int \frac{(c+d\sin(e+fx))^n}{\sqrt{\sin(e+fx)a+a}}dx-$$
$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{ 3267}$$

$$\frac{a^2(A-B)\cos(e+fx)\int \frac{(c+d\sin(e+fx))^n}{a(\sin(e+fx)+1)\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}-$$
$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{a(A-B)\cos(e+fx)\int \frac{(c+d\sin(e+fx))^n}{(\sin(e+fx)+1)\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}-$$
$$\frac{2B\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{ 154}$$

$$\frac{a(A-B)\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}\int \frac{(c+d\sin(e+fx))^n}{(\sin(e+fx)+1)\sqrt{\frac{d}{c+d}-\frac{d\sin(e+fx)}{c+d}}}d\sin(e+fx)}{f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}} - \frac{2B\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{a\sin(e+fx)+a}}$$

$\Big\downarrow$ 153

$$-\frac{a(A-B)\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}(c+d\sin(e+fx))^{n+1}\text{AppellF1}\left(n+1,\frac{1}{2},1,n+2,\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)}{f(n+1)(c-d)(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}} - \frac{2B\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{a\sin(e+fx)+a}}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/Sqrt[a + a*Sin[e + f*x]]
,x]
```

output
```
-((a*(A - B)*AppellF1[1 + n, 1/2, 1, 2 + n, (c + d*Sin[e + f*x])/(c + d),
(c + d*Sin[e + f*x])/(c - d)]*Cos[e + f*x]*Sqrt[(d*(1 - Sin[e + f*x]))/(c
+ d)]*(c + d*Sin[e + f*x])^(1 + n))/((c - d)*f*(1 + n)*(a - a*Sin[e + f*x]
)*Sqrt[a + a*Sin[e + f*x]])) - (2*B*Cos[e + f*x]*Hypergeometric2F1[1/2, -n
, 3/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(c + d*Sin[e + f*x])^n)/(f*Sqrt[a +
 a*Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !G
tQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3255
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e +
 f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x]
, x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !IntegerQ[2*n]
```

rule 3267
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m
, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 !IntegerQ[m]
```

rule 3466
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)/b   Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x
] + Simp[B/b   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [F]

$$\int \frac{(A+B\sin(fx+e))\,(c+d\sin(fx+e))^n}{\sqrt{a+a\sin(fx+e)}}dx$$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x)
```

output
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^n}{\sqrt{a\sin(fx+e)+a}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x, al
gorithm="fricas")
```

output
```
integral((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/sqrt(a*sin(f*x + e) +
 a), x)
```

Sympy [F]

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^n}{\sqrt{a + a\sin(e + fx)}}\,dx$$
$$= \int \frac{(A + B\sin{(e + fx)})\,(c + d\sin{(e + fx)})^n}{\sqrt{a\,(\sin{(e + fx)} + 1)}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**(1/2),x)
```

output
```
Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))**n/sqrt(a*(sin(e + f*x)
 + 1)), x)
```

Maxima [F]

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^n}{\sqrt{a + a\sin(e + fx)}}\,dx$$
$$= \int \frac{(B\sin{(fx + e)} + A)(d\sin{(fx + e)} + c)^n}{\sqrt{a\sin{(fx + e)} + a}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/sqrt(a*sin(f*x + e)
+ a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x, al
gorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(c+d\,\sin(e+f\,x))^n}{\sqrt{a+a\,\sin(e+f\,x)}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^(1/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^(1/
2), x)
```

Reduce [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{(\sin(fx+e)d+c)^n\sqrt{\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)+1}dx\right)b+\left(\int \frac{(\sin(fx+e)d+c)^n\sqrt{\sin(fx+e)+1}}{\sin(fx+e)+1}dx\right)a\right)}{a}$$

input

```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x)
```

output

```
(sqrt(a)*(int(((sin(e + f*x)*d + c)**n*sqrt(sin(e + f*x) + 1)*sin(e + f*x)
)/(sin(e + f*x) + 1),x)*b + int(((sin(e + f*x)*d + c)**n*sqrt(sin(e + f*x)
 + 1))/(sin(e + f*x) + 1),x)*a))/a
```

3.335 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 213

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx =$$

$$-\frac{B\,\text{AppellF1}\left(\frac{1}{2},-n,1,\frac{3}{2},\frac{d(1-\sin(e+fx))}{c+d},\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)(c+d\sin(e+fx))^n\Bigg(\frac{c+d\sin(e+fx)}{c+d}}{af\sqrt{a+a\sin(e+fx)}}$$

$$-\frac{(A-B)\,\text{AppellF1}\left(\frac{1}{2},-n,2,\frac{3}{2},\frac{d(1-\sin(e+fx))}{c+d},\frac{1}{2}(1-\sin(e+fx))\right)\cos(e+fx)(c+d\sin(e+fx))^n\Bigg(\frac{c+d\sin}{c}}{2af\sqrt{a+a\sin(e+fx)}}$$

output
```
-B*AppellF1(1/2,-n,1,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*cos(f*
x+e)*(c+d*sin(f*x+e))^n/a/f/(a+a*sin(f*x+e))^(1/2)/(((c+d*sin(f*x+e))/(c+d
))^n)-1/2*(A-B)*AppellF1(1/2,-n,2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f
*x+e))*cos(f*x+e)*(c+d*sin(f*x+e))^n/a/f/(a+a*sin(f*x+e))^(1/2)/(((c+d*sin
(f*x+e))/(c+d))^n)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 603 vs. $2(213) = 426$.

Time = 21.51 (sec) , antiderivative size = 603, normalized size of antiderivative = 2.83

$$\int \frac{(A + B\sin(e + fx))(c + d\sin(e + fx))^n}{(a + a\sin(e + fx))^{3/2}}\,dx = \frac{\sec(e + fx)(c + d\sin(e + fx))^n\left(aB(1 + \sin(e + fx))\left(\right.\right.}{}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x
])^(3/2),x]
```

output
```
(Sec[e + f*x]*(c + d*Sin[e + f*x])^n*(a*B*(1 + Sin[e + f*x])*((a*AppellF1[
1, 1/2, -n, 2, (1 + Sin[e + f*x])/2, (d*(1 + Sin[e + f*x]))/(-c + d)]*Sqrt
[2 - 2*Sin[e + f*x]]*(1 + Sin[e + f*x]))/((c + d*Sin[e + f*x])/(c - d))^n
- (4*Sqrt[(-1 + Sin[e + f*x])/(1 + Sin[e + f*x])]*(-2*a*(1 + 2*n)*AppellF1
[1/2 - n, -1/2, -n, 3/2 - n, 2/(1 + Sin[e + f*x]), (-c + d)/(d + d*Sin[e +
 f*x])] + a*(-1 + 2*n)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e
+ f*x]), (-c + d)/(d + d*Sin[e + f*x])]*(1 + Sin[e + f*x])))/((-1 + 4*n^2)
*(1 + (c - d)/(d + d*Sin[e + f*x]))^n)) + a*A*(1 + Sin[e + f*x])*((a*Appel
lF1[1, 1/2, -n, 2, (1 + Sin[e + f*x])/2, (d*(1 + Sin[e + f*x]))/(-c + d)]*
Sqrt[2 - 2*Sin[e + f*x]]*(1 + Sin[e + f*x]))/((c + d*Sin[e + f*x])/(c - d)
)^n - (4*Sqrt[(-1 + Sin[e + f*x])/(1 + Sin[e + f*x])]*(2*a*(1 + 2*n)*Appel
lF1[1/2 - n, -1/2, -n, 3/2 - n, 2/(1 + Sin[e + f*x]), (-c + d)/(d + d*Sin[
e + f*x])] + a*(-1 + 2*n)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin
[e + f*x]), (-c + d)/(d + d*Sin[e + f*x])]*(1 + Sin[e + f*x])))/((-1 + 4*n
^2)*(1 + (c - d)/(d + d*Sin[e + f*x]))^n))))/(8*a^3*f*Sqrt[a*(1 + Sin[e +
f*x])])
```

Rubi [A] (warning: unable to verify)

Time = 0.70 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.25, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {3042, 3466, 3042, 3267, 27, 154, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a\sin(e+fx)+a)^{3/2}}\,dx$$

$\downarrow$ 3042

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a\sin(e+fx)+a)^{3/2}}dx$$

$\downarrow$ 3466

$$(A-B)\int \frac{(c+d\sin(e+fx))^n}{(\sin(e+fx)a+a)^{3/2}}dx+\frac{B\int \frac{(c+d\sin(e+fx))^n}{\sqrt{\sin(e+fx)a+a}}dx}{a}$$

$\downarrow$ 3042

$$(A-B)\int \frac{(c+d\sin(e+fx))^n}{(\sin(e+fx)a+a)^{3/2}}dx+\frac{B\int \frac{(c+d\sin(e+fx))^n}{\sqrt{\sin(e+fx)a+a}}dx}{a}$$

$\downarrow$ 3267

$$\frac{a^2(A-B)\cos(e+fx)\int \frac{(c+d\sin(e+fx))^n}{a^2(\sin(e+fx)+1)^2\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}+\frac{aB\cos(e+fx)\int \frac{(c+d\sin(e+fx))^n}{a(\sin(e+fx)+1)\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}$$

$\downarrow$ 27

$$\frac{(A-B)\cos(e+fx)\int \frac{(c+d\sin(e+fx))^n}{(\sin(e+fx)+1)^2\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}+\frac{B\cos(e+fx)\int \frac{(c+d\sin(e+fx))^n}{(\sin(e+fx)+1)\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}$$

$\downarrow$ 154

$$\frac{(A-B)\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}\int \frac{(c+d\sin(e+fx))^n}{(\sin(e+fx)+1)^2\sqrt{\frac{d}{c+d}-\frac{d\sin(e+fx)}{c+d}}}d\sin(e+fx)}{f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}+\frac{B\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}\int \frac{(c+d\sin(e+fx))^n}{(\sin(e+fx)+1)\sqrt{\frac{d}{c+d}-\frac{d\sin(e+fx)}{c+d}}}d\sin(e+fx)}{f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}$$

$\downarrow$ 153

$$\frac{d(A-B)\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}(c+d\sin(e+fx))^{n+1}\operatorname{AppellF1}\left(n+1,\frac{1}{2},2,n+2,\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+f}{c-d}\right.}{f(n+1)(c-d)^2(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}$$

$$\frac{B\cos(e+fx)\sqrt{\frac{d(1-\sin(e+fx))}{c+d}}(c+d\sin(e+fx))^{n+1}\operatorname{AppellF1}\left(n+1,\frac{1}{2},1,n+2,\frac{c+d\sin(e+fx)}{c+d},\frac{c+d\sin(e+fx)}{c-d}\right)}{f(n+1)(c-d)(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^(3/
2),x]
```

output
```
-((B*AppellF1[1 + n, 1/2, 1, 2 + n, (c + d*Sin[e + f*x])/(c + d), (c + d*S
in[e + f*x])/(c - d)]*Cos[e + f*x]*Sqrt[(d*(1 - Sin[e + f*x]))/(c + d)]*(c
 + d*Sin[e + f*x])^(1 + n))/((c - d)*f*(1 + n)*(a - a*Sin[e + f*x])*Sqrt[a
 + a*Sin[e + f*x]])) + ((A - B)*d*AppellF1[1 + n, 1/2, 2, 2 + n, (c + d*Si
n[e + f*x])/(c + d), (c + d*Sin[e + f*x])/(c - d)]*Cos[e + f*x]*Sqrt[(d*(1
 - Sin[e + f*x]))/(c + d)]*(c + d*Sin[e + f*x])^(1 + n))/((c - d)^2*f*(1 +
 n)*(a - a*Sin[e + f*x])*Sqrt[a + a*Sin[e + f*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !G
tQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3267
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m
, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 !IntegerQ[m]
```

rule 3466
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)/b   Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x
] + Simp[B/b   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [F]

$$\int \frac{(A+B\sin(fx+e))\,(c+d\sin(fx+e))^{n}}{(a+a\sin(fx+e))^{\frac{3}{2}}}dx$$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x)
```

output
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^n}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x, al
gorithm="fricas")
```

output
```
integral(-(B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) +
c)^n/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

Sympy [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx=\int \frac{(A+B\sin(e+fx))\,(c+d\sin(e+fx))^n}{(a(\sin(e+fx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**(3/2),x)
```

output
```
Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))**n/(a*(sin(e + f*x) + 1
))**(3/2), x)
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(d\sin(fx+e)+c)^n}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)
^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x, al
gorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx=\int \frac{(A+B\,\sin(e+f\,x))\,\left(c+d\,\sin(e+f\,x)\right)^n}{\left(a+a\,\sin(e+f\,x)\right)^{3/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^(3/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^(3/
2), x)
```

Reduce [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^{3/2}}\,dx=\frac{\sqrt{a}\left(\left(\int\frac{(\sin(fx+e)d+c)^n\sqrt{\sin(fx+e)+1}\,\sin(fx+e)}{\sin(fx+e)^2+2\sin(fx+e)+1}dx\right)b+\left(\int\frac{(\sin(\cdots}{\text{si}\cdots}\right.\right.}{a^2}$$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x)
```

output

```
(sqrt(a)*(int(((sin(e + f*x)*d + c)**n*sqrt(sin(e + f*x) + 1)*sin(e + f*x)
)/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),x)*b + int(((sin(e + f*x)*d + c)*
*n*sqrt(sin(e + f*x) + 1))/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1),x)*a))/a
**2
```

3.336 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$

Optimal result

Integrand size = 35, antiderivative size = 351

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$

$$= \frac{(d(Ad(3+m)+B(2c+dm))-2(2+m)\left(Acd(3+m)+B(c^2+d^2+cdm)\right))\cos(e+fx)(a+a\sin(e+}{f(1+m)(2+m)(3+m)}$$

$$-\frac{2^{\frac{1}{2}+m}(A(3+m)\left(2cdm(2+m)+d^2(1+m+m^2)+c^2(2+3m+m^2)\right)+B(d^2m(5+3m+m^2)+c^2r}{}$$

$$-\frac{d(Ad(3+m)+B(2c+dm))\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(2+m)(3+m)}$$

$$-\frac{B\cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^2}{f(3+m)}$$

output
```
(d*(A*d*(3+m)+B*(d*m+2*c))-2*(2+m)*(A*c*d*(3+m)+B*(c*d*m+c^2+d^2)))*cos(f*
x+e)*(a+a*sin(f*x+e))^m/f/(1+m)/(2+m)/(3+m)-2^(1/2+m)*(A*(3+m)*(2*c*d*m*(2
+m)+d^2*(m^2+m+1)+c^2*(m^2+3*m+2))+B*(d^2*m*(m^2+3*m+5)+c^2*m*(m^2+5*m+6)+
2*c*d*(m^3+4*m^2+4*m+3)))*cos(f*x+e)*hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*
sin(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m/f/(1+m)/(2+m)/(3+m)
-d*(A*d*(3+m)+B*(d*m+2*c))*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)/a/f/(2+m)/(3+
m)-B*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^2/f/(3+m)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 27.68 (sec) , antiderivative size = 854, normalized size of antiderivative = 2.43

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$

$$=\frac{i(a(1+\sin(e+fx)))^m(1-i\cos(e+fx)+\sin(e+fx))^{-2m}\left(\frac{8Ac^2\,\text{Hypergeometric2F1}(-2m,-m,1-m,i\cos(e+fx)-\text{s}}{m}\right.}{}$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])
^2,x]
```

output
```
((I/8)*(a*(1 + Sin[e + f*x]))^m*((8*A*c^2*Hypergeometric2F1[-2*m, -m, 1 -
m, I*Cos[e + f*x] - Sin[e + f*x]])/m + (8*B*c*d*Hypergeometric2F1[-2*m, -m
, 1 - m, I*Cos[e + f*x] - Sin[e + f*x]])/m + (4*A*d^2*Hypergeometric2F1[-2
*m, -m, 1 - m, I*Cos[e + f*x] - Sin[e + f*x]])/m + (4*B*c^2*Hypergeometric
2F1[1 - m, -2*m, 2 - m, I*Cos[e + f*x] - Sin[e + f*x]]*((-I)*Cos[e + f*x]
+ Sin[e + f*x]))/(-1 + m) + (8*A*c*d*Hypergeometric2F1[1 - m, -2*m, 2 - m,
 I*Cos[e + f*x] - Sin[e + f*x]]*((-I)*Cos[e + f*x] + Sin[e + f*x]))/(-1 +
m) + (3*B*d^2*Hypergeometric2F1[1 - m, -2*m, 2 - m, I*Cos[e + f*x] - Sin[e
 + f*x]]*((-I)*Cos[e + f*x] + Sin[e + f*x]))/(-1 + m) + (4*B*c^2*Hypergeom
etric2F1[-1 - m, -2*m, -m, I*Cos[e + f*x] - Sin[e + f*x]]*(I*Cos[e + f*x]
+ Sin[e + f*x]))/(1 + m) + (8*A*c*d*Hypergeometric2F1[-1 - m, -2*m, -m, I*
Cos[e + f*x] - Sin[e + f*x]]*(I*Cos[e + f*x] + Sin[e + f*x]))/(1 + m) + (3
*B*d^2*Hypergeometric2F1[-1 - m, -2*m, -m, I*Cos[e + f*x] - Sin[e + f*x]]*
(I*Cos[e + f*x] + Sin[e + f*x]))/(1 + m) - (2*d*(2*B*c + A*d)*Hypergeometr
ic2F1[-2 - m, -2*m, -1 - m, I*Cos[e + f*x] - Sin[e + f*x]]*(Cos[2*(e + f*x
)] - I*Sin[2*(e + f*x)]))/(2 + m) - (4*B*c*d*Hypergeometric2F1[2 - m, -2*m
, 3 - m, I*Cos[e + f*x] - Sin[e + f*x]]*(Cos[2*(e + f*x)] + I*Sin[2*(e + f
*x)]))/(-2 + m) - (2*A*d^2*Hypergeometric2F1[2 - m, -2*m, 3 - m, I*Cos[e +
 f*x] - Sin[e + f*x]]*(Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)]))/(-2 + m) -
(I*B*d^2*Hypergeometric2F1[-3 - m, -2*m, -2 - m, I*Cos[e + f*x] - Sin[e...
```

Rubi [A] (verified)

Time = 1.72 (sec) , antiderivative size = 349, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {3042, 3462, 3042, 3447, 3042, 3502, 3042, 3230, 3042, 3131, 3042, 3130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c+d\sin(e+fx))^2dx$$

$$\Big\downarrow \text{3462}$$

$$\frac{\int (\sin(e+fx)a+a)^m(c+d\sin(e+fx))(a(Ac(m+3)+B(2d+cm))+a(Ad(m+3)+B(2c+dm))\sin(e+f}{a(m+3)}$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^2}{f(m+3)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (\sin(e+fx)a+a)^m(c+d\sin(e+fx))(a(Ac(m+3)+B(2d+cm))+a(Ad(m+3)+B(2c+dm))\sin(e+f}{a(m+3)}$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^2}{f(m+3)}$$

$$\Big\downarrow \text{3447}$$

$$\frac{\int (\sin(e+fx)a+a)^m\left(ad(Ad(m+3)+B(2c+dm))\sin^2(e+fx)+(ad(Ac(m+3)+B(2d+cm))+ac(Ad(m\right.}{a(m+3)}$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^2}{f(m+3)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (\sin(e+fx)a+a)^m\left(ad(Ad(m+3)+B(2c+dm))\sin(e+fx)^2+(ad(Ac(m+3)+B(2d+cm))+ac(Ad(m\right.}{a(m+3)}$$
$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^2}{f(m+3)}$$

$\downarrow$ 3502

$$\frac{\frac{\int (\sin(e+fx)a+a)^m \left(a^2(c(m+2)(Ac(m+3)+B(2d+cm))+d(m+1)(Ad(m+3)+B(2c+dm)))-a^2\left(d(Ad(m+3)+B(2c+dm))-2(m+2)\left(Acd(m+3\right.\right.\right.}{a(m+2)}}{a(m+3)}$$

$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^2}{f(m+3)}$$

$\downarrow$ 3042

$$\frac{\frac{\int (\sin(e+fx)a+a)^m \left(a^2(c(m+2)(Ac(m+3)+B(2d+cm))+d(m+1)(Ad(m+3)+B(2c+dm)))-a^2\left(d(Ad(m+3)+B(2c+dm))-2(m+2)\left(Acd(m+3\right.\right.\right.}{a(m+2)}}{a(m+3)}$$

$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^2}{f(m+3)}$$

$\downarrow$ 3230

$$\frac{\frac{\frac{a^2\left(A(m+3)\left(c^2\left(m^2+3m+2\right)+2cdm(m+2)+d^2\left(m^2+m+1\right)\right)+B\left(c^2m\left(m^2+5m+6\right)+2cd\left(m^3+4m^2+4m+3\right)+d^2m\left(m^2+3m+5\right)\right)\right)\int (\sin(e+fx)a+a)^m dx}{m+1}+\frac{a^2\ldots}{}}{a(m+2)}}{a(\ldots}$$

$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^2}{f(m+3)}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{a^2\left(A(m+3)\left(c^2\left(m^2+3m+2\right)+2cdm(m+2)+d^2\left(m^2+m+1\right)\right)+B\left(c^2m\left(m^2+5m+6\right)+2cd\left(m^3+4m^2+4m+3\right)+d^2m\left(m^2+3m+5\right)\right)\right)\int (\sin(e+fx)a+a)^m dx}{m+1}+\frac{a^2\ldots}{}}{a(m+2)}}{a(\ldots}$$

$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^2}{f(m+3)}$$

$\downarrow$ 3131

$$\frac{\frac{a^2\left(A(m+3)\left(c^2\left(m^2+3m+2\right)+2cdm(m+2)+d^2\left(m^2+m+1\right)\right)+B\left(c^2m\left(m^2+5m+6\right)+2cd\left(m^3+4m^2+4m+3\right)+d^2m\left(m^2+3m+5\right)\right)\right)(\sin(e+fx)+1)^{-m}(a\sin(e+f\ldots}{m+1}}{a(m+2)}$$

$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^2}{f(m+3)}$$

$\downarrow$ 3042

$$\frac{\frac{a^2\left(A(m+3)\left(c^2\left(m^2+3m+2\right)+2cdm(m+2)+d^2\left(m^2+m+1\right)\right)+B\left(c^2m\left(m^2+5m+6\right)+2cd\left(m^3+4m^2+4m+3\right)+d^2m\left(m^2+3m+5\right)\right)\right)(\sin(e+fx)+1)^{-m}(a\sin(e+f}{m+1}}{a(m+2)}$$

$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^2}{f(m+3)}$$

$\Big\downarrow$ 3130

$$\frac{\frac{a^2\cos(e+fx)\left(d(Ad(m+3)+B(2c+dm))-2(m+2)\left(Acd(m+3)+B\left(c^2+cdm+d^2\right)\right)\right)(a\sin(e+fx)+a)^m}{f(m+1)}-\frac{a^2 2^{m+\frac{1}{2}}\cos(e+fx)\left(A(m+3)\left(c^2\left(m^2+3m+2\right)+2cdm(m\right.\right.}{}}{}$$

$$\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^2}{f(m+3)}$$

input
```
Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
```

output
```
-((B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^2)/(f*(3 + m
))) + (-((d*(A*d*(3 + m) + B*(2*c + d*m))*Cos[e + f*x]*(a + a*Sin[e + f*x]
)^(1 + m))/(f*(2 + m))) + ((a^2*(d*(A*d*(3 + m) + B*(2*c + d*m)) - 2*(2 +
m)*(A*c*d*(3 + m) + B*(c^2 + d^2 + c*d*m)))*Cos[e + f*x]*(a + a*Sin[e + f*
x])^m)/(f*(1 + m)) - (2^(1/2 + m)*a^2*(A*(3 + m)*(2*c*d*m*(2 + m) + d^2*(1
 + m + m^2) + c^2*(2 + 3*m + m^2)) + B*(d^2*m*(5 + 3*m + m^2) + c^2*m*(6 +
 5*m + m^2) + 2*c*d*(3 + 4*m + 4*m^2 + m^3)))*Cos[e + f*x]*Hypergeometric2
F1[1/2, 1/2 - m, 3/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*
(a + a*Sin[e + f*x])^m)/(f*(1 + m)))/(a*(2 + m)))/(a*(3 + m))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3130
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

rule 3131
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3462
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Simp[1/(b*(m + n + 1))   Int[(a + b*Sin[e + f*x])^m*(c + d*S
in[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
 + n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int (a + a \sin (fx + e))^m (A + B \sin (fx + e)) (c + d \sin (fx + e))^2 dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x)
```

Fricas [F]

$$\int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c + d \sin(e + fx))^2 dx$$
$$= \int (B \sin (fx + e) + A)(d \sin (fx + e) + c)^2 (a \sin (fx + e) + a)^m dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
integral((A*c^2 + 2*B*c*d + A*d^2 - (2*B*c*d + A*d^2)*cos(f*x + e)^2 - (B*
d^2*cos(f*x + e)^2 - B*c^2 - 2*A*c*d - B*d^2)*sin(f*x + e))*(a*sin(f*x + e
) + a)^m, x)
```

Sympy [F]

$$\int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c + d \sin(e + fx))^2 dx$$
$$= \int (a(\sin (e + fx) + 1))^m (A + B \sin (e + fx)) (c + d \sin (e + fx))^2 dx$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))*(c + d*sin(e + f*x
))**2, x)
```

Maxima [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = \int (B\sin(fx+e)+A)(d\sin(fx+e)+c)^2(a\sin(fx+e)+a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algori
thm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^2*(a*sin(f*x + e) + a)
^m, x)
```

Giac [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = \int (B\sin(fx+e)+A)(d\sin(fx+e)+c)^2(a\sin(fx+e)+a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^2*(a*sin(f*x + e) + a)
^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx = \int (A+B\,\sin(e+f\,x))\,\left(a+a\,\sin(e+f\,x)\right)^m\,\left(c+d\,\sin(e+f\,x)\right)^2\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^2,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^2, x)
```

Reduce [F]

$$\begin{aligned}\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^2\,dx &= \left(\int (a+a\sin(fx+e))^m\,dx\right)a\,c^2+\left(\int (a+a\sin(fx+e))^m\sin(fx+e)^3\,dx\right)b\,d^2 \\ &\quad+\left(\int (a+a\sin(fx+e))^m\sin(fx+e)^2\,dx\right)a\,d^2 \\ &\quad+2\left(\int (a+a\sin(fx+e))^m\sin(fx+e)^2\,dx\right)bcd \\ &\quad+2\left(\int (a+a\sin(fx+e))^m\sin(fx+e)\,dx\right)acd \\ &\quad+\left(\int (a+a\sin(fx+e))^m\sin(fx+e)\,dx\right)b\,c^2\end{aligned}$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x)
```

output
```
int((sin(e + f*x)*a + a)**m,x)*a*c**2 + int((sin(e + f*x)*a + a)**m*sin(e
+ f*x)**3,x)*b*d**2 + int((sin(e + f*x)*a + a)**m*sin(e + f*x)**2,x)*a*d**
2 + 2*int((sin(e + f*x)*a + a)**m*sin(e + f*x)**2,x)*b*c*d + 2*int((sin(e
+ f*x)*a + a)**m*sin(e + f*x),x)*a*c*d + int((sin(e + f*x)*a + a)**m*sin(e
 + f*x),x)*b*c**2
```

3.337 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$

Optimal result

Integrand size = 33, antiderivative size = 199

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$= \frac{(Bd-(Bc+Ad)(2+m))\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)(2+m)}$$
$$- \frac{2^{\frac{1}{2}+m}(A(2+m)(c+cm+dm)+B(cm(2+m)+d(1+m+m^2)))\cos(e+fx)\,\text{Hypergeometric2F1}}{f(1+m)(2+m)}$$
$$- \frac{Bd\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(2+m)}$$

output
```
(B*d-(A*d+B*c)*(2+m))*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+m)/(2+m)-2^(1/2+m
)*(A*(2+m)*(c*m+d*m+c)+B*(c*m*(2+m)+d*(m^2+m+1)))*cos(f*x+e)*hypergeom([1/
2, 1/2-m],[3/2],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e
))^m/f/(1+m)/(2+m)-B*d*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)/a/f/(2+m)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 24.87 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.59

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c + d\sin(e + fx))\,dx =$$

$$-\frac{(a(1 + \sin(e + fx)))^m(\cos(e + fx) + i(1 + \sin(e + fx)))\left(-\frac{2(2Ac+Bd)\,\text{Hypergeometric2F1}(1,1+m,1-m,i\cos(e+}{m}\right.}{}$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])
,x]
```

output
```
-1/4*((a*(1 + Sin[e + f*x]))^m*(Cos[e + f*x] + I*(1 + Sin[e + f*x]))*((-2*
(2*A*c + B*d)*Hypergeometric2F1[1, 1 + m, 1 - m, I*Cos[e + f*x] - Sin[e +
f*x]])/m - ((2*I)*(B*c + A*d)*Hypergeometric2F1[1, m, -m, I*Cos[e + f*x] -
 Sin[e + f*x]]*(Cos[e + f*x] - I*Sin[e + f*x]))/(1 + m) + ((2*I)*(B*c + A*
d)*Hypergeometric2F1[1, 2 + m, 2 - m, I*Cos[e + f*x] - Sin[e + f*x]]*(Cos[
e + f*x] + I*Sin[e + f*x]))/(-1 + m) + (B*d*Hypergeometric2F1[1, -1 + m, -
1 - m, I*Cos[e + f*x] - Sin[e + f*x]]*(Cos[2*(e + f*x)] - I*Sin[2*(e + f*x
)]))/(2 + m) + (B*d*Hypergeometric2F1[1, 3 + m, 3 - m, I*Cos[e + f*x] - Si
n[e + f*x]]*(Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)]))/(-2 + m)))/f
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {3042, 3447, 3042, 3502, 3042, 3230, 3042, 3131, 3042, 3130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e + fx) + a)^m (A + B\sin(e + fx))(c + d\sin(e + fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c+d\sin(e+fx))dx$$

$$\Big\downarrow \text{ 3447}$$

$$\int (a\sin(e+fx)+a)^m\left((Ad+Bc)\sin(e+fx)+Ac+Bd\sin^2(e+fx)\right)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a\sin(e+fx)+a)^m\left((Ad+Bc)\sin(e+fx)+Ac+Bd\sin(e+fx)^2\right)dx$$

$$\Big\downarrow \text{ 3502}$$

$$\frac{\int (\sin(e+fx)a+a)^m(a(Bd(m+1)+Ac(m+2))-a(Bd-(Bc+Ad)(m+2))\sin(e+fx))dx}{a(m+2)}-\frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(m+2)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int (\sin(e+fx)a+a)^m(a(Bd(m+1)+Ac(m+2))-a(Bd-(Bc+Ad)(m+2))\sin(e+fx))dx}{a(m+2)}-\frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(m+2)}$$

$$\Big\downarrow \text{ 3230}$$

$$\frac{\frac{a\left(A(m+2)(cm+c+dm)+Bcm(m+2)+Bd\left(m^2+m+1\right)\right)\int (\sin(e+fx)a+a)^mdx}{m+1}+\frac{a\cos(e+fx)(Bd-(m+2)(Ad+Bc))(a\sin(e+fx)+a)^m}{f(m+1)}}{a(m+2)}-\frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(m+2)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{a\left(A(m+2)(cm+c+dm)+Bcm(m+2)+Bd\left(m^2+m+1\right)\right)\int (\sin(e+fx)a+a)^mdx}{m+1}+\frac{a\cos(e+fx)(Bd-(m+2)(Ad+Bc))(a\sin(e+fx)+a)^m}{f(m+1)}}{a(m+2)}-\frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(m+2)}$$

$$\Big\downarrow \text{ 3131}$$

$$\frac{\frac{a(A(m+2)(cm+c+dm)+Bcm(m+2)+Bd(m^2+m+1))(\sin(e+fx)+1)^{-m}(a\sin(e+fx)+a)^m\int(\sin(e+fx)+1)^m dx}{m+1}+\frac{a\cos(e+fx)(Bd-(m+2}{f}}{a(m+2)}$$
$$\frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(m+2)}$$

$\Big\downarrow$ 3042

$$\frac{\frac{a(A(m+2)(cm+c+dm)+Bcm(m+2)+Bd(m^2+m+1))(\sin(e+fx)+1)^{-m}(a\sin(e+fx)+a)^m\int(\sin(e+fx)+1)^m dx}{m+1}+\frac{a\cos(e+fx)(Bd-(m+2}{f}}{a(m+2)}$$
$$\frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(m+2)}$$

$\Big\downarrow$ 3130

$$\frac{\frac{a\cos(e+fx)(Bd-(m+2)(Ad+Bc))(a\sin(e+fx)+a)^m}{f(m+1)}-\frac{a2^{m+\frac{1}{2}}\cos(e+fx)(A(m+2)(cm+c+dm)+Bcm(m+2)+Bd(m^2+m+1))(\sin(e+fx)}{f(m+1}}{a(m+2)}$$
$$\frac{Bd\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(m+2)}$$

input
```
Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

output
```
-((B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(2 + m))) + ((a*(B*
d - (B*c + A*d)*(2 + m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + m))
- (2^(1/2 + m)*a*(B*c*m*(2 + m) + A*(2 + m)*(c + c*m + d*m) + B*d*(1 + m +
 m^2))*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e + f*x]
)/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 + m)))/(a
*(2 + m))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3130
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

rule 3131
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [F]

$$\int (a+a\sin(fx+e))^m (A+B\sin(fx+e))(c+d\sin(fx+e))\,dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

Fricas [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = \int (B\sin(fx+e)+A)(d\sin(fx+e)+c)(a\sin(fx+e)+a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
integral(-(B*d*cos(f*x + e)^2 - A*c - B*d - (B*c + A*d)*sin(f*x + e))*(a*s
in(f*x + e) + a)^m, x)
```

Sympy [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx = \int (a(\sin(e+fx)+1))^m (A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))*(c + d*sin(e + f*x
)), x)
```

Maxima [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c + d\sin(e + fx))\,dx$$
$$= \int (B\sin(fx + e) + A)(d\sin(fx + e) + c)(a\sin(fx + e) + a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorith
m="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m
, x)
```

Giac [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))(c + d\sin(e + fx))\,dx$$
$$= \int (B\sin(fx + e) + A)(d\sin(fx + e) + c)(a\sin(fx + e) + a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorith
m="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m
, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$=\int (A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m\,(c+d\,\sin(e+f\,x))\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x)),x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x)), x)
```

Reduce [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))\,dx$$
$$=\left(\int (a+a\sin(fx+e))^m\,dx\right)ac+\left(\int (a+a\sin(fx+e))^m\sin(fx+e)^2\,dx\right)bd$$
$$+\left(\int (a+a\sin(fx+e))^m\sin(fx+e)\,dx\right)ad$$
$$+\left(\int (a+a\sin(fx+e))^m\sin(fx+e)\,dx\right)bc$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

output
```
int((sin(e + f*x)*a + a)**m,x)*a*c + int((sin(e + f*x)*a + a)**m*sin(e + f
*x)**2,x)*b*d + int((sin(e + f*x)*a + a)**m*sin(e + f*x),x)*a*d + int((sin
(e + f*x)*a + a)**m*sin(e + f*x),x)*b*c
```

3.338 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 117

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = -\frac{B\cos(e+fx)(a+a\sin(e+fx))^m}{f(1+m)} - \frac{2^{\frac{1}{2}+m}(A+Am+Bm)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+}{f(1+m)}$$

output
```
-B*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+m)-2^(1/2+m)*(A*m+B*m+A)*cos(f*x+e)*
hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(
a+a*sin(f*x+e))^m/f/(1+m)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.28 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.86

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$
$$= \frac{2^m\left((A-B)B_{\frac{1}{2}(1+\sin(e+fx))}\left(\frac{1}{2}+m,\frac{1}{2}\right)+2BB_{\frac{1}{2}(1+\sin(e+fx))}\left(\frac{3}{2}+m,\frac{1}{2}\right)\right)\sqrt{\cos^2(e+fx)}\sec(e+fx)(1+\mathrm{s}}{f}$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

output
```
(2^m*((A - B)*Beta[(1 + Sin[e + f*x])/2, 1/2 + m, 1/2] + 2*B*Beta[(1 + Sin
[e + f*x])/2, 3/2 + m, 1/2])*Sqrt[Cos[e + f*x]^2]*Sec[e + f*x]*(a*(1 + Sin
[e + f*x]))^m)/(f*(1 + Sin[e + f*x])^m)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3230, 3042, 3131, 3042, 3130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))dx$$

$$\Big\downarrow \text{3230}$$

$$\frac{(Am+A+Bm)\int(\sin(e+fx)a+a)^mdx}{m+1}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(m+1)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(Am+A+Bm)\int(\sin(e+fx)a+a)^mdx}{m+1}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(m+1)}$$

$$\Big\downarrow \text{3131}$$

$$\frac{(Am+A+Bm)(\sin(e+fx)+1)^{-m}(a\sin(e+fx)+a)^m\int(\sin(e+fx)+1)^mdx}{m+1}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(m+1)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(Am+A+Bm)(\sin(e+fx)+1)^{-m}(a\sin(e+fx)+a)^m\int(\sin(e+fx)+1)^m dx}{m+1}-\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(m+1)}$$

$$\Big\downarrow \; 3130$$

$$-\frac{2^{m+\frac{1}{2}}(Am+A+Bm)\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m\right.}{f(m+1)}\frac{B\cos(e+fx)(a\sin(e+fx)+a)^m}{f(m+1)}$$

input
```
Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

output
```
-((B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + m))) - (2^(1/2 + m)*(A +
 A*m + B*m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e +
 f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 + m)
)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3130
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

rule 3131
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Maple [F]

$$\int (a + a\sin(fx + e))^m (A + B\sin(fx + e)) \, dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Fricas [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx)) \, dx$$
$$= \int (B\sin(fx + e) + A)(a\sin(fx + e) + a)^m \, dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fricas")
```

output
```
integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m, x)
```

Sympy [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$
$$=\int (a(\sin{(e+fx)}+1))^m\,(A+B\sin{(e+fx)})\;dx$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x)), x)
```

Maxima [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$
$$=\int (B\sin{(fx+e)}+A)(a\sin{(fx+e)}+a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m, x)
```

Giac [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$
$$=\int (B\sin{(fx+e)}+A)(a\sin{(fx+e)}+a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \int (A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m, x)
```

Reduce [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \left(\int (a+a\sin(fx+e))^m\,dx\right)a+\left(\int (a+a\sin(fx+e))^m\sin(fx+e)\,dx\right)b$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

output
```
int((sin(e + f*x)*a + a)**m,x)*a + int((sin(e + f*x)*a + a)**m*sin(e + f*x
),x)*b
```

3.339 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 189

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$

$$= \frac{2^{\frac{1}{2}+m}a(Bc-Ad)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2}-m,1,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1+\sin(e+f}{d(c+d)f}$$

$$-\frac{2^{\frac{1}{2}+m}B\cos(e+fx)\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{2}-m}(a+}{df}$$

output

```
2^(1/2+m)*a*(-A*d+B*c)*AppellF1(1/2,1,1/2-m,3/2,d*(1-sin(f*x+e))/(c+d),1/2
-1/2*sin(f*x+e))*cos(f*x+e)*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^(-1+m)
/d/(c+d)/f-2^(1/2+m)*B*cos(f*x+e)*hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*sin
(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m/d/f
```

Mathematica [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$
$$=\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$

input
```
Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x
]),x]
```

output
```
Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x
]), x]
```

Rubi [A] (warning: unable to verify)

Time = 0.72 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3465, 3042, 3131, 3042, 3130, 3267, 154, 27, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^m(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a\sin(e+fx)+a)^m(A+B\sin(e+fx))}{c+d\sin(e+fx)}dx$$

$$\Big\downarrow \; 3465$$

$$\frac{B\int(\sin(e+fx)a+a)^m dx}{d}-\frac{(Bc-Ad)\int\frac{(\sin(e+fx)a+a)^m}{c+d\sin(e+fx)}dx}{d}$$

$$\Big\downarrow \; 3042$$

$$\frac{B\int(\sin(e+fx)a+a)^m dx}{d}-\frac{(Bc-Ad)\int\frac{(\sin(e+fx)a+a)^m}{c+d\sin(e+fx)}dx}{d}$$

$\Big\downarrow$ 3131

$$\frac{B(\sin(e+fx)+1)^{-m}(a\sin(e+fx)+a)^m\int(\sin(e+fx)+1)^m dx}{d}-\frac{(Bc-Ad)\int\frac{(\sin(e+fx)a+a)^m}{c+d\sin(e+fx)}dx}{d}$$

$\Big\downarrow$ 3042

$$\frac{B(\sin(e+fx)+1)^{-m}(a\sin(e+fx)+a)^m\int(\sin(e+fx)+1)^m dx}{d}-\frac{(Bc-Ad)\int\frac{(\sin(e+fx)a+a)^m}{c+d\sin(e+fx)}dx}{d}$$

$\Big\downarrow$ 3130

$$-\frac{(Bc-Ad)\int\frac{(\sin(e+fx)a+a)^m}{c+d\sin(e+fx)}dx}{d}-\frac{B2^{m+\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sin(e+\right.}{df}$$

$\Big\downarrow$ 3267

$$-\frac{a^2(Bc-Ad)\cos(e+fx)\int\frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}}{\sqrt{a-a\sin(e+fx)}(c+d\sin(e+fx))}d\sin(e+fx)}{df\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}-\frac{B2^{m+\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sin(e+\right.}{df}$$

$\Big\downarrow$ 154

$$-\frac{a^2(Bc-Ad)\sqrt{1-\sin(e+fx)}\cos(e+fx)\int\frac{\sqrt{2}(\sin(e+fx)a+a)^{m-\frac{1}{2}}}{\sqrt{1-\sin(e+fx)}(c+d\sin(e+fx))}d\sin(e+fx)}{\sqrt{2}df(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}-\frac{B2^{m+\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sin(e+\right.}{df}$$

$\Big\downarrow$ 27

$$-\frac{a^2(Bc-Ad)\sqrt{1-\sin(e+fx)}\cos(e+fx)\int\frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}}{\sqrt{1-\sin(e+fx)}(c+d\sin(e+fx))}d\sin(e+fx)}{df(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}-\frac{B2^{m+\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sin(e+\right.}{df}$$

$\Big\downarrow$ 153

$$-\frac{\sqrt{2}a(Bc-Ad)\sqrt{1-\sin(e+fx)}\cos(e+fx)(a\sin(e+fx)+a)^m \operatorname{AppellF1}\left(m+\frac{1}{2},\frac{1}{2},1,m+\frac{3}{2},\frac{1}{2}(\sin(e+fx)\right.}{df(2m+1)(c-d)(a-a\sin(e+fx))}$$
$$\frac{B2^{m+\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sin(e+\right.}{df}$$

input
```
Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),x]
```

output
```
-((2^(1/2 + m)*B*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Si
n[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(d*f)
) - (Sqrt[2]*a*(B*c - A*d)*AppellF1[1/2 + m, 1/2, 1, 3/2 + m, (1 + Sin[e +
 f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*Sqrt[1 - Sin[e +
 f*x]]*(a + a*Sin[e + f*x])^m)/((c - d)*d*f*(1 + 2*m)*(a - a*Sin[e + f*x])
)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !G
tQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3130
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

rule 3131
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

rule 3267
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m
, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 !IntegerQ[m]
```

rule 3465
```
Int[(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*S
in[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 NeQ[m + 1/2, 0]
```

Maple [F]

$$\int \frac{(a+a\sin(fx+e))^m (A+B\sin(fx+e))}{c+d\sin(fx+e)}dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{d\sin(fx+e)+c}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorith
m="fricas")
```

output
```
integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c),
 x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{d\sin(fx+e)+c}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorith
m="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)
, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,0]%%%} / %%%{1,[0,0,1]%%%} Error: Bad Argument Valu
e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,\left(a+a\,\sin(e+f\,x)\right)^m}{c+d\,\sin(e+f\,x)}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x)),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x)), x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{c+d\sin(e+fx)}\,dx = \left(\int \frac{(a+a\sin(fx+e))^m}{\sin(fx+e)\,d+c}dx\right)a+\left(\int \frac{(a+a\sin(fx+e))^m\sin(fx+e)}{\sin(fx+e)\,d+c}dx\right)b$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

output
```
int((sin(e + f*x)*a + a)**m/(sin(e + f*x)*d + c),x)*a + int(((sin(e + f*x)
*a + a)**m*sin(e + f*x))/(sin(e + f*x)*d + c),x)*b
```

3.340 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 295

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = -\frac{2^{\frac{1}{2}+m}a(Ad(c(1-m)-dm)-B(d^2-c^2m-cdm))\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2}-m,1,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-}{}\right.}{(c-d)d(c+d)^2f}$$
$$+\frac{2^{\frac{1}{2}+m}(Bc-Ad)m\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx)}{d\,(c^2-d^2)\,f}$$
$$-\frac{(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^m}{(c^2-d^2)\,f(c+d\sin(e+fx))}$$

output
```
-2^(1/2+m)*a*(A*d*(c*(1-m)-d*m)-B*(-c^2*m-c*d*m+d^2))*AppellF1(1/2,1,1/2-m
,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*cos(f*x+e)*(1+sin(f*x+e))^
(1/2-m)*(a+a*sin(f*x+e))^(-1+m)/(c-d)/d/(c+d)^2/f+2^(1/2+m)*(-A*d+B*c)*m*c
os(f*x+e)*hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^
(-1/2-m)*(a+a*sin(f*x+e))^m/d/(c^2-d^2)/f-(-A*d+B*c)*cos(f*x+e)*(a+a*sin(f
*x+e))^m/(c^2-d^2)/f/(c+d*sin(f*x+e))
```

Mathematica [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$$

input
```
Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x
])^2,x]
```

output
```
Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x
])^2, x]
```

Rubi [A] (warning: unable to verify)

Time = 1.23 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3042, 3463, 25, 3042, 3465, 3042, 3131, 3042, 3130, 3267, 154, 27, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}dx$$

$$\Big\downarrow \text{3463}$$

$$-\frac{\int -\frac{(\sin(e+fx)a+a)^m(a(Ac+Bmc-Bd-Adm)-a(Bc-Ad)m\sin(e+fx))}{c+d\sin(e+fx)}dx}{a(c^2-d^2)}-\frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{f(c^2-d^2)(c+d\sin(e+fx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(\sin(e+fx)a+a)^m(a(Ac+Bmc-Bd-Adm)-a(Bc-Ad)m\sin(e+fx))}{c+d\sin(e+fx)}dx}{a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

$\downarrow$ 3042

$$\frac{\int \frac{(\sin(e+fx)a+a)^m(a(Ac+Bmc-Bd-Adm)-a(Bc-Ad)m\sin(e+fx))}{c+d\sin(e+fx)}dx}{a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

$\downarrow$ 3465

$$\frac{\frac{a\left(Ad(c(1-m)-dm)-B\left(c^2(-m)-cdm+d^2\right)\right)\int \frac{(\sin(e+fx)a+a)^m}{c+d\sin(e+fx)}dx}{d}-\frac{am(Bc-Ad)\int(\sin(e+fx)a+a)^mdx}{d}}{a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

$\downarrow$ 3042

$$\frac{\frac{a\left(Ad(c(1-m)-dm)-B\left(c^2(-m)-cdm+d^2\right)\right)\int \frac{(\sin(e+fx)a+a)^m}{c+d\sin(e+fx)}dx}{d}-\frac{am(Bc-Ad)\int(\sin(e+fx)a+a)^mdx}{d}}{a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

$\downarrow$ 3131

$$\frac{\frac{a\left(Ad(c(1-m)-dm)-B\left(c^2(-m)-cdm+d^2\right)\right)\int \frac{(\sin(e+fx)a+a)^m}{c+d\sin(e+fx)}dx}{d}-\frac{am(Bc-Ad)(\sin(e+fx)+1)^{-m}(a\sin(e+fx)+a)^m\int(\sin(e+fx)+1)^mdx}{d}}{a\left(c^2-d^2\right)}$$

$$\frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

$\downarrow$ 3042

$$\frac{\frac{a\left(Ad(c(1-m)-dm)-B\left(c^2(-m)-cdm+d^2\right)\right)\int \frac{(\sin(e+fx)a+a)^m}{c+d\sin(e+fx)}dx}{d}-\frac{am(Bc-Ad)(\sin(e+fx)+1)^{-m}(a\sin(e+fx)+a)^m\int(\sin(e+fx)+1)^mdx}{d}}{a\left(c^2-d^2\right)}$$

$$\frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

$\downarrow$ 3130

$$\frac{\frac{a\left(Ad(c(1-m)-dm)-B\left(c^2(-m)-cdm+d^2\right)\right)\int \frac{(\sin(e+fx)a+a)^m}{c+d\sin(e+fx)}dx}{d}+\frac{a2^{m+\frac{1}{2}}m(Bc-Ad)\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m \mathrm{I}}{df}}{a\left(c^2-d^2\right)}$$

$$\frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

$\downarrow$ 3267

$$\frac{\frac{a^3\cos(e+fx)\left(Ad(c(1-m)-dm)-B\left(c^2(-m)-cdm+d^2\right)\right)\int \frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}}{\sqrt{a-a\sin(e+fx)}(c+d\sin(e+fx))}d\sin(e+fx)}{df\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}+\frac{a2^{m+\frac{1}{2}}m(Bc-Ad)\cos(e+fx)(\sin(e+}{}}{a\left(c^2-d^2\right)}$$

$$\frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

$\downarrow$ 154

$$\frac{\frac{a^3\sqrt{1-\sin(e+fx)}\cos(e+fx)\left(Ad(c(1-m)-dm)-B\left(c^2(-m)-cdm+d^2\right)\right)\int \frac{\sqrt{2}(\sin(e+fx)a+a)^{m-\frac{1}{2}}}{\sqrt{1-\sin(e+fx)}(c+d\sin(e+fx))}d\sin(e+fx)}{\sqrt{2}df(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}+\frac{a2^{m+\frac{1}{2}}m(Bc-Ad)\cos(}{}}{a\left(c^2-d^2\right)}$$

$$\frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

$\downarrow$ 27

$$\frac{\frac{a^3\sqrt{1-\sin(e+fx)}\cos(e+fx)\left(Ad(c(1-m)-dm)-B\left(c^2(-m)-cdm+d^2\right)\right)\int \frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}}{\sqrt{1-\sin(e+fx)}(c+d\sin(e+fx))}d\sin(e+fx)}{df(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}+\frac{a2^{m+\frac{1}{2}}m(Bc-Ad)\cos(}{}}{a\left(c^2-d^2\right)}$$

$$\frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

$\downarrow$ 153

$$\frac{\frac{\sqrt{2}a^2\sqrt{1-\sin(e+fx)}\cos(e+fx)\left(Ad(c(1-m)-dm)-B\left(c^2(-m)-cdm+d^2\right)\right)(a\sin(e+fx)+a)^m \operatorname{AppellF1}\left(m+\frac{1}{2},\frac{1}{2},1,m+\frac{3}{2},\frac{1}{2}(\sin(e+fx)+1),-\frac{d}{}\right.}{df(2m+1)(c-d)(a-a\sin(e+fx))}}{a\left(c^2-\right.}$$

$$\frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

input
```
Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2,x
]
```

output
```
-(((B*c - A*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/((c^2 - d^2)*f*(c + d*
Sin[e + f*x]))) + ((2^(1/2 + m)*a*(B*c - A*d)*m*Cos[e + f*x]*Hypergeometri
c2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m
)*(a + a*Sin[e + f*x])^m)/(d*f) + (Sqrt[2]*a^2*(A*d*(c*(1 - m) - d*m) - B*
(d^2 - c^2*m - c*d*m))*AppellF1[1/2 + m, 1/2, 1, 3/2 + m, (1 + Sin[e + f*x
])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*Sqrt[1 - Sin[e + f*x
]]*(a + a*Sin[e + f*x])^m)/((c - d)*d*f*(1 + 2*m)*(a - a*Sin[e + f*x])))/(
a*(c^2 - d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !G
tQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3130
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

rule 3131
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

rule 3267
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m
, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 !IntegerQ[m]
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

rule 3465
```
Int[(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*S
in[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 NeQ[m + 1/2, 0]
```

Maple [F]

$$\int \frac{(a + a\sin(fx + e))^m (A + B\sin(fx + e))}{(c + d\sin(fx + e))^2} dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

Fricas [F]

$$\int \frac{(a + a\sin(e + fx))^m(A + B\sin(e + fx))}{(c + d\sin(e + fx))^2} dx = \int \frac{(B\sin(fx + e) + A)(a\sin(fx + e) + a)^m}{(d\sin(fx + e) + c)^2} dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
integral(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d^2*cos(f*x + e)^2
- 2*c*d*sin(f*x + e) - c^2 - d^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + a\sin(e + fx))^m(A + B\sin(e + fx))}{(c + d\sin(e + fx))^2} dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(d\sin(fx+e)+c)^2}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algori
thm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)
^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,0]%%%} / %%%{1,[0,0,2]%%%} Error: Bad Argument Valu
e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m}{(c+d\,\sin(e+f\,x))^2}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^2,x
)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^2,
x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \left(\int \frac{(a+a\sin(fx+e))^m}{\sin(fx+e)^2 d^2+2\sin(fx+e)\,cd+c^2}dx\right)a + \left(\int \frac{(a+a\sin(fx+e))^m \sin(fx+e)}{\sin(fx+e)^2 d^2+2\sin(fx+e)\,cd+c^2}dx\right)b$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

output
```
int((sin(e + f*x)*a + a)**m/(sin(e + f*x)**2*d**2 + 2*sin(e + f*x)*c*d + c
**2),x)*a + int(((sin(e + f*x)*a + a)**m*sin(e + f*x))/(sin(e + f*x)**2*d*
*2 + 2*sin(e + f*x)*c*d + c**2),x)*b
```

3.341 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 469

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx =$$

$$-\frac{2^{-\frac{1}{2}+m}a(B(2d^3m+c^3(1-m)m+2c^2d(1-m)m-cd^2(3-3m+m^2))-Ad(2cd(2-m)m-c^2(2-}{}$$

$$-\frac{2^{-\frac{1}{2}+m}m(Ad(c(3-m)-dm)-B(2d^2+c^2(1-m)-cdm))\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-\right.}{d\left(c^2-d^2\right)^2f}$$

$$-\frac{(Bc-Ad)\cos(e+fx)(a+a\sin(e+fx))^m}{2\left(c^2-d^2\right)f(c+d\sin(e+fx))^2}$$

$$+\frac{(Ad(c(3-m)-dm)-B(2d^2+c^2(1-m)-cdm))\cos(e+fx)(a+a\sin(e+fx))^m}{2\left(c^2-d^2\right)^2f(c+d\sin(e+fx))}$$

output

```
-2^(-1/2+m)*a*(B*(2*d^3*m+c^3*(1-m)*m+2*c^2*d*(1-m)*m-c*d^2*(m^2-3*m+3))-A
*d*(2*c*d*(2-m)*m-c^2*(m^2-3*m+2)-d^2*(m^2-m+1)))*AppellF1(1/2,1,1/2-m,3/2
,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*cos(f*x+e)*(1+sin(f*x+e))^(1/2
-m)*(a+a*sin(f*x+e))^(-1+m)/(c-d)^2/d/(c+d)^3/f-2^(-1/2+m)*m*(A*d*(c*(3-m)
-d*m)-B*(2*d^2+c^2*(1-m)-c*d*m))*cos(f*x+e)*hypergeom([1/2, 1/2-m],[3/2],1
/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m/d/(c^2-d^2)^
2/f-1/2*(-A*d+B*c)*cos(f*x+e)*(a+a*sin(f*x+e))^m/(c^2-d^2)/f/(c+d*sin(f*x+
e))^2+1/2*(A*d*(c*(3-m)-d*m)-B*(2*d^2+c^2*(1-m)-c*d*m))*cos(f*x+e)*(a+a*si
n(f*x+e))^m/(c^2-d^2)^2/f/(c+d*sin(f*x+e))
```

Mathematica [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$$

input
```
Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x
])^3,x]
```

output
```
Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x
])^3, x]
```

Rubi [A] (warning: unable to verify)

Time = 2.31 (sec) , antiderivative size = 504, normalized size of antiderivative = 1.07, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3463, 25, 3042, 3463, 3042, 3465, 3042, 3131, 3042, 3130, 3267, 154, 27, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}dx$$

$$\Big\downarrow \text{3463}$$

$$-\frac{\int -\frac{(\sin(e+fx)a+a)^m(a(2Ac+Bmc-2Bd-Adm)+a(Bc-Ad)(1-m)\sin(e+fx))}{(c+d\sin(e+fx))^2}dx}{2a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(\sin(e+fx)a+a)^m(a(2Ac+Bmc-2Bd-Adm)+a(Bc-Ad)(1-m)\sin(e+fx))}{(c+d\sin(e+fx))^2}dx}{2a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}$$

$\big\downarrow$ 3042

$$\frac{\int \frac{(\sin(e+fx)a+a)^m(a(2Ac+Bmc-2Bd-Adm)+a(Bc-Ad)(1-m)\sin(e+fx))}{(c+d\sin(e+fx))^2}dx}{2a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}$$

$\big\downarrow$ 3463

$$\frac{\frac{a\cos(e+fx)\left(Ad(c(3-m)-dm)-B\left(c^2(1-m)-cdm+2d^2\right)\right)(a\sin(e+fx)+a)^m}{f(c^2-d^2)(c+d\sin(e+fx))}-\frac{\int \frac{(\sin(e+fx)a+a)^m\left(a^2((Bc-Ad)(1-m)(d-cm)-(c-dm)(2Ac+Bm}{}}{}}{2a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}$$

$\big\downarrow$ 3042

$$\frac{\frac{a\cos(e+fx)\left(Ad(c(3-m)-dm)-B\left(c^2(1-m)-cdm+2d^2\right)\right)(a\sin(e+fx)+a)^m}{f(c^2-d^2)(c+d\sin(e+fx))}-\frac{\int \frac{(\sin(e+fx)a+a)^m\left(a^2((Bc-Ad)(1-m)(d-cm)-(c-dm)(2Ac+Bm}{}}{}}{2a\left(c^2-d^2\right)}-\frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}$$

$\big\downarrow$ 3465

$$\frac{\frac{a\cos(e+fx)\left(Ad(c(3-m)-dm)-B\left(c^2(1-m)-cdm+2d^2\right)\right)(a\sin(e+fx)+a)^m}{f(c^2-d^2)(c+d\sin(e+fx))}-\frac{-\frac{a^2m\left(Ad(c(3-m)-dm)-B\left(c^2(1-m)-cdm+2d^2\right)\right)\int(\sin(e+fx)a+}{d}}{}}{2a\left(c^2-\right.}-\frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}$$

$\big\downarrow$ 3042

$$\frac{\frac{a\cos(e+fx)\left(Ad(c(3-m)-dm)-B\left(c^2(1-m)-cdm+2d^2\right)\right)(a\sin(e+fx)+a)^m}{f(c^2-d^2)(c+d\sin(e+fx))}-\frac{-\frac{a^2m\left(Ad(c(3-m)-dm)-B\left(c^2(1-m)-cdm+2d^2\right)\right)\int(\sin(e+fx)a+}{d}}{}}{2a\left(c^2-\right.}-\frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}$$

$\Big\downarrow$ 3131

$$\frac{\frac{a\cos(e+fx)\left(Ad(c(3-m)-dm)-B\left(c^2(1-m)-cdm+2d^2\right)\right)(a\sin(e+fx)+a)^m}{f(c^2-d^2)(c+d\sin(e+fx))}-\frac{-\frac{a^2m\left(Ad(c(3-m)-dm)-B\left(c^2(1-m)-cdm+2d^2\right)\right)(\sin(e+fx)+1)^{-}}{d}}{}}{}$$

$$\frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{a\cos(e+fx)\left(Ad(c(3-m)-dm)-B\left(c^2(1-m)-cdm+2d^2\right)\right)(a\sin(e+fx)+a)^m}{f(c^2-d^2)(c+d\sin(e+fx))}-\frac{-\frac{a^2m\left(Ad(c(3-m)-dm)-B\left(c^2(1-m)-cdm+2d^2\right)\right)(\sin(e+fx)+1)^{-}}{d}}{}}{}$$

$$\frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}$$

$\Big\downarrow$ 3130

$$\frac{\frac{a\cos(e+fx)\left(Ad(c(3-m)-dm)-B\left(c^2(1-m)-cdm+2d^2\right)\right)(a\sin(e+fx)+a)^m}{f(c^2-d^2)(c+d\sin(e+fx))}-\frac{a^22^{m+\frac{1}{2}}m\cos(e+fx)\left(Ad(c(3-m)-dm)-B\left(c^2(1-m)-cdm+2d^2\right)\right)}{}}{}$$

$$\frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}$$

$\Big\downarrow$ 3267

$$\frac{\frac{a\cos(e+fx)\left(Ad(c(3-m)-dm)-B\left(c^2(1-m)-cdm+2d^2\right)\right)(a\sin(e+fx)+a)^m}{f(c^2-d^2)(c+d\sin(e+fx))}-\frac{a^22^{m+\frac{1}{2}}m\cos(e+fx)\left(Ad(c(3-m)-dm)-B\left(c^2(1-m)-cdm+2d^2\right)\right)}{}}{}$$

$$\frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}$$

$\Big\downarrow$ 154

$$\frac{\frac{a\cos(e+fx)\left(Ad(c(3-m)-dm)-B\left(c^2(1-m)-cdm+2d^2\right)\right)(a\sin(e+fx)+a)^m}{f(c^2-d^2)(c+d\sin(e+fx))}-\frac{a^22^{m+\frac{1}{2}}m\cos(e+fx)\left(Ad(c(3-m)-dm)-B\left(c^2(1-m)-cdm+2d^2\right)\right)}{}}{}$$

$$\frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{a\cos(e+fx)\left(Ad(c(3-m)-dm)-B\left(c^2(1-m)-cdm+2d^2\right)\right)(a\sin(e+fx)+a)^m}{f(c^2-d^2)(c+d\sin(e+fx))}-\frac{a^2 2^{m+\frac{1}{2}} m\cos(e+fx)\left(Ad(c(3-m)-dm)-B\left(c^2(1-m)-cdm+2d^2\right)\right)}{}}{}$$

$$\frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}$$

$$\Big\downarrow \ 153$$

$$\frac{\frac{a\cos(e+fx)\left(Ad(c(3-m)-dm)-B\left(c^2(1-m)-cdm+2d^2\right)\right)(a\sin(e+fx)+a)^m}{f(c^2-d^2)(c+d\sin(e+fx))}-\frac{a^2 2^{m+\frac{1}{2}} m\cos(e+fx)\left(Ad(c(3-m)-dm)-B\left(c^2(1-m)-cdm+2d^2\right)\right)}{}}{}$$

$$\frac{(Bc-Ad)\cos(e+fx)(a\sin(e+fx)+a)^m}{2f\left(c^2-d^2\right)(c+d\sin(e+fx))^2}$$

input
```
Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^3,x
]
```

output
```
-1/2*((B*c - A*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/((c^2 - d^2)*f*(c +
 d*Sin[e + f*x])^2) + ((a*(A*d*(c*(3 - m) - d*m) - B*(2*d^2 + c^2*(1 - m)
- c*d*m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/((c^2 - d^2)*f*(c + d*Sin[e
 + f*x])) - ((2^(1/2 + m)*a^2*m*(A*d*(c*(3 - m) - d*m) - B*(2*d^2 + c^2*(1
 - m) - c*d*m))*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin
[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(d*f)
- (Sqrt[2]*a^3*(B*(2*d^3*m + c^3*(1 - m)*m + 2*c^2*d*(1 - m)*m - c*d^2*(3
- 3*m + m^2)) - A*d*(2*c*d*(2 - m)*m - c^2*(2 - 3*m + m^2) - d^2*(1 - m +
m^2)))*AppellF1[1/2 + m, 1/2, 1, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 +
Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e
+ f*x])^m)/((c - d)*d*f*(1 + 2*m)*(a - a*Sin[e + f*x])))/(a*(c^2 - d^2)))/
(2*a*(c^2 - d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !G
tQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3130
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

rule 3131
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

rule 3267
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m
, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 !IntegerQ[m]
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

rule 3465
```
Int[(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[B
/d   Int[(a + b*Sin[e + f*x])^m, x], x] - Simp[(B*c - A*d)/d   Int[(a + b*S
in[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 NeQ[m + 1/2, 0]
```

Maple [F]

$$\int \frac{(a+a\sin(fx+e))^m (A+B\sin(fx+e))}{(c+d\sin(fx+e))^3}dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)
```

Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(d\sin(fx+e)+c)^3}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algori
thm="fricas")
```

output
```
integral(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(3*c*d^2*cos(f*x + e
)^2 - c^3 - 3*c*d^2 + (d^3*cos(f*x + e)^2 - 3*c^2*d - d^3)*sin(f*x + e)),
x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(d\sin(fx+e)+c)^3}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algori
thm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)
^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,0]%%%} / %%%{1,[0,0,3]%%%} Error: Bad Argument Valu
e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx = \int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m}{(c+d\,\sin(e+f\,x))^3}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^3,x
)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^3,
x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^3}\,dx$$
$$= \left(\int \frac{(a+a\sin(fx+e))^m}{\sin(fx+e)^3 d^3+3\sin(fx+e)^2 c d^2+3\sin(fx+e)c^2d+c^3}dx\right)a$$
$$+\left(\int \frac{(a+a\sin(fx+e))^m\sin(fx+e)}{\sin(fx+e)^3 d^3+3\sin(fx+e)^2 c d^2+3\sin(fx+e)c^2d+c^3}dx\right)b$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)
```

output
```
int((sin(e + f*x)*a + a)**m/(sin(e + f*x)**3*d**3 + 3*sin(e + f*x)**2*c*d*
*2 + 3*sin(e + f*x)*c**2*d + c**3),x)*a + int(((sin(e + f*x)*a + a)**m*sin
(e + f*x))/(sin(e + f*x)**3*d**3 + 3*sin(e + f*x)**2*c*d**2 + 3*sin(e + f*
x)*c**2*d + c**3),x)*b
```

3.342 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 272

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}\,dx =$$

$$-\frac{2^{\frac{1}{2}+m}a(A-B)(c+d)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2}-m,-\frac{3}{2},\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1+\sin}{f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

$$-\frac{2^{\frac{3}{2}+m}B(c+d)\operatorname{AppellF1}\left(\frac{1}{2},-\frac{1}{2}-m,-\frac{3}{2},\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1+\sin(e+}{f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

output

```
-2^(1/2+m)*a*(A-B)*(c+d)*AppellF1(1/2,-3/2,1/2-m,3/2,d*(1-sin(f*x+e))/(c+d
),1/2-1/2*sin(f*x+e))*cos(f*x+e)*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^(
-1+m)*(c+d*sin(f*x+e))^(1/2)/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-2^(3/2+m)*B*
(c+d)*AppellF1(1/2,-3/2,-1/2-m,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+
e))*cos(f*x+e)*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))
^(1/2)/f/((c+d*sin(f*x+e))/(c+d))^(1/2)
```

Mathematica [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}\,dx=\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}\,dx$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])
^(3/2),x]
```

output
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])
^(3/2), x]
```

Rubi [A] (warning: unable to verify)

Time = 0.77 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {3042, 3466, 3042, 3267, 157, 27, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}dx$$

$$\Big\downarrow \text{3466}$$

$$(A-B)\int (\sin(e+fx)a+a)^m(c+d\sin(e+fx))^{3/2}dx+\frac{B\int (\sin(e+fx)a+a)^{m+1}(c+d\sin(e+fx))^{3/2}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$(A-B)\int (\sin(e+fx)a+a)^m(c+d\sin(e+fx))^{3/2}dx+\frac{B\int (\sin(e+fx)a+a)^{m+1}(c+d\sin(e+fx))^{3/2}dx}{a}$$

$\Big\downarrow$ 3267

$$\frac{a^2(A-B)\cos(e+fx)\int \frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}(c+d\sin(e+fx))^{3/2}}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}+\frac{aB\cos(e+fx)\int \frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}(c+d\sin(e+fx))^{3/2}}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}$$

$\Big\downarrow$ 157

$$\frac{a^2(A-B)\sqrt{1-\sin(e+fx)}\cos(e+fx)\int \frac{\sqrt{2}(\sin(e+fx)a+a)^{m-\frac{1}{2}}(c+d\sin(e+fx))^{3/2}}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}+\frac{aB\sqrt{1-\sin(e+fx)}\cos(e+fx)\int \frac{\sqrt{2}(\sin(e+fx)a+a)^{m+\frac{1}{2}}(c+d\sin(e+fx))^{3/2}}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}$$

$\Big\downarrow$ 27

$$\frac{a^2(A-B)\sqrt{1-\sin(e+fx)}\cos(e+fx)\int \frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}(c+d\sin(e+fx))^{3/2}}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}+\frac{aB\sqrt{1-\sin(e+fx)}\cos(e+fx)\int \frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}(c+d\sin(e+fx))^{3/2}}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}$$

$\Big\downarrow$ 156

$$\frac{a^2(A-B)(c-d)\sqrt{1-\sin(e+fx)}\cos(e+fx)\sqrt{c+d\sin(e+fx)}\int \frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}\left(\frac{c}{c-d}+\frac{d\sin(e+fx)}{c-d}\right)^{3/2}}{\sqrt{1-\sin(e+fx)}}d\sin(e}{f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

$$\frac{aB(c-d)\sqrt{1-\sin(e+fx)}\cos(e+fx)\sqrt{c+d\sin(e+fx)}\int \frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}\left(\frac{c}{c-d}+\frac{d\sin(e+fx)}{c-d}\right)^{3/2}}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

$\Big\downarrow$ 155

$$\frac{\sqrt{2}a(A-B)(c-d)\sqrt{1-\sin(e+fx)}\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{c+d\sin(e+fx)}\,\text{AppellF1}\left(m+\frac{1}{2},\frac{1}{2},-\right.}{f(2m+1)(a-a\sin(e+fx))\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$
$$\frac{\sqrt{2}B(c-d)\sqrt{1-\sin(e+fx)}\cos(e+fx)(a\sin(e+fx)+a)^{m+1}\sqrt{c+d\sin(e+fx)}\,\text{AppellF1}\left(m+\frac{3}{2},\frac{1}{2},-\frac{3}{2},m\right.}{f(2m+3)(a-a\sin(e+fx))\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

input
```
Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2)
,x]
```

output
```
(Sqrt[2]*a*(A - B)*(c - d)*AppellF1[1/2 + m, 1/2, -3/2, 3/2 + m, (1 + Sin[
e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*Sqrt[1 - Sin[
e + f*x]]*(a + a*Sin[e + f*x])^m*Sqrt[c + d*Sin[e + f*x]])/(f*(1 + 2*m)*(a
 - a*Sin[e + f*x])*Sqrt[(c + d*Sin[e + f*x])/(c - d)]) + (Sqrt[2]*B*(c - d
)*AppellF1[3/2 + m, 1/2, -3/2, 5/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Si
n[e + f*x]))/(c - d))]*Cos[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e +
f*x])^(1 + m)*Sqrt[c + d*Sin[e + f*x]])/(f*(3 + 2*m)*(a - a*Sin[e + f*x])*
Sqrt[(c + d*Sin[e + f*x])/(c - d)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 157
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
&  !GtQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !Si
mplerQ[e + f*x, a + b*x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3267
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m
, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 !IntegerQ[m]
```

rule 3466
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)/b   Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x
] + Simp[B/b   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [F]

$$\int (a+a\sin(fx+e))^m (A+B\sin(fx+e)) (c+d\sin(fx+e))^{\frac{3}{2}} dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}\,dx = \int (B\sin(fx+e)+A)(d\sin(fx+e)+c)^{\frac{3}{2}}(a\sin(fx+e)+a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x, al
gorithm="fricas")
```

output
```
integral(-(B*d*cos(f*x + e)^2 - A*c - B*d - (B*c + A*d)*sin(f*x + e))*sqrt
(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}\,dx=\int (B\sin(fx+e)+A)(d\sin(fx+e)+c)^{\frac{3}{2}}(a\sin(fx+e)+a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^(3/2)*(a*sin(f*x + e)
+ a)^m, x)
```

Giac [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}\,dx=\int (B\sin(fx+e)+A)(d\sin(fx+e)+c)^{\frac{3}{2}}(a\sin(fx+e)+a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x, al
gorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^(3/2)*(a*sin(f*x + e)
+ a)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}\,dx = \int (A + B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m\,(c+d\,\sin(e+f\,x))^{3/2}\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^(3/2)
,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^(3/2)
, x)
```

Reduce [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}\,dx = \left(\int \sqrt{\sin(fx+e)\,d+c}\,(a+a\sin(fx+e))^m\sin(fx+e)^2\,dx\right)bd + \left(\int \sqrt{\sin(fx+e)\,d+c}\,(a+a\sin(fx+e))^m\sin(fx+e)\,dx\right)ad + \left(\int \sqrt{\sin(fx+e)\,d+c}\,(a+a\sin(fx+e))^m\sin(fx+e)\,dx\right)bc + \left(\int \sqrt{\sin(fx+e)\,d+c}\,(a+a\sin(fx+e))^m\,dx\right)ac$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x)
```

output
```
int(sqrt(sin(e + f*x)*d + c)*(sin(e + f*x)*a + a)**m*sin(e + f*x)**2,x)*b*
d + int(sqrt(sin(e + f*x)*d + c)*(sin(e + f*x)*a + a)**m*sin(e + f*x),x)*a
*d + int(sqrt(sin(e + f*x)*d + c)*(sin(e + f*x)*a + a)**m*sin(e + f*x),x)*
b*c + int(sqrt(sin(e + f*x)*d + c)*(sin(e + f*x)*a + a)**m,x)*a*c
```

3.343 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c+d\sin(e}$

Optimal result

Integrand size = 37, antiderivative size = 266

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}\,dx =$$

$$-\frac{2^{\frac{1}{2}+m}a(A-B)\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2}-m,-\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1+\sin(e+}{f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

$$-\frac{2^{\frac{3}{2}+m}B\,\text{AppellF1}\left(\frac{1}{2},-\frac{1}{2}-m,-\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1+\sin(e+fx))}{f\sqrt{\frac{c+d\sin(e+fx)}{c+d}}}$$

output
```
-2^(1/2+m)*a*(A-B)*AppellF1(1/2,-1/2,1/2-m,3/2,d*(1-sin(f*x+e))/(c+d),1/2-
1/2*sin(f*x+e))*cos(f*x+e)*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^(-1+m)*
(c+d*sin(f*x+e))^(1/2)/f/((c+d*sin(f*x+e))/(c+d))^(1/2)-2^(3/2+m)*B*Appell
F1(1/2,-1/2,-1/2-m,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*cos(f*x+
e)*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(1/2)/f/((c
+d*sin(f*x+e))/(c+d))^(1/2)
```

Mathematica [F]

$$\int (a + a\sin(e + fx))^m(A + B\sin(e + fx))\sqrt{c + d\sin(e + fx)}\,dx$$
$$= \int (a + a\sin(e + fx))^m(A + B\sin(e + fx))\sqrt{c + d\sin(e + fx)}\,dx$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*Sqrt[c + d*Sin[e + f
*x]],x]
```

output
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*Sqrt[c + d*Sin[e + f
*x]], x]
```

Rubi [A] (warning: unable to verify)

Time = 0.73 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.216$, Rules used = {3042, 3466, 3042, 3267, 157, 27, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e + fx) + a)^m(A + B\sin(e + fx))\sqrt{c + d\sin(e + fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e + fx) + a)^m(A + B\sin(e + fx))\sqrt{c + d\sin(e + fx)}dx$$

$$\Big\downarrow \text{3466}$$

$$(A - B)\int (\sin(e + fx)a + a)^m\sqrt{c + d\sin(e + fx)}dx + \frac{B\int (\sin(e + fx)a + a)^{m+1}\sqrt{c + d\sin(e + fx)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-B)\int (\sin(e+fx)a+a)^m\sqrt{c+d\sin(e+fx)}dx+\frac{B\int (\sin(e+fx)a+a)^{m+1}\sqrt{c+d\sin(e+fx)}dx}{a}}{}$$

$$\Big\downarrow \text{ 3267}$$

$$\frac{a^2(A-B)\cos(e+fx)\int \frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}\sqrt{c+d\sin(e+fx)}}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}+\frac{aB\cos(e+fx)\int \frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}\sqrt{c+d\sin(e+fx)}}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{ 157}$$

$$\frac{a^2(A-B)\sqrt{1-\sin(e+fx)}\cos(e+fx)\int \frac{\sqrt{2}(\sin(e+fx)a+a)^{m-\frac{1}{2}}\sqrt{c+d\sin(e+fx)}}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}+\frac{aB\sqrt{1-\sin(e+fx)}\cos(e+fx)\int \frac{\sqrt{2}(\sin(e+fx)a+a)^{m+\frac{1}{2}}\sqrt{c+d\sin(e+fx)}}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{a^2(A-B)\sqrt{1-\sin(e+fx)}\cos(e+fx)\int \frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}\sqrt{c+d\sin(e+fx)}}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}+\frac{aB\sqrt{1-\sin(e+fx)}\cos(e+fx)\int \frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}\sqrt{c+d\sin(e+fx)}}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{ 156}$$

$$\frac{a^2(A-B)\sqrt{1-\sin(e+fx)}\cos(e+fx)\sqrt{c+d\sin(e+fx)}\int \frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}\sqrt{\frac{c}{c-d}+\frac{d\sin(e+fx)}{c-d}}}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}+\frac{aB\sqrt{1-\sin(e+fx)}\cos(e+fx)\sqrt{c+d\sin(e+fx)}\int \frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}\sqrt{\frac{c}{c-d}+\frac{d\sin(e+fx)}{c-d}}}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

$$\Big\downarrow \text{ 155}$$

$$\frac{\sqrt{2}a(A-B)\sqrt{1-\sin(e+fx)}\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{c+d\sin(e+fx)}\operatorname{AppellF1}\left(m+\frac{1}{2},\frac{1}{2},-\frac{1}{2},m\right.}{f(2m+1)(a-a\sin(e+fx))\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$
$$\frac{\sqrt{2}B\sqrt{1-\sin(e+fx)}\cos(e+fx)(a\sin(e+fx)+a)^{m+1}\sqrt{c+d\sin(e+fx)}\operatorname{AppellF1}\left(m+\frac{3}{2},\frac{1}{2},-\frac{1}{2},m+\frac{5}{2},\frac{1}{2}\right.}{f(2m+3)(a-a\sin(e+fx))\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

input
```
Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]],x
]
```

output
```
(Sqrt[2]*a*(A - B)*AppellF1[1/2 + m, 1/2, -1/2, 3/2 + m, (1 + Sin[e + f*x]
)/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*Sqrt[1 - Sin[e + f*x]
]*(a + a*Sin[e + f*x])^m*Sqrt[c + d*Sin[e + f*x]])/(f*(1 + 2*m)*(a - a*Sin
[e + f*x])*Sqrt[(c + d*Sin[e + f*x])/(c - d)]) + (Sqrt[2]*B*AppellF1[3/2 +
 m, 1/2, -1/2, 5/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c
- d))]*Cos[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(1 + m)*Sq
rt[c + d*Sin[e + f*x]])/(f*(3 + 2*m)*(a - a*Sin[e + f*x])*Sqrt[(c + d*Sin[
e + f*x])/(c - d)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
 - e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 157
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
&  !GtQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !Si
mplerQ[e + f*x, a + b*x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3267
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m
, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 !IntegerQ[m]
```

rule 3466
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)/b   Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x
] + Simp[B/b   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [F]

$$\int (a + a\sin(fx + e))^m (A + B\sin(fx + e))\sqrt{c + d\sin(fx + e)}dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))\sqrt{c + d\sin(e + fx)}\,dx = \int (B\sin(fx + e) + A)\sqrt{d\sin(fx + e) + c}(a\sin(fx + e) + a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x, al
gorithm="fricas")
```

output
```
integral((B*sin(f*x + e) + A)*sqrt(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a
)^m, x)
```

Sympy [F]

$$\int (a + a\sin(e + fx))^m (A + B\sin(e + fx))\sqrt{c + d\sin(e + fx)}\,dx = \int (a(\sin(e + fx) + 1))^m (A + B\sin(e + fx))\sqrt{c + d\sin(e + fx)}\,dx$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**(1/2),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))*sqrt(c + d*sin(e +
 f*x)), x)
```

Maxima [F]

$$\int (a + a\sin(e + fx))^m(A + B\sin(e + fx))\sqrt{c + d\sin(e + fx)}\,dx$$
$$= \int (B\sin(fx + e) + A)\sqrt{d\sin(fx + e) + c}(a\sin(fx + e) + a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(d*sin(f*x + e) + c)*(a*sin(f*x + e) +
a)^m, x)
```

Giac [F]

$$\int (a + a\sin(e + fx))^m(A + B\sin(e + fx))\sqrt{c + d\sin(e + fx)}\,dx$$
$$= \int (B\sin(fx + e) + A)\sqrt{d\sin(fx + e) + c}(a\sin(fx + e) + a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x, al
gorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(d*sin(f*x + e) + c)*(a*sin(f*x + e) +
a)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}\,dx$$
$$=\int (A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m\,\sqrt{c+d\,\sin(e+f\,x)}\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^(1/2)
,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^(1/2)
, x)
```

Reduce [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}\,dx$$
$$=\left(\int \sqrt{\sin(fx+e)\,d+c}\,(a+a\sin(fx+e))^m\sin(fx+e)\,dx\right)b+\left(\int \sqrt{\sin(fx+e)\,d+c}\,(a+a\sin(fx+e))^m\,dx\right)a$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x)
```

output
```
int(sqrt(sin(e + f*x)*d + c)*(sin(e + f*x)*a + a)**m*sin(e + f*x),x)*b + i
nt(sqrt(sin(e + f*x)*d + c)*(sin(e + f*x)*a + a)**m,x)*a
```

3.344 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 266

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}\,dx = -\frac{2^{\frac{1}{2}+m}a(A-B)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1+\sin(e+f}{f\sqrt{c+d\sin(e+fx)}} - \frac{2^{\frac{3}{2}+m}B\operatorname{AppellF1}\left(\frac{1}{2},-\frac{1}{2}-m,\frac{1}{2},\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1+\sin(e+fx))^{-\frac{1}{2}}}{f\sqrt{c+d\sin(e+fx)}}$$

output
```
-2^(1/2+m)*a*(A-B)*AppellF1(1/2,1/2,1/2-m,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1
/2*sin(f*x+e))*cos(f*x+e)*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^(-1+m)*(
(c+d*sin(f*x+e))/(c+d))^(1/2)/f/(c+d*sin(f*x+e))^(1/2)-2^(3/2+m)*B*AppellF
1(1/2,1/2,-1/2-m,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*cos(f*x+e)
*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m*((c+d*sin(f*x+e))/(c+d))^(1/2)
/f/(c+d*sin(f*x+e))^(1/2)
```

Mathematica [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}\,dx$$
$$=\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}\,dx$$

input
```
Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/Sqrt[c + d*Sin[e +
 f*x]],x]
```

output
```
Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/Sqrt[c + d*Sin[e +
 f*x]], x]
```

Rubi [A] (warning: unable to verify)

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Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^m(A+B\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^m(A+B\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3466}$$

$$(A-B)\int \frac{(\sin(e+fx)a+a)^m}{\sqrt{c+d\sin(e+fx)}}dx+\frac{B\int \frac{(\sin(e+fx)a+a)^{m+1}}{\sqrt{c+d\sin(e+fx)}}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$(A-B)\int \frac{(\sin(e+fx)a+a)^m}{\sqrt{c+d\sin(e+fx)}}dx+\frac{B\int \frac{(\sin(e+fx)a+a)^{m+1}}{\sqrt{c+d\sin(e+fx)}}dx}{a}$$

$\Big\downarrow$ 3267

$$\frac{a^2(A-B)\cos(e+fx)\int \frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}}{\sqrt{a-a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}+\frac{aB\cos(e+fx)\int \frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}}{\sqrt{a-a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}$$

$\Big\downarrow$ 157

$$\frac{a^2(A-B)\sqrt{1-\sin(e+fx)}\cos(e+fx)\int \frac{\sqrt{2}(\sin(e+fx)a+a)^{m-\frac{1}{2}}}{\sqrt{1-\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}d\sin(e+fx)}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}+\frac{aB\sqrt{1-\sin(e+fx)}\cos(e+fx)\int \frac{\sqrt{2}(\sin(e+fx)a+a)^{m+\frac{1}{2}}}{\sqrt{1-\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}d\sin(e+fx)}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}$$

$\Big\downarrow$ 27

$$\frac{a^2(A-B)\sqrt{1-\sin(e+fx)}\cos(e+fx)\int \frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}}{\sqrt{1-\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}d\sin(e+fx)}{f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}+\frac{aB\sqrt{1-\sin(e+fx)}\cos(e+fx)\int \frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}}{\sqrt{1-\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}d\sin(e+fx)}{f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}$$

$\Big\downarrow$ 156

$$\frac{a^2(A-B)\sqrt{1-\sin(e+fx)}\cos(e+fx)\sqrt{\frac{c+d\sin(e+fx)}{c-d}}\int \frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}}{\sqrt{1-\sin(e+fx)}\sqrt{\frac{c}{c-d}+\frac{d\sin(e+fx)}{c-d}}}d\sin(e+fx)}{f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}+\frac{aB\sqrt{1-\sin(e+fx)}\cos(e+fx)\sqrt{\frac{c+d\sin(e+fx)}{c-d}}\int \frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}}{\sqrt{1-\sin(e+fx)}\sqrt{\frac{c}{c-d}+\frac{d\sin(e+fx)}{c-d}}}d\sin(e+fx)}{f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}$$

$\Big\downarrow$ 155

$$\frac{\sqrt{2}a(A-B)\sqrt{1-\sin(e+fx)}\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{\frac{c+d\sin(e+fx)}{c-d}}\operatorname{AppellF1}\left(m+\frac{1}{2},\frac{1}{2},\frac{1}{2},m+\frac{3}{2},\frac{1}{2}(s\right.}{f(2m+1)(a-a\sin(e+fx))\sqrt{c+d\sin(e+fx)}}$$
$$\frac{\sqrt{2}B\sqrt{1-\sin(e+fx)}\cos(e+fx)(a\sin(e+fx)+a)^{m+1}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}\operatorname{AppellF1}\left(m+\frac{3}{2},\frac{1}{2},\frac{1}{2},m+\frac{5}{2},\frac{1}{2}(\sin(e\right.}{f(2m+3)(a-a\sin(e+fx))\sqrt{c+d\sin(e+fx)}}$$

input
```
Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/Sqrt[c + d*Sin[e + f*x]]
,x]
```

output
```
(Sqrt[2]*a*(A - B)*AppellF1[1/2 + m, 1/2, 1/2, 3/2 + m, (1 + Sin[e + f*x])
/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*Sqrt[1 - Sin[e + f*x]]
*(a + a*Sin[e + f*x])^m*Sqrt[(c + d*Sin[e + f*x])/(c - d)])/(f*(1 + 2*m)*(
a - a*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]) + (Sqrt[2]*B*AppellF1[3/2 +
m, 1/2, 1/2, 5/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c -
d))]*Cos[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(1 + m)*Sqrt
[(c + d*Sin[e + f*x])/(c - d)])/(f*(3 + 2*m)*(a - a*Sin[e + f*x])*Sqrt[c +
 d*Sin[e + f*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 157
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
&  !GtQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !Si
mplerQ[e + f*x, a + b*x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3267
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m
, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 !IntegerQ[m]
```

rule 3466
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)/b   Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x
] + Simp[B/b   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [F]

$$\int \frac{(a + a\sin(fx + e))^m (A + B\sin(fx + e))}{\sqrt{c + d\sin(fx + e)}} dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int \frac{(a + a\sin(e + fx))^m(A + B\sin(e + fx))}{\sqrt{c + d\sin(e + fx)}}\, dx = \int \frac{(B\sin(fx + e) + A)(a\sin(fx + e) + a)^m}{\sqrt{d\sin(fx + e) + c}}\, dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, al
gorithm="fricas")
```

output
```
integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/sqrt(d*sin(f*x + e) +
 c), x)
```

Sympy [F]

$$\int \frac{(a + a\sin(e + fx))^m(A + B\sin(e + fx))}{\sqrt{c + d\sin(e + fx)}}\, dx = \int \frac{(a(\sin(e + fx) + 1))^m (A + B\sin(e + fx))}{\sqrt{c + d\sin(e + fx)}}\, dx$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**(1/2),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))/sqrt(c + d*sin(e +
 f*x)), x)
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{\sqrt{d\sin(fx+e)+c}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/sqrt(d*sin(f*x + e)
+ c), x)
```

Giac [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}\,dx = \int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{\sqrt{d\sin(fx+e)+c}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, al
gorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/sqrt(d*sin(f*x + e)
+ c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}\,dx = \int \frac{(A+B\sin(e+fx))\,(a+a\sin(e+fx))^m}{\sqrt{c+d\sin(e+fx)}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^(1/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^(1/
2), x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{\sqrt{c+d\sin(e+fx)}}\,dx = \left(\int \frac{\sqrt{\sin(fx+e)\,d+c}\,(a+a\sin(fx+e))^m\sin(fx+e)}{\sin(fx+e)\,d+c}dx\right)b + \left(\int \frac{\sqrt{\sin(fx+e)\,d+c}\,(a+a\sin(fx+e))^m}{\sin(fx+e)\,d+c}dx\right)a$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x)
```

output
```
int((sqrt(sin(e + f*x)*d + c)*(sin(e + f*x)*a + a)**m*sin(e + f*x))/(sin(e
 + f*x)*d + c),x)*b + int((sqrt(sin(e + f*x)*d + c)*(sin(e + f*x)*a + a)**
m)/(sin(e + f*x)*d + c),x)*a
```

3.345 $\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 276

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^{3/2}}\,dx =$$

$$-\frac{2^{\frac{1}{2}+m}a(A-B)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1+\sin(e+fx)}{(c+d)f\sqrt{c+d\sin(e+fx)}}$$

$$-\frac{2^{\frac{3}{2}+m}B\operatorname{AppellF1}\left(\frac{1}{2},-\frac{1}{2}-m,\frac{3}{2},\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1+\sin(e+fx))^{-\frac{1}{2}-m}}{(c+d)f\sqrt{c+d\sin(e+fx)}}$$

output
```
-2^(1/2+m)*a*(A-B)*AppellF1(1/2,3/2,1/2-m,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1
/2*sin(f*x+e))*cos(f*x+e)*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^(-1+m)*(
(c+d*sin(f*x+e))/(c+d))^(1/2)/(c+d)/f/(c+d*sin(f*x+e))^(1/2)-2^(3/2+m)*B*A
ppellF1(1/2,3/2,-1/2-m,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*cos(
f*x+e)*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m*((c+d*sin(f*x+e))/(c+d))
^(1/2)/(c+d)/f/(c+d*sin(f*x+e))^(1/2)
```

Mathematica [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^{3/2}}\,dx=\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^{3/2}}\,dx$$

input
```
Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x
])^(3/2),x]
```

output
```
Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x
])^(3/2), x]
```

Rubi [A] (warning: unable to verify)

Time = 0.75 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {3042, 3466, 3042, 3267, 157, 27, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\sin(e+fx)+a)^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3466}$$

$$(A-B)\int \frac{(\sin(e+fx)a+a)^m}{(c+d\sin(e+fx))^{3/2}}dx+\frac{B\int \frac{(\sin(e+fx)a+a)^{m+1}}{(c+d\sin(e+fx))^{3/2}}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$(A-B)\int \frac{(\sin(e+fx)a+a)^m}{(c+d\sin(e+fx))^{3/2}}dx+\frac{B\int \frac{(\sin(e+fx)a+a)^{m+1}}{(c+d\sin(e+fx))^{3/2}}dx}{a}$$

$$\Big\downarrow \text{3267}$$

$$\frac{a^2(A-B)\cos(e+fx)\int \frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}}{\sqrt{a-a\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}+\frac{aB\cos(e+fx)\int \frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}}{\sqrt{a-a\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{157}$$

$$\frac{a^2(A-B)\sqrt{1-\sin(e+fx)}\cos(e+fx)\int \frac{\sqrt{2}(\sin(e+fx)a+a)^{m-\frac{1}{2}}}{\sqrt{1-\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}d\sin(e+fx)}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}+\frac{aB\sqrt{1-\sin(e+fx)}\cos(e+fx)\int \frac{\sqrt{2}(\sin(e+fx)a+a)^{m+\frac{1}{2}}}{\sqrt{1-\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}d\sin(e+fx)}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{a^2(A-B)\sqrt{1-\sin(e+fx)}\cos(e+fx)\int \frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}}{\sqrt{1-\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}d\sin(e+fx)}{f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}+\frac{aB\sqrt{1-\sin(e+fx)}\cos(e+fx)\int \frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}}{\sqrt{1-\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}d\sin(e+fx)}{f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{156}$$

$$\frac{a^2(A-B)\sqrt{1-\sin(e+fx)}\cos(e+fx)\sqrt{\frac{c+d\sin(e+fx)}{c-d}}\int \frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}}{\sqrt{1-\sin(e+fx)}\left(\frac{c}{c-d}+\frac{d\sin(e+fx)}{c-d}\right)^{3/2}}d\sin(e+fx)}{f(c-d)(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}+\frac{aB\sqrt{1-\sin(e+fx)}\cos(e+fx)\sqrt{\frac{c+d\sin(e+fx)}{c-d}}\int \frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}}{\sqrt{1-\sin(e+fx)}\left(\frac{c}{c-d}+\frac{d\sin(e+fx)}{c-d}\right)^{3/2}}d\sin(e+fx)}{f(c-d)(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}$$

$$\Big\downarrow \text{155}$$

$$\frac{\sqrt{2}a(A-B)\sqrt{1-\sin(e+fx)}\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{\frac{c+d\sin(e+fx)}{c-d}}\operatorname{AppellF1}\left(m+\frac{1}{2},\frac{1}{2},\frac{3}{2},m+\frac{3}{2},\frac{1}{2}(\right.}{f(2m+1)(c-d)(a-a\sin(e+fx))\sqrt{c+d\sin(e+fx)}}$$

$$\frac{\sqrt{2}B\sqrt{1-\sin(e+fx)}\cos(e+fx)(a\sin(e+fx)+a)^{m+1}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}\operatorname{AppellF1}\left(m+\frac{3}{2},\frac{1}{2},\frac{3}{2},m+\frac{5}{2},\frac{1}{2}(\sin(e\right.}{f(2m+3)(c-d)(a-a\sin(e+fx))\sqrt{c+d\sin(e+fx)}}$$

input
```
Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^(3/
2),x]
```

output
```
(Sqrt[2]*a*(A - B)*AppellF1[1/2 + m, 1/2, 3/2, 3/2 + m, (1 + Sin[e + f*x])
/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*Sqrt[1 - Sin[e + f*x]]
*(a + a*Sin[e + f*x])^m*Sqrt[(c + d*Sin[e + f*x])/(c - d)])/((c - d)*f*(1
+ 2*m)*(a - a*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]) + (Sqrt[2]*B*AppellF
1[3/2 + m, 1/2, 3/2, 5/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]
))/(c - d))]*Cos[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(1 +
 m)*Sqrt[(c + d*Sin[e + f*x])/(c - d)])/((c - d)*f*(3 + 2*m)*(a - a*Sin[e
+ f*x])*Sqrt[c + d*Sin[e + f*x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 157
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& !GtQ[Simplify[b/(b*c - a*d)], 0] && !SimplerQ[c + d*x, a + b*x] && !Si
mplerQ[e + f*x, a + b*x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3267
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m
, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 !IntegerQ[m]
```

rule 3466
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)/b   Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x
] + Simp[B/b   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [F]

$$\int \frac{(a+a\sin(fx+e))^m (A+B\sin(fx+e))}{(c+d\sin(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^{3/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(d\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, al
gorithm="fricas")
```

output
```
integral(-(B*sin(f*x + e) + A)*sqrt(d*sin(f*x + e) + c)*(a*sin(f*x + e) +
a)^m/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2), x)
```

Sympy [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^{3/2}}\,dx=\int \frac{(a(\sin(e+fx)+1))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**(3/2),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))/(c + d*sin(e + f*x
))**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^{3/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(d\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, al
gorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)
^(3/2), x)
```

Giac [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^{3/2}}\,dx=\int \frac{(B\sin(fx+e)+A)(a\sin(fx+e)+a)^m}{(d\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, al
gorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)
^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^{3/2}}\,dx=\int \frac{(A+B\sin(e+fx))\,(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^{3/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^(3/
2),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^(3/
2), x)
```

Reduce [F]

$$\int \frac{(a+a\sin(e+fx))^m(A+B\sin(e+fx))}{(c+d\sin(e+fx))^{3/2}}\,dx=\left(\int \frac{\sqrt{\sin(fx+e)\,d+c}\,(a+a\sin(fx+e))^m\sin(fx+\right.$$
$$\left.\overline{\sin(fx+e)^2d^2+2\sin(fx+e)\,cd+c^2}\right.$$
$$+\left(\int \frac{\sqrt{\sin(fx+e)\,d+c}\,(a+a\sin(fx+e))^m}{\sin(fx+e)^2d^2+2\sin(fx+e)\,cd+c^2}dx\right)a$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x)
```

output
```
int((sqrt(sin(e + f*x)*d + c)*(sin(e + f*x)*a + a)**m*sin(e + f*x))/(sin(e
 + f*x)**2*d**2 + 2*sin(e + f*x)*c*d + c**2),x)*b + int((sqrt(sin(e + f*x)
*d + c)*(sin(e + f*x)*a + a)**m)/(sin(e + f*x)**2*d**2 + 2*sin(e + f*x)*c*
d + c**2),x)*a
```

3.346 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal result

Integrand size = 35, antiderivative size = 262

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx =$$
$$-\frac{2^{\frac{1}{2}+m}a(A-B)\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2}-m,-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1+\sin(e+\cdots}{f}$$
$$-\frac{2^{\frac{3}{2}+m}B\operatorname{AppellF1}\left(\frac{1}{2},-\frac{1}{2}-m,-n,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1+\sin(e+fx))\cdots}{f}$$

output
```
-2^(1/2+m)*a*(A-B)*AppellF1(1/2,-n,1/2-m,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/
2*sin(f*x+e))*cos(f*x+e)*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^(-1+m)*(c
+d*sin(f*x+e))^n/f/(((c+d*sin(f*x+e))/(c+d))^n)-2^(3/2+m)*B*AppellF1(1/2,-
n,-1/2-m,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*cos(f*x+e)*(1+sin(
f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n/f/(((c+d*sin(f*x+e)
)/(c+d))^n)
```

Mathematica [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$=\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])
^n,x]
```

output
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])
^n, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.66 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3042, 3466, 3042, 3267, 157, 27, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c+d\sin(e+fx))^ndx$$

$$\Big\downarrow \text{ 3466}$$

$$(A-B)\int (\sin(e+fx)a+a)^m(c+d\sin(e+fx))^ndx+\frac{B\int (\sin(e+fx)a+a)^{m+1}(c+d\sin(e+fx))^ndx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(A-B)\int (\sin(e+fx)a+a)^m(c+d\sin(e+fx))^ndx+B\int (\sin(e+fx)a+a)^{m+1}(c+d\sin(e+fx))^ndx}{a}$$

$\Big\downarrow$ 3267

$$\frac{a^2(A-B)\cos(e+fx)\int \frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}(c+d\sin(e+fx))^n}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}+\frac{aB\cos(e+fx)\int \frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}(c+d\sin(e+fx))^n}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}$$

$\Big\downarrow$ 157

$$\frac{a^2(A-B)\sqrt{1-\sin(e+fx)}\cos(e+fx)\int \frac{\sqrt{2}(\sin(e+fx)a+a)^{m-\frac{1}{2}}(c+d\sin(e+fx))^n}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}+\frac{aB\sqrt{1-\sin(e+fx)}\cos(e+fx)\int \frac{\sqrt{2}(\sin(e+fx)a+a)^{m+\frac{1}{2}}(c+d\sin(e+fx))^n}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}$$

$\Big\downarrow$ 27

$$\frac{a^2(A-B)\sqrt{1-\sin(e+fx)}\cos(e+fx)\int \frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}(c+d\sin(e+fx))^n}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}+\frac{aB\sqrt{1-\sin(e+fx)}\cos(e+fx)\int \frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}(c+d\sin(e+fx))^n}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}$$

$\Big\downarrow$ 156

$$\frac{a^2(A-B)\sqrt{1-\sin(e+fx)}\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}\int \frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}\left(\frac{c}{c-d}+\frac{d\sin(e+fx)}{c-d}\right.}{\sqrt{1-\sin(e+fx)}}}{f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}$$

$$\frac{aB\sqrt{1-\sin(e+fx)}\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}\int \frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}\left(\frac{c}{c-d}+\frac{d\sin(e+fx)}{c-d}\right)^n}{\sqrt{1-\sin(e+fx)}}d\sin}{f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}$$

$\Big\downarrow$ 155

$$\frac{\sqrt{2}a(A-B)\sqrt{1-\sin(e+fx)}\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}\text{AppellF1}}{f(2m+1)(a-a\sin(e+fx))}$$
$$\frac{\sqrt{2}B\sqrt{1-\sin(e+fx)}\cos(e+fx)(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}\text{AppellF1}\left(m+\right.}{f(2m+3)(a-a\sin(e+fx))}$$

input
```
Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

output
```
(Sqrt[2]*a*(A - B)*AppellF1[1/2 + m, 1/2, -n, 3/2 + m, (1 + Sin[e + f*x])/
2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*Sqrt[1 - Sin[e + f*x]]*
(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(1 + 2*m)*(a - a*Sin[e +
 f*x])*((c + d*Sin[e + f*x])/(c - d))^n) + (Sqrt[2]*B*AppellF1[3/2 + m, 1/
2, -n, 5/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*C
os[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(1 + m)*(c + d*Sin
[e + f*x])^n)/(f*(3 + 2*m)*(a - a*Sin[e + f*x])*((c + d*Sin[e + f*x])/(c -
 d))^n)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 157
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
&  !GtQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !Si
mplerQ[e + f*x, a + b*x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3267
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m
, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 !IntegerQ[m]
```

rule 3466
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)/b   Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x
] + Simp[B/b   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [F]

$$\int (a + a\sin(fx+e))^m (A + B\sin(fx+e)) (c + d\sin(fx+e))^n dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

Fricas [F]

$$\int (a + a\sin(e+fx))^m (A + B\sin(e+fx))(c + d\sin(e+fx))^n dx$$
$$= \int (B\sin(fx+e) + A)(a\sin(fx+e) + a)^m (d\sin(fx+e) + c)^n dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algori
thm="fricas")
```

output
```
integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^
n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\sin(e+fx))^m (A + B\sin(e+fx))(c + d\sin(e+fx))^n dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m(d\sin(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algori
thm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)
^n, x)
```

Giac [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$=\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m(d\sin(fx+e)+c)^n\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algori
thm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)
^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e+fx))^m (A + B\sin(e+fx))(c + d\sin(e+fx))^n\,dx$$
$$= \int (A + B\,\sin(e+f\,x))\,(a + a\,\sin(e+f\,x))^m\,(c + d\,\sin(e+f\,x))^n\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^n,x)
```

output
```
int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^n, x)
```

Reduce [F]

$$\int (a + a\sin(e+fx))^m (A + B\sin(e+fx))(c + d\sin(e+fx))^n\,dx$$
$$= \left(\int (\sin(fx+e)\,d + c)^n\,(a + a\sin(fx+e))^m \sin(fx+e)\,dx\right) b$$
$$+ \left(\int (\sin(fx+e)\,d + c)^n\,(a + a\sin(fx+e))^m\,dx\right) a$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

output
```
int((sin(e + f*x)*d + c)**n*(sin(e + f*x)*a + a)**m*sin(e + f*x),x)*b + in
t((sin(e + f*x)*d + c)**n*(sin(e + f*x)*a + a)**m,x)*a
```

3.347 $\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{-1-m}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 267

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{-1-m}\,dx =$$

$$-\frac{2^{\frac{1}{2}+m}a(A-B)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{(c-d)(1-\sin(e+fx))}{2(c+d\sin(e+fx))}\right)(a+a\sin(e+fx))^{-1}}{(c+d)f}$$

$$-\frac{2^{\frac{3}{2}+m}B\,\text{AppellF1}\left(\frac{1}{2},-\frac{1}{2}-m,1+m,\frac{3}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1+\sin(e+fx}{(c+d)f}$$

output
```
-2^(1/2+m)*a*(A-B)*cos(f*x+e)*hypergeom([1/2, 1/2-m],[3/2],(c-d)*(1-sin(f*
x+e))/(2*c+2*d*sin(f*x+e)))*(a+a*sin(f*x+e))^(-1+m)*((c+d)*(1+sin(f*x+e))/
(c+d*sin(f*x+e)))^(1/2-m)/(c+d)/f/((c+d*sin(f*x+e))^m)-2^(3/2+m)*B*AppellF
1(1/2,1+m,-1/2-m,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*cos(f*x+e)
*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m*((c+d*sin(f*x+e))/(c+d))^m/(c+
d)/f/((c+d*sin(f*x+e))^m)
```

Mathematica [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{-1-m}\,dx$$
$$=\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{-1-m}\,dx$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])
^(-1 - m),x]
```

output
```
Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])
^(-1 - m), x]
```

Rubi [A] (warning: unable to verify)

Time = 0.74 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3466, 3042, 3267, 142, 157, 27, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{-m-1}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{-m-1}dx$$

$$\Big\downarrow \text{3466}$$

$$(A-B)\int (\sin(e+fx)a+a)^m(c+d\sin(e+fx))^{-m-1}dx+\frac{B\int (\sin(e+fx)a+a)^{m+1}(c+d\sin(e+fx))^{-m-1}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$(A-B)\int (\sin(e+fx)a+a)^m (c+d\sin(e+fx))^{-m-1}dx+\frac{B\int (\sin(e+fx)a+a)^{m+1}(c+d\sin(e+fx))^{-m-1}dx}{a}$$

$\Big\downarrow$ 3267

$$\frac{a^2(A-B)\cos(e+fx)\int \frac{(\sin(e+fx)a+a)^{m-\frac{1}{2}}(c+d\sin(e+fx))^{-m-1}}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}+\frac{aB\cos(e+fx)\int \frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}(c+d\sin(e+fx))^{-m-1}}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}$$

$\Big\downarrow$ 142

$$\frac{aB\cos(e+fx)\int \frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}(c+d\sin(e+fx))^{-m-1}}{\sqrt{a-a\sin(e+fx)}}d\sin(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}}-\frac{a2^{m+\frac{1}{2}}(A-B)\cos(e+fx)(a\sin(e+fx)+a)^{m-1}\left(\frac{(c+d)(\sin(e+fx)+1)}{c+d\sin(e+fx)}\right)^{\frac{1}{2}-m}(c+d\sin(e+fx))^{-m}\text{Hypergeometr}}{f(c+d)}$$

$\Big\downarrow$ 157

$$\frac{aB\sqrt{1-\sin(e+fx)}\cos(e+fx)\int \frac{\sqrt{2}(\sin(e+fx)a+a)^{m+\frac{1}{2}}(c+d\sin(e+fx))^{-m-1}}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{\sqrt{2}f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}-\frac{a2^{m+\frac{1}{2}}(A-B)\cos(e+fx)(a\sin(e+fx)+a)^{m-1}\left(\frac{(c+d)(\sin(e+fx)+1)}{c+d\sin(e+fx)}\right)^{\frac{1}{2}-m}(c+d\sin(e+fx))^{-m}\text{Hypergeometr}}{f(c+d)}$$

$\Big\downarrow$ 27

$$\frac{aB\sqrt{1-\sin(e+fx)}\cos(e+fx)\int \frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}(c+d\sin(e+fx))^{-m-1}}{\sqrt{1-\sin(e+fx)}}d\sin(e+fx)}{f(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}-\frac{a2^{m+\frac{1}{2}}(A-B)\cos(e+fx)(a\sin(e+fx)+a)^{m-1}\left(\frac{(c+d)(\sin(e+fx)+1)}{c+d\sin(e+fx)}\right)^{\frac{1}{2}-m}(c+d\sin(e+fx))^{-m}\text{Hypergeometr}}{f(c+d)}$$

$\Big\downarrow$ 156

$$\frac{aB\sqrt{1-\sin(e+fx)}\cos(e+fx)(c+d\sin(e+fx))^{-m}\left(\frac{c+d\sin(e+fx)}{c-d}\right)^m\int \frac{(\sin(e+fx)a+a)^{m+\frac{1}{2}}\left(\frac{c}{c-d}+\frac{d\sin(e+fx)}{c-d}\right)^{-m-1}}{\sqrt{1-\sin(e+fx)}}}{f(c-d)(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}}$$
$$\frac{a2^{m+\frac{1}{2}}(A-B)\cos(e+fx)(a\sin(e+fx)+a)^{m-1}\left(\frac{(c+d)(\sin(e+fx)+1)}{c+d\sin(e+fx)}\right)^{\frac{1}{2}-m}(c+d\sin(e+fx))^{-m}\text{Hypergeometr}}{f(c+d)}$$

$\big\downarrow$ 155

$$\frac{\sqrt{2}B\sqrt{1-\sin(e+fx)}\cos(e+fx)(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^{-m}\left(\frac{c+d\sin(e+fx)}{c-d}\right)^m\text{AppellF1}\left(m\right.}{f(2m+3)(c-d)(a-a\sin(e+fx))}$$
$$\frac{a2^{m+\frac{1}{2}}(A-B)\cos(e+fx)(a\sin(e+fx)+a)^{m-1}\left(\frac{(c+d)(\sin(e+fx)+1)}{c+d\sin(e+fx)}\right)^{\frac{1}{2}-m}(c+d\sin(e+fx))^{-m}\text{Hypergeometr}}{f(c+d)}$$

input
```
Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^(-1 -
 m),x]
```

output
```
-((2^(1/2 + m)*a*(A - B)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2,
 ((c - d)*(1 - Sin[e + f*x]))/(2*(c + d*Sin[e + f*x]))]*(a + a*Sin[e + f*x
])^(-1 + m)*(((c + d)*(1 + Sin[e + f*x]))/(c + d*Sin[e + f*x]))^(1/2 - m))
/((c + d)*f*(c + d*Sin[e + f*x])^m)) + (Sqrt[2]*B*AppellF1[3/2 + m, 1/2, 1
 + m, 5/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Co
s[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(1 + m)*((c + d*Sin
[e + f*x])/(c - d))^m)/((c - d)*f*(3 + 2*m)*(a - a*Sin[e + f*x])*(c + d*Si
n[e + f*x])^m)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 142
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((b*e
 - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*((a +
b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*(e + f
*x)))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n + p + 2,
 0] &&  !IntegerQ[n]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 157
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
&  !GtQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !Si
mplerQ[e + f*x, a + b*x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3267
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_.), x_Symbol] :> Simp[a^2*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]))   Subst[Int[(a + b*x)^(m - 1/2)*((c + d*
x)^n/Sqrt[a - b*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m
, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 !IntegerQ[m]
```

rule 3466
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(A*b - a*B)/b   Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x
] + Simp[B/b   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Maple [F]

$$\int (a+a\sin(fx+e))^{m}(A+B\sin(fx+e))(c+d\sin(fx+e))^{-1-m}\,dx$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(-1-m),x)
```

output
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(-1-m),x)
```

Fricas [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{-1-m}\,dx = \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m(d\sin(fx+e)+c)^{-m-1}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(-1-m),x, a
lgorithm="fricas")
```

output
```
integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^
(-m - 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{-1-m}\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**(-1-m),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{-1-m}\,dx$$
$$= \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m(d\sin(fx+e)+c)^{-m-1}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(-1-m),x, a
lgorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)
^(-m - 1), x)
```

Giac [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{-1-m}\,dx$$
$$= \int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m(d\sin(fx+e)+c)^{-m-1}\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(-1-m),x, a
lgorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)
^(-m - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{-1-m}\,dx$$
$$=\int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m}{(c+d\,\sin(e+f\,x))^{m+1}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^(m
+ 1),x)
```

output
```
int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^(m
+ 1), x)
```

Reduce [F]

$$\int (a+a\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^{-1-m}\,dx$$
$$=\left(\int \frac{(a+a\sin(fx+e))^m}{(\sin(fx+e)\,d+c)^m\sin(fx+e)\,d+(\sin(fx+e)\,d+c)^m\,c}dx\right)a$$
$$+\left(\int \frac{(a+a\sin(fx+e))^m\sin(fx+e)}{(\sin(fx+e)\,d+c)^m\sin(fx+e)\,d+(\sin(fx+e)\,d+c)^m\,c}dx\right)b$$

input
```
int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(-1-m),x)
```

output
```
int((sin(e + f*x)*a + a)**m/((sin(e + f*x)*d + c)**m*sin(e + f*x)*d + (sin
(e + f*x)*d + c)**m*c),x)*a + int(((sin(e + f*x)*a + a)**m*sin(e + f*x))/(
(sin(e + f*x)*d + c)**m*sin(e + f*x)*d + (sin(e + f*x)*d + c)**m*c),x)*b
```

3.348 $\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$

Optimal result

Integrand size = 36, antiderivative size = 144

$$\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx = -\frac{2^{\frac{1}{2}+m}\operatorname{AppellF1}\left(\frac{3}{2},\frac{1}{2}-m,-n,\frac{5}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\sec(e+fx)(1+\sin(e+fx))^{\frac{1}{2}-m}}{3af}$$

output

```
-1/3*2^(1/2+m)*AppellF1(3/2,-n,1/2-m,5/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*si
n(f*x+e))*sec(f*x+e)*(1+sin(f*x+e))^(1/2-m)*(a-a*sin(f*x+e))^2*(a+a*sin(f*
x+e))^m*(c+d*sin(f*x+e))^n/a/f/(((c+d*sin(f*x+e))/(c+d))^n)
```

Mathematica [F]

$$\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx = \int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$$

input
```
Integrate[(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])
^n,x]
```

output
```
Integrate[(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])
^n, x]
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3487, 157, 27, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-a\sin(e+fx))(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a-a\sin(e+fx))(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^n dx$$

$$\Big\downarrow \text{3487}$$

$$\frac{\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\int \sqrt{a-a\sin(e+fx)}(\sin(e+fx)a+a)^{m-\frac{1}{2}}(c+d\sin(e+f}{f}$$

$$\Big\downarrow \text{157}$$

$$\frac{\sqrt{2}\sec(e+fx)(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}\int \frac{\sqrt{1-\sin(e+fx)}(\sin(e+fx)a+a)^{m-\frac{1}{2}}(c+d\sin(e+fx))^n}{\sqrt{2}}d\sin(e+fx}{f\sqrt{1-\sin(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sec(e+fx)(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}\int \sqrt{1-\sin(e+fx)}(\sin(e+fx)a+a)^{m-\frac{1}{2}}(c+d\sin(e+fx)}{f\sqrt{1-\sin(e+fx)}}$$

$$\Big\downarrow \text{156}$$

$$\frac{\sec(e+fx)(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}\int\sqrt{1-\sin(e+fx)}(\sin}{f\sqrt{1-\sin(e+fx)}}$$

$$\Big\downarrow \text{155}$$

$$\frac{2\sqrt{2}\sec(e+fx)(a-a\sin(e+fx))(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}\operatorname{AppellF1}\left(m+\right.}{af(2m+1)\sqrt{1-\sin(e+fx)}}$$

input
```
Int[(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
```

output
```
(2*Sqrt[2]*AppellF1[1/2 + m, -1/2, -n, 3/2 + m, (1 + Sin[e + f*x])/2, -((d
*(1 + Sin[e + f*x]))/(c - d))]*Sec[e + f*x]*(a - a*Sin[e + f*x])*(a + a*Si
n[e + f*x])^(1 + m)*(c + d*Sin[e + f*x])^n)/(a*f*(1 + 2*m)*Sqrt[1 - Sin[e
+ f*x]]*((c + d*Sin[e + f*x])/(c - d))^n)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 157
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
&  !GtQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !Si
mplerQ[e + f*x, a + b*x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3487
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])^(p_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol]
:> Simp[Sqrt[a + b*Sin[e + f*x]]*(Sqrt[c + d*Sin[e + f*x]]/(f*Cos[e + f*x])
)   Subst[Int[(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2)*(A + B*x)^p, x], x, S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n, p}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int (a-a\sin(fx+e))\,(a+a\sin(fx+e))^m\,(c+d\sin(fx+e))^n\,dx$$

input
```
int((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

output
```
int((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

Fricas [F]

$$\int (a - a\sin(e + fx))(a + a\sin(e + fx))^m(c + d\sin(e + fx))^n\,dx$$
$$= \int -(a\sin(fx + e) - a)(a\sin(fx + e) + a)^m(d\sin(fx + e) + c)^n\,dx$$

input
```
integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algori
thm="fricas")
```

output
```
integral(-(a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)
^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a - a\sin(e + fx))(a + a\sin(e + fx))^m(c + d\sin(e + fx))^n\,dx = \text{Timed out}$$

input
```
integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**n,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a - a\sin(e + fx))(a + a\sin(e + fx))^m(c + d\sin(e + fx))^n\,dx$$
$$= \int -(a\sin(fx + e) - a)(a\sin(fx + e) + a)^m(d\sin(fx + e) + c)^n\,dx$$

input
```
integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algori
thm="maxima")
```

output
```
-integrate((a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c
)^n, x)
```

Giac [F]

$$\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$$
$$=\int -(a\sin(fx+e)-a)(a\sin(fx+e)+a)^m(d\sin(fx+e)+c)^n\,dx$$

input
```
integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algori
thm="giac")
```

output
```
integrate(-(a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c
)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$$
$$=\int (a+a\,\sin(e+f\,x))^m\,(a-a\,\sin(e+f\,x))\,(c+d\,\sin(e+f\,x))^n\,dx$$

input
```
int((a + a*sin(e + f*x))^m*(a - a*sin(e + f*x))*(c + d*sin(e + f*x))^n,x)
```

output
```
int((a + a*sin(e + f*x))^m*(a - a*sin(e + f*x))*(c + d*sin(e + f*x))^n, x)
```

Reduce [F]

$$\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$$
$$= a\left(-\left(\int (\sin(fx+e)\,d+c)^n\,(a+a\sin(fx+e))^m\sin(fx+e)\,dx\right) + \int (\sin(fx+e)\,d+c)^n\,(a+a\sin(fx+e))^m\,dx\right)$$

input
```
int((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

output
```
a*( - int((sin(e + f*x)*d + c)**n*(sin(e + f*x)*a + a)**m*sin(e + f*x),x)
+ int((sin(e + f*x)*d + c)**n*(sin(e + f*x)*a + a)**m,x))
```

3.349 $\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 149

$$\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}\,dx = -\frac{2^{\frac{1}{2}+m}\operatorname{AppellF1}\left(\frac{3}{2},\frac{1}{2}-m,1+m,\frac{5}{2},\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\sec(e+fx)(1+\sin(e+fx))^{\frac{1}{2}}}{3a(c+d)f}$$

output
```
-1/3*2^(1/2+m)*AppellF1(3/2,1+m,1/2-m,5/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*s
in(f*x+e))*sec(f*x+e)*(1+sin(f*x+e))^(1/2-m)*(a-a*sin(f*x+e))^2*(a+a*sin(f
*x+e))^m*((c+d*sin(f*x+e))/(c+d))^m/a/(c+d)/f/((c+d*sin(f*x+e))^m)
```

Mathematica [F]

$$\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}\,dx$$
$$= \int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}\,dx$$

input
```
Integrate[(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])
^(-1 - m),x]
```

output
```
Integrate[(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])
^(-1 - m), x]
```

Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3487, 157, 27, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-a\sin(e+fx))(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^{-m-1}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a-a\sin(e+fx))(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^{-m-1}dx$$

$$\Big\downarrow \text{3487}$$

$$\frac{\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\int\sqrt{a-a\sin(e+fx)}(\sin(e+fx)a+a)^{m-\frac{1}{2}}(c+d\sin(e+f$$

$$\Big\downarrow \text{157}$$

$$\frac{\sqrt{2}\sec(e+fx)(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}\int\frac{\sqrt{1-\sin(e+fx)}(\sin(e+fx)a+a)^{m-\frac{1}{2}}(c+d\sin(e+fx))^{-m-1}}{\sqrt{2}}d\sin(e+}{f\sqrt{1-\sin(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sec(e+fx)(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}\int\sqrt{1-\sin(e+fx)}(\sin(e+fx)a+a)^{m-\frac{1}{2}}(c+d\sin(e+fx)}{f\sqrt{1-\sin(e+fx)}}$$

$$\Big\downarrow \text{156}$$

$$\frac{\sec(e+fx)(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}(c+d\sin(e+fx))^{-m}\left(\frac{c+d\sin(e+fx)}{c-d}\right)^m\int\sqrt{1-\sin(e+fx)}(\mathrm{si}}{f(c-d)\sqrt{1-\sin(e+fx)}}$$

$$\Big\downarrow \text{155}$$

$$\frac{2\sqrt{2}\sec(e+fx)(a-a\sin(e+fx))(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^{-m}\left(\frac{c+d\sin(e+fx)}{c-d}\right)^m\mathrm{AppellF1}\left(m\right.}{af(2m+1)(c-d)\sqrt{1-\sin(e+fx)}}$$

input
```
Int[(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-1 -
 m),x]
```

output
```
(2*Sqrt[2]*AppellF1[1/2 + m, -1/2, 1 + m, 3/2 + m, (1 + Sin[e + f*x])/2, -
((d*(1 + Sin[e + f*x]))/(c - d))]*Sec[e + f*x]*(a - a*Sin[e + f*x])*(a + a
*Sin[e + f*x])^(1 + m)*((c + d*Sin[e + f*x])/(c - d))^m)/(a*(c - d)*f*(1 +
 2*m)*Sqrt[1 - Sin[e + f*x]]*(c + d*Sin[e + f*x])^m)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] &&  !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) &&  !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] &&  !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 157
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &
&  !GtQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !Si
mplerQ[e + f*x, a + b*x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3487
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])^(p_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol]
:> Simp[Sqrt[a + b*Sin[e + f*x]]*(Sqrt[c + d*Sin[e + f*x]]/(f*Cos[e + f*x])
)   Subst[Int[(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2)*(A + B*x)^p, x], x, S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n, p}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0]
```

Maple [F]

$$\int (a-a\sin(fx+e))\,(a+a\sin(fx+e))^m\,(c+d\sin(fx+e))^{-1-m}\,dx$$

input
```
int((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x)
```

output
```
int((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x)
```

Fricas [F]

$$\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}\,dx$$
$$=\int -(a\sin(fx+e)-a)(a\sin(fx+e)+a)^m(d\sin(fx+e)+c)^{-m-1}\,dx$$

input
```
integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x, a
lgorithm="fricas")
```

output
```
integral(-(a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)
^(-m - 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}\,dx=\text{Timed out}$$

input
```
integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**(-1-m),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}\,dx$$
$$=\int -(a\sin(fx+e)-a)(a\sin(fx+e)+a)^m(d\sin(fx+e)+c)^{-m-1}\,dx$$

input
```
integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x, a
lgorithm="maxima")
```

output
```
-integrate((a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c
)^(-m - 1), x)
```

Giac [F]

$$\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}\,dx$$
$$=\int -(a\sin(fx+e)-a)(a\sin(fx+e)+a)^m(d\sin(fx+e)+c)^{-m-1}\,dx$$

input
```
integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x, a
lgorithm="giac")
```

output
```
integrate(-(a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c
)^(-m - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}\,dx$$
$$=\int \frac{(a+a\,\sin(e+f\,x))^m\,(a-a\,\sin(e+f\,x))}{(c+d\,\sin(e+f\,x))^{m+1}}\,dx$$

input
```
int(((a + a*sin(e + f*x))^m*(a - a*sin(e + f*x)))/(c + d*sin(e + f*x))^(m
+ 1),x)
```

output
```
int(((a + a*sin(e + f*x))^m*(a - a*sin(e + f*x)))/(c + d*sin(e + f*x))^(m
+ 1), x)
```

Reduce [F]

$$\int (a-a\sin(e+fx))(a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}\,dx$$
$$= a\left(\int \frac{(a+a\sin(fx+e))^m}{(\sin(fx+e)\,d+c)^m\sin(fx+e)\,d+(\sin(fx+e)\,d+c)^m c}dx - \left(\int \frac{(a+a\sin(fx+e))^m\sin(fx+e)}{(\sin(fx+e)\,d+c)^m\sin(fx+e)\,d+(\sin(fx+e)\,d+c)^m c}dx\right)\right)$$

input
```
int((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m),x)
```

output
```
a*(int((sin(e + f*x)*a + a)**m/((sin(e + f*x)*d + c)**m*sin(e + f*x)*d + (
sin(e + f*x)*d + c)**m*c),x) - int(((sin(e + f*x)*a + a)**m*sin(e + f*x))/
((sin(e + f*x)*d + c)**m*sin(e + f*x)*d + (sin(e + f*x)*d + c)**m*c),x))
```

3.350 $\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d-(c-d)m+(c+(c-d)m)\sin(e+fx))\,dx$

Optimal result

Integrand size = 55, antiderivative size = 39

$$\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d-(c-d)m+(c+(c-d)m)\sin(e+fx))\,dx = -\frac{\cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}}{f}$$

output
```
-cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m)/f
```

Mathematica [A] (verified)

Time = 4.34 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d-(c-d)m+(c+(c-d)m)\sin(e+fx))\,dx = -\frac{\cos(e+fx)(a(1+\sin(e+fx)))^m(c+d\sin(e+fx))^{-1-m}}{f}$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-2 - m)*(d - (c - d
)*m + (c + (c - d)*m)*Sin[e + f*x]),x]
```

output
```
-((Cos[e + f*x]*(a*(1 + Sin[e + f*x]))^m*(c + d*Sin[e + f*x])^(-1 - m))/f)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {3042, 3453}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^m(c+d\sin(e+fx))^{-m-2}((m(c-d)+c)\sin(e+fx)-m(c-d)+d)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^m(c+d\sin(e+fx))^{-m-2}((m(c-d)+c)\sin(e+fx)-m(c-d)+d)dx$$

$$\Big\downarrow \text{3453}$$

$$-\frac{\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^{-m-1}}{f}$$

input
```
Int[(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-2 - m)*(d - (c - d)*m +
(c + (c - d)*m)*Sin[e + f*x]),x]
```

output
```
-((Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-1 - m))/f)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3453
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n},
x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ[m
+ n + 2, 0] && EqQ[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)), 0]
```

Maple [F]

$$\int (a+a\sin(fx+e))^m (c+d\sin(fx+e))^{-2-m} (d-(c-d)m+(c+(c-d)m)\sin(fx+e))\,dx$$

input
```
int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d-(c-d)*m+(c+(c-d)*m)*sin(
f*x+e)),x)
```

output
```
int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d-(c-d)*m+(c+(c-d)*m)*sin(
f*x+e)),x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.44

$$\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d-(c-d)m+(c+(c-d)m)\sin(e+fx))\,dx$$
$$= -\frac{(d\cos(fx+e)\sin(fx+e)+c\cos(fx+e))(a\sin(fx+e)+a)^m(d\sin(fx+e)+c)^{-m-2}}{f}$$

input
```
integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d-(c-d)*m+(c+(c-d)*m
)*sin(f*x+e)),x, algorithm="fricas")
```

output
```
-(d*cos(f*x + e)*sin(f*x + e) + c*cos(f*x + e))*(a*sin(f*x + e) + a)^m*(d*
sin(f*x + e) + c)^(-m - 2)/f
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^m (c + d\sin(e + fx))^{-2-m} (d - (c - d)m + (c + (c - d)m)\sin(e + fx))\, dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**(-2-m)*(d-(c-d)*m+(c+(c-d)
*m)*sin(f*x+e)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+a\sin(e+fx))^m (c+d\sin(e+fx))^{-2-m} (d-(c-d)m+(c+(c-d)m)\sin(e+fx))\, dx = \int -((c-d)m - ((c-d)m+c)\sin(fx+e) - d)(a\sin(fx+e)+a)^m (d\sin(fx+e)+c)^{-m-2}\, dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d-(c-d)*m+(c+(c-d)*m
)*sin(f*x+e)),x, algorithm="maxima")
```

output
```
-integrate(((c - d)*m - ((c - d)*m + c)*sin(f*x + e) - d)*(a*sin(f*x + e)
+ a)^m*(d*sin(f*x + e) + c)^(-m - 2), x)
```

Giac [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d-(c-d)m+(c+(c-d)m)\sin(e+fx))\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d-(c-d)*m+(c+(c-d)*m
)*sin(f*x+e)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 37.92 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.51

$$\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d-(c-d)m+(c+(c-d)m)\sin(e+fx))\,dx = -\frac{(a\,(\sin(e+f\,x)+1))^m\,\left(d\,\sin(2\,e+2\,f\,x)-2\,c\,\left(2\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)\right)}{f\,(c+d\,\sin(e+f\,x))^m\,\left(d^2\,\left(2\sin(e+f\,x)^2-1\right)+2\,c^2+d^2+4\,c\,d\,\sin(e+f\,x)\right)}$$

input
```
int(((a + a*sin(e + f*x))^m*(d - m*(c - d) + sin(e + f*x)*(c + m*(c - d)))
)/(c + d*sin(e + f*x))^(m + 2),x)
```

output
```
-((a*(sin(e + f*x) + 1))^m*(d*sin(2*e + 2*f*x) - 2*c*(2*sin(e/2 + (f*x)/2)
^2 - 1)))/(f*(c + d*sin(e + f*x))^m*(d^2*(2*sin(e + f*x)^2 - 1) + 2*c^2 +
d^2 + 4*c*d*sin(e + f*x)))
```

Reduce [F]

$$\int (a+a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d-(c-d)m+(c+(c-d)m)\sin(e+fx))\,dx$$
$$= -\left(\int \frac{(a+a\sin(fx+e))^m}{(\sin(fx+e)\,d+c)^m\sin(fx+e)^2\,d^2+2(\sin(fx+e)\,d+c)^m\sin(fx+e)\,cd+(\sin(fx+e)\,d+\right.$$
$$+\left(\int \frac{(a+a\sin(fx+e))^m}{(\sin(fx+e)\,d+c)^m\sin(fx+e)^2\,d^2+2(\sin(fx+e)\,d+c)^m\sin(fx+e)\,cd+(\sin(fx+e)\,d+\right.$$
$$+\left(\int \frac{(a+a\sin(fx+e))^m}{(\sin(fx+e)\,d+c)^m\sin(fx+e)^2\,d^2+2(\sin(fx+e)\,d+c)^m\sin(fx+e)\,cd+(\sin(fx+e)\,d+\right.$$
$$+\left(\int \frac{(a+a\sin(fx+e))^m\sin(fx+e)}{(\sin(fx+e)\,d+c)^m\sin(fx+e)^2\,d^2+2(\sin(fx+e)\,d+c)^m\sin(fx+e)\,cd+(\sin(fx+e)\,d+\right.$$
$$+\left(\int \frac{(a+a\sin(fx+e))^m\sin(fx+e)}{(\sin(fx+e)\,d+c)^m\sin(fx+e)^2\,d^2+2(\sin(fx+e)\,d+c)^m\sin(fx+e)\,cd+(\sin(fx+e)\,d+\right.$$
$$-\left(\int \frac{(a+a\sin(fx+e))^m\sin(fx+e)}{(\sin(fx+e)\,d+c)^m\sin(fx+e)^2\,d^2+2(\sin(fx+e)\,d+c)^m\sin(fx+e)\,cd+(\sin(fx+e)\,d+\right.$$

input
```
int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d-(c-d)*m+(c+(c-d)*m)*sin(
f*x+e)),x)
```

output
```
 - int((sin(e + f*x)*a + a)**m/((sin(e + f*x)*d + c)**m*sin(e + f*x)**2*d*
*2 + 2*(sin(e + f*x)*d + c)**m*sin(e + f*x)*c*d + (sin(e + f*x)*d + c)**m*
c**2),x)*c*m + int((sin(e + f*x)*a + a)**m/((sin(e + f*x)*d + c)**m*sin(e
+ f*x)**2*d**2 + 2*(sin(e + f*x)*d + c)**m*sin(e + f*x)*c*d + (sin(e + f*x
)*d + c)**m*c**2),x)*d*m + int((sin(e + f*x)*a + a)**m/((sin(e + f*x)*d +
c)**m*sin(e + f*x)**2*d**2 + 2*(sin(e + f*x)*d + c)**m*sin(e + f*x)*c*d +
(sin(e + f*x)*d + c)**m*c**2),x)*d + int(((sin(e + f*x)*a + a)**m*sin(e +
f*x))/((sin(e + f*x)*d + c)**m*sin(e + f*x)**2*d**2 + 2*(sin(e + f*x)*d +
c)**m*sin(e + f*x)*c*d + (sin(e + f*x)*d + c)**m*c**2),x)*c*m + int(((sin(
e + f*x)*a + a)**m*sin(e + f*x))/((sin(e + f*x)*d + c)**m*sin(e + f*x)**2*
d**2 + 2*(sin(e + f*x)*d + c)**m*sin(e + f*x)*c*d + (sin(e + f*x)*d + c)**
m*c**2),x)*c - int(((sin(e + f*x)*a + a)**m*sin(e + f*x))/((sin(e + f*x)*d
 + c)**m*sin(e + f*x)**2*d**2 + 2*(sin(e + f*x)*d + c)**m*sin(e + f*x)*c*d
 + (sin(e + f*x)*d + c)**m*c**2),x)*d*m
```

3.351 $\int (a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d+(c+d)m+(c+(c+d)m)\sin(e+fx))\,dx$

Optimal result

Integrand size = 51, antiderivative size = 40

$$\int (a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d+(c+d)m+(c+(c+d)m)\sin(e+fx))\,dx = -\frac{\cos(e+fx)(a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}}{f}$$

output
```
-cos(f*x+e)*(a-a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-1-m)/f
```

Mathematica [A] (verified)

Time = 3.98 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int (a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d+(c+d)m+(c+(c+d)m)\sin(e+fx))\,dx = -\frac{\cos(e+fx)(a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-1-m}}{f}$$

input
```
Integrate[(a - a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-2 - m)*(d + (c + d
)*m + (c + (c + d)*m)*Sin[e + f*x]),x]
```

output
```
-((Cos[e + f*x]*(a - a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-1 - m))/f)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.039$, Rules used = {3042, 3453}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-m-2}((m(c+d)+c)\sin(e+fx)+m(c+d)+d)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-m-2}((m(c+d)+c)\sin(e+fx)+m(c+d)+d)dx$$

$$\Big\downarrow \text{3453}$$

$$-\frac{\cos(e+fx)(a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-m-1}}{f}$$

input
```
Int[(a - a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-2 - m)*(d + (c + d)*m +
(c + (c + d)*m)*Sin[e + f*x]),x]
```

output
```
-((Cos[e + f*x]*(a - a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-1 - m))/f)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3453
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n},
x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ[m
+ n + 2, 0] && EqQ[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)), 0]
```

Maple [F]

$$\int (a-a\sin(fx+e))^m (c+d\sin(fx+e))^{-2-m} (d+(c+d)m+(c+(c+d)m)\sin(fx+e))\,dx$$

input
```
int((a-a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d+(c+d)*m+(c+(c+d)*m)*sin(
f*x+e)),x)
```

output
```
int((a-a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d+(c+d)*m+(c+(c+d)*m)*sin(
f*x+e)),x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.42

$$\int (a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d+(c+d)m+(c+(c+d)m)\sin(e+fx))\,dx$$
$$= -\frac{(d\cos(fx+e)\sin(fx+e)+c\cos(fx+e))(-a\sin(fx+e)+a)^m(d\sin(fx+e)+c)^{-m-2}}{f}$$

input
```
integrate((a-a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d+(c+d)*m+(c+(c+d)*m
)*sin(f*x+e)),x, algorithm="fricas")
```

output
```
-(d*cos(f*x + e)*sin(f*x + e) + c*cos(f*x + e))*(-a*sin(f*x + e) + a)^m*(d
*sin(f*x + e) + c)^(-m - 2)/f
```

Sympy [F(-1)]

Timed out.

$$\int (a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d+(c+d)m+(c+(c+d)m)\sin(e+fx))\,dx = \text{Timed out}$$

input
```
integrate((a-a*sin(f*x+e))**m*(c+d*sin(f*x+e))**(-2-m)*(d+(c+d)*m+(c+(c+d)
*m)*sin(f*x+e)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d+(c+d)m+(c+(c+d)m)\sin(e+fx))\,dx = \int ((c+d)m+((c+d)m+c)\sin(fx+e)+d)(-a\sin(fx+e)+a)^m(d\sin(fx+e)+c)^{-m-2}\,dx$$

input
```
integrate((a-a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d+(c+d)*m+(c+(c+d)*m
)*sin(f*x+e)),x, algorithm="maxima")
```

output
```
integrate(((c + d)*m + ((c + d)*m + c)*sin(f*x + e) + d)*(-a*sin(f*x + e)
+ a)^m*(d*sin(f*x + e) + c)^(-m - 2), x)
```

Giac [F(-1)]

Timed out.

$$\int (a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d+(c+d)m+(c+(c+d)m)\sin(e+fx))\,dx=\text{Timed out}$$

input
```
integrate((a-a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d+(c+d)*m+(c+(c+d)*m
)*sin(f*x+e)),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 38.76 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.48

$$\int (a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d+(c+d)m+(c+(c+d)m)\sin(e+fx))\,dx$$
$$=-\frac{(-a\,(\sin(e+f\,x)-1))^m\,\left(d\,\sin(2\,e+2\,f\,x)-2\,c\,\left(2\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)\right)}{f\,(c+d\,\sin(e+f\,x))^m\,\left(d^2\,\left(2\sin(e+f\,x)^2-1\right)+2\,c^2+d^2+4\,c\,d\,\sin(e+f\,x)\right)}$$

input
```
int(((a - a*sin(e + f*x))^m*(d + sin(e + f*x)*(c + m*(c + d)) + m*(c + d))
)/(c + d*sin(e + f*x))^(m + 2),x)
```

output
```
-((-a*(sin(e + f*x) - 1))^m*(d*sin(2*e + 2*f*x) - 2*c*(2*sin(e/2 + (f*x)/2
)^2 - 1)))/(f*(c + d*sin(e + f*x))^m*(d^2*(2*sin(e + f*x)^2 - 1) + 2*c^2 +
 d^2 + 4*c*d*sin(e + f*x)))
```

Reduce [F]

$$\int (a-a\sin(e+fx))^m(c+d\sin(e+fx))^{-2-m}(d+(c+d)m+(c+(c+d)m)\sin(e+fx))\,dx$$
$$= \left(\int \frac{(-a\sin(fx+e)+a)^m}{(\sin(fx+e)\,d+c)^m\sin(fx+e)^2\,d^2+2(\sin(fx+e)\,d+c)^m\sin(fx+e)\,cd+(\sin(fx+e)\,d+c)\right.$$
$$+\left(\int \frac{(-a\sin(fx+e)+a)^m}{(\sin(fx+e)\,d+c)^m\sin(fx+e)^2\,d^2+2(\sin(fx+e)\,d+c)^m\sin(fx+e)\,cd+(\sin(fx+e)\,d+\right.$$
$$+\left(\int \frac{(-a\sin(fx+e)+a)^m}{(\sin(fx+e)\,d+c)^m\sin(fx+e)^2\,d^2+2(\sin(fx+e)\,d+c)^m\sin(fx+e)\,cd+(\sin(fx+e)\,d+\right.$$
$$+\left(\int \frac{(-a\sin(fx+e)+a)^m\sin(fx+e)}{(\sin(fx+e)\,d+c)^m\sin(fx+e)^2\,d^2+2(\sin(fx+e)\,d+c)^m\sin(fx+e)\,cd+(\sin(fx+e)\,d+\right.$$
$$+\left(\int \frac{(-a\sin(fx+e)+a)^m\sin(fx+e)}{(\sin(fx+e)\,d+c)^m\sin(fx+e)^2\,d^2+2(\sin(fx+e)\,d+c)^m\sin(fx+e)\,cd+(\sin(fx+e)\,d+\right.$$
$$+\left(\int \frac{(-a\sin(fx+e)+a)^m\sin(fx+e)}{(\sin(fx+e)\,d+c)^m\sin(fx+e)^2\,d^2+2(\sin(fx+e)\,d+c)^m\sin(fx+e)\,cd+(\sin(fx+e)\,d+\right.$$

input
```
int((a-a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(-2-m)*(d+(c+d)*m+(c+(c+d)*m)*sin(
f*x+e)),x)
```

output
```
int(( - sin(e + f*x)*a + a)**m/((sin(e + f*x)*d + c)**m*sin(e + f*x)**2*d*
*2 + 2*(sin(e + f*x)*d + c)**m*sin(e + f*x)*c*d + (sin(e + f*x)*d + c)**m*
c**2),x)*c*m + int(( - sin(e + f*x)*a + a)**m/((sin(e + f*x)*d + c)**m*sin
(e + f*x)**2*d**2 + 2*(sin(e + f*x)*d + c)**m*sin(e + f*x)*c*d + (sin(e +
f*x)*d + c)**m*c**2),x)*d*m + int(( - sin(e + f*x)*a + a)**m/((sin(e + f*x
)*d + c)**m*sin(e + f*x)**2*d**2 + 2*(sin(e + f*x)*d + c)**m*sin(e + f*x)*
c*d + (sin(e + f*x)*d + c)**m*c**2),x)*d + int((( - sin(e + f*x)*a + a)**m
*sin(e + f*x))/((sin(e + f*x)*d + c)**m*sin(e + f*x)**2*d**2 + 2*(sin(e +
f*x)*d + c)**m*sin(e + f*x)*c*d + (sin(e + f*x)*d + c)**m*c**2),x)*c*m + i
nt((( - sin(e + f*x)*a + a)**m*sin(e + f*x))/((sin(e + f*x)*d + c)**m*sin(
e + f*x)**2*d**2 + 2*(sin(e + f*x)*d + c)**m*sin(e + f*x)*c*d + (sin(e + f
*x)*d + c)**m*c**2),x)*c + int((( - sin(e + f*x)*a + a)**m*sin(e + f*x))/(
(sin(e + f*x)*d + c)**m*sin(e + f*x)**2*d**2 + 2*(sin(e + f*x)*d + c)**m*s
in(e + f*x)*c*d + (sin(e + f*x)*d + c)**m*c**2),x)*d*m
```

3.352 $\int \frac{(a+b\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 199

$$\begin{aligned}\int \frac{(a+b\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx &= -\frac{b(2bBc-Abd-2aBd)x}{d^3}\\ &\quad-\frac{2(bc-ad)\left(ad^2(Ac-Bd)-b(2Bc^3-Ac^2d-3Bcd^2+2Ad^3)\right)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{d^3\left(c^2-d^2\right)^{3/2}f}\\ &\quad-\frac{b^2B\cos(e+fx)}{d^2f}-\frac{(bc-ad)^2(Bc-Ad)\cos(e+fx)}{d^2\left(c^2-d^2\right)f(c+d\sin(e+fx))}\end{aligned}$$

output
```
-b*(-A*b*d-2*B*a*d+2*B*b*c)*x/d^3-2*(-a*d+b*c)*(a*d^2*(A*c-B*d)-b*(-A*c^2*
d+2*A*d^3+2*B*c^3-3*B*c*d^2))*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1
/2))/d^3/(c^2-d^2)^(3/2)/f-b^2*B*cos(f*x+e)/d^2/f-(-a*d+b*c)^2*(-A*d+B*c)*
cos(f*x+e)/d^2/(c^2-d^2)/f/(c+d*sin(f*x+e))
```

Mathematica [A] (verified)

Time = 4.16 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.94

$$\int \frac{(a+b\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \frac{b(-2bBc+Abd+2aBd)(e+fx)+\frac{2(bc-ad)\left(ad^2(-Ac+Bd)+b\left(2Bc^3-Ac^2d-3Bcd^2+2Ad^3\right)\right)\arctan\left(\frac{d+c\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c^2-d^2)^{3/2}}-}{d^3f}$$

input
```
Integrate[((a + b*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x
])^2,x]
```

output
```
(b*(-2*b*B*c + A*b*d + 2*a*B*d)*(e + f*x) + (2*(b*c - a*d)*(a*d^2*(-(A*c)
+ B*d) + b*(2*B*c^3 - A*c^2*d - 3*B*c*d^2 + 2*A*d^3))*ArcTan[(d + c*Tan[(e
 + f*x)/2])/Sqrt[c^2 - d^2]])/(c^2 - d^2)^(3/2) - b^2*B*d*Cos[e + f*x] + (
d*(b*c - a*d)^2*(-(B*c) + A*d)*Cos[e + f*x])/((c - d)*(c + d)*(c + d*Sin[e
 + f*x])))/(d^3*f)
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.18, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {3042, 3467, 25, 3042, 3502, 3042, 3214, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}dx$$

$$\Big\downarrow \text{3467}$$

$$\frac{\int -\frac{-b^2Bd(c^2-d^2)\sin^2(e+fx)+b(bBc-Abd-2aBd)(c^2-d^2)\sin(e+fx)+d(B(bc-ad)^2-Ad(ca^2-2bda+b^2c))}{c+d\sin(e+fx)}dx}{d^2(c^2-d^2)}-\frac{(bc-ad)^2(Bc-Ad)\cos(e+fx)}{d^2f(c^2-d^2)(c+d\sin(e+fx))}$$

$\Big\downarrow$ 25

$$-\frac{\int \frac{-b^2Bd(c^2-d^2)\sin^2(e+fx)+b(bBc-Abd-2aBd)(c^2-d^2)\sin(e+fx)+d(B(bc-ad)^2-Ad(ca^2-2bda+b^2c))}{c+d\sin(e+fx)}dx}{d^2(c^2-d^2)}-\frac{(bc-ad)^2(Bc-Ad)\cos(e+fx)}{d^2f(c^2-d^2)(c+d\sin(e+fx))}$$

$\Big\downarrow$ 3042

$$-\frac{\int \frac{-b^2Bd(c^2-d^2)\sin(e+fx)^2+b(bBc-Abd-2aBd)(c^2-d^2)\sin(e+fx)+d(B(bc-ad)^2-Ad(ca^2-2bda+b^2c))}{c+d\sin(e+fx)}dx}{d^2(c^2-d^2)}-\frac{(bc-ad)^2(Bc-Ad)\cos(e+fx)}{d^2f(c^2-d^2)(c+d\sin(e+fx))}$$

$\Big\downarrow$ 3502

$$-\frac{\frac{\int \frac{(B(bc-ad)^2-Ad(ca^2-2bda+b^2c))d^2+b(2bBc-Abd-2aBd)(c^2-d^2)\sin(e+fx)d}{c+d\sin(e+fx)}dx}{d}+\frac{b^2B(c^2-d^2)\cos(e+fx)}{f}}{d^2(c^2-d^2)}-\frac{(bc-ad)^2(Bc-Ad)\cos(e+fx)}{d^2f(c^2-d^2)(c+d\sin(e+fx))}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{\int \frac{(B(bc-ad)^2-Ad(ca^2-2bda+b^2c))d^2+b(2bBc-Abd-2aBd)(c^2-d^2)\sin(e+fx)d}{c+d\sin(e+fx)}dx}{d}+\frac{b^2B(c^2-d^2)\cos(e+fx)}{f}}{d^2(c^2-d^2)}-\frac{(bc-ad)^2(Bc-Ad)\cos(e+fx)}{d^2f(c^2-d^2)(c+d\sin(e+fx))}$$

$\Big\downarrow$ 3214

$$-\frac{\frac{(bc-ad)(ad^2(Ac-Bd)-b(-Ac^2d+2Ad^3+2Bc^3-3Bcd^2))\int \frac{1}{c+d\sin(e+fx)}dx+bx(c^2-d^2)(-2aBd-Abd+2bBc)}{d}+\frac{b^2B(c^2-d^2)\cos(e+fx)}{f}}{d^2(c^2-d^2)}-\frac{(bc-ad)^2(Bc-Ad)\cos(e+fx)}{d^2f(c^2-d^2)(c+d\sin(e+fx))}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{(bc-ad)\left(ad^2(Ac-Bd)-b\left(-Ac^2d+2Ad^3+2Bc^3-3Bcd^2\right)\right)\int \frac{1}{c+d\sin(e+fx)}dx+bx\left(c^2-d^2\right)(-2aBd-Abd+2bBc)}{d}+\frac{b^2B\left(c^2-d^2\right)\cos(e+fx)}{f}}{d^2\left(c^2-d^2\right)}-\frac{(bc-ad)^2(Bc-Ad)\cos(e+fx)}{d^2f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

$\downarrow$ 3139

$$-\frac{\frac{\frac{2(bc-ad)\left(ad^2(Ac-Bd)-b\left(-Ac^2d+2Ad^3+2Bc^3-3Bcd^2\right)\right)\int \frac{1}{c\tan^2\left(\frac{1}{2}(e+fx)\right)+2d\tan\left(\frac{1}{2}(e+fx)\right)+c}d\tan\left(\frac{1}{2}(e+fx)\right)}{f}+bx\left(c^2-d^2\right)(-2aBd-Abd+2bBc)}{d}}{d^2\left(c^2-d^2\right)}-\frac{(bc-ad)^2(Bc-Ad)\cos(e+fx)}{d^2f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

$\downarrow$ 1083

$$-\frac{\frac{bx\left(c^2-d^2\right)(-2aBd-Abd+2bBc)-\frac{4(bc-ad)\left(ad^2(Ac-Bd)-b\left(-Ac^2d+2Ad^3+2Bc^3-3Bcd^2\right)\right)\int \frac{1}{-\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)^2-4\left(c^2-d^2\right)}d\left(2d+2c\tan\left(\frac{1}{2}(e+fx)\right)\right)}{f}}{d}}{d^2\left(c^2-d^2\right)}-\frac{(bc-ad)^2(Bc-Ad)\cos(e+fx)}{d^2f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

$\downarrow$ 217

$$-\frac{\frac{\frac{2(bc-ad)\left(ad^2(Ac-Bd)-b\left(-Ac^2d+2Ad^3+2Bc^3-3Bcd^2\right)\right)\arctan\left(\frac{2c\tan\left(\frac{1}{2}(e+fx)\right)+2d}{2\sqrt{c^2-d^2}}\right)}{f\sqrt{c^2-d^2}}+bx\left(c^2-d^2\right)(-2aBd-Abd+2bBc)}{d}+\frac{b^2B\left(c^2-d^2\right)\cos(e+fx)}{f}}{d^2\left(c^2-d^2\right)}-\frac{(bc-ad)^2(Bc-Ad)\cos(e+fx)}{d^2f\left(c^2-d^2\right)(c+d\sin(e+fx))}$$

input
```
Int[((a + b*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2,x
]
```

output
```
-(((b*(2*b*B*c - A*b*d - 2*a*B*d)*(c^2 - d^2)*x + (2*(b*c - a*d)*(a*d^2*(A
*c - B*d) - b*(2*B*c^3 - A*c^2*d - 3*B*c*d^2 + 2*A*d^3))*ArcTan[(2*d + 2*c
*Tan[(e + f*x)/2])/(2*Sqrt[c^2 - d^2])])/(Sqrt[c^2 - d^2]*f))/d + (b^2*B*(
c^2 - d^2)*Cos[e + f*x])/f)/(d^2*(c^2 - d^2))) - ((b*c - a*d)^2*(B*c - A*d
)*Cos[e + f*x])/(d^2*(c^2 - d^2)*f*(c + d*Sin[e + f*x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3467
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*d^2
*(n + 1)*(c^2 - d^2))), x] - Simp[1/(d^2*(n + 1)*(c^2 - d^2))   Int[(c + d*
Sin[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c
- 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*
a*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n +
1)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[
n, -1]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.88

method	result
derivativedivides	$\frac{2\left(\frac{d^2\left(A a^2 d^3-2Aabc\,d^2+A b^2c^2d-B a^2c\,d^2+2Bab\,c^2d-c^3b^2B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^2-d^2\right)c}+\frac{d\left(A a^2 d^3-2Aabc\,d^2+A b^2c^2d-B a^2c\,d^2+2Bab\,c\right.}{c^2-d^2}\right.}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}$
default	$\frac{2\left(\frac{d^2\left(A a^2 d^3-2Aabc\,d^2+A b^2c^2d-B a^2c\,d^2+2Bab\,c^2d-c^3b^2B\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^2-d^2\right)c}+\frac{d\left(A a^2 d^3-2Aabc\,d^2+A b^2c^2d-B a^2c\,d^2+2Bab\,c\right.}{c^2-d^2}\right.}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c+2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+c}$
risch	Expression too large to display

input
```
int((a+b*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x,method=_RETUR
NVERBOSE)
```

output
```
1/f*(2/d^3*((d^2*(A*a^2*d^3-2*A*a*b*c*d^2+A*b^2*c^2*d-B*a^2*c*d^2+2*B*a*b*
c^2*d-B*b^2*c^3)/(c^2-d^2)/c*tan(1/2*f*x+1/2*e)+d*(A*a^2*d^3-2*A*a*b*c*d^2
+A*b^2*c^2*d-B*a^2*c*d^2+2*B*a*b*c^2*d-B*b^2*c^3)/(c^2-d^2))/(tan(1/2*f*x+
1/2*e)^2*c+2*d*tan(1/2*f*x+1/2*e)+c)+(A*a^2*c*d^3-2*A*a*b*d^4-A*b^2*c^3*d+
2*A*b^2*c*d^3-B*a^2*d^4-2*B*a*b*c^3*d+4*B*a*b*c*d^3+2*B*b^2*c^4-3*B*b^2*c^
2*d^2)/(c^2-d^2)^(3/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(
1/2)))+2*b/d^3*(-B*b*d/(1+tan(1/2*f*x+1/2*e)^2)+(A*b*d+2*B*a*d-2*B*b*c)*ar
ctan(tan(1/2*f*x+1/2*e))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 611 vs. $2(194) = 388$.

Time = 0.17 (sec) , antiderivative size = 1308, normalized size of antiderivative = 6.57

$$\int \frac{(a+b\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algori
thm="fricas")
```

output
```
[-1/2*(2*(2*B*b^2*c^6 - 4*B*b^2*c^4*d^2 + 2*B*b^2*c^2*d^4 - (2*B*a*b + A*b
^2)*c^5*d + 2*(2*B*a*b + A*b^2)*c^3*d^3 - (2*B*a*b + A*b^2)*c*d^5)*f*x + (
2*B*b^2*c^5 - 3*B*b^2*c^3*d^2 - (2*B*a*b + A*b^2)*c^4*d + (A*a^2 + 4*B*a*b
 + 2*A*b^2)*c^2*d^3 - (B*a^2 + 2*A*a*b)*c*d^4 + (2*B*b^2*c^4*d - 3*B*b^2*c
^2*d^3 - (2*B*a*b + A*b^2)*c^3*d^2 + (A*a^2 + 4*B*a*b + 2*A*b^2)*c*d^4 - (
B*a^2 + 2*A*a*b)*d^5)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*co
s(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x
+ e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f
*x + e) - c^2 - d^2)) + 2*(2*B*b^2*c^5*d + A*a^2*d^6 - (2*B*a*b + A*b^2)*c
^4*d^2 + (B*a^2 + 2*A*a*b - 3*B*b^2)*c^3*d^3 - (A*a^2 - 2*B*a*b - A*b^2)*c
^2*d^4 - (B*a^2 + 2*A*a*b - B*b^2)*c*d^5)*cos(f*x + e) + 2*((2*B*b^2*c^5*d
 - 4*B*b^2*c^3*d^3 + 2*B*b^2*c*d^5 - (2*B*a*b + A*b^2)*c^4*d^2 + 2*(2*B*a*
b + A*b^2)*c^2*d^4 - (2*B*a*b + A*b^2)*d^6)*f*x + (B*b^2*c^4*d^2 - 2*B*b^2
*c^2*d^4 + B*b^2*d^6)*cos(f*x + e))*sin(f*x + e))/((c^4*d^4 - 2*c^2*d^6 +
d^8)*f*sin(f*x + e) + (c^5*d^3 - 2*c^3*d^5 + c*d^7)*f), -((2*B*b^2*c^6 - 4
*B*b^2*c^4*d^2 + 2*B*b^2*c^2*d^4 - (2*B*a*b + A*b^2)*c^5*d + 2*(2*B*a*b +
A*b^2)*c^3*d^3 - (2*B*a*b + A*b^2)*c*d^5)*f*x + (2*B*b^2*c^5 - 3*B*b^2*c^3
*d^2 - (2*B*a*b + A*b^2)*c^4*d + (A*a^2 + 4*B*a*b + 2*A*b^2)*c^2*d^3 - (B*
a^2 + 2*A*a*b)*c*d^4 + (2*B*b^2*c^4*d - 3*B*b^2*c^2*d^3 - (2*B*a*b + A*b^2
)*c^3*d^2 + (A*a^2 + 4*B*a*b + 2*A*b^2)*c*d^4 - (B*a^2 + 2*A*a*b)*d^5)*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algori
thm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 749 vs. $2(194) = 388$.

Time $= 0.20$ (sec) , antiderivative size $= 749$, normalized size of antiderivative $= 3.76$

$$\int \frac{(a+b\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algori
thm="giac")
```

output
```
(2*(2*B*b^2*c^4 - 2*B*a*b*c^3*d - A*b^2*c^3*d - 3*B*b^2*c^2*d^2 + A*a^2*c*
d^3 + 4*B*a*b*c*d^3 + 2*A*b^2*c*d^3 - B*a^2*d^4 - 2*A*a*b*d^4)*(pi*floor(1
/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c
^2 - d^2)))/((c^2*d^3 - d^5)*sqrt(c^2 - d^2)) - 2*(B*b^2*c^3*d*tan(1/2*f*x
 + 1/2*e)^3 - 2*B*a*b*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 - A*b^2*c^2*d^2*tan(1
/2*f*x + 1/2*e)^3 + B*a^2*c*d^3*tan(1/2*f*x + 1/2*e)^3 + 2*A*a*b*c*d^3*tan
(1/2*f*x + 1/2*e)^3 - A*a^2*d^4*tan(1/2*f*x + 1/2*e)^3 + 2*B*b^2*c^4*tan(1
/2*f*x + 1/2*e)^2 - 2*B*a*b*c^3*d*tan(1/2*f*x + 1/2*e)^2 - A*b^2*c^3*d*tan
(1/2*f*x + 1/2*e)^2 + B*a^2*c^2*d^2*tan(1/2*f*x + 1/2*e)^2 + 2*A*a*b*c^2*d
^2*tan(1/2*f*x + 1/2*e)^2 - B*b^2*c^2*d^2*tan(1/2*f*x + 1/2*e)^2 - A*a^2*c
*d^3*tan(1/2*f*x + 1/2*e)^2 + 3*B*b^2*c^3*d*tan(1/2*f*x + 1/2*e) - 2*B*a*b
*c^2*d^2*tan(1/2*f*x + 1/2*e) - A*b^2*c^2*d^2*tan(1/2*f*x + 1/2*e) + B*a^2
*c*d^3*tan(1/2*f*x + 1/2*e) + 2*A*a*b*c*d^3*tan(1/2*f*x + 1/2*e) - 2*B*b^2
*c*d^3*tan(1/2*f*x + 1/2*e) - A*a^2*d^4*tan(1/2*f*x + 1/2*e) + 2*B*b^2*c^4
 - 2*B*a*b*c^3*d - A*b^2*c^3*d + B*a^2*c^2*d^2 + 2*A*a*b*c^2*d^2 - B*b^2*c
^2*d^2 - A*a^2*c*d^3)/((c^3*d^2 - c*d^4)*(c*tan(1/2*f*x + 1/2*e)^4 + 2*d*t
an(1/2*f*x + 1/2*e)^3 + 2*c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2
*e) + c)) - (2*B*b^2*c - 2*B*a*b*d - A*b^2*d)*(f*x + e)/d^3)/f
```

Mupad [B] (verification not implemented)

Time = 55.73 (sec) , antiderivative size = 16312, normalized size of antiderivative = 81.97

$$\int \frac{(a+b\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(((A + B*sin(e + f*x))*(a + b*sin(e + f*x))^2)/(c + d*sin(e + f*x))^2,x
)
```

output
```
((2*(A*a^2*d^3 - 2*B*b^2*c^3 + A*b^2*c^2*d - B*a^2*c*d^2 + B*b^2*c*d^2 - 2
*A*a*b*c*d^2 + 2*B*a*b*c^2*d))/(d^2*(c^2 - d^2)) + (2*tan(e/2 + (f*x)/2)^2
*(A*a^2*d^3 - 2*B*b^2*c^3 + A*b^2*c^2*d - B*a^2*c*d^2 + B*b^2*c*d^2 - 2*A*
a*b*c*d^2 + 2*B*a*b*c^2*d))/(d^2*(c^2 - d^2)) + (2*tan(e/2 + (f*x)/2)^3*(A
*a^2*d^3 - B*b^2*c^3 + A*b^2*c^2*d - B*a^2*c*d^2 - 2*A*a*b*c*d^2 + 2*B*a*b
*c^2*d))/(c*d*(c^2 - d^2)) + (2*tan(e/2 + (f*x)/2)*(A*a^2*d^3 - 3*B*b^2*c^
3 + A*b^2*c^2*d - B*a^2*c*d^2 + 2*B*b^2*c*d^2 - 2*A*a*b*c*d^2 + 2*B*a*b*c^
2*d))/(c*d*(c^2 - d^2)))/(f*(c + 2*d*tan(e/2 + (f*x)/2) + 2*c*tan(e/2 + (f
*x)/2)^2 + c*tan(e/2 + (f*x)/2)^4 + 2*d*tan(e/2 + (f*x)/2)^3)) + (atan((((
b*d*(A*b + 2*B*a)*1i - B*b^2*c*2i)*((32*(A^2*b^4*c^2*d^8 - 2*A^2*b^4*c^4*d
^6 + A^2*b^4*c^6*d^4 + 4*B^2*b^4*c^4*d^6 - 8*B^2*b^4*c^6*d^4 + 4*B^2*b^4*c
^8*d^2 + 4*B^2*a^2*b^2*c^2*d^8 - 8*B^2*a^2*b^2*c^4*d^6 + 4*B^2*a^2*b^2*c^6
*d^4 - 4*A*B*b^4*c^3*d^7 + 8*A*B*b^4*c^5*d^5 - 4*A*B*b^4*c^7*d^3 - 8*B^2*a
*b^3*c^3*d^7 + 16*B^2*a*b^3*c^5*d^5 - 8*B^2*a*b^3*c^7*d^3 + 4*A*B*a*b^3*c^
2*d^8 - 8*A*B*a*b^3*c^4*d^6 + 4*A*B*a*b^3*c^6*d^4))/(d^9 - 2*c^2*d^7 + c^4
*d^5) + ((b*d*(A*b + 2*B*a)*1i - B*b^2*c*2i)*((((32*(c^2*d^12 - 2*c^4*d^10
 + c^6*d^8))/(d^9 - 2*c^2*d^7 + c^4*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^1
4 - 8*c^3*d^12 + 7*c^5*d^10 - 2*c^7*d^8))/(d^10 - 2*c^2*d^8 + c^4*d^6))*(b
*d*(A*b + 2*B*a)*1i - B*b^2*c*2i))/d^3 - (32*(A*a^2*c^5*d^7 - A*a^2*c^3*d^
9 - A*b^2*c*d^11 + A*b^2*c^3*d^9 + B*a^2*c^2*d^10 - B*a^2*c^4*d^8 + 2*B...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1138, normalized size of antiderivative = 5.72

$$\int \frac{(a+b\sin(e+fx))^2(A+B\sin(e+fx))}{(c+d\sin(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int((a+b*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

output

```
(2*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*sin(
e + f*x)*a**3*c*d**4 - 6*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/s
qrt(c**2 - d**2))*sin(e + f*x)*a**2*b*d**5 - 6*sqrt(c**2 - d**2)*atan((tan
((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*sin(e + f*x)*a*b**2*c**3*d**2 + 12
*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*sin(e
+ f*x)*a*b**2*c*d**4 + 4*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/s
qrt(c**2 - d**2))*sin(e + f*x)*b**3*c**4*d - 6*sqrt(c**2 - d**2)*atan((tan
((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*sin(e + f*x)*b**3*c**2*d**3 + 2*sq
rt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*a**3*c**2
*d**3 - 6*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2
))*a**2*b*c*d**4 - 6*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*c + d)/sqrt(
c**2 - d**2))*a*b**2*c**4*d + 12*sqrt(c**2 - d**2)*atan((tan((e + f*x)/2)*
c + d)/sqrt(c**2 - d**2))*a*b**2*c**2*d**3 + 4*sqrt(c**2 - d**2)*atan((tan
((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*b**3*c**5 - 6*sqrt(c**2 - d**2)*at
an((tan((e + f*x)/2)*c + d)/sqrt(c**2 - d**2))*b**3*c**3*d**2 - cos(e + f*
x)*sin(e + f*x)*b**3*c**4*d**2 + 2*cos(e + f*x)*sin(e + f*x)*b**3*c**2*d**
4 - cos(e + f*x)*sin(e + f*x)*b**3*d**6 + cos(e + f*x)*a**3*c**2*d**4 - co
s(e + f*x)*a**3*d**6 - 3*cos(e + f*x)*a**2*b*c**3*d**3 + 3*cos(e + f*x)*a*
*2*b*c*d**5 + 3*cos(e + f*x)*a*b**2*c**4*d**2 - 3*cos(e + f*x)*a*b**2*c**2
*d**4 - 2*cos(e + f*x)*b**3*c**5*d + 3*cos(e + f*x)*b**3*c**3*d**3 - co...
```

3.353 $\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 840

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}\,dx = \frac{(c-d)\sqrt{c+d}(2Ab^2c-2abBc-2aAbd+3a^2Bd-b^2B \cdots}{\cdots}$$

$$+\frac{\sqrt{c+d}(3bBc+2Abd-3aBd)\,\text{EllipticPi}\left(\frac{b(c+d)}{(a+b)d},\arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sec(e+fx)\sqrt{- \cdots}}{b^3\sqrt{a+b}f}$$

$$+\frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b(a^2-b^2)f\sqrt{a+b\sin(e+fx)}}$$

$$-\frac{(2Ab(bc-ad)-B(2abc-3a^2d+b^2d))\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b(a^2-b^2)f\sqrt{a+b\sin(e+fx)}}$$

$$+\frac{\sqrt{a+b}(2Ab(b(c-2d)+ad)-B(3a^2d-6abd+b^2(2c+d)))\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right),\frac{(a+b \cdots}{(a-b \cdots}\right.}{(a-b)b^3\sqrt{c+d}f}$$

output
```
(c-d)*(c+d)^(1/2)*(-2*A*a*b*d+2*A*b^2*c+3*B*a^2*d-2*B*a*b*c-B*b^2*d)*Ellip
ticE((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2)
,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(-(-a*d+b*c)*(1-sin(f*x+e))/(
c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x
+e)))^(1/2)*(a+b*sin(f*x+e))/(a-b)/b^2/(a+b)^(1/2)/(-a*d+b*c)/f+(c+d)^(1/2
)*(2*A*b*d-3*B*a*d+3*B*b*c)*EllipticPi((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/
(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),b*(c+d)/(a+b)/d,((a-b)*(c+d)/(a+b)/(c-d
))^(1/2))*sec(f*x+e)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(
1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)*(a+b*sin(f*x
+e))/b^3/(a+b)^(1/2)/f+2*(A*b-B*a)*(-a*d+b*c)*cos(f*x+e)*(c+d*sin(f*x+e))^
(1/2)/b/(a^2-b^2)/f/(a+b*sin(f*x+e))^(1/2)-(2*A*b*(-a*d+b*c)-B*(-3*a^2*d+2
*a*b*c+b^2*d))*cos(f*x+e)*(c+d*sin(f*x+e))^(1/2)/b/(a^2-b^2)/f/(a+b*sin(f*
x+e))^(1/2)+(a+b)^(1/2)*(2*A*b*(b*(c-2*d)+a*d)-B*(3*a^2*d-6*a*b*d+b^2*(2*c
+d)))*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*
x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*((-a*d+b*c)*(1-sin
(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(
c+d*sin(f*x+e)))^(1/2)*(c+d*sin(f*x+e))/(a-b)/b^3/(c+d)^(1/2)/f
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2042 vs. 2(840) = 1680.

Time = 9.39 (sec) , antiderivative size = 2042, normalized size of antiderivative = 2.43

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2))/(a + b*Sin[e +
 f*x])^(3/2),x]
```

output
```
(-2*(A*b^2*c*Cos[e + f*x] - a*b*B*c*Cos[e + f*x] - a*A*b*d*Cos[e + f*x] +
a^2*B*d*Cos[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/(b*(-a^2 + b^2)*f*Sqrt[a +
 b*Sin[e + f*x]]) + ((-4*(-(b*c) + a*d)*(2*a*A*b*c^2 - 2*b^2*B*c^2 - 2*A*b
^2*c*d + 2*a*b*B*c*d + a^2*B*d^2 - b^2*B*d^2)*Sqrt[((c + d)*Cot[(-e + Pi/2
 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f
*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d)
)/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d
)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((
-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/
((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-
(b*c) + a*d)*(2*A*b^2*c^2 - 2*a*b*B*c^2 + 4*a^2*B*c*d - 4*b^2*B*c*d - 2*A*
b^2*d^2 + 2*a*b*B*d^2)*((Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d
)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e
 + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]
*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*
x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/
2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt
[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d)*Cot[(-e +
Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[
Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c)...
```

Rubi [A] (verified)

Time = 4.46 (sec) , antiderivative size = 851, normalized size of antiderivative = 1.01, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 3468, 27, 3042, 3540, 25, 3042, 3532, 25, 3042, 3290, 3477, 3042, 3297, 3475}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3468}$$

$$\frac{\frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{bf\left(a^2-b^2\right)\sqrt{a+b\sin(e+fx)}}-}{2\int \frac{c(Bc+2Ad)b^2-a\left(2Bcd+A\left(c^2+d^2\right)\right)b+a^2Bd^2-d\left(2Ab(bc-ad)-B\left(-3da^2+2bca+b^2d\right)\right)\sin^2(e+fx)-\left(A\left(c^2-d^2\right)b^2+B\left(2cda^2-b\left(c^2-d^2\right)a-\right.\right.}{2\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}}{b\left(a^2-b^2\right)}$$

$\downarrow$ 27

$$\frac{\frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{bf\left(a^2-b^2\right)\sqrt{a+b\sin(e+fx)}}-}{\int \frac{c(Bc+2Ad)b^2-a\left(2Bcd+A\left(c^2+d^2\right)\right)b+a^2Bd^2-d\left(2Ab(bc-ad)-B\left(-3da^2+2bca+b^2d\right)\right)\sin^2(e+fx)-\left(A\left(c^2-d^2\right)b^2+B\left(2cda^2-b\left(c^2-d^2\right)a-2b\right.\right.}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}}{b\left(a^2-b^2\right)}$$

$\downarrow$ 3042

$$\frac{\frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{bf\left(a^2-b^2\right)\sqrt{a+b\sin(e+fx)}}-}{\int \frac{c(Bc+2Ad)b^2-a\left(2Bcd+A\left(c^2+d^2\right)\right)b+a^2Bd^2-d\left(2Ab(bc-ad)-B\left(-3da^2+2bca+b^2d\right)\right)\sin(e+fx)^2-\left(A\left(c^2-d^2\right)b^2+B\left(2cda^2-b\left(c^2-d^2\right)a-2b\right.\right.}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}}{b\left(a^2-b^2\right)}$$

$\downarrow$ 3540

$$\frac{\frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{bf\left(a^2-b^2\right)\sqrt{a+b\sin(e+fx)}}-}{\frac{\int -\frac{\left(a^2-b^2\right)d^2(3bBc+2Abd-3aBd)\sin^2(e+fx)+2\left(a^2-b^2\right)cd(bBc+2Abd-aBd)\sin(e+fx)+\left(a^2-b^2\right)d\left(2Abc^2-Bd(bc-ad)\right)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}dx}{2d}+\frac{\cos(e+fx)(2Ab(bc-ad)}{}}{b\left(a^2-b^2\right)}$$

$\downarrow$ 25

$$\frac{\frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{bf\left(a^2-b^2\right)\sqrt{a+b\sin(e+fx)}}-}{\frac{\cos(e+fx)\left(2Ab(bc-ad)-B\left(-3a^2d+2abc+b^2d\right)\right)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+b\sin(e+fx)}}-\frac{\int \frac{\left(a^2-b^2\right)d^2(3bBc+2Abd-3aBd)\sin^2(e+fx)+2\left(a^2-b^2\right)cd(bBc+2Abd-aBd)\sin}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}}{2d}}{b\left(a^2-b^2\right)}$$

$\downarrow$ 3042

$$\frac{\frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{bf\left(a^2-b^2\right)\sqrt{a+b\sin(e+fx)}}-}{\frac{\cos(e+fx)\left(2Ab(bc-ad)-B\left(-3a^2d+2abc+b^2d\right)\right)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+b\sin(e+fx)}}-\frac{\int \frac{\left(a^2-b^2\right)d^2(3bBc+2Abd-3aBd)\sin(e+fx)^2+2\left(a^2-b^2\right)cd(bBc+2Abd-aBd)\sin}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}}{2d}}{b\left(a^2-b^2\right)}$$

$\downarrow$ 3532

$$\frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{bf\left(a^2-b^2\right)\sqrt{a+b\sin(e+fx)}}-$$

$$\frac{\frac{\cos(e+fx)\left(2Ab(bc-ad)-B\left(-3a^2d+2abc+b^2d\right)\right)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+b\sin(e+fx)}}-\frac{\frac{d^2\left(a^2-b^2\right)(-3aBd+2Abd+3bBc)\int\frac{\sqrt{a+b\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}dx}{b^2}+\int-\frac{\left(a^2-b^2\right)d(bc-ad)}{}}{}}{b\left(a^2-b^2\right)}$$

$\Big\downarrow$ 25

$$\frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{bf\left(a^2-b^2\right)\sqrt{a+b\sin(e+fx)}}-$$

$$\frac{\frac{\cos(e+fx)\left(2Ab(bc-ad)-B\left(-3a^2d+2abc+b^2d\right)\right)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+b\sin(e+fx)}}-\frac{\frac{d^2\left(a^2-b^2\right)(-3aBd+2Abd+3bBc)\int\frac{\sqrt{a+b\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}dx}{b^2}-\int\frac{\left(a^2-b^2\right)d(bc-ad)\left((\right.}{}}{2}}{b\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3042

$$\frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{bf\left(a^2-b^2\right)\sqrt{a+b\sin(e+fx)}}-$$

$$\frac{\frac{\cos(e+fx)\left(2Ab(bc-ad)-B\left(-3a^2d+2abc+b^2d\right)\right)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+b\sin(e+fx)}}-\frac{\frac{d^2\left(a^2-b^2\right)(-3aBd+2Abd+3bBc)\int\frac{\sqrt{a+b\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}dx}{b^2}-\int\frac{\left(a^2-b^2\right)d(bc-ad)\left((\right.}{}}{2}}{b\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3290

$$\frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{bf\left(a^2-b^2\right)\sqrt{a+b\sin(e+fx)}}-$$

$$\frac{\cos(e+fx)\left(2Ab(bc-ad)-B\left(-3a^2d+2abc+b^2d\right)\right)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+b\sin(e+fx)}}-\frac{2d\left(a^2-b^2\right)\sqrt{c+d}\sec(e+fx)(-3aBd+2Abd+3bBc)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-}{(c+d)(a+b}}}{}$$

$\Big\downarrow$ 3477

$$\frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{bf\left(a^2-b^2\right)\sqrt{a+b\sin(e+fx)}}-$$

$$\frac{\cos(e+fx)\left(2Ab(bc-ad)-B\left(-3a^2d+2abc+b^2d\right)\right)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+b\sin(e+fx)}}-\frac{2d\left(a^2-b^2\right)\sqrt{c+d}\sec(e+fx)(-3aBd+2Abd+3bBc)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-}{(c+d)(a+b}}}{}$$

$\Big\downarrow$ 3042

$$\frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{bf\left(a^2-b^2\right)\sqrt{a+b\sin(e+fx)}}-$$

$$\frac{\frac{\cos(e+fx)\left(2Ab(bc-ad)-B\left(-3a^2d+2abc+b^2d\right)\right)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+b\sin(e+fx)}}-\frac{2d\left(a^2-b^2\right)\sqrt{c+d}\sec(e+fx)(-3aBd+2Abd+3bBc)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-}{(c+d)(a+b}}}{}}{}$$

$\downarrow$ 3297

$$\frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{bf\left(a^2-b^2\right)\sqrt{a+b\sin(e+fx)}}-$$

$$\frac{\frac{\cos(e+fx)\left(2Ab(bc-ad)-B\left(-3a^2d+2abc+b^2d\right)\right)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+b\sin(e+fx)}}-\frac{2d\left(a^2-b^2\right)\sqrt{c+d}\sec(e+fx)(-3aBd+2Abd+3bBc)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-}{(c+d)(a+b}}}{}}{}$$

$\downarrow$ 3475

$$\frac{2(Ab-aB)(bc-ad)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{b\left(a^2-b^2\right)f\sqrt{a+b\sin(e+fx)}}-$$

$$\frac{\frac{\left(2Ab(bc-ad)-B\left(-3da^2+2bca+b^2d\right)\right)\cos(e+fx)\sqrt{c+d\sin(e+fx)}}{f\sqrt{a+b\sin(e+fx)}}-\frac{2\left(a^2-b^2\right)d\sqrt{c+d}(3bBc+2Abd-3aBd)\text{EllipticPi}\left(\frac{b(c+d)}{(a+b)d},\arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(}}{\sqrt{c+d}\sqrt{a+b\sin(}}\right.\right.}{}}{}$$

input
```
Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2))/(a + b*Sin[e + f*x])
^(3/2),x]
```

output
```
(2*(A*b - a*B)*(b*c - a*d)*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(b*(a^2
- b^2)*f*Sqrt[a + b*Sin[e + f*x]]) - (((2*A*b*(b*c - a*d) - B*(2*a*b*c - 3
*a^2*d + b^2*d))*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(f*Sqrt[a + b*Sin[
e + f*x]]) - ((2*(a^2 - b^2)*d*Sqrt[c + d]*(3*b*B*c + 2*A*b*d - 3*a*B*d)*E
llipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e +
f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)
*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(
a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a +
 b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/(b^2*Sqrt[a + b]*f) - ((-2*b*Sqrt
[a + b]*(c - d)*d*Sqrt[c + d]*(2*A*b*(b*c - a*d) - B*(2*a*b*c - 3*a^2*d +
b^2*d))*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c +
d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e
+ f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x
])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))
]*(a + b*Sin[e + f*x]))/((b*c - a*d)*f) - (2*(a + b)^(3/2)*d*(2*A*b*(b*(c
- 2*d) + a*d) - B*(3*a^2*d - 6*a*b*d + b^2*(2*c + d)))*EllipticF[ArcSin[(S
qrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]]
)], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1
 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 +
 Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3290
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

rule 3297
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

rule 3468
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n +
1)*(c^2 - d^2))   Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n
+ 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a
*B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
 - a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c -
a*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

rule 3475
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3532
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[C/b^2   Int[Sqrt[a + b*Sin[e + f*x]]
/Sqrt[c + d*Sin[e + f*x]], x], x] + Simp[1/b^2   Int[(A*b^2 - a^2*C + b*(b*
B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &
& NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3540
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(Sqrt[c + d*Sin[e + f
*x]]/(d*f*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[1/(2*d)   Int[(1/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]))*Simp[2*a*A*d - C*(b*c - a*d) -
 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e +
 f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 78.86 (sec) , antiderivative size = 1089305, normalized size of antiderivative = 1296.79

method	result	size
default	Expression too large to display	1089305
parts	Expression too large to display	1235992

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(3/2),x,metho
d=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(3/2),x
, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}\,dx = \int \frac{(A+B\sin{(e+fx)})\,(c+d\sin{(e+fx)})^{\frac{3}{2}}}{(a+b\sin{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e))**(3/2)
,x)
```

output
```
Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))**(3/2)/(a + b*sin(e + f
*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin{(fx+e)}+A)(d\sin{(fx+e)}+c)^{\frac{3}{2}}}{(b\sin{(fx+e)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(3/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e)
+ a)^(3/2), x)
```

Giac [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}\,dx = \int \frac{(B\sin{(fx+e)}+A)(d\sin{(fx+e)}+c)^{\frac{3}{2}}}{(b\sin{(fx+e)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(3/2),x
, algorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e)
+ a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}\,dx=\int \frac{\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{3/2}}{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^(3/2))/(a + b*sin(e + f*x))
^(3/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^(3/2))/(a + b*sin(e + f*x))
^(3/2), x)
```

Reduce [F]

$$\int \frac{(A+B\sin(e+fx))(c+d\sin(e+fx))^{3/2}}{(a+b\sin(e+fx))^{3/2}}\,dx=\left(\int \frac{\sqrt{\sin\left(fx+e\right)d+c}\,\sqrt{\sin\left(fx+e\right)b+a}\,\sin\left(fx+\right.}{\sin\left(fx+e\right)b+a}\right.$$
$$+\left(\int \frac{\sqrt{\sin\left(fx+e\right)d+c}\,\sqrt{\sin\left(fx+e\right)b+a}}{\sin\left(fx+e\right)b+a}dx\right)c$$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(3/2),x)
```

output
```
int((sqrt(sin(e + f*x)*d + c)*sqrt(sin(e + f*x)*b + a)*sin(e + f*x))/(sin(
e + f*x)*b + a),x)*d + int((sqrt(sin(e + f*x)*d + c)*sqrt(sin(e + f*x)*b +
 a))/(sin(e + f*x)*b + a),x)*c
```

3.354 $\int \frac{(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 630

$$\int \frac{(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx = \frac{2(Ab-aB)(c-d)\sqrt{c+d}E\left(\arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)|\right.}{}$$
$$+\frac{2\sqrt{a+b}(Ab-aB)(c-d)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right),\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+fx)\sqrt{\frac{(bc-ad)(1-\sin(e+}{(a+b)(c+d\sin(e+}}}{(a-b)b\sqrt{c+d}(bc-ad)f}$$
$$+\frac{2\sqrt{a+b}B\,\text{EllipticPi}\left(\frac{(a+b)d}{b(c+d)},\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right),\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+fx)\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-}}{b^2\sqrt{c+d}f}$$

output
```
2*(A*b-B*a)*(c-d)*(c+d)^(1/2)*EllipticE((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)
/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f
*x+e)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c
)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)*(a+b*sin(f*x+e))/(a-b)/b/(a
+b)^(1/2)/(-a*d+b*c)/f+2*(a+b)^(1/2)*(A*b-B*a)*(c-d)*EllipticF((c+d)^(1/2)
*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a
-b)/(c+d))^(1/2))*sec(f*x+e)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x
+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)*(c+d
*sin(f*x+e))/(a-b)/b/(c+d)^(1/2)/(-a*d+b*c)/f+2*(a+b)^(1/2)*B*EllipticPi((
c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),(a+b)
*d/b/(c+d),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*((-a*d+b*c)*(1-sin(
f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c
+d*sin(f*x+e)))^(1/2)*(c+d*sin(f*x+e))/b^2/(c+d)^(1/2)/f
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1901 vs. $2(630) = 1260$.

Time = 18.02 (sec) , antiderivative size = 1901, normalized size of antiderivative = 3.02

$$\int \frac{(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[((A + B*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/(a + b*Sin[e + f
*x])^(3/2),x]
```

output
```
(-2*(-(A*b*Cos[e + f*x]) + a*B*Cos[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/((a
^2 - b^2)*f*Sqrt[a + b*Sin[e + f*x]]) + ((-4*(a*A*c - b*B*c)*(-(b*c) + a*d
)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqr
t[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d
)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e +
 Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e +
f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Si
n[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sq
rt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(A*b*c - a*B*c + a*A*d - b*B*d)
*((Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sq
rt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*
d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e
+ Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e +
 f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*S
in[e + f*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*S
qrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c
+ d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e
 + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(
b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*S
qrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) +...
```

Rubi [A] (verified)

Time = 1.69 (sec) , antiderivative size = 621, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.205$, Rules used = {3042, 3471, 3042, 3274, 3042, 3290, 3297, 3475}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3471}$$

$$\frac{(Ab-aB)\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}dx}{b}+\frac{B\int \frac{\sqrt{c+d\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}}dx}{b}$$

$\Big\downarrow$ 3042

$$\frac{(Ab-aB)\int \frac{\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}dx}{b}+\frac{B\int \frac{\sqrt{c+d\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}}dx}{b}$$

$\Big\downarrow$ 3274

$$\frac{(Ab-aB)\left(\frac{(c-d)\int \frac{1}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}dx}{a-b}-\frac{(bc-ad)\int \frac{\sin(e+fx)+1}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}dx}{a-b}\right)}{b}+\frac{B\int \frac{\sqrt{c+d\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}}dx}{b}$$

$\Big\downarrow$ 3042

$$\frac{(Ab-aB)\left(\frac{(c-d)\int \frac{1}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}dx}{a-b}-\frac{(bc-ad)\int \frac{\sin(e+fx)+1}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}dx}{a-b}\right)}{b}+\frac{B\int \frac{\sqrt{c+d\sin(e+fx)}}{\sqrt{a+b\sin(e+fx)}}dx}{b}$$

$\Big\downarrow$ 3290

$$\frac{(Ab-aB)\left(\frac{(c-d)\int \frac{1}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}dx}{a-b}-\frac{(bc-ad)\int \frac{\sin(e+fx)+1}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}dx}{a-b}\right)}{b}+\frac{2B\sqrt{a+b}\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\text{EllipticPi}\left(\frac{(a+b)d}{b(c+d)},\arcsin\left(\right.\right.}{b^2f\sqrt{c+d}}$$

$\Big\downarrow$ 3297

$$\frac{(Ab-aB)\left(\frac{2\sqrt{a+b}(c-d)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+f}}{\sqrt{a+b}\sqrt{c+d\sin(e+f}}\right.\right.}{f(a-b)\sqrt{c+d}(bc-ad)}\right.}{b}$$
$$\frac{2B\sqrt{a+b}\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\text{EllipticPi}\left(\frac{(a+b)d}{b(c+d)},\arcsin\left(\right.\right.}{b^2f\sqrt{c+d}}$$

$\Big\downarrow$ 3475

$$(Ab-aB)\left(\frac{2\sqrt{a+b}(c-d)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+f}}{\sqrt{a+b}\sqrt{c+d\sin(e+f}}\right.\right.}{f(a-b)\sqrt{c+d}(bc-ad)}\right.$$

$$\frac{2B\sqrt{a+b}\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\,\text{EllipticPi}\left(\frac{(a+b)d}{b(c+d)},\arcsin\left(\right.\right.}{b^2f\sqrt{c+d}}$$

input

```
Int[((A + B*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/(a + b*Sin[e + f*x])^(
3/2),x]
```

output

```
(2*Sqrt[a + b]*B*EllipticPi[((a + b)*d)/(b*(c + d)), ArcSin[(Sqrt[c + d]*S
qrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)
*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f
*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x
]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/(b^2*Sqrt[c + d
]*f) + ((A*b - a*B)*((2*(c - d)*Sqrt[c + d]*EllipticE[ArcSin[(Sqrt[a + b]*
Sqrt[c + d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b
)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e
+ f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f
*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/((a - b)*Sqrt[
a + b]*(b*c - a*d)*f) + (2*Sqrt[a + b]*(c - d)*EllipticF[ArcSin[(Sqrt[c +
d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a
+ b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e
 + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e +
 f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/((a - b)*Sq
rt[c + d]*(b*c - a*d)*f)))/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3274
```
Int[Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(c - d)/(a - b)   Int[1/(Sqrt[a + b*Si
n[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Simp[(b*c - a*d)/(a - b)
Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3290
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[2*((a + b*Sin[e + f*x])/(d*f*Rt[(a + b)/
(c + d), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a
 + b*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a +
 b*Sin[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a + b))), ArcSin[Rt[(a + b)/(
c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((
c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

rule 3297
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

rule 3471
```
Int[(((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)]])/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2), x_Symbol] :> S
imp[B/b   Int[Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]], x], x] + S
imp[(A*b - a*B)/b   Int[Sqrt[c + d*Sin[e + f*x]]/(a + b*Sin[e + f*x])^(3/2)
, x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3475
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 37.22 (sec) , antiderivative size = 647398, normalized size of antiderivative = 1027.62

method	result	size
parts	Expression too large to display	647398
default	Expression too large to display	651747

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(3/2),x,metho
d=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx=\int \frac{(B\sin(fx+e)+A)\sqrt{d\sin(fx+e)+c}}{(b\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(3/2),x
, algorithm="fricas")
```

output
```
integral(-(B*sin(f*x + e) + A)*sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e
) + c)/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2), x)
```

Sympy [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx=\int \frac{(A+B\sin{(e+fx)})\sqrt{c+d\sin{(e+fx)}}}{(a+b\sin{(e+fx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e))**(3/2)
,x)
```

output
```
Integral((A + B*sin(e + f*x))*sqrt(c + d*sin(e + f*x))/(a + b*sin(e + f*x)
)**(3/2), x)
```

Maxima [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx=\int \frac{(B\sin{(fx+e)}+A)\sqrt{d\sin{(fx+e)}+c}}{(b\sin{(fx+e)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(3/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(d*sin(f*x + e) + c)/(b*sin(f*x + e) +
a)^(3/2), x)
```

Giac [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx=\int \frac{(B\sin(fx+e)+A)\sqrt{d\sin(fx+e)+c}}{(b\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(3/2),x
, algorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*sqrt(d*sin(f*x + e) + c)/(b*sin(f*x + e) +
a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx=\int \frac{(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx$$

input
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^(1/2))/(a + b*sin(e + f*x))
^(3/2),x)
```

output
```
int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^(1/2))/(a + b*sin(e + f*x))
^(3/2), x)
```

Reduce [F]

$$\int \frac{(A+B\sin(e+fx))\sqrt{c+d\sin(e+fx)}}{(a+b\sin(e+fx))^{3/2}}\,dx=\int \frac{\sqrt{\sin(fx+e)d+c}\,\sqrt{\sin(fx+e)b+a}}{\sin(fx+e)b+a}dx$$

input
```
int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(3/2),x)
```

output
```
int((sqrt(sin(e + f*x)*d + c)*sqrt(sin(e + f*x)*b + a))/(sin(e + f*x)*b +
a),x)
```

3.355 $\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 417

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx = \frac{2(Ab-aB)(c-d)\sqrt{c+d}E\left(\arcsin\left(\frac{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}\right)\right.}{} + \frac{2\sqrt{a+b}(A-B)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right),\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+fx)\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(b}{(}}}{(a-b)\sqrt{c+d}(bc-ad)f}$$

output
```
2*(A*b-B*a)*(c-d)*(c+d)^(1/2)*EllipticE((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)
/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f
*x+e)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c
)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)*(a+b*sin(f*x+e))/(a-b)/(a+b
)^(1/2)/(-a*d+b*c)^2/f+2*(a+b)^(1/2)*(A-B)*EllipticF((c+d)^(1/2)*(a+b*sin(
f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))
^(1/2))*sec(f*x+e)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2
)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)*(c+d*sin(f*x+e
))/(a-b)/(c+d)^(1/2)/(-a*d+b*c)/f
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1949 vs. $2(417) = 834$.

Time = 7.36 (sec) , antiderivative size = 1949, normalized size of antiderivative = 4.67

$$\int \frac{A + B\sin(e + fx)}{(a + b\sin(e + fx))^{3/2}\sqrt{c + d\sin(e + fx)}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[
e + f*x]]),x]
```

output
```
(-2*(A*b^2*Cos[e + f*x] - a*b*B*Cos[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/((
a^2 - b^2)*(-(b*c) + a*d)*f*Sqrt[a + b*Sin[e + f*x]]) + ((-4*(-(b*c) + a*d
)*(-(a*A*b*c) + b^2*B*c + a^2*A*d - A*b^2*d)*Sqrt[((c + d)*Cot[(-e + Pi/2
- f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*
x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))
/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)
*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-
a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)])/(
(a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(
b*c) + a*d)*(-(A*b^2*c) + a*b*B*c - a*A*b*d + a^2*B*d)*((Sqrt[((c + d)*Cot
[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e
 + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(
b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*S
qrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*
d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c
) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(
b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*
(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b
)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[...
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {3042, 3477, 3042, 3297, 3475}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}dx$$

$$\Big\downarrow \text{3477}$$

$$\frac{(A-B)\int \frac{1}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}dx}{a-b}-\frac{(Ab-aB)\int \frac{\sin(e+fx)+1}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}dx}{a-b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-B)\int \frac{1}{\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}dx}{a-b}-\frac{(Ab-aB)\int \frac{\sin(e+fx)+1}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}dx}{a-b}$$

$$\Big\downarrow \text{3297}$$

$$\frac{2\sqrt{a+b}(A-B)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\text{EllipticF}\left(\arcsin\left(\frac{\surd}{\surd}\right.\right.}{f(a-b)\sqrt{c+d}(bc-ad)}$$
$$\frac{(Ab-aB)\int \frac{\sin(e+fx)+1}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}dx}{a-b}$$

$$\Big\downarrow \text{3475}$$

$$\frac{2\sqrt{a+b}(A-B)\sec(e+fx)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d\sin(e+fx))}}\text{EllipticF}\left(\arcsin\left(\frac{\surd}{\surd}\right.\right.}{f(a-b)\sqrt{c+d}(bc-ad)}$$
$$\frac{2(c-d)\sqrt{c+d}(Ab-aB)\sec(e+fx)(a+b\sin(e+fx))\sqrt{-\frac{(bc-ad)(1-\sin(e+fx))}{(c+d)(a+b\sin(e+fx))}}\sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(c-d)(a+b\sin(e+fx))}}E\left(\arcsin\left(\right.\right.}{f(a-b)\sqrt{a+b}(bc-ad)^2}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*
x]]),x]
```

output
```
(2*(A*b - a*B)*(c - d)*Sqrt[c + d]*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c +
d*Sin[e + f*x]])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d)
)/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/
((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c
 - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x]))/((a - b)*Sqrt[a + b]*(b
*c - a*d)^2*f) + (2*Sqrt[a + b]*(A - B)*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt
[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c
 - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]
))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))
/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/((a - b)*Sqrt[c +
d]*(b*c - a*d)*f)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3297
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

rule 3475
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 88769 vs. 2(387) = 774.

Time = 23.30 (sec) , antiderivative size = 88770, normalized size of antiderivative = 212.88

method	result	size
parts	Expression too large to display	88770
default	Expression too large to display	88910

input
```
int((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx=\int \frac{B\sin(fx+e)+A}{(b\sin(fx+e)+a)^{\frac{3}{2}}\sqrt{d\sin(fx+e)+c}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x
, algorithm="fricas")
```

output
```
integral((B*sin(f*x + e) + A)*sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
 + c)/(2*a*b*d - (b^2*c + 2*a*b*d)*cos(f*x + e)^2 + (a^2 + b^2)*c - (b^2*d
*cos(f*x + e)^2 - 2*a*b*c - (a^2 + b^2)*d)*sin(f*x + e)), x)
```

Sympy [F]

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx=\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{\frac{3}{2}}\sqrt{c+d\sin(e+fx)}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(1/2)
,x)
```

output
```
Integral((A + B*sin(e + f*x))/((a + b*sin(e + f*x))**(3/2)*sqrt(c + d*sin(
e + f*x))), x)
```

Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx=\int \frac{B\sin(fx+e)+A}{(b\sin(fx+e)+a)^{\frac{3}{2}}\sqrt{d\sin(fx+e)+c}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/((b*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x
+ e) + c)), x)
```

Giac [F]

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx=\int \frac{B\sin\left(fx+e\right)+A}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\sqrt{d\sin\left(fx+e\right)+c}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x
, algorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)/((b*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x
+ e) + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx=\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^{3/2}\,\sqrt{c+d\,\sin\left(e+f\,x\right)}}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^
(1/2)),x)
```

output
```
int((A + B*sin(e + f*x))/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^
(1/2)), x)
```

Reduce [F]

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx=\int \frac{\sqrt{\sin(fx+e)\,d+c}\,\sqrt{\sin(fx+e)\,b+a}}{\sin(fx+e)^2\,bd+\sin(fx+e)\,ad+\sin(fx+e)\,bc+$$

input
```
int((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x)
```

output
```
int((sqrt(sin(e + f*x)*d + c)*sqrt(sin(e + f*x)*b + a))/(sin(e + f*x)**2*b
*d + sin(e + f*x)*a*d + sin(e + f*x)*b*c + a*c),x)
```

3.356 $\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 544

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx = \frac{2b(Ab-aB)\cos(e+fx)}{(a^2-b^2)\,(bc-ad)\,f\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e-}$$
$$-\frac{2(A(a^2d^2+b^2(c^2-2d^2))-B(a^2cd-b^2cd+ab(c^2-d^2)))\,E\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right)|\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec}{\sqrt{a+b}(c-d)\sqrt{c+d}(bc-ad)^3f}$$
$$+\frac{2(Abc+bBc-aAd-2Abd+aBd)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sin(e+fx)}}\right),\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sec(e+fx)\sqrt{\frac{(bc-}{(a+}}}{\sqrt{a+b}(c-d)\sqrt{c+d}(bc-ad)^2f}$$

output
```
2*b*(A*b-B*a)*cos(f*x+e)/(a^2-b^2)/(-a*d+b*c)/f/(a+b*sin(f*x+e))^(1/2)/(c+
d*sin(f*x+e))^(1/2)-2*(A*(a^2*d^2+b^2*(c^2-2*d^2))-B*(a^2*c*d-b^2*c*d+a*b*
(c^2-d^2)))*EllipticE((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*
sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*((-a*d+b*c)*
(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(
a-b)/(c+d*sin(f*x+e)))^(1/2)*(c+d*sin(f*x+e))/(a+b)^(1/2)/(c-d)/(c+d)^(1/2
)/(-a*d+b*c)^3/f+2*(-A*a*d+A*b*c-2*A*b*d+B*a*d+B*b*c)*EllipticF((c+d)^(1/2
)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(
a-b)/(c+d))^(1/2))*sec(f*x+e)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*
x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)*(c+
d*sin(f*x+e))/(a+b)^(1/2)/(c-d)/(c+d)^(1/2)/(-a*d+b*c)^2/f
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2266 vs. $2(544) = 1088$.

Time = 8.13 (sec) , antiderivative size = 2266, normalized size of antiderivative = 4.17

$$\int \frac{A + B\sin(e + fx)}{(a + b\sin(e + fx))^{3/2}(c + d\sin(e + fx))^{3/2}} dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e +
f*x])^(3/2)),x]
```

output
```
(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((2*(A*b^3*Cos[e + f*x]
 - a*b^2*B*Cos[e + f*x]))/((a^2 - b^2)*(-(b*c) + a*d)^2*(a + b*Sin[e + f*x
])) - (2*(B*c*d^2*Cos[e + f*x] - A*d^3*Cos[e + f*x]))/((b*c - a*d)^2*(c^2
- d^2)*(c + d*Sin[e + f*x]))))/f + ((-4*(-(b*c) + a*d)*(a*A*b^2*c^3 - b^3*
B*c^3 - 2*a^2*A*b*c^2*d + 2*A*b^3*c^2*d + a^3*A*c*d^2 - 2*a*A*b^2*c*d^2 +
b^3*B*c*d^2 + 2*a^2*A*b*d^3 - 2*A*b^3*d^3 - a^3*B*d^3 + a*b^2*B*d^3)*Sqrt[
((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a
- b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt
[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 -
 f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/
(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f
*x]))/(-(b*c) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c +
d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(A*b^3*c^3 - a*b^2*B*c^3 + a*A*b^2*c^2
*d - 2*a^2*b*B*c^2*d + b^3*B*c^2*d + a^2*A*b*c*d^2 - 2*A*b^3*c*d^2 - a^3*B
*c*d^2 + 2*a*b^2*B*c*d^2 + a^3*A*d^3 - 2*a*A*b^2*d^3 + a^2*b*B*d^3)*((Sqrt
[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a
 - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqr
t[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2
- f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))
/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e...
```

Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 578, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.205$, Rules used = {3042, 3479, 27, 3042, 3477, 3042, 3297, 3475}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}dx$$

$$\Big\downarrow \text{3479}$$

$$\frac{2b(Ab-aB)\cos(e+fx)}{f(a^2-b^2)(bc-ad)\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}-\frac{2\int \frac{Ada^2-(Abc-bBd)a+b^2(Bc-2Ad)-(Ab-aB)(bc+ad)\sin(e+fx)}{2\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}dx}{(a^2-b^2)(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{2b(Ab-aB)\cos(e+fx)}{f(a^2-b^2)(bc-ad)\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}-\frac{\int \frac{Ada^2-(Abc-bBd)a+b^2(Bc-2Ad)-(Ab-aB)(bc+ad)\sin(e+fx)}{\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}dx}{(a^2-b^2)(bc-ad)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b(Ab-aB)\cos(e+fx)}{f(a^2-b^2)(bc-ad)\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}-\frac{\int \frac{Ada^2-(Abc-bBd)a+b^2(Bc-2Ad)-(Ab-aB)(bc+ad)\sin(e+fx)}{\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}dx}{(a^2-b^2)(bc-ad)}$$

$$\Big\downarrow \text{3477}$$

$$\frac{2b(Ab-aB)\cos(e+fx)}{f(a^2-b^2)(bc-ad)\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}-\frac{\frac{\left(a^2(-A)d^2+a^2Bcd+abB\left(c^2-d^2\right)-Ab^2\left(c^2-2d^2\right)-b^2Bcd\right)\int \frac{\sin(e+fx)+1}{\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}dx}{c-d}-\frac{(a-b)(-aAd+aBd+Abc-2Abd+bBc)\int \frac{\quad}{\sqrt{\quad}}}{c-d}}{(a^2-b^2)(bc-ad)}$$

$\Big\downarrow$ 3042

$$\frac{2b(Ab-aB)\cos(e+fx)}{f\left(a^2-b^2\right)(bc-ad)\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}-\frac{\frac{\left(a^2(-A)d^2+a^2Bcd+abB\left(c^2-d^2\right)-Ab^2\left(c^2-2d^2\right)-b^2Bcd\right)\int\frac{\sin(e+fx)+1}{\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}dx}{c-d}-\frac{(a-b)(-aAd+aBd+Abc-2Abd+bBc)\int}{c-d}}{\left(a^2-b^2\right)(bc-ad)}$$

$\Big\downarrow$ 3297

$$\frac{2b(Ab-aB)\cos(e+fx)}{f\left(a^2-b^2\right)(bc-ad)\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}-\frac{\frac{\left(a^2(-A)d^2+a^2Bcd+abB\left(c^2-d^2\right)-Ab^2\left(c^2-2d^2\right)-b^2Bcd\right)\int\frac{\sin(e+fx)+1}{\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}dx}{c-d}-\frac{2(a-b)\sqrt{a+b}\sec(e+fx)(-aAd+aBd+Abc}{}}{\left(a^2-b^2\right)(bc}$$

$\Big\downarrow$ 3475

$$\frac{2b(Ab-aB)\cos(e+fx)}{f\left(a^2-b^2\right)(bc-ad)\sqrt{a+b\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}-\frac{-\frac{2(a-b)\sqrt{a+b}\sec(e+fx)\left(a^2(-A)d^2+a^2Bcd+abB\left(c^2-d^2\right)-Ab^2\left(c^2-2d^2\right)-b^2Bcd\right)(c+d\sin(e+fx))\sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d\sin(e+fx))}}\sqrt{-\frac{(bc-ad)(\sin(e+}{(a-b)(c+d\sin(}}}{f(c-d)\sqrt{c+d}(bc-ad)^2}}{}$$

input
```
Int[(A + B*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^
(3/2)),x]
```

output
```
(2*b*(A*b - a*B)*Cos[e + f*x])/((a^2 - b^2)*(b*c - a*d)*f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((-2*(a - b)*Sqrt[a + b]*(a^2*B*c*d -
 b^2*B*c*d - a^2*A*d^2 - A*b^2*(c^2 - 2*d^2) + a*b*B*(c^2 - d^2))*Elliptic
E[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Si
n[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*
c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c
- a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e +
 f*x]))/((c - d)*Sqrt[c + d]*(b*c - a*d)^2*f) - (2*(a - b)*Sqrt[a + b]*(A*
b*c + b*B*c - a*A*d - 2*A*b*d + a*B*d)*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[
a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c
- d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x])
)/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/
((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/((c - d)*Sqrt[c + d
]*(b*c - a*d)*f))/((a^2 - b^2)*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3297
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

rule 3475
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 177546 vs. 2(510) = 1020.

Time = 29.97 (sec) , antiderivative size = 177547, normalized size of antiderivative = 326.37

method	result	size
parts	Expression too large to display	177547
default	Expression too large to display	181372

input
```
int((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x,metho
d=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx=\int \frac{B\sin\left(fx+e\right)+A}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x
, algorithm="fricas")
```

output
```
integral((B*sin(f*x + e) + A)*sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
 + c)/(b^2*d^2*cos(f*x + e)^4 + 4*a*b*c*d + (a^2 + b^2)*c^2 + (a^2 + b^2)*
d^2 - (b^2*c^2 + 4*a*b*c*d + (a^2 + 2*b^2)*d^2)*cos(f*x + e)^2 + 2*(a*b*c^
2 + a*b*d^2 + (a^2 + b^2)*c*d - (b^2*c*d + a*b*d^2)*cos(f*x + e)^2)*sin(f*
x + e)), x)
```

Sympy [F]

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx=\int \frac{A+B\sin\left(e+fx\right)}{\left(a+b\sin\left(e+fx\right)\right)^{\frac{3}{2}}\left(c+d\sin\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(3/2)
,x)
```

output
```
Integral((A + B*sin(e + f*x))/((a + b*sin(e + f*x))**(3/2)*(c + d*sin(e +
f*x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx=\int \frac{B\sin\left(fx+e\right)+A}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e)
 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx=\int \frac{B\sin\left(fx+e\right)+A}{\left(b\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\left(d\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x
, algorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)/((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e)
 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx=\int \frac{A+B\,\sin\left(e+f\,x\right)}{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^{3/2}\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^
(3/2)),x)
```

output
```
int((A + B*sin(e + f*x))/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^
(3/2)), x)
```

Reduce [F]

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{\sin(fx+e)\,d+c}}{\sin(fx+e)^3\,b\,d^2+\sin(fx+e)^2\,a\,d^2+2\sin(fx+e)}$$

input
```
int((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x)
```

output
```
int((sqrt(sin(e + f*x)*d + c)*sqrt(sin(e + f*x)*b + a))/(sin(e + f*x)**3*b
*d**2 + sin(e + f*x)**2*a*d**2 + 2*sin(e + f*x)**2*b*c*d + 2*sin(e + f*x)*
a*c*d + sin(e + f*x)*b*c**2 + a*c**2),x)
```

3.357 $\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 858

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx = \frac{2b(Ab-aB)\cos(e+fx)}{(a^2-b^2)\,(bc-ad)f\sqrt{a+b\sin(e+fx)}(c+d\sin(e+}$$

$$+\frac{2d(A(a^2d^2+b^2(3c^2-4d^2))-B(a^2cd-b^2cd+3ab(c^2-d^2)))\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3\,(a^2-b^2)\,(bc-ad)^2\,(c^2-d^2)\,f(c+d\sin(e+fx))^{3/2}}$$

$$+\frac{2(B(2a^2bcd(3c^2-d^2)-2b^3cd(3c^2-d^2)-a^3d^2(c^2+3d^2)+ab^2(3c^4-5c^2d^2+6d^4))+A(4a^3cd^3-4ab^2c}{}$$

$$-\frac{2(B(a^2d^2(c+3d)-b^2c(3c^2+3cd-2d^2)-6abd(c^2-d^2))-A(a^2d^2(3c+d)-6abd(c^2-d^2)+b^2(3c^3-}{}$$

output
```
2*b*(A*b-B*a)*cos(f*x+e)/(a^2-b^2)/(-a*d+b*c)/f/(a+b*sin(f*x+e))^(1/2)/(c+
d*sin(f*x+e))^(3/2)+2/3*d*(A*(a^2*d^2+b^2*(3*c^2-4*d^2))-B*(a^2*c*d-b^2*c*
d+3*a*b*(c^2-d^2)))*cos(f*x+e)*(a+b*sin(f*x+e))^(1/2)/(a^2-b^2)/(-a*d+b*c)
^2/(c^2-d^2)/f/(c+d*sin(f*x+e))^(3/2)+2/3*(B*(2*a^2*b*c*d*(3*c^2-d^2)-2*b^
3*c*d*(3*c^2-d^2)-a^3*d^2*(c^2+3*d^2)+a*b^2*(3*c^4-5*c^2*d^2+6*d^4))+A*(4*
a^3*c*d^3-4*a*b^2*c*d^3-a^2*b*d^2*(9*c^2-5*d^2)-b^3*(3*c^4-15*c^2*d^2+8*d^
4)))*EllipticE((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x
+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*((-a*d+b*c)*(1-sin(
f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c
+d*sin(f*x+e)))^(1/2)*(c+d*sin(f*x+e))/(a+b)^(1/2)/(c-d)^2/(c+d)^(3/2)/(-a
*d+b*c)^4/f-2/3*(B*(a^2*d^2*(c+3*d)-b^2*c*(3*c^2+3*c*d-2*d^2)-6*a*b*d*(c^2
-d^2))-A*(a^2*d^2*(3*c+d)-6*a*b*d*(c^2-d^2)+b^2*(3*c^3-9*c^2*d-6*c*d^2+8*d
^3)))*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*
x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*((-a*d+b*c)*(1-sin
(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(
c+d*sin(f*x+e)))^(1/2)*(c+d*sin(f*x+e))/(a+b)^(1/2)/(c-d)^2/(c+d)^(3/2)/(-
a*d+b*c)^3/f
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2837 vs. $2(858) = 1716$.

Time = 9.30 (sec) , antiderivative size = 2837, normalized size of antiderivative = 3.31

$$\int \frac{A + B\sin(e + fx)}{(a + b\sin(e + fx))^{3/2}(c + d\sin(e + fx))^{5/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e +
f*x])^(5/2)),x]
```

output
```
(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((-2*(A*b^4*Cos[e + f*x
] - a*b^3*B*Cos[e + f*x]))/((a^2 - b^2)*(-(b*c) + a*d)^3*(a + b*Sin[e + f*
x])) + (2*(-(B*c*d^2*Cos[e + f*x]) + A*d^3*Cos[e + f*x]))/(3*(b*c - a*d)^2
*(c^2 - d^2)*(c + d*Sin[e + f*x])^2) - (2*(6*b*B*c^3*d^2*Cos[e + f*x] - 9*
A*b*c^2*d^3*Cos[e + f*x] - a*B*c^2*d^3*Cos[e + f*x] + 4*a*A*c*d^4*Cos[e +
f*x] - 2*b*B*c*d^4*Cos[e + f*x] + 5*A*b*d^5*Cos[e + f*x] - 3*a*B*d^5*Cos[e
 + f*x]))/(3*(b*c - a*d)^3*(c^2 - d^2)^2*(c + d*Sin[e + f*x]))))/f + ((-4*
(-(b*c) + a*d)*(-3*a*A*b^3*c^5 + 3*b^4*B*c^5 + 9*a^2*A*b^2*c^4*d - 9*A*b^4
*c^4*d - 9*a^3*A*b*c^3*d^2 + 15*a*A*b^3*c^3*d^2 - a^2*b^2*B*c^3*d^2 - 5*b^
4*B*c^3*d^2 + 3*a^4*A*c^2*d^3 - 20*a^2*A*b^2*c^2*d^3 + 17*A*b^4*c^2*d^3 +
10*a^3*b*B*c^2*d^3 - 10*a*b^3*B*c^2*d^3 + 5*a^3*A*b*c*d^4 - 8*a*A*b^3*c*d^
4 - 4*a^4*B*c*d^4 + 5*a^2*b^2*B*c*d^4 + 2*b^4*B*c*d^4 + a^4*A*d^5 + 7*a^2*
A*b^2*d^5 - 8*A*b^4*d^5 - 6*a^3*b*B*d^5 + 6*a*b^3*B*d^5)*Sqrt[((c + d)*Cot
[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e
 + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(
b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*S
qrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*
d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c
) + a*d)])/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]) - 4*(-(b*c) + a*d)*(-3*A*b^4*c^5 + 3*a*b^3*B*c^5 - 3*a*A*b^3*c^4*d...
```

Rubi [A] (verified)

Time = 3.42 (sec) , antiderivative size = 903, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.282$, Rules used = {3042, 3479, 27, 3042, 3534, 27, 3042, 3477, 3042, 3297, 3475}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}dx$$

$$\Big\downarrow \text{3479}$$

$$\frac{2b(Ab-aB)\cos(e+fx)}{f\left(a^2-b^2\right)(bc-ad)\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}-\frac{2\int\frac{Ada^2-(Abc-3bBd)a+2b(Ab-aB)d\sin^2(e+fx)+b^2(Bc-4Ad)-(Ab-aB)(bc+ad)\sin(e+fx)}{2\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{5/2}}dx}{\left(a^2-b^2\right)(bc-ad)}$$

$\big\downarrow$ 27

$$\frac{2b(Ab-aB)\cos(e+fx)}{f\left(a^2-b^2\right)(bc-ad)\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}-\frac{\int\frac{Ada^2-(Abc-3bBd)a+2b(Ab-aB)d\sin^2(e+fx)+b^2(Bc-4Ad)-(Ab-aB)(bc+ad)\sin(e+fx)}{\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{5/2}}dx}{\left(a^2-b^2\right)(bc-ad)}$$

$\big\downarrow$ 3042

$$\frac{2b(Ab-aB)\cos(e+fx)}{f\left(a^2-b^2\right)(bc-ad)\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}-\frac{\int\frac{Ada^2-(Abc-3bBd)a+2b(Ab-aB)d\sin(e+fx)^2+b^2(Bc-4Ad)-(Ab-aB)(bc+ad)\sin(e+fx)}{\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{5/2}}dx}{\left(a^2-b^2\right)(bc-ad)}$$

$\big\downarrow$ 3534

$$\frac{2b(Ab-aB)\cos(e+fx)}{f\left(a^2-b^2\right)(bc-ad)\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}-$$
$$2\int-\frac{3d^2(Ac-Bd)a^3-bd\left(6Ac^2-Bdc-5Ad^2\right)a^2+3b^2(Ac-Bd)\left(c^2-2d^2\right)a-b^3\left(3Bc^3-9Adc^2-2Bd^2c+8Ad^3\right)+\left(B\left(cd^2a^3-3bd\left(2c^2-d^2\right)a^2-b^2c\left(3c^2-2d^2\right)a+\right.\right.}{2\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}} \quad \frac{}{3(c^2-d^2)(bc-ad)} \quad (\ldots$$

$\big\downarrow$ 27

$$\frac{2b(Ab-aB)\cos(e+fx)}{f\left(a^2-b^2\right)(bc-ad)\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}-$$
$$\frac{2d\cos(e+fx)\left(a^2(-A)d^2+a^2Bcd+3abB\left(c^2-d^2\right)-Ab^2\left(3c^2-4d^2\right)-b^2Bcd\right)\sqrt{a+b\sin(e+fx)}}{3f(c^2-d^2)(bc-ad)(c+d\sin(e+fx))^{3/2}}-\frac{\int\frac{3d^2(Ac-Bd)a^3-bd\left(6Ac^2-Bdc-5Ad^2\right)a^2+3b^2(A\ldots}{}}{} \quad (a\ldots$$

$\big\downarrow$ 3042

$$\frac{2b(Ab-aB)\cos(e+fx)}{f\left(a^2-b^2\right)(bc-ad)\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}-$$
$$\frac{2d\cos(e+fx)\left(a^2(-A)d^2+a^2Bcd+3abB\left(c^2-d^2\right)-Ab^2\left(3c^2-4d^2\right)-b^2Bcd\right)\sqrt{a+b\sin(e+fx)}}{3f(c^2-d^2)(bc-ad)(c+d\sin(e+fx))^{3/2}}-\frac{\int\frac{3d^2(Ac-Bd)a^3-bd\left(6Ac^2-Bdc-5Ad^2\right)a^2+3b^2(A\ldots}{}}{} \quad (a\ldots$$

$\big\downarrow$ 3477

$$\frac{2b(Ab-aB)\cos(e+fx)}{f\left(a^2-b^2\right)(bc-ad)\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}-\frac{\frac{2d\cos(e+fx)\left(a^2(-A)d^2+a^2Bcd+3abB\left(c^2-d^2\right)-Ab^2\left(3c^2-4d^2\right)-b^2Bcd\right)\sqrt{a+b\sin(e+fx)}}{3f\left(c^2-d^2\right)(bc-ad)(c+d\sin(e+fx))^{3/2}}-\frac{-\frac{(a-b)\left(B\left(a^2d^2(c+3d)-6abd\left(c^2-d^2\right)+b^2(-c)\left(3c^2 \cdots\right.\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$\downarrow$ 3042

$$\frac{2b(Ab-aB)\cos(e+fx)}{f\left(a^2-b^2\right)(bc-ad)\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}-\frac{\frac{2d\cos(e+fx)\left(a^2(-A)d^2+a^2Bcd+3abB\left(c^2-d^2\right)-Ab^2\left(3c^2-4d^2\right)-b^2Bcd\right)\sqrt{a+b\sin(e+fx)}}{3f\left(c^2-d^2\right)(bc-ad)(c+d\sin(e+fx))^{3/2}}-\frac{-\frac{(a-b)\left(B\left(a^2d^2(c+3d)-6abd\left(c^2-d^2\right)+b^2(-c)\left(3c^2 \cdots\right.\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$\downarrow$ 3297

$$\frac{2b(Ab-aB)\cos(e+fx)}{f\left(a^2-b^2\right)(bc-ad)\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}-\frac{\frac{2d\cos(e+fx)\left(a^2(-A)d^2+a^2Bcd+3abB\left(c^2-d^2\right)-Ab^2\left(3c^2-4d^2\right)-b^2Bcd\right)\sqrt{a+b\sin(e+fx)}}{3f\left(c^2-d^2\right)(bc-ad)(c+d\sin(e+fx))^{3/2}}-\frac{-\frac{\left(A\left(4a^3cd^3-a^2bd^2\left(9c^2-5d^2\right)-4ab^2cd^3-\left(b^3\left(3c^4 \cdots\right.\right.\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$\downarrow$ 3475

$$\frac{2b(Ab-aB)\cos(e+fx)}{\left(a^2-b^2\right)(bc-ad)f\sqrt{a+b\sin(e+fx)}(c+d\sin(e+fx))^{3/2}}-\frac{\frac{2d\left(-Ad^2a^2+Bcda^2+3bB\left(c^2-d^2\right)a-b^2Bcd-Ab^2\left(3c^2-4d^2\right)\right)\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{3(bc-ad)\left(c^2-d^2\right)f(c+d\sin(e+fx))^{3/2}}-\frac{\frac{2(a-b)\sqrt{a+b}\left(B\left(-d^2\left(c^2+3d^2\right)a^3+2bcd\left(3c^2-d^2\right)a^2+ \cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

input

```
Int[(A + B*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^
(5/2)),x]
```

output
```
(2*b*(A*b - a*B)*Cos[e + f*x])/((a^2 - b^2)*(b*c - a*d)*f*Sqrt[a + b*Sin[e
 + f*x]]*(c + d*Sin[e + f*x])^(3/2)) - ((2*d*(a^2*B*c*d - b^2*B*c*d - a^2*
A*d^2 - A*b^2*(3*c^2 - 4*d^2) + 3*a*b*B*(c^2 - d^2))*Cos[e + f*x]*Sqrt[a +
 b*Sin[e + f*x]])/(3*(b*c - a*d)*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^(3/2))
 - ((2*(a - b)*Sqrt[a + b]*(B*(2*a^2*b*c*d*(3*c^2 - d^2) - 2*b^3*c*d*(3*c^
2 - d^2) - a^3*d^2*(c^2 + 3*d^2) + a*b^2*(3*c^4 - 5*c^2*d^2 + 6*d^4)) + A*
(4*a^3*c*d^3 - 4*a*b^2*c*d^3 - a^2*b*d^2*(9*c^2 - 5*d^2) - b^3*(3*c^4 - 15
*c^2*d^2 + 8*d^4)))*EllipticE[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]]
)/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c +
 d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*S
in[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin
[e + f*x])))]*(c + d*Sin[e + f*x]))/((c - d)*Sqrt[c + d]*(b*c - a*d)^2*f)
- (2*(a - b)*Sqrt[a + b]*(B*(a^2*d^2*(c + 3*d) - b^2*c*(3*c^2 + 3*c*d - 2*
d^2) - 6*a*b*d*(c^2 - d^2)) - A*(a^2*d^2*(3*c + d) - 6*a*b*d*(c^2 - d^2) +
 b^2*(3*c^3 - 9*c^2*d - 6*c*d^2 + 8*d^3)))*EllipticF[ArcSin[(Sqrt[c + d]*S
qrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)
*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f
*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x
]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x]))/((c - d)*Sqrt[c
 + d]*(b*c - a*d)*f))/(3*(b*c - a*d)*(c^2 - d^2)))/((a^2 - b^2)*(b*c - ...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3297
```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Simp[2*((c + d*Sin[e + f*x])/(f*(b*c - a*d
)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 - Sin[e + f*x]
)/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[(-(b*c - a*d))*((1 + Sin[e + f*x])/
((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(S
qrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], (a + b)*((c - d)/((a -
b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]
```

rule 3475
```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_) + (b_.)*sin[(e_.) + (f_.)
*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Sim
p[-2*A*(c - d)*((a + b*Sin[e + f*x])/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2
]*Cos[e + f*x]))*Sqrt[(b*c - a*d)*((1 + Sin[e + f*x])/((c - d)*(a + b*Sin[e
 + f*x])))]*Sqrt[(-(b*c - a*d))*((1 - Sin[e + f*x])/((c + d)*(a + b*Sin[e +
 f*x])))]*EllipticE[ArcSin[Rt[(a + b)/(c + d), 2]*(Sqrt[c + d*Sin[e + f*x]]
/Sqrt[a + b*Sin[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

rule 3477
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> S
imp[(A - B)/(a - b)   Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]), x], x] - Simp[(A*b - a*B)/(a - b)   Int[(1 + Sin[e + f*x])/((a + b*Si
n[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e
, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && NeQ[A, B]
```

rule 3479
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m +
1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
 + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n +
2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)
*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rat
ionalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(I
ntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])
))
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 397568 vs. 2(814) = 1628.

Time = 48.46 (sec) , antiderivative size = 397569, normalized size of antiderivative = 463.37

method	result	size
parts	Expression too large to display	397569
default	Expression too large to display	422919

input
```
int((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x,metho
d=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx = \int \frac{B\sin\left(fx+e\right)+A}{(b\sin\left(fx+e\right)+a)^{\frac{3}{2}}(d\sin\left(fx+e\right)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x
, algorithm="fricas")
```

output
```
integral((B*sin(f*x + e) + A)*sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
 + c)/(6*a*b*c^2*d + 2*a*b*d^3 + (3*b^2*c*d^2 + 2*a*b*d^3)*cos(f*x + e)^4
+ (a^2 + b^2)*c^3 + 3*(a^2 + b^2)*c*d^2 - (b^2*c^3 + 6*a*b*c^2*d + 4*a*b*d
^3 + 3*(a^2 + 2*b^2)*c*d^2)*cos(f*x + e)^2 + (b^2*d^3*cos(f*x + e)^4 + 2*a
*b*c^3 + 6*a*b*c*d^2 + 3*(a^2 + b^2)*c^2*d + (a^2 + b^2)*d^3 - (3*b^2*c^2*
d + 6*a*b*c*d^2 + (a^2 + 2*b^2)*d^3)*cos(f*x + e)^2)*sin(f*x + e)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(5/2)
,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx=\int \frac{B\sin(fx+e)+A}{(b\sin(fx+e)+a)^{\frac{3}{2}}(d\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x
, algorithm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)/((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e)
 + c)^(5/2)), x)
```

Giac [F]

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx=\int \frac{B\sin(fx+e)+A}{(b\sin(fx+e)+a)^{\frac{3}{2}}(d\sin(fx+e)+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x
, algorithm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)/((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e)
 + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx=\int \frac{A+B\,\sin(e+f\,x)}{(a+b\,\sin(e+f\,x))^{3/2}\,(c+d\,\sin(e+f\,x))^{5/2}}\,dx$$

input
```
int((A + B*sin(e + f*x))/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^
(5/2)),x)
```

output
```
int((A + B*sin(e + f*x))/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^
(5/2)), x)
```

Reduce [F]

$$\int \frac{A+B\sin(e+fx)}{(a+b\sin(e+fx))^{3/2}(c+d\sin(e+fx))^{5/2}}\,dx = \int \frac{}{\sin\left(fx+e\right)^{4}b\,d^{3}+\sin\left(fx+e\right)^{3}a\,d^{3}+3\sin\left(fx+\right.}$$

input
```
int((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x)
```

output
```
int((sqrt(sin(e + f*x)*d + c)*sqrt(sin(e + f*x)*b + a))/(sin(e + f*x)**4*b
*d**3 + sin(e + f*x)**3*a*d**3 + 3*sin(e + f*x)**3*b*c*d**2 + 3*sin(e + f*
x)**2*a*c*d**2 + 3*sin(e + f*x)**2*b*c**2*d + 3*sin(e + f*x)*a*c**2*d + si
n(e + f*x)*b*c**3 + a*c**3),x)
```

3.358 $\int (a+b\sin(e+fx))^m (A+B\sin(e+fx))(c+d\sin(e+fx))^n \, dx$

Optimal result

Integrand size = 35, antiderivative size = 35

$$\int (a+b\sin(e+fx))^m (A+B\sin(e+fx))(c+d\sin(e+fx))^n \, dx = \text{Int}((a+b\sin(e+fx))^m (A+B\sin(e+fx))(c+d\sin(e+fx))^n, x)$$

output
```
Defer(Int)((a+b*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

Mathematica [N/A]

Not integrable

Time = 20.92 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int (a+b\sin(e+fx))^m (A+B\sin(e+fx))(c+d\sin(e+fx))^n \, dx = \int (a+b\sin(e+fx))^m (A+B\sin(e+fx))(c+d\sin(e+fx))^n \, dx$$

input
```
Integrate[(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])
^n,x]
```

output
```
Integrate[(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])
^n, x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3486}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+B\sin(e+fx))(a+b\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (A+B\sin(e+fx))(a+b\sin(e+fx))^m(c+d\sin(e+fx))^ndx$$

$$\Big\downarrow \text{3486}$$

$$\int (A+B\sin(e+fx))(a+b\sin(e+fx))^m(c+d\sin(e+fx))^ndx$$

input
```
Int[(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3486
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> U
nintegrable[(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]
)^n, x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Maple [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \left(a+b\sin \left(fx+e\right)\right)^{m} \left(A+B\sin \left(fx+e\right)\right) \left(c+d\sin \left(fx+e\right)\right)^{n}dx$$

input
```
int((a+b*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

output
```
int((a+b*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

Fricas [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int (a+b\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx = \int (B\sin\left(fx+e\right)+A)(b\sin\left(fx+e\right)+a)^m(d\sin\left(fx+e\right)+c)^n\,dx$$

input
```
integrate((a+b*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algori
thm="fricas")
```

output
```
integral((B*sin(f*x + e) + A)*(b*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^
n, x)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+b\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$= \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((a+b*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [N/A]

Not integrable

Time = 26.56 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int (a+b\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$= \int (B\sin(fx+e)+A)(b\sin(fx+e)+a)^m(d\sin(fx+e)+c)^n\,dx$$

input
```
integrate((a+b*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algori
thm="maxima")
```

output
```
integrate((B*sin(f*x + e) + A)*(b*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)
^n, x)
```

Giac [N/A]

Not integrable

Time = 2.51 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int (a+b\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$=\int (B\sin(fx+e)+A)(b\sin(fx+e)+a)^m(d\sin(fx+e)+c)^n\,dx$$

input
```
integrate((a+b*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algori
thm="giac")
```

output
```
integrate((B*sin(f*x + e) + A)*(b*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)
^n, x)
```

Mupad [N/A]

Not integrable

Time = 44.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int (a+b\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$=\int (A+B\,\sin(e+f\,x))\,(a+b\,\sin(e+f\,x))^m\,(c+d\,\sin(e+f\,x))^n\,dx$$

input
```
int((A + B*sin(e + f*x))*(a + b*sin(e + f*x))^m*(c + d*sin(e + f*x))^n,x)
```

output
```
int((A + B*sin(e + f*x))*(a + b*sin(e + f*x))^m*(c + d*sin(e + f*x))^n, x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.86

$$\int (a+b\sin(e+fx))^m(A+B\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$
$$=\left(\int(\sin(fx+e)\,d+c)^n\,(\sin(fx+e)\,b+a)^m\sin(fx+e)\,dx\right)b$$
$$+\left(\int(\sin(fx+e)\,d+c)^n\,(\sin(fx+e)\,b+a)^m\,dx\right)a$$

input
```
int((a+b*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

output
```
int((sin(e + f*x)*d + c)**n*(sin(e + f*x)*b + a)**m*sin(e + f*x),x)*b + in
t((sin(e + f*x)*d + c)**n*(sin(e + f*x)*b + a)**m,x)*a
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                    convert(leaf_count_result,string)," vs. $2 (",
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):    #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])    #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#           issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file